Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.4-Linear-quadratic-binomial/71-1.2.1.2

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**477**]. This is test number [71].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (477)	0.00 (0)
Mathematica	98.95 (472)	1.05 (5)
Maple	77.15 (368)	22.85 (109)
Fricas	73.79 (352)	26.21 (125)
Mupad	56.81 (271)	43.19 (206)
Reduce	56.18 (268)	43.82 (209)
Giac	55.35 (264)	44.65 (213)
Maxima	48.43 (231)	51.57 (246)
Sympy	44.03 (210)	55.97 (267)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

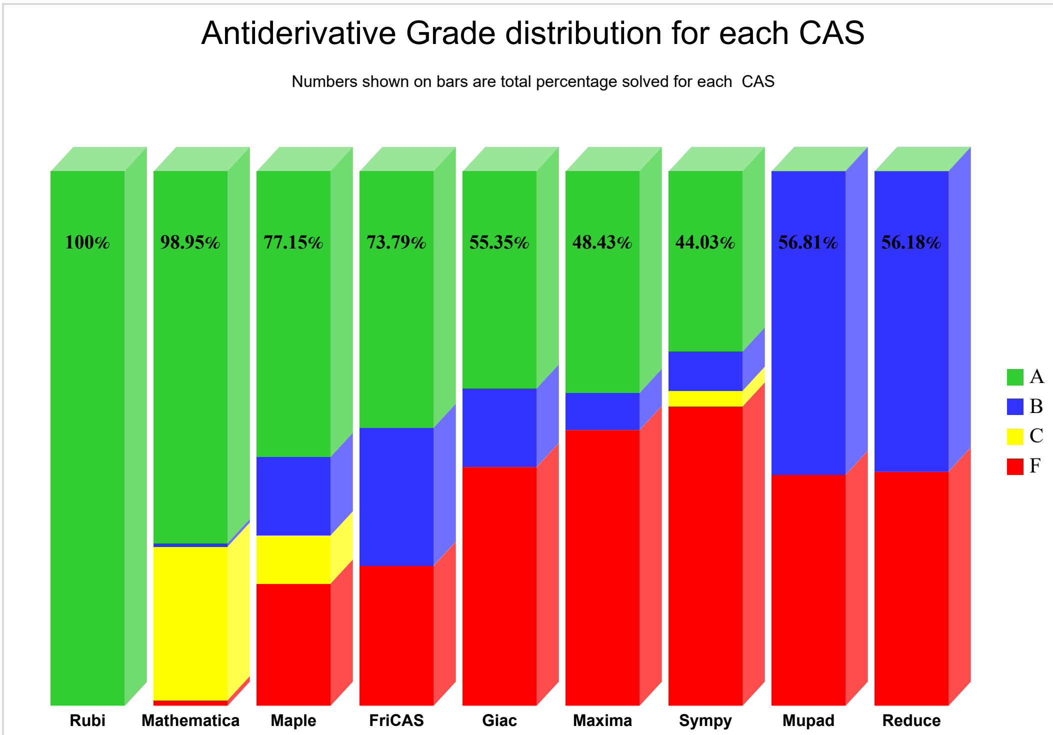
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

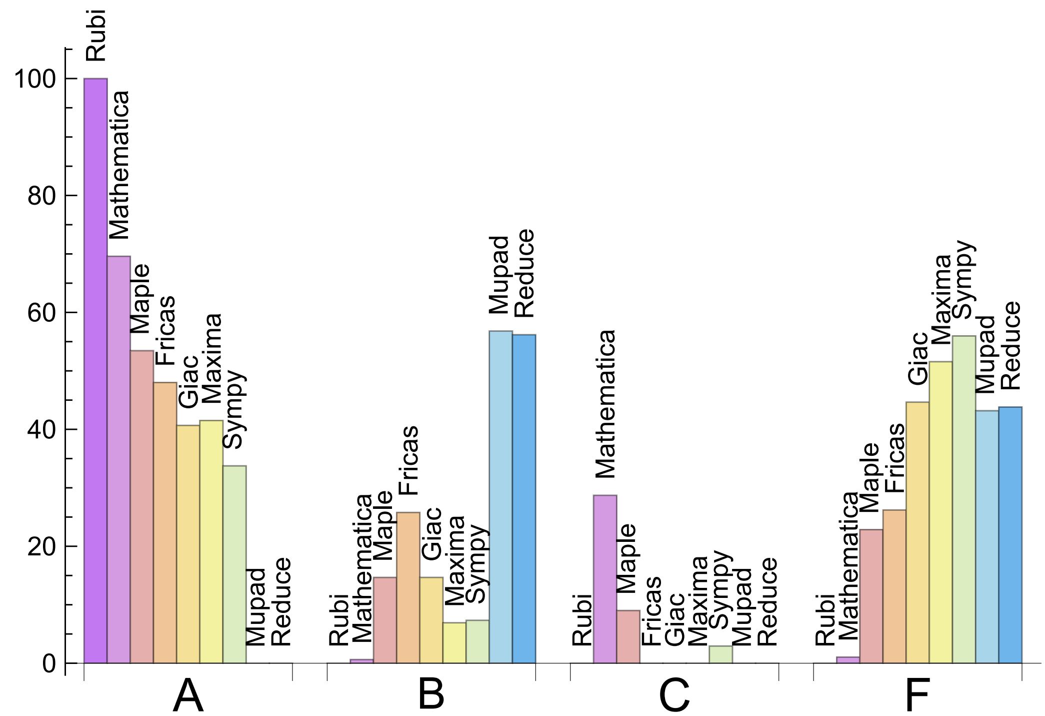
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	69.602	0.629	28.721	1.048
Maple	53.459	14.675	9.015	22.851
Fricas	48.008	25.786	0.000	26.205
Maxima	41.509	6.918	0.000	51.572
Giac	40.671	14.675	0.000	44.654
Sympy	33.753	7.338	2.935	55.975
Mupad	0.000	56.813	0.000	43.187
Reduce	0.000	56.184	0.000	43.816

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	5	100.00	0.00	0.00
Maple	109	96.33	3.67	0.00
Fricas	125	89.60	9.60	0.80
Mupad	206	0.00	100.00	0.00
Reduce	209	100.00	0.00	0.00
Giac	213	88.73	8.45	2.82
Maxima	246	99.59	0.00	0.41
Sympy	267	73.41	26.59	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Giac	0.15
Rubi	0.66
Maple	1.02
Sympy	1.09
Reduce	1.32
Fricas	2.05
Mathematica	3.04
Mupad	4.48

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	182.91	0.98	148.00	1.00
Rubi	226.10	1.06	161.00	1.00
Sympy	271.87	1.95	131.00	1.29
Maxima	320.10	1.77	140.00	1.13
Giac	336.10	1.75	178.50	1.15
Maple	540.14	2.38	148.00	1.05
Reduce	757.15	3.16	234.00	1.80
Fricas	1165.33	4.20	304.50	1.77
Mupad	1865.96	5.84	157.00	1.20

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

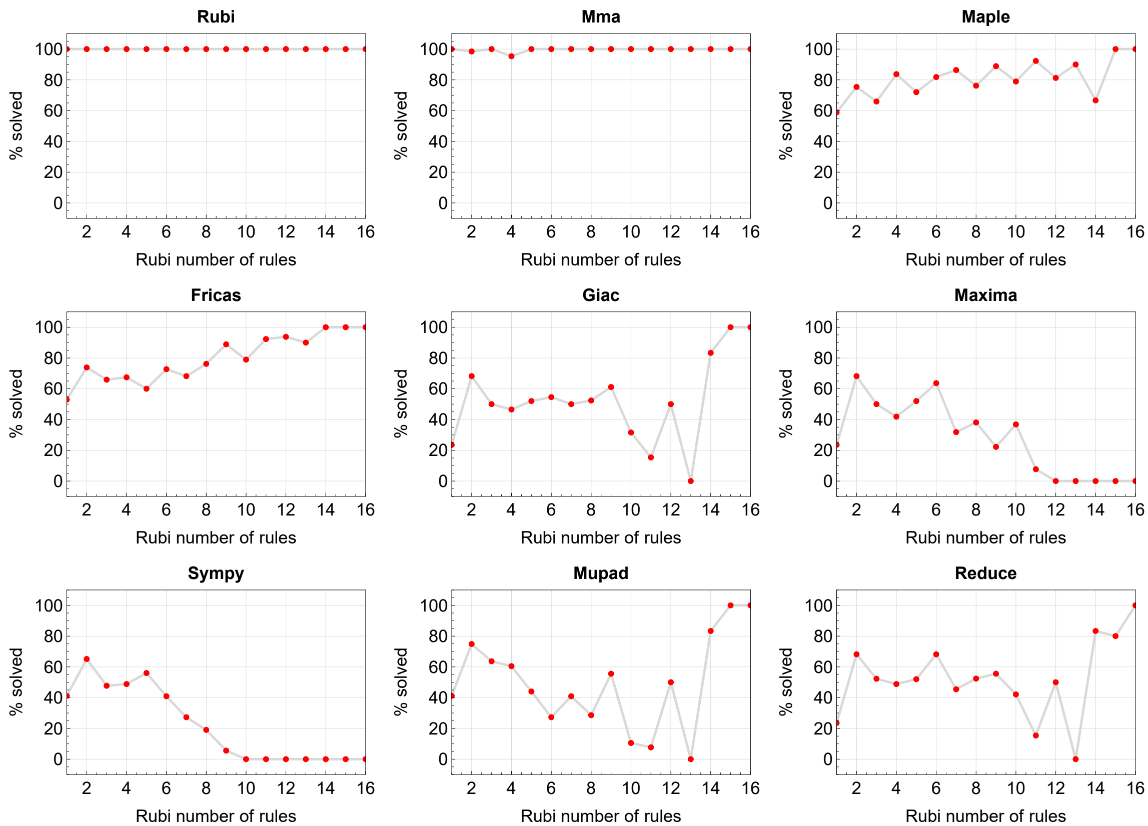


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

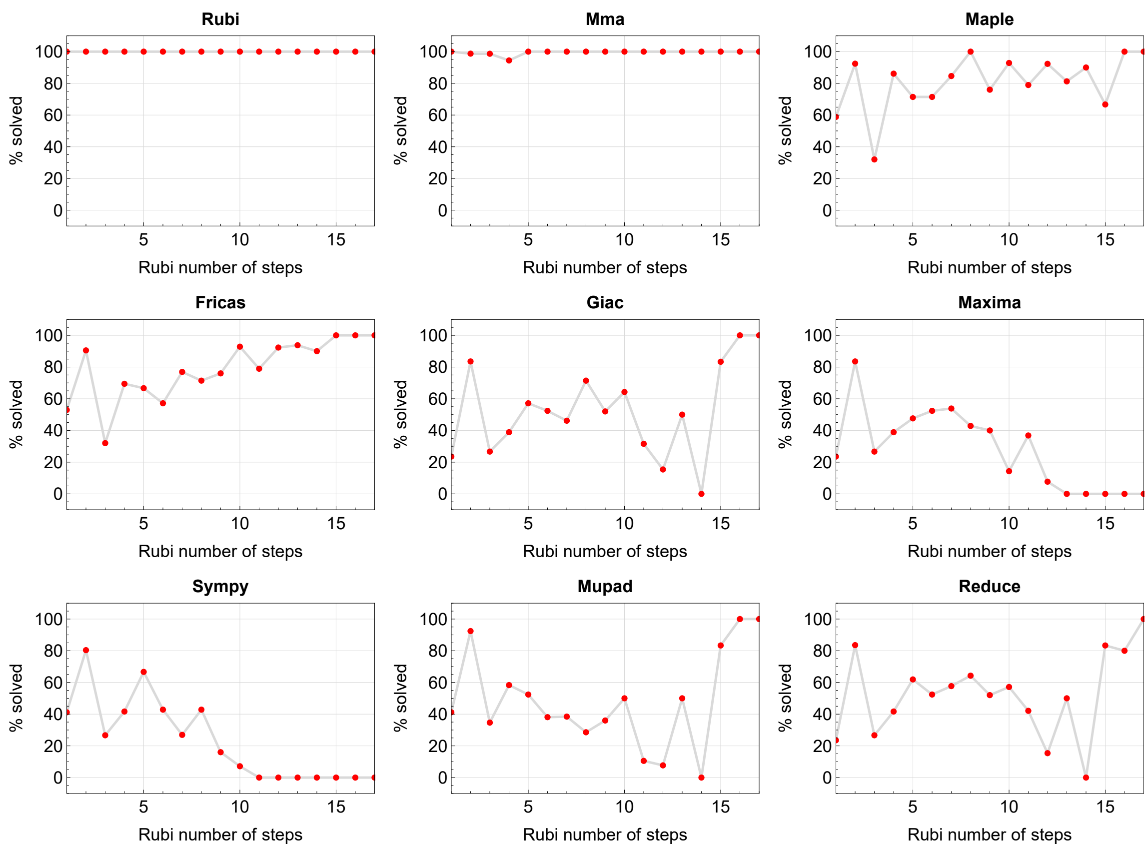


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

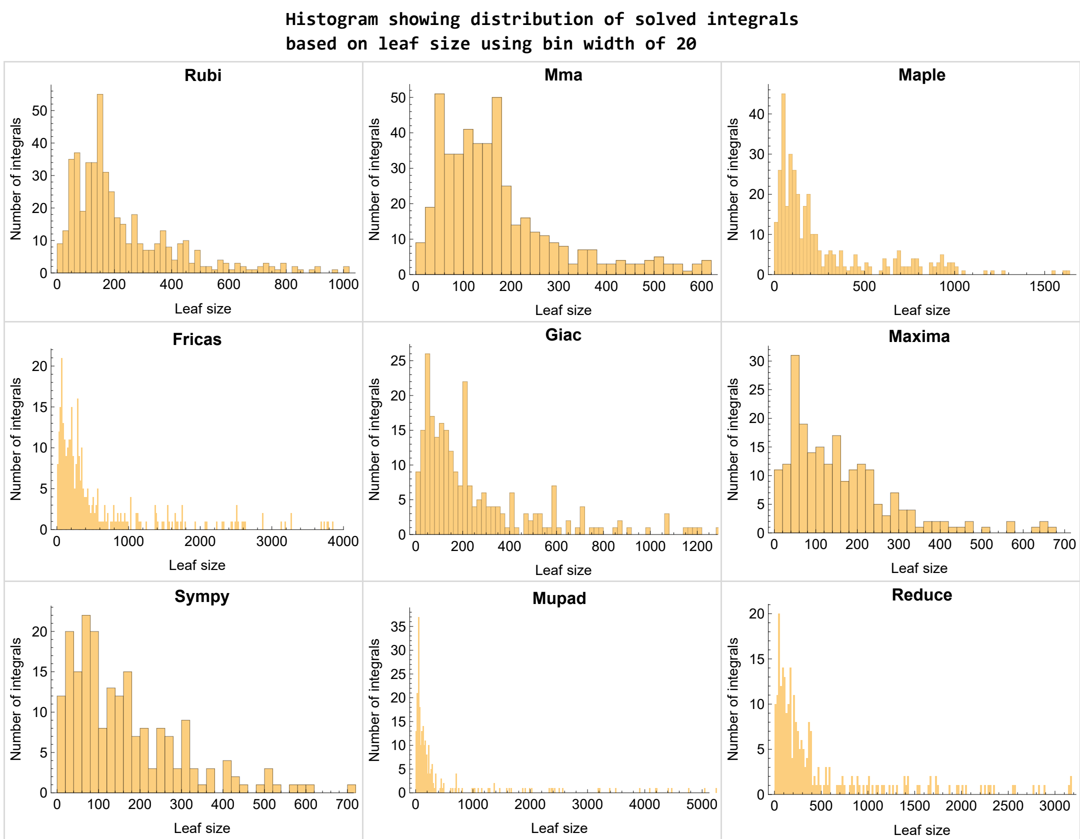


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

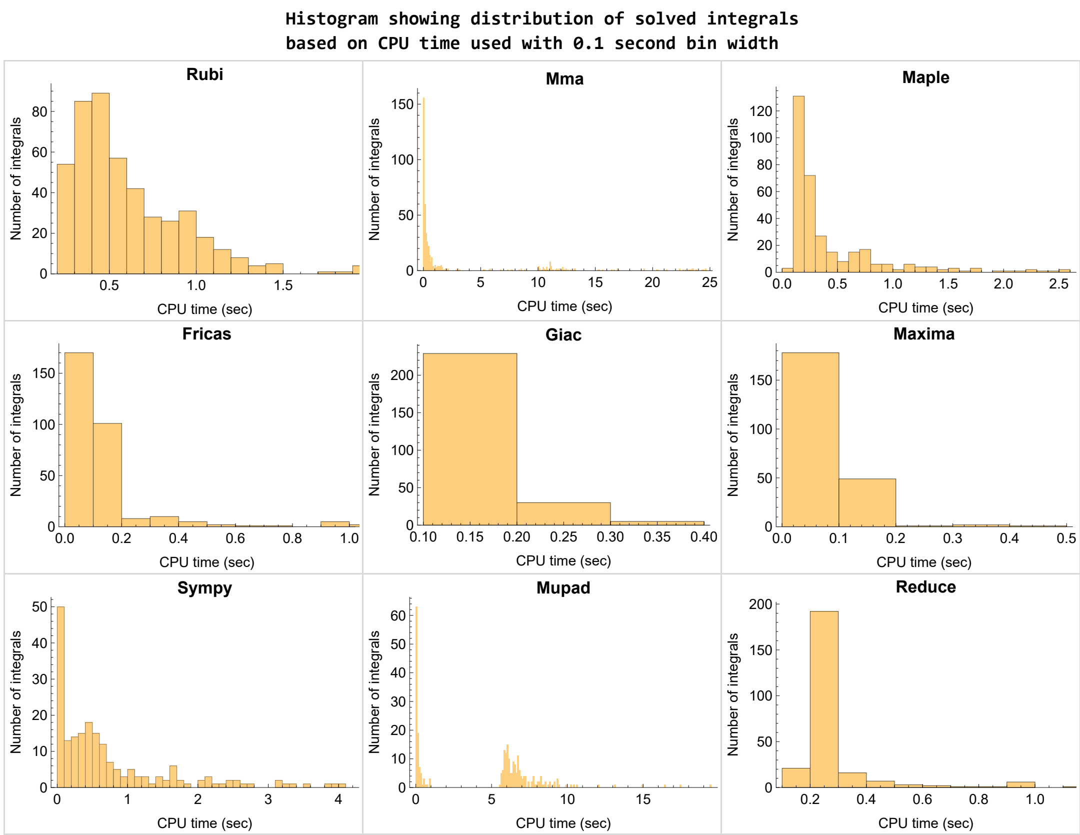


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

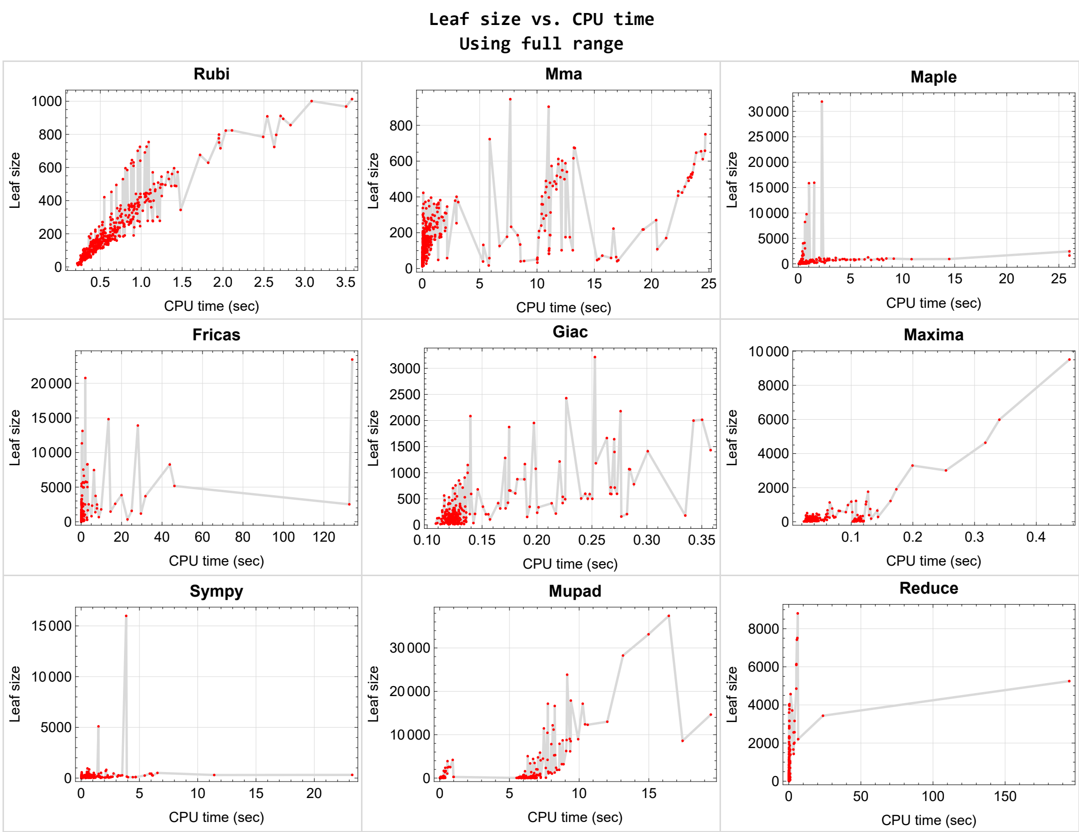


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {343, 344, 345, 347, 348, 349, 350, 351, 353, 354, 359, 391, 423, 454, 455, 456, 457, 458, 459, 460, 461, 471, 472, 476, 477}

Mathematica {347, 348, 353, 354, 365, 366, 379, 380, 399, 442, 443, 444, 445, 446, 458, 467, 475}

Maple $\{201, 355, 356, 374, 375, 376, 377\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

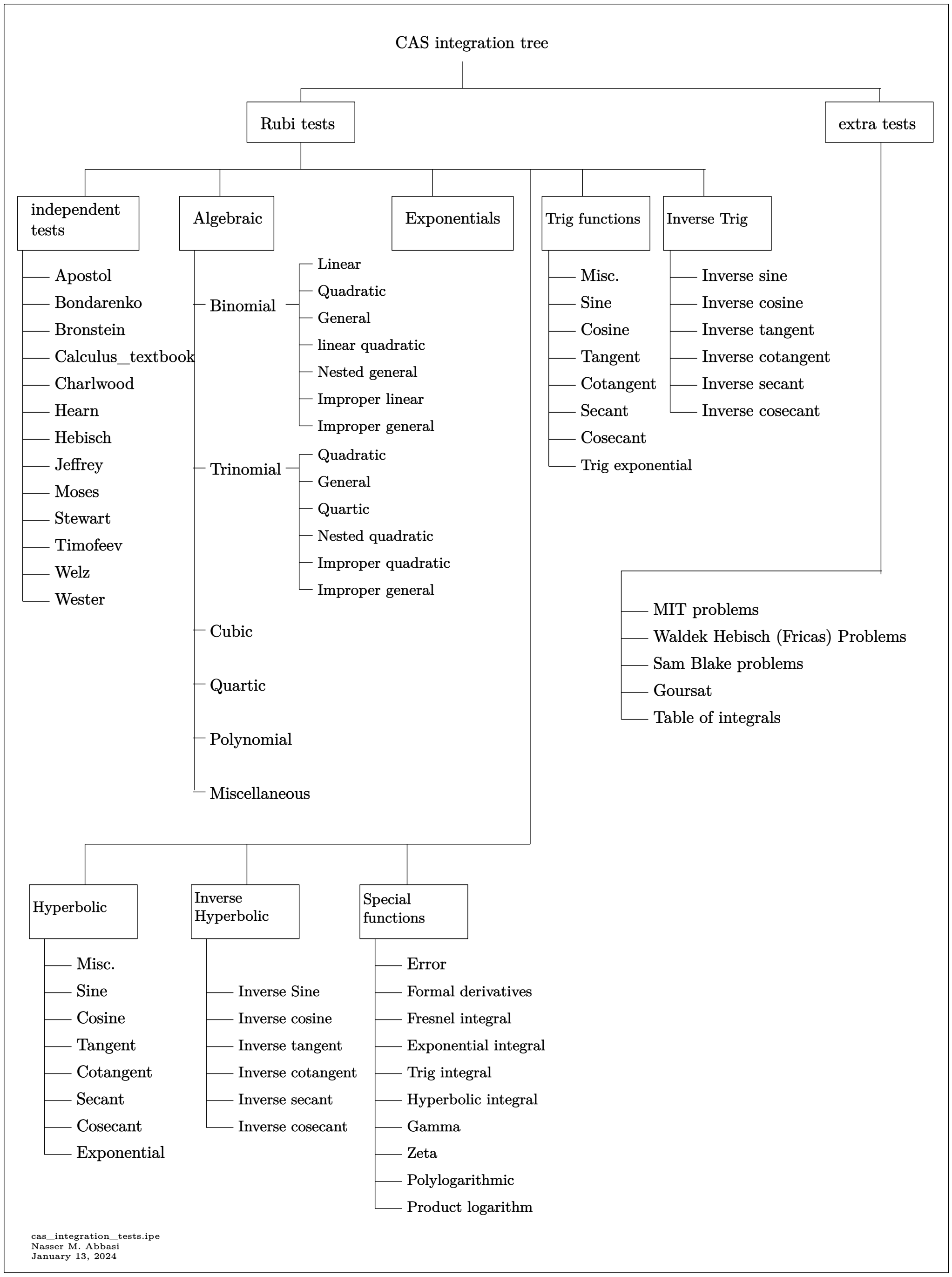
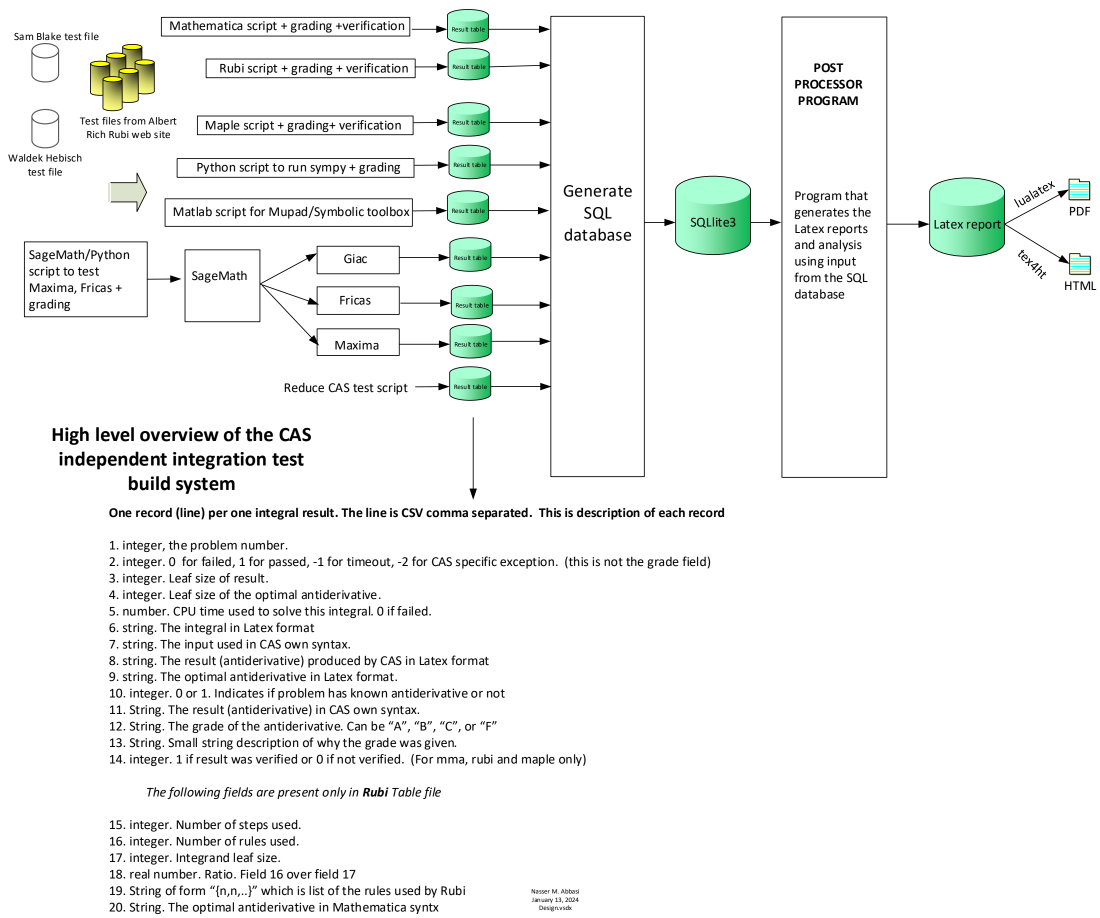


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 342, 346, 352, 355, 356, 357, 358, 389, 390, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 470, 471, 472, 473, 474, 475 }

B grade { 241, 250, 261 }

C grade { 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 347, 348, 349, 350, 351, 353, 354, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 421, 422, 423 }

F normal fail { 398, 468, 469, 476, 477 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 192, 196, 197, 198, 199, 208, 209, 210, 211, 212, 233, 234, 235, 236, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 277, 278, 279, 280, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 307, 319, 324, 331, 374, 375, 376, 377, 394, 417, 418, 419, 420, 431, 432, 433, 434 }

B grade { 191, 193, 200, 237, 238, 239, 240, 241, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 263, 264, 265, 273, 274, 275, 276, 281, 282, 283, 284, 290, 291, 292, 300, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 392, 393 }

C grade { 201, 202, 203, 204, 205, 206, 207, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 343, 344, 345, 349, 350, 351, 355, 356, 357, 358, 360, 361, 362, 363, 364, 378 }

F normal fail { 342, 346, 347, 348, 352, 353, 354, 359, 365, 366, 367, 368, 369, 370, 371, 372, 373, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477 }

F(-1) timedout fail { 186, 187, 194, 195 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 45, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 128, 131, 132, 133, 134, 153, 154, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 179, 185, 196, 197, 198, 199, 200, 201, 208, 209, 210, 211, 212, 233, 234, 235, 236, 237, 242, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 258, 259, 266, 267, 268, 269, 270, 271, 272, 277, 278, 279, 280, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 357, 358, 378 }

B grade { 34, 35, 36, 41, 42, 43, 44, 46, 49, 50, 51, 81, 82, 83, 108, 115, 121, 122, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 155, 156, 163, 170, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 238, 239, 240, 241, 248, 249, 250, 251, 252, 260, 261, 262, 263, 264, 265, 273, 274, 275, 276, 281, 282, 283, 284, 290, 291, 292, 299, 319, 334, 341, 342, 346, 355, 356, 364, 392, 393, 394 }

C grade { }

F normal fail { 343, 344, 345, 347, 348, 349, 350, 351, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 390, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477 }

F(-1) timedout fail { 352, 353, 354, 359, 371, 372, 373, 385, 386, 387, 388, 389 }

F(-2) exception fail { 391 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 131, 132, 133, 134, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 233, 234, 235, 236, 237, 238, 242, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 393, 394 }

B grade { 32, 35, 42, 43, 50, 51, 115, 122, 129, 130, 239, 240, 241, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 276, 282, 283, 284, 290, 291, 292, 392 }

C grade { }

F normal fail { 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 155, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477 }

F(-1) timedout fail { }

F(-2) exception fail { 154 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 131, 132, 133, 134, 153, 154, 159, 160, 161, 162, 166, 167, 168, 169, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 196, 198, 199, 200, 201, 210, 211, 212, 233, 234, 235, 236, 242, 243, 244, 245, 253, 254, 255, 256, 266, 268, 269, 270, 271, 272, 273, 277, 278, 279, 280, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299 }

B grade { 32, 34, 42, 43, 50, 51, 73, 89, 115, 122, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 155, 156, 157, 158, 163, 164, 165, 170, 171, 172, 182, 188, 189, 195, 208, 209, 239, 240, 248, 249, 251, 252, 259, 260, 262, 263, 264, 265, 267, 275, 276, 281, 283, 284, 290, 291, 292, 392, 393, 394 }

C grade { }

F normal fail { 202, 203, 204, 205, 206, 207, 213, 214, 215, 217, 219, 220, 221, 232, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477 }

F(-1) timedout fail { 197, 216, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 241, 247, 250, 258, 261, 282 }

F(-2) exception fail { 218, 237, 238, 246, 257, 274 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 236, 245, 256, 266, 267, 268, 272, 279, 280, 287, 288, 289, 294, 295, 296, 297, 298, 299, 345, 351, 362, 363, 369, 370, 376, 377, 383, 384, 392, 393, 394, 412, 413, 419, 420, 426, 427, 433, 434 }

C grade { }

F normal fail { }

F(-1) timedout fail { 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 273, 274, 275, 276, 277, 278, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 414, 415, 416, 417, 418, 421, 422, 423, 424, 425, 428, 429, 430, 431, 432, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 39, 40, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 112, 113, 119, 120, 123, 124, 126, 127, 128, 131, 132, 133, 134, 153, 154, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 233, 234, 235, 236, 244, 245, 256, 268, 269, 270, 271, 272, 280, 294, 295, 296, 297, 298, 299, 343, 344, 345, 349, 350, 351, 360, 368, 369, 374, 382, 383, 411, 412, 417, 418, 419, 425, 426, 431, 432, 433 }

B grade { 28, 29, 30, 32, 36, 37, 38, 73, 89, 102, 103, 104, 105, 109, 110, 111, 116, 117, 118, 125, 156, 161, 162, 169, 242, 243, 253, 254, 255, 289, 392, 393, 394, 410, 424 }

C grade { 361, 362, 363, 367, 370, 375, 376, 377, 381, 384, 413, 420, 427, 434 }

F normal fail { 135, 136, 137, 138, 139, 140, 155, 177, 178, 179, 180, 181, 182, 188, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 237, 238, 239, 240, 241, 246, 247, 248, 249, 250, 251, 252, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 342, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 359, 364, 365, 366, 371, 372, 373, 378, 379, 380, 385, 386, 387, 388, 389, 390, 391, 395, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 414, 415, 416, 421, 422, 423, 428, 429, 430, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 466, 474 }

F(-1) timedout fail { 33, 34, 35, 41, 42, 43, 49, 50, 51, 106, 107, 108, 114, 115, 121, 122, 129, 130, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 201, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 265, 335, 336, 397, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 208, 209, 210, 211, 212, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 392, 393, 394 }

C grade { }

F normal fail { 189, 202, 203, 204, 205, 206, 207, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 242, 253, 257, 269, 277, 285, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	88	89	89	94	89	91	89
N.S.	1	1.00	1.20	1.35	1.37	1.37	1.45	1.37	1.40	1.37
time (sec)	N/A	0.373	0.013	0.142	0.027	0.068	0.025	0.123	0.218	6.043

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	61	60	60	61	61	63	60
N.S.	1	1.00	0.86	0.94	0.92	0.92	0.94	0.94	0.97	0.92
time (sec)	N/A	0.347	0.015	0.137	0.032	0.064	0.024	0.122	0.214	0.028

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	41	40	35	34	34	36	34	35	34
N.S.	1	1.02	1.00	0.88	0.85	0.85	0.90	0.85	0.88	0.85
time (sec)	N/A	0.275	0.003	0.092	0.041	0.062	0.019	0.109	0.202	0.047

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	14	17	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	0.88	1.06	0.88
time (sec)	N/A	0.244	0.000	0.032	0.035	0.075	0.018	0.112	0.209	0.020

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	46	45	49	48	42	50	53	47
N.S.	1	0.98	0.92	0.90	0.98	0.96	0.84	1.00	1.06	0.94
time (sec)	N/A	0.324	0.012	0.149	0.029	0.063	0.087	0.121	0.205	0.073

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	54	55	71	48	71	75	59
N.S.	1	1.00	0.94	1.06	1.08	1.39	0.94	1.39	1.47	1.16
time (sec)	N/A	0.335	0.018	0.148	0.031	0.076	0.128	0.122	0.234	5.987

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	56	65	84	65	56	96	67
N.S.	1	1.00	0.88	0.93	1.08	1.40	1.08	0.93	1.60	1.12
time (sec)	N/A	0.345	0.017	0.164	0.039	0.069	0.167	0.118	0.223	6.002

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	48	72	72	73	47	53	71
N.S.	1	1.00	0.73	0.80	1.20	1.20	1.22	0.78	0.88	1.18
time (sec)	N/A	0.342	0.011	0.148	0.028	0.071	0.219	0.118	0.237	0.048

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	49	83	83	87	67	80	85
N.S.	1	1.00	0.68	0.75	1.28	1.28	1.34	1.03	1.23	1.31
time (sec)	N/A	0.343	0.016	0.155	0.039	0.089	0.279	0.122	0.212	0.059

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	121	144	147	147	155	147	149	143
N.S.	1	1.00	0.90	1.07	1.09	1.09	1.15	1.09	1.10	1.06
time (sec)	N/A	0.580	0.019	0.151	0.035	0.072	0.029	0.115	0.220	5.943

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	101	101	101	105	102	104	100
N.S.	1	1.00	0.90	1.03	1.03	1.03	1.07	1.04	1.06	1.02
time (sec)	N/A	0.402	0.017	0.148	0.034	0.098	0.024	0.122	0.208	0.049

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	59	58	58	65	58	59	58
N.S.	1	1.00	1.26	1.09	1.07	1.07	1.20	1.07	1.09	1.07
time (sec)	N/A	0.319	0.004	0.134	0.033	0.073	0.021	0.121	0.226	0.030

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	25	26	25	28	25
N.S.	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.97	0.86
time (sec)	N/A	0.262	0.000	0.106	0.036	0.097	0.018	0.108	0.222	0.031

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	115	117	117	99	118	128	119
N.S.	1	1.00	0.80	1.04	1.05	1.05	0.89	1.06	1.15	1.07
time (sec)	N/A	0.461	0.030	0.184	0.034	0.078	0.152	0.121	0.213	0.049

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	107	121	125	173	117	167	175	129
N.S.	1	1.00	0.96	1.09	1.13	1.56	1.05	1.50	1.58	1.16
time (sec)	N/A	0.472	0.046	0.168	0.028	0.073	0.249	0.116	0.208	0.060

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	124	134	200	134	126	213	136
N.S.	1	1.00	1.03	1.05	1.14	1.69	1.14	1.07	1.81	1.15
time (sec)	N/A	0.469	0.031	0.162	0.030	0.073	0.393	0.124	0.206	5.941

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	120	144	195	144	120	175	145
N.S.	1	1.00	0.97	1.01	1.21	1.64	1.21	1.01	1.47	1.22
time (sec)	N/A	0.500	0.032	0.162	0.032	0.116	0.568	0.124	0.209	5.927

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	104	124	156	204	160	177	223	156
N.S.	1	1.00	0.82	0.98	1.23	1.61	1.26	1.39	1.76	1.23
time (sec)	N/A	0.481	0.028	0.162	0.035	0.098	0.745	0.122	0.196	5.840

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	94	117	166	166	168	117	118	165
N.S.	1	1.00	0.73	0.91	1.30	1.30	1.31	0.91	0.92	1.29
time (sec)	N/A	0.465	0.028	0.169	0.034	0.102	0.910	0.118	0.213	0.083

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	94	117	174	174	178	116	171	176
N.S.	1	1.00	0.70	0.87	1.29	1.29	1.32	0.86	1.27	1.30
time (sec)	N/A	0.480	0.025	0.168	0.040	0.095	1.136	0.124	0.210	0.094

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	160	200	202	202	216	205	207	202
N.S.	1	1.00	0.86	1.07	1.08	1.08	1.16	1.10	1.11	1.08
time (sec)	N/A	0.636	0.046	0.155	0.036	0.105	0.032	0.116	0.210	0.082

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	120	140	140	140	150	142	145	140
N.S.	1	1.00	0.95	1.11	1.11	1.11	1.19	1.13	1.15	1.11
time (sec)	N/A	0.479	0.018	0.154	0.027	0.062	0.027	0.121	0.212	5.917

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	94	83	82	82	92	82	83	82
N.S.	1	0.98	1.42	1.26	1.24	1.24	1.39	1.24	1.26	1.24
time (sec)	N/A	0.335	0.003	0.141	0.033	0.092	0.023	0.123	0.207	0.035

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	36	36	36	36	39	36
N.S.	1	1.00	1.00	0.92	0.90	0.90	0.90	0.90	0.98	0.90
time (sec)	N/A	0.280	0.002	0.108	0.025	0.096	0.022	0.127	0.194	0.041

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	153	212	216	215	189	223	237	233
N.S.	1	1.00	0.77	1.07	1.09	1.09	0.95	1.13	1.20	1.18
time (sec)	N/A	0.654	0.046	0.163	0.036	0.079	0.227	0.117	0.224	5.870

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	202	221	223	296	206	287	309	293
N.S.	1	1.00	1.10	1.20	1.21	1.61	1.12	1.56	1.68	1.59
time (sec)	N/A	0.683	0.048	0.166	0.034	0.091	0.375	0.118	0.287	0.074

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	209	221	230	341	235	220	366	252
N.S.	1	1.00	1.09	1.16	1.20	1.79	1.23	1.15	1.92	1.32
time (sec)	N/A	0.653	0.061	0.162	0.037	0.073	0.641	0.123	0.224	5.958

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	117	177	179	174	214	183	237	176
N.S.	1	1.09	1.14	1.72	1.74	1.69	2.08	1.78	2.30	1.71
time (sec)	N/A	0.502	0.060	0.197	0.028	0.115	0.666	0.126	0.210	5.983

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	86	123	122	119	163	130	167	122
N.S.	1	1.08	1.13	1.62	1.61	1.57	2.14	1.71	2.20	1.61
time (sec)	N/A	0.415	0.032	0.184	0.028	0.070	0.510	0.130	0.216	0.149

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	65	55	82	82	76	112	84	107	81
N.S.	1	1.05	0.89	1.32	1.32	1.23	1.81	1.35	1.73	1.31
time (sec)	N/A	0.386	0.024	0.177	0.034	0.094	0.300	0.121	0.208	6.083

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	37	37	46	46	41	71	48	56	45
N.S.	1	0.80	0.80	1.00	1.00	0.89	1.54	1.04	1.22	0.98
time (sec)	N/A	0.277	0.007	0.157	0.034	0.082	0.156	0.115	0.200	6.017

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	26	31	25	20	33	27	14
N.S.	1	1.00	1.00	1.86	2.21	1.79	1.43	2.36	1.93	1.00
time (sec)	N/A	0.229	0.004	0.138	0.029	0.083	0.066	0.118	0.200	0.045

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	58	70	71	64	0	93	81	72
N.S.	1	1.04	0.78	0.95	0.96	0.86	0.00	1.26	1.09	0.97
time (sec)	N/A	0.412	0.030	0.224	0.035	0.211	0.000	0.118	0.232	6.706

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	113	118	107	157	244	0	285	350	147
N.S.	1	1.06	1.10	1.00	1.47	2.28	0.00	2.66	3.27	1.37
time (sec)	N/A	0.525	0.058	0.254	0.035	0.504	0.000	0.132	0.232	6.101

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	170	197	158	314	605	0	277	839	294
N.S.	1	1.06	1.22	0.98	1.95	3.76	0.00	1.72	5.21	1.83
time (sec)	N/A	0.680	0.096	0.296	0.042	2.812	0.000	0.114	0.221	6.605

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	134	159	203	195	298	264	198	426	193
N.S.	1	1.02	1.21	1.55	1.49	2.27	2.02	1.51	3.25	1.47
time (sec)	N/A	0.597	0.064	0.192	0.034	0.082	0.874	0.118	0.216	5.872

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	103	149	148	215	216	153	305	147
N.S.	1	1.00	0.84	1.21	1.20	1.75	1.76	1.24	2.48	1.20
time (sec)	N/A	0.536	0.053	0.193	0.038	0.098	0.649	0.121	0.228	5.890

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	89	85	109	111	155	156	114	202	115
N.S.	1	1.09	1.04	1.33	1.35	1.89	1.90	1.39	2.46	1.40
time (sec)	N/A	0.443	0.030	0.178	0.027	0.087	0.325	0.121	0.210	0.111

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	58	62	68	87	56	70	97	46
N.S.	1	1.00	1.05	1.13	1.24	1.58	1.02	1.27	1.76	0.84
time (sec)	N/A	0.285	0.012	0.156	0.032	0.079	0.150	0.125	0.218	5.840

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	51	52	70	39	54	84	36
N.S.	1	1.00	0.90	1.28	1.30	1.75	0.98	1.35	2.10	0.90
time (sec)	N/A	0.263	0.015	0.148	0.026	0.069	0.103	0.125	0.226	0.090

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	142	128	134	195	298	0	231	398	201
N.S.	1	1.02	0.92	0.96	1.40	2.14	0.00	1.66	2.86	1.45
time (sec)	N/A	0.599	0.077	0.276	0.038	1.353	0.000	0.127	0.224	6.560

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	184	188	174	410	799	0	528	1153	346
N.S.	1	1.03	1.06	0.98	2.30	4.49	0.00	2.97	6.48	1.94
time (sec)	N/A	0.740	0.130	0.346	0.046	5.136	0.000	0.120	0.231	6.730

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	246	267	232	671	1464	0	496	2057	653
N.S.	1	1.04	1.13	0.98	2.83	6.18	0.00	2.09	8.68	2.76
time (sec)	N/A	0.987	0.195	0.386	0.062	14.541	0.000	0.125	0.209	7.108

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	164	158	221	234	383	318	231	524	178
N.S.	1	0.89	0.86	1.20	1.27	2.08	1.73	1.26	2.85	0.97
time (sec)	N/A	0.678	0.052	0.199	0.037	0.104	1.162	0.124	0.210	0.156

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	136	167	184	285	258	178	353	185
N.S.	1	1.03	0.88	1.08	1.19	1.85	1.68	1.16	2.29	1.20
time (sec)	N/A	0.701	0.044	0.182	0.038	0.121	0.697	0.120	0.217	0.122

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	158	110	135	152	252	144	146	327	114
N.S.	1	1.25	0.87	1.07	1.21	2.00	1.14	1.16	2.60	0.90
time (sec)	N/A	0.626	0.040	0.191	0.036	0.109	0.436	0.125	0.225	0.108

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	85	76	74	92	132	88	84	163	68
N.S.	1	1.09	0.97	0.95	1.18	1.69	1.13	1.08	2.09	0.87
time (sec)	N/A	0.319	0.020	0.164	0.033	0.086	0.221	0.128	0.218	5.929

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	70	64	64	75	110	70	68	145	59
N.S.	1	1.13	1.03	1.03	1.21	1.77	1.13	1.10	2.34	0.95
time (sec)	N/A	0.292	0.010	0.153	0.032	0.093	0.148	0.121	0.201	5.814

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	239	210	225	409	663	0	408	890	448
N.S.	1	1.01	0.89	0.95	1.73	2.81	0.00	1.73	3.77	1.90
time (sec)	N/A	0.915	0.133	0.336	0.048	8.758	0.000	0.121	0.212	6.866

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	279	281	265	776	1573	0	714	2287	697
N.S.	1	1.02	1.03	0.97	2.84	5.76	0.00	2.62	8.38	2.55
time (sec)	N/A	1.140	0.153	0.419	0.069	25.006	0.000	0.132	0.200	7.195

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	344	372	326	1132	2519	0	757	3600	1096
N.S.	1	1.03	1.11	0.97	3.38	7.52	0.00	2.26	10.75	3.27
time (sec)	N/A	1.482	0.265	0.507	0.065	132.566	0.000	0.126	0.212	7.806

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	101	94	93	93	100	96	99	93
N.S.	1	1.00	1.77	1.65	1.63	1.63	1.75	1.68	1.74	1.63
time (sec)	N/A	0.384	0.019	0.140	0.026	0.073	0.029	0.128	0.222	0.051

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	72	72	72	80	73	75	71
N.S.	1	1.00	1.30	1.26	1.26	1.26	1.40	1.28	1.32	1.25
time (sec)	N/A	0.358	0.015	0.138	0.026	0.097	0.027	0.118	0.234	5.760

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	48	47	47	51	49	51	48
N.S.	1	1.00	0.93	0.84	0.82	0.82	0.89	0.86	0.89	0.84
time (sec)	N/A	0.344	0.009	0.135	0.033	0.073	0.022	0.120	0.246	0.026

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	32	27	26	26	29	26	27	26
N.S.	1	1.03	1.03	0.87	0.84	0.84	0.94	0.84	0.87	0.84
time (sec)	N/A	0.274	0.003	0.102	0.026	0.095	0.024	0.119	0.214	0.046

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	39	39	39	36	40	44	39
N.S.	1	1.00	0.93	0.95	0.95	0.95	0.88	0.98	1.07	0.95
time (sec)	N/A	0.311	0.013	0.143	0.026	0.073	0.084	0.120	0.220	5.510

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	44	46	59	42	66	64	49
N.S.	1	1.00	0.91	1.02	1.07	1.37	0.98	1.53	1.49	1.14
time (sec)	N/A	0.316	0.020	0.142	0.026	0.069	0.121	0.120	0.220	0.065

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	48	47	57	73	56	48	83	58
N.S.	1	0.96	0.91	0.89	1.08	1.38	1.06	0.91	1.57	1.09
time (sec)	N/A	0.329	0.017	0.143	0.034	0.088	0.161	0.120	0.225	5.722

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	39	39	63	63	66	38	48	63
N.S.	1	0.96	0.72	0.72	1.17	1.17	1.22	0.70	0.89	1.17
time (sec)	N/A	0.329	0.012	0.146	0.032	0.065	0.214	0.129	0.251	0.045

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	40	40	75	75	80	59	73	77
N.S.	1	0.97	0.68	0.68	1.27	1.27	1.36	1.00	1.24	1.31
time (sec)	N/A	0.325	0.013	0.149	0.037	0.067	0.269	0.126	0.259	5.611

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	167	165	163	163	182	172	174	160
N.S.	1	1.00	1.43	1.41	1.39	1.39	1.56	1.47	1.49	1.37
time (sec)	N/A	0.538	0.020	0.150	0.030	0.062	0.033	0.122	0.279	0.072

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	128	130	130	141	131	133	127
N.S.	1	1.00	1.00	1.09	1.11	1.11	1.21	1.12	1.14	1.09
time (sec)	N/A	0.502	0.017	0.146	0.026	0.088	0.028	0.111	0.224	5.656

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	91	88	87	87	95	89	92	87
N.S.	1	1.00	1.14	1.10	1.09	1.09	1.19	1.11	1.15	1.09
time (sec)	N/A	0.364	0.011	0.145	0.027	0.071	0.025	0.122	0.194	0.049

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	50	50	58	50	51	50
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.13	1.11
time (sec)	N/A	0.298	0.002	0.133	0.026	0.064	0.021	0.117	0.191	0.028

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	96	105	105	88	106	116	106
N.S.	1	1.00	0.84	1.02	1.12	1.12	0.94	1.13	1.23	1.13
time (sec)	N/A	0.419	0.024	0.193	0.061	0.065	0.150	0.125	0.214	0.047

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	100	112	150	107	153	159	116
N.S.	1	1.00	0.97	1.06	1.19	1.60	1.14	1.63	1.69	1.23
time (sec)	N/A	0.423	0.046	0.155	0.032	0.069	0.219	0.124	0.204	0.060

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	111	107	120	177	122	112	193	125
N.S.	1	1.00	1.11	1.07	1.20	1.77	1.22	1.12	1.93	1.25
time (sec)	N/A	0.445	0.030	0.155	0.035	0.069	0.374	0.113	0.217	0.070

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	110	106	130	183	138	106	171	133
N.S.	1	1.00	1.09	1.05	1.29	1.81	1.37	1.05	1.69	1.32
time (sec)	N/A	0.450	0.045	0.158	0.033	0.093	0.533	0.124	0.203	6.944

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	100	109	146	194	150	165	211	144
N.S.	1	1.00	0.92	1.00	1.34	1.78	1.38	1.51	1.94	1.32
time (sec)	N/A	0.446	0.031	0.161	0.029	0.072	0.692	0.128	0.195	5.843

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	90	105	151	151	162	104	107	148
N.S.	1	1.00	0.82	0.95	1.37	1.37	1.47	0.95	0.97	1.35
time (sec)	N/A	0.444	0.032	0.160	0.034	0.065	0.914	0.127	0.187	6.125

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	89	102	159	159	172	103	160	157
N.S.	1	1.00	0.76	0.87	1.36	1.36	1.47	0.88	1.37	1.34
time (sec)	N/A	0.443	0.026	0.160	0.037	0.090	1.109	0.121	0.200	5.750

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	90	105	173	173	185	104	171	171
N.S.	1	1.00	0.79	0.92	1.52	1.52	1.62	0.91	1.50	1.50
time (sec)	N/A	0.440	0.031	0.165	0.036	0.072	1.385	0.123	0.211	0.075

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	338	336	336	336	371	362	365	329
N.S.	1	1.00	1.78	1.77	1.77	1.77	1.95	1.91	1.92	1.73
time (sec)	N/A	0.908	0.041	0.160	0.029	0.091	0.041	0.116	0.200	5.824

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	252	284	292	292	321	303	307	281
N.S.	1	1.00	1.33	1.49	1.54	1.54	1.69	1.59	1.62	1.48
time (sec)	N/A	0.799	0.046	0.158	0.028	0.074	0.039	0.120	0.197	5.744

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	206	234	232	232	255	246	249	224
N.S.	1	1.00	1.10	1.24	1.23	1.23	1.36	1.31	1.32	1.19
time (sec)	N/A	0.729	0.042	0.156	0.032	0.069	0.038	0.117	0.197	6.731

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	155	183	181	181	202	188	191	174
N.S.	1	1.00	0.96	1.14	1.12	1.12	1.25	1.17	1.19	1.08
time (sec)	N/A	0.575	0.040	0.149	0.027	0.077	0.030	0.123	0.202	5.666

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	116	127	126	126	139	129	133	121
N.S.	1	1.00	1.12	1.22	1.21	1.21	1.34	1.24	1.28	1.16
time (sec)	N/A	0.415	0.021	0.148	0.028	0.073	0.026	0.127	0.210	0.053

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	85	74	73	73	85	73	75	73
N.S.	1	0.98	1.52	1.32	1.30	1.30	1.52	1.30	1.34	1.30
time (sec)	N/A	0.326	0.009	0.138	0.034	0.073	0.023	0.114	0.216	0.035

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	142	190	198	198	177	205	221	213
N.S.	1	1.00	0.82	1.10	1.14	1.14	1.02	1.18	1.28	1.23
time (sec)	N/A	0.610	0.042	0.159	0.032	0.105	0.188	0.122	0.213	0.058

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	193	194	206	272	192	270	289	274
N.S.	1	1.00	1.22	1.23	1.30	1.72	1.22	1.71	1.83	1.73
time (sec)	N/A	0.610	0.047	0.161	0.033	0.074	0.356	0.124	0.220	0.077

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	198	191	214	315	218	205	341	235
N.S.	1	1.00	1.21	1.17	1.31	1.93	1.34	1.26	2.09	1.44
time (sec)	N/A	0.617	0.054	0.161	0.041	0.073	0.593	0.124	0.223	5.707

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	197	198	226	334	238	205	332	228
N.S.	1	1.00	1.19	1.20	1.37	2.02	1.44	1.24	2.01	1.38
time (sec)	N/A	0.595	0.050	0.163	0.036	0.091	0.972	0.118	0.210	0.102

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	185	203	239	356	243	290	395	236
N.S.	1	1.00	1.08	1.19	1.40	2.08	1.42	1.70	2.31	1.38
time (sec)	N/A	0.616	0.049	0.164	0.038	0.073	1.497	0.118	0.209	5.748

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	182	198	246	326	264	200	301	247
N.S.	1	1.00	1.06	1.15	1.43	1.90	1.53	1.16	1.75	1.44
time (sec)	N/A	0.619	0.062	0.165	0.037	0.070	2.278	0.124	0.208	5.954

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	172	201	263	339	272	208	372	262
N.S.	1	1.00	0.93	1.09	1.43	1.84	1.48	1.13	2.02	1.42
time (sec)	N/A	0.608	0.049	0.165	0.037	0.071	3.524	0.124	0.223	7.246

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	161	191	263	263	286	202	203	254
N.S.	1	1.00	0.90	1.07	1.48	1.48	1.61	1.13	1.14	1.43
time (sec)	N/A	0.591	0.042	0.168	0.040	0.077	6.088	0.120	0.217	5.866

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	163	199	282	282	301	204	281	275
N.S.	1	1.00	0.87	1.06	1.50	1.50	1.60	1.09	1.49	1.46
time (sec)	N/A	0.591	0.045	0.171	0.039	0.099	11.416	0.128	0.213	0.109

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	163	199	293	293	313	204	292	287
N.S.	1	1.00	0.86	1.05	1.54	1.54	1.65	1.07	1.54	1.51
time (sec)	N/A	0.609	0.042	0.174	0.044	0.076	23.252	0.125	0.203	5.875

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	423	500	510	510	571	547	549	495
N.S.	1	1.00	1.52	1.80	1.83	1.83	2.05	1.97	1.97	1.78
time (sec)	N/A	1.228	0.080	0.172	0.029	0.085	0.052	0.127	0.206	0.284

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	361	435	441	441	486	472	474	425
N.S.	1	1.00	1.31	1.58	1.60	1.60	1.76	1.71	1.72	1.54
time (sec)	N/A	1.143	0.106	0.167	0.038	0.091	0.049	0.116	0.198	0.233

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	307	370	374	374	418	397	399	357
N.S.	1	1.00	1.10	1.33	1.35	1.35	1.50	1.43	1.44	1.28
time (sec)	N/A	1.080	0.085	0.165	0.027	0.065	0.043	0.119	0.218	6.033

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	300	305	306	306	340	322	324	288
N.S.	1	1.00	1.11	1.13	1.13	1.13	1.26	1.19	1.20	1.07
time (sec)	N/A	0.857	0.038	0.157	0.036	0.062	0.038	0.125	0.218	0.154

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	197	240	244	244	270	247	249	225
N.S.	1	1.00	0.94	1.15	1.17	1.17	1.29	1.18	1.19	1.08
time (sec)	N/A	0.694	0.062	0.159	0.029	0.064	0.035	0.114	0.218	0.124

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	148	168	169	169	187	171	174	161
N.S.	1	1.00	1.12	1.27	1.28	1.28	1.42	1.30	1.32	1.22
time (sec)	N/A	0.506	0.025	0.153	0.028	0.062	0.030	0.114	0.204	6.002

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	71	110	97	96	96	112	96	99	96
N.S.	1	0.97	1.51	1.33	1.32	1.32	1.53	1.32	1.36	1.32
time (sec)	N/A	0.336	0.003	0.142	0.026	0.064	0.024	0.122	0.209	0.057

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	227	307	319	319	292	338	360	357
N.S.	1	1.00	0.86	1.16	1.21	1.21	1.11	1.28	1.36	1.35
time (sec)	N/A	0.872	0.086	0.389	0.027	0.092	0.275	0.130	0.207	5.984

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	289	315	330	421	314	414	452	701
N.S.	1	1.00	1.13	1.24	1.29	1.65	1.23	1.62	1.77	2.75
time (sec)	N/A	0.849	0.076	0.165	0.033	0.068	0.484	0.120	0.203	5.995

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	14	16	15	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88	0.88
time (sec)	N/A	0.258	0.005	0.125	0.023	0.066	0.027	0.124	0.225	5.646

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	7	9	9	7	9	6	6
N.S.	1	1.00	1.00	0.64	0.82	0.82	0.64	0.82	0.55	0.55
time (sec)	N/A	0.228	0.001	0.107	0.028	0.071	0.022	0.120	0.216	0.018

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	16	12	13	13	12	14	13	11
N.S.	1	1.00	1.23	0.92	1.00	1.00	0.92	1.08	1.00	0.85
time (sec)	N/A	0.269	0.005	0.125	0.029	0.067	0.030	0.128	0.225	5.849

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88	0.88
time (sec)	N/A	0.263	0.007	0.115	0.033	0.068	0.026	0.131	0.225	0.034

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	111	117	114	283	401	119	166	127
N.S.	1	1.00	0.90	0.95	0.93	2.30	3.26	0.97	1.35	1.03
time (sec)	N/A	0.463	0.075	0.291	0.113	0.086	0.484	0.120	0.274	0.122

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	80	80	79	187	308	81	104	91
N.S.	1	1.00	0.89	0.89	0.88	2.08	3.42	0.90	1.16	1.01
time (sec)	N/A	0.405	0.041	0.220	0.108	0.077	0.387	0.120	0.208	0.139

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	53	53	137	185	53	71	62
N.S.	1	1.00	0.95	0.90	0.90	2.32	3.14	0.90	1.20	1.05
time (sec)	N/A	0.341	0.034	0.203	0.118	0.079	0.238	0.125	0.202	6.153

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	39	32
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.93	0.76
time (sec)	N/A	0.269	0.011	0.159	0.118	0.077	0.130	0.129	0.230	6.019

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	79	63	69	76	135	0	80	60	230
N.S.	1	0.92	0.73	0.80	0.88	1.57	0.00	0.93	0.70	2.67
time (sec)	N/A	0.335	0.028	0.221	0.116	0.088	0.000	0.121	0.227	6.640

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	137	113	104	167	350	0	193	241	452
N.S.	1	1.11	0.92	0.85	1.36	2.85	0.00	1.57	1.96	3.67
time (sec)	N/A	0.487	0.070	0.239	0.114	0.175	0.000	0.126	0.229	6.551

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	189	140	160	323	853	0	282	594	745
N.S.	1	1.07	0.80	0.91	1.84	4.85	0.00	1.60	3.38	4.23
time (sec)	N/A	0.614	0.220	0.312	0.115	0.762	0.000	0.123	0.211	6.938

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	183	164	183	181	561	515	184	367	191
N.S.	1	0.98	0.88	0.98	0.97	3.02	2.77	0.99	1.97	1.03
time (sec)	N/A	0.599	0.098	0.287	0.106	0.091	1.087	0.117	0.216	0.163

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	143	137	137	140	433	403	137	296	131
N.S.	1	0.99	0.95	0.95	0.97	3.01	2.80	0.95	2.06	0.91
time (sec)	N/A	0.514	0.073	0.259	0.109	0.111	0.719	0.121	0.221	6.284

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	107	107	107	108	311	298	107	197	143
N.S.	1	0.96	0.96	0.96	0.96	2.78	2.66	0.96	1.76	1.28
time (sec)	N/A	0.435	0.067	0.220	0.105	0.083	0.527	0.119	0.213	0.095

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	72	77	74	73	223	129	70	144	68
N.S.	1	0.91	0.97	0.94	0.92	2.82	1.63	0.89	1.82	0.86
time (sec)	N/A	0.300	0.081	0.184	0.105	0.098	0.269	0.120	0.222	6.089

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	49	48	140	90	47	71	44
N.S.	1	1.00	1.00	0.86	0.84	2.46	1.58	0.82	1.25	0.77
time (sec)	N/A	0.274	0.036	0.162	0.120	0.088	0.139	0.124	0.219	0.053

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	161	138	138	189	441	0	201	284	609
N.S.	1	1.13	0.97	0.97	1.33	3.11	0.00	1.42	2.00	4.29
time (sec)	N/A	0.549	0.119	0.247	0.116	0.390	0.000	0.121	0.210	6.704

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	219	162	181	394	1111	0	402	837	819
N.S.	1	1.12	0.83	0.92	2.01	5.67	0.00	2.05	4.27	4.18
time (sec)	N/A	0.677	0.253	0.286	0.110	1.207	0.000	0.130	0.215	7.157

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	210	199	215	236	707	520	217	479	467
N.S.	1	0.93	0.88	0.96	1.05	3.14	2.31	0.96	2.13	2.08
time (sec)	N/A	0.676	0.192	0.287	0.112	0.089	1.709	0.130	0.206	0.266

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	129	148	164	182	554	328	168	392	197
N.S.	1	0.78	0.90	0.99	1.10	3.36	1.99	1.02	2.38	1.19
time (sec)	N/A	0.378	0.197	0.199	0.132	0.094	1.059	0.124	0.217	6.194

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	105	127	119	144	406	272	128	266	125
N.S.	1	0.77	0.93	0.87	1.05	2.96	1.99	0.93	1.94	0.91
time (sec)	N/A	0.344	0.104	0.194	0.114	0.097	0.636	0.127	0.218	6.253

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	121	101	95	113	359	172	97	242	101
N.S.	1	1.08	0.90	0.85	1.01	3.21	1.54	0.87	2.16	0.90
time (sec)	N/A	0.379	0.060	0.191	0.107	0.083	0.398	0.124	0.214	5.980

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	71	70	74	212	124	60	124	64
N.S.	1	1.09	0.95	0.93	0.99	2.83	1.65	0.80	1.65	0.85
time (sec)	N/A	0.301	0.046	0.169	0.110	0.090	0.211	0.117	0.215	0.086

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	256	180	243	386	1020	0	361	679	987
N.S.	1	1.21	0.85	1.15	1.82	4.81	0.00	1.70	3.20	4.66
time (sec)	N/A	0.792	0.129	0.286	0.129	1.903	0.000	0.130	0.210	7.205

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	339	241	311	749	2322	0	598	1734	1296
N.S.	1	1.22	0.86	1.11	2.68	8.32	0.00	2.14	6.22	4.65
time (sec)	N/A	1.020	0.294	0.433	0.131	5.442	0.000	0.125	0.222	7.940

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	186	270	298	336	1026	508	322	750	342
N.S.	1	0.64	0.93	1.03	1.16	3.54	1.75	1.11	2.59	1.18
time (sec)	N/A	0.460	0.110	0.226	0.119	0.084	6.546	0.124	0.226	0.227

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	162	220	236	283	822	447	265	576	281
N.S.	1	0.68	0.92	0.98	1.18	3.42	1.86	1.10	2.40	1.17
time (sec)	N/A	0.417	0.183	0.203	0.144	0.132	2.783	0.118	0.225	6.461

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	162	197	198	235	746	413	214	551	263
N.S.	1	0.74	0.90	0.91	1.08	3.42	1.89	0.98	2.53	1.21
time (sec)	N/A	0.443	0.120	0.210	0.112	0.094	1.507	0.123	0.226	0.199

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	179	155	146	188	560	320	159	373	163
N.S.	1	1.03	0.89	0.84	1.08	3.22	1.84	0.91	2.14	0.94
time (sec)	N/A	0.516	0.132	0.201	0.115	0.119	0.851	0.122	0.221	6.050

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	146	127	116	151	490	214	126	344	132
N.S.	1	1.01	0.88	0.81	1.05	3.40	1.49	0.88	2.39	0.92
time (sec)	N/A	0.402	0.078	0.204	0.112	0.097	0.525	0.109	0.243	0.131

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	107	83	91	97	282	150	72	175	87
N.S.	1	1.15	0.89	0.98	1.04	3.03	1.61	0.77	1.88	0.94
time (sec)	N/A	0.326	0.039	0.174	0.111	0.104	0.280	0.129	0.267	6.246

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	360	265	390	655	1784	0	562	1226	1470
N.S.	1	1.22	0.90	1.32	2.22	6.05	0.00	1.91	4.16	4.98
time (sec)	N/A	1.123	0.197	0.434	0.142	9.914	0.000	0.115	0.223	7.781

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	490	336	473	1215	3843	0	906	2894	1876
N.S.	1	1.30	0.89	1.25	3.22	10.19	0.00	2.40	7.68	4.98
time (sec)	N/A	1.410	0.339	0.358	0.164	19.959	0.000	0.134	0.319	8.414

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	36	22	21	21	22	22	21	25
N.S.	1	1.16	1.16	0.71	0.68	0.68	0.71	0.71	0.68	0.81
time (sec)	N/A	0.268	0.007	0.164	0.115	0.104	0.064	0.116	0.229	0.062

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	28	25	20	19	19	20	20	19	25
N.S.	1	1.12	1.00	0.80	0.76	0.76	0.80	0.80	0.76	1.00
time (sec)	N/A	0.268	0.009	0.151	0.105	0.080	0.059	0.114	0.214	6.062

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	25	20	19	19	19	20	19	25
N.S.	1	1.04	1.00	0.80	0.76	0.76	0.76	0.80	0.76	1.00
time (sec)	N/A	0.264	0.006	0.151	0.108	0.089	0.058	0.127	0.205	0.054

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	27	19	18	17	17	17	17	18	17
N.S.	1	1.23	0.86	0.82	0.77	0.77	0.77	0.77	0.82	0.77
time (sec)	N/A	0.261	0.002	0.072	0.037	0.069	0.016	0.125	0.206	0.033

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	184	225	216	0	1617	0	416	425	3385
N.S.	1	1.10	1.35	1.29	0.00	9.68	0.00	2.49	2.54	20.27
time (sec)	N/A	0.782	0.553	0.953	0.000	0.145	0.000	0.165	0.209	6.486

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	167	208	168	0	974	0	314	268	1581
N.S.	1	1.12	1.40	1.13	0.00	6.54	0.00	2.11	1.80	10.61
time (sec)	N/A	0.648	0.367	0.451	0.000	0.117	0.000	0.167	0.202	6.118

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	167	156	127	0	343	0	203	123	302
N.S.	1	1.25	1.16	0.95	0.00	2.56	0.00	1.51	0.92	2.25
time (sec)	N/A	0.460	0.289	0.408	0.000	0.083	0.000	0.154	0.221	0.363

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	148	153	101	0	949	0	206	271	1366
N.S.	1	1.10	1.14	0.75	0.00	7.08	0.00	1.54	2.02	10.19
time (sec)	N/A	0.405	0.303	0.363	0.000	0.090	0.000	0.139	0.231	0.373

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	209	218	188	0	2861	0	656	499	4412
N.S.	1	1.31	1.36	1.18	0.00	17.88	0.00	4.10	3.12	27.58
time (sec)	N/A	0.535	0.477	0.452	0.000	0.128	0.000	0.176	0.215	6.697

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	270	241	224	0	5142	0	1165	1425	7831
N.S.	1	1.42	1.27	1.18	0.00	27.06	0.00	6.13	7.50	41.22
time (sec)	N/A	0.770	0.719	0.674	0.000	0.226	0.000	0.189	0.220	8.005

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	274	295	341	0	2073	0	589	1267	4090
N.S.	1	1.04	1.12	1.30	0.00	7.88	0.00	2.24	4.82	15.55
time (sec)	N/A	0.979	1.073	0.868	0.000	0.331	0.000	0.248	0.251	7.239

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	244	257	296	0	1370	0	506	897	1988
N.S.	1	1.06	1.11	1.28	0.00	5.93	0.00	2.19	3.88	8.61
time (sec)	N/A	0.796	1.231	0.701	0.000	0.137	0.000	0.240	0.329	7.013

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	239	233	250	0	679	0	414	580	704
N.S.	1	1.14	1.11	1.20	0.00	3.25	0.00	1.98	2.78	3.37
time (sec)	N/A	0.714	1.115	0.563	0.000	0.107	0.000	0.213	0.348	6.040

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	201	226	246	0	1385	0	347	908	2332
N.S.	1	1.04	1.16	1.27	0.00	7.14	0.00	1.79	4.68	12.02
time (sec)	N/A	0.536	0.585	0.675	0.000	0.117	0.000	0.193	0.313	7.608

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	262	274	305	0	3279	0	873	1305	5253
N.S.	1	1.18	1.23	1.37	0.00	14.77	0.00	3.93	5.88	23.66
time (sec)	N/A	0.784	0.790	0.701	0.000	0.363	0.000	0.189	0.439	8.208

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	342	311	347	0	5703	0	1411	1867	8700
N.S.	1	1.29	1.17	1.31	0.00	21.52	0.00	5.32	7.05	32.83
time (sec)	N/A	0.907	1.976	0.705	0.000	0.930	0.000	0.301	0.686	8.818

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	400	359	402	0	8308	0	1998	4569	12290
N.S.	1	1.29	1.15	1.29	0.00	26.71	0.00	6.42	14.69	39.52
time (sec)	N/A	1.152	1.402	1.178	0.000	3.168	0.000	0.342	1.255	10.614

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	323	346	377	0	1730	0	714	1967	2518
N.S.	1	1.10	1.18	1.28	0.00	5.88	0.00	2.43	6.69	8.56
time (sec)	N/A	0.937	2.156	0.987	0.000	0.211	0.000	0.271	3.546	0.758

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	312	289	329	0	1029	0	596	1430	1015
N.S.	1	1.12	1.04	1.18	0.00	3.69	0.00	2.14	5.13	3.64
time (sec)	N/A	0.854	1.684	0.835	0.000	0.132	0.000	0.267	3.401	8.050

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	284	268	281	0	1753	0	500	2012	3191
N.S.	1	1.06	1.00	1.05	0.00	6.54	0.00	1.87	7.51	11.91
time (sec)	N/A	0.871	1.876	0.684	0.000	0.143	0.000	0.250	3.905	8.883

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	344	340	369	0	3787	0	1064	2568	6163
N.S.	1	1.22	1.21	1.31	0.00	13.48	0.00	3.79	9.14	21.93
time (sec)	N/A	0.855	1.254	1.184	0.000	0.457	0.000	0.285	3.989	9.355

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	401	357	505	0	5776	0	1642	3161	8961
N.S.	1	1.27	1.13	1.60	0.00	18.34	0.00	5.21	10.03	28.45
time (sec)	N/A	1.065	1.274	1.532	0.000	2.306	0.000	0.270	4.009	9.931

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	42	35	29	45	40	48	48	40	28
N.S.	1	1.20	1.00	0.83	1.29	1.14	1.37	1.37	1.14	0.80
time (sec)	N/A	0.290	0.059	0.405	0.120	0.095	1.654	0.128	0.203	6.359

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	77	66	47	0	267	80	54	85	46
N.S.	1	1.33	1.14	0.81	0.00	4.60	1.38	0.93	1.47	0.79
time (sec)	N/A	0.370	0.080	0.497	0.000	0.124	1.813	0.124	0.209	0.177

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	157	150	114	0	299	0	216	121	255
N.S.	1	1.19	1.14	0.86	0.00	2.27	0.00	1.64	0.92	1.93
time (sec)	N/A	0.490	0.242	0.601	0.000	0.106	0.000	0.217	0.209	1.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	41	47	108	218	357	107	44
N.S.	1	1.00	0.70	0.65	0.75	1.71	3.46	5.67	1.70	0.70
time (sec)	N/A	0.330	0.051	0.234	0.034	0.068	0.354	0.130	0.199	6.275

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	41	47	85	76	229	83	44
N.S.	1	1.00	0.70	0.65	0.75	1.35	1.21	3.63	1.32	0.70
time (sec)	N/A	0.328	0.042	0.221	0.027	0.092	0.518	0.120	0.195	0.057

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	40	47	62	76	127	59	44
N.S.	1	1.00	0.70	0.63	0.75	0.98	1.21	2.02	0.94	0.70
time (sec)	N/A	0.329	0.037	0.168	0.036	0.098	0.479	0.125	0.191	6.099

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	40	53	40	70	53	39	44
N.S.	1	1.00	0.72	0.66	0.87	0.66	1.15	0.87	0.64	0.72
time (sec)	N/A	0.327	0.043	0.165	0.032	0.089	0.360	0.117	0.190	6.484

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	39	54	49	75	56	40	44
N.S.	1	1.00	0.73	0.66	0.92	0.83	1.27	0.95	0.68	0.75
time (sec)	N/A	0.329	0.043	0.181	0.028	0.090	0.615	0.122	0.203	0.057

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	46	38	52	61	168	48	48	44
N.S.	1	1.00	0.78	0.64	0.88	1.03	2.85	0.81	0.81	0.75
time (sec)	N/A	0.323	0.050	0.196	0.036	0.073	0.317	0.121	0.197	0.048

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	38	44	72	252	44	59	44
N.S.	1	1.00	0.72	0.62	0.72	1.18	4.13	0.72	0.97	0.72
time (sec)	N/A	0.319	0.044	0.180	0.041	0.135	0.461	0.122	0.212	0.061

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	88	113	218	177	704	223	114
N.S.	1	1.00	0.76	0.69	0.89	1.72	1.39	5.54	1.76	0.90
time (sec)	N/A	0.422	0.071	0.244	0.032	0.083	0.734	0.124	0.213	0.071

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	97	88	113	181	177	471	182	114
N.S.	1	1.00	0.76	0.69	0.89	1.43	1.39	3.71	1.43	0.90
time (sec)	N/A	0.425	0.081	0.226	0.036	0.072	0.648	0.123	0.212	0.040

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	88	113	143	177	274	141	114
N.S.	1	1.00	0.76	0.69	0.89	1.13	1.39	2.16	1.11	0.90
time (sec)	N/A	0.411	0.058	0.216	0.051	0.076	0.584	0.124	0.259	0.045

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	96	88	120	107	175	120	104	114
N.S.	1	1.00	0.77	0.70	0.96	0.86	1.40	0.96	0.83	0.91
time (sec)	N/A	0.401	0.057	0.195	0.045	0.091	0.554	0.121	0.233	0.041

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	97	88	121	117	160	140	106	128
N.S.	1	1.00	0.79	0.72	0.98	0.95	1.30	1.14	0.86	1.04
time (sec)	N/A	0.397	0.072	0.217	0.028	0.081	1.604	0.122	0.205	0.045

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	96	84	119	128	153	135	113	122
N.S.	1	1.00	0.78	0.68	0.97	1.04	1.24	1.10	0.92	0.99
time (sec)	N/A	0.428	0.076	0.256	0.043	0.100	1.622	0.116	0.204	6.440

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	96	91	121	137	592	131	124	105
N.S.	1	1.00	0.78	0.74	0.98	1.11	4.81	1.07	1.01	0.85
time (sec)	N/A	0.432	0.076	0.251	0.055	0.074	0.493	0.125	0.199	0.095

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	171	162	209	356	304	1147	373	187
N.S.	1	1.00	0.84	0.79	1.02	1.75	1.49	5.62	1.83	0.92
time (sec)	N/A	0.562	0.133	0.325	0.028	0.089	0.890	0.137	0.202	6.420

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	170	163	209	303	304	784	315	187
N.S.	1	1.00	0.83	0.80	1.02	1.49	1.49	3.84	1.54	0.92
time (sec)	N/A	0.564	0.134	0.273	0.034	0.084	0.772	0.130	0.196	6.399

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	170	162	209	252	304	469	257	187
N.S.	1	1.00	0.83	0.79	1.02	1.24	1.49	2.30	1.26	0.92
time (sec)	N/A	0.538	0.099	0.279	0.046	0.076	0.704	0.123	0.196	0.054

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	171	162	212	202	303	212	203	187
N.S.	1	1.00	0.86	0.81	1.06	1.01	1.52	1.06	1.02	0.94
time (sec)	N/A	0.520	0.104	0.237	0.031	0.105	0.672	0.127	0.191	0.054

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	171	163	217	211	272	265	205	215
N.S.	1	1.00	0.86	0.82	1.10	1.07	1.37	1.34	1.04	1.09
time (sec)	N/A	0.511	0.129	0.265	0.049	0.075	3.193	0.134	0.210	6.205

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	171	163	215	223	252	258	212	225
N.S.	1	1.00	0.86	0.82	1.08	1.12	1.26	1.29	1.06	1.12
time (sec)	N/A	0.524	0.106	0.279	0.039	0.104	3.228	0.129	0.209	6.402

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	170	163	215	234	241	253	223	222
N.S.	1	1.00	0.87	0.83	1.10	1.19	1.23	1.29	1.14	1.13
time (sec)	N/A	0.526	0.107	0.286	0.045	0.074	3.384	0.125	0.206	6.405

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	724	250	739	0	1641	0	424	1536	3481
N.S.	1	1.46	0.50	1.49	0.00	3.30	0.00	0.85	3.09	7.00
time (sec)	N/A	2.626	0.648	3.298	0.000	0.131	0.000	0.174	0.223	6.804

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	751	233	686	0	998	0	316	1113	1625
N.S.	1	1.68	0.52	1.53	0.00	2.23	0.00	0.71	2.49	3.64
time (sec)	N/A	1.952	0.450	1.348	0.000	0.100	0.000	0.171	0.207	0.432

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	545	176	448	0	355	0	203	714	308
N.S.	1	1.53	0.49	1.26	0.00	1.00	0.00	0.57	2.01	0.87
time (sec)	N/A	1.248	0.351	1.415	0.000	0.091	0.000	0.156	0.209	6.583

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	596	171	446	0	941	0	210	1091	1366
N.S.	1	1.49	0.43	1.12	0.00	2.35	0.00	0.52	2.73	3.42
time (sec)	N/A	1.402	0.375	0.891	0.000	0.093	0.000	0.144	0.216	6.562

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	716	242	678	0	2863	0	659	1650	4471
N.S.	1	1.43	0.48	1.35	0.00	5.71	0.00	1.32	3.29	8.92
time (sec)	N/A	1.969	0.617	1.130	0.000	0.166	0.000	0.175	0.977	7.404

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	823	265	761	0	5149	0	1215	3434	7908
N.S.	1	1.46	0.47	1.35	0.00	9.16	0.00	2.16	6.11	14.07
time (sec)	N/A	2.033	1.010	1.386	0.000	0.188	0.000	0.221	23.704	8.593

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	856	316	935	0	2091	0	596	4002	4192
N.S.	1	1.41	0.52	1.54	0.00	3.44	0.00	0.98	6.59	6.91
time (sec)	N/A	2.825	1.316	2.124	0.000	0.333	0.000	0.244	0.266	0.931

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	797	281	764	0	1383	0	492	3158	2031
N.S.	1	1.50	0.53	1.43	0.00	2.59	0.00	0.92	5.92	3.81
time (sec)	N/A	2.650	1.438	1.760	0.000	0.197	0.000	0.226	0.436	6.928

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	776	258	681	0	679	0	418	2303	717
N.S.	1	1.62	0.54	1.42	0.00	1.41	0.00	0.87	4.80	1.49
time (sec)	N/A	1.949	1.306	1.276	0.000	0.134	0.000	0.223	0.530	6.732

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	471	676	250	0	0	1371	0	337	3166	2380
N.S.	1	1.44	0.53	0.00	0.00	2.91	0.00	0.72	6.72	5.05
time (sec)	N/A	1.719	0.718	0.000	0.000	0.105	0.000	0.201	0.424	8.345

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	566	799	300	0	0	3267	0	875	4039	5300
N.S.	1	1.41	0.53	0.00	0.00	5.77	0.00	1.55	7.14	9.36
time (sec)	N/A	1.951	0.894	0.000	0.000	0.246	0.000	0.182	0.465	8.283

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	911	338	976	0	5698	0	1397	5249	8777
N.S.	1	1.43	0.53	1.53	0.00	8.93	0.00	2.19	8.23	13.76
time (sec)	N/A	2.704	1.433	2.947	0.000	0.914	0.000	0.271	194.482	9.101

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	1013	379	1162	0	8281	0	2014	19	12390
N.S.	1	1.39	0.52	1.59	0.00	11.33	0.00	2.76	0.03	16.95
time (sec)	N/A	3.576	2.859	3.586	0.000	3.023	0.000	0.350	200.024	10.424

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	968	401	991	0	2454	0	779	7420	4769
N.S.	1	1.39	0.58	1.43	0.00	3.54	0.00	1.12	10.69	6.87
time (sec)	N/A	3.504	2.980	2.474	0.000	0.594	0.000	0.288	5.617	7.049

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	626	894	371	1201	0	1751	0	719	6099	2569
N.S.	1	1.43	0.59	1.92	0.00	2.80	0.00	1.15	9.74	4.10
time (sec)	N/A	2.736	3.128	1.996	0.000	0.182	0.000	0.267	5.230	0.656

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	785	308	902	0	1037	0	595	4855	1028
N.S.	1	1.40	0.55	1.60	0.00	1.85	0.00	1.06	8.64	1.83
time (sec)	N/A	2.491	2.074	1.726	0.000	0.102	0.000	0.273	5.218	0.542

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	824	287	975	0	1752	0	502	6142	3204
N.S.	1	1.38	0.48	1.64	0.00	2.94	0.00	0.84	10.31	5.38
time (sec)	N/A	2.109	1.760	2.791	0.000	0.154	0.000	0.246	5.294	8.759

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	642	909	367	0	0	3770	0	1068	7511	6238
N.S.	1	1.42	0.57	0.00	0.00	5.87	0.00	1.66	11.70	9.72
time (sec)	N/A	2.541	1.406	0.000	0.000	0.418	0.000	0.284	5.956	9.069

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	721	1001	382	0	0	5779	0	1664	8803	9035
N.S.	1	1.39	0.53	0.00	0.00	8.02	0.00	2.31	12.21	12.53
time (sec)	N/A	3.083	1.558	0.000	0.000	2.177	0.000	0.264	6.201	9.324

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	226	61	195	0	186	0	179	278	179
N.S.	1	1.40	0.38	1.20	0.00	1.15	0.00	1.10	1.72	1.10
time (sec)	N/A	0.715	0.099	2.539	0.000	0.084	0.000	0.335	0.243	0.143

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	363	82	256	0	199	0	0	468	133
N.S.	1	1.52	0.34	1.07	0.00	0.83	0.00	0.00	1.96	0.56
time (sec)	N/A	0.968	0.131	1.799	0.000	0.085	0.000	0.000	0.241	6.485

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	488	180	363	0	311	0	233	588	261
N.S.	1	1.55	0.57	1.16	0.00	0.99	0.00	0.74	1.87	0.83
time (sec)	N/A	1.368	0.306	2.391	0.000	0.077	0.000	0.200	0.224	0.986

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	214	57	182	0	168	0	160	250	109
N.S.	1	1.36	0.36	1.16	0.00	1.07	0.00	1.02	1.59	0.69
time (sec)	N/A	0.705	0.102	0.935	0.000	0.090	0.000	0.277	0.298	0.148

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	226	65	269	0	153	0	152	251	226
N.S.	1	1.47	0.42	1.75	0.00	0.99	0.00	0.99	1.63	1.47
time (sec)	N/A	0.654	0.100	0.924	0.000	0.084	0.000	0.191	0.295	0.135

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	218	302	92	443	0	288	0	206	845	440
N.S.	1	1.39	0.42	2.03	0.00	1.32	0.00	0.94	3.88	2.02
time (sec)	N/A	0.906	0.245	2.204	0.000	0.093	0.000	0.282	0.248	6.002

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	629	129	88	0	2309	0	0	128	1906
N.S.	1	1.01	0.21	0.14	0.00	3.70	0.00	0.00	0.21	3.05
time (sec)	N/A	1.819	0.111	1.010	0.000	0.229	0.000	0.000	0.377	8.569

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	66	59	0	1402	0	0	19	2369
N.S.	1	1.00	0.15	0.13	0.00	3.18	0.00	0.00	0.04	5.37
time (sec)	N/A	1.098	0.068	0.573	0.000	0.116	0.000	0.000	0.257	0.394

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	64	57	0	640	0	0	19	1676
N.S.	1	1.00	0.15	0.13	0.00	1.45	0.00	0.00	0.04	3.80
time (sec)	N/A	1.037	0.060	0.569	0.000	0.084	0.000	0.000	0.256	0.287

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	65	59	0	1694	0	0	26	1270
N.S.	1	1.00	0.15	0.13	0.00	3.84	0.00	0.00	0.06	2.88
time (sec)	N/A	0.936	0.066	0.573	0.000	0.098	0.000	0.000	0.250	8.245

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	67	59	0	3194	0	0	26	2435
N.S.	1	1.00	0.15	0.13	0.00	7.24	0.00	0.00	0.06	5.52
time (sec)	N/A	0.991	0.059	0.573	0.000	0.222	0.000	0.000	0.286	0.559

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	571	114	95	0	6636	0	0	52	5055
N.S.	1	0.90	0.18	0.15	0.00	10.48	0.00	0.00	0.08	7.99
time (sec)	N/A	1.141	0.139	0.631	0.000	1.360	0.000	0.000	0.281	6.311

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	44	41	48	62	76	129	60	44
N.S.	1	1.00	0.69	0.64	0.75	0.97	1.19	2.02	0.94	0.69
time (sec)	N/A	0.328	0.056	0.185	0.047	0.071	0.593	0.119	0.289	0.073

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	44	41	48	62	76	127	60	44
N.S.	1	1.00	0.69	0.64	0.75	0.97	1.19	1.98	0.94	0.69
time (sec)	N/A	0.311	0.047	0.178	0.033	0.067	0.541	0.115	0.241	6.549

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	44	41	48	40	76	54	40	44
N.S.	1	1.00	0.69	0.64	0.75	0.62	1.19	0.84	0.62	0.69
time (sec)	N/A	0.310	0.041	0.177	0.056	0.070	0.491	0.117	0.248	0.056

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	44	41	54	40	70	54	40	44
N.S.	1	1.00	0.71	0.66	0.87	0.65	1.13	0.87	0.65	0.71
time (sec)	N/A	0.316	0.051	0.178	0.047	0.088	0.450	0.123	0.233	0.054

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	44	40	56	50	76	58	40	44
N.S.	1	1.00	0.71	0.65	0.90	0.81	1.23	0.94	0.65	0.71
time (sec)	N/A	0.318	0.042	0.185	0.028	0.071	0.708	0.125	0.256	5.602

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	302	170	108	0	5625	0	0	20	12163
N.S.	1	1.01	0.57	0.36	0.00	18.75	0.00	0.00	0.07	40.54
time (sec)	N/A	1.198	0.155	0.900	0.000	1.480	0.000	0.000	200.041	8.100

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	289	131	92	0	5710	0	0	20	11459
N.S.	1	1.02	0.46	0.32	0.00	20.11	0.00	0.00	0.07	40.35
time (sec)	N/A	0.899	0.165	0.685	0.000	2.910	0.000	0.000	200.026	7.472

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	287	135	90	0	3132	0	0	20	5862
N.S.	1	1.02	0.48	0.32	0.00	11.11	0.00	0.00	0.07	20.79
time (sec)	N/A	0.891	0.109	0.747	0.000	0.185	0.000	0.000	200.046	7.193

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	67	60	0	2521	0	0	20	3910
N.S.	1	1.00	0.25	0.22	0.00	9.44	0.00	0.00	0.07	14.64
time (sec)	N/A	0.750	0.100	0.602	0.000	0.226	0.000	0.000	200.026	0.557

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	65	58	0	797	0	0	20	703
N.S.	1	1.00	0.24	0.22	0.00	2.99	0.00	0.00	0.07	2.63
time (sec)	N/A	0.731	0.074	0.616	0.000	0.094	0.000	0.000	200.025	7.014

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	66	60	0	2611	0	0	20	4203
N.S.	1	1.00	0.25	0.22	0.00	9.78	0.00	0.00	0.07	15.74
time (sec)	N/A	0.692	0.083	0.619	0.000	0.120	0.000	0.000	200.038	7.735

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	68	60	0	5497	0	0	20	8501
N.S.	1	1.00	0.25	0.22	0.00	20.59	0.00	0.00	0.07	31.84
time (sec)	N/A	0.669	0.069	0.619	0.000	0.187	0.000	0.000	200.027	9.397

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	372	114	96	0	11325	0	0	20	17873
N.S.	1	1.27	0.39	0.33	0.00	38.65	0.00	0.00	0.07	61.00
time (sec)	N/A	0.882	0.146	0.648	0.000	0.370	0.000	0.000	200.051	9.405

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	374	116	96	0	13123	0	0	20	12990
N.S.	1	1.27	0.39	0.33	0.00	44.48	0.00	0.00	0.07	44.03
time (sec)	N/A	0.848	0.122	0.639	0.000	0.672	0.000	0.000	200.043	12.020

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	425	167	125	0	20772	0	0	20	33153
N.S.	1	1.32	0.52	0.39	0.00	64.31	0.00	0.00	0.06	102.64
time (sec)	N/A	1.054	0.270	0.701	0.000	2.087	0.000	0.000	200.044	14.980

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	428	361	179	0	7469	0	0	20	23842
N.S.	1	0.98	0.83	0.41	0.00	17.17	0.00	0.00	0.05	54.81
time (sec)	N/A	1.252	1.015	0.959	0.000	6.359	0.000	0.000	200.029	9.140

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	396	277	142	0	8266	0	0	20	16629
N.S.	1	0.98	0.69	0.35	0.00	20.51	0.00	0.00	0.05	41.26
time (sec)	N/A	1.100	0.516	0.812	0.000	43.817	0.000	0.000	200.030	8.249

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	396	282	141	0	4964	0	0	20	17148
N.S.	1	0.98	0.70	0.35	0.00	12.32	0.00	0.00	0.05	42.55
time (sec)	N/A	1.116	0.483	0.790	0.000	1.312	0.000	0.000	200.038	7.759

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	383	216	121	0	5011	0	0	20	11171
N.S.	1	1.00	0.57	0.32	0.00	13.12	0.00	0.00	0.05	29.24
time (sec)	N/A	0.956	0.403	0.760	0.000	3.125	0.000	0.000	200.051	8.167

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	381	220	121	0	2425	0	0	20	10441
N.S.	1	1.00	0.58	0.32	0.00	6.38	0.00	0.00	0.05	27.48
time (sec)	N/A	0.926	0.354	0.753	0.000	0.196	0.000	0.000	200.030	7.724

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	368	129	102	0	4973	0	0	20	8593
N.S.	1	1.02	0.36	0.28	0.00	13.81	0.00	0.00	0.06	23.87
time (sec)	N/A	0.886	0.214	0.749	0.000	0.998	0.000	0.000	200.042	17.423

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	370	131	102	0	7535	0	0	20	14637
N.S.	1	1.02	0.36	0.28	0.00	20.70	0.00	0.00	0.05	40.21
time (sec)	N/A	0.850	0.174	0.740	0.000	1.066	0.000	0.000	200.041	19.448

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	444	176	135	0	13905	0	0	20	17149
N.S.	1	1.13	0.45	0.34	0.00	35.29	0.00	0.00	0.05	43.53
time (sec)	N/A	1.074	0.310	0.747	0.000	28.049	0.000	0.000	200.039	10.257

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	446	178	135	0	14823	0	0	20	28255
N.S.	1	1.13	0.45	0.34	0.00	37.62	0.00	0.00	0.05	71.71
time (sec)	N/A	1.060	0.261	0.755	0.000	13.542	0.000	0.000	200.061	13.147

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	531	348	183	0	23435	0	0	20	37402
N.S.	1	1.22	0.80	0.42	0.00	53.63	0.00	0.00	0.05	85.59
time (sec)	N/A	1.339	0.598	0.810	0.000	133.993	0.000	0.000	200.059	16.432

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	211	176	181	238	400	270	205	19	0
N.S.	1	1.02	0.85	0.87	1.15	1.93	1.30	0.99	0.09	0.00
time (sec)	N/A	0.623	0.503	0.319	0.033	0.109	0.487	0.119	200.061	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	151	131	132	149	286	192	148	207	0
N.S.	1	1.05	0.91	0.92	1.03	1.99	1.33	1.03	1.44	0.00
time (sec)	N/A	0.435	0.343	0.276	0.034	0.091	0.462	0.119	0.974	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	99	87	107	214	139	97	147	0
N.S.	1	1.08	0.97	0.85	1.05	2.10	1.36	0.95	1.44	0.00
time (sec)	N/A	0.359	0.275	0.214	0.028	0.126	0.453	0.117	0.710	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	68	54	45	128	90	55	71	52
N.S.	1	1.01	1.01	0.81	0.67	1.91	1.34	0.82	1.06	0.78
time (sec)	N/A	0.285	0.160	0.190	0.028	0.102	0.322	0.118	0.961	6.472

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	108	110	181	84	574	0	0	1048	0
N.S.	1	1.05	1.07	1.76	0.82	5.57	0.00	0.00	10.17	0.00
time (sec)	N/A	0.432	0.267	0.272	0.043	0.163	0.000	0.000	0.905	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	119	536	94	884	0	0	394	0
N.S.	1	1.05	1.08	4.87	0.85	8.04	0.00	0.00	3.58	0.00
time (sec)	N/A	0.421	0.548	0.267	0.052	0.155	0.000	0.000	0.607	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	110	900	243	519	0	316	396	0
N.S.	1	1.00	1.07	8.74	2.36	5.04	0.00	3.07	3.84	0.00
time (sec)	N/A	0.369	0.739	0.317	0.065	0.182	0.000	0.131	0.964	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	156	173	988	428	861	0	529	699	0
N.S.	1	1.08	1.20	6.86	2.97	5.98	0.00	3.67	4.85	0.00
time (sec)	N/A	0.435	10.147	0.372	0.059	0.378	0.000	0.134	0.894	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	223	904	1991	977	1485	0	0	1557	0
N.S.	1	1.08	4.39	9.67	4.74	7.21	0.00	0.00	7.56	0.00
time (sec)	N/A	0.541	11.015	0.461	0.095	1.333	0.000	0.000	0.575	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	233	230	256	300	544	518	287	19	0
N.S.	1	0.91	0.90	1.00	1.18	2.13	2.03	1.13	0.07	0.00
time (sec)	N/A	0.622	0.649	0.319	0.036	0.182	0.595	0.129	200.055	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	173	173	189	190	402	366	217	297	0
N.S.	1	0.96	0.96	1.05	1.06	2.23	2.03	1.21	1.65	0.00
time (sec)	N/A	0.455	0.559	0.269	0.043	0.106	0.536	0.124	0.375	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	132	131	128	146	296	236	143	212	0
N.S.	1	0.96	0.96	0.93	1.07	2.16	1.72	1.04	1.55	0.00
time (sec)	N/A	0.386	0.425	0.217	0.036	0.159	0.495	0.125	0.490	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	90	87	70	61	176	119	76	111	54
N.S.	1	1.03	1.00	0.80	0.70	2.02	1.37	0.87	1.28	0.62
time (sec)	N/A	0.313	0.268	0.194	0.030	0.104	0.391	0.119	0.317	6.725

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	170	155	238	154	774	0	0	1948	0
N.S.	1	1.07	0.97	1.50	0.97	4.87	0.00	0.00	12.25	0.00
time (sec)	N/A	0.602	0.491	0.292	0.046	1.388	0.000	0.000	0.314	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	164	161	446	148	888	0	0	411	0
N.S.	1	0.94	0.92	2.55	0.85	5.07	0.00	0.00	2.35	0.00
time (sec)	N/A	0.554	1.097	0.334	0.058	0.319	0.000	0.000	0.305	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	168	162	883	341	1545	0	349	999	0
N.S.	1	0.92	0.89	4.85	1.87	8.49	0.00	1.92	5.49	0.00
time (sec)	N/A	0.499	1.048	0.395	0.071	0.419	0.000	0.151	0.321	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	234	210	2446	642	2513	0	604	1734	0
N.S.	1	1.06	0.95	11.12	2.92	11.42	0.00	2.75	7.88	0.00
time (sec)	N/A	0.634	1.972	0.385	0.080	3.753	0.000	0.180	0.352	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	167	723	4020	1223	1123	0	0	956	0
N.S.	1	1.09	4.73	26.27	7.99	7.34	0.00	0.00	6.25	0.00
time (sec)	N/A	0.449	5.872	0.450	0.108	1.189	0.000	0.000	0.951	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	220	253	4108	1762	1647	0	1283	1397	0
N.S.	1	1.13	1.30	21.07	9.04	8.45	0.00	6.58	7.16	0.00
time (sec)	N/A	0.511	2.976	0.592	0.127	3.295	0.000	0.171	1.194	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	287	358	8231	3297	2485	0	1875	2558	0
N.S.	1	1.07	1.33	30.60	12.26	9.24	0.00	6.97	9.51	0.00
time (sec)	N/A	0.650	10.501	0.638	0.199	7.863	0.000	0.175	3.316	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	256	283	332	364	692	858	372	19	0
N.S.	1	0.83	0.92	1.08	1.19	2.25	2.79	1.21	0.06	0.00
time (sec)	N/A	0.626	0.755	0.355	0.038	0.136	0.694	0.126	200.087	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	195	215	223	230	518	617	286	387	0
N.S.	1	0.90	1.00	1.03	1.06	2.40	2.86	1.32	1.79	0.00
time (sec)	N/A	0.512	0.649	0.294	0.035	0.117	0.623	0.125	0.313	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	154	161	169	185	380	371	191	277	0
N.S.	1	0.90	0.94	0.98	1.08	2.21	2.16	1.11	1.61	0.00
time (sec)	N/A	0.406	0.518	0.233	0.034	0.121	0.584	0.126	0.529	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	107	86	77	224	150	101	151	54
N.S.	1	1.05	1.00	0.80	0.72	2.09	1.40	0.94	1.41	0.50
time (sec)	N/A	0.333	0.411	0.201	0.032	0.093	0.445	0.130	0.225	6.580

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	245	221	331	272	1176	0	0	19	0
N.S.	1	1.08	0.98	1.46	1.20	5.20	0.00	0.00	0.08	0.00
time (sec)	N/A	0.787	0.971	0.296	0.074	29.546	0.000	0.000	200.032	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	237	229	529	246	1372	0	0	814	0
N.S.	1	0.91	0.88	2.03	0.95	5.28	0.00	0.00	3.13	0.00
time (sec)	N/A	0.737	0.911	0.348	0.072	7.031	0.000	0.000	0.280	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	222	229	967	561	1553	0	537	1005	0
N.S.	1	0.84	0.86	3.65	2.12	5.86	0.00	2.03	3.79	0.00
time (sec)	N/A	0.705	1.318	0.428	0.102	0.963	0.000	0.223	0.248	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	235	228	1726	1023	2501	0	590	1756	0
N.S.	1	0.85	0.82	6.23	3.69	9.03	0.00	2.13	6.34	0.00
time (sec)	N/A	0.680	2.024	0.506	0.124	1.972	0.000	0.268	0.291	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	311	946	3030	1903	3733	0	0	2824	0
N.S.	1	1.01	3.07	9.84	6.18	12.12	0.00	0.00	9.17	0.00
time (sec)	N/A	0.812	7.668	0.641	0.173	7.138	0.000	0.000	0.387	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	369	359	9762	3009	5169	0	1432	3958	0
N.S.	1	1.03	1.01	27.34	8.43	14.48	0.00	4.01	11.09	0.00
time (sec)	N/A	0.914	10.406	0.784	0.254	46.131	0.000	0.358	0.445	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	231	305	15868	4635	1929	0	1953	1662	0
N.S.	1	1.14	1.50	78.17	22.83	9.50	0.00	9.62	8.19	0.00
time (sec)	N/A	0.560	10.409	1.018	0.318	7.809	0.000	0.197	3.165	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	284	403	15956	5985	2593	0	2428	2204	0
N.S.	1	1.15	1.64	64.86	24.33	10.54	0.00	9.87	8.96	0.00
time (sec)	N/A	0.633	10.434	1.523	0.340	16.854	0.000	0.227	6.497	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	351	489	31927	9514	3693	0	3216	3709	0
N.S.	1	1.06	1.47	96.17	28.66	11.12	0.00	9.69	11.17	0.00
time (sec)	N/A	0.746	10.779	2.259	0.454	31.794	0.000	0.253	2.393	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	60	50	53	62	0	71	100	49
N.S.	1	1.09	1.07	0.89	0.95	1.11	0.00	1.27	1.79	0.88
time (sec)	N/A	0.306	0.162	0.291	0.111	0.083	0.000	0.129	0.232	6.395

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	79	68	56	54	75	0	105	121	48
N.S.	1	1.18	1.01	0.84	0.81	1.12	0.00	1.57	1.81	0.72
time (sec)	N/A	0.342	0.199	0.298	0.108	0.121	0.000	0.157	0.228	0.174

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	63	61	44	47	50	66	43	56	44
N.S.	1	1.15	1.11	0.80	0.85	0.91	1.20	0.78	1.02	0.80
time (sec)	N/A	0.287	0.304	0.280	0.104	0.106	0.085	0.123	0.245	0.394

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	185	127	113	176	270	167	139	19	0
N.S.	1	1.15	0.79	0.70	1.09	1.68	1.04	0.86	0.12	0.00
time (sec)	N/A	0.560	0.415	0.312	0.032	0.105	0.474	0.136	200.040	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	127	91	80	111	180	128	93	123	0
N.S.	1	1.15	0.83	0.73	1.01	1.64	1.16	0.85	1.12	0.00
time (sec)	N/A	0.415	0.329	0.280	0.033	0.122	0.462	0.135	0.336	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	86	69	57	69	138	99	64	87	0
N.S.	1	1.26	1.01	0.84	1.01	2.03	1.46	0.94	1.28	0.00
time (sec)	N/A	0.341	0.220	0.224	0.030	0.112	0.416	0.129	0.252	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	92	71	39	38	36
N.S.	1	1.00	1.07	0.86	0.67	2.14	1.65	0.91	0.88	0.84
time (sec)	N/A	0.269	0.140	0.189	0.044	0.112	0.317	0.132	0.256	6.773

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	64	127	52	211	0	60	1019	0
N.S.	1	1.00	1.19	2.35	0.96	3.91	0.00	1.11	18.87	0.00
time (sec)	N/A	0.266	0.197	0.238	0.040	0.128	0.000	0.136	0.245	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	101	215	93	381	0	0	231	0
N.S.	1	1.00	1.11	2.36	1.02	4.19	0.00	0.00	2.54	0.00
time (sec)	N/A	0.347	0.441	0.280	0.043	0.142	0.000	0.000	0.260	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	158	132	444	243	713	0	351	729	0
N.S.	1	1.09	0.91	3.06	1.68	4.92	0.00	2.42	5.03	0.00
time (sec)	N/A	0.456	0.812	0.352	0.058	0.264	0.000	0.135	0.253	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	227	209	762	479	1139	0	593	1162	0
N.S.	1	1.15	1.06	3.85	2.42	5.75	0.00	2.99	5.87	0.00
time (sec)	N/A	0.671	10.180	0.365	0.062	0.438	0.000	0.140	0.273	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	179	126	147	174	368	0	144	19	0
N.S.	1	1.18	0.83	0.97	1.14	2.42	0.00	0.95	0.12	0.00
time (sec)	N/A	0.519	0.530	0.355	0.030	0.115	0.000	0.133	200.036	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	123	90	110	110	246	0	103	205	0
N.S.	1	1.21	0.88	1.08	1.08	2.41	0.00	1.01	2.01	0.00
time (sec)	N/A	0.385	0.425	0.287	0.028	0.114	0.000	0.120	0.215	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	84	69	75	68	201	0	70	163	75
N.S.	1	1.22	1.00	1.09	0.99	2.91	0.00	1.01	2.36	1.09
time (sec)	N/A	0.336	0.367	0.224	0.031	0.117	0.000	0.134	0.196	7.617

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	27	31	35	46	23	54	24
N.S.	1	1.00	0.96	0.96	1.11	1.25	1.64	0.82	1.93	0.86
time (sec)	N/A	0.241	0.344	0.207	0.026	0.090	2.021	0.130	0.230	6.948

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	104	315	123	456	0	180	2247	0
N.S.	1	1.00	1.11	3.35	1.31	4.85	0.00	1.91	23.90	0.00
time (sec)	N/A	0.369	0.716	0.266	0.044	0.141	0.000	0.127	0.320	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	165	149	515	284	900	0	0	720	0
N.S.	1	1.15	1.04	3.60	1.99	6.29	0.00	0.00	5.03	0.00
time (sec)	N/A	0.480	0.725	0.292	0.060	0.193	0.000	0.000	0.263	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	245	240	933	630	1558	0	680	1666	0
N.S.	1	1.14	1.12	4.34	2.93	7.25	0.00	3.16	7.75	0.00
time (sec)	N/A	0.676	10.372	0.351	0.082	0.455	0.000	0.146	0.232	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	327	279	1551	1179	2226	0	1075	2350	0
N.S.	1	1.16	0.99	5.48	4.17	7.87	0.00	3.80	8.30	0.00
time (sec)	N/A	0.889	10.571	0.412	0.100	0.953	0.000	0.199	0.270	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	219	179	281	281	488	0	207	19	0
N.S.	1	1.07	0.87	1.37	1.37	2.38	0.00	1.01	0.09	0.00
time (sec)	N/A	0.650	0.823	0.442	0.036	0.118	0.000	0.130	200.045	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	174	130	217	213	401	0	155	377	0
N.S.	1	1.12	0.83	1.39	1.37	2.57	0.00	0.99	2.42	0.00
time (sec)	N/A	0.533	0.746	0.322	0.038	0.132	0.000	0.127	0.250	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	79	78	83	133	107	0	90	220	82
N.S.	1	0.76	0.75	0.80	1.28	1.03	0.00	0.87	2.12	0.79
time (sec)	N/A	0.319	0.587	0.309	0.026	0.141	0.000	0.123	0.249	6.758

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	58	57	55	92	75	0	55	175	68
N.S.	1	0.75	0.74	0.71	1.19	0.97	0.00	0.71	2.27	0.88
time (sec)	N/A	0.280	0.484	0.243	0.027	0.105	0.000	0.128	0.235	6.758

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	39	48	62	146	37	103	41
N.S.	1	1.00	0.84	0.76	0.94	1.22	2.86	0.73	2.02	0.80
time (sec)	N/A	0.267	0.374	0.197	0.026	0.088	3.919	0.120	0.277	6.547

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	173	163	601	283	858	0	998	3779	0
N.S.	1	1.12	1.06	3.90	1.84	5.57	0.00	6.48	24.54	0.00
time (sec)	N/A	0.521	0.768	0.266	0.062	0.167	0.000	0.137	0.480	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	280	247	899	563	1678	0	1180	1399	0
N.S.	1	1.29	1.14	4.14	2.59	7.73	0.00	5.44	6.45	0.00
time (sec)	N/A	0.771	1.286	0.291	0.091	0.355	0.000	0.254	0.329	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	374	296	1603	1176	2628	0	2179	2772	0
N.S.	1	1.25	0.99	5.34	3.92	8.76	0.00	7.26	9.24	0.00
time (sec)	N/A	0.978	10.795	0.365	0.124	1.030	0.000	0.276	0.302	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	92	106	90	70	98	0	60	141	0
N.S.	1	1.30	1.49	1.27	0.99	1.38	0.00	0.85	1.99	0.00
time (sec)	N/A	0.378	0.416	0.298	0.027	0.080	0.000	0.136	0.256	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	17	16	30	12	16	15	16
N.S.	1	1.00	1.89	0.94	0.89	1.67	0.67	0.89	0.83	0.89
time (sec)	N/A	0.242	0.098	0.207	0.120	0.073	0.076	0.134	0.234	0.035

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	17	16	30	12	16	15	16
N.S.	1	1.00	1.70	0.85	0.80	1.50	0.60	0.80	0.75	0.80
time (sec)	N/A	0.238	0.097	0.253	0.106	0.099	0.064	0.121	0.225	0.036

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	26	15	14	22	14	22	22	14
N.S.	1	1.00	1.44	0.83	0.78	1.22	0.78	1.22	1.22	0.78
time (sec)	N/A	0.226	0.072	0.173	0.103	0.082	0.063	0.121	0.215	6.149

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	54	52	38	42	49	42	35	44	35
N.S.	1	1.38	1.33	0.97	1.08	1.26	1.08	0.90	1.13	0.90
time (sec)	N/A	0.296	0.199	0.250	0.108	0.097	0.085	0.142	0.225	0.038

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	52	49	31	38	45	42	45	50	32
N.S.	1	1.33	1.26	0.79	0.97	1.15	1.08	1.15	1.28	0.82
time (sec)	N/A	0.286	0.164	0.213	0.104	0.091	0.085	0.122	0.245	0.034

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	13	20	25	20	12	31	12
N.S.	1	1.00	0.89	0.72	1.11	1.39	1.11	0.67	1.72	0.67
time (sec)	N/A	0.231	0.105	0.164	0.026	0.098	1.680	0.125	0.240	6.134

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	390	527	684	0	277	0	0	271	0
N.S.	1	0.94	1.27	1.65	0.00	0.67	0.00	0.00	0.65	0.00
time (sec)	N/A	1.046	23.448	2.765	0.000	0.115	0.000	0.000	1.404	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	351	490	489	0	232	0	0	251	0
N.S.	1	0.98	1.37	1.37	0.00	0.65	0.00	0.00	0.70	0.00
time (sec)	N/A	0.916	23.225	1.440	0.000	0.110	0.000	0.000	1.247	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	308	424	467	0	211	0	0	25	0
N.S.	1	0.97	1.34	1.48	0.00	0.67	0.00	0.00	0.08	0.00
time (sec)	N/A	0.747	22.658	1.330	0.000	0.097	0.000	0.000	0.606	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	293	407	375	0	219	0	0	36	0
N.S.	1	0.98	1.37	1.26	0.00	0.73	0.00	0.00	0.12	0.00
time (sec)	N/A	0.711	22.310	0.691	0.000	0.117	0.000	0.000	1.269	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	364	456	681	0	343	0	0	47	0
N.S.	1	1.01	1.26	1.88	0.00	0.95	0.00	0.00	0.13	0.00
time (sec)	N/A	0.862	10.821	1.115	0.000	0.121	0.000	0.000	0.542	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	446	486	784	0	572	0	0	58	0
N.S.	1	1.01	1.10	1.77	0.00	1.29	0.00	0.00	0.13	0.00
time (sec)	N/A	1.058	11.527	1.345	0.000	0.107	0.000	0.000	67.462	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	495	658	934	0	370	0	0	527	0
N.S.	1	0.97	1.29	1.82	0.00	0.72	0.00	0.00	1.03	0.00
time (sec)	N/A	1.296	24.651	4.425	0.000	0.097	0.000	0.000	2.340	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	446	611	715	0	313	0	0	479	0
N.S.	1	1.00	1.37	1.61	0.00	0.70	0.00	0.00	1.08	0.00
time (sec)	N/A	1.115	24.461	2.889	0.000	0.098	0.000	0.000	1.988	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	388	525	679	0	276	0	0	271	0
N.S.	1	0.99	1.35	1.74	0.00	0.71	0.00	0.00	0.69	0.00
time (sec)	N/A	0.955	23.531	3.582	0.000	0.105	0.000	0.000	1.866	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	362	507	751	0	297	0	0	682	0
N.S.	1	0.94	1.31	1.95	0.00	0.77	0.00	0.00	1.77	0.00
time (sec)	N/A	0.889	23.175	6.982	0.000	0.088	0.000	0.000	3.818	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	351	457	719	0	321	0	0	997	0
N.S.	1	0.94	1.22	1.92	0.00	0.86	0.00	0.00	2.67	0.00
time (sec)	N/A	0.849	22.897	8.138	0.000	0.129	0.000	0.000	49.299	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	420	518	781	0	528	0	0	0	0
N.S.	1	1.01	1.24	1.87	0.00	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.978	11.733	3.981	0.000	0.102	0.000	0.000	71.934	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	499	540	876	0	814	0	0	0	0
N.S.	1	1.00	1.08	1.75	0.00	1.62	0.00	0.00	0.00	0.00
time (sec)	N/A	1.259	12.607	5.825	0.000	0.135	0.000	0.000	5.319	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	573	750	981	0	422	0	0	769	0
N.S.	1	1.02	1.33	1.74	0.00	0.75	0.00	0.00	1.37	0.00
time (sec)	N/A	1.446	24.689	2.960	0.000	0.114	0.000	0.000	2.670	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	498	655	927	0	372	0	0	524	0
N.S.	1	1.01	1.33	1.88	0.00	0.76	0.00	0.00	1.07	0.00
time (sec)	N/A	1.240	24.317	4.231	0.000	0.134	0.000	0.000	10.348	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	458	647	1267	0	421	0	0	1148	0
N.S.	1	0.96	1.36	2.66	0.00	0.88	0.00	0.00	2.41	0.00
time (sec)	N/A	1.170	23.885	6.690	0.000	0.139	0.000	0.000	7.340	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	433	583	1041	0	474	0	0	0	0
N.S.	1	0.96	1.29	2.31	0.00	1.05	0.00	0.00	0.00	0.00
time (sec)	N/A	1.051	23.709	8.446	0.000	0.133	0.000	0.000	54.078	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	421	513	907	0	507	0	0	0	0
N.S.	1	0.86	1.04	1.84	0.00	1.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.985	23.353	10.864	0.000	0.135	0.000	0.000	71.678	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	501	602	939	0	781	0	0	0	0
N.S.	1	0.99	1.19	1.85	0.00	1.54	0.00	0.00	0.00	0.00
time (sec)	N/A	1.156	12.193	14.477	0.000	0.142	0.000	0.000	7.823	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	572	675	1008	0	1118	0	0	0	0
N.S.	1	0.94	1.11	1.66	0.00	1.84	0.00	0.00	0.00	0.00
time (sec)	N/A	1.326	13.228	9.155	0.000	0.183	0.000	0.000	12.079	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	536	737	0	276	0	0	356	0
N.S.	1	1.00	1.30	1.79	0.00	0.67	0.00	0.00	0.87	0.00
time (sec)	N/A	1.085	23.577	2.973	0.000	0.107	0.000	0.000	1.834	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	348	508	631	0	235	0	0	335	0
N.S.	1	0.98	1.44	1.78	0.00	0.66	0.00	0.00	0.95	0.00
time (sec)	N/A	0.948	23.458	2.027	0.000	0.096	0.000	0.000	1.584	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	303	431	582	0	212	0	0	168	0
N.S.	1	0.97	1.39	1.87	0.00	0.68	0.00	0.00	0.54	0.00
time (sec)	N/A	0.753	22.325	1.150	0.000	0.115	0.000	0.000	1.253	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	126	270	379	0	175	0	0	28	0
N.S.	1	0.95	2.05	2.87	0.00	1.33	0.00	0.00	0.21	0.00
time (sec)	N/A	0.374	20.393	0.369	0.000	0.094	0.000	0.000	0.815	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	126	170	194	0	67	0	0	42	0
N.S.	1	0.95	1.29	1.47	0.00	0.51	0.00	0.00	0.32	0.00
time (sec)	N/A	0.378	21.268	1.525	0.000	0.088	0.000	0.000	0.761	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	179	313	617	0	239	0	0	66	0
N.S.	1	0.97	1.69	3.34	0.00	1.29	0.00	0.00	0.36	0.00
time (sec)	N/A	0.463	11.180	2.701	0.000	0.094	0.000	0.000	1.344	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	381	461	698	0	384	0	0	90	0
N.S.	1	1.00	1.21	1.84	0.00	1.01	0.00	0.00	0.24	0.00
time (sec)	N/A	0.882	11.375	5.377	0.000	0.092	0.000	0.000	0.532	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	467	499	790	0	639	0	0	114	0
N.S.	1	1.04	1.12	1.77	0.00	1.43	0.00	0.00	0.26	0.00
time (sec)	N/A	1.173	12.396	7.754	0.000	0.131	0.000	0.000	69.556	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	426	549	782	0	373	0	0	1513	0
N.S.	1	1.01	1.30	1.86	0.00	0.89	0.00	0.00	3.59	0.00
time (sec)	N/A	1.060	12.199	5.481	0.000	0.085	0.000	0.000	2.610	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	363	510	678	0	306	0	0	1166	0
N.S.	1	1.02	1.44	1.91	0.00	0.86	0.00	0.00	3.28	0.00
time (sec)	N/A	0.918	12.039	2.533	0.000	0.092	0.000	0.000	2.170	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	320	442	629	0	271	0	0	1165	0
N.S.	1	1.02	1.41	2.01	0.00	0.87	0.00	0.00	3.72	0.00
time (sec)	N/A	0.762	11.693	0.750	0.000	0.117	0.000	0.000	1.372	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	295	425	380	0	229	0	0	531	0
N.S.	1	1.01	1.46	1.31	0.00	0.79	0.00	0.00	1.82	0.00
time (sec)	N/A	0.713	10.731	0.707	0.000	0.096	0.000	0.000	1.581	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	344	397	668	0	300	0	0	64	0
N.S.	1	1.05	1.21	2.04	0.00	0.91	0.00	0.00	0.20	0.00
time (sec)	N/A	0.803	11.037	3.339	0.000	0.094	0.000	0.000	1.744	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	422	479	803	0	557	0	0	380	0
N.S.	1	1.04	1.18	1.98	0.00	1.38	0.00	0.00	0.94	0.00
time (sec)	N/A	0.995	11.133	5.007	0.000	0.135	0.000	0.000	1.184	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	509	612	943	0	925	0	0	146	0
N.S.	1	1.05	1.26	1.95	0.00	1.91	0.00	0.00	0.30	0.00
time (sec)	N/A	1.207	11.860	7.593	0.000	0.167	0.000	0.000	71.091	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	478	673	871	0	565	0	0	0	0
N.S.	1	1.01	1.43	1.85	0.00	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	1.233	13.298	5.340	0.000	0.123	0.000	0.000	4.183	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	422	588	806	0	498	0	0	0	0
N.S.	1	1.02	1.42	1.95	0.00	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	1.048	12.480	4.667	0.000	0.129	0.000	0.000	4.194	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	389	549	746	0	423	0	0	0	0
N.S.	1	1.01	1.43	1.94	0.00	1.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.968	12.213	4.569	0.000	0.088	0.000	0.000	3.140	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	367	473	691	0	352	0	0	0	0
N.S.	1	1.01	1.31	1.91	0.00	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.959	11.668	1.236	0.000	0.089	0.000	0.000	2.726	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	406	573	772	0	467	0	0	50	0
N.S.	1	1.04	1.47	1.98	0.00	1.20	0.00	0.00	0.13	0.00
time (sec)	N/A	0.957	11.267	1.177	0.000	0.101	0.000	0.000	1.550	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	466	583	862	0	582	0	0	90	0
N.S.	1	1.04	1.30	1.92	0.00	1.30	0.00	0.00	0.20	0.00
time (sec)	N/A	1.110	11.855	6.353	0.000	0.115	0.000	0.000	1.502	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	561	616	1008	0	1158	0	0	590	0
N.S.	1	1.06	1.16	1.91	0.00	2.19	0.00	0.00	1.12	0.00
time (sec)	N/A	1.377	13.178	8.010	0.000	0.155	0.000	0.000	2.878	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	260	0	0	337	0	0	38	0
N.S.	1	1.00	1.72	0.00	0.00	2.23	0.00	0.00	0.25	0.00
time (sec)	N/A	0.387	0.358	0.000	0.000	22.922	0.000	0.000	0.370	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	631	53	40	0	0	85	0	63	0
N.S.	1	1.13	0.09	0.07	0.00	0.00	0.15	0.00	0.11	0.00
time (sec)	N/A	0.907	15.393	0.182	0.000	0.000	2.549	0.000	0.406	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	610	47	35	0	0	68	0	46	0
N.S.	1	1.14	0.09	0.07	0.00	0.00	0.13	0.00	0.09	0.00
time (sec)	N/A	0.907	15.245	0.184	0.000	0.000	1.681	0.000	0.302	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	585	40	29	0	0	36	0	29	27
N.S.	1	1.11	0.08	0.05	0.00	0.00	0.07	0.00	0.05	0.05
time (sec)	N/A	0.830	8.570	0.142	0.000	0.000	1.098	0.000	0.247	0.273

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	97	152	0	0	194	0	0	28	0
N.S.	1	0.97	1.52	0.00	0.00	1.94	0.00	0.00	0.28	0.00
time (sec)	N/A	0.305	0.241	0.000	0.000	1.577	0.000	0.000	0.260	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	637	701	211	0	0	0	0	0	42	0
N.S.	1	1.10	0.33	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.958	10.692	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	659	726	223	0	0	0	0	0	56	0
N.S.	1	1.10	0.34	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.066	16.670	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	645	60	49	0	0	148	0	65	0
N.S.	1	1.14	0.11	0.09	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.888	10.037	0.229	0.000	0.000	3.113	0.000	0.276	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	625	51	43	0	0	73	0	47	0
N.S.	1	1.16	0.09	0.08	0.00	0.00	0.14	0.00	0.09	0.00
time (sec)	N/A	0.877	10.031	0.187	0.000	0.000	2.171	0.000	0.289	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	596	42	37	0	0	39	0	30	28
N.S.	1	1.12	0.08	0.07	0.00	0.00	0.07	0.00	0.06	0.05
time (sec)	N/A	0.808	8.875	0.207	0.000	0.000	1.224	0.000	0.251	6.831

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	109	166	0	0	0	0	0	29	0
N.S.	1	0.97	1.48	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.304	0.341	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	653	725	132	0	0	0	0	0	43	0
N.S.	1	1.11	0.20	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.985	5.315	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	679	754	134	0	0	0	0	0	60	0
N.S.	1	1.11	0.20	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.091	8.532	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	106	104	172	2447	0	282	0	0	57	0
N.S.	1	0.98	1.62	23.08	0.00	2.66	0.00	0.00	0.54	0.00
time (sec)	N/A	0.293	0.287	26.010	0.000	2.925	0.000	0.000	0.309	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	103	101	168	1625	0	282	0	0	59	0
N.S.	1	0.98	1.63	15.78	0.00	2.74	0.00	0.00	0.57	0.00
time (sec)	N/A	0.292	0.311	26.013	0.000	2.962	0.000	0.000	0.294	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	78	107	617	0	113	0	0	29	0
N.S.	1	0.96	1.32	7.62	0.00	1.40	0.00	0.00	0.36	0.00
time (sec)	N/A	0.287	0.131	1.237	0.000	0.324	0.000	0.000	0.227	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	76	105	616	0	115	0	0	27	0
N.S.	1	0.94	1.30	7.60	0.00	1.42	0.00	0.00	0.33	0.00
time (sec)	N/A	0.294	0.115	1.250	0.000	0.327	0.000	0.000	0.244	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	271	0	0	0	0	0	38	0
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.416	0.803	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	80	46	44	0	0	129	0	54	0
N.S.	1	1.25	0.72	0.69	0.00	0.00	2.02	0.00	0.84	0.00
time (sec)	N/A	0.307	17.092	0.190	0.000	0.000	2.542	0.000	0.276	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	60	41	39	0	0	56	0	38	0
N.S.	1	1.36	0.93	0.89	0.00	0.00	1.27	0.00	0.86	0.00
time (sec)	N/A	0.290	16.990	0.174	0.000	0.000	1.675	0.000	0.259	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	31	0	0	31	0	23	24
N.S.	1	1.00	0.92	0.86	0.00	0.00	0.86	0.00	0.64	0.67
time (sec)	N/A	0.263	10.031	0.160	0.000	0.000	0.824	0.000	0.249	6.088

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	14	0	0	19	0	11	12
N.S.	1	1.00	0.85	0.70	0.00	0.00	0.95	0.00	0.55	0.60
time (sec)	N/A	0.219	5.791	0.132	0.000	0.000	0.372	0.000	0.215	5.866

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	92	102	211	0	350	0	0	25	0
N.S.	1	0.81	0.90	1.87	0.00	3.10	0.00	0.00	0.22	0.00
time (sec)	N/A	0.300	13.123	1.694	0.000	1.206	0.000	0.000	0.227	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	130	175	0	0	0	0	0	39	0
N.S.	1	0.87	1.17	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.381	12.863	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	158	218	0	0	0	0	0	53	0
N.S.	1	0.93	1.29	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.432	19.225	0.000	0.000	0.000	0.000	0.000	0.397	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	169	111	0	0	0	700	0	98	0
N.S.	1	1.17	0.77	0.00	0.00	0.00	4.86	0.00	0.68	0.00
time (sec)	N/A	0.455	11.066	0.000	0.000	0.000	2.106	0.000	0.272	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	137	100	0	0	0	131	0	73	0
N.S.	1	1.28	0.93	0.00	0.00	0.00	1.22	0.00	0.68	0.00
time (sec)	N/A	0.395	11.042	0.000	0.000	0.000	1.499	0.000	0.262	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	82	0	0	0	83	0	45	76
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.86	0.00	0.47	0.79
time (sec)	N/A	0.315	11.042	0.000	0.000	0.000	1.235	0.000	0.241	6.344

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	0	0	0	36	0	20	50
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.56	0.00	0.31	0.78
time (sec)	N/A	0.270	5.847	0.000	0.000	0.000	0.486	0.000	0.263	5.887

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	214	177	0	0	0	0	0	42	0
N.S.	1	0.85	0.70	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.440	7.383	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	354	321	186	0	0	0	0	0	67	0
N.S.	1	0.91	0.53	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.638	8.314	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	392	371	186	0	0	0	0	0	92	0
N.S.	1	0.95	0.47	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.749	11.157	0.000	0.000	0.000	0.000	0.000	0.574	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	204	244	64	59	0	0	129	0	239	0
N.S.	1	1.20	0.31	0.29	0.00	0.00	0.63	0.00	1.17	0.00
time (sec)	N/A	0.486	16.906	0.190	0.000	0.000	2.695	0.000	0.248	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	184	224	59	54	0	0	56	0	149	0
N.S.	1	1.22	0.32	0.29	0.00	0.00	0.30	0.00	0.81	0.00
time (sec)	N/A	0.440	16.460	0.174	0.000	0.000	1.769	0.000	0.259	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	200	51	48	0	0	31	0	134	42
N.S.	1	1.16	0.30	0.28	0.00	0.00	0.18	0.00	0.78	0.24
time (sec)	N/A	0.387	10.017	0.167	0.000	0.000	0.858	0.000	0.226	0.321

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	184	39	36	0	0	19	0	11	30
N.S.	1	1.17	0.25	0.23	0.00	0.00	0.12	0.00	0.07	0.19
time (sec)	N/A	0.362	5.292	0.135	0.000	0.000	0.377	0.000	0.224	6.094

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	102	145	0	93	0	0	25	0
N.S.	1	1.00	1.48	2.10	0.00	1.35	0.00	0.00	0.36	0.00
time (sec)	N/A	0.259	12.146	0.820	0.000	1.251	0.000	0.000	0.232	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	271	175	0	0	0	0	0	39	0
N.S.	1	1.12	0.72	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.526	12.485	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	300	218	0	0	0	0	0	53	0
N.S.	1	1.15	0.83	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.624	19.293	0.000	0.000	0.000	0.000	0.000	0.312	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	401	530	112	0	0	0	816	0	424	0
N.S.	1	1.32	0.28	0.00	0.00	0.00	2.03	0.00	1.06	0.00
time (sec)	N/A	0.777	11.068	0.000	0.000	0.000	2.161	0.000	0.312	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	495	101	0	0	0	131	0	269	0
N.S.	1	1.36	0.28	0.00	0.00	0.00	0.36	0.00	0.74	0.00
time (sec)	N/A	0.697	11.051	0.000	0.000	0.000	1.567	0.000	0.250	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	346	453	83	0	0	0	83	0	239	78
N.S.	1	1.31	0.24	0.00	0.00	0.00	0.24	0.00	0.69	0.23
time (sec)	N/A	0.637	11.042	0.000	0.000	0.000	1.233	0.000	0.235	6.606

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	421	59	0	0	0	36	0	22	51
N.S.	1	1.34	0.19	0.00	0.00	0.00	0.11	0.00	0.07	0.16
time (sec)	N/A	0.550	0.012	0.000	0.000	0.000	0.471	0.000	0.235	6.198

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	178	0	0	0	0	0	46	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.375	1.105	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	529	641	187	0	0	0	0	0	73	0
N.S.	1	1.21	0.35	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.990	1.110	0.000	0.000	0.000	0.000	0.000	0.271	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	569	691	187	0	0	0	0	0	100	0
N.S.	1	1.21	0.33	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.045	11.132	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	126	0	0	0	0	0	28	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.956	6.734	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	273	233	0	0	0	0	0	28	0
N.S.	1	1.02	0.87	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.877	7.740	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	200	108	0	0	0	0	0	800	0
N.S.	1	0.98	0.53	0.00	0.00	0.00	0.00	0.00	3.92	0.00
time (sec)	N/A	0.505	20.496	0.000	0.000	0.000	0.000	0.000	164.840	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	191	72	0	0	0	0	0	21	0
N.S.	1	1.16	0.44	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.657	15.696	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	379	954	472	1250	15990	2085	1411	1144
N.S.	1	1.00	1.70	4.28	2.12	5.61	71.70	9.35	6.33	5.13
time (sec)	N/A	0.692	0.543	0.302	0.044	0.102	3.865	0.139	0.241	6.773

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	176	420	235	520	5097	851	554	496
N.S.	1	1.00	1.26	3.00	1.68	3.71	36.41	6.08	3.96	3.54
time (sec)	N/A	0.483	0.250	0.234	0.043	0.089	1.489	0.129	0.256	6.190

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	100	89	149	952	237	142	163
N.S.	1	1.00	0.84	1.43	1.27	2.13	13.60	3.39	2.03	2.33
time (sec)	N/A	0.358	0.066	0.204	0.033	0.084	0.544	0.136	0.226	6.139

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	167	145	0	0	0	0	0	19	0
N.S.	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.609	0.186	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	299	253	0	0	0	0	0	30	0
N.S.	1	0.98	0.83	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.841	0.315	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	474	488	396	0	0	0	0	0	41	0
N.S.	1	1.03	0.84	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.428	0.757	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	0	0	0	0	0	0	19	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.513	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0	18	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.406	0.252	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0	20	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.420	0.288	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0	36	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.417	0.407	0.000	0.000	0.000	0.000	0.000	199.258	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	0	0	0	0	0	25	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.282	2.158	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	109	101	0	0	0	0	0	23	0
N.S.	1	1.33	1.23	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.335	0.267	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	0	0	0	0	0	20	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.283	1.379	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	104	94	0	0	0	0	0	18	0
N.S.	1	1.27	1.15	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.323	0.123	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	102	0	0	0	0	0	25	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.342	0.238	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	119	117	0	0	0	0	0	23	0
N.S.	1	1.32	1.30	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.377	0.303	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	149	0	0	0	0	0	21	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.433	0.264	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	136	160	0	0	0	0	0	20	0
N.S.	1	1.04	1.22	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.490	0.269	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	185	223	0	0	0	437	0	1314	0
N.S.	1	1.09	1.32	0.00	0.00	0.00	2.59	0.00	7.78	0.00
time (sec)	N/A	0.566	0.372	0.000	0.000	0.000	5.893	0.000	0.202	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	126	133	0	0	0	97	0	808	0
N.S.	1	1.18	1.24	0.00	0.00	0.00	0.91	0.00	7.55	0.00
time (sec)	N/A	0.418	0.203	0.000	0.000	0.000	4.666	0.000	0.205	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0	223	65
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.87	0.00	3.19	0.93
time (sec)	N/A	0.300	0.103	0.000	0.000	0.000	2.403	0.000	0.216	6.831

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	44	0	0	0	22	0	94	41
N.S.	1	1.26	1.26	0.00	0.00	0.00	0.63	0.00	2.69	1.17
time (sec)	N/A	0.259	0.002	0.000	0.000	0.000	1.069	0.000	0.212	6.796

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0	19	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.438	0.212	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	191	141	0	0	0	0	0	996	0
N.S.	1	1.27	0.94	0.00	0.00	0.00	0.00	0.00	6.64	0.00
time (sec)	N/A	0.621	0.232	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	322	142	0	0	0	0	0	41	0
N.S.	1	1.49	0.66	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.953	0.286	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	154	182	99	0	0	248	0	1112	0
N.S.	1	1.08	1.27	0.69	0.00	0.00	1.73	0.00	7.78	0.00
time (sec)	N/A	0.501	0.248	0.781	0.000	0.000	5.454	0.000	0.197	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	102	98	69	0	0	85	0	707	0
N.S.	1	1.12	1.08	0.76	0.00	0.00	0.93	0.00	7.77	0.00
time (sec)	N/A	0.391	0.166	0.451	0.000	0.000	4.017	0.000	0.213	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	72	44	0	0	51	0	203	39
N.S.	1	1.00	1.60	0.98	0.00	0.00	1.13	0.00	4.51	0.87
time (sec)	N/A	0.264	0.084	0.234	0.000	0.000	2.070	0.000	0.224	0.289

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	0	22	0	88	17
N.S.	1	1.00	1.00	0.86	0.00	0.00	1.00	0.00	4.00	0.77
time (sec)	N/A	0.220	0.023	0.152	0.000	0.000	0.625	0.000	0.202	6.856

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	139	0	0	0	0	0	19	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.383	0.223	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	146	149	0	0	0	0	0	924	0
N.S.	1	1.20	1.22	0.00	0.00	0.00	0.00	0.00	7.57	0.00
time (sec)	N/A	0.521	0.315	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	273	150	0	0	0	0	0	41	0
N.S.	1	1.47	0.81	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.830	0.301	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	186	228	0	0	0	434	0	1354	0
N.S.	1	1.16	1.42	0.00	0.00	0.00	2.71	0.00	8.46	0.00
time (sec)	N/A	0.548	0.317	0.000	0.000	0.000	6.021	0.000	0.212	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	128	135	0	0	0	102	0	831	0
N.S.	1	1.17	1.24	0.00	0.00	0.00	0.94	0.00	7.62	0.00
time (sec)	N/A	0.414	0.179	0.000	0.000	0.000	4.681	0.000	0.229	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	102	0	0	0	65	0	236	67
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.90	0.00	3.28	0.93
time (sec)	N/A	0.302	0.094	0.000	0.000	0.000	2.435	0.000	0.324	7.246

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	24	0	99	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	2.20	0.93
time (sec)	N/A	0.258	0.017	0.000	0.000	0.000	1.065	0.000	0.357	6.826

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	128	0	0	0	0	0	20	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.451	0.168	0.000	0.000	0.000	0.000	0.000	0.353	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	0	0	0	0	0	1021	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	6.81	0.00
time (sec)	N/A	0.462	0.213	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	152	139	0	0	0	0	0	42	0
N.S.	1	1.04	0.95	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.434	0.260	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	153	182	99	0	0	253	0	1112	0
N.S.	1	1.21	1.44	0.79	0.00	0.00	2.01	0.00	8.83	0.00
time (sec)	N/A	0.508	0.256	0.794	0.000	0.000	6.140	0.000	0.208	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	101	98	69	0	0	88	0	707	0
N.S.	1	1.12	1.09	0.77	0.00	0.00	0.98	0.00	7.86	0.00
time (sec)	N/A	0.370	0.175	0.420	0.000	0.000	4.434	0.000	0.217	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	65	44	0	0	53	0	203	57
N.S.	1	1.00	1.44	0.98	0.00	0.00	1.18	0.00	4.51	1.27
time (sec)	N/A	0.260	0.099	0.274	0.000	0.000	2.357	0.000	0.197	6.650

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	0	24	0	88	34
N.S.	1	1.00	1.00	0.86	0.00	0.00	1.09	0.00	4.00	1.55
time (sec)	N/A	0.225	0.022	0.155	0.000	0.000	0.745	0.000	0.198	6.508

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	127	0	0	0	0	0	19	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.395	0.222	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	147	137	0	0	0	0	0	924	0
N.S.	1	0.84	0.78	0.00	0.00	0.00	0.00	0.00	5.28	0.00
time (sec)	N/A	0.480	0.309	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	149	138	0	0	0	0	0	41	0
N.S.	1	0.85	0.79	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.463	0.322	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	173	0	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	1.501	0.000	0.000	0.000	0.000	0.000	0.957	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	155	0	0	0	0	0	863	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	5.75	0.00
time (sec)	N/A	0.406	0.467	0.000	0.000	0.000	0.000	0.000	0.543	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	153	0	0	0	0	0	25	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.390	0.487	0.000	0.000	0.000	0.000	0.000	0.281	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	153	0	0	0	0	0	36	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.421	0.676	0.000	0.000	0.000	0.000	0.000	0.357	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	194	0	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.571	1.536	0.000	0.000	0.000	0.000	0.000	0.977	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	181	0	0	0	0	0	810	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	5.40	0.00
time (sec)	N/A	0.475	0.443	0.000	0.000	0.000	0.000	0.000	0.514	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	179	0	0	0	0	0	25	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.455	0.533	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	179	0	0	0	0	0	36	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.487	0.745	0.000	0.000	0.000	0.000	0.000	0.349	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	191	0	0	0	0	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	1.634	0.000	0.000	0.000	0.000	0.000	0.771	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	148	0	0	0	0	0	755	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	5.32	0.00
time (sec)	N/A	0.410	0.555	0.000	0.000	0.000	0.000	0.000	0.456	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	146	0	0	0	0	0	25	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.397	0.538	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	146	0	0	0	0	0	36	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.417	0.729	0.000	0.000	0.000	0.000	0.000	0.406	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	180	0	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	1.522	0.000	0.000	0.000	0.000	0.000	0.948	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	150	0	0	0	0	0	875	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	6.12	0.00
time (sec)	N/A	0.392	0.444	0.000	0.000	0.000	0.000	0.000	0.590	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	148	0	0	0	0	0	26	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.383	0.486	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	148	0	0	0	0	0	37	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.387	0.667	0.000	0.000	0.000	0.000	0.000	0.348	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	170	176	0	0	0	0	0	0	0
N.S.	1	1.19	1.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	1.640	0.000	0.000	0.000	0.000	0.000	0.845	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	156	170	0	0	0	0	0	822	0
N.S.	1	1.09	1.19	0.00	0.00	0.00	0.00	0.00	5.75	0.00
time (sec)	N/A	0.540	0.448	0.000	0.000	0.000	0.000	0.000	0.481	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	156	168	0	0	0	0	0	26	0
N.S.	1	1.11	1.19	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.552	0.479	0.000	0.000	0.000	0.000	0.000	0.303	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	169	168	0	0	0	0	0	37	0
N.S.	1	1.20	1.19	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.530	0.689	0.000	0.000	0.000	0.000	0.000	0.344	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	145	182	0	0	0	0	0	0	0
N.S.	1	1.10	1.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	1.586	0.000	0.000	0.000	0.000	0.000	0.644	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	128	135	0	0	0	0	0	755	0
N.S.	1	0.97	1.02	0.00	0.00	0.00	0.00	0.00	5.72	0.00
time (sec)	N/A	0.489	0.488	0.000	0.000	0.000	0.000	0.000	0.372	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	128	133	0	0	0	0	0	25	0
N.S.	1	0.98	1.02	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.485	0.483	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	141	133	0	0	0	0	0	36	0
N.S.	1	1.08	1.02	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.506	0.685	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	157	0	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	0.139	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	183	0	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.160	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	150	0	0	0	0	0	1581	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	10.98	0.00
time (sec)	N/A	0.467	0.355	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	166	0	0	0	0	0	23	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.476	0.161	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	160	0	0	0	0	0	38	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.424	0.151	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	200	0	0	0	0	0	58	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.552	0.194	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	0	0	0	0	0	0	78	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.530	0.000	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	347	354	0	0	0	0	0	0	98	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.708	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	152	0	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	0.141	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	159	172	0	0	0	0	0	0	0
N.S.	1	1.10	1.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.574	0.154	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	137	137	0	0	0	0	0	1581	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	11.80	0.00
time (sec)	N/A	0.556	0.264	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	161	0	0	0	0	0	24	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.444	0.136	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	155	0	0	0	0	0	39	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.407	0.124	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	183	0	0	0	0	0	59	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.471	0.183	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	253	0	0	0	0	0	0	79	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.525	0.000	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	338	0	0	0	0	0	0	99	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.734	0.000	0.000	0.000	0.000	0.000	0.000	0.352	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [391] had the largest ratio of [.866666999999999965]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	20	0.100
2	A	2	2	1.00	20	0.100
3	A	2	2	1.02	18	0.111
4	A	1	1	1.00	12	0.083
5	A	2	2	0.98	20	0.100
6	A	2	2	1.00	20	0.100
7	A	2	2	1.00	20	0.100
8	A	2	2	1.00	20	0.100
9	A	2	2	1.00	20	0.100
10	A	2	2	1.00	22	0.091
11	A	2	2	1.00	22	0.091
12	A	3	3	1.00	20	0.150
13	A	2	2	1.00	14	0.143
14	A	2	2	1.00	22	0.091
15	A	2	2	1.00	22	0.091
16	A	2	2	1.00	22	0.091
17	A	2	2	1.00	22	0.091
18	A	2	2	1.00	22	0.091
19	A	2	2	1.00	22	0.091
20	A	2	2	1.00	22	0.091
21	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	2	2	1.00	22	0.091
23	A	3	3	0.98	20	0.150
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	22	0.091
26	A	2	2	1.00	22	0.091
27	A	2	2	1.00	22	0.091
28	A	2	2	1.09	22	0.091
29	A	2	2	1.08	22	0.091
30	A	2	2	1.05	22	0.091
31	A	3	3	0.80	20	0.150
32	A	1	1	1.00	14	0.071
33	A	2	2	1.04	22	0.091
34	A	2	2	1.06	22	0.091
35	A	2	2	1.06	22	0.091
36	A	2	2	1.02	22	0.091
37	A	2	2	1.00	22	0.091
38	A	2	2	1.09	22	0.091
39	A	2	2	1.00	20	0.100
40	A	2	2	1.00	14	0.143
41	A	2	2	1.02	22	0.091
42	A	2	2	1.03	22	0.091
43	A	2	2	1.04	22	0.091
44	A	2	2	0.89	22	0.091
45	A	2	2	1.03	22	0.091
46	A	2	2	1.25	22	0.091
47	A	3	3	1.09	20	0.150
48	A	3	3	1.13	14	0.214
49	A	2	2	1.01	22	0.091
50	A	2	2	1.02	22	0.091
51	A	2	2	1.03	22	0.091
52	A	2	2	1.00	15	0.133
53	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	15	0.133
55	A	2	2	1.03	13	0.154
56	A	2	2	1.00	15	0.133
57	A	2	2	1.00	15	0.133
58	A	2	2	0.96	15	0.133
59	A	2	2	0.96	15	0.133
60	A	2	2	0.97	15	0.133
61	A	2	2	1.00	17	0.118
62	A	2	2	1.00	17	0.118
63	A	2	2	1.00	17	0.118
64	A	3	3	1.00	15	0.200
65	A	2	2	1.00	17	0.118
66	A	2	2	1.00	17	0.118
67	A	2	2	1.00	17	0.118
68	A	2	2	1.00	17	0.118
69	A	2	2	1.00	17	0.118
70	A	2	2	1.00	17	0.118
71	A	2	2	1.00	17	0.118
72	A	2	2	1.00	17	0.118
73	A	2	2	1.00	17	0.118
74	A	2	2	1.00	17	0.118
75	A	2	2	1.00	17	0.118
76	A	2	2	1.00	17	0.118
77	A	2	2	1.00	17	0.118
78	A	3	3	0.98	15	0.200
79	A	2	2	1.00	17	0.118
80	A	2	2	1.00	17	0.118
81	A	2	2	1.00	17	0.118
82	A	2	2	1.00	17	0.118
83	A	2	2	1.00	17	0.118
84	A	2	2	1.00	17	0.118
85	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	17	0.118
87	A	2	2	1.00	17	0.118
88	A	2	2	1.00	17	0.118
89	A	2	2	1.00	17	0.118
90	A	2	2	1.00	17	0.118
91	A	2	2	1.00	17	0.118
92	A	2	2	1.00	17	0.118
93	A	2	2	1.00	17	0.118
94	A	2	2	1.00	17	0.118
95	A	3	3	0.97	15	0.200
96	A	2	2	1.00	17	0.118
97	A	2	2	1.00	17	0.118
98	A	2	2	1.00	11	0.182
99	A	2	2	1.00	11	0.182
100	A	2	2	1.00	15	0.133
101	A	2	2	1.00	11	0.182
102	A	2	2	1.00	17	0.118
103	A	2	2	1.00	17	0.118
104	A	2	2	1.00	17	0.118
105	A	3	3	1.00	15	0.200
106	A	4	4	0.92	17	0.235
107	A	3	3	1.11	17	0.176
108	A	3	3	1.07	17	0.176
109	A	3	3	0.98	17	0.176
110	A	3	3	0.99	17	0.176
111	A	3	3	0.96	17	0.176
112	A	2	2	0.91	17	0.118
113	A	2	2	1.00	15	0.133
114	A	4	4	1.13	17	0.235
115	A	4	4	1.12	17	0.235
116	A	4	4	0.93	17	0.235
117	A	3	3	0.78	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
118	A	3	3	0.77	17	0.176
119	A	3	3	1.08	17	0.176
120	A	3	3	1.09	15	0.200
121	A	7	7	1.21	17	0.412
122	A	6	6	1.22	17	0.353
123	A	4	4	0.64	17	0.235
124	A	4	4	0.68	17	0.235
125	A	5	5	0.74	17	0.294
126	A	5	5	1.03	17	0.294
127	A	4	4	1.01	17	0.235
128	A	4	4	1.15	15	0.267
129	A	9	9	1.22	17	0.529
130	A	9	9	1.30	17	0.529
131	A	5	5	1.16	13	0.385
132	A	4	4	1.12	13	0.308
133	A	4	4	1.04	13	0.308
134	A	2	2	1.23	11	0.182
135	A	10	9	1.10	20	0.450
136	A	7	6	1.12	20	0.300
137	A	5	4	1.25	20	0.200
138	A	4	3	1.10	20	0.150
139	A	7	6	1.31	20	0.300
140	A	9	8	1.42	20	0.400
141	A	13	12	1.04	20	0.600
142	A	11	10	1.06	20	0.500
143	A	8	7	1.14	20	0.350
144	A	8	7	1.04	20	0.350
145	A	8	7	1.18	20	0.350
146	A	11	10	1.29	20	0.500
147	A	13	12	1.29	20	0.600
148	A	10	9	1.10	20	0.450
149	A	10	9	1.12	20	0.450

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	10	9	1.06	20	0.450
151	A	10	9	1.22	20	0.450
152	A	10	9	1.27	20	0.450
153	A	5	4	1.20	19	0.211
154	A	5	4	1.33	19	0.211
155	A	5	4	1.19	20	0.200
156	A	2	2	1.00	17	0.118
157	A	2	2	1.00	17	0.118
158	A	2	2	1.00	17	0.118
159	A	2	2	1.00	17	0.118
160	A	2	2	1.00	17	0.118
161	A	2	2	1.00	17	0.118
162	A	2	2	1.00	17	0.118
163	A	2	2	1.00	19	0.105
164	A	2	2	1.00	19	0.105
165	A	2	2	1.00	19	0.105
166	A	2	2	1.00	19	0.105
167	A	2	2	1.00	19	0.105
168	A	2	2	1.00	19	0.105
169	A	2	2	1.00	19	0.105
170	A	2	2	1.00	19	0.105
171	A	2	2	1.00	19	0.105
172	A	2	2	1.00	19	0.105
173	A	2	2	1.00	19	0.105
174	A	2	2	1.00	19	0.105
175	A	2	2	1.00	19	0.105
176	A	2	2	1.00	19	0.105
177	A	16	15	1.46	19	0.789
178	A	13	12	1.68	19	0.632
179	A	9	8	1.53	19	0.421
180	A	10	9	1.49	19	0.474
181	A	13	12	1.43	19	0.632

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	13	12	1.46	19	0.632
183	A	16	15	1.41	19	0.789
184	A	16	15	1.50	19	0.789
185	A	13	12	1.62	19	0.632
186	A	13	12	1.44	19	0.632
187	A	13	12	1.41	19	0.632
188	A	15	14	1.43	19	0.737
189	A	16	15	1.39	19	0.789
190	A	17	16	1.39	19	0.842
191	A	15	14	1.43	19	0.737
192	A	16	15	1.40	19	0.789
193	A	15	14	1.38	19	0.737
194	A	15	14	1.42	19	0.737
195	A	15	14	1.39	19	0.737
196	A	9	8	1.40	17	0.471
197	A	9	8	1.52	17	0.471
198	A	9	8	1.55	19	0.421
199	A	9	8	1.36	15	0.533
200	A	8	7	1.47	15	0.467
201	A	12	11	1.39	15	0.733
202	A	3	3	1.01	19	0.158
203	A	2	2	1.00	19	0.105
204	A	2	2	1.00	19	0.105
205	A	2	2	1.00	19	0.105
206	A	2	2	1.00	19	0.105
207	A	3	3	0.90	19	0.158
208	A	2	2	1.00	18	0.111
209	A	2	2	1.00	18	0.111
210	A	2	2	1.00	18	0.111
211	A	2	2	1.00	18	0.111
212	A	2	2	1.00	18	0.111
213	A	7	7	1.01	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	4	4	1.02	20	0.200
215	A	4	4	1.02	20	0.200
216	A	2	2	1.00	20	0.100
217	A	2	2	1.00	20	0.100
218	A	2	2	1.00	20	0.100
219	A	2	2	1.00	20	0.100
220	A	3	3	1.27	20	0.150
221	A	3	3	1.27	20	0.150
222	A	5	5	1.32	20	0.250
223	A	9	9	0.98	20	0.450
224	A	7	7	0.98	20	0.350
225	A	7	7	0.98	20	0.350
226	A	4	4	1.00	20	0.200
227	A	4	4	1.00	20	0.200
228	A	4	4	1.02	20	0.200
229	A	4	4	1.02	20	0.200
230	A	4	4	1.13	20	0.200
231	A	4	4	1.13	20	0.200
232	A	7	7	1.22	20	0.350
233	A	9	8	1.02	19	0.421
234	A	6	5	1.05	19	0.263
235	A	6	5	1.08	19	0.263
236	A	5	4	1.01	17	0.235
237	A	7	6	1.05	19	0.316
238	A	7	6	1.05	19	0.316
239	A	4	3	1.00	19	0.158
240	A	5	4	1.08	19	0.211
241	A	7	6	1.08	19	0.316
242	A	9	8	0.91	19	0.421
243	A	7	6	0.96	19	0.316
244	A	7	6	0.96	19	0.316
245	A	6	5	1.03	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	9	8	1.07	19	0.421
247	A	9	8	0.94	19	0.421
248	A	9	8	0.92	19	0.421
249	A	9	8	1.06	19	0.421
250	A	5	4	1.09	19	0.211
251	A	6	5	1.13	19	0.263
252	A	8	7	1.07	19	0.368
253	A	10	9	0.83	19	0.474
254	A	8	7	0.90	19	0.368
255	A	8	7	0.90	19	0.368
256	A	7	6	1.05	17	0.353
257	A	11	10	1.08	19	0.526
258	A	11	10	0.91	19	0.526
259	A	11	10	0.84	19	0.526
260	A	11	10	0.85	19	0.526
261	A	11	10	1.01	19	0.526
262	A	11	10	1.03	19	0.526
263	A	6	5	1.14	19	0.263
264	A	7	6	1.15	19	0.316
265	A	9	8	1.06	19	0.421
266	A	6	5	1.09	17	0.294
267	A	7	6	1.18	19	0.316
268	A	5	4	1.15	17	0.235
269	A	7	6	1.15	19	0.316
270	A	5	4	1.15	19	0.211
271	A	5	4	1.26	19	0.211
272	A	4	3	1.00	17	0.176
273	A	3	2	1.00	19	0.105
274	A	4	3	1.00	19	0.158
275	A	6	5	1.09	19	0.263
276	A	8	7	1.15	19	0.368
277	A	8	7	1.18	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	6	5	1.21	19	0.263
279	A	6	5	1.22	19	0.263
280	A	1	1	1.00	17	0.059
281	A	6	5	1.00	19	0.263
282	A	7	6	1.15	19	0.316
283	A	10	9	1.14	19	0.474
284	A	12	11	1.16	19	0.579
285	A	9	8	1.07	19	0.421
286	A	7	6	1.12	19	0.316
287	A	2	2	0.76	19	0.105
288	A	2	2	0.75	19	0.105
289	A	2	2	1.00	17	0.118
290	A	7	6	1.12	19	0.316
291	A	8	7	1.29	19	0.368
292	A	11	10	1.25	19	0.526
293	A	7	6	1.30	23	0.261
294	A	2	2	1.00	15	0.133
295	A	2	2	1.00	15	0.133
296	A	2	2	1.00	13	0.154
297	A	4	4	1.38	19	0.211
298	A	3	3	1.33	17	0.176
299	A	1	1	1.00	15	0.067
300	A	12	11	0.94	22	0.500
301	A	11	10	0.98	22	0.455
302	A	9	8	0.97	22	0.364
303	A	9	8	0.98	22	0.364
304	A	11	10	1.01	22	0.455
305	A	13	12	1.01	22	0.545
306	A	14	13	0.97	22	0.591
307	A	13	12	1.00	22	0.545
308	A	11	10	0.99	22	0.455
309	A	11	10	0.94	22	0.455

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	11	10	0.94	22	0.455
311	A	11	10	1.01	22	0.455
312	A	13	12	1.00	22	0.545
313	A	15	14	1.02	22	0.636
314	A	13	12	1.01	22	0.545
315	A	13	12	0.96	22	0.545
316	A	13	12	0.96	22	0.545
317	A	14	13	0.86	22	0.591
318	A	13	12	0.99	22	0.545
319	A	14	13	0.94	22	0.591
320	A	14	13	1.00	22	0.591
321	A	12	11	0.98	22	0.500
322	A	10	9	0.97	22	0.409
323	A	4	3	0.95	22	0.136
324	A	4	3	0.95	22	0.136
325	A	6	5	0.97	22	0.227
326	A	12	11	1.00	22	0.500
327	A	14	13	1.04	22	0.591
328	A	14	13	1.01	22	0.591
329	A	12	11	1.02	22	0.500
330	A	10	9	1.02	22	0.409
331	A	10	9	1.01	22	0.409
332	A	10	9	1.05	22	0.409
333	A	12	11	1.04	22	0.500
334	A	14	13	1.05	22	0.591
335	A	14	13	1.01	22	0.591
336	A	12	11	1.02	22	0.500
337	A	12	11	1.01	22	0.500
338	A	12	11	1.01	22	0.500
339	A	12	11	1.04	22	0.500
340	A	12	11	1.04	22	0.500
341	A	14	13	1.06	22	0.591

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	1	1	1.00	24	0.042
343	A	8	7	1.13	19	0.368
344	A	8	7	1.14	19	0.368
345	A	6	5	1.11	17	0.294
346	A	1	1	0.97	19	0.053
347	A	9	8	1.10	19	0.421
348	A	11	10	1.10	19	0.526
349	A	8	7	1.14	21	0.333
350	A	8	7	1.16	21	0.333
351	A	6	5	1.12	19	0.263
352	A	1	1	0.97	21	0.048
353	A	9	8	1.11	21	0.381
354	A	11	10	1.11	21	0.476
355	A	1	1	0.98	19	0.053
356	A	1	1	0.98	21	0.048
357	A	2	2	0.96	19	0.105
358	A	2	2	0.94	17	0.118
359	A	3	3	1.00	24	0.125
360	A	4	4	1.25	17	0.235
361	A	4	4	1.36	17	0.235
362	A	2	2	1.00	15	0.133
363	A	1	1	1.00	11	0.091
364	A	2	2	0.81	17	0.118
365	A	6	6	0.87	17	0.353
366	A	7	7	0.93	17	0.412
367	A	6	6	1.17	27	0.222
368	A	6	6	1.28	27	0.222
369	A	3	3	1.00	25	0.120
370	A	2	2	1.00	19	0.105
371	A	2	2	0.85	27	0.074
372	A	7	7	0.91	27	0.259
373	A	9	9	0.95	27	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	7	6	1.20	17	0.353
375	A	7	6	1.22	17	0.353
376	A	6	5	1.16	15	0.333
377	A	5	4	1.17	11	0.364
378	A	1	1	1.00	17	0.059
379	A	9	8	1.12	17	0.471
380	A	11	10	1.15	17	0.588
381	A	9	8	1.32	28	0.286
382	A	9	8	1.36	28	0.286
383	A	7	6	1.31	26	0.231
384	A	6	5	1.34	20	0.250
385	A	1	1	1.00	28	0.036
386	A	10	9	1.21	28	0.321
387	A	13	12	1.21	28	0.429
388	A	11	10	1.00	19	0.526
389	A	12	11	1.02	19	0.579
390	A	1	1	0.98	21	0.048
391	A	14	13	1.16	15	0.867
392	A	2	2	1.00	17	0.118
393	A	2	2	1.00	17	0.118
394	A	2	2	1.00	15	0.133
395	A	2	2	1.08	17	0.118
396	A	4	4	0.98	17	0.235
397	A	7	7	1.03	17	0.412
398	A	3	2	1.00	19	0.105
399	A	3	2	1.00	19	0.105
400	A	3	2	1.00	19	0.105
401	A	3	2	1.00	19	0.105
402	A	3	3	1.00	19	0.158
403	A	3	2	1.33	19	0.105
404	A	2	2	1.00	19	0.105
405	A	3	2	1.27	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	4	4	1.00	19	0.211
407	A	3	2	1.32	19	0.105
408	A	4	4	1.00	20	0.200
409	A	3	2	1.04	19	0.105
410	A	5	5	1.09	17	0.294
411	A	5	5	1.18	17	0.294
412	A	3	3	1.00	15	0.200
413	A	2	2	1.26	9	0.222
414	A	6	5	1.00	17	0.294
415	A	2	2	1.27	17	0.118
416	A	2	2	1.49	17	0.118
417	A	4	4	1.08	17	0.235
418	A	4	4	1.12	17	0.235
419	A	2	2	1.00	15	0.133
420	A	1	1	1.00	9	0.111
421	A	5	4	1.00	17	0.235
422	A	2	2	1.20	17	0.118
423	A	2	2	1.47	17	0.118
424	A	5	5	1.16	18	0.278
425	A	5	5	1.17	18	0.278
426	A	3	3	1.00	16	0.188
427	A	2	2	1.00	10	0.200
428	A	6	5	1.00	18	0.278
429	A	3	2	1.00	18	0.111
430	A	3	2	1.04	18	0.111
431	A	4	4	1.21	17	0.235
432	A	4	4	1.12	17	0.235
433	A	2	2	1.00	15	0.133
434	A	1	1	1.00	9	0.111
435	A	5	4	1.00	17	0.235
436	A	3	2	0.84	17	0.118
437	A	3	2	0.85	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
438	A	3	2	1.00	19	0.105
439	A	3	2	1.00	19	0.105
440	A	3	2	1.00	19	0.105
441	A	3	2	1.00	19	0.105
442	A	3	2	1.00	19	0.105
443	A	3	2	1.00	19	0.105
444	A	3	2	1.00	19	0.105
445	A	3	2	1.00	19	0.105
446	A	3	2	1.00	19	0.105
447	A	3	2	1.00	19	0.105
448	A	3	2	1.00	19	0.105
449	A	3	2	1.00	19	0.105
450	A	3	2	1.00	20	0.100
451	A	3	2	1.00	20	0.100
452	A	3	2	1.00	20	0.100
453	A	3	2	1.00	20	0.100
454	A	3	3	1.19	20	0.150
455	A	3	3	1.09	20	0.150
456	A	3	3	1.11	20	0.150
457	A	3	3	1.20	20	0.150
458	A	3	3	1.10	19	0.158
459	A	3	3	0.97	19	0.158
460	A	3	3	0.98	19	0.158
461	A	3	3	1.08	19	0.158
462	A	3	2	1.00	17	0.118
463	A	3	2	1.00	17	0.118
464	A	3	2	1.00	17	0.118
465	A	3	2	1.00	19	0.105
466	A	3	2	1.00	21	0.095
467	A	1	1	1.00	21	0.048
468	A	2	2	1.00	21	0.095
469	A	4	4	1.02	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	3	2	1.00	18	0.111
471	A	3	3	1.10	18	0.167
472	A	3	3	1.02	17	0.176
473	A	3	2	1.00	20	0.100
474	A	3	2	1.00	22	0.091
475	A	1	1	1.00	22	0.045
476	A	2	2	0.99	22	0.091
477	A	4	4	1.01	22	0.182

CHAPTER 3

Listing of integrals

3.1 $\int (c+dx)^3 \left(a^2-b^2x^2\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int (c+dx)^3 \left(a^2-b^2x^2\right) \, dx = -\frac{(bc-ad)(bc+ad)(c+dx)^4}{4d^3}+\frac{2b^2c(c+dx)^5}{5d^3}-\frac{b^2(c+dx)^6}{6d^3}$$

output
```
-1/4*(-a*d+b*c)*(a*d+b*c)*(d*x+c)^4/d^3+2/5*b^2*c*(d*x+c)^5/d^3-1/6*b^2*(d
*x+c)^6/d^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int (c+dx)^3 \left(a^2-b^2x^2\right) \, dx = \frac{1}{4}a^2x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) - \frac{1}{60}b^2x^3\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right)$$

input
```
Integrate[(c + d*x)^3*(a^2 - b^2*x^2),x]
```

output
```
(a^2*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (b^2*x^3*(20*c^3 +
 45*c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3))/60
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 - b^2x^2)(c+dx)^3 \, dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{(c+dx)^3(a^2d^2-b^2c^2)}{d^2} - \frac{b^2(c+dx)^5}{d^2} + \frac{2b^2c(c+dx)^4}{d^2}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(c+dx)^4(bc-ad)(ad+bc)}{4d^3} - \frac{b^2(c+dx)^6}{6d^3} + \frac{2b^2c(c+dx)^5}{5d^3}$$

input
```
Int[(c + d*x)^3*(a^2 - b^2*x^2),x]
```

output
```
-1/4*((b*c - a*d)*(b*c + a*d)*(c + d*x)^4)/d^3 + (2*b^2*c*(c + d*x)^5)/(5*
d^3) - (b^2*(c + d*x)^6)/(6*d^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

method	result	s
norman	$-\frac{b^2 d^3 x^6}{6}-\frac{3 b^2 c\, d^2 x^5}{5}+\left(\frac{1}{4} a^2 d^3-\frac{3}{4} b^2 c^2 d\right) x^4+\left(a^2 c\, d^2-\frac{1}{3} c^3 b^2\right) x^3+\frac{3 a^2 c^2 d\, x^2}{2}+a^2 c^3 x$	8
default	$-\frac{b^2 d^3 x^6}{6}-\frac{3 b^2 c\, d^2 x^5}{5}+\frac{\left(a^2 d^3-3 b^2 c^2 d\right) x^4}{4}+\frac{\left(3 a^2 c\, d^2-c^3 b^2\right) x^3}{3}+\frac{3 a^2 c^2 d\, x^2}{2}+a^2 c^3 x$	9
risch	$-\frac{1}{6} b^2 d^3 x^6-\frac{3}{5} b^2 c\, d^2 x^5+\frac{1}{4} x^4 a^2 d^3-\frac{3}{4} x^4 b^2 c^2 d+x^3 a^2 c\, d^2-\frac{1}{3} x^3 c^3 b^2+\frac{3}{2} a^2 c^2 d\, x^2+a^2 c^3 x$	9
parallelrisch	$-\frac{1}{6} b^2 d^3 x^6-\frac{3}{5} b^2 c\, d^2 x^5+\frac{1}{4} x^4 a^2 d^3-\frac{3}{4} x^4 b^2 c^2 d+x^3 a^2 c\, d^2-\frac{1}{3} x^3 c^3 b^2+\frac{3}{2} a^2 c^2 d\, x^2+a^2 c^3 x$	9
gosper	$\frac{x\left(-10 b^2 d^3 x^5-36 b^2 c\, d^2 x^4+15 x^3 a^2 d^3-45 x^3 b^2 c^2 d+60 x^2 a^2 c\, d^2-20 x^2 c^3 b^2+90 a^2 c^2 d x+60 a^2 c^3\right)}{60}$	9
orering	$\frac{x\left(-10 b^2 d^3 x^5-36 b^2 c\, d^2 x^4+15 x^3 a^2 d^3-45 x^3 b^2 c^2 d+60 x^2 a^2 c\, d^2-20 x^2 c^3 b^2+90 a^2 c^2 d x+60 a^2 c^3\right)\left(-b^2 x^2+a^2\right)}{60 (b x+a) (-b x+a)}$	1

input
```
int((d*x+c)^3*(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b^2*d^3*x^6-3/5*b^2*c*d^2*x^5+(1/4*a^2*d^3-3/4*b^2*c^2*d)*x^4+(a^2*c*
d^2-1/3*c^3*b^2)*x^3+3/2*a^2*c^2*d*x^2+a^2*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int (c+dx)^3\left(a^2-b^2x^2\right)\,dx=-\frac{1}{6}\,b^2d^3x^6-\frac{3}{5}\,b^2cd^2x^5+\frac{3}{2}\,a^2c^2dx^2+a^2c^3x-\frac{1}{4}\left(3\,b^2c^2d-a^2d^3\right)x^4-\frac{1}{3}\left(b^2c^3-3\,a^2cd^2\right)x^3$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/6*b^2*d^3*x^6 - 3/5*b^2*c*d^2*x^5 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x - 1/4
*(3*b^2*c^2*d - a^2*d^3)*x^4 - 1/3*(b^2*c^3 - 3*a^2*c*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int (c+dx)^3 \left(a^2-b^2x^2\right) dx = a^2c^3x + \frac{3a^2c^2dx^2}{2} - \frac{3b^2cd^2x^5}{5} - \frac{b^2d^3x^6}{6} + x^4\left(\frac{a^2d^3}{4} - \frac{3b^2c^2d}{4}\right) + x^3\left(a^2cd^2 - \frac{b^2c^3}{3}\right)$$

input
```
integrate((d*x+c)**3*(-b**2*x**2+a**2),x)
```

output
```
a**2*c**3*x + 3*a**2*c**2*d*x**2/2 - 3*b**2*c*d**2*x**5/5 - b**2*d**3*x**6
/6 + x**4*(a**2*d**3/4 - 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 - b**2*c**3/
3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int (c+dx)^3 \left(a^2-b^2x^2\right) dx = -\frac{1}{6}\,b^2d^3x^6 - \frac{3}{5}\,b^2cd^2x^5 + \frac{3}{2}\,a^2c^2dx^2 + a^2c^3x - \frac{1}{4}\left(3\,b^2c^2d - a^2d^3\right)x^4 - \frac{1}{3}\left(b^2c^3 - 3\,a^2cd^2\right)x^3$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/6*b^2*d^3*x^6 - 3/5*b^2*c*d^2*x^5 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x - 1/4
*(3*b^2*c^2*d - a^2*d^3)*x^4 - 1/3*(b^2*c^3 - 3*a^2*c*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int (c+dx)^3 \left(a^2-b^2x^2\right) \, dx = -\frac{1}{6} b^2 d^3 x^6 - \frac{3}{5} b^2 c d^2 x^5 - \frac{3}{4} b^2 c^2 d x^4 + \frac{1}{4} a^2 d^3 x^4 - \frac{1}{3} b^2 c^3 x^3 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + a^2 c^3 x$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-1/6*b^2*d^3*x^6 - 3/5*b^2*c*d^2*x^5 - 3/4*b^2*c^2*d*x^4 + 1/4*a^2*d^3*x^4
 - 1/3*b^2*c^3*x^3 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int (c+dx)^3 \left(a^2-b^2x^2\right) \, dx = x^4 \left(\frac{a^2\,d^3}{4}-\frac{3\,b^2\,c^2\,d}{4}\right) - x^3 \left(\frac{b^2\,c^3}{3}-a^2\,c\,d^2\right) + a^2\,c^3\,x - \frac{b^2\,d^3\,x^6}{6} + \frac{3\,a^2\,c^2\,d\,x^2}{2} - \frac{3\,b^2\,c\,d^2\,x^5}{5}$$

input
```
int((a^2 - b^2*x^2)*(c + d*x)^3,x)
```

output
```
x^4*((a^2*d^3)/4 - (3*b^2*c^2*d)/4) - x^3*((b^2*c^3)/3 - a^2*c*d^2) + a^2*
c^3*x - (b^2*d^3*x^6)/6 + (3*a^2*c^2*d*x^2)/2 - (3*b^2*c*d^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int (c+dx)^3 \left(a^2-b^2x^2\right) \, dx = \frac{x(-10b^2d^3x^5-36b^2c\,d^2x^4+15a^2d^3x^3-45b^2c^2d\,x^3+60a^2c\,d^2x^2-20b^2c^3x^2+90a^2c^2dx+60a^2c^3)}{60}$$

input
```
int((d*x+c)^3*(-b^2*x^2+a^2),x)
```

output
```
(x*(60*a**2*c**3 + 90*a**2*c**2*d*x + 60*a**2*c*d**2*x**2 + 15*a**2*d**3*x
**3 - 20*b**2*c**3*x**2 - 45*b**2*c**2*d*x**3 - 36*b**2*c*d**2*x**4 - 10*b
**2*d**3*x**5))/60
```

3.2 $\int (c+dx)^2 (a^2-b^2x^2) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int (c+dx)^2 (a^2-b^2x^2) \, dx = -\frac{(bc-ad)(bc+ad)(c+dx)^3}{3d^3} + \frac{b^2c(c+dx)^4}{2d^3} - \frac{b^2(c+dx)^5}{5d^3}$$

output
```
-1/3*(-a*d+b*c)*(a*d+b*c)*(d*x+c)^3/d^3+1/2*b^2*c*(d*x+c)^4/d^3-1/5*b^2*(d
*x+c)^5/d^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int (c+dx)^2 (a^2-b^2x^2) \, dx = -\frac{1}{30}b^2x^3(10c^2+15cdx+6d^2x^2) + a^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)$$

input
```
Integrate[(c + d*x)^2*(a^2 - b^2*x^2),x]
```

output
```
-1/30*(b^2*x^3*(10*c^2 + 15*c*d*x + 6*d^2*x^2)) + a^2*(c^2*x + c*d*x^2 + (
d^2*x^3)/3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)(c+dx)^2\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(c+dx)^2\left(a^2d^2-b^2c^2\right)}{d^2}-\frac{b^2(c+dx)^4}{d^2}+\frac{2b^2c(c+dx)^3}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(c+dx)^3(bc-ad)(ad+bc)}{3d^3}-\frac{b^2(c+dx)^5}{5d^3}+\frac{b^2c(c+dx)^4}{2d^3}$$

input
```
Int[(c + d*x)^2*(a^2 - b^2*x^2),x]
```

output
```
-1/3*((b*c - a*d)*(b*c + a*d)*(c + d*x)^3)/d^3 + (b^2*c*(c + d*x)^4)/(2*d^
3) - (b^2*(c + d*x)^5)/(5*d^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{b^2d^2x^5}{5}-\frac{b^2cd\,x^4}{2}+\frac{(a^2d^2-b^2c^2)x^3}{3}+a^2cd\,x^2+a^2c^2x$	61
norman	$-\frac{b^2d^2x^5}{5}-\frac{b^2cd\,x^4}{2}+\left(\frac{a^2d^2}{3}-\frac{b^2c^2}{3}\right)x^3+a^2cd\,x^2+a^2c^2x$	61
risch	$-\frac{1}{5}b^2d^2x^5-\frac{1}{2}b^2cd\,x^4+\frac{1}{3}x^3a^2d^2-\frac{1}{3}x^3b^2c^2+a^2cd\,x^2+a^2c^2x$	62
parallelrisch	$-\frac{1}{5}b^2d^2x^5-\frac{1}{2}b^2cd\,x^4+\frac{1}{3}x^3a^2d^2-\frac{1}{3}x^3b^2c^2+a^2cd\,x^2+a^2c^2x$	62
gosper	$\frac{x(-6b^2d^2x^4-15b^2cd\,x^3+10x^2a^2d^2-10b^2c^2x^2+30a^2cdx+30a^2c^2)}{30}$	64
orering	$\frac{x(-6b^2d^2x^4-15b^2cd\,x^3+10x^2a^2d^2-10b^2c^2x^2+30a^2cdx+30a^2c^2)(-b^2x^2+a^2)}{30(bx+a)(-bx+a)}$	91

input
```
int((d*x+c)^2*(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^2*d^2*x^5-1/2*b^2*c*d*x^4+1/3*(a^2*d^2-b^2*c^2)*x^3+a^2*c*d*x^2+a^2
*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (c+dx)^2\left(a^2-b^2x^2\right)\,dx=-\frac{1}{5}\,b^2d^2x^5-\frac{1}{2}\,b^2cdx^4+a^2cdx^2+a^2c^2x-\frac{1}{3}\left(b^2c^2-a^2d^2\right)x^3$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/5*b^2*d^2*x^5 - 1/2*b^2*c*d*x^4 + a^2*c*d*x^2 + a^2*c^2*x - 1/3*(b^2*c^
2 - a^2*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (c+dx)^2\left(a^2-b^2x^2\right)\,dx=a^2c^2x+a^2cdx^2-\frac{b^2cdx^4}{2}-\frac{b^2d^2x^5}{5}+x^3\left(\frac{a^2d^2}{3}-\frac{b^2c^2}{3}\right)$$

input
```
integrate((d*x+c)**2*(-b**2*x**2+a**2),x)
```

output
```
a**2*c**2*x + a**2*c*d*x**2 - b**2*c*d*x**4/2 - b**2*d**2*x**5/5 + x**3*(a
**2*d**2/3 - b**2*c**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (c+dx)^2\left(a^2-b^2x^2\right)\,dx=-\frac{1}{5}\,b^2d^2x^5-\frac{1}{2}\,b^2cdx^4+a^2cdx^2+a^2c^2x-\frac{1}{3}\,\left(b^2c^2-a^2d^2\right)x^3$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/5*b^2*d^2*x^5 - 1/2*b^2*c*d*x^4 + a^2*c*d*x^2 + a^2*c^2*x - 1/3*(b^2*c^
2 - a^2*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (c+dx)^2\left(a^2-b^2x^2\right)\,dx=-\frac{1}{5}\,b^2d^2x^5-\frac{1}{2}\,b^2cdx^4-\frac{1}{3}\,b^2c^2x^3+\frac{1}{3}\,a^2d^2x^3+a^2cdx^2+a^2c^2x$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-1/5*b^2*d^2*x^5 - 1/2*b^2*c*d*x^4 - 1/3*b^2*c^2*x^3 + 1/3*a^2*d^2*x^3 + a
^2*c*d*x^2 + a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)\,dx = x^3\left(\frac{a^2\,d^2}{3}-\frac{b^2\,c^2}{3}\right)+a^2\,c^2\,x-\frac{b^2\,d^2\,x^5}{5}+a^2\,c\,d\,x^2-\frac{b^2\,c\,d\,x^4}{2}$$

input
```
int((a^2 - b^2*x^2)*(c + d*x)^2,x)
```

output
```
x^3*((a^2*d^2)/3 - (b^2*c^2)/3) + a^2*c^2*x - (b^2*d^2*x^5)/5 + a^2*c*d*x^
2 - (b^2*c*d*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)\,dx = \frac{x(-6b^2d^2x^4-15b^2cd\,x^3+10a^2d^2x^2-10b^2c^2x^2+30a^2cdx+30a^2c^2)}{30}$$

input
```
int((d*x+c)^2*(-b^2*x^2+a^2),x)
```

output
```
(x*(30*a**2*c**2 + 30*a**2*c*d*x + 10*a**2*d**2*x**2 - 10*b**2*c**2*x**2 -
 15*b**2*c*d*x**3 - 6*b**2*d**2*x**4))/30
```

3.3 $\int (c+dx)\left(a^2-b^2x^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx=a^2cx-\frac{1}{3}b^2cx^3-\frac{d(a^2-b^2x^2)^2}{4b^2}$$

output
```
a^2*c*x-1/3*b^2*c*x^3-1/4*d*(-b^2*x^2+a^2)^2/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx=a^2cx+\frac{1}{2}a^2dx^2-\frac{1}{3}b^2cx^3-\frac{1}{4}b^2dx^4$$

input
```
Integrate[(c + d*x)*(a^2 - b^2*x^2),x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 - (b^2*c*x^3)/3 - (b^2*d*x^4)/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {455, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 - b^2x^2)(c + dx)\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int (a^2 - b^2x^2)\,dx - \frac{d(a^2 - b^2x^2)^2}{4b^2}$$

$$\Big\downarrow \text{2009}$$

$$c\left(a^2x - \frac{b^2x^3}{3}\right) - \frac{d(a^2 - b^2x^2)^2}{4b^2}$$

input
```
Int[(c + d*x)*(a^2 - b^2*x^2),x]
```

output
```
-1/4*(d*(a^2 - b^2*x^2)^2)/b^2 + c*(a^2*x - (b^2*x^3)/3)
```

Defintions of rubi rules used

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{4}b^2 d\,x^4 - \frac{1}{3}b^2 c\,x^3 + \frac{1}{2}a^2 d\,x^2 + a^2 cx$	35
norman	$-\frac{1}{4}b^2 d\,x^4 - \frac{1}{3}b^2 c\,x^3 + \frac{1}{2}a^2 d\,x^2 + a^2 cx$	35
risch	$-\frac{1}{4}b^2 d\,x^4 - \frac{1}{3}b^2 c\,x^3 + \frac{1}{2}a^2 d\,x^2 + a^2 cx$	35
parallelrisch	$-\frac{1}{4}b^2 d\,x^4 - \frac{1}{3}b^2 c\,x^3 + \frac{1}{2}a^2 d\,x^2 + a^2 cx$	35
gosper	$\frac{x(-3b^2 d\,x^3 - 4b^2 c\,x^2 + 6a^2 dx + 12a^2 c)}{12}$	36
orering	$\frac{x(-3b^2 d\,x^3 - 4b^2 c\,x^2 + 6a^2 dx + 12a^2 c)(-b^2 x^2 + a^2)}{12(bx+a)(-bx+a)}$	63

input
```
int((d*x+c)*(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^2*d*x^4-1/3*b^2*c*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx = -\frac{1}{4}\,b^2dx^4 - \frac{1}{3}\,b^2cx^3 + \frac{1}{2}\,a^2dx^2 + a^2cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/4*b^2*d*x^4 - 1/3*b^2*c*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx=a^2cx+\frac{a^2dx^2}{2}-\frac{b^2cx^3}{3}-\frac{b^2dx^4}{4}$$

input
```
integrate((d*x+c)*(-b**2*x**2+a**2),x)
```

output
```
a**2*c*x + a**2*d*x**2/2 - b**2*c*x**3/3 - b**2*d*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx=-\frac{1}{4}\,b^2dx^4-\frac{1}{3}\,b^2cx^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/4*b^2*d*x^4 - 1/3*b^2*c*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx=-\frac{1}{4}\,b^2dx^4-\frac{1}{3}\,b^2cx^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-1/4*b^2*d*x^4 - 1/3*b^2*c*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx=\frac{d\,a^2\,x^2}{2}+c\,a^2\,x-\frac{d\,b^2\,x^4}{4}-\frac{c\,b^2\,x^3}{3}$$

input
```
int((a^2 - b^2*x^2)*(c + d*x),x)
```

output
```
(a^2*d*x^2)/2 - (b^2*c*x^3)/3 - (b^2*d*x^4)/4 + a^2*c*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int (c+dx)\left(a^2-b^2x^2\right)\,dx=\frac{x(-3b^2d\,x^3-4b^2c\,x^2+6a^2dx+12a^2c)}{12}$$

input
```
int((d*x+c)*(-b^2*x^2+a^2),x)
```

output
```
(x*(12*a**2*c + 6*a**2*d*x - 4*b**2*c*x**2 - 3*b**2*d*x**3))/12
```

3.4 $\int (a^2 - b^2x^2) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 16

$$\int (a^2 - b^2x^2) \, dx = a^2x - \frac{b^2x^3}{3}$$

output
```
a^2*x-1/3*b^2*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a^2 - b^2x^2) \, dx = a^2x - \frac{b^2x^3}{3}$$

input
```
Integrate[a^2 - b^2*x^2,x]
```

output
```
a^2*x - (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$a^2x-\frac{b^2x^3}{3}$$

input
```
Int[a^2 - b^2*x^2,x]
```

output
```
a^2*x - (b^2*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$a^2x - \frac{1}{3}b^2x^3$	15
norman	$a^2x - \frac{1}{3}b^2x^3$	15
risch	$a^2x - \frac{1}{3}b^2x^3$	15
parallelrisch	$a^2x - \frac{1}{3}b^2x^3$	15
parts	$a^2x - \frac{1}{3}b^2x^3$	15
gosper	$\frac{x(-b^2x^2+3a^2)}{3}$	18
orering	$\frac{x(-b^2x^2+3a^2)(-b^2x^2+a^2)}{3(bx+a)(-bx+a)}$	45

input
```
int(-b^2*x^2+a^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-1/3*b^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (a^2 - b^2x^2) \, dx = -\frac{1}{3} \, b^2x^3 + a^2x$$

input
```
integrate(-b^2*x^2+a^2,x, algorithm="fricas")
```

output
```
-1/3*b^2*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(a^2-b^2x^2\right)\,dx=a^2x-\frac{b^2x^3}{3}$$

input
```
integrate(-b**2*x**2+a**2,x)
```

output
```
a**2*x - b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(a^2-b^2x^2\right)\,dx=-\frac{1}{3}\,b^2x^3+a^2x$$

input
```
integrate(-b^2*x^2+a^2,x, algorithm="maxima")
```

output
```
-1/3*b^2*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(a^2-b^2x^2\right)\,dx=-\frac{1}{3}\,b^2x^3+a^2x$$

input
```
integrate(-b^2*x^2+a^2,x, algorithm="giac")
```

output
```
-1/3*b^2*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (a^2 - b^2x^2) \, dx = a^2 x - \frac{b^2 x^3}{3}$$

input
```
int(a^2 - b^2*x^2,x)
```

output
```
a^2*x - (b^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (a^2 - b^2x^2) \, dx = \frac{x(-b^2x^2 + 3a^2)}{3}$$

input
```
int(-b^2*x^2+a^2,x)
```

output
```
(x*(3*a**2 - b**2*x**2))/3
```

3.5 $\int \frac{a^2-b^2x^2}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{a^2-b^2x^2}{c+dx}\,dx=\frac{b^2cx}{d^2}-\frac{b^2x^2}{2d}-\frac{(b^2c^2-a^2d^2)\log(c+dx)}{d^3}$$

output
```
b^2*c*x/d^2-1/2*b^2*x^2/d-(-a^2*d^2+b^2*c^2)*ln(d*x+c)/d^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{a^2-b^2x^2}{c+dx}\,dx=\frac{b^2dx(2c-dx)+(-2b^2c^2+2a^2d^2)\log(c+dx)}{2d^3}$$

input
```
Integrate[(a^2 - b^2*x^2)/(c + d*x),x]
```

output
```
(b^2*d*x*(2*c - d*x) + (-2*b^2*c^2 + 2*a^2*d^2)*Log[c + d*x])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2-b^2x^2}{c+dx}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{a^2d^2-b^2c^2}{d^2(c+dx)}+\frac{b^2c}{d^2}-\frac{b^2x}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)(ad+bc)\log(c+dx)}{d^3}+\frac{b^2cx}{d^2}-\frac{b^2x^2}{2d}$$

input
```
Int[(a^2 - b^2*x^2)/(c + d*x),x]
```

output
```
(b^2*c*x)/d^2 - (b^2*x^2)/(2*d) - ((b*c - a*d)*(b*c + a*d)*Log[c + d*x])/d
^3
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{b^2\left(-\frac{1}{2}d\,x^2+cx\right)}{d^2}+\frac{\left(a^2d^2-b^2c^2\right)\ln\left(dx+c\right)}{d^3}$	45
norman	$\frac{b^2cx}{d^2}-\frac{b^2x^2}{2d}+\frac{\left(a^2d^2-b^2c^2\right)\ln\left(dx+c\right)}{d^3}$	48
risch	$\frac{b^2cx}{d^2}-\frac{b^2x^2}{2d}+\frac{\ln\left(dx+c\right)a^2}{d}-\frac{\ln\left(dx+c\right)b^2c^2}{d^3}$	52
parallelrisch	$\frac{-x^2b^2d^2+2\ln\left(dx+c\right)a^2d^2-2\ln\left(dx+c\right)b^2c^2+2b^2cxd}{2d^3}$	54

input
```
int((-b^2*x^2+a^2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
b^2/d^2*(-1/2*d*x^2+c*x)+(a^2*d^2-b^2*c^2)/d^3*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{a^2-b^2x^2}{c+dx}\,dx=-\frac{b^2d^2x^2-2\,b^2cdx+2\,(b^2c^2-a^2d^2)\log\left(dx+c\right)}{2\,d^3}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(b^2*d^2*x^2 - 2*b^2*c*d*x + 2*(b^2*c^2 - a^2*d^2)*log(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{a^2-b^2x^2}{c+dx}\,dx=\frac{b^2cx}{d^2}-\frac{b^2x^2}{2d}+\frac{(ad-bc)\,(ad+bc)\log\left(c+dx\right)}{d^3}$$

input
```
integrate((-b**2*x**2+a**2)/(d*x+c),x)
```

output
```
b**2*c*x/d**2 - b**2*x**2/(2*d) + (a*d - b*c)*(a*d + b*c)*log(c + d*x)/d**
3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{a^2 - b^2x^2}{c + dx}\,dx = -\frac{b^2dx^2 - 2\,b^2cx}{2\,d^2} - \frac{(b^2c^2 - a^2d^2)\log{(dx + c)}}{d^3}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(b^2*d*x^2 - 2*b^2*c*x)/d^2 - (b^2*c^2 - a^2*d^2)*log(d*x + c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{a^2 - b^2x^2}{c + dx}\,dx = -\frac{b^2dx^2 - 2\,b^2cx}{2\,d^2} - \frac{(b^2c^2 - a^2d^2)\log{(|dx + c|)}}{d^3}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c),x, algorithm="giac")
```

output
```
-1/2*(b^2*d*x^2 - 2*b^2*c*x)/d^2 - (b^2*c^2 - a^2*d^2)*log(abs(d*x + c))/d
^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{a^2-b^2x^2}{c+dx}\,dx=\frac{\ln\left(c+d\,x\right)\,\left(a^2\,d^2-b^2\,c^2\right)}{d^3}-\frac{b^2\,x^2}{2\,d}+\frac{b^2\,c\,x}{d^2}$$

input
```
int((a^2 - b^2*x^2)/(c + d*x),x)
```

output
```
(log(c + d*x)*(a^2*d^2 - b^2*c^2))/d^3 - (b^2*x^2)/(2*d) + (b^2*c*x)/d^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{a^2-b^2x^2}{c+dx}\,dx=\frac{2\log(dx+c)\,a^2d^2-2\log(dx+c)\,b^2c^2+2b^2cdx-b^2d^2x^2}{2d^3}$$

input
```
int((-b^2*x^2+a^2)/(d*x+c),x)
```

output
```
(2*log(c + d*x)*a**2*d**2 - 2*log(c + d*x)*b**2*c**2 + 2*b**2*c*d*x - b**2
*d**2*x**2)/(2*d**3)
```

3.6 $\int \frac{a^2-b^2x^2}{(c+dx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{a^2-b^2x^2}{(c+dx)^2}\,dx=-\frac{b^2x}{d^2}+\frac{(bc-ad)(bc+ad)}{d^3(c+dx)}+\frac{2b^2c\log(c+dx)}{d^3}$$

output
```
-b^2*x/d^2+(-a*d+b*c)*(a*d+b*c)/d^3/(d*x+c)+2*b^2*c*ln(d*x+c)/d^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{a^2-b^2x^2}{(c+dx)^2}\,dx=\frac{-b^2dx+\frac{b^2c^2-a^2d^2}{c+dx}+2b^2c\log(c+dx)}{d^3}$$

input
```
Integrate[(a^2 - b^2*x^2)/(c + d*x)^2,x]
```

output
```
(-(b^2*d*x) + (b^2*c^2 - a^2*d^2)/(c + d*x) + 2*b^2*c*Log[c + d*x])/d^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2-b^2x^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{a^2d^2-b^2c^2}{d^2(c+dx)^2}+\frac{2b^2c}{d^2(c+dx)}-\frac{b^2}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)(ad+bc)}{d^3(c+dx)}+\frac{2b^2c\log(c+dx)}{d^3}-\frac{b^2x}{d^2}$$

input
```
Int[(a^2 - b^2*x^2)/(c + d*x)^2,x]
```

output
```
-((b^2*x)/d^2) + ((b*c - a*d)*(b*c + a*d))/(d^3*(c + d*x)) + (2*b^2*c*Log[
c + d*x])/d^3
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{b^{2}x}{d^{2}}-\frac{a^{2}d^{2}-b^{2}c^{2}}{d^{3}(dx+c)}+\frac{2b^{2}c\ln(dx+c)}{d^{3}}$	54
norman	$\frac{-\frac{a^{2}d^{2}-2b^{2}c^{2}}{d^{3}}-\frac{b^{2}x^{2}}{d}}{dx+c}+\frac{2b^{2}c\ln(dx+c)}{d^{3}}$	58
risch	$-\frac{b^{2}x}{d^{2}}-\frac{a^{2}}{d(dx+c)}+\frac{b^{2}c^{2}}{d^{3}(dx+c)}+\frac{2b^{2}c\ln(dx+c)}{d^{3}}$	58
parallelrisch	$\frac{2\ln(dx+c)x\,b^{2}cd-x^{2}b^{2}d^{2}+2\ln(dx+c)b^{2}c^{2}-a^{2}d^{2}+2b^{2}c^{2}}{d^{3}(dx+c)}$	68

input
```
int((-b^2*x^2+a^2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2*x/d^2-(a^2*d^2-b^2*c^2)/d^3/(d*x+c)+2*b^2*c*ln(d*x+c)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{a^2-b^2x^2}{(c+dx)^2}\,dx=-\frac{b^2d^2x^2+b^2cdx-b^2c^2+a^2d^2-2\left(b^2cdx+b^2c^2\right)\log\left(dx+c\right)}{d^4x+cd^3}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + a^2*d^2 - 2*(b^2*c*d*x + b^2*c^2)*lo
g(d*x + c))/(d^4*x + c*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{a^2 - b^2x^2}{(c + dx)^2}\, dx = \frac{2b^2c\log{(c + dx)}}{d^3} - \frac{b^2x}{d^2} - \frac{a^2d^2 - b^2c^2}{cd^3 + d^4x}$$

input
```
integrate((-b**2*x**2+a**2)/(d*x+c)**2,x)
```

output
```
2*b**2*c*log(c + d*x)/d**3 - b**2*x/d**2 - (a**2*d**2 - b**2*c**2)/(c*d**3
 + d**4*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{a^2 - b^2x^2}{(c + dx)^2}\, dx = -\frac{b^2x}{d^2} + \frac{2\, b^2c\log{(dx + c)}}{d^3} + \frac{b^2c^2 - a^2d^2}{d^4x + cd^3}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-b^2*x/d^2 + 2*b^2*c*log(d*x + c)/d^3 + (b^2*c^2 - a^2*d^2)/(d^4*x + c*d^3
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{a^2 - b^2x^2}{(c + dx)^2}\, dx = -b^2\left(\frac{2\, c\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3} + \frac{dx + c}{d^3} - \frac{c^2}{(dx + c)d^3}\right) - \frac{a^2}{(dx + c)d}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-b^2*(2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2
/((d*x + c)*d^3)) - a^2/((d*x + c)*d)
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{a^2 - b^2x^2}{(c + dx)^2}\,dx = \frac{2\,b^2\,c\,\ln\left(c + d\,x\right)}{d^3} - \frac{b^2\,x}{d^2} - \frac{a^2\,d^2 - b^2\,c^2}{d\,\left(x\,d^3 + c\,d^2\right)}$$

input
```
int((a^2 - b^2*x^2)/(c + d*x)^2,x)
```

output
```
(2*b^2*c*log(c + d*x))/d^3 - (b^2*x)/d^2 - (a^2*d^2 - b^2*c^2)/(d*(c*d^2 +
 d^3*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \frac{a^2 - b^2x^2}{(c + dx)^2}\,dx = \frac{2\log(dx + c)\,b^2c^3 + 2\log(dx + c)\,b^2c^2dx + a^2d^3x - 2b^2c^2dx - b^2c\,d^2x^2}{c\,d^3\,(dx + c)}$$

input
```
int((-b^2*x^2+a^2)/(d*x+c)^2,x)
```

output
```
(2*log(c + d*x)*b**2*c**3 + 2*log(c + d*x)*b**2*c**2*d*x + a**2*d**3*x - 2
*b**2*c**2*d*x - b**2*c*d**2*x**2)/(c*d**3*(c + d*x))
```

3.7 $\int \frac{a^2-b^2x^2}{(c+dx)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{a^2-b^2x^2}{(c+dx)^3}\,dx=\frac{(bc-ad)(bc+ad)}{2d^3(c+dx)^2}-\frac{2b^2c}{d^3(c+dx)}-\frac{b^2\log(c+dx)}{d^3}$$

output
```
1/2*(-a*d+b*c)*(a*d+b*c)/d^3/(d*x+c)^2-2*b^2*c/d^3/(d*x+c)-b^2*ln(d*x+c)/d
^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{a^2-b^2x^2}{(c+dx)^3}\,dx=-\frac{a^2d^2+b^2c(3c+4dx)+2b^2(c+dx)^2\log(c+dx)}{2d^3(c+dx)^2}$$

input
```
Integrate[(a^2 - b^2*x^2)/(c + d*x)^3,x]
```

output
```
-1/2*(a^2*d^2 + b^2*c*(3*c + 4*d*x) + 2*b^2*(c + d*x)^2*Log[c + d*x])/(d^3
*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2-b^2x^2}{(c+dx)^3}\,dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{a^2d^2-b^2c^2}{d^2(c+dx)^3}-\frac{b^2}{d^2(c+dx)}+\frac{2b^2c}{d^2(c+dx)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(bc-ad)(ad+bc)}{2d^3(c+dx)^2}-\frac{2b^2c}{d^3(c+dx)}-\frac{b^2\log(c+dx)}{d^3}$$

input
```
Int[(a^2 - b^2*x^2)/(c + d*x)^3,x]
```

output
```
((b*c - a*d)*(b*c + a*d))/(2*d^3*(c + d*x)^2) - (2*b^2*c)/(d^3*(c + d*x))
- (b^2*Log[c + d*x])/d^3
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

method	result	size
norman	$\frac{-\frac{a^2d^2+3b^2c^2}{2d^3}-\frac{2b^2cx}{d^2}}{(dx+c)^2}-\frac{b^2\ln(dx+c)}{d^3}$	56
risch	$\frac{-\frac{a^2d^2+3b^2c^2}{2d^3}-\frac{2b^2cx}{d^2}}{(dx+c)^2}-\frac{b^2\ln(dx+c)}{d^3}$	56
default	$-\frac{2b^2c}{d^3(dx+c)}-\frac{a^2d^2-b^2c^2}{2d^3(dx+c)^2}-\frac{b^2\ln(dx+c)}{d^3}$	60
parallelrisch	$-\frac{2\ln(dx+c)x^2b^2d^2+4\ln(dx+c)x\,b^2cd+2\ln(dx+c)b^2c^2+4b^2cxd+a^2d^2+3b^2c^2}{2d^3(dx+c)^2}$	82

input
```
int((-b^2*x^2+a^2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(a^2*d^2+3*b^2*c^2)/d^3-2*b^2*c*x/d^2)/(d*x+c)^2-b^2*ln(d*x+c)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

$$\int \frac{a^2-b^2x^2}{(c+dx)^3}\,dx = -\frac{4\,b^2cdx+3\,b^2c^2+a^2d^2+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(dx+c)}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*b^2*c*d*x + 3*b^2*c^2 + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{a^2-b^2x^2}{(c+dx)^3}\,dx=-\frac{b^2\log{(c+dx)}}{d^3}-\frac{a^2d^2+3b^2c^2+4b^2cdx}{2c^2d^3+4cd^4x+2d^5x^2}$$

input
```
integrate((-b**2*x**2+a**2)/(d*x+c)**3,x)
```

output
```
-b**2*log(c + d*x)/d**3 - (a**2*d**2 + 3*b**2*c**2 + 4*b**2*c*d*x)/(2*c**2
*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{a^2-b^2x^2}{(c+dx)^3}\,dx=-\frac{4\,b^2cdx+3\,b^2c^2+a^2d^2}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}-\frac{b^2\log{(dx+c)}}{d^3}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(4*b^2*c*d*x + 3*b^2*c^2 + a^2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3) -
 b^2*log(d*x + c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{a^2-b^2x^2}{(c+dx)^3}\,dx=-\frac{b^2\log{(|dx+c|)}}{d^3}-\frac{4\,b^2cx+\frac{3\,b^2c^2+a^2d^2}{d}}{2\,(dx+c)^2d^2}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-b^2*log(abs(d*x + c))/d^3 - 1/2*(4*b^2*c*x + (3*b^2*c^2 + a^2*d^2)/d)/((d
*x + c)^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{a^2 - b^2x^2}{(c + dx)^3} \, dx = -\frac{\frac{a^2 \, d^2 + 3 \, b^2 \, c^2}{2 \, d^3} + \frac{2 \, b^2 \, c \, x}{d^2}}{c^2 + 2 \, c \, d \, x + d^2 \, x^2} - \frac{b^2 \, \ln (c + d \, x)}{d^3}$$

input
```
int((a^2 - b^2*x^2)/(c + d*x)^3,x)
```

output
```
- ((a^2*d^2 + 3*b^2*c^2)/(2*d^3) + (2*b^2*c*x)/d^2)/(c^2 + d^2*x^2 + 2*c*d
*x) - (b^2*log(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

$$\int \frac{a^2 - b^2x^2}{(c + dx)^3} \, dx = \frac{-2 \log(dx + c) \, b^2c^2 - 4 \log(dx + c) \, b^2cdx - 2 \log(dx + c) \, b^2d^2x^2 - a^2d^2 - b^2c^2 + 2b^2d^2x^2}{2d^3 \left(d^2x^2 + 2cdx + c^2\right)}$$

input
```
int((-b^2*x^2+a^2)/(d*x+c)^3,x)
```

output
```
( - 2*log(c + d*x)*b**2*c**2 - 4*log(c + d*x)*b**2*c*d*x - 2*log(c + d*x)*
b**2*d**2*x**2 - a**2*d**2 - b**2*c**2 + 2*b**2*d**2*x**2)/(2*d**3*(c**2 +
 2*c*d*x + d**2*x**2))
```

3.8 $\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx = \frac{(bc-ad)(bc+ad)}{3d^3(c+dx)^3} - \frac{b^2c}{d^3(c+dx)^2} + \frac{b^2}{d^3(c+dx)}$$

output
```
1/3*(-a*d+b*c)*(a*d+b*c)/d^3/(d*x+c)^3-b^2*c/d^3/(d*x+c)^2+b^2/d^3/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx = \frac{-a^2d^2+b^2(c^2+3cdx+3d^2x^2)}{3d^3(c+dx)^3}$$

input
```
Integrate[(a^2 - b^2*x^2)/(c + d*x)^4,x]
```

output
```
(-(a^2*d^2) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(3*d^3*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{a^2d^2-b^2c^2}{d^2(c+dx)^4}-\frac{b^2}{d^2(c+dx)^2}+\frac{2b^2c}{d^2(c+dx)^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(bc-ad)(ad+bc)}{3d^3(c+dx)^3}+\frac{b^2}{d^3(c+dx)}-\frac{b^2c}{d^3(c+dx)^2}$$

input
```
Int[(a^2 - b^2*x^2)/(c + d*x)^4,x]
```

output
```
((b*c - a*d)*(b*c + a*d))/(3*d^3*(c + d*x)^3) - (b^2*c)/(d^3*(c + d*x)^2)
+ b^2/(d^3*(c + d*x))
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{-3x^{2}b^{2}d^{2}-3b^{2}cxd+a^{2}d^{2}-b^{2}c^{2}}{3d^{3}\left(dx+c\right)^{3}}$	48
parallelrisch	$\frac{3x^{2}b^{2}d^{2}+3b^{2}cxd-a^{2}d^{2}+b^{2}c^{2}}{3d^{3}\left(dx+c\right)^{3}}$	48
norman	$\frac{\frac{b^{2}x^{2}}{d}+\frac{b^{2}cx}{d^{2}}-\frac{a^{2}d^{2}-b^{2}c^{2}}{3d^{3}}}{\left(dx+c\right)^{3}}$	50
risch	$\frac{\frac{b^{2}x^{2}}{d}+\frac{b^{2}cx}{d^{2}}-\frac{a^{2}d^{2}-b^{2}c^{2}}{3d^{3}}}{\left(dx+c\right)^{3}}$	50
default	$\frac{b^{2}}{d^{3}\left(dx+c\right)}-\frac{b^{2}c}{d^{3}\left(dx+c\right)^{2}}-\frac{a^{2}d^{2}-b^{2}c^{2}}{3d^{3}\left(dx+c\right)^{3}}$	60
orering	$-\frac{\left(-3x^{2}b^{2}d^{2}-3b^{2}cxd+a^{2}d^{2}-b^{2}c^{2}\right)\left(-b^{2}x^{2}+a^{2}\right)}{3d^{3}\left(bx+a\right)\left(dx+c\right)^{3}\left(-bx+a\right)}$	75

input
```
int((-b^2*x^2+a^2)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b^2*d^2*x^2-3*b^2*c*d*x+a^2*d^2-b^2*c^2)/d^3/(d*x+c)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx=\frac{3\,b^2d^2x^2+3\,b^2cdx+b^2c^2-a^2d^2}{3\,(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3)}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + b^2*c^2 - a^2*d^2)/(d^6*x^3 + 3*c*d^5*x
^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx = -\frac{a^2d^2-b^2c^2-3b^2cdx-3b^2d^2x^2}{3c^3d^3+9c^2d^4x+9cd^5x^2+3d^6x^3}$$

input
```
integrate((-b**2*x**2+a**2)/(d*x+c)**4,x)
```

output
```
-(a**2*d**2 - b**2*c**2 - 3*b**2*c*d*x - 3*b**2*d**2*x**2)/(3*c**3*d**3 +
9*c**2*d**4*x + 9*c*d**5*x**2 + 3*d**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx = \frac{3\,b^2d^2x^2+3\,b^2cdx+b^2c^2-a^2d^2}{3\,(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3)}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + b^2*c^2 - a^2*d^2)/(d^6*x^3 + 3*c*d^5*x
^2 + 3*c^2*d^4*x + c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{a^2-b^2x^2}{(c+dx)^4}\,dx = \frac{3\,b^2d^2x^2+3\,b^2cdx+b^2c^2-a^2d^2}{3\,(dx+c)^3d^3}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + b^2*c^2 - a^2*d^2)/((d*x + c)^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{a^2 - b^2x^2}{(c+dx)^4}\,dx = \frac{\frac{b^2\,x^2}{d} - \frac{a^2\,d^2-b^2\,c^2}{3\,d^3} + \frac{b^2\,c\,x}{d^2}}{c^3 + 3\,c^2\,d\,x + 3\,c\,d^2\,x^2 + d^3\,x^3}$$

input
```
int((a^2 - b^2*x^2)/(c + d*x)^4,x)
```

output
```
((b^2*x^2)/d - (a^2*d^2 - b^2*c^2)/(3*d^3) + (b^2*c*x)/d^2)/(c^3 + d^3*x^3
 + 3*c*d^2*x^2 + 3*c^2*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{a^2 - b^2x^2}{(c+dx)^4}\,dx = \frac{-b^2d\,x^3 - a^2c}{3cd\,(d^3x^3 + 3c\,d^2x^2 + 3c^2dx + c^3)}$$

input
```
int((-b^2*x^2+a^2)/(d*x+c)^4,x)
```

output
```
( - (a**2*c + b**2*d*x**3))/(3*c*d*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3))
```

3.9 $\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx=\frac{(bc-ad)(bc+ad)}{4d^3(c+dx)^4}-\frac{2b^2c}{3d^3(c+dx)^3}+\frac{b^2}{2d^3(c+dx)^2}$$

output
```
1/4*(-a*d+b*c)*(a*d+b*c)/d^3/(d*x+c)^4-2/3*b^2*c/d^3/(d*x+c)^3+1/2*b^2/d^3
/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx=\frac{-3a^2d^2+b^2(c^2+4cdx+6d^2x^2)}{12d^3(c+dx)^4}$$

input
```
Integrate[(a^2 - b^2*x^2)/(c + d*x)^5,x]
```

output
```
(-3*a^2*d^2 + b^2*(c^2 + 4*c*d*x + 6*d^2*x^2))/(12*d^3*(c + d*x)^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{a^2d^2-b^2c^2}{d^2(c+dx)^5}-\frac{b^2}{d^2(c+dx)^3}+\frac{2b^2c}{d^2(c+dx)^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(bc-ad)(ad+bc)}{4d^3(c+dx)^4}+\frac{b^2}{2d^3(c+dx)^2}-\frac{2b^2c}{3d^3(c+dx)^3}$$

input
```
Int[(a^2 - b^2*x^2)/(c + d*x)^5,x]
```

output
```
((b*c - a*d)*(b*c + a*d))/(4*d^3*(c + d*x)^4) - (2*b^2*c)/(3*d^3*(c + d*x)
^3) + b^2/(2*d^3*(c + d*x)^2)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{-6x^2b^2d^2-4b^2cxd+3a^2d^2-b^2c^2}{12d^3(dx+c)^4}$	49
parallelrisch	$\frac{6b^2x^2d^3+4b^2cx\,d^2-3a^2d^3+b^2c^2d}{12d^4(dx+c)^4}$	51
risch	$\frac{\frac{b^2x^2}{2d}+\frac{b^2cx}{3d^2}-\frac{3a^2d^2-b^2c^2}{12d^3}}{(dx+c)^4}$	53
norman	$\frac{\frac{b^2x^2}{2d}+\frac{b^2cx}{3d^2}-\frac{3a^2d^3-b^2c^2d}{12d^4}}{(dx+c)^4}$	54
default	$-\frac{a^2d^2-b^2c^2}{4d^3(dx+c)^4}+\frac{b^2}{2d^3(dx+c)^2}-\frac{2b^2c}{3d^3(dx+c)^3}$	61
orering	$-\frac{(-6x^2b^2d^2-4b^2cxd+3a^2d^2-b^2c^2)(-b^2x^2+a^2)}{12d^3(bx+a)(dx+c)^4(-bx+a)}$	76

input
```
int((-b^2*x^2+a^2)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/d^3*(-6*b^2*d^2*x^2-4*b^2*c*d*x+3*a^2*d^2-b^2*c^2)/(d*x+c)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx = \frac{6\,b^2d^2x^2+4\,b^2cdx+b^2c^2-3\,a^2d^2}{12\,(d^7x^4+4\,cd^6x^3+6\,c^2d^5x^2+4\,c^3d^4x+c^4d^3)}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/12*(6*b^2*d^2*x^2 + 4*b^2*c*d*x + b^2*c^2 - 3*a^2*d^2)/(d^7*x^4 + 4*c*d^
6*x^3 + 6*c^2*d^5*x^2 + 4*c^3*d^4*x + c^4*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx = -\frac{3a^2d^2-b^2c^2-4b^2cdx-6b^2d^2x^2}{12c^4d^3+48c^3d^4x+72c^2d^5x^2+48cd^6x^3+12d^7x^4}$$

input
```
integrate((-b**2*x**2+a**2)/(d*x+c)**5,x)
```

output
```
-(3*a**2*d**2 - b**2*c**2 - 4*b**2*c*d*x - 6*b**2*d**2*x**2)/(12*c**4*d**3
 + 48*c**3*d**4*x + 72*c**2*d**5*x**2 + 48*c*d**6*x**3 + 12*d**7*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx = \frac{6\,b^2d^2x^2+4\,b^2cdx+b^2c^2-3\,a^2d^2}{12\,(d^7x^4+4\,cd^6x^3+6\,c^2d^5x^2+4\,c^3d^4x+c^4d^3)}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/12*(6*b^2*d^2*x^2 + 4*b^2*c*d*x + b^2*c^2 - 3*a^2*d^2)/(d^7*x^4 + 4*c*d^
6*x^3 + 6*c^2*d^5*x^2 + 4*c^3*d^4*x + c^4*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx = -\frac{\frac{3\,a^2}{(dx+c)^4}-\frac{6\,b^2}{(dx+c)^2d^2}+\frac{8\,b^2c}{(dx+c)^3d^2}-\frac{3\,b^2c^2}{(dx+c)^4d^2}}{12\,d}$$

input
```
integrate((-b^2*x^2+a^2)/(d*x+c)^5,x, algorithm="giac")
```

output
```
-1/12*(3*a^2/(d*x + c)^4 - 6*b^2/((d*x + c)^2*d^2) + 8*b^2*c/((d*x + c)^3*
d^2) - 3*b^2*c^2/((d*x + c)^4*d^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx=\frac{\frac{b^2\,x^2}{2\,d}-\frac{3\,a^2\,d^2-b^2\,c^2}{12\,d^3}+\frac{b^2\,c\,x}{3\,d^2}}{c^4+4\,c^3\,d\,x+6\,c^2\,d^2\,x^2+4\,c\,d^3\,x^3+d^4\,x^4}$$

input
```
int((a^2 - b^2*x^2)/(c + d*x)^5,x)
```

output
```
((b^2*x^2)/(2*d) - (3*a^2*d^2 - b^2*c^2)/(12*d^3) + (b^2*c*x)/(3*d^2))/(c^
4 + d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{a^2-b^2x^2}{(c+dx)^5}\,dx=\frac{6b^2d^2x^2+4b^2cdx-3a^2d^2+b^2c^2}{12d^3\left(d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4\right)}$$

input
```
int((-b^2*x^2+a^2)/(d*x+c)^5,x)
```

output
```
( - 3*a**2*d**2 + b**2*c**2 + 4*b**2*c*d*x + 6*b**2*d**2*x**2)/(12*d**3*(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4))
```

3.10 $\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \, dx = \frac{\left(b^2c^2-a^2d^2\right)^2 (c+dx)^4}{4d^5} - \frac{4b^2c(bc-ad)(bc+ad)(c+dx)^5}{5d^5} + \frac{b^2\left(3b^2c^2-a^2d^2\right)(c+dx)^6}{3d^5} - \frac{4b^4c(c+dx)^7}{7d^5} + \frac{b^4(c+dx)^8}{8d^5}$$

output
```
1/4*(-a^2*d^2+b^2*c^2)^2*(d*x+c)^4/d^5-4/5*b^2*c*(-a*d+b*c)*(a*d+b*c)*(d*x
+c)^5/d^5+1/3*b^2*(-a^2*d^2+3*b^2*c^2)*(d*x+c)^6/d^5-4/7*b^4*c*(d*x+c)^7/d
^5+1/8*b^4*(d*x+c)^8/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{4}a^4x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) - \frac{1}{30}a^2b^2x^3\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right) + \frac{1}{280}b^4x^5\left(56c^3+140c^2dx+120cd^2x^2+35d^3x^3\right)$$

input
```
Integrate[(c + d*x)^3*(a^2 - b^2*x^2)^2,x]
```

output
```
(a^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (a^2*b^2*x^3*(20*c
^3 + 45*c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3))/30 + (b^4*x^5*(56*c^3 + 140*
c^2*d*x + 120*c*d^2*x^2 + 35*d^3*x^3))/280
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)^2 (c+dx)^3 \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(c+dx)^3 \left(a^2d^2-b^2c^2\right)^2}{d^4}-\frac{4(c+dx)^4 \left(b^4c^3-a^2b^2cd^2\right)}{d^4}-\frac{2(c+dx)^5 \left(a^2b^2d^2-3b^4c^2\right)}{d^4}+\frac{b^4(c+dx)^7}{d^4}-\frac{4b^4\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(c+dx)^6 \left(3b^2c^2-a^2d^2\right)}{3d^5}+\frac{(c+dx)^4 \left(b^2c^2-a^2d^2\right)^2}{4d^5}-\frac{4b^2c(c+dx)^5(bc-ad)(ad+bc)}{5d^5}+\frac{b^4(c+dx)^8}{8d^5}-\frac{4b^4c(c+dx)^7}{7d^5}$$

input
```
Int[(c + d*x)^3*(a^2 - b^2*x^2)^2,x]
```

output
```
((b^2*c^2 - a^2*d^2)^2*(c + d*x)^4)/(4*d^5) - (4*b^2*c*(b*c - a*d)*(b*c +
a*d)*(c + d*x)^5)/(5*d^5) + (b^2*(3*b^2*c^2 - a^2*d^2)*(c + d*x)^6)/(3*d^5
) - (4*b^4*c*(c + d*x)^7)/(7*d^5) + (b^4*(c + d*x)^8)/(8*d^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

method	result
norman	$\frac{b^4 d^3 x^8}{8} + \frac{3 b^4 c\, d^2 x^7}{7} + \left(-\frac{1}{3} a^2 b^2 d^3 + \frac{1}{2} b^4 c^2 d\right) x^6 + \left(-\frac{6}{5} a^2 b^2 c\, d^2 + \frac{1}{5} b^4 c^3\right) x^5 + \left(\frac{1}{4} a^4 d^3 - \frac{3}{2} a^2 b^2 c^2$
default	$\frac{b^4 d^3 x^8}{8} + \frac{3 b^4 c\, d^2 x^7}{7} + \frac{(-2 a^2 b^2 d^3 + 3 b^4 c^2 d) x^6}{6} + \frac{(-6 a^2 b^2 c\, d^2 + b^4 c^3) x^5}{5} + \frac{(a^4 d^3 - 6 a^2 b^2 c^2 d) x^4}{4} + \frac{(3 a^4 c\, d^2 - 2 a^2 b^2 c^3}{3}$
risch	$\frac{1}{8} b^4 d^3 x^8 + \frac{3}{7} b^4 c\, d^2 x^7 - \frac{1}{3} x^6 a^2 b^2 d^3 + \frac{1}{2} x^6 b^4 c^2 d - \frac{6}{5} x^5 a^2 b^2 c\, d^2 + \frac{1}{5} x^5 b^4 c^3 + \frac{1}{4} x^4 a^4 d^3 - \frac{3}{2} x^4 a^2 b$
parallelrisch	$\frac{1}{8} b^4 d^3 x^8 + \frac{3}{7} b^4 c\, d^2 x^7 - \frac{1}{3} x^6 a^2 b^2 d^3 + \frac{1}{2} x^6 b^4 c^2 d - \frac{6}{5} x^5 a^2 b^2 c\, d^2 + \frac{1}{5} x^5 b^4 c^3 + \frac{1}{4} x^4 a^4 d^3 - \frac{3}{2} x^4 a^2 b$
gosper	$\frac{x(105 b^4 d^3 x^7 + 360 b^4 c\, d^2 x^6 - 280 x^5 a^2 b^2 d^3 + 420 x^5 b^4 c^2 d - 1008 x^4 a^2 b^2 c\, d^2 + 168 x^4 b^4 c^3 + 210 x^3 a^4 d^3 - 1260 x^3 a^2 b^2 c^2 d + 840 x^2 a^4}{840}$
orering	$\frac{x(105 b^4 d^3 x^7 + 360 b^4 c\, d^2 x^6 - 280 x^5 a^2 b^2 d^3 + 420 x^5 b^4 c^2 d - 1008 x^4 a^2 b^2 c\, d^2 + 168 x^4 b^4 c^3 + 210 x^3 a^4 d^3 - 1260 x^3 a^2 b^2 c^2 d + 840 x^2 a^4}{840 (b x + a)^2 (-b x + a)^2}$

input
```
int((d*x+c)^3*(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^4*d^3*x^8+3/7*b^4*c*d^2*x^7+(-1/3*a^2*b^2*d^3+1/2*b^4*c^2*d)*x^6+(-6
/5*a^2*b^2*c*d^2+1/5*b^4*c^3)*x^5+(1/4*a^4*d^3-3/2*a^2*b^2*c^2*d)*x^4+(a^4
*c*d^2-2/3*a^2*b^2*c^3)*x^3+3/2*a^4*c^2*d*x^2+a^4*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{8} b^4 d^3 x^8 + \frac{3}{7} b^4 c d^2 x^7 + \frac{3}{2} a^4 c^2 d x^2 + a^4 c^3 x + \frac{1}{6} \left(3 b^4 c^2 d - 2 a^2 b^2 d^3\right) x^6 + \frac{1}{5} \left(b^4 c^3 - 6 a^2 b^2 c d^2\right) x^5 - \frac{1}{4} \left(6 a^2 b^2 c^2 d - a^4 d^3\right) x^4 - \frac{1}{3} \left(2 a^2 b^2 c^3 - 3 a^4 c d^2\right) x^3$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 3/2*a^4*c^2*d*x^2 + a^4*c^3*x + 1/6*
(3*b^4*c^2*d - 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 - 6*a^2*b^2*c*d^2)*x^5 -
1/4*(6*a^2*b^2*c^2*d - a^4*d^3)*x^4 - 1/3*(2*a^2*b^2*c^3 - 3*a^4*c*d^2)*x^
3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \, dx = a^4 c^3 x + \frac{3a^4c^2dx^2}{2} + \frac{3b^4cd^2x^7}{7} + \frac{b^4d^3x^8}{8} + x^6 \left(-\frac{a^2b^2d^3}{3} + \frac{b^4c^2d}{2}\right) + x^5 \left(-\frac{6a^2b^2cd^2}{5} + \frac{b^4c^3}{5}\right) + x^4 \left(\frac{a^4d^3}{4} - \frac{3a^2b^2c^2d}{2}\right) + x^3 \left(a^4cd^2 - \frac{2a^2b^2c^3}{3}\right)$$

input
```
integrate((d*x+c)**3*(-b**2*x**2+a**2)**2,x)
```

output
```
a**4*c**3*x + 3*a**4*c**2*d*x**2/2 + 3*b**4*c*d**2*x**7/7 + b**4*d**3*x**8
/8 + x**6*(-a**2*b**2*d**3/3 + b**4*c**2*d/2) + x**5*(-6*a**2*b**2*c*d**2/
5 + b**4*c**3/5) + x**4*(a**4*d**3/4 - 3*a**2*b**2*c**2*d/2) + x**3*(a**4*
c*d**2 - 2*a**2*b**2*c**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{8} b^4 d^3 x^8 + \frac{3}{7} b^4 c d^2 x^7 + \frac{3}{2} a^4 c^2 d x^2 + a^4 c^3 x + \frac{1}{6} \left(3 b^4 c^2 d - 2 a^2 b^2 d^3\right) x^6 + \frac{1}{5} \left(b^4 c^3 - 6 a^2 b^2 c d^2\right) x^5 - \frac{1}{4} \left(6 a^2 b^2 c^2 d - a^4 d^3\right) x^4 - \frac{1}{3} \left(2 a^2 b^2 c^3 - 3 a^4 c d^2\right) x^3$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 3/2*a^4*c^2*d*x^2 + a^4*c^3*x + 1/6*
(3*b^4*c^2*d - 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 - 6*a^2*b^2*c*d^2)*x^5 -
1/4*(6*a^2*b^2*c^2*d - a^4*d^3)*x^4 - 1/3*(2*a^2*b^2*c^3 - 3*a^4*c*d^2)*x^
3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{8} b^4 d^3 x^8 + \frac{3}{7} b^4 c d^2 x^7 + \frac{1}{2} b^4 c^2 d x^6 - \frac{1}{3} a^2 b^2 d^3 x^6 + \frac{1}{5} b^4 c^3 x^5 - \frac{6}{5} a^2 b^2 c d^2 x^5 - \frac{3}{2} a^2 b^2 c^2 d x^4 + \frac{1}{4} a^4 d^3 x^4 - \frac{2}{3} a^2 b^2 c^3 x^3 + a^4 c d^2 x^3 + \frac{3}{2} a^4 c^2 d x^2 + a^4 c^3 x$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 1/2*b^4*c^2*d*x^6 - 1/3*a^2*b^2*d^3*
x^6 + 1/5*b^4*c^3*x^5 - 6/5*a^2*b^2*c*d^2*x^5 - 3/2*a^2*b^2*c^2*d*x^4 + 1/
4*a^4*d^3*x^4 - 2/3*a^2*b^2*c^3*x^3 + a^4*c*d^2*x^3 + 3/2*a^4*c^2*d*x^2 +
a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \,dx = x^3\left(a^4\,c\,d^2-\frac{2\,a^2\,b^2\,c^3}{3}\right)+x^4\left(\frac{a^4\,d^3}{4}-\frac{3\,a^2\,b^2\,c^2\,d}{2}\right)+x^5\left(\frac{b^4\,c^3}{5}-\frac{6\,a^2\,b^2\,c\,d^2}{5}\right)+x^6\left(\frac{b^4\,c^2\,d}{2}-\frac{a^2\,b^2\,d^3}{3}\right)+a^4\,c^3\,x+\frac{b^4\,d^3\,x^8}{8}+\frac{3\,a^4\,c^2\,d\,x^2}{2}+\frac{3\,b^4\,c\,d^2\,x^7}{7}$$

input
```
int((a^2 - b^2*x^2)^2*(c + d*x)^3,x)
```

output
```
x^3*(a^4*c*d^2 - (2*a^2*b^2*c^3)/3) + x^4*((a^4*d^3)/4 - (3*a^2*b^2*c^2*d)
/2) + x^5*((b^4*c^3)/5 - (6*a^2*b^2*c*d^2)/5) + x^6*((b^4*c^2*d)/2 - (a^2*
b^2*d^3)/3) + a^4*c^3*x + (b^4*d^3*x^8)/8 + (3*a^4*c^2*d*x^2)/2 + (3*b^4*c
*d^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^2 \,dx = \frac{x(105b^4d^3x^7+360b^4c\,d^2x^6-280a^2b^2d^3x^5+420b^4c^2d\,x^5-1008a^2b^2c\,d^2x^4+168b^4c^3x^4+210a^4d^3x^3-1}{840}$$

input
```
int((d*x+c)^3*(-b^2*x^2+a^2)^2,x)
```

output
```
(x*(840*a**4*c**3 + 1260*a**4*c**2*d*x + 840*a**4*c*d**2*x**2 + 210*a**4*d
**3*x**3 - 560*a**2*b**2*c**3*x**2 - 1260*a**2*b**2*c**2*d*x**3 - 1008*a**
2*b**2*c*d**2*x**4 - 280*a**2*b**2*d**3*x**5 + 168*b**4*c**3*x**4 + 420*b*
*4*c**2*d*x**5 + 360*b**4*c*d**2*x**6 + 105*b**4*d**3*x**7))/840
```

3.11 $\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = a^4c^2x-\frac{1}{3}a^2\left(2b^2c^2-a^2d^2\right)x^3 + \frac{1}{5}b^2\left(b^2c^2-2a^2d^2\right)x^5+\frac{1}{7}b^4d^2x^7-\frac{cd(a^2-b^2x^2)^3}{3b^2}$$

output
```
a^4*c^2*x-1/3*a^2*(-a^2*d^2+2*b^2*c^2)*x^3+1/5*b^2*(-2*a^2*d^2+b^2*c^2)*x^
5+1/7*b^4*d^2*x^7-1/3*c*d*(-b^2*x^2+a^2)^3/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = -\frac{1}{15}a^2b^2x^3\left(10c^2+15cdx+6d^2x^2\right) + \frac{1}{105}b^4x^5\left(21c^2+35cdx+15d^2x^2\right) + a^4\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)$$

input
```
Integrate[(c + d*x)^2*(a^2 - b^2*x^2)^2,x]
```

output
```
-1/15*(a^2*b^2*x^3*(10*c^2 + 15*c*d*x + 6*d^2*x^2)) + (b^4*x^5*(21*c^2 + 3
5*c*d*x + 15*d^2*x^2))/105 + a^4*(c^2*x + c*d*x^2 + (d^2*x^3)/3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)^2 (c+dx)^2 \, dx$$

$$\Big\downarrow \text{475}$$

$$\int \left(b^4d^2x^6+b^2\left(b^2c^2-2a^2d^2\right)x^4+a^2\left(a^2d^2-2b^2c^2\right)x^2+a^4c^2\right)dx-\frac{cd\left(a^2-b^2x^2\right)^3}{3b^2}$$

$$\Big\downarrow \text{2009}$$

$$a^4c^2x+\frac{1}{5}b^2x^5\left(b^2c^2-2a^2d^2\right)-\frac{1}{3}a^2x^3\left(2b^2c^2-a^2d^2\right)-\frac{cd\left(a^2-b^2x^2\right)^3}{3b^2}+\frac{1}{7}b^4d^2x^7$$

input
```
Int[(c + d*x)^2*(a^2 - b^2*x^2)^2,x]
```

output
```
a^4*c^2*x - (a^2*(2*b^2*c^2 - a^2*d^2)*x^3)/3 + (b^2*(b^2*c^2 - 2*a^2*d^2)
*x^5)/5 + (b^4*d^2*x^7)/7 - (c*d*(a^2 - b^2*x^2)^3)/(3*b^2)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

method	result
default	$\frac{b^4 d^2 x^7}{7}+\frac{b^4 c d\,x^6}{3}+\frac{\left(-2 b^2 d^2 a^2+b^4 c^2\right) x^5}{5}-a^2 b^2 c d\,x^4+\frac{\left(a^4 d^2-2 a^2 c^2 b^2\right) x^3}{3}+a^4 c d\,x^2+a^4 c^2 x$
norman	$\frac{b^4 d^2 x^7}{7}+\frac{b^4 c d\,x^6}{3}+\left(-\frac{2}{5} b^2 d^2 a^2+\frac{1}{5} b^4 c^2\right) x^5-a^2 b^2 c d\,x^4+\left(\frac{1}{3} a^4 d^2-\frac{2}{3} a^2 c^2 b^2\right) x^3+a^4 c d\,x^2+a$
risch	$\frac{1}{7} b^4 d^2 x^7+\frac{1}{3} b^4 c d\,x^6-\frac{2}{5} x^5 b^2 d^2 a^2+\frac{1}{5} x^5 b^4 c^2-a^2 b^2 c d\,x^4+\frac{1}{3} x^3 a^4 d^2-\frac{2}{3} x^3 a^2 c^2 b^2+a^4 c d\,x^2+$
parallelrisch	$\frac{1}{7} b^4 d^2 x^7+\frac{1}{3} b^4 c d\,x^6-\frac{2}{5} x^5 b^2 d^2 a^2+\frac{1}{5} x^5 b^4 c^2-a^2 b^2 c d\,x^4+\frac{1}{3} x^3 a^4 d^2-\frac{2}{3} x^3 a^2 c^2 b^2+a^4 c d\,x^2+$
gosper	$\frac{x \left(15 b^4 d^2 x^6+35 b^4 c d\,x^5-42 x^4 b^2 d^2 a^2+21 x^4 b^4 c^2-105 a^2 b^2 c d\,x^3+35 x^2 a^4 d^2-70 x^2 a^2 c^2 b^2+105 a^4 c d x+105 a^4 c^2\right)}{105}$
orering	$\frac{x \left(15 b^4 d^2 x^6+35 b^4 c d\,x^5-42 x^4 b^2 d^2 a^2+21 x^4 b^4 c^2-105 a^2 b^2 c d\,x^3+35 x^2 a^4 d^2-70 x^2 a^2 c^2 b^2+105 a^4 c d x+105 a^4 c^2\right) \left(-b^2 x^2+a^2\right)^2}{105 (b x+a)^2 (-b x+a)^2}$

input
```
int((d*x+c)^2*(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^4*d^2*x^7+1/3*b^4*c*d*x^6+1/5*(-2*a^2*b^2*d^2+b^4*c^2)*x^5-a^2*b^2*c
*d*x^4+1/3*(a^4*d^2-2*a^2*b^2*c^2)*x^3+a^4*c*d*x^2+a^4*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{7} b^4 d^2 x^7 + \frac{1}{3} b^4 c d x^6 - a^2 b^2 c d x^4 + a^4 c d x^2 + a^4 c^2 x + \frac{1}{5} \left(b^4 c^2 - 2 a^2 b^2 d^2\right) x^5 - \frac{1}{3} \left(2 a^2 b^2 c^2 - a^4 d^2\right) x^3$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 - a^2*b^2*c*d*x^4 + a^4*c*d*x^2 + a^4*c^
2*x + 1/5*(b^4*c^2 - 2*a^2*b^2*d^2)*x^5 - 1/3*(2*a^2*b^2*c^2 - a^4*d^2)*x^
3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = a^4 c^2 x + a^4 c d x^2 - a^2 b^2 c d x^4 + \frac{b^4 c d x^6}{3} + \frac{b^4 d^2 x^7}{7} + x^5 \left(-\frac{2a^2 b^2 d^2}{5} + \frac{b^4 c^2}{5}\right) + x^3 \left(\frac{a^4 d^2}{3} - \frac{2a^2 b^2 c^2}{3}\right)$$

input
```
integrate((d*x+c)**2*(-b**2*x**2+a**2)**2,x)
```

output
```
a**4*c**2*x + a**4*c*d*x**2 - a**2*b**2*c*d*x**4 + b**4*c*d*x**6/3 + b**4*
d**2*x**7/7 + x**5*(-2*a**2*b**2*d**2/5 + b**4*c**2/5) + x**3*(a**4*d**2/3
 - 2*a**2*b**2*c**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{7}\,b^4d^2x^7 + \frac{1}{3}\,b^4cdx^6 - a^2b^2cdx^4 + a^4cdx^2 + a^4c^2x + \frac{1}{5}\,\left(b^4c^2 - 2\,a^2b^2d^2\right)x^5 - \frac{1}{3}\,\left(2\,a^2b^2c^2 - a^4d^2\right)x^3$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 - a^2*b^2*c*d*x^4 + a^4*c*d*x^2 + a^4*c^
2*x + 1/5*(b^4*c^2 - 2*a^2*b^2*d^2)*x^5 - 1/3*(2*a^2*b^2*c^2 - a^4*d^2)*x^
3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{7}\,b^4d^2x^7 + \frac{1}{3}\,b^4cdx^6 + \frac{1}{5}\,b^4c^2x^5 - \frac{2}{5}\,a^2b^2d^2x^5 - a^2b^2cdx^4 - \frac{2}{3}\,a^2b^2c^2x^3 + \frac{1}{3}\,a^4d^2x^3 + a^4cdx^2 + a^4c^2x$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 1/5*b^4*c^2*x^5 - 2/5*a^2*b^2*d^2*x^5
- a^2*b^2*c*d*x^4 - 2/3*a^2*b^2*c^2*x^3 + 1/3*a^4*d^2*x^3 + a^4*c*d*x^2 +
a^4*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = x^3 \left(\frac{a^4 d^2}{3} - \frac{2 a^2 b^2 c^2}{3}\right) + x^5 \left(\frac{b^4 c^2}{5} - \frac{2 a^2 b^2 d^2}{5}\right) + a^4 c^2 x + \frac{b^4 d^2 x^7}{7} + a^4 c d x^2 + \frac{b^4 c d x^6}{3} - a^2 b^2 c d x^4$$

input
```
int((a^2 - b^2*x^2)^2*(c + d*x)^2,x)
```

output
```
x^3*((a^4*d^2)/3 - (2*a^2*b^2*c^2)/3) + x^5*((b^4*c^2)/5 - (2*a^2*b^2*d^2)
/5) + a^4*c^2*x + (b^4*d^2*x^7)/7 + a^4*c*d*x^2 + (b^4*c*d*x^6)/3 - a^2*b^
2*c*d*x^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^2 \, dx = \frac{x(15b^4d^2x^6 + 35b^4cd\,x^5 - 42a^2b^2d^2x^4 + 21b^4c^2x^4 - 105a^2b^2cd\,x^3 + 35a^4d^2x^2 - 70a^2b^2c^2x^2 + 105a^4cdx +}{105}$$

input
```
int((d*x+c)^2*(-b^2*x^2+a^2)^2,x)
```

output
```
(x*(105*a**4*c**2 + 105*a**4*c*d*x + 35*a**4*d**2*x**2 - 70*a**2*b**2*c**2
*x**2 - 105*a**2*b**2*c*d*x**3 - 42*a**2*b**2*d**2*x**4 + 21*b**4*c**2*x**
4 + 35*b**4*c*d*x**5 + 15*b**4*d**2*x**6))/105
```

3.12 $\int (c+dx)\left(a^2-b^2x^2\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int (c+dx)\left(a^2-b^2x^2\right)^2 \, dx = a^4cx-\frac{2}{3}a^2b^2cx^3+\frac{1}{5}b^4cx^5-\frac{d(a^2-b^2x^2)^3}{6b^2}$$

output
```
a^4*c*x-2/3*a^2*b^2*c*x^3+1/5*b^4*c*x^5-1/6*d*(-b^2*x^2+a^2)^3/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int (c+dx)\left(a^2-b^2x^2\right)^2 \, dx = a^4cx+\frac{1}{2}a^4dx^2-\frac{2}{3}a^2b^2cx^3-\frac{1}{2}a^2b^2dx^4+\frac{1}{5}b^4cx^5+\frac{1}{6}b^4dx^6$$

input
```
Integrate[(c + d*x)*(a^2 - b^2*x^2)^2,x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 - (2*a^2*b^2*c*x^3)/3 - (a^2*b^2*d*x^4)/2 + (b^4*c
*x^5)/5 + (b^4*d*x^6)/6
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)^2 (c+dx)\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(a^2-b^2x^2\right)^2dx-\frac{d\left(a^2-b^2x^2\right)^3}{6b^2}$$

$$\Big\downarrow \text{210}$$

$$c\int \left(a^4-2b^2x^2a^2+b^4x^4\right)dx-\frac{d\left(a^2-b^2x^2\right)^3}{6b^2}$$

$$\Big\downarrow \text{2009}$$

$$c\left(a^4x-\frac{2}{3}a^2b^2x^3+\frac{b^4x^5}{5}\right)-\frac{d\left(a^2-b^2x^2\right)^3}{6b^2}$$

input
```
Int[(c + d*x)*(a^2 - b^2*x^2)^2,x]
```

output
```
-1/6*(d*(a^2 - b^2*x^2)^3)/b^2 + c*(a^4*x - (2*a^2*b^2*x^3)/3 + (b^4*x^5)/
5)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{1}{6}b^4d\,x^6+\frac{1}{5}b^4c\,x^5-\frac{1}{2}a^2b^2d\,x^4-\frac{2}{3}a^2b^2c\,x^3+\frac{1}{2}a^4d\,x^2+a^4cx$	59
norman	$\frac{1}{6}b^4d\,x^6+\frac{1}{5}b^4c\,x^5-\frac{1}{2}a^2b^2d\,x^4-\frac{2}{3}a^2b^2c\,x^3+\frac{1}{2}a^4d\,x^2+a^4cx$	59
risch	$\frac{1}{6}b^4d\,x^6+\frac{1}{5}b^4c\,x^5-\frac{1}{2}a^2b^2d\,x^4-\frac{2}{3}a^2b^2c\,x^3+\frac{1}{2}a^4d\,x^2+a^4cx$	59
parallelrisch	$\frac{1}{6}b^4d\,x^6+\frac{1}{5}b^4c\,x^5-\frac{1}{2}a^2b^2d\,x^4-\frac{2}{3}a^2b^2c\,x^3+\frac{1}{2}a^4d\,x^2+a^4cx$	59
gosper	$\frac{x\left(5b^4d\,x^5+6x^4b^4c-15a^2b^2d\,x^3-20a^2b^2c\,x^2+15a^4dx+30c\,a^4\right)}{30}$	60
orering	$\frac{x\left(5b^4d\,x^5+6x^4b^4c-15a^2b^2d\,x^3-20a^2b^2c\,x^2+15a^4dx+30c\,a^4\right)\left(-b^2x^2+a^2\right)^2}{30(bx+a)^2(-bx+a)^2}$	89

input
```
int((d*x+c)*(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^4*d*x^6+1/5*b^4*c*x^5-1/2*a^2*b^2*d*x^4-2/3*a^2*b^2*c*x^3+1/2*a^4*d*
x^2+a^4*c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int (c+dx)\left(a^2-b^2x^2\right)^2\,dx=\frac{1}{6}\,b^4dx^6+\frac{1}{5}\,b^4cx^5-\frac{1}{2}\,a^2b^2dx^4-\frac{2}{3}\,a^2b^2cx^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 - 1/2*a^2*b^2*d*x^4 - 2/3*a^2*b^2*c*x^3 + 1/
2*a^4*d*x^2 + a^4*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int (c+dx)\left(a^2-b^2x^2\right)^2\,dx=a^4cx+\frac{a^4dx^2}{2}-\frac{2a^2b^2cx^3}{3}-\frac{a^2b^2dx^4}{2}+\frac{b^4cx^5}{5}+\frac{b^4dx^6}{6}$$

input
```
integrate((d*x+c)*(-b**2*x**2+a**2)**2,x)
```

output
```
a**4*c*x + a**4*d*x**2/2 - 2*a**2*b**2*c*x**3/3 - a**2*b**2*d*x**4/2 + b**
4*c*x**5/5 + b**4*d*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int (c+dx)\left(a^2-b^2x^2\right)^2\,dx=\frac{1}{6}\,b^4dx^6+\frac{1}{5}\,b^4cx^5-\frac{1}{2}\,a^2b^2dx^4-\frac{2}{3}\,a^2b^2cx^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 - 1/2*a^2*b^2*d*x^4 - 2/3*a^2*b^2*c*x^3 + 1/
2*a^4*d*x^2 + a^4*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int (c+dx) \left(a^2-b^2x^2\right)^2 \, dx = \frac{1}{6} b^4dx^6 + \frac{1}{5} b^4cx^5 - \frac{1}{2} a^2b^2dx^4 - \frac{2}{3} a^2b^2cx^3 + \frac{1}{2} a^4dx^2 + a^4cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 - 1/2*a^2*b^2*d*x^4 - 2/3*a^2*b^2*c*x^3 + 1/
2*a^4*d*x^2 + a^4*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int (c+dx) \left(a^2-b^2x^2\right)^2 \, dx = \frac{d\,a^4\,x^2}{2} + c\,a^4\,x - \frac{d\,a^2\,b^2\,x^4}{2} - \frac{2\,c\,a^2\,b^2\,x^3}{3} + \frac{d\,b^4\,x^6}{6} + \frac{c\,b^4\,x^5}{5}$$

input
```
int((a^2 - b^2*x^2)^2*(c + d*x),x)
```

output
```
(a^4*d*x^2)/2 + (b^4*c*x^5)/5 + (b^4*d*x^6)/6 + a^4*c*x - (2*a^2*b^2*c*x^3
)/3 - (a^2*b^2*d*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int (c+dx) \left(a^2-b^2x^2\right)^2 \, dx = \frac{x(5b^4d\,x^5 + 6b^4c\,x^4 - 15a^2b^2d\,x^3 - 20a^2b^2c\,x^2 + 15a^4dx + 30a^4c)}{30}$$

input
```
int((d*x+c)*(-b^2*x^2+a^2)^2,x)
```

output

```
(x*(30*a**4*c + 15*a**4*d*x - 20*a**2*b**2*c*x**2 - 15*a**2*b**2*d*x**3 +
6*b**4*c*x**4 + 5*b**4*d*x**5))/30
```

3.13 $\int \left(a^2-b^2x^2\right)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \left(a^2-b^2x^2\right)^2 \, dx = a^4x-\frac{2}{3}a^2b^2x^3+\frac{b^4x^5}{5}$$

output
```
a^4*x-2/3*a^2*b^2*x^3+1/5*b^4*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \left(a^2-b^2x^2\right)^2 \, dx = a^4x-\frac{2}{3}a^2b^2x^3+\frac{b^4x^5}{5}$$

input
```
Integrate[(a^2 - b^2*x^2)^2,x]
```

output
```
a^4*x - (2*a^2*b^2*x^3)/3 + (b^4*x^5)/5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2 - b^2x^2)^2 \, dx$$

$$\Big\downarrow \text{ 210}$$

$$\int (a^4 - 2a^2b^2x^2 + b^4x^4) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^4x - \frac{2}{3}a^2b^2x^3 + \frac{b^4x^5}{5}$$

input
```
Int[(a^2 - b^2*x^2)^2,x]
```

output
```
a^4*x - (2*a^2*b^2*x^3)/3 + (b^4*x^5)/5
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$a^4x-\frac{2}{3}a^2b^2x^3+\frac{1}{5}b^4x^5$	26
norman	$a^4x-\frac{2}{3}a^2b^2x^3+\frac{1}{5}b^4x^5$	26
risch	$a^4x-\frac{2}{3}a^2b^2x^3+\frac{1}{5}b^4x^5$	26
parallelrisch	$a^4x-\frac{2}{3}a^2b^2x^3+\frac{1}{5}b^4x^5$	26
gosper	$\frac{x(3b^4x^4-10a^2b^2x^2+15a^4)}{15}$	29
orering	$\frac{x(3b^4x^4-10a^2b^2x^2+15a^4)(-b^2x^2+a^2)^2}{15(bx+a)^2(-bx+a)^2}$	58

input
```
int((-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^4*x-2/3*a^2*b^2*x^3+1/5*b^4*x^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \left(a^2-b^2x^2\right)^2\,dx=\frac{1}{5}\,b^4x^5-\frac{2}{3}\,a^2b^2x^3+a^4x$$

input
```
integrate((-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/5*b^4*x^5 - 2/3*a^2*b^2*x^3 + a^4*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \left(a^2-b^2x^2\right)^2\,dx=a^4x-\frac{2a^2b^2x^3}{3}+\frac{b^4x^5}{5}$$

input
```
integrate((-b**2*x**2+a**2)**2,x)
```

output
```
a**4*x - 2*a**2*b**2*x**3/3 + b**4*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \left(a^2-b^2x^2\right)^2\,dx=\frac{1}{5}\,b^4x^5-\frac{2}{3}\,a^2b^2x^3+a^4x$$

input
```
integrate((-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^4*x^5 - 2/3*a^2*b^2*x^3 + a^4*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \left(a^2-b^2x^2\right)^2\,dx=\frac{1}{5}\,b^4x^5-\frac{2}{3}\,a^2b^2x^3+a^4x$$

input
```
integrate((-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/5*b^4*x^5 - 2/3*a^2*b^2*x^3 + a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \left(a^2-b^2x^2\right)^2\,dx=a^4\,x-\frac{2\,a^2\,b^2\,x^3}{3}+\frac{b^4\,x^5}{5}$$

input
```
int((a^2 - b^2*x^2)^2,x)
```

output
```
a^4*x + (b^4*x^5)/5 - (2*a^2*b^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \left(a^2-b^2x^2\right)^2\,dx=\frac{x(3b^4x^4-10a^2b^2x^2+15a^4)}{15}$$

input
```
int((-b^2*x^2+a^2)^2,x)
```

output
```
(x*(15*a**4 - 10*a**2*b**2*x**2 + 3*b**4*x**4))/15
```

3.14 $\int \frac{(a^2-b^2x^2)^2}{c+dx} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{(a^2-b^2x^2)^2}{c+dx} \, dx = -\frac{b^2c(b^2c^2-2a^2d^2)\,x}{d^4}+\frac{b^2(b^2c^2-2a^2d^2)\,x^2}{2d^3} - \frac{b^4cx^3}{3d^2}+\frac{b^4x^4}{4d}+\frac{(b^2c^2-a^2d^2)^2\log(c+dx)}{d^5}$$

output
```
-b^2*c*(-2*a^2*d^2+b^2*c^2)*x/d^4+1/2*b^2*(-2*a^2*d^2+b^2*c^2)*x^2/d^3-1/3
*b^4*c*x^3/d^2+1/4*b^4*x^4/d+(-a^2*d^2+b^2*c^2)^2*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{(a^2-b^2x^2)^2}{c+dx} \, dx = \frac{-12a^2b^2d^3x(-2c+dx)+b^4dx(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3)+12(b^2c^2-a^2d^2)^2\log(c+dx)}{12d^5}$$

input
```
Integrate[(a^2 - b^2*x^2)^2/(c + d*x),x]
```

output
```
(-12*a^2*b^2*d^3*x*(-2*c + d*x) + b^4*d*x*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x
^2 + 3*d^3*x^3) + 12*(b^2*c^2 - a^2*d^2)^2*Log[c + d*x])/(12*d^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2-b^2x^2\right)^2}{c+dx}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{\left(a^2d^2-b^2c^2\right)^2}{d^4(c+dx)}-\frac{b^2c\left(b^2c^2-2a^2d^2\right)}{d^4}+\frac{b^2x\left(b^2c^2-2a^2d^2\right)}{d^3}-\frac{b^4cx^2}{d^2}+\frac{b^4x^3}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(b^2c^2-a^2d^2\right)^2\log(c+dx)}{d^5}-\frac{b^2cx\left(b^2c^2-2a^2d^2\right)}{d^4}+\frac{b^2x^2\left(b^2c^2-2a^2d^2\right)}{2d^3}-\frac{b^4cx^3}{3d^2}+\frac{b^4x^4}{4d}$$

input
```
Int[(a^2 - b^2*x^2)^2/(c + d*x),x]
```

output
```
-((b^2*c*(b^2*c^2 - 2*a^2*d^2)*x)/d^4) + (b^2*(b^2*c^2 - 2*a^2*d^2)*x^2)/(
2*d^3) - (b^4*c*x^3)/(3*d^2) + (b^4*x^4)/(4*d) + ((b^2*c^2 - a^2*d^2)^2*Lo
g[c + d*x])/d^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04

method	result
default	$\frac{b^2\left(\frac{b^2x^4d^3}{4}-\frac{b^2c\,x^3d^2}{3}-\frac{(2a^2d^2-b^2c^2)x^2d}{2}+c(2a^2d^2-b^2c^2)x\right)}{d^4}+\frac{(a^4d^4-2a^2b^2c^2d^2+c^4b^4)\ln(dx+c)}{d^5}$
norman	$\frac{b^2c(2a^2d^2-b^2c^2)x}{d^4}+\frac{b^4x^4}{4d}-\frac{b^2(2a^2d^2-b^2c^2)x^2}{2d^3}-\frac{b^4c\,x^3}{3d^2}+\frac{(a^4d^4-2a^2b^2c^2d^2+c^4b^4)\ln(dx+c)}{d^5}$
risch	$\frac{b^4x^4}{4d}-\frac{b^4c\,x^3}{3d^2}-\frac{b^2a^2x^2}{d}+\frac{b^4c^2x^2}{2d^3}+\frac{2b^2a^2cx}{d^2}-\frac{b^4c^3x}{d^4}+\frac{\ln(dx+c)a^4}{d}-\frac{2\ln(dx+c)a^2b^2c^2}{d^3}+\frac{\ln(dx+c)c^4b^4}{d^5}$
parallelrisch	$\frac{3b^4x^4d^4-4b^4c\,x^3d^3-12x^2a^2b^2d^4+6x^2b^4c^2d^2+12\ln(dx+c)a^4d^4-24\ln(dx+c)a^2b^2c^2d^2+12\ln(dx+c)b^4c^4+24x\,a^2b^2c\,d^3-12}{12d^5}$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
b^2/d^4*(1/4*b^2*x^4*d^3-1/3*b^2*c*x^3*d^2-1/2*(2*a^2*d^2-b^2*c^2)*x^2*d+c
*(2*a^2*d^2-b^2*c^2)*x)+(a^4*d^4-2*a^2*b^2*c^2*d^2+b^4*c^4)/d^5*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \frac{(a^2-b^2x^2)^2}{c+dx}\,dx$$
$$=\frac{3\,b^4d^4x^4-4\,b^4cd^3x^3+6\,(b^4c^2d^2-2\,a^2b^2d^4)x^2-12\,(b^4c^3d-2\,a^2b^2cd^3)x+12\,(b^4c^4-2\,a^2b^2c^2d^2+a^4d^4)}{12\,d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*d^4*x^4 - 4*b^4*c*d^3*x^3 + 6*(b^4*c^2*d^2 - 2*a^2*b^2*d^4)*x^
2 - 12*(b^4*c^3*d - 2*a^2*b^2*c*d^3)*x + 12*(b^4*c^4 - 2*a^2*b^2*c^2*d^2 +
 a^4*d^4)*log(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

$$\int \frac{(a^2-b^2x^2)^2}{c+dx}\,dx = -\frac{b^4cx^3}{3d^2}+\frac{b^4x^4}{4d}+x^2\left(-\frac{a^2b^2}{d}+\frac{b^4c^2}{2d^3}\right)+x\left(\frac{2a^2b^2c}{d^2}-\frac{b^4c^3}{d^4}\right)+\frac{(ad-bc)^2(ad+bc)^2\log(c+dx)}{d^5}$$

input
```
integrate((-b**2*x**2+a**2)**2/(d*x+c),x)
```

output
```
-b**4*c*x**3/(3*d**2) + b**4*x**4/(4*d) + x**2*(-a**2*b**2/d + b**4*c**2/(
2*d**3)) + x*(2*a**2*b**2*c/d**2 - b**4*c**3/d**4) + (a*d - b*c)**2*(a*d +
 b*c)**2*log(c + d*x)/d**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \frac{(a^2-b^2x^2)^2}{c+dx}\,dx = \frac{3\,b^4d^3x^4-4\,b^4cd^2x^3+6\,(b^4c^2d-2\,a^2b^2d^3)x^2-12\,(b^4c^3-2\,a^2b^2cd^2)x}{12\,d^4}+\frac{(b^4c^4-2\,a^2b^2c^2d^2+a^4d^4)\log(dx+c)}{d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - 2*a^2*b^2*d^3)*x^2
- 12*(b^4*c^3 - 2*a^2*b^2*c*d^2)*x)/d^4 + (b^4*c^4 - 2*a^2*b^2*c^2*d^2 + a
^4*d^4)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{(a^2-b^2x^2)^2}{c+dx}\,dx = \frac{3\,b^4d^3x^4-4\,b^4cd^2x^3+6\,b^4c^2dx^2-12\,a^2b^2d^3x^2-12\,b^4c^3x+24\,a^2b^2cd^2x}{12\,d^4} + \frac{(b^4c^4-2\,a^2b^2c^2d^2+a^4d^4)\log(|dx+c|)}{d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 - 12*a^2*b^2*d^3*x
^2 - 12*b^4*c^3*x + 24*a^2*b^2*c*d^2*x)/d^4 + (b^4*c^4 - 2*a^2*b^2*c^2*d^2
 + a^4*d^4)*log(abs(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{(a^2-b^2x^2)^2}{c+dx}\,dx = \frac{\ln(c+d\,x)\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}{d^5} - x^2\left(\frac{a^2\,b^2}{d}-\frac{b^4\,c^2}{2\,d^3}\right)+\frac{b^4\,x^4}{4\,d}-\frac{b^4\,c\,x^3}{3\,d^2}+\frac{c\,x\left(\frac{2\,a^2\,b^2}{d}-\frac{b^4\,c^2}{d^3}\right)}{d}$$

input
```
int((a^2 - b^2*x^2)^2/(c + d*x),x)
```

output
```
(log(c + d*x)*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2))/d^5 - x^2*((a^2*b^2
)/d - (b^4*c^2)/(2*d^3)) + (b^4*x^4)/(4*d) - (b^4*c*x^3)/(3*d^2) + (c*x*((
2*a^2*b^2)/d - (b^4*c^2)/d^3))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{(a^2 - b^2x^2)^2}{c + dx}\,dx$$
$$= \frac{12\log(dx+c)\,a^4d^4 - 24\log(dx+c)\,a^2b^2c^2d^2 + 12\log(dx+c)\,b^4c^4 + 24a^2b^2c\,d^3x - 12a^2b^2d^4x^2 - 12b^4c^3}{12d^5}$$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c),x)
```

output
```
(12*log(c + d*x)*a**4*d**4 - 24*log(c + d*x)*a**2*b**2*c**2*d**2 + 12*log(
c + d*x)*b**4*c**4 + 24*a**2*b**2*c*d**3*x - 12*a**2*b**2*d**4*x**2 - 12*b
**4*c**3*d*x + 6*b**4*c**2*d**2*x**2 - 4*b**4*c*d**3*x**3 + 3*b**4*d**4*x*
*4)/(12*d**5)
```

3.15 $\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx = \frac{b^2(3b^2c^2-2a^2d^2)\,x}{d^4}-\frac{b^4cx^2}{d^3}+\frac{b^4x^3}{3d^2}-\frac{(b^2c^2-a^2d^2)^2}{d^5(c+dx)} - \frac{4b^2c(bc-ad)(bc+ad)\log(c+dx)}{d^5}$$

output
```
b^2*(-2*a^2*d^2+3*b^2*c^2)*x/d^4-b^4*c*x^2/d^3+1/3*b^4*x^3/d^2-(-a^2*d^2+b
^2*c^2)^2/d^5/(d*x+c)-4*b^2*c*(-a*d+b*c)*(a*d+b*c)*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx = \frac{9b^4c^2dx-6a^2b^2d^3x-3b^4cd^2x^2+b^4d^3x^3-\frac{3(b^2c^2-a^2d^2)^2}{c+dx}-12(b^4c^3-a^2b^2cd^2)\log(c+dx)}{3d^5}$$

input
```
Integrate[(a^2 - b^2*x^2)^2/(c + d*x)^2,x]
```

output
```
(9*b^4*c^2*d*x - 6*a^2*b^2*d^3*x - 3*b^4*c*d^2*x^2 + b^4*d^3*x^3 - (3*(b^2
*c^2 - a^2*d^2)^2)/(c + d*x) - 12*(b^4*c^3 - a^2*b^2*c*d^2)*Log[c + d*x])/
(3*d^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2-b^2x^2\right)^2}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{\left(a^2d^2-b^2c^2\right)^2}{d^4(c+dx)^2}+\frac{4\left(a^2b^2cd^2-b^4c^3\right)}{d^4(c+dx)}+\frac{3b^4c^2-2a^2b^2d^2}{d^4}-\frac{2b^4cx}{d^3}+\frac{b^4x^2}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(b^2c^2-a^2d^2\right)^2}{d^5(c+dx)}+\frac{b^2x\left(3b^2c^2-2a^2d^2\right)}{d^4}-\frac{4b^2c(bc-ad)(ad+bc)\log(c+dx)}{d^5}-\frac{b^4cx^2}{d^3}+\frac{b^4x^3}{3d^2}$$

input
```
Int[(a^2 - b^2*x^2)^2/(c + d*x)^2,x]
```

output
```
(b^2*(3*b^2*c^2 - 2*a^2*d^2)*x)/d^4 - (b^4*c*x^2)/d^3 + (b^4*x^3)/(3*d^2)
- (b^2*c^2 - a^2*d^2)^2/(d^5*(c + d*x)) - (4*b^2*c*(b*c - a*d)*(b*c + a*d)
*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09

method	result
default	$-\frac{b^{2}\left(-\frac{1}{3}x^{3}b^{2}d^{2}+b^{2}c\,x^{2}d+2a^{2}d^{2}x-3x\,b^{2}c^{2}\right)}{d^{4}}-\frac{a^{4}d^{4}-2a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}}{d^{5}\left(dx+c\right)}+\frac{4b^{2}c\left(a^{2}d^{2}-b^{2}c^{2}\right)\ln\left(dx+c\right)}{d^{5}}$
norman	$\frac{-\frac{a^{4}d^{4}-4a^{2}b^{2}c^{2}d^{2}+4c^{4}b^{4}}{d^{5}}+\frac{b^{4}x^{4}}{3d}-\frac{2b^{2}\left(a^{2}d^{2}-b^{2}c^{2}\right)x^{2}}{d^{3}}-\frac{2b^{4}c\,x^{3}}{3d^{2}}}{dx+c}+\frac{4b^{2}c\left(a^{2}d^{2}-b^{2}c^{2}\right)\ln\left(dx+c\right)}{d^{5}}$
risch	$\frac{b^{4}x^{3}}{3d^{2}}-\frac{b^{4}c\,x^{2}}{d^{3}}-\frac{2b^{2}a^{2}x}{d^{2}}+\frac{3b^{4}x\,c^{2}}{d^{4}}-\frac{a^{4}}{d\left(dx+c\right)}+\frac{2a^{2}b^{2}c^{2}}{d^{3}\left(dx+c\right)}-\frac{c^{4}b^{4}}{d^{5}\left(dx+c\right)}+\frac{4b^{2}c\ln\left(dx+c\right)a^{2}}{d^{3}}-\frac{4b^{4}c^{3}\ln\left(dx+c\right)}{d^{5}}$
parallelrisch	$\frac{b^{4}x^{4}d^{4}-2b^{4}c\,x^{3}d^{3}+12\ln\left(dx+c\right)x\,a^{2}b^{2}c\,d^{3}-12\ln\left(dx+c\right)x\,b^{4}c^{3}d-6x^{2}a^{2}b^{2}d^{4}+6x^{2}b^{4}c^{2}d^{2}+12\ln\left(dx+c\right)a^{2}b^{2}c^{2}d^{2}-12\ln(dx+c}{3d^{5}\left(dx+c\right)}$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2/d^4*(-1/3*x^3*b^2*d^2+b^2*c*x^2*d+2*a^2*d^2*x-3*x*b^2*c^2)-(a^4*d^4-2
*a^2*b^2*c^2*d^2+b^4*c^4)/d^5/(d*x+c)+4*b^2*c/d^5*(a^2*d^2-b^2*c^2)*ln(d*x
+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.56

$$\int \frac{\left(a^2-b^2x^2\right)^2}{(c+dx)^2}\,dx$$
$$=\frac{b^4d^4x^4-2\,b^4cd^3x^3-3\,b^4c^4+6\,a^2b^2c^2d^2-3\,a^4d^4+6\,(b^4c^2d^2-a^2b^2d^4)x^2+3\,(3\,b^4c^3d-2\,a^2b^2cd^3)x-12}{3\,(d^6x+cd^5)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*d^4*x^4 - 2*b^4*c*d^3*x^3 - 3*b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 3*a^4
*d^4 + 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 2*a^2*b^2*c*d^
3)*x - 12*(b^4*c^4 - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a^2*b^2*c*d^3)*x)*log(
d*x + c))/(d^6*x + c*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx = -\frac{b^4cx^2}{d^3} + \frac{b^4x^3}{3d^2} + \frac{4b^2c(ad-bc)\,(ad+bc)\log{(c+dx)}}{d^5} + x\left(-\frac{2a^2b^2}{d^2} + \frac{3b^4c^2}{d^4}\right) + \frac{-a^4d^4 + 2a^2b^2c^2d^2 - b^4c^4}{cd^5 + d^6x}$$

input
```
integrate((-b**2*x**2+a**2)**2/(d*x+c)**2,x)
```

output
```
-b**4*c*x**2/d**3 + b**4*x**3/(3*d**2) + 4*b**2*c*(a*d - b*c)*(a*d + b*c)*
log(c + d*x)/d**5 + x*(-2*a**2*b**2/d**2 + 3*b**4*c**2/d**4) + (-a**4*d**4
 + 2*a**2*b**2*c**2*d**2 - b**4*c**4)/(c*d**5 + d**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx = -\frac{b^4c^4 - 2\,a^2b^2c^2d^2 + a^4d^4}{d^6x + cd^5} + \frac{b^4d^2x^3 - 3\,b^4cdx^2 + 3\,(3\,b^4c^2 - 2\,a^2b^2d^2)x}{3\,d^4} - \frac{4\,(b^4c^3 - a^2b^2cd^2)\log{(dx+c)}}{d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(b^4*c^4 - 2*a^2*b^2*c^2*d^2 + a^4*d^4)/(d^6*x + c*d^5) + 1/3*(b^4*d^2*x^
3 - 3*b^4*c*d*x^2 + 3*(3*b^4*c^2 - 2*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - a^
2*b^2*c*d^2)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.50

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx = \frac{\left(b^4-\frac{6b^4c}{dx+c}+\frac{6(3b^4c^2d^2-a^2b^2d^4)}{(dx+c)^2d^2}\right)(dx+c)^3}{3d^5} + \frac{4(b^4c^3-a^2b^2cd^2)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5} - \frac{\frac{b^4c^4d^3}{dx+c}-\frac{2a^2b^2c^2d^5}{dx+c}+\frac{a^4d^7}{dx+c}}{d^8}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/3*(b^4 - 6*b^4*c/(d*x + c) + 6*(3*b^4*c^2*d^2 - a^2*b^2*d^4)/((d*x + c)^
2*d^2))*(d*x + c)^3/d^5 + 4*(b^4*c^3 - a^2*b^2*c*d^2)*log(abs(d*x + c)/((d
*x + c)^2*abs(d)))/d^5 - (b^4*c^4*d^3/(d*x + c) - 2*a^2*b^2*c^2*d^5/(d*x +
 c) + a^4*d^7/(d*x + c))/d^8
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx = \frac{b^4x^3}{3d^2} - \frac{a^4d^4-2a^2b^2c^2d^2+b^4c^4}{d(xd^5+cd^4)} - x\left(\frac{2a^2b^2}{d^2}-\frac{3b^4c^2}{d^4}\right) - \frac{\ln(c+dx)(4b^4c^3-4a^2b^2cd^2)}{d^5} - \frac{b^4cx^2}{d^3}$$

input
```
int((a^2 - b^2*x^2)^2/(c + d*x)^2,x)
```

output
```
(b^4*x^3)/(3*d^2) - (a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)/(d*(c*d^4 + d^
5*x)) - x*((2*a^2*b^2)/d^2 - (3*b^4*c^2)/d^4) - (log(c + d*x)*(4*b^4*c^3 -
 4*a^2*b^2*c*d^2))/d^5 - (b^4*c*x^2)/d^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.58

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^2}\,dx$$
$$= \frac{12\log(dx+c)\,a^2b^2c^3d^2+12\log(dx+c)\,a^2b^2c^2d^3x-12\log(dx+c)\,b^4c^5-12\log(dx+c)\,b^4c^4dx+3a^4d^5}{3c\,d^5\,(dx+c)}$$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^2,x)
```

output
```
(12*log(c + d*x)*a**2*b**2*c**3*d**2 + 12*log(c + d*x)*a**2*b**2*c**2*d**3
*x - 12*log(c + d*x)*b**4*c**5 - 12*log(c + d*x)*b**4*c**4*d*x + 3*a**4*d*
*5*x - 12*a**2*b**2*c**2*d**3*x - 6*a**2*b**2*c*d**4*x**2 + 12*b**4*c**4*d
*x + 6*b**4*c**3*d**2*x**2 - 2*b**4*c**2*d**3*x**3 + b**4*c*d**4*x**4)/(3*
c*d**5*(c + d*x))
```

3.16 $\int \frac{(a^2-b^2x^2)^2}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^3}\,dx = -\frac{3b^4cx}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(b^2c^2-a^2d^2)^2}{2d^5(c+dx)^2} + \frac{4b^2c(bc-ad)(bc+ad)}{d^5(c+dx)}+\frac{2b^2(3b^2c^2-a^2d^2)\log(c+dx)}{d^5}$$

output
```
-3*b^4*c*x/d^4+1/2*b^4*x^2/d^3-1/2*(-a^2*d^2+b^2*c^2)^2/d^5/(d*x+c)^2+4*b^
2*c*(-a*d+b*c)*(a*d+b*c)/d^5/(d*x+c)+2*b^2*(-a^2*d^2+3*b^2*c^2)*ln(d*x+c)/
d^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^3}\,dx = \frac{-a^4d^4-2a^2b^2cd^2(3c+4dx)+b^4(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4)+4b^2(3b^2c^2-a^2d^2)(c+dx}{2d^5(c+dx)^2}$$

input
```
Integrate[(a^2 - b^2*x^2)^2/(c + d*x)^3,x]
```

output
```
(-(a^4*d^4) - 2*a^2*b^2*c*d^2*(3*c + 4*d*x) + b^4*(7*c^4 + 2*c^3*d*x - 11*
c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 4*b^2*(3*b^2*c^2 - a^2*d^2)*(c + d*
x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(a^2d^2-b^2c^2)^2}{d^4(c+dx)^3}+\frac{4(a^2b^2cd^2-b^4c^3)}{d^4(c+dx)^2}-\frac{2(a^2b^2d^2-3b^4c^2)}{d^4(c+dx)}-\frac{3b^4c}{d^4}+\frac{b^4x}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(b^2c^2-a^2d^2)^2}{2d^5(c+dx)^2}+\frac{2b^2(3b^2c^2-a^2d^2)\log(c+dx)}{d^5}+\frac{4b^2c(bc-ad)(ad+bc)}{d^5(c+dx)}-\frac{3b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

input
```
Int[(a^2 - b^2*x^2)^2/(c + d*x)^3,x]
```

output
```
(-3*b^4*c*x)/d^4 + (b^4*x^2)/(2*d^3) - (b^2*c^2 - a^2*d^2)^2/(2*d^5*(c + d
*x)^2) + (4*b^2*c*(b*c - a*d)*(b*c + a*d))/(d^5*(c + d*x)) + (2*b^2*(3*b^2
*c^2 - a^2*d^2)*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

method	result
default	$-\frac{b^4\left(-\frac{1}{2}d\,x^2+3cx\right)}{d^4}-\frac{4b^2c\left(a^2d^2-b^2c^2\right)}{d^5\left(dx+c\right)}-\frac{a^4d^4-2a^2b^2c^2d^2+c^4b^4}{2d^5\left(dx+c\right)^2}-\frac{2b^2\left(a^2d^2-3b^2c^2\right)\ln\left(dx+c\right)}{d^5}$
norman	$\frac{-\frac{a^4d^4+6a^2b^2c^2d^2-18c^4b^4}{2d^5}+\frac{b^4x^4}{2d}-\frac{2b^4c\,x^3}{d^2}-\frac{2c\left(2b^2d^2a^2-6b^4c^2\right)x}{d^4}}{\left(dx+c\right)^2}-\frac{2b^2\left(a^2d^2-3b^2c^2\right)\ln\left(dx+c\right)}{d^5}$
risch	$\frac{b^4x^2}{2d^3}-\frac{3b^4cx}{d^4}+\frac{\left(-4a^2b^2c\,d^2+4b^4c^3\right)x-\frac{a^4d^4+6a^2b^2c^2d^2-7c^4b^4}{2d}}{d^4\left(dx+c\right)^2}-\frac{2b^2\ln\left(dx+c\right)a^2}{d^3}+\frac{6b^4\ln\left(dx+c\right)c^2}{d^5}$
parallelrisch	$-\frac{-b^4x^4d^4+4\ln\left(dx+c\right)x^2a^2b^2d^4-12\ln\left(dx+c\right)x^2b^4c^2d^2+4b^4c\,x^3d^3+8\ln\left(dx+c\right)x\,a^2b^2c\,d^3-24\ln\left(dx+c\right)x\,b^4c^3d+4\ln\left(dx+c\right)}{2d^5\left(dx+c\right)^2}$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^4/d^4*(-1/2*d*x^2+3*c*x)-4*b^2*c/d^5*(a^2*d^2-b^2*c^2)/(d*x+c)-1/2*(a^4
*d^4-2*a^2*b^2*c^2*d^2+b^4*c^4)/d^5/(d*x+c)^2-2*b^2/d^5*(a^2*d^2-3*b^2*c^2
)*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.69

$$\int\frac{\left(a^2-b^2x^2\right)^2}{(c+dx)^3}\,dx$$
$$=\frac{b^4d^4x^4-4\,b^4cd^3x^3-11\,b^4c^2d^2x^2+7\,b^4c^4-6\,a^2b^2c^2d^2-a^4d^4+2\left(b^4c^3d-4\,a^2b^2cd^3\right)x+4\left(3\,b^4c^4-a^2b^2\right.}{2\left(d^7x^2+2\,cd^6x+c^2d^5\right)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(b^4*d^4*x^4 - 4*b^4*c*d^3*x^3 - 11*b^4*c^2*d^2*x^2 + 7*b^4*c^4 - 6*a^
2*b^2*c^2*d^2 - a^4*d^4 + 2*(b^4*c^3*d - 4*a^2*b^2*c*d^3)*x + 4*(3*b^4*c^4
 - a^2*b^2*c^2*d^2 + (3*b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 2*(3*b^4*c^3*d -
a^2*b^2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14

$$\int \frac{(a^2 - b^2x^2)^2}{(c+dx)^3}\,dx = -\frac{3b^4cx}{d^4} + \frac{b^4x^2}{2d^3} - \frac{2b^2(a^2d^2 - 3b^2c^2)\log(c+dx)}{d^5} + \frac{-a^4d^4 - 6a^2b^2c^2d^2 + 7b^4c^4 + x(-8a^2b^2cd^3 + 8b^4c^3d)}{2c^2d^5 + 4cd^6x + 2d^7x^2}$$

input
```
integrate((-b**2*x**2+a**2)**2/(d*x+c)**3,x)
```

output
```
-3*b**4*c*x/d**4 + b**4*x**2/(2*d**3) - 2*b**2*(a**2*d**2 - 3*b**2*c**2)*l
og(c + d*x)/d**5 + (-a**4*d**4 - 6*a**2*b**2*c**2*d**2 + 7*b**4*c**4 + x*(
-8*a**2*b**2*c*d**3 + 8*b**4*c**3*d))/(2*c**2*d**5 + 4*c*d**6*x + 2*d**7*x
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14

$$\int \frac{(a^2 - b^2x^2)^2}{(c+dx)^3}\,dx = \frac{7\,b^4c^4 - 6\,a^2b^2c^2d^2 - a^4d^4 + 8\,(b^4c^3d - a^2b^2cd^3)x}{2\,(d^7x^2 + 2\,cd^6x + c^2d^5)} + \frac{b^4dx^2 - 6\,b^4cx}{2\,d^4} + \frac{2\,(3\,b^4c^2 - a^2b^2d^2)\log(dx+c)}{d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*(7*b^4*c^4 - 6*a^2*b^2*c^2*d^2 - a^4*d^4 + 8*(b^4*c^3*d - a^2*b^2*c*d^
3)*x)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 6*b^4*c*x)/d^4 +
2*(3*b^4*c^2 - a^2*b^2*d^2)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^3}\,dx = \frac{2\,(3\,b^4c^2-a^2b^2d^2)\log(|dx+c|)}{d^5}+\frac{b^4d^3x^2-6\,b^4cd^2x}{2\,d^6}+\frac{7\,b^4c^4-6\,a^2b^2c^2d^2-a^4d^4+8\,(b^4c^3d-a^2b^2cd^3)x}{2\,(dx+c)^2d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
2*(3*b^4*c^2 - a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d^3*x^2 - 6*b
^4*c*d^2*x)/d^6 + 1/2*(7*b^4*c^4 - 6*a^2*b^2*c^2*d^2 - a^4*d^4 + 8*(b^4*c^
3*d - a^2*b^2*c*d^3)*x)/((d*x + c)^2*d^5)
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^3}\,dx = \frac{x\,(4\,b^4\,c^3-4\,a^2\,b^2\,c\,d^2)-\frac{a^4\,d^4+6\,a^2\,b^2\,c^2\,d^2-7\,b^4\,c^4}{2\,d}}{c^2\,d^4+2\,c\,d^5\,x+d^6\,x^2}+\frac{\ln(c+d\,x)\,(6\,b^4\,c^2-2\,a^2\,b^2\,d^2)}{d^5}+\frac{b^4\,x^2}{2\,d^3}-\frac{3\,b^4\,c\,x}{d^4}$$

input
```
int((a^2 - b^2*x^2)^2/(c + d*x)^3,x)
```

output
```
(x*(4*b^4*c^3 - 4*a^2*b^2*c*d^2) - (a^4*d^4 - 7*b^4*c^4 + 6*a^2*b^2*c^2*d^
2)/(2*d))/(c^2*d^4 + d^6*x^2 + 2*c*d^5*x) + (log(c + d*x)*(6*b^4*c^2 - 2*a
^2*b^2*d^2))/d^5 + (b^4*x^2)/(2*d^3) - (3*b^4*c*x)/d^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.81

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^3}\,dx$$
$$= \frac{-4\log(dx+c)\,a^2b^2c^2d^2 - 8\log(dx+c)\,a^2b^2c\,d^3x - 4\log(dx+c)\,a^2b^2d^4x^2 + 12\log(dx+c)\,b^4c^4 + 24\log}{2d^5\,(}$$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^3,x)
```

output
```
( - 4*log(c + d*x)*a**2*b**2*c**2*d**2 - 8*log(c + d*x)*a**2*b**2*c*d**3*x
 - 4*log(c + d*x)*a**2*b**2*d**4*x**2 + 12*log(c + d*x)*b**4*c**4 + 24*log
(c + d*x)*b**4*c**3*d*x + 12*log(c + d*x)*b**4*c**2*d**2*x**2 - a**4*d**4
- 2*a**2*b**2*c**2*d**2 + 4*a**2*b**2*d**4*x**2 + 6*b**4*c**4 - 12*b**4*c*
*2*d**2*x**2 - 4*b**4*c*d**3*x**3 + b**4*d**4*x**4)/(2*d**5*(c**2 + 2*c*d*
x + d**2*x**2))
```

3.17 $\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx = \frac{b^4x}{d^4} - \frac{(b^2c^2-a^2d^2)^2}{3d^5(c+dx)^3} + \frac{2b^2c(bc-ad)(bc+ad)}{d^5(c+dx)^2} - \frac{2b^2(3b^2c^2-a^2d^2)}{d^5(c+dx)} - \frac{4b^4c\log(c+dx)}{d^5}$$

output
```
b^4*x/d^4-1/3*(-a^2*d^2+b^2*c^2)^2/d^5/(d*x+c)^3+2*b^2*c*(-a*d+b*c)*(a*d+b
*c)/d^5/(d*x+c)^2-2*b^2*(-a^2*d^2+3*b^2*c^2)/d^5/(d*x+c)-4*b^4*c*ln(d*x+c)
/d^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx = \frac{-a^4d^4+2a^2b^2d^2(c^2+3cdx+3d^2x^2)+b^4(-13c^4-27c^3dx-9c^2d^2x^2+9cd^3x^3+3d^4x^4)-12b^4c(c+dx}{3d^5(c+dx)^3}$$

input
```
Integrate[(a^2 - b^2*x^2)^2/(c + d*x)^4,x]
```

output
```
(-(a^4*d^4) + 2*a^2*b^2*d^2*(c^2 + 3*c*d*x + 3*d^2*x^2) + b^4*(-13*c^4 - 2
7*c^3*d*x - 9*c^2*d^2*x^2 + 9*c*d^3*x^3 + 3*d^4*x^4) - 12*b^4*c*(c + d*x)^
3*Log[c + d*x])/(3*d^5*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 - b^2x^2)^2}{(c+dx)^4} \, dx$$

$$\Big\downarrow \text{ 476}$$

$$\int \left(\frac{(a^2d^2 - b^2c^2)^2}{d^4(c+dx)^4} + \frac{4(a^2b^2cd^2 - b^4c^3)}{d^4(c+dx)^3} - \frac{2(a^2b^2d^2 - 3b^4c^2)}{d^4(c+dx)^2} - \frac{4b^4c}{d^4(c+dx)} + \frac{b^4}{d^4}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b^2(3b^2c^2 - a^2d^2)}{d^5(c+dx)} - \frac{(b^2c^2 - a^2d^2)^2}{3d^5(c+dx)^3} + \frac{2b^2c(bc-ad)(ad+bc)}{d^5(c+dx)^2} - \frac{4b^4c\log(c+dx)}{d^5} + \frac{b^4x}{d^4}$$

input
```
Int[(a^2 - b^2*x^2)^2/(c + d*x)^4,x]
```

output
```
(b^4*x)/d^4 - (b^2*c^2 - a^2*d^2)^2/(3*d^5*(c + d*x)^3) + (2*b^2*c*(b*c -
a*d)*(b*c + a*d))/(d^5*(c + d*x)^2) - (2*b^2*(3*b^2*c^2 - a^2*d^2))/(d^5*(
c + d*x)) - (4*b^4*c*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result
risch	$\frac{b^4 x}{d^4}+\frac{(2a^2b^2d^3-6b^4c^2d)x^2+2b^2c(a^2d^2-5b^2c^2)x-\frac{a^4d^4-2a^2b^2c^2d^2+13c^4b^4}{3d}}{d^4(dx+c)^3}-\frac{4b^4c\ln(dx+c)}{d^5}$
norman	$\frac{\frac{b^4x^4}{d}+\frac{\left(2b^2d^2a^2-12b^4c^2\right)x^2}{d^3}+\frac{c\left(2b^2d^2a^2-18b^4c^2\right)x}{d^4}-\frac{a^4d^4-2a^2b^2c^2d^2+22c^4b^4}{3d^5}}{(dx+c)^3}-\frac{4b^4c\ln(dx+c)}{d^5}$
default	$\frac{b^4x}{d^4}+\frac{2b^2\left(a^2d^2-3b^2c^2\right)}{d^5(dx+c)}-\frac{2b^2c(a^2d^2-b^2c^2)}{d^5(dx+c)^2}-\frac{a^4d^4-2a^2b^2c^2d^2+c^4b^4}{3d^5(dx+c)^3}-\frac{4b^4c\ln(dx+c)}{d^5}$
parallelrisch	$-\frac{12\ln(dx+c)x^3b^4c\,d^3-3b^4x^4d^4+36\ln(dx+c)x^2b^4c^2d^2+36\ln(dx+c)x\,b^4c^3d-6x^2a^2b^2d^4+36x^2b^4c^2d^2+12\ln(dx+c)b^4c^4-6}{3d^5(dx+c)^3}$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
b^4*x/d^4+((2*a^2*b^2*d^3-6*b^4*c^2*d)*x^2+2*b^2*c*(a^2*d^2-5*b^2*c^2)*x-1
/3*(a^4*d^4-2*a^2*b^2*c^2*d^2+13*b^4*c^4)/d)/d^4/(d*x+c)^3-4*b^4*c*ln(d*x+
c)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.64

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx$$
$$=\frac{3\,b^4d^4x^4+9\,b^4cd^3x^3-13\,b^4c^4+2\,a^2b^2c^2d^2-a^4d^4-3\,(3\,b^4c^2d^2-2\,a^2b^2d^4)x^2-3\,(9\,b^4c^3d-2\,a^2b^2cd^3)x}{3\,(d^8x^3+3\,cd^7x^2+3\,c^2d^6x+c^3d^5)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^4*d^4*x^4 + 9*b^4*c*d^3*x^3 - 13*b^4*c^4 + 2*a^2*b^2*c^2*d^2 - a^
4*d^4 - 3*(3*b^4*c^2*d^2 - 2*a^2*b^2*d^4)*x^2 - 3*(9*b^4*c^3*d - 2*a^2*b^2
*c*d^3)*x - 12*(b^4*c*d^3*x^3 + 3*b^4*c^2*d^2*x^2 + 3*b^4*c^3*d*x + b^4*c^
4)*log(d*x + c))/(d^8*x^3 + 3*c*d^7*x^2 + 3*c^2*d^6*x + c^3*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx = -\frac{4b^4c\log{(c+dx)}}{d^5}+\frac{b^4x}{d^4} + \frac{-a^4d^4+2a^2b^2c^2d^2-13b^4c^4+x^2\cdot(6a^2b^2d^4-18b^4c^2d^2)+x(6a^2b^2cd^3-30b^4c^3d)}{3c^3d^5+9c^2d^6x+9cd^7x^2+3d^8x^3}$$

input
```
integrate((-b**2*x**2+a**2)**2/(d*x+c)**4,x)
```

output
```
-4*b**4*c*log(c + d*x)/d**5 + b**4*x/d**4 + (-a**4*d**4 + 2*a**2*b**2*c**2
*d**2 - 13*b**4*c**4 + x**2*(6*a**2*b**2*d**4 - 18*b**4*c**2*d**2) + x*(6*
a**2*b**2*c*d**3 - 30*b**4*c**3*d))/(3*c**3*d**5 + 9*c**2*d**6*x + 9*c*d**
7*x**2 + 3*d**8*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx = \frac{b^4x}{d^4}-\frac{4\,b^4c\log{(dx+c)}}{d^5} - \frac{13\,b^4c^4-2\,a^2b^2c^2d^2+a^4d^4+6\,(3\,b^4c^2d^2-a^2b^2d^4)x^2+6\,(5\,b^4c^3d-a^2b^2cd^3)x}{3\,(d^8x^3+3\,cd^7x^2+3\,c^2d^6x+c^3d^5)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
b^4*x/d^4 - 4*b^4*c*log(d*x + c)/d^5 - 1/3*(13*b^4*c^4 - 2*a^2*b^2*c^2*d^2
 + a^4*d^4 + 6*(3*b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 6*(5*b^4*c^3*d - a^2*b^
2*c*d^3)*x)/(d^8*x^3 + 3*c*d^7*x^2 + 3*c^2*d^6*x + c^3*d^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx = \frac{b^4x}{d^4} - \frac{4\,b^4c\log(|dx+c|)}{d^5} - \frac{13\,b^4c^4-2\,a^2b^2c^2d^2+a^4d^4+6\,(3\,b^4c^2d^2-a^2b^2d^4)x^2+6\,(5\,b^4c^3d-a^2b^2cd^3)x}{3\,(dx+c)^3d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
b^4*x/d^4 - 4*b^4*c*log(abs(d*x + c))/d^5 - 1/3*(13*b^4*c^4 - 2*a^2*b^2*c^
2*d^2 + a^4*d^4 + 6*(3*b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 6*(5*b^4*c^3*d - a
^2*b^2*c*d^3)*x)/((d*x + c)^3*d^5)
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx = \frac{b^4\,x}{d^4} - \frac{x\,(10\,b^4\,c^3-2\,a^2\,b^2\,c\,d^2)+x^2\,(6\,b^4\,c^2\,d-2\,a^2\,b^2\,d^3)+\frac{a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+13\,b^4\,c^4}{3\,d}}{c^3\,d^4+3\,c^2\,d^5\,x+3\,c\,d^6\,x^2+d^7\,x^3} - \frac{4\,b^4\,c\,\ln(c+d\,x)}{d^5}$$

input
```
int((a^2 - b^2*x^2)^2/(c + d*x)^4,x)
```

output
```
(b^4*x)/d^4 - (x*(10*b^4*c^3 - 2*a^2*b^2*c*d^2) + x^2*(6*b^4*c^2*d - 2*a^2
*b^2*d^3) + (a^4*d^4 + 13*b^4*c^4 - 2*a^2*b^2*c^2*d^2)/(3*d))/(c^3*d^4 + d
^7*x^3 + 3*c^2*d^5*x + 3*c*d^6*x^2) - (4*b^4*c*log(c + d*x))/d^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.47

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^4}\,dx = \frac{-12\log(dx+c)\,b^4c^5-36\log(dx+c)\,b^4c^4dx-36\log(dx+c)\,b^4c^3d^2x^2-12\log(dx+c)\,b^4c^2d^3x^3-a^4c\,d}{3c\,d^5\,(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}$$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^4,x)
```

output
```
( - 12*log(c + d*x)*b**4*c**5 - 36*log(c + d*x)*b**4*c**4*d*x - 36*log(c +
 d*x)*b**4*c**3*d**2*x**2 - 12*log(c + d*x)*b**4*c**2*d**3*x**3 - a**4*c*d
**4 - 2*a**2*b**2*d**5*x**3 - 10*b**4*c**5 - 18*b**4*c**4*d*x + 12*b**4*c*
*2*d**3*x**3 + 3*b**4*c*d**4*x**4)/(3*c*d**5*(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3))
```

3.18 $\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx = -\frac{(b^2c^2-a^2d^2)^2}{4d^5(c+dx)^4}+\frac{4b^2c(bc-ad)(bc+ad)}{3d^5(c+dx)^3} - \frac{b^2(3b^2c^2-a^2d^2)}{d^5(c+dx)^2}+\frac{4b^4c}{d^5(c+dx)}+\frac{b^4\log(c+dx)}{d^5}$$

output
```
-1/4*(-a^2*d^2+b^2*c^2)^2/d^5/(d*x+c)^4+4/3*b^2*c*(-a*d+b*c)*(a*d+b*c)/d^5
/(d*x+c)^3-b^2*(-a^2*d^2+3*b^2*c^2)/d^5/(d*x+c)^2+4*b^4*c/d^5/(d*x+c)+b^4*
ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.82

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx = \frac{-3a^4d^4+2a^2b^2d^2(c^2+4cdx+6d^2x^2)+b^4c(25c^3+88c^2dx+108cd^2x^2+48d^3x^3)+12b^4(c+dx)^4\log(c}{12d^5(c+dx)^4}$$

input
```
Integrate[(a^2 - b^2*x^2)^2/(c + d*x)^5,x]
```

output
```
(-3*a^4*d^4 + 2*a^2*b^2*d^2*(c^2 + 4*c*d*x + 6*d^2*x^2) + b^4*c*(25*c^3 +
88*c^2*d*x + 108*c*d^2*x^2 + 48*d^3*x^3) + 12*b^4*(c + d*x)^4*Log[c + d*x]
)/(12*d^5*(c + d*x)^4)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(a^2d^2-b^2c^2)^2}{d^4(c+dx)^5}+\frac{4(a^2b^2cd^2-b^4c^3)}{d^4(c+dx)^4}-\frac{2(a^2b^2d^2-3b^4c^2)}{d^4(c+dx)^3}+\frac{b^4}{d^4(c+dx)}-\frac{4b^4c}{d^4(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2(3b^2c^2-a^2d^2)}{d^5(c+dx)^2}-\frac{(b^2c^2-a^2d^2)^2}{4d^5(c+dx)^4}+\frac{4b^2c(bc-ad)(ad+bc)}{3d^5(c+dx)^3}+\frac{4b^4c}{d^5(c+dx)}+\frac{b^4\log(c+dx)}{d^5}$$

input
```
Int[(a^2 - b^2*x^2)^2/(c + d*x)^5,x]
```

output
```
-1/4*(b^2*c^2 - a^2*d^2)^2/(d^5*(c + d*x)^4) + (4*b^2*c*(b*c - a*d)*(b*c +
 a*d))/(3*d^5*(c + d*x)^3) - (b^2*(3*b^2*c^2 - a^2*d^2))/(d^5*(c + d*x)^2)
 + (4*b^4*c)/(d^5*(c + d*x)) + (b^4*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\frac{\left(b^2d^2a^2+9b^4c^2\right)x^2}{d^3}-\frac{3a^4d^4-2a^2b^2c^2d^2-25c^4b^4}{12d^5}+\frac{4b^4c\,x^3}{d^2}+\frac{2c\left(b^2d^2a^2+11b^4c^2\right)x}{3d^4}}{(dx+c)^4}+\frac{b^4\ln(dx+c)}{d^5}$
risch	$\frac{\frac{4b^4c\,x^3}{d^2}+\frac{b^2\left(a^2d^2+9b^2c^2\right)x^2}{d^3}+\frac{2b^2c\left(a^2d^2+11b^2c^2\right)x}{3d^4}-\frac{3a^4d^4-2a^2b^2c^2d^2-25c^4b^4}{12d^5}}{(dx+c)^4}+\frac{b^4\ln(dx+c)}{d^5}$
default	$\frac{4b^4c}{d^5(dx+c)}-\frac{a^4d^4-2a^2b^2c^2d^2+c^4b^4}{4d^5(dx+c)^4}+\frac{b^2\left(a^2d^2-3b^2c^2\right)}{d^5(dx+c)^2}-\frac{4b^2c\left(a^2d^2-b^2c^2\right)}{3d^5(dx+c)^3}+\frac{b^4\ln(dx+c)}{d^5}$
parallelrisch	$\frac{12\ln(dx+c)x^4b^4d^4+48\ln(dx+c)x^3b^4c\,d^3+72\ln(dx+c)x^2b^4c^2d^2+48b^4c\,x^3d^3+48\ln(dx+c)x\,b^4c^3d+12x^2a^2b^2d^4+108x^2b^4c^2}{12d^5(dx+c)^4}$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
((a^2*b^2*d^2+9*b^4*c^2)/d^3*x^2-1/12*(3*a^4*d^4-2*a^2*b^2*c^2*d^2-25*b^4*
c^4)/d^5+4*b^4*c*x^3/d^2+2/3*c*(a^2*b^2*d^2+11*b^4*c^2)/d^4*x)/(d*x+c)^4+b
^4*ln(d*x+c)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.61

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx$$
$$= \frac{48\,b^4cd^3x^3 + 25\,b^4c^4 + 2\,a^2b^2c^2d^2 - 3\,a^4d^4 + 12\,(9\,b^4c^2d^2 + a^2b^2d^4)x^2 + 8\,(11\,b^4c^3d + a^2b^2cd^3)x + 12\,(b^4d}{12\,(d^9x^4 + 4\,cd^8x^3 + 6\,c^2d^7x^2 + 4\,c^3d^6x + c^4d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/12*(48*b^4*c*d^3*x^3 + 25*b^4*c^4 + 2*a^2*b^2*c^2*d^2 - 3*a^4*d^4 + 12*(
9*b^4*c^2*d^2 + a^2*b^2*d^4)*x^2 + 8*(11*b^4*c^3*d + a^2*b^2*c*d^3)*x + 12
*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4)*log(d*x + c))/(d^9*x^4 + 4*c*d^8*x^3 + 6*c^2*d^7*x^2 + 4*c^3*d^6*x +
c^4*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx = \frac{b^4\log{(c+dx)}}{d^5}$$
$$+ \frac{-3a^4d^4 + 2a^2b^2c^2d^2 + 25b^4c^4 + 48b^4cd^3x^3 + x^2\cdot(12a^2b^2d^4 + 108b^4c^2d^2) + x(8a^2b^2cd^3 + 88b^4c^3d)}{12c^4d^5 + 48c^3d^6x + 72c^2d^7x^2 + 48cd^8x^3 + 12d^9x^4}$$

input
```
integrate((-b**2*x**2+a**2)**2/(d*x+c)**5,x)
```

output
```
b**4*log(c + d*x)/d**5 + (-3*a**4*d**4 + 2*a**2*b**2*c**2*d**2 + 25*b**4*c
**4 + 48*b**4*c*d**3*x**3 + x**2*(12*a**2*b**2*d**4 + 108*b**4*c**2*d**2)
+ x*(8*a**2*b**2*c*d**3 + 88*b**4*c**3*d))/(12*c**4*d**5 + 48*c**3*d**6*x
+ 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.23

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx = \frac{48\,b^4cd^3x^3+25\,b^4c^4+2\,a^2b^2c^2d^2-3\,a^4d^4+12\,(9\,b^4c^2d^2+a^2b^2d^4)x^2+8\,(11\,b^4c^3d+a^2b^2cd^3)x}{12\,(d^9x^4+4\,cd^8x^3+6\,c^2d^7x^2+4\,c^3d^6x+c^4d^5)} + \frac{b^4\log(dx+c)}{d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/12*(48*b^4*c*d^3*x^3 + 25*b^4*c^4 + 2*a^2*b^2*c^2*d^2 - 3*a^4*d^4 + 12*(
9*b^4*c^2*d^2 + a^2*b^2*d^4)*x^2 + 8*(11*b^4*c^3*d + a^2*b^2*c*d^3)*x)/(d^
9*x^4 + 4*c*d^8*x^3 + 6*c^2*d^7*x^2 + 4*c^3*d^6*x + c^4*d^5) + b^4*log(d*x
 + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx = -\frac{b^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5} + \frac{\frac{48\,b^4cd^{15}}{dx+c}-\frac{36\,b^4c^2d^{15}}{(dx+c)^2}+\frac{16\,b^4c^3d^{15}}{(dx+c)^3}-\frac{3\,b^4c^4d^{15}}{(dx+c)^4}+\frac{12\,a^2b^2d^{17}}{(dx+c)^2}-\frac{16\,a^2b^2cd^{17}}{(dx+c)^3}+\frac{6\,a^2b^2c^2d^{17}}{(dx+c)^4}-\frac{3\,a^4d^{19}}{(dx+c)^4}}{12\,d^{20}}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^5,x, algorithm="giac")
```

output
```
-b^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 1/12*(48*b^4*c*d^15/(d*x
 + c) - 36*b^4*c^2*d^15/(d*x + c)^2 + 16*b^4*c^3*d^15/(d*x + c)^3 - 3*b^4*
c^4*d^15/(d*x + c)^4 + 12*a^2*b^2*d^17/(d*x + c)^2 - 16*a^2*b^2*c*d^17/(d*
x + c)^3 + 6*a^2*b^2*c^2*d^17/(d*x + c)^4 - 3*a^4*d^19/(d*x + c)^4)/d^20
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.23

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx = \frac{\frac{-3\,a^4\,d^4+2\,a^2\,b^2\,c^2\,d^2+25\,b^4\,c^4}{12\,d^5}+\frac{2\,x\,(a^2\,b^2\,c\,d^2+11\,b^4\,c^3)}{3\,d^4}+\frac{4\,b^4\,c\,x^3}{d^2}+\frac{b^2\,x^2\,(a^2\,d^2+9\,b^2\,c^2)}{d^3}}{c^4+4\,c^3\,d\,x+6\,c^2\,d^2\,x^2+4\,c\,d^3\,x^3+d^4\,x^4} + \frac{b^4\,\ln(c+d\,x)}{d^5}$$

input
```
int((a^2 - b^2*x^2)^2/(c + d*x)^5,x)
```

output
```
((25*b^4*c^4 - 3*a^4*d^4 + 2*a^2*b^2*c^2*d^2)/(12*d^5) + (2*x*(11*b^4*c^3
+ a^2*b^2*c*d^2))/(3*d^4) + (4*b^4*c*x^3)/d^2 + (b^2*x^2*(a^2*d^2 + 9*b^2*
c^2))/d^3)/(c^4 + d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x) + (b^
4*log(c + d*x))/d^5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.76

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^5}\,dx = \frac{12\log(dx+c)\,b^4c^4+48\log(dx+c)\,b^4c^3dx+72\log(dx+c)\,b^4c^2d^2x^2+48\log(dx+c)\,b^4c\,d^3x^3+12\log(}{12d^5\,(d^4x^4+4c\,d^3x^3}$$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^5,x)
```

output
```
(12*log(c + d*x)*b**4*c**4 + 48*log(c + d*x)*b**4*c**3*d*x + 72*log(c + d*
x)*b**4*c**2*d**2*x**2 + 48*log(c + d*x)*b**4*c*d**3*x**3 + 12*log(c + d*x
)*b**4*d**4*x**4 - 3*a**4*d**4 + 2*a**2*b**2*c**2*d**2 + 8*a**2*b**2*c*d**
3*x + 12*a**2*b**2*d**4*x**2 + 13*b**4*c**4 + 40*b**4*c**3*d*x + 36*b**4*c
**2*d**2*x**2 - 12*b**4*d**4*x**4)/(12*d**5*(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4))
```

3.19 $\int \frac{(a^2-b^2x^2)^2}{(c+dx)^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^6}\,dx = -\frac{(b^2c^2-a^2d^2)^2}{5d^5(c+dx)^5}+\frac{b^2c(bc-ad)(bc+ad)}{d^5(c+dx)^4} - \frac{2b^2(3b^2c^2-a^2d^2)}{3d^5(c+dx)^3}+\frac{2b^4c}{d^5(c+dx)^2}-\frac{b^4}{d^5(c+dx)}$$

output
```
-1/5*(-a^2*d^2+b^2*c^2)^2/d^5/(d*x+c)^5+b^2*c*(-a*d+b*c)*(a*d+b*c)/d^5/(d*
x+c)^4-2/3*b^2*(-a^2*d^2+3*b^2*c^2)/d^5/(d*x+c)^3+2*b^4*c/d^5/(d*x+c)^2-b^
4/d^5/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^6}\,dx = \frac{-3a^4d^4+a^2b^2d^2(c^2+5cdx+10d^2x^2)-3b^4(c^4+5c^3dx+10c^2d^2x^2+10cd^3x^3+5d^4x^4)}{15d^5(c+dx)^5}$$

input
```
Integrate[(a^2 - b^2*x^2)^2/(c + d*x)^6,x]
```

output
```
(-3*a^4*d^4 + a^2*b^2*d^2*(c^2 + 5*c*d*x + 10*d^2*x^2) - 3*b^4*(c^4 + 5*c^
3*d*x + 10*c^2*d^2*x^2 + 10*c*d^3*x^3 + 5*d^4*x^4))/(15*d^5*(c + d*x)^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 - b^2x^2)^2}{(c+dx)^6}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(a^2d^2 - b^2c^2)^2}{d^4(c+dx)^6} + \frac{4(a^2b^2cd^2 - b^4c^3)}{d^4(c+dx)^5} - \frac{2(a^2b^2d^2 - 3b^4c^2)}{d^4(c+dx)^4} + \frac{b^4}{d^4(c+dx)^2} - \frac{4b^4c}{d^4(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^2(3b^2c^2 - a^2d^2)}{3d^5(c+dx)^3} - \frac{(b^2c^2 - a^2d^2)^2}{5d^5(c+dx)^5} + \frac{b^2c(bc-ad)(ad+bc)}{d^5(c+dx)^4} - \frac{b^4}{d^5(c+dx)} + \frac{2b^4c}{d^5(c+dx)^2}$$

input
```
Int[(a^2 - b^2*x^2)^2/(c + d*x)^6,x]
```

output
```
-1/5*(b^2*c^2 - a^2*d^2)^2/(d^5*(c + d*x)^5) + (b^2*c*(b*c - a*d)*(b*c + a
*d))/(d^5*(c + d*x)^4) - (2*b^2*(3*b^2*c^2 - a^2*d^2))/(3*d^5*(c + d*x)^3)
 + (2*b^4*c)/(d^5*(c + d*x)^2) - b^4/(d^5*(c + d*x))
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91

method	result	s
parallelrisch	$\frac{-15b^4x^4d^4-30b^4c\,x^3d^3+10x^2a^2b^2d^4-30x^2b^4c^2d^2+5x\,a^2b^2c\,d^3-15x\,b^4c^3d-3a^4d^4+a^2b^2c^2d^2-3c^4b^4}{15d^5(dx+c)^5}$	1
gosper	$-\frac{15b^4x^4d^4+30b^4c\,x^3d^3-10x^2a^2b^2d^4+30x^2b^4c^2d^2-5x\,a^2b^2c\,d^3+15x\,b^4c^3d+3a^4d^4-a^2b^2c^2d^2+3c^4b^4}{15d^5(dx+c)^5}$	1
norman	$\frac{-\frac{b^4x^4}{d}-\frac{2b^4c\,x^3}{d^2}+\frac{2\left(b^2d^2a^2-3b^4c^2\right)x^2}{3d^3}+\frac{c\left(b^2d^2a^2-3b^4c^2\right)x}{3d^4}-\frac{3a^4d^4-a^2b^2c^2d^2+3c^4b^4}{15d^5}}{(dx+c)^5}$	1
risch	$\frac{-\frac{b^4x^4}{d}-\frac{2b^4c\,x^3}{d^2}+\frac{2b^2\left(a^2d^2-3b^2c^2\right)x^2}{3d^3}+\frac{c\,b^2\left(a^2d^2-3b^2c^2\right)x}{3d^4}-\frac{3a^4d^4-a^2b^2c^2d^2+3c^4b^4}{15d^5}}{(dx+c)^5}$	1
default	$-\frac{b^4}{d^5(dx+c)}-\frac{b^2c(a^2d^2-b^2c^2)}{d^5(dx+c)^4}+\frac{2b^4c}{d^5(dx+c)^2}+\frac{2b^2(a^2d^2-3b^2c^2)}{3d^5(dx+c)^3}-\frac{a^4d^4-2a^2b^2c^2d^2+c^4b^4}{5d^5(dx+c)^5}$	1
orering	$-\frac{(15b^4x^4d^4+30b^4c\,x^3d^3-10x^2a^2b^2d^4+30x^2b^4c^2d^2-5x\,a^2b^2c\,d^3+15x\,b^4c^3d+3a^4d^4-a^2b^2c^2d^2+3c^4b^4)\left(-b^2x^2+a^2\right)^2}{15d^5(bx+a)^2(dx+c)^5(-bx+a)^2}$	1

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-15*b^4*d^4*x^4-30*b^4*c*d^3*x^3+10*a^2*b^2*d^4*x^2-30*b^4*c^2*d^2*x
^2+5*a^2*b^2*c*d^3*x-15*b^4*c^3*d*x-3*a^4*d^4+a^2*b^2*c^2*d^2-3*b^4*c^4)/d
^5/(d*x+c)^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.30

$$\int \frac{(a^2 - b^2x^2)^2}{(c + dx)^6}\, dx = -\frac{15\, b^4 d^4 x^4 + 30\, b^4 c d^3 x^3 + 3\, b^4 c^4 - a^2 b^2 c^2 d^2 + 3\, a^4 d^4 + 10\, (3\, b^4 c^2 d^2 - a^2 b^2 d^4) x^2 + 5\, (3\, b^4 c^3 d - a^2 b^2 c d^3)}{15\, (d^{10} x^5 + 5\, c d^9 x^4 + 10\, c^2 d^8 x^3 + 10\, c^3 d^7 x^2 + 5\, c^4 d^6 x + c^5 d^5)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/15*(15*b^4*d^4*x^4 + 30*b^4*c*d^3*x^3 + 3*b^4*c^4 - a^2*b^2*c^2*d^2 + 3
*a^4*d^4 + 10*(3*b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 5*(3*b^4*c^3*d - a^2*b^2
*c*d^3)*x)/(d^10*x^5 + 5*c*d^9*x^4 + 10*c^2*d^8*x^3 + 10*c^3*d^7*x^2 + 5*c
^4*d^6*x + c^5*d^5)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.31

$$\int \frac{(a^2 - b^2x^2)^2}{(c + dx)^6}\, dx = \frac{-3a^4d^4 + a^2b^2c^2d^2 - 3b^4c^4 - 30b^4cd^3x^3 - 15b^4d^4x^4 + x^2 \cdot (10a^2b^2d^4 - 30b^4c^2d^2) + x(5a^2b^2cd^3 - 15b^4c^3}{15c^5d^5 + 75c^4d^6x + 150c^3d^7x^2 + 150c^2d^8x^3 + 75cd^9x^4 + 15d^{10}x^5}$$

input
```
integrate((-b**2*x**2+a**2)**2/(d*x+c)**6,x)
```

output
```
(-3*a**4*d**4 + a**2*b**2*c**2*d**2 - 3*b**4*c**4 - 30*b**4*c*d**3*x**3 -
15*b**4*d**4*x**4 + x**2*(10*a**2*b**2*d**4 - 30*b**4*c**2*d**2) + x*(5*a*
*2*b**2*c*d**3 - 15*b**4*c**3*d))/(15*c**5*d**5 + 75*c**4*d**6*x + 150*c**
3*d**7*x**2 + 150*c**2*d**8*x**3 + 75*c*d**9*x**4 + 15*d**10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.30

$$\int \frac{(a^2 - b^2x^2)^2}{(c + dx)^6}\, dx = -\frac{15\, b^4d^4x^4 + 30\, b^4cd^3x^3 + 3\, b^4c^4 - a^2b^2c^2d^2 + 3\, a^4d^4 + 10\, (3\, b^4c^2d^2 - a^2b^2d^4)x^2 + 5\, (3\, b^4c^3d - a^2b^2cd^3)}{15\, (d^{10}x^5 + 5\, cd^9x^4 + 10\, c^2d^8x^3 + 10\, c^3d^7x^2 + 5\, c^4d^6x + c^5d^5)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^6,x, algorithm="maxima")
```

output
```
-1/15*(15*b^4*d^4*x^4 + 30*b^4*c*d^3*x^3 + 3*b^4*c^4 - a^2*b^2*c^2*d^2 + 3
*a^4*d^4 + 10*(3*b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 5*(3*b^4*c^3*d - a^2*b^2
*c*d^3)*x)/(d^10*x^5 + 5*c*d^9*x^4 + 10*c^2*d^8*x^3 + 10*c^3*d^7*x^2 + 5*c
^4*d^6*x + c^5*d^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 - b^2x^2)^2}{(c + dx)^6}\, dx = -\frac{15\, b^4d^4x^4 + 30\, b^4cd^3x^3 + 30\, b^4c^2d^2x^2 - 10\, a^2b^2d^4x^2 + 15\, b^4c^3dx - 5\, a^2b^2cd^3x + 3\, b^4c^4 - a^2b^2c^2d^2 + 3\, a}{15\, (dx + c)^5d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/15*(15*b^4*d^4*x^4 + 30*b^4*c*d^3*x^3 + 30*b^4*c^2*d^2*x^2 - 10*a^2*b^2
*d^4*x^2 + 15*b^4*c^3*d*x - 5*a^2*b^2*c*d^3*x + 3*b^4*c^4 - a^2*b^2*c^2*d^
2 + 3*a^4*d^4)/((d*x + c)^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^6}\,dx = -\frac{\frac{3\,a^4\,d^4-a^2\,b^2\,c^2\,d^2+3\,b^4\,c^4}{15\,d^5}+\frac{b^4\,x^4}{d}+\frac{2\,b^4\,c\,x^3}{d^2}-\frac{2\,b^2\,x^2\,\left(a^2\,d^2-3\,b^2\,c^2\right)}{3\,d^3}-\frac{b^2\,c\,x\,\left(a^2\,d^2-3\,b^2\,c^2\right)}{3\,d^4}}{c^5+5\,c^4\,d\,x+10\,c^3\,d^2\,x^2+10\,c^2\,d^3\,x^3+5\,c\,d^4\,x^4+d^5\,x^5}$$

input
```
int((a^2 - b^2*x^2)^2/(c + d*x)^6,x)
```

output
```
-((3*a^4*d^4 + 3*b^4*c^4 - a^2*b^2*c^2*d^2)/(15*d^5) + (b^4*x^4)/d + (2*b^
4*c*x^3)/d^2 - (2*b^2*x^2*(a^2*d^2 - 3*b^2*c^2))/(3*d^3) - (b^2*c*x*(a^2*d
^2 - 3*b^2*c^2))/(3*d^4))/(c^5 + d^5*x^5 + 5*c*d^4*x^4 + 10*c^3*d^2*x^2 +
10*c^2*d^3*x^3 + 5*c^4*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^6}\,dx = \frac{3b^4d^3x^5+10a^2b^2c\,d^2x^2+5a^2b^2c^2dx-3a^4c\,d^2+a^2b^2c^3}{15c\,d^3\left(d^5x^5+5c\,d^4x^4+10c^2d^3x^3+10c^3d^2x^2+5c^4dx+c^5\right)}$$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^6,x)
```

output
```
( - 3*a**4*c*d**2 + a**2*b**2*c**3 + 5*a**2*b**2*c**2*d*x + 10*a**2*b**2*c
*d**2*x**2 + 3*b**4*d**3*x**5)/(15*c*d**3*(c**5 + 5*c**4*d*x + 10*c**3*d**
2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5))
```

3.20 $\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = -\frac{(b^2c^2-a^2d^2)^2}{6d^5(c+dx)^6}+\frac{4b^2c(bc-ad)(bc+ad)}{5d^5(c+dx)^5} - \frac{b^2(3b^2c^2-a^2d^2)}{2d^5(c+dx)^4}+\frac{4b^4c}{3d^5(c+dx)^3}-\frac{b^4}{2d^5(c+dx)^2}$$

output
```
-1/6*(-a^2*d^2+b^2*c^2)^2/d^5/(d*x+c)^6+4/5*b^2*c*(-a*d+b*c)*(a*d+b*c)/d^5
/(d*x+c)^5-1/2*b^2*(-a^2*d^2+3*b^2*c^2)/d^5/(d*x+c)^4+4/3*b^4*c/d^5/(d*x+c
)^3-1/2*b^4/d^5/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = \frac{-5a^4d^4+a^2b^2d^2(c^2+6cdx+15d^2x^2)-b^4(c^4+6c^3dx+15c^2d^2x^2+20cd^3x^3+15d^4x^4)}{30d^5(c+dx)^6}$$

input
```
Integrate[(a^2 - b^2*x^2)^2/(c + d*x)^7,x]
```

output
```
(-5*a^4*d^4 + a^2*b^2*d^2*(c^2 + 6*c*d*x + 15*d^2*x^2) - b^4*(c^4 + 6*c^3*
d*x + 15*c^2*d^2*x^2 + 20*c*d^3*x^3 + 15*d^4*x^4))/(30*d^5*(c + d*x)^6)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(a^2d^2-b^2c^2)^2}{d^4(c+dx)^7}+\frac{4(a^2b^2cd^2-b^4c^3)}{d^4(c+dx)^6}-\frac{2(a^2b^2d^2-3b^4c^2)}{d^4(c+dx)^5}+\frac{b^4}{d^4(c+dx)^3}-\frac{4b^4c}{d^4(c+dx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2(3b^2c^2-a^2d^2)}{2d^5(c+dx)^4}-\frac{(b^2c^2-a^2d^2)^2}{6d^5(c+dx)^6}+\frac{4b^2c(bc-ad)(ad+bc)}{5d^5(c+dx)^5}-\frac{b^4}{2d^5(c+dx)^2}+\frac{4b^4c}{3d^5(c+dx)^3}$$

input
```
Int[(a^2 - b^2*x^2)^2/(c + d*x)^7,x]
```

output
```
-1/6*(b^2*c^2 - a^2*d^2)^2/(d^5*(c + d*x)^6) + (4*b^2*c*(b*c - a*d)*(b*c +
 a*d))/(5*d^5*(c + d*x)^5) - (b^2*(3*b^2*c^2 - a^2*d^2))/(2*d^5*(c + d*x)^
4) + (4*b^4*c)/(3*d^5*(c + d*x)^3) - b^4/(2*d^5*(c + d*x)^2)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

method	result	size
gosper	$-\frac{15 b^{4} x^{4} d^{4}+20 b^{4} c\,x^{3} d^{3}-15 x^{2} a^{2} b^{2} d^{4}+15 x^{2} b^{4} c^{2} d^{2}-6 x\,a^{2} b^{2} c\,d^{3}+6 x\,b^{4} c^{3} d+5 a^{4} d^{4}-a^{2} b^{2} c^{2} d^{2}+c^{4} b^{4}}{30 d^{5} \left(d x+c\right)^{6}}$	117
parallelrisch	$\frac{-15 b^{4} x^{4} d^{5}-20 b^{4} c\,x^{3} d^{4}+15 a^{2} b^{2} d^{5} x^{2}-15 b^{4} c^{2} d^{3} x^{2}+6 a^{2} b^{2} c\,d^{4} x-6 b^{4} c^{3} d^{2} x-5 a^{4} d^{5}+a^{2} b^{2} c^{2} d^{3}-b^{4} c^{4} d}{30 d^{6} \left(d x+c\right)^{6}}$	120
risch	$\frac{-\frac{b^{4} x^{4}}{2 d}-\frac{2 b^{4} c\,x^{3}}{3 d^{2}}+\frac{b^{2} \left(a^{2} d^{2}-b^{2} c^{2}\right) x^{2}}{2 d^{3}}+\frac{c\,b^{2} \left(a^{2} d^{2}-b^{2} c^{2}\right) x}{5 d^{4}}-\frac{5 a^{4} d^{4}-a^{2} b^{2} c^{2} d^{2}+c^{4} b^{4}}{30 d^{5}}}{\left(d x+c\right)^{6}}$	121
norman	$\frac{-\frac{b^{4} x^{4}}{2 d}-\frac{2 b^{4} c\,x^{3}}{3 d^{2}}+\frac{\left(a^{2} b^{2} d^{3}-b^{4} c^{2} d\right) x^{2}}{2 d^{4}}+\frac{c \left(a^{2} b^{2} d^{3}-b^{4} c^{2} d\right) x}{5 d^{5}}-\frac{5 a^{4} d^{5}-a^{2} b^{2} c^{2} d^{3}+b^{4} c^{4} d}{30 d^{6}}}{\left(d x+c\right)^{6}}$	124
default	$\frac{b^{2} \left(a^{2} d^{2}-3 b^{2} c^{2}\right)}{2 d^{5} \left(d x+c\right)^{4}}-\frac{b^{4}}{2 d^{5} \left(d x+c\right)^{2}}+\frac{4 b^{4} c}{3 d^{5} \left(d x+c\right)^{3}}-\frac{a^{4} d^{4}-2 a^{2} b^{2} c^{2} d^{2}+c^{4} b^{4}}{6 d^{5} \left(d x+c\right)^{6}}-\frac{4 b^{2} c \left(a^{2} d^{2}-b^{2} c^{2}\right)}{5 d^{5} \left(d x+c\right)^{5}}$	137
orering	$-\frac{\left(15 b^{4} x^{4} d^{4}+20 b^{4} c\,x^{3} d^{3}-15 x^{2} a^{2} b^{2} d^{4}+15 x^{2} b^{4} c^{2} d^{2}-6 x\,a^{2} b^{2} c\,d^{3}+6 x\,b^{4} c^{3} d+5 a^{4} d^{4}-a^{2} b^{2} c^{2} d^{2}+c^{4} b^{4}\right) \left(-b^{2} x^{2}+a^{2}\right)^{2}}{30 d^{5} \left(b x+a\right)^{2} \left(d x+c\right)^{6} \left(-b x+a\right)^{2}}$	146

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/30/d^5*(15*b^4*d^4*x^4+20*b^4*c*d^3*x^3-15*a^2*b^2*d^4*x^2+15*b^4*c^2*d
^2*x^2-6*a^2*b^2*c*d^3*x+6*b^4*c^3*d*x+5*a^4*d^4-a^2*b^2*c^2*d^2+b^4*c^4)/
(d*x+c)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = -\frac{15\,b^4d^4x^4+20\,b^4cd^3x^3+b^4c^4-a^2b^2c^2d^2+5\,a^4d^4+15\,(b^4c^2d^2-a^2b^2d^4)x^2+6\,(b^4c^3d-a^2b^2cd^3)x}{30\,(d^{11}x^6+6\,cd^{10}x^5+15\,c^2d^9x^4+20\,c^3d^8x^3+15\,c^4d^7x^2+6\,c^5d^6x+c^6d^5)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^7,x, algorithm="fricas")
```

output
```
-1/30*(15*b^4*d^4*x^4 + 20*b^4*c*d^3*x^3 + b^4*c^4 - a^2*b^2*c^2*d^2 + 5*a
^4*d^4 + 15*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 6*(b^4*c^3*d - a^2*b^2*c*d^3
)*x)/(d^11*x^6 + 6*c*d^10*x^5 + 15*c^2*d^9*x^4 + 20*c^3*d^8*x^3 + 15*c^4*d
^7*x^2 + 6*c^5*d^6*x + c^6*d^5)
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.32

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = \frac{-5a^4d^4+a^2b^2c^2d^2-b^4c^4-20b^4cd^3x^3-15b^4d^4x^4+x^2\cdot(15a^2b^2d^4-15b^4c^2d^2)+x(6a^2b^2cd^3-6b^4c^3d)}{30c^6d^5+180c^5d^6x+450c^4d^7x^2+600c^3d^8x^3+450c^2d^9x^4+180cd^{10}x^5+30d^{11}x^6}$$

input
```
integrate((-b**2*x**2+a**2)**2/(d*x+c)**7,x)
```

output
```
(-5*a**4*d**4 + a**2*b**2*c**2*d**2 - b**4*c**4 - 20*b**4*c*d**3*x**3 - 15
*b**4*d**4*x**4 + x**2*(15*a**2*b**2*d**4 - 15*b**4*c**2*d**2) + x*(6*a**2
*b**2*c*d**3 - 6*b**4*c**3*d))/(30*c**6*d**5 + 180*c**5*d**6*x + 450*c**4*
d**7*x**2 + 600*c**3*d**8*x**3 + 450*c**2*d**9*x**4 + 180*c*d**10*x**5 + 3
0*d**11*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = -\frac{15\,b^4d^4x^4+20\,b^4cd^3x^3+b^4c^4-a^2b^2c^2d^2+5\,a^4d^4+15\,(b^4c^2d^2-a^2b^2d^4)x^2+6\,(b^4c^3d-a^2b^2cd^3)x}{30\,(d^{11}x^6+6\,cd^{10}x^5+15\,c^2d^9x^4+20\,c^3d^8x^3+15\,c^4d^7x^2+6\,c^5d^6x+c^6d^5)}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^7,x, algorithm="maxima")
```

output
```
-1/30*(15*b^4*d^4*x^4 + 20*b^4*c*d^3*x^3 + b^4*c^4 - a^2*b^2*c^2*d^2 + 5*a
^4*d^4 + 15*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 + 6*(b^4*c^3*d - a^2*b^2*c*d^3
)*x)/(d^11*x^6 + 6*c*d^10*x^5 + 15*c^2*d^9*x^4 + 20*c^3*d^8*x^3 + 15*c^4*d
^7*x^2 + 6*c^5*d^6*x + c^6*d^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = -\frac{15\,b^4d^4x^4+20\,b^4cd^3x^3+15\,b^4c^2d^2x^2-15\,a^2b^2d^4x^2+6\,b^4c^3dx-6\,a^2b^2cd^3x+b^4c^4-a^2b^2c^2d^2+5\,a^4d}{30\,(dx+c)^6d^5}$$

input
```
integrate((-b^2*x^2+a^2)^2/(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/30*(15*b^4*d^4*x^4 + 20*b^4*c*d^3*x^3 + 15*b^4*c^2*d^2*x^2 - 15*a^2*b^2
*d^4*x^2 + 6*b^4*c^3*d*x - 6*a^2*b^2*c*d^3*x + b^4*c^4 - a^2*b^2*c^2*d^2 +
 5*a^4*d^4)/((d*x + c)^6*d^5)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = -\frac{\frac{5\,a^4\,d^4-a^2\,b^2\,c^2\,d^2+b^4\,c^4}{30\,d^5}+\frac{b^4\,x^4}{2\,d}+\frac{2\,b^4\,c\,x^3}{3\,d^2}-\frac{b^2\,x^2\,\left(a^2\,d^2-b^2\,c^2\right)}{2\,d^3}-\frac{b^2\,c\,x\,\left(a^2\,d^2-b^2\,c^2\right)}{5\,d^4}}{c^6+6\,c^5\,d\,x+15\,c^4\,d^2\,x^2+20\,c^3\,d^3\,x^3+15\,c^2\,d^4\,x^4+6\,c\,d^5\,x^5+d^6\,x^6}$$

input
```
int((a^2 - b^2*x^2)^2/(c + d*x)^7,x)
```

output
```
-((5*a^4*d^4 + b^4*c^4 - a^2*b^2*c^2*d^2)/(30*d^5) + (b^4*x^4)/(2*d) + (2*
b^4*c*x^3)/(3*d^2) - (b^2*x^2*(a^2*d^2 - b^2*c^2))/(2*d^3) - (b^2*c*x*(a^2
*d^2 - b^2*c^2))/(5*d^4))/(c^6 + d^6*x^6 + 6*c*d^5*x^5 + 15*c^4*d^2*x^2 +
20*c^3*d^3*x^3 + 15*c^2*d^4*x^4 + 6*c^5*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.27

$$\int \frac{(a^2-b^2x^2)^2}{(c+dx)^7}\,dx = \frac{-15b^4d^4x^4-20b^4c\,d^3x^3+15a^2b^2d^4x^2-15b^4c^2d^2x^2+6a^2b^2c\,d^3x-6b^4c^3dx-5a^4d^4+a^2b^2c^2d^2-b^4c^4}{30d^5\left(d^6x^6+6c\,d^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c^5dx+c^6\right)}$$

input
```
int((-b^2*x^2+a^2)^2/(d*x+c)^7,x)
```

output
```
( - 5*a**4*d**4 + a**2*b**2*c**2*d**2 + 6*a**2*b**2*c*d**3*x + 15*a**2*b**
2*d**4*x**2 - b**4*c**4 - 6*b**4*c**3*d*x - 15*b**4*c**2*d**2*x**2 - 20*b*
*4*c*d**3*x**3 - 15*b**4*d**4*x**4)/(30*d**5*(c**6 + 6*c**5*d*x + 15*c**4*
d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x
**6))
```

3.21 $\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 187

$$\begin{aligned}\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx = {} & a^6c^3x - a^4c(bc-ad)(bc+ad)x^3 + \frac{1}{4}a^6d^3x^4 \\ & + \frac{3}{5}a^2b^2c\left(b^2c^2-3a^2d^2\right)x^5 - \frac{1}{2}a^4b^2d^3x^6 \\ & - \frac{1}{7}b^4c\left(b^2c^2-9a^2d^2\right)x^7 + \frac{3}{8}a^2b^4d^3x^8 \\ & - \frac{1}{3}b^6cd^2x^9 - \frac{1}{10}b^6d^3x^{10} - \frac{3c^2d(a^2-b^2x^2)^4}{8b^2}\end{aligned}$$

output
```
a^6*c^3*x-a^4*c*(-a*d+b*c)*(a*d+b*c)*x^3+1/4*a^6*d^3*x^4+3/5*a^2*b^2*c*(-3
*a^2*d^2+b^2*c^2)*x^5-1/2*a^4*b^2*d^3*x^6-1/7*b^4*c*(-9*a^2*d^2+b^2*c^2)*x
^7+3/8*a^2*b^4*d^3*x^8-1/3*b^6*c*d^2*x^9-1/10*b^6*d^3*x^10-3/8*c^2*d*(-b^2
*x^2+a^2)^4/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int (c+dx)^3\left(a^2-b^2x^2\right)^3\,dx=\frac{1}{840}x\left(210a^6\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)-42a^4b^2x^2\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right)+9a^2b^4x^4\left(56c^3+140c^2dx+120cd^2x^2+35d^3x^3\right)-b^6x^6\left(120c^3+315c^2dx+280cd^2x^2+84d^3x^3\right)\right)$$

input
```
Integrate[(c + d*x)^3*(a^2 - b^2*x^2)^3,x]
```

output
```
(x*(210*a^6*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 42*a^4*b^2*x^2*(
20*c^3 + 45*c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 9*a^2*b^4*x^4*(56*c^3 +
 140*c^2*d*x + 120*c*d^2*x^2 + 35*d^3*x^3) - b^6*x^6*(120*c^3 + 315*c^2*d*
x + 280*c*d^2*x^2 + 84*d^3*x^3)))/840
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)^3(c+dx)^3\,dx$$

$$\Big\downarrow \text{475}$$

$$\int \left(-b^6d^3x^9-3b^6cd^2x^8+3a^2b^4d^3x^7-b^4c\left(b^2c^2-9a^2d^2\right)x^6-3a^4b^2d^3x^5-3a^2b^2c\left(3a^2d^2-b^2c^2\right)x^4+a^6d^3x^3+\right.$$

$$\frac{3c^2d\left(a^2-b^2x^2\right)^4}{8b^2}$$

$$\Big\downarrow \text{2009}$$

$$a^6c^3x + \frac{1}{4}a^6d^3x^4 - \frac{1}{2}a^4b^2d^3x^6 - a^4cx^3(bc-ad)(ad+bc) + \frac{3}{8}a^2b^4d^3x^8 + \frac{3}{5}a^2b^2cx^5\left(b^2c^2-3a^2d^2\right) - \frac{3c^2d\left(a^2-b^2x^2\right)^4}{8b^2} - \frac{1}{7}b^4cx^7\left(b^2c^2-9a^2d^2\right) - \frac{1}{3}b^6cd^2x^9 - \frac{1}{10}b^6d^3x^{10}$$

input
```
Int[(c + d*x)^3*(a^2 - b^2*x^2)^3,x]
```

output
```
a^6*c^3*x - a^4*c*(b*c - a*d)*(b*c + a*d)*x^3 + (a^6*d^3*x^4)/4 + (3*a^2*b
^2*c*(b^2*c^2 - 3*a^2*d^2)*x^5)/5 - (a^4*b^2*d^3*x^6)/2 - (b^4*c*(b^2*c^2
- 9*a^2*d^2)*x^7)/7 + (3*a^2*b^4*d^3*x^8)/8 - (b^6*c*d^2*x^9)/3 - (b^6*d^3
*x^10)/10 - (3*c^2*d*(a^2 - b^2*x^2)^4)/(8*b^2)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.07

method	result
norman	$-\frac{b^6d^3x^{10}}{10} - \frac{b^6c\,d^2x^9}{3} + \left(\frac{3}{8}b^4d^3a^2 - \frac{3}{8}b^6c^2d\right)x^8 + \left(\frac{9}{7}a^2b^4c\,d^2 - \frac{1}{7}b^6c^3\right)x^7 + \left(-\frac{1}{2}a^4b^2d^3 + \frac{3}{2}a^2b^4 \right.$
default	$-\frac{b^6d^3x^{10}}{10} - \frac{b^6c\,d^2x^9}{3} + \frac{\left(3b^4d^3a^2-3b^6c^2d\right)x^8}{8} + \frac{\left(9a^2b^4c\,d^2-b^6c^3\right)x^7}{7} + \frac{\left(-3a^4b^2d^3+9a^2b^4c^2d\right)x^6}{6} + \frac{\left(-9a^4b^2c\,d^2+\right.}{5}$
risch	$-\frac{1}{10}b^6d^3x^{10} - \frac{1}{3}b^6c\,d^2x^9 + \frac{3}{8}a^2b^4d^3x^8 - \frac{3}{8}x^8b^6c^2d + \frac{9}{7}x^7a^2b^4c\,d^2 - \frac{1}{7}x^7b^6c^3 - \frac{1}{2}a^4b^2d^3x^6 + \frac{3}{2}$
parallelrisch	$-\frac{1}{10}b^6d^3x^{10} - \frac{1}{3}b^6c\,d^2x^9 + \frac{3}{8}a^2b^4d^3x^8 - \frac{3}{8}x^8b^6c^2d + \frac{9}{7}x^7a^2b^4c\,d^2 - \frac{1}{7}x^7b^6c^3 - \frac{1}{2}a^4b^2d^3x^6 + \frac{3}{2}$
gosper	$\frac{x\left(-84b^6d^3x^9-280b^6c\,d^2x^8+315x^7b^4d^3a^2-315x^7b^6c^2d+1080x^6a^2b^4c\,d^2-120x^6b^6c^3-420x^5a^4b^2d^3+1260x^5a^2b^4c^2d-1512\right.}{840}$
orering	$\frac{x\left(-84b^6d^3x^9-280b^6c\,d^2x^8+315x^7b^4d^3a^2-315x^7b^6c^2d+1080x^6a^2b^4c\,d^2-120x^6b^6c^3-420x^5a^4b^2d^3+1260x^5a^2b^4c^2d-1512\right.}{840(bx+a)^3}$

input
```
int((d*x+c)^3*(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*b^6*d^3*x^10-1/3*b^6*c*d^2*x^9+(3/8*b^4*d^3*a^2-3/8*b^6*c^2*d)*x^8+(
9/7*a^2*b^4*c*d^2-1/7*b^6*c^3)*x^7+(-1/2*a^4*b^2*d^3+3/2*a^2*b^4*c^2*d)*x^
6+(-9/5*a^4*b^2*c*d^2+3/5*a^2*b^4*c^3)*x^5+(1/4*a^6*d^3-9/4*a^4*b^2*c^2*d)
*x^4+(a^6*c*d^2-a^4*b^2*c^3)*x^3+3/2*a^6*c^2*d*x^2+a^6*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx = -\frac{1}{10}\,b^6d^3x^{10} - \frac{1}{3}\,b^6cd^2x^9 + \frac{3}{2}\,a^6c^2dx^2 + a^6c^3x - \frac{3}{8}\left(b^6c^2d - a^2b^4d^3\right)x^8 - \frac{1}{7}\left(b^6c^3 - 9\,a^2b^4cd^2\right)x^7 + \frac{1}{2}\left(3\,a^2b^4c^2d - a^4b^2d^3\right)x^6 + \frac{3}{5}\left(a^2b^4c^3 - 3\,a^4b^2cd^2\right)x^5 - \frac{1}{4}\left(9\,a^4b^2c^2d - a^6d^3\right)x^4 - \left(a^4b^2c^3 - a^6cd^2\right)x^3$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/10*b^6*d^3*x^10 - 1/3*b^6*c*d^2*x^9 + 3/2*a^6*c^2*d*x^2 + a^6*c^3*x - 3
/8*(b^6*c^2*d - a^2*b^4*d^3)*x^8 - 1/7*(b^6*c^3 - 9*a^2*b^4*c*d^2)*x^7 + 1
/2*(3*a^2*b^4*c^2*d - a^4*b^2*d^3)*x^6 + 3/5*(a^2*b^4*c^3 - 3*a^4*b^2*c*d^
2)*x^5 - 1/4*(9*a^4*b^2*c^2*d - a^6*d^3)*x^4 - (a^4*b^2*c^3 - a^6*c*d^2)*x
^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.16

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx = a^6c^3x + \frac{3a^6c^2dx^2}{2} - \frac{b^6cd^2x^9}{3} - \frac{b^6d^3x^{10}}{10} + x^8 \cdot \left(\frac{3a^2b^4d^3}{8} - \frac{3b^6c^2d}{8}\right) + x^7 \cdot \left(\frac{9a^2b^4cd^2}{7} - \frac{b^6c^3}{7}\right) + x^6\left(-\frac{a^4b^2d^3}{2} + \frac{3a^2b^4c^2d}{2}\right) + x^5\left(-\frac{9a^4b^2cd^2}{5} + \frac{3a^2b^4c^3}{5}\right) + x^4\left(\frac{a^6d^3}{4} - \frac{9a^4b^2c^2d}{4}\right) + x^3\left(a^6cd^2 - a^4b^2c^3\right)$$

input
```
integrate((d*x+c)**3*(-b**2*x**2+a**2)**3,x)
```

output
```
a**6*c**3*x + 3*a**6*c**2*d*x**2/2 - b**6*c*d**2*x**9/3 - b**6*d**3*x**10/
10 + x**8*(3*a**2*b**4*d**3/8 - 3*b**6*c**2*d/8) + x**7*(9*a**2*b**4*c*d**
2/7 - b**6*c**3/7) + x**6*(-a**4*b**2*d**3/2 + 3*a**2*b**4*c**2*d/2) + x**
5*(-9*a**4*b**2*c*d**2/5 + 3*a**2*b**4*c**3/5) + x**4*(a**6*d**3/4 - 9*a**
4*b**2*c**2*d/4) + x**3*(a**6*c*d**2 - a**4*b**2*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx = -\frac{1}{10}\,b^6d^3x^{10} - \frac{1}{3}\,b^6cd^2x^9 + \frac{3}{2}\,a^6c^2dx^2 + a^6c^3x - \frac{3}{8}\left(b^6c^2d - a^2b^4d^3\right)x^8 - \frac{1}{7}\left(b^6c^3 - 9\,a^2b^4cd^2\right)x^7 + \frac{1}{2}\left(3\,a^2b^4c^2d - a^4b^2d^3\right)x^6 + \frac{3}{5}\left(a^2b^4c^3 - 3\,a^4b^2cd^2\right)x^5 - \frac{1}{4}\left(9\,a^4b^2c^2d - a^6d^3\right)x^4 - \left(a^4b^2c^3 - a^6cd^2\right)x^3$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/10*b^6*d^3*x^10 - 1/3*b^6*c*d^2*x^9 + 3/2*a^6*c^2*d*x^2 + a^6*c^3*x - 3
/8*(b^6*c^2*d - a^2*b^4*d^3)*x^8 - 1/7*(b^6*c^3 - 9*a^2*b^4*c*d^2)*x^7 + 1
/2*(3*a^2*b^4*c^2*d - a^4*b^2*d^3)*x^6 + 3/5*(a^2*b^4*c^3 - 3*a^4*b^2*c*d^
2)*x^5 - 1/4*(9*a^4*b^2*c^2*d - a^6*d^3)*x^4 - (a^4*b^2*c^3 - a^6*c*d^2)*x
^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx = -\frac{1}{10}b^6d^3x^{10} - \frac{1}{3}b^6cd^2x^9 - \frac{3}{8}b^6c^2dx^8 + \frac{3}{8}a^2b^4d^3x^8 - \frac{1}{7}b^6c^3x^7 + \frac{9}{7}a^2b^4cd^2x^7 + \frac{3}{2}a^2b^4c^2dx^6 - \frac{1}{2}a^4b^2d^3x^6 + \frac{3}{5}a^2b^4c^3x^5 - \frac{9}{5}a^4b^2cd^2x^5 - \frac{9}{4}a^4b^2c^2dx^4 + \frac{1}{4}a^6d^3x^4 - a^4b^2c^3x^3 + a^6cd^2x^3 + \frac{3}{2}a^6c^2dx^2 + a^6c^3x$$

input
```
integrate((d*x+c)^3*(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/10*b^6*d^3*x^10 - 1/3*b^6*c*d^2*x^9 - 3/8*b^6*c^2*d*x^8 + 3/8*a^2*b^4*d
^3*x^8 - 1/7*b^6*c^3*x^7 + 9/7*a^2*b^4*c*d^2*x^7 + 3/2*a^2*b^4*c^2*d*x^6 -
 1/2*a^4*b^2*d^3*x^6 + 3/5*a^2*b^4*c^3*x^5 - 9/5*a^4*b^2*c*d^2*x^5 - 9/4*a
^4*b^2*c^2*d*x^4 + 1/4*a^6*d^3*x^4 - a^4*b^2*c^3*x^3 + a^6*c*d^2*x^3 + 3/2
*a^6*c^2*d*x^2 + a^6*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx = x^5 \left(\frac{3\,a^2\,b^4\,c^3}{5} - \frac{9\,a^4\,b^2\,c\,d^2}{5}\right) - x^6 \left(\frac{a^4\,b^2\,d^3}{2} - \frac{3\,a^2\,b^4\,c^2\,d}{2}\right) + x^3 \left(a^6\,c\,d^2 - a^4\,b^2\,c^3\right) + x^4 \left(\frac{a^6\,d^3}{4} - \frac{9\,a^4\,b^2\,c^2\,d}{4}\right) - x^7 \left(\frac{b^6\,c^3}{7} - \frac{9\,a^2\,b^4\,c\,d^2}{7}\right) - x^8 \left(\frac{3\,b^6\,c^2\,d}{8} - \frac{3\,a^2\,b^4\,d^3}{8}\right) + a^6\,c^3\,x - \frac{b^6\,d^3\,x^{10}}{10} + \frac{3\,a^6\,c^2\,d\,x^2}{2} - \frac{b^6\,c\,d^2\,x^9}{3}$$

input
```
int((a^2 - b^2*x^2)^3*(c + d*x)^3,x)
```

output
```
x^5*((3*a^2*b^4*c^3)/5 - (9*a^4*b^2*c*d^2)/5) - x^6*((a^4*b^2*d^3)/2 - (3*
a^2*b^4*c^2*d)/2) + x^3*(a^6*c*d^2 - a^4*b^2*c^3) + x^4*((a^6*d^3)/4 - (9*
a^4*b^2*c^2*d)/4) - x^7*((b^6*c^3)/7 - (9*a^2*b^4*c*d^2)/7) - x^8*((3*b^6*
c^2*d)/8 - (3*a^2*b^4*d^3)/8) + a^6*c^3*x - (b^6*d^3*x^10)/10 + (3*a^6*c^2
*d*x^2)/2 - (b^6*c*d^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.11

$$\int (c+dx)^3 \left(a^2-b^2x^2\right)^3 \, dx$$
$$= \frac{x(-84b^6d^3x^9 - 280b^6c\,d^2x^8 + 315a^2b^4d^3x^7 - 315b^6c^2d\,x^7 + 1080a^2b^4c\,d^2x^6 - 120b^6c^3x^6 - 420a^4b^2d^3x^5 -}{}$$

input
```
int((d*x+c)^3*(-b^2*x^2+a^2)^3,x)
```

output
```
(x*(840*a**6*c**3 + 1260*a**6*c**2*d*x + 840*a**6*c*d**2*x**2 + 210*a**6*d
**3*x**3 - 840*a**4*b**2*c**3*x**2 - 1890*a**4*b**2*c**2*d*x**3 - 1512*a**
4*b**2*c*d**2*x**4 - 420*a**4*b**2*d**3*x**5 + 504*a**2*b**4*c**3*x**4 + 1
260*a**2*b**4*c**2*d*x**5 + 1080*a**2*b**4*c*d**2*x**6 + 315*a**2*b**4*d**
3*x**7 - 120*b**6*c**3*x**6 - 315*b**6*c**2*d*x**7 - 280*b**6*c*d**2*x**8
- 84*b**6*d**3*x**9))/840
```

3.22 $\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \, dx = a^6c^2x-\frac{1}{3}a^4\left(3b^2c^2-a^2d^2\right)x^3+\frac{3}{5}a^2b^2(bc-ad)(bc+ad)x^5 -\frac{1}{7}b^4\left(b^2c^2-3a^2d^2\right)x^7-\frac{1}{9}b^6d^2x^9-\frac{cd\left(a^2-b^2x^2\right)^4}{4b^2}$$

output
```
a^6*c^2*x-1/3*a^4*(-a^2*d^2+3*b^2*c^2)*x^3+3/5*a^2*b^2*(-a*d+b*c)*(a*d+b*c
)*x^5-1/7*b^4*(-3*a^2*d^2+b^2*c^2)*x^7-1/9*b^6*d^2*x^9-1/4*c*d*(-b^2*x^2+a
^2)^4/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \, dx = -\frac{1}{10}a^4b^2x^3\left(10c^2+15cdx+6d^2x^2\right) +\frac{1}{35}a^2b^4x^5\left(21c^2+35cdx+15d^2x^2\right) -\frac{1}{252}b^6x^7\left(36c^2+63cdx+28d^2x^2\right) +a^6\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)$$

input
```
Integrate[(c + d*x)^2*(a^2 - b^2*x^2)^3,x]
```

output
```
-1/10*(a^4*b^2*x^3*(10*c^2 + 15*c*d*x + 6*d^2*x^2)) + (a^2*b^4*x^5*(21*c^2
 + 35*c*d*x + 15*d^2*x^2))/35 - (b^6*x^7*(36*c^2 + 63*c*d*x + 28*d^2*x^2))
/252 + a^6*(c^2*x + c*d*x^2 + (d^2*x^3)/3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)^3 (c+dx)^2 \, dx$$

$$\Big\downarrow \text{475}$$

$$\int \left(-b^6d^2x^8-b^4\left(b^2c^2-3a^2d^2\right)x^6-3a^2b^2\left(a^2d^2-b^2c^2\right)x^4+a^4\left(a^2d^2-3b^2c^2\right)x^2+a^6c^2\right)dx-\frac{cd\left(a^2-b^2x^2\right)^4}{4b^2}$$

$$\Big\downarrow \text{2009}$$

$$a^6c^2x+\frac{3}{5}a^2b^2x^5(bc-ad)(ad+bc)-\frac{cd\left(a^2-b^2x^2\right)^4}{4b^2}-\frac{1}{7}b^4x^7\left(b^2c^2-3a^2d^2\right)-\frac{1}{3}a^4x^3\left(3b^2c^2-a^2d^2\right)-\frac{1}{9}b^6d^2x^9$$

input
```
Int[(c + d*x)^2*(a^2 - b^2*x^2)^3,x]
```

output
```
a^6*c^2*x - (a^4*(3*b^2*c^2 - a^2*d^2)*x^3)/3 + (3*a^2*b^2*(b*c - a*d)*(b*
c + a*d)*x^5)/5 - (b^4*(b^2*c^2 - 3*a^2*d^2)*x^7)/7 - (b^6*d^2*x^9)/9 - (c
*d*(a^2 - b^2*x^2)^4)/(4*b^2)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11

method	result
norman	$-\frac{b^6 d^2 x^9}{9}-\frac{b^6 cd\,x^8}{4}+\left(\frac{3}{7}d^2 b^4 a^2-\frac{1}{7}b^6 c^2\right)x^7+a^2 b^4 cd\,x^6+\left(-\frac{3}{5}a^4 b^2 d^2+\frac{3}{5}a^2 b^4 c^2\right)x^5-\frac{3a^4 b^2 cd\,x}{2}$
default	$-\frac{b^6 d^2 x^9}{9}-\frac{b^6 cd\,x^8}{4}+\frac{(3d^2 b^4 a^2-b^6 c^2)x^7}{7}+a^2 b^4 cd\,x^6+\frac{(-3a^4 b^2 d^2+3a^2 b^4 c^2)x^5}{5}-\frac{3a^4 b^2 cd\,x^4}{2}+\frac{(a^6 d^2-3c^2 a}{3}$
risch	$-\frac{1}{9}b^6 d^2 x^9-\frac{1}{4}b^6 cd\,x^8+\frac{3}{7}x^7 d^2 b^4 a^2-\frac{1}{7}x^7 b^6 c^2+a^2 b^4 cd\,x^6-\frac{3}{5}x^5 a^4 b^2 d^2+\frac{3}{5}x^5 a^2 b^4 c^2-\frac{3}{2}a^4 b$
parallelrisch	$-\frac{1}{9}b^6 d^2 x^9-\frac{1}{4}b^6 cd\,x^8+\frac{3}{7}x^7 d^2 b^4 a^2-\frac{1}{7}x^7 b^6 c^2+a^2 b^4 cd\,x^6-\frac{3}{5}x^5 a^4 b^2 d^2+\frac{3}{5}x^5 a^2 b^4 c^2-\frac{3}{2}a^4 b$
gosper	$\frac{x(-140b^6 d^2 x^8-315b^6 cd\,x^7+540x^6 d^2 b^4 a^2-180x^6 b^6 c^2+1260a^2 b^4 cd\,x^5-756x^4 a^4 b^2 d^2+756x^4 a^2 b^4 c^2-1890a^4 b^2 cd\,x^3+420x}{1260}$
orering	$\frac{x(-140b^6 d^2 x^8-315b^6 cd\,x^7+540x^6 d^2 b^4 a^2-180x^6 b^6 c^2+1260a^2 b^4 cd\,x^5-756x^4 a^4 b^2 d^2+756x^4 a^2 b^4 c^2-1890a^4 b^2 cd\,x^3+420x}{1260(bx+a)^3(-bx+a)^3}$

input
```
int((d*x+c)^2*(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/9*b^6*d^2*x^9-1/4*b^6*c*d*x^8+(3/7*d^2*b^4*a^2-1/7*b^6*c^2)*x^7+a^2*b^4
*c*d*x^6+(-3/5*a^4*b^2*d^2+3/5*a^2*b^4*c^2)*x^5-3/2*a^4*b^2*c*d*x^4+(1/3*a
^6*d^2-c^2*a^4*b^2)*x^3+a^6*c*d*x^2+a^6*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \,dx = -\frac{1}{9}\,b^6d^2x^9 - \frac{1}{4}\,b^6cdx^8 + a^2b^4cdx^6 - \frac{3}{2}\,a^4b^2cdx^4 + a^6cdx^2 + a^6c^2x - \frac{1}{7}\,\left(b^6c^2 - 3\,a^2b^4d^2\right)x^7 + \frac{3}{5}\,\left(a^2b^4c^2 - a^4b^2d^2\right)x^5 - \frac{1}{3}\,\left(3\,a^4b^2c^2 - a^6d^2\right)x^3$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/9*b^6*d^2*x^9 - 1/4*b^6*c*d*x^8 + a^2*b^4*c*d*x^6 - 3/2*a^4*b^2*c*d*x^4
 + a^6*c*d*x^2 + a^6*c^2*x - 1/7*(b^6*c^2 - 3*a^2*b^4*d^2)*x^7 + 3/5*(a^2*
b^4*c^2 - a^4*b^2*d^2)*x^5 - 1/3*(3*a^4*b^2*c^2 - a^6*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \,dx = a^6c^2x + a^6cdx^2 - \frac{3a^4b^2cdx^4}{2} + a^2b^4cdx^6 - \frac{b^6cdx^8}{4} - \frac{b^6d^2x^9}{9} + x^7\cdot\left(\frac{3a^2b^4d^2}{7} - \frac{b^6c^2}{7}\right) + x^5\left(-\frac{3a^4b^2d^2}{5} + \frac{3a^2b^4c^2}{5}\right) + x^3\left(\frac{a^6d^2}{3} - a^4b^2c^2\right)$$

input
```
integrate((d*x+c)**2*(-b**2*x**2+a**2)**3,x)
```

output
```
a**6*c**2*x + a**6*c*d*x**2 - 3*a**4*b**2*c*d*x**4/2 + a**2*b**4*c*d*x**6
- b**6*c*d*x**8/4 - b**6*d**2*x**9/9 + x**7*(3*a**2*b**4*d**2/7 - b**6*c**
2/7) + x**5*(-3*a**4*b**2*d**2/5 + 3*a**2*b**4*c**2/5) + x**3*(a**6*d**2/3
 - a**4*b**2*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \, dx = -\frac{1}{9} b^6 d^2 x^9 - \frac{1}{4} b^6 c d x^8 + a^2 b^4 c d x^6 - \frac{3}{2} a^4 b^2 c d x^4 + a^6 c d x^2 + a^6 c^2 x - \frac{1}{7} \left(b^6 c^2 - 3 a^2 b^4 d^2\right) x^7 + \frac{3}{5} \left(a^2 b^4 c^2 - a^4 b^2 d^2\right) x^5 - \frac{1}{3} \left(3 a^4 b^2 c^2 - a^6 d^2\right) x^3$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/9*b^6*d^2*x^9 - 1/4*b^6*c*d*x^8 + a^2*b^4*c*d*x^6 - 3/2*a^4*b^2*c*d*x^4
 + a^6*c*d*x^2 + a^6*c^2*x - 1/7*(b^6*c^2 - 3*a^2*b^4*d^2)*x^7 + 3/5*(a^2*
b^4*c^2 - a^4*b^2*d^2)*x^5 - 1/3*(3*a^4*b^2*c^2 - a^6*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \, dx = -\frac{1}{9} b^6 d^2 x^9 - \frac{1}{4} b^6 c d x^8 - \frac{1}{7} b^6 c^2 x^7 + \frac{3}{7} a^2 b^4 d^2 x^7 + a^2 b^4 c d x^6 + \frac{3}{5} a^2 b^4 c^2 x^5 - \frac{3}{5} a^4 b^2 d^2 x^5 - \frac{3}{2} a^4 b^2 c d x^4 - a^4 b^2 c^2 x^3 + \frac{1}{3} a^6 d^2 x^3 + a^6 c d x^2 + a^6 c^2 x$$

input
```
integrate((d*x+c)^2*(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/9*b^6*d^2*x^9 - 1/4*b^6*c*d*x^8 - 1/7*b^6*c^2*x^7 + 3/7*a^2*b^4*d^2*x^7
 + a^2*b^4*c*d*x^6 + 3/5*a^2*b^4*c^2*x^5 - 3/5*a^4*b^2*d^2*x^5 - 3/2*a^4*b
^2*c*d*x^4 - a^4*b^2*c^2*x^3 + 1/3*a^6*d^2*x^3 + a^6*c*d*x^2 + a^6*c^2*x
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \, dx = x^3 \left(\frac{a^6 d^2}{3} - a^4 b^2 c^2\right) - x^7 \left(\frac{b^6 c^2}{7} - \frac{3 a^2 b^4 d^2}{7}\right) + x^5 \left(\frac{3 a^2 b^4 c^2}{5} - \frac{3 a^4 b^2 d^2}{5}\right) + a^6 c^2 x - \frac{b^6 d^2 x^9}{9} + a^6 c d x^2 - \frac{b^6 c d x^8}{4} - \frac{3 a^4 b^2 c d x^4}{2} + a^2 b^4 c d x^6$$

input
```
int((a^2 - b^2*x^2)^3*(c + d*x)^2,x)
```

output
```
x^3*((a^6*d^2)/3 - a^4*b^2*c^2) - x^7*((b^6*c^2)/7 - (3*a^2*b^4*d^2)/7) +
x^5*((3*a^2*b^4*c^2)/5 - (3*a^4*b^2*d^2)/5) + a^6*c^2*x - (b^6*d^2*x^9)/9
+ a^6*c*d*x^2 - (b^6*c*d*x^8)/4 - (3*a^4*b^2*c*d*x^4)/2 + a^2*b^4*c*d*x^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15

$$\int (c+dx)^2 \left(a^2-b^2x^2\right)^3 \, dx = \frac{x(-140b^6d^2x^8 - 315b^6cd\,x^7 + 540a^2b^4d^2x^6 - 180b^6c^2x^6 + 1260a^2b^4cd\,x^5 - 756a^4b^2d^2x^4 + 756a^2b^4c^2x^4 -}{1260}$$

input
```
int((d*x+c)^2*(-b^2*x^2+a^2)^3,x)
```

output
```
(x*(1260*a**6*c**2 + 1260*a**6*c*d*x + 420*a**6*d**2*x**2 - 1260*a**4*b**2
*c**2*x**2 - 1890*a**4*b**2*c*d*x**3 - 756*a**4*b**2*d**2*x**4 + 756*a**2*
b**4*c**2*x**4 + 1260*a**2*b**4*c*d*x**5 + 540*a**2*b**4*d**2*x**6 - 180*b
**6*c**2*x**6 - 315*b**6*c*d*x**7 - 140*b**6*d**2*x**8))/1260
```

3.23 $\int (c+dx) (a^2-b^2x^2)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int (c+dx) \left(a^2-b^2x^2\right)^3 \, dx = a^6cx-a^4b^2cx^3+\frac{3}{5}a^2b^4cx^5-\frac{1}{7}b^6cx^7-\frac{d(a^2-b^2x^2)^4}{8b^2}$$

output
```
a^6*c*x-a^4*b^2*c*x^3+3/5*a^2*b^4*c*x^5-1/7*b^6*c*x^7-1/8*d*(-b^2*x^2+a^2)
^4/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int (c+dx) \left(a^2-b^2x^2\right)^3 \, dx = a^6cx+\frac{1}{2}a^6dx^2-a^4b^2cx^3-\frac{3}{4}a^4b^2dx^4 +\frac{3}{5}a^2b^4cx^5+\frac{1}{2}a^2b^4dx^6-\frac{1}{7}b^6cx^7-\frac{1}{8}b^6dx^8$$

input
```
Integrate[(c + d*x)*(a^2 - b^2*x^2)^3,x]
```

output
```
a^6*c*x + (a^6*d*x^2)/2 - a^4*b^2*c*x^3 - (3*a^4*b^2*d*x^4)/4 + (3*a^2*b^4
*c*x^5)/5 + (a^2*b^4*d*x^6)/2 - (b^6*c*x^7)/7 - (b^6*d*x^8)/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)^3 (c+dx)\, dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(a^2-b^2x^2\right)^3 dx-\frac{d\left(a^2-b^2x^2\right)^4}{8b^2}$$

$$\Big\downarrow \text{210}$$

$$c\int \left(a^6-3b^2x^2a^4+3b^4x^4a^2-b^6x^6\right) dx-\frac{d\left(a^2-b^2x^2\right)^4}{8b^2}$$

$$\Big\downarrow \text{2009}$$

$$c\left(a^6x-a^4b^2x^3+\frac{3}{5}a^2b^4x^5-\frac{1}{7}b^6x^7\right)-\frac{d\left(a^2-b^2x^2\right)^4}{8b^2}$$

input
```
Int[(c + d*x)*(a^2 - b^2*x^2)^3,x]
```

output
```
-1/8*(d*(a^2 - b^2*x^2)^4)/b^2 + c*(a^6*x - a^4*b^2*x^3 + (3*a^2*b^4*x^5)/
5 - (b^6*x^7)/7)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{1}{8}b^6dx^8-\frac{1}{7}b^6cx^7+\frac{1}{2}a^2b^4dx^6+\frac{3}{5}a^2b^4cx^5-\frac{3}{4}a^4b^2dx^4-a^4b^2cx^3+\frac{1}{2}a^6dx^2+a^6cx$	83
norman	$-\frac{1}{8}b^6dx^8-\frac{1}{7}b^6cx^7+\frac{1}{2}a^2b^4dx^6+\frac{3}{5}a^2b^4cx^5-\frac{3}{4}a^4b^2dx^4-a^4b^2cx^3+\frac{1}{2}a^6dx^2+a^6cx$	83
risch	$-\frac{1}{8}b^6dx^8-\frac{1}{7}b^6cx^7+\frac{1}{2}a^2b^4dx^6+\frac{3}{5}a^2b^4cx^5-\frac{3}{4}a^4b^2dx^4-a^4b^2cx^3+\frac{1}{2}a^6dx^2+a^6cx$	83
parallelrisch	$-\frac{1}{8}b^6dx^8-\frac{1}{7}b^6cx^7+\frac{1}{2}a^2b^4dx^6+\frac{3}{5}a^2b^4cx^5-\frac{3}{4}a^4b^2dx^4-a^4b^2cx^3+\frac{1}{2}a^6dx^2+a^6cx$	83
gosper	$\frac{x(-35b^6dx^7-40b^6cx^6+140a^2b^4dx^5+168a^2b^4cx^4-210a^4b^2dx^3-280a^4b^2cx^2+140a^6dx+280ca^6)}{280}$	84
orering	$\frac{x(-35b^6dx^7-40b^6cx^6+140a^2b^4dx^5+168a^2b^4cx^4-210a^4b^2dx^3-280a^4b^2cx^2+140a^6dx+280ca^6)(-b^2x^2+a^2)^3}{280(bx+a)^3(-bx+a)^3}$	113

input
```
int((d*x+c)*(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^6*d*x^8-1/7*b^6*c*x^7+1/2*a^2*b^4*d*x^6+3/5*a^2*b^4*c*x^5-3/4*a^4*b
^2*d*x^4-a^4*b^2*c*x^3+1/2*a^6*d*x^2+a^6*c*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int (c+dx)\left(a^2-b^2x^2\right)^3\,dx=-\frac{1}{8}\,b^6dx^8-\frac{1}{7}\,b^6cx^7+\frac{1}{2}\,a^2b^4dx^6+\frac{3}{5}\,a^2b^4cx^5-\frac{3}{4}\,a^4b^2dx^4-a^4b^2cx^3+\frac{1}{2}\,a^6dx^2+a^6cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/8*b^6*d*x^8 - 1/7*b^6*c*x^7 + 1/2*a^2*b^4*d*x^6 + 3/5*a^2*b^4*c*x^5 - 3
/4*a^4*b^2*d*x^4 - a^4*b^2*c*x^3 + 1/2*a^6*d*x^2 + a^6*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int (c+dx)\left(a^2-b^2x^2\right)^3\,dx = a^6cx+\frac{a^6dx^2}{2}-a^4b^2cx^3-\frac{3a^4b^2dx^4}{4}+\frac{3a^2b^4cx^5}{5}+\frac{a^2b^4dx^6}{2}-\frac{b^6cx^7}{7}-\frac{b^6dx^8}{8}$$

input
```
integrate((d*x+c)*(-b**2*x**2+a**2)**3,x)
```

output
```
a**6*c*x + a**6*d*x**2/2 - a**4*b**2*c*x**3 - 3*a**4*b**2*d*x**4/4 + 3*a**
2*b**4*c*x**5/5 + a**2*b**4*d*x**6/2 - b**6*c*x**7/7 - b**6*d*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int (c+dx)\left(a^2-b^2x^2\right)^3\,dx = -\frac{1}{8}\,b^6dx^8-\frac{1}{7}\,b^6cx^7+\frac{1}{2}\,a^2b^4dx^6+\frac{3}{5}\,a^2b^4cx^5-\frac{3}{4}\,a^4b^2dx^4-a^4b^2cx^3+\frac{1}{2}\,a^6dx^2+a^6cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/8*b^6*d*x^8 - 1/7*b^6*c*x^7 + 1/2*a^2*b^4*d*x^6 + 3/5*a^2*b^4*c*x^5 - 3
/4*a^4*b^2*d*x^4 - a^4*b^2*c*x^3 + 1/2*a^6*d*x^2 + a^6*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int (c+dx)\left(a^2-b^2x^2\right)^3\,dx = -\frac{1}{8}\,b^6dx^8-\frac{1}{7}\,b^6cx^7+\frac{1}{2}\,a^2b^4dx^6+\frac{3}{5}\,a^2b^4cx^5-\frac{3}{4}\,a^4b^2dx^4-a^4b^2cx^3+\frac{1}{2}\,a^6dx^2+a^6cx$$

input
```
integrate((d*x+c)*(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/8*b^6*d*x^8 - 1/7*b^6*c*x^7 + 1/2*a^2*b^4*d*x^6 + 3/5*a^2*b^4*c*x^5 - 3
/4*a^4*b^2*d*x^4 - a^4*b^2*c*x^3 + 1/2*a^6*d*x^2 + a^6*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int (c+dx)\left(a^2-b^2x^2\right)^3\,dx = \frac{d\,a^6\,x^2}{2}+c\,a^6\,x-\frac{3\,d\,a^4\,b^2\,x^4}{4}-c\,a^4\,b^2\,x^3+\frac{d\,a^2\,b^4\,x^6}{2}+\frac{3\,c\,a^2\,b^4\,x^5}{5}-\frac{d\,b^6\,x^8}{8}-\frac{c\,b^6\,x^7}{7}$$

input
```
int((a^2 - b^2*x^2)^3*(c + d*x),x)
```

output
```
(a^6*d*x^2)/2 - (b^6*c*x^7)/7 - (b^6*d*x^8)/8 + a^6*c*x - a^4*b^2*c*x^3 +
(3*a^2*b^4*c*x^5)/5 - (3*a^4*b^2*d*x^4)/4 + (a^2*b^4*d*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int (c+dx)\left(a^2-b^2x^2\right)^3\,dx$$
$$=\frac{x(-35b^6d\,x^7-40b^6c\,x^6+140a^2b^4d\,x^5+168a^2b^4c\,x^4-210a^4b^2d\,x^3-280a^4b^2c\,x^2+140a^6dx+280a^6c)}{280}$$

input
```
int((d*x+c)*(-b^2*x^2+a^2)^3,x)
```

output
```
(x*(280*a**6*c + 140*a**6*d*x - 280*a**4*b**2*c*x**2 - 210*a**4*b**2*d*x**
3 + 168*a**2*b**4*c*x**4 + 140*a**2*b**4*d*x**5 - 40*b**6*c*x**6 - 35*b**6
*d*x**7))/280
```

3.24 $\int (a^2 - b^2x^2)^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (a^2 - b^2x^2)^3 \, dx = a^6x - a^4b^2x^3 + \frac{3}{5}a^2b^4x^5 - \frac{b^6x^7}{7}$$

output
```
a^6*x-a^4*b^2*x^3+3/5*a^2*b^4*x^5-1/7*b^6*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (a^2 - b^2x^2)^3 \, dx = a^6x - a^4b^2x^3 + \frac{3}{5}a^2b^4x^5 - \frac{b^6x^7}{7}$$

input
```
Integrate[(a^2 - b^2*x^2)^3,x]
```

output
```
a^6*x - a^4*b^2*x^3 + (3*a^2*b^4*x^5)/5 - (b^6*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-b^2x^2\right)^3 \, dx$$
$$\Big\downarrow \text{ 210}$$
$$\int \left(a^6-3a^4b^2x^2+3a^2b^4x^4-b^6x^6\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$a^6x-a^4b^2x^3+\frac{3}{5}a^2b^4x^5-\frac{1}{7}b^6x^7$$

input
```
Int[(a^2 - b^2*x^2)^3,x]
```

output
```
a^6*x - a^4*b^2*x^3 + (3*a^2*b^4*x^5)/5 - (b^6*x^7)/7
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
default	$a^6x-a^4b^2x^3+\frac{3}{5}a^2b^4x^5-\frac{1}{7}b^6x^7$	37
norman	$a^6x-a^4b^2x^3+\frac{3}{5}a^2b^4x^5-\frac{1}{7}b^6x^7$	37
risch	$a^6x-a^4b^2x^3+\frac{3}{5}a^2b^4x^5-\frac{1}{7}b^6x^7$	37
parallelrisch	$a^6x-a^4b^2x^3+\frac{3}{5}a^2b^4x^5-\frac{1}{7}b^6x^7$	37
gosper	$\frac{x(-5b^6x^6+21a^2b^4x^4-35a^4b^2x^2+35a^6)}{35}$	40
orering	$\frac{x(-5b^6x^6+21a^2b^4x^4-35a^4b^2x^2+35a^6)(-b^2x^2+a^2)^3}{35(bx+a)^3(-bx+a)^3}$	69

input
```
int((-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
a^6*x-a^4*b^2*x^3+3/5*a^2*b^4*x^5-1/7*b^6*x^7
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a^2-b^2x^2\right)^3\,dx=-\frac{1}{7}\,b^6x^7+\frac{3}{5}\,a^2b^4x^5-a^4b^2x^3+a^6x$$

input
```
integrate((-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/7*b^6*x^7 + 3/5*a^2*b^4*x^5 - a^4*b^2*x^3 + a^6*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a^2-b^2x^2\right)^3\,dx = a^6x - a^4b^2x^3 + \frac{3a^2b^4x^5}{5} - \frac{b^6x^7}{7}$$

input
```
integrate((-b**2*x**2+a**2)**3,x)
```

output
```
a**6*x - a**4*b**2*x**3 + 3*a**2*b**4*x**5/5 - b**6*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a^2-b^2x^2\right)^3\,dx = -\frac{1}{7}\,b^6x^7 + \frac{3}{5}\,a^2b^4x^5 - a^4b^2x^3 + a^6x$$

input
```
integrate((-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/7*b^6*x^7 + 3/5*a^2*b^4*x^5 - a^4*b^2*x^3 + a^6*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a^2-b^2x^2\right)^3\,dx = -\frac{1}{7}\,b^6x^7 + \frac{3}{5}\,a^2b^4x^5 - a^4b^2x^3 + a^6x$$

input
```
integrate((-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/7*b^6*x^7 + 3/5*a^2*b^4*x^5 - a^4*b^2*x^3 + a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a^2-b^2x^2\right)^3\,dx=a^6\,x-a^4\,b^2\,x^3+\frac{3\,a^2\,b^4\,x^5}{5}-\frac{b^6\,x^7}{7}$$

input
```
int((a^2 - b^2*x^2)^3,x)
```

output
```
a^6*x - (b^6*x^7)/7 - a^4*b^2*x^3 + (3*a^2*b^4*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \left(a^2-b^2x^2\right)^3\,dx=\frac{x(-5b^6x^6+21a^2b^4x^4-35a^4b^2x^2+35a^6)}{35}$$

input
```
int((-b^2*x^2+a^2)^3,x)
```

output
```
(x*(35*a**6 - 35*a**4*b**2*x**2 + 21*a**2*b**4*x**4 - 5*b**6*x**6))/35
```

3.25 $\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 198

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = \frac{b^2c(b^4c^4-3a^2b^2c^2d^2+3a^4d^4)\,x}{d^6} - \frac{b^2(b^4c^4-3a^2b^2c^2d^2+3a^4d^4)\,x^2}{2d^5} + \frac{b^4c(b^2c^2-3a^2d^2)\,x^3}{3d^4} - \frac{b^4(b^2c^2-3a^2d^2)\,x^4}{4d^3} + \frac{b^6cx^5}{5d^2} - \frac{b^6x^6}{6d} - \frac{(b^2c^2-a^2d^2)^3\log(c+dx)}{d^7}$$

output

```
b^2*c*(3*a^4*d^4-3*a^2*b^2*c^2*d^2+b^4*c^4)*x/d^6-1/2*b^2*(3*a^4*d^4-3*a^2
*b^2*c^2*d^2+b^4*c^4)*x^2/d^5+1/3*b^4*c*(-3*a^2*d^2+b^2*c^2)*x^3/d^4-1/4*b
^4*(-3*a^2*d^2+b^2*c^2)*x^4/d^3+1/5*b^6*c*x^5/d^2-1/6*b^6*x^6/d-(-a^2*d^2+
b^2*c^2)^3*ln(d*x+c)/d^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.77

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = \frac{b^2dx(-90a^4d^4(-2c+dx)+15a^2b^2d^2(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3)+b^4(60c^5-30c^4dx+20c^3d^2x^2}{60d^7}$$

input
```
Integrate[(a^2 - b^2*x^2)^3/(c + d*x),x]
```

output
```
(b^2*d*x*(-90*a^4*d^4*(-2*c + d*x) + 15*a^2*b^2*d^2*(-12*c^3 + 6*c^2*d*x -
 4*c*d^2*x^2 + 3*d^3*x^3) + b^4*(60*c^5 - 30*c^4*d*x + 20*c^3*d^2*x^2 - 15
*c^2*d^3*x^3 + 12*c*d^4*x^4 - 10*d^5*x^5)) - 60*(b^2*c^2 - a^2*d^2)^3*Log[
c + d*x])/(60*d^7)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(a^2d^2-b^2c^2)^3}{d^6(c+dx)}+\frac{b^4cx^2(b^2c^2-3a^2d^2)}{d^4}-\frac{b^4x^3(b^2c^2-3a^2d^2)}{d^3}-\frac{b^2x(3a^4d^4-3a^2b^2c^2d^2+b^4c^4)}{d^5}+\frac{3a^4b^2cd^4}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(b^2c^2-a^2d^2)^3\log(c+dx)}{d^7}+\frac{b^4cx^3(b^2c^2-3a^2d^2)}{3d^4}-\frac{b^4x^4(b^2c^2-3a^2d^2)}{4d^3}+\frac{b^2cx(3a^4d^4-3a^2b^2c^2d^2+b^4c^4)}{d^6}-\frac{b^2x^2(3a^4d^4-3a^2b^2c^2d^2+b^4c^4)}{2d^5}+\frac{b^6cx^5}{5d^2}-\frac{b^6x^6}{6d}$$

input
```
Int[(a^2 - b^2*x^2)^3/(c + d*x),x]
```

output
```
(b^2*c*(b^4*c^4 - 3*a^2*b^2*c^2*d^2 + 3*a^4*d^4)*x)/d^6 - (b^2*(b^4*c^4 -
3*a^2*b^2*c^2*d^2 + 3*a^4*d^4)*x^2)/(2*d^5) + (b^4*c*(b^2*c^2 - 3*a^2*d^2)
*x^3)/(3*d^4) - (b^4*(b^2*c^2 - 3*a^2*d^2)*x^4)/(4*d^3) + (b^6*c*x^5)/(5*d
^2) - (b^6*x^6)/(6*d) - ((b^2*c^2 - a^2*d^2)^3*Log[c + d*x])/d^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^{2}\left(-\frac{b^{4}x^{6}d^{5}}{6}+\frac{b^{4}c\,x^{5}d^{4}}{5}-\frac{d\left(-3a^{2}b^{2}d^{4}+b^{4}c^{2}d^{2}\right)x^{4}}{4}+\frac{c\left(-3a^{2}b^{2}d^{4}+b^{4}c^{2}d^{2}\right)x^{3}}{3}-\frac{\left(3a^{4}d^{4}-3a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}\right)x^{2}d}{2}+c(3a^{4}d^{4}-3a^{2}b^{2}c^{2}\cdots}{d^{6}}$
norman	$\frac{b^{2}c\left(3a^{4}d^{4}-3a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}\right)x}{d^{6}}-\frac{b^{6}x^{6}}{6d}-\frac{b^{2}\left(3a^{4}d^{4}-3a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}\right)x^{2}}{2d^{5}}+\frac{b^{4}\left(3a^{2}d^{2}-b^{2}c^{2}\right)x^{4}}{4d^{3}}+\frac{b^{6}c\,x^{5}}{5d^{2}}-\frac{b^{4}c(3a^{2}\cdots}{\cdots}$
risch	$-\frac{b^{6}x^{6}}{6d}+\frac{b^{6}c\,x^{5}}{5d^{2}}+\frac{3b^{4}a^{2}x^{4}}{4d}-\frac{b^{6}c^{2}x^{4}}{4d^{3}}-\frac{b^{4}a^{2}c\,x^{3}}{d^{2}}+\frac{b^{6}c^{3}x^{3}}{3d^{4}}-\frac{3b^{2}a^{4}x^{2}}{2d}+\frac{3b^{4}a^{2}c^{2}x^{2}}{2d^{3}}-\frac{b^{6}c^{4}x^{2}}{2d^{5}}+\frac{3b^{2}a^{4}cx}{d^{2}}-\cdots$
parallelrisch	$\frac{-10x^{6}b^{6}d^{6}+12b^{6}c\,x^{5}d^{5}+45x^{4}a^{2}b^{4}d^{6}-15x^{4}b^{6}c^{2}d^{4}-60x^{3}a^{2}b^{4}c\,d^{5}+20x^{3}b^{6}c^{3}d^{3}-90x^{2}a^{4}b^{2}d^{6}+90x^{2}a^{2}b^{4}c^{2}d^{4}-30x^{2}b^{6}c^{4}d^{2}+\cdots}{6\cdots}$

input
```
int((-b^2*x^2+a^2)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
b^2/d^6*(-1/6*b^4*x^6*d^5+1/5*b^4*c*x^5*d^4-1/4*d*(-3*a^2*b^2*d^4+b^4*c^2*
d^2)*x^4+1/3*c*(-3*a^2*b^2*d^4+b^4*c^2*d^2)*x^3-1/2*(3*a^4*d^4-3*a^2*b^2*c
^2*d^2+b^4*c^4)*x^2*d+c*(3*a^4*d^4-3*a^2*b^2*c^2*d^2+b^4*c^4)*x)+(a^6*d^6-
3*a^4*b^2*c^2*d^4+3*a^2*b^4*c^4*d^2-b^6*c^6)/d^7*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = -\frac{10\,b^6d^6x^6-12\,b^6cd^5x^5+15\,(b^6c^2d^4-3\,a^2b^4d^6)x^4-20\,(b^6c^3d^3-3\,a^2b^4cd^5)x^3+30\,(b^6c^4d^2-3\,a^2b^4c^2d}{}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c),x, algorithm="fricas")
```

output
```
-1/60*(10*b^6*d^6*x^6 - 12*b^6*c*d^5*x^5 + 15*(b^6*c^2*d^4 - 3*a^2*b^4*d^6
)*x^4 - 20*(b^6*c^3*d^3 - 3*a^2*b^4*c*d^5)*x^3 + 30*(b^6*c^4*d^2 - 3*a^2*b
^4*c^2*d^4 + 3*a^4*b^2*d^6)*x^2 - 60*(b^6*c^5*d - 3*a^2*b^4*c^3*d^3 + 3*a^
4*b^2*c*d^5)*x + 60*(b^6*c^6 - 3*a^2*b^4*c^4*d^2 + 3*a^4*b^2*c^2*d^4 - a^6
*d^6)*log(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.95

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = \frac{b^6cx^5}{5d^2} - \frac{b^6x^6}{6d} - x^4\left(-\frac{3a^2b^4}{4d}+\frac{b^6c^2}{4d^3}\right) - x^3\left(\frac{a^2b^4c}{d^2}-\frac{b^6c^3}{3d^4}\right) - x^2 \cdot \left(\frac{3a^4b^2}{2d}-\frac{3a^2b^4c^2}{2d^3}+\frac{b^6c^4}{2d^5}\right) - x\left(-\frac{3a^4b^2c}{d^2}+\frac{3a^2b^4c^3}{d^4}-\frac{b^6c^5}{d^6}\right) + \frac{(ad-bc)^3\,(ad+bc)^3\log{(c+dx)}}{d^7}$$

input
```
integrate((-b**2*x**2+a**2)**3/(d*x+c),x)
```

output
```
b**6*c*x**5/(5*d**2) - b**6*x**6/(6*d) - x**4*(-3*a**2*b**4/(4*d) + b**6*c
**2/(4*d**3)) - x**3*(a**2*b**4*c/d**2 - b**6*c**3/(3*d**4)) - x**2*(3*a**
4*b**2/(2*d) - 3*a**2*b**4*c**2/(2*d**3) + b**6*c**4/(2*d**5)) - x*(-3*a**
4*b**2*c/d**2 + 3*a**2*b**4*c**3/d**4 - b**6*c**5/d**6) + (a*d - b*c)**3*(
a*d + b*c)**3*log(c + d*x)/d**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = -\frac{10\,b^6d^5x^6-12\,b^6cd^4x^5+15\,(b^6c^2d^3-3\,a^2b^4d^5)x^4-20\,(b^6c^3d^2-3\,a^2b^4cd^4)x^3+30\,(b^6c^4d-3\,a^2b^4c^2d^3}{60\,d^6} - \frac{(b^6c^6-3\,a^2b^4c^4d^2+3\,a^4b^2c^2d^4-a^6d^6)\log(dx+c)}{d^7}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/60*(10*b^6*d^5*x^6 - 12*b^6*c*d^4*x^5 + 15*(b^6*c^2*d^3 - 3*a^2*b^4*d^5
)*x^4 - 20*(b^6*c^3*d^2 - 3*a^2*b^4*c*d^4)*x^3 + 30*(b^6*c^4*d - 3*a^2*b^4
*c^2*d^3 + 3*a^4*b^2*d^5)*x^2 - 60*(b^6*c^5 - 3*a^2*b^4*c^3*d^2 + 3*a^4*b^
2*c*d^4)*x)/d^6 - (b^6*c^6 - 3*a^2*b^4*c^4*d^2 + 3*a^4*b^2*c^2*d^4 - a^6*d
^6)*log(d*x + c)/d^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = -\frac{10\,b^6d^5x^6-12\,b^6cd^4x^5+15\,b^6c^2d^3x^4-45\,a^2b^4d^5x^4-20\,b^6c^3d^2x^3+60\,a^2b^4cd^4x^3+30\,b^6c^4dx^2-90\,a^2}{60\,d^6} - \frac{(b^6c^6-3\,a^2b^4c^4d^2+3\,a^4b^2c^2d^4-a^6d^6)\log(|dx+c|)}{d^7}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c),x, algorithm="giac")
```

output
```
-1/60*(10*b^6*d^5*x^6 - 12*b^6*c*d^4*x^5 + 15*b^6*c^2*d^3*x^4 - 45*a^2*b^4
*d^5*x^4 - 20*b^6*c^3*d^2*x^3 + 60*a^2*b^4*c*d^4*x^3 + 30*b^6*c^4*d*x^2 -
90*a^2*b^4*c^2*d^3*x^2 + 90*a^4*b^2*d^5*x^2 - 60*b^6*c^5*x + 180*a^2*b^4*c
^3*d^2*x - 180*a^4*b^2*c*d^4*x)/d^6 - (b^6*c^6 - 3*a^2*b^4*c^4*d^2 + 3*a^4
*b^2*c^2*d^4 - a^6*d^6)*log(abs(d*x + c))/d^7
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.18

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = x^2\left(\frac{c^2\left(\frac{3\,a^2\,b^4}{d}-\frac{b^6\,c^2}{d^3}\right)}{2\,d^2}-\frac{3\,a^4\,b^2}{2\,d}\right)+x^4\left(\frac{3\,a^2\,b^4}{4\,d}-\frac{b^6\,c^2}{4\,d^3}\right)+\frac{\ln(c+d\,x)\,\left(a^6\,d^6-3\,a^4\,b^2\,c^2\,d^4+3\,a^2\,b^4\,c^4\,d^2-b^6\,c^6\right)}{d^7}-\frac{b^6\,x^6}{6\,d}-\frac{c\,x^3\left(\frac{3\,a^2\,b^4}{d}-\frac{b^6\,c^2}{d^3}\right)}{3\,d}+\frac{b^6\,c\,x^5}{5\,d^2}-\frac{c\,x\left(\frac{c^2\left(\frac{3\,a^2\,b^4}{d}-\frac{b^6\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^4\,b^2}{d}\right)}{d}$$

input
```
int((a^2 - b^2*x^2)^3/(c + d*x),x)
```

output
```
x^2*((c^2*((3*a^2*b^4)/d - (b^6*c^2)/d^3))/(2*d^2) - (3*a^4*b^2)/(2*d)) +
x^4*((3*a^2*b^4)/(4*d) - (b^6*c^2)/(4*d^3)) + (log(c + d*x)*(a^6*d^6 - b^6
*c^6 + 3*a^2*b^4*c^4*d^2 - 3*a^4*b^2*c^2*d^4))/d^7 - (b^6*x^6)/(6*d) - (c*
x^3*((3*a^2*b^4)/d - (b^6*c^2)/d^3))/(3*d) + (b^6*c*x^5)/(5*d^2) - (c*x*((
c^2*((3*a^2*b^4)/d - (b^6*c^2)/d^3))/d^2 - (3*a^4*b^2)/d))/d
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.20

$$\int \frac{(a^2-b^2x^2)^3}{c+dx}\,dx = \frac{60\log(dx+c)\,a^6d^6-180\log(dx+c)\,a^4b^2c^2d^4+180\log(dx+c)\,a^2b^4c^4d^2-60\log(dx+c)\,b^6c^6+180a^4b$$

input
```
int((-b^2*x^2+a^2)^3/(d*x+c),x)
```

output
```
(60*log(c + d*x)*a**6*d**6 - 180*log(c + d*x)*a**4*b**2*c**2*d**4 + 180*lo
g(c + d*x)*a**2*b**4*c**4*d**2 - 60*log(c + d*x)*b**6*c**6 + 180*a**4*b**2
*c*d**5*x - 90*a**4*b**2*d**6*x**2 - 180*a**2*b**4*c**3*d**3*x + 90*a**2*b
**4*c**2*d**4*x**2 - 60*a**2*b**4*c*d**5*x**3 + 45*a**2*b**4*d**6*x**4 + 6
0*b**6*c**5*d*x - 30*b**6*c**4*d**2*x**2 + 20*b**6*c**3*d**3*x**3 - 15*b**
6*c**2*d**4*x**4 + 12*b**6*c*d**5*x**5 - 10*b**6*d**6*x**6)/(60*d**7)
```

3.26 $\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2} \, dx = -\frac{b^2(5b^4c^4-9a^2b^2c^2d^2+3a^4d^4)\,x}{d^6}+\frac{b^4c(2b^2c^2-3a^2d^2)\,x^2}{d^5} -\frac{b^4(bc-ad)(bc+ad)x^3}{d^4}+\frac{b^6cx^4}{2d^3}-\frac{b^6x^5}{5d^2} +\frac{(b^2c^2-a^2d^2)^3}{d^7(c+dx)}+\frac{6b^2c(b^2c^2-a^2d^2)^2\log(c+dx)}{d^7}$$

output
```
-b^2*(3*a^4*d^4-9*a^2*b^2*c^2*d^2+5*b^4*c^4)*x/d^6+b^4*c*(-3*a^2*d^2+2*b^2
*c^2)*x^2/d^5-b^4*(-a*d+b*c)*(a*d+b*c)*x^3/d^4+1/2*b^6*c*x^4/d^3-1/5*b^6*x
^5/d^2+(-a^2*d^2+b^2*c^2)^3/d^7/(d*x+c)+6*b^2*c*(-a^2*d^2+b^2*c^2)^2*ln(d*
x+c)/d^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2}\,dx$$
$$= \frac{-10a^6d^6-30a^4b^2d^4(-c^2+cdx+d^2x^2)+10a^2b^4d^2(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4)+b^6(10c}{10d^7(}$$

input
```
Integrate[(a^2 - b^2*x^2)^3/(c + d*x)^2,x]
```

output
```
(-10*a^6*d^6 - 30*a^4*b^2*d^4*(-c^2 + c*d*x + d^2*x^2) + 10*a^2*b^4*d^2*(-
3*c^4 + 9*c^3*d*x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) + b^6*(10*c^6 -
 50*c^5*d*x - 30*c^4*d^2*x^2 + 10*c^3*d^3*x^3 - 5*c^2*d^4*x^4 + 3*c*d^5*x^
5 - 2*d^6*x^6) + 60*c*(b^3*c^2 - a^2*b*d^2)^2*(c + d*x)*Log[c + d*x])/(10*
d^7*(c + d*x))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{476}$$
$$\int \left(\frac{6c(b^3c^2-a^2bd^2)^2}{d^6(c+dx)}+\frac{(a^2d^2-b^2c^2)^3}{d^6(c+dx)^2}+\frac{2b^4cx(2b^2c^2-3a^2d^2)}{d^5}-\frac{3b^4x^2(b^2c^2-a^2d^2)}{d^4}+\frac{-3a^4b^2d^4+9a^2b^4c^2}{d^6}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{(b^2c^2-a^2d^2)^3}{d^7(c+dx)}+\frac{6b^2c(b^2c^2-a^2d^2)^2\log(c+dx)}{d^7}+\frac{b^4cx^2(2b^2c^2-3a^2d^2)}{d^5}-\frac{b^2x(3a^4d^4-9a^2b^2c^2d^2+5b^4c^4)}{d^6}-\frac{b^4x^3(bc-ad)(ad+bc)}{d^4}+\frac{b^6cx^4}{2d^3}-\frac{b^6x^5}{5d^2}$$

input
```
Int[(a^2 - b^2*x^2)^3/(c + d*x)^2,x]
```

output
```
-((b^2*(5*b^4*c^4 - 9*a^2*b^2*c^2*d^2 + 3*a^4*d^4)*x)/d^6) + (b^4*c*(2*b^2
*c^2 - 3*a^2*d^2)*x^2)/d^5 - (b^4*(b*c - a*d)*(b*c + a*d)*x^3)/d^4 + (b^6*
c*x^4)/(2*d^3) - (b^6*x^5)/(5*d^2) + (b^2*c^2 - a^2*d^2)^3/(d^7*(c + d*x))
 + (6*b^2*c*(b^2*c^2 - a^2*d^2)^2*Log[c + d*x])/d^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

method	result
default	$-\frac{b^2\left(\frac{1}{5}b^4x^5d^4-\frac{1}{2}b^4c\,x^4d^3-x^3a^2b^2d^4+x^3b^4c^2d^2+3x^2a^2b^2c\,d^3-2x^2b^4c^3d+3a^4d^4x-9a^2b^2c^2d^2x+5c^4b^4x\right)}{d^6}-\frac{a^6d^6-3a^4b^2c\ldots}{\ldots}$
norman	$\frac{\frac{\left(a^6d^6-6a^4b^2c^2d^4+12a^2b^4c^4d^2-6b^6c^6\right)x}{c\,d^6}-\frac{b^6x^6}{5d}-\frac{3b^2\left(a^4d^4-2a^2b^2c^2d^2+c^4b^4\right)x^2}{d^5}+\frac{b^4\left(2a^2d^2-b^2c^2\right)x^4}{2d^3}+\frac{3b^6c\,x^5}{10d^2}-\frac{c\,b^4\left(2a^2d^2-b^2c\ldots\right)}{d^4}\ldots}{dx+c}$
risch	$-\frac{b^6x^5}{5d^2}+\frac{b^6c\,x^4}{2d^3}+\frac{b^4x^3a^2}{d^2}-\frac{b^6x^3c^2}{d^4}-\frac{3b^4x^2a^2c}{d^3}+\frac{2b^6x^2c^3}{d^5}-\frac{3b^2a^4x}{d^2}+\frac{9b^4a^2c^2x}{d^4}-\frac{5b^6c^4x}{d^6}-\frac{a^6}{d(dx+c)}+\ldots$
parallelrisch	$\frac{-2x^6b^6d^6+3b^6c\,x^5d^5+10x^4a^2b^4d^6-5x^4b^6c^2d^4-20x^3a^2b^4c\,d^5+10x^3b^6c^3d^3+60\ln(dx+c)x\,a^4b^2c\,d^5-120\ln(dx+c)x\,a^2b^4c^3\ldots}{\ldots}$

input
```
int((-b^2*x^2+a^2)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2/d^6*(1/5*b^4*x^5*d^4-1/2*b^4*c*x^4*d^3-x^3*a^2*b^2*d^4+x^3*b^4*c^2*d^
2+3*x^2*a^2*b^2*c*d^3-2*x^2*b^4*c^3*d+3*a^4*d^4*x-9*a^2*b^2*c^2*d^2*x+5*c^
4*b^4*x)-(a^6*d^6-3*a^4*b^2*c^2*d^4+3*a^2*b^4*c^4*d^2-b^6*c^6)/d^7/(d*x+c)
+6*b^2*c/d^7*(a^4*d^4-2*a^2*b^2*c^2*d^2+b^4*c^4)*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.61

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2}\,dx =$$
$$-\frac{2\,b^6d^6x^6 - 3\,b^6cd^5x^5 - 10\,b^6c^6 + 30\,a^2b^4c^4d^2 - 30\,a^4b^2c^2d^4 + 10\,a^6d^6 + 5\,(b^6c^2d^4 - 2\,a^2b^4d^6)x^4 - 10\,(b^6}{}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/10*(2*b^6*d^6*x^6 - 3*b^6*c*d^5*x^5 - 10*b^6*c^6 + 30*a^2*b^4*c^4*d^2 -
 30*a^4*b^2*c^2*d^4 + 10*a^6*d^6 + 5*(b^6*c^2*d^4 - 2*a^2*b^4*d^6)*x^4 - 1
0*(b^6*c^3*d^3 - 2*a^2*b^4*c*d^5)*x^3 + 30*(b^6*c^4*d^2 - 2*a^2*b^4*c^2*d^
4 + a^4*b^2*d^6)*x^2 + 10*(5*b^6*c^5*d - 9*a^2*b^4*c^3*d^3 + 3*a^4*b^2*c*d
^5)*x - 60*(b^6*c^6 - 2*a^2*b^4*c^4*d^2 + a^4*b^2*c^2*d^4 + (b^6*c^5*d - 2
*a^2*b^4*c^3*d^3 + a^4*b^2*c*d^5)*x)*log(d*x + c))/(d^8*x + c*d^7)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2}\,dx = {} & \frac{b^6cx^4}{2d^3} - \frac{b^6x^5}{5d^2} + \frac{6b^2c(ad-bc)^2\,(ad+bc)^2\log{(c+dx)}}{d^7} \\ & - x^3\left(-\frac{a^2b^4}{d^2} + \frac{b^6c^2}{d^4}\right) - x^2\cdot\left(\frac{3a^2b^4c}{d^3} - \frac{2b^6c^3}{d^5}\right) \\ & - x\left(\frac{3a^4b^2}{d^2} - \frac{9a^2b^4c^2}{d^4} + \frac{5b^6c^4}{d^6}\right) \\ & - \frac{a^6d^6 - 3a^4b^2c^2d^4 + 3a^2b^4c^4d^2 - b^6c^6}{cd^7 + d^8x}\end{aligned}$$

input
```
integrate((-b**2*x**2+a**2)**3/(d*x+c)**2,x)
```

output
```
b**6*c*x**4/(2*d**3) - b**6*x**5/(5*d**2) + 6*b**2*c*(a*d - b*c)**2*(a*d +
 b*c)**2*log(c + d*x)/d**7 - x**3*(-a**2*b**4/d**2 + b**6*c**2/d**4) - x**
2*(3*a**2*b**4*c/d**3 - 2*b**6*c**3/d**5) - x*(3*a**4*b**2/d**2 - 9*a**2*b
**4*c**2/d**4 + 5*b**6*c**4/d**6) - (a**6*d**6 - 3*a**4*b**2*c**2*d**4 + 3
*a**2*b**4*c**4*d**2 - b**6*c**6)/(c*d**7 + d**8*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.21

$$\int \frac{(a^2 - b^2x^2)^3}{(c + dx)^2}\, dx = \frac{b^6c^6 - 3\,a^2b^4c^4d^2 + 3\,a^4b^2c^2d^4 - a^6d^6}{d^8x + cd^7} - \frac{2\,b^6d^4x^5 - 5\,b^6cd^3x^4 + 10\,(b^6c^2d^2 - a^2b^4d^4)x^3 - 10\,(2\,b^6c^3d - 3\,a^2b^4cd^3)x^2 + 10\,(5\,b^6c^4 - 9\,a^2b^4c^2d^2 +}{10\,d^6} + \frac{6\,(b^6c^5 - 2\,a^2b^4c^3d^2 + a^4b^2cd^4)\log{(dx + c)}}{d^7}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(b^6*c^6 - 3*a^2*b^4*c^4*d^2 + 3*a^4*b^2*c^2*d^4 - a^6*d^6)/(d^8*x + c*d^7
) - 1/10*(2*b^6*d^4*x^5 - 5*b^6*c*d^3*x^4 + 10*(b^6*c^2*d^2 - a^2*b^4*d^4)
*x^3 - 10*(2*b^6*c^3*d - 3*a^2*b^4*c*d^3)*x^2 + 10*(5*b^6*c^4 - 9*a^2*b^4*
c^2*d^2 + 3*a^4*b^2*d^4)*x)/d^6 + 6*(b^6*c^5 - 2*a^2*b^4*c^3*d^2 + a^4*b^2
*c*d^4)*log(d*x + c)/d^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.56

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2}\,dx =$$
$$-\frac{\left(2\,b^6-\frac{15\,b^6c}{dx+c}+\frac{10\,(5\,b^6c^2d^2-a^2b^4d^4)}{(dx+c)^2d^2}-\frac{20\,(5\,b^6c^3d^3-3\,a^2b^4cd^5)}{(dx+c)^3d^3}+\frac{30\,(5\,b^6c^4d^4-6\,a^2b^4c^2d^6+a^4b^2d^8)}{(dx+c)^4d^4}\right)(dx+c)^5}{10\,d^7}$$
$$-\frac{6\,(b^6c^5-2\,a^2b^4c^3d^2+a^4b^2cd^4)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^7}+\frac{\frac{b^6c^6d^5}{dx+c}-\frac{3\,a^2b^4c^4d^7}{dx+c}+\frac{3\,a^4b^2c^2d^9}{dx+c}-\frac{a^6d^{11}}{dx+c}}{d^{12}}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/10*(2*b^6 - 15*b^6*c/(d*x + c) + 10*(5*b^6*c^2*d^2 - a^2*b^4*d^4)/((d*x
 + c)^2*d^2) - 20*(5*b^6*c^3*d^3 - 3*a^2*b^4*c*d^5)/((d*x + c)^3*d^3) + 30
*(5*b^6*c^4*d^4 - 6*a^2*b^4*c^2*d^6 + a^4*b^2*d^8)/((d*x + c)^4*d^4))*(d*x
 + c)^5/d^7 - 6*(b^6*c^5 - 2*a^2*b^4*c^3*d^2 + a^4*b^2*c*d^4)*log(abs(d*x
+ c)/((d*x + c)^2*abs(d)))/d^7 + (b^6*c^6*d^5/(d*x + c) - 3*a^2*b^4*c^4*d^
7/(d*x + c) + 3*a^4*b^2*c^2*d^9/(d*x + c) - a^6*d^11/(d*x + c))/d^12
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.59

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2}\,dx = x^3\left(\frac{a^2\,b^4}{d^2}-\frac{b^6\,c^2}{d^4}\right)-x^2\left(\frac{c\left(\frac{3\,a^2\,b^4}{d^2}-\frac{3\,b^6\,c^2}{d^4}\right)}{d}+\frac{b^6\,c^3}{d^5}\right) - x\left(\frac{c^2\left(\frac{3\,a^2\,b^4}{d^2}-\frac{3\,b^6\,c^2}{d^4}\right)}{d^2}+\frac{3\,a^4\,b^2}{d^2} - \frac{2\,c\left(\frac{2\,c\left(\frac{3\,a^2\,b^4}{d^2}-\frac{3\,b^6\,c^2}{d^4}\right)}{d}+\frac{2\,b^6\,c^3}{d^5}\right)}{d}\right) + \frac{\ln\left(c+d\,x\right)\,\left(6\,a^4\,b^2\,c\,d^4-12\,a^2\,b^4\,c^3\,d^2+6\,b^6\,c^5\right)}{d^7}-\frac{b^6\,x^5}{5\,d^2} - \frac{a^6\,d^6-3\,a^4\,b^2\,c^2\,d^4+3\,a^2\,b^4\,c^4\,d^2-b^6\,c^6}{d\,\left(x\,d^7+c\,d^6\right)}+\frac{b^6\,c\,x^4}{2\,d^3}$$

input
```
int((a^2 - b^2*x^2)^3/(c + d*x)^2,x)
```

output
```
x^3*((a^2*b^4)/d^2 - (b^6*c^2)/d^4) - x^2*((c*((3*a^2*b^4)/d^2 - (3*b^6*c^
2)/d^4))/d + (b^6*c^3)/d^5) - x*((c^2*((3*a^2*b^4)/d^2 - (3*b^6*c^2)/d^4))
/d^2 + (3*a^4*b^2)/d^2 - (2*c*((2*c*((3*a^2*b^4)/d^2 - (3*b^6*c^2)/d^4))/d
 + (2*b^6*c^3)/d^5))/d) + (log(c + d*x)*(6*b^6*c^5 + 6*a^4*b^2*c*d^4 - 12*
a^2*b^4*c^3*d^2))/d^7 - (b^6*x^5)/(5*d^2) - (a^6*d^6 - b^6*c^6 + 3*a^2*b^4
*c^4*d^2 - 3*a^4*b^2*c^2*d^4)/(d*(c*d^6 + d^7*x)) + (b^6*c*x^4)/(2*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.68

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^2}\,dx = \frac{60\log(dx+c)\,a^4b^2c^3d^4+60\log(dx+c)\,a^4b^2c^2d^5x-120\log(dx+c)\,a^2b^4c^5d^2-120\log(dx+c)\,a^2b^4c^4d^3}{}$$

input
```
int((-b^2*x^2+a^2)^3/(d*x+c)^2,x)
```

output
```
(60*log(c + d*x)*a**4*b**2*c**3*d**4 + 60*log(c + d*x)*a**4*b**2*c**2*d**5
*x - 120*log(c + d*x)*a**2*b**4*c**5*d**2 - 120*log(c + d*x)*a**2*b**4*c**
4*d**3*x + 60*log(c + d*x)*b**6*c**7 + 60*log(c + d*x)*b**6*c**6*d*x + 10*
a**6*d**7*x - 60*a**4*b**2*c**2*d**5*x - 30*a**4*b**2*c*d**6*x**2 + 120*a*
*2*b**4*c**4*d**3*x + 60*a**2*b**4*c**3*d**4*x**2 - 20*a**2*b**4*c**2*d**5
*x**3 + 10*a**2*b**4*c*d**6*x**4 - 60*b**6*c**6*d*x - 30*b**6*c**5*d**2*x*
*2 + 10*b**6*c**4*d**3*x**3 - 5*b**6*c**3*d**4*x**4 + 3*b**6*c**2*d**5*x**
5 - 2*b**6*c*d**6*x**6)/(10*c*d**7*(c + d*x))
```

3.27 $\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 191

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3} dx = \frac{b^4c(10b^2c^2-9a^2d^2)\,x}{d^6} - \frac{3b^4(2b^2c^2-a^2d^2)\,x^2}{2d^5} + \frac{b^6cx^3}{d^4} - \frac{b^6x^4}{4d^3} + \frac{(b^2c^2-a^2d^2)^3}{2d^7(c+dx)^2} - \frac{6b^2c(b^2c^2-a^2d^2)^2}{d^7(c+dx)} - \frac{3b^2(5b^4c^4-6a^2b^2c^2d^2+a^4d^4)\log(c+dx)}{d^7}$$

output
```
b^4*c*(-9*a^2*d^2+10*b^2*c^2)*x/d^6-3/2*b^4*(-a^2*d^2+2*b^2*c^2)*x^2/d^5+b
^6*c*x^3/d^4-1/4*b^6*x^4/d^3+1/2*(-a^2*d^2+b^2*c^2)^3/d^7/(d*x+c)^2-6*b^2*
c*(-a^2*d^2+b^2*c^2)^2/d^7/(d*x+c)-3*b^2*(a^4*d^4-6*a^2*b^2*c^2*d^2+5*b^4*
c^4)*ln(d*x+c)/d^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.09

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3}\,dx = \frac{-2a^6d^6-6a^4b^2cd^4(3c+4dx)+6a^2b^4d^2(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4)+b^6(-22c^6+16c^5dx}{4d^7(}$$

input
```
Integrate[(a^2 - b^2*x^2)^3/(c + d*x)^3,x]
```

output
```
(-2*a^6*d^6 - 6*a^4*b^2*c*d^4*(3*c + 4*d*x) + 6*a^2*b^4*d^2*(7*c^4 + 2*c^3
*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^6*(-22*c^6 + 16*c^5*d*x
 + 68*c^4*d^2*x^2 + 20*c^3*d^3*x^3 - 5*c^2*d^4*x^4 + 2*c*d^5*x^5 - d^6*x^6
) - 12*b^2*(5*b^4*c^4 - 6*a^2*b^2*c^2*d^2 + a^4*d^4)*(c + d*x)^2*Log[c + d
*x])/(4*d^7*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{6c(b^3c^2-a^2bd^2)^2}{d^6(c+dx)^2}+\frac{(a^2d^2-b^2c^2)^3}{d^6(c+dx)^3}+\frac{10b^6c^3-9a^2b^4cd^2}{d^6}-\frac{3b^4x(2b^2c^2-a^2d^2)}{d^5}-\frac{3(a^4b^2d^4-6a^2b^4c^2d^2+}{d^6(c+dx)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b^2c\left(b^2c^2-a^2d^2\right)^2}{d^7(c+dx)}+\frac{\left(b^2c^2-a^2d^2\right)^3}{2d^7(c+dx)^2}+\frac{b^4cx\left(10b^2c^2-9a^2d^2\right)}{d^6}-\frac{3b^4x^2\left(2b^2c^2-a^2d^2\right)}{2d^5}-\frac{3b^2\left(a^4d^4-6a^2b^2c^2d^2+5b^4c^4\right)\log(c+dx)}{d^7}+\frac{b^6cx^3}{d^4}-\frac{b^6x^4}{4d^3}$$

input
```
Int[(a^2 - b^2*x^2)^3/(c + d*x)^3,x]
```

output
```
(b^4*c*(10*b^2*c^2 - 9*a^2*d^2)*x)/d^6 - (3*b^4*(2*b^2*c^2 - a^2*d^2)*x^2)
/(2*d^5) + (b^6*c*x^3)/d^4 - (b^6*x^4)/(4*d^3) + (b^2*c^2 - a^2*d^2)^3/(2*
d^7*(c + d*x)^2) - (6*b^2*c*(b^2*c^2 - a^2*d^2)^2)/(d^7*(c + d*x)) - (3*b^
2*(5*b^4*c^4 - 6*a^2*b^2*c^2*d^2 + a^4*d^4)*Log[c + d*x])/d^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

method	result
default	$-\frac{b^4\left(\frac{1}{4}b^2x^4d^3-b^2c\,x^3d^2-\frac{3}{2}a^2d^3x^2+3b^2c^2d\,x^2+9a^2c\,d^2x-10c^3b^2x\right)}{d^6}-\frac{6b^2c\left(a^4d^4-2a^2b^2c^2d^2+c^4b^4\right)}{d^7(dx+c)}-\frac{a^6d^6-3a^4b^2c^2d ...}{2d^7(...}$
norman	$\frac{-\frac{a^6d^6+9a^4b^2c^2d^4-54a^2b^4c^4d^2+45b^6c^6}{2d^7}-\frac{b^6x^6}{4d}+\frac{b^4\left(6a^2d^2-5b^2c^2\right)x^4}{4d^3}+\frac{b^6c\,x^5}{2d^2}-\frac{2c\left(3d^4a^4b^2-18a^2b^4c^2d^2+15b^6c^4\right)x}{d^6}-\frac{b^4c\left(6a^2d^2- ...\right)}{d^4 ...}}{(dx+c)^2}$
risch	$-\frac{b^6x^4}{4d^3}+\frac{b^6c\,x^3}{d^4}+\frac{3b^4a^2x^2}{2d^3}-\frac{3b^6c^2x^2}{d^5}-\frac{9b^4a^2cx}{d^4}+\frac{10b^6c^3x}{d^6}+\frac{\left(-6a^4b^2c\,d^4+12a^2b^4c^3d^2-6b^6c^5\right)x-\frac{a^6d^6+9a^4b^2c ...}{...}}{d^6(dx+c)^2}$
parallelrisch	$-\frac{12\ln(dx+c)x^2a^4b^2d^6+60\ln(dx+c)x^2b^6c^4d^2+90b^6c^6+120\ln(dx+c)x\,b^6c^5d-72\ln(dx+c)x^2a^2b^4c^2d^4+24\ln(dx+c)x\,a^4b^2c ...}{...}$

input
```
int((-b^2*x^2+a^2)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^4/d^6*(1/4*b^2*x^4*d^3-b^2*c*x^3*d^2-3/2*a^2*d^3*x^2+3*b^2*c^2*d*x^2+9*
a^2*c*d^2*x-10*c^3*b^2*x)-6*b^2*c/d^7*(a^4*d^4-2*a^2*b^2*c^2*d^2+b^4*c^4)/
(d*x+c)-1/2*(a^6*d^6-3*a^4*b^2*c^2*d^4+3*a^2*b^4*c^4*d^2-b^6*c^6)/d^7/(d*x
+c)^2-3*b^2*(a^4*d^4-6*a^2*b^2*c^2*d^2+5*b^4*c^4)*ln(d*x+c)/d^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.79

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3}\,dx = -\frac{b^6d^6x^6-2\,b^6cd^5x^5+22\,b^6c^6-42\,a^2b^4c^4d^2+18\,a^4b^2c^2d^4+2\,a^6d^6+(5\,b^6c^2d^4-6\,a^2b^4d^6)x^4-4\,(5\,b^6c^3}{}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^6*d^6*x^6 - 2*b^6*c*d^5*x^5 + 22*b^6*c^6 - 42*a^2*b^4*c^4*d^2 + 18
*a^4*b^2*c^2*d^4 + 2*a^6*d^6 + (5*b^6*c^2*d^4 - 6*a^2*b^4*d^6)*x^4 - 4*(5*
b^6*c^3*d^3 - 6*a^2*b^4*c*d^5)*x^3 - 2*(34*b^6*c^4*d^2 - 33*a^2*b^4*c^2*d^
4)*x^2 - 4*(4*b^6*c^5*d + 3*a^2*b^4*c^3*d^3 - 6*a^4*b^2*c*d^5)*x + 12*(5*b
^6*c^6 - 6*a^2*b^4*c^4*d^2 + a^4*b^2*c^2*d^4 + (5*b^6*c^4*d^2 - 6*a^2*b^4*
c^2*d^4 + a^4*b^2*d^6)*x^2 + 2*(5*b^6*c^5*d - 6*a^2*b^4*c^3*d^3 + a^4*b^2*
c*d^5)*x)*log(d*x + c))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.23

$$\begin{aligned}\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3}\,dx &= \frac{b^6cx^3}{d^4}-\frac{b^6x^4}{4d^3}-\frac{3b^2(ad-bc)\,(ad+bc)\,(a^2d^2-5b^2c^2)\log{(c+dx)}}{d^7} \\ &\quad - x^2\left(-\frac{3a^2b^4}{2d^3}+\frac{3b^6c^2}{d^5}\right)-x\left(\frac{9a^2b^4c}{d^4}-\frac{10b^6c^3}{d^6}\right) \\ &\quad - \frac{a^6d^6+9a^4b^2c^2d^4-21a^2b^4c^4d^2+11b^6c^6+x(12a^4b^2cd^5-24a^2b^4c^3d^3+12b^6c^5d)}{2c^2d^7+4cd^8x+2d^9x^2}\end{aligned}$$

input
```
integrate((-b**2*x**2+a**2)**3/(d*x+c)**3,x)
```

output
```
b**6*c*x**3/d**4 - b**6*x**4/(4*d**3) - 3*b**2*(a*d - b*c)*(a*d + b*c)*(a*
*2*d**2 - 5*b**2*c**2)*log(c + d*x)/d**7 - x**2*(-3*a**2*b**4/(2*d**3) + 3
*b**6*c**2/d**5) - x*(9*a**2*b**4*c/d**4 - 10*b**6*c**3/d**6) - (a**6*d**6
 + 9*a**4*b**2*c**2*d**4 - 21*a**2*b**4*c**4*d**2 + 11*b**6*c**6 + x*(12*a
**4*b**2*c*d**5 - 24*a**2*b**4*c**3*d**3 + 12*b**6*c**5*d))/(2*c**2*d**7 +
 4*c*d**8*x + 2*d**9*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.20

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3}\,dx = -\frac{11\,b^6c^6-21\,a^2b^4c^4d^2+9\,a^4b^2c^2d^4+a^6d^6+12\,(b^6c^5d-2\,a^2b^4c^3d^3+a^4b^2cd^5)x}{2\,(d^9x^2+2\,cd^8x+c^2d^7)} - \frac{b^6d^3x^4-4\,b^6cd^2x^3+6\,(2\,b^6c^2d-a^2b^4d^3)x^2-4\,(10\,b^6c^3-9\,a^2b^4cd^2)x}{4\,d^6} - \frac{3\,(5\,b^6c^4-6\,a^2b^4c^2d^2+a^4b^2d^4)\log(dx+c)}{d^7}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(11*b^6*c^6 - 21*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 + a^6*d^6 + 12*(
b^6*c^5*d - 2*a^2*b^4*c^3*d^3 + a^4*b^2*c*d^5)*x)/(d^9*x^2 + 2*c*d^8*x + c
^2*d^7) - 1/4*(b^6*d^3*x^4 - 4*b^6*c*d^2*x^3 + 6*(2*b^6*c^2*d - a^2*b^4*d^
3)*x^2 - 4*(10*b^6*c^3 - 9*a^2*b^4*c*d^2)*x)/d^6 - 3*(5*b^6*c^4 - 6*a^2*b^
4*c^2*d^2 + a^4*b^2*d^4)*log(d*x + c)/d^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.15

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3}\,dx$$
$$= -\frac{3\,(5\,b^6c^4-6\,a^2b^4c^2d^2+a^4b^2d^4)\log{(|dx+c|)}}{d^7}$$
$$-\frac{11\,b^6c^6-21\,a^2b^4c^4d^2+9\,a^4b^2c^2d^4+a^6d^6+12\,(b^6c^5d-2\,a^2b^4c^3d^3+a^4b^2cd^5)x}{2\,(dx+c)^2d^7}$$
$$-\frac{b^6d^9x^4-4\,b^6cd^8x^3+12\,b^6c^2d^7x^2-6\,a^2b^4d^9x^2-40\,b^6c^3d^6x+36\,a^2b^4cd^8x}{4\,d^{12}}$$

input
```
integrate((-b^2*x^2+a^2)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
-3*(5*b^6*c^4 - 6*a^2*b^4*c^2*d^2 + a^4*b^2*d^4)*log(abs(d*x + c))/d^7 - 1
/2*(11*b^6*c^6 - 21*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 + a^6*d^6 + 12*(b^
6*c^5*d - 2*a^2*b^4*c^3*d^3 + a^4*b^2*c*d^5)*x)/((d*x + c)^2*d^7) - 1/4*(b
^6*d^9*x^4 - 4*b^6*c*d^8*x^3 + 12*b^6*c^2*d^7*x^2 - 6*a^2*b^4*d^9*x^2 - 40
*b^6*c^3*d^6*x + 36*a^2*b^4*c*d^8*x)/d^12
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.32

$$\int \frac{(a^2-b^2x^2)^3}{(c+dx)^3}\,dx$$
$$= x^2\left(\frac{3\,a^2\,b^4}{2\,d^3}-\frac{3\,b^6\,c^2}{d^5}\right)-x\left(\frac{3\,c\left(\frac{3\,a^2\,b^4}{d^3}-\frac{6\,b^6\,c^2}{d^5}\right)}{d}+\frac{8\,b^6\,c^3}{d^6}\right)$$
$$-\frac{\frac{a^6\,d^6+9\,a^4\,b^2\,c^2\,d^4-21\,a^2\,b^4\,c^4\,d^2+11\,b^6\,c^6}{2\,d}+x\,(6\,a^4\,b^2\,c\,d^4-12\,a^2\,b^4\,c^3\,d^2+6\,b^6\,c^5)}{c^2\,d^6+2\,c\,d^7\,x+d^8\,x^2}$$
$$-\frac{b^6\,x^4}{4\,d^3}-\frac{\ln{(c+d\,x)}\,(3\,a^4\,b^2\,d^4-18\,a^2\,b^4\,c^2\,d^2+15\,b^6\,c^4)}{d^7}+\frac{b^6\,c\,x^3}{d^4}$$

input
```
int((a^2 - b^2*x^2)^3/(c + d*x)^3,x)
```

output
```
x^2*((3*a^2*b^4)/(2*d^3) - (3*b^6*c^2)/d^5) - x*((3*c*((3*a^2*b^4)/d^3 - (
6*b^6*c^2)/d^5))/d + (8*b^6*c^3)/d^6) - ((a^6*d^6 + 11*b^6*c^6 - 21*a^2*b^
4*c^4*d^2 + 9*a^4*b^2*c^2*d^4)/(2*d) + x*(6*b^6*c^5 + 6*a^4*b^2*c*d^4 - 12
*a^2*b^4*c^3*d^2))/(c^2*d^6 + d^8*x^2 + 2*c*d^7*x) - (b^6*x^4)/(4*d^3) - (
log(c + d*x)*(15*b^6*c^4 + 3*a^4*b^2*d^4 - 18*a^2*b^4*c^2*d^2))/d^7 + (b^6
*c*x^3)/d^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.92

$$\int \frac{(a^2 - b^2x^2)^3}{(c + dx)^3}\,dx$$
$$= \frac{-12\log(dx+c)\,a^4b^2c^2d^4 - 24\log(dx+c)\,a^4b^2c\,d^5x - 12\log(dx+c)\,a^4b^2d^6x^2 + 72\log(dx+c)\,a^2b^4c^4d^2}{}$$

input
```
int((-b^2*x^2+a^2)^3/(d*x+c)^3,x)
```

output
```
( - 12*log(c + d*x)*a**4*b**2*c**2*d**4 - 24*log(c + d*x)*a**4*b**2*c*d**5
*x - 12*log(c + d*x)*a**4*b**2*d**6*x**2 + 72*log(c + d*x)*a**2*b**4*c**4*
d**2 + 144*log(c + d*x)*a**2*b**4*c**3*d**3*x + 72*log(c + d*x)*a**2*b**4*
c**2*d**4*x**2 - 60*log(c + d*x)*b**6*c**6 - 120*log(c + d*x)*b**6*c**5*d*
x - 60*log(c + d*x)*b**6*c**4*d**2*x**2 - 2*a**6*d**6 - 6*a**4*b**2*c**2*d
**4 + 12*a**4*b**2*d**6*x**2 + 36*a**2*b**4*c**4*d**2 - 72*a**2*b**4*c**2*
d**4*x**2 - 24*a**2*b**4*c*d**5*x**3 + 6*a**2*b**4*d**6*x**4 - 30*b**6*c**
6 + 60*b**6*c**4*d**2*x**2 + 20*b**6*c**3*d**3*x**3 - 5*b**6*c**2*d**4*x**
4 + 2*b**6*c*d**5*x**5 - b**6*d**6*x**6)/(4*d**7*(c**2 + 2*c*d*x + d**2*x*
*2))
```

3.28 $\int \frac{(c+dx)^4}{a^2-b^2x^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{(c+dx)^4}{a^2-b^2x^2} \, dx = -\frac{d^2(6b^2c^2+a^2d^2)\,x}{b^4} - \frac{2cd^3x^2}{b^2} - \frac{d^4x^3}{3b^2} - \frac{(bc+ad)^4\log(a-bx)}{2ab^5} + \frac{(bc-ad)^4\log(a+bx)}{2ab^5}$$

output
```
-d^2*(a^2*d^2+6*b^2*c^2)*x/b^4-2*c*d^3*x^2/b^2-1/3*d^4*x^3/b^2-1/2*(a*d+b*
c)^4*ln(-b*x+a)/a/b^5+1/2*(-a*d+b*c)^4*ln(b*x+a)/a/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)^4}{a^2-b^2x^2} \, dx = \frac{(b^4c^4+6a^2b^2c^2d^2+a^4d^4)\,\text{arctanh}\left(\frac{bx}{a}\right)}{ab^5} - \frac{d(3a^2d^3x+b^2dx(18c^2+6cdx+d^2x^2)+6(b^2c^3+a^2cd^2)\log(a^2-b^2x^2))}{3b^4}$$

input
```
Integrate[(c + d*x)^4/(a^2 - b^2*x^2),x]
```

output
```
((b^4*c^4 + 6*a^2*b^2*c^2*d^2 + a^4*d^4)*ArcTanh[(b*x)/a])/(a*b^5) - (d*(3
*a^2*d^3*x + b^2*d*x*(18*c^2 + 6*c*d*x + d^2*x^2) + 6*(b^2*c^3 + a^2*c*d^2
)*Log[a^2 - b^2*x^2]))/(3*b^4)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{a^2-b^2x^2}\,dx$$
$$\Big\downarrow \; 477$$
$$\frac{\int \left(-\frac{a^2x^2d^4}{b^2}-\frac{4a^2cxd^3}{b^2}-\frac{a^2\left(6b^2c^2+a^2d^2\right)d^2}{b^4}+\frac{a(bc+ad)^4}{2b^4(a-bx)}+\frac{a(bc-ad)^4}{2b^4(a+bx)}\right)dx}{a^2}$$
$$\Big\downarrow \; 2009$$
$$\frac{-\frac{2a^2cd^3x^2}{b^2}-\frac{a^2d^4x^3}{3b^2}-\frac{a^2d^2x\left(a^2d^2+6b^2c^2\right)}{b^4}-\frac{a(ad+bc)^4\log(a-bx)}{2b^5}+\frac{a(bc-ad)^4\log(a+bx)}{2b^5}}{a^2}$$

input
```
Int[(c + d*x)^4/(a^2 - b^2*x^2),x]
```

output
```
(-((a^2*d^2*(6*b^2*c^2 + a^2*d^2)*x)/b^4) - (2*a^2*c*d^3*x^2)/b^2 - (a^2*d
^4*x^3)/(3*b^2) - (a*(b*c + a*d)^4*Log[a - b*x])/(2*b^5) + (a*(b*c - a*d)^
4*Log[a + b*x])/(2*b^5))/a^2
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.72

method	result
norman	$-\frac{d^4x^3}{3b^2}-\frac{2c\,d^3x^2}{b^2}-\frac{d^2\left(a^2d^2+6b^2c^2\right)x}{b^4}+\frac{\left(a^4d^4-4c\,a^3b\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)\ln(bx+a)}{2b^5a}-\frac{(a^4d^4+4c\,a^3b\,d^3}{}$
default	$-\frac{d^2\left(\frac{1}{3}x^3b^2d^2+2b^2c\,x^2d+a^2d^2x+6x\,b^2c^2\right)}{b^4}+\frac{\left(-a^4d^4-4c\,a^3b\,d^3-6a^2b^2c^2d^2-4a\,b^3c^3d-c^4b^4\right)\ln(-bx+a)}{2a\,b^5}+\frac{(a^4d^4-4c\,a}{}$
parallelrisch	$-\frac{2d^4x^3a\,b^3+12c\,d^3x^2a\,b^3+3\ln(bx-a)a^4d^4+12\ln(bx-a)a^3bc\,d^3+18\ln(bx-a)a^2b^2c^2d^2+12\ln(bx-a)a\,b^3c^3d+3\ln(bx-a)b^4}{}$
risch	$-\frac{d^4x^3}{3b^2}-\frac{2c\,d^3x^2}{b^2}-\frac{d^4a^2x}{b^4}-\frac{6d^2x\,c^2}{b^2}+\frac{a^3\ln(-bx-a)d^4}{2b^5}-\frac{2a^2\ln(-bx-a)c\,d^3}{b^4}+\frac{3a\ln(-bx-a)c^2d^2}{b^3}-\frac{2\ln(-bx}{b^2}$

input
```
int((d*x+c)^4/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^4*x^3/b^2-2*c*d^3*x^2/b^2-d^2*(a^2*d^2+6*b^2*c^2)*x/b^4+1/2/b^5*(a^
4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/a*ln(b*x+a)-1
/2*(a^4*d^4+4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d+b^4*c^4)/a/b^5*l
n(-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)^4}{a^2-b^2x^2}\,dx = -\frac{2\,ab^3d^4x^3 + 12\,ab^3cd^3x^2 + 6\,(6\,ab^3c^2d^2 + a^3bd^4)x - 3\,(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)}{6\,ab^5}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/6*(2*a*b^3*d^4*x^3 + 12*a*b^3*c*d^3*x^2 + 6*(6*a*b^3*c^2*d^2 + a^3*b*d^
4)*x - 3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*log(b*x + a) + 3*(b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 4*a
^3*b*c*d^3 + a^4*d^4)*log(b*x - a))/(a*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(92) = 184.

Time = 0.67 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx)^4}{a^2-b^2x^2}\,dx = -x\left(\frac{a^2d^4}{b^4}+\frac{6c^2d^2}{b^2}\right)-\frac{2cd^3x^2}{b^2}-\frac{d^4x^3}{3b^2} + \frac{(ad-bc)^4\log\left(x+\frac{4a^4cd^3+4a^2b^2c^3d+\frac{a(ad-bc)^4}{b}}{a^4d^4+6a^2b^2c^2d^2+b^4c^4}\right)}{2ab^5} - \frac{(ad+bc)^4\log\left(x+\frac{4a^4cd^3+4a^2b^2c^3d-\frac{a(ad+bc)^4}{b}}{a^4d^4+6a^2b^2c^2d^2+b^4c^4}\right)}{2ab^5}$$

input
```
integrate((d*x+c)**4/(-b**2*x**2+a**2),x)
```

output
```
-x*(a**2*d**4/b**4 + 6*c**2*d**2/b**2) - 2*c*d**3*x**2/b**2 - d**4*x**3/(3
*b**2) + (a*d - b*c)**4*log(x + (4*a**4*c*d**3 + 4*a**2*b**2*c**3*d + a*(a
*d - b*c)**4/b)/(a**4*d**4 + 6*a**2*b**2*c**2*d**2 + b**4*c**4))/(2*a*b**5
) - (a*d + b*c)**4*log(x + (4*a**4*c*d**3 + 4*a**2*b**2*c**3*d - a*(a*d +
b*c)**4/b)/(a**4*d**4 + 6*a**2*b**2*c**2*d**2 + b**4*c**4))/(2*a*b**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.74

$$\int \frac{(c+dx)^4}{a^2-b^2x^2}\,dx = -\frac{b^2d^4x^3+6\,b^2cd^3x^2+3\,(6\,b^2c^2d^2+a^2d^4)x}{3\,b^4} + \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log(bx+a)}{2\,ab^5} - \frac{(b^4c^4+4\,ab^3c^3d+6\,a^2b^2c^2d^2+4\,a^3bcd^3+a^4d^4)\log(bx-a)}{2\,ab^5}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/3*(b^2*d^4*x^3 + 6*b^2*c*d^3*x^2 + 3*(6*b^2*c^2*d^2 + a^2*d^4)*x)/b^4 +
 1/2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*log(b*x + a)/(a*b^5) - 1/2*(b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 + 4*a^3*b*c*d^3 + a^4*d^4)*log(b*x - a)/(a*b^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx)^4}{a^2-b^2x^2}\,dx = \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log(|bx+a|)}{2\,ab^5} - \frac{(b^4c^4+4\,ab^3c^3d+6\,a^2b^2c^2d^2+4\,a^3bcd^3+a^4d^4)\log(|bx-a|)}{2\,ab^5} - \frac{b^4d^4x^3+6\,b^4cd^3x^2+18\,b^4c^2d^2x+3\,a^2b^2d^4x}{3\,b^6}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*log(abs(b*x + a))/(a*b^5) - 1/2*(b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 + 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(b*x - a))/(a*b^5) - 1/3*(b^4*d^4*x
^3 + 6*b^4*c*d^3*x^2 + 18*b^4*c^2*d^2*x + 3*a^2*b^2*d^4*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^4}{a^2-b^2x^2}\,dx = \frac{\ln(a+b\,x)\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{2\,a\,b^5} - \frac{d^4\,x^3}{3\,b^2} - \frac{2\,c\,d^3\,x^2}{b^2} - x\left(\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right) - \frac{\ln(a-b\,x)\,\left(a^4\,d^4+4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{2\,a\,b^5}$$

input
```
int((c + d*x)^4/(a^2 - b^2*x^2),x)
```

output
```
(log(a + b*x)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3))/(2*a*b^5) - (d^4*x^3)/(3*b^2) - (2*c*d^3*x^2)/b^2 - x*((a^2*d
^4)/b^4 + (6*c^2*d^2)/b^2) - (log(a - b*x)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 + 4*a*b^3*c^3*d + 4*a^3*b*c*d^3))/(2*a*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx)^4}{a^2-b^2x^2}\,dx = \frac{3\log(-bx-a)\,a^4d^4-12\log(-bx-a)\,a^3bc\,d^3+18\log(-bx-a)\,a^2b^2c^2d^2-12\log(-bx-a)\,a\,b^3c^3d+3$$

input
```
int((d*x+c)^4/(-b^2*x^2+a^2),x)
```

output
```
(3*log( - a - b*x)*a**4*d**4 - 12*log( - a - b*x)*a**3*b*c*d**3 + 18*log(
- a - b*x)*a**2*b**2*c**2*d**2 - 12*log( - a - b*x)*a*b**3*c**3*d + 3*log(
 - a - b*x)*b**4*c**4 - 3*log(a - b*x)*a**4*d**4 - 12*log(a - b*x)*a**3*b*
c*d**3 - 18*log(a - b*x)*a**2*b**2*c**2*d**2 - 12*log(a - b*x)*a*b**3*c**3
*d - 3*log(a - b*x)*b**4*c**4 - 6*a**3*b*d**4*x - 36*a*b**3*c**2*d**2*x -
12*a*b**3*c*d**3*x**2 - 2*a*b**3*d**4*x**3)/(6*a*b**5)
```

3.29 $\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = -\frac{3cd^2x}{b^2} - \frac{d^3x^2}{2b^2} - \frac{(bc+ad)^3\log(a-bx)}{2ab^4} + \frac{(bc-ad)^3\log(a+bx)}{2ab^4}$$

output
```
-3*c*d^2*x/b^2-1/2*d^3*x^2/b^2-1/2*(a*d+b*c)^3*ln(-b*x+a)/a/b^4+1/2*(-a*d+
b*c)^3*ln(b*x+a)/a/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = \frac{2(b^3c^3+3a^2bcd^2)\,\text{arctanh}\left(\frac{bx}{a}\right) - ad\left(b^2dx(6c+dx)+(3b^2c^2+a^2d^2)\log\left(a^2-b^2x^2\right)\right)}{2ab^4}$$

input
```
Integrate[(c + d*x)^3/(a^2 - b^2*x^2),x]
```

output
```
(2*(b^3*c^3 + 3*a^2*b*c*d^2)*ArcTanh[(b*x)/a] - a*d*(b^2*d*x*(6*c + d*x) +
 (3*b^2*c^2 + a^2*d^2)*Log[a^2 - b^2*x^2]))/(2*a*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a^2-b^2x^2} \, dx$$

$$\Big\downarrow \; 477$$

$$\frac{\int \left(-\frac{a^2xd^3}{b^2}-\frac{3a^2cd^2}{b^2}+\frac{a(bc+ad)^3}{2b^3(a-bx)}+\frac{a(bc-ad)^3}{2b^3(a+bx)}\right)dx}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{3a^2cd^2x}{b^2}-\frac{a^2d^3x^2}{2b^2}-\frac{a(ad+bc)^3\log(a-bx)}{2b^4}+\frac{a(bc-ad)^3\log(a+bx)}{2b^4}}{a^2}$$

input
```
Int[(c + d*x)^3/(a^2 - b^2*x^2),x]
```

output
```
((-3*a^2*c*d^2*x)/b^2 - (a^2*d^3*x^2)/(2*b^2) - (a*(b*c + a*d)^3*Log[a - b
*x])/(2*b^4) + (a*(b*c - a*d)^3*Log[a + b*x])/(2*b^4))/a^2
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.62

method	result
default	$-\frac{d^{2}\left(\frac{1}{2}d\,x^{2}+3cx\right)}{b^{2}}+\frac{\left(-a^{3}d^{3}-3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\ln\left(-bx+a\right)}{2a\,b^{4}}+\frac{\left(-a^{3}d^{3}+3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d+b^{3}c^{3}\right)\ln\left(bx+a\right)}{2a\,b^{4}}$
norman	$-\frac{d^{3}x^{2}}{2b^{2}}-\frac{3c\,d^{2}x}{b^{2}}-\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\ln\left(bx+a\right)}{2a\,b^{4}}-\frac{\left(a^{3}d^{3}+3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d+b^{3}c^{3}\right)\ln\left(-bx+a\right)}{2a\,b^{4}}$
risch	$-\frac{d^{3}x^{2}}{2b^{2}}-\frac{3c\,d^{2}x}{b^{2}}-\frac{a^{2}\ln\left(bx+a\right)d^{3}}{2b^{4}}+\frac{3a\ln\left(bx+a\right)c\,d^{2}}{2b^{3}}-\frac{3\ln\left(bx+a\right)c^{2}d}{2b^{2}}+\frac{\ln\left(bx+a\right)c^{3}}{2ab}-\frac{a^{2}\ln\left(-bx+a\right)d^{3}}{2b^{4}}-3a\ln$
parallelrisch	$-\frac{d^{3}x^{2}a\,b^{2}+\ln\left(bx-a\right)a^{3}d^{3}+3\ln\left(bx-a\right)a^{2}bc\,d^{2}+3\ln\left(bx-a\right)a\,b^{2}c^{2}d+\ln\left(bx-a\right)b^{3}c^{3}+\ln\left(bx+a\right)a^{3}d^{3}-3\ln\left(bx+a\right)a^{2}bc\,d^{2}+3\ln(}{2a\,b^{4}}$

input
```
int((d*x+c)^3/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-d^2/b^2*(1/2*d*x^2+3*c*x)+1/2*(-a^3*d^3-3*a^2*b*c*d^2-3*a*b^2*c^2*d-b^3*c
^3)/a/b^4*ln(-b*x+a)+1/2*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/a/
b^4*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = -\frac{ab^2d^3x^2+6\,ab^2cd^2x-\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx+a\right)+\left(b^3c^3+3\,ab^2c^2d+3\,a^2bcd^2+\right.}{2\,ab^4}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/2*(a*b^2*d^3*x^2 + 6*a*b^2*c*d^2*x - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)*log(b*x + a) + (b^3*c^3 + 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
+ a^3*d^3)*log(b*x - a))/(a*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(66) = 132$.

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.14

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = -\frac{3cd^2x}{b^2} - \frac{d^3x^2}{2b^2} - \frac{(ad-bc)^3\log\left(x + \frac{a^4d^3+3a^2b^2c^2d-a(ad-bc)^3}{3a^2b^2cd^2+b^4c^3}\right)}{2ab^4} - \frac{(ad+bc)^3\log\left(x + \frac{a^4d^3+3a^2b^2c^2d-a(ad+bc)^3}{3a^2b^2cd^2+b^4c^3}\right)}{2ab^4}$$

input
```
integrate((d*x+c)**3/(-b**2*x**2+a**2),x)
```

output
```
-3*c*d**2*x/b**2 - d**3*x**2/(2*b**2) - (a*d - b*c)**3*log(x + (a**4*d**3
+ 3*a**2*b**2*c**2*d - a*(a*d - b*c)**3)/(3*a**2*b**2*c*d**2 + b**4*c**3))
/(2*a*b**4) - (a*d + b*c)**3*log(x + (a**4*d**3 + 3*a**2*b**2*c**2*d - a*(
a*d + b*c)**3)/(3*a**2*b**2*c*d**2 + b**4*c**3))/(2*a*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = -\frac{d^3x^2+6\,cd^2x}{2\,b^2} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(bx+a)}{2\,ab^4} - \frac{(b^3c^3+3\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\log(bx-a)}{2\,ab^4}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*(d^3*x^2 + 6*c*d^2*x)/b^2 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*log(b*x + a)/(a*b^4) - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 + a^3*d^3)*log(b*x - a)/(a*b^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = -\frac{b^2d^3x^2+6\,b^2cd^2x}{2\,b^4} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|bx+a|)}{2\,ab^4} - \frac{(b^3c^3+3\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\log(|bx-a|)}{2\,ab^4}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-1/2*(b^2*d^3*x^2 + 6*b^2*c*d^2*x)/b^4 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x + a))/(a*b^4) - 1/2*(b^3*c^3 + 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*log(abs(b*x - a))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = -\frac{d^3\,x^2}{2\,b^2} - \frac{\ln(a+b\,x)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,b^4} - \frac{\ln(a-b\,x)\,\left(a^3\,d^3+3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{2\,a\,b^4} - \frac{3\,c\,d^2\,x}{b^2}$$

input
```
int((c + d*x)^3/(a^2 - b^2*x^2),x)
```

output
```
- (d^3*x^2)/(2*b^2) - (log(a + b*x)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(2*a*b^4) - (log(a - b*x)*(a^3*d^3 + b^3*c^3 + 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2))/(2*a*b^4) - (3*c*d^2*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.20

$$\int \frac{(c+dx)^3}{a^2-b^2x^2}\,dx = \frac{-\log(-bx-a)\,a^3d^3+3\log(-bx-a)\,a^2bc\,d^2-3\log(-bx-a)\,a\,b^2c^2d+\log(-bx-a)\,b^3c^3-\log(-bx+\ldots}{2a\,\ldots}$$

input
```
int((d*x+c)^3/(-b^2*x^2+a^2),x)
```

output
```
( - log( - a - b*x)*a**3*d**3 + 3*log( - a - b*x)*a**2*b*c*d**2 - 3*log( -
 a - b*x)*a*b**2*c**2*d + log( - a - b*x)*b**3*c**3 - log(a - b*x)*a**3*d*
*3 - 3*log(a - b*x)*a**2*b*c*d**2 - 3*log(a - b*x)*a*b**2*c**2*d - log(a -
 b*x)*b**3*c**3 - 6*a*b**2*c*d**2*x - a*b**2*d**3*x**2)/(2*a*b**4)
```

3.30 $\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx = -\frac{d^2x}{b^2} - \frac{(bc+ad)^2\log(a-bx)}{2ab^3} + \frac{(bc-ad)^2\log(a+bx)}{2ab^3}$$

output
```
-d^2*x/b^2-1/2*(a*d+b*c)^2*ln(-b*x+a)/a/b^3+1/2*(-a*d+b*c)^2*ln(b*x+a)/a/b
^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx = \frac{(b^2c^2+a^2d^2)\,\text{arctanh}\left(\frac{bx}{a}\right) - abd(dx + c\log(a^2-b^2x^2))}{ab^3}$$

input
```
Integrate[(c + d*x)^2/(a^2 - b^2*x^2),x]
```

output
```
((b^2*c^2 + a^2*d^2)*ArcTanh[(b*x)/a] - a*b*d*(d*x + c*Log[a^2 - b^2*x^2])
)/(a*b^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(-\frac{a^2d^2}{b^2}+\frac{a(bc+ad)^2}{2b^2(a-bx)}+\frac{a(bc-ad)^2}{2b^2(a+bx)}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2d^2x}{b^2}-\frac{a(ad+bc)^2\log(a-bx)}{2b^3}+\frac{a(bc-ad)^2\log(a+bx)}{2b^3}}{a^2}$$

input
```
Int[(c + d*x)^2/(a^2 - b^2*x^2),x]
```

output
```
(-((a^2*d^2*x)/b^2) - (a*(b*c + a*d)^2*Log[a - b*x])/(2*b^3) + (a*(b*c - a
*d)^2*Log[a + b*x])/(2*b^3))/a^2
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

method	result	size
norman	$-\frac{d^{2}x}{b^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(bx+a\right)}{2b^{3}a}-\frac{\left(a^{2}d^{2}+2abcd+b^{2}c^{2}\right)\ln\left(-bx+a\right)}{2a\,b^{3}}$	82
default	$-\frac{d^{2}x}{b^{2}}+\frac{\left(-a^{2}d^{2}-2abcd-b^{2}c^{2}\right)\ln\left(-bx+a\right)}{2a\,b^{3}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(bx+a\right)}{2b^{3}a}$	84
parallelrisch	$-\frac{\ln\left(bx-a\right)a^{2}d^{2}+2\ln\left(bx-a\right)abcd+\ln\left(bx-a\right)b^{2}c^{2}-\ln\left(bx+a\right)a^{2}d^{2}+2\ln\left(bx+a\right)abcd-\ln\left(bx+a\right)b^{2}c^{2}+2d^{2}xab}{2a\,b^{3}}$	102
risch	$-\frac{d^{2}x}{b^{2}}-\frac{a\ln\left(bx-a\right)d^{2}}{2b^{3}}-\frac{\ln\left(bx-a\right)cd}{b^{2}}-\frac{\ln\left(bx-a\right)c^{2}}{2ba}+\frac{a\ln\left(-bx-a\right)d^{2}}{2b^{3}}-\frac{\ln\left(-bx-a\right)cd}{b^{2}}+\frac{\ln\left(-bx-a\right)c^{2}}{2ba}$	116

input
```
int((d*x+c)^2/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-d^2*x/b^2+1/2/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*ln(b*x+a)-1/2*(a^2*d^2+2*
a*b*c*d+b^2*c^2)/a/b^3*ln(-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx = -\frac{2\,abd^2x-(b^2c^2-2\,abcd+a^2d^2)\log(bx+a)+(b^2c^2+2\,abcd+a^2d^2)\log(bx-a)}{2\,ab^3}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*d^2*x - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(b*x + a) + (b^2*c^
2 + 2*a*b*c*d + a^2*d^2)*log(b*x - a))/(a*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx = -\frac{d^2x}{b^2} + \frac{(ad-bc)^2\log\left(x+\frac{2a^2cd+\frac{a(ad-bc)^2}{b}}{a^2d^2+b^2c^2}\right)}{2ab^3} - \frac{(ad+bc)^2\log\left(x+\frac{2a^2cd-\frac{a(ad+bc)^2}{b}}{a^2d^2+b^2c^2}\right)}{2ab^3}$$

input
```
integrate((d*x+c)**2/(-b**2*x**2+a**2),x)
```

output
```
-d**2*x/b**2 + (a*d - b*c)**2*log(x + (2*a**2*c*d + a*(a*d - b*c)**2/b)/(a
**2*d**2 + b**2*c**2))/(2*a*b**3) - (a*d + b*c)**2*log(x + (2*a**2*c*d - a
*(a*d + b*c)**2/b)/(a**2*d**2 + b**2*c**2))/(2*a*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx = -\frac{d^2x}{b^2} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log(bx+a)}{2\,ab^3} - \frac{(b^2c^2+2\,abcd+a^2d^2)\log(bx-a)}{2\,ab^3}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-d^2*x/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(b*x + a)/(a*b^3) - 1/
2*(b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(b*x - a)/(a*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx = -\frac{d^2x}{b^2} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log(|bx+a|)}{2\,ab^3} - \frac{(b^2c^2+2\,abcd+a^2d^2)\log(|bx-a|)}{2\,ab^3}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-d^2*x/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(b*x + a))/(a*b^3)
 - 1/2*(b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(abs(b*x - a))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx = \frac{\ln(a+b\,x)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,a\,b^3} - \frac{d^2\,x}{b^2} - \frac{\ln(a-b\,x)\,(a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}{2\,a\,b^3}$$

input
```
int((c + d*x)^2/(a^2 - b^2*x^2),x)
```

output
```
(log(a + b*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*b^3) - (d^2*x)/b^2 - (
log(a - b*x)*(a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(2*a*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)^2}{a^2-b^2x^2}\,dx$$
$$= \frac{\log(-bx-a)\,a^2d^2 - 2\log(-bx-a)\,abcd + \log(-bx-a)\,b^2c^2 - \log(-bx+a)\,a^2d^2 - 2\log(-bx+a)\,abc}{2a\,b^3}$$

input
```
int((d*x+c)^2/(-b^2*x^2+a^2),x)
```

output
```
(log( - a - b*x)*a**2*d**2 - 2*log( - a - b*x)*a*b*c*d + log( - a - b*x)*b
**2*c**2 - log(a - b*x)*a**2*d**2 - 2*log(a - b*x)*a*b*c*d - log(a - b*x)*
b**2*c**2 - 2*a*b*d**2*x)/(2*a*b**3)
```

3.31 $\int \frac{c+dx}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx = -\frac{\left(\frac{bc}{a}+d\right)\log(a-bx)}{2b^2}+\frac{\left(\frac{bc}{a}-d\right)\log(a+bx)}{2b^2}$$

output
```
-1/2*(b*c/a+d)*ln(-b*x+a)/b^2+1/2*(b*c/a-d)*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx = \frac{c\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}-\frac{d\log\left(a^2-b^2x^2\right)}{2b^2}$$

input
```
Integrate[(c + d*x)/(a^2 - b^2*x^2),x]
```

output
```
(c*ArcTanh[(b*x)/a])/(a*b) - (d*Log[a^2 - b^2*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {452, 221, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \; 452$$

$$c\int \frac{1}{a^2-b^2x^2}dx+d\int \frac{x}{a^2-b^2x^2}dx$$

$$\Big\downarrow \; 221$$

$$d\int \frac{x}{a^2-b^2x^2}dx+\frac{c\,\text{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

$$\Big\downarrow \; 240$$

$$\frac{c\,\text{arctanh}\left(\frac{bx}{a}\right)}{ab}-\frac{d\log\left(a^2-b^2x^2\right)}{2b^2}$$

input
```
Int[(c + d*x)/(a^2 - b^2*x^2),x]
```

output
```
(c*ArcTanh[(b*x)/a])/(a*b) - (d*Log[a^2 - b^2*x^2])/(2*b^2)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

method	result	size
norman	$-\frac{(ad-bc)\ln(bx+a)}{2a\,b^2}-\frac{(ad+bc)\ln(-bx+a)}{2a\,b^2}$	46
default	$\frac{(-ad-bc)\ln(-bx+a)}{2a\,b^2}+\frac{(-ad+bc)\ln(bx+a)}{2a\,b^2}$	48
parallelrisch	$-\frac{\ln(bx-a)ad+\ln(bx-a)bc+\ln(bx+a)ad-\ln(bx+a)bc}{2a\,b^2}$	51
risch	$-\frac{\ln(bx+a)d}{2b^2}+\frac{\ln(bx+a)c}{2ab}-\frac{\ln(-bx+a)d}{2b^2}-\frac{\ln(-bx+a)c}{2ab}$	58

input
```
int((d*x+c)/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*d-b*c)/a/b^2*ln(b*x+a)-1/2*(a*d+b*c)/a/b^2*ln(-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx=\frac{(bc-ad)\log(bx+a)-(bc+ad)\log(bx-a)}{2\,ab^2}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*((b*c - a*d)*log(b*x + a) - (b*c + a*d)*log(b*x - a))/(a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx = -\frac{(ad-bc)\log\left(x+\frac{a^2d-a(ad-bc)}{b^2c}\right)}{2ab^2} - \frac{(ad+bc)\log\left(x+\frac{a^2d-a(ad+bc)}{b^2c}\right)}{2ab^2}$$

input
```
integrate((d*x+c)/(-b**2*x**2+a**2),x)
```

output
```
-(a*d - b*c)*log(x + (a**2*d - a*(a*d - b*c))/(b**2*c))/(2*a*b**2) - (a*d
+ b*c)*log(x + (a**2*d - a*(a*d + b*c))/(b**2*c))/(2*a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx = \frac{(bc-ad)\log(bx+a)}{2\,ab^2} - \frac{(bc+ad)\log(bx-a)}{2\,ab^2}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*(b*c - a*d)*log(b*x + a)/(a*b^2) - 1/2*(b*c + a*d)*log(b*x - a)/(a*b^2
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx = \frac{(bc-ad)\log(|bx+a|)}{2\,ab^2} - \frac{(bc+ad)\log(|bx-a|)}{2\,ab^2}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*(b*c - a*d)*log(abs(b*x + a))/(a*b^2) - 1/2*(b*c + a*d)*log(abs(b*x -
a))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx = -\frac{\ln(a+b\,x)\,(a\,d-b\,c)}{2\,a\,b^2} - \frac{\ln(a-b\,x)\,(a\,d+b\,c)}{2\,a\,b^2}$$

input
```
int((c + d*x)/(a^2 - b^2*x^2),x)
```

output
```
- (log(a + b*x)*(a*d - b*c))/(2*a*b^2) - (log(a - b*x)*(a*d + b*c))/(2*a*b
^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{c+dx}{a^2-b^2x^2}\,dx = \frac{-\log(-bx-a)\,ad+\log(-bx-a)\,bc-\log(-bx+a)\,ad-\log(-bx+a)\,bc}{2a\,b^2}$$

input
```
int((d*x+c)/(-b^2*x^2+a^2),x)
```

output
```
( - log( - a - b*x)*a*d + log( - a - b*x)*b*c - log(a - b*x)*a*d - log(a -
 b*x)*b*c)/(2*a*b**2)
```

3.32 $\int \frac{1}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

output
```
arctanh(b*x/a)/a/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

input
```
Integrate[(a^2 - b^2*x^2)^(-1),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \; 221$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

input
```
Int[(a^2 - b^2*x^2)^(-1),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

method	result	size
parallelrisch	$-\frac{\ln(bx-a)-\ln(bx+a)}{2ab}$	26
default	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ba}$	31
norman	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ba}$	31
risch	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ba}$	31

input
```
int(1/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(b*x-a)-ln(b*x+a))/a/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\log\left(bx+a\right)-\log\left(bx-a\right)}{2\,ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*(log(b*x + a) - log(b*x - a))/(a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{a^2-b^2x^2}\,dx=-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{ab}$$

input
```
integrate(1/(-b**2*x**2+a**2),x)
```

output
```
-(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{a^2 - b^2x^2}\, dx = \frac{\log(bx + a)}{2\, ab} - \frac{\log(bx - a)}{2\, ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)/(a*b) - 1/2*log(b*x - a)/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{a^2 - b^2x^2}\, dx = \frac{\log(|bx + a|)}{2\, ab} - \frac{\log(|bx - a|)}{2\, ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x + a))/(a*b) - 1/2*log(abs(b*x - a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 - b^2x^2}\, dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b}$$

input
```
int(1/(a^2 - b^2*x^2),x)
```

output
```
atanh((b*x)/a)/(a*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{1}{a^2 - b^2x^2} \, dx = \frac{\log(-bx - a) - \log(-bx + a)}{2ab}$$

input
```
int(1/(-b^2*x^2+a^2),x)
```

output
```
(log( - a - b*x) - log(a - b*x))/(2*a*b)
```

3.33 $\int \frac{1}{(c+dx)(a^2-b^2x^2)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)} \, dx = -\frac{\log(a-bx)}{2a(bc+ad)} + \frac{\log(a+bx)}{2a(bc-ad)} - \frac{d\log(c+dx)}{b^2c^2-a^2d^2}$$

output
```
-1/2*ln(-b*x+a)/a/(a*d+b*c)+1/2*ln(b*x+a)/a/(-a*d+b*c)-d*ln(d*x+c)/(-a^2*d
^2+b^2*c^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)} \, dx = \frac{2bc\text{arctanh}\left(\frac{bx}{a}\right) + ad(-2\log(c+dx) + \log(a^2-b^2x^2))}{2ab^2c^2-2a^3d^2}$$

input
```
Integrate[1/((c + d*x)*(a^2 - b^2*x^2)),x]
```

output
```
(2*b*c*ArcTanh[(b*x)/a] + a*d*(-2*Log[c + d*x] + Log[a^2 - b^2*x^2]))/(2*a
*b^2*c^2 - 2*a^3*d^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)(c+dx)}dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(-\frac{a^2d^2}{(b^2c^2-a^2d^2)(c+dx)}+\frac{ab}{2(bc+ad)(a-bx)}+\frac{ab}{2(bc-ad)(a+bx)}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2d\log(c+dx)}{b^2c^2-a^2d^2}-\frac{a\log(a-bx)}{2(ad+bc)}+\frac{a\log(a+bx)}{2(bc-ad)}}{a^2}$$

input
```
Int[1/((c + d*x)*(a^2 - b^2*x^2)),x]
```

output
```
(-1/2*(a*Log[a - b*x])/(b*c + a*d) + (a*Log[a + b*x])/(2*(b*c - a*d)) - (a
^2*d*Log[c + d*x])/(b^2*c^2 - a^2*d^2))/a^2
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{d\ln(dx+c)}{a^2d^2-b^2c^2}-\frac{\ln(bx+a)}{2a(ad-bc)}-\frac{\ln(-bx+a)}{2a(ad+bc)}$	70
default	$-\frac{\ln(-bx+a)}{2a(ad+bc)}-\frac{\ln(bx+a)}{2a(ad-bc)}+\frac{d\ln(dx+c)}{(ad-bc)(ad+bc)}$	71
risch	$\frac{d\ln(dx+c)}{a^2d^2-b^2c^2}-\frac{\ln(bx-a)}{2a(ad+bc)}-\frac{\ln(-bx-a)}{2a(ad-bc)}$	74
parallelrisch	$-\frac{\ln(bx-a)ad-\ln(bx-a)bc+\ln(bx+a)ad+\ln(bx+a)bc-2d\ln(dx+c)a}{2a(a^2d^2-b^2c^2)}$	76

input
```
int(1/(d*x+c)/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
d/(a^2*d^2-b^2*c^2)*ln(d*x+c)-1/2/a/(a*d-b*c)*ln(b*x+a)-1/2*ln(-b*x+a)/a/(
a*d+b*c)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)}\,dx = -\frac{2\,ad\log(dx+c)-(bc+ad)\log(bx+a)+(bc-ad)\log(bx-a)}{2\,(ab^2c^2-a^3d^2)}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*log(d*x + c) - (b*c + a*d)*log(b*x + a) + (b*c - a*d)*log(b*x
- a))/(a*b^2*c^2 - a^3*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*x+c)/(-b**2*x**2+a**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)}\,dx=-\frac{d\log(dx+c)}{b^2c^2-a^2d^2}+\frac{\log(bx+a)}{2(abc-a^2d)}-\frac{\log(bx-a)}{2(abc+a^2d)}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-d*log(d*x + c)/(b^2*c^2 - a^2*d^2) + 1/2*log(b*x + a)/(a*b*c - a^2*d) - 1
/2*log(b*x - a)/(a*b*c + a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)}\,dx=\frac{b^2\log(|bx+a|)}{2(ab^3c-a^2b^2d)}-\frac{b^2\log(|bx-a|)}{2(ab^3c+a^2b^2d)}-\frac{d^2\log(|dx+c|)}{b^2c^2d-a^2d^3}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*b^2*log(abs(b*x + a))/(a*b^3*c - a^2*b^2*d) - 1/2*b^2*log(abs(b*x - a)
)/(a*b^3*c + a^2*b^2*d) - d^2*log(abs(d*x + c))/(b^2*c^2*d - a^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)}\,dx = \frac{d\,\ln(c+d\,x)}{a^2\,d^2-b^2\,c^2}-\frac{\ln(a-b\,x)}{2\,d\,a^2+2\,b\,c\,a}-\frac{\ln(a+b\,x)}{2\,a^2\,d-2\,a\,b\,c}$$

input
```
int(1/((a^2 - b^2*x^2)*(c + d*x)),x)
```

output
```
(d*log(c + d*x))/(a^2*d^2 - b^2*c^2) - log(a - b*x)/(2*a^2*d + 2*a*b*c) -
log(a + b*x)/(2*a^2*d - 2*a*b*c)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)}\,dx = \frac{-\log(-bx-a)\,ad-\log(-bx-a)\,bc-\log(-bx+a)\,ad+\log(-bx+a)\,bc+2\log(dx+c)\,ad}{2a\,(a^2d^2-b^2c^2)}$$

input
```
int(1/(d*x+c)/(-b^2*x^2+a^2),x)
```

output
```
( - log( - a - b*x)*a*d - log( - a - b*x)*b*c - log(a - b*x)*a*d + log(a -
 b*x)*b*c + 2*log(c + d*x)*a*d)/(2*a*(a**2*d**2 - b**2*c**2))
```

3.34 $\int \frac{1}{(c+dx)^2\left(a^2-b^2x^2\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{1}{(c+dx)^2\left(a^2-b^2x^2\right)}\,dx = \frac{d}{\left(b^2c^2-a^2d^2\right)(c+dx)}-\frac{b\log(a-bx)}{2a(bc+ad)^2}+\frac{b\log(a+bx)}{2a(bc-ad)^2}-\frac{2b^2cd\log(c+dx)}{\left(b^2c^2-a^2d^2\right)^2}$$

output
```
d/(-a^2*d^2+b^2*c^2)/(d*x+c)-1/2*b*ln(-b*x+a)/a/(a*d+b*c)^2+1/2*b*ln(b*x+a
)/a/(-a*d+b*c)^2-2*b^2*c*d*ln(d*x+c)/(-a^2*d^2+b^2*c^2)^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2\left(a^2-b^2x^2\right)}\,dx = \frac{b\left(b^2c^2+a^2d^2\right)(c+dx)\text{arctanh}\left(\frac{bx}{a}\right)+ad\left(b^2c^2-a^2d^2-2b^2c(c+dx)\log(c+dx)+b^2c(c+dx)\log\left(a^2-\right.\right.}{a\left(b^2c^2-a^2d^2\right)^2(c+dx)}$$

input
```
Integrate[1/((c + d*x)^2*(a^2 - b^2*x^2)),x]
```

output
```
(b*(b^2*c^2 + a^2*d^2)*(c + d*x)*ArcTanh[(b*x)/a] + a*d*(b^2*c^2 - a^2*d^2
 - 2*b^2*c*(c + d*x)*Log[c + d*x] + b^2*c*(c + d*x)*Log[a^2 - b^2*x^2]))/(
a*(b^2*c^2 - a^2*d^2)^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)(c+dx)^2}dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(\frac{ab^2}{2(bc+ad)^2(a-bx)}+\frac{ab^2}{2(bc-ad)^2(a+bx)}-\frac{2a^2cd^2b^2}{(b^2c^2-a^2d^2)^2(c+dx)}-\frac{a^2d^2}{(b^2c^2-a^2d^2)(c+dx)^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2d}{(c+dx)(b^2c^2-a^2d^2)}-\frac{2a^2b^2cd\log(c+dx)}{(b^2c^2-a^2d^2)^2}-\frac{ab\log(a-bx)}{2(ad+bc)^2}+\frac{ab\log(a+bx)}{2(bc-ad)^2}}{a^2}$$

input
```
Int[1/((c + d*x)^2*(a^2 - b^2*x^2)),x]
```

output
```
((a^2*d)/((b^2*c^2 - a^2*d^2)*(c + d*x)) - (a*b*Log[a - b*x])/(2*(b*c + a*
d)^2) + (a*b*Log[a + b*x])/(2*(b*c - a*d)^2) - (2*a^2*b^2*c*d*Log[c + d*x]
)/(b^2*c^2 - a^2*d^2)^2)/a^2
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

method	result
default	$\frac{b\ln(bx+a)}{2(ad-bc)^2 a}-\frac{d}{(ad-bc)(ad+bc)(dx+c)}-\frac{2b^2cd\ln(dx+c)}{(ad-bc)^2(ad+bc)^2}-\frac{b\ln(-bx+a)}{2a(ad+bc)^2}$
risch	$-\frac{d}{(a^2d^2-b^2c^2)(dx+c)}+\frac{b\ln(bx+a)}{2(a^2d^2-2abcd+b^2c^2)a}-\frac{2b^2cd\ln(-dx-c)}{a^4d^4-2a^2b^2c^2d^2+c^4b^4}-\frac{b\ln(bx-a)}{2(a^2d^2+2abcd+b^2c^2)a}$
norman	$\frac{d^2x}{c(a^2d^2-b^2c^2)(dx+c)}+\frac{b\ln(bx+a)}{2(a^2d^2-2abcd+b^2c^2)a}-\frac{b\ln(-bx+a)}{2(a^2d^2+2abcd+b^2c^2)a}-\frac{2b^2cd\ln(dx+c)}{a^4d^4-2a^2b^2c^2d^2+c^4b^4}$
parallelrisch	$-\frac{\ln(bx-a)x a^2bc d^3-2\ln(bx-a)xa b^2c^2d^2+\ln(bx-a)x b^3c^3d-\ln(bx+a)x a^2bc d^3-2\ln(bx+a)xa b^2c^2d^2-\ln(bx+a)x b^3c^3d+}{}$

input
```
int(1/(d*x+c)^2/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/(a*d-b*c)^2/a*ln(b*x+a)-d/(a*d-b*c)/(a*d+b*c)/(d*x+c)-2*b^2*c*d/(a*d
-b*c)^2/(a*d+b*c)^2*ln(d*x+c)-1/2*b*ln(-b*x+a)/a/(a*d+b*c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(103) = 206.

Time = 0.50 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.28

$$\int \frac{1}{(c+dx)^2(a^2-b^2x^2)}\,dx$$
$$=\frac{2\,ab^2c^2d-2\,a^3d^3+(b^3c^3+2\,ab^2c^2d+a^2bcd^2+(b^3c^2d+2\,ab^2cd^2+a^2bd^3)x)\log(bx+a)-(b^3c^3-2\,ab^2}{2\,(ab^4c^5-2\,a^3b^2c^3d^2+a^5cd^4+(ab^4c}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*(2*a*b^2*c^2*d - 2*a^3*d^3 + (b^3*c^3 + 2*a*b^2*c^2*d + a^2*b*c*d^2 +
(b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a) - (b^3*c^3 - 2*a*b
^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*
x - a) - 4*(a*b^2*c*d^2*x + a*b^2*c^2*d)*log(d*x + c))/(a*b^4*c^5 - 2*a^3*
b^2*c^3*d^2 + a^5*c*d^4 + (a*b^4*c^4*d - 2*a^3*b^2*c^2*d^3 + a^5*d^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\,(a^2-b^2x^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(-b**2*x**2+a**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{1}{(c+dx)^2\,(a^2-b^2x^2)}\,dx = & -\frac{2\,b^2cd\log\left(dx+c\right)}{b^4c^4-2\,a^2b^2c^2d^2+a^4d^4}+\frac{b\log\left(bx+a\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)}\\ & -\frac{b\log\left(bx-a\right)}{2\,(ab^2c^2+2\,a^2bcd+a^3d^2)}\\ & +\frac{d}{b^2c^3-a^2cd^2+(b^2c^2d-a^2d^3)x}\end{aligned}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-2*b^2*c*d*log(d*x + c)/(b^4*c^4 - 2*a^2*b^2*c^2*d^2 + a^4*d^4) + 1/2*b*lo
g(b*x + a)/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) - 1/2*b*log(b*x - a)/(a*b^2
*c^2 + 2*a^2*b*c*d + a^3*d^2) + d/(b^2*c^3 - a^2*c*d^2 + (b^2*c^2*d - a^2*
d^3)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(103) = 206$.

Time = 0.13 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.66

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)} \, dx = \frac{b^2cd \log \left(\left| b^2 - \frac{2\,b^2c}{dx+c} + \frac{b^2c^2}{(dx+c)^2} - \frac{a^2d^2}{(dx+c)^2} \right| \right)}{b^4c^4 - 2\,a^2b^2c^2d^2 + a^4d^4} + \frac{d^3}{(b^2c^2d^2 - a^2d^4)(dx+c)} - \frac{(b^4c^2d^2 + a^2b^2d^4) \log \left(\frac{\left| 2\,b^2cd - \frac{2\,b^2c^2d}{dx+c} + \frac{2\,a^2d^3}{dx+c} - 2\,d^2|a||b| \right|}{\left| 2\,b^2cd - \frac{2\,b^2c^2d}{dx+c} + \frac{2\,a^2d^3}{dx+c} + 2\,d^2|a||b| \right|} \right)}{2\,(b^4c^4 - 2\,a^2b^2c^2d^2 + a^4d^4)d^2|a||b|}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
b^2*c*d*log(abs(b^2 - 2*b^2*c/(d*x + c) + b^2*c^2/(d*x + c)^2 - a^2*d^2/(d
*x + c)^2))/(b^4*c^4 - 2*a^2*b^2*c^2*d^2 + a^4*d^4) + d^3/((b^2*c^2*d^2 -
a^2*d^4)*(d*x + c)) - 1/2*(b^4*c^2*d^2 + a^2*b^2*d^4)*log(abs(2*b^2*c*d -
2*b^2*c^2*d/(d*x + c) + 2*a^2*d^3/(d*x + c) - 2*d^2*abs(a)*abs(b))/abs(2*b
^2*c*d - 2*b^2*c^2*d/(d*x + c) + 2*a^2*d^3/(d*x + c) + 2*d^2*abs(a)*abs(b)
))/((b^4*c^4 - 2*a^2*b^2*c^2*d^2 + a^4*d^4)*d^2*abs(a)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.37

$$\int \frac{1}{(c+dx)^2(a^2-b^2x^2)}\,dx = \frac{b\,\ln(a+b\,x)}{2\,\left(a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2\right)} - \frac{b\,\ln(a-b\,x)}{2\,\left(a^3\,d^2+2\,a^2\,b\,c\,d+a\,b^2\,c^2\right)} - \frac{d}{\left(a^2\,d^2-b^2\,c^2\right)\,\left(c+d\,x\right)} - \frac{2\,b^2\,c\,d\,\ln(c+d\,x)}{a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4}$$

input
```
int(1/((a^2 - b^2*x^2)*(c + d*x)^2),x)
```

output
```
(b*log(a + b*x))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) - (b*log(a - b*x)
)/(2*(a^3*d^2 + a*b^2*c^2 + 2*a^2*b*c*d)) - d/((a^2*d^2 - b^2*c^2)*(c + d*
x)) - (2*b^2*c*d*log(c + d*x))/(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.27

$$\int \frac{1}{(c+dx)^2(a^2-b^2x^2)}\,dx = \frac{\log(-bx-a)\,a^2b\,c^2d^2+\log(-bx-a)\,a^2bc\,d^3x+2\log(-bx-a)\,a\,b^2c^3d+2\log(-bx-a)\,a\,b^2c^2d^2x+\log}{}$$

input
```
int(1/(d*x+c)^2/(-b^2*x^2+a^2),x)
```

output
```
(log( - a - b*x)*a**2*b*c**2*d**2 + log( - a - b*x)*a**2*b*c*d**3*x + 2*lo
g( - a - b*x)*a*b**2*c**3*d + 2*log( - a - b*x)*a*b**2*c**2*d**2*x + log(
- a - b*x)*b**3*c**4 + log( - a - b*x)*b**3*c**3*d*x - log(a - b*x)*a**2*b
*c**2*d**2 - log(a - b*x)*a**2*b*c*d**3*x + 2*log(a - b*x)*a*b**2*c**3*d +
 2*log(a - b*x)*a*b**2*c**2*d**2*x - log(a - b*x)*b**3*c**4 - log(a - b*x)
*b**3*c**3*d*x - 4*log(c + d*x)*a*b**2*c**3*d - 4*log(c + d*x)*a*b**2*c**2
*d**2*x + 2*a**3*d**4*x - 2*a*b**2*c**2*d**2*x)/(2*a*c*(a**4*c*d**4 + a**4
*d**5*x - 2*a**2*b**2*c**3*d**2 - 2*a**2*b**2*c**2*d**3*x + b**4*c**5 + b*
*4*c**4*d*x))
```

3.35 $\int \frac{1}{(c+dx)^3(a^2-b^2x^2)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)} \, dx = \frac{d}{2 (b^2c^2-a^2d^2) (c+dx)^2} + \frac{2b^2cd}{(b^2c^2-a^2d^2)^2 (c+dx)} - \frac{b^2 \log(a-bx)}{2a(bc+ad)^3} + \frac{b^2 \log(a+bx)}{2a(bc-ad)^3} - \frac{b^2d(3b^2c^2+a^2d^2) \log(c+dx)}{(b^2c^2-a^2d^2)^3}$$

output
```
1/2*d/(-a^2*d^2+b^2*c^2)/(d*x+c)^2+2*b^2*c*d/(-a^2*d^2+b^2*c^2)^2/(d*x+c)-
1/2*b^2*ln(-b*x+a)/a/(a*d+b*c)^3+1/2*b^2*ln(b*x+a)/a/(-a*d+b*c)^3-b^2*d*(a
^2*d^2+3*b^2*c^2)*ln(d*x+c)/(-a^2*d^2+b^2*c^2)^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)} \, dx = \frac{ad(b^2c^2-a^2d^2)^2 + 4ab^2cd(b^2c^2-a^2d^2) (c+dx) + 2b^3(b^2c^3+3a^2cd^2) (c+dx)^2 \text{arctanh}\left(\frac{bx}{a}\right) - 2a(3b^4c^2d}{2a (b^2c^2-a^2d^2)^3 (c+dx)^2}$$

input
```
Integrate[1/((c + d*x)^3*(a^2 - b^2*x^2)),x]
```

output
```
(a*d*(b^2*c^2 - a^2*d^2)^2 + 4*a*b^2*c*d*(b^2*c^2 - a^2*d^2)*(c + d*x) + 2
*b^3*(b^2*c^3 + 3*a^2*c*d^2)*(c + d*x)^2*ArcTanh[(b*x)/a] - 2*a*(3*b^4*c^2
*d + a^2*b^2*d^3)*(c + d*x)^2*Log[c + d*x] + a*b^2*(3*b^2*c^2*d + a^2*d^3)
*(c + d*x)^2*Log[a^2 - b^2*x^2])/(2*a*(b^2*c^2 - a^2*d^2)^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)(c+dx)^3}dx$$

$$\Big\downarrow \ 477$$

$$\frac{\int \left(\frac{ab^3}{2(bc+ad)^3(a-bx)}+\frac{ab^3}{2(bc-ad)^3(a+bx)}-\frac{a^2d^2(3b^2c^2+a^2d^2)b^2}{(b^2c^2-a^2d^2)^3(c+dx)}-\frac{2a^2cd^2b^2}{(b^2c^2-a^2d^2)^2(c+dx)^2}-\frac{a^2d^2}{(b^2c^2-a^2d^2)(c+dx)^3}\right)dx}{a^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2a^2b^2cd}{(c+dx)(b^2c^2-a^2d^2)^2}+\frac{a^2d}{2(c+dx)^2(b^2c^2-a^2d^2)}-\frac{a^2b^2d(a^2d^2+3b^2c^2)\log(c+dx)}{(b^2c^2-a^2d^2)^3}-\frac{ab^2\log(a-bx)}{2(ad+bc)^3}+\frac{ab^2\log(a+bx)}{2(bc-ad)^3}}{a^2}$$

input
```
Int[1/((c + d*x)^3*(a^2 - b^2*x^2)),x]
```

output
```
((a^2*d)/(2*(b^2*c^2 - a^2*d^2)*(c + d*x)^2) + (2*a^2*b^2*c*d)/((b^2*c^2 -
 a^2*d^2)^2*(c + d*x)) - (a*b^2*Log[a - b*x])/(2*(b*c + a*d)^3) + (a*b^2*L
og[a + b*x])/(2*(b*c - a*d)^3) - (a^2*b^2*d*(3*b^2*c^2 + a^2*d^2)*Log[c +
d*x])/(b^2*c^2 - a^2*d^2)^3)/a^2
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

method	result
default	$-\frac{b^2 \ln(bx+a)}{2(ad-bc)^3 a}-\frac{d}{2(ad-bc)(ad+bc)(dx+c)^2}+\frac{b^2 d(a^2d^2+3b^2c^2)\ln(dx+c)}{(ad-bc)^3(ad+bc)^3}+\frac{2b^2cd}{(ad-bc)^2(ad+bc)^2(dx+c)}-\frac{b^2\ln(-bx+\cdots}{2a(ad+bc)\cdots}$
norman	$\frac{\frac{-a^2d^5+5b^2c^2d^3}{2d^2\left(a^4d^4-2a^2b^2c^2d^2+c^4b^4\right)}+\frac{2c\,b^2d^2x}{a^4d^4-2a^2b^2c^2d^2+c^4b^4}}{(dx+c)^2}+\frac{b^2d(a^2d^2+3b^2c^2)\ln(dx+c)}{a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6}-\frac{b^2\ln(bx+a)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d\cdots}$
risch	$\frac{\frac{2c\,b^2d^2x}{a^4d^4-2a^2b^2c^2d^2+c^4b^4}-\frac{\left(a^2d^2-5b^2c^2\right)d}{2\left(a^4d^4-2a^2b^2c^2d^2+c^4b^4\right)}}{(dx+c)^2}+\frac{b^2d^3\ln(dx+c)a^2}{a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6}+\frac{3b^4d\ln(dx+c)c^2}{a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2\cdots}$
parallelrisch	$-\frac{a^5d^7+3\ln(bx+a)x^2a^2b^3c\,d^6+3\ln(bx+a)x^2a\,b^4c^2d^5-4\ln(dx+c)x\,a^3b^2c\,d^6+6\ln(bx-a)xa\,b^4c^3d^4+6\ln(bx+a)x\,a^2b^3c^2d^5+\cdots}{\cdots}$

input
```
int(1/(d*x+c)^3/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/(a*d-b*c)^3/a*ln(b*x+a)-1/2*d/(a*d-b*c)/(a*d+b*c)/(d*x+c)^2+b^2*d
*(a^2*d^2+3*b^2*c^2)/(a*d-b*c)^3/(a*d+b*c)^3*ln(d*x+c)+2*b^2*c*d/(a*d-b*c)
^2/(a*d+b*c)^2/(d*x+c)-1/2*b^2*ln(-b*x+a)/a/(a*d+b*c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(155) = 310$.

Time = 2.81 (sec) , antiderivative size = 605, normalized size of antiderivative = 3.76

$$\int \frac{1}{(c+dx)^3(a^2-b^2x^2)}\,dx = \frac{5\,ab^4c^4d - 6\,a^3b^2c^2d^3 + a^5d^5 + 4\,(ab^4c^3d^2 - a^3b^2cd^4)x + (b^5c^5 + 3\,ab^4c^4d + 3\,a^2b^3c^3d^2 + a^3b^2c^2d^3 + (b^5c^3}{}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*(5*a*b^4*c^4*d - 6*a^3*b^2*c^2*d^3 + a^5*d^5 + 4*(a*b^4*c^3*d^2 - a^3*
b^2*c*d^4)*x + (b^5*c^5 + 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + a^3*b^2*c^2*
d^3 + (b^5*c^3*d^2 + 3*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2
+ 2*(b^5*c^4*d + 3*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*x)*l
og(b*x + a) - (b^5*c^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d
^3 + (b^5*c^3*d^2 - 3*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 +
 2*(b^5*c^4*d - 3*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*lo
g(b*x - a) - 2*(3*a*b^4*c^4*d + a^3*b^2*c^2*d^3 + (3*a*b^4*c^2*d^3 + a^3*b
^2*d^5)*x^2 + 2*(3*a*b^4*c^3*d^2 + a^3*b^2*c*d^4)*x)*log(d*x + c))/(a*b^6*
c^8 - 3*a^3*b^4*c^6*d^2 + 3*a^5*b^2*c^4*d^4 - a^7*c^2*d^6 + (a*b^6*c^6*d^2
 - 3*a^3*b^4*c^4*d^4 + 3*a^5*b^2*c^2*d^6 - a^7*d^8)*x^2 + 2*(a*b^6*c^7*d -
 3*a^3*b^4*c^5*d^3 + 3*a^5*b^2*c^3*d^5 - a^7*c*d^7)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3(a^2-b^2x^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**3/(-b**2*x**2+a**2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(155) = 310$.

Time = 0.04 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.95

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)}\,dx = \frac{b^2 \log(bx+a)}{2\,(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)} - \frac{b^2 \log(bx-a)}{2\,(ab^3c^3 + 3\,a^2b^2c^2d + 3\,a^3bcd^2 + a^4d^3)} - \frac{(3\,b^4c^2d + a^2b^2d^3)\log(dx+c)}{b^6c^6 - 3\,a^2b^4c^4d^2 + 3\,a^4b^2c^2d^4 - a^6d^6} + \frac{4\,b^2cd^2x + 5\,b^2c^2d - a^2d^3}{2\,(b^4c^6 - 2\,a^2b^2c^4d^2 + a^4c^2d^4 + (b^4c^4d^2 - 2\,a^2b^2c^2d^4 + a^4d^6)x^2 + 2\,(b^4c^5d - 2\,a^2b^2c^3d^3 + a^4cd^5)x)}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*b^2*log(b*x + a)/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^
3) - 1/2*b^2*log(b*x - a)/(a*b^3*c^3 + 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 + a
^4*d^3) - (3*b^4*c^2*d + a^2*b^2*d^3)*log(d*x + c)/(b^6*c^6 - 3*a^2*b^4*c^
4*d^2 + 3*a^4*b^2*c^2*d^4 - a^6*d^6) + 1/2*(4*b^2*c*d^2*x + 5*b^2*c^2*d -
a^2*d^3)/(b^4*c^6 - 2*a^2*b^2*c^4*d^2 + a^4*c^2*d^4 + (b^4*c^4*d^2 - 2*a^2
*b^2*c^2*d^4 + a^4*d^6)*x^2 + 2*(b^4*c^5*d - 2*a^2*b^2*c^3*d^3 + a^4*c*d^5
)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.72

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)}\,dx = \frac{b^3 \log(|bx+a|)}{2\,(ab^4c^3 - 3\,a^2b^3c^2d + 3\,a^3b^2cd^2 - a^4bd^3)} - \frac{b^3 \log(|bx-a|)}{2\,(ab^4c^3 + 3\,a^2b^3c^2d + 3\,a^3b^2cd^2 + a^4bd^3)} - \frac{(3\,b^4c^2d^2 + a^2b^2d^4)\log(|dx+c|)}{b^6c^6d - 3\,a^2b^4c^4d^3 + 3\,a^4b^2c^2d^5 - a^6d^7} + \frac{5\,b^4c^4d - 6\,a^2b^2c^2d^3 + a^4d^5 + 4\,(b^4c^3d^2 - a^2b^2cd^4)x}{2\,(bc+ad)^3(bc-ad)^3(dx+c)^2}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*b^3*log(abs(b*x + a))/(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 -
 a^4*b*d^3) - 1/2*b^3*log(abs(b*x - a))/(a*b^4*c^3 + 3*a^2*b^3*c^2*d + 3*a
^3*b^2*c*d^2 + a^4*b*d^3) - (3*b^4*c^2*d^2 + a^2*b^2*d^4)*log(abs(d*x + c)
)/(b^6*c^6*d - 3*a^2*b^4*c^4*d^3 + 3*a^4*b^2*c^2*d^5 - a^6*d^7) + 1/2*(5*b
^4*c^4*d - 6*a^2*b^2*c^2*d^3 + a^4*d^5 + 4*(b^4*c^3*d^2 - a^2*b^2*c*d^4)*x
)/((b*c + a*d)^3*(b*c - a*d)^3*(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.83

$$\int \frac{1}{(c+dx)^3\,(a^2-b^2x^2)}\,dx = \frac{\ln(c+d\,x)\,\left(a^2\,b^2\,d^3+3\,b^4\,c^2\,d\right)}{a^6\,d^6-3\,a^4\,b^2\,c^2\,d^4+3\,a^2\,b^4\,c^4\,d^2-b^6\,c^6} - \frac{b^2\,\ln(a+b\,x)}{2\,\left(a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3\right)} - \frac{b^2\,\ln(a-b\,x)}{2\,\left(a^4\,d^3+3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d+a\,b^3\,c^3\right)} - \frac{\frac{a^2\,d^3-5\,b^2\,c^2\,d}{2\,\left(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4\right)}-\frac{2\,b^2\,c\,d^2\,x}{a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

input
```
int(1/((a^2 - b^2*x^2)*(c + d*x)^3),x)
```

output
```
(log(c + d*x)*(3*b^4*c^2*d + a^2*b^2*d^3))/(a^6*d^6 - b^6*c^6 + 3*a^2*b^4*
c^4*d^2 - 3*a^4*b^2*c^2*d^4) - (b^2*log(a + b*x))/(2*(a^4*d^3 - a*b^3*c^3
+ 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) - (b^2*log(a - b*x))/(2*(a^4*d^3 + a*b
^3*c^3 + 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2)) - ((a^2*d^3 - 5*b^2*c^2*d)/(2*(
a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) - (2*b^2*c*d^2*x)/(a^4*d^4 + b^4*c
^4 - 2*a^2*b^2*c^2*d^2))/(c^2 + d^2*x^2 + 2*c*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 839, normalized size of antiderivative = 5.21

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^3/(-b^2*x^2+a^2),x)
```

output
```
( - log( - a - b*x)*a**3*b**2*c**2*d**3 - 2*log( - a - b*x)*a**3*b**2*c*d*
*4*x - log( - a - b*x)*a**3*b**2*d**5*x**2 - 3*log( - a - b*x)*a**2*b**3*c
**3*d**2 - 6*log( - a - b*x)*a**2*b**3*c**2*d**3*x - 3*log( - a - b*x)*a**
2*b**3*c*d**4*x**2 - 3*log( - a - b*x)*a*b**4*c**4*d - 6*log( - a - b*x)*a
*b**4*c**3*d**2*x - 3*log( - a - b*x)*a*b**4*c**2*d**3*x**2 - log( - a - b
*x)*b**5*c**5 - 2*log( - a - b*x)*b**5*c**4*d*x - log( - a - b*x)*b**5*c**
3*d**2*x**2 - log(a - b*x)*a**3*b**2*c**2*d**3 - 2*log(a - b*x)*a**3*b**2*
c*d**4*x - log(a - b*x)*a**3*b**2*d**5*x**2 + 3*log(a - b*x)*a**2*b**3*c**
3*d**2 + 6*log(a - b*x)*a**2*b**3*c**2*d**3*x + 3*log(a - b*x)*a**2*b**3*c
*d**4*x**2 - 3*log(a - b*x)*a*b**4*c**4*d - 6*log(a - b*x)*a*b**4*c**3*d**
2*x - 3*log(a - b*x)*a*b**4*c**2*d**3*x**2 + log(a - b*x)*b**5*c**5 + 2*lo
g(a - b*x)*b**5*c**4*d*x + log(a - b*x)*b**5*c**3*d**2*x**2 + 2*log(c + d*
x)*a**3*b**2*c**2*d**3 + 4*log(c + d*x)*a**3*b**2*c*d**4*x + 2*log(c + d*x
)*a**3*b**2*d**5*x**2 + 6*log(c + d*x)*a*b**4*c**4*d + 12*log(c + d*x)*a*b
**4*c**3*d**2*x + 6*log(c + d*x)*a*b**4*c**2*d**3*x**2 - a**5*d**5 + 4*a**
3*b**2*c**2*d**3 - 2*a**3*b**2*d**5*x**2 - 3*a*b**4*c**4*d + 2*a*b**4*c**2
*d**3*x**2)/(2*a*(a**6*c**2*d**6 + 2*a**6*c*d**7*x + a**6*d**8*x**2 - 3*a*
*4*b**2*c**4*d**4 - 6*a**4*b**2*c**3*d**5*x - 3*a**4*b**2*c**2*d**6*x**2 +
 3*a**2*b**4*c**6*d**2 + 6*a**2*b**4*c**5*d**3*x + 3*a**2*b**4*c**4*d**4*x
**2 - b**6*c**8 - 2*b**6*c**7*d*x - b**6*c**6*d**2*x**2))
```

3.36 $\int \frac{(c+dx)^4}{\left(a^2-b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 131

$$\int \frac{(c+dx)^4}{\left(a^2-b^2x^2\right)^2}\,dx = \frac{d^4x}{b^4}+\frac{(bc+ad)^4}{4a^2b^5(a-bx)}-\frac{(bc-ad)^4}{4a^2b^5(a+bx)}-\frac{(bc-3ad)(bc+ad)^3\log(a-bx)}{4a^3b^5}+\frac{(bc-ad)^3(bc+3ad)\log(a+bx)}{4a^3b^5}$$

output
```
d^4*x/b^4+1/4*(a*d+b*c)^4/a^2/b^5/(-b*x+a)-1/4*(-a*d+b*c)^4/a^2/b^5/(b*x+a
)-1/4*(-3*a*d+b*c)*(a*d+b*c)^3*ln(-b*x+a)/a^3/b^5+1/4*(-a*d+b*c)^3*(3*a*d+
b*c)*ln(b*x+a)/a^3/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^4}{\left(a^2-b^2x^2\right)^2}\,dx = \frac{d^4x}{b^4}+\frac{-4a^2b^2c^3d-4a^4cd^3-b^4c^4x-6a^2b^2c^2d^2x-a^4d^4x}{2a^2b^4\left(-a^2+b^2x^2\right)}-\frac{\left(-b^4c^4+6a^2b^2c^2d^2+3a^4d^4\right)\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^3b^5}+\frac{2cd^3\log\left(a^2-b^2x^2\right)}{b^4}$$

input
```
Integrate[(c + d*x)^4/(a^2 - b^2*x^2)^2,x]
```

output
```
(d^4*x)/b^4 + (-4*a^2*b^2*c^3*d - 4*a^4*c*d^3 - b^4*c^4*x - 6*a^2*b^2*c^2*
d^2*x - a^4*d^4*x)/(2*a^2*b^4*(-a^2 + b^2*x^2)) - ((-(b^4*c^4) + 6*a^2*b^2
*c^2*d^2 + 3*a^4*d^4)*ArcTanh[(b*x)/a])/(2*a^3*b^5) + (2*c*d^3*Log[a^2 - b
^2*x^2])/b^4
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^2}\,dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(\frac{a^4d^4}{b^4}+\frac{a(bc-3ad)(bc+ad)^3}{4b^4(a-bx)}+\frac{a(bc-ad)^3(bc+3ad)}{4b^4(a+bx)}+\frac{a^2(bc+ad)^4}{4b^4(a-bx)^2}+\frac{a^2(bc-ad)^4}{4b^4(a+bx)^2}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^4d^4x}{b^4}+\frac{a^2(ad+bc)^4}{4b^5(a-bx)}-\frac{a^2(bc-ad)^4}{4b^5(a+bx)}-\frac{a(bc-3ad)(ad+bc)^3\log(a-bx)}{4b^5}+\frac{a(bc-ad)^3(3ad+bc)\log(a+bx)}{4b^5}}{a^4}$$

input
```
Int[(c + d*x)^4/(a^2 - b^2*x^2)^2,x]
```

output
```
((a^4*d^4*x)/b^4 + (a^2*(b*c + a*d)^4)/(4*b^5*(a - b*x)) - (a^2*(b*c - a*d
)^4)/(4*b^5*(a + b*x)) - (a*(b*c - 3*a*d)*(b*c + a*d)^3*Log[a - b*x])/(4*b
^5) + (a*(b*c - a*d)^3*(b*c + 3*a*d)*Log[a + b*x])/(4*b^5))/a^4
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.55

method	result
norman	$\frac{-\frac{-2d^3c\,a^2-2b^2c^3d}{b^4}-\frac{d^4x^3}{b^2}+\frac{\left(3a^4d^4+6a^2b^2c^2d^2+c^4b^4\right)x}{2a^2b^4}}{-b^2x^2+a^2}+\frac{\left(3a^4d^4+8c\,a^3b\,d^3+6a^2b^2c^2d^2-c^4b^4\right)\ln\left(-bx+a\right)}{4b^5a^3}-\frac{(3a^4d^4-8c}{}$
risch	$\frac{d^4x}{b^4}+\frac{\frac{\left(a^4d^4+6a^2b^2c^2d^2+c^4b^4\right)x}{2a^2}+2cd\left(a^2d^2+b^2c^2\right)}{b^4\left(-b^2x^2+a^2\right)}-\frac{3a\ln\left(-bx-a\right)d^4}{4b^5}+\frac{2\ln\left(-bx-a\right)c\,d^3}{b^4}-\frac{3\ln\left(-bx-a\right)c^2d^2}{2b^3a}+\ln($
default	$\frac{d^4x}{b^4}+\frac{\left(-3a^4d^4+8c\,a^3b\,d^3-6a^2b^2c^2d^2+c^4b^4\right)\ln\left(bx+a\right)}{4a^3b^5}-\frac{a^4d^4-4c\,a^3b\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4}{4b^5a^2\left(bx+a\right)}+\frac{(3a^4d^4+8c\,a^3}{}$
parallelrisch	$\frac{3\ln\left(bx-a\right)x^2a^4b^2d^4+8\ln\left(bx-a\right)x^2a^3b^3c\,d^3+6\ln\left(bx-a\right)x^2a^2b^4c^2d^2-\ln\left(bx-a\right)x^2b^6c^4-3\ln\left(bx+a\right)x^2a^4b^2d^4+8\ln\left(bx+a\right)x^2a}{}$

input
```
int((d*x+c)^4/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(-2*a^2*c*d^3-2*b^2*c^3*d)/b^4-d^4*x^3/b^2+1/2*(3*a^4*d^4+6*a^2*b^2*c^2*
d^2+b^4*c^4)/a^2/b^4*x)/(-b^2*x^2+a^2)+1/4/b^5*(3*a^4*d^4+8*a^3*b*c*d^3+6*
a^2*b^2*c^2*d^2-b^4*c^4)/a^3*ln(-b*x+a)-1/4*(3*a^4*d^4-8*a^3*b*c*d^3+6*a^2
*b^2*c^2*d^2-b^4*c^4)/a^3/b^5*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(124) = 248$.

Time = 0.08 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.27

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^2}\,dx$$
$$= 4\,a^3b^3d^4x^3 - 8\,a^3b^3c^3d - 8\,a^5bcd^3 - 2\,(ab^5c^4 + 6\,a^3b^3c^2d^2 + 3\,a^5bd^4)x - (a^2b^4c^4 - 6\,a^4b^2c^2d^2 + 8\,a^5bcd^3 -$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/4*(4*a^3*b^3*d^4*x^3 - 8*a^3*b^3*c^3*d - 8*a^5*b*c*d^3 - 2*(a*b^5*c^4 +
6*a^3*b^3*c^2*d^2 + 3*a^5*b*d^4)*x - (a^2*b^4*c^4 - 6*a^4*b^2*c^2*d^2 + 8*
a^5*b*c*d^3 - 3*a^6*d^4 - (b^6*c^4 - 6*a^2*b^4*c^2*d^2 + 8*a^3*b^3*c*d^3 -
 3*a^4*b^2*d^4)*x^2)*log(b*x + a) + (a^2*b^4*c^4 - 6*a^4*b^2*c^2*d^2 - 8*a
^5*b*c*d^3 - 3*a^6*d^4 - (b^6*c^4 - 6*a^2*b^4*c^2*d^2 - 8*a^3*b^3*c*d^3 -
3*a^4*b^2*d^4)*x^2)*log(b*x - a))/(a^3*b^7*x^2 - a^5*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(117) = 234$.

Time = 0.87 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.02

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^2}\,dx = \frac{-4a^4cd^3 - 4a^2b^2c^3d + x(-a^4d^4 - 6a^2b^2c^2d^2 - b^4c^4)}{-2a^4b^4 + 2a^2b^6x^2} + \frac{d^4x}{b^4} - \frac{(ad-bc)^3\cdot(3ad+bc)\log\left(x + \frac{8a^4cd^3 + \frac{a(ad-bc)^3\cdot(3ad+bc)}{b}}{3a^4d^4+6a^2b^2c^2d^2-b^4c^4}\right)}{4a^3b^5} + \frac{(ad+bc)^3\cdot(3ad-bc)\log\left(x + \frac{8a^4cd^3 - \frac{a(ad+bc)^3\cdot(3ad-bc)}{b}}{3a^4d^4+6a^2b^2c^2d^2-b^4c^4}\right)}{4a^3b^5}$$

input
```
integrate((d*x+c)**4/(-b**2*x**2+a**2)**2,x)
```

output
```
(-4*a**4*c*d**3 - 4*a**2*b**2*c**3*d + x*(-a**4*d**4 - 6*a**2*b**2*c**2*d*
*2 - b**4*c**4))/(-2*a**4*b**4 + 2*a**2*b**6*x**2) + d**4*x/b**4 - (a*d -
b*c)**3*(3*a*d + b*c)*log(x + (8*a**4*c*d**3 + a*(a*d - b*c)**3*(3*a*d + b
*c)/b)/(3*a**4*d**4 + 6*a**2*b**2*c**2*d**2 - b**4*c**4))/(4*a**3*b**5) +
(a*d + b*c)**3*(3*a*d - b*c)*log(x + (8*a**4*c*d**3 - a*(a*d + b*c)**3*(3*
a*d - b*c)/b)/(3*a**4*d**4 + 6*a**2*b**2*c**2*d**2 - b**4*c**4))/(4*a**3*b
**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^2}\,dx = \frac{d^4x}{b^4} - \frac{4\,a^2b^2c^3d + 4\,a^4cd^3 + (b^4c^4 + 6\,a^2b^2c^2d^2 + a^4d^4)x}{2\,(a^2b^6x^2 - a^4b^4)} + \frac{(b^4c^4 - 6\,a^2b^2c^2d^2 + 8\,a^3bcd^3 - 3\,a^4d^4)\log(bx+a)}{4\,a^3b^5} - \frac{(b^4c^4 - 6\,a^2b^2c^2d^2 - 8\,a^3bcd^3 - 3\,a^4d^4)\log(bx-a)}{4\,a^3b^5}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
d^4*x/b^4 - 1/2*(4*a^2*b^2*c^3*d + 4*a^4*c*d^3 + (b^4*c^4 + 6*a^2*b^2*c^2*
d^2 + a^4*d^4)*x)/(a^2*b^6*x^2 - a^4*b^4) + 1/4*(b^4*c^4 - 6*a^2*b^2*c^2*d
^2 + 8*a^3*b*c*d^3 - 3*a^4*d^4)*log(b*x + a)/(a^3*b^5) - 1/4*(b^4*c^4 - 6*
a^2*b^2*c^2*d^2 - 8*a^3*b*c*d^3 - 3*a^4*d^4)*log(b*x - a)/(a^3*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^2}\,dx = \frac{d^4x}{b^4} + \frac{(b^4c^4 - 6\,a^2b^2c^2d^2 + 8\,a^3bcd^3 - 3\,a^4d^4)\log(|bx+a|)}{4\,a^3b^5} - \frac{(b^4c^4 - 6\,a^2b^2c^2d^2 - 8\,a^3bcd^3 - 3\,a^4d^4)\log(|bx-a|)}{4\,a^3b^5} - \frac{4\,a^2b^2c^3d + 4\,a^4cd^3 + (b^4c^4 + 6\,a^2b^2c^2d^2 + a^4d^4)x}{2\,(bx+a)(bx-a)a^2b^4}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
d^4*x/b^4 + 1/4*(b^4*c^4 - 6*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 3*a^4*d^4)*
log(abs(b*x + a))/(a^3*b^5) - 1/4*(b^4*c^4 - 6*a^2*b^2*c^2*d^2 - 8*a^3*b*c
*d^3 - 3*a^4*d^4)*log(abs(b*x - a))/(a^3*b^5) - 1/2*(4*a^2*b^2*c^3*d + 4*a
^4*c*d^3 + (b^4*c^4 + 6*a^2*b^2*c^2*d^2 + a^4*d^4)*x)/((b*x + a)*(b*x - a)
*a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^2}\,dx = \frac{\frac{x\,(a^4\,d^4+6\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}{2\,a^2}+2\,a^2\,c\,d^3+2\,b^2\,c^3\,d}{a^2\,b^4-b^6\,x^2}+\frac{d^4\,x}{b^4} - \frac{\ln(a+b\,x)\,\left(3\,a^4\,d^4-8\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-b^4\,c^4\right)}{4\,a^3\,b^5} + \frac{\ln(a-b\,x)\,\left(3\,a^4\,d^4+8\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-b^4\,c^4\right)}{4\,a^3\,b^5}$$

input
```
int((c + d*x)^4/(a^2 - b^2*x^2)^2,x)
```

output
```
((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2))/(2*a^2) + 2*a^2*c*d^3 + 2*b^2
*c^3*d)/(a^2*b^4 - b^6*x^2) + (d^4*x)/b^4 - (log(a + b*x)*(3*a^4*d^4 - b^4
*c^4 + 6*a^2*b^2*c^2*d^2 - 8*a^3*b*c*d^3))/(4*a^3*b^5) + (log(a - b*x)*(3*
a^4*d^4 - b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3))/(4*a^3*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.25

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^2}\,dx = \frac{-3\log(-bx-a)\,a^6d^4+8\log(-bx-a)\,a^5bc\,d^3-6\log(-bx-a)\,a^4b^2c^2d^2+3\log(-bx-a)\,a^4b^2d^4x^2-8}{}$$

input
```
int((d*x+c)^4/(-b^2*x^2+a^2)^2,x)
```

output
```
( - 3*log( - a - b*x)*a**6*d**4 + 8*log( - a - b*x)*a**5*b*c*d**3 - 6*log(
 - a - b*x)*a**4*b**2*c**2*d**2 + 3*log( - a - b*x)*a**4*b**2*d**4*x**2 -
8*log( - a - b*x)*a**3*b**3*c*d**3*x**2 + log( - a - b*x)*a**2*b**4*c**4 +
 6*log( - a - b*x)*a**2*b**4*c**2*d**2*x**2 - log( - a - b*x)*b**6*c**4*x*
*2 + 3*log(a - b*x)*a**6*d**4 + 8*log(a - b*x)*a**5*b*c*d**3 + 6*log(a - b
*x)*a**4*b**2*c**2*d**2 - 3*log(a - b*x)*a**4*b**2*d**4*x**2 - 8*log(a - b
*x)*a**3*b**3*c*d**3*x**2 - log(a - b*x)*a**2*b**4*c**4 - 6*log(a - b*x)*a
**2*b**4*c**2*d**2*x**2 + log(a - b*x)*b**6*c**4*x**2 + 6*a**5*b*d**4*x +
12*a**3*b**3*c**2*d**2*x + 8*a**3*b**3*c*d**3*x**2 - 4*a**3*b**3*d**4*x**3
 + 2*a*b**5*c**4*x + 8*a*b**5*c**3*d*x**2)/(4*a**3*b**5*(a**2 - b**2*x**2)
)
```

3.37 $\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx = \frac{(bc+ad)^3}{4a^2b^4(a-bx)} - \frac{(bc-ad)^3}{4a^2b^4(a+bx)} - \frac{(bc-2ad)(bc+ad)^2\log(a-bx)}{4a^3b^4} + \frac{(bc-ad)^2(bc+2ad)\log(a+bx)}{4a^3b^4}$$

output
```
1/4*(a*d+b*c)^3/a^2/b^4/(-b*x+a)-1/4*(-a*d+b*c)^3/a^2/b^4/(b*x+a)-1/4*(-2*
a*d+b*c)*(a*d+b*c)^2*ln(-b*x+a)/a^3/b^4+1/4*(-a*d+b*c)^2*(2*a*d+b*c)*ln(b*
x+a)/a^3/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx = \frac{\frac{a^4d^3+b^4c^3x+3a^2b^2cd(c+dx)}{a^4-a^2b^2x^2} + \frac{b(b^2c^3-3a^2cd^2)\operatorname{arctanh}\left(\frac{bx}{a}\right)}{a^3} + d^3\log\left(a^2-b^2x^2\right)}{2b^4}$$

input
```
Integrate[(c + d*x)^3/(a^2 - b^2*x^2)^2,x]
```

output
```
((a^4*d^3 + b^4*c^3*x + 3*a^2*b^2*c*d*(c + d*x))/(a^4 - a^2*b^2*x^2) + (b*
(b^2*c^3 - 3*a^2*c*d^2)*ArcTanh[(b*x)/a])/a^3 + d^3*Log[a^2 - b^2*x^2])/(2
*b^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(\frac{a^2(bc-ad)^3}{4b^3(a+bx)^2}+\frac{a(bc+2ad)(bc-ad)^2}{4b^3(a+bx)}+\frac{a(bc-2ad)(bc+ad)^2}{4b^3(a-bx)}+\frac{a^2(bc+ad)^3}{4b^3(a-bx)^2}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2(bc-ad)^3}{4b^4(a+bx)}+\frac{a^2(ad+bc)^3}{4b^4(a-bx)}+\frac{a(2ad+bc)(bc-ad)^2\log(a+bx)}{4b^4}-\frac{a(bc-2ad)(ad+bc)^2\log(a-bx)}{4b^4}}{a^4}$$

input
```
Int[(c + d*x)^3/(a^2 - b^2*x^2)^2,x]
```

output
```
((a^2*(b*c + a*d)^3)/(4*b^4*(a - b*x)) - (a^2*(b*c - a*d)^3)/(4*b^4*(a + b
*x)) - (a*(b*c - 2*a*d)*(b*c + a*d)^2*Log[a - b*x])/(4*b^4) + (a*(b*c - a*
d)^2*(b*c + 2*a*d)*Log[a + b*x])/(4*b^4))/a^4
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.21

method	result
norman	$\frac{-\frac{-a^2d^3-3b^2c^2d}{2b^4}+\frac{c\left(3a^2d^2+b^2c^2\right)x}{2a^2b^2}}{-b^2x^2+a^2}+\frac{\left(2a^3d^3-3a^2bc\,d^2+b^3c^3\right)\ln(bx+a)}{4b^4a^3}+\frac{\left(2a^3d^3+3a^2bc\,d^2-b^3c^3\right)\ln(-bx+a)}{4b^4a^3}$
risch	$\frac{\frac{c\left(3a^2d^2+b^2c^2\right)x}{2a^2b^2}+\frac{d\left(a^2d^2+3b^2c^2\right)}{2b^4}}{-b^2x^2+a^2}+\frac{\ln(bx-a)d^3}{2b^4}+\frac{3\ln(bx-a)c\,d^2}{4b^3a}-\frac{\ln(bx-a)c^3}{4b\,a^3}+\frac{\ln(-bx-a)d^3}{2b^4}-\frac{3\ln(-bx-a)c\,d^2}{4b^3a}$
default	$\frac{\left(2a^3d^3-3a^2bc\,d^2+b^3c^3\right)\ln(bx+a)}{4b^4a^3}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4a^2b^4(bx+a)}+\frac{\left(2a^3d^3+3a^2bc\,d^2-b^3c^3\right)\ln(-bx+a)}{4b^4a^3}+\frac{a^3d^3+}{}$
parallelrisch	$\frac{2\ln(bx-a)x^2a^3b^2d^3+3\ln(bx-a)x^2a^2b^3c\,d^2-\ln(bx-a)x^2b^5c^3+2\ln(bx+a)x^2a^3b^2d^3-3\ln(bx+a)x^2a^2b^3c\,d^2+\ln(bx+a)x^2b^5c}{}$

input
```
int((d*x+c)^3/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(-a^2*d^3-3*b^2*c^2*d)/b^4+1/2*c*(3*a^2*d^2+b^2*c^2)/a^2/b^2*x)/(-b^
2*x^2+a^2)+1/4/b^4*(2*a^3*d^3-3*a^2*b*c*d^2+b^3*c^3)/a^3*ln(b*x+a)+1/4/b^4
*(2*a^3*d^3+3*a^2*b*c*d^2-b^3*c^3)/a^3*ln(-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.75

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx = -\frac{6\,a^3b^2c^2d+2\,a^5d^3+2\,(ab^4c^3+3\,a^3b^2cd^2)x+(a^2b^3c^3-3\,a^4bcd^2+2\,a^5d^3-(b^5c^3-3\,a^2b^3cd^2+2\,a^3b^2}{4\,(a^3b^6x^2-}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(6*a^3*b^2*c^2*d + 2*a^5*d^3 + 2*(a*b^4*c^3 + 3*a^3*b^2*c*d^2)*x + (a
^2*b^3*c^3 - 3*a^4*b*c*d^2 + 2*a^5*d^3 - (b^5*c^3 - 3*a^2*b^3*c*d^2 + 2*a^
3*b^2*d^3)*x^2)*log(b*x + a) - (a^2*b^3*c^3 - 3*a^4*b*c*d^2 - 2*a^5*d^3 -
(b^5*c^3 - 3*a^2*b^3*c*d^2 - 2*a^3*b^2*d^3)*x^2)*log(b*x - a))/(a^3*b^6*x^
2 - a^5*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(107) = 214.

Time = 0.65 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx = \frac{-a^4d^3-3a^2b^2c^2d+x(-3a^2b^2cd^2-b^4c^3)}{-2a^4b^4+2a^2b^6x^2} + \frac{(ad-bc)^2\cdot(2ad+bc)\log\left(x+\frac{2a^4d^3-a(ad-bc)^2\cdot(2ad+bc)}{3a^2b^2cd^2-b^4c^3}\right)}{4a^3b^4} + \frac{(ad+bc)^2\cdot(2ad-bc)\log\left(x+\frac{2a^4d^3-a(ad+bc)^2\cdot(2ad-bc)}{3a^2b^2cd^2-b^4c^3}\right)}{4a^3b^4}$$

input
```
integrate((d*x+c)**3/(-b**2*x**2+a**2)**2,x)
```

output
```
(-a**4*d**3 - 3*a**2*b**2*c**2*d + x*(-3*a**2*b**2*c*d**2 - b**4*c**3))/(-
2*a**4*b**4 + 2*a**2*b**6*x**2) + (a*d - b*c)**2*(2*a*d + b*c)*log(x + (2*
a**4*d**3 - a*(a*d - b*c)**2*(2*a*d + b*c))/(3*a**2*b**2*c*d**2 - b**4*c**
3))/(4*a**3*b**4) + (a*d + b*c)**2*(2*a*d - b*c)*log(x + (2*a**4*d**3 - a*
(a*d + b*c)**2*(2*a*d - b*c))/(3*a**2*b**2*c*d**2 - b**4*c**3))/(4*a**3*b*
*4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx = -\frac{3\,a^2b^2c^2d + a^4d^3 + (b^4c^3 + 3\,a^2b^2cd^2)x}{2\,(a^2b^6x^2 - a^4b^4)} + \frac{(b^3c^3 - 3\,a^2bcd^2 + 2\,a^3d^3)\log(bx+a)}{4\,a^3b^4} - \frac{(b^3c^3 - 3\,a^2bcd^2 - 2\,a^3d^3)\log(bx-a)}{4\,a^3b^4}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*a^2*b^2*c^2*d + a^4*d^3 + (b^4*c^3 + 3*a^2*b^2*c*d^2)*x)/(a^2*b^6*
x^2 - a^4*b^4) + 1/4*(b^3*c^3 - 3*a^2*b*c*d^2 + 2*a^3*d^3)*log(b*x + a)/(a
^3*b^4) - 1/4*(b^3*c^3 - 3*a^2*b*c*d^2 - 2*a^3*d^3)*log(b*x - a)/(a^3*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^2}\,dx = -\frac{(b^2c^3 + 3\,a^2cd^2)x + \frac{3\,a^2b^2c^2d + a^4d^3}{b^2}}{2\,(bx+a)(bx-a)a^2b^2} + \frac{(b^3c^3 - 3\,a^2bcd^2 + 2\,a^3d^3)\log(|bx+a|)}{4\,a^3b^4} - \frac{(b^3c^3 - 3\,a^2bcd^2 - 2\,a^3d^3)\log(|bx-a|)}{4\,a^3b^4}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/2*((b^2*c^3 + 3*a^2*c*d^2)*x + (3*a^2*b^2*c^2*d + a^4*d^3)/b^2)/((b*x +
 a)*(b*x - a)*a^2*b^2) + 1/4*(b^3*c^3 - 3*a^2*b*c*d^2 + 2*a^3*d^3)*log(abs
(b*x + a))/(a^3*b^4) - 1/4*(b^3*c^3 - 3*a^2*b*c*d^2 - 2*a^3*d^3)*log(abs(b
*x - a))/(a^3*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^3}{\left(a^2-b^2x^2\right)^2}\,dx = \frac{\frac{d\,\left(a^2\,d^2+3\,b^2\,c^2\right)}{2\,b^4}+\frac{c\,x\,\left(3\,a^2\,d^2+b^2\,c^2\right)}{2\,a^2\,b^2}}{a^2-b^2\,x^2} + \frac{\ln\left(a+b\,x\right)\,\left(2\,a^3\,d^3-3\,a^2\,b\,c\,d^2+b^3\,c^3\right)}{4\,a^3\,b^4} + \frac{\ln\left(a-b\,x\right)\,\left(2\,a^3\,d^3+3\,a^2\,b\,c\,d^2-b^3\,c^3\right)}{4\,a^3\,b^4}$$

input
```
int((c + d*x)^3/(a^2 - b^2*x^2)^2,x)
```

output
```
((d*(a^2*d^2 + 3*b^2*c^2))/(2*b^4) + (c*x*(3*a^2*d^2 + b^2*c^2))/(2*a^2*b^
2))/(a^2 - b^2*x^2) + (log(a + b*x)*(2*a^3*d^3 + b^3*c^3 - 3*a^2*b*c*d^2))
/(4*a^3*b^4) + (log(a - b*x)*(2*a^3*d^3 - b^3*c^3 + 3*a^2*b*c*d^2))/(4*a^3
*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.48

$$\int \frac{(c+dx)^3}{\left(a^2-b^2x^2\right)^2}\,dx = \frac{2\log(-bx-a)\,a^5d^3-3\log(-bx-a)\,a^4bc\,d^2-2\log(-bx-a)\,a^3b^2d^3x^2+\log(-bx-a)\,a^2b^3c^3+3\log(-}{}$$

input
```
int((d*x+c)^3/(-b^2*x^2+a^2)^2,x)
```

output
```
(2*log( - a - b*x)*a**5*d**3 - 3*log( - a - b*x)*a**4*b*c*d**2 - 2*log( -
a - b*x)*a**3*b**2*d**3*x**2 + log( - a - b*x)*a**2*b**3*c**3 + 3*log( - a
 - b*x)*a**2*b**3*c*d**2*x**2 - log( - a - b*x)*b**5*c**3*x**2 + 2*log(a -
 b*x)*a**5*d**3 + 3*log(a - b*x)*a**4*b*c*d**2 - 2*log(a - b*x)*a**3*b**2*
d**3*x**2 - log(a - b*x)*a**2*b**3*c**3 - 3*log(a - b*x)*a**2*b**3*c*d**2*
x**2 + log(a - b*x)*b**5*c**3*x**2 + 6*a**3*b**2*c*d**2*x + 2*a**3*b**2*d*
*3*x**2 + 2*a*b**4*c**3*x + 6*a*b**4*c**2*d*x**2)/(4*a**3*b**4*(a**2 - b**
2*x**2))
```

3.38 $\int \frac{(c+dx)^2}{\left(a^2-b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{(c+dx)^2}{\left(a^2-b^2x^2\right)^2}\,dx=\frac{2a^2cd+\left(b^2c^2+a^2d^2\right)x}{2a^2b^2\left(a^2-b^2x^2\right)}+\frac{(bc-ad)(bc+ad)\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3b^3}$$

output
```
1/2*(2*a^2*c*d+(a^2*d^2+b^2*c^2)*x)/a^2/b^2/(-b^2*x^2+a^2)+1/2*(-a*d+b*c)*
(a*d+b*c)*arctanh(b*x/a)/a^3/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^2}{\left(a^2-b^2x^2\right)^2}\,dx=\frac{-2a^2cd-b^2c^2x-a^2d^2x}{2a^2b^2\left(-a^2+b^2x^2\right)}-\frac{\left(-b^2c^2+a^2d^2\right)\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3b^3}$$

input
```
Integrate[(c + d*x)^2/(a^2 - b^2*x^2)^2,x]
```

output
```
(-2*a^2*c*d - b^2*c^2*x - a^2*d^2*x)/(2*a^2*b^2*(-a^2 + b^2*x^2)) - ((-(b^
2*c^2) + a^2*d^2)*ArcTanh[(b*x)/a])/(2*a^3*b^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^2}dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(\frac{\left(c^2-\frac{a^2d^2}{b^2}\right)a^2}{2(a^2-b^2x^2)}+\frac{(bc+ad)^2a^2}{4b^2(a-bx)^2}+\frac{(bc-ad)^2a^2}{4b^2(a+bx)^2}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a\text{arctanh}\left(\frac{bx}{a}\right)\left(c^2-\frac{a^2d^2}{b^2}\right)}{2b}-\frac{a^2(bc-ad)^2}{4b^3(a+bx)}+\frac{a^2(ad+bc)^2}{4b^3(a-bx)}}{a^4}$$

input
```
Int[(c + d*x)^2/(a^2 - b^2*x^2)^2,x]
```

output
```
((a^2*(b*c + a*d)^2)/(4*b^3*(a - b*x)) - (a^2*(b*c - a*d)^2)/(4*b^3*(a + b
*x)) + (a*(c^2 - (a^2*d^2)/b^2)*ArcTanh[(b*x)/a])/(2*b))/a^4
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

method	result
norman	$\frac{\frac{cd}{b^2}+\frac{\left(a^2d^2+b^2c^2\right)x}{2a^2b^2}}{-b^2x^2+a^2}+\frac{\left(a^2d^2-b^2c^2\right)\ln\left(-bx+a\right)}{4b^3a^3}-\frac{\left(a^2d^2-b^2c^2\right)\ln\left(bx+a\right)}{4b^3a^3}$
risch	$\frac{\frac{cd}{b^2}+\frac{\left(a^2d^2+b^2c^2\right)x}{2a^2b^2}}{-b^2x^2+a^2}+\frac{\ln\left(bx-a\right)d^2}{4b^3a}-\frac{\ln\left(bx-a\right)c^2}{4b\,a^3}-\frac{\ln\left(-bx-a\right)d^2}{4b^3a}+\frac{\ln\left(-bx-a\right)c^2}{4b\,a^3}$
default	$\frac{\left(-a^2d^2+b^2c^2\right)\ln\left(bx+a\right)}{4b^3a^3}-\frac{a^2d^2-2abcd+b^2c^2}{4a^2b^3\left(bx+a\right)}+\frac{\left(a^2d^2-b^2c^2\right)\ln\left(-bx+a\right)}{4b^3a^3}+\frac{a^2d^2+2abcd+b^2c^2}{4a^2b^3\left(-bx+a\right)}$
parallelrisch	$\frac{\ln\left(bx-a\right)x^2a^2b^2d^2-\ln\left(bx-a\right)x^2b^4c^2-\ln\left(bx+a\right)x^2a^2b^2d^2+\ln\left(bx+a\right)x^2b^4c^2-\ln\left(bx-a\right)a^4d^2+\ln\left(bx-a\right)a^2b^2c^2+\ln\left(bx+a\right)a^4d^2}{4a^3b^3\left(b^2x^2-a^2\right)}$

input
```
int((d*x+c)^2/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*d/b^2+1/2*(a^2*d^2+b^2*c^2)/a^2/b^2*x)/(-b^2*x^2+a^2)+1/4/b^3*(a^2*d^2-
b^2*c^2)/a^3*ln(-b*x+a)-1/4/b^3*(a^2*d^2-b^2*c^2)/a^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)^2}{\left(a^2-b^2x^2\right)^2}\,dx =$$
$$-\frac{4\,a^3bcd+2\left(ab^3c^2+a^3bd^2\right)x+\left(a^2b^2c^2-a^4d^2-\left(b^4c^2-a^2b^2d^2\right)x^2\right)\log\left(bx+a\right)-\left(a^2b^2c^2-a^4d^2-(b\right.}{4\left(a^3b^5x^2-a^5b^3\right)}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a^3*b*c*d + 2*(a*b^3*c^2 + a^3*b*d^2)*x + (a^2*b^2*c^2 - a^4*d^2 -
 (b^4*c^2 - a^2*b^2*d^2)*x^2)*log(b*x + a) - (a^2*b^2*c^2 - a^4*d^2 - (b^4
*c^2 - a^2*b^2*d^2)*x^2)*log(b*x - a))/(a^3*b^5*x^2 - a^5*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(70) = 140$.

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^2}\,dx = \frac{-2a^2cd + x(-a^2d^2-b^2c^2)}{-2a^4b^2+2a^2b^4x^2} + \frac{(ad-bc)(ad+bc)\log\left(-\frac{a(ad-bc)(ad+bc)}{b(a^2d^2-b^2c^2)}+x\right)}{4a^3b^3} - \frac{(ad-bc)(ad+bc)\log\left(\frac{a(ad-bc)(ad+bc)}{b(a^2d^2-b^2c^2)}+x\right)}{4a^3b^3}$$

input
```
integrate((d*x+c)**2/(-b**2*x**2+a**2)**2,x)
```

output
```
(-2*a**2*c*d + x*(-a**2*d**2 - b**2*c**2))/(-2*a**4*b**2 + 2*a**2*b**4*x**
2) + (a*d - b*c)*(a*d + b*c)*log(-a*(a*d - b*c)*(a*d + b*c)/(b*(a**2*d**2
- b**2*c**2)) + x)/(4*a**3*b**3) - (a*d - b*c)*(a*d + b*c)*log(a*(a*d - b*
c)*(a*d + b*c)/(b*(a**2*d**2 - b**2*c**2)) + x)/(4*a**3*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^2}\,dx = -\frac{2\,a^2cd + (b^2c^2+a^2d^2)x}{2\,(a^2b^4x^2-a^4b^2)} + \frac{(b^2c^2-a^2d^2)\log(bx+a)}{4\,a^3b^3} - \frac{(b^2c^2-a^2d^2)\log(bx-a)}{4\,a^3b^3}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a^2*c*d + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^4*x^2 - a^4*b^2) + 1/4*(b^
2*c^2 - a^2*d^2)*log(b*x + a)/(a^3*b^3) - 1/4*(b^2*c^2 - a^2*d^2)*log(b*x
- a)/(a^3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^2}\,dx = -\frac{b^2c^2x + a^2d^2x + 2\,a^2cd}{2\,(b^2x^2-a^2)a^2b^2} + \frac{(b^3c^2-a^2bd^2)\log\left(|bx+a|\right)}{4\,a^3b^4} - \frac{(b^3c^2-a^2bd^2)\log\left(|bx-a|\right)}{4\,a^3b^4}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2*x + a^2*d^2*x + 2*a^2*c*d)/((b^2*x^2 - a^2)*a^2*b^2) + 1/4*(
b^3*c^2 - a^2*b*d^2)*log(abs(b*x + a))/(a^3*b^4) - 1/4*(b^3*c^2 - a^2*b*d^
2)*log(abs(b*x - a))/(a^3*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^2}\,dx = \frac{\frac{c\,d}{b^2} + \frac{x\,(a^2\,d^2+b^2\,c^2)}{2\,a^2\,b^2}}{a^2-b^2\,x^2} - \frac{2\,\mathrm{atanh}\left(\frac{4\,b\,x\,\left(\frac{a^2\,d^2}{4}-\frac{b^2\,c^2}{4}\right)}{a\,(a^2\,d^2-b^2\,c^2)}\right)\left(\frac{a^2\,d^2}{4}-\frac{b^2\,c^2}{4}\right)}{a^3\,b^3}$$

input
```
int((c + d*x)^2/(a^2 - b^2*x^2)^2,x)
```

output
```
((c*d)/b^2 + (x*(a^2*d^2 + b^2*c^2))/(2*a^2*b^2))/(a^2 - b^2*x^2) - (2*ata
nh((4*b*x*((a^2*d^2)/4 - (b^2*c^2)/4))/(a*(a^2*d^2 - b^2*c^2)))*((a^2*d^2)
/4 - (b^2*c^2)/4))/(a^3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.46

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^2}\,dx$$
$$= \frac{-\log(-bx-a)\,a^4d^2+\log(-bx-a)\,a^2b^2c^2+\log(-bx-a)\,a^2b^2d^2x^2-\log(-bx-a)\,b^4c^2x^2+\log(-bx+}{4a^3b}$$

input
```
int((d*x+c)^2/(-b^2*x^2+a^2)^2,x)
```

output
```
( - log( - a - b*x)*a**4*d**2 + log( - a - b*x)*a**2*b**2*c**2 + log( - a
- b*x)*a**2*b**2*d**2*x**2 - log( - a - b*x)*b**4*c**2*x**2 + log(a - b*x)
*a**4*d**2 - log(a - b*x)*a**2*b**2*c**2 - log(a - b*x)*a**2*b**2*d**2*x**
2 + log(a - b*x)*b**4*c**2*x**2 + 2*a**3*b*d**2*x + 2*a*b**3*c**2*x + 4*a*
b**3*c*d*x**2)/(4*a**3*b**3*(a**2 - b**2*x**2))
```

3.39 $\int \frac{c+dx}{(a^2-b^2x^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{c+dx}{(a^2-b^2x^2)^2} dx = \frac{a^2d+b^2cx}{2a^2b^2(a^2-b^2x^2)} + \frac{c\,\mathrm{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}$$

output
```
1/2*(b^2*c*x+a^2*d)/a^2/b^2/(-b^2*x^2+a^2)+1/2*c*arctanh(b*x/a)/a^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{c+dx}{(a^2-b^2x^2)^2} dx = \frac{-a^2d-b^2cx}{2a^2b^2(-a^2+b^2x^2)} + \frac{c\,\mathrm{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}$$

input
```
Integrate[(c + d*x)/(a^2 - b^2*x^2)^2,x]
```

output
```
(-(a^2*d) - b^2*c*x)/(2*a^2*b^2*(-a^2 + b^2*x^2)) + (c*ArcTanh[(b*x)/a])/(
2*a^3*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {454, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a^2-b^2x^2)^2}dx$$

$$\Big\downarrow \text{454}$$

$$\frac{c\int \frac{1}{a^2-b^2x^2}dx}{2a^2}+\frac{a^2d+b^2cx}{2a^2b^2(a^2-b^2x^2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{c\,\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}+\frac{a^2d+b^2cx}{2a^2b^2(a^2-b^2x^2)}$$

input
```
Int[(c + d*x)/(a^2 - b^2*x^2)^2,x]
```

output
```
(a^2*d + b^2*c*x)/(2*a^2*b^2*(a^2 - b^2*x^2)) + (c*ArcTanh[(b*x)/a])/(2*a^
3*b)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

method	result	size
norman	$\frac{\frac{d}{2b^2}+\frac{cx}{2a^2}}{-b^2x^2+a^2}-\frac{c\ln(-bx+a)}{4b\,a^3}+\frac{c\ln(bx+a)}{4b\,a^3}$	62
risch	$\frac{\frac{d}{2b^2}+\frac{cx}{2a^2}}{-b^2x^2+a^2}-\frac{c\ln(-bx+a)}{4b\,a^3}+\frac{c\ln(bx+a)}{4b\,a^3}$	62
default	$\frac{c\ln(bx+a)}{4b\,a^3}-\frac{-ad+bc}{4a^2b^2(bx+a)}-\frac{c\ln(-bx+a)}{4b\,a^3}+\frac{ad+bc}{4b^2a^2(-bx+a)}$	79
parallelrisch	$-\frac{\ln(bx-a)x^2b^3c-\ln(bx+a)x^2b^3c-\ln(bx-a)a^2bc+\ln(bx+a)a^2bc+2a\,b^2cx+2a^3d}{4a^3b^2(b^2x^2-a^2)}$	97

input
```
int((d*x+c)/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*d/b^2+1/2*c/a^2*x)/(-b^2*x^2+a^2)-1/4/b/a^3*c*ln(-b*x+a)+1/4/b/a^3*c*
ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{c+dx}{(a^2-b^2x^2)^2}\,dx$$
$$=-\frac{2\,ab^2cx+2\,a^3d-(b^3cx^2-a^2bc)\log(bx+a)+(b^3cx^2-a^2bc)\log(bx-a)}{4\,(a^3b^4x^2-a^5b^2)}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a*b^2*c*x + 2*a^3*d - (b^3*c*x^2 - a^2*b*c)*log(b*x + a) + (b^3*c*
x^2 - a^2*b*c)*log(b*x - a))/(a^3*b^4*x^2 - a^5*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{c+dx}{(a^2-b^2x^2)^2}\,dx = \frac{-a^2d-b^2cx}{-2a^4b^2+2a^2b^4x^2} + \frac{c\left(-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}\right)}{a^3b}$$

input
```
integrate((d*x+c)/(-b**2*x**2+a**2)**2,x)
```

output
```
(-a**2*d - b**2*c*x)/(-2*a**4*b**2 + 2*a**2*b**4*x**2) + c*(-log(-a/b + x)
/4 + log(a/b + x)/4)/(a**3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{c+dx}{(a^2-b^2x^2)^2}\,dx = -\frac{b^2cx+a^2d}{2\,(a^2b^4x^2-a^4b^2)} + \frac{c\log(bx+a)}{4\,a^3b} - \frac{c\log(bx-a)}{4\,a^3b}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*c*x + a^2*d)/(a^2*b^4*x^2 - a^4*b^2) + 1/4*c*log(b*x + a)/(a^3*b
) - 1/4*c*log(b*x - a)/(a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{c+dx}{(a^2-b^2x^2)^2}\,dx = \frac{c\log(|bx+a|)}{4\,a^3b} - \frac{c\log(|bx-a|)}{4\,a^3b} - \frac{b^2cx+a^2d}{2\,(b^2x^2-a^2)a^2b^2}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/4*c*log(abs(b*x + a))/(a^3*b) - 1/4*c*log(abs(b*x - a))/(a^3*b) - 1/2*(b
^2*c*x + a^2*d)/((b^2*x^2 - a^2)*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{(a^2-b^2x^2)^2}\,dx=\frac{\frac{d}{2\,b^2}+\frac{c\,x}{2\,a^2}}{a^2-b^2\,x^2}+\frac{c\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^3\,b}$$

input
```
int((c + d*x)/(a^2 - b^2*x^2)^2,x)
```

output
```
(d/(2*b^2) + (c*x)/(2*a^2))/(a^2 - b^2*x^2) + (c*atanh((b*x)/a))/(2*a^3*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int \frac{c+dx}{(a^2-b^2x^2)^2}\,dx$$
$$=\frac{\log(-bx-a)\,a^2c-\log(-bx-a)\,b^2c\,x^2-\log(-bx+a)\,a^2c+\log(-bx+a)\,b^2c\,x^2+2abcx+2abd\,x^2}{4a^3b\,(-b^2x^2+a^2)}$$

input
```
int((d*x+c)/(-b^2*x^2+a^2)^2,x)
```

output
```
(log( - a - b*x)*a**2*c - log( - a - b*x)*b**2*c*x**2 - log(a - b*x)*a**2*
c + log(a - b*x)*b**2*c*x**2 + 2*a*b*c*x + 2*a*b*d*x**2)/(4*a**3*b*(a**2 -
 b**2*x**2))
```

3.40 $\int \frac{1}{(a^2-b^2x^2)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{1}{(a^2-b^2x^2)^2} dx = \frac{x}{2a^2(a^2-b^2x^2)} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}$$

output
```
1/2*x/a^2/(-b^2*x^2+a^2)+1/2*arctanh(b*x/a)/a^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a^2-b^2x^2)^2} dx = \frac{\frac{ax}{a^2-b^2x^2} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{b}}{2a^3}$$

input
```
Integrate[(a^2 - b^2*x^2)^(-2),x]
```

output
```
((a*x)/(a^2 - b^2*x^2) + ArcTanh[(b*x)/a]/b)/(2*a^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)^2}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{1}{a^2-b^2x^2}dx}{2a^2}+\frac{x}{2a^2(a^2-b^2x^2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}+\frac{x}{2a^2(a^2-b^2x^2)}$$

input
```
Int[(a^2 - b^2*x^2)^(-2),x]
```

output
```
x/(2*a^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

method	result	size
norman	$\frac{x}{2a^2(-b^2x^2+a^2)}-\frac{\ln(-bx+a)}{4b\,a^3}+\frac{\ln(bx+a)}{4b\,a^3}$	51
risch	$\frac{x}{2a^2(-b^2x^2+a^2)}-\frac{\ln(-bx+a)}{4b\,a^3}+\frac{\ln(bx+a)}{4b\,a^3}$	51
default	$\frac{\ln(bx+a)}{4b\,a^3}-\frac{1}{4a^2b(bx+a)}-\frac{\ln(-bx+a)}{4b\,a^3}+\frac{1}{4a^2b(-bx+a)}$	62
parallelrisch	$-\frac{\ln(bx-a)x^2b^3-\ln(bx+a)x^2b^3-\ln(bx-a)a^2b+\ln(bx+a)a^2b+2xa\,b^2}{4a^3b^2(b^2x^2-a^2)}$	86

input
```
int(1/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a^2/(-b^2*x^2+a^2)-1/4/b/a^3*ln(-b*x+a)+1/4/b/a^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \frac{1}{(a^2-b^2x^2)^2}\,dx=-\frac{2\,abx-(b^2x^2-a^2)\log(bx+a)+(b^2x^2-a^2)\log(bx-a)}{4\,(a^3b^3x^2-a^5b)}$$

input
```
integrate(1/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a*b*x - (b^2*x^2 - a^2)*log(b*x + a) + (b^2*x^2 - a^2)*log(b*x - a
))/(a^3*b^3*x^2 - a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a^2-b^2x^2\right)^2}\,dx = -\frac{x}{-2a^4+2a^2b^2x^2} + \frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^3b}$$

input
```
integrate(1/(-b**2*x**2+a**2)**2,x)
```

output
```
-x/(-2*a**4 + 2*a**2*b**2*x**2) + (-log(-a/b + x)/4 + log(a/b + x)/4)/(a**
3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a^2-b^2x^2\right)^2}\,dx = -\frac{x}{2\left(a^2b^2x^2-a^4\right)} + \frac{\log\left(bx+a\right)}{4\,a^3b} - \frac{\log\left(bx-a\right)}{4\,a^3b}$$

input
```
integrate(1/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(a^2*b^2*x^2 - a^4) + 1/4*log(b*x + a)/(a^3*b) - 1/4*log(b*x - a)/(
a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a^2-b^2x^2\right)^2}\,dx = -\frac{x}{2\left(b^2x^2-a^2\right)a^2} + \frac{\log\left(|bx+a|\right)}{4\,a^3b} - \frac{\log\left(|bx-a|\right)}{4\,a^3b}$$

input
```
integrate(1/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/2*x/((b^2*x^2 - a^2)*a^2) + 1/4*log(abs(b*x + a))/(a^3*b) - 1/4*log(abs
(b*x - a))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a^2-b^2x^2)^2}\,dx=\frac{x}{2\,a^2\,\left(a^2-b^2\,x^2\right)}+\frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^3\,b}$$

input
```
int(1/(a^2 - b^2*x^2)^2,x)
```

output
```
x/(2*a^2*(a^2 - b^2*x^2)) + atanh((b*x)/a)/(2*a^3*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \frac{1}{(a^2-b^2x^2)^2}\,dx = \frac{\log(-bx-a)\,a^2-\log(-bx-a)\,b^2x^2-\log(-bx+a)\,a^2+\log(-bx+a)\,b^2x^2+2abx}{4a^3b\,(-b^2x^2+a^2)}$$

input
```
int(1/(-b^2*x^2+a^2)^2,x)
```

output
```
(log( - a - b*x)*a**2 - log( - a - b*x)*b**2*x**2 - log(a - b*x)*a**2 + lo
g(a - b*x)*b**2*x**2 + 2*a*b*x)/(4*a**3*b*(a**2 - b**2*x**2))
```

3.41 $\int \frac{1}{(c+dx)(a^2-b^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)^2} dx = \frac{1}{4a^2(bc+ad)(a-bx)} - \frac{1}{4a^2(bc-ad)(a+bx)} - \frac{(bc+2ad)\log(a-bx)}{4a^3(bc+ad)^2} + \frac{(bc-2ad)\log(a+bx)}{4a^3(bc-ad)^2} + \frac{d^3\log(c+dx)}{(b^2c^2-a^2d^2)^2}$$

output
```
1/4/a^2/(a*d+b*c)/(-b*x+a)-1/4/a^2/(-a*d+b*c)/(b*x+a)-1/4*(2*a*d+b*c)*ln(-
b*x+a)/a^3/(a*d+b*c)^2+1/4*(-2*a*d+b*c)*ln(b*x+a)/a^3/(-a*d+b*c)^2+d^3*ln(
d*x+c)/(-a^2*d^2+b^2*c^2)^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c+dx)(a^2-b^2x^2)^2} dx = \frac{\frac{(b^2c^2-a^2d^2)(-a^2d+b^2cx)}{a^4-a^2b^2x^2} + \frac{b(b^2c^3-3a^2cd^2)\text{arctanh}\left(\frac{bx}{a}\right)}{a^3} + 2d^3\log(c+dx) - d^3\log\left(a^2-b^2x^2\right)}{2\left(b^2c^2-a^2d^2\right)^2}$$

input
```
Integrate[1/((c + d*x)*(a^2 - b^2*x^2)^2),x]
```

output
```
(((b^2*c^2 - a^2*d^2)*(-(a^2*d) + b^2*c*x))/(a^4 - a^2*b^2*x^2) + (b*(b^2*
c^3 - 3*a^2*c*d^2)*ArcTanh[(b*x)/a])/a^3 + 2*d^3*Log[c + d*x] - d^3*Log[a^
2 - b^2*x^2])/(2*(b^2*c^2 - a^2*d^2)^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2 - b^2x^2)^2 (c + dx)} dx$$

$$\Big\downarrow \text{ 477}$$

$$\frac{\int \left(\frac{a^4d^4}{(b^2c^2-a^2d^2)^2(c+dx)} + \frac{ab(bc+2ad)}{4(bc+ad)^2(a-bx)} + \frac{ab(bc-2ad)}{4(bc-ad)^2(a+bx)} + \frac{a^2b}{4(bc+ad)(a-bx)^2} + \frac{a^2b}{4(bc-ad)(a+bx)^2}\right) dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2}{4(a-bx)(ad+bc)} - \frac{a^2}{4(a+bx)(bc-ad)} + \frac{a^4d^3\log(c+dx)}{(b^2c^2-a^2d^2)^2} - \frac{a(2ad+bc)\log(a-bx)}{4(ad+bc)^2} + \frac{a(bc-2ad)\log(a+bx)}{4(bc-ad)^2}}{a^4}$$

input
```
Int[1/((c + d*x)*(a^2 - b^2*x^2)^2),x]
```

output
```
(a^2/(4*(b*c + a*d)*(a - b*x)) - a^2/(4*(b*c - a*d)*(a + b*x)) - (a*(b*c +
 2*a*d)*Log[a - b*x])/(4*(b*c + a*d)^2) + (a*(b*c - 2*a*d)*Log[a + b*x])/(
4*(b*c - a*d)^2) + (a^4*d^3*Log[c + d*x])/(b^2*c^2 - a^2*d^2)^2)/a^4
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

method	result
default	$\frac{1}{4a^2(ad-bc)(bx+a)}-\frac{(2ad-bc)\ln(bx+a)}{4a^3(ad-bc)^2}+\frac{d^3\ln(dx+c)}{(ad+bc)^2(ad-bc)^2}+\frac{1}{4a^2(ad+bc)(-bx+a)}-\frac{(2ad+bc)\ln(-bx+a)}{4a^3(ad+bc)^2}$
norman	$\frac{\frac{d}{2a^2d^2-2b^2c^2}-\frac{b^2cx}{2\left(a^2d^2-b^2c^2\right)a^2}}{-b^2x^2+a^2}+\frac{d^3\ln(dx+c)}{a^4d^4-2a^2b^2c^2d^2+c^4b^4}-\frac{(2ad-bc)\ln(bx+a)}{4(a^2d^2-2abcd+b^2c^2)a^3}-\frac{(2ad+bc)\ln(-bx+a)}{4(a^2d^2+2abcd+b^2c^2)a^3}$
risch	$\frac{\frac{d}{2a^2d^2-2b^2c^2}-\frac{b^2cx}{2\left(a^2d^2-b^2c^2\right)a^2}}{-b^2x^2+a^2}+\frac{d^3\ln(dx+c)}{a^4d^4-2a^2b^2c^2d^2+c^4b^4}-\frac{\ln(bx+a)d}{2(a^2d^2-2abcd+b^2c^2)a^2}+\frac{\ln(bx+a)bc}{4(a^2d^2-2abcd+b^2c^2)a^3}-$
parallelrisch	$-\frac{2\ln(bx-a)x^2a^3b^4d^3-3\ln(bx-a)x^2a^2b^5c\,d^2+\ln(bx-a)x^2b^7c^3+2\ln(bx+a)x^2a^3b^4d^3+3\ln(bx+a)x^2a^2b^5c\,d^2-\ln(bx+a)x^2}{}$

input
```
int(1/(d*x+c)/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2/(a*d-b*c)/(b*x+a)-1/4*(2*a*d-b*c)/a^3/(a*d-b*c)^2*ln(b*x+a)+d^3/(a
*d+b*c)^2/(a*d-b*c)^2*ln(d*x+c)+1/4/a^2/(a*d+b*c)/(-b*x+a)-1/4*(2*a*d+b*c)
*ln(-b*x+a)/a^3/(a*d+b*c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(132) = 264$.

Time = 1.35 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.14

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^2}\,dx = -\frac{2\,a^3b^2c^2d-2\,a^5d^3-2\left(ab^4c^3-a^3b^2cd^2\right)x-\left(a^2b^3c^3-3\,a^4bcd^2-2\,a^5d^3-\left(b^5c^3-3\,a^2b^3cd^2-2\,a^3b^2d^3\right.\right.}{4\left(a^5b^4c^4-2\,a^7b^2c^2\right.}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a^3*b^2*c^2*d - 2*a^5*d^3 - 2*(a*b^4*c^3 - a^3*b^2*c*d^2)*x - (a^2
*b^3*c^3 - 3*a^4*b*c*d^2 - 2*a^5*d^3 - (b^5*c^3 - 3*a^2*b^3*c*d^2 - 2*a^3*
b^2*d^3)*x^2)*log(b*x + a) + (a^2*b^3*c^3 - 3*a^4*b*c*d^2 + 2*a^5*d^3 - (b
^5*c^3 - 3*a^2*b^3*c*d^2 + 2*a^3*b^2*d^3)*x^2)*log(b*x - a) + 4*(a^3*b^2*d
^3*x^2 - a^5*d^3)*log(d*x + c))/(a^5*b^4*c^4 - 2*a^7*b^2*c^2*d^2 + a^9*d^4
 - (a^3*b^6*c^4 - 2*a^5*b^4*c^2*d^2 + a^7*b^2*d^4)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(-b**2*x**2+a**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.40

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^2}\,dx = \frac{d^3\log(dx+c)}{b^4c^4-2\,a^2b^2c^2d^2+a^4d^4}+\frac{(bc-2\,ad)\log(bx+a)}{4\,(a^3b^2c^2-2\,a^4bcd+a^5d^2)} - \frac{(bc+2\,ad)\log(bx-a)}{4\,(a^3b^2c^2+2\,a^4bcd+a^5d^2)} + \frac{b^2cx-a^2d}{2\,(a^4b^2c^2-a^6d^2-(a^2b^4c^2-a^4b^2d^2)x^2)}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
d^3*log(d*x + c)/(b^4*c^4 - 2*a^2*b^2*c^2*d^2 + a^4*d^4) + 1/4*(b*c - 2*a*
d)*log(b*x + a)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2) - 1/4*(b*c + 2*a*d)*
log(b*x - a)/(a^3*b^2*c^2 + 2*a^4*b*c*d + a^5*d^2) + 1/2*(b^2*c*x - a^2*d)
/(a^4*b^2*c^2 - a^6*d^2 - (a^2*b^4*c^2 - a^4*b^2*d^2)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.66

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^2}\,dx = \frac{d^4\log(|dx+c|)}{b^4c^4d-2\,a^2b^2c^2d^3+a^4d^5}+\frac{(b^2c-2\,abd)\log(|bx+a|)}{4\,(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)} - \frac{(b^2c+2\,abd)\log(|bx-a|)}{4\,(a^3b^3c^2+2\,a^4b^2cd+a^5bd^2)} + \frac{a^2b^2c^2d-a^4d^3-(b^4c^3-a^2b^2cd^2)x}{2\,(bc+ad)^2(bc-ad)^2(bx+a)(bx-a)a^2}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
d^4*log(abs(d*x + c))/(b^4*c^4*d - 2*a^2*b^2*c^2*d^3 + a^4*d^5) + 1/4*(b^2
*c - 2*a*b*d)*log(abs(b*x + a))/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
- 1/4*(b^2*c + 2*a*b*d)*log(abs(b*x - a))/(a^3*b^3*c^2 + 2*a^4*b^2*c*d + a
^5*b*d^2) + 1/2*(a^2*b^2*c^2*d - a^4*d^3 - (b^4*c^3 - a^2*b^2*c*d^2)*x)/((
b*c + a*d)^2*(b*c - a*d)^2*(b*x + a)*(b*x - a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.56 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.45

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^2}\,dx = \frac{\frac{d}{2\,(a^2\,d^2-b^2\,c^2)}-\frac{b^2\,c\,x}{2\,a^2\,(a^2\,d^2-b^2\,c^2)}}{a^2-b^2\,x^2}+\frac{d^3\,\ln\left(c+d\,x\right)}{a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4} - \frac{\ln\left(a+b\,x\right)\,\left(2\,a\,d-b\,c\right)}{4\,a^5\,d^2-8\,a^4\,b\,c\,d+4\,a^3\,b^2\,c^2} - \frac{\ln\left(a-b\,x\right)\,\left(2\,a\,d+b\,c\right)}{4\,a^5\,d^2+8\,a^4\,b\,c\,d+4\,a^3\,b^2\,c^2}$$

input
```
int(1/((a^2 - b^2*x^2)^2*(c + d*x)),x)
```

output
```
(d/(2*(a^2*d^2 - b^2*c^2)) - (b^2*c*x)/(2*a^2*(a^2*d^2 - b^2*c^2)))/(a^2 -
 b^2*x^2) + (d^3*log(c + d*x))/(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2) - (
log(a + b*x)*(2*a*d - b*c))/(4*a^5*d^2 + 4*a^3*b^2*c^2 - 8*a^4*b*c*d) - (l
og(a - b*x)*(2*a*d + b*c))/(4*a^5*d^2 + 4*a^3*b^2*c^2 + 8*a^4*b*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.86

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^2}\,dx$$
$$= \frac{-2\log(-bx-a)\,a^5d^3-3\log(-bx-a)\,a^4bc\,d^2+2\log(-bx-a)\,a^3b^2d^3x^2+\log(-bx-a)\,a^2b^3c^3+3\log}{}$$

input
```
int(1/(d*x+c)/(-b^2*x^2+a^2)^2,x)
```

output
```
( - 2*log( - a - b*x)*a**5*d**3 - 3*log( - a - b*x)*a**4*b*c*d**2 + 2*log(
 - a - b*x)*a**3*b**2*d**3*x**2 + log( - a - b*x)*a**2*b**3*c**3 + 3*log(
- a - b*x)*a**2*b**3*c*d**2*x**2 - log( - a - b*x)*b**5*c**3*x**2 - 2*log(
a - b*x)*a**5*d**3 + 3*log(a - b*x)*a**4*b*c*d**2 + 2*log(a - b*x)*a**3*b*
*2*d**3*x**2 - log(a - b*x)*a**2*b**3*c**3 - 3*log(a - b*x)*a**2*b**3*c*d*
*2*x**2 + log(a - b*x)*b**5*c**3*x**2 + 4*log(c + d*x)*a**5*d**3 - 4*log(c
 + d*x)*a**3*b**2*d**3*x**2 - 2*a**3*b**2*c*d**2*x + 2*a**3*b**2*d**3*x**2
 + 2*a*b**4*c**3*x - 2*a*b**4*c**2*d*x**2)/(4*a**3*(a**6*d**4 - 2*a**4*b**
2*c**2*d**2 - a**4*b**2*d**4*x**2 + a**2*b**4*c**4 + 2*a**2*b**4*c**2*d**2
*x**2 - b**6*c**4*x**2))
```

3.42 $\int \frac{1}{(c+dx)^2(a^2-b^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 178

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a^2-b^2x^2)^2} dx &= \frac{b}{4a^2(bc+ad)^2(a-bx)} - \frac{b}{4a^2(bc-ad)^2(a+bx)} \\ &\quad - \frac{d^3}{(b^2c^2-a^2d^2)^2(c+dx)} - \frac{b(bc+3ad)\log(a-bx)}{4a^3(bc+ad)^3} \\ &\quad + \frac{b(bc-3ad)\log(a+bx)}{4a^3(bc-ad)^3} + \frac{4b^2cd^3\log(c+dx)}{(b^2c^2-a^2d^2)^3}\end{aligned}$$

output
```
1/4*b/a^2/(a*d+b*c)^2/(-b*x+a)-1/4*b/a^2/(-a*d+b*c)^2/(b*x+a)-d^3/(-a^2*d^
2+b^2*c^2)^2/(d*x+c)-1/4*b*(3*a*d+b*c)*ln(-b*x+a)/a^3/(a*d+b*c)^3+1/4*b*(-
3*a*d+b*c)*ln(b*x+a)/a^3/(-a*d+b*c)^3+4*b^2*c*d^3*ln(d*x+c)/(-a^2*d^2+b^2*
c^2)^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06

$$\int \frac{1}{(c+dx)^2\left(a^2-b^2x^2\right)^2}\,dx$$
$$=\frac{\frac{2b^2c^2d^3-2a^2d^5}{c+dx}+\frac{2a^2b^4c^3d-b^6c^4x+a^4b^2d^3(-2c+dx)}{a^4-a^2b^2x^2}+\frac{b(-b^4c^4+6a^2b^2c^2d^2+3a^4d^4)\operatorname{arctanh}\left(\frac{bx}{a}\right)}{a^3}-8b^2cd^3\log(c+dx)+4b}{2\left(-b^2c^2+a^2d^2\right)^3}$$

input
```
Integrate[1/((c + d*x)^2*(a^2 - b^2*x^2)^2),x]
```

output
```
((2*b^2*c^2*d^3 - 2*a^2*d^5)/(c + d*x) + (2*a^2*b^4*c^3*d - b^6*c^4*x + a^
4*b^2*d^3*(-2*c + d*x))/(a^4 - a^2*b^2*x^2) + (b*(-(b^4*c^4) + 6*a^2*b^2*c
^2*d^2 + 3*a^4*d^4)*ArcTanh[(b*x)/a])/a^3 - 8*b^2*c*d^3*Log[c + d*x] + 4*b
^2*c*d^3*Log[a^2 - b^2*x^2])/(2*(-(b^2*c^2) + a^2*d^2)^3)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2-b^2x^2\right)^2(c+dx)^2}\,dx$$
$$\Big\downarrow \text{477}$$
$$\frac{\int \left(\frac{4a^4b^2cd^4}{(b^2c^2-a^2d^2)^3(c+dx)}+\frac{a^4d^4}{(b^2c^2-a^2d^2)^2(c+dx)^2}+\frac{ab^2(bc+3ad)}{4(bc+ad)^3(a-bx)}+\frac{ab^2(bc-3ad)}{4(bc-ad)^3(a+bx)}+\frac{a^2b^2}{4(bc+ad)^2(a-bx)^2}+\frac{a^2b^2}{4(bc-ad)^2(a+bx)^2}\right)}{a^4}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{a^2b}{4(a-bx)(ad+bc)^2}-\frac{a^2b}{4(a+bx)(bc-ad)^2}-\frac{a^4d^3}{(c+dx)(b^2c^2-a^2d^2)^2}+\frac{4a^4b^2cd^3\log(c+dx)}{(b^2c^2-a^2d^2)^3}-\frac{ab(3ad+bc)\log(a-bx)}{4(ad+bc)^3}+\frac{ab(bc-3ad)\log(a+bx)}{4(bc-ad)^3}}{a^4}$$

input
```
Int[1/((c + d*x)^2*(a^2 - b^2*x^2)^2),x]
```

output
```
((a^2*b)/(4*(b*c + a*d)^2*(a - b*x)) - (a^2*b)/(4*(b*c - a*d)^2*(a + b*x))
 - (a^4*d^3)/((b^2*c^2 - a^2*d^2)^2*(c + d*x)) - (a*b*(b*c + 3*a*d)*Log[a
- b*x])/(4*(b*c + a*d)^3) + (a*b*(b*c - 3*a*d)*Log[a + b*x])/(4*(b*c - a*d
)^3) + (4*a^4*b^2*c*d^3*Log[c + d*x])/(b^2*c^2 - a^2*d^2)^3)/a^4
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98

method	result
default	$-\frac{b}{4a^{2}(ad-bc)^{2}(bx+a)}+\frac{b(3ad-bc)\ln(bx+a)}{4a^{3}(ad-bc)^{3}}-\frac{d^{3}}{(ad+bc)^{2}(ad-bc)^{2}(dx+c)}-\frac{4d^{3}b^{2}c\ln(dx+c)}{(ad+bc)^{3}(ad-bc)^{3}}+\frac{b}{4a^{2}(ad+bc)^{2}(-bx}$
norman	$\frac{\frac{d}{2a^{2}d^{2}-2b^{2}c^{2}}+\frac{d\left(-3a^{2}b^{2}d^{3}-b^{4}c^{2}d\right)x^{3}}{2c\left(a^{4}d^{4}-2a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}\right)a^{2}}-\frac{\left(-3d^{4}a^{4}b^{2}-b^{6}c^{4}\right)x}{2c\,a^{2}\left(a^{4}d^{4}-2a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}\right)b^{2}}}{(dx+c)(-b^{2}x^{2}+a^{2})}+\frac{b(3ad-bc)\ln(bx+a)}{4a^{3}(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}-\frac{}{4}$
risch	$\frac{\frac{\left(3a^{2}d^{2}+b^{2}c^{2}\right)b^{2}d\,x^{2}}{2\left(a^{4}d^{4}-2a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}\right)a^{2}}-\frac{b^{2}cx}{2\left(a^{2}d^{2}-b^{2}c^{2}\right)a^{2}}-\frac{d\left(a^{2}d^{2}+b^{2}c^{2}\right)}{a^{4}d^{4}-2a^{2}b^{2}c^{2}d^{2}+c^{4}b^{4}}}{(dx+c)(-b^{2}x^{2}+a^{2})}+\frac{3b\ln(bx+a)d}{4a^{2}(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}-\frac{}{4a^{3}(a^{3}}$
parallelrisch	$-\frac{-8\ln(bx-a)x^{2}a^{3}b^{6}c^{3}d^{3}+6\ln(bx-a)x^{2}a^{2}b^{7}c^{4}d^{2}-6x^{3}a^{5}b^{4}d^{6}+6x\,a^{7}b^{2}d^{6}-2xa\,b^{8}c^{6}-\ln(bx-a)x^{2}b^{9}c^{6}+\ln(bx+a)x^{2}b^{9}c^{6}+}{}$

input
```
int(1/(d*x+c)^2/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2*b/(a*d-b*c)^2/(b*x+a)+1/4*b*(3*a*d-b*c)/a^3/(a*d-b*c)^3*ln(b*x+a)
-d^3/(a*d+b*c)^2/(a*d-b*c)^2/(d*x+c)-4*d^3*b^2*c/(a*d+b*c)^3/(a*d-b*c)^3*l
n(d*x+c)+1/4*b/a^2/(a*d+b*c)^2/(-b*x+a)-1/4*b*(3*a*d+b*c)*ln(-b*x+a)/a^3/(
a*d+b*c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(171) = 342$.

Time = 5.14 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.49

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^2}\,dx = -\frac{4\,a^3b^4c^4d - 4\,a^7d^5 - 2\,(ab^6c^4d + 2\,a^3b^4c^2d^3 - 3\,a^5b^2d^5)x^2 - 2\,(ab^6c^5 - 2\,a^3b^4c^3d^2 + a^5b^2cd^4)x - (a^2b^5c}{}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a^3*b^4*c^4*d - 4*a^7*d^5 - 2*(a*b^6*c^4*d + 2*a^3*b^4*c^2*d^3 - 3
*a^5*b^2*d^5)*x^2 - 2*(a*b^6*c^5 - 2*a^3*b^4*c^3*d^2 + a^5*b^2*c*d^4)*x -
(a^2*b^5*c^5 - 6*a^4*b^3*c^3*d^2 - 8*a^5*b^2*c^2*d^3 - 3*a^6*b*c*d^4 - (b^
7*c^4*d - 6*a^2*b^5*c^2*d^3 - 8*a^3*b^4*c*d^4 - 3*a^4*b^3*d^5)*x^3 - (b^7*
c^5 - 6*a^2*b^5*c^3*d^2 - 8*a^3*b^4*c^2*d^3 - 3*a^4*b^3*c*d^4)*x^2 + (a^2*
b^5*c^4*d - 6*a^4*b^3*c^2*d^3 - 8*a^5*b^2*c*d^4 - 3*a^6*b*d^5)*x)*log(b*x
+ a) + (a^2*b^5*c^5 - 6*a^4*b^3*c^3*d^2 + 8*a^5*b^2*c^2*d^3 - 3*a^6*b*c*d^
4 - (b^7*c^4*d - 6*a^2*b^5*c^2*d^3 + 8*a^3*b^4*c*d^4 - 3*a^4*b^3*d^5)*x^3
- (b^7*c^5 - 6*a^2*b^5*c^3*d^2 + 8*a^3*b^4*c^2*d^3 - 3*a^4*b^3*c*d^4)*x^2
+ (a^2*b^5*c^4*d - 6*a^4*b^3*c^2*d^3 + 8*a^5*b^2*c*d^4 - 3*a^6*b*d^5)*x)*l
og(b*x - a) + 16*(a^3*b^4*c*d^4*x^3 + a^3*b^4*c^2*d^3*x^2 - a^5*b^2*c*d^4*
x - a^5*b^2*c^2*d^3)*log(d*x + c))/(a^5*b^6*c^7 - 3*a^7*b^4*c^5*d^2 + 3*a^
9*b^2*c^3*d^4 - a^11*c*d^6 - (a^3*b^8*c^6*d - 3*a^5*b^6*c^4*d^3 + 3*a^7*b^
4*c^2*d^5 - a^9*b^2*d^7)*x^3 - (a^3*b^8*c^7 - 3*a^5*b^6*c^5*d^2 + 3*a^7*b^
4*c^3*d^4 - a^9*b^2*c*d^6)*x^2 + (a^5*b^6*c^6*d - 3*a^7*b^4*c^4*d^3 + 3*a^
9*b^2*c^2*d^5 - a^11*d^7)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^2}\, dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(-b**2*x**2+a**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(171) = 342$.

Time = 0.05 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.30

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^2}\, dx = \frac{4\,b^2cd^3\log(dx+c)}{b^6c^6-3\,a^2b^4c^4d^2+3\,a^4b^2c^2d^4-a^6d^6} + \frac{(b^2c-3\,abd)\log(bx+a)}{4\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3)} - \frac{(b^2c+3\,abd)\log(bx-a)}{4\,(a^3b^3c^3+3\,a^4b^2c^2d+3\,a^5bcd^2+a^6d^3)} - \frac{2\,a^2b^2c^2d+2\,a^4d^3-(b^4c^2d+3\,a^2b^2d^3)x^2-(b^4c^3-a^2b^2cd^2)x}{2\,(a^4b^4c^5-2\,a^6b^2c^3d^2+a^8cd^4-(a^2b^6c^4d-2\,a^4b^4c^2d^3+a^6b^2d^5)x^3-(a^2b^6c^5-2\,a^4b^4c^3d^2+a^6b^2cd^4)x}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
4*b^2*c*d^3*log(d*x + c)/(b^6*c^6 - 3*a^2*b^4*c^4*d^2 + 3*a^4*b^2*c^2*d^4
- a^6*d^6) + 1/4*(b^2*c - 3*a*b*d)*log(b*x + a)/(a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3) - 1/4*(b^2*c + 3*a*b*d)*log(b*x - a)/(a^3*
b^3*c^3 + 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 + a^6*d^3) - 1/2*(2*a^2*b^2*c^2*
d + 2*a^4*d^3 - (b^4*c^2*d + 3*a^2*b^2*d^3)*x^2 - (b^4*c^3 - a^2*b^2*c*d^2
)*x)/(a^4*b^4*c^5 - 2*a^6*b^2*c^3*d^2 + a^8*c*d^4 - (a^2*b^6*c^4*d - 2*a^4
*b^4*c^2*d^3 + a^6*b^2*d^5)*x^3 - (a^2*b^6*c^5 - 2*a^4*b^4*c^3*d^2 + a^6*b
^2*c*d^4)*x^2 + (a^4*b^4*c^4*d - 2*a^6*b^2*c^2*d^3 + a^8*d^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(171) = 342$.

Time = 0.12 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.97

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^2} \, dx$$
$$= -\frac{2\, b^2 c d^3 \log \left(\left| b^2 - \frac{2\, b^2 c}{dx+c} + \frac{b^2 c^2}{(dx+c)^2} - \frac{a^2 d^2}{(dx+c)^2} \right| \right)}{b^6 c^6 - 3\, a^2 b^4 c^4 d^2 + 3\, a^4 b^2 c^2 d^4 - a^6 d^6} - \frac{d^7}{(b^4 c^4 d^4 - 2\, a^2 b^2 c^2 d^6 + a^4 d^8)(dx+c)}$$
$$- \frac{(b^6 c^4 d^2 - 6\, a^2 b^4 c^2 d^4 - 3\, a^4 b^2 d^6) \log \left(\frac{\left| 2\, b^2 c d - \frac{2\, b^2 c^2 d}{dx+c} + \frac{2\, a^2 d^3}{dx+c} - 2\, d^2 |a||b| \right|}{\left| 2\, b^2 c d - \frac{2\, b^2 c^2 d}{dx+c} + \frac{2\, a^2 d^3}{dx+c} + 2\, d^2 |a||b| \right|} \right)}{4\, (a^2 b^6 c^6 - 3\, a^4 b^4 c^4 d^2 + 3\, a^6 b^2 c^2 d^4 - a^8 d^6) d^2 |a||b|}$$
$$- \frac{\frac{b^6 c^3 d + 3\, a^2 b^4 c d^3}{b^2 c^2 - a^2 d^2} - \frac{b^6 c^4 d^2 + 6\, a^2 b^4 c^2 d^4 + a^4 b^2 d^6}{(b^2 c^2 - a^2 d^2)(dx+c) d}}{2 \left(b^2 - \frac{2\, b^2 c}{dx+c} + \frac{b^2 c^2}{(dx+c)^2} - \frac{a^2 d^2}{(dx+c)^2} \right) (bc+ad)^2 (bc-ad)^2 a^2}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-2*b^2*c*d^3*log(abs(b^2 - 2*b^2*c/(d*x + c) + b^2*c^2/(d*x + c)^2 - a^2*d
^2/(d*x + c)^2))/(b^6*c^6 - 3*a^2*b^4*c^4*d^2 + 3*a^4*b^2*c^2*d^4 - a^6*d^
6) - d^7/((b^4*c^4*d^4 - 2*a^2*b^2*c^2*d^6 + a^4*d^8)*(d*x + c)) - 1/4*(b^
6*c^4*d^2 - 6*a^2*b^4*c^2*d^4 - 3*a^4*b^2*d^6)*log(abs(2*b^2*c*d - 2*b^2*c
^2*d/(d*x + c) + 2*a^2*d^3/(d*x + c) - 2*d^2*abs(a)*abs(b))/abs(2*b^2*c*d
- 2*b^2*c^2*d/(d*x + c) + 2*a^2*d^3/(d*x + c) + 2*d^2*abs(a)*abs(b)))/((a^
2*b^6*c^6 - 3*a^4*b^4*c^4*d^2 + 3*a^6*b^2*c^2*d^4 - a^8*d^6)*d^2*abs(a)*ab
s(b)) - 1/2*((b^6*c^3*d + 3*a^2*b^4*c*d^3)/(b^2*c^2 - a^2*d^2) - (b^6*c^4*
d^2 + 6*a^2*b^4*c^2*d^4 + a^4*b^2*d^6)/((b^2*c^2 - a^2*d^2)*(d*x + c)*d))/
((b^2 - 2*b^2*c/(d*x + c) + b^2*c^2/(d*x + c)^2 - a^2*d^2/(d*x + c)^2)*(b*
c + a*d)^2*(b*c - a*d)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.94

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^2} \, dx = -\frac{\frac{a^2 d^3+b^2 c^2 d}{(a^2 d^2-b^2 c^2)^2} - \frac{b^2 x^2 (3 a^2 d^3+b^2 c^2 d)}{2 a^2 (a^4 d^4-2 a^2 b^2 c^2 d^2+b^4 c^4)} + \frac{b^2 c x}{2 a^2 (a^2 d^2-b^2 c^2)}}{d a^2 x + c a^2 - d b^2 x^3 - c b^2 x^2} - \frac{\ln(a+b x) (b^2 c - 3 a b d)}{4 a^6 d^3 - 12 a^5 b c d^2 + 12 a^4 b^2 c^2 d - 4 a^3 b^3 c^3} - \frac{\ln(a-b x) (c b^2 + 3 a d b)}{4 a^6 d^3 + 12 a^5 b c d^2 + 12 a^4 b^2 c^2 d + 4 a^3 b^3 c^3} - \frac{4 b^2 c d^3 \ln(c+d x)}{a^6 d^6 - 3 a^4 b^2 c^2 d^4 + 3 a^2 b^4 c^4 d^2 - b^6 c^6}$$

input
```
int(1/((a^2 - b^2*x^2)^2*(c + d*x)^2),x)
```

output
```
- ((a^2*d^3 + b^2*c^2*d)/(a^2*d^2 - b^2*c^2)^2 - (b^2*x^2*(3*a^2*d^3 + b^2
*c^2*d))/(2*a^2*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) + (b^2*c*x)/(2*a^
2*(a^2*d^2 - b^2*c^2)))/(a^2*c - b^2*c*x^2 - b^2*d*x^3 + a^2*d*x) - (log(a
 + b*x)*(b^2*c - 3*a*b*d))/(4*a^6*d^3 - 4*a^3*b^3*c^3 + 12*a^4*b^2*c^2*d -
 12*a^5*b*c*d^2) - (log(a - b*x)*(b^2*c + 3*a*b*d))/(4*a^6*d^3 + 4*a^3*b^3
*c^3 + 12*a^4*b^2*c^2*d + 12*a^5*b*c*d^2) - (4*b^2*c*d^3*log(c + d*x))/(a^
6*d^6 - b^6*c^6 + 3*a^2*b^4*c^4*d^2 - 3*a^4*b^2*c^2*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1153, normalized size of antiderivative = 6.48

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(-b^2*x^2+a^2)^2,x)
```

output
```
(3*log( - a - b*x)*a**6*b*c**2*d**4 + 3*log( - a - b*x)*a**6*b*c*d**5*x +
8*log( - a - b*x)*a**5*b**2*c**3*d**3 + 8*log( - a - b*x)*a**5*b**2*c**2*d
**4*x + 6*log( - a - b*x)*a**4*b**3*c**4*d**2 + 6*log( - a - b*x)*a**4*b**
3*c**3*d**3*x - 3*log( - a - b*x)*a**4*b**3*c**2*d**4*x**2 - 3*log( - a -
b*x)*a**4*b**3*c*d**5*x**3 - 8*log( - a - b*x)*a**3*b**4*c**3*d**3*x**2 -
8*log( - a - b*x)*a**3*b**4*c**2*d**4*x**3 - log( - a - b*x)*a**2*b**5*c**
6 - log( - a - b*x)*a**2*b**5*c**5*d*x - 6*log( - a - b*x)*a**2*b**5*c**4*
d**2*x**2 - 6*log( - a - b*x)*a**2*b**5*c**3*d**3*x**3 + log( - a - b*x)*b
**7*c**6*x**2 + log( - a - b*x)*b**7*c**5*d*x**3 - 3*log(a - b*x)*a**6*b*c
**2*d**4 - 3*log(a - b*x)*a**6*b*c*d**5*x + 8*log(a - b*x)*a**5*b**2*c**3*
d**3 + 8*log(a - b*x)*a**5*b**2*c**2*d**4*x - 6*log(a - b*x)*a**4*b**3*c**
4*d**2 - 6*log(a - b*x)*a**4*b**3*c**3*d**3*x + 3*log(a - b*x)*a**4*b**3*c
**2*d**4*x**2 + 3*log(a - b*x)*a**4*b**3*c*d**5*x**3 - 8*log(a - b*x)*a**3
*b**4*c**3*d**3*x**2 - 8*log(a - b*x)*a**3*b**4*c**2*d**4*x**3 + log(a - b
*x)*a**2*b**5*c**6 + log(a - b*x)*a**2*b**5*c**5*d*x + 6*log(a - b*x)*a**2
*b**5*c**4*d**2*x**2 + 6*log(a - b*x)*a**2*b**5*c**3*d**3*x**3 - log(a - b
*x)*b**7*c**6*x**2 - log(a - b*x)*b**7*c**5*d*x**3 - 16*log(c + d*x)*a**5*
b**2*c**3*d**3 - 16*log(c + d*x)*a**5*b**2*c**2*d**4*x + 16*log(c + d*x)*a
**3*b**4*c**3*d**3*x**2 + 16*log(c + d*x)*a**3*b**4*c**2*d**4*x**3 + 2*a**
7*c*d**5 + 6*a**7*d**6*x - 4*a**5*b**2*c**3*d**3 - 6*a**5*b**2*c**2*d**...
```

3.43 $\int \frac{1}{(c+dx)^3\left(a^2-b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 237

$$\begin{aligned}\int \frac{1}{(c+dx)^3\left(a^2-b^2x^2\right)^2}\,dx = {} & \frac{b^2}{4a^2(bc+ad)^3(a-bx)}-\frac{b^2}{4a^2(bc-ad)^3(a+bx)}\\ & -\frac{d^3}{2\left(b^2c^2-a^2d^2\right)^2(c+dx)^2}-\frac{4b^2cd^3}{\left(b^2c^2-a^2d^2\right)^3(c+dx)}\\ & -\frac{b^2(bc+4ad)\log(a-bx)}{4a^3(bc+ad)^4}+\frac{b^2(bc-4ad)\log(a+bx)}{4a^3(bc-ad)^4}\\ & +\frac{2b^2d^3\left(5b^2c^2+a^2d^2\right)\log(c+dx)}{\left(b^2c^2-a^2d^2\right)^4}\end{aligned}$$

output
```
1/4*b^2/a^2/(a*d+b*c)^3/(-b*x+a)-1/4*b^2/a^2/(-a*d+b*c)^3/(b*x+a)-1/2*d^3/
(-a^2*d^2+b^2*c^2)^2/(d*x+c)^2-4*b^2*c*d^3/(-a^2*d^2+b^2*c^2)^3/(d*x+c)-1/
4*b^2*(4*a*d+b*c)*ln(-b*x+a)/a^3/(a*d+b*c)^4+1/4*b^2*(-4*a*d+b*c)*ln(b*x+a
)/a^3/(-a*d+b*c)^4+2*b^2*d^3*(a^2*d^2+5*b^2*c^2)*ln(d*x+c)/(-a^2*d^2+b^2*c
^2)^4
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^2} dx$$
$$= \frac{-\frac{d^3(b^2c^2-a^2d^2)^2}{(c+dx)^2} + \frac{8b^2cd^3(-b^2c^2+a^2d^2)}{c+dx} + \frac{(b^2c^2-a^2d^2)(-a^4b^2d^3+b^6c^3x-3a^2b^4cd(c-dx))}{a^4-a^2b^2x^2} + \frac{b^3(b^4c^5-10a^2b^2c^3d^2-15a^4cd^4)\text{arcta}}{a^3}}{2(b^2c^2-a^2d^2)^4}$$

input
```
Integrate[1/((c + d*x)^3*(a^2 - b^2*x^2)^2),x]
```

output
```
(-((d^3*(b^2*c^2 - a^2*d^2)^2)/(c + d*x)^2) + (8*b^2*c*d^3*(-(b^2*c^2) + a
^2*d^2))/(c + d*x) + ((b^2*c^2 - a^2*d^2)*(-(a^4*b^2*d^3) + b^6*c^3*x - 3*
a^2*b^4*c*d*(c - d*x)))/(a^4 - a^2*b^2*x^2) + (b^3*(b^4*c^5 - 10*a^2*b^2*c
^3*d^2 - 15*a^4*c*d^4)*ArcTanh[(b*x)/a])/a^3 + 4*(5*b^4*c^2*d^3 + a^2*b^2*
d^5)*Log[c + d*x] - 2*b^2*(5*b^2*c^2*d^3 + a^2*d^5)*Log[a^2 - b^2*x^2])/(2
*(b^2*c^2 - a^2*d^2)^4)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)^2 (c+dx)^3} dx$$
$$\Big\downarrow \text{477}$$
$$\frac{\int \left(\frac{2a^4b^2(5b^2c^2+a^2d^2)d^4}{(b^2c^2-a^2d^2)^4(c+dx)} + \frac{4a^4b^2cd^4}{(b^2c^2-a^2d^2)^3(c+dx)^2} + \frac{a^4d^4}{(b^2c^2-a^2d^2)^2(c+dx)^3} + \frac{ab^3(bc+4ad)}{4(bc+ad)^4(a-bx)} + \frac{ab^3(bc-4ad)}{4(bc-ad)^4(a+bx)} + \frac{a^2b^3}{4(bc+ad)^3(a-b}\right.}{a^4}$$
$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2b^2}{4(a-bx)(ad+bc)^3}-\frac{a^2b^2}{4(a+bx)(bc-ad)^3}-\frac{4a^4b^2cd^3}{(c+dx)(b^2c^2-a^2d^2)^3}-\frac{a^4d^3}{2(c+dx)^2(b^2c^2-a^2d^2)^2}+\frac{2a^4b^2d^3(a^2d^2+5b^2c^2)\log(c+dx)}{(b^2c^2-a^2d^2)^4}-\frac{ab^2(4ad+\ldots}{4(\ldots}}{a^4}$$

input
```
Int[1/((c + d*x)^3*(a^2 - b^2*x^2)^2),x]
```

output
```
((a^2*b^2)/(4*(b*c + a*d)^3*(a - b*x)) - (a^2*b^2)/(4*(b*c - a*d)^3*(a + b
*x)) - (a^4*d^3)/(2*(b^2*c^2 - a^2*d^2)^2*(c + d*x)^2) - (4*a^4*b^2*c*d^3)
/((b^2*c^2 - a^2*d^2)^3*(c + d*x)) - (a*b^2*(b*c + 4*a*d)*Log[a - b*x])/(4
*(b*c + a*d)^4) + (a*b^2*(b*c - 4*a*d)*Log[a + b*x])/(4*(b*c - a*d)^4) + (
2*a^4*b^2*d^3*(5*b^2*c^2 + a^2*d^2)*Log[c + d*x])/(b^2*c^2 - a^2*d^2)^4)/a
^4
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.98

method	result
default	$\frac{b^2}{4(ad-bc)^3a^2(bx+a)}-\frac{b^2(4ad-bc)\ln(bx+a)}{4(ad-bc)^4a^3}-\frac{d^3}{2(ad-bc)^2(ad+bc)^2(dx+c)^2}+\frac{4d^3b^2c}{(ad-bc)^3(ad+bc)^3(dx+c)}+\frac{2d^3b^2(a^2\ldots}{(ad-\ldots}$
norman	$\frac{\frac{\left(2a^6d^6-10a^4b^2c^2d^4-3a^2b^4c^4d^2-b^6c^6\right)x}{2c\,a^2\left(a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6\right)}+\frac{\left(a^6d^7-9a^4b^2c^2d^5-5a^2b^4c^4d^3+b^6c^6d\right)x^2}{2a^2c^2\left(a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6\right)}-\frac{b^2d^2\left(a^4d^5-10a^2b^2c^2d^3-3b^4c^4d\right)x^4}{2a^2c^2\left(a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c\ldots\right)}\ldots}{(dx+c)^2(-b^2x^2+a^2)}$
risch	$\frac{-\frac{b^4c\,d^2\left(11a^2d^2+b^2c^2\right)x^3}{2a^2\left(a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6\right)}+\frac{d\,b^2\left(a^4d^4-6a^2b^2c^2d^2-c^4b^4\right)x^2}{a^2\left(a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6\right)}+\frac{b^2c\left(10a^4d^4+3a^2b^2c^2d^2-c^4b^4\right)x}{2a^2\left(a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6\right)}-\ldots}{(dx+c)^2(-b^2x^2+a^2)}$
parallelrisch	Expression too large to display

input
```
int(1/(d*x+c)^3/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2/(a*d-b*c)^3/a^2/(b*x+a)-1/4*b^2*(4*a*d-b*c)/(a*d-b*c)^4/a^3*ln(b*x
+a)-1/2*d^3/(a*d-b*c)^2/(a*d+b*c)^2/(d*x+c)^2+4*d^3*b^2*c/(a*d-b*c)^3/(a*d
+b*c)^3/(d*x+c)+2*d^3*b^2*(a^2*d^2+5*b^2*c^2)/(a*d-b*c)^4/(a*d+b*c)^4*ln(d
*x+c)+1/4*b^2/a^2/(a*d+b*c)^3/(-b*x+a)-1/4*b^2*(4*a*d+b*c)*ln(-b*x+a)/a^3/
(a*d+b*c)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. $2(228) = 456$.

Time = 14.54 (sec) , antiderivative size = 1464, normalized size of antiderivative = 6.18

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(6*a^3*b^6*c^6*d + 14*a^5*b^4*c^4*d^3 - 22*a^7*b^2*c^2*d^5 + 2*a^9*d^
7 - 2*(a*b^8*c^5*d^2 + 10*a^3*b^6*c^3*d^4 - 11*a^5*b^4*c*d^6)*x^3 - 4*(a*b
^8*c^6*d + 5*a^3*b^6*c^4*d^3 - 7*a^5*b^4*c^2*d^5 + a^7*b^2*d^7)*x^2 - 2*(a
*b^8*c^7 - 4*a^3*b^6*c^5*d^2 - 7*a^5*b^4*c^3*d^4 + 10*a^7*b^2*c*d^6)*x - (
a^2*b^7*c^7 - 10*a^4*b^5*c^5*d^2 - 20*a^5*b^4*c^4*d^3 - 15*a^6*b^3*c^3*d^4
 - 4*a^7*b^2*c^2*d^5 - (b^9*c^5*d^2 - 10*a^2*b^7*c^3*d^4 - 20*a^3*b^6*c^2*
d^5 - 15*a^4*b^5*c*d^6 - 4*a^5*b^4*d^7)*x^4 - 2*(b^9*c^6*d - 10*a^2*b^7*c^
4*d^3 - 20*a^3*b^6*c^3*d^4 - 15*a^4*b^5*c^2*d^5 - 4*a^5*b^4*c*d^6)*x^3 - (
b^9*c^7 - 11*a^2*b^7*c^5*d^2 - 20*a^3*b^6*c^4*d^3 - 5*a^4*b^5*c^3*d^4 + 16
*a^5*b^4*c^2*d^5 + 15*a^6*b^3*c*d^6 + 4*a^7*b^2*d^7)*x^2 + 2*(a^2*b^7*c^6*
d - 10*a^4*b^5*c^4*d^3 - 20*a^5*b^4*c^3*d^4 - 15*a^6*b^3*c^2*d^5 - 4*a^7*b
^2*c*d^6)*x)*log(b*x + a) + (a^2*b^7*c^7 - 10*a^4*b^5*c^5*d^2 + 20*a^5*b^4
*c^4*d^3 - 15*a^6*b^3*c^3*d^4 + 4*a^7*b^2*c^2*d^5 - (b^9*c^5*d^2 - 10*a^2*
b^7*c^3*d^4 + 20*a^3*b^6*c^2*d^5 - 15*a^4*b^5*c*d^6 + 4*a^5*b^4*d^7)*x^4 -
 2*(b^9*c^6*d - 10*a^2*b^7*c^4*d^3 + 20*a^3*b^6*c^3*d^4 - 15*a^4*b^5*c^2*d
^5 + 4*a^5*b^4*c*d^6)*x^3 - (b^9*c^7 - 11*a^2*b^7*c^5*d^2 + 20*a^3*b^6*c^4
*d^3 - 5*a^4*b^5*c^3*d^4 - 16*a^5*b^4*c^2*d^5 + 15*a^6*b^3*c*d^6 - 4*a^7*b
^2*d^7)*x^2 + 2*(a^2*b^7*c^6*d - 10*a^4*b^5*c^4*d^3 + 20*a^5*b^4*c^3*d^4 -
 15*a^6*b^3*c^2*d^5 + 4*a^7*b^2*c*d^6)*x)*log(b*x - a) - 8*(5*a^5*b^4*c^4*
d^3 + a^7*b^2*c^2*d^5 - (5*a^3*b^6*c^2*d^5 + a^5*b^4*d^7)*x^4 - 2*(5*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 \left(a^2-b^2x^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**3/(-b**2*x**2+a**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(228) = 456$.

Time = 0.06 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.83

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^2} dx = \frac{(b^3c - 4ab^2d)\log(bx+a)}{4(a^3b^4c^4 - 4a^4b^3c^3d + 6a^5b^2c^2d^2 - 4a^6bcd^3 + a^7d^4)} - \frac{(b^3c + 4ab^2d)\log(bx-a)}{4(a^3b^4c^4 + 4a^4b^3c^3d + 6a^5b^2c^2d^2 + 4a^6bcd^3 + a^7d^4)} + \frac{2(5b^4c^2d^3 + a^2b^2d^5)\log(dx+c)}{b^8c^8 - 4a^2b^6c^6d^2 + 6a^4b^4c^4d^4 - 4a^6b^2c^2d^6 + a^8d^8} - \frac{3a^2b^4c^4d + 10a^4b^2c^2d^3 - a^6d^5 - (b^6c}{2(a^4b^6c^8 - 3a^6b^4c^6d^2 + 3a^8b^2c^4d^4 - a^{10}c^2d^6 - (a^2b^8c^6d^2 - 3a^4b^6c^4d^4 + 3a^6b^4c^2d^6 - a^8b^2d^8)x^4 - 2(a}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/4*(b^3*c - 4*a*b^2*d)*log(b*x + a)/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^
5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4) - 1/4*(b^3*c + 4*a*b^2*d)*log(b*x
 - a)/(a^3*b^4*c^4 + 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 + 4*a^6*b*c*d^3 +
 a^7*d^4) + 2*(5*b^4*c^2*d^3 + a^2*b^2*d^5)*log(d*x + c)/(b^8*c^8 - 4*a^2*
b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^4 - 4*a^6*b^2*c^2*d^6 + a^8*d^8) - 1/2*(3*a^
2*b^4*c^4*d + 10*a^4*b^2*c^2*d^3 - a^6*d^5 - (b^6*c^3*d^2 + 11*a^2*b^4*c*d
^4)*x^3 - 2*(b^6*c^4*d + 6*a^2*b^4*c^2*d^3 - a^4*b^2*d^5)*x^2 - (b^6*c^5 -
 3*a^2*b^4*c^3*d^2 - 10*a^4*b^2*c*d^4)*x)/(a^4*b^6*c^8 - 3*a^6*b^4*c^6*d^2
 + 3*a^8*b^2*c^4*d^4 - a^10*c^2*d^6 - (a^2*b^8*c^6*d^2 - 3*a^4*b^6*c^4*d^4
 + 3*a^6*b^4*c^2*d^6 - a^8*b^2*d^8)*x^4 - 2*(a^2*b^8*c^7*d - 3*a^4*b^6*c^5
*d^3 + 3*a^6*b^4*c^3*d^5 - a^8*b^2*c*d^7)*x^3 - (a^2*b^8*c^8 - 4*a^4*b^6*c
^6*d^2 + 6*a^6*b^4*c^4*d^4 - 4*a^8*b^2*c^2*d^6 + a^10*d^8)*x^2 + 2*(a^4*b^
6*c^7*d - 3*a^6*b^4*c^5*d^3 + 3*a^8*b^2*c^3*d^5 - a^10*c*d^7)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(228) = 456$.

Time = 0.13 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.09

$$\int \frac{1}{(c+dx)^3\left(a^2-b^2x^2\right)^2}\,dx = \frac{(b^4c-4\,ab^3d)\log(|bx+a|)}{4\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4)} - \frac{(b^4c+4\,ab^3d)\log(|bx-a|)}{4\,(a^3b^5c^4+4\,a^4b^4c^3d+6\,a^5b^3c^2d^2+4\,a^6b^2cd^3+a^7bd^4)} + \frac{2\,(5\,b^4c^2d^4+a^2b^2d^6)\log(|dx+c|)}{b^8c^8d-4\,a^2b^6c^6d^3+6\,a^4b^4c^4d^5-4\,a^6b^2c^2d^7+a^8d^9} + \frac{3\,a^2b^6c^6d+7\,a^4b^4c^4d^3-11\,a^6b^2c^2d^5+a^8d^7-(b^8c^5d^2+10\,a^2b^6c^3d^4-11\,a^4b^4cd^6)x^3-2\,(b^8c^6d+5\,a^2b}{2\,(bc+ad)^4(bc-ad)^4(bx+a)(bx-}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/4*(b^4*c - 4*a*b^3*d)*log(abs(b*x + a))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d +
 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4) - 1/4*(b^4*c + 4*a*b^3*d
)*log(abs(b*x - a))/(a^3*b^5*c^4 + 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 + 4
*a^6*b^2*c*d^3 + a^7*b*d^4) + 2*(5*b^4*c^2*d^4 + a^2*b^2*d^6)*log(abs(d*x
+ c))/(b^8*c^8*d - 4*a^2*b^6*c^6*d^3 + 6*a^4*b^4*c^4*d^5 - 4*a^6*b^2*c^2*d
^7 + a^8*d^9) + 1/2*(3*a^2*b^6*c^6*d + 7*a^4*b^4*c^4*d^3 - 11*a^6*b^2*c^2*
d^5 + a^8*d^7 - (b^8*c^5*d^2 + 10*a^2*b^6*c^3*d^4 - 11*a^4*b^4*c*d^6)*x^3
- 2*(b^8*c^6*d + 5*a^2*b^6*c^4*d^3 - 7*a^4*b^4*c^2*d^5 + a^6*b^2*d^7)*x^2
- (b^8*c^7 - 4*a^2*b^6*c^5*d^2 - 7*a^4*b^4*c^3*d^4 + 10*a^6*b^2*c*d^6)*x)/
((b*c + a*d)^4*(b*c - a*d)^4*(b*x + a)*(b*x - a)*(d*x + c)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 7.11 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.76

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^2}\,dx$$
$$= \frac{\frac{-a^4\,d^5+10\,a^2\,b^2\,c^2\,d^3+3\,b^4\,c^4\,d}{2\,(a^2\,d^2-b^2\,c^2)\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}-\frac{b^2\,x^3\,(11\,a^2\,b^2\,c\,d^4+b^4\,c^3\,d^2)}{2\,a^2\,(a^6\,d^6-3\,a^4\,b^2\,c^2\,d^4+3\,a^2\,b^4\,c^4\,d^2-b^6\,c^6)}-\frac{x^2\,(-a^4\,b^2\,d^5+6\,a^2\,b^4\,c^2\,d^3+b^6\,c^4\,d)}{a^2\,(a^2\,d^2-b^2\,c^2)\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}}{x^2\,(a^2\,d^2-b^2\,c^2)+a^2\,c^2-b^2\,d^2\,x^4+2\,a^2\,c\,d\,x-2\,b^2\,c\,d\,x^3}$$
$$+\frac{\ln(c+d\,x)\,(2\,a^2\,b^2\,d^5+10\,b^4\,c^2\,d^3)}{a^8\,d^8-4\,a^6\,b^2\,c^2\,d^6+6\,a^4\,b^4\,c^4\,d^4-4\,a^2\,b^6\,c^6\,d^2+b^8\,c^8}$$
$$+\frac{\ln(a+b\,x)\,(b^3\,c-4\,a\,b^2\,d)}{4\,(a^7\,d^4-4\,a^6\,b\,c\,d^3+6\,a^5\,b^2\,c^2\,d^2-4\,a^4\,b^3\,c^3\,d+a^3\,b^4\,c^4)}$$
$$-\frac{\ln(a-b\,x)\,(c\,b^3+4\,a\,d\,b^2)}{4\,(a^7\,d^4+4\,a^6\,b\,c\,d^3+6\,a^5\,b^2\,c^2\,d^2+4\,a^4\,b^3\,c^3\,d+a^3\,b^4\,c^4)}$$

input
```
int(1/((a^2 - b^2*x^2)^2*(c + d*x)^3),x)
```

output
```
((3*b^4*c^4*d - a^4*d^5 + 10*a^2*b^2*c^2*d^3)/(2*(a^2*d^2 - b^2*c^2)*(a^4*
d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) - (b^2*x^3*(b^4*c^3*d^2 + 11*a^2*b^2*c
*d^4))/(2*a^2*(a^6*d^6 - b^6*c^6 + 3*a^2*b^4*c^4*d^2 - 3*a^4*b^2*c^2*d^4))
 - (x^2*(b^6*c^4*d - a^4*b^2*d^5 + 6*a^2*b^4*c^2*d^3))/(a^2*(a^2*d^2 - b^2
*c^2)*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) + (c*x*(10*a^4*b^2*d^4 - b^
6*c^4 + 3*a^2*b^4*c^2*d^2))/(2*a^2*(a^2*d^2 - b^2*c^2)*(a^4*d^4 + b^4*c^4
- 2*a^2*b^2*c^2*d^2)))/(x^2*(a^2*d^2 - b^2*c^2) + a^2*c^2 - b^2*d^2*x^4 +
2*a^2*c*d*x - 2*b^2*c*d*x^3) + (log(c + d*x)*(2*a^2*b^2*d^5 + 10*b^4*c^2*d
^3))/(a^8*d^8 + b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^4 - 4*a^6*b^
2*c^2*d^6) + (log(a + b*x)*(b^3*c - 4*a*b^2*d))/(4*(a^7*d^4 + a^3*b^4*c^4
- 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)) - (log(a - b*x)*(b
^3*c + 4*a*b^2*d))/(4*(a^7*d^4 + a^3*b^4*c^4 + 4*a^4*b^3*c^3*d + 6*a^5*b^2
*c^2*d^2 + 4*a^6*b*c*d^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2057, normalized size of antiderivative = 8.68

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^3/(-b^2*x^2+a^2)^2,x)
```

output
```
( - 4*log( - a - b*x)*a**7*b**2*c**2*d**5 - 8*log( - a - b*x)*a**7*b**2*c*
d**6*x - 4*log( - a - b*x)*a**7*b**2*d**7*x**2 - 15*log( - a - b*x)*a**6*b
**3*c**3*d**4 - 30*log( - a - b*x)*a**6*b**3*c**2*d**5*x - 15*log( - a - b
*x)*a**6*b**3*c*d**6*x**2 - 20*log( - a - b*x)*a**5*b**4*c**4*d**3 - 40*lo
g( - a - b*x)*a**5*b**4*c**3*d**4*x - 16*log( - a - b*x)*a**5*b**4*c**2*d*
*5*x**2 + 8*log( - a - b*x)*a**5*b**4*c*d**6*x**3 + 4*log( - a - b*x)*a**5
*b**4*d**7*x**4 - 10*log( - a - b*x)*a**4*b**5*c**5*d**2 - 20*log( - a - b
*x)*a**4*b**5*c**4*d**3*x + 5*log( - a - b*x)*a**4*b**5*c**3*d**4*x**2 + 3
0*log( - a - b*x)*a**4*b**5*c**2*d**5*x**3 + 15*log( - a - b*x)*a**4*b**5*
c*d**6*x**4 + 20*log( - a - b*x)*a**3*b**6*c**4*d**3*x**2 + 40*log( - a -
b*x)*a**3*b**6*c**3*d**4*x**3 + 20*log( - a - b*x)*a**3*b**6*c**2*d**5*x**
4 + log( - a - b*x)*a**2*b**7*c**7 + 2*log( - a - b*x)*a**2*b**7*c**6*d*x
+ 11*log( - a - b*x)*a**2*b**7*c**5*d**2*x**2 + 20*log( - a - b*x)*a**2*b*
*7*c**4*d**3*x**3 + 10*log( - a - b*x)*a**2*b**7*c**3*d**4*x**4 - log( - a
 - b*x)*b**9*c**7*x**2 - 2*log( - a - b*x)*b**9*c**6*d*x**3 - log( - a - b
*x)*b**9*c**5*d**2*x**4 - 4*log(a - b*x)*a**7*b**2*c**2*d**5 - 8*log(a - b
*x)*a**7*b**2*c*d**6*x - 4*log(a - b*x)*a**7*b**2*d**7*x**2 + 15*log(a - b
*x)*a**6*b**3*c**3*d**4 + 30*log(a - b*x)*a**6*b**3*c**2*d**5*x + 15*log(a
 - b*x)*a**6*b**3*c*d**6*x**2 - 20*log(a - b*x)*a**5*b**4*c**4*d**3 - 40*l
og(a - b*x)*a**5*b**4*c**3*d**4*x - 16*log(a - b*x)*a**5*b**4*c**2*d**5...
```

3.44 $\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{4a^2b^2cd\left(c^2+\frac{a^2d^2}{b^2}\right)+(b^4c^4+6a^2b^2c^2d^2+a^4d^4)\,x}{4a^2b^4\,(a^2-b^2x^2)^2} - \frac{16a^4cd^3-(3b^4c^4-6a^2b^2c^2d^2-5a^4d^4)\,x}{8a^4b^4\,(a^2-b^2x^2)} + \frac{3(b^2c^2-a^2d^2)^2\,\text{arctanh}\left(\frac{bx}{a}\right)}{8a^5b^5}$$

output
```
1/4*(4*a^2*b^2*c*d*(c^2+a^2*d^2/b^2)+(a^4*d^4+6*a^2*b^2*c^2*d^2+b^4*c^4)*x
)/a^2/b^4/(-b^2*x^2+a^2)^2-1/8*(16*a^4*c*d^3-(-5*a^4*d^4-6*a^2*b^2*c^2*d^2
+3*b^4*c^4)*x)/a^4/b^4/(-b^2*x^2+a^2)+3/8*(-a^2*d^2+b^2*c^2)^2*arctanh(b*x
/a)/a^5/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{-3ab^7c^4x^3 - a^7bd^3(8c+3dx) + a^3b^5c^2x(5c^2+6d^2x^2) + a^5b^3d(8c^3+6c^2dx+16cd^2x^2+5d^3x^3) + 3(b^2c^2}{8a^5b^5\left(a^2-b^2x^2\right)^2}$$

input
```
Integrate[(c + d*x)^4/(a^2 - b^2*x^2)^3,x]
```

output
```
(-3*a*b^7*c^4*x^3 - a^7*b*d^3*(8*c + 3*d*x) + a^3*b^5*c^2*x*(5*c^2 + 6*d^2
*x^2) + a^5*b^3*d*(8*c^3 + 6*c^2*d*x + 16*c*d^2*x^2 + 5*d^3*x^3) + 3*(b^2*
c^2 - a^2*d^2)^2*(a^2 - b^2*x^2)^2*ArcTanh[(b*x)/a])/(8*a^5*b^5*(a^2 - b^2
*x^2)^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx$$
$$\Big\downarrow \; 477$$
$$\frac{\int \left(\frac{a^3(bc-ad)^4}{8b^4(a+bx)^3}+\frac{a^2(3bc+5ad)(bc-ad)^3}{16b^4(a+bx)^2}+\frac{3a^2\left(b^2c^2-a^2d^2\right)^2}{8b^4(a^2-b^2x^2)}+\frac{a^2(3bc-5ad)(bc+ad)^3}{16b^4(a-bx)^2}+\frac{a^3(bc+ad)^4}{8b^4(a-bx)^3}\right)dx}{a^6}$$
$$\Big\downarrow \; 2009$$
$$\frac{-\frac{a^3(bc-ad)^4}{16b^5(a+bx)^2}+\frac{a^3(ad+bc)^4}{16b^5(a-bx)^2}+\frac{3a\,\text{arctanh}\left(\frac{bx}{a}\right)\left(b^2c^2-a^2d^2\right)^2}{8b^5}-\frac{a^2(5ad+3bc)(bc-ad)^3}{16b^5(a+bx)}+\frac{a^2(3bc-5ad)(ad+bc)^3}{16b^5(a-bx)}}{a^6}$$

input
```
Int[(c + d*x)^4/(a^2 - b^2*x^2)^3,x]
```

output
```
((a^3*(b*c + a*d)^4)/(16*b^5*(a - b*x)^2) + (a^2*(3*b*c - 5*a*d)*(b*c + a*
d)^3)/(16*b^5*(a - b*x)) - (a^3*(b*c - a*d)^4)/(16*b^5*(a + b*x)^2) - (a^2
*(b*c - a*d)^3*(3*b*c + 5*a*d))/(16*b^5*(a + b*x)) + (3*a*(b^2*c^2 - a^2*d
^2)^2*ArcTanh[(b*x)/a])/(8*b^5))/a^6
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\frac{-d^3 c a^2+b^2 c^3 d}{b^4}+\frac{2c d^3 x^2}{b^2}-\frac{\left(3a^4 d^4-6a^2 b^2 c^2 d^2-5c^4 b^4\right)x}{8a^2 b^4}+\frac{\left(5a^4 d^4+6a^2 b^2 c^2 d^2-3c^4 b^4\right)x^3}{8a^4 b^2}}{\left(-b^2 x^2+a^2\right)^2}-\frac{3\left(a^4 d^4-2a^2 b^2 c^2 d^2+c^4 b^4\right)\ln(-bx-}{16b^5 a^5}$
risch	$\frac{\frac{\left(5a^4 d^4+6a^2 b^2 c^2 d^2-3c^4 b^4\right)x^3}{8a^4 b^2}+\frac{2c d^3 x^2}{b^2}-\frac{\left(3a^4 d^4-6a^2 b^2 c^2 d^2-5c^4 b^4\right)x}{8a^2 b^4}-\frac{cd\left(a^2 d^2-b^2 c^2\right)}{b^4}}{\left(-b^2 x^2+a^2\right)^2}-\frac{3\ln(-bx+a)d^4}{16b^5 a}+\frac{3\ln(-bx+a)c^2}{8b^3 a^3}$
default	$\frac{\left(3a^4 d^4-6a^2 b^2 c^2 d^2+3c^4 b^4\right)\ln(bx+a)}{16b^5 a^5}-\frac{-5a^4 d^4+12c a^3 b d^3-6a^2 b^2 c^2 d^2-4a b^3 c^3 d+3c^4 b^4}{16a^4 b^5 (bx+a)}-\frac{a^4 d^4-4c a^3 b d^3+6a^2 b^2 c^2 d^2}{16a^3 b^5 (bx+a)}$
parallelrisch	$-\frac{6x^3 a b^7 c^4+6x a^7 b d^4-10x a^3 b^5 c^4+3\ln(bx-a)x^4 b^8 c^4-3\ln(bx+a)x^4 b^8 c^4+3\ln(bx-a)a^4 b^4 c^4-3\ln(bx+a)a^4 b^4 c^4-6\ln(bx-}{}$

input
```
int((d*x+c)^4/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
((-a^2*c*d^3+b^2*c^3*d)/b^4+2*c*d^3*x^2/b^2-1/8*(3*a^4*d^4-6*a^2*b^2*c^2*d
^2-5*b^4*c^4)/a^2/b^4*x+1/8*(5*a^4*d^4+6*a^2*b^2*c^2*d^2-3*b^4*c^4)/a^4/b^
2*x^3)/(-b^2*x^2+a^2)^2-3/16*(a^4*d^4-2*a^2*b^2*c^2*d^2+b^4*c^4)/b^5/a^5*l
n(-b*x+a)+3/16*(a^4*d^4-2*a^2*b^2*c^2*d^2+b^4*c^4)/b^5/a^5*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(180) = 360$.

Time = 0.10 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{32\,a^5b^3cd^3x^2 + 16\,a^5b^3c^3d - 16\,a^7bcd^3 - 2\,(3\,ab^7c^4 - 6\,a^3b^5c^2d^2 - 5\,a^5b^3d^4)x^3 + 2\,(5\,a^3b^5c^4 + 6\,a^5b^3c^2d^2}{}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/16*(32*a^5*b^3*c*d^3*x^2 + 16*a^5*b^3*c^3*d - 16*a^7*b*c*d^3 - 2*(3*a*b^
7*c^4 - 6*a^3*b^5*c^2*d^2 - 5*a^5*b^3*d^4)*x^3 + 2*(5*a^3*b^5*c^4 + 6*a^5*
b^3*c^2*d^2 - 3*a^7*b*d^4)*x + 3*(a^4*b^4*c^4 - 2*a^6*b^2*c^2*d^2 + a^8*d^
4 + (b^8*c^4 - 2*a^2*b^6*c^2*d^2 + a^4*b^4*d^4)*x^4 - 2*(a^2*b^6*c^4 - 2*a
^4*b^4*c^2*d^2 + a^6*b^2*d^4)*x^2)*log(b*x + a) - 3*(a^4*b^4*c^4 - 2*a^6*b
^2*c^2*d^2 + a^8*d^4 + (b^8*c^4 - 2*a^2*b^6*c^2*d^2 + a^4*b^4*d^4)*x^4 - 2
*(a^2*b^6*c^4 - 2*a^4*b^4*c^2*d^2 + a^6*b^2*d^4)*x^2)*log(b*x - a))/(a^5*b
^9*x^4 - 2*a^7*b^7*x^2 + a^9*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx = -\frac{8a^6cd^3-8a^4b^2c^3d-16a^4b^2cd^3x^2+x^3(-5a^4b^2d^4-6a^2b^4c^2d^2+3b^6c^4)+x(3a^6d^4-6a^4b^2c^2d^2-5a^2b^4}{8a^8b^4-16a^6b^6x^2+8a^4b^8x^4} - \frac{3(ad-bc)^2(ad+bc)^2\log\left(-\frac{3a(ad-bc)^2(ad+bc)^2}{b(3a^4d^4-6a^2b^2c^2d^2+3b^4c^4)}+x\right)}{16a^5b^5} + \frac{3(ad-bc)^2(ad+bc)^2\log\left(\frac{3a(ad-bc)^2(ad+bc)^2}{b(3a^4d^4-6a^2b^2c^2d^2+3b^4c^4)}+x\right)}{16a^5b^5}$$

input
```
integrate((d*x+c)**4/(-b**2*x**2+a**2)**3,x)
```

output
```
-(8*a**6*c*d**3 - 8*a**4*b**2*c**3*d - 16*a**4*b**2*c*d**3*x**2 + x**3*(-5
*a**4*b**2*d**4 - 6*a**2*b**4*c**2*d**2 + 3*b**6*c**4) + x*(3*a**6*d**4 -
6*a**4*b**2*c**2*d**2 - 5*a**2*b**4*c**4))/(8*a**8*b**4 - 16*a**6*b**6*x**
2 + 8*a**4*b**8*x**4) - 3*(a*d - b*c)**2*(a*d + b*c)**2*log(-3*a*(a*d - b*
c)**2*(a*d + b*c)**2/(b*(3*a**4*d**4 - 6*a**2*b**2*c**2*d**2 + 3*b**4*c**4
)) + x)/(16*a**5*b**5) + 3*(a*d - b*c)**2*(a*d + b*c)**2*log(3*a*(a*d - b*
c)**2*(a*d + b*c)**2/(b*(3*a**4*d**4 - 6*a**2*b**2*c**2*d**2 + 3*b**4*c**4
)) + x)/(16*a**5*b**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{16\,a^4b^2cd^3x^2+8\,a^4b^2c^3d-8\,a^6cd^3-(3\,b^6c^4-6\,a^2b^4c^2d^2-5\,a^4b^2d^4)x^3+(5\,a^2b^4c^4+6\,a^4b^2c^2d^2-3\,a^6d^4}{8\,(a^4b^8x^4-2\,a^6b^6x^2+a^8b^4)} + \frac{3\,(b^4c^4-2\,a^2b^2c^2d^2+a^4d^4)\log(bx+a)}{16\,a^5b^5} - \frac{3\,(b^4c^4-2\,a^2b^2c^2d^2+a^4d^4)\log(bx-a)}{16\,a^5b^5}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/8*(16*a^4*b^2*c*d^3*x^2 + 8*a^4*b^2*c^3*d - 8*a^6*c*d^3 - (3*b^6*c^4 - 6
*a^2*b^4*c^2*d^2 - 5*a^4*b^2*d^4)*x^3 + (5*a^2*b^4*c^4 + 6*a^4*b^2*c^2*d^2
 - 3*a^6*d^4)*x)/(a^4*b^8*x^4 - 2*a^6*b^6*x^2 + a^8*b^4) + 3/16*(b^4*c^4 -
 2*a^2*b^2*c^2*d^2 + a^4*d^4)*log(b*x + a)/(a^5*b^5) - 3/16*(b^4*c^4 - 2*a
^2*b^2*c^2*d^2 + a^4*d^4)*log(b*x - a)/(a^5*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx = -\frac{3\,b^6c^4x^3 - 6\,a^2b^4c^2d^2x^3 - 5\,a^4b^2d^4x^3 - 16\,a^4b^2cd^3x^2 - 5\,a^2b^4c^4x - 6\,a^4b^2c^2d^2x + 3\,a^6d^4x - 8\,a^4b^2c^3d -}{8\,(b^2x^2-a^2)^2a^4b^4} + \frac{3\,(b^5c^4 - 2\,a^2b^3c^2d^2 + a^4bd^4)\log(|bx+a|)}{16\,a^5b^6} - \frac{3\,(b^5c^4 - 2\,a^2b^3c^2d^2 + a^4bd^4)\log(|bx-a|)}{16\,a^5b^6}$$

input
```
integrate((d*x+c)^4/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*b^6*c^4*x^3 - 6*a^2*b^4*c^2*d^2*x^3 - 5*a^4*b^2*d^4*x^3 - 16*a^4*b
^2*c*d^3*x^2 - 5*a^2*b^4*c^4*x - 6*a^4*b^2*c^2*d^2*x + 3*a^6*d^4*x - 8*a^4
*b^2*c^3*d + 8*a^6*c*d^3)/((b^2*x^2 - a^2)^2*a^4*b^4) + 3/16*(b^5*c^4 - 2*
a^2*b^3*c^2*d^2 + a^4*b*d^4)*log(abs(b*x + a))/(a^5*b^6) - 3/16*(b^5*c^4 -
 2*a^2*b^3*c^2*d^2 + a^4*b*d^4)*log(abs(b*x - a))/(a^5*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{\frac{2\,c\,d^3\,x^2}{b^2} - \frac{c\,d\,(a^2\,d^2-b^2\,c^2)}{b^4} + \frac{x^3\,(5\,a^4\,d^4+6\,a^2\,b^2\,c^2\,d^2-3\,b^4\,c^4)}{8\,a^4\,b^2} + \frac{x\,(-3\,a^4\,d^4+6\,a^2\,b^2\,c^2\,d^2+5\,b^4\,c^4)}{8\,a^2\,b^4}}{a^4-2\,a^2\,b^2\,x^2+b^4\,x^4} + \frac{3\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)\,(a\,d+b\,c)^2\,(a\,d-b\,c)^2}{8\,a^5\,b^5}$$

input
```
int((c + d*x)^4/(a^2 - b^2*x^2)^3,x)
```

output
```
((2*c*d^3*x^2)/b^2 - (c*d*(a^2*d^2 - b^2*c^2))/b^4 + (x^3*(5*a^4*d^4 - 3*b
^4*c^4 + 6*a^2*b^2*c^2*d^2))/(8*a^4*b^2) + (x*(5*b^4*c^4 - 3*a^4*d^4 + 6*a
^2*b^2*c^2*d^2))/(8*a^2*b^4))/(a^4 + b^4*x^4 - 2*a^2*b^2*x^2) + (3*atanh((
b*x)/a)*(a*d + b*c)^2*(a*d - b*c)^2)/(8*a^5*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.85

$$\int \frac{(c+dx)^4}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{3\log(-bx-a)\,a^4b^4c^4 + 3\log(-bx-a)\,b^8c^4x^4 - 3\log(-bx+a)\,a^4b^4c^4 - 3\log(-bx+a)\,b^8c^4x^4 - 6a^7b\,d}{}$$

input
```
int((d*x+c)^4/(-b^2*x^2+a^2)^3,x)
```

output
```
(3*log( - a - b*x)*a**8*d**4 - 6*log( - a - b*x)*a**6*b**2*c**2*d**2 - 6*l
og( - a - b*x)*a**6*b**2*d**4*x**2 + 3*log( - a - b*x)*a**4*b**4*c**4 + 12
*log( - a - b*x)*a**4*b**4*c**2*d**2*x**2 + 3*log( - a - b*x)*a**4*b**4*d*
*4*x**4 - 6*log( - a - b*x)*a**2*b**6*c**4*x**2 - 6*log( - a - b*x)*a**2*b
**6*c**2*d**2*x**4 + 3*log( - a - b*x)*b**8*c**4*x**4 - 3*log(a - b*x)*a**
8*d**4 + 6*log(a - b*x)*a**6*b**2*c**2*d**2 + 6*log(a - b*x)*a**6*b**2*d**
4*x**2 - 3*log(a - b*x)*a**4*b**4*c**4 - 12*log(a - b*x)*a**4*b**4*c**2*d*
*2*x**2 - 3*log(a - b*x)*a**4*b**4*d**4*x**4 + 6*log(a - b*x)*a**2*b**6*c*
*4*x**2 + 6*log(a - b*x)*a**2*b**6*c**2*d**2*x**4 - 3*log(a - b*x)*b**8*c*
*4*x**4 - 6*a**7*b*d**4*x + 16*a**5*b**3*c**3*d + 12*a**5*b**3*c**2*d**2*x
 + 10*a**5*b**3*d**4*x**3 + 10*a**3*b**5*c**4*x + 12*a**3*b**5*c**2*d**2*x
**3 + 16*a**3*b**5*c*d**3*x**4 - 6*a*b**7*c**4*x**3)/(16*a**5*b**5*(a**4 -
 2*a**2*b**2*x**2 + b**4*x**4))
```

3.45 $\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{a^2d\left(3c^2+\frac{a^2d^2}{b^2}\right)+c(b^2c^2+3a^2d^2)\,x}{4a^2b^2\left(a^2-b^2x^2\right)^2} - \frac{4a^4d^3-3b^2c(bc-ad)(bc+ad)x}{8a^4b^4\left(a^2-b^2x^2\right)} + \frac{3c\left(c^2-\frac{a^2d^2}{b^2}\right)\operatorname{arctanh}\left(\frac{bx}{a}\right)}{8a^5b}$$

output
```
1/4*(a^2*d*(3*c^2+a^2*d^2/b^2)+c*(3*a^2*d^2+b^2*c^2)*x)/a^2/b^2/(-b^2*x^2+
a^2)^2-1/8*(4*a^4*d^3-3*b^2*c*(-a*d+b*c)*(a*d+b*c)*x)/a^4/b^4/(-b^2*x^2+a^
2)+3/8*c*(c^2-a^2*d^2/b^2)*arctanh(b*x/a)/a^5/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{-2a^7d^3-3ab^6c^3x^3+a^3b^4cx(5c^2+3d^2x^2)+a^5b^2d(6c^2+3cdx+4d^2x^2)+3bc(b^2c^2-a^2d^2)\left(a^2-b^2x^2\right)^2}{8a^5b^4\left(a^2-b^2x^2\right)^2}$$

input
```
Integrate[(c + d*x)^3/(a^2 - b^2*x^2)^3,x]
```

output
```
(-2*a^7*d^3 - 3*a*b^6*c^3*x^3 + a^3*b^4*c*x*(5*c^2 + 3*d^2*x^2) + a^5*b^2*
d*(6*c^2 + 3*c*d*x + 4*d^2*x^2) + 3*b*c*(b^2*c^2 - a^2*d^2)*(a^2 - b^2*x^2
)^2*ArcTanh[(b*x)/a])/(8*a^5*b^4*(a^2 - b^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \; 477$$

$$\frac{\int \left(\frac{a^3(bc-ad)^3}{8b^3(a+bx)^3}+\frac{3a^2(bc+ad)(bc-ad)^2}{16b^3(a+bx)^2}+\frac{3a^2(bc+ad)^2(bc-ad)}{16b^3(a-bx)^2}+\frac{3a^2c\left(c^2-\frac{a^2d^2}{b^2}\right)}{8(a^2-b^2x^2)}+\frac{a^3(bc+ad)^3}{8b^3(a-bx)^3}\right)dx}{a^6}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{a^3(bc-ad)^3}{16b^4(a+bx)^2}+\frac{a^3(ad+bc)^3}{16b^4(a-bx)^2}+\frac{3ac\,\text{arctanh}\left(\frac{bx}{a}\right)\left(c^2-\frac{a^2d^2}{b^2}\right)}{8b}-\frac{3a^2(ad+bc)(bc-ad)^2}{16b^4(a+bx)}+\frac{3a^2(ad+bc)^2(bc-ad)}{16b^4(a-bx)}}{a^6}$$

input
```
Int[(c + d*x)^3/(a^2 - b^2*x^2)^3,x]
```

output
```
((a^3*(b*c + a*d)^3)/(16*b^4*(a - b*x)^2) + (3*a^2*(b*c - a*d)*(b*c + a*d)
^2)/(16*b^4*(a - b*x)) - (a^3*(b*c - a*d)^3)/(16*b^4*(a + b*x)^2) - (3*a^2
*(b*c - a*d)^2*(b*c + a*d))/(16*b^4*(a + b*x)) + (3*a*c*(c^2 - (a^2*d^2)/b
^2)*ArcTanh[(b*x)/a])/(8*b))/a^6
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

method	result
norman	$\frac{\frac{-a^2d^3+3b^2c^2d}{4b^4}+\frac{d^3x^2}{2b^2}+\frac{3c\left(a^2d^2-b^2c^2\right)x^3}{8a^4}+\frac{c\left(3a^2d^2+5b^2c^2\right)x}{8a^2b^2}}{\left(-b^2x^2+a^2\right)^2}+\frac{3c\left(a^2d^2-b^2c^2\right)\ln\left(-bx+a\right)}{16a^5b^3}-\frac{3c\left(a^2d^2-b^2c^2\right)\ln\left(bx+a\right)}{16a^5b^3}$
risch	$\frac{\frac{3c\left(a^2d^2-b^2c^2\right)x^3}{8a^4}+\frac{d^3x^2}{2b^2}+\frac{c\left(3a^2d^2+5b^2c^2\right)x}{8a^2b^2}-\frac{d\left(a^2d^2-3b^2c^2\right)}{4b^4}}{\left(-b^2x^2+a^2\right)^2}+\frac{3c\ln\left(bx-a\right)d^2}{16a^3b^3}-\frac{3c^3\ln\left(bx-a\right)}{16a^5b}-\frac{3c\ln\left(-bx-a\right)d^2}{16a^3b^3}+$ 3
default	$-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{16a^3b^4\left(bx+a\right)^2}-\frac{3a^3d^3-3a^2bc\,d^2-3a\,b^2c^2d+3b^3c^3}{16a^4b^4\left(bx+a\right)}-\frac{3c\left(a^2d^2-b^2c^2\right)\ln\left(bx+a\right)}{16a^5b^3}+\frac{-3a^3d^3-3a^2bc}{16a^4b}$
parallelrisch	$3\ln\left(bx-a\right)x^4a^2b^5c\,d^2-3\ln\left(bx-a\right)x^4b^7c^3-3\ln\left(bx+a\right)x^4a^2b^5c\,d^2+3\ln\left(bx+a\right)x^4b^7c^3-6\ln\left(bx-a\right)x^2a^4b^3c\,d^2+6\ln\left(bx-a\right)x^2a$

input
```
int((d*x+c)^3/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4*(-a^2*d^3+3*b^2*c^2*d)/b^4+1/2*d^3*x^2/b^2+3/8/a^4*c*(a^2*d^2-b^2*c^2
)*x^3+1/8*c*(3*a^2*d^2+5*b^2*c^2)/a^2/b^2*x)/(-b^2*x^2+a^2)^2+3/16*c*(a^2*
d^2-b^2*c^2)/a^5/b^3*ln(-b*x+a)-3/16*c*(a^2*d^2-b^2*c^2)/a^5/b^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.85

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{8\,a^5b^2d^3x^2 + 12\,a^5b^2c^2d - 4\,a^7d^3 - 6\,(ab^6c^3 - a^3b^4cd^2)x^3 + 2\,(5\,a^3b^4c^3 + 3\,a^5b^2cd^2)x + 3\,(a^4b^3c^3 - a^6bcd}{}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/16*(8*a^5*b^2*d^3*x^2 + 12*a^5*b^2*c^2*d - 4*a^7*d^3 - 6*(a*b^6*c^3 - a^
3*b^4*c*d^2)*x^3 + 2*(5*a^3*b^4*c^3 + 3*a^5*b^2*c*d^2)*x + 3*(a^4*b^3*c^3
- a^6*b*c*d^2 + (b^7*c^3 - a^2*b^5*c*d^2)*x^4 - 2*(a^2*b^5*c^3 - a^4*b^3*c
*d^2)*x^2)*log(b*x + a) - 3*(a^4*b^3*c^3 - a^6*b*c*d^2 + (b^7*c^3 - a^2*b^
5*c*d^2)*x^4 - 2*(a^2*b^5*c^3 - a^4*b^3*c*d^2)*x^2)*log(b*x - a))/(a^5*b^8
*x^4 - 2*a^7*b^6*x^2 + a^9*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx$$
$$= -\frac{2a^6d^3 - 6a^4b^2c^2d - 4a^4b^2d^3x^2 + x^3(-3a^2b^4cd^2 + 3b^6c^3) + x(-3a^4b^2cd^2 - 5a^2b^4c^3)}{8a^8b^4 - 16a^6b^6x^2 + 8a^4b^8x^4}$$
$$+ \frac{3c(ad-bc)\,(ad+bc)\log\left(-\frac{3ac(ad-bc)(ad+bc)}{b(3a^2cd^2-3b^2c^3)} + x\right)}{16a^5b^3}$$
$$- \frac{3c(ad-bc)\,(ad+bc)\log\left(\frac{3ac(ad-bc)(ad+bc)}{b(3a^2cd^2-3b^2c^3)} + x\right)}{16a^5b^3}$$

input
```
integrate((d*x+c)**3/(-b**2*x**2+a**2)**3,x)
```

output
```
-(2*a**6*d**3 - 6*a**4*b**2*c**2*d - 4*a**4*b**2*d**3*x**2 + x**3*(-3*a**2
*b**4*c*d**2 + 3*b**6*c**3) + x*(-3*a**4*b**2*c*d**2 - 5*a**2*b**4*c**3))/
(8*a**8*b**4 - 16*a**6*b**6*x**2 + 8*a**4*b**8*x**4) + 3*c*(a*d - b*c)*(a*
d + b*c)*log(-3*a*c*(a*d - b*c)*(a*d + b*c)/(b*(3*a**2*c*d**2 - 3*b**2*c**
3)) + x)/(16*a**5*b**3) - 3*c*(a*d - b*c)*(a*d + b*c)*log(3*a*c*(a*d - b*c
)*(a*d + b*c)/(b*(3*a**2*c*d**2 - 3*b**2*c**3)) + x)/(16*a**5*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{4\,a^4b^2d^3x^2+6\,a^4b^2c^2d-2\,a^6d^3-3\,(b^6c^3-a^2b^4cd^2)x^3+(5\,a^2b^4c^3+3\,a^4b^2cd^2)x}{8\,(a^4b^8x^4-2\,a^6b^6x^2+a^8b^4)} + \frac{3\,(b^2c^3-a^2cd^2)\log(bx+a)}{16\,a^5b^3} - \frac{3\,(b^2c^3-a^2cd^2)\log(bx-a)}{16\,a^5b^3}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/8*(4*a^4*b^2*d^3*x^2 + 6*a^4*b^2*c^2*d - 2*a^6*d^3 - 3*(b^6*c^3 - a^2*b^
4*c*d^2)*x^3 + (5*a^2*b^4*c^3 + 3*a^4*b^2*c*d^2)*x)/(a^4*b^8*x^4 - 2*a^6*b
^6*x^2 + a^8*b^4) + 3/16*(b^2*c^3 - a^2*c*d^2)*log(b*x + a)/(a^5*b^3) - 3/
16*(b^2*c^3 - a^2*c*d^2)*log(b*x - a)/(a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{3\,(b^3c^3-a^2bcd^2)\log(|bx+a|)}{16\,a^5b^4} - \frac{3\,(b^3c^3-a^2bcd^2)\log(|bx-a|)}{16\,a^5b^4} - \frac{3\,b^6c^3x^3-3\,a^2b^4cd^2x^3-4\,a^4b^2d^3x^2-5\,a^2b^4c^3x-3\,a^4b^2cd^2x-6\,a^4b^2c^2d+2\,a^6d^3}{8\,(b^2x^2-a^2)^2a^4b^4}$$

input
```
integrate((d*x+c)^3/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3/16*(b^3*c^3 - a^2*b*c*d^2)*log(abs(b*x + a))/(a^5*b^4) - 3/16*(b^3*c^3 -
 a^2*b*c*d^2)*log(abs(b*x - a))/(a^5*b^4) - 1/8*(3*b^6*c^3*x^3 - 3*a^2*b^4
*c*d^2*x^3 - 4*a^4*b^2*d^3*x^2 - 5*a^2*b^4*c^3*x - 3*a^4*b^2*c*d^2*x - 6*a
^4*b^2*c^2*d + 2*a^6*d^3)/((b^2*x^2 - a^2)^2*a^4*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{\frac{d^3\,x^2}{2\,b^2}-\frac{d\,(a^2\,d^2-3\,b^2\,c^2)}{4\,b^4}+\frac{3\,c\,x^3\,(a^2\,d^2-b^2\,c^2)}{8\,a^4}+\frac{c\,x\,(3\,a^2\,d^2+5\,b^2\,c^2)}{8\,a^2\,b^2}}{a^4-2\,a^2\,b^2\,x^2+b^4\,x^4} - \frac{2\,\mathrm{atanh}\left(\frac{16\,b\,x\,\left(\frac{3\,b^2\,c^3}{16}-\frac{3\,a^2\,c\,d^2}{16}\right)}{3\,a\,c\,(a^2\,d^2-b^2\,c^2)}\right)\left(\frac{3\,b^2\,c^3}{16}-\frac{3\,a^2\,c\,d^2}{16}\right)}{a^5\,b^3}$$

input
```
int((c + d*x)^3/(a^2 - b^2*x^2)^3,x)
```

output
```
((d^3*x^2)/(2*b^2) - (d*(a^2*d^2 - 3*b^2*c^2))/(4*b^4) + (3*c*x^3*(a^2*d^2
 - b^2*c^2))/(8*a^4) + (c*x*(3*a^2*d^2 + 5*b^2*c^2))/(8*a^2*b^2))/(a^4 + b
^4*x^4 - 2*a^2*b^2*x^2) - (2*atanh((16*b*x*((3*b^2*c^3)/16 - (3*a^2*c*d^2)
/16))/(3*a*c*(a^2*d^2 - b^2*c^2)))*((3*b^2*c^3)/16 - (3*a^2*c*d^2)/16))/(a
^5*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.29

$$\int \frac{(c+dx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{-3\log(-bx-a)\,a^6c\,d^2+3\log(-bx-a)\,a^4b^2c^3+6\log(-bx-a)\,a^4b^2c\,d^2x^2-6\log(-bx-a)\,a^2b^4c^3x^2}{}$$

input
```
int((d*x+c)^3/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 3*log( - a - b*x)*a**6*c*d**2 + 3*log( - a - b*x)*a**4*b**2*c**3 + 6*l
og( - a - b*x)*a**4*b**2*c*d**2*x**2 - 6*log( - a - b*x)*a**2*b**4*c**3*x*
*2 - 3*log( - a - b*x)*a**2*b**4*c*d**2*x**4 + 3*log( - a - b*x)*b**6*c**3
*x**4 + 3*log(a - b*x)*a**6*c*d**2 - 3*log(a - b*x)*a**4*b**2*c**3 - 6*log
(a - b*x)*a**4*b**2*c*d**2*x**2 + 6*log(a - b*x)*a**2*b**4*c**3*x**2 + 3*l
og(a - b*x)*a**2*b**4*c*d**2*x**4 - 3*log(a - b*x)*b**6*c**3*x**4 + 12*a**
5*b*c**2*d + 6*a**5*b*c*d**2*x + 10*a**3*b**3*c**3*x + 6*a**3*b**3*c*d**2*
x**3 + 4*a**3*b**3*d**3*x**4 - 6*a*b**5*c**3*x**3)/(16*a**5*b**3*(a**4 - 2
*a**2*b**2*x**2 + b**4*x**4))
```

3.46 $\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx = \frac{2a^2cd+(b^2c^2+a^2d^2)\,x}{4a^2b^2\,(a^2-b^2x^2)^2}+\frac{(3b^2c^2-a^2d^2)\,x}{8a^4b^2\,(a^2-b^2x^2)} + \frac{(3b^2c^2-a^2d^2)\,\text{arctanh}\left(\frac{bx}{a}\right)}{8a^5b^3}$$

output
```
1/4*(2*a^2*c*d+(a^2*d^2+b^2*c^2)*x)/a^2/b^2/(-b^2*x^2+a^2)^2+1/8*(-a^2*d^2
+3*b^2*c^2)*x/a^4/b^2/(-b^2*x^2+a^2)+1/8*(-a^2*d^2+3*b^2*c^2)*arctanh(b*x/
a)/a^5/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx = \frac{-3ab^5c^2x^3+a^5bd(4c+dx)+a^3b^3x(5c^2+d^2x^2)+(3b^2c^2-a^2d^2)\,(a^2-b^2x^2)^2\,\text{arctanh}\left(\frac{bx}{a}\right)}{8a^5b^3\,(a^2-b^2x^2)^2}$$

input
```
Integrate[(c + d*x)^2/(a^2 - b^2*x^2)^3,x]
```

output
```
(-3*a*b^5*c^2*x^3 + a^5*b*d*(4*c + d*x) + a^3*b^3*x*(5*c^2 + d^2*x^2) + (3
*b^2*c^2 - a^2*d^2)*(a^2 - b^2*x^2)^2*ArcTanh[(b*x)/a])/(8*a^5*b^3*(a^2 -
b^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3} \, dx$$

$$\Big\downarrow \; 477$$

$$\frac{\int \left(\frac{(bc+ad)^2a^3}{8b^2(a-bx)^3}+\frac{(bc-ad)^2a^3}{8b^2(a+bx)^3}+\frac{\left(3c^2-\frac{a^2d^2}{b^2}\right)a^2}{8(a^2-b^2x^2)}+\frac{(3bc-ad)(bc+ad)a^2}{16b^2(a-bx)^2}+\frac{(bc-ad)(3bc+ad)a^2}{16b^2(a+bx)^2}\right)dx}{a^6}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{a^3(ad+bc)^2}{16b^3(a-bx)^2}-\frac{a^3(bc-ad)^2}{16b^3(a+bx)^2}+\frac{a\text{arctanh}\left(\frac{bx}{a}\right)(3b^2c^2-a^2d^2)}{8b^3}+\frac{a^2(3bc-ad)(ad+bc)}{16b^3(a-bx)}-\frac{a^2(bc-ad)(ad+3bc)}{16b^3(a+bx)}}{a^6}$$

input
```
Int[(c + d*x)^2/(a^2 - b^2*x^2)^3,x]
```

output
```
((a^3*(b*c + a*d)^2)/(16*b^3*(a - b*x)^2) + (a^2*(3*b*c - a*d)*(b*c + a*d)
)/(16*b^3*(a - b*x)) - (a^3*(b*c - a*d)^2)/(16*b^3*(a + b*x)^2) - (a^2*(b*
c - a*d)*(3*b*c + a*d))/(16*b^3*(a + b*x)) + (a*(3*b^2*c^2 - a^2*d^2)*ArcT
anh[(b*x)/a])/(8*b^3))/a^6
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07

method	result
norman	$\frac{\frac{cd}{2b^{2}}+\frac{\left(a^{2}d^{2}-3b^{2}c^{2}\right)x^{3}}{8a^{4}}+\frac{\left(a^{2}d^{2}+5b^{2}c^{2}\right)x}{8a^{2}b^{2}}}{\left(-b^{2}x^{2}+a^{2}\right)^{2}}+\frac{\left(a^{2}d^{2}-3b^{2}c^{2}\right)\ln\left(-bx+a\right)}{16a^{5}b^{3}}-\frac{\left(a^{2}d^{2}-3b^{2}c^{2}\right)\ln\left(bx+a\right)}{16a^{5}b^{3}}$
risch	$\frac{\frac{cd}{2b^{2}}+\frac{\left(a^{2}d^{2}-3b^{2}c^{2}\right)x^{3}}{8a^{4}}+\frac{\left(a^{2}d^{2}+5b^{2}c^{2}\right)x}{8a^{2}b^{2}}}{\left(-b^{2}x^{2}+a^{2}\right)^{2}}-\frac{\ln\left(-bx-a\right)d^{2}}{16a^{3}b^{3}}+\frac{3\ln\left(-bx-a\right)c^{2}}{16a^{5}b}+\frac{\ln\left(bx-a\right)d^{2}}{16a^{3}b^{3}}-\frac{3\ln\left(bx-a\right)c^{2}}{16a^{5}b}$
default	$\frac{\left(-a^{2}d^{2}+3b^{2}c^{2}\right)\ln\left(bx+a\right)}{16a^{5}b^{3}}-\frac{-a^{2}d^{2}-2abcd+3b^{2}c^{2}}{16a^{4}b^{3}\left(bx+a\right)}-\frac{a^{2}d^{2}-2abcd+b^{2}c^{2}}{16a^{3}b^{3}\left(bx+a\right)^{2}}+\frac{\left(a^{2}d^{2}-3b^{2}c^{2}\right)\ln\left(-bx+a\right)}{16a^{5}b^{3}}-\frac{-a^{2}d^{2}-2abc\ldots}{16a^{3}b^{3}(-bx\ldots}$
parallelrisch	$\ln\left(bx-a\right)x^{4}a^{2}b^{5}d^{2}-3\ln\left(bx-a\right)x^{4}b^{7}c^{2}-\ln\left(bx+a\right)x^{4}a^{2}b^{5}d^{2}+3\ln\left(bx+a\right)x^{4}b^{7}c^{2}-2\ln\left(bx-a\right)x^{2}a^{4}b^{3}d^{2}+6\ln\left(bx-a\right)x^{2}a^{2}b^{5}c^{2}+\ldots$

input
```
int((d*x+c)^2/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c*d/b^2+1/8/a^4*(a^2*d^2-3*b^2*c^2)*x^3+1/8*(a^2*d^2+5*b^2*c^2)/a^2/b
^2*x)/(-b^2*x^2+a^2)^2+1/16*(a^2*d^2-3*b^2*c^2)/a^5/b^3*ln(-b*x+a)-1/16*(a
^2*d^2-3*b^2*c^2)/a^5/b^3*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(122) = 244$.

Time = 0.11 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{8\,a^5bcd - 2\,(3\,ab^5c^2 - a^3b^3d^2)x^3 + 2\,(5\,a^3b^3c^2 + a^5bd^2)x + (3\,a^4b^2c^2 - a^6d^2 + (3\,b^6c^2 - a^2b^4d^2)x^4 - 2\,(3\,a}{16\,(a^5b^7x^4}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/16*(8*a^5*b*c*d - 2*(3*a*b^5*c^2 - a^3*b^3*d^2)*x^3 + 2*(5*a^3*b^3*c^2 +
 a^5*b*d^2)*x + (3*a^4*b^2*c^2 - a^6*d^2 + (3*b^6*c^2 - a^2*b^4*d^2)*x^4 -
 2*(3*a^2*b^4*c^2 - a^4*b^2*d^2)*x^2)*log(b*x + a) - (3*a^4*b^2*c^2 - a^6*
d^2 + (3*b^6*c^2 - a^2*b^4*d^2)*x^4 - 2*(3*a^2*b^4*c^2 - a^4*b^2*d^2)*x^2)
*log(b*x - a))/(a^5*b^7*x^4 - 2*a^7*b^5*x^2 + a^9*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx = -\frac{-4a^4cd + x^3(-a^2b^2d^2 + 3b^4c^2) + x(-a^4d^2 - 5a^2b^2c^2)}{8a^8b^2 - 16a^6b^4x^2 + 8a^4b^6x^4} + \frac{(a^2d^2 - 3b^2c^2)\log\left(-\frac{a}{b} + x\right)}{16a^5b^3} - \frac{(a^2d^2 - 3b^2c^2)\log\left(\frac{a}{b} + x\right)}{16a^5b^3}$$

input
```
integrate((d*x+c)**2/(-b**2*x**2+a**2)**3,x)
```

output
```
-(-4*a**4*c*d + x**3*(-a**2*b**2*d**2 + 3*b**4*c**2) + x*(-a**4*d**2 - 5*a
**2*b**2*c**2))/(8*a**8*b**2 - 16*a**6*b**4*x**2 + 8*a**4*b**6*x**4) + (a*
*2*d**2 - 3*b**2*c**2)*log(-a/b + x)/(16*a**5*b**3) - (a**2*d**2 - 3*b**2*
c**2)*log(a/b + x)/(16*a**5*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx = \frac{4\,a^4cd-(3\,b^4c^2-a^2b^2d^2)x^3+(5\,a^2b^2c^2+a^4d^2)x}{8\,(a^4b^6x^4-2\,a^6b^4x^2+a^8b^2)} + \frac{(3\,b^2c^2-a^2d^2)\log(bx+a)}{16\,a^5b^3} - \frac{(3\,b^2c^2-a^2d^2)\log(bx-a)}{16\,a^5b^3}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/8*(4*a^4*c*d - (3*b^4*c^2 - a^2*b^2*d^2)*x^3 + (5*a^2*b^2*c^2 + a^4*d^2)
*x)/(a^4*b^6*x^4 - 2*a^6*b^4*x^2 + a^8*b^2) + 1/16*(3*b^2*c^2 - a^2*d^2)*l
og(b*x + a)/(a^5*b^3) - 1/16*(3*b^2*c^2 - a^2*d^2)*log(b*x - a)/(a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx = -\frac{3\,b^4c^2x^3-a^2b^2d^2x^3-5\,a^2b^2c^2x-a^4d^2x-4\,a^4cd}{8\,(b^2x^2-a^2)^2a^4b^2} + \frac{(3\,b^3c^2-a^2bd^2)\log(|bx+a|)}{16\,a^5b^4} - \frac{(3\,b^3c^2-a^2bd^2)\log(|bx-a|)}{16\,a^5b^4}$$

input
```
integrate((d*x+c)^2/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*b^4*c^2*x^3 - a^2*b^2*d^2*x^3 - 5*a^2*b^2*c^2*x - a^4*d^2*x - 4*a^
4*c*d)/((b^2*x^2 - a^2)^2*a^4*b^2) + 1/16*(3*b^3*c^2 - a^2*b*d^2)*log(abs(
b*x + a))/(a^5*b^4) - 1/16*(3*b^3*c^2 - a^2*b*d^2)*log(abs(b*x - a))/(a^5*
b^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx = \frac{\frac{x^3\,(a^2\,d^2-3\,b^2\,c^2)}{8\,a^4}+\frac{c\,d}{2\,b^2}+\frac{x\,(a^2\,d^2+5\,b^2\,c^2)}{8\,a^2\,b^2}}{a^4-2\,a^2\,b^2\,x^2+b^4\,x^4}-\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)\,(a^2\,d^2-3\,b^2\,c^2)}{8\,a^5\,b^3}$$

input
```
int((c + d*x)^2/(a^2 - b^2*x^2)^3,x)
```

output
```
((x^3*(a^2*d^2 - 3*b^2*c^2))/(8*a^4) + (c*d)/(2*b^2) + (x*(a^2*d^2 + 5*b^2
*c^2))/(8*a^2*b^2))/(a^4 + b^4*x^4 - 2*a^2*b^2*x^2) - (atanh((b*x)/a)*(a^2
*d^2 - 3*b^2*c^2))/(8*a^5*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.60

$$\int \frac{(c+dx)^2}{(a^2-b^2x^2)^3}\,dx = \frac{-\log(-bx-a)\,a^6d^2+3\log(-bx-a)\,a^4b^2c^2+2\log(-bx-a)\,a^4b^2d^2x^2-6\log(-bx-a)\,a^2b^4c^2x^2-\log}{}$$

input
```
int((d*x+c)^2/(-b^2*x^2+a^2)^3,x)
```

output
```
( - log( - a - b*x)*a**6*d**2 + 3*log( - a - b*x)*a**4*b**2*c**2 + 2*log(
- a - b*x)*a**4*b**2*d**2*x**2 - 6*log( - a - b*x)*a**2*b**4*c**2*x**2 - l
og( - a - b*x)*a**2*b**4*d**2*x**4 + 3*log( - a - b*x)*b**6*c**2*x**4 + lo
g(a - b*x)*a**6*d**2 - 3*log(a - b*x)*a**4*b**2*c**2 - 2*log(a - b*x)*a**4
*b**2*d**2*x**2 + 6*log(a - b*x)*a**2*b**4*c**2*x**2 + log(a - b*x)*a**2*b
**4*d**2*x**4 - 3*log(a - b*x)*b**6*c**2*x**4 + 8*a**5*b*c*d + 2*a**5*b*d*
*2*x + 10*a**3*b**3*c**2*x + 2*a**3*b**3*d**2*x**3 - 6*a*b**5*c**2*x**3)/(
16*a**5*b**3*(a**4 - 2*a**2*b**2*x**2 + b**4*x**4))
```

3.47 $\int \frac{c+dx}{\left(a^2-b^2x^2\right)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{c+dx}{\left(a^2-b^2x^2\right)^3} dx = \frac{a^2d+b^2cx}{4a^2b^2\left(a^2-b^2x^2\right)^2}+\frac{3cx}{8a^4\left(a^2-b^2x^2\right)}+\frac{3c\text{arctanh}\left(\frac{bx}{a}\right)}{8a^5b}$$

output
```
1/4*(b^2*c*x+a^2*d)/a^2/b^2/(-b^2*x^2+a^2)^2+3/8*c*x/a^4/(-b^2*x^2+a^2)+3/
8*c*arctanh(b*x/a)/a^5/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{\left(a^2-b^2x^2\right)^3} dx = \frac{2a^5d+5a^3b^2cx-3ab^4cx^3+3bc\left(a^2-b^2x^2\right)^2 \text{arctanh}\left(\frac{bx}{a}\right)}{8a^5b^2\left(a^2-b^2x^2\right)^2}$$

input
```
Integrate[(c + d*x)/(a^2 - b^2*x^2)^3,x]
```

output
```
(2*a^5*d + 5*a^3*b^2*c*x - 3*a*b^4*c*x^3 + 3*b*c*(a^2 - b^2*x^2)^2*ArcTanh
[(b*x)/a])/(8*a^5*b^2*(a^2 - b^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {454, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\left(a^2-b^2x^2\right)^3}dx$$

$$\Big\downarrow \text{454}$$

$$\frac{3c\int \frac{1}{\left(a^2-b^2x^2\right)^2}dx}{4a^2}+\frac{a^2d+b^2cx}{4a^2b^2\left(a^2-b^2x^2\right)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3c\left(\frac{\int \frac{1}{a^2-b^2x^2}dx}{2a^2}+\frac{x}{2a^2\left(a^2-b^2x^2\right)}\right)}{4a^2}+\frac{a^2d+b^2cx}{4a^2b^2\left(a^2-b^2x^2\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{a^2d+b^2cx}{4a^2b^2\left(a^2-b^2x^2\right)^2}+\frac{3c\left(\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}+\frac{x}{2a^2\left(a^2-b^2x^2\right)}\right)}{4a^2}$$

input
```
Int[(c + d*x)/(a^2 - b^2*x^2)^3,x]
```

output
```
(a^2*d + b^2*c*x)/(4*a^2*b^2*(a^2 - b^2*x^2)^2) + (3*c*(x/(2*a^2*(a^2 - b^
2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b)))/(4*a^2)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\frac{d}{4b^{2}}+\frac{5cx}{8a^{2}}-\frac{3b^{2}c\,x^{3}}{8a^{4}}}{\left(-b^{2}x^{2}+a^{2}\right)^{2}}-\frac{3c\ln\left(-bx+a\right)}{16a^{5}b}+\frac{3c\ln\left(bx+a\right)}{16a^{5}b}$
risch	$\frac{\frac{d}{4b^{2}}+\frac{5cx}{8a^{2}}-\frac{3b^{2}c\,x^{3}}{8a^{4}}}{\left(-b^{2}x^{2}+a^{2}\right)^{2}}-\frac{3c\ln\left(-bx+a\right)}{16a^{5}b}+\frac{3c\ln\left(bx+a\right)}{16a^{5}b}$
default	$\frac{3c\ln\left(bx+a\right)}{16a^{5}b}-\frac{-ad+bc}{16a^{3}b^{2}\left(bx+a\right)^{2}}-\frac{-ad+3bc}{16a^{4}b^{2}\left(bx+a\right)}-\frac{3c\ln\left(-bx+a\right)}{16a^{5}b}-\frac{-ad-bc}{16a^{3}b^{2}\left(-bx+a\right)^{2}}+\frac{ad+3bc}{16a^{4}b^{2}\left(-bx+a\right)}$
parallelrisch	$-\frac{3\ln\left(bx-a\right)x^{4}b^{7}c-3\ln\left(bx+a\right)x^{4}b^{7}c-6\ln\left(bx-a\right)x^{2}a^{2}b^{5}c+6\ln\left(bx+a\right)x^{2}a^{2}b^{5}c+6x^{3}a\,b^{6}c+3\ln\left(bx-a\right)a^{4}b^{3}c-3\ln\left(bx+a\right)a^{4}b^{3}}{16a^{5}b^{4}\left(b^{2}x^{2}-a^{2}\right)^{2}}$

input
```
int((d*x+c)/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4*d/b^2+5/8*c/a^2*x-3/8/a^4*b^2*c*x^3)/(-b^2*x^2+a^2)^2-3/16*c/a^5/b*ln
(-b*x+a)+3/16*c/a^5/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\int \frac{c+dx}{\left(a^2-b^2x^2\right)^3}\,dx = -\frac{6\,ab^4cx^3 - 10\,a^3b^2cx - 4\,a^5d - 3\,(b^5cx^4 - 2\,a^2b^3cx^2 + a^4bc)\log\left(bx+a\right) + 3\,(b^5cx^4 - 2\,a^2b^3cx^2 + a^4bc}{16\,(a^5b^6x^4 - 2\,a^7b^4x^2 + a^9b^2)}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*a*b^4*c*x^3 - 10*a^3*b^2*c*x - 4*a^5*d - 3*(b^5*c*x^4 - 2*a^2*b^3
*c*x^2 + a^4*b*c)*log(b*x + a) + 3*(b^5*c*x^4 - 2*a^2*b^3*c*x^2 + a^4*b*c)
*log(b*x - a))/(a^5*b^6*x^4 - 2*a^7*b^4*x^2 + a^9*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{c+dx}{\left(a^2-b^2x^2\right)^3}\,dx = -\frac{-2a^4d-5a^2b^2cx+3b^4cx^3}{8a^8b^2-16a^6b^4x^2+8a^4b^6x^4} - \frac{c\left(\frac{3\log\left(-\frac{a}{b}+x\right)}{16}-\frac{3\log\left(\frac{a}{b}+x\right)}{16}\right)}{a^5b}$$

input
```
integrate((d*x+c)/(-b**2*x**2+a**2)**3,x)
```

output
```
-(-2*a**4*d - 5*a**2*b**2*c*x + 3*b**4*c*x**3)/(8*a**8*b**2 - 16*a**6*b**4
*x**2 + 8*a**4*b**6*x**4) - c*(3*log(-a/b + x)/16 - 3*log(a/b + x)/16)/(a*
*5*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{(a^2-b^2x^2)^3}\,dx = -\frac{3\,b^4cx^3-5\,a^2b^2cx-2\,a^4d}{8\,(a^4b^6x^4-2\,a^6b^4x^2+a^8b^2)}+\frac{3\,c\log\left(bx+a\right)}{16\,a^5b}-\frac{3\,c\log\left(bx-a\right)}{16\,a^5b}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b^4*c*x^3 - 5*a^2*b^2*c*x - 2*a^4*d)/(a^4*b^6*x^4 - 2*a^6*b^4*x^2
+ a^8*b^2) + 3/16*c*log(b*x + a)/(a^5*b) - 3/16*c*log(b*x - a)/(a^5*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \frac{c+dx}{(a^2-b^2x^2)^3}\,dx = \frac{3\,c\log\left(|bx+a|\right)}{16\,a^5b}-\frac{3\,c\log\left(|bx-a|\right)}{16\,a^5b}-\frac{3\,b^4cx^3-5\,a^2b^2cx-2\,a^4d}{8\,(b^2x^2-a^2)^2a^4b^2}$$

input
```
integrate((d*x+c)/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3/16*c*log(abs(b*x + a))/(a^5*b) - 3/16*c*log(abs(b*x - a))/(a^5*b) - 1/8*
(3*b^4*c*x^3 - 5*a^2*b^2*c*x - 2*a^4*d)/((b^2*x^2 - a^2)^2*a^4*b^2)
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{c+dx}{(a^2-b^2x^2)^3}\,dx = \frac{\frac{d}{4\,b^2}+\frac{5\,c\,x}{8\,a^2}-\frac{3\,b^2\,c\,x^3}{8\,a^4}}{a^4-2\,a^2\,b^2\,x^2+b^4\,x^4}+\frac{3\,c\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{8\,a^5\,b}$$

input
```
int((c + d*x)/(a^2 - b^2*x^2)^3,x)
```

output
```
(d/(4*b^2) + (5*c*x)/(8*a^2) - (3*b^2*c*x^3)/(8*a^4))/(a^4 + b^4*x^4 - 2*a
^2*b^2*x^2) + (3*c*atanh((b*x)/a))/(8*a^5*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.09

$$\int \frac{c+dx}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{3\log(-bx-a)\,a^4bc-6\log(-bx-a)\,a^2b^3c\,x^2+3\log(-bx-a)\,b^5c\,x^4-3\log(-bx+a)\,a^4bc+6\log(-bx}{16a^5b^2\,(b^4x^4-2a^2b^2x^2+a^4)}$$

input
```
int((d*x+c)/(-b^2*x^2+a^2)^3,x)
```

output
```
(3*log( - a - b*x)*a**4*b*c - 6*log( - a - b*x)*a**2*b**3*c*x**2 + 3*log(
- a - b*x)*b**5*c*x**4 - 3*log(a - b*x)*a**4*b*c + 6*log(a - b*x)*a**2*b**
3*c*x**2 - 3*log(a - b*x)*b**5*c*x**4 + 4*a**5*d + 10*a**3*b**2*c*x - 6*a*
b**4*c*x**3)/(16*a**5*b**2*(a**4 - 2*a**2*b**2*x**2 + b**4*x**4))
```

3.48 $\int \frac{1}{(a^2-b^2x^2)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{(a^2-b^2x^2)^3} dx = \frac{x}{4a^2(a^2-b^2x^2)^2} + \frac{3x}{8a^4(a^2-b^2x^2)} + \frac{3\text{arctanh}\left(\frac{bx}{a}\right)}{8a^5b}$$

output
```
1/4*x/a^2/(-b^2*x^2+a^2)^2+3/8*x/a^4/(-b^2*x^2+a^2)+3/8*arctanh(b*x/a)/a^5
/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a^2-b^2x^2)^3} dx = \frac{x}{4a^2(-a^2+b^2x^2)^2} - \frac{3x}{8a^4(-a^2+b^2x^2)} + \frac{3\text{arctanh}\left(\frac{bx}{a}\right)}{8a^5b}$$

input
```
Integrate[(a^2 - b^2*x^2)^(-3),x]
```

output
```
x/(4*a^2*(-a^2 + b^2*x^2)^2) - (3*x)/(8*a^4*(-a^2 + b^2*x^2)) + (3*ArcTanh
[(b*x)/a])/(8*a^5*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)^3}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{1}{(a^2-b^2x^2)^2}dx}{4a^2}+\frac{x}{4a^2(a^2-b^2x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{\int \frac{1}{a^2-b^2x^2}dx}{2a^2}+\frac{x}{2a^2(a^2-b^2x^2)}\right)}{4a^2}+\frac{x}{4a^2(a^2-b^2x^2)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{4a^2(a^2-b^2x^2)^2}+\frac{3\left(\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}+\frac{x}{2a^2(a^2-b^2x^2)}\right)}{4a^2}$$

input
```
Int[(a^2 - b^2*x^2)^(-3),x]
```

output
```
x/(4*a^2*(a^2 - b^2*x^2)^2) + (3*(x/(2*a^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x
)/a]/(2*a^3*b)))/(4*a^2)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

method	result
norman	$\frac{\frac{5x}{8a^2}-\frac{3b^2x^3}{8a^4}}{(-b^2x^2+a^2)^2}-\frac{3\ln(-bx+a)}{16a^5b}+\frac{3\ln(bx+a)}{16a^5b}$
risch	$\frac{\frac{5x}{8a^2}-\frac{3b^2x^3}{8a^4}}{(-b^2x^2+a^2)^2}-\frac{3\ln(-bx+a)}{16a^5b}+\frac{3\ln(bx+a)}{16a^5b}$
default	$\frac{3\ln(bx+a)}{16a^5b}-\frac{3}{16a^4b(bx+a)}-\frac{1}{16a^3b(bx+a)^2}-\frac{3\ln(-bx+a)}{16a^5b}+\frac{3}{16a^4b(-bx+a)}+\frac{1}{16b\,a^3(-bx+a)^2}$
parallelrisch	$-\frac{3\ln(bx-a)x^4b^7-3\ln(bx+a)x^4b^7-6\ln(bx-a)x^2a^2b^5+6\ln(bx+a)x^2a^2b^5+6x^3a\,b^6+3\ln(bx-a)a^4b^3-3\ln(bx+a)a^4b^3-10x}{16a^5b^4(b^2x^2-a^2)^2}$

input
```
int(1/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/8/a^2*x-3/8*b^2/a^4*x^3)/(-b^2*x^2+a^2)^2-3/16/a^5/b*ln(-b*x+a)+3/16/a^
5/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{1}{(a^2 - b^2x^2)^3} \, dx = -\frac{6\,ab^3x^3 - 10\,a^3bx - 3\,(b^4x^4 - 2\,a^2b^2x^2 + a^4)\log(bx + a) + 3\,(b^4x^4 - 2\,a^2b^2x^2 + a^4)\log(bx - a)}{16\,(a^5b^5x^4 - 2\,a^7b^3x^2 + a^9b)}$$

input
```
integrate(1/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*a*b^3*x^3 - 10*a^3*b*x - 3*(b^4*x^4 - 2*a^2*b^2*x^2 + a^4)*log(b*
x + a) + 3*(b^4*x^4 - 2*a^2*b^2*x^2 + a^4)*log(b*x - a))/(a^5*b^5*x^4 - 2*
a^7*b^3*x^2 + a^9*b)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a^2 - b^2x^2)^3} \, dx = -\frac{-5a^2x + 3b^2x^3}{8a^8 - 16a^6b^2x^2 + 8a^4b^4x^4} - \frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16} - \frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^5b}$$

input
```
integrate(1/(-b**2*x**2+a**2)**3,x)
```

output
```
-(-5*a**2*x + 3*b**2*x**3)/(8*a**8 - 16*a**6*b**2*x**2 + 8*a**4*b**4*x**4)
 - (3*log(-a/b + x)/16 - 3*log(a/b + x)/16)/(a**5*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a^2-b^2x^2)^3}\,dx = -\frac{3\,b^2x^3-5\,a^2x}{8\,(a^4b^4x^4-2\,a^6b^2x^2+a^8)}+\frac{3\,\log\left(bx+a\right)}{16\,a^5b}-\frac{3\,\log\left(bx-a\right)}{16\,a^5b}$$

input
```
integrate(1/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b^2*x^3 - 5*a^2*x)/(a^4*b^4*x^4 - 2*a^6*b^2*x^2 + a^8) + 3/16*log(
b*x + a)/(a^5*b) - 3/16*log(b*x - a)/(a^5*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a^2-b^2x^2)^3}\,dx = -\frac{3\,b^2x^3-5\,a^2x}{8\,(b^2x^2-a^2)^2a^4}+\frac{3\,\log\left(|bx+a|\right)}{16\,a^5b}-\frac{3\,\log\left(|bx-a|\right)}{16\,a^5b}$$

input
```
integrate(1/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*b^2*x^3 - 5*a^2*x)/((b^2*x^2 - a^2)^2*a^4) + 3/16*log(abs(b*x + a)
)/(a^5*b) - 3/16*log(abs(b*x - a))/(a^5*b)
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a^2-b^2x^2)^3}\,dx = \frac{\frac{5\,x}{8\,a^2}-\frac{3\,b^2\,x^3}{8\,a^4}}{a^4-2\,a^2\,b^2\,x^2+b^4\,x^4}+\frac{3\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{8\,a^5\,b}$$

input
```
int(1/(a^2 - b^2*x^2)^3,x)
```

output
```
((5*x)/(8*a^2) - (3*b^2*x^3)/(8*a^4))/(a^4 + b^4*x^4 - 2*a^2*b^2*x^2) + (3
*atanh((b*x)/a))/(8*a^5*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.34

$$\int \frac{1}{(a^2-b^2x^2)^3}\,dx = \frac{3\log(-bx-a)\,a^4-6\log(-bx-a)\,a^2b^2x^2+3\log(-bx-a)\,b^4x^4-3\log(-bx+a)\,a^4+6\log(-bx+a)\,a}{16a^5b\,(b^4x^4-2a^2b^2x^2+a^4)}$$

input

```
int(1/(-b^2*x^2+a^2)^3,x)
```

output

```
(3*log( - a - b*x)*a**4 - 6*log( - a - b*x)*a**2*b**2*x**2 + 3*log( - a -
b*x)*b**4*x**4 - 3*log(a - b*x)*a**4 + 6*log(a - b*x)*a**2*b**2*x**2 - 3*l
og(a - b*x)*b**4*x**4 + 10*a**3*b*x - 6*a*b**3*x**3)/(16*a**5*b*(a**4 - 2*
a**2*b**2*x**2 + b**4*x**4))
```

3.49 $\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 236

$$\begin{aligned}\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^3}\,dx &= \frac{1}{16a^3(bc+ad)(a-bx)^2}+\frac{3bc+5ad}{16a^4(bc+ad)^2(a-bx)} \\ &\quad-\frac{1}{16a^3(bc-ad)(a+bx)^2}-\frac{3bc-5ad}{16a^4(bc-ad)^2(a+bx)} \\ &\quad-\frac{(3b^2c^2+9abcd+8a^2d^2)\log(a-bx)}{16a^5(bc+ad)^3} \\ &\quad+\frac{(3b^2c^2-9abcd+8a^2d^2)\log(a+bx)}{16a^5(bc-ad)^3}-\frac{d^5\log(c+dx)}{\left(b^2c^2-a^2d^2\right)^3}\end{aligned}$$

output
```
1/16/a^3/(a*d+b*c)/(-b*x+a)^2+1/16*(5*a*d+3*b*c)/a^4/(a*d+b*c)^2/(-b*x+a)-
1/16/a^3/(-a*d+b*c)/(b*x+a)^2-1/16*(-5*a*d+3*b*c)/a^4/(-a*d+b*c)^2/(b*x+a)
-1/16*(8*a^2*d^2+9*a*b*c*d+3*b^2*c^2)*ln(-b*x+a)/a^5/(a*d+b*c)^3+1/16*(8*a
^2*d^2-9*a*b*c*d+3*b^2*c^2)*ln(b*x+a)/a^5/(-a*d+b*c)^3-d^5*ln(d*x+c)/(-a^2
*d^2+b^2*c^2)^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89

$$\int \frac{1}{(c+dx)\,(a^2-b^2x^2)^3}\,dx$$
$$= \frac{\frac{2(b^2c^2-a^2d^2)^2(a^2d-b^2cx)}{(a^3-ab^2x^2)^2} + \frac{(-b^2c^2+a^2d^2)(4a^4d^3+3b^4c^3x-7a^2b^2cd^2x)}{a^6-a^4b^2x^2} - \frac{b(3b^4c^5-10a^2b^2c^3d^2+15a^4cd^4)\operatorname{arctanh}\left(\frac{bx}{a}\right)}{a^5} + 8d^5\log}{8\,(-b^2c^2+a^2d^2)^3}$$

input
```
Integrate[1/((c + d*x)*(a^2 - b^2*x^2)^3),x]
```

output
```
((2*(b^2*c^2 - a^2*d^2)^2*(a^2*d - b^2*c*x))/(a^3 - a*b^2*x^2)^2 + ((-(b^2
*c^2) + a^2*d^2)*(4*a^4*d^3 + 3*b^4*c^3*x - 7*a^2*b^2*c*d^2*x))/(a^6 - a^4
*b^2*x^2) - (b*(3*b^4*c^5 - 10*a^2*b^2*c^3*d^2 + 15*a^4*c*d^4)*ArcTanh[(b*
x)/a])/a^5 + 8*d^5*Log[c + d*x] - 4*d^5*Log[a^2 - b^2*x^2])/(8*(-(b^2*c^2)
 + a^2*d^2)^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)^3\,(c+dx)}\,dx$$

$$\Big\downarrow \; 477$$

$$\frac{\int \left(-\frac{a^6d^6}{(b^2c^2-a^2d^2)^3(c+dx)} + \frac{ab(3b^2c^2+9abdc+8a^2d^2)}{16(bc+ad)^3(a-bx)} + \frac{ab(3b^2c^2-9abdc+8a^2d^2)}{16(bc-ad)^3(a+bx)} + \frac{a^2b(3bc+5ad)}{16(bc+ad)^2(a-bx)^2} + \frac{a^2b(3bc-5ad)}{16(bc-ad)^2(a+bx)^2} + \frac{}{8(bc}\right.}{a^6}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{a^3}{16(a-bx)^2(ad+bc)}-\frac{a^3}{16(a+bx)^2(bc-ad)}-\frac{a(8a^2d^2+9abcd+3b^2c^2)\log(a-bx)}{16(ad+bc)^3}+\frac{a(8a^2d^2-9abcd+3b^2c^2)\log(a+bx)}{16(bc-ad)^3}+\frac{a^2(5ad+3bc)}{16(a-bx)(ad+bc)^2}\cdots}{a^6}$$

input
```
Int[1/((c + d*x)*(a^2 - b^2*x^2)^3),x]
```

output
```
(a^3/(16*(b*c + a*d)*(a - b*x)^2) + (a^2*(3*b*c + 5*a*d))/(16*(b*c + a*d)^
2*(a - b*x)) - a^3/(16*(b*c - a*d)*(a + b*x)^2) - (a^2*(3*b*c - 5*a*d))/(1
6*(b*c - a*d)^2*(a + b*x)) - (a*(3*b^2*c^2 + 9*a*b*c*d + 8*a^2*d^2)*Log[a
- b*x])/(16*(b*c + a*d)^3) + (a*(3*b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*Log[a
+ b*x])/(16*(b*c - a*d)^3) - (a^6*d^5*Log[c + d*x])/(b^2*c^2 - a^2*d^2)^3)
/a^6
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95

method	result
default	$\frac{1}{16(ad-bc)a^3(bx+a)^2}+\frac{5ad-3bc}{16(ad-bc)^2a^4(bx+a)}-\frac{(8a^2d^2-9abcd+3b^2c^2)\ln(bx+a)}{16(ad-bc)^3a^5}+\frac{d^5\ln(dx+c)}{(ad-bc)^3(ad+bc)^3}+\frac{\cdots}{16a^3(ad+b\cdots}$
norman	$\frac{\frac{3b^4d^3a^2-b^6c^2d}{4b^4(a^4d^4-2a^2b^2c^2d^2+c^4b^4)}-\frac{b^2d^3x^2}{2(a^4d^4-2a^2b^2c^2d^2+c^4b^4)}-\frac{cb^2(9a^2d^2-5b^2c^2)x}{8a^2(a^4d^4-2a^2b^2c^2d^2+c^4b^4)}+\frac{cb^4(7a^2d^2-3b^2c^2)x^3}{8(a^4d^4-2a^2b^2c^2d^2+c^4b^4)a^4}}{(-b^2x^2+a^2)^2}+\frac{\cdots}{a^6d^6\cdots}$
risch	$\frac{\frac{cb^4(7a^2d^2-3b^2c^2)x^3}{8(a^4d^4-2a^2b^2c^2d^2+c^4b^4)a^4}-\frac{b^2d^3x^2}{2(a^4d^4-2a^2b^2c^2d^2+c^4b^4)}-\frac{cb^2(9a^2d^2-5b^2c^2)x}{8a^2(a^4d^4-2a^2b^2c^2d^2+c^4b^4)}+\frac{d(3a^2d^2-b^2c^2)}{4a^4d^4-8a^2b^2c^2d^2+4c^4b^4}}{(-b^2x^2+a^2)^2}+\frac{\cdots}{a^6d^6-3\cdots}$
parallelrisch	$-\frac{10\ln(bx-a)x^4a^2b^{11}c^3d^2+15\ln(bx+a)x^4a^4b^9cd^4-10\ln(bx+a)x^4a^2b^{11}c^3d^2+30\ln(bx-a)x^2a^6b^7cd^4-20\ln(bx-a)x^2a^4b^9\cdots}{\cdots}$

input
```
int(1/(d*x+c)/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/(a*d-b*c)/a^3/(b*x+a)^2+1/16*(5*a*d-3*b*c)/(a*d-b*c)^2/a^4/(b*x+a)-1/
16*(8*a^2*d^2-9*a*b*c*d+3*b^2*c^2)/(a*d-b*c)^3/a^5*ln(b*x+a)+d^5/(a*d-b*c)
^3/(a*d+b*c)^3*ln(d*x+c)+1/16/a^3/(a*d+b*c)/(-b*x+a)^2+1/16*(5*a*d+3*b*c)/
a^4/(a*d+b*c)^2/(-b*x+a)-1/16*(8*a^2*d^2+9*a*b*c*d+3*b^2*c^2)*ln(-b*x+a)/a
^5/(a*d+b*c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(226) = 452$.

Time = 8.76 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.81

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^3}\,dx = -\frac{4\,a^5b^4c^4d-16\,a^7b^2c^2d^3+12\,a^9d^5+2\,(3\,ab^8c^5-10\,a^3b^6c^3d^2+7\,a^5b^4cd^4)x^3+8\,(a^5b^4c^2d^3-a^7b^2d^5)x^2}{}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(4*a^5*b^4*c^4*d - 16*a^7*b^2*c^2*d^3 + 12*a^9*d^5 + 2*(3*a*b^8*c^5
- 10*a^3*b^6*c^3*d^2 + 7*a^5*b^4*c*d^4)*x^3 + 8*(a^5*b^4*c^2*d^3 - a^7*b^2
*d^5)*x^2 - 2*(5*a^3*b^6*c^5 - 14*a^5*b^4*c^3*d^2 + 9*a^7*b^2*c*d^4)*x - (
3*a^4*b^5*c^5 - 10*a^6*b^3*c^3*d^2 + 15*a^8*b*c*d^4 + 8*a^9*d^5 + (3*b^9*c
^5 - 10*a^2*b^7*c^3*d^2 + 15*a^4*b^5*c*d^4 + 8*a^5*b^4*d^5)*x^4 - 2*(3*a^2
*b^7*c^5 - 10*a^4*b^5*c^3*d^2 + 15*a^6*b^3*c*d^4 + 8*a^7*b^2*d^5)*x^2)*log
(b*x + a) + (3*a^4*b^5*c^5 - 10*a^6*b^3*c^3*d^2 + 15*a^8*b*c*d^4 - 8*a^9*d
^5 + (3*b^9*c^5 - 10*a^2*b^7*c^3*d^2 + 15*a^4*b^5*c*d^4 - 8*a^5*b^4*d^5)*x
^4 - 2*(3*a^2*b^7*c^5 - 10*a^4*b^5*c^3*d^2 + 15*a^6*b^3*c*d^4 - 8*a^7*b^2*
d^5)*x^2)*log(b*x - a) + 16*(a^5*b^4*d^5*x^4 - 2*a^7*b^2*d^5*x^2 + a^9*d^5
)*log(d*x + c))/(a^9*b^6*c^6 - 3*a^11*b^4*c^4*d^2 + 3*a^13*b^2*c^2*d^4 - a
^15*d^6 + (a^5*b^10*c^6 - 3*a^7*b^8*c^4*d^2 + 3*a^9*b^6*c^2*d^4 - a^11*b^4
*d^6)*x^4 - 2*(a^7*b^8*c^6 - 3*a^9*b^6*c^4*d^2 + 3*a^11*b^4*c^2*d^4 - a^13
*b^2*d^6)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\,(a^2-b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(-b**2*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.73

$$\begin{aligned}\int \frac{1}{(c+dx)\,(a^2-b^2x^2)^3}\,dx = & -\frac{d^5\log(dx+c)}{b^6c^6-3\,a^2b^4c^4d^2+3\,a^4b^2c^2d^4-a^6d^6} \\ & +\frac{(3\,b^2c^2-9\,abcd+8\,a^2d^2)\log(bx+a)}{16\,(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3)} \\ & -\frac{(3\,b^2c^2+9\,abcd+8\,a^2d^2)\log(bx-a)}{16\,(a^5b^3c^3+3\,a^6b^2c^2d+3\,a^7bcd^2+a^8d^3)} \\ & -\frac{4\,a^4b^2d^3x^2+2\,a^4b^2c^2d-6\,a^6d^3+(3\,b^6c^3-7\,a^2b^4cd^2)x^3-(5\,a^2b^4c^3-9\,a^4b^2cd^2)x}{8\,(a^8b^4c^4-2\,a^{10}b^2c^2d^2+a^{12}d^4+(a^4b^8c^4-2\,a^6b^6c^2d^2+a^8b^4d^4)x^4-2\,(a^6b^6c^4-2\,a^8b^4c^2d^2+a^{10}b^2d^4)}\end{aligned}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-d^5*log(d*x + c)/(b^6*c^6 - 3*a^2*b^4*c^4*d^2 + 3*a^4*b^2*c^2*d^4 - a^6*d
^6) + 1/16*(3*b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*log(b*x + a)/(a^5*b^3*c^3 -
 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3) - 1/16*(3*b^2*c^2 + 9*a*b*c*d
+ 8*a^2*d^2)*log(b*x - a)/(a^5*b^3*c^3 + 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 +
 a^8*d^3) - 1/8*(4*a^4*b^2*d^3*x^2 + 2*a^4*b^2*c^2*d - 6*a^6*d^3 + (3*b^6*
c^3 - 7*a^2*b^4*c*d^2)*x^3 - (5*a^2*b^4*c^3 - 9*a^4*b^2*c*d^2)*x)/(a^8*b^4
*c^4 - 2*a^10*b^2*c^2*d^2 + a^12*d^4 + (a^4*b^8*c^4 - 2*a^6*b^6*c^2*d^2 +
a^8*b^4*d^4)*x^4 - 2*(a^6*b^6*c^4 - 2*a^8*b^4*c^2*d^2 + a^10*b^2*d^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.73

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^3}\,dx$$
$$= -\frac{d^6\log(|dx+c|)}{b^6c^6d-3\,a^2b^4c^4d^3+3\,a^4b^2c^2d^5-a^6d^7}+\frac{(3\,b^3c^2-9\,ab^2cd+8\,a^2bd^2)\log(|bx+a|)}{16\,(a^5b^4c^3-3\,a^6b^3c^2d+3\,a^7b^2cd^2-a^8bd^3)}$$
$$-\frac{(3\,b^3c^2+9\,ab^2cd+8\,a^2bd^2)\log(|bx-a|)}{16\,(a^5b^4c^3+3\,a^6b^3c^2d+3\,a^7b^2cd^2+a^8bd^3)}$$
$$-\frac{2\,a^4b^4c^4d-8\,a^6b^2c^2d^3+6\,a^8d^5+(3\,b^8c^5-10\,a^2b^6c^3d^2+7\,a^4b^4cd^4)x^3+4\,(a^4b^4c^2d^3-a^6b^2d^5)x^2-(5\,a}{8\,(bc+ad)^3(bc-ad)^3(bx+a)^2(bx-a)^2a^4}$$

input
```
integrate(1/(d*x+c)/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-d^6*log(abs(d*x + c))/(b^6*c^6*d - 3*a^2*b^4*c^4*d^3 + 3*a^4*b^2*c^2*d^5
- a^6*d^7) + 1/16*(3*b^3*c^2 - 9*a*b^2*c*d + 8*a^2*b*d^2)*log(abs(b*x + a)
)/(a^5*b^4*c^3 - 3*a^6*b^3*c^2*d + 3*a^7*b^2*c*d^2 - a^8*b*d^3) - 1/16*(3*
b^3*c^2 + 9*a*b^2*c*d + 8*a^2*b*d^2)*log(abs(b*x - a))/(a^5*b^4*c^3 + 3*a^
6*b^3*c^2*d + 3*a^7*b^2*c*d^2 + a^8*b*d^3) - 1/8*(2*a^4*b^4*c^4*d - 8*a^6*
b^2*c^2*d^3 + 6*a^8*d^5 + (3*b^8*c^5 - 10*a^2*b^6*c^3*d^2 + 7*a^4*b^4*c*d^
4)*x^3 + 4*(a^4*b^4*c^2*d^3 - a^6*b^2*d^5)*x^2 - (5*a^2*b^6*c^5 - 14*a^4*b
^4*c^3*d^2 + 9*a^6*b^2*c*d^4)*x)/((b*c + a*d)^3*(b*c - a*d)^3*(b*x + a)^2*
(b*x - a)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.90

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^3}\,dx$$
$$= \frac{\frac{3\,a^2\,d^3-b^2\,c^2\,d}{4\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}-\frac{x^3\,(3\,b^6\,c^3-7\,a^2\,b^4\,c\,d^2)}{8\,a^4\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}-\frac{b^2\,d^3\,x^2}{2\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}+\frac{x\,(5\,b^4\,c^3-9\,a^2\,b^2\,c\,d^2)}{8\,a^2\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}}{a^4-2\,a^2\,b^2\,x^2+b^4\,x^4}$$
$$-\frac{\ln(a+b\,x)\,(8\,a^2\,d^2-9\,a\,b\,c\,d+3\,b^2\,c^2)}{16\,a^8\,d^3-48\,a^7\,b\,c\,d^2+48\,a^6\,b^2\,c^2\,d-16\,a^5\,b^3\,c^3}$$
$$-\frac{\ln(a-b\,x)\,(8\,a^2\,d^2+9\,a\,b\,c\,d+3\,b^2\,c^2)}{16\,a^8\,d^3+48\,a^7\,b\,c\,d^2+48\,a^6\,b^2\,c^2\,d+16\,a^5\,b^3\,c^3}$$
$$+\frac{d^5\,\ln(c+d\,x)}{a^6\,d^6-3\,a^4\,b^2\,c^2\,d^4+3\,a^2\,b^4\,c^4\,d^2-b^6\,c^6}$$

input
```
int(1/((a^2 - b^2*x^2)^3*(c + d*x)),x)
```

output
```
((3*a^2*d^3 - b^2*c^2*d)/(4*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) - (x^
3*(3*b^6*c^3 - 7*a^2*b^4*c*d^2))/(8*a^4*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2
*d^2)) - (b^2*d^3*x^2)/(2*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) + (x*(5
*b^4*c^3 - 9*a^2*b^2*c*d^2))/(8*a^2*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2
)))/(a^4 + b^4*x^4 - 2*a^2*b^2*x^2) - (log(a + b*x)*(8*a^2*d^2 + 3*b^2*c^2
 - 9*a*b*c*d))/(16*a^8*d^3 - 16*a^5*b^3*c^3 + 48*a^6*b^2*c^2*d - 48*a^7*b*
c*d^2) - (log(a - b*x)*(8*a^2*d^2 + 3*b^2*c^2 + 9*a*b*c*d))/(16*a^8*d^3 +
16*a^5*b^3*c^3 + 48*a^6*b^2*c^2*d + 48*a^7*b*c*d^2) + (d^5*log(c + d*x))/(
a^6*d^6 - b^6*c^6 + 3*a^2*b^4*c^4*d^2 - 3*a^4*b^2*c^2*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.77

$$\int \frac{1}{(c+dx)\left(a^2-b^2x^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 8*log( - a - b*x)*a**9*d**5 - 15*log( - a - b*x)*a**8*b*c*d**4 + 16*lo
g( - a - b*x)*a**7*b**2*d**5*x**2 + 10*log( - a - b*x)*a**6*b**3*c**3*d**2
 + 30*log( - a - b*x)*a**6*b**3*c*d**4*x**2 - 8*log( - a - b*x)*a**5*b**4*
d**5*x**4 - 3*log( - a - b*x)*a**4*b**5*c**5 - 20*log( - a - b*x)*a**4*b**
5*c**3*d**2*x**2 - 15*log( - a - b*x)*a**4*b**5*c*d**4*x**4 + 6*log( - a -
 b*x)*a**2*b**7*c**5*x**2 + 10*log( - a - b*x)*a**2*b**7*c**3*d**2*x**4 -
3*log( - a - b*x)*b**9*c**5*x**4 - 8*log(a - b*x)*a**9*d**5 + 15*log(a - b
*x)*a**8*b*c*d**4 + 16*log(a - b*x)*a**7*b**2*d**5*x**2 - 10*log(a - b*x)*
a**6*b**3*c**3*d**2 - 30*log(a - b*x)*a**6*b**3*c*d**4*x**2 - 8*log(a - b*
x)*a**5*b**4*d**5*x**4 + 3*log(a - b*x)*a**4*b**5*c**5 + 20*log(a - b*x)*a
**4*b**5*c**3*d**2*x**2 + 15*log(a - b*x)*a**4*b**5*c*d**4*x**4 - 6*log(a
- b*x)*a**2*b**7*c**5*x**2 - 10*log(a - b*x)*a**2*b**7*c**3*d**2*x**4 + 3*
log(a - b*x)*b**9*c**5*x**4 + 16*log(c + d*x)*a**9*d**5 - 32*log(c + d*x)*
a**7*b**2*d**5*x**2 + 16*log(c + d*x)*a**5*b**4*d**5*x**4 + 8*a**9*d**5 -
12*a**7*b**2*c**2*d**3 - 18*a**7*b**2*c*d**4*x + 4*a**5*b**4*c**4*d + 28*a
**5*b**4*c**3*d**2*x + 14*a**5*b**4*c*d**4*x**3 - 4*a**5*b**4*d**5*x**4 -
10*a**3*b**6*c**5*x - 20*a**3*b**6*c**3*d**2*x**3 + 4*a**3*b**6*c**2*d**3*
x**4 + 6*a*b**8*c**5*x**3)/(16*a**5*(a**10*d**6 - 3*a**8*b**2*c**2*d**4 -
2*a**8*b**2*d**6*x**2 + 3*a**6*b**4*c**4*d**2 + 6*a**6*b**4*c**2*d**4*x**2
 + a**6*b**4*d**6*x**4 - a**4*b**6*c**6 - 6*a**4*b**6*c**4*d**2*x**2 - ...
```

3.50 $\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 273

$$\begin{aligned}\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^3} dx &= \frac{b}{16a^3(bc+ad)^2(a-bx)^2} + \frac{b(3bc+7ad)}{16a^4(bc+ad)^3(a-bx)} \\ &- \frac{b}{16a^3(bc-ad)^2(a+bx)^2} \\ &- \frac{b(3bc-7ad)}{16a^4(bc-ad)^3(a+bx)} + \frac{d^5}{(b^2c^2-a^2d^2)^3 (c+dx)} \\ &- \frac{3b(b^2c^2+4abcd+5a^2d^2)\log(a-bx)}{16a^5(bc+ad)^4} \\ &+ \frac{3b(b^2c^2-4abcd+5a^2d^2)\log(a+bx)}{16a^5(bc-ad)^4} \\ &- \frac{6b^2cd^5\log(c+dx)}{(b^2c^2-a^2d^2)^4}\end{aligned}$$

output

```
1/16*b/a^3/(a*d+b*c)^2/(-b*x+a)^2+1/16*b*(7*a*d+3*b*c)/a^4/(a*d+b*c)^3/(-b
*x+a)-1/16*b/a^3/(-a*d+b*c)^2/(b*x+a)^2-1/16*b*(-7*a*d+3*b*c)/a^4/(-a*d+b*
c)^3/(b*x+a)+d^5/(-a^2*d^2+b^2*c^2)^3/(d*x+c)-3/16*b*(5*a^2*d^2+4*a*b*c*d+
b^2*c^2)*ln(-b*x+a)/a^5/(a*d+b*c)^4+3/16*b*(5*a^2*d^2-4*a*b*c*d+b^2*c^2)*l
n(b*x+a)/a^5/(-a*d+b*c)^4-6*b^2*c*d^5*ln(d*x+c)/(-a^2*d^2+b^2*c^2)^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.03

$$\int \frac{1}{(c+dx)^2 \left(a^2-b^2x^2\right)^3} \, dx$$
$$= \frac{\frac{8b^2c^2d^5-8a^2d^7}{c+dx} + \frac{(b^2c^2-a^2d^2)(3b^6c^4x-12a^2b^4c^2d^2x+a^4b^2d^3(16c-7dx))}{a^6-a^4b^2x^2} + \frac{2(b^2c^2-a^2d^2)^2(b^4c^2x+a^2b^2d(-2c+dx))}{(a^3-ab^2x^2)^2} + \frac{3b(b^6c^6-5a^2b^4}{}}{8\left(b^2c^2-a^2d^2\right)^4}$$

input
```
Integrate[1/((c + d*x)^2*(a^2 - b^2*x^2)^3),x]
```

output
```
((8*b^2*c^2*d^5 - 8*a^2*d^7)/(c + d*x) + ((b^2*c^2 - a^2*d^2)*(3*b^6*c^4*x
 - 12*a^2*b^4*c^2*d^2*x + a^4*b^2*d^3*(16*c - 7*d*x)))/(a^6 - a^4*b^2*x^2)
 + (2*(b^2*c^2 - a^2*d^2)^2*(b^4*c^2*x + a^2*b^2*d*(-2*c + d*x)))/(a^3 - a
*b^2*x^2)^2 + (3*b*(b^6*c^6 - 5*a^2*b^4*c^4*d^2 + 15*a^4*b^2*c^2*d^4 + 5*a
^6*d^6)*ArcTanh[(b*x)/a])/a^5 - 48*b^2*c*d^5*Log[c + d*x] + 24*b^2*c*d^5*L
og[a^2 - b^2*x^2])/(8*(b^2*c^2 - a^2*d^2)^4)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2-b^2x^2\right)^3 (c+dx)^2} \, dx$$
$$\Big\downarrow \text{477}$$
$$\frac{\int \left(-\frac{6a^6b^2cd^6}{(b^2c^2-a^2d^2)^4(c+dx)} - \frac{a^6d^6}{(b^2c^2-a^2d^2)^3(c+dx)^2} + \frac{3ab^2(b^2c^2+4abdc+5a^2d^2)}{16(bc+ad)^4(a-bx)} + \frac{3ab^2(b^2c^2-4abdc+5a^2d^2)}{16(bc-ad)^4(a+bx)} + \frac{a^2b^2(3bc+7ad)}{16(bc+ad)^3(a-bx)^2} + \right.}{a^6}$$
$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3b}{16(a-bx)^2(ad+bc)^2}-\frac{a^3b}{16(a+bx)^2(bc-ad)^2}-\frac{3ab(5a^2d^2+4abcd+b^2c^2)\log(a-bx)}{16(ad+bc)^4}+\frac{3ab(5a^2d^2-4abcd+b^2c^2)\log(a+bx)}{16(bc-ad)^4}+\frac{a^2b(7ad+3b}{16(a-bx)(ad+}}{a^6}$$

input
```
Int[1/((c + d*x)^2*(a^2 - b^2*x^2)^3),x]
```

output
```
((a^3*b)/(16*(b*c + a*d)^2*(a - b*x)^2) + (a^2*b*(3*b*c + 7*a*d))/(16*(b*c
 + a*d)^3*(a - b*x)) - (a^3*b)/(16*(b*c - a*d)^2*(a + b*x)^2) - (a^2*b*(3*
b*c - 7*a*d))/(16*(b*c - a*d)^3*(a + b*x)) + (a^6*d^5)/((b^2*c^2 - a^2*d^2
)^3*(c + d*x)) - (3*a*b*(b^2*c^2 + 4*a*b*c*d + 5*a^2*d^2)*Log[a - b*x])/(1
6*(b*c + a*d)^4) + (3*a*b*(b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)*Log[a + b*x])/
(16*(b*c - a*d)^4) - (6*a^6*b^2*c*d^5*Log[c + d*x])/(b^2*c^2 - a^2*d^2)^4)
/a^6
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.97

method	result
default	$-\frac{b}{16(ad-bc)^2 a^3 (bx+a)^2}-\frac{b(7ad-3bc)}{16(ad-bc)^3 a^4 (bx+a)}+\frac{3b(5a^2d^2-4abcd+b^2c^2)\ln(bx+a)}{16(ad-bc)^4 a^5}-\frac{d^5}{(ad-bc)^3(ad+bc)^3(dx+c)}-\cdots$
norman	$\frac{\frac{7b^4d^3a^2-b^6c^2d}{8(a^4d^4-2a^2b^2c^2d^2+c^4b^4)b^4}-\frac{(5b^4d^3a^2+b^6c^2d)x^2}{8a^2b^2(a^4d^4-2a^2b^2c^2d^2+c^4b^4)}-\frac{(30a^6b^4d^6+15a^4b^6c^2d^4+6a^2b^8c^4d^2-3b^{10}c^6)x^3}{8cb^2a^4(a^6d^6-3a^4b^2c^2d^4+3a^2b^4c^4d^2-b^6c^6)}+\frac{d(15a^4b^4d^5+\cdots}{8ca^4(a^6d^6-3a^4\cdots}}{(dx+c)(-b^2x^2+a^2)^2}$
risch	$\frac{-\frac{3b^4d(5a^4d^4+4a^2b^2c^2d^2-c^4b^4)x^4}{8(a^2d^2-b^2c^2)(a^4d^4-2a^2b^2c^2d^2+c^4b^4)a^4}+\frac{3(3a^2d^2-b^2c^2)b^4cx^3}{8(a^4d^4-2a^2b^2c^2d^2+c^4b^4)a^4}+\frac{db^2(25a^4d^4+28a^2b^2c^2d^2-5c^4b^4)x^2}{8(a^2d^2-b^2c^2)(a^4d^4-2a^2b^2c^2d^2+c^4b^4)a^2}-\frac{b^2c(\cdots}{8a^2(a^4d\cdots}}{(dx+c)(-b^2x^2+a^2)^2}$
parallelrisch	Expression too large to display

input
```
int(1/(d*x+c)^2/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*b/(a*d-b*c)^2/a^3/(b*x+a)^2-1/16*b*(7*a*d-3*b*c)/(a*d-b*c)^3/a^4/(b*
x+a)+3/16*b*(5*a^2*d^2-4*a*b*c*d+b^2*c^2)/(a*d-b*c)^4/a^5*ln(b*x+a)-d^5/(a
*d-b*c)^3/(a*d+b*c)^3/(d*x+c)-6*d^5*b^2*c/(a*d-b*c)^4/(a*d+b*c)^4*ln(d*x+c
)+1/16*b/a^3/(a*d+b*c)^2/(-b*x+a)^2+1/16*b*(7*a*d+3*b*c)/a^4/(a*d+b*c)^3/(
-b*x+a)-3/16*b*(5*a^2*d^2+4*a*b*c*d+b^2*c^2)*ln(-b*x+a)/a^5/(a*d+b*c)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1573 vs. 2(263) = 526.

Time = 25.01 (sec) , antiderivative size = 1573, normalized size of antiderivative = 5.76

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(8*a^5*b^6*c^6*d - 48*a^7*b^4*c^4*d^3 + 24*a^9*b^2*c^2*d^5 + 16*a^11
*d^7 + 6*(a*b^10*c^6*d - 5*a^3*b^8*c^4*d^3 - a^5*b^6*c^2*d^5 + 5*a^7*b^4*d
^7)*x^4 + 6*(a*b^10*c^7 - 5*a^3*b^8*c^5*d^2 + 7*a^5*b^6*c^3*d^4 - 3*a^7*b^
4*c*d^6)*x^3 - 2*(5*a^3*b^8*c^6*d - 33*a^5*b^6*c^4*d^3 + 3*a^7*b^4*c^2*d^5
 + 25*a^9*b^2*d^7)*x^2 - 2*(5*a^3*b^8*c^7 - 21*a^5*b^6*c^5*d^2 + 27*a^7*b^
4*c^3*d^4 - 11*a^9*b^2*c*d^6)*x - 3*(a^4*b^7*c^7 - 5*a^6*b^5*c^5*d^2 + 15*
a^8*b^3*c^3*d^4 + 16*a^9*b^2*c^2*d^5 + 5*a^10*b*c*d^6 + (b^11*c^6*d - 5*a^
2*b^9*c^4*d^3 + 15*a^4*b^7*c^2*d^5 + 16*a^5*b^6*c*d^6 + 5*a^6*b^5*d^7)*x^5
 + (b^11*c^7 - 5*a^2*b^9*c^5*d^2 + 15*a^4*b^7*c^3*d^4 + 16*a^5*b^6*c^2*d^5
 + 5*a^6*b^5*c*d^6)*x^4 - 2*(a^2*b^9*c^6*d - 5*a^4*b^7*c^4*d^3 + 15*a^6*b^
5*c^2*d^5 + 16*a^7*b^4*c*d^6 + 5*a^8*b^3*d^7)*x^3 - 2*(a^2*b^9*c^7 - 5*a^4
*b^7*c^5*d^2 + 15*a^6*b^5*c^3*d^4 + 16*a^7*b^4*c^2*d^5 + 5*a^8*b^3*c*d^6)*
x^2 + (a^4*b^7*c^6*d - 5*a^6*b^5*c^4*d^3 + 15*a^8*b^3*c^2*d^5 + 16*a^9*b^2
*c*d^6 + 5*a^10*b*d^7)*x)*log(b*x + a) + 3*(a^4*b^7*c^7 - 5*a^6*b^5*c^5*d^
2 + 15*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5 + 5*a^10*b*c*d^6 + (b^11*c^6*d
 - 5*a^2*b^9*c^4*d^3 + 15*a^4*b^7*c^2*d^5 - 16*a^5*b^6*c*d^6 + 5*a^6*b^5*d
^7)*x^5 + (b^11*c^7 - 5*a^2*b^9*c^5*d^2 + 15*a^4*b^7*c^3*d^4 - 16*a^5*b^6*
c^2*d^5 + 5*a^6*b^5*c*d^6)*x^4 - 2*(a^2*b^9*c^6*d - 5*a^4*b^7*c^4*d^3 + 15
*a^6*b^5*c^2*d^5 - 16*a^7*b^4*c*d^6 + 5*a^8*b^3*d^7)*x^3 - 2*(a^2*b^9*c^7
- 5*a^4*b^7*c^5*d^2 + 15*a^6*b^5*c^3*d^4 - 16*a^7*b^4*c^2*d^5 + 5*a^8*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(-b**2*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(263) = 526$.

Time = 0.07 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.84

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^3} dx = -\frac{6 b^2cd^5 \log(dx+c)}{b^8c^8 - 4 a^2b^6c^6d^2 + 6 a^4b^4c^4d^4 - 4 a^6b^2c^2d^6 + a^8d^8} + \frac{3 (b^3c^2 - 4 ab^2cd + 5 a^2bd^2) \log(bx+a)}{16 (a^5b^4c^4 - 4 a^6b^3c^3d + 6 a^7b^2c^2d^2 - 4 a^8bcd^3 + a^9d^4)} - \frac{3 (b^3c^2 + 4 ab^2cd + 5 a^2bd^2) \log(bx-a)}{16 (a^5b^4c^4 + 4 a^6b^3c^3d + 6 a^7b^2c^2d^2 + 4 a^8bcd^3 + a^9d^4)} - \frac{4 a^4b^4c^4d - 20 a^6b^2c^2d^3 - 8 a^8d^5 + 3 (b^8c^4d -}{8 (a^8b^6c^7 - 3 a^{10}b^4c^5d^2 + 3 a^{12}b^2c^3d^4 - a^{14}cd^6 + (a^4b^{10}c^6d - 3 a^6b^8c^4d^3 + 3 a^8b^6c^2d^5 - a^{10}b^4d^7)x^5 + (a}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-6*b^2*c*d^5*log(d*x + c)/(b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^4
 - 4*a^6*b^2*c^2*d^6 + a^8*d^8) + 3/16*(b^3*c^2 - 4*a*b^2*c*d + 5*a^2*b*d^
2)*log(b*x + a)/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8
*b*c*d^3 + a^9*d^4) - 3/16*(b^3*c^2 + 4*a*b^2*c*d + 5*a^2*b*d^2)*log(b*x -
 a)/(a^5*b^4*c^4 + 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 + 4*a^8*b*c*d^3 + a
^9*d^4) - 1/8*(4*a^4*b^4*c^4*d - 20*a^6*b^2*c^2*d^3 - 8*a^8*d^5 + 3*(b^8*c
^4*d - 4*a^2*b^6*c^2*d^3 - 5*a^4*b^4*d^5)*x^4 + 3*(b^8*c^5 - 4*a^2*b^6*c^3
*d^2 + 3*a^4*b^4*c*d^4)*x^3 - (5*a^2*b^6*c^4*d - 28*a^4*b^4*c^2*d^3 - 25*a
^6*b^2*d^5)*x^2 - (5*a^2*b^6*c^5 - 16*a^4*b^4*c^3*d^2 + 11*a^6*b^2*c*d^4)*
x)/(a^8*b^6*c^7 - 3*a^10*b^4*c^5*d^2 + 3*a^12*b^2*c^3*d^4 - a^14*c*d^6 + (
a^4*b^10*c^6*d - 3*a^6*b^8*c^4*d^3 + 3*a^8*b^6*c^2*d^5 - a^10*b^4*d^7)*x^5
 + (a^4*b^10*c^7 - 3*a^6*b^8*c^5*d^2 + 3*a^8*b^6*c^3*d^4 - a^10*b^4*c*d^6)
*x^4 - 2*(a^6*b^8*c^6*d - 3*a^8*b^6*c^4*d^3 + 3*a^10*b^4*c^2*d^5 - a^12*b^
2*d^7)*x^3 - 2*(a^6*b^8*c^7 - 3*a^8*b^6*c^5*d^2 + 3*a^10*b^4*c^3*d^4 - a^1
2*b^2*c*d^6)*x^2 + (a^8*b^6*c^6*d - 3*a^10*b^4*c^4*d^3 + 3*a^12*b^2*c^2*d^
5 - a^14*d^7)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(263) = 526$.

Time = 0.13 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.62

$$\int \frac{1}{(c+dx)^2(a^2-b^2x^2)^3}\,dx = \frac{d^{11}}{(b^6c^6d^6-3a^2b^4c^4d^8+3a^4b^2c^2d^{10}-a^6d^{12})(dx+c)} + \frac{3b^2cd^5\log\left(\left|b^2-\frac{2b^2c}{dx+c}+\frac{b^2c^2}{(dx+c)^2}-\frac{a^2d^2}{(dx+c)^2}\right|\right)}{b^8c^8-4a^2b^6c^6d^2+6a^4b^4c^4d^4-4a^6b^2c^2d^6+a^8d^8} - \frac{3(b^8c^6d^2-5a^2b^6c^4d^4+15a^4b^4c^2d^6+5a^6b^2d^8)\log\left(\frac{\left|2b^2cd-\frac{2b^2c^2d}{dx+c}+\frac{2a^2d^3}{dx+c}-2d^2|a||b|\right|}{\left|2b^2cd-\frac{2b^2c^2d}{dx+c}+\frac{2a^2d^3}{dx+c}+2d^2|a||b|\right|}\right)}{16(a^4b^8c^8-4a^6b^6c^6d^2+6a^8b^4c^4d^4-4a^{10}b^2c^2d^6+a^{12}d^8)d^2|a||b|} - \frac{3b^{10}c^5d-14a^2b^8c^3d^3-29a^4b^6cd^5-\frac{9b^{10}c^6d^2-41a^2b^8c^4d^4-121a^4b^6c^2d^6-7a^6b^4d^8}{(dx+c)d}+\frac{9b^{10}c^7d^3-45a^2b^8c^5d^5-145a^4b^6c}{(dx+c)^2d^2}}{8\left(b^2-\frac{2b^2c}{dx+c}+\frac{b^2c^2}{(dx+c)^2}-\frac{a^2d^2}{(dx+c)^2}\right)^2(bc+ad)^4(bc-}$$

input
```
integrate(1/(d*x+c)^2/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
d^11/((b^6*c^6*d^6 - 3*a^2*b^4*c^4*d^8 + 3*a^4*b^2*c^2*d^10 - a^6*d^12)*(d
*x + c)) + 3*b^2*c*d^5*log(abs(b^2 - 2*b^2*c/(d*x + c) + b^2*c^2/(d*x + c)
^2 - a^2*d^2/(d*x + c)^2))/(b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^
4 - 4*a^6*b^2*c^2*d^6 + a^8*d^8) - 3/16*(b^8*c^6*d^2 - 5*a^2*b^6*c^4*d^4 +
 15*a^4*b^4*c^2*d^6 + 5*a^6*b^2*d^8)*log(abs(2*b^2*c*d - 2*b^2*c^2*d/(d*x
+ c) + 2*a^2*d^3/(d*x + c) - 2*d^2*abs(a)*abs(b))/abs(2*b^2*c*d - 2*b^2*c^
2*d/(d*x + c) + 2*a^2*d^3/(d*x + c) + 2*d^2*abs(a)*abs(b)))/((a^4*b^8*c^8
- 4*a^6*b^6*c^6*d^2 + 6*a^8*b^4*c^4*d^4 - 4*a^10*b^2*c^2*d^6 + a^12*d^8)*d
^2*abs(a)*abs(b)) - 1/8*(3*b^10*c^5*d - 14*a^2*b^8*c^3*d^3 - 29*a^4*b^6*c*
d^5 - (9*b^10*c^6*d^2 - 41*a^2*b^8*c^4*d^4 - 121*a^4*b^6*c^2*d^6 - 7*a^6*b
^4*d^8)/((d*x + c)*d) + (9*b^10*c^7*d^3 - 45*a^2*b^8*c^5*d^5 - 145*a^4*b^6
*c^3*d^7 + 21*a^6*b^4*c*d^9)/((d*x + c)^2*d^2) - 3*(b^10*c^8*d^4 - 6*a^2*b
^8*c^6*d^6 - 20*a^4*b^6*c^4*d^8 + 22*a^6*b^4*c^2*d^10 + 3*a^8*b^2*d^12)/((
d*x + c)^3*d^3))/((b^2 - 2*b^2*c/(d*x + c) + b^2*c^2/(d*x + c)^2 - a^2*d^2
/(d*x + c)^2)^2*(b*c + a*d)^4*(b*c - a*d)^4*a^4)
```

Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 697, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^3}\,dx$$

$$= \frac{\ln(a+b\,x)\,\left(15\,a^2\,b\,d^2-12\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}{16\,\left(a^9\,d^4-4\,a^8\,b\,c\,d^3+6\,a^7\,b^2\,c^2\,d^2-4\,a^6\,b^3\,c^3\,d+a^5\,b^4\,c^4\right)}$$

$$-\frac{\frac{2\,a^4\,d^5+5\,a^2\,b^2\,c^2\,d^3-b^4\,c^4\,d}{2\,(a^2\,d^2-b^2\,c^2)\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}+\frac{3\,x^3\,(b^6\,c^3-3\,a^2\,b^4\,c\,d^2)}{8\,a^4\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}-\frac{x\,(5\,b^4\,c^3-11\,a^2\,b^2\,c\,d^2)}{8\,a^2\,(a^4\,d^4-2\,a^2\,b^2\,c^2\,d^2+b^4\,c^4)}+\frac{3\,x^4\,(5\,a^4\,b^4\,d^5-}{8\,a^4\,(a^6\,d^6-3\,a^4\,b^2}}{d\,a^4\,x+c\,a^4-2\,d\,a^2\,b^2\,x^3-2\,c\,a^2\,b^2\,x^2+d\,b^4\,x^5+c}$$

$$-\frac{\ln(a-b\,x)\,\left(15\,a^2\,b\,d^2+12\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}{16\,a^9\,d^4+64\,a^8\,b\,c\,d^3+96\,a^7\,b^2\,c^2\,d^2+64\,a^6\,b^3\,c^3\,d+16\,a^5\,b^4\,c^4}$$

$$-\frac{6\,b^2\,c\,d^5\,\ln(c+d\,x)}{a^8\,d^8-4\,a^6\,b^2\,c^2\,d^6+6\,a^4\,b^4\,c^4\,d^4-4\,a^2\,b^6\,c^6\,d^2+b^8\,c^8}$$

input

```
int(1/((a^2 - b^2*x^2)^3*(c + d*x)^2),x)
```

output

```
(log(a + b*x)*(3*b^3*c^2 + 15*a^2*b*d^2 - 12*a*b^2*c*d))/(16*(a^9*d^4 + a^
5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) - ((2*a^
4*d^5 - b^4*c^4*d + 5*a^2*b^2*c^2*d^3)/(2*(a^2*d^2 - b^2*c^2)*(a^4*d^4 + b
^4*c^4 - 2*a^2*b^2*c^2*d^2)) + (3*x^3*(b^6*c^3 - 3*a^2*b^4*c*d^2))/(8*a^4*
(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) - (x*(5*b^4*c^3 - 11*a^2*b^2*c*d^
2))/(8*a^2*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2)) + (3*x^4*(5*a^4*b^4*d^
5 - b^8*c^4*d + 4*a^2*b^6*c^2*d^3))/(8*a^4*(a^6*d^6 - b^6*c^6 + 3*a^2*b^4*
c^4*d^2 - 3*a^4*b^2*c^2*d^4)) - (x^2*(25*a^4*b^2*d^5 - 5*b^6*c^4*d + 28*a^
2*b^4*c^2*d^3))/(8*a^2*(a^2*d^2 - b^2*c^2)*(a^4*d^4 + b^4*c^4 - 2*a^2*b^2*
c^2*d^2)))/(a^4*c + b^4*c*x^4 + b^4*d*x^5 + a^4*d*x - 2*a^2*b^2*c*x^2 - 2*
a^2*b^2*d*x^3) - (log(a - b*x)*(3*b^3*c^2 + 15*a^2*b*d^2 + 12*a*b^2*c*d))/
(16*a^9*d^4 + 16*a^5*b^4*c^4 + 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 + 64*
a^8*b*c*d^3) - (6*b^2*c*d^5*log(c + d*x))/(a^8*d^8 + b^8*c^8 - 4*a^2*b^6*c
^6*d^2 + 6*a^4*b^4*c^4*d^4 - 4*a^6*b^2*c^2*d^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2287, normalized size of antiderivative = 8.38

$$\int \frac{1}{(c+dx)^2 (a^2-b^2x^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(-b^2*x^2+a^2)^3,x)
```

output
```
(15*log( - a - b*x)*a**10*b*c**2*d**6 + 15*log( - a - b*x)*a**10*b*c*d**7*
x + 48*log( - a - b*x)*a**9*b**2*c**3*d**5 + 48*log( - a - b*x)*a**9*b**2*
c**2*d**6*x + 45*log( - a - b*x)*a**8*b**3*c**4*d**4 + 45*log( - a - b*x)*
a**8*b**3*c**3*d**5*x - 30*log( - a - b*x)*a**8*b**3*c**2*d**6*x**2 - 30*l
og( - a - b*x)*a**8*b**3*c*d**7*x**3 - 96*log( - a - b*x)*a**7*b**4*c**3*d
**5*x**2 - 96*log( - a - b*x)*a**7*b**4*c**2*d**6*x**3 - 15*log( - a - b*x
)*a**6*b**5*c**6*d**2 - 15*log( - a - b*x)*a**6*b**5*c**5*d**3*x - 90*log(
 - a - b*x)*a**6*b**5*c**4*d**4*x**2 - 90*log( - a - b*x)*a**6*b**5*c**3*d
**5*x**3 + 15*log( - a - b*x)*a**6*b**5*c**2*d**6*x**4 + 15*log( - a - b*x
)*a**6*b**5*c*d**7*x**5 + 48*log( - a - b*x)*a**5*b**6*c**3*d**5*x**4 + 48
*log( - a - b*x)*a**5*b**6*c**2*d**6*x**5 + 3*log( - a - b*x)*a**4*b**7*c*
*8 + 3*log( - a - b*x)*a**4*b**7*c**7*d*x + 30*log( - a - b*x)*a**4*b**7*c
**6*d**2*x**2 + 30*log( - a - b*x)*a**4*b**7*c**5*d**3*x**3 + 45*log( - a
- b*x)*a**4*b**7*c**4*d**4*x**4 + 45*log( - a - b*x)*a**4*b**7*c**3*d**5*x
**5 - 6*log( - a - b*x)*a**2*b**9*c**8*x**2 - 6*log( - a - b*x)*a**2*b**9*
c**7*d*x**3 - 15*log( - a - b*x)*a**2*b**9*c**6*d**2*x**4 - 15*log( - a -
b*x)*a**2*b**9*c**5*d**3*x**5 + 3*log( - a - b*x)*b**11*c**8*x**4 + 3*log(
 - a - b*x)*b**11*c**7*d*x**5 - 15*log(a - b*x)*a**10*b*c**2*d**6 - 15*log
(a - b*x)*a**10*b*c*d**7*x + 48*log(a - b*x)*a**9*b**2*c**3*d**5 + 48*log(
a - b*x)*a**9*b**2*c**2*d**6*x - 45*log(a - b*x)*a**8*b**3*c**4*d**4 - ...
```

3.51 $\int \frac{1}{(c+dx)^3(a^2-b^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 335

$$\begin{aligned}\int \frac{1}{(c+dx)^3(a^2-b^2x^2)^3}\,dx = {} & \frac{b^2}{16a^3(bc+ad)^3(a-bx)^2}+\frac{3b^2(bc+3ad)}{16a^4(bc+ad)^4(a-bx)} \\ & -\frac{b^2}{16a^3(bc-ad)^3(a+bx)^2}-\frac{3b^2(bc-3ad)}{16a^4(bc-ad)^4(a+bx)} \\ & +\frac{d^5}{2(b^2c^2-a^2d^2)^3(c+dx)^2}+\frac{6b^2cd^5}{(b^2c^2-a^2d^2)^4(c+dx)} \\ & -\frac{3b^2(b^2c^2+5abcd+8a^2d^2)\log(a-bx)}{16a^5(bc+ad)^5} \\ & +\frac{3b^2(b^2c^2-5abcd+8a^2d^2)\log(a+bx)}{16a^5(bc-ad)^5} \\ & -\frac{3b^2d^5(7b^2c^2+a^2d^2)\log(c+dx)}{(b^2c^2-a^2d^2)^5}\end{aligned}$$

output
```
1/16*b^2/a^3/(a*d+b*c)^3/(-b*x+a)^2+3/16*b^2*(3*a*d+b*c)/a^4/(a*d+b*c)^4/(
-b*x+a)-1/16*b^2/a^3/(-a*d+b*c)^3/(b*x+a)^2-3/16*b^2*(-3*a*d+b*c)/a^4/(-a*
d+b*c)^4/(b*x+a)+1/2*d^5/(-a^2*d^2+b^2*c^2)^3/(d*x+c)^2+6*b^2*c*d^5/(-a^2*
d^2+b^2*c^2)^4/(d*x+c)-3/16*b^2*(8*a^2*d^2+5*a*b*c*d+b^2*c^2)*ln(-b*x+a)/a
^5/(a*d+b*c)^5+3/16*b^2*(8*a^2*d^2-5*a*b*c*d+b^2*c^2)*ln(b*x+a)/a^5/(-a*d+
b*c)^5-3*b^2*d^5*(a^2*d^2+7*b^2*c^2)*ln(d*x+c)/(-a^2*d^2+b^2*c^2)^5
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^3} dx$$
$$= \frac{-\frac{4d^5(b^2c^2-a^2d^2)^2}{(c+dx)^2} + \frac{48b^2cd^5(-b^2c^2+a^2d^2)}{c+dx} + \frac{(-b^2c^2+a^2d^2)(8a^6b^2d^5+3b^8c^5x-18a^2b^6c^3d^2x+a^4b^4cd^3(40c-33dx))}{a^6-a^4b^2x^2} + \frac{2(b^2c^2-a^2d^2)^2}{}}{}$$

input
```
Integrate[1/((c + d*x)^3*(a^2 - b^2*x^2)^3),x]
```

output
```
((-4*d^5*(b^2*c^2 - a^2*d^2)^2)/(c + d*x)^2 + (48*b^2*c*d^5*(-(b^2*c^2) +
a^2*d^2))/(c + d*x) + ((-(b^2*c^2) + a^2*d^2)*(8*a^6*b^2*d^5 + 3*b^8*c^5*x
 - 18*a^2*b^6*c^3*d^2*x + a^4*b^4*c*d^3*(40*c - 33*d*x)))/(a^6 - a^4*b^2*x
^2) + (2*(b^2*c^2 - a^2*d^2)^2*(a^4*b^2*d^3 - b^6*c^3*x + 3*a^2*b^4*c*d*(c
 - d*x)))/(a^3 - a*b^2*x^2)^2 - (3*b^3*(b^6*c^7 - 7*a^2*b^4*c^5*d^2 + 35*a
^4*b^2*c^3*d^4 + 35*a^6*c*d^6)*ArcTanh[(b*x)/a])/a^5 + 24*(7*b^4*c^2*d^5 +
 a^2*b^2*d^7)*Log[c + d*x] - 12*b^2*(7*b^2*c^2*d^5 + a^2*d^7)*Log[a^2 - b^
2*x^2])/(8*(-(b^2*c^2) + a^2*d^2)^5)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2-b^2x^2)^3 (c+dx)^3} dx$$
$$\Big\downarrow \text{477}$$
$$\frac{\int \left(-\frac{3a^6b^2(7b^2c^2+a^2d^2)d^6}{(b^2c^2-a^2d^2)^5(c+dx)} - \frac{6a^6b^2cd^6}{(b^2c^2-a^2d^2)^4(c+dx)^2} - \frac{a^6d^6}{(b^2c^2-a^2d^2)^3(c+dx)^3} + \frac{3ab^3(b^2c^2+5abdc+8a^2d^2)}{16(bc+ad)^5(a-bx)} + \frac{3ab^3(b^2c^2-5abdc+8a^2d^2)}{16(bc-ad)^5(a+bx)}\right.}{a^6}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{a^3b^2}{16(a-bx)^2(ad+bc)^3} - \frac{a^3b^2}{16(a+bx)^2(bc-ad)^3} - \frac{3ab^2(8a^2d^2+5abcd+b^2c^2)\log(a-bx)}{16(ad+bc)^5} + \frac{3ab^2(8a^2d^2-5abcd+b^2c^2)\log(a+bx)}{16(bc-ad)^5} + \frac{3a^2b^2(3ad}{16(a-bx)(ad}}{a^6}$$

input
```
Int[1/((c + d*x)^3*(a^2 - b^2*x^2)^3),x]
```

output
```
((a^3*b^2)/(16*(b*c + a*d)^3*(a - b*x)^2) + (3*a^2*b^2*(b*c + 3*a*d))/(16*
(b*c + a*d)^4*(a - b*x)) - (a^3*b^2)/(16*(b*c - a*d)^3*(a + b*x)^2) - (3*a
^2*b^2*(b*c - 3*a*d))/(16*(b*c - a*d)^4*(a + b*x)) + (a^6*d^5)/(2*(b^2*c^2
 - a^2*d^2)^3*(c + d*x)^2) + (6*a^6*b^2*c*d^5)/((b^2*c^2 - a^2*d^2)^4*(c +
 d*x)) - (3*a*b^2*(b^2*c^2 + 5*a*b*c*d + 8*a^2*d^2)*Log[a - b*x])/(16*(b*c
 + a*d)^5) + (3*a*b^2*(b^2*c^2 - 5*a*b*c*d + 8*a^2*d^2)*Log[a + b*x])/(16*
(b*c - a*d)^5) - (3*a^6*b^2*d^5*(7*b^2*c^2 + a^2*d^2)*Log[c + d*x])/(b^2*c
^2 - a^2*d^2)^5)/a^6
```

Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2}{16(ad-bc)^3a^3(bx+a)^2} + \frac{3b^2(3ad-bc)}{16(ad-bc)^4a^4(bx+a)} - \frac{3b^2(8a^2d^2-5abcd+b^2c^2)\ln(bx+a)}{16(ad-bc)^5a^5} - \frac{d^5}{2(ad-bc)^3(ad+bc)^3(dx+c)^2} +$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(1/(d*x+c)^3/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*b^2/(a*d-b*c)^3/a^3/(b*x+a)^2+3/16*b^2*(3*a*d-b*c)/(a*d-b*c)^4/a^4/(b
*x+a)-3/16*b^2*(8*a^2*d^2-5*a*b*c*d+b^2*c^2)/(a*d-b*c)^5/a^5*ln(b*x+a)-1/2
*d^5/(a*d-b*c)^3/(a*d+b*c)^3/(d*x+c)^2+6*d^5*b^2*c/(a*d-b*c)^4/(a*d+b*c)^4
/(d*x+c)+3*d^5*b^2*(a^2*d^2+7*b^2*c^2)/(a*d-b*c)^5/(a*d+b*c)^5*ln(d*x+c)+1
/16*b^2/a^3/(a*d+b*c)^3/(-b*x+a)^2+3/16*b^2*(3*a*d+b*c)/a^4/(a*d+b*c)^4/(-
b*x+a)-3/16*b^2*(8*a^2*d^2+5*a*b*c*d+b^2*c^2)*ln(-b*x+a)/a^5/(a*d+b*c)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2519 vs. $2(323) = 646$.

Time = 132.57 (sec) , antiderivative size = 2519, normalized size of antiderivative = 7.52

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(12*a^5*b^8*c^8*d - 100*a^7*b^6*c^6*d^3 - 36*a^9*b^4*c^4*d^5 + 132*a
^11*b^2*c^2*d^7 - 8*a^13*d^9 + 6*(a*b^12*c^7*d^2 - 7*a^3*b^10*c^5*d^4 - 21
*a^5*b^8*c^3*d^6 + 27*a^7*b^6*c*d^8)*x^5 + 12*(a*b^12*c^8*d - 7*a^3*b^10*c
^6*d^3 - 7*a^5*b^8*c^4*d^5 + 15*a^7*b^6*c^2*d^7 - 2*a^9*b^4*d^9)*x^4 + 2*(
3*a*b^12*c^9 - 26*a^3*b^10*c^7*d^2 + 84*a^5*b^8*c^5*d^4 + 90*a^7*b^6*c^3*d
^6 - 151*a^9*b^4*c*d^8)*x^3 - 4*(5*a^3*b^10*c^8*d - 42*a^5*b^8*c^6*d^3 - 3
6*a^7*b^6*c^4*d^5 + 82*a^9*b^4*c^2*d^7 - 9*a^11*b^2*d^9)*x^2 - 2*(5*a^3*b^
10*c^9 - 31*a^5*b^8*c^7*d^2 + 75*a^7*b^6*c^5*d^4 + 19*a^9*b^4*c^3*d^6 - 68
*a^11*b^2*c*d^8)*x - 3*(a^4*b^9*c^9 - 7*a^6*b^7*c^7*d^2 + 35*a^8*b^5*c^5*d
^4 + 56*a^9*b^4*c^4*d^5 + 35*a^10*b^3*c^3*d^6 + 8*a^11*b^2*c^2*d^7 + (b^13
*c^7*d^2 - 7*a^2*b^11*c^5*d^4 + 35*a^4*b^9*c^3*d^6 + 56*a^5*b^8*c^2*d^7 +
35*a^6*b^7*c*d^8 + 8*a^7*b^6*d^9)*x^6 + 2*(b^13*c^8*d - 7*a^2*b^11*c^6*d^3
 + 35*a^4*b^9*c^4*d^5 + 56*a^5*b^8*c^3*d^6 + 35*a^6*b^7*c^2*d^7 + 8*a^7*b^
6*c*d^8)*x^5 + (b^13*c^9 - 9*a^2*b^11*c^7*d^2 + 49*a^4*b^9*c^5*d^4 + 56*a^
5*b^8*c^4*d^5 - 35*a^6*b^7*c^3*d^6 - 104*a^7*b^6*c^2*d^7 - 70*a^8*b^5*c*d^
8 - 16*a^9*b^4*d^9)*x^4 - 4*(a^2*b^11*c^8*d - 7*a^4*b^9*c^6*d^3 + 35*a^6*b
^7*c^4*d^5 + 56*a^7*b^6*c^3*d^6 + 35*a^8*b^5*c^2*d^7 + 8*a^9*b^4*c*d^8)*x^
3 - (2*a^2*b^11*c^9 - 15*a^4*b^9*c^7*d^2 + 77*a^6*b^7*c^5*d^4 + 112*a^7*b^
6*c^4*d^5 + 35*a^8*b^5*c^3*d^6 - 40*a^9*b^4*c^2*d^7 - 35*a^10*b^3*c*d^8 -
8*a^11*b^2*d^9)*x^2 + 2*(a^4*b^9*c^8*d - 7*a^6*b^7*c^6*d^3 + 35*a^8*b^5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**3/(-b**2*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(323) = 646$.

Time = 0.07 (sec) , antiderivative size = 1132, normalized size of antiderivative = 3.38

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
3/16*(b^4*c^2 - 5*a*b^3*c*d + 8*a^2*b^2*d^2)*log(b*x + a)/(a^5*b^5*c^5 - 5
*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3 + 5*a^9*b*c*d^4 -
 a^10*d^5) - 3/16*(b^4*c^2 + 5*a*b^3*c*d + 8*a^2*b^2*d^2)*log(b*x - a)/(a^
5*b^5*c^5 + 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 + 5*
a^9*b*c*d^4 + a^10*d^5) - 3*(7*b^4*c^2*d^5 + a^2*b^2*d^7)*log(d*x + c)/(b^
10*c^10 - 5*a^2*b^8*c^8*d^2 + 10*a^4*b^6*c^6*d^4 - 10*a^6*b^4*c^4*d^6 + 5*
a^8*b^2*c^2*d^8 - a^10*d^10) - 1/8*(6*a^4*b^6*c^6*d - 44*a^6*b^4*c^4*d^3 -
 62*a^8*b^2*c^2*d^5 + 4*a^10*d^7 + 3*(b^10*c^5*d^2 - 6*a^2*b^8*c^3*d^4 - 2
7*a^4*b^6*c*d^6)*x^5 + 6*(b^10*c^6*d - 6*a^2*b^8*c^4*d^3 - 13*a^4*b^6*c^2*
d^5 + 2*a^6*b^4*d^7)*x^4 + (3*b^10*c^7 - 23*a^2*b^8*c^5*d^2 + 61*a^4*b^6*c
^3*d^4 + 151*a^6*b^4*c*d^6)*x^3 - 2*(5*a^2*b^8*c^6*d - 37*a^4*b^6*c^4*d^3
- 73*a^6*b^4*c^2*d^5 + 9*a^8*b^2*d^7)*x^2 - (5*a^2*b^8*c^7 - 26*a^4*b^6*c^
5*d^2 + 49*a^6*b^4*c^3*d^4 + 68*a^8*b^2*c*d^6)*x)/(a^8*b^8*c^10 - 4*a^10*b
^6*c^8*d^2 + 6*a^12*b^4*c^6*d^4 - 4*a^14*b^2*c^4*d^6 + a^16*c^2*d^8 + (a^4
*b^12*c^8*d^2 - 4*a^6*b^10*c^6*d^4 + 6*a^8*b^8*c^4*d^6 - 4*a^10*b^6*c^2*d^
8 + a^12*b^4*d^10)*x^6 + 2*(a^4*b^12*c^9*d - 4*a^6*b^10*c^7*d^3 + 6*a^8*b^
8*c^5*d^5 - 4*a^10*b^6*c^3*d^7 + a^12*b^4*c*d^9)*x^5 + (a^4*b^12*c^10 - 6*
a^6*b^10*c^8*d^2 + 14*a^8*b^8*c^6*d^4 - 16*a^10*b^6*c^4*d^6 + 9*a^12*b^4*c
^2*d^8 - 2*a^14*b^2*d^10)*x^4 - 4*(a^6*b^10*c^9*d - 4*a^8*b^8*c^7*d^3 + 6*
a^10*b^6*c^5*d^5 - 4*a^12*b^4*c^3*d^7 + a^14*b^2*c*d^9)*x^3 - (2*a^6*b^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(323) = 646$.

Time = 0.13 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.26

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^3}\,dx = \frac{3\,(b^5c^2-5\,ab^4cd+8\,a^2b^3d^2)\log(|bx+a|)}{16\,(a^5b^6c^5-5\,a^6b^5c^4d+10\,a^7b^4c^3d^2-10\,a^8b^3c^2d^3+5\,a^9b^2cd^4-a^{10}bd^5)} - \frac{3\,(b^5c^2+5\,ab^4cd+8\,a^2b^3d^2)\log(|bx-a|)}{16\,(a^5b^6c^5+5\,a^6b^5c^4d+10\,a^7b^4c^3d^2+10\,a^8b^3c^2d^3+5\,a^9b^2cd^4+a^{10}bd^5)} - \frac{3\,(7\,b^4c^2d^6+a^2b^2d^8)\log(|dx+c|)}{b^{10}c^{10}d-5\,a^2b^8c^8d^3+10\,a^4b^6c^6d^5-10\,a^6b^4c^4d^7+5\,a^8b^2c^2d^9-a^{10}d^{11}} - \frac{3\,b^{10}c^5d^2x^5-18\,a^2b^8c^3d^4x^5-81\,a^4b^6cd^6x^5+6\,b^{10}c^6dx^4-36\,a^2b^8c^4d^3x^4-78\,a^4b^6c^2d^5x^4+12\,a^6b^4d^7x}{}$$

input
```
integrate(1/(d*x+c)^3/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3/16*(b^5*c^2 - 5*a*b^4*c*d + 8*a^2*b^3*d^2)*log(abs(b*x + a))/(a^5*b^6*c^
5 - 5*a^6*b^5*c^4*d + 10*a^7*b^4*c^3*d^2 - 10*a^8*b^3*c^2*d^3 + 5*a^9*b^2*
c*d^4 - a^10*b*d^5) - 3/16*(b^5*c^2 + 5*a*b^4*c*d + 8*a^2*b^3*d^2)*log(abs
(b*x - a))/(a^5*b^6*c^5 + 5*a^6*b^5*c^4*d + 10*a^7*b^4*c^3*d^2 + 10*a^8*b^
3*c^2*d^3 + 5*a^9*b^2*c*d^4 + a^10*b*d^5) - 3*(7*b^4*c^2*d^6 + a^2*b^2*d^8
)*log(abs(d*x + c))/(b^10*c^10*d - 5*a^2*b^8*c^8*d^3 + 10*a^4*b^6*c^6*d^5
- 10*a^6*b^4*c^4*d^7 + 5*a^8*b^2*c^2*d^9 - a^10*d^11) - 1/8*(3*b^10*c^5*d^
2*x^5 - 18*a^2*b^8*c^3*d^4*x^5 - 81*a^4*b^6*c*d^6*x^5 + 6*b^10*c^6*d*x^4 -
 36*a^2*b^8*c^4*d^3*x^4 - 78*a^4*b^6*c^2*d^5*x^4 + 12*a^6*b^4*d^7*x^4 + 3*
b^10*c^7*x^3 - 23*a^2*b^8*c^5*d^2*x^3 + 61*a^4*b^6*c^3*d^4*x^3 + 151*a^6*b
^4*c*d^6*x^3 - 10*a^2*b^8*c^6*d*x^2 + 74*a^4*b^6*c^4*d^3*x^2 + 146*a^6*b^4
*c^2*d^5*x^2 - 18*a^8*b^2*d^7*x^2 - 5*a^2*b^8*c^7*x + 26*a^4*b^6*c^5*d^2*x
 - 49*a^6*b^4*c^3*d^4*x - 68*a^8*b^2*c*d^6*x + 6*a^4*b^6*c^6*d - 44*a^6*b^
4*c^4*d^3 - 62*a^8*b^2*c^2*d^5 + 4*a^10*d^7)/((a^4*b^8*c^8 - 4*a^6*b^6*c^6
*d^2 + 6*a^8*b^4*c^4*d^4 - 4*a^10*b^2*c^2*d^6 + a^12*d^8)*(b^2*d*x^3 + b^2
*c*x^2 - a^2*d*x - a^2*c)^2)
```

Mupad [B] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 1096, normalized size of antiderivative = 3.27

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/((a^2 - b^2*x^2)^3*(c + d*x)^3),x)
```

output
```
(log(c + d*x)*(3*a^2*b^2*d^7 + 21*b^4*c^2*d^5))/(a^10*d^10 - b^10*c^10 + 5
*a^2*b^8*c^8*d^2 - 10*a^4*b^6*c^6*d^4 + 10*a^6*b^4*c^4*d^6 - 5*a^8*b^2*c^2
*d^8) - ((2*a^6*d^7 + 3*b^6*c^6*d - 22*a^2*b^4*c^4*d^3 - 31*a^4*b^2*c^2*d^
5)/(4*(a^8*d^8 + b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^4 - 4*a^6*b
^2*c^2*d^6)) - (3*x^5*(27*a^4*b^6*c*d^6 - b^10*c^5*d^2 + 6*a^2*b^8*c^3*d^4
))/(8*a^4*(a^8*d^8 + b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^4 - 4*a
^6*b^2*c^2*d^6)) - (x*(5*b^8*c^7 + 68*a^6*b^2*c*d^6 - 26*a^2*b^6*c^5*d^2 +
 49*a^4*b^4*c^3*d^4))/(8*a^2*(a^8*d^8 + b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^
4*b^4*c^4*d^4 - 4*a^6*b^2*c^2*d^6)) + (x^3*(3*b^10*c^7 + 151*a^6*b^4*c*d^6
 - 23*a^2*b^8*c^5*d^2 + 61*a^4*b^6*c^3*d^4))/(8*a^4*(a^8*d^8 + b^8*c^8 - 4
*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^4 - 4*a^6*b^2*c^2*d^6)) + (3*x^4*(b^10*
c^6*d + 2*a^6*b^4*d^7 - 6*a^2*b^8*c^4*d^3 - 13*a^4*b^6*c^2*d^5))/(4*a^4*(a
^8*d^8 + b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d^4 - 4*a^6*b^2*c^2*d
^6)) - (x^2*(5*b^8*c^6*d + 9*a^6*b^2*d^7 - 37*a^2*b^6*c^4*d^3 - 73*a^4*b^4
*c^2*d^5))/(4*a^2*(a^8*d^8 + b^8*c^8 - 4*a^2*b^6*c^6*d^2 + 6*a^4*b^4*c^4*d
^4 - 4*a^6*b^2*c^2*d^6)))/(x^2*(a^4*d^2 - 2*a^2*b^2*c^2) + x^4*(b^4*c^2 -
2*a^2*b^2*d^2) + a^4*c^2 + b^4*d^2*x^6 + 2*a^4*c*d*x + 2*b^4*c*d*x^5 - 4*a
^2*b^2*c*d*x^3) - (log(a + b*x)*(3*b^4*c^2 + 24*a^2*b^2*d^2 - 15*a*b^3*c*d
))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10
*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)) - (log(a - b*x)*(3*b^4*c^2 + 24*a^2*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3600, normalized size of antiderivative = 10.75

$$\int \frac{1}{(c+dx)^3 (a^2-b^2x^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^3/(-b^2*x^2+a^2)^3,x)
```

output

```
( - 24*log( - a - b*x)*a**11*b**2*c**2*d**7 - 48*log( - a - b*x)*a**11*b**
2*c*d**8*x - 24*log( - a - b*x)*a**11*b**2*d**9*x**2 - 105*log( - a - b*x)
*a**10*b**3*c**3*d**6 - 210*log( - a - b*x)*a**10*b**3*c**2*d**7*x - 105*l
og( - a - b*x)*a**10*b**3*c*d**8*x**2 - 168*log( - a - b*x)*a**9*b**4*c**4
*d**5 - 336*log( - a - b*x)*a**9*b**4*c**3*d**6*x - 120*log( - a - b*x)*a*
*9*b**4*c**2*d**7*x**2 + 96*log( - a - b*x)*a**9*b**4*c*d**8*x**3 + 48*log
( - a - b*x)*a**9*b**4*d**9*x**4 - 105*log( - a - b*x)*a**8*b**5*c**5*d**4
 - 210*log( - a - b*x)*a**8*b**5*c**4*d**5*x + 105*log( - a - b*x)*a**8*b*
*5*c**3*d**6*x**2 + 420*log( - a - b*x)*a**8*b**5*c**2*d**7*x**3 + 210*log
( - a - b*x)*a**8*b**5*c*d**8*x**4 + 336*log( - a - b*x)*a**7*b**6*c**4*d*
*5*x**2 + 672*log( - a - b*x)*a**7*b**6*c**3*d**6*x**3 + 312*log( - a - b*
x)*a**7*b**6*c**2*d**7*x**4 - 48*log( - a - b*x)*a**7*b**6*c*d**8*x**5 - 2
4*log( - a - b*x)*a**7*b**6*d**9*x**6 + 21*log( - a - b*x)*a**6*b**7*c**7*
d**2 + 42*log( - a - b*x)*a**6*b**7*c**6*d**3*x + 231*log( - a - b*x)*a**6
*b**7*c**5*d**4*x**2 + 420*log( - a - b*x)*a**6*b**7*c**4*d**5*x**3 + 105*
log( - a - b*x)*a**6*b**7*c**3*d**6*x**4 - 210*log( - a - b*x)*a**6*b**7*c
**2*d**7*x**5 - 105*log( - a - b*x)*a**6*b**7*c*d**8*x**6 - 168*log( - a -
 b*x)*a**5*b**8*c**4*d**5*x**4 - 336*log( - a - b*x)*a**5*b**8*c**3*d**6*x
**5 - 168*log( - a - b*x)*a**5*b**8*c**2*d**7*x**6 - 3*log( - a - b*x)*a**
4*b**9*c**9 - 6*log( - a - b*x)*a**4*b**9*c**8*d*x - 45*log( - a - b*x)...
```

3.52 $\int (d+ex)^4 (a+cx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int (d+ex)^4 (a+cx^2)\,dx = \frac{(cd^2+ae^2)(d+ex)^5}{5e^3} - \frac{cd(d+ex)^6}{3e^3} + \frac{c(d+ex)^7}{7e^3}$$

output
```
1/5*(a*e^2+c*d^2)*(e*x+d)^5/e^3-1/3*c*d*(e*x+d)^6/e^3+1/7*c*(e*x+d)^7/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.77

$$\int (d+ex)^4 (a+cx^2)\,dx = ad^4x + 2ad^3ex^2 + \frac{1}{3}d^2(cd^2+6ae^2)x^3 + de(cd^2+ae^2)x^4 + \frac{1}{5}e^2(6cd^2+ae^2)x^5 + \frac{2}{3}cde^3x^6 + \frac{1}{7}ce^4x^7$$

input
```
Integrate[(d + e*x)^4*(a + c*x^2),x]
```

output
```
a*d^4*x + 2*a*d^3*e*x^2 + (d^2*(c*d^2 + 6*a*e^2)*x^3)/3 + d*e*(c*d^2 + a*e
^2)*x^4 + (e^2*(6*c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e^3*x^6)/3 + (c*e^4*x^7)/
7
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^4\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(d+ex)^4\left(ae^2+cd^2\right)}{e^2}+\frac{c(d+ex)^6}{e^2}-\frac{2cd(d+ex)^5}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^5\left(ae^2+cd^2\right)}{5e^3}+\frac{c(d+ex)^7}{7e^3}-\frac{cd(d+ex)^6}{3e^3}$$

input
```
Int[(d + e*x)^4*(a + c*x^2),x]
```

output
```
((c*d^2 + a*e^2)*(d + e*x)^5)/(5*e^3) - (c*d*(d + e*x)^6)/(3*e^3) + (c*(d
+ e*x)^7)/(7*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.65

method	result
norman	$\frac{e^4 c x^7}{7}+\frac{2d e^3 c x^6}{3}+\left(\frac{1}{5}e^4 a+\frac{6}{5}d^2e^2c\right)x^5+\left(d e^3 a+d^3 ec\right)x^4+\left(2d^2e^2a+\frac{1}{3}d^4c\right)x^3+2d^3ea\,x^2$
gosper	$\frac{1}{7}e^4c\,x^7+\frac{2}{3}d\,e^3c\,x^6+\frac{1}{5}x^5e^4a+\frac{6}{5}x^5d^2e^2c+ad\,e^3x^4+c\,d^3e\,x^4+2x^3d^2e^2a+\frac{1}{3}x^3d^4c+2d^3e$
default	$\frac{e^4 c x^7}{7}+\frac{2d e^3 c x^6}{3}+\frac{(e^4a+6d^2e^2c)x^5}{5}+\frac{(4d\,e^3a+4d^3ec)x^4}{4}+\frac{(6d^2e^2a+d^4c)x^3}{3}+2d^3ea\,x^2+xa\,d^4$
risch	$\frac{1}{7}e^4c\,x^7+\frac{2}{3}d\,e^3c\,x^6+\frac{1}{5}x^5e^4a+\frac{6}{5}x^5d^2e^2c+ad\,e^3x^4+c\,d^3e\,x^4+2x^3d^2e^2a+\frac{1}{3}x^3d^4c+2d^3e$
parallelrisch	$\frac{1}{7}e^4c\,x^7+\frac{2}{3}d\,e^3c\,x^6+\frac{1}{5}x^5e^4a+\frac{6}{5}x^5d^2e^2c+ad\,e^3x^4+c\,d^3e\,x^4+2x^3d^2e^2a+\frac{1}{3}x^3d^4c+2d^3e$
orering	$\frac{x(15e^4c\,x^6+70d\,e^3c\,x^5+21a\,e^4x^4+126c\,d^2e^2x^4+105ad\,e^3x^3+105c\,d^3e\,x^3+210a\,d^2e^2x^2+35c\,d^4x^2+210d^3eax+105a\,d^4)}{105}$

input
```
int((e*x+d)^4*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*e^4*c*x^7+2/3*d*e^3*c*x^6+(1/5*e^4*a+6/5*d^2*e^2*c)*x^5+(a*d*e^3+c*d^3
*e)*x^4+(2*d^2*e^2*a+1/3*d^4*c)*x^3+2*d^3*e*a*x^2+x*a*d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int (d+ex)^4\left(a+cx^2\right)\,dx=\frac{1}{7}\,ce^4x^7+\frac{2}{3}\,cde^3x^6+2\,ad^3ex^2+ad^4x+\frac{1}{5}\left(6\,cd^2e^2+ae^4\right)x^5+\left(cd^3e+ade^3\right)x^4+\frac{1}{3}\left(cd^4+6\,ad^2e^2\right)x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/7*c*e^4*x^7 + 2/3*c*d*e^3*x^6 + 2*a*d^3*e*x^2 + a*d^4*x + 1/5*(6*c*d^2*e
^2 + a*e^4)*x^5 + (c*d^3*e + a*d*e^3)*x^4 + 1/3*(c*d^4 + 6*a*d^2*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int (d+ex)^4 \left(a+cx^2\right) dx = ad^4x + 2ad^3ex^2 + \frac{2cde^3x^6}{3} + \frac{ce^4x^7}{7} + x^5\left(\frac{ae^4}{5} + \frac{6cd^2e^2}{5}\right) + x^4\left(ade^3 + cd^3e\right) + x^3 \cdot \left(2ad^2e^2 + \frac{cd^4}{3}\right)$$

input
```
integrate((e*x+d)**4*(c*x**2+a),x)
```

output
```
a*d**4*x + 2*a*d**3*e*x**2 + 2*c*d*e**3*x**6/3 + c*e**4*x**7/7 + x**5*(a*e
**4/5 + 6*c*d**2*e**2/5) + x**4*(a*d*e**3 + c*d**3*e) + x**3*(2*a*d**2*e**
2 + c*d**4/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int (d+ex)^4 \left(a+cx^2\right) dx = \frac{1}{7}ce^4x^7 + \frac{2}{3}cde^3x^6 + 2ad^3ex^2 + ad^4x + \frac{1}{5}\left(6cd^2e^2 + ae^4\right)x^5 + \left(cd^3e + ade^3\right)x^4 + \frac{1}{3}\left(cd^4 + 6ad^2e^2\right)x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/7*c*e^4*x^7 + 2/3*c*d*e^3*x^6 + 2*a*d^3*e*x^2 + a*d^4*x + 1/5*(6*c*d^2*e
^2 + a*e^4)*x^5 + (c*d^3*e + a*d*e^3)*x^4 + 1/3*(c*d^4 + 6*a*d^2*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int (d+ex)^4 \left(a+cx^2\right) dx = \frac{1}{7} ce^4x^7 + \frac{2}{3} cde^3x^6 + \frac{6}{5} cd^2e^2x^5 + \frac{1}{5} ae^4x^5 + cd^3ex^4 + ade^3x^4 + \frac{1}{3} cd^4x^3 + 2\, ad^2e^2x^3 + 2\, ad^3ex^2 + ad^4x$$

input
```
integrate((e*x+d)^4*(c*x^2+a),x, algorithm="giac")
```

output
```
1/7*c*e^4*x^7 + 2/3*c*d*e^3*x^6 + 6/5*c*d^2*e^2*x^5 + 1/5*a*e^4*x^5 + c*d^
3*e*x^4 + a*d*e^3*x^4 + 1/3*c*d^4*x^3 + 2*a*d^2*e^2*x^3 + 2*a*d^3*e*x^2 +
a*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int (d+ex)^4 \left(a+cx^2\right) dx = x^3 \left(\frac{c\, d^4}{3} + 2\, a\, d^2\, e^2\right) + x^5 \left(\frac{6\, c\, d^2\, e^2}{5} + \frac{a\, e^4}{5}\right) + x^4 \left(c\, d^3\, e + a\, d\, e^3\right) + \frac{c\, e^4\, x^7}{7} + a\, d^4\, x + 2\, a\, d^3\, e\, x^2 + \frac{2\, c\, d\, e^3\, x^6}{3}$$

input
```
int((a + c*x^2)*(d + e*x)^4,x)
```

output
```
x^3*((c*d^4)/3 + 2*a*d^2*e^2) + x^5*((a*e^4)/5 + (6*c*d^2*e^2)/5) + x^4*(a
*d*e^3 + c*d^3*e) + (c*e^4*x^7)/7 + a*d^4*x + 2*a*d^3*e*x^2 + (2*c*d*e^3*x
^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int (d+ex)^4\left(a+cx^2\right)\,dx$$
$$=\frac{x(15c\,e^4x^6+70cd\,e^3x^5+21a\,e^4x^4+126c\,d^2e^2x^4+105ad\,e^3x^3+105c\,d^3e\,x^3+210a\,d^2e^2x^2+35c\,d^4x^2+}{105}$$

input
```
int((e*x+d)^4*(c*x^2+a),x)
```

output
```
(x*(105*a*d**4 + 210*a*d**3*e*x + 210*a*d**2*e**2*x**2 + 105*a*d*e**3*x**3
 + 21*a*e**4*x**4 + 35*c*d**4*x**2 + 105*c*d**3*e*x**3 + 126*c*d**2*e**2*x
**4 + 70*c*d*e**3*x**5 + 15*c*e**4*x**6))/105
```

3.53 $\int (d+ex)^3 (a+cx^2) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int (d+ex)^3 (a+cx^2) \, dx = \frac{(cd^2+ae^2)(d+ex)^4}{4e^3} - \frac{2cd(d+ex)^5}{5e^3} + \frac{c(d+ex)^6}{6e^3}$$

output
```
1/4*(a*e^2+c*d^2)*(e*x+d)^4/e^3-2/5*c*d*(e*x+d)^5/e^3+1/6*c*(e*x+d)^6/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int (d+ex)^3 (a+cx^2) \, dx = \frac{1}{4}ax(4d^3+6d^2ex+4de^2x^2+e^3x^3) + \frac{1}{60}cx^3(20d^3+45d^2ex+36de^2x^2+10e^3x^3)$$

input
```
Integrate[(d + e*x)^3*(a + c*x^2),x]
```

output
```
(a*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))/4 + (c*x^3*(20*d^3 + 45*
d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3))/60
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^3dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(d+ex)^3\left(ae^2+cd^2\right)}{e^2}+\frac{c(d+ex)^5}{e^2}-\frac{2cd(d+ex)^4}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^4\left(ae^2+cd^2\right)}{4e^3}+\frac{c(d+ex)^6}{6e^3}-\frac{2cd(d+ex)^5}{5e^3}$$

input
```
Int[(d + e*x)^3*(a + c*x^2),x]
```

output
```
((c*d^2 + a*e^2)*(d + e*x)^4)/(4*e^3) - (2*c*d*(d + e*x)^5)/(5*e^3) + (c*(
d + e*x)^6)/(6*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{c\,e^{3} x^{6}}{6}+\frac{3 d\,e^{2} c\,x^{5}}{5}+\left(\frac{1}{4} a\,e^{3}+\frac{3}{4} d^{2} e c\right) x^{4}+\left(d\,e^{2} a+\frac{1}{3} d^{3} c\right) x^{3}+\frac{3 d^{2} e a\,x^{2}}{2}+a x\,d^{3}$	72
default	$\frac{c\,e^{3} x^{6}}{6}+\frac{3 d\,e^{2} c\,x^{5}}{5}+\frac{\left(a\,e^{3}+3 d^{2} e c\right) x^{4}}{4}+\frac{\left(3 d\,e^{2} a+d^{3} c\right) x^{3}}{3}+\frac{3 d^{2} e a\,x^{2}}{2}+a x\,d^{3}$	73
gosper	$\frac{1}{6} c\,e^{3} x^{6}+\frac{3}{5} d\,e^{2} c\,x^{5}+\frac{1}{4} x^{4} a\,e^{3}+\frac{3}{4} x^{4} d^{2} e c+x^{3} d\,e^{2} a+\frac{1}{3} d^{3} c\,x^{3}+\frac{3}{2} d^{2} e a\,x^{2}+a x\,d^{3}$	74
risch	$\frac{1}{6} c\,e^{3} x^{6}+\frac{3}{5} d\,e^{2} c\,x^{5}+\frac{1}{4} x^{4} a\,e^{3}+\frac{3}{4} x^{4} d^{2} e c+x^{3} d\,e^{2} a+\frac{1}{3} d^{3} c\,x^{3}+\frac{3}{2} d^{2} e a\,x^{2}+a x\,d^{3}$	74
parallelrisch	$\frac{1}{6} c\,e^{3} x^{6}+\frac{3}{5} d\,e^{2} c\,x^{5}+\frac{1}{4} x^{4} a\,e^{3}+\frac{3}{4} x^{4} d^{2} e c+x^{3} d\,e^{2} a+\frac{1}{3} d^{3} c\,x^{3}+\frac{3}{2} d^{2} e a\,x^{2}+a x\,d^{3}$	74
orering	$\frac{x\left(10 c\,e^{3} x^{5}+36 d\,e^{2} c\,x^{4}+15 a\,e^{3} x^{3}+45 c\,d^{2} e\,x^{3}+60 a d\,e^{2} x^{2}+20 c\,d^{3} x^{2}+90 d^{2} e a x+60 a\,d^{3}\right)}{60}$	76

input
```
int((e*x+d)^3*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*c*e^3*x^6+3/5*d*e^2*c*x^5+(1/4*a*e^3+3/4*d^2*e*c)*x^4+(d*e^2*a+1/3*d^3
*c)*x^3+3/2*d^2*e*a*x^2+a*x*d^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int (d+ex)^3\left(a+cx^2\right)\,dx=\frac{1}{6}\,ce^3x^6+\frac{3}{5}\,cde^2x^5+\frac{3}{2}\,ad^2ex^2+ad^3x+\frac{1}{4}\left(3\,cd^2e+ae^3\right)x^4+\frac{1}{3}\left(cd^3+3\,ade^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/6*c*e^3*x^6 + 3/5*c*d*e^2*x^5 + 3/2*a*d^2*e*x^2 + a*d^3*x + 1/4*(3*c*d^2
*e + a*e^3)*x^4 + 1/3*(c*d^3 + 3*a*d*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int (d+ex)^3\left(a+cx^2\right)\,dx = ad^3x + \frac{3ad^2ex^2}{2} + \frac{3cde^2x^5}{5} + \frac{ce^3x^6}{6} + x^4\left(\frac{ae^3}{4} + \frac{3cd^2e}{4}\right) + x^3\left(ade^2 + \frac{cd^3}{3}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+a),x)
```

output
```
a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a
*e**3/4 + 3*c*d**2*e/4) + x**3*(a*d*e**2 + c*d**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int (d+ex)^3\left(a+cx^2\right)\,dx = \frac{1}{6}\,ce^3x^6 + \frac{3}{5}\,cde^2x^5 + \frac{3}{2}\,ad^2ex^2 + ad^3x + \frac{1}{4}\left(3\,cd^2e + ae^3\right)x^4 + \frac{1}{3}\left(cd^3 + 3\,ade^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/6*c*e^3*x^6 + 3/5*c*d*e^2*x^5 + 3/2*a*d^2*e*x^2 + a*d^3*x + 1/4*(3*c*d^2
*e + a*e^3)*x^4 + 1/3*(c*d^3 + 3*a*d*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int (d+ex)^3 \left(a+cx^2\right) \, dx = \frac{1}{6}ce^3x^6 + \frac{3}{5}cde^2x^5 + \frac{3}{4}cd^2ex^4 + \frac{1}{4}ae^3x^4 + \frac{1}{3}cd^3x^3 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + ad^3x$$

input
```
integrate((e*x+d)^3*(c*x^2+a),x, algorithm="giac")
```

output
```
1/6*c*e^3*x^6 + 3/5*c*d*e^2*x^5 + 3/4*c*d^2*e*x^4 + 1/4*a*e^3*x^4 + 1/3*c*
d^3*x^3 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int (d+ex)^3 \left(a+cx^2\right) \, dx = x^3 \left(\frac{c\,d^3}{3} + a\,d\,e^2\right) + x^4 \left(\frac{3\,c\,d^2\,e}{4} + \frac{a\,e^3}{4}\right) + \frac{c\,e^3\,x^6}{6} + a\,d^3\,x + \frac{3\,a\,d^2\,e\,x^2}{2} + \frac{3\,c\,d\,e^2\,x^5}{5}$$

input
```
int((a + c*x^2)*(d + e*x)^3,x)
```

output
```
x^3*((c*d^3)/3 + a*d*e^2) + x^4*((a*e^3)/4 + (3*c*d^2*e)/4) + (c*e^3*x^6)/
6 + a*d^3*x + (3*a*d^2*e*x^2)/2 + (3*c*d*e^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int (d+ex)^3 \left(a+cx^2\right) dx$$
$$= \frac{x(10c\,e^3x^5+36cd\,e^2x^4+15a\,e^3x^3+45c\,d^2e\,x^3+60ad\,e^2x^2+20c\,d^3x^2+90a\,d^2ex+60a\,d^3)}{60}$$

input
```
int((e*x+d)^3*(c*x^2+a),x)
```

output
```
(x*(60*a*d**3 + 90*a*d**2*e*x + 60*a*d*e**2*x**2 + 15*a*e**3*x**3 + 20*c*d
**3*x**2 + 45*c*d**2*e*x**3 + 36*c*d*e**2*x**4 + 10*c*e**3*x**5))/60
```

3.54 $\int (d+ex)^2 (a+cx^2)\, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int (d+ex)^2 (a+cx^2)\, dx = \frac{(cd^2+ae^2)(d+ex)^3}{3e^3} - \frac{cd(d+ex)^4}{2e^3} + \frac{c(d+ex)^5}{5e^3}$$

output
```
1/3*(a*e^2+c*d^2)*(e*x+d)^3/e^3-1/2*c*d*(e*x+d)^4/e^3+1/5*c*(e*x+d)^5/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int (d+ex)^2 (a+cx^2)\, dx = ad^2x + adex^2 + \frac{1}{3}(cd^2+ae^2)x^3 + \frac{1}{2}cdex^4 + \frac{1}{5}ce^2x^5$$

input
```
Integrate[(d + e*x)^2*(a + c*x^2),x]
```

output
```
a*d^2*x + a*d*e*x^2 + ((c*d^2 + a*e^2)*x^3)/3 + (c*d*e*x^4)/2 + (c*e^2*x^5
)/5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)(d+ex)^2\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(d+ex)^2(ae^2+cd^2)}{e^2}+\frac{c(d+ex)^4}{e^2}-\frac{2cd(d+ex)^3}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^3(ae^2+cd^2)}{3e^3}+\frac{c(d+ex)^5}{5e^3}-\frac{cd(d+ex)^4}{2e^3}$$

input
```
Int[(d + e*x)^2*(a + c*x^2),x]
```

output
```
((c*d^2 + a*e^2)*(d + e*x)^3)/(3*e^3) - (c*d*(d + e*x)^4)/(2*e^3) + (c*(d
+ e*x)^5)/(5*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{c\,e^{2} x^{5}}{5}+\frac{d e c\,x^{4}}{2}+\frac{\left(a\,e^{2}+c\,d^{2}\right) x^{3}}{3}+a d e\,x^{2}+a\,d^{2} x$	48
norman	$\frac{c\,e^{2} x^{5}}{5}+\frac{d e c\,x^{4}}{2}+\left(\frac{a\,e^{2}}{3}+\frac{c\,d^{2}}{3}\right) x^{3}+a d e\,x^{2}+a\,d^{2} x$	49
gosper	$\frac{1}{5} c\,e^{2} x^{5}+\frac{1}{2} d e c\,x^{4}+\frac{1}{3} x^{3} a\,e^{2}+\frac{1}{3} x^{3} c\,d^{2}+a d e\,x^{2}+a\,d^{2} x$	50
risch	$\frac{1}{5} c\,e^{2} x^{5}+\frac{1}{2} d e c\,x^{4}+\frac{1}{3} x^{3} a\,e^{2}+\frac{1}{3} x^{3} c\,d^{2}+a d e\,x^{2}+a\,d^{2} x$	50
parallelrisch	$\frac{1}{5} c\,e^{2} x^{5}+\frac{1}{2} d e c\,x^{4}+\frac{1}{3} x^{3} a\,e^{2}+\frac{1}{3} x^{3} c\,d^{2}+a d e\,x^{2}+a\,d^{2} x$	50
orering	$\frac{x \left(6 c\,e^{2} x^{4}+15 d e c\,x^{3}+10 a\,e^{2} x^{2}+10 c\,d^{2} x^{2}+30 a d e x+30 a\,d^{2}\right)}{30}$	52

input
```
int((e*x+d)^2*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*c*e^2*x^5+1/2*d*e*c*x^4+1/3*(a*e^2+c*d^2)*x^3+a*d*e*x^2+a*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int (d+ex)^2\left(a+cx^2\right)\,dx=\frac{1}{5}\,ce^2x^5+\frac{1}{2}\,cdex^4+adex^2+ad^2x+\frac{1}{3}\,(cd^2+ae^2)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/5*c*e^2*x^5 + 1/2*c*d*e*x^4 + a*d*e*x^2 + a*d^2*x + 1/3*(c*d^2 + a*e^2)*
x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int (d+ex)^2 \left(a+cx^2\right) dx = ad^2x + adex^2 + \frac{cdex^4}{2} + \frac{ce^2x^5}{5} + x^3\left(\frac{ae^2}{3} + \frac{cd^2}{3}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+a),x)
```

output
```
a*d**2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2/3 + c*
d**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int (d+ex)^2 \left(a+cx^2\right) dx = \frac{1}{5}\,ce^2x^5 + \frac{1}{2}\,cdex^4 + adex^2 + ad^2x + \frac{1}{3}\,(cd^2 + ae^2)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/5*c*e^2*x^5 + 1/2*c*d*e*x^4 + a*d*e*x^2 + a*d^2*x + 1/3*(c*d^2 + a*e^2)*
x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int (d+ex)^2 \left(a+cx^2\right) dx = \frac{1}{5}\,ce^2x^5 + \frac{1}{2}\,cdex^4 + \frac{1}{3}\,cd^2x^3 + \frac{1}{3}\,ae^2x^3 + adex^2 + ad^2x$$

input
```
integrate((e*x+d)^2*(c*x^2+a),x, algorithm="giac")
```

output
```
1/5*c*e^2*x^5 + 1/2*c*d*e*x^4 + 1/3*c*d^2*x^3 + 1/3*a*e^2*x^3 + a*d*e*x^2
+ a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int (d+ex)^2\left(a+cx^2\right)\,dx=x^3\left(\frac{c\,d^2}{3}+\frac{a\,e^2}{3}\right)+\frac{c\,e^2\,x^5}{5}+a\,d^2\,x+a\,d\,e\,x^2+\frac{c\,d\,e\,x^4}{2}$$

input
```
int((a + c*x^2)*(d + e*x)^2,x)
```

output
```
x^3*((a*e^2)/3 + (c*d^2)/3) + (c*e^2*x^5)/5 + a*d^2*x + a*d*e*x^2 + (c*d*e
*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int (d+ex)^2\left(a+cx^2\right)\,dx$$
$$=\frac{x(6c\,e^2x^4+15cde\,x^3+10a\,e^2x^2+10c\,d^2x^2+30adex+30a\,d^2)}{30}$$

input
```
int((e*x+d)^2*(c*x^2+a),x)
```

output
```
(x*(30*a*d**2 + 30*a*d*e*x + 10*a*e**2*x**2 + 10*c*d**2*x**2 + 15*c*d*e*x*
*3 + 6*c*e**2*x**4))/30
```

3.55 $\int (d+ex)\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int (d+ex)\left(a+cx^2\right)\,dx=adx+\frac{1}{3}cdx^3+\frac{e\left(a+cx^2\right)^2}{4c}$$

output
```
a*d*x+1/3*c*d*x^3+1/4*e*(c*x^2+a)^2/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (d+ex)\left(a+cx^2\right)\,dx=adx+\frac{1}{2}aex^2+\frac{1}{3}cdx^3+\frac{1}{4}cex^4$$

input
```
Integrate[(d + e*x)*(a + c*x^2),x]
```

output
```
a*d*x + (a*e*x^2)/2 + (c*d*x^3)/3 + (c*e*x^4)/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {455, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)(d+ex)\,dx$$

$$\Big\downarrow \text{455}$$

$$d\int (cx^2+a)\,dx+\frac{e(a+cx^2)^2}{4c}$$

$$\Big\downarrow \text{2009}$$

$$d\left(ax+\frac{cx^3}{3}\right)+\frac{e(a+cx^2)^2}{4c}$$

input
```
Int[(d + e*x)*(a + c*x^2),x]
```

output
```
(e*(a + c*x^2)^2)/(4*c) + d*(a*x + (c*x^3)/3)
```

Defintions of rubi rules used

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
gosper	$\frac{1}{4}ce\,x^4+\frac{1}{3}cd\,x^3+\frac{1}{2}ae\,x^2+adx$	27
default	$\frac{1}{4}ce\,x^4+\frac{1}{3}cd\,x^3+\frac{1}{2}ae\,x^2+adx$	27
norman	$\frac{1}{4}ce\,x^4+\frac{1}{3}cd\,x^3+\frac{1}{2}ae\,x^2+adx$	27
risch	$\frac{1}{4}ce\,x^4+\frac{1}{3}cd\,x^3+\frac{1}{2}ae\,x^2+adx$	27
parallelrisch	$\frac{1}{4}ce\,x^4+\frac{1}{3}cd\,x^3+\frac{1}{2}ae\,x^2+adx$	27
orering	$\frac{x(3ce\,x^3+4cd\,x^2+6aex+12ad)}{12}$	28

input
```
int((e*x+d)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*e*x^4+1/3*c*d*x^3+1/2*a*e*x^2+a*d*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (d+ex)\left(a+cx^2\right)\,dx=\frac{1}{4}\,cex^4+\frac{1}{3}\,cdx^3+\frac{1}{2}\,aex^2+adx$$

input
```
integrate((e*x+d)*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*c*e*x^4 + 1/3*c*d*x^3 + 1/2*a*e*x^2 + a*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (d+ex)\left(a+cx^2\right)\,dx = adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}$$

input
```
integrate((e*x+d)*(c*x**2+a),x)
```

output
```
a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (d+ex)\left(a+cx^2\right)\,dx = \frac{1}{4}\,cex^4 + \frac{1}{3}\,cdx^3 + \frac{1}{2}\,aex^2 + adx$$

input
```
integrate((e*x+d)*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/4*c*e*x^4 + 1/3*c*d*x^3 + 1/2*a*e*x^2 + a*d*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (d+ex)\left(a+cx^2\right)\,dx = \frac{1}{4}\,cex^4 + \frac{1}{3}\,cdx^3 + \frac{1}{2}\,aex^2 + adx$$

input
```
integrate((e*x+d)*(c*x^2+a),x, algorithm="giac")
```

output
```
1/4*c*e*x^4 + 1/3*c*d*x^3 + 1/2*a*e*x^2 + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (d+ex)\left(a+cx^2\right)\,dx=\frac{c\,e\,x^4}{4}+\frac{c\,d\,x^3}{3}+\frac{a\,e\,x^2}{2}+a\,d\,x$$

input
```
int((a + c*x^2)*(d + e*x),x)
```

output
```
a*d*x + (a*e*x^2)/2 + (c*d*x^3)/3 + (c*e*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(a+cx^2\right)\,dx=\frac{x(3ce\,x^3+4cd\,x^2+6aex+12ad)}{12}$$

input
```
int((e*x+d)*(c*x^2+a),x)
```

output
```
(x*(12*a*d + 6*a*e*x + 4*c*d*x**2 + 3*c*e*x**3))/12
```

3.56 $\int \frac{a+cx^2}{d+ex} dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{a+cx^2}{d+ex} dx = -\frac{cdx}{e^2} + \frac{cx^2}{2e} + \frac{(cd^2+ae^2)\log(d+ex)}{e^3}$$

output
```
-c*d*x/e^2+1/2*c*x^2/e+(a*e^2+c*d^2)*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^2}{d+ex} dx = \frac{cex(-2d+ex)+2(cd^2+ae^2)\log(d+ex)}{2e^3}$$

input
```
Integrate[(a + c*x^2)/(d + e*x),x]
```

output
```
(c*e*x*(-2*d + e*x) + 2*(c*d^2 + a*e^2)*Log[d + e*x])/(2*e^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{d+ex}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)}-\frac{cd}{e^2}+\frac{cx}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ae^2+cd^2)\log(d+ex)}{e^3}-\frac{cdx}{e^2}+\frac{cx^2}{2e}$$

input
```
Int[(a + c*x^2)/(d + e*x),x]
```

output
```
-((c*d*x)/e^2) + (c*x^2)/(2*e) + ((c*d^2 + a*e^2)*Log[d + e*x])/e^3
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{c\left(-\frac{1}{2} e\, x^{2}+d x \right)}{e^{2}}+\frac{\left(a\, e^{2}+c\, d^{2}\right) \ln \left(e x +d \right)}{e^{3}}$	39
norman	$-\frac{c d x}{e^{2}}+\frac{c\, x^{2}}{2 e}+\frac{\left(a\, e^{2}+c\, d^{2}\right) \ln \left(e x +d \right)}{e^{3}}$	40
risch	$\frac{c\, x^{2}}{2 e}-\frac{c d x}{e^{2}}+\frac{\ln \left(e x +d \right) a}{e}+\frac{\ln \left(e x +d \right) c\, d^{2}}{e^{3}}$	44
parallelrisch	$\frac{x^{2} c\, e^{2}+2 \ln \left(e x +d \right) a\, e^{2}+2 \ln \left(e x +d \right) c\, d^{2}-2 c d x e}{2 e^{3}}$	45

input
```
int((c*x^2+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^2*(-1/2*e*x^2+d*x)+(a*e^2+c*d^2)*ln(e*x+d)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{a+cx^2}{d+ex}\,dx = \frac{ce^2x^2 - 2\,cdex + 2\,(cd^2 + ae^2)\log\left(ex+d\right)}{2\,e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/2*(c*e^2*x^2 - 2*c*d*e*x + 2*(c*d^2 + a*e^2)*log(e*x + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{a+cx^2}{d+ex}\,dx = -\frac{cdx}{e^2} + \frac{cx^2}{2e} + \frac{(ae^2+cd^2)\log\left(d+ex\right)}{e^3}$$

input
```
integrate((c*x**2+a)/(e*x+d),x)
```

output
```
-c*d*x/e**2 + c*x**2/(2*e) + (a*e**2 + c*d**2)*log(d + e*x)/e**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{a+cx^2}{d+ex}\,dx = \frac{cex^2-2\,cdx}{2\,e^2}+\frac{(cd^2+ae^2)\log(ex+d)}{e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*(c*e*x^2 - 2*c*d*x)/e^2 + (c*d^2 + a*e^2)*log(e*x + d)/e^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{a+cx^2}{d+ex}\,dx = \frac{cex^2-2\,cdx}{2\,e^2}+\frac{(cd^2+ae^2)\log(|ex+d|)}{e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/2*(c*e*x^2 - 2*c*d*x)/e^2 + (c*d^2 + a*e^2)*log(abs(e*x + d))/e^3
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{a+cx^2}{d+ex}\,dx = \frac{c\,x^2}{2\,e}+\frac{\ln(d+e\,x)\,\left(c\,d^2+a\,e^2\right)}{e^3}-\frac{c\,d\,x}{e^2}$$

input
```
int((a + c*x^2)/(d + e*x),x)
```

output
```
(c*x^2)/(2*e) + (log(d + e*x)*(a*e^2 + c*d^2))/e^3 - (c*d*x)/e^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{a+cx^2}{d+ex}\,dx = \frac{2\log(ex+d)\,a\,e^2+2\log(ex+d)\,c\,d^2-2cdex+c\,e^2x^2}{2e^3}$$

input
```
int((c*x^2+a)/(e*x+d),x)
```

output
```
(2*log(d + e*x)*a*e**2 + 2*log(d + e*x)*c*d**2 - 2*c*d*e*x + c*e**2*x**2)/
(2*e**3)
```

3.57 $\int \frac{a+cx^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx=\frac{cx}{e^2}-\frac{cd^2+ae^2}{e^3(d+ex)}-\frac{2cd\log(d+ex)}{e^3}$$

output
```
c*x/e^2-(a*e^2+c*d^2)/e^3/(e*x+d)-2*c*d*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx=\frac{cex-\frac{cd^2+ae^2}{d+ex}-2cd\log(d+ex)}{e^3}$$

input
```
Integrate[(a + c*x^2)/(d + e*x)^2,x]
```

output
```
(c*e*x - (c*d^2 + a*e^2)/(d + e*x) - 2*c*d*Log[d + e*x])/e^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)^2}-\frac{2cd}{e^2(d+ex)}+\frac{c}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ae^2+cd^2}{e^3(d+ex)}-\frac{2cd\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

input
```
Int[(a + c*x^2)/(d + e*x)^2,x]
```

output
```
(c*x)/e^2 - (c*d^2 + a*e^2)/(e^3*(d + e*x)) - (2*c*d*Log[d + e*x])/e^3
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{cx}{e^2}-\frac{a\,e^2+c\,d^2}{e^3(ex+d)}-\frac{2cd\ln(ex+d)}{e^3}$	44
norman	$\frac{\frac{c\,x^2}{e}-\frac{a\,e^2+2c\,d^2}{e^3}}{ex+d}-\frac{2cd\ln(ex+d)}{e^3}$	49
risch	$\frac{cx}{e^2}-\frac{a}{e(ex+d)}-\frac{c\,d^2}{e^3(ex+d)}-\frac{2cd\ln(ex+d)}{e^3}$	50
parallelrisch	$-\frac{2\ln(ex+d)xcde-x^2c\,e^2+2\ln(ex+d)c\,d^2+a\,e^2+2c\,d^2}{e^3(ex+d)}$	58

input
```
int((c*x^2+a)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2-(a*e^2+c*d^2)/e^3/(e*x+d)-2*c*d*ln(e*x+d)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx=\frac{ce^2x^2+cdex-cd^2-ae^2-2\,(cdex+cd^2)\log{(ex+d)}}{e^4x+de^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
(c*e^2*x^2 + c*d*e*x - c*d^2 - a*e^2 - 2*(c*d*e*x + c*d^2)*log(e*x + d))/(
e^4*x + d*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx = -\frac{2cd\log(d+ex)}{e^3}+\frac{cx}{e^2}+\frac{-ae^2-cd^2}{de^3+e^4x}$$

input
```
integrate((c*x**2+a)/(e*x+d)**2,x)
```

output
```
-2*c*d*log(d + e*x)/e**3 + c*x/e**2 + (-a*e**2 - c*d**2)/(d*e**3 + e**4*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx = -\frac{cd^2+ae^2}{e^4x+de^3}+\frac{cx}{e^2}-\frac{2\,cd\log(ex+d)}{e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c*d^2 + a*e^2)/(e^4*x + d*e^3) + c*x/e^2 - 2*c*d*log(e*x + d)/e^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx = c\left(\frac{2\,d\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^3}+\frac{ex+d}{e^3}-\frac{d^2}{(ex+d)e^3}\right)-\frac{a}{(ex+d)e}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
c*(2*d*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + (e*x + d)/e^3 - d^2/((
e*x + d)*e^3)) - a/((e*x + d)*e)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx = \frac{c\,x}{e^2} - \frac{c\,d^2+a\,e^2}{e\,\left(x\,e^3+d\,e^2\right)} - \frac{2\,c\,d\,\ln\left(d+e\,x\right)}{e^3}$$

input
```
int((a + c*x^2)/(d + e*x)^2,x)
```

output
```
(c*x)/e^2 - (a*e^2 + c*d^2)/(e*(d*e^2 + e^3*x)) - (2*c*d*log(d + e*x))/e^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{a+cx^2}{(d+ex)^2}\,dx = \frac{-2\log(ex+d)\,c\,d^3-2\log(ex+d)\,c\,d^2ex+a\,e^3x+2c\,d^2ex+cd\,e^2x^2}{d\,e^3\,(ex+d)}$$

input
```
int((c*x^2+a)/(e*x+d)^2,x)
```

output
```
( - 2*log(d + e*x)*c*d**3 - 2*log(d + e*x)*c*d**2*e*x + a*e**3*x + 2*c*d**
2*e*x + c*d*e**2*x**2)/(d*e**3*(d + e*x))
```

3.58 $\int \frac{a+cx^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{a+cx^2}{(d+ex)^3}\,dx = \frac{-cd^2-ae^2}{2e^3(d+ex)^2} + \frac{2cd}{e^3(d+ex)} + \frac{c\log(d+ex)}{e^3}$$

output
```
1/2*(-a*e^2-c*d^2)/e^3/(e*x+d)^2+2*c*d/e^3/(e*x+d)+c*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^2}{(d+ex)^3}\,dx = \frac{-ae^2+cd(3d+4ex)+2c(d+ex)^2\log(d+ex)}{2e^3(d+ex)^2}$$

input
```
Integrate[(a + c*x^2)/(d + e*x)^3,x]
```

output
```
(-(a*e^2) + c*d*(3*d + 4*e*x) + 2*c*(d + e*x)^2*Log[d + e*x])/(2*e^3*(d + 
e*x)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)^3}+\frac{c}{e^2(d+ex)}-\frac{2cd}{e^2(d+ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ae^2+cd^2}{2e^3(d+ex)^2}+\frac{2cd}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

input
```
Int[(a + c*x^2)/(d + e*x)^3,x]
```

output
```
-1/2*(c*d^2 + a*e^2)/(e^3*(d + e*x)^2) + (2*c*d)/(e^3*(d + e*x)) + (c*Log[
d + e*x])/e^3
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
norman	$\frac{-\frac{a\,e^{2}-3c\,d^{2}}{2e^{3}}+\frac{2cdx}{e^{2}}}{\left(ex+d\right)^{2}}+\frac{c\ln\left(ex+d\right)}{e^{3}}$	47
risch	$\frac{-\frac{a\,e^{2}-3c\,d^{2}}{2e^{3}}+\frac{2cdx}{e^{2}}}{\left(ex+d\right)^{2}}+\frac{c\ln\left(ex+d\right)}{e^{3}}$	47
default	$\frac{c\ln\left(ex+d\right)}{e^{3}}+\frac{2cd}{e^{3}\left(ex+d\right)}-\frac{a\,e^{2}+c\,d^{2}}{2e^{3}\left(ex+d\right)^{2}}$	50
parallelrisch	$\frac{2\ln\left(ex+d\right)x^{2}c\,e^{2}+4\ln\left(ex+d\right)xcde+2\ln\left(ex+d\right)c\,d^{2}+4cdxe-a\,e^{2}+3c\,d^{2}}{2e^{3}\left(ex+d\right)^{2}}$	71

input
```
int((c*x^2+a)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(a*e^2-3*c*d^2)/e^3+2*c*d*x/e^2)/(e*x+d)^2+c*ln(e*x+d)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{a+cx^2}{(d+ex)^3}\,dx = \frac{4\,cdex+3\,cd^2-ae^2+2\,(ce^2x^2+2\,cdex+cd^2)\log{(ex+d)}}{2\,(e^5x^2+2\,de^4x+d^2e^3)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(4*c*d*e*x + 3*c*d^2 - a*e^2 + 2*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*log(e
*x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{a+cx^2}{(d+ex)^3}\,dx = \frac{c\log(d+ex)}{e^3} + \frac{-ae^2+3cd^2+4cdex}{2d^2e^3+4de^4x+2e^5x^2}$$

input
```
integrate((c*x**2+a)/(e*x+d)**3,x)
```

output
```
c*log(d + e*x)/e**3 + (-a*e**2 + 3*c*d**2 + 4*c*d*e*x)/(2*d**2*e**3 + 4*d*
e**4*x + 2*e**5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{a+cx^2}{(d+ex)^3}\,dx = \frac{4\,cdex+3\,cd^2-ae^2}{2\,(e^5x^2+2\,de^4x+d^2e^3)} + \frac{c\log(ex+d)}{e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(4*c*d*e*x + 3*c*d^2 - a*e^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) + c*log(
e*x + d)/e^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^2}{(d+ex)^3}\,dx = \frac{c\log(|ex+d|)}{e^3} + \frac{4\,cdx+\frac{3\,cd^2-ae^2}{e}}{2\,(ex+d)^2e^2}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3,x, algorithm="giac")
```

output
```
c*log(abs(e*x + d))/e^3 + 1/2*(4*c*d*x + (3*c*d^2 - a*e^2)/e)/((e*x + d)^2
*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{a + cx^2}{(d + ex)^3} dx = \frac{c \ln (d + e x)}{e^3} - \frac{\frac{a e^2 - 3 c d^2}{2 e^3} - \frac{2 c d x}{e^2}}{d^2 + 2 d e x + e^2 x^2}$$

input
```
int((a + c*x^2)/(d + e*x)^3,x)
```

output
```
(c*log(d + e*x))/e^3 - ((a*e^2 - 3*c*d^2)/(2*e^3) - (2*c*d*x)/e^2)/(d^2 +
e^2*x^2 + 2*d*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{a + cx^2}{(d + ex)^3} dx = \frac{2 \log(ex + d) c d^2 + 4 \log(ex + d) cdex + 2 \log(ex + d) c e^2 x^2 - a e^2 + c d^2 - 2c e^2 x^2}{2e^3 (e^2 x^2 + 2dex + d^2)}$$

input
```
int((c*x^2+a)/(e*x+d)^3,x)
```

output
```
(2*log(d + e*x)*c*d**2 + 4*log(d + e*x)*c*d*e*x + 2*log(d + e*x)*c*e**2*x*
*2 - a*e**2 + c*d**2 - 2*c*e**2*x**2)/(2*e**3*(d**2 + 2*d*e*x + e**2*x**2)
)
```

3.59 $\int \frac{a+cx^2}{(d+ex)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{a+cx^2}{(d+ex)^4} dx = \frac{-cd^2-ae^2}{3e^3(d+ex)^3} + \frac{cd}{e^3(d+ex)^2} - \frac{c}{e^3(d+ex)}$$

output
```
1/3*(-a*e^2-c*d^2)/e^3/(e*x+d)^3+c*d/e^3/(e*x+d)^2-c/e^3/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{(d+ex)^4} dx = -\frac{ae^2+c(d^2+3dex+3e^2x^2)}{3e^3(d+ex)^3}$$

input
```
Integrate[(a + c*x^2)/(d + e*x)^4,x]
```

output
```
-1/3*(a*e^2 + c*(d^2 + 3*d*e*x + 3*e^2*x^2))/(e^3*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^4} \, dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)^4}+\frac{c}{e^2(d+ex)^2}-\frac{2cd}{e^2(d+ex)^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{ae^2+cd^2}{3e^3(d+ex)^3}-\frac{c}{e^3(d+ex)}+\frac{cd}{e^3(d+ex)^2}$$

input
```
Int[(a + c*x^2)/(d + e*x)^4,x]
```

output
```
-1/3*(c*d^2 + a*e^2)/(e^3*(d + e*x)^3) + (c*d)/(e^3*(d + e*x)^2) - c/(e^3*
(d + e*x))
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{3x^{2}c\,e^{2}+3cdxe+a\,e^{2}+c\,d^{2}}{3\left(ex+d\right)^{3}e^{3}}$	39
orering	$-\frac{3x^{2}c\,e^{2}+3cdxe+a\,e^{2}+c\,d^{2}}{3\left(ex+d\right)^{3}e^{3}}$	39
parallelrisch	$\frac{-3x^{2}c\,e^{2}-3cdxe-a\,e^{2}-c\,d^{2}}{3e^{3}\left(ex+d\right)^{3}}$	41
norman	$\frac{-\frac{c\,x^{2}}{e}-\frac{cdx}{e^{2}}-\frac{a\,e^{2}+c\,d^{2}}{3e^{3}}}{\left(ex+d\right)^{3}}$	43
risch	$\frac{-\frac{c\,x^{2}}{e}-\frac{cdx}{e^{2}}-\frac{a\,e^{2}+c\,d^{2}}{3e^{3}}}{\left(ex+d\right)^{3}}$	43
default	$-\frac{a\,e^{2}+c\,d^{2}}{3e^{3}\left(ex+d\right)^{3}}-\frac{c}{e^{3}\left(ex+d\right)}+\frac{cd}{e^{3}\left(ex+d\right)^{2}}$	51

input
```
int((c*x^2+a)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*c*e^2*x^2+3*c*d*e*x+a*e^2+c*d^2)/(e*x+d)^3/e^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{a+cx^2}{(d+ex)^4}\,dx=-\frac{3\,ce^2x^2+3\,cdex+cd^2+ae^2}{3\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/3*(3*c*e^2*x^2 + 3*c*d*e*x + c*d^2 + a*e^2)/(e^6*x^3 + 3*d*e^5*x^2 + 3*
d^2*e^4*x + d^3*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{a+cx^2}{(d+ex)^4}\,dx = \frac{-ae^2-cd^2-3cdex-3ce^2x^2}{3d^3e^3+9d^2e^4x+9de^5x^2+3e^6x^3}$$

input
```
integrate((c*x**2+a)/(e*x+d)**4,x)
```

output
```
(-a*e**2 - c*d**2 - 3*c*d*e*x - 3*c*e**2*x**2)/(3*d**3*e**3 + 9*d**2*e**4*
x + 9*d*e**5*x**2 + 3*e**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{a+cx^2}{(d+ex)^4}\,dx = -\frac{3\,ce^2x^2+3\,cdex+cd^2+ae^2}{3\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/3*(3*c*e^2*x^2 + 3*c*d*e*x + c*d^2 + a*e^2)/(e^6*x^3 + 3*d*e^5*x^2 + 3*
d^2*e^4*x + d^3*e^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{a+cx^2}{(d+ex)^4}\,dx = -\frac{3\,ce^2x^2+3\,cdex+cd^2+ae^2}{3\,(ex+d)^3e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^4,x, algorithm="giac")
```

output
```
-1/3*(3*c*e^2*x^2 + 3*c*d*e*x + c*d^2 + a*e^2)/((e*x + d)^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{a+cx^2}{(d+ex)^4} \, dx = -\frac{\frac{c\,d^2+a\,e^2}{3\,e^3}+\frac{c\,x^2}{e}+\frac{c\,d\,x}{e^2}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

input
```
int((a + c*x^2)/(d + e*x)^4,x)
```

output
```
-((a*e^2 + c*d^2)/(3*e^3) + (c*x^2)/e + (c*d*x)/e^2)/(d^3 + e^3*x^3 + 3*d*
e^2*x^2 + 3*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{a+cx^2}{(d+ex)^4} \, dx = \frac{ce\,x^3-ad}{3de\,(e^3x^3+3d\,e^2x^2+3d^2ex+d^3)}$$

input
```
int((c*x^2+a)/(e*x+d)^4,x)
```

output
```
( - a*d + c*e*x**3)/(3*d*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3)
)
```

3.60 $\int \frac{a+cx^2}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx=\frac{-cd^2-ae^2}{4e^3(d+ex)^4}+\frac{2cd}{3e^3(d+ex)^3}-\frac{c}{2e^3(d+ex)^2}$$

output
```
1/4*(-a*e^2-c*d^2)/e^3/(e*x+d)^4+2/3*c*d/e^3/(e*x+d)^3-1/2*c/e^3/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx=-\frac{3ae^2+c(d^2+4dex+6e^2x^2)}{12e^3(d+ex)^4}$$

input
```
Integrate[(a + c*x^2)/(d + e*x)^5,x]
```

output
```
-1/12*(3*a*e^2 + c*(d^2 + 4*d*e*x + 6*e^2*x^2))/(e^3*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^5} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)^5}+\frac{c}{e^2(d+ex)^3}-\frac{2cd}{e^2(d+ex)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ae^2+cd^2}{4e^3(d+ex)^4}-\frac{c}{2e^3(d+ex)^2}+\frac{2cd}{3e^3(d+ex)^3}$$

input
```
Int[(a + c*x^2)/(d + e*x)^5,x]
```

output
```
-1/4*(c*d^2 + a*e^2)/(e^3*(d + e*x)^4) + (2*c*d)/(3*e^3*(d + e*x)^3) - c/(
2*e^3*(d + e*x)^2)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{6x^{2} c \,e^{2}+4 c d x e+3 a \,e^{2}+c \,d^{2}}{12 e^{3} \left(e x+d\right)^{4}}$	40
orering	$-\frac{6x^{2} c \,e^{2}+4 c d x e+3 a \,e^{2}+c \,d^{2}}{12 e^{3} \left(e x+d\right)^{4}}$	40
risch	$\frac{-\frac{c \,x^{2}}{2 e}-\frac{c d x}{3 e^{2}}-\frac{3 a \,e^{2}+c \,d^{2}}{12 e^{3}}}{\left(e x+d\right)^{4}}$	44
parallelrisch	$\frac{-6 c \,x^{2} e^{3}-4 c d x \,e^{2}-3 a \,e^{3}-d^{2} e c}{12 e^{4} \left(e x+d\right)^{4}}$	44
norman	$\frac{-\frac{c \,x^{2}}{2 e}-\frac{c d x}{3 e^{2}}-\frac{3 a \,e^{3}+d^{2} e c}{12 e^{4}}}{\left(e x+d\right)^{4}}$	45
default	$\frac{2 c d}{3 e^{3} \left(e x+d\right)^{3}}-\frac{a \,e^{2}+c \,d^{2}}{4 e^{3} \left(e x+d\right)^{4}}-\frac{c}{2 e^{3} \left(e x+d\right)^{2}}$	52

input
```
int((c*x^2+a)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/e^3*(6*c*e^2*x^2+4*c*d*e*x+3*a*e^2+c*d^2)/(e*x+d)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx=-\frac{6\,ce^2x^2+4\,cdex+cd^2+3\,ae^2}{12\,(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
-1/12*(6*c*e^2*x^2 + 4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^7*x^4 + 4*d*e^6*x^3 +
 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx = \frac{-3ae^2-cd^2-4cdex-6ce^2x^2}{12d^4e^3+48d^3e^4x+72d^2e^5x^2+48de^6x^3+12e^7x^4}$$

input
```
integrate((c*x**2+a)/(e*x+d)**5,x)
```

output
```
(-3*a*e**2 - c*d**2 - 4*c*d*e*x - 6*c*e**2*x**2)/(12*d**4*e**3 + 48*d**3*e
**4*x + 72*d**2*e**5*x**2 + 48*d*e**6*x**3 + 12*e**7*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx = -\frac{6\,ce^2x^2+4\,cdex+cd^2+3\,ae^2}{12\,(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
-1/12*(6*c*e^2*x^2 + 4*c*d*e*x + c*d^2 + 3*a*e^2)/(e^7*x^4 + 4*d*e^6*x^3 +
 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx = -\frac{\frac{3\,a}{(ex+d)^4}+\frac{6\,c}{(ex+d)^2e^2}-\frac{8\,cd}{(ex+d)^3e^2}+\frac{3\,cd^2}{(ex+d)^4e^2}}{12\,e}$$

input
```
integrate((c*x^2+a)/(e*x+d)^5,x, algorithm="giac")
```

output
```
-1/12*(3*a/(e*x + d)^4 + 6*c/((e*x + d)^2*e^2) - 8*c*d/((e*x + d)^3*e^2) +
 3*c*d^2/((e*x + d)^4*e^2))/e
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx = -\frac{\frac{c\,d^2+3\,a\,e^2}{12\,e^3}+\frac{c\,x^2}{2\,e}+\frac{c\,d\,x}{3\,e^2}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int((a + c*x^2)/(d + e*x)^5,x)
```

output
```
-((3*a*e^2 + c*d^2)/(12*e^3) + (c*x^2)/(2*e) + (c*d*x)/(3*e^2))/(d^4 + e^4
*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{a+cx^2}{(d+ex)^5}\,dx = \frac{-6c\,e^2x^2-4cdex-3a\,e^2-c\,d^2}{12e^3\,(e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4)}$$

input
```
int((c*x^2+a)/(e*x+d)^5,x)
```

output
```
( - 3*a*e**2 - c*d**2 - 4*c*d*e*x - 6*c*e**2*x**2)/(12*e**3*(d**4 + 4*d**3
*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.61 $\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 117

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx = \frac{\left(cd^2+ae^2\right)^2 (d+ex)^5}{5e^5} - \frac{2cd\left(cd^2+ae^2\right)(d+ex)^6}{3e^5} + \frac{2c\left(3cd^2+ae^2\right)(d+ex)^7}{7e^5} - \frac{c^2d(d+ex)^8}{2e^5} + \frac{c^2(d+ex)^9}{9e^5}$$

output
```
1/5*(a*e^2+c*d^2)^2*(e*x+d)^5/e^5-2/3*c*d*(a*e^2+c*d^2)*(e*x+d)^6/e^5+2/7*
c*(a*e^2+3*c*d^2)*(e*x+d)^7/e^5-1/2*c^2*d*(e*x+d)^8/e^5+1/9*c^2*(e*x+d)^9/
e^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.43

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx = a^2d^4x + 2a^2d^3ex^2 + \frac{2}{3}ad^2\left(cd^2+3ae^2\right)x^3 + ade\left(2cd^2+ae^2\right)x^4 + \frac{1}{5}\left(c^2d^4+12acd^2e^2+a^2e^4\right)x^5 + \frac{2}{3}cde\left(cd^2+2ae^2\right)x^6 + \frac{2}{7}ce^2\left(3cd^2+ae^2\right)x^7 + \frac{1}{2}c^2de^3x^8 + \frac{1}{9}c^2e^4x^9$$

input
```
Integrate[(d + e*x)^4*(a + c*x^2)^2,x]
```

output
```
a^2*d^4*x + 2*a^2*d^3*e*x^2 + (2*a*d^2*(c*d^2 + 3*a*e^2)*x^3)/3 + a*d*e*(2
*c*d^2 + a*e^2)*x^4 + ((c^2*d^4 + 12*a*c*d^2*e^2 + a^2*e^4)*x^5)/5 + (2*c*
d*e*(c*d^2 + 2*a*e^2)*x^6)/3 + (2*c*e^2*(3*c*d^2 + a*e^2)*x^7)/7 + (c^2*d*
e^3*x^8)/2 + (c^2*e^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (d+ex)^4 \, dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{2c(d+ex)^6 (ae^2+3cd^2)}{e^4} - \frac{4cd(d+ex)^5 (ae^2+cd^2)}{e^4} + \frac{(d+ex)^4 (ae^2+cd^2)^2}{e^4} + \frac{c^2(d+ex)^8}{e^4} - \frac{4c^2d(d+e}{e^4}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2c(d+ex)^7 (ae^2+3cd^2)}{7e^5} - \frac{2cd(d+ex)^6 (ae^2+cd^2)}{3e^5} + \frac{(d+ex)^5 (ae^2+cd^2)^2}{5e^5} + \frac{c^2(d+ex)^9}{9e^5} - \frac{c^2d(d+ex)^8}{2e^5}$$

input
```
Int[(d + e*x)^4*(a + c*x^2)^2,x]
```

output
```
((c*d^2 + a*e^2)^2*(d + e*x)^5)/(5*e^5) - (2*c*d*(c*d^2 + a*e^2)*(d + e*x)
^6)/(3*e^5) + (2*c*(3*c*d^2 + a*e^2)*(d + e*x)^7)/(7*e^5) - (c^2*d*(d + e*
x)^8)/(2*e^5) + (c^2*(d + e*x)^9)/(9*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

method	result
norman	$\frac{e^4c^2x^9}{9}+\frac{d\,e^3c^2x^8}{2}+\left(\frac{2}{7}e^4ac+\frac{6}{7}d^2e^2c^2\right)x^7+\left(\frac{4}{3}d\,e^3ac+\frac{2}{3}d^3e\,c^2\right)x^6+\left(\frac{1}{5}a^2e^4+\frac{12}{5}ac\,d^2e^2+\frac{1}{5}\right.$
default	$\frac{e^4c^2x^9}{9}+\frac{d\,e^3c^2x^8}{2}+\frac{(2e^4ac+6d^2e^2c^2)x^7}{7}+\frac{(8d\,e^3ac+4d^3e\,c^2)x^6}{6}+\frac{(a^2e^4+12ac\,d^2e^2+c^2d^4)x^5}{5}+\frac{(4a^2d\,e^3+8d^3ea}{4}$
gosper	$\frac{1}{9}e^4c^2x^9+\frac{1}{2}d\,e^3c^2x^8+\frac{2}{7}x^7e^4ac+\frac{6}{7}x^7d^2e^2c^2+\frac{4}{3}x^6d\,e^3ac+\frac{2}{3}x^6d^3e\,c^2+\frac{1}{5}x^5a^2e^4+\frac{12}{5}x^5ac\,d$
risch	$\frac{1}{9}e^4c^2x^9+\frac{1}{2}d\,e^3c^2x^8+\frac{2}{7}x^7e^4ac+\frac{6}{7}x^7d^2e^2c^2+\frac{4}{3}x^6d\,e^3ac+\frac{2}{3}x^6d^3e\,c^2+\frac{1}{5}x^5a^2e^4+\frac{12}{5}x^5ac\,d$
parallelrisch	$\frac{1}{9}e^4c^2x^9+\frac{1}{2}d\,e^3c^2x^8+\frac{2}{7}x^7e^4ac+\frac{6}{7}x^7d^2e^2c^2+\frac{4}{3}x^6d\,e^3ac+\frac{2}{3}x^6d^3e\,c^2+\frac{1}{5}x^5a^2e^4+\frac{12}{5}x^5ac\,d$
orering	$\frac{x(70e^4c^2x^8+315d\,e^3c^2x^7+180ac\,e^4x^6+540c^2d^2e^2x^6+840acd\,e^3x^5+420c^2d^3e\,x^5+126a^2e^4x^4+1512ac\,d^2e^2x^4+126c^2d^4x^4+}{630}$

input
```
int((e*x+d)^4*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*e^4*c^2*x^9+1/2*d*e^3*c^2*x^8+(2/7*e^4*a*c+6/7*d^2*e^2*c^2)*x^7+(4/3*d
*e^3*a*c+2/3*d^3*e*c^2)*x^6+(1/5*a^2*e^4+12/5*a*c*d^2*e^2+1/5*c^2*d^4)*x^5
+(a^2*d*e^3+2*a*c*d^3*e)*x^4+(2*d^2*e^2*a^2+2/3*a*c*d^4)*x^3+2*d^3*e*a^2*x
^2+a^2*d^4*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.39

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \,dx = \frac{1}{9}\,c^2e^4x^9 + \frac{1}{2}\,c^2de^3x^8 + 2\,a^2d^3ex^2 + \frac{2}{7}\left(3\,c^2d^2e^2 + ace^4\right)x^7 + a^2d^4x + \frac{2}{3}\left(c^2d^3e + 2\,acde^3\right)x^6 + \frac{1}{5}\left(c^2d^4 + 12\,acd^2e^2 + a^2e^4\right)x^5 + \left(2\,acd^3e + a^2de^3\right)x^4 + \frac{2}{3}\left(acd^4 + 3\,a^2d^2e^2\right)x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*e^4*x^9 + 1/2*c^2*d*e^3*x^8 + 2*a^2*d^3*e*x^2 + 2/7*(3*c^2*d^2*e^2
 + a*c*e^4)*x^7 + a^2*d^4*x + 2/3*(c^2*d^3*e + 2*a*c*d*e^3)*x^6 + 1/5*(c^2
*d^4 + 12*a*c*d^2*e^2 + a^2*e^4)*x^5 + (2*a*c*d^3*e + a^2*d*e^3)*x^4 + 2/3
*(a*c*d^4 + 3*a^2*d^2*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.56

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \,dx = a^2d^4x + 2a^2d^3ex^2 + \frac{c^2de^3x^8}{2} + \frac{c^2e^4x^9}{9} + x^7\cdot\left(\frac{2ace^4}{7} + \frac{6c^2d^2e^2}{7}\right) + x^6\cdot\left(\frac{4acde^3}{3} + \frac{2c^2d^3e}{3}\right) + x^5\left(\frac{a^2e^4}{5} + \frac{12acd^2e^2}{5} + \frac{c^2d^4}{5}\right) + x^4\left(a^2de^3 + 2acd^3e\right) + x^3\cdot\left(2a^2d^2e^2 + \frac{2acd^4}{3}\right)$$

input
```
integrate((e*x+d)**4*(c*x**2+a)**2,x)
```

output
```
a**2*d**4*x + 2*a**2*d**3*e*x**2 + c**2*d*e**3*x**8/2 + c**2*e**4*x**9/9 +
 x**7*(2*a*c*e**4/7 + 6*c**2*d**2*e**2/7) + x**6*(4*a*c*d*e**3/3 + 2*c**2*
d**3*e/3) + x**5*(a**2*e**4/5 + 12*a*c*d**2*e**2/5 + c**2*d**4/5) + x**4*(
a**2*d*e**3 + 2*a*c*d**3*e) + x**3*(2*a**2*d**2*e**2 + 2*a*c*d**4/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.39

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx = \frac{1}{9}\,c^2e^4x^9 + \frac{1}{2}\,c^2de^3x^8 + 2\,a^2d^3ex^2 + \frac{2}{7}\left(3\,c^2d^2e^2 + ace^4\right)x^7 + a^2d^4x + \frac{2}{3}\left(c^2d^3e + 2\,acde^3\right)x^6 + \frac{1}{5}\left(c^2d^4 + 12\,acd^2e^2 + a^2e^4\right)x^5 + \left(2\,acd^3e + a^2de^3\right)x^4 + \frac{2}{3}\left(acd^4 + 3\,a^2d^2e^2\right)x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*e^4*x^9 + 1/2*c^2*d*e^3*x^8 + 2*a^2*d^3*e*x^2 + 2/7*(3*c^2*d^2*e^2
 + a*c*e^4)*x^7 + a^2*d^4*x + 2/3*(c^2*d^3*e + 2*a*c*d*e^3)*x^6 + 1/5*(c^2
*d^4 + 12*a*c*d^2*e^2 + a^2*e^4)*x^5 + (2*a*c*d^3*e + a^2*d*e^3)*x^4 + 2/3
*(a*c*d^4 + 3*a^2*d^2*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.47

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx = \frac{1}{9}\,c^2e^4x^9 + \frac{1}{2}\,c^2de^3x^8 + \frac{6}{7}\,c^2d^2e^2x^7 + \frac{2}{7}\,ace^4x^7 + \frac{2}{3}\,c^2d^3ex^6 + \frac{4}{3}\,acde^3x^6 + \frac{1}{5}\,c^2d^4x^5 + \frac{12}{5}\,acd^2e^2x^5 + \frac{1}{5}\,a^2e^4x^5 + 2\,acd^3ex^4 + a^2de^3x^4 + \frac{2}{3}\,acd^4x^3 + 2\,a^2d^2e^2x^3 + 2\,a^2d^3ex^2 + a^2d^4x$$

input
```
integrate((e*x+d)^4*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*e^4*x^9 + 1/2*c^2*d*e^3*x^8 + 6/7*c^2*d^2*e^2*x^7 + 2/7*a*c*e^4*x^
7 + 2/3*c^2*d^3*e*x^6 + 4/3*a*c*d*e^3*x^6 + 1/5*c^2*d^4*x^5 + 12/5*a*c*d^2
*e^2*x^5 + 1/5*a^2*e^4*x^5 + 2*a*c*d^3*e*x^4 + a^2*d*e^3*x^4 + 2/3*a*c*d^4
*x^3 + 2*a^2*d^2*e^2*x^3 + 2*a^2*d^3*e*x^2 + a^2*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx = x^5 \left(\frac{a^2 e^4}{5} + \frac{12 a c d^2 e^2}{5} + \frac{c^2 d^4}{5}\right) + x^3 \left(2 a^2 d^2 e^2 + \frac{2 c a d^4}{3}\right) + x^7 \left(\frac{6 c^2 d^2 e^2}{7} + \frac{2 a c e^4}{7}\right) + a^2 d^4 x + \frac{c^2 e^4 x^9}{9} + 2 a^2 d^3 e x^2 + \frac{c^2 d e^3 x^8}{2} + a d e x^4 \left(2 c d^2 + a e^2\right) + \frac{2 c d e x^6 \left(c d^2 + 2 a e^2\right)}{3}$$

input
```
int((a + c*x^2)^2*(d + e*x)^4,x)
```

output
```
x^5*((a^2*e^4)/5 + (c^2*d^4)/5 + (12*a*c*d^2*e^2)/5) + x^3*(2*a^2*d^2*e^2
+ (2*a*c*d^4)/3) + x^7*((6*c^2*d^2*e^2)/7 + (2*a*c*e^4)/7) + a^2*d^4*x + (
c^2*e^4*x^9)/9 + 2*a^2*d^3*e*x^2 + (c^2*d*e^3*x^8)/2 + a*d*e*x^4*(a*e^2 +
2*c*d^2) + (2*c*d*e*x^6*(2*a*e^2 + c*d^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.49

$$\int (d+ex)^4 \left(a+cx^2\right)^2 \, dx = \frac{x(70c^2e^4x^8 + 315c^2d e^3x^7 + 180ac e^4x^6 + 540c^2d^2e^2x^6 + 840acd e^3x^5 + 420c^2d^3e x^5 + 126a^2e^4x^4 + 1512}{}$$

input
```
int((e*x+d)^4*(c*x^2+a)^2,x)
```

output
```
(x*(630*a**2*d**4 + 1260*a**2*d**3*e*x + 1260*a**2*d**2*e**2*x**2 + 630*a*
*2*d*e**3*x**3 + 126*a**2*e**4*x**4 + 420*a*c*d**4*x**2 + 1260*a*c*d**3*e*
x**3 + 1512*a*c*d**2*e**2*x**4 + 840*a*c*d*e**3*x**5 + 180*a*c*e**4*x**6 +
 126*c**2*d**4*x**4 + 420*c**2*d**3*e*x**5 + 540*c**2*d**2*e**2*x**6 + 315
*c**2*d*e**3*x**7 + 70*c**2*e**4*x**8))/630
```

3.62 $\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 117

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = \frac{\left(cd^2+ae^2\right)^2 (d+ex)^4}{4e^5} - \frac{4cd\left(cd^2+ae^2\right)(d+ex)^5}{5e^5} + \frac{c\left(3cd^2+ae^2\right)(d+ex)^6}{3e^5} - \frac{4c^2d(d+ex)^7}{7e^5} + \frac{c^2(d+ex)^8}{8e^5}$$

output
```
1/4*(a*e^2+c*d^2)^2*(e*x+d)^4/e^5-4/5*c*d*(a*e^2+c*d^2)*(e*x+d)^5/e^5+1/3*
c*(a*e^2+3*c*d^2)*(e*x+d)^6/e^5-4/7*c^2*d*(e*x+d)^7/e^5+1/8*c^2*(e*x+d)^8/
e^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = \frac{1}{4}a^2x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right) + \frac{1}{30}acx^3\left(20d^3+45d^2ex+36de^2x^2+10e^3x^3\right) + \frac{1}{280}c^2x^5\left(56d^3+140d^2ex+120de^2x^2+35e^3x^3\right)$$

input
```
Integrate[(d + e*x)^3*(a + c*x^2)^2,x]
```

output
```
(a^2*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))/4 + (a*c*x^3*(20*d^3 +
 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3))/30 + (c^2*x^5*(56*d^3 + 140*d^2*
e*x + 120*d*e^2*x^2 + 35*e^3*x^3))/280
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (d+ex)^3 \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(d+ex)^5 \left(ae^2+3cd^2\right)}{e^4}-\frac{4cd(d+ex)^4 \left(ae^2+cd^2\right)}{e^4}+\frac{(d+ex)^3 \left(ae^2+cd^2\right)^2}{e^4}+\frac{c^2(d+ex)^7}{e^4}-\frac{4c^2d(d+e}{e^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(d+ex)^6 \left(ae^2+3cd^2\right)}{3e^5}-\frac{4cd(d+ex)^5 \left(ae^2+cd^2\right)}{5e^5}+\frac{(d+ex)^4 \left(ae^2+cd^2\right)^2}{4e^5}+\frac{c^2(d+ex)^8}{8e^5}-\frac{4c^2d(d+ex)^7}{7e^5}$$

input
```
Int[(d + e*x)^3*(a + c*x^2)^2,x]
```

output
```
((c*d^2 + a*e^2)^2*(d + e*x)^4)/(4*e^5) - (4*c*d*(c*d^2 + a*e^2)*(d + e*x)
^5)/(5*e^5) + (c*(3*c*d^2 + a*e^2)*(d + e*x)^6)/(3*e^5) - (4*c^2*d*(d + e*
x)^7)/(7*e^5) + (c^2*(d + e*x)^8)/(8*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

method	result
norman	$\frac{e^3c^2x^8}{8}+\frac{3d\,e^2c^2x^7}{7}+\left(\frac{1}{3}e^3ac+\frac{1}{2}d^2e\,c^2\right)x^6+\left(\frac{6}{5}d\,e^2ac+\frac{1}{5}d^3c^2\right)x^5+\left(\frac{1}{4}a^2e^3+\frac{3}{2}d^2eac\right)x^4+($
default	$\frac{e^3c^2x^8}{8}+\frac{3d\,e^2c^2x^7}{7}+\frac{(2e^3ac+3d^2e\,c^2)x^6}{6}+\frac{(6d\,e^2ac+d^3c^2)x^5}{5}+\frac{(a^2e^3+6d^2eac)x^4}{4}+\frac{(3a^2d\,e^2+2ac\,d^3)x^3}{3}+\frac{3d^2}{}$
gosper	$\frac{1}{8}e^3c^2x^8+\frac{3}{7}d\,e^2c^2x^7+\frac{1}{3}x^6e^3ac+\frac{1}{2}x^6d^2e\,c^2+\frac{6}{5}x^5d\,e^2ac+\frac{1}{5}x^5d^3c^2+\frac{1}{4}x^4a^2e^3+\frac{3}{2}x^4d^2eac$
risch	$\frac{1}{8}e^3c^2x^8+\frac{3}{7}d\,e^2c^2x^7+\frac{1}{3}x^6e^3ac+\frac{1}{2}x^6d^2e\,c^2+\frac{6}{5}x^5d\,e^2ac+\frac{1}{5}x^5d^3c^2+\frac{1}{4}x^4a^2e^3+\frac{3}{2}x^4d^2eac$
parallelrisch	$\frac{1}{8}e^3c^2x^8+\frac{3}{7}d\,e^2c^2x^7+\frac{1}{3}x^6e^3ac+\frac{1}{2}x^6d^2e\,c^2+\frac{6}{5}x^5d\,e^2ac+\frac{1}{5}x^5d^3c^2+\frac{1}{4}x^4a^2e^3+\frac{3}{2}x^4d^2eac$
orering	$\frac{x(105e^3c^2x^7+360d\,e^2c^2x^6+280ac\,e^3x^5+420c^2d^2e\,x^5+1008acd\,e^2x^4+168c^2d^3x^4+210a^2e^3x^3+1260ac\,d^2e\,x^3+840a^2d\,e^2x^2}{840}$

input
```
int((e*x+d)^3*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*e^3*c^2*x^8+3/7*d*e^2*c^2*x^7+(1/3*e^3*a*c+1/2*d^2*e*c^2)*x^6+(6/5*d*e
^2*a*c+1/5*d^3*c^2)*x^5+(1/4*a^2*e^3+3/2*d^2*e*a*c)*x^4+(a^2*d*e^2+2/3*a*c
*d^3)*x^3+3/2*d^2*e*a^2*x^2+a^2*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = \frac{1}{8}\,c^2e^3x^8 + \frac{3}{7}\,c^2de^2x^7 + \frac{3}{2}\,a^2d^2ex^2 + \frac{1}{6}\left(3\,c^2d^2e + 2\,ace^3\right)x^6 + a^2d^3x + \frac{1}{5}\left(c^2d^3 + 6\,acde^2\right)x^5 + \frac{1}{4}\left(6\,acd^2e + a^2e^3\right)x^4 + \frac{1}{3}\left(2\,acd^3 + 3\,a^2de^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*c^2*e^3*x^8 + 3/7*c^2*d*e^2*x^7 + 3/2*a^2*d^2*e*x^2 + 1/6*(3*c^2*d^2*e
 + 2*a*c*e^3)*x^6 + a^2*d^3*x + 1/5*(c^2*d^3 + 6*a*c*d*e^2)*x^5 + 1/4*(6*a
*c*d^2*e + a^2*e^3)*x^4 + 1/3*(2*a*c*d^3 + 3*a^2*d*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = a^2d^3x + \frac{3a^2d^2ex^2}{2} + \frac{3c^2de^2x^7}{7} + \frac{c^2e^3x^8}{8} + x^6\left(\frac{ace^3}{3} + \frac{c^2d^2e}{2}\right) + x^5\cdot\left(\frac{6acde^2}{5} + \frac{c^2d^3}{5}\right) + x^4\left(\frac{a^2e^3}{4} + \frac{3acd^2e}{2}\right) + x^3\left(a^2de^2 + \frac{2acd^3}{3}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**2,x)
```

output
```
a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + 3*c**2*d*e**2*x**7/7 + c**2*e**3*x**8
/8 + x**6*(a*c*e**3/3 + c**2*d**2*e/2) + x**5*(6*a*c*d*e**2/5 + c**2*d**3/
5) + x**4*(a**2*e**3/4 + 3*a*c*d**2*e/2) + x**3*(a**2*d*e**2 + 2*a*c*d**3/
3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = \frac{1}{8}\,c^2e^3x^8 + \frac{3}{7}\,c^2de^2x^7 + \frac{3}{2}\,a^2d^2ex^2 + \frac{1}{6}\left(3\,c^2d^2e + 2\,ace^3\right)x^6 + a^2d^3x + \frac{1}{5}\left(c^2d^3 + 6\,acde^2\right)x^5 + \frac{1}{4}\left(6\,acd^2e + a^2e^3\right)x^4 + \frac{1}{3}\left(2\,acd^3 + 3\,a^2de^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/8*c^2*e^3*x^8 + 3/7*c^2*d*e^2*x^7 + 3/2*a^2*d^2*e*x^2 + 1/6*(3*c^2*d^2*e
 + 2*a*c*e^3)*x^6 + a^2*d^3*x + 1/5*(c^2*d^3 + 6*a*c*d*e^2)*x^5 + 1/4*(6*a
*c*d^2*e + a^2*e^3)*x^4 + 1/3*(2*a*c*d^3 + 3*a^2*d*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = \frac{1}{8}\,c^2e^3x^8 + \frac{3}{7}\,c^2de^2x^7 + \frac{1}{2}\,c^2d^2ex^6 + \frac{1}{3}\,ace^3x^6 + \frac{1}{5}\,c^2d^3x^5 + \frac{6}{5}\,acde^2x^5 + \frac{3}{2}\,acd^2ex^4 + \frac{1}{4}\,a^2e^3x^4 + \frac{2}{3}\,acd^3x^3 + a^2de^2x^3 + \frac{3}{2}\,a^2d^2ex^2 + a^2d^3x$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*c^2*e^3*x^8 + 3/7*c^2*d*e^2*x^7 + 1/2*c^2*d^2*e*x^6 + 1/3*a*c*e^3*x^6
+ 1/5*c^2*d^3*x^5 + 6/5*a*c*d*e^2*x^5 + 3/2*a*c*d^2*e*x^4 + 1/4*a^2*e^3*x^
4 + 2/3*a*c*d^3*x^3 + a^2*d*e^2*x^3 + 3/2*a^2*d^2*e*x^2 + a^2*d^3*x
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = x^3 \left(a^2 d e^2 + \frac{2 c a d^3}{3}\right) + x^4 \left(\frac{a^2 e^3}{4} + \frac{3 c a d^2 e}{2}\right) + x^5 \left(\frac{c^2 d^3}{5} + \frac{6 a c d e^2}{5}\right) + x^6 \left(\frac{c^2 d^2 e}{2} + \frac{a c e^3}{3}\right) + a^2 d^3 x + \frac{c^2 e^3 x^8}{8} + \frac{3 a^2 d^2 e x^2}{2} + \frac{3 c^2 d e^2 x^7}{7}$$

input
```
int((a + c*x^2)^2*(d + e*x)^3,x)
```

output
```
x^3*(a^2*d*e^2 + (2*a*c*d^3)/3) + x^4*((a^2*e^3)/4 + (3*a*c*d^2*e)/2) + x^
5*((c^2*d^3)/5 + (6*a*c*d*e^2)/5) + x^6*((c^2*d^2*e)/2 + (a*c*e^3)/3) + a^
2*d^3*x + (c^2*e^3*x^8)/8 + (3*a^2*d^2*e*x^2)/2 + (3*c^2*d*e^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \, dx = \frac{x(105c^2e^3x^7 + 360c^2d e^2x^6 + 280ac e^3x^5 + 420c^2d^2e x^5 + 1008acd e^2x^4 + 168c^2d^3x^4 + 210a^2e^3x^3 + 1260}{840}$$

input
```
int((e*x+d)^3*(c*x^2+a)^2,x)
```

output
```
(x*(840*a**2*d**3 + 1260*a**2*d**2*e*x + 840*a**2*d*e**2*x**2 + 210*a**2*e
**3*x**3 + 560*a*c*d**3*x**2 + 1260*a*c*d**2*e*x**3 + 1008*a*c*d*e**2*x**4
 + 280*a*c*e**3*x**5 + 168*c**2*d**3*x**4 + 420*c**2*d**2*e*x**5 + 360*c**
2*d*e**2*x**6 + 105*c**2*e**3*x**7))/840
```

3.63 $\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx = a^2d^2x + \frac{1}{3}a\left(2cd^2+ae^2\right)x^3 + \frac{1}{5}c\left(cd^2+2ae^2\right)x^5 + \frac{1}{7}c^2e^2x^7 + \frac{de\left(a+cx^2\right)^3}{3c}$$

output
```
a^2*d^2*x+1/3*a*(a*e^2+2*c*d^2)*x^3+1/5*c*(2*a*e^2+c*d^2)*x^5+1/7*c^2*e^2*
x^7+1/3*d*e*(c*x^2+a)^3/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx = a^2d^2x + a^2dex^2 + \frac{1}{3}a\left(2cd^2+ae^2\right)x^3 + acdex^4 + \frac{1}{5}c\left(cd^2+2ae^2\right)x^5 + \frac{1}{3}c^2dex^6 + \frac{1}{7}c^2e^2x^7$$

input
```
Integrate[(d + e*x)^2*(a + c*x^2)^2,x]
```

output
```
a^2*d^2*x + a^2*d*e*x^2 + (a*(2*c*d^2 + a*e^2)*x^3)/3 + a*c*d*e*x^4 + (c*(
c*d^2 + 2*a*e^2)*x^5)/5 + (c^2*d*e*x^6)/3 + (c^2*e^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (d+ex)^2 \, dx$$

$$\Big\downarrow \text{475}$$

$$\int \left(c^2e^2x^6+c\left(cd^2+2ae^2\right)x^4+a\left(2cd^2+ae^2\right)x^2+a^2d^2\right)dx+\frac{de\left(a+cx^2\right)^3}{3c}$$

$$\Big\downarrow \text{2009}$$

$$a^2d^2x+\frac{1}{5}cx^5\left(2ae^2+cd^2\right)+\frac{1}{3}ax^3\left(ae^2+2cd^2\right)+\frac{de\left(a+cx^2\right)^3}{3c}+\frac{1}{7}c^2e^2x^7$$

input
```
Int[(d + e*x)^2*(a + c*x^2)^2,x]
```

output
```
a^2*d^2*x + (a*(2*c*d^2 + a*e^2)*x^3)/3 + (c*(c*d^2 + 2*a*e^2)*x^5)/5 + (c
^2*e^2*x^7)/7 + (d*e*(a + c*x^2)^3)/(3*c)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

method	result
default	$\frac{c^2e^2x^7}{7}+\frac{de\,c^2x^6}{3}+\frac{(2ac\,e^2+d^2c^2)x^5}{5}+acde\,x^4+\frac{(e^2a^2+2a\,d^2c)x^3}{3}+de\,a^2x^2+a^2d^2x$
norman	$\frac{c^2e^2x^7}{7}+\frac{de\,c^2x^6}{3}+\left(\frac{2}{5}ac\,e^2+\frac{1}{5}d^2c^2\right)x^5+acde\,x^4+\left(\frac{1}{3}e^2a^2+\frac{2}{3}a\,d^2c\right)x^3+de\,a^2x^2+a^2d^2x$
gosper	$\frac{1}{7}c^2e^2x^7+\frac{1}{3}de\,c^2x^6+\frac{2}{5}x^5ac\,e^2+\frac{1}{5}x^5d^2c^2+acde\,x^4+\frac{1}{3}x^3e^2a^2+\frac{2}{3}ac\,d^2x^3+de\,a^2x^2+a^2d$
risch	$\frac{1}{7}c^2e^2x^7+\frac{1}{3}de\,c^2x^6+\frac{2}{5}x^5ac\,e^2+\frac{1}{5}x^5d^2c^2+acde\,x^4+\frac{1}{3}x^3e^2a^2+\frac{2}{3}ac\,d^2x^3+de\,a^2x^2+a^2d$
parallelrisch	$\frac{1}{7}c^2e^2x^7+\frac{1}{3}de\,c^2x^6+\frac{2}{5}x^5ac\,e^2+\frac{1}{5}x^5d^2c^2+acde\,x^4+\frac{1}{3}x^3e^2a^2+\frac{2}{3}ac\,d^2x^3+de\,a^2x^2+a^2d$
orering	$\frac{x\left(15e^2c^2x^6+35de\,c^2x^5+42ac\,e^2x^4+21c^2d^2x^4+105acde\,x^3+35a^2e^2x^2+70a\,d^2x^2c+105de\,a^2x+105a^2d^2\right)}{105}$

input
```
int((e*x+d)^2*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*c^2*e^2*x^7+1/3*d*e*c^2*x^6+1/5*(2*a*c*e^2+c^2*d^2)*x^5+a*c*d*e*x^4+1/
3*(a^2*e^2+2*a*c*d^2)*x^3+d*e*a^2*x^2+a^2*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx = \frac{1}{7}\,c^2e^2x^7 + \frac{1}{3}\,c^2dex^6 + acdex^4 + a^2dex^2 + \frac{1}{5}\left(c^2d^2 + 2\,ace^2\right)x^5 + a^2d^2x + \frac{1}{3}\left(2\,acd^2 + a^2e^2\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/7*c^2*e^2*x^7 + 1/3*c^2*d*e*x^6 + a*c*d*e*x^4 + a^2*d*e*x^2 + 1/5*(c^2*d
^2 + 2*a*c*e^2)*x^5 + a^2*d^2*x + 1/3*(2*a*c*d^2 + a^2*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx = a^2d^2x + a^2dex^2 + acdex^4 + \frac{c^2dex^6}{3} + \frac{c^2e^2x^7}{7} + x^5 \cdot \left(\frac{2ace^2}{5} + \frac{c^2d^2}{5}\right) + x^3\left(\frac{a^2e^2}{3} + \frac{2acd^2}{3}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**2,x)
```

output
```
a**2*d**2*x + a**2*d*e*x**2 + a*c*d*e*x**4 + c**2*d*e*x**6/3 + c**2*e**2*x
**7/7 + x**5*(2*a*c*e**2/5 + c**2*d**2/5) + x**3*(a**2*e**2/3 + 2*a*c*d**2
/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int (d+ex)^2\left(a+cx^2\right)^2\,dx=\frac{1}{7}\,c^2e^2x^7+\frac{1}{3}\,c^2dex^6+acdex^4+a^2dex^2+\frac{1}{5}\,\left(c^2d^2+2\,ace^2\right)x^5+a^2d^2x+\frac{1}{3}\,\left(2\,acd^2+a^2e^2\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/7*c^2*e^2*x^7 + 1/3*c^2*d*e*x^6 + a*c*d*e*x^4 + a^2*d*e*x^2 + 1/5*(c^2*d
^2 + 2*a*c*e^2)*x^5 + a^2*d^2*x + 1/3*(2*a*c*d^2 + a^2*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int (d+ex)^2\left(a+cx^2\right)^2\,dx=\frac{1}{7}\,c^2e^2x^7+\frac{1}{3}\,c^2dex^6+\frac{1}{5}\,c^2d^2x^5+\frac{2}{5}\,ace^2x^5+acdex^4+\frac{2}{3}\,acd^2x^3+\frac{1}{3}\,a^2e^2x^3+a^2dex^2+a^2d^2x$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/7*c^2*e^2*x^7 + 1/3*c^2*d*e*x^6 + 1/5*c^2*d^2*x^5 + 2/5*a*c*e^2*x^5 + a*
c*d*e*x^4 + 2/3*a*c*d^2*x^3 + 1/3*a^2*e^2*x^3 + a^2*d*e*x^2 + a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int (d+ex)^2\left(a+cx^2\right)^2\,dx=x^3\,\left(\frac{a^2\,e^2}{3}+\frac{2\,c\,a\,d^2}{3}\right)+x^5\,\left(\frac{c^2\,d^2}{5}+\frac{2\,a\,c\,e^2}{5}\right)+a^2\,d^2\,x+\frac{c^2\,e^2\,x^7}{7}+a^2\,d\,e\,x^2+\frac{c^2\,d\,e\,x^6}{3}+a\,c\,d\,e\,x^4$$

input
```
int((a + c*x^2)^2*(d + e*x)^2,x)
```

output
```
x^3*((a^2*e^2)/3 + (2*a*c*d^2)/3) + x^5*((c^2*d^2)/5 + (2*a*c*e^2)/5) + a^
2*d^2*x + (c^2*e^2*x^7)/7 + a^2*d*e*x^2 + (c^2*d*e*x^6)/3 + a*c*d*e*x^4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \, dx = \frac{x(15c^2e^2x^6 + 35c^2de\,x^5 + 42ac\,e^2x^4 + 21c^2d^2x^4 + 105acde\,x^3 + 35a^2e^2x^2 + 70ac\,d^2x^2 + 105a^2dex + 105}{105}$$

input
```
int((e*x+d)^2*(c*x^2+a)^2,x)
```

output
```
(x*(105*a**2*d**2 + 105*a**2*d*e*x + 35*a**2*e**2*x**2 + 70*a*c*d**2*x**2
+ 105*a*c*d*e*x**3 + 42*a*c*e**2*x**4 + 21*c**2*d**2*x**4 + 35*c**2*d*e*x*
*5 + 15*c**2*e**2*x**6))/105
```

3.64 $\int (d+ex) \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int (d+ex) \left(a+cx^2\right)^2 \, dx = a^2dx + \frac{2}{3}acdx^3 + \frac{1}{5}c^2dx^5 + \frac{e(a+cx^2)^3}{6c}$$

output
```
a^2*d*x+2/3*a*c*d*x^3+1/5*c^2*d*x^5+1/6*e*(c*x^2+a)^3/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int (d+ex) \left(a+cx^2\right)^2 \, dx = a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}acdx^3 + \frac{1}{2}acex^4 + \frac{1}{5}c^2dx^5 + \frac{1}{6}c^2ex^6$$

input
```
Integrate[(d + e*x)*(a + c*x^2)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (2*a*c*d*x^3)/3 + (a*c*e*x^4)/2 + (c^2*d*x^5)/5
+ (c^2*e*x^6)/6
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + cx^2)^2 (d + ex) \, dx$$

$$\Big\downarrow \ 455$$

$$d \int (cx^2 + a)^2 \, dx + \frac{e(a + cx^2)^3}{6c}$$

$$\Big\downarrow \ 210$$

$$d \int (c^2x^4 + 2acx^2 + a^2) \, dx + \frac{e(a + cx^2)^3}{6c}$$

$$\Big\downarrow \ 2009$$

$$d\left(a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}\right) + \frac{e(a + cx^2)^3}{6c}$$

input
```
Int[(d + e*x)*(a + c*x^2)^2,x]
```

output
```
(e*(a + c*x^2)^3)/(6*c) + d*(a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{1}{6}c^2e\,x^6+\frac{1}{5}c^2d\,x^5+\frac{1}{2}ace\,x^4+\frac{2}{3}acd\,x^3+\frac{1}{2}a^2e\,x^2+a^2dx$	51
default	$\frac{1}{6}c^2e\,x^6+\frac{1}{5}c^2d\,x^5+\frac{1}{2}ace\,x^4+\frac{2}{3}acd\,x^3+\frac{1}{2}a^2e\,x^2+a^2dx$	51
norman	$\frac{1}{6}c^2e\,x^6+\frac{1}{5}c^2d\,x^5+\frac{1}{2}ace\,x^4+\frac{2}{3}acd\,x^3+\frac{1}{2}a^2e\,x^2+a^2dx$	51
risch	$\frac{1}{6}c^2e\,x^6+\frac{1}{5}c^2d\,x^5+\frac{1}{2}ace\,x^4+\frac{2}{3}acd\,x^3+\frac{1}{2}a^2e\,x^2+a^2dx$	51
parallelrisch	$\frac{1}{6}c^2e\,x^6+\frac{1}{5}c^2d\,x^5+\frac{1}{2}ace\,x^4+\frac{2}{3}acd\,x^3+\frac{1}{2}a^2e\,x^2+a^2dx$	51
orering	$\frac{x(5e\,c^2x^5+6c^2d\,x^4+15ace\,x^3+20ad\,x^2c+15a^2ex+30a^2d)}{30}$	52

input
```
int((e*x+d)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*e*x^6+1/5*c^2*d*x^5+1/2*a*c*e*x^4+2/3*a*c*d*x^3+1/2*a^2*e*x^2+a^2*
d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+cx^2\right)^2\,dx=\frac{1}{6}\,c^2ex^6+\frac{1}{5}\,c^2dx^5+\frac{1}{2}\,acex^4+\frac{2}{3}\,acdx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/6*c^2*e*x^6 + 1/5*c^2*d*x^5 + 1/2*a*c*e*x^4 + 2/3*a*c*d*x^3 + 1/2*a^2*e*
x^2 + a^2*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int (d+ex)\left(a+cx^2\right)^2\,dx = a^2dx + \frac{a^2ex^2}{2} + \frac{2acdx^3}{3} + \frac{acex^4}{2} + \frac{c^2dx^5}{5} + \frac{c^2ex^6}{6}$$

input
```
integrate((e*x+d)*(c*x**2+a)**2,x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + 2*a*c*d*x**3/3 + a*c*e*x**4/2 + c**2*d*x**5/5 +
 c**2*e*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+cx^2\right)^2\,dx = \frac{1}{6}\,c^2ex^6 + \frac{1}{5}\,c^2dx^5 + \frac{1}{2}\,acex^4 + \frac{2}{3}\,acdx^3 + \frac{1}{2}\,a^2ex^2 + a^2dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*c^2*e*x^6 + 1/5*c^2*d*x^5 + 1/2*a*c*e*x^4 + 2/3*a*c*d*x^3 + 1/2*a^2*e*
x^2 + a^2*d*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+cx^2\right)^2\,dx = \frac{1}{6}\,c^2ex^6 + \frac{1}{5}\,c^2dx^5 + \frac{1}{2}\,acex^4 + \frac{2}{3}\,acdx^3 + \frac{1}{2}\,a^2ex^2 + a^2dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/6*c^2*e*x^6 + 1/5*c^2*d*x^5 + 1/2*a*c*e*x^4 + 2/3*a*c*d*x^3 + 1/2*a^2*e*
x^2 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+cx^2\right)^2\,dx=\frac{e\,a^2\,x^2}{2}+d\,a^2\,x+\frac{e\,a\,c\,x^4}{2}+\frac{2\,d\,a\,c\,x^3}{3}+\frac{e\,c^2\,x^6}{6}+\frac{d\,c^2\,x^5}{5}$$

input
```
int((a + c*x^2)^2*(d + e*x),x)
```

output
```
(a^2*e*x^2)/2 + (c^2*d*x^5)/5 + (c^2*e*x^6)/6 + a^2*d*x + (2*a*c*d*x^3)/3
+ (a*c*e*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (d+ex)\left(a+cx^2\right)^2\,dx=\frac{x(5c^2e\,x^5+6c^2d\,x^4+15ace\,x^3+20acd\,x^2+15a^2ex+30a^2d)}{30}$$

input
```
int((e*x+d)*(c*x^2+a)^2,x)
```

output
```
(x*(30*a**2*d + 15*a**2*e*x + 20*a*c*d*x**2 + 15*a*c*e*x**3 + 6*c**2*d*x**
4 + 5*c**2*e*x**5))/30
```

3.65 $\int \frac{(a+cx^2)^2}{d+ex}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx = -\frac{cd(cd^2+2ae^2)\,x}{e^4}+\frac{c(cd^2+2ae^2)\,x^2}{2e^3} -\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}+\frac{(cd^2+ae^2)^2\log(d+ex)}{e^5}$$

output
```
-c*d*(2*a*e^2+c*d^2)*x/e^4+1/2*c*(2*a*e^2+c*d^2)*x^2/e^3-1/3*c^2*d*x^3/e^2
+1/4*c^2*x^4/e+(a*e^2+c*d^2)^2*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx = \frac{cex(12ae^2(-2d+ex)+c(-12d^3+6d^2ex-4de^2x^2+3e^3x^3))+12(cd^2+ae^2)^2\log(d+ex)}{12e^5}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x),x]
```

output
```
(c*e*x*(12*a*e^2*(-2*d + e*x) + c*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e
^3*x^3)) + 12*(c*d^2 + a*e^2)^2*Log[d + e*x])/(12*e^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{(ae^2+cd^2)^2}{e^4(d+ex)}-\frac{cd(2ae^2+cd^2)}{e^4}+\frac{cx(2ae^2+cd^2)}{e^3}-\frac{c^2dx^2}{e^2}+\frac{c^2x^3}{e}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(ae^2+cd^2)^2\log(d+ex)}{e^5}-\frac{cdx(2ae^2+cd^2)}{e^4}+\frac{cx^2(2ae^2+cd^2)}{2e^3}-\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}$$

input
```
Int[(a + c*x^2)^2/(d + e*x),x]
```

output
```
-((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^2)/(2*e^3) - (c^
2*d*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + ((c*d^2 + a*e^2)^2*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

method	result
default	$-\frac{c\left(-\frac{c\,x^4 e^3}{4}+\frac{cd\,x^3 e^2}{3}-\frac{\left(2a\,e^2+c\,d^2\right)x^2 e}{2}+d\left(2a\,e^2+c\,d^2\right)x\right)}{e^4}+\frac{\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)\ln(ex+d)}{e^5}$
norman	$\frac{c^2x^4}{4e}+\frac{c\left(2a\,e^2+c\,d^2\right)x^2}{2e^3}-\frac{c^2d\,x^3}{3e^2}-\frac{cd\left(2a\,e^2+c\,d^2\right)x}{e^4}+\frac{\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)\ln(ex+d)}{e^5}$
risch	$\frac{c^2x^4}{4e}-\frac{c^2d\,x^3}{3e^2}+\frac{ca\,x^2}{e}+\frac{c^2d^2x^2}{2e^3}-\frac{2cadx}{e^2}-\frac{c^2d^3x}{e^4}+\frac{\ln(ex+d)a^2}{e}+\frac{2\ln(ex+d)ac\,d^2}{e^3}+\frac{\ln(ex+d)c^2d^4}{e^5}$
parallelrisch	$\frac{3c^2x^4e^4-4d\,c^2x^3e^3+12x^2ac\,e^4+6x^2c^2d^2e^2+12\ln(ex+d)a^2e^4+24\ln(ex+d)ac\,d^2e^2+12\ln(ex+d)c^2d^4-24xacd\,e^3-12x\,c^2d^3}{12e^5}$

input
```
int((c*x^2+a)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^4*(-1/4*c*x^4*e^3+1/3*c*d*x^3*e^2-1/2*(2*a*e^2+c*d^2)*x^2*e+d*(2*a*e^
2+c*d^2)*x)+(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^5*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+cx^2\right)^2}{d+ex}\,dx$$
$$=\frac{3\,c^2e^4x^4-4\,c^2de^3x^3+6\,(c^2d^2e^2+2\,ace^4)x^2-12\,(c^2d^3e+2\,acde^3)x+12\,(c^2d^4+2\,acd^2e^2+a^2e^4)\log{(e}}{12\,e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d),x, algorithm="fricas")
```

output
```
1/12*(3*c^2*e^4*x^4 - 4*c^2*d*e^3*x^3 + 6*(c^2*d^2*e^2 + 2*a*c*e^4)*x^2 -
12*(c^2*d^3*e + 2*a*c*d*e^3)*x + 12*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*lo
g(e*x + d))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx = -\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}+x^2\left(\frac{ac}{e}+\frac{c^2d^2}{2e^3}\right)+x\left(-\frac{2acd}{e^2}-\frac{c^2d^3}{e^4}\right)+\frac{(ae^2+cd^2)^2\log(d+ex)}{e^5}$$

input
```
integrate((c*x**2+a)**2/(e*x+d),x)
```

output
```
-c**2*d*x**3/(3*e**2) + c**2*x**4/(4*e) + x**2*(a*c/e + c**2*d**2/(2*e**3)
) + x*(-2*a*c*d/e**2 - c**2*d**3/e**4) + (a*e**2 + c*d**2)**2*log(d + e*x)
/e**5
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx = \frac{3\,c^2e^3x^4-4\,c^2de^2x^3+6\,(c^2d^2e+2\,ace^3)x^2-12\,(c^2d^3+2\,acde^2)x}{12\,e^4}+\frac{(c^2d^4+2\,acd^2e^2+a^2e^4)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/12*(3*c^2*e^3*x^4 - 4*c^2*d*e^2*x^3 + 6*(c^2*d^2*e + 2*a*c*e^3)*x^2 - 12
*(c^2*d^3 + 2*a*c*d*e^2)*x)/e^4 + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*log(
e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx = \frac{3\,c^2e^3x^4-4\,c^2de^2x^3+6\,c^2d^2ex^2+12\,ace^3x^2-12\,c^2d^3x-24\,acde^2x}{12\,e^4} + \frac{(c^2d^4+2\,acd^2e^2+a^2e^4)\log(|ex+d|)}{e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d),x, algorithm="giac")
```

output
```
1/12*(3*c^2*e^3*x^4 - 4*c^2*d*e^2*x^3 + 6*c^2*d^2*e*x^2 + 12*a*c*e^3*x^2 -
 12*c^2*d^3*x - 24*a*c*d*e^2*x)/e^4 + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*
log(abs(e*x + d))/e^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx = x^2\left(\frac{c^2\,d^2}{2\,e^3}+\frac{a\,c}{e}\right)+\frac{\ln(d+e\,x)\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}{e^5} + \frac{c^2\,x^4}{4\,e}-\frac{c^2\,d\,x^3}{3\,e^2}-\frac{d\,x\left(\frac{c^2\,d^2}{e^3}+\frac{2\,a\,c}{e}\right)}{e}$$

input
```
int((a + c*x^2)^2/(d + e*x),x)
```

output
```
x^2*((c^2*d^2)/(2*e^3) + (a*c)/e) + (log(d + e*x)*(a^2*e^4 + c^2*d^4 + 2*a
*c*d^2*e^2))/e^5 + (c^2*x^4)/(4*e) - (c^2*d*x^3)/(3*e^2) - (d*x*((c^2*d^2)
/e^3 + (2*a*c)/e))/e
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int \frac{(a+cx^2)^2}{d+ex}\,dx = \frac{12\log(ex+d)\,a^2e^4+24\log(ex+d)\,ac\,d^2e^2+12\log(ex+d)\,c^2d^4-24acd\,e^3x+12ac\,e^4x^2-12c^2d^3ex+}{12e^5}$$

input
```
int((c*x^2+a)^2/(e*x+d),x)
```

output
```
(12*log(d + e*x)*a**2*e**4 + 24*log(d + e*x)*a*c*d**2*e**2 + 12*log(d + e*
x)*c**2*d**4 - 24*a*c*d*e**3*x + 12*a*c*e**4*x**2 - 12*c**2*d**3*e*x + 6*c
**2*d**2*e**2*x**2 - 4*c**2*d*e**3*x**3 + 3*c**2*e**4*x**4)/(12*e**5)
```

3.66 $\int \frac{(a+cx^2)^2}{(d+ex)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \frac{(a+cx^2)^2}{(d+ex)^2} dx = \frac{c(3cd^2+2ae^2)\,x}{e^4} - \frac{c^2dx^2}{e^3} + \frac{c^2x^3}{3e^2} - \frac{(cd^2+ae^2)^2}{e^5(d+ex)} - \frac{4cd(cd^2+ae^2)\log(d+ex)}{e^5}$$

output
```
c*(2*a*e^2+3*c*d^2)*x/e^4-c^2*d*x^2/e^3+1/3*c^2*x^3/e^2-(a*e^2+c*d^2)^2/e^
5/(e*x+d)-4*c*d*(a*e^2+c*d^2)*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{(a+cx^2)^2}{(d+ex)^2} dx = \frac{3ce(3cd^2+2ae^2)\,x - 3c^2de^2x^2 + c^2e^3x^3 - \frac{3(cd^2+ae^2)^2}{d+ex} - 12cd(cd^2+ae^2)\log(d+ex)}{3e^5}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^2,x]
```

output
```
(3*c*e*(3*c*d^2 + 2*a*e^2)*x - 3*c^2*d*e^2*x^2 + c^2*e^3*x^3 - (3*(c*d^2 +
 a*e^2)^2)/(d + e*x) - 12*c*d*(c*d^2 + a*e^2)*Log[d + e*x])/(3*e^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(-\frac{4cd(ae^2+cd^2)}{e^4(d+ex)}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^2}+\frac{c(2ae^2+3cd^2)}{e^4}-\frac{2c^2dx}{e^3}+\frac{c^2x^2}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(ae^2+cd^2)^2}{e^5(d+ex)}-\frac{4cd(ae^2+cd^2)\log(d+ex)}{e^5}+\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{c^2dx^2}{e^3}+\frac{c^2x^3}{3e^2}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^2,x]
```

output
```
(c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (c^2*d*x^2)/e^3 + (c^2*x^3)/(3*e^2) - (c*d
^2 + a*e^2)^2/(e^5*(d + e*x)) - (4*c*d*(c*d^2 + a*e^2)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

method	result
default	$\frac{c\left(\frac{1}{3}c\,e^{2}x^{3}-cd\,x^{2}e+2a\,e^{2}x+3c\,d^{2}x\right)}{e^{4}}-\frac{4cd\left(a\,e^{2}+c\,d^{2}\right)\ln\left(ex+d\right)}{e^{5}}-\frac{a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}{e^{5}\left(ex+d\right)}$
norman	$\frac{-\frac{a^{2}e^{4}+4ac\,d^{2}e^{2}+4c^{2}d^{4}}{e^{5}}+\frac{c^{2}x^{4}}{3e}-\frac{2c^{2}d\,x^{3}}{3e^{2}}+\frac{2c\left(a\,e^{2}+c\,d^{2}\right)x^{2}}{e^{3}}}{ex+d}-\frac{4cd\left(a\,e^{2}+c\,d^{2}\right)\ln\left(ex+d\right)}{e^{5}}$
risch	$\frac{c^{2}x^{3}}{3e^{2}}-\frac{c^{2}d\,x^{2}}{e^{3}}+\frac{2cax}{e^{2}}+\frac{3c^{2}d^{2}x}{e^{4}}-\frac{4cd\ln\left(ex+d\right)a}{e^{3}}-\frac{4c^{2}d^{3}\ln\left(ex+d\right)}{e^{5}}-\frac{a^{2}}{e\left(ex+d\right)}-\frac{2ac\,d^{2}}{e^{3}\left(ex+d\right)}-\frac{c^{2}d^{4}}{e^{5}\left(ex+d\right)}$
parallelrisch	$-\frac{-c^{2}x^{4}e^{4}+2d\,c^{2}x^{3}e^{3}+12\ln\left(ex+d\right)xacd\,e^{3}+12\ln\left(ex+d\right)x\,c^{2}d^{3}e-6x^{2}ac\,e^{4}-6x^{2}c^{2}d^{2}e^{2}+12\ln\left(ex+d\right)ac\,d^{2}e^{2}+12\ln\left(ex+d\right)c}{3e^{5}\left(ex+d\right)}$

input
```
int((c*x^2+a)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^4*(1/3*c*e^2*x^3-c*d*x^2*e+2*a*e^2*x+3*c*d^2*x)-4*c*d*(a*e^2+c*d^2)*ln
(e*x+d)/e^5-(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^5/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \frac{(a+cx^2)^2}{(d+ex)^2}\,dx$$
$$=\frac{c^2e^4x^4-2\,c^2de^3x^3-3\,c^2d^4-6\,acd^2e^2-3\,a^2e^4+6\,(c^2d^2e^2+ace^4)x^2+3\,(3\,c^2d^3e+2\,acde^3)x-12\,(c^2d}{3\,(e^6x+de^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/3*(c^2*e^4*x^4 - 2*c^2*d*e^3*x^3 - 3*c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4
 + 6*(c^2*d^2*e^2 + a*c*e^4)*x^2 + 3*(3*c^2*d^3*e + 2*a*c*d*e^3)*x - 12*(c
^2*d^4 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*log(e*x + d))/(e^6*x + d
*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)^2}{(d+ex)^2}\,dx = -\frac{c^2dx^2}{e^3} + \frac{c^2x^3}{3e^2} - \frac{4cd(ae^2+cd^2)\log(d+ex)}{e^5} + x\left(\frac{2ac}{e^2} + \frac{3c^2d^2}{e^4}\right) + \frac{-a^2e^4 - 2acd^2e^2 - c^2d^4}{de^5 + e^6x}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**2,x)
```

output
```
-c**2*d*x**2/e**3 + c**2*x**3/(3*e**2) - 4*c*d*(a*e**2 + c*d**2)*log(d + e
*x)/e**5 + x*(2*a*c/e**2 + 3*c**2*d**2/e**4) + (-a**2*e**4 - 2*a*c*d**2*e*
*2 - c**2*d**4)/(d*e**5 + e**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{(a+cx^2)^2}{(d+ex)^2}\,dx = -\frac{c^2d^4 + 2\,acd^2e^2 + a^2e^4}{e^6x + de^5} + \frac{c^2e^2x^3 - 3\,c^2dex^2 + 3\,(3\,c^2d^2 + 2\,ace^2)x}{3\,e^4} - \frac{4\,(c^2d^3 + acde^2)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)/(e^6*x + d*e^5) + 1/3*(c^2*e^2*x^3 -
3*c^2*d*e*x^2 + 3*(3*c^2*d^2 + 2*a*c*e^2)*x)/e^4 - 4*(c^2*d^3 + a*c*d*e^2)
*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int \frac{(a+cx^2)^2}{(d+ex)^2}\,dx = \frac{\left(c^2 - \frac{6\,c^2 d}{ex+d} + \frac{6\,(3\,c^2d^2e^2+ace^4)}{(ex+d)^2e^2}\right)(ex+d)^3}{3\,e^5} + \frac{4\,(c^2d^3+acde^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5} - \frac{\frac{c^2d^4e^3}{ex+d} + \frac{2\,acd^2e^5}{ex+d} + \frac{a^2e^7}{ex+d}}{e^8}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/3*(c^2 - 6*c^2*d/(e*x + d) + 6*(3*c^2*d^2*e^2 + a*c*e^4)/((e*x + d)^2*e^
2))*(e*x + d)^3/e^5 + 4*(c^2*d^3 + a*c*d*e^2)*log(abs(e*x + d)/((e*x + d)^
2*abs(e)))/e^5 - (c^2*d^4*e^3/(e*x + d) + 2*a*c*d^2*e^5/(e*x + d) + a^2*e^
7/(e*x + d))/e^8
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int \frac{(a+cx^2)^2}{(d+ex)^2}\,dx = x\left(\frac{3\,c^2\,d^2}{e^4} + \frac{2\,a\,c}{e^2}\right) - \frac{\ln(d+e\,x)\,\left(4\,c^2\,d^3 + 4\,a\,c\,d\,e^2\right)}{e^5} + \frac{c^2\,x^3}{3\,e^2} - \frac{a^2\,e^4 + 2\,a\,c\,d^2\,e^2 + c^2\,d^4}{e\,\left(x\,e^5 + d\,e^4\right)} - \frac{c^2\,d\,x^2}{e^3}$$

input
```
int((a + c*x^2)^2/(d + e*x)^2,x)
```

output
```
x*((3*c^2*d^2)/e^4 + (2*a*c)/e^2) - (log(d + e*x)*(4*c^2*d^3 + 4*a*c*d*e^2
))/e^5 + (c^2*x^3)/(3*e^2) - (a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)/(e*(d*e^4
 + e^5*x)) - (c^2*d*x^2)/e^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.69

$$\int \frac{(a+cx^2)^2}{(d+ex)^2}\,dx$$
$$= \frac{-12\log(ex+d)\,ac\,d^3e^2-12\log(ex+d)\,ac\,d^2e^3x-12\log(ex+d)\,c^2d^5-12\log(ex+d)\,c^2d^4ex+3a^2e^5}{3d\,e^5\,(ex+d)}$$

input
```
int((c*x^2+a)^2/(e*x+d)^2,x)
```

output
```
( - 12*log(d + e*x)*a*c*d**3*e**2 - 12*log(d + e*x)*a*c*d**2*e**3*x - 12*l
og(d + e*x)*c**2*d**5 - 12*log(d + e*x)*c**2*d**4*e*x + 3*a**2*e**5*x + 12
*a*c*d**2*e**3*x + 6*a*c*d*e**4*x**2 + 12*c**2*d**4*e*x + 6*c**2*d**3*e**2
*x**2 - 2*c**2*d**2*e**3*x**3 + c**2*d*e**4*x**4)/(3*d*e**5*(d + e*x))
```

3.67 $\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx = -\frac{3c^2dx}{e^4}+\frac{c^2x^2}{2e^3}-\frac{(cd^2+ae^2)^2}{2e^5(d+ex)^2}+\frac{4cd(cd^2+ae^2)}{e^5(d+ex)}+\frac{2c(3cd^2+ae^2)\log(d+ex)}{e^5}$$

output
```
-3*c^2*d*x/e^4+1/2*c^2*x^2/e^3-1/2*(a*e^2+c*d^2)^2/e^5/(e*x+d)^2+4*c*d*(a*
e^2+c*d^2)/e^5/(e*x+d)+2*c*(a*e^2+3*c*d^2)*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx = \frac{-a^2e^4+2acde^2(3d+4ex)+c^2(7d^4+2d^3ex-11d^2e^2x^2-4de^3x^3+e^4x^4)+4c(3cd^2+ae^2)(d+ex)^2\,\mathrm{lo}}{2e^5(d+ex)^2}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^3,x]
```

output
```
(-(a^2*e^4) + 2*a*c*d*e^2*(3*d + 4*e*x) + c^2*(7*d^4 + 2*d^3*e*x - 11*d^2*
e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + 4*c*(3*c*d^2 + a*e^2)*(d + e*x)^2*Log[d
 + e*x])/(2*e^5*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4(d+ex)}-\frac{4cd(ae^2+cd^2)}{e^4(d+ex)^2}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^3}-\frac{3c^2d}{e^4}+\frac{c^2x}{e^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4cd(ae^2+cd^2)}{e^5(d+ex)}-\frac{(ae^2+cd^2)^2}{2e^5(d+ex)^2}+\frac{2c(ae^2+3cd^2)\log(d+ex)}{e^5}-\frac{3c^2dx}{e^4}+\frac{c^2x^2}{2e^3}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^3,x]
```

output
```
(-3*c^2*d*x)/e^4 + (c^2*x^2)/(2*e^3) - (c*d^2 + a*e^2)^2/(2*e^5*(d + e*x)^
2) + (4*c*d*(c*d^2 + a*e^2))/(e^5*(d + e*x)) + (2*c*(3*c*d^2 + a*e^2)*Log[
d + e*x])/e^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

method	result
default	$-\frac{c^2\left(-\frac{1}{2}e\,x^2+3dx\right)}{e^4}+\frac{2c\left(a\,e^2+3c\,d^2\right)\ln(ex+d)}{e^5}+\frac{4cd\left(a\,e^2+c\,d^2\right)}{e^5(ex+d)}-\frac{a^2e^4+2ac\,d^2e^2+c^2d^4}{2e^5(ex+d)^2}$
norman	$\frac{-\frac{a^2e^4-6ac\,d^2e^2-18c^2d^4}{2e^5}+\frac{c^2x^4}{2e}-\frac{2c^2d\,x^3}{e^2}+\frac{2d\left(2ac\,e^2+6d^2c^2\right)x}{e^4}}{(ex+d)^2}+\frac{2c\left(a\,e^2+3c\,d^2\right)\ln(ex+d)}{e^5}$
risch	$\frac{c^2x^2}{2e^3}-\frac{3c^2dx}{e^4}+\frac{\left(4d\,e^2ac+4d^3c^2\right)x-\frac{a^2e^4-6ac\,d^2e^2-7c^2d^4}{2e}}{e^4(ex+d)^2}+\frac{2c\ln(ex+d)a}{e^3}+\frac{6c^2\ln(ex+d)d^2}{e^5}$
parallelrisch	$\frac{c^2x^4e^4+4\ln(ex+d)x^2ac\,e^4+12\ln(ex+d)x^2c^2d^2e^2-4d\,c^2x^3e^3+8\ln(ex+d)xacd\,e^3+24\ln(ex+d)x\,c^2d^3e+4\ln(ex+d)ac\,d^2e^2-}{2e^5(ex+d)^2}$

input
```
int((c*x^2+a)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-c^2/e^4*(-1/2*e*x^2+3*d*x)+2*c*(a*e^2+3*c*d^2)*ln(e*x+d)/e^5+4*c*d*(a*e^2
+c*d^2)/e^5/(e*x+d)-1/2*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^5/(e*x+d)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx$$
$$=\frac{c^2e^4x^4-4\,c^2de^3x^3-11\,c^2d^2e^2x^2+7\,c^2d^4+6\,acd^2e^2-a^2e^4+2\,(c^2d^3e+4\,acde^3)x+4\,(3\,c^2d^4+acd^2e^2}{2\,(e^7x^2+2\,de^6x+d^2e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(c^2*e^4*x^4 - 4*c^2*d*e^3*x^3 - 11*c^2*d^2*e^2*x^2 + 7*c^2*d^4 + 6*a*
c*d^2*e^2 - a^2*e^4 + 2*(c^2*d^3*e + 4*a*c*d*e^3)*x + 4*(3*c^2*d^4 + a*c*d
^2*e^2 + (3*c^2*d^2*e^2 + a*c*e^4)*x^2 + 2*(3*c^2*d^3*e + a*c*d*e^3)*x)*lo
g(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx = -\frac{3c^2dx}{e^4} + \frac{c^2x^2}{2e^3} + \frac{2c(ae^2+3cd^2)\log(d+ex)}{e^5} + \frac{-a^2e^4+6acd^2e^2+7c^2d^4+x(8acde^3+8c^2d^3e)}{2d^2e^5+4de^6x+2e^7x^2}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**3,x)
```

output
```
-3*c**2*d*x/e**4 + c**2*x**2/(2*e**3) + 2*c*(a*e**2 + 3*c*d**2)*log(d + e*
x)/e**5 + (-a**2*e**4 + 6*a*c*d**2*e**2 + 7*c**2*d**4 + x*(8*a*c*d*e**3 +
8*c**2*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx = \frac{7\,c^2d^4+6\,acd^2e^2-a^2e^4+8\,(c^2d^3e+acde^3)x}{2\,(e^7x^2+2\,de^6x+d^2e^5)} + \frac{c^2ex^2-6\,c^2dx}{2\,e^4} + \frac{2\,(3\,c^2d^2+ace^2)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x)/(e
^7*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(c^2*e*x^2 - 6*c^2*d*x)/e^4 + 2*(3*c^2
*d^2 + a*c*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx = \frac{2\,(3\,c^2d^2+ace^2)\log(|ex+d|)}{e^5}+\frac{c^2e^3x^2-6\,c^2de^2x}{2\,e^6}+\frac{7\,c^2d^4+6\,acd^2e^2-a^2e^4+8\,(c^2d^3e+acde^3)x}{2\,(ex+d)^2e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
2*(3*c^2*d^2 + a*c*e^2)*log(abs(e*x + d))/e^5 + 1/2*(c^2*e^3*x^2 - 6*c^2*d
*e^2*x)/e^6 + 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 8*(c^2*d^3*e + a*
c*d*e^3)*x)/((e*x + d)^2*e^5)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx = \frac{x\,(4\,c^2\,d^3+4\,a\,c\,d\,e^2)+\frac{-a^2\,e^4+6\,a\,c\,d^2\,e^2+7\,c^2\,d^4}{2\,e}}{d^2\,e^4+2\,d\,e^5\,x+e^6\,x^2}+\frac{\ln(d+e\,x)\,(6\,c^2\,d^2+2\,a\,c\,e^2)}{e^5}+\frac{c^2\,x^2}{2\,e^3}-\frac{3\,c^2\,d\,x}{e^4}$$

input
```
int((a + c*x^2)^2/(d + e*x)^3,x)
```

output
```
(x*(4*c^2*d^3 + 4*a*c*d*e^2) + (7*c^2*d^4 - a^2*e^4 + 6*a*c*d^2*e^2)/(2*e)
)/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) + (log(d + e*x)*(6*c^2*d^2 + 2*a*c*e^2))
/e^5 + (c^2*x^2)/(2*e^3) - (3*c^2*d*x)/e^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.93

$$\int \frac{(a+cx^2)^2}{(d+ex)^3}\,dx$$
$$= \frac{4\log(ex+d)\,ac\,d^2e^2 + 8\log(ex+d)\,acd\,e^3x + 4\log(ex+d)\,ac\,e^4x^2 + 12\log(ex+d)\,c^2d^4 + 24\log(ex+}{2e^5\,(e^2x^2}$$

input
```
int((c*x^2+a)^2/(e*x+d)^3,x)
```

output
```
(4*log(d + e*x)*a*c*d**2*e**2 + 8*log(d + e*x)*a*c*d*e**3*x + 4*log(d + e*
x)*a*c*e**4*x**2 + 12*log(d + e*x)*c**2*d**4 + 24*log(d + e*x)*c**2*d**3*e
*x + 12*log(d + e*x)*c**2*d**2*e**2*x**2 - a**2*e**4 + 2*a*c*d**2*e**2 - 4
*a*c*e**4*x**2 + 6*c**2*d**4 - 12*c**2*d**2*e**2*x**2 - 4*c**2*d*e**3*x**3
 + c**2*e**4*x**4)/(2*e**5*(d**2 + 2*d*e*x + e**2*x**2))
```

3.68 $\int \frac{(a+cx^2)^2}{(d+ex)^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{(a+cx^2)^2}{(d+ex)^4} \, dx = \frac{c^2x}{e^4} - \frac{(cd^2+ae^2)^2}{3e^5(d+ex)^3} + \frac{2cd(cd^2+ae^2)}{e^5(d+ex)^2} - \frac{2c(3cd^2+ae^2)}{e^5(d+ex)} - \frac{4c^2d\log(d+ex)}{e^5}$$

output
```
c^2*x/e^4-1/3*(a*e^2+c*d^2)^2/e^5/(e*x+d)^3+2*c*d*(a*e^2+c*d^2)/e^5/(e*x+d
)^2-2*c*(a*e^2+3*c*d^2)/e^5/(e*x+d)-4*c^2*d*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^2)^2}{(d+ex)^4} \, dx = -\frac{a^2e^4 + 2ace^2(d^2+3dex+3e^2x^2) + c^2(13d^4+27d^3ex+9d^2e^2x^2-9de^3x^3-3e^4x^4) + 12c^2d(d+ex)^3}{3e^5(d+ex)^3}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^4,x]
```

output
```
-1/3*(a^2*e^4 + 2*a*c*e^2*(d^2 + 3*d*e*x + 3*e^2*x^2) + c^2*(13*d^4 + 27*d
^3*e*x + 9*d^2*e^2*x^2 - 9*d*e^3*x^3 - 3*e^4*x^4) + 12*c^2*d*(d + e*x)^3*L
og[d + e*x])/(e^5*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx$$
$$\Big\downarrow \text{476}$$
$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4(d+ex)^2}-\frac{4cd(ae^2+cd^2)}{e^4(d+ex)^3}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^4}-\frac{4c^2d}{e^4(d+ex)}+\frac{c^2}{e^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2c(ae^2+3cd^2)}{e^5(d+ex)}+\frac{2cd(ae^2+cd^2)}{e^5(d+ex)^2}-\frac{(ae^2+cd^2)^2}{3e^5(d+ex)^3}-\frac{4c^2d\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^4,x]
```

output
```
(c^2*x)/e^4 - (c*d^2 + a*e^2)^2/(3*e^5*(d + e*x)^3) + (2*c*d*(c*d^2 + a*e^
2))/(e^5*(d + e*x)^2) - (2*c*(3*c*d^2 + a*e^2))/(e^5*(d + e*x)) - (4*c^2*d
*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05

method	result
risch	$\frac{c^2x}{e^4}+\frac{\left(-2e^3ac-6d^2e\,c^2\right)x^2-2dc\left(a\,e^2+5c\,d^2\right)x-\frac{a^2e^4+2ac\,d^2e^2+13c^2d^4}{3e}}{e^4(ex+d)^3}-\frac{4c^2d\ln(ex+d)}{e^5}$
default	$\frac{c^2x}{e^4}-\frac{a^2e^4+2ac\,d^2e^2+c^2d^4}{3e^5(ex+d)^3}-\frac{4c^2d\ln(ex+d)}{e^5}-\frac{2c\left(a\,e^2+3c\,d^2\right)}{e^5(ex+d)}+\frac{2cd\left(a\,e^2+c\,d^2\right)}{e^5(ex+d)^2}$
norman	$\frac{\frac{c^2x^4}{e}-\frac{a^2e^4+2ac\,d^2e^2+22c^2d^4}{3e^5}-\frac{\left(2ac\,e^2+12d^2c^2\right)x^2}{e^3}-\frac{d\left(2ac\,e^2+18d^2c^2\right)x}{e^4}}{(ex+d)^3}-\frac{4c^2d\ln(ex+d)}{e^5}$
parallelrisch	$-\frac{12\ln(ex+d)x^3c^2d\,e^3-3c^2x^4e^4+36\ln(ex+d)x^2c^2d^2e^2+36\ln(ex+d)x\,c^2d^3e+6x^2ac\,e^4+36x^2c^2d^2e^2+12\ln(ex+d)c^2d^4+6x}{3e^5(ex+d)^3}$

input
```
int((c*x^2+a)^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c^2*x/e^4+((-2*a*c*e^3-6*c^2*d^2*e)*x^2-2*d*c*(a*e^2+5*c*d^2)*x-1/3*(a^2*e
^4+2*a*c*d^2*e^2+13*c^2*d^4)/e)/e^4/(e*x+d)^3-4*c^2*d*ln(e*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.81

$$\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx = \frac{3\,c^2e^4x^4+9\,c^2de^3x^3-13\,c^2d^4-2\,acd^2e^2-a^2e^4-3\,(3\,c^2d^2e^2+2\,ace^4)x^2-3\,(9\,c^2d^3e+2\,acde^3)x-12}{3\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/3*(3*c^2*e^4*x^4 + 9*c^2*d*e^3*x^3 - 13*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^
4 - 3*(3*c^2*d^2*e^2 + 2*a*c*e^4)*x^2 - 3*(9*c^2*d^3*e + 2*a*c*d*e^3)*x -
12*(c^2*d*e^3*x^3 + 3*c^2*d^2*e^2*x^2 + 3*c^2*d^3*e*x + c^2*d^4)*log(e*x +
 d))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx = -\frac{4c^2d\log(d+ex)}{e^5}+\frac{c^2x}{e^4} + \frac{-a^2e^4-2acd^2e^2-13c^2d^4+x^2(-6ace^4-18c^2d^2e^2)+x(-6acde^3-30c^2d^3e)}{3d^3e^5+9d^2e^6x+9de^7x^2+3e^8x^3}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**4,x)
```

output
```
-4*c**2*d*log(d + e*x)/e**5 + c**2*x/e**4 + (-a**2*e**4 - 2*a*c*d**2*e**2
- 13*c**2*d**4 + x**2*(-6*a*c*e**4 - 18*c**2*d**2*e**2) + x*(-6*a*c*d*e**3
 - 30*c**2*d**3*e))/(3*d**3*e**5 + 9*d**2*e**6*x + 9*d*e**7*x**2 + 3*e**8*
x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.29

$$\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx = -\frac{13\,c^2d^4+2\,acd^2e^2+a^2e^4+6\,(3\,c^2d^2e^2+ace^4)x^2+6\,(5\,c^2d^3e+acde^3)x}{3\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)} + \frac{c^2x}{e^4}-\frac{4\,c^2d\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/3*(13*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4 + 6*(3*c^2*d^2*e^2 + a*c*e^4)*x
^2 + 6*(5*c^2*d^3*e + a*c*d*e^3)*x)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x +
 d^3*e^5) + c^2*x/e^4 - 4*c^2*d*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05

$$\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx = \frac{c^2x}{e^4} - \frac{4\,c^2d\log(|ex+d|)}{e^5} - \frac{13\,c^2d^4 + 2\,acd^2e^2 + a^2e^4 + 6\,(3\,c^2d^2e^2 + ace^4)x^2 + 6\,(5\,c^2d^3e + acde^3)x}{3\,(ex+d)^3e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^4,x, algorithm="giac")
```

output
```
c^2*x/e^4 - 4*c^2*d*log(abs(e*x + d))/e^5 - 1/3*(13*c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4 + 6*(3*c^2*d^2*e^2 + a*c*e^4)*x^2 + 6*(5*c^2*d^3*e + a*c*d*e^3
)*x)/((e*x + d)^3*e^5)
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx = \frac{c^2\,x}{e^4} - \frac{x\,(10\,c^2\,d^3 + 2\,a\,c\,d\,e^2) + x^2\,(6\,c^2\,d^2\,e + 2\,a\,c\,e^3) + \frac{a^2\,e^4 + 2\,a\,c\,d^2\,e^2 + 13\,c^2\,d^4}{3\,e}}{d^3\,e^4 + 3\,d^2\,e^5\,x + 3\,d\,e^6\,x^2 + e^7\,x^3} - \frac{4\,c^2\,d\,\ln(d+e\,x)}{e^5}$$

input
```
int((a + c*x^2)^2/(d + e*x)^4,x)
```

output
```
(c^2*x)/e^4 - (x*(10*c^2*d^3 + 2*a*c*d*e^2) + x^2*(6*c^2*d^2*e + 2*a*c*e^3
) + (a^2*e^4 + 13*c^2*d^4 + 2*a*c*d^2*e^2)/(3*e))/(d^3*e^4 + e^7*x^3 + 3*d
^2*e^5*x + 3*d*e^6*x^2) - (4*c^2*d*log(d + e*x))/e^5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.69

$$\int \frac{(a+cx^2)^2}{(d+ex)^4}\,dx = \frac{-12\log(ex+d)\,c^2d^5-36\log(ex+d)\,c^2d^4ex-36\log(ex+d)\,c^2d^3e^2x^2-12\log(ex+d)\,c^2d^2e^3x^3-a^2d}{3d\,e^5\,(e^3x^3+3d\,e^2x^2+3d^2ex+d^3)}$$

input
```
int((c*x^2+a)^2/(e*x+d)^4,x)
```

output
```
( - 12*log(d + e*x)*c**2*d**5 - 36*log(d + e*x)*c**2*d**4*e*x - 36*log(d +
 e*x)*c**2*d**3*e**2*x**2 - 12*log(d + e*x)*c**2*d**2*e**3*x**3 - a**2*d*e
**4 + 2*a*c*e**5*x**3 - 10*c**2*d**5 - 18*c**2*d**4*e*x + 12*c**2*d**2*e**
3*x**3 + 3*c**2*d*e**4*x**4)/(3*d*e**5*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2
+ e**3*x**3))
```

3.69 $\int \frac{(a+cx^2)^2}{(d+ex)^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int \frac{(a+cx^2)^2}{(d+ex)^5} dx = -\frac{(cd^2+ae^2)^2}{4e^5(d+ex)^4} + \frac{4cd(cd^2+ae^2)}{3e^5(d+ex)^3} - \frac{c(3cd^2+ae^2)}{e^5(d+ex)^2} + \frac{4c^2d}{e^5(d+ex)} + \frac{c^2\log(d+ex)}{e^5}$$

output
```
-1/4*(a*e^2+c*d^2)^2/e^5/(e*x+d)^4+4/3*c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^3-c*(
a*e^2+3*c*d^2)/e^5/(e*x+d)^2+4*c^2*d/e^5/(e*x+d)+c^2*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{(a+cx^2)^2}{(d+ex)^5} dx = \frac{-3a^2e^4 - 2ace^2(d^2+4dex+6e^2x^2) + c^2d(25d^3+88d^2ex+108de^2x^2+48e^3x^3) + 12c^2(d+ex)^4\log(d+}{12e^5(d+ex)^4}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^5,x]
```

output
```
(-3*a^2*e^4 - 2*a*c*e^2*(d^2 + 4*d*e*x + 6*e^2*x^2) + c^2*d*(25*d^3 + 88*d
^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) + 12*c^2*(d + e*x)^4*Log[d + e*x])/(1
2*e^5*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^5}\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4(d+ex)^3}-\frac{4cd(ae^2+cd^2)}{e^4(d+ex)^4}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^5}+\frac{c^2}{e^4(d+ex)}-\frac{4c^2d}{e^4(d+ex)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{c(ae^2+3cd^2)}{e^5(d+ex)^2}+\frac{4cd(ae^2+cd^2)}{3e^5(d+ex)^3}-\frac{(ae^2+cd^2)^2}{4e^5(d+ex)^4}+\frac{4c^2d}{e^5(d+ex)}+\frac{c^2\log(d+ex)}{e^5}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^5,x]
```

output
```
-1/4*(c*d^2 + a*e^2)^2/(e^5*(d + e*x)^4) + (4*c*d*(c*d^2 + a*e^2))/(3*e^5*
(d + e*x)^3) - (c*(3*c*d^2 + a*e^2))/(e^5*(d + e*x)^2) + (4*c^2*d)/(e^5*(d
 + e*x)) + (c^2*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\frac{4c^2 d x^3}{e^2}-\frac{c\left(a e^2-9c d^2\right)x^2}{e^3}-\frac{2cd\left(a e^2-11c d^2\right)x}{3e^4}-\frac{3a^2e^4+2ac d^2e^2-25c^2d^4}{12e^5}}{(ex+d)^4}+\frac{c^2\ln(ex+d)}{e^5}$
norman	$\frac{-\frac{3a^2e^4+2ac d^2e^2-25c^2d^4}{12e^5}-\frac{\left(ac e^2-9d^2c^2\right)x^2}{e^3}+\frac{4c^2 d x^3}{e^2}-\frac{2d\left(ac e^2-11d^2c^2\right)x}{3e^4}}{(ex+d)^4}+\frac{c^2\ln(ex+d)}{e^5}$
default	$\frac{4cd(a e^2+c d^2)}{3e^5(ex+d)^3}-\frac{a^2e^4+2ac d^2e^2+c^2d^4}{4e^5(ex+d)^4}+\frac{c^2\ln(ex+d)}{e^5}+\frac{4c^2d}{e^5(ex+d)}-\frac{c(a e^2+3c d^2)}{e^5(ex+d)^2}$
parallelrisch	$\frac{12\ln(ex+d)x^4c^2e^4+48\ln(ex+d)x^3c^2d e^3+72\ln(ex+d)x^2c^2d^2e^2+48d c^2x^3e^3+48\ln(ex+d)x c^2d^3e-12x^2ac e^4+108x^2c^2d^2e^2}{12e^5(ex+d)^4}$

input
```
int((c*x^2+a)^2/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
(4*c^2*d*x^3/e^2-c*(a*e^2-9*c*d^2)/e^3*x^2-2/3*c*d*(a*e^2-11*c*d^2)/e^4*x-
1/12*(3*a^2*e^4+2*a*c*d^2*e^2-25*c^2*d^4)/e^5)/(e*x+d)^4+c^2*ln(e*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.78

$$\int \frac{(a+cx^2)^2}{(d+ex)^5}\,dx$$
$$=\frac{48\,c^2de^3x^3+25\,c^2d^4-2\,acd^2e^2-3\,a^2e^4+12\,(9\,c^2d^2e^2-ace^4)x^2+8\,(11\,c^2d^3e-acde^3)x+12\,(c^2e^4x^4+}{12\,(e^9x^4+4\,de^8x^3+6\,d^2e^7x^2+4\,d^3e^6x+d^4e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^5,x, algorithm="fricas")
```

output
```
1/12*(48*c^2*d*e^3*x^3 + 25*c^2*d^4 - 2*a*c*d^2*e^2 - 3*a^2*e^4 + 12*(9*c^
2*d^2*e^2 - a*c*e^4)*x^2 + 8*(11*c^2*d^3*e - a*c*d*e^3)*x + 12*(c^2*e^4*x^
4 + 4*c^2*d*e^3*x^3 + 6*c^2*d^2*e^2*x^2 + 4*c^2*d^3*e*x + c^2*d^4)*log(e*x
 + d))/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int \frac{(a+cx^2)^2}{(d+ex)^5}\,dx = \frac{c^2\log{(d+ex)}}{e^5} + \frac{-3a^2e^4-2acd^2e^2+25c^2d^4+48c^2de^3x^3+x^2(-12ace^4+108c^2d^2e^2)+x(-8acde^3+88c^2d^3e)}{12d^4e^5+48d^3e^6x+72d^2e^7x^2+48de^8x^3+12e^9x^4}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**5,x)
```

output
```
c**2*log(d + e*x)/e**5 + (-3*a**2*e**4 - 2*a*c*d**2*e**2 + 25*c**2*d**4 +
48*c**2*d*e**3*x**3 + x**2*(-12*a*c*e**4 + 108*c**2*d**2*e**2) + x*(-8*a*c
*d*e**3 + 88*c**2*d**3*e))/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x
**2 + 48*d*e**8*x**3 + 12*e**9*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.34

$$\int \frac{(a+cx^2)^2}{(d+ex)^5}\,dx = \frac{48\,c^2de^3x^3+25\,c^2d^4-2\,acd^2e^2-3\,a^2e^4+12\,(9\,c^2d^2e^2-ace^4)x^2+8\,(11\,c^2d^3e-acde^3)x}{12\,(e^9x^4+4\,de^8x^3+6\,d^2e^7x^2+4\,d^3e^6x+d^4e^5)} + \frac{c^2\log{(ex+d)}}{e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^5,x, algorithm="maxima")
```

output
```
1/12*(48*c^2*d*e^3*x^3 + 25*c^2*d^4 - 2*a*c*d^2*e^2 - 3*a^2*e^4 + 12*(9*c^
2*d^2*e^2 - a*c*e^4)*x^2 + 8*(11*c^2*d^3*e - a*c*d*e^3)*x)/(e^9*x^4 + 4*d*
e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5) + c^2*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int \frac{(a+cx^2)^2}{(d+ex)^5}\,dx = -\frac{c^2\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5} + \frac{\frac{48\,c^2de^{15}}{ex+d} - \frac{36\,c^2d^2e^{15}}{(ex+d)^2} + \frac{16\,c^2d^3e^{15}}{(ex+d)^3} - \frac{3\,c^2d^4e^{15}}{(ex+d)^4} - \frac{12\,ace^{17}}{(ex+d)^2} + \frac{16\,acde^{17}}{(ex+d)^3} - \frac{6\,acd^2e^{17}}{(ex+d)^4} - \frac{3\,a^2e^{19}}{(ex+d)^4}}{12\,e^{20}}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^5,x, algorithm="giac")
```

output
```
-c^2*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^5 + 1/12*(48*c^2*d*e^15/(e*x
 + d) - 36*c^2*d^2*e^15/(e*x + d)^2 + 16*c^2*d^3*e^15/(e*x + d)^3 - 3*c^2*
d^4*e^15/(e*x + d)^4 - 12*a*c*e^17/(e*x + d)^2 + 16*a*c*d*e^17/(e*x + d)^3
 - 6*a*c*d^2*e^17/(e*x + d)^4 - 3*a^2*e^19/(e*x + d)^4)/e^20
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

$$\int \frac{(a+cx^2)^2}{(d+ex)^5}\,dx = \frac{c^2\,\ln(d+e\,x)}{e^5} - \frac{\frac{3\,a^2\,e^4+2\,a\,c\,d^2\,e^2-25\,c^2\,d^4}{12\,e^5} - \frac{2\,x\,(11\,c^2\,d^3-a\,c\,d\,e^2)}{3\,e^4} - \frac{4\,c^2\,d\,x^3}{e^2} + \frac{c\,x^2\,(a\,e^2-9\,c\,d^2)}{e^3}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int((a + c*x^2)^2/(d + e*x)^5,x)
```

output
```
(c^2*log(d + e*x))/e^5 - ((3*a^2*e^4 - 25*c^2*d^4 + 2*a*c*d^2*e^2)/(12*e^5
) - (2*x*(11*c^2*d^3 - a*c*d*e^2))/(3*e^4) - (4*c^2*d*x^3)/e^2 + (c*x^2*(a
*e^2 - 9*c*d^2))/e^3)/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3
*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.94

$$\int \frac{(a+cx^2)^2}{(d+ex)^5}\,dx$$
$$= \frac{12\log(ex+d)\,c^2d^4+48\log(ex+d)\,c^2d^3ex+72\log(ex+d)\,c^2d^2e^2x^2+48\log(ex+d)\,c^2d\,e^3x^3+12\log(}{12e^5\,(e^4x^4+4d\,e^3x^3+}$$

input
```
int((c*x^2+a)^2/(e*x+d)^5,x)
```

output
```
(12*log(d + e*x)*c**2*d**4 + 48*log(d + e*x)*c**2*d**3*e*x + 72*log(d + e*
x)*c**2*d**2*e**2*x**2 + 48*log(d + e*x)*c**2*d*e**3*x**3 + 12*log(d + e*x
)*c**2*e**4*x**4 - 3*a**2*e**4 - 2*a*c*d**2*e**2 - 8*a*c*d*e**3*x - 12*a*c
*e**4*x**2 + 13*c**2*d**4 + 40*c**2*d**3*e*x + 36*c**2*d**2*e**2*x**2 - 12
*c**2*e**4*x**4)/(12*e**5*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3
*x**3 + e**4*x**4))
```

3.70 $\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = -\frac{(cd^2+ae^2)^2}{5e^5(d+ex)^5} + \frac{cd(cd^2+ae^2)}{e^5(d+ex)^4} - \frac{2c(3cd^2+ae^2)}{3e^5(d+ex)^3} + \frac{2c^2d}{e^5(d+ex)^2} - \frac{c^2}{e^5(d+ex)}$$

output
```
-1/5*(a*e^2+c*d^2)^2/e^5/(e*x+d)^5+c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^4-2/3*c*(
a*e^2+3*c*d^2)/e^5/(e*x+d)^3+2*c^2*d/e^5/(e*x+d)^2-c^2/e^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = -\frac{3a^2e^4+ace^2(d^2+5dex+10e^2x^2)+3c^2(d^4+5d^3ex+10d^2e^2x^2+10de^3x^3+5e^4x^4)}{15e^5(d+ex)^5}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^6,x]
```

output
```
-1/15*(3*a^2*e^4 + a*c*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3*c^2*(d^4 + 5*d
^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4))/(e^5*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^6} \, dx$$

$$\Big\downarrow \text{ 476}$$

$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4(d+ex)^4} - \frac{4cd(ae^2+cd^2)}{e^4(d+ex)^5} + \frac{(ae^2+cd^2)^2}{e^4(d+ex)^6} + \frac{c^2}{e^4(d+ex)^2} - \frac{4c^2d}{e^4(d+ex)^3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2c(ae^2+3cd^2)}{3e^5(d+ex)^3} + \frac{cd(ae^2+cd^2)}{e^5(d+ex)^4} - \frac{(ae^2+cd^2)^2}{5e^5(d+ex)^5} - \frac{c^2}{e^5(d+ex)} + \frac{2c^2d}{e^5(d+ex)^2}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^6,x]
```

output
```
-1/5*(c*d^2 + a*e^2)^2/(e^5*(d + e*x)^5) + (c*d*(c*d^2 + a*e^2))/(e^5*(d +
 e*x)^4) - (2*c*(3*c*d^2 + a*e^2))/(3*e^5*(d + e*x)^3) + (2*c^2*d)/(e^5*(d
 + e*x)^2) - c^2/(e^5*(d + e*x))
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{15c^2x^4e^4+30d\,c^2x^3e^3+10x^2ac\,e^4+30x^2c^2d^2e^2+5xacd\,e^3+15x\,c^2d^3e+3a^2e^4+ac\,d^2e^2+3c^2d^4}{15(ex+d)^5e^5}$	105
risch	$\frac{-\frac{c^2x^4}{e}-\frac{2c^2d\,x^3}{e^2}-\frac{2c\left(a\,e^2+3c\,d^2\right)x^2}{3e^3}-\frac{dc\left(a\,e^2+3c\,d^2\right)x}{3e^4}-\frac{3a^2e^4+ac\,d^2e^2+3c^2d^4}{15e^5}}{(ex+d)^5}$	105
orering	$-\frac{15c^2x^4e^4+30d\,c^2x^3e^3+10x^2ac\,e^4+30x^2c^2d^2e^2+5xacd\,e^3+15x\,c^2d^3e+3a^2e^4+ac\,d^2e^2+3c^2d^4}{15(ex+d)^5e^5}$	105
parallelrisch	$\frac{-15c^2x^4e^4-30d\,c^2x^3e^3-10x^2ac\,e^4-30x^2c^2d^2e^2-5xacd\,e^3-15x\,c^2d^3e-3a^2e^4-ac\,d^2e^2-3c^2d^4}{15e^5(ex+d)^5}$	106
norman	$\frac{-\frac{c^2x^4}{e}-\frac{2c^2d\,x^3}{e^2}-\frac{2\left(ac\,e^2+3d^2c^2\right)x^2}{3e^3}-\frac{d\left(ac\,e^2+3d^2c^2\right)x}{3e^4}-\frac{3a^2e^4+ac\,d^2e^2+3c^2d^4}{15e^5}}{(ex+d)^5}$	109
default	$-\frac{2c(a\,e^2+3c\,d^2)}{3e^5(ex+d)^3}+\frac{cd(a\,e^2+c\,d^2)}{e^5(ex+d)^4}-\frac{a^2e^4+2ac\,d^2e^2+c^2d^4}{5e^5(ex+d)^5}-\frac{c^2}{e^5(ex+d)}+\frac{2c^2d}{e^5(ex+d)^2}$	119

input
```
int((c*x^2+a)^2/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(15*c^2*e^4*x^4+30*c^2*d*e^3*x^3+10*a*c*e^4*x^2+30*c^2*d^2*e^2*x^2+5
*a*c*d*e^3*x+15*c^2*d^3*e*x+3*a^2*e^4+a*c*d^2*e^2+3*c^2*d^4)/(e*x+d)^5/e^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = -\frac{15\,c^2e^4x^4+30\,c^2de^3x^3+3\,c^2d^4+acd^2e^2+3\,a^2e^4+10\,(3\,c^2d^2e^2+ace^4)x^2+5\,(3\,c^2d^3e+acde^3)x}{15\,(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^6,x, algorithm="fricas")
```

output
```
-1/15*(15*c^2*e^4*x^4 + 30*c^2*d*e^3*x^3 + 3*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4 + 10*(3*c^2*d^2*e^2 + a*c*e^4)*x^2 + 5*(3*c^2*d^3*e + a*c*d*e^3)*x)/(
e^10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d
^5*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = \frac{-3a^2e^4-acd^2e^2-3c^2d^4-30c^2de^3x^3-15c^2e^4x^4+x^2(-10ace^4-30c^2d^2e^2)+x(-5acde^3-15c^2d^3e)}{15d^5e^5+75d^4e^6x+150d^3e^7x^2+150d^2e^8x^3+75de^9x^4+15e^{10}x^5}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**6,x)
```

output
```
(-3*a**2*e**4 - a*c*d**2*e**2 - 3*c**2*d**4 - 30*c**2*d*e**3*x**3 - 15*c**
2*e**4*x**4 + x**2*(-10*a*c*e**4 - 30*c**2*d**2*e**2) + x*(-5*a*c*d*e**3 -
 15*c**2*d**3*e))/(15*d**5*e**5 + 75*d**4*e**6*x + 150*d**3*e**7*x**2 + 15
0*d**2*e**8*x**3 + 75*d*e**9*x**4 + 15*e**10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = -\frac{15\,c^2e^4x^4+30\,c^2de^3x^3+3\,c^2d^4+acd^2e^2+3\,a^2e^4+10\,(3\,c^2d^2e^2+ace^4)x^2+5\,(3\,c^2d^3e+acde^3)x}{15\,(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^6,x, algorithm="maxima")
```

output
```
-1/15*(15*c^2*e^4*x^4 + 30*c^2*d*e^3*x^3 + 3*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4 + 10*(3*c^2*d^2*e^2 + a*c*e^4)*x^2 + 5*(3*c^2*d^3*e + a*c*d*e^3)*x)/(
e^10*x^5 + 5*d*e^9*x^4 + 10*d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d
^5*e^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = -\frac{15\,c^2e^4x^4+30\,c^2de^3x^3+30\,c^2d^2e^2x^2+10\,ace^4x^2+15\,c^2d^3ex+5\,acde^3x+3\,c^2d^4+acd^2e^2+3\,a^2e^4}{15\,(ex+d)^5e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^6,x, algorithm="giac")
```

output
```
-1/15*(15*c^2*e^4*x^4 + 30*c^2*d*e^3*x^3 + 30*c^2*d^2*e^2*x^2 + 10*a*c*e^4
*x^2 + 15*c^2*d^3*e*x + 5*a*c*d*e^3*x + 3*c^2*d^4 + a*c*d^2*e^2 + 3*a^2*e^
4)/((e*x + d)^5*e^5)
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = -\frac{\frac{3\,a^2\,e^4+a\,c\,d^2\,e^2+3\,c^2\,d^4}{15\,e^5}+\frac{c^2\,x^4}{e}+\frac{2\,c^2\,d\,x^3}{e^2}+\frac{2\,c\,x^2\,\left(3\,c\,d^2+a\,e^2\right)}{3\,e^3}+\frac{c\,d\,x\,\left(3\,c\,d^2+a\,e^2\right)}{3\,e^4}}{d^5+5\,d^4\,e\,x+10\,d^3\,e^2\,x^2+10\,d^2\,e^3\,x^3+5\,d\,e^4\,x^4+e^5\,x^5}$$

input
```
int((a + c*x^2)^2/(d + e*x)^6,x)
```

output
```
-((3*a^2*e^4 + 3*c^2*d^4 + a*c*d^2*e^2)/(15*e^5) + (c^2*x^4)/e + (2*c^2*d*
x^3)/e^2 + (2*c*x^2*(a*e^2 + 3*c*d^2))/(3*e^3) + (c*d*x*(a*e^2 + 3*c*d^2))
/(3*e^4))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 +
 5*d^4*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int \frac{(a+cx^2)^2}{(d+ex)^6}\,dx = \frac{3c^2e^3x^5-10acd\,e^2x^2-5ac\,d^2ex-3a^2d\,e^2-ac\,d^3}{15d\,e^3\left(e^5x^5+5d\,e^4x^4+10d^2e^3x^3+10d^3e^2x^2+5d^4ex+d^5\right)}$$

input
```
int((c*x^2+a)^2/(e*x+d)^6,x)
```

output
```
( - 3*a**2*d*e**2 - a*c*d**3 - 5*a*c*d**2*e*x - 10*a*c*d*e**2*x**2 + 3*c**
2*e**3*x**5)/(15*d*e**3*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e
**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))
```

3.71 $\int \frac{(a+cx^2)^2}{(d+ex)^7} dx$

Optimal result

Integrand size = 17, antiderivative size = 117

$$\int \frac{(a+cx^2)^2}{(d+ex)^7} dx = -\frac{(cd^2+ae^2)^2}{6e^5(d+ex)^6} + \frac{4cd(cd^2+ae^2)}{5e^5(d+ex)^5} - \frac{c(3cd^2+ae^2)}{2e^5(d+ex)^4} + \frac{4c^2d}{3e^5(d+ex)^3} - \frac{c^2}{2e^5(d+ex)^2}$$

output
```
-1/6*(a*e^2+c*d^2)^2/e^5/(e*x+d)^6+4/5*c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^5-1/2
*c*(a*e^2+3*c*d^2)/e^5/(e*x+d)^4+4/3*c^2*d/e^5/(e*x+d)^3-1/2*c^2/e^5/(e*x+
d)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^2)^2}{(d+ex)^7} dx = -\frac{5a^2e^4+ace^2(d^2+6dex+15e^2x^2)+c^2(d^4+6d^3ex+15d^2e^2x^2+20de^3x^3+15e^4x^4)}{30e^5(d+ex)^6}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^7,x]
```

output
```
-1/30*(5*a^2*e^4 + a*c*e^2*(d^2 + 6*d*e*x + 15*e^2*x^2) + c^2*(d^4 + 6*d^3
*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4))/(e^5*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^7}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4(d+ex)^5}-\frac{4cd(ae^2+cd^2)}{e^4(d+ex)^6}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^7}+\frac{c^2}{e^4(d+ex)^3}-\frac{4c^2d}{e^4(d+ex)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(ae^2+3cd^2)}{2e^5(d+ex)^4}+\frac{4cd(ae^2+cd^2)}{5e^5(d+ex)^5}-\frac{(ae^2+cd^2)^2}{6e^5(d+ex)^6}-\frac{c^2}{2e^5(d+ex)^2}+\frac{4c^2d}{3e^5(d+ex)^3}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^7,x]
```

output
```
-1/6*(c*d^2 + a*e^2)^2/(e^5*(d + e*x)^6) + (4*c*d*(c*d^2 + a*e^2))/(5*e^5*
(d + e*x)^5) - (c*(3*c*d^2 + a*e^2))/(2*e^5*(d + e*x)^4) + (4*c^2*d)/(3*e^
5*(d + e*x)^3) - c^2/(2*e^5*(d + e*x)^2)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{-\frac{c^2 x^4}{2e}-\frac{2c^2 d\,x^3}{3e^2}-\frac{c\left(a\,e^2+c\,d^2\right)x^2}{2e^3}-\frac{dc\left(a\,e^2+c\,d^2\right)x}{5e^4}-\frac{5a^2 e^4+ac\,d^2 e^2+c^2 d^4}{30e^5}}{(ex+d)^6}$	102
gosper	$-\frac{15c^2x^4e^4+20d\,c^2x^3e^3+15x^2ac\,e^4+15x^2c^2d^2e^2+6xacd\,e^3+6x\,c^2d^3e+5a^2e^4+ac\,d^2e^2+c^2d^4}{30e^5(ex+d)^6}$	104
orering	$-\frac{15c^2x^4e^4+20d\,c^2x^3e^3+15x^2ac\,e^4+15x^2c^2d^2e^2+6xacd\,e^3+6x\,c^2d^3e+5a^2e^4+ac\,d^2e^2+c^2d^4}{30e^5(ex+d)^6}$	104
norman	$\frac{-\frac{c^2 x^4}{2e}-\frac{2c^2 d\,x^3}{3e^2}-\frac{\left(e^3 ac+d^2 e\,c^2\right)x^2}{2e^4}-\frac{d\left(e^3 ac+d^2 e\,c^2\right)x}{5e^5}-\frac{5a^2 e^5+ac\,d^2 e^3+c^2 d^4 e}{30e^6}}{(ex+d)^6}$	109
parallelrisch	$\frac{-15c^2x^4e^5-20c^2d\,x^3e^4-15ac\,e^5x^2-15c^2d^2e^3x^2-6acd\,e^4x-6c^2d^3e^2x-5a^2e^5-ac\,d^2e^3-c^2d^4e}{30e^6(ex+d)^6}$	109
default	$\frac{4c^2d}{3e^5(ex+d)^3}-\frac{c\left(a\,e^2+3c\,d^2\right)}{2e^5(ex+d)^4}+\frac{4cd\left(a\,e^2+c\,d^2\right)}{5e^5(ex+d)^5}-\frac{c^2}{2e^5(ex+d)^2}-\frac{a^2e^4+2ac\,d^2e^2+c^2d^4}{6e^5(ex+d)^6}$	120

input
```
int((c*x^2+a)^2/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c^2*x^4/e-2/3*c^2*d*x^3/e^2-1/2/e^3*c*(a*e^2+c*d^2)*x^2-1/5*d*c/e^4*
(a*e^2+c*d^2)*x-1/30/e^5*(5*a^2*e^4+a*c*d^2*e^2+c^2*d^4))/(e*x+d)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^2)^2}{(d+ex)^7}\,dx = -\frac{15\,c^2e^4x^4+20\,c^2de^3x^3+c^2d^4+acd^2e^2+5\,a^2e^4+15\,(c^2d^2e^2+ace^4)x^2+6\,(c^2d^3e+acde^3)x}{30\,(e^{11}x^6+6\,de^{10}x^5+15\,d^2e^9x^4+20\,d^3e^8x^3+15\,d^4e^7x^2+6\,d^5e^6x+d^6e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^7,x, algorithm="fricas")
```

output
```
-1/30*(15*c^2*e^4*x^4 + 20*c^2*d*e^3*x^3 + c^2*d^4 + a*c*d^2*e^2 + 5*a^2*e
^4 + 15*(c^2*d^2*e^2 + a*c*e^4)*x^2 + 6*(c^2*d^3*e + a*c*d*e^3)*x)/(e^11*x
^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d
^5*e^6*x + d^6*e^5)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.47

$$\int \frac{(a+cx^2)^2}{(d+ex)^7}\,dx = \frac{-5a^2e^4-acd^2e^2-c^2d^4-20c^2de^3x^3-15c^2e^4x^4+x^2(-15ace^4-15c^2d^2e^2)+x(-6acde^3-6c^2d^3e)}{30d^6e^5+180d^5e^6x+450d^4e^7x^2+600d^3e^8x^3+450d^2e^9x^4+180de^{10}x^5+30e^{11}x^6}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**7,x)
```

output
```
(-5*a**2*e**4 - a*c*d**2*e**2 - c**2*d**4 - 20*c**2*d*e**3*x**3 - 15*c**2*
e**4*x**4 + x**2*(-15*a*c*e**4 - 15*c**2*d**2*e**2) + x*(-6*a*c*d*e**3 - 6
*c**2*d**3*e))/(30*d**6*e**5 + 180*d**5*e**6*x + 450*d**4*e**7*x**2 + 600*
d**3*e**8*x**3 + 450*d**2*e**9*x**4 + 180*d*e**10*x**5 + 30*e**11*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^2)^2}{(d+ex)^7}\,dx = -\frac{15\,c^2e^4x^4+20\,c^2de^3x^3+c^2d^4+acd^2e^2+5\,a^2e^4+15\,(c^2d^2e^2+ace^4)x^2+6\,(c^2d^3e+acde^3)x}{30\,(e^{11}x^6+6\,de^{10}x^5+15\,d^2e^9x^4+20\,d^3e^8x^3+15\,d^4e^7x^2+6\,d^5e^6x+d^6e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^7,x, algorithm="maxima")
```

output
```
-1/30*(15*c^2*e^4*x^4 + 20*c^2*d*e^3*x^3 + c^2*d^4 + a*c*d^2*e^2 + 5*a^2*e
^4 + 15*(c^2*d^2*e^2 + a*c*e^4)*x^2 + 6*(c^2*d^3*e + a*c*d*e^3)*x)/(e^11*x
^6 + 6*d*e^10*x^5 + 15*d^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d
^5*e^6*x + d^6*e^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88

$$\int \frac{(a+cx^2)^2}{(d+ex)^7}\,dx = -\frac{15\,c^2e^4x^4+20\,c^2de^3x^3+15\,c^2d^2e^2x^2+15\,ace^4x^2+6\,c^2d^3ex+6\,acde^3x+c^2d^4+acd^2e^2+5\,a^2e^4}{30\,(ex+d)^6e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^7,x, algorithm="giac")
```

output
```
-1/30*(15*c^2*e^4*x^4 + 20*c^2*d*e^3*x^3 + 15*c^2*d^2*e^2*x^2 + 15*a*c*e^4
*x^2 + 6*c^2*d^3*e*x + 6*a*c*d*e^3*x + c^2*d^4 + a*c*d^2*e^2 + 5*a^2*e^4)/
((e*x + d)^6*e^5)
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

$$\int \frac{(a+cx^2)^2}{(d+ex)^7}\,dx = -\frac{\frac{5\,a^2\,e^4+a\,c\,d^2\,e^2+c^2\,d^4}{30\,e^5}+\frac{c^2\,x^4}{2\,e}+\frac{2\,c^2\,d\,x^3}{3\,e^2}+\frac{c\,x^2\,\left(c\,d^2+a\,e^2\right)}{2\,e^3}+\frac{c\,d\,x\,\left(c\,d^2+a\,e^2\right)}{5\,e^4}}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6}$$

input
```
int((a + c*x^2)^2/(d + e*x)^7,x)
```

output
```
-((5*a^2*e^4 + c^2*d^4 + a*c*d^2*e^2)/(30*e^5) + (c^2*x^4)/(2*e) + (2*c^2*
d*x^3)/(3*e^2) + (c*x^2*(a*e^2 + c*d^2))/(2*e^3) + (c*d*x*(a*e^2 + c*d^2))
/(5*e^4))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 +
 15*d^2*e^4*x^4 + 6*d^5*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^2}{(d+ex)^7}\,dx = \frac{-15c^2e^4x^4-20c^2d\,e^3x^3-15ac\,e^4x^2-15c^2d^2e^2x^2-6acd\,e^3x-6c^2d^3ex-5a^2e^4-ac\,d^2e^2-c^2d^4}{30e^5\left(e^6x^6+6d\,e^5x^5+15d^2e^4x^4+20d^3e^3x^3+15d^4e^2x^2+6d^5ex+d^6\right)}$$

input
```
int((c*x^2+a)^2/(e*x+d)^7,x)
```

output
```
( - 5*a**2*e**4 - a*c*d**2*e**2 - 6*a*c*d*e**3*x - 15*a*c*e**4*x**2 - c**2
*d**4 - 6*c**2*d**3*e*x - 15*c**2*d**2*e**2*x**2 - 20*c**2*d*e**3*x**3 - 1
5*c**2*e**4*x**4)/(30*e**5*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**
3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.72 $\int \frac{(a+cx^2)^2}{(d+ex)^8} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{(a+cx^2)^2}{(d+ex)^8} \, dx = -\frac{(cd^2+ae^2)^2}{7e^5(d+ex)^7} + \frac{2cd(cd^2+ae^2)}{3e^5(d+ex)^6} - \frac{2c(3cd^2+ae^2)}{5e^5(d+ex)^5} + \frac{c^2d}{e^5(d+ex)^4} - \frac{c^2}{3e^5(d+ex)^3}$$

output
```
-1/7*(a*e^2+c*d^2)^2/e^5/(e*x+d)^7+2/3*c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^6-2/5
*c*(a*e^2+3*c*d^2)/e^5/(e*x+d)^5+c^2*d/e^5/(e*x+d)^4-1/3*c^2/e^5/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

$$\int \frac{(a+cx^2)^2}{(d+ex)^8} \, dx = -\frac{15a^2e^4 + 2ace^2(d^2+7dex+21e^2x^2) + c^2(d^4+7d^3ex+21d^2e^2x^2+35de^3x^3+35e^4x^4)}{105e^5(d+ex)^7}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^8,x]
```

output
```
-1/105*(15*a^2*e^4 + 2*a*c*e^2*(d^2 + 7*d*e*x + 21*e^2*x^2) + c^2*(d^4 + 7
*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4))/(e^5*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4(d+ex)^6}-\frac{4cd(ae^2+cd^2)}{e^4(d+ex)^7}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^8}+\frac{c^2}{e^4(d+ex)^4}-\frac{4c^2d}{e^4(d+ex)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c(ae^2+3cd^2)}{5e^5(d+ex)^5}+\frac{2cd(ae^2+cd^2)}{3e^5(d+ex)^6}-\frac{(ae^2+cd^2)^2}{7e^5(d+ex)^7}-\frac{c^2}{3e^5(d+ex)^3}+\frac{c^2d}{e^5(d+ex)^4}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^8,x]
```

output
```
-1/7*(c*d^2 + a*e^2)^2/(e^5*(d + e*x)^7) + (2*c*d*(c*d^2 + a*e^2))/(3*e^5*
(d + e*x)^6) - (2*c*(3*c*d^2 + a*e^2))/(5*e^5*(d + e*x)^5) + (c^2*d)/(e^5*
(d + e*x)^4) - c^2/(3*e^5*(d + e*x)^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{35c^{2}x^{4}e^{4}+35d\,c^{2}x^{3}e^{3}+42x^{2}ac\,e^{4}+21x^{2}c^{2}d^{2}e^{2}+14xacd\,e^{3}+7x\,c^{2}d^{3}e+15a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}{105e^{5}\left(ex+d\right)^{7}}$	105
risch	$\frac{-\frac{c^{2}x^{4}}{3e}-\frac{c^{2}d\,x^{3}}{3e^{2}}-\frac{c\left(2a\,e^{2}+c\,d^{2}\right)x^{2}}{5e^{3}}-\frac{cd\left(2a\,e^{2}+c\,d^{2}\right)x}{15e^{4}}-\frac{15a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}{105e^{5}}}{\left(ex+d\right)^{7}}$	105
orering	$-\frac{35c^{2}x^{4}e^{4}+35d\,c^{2}x^{3}e^{3}+42x^{2}ac\,e^{4}+21x^{2}c^{2}d^{2}e^{2}+14xacd\,e^{3}+7x\,c^{2}d^{3}e+15a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}{105e^{5}\left(ex+d\right)^{7}}$	105
parallelrisch	$\frac{-35c^{2}x^{4}e^{6}-35c^{2}d\,x^{3}e^{5}-42ac\,e^{6}x^{2}-21c^{2}d^{2}e^{4}x^{2}-14acd\,e^{5}x-7c^{2}d^{3}e^{3}x-15a^{2}e^{6}-2ac\,d^{2}e^{4}-c^{2}d^{4}e^{2}}{105e^{7}\left(ex+d\right)^{7}}$	111
norman	$\frac{-\frac{c^{2}x^{4}}{3e}-\frac{c^{2}d\,x^{3}}{3e^{2}}-\frac{\left(2e^{4}ac+d^{2}e^{2}c^{2}\right)x^{2}}{5e^{5}}-\frac{d\left(2e^{4}ac+d^{2}e^{2}c^{2}\right)x}{15e^{6}}-\frac{15a^{2}e^{6}+2ac\,d^{2}e^{4}+c^{2}d^{4}e^{2}}{105e^{7}}}{\left(ex+d\right)^{7}}$	118
default	$-\frac{c^{2}}{3e^{5}\left(ex+d\right)^{3}}+\frac{c^{2}d}{e^{5}\left(ex+d\right)^{4}}-\frac{a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}{7e^{5}\left(ex+d\right)^{7}}-\frac{2c\left(a\,e^{2}+3c\,d^{2}\right)}{5e^{5}\left(ex+d\right)^{5}}+\frac{2cd\left(a\,e^{2}+c\,d^{2}\right)}{3e^{5}\left(ex+d\right)^{6}}$	119

input
```
int((c*x^2+a)^2/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/105/e^5*(35*c^2*e^4*x^4+35*c^2*d*e^3*x^3+42*a*c*e^4*x^2+21*c^2*d^2*e^2*
x^2+14*a*c*d*e^3*x+7*c^2*d^3*e*x+15*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/(e*x+d)
^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.52

$$\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx = -\frac{35\,c^2e^4x^4+35\,c^2de^3x^3+c^2d^4+2\,acd^2e^2+15\,a^2e^4+21\,(c^2d^2e^2+2\,ace^4)x^2+7\,(c^2d^3e+2\,acde^3)x}{105\,(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+7\,d^6e^6x+d^7e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^8,x, algorithm="fricas")
```

output
```
-1/105*(35*c^2*e^4*x^4 + 35*c^2*d*e^3*x^3 + c^2*d^4 + 2*a*c*d^2*e^2 + 15*a
^2*e^4 + 21*(c^2*d^2*e^2 + 2*a*c*e^4)*x^2 + 7*(c^2*d^3*e + 2*a*c*d*e^3)*x)
/(e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*
x^3 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d^7*e^5)
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.62

$$\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx = \frac{-15a^2e^4-2acd^2e^2-c^2d^4-35c^2de^3x^3-35c^2e^4x^4+x^2(-42ace^4-21c^2d^2e^2)+x(-14acde^3-7c^2d^3e)}{105d^7e^5+735d^6e^6x+2205d^5e^7x^2+3675d^4e^8x^3+3675d^3e^9x^4+2205d^2e^{10}x^5+735de^{11}x^6+105e^{12}x^7}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**8,x)
```

output
```
(-15*a**2*e**4 - 2*a*c*d**2*e**2 - c**2*d**4 - 35*c**2*d*e**3*x**3 - 35*c*
*2*e**4*x**4 + x**2*(-42*a*c*e**4 - 21*c**2*d**2*e**2) + x*(-14*a*c*d*e**3
 - 7*c**2*d**3*e))/(105*d**7*e**5 + 735*d**6*e**6*x + 2205*d**5*e**7*x**2
+ 3675*d**4*e**8*x**3 + 3675*d**3*e**9*x**4 + 2205*d**2*e**10*x**5 + 735*d
*e**11*x**6 + 105*e**12*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.52

$$\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx = -\frac{35\,c^2e^4x^4+35\,c^2de^3x^3+c^2d^4+2\,acd^2e^2+15\,a^2e^4+21\,(c^2d^2e^2+2\,ace^4)x^2+7\,(c^2d^3e+2\,acde^3)x}{105\,(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+7\,d^6e^6x+d^7e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^8,x, algorithm="maxima")
```

output
```
-1/105*(35*c^2*e^4*x^4 + 35*c^2*d*e^3*x^3 + c^2*d^4 + 2*a*c*d^2*e^2 + 15*a
^2*e^4 + 21*(c^2*d^2*e^2 + 2*a*c*e^4)*x^2 + 7*(c^2*d^3*e + 2*a*c*d*e^3)*x)
/(e^12*x^7 + 7*d*e^11*x^6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*
x^3 + 21*d^5*e^7*x^2 + 7*d^6*e^6*x + d^7*e^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx = -\frac{35\,c^2e^4x^4+35\,c^2de^3x^3+21\,c^2d^2e^2x^2+42\,ace^4x^2+7\,c^2d^3ex+14\,acde^3x+c^2d^4+2\,acd^2e^2+15\,a^2e^4}{105\,(ex+d)^7e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^8,x, algorithm="giac")
```

output
```
-1/105*(35*c^2*e^4*x^4 + 35*c^2*d*e^3*x^3 + 21*c^2*d^2*e^2*x^2 + 42*a*c*e^
4*x^2 + 7*c^2*d^3*e*x + 14*a*c*d*e^3*x + c^2*d^4 + 2*a*c*d^2*e^2 + 15*a^2*
e^4)/((e*x + d)^7*e^5)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx = -\frac{\frac{15\,a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4}{105\,e^5}+\frac{c^2\,x^4}{3\,e}+\frac{c^2\,d\,x^3}{3\,e^2}+\frac{c\,x^2\,\left(c\,d^2+2\,a\,e^2\right)}{5\,e^3}+\frac{c\,d\,x\,\left(c\,d^2+2\,a\,e^2\right)}{15\,e^4}}{d^7+7\,d^6\,e\,x+21\,d^5\,e^2\,x^2+35\,d^4\,e^3\,x^3+35\,d^3\,e^4\,x^4+21\,d^2\,e^5\,x^5+7\,d\,e^6\,x^6+e^7\,x^7}$$

input
```
int((a + c*x^2)^2/(d + e*x)^8,x)
```

output
```
-((15*a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)/(105*e^5) + (c^2*x^4)/(3*e) + (c^
2*d*x^3)/(3*e^2) + (c*x^2*(2*a*e^2 + c*d^2))/(5*e^3) + (c*d*x*(2*a*e^2 + c
*d^2))/(15*e^4))/(d^7 + e^7*x^7 + 7*d*e^6*x^6 + 21*d^5*e^2*x^2 + 35*d^4*e^
3*x^3 + 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 + 7*d^6*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^2}{(d+ex)^8}\,dx = \frac{-35c^2e^4x^4-35c^2d\,e^3x^3-42ac\,e^4x^2-21c^2d^2e^2x^2-14acd\,e^3x-7c^2d^3ex-15a^2e^4-2ac\,d^2e^2-c^2d^4}{105e^5\left(e^7x^7+7d\,e^6x^6+21d^2e^5x^5+35d^3e^4x^4+35d^4e^3x^3+21d^5e^2x^2+7d^6ex+d^7\right)}$$

input
```
int((c*x^2+a)^2/(e*x+d)^8,x)
```

output
```
( - 15*a**2*e**4 - 2*a*c*d**2*e**2 - 14*a*c*d*e**3*x - 42*a*c*e**4*x**2 -
c**2*d**4 - 7*c**2*d**3*e*x - 21*c**2*d**2*e**2*x**2 - 35*c**2*d*e**3*x**3
 - 35*c**2*e**4*x**4)/(105*e**5*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 3
5*d**4*e**3*x**3 + 35*d**3*e**4*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 +
 e**7*x**7))
```

3.73 $\int (d+ex)^6 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 190

$$\int (d+ex)^6 \left(a+cx^2\right)^3 \, dx = \frac{(cd^2+ae^2)^3 (d+ex)^7}{7e^7} - \frac{3cd(cd^2+ae^2)^2 (d+ex)^8}{4e^7} + \frac{c(cd^2+ae^2)(5cd^2+ae^2)(d+ex)^9}{3e^7} - \frac{2c^2d(5cd^2+3ae^2)(d+ex)^{10}}{5e^7} + \frac{3c^2(5cd^2+ae^2)(d+ex)^{11}}{11e^7} - \frac{c^3d(d+ex)^{12}}{2e^7} + \frac{c^3(d+ex)^{13}}{13e^7}$$

output
```
1/7*(a*e^2+c*d^2)^3*(e*x+d)^7/e^7-3/4*c*d*(a*e^2+c*d^2)^2*(e*x+d)^8/e^7+1/
3*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^9/e^7-2/5*c^2*d*(3*a*e^2+5*c*d^2
)*(e*x+d)^10/e^7+3/11*c^2*(a*e^2+5*c*d^2)*(e*x+d)^11/e^7-1/2*c^3*d*(e*x+d)
^12/e^7+1/13*c^3*(e*x+d)^13/e^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.78

$$\int (d+ex)^6 \left(a+cx^2\right)^3 \, dx = a^3d^6x + 3a^3d^5ex^2 + a^2d^4\left(cd^2+5ae^2\right)x^3 + \frac{1}{2}a^2d^3e\left(9cd^2+10ae^2\right)x^4 + \frac{3}{5}ad^2\left(c^2d^4+15acd^2e^2+5a^2e^4\right)x^5 + ade\left(3c^2d^4+10acd^2e^2+a^2e^4\right)x^6 + \frac{1}{7}\left(c^3d^6+45ac^2d^4e^2+45a^2cd^2e^4+a^3e^6\right)x^7 + \frac{3}{4}cde\left(c^2d^4+10acd^2e^2+3a^2e^4\right)x^8 + \frac{1}{3}ce^2\left(5c^2d^4+15acd^2e^2+a^2e^4\right)x^9 + \frac{1}{5}c^2de^3\left(10cd^2+9ae^2\right)x^{10} + \frac{3}{11}c^2e^4\left(5cd^2+ae^2\right)x^{11} + \frac{1}{2}c^3de^5x^{12} + \frac{1}{13}c^3e^6x^{13}$$

input
```
Integrate[(d + e*x)^6*(a + c*x^2)^3,x]
```

output
```
a^3*d^6*x + 3*a^3*d^5*e*x^2 + a^2*d^4*(c*d^2 + 5*a*e^2)*x^3 + (a^2*d^3*e*(
9*c*d^2 + 10*a*e^2)*x^4)/2 + (3*a*d^2*(c^2*d^4 + 15*a*c*d^2*e^2 + 5*a^2*e^
4)*x^5)/5 + a*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^6 + ((c^3*d^6 +
 45*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + a^3*e^6)*x^7)/7 + (3*c*d*e*(c^2*d^4
 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x^8)/4 + (c*e^2*(5*c^2*d^4 + 15*a*c*d^2*e^2
 + a^2*e^4)*x^9)/3 + (c^2*d*e^3*(10*c*d^2 + 9*a*e^2)*x^10)/5 + (3*c^2*e^4*
(5*c*d^2 + a*e^2)*x^11)/11 + (c^3*d*e^5*x^12)/2 + (c^3*e^6*x^13)/13
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex)^6 \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(d+ex)^{10}\left(ae^2+5cd^2\right)}{e^6}-\frac{4c^2d(d+ex)^9\left(3ae^2+5cd^2\right)}{e^6}+\frac{3c(d+ex)^8\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^6}-\frac{6cd(d\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3c^2(d+ex)^{11}\left(ae^2+5cd^2\right)}{11e^7}-\frac{2c^2d(d+ex)^{10}\left(3ae^2+5cd^2\right)}{5e^7}+\frac{c(d+ex)^9\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{3e^7}-\frac{3cd(d+ex)^8\left(ae^2+cd^2\right)^2}{4e^7}+\frac{(d+ex)^7\left(ae^2+cd^2\right)^3}{7e^7}+\frac{c^3(d+ex)^{13}}{13e^7}-\frac{c^3d(d+ex)^{12}}{2e^7}$$

input
```
Int[(d + e*x)^6*(a + c*x^2)^3,x]
```

output
```
((c*d^2 + a*e^2)^3*(d + e*x)^7)/(7*e^7) - (3*c*d*(c*d^2 + a*e^2)^2*(d + e*
x)^8)/(4*e^7) + (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^9)/(3*e^7)
- (2*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^10)/(5*e^7) + (3*c^2*(5*c*d^2 + a
*e^2)*(d + e*x)^11)/(11*e^7) - (c^3*d*(d + e*x)^12)/(2*e^7) + (c^3*(d + e*
x)^13)/(13*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.77

method	result
norman	$\frac{e^6c^3x^{13}}{13}+\frac{d\,e^5c^3x^{12}}{2}+\left(\frac{3}{11}e^6a\,c^2+\frac{15}{11}d^2e^4c^3\right)x^{11}+\left(\frac{9}{5}d\,e^5a\,c^2+2d^3e^3c^3\right)x^{10}+\left(\frac{1}{3}e^6a^2c+5d^2e\right.$
default	$\frac{e^6c^3x^{13}}{13}+\frac{d\,e^5c^3x^{12}}{2}+\frac{(3e^6a\,c^2+15d^2e^4c^3)x^{11}}{11}+\frac{(18d\,e^5a\,c^2+20d^3e^3c^3)x^{10}}{10}+\frac{(3e^6a^2c+45d^2e^4a\,c^2+15d^4e^2c^3)x^9}{9}$
gosper	$\frac{9}{5}x^{10}d\,e^5a\,c^2+5x^9d^2e^4a\,c^2+\frac{9}{4}x^8d\,e^5a^2c+\frac{15}{2}x^8d^3e^3a\,c^2+\frac{45}{7}x^7d^2e^4a^2c+\frac{45}{7}x^7d^4e^2a\,c^2+9x$
risch	$\frac{9}{5}x^{10}d\,e^5a\,c^2+5x^9d^2e^4a\,c^2+\frac{9}{4}x^8d\,e^5a^2c+\frac{15}{2}x^8d^3e^3a\,c^2+\frac{45}{7}x^7d^2e^4a^2c+\frac{45}{7}x^7d^4e^2a\,c^2+9x$
parallelrisch	$\frac{9}{5}x^{10}d\,e^5a\,c^2+5x^9d^2e^4a\,c^2+\frac{9}{4}x^8d\,e^5a^2c+\frac{15}{2}x^8d^3e^3a\,c^2+\frac{45}{7}x^7d^2e^4a^2c+\frac{45}{7}x^7d^4e^2a\,c^2+9x$
orering	$x\left(4620e^6c^3x^{12}+30030d\,e^5c^3x^{11}+16380a\,c^2e^6x^{10}+81900c^3d^2e^4x^{10}+108108a\,c^2d\,e^5x^9+120120c^3d^3e^3x^9+20020a^2c\,e^6x^8+3\right.$

input
```
int((e*x+d)^6*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*e^6*c^3*x^13+1/2*d*e^5*c^3*x^12+(3/11*e^6*a*c^2+15/11*d^2*e^4*c^3)*x^
11+(9/5*d*e^5*a*c^2+2*d^3*e^3*c^3)*x^10+(1/3*e^6*a^2*c+5*d^2*e^4*a*c^2+5/3
*d^4*e^2*c^3)*x^9+(9/4*d*e^5*a^2*c+15/2*d^3*e^3*a*c^2+3/4*d^5*e*c^3)*x^8+(
1/7*e^6*a^3+45/7*d^2*e^4*a^2*c+45/7*d^4*e^2*a*c^2+1/7*d^6*c^3)*x^7+(a^3*d*
e^5+10*a^2*c*d^3*e^3+3*a*c^2*d^5*e)*x^6+(3*d^2*e^4*a^3+9*d^4*e^2*a^2*c+3/5
*d^6*a*c^2)*x^5+(5*d^3*e^3*a^3+9/2*d^5*e*a^2*c)*x^4+(5*a^3*d^4*e^2+a^2*c*d
^6)*x^3+3*d^5*e*a^3*x^2+d^6*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.77

$$\int (d+ex)^6 \left(a+cx^2\right)^3 dx = \frac{1}{13}c^3e^6x^{13} + \frac{1}{2}c^3de^5x^{12} + \frac{3}{11}\left(5c^3d^2e^4 + ac^2e^6\right)x^{11} + 3a^3d^5ex^2 + \frac{1}{5}\left(10c^3d^3e^3 + 9ac^2de^5\right)x^{10} + a^3d^6x + \frac{1}{3}\left(5c^3d^4e^2 + 15ac^2d^2e^4 + a^2ce^6\right)x^9 + \frac{3}{4}\left(c^3d^5e + 10ac^2d^3e^3 + 3a^2cde^5\right)x^8 + \frac{1}{7}\left(c^3d^6 + 45ac^2d^4e^2 + 45a^2cd^2e^4 + a^3e^6\right)x^7 + \left(3ac^2d^5e + 10a^2cd^3e^3 + a^3de^5\right)x^6 + \frac{3}{5}\left(ac^2d^6 + 15a^2cd^4e^2 + 5a^3d^2e^4\right)x^5 + \frac{1}{2}\left(9a^2cd^5e + 10a^3d^3e^3\right)x^4 + \left(a^2cd^6 + 5a^3d^4e^2\right)x^3$$

input

```
integrate((e*x+d)^6*(c*x^2+a)^3,x, algorithm="fricas")
```

output

```
1/13*c^3*e^6*x^13 + 1/2*c^3*d*e^5*x^12 + 3/11*(5*c^3*d^2*e^4 + a*c^2*e^6)*
x^11 + 3*a^3*d^5*e*x^2 + 1/5*(10*c^3*d^3*e^3 + 9*a*c^2*d*e^5)*x^10 + a^3*d
^6*x + 1/3*(5*c^3*d^4*e^2 + 15*a*c^2*d^2*e^4 + a^2*c*e^6)*x^9 + 3/4*(c^3*d
^5*e + 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x^8 + 1/7*(c^3*d^6 + 45*a*c^2*d^4
*e^2 + 45*a^2*c*d^2*e^4 + a^3*e^6)*x^7 + (3*a*c^2*d^5*e + 10*a^2*c*d^3*e^3
 + a^3*d*e^5)*x^6 + 3/5*(a*c^2*d^6 + 15*a^2*c*d^4*e^2 + 5*a^3*d^2*e^4)*x^5
 + 1/2*(9*a^2*c*d^5*e + 10*a^3*d^3*e^3)*x^4 + (a^2*c*d^6 + 5*a^3*d^4*e^2)*
x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(180) = 360$.

Time = 0.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.95

$$\int (d+ex)^6 (a+cx^2)^3 \, dx = a^3d^6x + 3a^3d^5ex^2 + \frac{c^3de^5x^{12}}{2} + \frac{c^3e^6x^{13}}{13} + x^{11} \cdot \left(\frac{3ac^2e^6}{11} + \frac{15c^3d^2e^4}{11}\right) + x^{10} \cdot \left(\frac{9ac^2de^5}{5} + 2c^3d^3e^3\right) + x^9\left(\frac{a^2ce^6}{3} + 5ac^2d^2e^4 + \frac{5c^3d^4e^2}{3}\right) + x^8 \cdot \left(\frac{9a^2cde^5}{4} + \frac{15ac^2d^3e^3}{2} + \frac{3c^3d^5e}{4}\right) + x^7\left(\frac{a^3e^6}{7} + \frac{45a^2cd^2e^4}{7} + \frac{45ac^2d^4e^2}{7} + \frac{c^3d^6}{7}\right) + x^6\left(a^3de^5 + 10a^2cd^3e^3 + 3ac^2d^5e\right) + x^5 \cdot \left(3a^3d^2e^4 + 9a^2cd^4e^2 + \frac{3ac^2d^6}{5}\right) + x^4 \cdot \left(5a^3d^3e^3 + \frac{9a^2cd^5e}{2}\right) + x^3 \cdot \left(5a^3d^4e^2 + a^2cd^6\right)$$

input
```
integrate((e*x+d)**6*(c*x**2+a)**3,x)
```

output
```
a**3*d**6*x + 3*a**3*d**5*e*x**2 + c**3*d*e**5*x**12/2 + c**3*e**6*x**13/1
3 + x**11*(3*a*c**2*e**6/11 + 15*c**3*d**2*e**4/11) + x**10*(9*a*c**2*d*e*
*5/5 + 2*c**3*d**3*e**3) + x**9*(a**2*c*e**6/3 + 5*a*c**2*d**2*e**4 + 5*c*
*3*d**4*e**2/3) + x**8*(9*a**2*c*d*e**5/4 + 15*a*c**2*d**3*e**3/2 + 3*c**3
*d**5*e/4) + x**7*(a**3*e**6/7 + 45*a**2*c*d**2*e**4/7 + 45*a*c**2*d**4*e*
*2/7 + c**3*d**6/7) + x**6*(a**3*d*e**5 + 10*a**2*c*d**3*e**3 + 3*a*c**2*d
**5*e) + x**5*(3*a**3*d**2*e**4 + 9*a**2*c*d**4*e**2 + 3*a*c**2*d**6/5) +
x**4*(5*a**3*d**3*e**3 + 9*a**2*c*d**5*e/2) + x**3*(5*a**3*d**4*e**2 + a**
2*c*d**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.77

$$\begin{aligned}\int (d+ex)^6 \left(a+cx^2\right)^3 dx &= \frac{1}{13}c^3e^6x^{13} + \frac{1}{2}c^3de^5x^{12} + \frac{3}{11}\left(5c^3d^2e^4 + ac^2e^6\right)x^{11} \\ &\quad + 3a^3d^5ex^2 + \frac{1}{5}\left(10c^3d^3e^3 + 9ac^2de^5\right)x^{10} \\ &\quad + a^3d^6x + \frac{1}{3}\left(5c^3d^4e^2 + 15ac^2d^2e^4 + a^2ce^6\right)x^9 \\ &\quad + \frac{3}{4}\left(c^3d^5e + 10ac^2d^3e^3 + 3a^2cde^5\right)x^8 \\ &\quad + \frac{1}{7}\left(c^3d^6 + 45ac^2d^4e^2 + 45a^2cd^2e^4 + a^3e^6\right)x^7 \\ &\quad + \left(3ac^2d^5e + 10a^2cd^3e^3 + a^3de^5\right)x^6 \\ &\quad + \frac{3}{5}\left(ac^2d^6 + 15a^2cd^4e^2 + 5a^3d^2e^4\right)x^5 \\ &\quad + \frac{1}{2}\left(9a^2cd^5e + 10a^3d^3e^3\right)x^4 + \left(a^2cd^6 + 5a^3d^4e^2\right)x^3\end{aligned}$$

input
```
integrate((e*x+d)^6*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/13*c^3*e^6*x^13 + 1/2*c^3*d*e^5*x^12 + 3/11*(5*c^3*d^2*e^4 + a*c^2*e^6)*
x^11 + 3*a^3*d^5*e*x^2 + 1/5*(10*c^3*d^3*e^3 + 9*a*c^2*d*e^5)*x^10 + a^3*d
^6*x + 1/3*(5*c^3*d^4*e^2 + 15*a*c^2*d^2*e^4 + a^2*c*e^6)*x^9 + 3/4*(c^3*d
^5*e + 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x^8 + 1/7*(c^3*d^6 + 45*a*c^2*d^4
*e^2 + 45*a^2*c*d^2*e^4 + a^3*e^6)*x^7 + (3*a*c^2*d^5*e + 10*a^2*c*d^3*e^3
 + a^3*d*e^5)*x^6 + 3/5*(a*c^2*d^6 + 15*a^2*c*d^4*e^2 + 5*a^3*d^2*e^4)*x^5
 + 1/2*(9*a^2*c*d^5*e + 10*a^3*d^3*e^3)*x^4 + (a^2*c*d^6 + 5*a^3*d^4*e^2)*
x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(176) = 352.

Time = 0.12 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.91

$$\int (d+ex)^6 \left(a+cx^2\right)^3 \, dx = \frac{1}{13}c^3e^6x^{13} + \frac{1}{2}c^3de^5x^{12} + \frac{15}{11}c^3d^2e^4x^{11} + \frac{3}{11}ac^2e^6x^{11} + 2c^3d^3e^3x^{10} + \frac{9}{5}ac^2de^5x^{10} + \frac{5}{3}c^3d^4e^2x^9 + 5ac^2d^2e^4x^9 + \frac{1}{3}a^2ce^6x^9 + \frac{3}{4}c^3d^5ex^8 + \frac{15}{2}ac^2d^3e^3x^8 + \frac{9}{4}a^2cde^5x^8 + \frac{1}{7}c^3d^6x^7 + \frac{45}{7}ac^2d^4e^2x^7 + \frac{45}{7}a^2cd^2e^4x^7 + \frac{1}{7}a^3e^6x^7 + 3ac^2d^5ex^6 + 10a^2cd^3e^3x^6 + a^3de^5x^6 + \frac{3}{5}ac^2d^6x^5 + 9a^2cd^4e^2x^5 + 3a^3d^2e^4x^5 + \frac{9}{2}a^2cd^5ex^4 + 5a^3d^3e^3x^4 + a^2cd^6x^3 + 5a^3d^4e^2x^3 + 3a^3d^5ex^2 + a^3d^6x$$

input
```
integrate((e*x+d)^6*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/13*c^3*e^6*x^13 + 1/2*c^3*d*e^5*x^12 + 15/11*c^3*d^2*e^4*x^11 + 3/11*a*c
^2*e^6*x^11 + 2*c^3*d^3*e^3*x^10 + 9/5*a*c^2*d*e^5*x^10 + 5/3*c^3*d^4*e^2*
x^9 + 5*a*c^2*d^2*e^4*x^9 + 1/3*a^2*c*e^6*x^9 + 3/4*c^3*d^5*e*x^8 + 15/2*a
*c^2*d^3*e^3*x^8 + 9/4*a^2*c*d*e^5*x^8 + 1/7*c^3*d^6*x^7 + 45/7*a*c^2*d^4*
e^2*x^7 + 45/7*a^2*c*d^2*e^4*x^7 + 1/7*a^3*e^6*x^7 + 3*a*c^2*d^5*e*x^6 + 1
0*a^2*c*d^3*e^3*x^6 + a^3*d*e^5*x^6 + 3/5*a*c^2*d^6*x^5 + 9*a^2*c*d^4*e^2*
x^5 + 3*a^3*d^2*e^4*x^5 + 9/2*a^2*c*d^5*e*x^4 + 5*a^3*d^3*e^3*x^4 + a^2*c*
d^6*x^3 + 5*a^3*d^4*e^2*x^3 + 3*a^3*d^5*e*x^2 + a^3*d^6*x
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.73

$$\int (d+ex)^6 \left(a+cx^2\right)^3 \, dx = x^7 \left(\frac{a^3 e^6}{7} + \frac{45 a^2 c d^2 e^4}{7} + \frac{45 a c^2 d^4 e^2}{7} + \frac{c^3 d^6}{7}\right) + x^3 \left(5 a^3 d^4 e^2 + c a^2 d^6\right) + x^{11} \left(\frac{15 c^3 d^2 e^4}{11} + \frac{3 a c^2 e^6}{11}\right) + x^5 \left(3 a^3 d^2 e^4 + 9 a^2 c d^4 e^2 + \frac{3 a c^2 d^6}{5}\right) + x^9 \left(\frac{a^2 c e^6}{3} + 5 a c^2 d^2 e^4 + \frac{5 c^3 d^4 e^2}{3}\right) + a^3 d^6 x + \frac{c^3 e^6 x^{13}}{13} + 3 a^3 d^5 e x^2 + \frac{c^3 d e^5 x^{12}}{2} + a d e x^6 \left(a^2 e^4 + 10 a c d^2 e^2 + 3 c^2 d^4\right) + \frac{3 c d e x^8 \left(3 a^2 e^4 + 10 a c d^2 e^2 + c^2 d^4\right)}{4} + \frac{a^2 d^3 e x^4 \left(9 c d^2 + 10 a e^2\right)}{2} + \frac{c^2 d e^3 x^{10} \left(10 c d^2 + 9 a e^2\right)}{5}$$

input
```
int((a + c*x^2)^3*(d + e*x)^6,x)
```

output
```
x^7*((a^3*e^6)/7 + (c^3*d^6)/7 + (45*a*c^2*d^4*e^2)/7 + (45*a^2*c*d^2*e^4)
/7) + x^3*(a^2*c*d^6 + 5*a^3*d^4*e^2) + x^11*((3*a*c^2*e^6)/11 + (15*c^3*d
^2*e^4)/11) + x^5*((3*a*c^2*d^6)/5 + 3*a^3*d^2*e^4 + 9*a^2*c*d^4*e^2) + x^
9*((a^2*c*e^6)/3 + (5*c^3*d^4*e^2)/3 + 5*a*c^2*d^2*e^4) + a^3*d^6*x + (c^3
*e^6*x^13)/13 + 3*a^3*d^5*e*x^2 + (c^3*d*e^5*x^12)/2 + a*d*e*x^6*(a^2*e^4
+ 3*c^2*d^4 + 10*a*c*d^2*e^2) + (3*c*d*e*x^8*(3*a^2*e^4 + c^2*d^4 + 10*a*c
*d^2*e^2))/4 + (a^2*d^3*e*x^4*(10*a*e^2 + 9*c*d^2))/2 + (c^2*d*e^3*x^10*(9
*a*e^2 + 10*c*d^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.92

$$\int (d+ex)^6 \left(a+cx^2\right)^3 dx$$
$$= \frac{x(4620c^3e^6x^{12} + 30030c^3d\,e^5x^{11} + 16380a\,c^2e^6x^{10} + 81900c^3d^2e^4x^{10} + 108108a\,c^2d\,e^5x^9 + 120120c^3d^3e^3}{}$$

input
```
int((e*x+d)^6*(c*x^2+a)^3,x)
```

output
```
(x*(60060*a**3*d**6 + 180180*a**3*d**5*e*x + 300300*a**3*d**4*e**2*x**2 +
300300*a**3*d**3*e**3*x**3 + 180180*a**3*d**2*e**4*x**4 + 60060*a**3*d*e**
5*x**5 + 8580*a**3*e**6*x**6 + 60060*a**2*c*d**6*x**2 + 270270*a**2*c*d**5
*e*x**3 + 540540*a**2*c*d**4*e**2*x**4 + 600600*a**2*c*d**3*e**3*x**5 + 38
6100*a**2*c*d**2*e**4*x**6 + 135135*a**2*c*d*e**5*x**7 + 20020*a**2*c*e**6
*x**8 + 36036*a*c**2*d**6*x**4 + 180180*a*c**2*d**5*e*x**5 + 386100*a*c**2
*d**4*e**2*x**6 + 450450*a*c**2*d**3*e**3*x**7 + 300300*a*c**2*d**2*e**4*x
**8 + 108108*a*c**2*d*e**5*x**9 + 16380*a*c**2*e**6*x**10 + 8580*c**3*d**6
*x**6 + 45045*c**3*d**5*e*x**7 + 100100*c**3*d**4*e**2*x**8 + 120120*c**3*
d**3*e**3*x**9 + 81900*c**3*d**2*e**4*x**10 + 30030*c**3*d*e**5*x**11 + 46
20*c**3*e**6*x**12))/60060
```

3.74 $\int (d+ex)^5 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 190

$$\int (d+ex)^5 \left(a+cx^2\right)^3 \, dx = \frac{\left(cd^2+ae^2\right)^3 (d+ex)^6}{6e^7} - \frac{6cd\left(cd^2+ae^2\right)^2 (d+ex)^7}{7e^7} + \frac{3c\left(cd^2+ae^2\right)\left(5cd^2+ae^2\right)(d+ex)^8}{8e^7} - \frac{4c^2d\left(5cd^2+3ae^2\right)(d+ex)^9}{9e^7} + \frac{3c^2\left(5cd^2+ae^2\right)(d+ex)^{10}}{10e^7} - \frac{6c^3d(d+ex)^{11}}{11e^7} + \frac{c^3(d+ex)^{12}}{12e^7}$$

output
```
1/6*(a*e^2+c*d^2)^3*(e*x+d)^6/e^7-6/7*c*d*(a*e^2+c*d^2)^2*(e*x+d)^7/e^7+3/
8*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^8/e^7-4/9*c^2*d*(3*a*e^2+5*c*d^2
)*(e*x+d)^9/e^7+3/10*c^2*(a*e^2+5*c*d^2)*(e*x+d)^10/e^7-6/11*c^3*d*(e*x+d)
^11/e^7+1/12*c^3*(e*x+d)^12/e^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.33

$$\int (d+ex)^5 \left(a+cx^2\right)^3 \, dx$$
$$= \frac{1}{6}a^3x\left(6d^5+15d^4ex+20d^3e^2x^2+15d^2e^3x^3+6de^4x^4+e^5x^5\right)$$
$$+ \frac{1}{420}ac^2x^5\left(252d^5+1050d^4ex+1800d^3e^2x^2+1575d^2e^3x^3+700de^4x^4+126e^5x^5\right)$$
$$+ \frac{c^3x^7(792d^5+3465d^4ex+6160d^3e^2x^2+5544d^2e^3x^3+2520de^4x^4+462e^5x^5)}{5544}$$
$$+ a^2c\left(d^5x^3+\frac{15}{4}d^4ex^4+6d^3e^2x^5+5d^2e^3x^6+\frac{15}{7}de^4x^7+\frac{3e^5x^8}{8}\right)$$

input
```
Integrate[(d + e*x)^5*(a + c*x^2)^3,x]
```

output
```
(a^3*x*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*e^4*x^4
 + e^5*x^5))/6 + (a*c^2*x^5*(252*d^5 + 1050*d^4*e*x + 1800*d^3*e^2*x^2 + 1
575*d^2*e^3*x^3 + 700*d*e^4*x^4 + 126*e^5*x^5))/420 + (c^3*x^7*(792*d^5 +
3465*d^4*e*x + 6160*d^3*e^2*x^2 + 5544*d^2*e^3*x^3 + 2520*d*e^4*x^4 + 462*
e^5*x^5))/5544 + a^2*c*(d^5*x^3 + (15*d^4*e*x^4)/4 + 6*d^3*e^2*x^5 + 5*d^2
*e^3*x^6 + (15*d*e^4*x^7)/7 + (3*e^5*x^8)/8)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex)^5 \, dx$$
$$\Big\downarrow \; 476$$

$$\int \left(\frac{3c^2(d+ex)^9\left(ae^2+5cd^2\right)}{e^6}-\frac{4c^2d(d+ex)^8\left(3ae^2+5cd^2\right)}{e^6}+\frac{3c(d+ex)^7\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^6}-\frac{6cd(d\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3c^2(d+ex)^{10}\left(ae^2+5cd^2\right)}{10e^7}-\frac{4c^2d(d+ex)^9\left(3ae^2+5cd^2\right)}{9e^7}+\frac{3c(d+ex)^8\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{8e^7}-\frac{6cd(d+ex)^7\left(ae^2+cd^2\right)^2}{7e^7}+\frac{(d+ex)^6\left(ae^2+cd^2\right)^3}{6e^7}+\frac{c^3(d+ex)^{12}}{12e^7}-\frac{6c^3d(d+ex)^{11}}{11e^7}$$

input
```
Int[(d + e*x)^5*(a + c*x^2)^3,x]
```

output
```
((c*d^2 + a*e^2)^3*(d + e*x)^6)/(6*e^7) - (6*c*d*(c*d^2 + a*e^2)^2*(d + e*
x)^7)/(7*e^7) + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^8)/(8*e^7
) - (4*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^9)/(9*e^7) + (3*c^2*(5*c*d^2 +
a*e^2)*(d + e*x)^10)/(10*e^7) - (6*c^3*d*(d + e*x)^11)/(11*e^7) + (c^3*(d
+ e*x)^12)/(12*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.49

method	result
norman	$\frac{e^5c^3x^{12}}{12}+\frac{5d\,e^4c^3x^{11}}{11}+\left(\frac{3}{10}e^5a\,c^2+d^2e^3c^3\right)x^{10}+\left(\frac{5}{3}d\,e^4a\,c^2+\frac{10}{9}d^3e^2c^3\right)x^9+\left(\frac{3}{8}e^5a^2c+\frac{15}{4}d^2e\right.$
default	$\frac{e^5c^3x^{12}}{12}+\frac{5d\,e^4c^3x^{11}}{11}+\frac{(3e^5a\,c^2+10d^2e^3c^3)x^{10}}{10}+\frac{(15d\,e^4a\,c^2+10d^3e^2c^3)x^9}{9}+\frac{(3e^5a^2c+30d^2e^3a\,c^2+5d^4e\,c^3)x^8}{8}+$
gosper	$\frac{1}{12}e^5c^3x^{12}+\frac{5}{11}d\,e^4c^3x^{11}+\frac{3}{10}x^{10}e^5a\,c^2+x^{10}d^2e^3c^3+\frac{5}{3}x^9d\,e^4a\,c^2+\frac{10}{9}x^9d^3e^2c^3+\frac{3}{8}x^8e^5a^2c$ -
risch	$\frac{1}{12}e^5c^3x^{12}+\frac{5}{11}d\,e^4c^3x^{11}+\frac{3}{10}x^{10}e^5a\,c^2+x^{10}d^2e^3c^3+\frac{5}{3}x^9d\,e^4a\,c^2+\frac{10}{9}x^9d^3e^2c^3+\frac{3}{8}x^8e^5a^2c$ -
parallelrisch	$\frac{1}{12}e^5c^3x^{12}+\frac{5}{11}d\,e^4c^3x^{11}+\frac{3}{10}x^{10}e^5a\,c^2+x^{10}d^2e^3c^3+\frac{5}{3}x^9d\,e^4a\,c^2+\frac{10}{9}x^9d^3e^2c^3+\frac{3}{8}x^8e^5a^2c$ -
orering	$\frac{x(2310e^5c^3x^{11}+12600d\,e^4c^3x^{10}+8316a\,c^2e^5x^9+27720c^3d^2e^3x^9+46200a\,c^2d\,e^4x^8+30800c^3d^3e^2x^8+10395a^2c\,e^5x^7+103950}{}$

input
```
int((e*x+d)^5*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*e^5*c^3*x^12+5/11*d*e^4*c^3*x^11+(3/10*e^5*a*c^2+d^2*e^3*c^3)*x^10+(5
/3*d*e^4*a*c^2+10/9*d^3*e^2*c^3)*x^9+(3/8*e^5*a^2*c+15/4*d^2*e^3*a*c^2+5/8
*d^4*e*c^3)*x^8+(15/7*d*e^4*a^2*c+30/7*d^3*e^2*a*c^2+1/7*d^5*c^3)*x^7+(1/6
*a^3*e^5+5*d^2*e^3*a^2*c+5/2*d^4*e*a*c^2)*x^6+(d*e^4*a^3+6*d^3*e^2*a^2*c+3
/5*d^5*a*c^2)*x^5+(5/2*d^2*e^3*a^3+15/4*d^4*e*a^2*c)*x^4+(10/3*d^3*e^2*a^3
+d^5*a^2*c)*x^3+5/2*d^4*e*a^3*x^2+d^5*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.54

$$\int (d+ex)^5\left(a+cx^2\right)^3\,dx=\frac{1}{12}c^3e^5x^{12}+\frac{5}{11}c^3de^4x^{11}+\frac{1}{10}\left(10c^3d^2e^3+3ac^2e^5\right)x^{10}+\frac{5}{2}a^3d^4ex^2+\frac{5}{9}\left(2c^3d^3e^2+3ac^2de^4\right)x^9+a^3d^5x+\frac{1}{8}\left(5c^3d^4e+30ac^2d^2e^3+3a^2ce^5\right)x^8+\frac{1}{7}\left(c^3d^5+30ac^2d^3e^2+15a^2cde^4\right)x^7+\frac{1}{6}\left(15ac^2d^4e+30a^2cd^2e^3+a^3e^5\right)x^6+\frac{1}{5}\left(3ac^2d^5+30a^2cd^3e^2+5a^3de^4\right)x^5+\frac{5}{4}\left(3a^2cd^4e+2a^3d^2e^3\right)x^4+\frac{1}{3}\left(3a^2cd^5+10a^3d^3e^2\right)x^3$$

input

```
integrate((e*x+d)^5*(c*x^2+a)^3,x, algorithm="fricas")
```

output

```
1/12*c^3*e^5*x^12 + 5/11*c^3*d*e^4*x^11 + 1/10*(10*c^3*d^2*e^3 + 3*a*c^2*e
^5)*x^10 + 5/2*a^3*d^4*e*x^2 + 5/9*(2*c^3*d^3*e^2 + 3*a*c^2*d*e^4)*x^9 + a
^3*d^5*x + 1/8*(5*c^3*d^4*e + 30*a*c^2*d^2*e^3 + 3*a^2*c*e^5)*x^8 + 1/7*(c
^3*d^5 + 30*a*c^2*d^3*e^2 + 15*a^2*c*d*e^4)*x^7 + 1/6*(15*a*c^2*d^4*e + 30
*a^2*c*d^2*e^3 + a^3*e^5)*x^6 + 1/5*(3*a*c^2*d^5 + 30*a^2*c*d^3*e^2 + 5*a^
3*d*e^4)*x^5 + 5/4*(3*a^2*c*d^4*e + 2*a^3*d^2*e^3)*x^4 + 1/3*(3*a^2*c*d^5
+ 10*a^3*d^3*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.69

$$\int (d+ex)^5 \left(a+cx^2\right)^3 \, dx = a^3d^5x + \frac{5a^3d^4ex^2}{2} + \frac{5c^3de^4x^{11}}{11} + \frac{c^3e^5x^{12}}{12} + x^{10} \cdot \left(\frac{3ac^2e^5}{10} + c^3d^2e^3\right) + x^9 \cdot \left(\frac{5ac^2de^4}{3} + \frac{10c^3d^3e^2}{9}\right) + x^8 \cdot \left(\frac{3a^2ce^5}{8} + \frac{15ac^2d^2e^3}{4} + \frac{5c^3d^4e}{8}\right) + x^7 \cdot \left(\frac{15a^2cde^4}{7} + \frac{30ac^2d^3e^2}{7} + \frac{c^3d^5}{7}\right) + x^6\left(\frac{a^3e^5}{6} + 5a^2cd^2e^3 + \frac{5ac^2d^4e}{2}\right) + x^5\left(a^3de^4 + 6a^2cd^3e^2 + \frac{3ac^2d^5}{5}\right) + x^4 \cdot \left(\frac{5a^3d^2e^3}{2} + \frac{15a^2cd^4e}{4}\right) + x^3 \cdot \left(\frac{10a^3d^3e^2}{3} + a^2cd^5\right)$$

input
```
integrate((e*x+d)**5*(c*x**2+a)**3,x)
```

output
```
a**3*d**5*x + 5*a**3*d**4*e*x**2/2 + 5*c**3*d*e**4*x**11/11 + c**3*e**5*x*
*12/12 + x**10*(3*a*c**2*e**5/10 + c**3*d**2*e**3) + x**9*(5*a*c**2*d*e**4
/3 + 10*c**3*d**3*e**2/9) + x**8*(3*a**2*c*e**5/8 + 15*a*c**2*d**2*e**3/4
+ 5*c**3*d**4*e/8) + x**7*(15*a**2*c*d*e**4/7 + 30*a*c**2*d**3*e**2/7 + c*
*3*d**5/7) + x**6*(a**3*e**5/6 + 5*a**2*c*d**2*e**3 + 5*a*c**2*d**4*e/2) +
 x**5*(a**3*d*e**4 + 6*a**2*c*d**3*e**2 + 3*a*c**2*d**5/5) + x**4*(5*a**3*
d**2*e**3/2 + 15*a**2*c*d**4*e/4) + x**3*(10*a**3*d**3*e**2/3 + a**2*c*d**
5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.54

$$\int (d+ex)^5 (a+cx^2)^3 \, dx = \frac{1}{12} c^3 e^5 x^{12} + \frac{5}{11} c^3 d e^4 x^{11} + \frac{1}{10} (10 c^3 d^2 e^3 + 3 a c^2 e^5) x^{10} + \frac{5}{2} a^3 d^4 e x^2 + \frac{5}{9} (2 c^3 d^3 e^2 + 3 a c^2 d e^4) x^9 + a^3 d^5 x + \frac{1}{8} (5 c^3 d^4 e + 30 a c^2 d^2 e^3 + 3 a^2 c e^5) x^8 + \frac{1}{7} (c^3 d^5 + 30 a c^2 d^3 e^2 + 15 a^2 c d e^4) x^7 + \frac{1}{6} (15 a c^2 d^4 e + 30 a^2 c d^2 e^3 + a^3 e^5) x^6 + \frac{1}{5} (3 a c^2 d^5 + 30 a^2 c d^3 e^2 + 5 a^3 d e^4) x^5 + \frac{5}{4} (3 a^2 c d^4 e + 2 a^3 d^2 e^3) x^4 + \frac{1}{3} (3 a^2 c d^5 + 10 a^3 d^3 e^2) x^3$$

input
```
integrate((e*x+d)^5*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/12*c^3*e^5*x^12 + 5/11*c^3*d*e^4*x^11 + 1/10*(10*c^3*d^2*e^3 + 3*a*c^2*e
^5)*x^10 + 5/2*a^3*d^4*e*x^2 + 5/9*(2*c^3*d^3*e^2 + 3*a*c^2*d*e^4)*x^9 + a
^3*d^5*x + 1/8*(5*c^3*d^4*e + 30*a*c^2*d^2*e^3 + 3*a^2*c*e^5)*x^8 + 1/7*(c
^3*d^5 + 30*a*c^2*d^3*e^2 + 15*a^2*c*d*e^4)*x^7 + 1/6*(15*a*c^2*d^4*e + 30
*a^2*c*d^2*e^3 + a^3*e^5)*x^6 + 1/5*(3*a*c^2*d^5 + 30*a^2*c*d^3*e^2 + 5*a^
3*d*e^4)*x^5 + 5/4*(3*a^2*c*d^4*e + 2*a^3*d^2*e^3)*x^4 + 1/3*(3*a^2*c*d^5
+ 10*a^3*d^3*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.59

$$\int (d+ex)^5 \left(a+cx^2\right)^3 \, dx = \frac{1}{12} c^3 e^5 x^{12} + \frac{5}{11} c^3 d e^4 x^{11} + c^3 d^2 e^3 x^{10} + \frac{3}{10} a c^2 e^5 x^{10} + \frac{10}{9} c^3 d^3 e^2 x^9 + \frac{5}{3} a c^2 d e^4 x^9 + \frac{5}{8} c^3 d^4 e x^8 + \frac{15}{4} a c^2 d^2 e^3 x^8 + \frac{3}{8} a^2 c e^5 x^8 + \frac{1}{7} c^3 d^5 x^7 + \frac{30}{7} a c^2 d^3 e^2 x^7 + \frac{15}{7} a^2 c d e^4 x^7 + \frac{5}{2} a c^2 d^4 e x^6 + 5 a^2 c d^2 e^3 x^6 + \frac{1}{6} a^3 e^5 x^6 + \frac{3}{5} a c^2 d^5 x^5 + 6 a^2 c d^3 e^2 x^5 + a^3 d e^4 x^5 + \frac{15}{4} a^2 c d^4 e x^4 + \frac{5}{2} a^3 d^2 e^3 x^4 + a^2 c d^5 x^3 + \frac{10}{3} a^3 d^3 e^2 x^3 + \frac{5}{2} a^3 d^4 e x^2 + a^3 d^5 x$$

input

```
integrate((e*x+d)^5*(c*x^2+a)^3,x, algorithm="giac")
```

output

```
1/12*c^3*e^5*x^12 + 5/11*c^3*d*e^4*x^11 + c^3*d^2*e^3*x^10 + 3/10*a*c^2*e^
5*x^10 + 10/9*c^3*d^3*e^2*x^9 + 5/3*a*c^2*d*e^4*x^9 + 5/8*c^3*d^4*e*x^8 +
15/4*a*c^2*d^2*e^3*x^8 + 3/8*a^2*c*e^5*x^8 + 1/7*c^3*d^5*x^7 + 30/7*a*c^2*
d^3*e^2*x^7 + 15/7*a^2*c*d*e^4*x^7 + 5/2*a*c^2*d^4*e*x^6 + 5*a^2*c*d^2*e^3
*x^6 + 1/6*a^3*e^5*x^6 + 3/5*a*c^2*d^5*x^5 + 6*a^2*c*d^3*e^2*x^5 + a^3*d*e
^4*x^5 + 15/4*a^2*c*d^4*e*x^4 + 5/2*a^3*d^2*e^3*x^4 + a^2*c*d^5*x^3 + 10/3
*a^3*d^3*e^2*x^3 + 5/2*a^3*d^4*e*x^2 + a^3*d^5*x
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.48

$$\int (d+ex)^5 \left(a+cx^2\right)^3 \,dx = x^5 \left(a^3\,d\,e^4+6\,a^2\,c\,d^3\,e^2+\frac{3\,a\,c^2\,d^5}{5}\right) + x^6 \left(\frac{a^3\,e^5}{6}+5\,a^2\,c\,d^2\,e^3+\frac{5\,a\,c^2\,d^4\,e}{2}\right) + x^7 \left(\frac{15\,a^2\,c\,d\,e^4}{7}+\frac{30\,a\,c^2\,d^3\,e^2}{7}+\frac{c^3\,d^5}{7}\right) + x^8 \left(\frac{3\,a^2\,c\,e^5}{8}+\frac{15\,a\,c^2\,d^2\,e^3}{4}+\frac{5\,c^3\,d^4\,e}{8}\right) + x^3 \left(\frac{10\,a^3\,d^3\,e^2}{3}+c\,a^2\,d^5\right) + x^{10} \left(c^3\,d^2\,e^3+\frac{3\,a\,c^2\,e^5}{10}\right) + a^3\,d^5\,x + \frac{c^3\,e^5\,x^{12}}{12} + \frac{5\,a^3\,d^4\,e\,x^2}{2} + \frac{5\,c^3\,d\,e^4\,x^{11}}{11} + \frac{5\,a^2\,d^2\,e\,x^4\,(3\,c\,d^2+2\,a\,e^2)}{4} + \frac{5\,c^2\,d\,e^2\,x^9\,(2\,c\,d^2+3\,a\,e^2)}{9}$$

input
```
int((a + c*x^2)^3*(d + e*x)^5,x)
```

output
```
x^5*((3*a*c^2*d^5)/5 + a^3*d*e^4 + 6*a^2*c*d^3*e^2) + x^6*((a^3*e^5)/6 + 5
*a^2*c*d^2*e^3 + (5*a*c^2*d^4*e)/2) + x^7*((c^3*d^5)/7 + (30*a*c^2*d^3*e^2
)/7 + (15*a^2*c*d*e^4)/7) + x^8*((3*a^2*c*e^5)/8 + (5*c^3*d^4*e)/8 + (15*a
*c^2*d^2*e^3)/4) + x^3*(a^2*c*d^5 + (10*a^3*d^3*e^2)/3) + x^10*((3*a*c^2*e
^5)/10 + c^3*d^2*e^3) + a^3*d^5*x + (c^3*e^5*x^12)/12 + (5*a^3*d^4*e*x^2)/
2 + (5*c^3*d*e^4*x^11)/11 + (5*a^2*d^2*e*x^4*(2*a*e^2 + 3*c*d^2))/4 + (5*c
^2*d*e^2*x^9*(3*a*e^2 + 2*c*d^2))/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.62

$$\int (d+ex)^5 \left(a+cx^2\right)^3 \,dx = \frac{x(2310c^3e^5x^{11}+12600c^3d\,e^4x^{10}+8316a\,c^2e^5x^9+27720c^3d^2e^3x^9+46200a\,c^2d\,e^4x^8+30800c^3d^3e^2x^8+}{}$$

input
```
int((e*x+d)^5*(c*x^2+a)^3,x)
```

output
```
(x*(27720*a**3*d**5 + 69300*a**3*d**4*e*x + 92400*a**3*d**3*e**2*x**2 + 69
300*a**3*d**2*e**3*x**3 + 27720*a**3*d*e**4*x**4 + 4620*a**3*e**5*x**5 + 2
7720*a**2*c*d**5*x**2 + 103950*a**2*c*d**4*e*x**3 + 166320*a**2*c*d**3*e**
2*x**4 + 138600*a**2*c*d**2*e**3*x**5 + 59400*a**2*c*d*e**4*x**6 + 10395*a
**2*c*e**5*x**7 + 16632*a*c**2*d**5*x**4 + 69300*a*c**2*d**4*e*x**5 + 1188
00*a*c**2*d**3*e**2*x**6 + 103950*a*c**2*d**2*e**3*x**7 + 46200*a*c**2*d*e
**4*x**8 + 8316*a*c**2*e**5*x**9 + 3960*c**3*d**5*x**6 + 17325*c**3*d**4*e
*x**7 + 30800*c**3*d**3*e**2*x**8 + 27720*c**3*d**2*e**3*x**9 + 12600*c**3
*d*e**4*x**10 + 2310*c**3*e**5*x**11))/27720
```

3.75 $\int (d+ex)^4 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 188

$$\int (d+ex)^4 \left(a+cx^2\right)^3 \, dx = \frac{(cd^2+ae^2)^3 (d+ex)^5}{5e^7} - \frac{cd(cd^2+ae^2)^2 (d+ex)^6}{e^7} + \frac{3c(cd^2+ae^2)(5cd^2+ae^2)(d+ex)^7}{7e^7} - \frac{c^2d(5cd^2+3ae^2)(d+ex)^8}{2e^7} + \frac{c^2(5cd^2+ae^2)(d+ex)^9}{3e^7} - \frac{3c^3d(d+ex)^{10}}{5e^7} + \frac{c^3(d+ex)^{11}}{11e^7}$$

output

```
1/5*(a*e^2+c*d^2)^3*(e*x+d)^5/e^7-c*d*(a*e^2+c*d^2)^2*(e*x+d)^6/e^7+3/7*c*
(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^7/e^7-1/2*c^2*d*(3*a*e^2+5*c*d^2)*(e
*x+d)^8/e^7+1/3*c^2*(a*e^2+5*c*d^2)*(e*x+d)^9/e^7-3/5*c^3*d*(e*x+d)^10/e^7
+1/11*c^3*(e*x+d)^11/e^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10

$$\begin{aligned}&\int (d+ex)^4\left(a+cx^2\right)^3\,dx\\&=\frac{1}{210}ac^2x^5\left(126d^4+420d^3ex+540d^2e^2x^2+315de^3x^3+70e^4x^4\right)\\&\quad+\frac{c^3x^7\left(330d^4+1155d^3ex+1540d^2e^2x^2+924de^3x^3+210e^4x^4\right)}{2310}\\&\quad+a^3\left(d^4x+2d^3ex^2+2d^2e^2x^3+de^3x^4+\frac{e^4x^5}{5}\right)\\&\quad+a^2c\left(d^4x^3+3d^3ex^4+\frac{18}{5}d^2e^2x^5+2de^3x^6+\frac{3e^4x^7}{7}\right)\end{aligned}$$

input
```
Integrate[(d + e*x)^4*(a + c*x^2)^3,x]
```

output
```
(a*c^2*x^5*(126*d^4 + 420*d^3*e*x + 540*d^2*e^2*x^2 + 315*d*e^3*x^3 + 70*e
^4*x^4))/210 + (c^3*x^7*(330*d^4 + 1155*d^3*e*x + 1540*d^2*e^2*x^2 + 924*d
*e^3*x^3 + 210*e^4*x^4))/2310 + a^3*(d^4*x + 2*d^3*e*x^2 + 2*d^2*e^2*x^3 +
 d*e^3*x^4 + (e^4*x^5)/5) + a^2*c*(d^4*x^3 + 3*d^3*e*x^4 + (18*d^2*e^2*x^5
)/5 + 2*d*e^3*x^6 + (3*e^4*x^7)/7)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3(d+ex)^4\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{3c^2(d+ex)^8(ae^2+5cd^2)}{e^6} - \frac{4c^2d(d+ex)^7(3ae^2+5cd^2)}{e^6} + \frac{3c(d+ex)^6(ae^2+cd^2)(ae^2+5cd^2)}{e^6} - \frac{6cd(d\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2(d+ex)^9(ae^2+5cd^2)}{3e^7} - \frac{c^2d(d+ex)^8(3ae^2+5cd^2)}{2e^7} + \frac{3c(d+ex)^7(ae^2+cd^2)(ae^2+5cd^2)}{7e^7} - \frac{cd(d+ex)^6(ae^2+cd^2)^2}{e^7} + \frac{(d+ex)^5(ae^2+cd^2)^3}{5e^7} + \frac{c^3(d+ex)^{11}}{11e^7} - \frac{3c^3d(d+ex)^{10}}{5e^7}$$

input
```
Int[(d + e*x)^4*(a + c*x^2)^3,x]
```

output
```
((c*d^2 + a*e^2)^3*(d + e*x)^5)/(5*e^7) - (c*d*(c*d^2 + a*e^2)^2*(d + e*x)
^6)/e^7 + (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^7)/(7*e^7) - (c
^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^8)/(2*e^7) + (c^2*(5*c*d^2 + a*e^2)*(d
+ e*x)^9)/(3*e^7) - (3*c^3*d*(d + e*x)^10)/(5*e^7) + (c^3*(d + e*x)^11)/(1
1*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.24

method	result
norman	$\frac{e^4c^3x^{11}}{11}+\frac{2d\,e^3c^3x^{10}}{5}+\left(\frac{1}{3}e^4a\,c^2+\frac{2}{3}d^2e^2c^3\right)x^9+\left(\frac{3}{2}d\,e^3a\,c^2+\frac{1}{2}d^3e\,c^3\right)x^8+\left(\frac{3}{7}e^4a^2c+\frac{18}{7}d^2e^2a\right.$
default	$\frac{e^4c^3x^{11}}{11}+\frac{2d\,e^3c^3x^{10}}{5}+\frac{(3e^4a\,c^2+6d^2e^2c^3)x^9}{9}+\frac{(12d\,e^3a\,c^2+4d^3e\,c^3)x^8}{8}+\frac{(3e^4a^2c+18d^2e^2a\,c^2+d^4c^3)x^7}{7}+\frac{(12d\,e}{}$
gosper	$\frac{1}{11}e^4c^3x^{11}+\frac{2}{5}d\,e^3c^3x^{10}+\frac{1}{3}x^9e^4a\,c^2+\frac{2}{3}x^9d^2e^2c^3+\frac{3}{2}x^8d\,e^3a\,c^2+\frac{1}{2}x^8d^3e\,c^3+\frac{3}{7}x^7e^4a^2c+\frac{18}{7}$
risch	$\frac{1}{11}e^4c^3x^{11}+\frac{2}{5}d\,e^3c^3x^{10}+\frac{1}{3}x^9e^4a\,c^2+\frac{2}{3}x^9d^2e^2c^3+\frac{3}{2}x^8d\,e^3a\,c^2+\frac{1}{2}x^8d^3e\,c^3+\frac{3}{7}x^7e^4a^2c+\frac{18}{7}$
parallelrisch	$\frac{1}{11}e^4c^3x^{11}+\frac{2}{5}d\,e^3c^3x^{10}+\frac{1}{3}x^9e^4a\,c^2+\frac{2}{3}x^9d^2e^2c^3+\frac{3}{2}x^8d\,e^3a\,c^2+\frac{1}{2}x^8d^3e\,c^3+\frac{3}{7}x^7e^4a^2c+\frac{18}{7}$
orering	$\frac{x(210e^4c^3x^{10}+924d\,e^3c^3x^9+770a\,c^2e^4x^8+1540c^3d^2e^2x^8+3465a\,c^2d\,e^3x^7+1155c^3d^3e\,x^7+990a^2c\,e^4x^6+5940a\,c^2d^2e^2x^6+3}{}$

input
```
int((e*x+d)^4*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*e^4*c^3*x^11+2/5*d*e^3*c^3*x^10+(1/3*e^4*a*c^2+2/3*d^2*e^2*c^3)*x^9+(
3/2*d*e^3*a*c^2+1/2*d^3*e*c^3)*x^8+(3/7*e^4*a^2*c+18/7*d^2*e^2*a*c^2+1/7*d
^4*c^3)*x^7+(2*a^2*c*d*e^3+2*a*c^2*d^3*e)*x^6+(1/5*e^4*a^3+18/5*d^2*e^2*a^
2*c+3/5*d^4*a*c^2)*x^5+(a^3*d*e^3+3*a^2*c*d^3*e)*x^4+(2*a^3*d^2*e^2+a^2*c*
d^4)*x^3+2*d^3*e*a^3*x^2+d^4*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.23

$$\begin{aligned}\int (d+ex)^4\left(a+cx^2\right)^3\,dx &= \frac{1}{11}c^3e^4x^{11}+\frac{2}{5}c^3de^3x^{10}+\frac{1}{3}\left(2c^3d^2e^2+ac^2e^4\right)x^9\\&\quad+2a^3d^3ex^2+\frac{1}{2}\left(c^3d^3e+3ac^2de^3\right)x^8\\&\quad+a^3d^4x+\frac{1}{7}\left(c^3d^4+18ac^2d^2e^2+3a^2ce^4\right)x^7\\&\quad+2\left(ac^2d^3e+a^2cde^3\right)x^6\\&\quad+\frac{1}{5}\left(3ac^2d^4+18a^2cd^2e^2+a^3e^4\right)x^5\\&\quad+\left(3a^2cd^3e+a^3de^3\right)x^4+\left(a^2cd^4+2a^3d^2e^2\right)x^3\end{aligned}$$

input
```
integrate((e*x+d)^4*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/11*c^3*e^4*x^11 + 2/5*c^3*d*e^3*x^10 + 1/3*(2*c^3*d^2*e^2 + a*c^2*e^4)*x
^9 + 2*a^3*d^3*e*x^2 + 1/2*(c^3*d^3*e + 3*a*c^2*d*e^3)*x^8 + a^3*d^4*x + 1
/7*(c^3*d^4 + 18*a*c^2*d^2*e^2 + 3*a^2*c*e^4)*x^7 + 2*(a*c^2*d^3*e + a^2*c
*d*e^3)*x^6 + 1/5*(3*a*c^2*d^4 + 18*a^2*c*d^2*e^2 + a^3*e^4)*x^5 + (3*a^2*
c*d^3*e + a^3*d*e^3)*x^4 + (a^2*c*d^4 + 2*a^3*d^2*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.36

$$\int (d+ex)^4 \left(a+cx^2\right)^3 \, dx = a^3d^4x + 2a^3d^3ex^2 + \frac{2c^3de^3x^{10}}{5} + \frac{c^3e^4x^{11}}{11} + x^9\left(\frac{ac^2e^4}{3} + \frac{2c^3d^2e^2}{3}\right) + x^8\cdot\left(\frac{3ac^2de^3}{2} + \frac{c^3d^3e}{2}\right) + x^7\cdot\left(\frac{3a^2ce^4}{7} + \frac{18ac^2d^2e^2}{7} + \frac{c^3d^4}{7}\right) + x^6 \cdot\left(2a^2cde^3 + 2ac^2d^3e\right) + x^5\left(\frac{a^3e^4}{5} + \frac{18a^2cd^2e^2}{5} + \frac{3ac^2d^4}{5}\right) + x^4\left(a^3de^3 + 3a^2cd^3e\right) + x^3\cdot\left(2a^3d^2e^2 + a^2cd^4\right)$$

input
```
integrate((e*x+d)**4*(c*x**2+a)**3,x)
```

output
```
a**3*d**4*x + 2*a**3*d**3*e*x**2 + 2*c**3*d*e**3*x**10/5 + c**3*e**4*x**11
/11 + x**9*(a*c**2*e**4/3 + 2*c**3*d**2*e**2/3) + x**8*(3*a*c**2*d*e**3/2
+ c**3*d**3*e/2) + x**7*(3*a**2*c*e**4/7 + 18*a*c**2*d**2*e**2/7 + c**3*d*
*4/7) + x**6*(2*a**2*c*d*e**3 + 2*a*c**2*d**3*e) + x**5*(a**3*e**4/5 + 18*
a**2*c*d**2*e**2/5 + 3*a*c**2*d**4/5) + x**4*(a**3*d*e**3 + 3*a**2*c*d**3*
e) + x**3*(2*a**3*d**2*e**2 + a**2*c*d**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.23

$$\int (d+ex)^4 \left(a+cx^2\right)^3 \, dx = \frac{1}{11} c^3 e^4 x^{11} + \frac{2}{5} c^3 d e^3 x^{10} + \frac{1}{3} \left(2 c^3 d^2 e^2 + a c^2 e^4\right) x^9 + 2 a^3 d^3 e x^2 + \frac{1}{2} \left(c^3 d^3 e + 3 a c^2 d e^3\right) x^8 + a^3 d^4 x + \frac{1}{7} \left(c^3 d^4 + 18 a c^2 d^2 e^2 + 3 a^2 c e^4\right) x^7 + 2 \left(a c^2 d^3 e + a^2 c d e^3\right) x^6 + \frac{1}{5} \left(3 a c^2 d^4 + 18 a^2 c d^2 e^2 + a^3 e^4\right) x^5 + \left(3 a^2 c d^3 e + a^3 d e^3\right) x^4 + \left(a^2 c d^4 + 2 a^3 d^2 e^2\right) x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/11*c^3*e^4*x^11 + 2/5*c^3*d*e^3*x^10 + 1/3*(2*c^3*d^2*e^2 + a*c^2*e^4)*x
^9 + 2*a^3*d^3*e*x^2 + 1/2*(c^3*d^3*e + 3*a*c^2*d*e^3)*x^8 + a^3*d^4*x + 1
/7*(c^3*d^4 + 18*a*c^2*d^2*e^2 + 3*a^2*c*e^4)*x^7 + 2*(a*c^2*d^3*e + a^2*c
*d*e^3)*x^6 + 1/5*(3*a*c^2*d^4 + 18*a^2*c*d^2*e^2 + a^3*e^4)*x^5 + (3*a^2*
c*d^3*e + a^3*d*e^3)*x^4 + (a^2*c*d^4 + 2*a^3*d^2*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.31

$$\int (d+ex)^4 \left(a+cx^2\right)^3 \, dx = \frac{1}{11} c^3 e^4 x^{11} + \frac{2}{5} c^3 d e^3 x^{10} + \frac{2}{3} c^3 d^2 e^2 x^9 + \frac{1}{3} a c^2 e^4 x^9 + \frac{1}{2} c^3 d^3 e x^8 + \frac{3}{2} a c^2 d e^3 x^8 + \frac{1}{7} c^3 d^4 x^7 + \frac{18}{7} a c^2 d^2 e^2 x^7 + \frac{3}{7} a^2 c e^4 x^7 + 2 a c^2 d^3 e x^6 + 2 a^2 c d e^3 x^6 + \frac{3}{5} a c^2 d^4 x^5 + \frac{18}{5} a^2 c d^2 e^2 x^5 + \frac{1}{5} a^3 e^4 x^5 + 3 a^2 c d^3 e x^4 + a^3 d e^3 x^4 + a^2 c d^4 x^3 + 2 a^3 d^2 e^2 x^3 + 2 a^3 d^3 e x^2 + a^3 d^4 x$$

input
```
integrate((e*x+d)^4*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/11*c^3*e^4*x^11 + 2/5*c^3*d*e^3*x^10 + 2/3*c^3*d^2*e^2*x^9 + 1/3*a*c^2*e
^4*x^9 + 1/2*c^3*d^3*e*x^8 + 3/2*a*c^2*d*e^3*x^8 + 1/7*c^3*d^4*x^7 + 18/7*
a*c^2*d^2*e^2*x^7 + 3/7*a^2*c*e^4*x^7 + 2*a*c^2*d^3*e*x^6 + 2*a^2*c*d*e^3*
x^6 + 3/5*a*c^2*d^4*x^5 + 18/5*a^2*c*d^2*e^2*x^5 + 1/5*a^3*e^4*x^5 + 3*a^2
*c*d^3*e*x^4 + a^3*d*e^3*x^4 + a^2*c*d^4*x^3 + 2*a^3*d^2*e^2*x^3 + 2*a^3*d
^3*e*x^2 + a^3*d^4*x
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.19

$$\int (d+ex)^4 \left(a+cx^2\right)^3 \, dx = x^3 \left(2\,a^3\,d^2\,e^2+c\,a^2\,d^4\right)+x^9\left(\frac{2\,c^3\,d^2\,e^2}{3}+\frac{a\,c^2\,e^4}{3}\right) + x^5\left(\frac{a^3\,e^4}{5}+\frac{18\,a^2\,c\,d^2\,e^2}{5}+\frac{3\,a\,c^2\,d^4}{5}\right) + x^7\left(\frac{3\,a^2\,c\,e^4}{7}+\frac{18\,a\,c^2\,d^2\,e^2}{7}+\frac{c^3\,d^4}{7}\right)+a^3\,d^4\,x+\frac{c^3\,e^4\,x^{11}}{11} + 2\,a^3\,d^3\,e\,x^2+\frac{2\,c^3\,d\,e^3\,x^{10}}{5}+a^2\,d\,e\,x^4\left(3\,c\,d^2+a\,e^2\right) + \frac{c^2\,d\,e\,x^8\left(c\,d^2+3\,a\,e^2\right)}{2}+2\,a\,c\,d\,e\,x^6\left(c\,d^2+a\,e^2\right)$$

input
```
int((a + c*x^2)^3*(d + e*x)^4,x)
```

output
```
x^3*(a^2*c*d^4 + 2*a^3*d^2*e^2) + x^9*((a*c^2*e^4)/3 + (2*c^3*d^2*e^2)/3)
+ x^5*((a^3*e^4)/5 + (3*a*c^2*d^4)/5 + (18*a^2*c*d^2*e^2)/5) + x^7*((c^3*d
^4)/7 + (3*a^2*c*e^4)/7 + (18*a*c^2*d^2*e^2)/7) + a^3*d^4*x + (c^3*e^4*x^1
1)/11 + 2*a^3*d^3*e*x^2 + (2*c^3*d*e^3*x^10)/5 + a^2*d*e*x^4*(a*e^2 + 3*c*
d^2) + (c^2*d*e*x^8*(3*a*e^2 + c*d^2))/2 + 2*a*c*d*e*x^6*(a*e^2 + c*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.32

$$\int (d+ex)^4 \left(a+cx^2\right)^3 \, dx$$
$$= \frac{x(210c^3e^4x^{10} + 924c^3d\,e^3x^9 + 770a\,c^2e^4x^8 + 1540c^3d^2e^2x^8 + 3465a\,c^2d\,e^3x^7 + 1155c^3d^3e\,x^7 + 990a^2c\,e^4x}{}$$

input
```
int((e*x+d)^4*(c*x^2+a)^3,x)
```

output
```
(x*(2310*a**3*d**4 + 4620*a**3*d**3*e*x + 4620*a**3*d**2*e**2*x**2 + 2310*
a**3*d*e**3*x**3 + 462*a**3*e**4*x**4 + 2310*a**2*c*d**4*x**2 + 6930*a**2*
c*d**3*e*x**3 + 8316*a**2*c*d**2*e**2*x**4 + 4620*a**2*c*d*e**3*x**5 + 990
*a**2*c*e**4*x**6 + 1386*a*c**2*d**4*x**4 + 4620*a*c**2*d**3*e*x**5 + 5940
*a*c**2*d**2*e**2*x**6 + 3465*a*c**2*d*e**3*x**7 + 770*a*c**2*e**4*x**8 +
330*c**3*d**4*x**6 + 1155*c**3*d**3*e*x**7 + 1540*c**3*d**2*e**2*x**8 + 92
4*c**3*d*e**3*x**9 + 210*c**3*e**4*x**10))/2310
```

3.76 $\int (d+ex)^3 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 161

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \, dx = a^3d^3x + a^2d\left(cd^2+ae^2\right)x^3 + \frac{1}{4}a^3e^3x^4 + \frac{3}{5}acd\left(cd^2+3ae^2\right)x^5 + \frac{1}{2}a^2ce^3x^6 + \frac{1}{7}c^2d\left(cd^2+9ae^2\right)x^7 + \frac{3}{8}ac^2e^3x^8 + \frac{1}{3}c^3de^2x^9 + \frac{1}{10}c^3e^3x^{10} + \frac{3d^2e\left(a+cx^2\right)^4}{8c}$$

output
```
a^3*d^3*x+a^2*d*(a*e^2+c*d^2)*x^3+1/4*a^3*e^3*x^4+3/5*a*c*d*(3*a*e^2+c*d^2
)*x^5+1/2*a^2*c*e^3*x^6+1/7*c^2*d*(9*a*e^2+c*d^2)*x^7+3/8*a*c^2*e^3*x^8+1/
3*c^3*d*e^2*x^9+1/10*c^3*e^3*x^10+3/8*d^2*e*(c*x^2+a)^4/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \, dx = \frac{1}{840}x\left(210a^3\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right) + 42a^2cx^2\left(20d^3+45d^2ex+36de^2x^2+10e^3x^3\right) + 9ac^2x^4\left(56d^3+140d^2ex+120de^2x^2+35e^3x^3\right) + c^3x^6\left(120d^3+315d^2ex+280de^2x^2+84e^3x^3\right)\right)$$

input
```
Integrate[(d + e*x)^3*(a + c*x^2)^3,x]
```

output
```
(x*(210*a^3*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 42*a^2*c*x^2*(20
*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 9*a*c^2*x^4*(56*d^3 + 140
*d^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + c^3*x^6*(120*d^3 + 315*d^2*e*x +
280*d*e^2*x^2 + 84*e^3*x^3)))/840
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex)^3 \, dx$$

$$\Big\downarrow \; 475$$

$$\int \left(c^3e^3x^9+3c^3de^2x^8+3ac^2e^3x^7+c^2d\left(cd^2+9ae^2\right)x^6+3a^2ce^3x^5+3acd\left(cd^2+3ae^2\right)x^4+a^3e^3x^3+3a^2d\left(cd^2\right.\right.$$

$$\frac{3d^2e\left(a+cx^2\right)^4}{8c}$$

$$\Big\downarrow \; 2009$$

$$a^3d^3x+\frac{1}{4}a^3e^3x^4+a^2dx^3\left(ae^2+cd^2\right)+\frac{1}{2}a^2ce^3x^6+\frac{1}{7}c^2dx^7\left(9ae^2+cd^2\right)+\frac{3}{8}ac^2e^3x^8+\frac{3}{5}acdx^5\left(3ae^2+cd^2\right)+\frac{3d^2e\left(a+cx^2\right)^4}{8c}+\frac{1}{3}c^3de^2x^9+\frac{1}{10}c^3e^3x^{10}$$

input
```
Int[(d + e*x)^3*(a + c*x^2)^3,x]
```

output
```
a^3*d^3*x + a^2*d*(c*d^2 + a*e^2)*x^3 + (a^3*e^3*x^4)/4 + (3*a*c*d*(c*d^2
+ 3*a*e^2)*x^5)/5 + (a^2*c*e^3*x^6)/2 + (c^2*d*(c*d^2 + 9*a*e^2)*x^7)/7 +
(3*a*c^2*e^3*x^8)/8 + (c^3*d*e^2*x^9)/3 + (c^3*e^3*x^10)/10 + (3*d^2*e*(a
+ c*x^2)^4)/(8*c)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

method	result
norman	$\frac{c^3e^3x^{10}}{10}+\frac{c^3d\,e^2x^9}{3}+\left(\frac{3}{8}e^3a\,c^2+\frac{3}{8}d^2e\,c^3\right)x^8+\left(\frac{9}{7}d\,e^2a\,c^2+\frac{1}{7}d^3c^3\right)x^7+\left(\frac{1}{2}a^2c\,e^3+\frac{3}{2}c^2d^2ae\right)x$
gosper	$\frac{1}{10}c^3e^3x^{10}+\frac{1}{3}c^3d\,e^2x^9+\frac{3}{8}a\,c^2e^3x^8+\frac{3}{8}x^8d^2e\,c^3+\frac{9}{7}x^7d\,e^2a\,c^2+\frac{1}{7}x^7d^3c^3+\frac{1}{2}a^2c\,e^3x^6+\frac{3}{2}x^6$
default	$\frac{c^3e^3x^{10}}{10}+\frac{c^3d\,e^2x^9}{3}+\frac{(3e^3a\,c^2+3d^2e\,c^3)x^8}{8}+\frac{(9d\,e^2a\,c^2+d^3c^3)x^7}{7}+\frac{(3a^2c\,e^3+9c^2d^2ae)x^6}{6}+\frac{(9d\,e^2a^2c+3d^3a\,c^2)x}{5}$
risch	$\frac{1}{10}c^3e^3x^{10}+\frac{1}{3}c^3d\,e^2x^9+\frac{3}{8}a\,c^2e^3x^8+\frac{3}{8}x^8d^2e\,c^3+\frac{9}{7}x^7d\,e^2a\,c^2+\frac{1}{7}x^7d^3c^3+\frac{1}{2}a^2c\,e^3x^6+\frac{3}{2}x^6$
parallelrisch	$\frac{1}{10}c^3e^3x^{10}+\frac{1}{3}c^3d\,e^2x^9+\frac{3}{8}a\,c^2e^3x^8+\frac{3}{8}x^8d^2e\,c^3+\frac{9}{7}x^7d\,e^2a\,c^2+\frac{1}{7}x^7d^3c^3+\frac{1}{2}a^2c\,e^3x^6+\frac{3}{2}x^6$
orering	$\frac{x(84e^3c^3x^9+280d\,e^2c^3x^8+315a\,c^2e^3x^7+315c^3d^2e\,x^7+1080a\,c^2d\,e^2x^6+120c^3d^3x^6+420a^2c\,e^3x^5+1260a\,c^2d^2e\,x^5+1512a^2c}{840}$

input
```
int((e*x+d)^3*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^3*e^3*x^10+1/3*c^3*d*e^2*x^9+(3/8*e^3*a*c^2+3/8*d^2*e*c^3)*x^8+(9/7
*d*e^2*a*c^2+1/7*d^3*c^3)*x^7+(1/2*a^2*c*e^3+3/2*c^2*d^2*a*e)*x^6+(9/5*d*e
^2*a^2*c+3/5*d^3*a*c^2)*x^5+(1/4*e^3*a^3+9/4*d^2*e*a^2*c)*x^4+(a^3*d*e^2+a
^2*c*d^3)*x^3+3/2*d^2*e*a^3*x^2+a^3*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \,dx = \frac{1}{10}\,c^3e^3x^{10} + \frac{1}{3}\,c^3de^2x^9 + \frac{3}{8}\,\left(c^3d^2e + ac^2e^3\right)x^8 + \frac{3}{2}\,a^3d^2ex^2 + \frac{1}{7}\,\left(c^3d^3 + 9\,ac^2de^2\right)x^7 + a^3d^3x + \frac{1}{2}\,\left(3\,ac^2d^2e + a^2ce^3\right)x^6 + \frac{3}{5}\,\left(ac^2d^3 + 3\,a^2cde^2\right)x^5 + \frac{1}{4}\,\left(9\,a^2cd^2e + a^3e^3\right)x^4 + \left(a^2cd^3 + a^3de^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/10*c^3*e^3*x^10 + 1/3*c^3*d*e^2*x^9 + 3/8*(c^3*d^2*e + a*c^2*e^3)*x^8 +
3/2*a^3*d^2*e*x^2 + 1/7*(c^3*d^3 + 9*a*c^2*d*e^2)*x^7 + a^3*d^3*x + 1/2*(3
*a*c^2*d^2*e + a^2*c*e^3)*x^6 + 3/5*(a*c^2*d^3 + 3*a^2*c*d*e^2)*x^5 + 1/4*
(9*a^2*c*d^2*e + a^3*e^3)*x^4 + (a^2*c*d^3 + a^3*d*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \,dx = a^3d^3x + \frac{3a^3d^2ex^2}{2} + \frac{c^3de^2x^9}{3} + \frac{c^3e^3x^{10}}{10} + x^8 \cdot \left(\frac{3ac^2e^3}{8} + \frac{3c^3d^2e}{8}\right) + x^7 \cdot \left(\frac{9ac^2de^2}{7} + \frac{c^3d^3}{7}\right) + x^6\left(\frac{a^2ce^3}{2} + \frac{3ac^2d^2e}{2}\right) + x^5 \cdot \left(\frac{9a^2cde^2}{5} + \frac{3ac^2d^3}{5}\right) + x^4\left(\frac{a^3e^3}{4} + \frac{9a^2cd^2e}{4}\right) + x^3\left(a^3de^2 + a^2cd^3\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**3,x)
```

output
```
a**3*d**3*x + 3*a**3*d**2*e*x**2/2 + c**3*d*e**2*x**9/3 + c**3*e**3*x**10/
10 + x**8*(3*a*c**2*e**3/8 + 3*c**3*d**2*e/8) + x**7*(9*a*c**2*d*e**2/7 +
c**3*d**3/7) + x**6*(a**2*c*e**3/2 + 3*a*c**2*d**2*e/2) + x**5*(9*a**2*c*d
*e**2/5 + 3*a*c**2*d**3/5) + x**4*(a**3*e**3/4 + 9*a**2*c*d**2*e/4) + x**3
*(a**3*d*e**2 + a**2*c*d**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12

$$\int (d+ex)^3\left(a+cx^2\right)^3\,dx=\frac{1}{10}\,c^3e^3x^{10}+\frac{1}{3}\,c^3de^2x^9+\frac{3}{8}\left(c^3d^2e+ac^2e^3\right)x^8+\frac{3}{2}\,a^3d^2ex^2+\frac{1}{7}\left(c^3d^3+9\,ac^2de^2\right)x^7+a^3d^3x+\frac{1}{2}\left(3\,ac^2d^2e+a^2ce^3\right)x^6+\frac{3}{5}\left(ac^2d^3+3\,a^2cde^2\right)x^5+\frac{1}{4}\left(9\,a^2cd^2e+a^3e^3\right)x^4+\left(a^2cd^3+a^3de^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/10*c^3*e^3*x^10 + 1/3*c^3*d*e^2*x^9 + 3/8*(c^3*d^2*e + a*c^2*e^3)*x^8 +
3/2*a^3*d^2*e*x^2 + 1/7*(c^3*d^3 + 9*a*c^2*d*e^2)*x^7 + a^3*d^3*x + 1/2*(3
*a*c^2*d^2*e + a^2*c*e^3)*x^6 + 3/5*(a*c^2*d^3 + 3*a^2*c*d*e^2)*x^5 + 1/4*
(9*a^2*c*d^2*e + a^3*e^3)*x^4 + (a^2*c*d^3 + a^3*d*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int (d+ex)^3\left(a+cx^2\right)^3\,dx=\frac{1}{10}\,c^3e^3x^{10}+\frac{1}{3}\,c^3de^2x^9+\frac{3}{8}\,c^3d^2ex^8+\frac{3}{8}\,ac^2e^3x^8+\frac{1}{7}\,c^3d^3x^7+\frac{9}{7}\,ac^2de^2x^7+\frac{3}{2}\,ac^2d^2ex^6+\frac{1}{2}\,a^2ce^3x^6+\frac{3}{5}\,ac^2d^3x^5+\frac{9}{5}\,a^2cde^2x^5+\frac{9}{4}\,a^2cd^2ex^4+\frac{1}{4}\,a^3e^3x^4+a^2cd^3x^3+a^3de^2x^3+\frac{3}{2}\,a^3d^2ex^2+a^3d^3x$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/10*c^3*e^3*x^10 + 1/3*c^3*d*e^2*x^9 + 3/8*c^3*d^2*e*x^8 + 3/8*a*c^2*e^3*
x^8 + 1/7*c^3*d^3*x^7 + 9/7*a*c^2*d*e^2*x^7 + 3/2*a*c^2*d^2*e*x^6 + 1/2*a^
2*c*e^3*x^6 + 3/5*a*c^2*d^3*x^5 + 9/5*a^2*c*d*e^2*x^5 + 9/4*a^2*c*d^2*e*x^
4 + 1/4*a^3*e^3*x^4 + a^2*c*d^3*x^3 + a^3*d*e^2*x^3 + 3/2*a^3*d^2*e*x^2 +
a^3*d^3*x
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.08

$$\int (d+ex)^3\left(a+cx^2\right)^3\,dx = x^3\left(a^3\,d\,e^2+c\,a^2\,d^3\right)+x^4\left(\frac{a^3\,e^3}{4}+\frac{9\,c\,a^2\,d^2\,e}{4}\right)+x^7\left(\frac{c^3\,d^3}{7}+\frac{9\,a\,c^2\,d\,e^2}{7}\right)+x^8\left(\frac{3\,c^3\,d^2\,e}{8}+\frac{3\,a\,c^2\,e^3}{8}\right)+a^3\,d^3\,x+\frac{c^3\,e^3\,x^{10}}{10}+\frac{3\,a^3\,d^2\,e\,x^2}{2}+\frac{c^3\,d\,e^2\,x^9}{3}+\frac{3\,a\,c\,d\,x^5\,\left(c\,d^2+3\,a\,e^2\right)}{5}+\frac{a\,c\,e\,x^6\,\left(3\,c\,d^2+a\,e^2\right)}{2}$$

input
```
int((a + c*x^2)^3*(d + e*x)^3,x)
```

output
```
x^3*(a^2*c*d^3 + a^3*d*e^2) + x^4*((a^3*e^3)/4 + (9*a^2*c*d^2*e)/4) + x^7*
((c^3*d^3)/7 + (9*a*c^2*d*e^2)/7) + x^8*((3*a*c^2*e^3)/8 + (3*c^3*d^2*e)/8
) + a^3*d^3*x + (c^3*e^3*x^10)/10 + (3*a^3*d^2*e*x^2)/2 + (c^3*d*e^2*x^9)/
3 + (3*a*c*d*x^5*(3*a*e^2 + c*d^2))/5 + (a*c*e*x^6*(a*e^2 + 3*c*d^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \, dx$$
$$= \frac{x(84c^3e^3x^9 + 280c^3d\,e^2x^8 + 315a\,c^2e^3x^7 + 315c^3d^2e\,x^7 + 1080a\,c^2d\,e^2x^6 + 120c^3d^3x^6 + 420a^2c\,e^3x^5 + 12}{}$$

input
```
int((e*x+d)^3*(c*x^2+a)^3,x)
```

output
```
(x*(840*a**3*d**3 + 1260*a**3*d**2*e*x + 840*a**3*d*e**2*x**2 + 210*a**3*e
**3*x**3 + 840*a**2*c*d**3*x**2 + 1890*a**2*c*d**2*e*x**3 + 1512*a**2*c*d*
e**2*x**4 + 420*a**2*c*e**3*x**5 + 504*a*c**2*d**3*x**4 + 1260*a*c**2*d**2
*e*x**5 + 1080*a*c**2*d*e**2*x**6 + 315*a*c**2*e**3*x**7 + 120*c**3*d**3*x
**6 + 315*c**3*d**2*e*x**7 + 280*c**3*d*e**2*x**8 + 84*c**3*e**3*x**9))/84
0
```

3.77 $\int (d+ex)^2 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \, dx = a^3d^2x+\frac{1}{3}a^2\left(3cd^2+ae^2\right)x^3+\frac{3}{5}ac\left(cd^2+ae^2\right)x^5+\frac{1}{7}c^2\left(cd^2+3ae^2\right)x^7+\frac{1}{9}c^3e^2x^9+\frac{de\left(a+cx^2\right)^4}{4c}$$

output
```
a^3*d^2*x+1/3*a^2*(a*e^2+3*c*d^2)*x^3+3/5*a*c*(a*e^2+c*d^2)*x^5+1/7*c^2*(3
*a*e^2+c*d^2)*x^7+1/9*c^3*e^2*x^9+1/4*d*e*(c*x^2+a)^4/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \, dx = \frac{1}{10}a^2cx^3\left(10d^2+15dex+6e^2x^2\right)+\frac{1}{35}ac^2x^5\left(21d^2+35dex+15e^2x^2\right)+\frac{1}{252}c^3x^7\left(36d^2+63dex+28e^2x^2\right)+a^3\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)$$

input
```
Integrate[(d + e*x)^2*(a + c*x^2)^3,x]
```

output
```
(a^2*c*x^3*(10*d^2 + 15*d*e*x + 6*e^2*x^2))/10 + (a*c^2*x^5*(21*d^2 + 35*d
*e*x + 15*e^2*x^2))/35 + (c^3*x^7*(36*d^2 + 63*d*e*x + 28*e^2*x^2))/252 +
a^3*(d^2*x + d*e*x^2 + (e^2*x^3)/3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex)^2 \, dx$$

$$\Big\downarrow \text{475}$$

$$\int \left(c^3e^2x^8+c^2\left(cd^2+3ae^2\right)x^6+3ac\left(cd^2+ae^2\right)x^4+a^2\left(3cd^2+ae^2\right)x^2+a^3d^2\right)dx+\frac{de\left(a+cx^2\right)^4}{4c}$$

$$\Big\downarrow \text{2009}$$

$$a^3d^2x+\frac{1}{3}a^2x^3\left(ae^2+3cd^2\right)+\frac{1}{7}c^2x^7\left(3ae^2+cd^2\right)+\frac{3}{5}acx^5\left(ae^2+cd^2\right)+\frac{de\left(a+cx^2\right)^4}{4c}+\frac{1}{9}c^3e^2x^9$$

input
```
Int[(d + e*x)^2*(a + c*x^2)^3,x]
```

output
```
a^3*d^2*x + (a^2*(3*c*d^2 + a*e^2)*x^3)/3 + (3*a*c*(c*d^2 + a*e^2)*x^5)/5
+ (c^2*(c*d^2 + 3*a*e^2)*x^7)/7 + (c^3*e^2*x^9)/9 + (d*e*(a + c*x^2)^4)/(4
*c)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.22

method	result
norman	$\frac{c^3e^2x^9}{9}+\frac{de\,c^3x^8}{4}+\left(\frac{3}{7}e^2a\,c^2+\frac{1}{7}c^3d^2\right)x^7+dea\,c^2x^6+\left(\frac{3}{5}e^2a^2c+\frac{3}{5}a\,c^2d^2\right)x^5+\frac{3de\,a^2c\,x^4}{2}+\left(\frac{1}{3}a\right.$
default	$\frac{c^3e^2x^9}{9}+\frac{de\,c^3x^8}{4}+\frac{(3e^2a\,c^2+c^3d^2)x^7}{7}+dea\,c^2x^6+\frac{(3e^2a^2c+3a\,c^2d^2)x^5}{5}+\frac{3de\,a^2c\,x^4}{2}+\frac{(a^3e^2+3a^2c\,d^2)x^3}{3}+$
gosper	$\frac{1}{9}c^3e^2x^9+\frac{1}{4}de\,c^3x^8+\frac{3}{7}x^7e^2a\,c^2+\frac{1}{7}x^7c^3d^2+dea\,c^2x^6+\frac{3}{5}x^5e^2a^2c+\frac{3}{5}x^5a\,c^2d^2+\frac{3}{2}de\,a^2c\,x^4$
risch	$\frac{1}{9}c^3e^2x^9+\frac{1}{4}de\,c^3x^8+\frac{3}{7}x^7e^2a\,c^2+\frac{1}{7}x^7c^3d^2+dea\,c^2x^6+\frac{3}{5}x^5e^2a^2c+\frac{3}{5}x^5a\,c^2d^2+\frac{3}{2}de\,a^2c\,x^4$
parallelrisch	$\frac{1}{9}c^3e^2x^9+\frac{1}{4}de\,c^3x^8+\frac{3}{7}x^7e^2a\,c^2+\frac{1}{7}x^7c^3d^2+dea\,c^2x^6+\frac{3}{5}x^5e^2a^2c+\frac{3}{5}x^5a\,c^2d^2+\frac{3}{2}de\,a^2c\,x^4$
orering	$\frac{x\left(140e^2c^3x^8+315de\,c^3x^7+540a\,c^2e^2x^6+180c^3d^2x^6+1260dea\,c^2x^5+756a^2c\,e^2x^4+756a\,c^2d^2x^4+1890de\,a^2c\,x^3+420a^3e^2x^2\right.}{1260}$

input
```
int((e*x+d)^2*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*c^3*e^2*x^9+1/4*d*e*c^3*x^8+(3/7*e^2*a*c^2+1/7*c^3*d^2)*x^7+d*e*a*c^2*
x^6+(3/5*e^2*a^2*c+3/5*a*c^2*d^2)*x^5+3/2*d*e*a^2*c*x^4+(1/3*a^3*e^2+a^2*c
*d^2)*x^3+d*e*a^3*x^2+a^3*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

$$\int (d+ex)^2\left(a+cx^2\right)^3\,dx=\frac{1}{9}\,c^3e^2x^9+\frac{1}{4}\,c^3dex^8+ac^2dex^6+\frac{3}{2}\,a^2cdex^4+\frac{1}{7}\,\left(c^3d^2+3\,ac^2e^2\right)x^7+a^3dex^2+a^3d^2x+\frac{3}{5}\,\left(ac^2d^2+a^2ce^2\right)x^5+\frac{1}{3}\,\left(3\,a^2cd^2+a^3e^2\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/9*c^3*e^2*x^9 + 1/4*c^3*d*e*x^8 + a*c^2*d*e*x^6 + 3/2*a^2*c*d*e*x^4 + 1/
7*(c^3*d^2 + 3*a*c^2*e^2)*x^7 + a^3*d*e*x^2 + a^3*d^2*x + 3/5*(a*c^2*d^2 +
 a^2*c*e^2)*x^5 + 1/3*(3*a^2*c*d^2 + a^3*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.34

$$\int (d+ex)^2\left(a+cx^2\right)^3\,dx=a^3d^2x+a^3dex^2+\frac{3a^2cdex^4}{2}+ac^2dex^6+\frac{c^3dex^8}{4}+\frac{c^3e^2x^9}{9}+x^7\cdot\left(\frac{3ac^2e^2}{7}+\frac{c^3d^2}{7}\right)+x^5\cdot\left(\frac{3a^2ce^2}{5}+\frac{3ac^2d^2}{5}\right)+x^3\left(\frac{a^3e^2}{3}+a^2cd^2\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**3,x)
```

output
```
a**3*d**2*x + a**3*d*e*x**2 + 3*a**2*c*d*e*x**4/2 + a*c**2*d*e*x**6 + c**3
*d*e*x**8/4 + c**3*e**2*x**9/9 + x**7*(3*a*c**2*e**2/7 + c**3*d**2/7) + x*
*5*(3*a**2*c*e**2/5 + 3*a*c**2*d**2/5) + x**3*(a**3*e**2/3 + a**2*c*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

$$\int (d+ex)^2 (a+cx^2)^3 \, dx = \frac{1}{9}c^3e^2x^9 + \frac{1}{4}c^3dex^8 + ac^2dex^6 + \frac{3}{2}a^2cdex^4 + \frac{1}{7}(c^3d^2 + 3ac^2e^2)x^7 + a^3dex^2 + a^3d^2x + \frac{3}{5}(ac^2d^2 + a^2ce^2)x^5 + \frac{1}{3}(3a^2cd^2 + a^3e^2)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/9*c^3*e^2*x^9 + 1/4*c^3*d*e*x^8 + a*c^2*d*e*x^6 + 3/2*a^2*c*d*e*x^4 + 1/
7*(c^3*d^2 + 3*a*c^2*e^2)*x^7 + a^3*d*e*x^2 + a^3*d^2*x + 3/5*(a*c^2*d^2 +
 a^2*c*e^2)*x^5 + 1/3*(3*a^2*c*d^2 + a^3*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.24

$$\int (d+ex)^2 (a+cx^2)^3 \, dx = \frac{1}{9}c^3e^2x^9 + \frac{1}{4}c^3dex^8 + \frac{1}{7}c^3d^2x^7 + \frac{3}{7}ac^2e^2x^7 + ac^2dex^6 + \frac{3}{5}ac^2d^2x^5 + \frac{3}{5}a^2ce^2x^5 + \frac{3}{2}a^2cdex^4 + a^2cd^2x^3 + \frac{1}{3}a^3e^2x^3 + a^3dex^2 + a^3d^2x$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/9*c^3*e^2*x^9 + 1/4*c^3*d*e*x^8 + 1/7*c^3*d^2*x^7 + 3/7*a*c^2*e^2*x^7 +
a*c^2*d*e*x^6 + 3/5*a*c^2*d^2*x^5 + 3/5*a^2*c*e^2*x^5 + 3/2*a^2*c*d*e*x^4
+ a^2*c*d^2*x^3 + 1/3*a^3*e^2*x^3 + a^3*d*e*x^2 + a^3*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \, dx = x^3 \left(\frac{a^3 e^2}{3} + c a^2 d^2\right) + x^7 \left(\frac{c^3 d^2}{7} + \frac{3 a c^2 e^2}{7}\right) + a^3 d^2 x + \frac{c^3 e^2 x^9}{9} + \frac{3 a c x^5 (c d^2 + a e^2)}{5} + a^3 d e x^2 + \frac{c^3 d e x^8}{4} + \frac{3 a^2 c d e x^4}{2} + a c^2 d e x^6$$

input
```
int((a + c*x^2)^3*(d + e*x)^2,x)
```

output
```
x^3*((a^3*e^2)/3 + a^2*c*d^2) + x^7*((c^3*d^2)/7 + (3*a*c^2*e^2)/7) + a^3*
d^2*x + (c^3*e^2*x^9)/9 + (3*a*c*x^5*(a*e^2 + c*d^2))/5 + a^3*d*e*x^2 + (c
^3*d*e*x^8)/4 + (3*a^2*c*d*e*x^4)/2 + a*c^2*d*e*x^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \, dx = \frac{x(140c^3e^2x^8 + 315c^3de\,x^7 + 540a\,c^2e^2x^6 + 180c^3d^2x^6 + 1260a\,c^2de\,x^5 + 756a^2c\,e^2x^4 + 756a\,c^2d^2x^4 + 18}{1260}$$

input
```
int((e*x+d)^2*(c*x^2+a)^3,x)
```

output
```
(x*(1260*a**3*d**2 + 1260*a**3*d*e*x + 420*a**3*e**2*x**2 + 1260*a**2*c*d*
*2*x**2 + 1890*a**2*c*d*e*x**3 + 756*a**2*c*e**2*x**4 + 756*a*c**2*d**2*x*
*4 + 1260*a*c**2*d*e*x**5 + 540*a*c**2*e**2*x**6 + 180*c**3*d**2*x**6 + 31
5*c**3*d*e*x**7 + 140*c**3*e**2*x**8))/1260
```

3.78 $\int (d+ex)\left(a+cx^2\right)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int (d+ex)\left(a+cx^2\right)^3\,dx=a^3dx+a^2cdx^3+\frac{3}{5}ac^2dx^5+\frac{1}{7}c^3dx^7+\frac{e\left(a+cx^2\right)^4}{8c}$$

output
```
a^3*d*x+a^2*c*d*x^3+3/5*a*c^2*d*x^5+1/7*c^3*d*x^7+1/8*e*(c*x^2+a)^4/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (d+ex)\left(a+cx^2\right)^3\,dx=a^3dx+\frac{1}{2}a^3ex^2+a^2cdx^3+\frac{3}{4}a^2cex^4+\frac{3}{5}ac^2dx^5+\frac{1}{2}ac^2ex^6+\frac{1}{7}c^3dx^7+\frac{1}{8}c^3ex^8$$

input
```
Integrate[(d + e*x)*(a + c*x^2)^3,x]
```

output
```
a^3*d*x + (a^3*e*x^2)/2 + a^2*c*d*x^3 + (3*a^2*c*e*x^4)/4 + (3*a*c^2*d*x^5
)/5 + (a*c^2*e*x^6)/2 + (c^3*d*x^7)/7 + (c^3*e*x^8)/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex) \, dx$$

$$\Big\downarrow \text{455}$$

$$d\int \left(cx^2+a\right)^3dx+\frac{e\left(a+cx^2\right)^4}{8c}$$

$$\Big\downarrow \text{210}$$

$$d\int \left(c^3x^6+3ac^2x^4+3a^2cx^2+a^3\right)dx+\frac{e\left(a+cx^2\right)^4}{8c}$$

$$\Big\downarrow \text{2009}$$

$$d\left(a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}\right)+\frac{e\left(a+cx^2\right)^4}{8c}$$

input
```
Int[(d + e*x)*(a + c*x^2)^3,x]
```

output
```
(e*(a + c*x^2)^4)/(8*c) + d*(a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^
7)/7)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

method	result	size
gosper	$\frac{1}{8}c^3e\,x^8+\frac{1}{7}c^3d\,x^7+\frac{1}{2}a\,c^2e\,x^6+\frac{3}{5}a\,c^2d\,x^5+\frac{3}{4}a^2ce\,x^4+a^2cd\,x^3+\frac{1}{2}a^3e\,x^2+a^3dx$	74
default	$\frac{1}{8}c^3e\,x^8+\frac{1}{7}c^3d\,x^7+\frac{1}{2}a\,c^2e\,x^6+\frac{3}{5}a\,c^2d\,x^5+\frac{3}{4}a^2ce\,x^4+a^2cd\,x^3+\frac{1}{2}a^3e\,x^2+a^3dx$	74
norman	$\frac{1}{8}c^3e\,x^8+\frac{1}{7}c^3d\,x^7+\frac{1}{2}a\,c^2e\,x^6+\frac{3}{5}a\,c^2d\,x^5+\frac{3}{4}a^2ce\,x^4+a^2cd\,x^3+\frac{1}{2}a^3e\,x^2+a^3dx$	74
risch	$\frac{1}{8}c^3e\,x^8+\frac{1}{7}c^3d\,x^7+\frac{1}{2}a\,c^2e\,x^6+\frac{3}{5}a\,c^2d\,x^5+\frac{3}{4}a^2ce\,x^4+a^2cd\,x^3+\frac{1}{2}a^3e\,x^2+a^3dx$	74
parallelrisch	$\frac{1}{8}c^3e\,x^8+\frac{1}{7}c^3d\,x^7+\frac{1}{2}a\,c^2e\,x^6+\frac{3}{5}a\,c^2d\,x^5+\frac{3}{4}a^2ce\,x^4+a^2cd\,x^3+\frac{1}{2}a^3e\,x^2+a^3dx$	74
orering	$\frac{x\left(35e\,c^3x^7+40d\,c^3x^6+140ea\,c^2x^5+168a\,c^2d\,x^4+210e\,a^2c\,x^3+280a^2cd\,x^2+140a^3ex+280a^3d\right)}{280}$	76

input
```
int((e*x+d)*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*e*x^8+1/7*c^3*d*x^7+1/2*a*c^2*e*x^6+3/5*a*c^2*d*x^5+3/4*a^2*c*e*x^
4+a^2*c*d*x^3+1/2*a^3*e*x^2+a^3*d*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (d+ex)\left(a+cx^2\right)^3\,dx=\frac{1}{8}\,c^3ex^8+\frac{1}{7}\,c^3dx^7+\frac{1}{2}\,ac^2ex^6+\frac{3}{5}\,ac^2dx^5+\frac{3}{4}\,a^2cex^4+a^2cdx^3+\frac{1}{2}\,a^3ex^2+a^3dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/8*c^3*e*x^8 + 1/7*c^3*d*x^7 + 1/2*a*c^2*e*x^6 + 3/5*a*c^2*d*x^5 + 3/4*a^
2*c*e*x^4 + a^2*c*d*x^3 + 1/2*a^3*e*x^2 + a^3*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (d+ex)\left(a+cx^2\right)^3 dx = a^3dx + \frac{a^3ex^2}{2} + a^2cdx^3 + \frac{3a^2cex^4}{4} + \frac{3ac^2dx^5}{5} + \frac{ac^2ex^6}{2} + \frac{c^3dx^7}{7} + \frac{c^3ex^8}{8}$$

input
```
integrate((e*x+d)*(c*x**2+a)**3,x)
```

output
```
a**3*d*x + a**3*e*x**2/2 + a**2*c*d*x**3 + 3*a**2*c*e*x**4/4 + 3*a*c**2*d*
x**5/5 + a*c**2*e*x**6/2 + c**3*d*x**7/7 + c**3*e*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (d+ex)\left(a+cx^2\right)^3 dx = \frac{1}{8}c^3ex^8 + \frac{1}{7}c^3dx^7 + \frac{1}{2}ac^2ex^6 + \frac{3}{5}ac^2dx^5 + \frac{3}{4}a^2cex^4 + a^2cdx^3 + \frac{1}{2}a^3ex^2 + a^3dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*c^3*e*x^8 + 1/7*c^3*d*x^7 + 1/2*a*c^2*e*x^6 + 3/5*a*c^2*d*x^5 + 3/4*a^
2*c*e*x^4 + a^2*c*d*x^3 + 1/2*a^3*e*x^2 + a^3*d*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (d+ex)\left(a+cx^2\right)^3\,dx=\frac{1}{8}c^3ex^8+\frac{1}{7}c^3dx^7+\frac{1}{2}ac^2ex^6+\frac{3}{5}ac^2dx^5+\frac{3}{4}a^2cex^4+a^2cdx^3+\frac{1}{2}a^3ex^2+a^3dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*c^3*e*x^8 + 1/7*c^3*d*x^7 + 1/2*a*c^2*e*x^6 + 3/5*a*c^2*d*x^5 + 3/4*a^
2*c*e*x^4 + a^2*c*d*x^3 + 1/2*a^3*e*x^2 + a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (d+ex)\left(a+cx^2\right)^3\,dx=\frac{e\,a^3\,x^2}{2}+d\,a^3\,x+\frac{3\,e\,a^2\,c\,x^4}{4}+d\,a^2\,c\,x^3+\frac{e\,a\,c^2\,x^6}{2}+\frac{3\,d\,a\,c^2\,x^5}{5}+\frac{e\,c^3\,x^8}{8}+\frac{d\,c^3\,x^7}{7}$$

input
```
int((a + c*x^2)^3*(d + e*x),x)
```

output
```
(a^3*e*x^2)/2 + (c^3*d*x^7)/7 + (c^3*e*x^8)/8 + a^3*d*x + a^2*c*d*x^3 + (3
*a*c^2*d*x^5)/5 + (3*a^2*c*e*x^4)/4 + (a*c^2*e*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int (d+ex)\left(a+cx^2\right)^3\,dx$$
$$=\frac{x(35c^3e\,x^7+40c^3d\,x^6+140a\,c^2e\,x^5+168a\,c^2d\,x^4+210a^2ce\,x^3+280a^2cd\,x^2+140a^3ex+280a^3d)}{280}$$

input
```
int((e*x+d)*(c*x^2+a)^3,x)
```

output
```
(x*(280*a**3*d + 140*a**3*e*x + 280*a**2*c*d*x**2 + 210*a**2*c*e*x**3 + 16
8*a*c**2*d*x**4 + 140*a*c**2*e*x**5 + 40*c**3*d*x**6 + 35*c**3*e*x**7))/28
0
```

3.79 $\int \frac{(a+cx^2)^3}{d+ex}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 173

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx = -\frac{cd(c^2d^4+3acd^2e^2+3a^2e^4)\,x}{e^6}+\frac{c(c^2d^4+3acd^2e^2+3a^2e^4)\,x^2}{2e^5} - \frac{c^2d(cd^2+3ae^2)\,x^3}{3e^4}+\frac{c^2(cd^2+3ae^2)\,x^4}{4e^3} - \frac{c^3dx^5}{5e^2}+\frac{c^3x^6}{6e}+\frac{(cd^2+ae^2)^3\log(d+ex)}{e^7}$$

output
```
-c*d*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d^4)*x/e^6+1/2*c*(3*a^2*e^4+3*a*c*d^2*e^
2+c^2*d^4)*x^2/e^5-1/3*c^2*d*(3*a*e^2+c*d^2)*x^3/e^4+1/4*c^2*(3*a*e^2+c*d^
2)*x^4/e^3-1/5*c^3*d*x^5/e^2+1/6*c^3*x^6/e+(a*e^2+c*d^2)^3*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.82

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx = \frac{cex(90a^2e^4(-2d+ex)+15ace^2(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+c^2(-60d^5+30d^4ex-20d^3e^2x^2+}{60e^7}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x),x]
```

output
```
(c*e*x*(90*a^2*e^4*(-2*d + e*x) + 15*a*c*e^2*(-12*d^3 + 6*d^2*e*x - 4*d*e^
2*x^2 + 3*e^3*x^3) + c^2*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*e
^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5)) + 60*(c*d^2 + a*e^2)^3*Log[d + e*x])/
(60*e^7)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(-\frac{cd(3a^2e^4+3acd^2e^2+c^2d^4)}{e^6}+\frac{cx(3a^2e^4+3acd^2e^2+c^2d^4)}{e^5}-\frac{c^2dx^2(3ae^2+cd^2)}{e^4}+\frac{c^2x^3(3ae^2+cd^2)}{e^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cdx(3a^2e^4+3acd^2e^2+c^2d^4)}{e^6}+\frac{cx^2(3a^2e^4+3acd^2e^2+c^2d^4)}{2e^5}-\frac{c^2dx^3(3ae^2+cd^2)}{3e^4}+\frac{c^2x^4(3ae^2+cd^2)}{4e^3}+\frac{(ae^2+cd^2)^3\log(d+ex)}{e^7}-\frac{c^3dx^5}{5e^2}+\frac{c^3x^6}{6e}$$

input
```
Int[(a + c*x^2)^3/(d + e*x),x]
```

output
```
-((c*d*(c^2*d^4 + 3*a*c*d^2*e^2 + 3*a^2*e^4)*x)/e^6) + (c*(c^2*d^4 + 3*a*c
*d^2*e^2 + 3*a^2*e^4)*x^2)/(2*e^5) - (c^2*d*(c*d^2 + 3*a*e^2)*x^3)/(3*e^4)
 + (c^2*(c*d^2 + 3*a*e^2)*x^4)/(4*e^3) - (c^3*d*x^5)/(5*e^2) + (c^3*x^6)/(
6*e) + ((c*d^2 + a*e^2)^3*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10

method	result
default	$-\frac{c\left(-\frac{c^2x^6e^5}{6}+\frac{d\,c^2x^5e^4}{5}-\frac{e\left(3e^4ac+d^2e^2c^2\right)x^4}{4}+\frac{d\left(3e^4ac+d^2e^2c^2\right)x^3}{3}-\frac{\left(3a^2e^4+3ac\,d^2e^2+c^2d^4\right)x^2e}{2}+d\left(3a^2e^4+3ac\,d^2e^2+c^2d^4\right)\right.}{e^6}$
norman	$\frac{c^3x^6}{6e}+\frac{c(3a^2e^4+3ac\,d^2e^2+c^2d^4)x^2}{2e^5}+\frac{c^2(3a\,e^2+c\,d^2)x^4}{4e^3}-\frac{c^3d\,x^5}{5e^2}-\frac{cd(3a^2e^4+3ac\,d^2e^2+c^2d^4)x}{e^6}-\frac{c^2d(3a\,e^2+c\,d^2}{3e^4}$
risch	$\frac{c^3x^6}{6e}-\frac{c^3d\,x^5}{5e^2}+\frac{3c^2a\,x^4}{4e}+\frac{c^3d^2x^4}{4e^3}-\frac{c^2ad\,x^3}{e^2}-\frac{c^3d^3x^3}{3e^4}+\frac{3c\,a^2x^2}{2e}+\frac{3c^2a\,d^2x^2}{2e^3}+\frac{c^3d^4x^2}{2e^5}-\frac{3c\,a^2dx}{e^2}-\frac{3c^2a}{e}$
parallelrisch	$\frac{10x^6c^3e^6-12d\,c^3x^5e^5+45x^4a\,c^2e^6+15x^4c^3d^2e^4-60x^3a\,c^2d\,e^5-20x^3c^3d^3e^3+90x^2a^2c\,e^6+90x^2a\,c^2d^2e^4+30x^2c^3d^4e^2+60\ln}{60e^7}$

input
```
int((c*x^2+a)^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^6*(-1/6*c^2*x^6*e^5+1/5*d*c^2*x^5*e^4-1/4*e*(3*a*c*e^4+c^2*d^2*e^2)*x
^4+1/3*d*(3*a*c*e^4+c^2*d^2*e^2)*x^3-1/2*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d^4)
*x^2*e+d*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d^4)*x)+(a^3*e^6+3*a^2*c*d^2*e^4+3*a
*c^2*d^4*e^2+c^3*d^6)/e^7*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx$$
$$= \frac{10\,c^3e^6x^6 - 12\,c^3de^5x^5 + 15\,(c^3d^2e^4 + 3\,ac^2e^6)x^4 - 20\,(c^3d^3e^3 + 3\,ac^2de^5)x^3 + 30\,(c^3d^4e^2 + 3\,ac^2d^2e^4 + 3}{60\,e}$$

input
```
integrate((c*x^2+a)^3/(e*x+d),x, algorithm="fricas")
```

output
```
1/60*(10*c^3*e^6*x^6 - 12*c^3*d*e^5*x^5 + 15*(c^3*d^2*e^4 + 3*a*c^2*e^6)*x
^4 - 20*(c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 30*(c^3*d^4*e^2 + 3*a*c^2*d^2*
e^4 + 3*a^2*c*e^6)*x^2 - 60*(c^3*d^5*e + 3*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*
x + 60*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*log(e*x + d
))/e^7
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx = -\frac{c^3dx^5}{5e^2} + \frac{c^3x^6}{6e} + x^4\cdot\left(\frac{3ac^2}{4e} + \frac{c^3d^2}{4e^3}\right) + x^3\left(-\frac{ac^2d}{e^2} - \frac{c^3d^3}{3e^4}\right) + x^2\cdot\left(\frac{3a^2c}{2e} + \frac{3ac^2d^2}{2e^3} + \frac{c^3d^4}{2e^5}\right) + x\left(-\frac{3a^2cd}{e^2} - \frac{3ac^2d^3}{e^4} - \frac{c^3d^5}{e^6}\right) + \frac{(ae^2+cd^2)^3\log(d+ex)}{e^7}$$

input
```
integrate((c*x**2+a)**3/(e*x+d),x)
```

output
```
-c**3*d*x**5/(5*e**2) + c**3*x**6/(6*e) + x**4*(3*a*c**2/(4*e) + c**3*d**2
/(4*e**3)) + x**3*(-a*c**2*d/e**2 - c**3*d**3/(3*e**4)) + x**2*(3*a**2*c/(
2*e) + 3*a*c**2*d**2/(2*e**3) + c**3*d**4/(2*e**5)) + x*(-3*a**2*c*d/e**2
- 3*a*c**2*d**3/e**4 - c**3*d**5/e**6) + (a*e**2 + c*d**2)**3*log(d + e*x)
/e**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx$$
$$= \frac{10\,c^3e^5x^6 - 12\,c^3de^4x^5 + 15\,(c^3d^2e^3 + 3\,ac^2e^5)x^4 - 20\,(c^3d^3e^2 + 3\,ac^2de^4)x^3 + 30\,(c^3d^4e + 3\,ac^2d^2e^3 + 3\,\cdots}{60\,e^6}$$
$$+ \frac{(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d),x, algorithm="maxima")
```

output
```
1/60*(10*c^3*e^5*x^6 - 12*c^3*d*e^4*x^5 + 15*(c^3*d^2*e^3 + 3*a*c^2*e^5)*x
^4 - 20*(c^3*d^3*e^2 + 3*a*c^2*d*e^4)*x^3 + 30*(c^3*d^4*e + 3*a*c^2*d^2*e^
3 + 3*a^2*c*e^5)*x^2 - 60*(c^3*d^5 + 3*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4)*x)/e
^6 + (c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*log(e*x + d)/
e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx$$
$$= \frac{10\,c^3e^5x^6 - 12\,c^3de^4x^5 + 15\,c^3d^2e^3x^4 + 45\,ac^2e^5x^4 - 20\,c^3d^3e^2x^3 - 60\,ac^2de^4x^3 + 30\,c^3d^4ex^2 + 90\,ac^2d^2e\cdots}{60\,e^6}$$
$$+ \frac{(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)\log(|ex+d|)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d),x, algorithm="giac")
```

output
```
1/60*(10*c^3*e^5*x^6 - 12*c^3*d*e^4*x^5 + 15*c^3*d^2*e^3*x^4 + 45*a*c^2*e^
5*x^4 - 20*c^3*d^3*e^2*x^3 - 60*a*c^2*d*e^4*x^3 + 30*c^3*d^4*e*x^2 + 90*a*
c^2*d^2*e^3*x^2 + 90*a^2*c*e^5*x^2 - 60*c^3*d^5*x - 180*a*c^2*d^3*e^2*x -
180*a^2*c*d*e^4*x)/e^6 + (c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^
3*e^6)*log(abs(e*x + d))/e^7
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.23

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx = x^2\left(\frac{d^2\left(\frac{3\,a\,c^2}{e}+\frac{c^3\,d^2}{e^3}\right)}{2\,e^2}+\frac{3\,a^2\,c}{2\,e}\right)+x^4\left(\frac{3\,a\,c^2}{4\,e}+\frac{c^3\,d^2}{4\,e^3}\right)+\frac{c^3\,x^6}{6\,e}+\frac{\ln(d+e\,x)\,\left(a^3\,e^6+3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+c^3\,d^6\right)}{e^7}-\frac{c^3\,d\,x^5}{5\,e^2}-\frac{d\,x^3\left(\frac{3\,a\,c^2}{e}+\frac{c^3\,d^2}{e^3}\right)}{3\,e}-\frac{d\,x\left(\frac{d^2\left(\frac{3\,a\,c^2}{e}+\frac{c^3\,d^2}{e^3}\right)}{e^2}+\frac{3\,a^2\,c}{e}\right)}{e}$$

input
```
int((a + c*x^2)^3/(d + e*x),x)
```

output
```
x^2*((d^2*((3*a*c^2)/e + (c^3*d^2)/e^3))/(2*e^2) + (3*a^2*c)/(2*e)) + x^4*
((3*a*c^2)/(4*e) + (c^3*d^2)/(4*e^3)) + (c^3*x^6)/(6*e) + (log(d + e*x)*(a
^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4))/e^7 - (c^3*d*x^5)/(
5*e^2) - (d*x^3*((3*a*c^2)/e + (c^3*d^2)/e^3))/(3*e) - (d*x*((d^2*((3*a*c^
2)/e + (c^3*d^2)/e^3))/e^2 + (3*a^2*c)/e))/e
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{(a+cx^2)^3}{d+ex}\,dx = \frac{60\log(ex+d)\,a^3e^6+180\log(ex+d)\,a^2c\,d^2e^4+180\log(ex+d)\,a\,c^2d^4e^2+60\log(ex+d)\,c^3d^6-180a^2c}{}$$

input
```
int((c*x^2+a)^3/(e*x+d),x)
```

output
```
(60*log(d + e*x)*a**3*e**6 + 180*log(d + e*x)*a**2*c*d**2*e**4 + 180*log(d
 + e*x)*a*c**2*d**4*e**2 + 60*log(d + e*x)*c**3*d**6 - 180*a**2*c*d*e**5*x
 + 90*a**2*c*e**6*x**2 - 180*a*c**2*d**3*e**3*x + 90*a*c**2*d**2*e**4*x**2
 - 60*a*c**2*d*e**5*x**3 + 45*a*c**2*e**6*x**4 - 60*c**3*d**5*e*x + 30*c**
3*d**4*e**2*x**2 - 20*c**3*d**3*e**3*x**3 + 15*c**3*d**2*e**4*x**4 - 12*c*
*3*d*e**5*x**5 + 10*c**3*e**6*x**6)/(60*e**7)
```

3.80 $\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx = \frac{c(5c^2d^4+9acd^2e^2+3a^2e^4)\,x}{e^6} - \frac{c^2d(2cd^2+3ae^2)\,x^2}{e^5} + \frac{c^2(cd^2+ae^2)\,x^3}{e^4} - \frac{c^3dx^4}{2e^3} + \frac{c^3x^5}{5e^2} - \frac{(cd^2+ae^2)^3}{e^7(d+ex)} - \frac{6cd(cd^2+ae^2)^2\log(d+ex)}{e^7}$$

output
```
c*(3*a^2*e^4+9*a*c*d^2*e^2+5*c^2*d^4)*x/e^6-c^2*d*(3*a*e^2+2*c*d^2)*x^2/e^
5+c^2*(a*e^2+c*d^2)*x^3/e^4-1/2*c^3*d*x^4/e^3+1/5*c^3*x^5/e^2-(a*e^2+c*d^2
)^3/e^7/(e*x+d)-6*c*d*(a*e^2+c*d^2)^2*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.22

$$\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx = \frac{-10a^3e^6+30a^2ce^4(-d^2+dex+e^2x^2)+10ac^2e^2(-3d^4+9d^3ex+6d^2e^2x^2-2de^3x^3+e^4x^4)+c^3(-10d}{10e^7(d}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^2,x]
```

output
```
(-10*a^3*e^6 + 30*a^2*c*e^4*(-d^2 + d*e*x + e^2*x^2) + 10*a*c^2*e^2*(-3*d^
4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + c^3*(-10*d^6 + 50
*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d*e^5*x^5 +
 2*e^6*x^6) - 60*c*d*(c*d^2 + a*e^2)^2*(d + e*x)*Log[d + e*x])/(10*e^7*(d
+ e*x))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{c(3a^2e^4+9acd^2e^2+5c^2d^4)}{e^6}-\frac{2c^2dx(3ae^2+2cd^2)}{e^5}+\frac{3c^2x^2(ae^2+cd^2)}{e^4}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^2}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{cx(3a^2e^4+9acd^2e^2+5c^2d^4)}{e^6}-\frac{c^2dx^2(3ae^2+2cd^2)}{e^5}+\frac{c^2x^3(ae^2+cd^2)}{e^4}-\frac{(ae^2+cd^2)^3}{e^7(d+ex)}-\frac{6cd(ae^2+cd^2)^2\log(d+ex)}{e^7}-\frac{c^3dx^4}{2e^3}+\frac{c^3x^5}{5e^2}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^2,x]
```

output
```
(c*(5*c^2*d^4 + 9*a*c*d^2*e^2 + 3*a^2*e^4)*x)/e^6 - (c^2*d*(2*c*d^2 + 3*a*
e^2)*x^2)/e^5 + (c^2*(c*d^2 + a*e^2)*x^3)/e^4 - (c^3*d*x^4)/(2*e^3) + (c^3
*x^5)/(5*e^2) - (c*d^2 + a*e^2)^3/(e^7*(d + e*x)) - (6*c*d*(c*d^2 + a*e^2)
^2*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.23

method	result
default	$\frac{c\left(\frac{1}{5}c^{2}x^{5}e^{4}-\frac{1}{2}d\,c^{2}x^{4}e^{3}+x^{3}ac\,e^{4}+x^{3}c^{2}d^{2}e^{2}-3x^{2}acd\,e^{3}-2x^{2}c^{2}d^{3}e+3a^{2}e^{4}x+9ac\,d^{2}e^{2}x+5c^{2}d^{4}x\right)}{e^{6}}-\frac{6dc\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c\right.}{e^{7}}$
norman	$\frac{\frac{\left(e^{6}a^{3}+6d^{2}e^{4}a^{2}c+12d^{4}e^{2}a\,c^{2}+6d^{6}c^{3}\right)x}{d\,e^{6}}+\frac{c^{3}x^{6}}{5e}+\frac{c^{2}\left(2a\,e^{2}+c\,d^{2}\right)x^{4}}{2e^{3}}-\frac{3c^{3}d\,x^{5}}{10e^{2}}+\frac{3c\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}\right)x^{2}}{e^{5}}-\frac{d\,c^{2}\left(2a\,e^{2}+c\,d^{2}\right)x^{3}}{e^{4}}}{ex+d}-$
risch	$\frac{c^{3}x^{5}}{5e^{2}}-\frac{c^{3}d\,x^{4}}{2e^{3}}+\frac{c^{2}x^{3}a}{e^{2}}+\frac{c^{3}x^{3}d^{2}}{e^{4}}-\frac{3c^{2}x^{2}ad}{e^{3}}-\frac{2c^{3}x^{2}d^{3}}{e^{5}}+\frac{3c\,a^{2}x}{e^{2}}+\frac{9c^{2}a\,d^{2}x}{e^{4}}+\frac{5c^{3}d^{4}x}{e^{6}}-\frac{6dc\ln(ex+d)a^{2}}{e^{3}}-$
parallelrisch	$-\frac{-2x^{6}c^{3}e^{6}+3d\,c^{3}x^{5}e^{5}-10x^{4}a\,c^{2}e^{6}-5x^{4}c^{3}d^{2}e^{4}+20x^{3}a\,c^{2}d\,e^{5}+10x^{3}c^{3}d^{3}e^{3}+60\ln(ex+d)x\,a^{2}cd\,e^{5}+120\ln(ex+d)xa\,c^{2}d^{3}e}{}$

input
```
int((c*x^2+a)^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^6*(1/5*c^2*x^5*e^4-1/2*d*c^2*x^4*e^3+x^3*a*c*e^4+x^3*c^2*d^2*e^2-3*x^2
*a*c*d*e^3-2*x^2*c^2*d^3*e+3*a^2*e^4*x+9*a*c*d^2*e^2*x+5*c^2*d^4*x)-6*d/e^
7*c*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*ln(e*x+d)-(a^3*e^6+3*a^2*c*d^2*e^4+3*a
*c^2*d^4*e^2+c^3*d^6)/e^7/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.72

$$\int \frac{(a + cx^2)^3}{(d + ex)^2} \, dx = \frac{2\,c^3e^6x^6 - 3\,c^3de^5x^5 - 10\,c^3d^6 - 30\,ac^2d^4e^2 - 30\,a^2cd^2e^4 - 10\,a^3e^6 + 5\,(c^3d^2e^4 + 2\,ac^2e^6)x^4 - 10\,(c^3d^3e^3}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/10*(2*c^3*e^6*x^6 - 3*c^3*d*e^5*x^5 - 10*c^3*d^6 - 30*a*c^2*d^4*e^2 - 30
*a^2*c*d^2*e^4 - 10*a^3*e^6 + 5*(c^3*d^2*e^4 + 2*a*c^2*e^6)*x^4 - 10*(c^3*
d^3*e^3 + 2*a*c^2*d*e^5)*x^3 + 30*(c^3*d^4*e^2 + 2*a*c^2*d^2*e^4 + a^2*c*e
^6)*x^2 + 10*(5*c^3*d^5*e + 9*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x - 60*(c^3*d
^6 + 2*a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + (c^3*d^5*e + 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.22

$$\int \frac{(a + cx^2)^3}{(d + ex)^2} \, dx = -\frac{c^3dx^4}{2e^3} + \frac{c^3x^5}{5e^2} - \frac{6cd(ae^2 + cd^2)^2 \log(d + ex)}{e^7} + x^3\left(\frac{ac^2}{e^2} + \frac{c^3d^2}{e^4}\right) + x^2\left(-\frac{3ac^2d}{e^3} - \frac{2c^3d^3}{e^5}\right) + x\left(\frac{3a^2c}{e^2} + \frac{9ac^2d^2}{e^4} + \frac{5c^3d^4}{e^6}\right) + \frac{-a^3e^6 - 3a^2cd^2e^4 - 3ac^2d^4e^2 - c^3d^6}{de^7 + e^8x}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**2,x)
```

output
```
-c**3*d*x**4/(2*e**3) + c**3*x**5/(5*e**2) - 6*c*d*(a*e**2 + c*d**2)**2*lo
g(d + e*x)/e**7 + x**3*(a*c**2/e**2 + c**3*d**2/e**4) + x**2*(-3*a*c**2*d/
e**3 - 2*c**3*d**3/e**5) + x*(3*a**2*c/e**2 + 9*a*c**2*d**2/e**4 + 5*c**3*
d**4/e**6) + (-a**3*e**6 - 3*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 - c**3*
d**6)/(d*e**7 + e**8*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx = -\frac{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6}{e^8x+de^7} + \frac{2\,c^3e^4x^5-5\,c^3de^3x^4+10\,(c^3d^2e^2+ac^2e^4)x^3-10\,(2\,c^3d^3e+3\,ac^2de^3)x^2+10\,(5\,c^3d^4+9\,ac^2d^2e^2+3\,a}{10\,e^6} - \frac{6\,(c^3d^5+2\,ac^2d^3e^2+a^2cde^4)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)/(e^8*x + d*e^7) +
 1/10*(2*c^3*e^4*x^5 - 5*c^3*d*e^3*x^4 + 10*(c^3*d^2*e^2 + a*c^2*e^4)*x^3
- 10*(2*c^3*d^3*e + 3*a*c^2*d*e^3)*x^2 + 10*(5*c^3*d^4 + 9*a*c^2*d^2*e^2 +
 3*a^2*c*e^4)*x)/e^6 - 6*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*log(e*x
 + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.71

$$\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx = \frac{\left(2\,c^3-\frac{15\,c^3d}{ex+d}+\frac{10\,(5\,c^3d^2e^2+ac^2e^4)}{(ex+d)^2e^2}-\frac{20\,(5\,c^3d^3e^3+3\,ac^2de^5)}{(ex+d)^3e^3}+\frac{30\,(5\,c^3d^4e^4+6\,ac^2d^2e^6+a^2ce^8)}{(ex+d)^4e^4}\right)(ex+d)^5}{10\,e^7} + \frac{6\,(c^3d^5+2\,ac^2d^3e^2+a^2cde^4)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7} - \frac{\frac{c^3d^6e^5}{ex+d}+\frac{3\,ac^2d^4e^7}{ex+d}+\frac{3\,a^2cd^2e^9}{ex+d}+\frac{a^3e^{11}}{ex+d}}{e^{12}}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/10*(2*c^3 - 15*c^3*d/(e*x + d) + 10*(5*c^3*d^2*e^2 + a*c^2*e^4)/((e*x +
d)^2*e^2) - 20*(5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)/((e*x + d)^3*e^3) + 30*(5*c
^3*d^4*e^4 + 6*a*c^2*d^2*e^6 + a^2*c*e^8)/((e*x + d)^4*e^4))*(e*x + d)^5/e
^7 + 6*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*log(abs(e*x + d)/((e*x +
d)^2*abs(e)))/e^7 - (c^3*d^6*e^5/(e*x + d) + 3*a*c^2*d^4*e^7/(e*x + d) + 3
*a^2*c*d^2*e^9/(e*x + d) + a^3*e^11/(e*x + d))/e^12
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.73

$$\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx = x^2\left(\frac{c^3\,d^3}{e^5}-\frac{d\left(\frac{3\,a\,c^2}{e^2}+\frac{3\,c^3\,d^2}{e^4}\right)}{e}\right) - x\left(\frac{d^2\left(\frac{3\,a\,c^2}{e^2}+\frac{3\,c^3\,d^2}{e^4}\right)}{e^2}-\frac{3\,a^2\,c}{e^2}+\frac{2\,d\left(\frac{2\,c^3\,d^3}{e^5}-\frac{2\,d\left(\frac{3\,a\,c^2}{e^2}+\frac{3\,c^3\,d^2}{e^4}\right)}{e}\right)}{e}\right) + x^3\left(\frac{a\,c^2}{e^2}+\frac{c^3\,d^2}{e^4}\right)-\frac{a^3\,e^6+3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+c^3\,d^6}{e\,\left(x\,e^7+d\,e^6\right)} - \frac{\ln\left(d+e\,x\right)\,\left(6\,a^2\,c\,d\,e^4+12\,a\,c^2\,d^3\,e^2+6\,c^3\,d^5\right)}{e^7}+\frac{c^3\,x^5}{5\,e^2}-\frac{c^3\,d\,x^4}{2\,e^3}$$

input
```
int((a + c*x^2)^3/(d + e*x)^2,x)
```

output
```
x^2*((c^3*d^3)/e^5 - (d*((3*a*c^2)/e^2 + (3*c^3*d^2)/e^4))/e) - x*((d^2*((
3*a*c^2)/e^2 + (3*c^3*d^2)/e^4))/e^2 - (3*a^2*c)/e^2 + (2*d*((2*c^3*d^3)/e
^5 - (2*d*((3*a*c^2)/e^2 + (3*c^3*d^2)/e^4))/e))/e) + x^3*((a*c^2)/e^2 + (
c^3*d^2)/e^4) - (a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)/(e
*(d*e^6 + e^7*x)) - (log(d + e*x)*(6*c^3*d^5 + 12*a*c^2*d^3*e^2 + 6*a^2*c*
d*e^4))/e^7 + (c^3*x^5)/(5*e^2) - (c^3*d*x^4)/(2*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.83

$$\int \frac{(a+cx^2)^3}{(d+ex)^2}\,dx$$
$$= \frac{-60\log(ex+d)\,a^2c\,d^3e^4 - 60\log(ex+d)\,a^2c\,d^2e^5x - 120\log(ex+d)\,a\,c^2d^5e^2 - 120\log(ex+d)\,a\,c^2d^4e^3}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^2,x)
```

output
```
( - 60*log(d + e*x)*a**2*c*d**3*e**4 - 60*log(d + e*x)*a**2*c*d**2*e**5*x
- 120*log(d + e*x)*a*c**2*d**5*e**2 - 120*log(d + e*x)*a*c**2*d**4*e**3*x
- 60*log(d + e*x)*c**3*d**7 - 60*log(d + e*x)*c**3*d**6*e*x + 10*a**3*e**7
*x + 60*a**2*c*d**2*e**5*x + 30*a**2*c*d*e**6*x**2 + 120*a*c**2*d**4*e**3*
x + 60*a*c**2*d**3*e**4*x**2 - 20*a*c**2*d**2*e**5*x**3 + 10*a*c**2*d*e**6
*x**4 + 60*c**3*d**6*e*x + 30*c**3*d**5*e**2*x**2 - 10*c**3*d**4*e**3*x**3
 + 5*c**3*d**3*e**4*x**4 - 3*c**3*d**2*e**5*x**5 + 2*c**3*d*e**6*x**6)/(10
*d*e**7*(d + e*x))
```

3.81 $\int \frac{(a+cx^2)^3}{(d+ex)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 163

$$\int \frac{(a+cx^2)^3}{(d+ex)^3} dx = -\frac{c^2d(10cd^2+9ae^2)\,x}{e^6} + \frac{3c^2(2cd^2+ae^2)\,x^2}{2e^5} - \frac{c^3dx^3}{e^4} + \frac{c^3x^4}{4e^3} - \frac{(cd^2+ae^2)^3}{2e^7(d+ex)^2} + \frac{6cd(cd^2+ae^2)^2}{e^7(d+ex)} + \frac{3c(cd^2+ae^2)\,(5cd^2+ae^2)\log(d+ex)}{e^7}$$

output
```
-c^2*d*(9*a*e^2+10*c*d^2)*x/e^6+3/2*c^2*(a*e^2+2*c*d^2)*x^2/e^5-c^3*d*x^3/
e^4+1/4*c^3*x^4/e^3-1/2*(a*e^2+c*d^2)^3/e^7/(e*x+d)^2+6*c*d*(a*e^2+c*d^2)^
2/e^7/(e*x+d)+3*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.21

$$\int \frac{(a+cx^2)^3}{(d+ex)^3} dx = \frac{-2a^3e^6+6a^2cde^4(3d+4ex)+6ac^2e^2(7d^4+2d^3ex-11d^2e^2x^2-4de^3x^3+e^4x^4)+c^3(22d^6-16d^5ex-}{4e^7(d+}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^3,x]
```

output
```
(-2*a^3*e^6 + 6*a^2*c*d*e^4*(3*d + 4*e*x) + 6*a*c^2*e^2*(7*d^4 + 2*d^3*e*x
 - 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + c^3*(22*d^6 - 16*d^5*e*x - 68
*d^4*e^2*x^2 - 20*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 2*d*e^5*x^5 + e^6*x^6) + 1
2*c*(5*c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*(d + e*x)^2*Log[d + e*x])/(4*e^7
*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^3}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(-\frac{c^2d(9ae^2+10cd^2)}{e^6}+\frac{3c^2x(ae^2+2cd^2)}{e^5}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^2}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2dx(9ae^2+10cd^2)}{e^6}+\frac{3c^2x^2(ae^2+2cd^2)}{2e^5}+\frac{6cd(ae^2+cd^2)^2}{e^7(d+ex)}-\frac{(ae^2+cd^2)^3}{2e^7(d+ex)^2}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)\log(d+ex)}{e^7}-\frac{c^3dx^3}{e^4}+\frac{c^3x^4}{4e^3}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^3,x]
```

output
```
-((c^2*d*(10*c*d^2 + 9*a*e^2)*x)/e^6) + (3*c^2*(2*c*d^2 + a*e^2)*x^2)/(2*e
^5) - (c^3*d*x^3)/e^4 + (c^3*x^4)/(4*e^3) - (c*d^2 + a*e^2)^3/(2*e^7*(d +
e*x)^2) + (6*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d + e*x)) + (3*c*(c*d^2 + a*e^2)
*(5*c*d^2 + a*e^2)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.17

method	result
default	$-\frac{c^{2}\left(-\frac{1}{4} c\,x^{4} e^{3}+c d\,x^{3} e^{2}-\frac{3}{2} a\,e^{3} x^{2}-3 c\,d^{2} e\,x^{2}+9 d\,e^{2} a x+10 d^{3} c x\right)}{e^{6}}+\frac{3 c\left(a^{2} e^{4}+6 a c\,d^{2} e^{2}+5 c^{2} d^{4}\right) \ln \left(e x+d\right)}{e^{7}}+\frac{6 d c\left(a^{2} e^{4}+2 a \cdots\right.}{e^{7}\left(e \cdots\right.}$
norman	$\frac{-\frac{e^{6} a^{3}-9 d^{2} e^{4} a^{2} c-54 d^{4} e^{2} a\,c^{2}-45 d^{6} c^{3}}{2 e^{7}}+\frac{c^{3} x^{6}}{4 e}+\frac{c^{2}\left(6 a\,e^{2}+5 c\,d^{2}\right) x^{4}}{4 e^{3}}-\frac{c^{3} d\,x^{5}}{2 e^{2}}+\frac{2 d\left(3 e^{4} a^{2} c+18 d^{2} e^{2} a\,c^{2}+15 d^{4} c^{3}\right) x}{e^{6}}-\frac{c^{2} d\left(6 a\,e^{2}+5 c\,d^{2}\right)\cdots}{e^{4}}}{\left(e x+d\right)^{2}}$
risch	$\frac{c^{3} x^{4}}{4 e^{3}}-\frac{c^{3} d\,x^{3}}{e^{4}}+\frac{3 c^{2} a\,x^{2}}{2 e^{3}}+\frac{3 c^{3} d^{2} x^{2}}{e^{5}}-\frac{9 c^{2} d a x}{e^{4}}-\frac{10 c^{3} d^{3} x}{e^{6}}+\frac{\left(6 d\,e^{4} a^{2} c+12 d^{3} e^{2} a\,c^{2}+6 d^{5} c^{3}\right) x-\frac{e^{6} a^{3}-9 d^{2} e^{4} a^{2} c-21 d \cdots}{2 e \cdots}}{e^{6} \left(e x+d\right)^{2}}$
parallelrisch	$120 \ln \left(e x+d\right) x\,c^{3} d^{5} e+x^{6} c^{3} e^{6}+24 \ln \left(e x+d\right) x\,a^{2} c d\,e^{5}+144 \ln \left(e x+d\right) x a\,c^{2} d^{3} e^{3}+108 d^{4} e^{2} a\,c^{2}+72 \ln \left(e x+d\right) a\,c^{2} d^{4} e^{2}+24 x\,a^{2} c d \cdots$

input
```
int((c*x^2+a)^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-c^2/e^6*(-1/4*c*x^4*e^3+c*d*x^3*e^2-3/2*a*e^3*x^2-3*c*d^2*e*x^2+9*d*e^2*a
*x+10*d^3*c*x)+3*c/e^7*(a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)*ln(e*x+d)+6*d/e^7
*c*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/(e*x+d)-1/2*(a^3*e^6+3*a^2*c*d^2*e^4+3*
a*c^2*d^4*e^2+c^3*d^6)/e^7/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(157) = 314$.

Time = 0.07 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.93

$$\int \frac{(a+cx^2)^3}{(d+ex)^3}\,dx = \frac{c^3e^6x^6 - 2\,c^3de^5x^5 + 22\,c^3d^6 + 42\,ac^2d^4e^2 + 18\,a^2cd^2e^4 - 2\,a^3e^6 + (5\,c^3d^2e^4 + 6\,ac^2e^6)x^4 - 4\,(5\,c^3d^3e^3 +}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/4*(c^3*e^6*x^6 - 2*c^3*d*e^5*x^5 + 22*c^3*d^6 + 42*a*c^2*d^4*e^2 + 18*a^
2*c*d^2*e^4 - 2*a^3*e^6 + (5*c^3*d^2*e^4 + 6*a*c^2*e^6)*x^4 - 4*(5*c^3*d^3
*e^3 + 6*a*c^2*d*e^5)*x^3 - 2*(34*c^3*d^4*e^2 + 33*a*c^2*d^2*e^4)*x^2 - 4*
(4*c^3*d^5*e - 3*a*c^2*d^3*e^3 - 6*a^2*c*d*e^5)*x + 12*(5*c^3*d^6 + 6*a*c^
2*d^4*e^2 + a^2*c*d^2*e^4 + (5*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)*
x^2 + 2*(5*c^3*d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)*log(e*x + d))/(e^
9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{(a+cx^2)^3}{(d+ex)^3}\,dx &= -\frac{c^3dx^3}{e^4} + \frac{c^3x^4}{4e^3} + \frac{3c(ae^2+cd^2)\,(ae^2+5cd^2)\log{(d+ex)}}{e^7} \\ &\quad + x^2\cdot\left(\frac{3ac^2}{2e^3} + \frac{3c^3d^2}{e^5}\right) + x\left(-\frac{9ac^2d}{e^4} - \frac{10c^3d^3}{e^6}\right) \\ &\quad + \frac{-a^3e^6 + 9a^2cd^2e^4 + 21ac^2d^4e^2 + 11c^3d^6 + x(12a^2cde^5 + 24ac^2d^3e^3 + 12c^3d^5e)}{2d^2e^7 + 4de^8x + 2e^9x^2}\end{aligned}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**3,x)
```

output
```
-c**3*d*x**3/e**4 + c**3*x**4/(4*e**3) + 3*c*(a*e**2 + c*d**2)*(a*e**2 + 5
*c*d**2)*log(d + e*x)/e**7 + x**2*(3*a*c**2/(2*e**3) + 3*c**3*d**2/e**5) +
 x*(-9*a*c**2*d/e**4 - 10*c**3*d**3/e**6) + (-a**3*e**6 + 9*a**2*c*d**2*e*
*4 + 21*a*c**2*d**4*e**2 + 11*c**3*d**6 + x*(12*a**2*c*d*e**5 + 24*a*c**2*
d**3*e**3 + 12*c**3*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.31

$$\int \frac{(a+cx^2)^3}{(d+ex)^3}\,dx = \frac{11\,c^3d^6 + 21\,ac^2d^4e^2 + 9\,a^2cd^2e^4 - a^3e^6 + 12\,(c^3d^5e + 2\,ac^2d^3e^3 + a^2cde^5)x}{2\,(e^9x^2 + 2\,de^8x + d^2e^7)} + \frac{c^3e^3x^4 - 4\,c^3de^2x^3 + 6\,(2\,c^3d^2e + ac^2e^3)x^2 - 4\,(10\,c^3d^3 + 9\,ac^2de^2)x}{4\,e^6} + \frac{3\,(5\,c^3d^4 + 6\,ac^2d^2e^2 + a^2ce^4)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(11*c^3*d^6 + 21*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - a^3*e^6 + 12*(c^3*d
^5*e + 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) +
 1/4*(c^3*e^3*x^4 - 4*c^3*d*e^2*x^3 + 6*(2*c^3*d^2*e + a*c^2*e^3)*x^2 - 4*
(10*c^3*d^3 + 9*a*c^2*d*e^2)*x)/e^6 + 3*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2
*c*e^4)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.26

$$\int \frac{(a+cx^2)^3}{(d+ex)^3}\,dx = \frac{3\,(5\,c^3d^4+6\,ac^2d^2e^2+a^2ce^4)\log(|ex+d|)}{e^7} + \frac{11\,c^3d^6+21\,ac^2d^4e^2+9\,a^2cd^2e^4-a^3e^6+12\,(c^3d^5e+2\,ac^2d^3e^3+a^2cde^5)x}{2\,(ex+d)^2e^7} + \frac{c^3e^9x^4-4\,c^3de^8x^3+12\,c^3d^2e^7x^2+6\,ac^2e^9x^2-40\,c^3d^3e^6x-36\,ac^2de^8x}{4\,e^{12}}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^3,x, algorithm="giac")
```

output
```
3*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*log(abs(e*x + d))/e^7 + 1/2*(1
1*c^3*d^6 + 21*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - a^3*e^6 + 12*(c^3*d^5*e +
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/((e*x + d)^2*e^7) + 1/4*(c^3*e^9*x^4 -
4*c^3*d*e^8*x^3 + 12*c^3*d^2*e^7*x^2 + 6*a*c^2*e^9*x^2 - 40*c^3*d^3*e^6*x
- 36*a*c^2*d*e^8*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.44

$$\int \frac{(a+cx^2)^3}{(d+ex)^3}\,dx = x^2\left(\frac{3\,a\,c^2}{2\,e^3}+\frac{3\,c^3\,d^2}{e^5}\right) + \frac{\frac{-a^3\,e^6+9\,a^2\,c\,d^2\,e^4+21\,a\,c^2\,d^4\,e^2+11\,c^3\,d^6}{2\,e}+x\,(6\,a^2\,c\,d\,e^4+12\,a\,c^2\,d^3\,e^2+6\,c^3\,d^5)}{d^2\,e^6+2\,d\,e^7\,x+e^8\,x^2} + x\left(\frac{8\,c^3\,d^3}{e^6}-\frac{3\,d\left(\frac{3\,a\,c^2}{e^3}+\frac{6\,c^3\,d^2}{e^5}\right)}{e}\right) + \frac{\ln(d+e\,x)\,\left(3\,a^2\,c\,e^4+18\,a\,c^2\,d^2\,e^2+15\,c^3\,d^4\right)}{e^7} + \frac{c^3\,x^4}{4\,e^3} - \frac{c^3\,d\,x^3}{e^4}$$

input
```
int((a + c*x^2)^3/(d + e*x)^3,x)
```

output
```
x^2*((3*a*c^2)/(2*e^3) + (3*c^3*d^2)/e^5) + ((11*c^3*d^6 - a^3*e^6 + 21*a*
c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)/(2*e) + x*(6*c^3*d^5 + 12*a*c^2*d^3*e^2 + 6
*a^2*c*d*e^4))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) + x*((8*c^3*d^3)/e^6 - (3*d
*((3*a*c^2)/e^3 + (6*c^3*d^2)/e^5))/e) + (log(d + e*x)*(15*c^3*d^4 + 3*a^2
*c*e^4 + 18*a*c^2*d^2*e^2))/e^7 + (c^3*x^4)/(4*e^3) - (c^3*d*x^3)/e^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.09

$$\int \frac{(a+cx^2)^3}{(d+ex)^3}\,dx = \frac{12\log(ex+d)\,a^2c\,d^2e^4 + 24\log(ex+d)\,a^2cd\,e^5x + 12\log(ex+d)\,a^2c\,e^6x^2 + 72\log(ex+d)\,a\,c^2d^4e^2 + 14\cdots}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^3,x)
```

output
```
(12*log(d + e*x)*a**2*c*d**2*e**4 + 24*log(d + e*x)*a**2*c*d*e**5*x + 12*l
og(d + e*x)*a**2*c*e**6*x**2 + 72*log(d + e*x)*a*c**2*d**4*e**2 + 144*log(
d + e*x)*a*c**2*d**3*e**3*x + 72*log(d + e*x)*a*c**2*d**2*e**4*x**2 + 60*l
og(d + e*x)*c**3*d**6 + 120*log(d + e*x)*c**3*d**5*e*x + 60*log(d + e*x)*c
**3*d**4*e**2*x**2 - 2*a**3*e**6 + 6*a**2*c*d**2*e**4 - 12*a**2*c*e**6*x**
2 + 36*a*c**2*d**4*e**2 - 72*a*c**2*d**2*e**4*x**2 - 24*a*c**2*d*e**5*x**3
 + 6*a*c**2*e**6*x**4 + 30*c**3*d**6 - 60*c**3*d**4*e**2*x**2 - 20*c**3*d*
*3*e**3*x**3 + 5*c**3*d**2*e**4*x**4 - 2*c**3*d*e**5*x**5 + c**3*e**6*x**6
)/(4*e**7*(d**2 + 2*d*e*x + e**2*x**2))
```

3.82 $\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 165

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = \frac{c^2(10cd^2+3ae^2)\,x}{e^6} - \frac{2c^3dx^2}{e^5} + \frac{c^3x^3}{3e^4} - \frac{(cd^2+ae^2)^3}{3e^7(d+ex)^3} + \frac{3cd(cd^2+ae^2)^2}{e^7(d+ex)^2} - \frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{e^7(d+ex)} - \frac{4c^2d(5cd^2+3ae^2)\log(d+ex)}{e^7}$$

output
```
c^2*(3*a*e^2+10*c*d^2)*x/e^6-2*c^3*d*x^2/e^5+1/3*c^3*x^3/e^4-1/3*(a*e^2+c*
d^2)^3/e^7/(e*x+d)^3+3*c*d*(a*e^2+c*d^2)^2/e^7/(e*x+d)^2-3*c*(a*e^2+c*d^2)
*(a*e^2+5*c*d^2)/e^7/(e*x+d)-4*c^2*d*(3*a*e^2+5*c*d^2)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = \frac{-a^3e^6-3a^2ce^4(d^2+3dex+3e^2x^2)+3ac^2e^2(-13d^4-27d^3ex-9d^2e^2x^2+9de^3x^3+3e^4x^4)+c^3(-37d^{6}\cdots}{3e^7(\cdots}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^4,x]
```

output
```
(-(a^3*e^6) - 3*a^2*c*e^4*(d^2 + 3*d*e*x + 3*e^2*x^2) + 3*a*c^2*e^2*(-13*d
^4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^4*x^4) + c^3*(-37*d^6
- 51*d^5*e*x + 39*d^4*e^2*x^2 + 73*d^3*e^3*x^3 + 15*d^2*e^4*x^4 - 3*d*e^5*
x^5 + e^6*x^6) - 12*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^3*Log[d + e*x])/(3
*e^7*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(-\frac{4c^2d(3ae^2+5cd^2)}{e^6(d+ex)}+\frac{c^2(3ae^2+10cd^2)}{e^6}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^2}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^3}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c^2d(3ae^2+5cd^2)\log(d+ex)}{e^7}+\frac{c^2x(3ae^2+10cd^2)}{e^6}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^7(d+ex)}+\frac{3cd(ae^2+cd^2)^2}{e^7(d+ex)^2}-\frac{(ae^2+cd^2)^3}{3e^7(d+ex)^3}-\frac{2c^3dx^2}{e^5}+\frac{c^3x^3}{3e^4}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^4,x]
```

output
```
(c^2*(10*c*d^2 + 3*a*e^2)*x)/e^6 - (2*c^3*d*x^2)/e^5 + (c^3*x^3)/(3*e^4) -
 (c*d^2 + a*e^2)^3/(3*e^7*(d + e*x)^3) + (3*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d
 + e*x)^2) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*(d + e*x)) - (4*
c^2*d*(5*c*d^2 + 3*a*e^2)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.20

method	result
default	$\frac{c^{2}\left(\frac{1}{3} c\,e^{2} x^{3}-2 c d\,x^{2} e+3 a\,e^{2} x+10 c\,d^{2} x\right)}{e^{6}}-\frac{e^{6} a^{3}+3 d^{2} e^{4} a^{2} c+3 d^{4} e^{2} a\,c^{2}+d^{6} c^{3}}{3 e^{7} \left(e x+d\right)^{3}}-\frac{4 c^{2} d \left(3 a\,e^{2}+5 c\,d^{2}\right) \ln \left(e x+d\right)}{e^{7}}-\frac{3 c \left(a^{2} e^{4}-\cdots\right.}{\cdots}$
norman	$\frac{\frac{c^{2}\left(3 a\,e^{2}+5 c\,d^{2}\right) x^{4}}{e^{3}}-\frac{e^{6} a^{3}+3 d^{2} e^{4} a^{2} c+66 d^{4} e^{2} a\,c^{2}+110 d^{6} c^{3}}{3 e^{7}}+\frac{c^{3} x^{6}}{3 e}-\frac{3\left(e^{4} a^{2} c+12 d^{2} e^{2} a\,c^{2}+20 d^{4} c^{3}\right) x^{2}}{e^{5}}-\frac{c^{3} d\,x^{5}}{e^{2}}-\frac{3 d \left(e^{4} a^{2} c+18 d^{2} e^{2} a \cdots\right.}{e^{6}}\cdots}{\left(e x+d\right)^{3}}$
risch	$\frac{c^{3} x^{3}}{3 e^{4}}-\frac{2 c^{3} d\,x^{2}}{e^{5}}+\frac{3 c^{2} a x}{e^{4}}+\frac{10 c^{3} d^{2} x}{e^{6}}+\frac{\left(-3 e^{5} a^{2} c-18 d^{2} e^{3} a\,c^{2}-15 d^{4} e\,c^{3}\right) x^{2}-3 d c \left(a^{2} e^{4}+10 a c\,d^{2} e^{2}+9 c^{2} d^{4}\right) x-\frac{e^{6} a^{3}+3 d \cdots}{\cdots}}{e^{6} \left(e x+d\right)^{3}}$
parallelrisch	$-\frac{108 x^{2} a\,c^{2} d^{2} e^{4}+180 \ln \left(e x+d\right) x\,c^{3} d^{5} e-x^{6} c^{3} e^{6}+108 \ln \left(e x+d\right) x a\,c^{2} d^{3} e^{3}+66 d^{4} e^{2} a\,c^{2}+36 \ln \left(e x+d\right) x^{3} a\,c^{2} d\,e^{5}+36 \ln \left(e x+d\right) \cdots}{\cdots}$

input
```
int((c*x^2+a)^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c^2/e^6*(1/3*c*e^2*x^3-2*c*d*x^2*e+3*a*e^2*x+10*c*d^2*x)-1/3*(a^3*e^6+3*a^
2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6)/e^7/(e*x+d)^3-4*c^2*d*(3*a*e^2+5*c*d^
2)*ln(e*x+d)/e^7-3*c/e^7*(a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)/(e*x+d)+3*d/e^7
*c*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(161) = 322.

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.02

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = \frac{c^3e^6x^6 - 3\,c^3de^5x^5 - 37\,c^3d^6 - 39\,ac^2d^4e^2 - 3\,a^2cd^2e^4 - a^3e^6 + 3\,(5\,c^3d^2e^4 + 3\,ac^2e^6)x^4 + (73\,c^3d^3e^3 + 2}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/3*(c^3*e^6*x^6 - 3*c^3*d*e^5*x^5 - 37*c^3*d^6 - 39*a*c^2*d^4*e^2 - 3*a^2
*c*d^2*e^4 - a^3*e^6 + 3*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 + (73*c^3*d^3*e
^3 + 27*a*c^2*d*e^5)*x^3 + 3*(13*c^3*d^4*e^2 - 9*a*c^2*d^2*e^4 - 3*a^2*c*e
^6)*x^2 - 3*(17*c^3*d^5*e + 27*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*x - 12*(5*c^
3*d^6 + 3*a*c^2*d^4*e^2 + (5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 3*(5*c^3*d
^4*e^2 + 3*a*c^2*d^2*e^4)*x^2 + 3*(5*c^3*d^5*e + 3*a*c^2*d^3*e^3)*x)*log(e
*x + d))/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.44

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = -\frac{2c^3dx^2}{e^5} + \frac{c^3x^3}{3e^4} - \frac{4c^2d(3ae^2+5cd^2)\log(d+ex)}{e^7} + x\left(\frac{3ac^2}{e^4} + \frac{10c^3d^2}{e^6}\right) + \frac{-a^3e^6 - 3a^2cd^2e^4 - 39ac^2d^4e^2 - 37c^3d^6 + x^2(-9a^2ce^6 - 54ac^2d^2e^4 - 45c^3d^4e^2) + x(-9a^2cde^5 - 90ac}{3d^3e^7 + 9d^2e^8x + 9de^9x^2 + 3e^{10}x^3}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**4,x)
```

output
```
-2*c**3*d*x**2/e**5 + c**3*x**3/(3*e**4) - 4*c**2*d*(3*a*e**2 + 5*c*d**2)*
log(d + e*x)/e**7 + x*(3*a*c**2/e**4 + 10*c**3*d**2/e**6) + (-a**3*e**6 -
3*a**2*c*d**2*e**4 - 39*a*c**2*d**4*e**2 - 37*c**3*d**6 + x**2*(-9*a**2*c*
e**6 - 54*a*c**2*d**2*e**4 - 45*c**3*d**4*e**2) + x*(-9*a**2*c*d*e**5 - 90
*a*c**2*d**3*e**3 - 81*c**3*d**5*e))/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e*
*9*x**2 + 3*e**10*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = -\frac{37\,c^3d^6+39\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6+9\,(5\,c^3d^4e^2+6\,ac^2d^2e^4+a^2ce^6)x^2+9\,(9\,c^3d^5e+10\,ac^2d^3e^3}{3\,(e^{10}x^3+3\,de^9x^2+3\,d^2e^8x+d^3e^7)} + \frac{c^3e^2x^3-6\,c^3dex^2+3\,(10\,c^3d^2+3\,ac^2e^2)x}{3\,e^6} - \frac{4\,(5\,c^3d^3+3\,ac^2de^2)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/3*(37*c^3*d^6 + 39*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 + 9*(5*c^3
*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)*x^2 + 9*(9*c^3*d^5*e + 10*a*c^2*d^
3*e^3 + a^2*c*d*e^5)*x)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7) +
 1/3*(c^3*e^2*x^3 - 6*c^3*d*e*x^2 + 3*(10*c^3*d^2 + 3*a*c^2*e^2)*x)/e^6 -
4*(5*c^3*d^3 + 3*a*c^2*d*e^2)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.24

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = -\frac{4\,(5\,c^3d^3+3\,ac^2de^2)\log(|ex+d|)}{e^7} - \frac{37\,c^3d^6+39\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6+9\,(5\,c^3d^4e^2+6\,ac^2d^2e^4+a^2ce^6)x^2+9\,(9\,c^3d^5e+10\,ac^2d^3e^3}{3\,(ex+d)^3e^7} + \frac{c^3e^8x^3-6\,c^3de^7x^2+30\,c^3d^2e^6x+9\,ac^2e^8x}{3\,e^{12}}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^4,x, algorithm="giac")
```

output
```
-4*(5*c^3*d^3 + 3*a*c^2*d*e^2)*log(abs(e*x + d))/e^7 - 1/3*(37*c^3*d^6 + 3
9*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 + 9*(5*c^3*d^4*e^2 + 6*a*c^2*d
^2*e^4 + a^2*c*e^6)*x^2 + 9*(9*c^3*d^5*e + 10*a*c^2*d^3*e^3 + a^2*c*d*e^5)
*x)/((e*x + d)^3*e^7) + 1/3*(c^3*e^8*x^3 - 6*c^3*d*e^7*x^2 + 30*c^3*d^2*e^
6*x + 9*a*c^2*e^8*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.38

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = x\left(\frac{3\,a\,c^2}{e^4}+\frac{10\,c^3\,d^2}{e^6}\right) - \frac{x^2\,(3\,a^2\,c\,e^5+18\,a\,c^2\,d^2\,e^3+15\,c^3\,d^4\,e)+\frac{a^3\,e^6+3\,a^2\,c\,d^2\,e^4+39\,a\,c^2\,d^4\,e^2+37\,c^3\,d^6}{3\,e}+x\,(3\,a^2\,c\,d\,e^4+30\,a\,c^2\,d^3\,e^2}{d^3\,e^6+3\,d^2\,e^7\,x+3\,d\,e^8\,x^2+e^9\,x^3} - \frac{\ln(d+e\,x)\,(20\,c^3\,d^3+12\,a\,c^2\,d\,e^2)}{e^7}+\frac{c^3\,x^3}{3\,e^4}-\frac{2\,c^3\,d\,x^2}{e^5}$$

input
```
int((a + c*x^2)^3/(d + e*x)^4,x)
```

output
```
x*((3*a*c^2)/e^4 + (10*c^3*d^2)/e^6) - (x^2*(3*a^2*c*e^5 + 15*c^3*d^4*e +
18*a*c^2*d^2*e^3) + (a^3*e^6 + 37*c^3*d^6 + 39*a*c^2*d^4*e^2 + 3*a^2*c*d^2
*e^4)/(3*e) + x*(27*c^3*d^5 + 30*a*c^2*d^3*e^2 + 3*a^2*c*d*e^4))/(d^3*e^6
+ e^9*x^3 + 3*d^2*e^7*x + 3*d*e^8*x^2) - (log(d + e*x)*(20*c^3*d^3 + 12*a*
c^2*d*e^2))/e^7 + (c^3*x^3)/(3*e^4) - (2*c^3*d*x^2)/e^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.01

$$\int \frac{(a+cx^2)^3}{(d+ex)^4}\,dx = \frac{-36\log(ex+d)\,a\,c^2d^5e^2-108\log(ex+d)\,a\,c^2d^4e^3x-108\log(ex+d)\,a\,c^2d^3e^4x^2-36\log(ex+d)\,a\,c^2d}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^4,x)
```

output
```
( - 36*log(d + e*x)*a*c**2*d**5*e**2 - 108*log(d + e*x)*a*c**2*d**4*e**3*x
 - 108*log(d + e*x)*a*c**2*d**3*e**4*x**2 - 36*log(d + e*x)*a*c**2*d**2*e*
*5*x**3 - 60*log(d + e*x)*c**3*d**7 - 180*log(d + e*x)*c**3*d**6*e*x - 180
*log(d + e*x)*c**3*d**5*e**2*x**2 - 60*log(d + e*x)*c**3*d**4*e**3*x**3 -
a**3*d*e**6 + 3*a**2*c*e**7*x**3 - 30*a*c**2*d**5*e**2 - 54*a*c**2*d**4*e*
*3*x + 36*a*c**2*d**2*e**5*x**3 + 9*a*c**2*d*e**6*x**4 - 50*c**3*d**7 - 90
*c**3*d**6*e*x + 60*c**3*d**4*e**3*x**3 + 15*c**3*d**3*e**4*x**4 - 3*c**3*
d**2*e**5*x**5 + c**3*d*e**6*x**6)/(3*d*e**7*(d**3 + 3*d**2*e*x + 3*d*e**2
*x**2 + e**3*x**3))
```

3.83 $\int \frac{(a+cx^2)^3}{(d+ex)^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 171

$$\begin{aligned}\int \frac{(a+cx^2)^3}{(d+ex)^5}\,dx = &-\frac{5c^3dx}{e^6}+\frac{c^3x^2}{2e^5}-\frac{(cd^2+ae^2)^3}{4e^7(d+ex)^4}\\ &+\frac{2cd(cd^2+ae^2)^2}{e^7(d+ex)^3}-\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{2e^7(d+ex)^2}\\ &+\frac{4c^2d(5cd^2+3ae^2)}{e^7(d+ex)}+\frac{3c^2(5cd^2+ae^2)\log(d+ex)}{e^7}\end{aligned}$$

output

```
-5*c^3*d*x/e^6+1/2*c^3*x^2/e^5-1/4*(a*e^2+c*d^2)^3/e^7/(e*x+d)^4+2*c*d*(a*
e^2+c*d^2)^2/e^7/(e*x+d)^3-3/2*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)/e^7/(e*x+d)
^2+4*c^2*d*(3*a*e^2+5*c*d^2)/e^7/(e*x+d)+3*c^2*(a*e^2+5*c*d^2)*ln(e*x+d)/e
^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2)^3}{(d+ex)^5} dx = \frac{-a^3e^6 - a^2ce^4(d^2+4dex+6e^2x^2) + ac^2de^2(25d^3+88d^2ex+108de^2x^2+48e^3x^3) + c^3(57d^6+168d^5ex}{4e^7(d+e}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^5,x]
```

output
```
(-(a^3*e^6) - a^2*c*e^4*(d^2 + 4*d*e*x + 6*e^2*x^2) + a*c^2*d*e^2*(25*d^3
+ 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) + c^3*(57*d^6 + 168*d^5*e*x + 1
32*d^4*e^2*x^2 - 32*d^3*e^3*x^3 - 68*d^2*e^4*x^4 - 12*d*e^5*x^5 + 2*e^6*x^
6) + 12*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^4*Log[d + e*x])/(4*e^7*(d + e*x)^4
)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^5} dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(ae^2+5cd^2)}{e^6(d+ex)} - \frac{4c^2d(3ae^2+5cd^2)}{e^6(d+ex)^2} + \frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^3} - \frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^4} + \frac{(ae^2+cd^2)^3}{e^6(d+ex)^5} - \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c^2d(3ae^2+5cd^2)}{e^7(d+ex)}+\frac{3c^2(ae^2+5cd^2)\log(d+ex)}{e^7}-\frac{3c(ae^2+cd^2)\,(ae^2+5cd^2)}{2e^7(d+ex)^2}+\frac{2cd(ae^2+cd^2)^2}{e^7(d+ex)^3}-\frac{(ae^2+cd^2)^3}{4e^7(d+ex)^4}-\frac{5c^3dx}{e^6}+\frac{c^3x^2}{2e^5}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^5,x]
```

output
```
(-5*c^3*d*x)/e^6 + (c^3*x^2)/(2*e^5) - (c*d^2 + a*e^2)^3/(4*e^7*(d + e*x)^
4) + (2*c*d*(c*d^2 + a*e^2)^2)/(e^7*(d + e*x)^3) - (3*c*(c*d^2 + a*e^2)*(5
*c*d^2 + a*e^2))/(2*e^7*(d + e*x)^2) + (4*c^2*d*(5*c*d^2 + 3*a*e^2))/(e^7*
(d + e*x)) + (3*c^2*(5*c*d^2 + a*e^2)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.19

method	result
risch	$\frac{c^3x^2}{2e^5}-\frac{5c^3dx}{e^6}+\frac{(12d\,e^4a\,c^2+20d^3e^2c^3)x^3-\frac{3ce\left(a^2e^4-18ac\,d^2e^2-35c^2d^4\right)x^2}{2}+(-d\,e^4a^2c+22d^3e^2a\,c^2+47d^5c^3)x-\frac{e^6a^3+d}{}}{e^6(ex+d)^4}$
norman	$\frac{-\frac{e^6a^3+d^2e^4a^2c-25d^4e^2a\,c^2-125d^6c^3}{4e^7}+\frac{c^3x^6}{2e}-\frac{3\left(e^4a^2c-18d^2e^2a\,c^2-90d^4c^3\right)x^2}{2e^5}-\frac{3c^3d\,x^5}{e^2}+\frac{4d\left(3e^2a\,c^2+15c^3d^2\right)x^3}{e^4}-\frac{d\left(e^4a^2c-22c\right)}{}}{(ex+d)^4}$
default	$-\frac{c^3\left(-\frac{1}{2}e\,x^2+5dx\right)}{e^6}+\frac{2dc\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)}{e^7(ex+d)^3}-\frac{e^6a^3+3d^2e^4a^2c+3d^4e^2a\,c^2+d^6c^3}{4e^7(ex+d)^4}+\frac{3c^2\left(a\,e^2+5c\,d^2\right)\ln(ex+d)}{e^7}+$
parallelrisch	$\frac{60\ln(ex+d)x^4c^3d^2e^4+108x^2a\,c^2d^2e^4+240\ln(ex+d)x\,c^3d^5e+2x^6c^3e^6+48\ln(ex+d)xa\,c^2d^3e^3+25d^4e^2a\,c^2+48\ln(ex+d)x^3a}{}$

input
```
int((c*x^2+a)^3/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*c^3*x^2/e^5-5*c^3*d*x/e^6+((12*a*c^2*d*e^4+20*c^3*d^3*e^2)*x^3-3/2*c*e
*(a^2*e^4-18*a*c*d^2*e^2-35*c^2*d^4)*x^2+(-a^2*c*d*e^4+22*a*c^2*d^3*e^2+47
*c^3*d^5)*x-1/4*(a^3*e^6+a^2*c*d^2*e^4-25*a*c^2*d^4*e^2-57*c^3*d^6)/e)/e^6
/(e*x+d)^4+3*c^2*ln(e*x+d)/e^5*a+15*c^3*ln(e*x+d)/e^7*d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(165) = 330$.

Time = 0.07 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.08

$$\int \frac{(a+cx^2)^3}{(d+ex)^5}\,dx = \frac{2\,c^3e^6x^6 - 12\,c^3de^5x^5 - 68\,c^3d^2e^4x^4 + 57\,c^3d^6 + 25\,ac^2d^4e^2 - a^2cd^2e^4 - a^3e^6 - 16\,(2\,c^3d^3e^3 - 3\,ac^2de^5)x}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^5,x, algorithm="fricas")
```

output
```
1/4*(2*c^3*e^6*x^6 - 12*c^3*d*e^5*x^5 - 68*c^3*d^2*e^4*x^4 + 57*c^3*d^6 +
25*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6 - 16*(2*c^3*d^3*e^3 - 3*a*c^2*d
*e^5)*x^3 + 6*(22*c^3*d^4*e^2 + 18*a*c^2*d^2*e^4 - a^2*c*e^6)*x^2 + 4*(42*
c^3*d^5*e + 22*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x + 12*(5*c^3*d^6 + a*c^2*d^4*
e^2 + (5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 4*(5*c^3*d^3*e^3 + a*c^2*d*e^5)*x^
3 + 6*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 4*(5*c^3*d^5*e + a*c^2*d^3*e^3
)*x)*log(e*x + d))/(e^11*x^4 + 4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x
+ d^4*e^7)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^2)^3}{(d+ex)^5}\,dx = -\frac{5c^3dx}{e^6} + \frac{c^3x^2}{2e^5} + \frac{3c^2(ae^2+5cd^2)\log(d+ex)}{e^7} + \frac{-a^3e^6 - a^2cd^2e^4 + 25ac^2d^4e^2 + 57c^3d^6 + x^3\cdot(48ac^2de^5 + 80c^3d^3e^3) + x^2(-6a^2ce^6 + 108ac^2d^2e^4 + 21}{4d^4e^7 + 16d^3e^8x + 24d^2e^9x^2 + 16de^{10}x^3 + 4e^{11}x^4}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**5,x)
```

output
```
-5*c**3*d*x/e**6 + c**3*x**2/(2*e**5) + 3*c**2*(a*e**2 + 5*c*d**2)*log(d +
 e*x)/e**7 + (-a**3*e**6 - a**2*c*d**2*e**4 + 25*a*c**2*d**4*e**2 + 57*c**
3*d**6 + x**3*(48*a*c**2*d*e**5 + 80*c**3*d**3*e**3) + x**2*(-6*a**2*c*e**
6 + 108*a*c**2*d**2*e**4 + 210*c**3*d**4*e**2) + x*(-4*a**2*c*d*e**5 + 88*
a*c**2*d**3*e**3 + 188*c**3*d**5*e))/(4*d**4*e**7 + 16*d**3*e**8*x + 24*d*
*2*e**9*x**2 + 16*d*e**10*x**3 + 4*e**11*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^2)^3}{(d+ex)^5}\,dx = \frac{57\,c^3d^6 + 25\,ac^2d^4e^2 - a^2cd^2e^4 - a^3e^6 + 16\,(5\,c^3d^3e^3 + 3\,ac^2de^5)x^3 + 6\,(35\,c^3d^4e^2 + 18\,ac^2d^2e^4 - a^2ce^6)x}{4\,(e^{11}x^4 + 4\,de^{10}x^3 + 6\,d^2e^9x^2 + 4\,d^3e^8x + d^4e^7)} + \frac{c^3ex^2 - 10\,c^3dx}{2\,e^6} + \frac{3\,(5\,c^3d^2 + ac^2e^2)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^5,x, algorithm="maxima")
```

output
```
1/4*(57*c^3*d^6 + 25*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6 + 16*(5*c^3*d
^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 6*(35*c^3*d^4*e^2 + 18*a*c^2*d^2*e^4 - a^2*c
*e^6)*x^2 + 4*(47*c^3*d^5*e + 22*a*c^2*d^3*e^3 - a^2*c*d*e^5)*x)/(e^11*x^4
 + 4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7) + 1/2*(c^3*e*x^2
- 10*c^3*d*x)/e^6 + 3*(5*c^3*d^2 + a*c^2*e^2)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.70

$$\int \frac{(a+cx^2)^3}{(d+ex)^5}\,dx = \frac{\left(c^3-\frac{12c^3d}{ex+d}\right)(ex+d)^2}{2e^7} - \frac{3(5c^3d^2+ac^2e^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7} + \frac{\frac{80c^3d^3e^{29}}{ex+d} - \frac{30c^3d^4e^{29}}{(ex+d)^2} + \frac{8c^3d^5e^{29}}{(ex+d)^3} - \frac{c^3d^6e^{29}}{(ex+d)^4} + \frac{48ac^2de^{31}}{ex+d} - \frac{36ac^2d^2e^{31}}{(ex+d)^2} + \frac{16ac^2d^3e^{31}}{(ex+d)^3} - \frac{3ac^2d^4e^{31}}{(ex+d)^4} - \frac{6a^2ce^{33}}{(ex+d)^2} + \frac{8a^2c}{(ex+}}{4e^{36}}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^5,x, algorithm="giac")
```

output
```
1/2*(c^3 - 12*c^3*d/(e*x + d))*(e*x + d)^2/e^7 - 3*(5*c^3*d^2 + a*c^2*e^2)
*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^7 + 1/4*(80*c^3*d^3*e^29/(e*x +
d) - 30*c^3*d^4*e^29/(e*x + d)^2 + 8*c^3*d^5*e^29/(e*x + d)^3 - c^3*d^6*e^
29/(e*x + d)^4 + 48*a*c^2*d*e^31/(e*x + d) - 36*a*c^2*d^2*e^31/(e*x + d)^2
 + 16*a*c^2*d^3*e^31/(e*x + d)^3 - 3*a*c^2*d^4*e^31/(e*x + d)^4 - 6*a^2*c*
e^33/(e*x + d)^2 + 8*a^2*c*d*e^33/(e*x + d)^3 - 3*a^2*c*d^2*e^33/(e*x + d)
^4 - a^3*e^35/(e*x + d)^4)/e^36
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.38

$$\int \frac{(a+cx^2)^3}{(d+ex)^5}\,dx = \frac{x^2\left(-\frac{3a^2ce^5}{2}+27ac^2d^2e^3+\frac{105c^3d^4e}{2}\right) - \frac{a^3e^6+a^2cd^2e^4-25ac^2d^4e^2-57c^3d^6}{4e} + x\left(-a^2cde^4+22ac^2d^3e^2+\right.}{d^4e^6+4d^3e^7x+6d^2e^8x^2+4de^9x^3+e^{10}x^4} + \frac{\ln(d+ex)\,(15c^3d^2+3ac^2e^2)}{e^7} + \frac{c^3x^2}{2e^5} - \frac{5c^3dx}{e^6}$$

input
```
int((a + c*x^2)^3/(d + e*x)^5,x)
```

output
```
(x^2*((105*c^3*d^4*e)/2 - (3*a^2*c*e^5)/2 + 27*a*c^2*d^2*e^3) - (a^3*e^6 -
 57*c^3*d^6 - 25*a*c^2*d^4*e^2 + a^2*c*d^2*e^4)/(4*e) + x*(47*c^3*d^5 + 22
*a*c^2*d^3*e^2 - a^2*c*d*e^4) + x^3*(20*c^3*d^3*e^2 + 12*a*c^2*d*e^4))/(d^
4*e^6 + e^10*x^4 + 4*d^3*e^7*x + 4*d*e^9*x^3 + 6*d^2*e^8*x^2) + (log(d + e
*x)*(15*c^3*d^2 + 3*a*c^2*e^2))/e^7 + (c^3*x^2)/(2*e^5) - (5*c^3*d*x)/e^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.31

$$\int \frac{(a+cx^2)^3}{(d+ex)^5}\,dx$$
$$= \frac{12\log(ex+d)\,a\,c^2d^4e^2 + 48\log(ex+d)\,a\,c^2d^3e^3x + 72\log(ex+d)\,a\,c^2d^2e^4x^2 + 48\log(ex+d)\,a\,c^2d\,e^5x^3}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^5,x)
```

output
```
(12*log(d + e*x)*a*c**2*d**4*e**2 + 48*log(d + e*x)*a*c**2*d**3*e**3*x + 7
2*log(d + e*x)*a*c**2*d**2*e**4*x**2 + 48*log(d + e*x)*a*c**2*d*e**5*x**3
+ 12*log(d + e*x)*a*c**2*e**6*x**4 + 60*log(d + e*x)*c**3*d**6 + 240*log(d
 + e*x)*c**3*d**5*e*x + 360*log(d + e*x)*c**3*d**4*e**2*x**2 + 240*log(d +
 e*x)*c**3*d**3*e**3*x**3 + 60*log(d + e*x)*c**3*d**2*e**4*x**4 - a**3*e**
6 - a**2*c*d**2*e**4 - 4*a**2*c*d*e**5*x - 6*a**2*c*e**6*x**2 + 13*a*c**2*
d**4*e**2 + 40*a*c**2*d**3*e**3*x + 36*a*c**2*d**2*e**4*x**2 - 12*a*c**2*e
**6*x**4 + 65*c**3*d**6 + 200*c**3*d**5*e*x + 180*c**3*d**4*e**2*x**2 - 60
*c**3*d**2*e**4*x**4 - 12*c**3*d*e**5*x**5 + 2*c**3*e**6*x**6)/(4*e**7*(d*
*4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.84 $\int \frac{(a+cx^2)^3}{(d+ex)^6} dx$

Optimal result

Integrand size = 17, antiderivative size = 172

$$\int \frac{(a+cx^2)^3}{(d+ex)^6} dx = \frac{c^3x}{e^6} - \frac{(cd^2+ae^2)^3}{5e^7(d+ex)^5} + \frac{3cd(cd^2+ae^2)^2}{2e^7(d+ex)^4} - \frac{c(cd^2+ae^2)(5cd^2+ae^2)}{e^7(d+ex)^3} + \frac{2c^2d(5cd^2+3ae^2)}{e^7(d+ex)^2} - \frac{3c^2(5cd^2+ae^2)}{e^7(d+ex)} - \frac{6c^3d\log(d+ex)}{e^7}$$

output
```
c^3*x/e^6-1/5*(a*e^2+c*d^2)^3/e^7/(e*x+d)^5+3/2*c*d*(a*e^2+c*d^2)^2/e^7/(e
*x+d)^4-c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)/e^7/(e*x+d)^3+2*c^2*d*(3*a*e^2+5*c
*d^2)/e^7/(e*x+d)^2-3*c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)-6*c^3*d*ln(e*x+d)/e^
7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^2)^3}{(d+ex)^6} dx = -\frac{2a^3e^6 + a^2ce^4(d^2+5dex+10e^2x^2) + 6ac^2e^2(d^4+5d^3ex+10d^2e^2x^2+10de^3x^3+5e^4x^4) + c^3(87d^6 + \cdots}{10e^7(d+ \cdots}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^6,x]
```

output
```
-1/10*(2*a^3*e^6 + a^2*c*e^4*(d^2 + 5*d*e*x + 10*e^2*x^2) + 6*a*c^2*e^2*(d
^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) + c^3*(87*d^6
+ 375*d^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 + 50*d^2*e^4*x^4 - 50*d*
e^5*x^5 - 10*e^6*x^6) + 60*c^3*d*(d + e*x)^5*Log[d + e*x])/(e^7*(d + e*x)^
5)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(ae^2+5cd^2)}{e^6(d+ex)^2}-\frac{4c^2d(3ae^2+5cd^2)}{e^6(d+ex)^3}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^4}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^5}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^6}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c^2(ae^2+5cd^2)}{e^7(d+ex)}+\frac{2c^2d(3ae^2+5cd^2)}{e^7(d+ex)^2}-\frac{c(ae^2+cd^2)(ae^2+5cd^2)}{e^7(d+ex)^3}+\frac{3cd(ae^2+cd^2)^2}{2e^7(d+ex)^4}-\frac{(ae^2+cd^2)^3}{5e^7(d+ex)^5}-\frac{6c^3d\log(d+ex)}{e^7}+\frac{c^3x}{e^6}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^6,x]
```

output
```
(c^3*x)/e^6 - (c*d^2 + a*e^2)^3/(5*e^7*(d + e*x)^5) + (3*c*d*(c*d^2 + a*e^
2)^2)/(2*e^7*(d + e*x)^4) - (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*(d
+ e*x)^3) + (2*c^2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d + e*x)^2) - (3*c^2*(5*c*
d^2 + a*e^2))/(e^7*(d + e*x)) - (6*c^3*d*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.15

method	result
risch	$\frac{c^3 x}{e^6}+\frac{\left(-3e^5 a\, c^2-15d^2 e^3 c^3\right)x^4-2d\, e^2 c^2\left(3a\, e^2+25c\, d^2\right)x^3+\left(-e^5 a^2 c-6d^2 e^3 a\, c^2-65d^4 e\, c^3\right)x^2-\frac{dc\left(a^2 e^4+6ac\, d^2 e^2+77c^2 d^4\right)\cdots}{2}\cdots}{e^6 (ex+d)^5}$
norman	$\frac{\frac{c^3 x^6}{e}-\frac{2e^6 a^3+d^2 e^4 a^2 c+6d^4 e^2 a\, c^2+137d^6 c^3}{10e^7}-\frac{\left(3e^2 a\, c^2+30c^3 d^2\right)x^4}{e^3}-\frac{\left(e^4 a^2 c+6d^2 e^2 a\, c^2+110d^4 c^3\right)x^2}{e^5}-\frac{2d\left(3e^2 a\, c^2+45c^3 d^2\right)x^3}{e^4}-\frac{d(\cdots}{\cdots}\cdots}{(ex+d)^5}$
default	$\frac{c^3 x}{e^6}-\frac{c\left(a^2 e^4+6ac\, d^2 e^2+5c^2 d^4\right)}{e^7 (ex+d)^3}+\frac{3dc\left(a^2 e^4+2ac\, d^2 e^2+c^2 d^4\right)}{2e^7 (ex+d)^4}-\frac{6c^3 d\ln(ex+d)}{e^7}-\frac{e^6 a^3+3d^2 e^4 a^2 c+3d^4 e^2 a\, c^2+d^6 c^3}{5e^7 (ex+d)^5}\cdots$
parallelrisch	$-\frac{300\ln(ex+d)x^4 c^3 d^2 e^4+60\ln(ex+d)x^5 c^3 d\, e^5+60x^2 a\, c^2 d^2 e^4+300\ln(ex+d)x\, c^3 d^5 e-10x^6 c^3 e^6+6d^4 e^2 a\, c^2+5x\, a^2 cd\, e^5+d^2\cdots}{\cdots}$

input
```
int((c*x^2+a)^3/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
c^3*x/e^6+((-3*a*c^2*e^5-15*c^3*d^2*e^3)*x^4-2*d*e^2*c^2*(3*a*e^2+25*c*d^2
)*x^3+(-a^2*c*e^5-6*a*c^2*d^2*e^3-65*c^3*d^4*e)*x^2-1/2*d*c*(a^2*e^4+6*a*c
*d^2*e^2+77*c^2*d^4)*x-1/10/e*(2*a^3*e^6+a^2*c*d^2*e^4+6*a*c^2*d^4*e^2+87*
c^3*d^6))/e^6/(e*x+d)^5-6*c^3*d*ln(e*x+d)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.90

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx$$
$$= \frac{10\,c^3e^6x^6+50\,c^3de^5x^5-87\,c^3d^6-6\,ac^2d^4e^2-a^2cd^2e^4-2\,a^3e^6-10\,(5\,c^3d^2e^4+3\,ac^2e^6)x^4-20\,(20\,c^3d}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^6,x, algorithm="fricas")
```

output
```
1/10*(10*c^3*e^6*x^6 + 50*c^3*d*e^5*x^5 - 87*c^3*d^6 - 6*a*c^2*d^4*e^2 - a
^2*c*d^2*e^4 - 2*a^3*e^6 - 10*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 - 20*(20*c
^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 - 10*(60*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a
^2*c*e^6)*x^2 - 5*(75*c^3*d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x - 60*(c
^3*d*e^5*x^5 + 5*c^3*d^2*e^4*x^4 + 10*c^3*d^3*e^3*x^3 + 10*c^3*d^4*e^2*x^2
 + 5*c^3*d^5*e*x + c^3*d^6)*log(e*x + d))/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^
2*e^10*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7)
```

Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.53

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx = -\frac{6c^3d\log{(d+ex)}}{e^7}+\frac{c^3x}{e^6}$$
$$+\frac{-2a^3e^6-a^2cd^2e^4-6ac^2d^4e^2-87c^3d^6+x^4(-30ac^2e^6-150c^3d^2e^4)+x^3(-60ac^2de^5-500c^3d^3e^3)+}{10d^5e^7+50d^4e^8x+100d^3e^9x^2+100d^2e^{10}x^3}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**6,x)
```

output
```
-6*c**3*d*log(d + e*x)/e**7 + c**3*x/e**6 + (-2*a**3*e**6 - a**2*c*d**2*e*
*4 - 6*a*c**2*d**4*e**2 - 87*c**3*d**6 + x**4*(-30*a*c**2*e**6 - 150*c**3*
d**2*e**4) + x**3*(-60*a*c**2*d*e**5 - 500*c**3*d**3*e**3) + x**2*(-10*a**
2*c*e**6 - 60*a*c**2*d**2*e**4 - 650*c**3*d**4*e**2) + x*(-5*a**2*c*d*e**5
 - 30*a*c**2*d**3*e**3 - 385*c**3*d**5*e))/(10*d**5*e**7 + 50*d**4*e**8*x
+ 100*d**3*e**9*x**2 + 100*d**2*e**10*x**3 + 50*d*e**11*x**4 + 10*e**12*x*
*5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.43

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx = -\frac{87\,c^3d^6+6\,ac^2d^4e^2+a^2cd^2e^4+2\,a^3e^6+30\,(5\,c^3d^2e^4+ac^2e^6)x^4+20\,(25\,c^3d^3e^3+3\,ac^2de^5)x^3+10\,(6}{10\,(e^{12}x^5+5\,de^{11}x^4+10\,d^2e^{10}x^3+10\,d^3e^9x^2+} + \frac{c^3x}{e^6} - \frac{6\,c^3d\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^6,x, algorithm="maxima")
```

output
```
-1/10*(87*c^3*d^6 + 6*a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + 2*a^3*e^6 + 30*(5*c^
3*d^2*e^4 + a*c^2*e^6)*x^4 + 20*(25*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 10*
(65*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)*x^2 + 5*(77*c^3*d^5*e + 6*a
*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10*x^3
+ 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7) + c^3*x/e^6 - 6*c^3*d*log(e*x +
d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.16

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx = \frac{c^3x}{e^6} - \frac{6\,c^3d\log(|ex+d|)}{e^7} - \frac{87\,c^3d^6+6\,ac^2d^4e^2+a^2cd^2e^4+2\,a^3e^6+30\,(5\,c^3d^2e^4+ac^2e^6)x^4+20\,(25\,c^3d^3e^3+3\,ac^2de^5)x^3+10\,(6}{10\,(ex+d)^5e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^6,x, algorithm="giac")
```

output
```
c^3*x/e^6 - 6*c^3*d*log(abs(e*x + d))/e^7 - 1/10*(87*c^3*d^6 + 6*a*c^2*d^4
*e^2 + a^2*c*d^2*e^4 + 2*a^3*e^6 + 30*(5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 20
*(25*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 10*(65*c^3*d^4*e^2 + 6*a*c^2*d^2*e
^4 + a^2*c*e^6)*x^2 + 5*(77*c^3*d^5*e + 6*a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/
((e*x + d)^5*e^7)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx = \frac{c^3\,x}{e^6} - \frac{x^2\,(a^2\,c\,e^5+6\,a\,c^2\,d^2\,e^3+65\,c^3\,d^4\,e)+\frac{2\,a^3\,e^6+a^2\,c\,d^2\,e^4+6\,a\,c^2\,d^4\,e^2+87\,c^3\,d^6}{10\,e}+x^4\,(15\,c^3\,d^2\,e^3+3\,a\,c^2\,e^5)+x\,\big(}{d^5\,e^6+5\,d^4\,e^7\,x+10\,d^3\,e^8\,x^2+10\,d^2\,e^9\,x^3+5\,d\,e^{10}\,x} - \frac{6\,c^3\,d\,\ln(d+e\,x)}{e^7}$$

input
```
int((a + c*x^2)^3/(d + e*x)^6,x)
```

output
```
(c^3*x)/e^6 - (x^2*(a^2*c*e^5 + 65*c^3*d^4*e + 6*a*c^2*d^2*e^3) + (2*a^3*e
^6 + 87*c^3*d^6 + 6*a*c^2*d^4*e^2 + a^2*c*d^2*e^4)/(10*e) + x^4*(3*a*c^2*e
^5 + 15*c^3*d^2*e^3) + x*((77*c^3*d^5)/2 + 3*a*c^2*d^3*e^2 + (a^2*c*d*e^4)
/2) + x^3*(50*c^3*d^3*e^2 + 6*a*c^2*d*e^4))/(d^5*e^6 + e^11*x^5 + 5*d^4*e^
7*x + 5*d*e^10*x^4 + 10*d^3*e^8*x^2 + 10*d^2*e^9*x^3) - (6*c^3*d*log(d + e
*x))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.75

$$\int \frac{(a+cx^2)^3}{(d+ex)^6}\,dx = \frac{-60\log(ex+d)\,c^3d^7-300\log(ex+d)\,c^3d^6ex-600\log(ex+d)\,c^3d^5e^2x^2-600\log(ex+d)\,c^3d^4e^3x^3-}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^6,x)
```

output
```
( - 60*log(d + e*x)*c**3*d**7 - 300*log(d + e*x)*c**3*d**6*e*x - 600*log(d
 + e*x)*c**3*d**5*e**2*x**2 - 600*log(d + e*x)*c**3*d**4*e**3*x**3 - 300*l
og(d + e*x)*c**3*d**3*e**4*x**4 - 60*log(d + e*x)*c**3*d**2*e**5*x**5 - 2*
a**3*d*e**6 - a**2*c*d**3*e**4 - 5*a**2*c*d**2*e**5*x - 10*a**2*c*d*e**6*x
**2 + 6*a*c**2*e**7*x**5 - 77*c**3*d**7 - 325*c**3*d**6*e*x - 500*c**3*d**
5*e**2*x**2 - 300*c**3*d**4*e**3*x**3 + 60*c**3*d**2*e**5*x**5 + 10*c**3*d
*e**6*x**6)/(10*d*e**7*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e*
*3*x**3 + 5*d*e**4*x**4 + e**5*x**5))
```

3.85 $\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 184

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx = -\frac{(cd^2+ae^2)^3}{6e^7(d+ex)^6}+\frac{6cd(cd^2+ae^2)^2}{5e^7(d+ex)^5}-\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{4e^7(d+ex)^4}+\frac{4c^2d(5cd^2+3ae^2)}{3e^7(d+ex)^3}-\frac{3c^2(5cd^2+ae^2)}{2e^7(d+ex)^2}+\frac{6c^3d}{e^7(d+ex)}+\frac{c^3\log(d+ex)}{e^7}$$

output
```
-1/6*(a*e^2+c*d^2)^3/e^7/(e*x+d)^6+6/5*c*d*(a*e^2+c*d^2)^2/e^7/(e*x+d)^5-3
/4*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)/e^7/(e*x+d)^4+4/3*c^2*d*(3*a*e^2+5*c*d^
2)/e^7/(e*x+d)^3-3/2*c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)^2+6*c^3*d/e^7/(e*x+d)
+c^3*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx = \frac{-10a^3e^6-3a^2ce^4(d^2+6dex+15e^2x^2)-6ac^2e^2(d^4+6d^3ex+15d^2e^2x^2+20de^3x^3+15e^4x^4)+c^3d(147\cdots}{60e^7(d+e\cdots}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^7,x]
```

output
```
(-10*a^3*e^6 - 3*a^2*c*e^4*(d^2 + 6*d*e*x + 15*e^2*x^2) - 6*a*c^2*e^2*(d^4
 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) + c^3*d*(147*d^
5 + 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3 + 1350*d*e^4*x^4 + 3
60*e^5*x^5) + 60*c^3*(d + e*x)^6*Log[d + e*x])/(60*e^7*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(ae^2+5cd^2)}{e^6(d+ex)^3}-\frac{4c^2d(3ae^2+5cd^2)}{e^6(d+ex)^4}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^5}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^6}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^7}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c^2(ae^2+5cd^2)}{2e^7(d+ex)^2}+\frac{4c^2d(3ae^2+5cd^2)}{3e^7(d+ex)^3}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{4e^7(d+ex)^4}+\frac{6cd(ae^2+cd^2)^2}{5e^7(d+ex)^5}-\frac{(ae^2+cd^2)^3}{6e^7(d+ex)^6}+\frac{6c^3d}{e^7(d+ex)}+\frac{c^3\log(d+ex)}{e^7}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^7,x]
```

output
```
-1/6*(c*d^2 + a*e^2)^3/(e^7*(d + e*x)^6) + (6*c*d*(c*d^2 + a*e^2)^2)/(5*e^
7*(d + e*x)^5) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(4*e^7*(d + e*x)^
4) + (4*c^2*d*(5*c*d^2 + 3*a*e^2))/(3*e^7*(d + e*x)^3) - (3*c^2*(5*c*d^2 +
 a*e^2))/(2*e^7*(d + e*x)^2) + (6*c^3*d)/(e^7*(d + e*x)) + (c^3*Log[d + e*
x])/e^7
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.09

method	result
risch	$\frac{\frac{6c^3 d x^5}{e^2}-\frac{3c^2\left(a e^2-15c d^2\right)x^4}{2e^3}-\frac{2c^2 d\left(3a e^2-55c d^2\right)x^3}{3e^4}-\frac{c\left(3a^2e^4+6ac d^2e^2-125c^2d^4\right)x^2}{4e^5}-\frac{cd\left(3a^2e^4+6ac d^2e^2-137c^2d^4\right)x}{10e^6}-10e^6a^3 \cdots}{(ex+d)^6}$
norman	$\frac{-\frac{10e^6a^3+3d^2e^4a^2c+6d^4e^2a c^2-147d^6c^3}{60e^7}-\frac{3\left(e^2a c^2-15c^3d^2\right)x^4}{2e^3}-\frac{\left(3e^4a^2c+6d^2e^2a c^2-125d^4c^3\right)x^2}{4e^5}+\frac{6c^3d x^5}{e^2}-\frac{2d\left(3e^2a c^2-55c^3d^2\right) \cdots}{3e^4}}{(ex+d)^6}$
default	$\frac{4c^2d\left(3a e^2+5c d^2\right)}{3e^7(ex+d)^3}-\frac{3c\left(a^2e^4+6ac d^2e^2+5c^2d^4\right)}{4e^7(ex+d)^4}+\frac{c^3\ln(ex+d)}{e^7}+\frac{6dc\left(a^2e^4+2ac d^2e^2+c^2d^4\right)}{5e^7(ex+d)^5}+\frac{6c^3d}{e^7(ex+d)}-\frac{3c^2(a \cdots}{2e^7(\cdots}$
parallelrisch	$900\ln(ex+d)x^4c^3d^2e^4+60\ln(ex+d)x^6c^3e^6+360\ln(ex+d)x^5c^3d e^5-90x^2a c^2d^2e^4+360\ln(ex+d)x c^3d^5e-6d^4e^2a c^2-18x a^2 \cdots$

input
```
int((c*x^2+a)^3/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
(6*c^3*d*x^5/e^2-3/2*c^2*(a*e^2-15*c*d^2)/e^3*x^4-2/3*c^2*d*(3*a*e^2-55*c*
d^2)/e^4*x^3-1/4*c*(3*a^2*e^4+6*a*c*d^2*e^2-125*c^2*d^4)/e^5*x^2-1/10*c*d*
(3*a^2*e^4+6*a*c*d^2*e^2-137*c^2*d^4)/e^6*x-1/60*(10*a^3*e^6+3*a^2*c*d^2*e
^4+6*a*c^2*d^4*e^2-147*c^3*d^6)/e^7)/(e*x+d)^6+c^3*ln(e*x+d)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.84

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx$$
$$= \frac{360\,c^3de^5x^5 + 147\,c^3d^6 - 6\,ac^2d^4e^2 - 3\,a^2cd^2e^4 - 10\,a^3e^6 + 90\,(15\,c^3d^2e^4 - ac^2e^6)x^4 + 40\,(55\,c^3d^3e^3 - 3\,\ldots}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^7,x, algorithm="fricas")
```

output
```
1/60*(360*c^3*d*e^5*x^5 + 147*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4
- 10*a^3*e^6 + 90*(15*c^3*d^2*e^4 - a*c^2*e^6)*x^4 + 40*(55*c^3*d^3*e^3 -
3*a*c^2*d*e^5)*x^3 + 15*(125*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 - 3*a^2*c*e^6)*
x^2 + 6*(137*c^3*d^5*e - 6*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5)*x + 60*(c^3*e^6*
x^6 + 6*c^3*d*e^5*x^5 + 15*c^3*d^2*e^4*x^4 + 20*c^3*d^3*e^3*x^3 + 15*c^3*d
^4*e^2*x^2 + 6*c^3*d^5*e*x + c^3*d^6)*log(e*x + d))/(e^13*x^6 + 6*d*e^12*x
^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e^8*x + d^
6*e^7)
```

Sympy [A] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.48

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx = \frac{c^3\log(d+ex)}{e^7}$$
$$+ \frac{-10a^3e^6 - 3a^2cd^2e^4 - 6ac^2d^4e^2 + 147c^3d^6 + 360c^3de^5x^5 + x^4(-90ac^2e^6 + 1350c^3d^2e^4) + x^3(-120ac^2\ldots}{60d^6e^7 + 360d^5e^8x + 900d^4e^9x^2 + 1200d^3\ldots}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**7,x)
```

output
```
c**3*log(d + e*x)/e**7 + (-10*a**3*e**6 - 3*a**2*c*d**2*e**4 - 6*a*c**2*d*
*4*e**2 + 147*c**3*d**6 + 360*c**3*d*e**5*x**5 + x**4*(-90*a*c**2*e**6 + 1
350*c**3*d**2*e**4) + x**3*(-120*a*c**2*d*e**5 + 2200*c**3*d**3*e**3) + x*
*2*(-45*a**2*c*e**6 - 90*a*c**2*d**2*e**4 + 1875*c**3*d**4*e**2) + x*(-18*
a**2*c*d*e**5 - 36*a*c**2*d**3*e**3 + 822*c**3*d**5*e))/(60*d**6*e**7 + 36
0*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11
*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.43

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx$$
$$= \frac{360\,c^3de^5x^5 + 147\,c^3d^6 - 6\,ac^2d^4e^2 - 3\,a^2cd^2e^4 - 10\,a^3e^6 + 90\,(15\,c^3d^2e^4 - ac^2e^6)x^4 + 40\,(55\,c^3d^3e^3 - 3\,}{60\,(e^{13}x^6 + 6\,de^{12}x^5 + 15\,d^2e^{11}x^4 + 20\,d^3e^{10}x}$$
$$+ \frac{c^3\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^7,x, algorithm="maxima")
```

output
```
1/60*(360*c^3*d*e^5*x^5 + 147*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4
- 10*a^3*e^6 + 90*(15*c^3*d^2*e^4 - a*c^2*e^6)*x^4 + 40*(55*c^3*d^3*e^3 -
3*a*c^2*d*e^5)*x^3 + 15*(125*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 - 3*a^2*c*e^6)*
x^2 + 6*(137*c^3*d^5*e - 6*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5)*x)/(e^13*x^6 + 6
*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*x^2 + 6*d^5*e
^8*x + d^6*e^7) + c^3*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx = \frac{c^3\log(|ex+d|)}{e^7}$$
$$+ \frac{360\,c^3de^4x^5 + 90\,(15\,c^3d^2e^3 - ac^2e^5)x^4 + 40\,(55\,c^3d^3e^2 - 3\,ac^2de^4)x^3 + 15\,(125\,c^3d^4e - 6\,ac^2d^2e^3 - 3\,a}{60\,(ex+d)^6e^6}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^7,x, algorithm="giac")
```

output
```
c^3*log(abs(e*x + d))/e^7 + 1/60*(360*c^3*d*e^4*x^5 + 90*(15*c^3*d^2*e^3 -
 a*c^2*e^5)*x^4 + 40*(55*c^3*d^3*e^2 - 3*a*c^2*d*e^4)*x^3 + 15*(125*c^3*d^
4*e - 6*a*c^2*d^2*e^3 - 3*a^2*c*e^5)*x^2 + 6*(137*c^3*d^5 - 6*a*c^2*d^3*e^
2 - 3*a^2*c*d*e^4)*x + (147*c^3*d^6 - 6*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 -
10*a^3*e^6)/e)/((e*x + d)^6*e^6)
```

Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx = \frac{c^3\,\ln(d+e\,x)}{e^7} - \frac{\frac{10\,a^3\,e^6+3\,a^2\,c\,d^2\,e^4+6\,a\,c^2\,d^4\,e^2-147\,c^3\,d^6}{60\,e^7}+\frac{x^2\,(3\,a^2\,c\,e^4+6\,a\,c^2\,d^2\,e^2-125\,c^3\,d^4)}{4\,e^5}+\frac{x\,(3\,a^2\,c\,d\,e^4+6\,a\,c^2\,d^3\,e^2-137\,c^3\,d^5)}{10\,e^6}-\frac{2\,x^3\,(55}{}}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+}$$

input
```
int((a + c*x^2)^3/(d + e*x)^7,x)
```

output
```
(c^3*log(d + e*x))/e^7 - ((10*a^3*e^6 - 147*c^3*d^6 + 6*a*c^2*d^4*e^2 + 3*
a^2*c*d^2*e^4)/(60*e^7) + (x^2*(3*a^2*c*e^4 - 125*c^3*d^4 + 6*a*c^2*d^2*e^
2))/(4*e^5) + (x*(6*a*c^2*d^3*e^2 - 137*c^3*d^5 + 3*a^2*c*d*e^4))/(10*e^6)
 - (2*x^3*(55*c^3*d^3 - 3*a*c^2*d*e^2))/(3*e^4) - (6*c^3*d*x^5)/e^2 + (3*c
^2*x^4*(a*e^2 - 15*c*d^2))/(2*e^3))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*
e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.02

$$\int \frac{(a+cx^2)^3}{(d+ex)^7}\,dx = \frac{60\log(ex+d)\,c^3d^6+360\log(ex+d)\,c^3d^5ex+900\log(ex+d)\,c^3d^4e^2x^2+1200\log(ex+d)\,c^3d^3e^3x^3+9}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^7,x)
```

output
```
(60*log(d + e*x)*c**3*d**6 + 360*log(d + e*x)*c**3*d**5*e*x + 900*log(d +
e*x)*c**3*d**4*e**2*x**2 + 1200*log(d + e*x)*c**3*d**3*e**3*x**3 + 900*log
(d + e*x)*c**3*d**2*e**4*x**4 + 360*log(d + e*x)*c**3*d*e**5*x**5 + 60*log
(d + e*x)*c**3*e**6*x**6 - 10*a**3*e**6 - 3*a**2*c*d**2*e**4 - 18*a**2*c*d
*e**5*x - 45*a**2*c*e**6*x**2 - 6*a*c**2*d**4*e**2 - 36*a*c**2*d**3*e**3*x
 - 90*a*c**2*d**2*e**4*x**2 - 120*a*c**2*d*e**5*x**3 - 90*a*c**2*e**6*x**4
 + 87*c**3*d**6 + 462*c**3*d**5*e*x + 975*c**3*d**4*e**2*x**2 + 1000*c**3*
d**3*e**3*x**3 + 450*c**3*d**2*e**4*x**4 - 60*c**3*e**6*x**6)/(60*e**7*(d*
*6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**3*e**3*x**3 + 15*d**2*e**4*x**
4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.86 $\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 178

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx = -\frac{(cd^2+ae^2)^3}{7e^7(d+ex)^7}+\frac{cd(cd^2+ae^2)^2}{e^7(d+ex)^6}-\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{5e^7(d+ex)^5} + \frac{c^2d(5cd^2+3ae^2)}{e^7(d+ex)^4}-\frac{c^2(5cd^2+ae^2)}{e^7(d+ex)^3}+\frac{3c^3d}{e^7(d+ex)^2}-\frac{c^3}{e^7(d+ex)}$$

output
```
-1/7*(a*e^2+c*d^2)^3/e^7/(e*x+d)^7+c*d*(a*e^2+c*d^2)^2/e^7/(e*x+d)^6-3/5*c
*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)/e^7/(e*x+d)^5+c^2*d*(3*a*e^2+5*c*d^2)/e^7/(
e*x+d)^4-c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)^3+3*c^3*d/e^7/(e*x+d)^2-c^3/e^7/(
e*x+d)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.90

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx = -\frac{5a^3e^6+a^2ce^4(d^2+7dex+21e^2x^2)+ac^2e^2(d^4+7d^3ex+21d^2e^2x^2+35de^3x^3+35e^4x^4)+5c^3(d^6+7}{35e^7(d+ex)^7}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^8,x]
```

output
```
-1/35*(5*a^3*e^6 + a^2*c*e^4*(d^2 + 7*d*e*x + 21*e^2*x^2) + a*c^2*e^2*(d^4
 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 5*c^3*(d^6 +
7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x^4 + 21*d*e^5*x^
5 + 7*e^6*x^6))/(e^7*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(ae^2+5cd^2)}{e^6(d+ex)^4}-\frac{4c^2d(3ae^2+5cd^2)}{e^6(d+ex)^5}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^6}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^7}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^8}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2(ae^2+5cd^2)}{e^7(d+ex)^3}+\frac{c^2d(3ae^2+5cd^2)}{e^7(d+ex)^4}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{5e^7(d+ex)^5}+\frac{cd(ae^2+cd^2)^2}{e^7(d+ex)^6}-\frac{(ae^2+cd^2)^3}{7e^7(d+ex)^7}-\frac{c^3}{e^7(d+ex)}+\frac{3c^3d}{e^7(d+ex)^2}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^8,x]
```

output

```
-1/7*(c*d^2 + a*e^2)^3/(e^7*(d + e*x)^7) + (c*d*(c*d^2 + a*e^2)^2)/(e^7*(d
 + e*x)^6) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(5*e^7*(d + e*x)^5) +
 (c^2*d*(5*c*d^2 + 3*a*e^2))/(e^7*(d + e*x)^4) - (c^2*(5*c*d^2 + a*e^2))/(
e^7*(d + e*x)^3) + (3*c^3*d)/(e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Defintions of rubi rules used

rule 476

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07

method	result
risch	$\frac{-\frac{c^{3} x^{6}}{e}-\frac{3 c^{3} d \,x^{5}}{e^{2}}-\frac{c^{2} \left(a \,e^{2}+5 c \,d^{2}\right) x^{4}}{e^{3}}-\frac{d \,c^{2} \left(a \,e^{2}+5 c \,d^{2}\right) x^{3}}{e^{4}}-\frac{3 c \left(a^{2} e^{4}+a c \,d^{2} e^{2}+5 c^{2} d^{4}\right) x^{2}}{5 e^{5}}-\frac{d c \left(a^{2} e^{4}+a c \,d^{2} e^{2}+5 c^{2} d^{4}\right) x}{5 e^{6}}-5 e^{6} a^{3}+d^{2} e}{\left(e x +d \right)^{7}}$
norman	$\frac{-\frac{c^{3} x^{6}}{e}-\frac{3 c^{3} d \,x^{5}}{e^{2}}-\frac{\left(e^{2} a \,c^{2}+5 c^{3} d^{2}\right) x^{4}}{e^{3}}-\frac{d \left(e^{2} a \,c^{2}+5 c^{3} d^{2}\right) x^{3}}{e^{4}}-\frac{3 \left(e^{4} a^{2} c +d^{2} e^{2} a \,c^{2}+5 d^{4} c^{3}\right) x^{2}}{5 e^{5}}-\frac{d \left(e^{4} a^{2} c +d^{2} e^{2} a \,c^{2}+5 d^{4} c^{3}\right) x}{5 e^{6}}-5 e^{6} a^{3}}{\left(e x +d \right)^{7}}$
gosper	$-\frac{35 x^{6} c^{3} e^{6}+105 d \,c^{3} x^{5} e^{5}+35 x^{4} a \,c^{2} e^{6}+175 x^{4} c^{3} d^{2} e^{4}+35 x^{3} a \,c^{2} d \,e^{5}+175 x^{3} c^{3} d^{3} e^{3}+21 x^{2} a^{2} c \,e^{6}+21 x^{2} a \,c^{2} d^{2} e^{4}+105 x^{2} c^{3} d^{4} e}{35 \left(e x +d \right)^{7} e^{7}}$
orering	$-\frac{35 x^{6} c^{3} e^{6}+105 d \,c^{3} x^{5} e^{5}+35 x^{4} a \,c^{2} e^{6}+175 x^{4} c^{3} d^{2} e^{4}+35 x^{3} a \,c^{2} d \,e^{5}+175 x^{3} c^{3} d^{3} e^{3}+21 x^{2} a^{2} c \,e^{6}+21 x^{2} a \,c^{2} d^{2} e^{4}+105 x^{2} c^{3} d^{4} e}{35 \left(e x +d \right)^{7} e^{7}}$
parallelrisch	$\frac{-35 x^{6} c^{3} e^{6}-105 d \,c^{3} x^{5} e^{5}-35 x^{4} a \,c^{2} e^{6}-175 x^{4} c^{3} d^{2} e^{4}-35 x^{3} a \,c^{2} d \,e^{5}-175 x^{3} c^{3} d^{3} e^{3}-21 x^{2} a^{2} c \,e^{6}-21 x^{2} a \,c^{2} d^{2} e^{4}-105 x^{2} c^{3} d^{4} e^{2}}{35 e^{7} \left(e x +d \right)^{7}}$
default	$-\frac{c^{2} \left(a \,e^{2}+5 c \,d^{2}\right)}{e^{7} \left(e x +d \right)^{3}}+\frac{c^{2} d \left(3 a \,e^{2}+5 c \,d^{2}\right)}{e^{7} \left(e x +d \right)^{4}}-\frac{e^{6} a^{3}+3 d^{2} e^{4} a^{2} c +3 d^{4} e^{2} a \,c^{2}+d^{6} c^{3}}{7 e^{7} \left(e x +d \right)^{7}}-\frac{3 c \left(a^{2} e^{4}+6 a c \,d^{2} e^{2}+5 c^{2} d^{4}\right)}{5 e^{7} \left(e x +d \right)^{5}}-\frac{c^{3}}{e^{7} \left(e x +d \right)}$

input

```
int((c*x^2+a)^3/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
(-c^3*x^6/e-3*c^3*d*x^5/e^2-c^2*(a*e^2+5*c*d^2)/e^3*x^4-d*c^2*(a*e^2+5*c*d
^2)/e^4*x^3-3/5*c*(a^2*e^4+a*c*d^2*e^2+5*c^2*d^4)/e^5*x^2-1/5*d*c*(a^2*e^4
+a*c*d^2*e^2+5*c^2*d^4)/e^6*x-1/35*(5*a^3*e^6+a^2*c*d^2*e^4+a*c^2*d^4*e^2+
5*c^3*d^6)/e^7)/(e*x+d)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.48

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx = -\frac{35\,c^3e^6x^6 + 105\,c^3de^5x^5 + 5\,c^3d^6 + ac^2d^4e^2 + a^2cd^2e^4 + 5\,a^3e^6 + 35\,(5\,c^3d^2e^4 + ac^2e^6)x^4 + 35\,(5\,c^3d^3e^3}{35\,(e^{14}x^7 + 7\,de^{13}x^6 + 21\,d^2e^{12}x^5 + 35\,d^3e^{11}x^4 + 35\,d^4}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^8,x, algorithm="fricas")
```

output
```
-1/35*(35*c^3*e^6*x^6 + 105*c^3*d*e^5*x^5 + 5*c^3*d^6 + a*c^2*d^4*e^2 + a^
2*c*d^2*e^4 + 5*a^3*e^6 + 35*(5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 35*(5*c^3*d
^3*e^3 + a*c^2*d*e^5)*x^3 + 21*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4 + a^2*c*e^6)
*x^2 + 7*(5*c^3*d^5*e + a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^14*x^7 + 7*d*e^
13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*
x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Sympy [A] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.61

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx = \frac{-5a^3e^6 - a^2cd^2e^4 - ac^2d^4e^2 - 5c^3d^6 - 105c^3de^5x^5 - 35c^3e^6x^6 + x^4(-35ac^2e^6 - 175c^3d^2e^4) + x^3(-35ac}{35d^7e^7 + 245d^6e^8x + 735d^5e^9x^2 + 1225d^4e^{10}x^3 + 122}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**8,x)
```

output
```
(-5*a**3*e**6 - a**2*c*d**2*e**4 - a*c**2*d**4*e**2 - 5*c**3*d**6 - 105*c*
*3*d*e**5*x**5 - 35*c**3*e**6*x**6 + x**4*(-35*a*c**2*e**6 - 175*c**3*d**2
*e**4) + x**3*(-35*a*c**2*d*e**5 - 175*c**3*d**3*e**3) + x**2*(-21*a**2*c*
e**6 - 21*a*c**2*d**2*e**4 - 105*c**3*d**4*e**2) + x*(-7*a**2*c*d*e**5 - 7
*a*c**2*d**3*e**3 - 35*c**3*d**5*e))/(35*d**7*e**7 + 245*d**6*e**8*x + 735
*d**5*e**9*x**2 + 1225*d**4*e**10*x**3 + 1225*d**3*e**11*x**4 + 735*d**2*e
**12*x**5 + 245*d*e**13*x**6 + 35*e**14*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.48

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx = -\frac{35\,c^3e^6x^6+105\,c^3de^5x^5+5\,c^3d^6+ac^2d^4e^2+a^2cd^2e^4+5\,a^3e^6+35\,(5\,c^3d^2e^4+ac^2e^6)x^4+35\,(5\,c^3d^3e^3 \cdots}{35\,(e^{14}x^7+7\,de^{13}x^6+21\,d^2e^{12}x^5+35\,d^3e^{11}x^4+35\,d^4 \cdots}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^8,x, algorithm="maxima")
```

output
```
-1/35*(35*c^3*e^6*x^6 + 105*c^3*d*e^5*x^5 + 5*c^3*d^6 + a*c^2*d^4*e^2 + a^
2*c*d^2*e^4 + 5*a^3*e^6 + 35*(5*c^3*d^2*e^4 + a*c^2*e^6)*x^4 + 35*(5*c^3*d
^3*e^3 + a*c^2*d*e^5)*x^3 + 21*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4 + a^2*c*e^6)
*x^2 + 7*(5*c^3*d^5*e + a*c^2*d^3*e^3 + a^2*c*d*e^5)*x)/(e^14*x^7 + 7*d*e^
13*x^6 + 21*d^2*e^12*x^5 + 35*d^3*e^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*
x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx = -\frac{35\,c^3e^6x^6+105\,c^3de^5x^5+175\,c^3d^2e^4x^4+35\,ac^2e^6x^4+175\,c^3d^3e^3x^3+35\,ac^2de^5x^3+105\,c^3d^4e^2x^2+2 \cdots}{35\,(ex \cdots}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^8,x, algorithm="giac")
```

output
```
-1/35*(35*c^3*e^6*x^6 + 105*c^3*d*e^5*x^5 + 175*c^3*d^2*e^4*x^4 + 35*a*c^2
*e^6*x^4 + 175*c^3*d^3*e^3*x^3 + 35*a*c^2*d*e^5*x^3 + 105*c^3*d^4*e^2*x^2
+ 21*a*c^2*d^2*e^4*x^2 + 21*a^2*c*e^6*x^2 + 35*c^3*d^5*e*x + 7*a*c^2*d^3*e
^3*x + 7*a^2*c*d*e^5*x + 5*c^3*d^6 + a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + 5*a^3
*e^6)/((e*x + d)^7*e^7)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.43

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx =$$
$$-\frac{\frac{5\,a^3\,e^6+a^2\,c\,d^2\,e^4+a\,c^2\,d^4\,e^2+5\,c^3\,d^6}{35\,e^7}+\frac{c^3\,x^6}{e}+\frac{3\,c^3\,d\,x^5}{e^2}+\frac{c^2\,x^4\,\left(5\,c\,d^2+a\,e^2\right)}{e^3}+\frac{3\,c\,x^2\,\left(a^2\,e^4+a\,c\,d^2\,e^2+5\,c^2\,d^4\right)}{5\,e^5}+\frac{c\,d\,x\,\left(a^2\,e^4+a\,c\right.}{5\,e}}{d^7+7\,d^6\,e\,x+21\,d^5\,e^2\,x^2+35\,d^4\,e^3\,x^3+35\,d^3\,e^4\,x^4+21\,d^2\,e^5\,x^5+7\,d\,e^6\,x^6+}$$

input
```
int((a + c*x^2)^3/(d + e*x)^8,x)
```

output
```
-((5*a^3*e^6 + 5*c^3*d^6 + a*c^2*d^4*e^2 + a^2*c*d^2*e^4)/(35*e^7) + (c^3*
x^6)/e + (3*c^3*d*x^5)/e^2 + (c^2*x^4*(a*e^2 + 5*c*d^2))/e^3 + (3*c*x^2*(a
^2*e^4 + 5*c^2*d^4 + a*c*d^2*e^2))/(5*e^5) + (c*d*x*(a^2*e^4 + 5*c^2*d^4 +
 a*c*d^2*e^2))/(5*e^6) + (c^2*d*x^3*(a*e^2 + 5*c*d^2))/e^4)/(d^7 + e^7*x^7
 + 7*d*e^6*x^6 + 21*d^5*e^2*x^2 + 35*d^4*e^3*x^3 + 35*d^3*e^4*x^4 + 21*d^2
*e^5*x^5 + 7*d^6*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)^3}{(d+ex)^8}\,dx$$
$$= \frac{5c^3e^5x^7-35a\,c^2d\,e^4x^4-35a\,c^2d^2e^3x^3-21a^2cd\,e^4x^2-21a\,c^2d^3e^2x^2-7a^2c\,d^2e^3x-7a\,c^2d^4ex-5a^3d\,e^4}{35d\,e^5\left(e^7x^7+7d\,e^6x^6+21d^2e^5x^5+35d^3e^4x^4+35d^4e^3x^3+21d^5e^2x^2+7d^6ex+d^7\right)}$$

input
```
int((c*x^2+a)^3/(e*x+d)^8,x)
```

output
```
( - 5*a**3*d*e**4 - a**2*c*d**3*e**2 - 7*a**2*c*d**2*e**3*x - 21*a**2*c*d*
e**4*x**2 - a*c**2*d**5 - 7*a*c**2*d**4*e*x - 21*a*c**2*d**3*e**2*x**2 - 3
5*a*c**2*d**2*e**3*x**3 - 35*a*c**2*d*e**4*x**4 + 5*c**3*e**5*x**7)/(35*d*
e**5*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 35*d**4*e**3*x**3 + 35*d**3*
e**4*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 + e**7*x**7))
```

3.87 $\int \frac{(a+cx^2)^3}{(d+ex)^9} dx$

Optimal result

Integrand size = 17, antiderivative size = 188

$$\int \frac{(a+cx^2)^3}{(d+ex)^9} dx = -\frac{(cd^2+ae^2)^3}{8e^7(d+ex)^8} + \frac{6cd(cd^2+ae^2)^2}{7e^7(d+ex)^7} - \frac{c(cd^2+ae^2)(5cd^2+ae^2)}{2e^7(d+ex)^6} + \frac{4c^2d(5cd^2+3ae^2)}{5e^7(d+ex)^5} - \frac{3c^2(5cd^2+ae^2)}{4e^7(d+ex)^4} + \frac{2c^3d}{e^7(d+ex)^3} - \frac{c^3}{2e^7(d+ex)^2}$$

output
```
-1/8*(a*e^2+c*d^2)^3/e^7/(e*x+d)^8+6/7*c*d*(a*e^2+c*d^2)^2/e^7/(e*x+d)^7-1
/2*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)/e^7/(e*x+d)^6+4/5*c^2*d*(3*a*e^2+5*c*d^
2)/e^7/(e*x+d)^5-3/4*c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)^4+2*c^3*d/e^7/(e*x+d)
^3-1/2*c^3/e^7/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.87

$$\int \frac{(a+cx^2)^3}{(d+ex)^9} dx = -\frac{35a^3e^6 + 5a^2ce^4(d^2+8dex+28e^2x^2) + 3ac^2e^2(d^4+8d^3ex+28d^2e^2x^2+56de^3x^3+70e^4x^4) + 5c^3(d^6}{280e^7(d+ex)^8}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^9,x]
```

output
```
-1/280*(35*a^3*e^6 + 5*a^2*c*e^4*(d^2 + 8*d*e*x + 28*e^2*x^2) + 3*a*c^2*e^
2*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + 5*c^3*(
d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 + 70*d^2*e^4*x^4 + 56*d*
e^5*x^5 + 28*e^6*x^6))/(e^7*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^9}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(ae^2+5cd^2)}{e^6(d+ex)^5}-\frac{4c^2d(3ae^2+5cd^2)}{e^6(d+ex)^6}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^7}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^8}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^9}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c^2(ae^2+5cd^2)}{4e^7(d+ex)^4}+\frac{4c^2d(3ae^2+5cd^2)}{5e^7(d+ex)^5}-\frac{c(ae^2+cd^2)(ae^2+5cd^2)}{2e^7(d+ex)^6}+\frac{6cd(ae^2+cd^2)^2}{7e^7(d+ex)^7}-\frac{(ae^2+cd^2)^3}{8e^7(d+ex)^8}-\frac{c^3}{2e^7(d+ex)^2}+\frac{2c^3d}{e^7(d+ex)^3}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^9,x]
```

output
```
-1/8*(c*d^2 + a*e^2)^3/(e^7*(d + e*x)^8) + (6*c*d*(c*d^2 + a*e^2)^2)/(7*e^
7*(d + e*x)^7) - (c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(2*e^7*(d + e*x)^6)
 + (4*c^2*d*(5*c*d^2 + 3*a*e^2))/(5*e^7*(d + e*x)^5) - (3*c^2*(5*c*d^2 + a
*e^2))/(4*e^7*(d + e*x)^4) + (2*c^3*d)/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d +
 e*x)^2)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

method	result
risch	$\frac{-\frac{c^3x^6}{2e}-\frac{c^3d\,x^5}{e^2}-\frac{c^2\left(3a\,e^2+5c\,d^2\right)x^4}{4e^3}-\frac{d\,c^2\left(3a\,e^2+5c\,d^2\right)x^3}{5e^4}-\frac{c\left(5a^2e^4+3ac\,d^2e^2+5c^2d^4\right)x^2}{10e^5}-\frac{dc\left(5a^2e^4+3ac\,d^2e^2+5c^2d^4\right)x}{35e^6}-\frac{35e^6a^3}{}}{(ex+d)^8}$
gosper	$-\frac{140x^6c^3e^6+280d\,c^3x^5e^5+210x^4a\,c^2e^6+350x^4c^3d^2e^4+168x^3a\,c^2d\,e^5+280x^3c^3d^3e^3+140x^2a^2c\,e^6+84x^2a\,c^2d^2e^4+140x^2c}{280e^7(ex+d)^8}$
orering	$-\frac{140x^6c^3e^6+280d\,c^3x^5e^5+210x^4a\,c^2e^6+350x^4c^3d^2e^4+168x^3a\,c^2d\,e^5+280x^3c^3d^3e^3+140x^2a^2c\,e^6+84x^2a\,c^2d^2e^4+140x^2c}{280e^7(ex+d)^8}$
parallelrisch	$\frac{-140c^3x^6e^7-280c^3d\,x^5e^6-210a\,c^2e^7x^4-350c^3d^2e^5x^4-168a\,c^2d\,e^6x^3-280c^3d^3e^4x^3-140a^2c\,e^7x^2-84a\,c^2d^2e^5x^2-140c^3d^4}{280e^8(ex+d)^8}$
norman	$\frac{-\frac{c^3x^6}{2e}-\frac{c^3d\,x^5}{e^2}-\frac{\left(3e^3a\,c^2+5d^2e\,c^3\right)x^4}{4e^4}-\frac{d\left(3e^3a\,c^2+5d^2e\,c^3\right)x^3}{5e^5}-\frac{\left(5e^5a^2c+3d^2e^3a\,c^2+5d^4e\,c^3\right)x^2}{10e^6}-\frac{d\left(5e^5a^2c+3d^2e^3a\,c^2+5d^4e\,c^3\right)}{35e^7}}{(ex+d)^8}$
default	$\frac{2c^3d}{e^7(ex+d)^3}-\frac{3c^2(a\,e^2+5c\,d^2)}{4e^7(ex+d)^4}+\frac{6dc(a^2e^4+2ac\,d^2e^2+c^2d^4)}{7e^7(ex+d)^7}+\frac{4c^2d(3a\,e^2+5c\,d^2)}{5e^7(ex+d)^5}-\frac{e^6a^3+3d^2e^4a^2c+3d^4e^2a\,c^2+d^6c}{8e^7(ex+d)^8}$

input
```
int((c*x^2+a)^3/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c^3*x^6/e-c^3*d*x^5/e^2-1/4/e^3*c^2*(3*a*e^2+5*c*d^2)*x^4-1/5*d/e^4*
c^2*(3*a*e^2+5*c*d^2)*x^3-1/10*c/e^5*(5*a^2*e^4+3*a*c*d^2*e^2+5*c^2*d^4)*x
^2-1/35*d*c/e^6*(5*a^2*e^4+3*a*c*d^2*e^2+5*c^2*d^4)*x-1/280/e^7*(35*a^3*e^
6+5*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+5*c^3*d^6))/(e*x+d)^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^3}{(d+ex)^9}\,dx = -\frac{140\,c^3e^6x^6 + 280\,c^3de^5x^5 + 5\,c^3d^6 + 3\,ac^2d^4e^2 + 5\,a^2cd^2e^4 + 35\,a^3e^6 + 70\,(5\,c^3d^2e^4 + 3\,ac^2e^6)x^4 + 56\,(}{280\,(e^{15}x^8 + 8\,de^{14}x^7 + 28\,d^2e^{13}x^6 + 56\,d^3e^{12}x^5 + 70\,}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^9,x, algorithm="fricas")
```

output
```
-1/280*(140*c^3*e^6*x^6 + 280*c^3*d*e^5*x^5 + 5*c^3*d^6 + 3*a*c^2*d^4*e^2
+ 5*a^2*c*d^2*e^4 + 35*a^3*e^6 + 70*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 + 56
*(5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 28*(5*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4
 + 5*a^2*c*e^6)*x^2 + 8*(5*c^3*d^5*e + 3*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*x)
/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^1
1*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Sympy [A] (verification not implemented)

Time = 11.42 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.60

$$\int \frac{(a+cx^2)^3}{(d+ex)^9}\,dx = \frac{-35a^3e^6 - 5a^2cd^2e^4 - 3ac^2d^4e^2 - 5c^3d^6 - 280c^3de^5x^5 - 140c^3e^6x^6 + x^4(-210ac^2e^6 - 350c^3d^2e^4) + x^3(}{280d^8e^7 + 2240d^7e^8x + 7840d^6e^9x^2 + 15680d^5e^{10}x^3 + 19600}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**9,x)
```

output
```
(-35*a**3*e**6 - 5*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 - 5*c**3*d**6 - 2
80*c**3*d*e**5*x**5 - 140*c**3*e**6*x**6 + x**4*(-210*a*c**2*e**6 - 350*c*
*3*d**2*e**4) + x**3*(-168*a*c**2*d*e**5 - 280*c**3*d**3*e**3) + x**2*(-14
0*a**2*c*e**6 - 84*a*c**2*d**2*e**4 - 140*c**3*d**4*e**2) + x*(-40*a**2*c*
d*e**5 - 24*a*c**2*d**3*e**3 - 40*c**3*d**5*e))/(280*d**8*e**7 + 2240*d**7
*e**8*x + 7840*d**6*e**9*x**2 + 15680*d**5*e**10*x**3 + 19600*d**4*e**11*x
**4 + 15680*d**3*e**12*x**5 + 7840*d**2*e**13*x**6 + 2240*d*e**14*x**7 + 2
80*e**15*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^3}{(d+ex)^9} dx = -\frac{140 c^3e^6x^6 + 280 c^3de^5x^5 + 5 c^3d^6 + 3 ac^2d^4e^2 + 5 a^2cd^2e^4 + 35 a^3e^6 + 70 (5 c^3d^2e^4 + 3 ac^2e^6)x^4 + 56 (}{280 (e^{15}x^8 + 8 de^{14}x^7 + 28 d^2e^{13}x^6 + 56 d^3e^{12}x^5 + 70 }$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^9,x, algorithm="maxima")
```

output
```
-1/280*(140*c^3*e^6*x^6 + 280*c^3*d*e^5*x^5 + 5*c^3*d^6 + 3*a*c^2*d^4*e^2
+ 5*a^2*c*d^2*e^4 + 35*a^3*e^6 + 70*(5*c^3*d^2*e^4 + 3*a*c^2*e^6)*x^4 + 56
*(5*c^3*d^3*e^3 + 3*a*c^2*d*e^5)*x^3 + 28*(5*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4
 + 5*a^2*c*e^6)*x^2 + 8*(5*c^3*d^5*e + 3*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*x)
/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x^5 + 70*d^4*e^1
1*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^8*e^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^2)^3}{(d+ex)^9} dx = -\frac{140 c^3e^6x^6 + 280 c^3de^5x^5 + 350 c^3d^2e^4x^4 + 210 ac^2e^6x^4 + 280 c^3d^3e^3x^3 + 168 ac^2de^5x^3 + 140 c^3d^4e^2x^2}{28}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^9,x, algorithm="giac")
```

output
```
-1/280*(140*c^3*e^6*x^6 + 280*c^3*d*e^5*x^5 + 350*c^3*d^2*e^4*x^4 + 210*a*
c^2*e^6*x^4 + 280*c^3*d^3*e^3*x^3 + 168*a*c^2*d*e^5*x^3 + 140*c^3*d^4*e^2*
x^2 + 84*a*c^2*d^2*e^4*x^2 + 140*a^2*c*e^6*x^2 + 40*c^3*d^5*e*x + 24*a*c^2
*d^3*e^3*x + 40*a^2*c*d*e^5*x + 5*c^3*d^6 + 3*a*c^2*d^4*e^2 + 5*a^2*c*d^2*
e^4 + 35*a^3*e^6)/((e*x + d)^8*e^7)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.46

$$\int \frac{(a+cx^2)^3}{(d+ex)^9}\,dx =$$

$$-\frac{\frac{35\,a^3\,e^6+5\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+5\,c^3\,d^6}{280\,e^7}+\frac{c^3\,x^6}{2\,e}+\frac{c^3\,d\,x^5}{e^2}+\frac{c^2\,x^4\,\left(5\,c\,d^2+3\,a\,e^2\right)}{4\,e^3}+\frac{c\,x^2\,\left(5\,a^2\,e^4+3\,a\,c\,d^2\,e^2+5\,c^2\,d^4\right)}{10\,e^5}+\frac{c\,d\,x\,(5\,a^2}{}}{d^8+8\,d^7\,e\,x+28\,d^6\,e^2\,x^2+56\,d^5\,e^3\,x^3+70\,d^4\,e^4\,x^4+56\,d^3\,e^5\,x^5+28\,d^2\,e^6\,x^6+}$$

input
```
int((a + c*x^2)^3/(d + e*x)^9,x)
```

output
```
-((35*a^3*e^6 + 5*c^3*d^6 + 3*a*c^2*d^4*e^2 + 5*a^2*c*d^2*e^4)/(280*e^7) +
 (c^3*x^6)/(2*e) + (c^3*d*x^5)/e^2 + (c^2*x^4*(3*a*e^2 + 5*c*d^2))/(4*e^3)
 + (c*x^2*(5*a^2*e^4 + 5*c^2*d^4 + 3*a*c*d^2*e^2))/(10*e^5) + (c*d*x*(5*a^
2*e^4 + 5*c^2*d^4 + 3*a*c*d^2*e^2))/(35*e^6) + (c^2*d*x^3*(3*a*e^2 + 5*c*d
^2))/(5*e^4))/(d^8 + e^8*x^8 + 8*d*e^7*x^7 + 28*d^6*e^2*x^2 + 56*d^5*e^3*x
^3 + 70*d^4*e^4*x^4 + 56*d^3*e^5*x^5 + 28*d^2*e^6*x^6 + 8*d^7*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.49

$$\int \frac{(a+cx^2)^3}{(d+ex)^9}\,dx$$

$$= \frac{-140c^3e^6x^6-280c^3d\,e^5x^5-210a\,c^2e^6x^4-350c^3d^2e^4x^4-168a\,c^2d\,e^5x^3-280c^3d^3e^3x^3-140a^2c\,e^6x^2-}{280e^7\left(e^8x^8+8d\,e^7x^7+28d^2e^6x^6+56d^3e^5x^5\right.}$$

input
```
int((c*x^2+a)^3/(e*x+d)^9,x)
```

output
```
( - 35*a**3*e**6 - 5*a**2*c*d**2*e**4 - 40*a**2*c*d*e**5*x - 140*a**2*c*e*
*6*x**2 - 3*a*c**2*d**4*e**2 - 24*a*c**2*d**3*e**3*x - 84*a*c**2*d**2*e**4
*x**2 - 168*a*c**2*d*e**5*x**3 - 210*a*c**2*e**6*x**4 - 5*c**3*d**6 - 40*c
**3*d**5*e*x - 140*c**3*d**4*e**2*x**2 - 280*c**3*d**3*e**3*x**3 - 350*c**
3*d**2*e**4*x**4 - 280*c**3*d*e**5*x**5 - 140*c**3*e**6*x**6)/(280*e**7*(d
**8 + 8*d**7*e*x + 28*d**6*e**2*x**2 + 56*d**5*e**3*x**3 + 70*d**4*e**4*x*
*4 + 56*d**3*e**5*x**5 + 28*d**2*e**6*x**6 + 8*d*e**7*x**7 + e**8*x**8))
```

3.88 $\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 190

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{(cd^2+ae^2)^3}{9e^7(d+ex)^9}+\frac{3cd(cd^2+ae^2)^2}{4e^7(d+ex)^8}-\frac{3c(cd^2+ae^2)(5cd^2+ae^2)}{7e^7(d+ex)^7}+\frac{2c^2d(5cd^2+3ae^2)}{3e^7(d+ex)^6}-\frac{3c^2(5cd^2+ae^2)}{5e^7(d+ex)^5}+\frac{3c^3d}{2e^7(d+ex)^4}-\frac{c^3}{3e^7(d+ex)^3}$$

output
```
-1/9*(a*e^2+c*d^2)^3/e^7/(e*x+d)^9+3/4*c*d*(a*e^2+c*d^2)^2/e^7/(e*x+d)^8-3
/7*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)/e^7/(e*x+d)^7+2/3*c^2*d*(3*a*e^2+5*c*d^
2)/e^7/(e*x+d)^6-3/5*c^2*(a*e^2+5*c*d^2)/e^7/(e*x+d)^5+3/2*c^3*d/e^7/(e*x+
d)^4-1/3*c^3/e^7/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{140a^3e^6+15a^2ce^4(d^2+9dex+36e^2x^2)+6ac^2e^2(d^4+9d^3ex+36d^2e^2x^2+84de^3x^3+126e^4x^4)+5c^3}{1260e^7(d+ex)^9}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^10,x]
```

output
```
-1/1260*(140*a^3*e^6 + 15*a^2*c*e^4*(d^2 + 9*d*e*x + 36*e^2*x^2) + 6*a*c^2
*e^2*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4) + 5*c
^3*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x^3 + 126*d^2*e^4*x^4 +
126*d*e^5*x^5 + 84*e^6*x^6))/(e^7*(d + e*x)^9)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(ae^2+5cd^2)}{e^6(d+ex)^6}-\frac{4c^2d(3ae^2+5cd^2)}{e^6(d+ex)^7}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^8}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^9}+\frac{(ae^2+cd^2)^3}{e^6(d+ex)^{10}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c^2(ae^2+5cd^2)}{5e^7(d+ex)^5}+\frac{2c^2d(3ae^2+5cd^2)}{3e^7(d+ex)^6}-\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{7e^7(d+ex)^7}+\frac{3cd(ae^2+cd^2)^2}{4e^7(d+ex)^8}-\frac{(ae^2+cd^2)^3}{9e^7(d+ex)^9}-\frac{c^3}{3e^7(d+ex)^3}+\frac{3c^3d}{2e^7(d+ex)^4}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^10,x]
```

output
```
-1/9*(c*d^2 + a*e^2)^3/(e^7*(d + e*x)^9) + (3*c*d*(c*d^2 + a*e^2)^2)/(4*e^
7*(d + e*x)^8) - (3*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(7*e^7*(d + e*x)^
7) + (2*c^2*d*(5*c*d^2 + 3*a*e^2))/(3*e^7*(d + e*x)^6) - (3*c^2*(5*c*d^2 +
 a*e^2))/(5*e^7*(d + e*x)^5) + (3*c^3*d)/(2*e^7*(d + e*x)^4) - c^3/(3*e^7*
(d + e*x)^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.05

method	result
risch	$\frac{-\frac{c^3x^6}{3e}-\frac{c^3d\,x^5}{2e^2}-\frac{c^2\left(6a\,e^2+5c\,d^2\right)x^4}{10e^3}-\frac{c^2d\left(6a\,e^2+5c\,d^2\right)x^3}{15e^4}-\frac{c\left(15a^2e^4+6ac\,d^2e^2+5c^2d^4\right)x^2}{35e^5}-\frac{dc\left(15a^2e^4+6ac\,d^2e^2+5c^2d^4\right)x}{140e^6}-\frac{140e}{}}{(ex+d)^9}$
gosper	$-\frac{420x^6c^3e^6+630d\,c^3x^5e^5+756x^4a\,c^2e^6+630x^4c^3d^2e^4+504x^3a\,c^2d\,e^5+420x^3c^3d^3e^3+540x^2a^2c\,e^6+216x^2a\,c^2d^2e^4+180x^2}{1260e^7(ex+d)^9}$
orering	$-\frac{420x^6c^3e^6+630d\,c^3x^5e^5+756x^4a\,c^2e^6+630x^4c^3d^2e^4+504x^3a\,c^2d\,e^5+420x^3c^3d^3e^3+540x^2a^2c\,e^6+216x^2a\,c^2d^2e^4+180x^2}{1260e^7(ex+d)^9}$
parallelrisch	$\frac{-420c^3x^6e^8-630c^3d\,x^5e^7-756a\,c^2e^8x^4-630c^3d^2e^6x^4-504a\,c^2d\,e^7x^3-420c^3d^3e^5x^3-540a^2c\,e^8x^2-216a\,c^2d^2e^6x^2-180c^3d}{1260e^9(ex+d)^9}$
default	$-\frac{c^3}{3e^7(ex+d)^3}+\frac{3c^3d}{2e^7(ex+d)^4}-\frac{e^6a^3+3d^2e^4a^2c+3d^4e^2a\,c^2+d^6c^3}{9e^7(ex+d)^9}-\frac{3c\left(a^2e^4+6ac\,d^2e^2+5c^2d^4\right)}{7e^7(ex+d)^7}-\frac{3c^2\left(a\,e^2+5c\,d^2\right)}{5e^7(ex+d)^5}-$
norman	$\frac{-\frac{c^3x^6}{3e}-\frac{c^3d\,x^5}{2e^2}-\frac{\left(6e^4a\,c^2+5d^2e^2c^3\right)x^4}{10e^5}-\frac{d\left(6e^4a\,c^2+5d^2e^2c^3\right)x^3}{15e^6}-\frac{\left(15e^6a^2c+6d^2e^4a\,c^2+5d^4e^2c^3\right)x^2}{35e^7}-\frac{d\left(15e^6a^2c+6d^2e^4a\,c^2+5d^4e\right.}{140e^8}}{(ex+d)^9}$

input
```
int((c*x^2+a)^3/(e*x+d)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*c^3*x^6/e-1/2*c^3*d*x^5/e^2-1/10*c^2*(6*a*e^2+5*c*d^2)/e^3*x^4-1/15*
c^2*d*(6*a*e^2+5*c*d^2)/e^4*x^3-1/35*c/e^5*(15*a^2*e^4+6*a*c*d^2*e^2+5*c^2
*d^4)*x^2-1/140*d*c/e^6*(15*a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)*x-1/1260/e^7*
(140*a^3*e^6+15*a^2*c*d^2*e^4+6*a*c^2*d^4*e^2+5*c^3*d^6))/(e*x+d)^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+cx^2\right)^3}{(d+ex)^{10}}\,dx = -\frac{420\,c^3e^6x^6+630\,c^3de^5x^5+5\,c^3d^6+6\,ac^2d^4e^2+15\,a^2cd^2e^4+140\,a^3e^6+126\,(5\,c^3d^2e^4+6\,ac^2e^6)x^4+}{1260\,(e^{16}x^9+9\,de^{15}x^8+36\,d^2e^{14}x^7+84\,d^3e^{13}x^6+126\,d^4e}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^10,x, algorithm="fricas")
```

output
```
-1/1260*(420*c^3*e^6*x^6 + 630*c^3*d*e^5*x^5 + 5*c^3*d^6 + 6*a*c^2*d^4*e^2
 + 15*a^2*c*d^2*e^4 + 140*a^3*e^6 + 126*(5*c^3*d^2*e^4 + 6*a*c^2*e^6)*x^4
+ 84*(5*c^3*d^3*e^3 + 6*a*c^2*d*e^5)*x^3 + 36*(5*c^3*d^4*e^2 + 6*a*c^2*d^2
*e^4 + 15*a^2*c*e^6)*x^2 + 9*(5*c^3*d^5*e + 6*a*c^2*d^3*e^3 + 15*a^2*c*d*e
^5)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*
d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8
*e^8*x + d^9*e^7)
```

Sympy [A] (verification not implemented)

Time = 23.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.65

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx = \frac{-140a^3e^6 - 15a^2cd^2e^4 - 6ac^2d^4e^2 - 5c^3d^6 - 630c^3de^5x^5 - 420c^3e^6x^6 + x^4(-756ac^2e^6 - 630c^3d^2e^4) + x}{1260d^9e^7 + 11340d^8e^8x + 45360d^7e^9x^2 + 105840d^6e^{10}x^3 + 158760d^5e^{11}}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**10,x)
```

output
```
(-140*a**3*e**6 - 15*a**2*c*d**2*e**4 - 6*a*c**2*d**4*e**2 - 5*c**3*d**6 -
 630*c**3*d*e**5*x**5 - 420*c**3*e**6*x**6 + x**4*(-756*a*c**2*e**6 - 630*
c**3*d**2*e**4) + x**3*(-504*a*c**2*d*e**5 - 420*c**3*d**3*e**3) + x**2*(-
540*a**2*c*e**6 - 216*a*c**2*d**2*e**4 - 180*c**3*d**4*e**2) + x*(-135*a**
2*c*d*e**5 - 54*a*c**2*d**3*e**3 - 45*c**3*d**5*e))/(1260*d**9*e**7 + 1134
0*d**8*e**8*x + 45360*d**7*e**9*x**2 + 105840*d**6*e**10*x**3 + 158760*d**
5*e**11*x**4 + 158760*d**4*e**12*x**5 + 105840*d**3*e**13*x**6 + 45360*d**
2*e**14*x**7 + 11340*d*e**15*x**8 + 1260*e**16*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.54

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{420\,c^3e^6x^6 + 630\,c^3de^5x^5 + 5\,c^3d^6 + 6\,ac^2d^4e^2 + 15\,a^2cd^2e^4 + 140\,a^3e^6 + 126\,(5\,c^3d^2e^4 + 6\,ac^2e^6)x^4 + 8}{1260\,(e^{16}x^9 + 9\,de^{15}x^8 + 36\,d^2e^{14}x^7 + 84\,d^3e^{13}x^6 + 126\,d^4e}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^10,x, algorithm="maxima")
```

output
```
-1/1260*(420*c^3*e^6*x^6 + 630*c^3*d*e^5*x^5 + 5*c^3*d^6 + 6*a*c^2*d^4*e^2
 + 15*a^2*c*d^2*e^4 + 140*a^3*e^6 + 126*(5*c^3*d^2*e^4 + 6*a*c^2*e^6)*x^4
+ 84*(5*c^3*d^3*e^3 + 6*a*c^2*d*e^5)*x^3 + 36*(5*c^3*d^4*e^2 + 6*a*c^2*d^2
*e^4 + 15*a^2*c*e^6)*x^2 + 9*(5*c^3*d^5*e + 6*a*c^2*d^3*e^3 + 15*a^2*c*d*e
^5)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*
d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8
*e^8*x + d^9*e^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{420\,c^3e^6x^6+630\,c^3de^5x^5+630\,c^3d^2e^4x^4+756\,ac^2e^6x^4+420\,c^3d^3e^3x^3+504\,ac^2de^5x^3+180\,c^3d^4e^2x^2}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^10,x, algorithm="giac")
```

output
```
-1/1260*(420*c^3*e^6*x^6 + 630*c^3*d*e^5*x^5 + 630*c^3*d^2*e^4*x^4 + 756*a
*c^2*e^6*x^4 + 420*c^3*d^3*e^3*x^3 + 504*a*c^2*d*e^5*x^3 + 180*c^3*d^4*e^2
*x^2 + 216*a*c^2*d^2*e^4*x^2 + 540*a^2*c*e^6*x^2 + 45*c^3*d^5*e*x + 54*a*c
^2*d^3*e^3*x + 135*a^2*c*d*e^5*x + 5*c^3*d^6 + 6*a*c^2*d^4*e^2 + 15*a^2*c*
d^2*e^4 + 140*a^3*e^6)/((e*x + d)^9*e^7)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.51

$$\int \frac{(a+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{\frac{140\,a^3\,e^6+15\,a^2\,c\,d^2\,e^4+6\,a\,c^2\,d^4\,e^2+5\,c^3\,d^6}{1260\,e^7}+\frac{c^3\,x^6}{3\,e}+\frac{c^3\,d\,x^5}{2\,e^2}+\frac{c^2\,x^4\,\left(5\,c\,d^2+6\,a\,e^2\right)}{10\,e^3}+\frac{c\,x^2\,\left(15\,a^2\,e^4+6\,a\,c\,d^2\,e^2+5\,c^2\,d^4\right)}{35\,e^5}+\frac{c\,d\,x\,(}{}}{d^9+9\,d^8\,e\,x+36\,d^7\,e^2\,x^2+84\,d^6\,e^3\,x^3+126\,d^5\,e^4\,x^4+126\,d^4\,e^5\,x^5+84\,d^3\,e^6\,x^6+36\,d}$$

input
```
int((a + c*x^2)^3/(d + e*x)^10,x)
```

output
```
-((140*a^3*e^6 + 5*c^3*d^6 + 6*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4)/(1260*e^7
) + (c^3*x^6)/(3*e) + (c^3*d*x^5)/(2*e^2) + (c^2*x^4*(6*a*e^2 + 5*c*d^2))/
(10*e^3) + (c*x^2*(15*a^2*e^4 + 5*c^2*d^4 + 6*a*c*d^2*e^2))/(35*e^5) + (c*
d*x*(15*a^2*e^4 + 5*c^2*d^4 + 6*a*c*d^2*e^2))/(140*e^6) + (c^2*d*x^3*(6*a*
e^2 + 5*c*d^2))/(15*e^4))/(d^9 + e^9*x^9 + 9*d*e^8*x^8 + 36*d^7*e^2*x^2 +
84*d^6*e^3*x^3 + 126*d^5*e^4*x^4 + 126*d^4*e^5*x^5 + 84*d^3*e^6*x^6 + 36*d
^2*e^7*x^7 + 9*d^8*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+cx^2\right)^3}{(d+ex)^{10}}\,dx$$
$$= \frac{-420c^3e^6x^6-630c^3d\,e^5x^5-756a\,c^2e^6x^4-630c^3d^2e^4x^4-504a\,c^2d\,e^5x^3-420c^3d^3e^3x^3-540a^2c\,e^6x^2-}{1260e^7\left(e^9x^9+9d\,e^8x^8+36d^2e^7x^7+84d^3e^6x^6+1\right.}$$

input
```
int((c*x^2+a)^3/(e*x+d)^10,x)
```

output
```
( - 140*a**3*e**6 - 15*a**2*c*d**2*e**4 - 135*a**2*c*d*e**5*x - 540*a**2*c
*e**6*x**2 - 6*a*c**2*d**4*e**2 - 54*a*c**2*d**3*e**3*x - 216*a*c**2*d**2*
e**4*x**2 - 504*a*c**2*d*e**5*x**3 - 756*a*c**2*e**6*x**4 - 5*c**3*d**6 -
45*c**3*d**5*e*x - 180*c**3*d**4*e**2*x**2 - 420*c**3*d**3*e**3*x**3 - 630
*c**3*d**2*e**4*x**4 - 630*c**3*d*e**5*x**5 - 420*c**3*e**6*x**6)/(1260*e*
*7*(d**9 + 9*d**8*e*x + 36*d**7*e**2*x**2 + 84*d**6*e**3*x**3 + 126*d**5*e
**4*x**4 + 126*d**4*e**5*x**5 + 84*d**3*e**6*x**6 + 36*d**2*e**7*x**7 + 9*
d*e**8*x**8 + e**9*x**9))
```

3.89 $\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 278

$$\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx = \frac{(cd^2+ae^2)^4 (d+ex)^8}{8e^9} - \frac{8cd(cd^2+ae^2)^3 (d+ex)^9}{9e^9} + \frac{2c(cd^2+ae^2)^2 (7cd^2+ae^2)(d+ex)^{10}}{5e^9} - \frac{8c^2d(cd^2+ae^2)(7cd^2+3ae^2)(d+ex)^{11}}{11e^9} + \frac{c^2(35c^2d^4+30acd^2e^2+3a^2e^4)(d+ex)^{12}}{6e^9} - \frac{8c^3d(7cd^2+3ae^2)(d+ex)^{13}}{13e^9} + \frac{2c^3(7cd^2+ae^2)(d+ex)^{14}}{7e^9} - \frac{8c^4d(d+ex)^{15}}{15e^9} + \frac{c^4(d+ex)^{16}}{16e^9}$$

output
```
1/8*(a*e^2+c*d^2)^4*(e*x+d)^8/e^9-8/9*c*d*(a*e^2+c*d^2)^3*(e*x+d)^9/e^9+2/
5*c*(a*e^2+c*d^2)^2*(a*e^2+7*c*d^2)*(e*x+d)^10/e^9-8/11*c^2*d*(a*e^2+c*d^2
)*(3*a*e^2+7*c*d^2)*(e*x+d)^11/e^9+1/6*c^2*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^
2*d^4)*(e*x+d)^12/e^9-8/13*c^3*d*(3*a*e^2+7*c*d^2)*(e*x+d)^13/e^9+2/7*c^3*
(a*e^2+7*c*d^2)*(e*x+d)^14/e^9-8/15*c^4*d*(e*x+d)^15/e^9+1/16*c^4*(e*x+d)^
16/e^9
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.52

$$\int (d+ex)^7\left(a+cx^2\right)^4\,dx$$
$$=\frac{1}{8}a^4x\left(8d^7+28d^6ex+56d^5e^2x^2+70d^4e^3x^3+56d^3e^4x^4+28d^2e^5x^5+8de^6x^6+e^7x^7\right)$$
$$+\frac{1}{90}a^3cx^3\left(120d^7+630d^6ex+1512d^5e^2x^2+2100d^4e^3x^3+1800d^3e^4x^4+945d^2e^5x^5\right.$$
$$\left.+280de^6x^6+36e^7x^7\right)+\frac{1}{660}a^2c^2x^5\left(792d^7+4620d^6ex+11880d^5e^2x^2+17325d^4e^3x^3\right.$$
$$\left.+15400d^3e^4x^4+8316d^2e^5x^5+2520de^6x^6+330e^7x^7\right)$$
$$+\frac{ac^3x^7(3432d^7+21021d^6ex+56056d^5e^2x^2+84084d^4e^3x^3+76440d^3e^4x^4+42042d^2e^5x^5+12936de^6x}{6006}$$
$$+\frac{c^4x^9(11440d^7+72072d^6ex+196560d^5e^2x^2+300300d^4e^3x^3+277200d^3e^4x^4+154440d^2e^5x^5+48048}{102960}$$

input
```
Integrate[(d + e*x)^7*(a + c*x^2)^4,x]
```

output
```
(a^4*x*(8*d^7 + 28*d^6*e*x + 56*d^5*e^2*x^2 + 70*d^4*e^3*x^3 + 56*d^3*e^4*
x^4 + 28*d^2*e^5*x^5 + 8*d*e^6*x^6 + e^7*x^7))/8 + (a^3*c*x^3*(120*d^7 + 6
30*d^6*e*x + 1512*d^5*e^2*x^2 + 2100*d^4*e^3*x^3 + 1800*d^3*e^4*x^4 + 945*
d^2*e^5*x^5 + 280*d*e^6*x^6 + 36*e^7*x^7))/90 + (a^2*c^2*x^5*(792*d^7 + 46
20*d^6*e*x + 11880*d^5*e^2*x^2 + 17325*d^4*e^3*x^3 + 15400*d^3*e^4*x^4 + 8
316*d^2*e^5*x^5 + 2520*d*e^6*x^6 + 330*e^7*x^7))/660 + (a*c^3*x^7*(3432*d^
7 + 21021*d^6*e*x + 56056*d^5*e^2*x^2 + 84084*d^4*e^3*x^3 + 76440*d^3*e^4*
x^4 + 42042*d^2*e^5*x^5 + 12936*d*e^6*x^6 + 1716*e^7*x^7))/6006 + (c^4*x^9
*(11440*d^7 + 72072*d^6*e*x + 196560*d^5*e^2*x^2 + 300300*d^4*e^3*x^3 + 27
7200*d^3*e^4*x^4 + 154440*d^2*e^5*x^5 + 48048*d*e^6*x^6 + 6435*e^7*x^7))/1
02960
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^4 (d+ex)^7 \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c^2(d+ex)^{11}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{e^8}+\frac{4c^3(d+ex)^{13}\left(ae^2+7cd^2\right)}{e^8}-\frac{8c^3d(d+ex)^{12}\left(3ae^2+7cd^2\right)}{e^8}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2(d+ex)^{12}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{6e^9}+\frac{2c^3(d+ex)^{14}\left(ae^2+7cd^2\right)}{7e^9}-\frac{8c^3d(d+ex)^{13}\left(3ae^2+7cd^2\right)}{13e^9}-\frac{8c^2d(d+ex)^{11}\left(ae^2+cd^2\right)\left(3ae^2+7cd^2\right)}{11e^9}+\frac{2c(d+ex)^{10}\left(ae^2+cd^2\right)^2\left(ae^2+7cd^2\right)}{5e^9}-\frac{8cd(d+ex)^9\left(ae^2+cd^2\right)^3}{9e^9}+\frac{(d+ex)^8\left(ae^2+cd^2\right)^4}{8e^9}+\frac{c^4(d+ex)^{16}}{16e^9}-\frac{8c^4d(d+ex)^{15}}{15e^9}$$

input
```
Int[(d + e*x)^7*(a + c*x^2)^4,x]
```

output
```
((c*d^2 + a*e^2)^4*(d + e*x)^8)/(8*e^9) - (8*c*d*(c*d^2 + a*e^2)^3*(d + e*
x)^9)/(9*e^9) + (2*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^10)/(5*
e^9) - (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^11)/(11*e^9)
 + (c^2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^12)/(6*e^9) -
(8*c^3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^13)/(13*e^9) + (2*c^3*(7*c*d^2 + a*
e^2)*(d + e*x)^14)/(7*e^9) - (8*c^4*d*(d + e*x)^15)/(15*e^9) + (c^4*(d + e
*x)^16)/(16*e^9)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.80

method	result
norman	$d^7a^4x+\frac{7d^6e\,a^4x^2}{2}+\left(7d^5e^2a^4+\frac{4}{3}d^7c\,a^3\right)x^3+\left(\frac{35}{4}d^4e^3a^4+7d^6ec\,a^3\right)x^4+\left(7d^3e^4a^4+\frac{84}{5}d^5e^2\right.$
default	$\frac{e^7c^4x^{16}}{16}+\frac{7d\,e^6c^4x^{15}}{15}+\frac{\left(4e^7a\,c^3+21d^2e^5c^4\right)x^{14}}{14}+\frac{\left(28d\,e^6a\,c^3+35d^3e^4c^4\right)x^{13}}{13}+\frac{\left(6e^7a^2c^2+84d^2e^5a\,c^3+35d^4e^3c^4\right)x^{1}}{12}$
gosper	$\frac{28}{13}x^{13}d\,e^6a\,c^3+\frac{2}{5}x^{10}e^7c\,a^3+\frac{7}{10}x^{10}d^6e\,c^4+\frac{21}{11}x^{11}d^5e^2c^4+\frac{1}{2}x^{12}e^7a^2c^2+\frac{35}{12}x^{12}d^4e^3c^4+\frac{35}{13}x^{1}$
risch	$\frac{28}{13}x^{13}d\,e^6a\,c^3+\frac{2}{5}x^{10}e^7c\,a^3+\frac{7}{10}x^{10}d^6e\,c^4+\frac{21}{11}x^{11}d^5e^2c^4+\frac{1}{2}x^{12}e^7a^2c^2+\frac{35}{12}x^{12}d^4e^3c^4+\frac{35}{13}x^{1}$
parallelrisch	$\frac{28}{13}x^{13}d\,e^6a\,c^3+\frac{2}{5}x^{10}e^7c\,a^3+\frac{7}{10}x^{10}d^6e\,c^4+\frac{21}{11}x^{11}d^5e^2c^4+\frac{1}{2}x^{12}e^7a^2c^2+\frac{35}{12}x^{12}d^4e^3c^4+\frac{35}{13}x^{1}$
orering	$x\left(45045e^7c^4x^{15}+336336d\,e^6c^4x^{14}+205920a\,c^3e^7x^{13}+1081080c^4d^2e^5x^{13}+1552320a\,c^3d\,e^6x^{12}+1940400c^4d^3e^4x^{12}+360360\right.$

input
```
int((e*x+d)^7*(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
d^7*a^4*x+7/2*d^6*e*a^4*x^2+(7*d^5*e^2*a^4+4/3*d^7*c*a^3)*x^3+(35/4*d^4*e^
3*a^4+7*d^6*e*c*a^3)*x^4+(7*d^3*e^4*a^4+84/5*d^5*e^2*c*a^3+6/5*d^7*a^2*c^2
)*x^5+(7/2*d^2*e^5*a^4+70/3*d^4*e^3*c*a^3+7*d^6*e*a^2*c^2)*x^6+(d*e^6*a^4+
20*d^3*e^4*c*a^3+18*d^5*e^2*a^2*c^2+4/7*d^7*a*c^3)*x^7+(1/8*e^7*a^4+21/2*d
^2*e^5*c*a^3+105/4*d^4*e^3*a^2*c^2+7/2*d^6*e*a*c^3)*x^8+(28/9*d*e^6*c*a^3+
70/3*d^3*e^4*a^2*c^2+28/3*d^5*e^2*a*c^3+1/9*d^7*c^4)*x^9+(2/5*e^7*c*a^3+63
/5*d^2*e^5*a^2*c^2+14*d^4*e^3*a*c^3+7/10*d^6*e*c^4)*x^10+(42/11*d*e^6*a^2*
c^2+140/11*d^3*e^4*a*c^3+21/11*d^5*e^2*c^4)*x^11+(1/2*e^7*a^2*c^2+7*d^2*e^
5*a*c^3+35/12*d^4*e^3*c^4)*x^12+(28/13*d*e^6*a*c^3+35/13*d^3*e^4*c^4)*x^13
+(2/7*e^7*a*c^3+3/2*d^2*e^5*c^4)*x^14+7/15*d*e^6*c^4*x^15+1/16*e^7*c^4*x^1
6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.83

$$
\begin{aligned}
\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx = {} & \frac{1}{16}\,c^4e^7x^{16} + \frac{7}{15}\,c^4de^6x^{15} + \frac{1}{14}\left(21\,c^4d^2e^5 + 4\,ac^3e^7\right)x^{14} \\
& + \frac{7}{13}\left(5\,c^4d^3e^4 + 4\,ac^3de^6\right)x^{13} + \frac{7}{2}\,a^4d^6ex^2 \\
& + \frac{1}{12}\left(35\,c^4d^4e^3 + 84\,ac^3d^2e^5 + 6\,a^2c^2e^7\right)x^{12} \\
& + a^4d^7x + \frac{7}{11}\left(3\,c^4d^5e^2 + 20\,ac^3d^3e^4 + 6\,a^2c^2de^6\right)x^{11} \\
& + \frac{1}{10}\left(7\,c^4d^6e + 140\,ac^3d^4e^3 + 126\,a^2c^2d^2e^5 + 4\,a^3ce^7\right)x^{10} \\
& + \frac{1}{9}\left(c^4d^7 + 84\,ac^3d^5e^2 + 210\,a^2c^2d^3e^4 + 28\,a^3cde^6\right)x^9 \\
& + \frac{1}{8}\left(28\,ac^3d^6e + 210\,a^2c^2d^4e^3 + 84\,a^3cd^2e^5 + a^4e^7\right)x^8 \\
& + \frac{1}{7}\left(4\,ac^3d^7 + 126\,a^2c^2d^5e^2 + 140\,a^3cd^3e^4 + 7\,a^4de^6\right)x^7 \\
& + \frac{7}{6}\left(6\,a^2c^2d^6e + 20\,a^3cd^4e^3 + 3\,a^4d^2e^5\right)x^6 \\
& + \frac{1}{5}\left(6\,a^2c^2d^7 + 84\,a^3cd^5e^2 + 35\,a^4d^3e^4\right)x^5 \\
& + \frac{7}{4}\left(4\,a^3cd^6e + 5\,a^4d^4e^3\right)x^4 + \frac{1}{3}\left(4\,a^3cd^7 + 21\,a^4d^5e^2\right)x^3
\end{aligned}
$$

input
```
integrate((e*x+d)^7*(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/16*c^4*e^7*x^16 + 7/15*c^4*d*e^6*x^15 + 1/14*(21*c^4*d^2*e^5 + 4*a*c^3*e
^7)*x^14 + 7/13*(5*c^4*d^3*e^4 + 4*a*c^3*d*e^6)*x^13 + 7/2*a^4*d^6*e*x^2 +
 1/12*(35*c^4*d^4*e^3 + 84*a*c^3*d^2*e^5 + 6*a^2*c^2*e^7)*x^12 + a^4*d^7*x
 + 7/11*(3*c^4*d^5*e^2 + 20*a*c^3*d^3*e^4 + 6*a^2*c^2*d*e^6)*x^11 + 1/10*(
7*c^4*d^6*e + 140*a*c^3*d^4*e^3 + 126*a^2*c^2*d^2*e^5 + 4*a^3*c*e^7)*x^10
+ 1/9*(c^4*d^7 + 84*a*c^3*d^5*e^2 + 210*a^2*c^2*d^3*e^4 + 28*a^3*c*d*e^6)*
x^9 + 1/8*(28*a*c^3*d^6*e + 210*a^2*c^2*d^4*e^3 + 84*a^3*c*d^2*e^5 + a^4*e
^7)*x^8 + 1/7*(4*a*c^3*d^7 + 126*a^2*c^2*d^5*e^2 + 140*a^3*c*d^3*e^4 + 7*a
^4*d*e^6)*x^7 + 7/6*(6*a^2*c^2*d^6*e + 20*a^3*c*d^4*e^3 + 3*a^4*d^2*e^5)*x
^6 + 1/5*(6*a^2*c^2*d^7 + 84*a^3*c*d^5*e^2 + 35*a^4*d^3*e^4)*x^5 + 7/4*(4*
a^3*c*d^6*e + 5*a^4*d^4*e^3)*x^4 + 1/3*(4*a^3*c*d^7 + 21*a^4*d^5*e^2)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(272) = 544.

Time = 0.05 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.05

$$\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx = a^4d^7x + \frac{7a^4d^6ex^2}{2} + \frac{7c^4de^6x^{15}}{15} + \frac{c^4e^7x^{16}}{16} + x^{14} \cdot \left(\frac{2ac^3e^7}{7} + \frac{3c^4d^2e^5}{2}\right) + x^{13} \cdot \left(\frac{28ac^3de^6}{13} + \frac{35c^4d^3e^4}{13}\right) + x^{12}\left(\frac{a^2c^2e^7}{2} + 7ac^3d^2e^5 + \frac{35c^4d^4e^3}{12}\right) + x^{11} \cdot \left(\frac{42a^2c^2de^6}{11} + \frac{140ac^3d^3e^4}{11} + \frac{21c^4d^5e^2}{11}\right) + x^{10} \cdot \left(\frac{2a^3ce^7}{5} + \frac{63a^2c^2d^2e^5}{5} + 14ac^3d^4e^3 + \frac{7c^4d^6e}{10}\right) + x^9 \cdot \left(\frac{28a^3cde^6}{9} + \frac{70a^2c^2d^3e^4}{3} + \frac{28ac^3d^5e^2}{3} + \frac{c^4d^7}{9}\right) + x^8\left(\frac{a^4e^7}{8} + \frac{21a^3cd^2e^5}{2} + \frac{105a^2c^2d^4e^3}{4} + \frac{7ac^3d^6e}{2}\right) + x^7\left(a^4de^6 + 20a^3cd^3e^4 + 18a^2c^2d^5e^2 + \frac{4ac^3d^7}{7}\right) + x^6 \cdot \left(\frac{7a^4d^2e^5}{2} + \frac{70a^3cd^4e^3}{3} + 7a^2c^2d^6e\right) + x^5 \cdot \left(7a^4d^3e^4 + \frac{84a^3cd^5e^2}{5} + \frac{6a^2c^2d^7}{5}\right) + x^4 \cdot \left(\frac{35a^4d^4e^3}{4} + 7a^3cd^6e\right) + x^3 \cdot \left(7a^4d^5e^2 + \frac{4a^3cd^7}{3}\right)$$

input

```
integrate((e*x+d)**7*(c*x**2+a)**4,x)
```

output
```
a**4*d**7*x + 7*a**4*d**6*e*x**2/2 + 7*c**4*d*e**6*x**15/15 + c**4*e**7*x*
*16/16 + x**14*(2*a*c**3*e**7/7 + 3*c**4*d**2*e**5/2) + x**13*(28*a*c**3*d
*e**6/13 + 35*c**4*d**3*e**4/13) + x**12*(a**2*c**2*e**7/2 + 7*a*c**3*d**2
*e**5 + 35*c**4*d**4*e**3/12) + x**11*(42*a**2*c**2*d*e**6/11 + 140*a*c**3
*d**3*e**4/11 + 21*c**4*d**5*e**2/11) + x**10*(2*a**3*c*e**7/5 + 63*a**2*c
**2*d**2*e**5/5 + 14*a*c**3*d**4*e**3 + 7*c**4*d**6*e/10) + x**9*(28*a**3*
c*d*e**6/9 + 70*a**2*c**2*d**3*e**4/3 + 28*a*c**3*d**5*e**2/3 + c**4*d**7/
9) + x**8*(a**4*e**7/8 + 21*a**3*c*d**2*e**5/2 + 105*a**2*c**2*d**4*e**3/4
 + 7*a*c**3*d**6*e/2) + x**7*(a**4*d*e**6 + 20*a**3*c*d**3*e**4 + 18*a**2*
c**2*d**5*e**2 + 4*a*c**3*d**7/7) + x**6*(7*a**4*d**2*e**5/2 + 70*a**3*c*d
**4*e**3/3 + 7*a**2*c**2*d**6*e) + x**5*(7*a**4*d**3*e**4 + 84*a**3*c*d**5
*e**2/5 + 6*a**2*c**2*d**7/5) + x**4*(35*a**4*d**4*e**3/4 + 7*a**3*c*d**6*
e) + x**3*(7*a**4*d**5*e**2 + 4*a**3*c*d**7/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.83

$$\begin{aligned}\int (d+ex)^7\left(a+cx^2\right)^4\,dx &= \frac{1}{16}\,c^4e^7x^{16}+\frac{7}{15}\,c^4de^6x^{15}+\frac{1}{14}\,\left(21\,c^4d^2e^5+4\,ac^3e^7\right)x^{14}\\&\quad+\frac{7}{13}\,\left(5\,c^4d^3e^4+4\,ac^3de^6\right)x^{13}+\frac{7}{2}\,a^4d^6ex^2\\&\quad+\frac{1}{12}\,\left(35\,c^4d^4e^3+84\,ac^3d^2e^5+6\,a^2c^2e^7\right)x^{12}\\&\quad+a^4d^7x+\frac{7}{11}\,\left(3\,c^4d^5e^2+20\,ac^3d^3e^4+6\,a^2c^2de^6\right)x^{11}\\&\quad+\frac{1}{10}\,\left(7\,c^4d^6e+140\,ac^3d^4e^3+126\,a^2c^2d^2e^5+4\,a^3ce^7\right)x^{10}\\&\quad+\frac{1}{9}\,\left(c^4d^7+84\,ac^3d^5e^2+210\,a^2c^2d^3e^4+28\,a^3cde^6\right)x^9\\&\quad+\frac{1}{8}\,\left(28\,ac^3d^6e+210\,a^2c^2d^4e^3+84\,a^3cd^2e^5+a^4e^7\right)x^8\\&\quad+\frac{1}{7}\,\left(4\,ac^3d^7+126\,a^2c^2d^5e^2+140\,a^3cd^3e^4+7\,a^4de^6\right)x^7\\&\quad+\frac{7}{6}\,\left(6\,a^2c^2d^6e+20\,a^3cd^4e^3+3\,a^4d^2e^5\right)x^6\\&\quad+\frac{1}{5}\,\left(6\,a^2c^2d^7+84\,a^3cd^5e^2+35\,a^4d^3e^4\right)x^5\\&\quad+\frac{7}{4}\,\left(4\,a^3cd^6e+5\,a^4d^4e^3\right)x^4+\frac{1}{3}\,\left(4\,a^3cd^7+21\,a^4d^5e^2\right)x^3\end{aligned}$$

input
```
integrate((e*x+d)^7*(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/16*c^4*e^7*x^16 + 7/15*c^4*d*e^6*x^15 + 1/14*(21*c^4*d^2*e^5 + 4*a*c^3*e
^7)*x^14 + 7/13*(5*c^4*d^3*e^4 + 4*a*c^3*d*e^6)*x^13 + 7/2*a^4*d^6*e*x^2 +
 1/12*(35*c^4*d^4*e^3 + 84*a*c^3*d^2*e^5 + 6*a^2*c^2*e^7)*x^12 + a^4*d^7*x
 + 7/11*(3*c^4*d^5*e^2 + 20*a*c^3*d^3*e^4 + 6*a^2*c^2*d*e^6)*x^11 + 1/10*(
7*c^4*d^6*e + 140*a*c^3*d^4*e^3 + 126*a^2*c^2*d^2*e^5 + 4*a^3*c*e^7)*x^10
+ 1/9*(c^4*d^7 + 84*a*c^3*d^5*e^2 + 210*a^2*c^2*d^3*e^4 + 28*a^3*c*d*e^6)*
x^9 + 1/8*(28*a*c^3*d^6*e + 210*a^2*c^2*d^4*e^3 + 84*a^3*c*d^2*e^5 + a^4*e
^7)*x^8 + 1/7*(4*a*c^3*d^7 + 126*a^2*c^2*d^5*e^2 + 140*a^3*c*d^3*e^4 + 7*a
^4*d*e^6)*x^7 + 7/6*(6*a^2*c^2*d^6*e + 20*a^3*c*d^4*e^3 + 3*a^4*d^2*e^5)*x
^6 + 1/5*(6*a^2*c^2*d^7 + 84*a^3*c*d^5*e^2 + 35*a^4*d^3*e^4)*x^5 + 7/4*(4*
a^3*c*d^6*e + 5*a^4*d^4*e^3)*x^4 + 1/3*(4*a^3*c*d^7 + 21*a^4*d^5*e^2)*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(260) = 520$.

Time = 0.13 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.97

$$\begin{aligned}\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx &= \frac{1}{16}\,c^4e^7x^{16} + \frac{7}{15}\,c^4de^6x^{15} + \frac{3}{2}\,c^4d^2e^5x^{14} + \frac{2}{7}\,ac^3e^7x^{14} \\ &+ \frac{35}{13}\,c^4d^3e^4x^{13} + \frac{28}{13}\,ac^3de^6x^{13} + \frac{35}{12}\,c^4d^4e^3x^{12} \\ &+ 7\,ac^3d^2e^5x^{12} + \frac{1}{2}\,a^2c^2e^7x^{12} + \frac{21}{11}\,c^4d^5e^2x^{11} \\ &+ \frac{140}{11}\,ac^3d^3e^4x^{11} + \frac{42}{11}\,a^2c^2de^6x^{11} + \frac{7}{10}\,c^4d^6ex^{10} \\ &+ 14\,ac^3d^4e^3x^{10} + \frac{63}{5}\,a^2c^2d^2e^5x^{10} + \frac{2}{5}\,a^3ce^7x^{10} + \frac{1}{9}\,c^4d^7x^9 \\ &+ \frac{28}{3}\,ac^3d^5e^2x^9 + \frac{70}{3}\,a^2c^2d^3e^4x^9 + \frac{28}{9}\,a^3cde^6x^9 \\ &+ \frac{7}{2}\,ac^3d^6ex^8 + \frac{105}{4}\,a^2c^2d^4e^3x^8 + \frac{21}{2}\,a^3cd^2e^5x^8 + \frac{1}{8}\,a^4e^7x^8 \\ &+ \frac{4}{7}\,ac^3d^7x^7 + 18\,a^2c^2d^5e^2x^7 + 20\,a^3cd^3e^4x^7 + a^4de^6x^7 \\ &+ 7\,a^2c^2d^6ex^6 + \frac{70}{3}\,a^3cd^4e^3x^6 + \frac{7}{2}\,a^4d^2e^5x^6 + \frac{6}{5}\,a^2c^2d^7x^5 \\ &+ \frac{84}{5}\,a^3cd^5e^2x^5 + 7\,a^4d^3e^4x^5 + 7\,a^3cd^6ex^4 + \frac{35}{4}\,a^4d^4e^3x^4 \\ &+ \frac{4}{3}\,a^3cd^7x^3 + 7\,a^4d^5e^2x^3 + \frac{7}{2}\,a^4d^6ex^2 + a^4d^7x\end{aligned}$$

input
```
integrate((e*x+d)^7*(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*c^4*e^7*x^16 + 7/15*c^4*d*e^6*x^15 + 3/2*c^4*d^2*e^5*x^14 + 2/7*a*c^3
*e^7*x^14 + 35/13*c^4*d^3*e^4*x^13 + 28/13*a*c^3*d*e^6*x^13 + 35/12*c^4*d^
4*e^3*x^12 + 7*a*c^3*d^2*e^5*x^12 + 1/2*a^2*c^2*e^7*x^12 + 21/11*c^4*d^5*e
^2*x^11 + 140/11*a*c^3*d^3*e^4*x^11 + 42/11*a^2*c^2*d*e^6*x^11 + 7/10*c^4*
d^6*e*x^10 + 14*a*c^3*d^4*e^3*x^10 + 63/5*a^2*c^2*d^2*e^5*x^10 + 2/5*a^3*c
*e^7*x^10 + 1/9*c^4*d^7*x^9 + 28/3*a*c^3*d^5*e^2*x^9 + 70/3*a^2*c^2*d^3*e^
4*x^9 + 28/9*a^3*c*d*e^6*x^9 + 7/2*a*c^3*d^6*e*x^8 + 105/4*a^2*c^2*d^4*e^3
*x^8 + 21/2*a^3*c*d^2*e^5*x^8 + 1/8*a^4*e^7*x^8 + 4/7*a*c^3*d^7*x^7 + 18*a
^2*c^2*d^5*e^2*x^7 + 20*a^3*c*d^3*e^4*x^7 + a^4*d*e^6*x^7 + 7*a^2*c^2*d^6*
e*x^6 + 70/3*a^3*c*d^4*e^3*x^6 + 7/2*a^4*d^2*e^5*x^6 + 6/5*a^2*c^2*d^7*x^5
 + 84/5*a^3*c*d^5*e^2*x^5 + 7*a^4*d^3*e^4*x^5 + 7*a^3*c*d^6*e*x^4 + 35/4*a
^4*d^4*e^3*x^4 + 4/3*a^3*c*d^7*x^3 + 7*a^4*d^5*e^2*x^3 + 7/2*a^4*d^6*e*x^2
 + a^4*d^7*x
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.78

$$\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx = x^3 \left(7 a^4 d^5 e^2 + \frac{4 c a^3 d^7}{3}\right) + x^{14} \left(\frac{3 c^4 d^2 e^5}{2} + \frac{2 a c^3 e^7}{7}\right) + x^7 \left(a^4 d e^6 + 20 a^3 c d^3 e^4 + 18 a^2 c^2 d^5 e^2 + \frac{4 a c^3 d^7}{7}\right) + x^8 \left(\frac{a^4 e^7}{8} + \frac{21 a^3 c d^2 e^5}{2} + \frac{105 a^2 c^2 d^4 e^3}{4} + \frac{7 a c^3 d^6 e}{2}\right) + x^9 \left(\frac{28 a^3 c d e^6}{9} + \frac{70 a^2 c^2 d^3 e^4}{3} + \frac{28 a c^3 d^5 e^2}{3} + \frac{c^4 d^7}{9}\right) + x^{10} \left(\frac{2 a^3 c e^7}{5} + \frac{63 a^2 c^2 d^2 e^5}{5} + 14 a c^3 d^4 e^3 + \frac{7 c^4 d^6 e}{10}\right) + x^5 \left(7 a^4 d^3 e^4 + \frac{84 a^3 c d^5 e^2}{5} + \frac{6 a^2 c^2 d^7}{5}\right) + x^{12} \left(\frac{a^2 c^2 e^7}{2} + 7 a c^3 d^2 e^5 + \frac{35 c^4 d^4 e^3}{12}\right) + a^4 d^7 x + \frac{c^4 e^7 x^{16}}{16} + \frac{7 a^4 d^6 e x^2}{2} + \frac{7 c^4 d e^6 x^{15}}{15} + \frac{7 a^3 d^4 e x^4 \left(4 c d^2 + 5 a e^2\right)}{4} + \frac{7 c^3 d e^4 x^{13} \left(5 c d^2 + 4 a e^2\right)}{13} + \frac{7 a^2 d^2 e x^6 \left(3 a^2 e^4 + 20 a c d^2 e^2 + 6 c^2 d^4\right)}{6} + \frac{7 c^2 d e^2 x^{11} \left(6 a^2 e^4 + 20 a c d^2 e^2 + 3 c^2 d^4\right)}{11}$$

input
```
int((a + c*x^2)^4*(d + e*x)^7,x)
```

output
```
x^3*((4*a^3*c*d^7)/3 + 7*a^4*d^5*e^2) + x^14*((2*a*c^3*e^7)/7 + (3*c^4*d^2
*e^5)/2) + x^7*((4*a*c^3*d^7)/7 + a^4*d*e^6 + 20*a^3*c*d^3*e^4 + 18*a^2*c^
2*d^5*e^2) + x^8*((a^4*e^7)/8 + (21*a^3*c*d^2*e^5)/2 + (105*a^2*c^2*d^4*e^
3)/4 + (7*a*c^3*d^6*e)/2) + x^9*((c^4*d^7)/9 + (28*a*c^3*d^5*e^2)/3 + (70*
a^2*c^2*d^3*e^4)/3 + (28*a^3*c*d*e^6)/9) + x^10*((2*a^3*c*e^7)/5 + (7*c^4*
d^6*e)/10 + 14*a*c^3*d^4*e^3 + (63*a^2*c^2*d^2*e^5)/5) + x^5*((6*a^2*c^2*d
^7)/5 + 7*a^4*d^3*e^4 + (84*a^3*c*d^5*e^2)/5) + x^12*((a^2*c^2*e^7)/2 + (3
5*c^4*d^4*e^3)/12 + 7*a*c^3*d^2*e^5) + a^4*d^7*x + (c^4*e^7*x^16)/16 + (7*
a^4*d^6*e*x^2)/2 + (7*c^4*d*e^6*x^15)/15 + (7*a^3*d^4*e*x^4*(5*a*e^2 + 4*c
*d^2))/4 + (7*c^3*d*e^4*x^13*(4*a*e^2 + 5*c*d^2))/13 + (7*a^2*d^2*e*x^6*(3
*a^2*e^4 + 6*c^2*d^4 + 20*a*c*d^2*e^2))/6 + (7*c^2*d*e^2*x^11*(6*a^2*e^4 +
 3*c^2*d^4 + 20*a*c*d^2*e^2))/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.97

$$\int (d+ex)^7 \left(a+cx^2\right)^4 \, dx = \frac{x(45045c^4e^7x^{15} + 336336c^4d\,e^6x^{14} + 205920a\,c^3e^7x^{13} + 1081080c^4d^2e^5x^{13} + 1552320a\,c^3d\,e^6x^{12} + 19404}{}$$

input
```
int((e*x+d)^7*(c*x^2+a)^4,x)
```

output

```
(x*(720720*a**4*d**7 + 2522520*a**4*d**6*e*x + 5045040*a**4*d**5*e**2*x**2
 + 6306300*a**4*d**4*e**3*x**3 + 5045040*a**4*d**3*e**4*x**4 + 2522520*a**
4*d**2*e**5*x**5 + 720720*a**4*d*e**6*x**6 + 90090*a**4*e**7*x**7 + 960960
*a**3*c*d**7*x**2 + 5045040*a**3*c*d**6*e*x**3 + 12108096*a**3*c*d**5*e**2
*x**4 + 16816800*a**3*c*d**4*e**3*x**5 + 14414400*a**3*c*d**3*e**4*x**6 +
7567560*a**3*c*d**2*e**5*x**7 + 2242240*a**3*c*d*e**6*x**8 + 288288*a**3*c
*e**7*x**9 + 864864*a**2*c**2*d**7*x**4 + 5045040*a**2*c**2*d**6*e*x**5 +
12972960*a**2*c**2*d**5*e**2*x**6 + 18918900*a**2*c**2*d**4*e**3*x**7 + 16
816800*a**2*c**2*d**3*e**4*x**8 + 9081072*a**2*c**2*d**2*e**5*x**9 + 27518
40*a**2*c**2*d*e**6*x**10 + 360360*a**2*c**2*e**7*x**11 + 411840*a*c**3*d*
*7*x**6 + 2522520*a*c**3*d**6*e*x**7 + 6726720*a*c**3*d**5*e**2*x**8 + 100
90080*a*c**3*d**4*e**3*x**9 + 9172800*a*c**3*d**3*e**4*x**10 + 5045040*a*c
**3*d**2*e**5*x**11 + 1552320*a*c**3*d*e**6*x**12 + 205920*a*c**3*e**7*x**
13 + 80080*c**4*d**7*x**8 + 504504*c**4*d**6*e*x**9 + 1375920*c**4*d**5*e*
*2*x**10 + 2102100*c**4*d**4*e**3*x**11 + 1940400*c**4*d**3*e**4*x**12 + 1
081080*c**4*d**2*e**5*x**13 + 336336*c**4*d*e**6*x**14 + 45045*c**4*e**7*x
**15))/720720
```

3.90 $\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 276

$$\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx = \frac{(cd^2+ae^2)^4 (d+ex)^7}{7e^9} - \frac{cd(cd^2+ae^2)^3 (d+ex)^8}{e^9} + \frac{4c(cd^2+ae^2)^2 (7cd^2+ae^2) (d+ex)^9}{9e^9} - \frac{4c^2d(cd^2+ae^2) (7cd^2+3ae^2) (d+ex)^{10}}{5e^9} + \frac{2c^2(35c^2d^4+30acd^2e^2+3a^2e^4) (d+ex)^{11}}{11e^9} - \frac{2c^3d(7cd^2+3ae^2) (d+ex)^{12}}{3e^9} + \frac{4c^3(7cd^2+ae^2) (d+ex)^{13}}{13e^9} - \frac{4c^4d(d+ex)^{14}}{7e^9} + \frac{c^4(d+ex)^{15}}{15e^9}$$

output
```
1/7*(a*e^2+c*d^2)^4*(e*x+d)^7/e^9-c*d*(a*e^2+c*d^2)^3*(e*x+d)^8/e^9+4/9*c*
(a*e^2+c*d^2)^2*(a*e^2+7*c*d^2)*(e*x+d)^9/e^9-4/5*c^2*d*(a*e^2+c*d^2)*(3*a
*e^2+7*c*d^2)*(e*x+d)^10/e^9+2/11*c^2*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4
)*(e*x+d)^11/e^9-2/3*c^3*d*(3*a*e^2+7*c*d^2)*(e*x+d)^12/e^9+4/13*c^3*(a*e^
2+7*c*d^2)*(e*x+d)^13/e^9-4/7*c^4*d*(e*x+d)^14/e^9+1/15*c^4*(e*x+d)^15/e^9
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.31

$$\int (d+ex)^6 \left(a+cx^2\right)^4 dx$$
$$= \frac{6435a^4x(7d^6+21d^5ex+35d^4e^2x^2+35d^3e^3x^3+21d^2e^4x^4+7de^5x^5+e^6x^6)+715a^3cx^3(84d^6+378d^5ex}{}$$

input
```
Integrate[(d + e*x)^6*(a + c*x^2)^4,x]
```

output
```
(6435*a^4*x*(7*d^6 + 21*d^5*e*x + 35*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 21*d^2
*e^4*x^4 + 7*d*e^5*x^5 + e^6*x^6) + 715*a^3*c*x^3*(84*d^6 + 378*d^5*e*x +
756*d^4*e^2*x^2 + 840*d^3*e^3*x^3 + 540*d^2*e^4*x^4 + 189*d*e^5*x^5 + 28*e
^6*x^6) + 117*a^2*c^2*x^5*(462*d^6 + 2310*d^5*e*x + 4950*d^4*e^2*x^2 + 577
5*d^3*e^3*x^3 + 3850*d^2*e^4*x^4 + 1386*d*e^5*x^5 + 210*e^6*x^6) + 15*a*c^
3*x^7*(1716*d^6 + 9009*d^5*e*x + 20020*d^4*e^2*x^2 + 24024*d^3*e^3*x^3 + 1
6380*d^2*e^4*x^4 + 6006*d*e^5*x^5 + 924*e^6*x^6) + c^4*x^9*(5005*d^6 + 270
27*d^5*e*x + 61425*d^4*e^2*x^2 + 75075*d^3*e^3*x^3 + 51975*d^2*e^4*x^4 + 1
9305*d*e^5*x^5 + 3003*e^6*x^6))/45045
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^4 (d+ex)^6 dx$$
$$\Big\downarrow \; 476$$
$$\int \left(\frac{2c^2(d+ex)^{10}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{e^8}+\frac{4c^3(d+ex)^{12}\left(ae^2+7cd^2\right)}{e^8}-\frac{8c^3d(d+ex)^{11}\left(3ae^2+7cd^2\right)}{e^8}\right.$$

$$\downarrow \text{2009}$$

$$\frac{2c^2(d+ex)^{11}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{11e^9}+\frac{4c^3(d+ex)^{13}\left(ae^2+7cd^2\right)}{13e^9}-\frac{2c^3d(d+ex)^{12}\left(3ae^2+7cd^2\right)}{3e^9}-\frac{4c^2d(d+ex)^{10}\left(ae^2+cd^2\right)\left(3ae^2+7cd^2\right)}{5e^9}+\frac{4c(d+ex)^9\left(ae^2+cd^2\right)^2\left(ae^2+7cd^2\right)}{9e^9}-\frac{cd(d+ex)^8\left(ae^2+cd^2\right)^3}{e^9}+\frac{(d+ex)^7\left(ae^2+cd^2\right)^4}{7e^9}+\frac{c^4(d+ex)^{15}}{15e^9}-\frac{4c^4d(d+ex)^{14}}{7e^9}$$

input
```
Int[(d + e*x)^6*(a + c*x^2)^4,x]
```

output
```
((c*d^2 + a*e^2)^4*(d + e*x)^7)/(7*e^9) - (c*d*(c*d^2 + a*e^2)^3*(d + e*x)
^8)/e^9 + (4*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^9)/(9*e^9) -
(4*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^10)/(5*e^9) + (2*c^
2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^11)/(11*e^9) - (2*c^
3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^12)/(3*e^9) + (4*c^3*(7*c*d^2 + a*e^2)*(
d + e*x)^13)/(13*e^9) - (4*c^4*d*(d + e*x)^14)/(7*e^9) + (c^4*(d + e*x)^15
)/(15*e^9)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.58

method	result
norman	$d^6a^4x + 3d^5e\,a^4x^2 + \left(5d^4e^2a^4 + \frac{4}{3}d^6c\,a^3\right)x^3 + \left(5d^3e^3a^4 + 6d^5ec\,a^3\right)x^4 + \left(3d^2e^4a^4 + 12d^4e^2\right.$
default	$\frac{e^6c^4x^{15}}{15} + \frac{3d\,e^5c^4x^{14}}{7} + \frac{(4e^6a\,c^3+15d^2e^4c^4)x^{13}}{13} + \frac{(24d\,e^5a\,c^3+20d^3e^3c^4)x^{12}}{12} + \frac{(6e^6a^2c^2+60d^2e^4a\,c^3+15d^4e^2c^4)x^{11}}{11}$
gosper	$\frac{6}{11}x^{11}e^6a^2c^2 + \frac{15}{11}x^{11}d^4e^2c^4 + 3x^5d^2e^4a^4 + \frac{6}{5}x^5d^6a^2c^2 + x^6d\,e^5a^4 + \frac{4}{7}x^7d^6a\,c^3 + 12x^5d^4e^2c\,a^3$
risch	$\frac{6}{11}x^{11}e^6a^2c^2 + \frac{15}{11}x^{11}d^4e^2c^4 + 3x^5d^2e^4a^4 + \frac{6}{5}x^5d^6a^2c^2 + x^6d\,e^5a^4 + \frac{4}{7}x^7d^6a\,c^3 + 12x^5d^4e^2c\,a^3$
parallelrisch	$\frac{6}{11}x^{11}e^6a^2c^2 + \frac{15}{11}x^{11}d^4e^2c^4 + 3x^5d^2e^4a^4 + \frac{6}{5}x^5d^6a^2c^2 + x^6d\,e^5a^4 + \frac{4}{7}x^7d^6a\,c^3 + 12x^5d^4e^2c\,a^3$
orering	$\frac{x(3003e^6c^4x^{14}+19305d\,e^5c^4x^{13}+13860a\,c^3e^6x^{12}+51975c^4d^2e^4x^{12}+90090a\,c^3d\,e^5x^{11}+75075c^4d^3e^3x^{11}+24570a^2c^2e^6x^{10}+}{}$

input
```
int((e*x+d)^6*(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
d^6*a^4*x+3*d^5*e*a^4*x^2+(5*d^4*e^2*a^4+4/3*d^6*c*a^3)*x^3+(5*a^4*d^3*e^3
+6*a^3*c*d^5*e)*x^4+(3*d^2*e^4*a^4+12*d^4*e^2*c*a^3+6/5*d^6*a^2*c^2)*x^5+(
d*e^5*a^4+40/3*d^3*e^3*c*a^3+6*d^5*e*a^2*c^2)*x^6+(1/7*e^6*a^4+60/7*d^2*e^
4*c*a^3+90/7*d^4*e^2*a^2*c^2+4/7*d^6*a*c^3)*x^7+(3*a^3*c*d*e^5+15*a^2*c^2*
d^3*e^3+3*a*c^3*d^5*e)*x^8+(4/9*e^6*c*a^3+10*d^2*e^4*a^2*c^2+20/3*d^4*e^2*
a*c^3+1/9*d^6*c^4)*x^9+(18/5*d*e^5*a^2*c^2+8*d^3*e^3*a*c^3+3/5*d^5*e*c^4)*
x^10+(6/11*e^6*a^2*c^2+60/11*d^2*e^4*a*c^3+15/11*d^4*e^2*c^4)*x^11+(2*d*e^
5*a*c^3+5/3*d^3*e^3*c^4)*x^12+(4/13*e^6*a*c^3+15/13*d^2*e^4*c^4)*x^13+3/7*
d*e^5*c^4*x^14+1/15*e^6*c^4*x^15
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx = {} & \frac{1}{15}\, c^4 e^6 x^{15} + \frac{3}{7}\, c^4 d e^5 x^{14} + \frac{1}{13}\left(15\, c^4 d^2 e^4 + 4\, a c^3 e^6\right) x^{13} \\
& + \frac{1}{3}\left(5\, c^4 d^3 e^3 + 6\, a c^3 d e^5\right) x^{12} + 3\, a^4 d^5 e x^2 \\
& + \frac{3}{11}\left(5\, c^4 d^4 e^2 + 20\, a c^3 d^2 e^4 + 2\, a^2 c^2 e^6\right) x^{11} + a^4 d^6 x \\
& + \frac{1}{5}\left(3\, c^4 d^5 e + 40\, a c^3 d^3 e^3 + 18\, a^2 c^2 d e^5\right) x^{10} \\
& + \frac{1}{9}\left(c^4 d^6 + 60\, a c^3 d^4 e^2 + 90\, a^2 c^2 d^2 e^4 + 4\, a^3 c e^6\right) x^9 \\
& + 3\left(a c^3 d^5 e + 5\, a^2 c^2 d^3 e^3 + a^3 c d e^5\right) x^8 \\
& + \frac{1}{7}\left(4\, a c^3 d^6 + 90\, a^2 c^2 d^4 e^2 + 60\, a^3 c d^2 e^4 + a^4 e^6\right) x^7 \\
& + \frac{1}{3}\left(18\, a^2 c^2 d^5 e + 40\, a^3 c d^3 e^3 + 3\, a^4 d e^5\right) x^6 \\
& + \frac{3}{5}\left(2\, a^2 c^2 d^6 + 20\, a^3 c d^4 e^2 + 5\, a^4 d^2 e^4\right) x^5 \\
& + \left(6\, a^3 c d^5 e + 5\, a^4 d^3 e^3\right) x^4 + \frac{1}{3}\left(4\, a^3 c d^6 + 15\, a^4 d^4 e^2\right) x^3
\end{aligned}
$$

input
```
integrate((e*x+d)^6*(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/15*c^4*e^6*x^15 + 3/7*c^4*d*e^5*x^14 + 1/13*(15*c^4*d^2*e^4 + 4*a*c^3*e^
6)*x^13 + 1/3*(5*c^4*d^3*e^3 + 6*a*c^3*d*e^5)*x^12 + 3*a^4*d^5*e*x^2 + 3/1
1*(5*c^4*d^4*e^2 + 20*a*c^3*d^2*e^4 + 2*a^2*c^2*e^6)*x^11 + a^4*d^6*x + 1/
5*(3*c^4*d^5*e + 40*a*c^3*d^3*e^3 + 18*a^2*c^2*d*e^5)*x^10 + 1/9*(c^4*d^6
+ 60*a*c^3*d^4*e^2 + 90*a^2*c^2*d^2*e^4 + 4*a^3*c*e^6)*x^9 + 3*(a*c^3*d^5*
e + 5*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^8 + 1/7*(4*a*c^3*d^6 + 90*a^2*c^2*d
^4*e^2 + 60*a^3*c*d^2*e^4 + a^4*e^6)*x^7 + 1/3*(18*a^2*c^2*d^5*e + 40*a^3*
c*d^3*e^3 + 3*a^4*d*e^5)*x^6 + 3/5*(2*a^2*c^2*d^6 + 20*a^3*c*d^4*e^2 + 5*a
^4*d^2*e^4)*x^5 + (6*a^3*c*d^5*e + 5*a^4*d^3*e^3)*x^4 + 1/3*(4*a^3*c*d^6 +
 15*a^4*d^4*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.76

$$\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx = a^4d^6x + 3a^4d^5ex^2 + \frac{3c^4de^5x^{14}}{7} + \frac{c^4e^6x^{15}}{15} + x^{13} \cdot \left(\frac{4ac^3e^6}{13} + \frac{15c^4d^2e^4}{13}\right) + x^{12} \cdot \left(2ac^3de^5 + \frac{5c^4d^3e^3}{3}\right) + x^{11} \cdot \left(\frac{6a^2c^2e^6}{11} + \frac{60ac^3d^2e^4}{11} + \frac{15c^4d^4e^2}{11}\right) + x^{10} \cdot \left(\frac{18a^2c^2de^5}{5} + 8ac^3d^3e^3 + \frac{3c^4d^5e}{5}\right) + x^9 \cdot \left(\frac{4a^3ce^6}{9} + 10a^2c^2d^2e^4 + \frac{20ac^3d^4e^2}{3} + \frac{c^4d^6}{9}\right) + x^8 \cdot \left(3a^3cde^5 + 15a^2c^2d^3e^3 + 3ac^3d^5e\right) + x^7\left(\frac{a^4e^6}{7} + \frac{60a^3cd^2e^4}{7} + \frac{90a^2c^2d^4e^2}{7} + \frac{4ac^3d^6}{7}\right) + x^6\left(a^4de^5 + \frac{40a^3cd^3e^3}{3} + 6a^2c^2d^5e\right) + x^5 \cdot \left(3a^4d^2e^4 + 12a^3cd^4e^2 + \frac{6a^2c^2d^6}{5}\right) + x^4 \cdot \left(5a^4d^3e^3 + 6a^3cd^5e\right) + x^3 \cdot \left(5a^4d^4e^2 + \frac{4a^3cd^6}{3}\right)$$

input

```
integrate((e*x+d)**6*(c*x**2+a)**4,x)
```

output

```
a**4*d**6*x + 3*a**4*d**5*e*x**2 + 3*c**4*d*e**5*x**14/7 + c**4*e**6*x**15
/15 + x**13*(4*a*c**3*e**6/13 + 15*c**4*d**2*e**4/13) + x**12*(2*a*c**3*d*
e**5 + 5*c**4*d**3*e**3/3) + x**11*(6*a**2*c**2*e**6/11 + 60*a*c**3*d**2*e
**4/11 + 15*c**4*d**4*e**2/11) + x**10*(18*a**2*c**2*d*e**5/5 + 8*a*c**3*d
**3*e**3 + 3*c**4*d**5*e/5) + x**9*(4*a**3*c*e**6/9 + 10*a**2*c**2*d**2*e*
*4 + 20*a*c**3*d**4*e**2/3 + c**4*d**6/9) + x**8*(3*a**3*c*d*e**5 + 15*a**
2*c**2*d**3*e**3 + 3*a*c**3*d**5*e) + x**7*(a**4*e**6/7 + 60*a**3*c*d**2*e
**4/7 + 90*a**2*c**2*d**4*e**2/7 + 4*a*c**3*d**6/7) + x**6*(a**4*d*e**5 +
40*a**3*c*d**3*e**3/3 + 6*a**2*c**2*d**5*e) + x**5*(3*a**4*d**2*e**4 + 12*
a**3*c*d**4*e**2 + 6*a**2*c**2*d**6/5) + x**4*(5*a**4*d**3*e**3 + 6*a**3*c
*d**5*e) + x**3*(5*a**4*d**4*e**2 + 4*a**3*c*d**6/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.60

$$\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx = \frac{1}{15} c^4 e^6 x^{15} + \frac{3}{7} c^4 d e^5 x^{14} + \frac{1}{13} \left(15 c^4 d^2 e^4 + 4 a c^3 e^6\right) x^{13} + \frac{1}{3} \left(5 c^4 d^3 e^3 + 6 a c^3 d e^5\right) x^{12} + 3 a^4 d^5 e x^2 + \frac{3}{11} \left(5 c^4 d^4 e^2 + 20 a c^3 d^2 e^4 + 2 a^2 c^2 e^6\right) x^{11} + a^4 d^6 x + \frac{1}{5} \left(3 c^4 d^5 e + 40 a c^3 d^3 e^3 + 18 a^2 c^2 d e^5\right) x^{10} + \frac{1}{9} \left(c^4 d^6 + 60 a c^3 d^4 e^2 + 90 a^2 c^2 d^2 e^4 + 4 a^3 c e^6\right) x^9 + 3 \left(a c^3 d^5 e + 5 a^2 c^2 d^3 e^3 + a^3 c d e^5\right) x^8 + \frac{1}{7} \left(4 a c^3 d^6 + 90 a^2 c^2 d^4 e^2 + 60 a^3 c d^2 e^4 + a^4 e^6\right) x^7 + \frac{1}{3} \left(18 a^2 c^2 d^5 e + 40 a^3 c d^3 e^3 + 3 a^4 d e^5\right) x^6 + \frac{3}{5} \left(2 a^2 c^2 d^6 + 20 a^3 c d^4 e^2 + 5 a^4 d^2 e^4\right) x^5 + \left(6 a^3 c d^5 e + 5 a^4 d^3 e^3\right) x^4 + \frac{1}{3} \left(4 a^3 c d^6 + 15 a^4 d^4 e^2\right) x^3$$

input
```
integrate((e*x+d)^6*(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/15*c^4*e^6*x^15 + 3/7*c^4*d*e^5*x^14 + 1/13*(15*c^4*d^2*e^4 + 4*a*c^3*e^
6)*x^13 + 1/3*(5*c^4*d^3*e^3 + 6*a*c^3*d*e^5)*x^12 + 3*a^4*d^5*e*x^2 + 3/1
1*(5*c^4*d^4*e^2 + 20*a*c^3*d^2*e^4 + 2*a^2*c^2*e^6)*x^11 + a^4*d^6*x + 1/
5*(3*c^4*d^5*e + 40*a*c^3*d^3*e^3 + 18*a^2*c^2*d*e^5)*x^10 + 1/9*(c^4*d^6
+ 60*a*c^3*d^4*e^2 + 90*a^2*c^2*d^2*e^4 + 4*a^3*c*e^6)*x^9 + 3*(a*c^3*d^5*
e + 5*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^8 + 1/7*(4*a*c^3*d^6 + 90*a^2*c^2*d
^4*e^2 + 60*a^3*c*d^2*e^4 + a^4*e^6)*x^7 + 1/3*(18*a^2*c^2*d^5*e + 40*a^3*
c*d^3*e^3 + 3*a^4*d*e^5)*x^6 + 3/5*(2*a^2*c^2*d^6 + 20*a^3*c*d^4*e^2 + 5*a
^4*d^2*e^4)*x^5 + (6*a^3*c*d^5*e + 5*a^4*d^3*e^3)*x^4 + 1/3*(4*a^3*c*d^6 +
 15*a^4*d^4*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.71

$$\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx = \frac{1}{15}c^4e^6x^{15} + \frac{3}{7}c^4de^5x^{14} + \frac{15}{13}c^4d^2e^4x^{13} + \frac{4}{13}ac^3e^6x^{13} + \frac{5}{3}c^4d^3e^3x^{12} + 2ac^3de^5x^{12} + \frac{15}{11}c^4d^4e^2x^{11} + \frac{60}{11}ac^3d^2e^4x^{11} + \frac{6}{11}a^2c^2e^6x^{11} + \frac{3}{5}c^4d^5ex^{10} + 8ac^3d^3e^3x^{10} + \frac{18}{5}a^2c^2de^5x^{10} + \frac{1}{9}c^4d^6x^9 + \frac{20}{3}ac^3d^4e^2x^9 + 10a^2c^2d^2e^4x^9 + \frac{4}{9}a^3ce^6x^9 + 3ac^3d^5ex^8 + 15a^2c^2d^3e^3x^8 + 3a^3cde^5x^8 + \frac{4}{7}ac^3d^6x^7 + \frac{90}{7}a^2c^2d^4e^2x^7 + \frac{60}{7}a^3cd^2e^4x^7 + \frac{1}{7}a^4e^6x^7 + 6a^2c^2d^5ex^6 + \frac{40}{3}a^3cd^3e^3x^6 + a^4de^5x^6 + \frac{6}{5}a^2c^2d^6x^5 + 12a^3cd^4e^2x^5 + 3a^4d^2e^4x^5 + 6a^3cd^5ex^4 + 5a^4d^3e^3x^4 + \frac{4}{3}a^3cd^6x^3 + 5a^4d^4e^2x^3 + 3a^4d^5ex^2 + a^4d^6x$$

input
```
integrate((e*x+d)^6*(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/15*c^4*e^6*x^15 + 3/7*c^4*d*e^5*x^14 + 15/13*c^4*d^2*e^4*x^13 + 4/13*a*c
^3*e^6*x^13 + 5/3*c^4*d^3*e^3*x^12 + 2*a*c^3*d*e^5*x^12 + 15/11*c^4*d^4*e^
2*x^11 + 60/11*a*c^3*d^2*e^4*x^11 + 6/11*a^2*c^2*e^6*x^11 + 3/5*c^4*d^5*e*
x^10 + 8*a*c^3*d^3*e^3*x^10 + 18/5*a^2*c^2*d*e^5*x^10 + 1/9*c^4*d^6*x^9 +
20/3*a*c^3*d^4*e^2*x^9 + 10*a^2*c^2*d^2*e^4*x^9 + 4/9*a^3*c*e^6*x^9 + 3*a*
c^3*d^5*e*x^8 + 15*a^2*c^2*d^3*e^3*x^8 + 3*a^3*c*d*e^5*x^8 + 4/7*a*c^3*d^6
*x^7 + 90/7*a^2*c^2*d^4*e^2*x^7 + 60/7*a^3*c*d^2*e^4*x^7 + 1/7*a^4*e^6*x^7
 + 6*a^2*c^2*d^5*e*x^6 + 40/3*a^3*c*d^3*e^3*x^6 + a^4*d*e^5*x^6 + 6/5*a^2*
c^2*d^6*x^5 + 12*a^3*c*d^4*e^2*x^5 + 3*a^4*d^2*e^4*x^5 + 6*a^3*c*d^5*e*x^4
 + 5*a^4*d^3*e^3*x^4 + 4/3*a^3*c*d^6*x^3 + 5*a^4*d^4*e^2*x^3 + 3*a^4*d^5*e
*x^2 + a^4*d^6*x
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.54

$$\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx = x^7 \left(\frac{a^4 e^6}{7} + \frac{60 a^3 c d^2 e^4}{7} + \frac{90 a^2 c^2 d^4 e^2}{7} + \frac{4 a c^3 d^6}{7}\right) + x^9 \left(\frac{4 a^3 c e^6}{9} + 10 a^2 c^2 d^2 e^4 + \frac{20 a c^3 d^4 e^2}{3} + \frac{c^4 d^6}{9}\right) + x^3 \left(5 a^4 d^4 e^2 + \frac{4 c a^3 d^6}{3}\right) + x^{13} \left(\frac{15 c^4 d^2 e^4}{13} + \frac{4 a c^3 e^6}{13}\right) + x^5 \left(3 a^4 d^2 e^4 + 12 a^3 c d^4 e^2 + \frac{6 a^2 c^2 d^6}{5}\right) + x^{11} \left(\frac{6 a^2 c^2 e^6}{11} + \frac{60 a c^3 d^2 e^4}{11} + \frac{15 c^4 d^4 e^2}{11}\right) + a^4 d^6 x + \frac{c^4 e^6 x^{15}}{15} + 3 a^4 d^5 e x^2 + \frac{3 c^4 d e^5 x^{14}}{7} + a^3 d^3 e x^4 \left(6 c d^2 + 5 a e^2\right) + \frac{c^3 d e^3 x^{12} \left(5 c d^2 + 6 a e^2\right)}{3} + \frac{a^2 d e x^6 \left(3 a^2 e^4 + 40 a c d^2 e^2 + 18 c^2 d^4\right)}{3} + \frac{c^2 d e x^{10} \left(18 a^2 e^4 + 40 a c d^2 e^2 + 3 c^2 d^4\right)}{5} + 3 a c d e x^8 \left(a^2 e^4 + 5 a c d^2 e^2 + c^2 d^4\right)$$

input
```
int((a + c*x^2)^4*(d + e*x)^6,x)
```

output
```
x^7*((a^4*e^6)/7 + (4*a*c^3*d^6)/7 + (60*a^3*c*d^2*e^4)/7 + (90*a^2*c^2*d^
4*e^2)/7) + x^9*((c^4*d^6)/9 + (4*a^3*c*e^6)/9 + (20*a*c^3*d^4*e^2)/3 + 10
*a^2*c^2*d^2*e^4) + x^3*((4*a^3*c*d^6)/3 + 5*a^4*d^4*e^2) + x^13*((4*a*c^3
*e^6)/13 + (15*c^4*d^2*e^4)/13) + x^5*((6*a^2*c^2*d^6)/5 + 3*a^4*d^2*e^4 +
 12*a^3*c*d^4*e^2) + x^11*((6*a^2*c^2*e^6)/11 + (15*c^4*d^4*e^2)/11 + (60*
a*c^3*d^2*e^4)/11) + a^4*d^6*x + (c^4*e^6*x^15)/15 + 3*a^4*d^5*e*x^2 + (3*
c^4*d*e^5*x^14)/7 + a^3*d^3*e*x^4*(5*a*e^2 + 6*c*d^2) + (c^3*d*e^3*x^12*(6
*a*e^2 + 5*c*d^2))/3 + (a^2*d*e*x^6*(3*a^2*e^4 + 18*c^2*d^4 + 40*a*c*d^2*e
^2))/3 + (c^2*d*e*x^10*(18*a^2*e^4 + 3*c^2*d^4 + 40*a*c*d^2*e^2))/5 + 3*a*
c*d*e*x^8*(a^2*e^4 + c^2*d^4 + 5*a*c*d^2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.72

$$\int (d+ex)^6 \left(a+cx^2\right)^4 \, dx$$
$$= \frac{x(3003c^4e^6x^{14} + 19305c^4d\,e^5x^{13} + 13860a\,c^3e^6x^{12} + 51975c^4d^2e^4x^{12} + 90090a\,c^3d\,e^5x^{11} + 75075c^4d^3e^3x^{1}}{}$$

input
```
int((e*x+d)^6*(c*x^2+a)^4,x)
```

output
```
(x*(45045*a**4*d**6 + 135135*a**4*d**5*e*x + 225225*a**4*d**4*e**2*x**2 +
225225*a**4*d**3*e**3*x**3 + 135135*a**4*d**2*e**4*x**4 + 45045*a**4*d*e**
5*x**5 + 6435*a**4*e**6*x**6 + 60060*a**3*c*d**6*x**2 + 270270*a**3*c*d**5
*e*x**3 + 540540*a**3*c*d**4*e**2*x**4 + 600600*a**3*c*d**3*e**3*x**5 + 38
6100*a**3*c*d**2*e**4*x**6 + 135135*a**3*c*d*e**5*x**7 + 20020*a**3*c*e**6
*x**8 + 54054*a**2*c**2*d**6*x**4 + 270270*a**2*c**2*d**5*e*x**5 + 579150*
a**2*c**2*d**4*e**2*x**6 + 675675*a**2*c**2*d**3*e**3*x**7 + 450450*a**2*c
**2*d**2*e**4*x**8 + 162162*a**2*c**2*d*e**5*x**9 + 24570*a**2*c**2*e**6*x
**10 + 25740*a*c**3*d**6*x**6 + 135135*a*c**3*d**5*e*x**7 + 300300*a*c**3*
d**4*e**2*x**8 + 360360*a*c**3*d**3*e**3*x**9 + 245700*a*c**3*d**2*e**4*x*
*10 + 90090*a*c**3*d*e**5*x**11 + 13860*a*c**3*e**6*x**12 + 5005*c**4*d**6
*x**8 + 27027*c**4*d**5*e*x**9 + 61425*c**4*d**4*e**2*x**10 + 75075*c**4*d
**3*e**3*x**11 + 51975*c**4*d**2*e**4*x**12 + 19305*c**4*d*e**5*x**13 + 30
03*c**4*e**6*x**14))/45045
```

3.91 $\int (d+ex)^5 (a+cx^2)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 278

$$\int (d+ex)^5 (a+cx^2)^4 \, dx = \frac{(cd^2+ae^2)^4 (d+ex)^6}{6e^9} - \frac{8cd(cd^2+ae^2)^3 (d+ex)^7}{7e^9} + \frac{c(cd^2+ae^2)^2 (7cd^2+ae^2) (d+ex)^8}{2e^9} - \frac{8c^2d(cd^2+ae^2) (7cd^2+3ae^2) (d+ex)^9}{9e^9} + \frac{c^2(35c^2d^4+30acd^2e^2+3a^2e^4) (d+ex)^{10}}{5e^9} - \frac{8c^3d(7cd^2+3ae^2) (d+ex)^{11}}{11e^9} + \frac{c^3(7cd^2+ae^2) (d+ex)^{12}}{3e^9} - \frac{8c^4d(d+ex)^{13}}{13e^9} + \frac{c^4(d+ex)^{14}}{14e^9}$$

output
```
1/6*(a*e^2+c*d^2)^4*(e*x+d)^6/e^9-8/7*c*d*(a*e^2+c*d^2)^3*(e*x+d)^7/e^9+1/
2*c*(a*e^2+c*d^2)^2*(a*e^2+7*c*d^2)*(e*x+d)^8/e^9-8/9*c^2*d*(a*e^2+c*d^2)*
(3*a*e^2+7*c*d^2)*(e*x+d)^9/e^9+1/5*c^2*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d
^4)*(e*x+d)^10/e^9-8/11*c^3*d*(3*a*e^2+7*c*d^2)*(e*x+d)^11/e^9+1/3*c^3*(a*
e^2+7*c*d^2)*(e*x+d)^12/e^9-8/13*c^4*d*(e*x+d)^13/e^9+1/14*c^4*(e*x+d)^14/
e^9
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.10

$$\int (d+ex)^5 (a+cx^2)^4 \, dx = \frac{x(15015a^4(6d^5+15d^4ex+20d^3e^2x^2+15d^2e^3x^3+6de^4x^4+e^5x^5)+2145a^3cx^2(56d^5+210d^4ex+336d^3$$

input
```
Integrate[(d + e*x)^5*(a + c*x^2)^4,x]
```

output
```
(x*(15015*a^4*(6*d^5 + 15*d^4*e*x + 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 + 6*d*
e^4*x^4 + e^5*x^5) + 2145*a^3*c*x^2*(56*d^5 + 210*d^4*e*x + 336*d^3*e^2*x^
2 + 280*d^2*e^3*x^3 + 120*d*e^4*x^4 + 21*e^5*x^5) + 429*a^2*c^2*x^4*(252*d
^5 + 1050*d^4*e*x + 1800*d^3*e^2*x^2 + 1575*d^2*e^3*x^3 + 700*d*e^4*x^4 +
126*e^5*x^5) + 65*a*c^3*x^6*(792*d^5 + 3465*d^4*e*x + 6160*d^3*e^2*x^2 + 5
544*d^2*e^3*x^3 + 2520*d*e^4*x^4 + 462*e^5*x^5) + 5*c^4*x^8*(2002*d^5 + 90
09*d^4*e*x + 16380*d^3*e^2*x^2 + 15015*d^2*e^3*x^3 + 6930*d*e^4*x^4 + 1287
*e^5*x^5)))/90090
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^4 (d+ex)^5 \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c^2(d+ex)^9(3a^2e^4+30acd^2e^2+35c^2d^4)}{e^8}+\frac{4c^3(d+ex)^{11}(ae^2+7cd^2)}{e^8}-\frac{8c^3d(d+ex)^{10}(3ae^2+7cd^2)}{e^8}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2(d+ex)^{10}\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{5e^9}+\frac{c^3(d+ex)^{12}\left(ae^2+7cd^2\right)}{3e^9}-\frac{8c^3d(d+ex)^{11}\left(3ae^2+7cd^2\right)}{11e^9}-\frac{8c^2d(d+ex)^9\left(ae^2+cd^2\right)\left(3ae^2+7cd^2\right)}{9e^9}+\frac{c(d+ex)^8\left(ae^2+cd^2\right)^2\left(ae^2+7cd^2\right)}{2e^9}-\frac{8cd(d+ex)^7\left(ae^2+cd^2\right)^3}{7e^9}+\frac{(d+ex)^6\left(ae^2+cd^2\right)^4}{6e^9}+\frac{c^4(d+ex)^{14}}{14e^9}-\frac{8c^4d(d+ex)^{13}}{13e^9}$$

input
```
Int[(d + e*x)^5*(a + c*x^2)^4,x]
```

output
```
((c*d^2 + a*e^2)^4*(d + e*x)^6)/(6*e^9) - (8*c*d*(c*d^2 + a*e^2)^3*(d + e*
x)^7)/(7*e^9) + (c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2)*(d + e*x)^8)/(2*e^9
) - (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x)^9)/(9*e^9) + (c
^2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^10)/(5*e^9) - (8*c^
3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^11)/(11*e^9) + (c^3*(7*c*d^2 + a*e^2)*(d
 + e*x)^12)/(3*e^9) - (8*c^4*d*(d + e*x)^13)/(13*e^9) + (c^4*(d + e*x)^14)
/(14*e^9)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.33

method	result
norman	$d^5a^4x + \frac{5d^4e\,a^4x^2}{2} + \left(\frac{10}{3}d^3e^2a^4 + \frac{4}{3}d^5c\,a^3\right)x^3 + \left(\frac{5}{2}d^2e^3a^4 + 5d^4ec\,a^3\right)x^4 + \left(d\,e^4a^4 + 8d^3e^2c\,a\right.$
default	$\frac{e^5c^4x^{14}}{14} + \frac{5d\,e^4c^4x^{13}}{13} + \frac{(4e^5a\,c^3+10d^2e^3c^4)x^{12}}{12} + \frac{(20d\,e^4a\,c^3+10d^3e^2c^4)x^{11}}{11} + \frac{(6e^5a^2c^2+40d^2e^3a\,c^3+5d^4e\,c^4)x^{10}}{10}$
gosper	$\frac{1}{2}x^{10}d^4e\,c^4 + \frac{10}{11}x^{11}d^3e^2c^4 + 5x^4d^4ec\,a^3 + 8x^5d^3e^2c\,a^3 + \frac{20}{3}x^6d^2e^3c\,a^3 + \frac{1}{3}x^{12}e^5a\,c^3 + \frac{5}{6}x^{12}d^2$
risch	$\frac{1}{2}x^{10}d^4e\,c^4 + \frac{10}{11}x^{11}d^3e^2c^4 + 5x^4d^4ec\,a^3 + 8x^5d^3e^2c\,a^3 + \frac{20}{3}x^6d^2e^3c\,a^3 + \frac{1}{3}x^{12}e^5a\,c^3 + \frac{5}{6}x^{12}d^2$
parallelrisch	$\frac{1}{2}x^{10}d^4e\,c^4 + \frac{10}{11}x^{11}d^3e^2c^4 + 5x^4d^4ec\,a^3 + 8x^5d^3e^2c\,a^3 + \frac{20}{3}x^6d^2e^3c\,a^3 + \frac{1}{3}x^{12}e^5a\,c^3 + \frac{5}{6}x^{12}d^2$
orering	$x(6435e^5c^4x^{13}+34650d\,e^4c^4x^{12}+30030a\,c^3e^5x^{11}+75075c^4d^2e^3x^{11}+163800a\,c^3d\,e^4x^{10}+81900c^4d^3e^2x^{10}+54054a^2c^2e^5x^9+$

input
```
int((e*x+d)^5*(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
d^5*a^4*x+5/2*d^4*e*a^4*x^2+(10/3*d^3*e^2*a^4+4/3*d^5*c*a^3)*x^3+(5/2*d^2*
e^3*a^4+5*d^4*e*c*a^3)*x^4+(d*e^4*a^4+8*d^3*e^2*c*a^3+6/5*d^5*a^2*c^2)*x^5
+(1/6*e^5*a^4+20/3*d^2*e^3*c*a^3+5*d^4*e*a^2*c^2)*x^6+(20/7*d*e^4*c*a^3+60
/7*d^3*e^2*a^2*c^2+4/7*d^5*a*c^3)*x^7+(1/2*e^5*c*a^3+15/2*d^2*e^3*a^2*c^2+
5/2*d^4*e*a*c^3)*x^8+(10/3*d*e^4*a^2*c^2+40/9*d^3*e^2*a*c^3+1/9*d^5*c^4)*x
^9+(3/5*e^5*a^2*c^2+4*d^2*e^3*a*c^3+1/2*d^4*e*c^4)*x^10+(20/11*d*e^4*a*c^3
+10/11*d^3*e^2*c^4)*x^11+(1/3*e^5*a*c^3+5/6*d^2*e^3*c^4)*x^12+5/13*d*e^4*c
^4*x^13+1/14*e^5*c^4*x^14
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.35

$$\int (d+ex)^5 \left(a+cx^2\right)^4 \, dx = \frac{1}{14}\,c^4e^5x^{14} + \frac{5}{13}\,c^4de^4x^{13} + \frac{1}{6}\,\left(5\,c^4d^2e^3 + 2\,ac^3e^5\right)x^{12} + \frac{10}{11}\,\left(c^4d^3e^2 + 2\,ac^3de^4\right)x^{11} + \frac{5}{2}\,a^4d^4ex^2 + \frac{1}{10}\,\left(5\,c^4d^4e + 40\,ac^3d^2e^3 + 6\,a^2c^2e^5\right)x^{10} + a^4d^5x + \frac{1}{9}\,\left(c^4d^5 + 40\,ac^3d^3e^2 + 30\,a^2c^2de^4\right)x^9 + \frac{1}{2}\,\left(5\,ac^3d^4e + 15\,a^2c^2d^2e^3 + a^3ce^5\right)x^8 + \frac{4}{7}\,\left(ac^3d^5 + 15\,a^2c^2d^3e^2 + 5\,a^3cde^4\right)x^7 + \frac{1}{6}\,\left(30\,a^2c^2d^4e + 40\,a^3cd^2e^3 + a^4e^5\right)x^6 + \frac{1}{5}\,\left(6\,a^2c^2d^5 + 40\,a^3cd^3e^2 + 5\,a^4de^4\right)x^5 + \frac{5}{2}\,\left(2\,a^3cd^4e + a^4d^2e^3\right)x^4 + \frac{2}{3}\,\left(2\,a^3cd^5 + 5\,a^4d^3e^2\right)x^3$$

input
```
integrate((e*x+d)^5*(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/14*c^4*e^5*x^14 + 5/13*c^4*d*e^4*x^13 + 1/6*(5*c^4*d^2*e^3 + 2*a*c^3*e^5
)*x^12 + 10/11*(c^4*d^3*e^2 + 2*a*c^3*d*e^4)*x^11 + 5/2*a^4*d^4*e*x^2 + 1/
10*(5*c^4*d^4*e + 40*a*c^3*d^2*e^3 + 6*a^2*c^2*e^5)*x^10 + a^4*d^5*x + 1/9
*(c^4*d^5 + 40*a*c^3*d^3*e^2 + 30*a^2*c^2*d*e^4)*x^9 + 1/2*(5*a*c^3*d^4*e
+ 15*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^8 + 4/7*(a*c^3*d^5 + 15*a^2*c^2*d^3*e^
2 + 5*a^3*c*d*e^4)*x^7 + 1/6*(30*a^2*c^2*d^4*e + 40*a^3*c*d^2*e^3 + a^4*e^
5)*x^6 + 1/5*(6*a^2*c^2*d^5 + 40*a^3*c*d^3*e^2 + 5*a^4*d*e^4)*x^5 + 5/2*(2
*a^3*c*d^4*e + a^4*d^2*e^3)*x^4 + 2/3*(2*a^3*c*d^5 + 5*a^4*d^3*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.50

$$\int (d+ex)^5 \left(a+cx^2\right)^4 dx = a^4d^5x + \frac{5a^4d^4ex^2}{2} + \frac{5c^4de^4x^{13}}{13} + \frac{c^4e^5x^{14}}{14} + x^{12}\left(\frac{ac^3e^5}{3} + \frac{5c^4d^2e^3}{6}\right) + x^{11}\cdot\left(\frac{20ac^3de^4}{11} + \frac{10c^4d^3e^2}{11}\right) + x^{10}\cdot\left(\frac{3a^2c^2e^5}{5} + 4ac^3d^2e^3 + \frac{c^4d^4e}{2}\right) + x^9\cdot\left(\frac{10a^2c^2de^4}{3} + \frac{40ac^3d^3e^2}{9} + \frac{c^4d^5}{9}\right) + x^8\left(\frac{a^3ce^5}{2} + \frac{15a^2c^2d^2e^3}{2} + \frac{5ac^3d^4e}{2}\right) + x^7\cdot\left(\frac{20a^3cde^4}{7} + \frac{60a^2c^2d^3e^2}{7} + \frac{4ac^3d^5}{7}\right) + x^6\left(\frac{a^4e^5}{6} + \frac{20a^3cd^2e^3}{3} + 5a^2c^2d^4e\right) + x^5\left(a^4de^4 + 8a^3cd^3e^2 + \frac{6a^2c^2d^5}{5}\right) + x^4 \cdot\left(\frac{5a^4d^2e^3}{2} + 5a^3cd^4e\right) + x^3\cdot\left(\frac{10a^4d^3e^2}{3} + \frac{4a^3cd^5}{3}\right)$$

input
```
integrate((e*x+d)**5*(c*x**2+a)**4,x)
```

output
```
a**4*d**5*x + 5*a**4*d**4*e*x**2/2 + 5*c**4*d*e**4*x**13/13 + c**4*e**5*x*
*14/14 + x**12*(a*c**3*e**5/3 + 5*c**4*d**2*e**3/6) + x**11*(20*a*c**3*d*e
**4/11 + 10*c**4*d**3*e**2/11) + x**10*(3*a**2*c**2*e**5/5 + 4*a*c**3*d**2
*e**3 + c**4*d**4*e/2) + x**9*(10*a**2*c**2*d*e**4/3 + 40*a*c**3*d**3*e**2
/9 + c**4*d**5/9) + x**8*(a**3*c*e**5/2 + 15*a**2*c**2*d**2*e**3/2 + 5*a*c
**3*d**4*e/2) + x**7*(20*a**3*c*d*e**4/7 + 60*a**2*c**2*d**3*e**2/7 + 4*a*
c**3*d**5/7) + x**6*(a**4*e**5/6 + 20*a**3*c*d**2*e**3/3 + 5*a**2*c**2*d**
4*e) + x**5*(a**4*d*e**4 + 8*a**3*c*d**3*e**2 + 6*a**2*c**2*d**5/5) + x**4
*(5*a**4*d**2*e**3/2 + 5*a**3*c*d**4*e) + x**3*(10*a**4*d**3*e**2/3 + 4*a*
*3*c*d**5/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.35

$$\int (d+ex)^5 \left(a+cx^2\right)^4 dx = \frac{1}{14}c^4e^5x^{14} + \frac{5}{13}c^4de^4x^{13} + \frac{1}{6}\left(5c^4d^2e^3 + 2ac^3e^5\right)x^{12} + \frac{10}{11}\left(c^4d^3e^2 + 2ac^3de^4\right)x^{11} + \frac{5}{2}a^4d^4ex^2 + \frac{1}{10}\left(5c^4d^4e + 40ac^3d^2e^3 + 6a^2c^2e^5\right)x^{10} + a^4d^5x + \frac{1}{9}\left(c^4d^5 + 40ac^3d^3e^2 + 30a^2c^2de^4\right)x^9 + \frac{1}{2}\left(5ac^3d^4e + 15a^2c^2d^2e^3 + a^3ce^5\right)x^8 + \frac{4}{7}\left(ac^3d^5 + 15a^2c^2d^3e^2 + 5a^3cde^4\right)x^7 + \frac{1}{6}\left(30a^2c^2d^4e + 40a^3cd^2e^3 + a^4e^5\right)x^6 + \frac{1}{5}\left(6a^2c^2d^5 + 40a^3cd^3e^2 + 5a^4de^4\right)x^5 + \frac{5}{2}\left(2a^3cd^4e + a^4d^2e^3\right)x^4 + \frac{2}{3}\left(2a^3cd^5 + 5a^4d^3e^2\right)x^3$$

input
```
integrate((e*x+d)^5*(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/14*c^4*e^5*x^14 + 5/13*c^4*d*e^4*x^13 + 1/6*(5*c^4*d^2*e^3 + 2*a*c^3*e^5
)*x^12 + 10/11*(c^4*d^3*e^2 + 2*a*c^3*d*e^4)*x^11 + 5/2*a^4*d^4*e*x^2 + 1/
10*(5*c^4*d^4*e + 40*a*c^3*d^2*e^3 + 6*a^2*c^2*e^5)*x^10 + a^4*d^5*x + 1/9
*(c^4*d^5 + 40*a*c^3*d^3*e^2 + 30*a^2*c^2*d*e^4)*x^9 + 1/2*(5*a*c^3*d^4*e
+ 15*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^8 + 4/7*(a*c^3*d^5 + 15*a^2*c^2*d^3*e^
2 + 5*a^3*c*d*e^4)*x^7 + 1/6*(30*a^2*c^2*d^4*e + 40*a^3*c*d^2*e^3 + a^4*e^
5)*x^6 + 1/5*(6*a^2*c^2*d^5 + 40*a^3*c*d^3*e^2 + 5*a^4*d*e^4)*x^5 + 5/2*(2
*a^3*c*d^4*e + a^4*d^2*e^3)*x^4 + 2/3*(2*a^3*c*d^5 + 5*a^4*d^3*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.43

$$\int (d+ex)^5 \left(a+cx^2\right)^4 \, dx = \frac{1}{14} c^4 e^5 x^{14} + \frac{5}{13} c^4 d e^4 x^{13} + \frac{5}{6} c^4 d^2 e^3 x^{12} + \frac{1}{3} a c^3 e^5 x^{12} + \frac{10}{11} c^4 d^3 e^2 x^{11} + \frac{20}{11} a c^3 d e^4 x^{11} + \frac{1}{2} c^4 d^4 e x^{10} + 4 a c^3 d^2 e^3 x^{10} + \frac{3}{5} a^2 c^2 e^5 x^{10} + \frac{1}{9} c^4 d^5 x^9 + \frac{40}{9} a c^3 d^3 e^2 x^9 + \frac{10}{3} a^2 c^2 d e^4 x^9 + \frac{5}{2} a c^3 d^4 e x^8 + \frac{15}{2} a^2 c^2 d^2 e^3 x^8 + \frac{1}{2} a^3 c e^5 x^8 + \frac{4}{7} a c^3 d^5 x^7 + \frac{60}{7} a^2 c^2 d^3 e^2 x^7 + \frac{20}{7} a^3 c d e^4 x^7 + 5 a^2 c^2 d^4 e x^6 + \frac{20}{3} a^3 c d^2 e^3 x^6 + \frac{1}{6} a^4 e^5 x^6 + \frac{6}{5} a^2 c^2 d^5 x^5 + 8 a^3 c d^3 e^2 x^5 + a^4 d e^4 x^5 + 5 a^3 c d^4 e x^4 + \frac{5}{2} a^4 d^2 e^3 x^4 + \frac{4}{3} a^3 c d^5 x^3 + \frac{10}{3} a^4 d^3 e^2 x^3 + \frac{5}{2} a^4 d^4 e x^2 + a^4 d^5 x$$

input
```
integrate((e*x+d)^5*(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/14*c^4*e^5*x^14 + 5/13*c^4*d*e^4*x^13 + 5/6*c^4*d^2*e^3*x^12 + 1/3*a*c^3
*e^5*x^12 + 10/11*c^4*d^3*e^2*x^11 + 20/11*a*c^3*d*e^4*x^11 + 1/2*c^4*d^4*
e*x^10 + 4*a*c^3*d^2*e^3*x^10 + 3/5*a^2*c^2*e^5*x^10 + 1/9*c^4*d^5*x^9 + 4
0/9*a*c^3*d^3*e^2*x^9 + 10/3*a^2*c^2*d*e^4*x^9 + 5/2*a*c^3*d^4*e*x^8 + 15/
2*a^2*c^2*d^2*e^3*x^8 + 1/2*a^3*c*e^5*x^8 + 4/7*a*c^3*d^5*x^7 + 60/7*a^2*c
^2*d^3*e^2*x^7 + 20/7*a^3*c*d*e^4*x^7 + 5*a^2*c^2*d^4*e*x^6 + 20/3*a^3*c*d
^2*e^3*x^6 + 1/6*a^4*e^5*x^6 + 6/5*a^2*c^2*d^5*x^5 + 8*a^3*c*d^3*e^2*x^5 +
 a^4*d*e^4*x^5 + 5*a^3*c*d^4*e*x^4 + 5/2*a^4*d^2*e^3*x^4 + 4/3*a^3*c*d^5*x
^3 + 10/3*a^4*d^3*e^2*x^3 + 5/2*a^4*d^4*e*x^2 + a^4*d^5*x
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.28

$$\int (d+ex)^5 \left(a+cx^2\right)^4 dx = x^3 \left(\frac{10 a^4 d^3 e^2}{3} + \frac{4 c a^3 d^5}{3}\right) + x^{12} \left(\frac{5 c^4 d^2 e^3}{6} + \frac{a c^3 e^5}{3}\right) + x^5 \left(a^4 d e^4 + 8 a^3 c d^3 e^2 + \frac{6 a^2 c^2 d^5}{5}\right) + x^6 \left(\frac{a^4 e^5}{6} + \frac{20 a^3 c d^2 e^3}{3} + 5 a^2 c^2 d^4 e\right) + x^9 \left(\frac{10 a^2 c^2 d e^4}{3} + \frac{40 a c^3 d^3 e^2}{9} + \frac{c^4 d^5}{9}\right) + x^{10} \left(\frac{3 a^2 c^2 e^5}{5} + 4 a c^3 d^2 e^3 + \frac{c^4 d^4 e}{2}\right) + a^4 d^5 x + \frac{c^4 e^5 x^{14}}{14} + \frac{5 a^4 d^4 e x^2}{2} + \frac{5 c^4 d e^4 x^{13}}{13} + \frac{4 a c d x^7 (5 a^2 e^4 + 15 a c d^2 e^2 + c^2 d^4)}{7} + \frac{a c e x^8 (a^2 e^4 + 15 a c d^2 e^2 + 5 c^2 d^4)}{2} + \frac{5 a^3 d^2 e x^4 (2 c d^2 + a e^2)}{2} + \frac{10 c^3 d e^2 x^{11} (c d^2 + 2 a e^2)}{11}$$

input
```
int((a + c*x^2)^4*(d + e*x)^5,x)
```

output
```
x^3*((4*a^3*c*d^5)/3 + (10*a^4*d^3*e^2)/3) + x^12*((a*c^3*e^5)/3 + (5*c^4*
d^2*e^3)/6) + x^5*(a^4*d*e^4 + (6*a^2*c^2*d^5)/5 + 8*a^3*c*d^3*e^2) + x^6*
((a^4*e^5)/6 + 5*a^2*c^2*d^4*e + (20*a^3*c*d^2*e^3)/3) + x^9*((c^4*d^5)/9
+ (40*a*c^3*d^3*e^2)/9 + (10*a^2*c^2*d*e^4)/3) + x^10*((c^4*d^4*e)/2 + (3*
a^2*c^2*e^5)/5 + 4*a*c^3*d^2*e^3) + a^4*d^5*x + (c^4*e^5*x^14)/14 + (5*a^4
*d^4*e*x^2)/2 + (5*c^4*d*e^4*x^13)/13 + (4*a*c*d*x^7*(5*a^2*e^4 + c^2*d^4
+ 15*a*c*d^2*e^2))/7 + (a*c*e*x^8*(a^2*e^4 + 5*c^2*d^4 + 15*a*c*d^2*e^2))/
2 + (5*a^3*d^2*e*x^4*(a*e^2 + 2*c*d^2))/2 + (10*c^3*d*e^2*x^11*(2*a*e^2 +
c*d^2))/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.44

$$\int (d+ex)^5 \left(a+cx^2\right)^4 \, dx = \frac{x(6435c^4e^5x^{13} + 34650c^4d\,e^4x^{12} + 30030a\,c^3e^5x^{11} + 75075c^4d^2e^3x^{11} + 163800a\,c^3d\,e^4x^{10} + 81900c^4d^3e^2x}{}$$

input
```
int((e*x+d)^5*(c*x^2+a)^4,x)
```

output
```
(x*(90090*a**4*d**5 + 225225*a**4*d**4*e*x + 300300*a**4*d**3*e**2*x**2 +
225225*a**4*d**2*e**3*x**3 + 90090*a**4*d*e**4*x**4 + 15015*a**4*e**5*x**5
 + 120120*a**3*c*d**5*x**2 + 450450*a**3*c*d**4*e*x**3 + 720720*a**3*c*d**
3*e**2*x**4 + 600600*a**3*c*d**2*e**3*x**5 + 257400*a**3*c*d*e**4*x**6 + 4
5045*a**3*c*e**5*x**7 + 108108*a**2*c**2*d**5*x**4 + 450450*a**2*c**2*d**4
*e*x**5 + 772200*a**2*c**2*d**3*e**2*x**6 + 675675*a**2*c**2*d**2*e**3*x**
7 + 300300*a**2*c**2*d*e**4*x**8 + 54054*a**2*c**2*e**5*x**9 + 51480*a*c**
3*d**5*x**6 + 225225*a*c**3*d**4*e*x**7 + 400400*a*c**3*d**3*e**2*x**8 + 3
60360*a*c**3*d**2*e**3*x**9 + 163800*a*c**3*d*e**4*x**10 + 30030*a*c**3*e*
*5*x**11 + 10010*c**4*d**5*x**8 + 45045*c**4*d**4*e*x**9 + 81900*c**4*d**3
*e**2*x**10 + 75075*c**4*d**2*e**3*x**11 + 34650*c**4*d*e**4*x**12 + 6435*
c**4*e**5*x**13))/90090
```

3.92 $\int (d+ex)^4 \left(a+cx^2\right)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 270

$$\begin{aligned}\int (d+ex)^4 \left(a+cx^2\right)^4 \, dx = {} & a^4d^4x+\frac{2}{3}a^3d^2\left(2cd^2+3ae^2\right)x^3+a^4de^3x^4 \\ & +\frac{1}{5}a^2\left(6c^2d^4+24acd^2e^2+a^2e^4\right)x^5 \\ & +\frac{8}{3}a^3cde^3x^6+\frac{4}{7}ac\left(c^2d^4+9acd^2e^2+a^2e^4\right)x^7 \\ & +3a^2c^2de^3x^8+\frac{1}{9}c^2\left(c^2d^4+24acd^2e^2+6a^2e^4\right)x^9 \\ & +\frac{8}{5}ac^3de^3x^{10}+\frac{2}{11}c^3e^2\left(3cd^2+2ae^2\right)x^{11} \\ & +\frac{1}{3}c^4de^3x^{12}+\frac{1}{13}c^4e^4x^{13}+\frac{2d^3e\left(a+cx^2\right)^5}{5c}\end{aligned}$$

output
```
a^4*d^4*x+2/3*a^3*d^2*(3*a*e^2+2*c*d^2)*x^3+a^4*d*e^3*x^4+1/5*a^2*(a^2*e^4
+24*a*c*d^2*e^2+6*c^2*d^4)*x^5+8/3*a^3*c*d*e^3*x^6+4/7*a*c*(a^2*e^4+9*a*c*
d^2*e^2+c^2*d^4)*x^7+3*a^2*c^2*d*e^3*x^8+1/9*c^2*(6*a^2*e^4+24*a*c*d^2*e^2
+c^2*d^4)*x^9+8/5*a*c^3*d*e^3*x^10+2/11*c^3*e^2*(2*a*e^2+3*c*d^2)*x^11+1/3
*c^4*d*e^3*x^12+1/13*c^4*e^4*x^13+2/5*d^3*e*(c*x^2+a)^5/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11

$$\int (d+ex)^4 \left(a+cx^2\right)^4 dx = a^4d^4x + 2a^4d^3ex^2 + \frac{2}{3}a^3d^2\left(2cd^2+3ae^2\right)x^3 + a^3de\left(4cd^2 + ae^2\right)x^4 + \frac{1}{5}a^2\left(6c^2d^4+24acd^2e^2+a^2e^4\right)x^5 + \frac{4}{3}a^2cde\left(3cd^2+2ae^2\right)x^6 + \frac{4}{7}ac\left(c^2d^4+9acd^2e^2+a^2e^4\right)x^7 + ac^2de\left(2cd^2+3ae^2\right)x^8 + \frac{1}{9}c^2\left(c^2d^4+24acd^2e^2+6a^2e^4\right)x^9 + \frac{2}{5}c^3de\left(cd^2+4ae^2\right)x^{10} + \frac{2}{11}c^3e^2\left(3cd^2+2ae^2\right)x^{11} + \frac{1}{3}c^4de^3x^{12} + \frac{1}{13}c^4e^4x^{13}$$

input
```
Integrate[(d + e*x)^4*(a + c*x^2)^4,x]
```

output
```
a^4*d^4*x + 2*a^4*d^3*e*x^2 + (2*a^3*d^2*(2*c*d^2 + 3*a*e^2)*x^3)/3 + a^3*
d*e*(4*c*d^2 + a*e^2)*x^4 + (a^2*(6*c^2*d^4 + 24*a*c*d^2*e^2 + a^2*e^4)*x^
5)/5 + (4*a^2*c*d*e*(3*c*d^2 + 2*a*e^2)*x^6)/3 + (4*a*c*(c^2*d^4 + 9*a*c*d
^2*e^2 + a^2*e^4)*x^7)/7 + a*c^2*d*e*(2*c*d^2 + 3*a*e^2)*x^8 + (c^2*(c^2*d
^4 + 24*a*c*d^2*e^2 + 6*a^2*e^4)*x^9)/9 + (2*c^3*d*e*(c*d^2 + 4*a*e^2)*x^1
0)/5 + (2*c^3*e^2*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (c^4*d*e^3*x^12)/3 + (c^4
*e^4*x^13)/13
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^4 (d+ex)^4 dx$$

$$\Big\downarrow \ 475$$

$$\int \left(c^4e^4x^{12}+4c^4de^3x^{11}+2c^3e^2\left(3cd^2+2ae^2\right)x^{10}+16ac^3de^3x^9+c^2\left(c^2d^4+24ace^2d^2+6a^2e^4\right)x^8+24a^2c^2de^3x$$

$$\frac{2d^3e\left(a+cx^2\right)^5}{5c}$$

$$\Big\downarrow \; 2009$$

$$a^4d^4x+a^4de^3x^4+\frac{2}{3}a^3d^2x^3\left(3ae^2+2cd^2\right)+\frac{8}{3}a^3cde^3x^6+\frac{1}{9}c^2x^9\left(6a^2e^4+24acd^2e^2+c^2d^4\right)+\frac{4}{7}acx^7\left(a^2e^4+9acd^2e^2+c^2d^4\right)+\frac{1}{5}a^2x^5\left(a^2e^4+24acd^2e^2+6c^2d^4\right)+3a^2c^2de^3x^8+\frac{2}{11}c^3e^2x^{11}\left(2ae^2+3cd^2\right)+\frac{8}{5}ac^3de^3x^{10}+\frac{2d^3e\left(a+cx^2\right)^5}{5c}+\frac{1}{3}c^4de^3x^{12}+\frac{1}{13}c^4e^4x^{13}$$

input
```
Int[(d + e*x)^4*(a + c*x^2)^4,x]
```

output
```
a^4*d^4*x + (2*a^3*d^2*(2*c*d^2 + 3*a*e^2)*x^3)/3 + a^4*d*e^3*x^4 + (a^2*(
6*c^2*d^4 + 24*a*c*d^2*e^2 + a^2*e^4)*x^5)/5 + (8*a^3*c*d*e^3*x^6)/3 + (4*
a*c*(c^2*d^4 + 9*a*c*d^2*e^2 + a^2*e^4)*x^7)/7 + 3*a^2*c^2*d*e^3*x^8 + (c^
2*(c^2*d^4 + 24*a*c*d^2*e^2 + 6*a^2*e^4)*x^9)/9 + (8*a*c^3*d*e^3*x^10)/5 +
 (2*c^3*e^2*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (c^4*d*e^3*x^12)/3 + (c^4*e^4*x
^13)/13 + (2*d^3*e*(a + c*x^2)^5)/(5*c)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.13

method	result
norman	$\frac{c^4 e^4 x^{13}}{13}+\frac{c^4 d e^3 x^{12}}{3}+\left(\frac{4}{11}e^4 a c^3+\frac{6}{11}d^2 e^2 c^4\right)x^{11}+\left(\frac{8}{5}d e^3 a c^3+\frac{2}{5}d^3 e c^4\right)x^{10}+\left(\frac{2}{3}e^4 a^2 c^2+\frac{8}{3}d^2 e\right.$
default	$\frac{c^4 e^4 x^{13}}{13}+\frac{c^4 d e^3 x^{12}}{3}+\frac{(4e^4 a c^3+6d^2 e^2 c^4)x^{11}}{11}+\frac{(16 d e^3 a c^3+4d^3 e c^4)x^{10}}{10}+\frac{(6e^4 a^2 c^2+24d^2 e^2 a c^3+d^4 c^4)x^9}{9}+\frac{(24}{}$
gosper	$\frac{24}{5}x^5 d^2 e^2 c a^3+2a c^3 d^3 e x^8+4a^3 c d^3 e x^4+\frac{1}{5}x^5 e^4 a^4+\frac{8}{3}x^9 d^2 e^2 a c^3+\frac{36}{7}x^7 d^2 e^2 a^2 c^2+4x^6 d^3 e$
risch	$\frac{24}{5}x^5 d^2 e^2 c a^3+2a c^3 d^3 e x^8+4a^3 c d^3 e x^4+\frac{1}{5}x^5 e^4 a^4+\frac{8}{3}x^9 d^2 e^2 a c^3+\frac{36}{7}x^7 d^2 e^2 a^2 c^2+4x^6 d^3 e$
parallelrisch	$\frac{24}{5}x^5 d^2 e^2 c a^3+2a c^3 d^3 e x^8+4a^3 c d^3 e x^4+\frac{1}{5}x^5 e^4 a^4+\frac{8}{3}x^9 d^2 e^2 a c^3+\frac{36}{7}x^7 d^2 e^2 a^2 c^2+4x^6 d^3 e$
orering	$\frac{x(3465e^4 c^4 x^{12}+15015 d e^3 c^4 x^{11}+16380 a c^3 e^4 x^{10}+24570 c^4 d^2 e^2 x^{10}+72072 a c^3 d e^3 x^9+18018 c^4 d^3 e x^9+30030 a^2 c^2 e^4 x^8+120}{}$

input
```
int((e*x+d)^4*(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^4*e^4*x^13+1/3*c^4*d*e^3*x^12+(4/11*e^4*a*c^3+6/11*d^2*e^2*c^4)*x^1
1+(8/5*d*e^3*a*c^3+2/5*d^3*e*c^4)*x^10+(2/3*e^4*a^2*c^2+8/3*d^2*e^2*a*c^3+
1/9*d^4*c^4)*x^9+(3*a^2*c^2*d*e^3+2*a*c^3*d^3*e)*x^8+(4/7*e^4*c*a^3+36/7*d
^2*e^2*a^2*c^2+4/7*d^4*a*c^3)*x^7+(8/3*d*e^3*c*a^3+4*d^3*e*a^2*c^2)*x^6+(1
/5*e^4*a^4+24/5*d^2*e^2*c*a^3+6/5*d^4*a^2*c^2)*x^5+(a^4*d*e^3+4*a^3*c*d^3*
e)*x^4+(2*d^2*e^2*a^4+4/3*d^4*c*a^3)*x^3+2*d^3*e*a^4*x^2+a^4*d^4*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.13

$$\begin{aligned}\int (d+ex)^4 (a+cx^2)^4 \, dx = {} & \frac{1}{13} c^4 e^4 x^{13} + \frac{1}{3} c^4 d e^3 x^{12} + \frac{2}{11} \left(3 c^4 d^2 e^2 + 2 a c^3 e^4\right) x^{11} \\ & + \frac{2}{5} \left(c^4 d^3 e + 4 a c^3 d e^3\right) x^{10} + 2 a^4 d^3 e x^2 \\ & + \frac{1}{9} \left(c^4 d^4 + 24 a c^3 d^2 e^2 + 6 a^2 c^2 e^4\right) x^9 \\ & + a^4 d^4 x + \left(2 a c^3 d^3 e + 3 a^2 c^2 d e^3\right) x^8 \\ & + \frac{4}{7} \left(a c^3 d^4 + 9 a^2 c^2 d^2 e^2 + a^3 c e^4\right) x^7 \\ & + \frac{4}{3} \left(3 a^2 c^2 d^3 e + 2 a^3 c d e^3\right) x^6 \\ & + \frac{1}{5} \left(6 a^2 c^2 d^4 + 24 a^3 c d^2 e^2 + a^4 e^4\right) x^5 \\ & + \left(4 a^3 c d^3 e + a^4 d e^3\right) x^4 + \frac{2}{3} \left(2 a^3 c d^4 + 3 a^4 d^2 e^2\right) x^3\end{aligned}$$

input

```
integrate((e*x+d)^4*(c*x^2+a)^4,x, algorithm="fricas")
```

output

```
1/13*c^4*e^4*x^13 + 1/3*c^4*d*e^3*x^12 + 2/11*(3*c^4*d^2*e^2 + 2*a*c^3*e^4
)*x^11 + 2/5*(c^4*d^3*e + 4*a*c^3*d*e^3)*x^10 + 2*a^4*d^3*e*x^2 + 1/9*(c^4
*d^4 + 24*a*c^3*d^2*e^2 + 6*a^2*c^2*e^4)*x^9 + a^4*d^4*x + (2*a*c^3*d^3*e
+ 3*a^2*c^2*d*e^3)*x^8 + 4/7*(a*c^3*d^4 + 9*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x
^7 + 4/3*(3*a^2*c^2*d^3*e + 2*a^3*c*d*e^3)*x^6 + 1/5*(6*a^2*c^2*d^4 + 24*a
^3*c*d^2*e^2 + a^4*e^4)*x^5 + (4*a^3*c*d^3*e + a^4*d*e^3)*x^4 + 2/3*(2*a^3
*c*d^4 + 3*a^4*d^2*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.26

$$\int (d+ex)^4\left(a+cx^2\right)^4\,dx = a^4d^4x + 2a^4d^3ex^2 + \frac{c^4de^3x^{12}}{3} + \frac{c^4e^4x^{13}}{13} + x^{11}\cdot\left(\frac{4ac^3e^4}{11} + \frac{6c^4d^2e^2}{11}\right) + x^{10}\cdot\left(\frac{8ac^3de^3}{5} + \frac{2c^4d^3e}{5}\right) + x^9\cdot\left(\frac{2a^2c^2e^4}{3} + \frac{8ac^3d^2e^2}{3} + \frac{c^4d^4}{9}\right) + x^8\cdot\left(3a^2c^2de^3 + 2ac^3d^3e\right) + x^7\cdot\left(\frac{4a^3ce^4}{7} + \frac{36a^2c^2d^2e^2}{7} + \frac{4ac^3d^4}{7}\right) + x^6\cdot\left(\frac{8a^3cde^3}{3} + 4a^2c^2d^3e\right) + x^5\left(\frac{a^4e^4}{5} + \frac{24a^3cd^2e^2}{5} + \frac{6a^2c^2d^4}{5}\right) + x^4\left(a^4de^3 + 4a^3cd^3e\right) + x^3\cdot\left(2a^4d^2e^2 + \frac{4a^3cd^4}{3}\right)$$

input
```
integrate((e*x+d)**4*(c*x**2+a)**4,x)
```

output
```
a**4*d**4*x + 2*a**4*d**3*e*x**2 + c**4*d*e**3*x**12/3 + c**4*e**4*x**13/1
3 + x**11*(4*a*c**3*e**4/11 + 6*c**4*d**2*e**2/11) + x**10*(8*a*c**3*d*e**
3/5 + 2*c**4*d**3*e/5) + x**9*(2*a**2*c**2*e**4/3 + 8*a*c**3*d**2*e**2/3 +
 c**4*d**4/9) + x**8*(3*a**2*c**2*d*e**3 + 2*a*c**3*d**3*e) + x**7*(4*a**3
*c*e**4/7 + 36*a**2*c**2*d**2*e**2/7 + 4*a*c**3*d**4/7) + x**6*(8*a**3*c*d
*e**3/3 + 4*a**2*c**2*d**3*e) + x**5*(a**4*e**4/5 + 24*a**3*c*d**2*e**2/5
+ 6*a**2*c**2*d**4/5) + x**4*(a**4*d*e**3 + 4*a**3*c*d**3*e) + x**3*(2*a**
4*d**2*e**2 + 4*a**3*c*d**4/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.13

$$\int (d+ex)^4 \left(a+cx^2\right)^4 dx = \frac{1}{13}c^4e^4x^{13} + \frac{1}{3}c^4de^3x^{12} + \frac{2}{11}\left(3c^4d^2e^2 + 2ac^3e^4\right)x^{11} + \frac{2}{5}\left(c^4d^3e + 4ac^3de^3\right)x^{10} + 2a^4d^3ex^2 + \frac{1}{9}\left(c^4d^4 + 24ac^3d^2e^2 + 6a^2c^2e^4\right)x^9 + a^4d^4x + \left(2ac^3d^3e + 3a^2c^2de^3\right)x^8 + \frac{4}{7}\left(ac^3d^4 + 9a^2c^2d^2e^2 + a^3ce^4\right)x^7 + \frac{4}{3}\left(3a^2c^2d^3e + 2a^3cde^3\right)x^6 + \frac{1}{5}\left(6a^2c^2d^4 + 24a^3cd^2e^2 + a^4e^4\right)x^5 + \left(4a^3cd^3e + a^4de^3\right)x^4 + \frac{2}{3}\left(2a^3cd^4 + 3a^4d^2e^2\right)x^3$$

input

```
integrate((e*x+d)^4*(c*x^2+a)^4,x, algorithm="maxima")
```

output

```
1/13*c^4*e^4*x^13 + 1/3*c^4*d*e^3*x^12 + 2/11*(3*c^4*d^2*e^2 + 2*a*c^3*e^4
)*x^11 + 2/5*(c^4*d^3*e + 4*a*c^3*d*e^3)*x^10 + 2*a^4*d^3*e*x^2 + 1/9*(c^4
*d^4 + 24*a*c^3*d^2*e^2 + 6*a^2*c^2*e^4)*x^9 + a^4*d^4*x + (2*a*c^3*d^3*e
+ 3*a^2*c^2*d*e^3)*x^8 + 4/7*(a*c^3*d^4 + 9*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x
^7 + 4/3*(3*a^2*c^2*d^3*e + 2*a^3*c*d*e^3)*x^6 + 1/5*(6*a^2*c^2*d^4 + 24*a
^3*c*d^2*e^2 + a^4*e^4)*x^5 + (4*a^3*c*d^3*e + a^4*d*e^3)*x^4 + 2/3*(2*a^3
*c*d^4 + 3*a^4*d^2*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.19

$$\int (d+ex)^4 \left(a+cx^2\right)^4 \, dx = \frac{1}{13} c^4 e^4 x^{13} + \frac{1}{3} c^4 d e^3 x^{12} + \frac{6}{11} c^4 d^2 e^2 x^{11} + \frac{4}{11} a c^3 e^4 x^{11} + \frac{2}{5} c^4 d^3 e x^{10} + \frac{8}{5} a c^3 d e^3 x^{10} + \frac{1}{9} c^4 d^4 x^9 + \frac{8}{3} a c^3 d^2 e^2 x^9 + \frac{2}{3} a^2 c^2 e^4 x^9 + 2 a c^3 d^3 e x^8 + 3 a^2 c^2 d e^3 x^8 + \frac{4}{7} a c^3 d^4 x^7 + \frac{36}{7} a^2 c^2 d^2 e^2 x^7 + \frac{4}{7} a^3 c e^4 x^7 + 4 a^2 c^2 d^3 e x^6 + \frac{8}{3} a^3 c d e^3 x^6 + \frac{6}{5} a^2 c^2 d^4 x^5 + \frac{24}{5} a^3 c d^2 e^2 x^5 + \frac{1}{5} a^4 e^4 x^5 + 4 a^3 c d^3 e x^4 + a^4 d e^3 x^4 + \frac{4}{3} a^3 c d^4 x^3 + 2 a^4 d^2 e^2 x^3 + 2 a^4 d^3 e x^2 + a^4 d^4 x$$

input
```
integrate((e*x+d)^4*(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/13*c^4*e^4*x^13 + 1/3*c^4*d*e^3*x^12 + 6/11*c^4*d^2*e^2*x^11 + 4/11*a*c^
3*e^4*x^11 + 2/5*c^4*d^3*e*x^10 + 8/5*a*c^3*d*e^3*x^10 + 1/9*c^4*d^4*x^9 +
 8/3*a*c^3*d^2*e^2*x^9 + 2/3*a^2*c^2*e^4*x^9 + 2*a*c^3*d^3*e*x^8 + 3*a^2*c
^2*d*e^3*x^8 + 4/7*a*c^3*d^4*x^7 + 36/7*a^2*c^2*d^2*e^2*x^7 + 4/7*a^3*c*e^
4*x^7 + 4*a^2*c^2*d^3*e*x^6 + 8/3*a^3*c*d*e^3*x^6 + 6/5*a^2*c^2*d^4*x^5 +
24/5*a^3*c*d^2*e^2*x^5 + 1/5*a^4*e^4*x^5 + 4*a^3*c*d^3*e*x^4 + a^4*d*e^3*x
^4 + 4/3*a^3*c*d^4*x^3 + 2*a^4*d^2*e^2*x^3 + 2*a^4*d^3*e*x^2 + a^4*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.07

$$\int (d+ex)^4 \left(a+cx^2\right)^4 \, dx = x^5 \left(\frac{a^4 e^4}{5} + \frac{24 a^3 c d^2 e^2}{5} + \frac{6 a^2 c^2 d^4}{5}\right) + x^9 \left(\frac{2 a^2 c^2 e^4}{3} + \frac{8 a c^3 d^2 e^2}{3} + \frac{c^4 d^4}{9}\right) + x^3 \left(2 a^4 d^2 e^2 + \frac{4 c a^3 d^4}{3}\right) + x^{11} \left(\frac{6 c^4 d^2 e^2}{11} + \frac{4 a c^3 e^4}{11}\right) + a^4 d^4 x + \frac{c^4 e^4 x^{13}}{13} + 2 a^4 d^3 e x^2 + \frac{c^4 d e^3 x^{12}}{3} + \frac{4 a c x^7 (a^2 e^4 + 9 a c d^2 e^2 + c^2 d^4)}{7} + a^3 d e x^4 \left(4 c d^2 + a e^2\right) + \frac{2 c^3 d e x^{10} (c d^2 + 4 a e^2)}{5} + \frac{4 a^2 c d e x^6 (3 c d^2 + 2 a e^2)}{3} + a c^2 d e x^8 \left(2 c d^2 + 3 a e^2\right)$$

input
```
int((a + c*x^2)^4*(d + e*x)^4,x)
```

output
```
x^5*((a^4*e^4)/5 + (6*a^2*c^2*d^4)/5 + (24*a^3*c*d^2*e^2)/5) + x^9*((c^4*d
^4)/9 + (2*a^2*c^2*e^4)/3 + (8*a*c^3*d^2*e^2)/3) + x^3*((4*a^3*c*d^4)/3 +
2*a^4*d^2*e^2) + x^11*((4*a*c^3*e^4)/11 + (6*c^4*d^2*e^2)/11) + a^4*d^4*x
+ (c^4*e^4*x^13)/13 + 2*a^4*d^3*e*x^2 + (c^4*d*e^3*x^12)/3 + (4*a*c*x^7*(a
^2*e^4 + c^2*d^4 + 9*a*c*d^2*e^2))/7 + a^3*d*e*x^4*(a*e^2 + 4*c*d^2) + (2*
c^3*d*e*x^10*(4*a*e^2 + c*d^2))/5 + (4*a^2*c*d*e*x^6*(2*a*e^2 + 3*c*d^2))/
3 + a*c^2*d*e*x^8*(3*a*e^2 + 2*c*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.20

$$\int (d+ex)^4 \left(a+cx^2\right)^4 \, dx = \frac{x(3465c^4e^4x^{12} + 15015c^4 d e^3 x^{11} + 16380 a c^3 e^4 x^{10} + 24570 c^4 d^2 e^2 x^{10} + 72072 a c^3 d e^3 x^9 + 18018 c^4 d^3 e x^9 +}{}$$

input
```
int((e*x+d)^4*(c*x^2+a)^4,x)
```

output
```
(x*(45045*a**4*d**4 + 90090*a**4*d**3*e*x + 90090*a**4*d**2*e**2*x**2 + 45
045*a**4*d*e**3*x**3 + 9009*a**4*e**4*x**4 + 60060*a**3*c*d**4*x**2 + 1801
80*a**3*c*d**3*e*x**3 + 216216*a**3*c*d**2*e**2*x**4 + 120120*a**3*c*d*e**
3*x**5 + 25740*a**3*c*e**4*x**6 + 54054*a**2*c**2*d**4*x**4 + 180180*a**2*
c**2*d**3*e*x**5 + 231660*a**2*c**2*d**2*e**2*x**6 + 135135*a**2*c**2*d*e*
*3*x**7 + 30030*a**2*c**2*e**4*x**8 + 25740*a*c**3*d**4*x**6 + 90090*a*c**
3*d**3*e*x**7 + 120120*a*c**3*d**2*e**2*x**8 + 72072*a*c**3*d*e**3*x**9 +
16380*a*c**3*e**4*x**10 + 5005*c**4*d**4*x**8 + 18018*c**4*d**3*e*x**9 + 2
4570*c**4*d**2*e**2*x**10 + 15015*c**4*d*e**3*x**11 + 3465*c**4*e**4*x**12
))/45045
```

3.93 $\int (d+ex)^3 \left(a+cx^2\right)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 209

$$\begin{aligned}\int (d+ex)^3 \left(a+cx^2\right)^4 \, dx = {} & a^4d^3x+\frac{1}{3}a^3d\left(4cd^2+3ae^2\right)x^3+\frac{1}{4}a^4e^3x^4 \\ & +\frac{6}{5}a^2cd\left(cd^2+2ae^2\right)x^5+\frac{2}{3}a^3ce^3x^6+\frac{2}{7}ac^2d\left(2cd^2+9ae^2\right)x^7 \\ & +\frac{3}{4}a^2c^2e^3x^8+\frac{1}{9}c^3d\left(cd^2+12ae^2\right)x^9+\frac{2}{5}ac^3e^3x^{10} \\ & +\frac{3}{11}c^4de^2x^{11}+\frac{1}{12}c^4e^3x^{12}+\frac{3d^2e(a+cx^2)^5}{10c}\end{aligned}$$

output
```
a^4*d^3*x+1/3*a^3*d*(3*a*e^2+4*c*d^2)*x^3+1/4*a^4*e^3*x^4+6/5*a^2*c*d*(2*a
*e^2+c*d^2)*x^5+2/3*a^3*c*e^3*x^6+2/7*a*c^2*d*(9*a*e^2+2*c*d^2)*x^7+3/4*a^
2*c^2*e^3*x^8+1/9*c^3*d*(12*a*e^2+c*d^2)*x^9+2/5*a*c^3*e^3*x^10+3/11*c^4*d
*e^2*x^11+1/12*c^4*e^3*x^12+3/10*d^2*e*(c*x^2+a)^5/c
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.94

$$\int (d+ex)^3 \left(a+cx^2\right)^4 \, dx$$
$$= \frac{x(3465a^4(4d^3+6d^2ex+4de^2x^2+e^3x^3)+924a^3cx^2(20d^3+45d^2ex+36de^2x^2+10e^3x^3)+297a^2c^2x^4(5}{}$$

input
```
Integrate[(d + e*x)^3*(a + c*x^2)^4,x]
```

output
```
(x*(3465*a^4*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 924*a^3*c*x^2*(
20*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + 297*a^2*c^2*x^4*(56*d^3
 + 140*d^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + 66*a*c^3*x^6*(120*d^3 + 315
*d^2*e*x + 280*d*e^2*x^2 + 84*e^3*x^3) + 7*c^4*x^8*(220*d^3 + 594*d^2*e*x
+ 540*d*e^2*x^2 + 165*e^3*x^3)))/13860
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^4 (d+ex)^3 \, dx$$
$$\Big\downarrow \text{475}$$
$$\int \left(c^4e^3x^{11}+3c^4de^2x^{10}+4ac^3e^3x^9+c^3d\left(cd^2+12ae^2\right)x^8+6a^2c^2e^3x^7+2ac^2d\left(2cd^2+9ae^2\right)x^6+4a^3ce^3x^5+6\right.$$
$$\frac{3d^2e\left(a+cx^2\right)^5}{10c}$$
$$\Big\downarrow \text{2009}$$

$$a^4d^3x+\frac{1}{4}a^4e^3x^4+\frac{1}{3}a^3dx^3\left(3ae^2+4cd^2\right)+\frac{2}{3}a^3ce^3x^6+\frac{3}{4}a^2c^2e^3x^8+\frac{6}{5}a^2cdx^5\left(2ae^2+cd^2\right)+\frac{1}{9}c^3dx^9\left(12ae^2+cd^2\right)+\frac{2}{5}ac^3e^3x^{10}+\frac{2}{7}ac^2dx^7\left(9ae^2+2cd^2\right)+\frac{3d^2e\left(a+cx^2\right)^5}{10c}+\frac{3}{11}c^4de^2x^{11}+\frac{1}{12}c^4e^3x^{12}$$

input
```
Int[(d + e*x)^3*(a + c*x^2)^4,x]
```

output
```
a^4*d^3*x + (a^3*d*(4*c*d^2 + 3*a*e^2)*x^3)/3 + (a^4*e^3*x^4)/4 + (6*a^2*c
*d*(c*d^2 + 2*a*e^2)*x^5)/5 + (2*a^3*c*e^3*x^6)/3 + (2*a*c^2*d*(2*c*d^2 +
9*a*e^2)*x^7)/7 + (3*a^2*c^2*e^3*x^8)/4 + (c^3*d*(c*d^2 + 12*a*e^2)*x^9)/9
 + (2*a*c^3*e^3*x^10)/5 + (3*c^4*d*e^2*x^11)/11 + (c^4*e^3*x^12)/12 + (3*d
^2*e*(a + c*x^2)^5)/(10*c)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.15

method	result
norman	$\frac{c^4 e^3 x^{12}}{12}+\frac{3c^4 d\,e^2 x^{11}}{11}+\left(\frac{2}{5}e^3 a\,c^3+\frac{3}{10}d^2 e\,c^4\right)x^{10}+\left(\frac{4}{3}d\,e^2 a\,c^3+\frac{1}{9}d^3 c^4\right)x^9+\left(\frac{3}{4}e^3 a^2 c^2+\frac{3}{2}d^2 ea\,c$
default	$\frac{c^4 e^3 x^{12}}{12}+\frac{3c^4 d\,e^2 x^{11}}{11}+\frac{(4e^3 a\,c^3+3d^2 e\,c^4)x^{10}}{10}+\frac{(12d\,e^2 a\,c^3+d^3 c^4)x^9}{9}+\frac{(6e^3 a^2 c^2+12d^2 ea\,c^3)x^8}{8}+\frac{(18a^2 c^2 d\,e^2+4}{7}$
gosper	$\frac{1}{12}c^4 e^3 x^{12}+\frac{3}{11}c^4 d\,e^2 x^{11}+\frac{2}{5}a\,c^3 e^3 x^{10}+\frac{3}{10}x^{10} d^2 e\,c^4+\frac{4}{3}x^9 d\,e^2 a\,c^3+\frac{1}{9}x^9 d^3 c^4+\frac{3}{4}a^2 c^2 e^3 x^8+$
risch	$\frac{1}{12}c^4 e^3 x^{12}+\frac{3}{11}c^4 d\,e^2 x^{11}+\frac{2}{5}a\,c^3 e^3 x^{10}+\frac{3}{10}x^{10} d^2 e\,c^4+\frac{4}{3}x^9 d\,e^2 a\,c^3+\frac{1}{9}x^9 d^3 c^4+\frac{3}{4}a^2 c^2 e^3 x^8+$
parallelrisch	$\frac{1}{12}c^4 e^3 x^{12}+\frac{3}{11}c^4 d\,e^2 x^{11}+\frac{2}{5}a\,c^3 e^3 x^{10}+\frac{3}{10}x^{10} d^2 e\,c^4+\frac{4}{3}x^9 d\,e^2 a\,c^3+\frac{1}{9}x^9 d^3 c^4+\frac{3}{4}a^2 c^2 e^3 x^8+$
orering	$\underline{x(1155e^3 c^4 x^{11}+3780d\,e^2 c^4 x^{10}+5544a\,c^3 e^3 x^9+4158c^4 d^2 e\,x^9+18480a\,c^3 d\,e^2 x^8+1540c^4 d^3 x^8+10395a^2 c^2 e^3 x^7+20790a\,c^3 d^2}$

input
```
int((e*x+d)^3*(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*c^4*e^3*x^12+3/11*c^4*d*e^2*x^11+(2/5*e^3*a*c^3+3/10*d^2*e*c^4)*x^10+
(4/3*d*e^2*a*c^3+1/9*d^3*c^4)*x^9+(3/4*e^3*a^2*c^2+3/2*d^2*e*a*c^3)*x^8+(1
8/7*a^2*c^2*d*e^2+4/7*d^3*a*c^3)*x^7+(2/3*e^3*c*a^3+3*d^2*e*a^2*c^2)*x^6+(
12/5*d*e^2*c*a^3+6/5*d^3*a^2*c^2)*x^5+(1/4*e^3*a^4+3*d^2*e*c*a^3)*x^4+(d*e
^2*a^4+4/3*d^3*c*a^3)*x^3+3/2*d^2*e*a^4*x^2+a^4*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.17

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^4\,dx &= \frac{1}{12}\,c^4e^3x^{12}+\frac{3}{11}\,c^4de^2x^{11}+\frac{1}{10}\,(3\,c^4d^2e+4\,ac^3e^3)x^{10}\\&\quad+\frac{1}{9}\,(c^4d^3+12\,ac^3de^2)x^9+\frac{3}{2}\,a^4d^2ex^2\\&\quad+\frac{3}{4}\,(2\,ac^3d^2e+a^2c^2e^3)x^8\\&\quad+a^4d^3x+\frac{2}{7}\,(2\,ac^3d^3+9\,a^2c^2de^2)x^7\\&\quad+\frac{1}{3}\,(9\,a^2c^2d^2e+2\,a^3ce^3)x^6+\frac{6}{5}\,(a^2c^2d^3+2\,a^3cde^2)x^5\\&\quad+\frac{1}{4}\,(12\,a^3cd^2e+a^4e^3)x^4+\frac{1}{3}\,(4\,a^3cd^3+3\,a^4de^2)x^3\end{aligned}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/12*c^4*e^3*x^12 + 3/11*c^4*d*e^2*x^11 + 1/10*(3*c^4*d^2*e + 4*a*c^3*e^3)
*x^10 + 1/9*(c^4*d^3 + 12*a*c^3*d*e^2)*x^9 + 3/2*a^4*d^2*e*x^2 + 3/4*(2*a*
c^3*d^2*e + a^2*c^2*e^3)*x^8 + a^4*d^3*x + 2/7*(2*a*c^3*d^3 + 9*a^2*c^2*d*
e^2)*x^7 + 1/3*(9*a^2*c^2*d^2*e + 2*a^3*c*e^3)*x^6 + 6/5*(a^2*c^2*d^3 + 2*
a^3*c*d*e^2)*x^5 + 1/4*(12*a^3*c*d^2*e + a^4*e^3)*x^4 + 1/3*(4*a^3*c*d^3 +
 3*a^4*d*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.29

$$\int (d+ex)^3 \left(a+cx^2\right)^4 \, dx = a^4d^3x + \frac{3a^4d^2ex^2}{2} + \frac{3c^4de^2x^{11}}{11} + \frac{c^4e^3x^{12}}{12} + x^{10} \cdot \left(\frac{2ac^3e^3}{5} + \frac{3c^4d^2e}{10}\right) + x^9 \cdot \left(\frac{4ac^3de^2}{3} + \frac{c^4d^3}{9}\right) + x^8 \cdot \left(\frac{3a^2c^2e^3}{4} + \frac{3ac^3d^2e}{2}\right) + x^7 \cdot \left(\frac{18a^2c^2de^2}{7} + \frac{4ac^3d^3}{7}\right) + x^6 \cdot \left(\frac{2a^3ce^3}{3} + 3a^2c^2d^2e\right) + x^5 \cdot \left(\frac{12a^3cde^2}{5} + \frac{6a^2c^2d^3}{5}\right) + x^4\left(\frac{a^4e^3}{4} + 3a^3cd^2e\right) + x^3\left(a^4de^2 + \frac{4a^3cd^3}{3}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**4,x)
```

output
```
a**4*d**3*x + 3*a**4*d**2*e*x**2/2 + 3*c**4*d*e**2*x**11/11 + c**4*e**3*x*
*12/12 + x**10*(2*a*c**3*e**3/5 + 3*c**4*d**2*e/10) + x**9*(4*a*c**3*d*e**
2/3 + c**4*d**3/9) + x**8*(3*a**2*c**2*e**3/4 + 3*a*c**3*d**2*e/2) + x**7*
(18*a**2*c**2*d*e**2/7 + 4*a*c**3*d**3/7) + x**6*(2*a**3*c*e**3/3 + 3*a**2
*c**2*d**2*e) + x**5*(12*a**3*c*d*e**2/5 + 6*a**2*c**2*d**3/5) + x**4*(a**
4*e**3/4 + 3*a**3*c*d**2*e) + x**3*(a**4*d*e**2 + 4*a**3*c*d**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.17

$$\int (d+ex)^3 \left(a+cx^2\right)^4 \, dx = \frac{1}{12}\,c^4e^3x^{12} + \frac{3}{11}\,c^4de^2x^{11} + \frac{1}{10}\,\left(3\,c^4d^2e + 4\,ac^3e^3\right)x^{10} + \frac{1}{9}\,\left(c^4d^3 + 12\,ac^3de^2\right)x^9 + \frac{3}{2}\,a^4d^2ex^2 + \frac{3}{4}\,\left(2\,ac^3d^2e + a^2c^2e^3\right)x^8 + a^4d^3x + \frac{2}{7}\,\left(2\,ac^3d^3 + 9\,a^2c^2de^2\right)x^7 + \frac{1}{3}\,\left(9\,a^2c^2d^2e + 2\,a^3ce^3\right)x^6 + \frac{6}{5}\,\left(a^2c^2d^3 + 2\,a^3cde^2\right)x^5 + \frac{1}{4}\,\left(12\,a^3cd^2e + a^4e^3\right)x^4 + \frac{1}{3}\,\left(4\,a^3cd^3 + 3\,a^4de^2\right)x^3$$

input

```
integrate((e*x+d)^3*(c*x^2+a)^4,x, algorithm="maxima")
```

output

```
1/12*c^4*e^3*x^12 + 3/11*c^4*d*e^2*x^11 + 1/10*(3*c^4*d^2*e + 4*a*c^3*e^3)
*x^10 + 1/9*(c^4*d^3 + 12*a*c^3*d*e^2)*x^9 + 3/2*a^4*d^2*e*x^2 + 3/4*(2*a*
c^3*d^2*e + a^2*c^2*e^3)*x^8 + a^4*d^3*x + 2/7*(2*a*c^3*d^3 + 9*a^2*c^2*d*
e^2)*x^7 + 1/3*(9*a^2*c^2*d^2*e + 2*a^3*c*e^3)*x^6 + 6/5*(a^2*c^2*d^3 + 2*
a^3*c*d*e^2)*x^5 + 1/4*(12*a^3*c*d^2*e + a^4*e^3)*x^4 + 1/3*(4*a^3*c*d^3 +
 3*a^4*d*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.18

$$\int (d+ex)^3 \left(a+cx^2\right)^4 \, dx = \frac{1}{12}\,c^4e^3x^{12} + \frac{3}{11}\,c^4de^2x^{11} + \frac{3}{10}\,c^4d^2ex^{10} + \frac{2}{5}\,ac^3e^3x^{10} + \frac{1}{9}\,c^4d^3x^9 + \frac{4}{3}\,ac^3de^2x^9 + \frac{3}{2}\,ac^3d^2ex^8 + \frac{3}{4}\,a^2c^2e^3x^8 + \frac{4}{7}\,ac^3d^3x^7 + \frac{18}{7}\,a^2c^2de^2x^7 + 3\,a^2c^2d^2ex^6 + \frac{2}{3}\,a^3ce^3x^6 + \frac{6}{5}\,a^2c^2d^3x^5 + \frac{12}{5}\,a^3cde^2x^5 + 3\,a^3cd^2ex^4 + \frac{1}{4}\,a^4e^3x^4 + \frac{4}{3}\,a^3cd^3x^3 + a^4de^2x^3 + \frac{3}{2}\,a^4d^2ex^2 + a^4d^3x$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/12*c^4*e^3*x^12 + 3/11*c^4*d*e^2*x^11 + 3/10*c^4*d^2*e*x^10 + 2/5*a*c^3*
e^3*x^10 + 1/9*c^4*d^3*x^9 + 4/3*a*c^3*d*e^2*x^9 + 3/2*a*c^3*d^2*e*x^8 + 3
/4*a^2*c^2*e^3*x^8 + 4/7*a*c^3*d^3*x^7 + 18/7*a^2*c^2*d*e^2*x^7 + 3*a^2*c^
2*d^2*e*x^6 + 2/3*a^3*c*e^3*x^6 + 6/5*a^2*c^2*d^3*x^5 + 12/5*a^3*c*d*e^2*x
^5 + 3*a^3*c*d^2*e*x^4 + 1/4*a^4*e^3*x^4 + 4/3*a^3*c*d^3*x^3 + a^4*d*e^2*x
^3 + 3/2*a^4*d^2*e*x^2 + a^4*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08

$$\int (d+ex)^3\left(a+cx^2\right)^4\,dx = x^3\left(a^4\,d\,e^2+\frac{4\,c\,a^3\,d^3}{3}\right)+x^4\left(\frac{a^4\,e^3}{4}+3\,c\,a^3\,d^2\,e\right)+x^9\left(\frac{c^4\,d^3}{9}+\frac{4\,a\,c^3\,d\,e^2}{3}\right)+x^{10}\left(\frac{3\,c^4\,d^2\,e}{10}+\frac{2\,a\,c^3\,e^3}{5}\right)+a^4\,d^3\,x+\frac{c^4\,e^3\,x^{12}}{12}+\frac{3\,a^4\,d^2\,e\,x^2}{2}+\frac{3\,c^4\,d\,e^2\,x^{11}}{11}+\frac{6\,a^2\,c\,d\,x^5\,\left(c\,d^2+2\,a\,e^2\right)}{5}+\frac{2\,a\,c^2\,d\,x^7\,\left(2\,c\,d^2+9\,a\,e^2\right)}{7}+\frac{3\,a\,c^2\,e\,x^8\,\left(2\,c\,d^2+a\,e^2\right)}{4}+\frac{a^2\,c\,e\,x^6\,\left(9\,c\,d^2+2\,a\,e^2\right)}{3}$$

input
```
int((a + c*x^2)^4*(d + e*x)^3,x)
```

output
```
x^3*((4*a^3*c*d^3)/3 + a^4*d*e^2) + x^4*((a^4*e^3)/4 + 3*a^3*c*d^2*e) + x^
9*((c^4*d^3)/9 + (4*a*c^3*d*e^2)/3) + x^10*((2*a*c^3*e^3)/5 + (3*c^4*d^2*e
)/10) + a^4*d^3*x + (c^4*e^3*x^12)/12 + (3*a^4*d^2*e*x^2)/2 + (3*c^4*d*e^2
*x^11)/11 + (6*a^2*c*d*x^5*(2*a*e^2 + c*d^2))/5 + (2*a*c^2*d*x^7*(9*a*e^2
+ 2*c*d^2))/7 + (3*a*c^2*e*x^8*(a*e^2 + 2*c*d^2))/4 + (a^2*c*e*x^6*(2*a*e^
2 + 9*c*d^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.19

$$\int (d+ex)^3 \left(a+cx^2\right)^4 dx$$
$$= \frac{x(1155c^4e^3x^{11} + 3780c^4d\,e^2x^{10} + 5544a\,c^3e^3x^9 + 4158c^4d^2e\,x^9 + 18480a\,c^3d\,e^2x^8 + 1540c^4d^3x^8 + 10395a}{}$$

input
```
int((e*x+d)^3*(c*x^2+a)^4,x)
```

output
```
(x*(13860*a**4*d**3 + 20790*a**4*d**2*e*x + 13860*a**4*d*e**2*x**2 + 3465*
a**4*e**3*x**3 + 18480*a**3*c*d**3*x**2 + 41580*a**3*c*d**2*e*x**3 + 33264
*a**3*c*d*e**2*x**4 + 9240*a**3*c*e**3*x**5 + 16632*a**2*c**2*d**3*x**4 +
41580*a**2*c**2*d**2*e*x**5 + 35640*a**2*c**2*d*e**2*x**6 + 10395*a**2*c**
2*e**3*x**7 + 7920*a*c**3*d**3*x**6 + 20790*a*c**3*d**2*e*x**7 + 18480*a*c
**3*d*e**2*x**8 + 5544*a*c**3*e**3*x**9 + 1540*c**4*d**3*x**8 + 4158*c**4*
d**2*e*x**9 + 3780*c**4*d*e**2*x**10 + 1155*c**4*e**3*x**11))/13860
```

3.94 $\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 132

$$\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx = a^4d^2x + \frac{1}{3}a^3\left(4cd^2+ae^2\right)x^3 + \frac{2}{5}a^2c\left(3cd^2+2ae^2\right)x^5 + \frac{2}{7}ac^2\left(2cd^2+3ae^2\right)x^7 + \frac{1}{9}c^3\left(cd^2+4ae^2\right)x^9 + \frac{1}{11}c^4e^2x^{11} + \frac{de\left(a+cx^2\right)^5}{5c}$$

output
```
a^4*d^2*x+1/3*a^3*(a*e^2+4*c*d^2)*x^3+2/5*a^2*c*(2*a*e^2+3*c*d^2)*x^5+2/7*
a*c^2*(3*a*e^2+2*c*d^2)*x^7+1/9*c^3*(4*a*e^2+c*d^2)*x^9+1/11*c^4*e^2*x^11+
1/5*d*e*(c*x^2+a)^5/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx = \frac{2}{15}a^3cx^3\left(10d^2+15dex+6e^2x^2\right) + \frac{2}{35}a^2c^2x^5\left(21d^2+35dex+15e^2x^2\right) + \frac{1}{63}ac^3x^7\left(36d^2+63dex+28e^2x^2\right) + \frac{1}{495}c^4x^9\left(55d^2+99dex+45e^2x^2\right) + a^4\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)$$

input
```
Integrate[(d + e*x)^2*(a + c*x^2)^4,x]
```

output
```
(2*a^3*c*x^3*(10*d^2 + 15*d*e*x + 6*e^2*x^2))/15 + (2*a^2*c^2*x^5*(21*d^2
+ 35*d*e*x + 15*e^2*x^2))/35 + (a*c^3*x^7*(36*d^2 + 63*d*e*x + 28*e^2*x^2)
)/63 + (c^4*x^9*(55*d^2 + 99*d*e*x + 45*e^2*x^2))/495 + a^4*(d^2*x + d*e*x
^2 + (e^2*x^3)/3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^4 (d+ex)^2 \, dx$$

$$\Big\downarrow \text{475}$$

$$\int \left(c^4e^2x^{10}+c^3\left(cd^2+4ae^2\right)x^8+2ac^2\left(2cd^2+3ae^2\right)x^6+2a^2c\left(3cd^2+2ae^2\right)x^4+a^3\left(4cd^2+ae^2\right)x^2+a^4d^2\right)dx \frac{de(a+cx^2)^5}{5c}$$

$$\Big\downarrow \text{2009}$$

$$a^4d^2x+\frac{1}{3}a^3x^3\left(ae^2+4cd^2\right)+\frac{2}{5}a^2cx^5\left(2ae^2+3cd^2\right)+\frac{1}{9}c^3x^9\left(4ae^2+cd^2\right)+\frac{2}{7}ac^2x^7\left(3ae^2+2cd^2\right)+\frac{de(a+cx^2)^5}{5c}+\frac{1}{11}c^4e^2x^{11}$$

input
```
Int[(d + e*x)^2*(a + c*x^2)^4,x]
```

output
```
a^4*d^2*x + (a^3*(4*c*d^2 + a*e^2)*x^3)/3 + (2*a^2*c*(3*c*d^2 + 2*a*e^2)*x
^5)/5 + (2*a*c^2*(2*c*d^2 + 3*a*e^2)*x^7)/7 + (c^3*(c*d^2 + 4*a*e^2)*x^9)/
9 + (c^4*e^2*x^11)/11 + (d*e*(a + c*x^2)^5)/(5*c)
```

Defintions of rubi rules used

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.27

method	result
norman	$\frac{c^4e^2x^{11}}{11}+\frac{de\,c^4x^{10}}{5}+\left(\frac{4}{9}e^2a\,c^3+\frac{1}{9}c^4d^2\right)x^9+dea\,c^3x^8+\left(\frac{6}{7}a^2c^2e^2+\frac{4}{7}a\,c^3d^2\right)x^7+2a^2c^2de\,x^6$
default	$\frac{c^4e^2x^{11}}{11}+\frac{de\,c^4x^{10}}{5}+\frac{(4e^2a\,c^3+c^4d^2)x^9}{9}+dea\,c^3x^8+\frac{(6a^2c^2e^2+4a\,c^3d^2)x^7}{7}+2a^2c^2de\,x^6+\frac{(4e^2c\,a^3+6d^2a}{5}$
gosper	$\frac{1}{11}c^4e^2x^{11}+\frac{1}{5}de\,c^4x^{10}+\frac{4}{9}x^9e^2a\,c^3+\frac{1}{9}x^9c^4d^2+dea\,c^3x^8+\frac{6}{7}x^7a^2c^2e^2+\frac{4}{7}x^7a\,c^3d^2+2a^2c^2d$
risch	$\frac{1}{11}c^4e^2x^{11}+\frac{1}{5}de\,c^4x^{10}+\frac{4}{9}x^9e^2a\,c^3+\frac{1}{9}x^9c^4d^2+dea\,c^3x^8+\frac{6}{7}x^7a^2c^2e^2+\frac{4}{7}x^7a\,c^3d^2+2a^2c^2d$
parallelrisch	$\frac{1}{11}c^4e^2x^{11}+\frac{1}{5}de\,c^4x^{10}+\frac{4}{9}x^9e^2a\,c^3+\frac{1}{9}x^9c^4d^2+dea\,c^3x^8+\frac{6}{7}x^7a^2c^2e^2+\frac{4}{7}x^7a\,c^3d^2+2a^2c^2d$
orering	$\frac{x(315e^2c^4x^{10}+693de\,c^4x^9+1540a\,c^3e^2x^8+385c^4d^2x^8+3465dea\,c^3x^7+2970a^2c^2e^2x^6+1980a\,c^3d^2x^6+6930a^2c^2de\,x^5+2772}{3465}$

input
```
int((e*x+d)^2*(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^4*e^2*x^11+1/5*d*e*c^4*x^10+(4/9*e^2*a*c^3+1/9*c^4*d^2)*x^9+d*e*a*c
^3*x^8+(6/7*a^2*c^2*e^2+4/7*a*c^3*d^2)*x^7+2*a^2*c^2*d*e*x^6+(4/5*e^2*c*a^
3+6/5*d^2*a^2*c^2)*x^5+2*d*e*c*a^3*x^4+(1/3*a^4*e^2+4/3*a^3*c*d^2)*x^3+d*e
*a^4*x^2+a^4*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.28

$$\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx = \frac{1}{11} c^4 e^2 x^{11} + \frac{1}{5} c^4 dex^{10} + ac^3 dex^8 + 2 a^2 c^2 dex^6 + 2 a^3 cdex^4 + \frac{1}{9} \left(c^4 d^2 + 4 ac^3 e^2\right) x^9 + a^4 dex^2 + \frac{2}{7} \left(2 ac^3 d^2 + 3 a^2 c^2 e^2\right) x^7 + a^4 d^2 x + \frac{2}{5} \left(3 a^2 c^2 d^2 + 2 a^3 ce^2\right) x^5 + \frac{1}{3} \left(4 a^3 cd^2 + a^4 e^2\right) x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/11*c^4*e^2*x^11 + 1/5*c^4*d*e*x^10 + a*c^3*d*e*x^8 + 2*a^2*c^2*d*e*x^6 +
 2*a^3*c*d*e*x^4 + 1/9*(c^4*d^2 + 4*a*c^3*e^2)*x^9 + a^4*d*e*x^2 + 2/7*(2*
a*c^3*d^2 + 3*a^2*c^2*e^2)*x^7 + a^4*d^2*x + 2/5*(3*a^2*c^2*d^2 + 2*a^3*c*
e^2)*x^5 + 1/3*(4*a^3*c*d^2 + a^4*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.42

$$\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx = a^4 d^2 x + a^4 dex^2 + 2a^3 cdex^4 + 2a^2 c^2 dex^6 + ac^3 dex^8 + \frac{c^4 dex^{10}}{5} + \frac{c^4 e^2 x^{11}}{11} + x^9 \cdot \left(\frac{4ac^3 e^2}{9} + \frac{c^4 d^2}{9}\right) + x^7 \cdot \left(\frac{6a^2 c^2 e^2}{7} + \frac{4ac^3 d^2}{7}\right) + x^5 \cdot \left(\frac{4a^3 ce^2}{5} + \frac{6a^2 c^2 d^2}{5}\right) + x^3 \left(\frac{a^4 e^2}{3} + \frac{4a^3 cd^2}{3}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**4,x)
```

output
```
a**4*d**2*x + a**4*d*e*x**2 + 2*a**3*c*d*e*x**4 + 2*a**2*c**2*d*e*x**6 + a
*c**3*d*e*x**8 + c**4*d*e*x**10/5 + c**4*e**2*x**11/11 + x**9*(4*a*c**3*e*
*2/9 + c**4*d**2/9) + x**7*(6*a**2*c**2*e**2/7 + 4*a*c**3*d**2/7) + x**5*(
4*a**3*c*e**2/5 + 6*a**2*c**2*d**2/5) + x**3*(a**4*e**2/3 + 4*a**3*c*d**2/
3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.28

$$\int (d+ex)^2\left(a+cx^2\right)^4\,dx=\frac{1}{11}\,c^4e^2x^{11}+\frac{1}{5}\,c^4dex^{10}+ac^3dex^8+2\,a^2c^2dex^6+2\,a^3cdex^4+\frac{1}{9}\left(c^4d^2+4\,ac^3e^2\right)x^9+a^4dex^2+\frac{2}{7}\left(2\,ac^3d^2+3\,a^2c^2e^2\right)x^7+a^4d^2x+\frac{2}{5}\left(3\,a^2c^2d^2+2\,a^3ce^2\right)x^5+\frac{1}{3}\left(4\,a^3cd^2+a^4e^2\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/11*c^4*e^2*x^11 + 1/5*c^4*d*e*x^10 + a*c^3*d*e*x^8 + 2*a^2*c^2*d*e*x^6 +
 2*a^3*c*d*e*x^4 + 1/9*(c^4*d^2 + 4*a*c^3*e^2)*x^9 + a^4*d*e*x^2 + 2/7*(2*
a*c^3*d^2 + 3*a^2*c^2*e^2)*x^7 + a^4*d^2*x + 2/5*(3*a^2*c^2*d^2 + 2*a^3*c*
e^2)*x^5 + 1/3*(4*a^3*c*d^2 + a^4*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

$$\int (d+ex)^2\left(a+cx^2\right)^4\,dx=\frac{1}{11}\,c^4e^2x^{11}+\frac{1}{5}\,c^4dex^{10}+\frac{1}{9}\,c^4d^2x^9+\frac{4}{9}\,ac^3e^2x^9+ac^3dex^8+\frac{4}{7}\,ac^3d^2x^7+\frac{6}{7}\,a^2c^2e^2x^7+2\,a^2c^2dex^6+\frac{6}{5}\,a^2c^2d^2x^5+\frac{4}{5}\,a^3ce^2x^5+2\,a^3cdex^4+\frac{4}{3}\,a^3cd^2x^3+\frac{1}{3}\,a^4e^2x^3+a^4dex^2+a^4d^2x$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/11*c^4*e^2*x^11 + 1/5*c^4*d*e*x^10 + 1/9*c^4*d^2*x^9 + 4/9*a*c^3*e^2*x^9
 + a*c^3*d*e*x^8 + 4/7*a*c^3*d^2*x^7 + 6/7*a^2*c^2*e^2*x^7 + 2*a^2*c^2*d*e
*x^6 + 6/5*a^2*c^2*d^2*x^5 + 4/5*a^3*c*e^2*x^5 + 2*a^3*c*d*e*x^4 + 4/3*a^3
*c*d^2*x^3 + 1/3*a^4*e^2*x^3 + a^4*d*e*x^2 + a^4*d^2*x
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.22

$$\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx = x^3 \left(\frac{a^4 e^2}{3} + \frac{4 c a^3 d^2}{3}\right) + x^9 \left(\frac{c^4 d^2}{9} + \frac{4 a c^3 e^2}{9}\right) + a^4 d^2 x + \frac{c^4 e^2 x^{11}}{11} + a^4 d e x^2 + \frac{c^4 d e x^{10}}{5} + \frac{2 a^2 c x^5 (3 c d^2 + 2 a e^2)}{5} + \frac{2 a c^2 x^7 (2 c d^2 + 3 a e^2)}{7} + 2 a^3 c d e x^4 + a c^3 d e x^8 + 2 a^2 c^2 d e x^6$$

input
```
int((a + c*x^2)^4*(d + e*x)^2,x)
```

output
```
x^3*((a^4*e^2)/3 + (4*a^3*c*d^2)/3) + x^9*((c^4*d^2)/9 + (4*a*c^3*e^2)/9)
+ a^4*d^2*x + (c^4*e^2*x^11)/11 + a^4*d*e*x^2 + (c^4*d*e*x^10)/5 + (2*a^2*
c*x^5*(2*a*e^2 + 3*c*d^2))/5 + (2*a*c^2*x^7*(3*a*e^2 + 2*c*d^2))/7 + 2*a^3
*c*d*e*x^4 + a*c^3*d*e*x^8 + 2*a^2*c^2*d*e*x^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.32

$$\int (d+ex)^2 \left(a+cx^2\right)^4 \, dx = \frac{x(315c^4e^2x^{10} + 693c^4de\,x^9 + 1540a\,c^3e^2x^8 + 385c^4d^2x^8 + 3465a\,c^3de\,x^7 + 2970a^2c^2e^2x^6 + 1980a\,c^3d^2x^6}{}$$

input
```
int((e*x+d)^2*(c*x^2+a)^4,x)
```

output
```
(x*(3465*a**4*d**2 + 3465*a**4*d*e*x + 1155*a**4*e**2*x**2 + 4620*a**3*c*d
**2*x**2 + 6930*a**3*c*d*e*x**3 + 2772*a**3*c*e**2*x**4 + 4158*a**2*c**2*d
**2*x**4 + 6930*a**2*c**2*d*e*x**5 + 2970*a**2*c**2*e**2*x**6 + 1980*a*c**
3*d**2*x**6 + 3465*a*c**3*d*e*x**7 + 1540*a*c**3*e**2*x**8 + 385*c**4*d**2
*x**8 + 693*c**4*d*e*x**9 + 315*c**4*e**2*x**10))/3465
```

3.95 $\int (d+ex) \left(a+cx^2\right)^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int (d+ex) \left(a+cx^2\right)^4 \, dx = a^4dx+\frac{4}{3}a^3cdx^3+\frac{6}{5}a^2c^2dx^5+\frac{4}{7}ac^3dx^7+\frac{1}{9}c^4dx^9+\frac{e(a+cx^2)^5}{10c}$$

output
```
a^4*d*x+4/3*a^3*c*d*x^3+6/5*a^2*c^2*d*x^5+4/7*a*c^3*d*x^7+1/9*c^4*d*x^9+1/
10*e*(c*x^2+a)^5/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int (d+ex) \left(a+cx^2\right)^4 \, dx = a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3cdx^3+a^3cex^4+\frac{6}{5}a^2c^2dx^5 +a^2c^2ex^6+\frac{4}{7}ac^3dx^7+\frac{1}{2}ac^3ex^8+\frac{1}{9}c^4dx^9+\frac{1}{10}c^4ex^{10}$$

input
```
Integrate[(d + e*x)*(a + c*x^2)^4,x]
```

output
```
a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*c*d*x^3)/3 + a^3*c*e*x^4 + (6*a^2*c^2*d*x
^5)/5 + a^2*c^2*e*x^6 + (4*a*c^3*d*x^7)/7 + (a*c^3*e*x^8)/2 + (c^4*d*x^9)/
9 + (c^4*e*x^10)/10
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^4 (d+ex) \, dx$$

$$\Big\downarrow \text{455}$$

$$d\int \left(cx^2+a\right)^4dx+\frac{e\left(a+cx^2\right)^5}{10c}$$

$$\Big\downarrow \text{210}$$

$$d\int \left(c^4x^8+4ac^3x^6+6a^2c^2x^4+4a^3cx^2+a^4\right)dx+\frac{e\left(a+cx^2\right)^5}{10c}$$

$$\Big\downarrow \text{2009}$$

$$d\left(a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}\right)+\frac{e\left(a+cx^2\right)^5}{10c}$$

input
```
Int[(d + e*x)*(a + c*x^2)^4,x]
```

output
```
(e*(a + c*x^2)^5)/(10*c) + d*(a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5
+ (4*a*c^3*x^7)/7 + (c^4*x^9)/9)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

method	result
gosper	$\frac{1}{10}c^4e\,x^{10}+\frac{1}{9}c^4d\,x^9+\frac{1}{2}a\,c^3e\,x^8+\frac{4}{7}a\,c^3d\,x^7+a^2c^2e\,x^6+\frac{6}{5}a^2c^2d\,x^5+a^3ce\,x^4+\frac{4}{3}a^3cd\,x^3+$
default	$\frac{1}{10}c^4e\,x^{10}+\frac{1}{9}c^4d\,x^9+\frac{1}{2}a\,c^3e\,x^8+\frac{4}{7}a\,c^3d\,x^7+a^2c^2e\,x^6+\frac{6}{5}a^2c^2d\,x^5+a^3ce\,x^4+\frac{4}{3}a^3cd\,x^3+$
norman	$\frac{1}{10}c^4e\,x^{10}+\frac{1}{9}c^4d\,x^9+\frac{1}{2}a\,c^3e\,x^8+\frac{4}{7}a\,c^3d\,x^7+a^2c^2e\,x^6+\frac{6}{5}a^2c^2d\,x^5+a^3ce\,x^4+\frac{4}{3}a^3cd\,x^3+$
risch	$\frac{1}{10}c^4e\,x^{10}+\frac{1}{9}c^4d\,x^9+\frac{1}{2}a\,c^3e\,x^8+\frac{4}{7}a\,c^3d\,x^7+a^2c^2e\,x^6+\frac{6}{5}a^2c^2d\,x^5+a^3ce\,x^4+\frac{4}{3}a^3cd\,x^3+$
parallelrisch	$\frac{1}{10}c^4e\,x^{10}+\frac{1}{9}c^4d\,x^9+\frac{1}{2}a\,c^3e\,x^8+\frac{4}{7}a\,c^3d\,x^7+a^2c^2e\,x^6+\frac{6}{5}a^2c^2d\,x^5+a^3ce\,x^4+\frac{4}{3}a^3cd\,x^3+$
orering	$\frac{x\left(63e\,c^4x^9+70d\,c^4x^8+315ea\,c^3x^7+360c^3ad\,x^6+630e\,a^2c^2x^5+756d\,a^2c^2x^4+630ec\,a^3x^3+840dc\,a^3x^2+315a^4ex+630a^4d\right)}{630}$

input
```
int((e*x+d)*(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^4*e*x^10+1/9*c^4*d*x^9+1/2*a*c^3*e*x^8+4/7*a*c^3*d*x^7+a^2*c^2*e*x^
6+6/5*a^2*c^2*d*x^5+a^3*c*e*x^4+4/3*a^3*c*d*x^3+1/2*a^4*e*x^2+a^4*d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int (d+ex)\left(a+cx^2\right)^4\,dx=\frac{1}{10}\,c^4ex^{10}+\frac{1}{9}\,c^4dx^9+\frac{1}{2}\,ac^3ex^8+\frac{4}{7}\,ac^3dx^7+a^2c^2ex^6+\frac{6}{5}\,a^2c^2dx^5+a^3cex^4+\frac{4}{3}\,a^3cdx^3+\frac{1}{2}\,a^4ex^2+a^4dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/10*c^4*e*x^10 + 1/9*c^4*d*x^9 + 1/2*a*c^3*e*x^8 + 4/7*a*c^3*d*x^7 + a^2*
c^2*e*x^6 + 6/5*a^2*c^2*d*x^5 + a^3*c*e*x^4 + 4/3*a^3*c*d*x^3 + 1/2*a^4*e*
x^2 + a^4*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int (d+ex)\left(a+cx^2\right)^4\,dx=a^4dx+\frac{a^4ex^2}{2}+\frac{4a^3cdx^3}{3}+a^3cex^4+\frac{6a^2c^2dx^5}{5}+a^2c^2ex^6+\frac{4ac^3dx^7}{7}+\frac{ac^3ex^8}{2}+\frac{c^4dx^9}{9}+\frac{c^4ex^{10}}{10}$$

input
```
integrate((e*x+d)*(c*x**2+a)**4,x)
```

output
```
a**4*d*x + a**4*e*x**2/2 + 4*a**3*c*d*x**3/3 + a**3*c*e*x**4 + 6*a**2*c**2
*d*x**5/5 + a**2*c**2*e*x**6 + 4*a*c**3*d*x**7/7 + a*c**3*e*x**8/2 + c**4*
d*x**9/9 + c**4*e*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int (d+ex)\left(a+cx^2\right)^4\,dx=\frac{1}{10}\,c^4ex^{10}+\frac{1}{9}\,c^4dx^9+\frac{1}{2}\,ac^3ex^8+\frac{4}{7}\,ac^3dx^7+a^2c^2ex^6+\frac{6}{5}\,a^2c^2dx^5+a^3cex^4+\frac{4}{3}\,a^3cdx^3+\frac{1}{2}\,a^4ex^2+a^4dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/10*c^4*e*x^10 + 1/9*c^4*d*x^9 + 1/2*a*c^3*e*x^8 + 4/7*a*c^3*d*x^7 + a^2*
c^2*e*x^6 + 6/5*a^2*c^2*d*x^5 + a^3*c*e*x^4 + 4/3*a^3*c*d*x^3 + 1/2*a^4*e*
x^2 + a^4*d*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int (d+ex)\left(a+cx^2\right)^4\,dx = \frac{1}{10}c^4ex^{10} + \frac{1}{9}c^4dx^9 + \frac{1}{2}ac^3ex^8 + \frac{4}{7}ac^3dx^7 + a^2c^2ex^6 + \frac{6}{5}a^2c^2dx^5 + a^3cex^4 + \frac{4}{3}a^3cdx^3 + \frac{1}{2}a^4ex^2 + a^4dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/10*c^4*e*x^10 + 1/9*c^4*d*x^9 + 1/2*a*c^3*e*x^8 + 4/7*a*c^3*d*x^7 + a^2*
c^2*e*x^6 + 6/5*a^2*c^2*d*x^5 + a^3*c*e*x^4 + 4/3*a^3*c*d*x^3 + 1/2*a^4*e*
x^2 + a^4*d*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int (d+ex)\left(a+cx^2\right)^4\,dx = \frac{e\,a^4\,x^2}{2} + d\,a^4\,x + e\,a^3\,c\,x^4 + \frac{4\,d\,a^3\,c\,x^3}{3} + e\,a^2\,c^2\,x^6 + \frac{6\,d\,a^2\,c^2\,x^5}{5} + \frac{e\,a\,c^3\,x^8}{2} + \frac{4\,d\,a\,c^3\,x^7}{7} + \frac{e\,c^4\,x^{10}}{10} + \frac{d\,c^4\,x^9}{9}$$

input
```
int((a + c*x^2)^4*(d + e*x),x)
```

output
```
(a^4*e*x^2)/2 + (c^4*d*x^9)/9 + (c^4*e*x^10)/10 + a^4*d*x + (6*a^2*c^2*d*x
^5)/5 + a^2*c^2*e*x^6 + (4*a^3*c*d*x^3)/3 + (4*a*c^3*d*x^7)/7 + a^3*c*e*x^
4 + (a*c^3*e*x^8)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int (d+ex)\left(a+cx^2\right)^4\,dx$$
$$=\frac{x(63c^4e\,x^9+70c^4d\,x^8+315a\,c^3e\,x^7+360a\,c^3d\,x^6+630a^2c^2e\,x^5+756a^2c^2d\,x^4+630a^3ce\,x^3+840a^3cd\,}{630}$$

input
```
int((e*x+d)*(c*x^2+a)^4,x)
```

output
```
(x*(630*a**4*d + 315*a**4*e*x + 840*a**3*c*d*x**2 + 630*a**3*c*e*x**3 + 75
6*a**2*c**2*d*x**4 + 630*a**2*c**2*e*x**5 + 360*a*c**3*d*x**6 + 315*a*c**3
*e*x**7 + 70*c**4*d*x**8 + 63*c**4*e*x**9))/630
```

3.96 $\int \frac{\left(a+cx^2\right)^4}{d+ex}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 264

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx = -\frac{8cd(cd^2+ae^2)^3x}{e^8}+\frac{2c(cd^2+ae^2)^2(7cd^2+ae^2)(d+ex)^2}{e^9} - \frac{8c^2d(cd^2+ae^2)(7cd^2+3ae^2)(d+ex)^3}{3e^9} + \frac{c^2(35c^2d^4+30acd^2e^2+3a^2e^4)(d+ex)^4}{2e^9} - \frac{8c^3d(7cd^2+3ae^2)(d+ex)^5}{5e^9}+\frac{2c^3(7cd^2+ae^2)(d+ex)^6}{3e^9} - \frac{8c^4d(d+ex)^7}{7e^9}+\frac{c^4(d+ex)^8}{8e^9}+\frac{(cd^2+ae^2)^4\log(d+ex)}{e^9}$$

output
```
-8*c*d*(a*e^2+c*d^2)^3*x/e^8+2*c*(a*e^2+c*d^2)^2*(a*e^2+7*c*d^2)*(e*x+d)^2
/e^9-8/3*c^2*d*(a*e^2+c*d^2)*(3*a*e^2+7*c*d^2)*(e*x+d)^3/e^9+1/2*c^2*(3*a^
2*e^4+30*a*c*d^2*e^2+35*c^2*d^4)*(e*x+d)^4/e^9-8/5*c^3*d*(3*a*e^2+7*c*d^2)
*(e*x+d)^5/e^9+2/3*c^3*(a*e^2+7*c*d^2)*(e*x+d)^6/e^9-8/7*c^4*d*(e*x+d)^7/e
^9+1/8*c^4*(e*x+d)^8/e^9+(a*e^2+c*d^2)^4*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx$$
$$= \frac{cx(1680a^3e^6(-2d+ex)+420a^2ce^4(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+56ac^2e^2(-60d^5+30d^4ex-20\cdots}{\cdots}$$
$$+\frac{(cd^2+ae^2)^4\log(d+ex)}{e^9}$$

input
```
Integrate[(a + c*x^2)^4/(d + e*x),x]
```

output
```
(c*x*(1680*a^3*e^6*(-2*d + e*x) + 420*a^2*c*e^4*(-12*d^3 + 6*d^2*e*x - 4*d
*e^2*x^2 + 3*e^3*x^3) + 56*a*c^2*e^2*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^
2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + c^3*(-840*d^7 + 420*d^6*
e*x - 280*d^5*e^2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^
5 - 120*d*e^6*x^6 + 105*e^7*x^7)))/(840*e^8) + ((c*d^2 + a*e^2)^4*Log[d +
e*x])/e^9
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx$$
$$\Big\downarrow \text{476}$$
$$\int \left(\frac{2c^2(d+ex)^3(3a^2e^4+30acd^2e^2+35c^2d^4)}{e^8}+\frac{4c^3(d+ex)^5(ae^2+7cd^2)}{e^8}-\frac{8c^3d(d+ex)^4(3ae^2+7cd^2)}{e^8}+\cdots\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{c^2(d+ex)^4\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)}{2e^9}+\frac{2c^3(d+ex)^6\left(ae^2+7cd^2\right)}{3e^9}-\frac{8c^3d(d+ex)^5\left(3ae^2+7cd^2\right)}{5e^9}-\frac{8c^2d(d+ex)^3\left(ae^2+cd^2\right)\left(3ae^2+7cd^2\right)}{3e^9}+\frac{2c(d+ex)^2\left(ae^2+cd^2\right)^2\left(ae^2+7cd^2\right)}{e^9}+\frac{\left(ae^2+cd^2\right)^4\log(d+ex)}{e^9}-\frac{8cdx\left(ae^2+cd^2\right)^3}{e^8}+\frac{c^4(d+ex)^8}{8e^9}-\frac{8c^4d(d+ex)^7}{7e^9}$$

input
```
Int[(a + c*x^2)^4/(d + e*x),x]
```

output
```
(-8*c*d*(c*d^2 + a*e^2)^3*x)/e^8 + (2*c*(c*d^2 + a*e^2)^2*(7*c*d^2 + a*e^2
)*(d + e*x)^2)/e^9 - (8*c^2*d*(c*d^2 + a*e^2)*(7*c*d^2 + 3*a*e^2)*(d + e*x
)^3)/(3*e^9) + (c^2*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)*(d + e*x)^4)
/(2*e^9) - (8*c^3*d*(7*c*d^2 + 3*a*e^2)*(d + e*x)^5)/(5*e^9) + (2*c^3*(7*c
*d^2 + a*e^2)*(d + e*x)^6)/(3*e^9) - (8*c^4*d*(d + e*x)^7)/(7*e^9) + (c^4*
(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^4*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.16

method	result
norman	$\frac{c^4x^8}{8e}+\frac{c(4e^6a^3+6d^2e^4a^2c+4d^4e^2a\,c^2+d^6c^3)x^2}{2e^7}-\frac{d\,c^4x^7}{7e^2}+\frac{c^2(6a^2e^4+4ac\,d^2e^2+c^2d^4)x^4}{4e^5}+\frac{c^3(4a\,e^2+c\,d^2)x^6}{6e^3}-\frac{cd}{}$
risch	$\frac{\ln(ex+d)a^4}{e}+\frac{c^4x^8}{8e}+\frac{c^4x^6d^2}{6e^3}-\frac{c^4x^5d^3}{5e^4}+\frac{3c^2x^4a^2}{2e}+\frac{c^4x^4d^4}{4e^5}-\frac{c^4x^3d^5}{3e^6}+\frac{2c\,a^3x^2}{e}+\frac{c^4d^6x^2}{2e^7}-\frac{c^4d^7x}{e^8}+\frac{\ln(e}{}$
parallelrisch	$840\ln(ex+d)c^4d^8+105x^8c^4e^8+840\ln(ex+d)a^4e^8-120d\,c^4x^7e^7+560x^6a\,c^3e^8+140x^6c^4d^2e^6-168x^5c^4d^3e^5+1260x^4a^2c^2e^8+$
default	$-\frac{c\left(-\frac{c^3x^8e^7}{8}+\frac{d\,c^3x^7e^6}{7}+\frac{\left(-e^5\left(2a\,e^2+c\,d^2\right)c^2-2c^2e^7a\right)x^6}{6}+\frac{\left(d\left(2a\,e^2+c\,d^2\right)e^4c^2+2d\,e^6c^2a\right)x^5}{5}+\frac{\left(-2e^5\left(2a\,e^2+c\,d^2\right)ac-c\,e^3\left(2a^2\right.\right.}{4}\right.}{}$

input
```
int((c*x^2+a)^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/8/e*c^4*x^8+1/2*c/e^7*(4*a^3*e^6+6*a^2*c*d^2*e^4+4*a*c^2*d^4*e^2+c^3*d^6
)*x^2-1/7*d/e^2*c^4*x^7+1/4/e^5*c^2*(6*a^2*e^4+4*a*c*d^2*e^2+c^2*d^4)*x^4+
1/6/e^3*c^3*(4*a*e^2+c*d^2)*x^6-c*d*(4*a^3*e^6+6*a^2*c*d^2*e^4+4*a*c^2*d^4
*e^2+c^3*d^6)/e^8*x-1/3*d/e^6*c^2*(6*a^2*e^4+4*a*c*d^2*e^2+c^2*d^4)*x^3-1/
5*d/e^4*c^3*(4*a*e^2+c*d^2)*x^5+(a^4*e^8+4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4
+4*a*c^3*d^6*e^2+c^4*d^8)/e^9*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.21

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx$$
$$=\frac{105\,c^4e^8x^8-120\,c^4de^7x^7+140\,(c^4d^2e^6+4\,ac^3e^8)x^6-168\,(c^4d^3e^5+4\,ac^3de^7)x^5+210\,(c^4d^4e^4+4\,ac^3d^2}{}$$

input
```
integrate((c*x^2+a)^4/(e*x+d),x, algorithm="fricas")
```

output
```
1/840*(105*c^4*e^8*x^8 - 120*c^4*d*e^7*x^7 + 140*(c^4*d^2*e^6 + 4*a*c^3*e^
8)*x^6 - 168*(c^4*d^3*e^5 + 4*a*c^3*d*e^7)*x^5 + 210*(c^4*d^4*e^4 + 4*a*c^
3*d^2*e^6 + 6*a^2*c^2*e^8)*x^4 - 280*(c^4*d^5*e^3 + 4*a*c^3*d^3*e^5 + 6*a^
2*c^2*d*e^7)*x^3 + 420*(c^4*d^6*e^2 + 4*a*c^3*d^4*e^4 + 6*a^2*c^2*d^2*e^6
+ 4*a^3*c*e^8)*x^2 - 840*(c^4*d^7*e + 4*a*c^3*d^5*e^3 + 6*a^2*c^2*d^3*e^5
+ 4*a^3*c*d*e^7)*x + 840*(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 +
4*a^3*c*d^2*e^6 + a^4*e^8)*log(e*x + d))/e^9
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx = -\frac{c^4dx^7}{7e^2} + \frac{c^4x^8}{8e} + x^6\cdot\left(\frac{2ac^3}{3e} + \frac{c^4d^2}{6e^3}\right) + x^5\left(-\frac{4ac^3d}{5e^2} - \frac{c^4d^3}{5e^4}\right) + x^4\cdot\left(\frac{3a^2c^2}{2e} + \frac{ac^3d^2}{e^3} + \frac{c^4d^4}{4e^5}\right) + x^3\left(-\frac{2a^2c^2d}{e^2} - \frac{4ac^3d^3}{3e^4} - \frac{c^4d^5}{3e^6}\right) + x^2\cdot\left(\frac{2a^3c}{e} + \frac{3a^2c^2d^2}{e^3} + \frac{2ac^3d^4}{e^5} + \frac{c^4d^6}{2e^7}\right) + x\left(-\frac{4a^3cd}{e^2} - \frac{6a^2c^2d^3}{e^4} - \frac{4ac^3d^5}{e^6} - \frac{c^4d^7}{e^8}\right) + \frac{(ae^2+cd^2)^4\log(d+ex)}{e^9}$$

input
```
integrate((c*x**2+a)**4/(e*x+d),x)
```

output
```
-c**4*d*x**7/(7*e**2) + c**4*x**8/(8*e) + x**6*(2*a*c**3/(3*e) + c**4*d**2
/(6*e**3)) + x**5*(-4*a*c**3*d/(5*e**2) - c**4*d**3/(5*e**4)) + x**4*(3*a*
*2*c**2/(2*e) + a*c**3*d**2/e**3 + c**4*d**4/(4*e**5)) + x**3*(-2*a**2*c**
2*d/e**2 - 4*a*c**3*d**3/(3*e**4) - c**4*d**5/(3*e**6)) + x**2*(2*a**3*c/e
 + 3*a**2*c**2*d**2/e**3 + 2*a*c**3*d**4/e**5 + c**4*d**6/(2*e**7)) + x*(-
4*a**3*c*d/e**2 - 6*a**2*c**2*d**3/e**4 - 4*a*c**3*d**5/e**6 - c**4*d**7/e
**8) + (a*e**2 + c*d**2)**4*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.21

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx = \frac{105\,c^4e^7x^8 - 120\,c^4de^6x^7 + 140\,(c^4d^2e^5 + 4\,ac^3e^7)x^6 - 168\,(c^4d^3e^4 + 4\,ac^3de^6)x^5 + 210\,(c^4d^4e^3 + 4\,ac^3d^2\cdots}{\cdots} + \frac{(c^4d^8 + 4\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 + 4\,a^3cd^2e^6 + a^4e^8)\log(ex+d)}{e^9}$$

input
```
integrate((c*x^2+a)^4/(e*x+d),x, algorithm="maxima")
```

output
```
1/840*(105*c^4*e^7*x^8 - 120*c^4*d*e^6*x^7 + 140*(c^4*d^2*e^5 + 4*a*c^3*e^
7)*x^6 - 168*(c^4*d^3*e^4 + 4*a*c^3*d*e^6)*x^5 + 210*(c^4*d^4*e^3 + 4*a*c^
3*d^2*e^5 + 6*a^2*c^2*e^7)*x^4 - 280*(c^4*d^5*e^2 + 4*a*c^3*d^3*e^4 + 6*a^
2*c^2*d*e^6)*x^3 + 420*(c^4*d^6*e + 4*a*c^3*d^4*e^3 + 6*a^2*c^2*d^2*e^5 +
4*a^3*c*e^7)*x^2 - 840*(c^4*d^7 + 4*a*c^3*d^5*e^2 + 6*a^2*c^2*d^3*e^4 + 4*
a^3*c*d*e^6)*x)/e^8 + (c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a
^3*c*d^2*e^6 + a^4*e^8)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.28

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx = \frac{105\,c^4e^7x^8 - 120\,c^4de^6x^7 + 140\,c^4d^2e^5x^6 + 560\,ac^3e^7x^6 - 168\,c^4d^3e^4x^5 - 672\,ac^3de^6x^5 + 210\,c^4d^4e^3x^4 + \cdots}{\cdots} + \frac{(c^4d^8 + 4\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 + 4\,a^3cd^2e^6 + a^4e^8)\log(|ex+d|)}{e^9}$$

input
```
integrate((c*x^2+a)^4/(e*x+d),x, algorithm="giac")
```

output
```
1/840*(105*c^4*e^7*x^8 - 120*c^4*d*e^6*x^7 + 140*c^4*d^2*e^5*x^6 + 560*a*c
^3*e^7*x^6 - 168*c^4*d^3*e^4*x^5 - 672*a*c^3*d*e^6*x^5 + 210*c^4*d^4*e^3*x
^4 + 840*a*c^3*d^2*e^5*x^4 + 1260*a^2*c^2*e^7*x^4 - 280*c^4*d^5*e^2*x^3 -
1120*a*c^3*d^3*e^4*x^3 - 1680*a^2*c^2*d*e^6*x^3 + 420*c^4*d^6*e*x^2 + 1680
*a*c^3*d^4*e^3*x^2 + 2520*a^2*c^2*d^2*e^5*x^2 + 1680*a^3*c*e^7*x^2 - 840*c
^4*d^7*x - 3360*a*c^3*d^5*e^2*x - 5040*a^2*c^2*d^3*e^4*x - 3360*a^3*c*d*e^
6*x)/e^8 + (c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^
6 + a^4*e^8)*log(abs(e*x + d))/e^9
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.35

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx = x^2\left(\frac{d^2\left(\frac{d^2\left(\frac{4\,a\,c^3}{e}+\frac{c^4\,d^2}{e^3}\right)}{e^2}+\frac{6\,a^2\,c^2}{e}\right)}{2\,e^2}+\frac{2\,a^3\,c}{e}\right) + x^4\left(\frac{d^2\left(\frac{4\,a\,c^3}{e}+\frac{c^4\,d^2}{e^3}\right)}{4\,e^2}+\frac{3\,a^2\,c^2}{2\,e}\right)+x^6\left(\frac{2\,a\,c^3}{3\,e}+\frac{c^4\,d^2}{6\,e^3}\right) + \frac{\ln\left(d+e\,x\right)\,\left(a^4\,e^8+4\,a^3\,c\,d^2\,e^6+6\,a^2\,c^2\,d^4\,e^4+4\,a\,c^3\,d^6\,e^2+c^4\,d^8\right)}{e^9} + \frac{c^4\,x^8}{8\,e}-\frac{c^4\,d\,x^7}{7\,e^2}-\frac{d\,x^3\left(\frac{d^2\left(\frac{4\,a\,c^3}{e}+\frac{c^4\,d^2}{e^3}\right)}{e^2}+\frac{6\,a^2\,c^2}{e}\right)}{3\,e} - \frac{d\,x^5\left(\frac{4\,a\,c^3}{e}+\frac{c^4\,d^2}{e^3}\right)}{5\,e}-\frac{d\,x\left(\frac{d^2\left(\frac{d^2\left(\frac{4\,a\,c^3}{e}+\frac{c^4\,d^2}{e^3}\right)}{e^2}+\frac{6\,a^2\,c^2}{e}\right)}{e^2}+\frac{4\,a^3\,c}{e}\right)}{e}$$

input
```
int((a + c*x^2)^4/(d + e*x),x)
```

output
```
x^2*((d^2*((d^2*((4*a*c^3)/e + (c^4*d^2)/e^3))/e^2 + (6*a^2*c^2)/e))/(2*e^
2) + (2*a^3*c)/e) + x^4*((d^2*((4*a*c^3)/e + (c^4*d^2)/e^3))/(4*e^2) + (3*
a^2*c^2)/(2*e)) + x^6*((2*a*c^3)/(3*e) + (c^4*d^2)/(6*e^3)) + (log(d + e*x
)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4))/e^9 + (c^4*x^8)/(8*e) - (c^4*d*x^7)/(7*e^2) - (d*x^3*((d^2*((4*a*c^3)
/e + (c^4*d^2)/e^3))/e^2 + (6*a^2*c^2)/e))/(3*e) - (d*x^5*((4*a*c^3)/e + (
c^4*d^2)/e^3))/(5*e) - (d*x*((d^2*((d^2*((4*a*c^3)/e + (c^4*d^2)/e^3))/e^2
 + (6*a^2*c^2)/e))/e^2 + (4*a^3*c)/e))/e
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^2)^4}{d+ex}\,dx$$
$$= 840\log(ex+d)\,a^4e^8 + 840\log(ex+d)\,c^4d^8 + 105c^4e^8x^8 + 3360\log(ex+d)\,a^3c\,d^2e^6 + 5040\log(ex+d)\ldots$$

input
```
int((c*x^2+a)^4/(e*x+d),x)
```

output
```
(840*log(d + e*x)*a**4*e**8 + 3360*log(d + e*x)*a**3*c*d**2*e**6 + 5040*lo
g(d + e*x)*a**2*c**2*d**4*e**4 + 3360*log(d + e*x)*a*c**3*d**6*e**2 + 840*
log(d + e*x)*c**4*d**8 - 3360*a**3*c*d*e**7*x + 1680*a**3*c*e**8*x**2 - 50
40*a**2*c**2*d**3*e**5*x + 2520*a**2*c**2*d**2*e**6*x**2 - 1680*a**2*c**2*
d*e**7*x**3 + 1260*a**2*c**2*e**8*x**4 - 3360*a*c**3*d**5*e**3*x + 1680*a*
c**3*d**4*e**4*x**2 - 1120*a*c**3*d**3*e**5*x**3 + 840*a*c**3*d**2*e**6*x*
*4 - 672*a*c**3*d*e**7*x**5 + 560*a*c**3*e**8*x**6 - 840*c**4*d**7*e*x + 4
20*c**4*d**6*e**2*x**2 - 280*c**4*d**5*e**3*x**3 + 210*c**4*d**4*e**4*x**4
 - 168*c**4*d**3*e**5*x**5 + 140*c**4*d**2*e**6*x**6 - 120*c**4*d*e**7*x**
7 + 105*c**4*e**8*x**8)/(840*e**9)
```

3.97 $\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 255

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx = \frac{c(7c^3d^6+20ac^2d^4e^2+18a^2cd^2e^4+4a^3e^6)\,x}{e^8} - \frac{c^2d(3c^2d^4+8acd^2e^2+6a^2e^4)\,x^2}{e^7} + \frac{c^2(5c^2d^4+12acd^2e^2+6a^2e^4)\,x^3}{3e^6} - \frac{c^3d(cd^2+2ae^2)\,x^4}{e^5} + \frac{c^3(3cd^2+4ae^2)\,x^5}{5e^4} - \frac{c^4dx^6}{3e^3} + \frac{c^4x^7}{7e^2} - \frac{(cd^2+ae^2)^4}{e^9(d+ex)} - \frac{8cd(cd^2+ae^2)^3\log(d+ex)}{e^9}$$

output
```
c*(4*a^3*e^6+18*a^2*c*d^2*e^4+20*a*c^2*d^4*e^2+7*c^3*d^6)*x/e^8-c^2*d*(6*a
^2*e^4+8*a*c*d^2*e^2+3*c^2*d^4)*x^2/e^7+1/3*c^2*(6*a^2*e^4+12*a*c*d^2*e^2+
5*c^2*d^4)*x^3/e^6-c^3*d*(2*a*e^2+c*d^2)*x^4/e^5+1/5*c^3*(4*a*e^2+3*c*d^2)
*x^5/e^4-1/3*c^4*d*x^6/e^3+1/7*c^4*x^7/e^2-(a*e^2+c*d^2)^4/e^9/(e*x+d)-8*c
*d*(a*e^2+c*d^2)^3*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx$$
$$= \frac{-105a^4e^8+420a^3ce^6(-d^2+dex+e^2x^2)+210a^2c^2e^4(-3d^4+9d^3ex+6d^2e^2x^2-2de^3x^3+e^4x^4)+42a}{}$$

input
```
Integrate[(a + c*x^2)^4/(d + e*x)^2,x]
```

output
```
(-105*a^4*e^8 + 420*a^3*c*e^6*(-d^2 + d*e*x + e^2*x^2) + 210*a^2*c^2*e^4*(
-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 42*a*c^3*e^2
*(-10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 -
 3*d*e^5*x^5 + 2*e^6*x^6) + c^4*(-105*d^8 + 735*d^7*e*x + 420*d^6*e^2*x^2
- 140*d^5*e^3*x^3 + 70*d^4*e^4*x^4 - 42*d^3*e^5*x^5 + 28*d^2*e^6*x^6 - 20*
d*e^7*x^7 + 15*e^8*x^8) - 840*c*d*(c*d^2 + a*e^2)^3*(d + e*x)*Log[d + e*x]
)/(105*e^9*(d + e*x))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx$$
$$\Big\downarrow \; 476$$
$$\int \left(-\frac{2c^2dx(6a^2e^4+8acd^2e^2+3c^2d^4)}{e^7}+\frac{c^2x^2(6a^2e^4+12acd^2e^2+5c^2d^4)}{e^6}+\frac{c(4a^3e^6+18a^2cd^2e^4+20ac^2d^4e^2}{e^8}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{c^2dx^2(6a^2e^4+8acd^2e^2+3c^2d^4)}{e^7}+\frac{c^2x^3(6a^2e^4+12acd^2e^2+5c^2d^4)}{3e^6}+\frac{cx(4a^3e^6+18a^2cd^2e^4+20ac^2d^4e^2+7c^3d^6)}{e^8}-\frac{c^3dx^4(2ae^2+cd^2)}{e^5}+\frac{c^3x^5(4ae^2+3cd^2)}{5e^4}-\frac{(ae^2+cd^2)^4}{e^9(d+ex)}-\frac{8cd(ae^2+cd^2)^3\log(d+ex)}{e^9}-\frac{c^4dx^6}{3e^3}+\frac{c^4x^7}{7e^2}$$

input
```
Int[(a + c*x^2)^4/(d + e*x)^2,x]
```

output
```
(c*(7*c^3*d^6 + 20*a*c^2*d^4*e^2 + 18*a^2*c*d^2*e^4 + 4*a^3*e^6)*x)/e^8 -
(c^2*d*(3*c^2*d^4 + 8*a*c*d^2*e^2 + 6*a^2*e^4)*x^2)/e^7 + (c^2*(5*c^2*d^4
+ 12*a*c*d^2*e^2 + 6*a^2*e^4)*x^3)/(3*e^6) - (c^3*d*(c*d^2 + 2*a*e^2)*x^4)
/e^5 + (c^3*(3*c*d^2 + 4*a*e^2)*x^5)/(5*e^4) - (c^4*d*x^6)/(3*e^3) + (c^4*
x^7)/(7*e^2) - (c*d^2 + a*e^2)^4/(e^9*(d + e*x)) - (8*c*d*(c*d^2 + a*e^2)^
3*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.24

method	result
norman	$\frac{-\frac{a^4e^8+8a^3cd^2e^6+24a^2c^2d^4e^4+24ac^3d^6e^2+8c^4d^8}{e^9}+\frac{c^4x^8}{7e}+\frac{2c^2(3a^2e^4+3acd^2e^2+c^2d^4)x^4}{3e^5}+\frac{4c^3(3ae^2+cd^2)x^6}{15e^3}-\frac{4dc^4x^7}{21e^2}+\frac{4c(e^6a}{}}{ex+d}$
default	$\frac{c(\frac{1}{7}x^7c^3e^6-\frac{1}{3}dc^3x^6e^5+\frac{4}{5}x^5ac^2e^6+\frac{3}{5}x^5c^3d^2e^4-2x^4ac^2de^5-x^4c^3d^3e^3+2x^3a^2ce^6+4x^3ac^2d^2e^4+\frac{5}{3}x^3c^3d^4e^2-6x^2a^2cde^5-}{e^8}$
risch	$-\frac{6c^2x^2a^2d}{e^3}-\frac{8c^3x^2ad^3}{e^5}+\frac{18c^2d^2a^2x}{e^4}+\frac{20c^3d^4ax}{e^6}-\frac{8dc\ln(ex+d)a^3}{e^3}-\frac{2c^3x^4ad}{e^3}+\frac{4c^3x^3ad^2}{e^4}-\frac{c^4dx^6}{3e^3}-24d$
parallelrisch	$-\frac{840\ln(ex+d)c^4d^8-15x^8c^4e^8+105a^4e^8+840\ln(ex+d)xa^3cde^7+2520\ln(ex+d)xa^2c^2d^3e^5+2520\ln(ex+d)xac^3d^5e^3+840}{}$

input
```
int((c*x^2+a)^4/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(a^4*e^8+8*a^3*c*d^2*e^6+24*a^2*c^2*d^4*e^4+24*a*c^3*d^6*e^2+8*c^4*d^8)/
e^9+1/7/e*c^4*x^8+2/3*c^2*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d^4)/e^5*x^4+4/15*c
^3*(3*a*e^2+c*d^2)/e^3*x^6-4/21*d/e^2*c^4*x^7+4/e^7*c*(a^3*e^6+3*a^2*c*d^2
*e^4+3*a*c^2*d^4*e^2+c^3*d^6)*x^2-4/3*d*c^2*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d
^4)/e^6*x^3-2/5*d*c^3*(3*a*e^2+c*d^2)/e^4*x^5)/(e*x+d)-8*d/e^9*c*(a^3*e^6+
3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6)*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.65

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx = \frac{15\,c^4e^8x^8 - 20\,c^4de^7x^7 - 105\,c^4d^8 - 420\,ac^3d^6e^2 - 630\,a^2c^2d^4e^4 - 420\,a^3cd^2e^6 - 105\,a^4e^8 + 28\,(c^4d^2e^6 +$$

input
```
integrate((c*x^2+a)^4/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/105*(15*c^4*e^8*x^8 - 20*c^4*d*e^7*x^7 - 105*c^4*d^8 - 420*a*c^3*d^6*e^2
 - 630*a^2*c^2*d^4*e^4 - 420*a^3*c*d^2*e^6 - 105*a^4*e^8 + 28*(c^4*d^2*e^6
 + 3*a*c^3*e^8)*x^6 - 42*(c^4*d^3*e^5 + 3*a*c^3*d*e^7)*x^5 + 70*(c^4*d^4*e
^4 + 3*a*c^3*d^2*e^6 + 3*a^2*c^2*e^8)*x^4 - 140*(c^4*d^5*e^3 + 3*a*c^3*d^3
*e^5 + 3*a^2*c^2*d*e^7)*x^3 + 420*(c^4*d^6*e^2 + 3*a*c^3*d^4*e^4 + 3*a^2*c
^2*d^2*e^6 + a^3*c*e^8)*x^2 + 105*(7*c^4*d^7*e + 20*a*c^3*d^5*e^3 + 18*a^2
*c^2*d^3*e^5 + 4*a^3*c*d*e^7)*x - 840*(c^4*d^8 + 3*a*c^3*d^6*e^2 + 3*a^2*c
^2*d^4*e^4 + a^3*c*d^2*e^6 + (c^4*d^7*e + 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*
e^5 + a^3*c*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.23

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx = -\frac{c^4dx^6}{3e^3}+\frac{c^4x^7}{7e^2}-\frac{8cd(ae^2+cd^2)^3\log(d+ex)}{e^9}+x^5\cdot\left(\frac{4ac^3}{5e^2}+\frac{3c^4d^2}{5e^4}\right)+x^4\left(-\frac{2ac^3d}{e^3}-\frac{c^4d^3}{e^5}\right)+x^3\cdot\left(\frac{2a^2c^2}{e^2}+\frac{4ac^3d^2}{e^4}+\frac{5c^4d^4}{3e^6}\right)+x^2\left(-\frac{6a^2c^2d}{e^3}-\frac{8ac^3d^3}{e^5}-\frac{3c^4d^5}{e^7}\right)+x\left(\frac{4a^3c}{e^2}+\frac{18a^2c^2d^2}{e^4}+\frac{20ac^3d^4}{e^6}+\frac{7c^4d^6}{e^8}\right)+\frac{-a^4e^8-4a^3cd^2e^6-6a^2c^2d^4e^4-4ac^3d^6e^2-c^4d^8}{de^9+e^{10}x}$$

input
```
integrate((c*x**2+a)**4/(e*x+d)**2,x)
```

output
```
-c**4*d*x**6/(3*e**3) + c**4*x**7/(7*e**2) - 8*c*d*(a*e**2 + c*d**2)**3*lo
g(d + e*x)/e**9 + x**5*(4*a*c**3/(5*e**2) + 3*c**4*d**2/(5*e**4)) + x**4*(
-2*a*c**3*d/e**3 - c**4*d**3/e**5) + x**3*(2*a**2*c**2/e**2 + 4*a*c**3*d**
2/e**4 + 5*c**4*d**4/(3*e**6)) + x**2*(-6*a**2*c**2*d/e**3 - 8*a*c**3*d**3
/e**5 - 3*c**4*d**5/e**7) + x*(4*a**3*c/e**2 + 18*a**2*c**2*d**2/e**4 + 20
*a*c**3*d**4/e**6 + 7*c**4*d**6/e**8) + (-a**4*e**8 - 4*a**3*c*d**2*e**6 -
 6*a**2*c**2*d**4*e**4 - 4*a*c**3*d**6*e**2 - c**4*d**8)/(d*e**9 + e**10*x
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.29

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx = -\frac{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8}{e^{10}x+de^9}$$

$$+\frac{15\,c^4e^6x^7-35\,c^4de^5x^6+21\,(3\,c^4d^2e^4+4\,ac^3e^6)x^5-105\,(c^4d^3e^3+2\,ac^3de^5)x^4+35\,(5\,c^4d^4e^2+12\,ac^3\ldots}{}$$

$$-\frac{8\,(c^4d^7+3\,ac^3d^5e^2+3\,a^2c^2d^3e^4+a^3cde^6)\log(ex+d)}{e^9}$$

input
```
integrate((c*x^2+a)^4/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^
8)/(e^10*x + d*e^9) + 1/105*(15*c^4*e^6*x^7 - 35*c^4*d*e^5*x^6 + 21*(3*c^4
*d^2*e^4 + 4*a*c^3*e^6)*x^5 - 105*(c^4*d^3*e^3 + 2*a*c^3*d*e^5)*x^4 + 35*(
5*c^4*d^4*e^2 + 12*a*c^3*d^2*e^4 + 6*a^2*c^2*e^6)*x^3 - 105*(3*c^4*d^5*e +
 8*a*c^3*d^3*e^3 + 6*a^2*c^2*d*e^5)*x^2 + 105*(7*c^4*d^6 + 20*a*c^3*d^4*e^
2 + 18*a^2*c^2*d^2*e^4 + 4*a^3*c*e^6)*x)/e^8 - 8*(c^4*d^7 + 3*a*c^3*d^5*e^
2 + 3*a^2*c^2*d^3*e^4 + a^3*c*d*e^6)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.62

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx$$
$$= \frac{\left(15\,c^4 - \frac{140\,c^4 d}{ex+d} + \frac{84\,(7\,c^4d^2e^2+ac^3e^4)}{(ex+d)^2e^2} - \frac{210\,(7\,c^4d^3e^3+3\,ac^3de^5)}{(ex+d)^3e^3} + \frac{70\,(35\,c^4d^4e^4+30\,ac^3d^2e^6+3\,a^2c^2e^8)}{(ex+d)^4e^4} - \frac{420\,(7\,c^4d^5e^5+10\,ac^3}{(ex+d)}\right.}{105\,e^9}$$
$$+ \frac{8\,(c^4d^7+3\,ac^3d^5e^2+3\,a^2c^2d^3e^4+a^3cde^6)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^9}$$
$$- \frac{\frac{c^4d^8e^7}{ex+d} + \frac{4\,ac^3d^6e^9}{ex+d} + \frac{6\,a^2c^2d^4e^{11}}{ex+d} + \frac{4\,a^3cd^2e^{13}}{ex+d} + \frac{a^4e^{15}}{ex+d}}{e^{16}}$$

input
```
integrate((c*x^2+a)^4/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/105*(15*c^4 - 140*c^4*d/(e*x + d) + 84*(7*c^4*d^2*e^2 + a*c^3*e^4)/((e*x
 + d)^2*e^2) - 210*(7*c^4*d^3*e^3 + 3*a*c^3*d*e^5)/((e*x + d)^3*e^3) + 70*
(35*c^4*d^4*e^4 + 30*a*c^3*d^2*e^6 + 3*a^2*c^2*e^8)/((e*x + d)^4*e^4) - 42
0*(7*c^4*d^5*e^5 + 10*a*c^3*d^3*e^7 + 3*a^2*c^2*d*e^9)/((e*x + d)^5*e^5) +
 420*(7*c^4*d^6*e^6 + 15*a*c^3*d^4*e^8 + 9*a^2*c^2*d^2*e^10 + a^3*c*e^12)/
((e*x + d)^6*e^6))*(e*x + d)^7/e^9 + 8*(c^4*d^7 + 3*a*c^3*d^5*e^2 + 3*a^2*
c^2*d^3*e^4 + a^3*c*d*e^6)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^9 - (c
^4*d^8*e^7/(e*x + d) + 4*a*c^3*d^6*e^9/(e*x + d) + 6*a^2*c^2*d^4*e^11/(e*x
 + d) + 4*a^3*c*d^2*e^13/(e*x + d) + a^4*e^15/(e*x + d))/e^16
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.75

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx = x^4\left(\frac{c^4\,d^3}{2\,e^5}-\frac{d\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{2\,e}\right)+x^5\left(\frac{4\,a\,c^3}{5\,e^2}+\frac{3\,c^4\,d^2}{5\,e^4}\right)$$
$$+x^2\left(\frac{d\,\left(\frac{d^2\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e^2}+\frac{2\,d\,\left(\frac{2\,c^4\,d^3}{e^5}-\frac{2\,d\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e}\right)}{e}-\frac{6\,a^2\,c^2}{e^2}\right)}{e}-\frac{d^2\,\left(\frac{2\,c^4\,d^3}{e^5}-\frac{2\,d\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e}\right)}{2\,e^2}\right)$$
$$-x^3\left(\frac{d^2\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{3\,e^2}+\frac{2\,d\,\left(\frac{2\,c^4\,d^3}{e^5}-\frac{2\,d\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e}\right)}{3\,e}-\frac{2\,a^2\,c^2}{e^2}\right)$$
$$+x\left(\frac{4\,a^3\,c}{e^2}+\frac{d^2\,\left(\frac{d^2\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e^2}+\frac{2\,d\,\left(\frac{2\,c^4\,d^3}{e^5}-\frac{2\,d\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e}\right)}{e}-\frac{6\,a^2\,c^2}{e^2}\right)}{e^2}\right.$$
$$\left(2\,d\left(\frac{d^2\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e^2}+\frac{2\,d\,\left(\frac{2\,c^4\,d^3}{e^5}-\frac{2\,d\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{e}\right)}{e}-\frac{6\,a^2\,c^2}{e^2}\right)\quad d^2\left(\frac{2\,c^4\,d^3}{e^5}-\frac{2\,d\,\left(\frac{4\,a\,c^3}{e^2}+\frac{3\,c^4\,d^2}{e^4}\right)}{\dots}\right)\right.$$

input
```
int((a + c*x^2)^4/(d + e*x)^2,x)
```

output
```
x^4*((c^4*d^3)/(2*e^5) - (d*((4*a*c^3)/e^2 + (3*c^4*d^2)/e^4))/(2*e)) + x^
5*((4*a*c^3)/(5*e^2) + (3*c^4*d^2)/(5*e^4)) + x^2*((d*((d^2*((4*a*c^3)/e^2
 + (3*c^4*d^2)/e^4))/e^2 + (2*d*((2*c^4*d^3)/e^5 - (2*d*((4*a*c^3)/e^2 + (
3*c^4*d^2)/e^4))/e))/e - (6*a^2*c^2)/e^2))/e - (d^2*((2*c^4*d^3)/e^5 - (2*
d*((4*a*c^3)/e^2 + (3*c^4*d^2)/e^4))/e))/(2*e^2)) - x^3*((d^2*((4*a*c^3)/e
^2 + (3*c^4*d^2)/e^4))/(3*e^2) + (2*d*((2*c^4*d^3)/e^5 - (2*d*((4*a*c^3)/e
^2 + (3*c^4*d^2)/e^4))/e))/(3*e) - (2*a^2*c^2)/e^2) + x*((4*a^3*c)/e^2 + (
d^2*((d^2*((4*a*c^3)/e^2 + (3*c^4*d^2)/e^4))/e^2 + (2*d*((2*c^4*d^3)/e^5 -
 (2*d*((4*a*c^3)/e^2 + (3*c^4*d^2)/e^4))/e))/e - (6*a^2*c^2)/e^2))/e^2 - (
2*d*((2*d*((d^2*((4*a*c^3)/e^2 + (3*c^4*d^2)/e^4))/e^2 + (2*d*((2*c^4*d^3)
/e^5 - (2*d*((4*a*c^3)/e^2 + (3*c^4*d^2)/e^4))/e))/e - (6*a^2*c^2)/e^2))/e
 - (d^2*((2*c^4*d^3)/e^5 - (2*d*((4*a*c^3)/e^2 + (3*c^4*d^2)/e^4))/e))/e^2
))/e) - (a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2
*d^4*e^4)/(e*(d*e^8 + e^9*x)) + (c^4*x^7)/(7*e^2) - (log(d + e*x)*(8*c^4*d
^7 + 24*a*c^3*d^5*e^2 + 24*a^2*c^2*d^3*e^4 + 8*a^3*c*d*e^6))/e^9 - (c^4*d*
x^6)/(3*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.77

$$\int \frac{(a+cx^2)^4}{(d+ex)^2}\,dx$$
$$= \frac{-840\log(ex+d)\,a^3c\,d^2e^7x - 2520\log(ex+d)\,a^2c^2d^4e^5x - 2520\log(ex+d)\,a\,c^3d^6e^3x + 840c^4d^8ex + 42}{}$$

input
```
int((c*x^2+a)^4/(e*x+d)^2,x)
```

output
```
( - 840*log(d + e*x)*a**3*c*d**3*e**6 - 840*log(d + e*x)*a**3*c*d**2*e**7*
x - 2520*log(d + e*x)*a**2*c**2*d**5*e**4 - 2520*log(d + e*x)*a**2*c**2*d*
*4*e**5*x - 2520*log(d + e*x)*a*c**3*d**7*e**2 - 2520*log(d + e*x)*a*c**3*
d**6*e**3*x - 840*log(d + e*x)*c**4*d**9 - 840*log(d + e*x)*c**4*d**8*e*x
+ 105*a**4*e**9*x + 840*a**3*c*d**2*e**7*x + 420*a**3*c*d*e**8*x**2 + 2520
*a**2*c**2*d**4*e**5*x + 1260*a**2*c**2*d**3*e**6*x**2 - 420*a**2*c**2*d**
2*e**7*x**3 + 210*a**2*c**2*d*e**8*x**4 + 2520*a*c**3*d**6*e**3*x + 1260*a
*c**3*d**5*e**4*x**2 - 420*a*c**3*d**4*e**5*x**3 + 210*a*c**3*d**3*e**6*x*
*4 - 126*a*c**3*d**2*e**7*x**5 + 84*a*c**3*d*e**8*x**6 + 840*c**4*d**8*e*x
 + 420*c**4*d**7*e**2*x**2 - 140*c**4*d**6*e**3*x**3 + 70*c**4*d**5*e**4*x
**4 - 42*c**4*d**4*e**5*x**5 + 28*c**4*d**3*e**6*x**6 - 20*c**4*d**2*e**7*
x**7 + 15*c**4*d*e**8*x**8)/(105*d*e**9*(d + e*x))
```

3.98 $\int \frac{2+x^2}{2+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{2+x^2}{2+x} \, dx = -2x + \frac{x^2}{2} + 6\log(2+x)$$

output
```
-2*x+1/2*x^2+6*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{2+x^2}{2+x} \, dx = -6 - 2x + \frac{x^2}{2} + 6\log(2+x)$$

input
```
Integrate[(2 + x^2)/(2 + x),x]
```

output
```
-6 - 2*x + x^2/2 + 6*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2}{x+2} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(x+\frac{6}{x+2}-2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-2x+6\log(x+2)$$

input
```
Int[(2 + x^2)/(2 + x),x]
```

output
```
-2*x + x^2/2 + 6*Log[2 + x]
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-2x+\frac{x^2}{2}+6\ln\left(2+x\right)$	16
norman	$-2x+\frac{x^2}{2}+6\ln\left(2+x\right)$	16
risch	$-2x+\frac{x^2}{2}+6\ln\left(2+x\right)$	16
parallelrisch	$-2x+\frac{x^2}{2}+6\ln\left(2+x\right)$	16
meijerg	$6\ln\left(1+\frac{x}{2}\right)-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	18

input
```
int((x^2+2)/(2+x),x,method=_RETURNVERBOSE)
```

output
```
-2*x+1/2*x^2+6*ln(2+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx=\frac{1}{2}\,x^2-2\,x+6\,\log\left(x+2\right)$$

input
```
integrate((x^2+2)/(2+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - 2*x + 6*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{2+x^2}{2+x}\,dx = \frac{x^2}{2} - 2x + 6\log{(x+2)}$$

input
```
integrate((x**2+2)/(2+x),x)
```

output
```
x**2/2 - 2*x + 6*log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(x+2\right)$$

input
```
integrate((x^2+2)/(2+x),x, algorithm="maxima")
```

output
```
1/2*x^2 - 2*x + 6*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(|x+2|\right)$$

input
```
integrate((x^2+2)/(2+x),x, algorithm="giac")
```

output
```
1/2*x^2 - 2*x + 6*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = 6\ \ln\left(x+2\right)-2\,x+\frac{x^2}{2}$$

input
```
int((x^2 + 2)/(x + 2),x)
```

output
```
6*log(x + 2) - 2*x + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = 6\log(x+2)+\frac{x^2}{2}-2x$$

input
```
int((x^2+2)/(2+x),x)
```

output
```
(12*log(x + 2) + x**2 - 4*x)/2
```

3.99 $\int \frac{-4+x^2}{2+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{-4+x^2}{2+x} \, dx = \frac{1}{2}(2-x)^2$$

output
```
1/2*(2-x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-4+x^2}{2+x} \, dx = -2x+\frac{x^2}{2}$$

input
```
Integrate[(-4 + x^2)/(2 + x),x]
```

output
```
-2*x + x^2/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-4}{x+2}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int (x-2)dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{2}(2-x)^2$$

input
```
Int[(-4 + x^2)/(2 + x),x]
```

output
```
(2 - x)^2/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(x-4)}{2}$	7
meijerg	$-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	9
default	$-2x+\frac{1}{2}x^2$	10
norman	$-2x+\frac{1}{2}x^2$	10
risch	$-2x+\frac{1}{2}x^2$	10
parallelrisch	$-2x+\frac{1}{2}x^2$	10
parts	$-2x+\frac{1}{2}x^2$	10
orering	$\frac{x(x-4)(x^2-4)}{2(x-2)(2+x)}$	22

input
```
int((x^2-4)/(2+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x-4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x$$

input
```
integrate((x^2-4)/(2+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{x^2}{2} - 2x$$

input
```
integrate((x**2-4)/(2+x),x)
```

output
```
x**2/2 - 2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x$$

input
```
integrate((x^2-4)/(2+x),x, algorithm="maxima")
```

output
```
1/2*x^2 - 2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x$$

input
```
integrate((x^2-4)/(2+x),x, algorithm="giac")
```

output
```
1/2*x^2 - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-4+x^2}{2+x}\,dx=\frac{x\,(x-4)}{2}$$

input
```
int((x^2 - 4)/(x + 2),x)
```

output
```
(x*(x - 4))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-4+x^2}{2+x}\,dx=\frac{x(x-4)}{2}$$

input
```
int((x^2-4)/(2+x),x)
```

output
```
(x*(x - 4))/2
```

3.100 $\int \frac{-7+4x^2}{3+2x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{-7+4x^2}{3+2x}\,dx = -3x+x^2+\log(3+2x)$$

output
```
-3*x+x^2+ln(3+2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{-7+4x^2}{3+2x}\,dx = -\frac{27}{4}-3x+x^2+\log(3+2x)$$

input
```
Integrate[(-7 + 4*x^2)/(3 + 2*x),x]
```

output
```
-27/4 - 3*x + x^2 + Log[3 + 2*x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-7}{2x+3}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(2x+\frac{2}{2x+3}-3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^2-3x+\log(2x+3)$$

input
```
Int[(-7 + 4*x^2)/(3 + 2*x),x]
```

output
```
-3*x + x^2 + Log[3 + 2*x]
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x^2-3x+\ln\left(x+\frac{3}{2}\right)$	12
default	$-3x+x^2+\ln\left(2x+3\right)$	14
norman	$-3x+x^2+\ln\left(2x+3\right)$	14
risch	$-3x+x^2+\ln\left(2x+3\right)$	14
meijerg	$\ln\left(1+\frac{2x}{3}\right)-\frac{x(-2x+6)}{2}$	16

input
```
int((4*x^2-7)/(2*x+3),x,method=_RETURNVERBOSE)
```

output
```
x^2-3*x+ln(x+3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2-3\,x+\log\left(2\,x+3\right)$$

input
```
integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")
```

output
```
x^2 - 3*x + log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2-3x+\log\left(2x+3\right)$$

input
```
integrate((4*x**2-7)/(3+2*x),x)
```

output
```
x**2 - 3*x + log(2*x + 3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2-3\,x+\log\left(2\,x+3\right)$$

input
```
integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")
```

output
```
x^2 - 3*x + log(2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2-3\,x+\log\left(|2\,x+3|\right)$$

input
```
integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")
```

output
```
x^2 - 3*x + log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{-7+4x^2}{3+2x}\,dx = \ln\left(x+\frac{3}{2}\right)-3\,x+x^2$$

input
```
int((4*x^2 - 7)/(2*x + 3),x)
```

output
```
log(x + 3/2) - 3*x + x^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-7+4x^2}{3+2x}\,dx = \log(2x+3)+x^2-3x$$

input
```
int((4*x^2-7)/(3+2*x),x)
```

output
```
log(2*x + 3) + x**2 - 3*x
```

3.101 $\int \frac{1+x^2}{1+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1+x^2}{1+x} \, dx = -x + \frac{x^2}{2} + 2\log(1+x)$$

output
```
-x+1/2*x^2+2*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x^2}{1+x} \, dx = \frac{1}{2}\left(-3-2x+x^2+4\log(1+x)\right)$$

input
```
Integrate[(1 + x^2)/(1 + x),x]
```

output
```
(-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x+1} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(x+\frac{2}{x+1}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-x+2\log(x+1)$$

input
```
Int[(1 + x^2)/(1 + x),x]
```

output
```
-x + x^2/2 + 2*Log[1 + x]
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-x+\frac{x^{2}}{2}+2\ln \left(x +1\right)$	16
norman	$-x+\frac{x^{2}}{2}+2\ln \left(x +1\right)$	16
meijerg	$-\frac{x \left(-3 x +6\right)}{6}+2\ln \left(x +1\right)$	16
risch	$-x+\frac{x^{2}}{2}+2\ln \left(x +1\right)$	16
parallelrisch	$-x+\frac{x^{2}}{2}+2\ln \left(x +1\right)$	16

input
```
int((x^2+1)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*x^2+2*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx=\frac{1}{2}\,x^{2}-x+2\,\log \left(x+1\right)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - x + 2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{1+x}\,dx = \frac{x^2}{2} - x + 2\log(x+1)$$

input
```
integrate((x**2+1)/(1+x),x)
```

output
```
x**2/2 - x + 2*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}x^2 - x + 2\log(x+1)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="maxima")
```

output
```
1/2*x^2 - x + 2*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}x^2 - x + 2\log(|x+1|)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="giac")
```

output
```
1/2*x^2 - x + 2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = 2\,\ln(x+1) - x + \frac{x^2}{2}$$

input
```
int((x^2 + 1)/(x + 1),x)
```

output
```
2*log(x + 1) - x + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = 2\log(x+1) + \frac{x^2}{2} - x$$

input
```
int((x^2+1)/(1+x),x)
```

output
```
(4*log(x + 1) + x**2 - 2*x)/2
```

3.102 $\int \frac{(d+ex)^4}{a+cx^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{(d+ex)^4}{a+cx^2} \, dx = \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{2de^3x^2}{c}+\frac{e^4x^3}{3c}+\frac{(c^2d^4-6acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}+\frac{2de(cd^2-ae^2)\log(a+cx^2)}{c^2}$$

output
```
e^2*(-a*e^2+6*c*d^2)*x/c^2+2*d*e^3*x^2/c+1/3*e^4*x^3/c+(a^2*e^4-6*a*c*d^2*
e^2+c^2*d^4)*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/c^(5/2)+2*d*e*(-a*e^2+c*d^2
)*ln(c*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^4}{a+cx^2} \, dx = \frac{(c^2d^4-6acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}+\frac{e(-3ae^3x+cex(18d^2+6dex+e^2x^2)+6(cd^3-ade^2)\log(a+cx^2))}{3c^2}$$

input
```
Integrate[(d + e*x)^4/(a + c*x^2),x]
```

output
```
((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]
*c^(5/2)) + (e*(-3*a*e^3*x + c*e*x*(18*d^2 + 6*d*e*x + e^2*x^2) + 6*(c*d^3
 - a*d*e^2)*Log[a + c*x^2]))/(3*c^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{a+cx^2}\,dx$$

$$\Big\downarrow \text{478}$$

$$\int \left(\frac{a^2e^4+4cdex\left(cd^2-ae^2\right)-6acd^2e^2+c^2d^4}{c^2\left(a+cx^2\right)}+\frac{e^2\left(6cd^2-ae^2\right)}{c^2}+\frac{4de^3x}{c}+\frac{e^4x^2}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(a^2e^4-6acd^2e^2+c^2d^4\right)}{\sqrt{a}c^{5/2}}+\frac{2de\left(cd^2-ae^2\right)\log\left(a+cx^2\right)}{c^2}+\frac{e^2x\left(6cd^2-ae^2\right)}{c^2}+\frac{2de^3x^2}{c}+\frac{e^4x^3}{3c}$$

input
```
Int[(d + e*x)^4/(a + c*x^2),x]
```

output
```
(e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (2*d*e^3*x^2)/c + (e^4*x^3)/(3*c) + ((c^2*
d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/
2)) + (2*d*e*(c*d^2 - a*e^2)*Log[a + c*x^2])/c^2
```

Defintions of rubi rules used

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

method	result
default	$-\frac{e^{2}\left(-\frac{1}{3}c\,e^{2}x^{3}-2cd\,x^{2}e+a\,e^{2}x-6c\,d^{2}x\right)}{c^{2}}+\frac{\frac{\left(-4d\,e^{3}ac+4d^{3}e\,c^{2}\right)\ln\left(c\,x^{2}+a\right)}{2c}+\frac{\left(a^{2}e^{4}-6ac\,d^{2}e^{2}+c^{2}d^{4}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c^{2}}$
risch	$\frac{e^{4}x^{3}}{3c}+\frac{2d\,e^{3}x^{2}}{c}-\frac{e^{4}ax}{c^{2}}+\frac{6e^{2}d^{2}x}{c}-\frac{2a\ln\left(e^{4}a^{3}-6d^{2}e^{2}a^{2}c+d^{4}a\,c^{2}-\sqrt{-ac(a^{2}e^{4}-6ac\,d^{2}e^{2}+c^{2}d^{4})^{2}}\,x\right)d\,e^{3}}{c^{2}}+\frac{2\ln\left(e^{4}a^{3}-\right.}{}$

input
```
int((e*x+d)^4/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-e^2/c^2*(-1/3*c*e^2*x^3-2*c*d*x^2*e+a*e^2*x-6*c*d^2*x)+1/c^2*(1/2*(-4*a*c
*d*e^3+4*c^2*d^3*e)/c*ln(c*x^2+a)+(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)/(a*c)^(1
/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.30

$$\int \frac{(d+ex)^4}{a+cx^2}\,dx$$
$$=\left[\frac{2\,ac^2e^4x^3+12\,ac^2de^3x^2-3\,(c^2d^4-6\,acd^2e^2+a^2e^4)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)+6\,(6\,ac^2d^2e^2-a^2ce^4)\ldots}{6\,ac^3}\right.$$

input
```
integrate((e*x+d)^4/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*a*c^2*e^4*x^3 + 12*a*c^2*d*e^3*x^2 - 3*(c^2*d^4 - 6*a*c*d^2*e^2 +
a^2*e^4)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(6*a
*c^2*d^2*e^2 - a^2*c*e^4)*x + 12*(a*c^2*d^3*e - a^2*c*d*e^3)*log(c*x^2 + a
))/(a*c^3), 1/3*(a*c^2*e^4*x^3 + 6*a*c^2*d*e^3*x^2 + 3*(c^2*d^4 - 6*a*c*d^
2*e^2 + a^2*e^4)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(6*a*c^2*d^2*e^2 - a^
2*c*e^4)*x + 6*(a*c^2*d^3*e - a^2*c*d*e^3)*log(c*x^2 + a))/(a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(117) = 234$.

Time = 0.48 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.26

$$\int \frac{(d+ex)^4}{a+cx^2}\,dx = x\left(-\frac{ae^4}{c^2}+\frac{6d^2e^2}{c}\right)+\left(-\frac{2de(ae^2-cd^2)}{c^2} - \frac{\sqrt{-ac^5}(a^2e^4-6acd^2e^2+c^2d^4)}{2ac^5}\right)\log\left(x+\frac{4a^2de^3+2ac^2\left(-\frac{2de(ae^2-cd^2)}{c^2}-\frac{\sqrt{-ac^5}(a^2e^4-6acd^2e^2+c^2d^4)}{2ac^5}\right)-}{a^2e^4-6acd^2e^2+c^2d^4}\right. + \left(-\frac{2de(ae^2-cd^2)}{c^2} + \frac{\sqrt{-ac^5}(a^2e^4-6acd^2e^2+c^2d^4)}{2ac^5}\right)\log\left(x+\frac{4a^2de^3+2ac^2\left(-\frac{2de(ae^2-cd^2)}{c^2}+\frac{\sqrt{-ac^5}(a^2e^4-6acd^2e^2+c^2d^4)}{2ac^5}\right)-}{a^2e^4-6acd^2e^2+c^2d^4}\right. + \frac{2de^3x^2}{c}+\frac{e^4x^3}{3c}$$

input
```
integrate((e*x+d)**4/(c*x**2+a),x)
```

output
```
x*(-a*e**4/c**2 + 6*d**2*e**2/c) + (-2*d*e*(a*e**2 - c*d**2)/c**2 - sqrt(-
a*c**5)*(a**2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)/(2*a*c**5))*log(x + (4*a
**2*d*e**3 + 2*a*c**2*(-2*d*e*(a*e**2 - c*d**2)/c**2 - sqrt(-a*c**5)*(a**2
*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)/(2*a*c**5)) - 4*a*c*d**3*e)/(a**2*e**
4 - 6*a*c*d**2*e**2 + c**2*d**4)) + (-2*d*e*(a*e**2 - c*d**2)/c**2 + sqrt(
-a*c**5)*(a**2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)/(2*a*c**5))*log(x + (4*
a**2*d*e**3 + 2*a*c**2*(-2*d*e*(a*e**2 - c*d**2)/c**2 + sqrt(-a*c**5)*(a**
2*e**4 - 6*a*c*d**2*e**2 + c**2*d**4)/(2*a*c**5)) - 4*a*c*d**3*e)/(a**2*e*
*4 - 6*a*c*d**2*e**2 + c**2*d**4)) + 2*d*e**3*x**2/c + e**4*x**3/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^4}{a+cx^2}\,dx = \frac{2\,(cd^3e-ade^3)\log\left(cx^2+a\right)}{c^2} + \frac{\left(c^2d^4-6\,acd^2e^2+a^2e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{ce^4x^3+6\,cde^3x^2+3\,(6\,cd^2e^2-ae^4)x}{3\,c^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+a),x, algorithm="maxima")
```

output
```
2*(c*d^3*e - a*d*e^3)*log(c*x^2 + a)/c^2 + (c^2*d^4 - 6*a*c*d^2*e^2 + a^2*
e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/3*(c*e^4*x^3 + 6*c*d*e^3*x^
2 + 3*(6*c*d^2*e^2 - a*e^4)*x)/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^4}{a+cx^2}\,dx = \frac{2\,(cd^3e-ade^3)\log\left(cx^2+a\right)}{c^2} + \frac{\left(c^2d^4-6\,acd^2e^2+a^2e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{c^2e^4x^3+6\,c^2de^3x^2+18\,c^2d^2e^2x-3\,ace^4x}{3\,c^3}$$

input
```
integrate((e*x+d)^4/(c*x^2+a),x, algorithm="giac")
```

output
```
2*(c*d^3*e - a*d*e^3)*log(c*x^2 + a)/c^2 + (c^2*d^4 - 6*a*c*d^2*e^2 + a^2*
e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/3*(c^2*e^4*x^3 + 6*c^2*d*e^
3*x^2 + 18*c^2*d^2*e^2*x - 3*a*c*e^4*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^4}{a+cx^2} \, dx = \frac{e^4 x^3}{3 c} - x \left(\frac{a e^4}{c^2} - \frac{6 d^2 e^2}{c}\right) + \frac{2 d e^3 x^2}{c} + \frac{\operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right) (a^2 e^4 - 6 a c d^2 e^2 + c^2 d^4)}{\sqrt{a} c^{5/2}} - \frac{\ln (c x^2 + a) (16 a^2 c^3 d e^3 - 16 a c^4 d^3 e)}{8 a c^5}$$

input
```
int((d + e*x)^4/(a + c*x^2),x)
```

output
```
(e^4*x^3)/(3*c) - x*((a*e^4)/c^2 - (6*d^2*e^2)/c) + (2*d*e^3*x^2)/c + (ata
n((c^(1/2)*x)/a^(1/2))*(a^2*e^4 + c^2*d^4 - 6*a*c*d^2*e^2))/(a^(1/2)*c^(5/
2)) - (log(a + c*x^2)*(16*a^2*c^3*d*e^3 - 16*a*c^4*d^3*e))/(8*a*c^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^4}{a+cx^2} \, dx = \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right) a^2e^4 - 18\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right) ac\,d^2e^2 + 3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right) c^2d^4 - 6\log(c\,x^2+a)}{3a\,c^3}$$

input
```
int((e*x+d)^4/(c*x^2+a),x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*e**4 - 18*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d**2*e**2 + 3*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*c**2*d**4 - 6*log(a + c*x**2)*a**2*c*d*e**3 + 6*
log(a + c*x**2)*a*c**2*d**3*e - 3*a**2*c*e**4*x + 18*a*c**2*d**2*e**2*x +
6*a*c**2*d*e**3*x**2 + a*c**2*e**4*x**3)/(3*a*c**3)
```

3.103 $\int \frac{(d+ex)^3}{a+cx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \frac{3de^2x}{c}+\frac{e^3x^2}{2c}+\frac{d(cd^2-3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{e(3cd^2-ae^2)\log(a+cx^2)}{2c^2}$$

output
```
3*d*e^2*x/c+1/2*e^3*x^2/c+d*(-3*a*e^2+c*d^2)*arctan(c^(1/2)*x/a^(1/2))/a^(
1/2)/c^(3/2)+1/2*e*(-a*e^2+3*c*d^2)*ln(c*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \frac{d(cd^2-3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{e(cex(6d+ex)+(3cd^2-ae^2)\log(a+cx^2))}{2c^2}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^2),x]
```

output
```
(d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (e*(
c*e*x*(6*d + e*x) + (3*c*d^2 - a*e^2)*Log[a + c*x^2]))/(2*c^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{a+cx^2} \, dx$$

$$\Big\downarrow \ 478$$

$$\int \left(\frac{ex(3cd^2-ae^2)-3ade^2+cd^3}{c(a+cx^2)}+\frac{3de^2}{c}+\frac{e^3x}{c}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2-3ae^2)}{\sqrt{a}c^{3/2}}+\frac{e(3cd^2-ae^2)\log\left(a+cx^2\right)}{2c^2}+\frac{3de^2x}{c}+\frac{e^3x^2}{2c}$$

input
```
Int[(d + e*x)^3/(a + c*x^2),x]
```

output
```
(3*d*e^2*x)/c + (e^3*x^2)/(2*c) + (d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/
Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (e*(3*c*d^2 - a*e^2)*Log[a + c*x^2])/(2*c^2)
```

Defintions of rubi rules used

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

method	result
default	$\frac{e^2\left(\frac{1}{2}e\,x^2+3dx\right)}{c}+\frac{\frac{\left(-a\,e^3+3c\,d^2e\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(-3ad\,e^2+c\,d^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{e^3x^2}{2c}+\frac{3d\,e^2x}{c}-\frac{a\ln\left(-3a^2d\,e^2+ac\,d^3-\sqrt{-a\,d^2c(3a\,e^2-c\,d^2)^2}\,x\right)e^3}{2c^2}+\frac{3\ln\left(-3a^2d\,e^2+ac\,d^3-\sqrt{-a\,d^2c(3a\,e^2-c\,d^2)^2}\,x\right)d^2e}{2c}$

input
```
int((e*x+d)^3/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
e^2/c*(1/2*e*x^2+3*d*x)+1/c*(1/2*(-a*e^3+3*c*d^2*e)/c*ln(c*x^2+a)+(-3*a*d*
e^2+c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx$$
$$=\left[\frac{ace^3x^2+6\,acde^2x+(cd^3-3\,ade^2)\sqrt{-ac}\log\left(\frac{cx^2+2\,\sqrt{-ac}x-a}{cx^2+a}\right)+(3\,acd^2e-a^2e^3)\log\left(cx^2+a\right)}{2\,ac^2},\ \frac{ace^3x^2}{}\right.$$

input
```
integrate((e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(a*c*e^3*x^2 + 6*a*c*d*e^2*x + (c*d^3 - 3*a*d*e^2)*sqrt(-a*c)*log((c*
x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + (3*a*c*d^2*e - a^2*e^3)*log(c*x^2
 + a))/(a*c^2), 1/2*(a*c*e^3*x^2 + 6*a*c*d*e^2*x + 2*(c*d^3 - 3*a*d*e^2)*s
qrt(a*c)*arctan(sqrt(a*c)*x/a) + (3*a*c*d^2*e - a^2*e^3)*log(c*x^2 + a))/(
a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(83) = 166$.

Time = 0.39 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \left(-\frac{e(ae^2-3cd^2)}{2c^2} - \frac{d\sqrt{-ac^5}\cdot(3ae^2-cd^2)}{2ac^4}\right)\log\left(x + \frac{-a^2e^3 - 2ac^2\left(-\frac{e(ae^2-3cd^2)}{2c^2} - \frac{d\sqrt{-ac^5}\cdot(3ae^2-cd^2)}{2ac^4}\right) + 3acd^2e}{3acde^2 - c^2d^3}\right) + \left(-\frac{e(ae^2-3cd^2)}{2c^2} + \frac{d\sqrt{-ac^5}\cdot(3ae^2-cd^2)}{2ac^4}\right)\log\left(x + \frac{-a^2e^3 - 2ac^2\left(-\frac{e(ae^2-3cd^2)}{2c^2} + \frac{d\sqrt{-ac^5}\cdot(3ae^2-cd^2)}{2ac^4}\right) + 3acd^2e}{3acde^2 - c^2d^3}\right) + \frac{3de^2x}{c} + \frac{e^3x^2}{2c}$$

input
```
integrate((e*x+d)**3/(c*x**2+a),x)
```

output
```
(-e*(a*e**2 - 3*c*d**2)/(2*c**2) - d*sqrt(-a*c**5)*(3*a*e**2 - c*d**2)/(2*
a*c**4))*log(x + (-a**2*e**3 - 2*a*c**2*(-e*(a*e**2 - 3*c*d**2)/(2*c**2) -
 d*sqrt(-a*c**5)*(3*a*e**2 - c*d**2)/(2*a*c**4)) + 3*a*c*d**2*e)/(3*a*c*d*
e**2 - c**2*d**3)) + (-e*(a*e**2 - 3*c*d**2)/(2*c**2) + d*sqrt(-a*c**5)*(3
*a*e**2 - c*d**2)/(2*a*c**4))*log(x + (-a**2*e**3 - 2*a*c**2*(-e*(a*e**2 -
 3*c*d**2)/(2*c**2) + d*sqrt(-a*c**5)*(3*a*e**2 - c*d**2)/(2*a*c**4)) + 3*
a*c*d**2*e)/(3*a*c*d*e**2 - c**2*d**3)) + 3*d*e**2*x/c + e**3*x**2/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \frac{(cd^3-3\,ade^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{e^3x^2+6\,de^2x}{2\,c} + \frac{(3\,cd^2e-ae^3)\log\left(cx^2+a\right)}{2\,c^2}$$

input
```
integrate((e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

output
```
(c*d^3 - 3*a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(e^3*x^2 + 6
*d*e^2*x)/c + 1/2*(3*c*d^2*e - a*e^3)*log(c*x^2 + a)/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \frac{(cd^3-3\,ade^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{(3\,cd^2e-ae^3)\log\left(cx^2+a\right)}{2\,c^2} + \frac{ce^3x^2+6\,cde^2x}{2\,c^2}$$

input
```
integrate((e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

output
```
(c*d^3 - 3*a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(3*c*d^2*e -
 a*e^3)*log(c*x^2 + a)/c^2 + 1/2*(c*e^3*x^2 + 6*c*d*e^2*x)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \frac{e^3\,x^2}{2\,c} - \frac{\ln\left(c\,x^2+a\right)\,\left(4\,a^2\,c^2\,e^3-12\,a\,c^3\,d^2\,e\right)}{8\,a\,c^4} + \frac{3\,d\,e^2\,x}{c} - \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,a\,e^2-c\,d^2\right)}{\sqrt{a}\,c^{3/2}}$$

input
```
int((d + e*x)^3/(a + c*x^2),x)
```

output
```
(e^3*x^2)/(2*c) - (log(a + c*x^2)*(4*a^2*c^2*e^3 - 12*a*c^3*d^2*e))/(8*a*c
^4) + (3*d*e^2*x)/c - (d*atan((c^(1/2)*x)/a^(1/2))*(3*a*e^2 - c*d^2))/(a^(
1/2)*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^3}{a+cx^2}\,dx = \frac{-6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ad\,e^2+2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)c\,d^3-\log(c\,x^2+a)\,a^2e^3+3\log(c\,x^2+a)\,ac\,d^2e+6}{2a\,c^2}$$

input
```
int((e*x+d)^3/(c*x^2+a),x)
```

output
```
( - 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*d*e**2 + 2*sqrt(c)*s
qrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c*d**3 - log(a + c*x**2)*a**2*e**3 +
3*log(a + c*x**2)*a*c*d**2*e + 6*a*c*d*e**2*x + a*c*e**3*x**2)/(2*a*c**2)
```

3.104 $\int \frac{(d+ex)^2}{a+cx^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{(d+ex)^2}{a+cx^2} \, dx = \frac{e^2x}{c} + \frac{(cd^2-ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{de\log(a+cx^2)}{c}$$

output
```
e^2*x/c+(-a*e^2+c*d^2)*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/c^(3/2)+d*e*ln(c*
x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^2}{a+cx^2} \, dx = \frac{(cd^2-ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{e(ex+d\log(a+cx^2))}{c}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^2),x]
```

output
```
((c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (e*(e*x
+ d*Log[a + c*x^2]))/c
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+cx^2}\,dx$$

$$\Big\downarrow \text{478}$$

$$\int \left(\frac{-ae^2+cd^2+2cdex}{c\,(a+cx^2)}+\frac{e^2}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2-ae^2)}{\sqrt{a}c^{3/2}}+\frac{de\log(a+cx^2)}{c}+\frac{e^2x}{c}$$

input
```
Int[(d + e*x)^2/(a + c*x^2),x]
```

output
```
(e^2*x)/c + ((c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)
) + (d*e*Log[a + c*x^2])/c
```

Defintions of rubi rules used

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result
default	$\frac{e^2x}{c}+\frac{de\ln(c\,x^2+a)+\frac{(-a\,e^2+c\,d^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{e^2x}{c}+\frac{\ln\left(-e^2a^2+a\,d^2c-\sqrt{-ac(a\,e^2-c\,d^2)^2}\,x\right)de}{c}+\frac{\ln\left(-e^2a^2+a\,d^2c-\sqrt{-ac(a\,e^2-c\,d^2)^2}\,x\right)\sqrt{-ac(a\,e^2-c\,d^2)^2}}{2c^2a}+\frac{\ln\left(-e^2a\right.}{}$

input
```
int((e*x+d)^2/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
e^2*x/c+1/c*(d*e*ln(c*x^2+a)+(-a*e^2+c*d^2)/(a*c)^(1/2)*arctan(c*x/(a*c)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.32

$$\int\frac{(d+ex)^2}{a+cx^2}\,dx$$
$$=\left[\frac{2\,ace^2x+2\,acde\log\left(cx^2+a\right)+(cd^2-ae^2)\sqrt{-ac}\log\left(\frac{cx^2+2\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac^2},\frac{ace^2x+acde\log\left(cx^2+a\right)+(c}{ac^2}\right.$$

input
```
integrate((e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*e^2*x + 2*a*c*d*e*log(c*x^2 + a) + (c*d^2 - a*e^2)*sqrt(-a*c)*
log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), (a*c*e^2*x + a*c*d
*e*log(c*x^2 + a) + (c*d^2 - a*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^
2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(53) = 106$.

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.14

$$\int \frac{(d+ex)^2}{a+cx^2}\,dx = \left(\frac{de}{c} - \frac{\sqrt{-ac^3}(ae^2-cd^2)}{2ac^3}\right)\log\left(x + \frac{-2ac\left(\frac{de}{c} - \frac{\sqrt{-ac^3}(ae^2-cd^2)}{2ac^3}\right) + 2ade}{ae^2-cd^2}\right) + \left(\frac{de}{c} + \frac{\sqrt{-ac^3}(ae^2-cd^2)}{2ac^3}\right)\log\left(x + \frac{-2ac\left(\frac{de}{c} + \frac{\sqrt{-ac^3}(ae^2-cd^2)}{2ac^3}\right) + 2ade}{ae^2-cd^2}\right) + \frac{e^2x}{c}$$

input
```
integrate((e*x+d)**2/(c*x**2+a),x)
```

output
```
(d*e/c - sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3))*log(x + (-2*a*c*(d*e/
c - sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3)) + 2*a*d*e)/(a*e**2 - c*d**
2)) + (d*e/c + sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3))*log(x + (-2*a*c
*(d*e/c + sqrt(-a*c**3)*(a*e**2 - c*d**2)/(2*a*c**3)) + 2*a*d*e)/(a*e**2 -
 c*d**2)) + e**2*x/c
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2}{a+cx^2}\,dx = \frac{e^2x}{c} + \frac{de\log(cx^2+a)}{c} + \frac{(cd^2-ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

input
```
integrate((e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

output
```
e^2*x/c + d*e*log(c*x^2 + a)/c + (c*d^2 - a*e^2)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2}{a+cx^2}\,dx = \frac{e^2x}{c} + \frac{de\log(cx^2+a)}{c} + \frac{(cd^2-ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

input
```
integrate((e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

output
```
e^2*x/c + d*e*log(c*x^2 + a)/c + (c*d^2 - a*e^2)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2}{a+cx^2}\,dx = \frac{e^2\,x}{c} + \frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{\sqrt{a}\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}} + \frac{d\,e\,\ln\left(c\,x^2+a\right)}{c}$$

input
```
int((d + e*x)^2/(a + c*x^2),x)
```

output
```
(e^2*x)/c + (d^2*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)*c^(1/2)) - (a^(1/2)*e
^2*atan((c^(1/2)*x)/a^(1/2)))/c^(3/2) + (d*e*log(a + c*x^2))/c
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2}{a+cx^2}\,dx = \frac{-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a\,e^2+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)c\,d^2+\log(c\,x^2+a)\,acde+ac\,e^2x}{a\,c^2}$$

input
```
int((e*x+d)^2/(c*x^2+a),x)
```

output
```
( - sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*e**2 + sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*c*d**2 + log(a + c*x**2)*a*c*d*e + a*c*e**2
*x)/(a*c**2)
```

3.105 $\int \frac{d+ex}{a+cx^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{d+ex}{a+cx^2} dx = \frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + \frac{e\log(a+cx^2)}{2c}$$

output
```
d*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/c^(1/2)+1/2*e*ln(c*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{a+cx^2} dx = \frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + \frac{e\log(a+cx^2)}{2c}$$

input
```
Integrate[(d + e*x)/(a + c*x^2),x]
```

output
```
(d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (e*Log[a + c*x^2])/(2*
c)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+cx^2} \, dx$$

$$\Big\downarrow \text{452}$$

$$d\int \frac{1}{cx^2+a}dx+e\int \frac{x}{cx^2+a}dx$$

$$\Big\downarrow \text{218}$$

$$e\int \frac{x}{cx^2+a}dx+\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

$$\Big\downarrow \text{240}$$

$$\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{e\log\left(a+cx^2\right)}{2c}$$

input
```
Int[(d + e*x)/(a + c*x^2),x]
```

output
```
(d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (e*Log[a + c*x^2])/(2*
c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{e\ln\left(c\,x^{2}+a\right)}{2c}+\frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$	32
risch	$\frac{\ln\left(-\sqrt{-ac}\,x+a\right)d\sqrt{-ac}}{2ac}+\frac{\ln\left(-\sqrt{-ac}\,x+a\right)e}{2c}-\frac{\ln\left(\sqrt{-ac}\,x+a\right)d\sqrt{-ac}}{2ac}+\frac{\ln\left(\sqrt{-ac}\,x+a\right)e}{2c}$	90

input
```
int((e*x+d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*ln(c*x^2+a)/c+d/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{d+ex}{a+cx^2}\,dx = \left[\frac{ae\log\left(cx^2+a\right)-\sqrt{-ac}d\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac},\frac{ae\log\left(cx^2+a\right)+2\sqrt{ac}d\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2\,ac}\right]$$

input
```
integrate((e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(a*e*log(c*x^2 + a) - sqrt(-a*c)*d*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(
c*x^2 + a)))/(a*c), 1/2*(a*e*log(c*x^2 + a) + 2*sqrt(a*c)*d*arctan(sqrt(a*
c)*x/a))/(a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int \frac{d+ex}{a+cx^2}\,dx = \left(\frac{e}{2c} - \frac{d\sqrt{-ac^3}}{2ac^2}\right)\log\left(x + \frac{2ac\left(\frac{e}{2c} - \frac{d\sqrt{-ac^3}}{2ac^2}\right) - ae}{cd}\right) + \left(\frac{e}{2c} + \frac{d\sqrt{-ac^3}}{2ac^2}\right)\log\left(x + \frac{2ac\left(\frac{e}{2c} + \frac{d\sqrt{-ac^3}}{2ac^2}\right) - ae}{cd}\right)$$

input
```
integrate((e*x+d)/(c*x**2+a),x)
```

output
```
(e/(2*c) - d*sqrt(-a*c**3)/(2*a*c**2))*log(x + (2*a*c*(e/(2*c) - d*sqrt(-a
*c**3)/(2*a*c**2)) - a*e)/(c*d)) + (e/(2*c) + d*sqrt(-a*c**3)/(2*a*c**2))*
log(x + (2*a*c*(e/(2*c) + d*sqrt(-a*c**3)/(2*a*c**2)) - a*e)/(c*d))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{d+ex}{a+cx^2}\,dx = \frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{e\log\left(cx^2+a\right)}{2\,c}$$

input
```
integrate((e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

output
```
d*arctan(c*x/sqrt(a*c))/sqrt(a*c) + 1/2*e*log(c*x^2 + a)/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{d+ex}{a+cx^2}\,dx = \frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{e\log\left(cx^2+a\right)}{2\,c}$$

input
```
integrate((e*x+d)/(c*x^2+a),x, algorithm="giac")
```

output
```
d*arctan(c*x/sqrt(a*c))/sqrt(a*c) + 1/2*e*log(c*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{d+ex}{a+cx^2}\,dx = \frac{e\,\ln\left(c\,x^2+a\right)}{2\,c} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

input
```
int((d + e*x)/(a + c*x^2),x)
```

output
```
(e*log(a + c*x^2))/(2*c) + (d*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{a+cx^2}\,dx = \frac{2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)d+\log\left(c\,x^2+a\right)ae}{2ac}$$

input
```
int((e*x+d)/(c*x^2+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*d + log(a + c*x**2)*a*e)/
(2*a*c)
```

3.106 $\int \frac{1}{(d+ex)(a+cx^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx=\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^2+ae^2)}+\frac{e\log(d+ex)}{cd^2+ae^2}-\frac{e\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)}$$

output
```
c^(1/2)*d*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/(a*e^2+c*d^2)+e*ln(e*x+d)/(a*e
^2+c*d^2)-e*ln(c*x^2+a)/(2*a*e^2+2*c*d^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx=\frac{\frac{2\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}}+2e\log(d+ex)-e\log\left(a+cx^2\right)}{2cd^2+2ae^2}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^2)),x]
```

output
```
((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x] - e*
Log[a + c*x^2])/(2*c*d^2 + 2*a*e^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {479, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)(d+ex)}dx$$

$$\Big\downarrow \text{479}$$

$$\frac{c\int \frac{d-ex}{cx^2+a}dx}{ae^2+cd^2}+\frac{e\log(d+ex)}{ae^2+cd^2}$$

$$\Big\downarrow \text{452}$$

$$\frac{c\left(d\int \frac{1}{cx^2+a}dx-e\int \frac{x}{cx^2+a}dx\right)}{ae^2+cd^2}+\frac{e\log(d+ex)}{ae^2+cd^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{c\left(\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-e\int \frac{x}{cx^2+a}dx\right)}{ae^2+cd^2}+\frac{e\log(d+ex)}{ae^2+cd^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{c\left(\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-\frac{e\log(a+cx^2)}{2c}\right)}{ae^2+cd^2}+\frac{e\log(d+ex)}{ae^2+cd^2}$$

input
```
Int[1/((d + e*x)*(a + c*x^2)),x]
```

output
```
(e*Log[d + e*x])/(c*d^2 + a*e^2) + (c*((d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sq
rt[a]*Sqrt[c]) - (e*Log[a + c*x^2])/(2*c)))/(c*d^2 + a*e^2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{e\ln(ex+d)}{a\,e^2+c\,d^2}+\frac{c\left(-\frac{e\ln\left(c\,x^2+a\right)}{2c}+\frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{a\,e^2+c\,d^2}$	69
risch	$\frac{e\ln(ex+d)}{a\,e^2+c\,d^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(1+\left(e^2a^2+a\,d^2c\right)\_Z^2+2ae\_Z\right)}\_R\ln\left(\left(\left(3a\,e^2-c\,d^2\right)\_R+3e\right)x+4ade\_R+d\right)\right)}{2}$	86

input
```
int(1/(e*x+d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
e*ln(e*x+d)/(a*e^2+c*d^2)+c/(a*e^2+c*d^2)*(-1/2*e*ln(c*x^2+a)/c+d/(a*c)^(1
/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.57

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx = \left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^2+2\,ax\sqrt{-\frac{c}{a}}-a}{cx^2+a}\right)-e\log\left(cx^2+a\right)+2\,e\log\left(ex+d\right)}{2\,(cd^2+ae^2)}, \frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(x\sqrt{\frac{c}{a}}\right)-e\log\left(cx^2+a\right.}{2\,(cd^2+ae^2)}\right.$$

input
```
integrate(1/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(d*sqrt(-c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) - e*log
(c*x^2 + a) + 2*e*log(e*x + d))/(c*d^2 + a*e^2), 1/2*(2*d*sqrt(c/a)*arctan
(x*sqrt(c/a)) - e*log(c*x^2 + a) + 2*e*log(e*x + d))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx = \frac{cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2+ae^2)\sqrt{ac}} - \frac{e\log\left(cx^2+a\right)}{2\left(cd^2+ae^2\right)} + \frac{e\log\left(ex+d\right)}{cd^2+ae^2}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

output
```
c*d*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e*log(c*x^2 +
a)/(c*d^2 + a*e^2) + e*log(e*x + d)/(c*d^2 + a*e^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx = \frac{e^2\log\left(|ex+d|\right)}{cd^2e+ae^3} + \frac{cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2+ae^2)\sqrt{ac}} - \frac{e\log\left(cx^2+a\right)}{2\left(cd^2+ae^2\right)}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

output
```
e^2*log(abs(e*x + d))/(c*d^2*e + a*e^3) + c*d*arctan(c*x/sqrt(a*c))/((c*d^
2 + a*e^2)*sqrt(a*c)) - 1/2*e*log(c*x^2 + a)/(c*d^2 + a*e^2)
```

Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.67

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx = \frac{e\ln(d+e\,x)}{c\,d^2+a\,e^2} - \frac{\ln\left(3\,c^2\,e^2\,x+c^2\,d\,e-\frac{c^2\,e\,\left(a\,e-d\,\sqrt{-a\,c}\right)\,\left(-c\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{a^2\,e^2+c\,a\,d^2}\right)\,\left(a\,e-d\,\sqrt{-a\,c}\right)}{2\,\left(a^2\,e^2+c\,a\,d^2\right)} - \frac{\ln\left(3\,c^2\,e^2\,x+c^2\,d\,e-\frac{c^2\,e\,\left(a\,e+d\,\sqrt{-a\,c}\right)\,\left(-c\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{a^2\,e^2+c\,a\,d^2}\right)\,\left(a\,e+d\,\sqrt{-a\,c}\right)}{2\,\left(a^2\,e^2+c\,a\,d^2\right)}$$

input
```
int(1/((a + c*x^2)*(d + e*x)),x)
```

output
```
(e*log(d + e*x))/(a*e^2 + c*d^2) - (log(3*c^2*e^2*x + c^2*d*e - (c^2*e*(a*
e - d*(-a*c)^(1/2))*(4*a*d*e + 3*a*e^2*x - c*d^2*x))/(a^2*e^2 + a*c*d^2))*
(a*e - d*(-a*c)^(1/2)))/(2*(a^2*e^2 + a*c*d^2)) - (log(3*c^2*e^2*x + c^2*d
*e - (c^2*e*(a*e + d*(-a*c)^(1/2))*(4*a*d*e + 3*a*e^2*x - c*d^2*x))/(a^2*e
^2 + a*c*d^2))*(a*e + d*(-a*c)^(1/2)))/(2*(a^2*e^2 + a*c*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex)(a+cx^2)}\,dx = \frac{2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)d-\log(c\,x^2+a)\,ae+2\log(ex+d)\,ae}{2a\,(a\,e^2+c\,d^2)}$$

input
```
int(1/(e*x+d)/(c*x^2+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*d - log(a + c*x**2)*a*e +
 2*log(d + e*x)*a*e)/(2*a*(a*e**2 + c*d**2))
```

3.107 $\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx = -\frac{e}{(cd^2+ae^2)(d+ex)} + \frac{\sqrt{c}(cd^2-ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^2+ae^2)^2} + \frac{2cde\log(d+ex)}{(cd^2+ae^2)^2} - \frac{cde\log(a+cx^2)}{(cd^2+ae^2)^2}$$

output
```
-e/(a*e^2+c*d^2)/(e*x+d)+c^(1/2)*(-a*e^2+c*d^2)*arctan(c^(1/2)*x/a^(1/2))/
a^(1/2)/(a*e^2+c*d^2)^2+2*c*d*e*ln(e*x+d)/(a*e^2+c*d^2)^2-c*d*e*ln(c*x^2+a
)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx = \frac{\sqrt{c}(cd^2-ae^2)(d+ex)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) - \sqrt{a}e(cd^2+ae^2-2cd(d+ex)\log(d+ex)+cd(d+ex)\log(a+}{\sqrt{a}\,(cd^2+ae^2)^2(d+ex)}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^2)),x]
```

output
```
(Sqrt[c]*(c*d^2 - a*e^2)*(d + e*x)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] - Sqrt[a]*e
*(c*d^2 + a*e^2 - 2*c*d*(d + e*x)*Log[d + e*x] + c*d*(d + e*x)*Log[a + c*x
^2]))/(Sqrt[a]*(c*d^2 + a*e^2)^2*(d + e*x))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)(d+ex)^2}dx$$

$$\Big\downarrow \text{480}$$

$$\frac{c\int \frac{d-ex}{(d+ex)(cx^2+a)}dx}{ae^2+cd^2}-\frac{e}{(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{657}$$

$$\frac{c\int \left(\frac{2de^2}{(cd^2+ae^2)(d+ex)}+\frac{cd^2-2cexd-ae^2}{(cd^2+ae^2)(cx^2+a)}\right)dx}{ae^2+cd^2}-\frac{e}{(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2-ae^2)}{\sqrt{a}\sqrt{c}(ae^2+cd^2)}-\frac{de\log(a+cx^2)}{ae^2+cd^2}+\frac{2de\log(d+ex)}{ae^2+cd^2}\right)}{ae^2+cd^2}-\frac{e}{(d+ex)(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^2)),x]
```

output
```
-(e/((c*d^2 + a*e^2)*(d + e*x))) + (c*(((c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x)
/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)) + (2*d*e*Log[d + e*x])/(c*d^2
 + a*e^2) - (d*e*Log[a + c*x^2])/(c*d^2 + a*e^2)))/(c*d^2 + a*e^2)
```

Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

method	result
default	$-\frac{e}{(a\,e^2+c\,d^2)(ex+d)}+\frac{2cde\ln(ex+d)}{(a\,e^2+c\,d^2)^2}-\frac{c\left(de\ln(c\,x^2+a)+\frac{(a\,e^2-c\,d^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{(a\,e^2+c\,d^2)^2}$
risch	$-\frac{e}{(a\,e^2+c\,d^2)(ex+d)}+\frac{2dec\ln(ex+d)}{a^2e^4+2ac\,d^2e^2+c^2d^4}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(e^4a^3+2d^2e^2a^2c+d^4a\,c^2\right)\_Z^2+4acde\_Z+c\right)}\_R\ln\left(\left(\left(3e^6a^3+5a\right.\right.\right.\right.}{}$

input
```
int(1/(e*x+d)^2/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-e/(a*e^2+c*d^2)/(e*x+d)+2*c*d*e*ln(e*x+d)/(a*e^2+c*d^2)^2-c/(a*e^2+c*d^2)
^2*(d*e*ln(c*x^2+a)+(a*e^2-c*d^2)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx$$
$$= \left[-\frac{2\,cd^2e + 2\,ae^3 + (cd^3 - ade^2 + (cd^2e - ae^3)x)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^2 - 2\,ax\sqrt{-\frac{c}{a}} - a}{cx^2+a}\right) + 2\,(cde^2x + cd^2e)\log\left(cx^2 \right.}{2\,(c^2d^5 + 2\,acd^3e^2 + a^2de^4 + (c^2d^4e + 2\,acd^2e^3 + a^2e^5)x)}\right.$$
$$-\frac{cd^2e + ae^3 - (cd^3 - ade^2 + (cd^2e - ae^3)x)\sqrt{\frac{c}{a}}\arctan\left(x\sqrt{\frac{c}{a}}\right) + (cde^2x + cd^2e)\log\left(cx^2 + a\right) - 2\,(cde^2}{c^2d^5 + 2\,acd^3e^2 + a^2de^4 + (c^2d^4e + 2\,acd^2e^3 + a^2e^5)x}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*(2*c*d^2*e + 2*a*e^3 + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(
-c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(c*d*e^2*x + c*d
^2*e)*log(c*x^2 + a) - 4*(c*d*e^2*x + c*d^2*e)*log(e*x + d))/(c^2*d^5 + 2*
a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x), -(c*d^
2*e + a*e^3 - (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c/a)*arctan(x*s
qrt(c/a)) + (c*d*e^2*x + c*d^2*e)*log(c*x^2 + a) - 2*(c*d*e^2*x + c*d^2*e)
*log(e*x + d))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d
^2*e^3 + a^2*e^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+ex)^2\,(a+cx^2)}\,dx = -\frac{cde\log\left(cx^2+a\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{2\,cde\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{\left(c^2d^2-ace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}-\frac{e}{cd^3+ade^2+\left(cd^2e+ae^3\right)x}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

output
```
-c*d*e*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 2*c*d*e*log(e*
x + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (c^2*d^2 - a*c*e^2)*arctan(c*
x/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) - e/(c*d^3 +
a*d*e^2 + (c*d^2*e + a*e^3)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.57

$$\int \frac{1}{(d+ex)^2\,(a+cx^2)}\,dx = -\frac{cde\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{e^3}{(cd^2e^2+ae^4)(ex+d)}+\frac{\left(c^2d^2e^2-ace^4\right)\arctan\left(\frac{cd-\frac{cd^2}{ex+d}-\frac{ae^2}{ex+d}}{\sqrt{ac}e}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}e^2}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

output
```
-c*d*e*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - e^3/((c*d^2*e^2 + a*e^4)*(e*x + d)) +
(c^2*d^2*e^2 - a*c*e^4)*arctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(
sqrt(a*c)*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)*e^2)
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.67

$$\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx$$
$$= \frac{\ln\left(a^5 e^8 \sqrt{-a c} - c^3 d^8 (-a c)^{3/2} - 36 a^3 d^2 e^6 (-a c)^{3/2} + a c^5 d^8 x + a^5 c e^8 x + 70 a d^4 e^4 (-a c)^{5/2} + 36 \right.}{a^3 e^4 + 2 a^2 c d}$$
$$+ \frac{\ln\left(c^3 d^8 (-a c)^{3/2} - a^5 e^8 \sqrt{-a c} + 36 a^3 d^2 e^6 (-a c)^{3/2} + a c^5 d^8 x + a^5 c e^8 x - 70 a d^4 e^4 (-a c)^{5/2} - 3\right.}{a^3 e^4 + 2 a^2 c}$$
$$- \frac{e}{(c d^2 + a e^2)(d + e x)} + \frac{2 c d e \ln(d + e x)}{(c d^2 + a e^2)^2}$$

input
```
int(1/((a + c*x^2)*(d + e*x)^2),x)
```

output
```
(log(a^5*e^8*(-a*c)^(1/2) - c^3*d^8*(-a*c)^(3/2) - 36*a^3*d^2*e^6*(-a*c)^(
3/2) + a*c^5*d^8*x + a^5*c*e^8*x + 70*a*d^4*e^4*(-a*c)^(5/2) + 36*c*d^6*e^
2*(-a*c)^(5/2) + 36*a^2*c^4*d^6*e^2*x + 70*a^3*c^3*d^4*e^4*x + 36*a^4*c^2*
d^2*e^6*x)*(c*((d^2*(-a*c)^(1/2))/2 - a*d*e) - (a*e^2*(-a*c)^(1/2))/2))/(a
^3*e^4 + a*c^2*d^4 + 2*a^2*c*d^2*e^2) + (log(c^3*d^8*(-a*c)^(3/2) - a^5*e^
8*(-a*c)^(1/2) + 36*a^3*d^2*e^6*(-a*c)^(3/2) + a*c^5*d^8*x + a^5*c*e^8*x -
 70*a*d^4*e^4*(-a*c)^(5/2) - 36*c*d^6*e^2*(-a*c)^(5/2) + 36*a^2*c^4*d^6*e^
2*x + 70*a^3*c^3*d^4*e^4*x + 36*a^4*c^2*d^2*e^6*x)*(a*((e^2*(-a*c)^(1/2))/
2 - c*d*e) - (c*d^2*(-a*c)^(1/2))/2))/(a^3*e^4 + a*c^2*d^4 + 2*a^2*c*d^2*e
^2) - e/((a*e^2 + c*d^2)*(d + e*x)) + (2*c*d*e*log(d + e*x))/(a*e^2 + c*d^
2)^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.96

$$\int \frac{1}{(d+ex)^2(a+cx^2)}\,dx$$
$$= \frac{-\sqrt{c}\sqrt{a}\, atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right) a d^2 e^2 - \sqrt{c}\sqrt{a}\, atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right) a d e^3 x + \sqrt{c}\sqrt{a}\, atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right) c d^4 + \sqrt{c}\sqrt{a}\, atan\left(\frac{cx}{\sqrt{c}\sqrt{}}\right.}{ad\,(a^2 e^5 x + 2ac\,d^2 e^3}$$

input
```
int(1/(e*x+d)^2/(c*x^2+a),x)
```

output
```
( - sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*d**2*e**2 - sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*d*e**3*x + sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*c*d**4 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))
)*c*d**3*e*x - log(a + c*x**2)*a*c*d**3*e - log(a + c*x**2)*a*c*d**2*e**2*
x + 2*log(d + e*x)*a*c*d**3*e + 2*log(d + e*x)*a*c*d**2*e**2*x + a**2*e**4
*x + a*c*d**2*e**2*x)/(a*d*(a**2*d*e**4 + a**2*e**5*x + 2*a*c*d**3*e**2 +
2*a*c*d**2*e**3*x + c**2*d**5 + c**2*d**4*e*x))
```

3.108 $\int \frac{1}{(d+ex)^3(a+cx^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 176

$$\int \frac{1}{(d+ex)^3(a+cx^2)}\,dx = -\frac{e}{2(cd^2+ae^2)(d+ex)^2} - \frac{2cde}{(cd^2+ae^2)^2(d+ex)} + \frac{c^{3/2}d(cd^2-3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}(cd^2+ae^2)^3} + \frac{ce(3cd^2-ae^2)\log(d+ex)}{(cd^2+ae^2)^3} - \frac{ce(3cd^2-ae^2)\log(a+cx^2)}{2(cd^2+ae^2)^3}$$

output

```
-1/2*e/(a*e^2+c*d^2)/(e*x+d)^2-2*c*d*e/(a*e^2+c*d^2)^2/(e*x+d)+c^(3/2)*d*(
-3*a*e^2+c*d^2)*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/(a*e^2+c*d^2)^3+c*e*(-a*
e^2+3*c*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3-1/2*c*e*(-a*e^2+3*c*d^2)*ln(c*x^2+a
)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

$$\int \frac{1}{(d+ex)^3 (a+cx^2)} dx$$
$$= \frac{\frac{2c^{3/2}d(cd^2-3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}} + e\left(-\frac{(cd^2+ae^2)(ae^2+cd(5d+4ex))}{(d+ex)^2} + 2c(3cd^2-ae^2)\log(d+ex) + c(-3cd^2+ae^2)\log\right.}{2(cd^2+ae^2)^3}$$

input
```
Integrate[1/((d + e*x)^3*(a + c*x^2)),x]
```

output
```
((2*c^(3/2)*d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + e*(
-(((c*d^2 + a*e^2)*(a*e^2 + c*d*(5*d + 4*e*x)))/(d + e*x)^2) + 2*c*(3*c*d^
2 - a*e^2)*Log[d + e*x] + c*(-3*c*d^2 + a*e^2)*Log[a + c*x^2]))/(2*(c*d^2
+ a*e^2)^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)(d+ex)^3} dx$$
$$\Big\downarrow \text{480}$$
$$\frac{c\int \frac{d-ex}{(d+ex)^2(cx^2+a)}dx}{ae^2+cd^2} - \frac{e}{2(d+ex)^2(ae^2+cd^2)}$$
$$\Big\downarrow \text{657}$$
$$\frac{c\int \left(\frac{2de^2}{(cd^2+ae^2)(d+ex)^2} + \frac{3cd^2e^2-ae^4}{(cd^2+ae^2)^2(d+ex)} + \frac{c(d(cd^2-3ae^2)-e(3cd^2-ae^2)x)}{(cd^2+ae^2)^2(cx^2+a)}\right)dx}{ae^2+cd^2} - \frac{e}{2(d+ex)^2(ae^2+cd^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2-3ae^2)}{\sqrt{a}(ae^2+cd^2)^2}-\frac{e(3cd^2-ae^2)\log(a+cx^2)}{2(ae^2+cd^2)^2}-\frac{2de}{(d+ex)(ae^2+cd^2)}+\frac{e(3cd^2-ae^2)\log(d+ex)}{(ae^2+cd^2)^2}\right)}{ae^2+cd^2}-\frac{e}{2(d+ex)^2\,(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x)^3*(a + c*x^2)),x]
```

output
```
-1/2*e/((c*d^2 + a*e^2)*(d + e*x)^2) + (c*((-2*d*e)/((c*d^2 + a*e^2)*(d +
e*x)) + (Sqrt[c]*d*(c*d^2 - 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]
*(c*d^2 + a*e^2)^2) + (e*(3*c*d^2 - a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2
 - (e*(3*c*d^2 - a*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)))/(c*d^2 + a
*e^2)
```

Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

method	result
default	$-\frac{e}{2(a\,e^2+c\,d^2)(ex+d)^2}-\frac{ce(a\,e^2-3c\,d^2)\ln(ex+d)}{(a\,e^2+c\,d^2)^3}-\frac{2cde}{(a\,e^2+c\,d^2)^2(ex+d)}-\frac{c^2\left(\frac{(-a\,e^3+3c\,d^2e)\ln(c\,x^2+a)}{2c}+\frac{(3ad\,e^2-c\,d^3)\,\text{arc}}{\sqrt{ac}}\right.}{(a\,e^2+c\,d^2)^3}$
risch	Expression too large to display

input
```
int(1/(e*x+d)^3/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e/(a*e^2+c*d^2)/(e*x+d)^2-c*e*(a*e^2-3*c*d^2)/(a*e^2+c*d^2)^3*ln(e*x+
d)-2*c*d*e/(a*e^2+c*d^2)^2/(e*x+d)-c^2/(a*e^2+c*d^2)^3*(1/2*(-a*e^3+3*c*d^
2*e)/c*ln(c*x^2+a)+(3*a*d*e^2-c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(164) = 328.

Time = 0.76 (sec) , antiderivative size = 853, normalized size of antiderivative = 4.85

$$\int \frac{1}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*(5*c^2*d^4*e + 6*a*c*d^2*e^3 + a^2*e^5 + (c^2*d^5 - 3*a*c*d^3*e^2 +
(c^2*d^3*e^2 - 3*a*c*d*e^4)*x^2 + 2*(c^2*d^4*e - 3*a*c*d^2*e^3)*x)*sqrt(-c
/a)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 4*(c^2*d^3*e^2 + a*c
*d*e^4)*x + (3*c^2*d^4*e - a*c*d^2*e^3 + (3*c^2*d^2*e^3 - a*c*e^5)*x^2 + 2
*(3*c^2*d^3*e^2 - a*c*d*e^4)*x)*log(c*x^2 + a) - 2*(3*c^2*d^4*e - a*c*d^2*
e^3 + (3*c^2*d^2*e^3 - a*c*e^5)*x^2 + 2*(3*c^2*d^3*e^2 - a*c*d*e^4)*x)*log
(e*x + d))/(c^3*d^8 + 3*a*c^2*d^6*e^2 + 3*a^2*c*d^4*e^4 + a^3*d^2*e^6 + (c
^3*d^6*e^2 + 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6 + a^3*e^8)*x^2 + 2*(c^3*d^7
*e + 3*a*c^2*d^5*e^3 + 3*a^2*c*d^3*e^5 + a^3*d*e^7)*x), -1/2*(5*c^2*d^4*e
+ 6*a*c*d^2*e^3 + a^2*e^5 - 2*(c^2*d^5 - 3*a*c*d^3*e^2 + (c^2*d^3*e^2 - 3*
a*c*d*e^4)*x^2 + 2*(c^2*d^4*e - 3*a*c*d^2*e^3)*x)*sqrt(c/a)*arctan(x*sqrt(
c/a)) + 4*(c^2*d^3*e^2 + a*c*d*e^4)*x + (3*c^2*d^4*e - a*c*d^2*e^3 + (3*c^
2*d^2*e^3 - a*c*e^5)*x^2 + 2*(3*c^2*d^3*e^2 - a*c*d*e^4)*x)*log(c*x^2 + a)
 - 2*(3*c^2*d^4*e - a*c*d^2*e^3 + (3*c^2*d^2*e^3 - a*c*e^5)*x^2 + 2*(3*c^2
*d^3*e^2 - a*c*d*e^4)*x)*log(e*x + d))/(c^3*d^8 + 3*a*c^2*d^6*e^2 + 3*a^2*
c*d^4*e^4 + a^3*d^2*e^6 + (c^3*d^6*e^2 + 3*a*c^2*d^4*e^4 + 3*a^2*c*d^2*e^6
 + a^3*e^8)*x^2 + 2*(c^3*d^7*e + 3*a*c^2*d^5*e^3 + 3*a^2*c*d^3*e^5 + a^3*d
*e^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(c*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.84

$$\int \frac{1}{(d+ex)^3\,(a+cx^2)}\,dx = -\frac{(3\,c^2d^2e-ace^3)\log\left(cx^2+a\right)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} + \frac{(3\,c^2d^2e-ace^3)\log\left(ex+d\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{(c^3d^3-3\,ac^2de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\sqrt{ac}} - \frac{4\,cde^2x+5\,cd^2e+ae^3}{2\,(c^2d^6+2\,acd^4e^2+a^2d^2e^4+(c^2d^4e^2+2\,acd^2e^4+a^2e^6)x^2+2\,(c^2d^5e+2\,acd^3e^3+a^2de^5)x)}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(3*c^2*d^2*e - a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3
*a^2*c*d^2*e^4 + a^3*e^6) + (3*c^2*d^2*e - a*c*e^3)*log(e*x + d)/(c^3*d^6
+ 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (c^3*d^3 - 3*a*c^2*d*e^2)
*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3
*e^6)*sqrt(a*c)) - 1/2*(4*c*d*e^2*x + 5*c*d^2*e + a*e^3)/(c^2*d^6 + 2*a*c*
d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + 2*(c
^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.60

$$\int \frac{1}{(d+ex)^3\,(a+cx^2)}\,dx = -\frac{(3\,c^2d^2e-ace^3)\log\left(cx^2+a\right)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} + \frac{(3\,c^2d^2e^2-ace^4)\log\left(|ex+d|\right)}{c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7} + \frac{(c^3d^3-3\,ac^2de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\sqrt{ac}} - \frac{5\,c^2d^4e+6\,acd^2e^3+a^2e^5+4\,(c^2d^3e^2+acde^4)x}{2\,(cd^2+ae^2)^3(ex+d)^2}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(3*c^2*d^2*e - a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3
*a^2*c*d^2*e^4 + a^3*e^6) + (3*c^2*d^2*e^2 - a*c*e^4)*log(abs(e*x + d))/(c
^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + (c^3*d^3 - 3*a*c
^2*d*e^2)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(5*c^2*d^4*e + 6*a*c*d^2*e^3 + a^2*e^5 + 4
*(c^2*d^3*e^2 + a*c*d*e^4)*x)/((c*d^2 + a*e^2)^3*(e*x + d)^2)
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 745, normalized size of antiderivative = 4.23

$$\int \frac{1}{(d+ex)^3(a+cx^2)}\,dx = \frac{\ln(d+e\,x)\,\left(3\,c^2\,d^2\,e-a\,c\,e^3\right)}{a^3\,e^6+3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+c^3\,d^6}$$
$$-\frac{\ln\left(c^2\,d^{10}\,\left(-a\,c^3\right)^{3/2}-9\,a^6\,e^{10}\,\sqrt{-a\,c^3}+9\,a^6\,c^2\,e^{10}\,x+106\,a^2\,d^6\,e^4\,\left(-a\,c^3\right)^{3/2}+a\,c^7\,d^{10}\,x+6\,a^4\,c^2\,d^4\,e^{6}\right.}{}$$
$$-\frac{\ln\left(c^2\,d^{10}\,\left(-a\,c^3\right)^{3/2}-9\,a^6\,e^{10}\,\sqrt{-a\,c^3}-9\,a^6\,c^2\,e^{10}\,x+106\,a^2\,d^6\,e^4\,\left(-a\,c^3\right)^{3/2}-a\,c^7\,d^{10}\,x+6\,a^4\,c^2\,d^4\,e^{6}\right.}{}$$
$$-\frac{\frac{5\,c\,d^2\,e+a\,e^3}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}+\frac{2\,c\,d\,e^2\,x}{a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4}}{d^2+2\,d\,e\,x+e^2\,x^2}$$

input
```
int(1/((a + c*x^2)*(d + e*x)^3),x)
```

output
```
(log(d + e*x)*(3*c^2*d^2*e - a*c*e^3))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4) - (log(c^2*d^10*(-a*c^3)^(3/2) - 9*a^6*e^10*(-a*c^3)^
(1/2) + 9*a^6*c^2*e^10*x + 106*a^2*d^6*e^4*(-a*c^3)^(3/2) + a*c^7*d^10*x +
 6*a^4*c^2*d^4*e^6*(-a*c^3)^(1/2) + 77*a*c*d^8*e^2*(-a*c^3)^(3/2) + 77*a^2
*c^6*d^8*e^2*x + 106*a^3*c^5*d^6*e^4*x - 6*a^4*c^4*d^4*e^6*x - 27*a^5*c^3*
d^2*e^8*x + 27*a^5*c*d^2*e^8*(-a*c^3)^(1/2))*(c*((d^3*(-a*c^3)^(1/2))/2 -
(a^2*e^3)/2) + (3*a*c^2*d^2*e)/2 - (3*a*d*e^2*(-a*c^3)^(1/2))/2))/(a^4*e^6
 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - (log(c^2*d^10*(-a*c^
3)^(3/2) - 9*a^6*e^10*(-a*c^3)^(1/2) - 9*a^6*c^2*e^10*x + 106*a^2*d^6*e^4*
(-a*c^3)^(3/2) - a*c^7*d^10*x + 6*a^4*c^2*d^4*e^6*(-a*c^3)^(1/2) + 77*a*c*
d^8*e^2*(-a*c^3)^(3/2) - 77*a^2*c^6*d^8*e^2*x - 106*a^3*c^5*d^6*e^4*x + 6*
a^4*c^4*d^4*e^6*x + 27*a^5*c^3*d^2*e^8*x + 27*a^5*c*d^2*e^8*(-a*c^3)^(1/2)
)*((3*a*c^2*d^2*e)/2 - c*((d^3*(-a*c^3)^(1/2))/2 + (a^2*e^3)/2) + (3*a*d*e
^2*(-a*c^3)^(1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*
d^4*e^2) - ((a*e^3 + 5*c*d^2*e)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) +
(2*c*d*e^2*x)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*
x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.38

$$\int \frac{1}{(d+ex)^3\,(a+cx^2)}\,dx$$
$$= \frac{-6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,d^3e^2 - 12\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,d^2e^3x - 6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)acd\,e^4x^2 + 2\sqrt{c}}{}$$

input
```
int(1/(e*x+d)^3/(c*x^2+a),x)
```

output

```
( - 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d**3*e**2 - 12*sqr
t(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d**2*e**3*x - 6*sqrt(c)*sqr
t(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d*e**4*x**2 + 2*sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*c**2*d**5 + 4*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(
c)*sqrt(a)))*c**2*d**4*e*x + 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)
))*c**2*d**3*e**2*x**2 + log(a + c*x**2)*a**2*c*d**2*e**3 + 2*log(a + c*x*
*2)*a**2*c*d*e**4*x + log(a + c*x**2)*a**2*c*e**5*x**2 - 3*log(a + c*x**2)
*a*c**2*d**4*e - 6*log(a + c*x**2)*a*c**2*d**3*e**2*x - 3*log(a + c*x**2)*
a*c**2*d**2*e**3*x**2 - 2*log(d + e*x)*a**2*c*d**2*e**3 - 4*log(d + e*x)*a
**2*c*d*e**4*x - 2*log(d + e*x)*a**2*c*e**5*x**2 + 6*log(d + e*x)*a*c**2*d
**4*e + 12*log(d + e*x)*a*c**2*d**3*e**2*x + 6*log(d + e*x)*a*c**2*d**2*e*
*3*x**2 - a**3*e**5 - 4*a**2*c*d**2*e**3 + 2*a**2*c*e**5*x**2 - 3*a*c**2*d
**4*e + 2*a*c**2*d**2*e**3*x**2)/(2*a*(a**3*d**2*e**6 + 2*a**3*d*e**7*x +
a**3*e**8*x**2 + 3*a**2*c*d**4*e**4 + 6*a**2*c*d**3*e**5*x + 3*a**2*c*d**2
*e**6*x**2 + 3*a*c**2*d**6*e**2 + 6*a*c**2*d**5*e**3*x + 3*a*c**2*d**4*e**
4*x**2 + c**3*d**8 + 2*c**3*d**7*e*x + c**3*d**6*e**2*x**2))
```

3.109 $\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx = \frac{5de^4x}{c^2} + \frac{e^5x^2}{2c^2} - \frac{ae(5c^2d^4-10acd^2e^2+a^2e^4)-cd(c^2d^4-10acd^2e^2+5a^2e^4)\,x}{2ac^3\,(a+cx^2)} + \frac{d(c^2d^4+10acd^2e^2-15a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} + \frac{e^3(5cd^2-ae^2)\log(a+cx^2)}{c^3}$$

output
```
5*d*e^4*x/c^2+1/2*e^5*x^2/c^2-1/2*(a*e*(a^2*e^4-10*a*c*d^2*e^2+5*c^2*d^4)-
c*d*(5*a^2*e^4-10*a*c*d^2*e^2+c^2*d^4)*x)/a/c^3/(c*x^2+a)+1/2*d*(-15*a^2*e
^4+10*a*c*d^2*e^2+c^2*d^4)*arctan(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(5/2)+e^3*(
-a*e^2+5*c*d^2)*ln(c*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx$$
$$= \frac{10cde^4x + ce^5x^2 + \frac{-a^3e^5+c^3d^5x+5a^2cde^3(2d+ex)-5ac^2d^3e(d+2ex)}{a(a+cx^2)} + \frac{\sqrt{c}d\left(c^2d^4+10acd^2e^2-15a^2e^4\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}} + 2(5cd^2e^3}{2c^3}$$

input
```
Integrate[(d + e*x)^5/(a + c*x^2)^2,x]
```

output
```
(10*c*d*e^4*x + c*e^5*x^2 + (-(a^3*e^5) + c^3*d^5*x + 5*a^2*c*d*e^3*(2*d +
 e*x) - 5*a*c^2*d^3*e*(d + 2*e*x))/(a*(a + c*x^2)) + (Sqrt[c]*d*(c^2*d^4 +
 10*a*c*d^2*e^2 - 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*(5*
c*d^2*e^3 - a*e^5)*Log[a + c*x^2])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {495, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \; 495$$
$$\frac{\int \frac{(d+ex)^3\left(cd^2-3cexd+4ae^2\right)}{cx^2+a}dx}{2ac} - \frac{(d+ex)^4(ae-cdx)}{2ac\left(a+cx^2\right)}$$
$$\Big\downarrow \; 657$$

$$\frac{\int \left(-3dx^2e^4-\frac{4(2cd^2-ae^2)xe^3}{c}-3d\left(2d^2-\frac{5ae^2}{c}\right)e^2+\frac{c^2d^5+10ace^2d^3-15a^2e^4d+4ae^3(5cd^2-ae^2)x}{c(cx^2+a)}\right)dx}{2ac}-\frac{(d+ex)^4(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(-15a^2e^4+10acd^2e^2+c^2d^4)}{\sqrt{a}c^{3/2}}+\frac{2ae^3(5cd^2-ae^2)\log(a+cx^2)}{c^2}-3de^2x\left(2d^2-\frac{5ae^2}{c}\right)-\frac{2e^3x^2(2cd^2-ae^2)}{c}-de^4x^3}{2ac}-\frac{(d+ex)^4(ae-cdx)}{2ac\,(a+cx^2)}$$

input
```
Int[(d + e*x)^5/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*e - c*d*x)*(d + e*x)^4)/(a*c*(a + c*x^2)) + (-3*d*e^2*(2*d^2 - (5
*a*e^2)/c)*x - (2*e^3*(2*c*d^2 - a*e^2)*x^2)/c - d*e^4*x^3 + (d*(c^2*d^4 +
 10*a*c*d^2*e^2 - 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2
)) + (2*a*e^3*(5*c*d^2 - a*e^2)*Log[a + c*x^2])/c^2)/(2*a*c)
```

Defintions of rubi rules used

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

method	result
default	$\frac{e^{4}\left(\frac{1}{2} e\,x^{2}+5 d x\right)}{c^{2}}-\frac{\frac{-\frac{d\left(5 a^{2} e^{4}-10 a c\,d^{2} e^{2}+c^{2} d^{4}\right) x}{2 a}+\frac{e\left(a^{2} e^{4}-10 a c\,d^{2} e^{2}+5 c^{2} d^{4}\right)}{2 c}}{c\,x^{2}+a}+\frac{\frac{\left(4 a^{2} e^{5}-20 a c\,d^{2} e^{3}\right) \ln\left(c\,x^{2}+a\right)}{2 c}+\frac{\left(15 a^{2} d\,e^{4}-10 a c\,d^{3} e^{2}\right.}{\surd}}{2 a}}{c^{2}}$
risch	$\frac{e^{5} x^{2}}{2 c^{2}}+\frac{5 d\,e^{4} x}{c^{2}}+\frac{\frac{d\left(5 a^{2} e^{4}-10 a c\,d^{2} e^{2}+c^{2} d^{4}\right) x}{2 a}-\frac{e\left(a^{2} e^{4}-10 a c\,d^{2} e^{2}+5 c^{2} d^{4}\right)}{2 c}}{c^{2}\left(c\,x^{2}+a\right)}-\frac{a \ln\left(-15 d\,e^{4} a^{3}+10 d^{3} e^{2} a^{2} c+d^{5} a\,c^{2}-\sqrt{-a\,d^{2} c(15}\right.}{c^{3}}$

input
```
int((e*x+d)^5/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^4/c^2*(1/2*e*x^2+5*d*x)-1/c^2*((-1/2*d*(5*a^2*e^4-10*a*c*d^2*e^2+c^2*d^4
)/a*x+1/2*e*(a^2*e^4-10*a*c*d^2*e^2+5*c^2*d^4)/c)/(c*x^2+a)+1/2/a*(1/2*(4*
a^2*e^5-20*a*c*d^2*e^3)/c*ln(c*x^2+a)+(15*a^2*d*e^4-10*a*c*d^3*e^2-c^2*d^5
)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.02

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx$$
$$= \left[\frac{2\,a^2c^2e^5x^4+20\,a^2c^2de^4x^3+2\,a^3ce^5x^2-10\,a^2c^2d^4e+20\,a^3cd^2e^3-2\,a^4e^5+(ac^2d^5+10\,a^2cd^3e^2-15\,a}{}\right.$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a^2*c^2*e^5*x^4 + 20*a^2*c^2*d*e^4*x^3 + 2*a^3*c*e^5*x^2 - 10*a^2*
c^2*d^4*e + 20*a^3*c*d^2*e^3 - 2*a^4*e^5 + (a*c^2*d^5 + 10*a^2*c*d^3*e^2 -
 15*a^3*d*e^4 + (c^3*d^5 + 10*a*c^2*d^3*e^2 - 15*a^2*c*d*e^4)*x^2)*sqrt(-a
*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(a*c^3*d^5 - 10*a^2*
c^2*d^3*e^2 + 15*a^3*c*d*e^4)*x + 4*(5*a^3*c*d^2*e^3 - a^4*e^5 + (5*a^2*c^
2*d^2*e^3 - a^3*c*e^5)*x^2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(
a^2*c^2*e^5*x^4 + 10*a^2*c^2*d*e^4*x^3 + a^3*c*e^5*x^2 - 5*a^2*c^2*d^4*e +
 10*a^3*c*d^2*e^3 - a^4*e^5 + (a*c^2*d^5 + 10*a^2*c*d^3*e^2 - 15*a^3*d*e^4
 + (c^3*d^5 + 10*a*c^2*d^3*e^2 - 15*a^2*c*d*e^4)*x^2)*sqrt(a*c)*arctan(sqr
t(a*c)*x/a) + (a*c^3*d^5 - 10*a^2*c^2*d^3*e^2 + 15*a^3*c*d*e^4)*x + 2*(5*a
^3*c*d^2*e^3 - a^4*e^5 + (5*a^2*c^2*d^2*e^3 - a^3*c*e^5)*x^2)*log(c*x^2 +
a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(182) = 364$.

Time = 1.09 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx = \left(-\frac{e^3(ae^2-5cd^2)}{c^3} - \frac{d\sqrt{-a^3c^7}\cdot(15a^2e^4-10acd^2e^2-c^2d^4)}{4a^3c^6}\right)\log\left(x + \frac{-4a^3e^5-4a^2c^3\left(-\frac{e^3(ae^2-5cd^2)}{c^3} - \frac{d\sqrt{-a^3c^7}\cdot(15a^2e^4-10a}{4a^3c^6}\right.}{15a^2cde^4-10ac^2d^3e^2-c^3a}\right. + \left(-\frac{e^3(ae^2-5cd^2)}{c^3} + \frac{d\sqrt{-a^3c^7}\cdot(15a^2e^4-10acd^2e^2-c^2d^4)}{4a^3c^6}\right)\log\left(x + \frac{-4a^3e^5-4a^2c^3\left(-\frac{e^3(ae^2-5cd^2)}{c^3} + \frac{d\sqrt{-a^3c^7}\cdot(15a^2e^4-10a}{4a^3c^6}\right.}{15a^2cde^4-10ac^2d^3e^2-c^3a}\right. + \frac{-a^3e^5+10a^2cd^2e^3-5ac^2d^4e+x(5a^2cde^4-10ac^2d^3e^2+c^3d^5)}{2a^2c^3+2ac^4x^2} + \frac{5de^4x}{c^2} + \frac{e^5x^2}{2c^2}$$

input
```
integrate((e*x+d)**5/(c*x**2+a)**2,x)
```

output
```
(-e**3*(a*e**2 - 5*c*d**2)/c**3 - d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*a*
c*d**2*e**2 - c**2*d**4)/(4*a**3*c**6))*log(x + (-4*a**3*e**5 - 4*a**2*c**
3*(-e**3*(a*e**2 - 5*c*d**2)/c**3 - d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*
a*c*d**2*e**2 - c**2*d**4)/(4*a**3*c**6)) + 20*a**2*c*d**2*e**3)/(15*a**2*
c*d*e**4 - 10*a*c**2*d**3*e**2 - c**3*d**5)) + (-e**3*(a*e**2 - 5*c*d**2)/
c**3 + d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*a*c*d**2*e**2 - c**2*d**4)/(4
*a**3*c**6))*log(x + (-4*a**3*e**5 - 4*a**2*c**3*(-e**3*(a*e**2 - 5*c*d**2
)/c**3 + d*sqrt(-a**3*c**7)*(15*a**2*e**4 - 10*a*c*d**2*e**2 - c**2*d**4)/
(4*a**3*c**6)) + 20*a**2*c*d**2*e**3)/(15*a**2*c*d*e**4 - 10*a*c**2*d**3*e
**2 - c**3*d**5)) + (-a**3*e**5 + 10*a**2*c*d**2*e**3 - 5*a*c**2*d**4*e +
x*(5*a**2*c*d*e**4 - 10*a*c**2*d**3*e**2 + c**3*d**5))/(2*a**2*c**3 + 2*a*
c**4*x**2) + 5*d*e**4*x/c**2 + e**5*x**2/(2*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx = -\frac{5\,ac^2d^4e - 10\,a^2cd^2e^3 + a^3e^5 - (c^3d^5 - 10\,ac^2d^3e^2 + 5\,a^2cde^4)x}{2\,(ac^4x^2 + a^2c^3)} + \frac{e^5x^2 + 10\,de^4x}{2\,c^2} + \frac{(5\,cd^2e^3 - ae^5)\log(cx^2+a)}{c^3} + \frac{(c^2d^5 + 10\,acd^3e^2 - 15\,a^2de^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(5*a*c^2*d^4*e - 10*a^2*c*d^2*e^3 + a^3*e^5 - (c^3*d^5 - 10*a*c^2*d^3
*e^2 + 5*a^2*c*d*e^4)*x)/(a*c^4*x^2 + a^2*c^3) + 1/2*(e^5*x^2 + 10*d*e^4*x
)/c^2 + (5*c*d^2*e^3 - a*e^5)*log(c*x^2 + a)/c^3 + 1/2*(c^2*d^5 + 10*a*c*d
^3*e^2 - 15*a^2*d*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx = \frac{(5\,cd^2e^3-ae^5)\log\left(cx^2+a\right)}{c^3} + \frac{(c^2d^5+10\,acd^3e^2-15\,a^2de^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} + \frac{c^2e^5x^2+10\,c^2de^4x}{2\,c^4} - \frac{5\,ac^2d^4e-10\,a^2cd^2e^3+a^3e^5-(c^3d^5-10\,ac^2d^3e^2+5\,a^2cde^4)x}{2\,(cx^2+a)ac^3}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
(5*c*d^2*e^3 - a*e^5)*log(c*x^2 + a)/c^3 + 1/2*(c^2*d^5 + 10*a*c*d^3*e^2 -
 15*a^2*d*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(c^2*e^5*x^2
+ 10*c^2*d*e^4*x)/c^4 - 1/2*(5*a*c^2*d^4*e - 10*a^2*c*d^2*e^3 + a^3*e^5 -
(c^3*d^5 - 10*a*c^2*d^3*e^2 + 5*a^2*c*d*e^4)*x)/((c*x^2 + a)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx = \frac{e^5\,x^2}{2\,c^2} - \frac{\frac{a^2\,e^5-10\,a\,c\,d^2\,e^3+5\,c^2\,d^4\,e}{2\,c} - \frac{x\,\left(5\,a^2\,d\,e^4-10\,a\,c\,d^3\,e^2+c^2\,d^5\right)}{2\,a}}{c^3\,x^2+a\,c^2} - \frac{\ln\left(c\,x^2+a\right)\,\left(32\,a^4\,c^3\,e^5-160\,a^3\,c^4\,d^2\,e^3\right)}{32\,a^3\,c^6} + \frac{5\,d\,e^4\,x}{c^2} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-15\,a^2\,e^4+10\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{2\,a^{3/2}\,c^{5/2}}$$

input
```
int((d + e*x)^5/(a + c*x^2)^2,x)
```

output
```
(e^5*x^2)/(2*c^2) - ((a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)/(2*c) - (x*(
c^2*d^5 + 5*a^2*d*e^4 - 10*a*c*d^3*e^2))/(2*a))/(a*c^2 + c^3*x^2) - (log(a
 + c*x^2)*(32*a^4*c^3*e^5 - 160*a^3*c^4*d^2*e^3))/(32*a^3*c^6) + (5*d*e^4*
x)/c^2 + (d*atan((c^(1/2)*x)/a^(1/2))*(c^2*d^4 - 15*a^2*e^4 + 10*a*c*d^2*e
^2))/(2*a^(3/2)*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^5}{(a+cx^2)^2}\,dx$$
$$= \frac{-15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3d\,e^4+10\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2c\,d^3e^2-15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2cd\,e^4x^2+\sqrt{}}{}$$

input
```
int((e*x+d)^5/(c*x^2+a)^2,x)
```

output
```
( - 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*d*e**4 + 10*sqrt
(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d**3*e**2 - 15*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d*e**4*x**2 + sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**5 + 10*sqrt(c)*sqrt(a)*atan((c*x)/(
sqrt(c)*sqrt(a)))*a*c**2*d**3*e**2*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*c**3*d**5*x**2 - 2*log(a + c*x**2)*a**4*e**5 + 10*log(a + c*
x**2)*a**3*c*d**2*e**3 - 2*log(a + c*x**2)*a**3*c*e**5*x**2 + 10*log(a + c
*x**2)*a**2*c**2*d**2*e**3*x**2 + 15*a**3*c*d*e**4*x + 2*a**3*c*e**5*x**2
 - 10*a**2*c**2*d**3*e**2*x - 10*a**2*c**2*d**2*e**3*x**2 + 10*a**2*c**2*d*
e**4*x**3 + a**2*c**2*e**5*x**4 + a*c**3*d**5*x + 5*a*c**3*d**4*e*x**2)/(2
*a**2*c**3*(a + c*x**2))
```

3.110 $\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{e^4x}{c^2} - \frac{4ade(cd^2-ae^2)-(c^2d^4-6acd^2e^2+a^2e^4)\,x}{2ac^2\,(a+cx^2)} + \frac{(c^2d^4+6acd^2e^2-3a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} + \frac{2de^3\log(a+cx^2)}{c^2}$$

output
```
e^4*x/c^2-1/2*(4*a*d*e*(-a*e^2+c*d^2)-(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*x)/a
/c^2/(c*x^2+a)+1/2*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)*arctan(c^(1/2)*x/a^(
1/2))/a^(3/2)/c^(5/2)+2*d*e^3*ln(c*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{e^4x}{c^2} + \frac{c^2d^4x+a^2e^3(4d+ex)-2acd^2e(2d+3ex)}{2ac^2\,(a+cx^2)} + \frac{(c^2d^4+6acd^2e^2-3a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} + \frac{2de^3\log(a+cx^2)}{c^2}$$

input
```
Integrate[(d + e*x)^4/(a + c*x^2)^2,x]
```

output
```
(e^4*x)/c^2 + (c^2*d^4*x + a^2*e^3*(4*d + e*x) - 2*a*c*d^2*e*(2*d + 3*e*x)
)/(2*a*c^2*(a + c*x^2)) + ((c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTan[(S
qrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (2*d*e^3*Log[a + c*x^2])/c^2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {495, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx$$

$$\Big\downarrow \; 495$$

$$\frac{\int \frac{(d+ex)^2(cd^2-2cexd+3ae^2)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^3(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(-2dxe^3-3\left(d^2-\frac{ae^2}{c}\right)e^2+\frac{c^2d^4+6ace^2d^2+8ace^3xd-3a^2e^4}{c(cx^2+a)}\right)dx}{2ac}-\frac{(d+ex)^3(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(-3a^2e^4+6acd^2e^2+c^2d^4)}{\sqrt{a}c^{3/2}}-3e^2x\left(d^2-\frac{ae^2}{c}\right)+\frac{4ade^3\log(a+cx^2)}{c}-de^3x^2}{2ac}-\frac{(d+ex)^3(ae-cdx)}{2ac\,(a+cx^2)}$$

input
```
Int[(d + e*x)^4/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*e - c*d*x)*(d + e*x)^3)/(a*c*(a + c*x^2)) + (-3*e^2*(d^2 - (a*e^2
)/c)*x - d*e^3*x^2 + ((c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (4*a*d*e^3*Log[a + c*x^2])/c)/(2*a*c)
```

Defintions of rubi rules used

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

method	result
default	$\frac{e^{4} x}{c^{2}}-\frac{\frac{-\frac{\left(a^{2} e^{4}-6 a c \,d^{2} e^{2}+c^{2} d^{4}\right) x}{2 a}-2 d e \left(a \,e^{2}-c \,d^{2}\right)}{c \,x^{2}+a}+\frac{-4 d \,e^{3} a \ln \left(c \,x^{2}+a \right)+\frac{\left(3 a^{2} e^{4}-6 a c \,d^{2} e^{2}-c^{2} d^{4}\right) \arctan \left(\frac{c x}{\sqrt{a c}}\right)}{\sqrt{a c}}}{2 a}}{c^{2}}$
risch	$\frac{e^{4} x}{c^{2}}+\frac{\frac{\left(a^{2} e^{4}-6 a c \,d^{2} e^{2}+c^{2} d^{4}\right) x}{2 a}+2 d e \left(a \,e^{2}-c \,d^{2}\right)}{c^{2} \left(c \,x^{2}+a \right)}+\frac{2 \ln \left(-3 e^{4} a^{3}+6 d^{2} e^{2} a^{2} c +d^{4} a \,c^{2}-\sqrt{-a c \left(3 a^{2} e^{4}-6 a c \,d^{2} e^{2}-c^{2} d^{4}\right)^{2}}\, x \right) d \,e^{3}}{c^{2}}+$

input
```
int((e*x+d)^4/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^4*x/c^2-1/c^2*((-1/2*(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)/a*x-2*d*e*(a*e^2-c*
d^2))/(c*x^2+a)+1/2/a*(-4*d*e^3*a*ln(c*x^2+a)+(3*a^2*e^4-6*a*c*d^2*e^2-c^2
*d^4)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 433, normalized size of antiderivative = 3.01

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx$$
$$= \left[\frac{4\,a^2c^2e^4x^3 - 8\,a^2c^2d^3e + 8\,a^3cde^3 + (ac^2d^4 + 6\,a^2cd^2e^2 - 3\,a^3e^4 + (c^3d^4 + 6\,ac^2d^2e^2 - 3\,a^2ce^4)x^2)\sqrt{-a}}{4\,(a^2c^4x^2 +} \right.$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^2*c^2*e^4*x^3 - 8*a^2*c^2*d^3*e + 8*a^3*c*d*e^3 + (a*c^2*d^4 + 6
*a^2*c*d^2*e^2 - 3*a^3*e^4 + (c^3*d^4 + 6*a*c^2*d^2*e^2 - 3*a^2*c*e^4)*x^2
)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(a*c^3*d^4
- 6*a^2*c^2*d^2*e^2 + 3*a^3*c*e^4)*x + 8*(a^2*c^2*d*e^3*x^2 + a^3*c*d*e^3)
*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*a^2*c^2*e^4*x^3 - 4*a^2*c
^2*d^3*e + 4*a^3*c*d*e^3 + (a*c^2*d^4 + 6*a^2*c*d^2*e^2 - 3*a^3*e^4 + (c^3
*d^4 + 6*a*c^2*d^2*e^2 - 3*a^2*c*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a)
 + (a*c^3*d^4 - 6*a^2*c^2*d^2*e^2 + 3*a^3*c*e^4)*x + 4*(a^2*c^2*d*e^3*x^2
+ a^3*c*d*e^3)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(138) = 276$.

Time = 0.72 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.80

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx = \left(\frac{2de^3}{c^2} - \frac{\sqrt{-a^3c^5}\cdot(3a^2e^4-6acd^2e^2-c^2d^4)}{4a^3c^5}\right)\log\left(x + \frac{-4a^2c^2\cdot\left(\frac{2de^3}{c^2}-\frac{\sqrt{-a^3c^5}\cdot(3a^2e^4-6acd^2e^2-c^2d^4)}{4a^3c^5}\right)+8a^2de^3}{3a^2e^4-6acd^2e^2-c^2d^4}\right) + \left(\frac{2de^3}{c^2} + \frac{\sqrt{-a^3c^5}\cdot(3a^2e^4-6acd^2e^2-c^2d^4)}{4a^3c^5}\right)\log\left(x + \frac{-4a^2c^2\cdot\left(\frac{2de^3}{c^2}+\frac{\sqrt{-a^3c^5}\cdot(3a^2e^4-6acd^2e^2-c^2d^4)}{4a^3c^5}\right)+8a^2de^3}{3a^2e^4-6acd^2e^2-c^2d^4}\right) + \frac{4a^2de^3-4acd^3e+x(a^2e^4-6acd^2e^2+c^2d^4)}{2a^2c^2+2ac^3x^2} + \frac{e^4x}{c^2}$$

input
```
integrate((e*x+d)**4/(c*x**2+a)**2,x)
```

output
```
(2*d*e**3/c**2 - sqrt(-a**3*c**5)*(3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d*
*4)/(4*a**3*c**5))*log(x + (-4*a**2*c**2*(2*d*e**3/c**2 - sqrt(-a**3*c**5)
*(3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d**4)/(4*a**3*c**5)) + 8*a**2*d*e**
3)/(3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d**4)) + (2*d*e**3/c**2 + sqrt(-a
**3*c**5)*(3*a**2*e**4 - 6*a*c*d**2*e**2 - c**2*d**4)/(4*a**3*c**5))*log(x
 + (-4*a**2*c**2*(2*d*e**3/c**2 + sqrt(-a**3*c**5)*(3*a**2*e**4 - 6*a*c*d*
*2*e**2 - c**2*d**4)/(4*a**3*c**5)) + 8*a**2*d*e**3)/(3*a**2*e**4 - 6*a*c*
d**2*e**2 - c**2*d**4)) + (4*a**2*d*e**3 - 4*a*c*d**3*e + x*(a**2*e**4 - 6
*a*c*d**2*e**2 + c**2*d**4))/(2*a**2*c**2 + 2*a*c**3*x**2) + e**4*x/c**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{e^4x}{c^2} + \frac{2\,de^3\log\left(cx^2+a\right)}{c^2} - \frac{4\,acd^3e - 4\,a^2de^3 - (c^2d^4 - 6\,acd^2e^2 + a^2e^4)x}{2\,(ac^3x^2 + a^2c^2)} + \frac{(c^2d^4 + 6\,acd^2e^2 - 3\,a^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
e^4*x/c^2 + 2*d*e^3*log(c*x^2 + a)/c^2 - 1/2*(4*a*c*d^3*e - 4*a^2*d*e^3 -
(c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(c^2*d^
4 + 6*a*c*d^2*e^2 - 3*a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{e^4x}{c^2} + \frac{2\,de^3\log\left(cx^2+a\right)}{c^2} + \frac{(c^2d^4 + 6\,acd^2e^2 - 3\,a^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} - \frac{4\,acd^3e - 4\,a^2de^3 - (c^2d^4 - 6\,acd^2e^2 + a^2e^4)x}{2\,(cx^2+a)ac^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
e^4*x/c^2 + 2*d*e^3*log(c*x^2 + a)/c^2 + 1/2*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*
a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) - 1/2*(4*a*c*d^3*e - 4*a^
2*d*e^3 - (c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*x)/((c*x^2 + a)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{\frac{x\,(a^2\,e^4-6\,a\,c\,d^2\,e^2+c^2\,d^4)}{2\,a}+2\,a\,d\,e^3-2\,c\,d^3\,e}{c^3\,x^2+a\,c^2}+\frac{e^4\,x}{c^2} + \frac{2\,d\,e^3\,\ln\left(c\,x^2+a\right)}{c^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-3\,a^2\,e^4+6\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{2\,a^{3/2}\,c^{5/2}}$$

input
```
int((d + e*x)^4/(a + c*x^2)^2,x)
```

output
```
((x*(a^2*e^4 + c^2*d^4 - 6*a*c*d^2*e^2))/(2*a) + 2*a*d*e^3 - 2*c*d^3*e)/(a
*c^2 + c^3*x^2) + (e^4*x)/c^2 + (2*d*e^3*log(a + c*x^2))/c^2 + (atan((c^(1
/2)*x)/a^(1/2))*(c^2*d^4 - 3*a^2*e^4 + 6*a*c*d^2*e^2))/(2*a^(3/2)*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.06

$$\int \frac{(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3e^4+6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2c\,d^2e^2-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2c\,e^4x^2+\sqrt{c}\,\sqrt{a}\,a}{}$$

input
```
int((e*x+d)^4/(c*x^2+a)^2,x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*e**4 + 6*sqrt(c)*
sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d**2*e**2 - 3*sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*e**4*x**2 + sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a*c**2*d**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*a*c**2*d**2*e**2*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt
(a)))*c**3*d**4*x**2 + 4*log(a + c*x**2)*a**3*c*d*e**3 + 4*log(a + c*x**2)
*a**2*c**2*d*e**3*x**2 + 3*a**3*c*e**4*x - 6*a**2*c**2*d**2*e**2*x - 4*a**
2*c**2*d*e**3*x**2 + 2*a**2*c**2*e**4*x**3 + a*c**3*d**4*x + 4*a*c**3*d**3
*e*x**2)/(2*a**2*c**3*(a + c*x**2))
```

3.111 $\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 112

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx = -\frac{ae(3cd^2-ae^2)-cd(cd^2-3ae^2)\,x}{2ac^2\,(a+cx^2)} + \frac{d(cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{e^3\log\left(a+cx^2\right)}{2c^2}$$

output
```
-1/2*(a*e*(-a*e^2+3*c*d^2)-c*d*(-3*a*e^2+c*d^2)*x)/a/c^2/(c*x^2+a)+1/2*d*(
3*a*e^2+c*d^2)*arctan(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(3/2)+1/2*e^3*ln(c*x^2+
a)/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{\sqrt{c}d(cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) + \frac{\sqrt{a}\left(a^2e^3+c^2d^3x-3acde(d+ex)+ae^3(a+cx^2)\log(a+cx^2)\right)}{a+cx^2}}{2a^{3/2}c^2}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^2)^2,x]
```

output
```
(Sqrt[c]*d*(c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + (Sqrt[a]*(a^2*e
^3 + c^2*d^3*x - 3*a*c*d*e*(d + e*x) + a*e^3*(a + c*x^2)*Log[a + c*x^2]))/
(a + c*x^2))/(2*a^(3/2)*c^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {495, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^2)^2} dx$$

$$\Big\downarrow \; 495$$

$$\frac{\int \frac{(d+ex)(cd^2-cexd+2ae^2)}{cx^2+a}dx}{2ac} - \frac{(d+ex)^2(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{cd^3+3ae^2d+2ae^3x}{cx^2+a} - de^2\right) dx}{2ac} - \frac{(d+ex)^2(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3ae^2+cd^2)}{\sqrt{a}\sqrt{c}} + \frac{ae^3\log(a+cx^2)}{c} - de^2x}{2ac} - \frac{(d+ex)^2(ae-cdx)}{2ac(a+cx^2)}$$

input
```
Int[(d + e*x)^3/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*e - c*d*x)*(d + e*x)^2)/(a*c*(a + c*x^2)) + (-(d*e^2*x) + (d*(c*d
^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (a*e^3*Log[
a + c*x^2])/c)/(2*a*c)
```

Defintions of rubi rules used

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

method	result
default	$\frac{-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{2ac}+\frac{e\left(a\,e^{2}-3c\,d^{2}\right)}{2c^{2}}}{c\,x^{2}+a}+\frac{\frac{a\,e^{3}\ln\left(c\,x^{2}+a\right)}{c}+\frac{\left(3ad\,e^{2}+c\,d^{3}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2ac}$
risch	$\frac{-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{2ac}+\frac{e\left(a\,e^{2}-3c\,d^{2}\right)}{2c^{2}}}{c\,x^{2}+a}+\frac{\ln\left(3a^{2}d\,e^{2}+ac\,d^{3}-\sqrt{-a\,d^{2}c(3a\,e^{2}+c\,d^{2})^{2}}\,x\right)e^{3}}{2c^{2}}+\frac{\ln\left(3a^{2}d\,e^{2}+ac\,d^{3}-\sqrt{-a\,d^{2}c(3a\,e^{2}+c\,d \cdots}\right.}{4a^{2}c^{2}}$

input
```
int((e*x+d)^3/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*d*(3*a*e^2-c*d^2)/a/c*x+1/2*e*(a*e^2-3*c*d^2)/c^2)/(c*x^2+a)+1/2/a/c
*(a*e^3/c*ln(c*x^2+a)+(3*a*d*e^2+c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx$$
$$= \left[-\frac{6\,a^2cd^2e-2\,a^3e^3+(acd^3+3\,a^2de^2+(c^2d^3+3\,acde^2)x^2)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)-2\,(ac^2d^3-3\,a^2}{4\,(a^2c^3x^2+a^3c^2)}\right.$$
$$-\frac{3\,a^2cd^2e-a^3e^3-(acd^3+3\,a^2de^2+(c^2d^3+3\,acde^2)x^2)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)-(ac^2d^3-3\,a^2cde^2)x-(}{2\,(a^2c^3x^2+a^3c^2)}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(6*a^2*c*d^2*e - 2*a^3*e^3 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a
*c*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) -
2*(a*c^2*d^3 - 3*a^2*c*d*e^2)*x - 2*(a^2*c*e^3*x^2 + a^3*e^3)*log(c*x^2 +
a))/(a^2*c^3*x^2 + a^3*c^2), -1/2*(3*a^2*c*d^2*e - a^3*e^3 - (a*c*d^3 + 3*
a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) -
 (a*c^2*d^3 - 3*a^2*c*d*e^2)*x - (a^2*c*e^3*x^2 + a^3*e^3)*log(c*x^2 + a))
/(a^2*c^3*x^2 + a^3*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(102) = 204.

Time = 0.53 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.66

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx = \left(\frac{e^3}{2c^2} - \frac{d\sqrt{-a^3c^5}\cdot(3ae^2+cd^2)}{4a^3c^4}\right)\log\left(x + \frac{4a^2c^2\left(\frac{e^3}{2c^2} - \frac{d\sqrt{-a^3c^5}\cdot(3ae^2+cd^2)}{4a^3c^4}\right) - 2a^2e^3}{3acde^2+c^2d^3}\right) + \left(\frac{e^3}{2c^2} + \frac{d\sqrt{-a^3c^5}\cdot(3ae^2+cd^2)}{4a^3c^4}\right)\log\left(x + \frac{4a^2c^2\left(\frac{e^3}{2c^2} + \frac{d\sqrt{-a^3c^5}\cdot(3ae^2+cd^2)}{4a^3c^4}\right) - 2a^2e^3}{3acde^2+c^2d^3}\right) + \frac{a^2e^3 - 3acd^2e + x(-3acde^2+c^2d^3)}{2a^2c^2+2ac^3x^2}$$

input
```
integrate((e*x+d)**3/(c*x**2+a)**2,x)
```

output
```
(e**3/(2*c**2) - d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/(4*a**3*c**4))*log
(x + (4*a**2*c**2*(e**3/(2*c**2) - d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/
(4*a**3*c**4)) - 2*a**2*e**3)/(3*a*c*d*e**2 + c**2*d**3)) + (e**3/(2*c**2)
 + d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/(4*a**3*c**4))*log(x + (4*a**2*c
**2*(e**3/(2*c**2) + d*sqrt(-a**3*c**5)*(3*a*e**2 + c*d**2)/(4*a**3*c**4))
 - 2*a**2*e**3)/(3*a*c*d*e**2 + c**2*d**3)) + (a**2*e**3 - 3*a*c*d**2*e +
x*(-3*a*c*d*e**2 + c**2*d**3))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{e^3\log(cx^2+a)}{2\,c^2} - \frac{3\,acd^2e - a^2e^3 - (c^2d^3 - 3\,acde^2)x}{2\,(ac^3x^2+a^2c^2)} + \frac{(cd^3+3\,ade^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*e^3*log(c*x^2 + a)/c^2 - 1/2*(3*a*c*d^2*e - a^2*e^3 - (c^2*d^3 - 3*a*c
*d*e^2)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(c*d^3 + 3*a*d*e^2)*arctan(c*x/sqrt
(a*c))/(sqrt(a*c)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{e^3\log\left(cx^2+a\right)}{2\,c^2} + \frac{(cd^3+3\,ade^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac} + \frac{(cd^3-3\,ade^2)x - \frac{3\,acd^2e-a^2e^3}{c}}{2\,(cx^2+a)ac}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*e^3*log(c*x^2 + a)/c^2 + 1/2*(c*d^3 + 3*a*d*e^2)*arctan(c*x/sqrt(a*c))
/(sqrt(a*c)*a*c) + 1/2*((c*d^3 - 3*a*d*e^2)*x - (3*a*c*d^2*e - a^2*e^3)/c)
/((c*x^2 + a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{d^3\,x}{2\,\left(a^2+c\,a\,x^2\right)} - \frac{3\,d^2\,e}{2\,\left(c^2\,x^2+a\,c\right)} + \frac{e^3\,\ln\left(c\,x^2+a\right)}{2\,c^2} + \frac{a\,e^3}{2\,\left(c^3\,x^2+a\,c^2\right)} + \frac{d^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}} - \frac{3\,d\,e^2\,x}{2\,\left(c^2\,x^2+a\,c\right)} + \frac{3\,d\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}}$$

input
```
int((d + e*x)^3/(a + c*x^2)^2,x)
```

output
```
(d^3*x)/(2*(a^2 + a*c*x^2)) - (3*d^2*e)/(2*(a*c + c^2*x^2)) + (e^3*log(a +
 c*x^2))/(2*c^2) + (a*e^3)/(2*(a*c^2 + c^3*x^2)) + (d^3*atan((c^(1/2)*x)/a
^(1/2)))/(2*a^(3/2)*c^(1/2)) - (3*d*e^2*x)/(2*(a*c + c^2*x^2)) + (3*d*e^2*
atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3}{(a+cx^2)^2}\,dx$$
$$= \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2d\,e^2+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,d^3+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)acd\,e^2x^2+\sqrt{c}\,\sqrt{a}\,atan\left(\right.}{2a^2c^2\left(\right.}$$

input
```
int((e*x+d)^3/(c*x^2+a)^2,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*d*e**2 + sqrt(c)*sqr
t(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d**3 + 3*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a*c*d*e**2*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*c**2*d**3*x**2 + log(a + c*x**2)*a**3*e**3 + log(a + c*x**2)*a**
2*c*e**3*x**2 - 3*a**2*c*d*e**2*x - a**2*c*e**3*x**2 + a*c**2*d**3*x + 3*a
*c**2*d**2*e*x**2)/(2*a**2*c**2*(a + c*x**2))
```

3.112 $\int \frac{(d+ex)^2}{(a+cx^2)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 79

$$\int \frac{(d+ex)^2}{(a+cx^2)^2} dx = -\frac{2ade-(cd^2-ae^2)\,x}{2ac\,(a+cx^2)}+\frac{(cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

output
```
-1/2*(2*a*d*e-(-a*e^2+c*d^2)*x)/a/c/(c*x^2+a)+1/2*(a*e^2+c*d^2)*arctan(c^(
1/2)*x/a^(1/2))/a^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^2}{(a+cx^2)^2} dx = \frac{-2ade+cd^2x-ae^2x}{2ac\,(a+cx^2)}+\frac{(cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^2)^2,x]
```

output
```
(-2*a*d*e + c*d^2*x - a*e^2*x)/(2*a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^2)^2}dx$$

$$\Big\downarrow \text{ 487}$$

$$\frac{(ae^2+cd^2)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+cd^2)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac\,(a+cx^2)}$$

input
```
Int[(d + e*x)^2/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*e - c*d*x)*(d + e*x))/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{-\frac{\left(a\,e^{2}-c\,d^{2}\right)x}{2ac}-\frac{de}{c}}{c\,x^{2}+a}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2ac\sqrt{ac}}$	74
risch	$\frac{-\frac{\left(a\,e^{2}-c\,d^{2}\right)x}{2ac}-\frac{de}{c}}{c\,x^{2}+a}-\frac{\ln\left(cx+\sqrt{-ac}\right)e^{2}}{4\sqrt{-ac}\,c}-\frac{\ln\left(cx+\sqrt{-ac}\right)d^{2}}{4\sqrt{-ac}\,a}+\frac{\ln\left(-cx+\sqrt{-ac}\right)e^{2}}{4\sqrt{-ac}\,c}+\frac{\ln\left(-cx+\sqrt{-ac}\right)d^{2}}{4\sqrt{-ac}\,a}$	143

input
```
int((e*x+d)^2/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(a*e^2-c*d^2)/a/c*x-d*e/c)/(c*x^2+a)+1/2*(a*e^2+c*d^2)/a/c/(a*c)^(1/
2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.82

$$\int \frac{(d+ex)^2}{(a+cx^2)^2}\,dx$$
$$= \left[-\frac{4\,a^2cde+(acd^2+a^2e^2+(c^2d^2+ace^2)x^2)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)-2\,(ac^2d^2-a^2ce^2)x}{4\,(a^2c^3x^2+a^3c^2)},\right.$$
$$\left.-\frac{2\,a^2cde-(acd^2+a^2e^2+(c^2d^2+ace^2)x^2)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)-(ac^2d^2-a^2ce^2)x}{2\,(a^2c^3x^2+a^3c^2)}\right]$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*c*d*e + (a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)*sqrt(-a
*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(a*c^2*d^2 - a^2*c*e
^2)*x)/(a^2*c^3*x^2 + a^3*c^2), -1/2*(2*a^2*c*d*e - (a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (a*c^2*d^2 - a^2
*c*e^2)*x)/(a^2*c^3*x^2 + a^3*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^2}{(a+cx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3c^3}}\,(ae^2+cd^2)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c^3}}\,(ae^2+cd^2)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4} + \frac{-2ade+x(-ae^2+cd^2)}{2a^2c+2ac^2x^2}$$

input
```
integrate((e*x+d)**2/(c*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**3*c**3))*(a*e**2 + c*d**2)*log(-a**2*c*sqrt(-1/(a**3*c**3)) +
 x)/4 + sqrt(-1/(a**3*c**3))*(a*e**2 + c*d**2)*log(a**2*c*sqrt(-1/(a**3*c*
*3)) + x)/4 + (-2*a*d*e + x*(-a*e**2 + c*d**2))/(2*a**2*c + 2*a*c**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{(a+cx^2)^2}\,dx = -\frac{2\,ade-(cd^2-ae^2)x}{2\,(ac^2x^2+a^2c)} + \frac{(cd^2+ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*d*e - (c*d^2 - a*e^2)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(c*d^2 + a*e^
2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^2}{(a+cx^2)^2}\,dx = \frac{(cd^2+ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac} + \frac{cd^2x-ae^2x-2\,ade}{2\,(cx^2+a)ac}$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(c*d^2 + a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) + 1/2*(c*d^2*x -
 a*e^2*x - 2*a*d*e)/((c*x^2 + a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^2}{(a+cx^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(c\,d^2+a\,e^2)}{2\,a^{3/2}\,c^{3/2}} - \frac{\frac{d\,e}{c}+\frac{x\,(a\,e^2-c\,d^2)}{2\,a\,c}}{c\,x^2+a}$$

input
```
int((d + e*x)^2/(a + c*x^2)^2,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(a*e^2 + c*d^2))/(2*a^(3/2)*c^(3/2)) - ((d*e)/c
 + (x*(a*e^2 - c*d^2))/(2*a*c))/(a + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^2}{(a+cx^2)^2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2e^2+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,d^2+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,e^2x^2+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right.}{2a^2c^2\,(c\,x^2+a)}$$

input
```
int((e*x+d)^2/(c*x^2+a)^2,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*e**2 + sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a*c*e**2*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))
)*c**2*d**2*x**2 - a**2*c*e**2*x + a*c**2*d**2*x + 2*a*c**2*d*e*x**2)/(2*a
**2*c**2*(a + c*x**2))
```

3.113 $\int \frac{d+ex}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx=\frac{-ae+cdx}{2ac\,(a+cx^2)}+\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

output
```
1/2*(c*d*x-a*e)/a/c/(c*x^2+a)+1/2*d*arctan(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(1
/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx=\frac{-ae+cdx}{2ac\,(a+cx^2)}+\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

input
```
Integrate[(d + e*x)/(a + c*x^2)^2,x]
```

output
```
(-(a*e) + c*d*x)/(2*a*c*(a + c*x^2)) + (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^2)^2}dx$$

$$\Big\downarrow \text{454}$$

$$\frac{d\int \frac{1}{cx^2+a}dx}{2a}-\frac{ae-cdx}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}-\frac{ae-cdx}{2ac\,(a+cx^2)}$$

input
```
Int[(d + e*x)/(a + c*x^2)^2,x]
```

output
```
-1/2*(a*e - c*d*x)/(a*c*(a + c*x^2)) + (d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2cdx-2ae}{4ac(c\,x^2+a)}+\frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2a\sqrt{ac}}$	49
risch	$\frac{\frac{dx}{2a}-\frac{e}{2c}}{c\,x^2+a}-\frac{d\ln\left(cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\,a}+\frac{d\ln\left(-cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\,a}$	73

input
```
int((e*x+d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*d*x-2*a*e)/a/c/(c*x^2+a)+1/2*d/a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.46

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx$$
$$=\left[\frac{2\,acdx-2\,a^2e-(cdx^2+ad)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{4\,(a^2c^2x^2+a^3c)},\frac{acdx-a^2e+(cdx^2+ad)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2\,(a^2c^2x^2+a^3c)}\right]$$

input
```
integrate((e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*c*d*x - 2*a^2*e - (c*d*x^2 + a*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt
(-a*c)*x - a)/(c*x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(a*c*d*x - a^2*e +
(c*d*x^2 + a*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx = d\left(-\frac{\sqrt{-\frac{1}{a^3c}}\log\left(-a^2\sqrt{-\frac{1}{a^3c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3c}}\log\left(a^2\sqrt{-\frac{1}{a^3c}}+x\right)}{4}\right) + \frac{-ae+cdx}{2a^2c+2ac^2x^2}$$

input
```
integrate((e*x+d)/(c*x**2+a)**2,x)
```

output
```
d*(-sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c)) + x)/4 + sqrt(-1/(a**3*c
))*log(a**2*sqrt(-1/(a**3*c)) + x)/4) + (-a*e + c*d*x)/(2*a**2*c + 2*a*c**
2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx = \frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a}+\frac{cdx-ae}{2(ac^2x^2+a^2c)}$$

input
```
integrate((e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*(c*d*x - a*e)/(a*c^2*x^2 +
 a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx = \frac{d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{cdx-ae}{2\,(cx^2+a)ac}$$

input
```
integrate((e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*(c*d*x - a*e)/((c*x^2 + a)
*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx = \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}} - \frac{\frac{e}{2\,c}-\frac{d\,x}{2\,a}}{c\,x^2+a}$$

input
```
int((d + e*x)/(a + c*x^2)^2,x)
```

output
```
(d*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*c^(1/2)) - (e/(2*c) - (d*x)/(2*a)
)/(a + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{d+ex}{(a+cx^2)^2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ad+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)cd\,x^2+acdx+ace\,x^2}{2a^2c\,(c\,x^2+a)}$$

input
```
int((e*x+d)/(c*x^2+a)^2,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*d + sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*c*d*x**2 + a*c*d*x + a*c*e*x**2)/(2*a**2*c*(a + c
*x**2))
```

3.114 $\int \frac{1}{(d+ex)(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 142

$$\int \frac{1}{(d+ex)(a+cx^2)^2}\,dx = \frac{ae+cdx}{2a(cd^2+ae^2)(a+cx^2)} + \frac{\sqrt{c}d(cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}(cd^2+ae^2)^2} + \frac{e^3\log(d+ex)}{(cd^2+ae^2)^2} - \frac{e^3\log(a+cx^2)}{2(cd^2+ae^2)^2}$$

output
```
1/2*(c*d*x+a*e)/a/(a*e^2+c*d^2)/(c*x^2+a)+1/2*c^(1/2)*d*(3*a*e^2+c*d^2)*ar
ctan(c^(1/2)*x/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^2+e^3*ln(e*x+d)/(a*e^2+c*d^2
)^2-1/2*e^3*ln(c*x^2+a)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)(a+cx^2)^2}\,dx$$
$$= \frac{\sqrt{c}d(cd^2+3ae^2)(a+cx^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) + \sqrt{a}((cd^2+ae^2)(ae+cdx) + 2ae^3(a+cx^2)\log(d+ex) - ae}{2a^{3/2}(cd^2+ae^2)^2(a+cx^2)}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^2)^2),x]
```

output
```
(Sqrt[c]*d*(c*d^2 + 3*a*e^2)*(a + c*x^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqr
t[a]*((c*d^2 + a*e^2)*(a*e + c*d*x) + 2*a*e^3*(a + c*x^2)*Log[d + e*x] - a
*e^3*(a + c*x^2)*Log[a + c*x^2]))/(2*a^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x^2)
)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^2(d+ex)}dx$$

$$\Big\downarrow \ 496$$

$$\frac{ae+cdx}{2a(a+cx^2)(ae^2+cd^2)}-\frac{\int -\frac{cd^2+cexd+2ae^2}{(d+ex)(cx^2+a)}dx}{2a(ae^2+cd^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{cd^2+cexd+2ae^2}{(d+ex)(cx^2+a)}dx}{2a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \ 657$$

$$\frac{\int \left(\frac{2ae^4}{(cd^2+ae^2)(d+ex)}+\frac{c(cd^3+3ae^2d-2ae^3x)}{(cd^2+ae^2)(cx^2+a)}\right)dx}{2a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3ae^2+cd^2)}{\sqrt{a}(ae^2+cd^2)}-\frac{ae^3\log(a+cx^2)}{ae^2+cd^2}+\frac{2ae^3\log(d+ex)}{ae^2+cd^2}}{2a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x)*(a + c*x^2)^2),x]
```

output
```
(a*e + c*d*x)/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + ((Sqrt[c]*d*(c*d^2 + 3*a
*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)) + (2*a*e^3*Lo
g[d + e*x])/(c*d^2 + a*e^2) - (a*e^3*Log[a + c*x^2])/(c*d^2 + a*e^2))/(2*a
*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

method	result
default	$\frac{e^{3}\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}+\frac{c\left(\frac{\frac{d\left(a\,e^{2}+c\,d^{2}\right)x}{2a}+\frac{e\left(a\,e^{2}+c\,d^{2}\right)}{2c}}{c\,x^{2}+a}+\frac{-\frac{a\,e^{3}\ln\left(c\,x^{2}+a\right)}{c}+\frac{\left(3ad\,e^{2}+c\,d^{3}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2a}\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	$\frac{\frac{cdx}{2a\left(a\,e^{2}+c\,d^{2}\right)}+\frac{e}{2a\,e^{2}+2c\,d^{2}}}{c\,x^{2}+a}+\frac{e^{3}\ln\left(ex+d\right)}{a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{5}e^{4}+2a^{4}c\,d^{2}e^{2}+a^{3}c^{2}d^{4}\right)\_Z^{2}+4a^{3}e^{3}\_Z+4a\,e^{2}+c\,d^{2}\right)}\_R$

input
```
int(1/(e*x+d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^3*ln(e*x+d)/(a*e^2+c*d^2)^2+c/(a*e^2+c*d^2)^2*((1/2*d*(a*e^2+c*d^2)/a*x+
1/2*e*(a*e^2+c*d^2)/c)/(c*x^2+a)+1/2/a*(-a*e^3/c*ln(c*x^2+a)+(3*a*d*e^2+c*
d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.11

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx$$
$$= \left[\frac{2\,acd^2e+2\,a^2e^3+\left(acd^3+3\,a^2de^2+\left(c^2d^3+3\,acde^2\right)x^2\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^2+2\,ax\sqrt{-\frac{c}{a}}-a}{cx^2+a}\right)+2\left(c^2d^3+acde^2\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+\left(ac^3d^4+2\,a^2c^2d^2e\right.\right.}\right.$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*c*d^2*e + 2*a^2*e^3 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*
d*e^2)*x^2)*sqrt(-c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2
*(c^2*d^3 + a*c*d*e^2)*x - 2*(a*c*e^3*x^2 + a^2*e^3)*log(c*x^2 + a) + 4*(a
*c*e^3*x^2 + a^2*e^3)*log(e*x + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), 1/2*(a*c*d^2*e + a^
2*e^3 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^2)*sqrt(c/a)*ar
ctan(x*sqrt(c/a)) + (c^2*d^3 + a*c*d*e^2)*x - (a*c*e^3*x^2 + a^2*e^3)*log(
c*x^2 + a) + 2*(a*c*e^3*x^2 + a^2*e^3)*log(e*x + d))/(a^2*c^2*d^4 + 2*a^3*
c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.33

$$\begin{aligned}\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx = &-\frac{e^3\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{e^3\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}\\&+\frac{\left(c^2d^3+3\,acde^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}\\&+\frac{cdx+ae}{2\left(a^2cd^2+a^3e^2+\left(ac^2d^2+a^2ce^2\right)x^2\right)}\end{aligned}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*e^3*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + e^3*log(e*x
+ d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*(c^2*d^3 + 3*a*c*d*e^2)*arc
tan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1
/2*(c*d*x + a*e)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.42

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx = \frac{e^4\log\left(|ex+d|\right)}{c^2d^4e+2\,acd^2e^3+a^2e^5} - \frac{e^3\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)} + \frac{\left(c^2d^3+3\,acde^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}} + \frac{acd^2e+a^2e^3+\left(c^2d^3+acde^2\right)x}{2\left(cd^2+ae^2\right)^2\left(cx^2+a\right)a}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
e^4*log(abs(e*x + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/2*e^3*log(
c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*(c^2*d^3 + 3*a*c*d*e^
2)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c
)) + 1/2*(a*c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)/((c*d^2 + a*e^2)^
2*(c*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 609, normalized size of antiderivative = 4.29

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx = \frac{\frac{e}{2\left(c\,d^2+a\,e^2\right)}+\frac{c\,d\,x}{2\,a\left(c\,d^2+a\,e^2\right)}}{c\,x^2+a} + \frac{e^3\,\ln\left(d+e\,x\right)}{\left(c\,d^2+a\,e^2\right)^2} + \frac{\ln\left(36\,a^7\,e^{10}\,\sqrt{-a^3\,c}+a^3\,c^6\,d^{10}\,x+a^2\,c^5\,d^{10}\,\sqrt{-a^3\,c}-81\,a^3\,d^2\,e^8\,\left(-a^3\,c\right)^{3/2}-8\,c^3\,d^8\,e^2\,\left(-a^3\,c\right)^{3/2}+36\right.}{} - \frac{\ln\left(a^3\,c^6\,d^{10}\,x-36\,a^7\,e^{10}\,\sqrt{-a^3\,c}-a^2\,c^5\,d^{10}\,\sqrt{-a^3\,c}+81\,a^3\,d^2\,e^8\,\left(-a^3\,c\right)^{3/2}+8\,c^3\,d^8\,e^2\,\left(-a^3\,c\right)^{3/2}+36\right.}{}$$

input
```
int(1/((a + c*x^2)^2*(d + e*x)),x)
```

output
```
(e/(2*(a*e^2 + c*d^2)) + (c*d*x)/(2*a*(a*e^2 + c*d^2)))/(a + c*x^2) + (e^3
*log(d + e*x))/(a*e^2 + c*d^2)^2 + (log(36*a^7*e^10*(-a^3*c)^(1/2) + a^3*c
^6*d^10*x + a^2*c^5*d^10*(-a^3*c)^(1/2) - 81*a^3*d^2*e^8*(-a^3*c)^(3/2) -
8*c^3*d^8*e^2*(-a^3*c)^(3/2) + 36*a^8*c*e^10*x + 8*a^4*c^5*d^8*e^2*x + 22*
a^5*c^4*d^6*e^4*x + 60*a^6*c^3*d^4*e^6*x + 81*a^7*c^2*d^2*e^8*x - 22*a*c^2
*d^6*e^4*(-a^3*c)^(3/2) - 60*a^2*c*d^4*e^6*(-a^3*c)^(3/2))*(c*d^3*(-a^3*c)
^(1/2) - 2*a^3*e^3 + 3*a*d*e^2*(-a^3*c)^(1/2)))/(4*(a^5*e^4 + a^3*c^2*d^4
+ 2*a^4*c*d^2*e^2)) - (log(a^3*c^6*d^10*x - 36*a^7*e^10*(-a^3*c)^(1/2) - a
^2*c^5*d^10*(-a^3*c)^(1/2) + 81*a^3*d^2*e^8*(-a^3*c)^(3/2) + 8*c^3*d^8*e^2
*(-a^3*c)^(3/2) + 36*a^8*c*e^10*x + 8*a^4*c^5*d^8*e^2*x + 22*a^5*c^4*d^6*e
^4*x + 60*a^6*c^3*d^4*e^6*x + 81*a^7*c^2*d^2*e^8*x + 22*a*c^2*d^6*e^4*(-a^
3*c)^(3/2) + 60*a^2*c*d^4*e^6*(-a^3*c)^(3/2))*(2*a^3*e^3 + c*d^3*(-a^3*c)^
(1/2) + 3*a*d*e^2*(-a^3*c)^(1/2)))/(4*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2
*e^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.00

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^2}\,dx$$

$$= \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2d\,e^2 + \sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,d^3 + 3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)acd\,e^2x^2 + \sqrt{c}\,\sqrt{a}\,atan\Big(}{2a^2\,(a^2c\,e^4}$$

input
```
int(1/(e*x+d)/(c*x^2+a)^2,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*d*e**2 + sqrt(c)*sqr
t(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d**3 + 3*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a*c*d*e**2*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*c**2*d**3*x**2 - log(a + c*x**2)*a**3*e**3 - log(a + c*x**2)*a**
2*c*e**3*x**2 + 2*log(d + e*x)*a**3*e**3 + 2*log(d + e*x)*a**2*c*e**3*x**2
 + a**2*c*d*e**2*x - a**2*c*e**3*x**2 + a*c**2*d**3*x - a*c**2*d**2*e*x**2
)/(2*a**2*(a**3*e**4 + 2*a**2*c*d**2*e**2 + a**2*c*e**4*x**2 + a*c**2*d**4
 + 2*a*c**2*d**2*e**2*x**2 + c**3*d**4*x**2))
```

3.115 $\int \frac{1}{(d+ex)^2(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 196

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = -\frac{e^3}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{c(2ade+\left(cd^2-ae^2\right)x)}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\sqrt{c}(c^2d^4+6acd^2e^2-3a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)^3} + \frac{4cde^3\log(d+ex)}{\left(cd^2+ae^2\right)^3}-\frac{2cde^3\log\left(a+cx^2\right)}{\left(cd^2+ae^2\right)^3}$$

output
```
-e^3/(a*e^2+c*d^2)^2/(e*x+d)+1/2*c*(2*a*d*e+(-a*e^2+c*d^2)*x)/a/(a*e^2+c*d
^2)^2/(c*x^2+a)+1/2*c^(1/2)*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)*arctan(c^(1
/2)*x/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^3+4*c*d*e^3*ln(e*x+d)/(a*e^2+c*d^2)^3
-2*c*d*e^3*ln(c*x^2+a)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^2} \, dx = \frac{-\frac{2e^3(cd^2+ae^2)}{d+ex} + \frac{c(cd^2+ae^2)(cd^2x+ae(2d-ex))}{a(a+cx^2)} + \frac{\sqrt{c}(c^2d^4+6acd^2e^2-3a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}} + 8cde^3\log(d+ex) - 4cde^3\log}{2(cd^2+ae^2)^3}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^2)^2),x]
```

output
```
((-2*e^3*(c*d^2 + a*e^2))/(d + e*x) + (c*(c*d^2 + a*e^2)*(c*d^2*x + a*e*(2
*d - e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^
4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 8*c*d*e^3*Log[d + e*x] - 4*c*d*e
^3*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^2 (d+ex)^2} \, dx$$

$$\Big\downarrow \; 496$$

$$\frac{ae+cdx}{2a(a+cx^2)(d+ex)(ae^2+cd^2)} - \frac{\int -\frac{cd^2+2cexd+3ae^2}{(d+ex)^2(cx^2+a)}dx}{2a(ae^2+cd^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{cd^2+2cexd+3ae^2}{(d+ex)^2(cx^2+a)}dx}{2a(ae^2+cd^2)} + \frac{ae+cdx}{2a(a+cx^2)(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{8acde^4}{(cd^2+ae^2)^2(d+ex)}+\frac{(3ae^2-cd^2)e^2}{(cd^2+ae^2)(d+ex)^2}+\frac{c(c^2d^4+6ace^2d^2-8ace^3xd-3a^2e^4)}{(cd^2+ae^2)^2(cx^2+a)}\right)dx}{2a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{2a\left(a+cx^2\right)\left(d+ex\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(-3a^2e^4+6acd^2e^2+c^2d^4)}{\sqrt{a}(ae^2+cd^2)^2}+\frac{e(cd^2-3ae^2)}{(d+ex)(ae^2+cd^2)}-\frac{4acde^3\log(a+cx^2)}{(ae^2+cd^2)^2}+\frac{8acde^3\log(d+ex)}{(ae^2+cd^2)^2}}{2a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{2a\left(a+cx^2\right)\left(d+ex\right)\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^2)^2),x]
```

output
```
(a*e + c*d*x)/(2*a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)) + ((e*(c*d^2 - 3
*a*e^2))/((c*d^2 + a*e^2)*(d + e*x)) + (Sqrt[c]*(c^2*d^4 + 6*a*c*d^2*e^2 -
 3*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) + (8*
a*c*d*e^3*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (4*a*c*d*e^3*Log[a + c*x^2])/(
c*d^2 + a*e^2)^2)/(2*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.92

method	result
default	$-\frac{e^{3}}{\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(ex+d\right)}+\frac{4cd\,e^{3}\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{3}}-\frac{c\left(\frac{\frac{\left(a^{2}e^{4}-c^{2}d^{4}\right)x}{2a}-de\left(a\,e^{2}+c\,d^{2}\right)}{c\,x^{2}+a}+\frac{4d\,e^{3}a\ln\left(c\,x^{2}+a\right)+\frac{\left(3a^{2}e^{4}-6ac\,d^{2}e^{2}-c^{2}d^{4}\right)\arctan}{\sqrt{ac}}}{2a}\right.}{\left(a\,e^{2}+c\,d^{2}\right)^{3}}$
risch	$\frac{-\frac{ec\left(3a\,e^{2}-c\,d^{2}\right)x^{2}}{2\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}\right)a}+\frac{cdx}{2a\left(a\,e^{2}+c\,d^{2}\right)}-\frac{e\left(a\,e^{2}-c\,d^{2}\right)}{a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}}{\left(ex+d\right)\left(c\,x^{2}+a\right)}+\frac{4d\,e^{3}c\ln\left(ex+d\right)}{e^{6}a^{3}+3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}+d^{6}c^{3}}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(a^{6}e^{6}+3\right.\right.\right.}{}$

input
```
int(1/(e*x+d)^2/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-e^3/(a*e^2+c*d^2)^2/(e*x+d)+4*c*d*e^3*ln(e*x+d)/(a*e^2+c*d^2)^3-c/(a*e^2+
c*d^2)^3*((1/2*(a^2*e^4-c^2*d^4)/a*x-d*e*(a*e^2+c*d^2))/(c*x^2+a)+1/2/a*(4
*d*e^3*a*ln(c*x^2+a)+(3*a^2*e^4-6*a*c*d^2*e^2-c^2*d^4)/(a*c)^(1/2)*arctan(
c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(184) = 368.

Time = 1.21 (sec) , antiderivative size = 1111, normalized size of antiderivative = 5.67

$$\int \frac{1}{(d+ex)^2\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*c^2*d^4*e - 4*a^3*e^5 + 2*(c^3*d^4*e - 2*a*c^2*d^2*e^3 - 3*a^2*c
*e^5)*x^2 - (a*c^2*d^5 + 6*a^2*c*d^3*e^2 - 3*a^3*d*e^4 + (c^3*d^4*e + 6*a*
c^2*d^2*e^3 - 3*a^2*c*e^5)*x^3 + (c^3*d^5 + 6*a*c^2*d^3*e^2 - 3*a^2*c*d*e^
4)*x^2 + (a*c^2*d^4*e + 6*a^2*c*d^2*e^3 - 3*a^3*e^5)*x)*sqrt(-c/a)*log((c*
x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(c^3*d^5 + 2*a*c^2*d^3*e^2 +
a^2*c*d*e^4)*x - 8*(a*c^2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 + a^2*c*d*e^4*x +
a^2*c*d^2*e^3)*log(c*x^2 + a) + 16*(a*c^2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 +
a^2*c*d*e^4*x + a^2*c*d^2*e^3)*log(e*x + d))/(a^2*c^3*d^7 + 3*a^3*c^2*d^5*
e^2 + 3*a^4*c*d^3*e^4 + a^5*d*e^6 + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a
^3*c^2*d^2*e^5 + a^4*c*e^7)*x^3 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c
^2*d^3*e^4 + a^4*c*d*e^6)*x^2 + (a^2*c^3*d^6*e + 3*a^3*c^2*d^4*e^3 + 3*a^4
*c*d^2*e^5 + a^5*e^7)*x), 1/2*(2*a*c^2*d^4*e - 2*a^3*e^5 + (c^3*d^4*e - 2*
a*c^2*d^2*e^3 - 3*a^2*c*e^5)*x^2 + (a*c^2*d^5 + 6*a^2*c*d^3*e^2 - 3*a^3*d*
e^4 + (c^3*d^4*e + 6*a*c^2*d^2*e^3 - 3*a^2*c*e^5)*x^3 + (c^3*d^5 + 6*a*c^2
*d^3*e^2 - 3*a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + 6*a^2*c*d^2*e^3 - 3*a^3*e^5
)*x)*sqrt(c/a)*arctan(x*sqrt(c/a)) + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*
e^4)*x - 4*(a*c^2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 + a^2*c*d*e^4*x + a^2*c*d^
2*e^3)*log(c*x^2 + a) + 8*(a*c^2*d*e^4*x^3 + a*c^2*d^2*e^3*x^2 + a^2*c*d*e
^4*x + a^2*c*d^2*e^3)*log(e*x + d))/(a^2*c^3*d^7 + 3*a^3*c^2*d^5*e^2 + 3*a
^4*c*d^3*e^4 + a^5*d*e^6 + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a^3*c^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(184) = 368$.

Time = 0.11 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.01

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^2} dx = -\frac{2\,cde^3 \log(cx^2+a)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{4\,cde^3 \log(ex+d)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{(c^3d^4+6\,ac^2d^2e^2-3\,a^2ce^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6)\sqrt{ac}} + \frac{2\,acd^2e-2\,a^2e^3+(c^2d^2e-3\,ace^3)x^2+(c^2d^3+acde^2)x}{2\,(a^2c^2d^5+2\,a^3cd^3e^2+a^4de^4+(ac^3d^4e+2\,a^2c^2d^2e^3+a^3ce^5)x^3+(ac^3d^5+2\,a^2c^2d^3e^2+a^3cde^4)x^2+}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-2*c*d*e^3*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a
^3*e^6) + 4*c*d*e^3*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + a^3*e^6) + 1/2*(c^3*d^4 + 6*a*c^2*d^2*e^2 - 3*a^2*c*e^4)*arctan(c*x/
sqrt(a*c))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sq
rt(a*c)) + 1/2*(2*a*c*d^2*e - 2*a^2*e^3 + (c^2*d^2*e - 3*a*c*e^3)*x^2 + (c
^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^5 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3
*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e
^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(184) = 368$.

Time = 0.13 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.05

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = -\frac{e^7}{(c^2d^4e^4+2\,acd^2e^6+a^2e^8)(ex+d)} - \frac{2\,cde^3\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{(c^3d^4e^2+6\,ac^2d^2e^4-3\,a^2ce^6)\arctan\left(\frac{cd-\frac{cd^2}{ex+d}-\frac{ae^2}{ex+d}}{\sqrt{ac}e}\right)}{2\,(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6)\sqrt{ac}e^2} + \frac{\frac{c^3d^3e-3\,ac^2de^3}{cd^2+ae^2}-\frac{c^3d^4e^2-6\,ac^2d^2e^4+a^2ce^6}{(cd^2+ae^2)(ex+d)e}}{2\,(cd^2+ae^2)^2a\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-e^7/((c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8)*(e*x + d)) - 2*c*d*e^3*log(c
 - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^3*d^6 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + 1/2*(c^3*d^4*e^2 + 6*a*c^2*d^2
*e^4 - 3*a^2*c*e^6)*arctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt
(a*c)*e))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqr
t(a*c)*e^2) + 1/2*((c^3*d^3*e - 3*a*c^2*d*e^3)/(c*d^2 + a*e^2) - (c^3*d^4*
e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/((c*d^2 + a*e^2)*(e*x + d)*e))/((c*d^2
+ a*e^2)^2*a*(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)
)
```

Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 819, normalized size of antiderivative = 4.18

$$\int \frac{1}{(d+ex)^2(a+cx^2)^2}\,dx = \frac{4\,c\,d\,e^3\,\ln(d+e\,x)}{(c\,d^2+a\,e^2)^3}$$
$$-\frac{\ln\left(c^5\,d^{12}\,(-a^3\,c)^{3/2}-9\,a^9\,e^{12}\,\sqrt{-a^3\,c}+a^4\,c^7\,d^{12}\,x-1119\,a\,d^4\,e^8\,(-a^3\,c)^{5/2}-612\,c\,d^6\,e^6\,(-a^3\,c)^{5/2}+\right.}{}$$
$$-\frac{\ln\left(9\,a^9\,e^{12}\,\sqrt{-a^3\,c}-c^5\,d^{12}\,(-a^3\,c)^{3/2}+a^4\,c^7\,d^{12}\,x+1119\,a\,d^4\,e^8\,(-a^3\,c)^{5/2}+612\,c\,d^6\,e^6\,(-a^3\,c)^{5/2}-\right.}{}$$
$$-\frac{\frac{a\,e^3-c\,d^2\,e}{(c\,d^2+a\,e^2)^2}-\frac{c\,d\,x}{2\,a\,(c\,d^2+a\,e^2)}+\frac{c\,x^2\,(3\,a\,e^3-c\,d^2\,e)}{2\,a\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}}{c\,e\,x^3+c\,d\,x^2+a\,e\,x+a\,d}$$

input
```
int(1/((a + c*x^2)^2*(d + e*x)^2),x)
```

output
```
(4*c*d*e^3*log(d + e*x))/(a*e^2 + c*d^2)^3 - (log(c^5*d^12*(-a^3*c)^(3/2)
- 9*a^9*e^12*(-a^3*c)^(1/2) + a^4*c^7*d^12*x - 1119*a*d^4*e^8*(-a^3*c)^(5/
2) - 612*c*d^6*e^6*(-a^3*c)^(5/2) + 558*a^5*d^2*e^10*(-a^3*c)^(3/2) + 9*a^
10*c*e^12*x + 55*a^2*c^3*d^8*e^4*(-a^3*c)^(3/2) + 14*a^5*c^6*d^10*e^2*x +
55*a^6*c^5*d^8*e^4*x + 612*a^7*c^4*d^6*e^6*x + 1119*a^8*c^3*d^4*e^8*x + 55
8*a^9*c^2*d^2*e^10*x + 14*a*c^4*d^10*e^2*(-a^3*c)^(3/2))*(c*(2*a^3*d*e^3 +
 (3*a*d^2*e^2*(-a^3*c)^(1/2))/2) - (3*a^2*e^4*(-a^3*c)^(1/2))/4 + (c^2*d^4
*(-a^3*c)^(1/2))/4))/(a^6*e^6 + a^3*c^3*d^6 + 3*a^5*c*d^2*e^4 + 3*a^4*c^2*
d^4*e^2) - (log(9*a^9*e^12*(-a^3*c)^(1/2) - c^5*d^12*(-a^3*c)^(3/2) + a^4*
c^7*d^12*x + 1119*a*d^4*e^8*(-a^3*c)^(5/2) + 612*c*d^6*e^6*(-a^3*c)^(5/2)
- 558*a^5*d^2*e^10*(-a^3*c)^(3/2) + 9*a^10*c*e^12*x - 55*a^2*c^3*d^8*e^4*(
-a^3*c)^(3/2) + 14*a^5*c^6*d^10*e^2*x + 55*a^6*c^5*d^8*e^4*x + 612*a^7*c^4
*d^6*e^6*x + 1119*a^8*c^3*d^4*e^8*x + 558*a^9*c^2*d^2*e^10*x - 14*a*c^4*d^
10*e^2*(-a^3*c)^(3/2))*(c*(2*a^3*d*e^3 - (3*a*d^2*e^2*(-a^3*c)^(1/2))/2) +
 (3*a^2*e^4*(-a^3*c)^(1/2))/4 - (c^2*d^4*(-a^3*c)^(1/2))/4))/(a^6*e^6 + a^
3*c^3*d^6 + 3*a^5*c*d^2*e^4 + 3*a^4*c^2*d^4*e^2) - ((a*e^3 - c*d^2*e)/(a*e
^2 + c*d^2)^2 - (c*d*x)/(2*a*(a*e^2 + c*d^2)) + (c*x^2*(3*a*e^3 - c*d^2*e)
)/(2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e*x + c*d*x^2 + c*e*
x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 837, normalized size of antiderivative = 4.27

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^2} \, dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^2/(c*x^2+a)^2,x)
```

output

```
( - 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*d**2*e**4 - 3*sqr
t(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*d*e**5*x + 6*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d**4*e**2 + 6*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d**3*e**3*x - 3*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**2*c*d**2*e**4*x**2 - 3*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a**2*c*d*e**5*x**3 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*a*c**2*d**6 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*
c**2*d**5*e*x + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**
4*e**2*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**3*
e**3*x**3 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**3*d**6*x**2 +
 sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**3*d**5*e*x**3 - 4*log(a
+ c*x**2)*a**3*c*d**3*e**3 - 4*log(a + c*x**2)*a**3*c*d**2*e**4*x - 4*log(
a + c*x**2)*a**2*c**2*d**3*e**3*x**2 - 4*log(a + c*x**2)*a**2*c**2*d**2*e*
*4*x**3 + 8*log(d + e*x)*a**3*c*d**3*e**3 + 8*log(d + e*x)*a**3*c*d**2*e**
4*x + 8*log(d + e*x)*a**2*c**2*d**3*e**3*x**2 + 8*log(d + e*x)*a**2*c**2*d
**2*e**4*x**3 + a**4*d*e**5 + 3*a**4*e**6*x + 2*a**3*c*d**3*e**3 + 3*a**3*
c*d**2*e**4*x + 3*a**3*c*e**6*x**3 + a**2*c**2*d**5*e + a**2*c**2*d**4*e**
2*x + 2*a**2*c**2*d**2*e**4*x**3 + a*c**3*d**6*x - a*c**3*d**4*e**2*x**3)/
(2*a**2*d*(a**4*d*e**6 + a**4*e**7*x + 3*a**3*c*d**3*e**4 + 3*a**3*c*d**2*
e**5*x + a**3*c*d*e**6*x**2 + a**3*c*e**7*x**3 + 3*a**2*c**2*d**5*e**2 ...
```

3.116 $\int \frac{(d+ex)^5}{(a+cx^2)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 225

$$\int \frac{(d+ex)^5}{(a+cx^2)^3} dx = -\frac{ae(5c^2d^4-10acd^2e^2+a^2e^4)-cd(c^2d^4-10acd^2e^2+5a^2e^4)x}{4ac^3(a+cx^2)^2} - \frac{8a^2e^3(5cd^2-ae^2)-cd(3cd^2-5ae^2)(cd^2+5ae^2)x}{8a^2c^3(a+cx^2)} + \frac{d(3c^2d^4+10acd^2e^2+15a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}} + \frac{e^5\log(a+cx^2)}{2c^3}$$

output
```
-1/4*(a*e*(a^2*e^4-10*a*c*d^2*e^2+5*c^2*d^4)-c*d*(5*a^2*e^4-10*a*c*d^2*e^2
+c^2*d^4)*x)/a/c^3/(c*x^2+a)^2-1/8*(8*a^2*e^3*(-a*e^2+5*c*d^2)-c*d*(-5*a*e
^2+3*c*d^2)*(5*a*e^2+c*d^2)*x)/a^2/c^3/(c*x^2+a)+1/8*d*(15*a^2*e^4+10*a*c*
d^2*e^2+3*c^2*d^4)*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/c^(5/2)+1/2*e^5*ln(c*
x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx$$

$$= \frac{-\frac{2(a^3e^5-c^3d^5x-5a^2cde^3(2d+ex)+5ac^2d^3e(d+2ex))}{a(a+cx^2)^2}+\frac{8a^3e^5+3c^3d^5x+10ac^2d^3e^2x-5a^2cde^3(8d+5ex)}{a^2(a+cx^2)}+\frac{\sqrt{c}d(3c^2d^4+10acd^2e^2+15a^2e^4}{a^{5/2}}}{8c^3}$$

input
```
Integrate[(d + e*x)^5/(a + c*x^2)^3,x]
```

output
```
((-2*(a^3*e^5 - c^3*d^5*x - 5*a^2*c*d*e^3*(2*d + e*x) + 5*a*c^2*d^3*e*(d +
 2*e*x)))/(a*(a + c*x^2)^2) + (8*a^3*e^5 + 3*c^3*d^5*x + 10*a*c^2*d^3*e^2*
x - 5*a^2*c*d*e^3*(8*d + 5*e*x))/(a^2*(a + c*x^2)) + (Sqrt[c]*d*(3*c^2*d^4
 + 10*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*e
^5*Log[a + c*x^2])/(8*c^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {495, 684, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx$$

$$\Big\downarrow \ 495$$

$$\frac{\int \frac{(d+ex)^3(3cd^2-cexd+4ae^2)}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^4(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 684$$

$$\frac{\frac{\int \frac{(d+ex)\left(3c^2d^4+7ace^2d^2-ce\left(3cd^2+7ae^2\right)xd+8a^2e^4\right)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^2\left(2ae\left(2ae^2+cd^2\right)-cdx\left(5ae^2+3cd^2\right)\right)}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^4(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{ 657}$$

$$\frac{\frac{\int \left(-7ade^4-3cd^3e^2+\frac{3c^2d^5+10ace^2d^3+15a^2e^4d+8a^2e^5x}{cx^2+a}\right)dx}{2ac}-\frac{(d+ex)^2\left(2ae\left(2ae^2+cd^2\right)-cdx\left(5ae^2+3cd^2\right)\right)}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^4(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\frac{d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)}{\sqrt{a}\sqrt{c}}+\frac{4a^2e^5\log\left(a+cx^2\right)}{c}-de^2x\left(7ae^2+3cd^2\right)}{2ac}-\frac{(d+ex)^2\left(2ae\left(2ae^2+cd^2\right)-cdx\left(5ae^2+3cd^2\right)\right)}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^4(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

input
```
Int[(d + e*x)^5/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*e - c*d*x)*(d + e*x)^4)/(a*c*(a + c*x^2)^2) + (-1/2*((d + e*x)^2*
(2*a*e*(c*d^2 + 2*a*e^2) - c*d*(3*c*d^2 + 5*a*e^2)*x))/(a*c*(a + c*x^2)) +
 (-(d*e^2*(3*c*d^2 + 7*a*e^2)*x) + (d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2
*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (4*a^2*e^5*Log[a +
c*x^2])/c)/(2*a*c))/(4*a*c)
```

Defintions of rubi rules used

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

method	result
default	$\frac{-\frac{d\left(25a^2e^4-10ac\,d^2e^2-3c^2d^4\right)x^3}{8a^2c}+\frac{e^3\left(a\,e^2-5c\,d^2\right)x^2}{c^2}-\frac{5d\left(3a^2e^4+2ac\,d^2e^2-c^2d^4\right)x}{8a\,c^2}+\frac{e\left(3a^2e^4-10ac\,d^2e^2-5c^2d^4\right)}{4c^3}}{\left(c\,x^2+a\right)^2}+\frac{\frac{4a^2e^5\ln\left(c\,x^2+a\right)}{c}}{}$
risch	$\frac{-\frac{d\left(25a^2e^4-10ac\,d^2e^2-3c^2d^4\right)x^3}{8a^2c}+\frac{e^3\left(a\,e^2-5c\,d^2\right)x^2}{c^2}-\frac{5d\left(3a^2e^4+2ac\,d^2e^2-c^2d^4\right)x}{8a\,c^2}+\frac{e\left(3a^2e^4-10ac\,d^2e^2-5c^2d^4\right)}{4c^3}}{\left(c\,x^2+a\right)^2}+\frac{\ln\left(15d\,e^4a^3+10\right.}{}$

input
```
int((e*x+d)^5/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*d*(25*a^2*e^4-10*a*c*d^2*e^2-3*c^2*d^4)/a^2/c*x^3+e^3*(a*e^2-5*c*d^2
)/c^2*x^2-5/8*d*(3*a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/a/c^2*x+1/4*e*(3*a^2*e^4
-10*a*c*d^2*e^2-5*c^2*d^4)/c^3)/(c*x^2+a)^2+1/8/a^2/c^2*(4*a^2*e^5/c*ln(c*
x^2+a)+(15*a^2*d*e^4+10*a*c*d^3*e^2+3*c^2*d^5)/(a*c)^(1/2)*arctan(c*x/(a*c
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.14

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{20\,a^3c^2d^4e + 40\,a^4cd^2e^3 - 12\,a^5e^5 - 2\,(3\,ac^4d^5 + 10\,a^2c^3d^3e^2 - 25\,a^3c^2de^4)x^3 + 16\,(5\,a^3c^2d^2e^3 - a^4ce\ldots}{\ldots}\right.$$
$$-\frac{10\,a^3c^2d^4e + 20\,a^4cd^2e^3 - 6\,a^5e^5 - (3\,ac^4d^5 + 10\,a^2c^3d^3e^2 - 25\,a^3c^2de^4)x^3 + 8\,(5\,a^3c^2d^2e^3 - a^4ce^5)x^2\ldots}{\ldots}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(20*a^3*c^2*d^4*e + 40*a^4*c*d^2*e^3 - 12*a^5*e^5 - 2*(3*a*c^4*d^5
+ 10*a^2*c^3*d^3*e^2 - 25*a^3*c^2*d*e^4)*x^3 + 16*(5*a^3*c^2*d^2*e^3 - a^4
*c*e^5)*x^2 + (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 + 15*a^4*d*e^4 + (3*c^4*d^
5 + 10*a*c^3*d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*(3*a*c^3*d^5 + 10*a^2*c^2
*d^3*e^2 + 15*a^3*c*d*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a
)/(c*x^2 + a)) - 10*(a^2*c^3*d^5 - 2*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4)*x -
8*(a^3*c^2*e^5*x^4 + 2*a^4*c*e^5*x^2 + a^5*e^5)*log(c*x^2 + a))/(a^3*c^5*x
^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(10*a^3*c^2*d^4*e + 20*a^4*c*d^2*e^3 -
 6*a^5*e^5 - (3*a*c^4*d^5 + 10*a^2*c^3*d^3*e^2 - 25*a^3*c^2*d*e^4)*x^3 + 8
*(5*a^3*c^2*d^2*e^3 - a^4*c*e^5)*x^2 - (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 +
 15*a^4*d*e^4 + (3*c^4*d^5 + 10*a*c^3*d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*
(3*a*c^3*d^5 + 10*a^2*c^2*d^3*e^2 + 15*a^3*c*d*e^4)*x^2)*sqrt(a*c)*arctan(
sqrt(a*c)*x/a) - 5*(a^2*c^3*d^5 - 2*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4)*x - 4
*(a^3*c^2*e^5*x^4 + 2*a^4*c*e^5*x^2 + a^5*e^5)*log(c*x^2 + a))/(a^3*c^5*x^
4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(216) = 432$.

Time = 1.71 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.31

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx = \left(\frac{e^5}{2c^3} - \frac{d\sqrt{-a^5c^7}\cdot(15a^2e^4+10acd^2e^2+3c^2d^4)}{16a^5c^6}\right)\log\left(x + \frac{16a^3c^3\left(\frac{e^5}{2c^3} - \frac{d\sqrt{-a^5c^7}\cdot(15a^2e^4+10acd^2e^2+3c^2d^4)}{16a^5c^6}\right) - 8a^3\cdots}{15a^2cde^4+10ac^2d^3e^2+3c^3d^5}\right) + \left(\frac{e^5}{2c^3} + \frac{d\sqrt{-a^5c^7}\cdot(15a^2e^4+10acd^2e^2+3c^2d^4)}{16a^5c^6}\right)\log\left(x + \frac{16a^3c^3\left(\frac{e^5}{2c^3} + \frac{d\sqrt{-a^5c^7}\cdot(15a^2e^4+10acd^2e^2+3c^2d^4)}{16a^5c^6}\right) - 8a^3\cdots}{15a^2cde^4+10ac^2d^3e^2+3c^3d^5}\right) + \frac{6a^4e^5-20a^3cd^2e^3-10a^2c^2d^4e+x^3(-25a^2c^2de^4+10ac^3d^3e^2+3c^4d^5)+x^2\cdot(8a^3ce^5-40a^2c^2d^2e^3)+\cdots}{8a^4c^3+16a^3c^4x^2+8a^2c^5x^4}$$

input
```
integrate((e*x+d)**5/(c*x**2+a)**3,x)
```

output
```
(e**5/(2*c**3) - d*sqrt(-a**5*c**7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c
**2*d**4)/(16*a**5*c**6))*log(x + (16*a**3*c**3*(e**5/(2*c**3) - d*sqrt(-a
**5*c**7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c**2*d**4)/(16*a**5*c**6))
- 8*a**3*e**5)/(15*a**2*c*d*e**4 + 10*a*c**2*d**3*e**2 + 3*c**3*d**5)) + (
e**5/(2*c**3) + d*sqrt(-a**5*c**7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c*
*2*d**4)/(16*a**5*c**6))*log(x + (16*a**3*c**3*(e**5/(2*c**3) + d*sqrt(-a*
*5*c**7)*(15*a**2*e**4 + 10*a*c*d**2*e**2 + 3*c**2*d**4)/(16*a**5*c**6)) -
 8*a**3*e**5)/(15*a**2*c*d*e**4 + 10*a*c**2*d**3*e**2 + 3*c**3*d**5)) + (6
*a**4*e**5 - 20*a**3*c*d**2*e**3 - 10*a**2*c**2*d**4*e + x**3*(-25*a**2*c*
*2*d*e**4 + 10*a*c**3*d**3*e**2 + 3*c**4*d**5) + x**2*(8*a**3*c*e**5 - 40*
a**2*c**2*d**2*e**3) + x*(-15*a**3*c*d*e**4 - 10*a**2*c**2*d**3*e**2 + 5*a
*c**3*d**5))/(8*a**4*c**3 + 16*a**3*c**4*x**2 + 8*a**2*c**5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx = \frac{e^5\log(cx^2+a)}{2\,c^3} - \frac{10\,a^2c^2d^4e + 20\,a^3cd^2e^3 - 6\,a^4e^5 - (3\,c^4d^5 + 10\,ac^3d^3e^2 - 25\,a^2c^2de^4)x^3 + 8\,(5\,a^2c^2d^2e^3 - a^3ce^5)x^2 - 5}{8\,(a^2c^5x^4 + 2\,a^3c^4x^2 + a^4c^3)} + \frac{(3\,c^2d^5 + 10\,acd^3e^2 + 15\,a^2de^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*e^5*log(c*x^2 + a)/c^3 - 1/8*(10*a^2*c^2*d^4*e + 20*a^3*c*d^2*e^3 - 6*
a^4*e^5 - (3*c^4*d^5 + 10*a*c^3*d^3*e^2 - 25*a^2*c^2*d*e^4)*x^3 + 8*(5*a^2
*c^2*d^2*e^3 - a^3*c*e^5)*x^2 - 5*(a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 - 3*a^3*c
*d*e^4)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*c^3) + 1/8*(3*c^2*d^5 + 10*a
*c*d^3*e^2 + 15*a^2*d*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx = \frac{e^5\log(cx^2+a)}{2\,c^3} + \frac{(3\,c^2d^5 + 10\,acd^3e^2 + 15\,a^2de^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2} + \frac{(3\,c^3d^5 + 10\,ac^2d^3e^2 - 25\,a^2cde^4)x^3 - 8\,(5\,a^2cd^2e^3 - a^3e^5)x^2 + 5\,(ac^2d^5 - 2\,a^2cd^3e^2 - 3\,a^3de^4)x - \frac{2\,(5}{}}{8\,(cx^2+a)^2a^2c^2}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*e^5*log(c*x^2 + a)/c^3 + 1/8*(3*c^2*d^5 + 10*a*c*d^3*e^2 + 15*a^2*d*e^
4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*((3*c^3*d^5 + 10*a*c^2*
d^3*e^2 - 25*a^2*c*d*e^4)*x^3 - 8*(5*a^2*c*d^2*e^3 - a^3*e^5)*x^2 + 5*(a*c
^2*d^5 - 2*a^2*c*d^3*e^2 - 3*a^3*d*e^4)*x - 2*(5*a^2*c^2*d^4*e + 10*a^3*c*
d^2*e^3 - 3*a^4*e^5)/c)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.08

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx = \frac{e^5\ln(c\,x^2+a)}{2\,c^3} - \frac{5\,d^4\,e}{4\,(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4)} + \frac{5\,d^5\,x}{8\,(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4)} + \frac{3\,a^2\,e^5}{4\,(a^2\,c^3+2\,a\,c^4\,x^2+c^5\,x^4)} - \frac{5\,a\,d^2\,e^3}{2\,(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4)} + \frac{a\,e^5\,x^2}{a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4} - \frac{5\,d^2\,e^3\,x^2}{a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4} + \frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}} + \frac{3\,c\,d^5\,x^3}{8\,(a^4+2\,a^3\,c\,x^2+a^2\,c^2\,x^4)} + \frac{5\,d^3\,e^2\,x^3}{4\,(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4)} - \frac{5\,d^3\,e^2\,x}{4\,(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4)} - \frac{25\,d\,e^4\,x^3}{8\,(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4)} + \frac{5\,d^3\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{4\,a^{3/2}\,c^{3/2}} - \frac{15\,a\,d\,e^4\,x}{8\,(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4)} + \frac{15\,d\,e^4\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,c^{5/2}}$$

input
```
int((d + e*x)^5/(a + c*x^2)^3,x)
```

output
```
(e^5*log(a + c*x^2))/(2*c^3) - (5*d^4*e)/(4*(a^2*c + c^3*x^4 + 2*a*c^2*x^2
)) + (5*d^5*x)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) + (3*a^2*e^5)/(4*(a^2*c
^3 + c^5*x^4 + 2*a*c^4*x^2)) - (5*a*d^2*e^3)/(2*(a^2*c^2 + c^4*x^4 + 2*a*c
^3*x^2)) + (a*e^5*x^2)/(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2) - (5*d^2*e^3*x^2)
/(a^2*c + c^3*x^4 + 2*a*c^2*x^2) + (3*d^5*atan((c^(1/2)*x)/a^(1/2)))/(8*a^
(5/2)*c^(1/2)) + (3*c*d^5*x^3)/(8*(a^4 + 2*a^3*c*x^2 + a^2*c^2*x^4)) + (5*
d^3*e^2*x^3)/(4*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (5*d^3*e^2*x)/(4*(a^2*c
 + c^3*x^4 + 2*a*c^2*x^2)) - (25*d*e^4*x^3)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*
x^2)) + (5*d^3*e^2*atan((c^(1/2)*x)/a^(1/2)))/(4*a^(3/2)*c^(3/2)) - (15*a*
d*e^4*x)/(8*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (15*d*e^4*atan((c^(1/2)*x
)/a^(1/2)))/(8*a^(1/2)*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex)^5}{(a+cx^2)^3}\,dx$$
$$= \frac{15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^4d\,e^4+10\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3c\,d^3e^2+30\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3cd\,e^4x^2+3\sqrt{c}}{}$$

input
```
int((e*x+d)^5/(c*x^2+a)^3,x)
```

output
```
(15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*d*e**4 + 10*sqrt(c)
*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d**3*e**2 + 30*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d*e**4*x**2 + 3*sqrt(c)*sqrt(a)*at
an((c*x)/(sqrt(c)*sqrt(a)))*a**2*c**2*d**5 + 20*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**2*c**2*d**3*e**2*x**2 + 15*sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a**2*c**2*d*e**4*x**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a*c**3*d**5*x**2 + 10*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a*c**3*d**3*e**2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*c**4*d**5*x**4 + 4*log(a + c*x**2)*a**5*e**5 + 8*log(a + c*x**
2)*a**4*c*e**5*x**2 + 4*log(a + c*x**2)*a**3*c**2*e**5*x**4 + 2*a**5*e**5
- 15*a**4*c*d*e**4*x - 10*a**3*c**2*d**4*e - 10*a**3*c**2*d**3*e**2*x - 25
*a**3*c**2*d*e**4*x**3 - 4*a**3*c**2*e**5*x**4 + 5*a**2*c**3*d**5*x + 10*a
**2*c**3*d**3*e**2*x**3 + 20*a**2*c**3*d**2*e**3*x**4 + 3*a*c**4*d**5*x**3
)/(8*a**3*c**3*(a**2 + 2*a*c*x**2 + c**2*x**4))
```

3.117 $\int \frac{(d+ex)^4}{(a+cx^2)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 165

$$\int \frac{(d+ex)^4}{(a+cx^2)^3} dx = -\frac{4ade(cd^2-ae^2)-(c^2d^4-6acd^2e^2+a^2e^4)\,x}{4ac^2\,(a+cx^2)^2} - \frac{16a^2de^3-(3c^2d^4+6acd^2e^2-5a^2e^4)\,x}{8a^2c^2\,(a+cx^2)} + \frac{3(cd^2+ae^2)^2\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

output

```
-1/4*(4*a*d*e*(-a*e^2+c*d^2)-(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*x)/a/c^2/(c*x
^2+a)^2-1/8*(16*a^2*d*e^3-(-5*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*x)/a^2/c^2/
(c*x^2+a)+3/8*(a*e^2+c*d^2)^2*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx = \frac{3c^3d^4x^3-a^3e^3(8d+3ex)+ac^2d^2x(5d^2+6e^2x^2)-a^2ce(8d^3+6d^2ex+16de^2x^2+5e^3x^3)}{8a^2c^2\left(a+cx^2\right)^2} + \frac{3\left(cd^2+ae^2\right)^2\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

input
```
Integrate[(d + e*x)^4/(a + c*x^2)^3,x]
```

output
```
(3*c^3*d^4*x^3 - a^3*e^3*(8*d + 3*e*x) + a*c^2*d^2*x*(5*d^2 + 6*e^2*x^2) -
 a^2*c*e*(8*d^3 + 6*d^2*e*x + 16*d*e^2*x^2 + 5*e^3*x^3))/(8*a^2*c^2*(a + c
*x^2)^2) + (3*(c*d^2 + a*e^2)^2*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^
(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {487, 487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx$$

$$\Big\downarrow \text{487}$$

$$\frac{3\left(ae^2+cd^2\right)\int \frac{(d+ex)^2}{(cx^2+a)^2}dx}{4ac} - \frac{(d+ex)^3(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{487}$$

$$\frac{3(ae^2+cd^2)\left(\frac{(ae^2+cd^2)\int\frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3(ae^2+cd^2)\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+cd^2)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac\,(a+cx^2)^2}$$

input
```
Int[(d + e*x)^4/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*e - c*d*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^2) + (3*(c*d^2 + a*e^2)*
(-1/2*((a*e - c*d*x)*(d + e*x))/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))))/(4*a*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99

method	result
default	$\frac{-\frac{\left(5a^2e^4-6acd^2e^2-3c^2d^4\right)x^3}{8a^2c}-\frac{2de^3x^2}{c}-\frac{\left(3a^2e^4+6acd^2e^2-5c^2d^4\right)x}{8ac^2}-\frac{de\left(ae^2+cd^2\right)}{c^2}}{\left(cx^2+a\right)^2}+\frac{3\left(a^2e^4+2acd^2e^2+c^2d^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c^2\sqrt{ac}}$
risch	$\frac{-\frac{\left(5a^2e^4-6acd^2e^2-3c^2d^4\right)x^3}{8a^2c}-\frac{2de^3x^2}{c}-\frac{\left(3a^2e^4+6acd^2e^2-5c^2d^4\right)x}{8ac^2}-\frac{de\left(ae^2+cd^2\right)}{c^2}}{\left(cx^2+a\right)^2}-\frac{3\ln\left(cx+\sqrt{-ac}\right)e^4}{16\sqrt{-ac}\,c^2}-\frac{3\ln\left(cx+\sqrt{-ac}\right)d^2e^2}{8\sqrt{-ac}\,ca}$

input
```
int((e*x+d)^4/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*(5*a^2*e^4-6*a*c*d^2*e^2-3*c^2*d^4)/a^2/c*x^3-2*d*e^3*x^2/c-1/8*(3*a
^2*e^4+6*a*c*d^2*e^2-5*c^2*d^4)/a/c^2*x-d*e*(a*e^2+c*d^2)/c^2)/(c*x^2+a)^2
+3/8*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/a^2/c^2/(a*c)^(1/2)*arctan(c*x/(a*c)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.36

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{32\,a^3c^2de^3x^2+16\,a^3c^2d^3e+16\,a^4cde^3-2\,(3\,ac^4d^4+6\,a^2c^3d^2e^2-5\,a^3c^2e^4)x^3+3\,(a^2c^2d^4+2\,a^3cd^2e^2}{}\right.$$
$$-\frac{16\,a^3c^2de^3x^2+8\,a^3c^2d^3e+8\,a^4cde^3-(3\,ac^4d^4+6\,a^2c^3d^2e^2-5\,a^3c^2e^4)x^3-3\,(a^2c^2d^4+2\,a^3cd^2e^2+a}{8}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(32*a^3*c^2*d*e^3*x^2 + 16*a^3*c^2*d^3*e + 16*a^4*c*d*e^3 - 2*(3*a*
c^4*d^4 + 6*a^2*c^3*d^2*e^2 - 5*a^3*c^2*e^4)*x^3 + 3*(a^2*c^2*d^4 + 2*a^3*
c*d^2*e^2 + a^4*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4 + 2*(a
*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*s
qrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(5*a^2*c^3*d^4 - 6*a^3*c^2*d^2*e^2 - 3*a
^4*c*e^4)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(16*a^3*c^2*d*e
^3*x^2 + 8*a^3*c^2*d^3*e + 8*a^4*c*d*e^3 - (3*a*c^4*d^4 + 6*a^2*c^3*d^2*e^
2 - 5*a^3*c^2*e^4)*x^3 - 3*(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4 + 2*(a*c^3*d^4 + 2*a^2*c^2*d^2*e
^2 + a^3*c*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (5*a^2*c^3*d^4 - 6*
a^3*c^2*d^2*e^2 - 3*a^4*c*e^4)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(151) = 302$.

Time = 1.06 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.99

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2\log\left(-\frac{3a^3c^2\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2}{3a^2e^4+6acd^2e^2+3c^2d^4}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2\log\left(\frac{3a^3c^2\sqrt{-\frac{1}{a^5c^5}}\left(ae^2+cd^2\right)^2}{3a^2e^4+6acd^2e^2+3c^2d^4}+x\right)}{16} + \frac{-8a^3de^3-8a^2cd^3e-16a^2cde^3x^2+x^3\left(-5a^2ce^4+6ac^2d^2e^2+3c^3d^4\right)+x\left(-3a^3e^4-6a^2cd^2e^2+5ac^2d^4\right)}{8a^4c^2+16a^3c^3x^2+8a^2c^4x^4}$$

input
```
integrate((e*x+d)**4/(c*x**2+a)**3,x)
```

output
```
-3*sqrt(-1/(a**5*c**5))*(a*e**2 + c*d**2)**2*log(-3*a**3*c**2*sqrt(-1/(a**
5*c**5))*(a*e**2 + c*d**2)**2/(3*a**2*e**4 + 6*a*c*d**2*e**2 + 3*c**2*d**4
) + x)/16 + 3*sqrt(-1/(a**5*c**5))*(a*e**2 + c*d**2)**2*log(3*a**3*c**2*sq
rt(-1/(a**5*c**5))*(a*e**2 + c*d**2)**2/(3*a**2*e**4 + 6*a*c*d**2*e**2 + 3
*c**2*d**4) + x)/16 + (-8*a**3*d*e**3 - 8*a**2*c*d**3*e - 16*a**2*c*d*e**3
*x**2 + x**3*(-5*a**2*c*e**4 + 6*a*c**2*d**2*e**2 + 3*c**3*d**4) + x*(-3*a
**3*e**4 - 6*a**2*c*d**2*e**2 + 5*a*c**2*d**4))/(8*a**4*c**2 + 16*a**3*c**
3*x**2 + 8*a**2*c**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx = -\frac{16\,a^2cde^3x^2 + 8\,a^2cd^3e + 8\,a^3de^3 - (3\,c^3d^4 + 6\,ac^2d^2e^2 - 5\,a^2ce^4)x^3 - (5\,ac^2d^4 - 6\,a^2cd^2e^2 - 3\,a^3e^4)x}{8\,(a^2c^4x^4 + 2\,a^3c^3x^2 + a^4c^2)} + \frac{3\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(16*a^2*c*d*e^3*x^2 + 8*a^2*c*d^3*e + 8*a^3*d*e^3 - (3*c^3*d^4 + 6*a*
c^2*d^2*e^2 - 5*a^2*c*e^4)*x^3 - (5*a*c^2*d^4 - 6*a^2*c*d^2*e^2 - 3*a^3*e^
4)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 3/8*(c^2*d^4 + 2*a*c*d^2*e
^2 + a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx = \frac{3\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2} + \frac{3\,c^3d^4x^3 + 6\,ac^2d^2e^2x^3 - 5\,a^2ce^4x^3 - 16\,a^2cde^3x^2 + 5\,ac^2d^4x - 6\,a^2cd^2e^2x - 3\,a^3e^4x - 8\,a^2cd^3e - 8\,a^3}{8\,(cx^2+a)^2a^2c^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a
^2*c^2) + 1/8*(3*c^3*d^4*x^3 + 6*a*c^2*d^2*e^2*x^3 - 5*a^2*c*e^4*x^3 - 16*
a^2*c*d*e^3*x^2 + 5*a*c^2*d^4*x - 6*a^2*c*d^2*e^2*x - 3*a^3*e^4*x - 8*a^2*
c*d^3*e - 8*a^3*d*e^3)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x\,\left(c\,d^2+a\,e^2\right)^2}{\sqrt{a}\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)^2}{8\,a^{5/2}\,c^{5/2}} - \frac{\frac{2\,d\,e^3\,x^2}{c}+\frac{x\,\left(3\,a^2\,e^4+6\,a\,c\,d^2\,e^2-5\,c^2\,d^4\right)}{8\,a\,c^2}+\frac{d\,e\,\left(c\,d^2+a\,e^2\right)}{c^2}-\frac{x^3\,\left(-5\,a^2\,e^4+6\,a\,c\,d^2\,e^2+3\,c^2\,d^4\right)}{8\,a^2\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

input
```
int((d + e*x)^4/(a + c*x^2)^3,x)
```

output
```
(3*atan((c^(1/2)*x*(a*e^2 + c*d^2)^2)/(a^(1/2)*(a^2*e^4 + c^2*d^4 + 2*a*c*
d^2*e^2)))*(a*e^2 + c*d^2)^2)/(8*a^(5/2)*c^(5/2)) - ((2*d*e^3*x^2)/c + (x*
(3*a^2*e^4 - 5*c^2*d^4 + 6*a*c*d^2*e^2))/(8*a*c^2) + (d*e*(a*e^2 + c*d^2))
/c^2 - (x^3*(3*c^2*d^4 - 5*a^2*e^4 + 6*a*c*d^2*e^2))/(8*a^2*c))/(a^2 + c^2
*x^4 + 2*a*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)^4}{(a+cx^2)^3}\,dx = \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^4e^4+6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3c\,d^2e^2+6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3c\,e^4x^2+3\sqrt{c}\,\sqrt{a}\,at}{}$$

input
```
int((e*x+d)^4/(c*x^2+a)^3,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*e**4 + 6*sqrt(c)*sqr
t(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d**2*e**2 + 6*sqrt(c)*sqrt(a)*at
an((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*e**4*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**2*c**2*d**4 + 12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(
c)*sqrt(a)))*a**2*c**2*d**2*e**2*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a**2*c**2*e**4*x**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*a*c**3*d**4*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)
))*a*c**3*d**2*e**2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))
*c**4*d**4*x**4 - 3*a**4*c*e**4*x - 8*a**3*c**2*d**3*e - 6*a**3*c**2*d**2*
e**2*x - 5*a**3*c**2*e**4*x**3 + 5*a**2*c**3*d**4*x + 6*a**2*c**3*d**2*e**
2*x**3 + 8*a**2*c**3*d*e**3*x**4 + 3*a*c**4*d**4*x**3)/(8*a**3*c**3*(a**2
+ 2*a*c*x**2 + c**2*x**4))
```

3.118 $\int \frac{(d+ex)^3}{(a+cx^2)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{(d+ex)^3}{(a+cx^2)^3} \, dx = -\frac{ae(3cd^2-ae^2)-cd(cd^2-3ae^2)\,x}{4ac^2\,(a+cx^2)^2} - \frac{4a^2e^3-3cd(cd^2+ae^2)\,x}{8a^2c^2\,(a+cx^2)} + \frac{3d(cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

output
```
-1/4*(a*e*(-a*e^2+3*c*d^2)-c*d*(-3*a*e^2+c*d^2)*x)/a/c^2/(c*x^2+a)^2-1/8*(
4*a^2*e^3-3*c*d*(a*e^2+c*d^2)*x)/a^2/c^2/(c*x^2+a)+3/8*d*(a*e^2+c*d^2)*arc
tan(c^(1/2)*x/a^(1/2))/a^(5/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3}{(a+cx^2)^3} \, dx = \frac{\frac{\sqrt{a}\left(-2a^3e^3+3c^3d^3x^3+ac^2dx(5d^2+3e^2x^2)-a^2ce(6d^2+3dex+4e^2x^2)\right)}{(a+cx^2)^2} + 3\sqrt{c}d(cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^2}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^3*e^3 + 3*c^3*d^3*x^3 + a*c^2*d*x*(5*d^2 + 3*e^2*x^2) - a^
2*c*e*(6*d^2 + 3*d*e*x + 4*e^2*x^2)))/(a + c*x^2)^2 + 3*Sqrt[c]*d*(c*d^2 +
 a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {490, 487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^2)^3} dx$$

$$\Big\downarrow \text{490}$$

$$\frac{3d\int \frac{(d+ex)^2}{(cx^2+a)^2}dx}{4a}+\frac{x(d+ex)^3}{4a(a+cx^2)^2}$$

$$\Big\downarrow \text{487}$$

$$\frac{3d\left(\frac{(ae^2+cd^2)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4a}+\frac{x(d+ex)^3}{4a(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3d\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+cd^2)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4a}+\frac{x(d+ex)^3}{4a(a+cx^2)^2}$$

input
```
Int[(d + e*x)^3/(a + c*x^2)^3,x]
```

output
```
(x*(d + e*x)^3)/(4*a*(a + c*x^2)^2) + (3*d*(-1/2*((a*e - c*d*x)*(d + e*x))
/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3
/2)*c^(3/2))))/(4*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 490
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] - Simp[c*(n/(2*a*(
p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{3d\left(a e^2+c d^2\right)x^3}{8a^2}-\frac{e^3x^2}{2c}-\frac{d\left(3a e^2-5c d^2\right)x}{8ac}-\frac{e\left(a e^2+3c d^2\right)}{4c^2}}{\left(c x^2+a\right)^2}+\frac{3d\left(a e^2+c d^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$
risch	$\frac{\frac{3d\left(a e^2+c d^2\right)x^3}{8a^2}-\frac{e^3x^2}{2c}-\frac{d\left(3a e^2-5c d^2\right)x}{8ac}-\frac{e\left(a e^2+3c d^2\right)}{4c^2}}{\left(c x^2+a\right)^2}-\frac{3d\ln\left(cx+\sqrt{-ac}\right)e^2}{16\sqrt{-ac}\,ca}-\frac{3d^3\ln\left(cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^2}+\frac{3d\ln\left(-cx+\sqrt{-ac}\right)e^2}{16\sqrt{-ac}\,ca}$

input
```
int((e*x+d)^3/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/8*d*(a*e^2+c*d^2)/a^2*x^3-1/2*e^3*x^2/c-1/8*d*(3*a*e^2-5*c*d^2)/a/c*x-1
/4*e*(a*e^2+3*c*d^2)/c^2)/(c*x^2+a)^2+3/8*d*(a*e^2+c*d^2)/a^2/c/(a*c)^(1/2
)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.96

$$\int \frac{(d+ex)^3}{(a+cx^2)^3}\,dx$$

$$= \left[-\frac{8\,a^3ce^3x^2 + 12\,a^3cd^2e + 4\,a^4e^3 - 6\,(ac^3d^3 + a^2c^2de^2)x^3 + 3\,(a^2cd^3 + a^3de^2 + (c^3d^3 + ac^2de^2)x^4 + 2\,(a}{16\,(a^3c^4x^4 + 2\,a^4c^3x^2 + a^5c^2)} \right.$$

$$-\frac{4\,a^3ce^3x^2 + 6\,a^3cd^2e + 2\,a^4e^3 - 3\,(ac^3d^3 + a^2c^2de^2)x^3 - 3\,(a^2cd^3 + a^3de^2 + (c^3d^3 + ac^2de^2)x^4 + 2\,(ac}{8\,(a^3c^4x^4 + 2\,a^4c^3x^2 + a^5c^2)}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*a^3*c*e^3*x^2 + 12*a^3*c*d^2*e + 4*a^4*e^3 - 6*(a*c^3*d^3 + a^2*
c^2*d*e^2)*x^3 + 3*(a^2*c*d^3 + a^3*d*e^2 + (c^3*d^3 + a*c^2*d*e^2)*x^4 +
2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x -
a)/(c*x^2 + a)) - 2*(5*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*x)/(a^3*c^4*x^4 + 2*a^
4*c^3*x^2 + a^5*c^2), -1/8*(4*a^3*c*e^3*x^2 + 6*a^3*c*d^2*e + 2*a^4*e^3 -
3*(a*c^3*d^3 + a^2*c^2*d*e^2)*x^3 - 3*(a^2*c*d^3 + a^3*d*e^2 + (c^3*d^3 +
a*c^2*d*e^2)*x^4 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(
a*c)*x/a) - (5*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^
2 + a^5*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(128) = 256.

Time = 0.64 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.99

$$
\begin{aligned}
&\int \frac{(d+ex)^3}{(a+cx^2)^3}\,dx \\
&= -\frac{3d\sqrt{-\frac{1}{a^5c^3}}(ae^2+cd^2)\log\left(-\frac{3a^3cd\sqrt{-\frac{1}{a^5c^3}}(ae^2+cd^2)}{3ade^2+3cd^3}+x\right)}{16} \\
&\quad+\frac{3d\sqrt{-\frac{1}{a^5c^3}}(ae^2+cd^2)\log\left(\frac{3a^3cd\sqrt{-\frac{1}{a^5c^3}}(ae^2+cd^2)}{3ade^2+3cd^3}+x\right)}{16} \\
&\quad+\frac{-2a^3e^3-6a^2cd^2e-4a^2ce^3x^2+x^3\cdot(3ac^2de^2+3c^3d^3)+x(-3a^2cde^2+5ac^2d^3)}{8a^4c^2+16a^3c^3x^2+8a^2c^4x^4}
\end{aligned}
$$

input
```
integrate((e*x+d)**3/(c*x**2+a)**3,x)
```

output
```
-3*d*sqrt(-1/(a**5*c**3))*(a*e**2 + c*d**2)*log(-3*a**3*c*d*sqrt(-1/(a**5*
c**3))*(a*e**2 + c*d**2)/(3*a*d*e**2 + 3*c*d**3) + x)/16 + 3*d*sqrt(-1/(a*
*5*c**3))*(a*e**2 + c*d**2)*log(3*a**3*c*d*sqrt(-1/(a**5*c**3))*(a*e**2 +
c*d**2)/(3*a*d*e**2 + 3*c*d**3) + x)/16 + (-2*a**3*e**3 - 6*a**2*c*d**2*e
- 4*a**2*c*e**3*x**2 + x**3*(3*a*c**2*d*e**2 + 3*c**3*d**3) + x*(-3*a**2*c
*d*e**2 + 5*a*c**2*d**3))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**2*c**4*x
**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

$$
\begin{aligned}
&\int \frac{(d+ex)^3}{(a+cx^2)^3}\,dx \\
&= -\frac{4\,a^2ce^3x^2+6\,a^2cd^2e+2\,a^3e^3-3\,(c^3d^3+ac^2de^2)x^3-(5\,ac^2d^3-3\,a^2cde^2)x}{8\,(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2)} \\
&\quad+\frac{3\,(cd^3+ade^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}
\end{aligned}
$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*a^2*c*e^3*x^2 + 6*a^2*c*d^2*e + 2*a^3*e^3 - 3*(c^3*d^3 + a*c^2*d*e
^2)*x^3 - (5*a*c^2*d^3 - 3*a^2*c*d*e^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 +
a^4*c^2) + 3/8*(c*d^3 + a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3}{(a+cx^2)^3} dx = \frac{3 (cd^3 + ade^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8 \sqrt{ac} a^2 c} + \frac{3 c^3 d^3 x^3 + 3 a c^2 d e^2 x^3 - 4 a^2 c e^3 x^2 + 5 a c^2 d^3 x - 3 a^2 c d e^2 x - 6 a^2 c d^2 e - 2 a^3 e^3}{8 (cx^2 + a)^2 a^2 c^2}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*(c*d^3 + a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(3*c^3
*d^3*x^3 + 3*a*c^2*d*e^2*x^3 - 4*a^2*c*e^3*x^2 + 5*a*c^2*d^3*x - 3*a^2*c*d
*e^2*x - 6*a^2*c*d^2*e - 2*a^3*e^3)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3}{(a+cx^2)^3} dx = \frac{3 d \operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right) (c d^2 + a e^2)}{8 a^{5/2} c^{3/2}} - \frac{\frac{e^3 x^2}{2 c} + \frac{e (3 c d^2 + a e^2)}{4 c^2} - \frac{3 d x^3 (c d^2 + a e^2)}{8 a^2} + \frac{d x (3 a e^2 - 5 c d^2)}{8 a c}}{a^2 + 2 a c x^2 + c^2 x^4}$$

input
```
int((d + e*x)^3/(a + c*x^2)^3,x)
```

output
```
(3*d*atan((c^(1/2)*x)/a^(1/2))*(a*e^2 + c*d^2))/(8*a^(5/2)*c^(3/2)) - ((e^
3*x^2)/(2*c) + (e*(a*e^2 + 3*c*d^2))/(4*c^2) - (3*d*x^3*(a*e^2 + c*d^2))/(
8*a^2) + (d*x*(3*a*e^2 - 5*c*d^2))/(8*a*c))/(a^2 + c^2*x^4 + 2*a*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^3}{(a+cx^2)^3}\,dx$$
$$= \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3d\,e^2 + 3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2c\,d^3 + 6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2cd\,e^2x^2 + 6\sqrt{c}\,\sqrt{a}\,a}{}$$

input
```
int((e*x+d)^3/(c*x^2+a)^3,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*d*e**2 + 3*sqrt(c)*s
qrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d**3 + 6*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d*e**2*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a*c**2*d**3*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(
c)*sqrt(a)))*a*c**2*d*e**2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sq
rt(a)))*c**3*d**3*x**4 - 6*a**3*c*d**2*e - 3*a**3*c*d*e**2*x + 5*a**2*c**2
*d**3*x + 3*a**2*c**2*d*e**2*x**3 + 2*a**2*c**2*e**3*x**4 + 3*a*c**3*d**3*
x**3)/(8*a**3*c**2*(a**2 + 2*a*c*x**2 + c**2*x**4))
```

3.119 $\int \frac{(d+ex)^2}{(a+cx^2)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 112

$$\int \frac{(d+ex)^2}{(a+cx^2)^3} \, dx = -\frac{2ade-(cd^2-ae^2)\,x}{4ac\,(a+cx^2)^2}+\frac{(3cd^2+ae^2)\,x}{8a^2c\,(a+cx^2)}+\frac{(3cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

output
```
-1/4*(2*a*d*e-(-a*e^2+c*d^2)*x)/a/c/(c*x^2+a)^2+1/8*(a*e^2+3*c*d^2)*x/a^2/
c/(c*x^2+a)+1/8*(a*e^2+3*c*d^2)*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2}{(a+cx^2)^3} \, dx = \frac{3c^2d^2x^3-a^2e(4d+ex)+acx(5d^2+e^2x^2)}{8a^2c\,(a+cx^2)^2}+\frac{(3cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^2)^3,x]
```

output
```
(3*c^2*d^2*x^3 - a^2*e*(4*d + e*x) + a*c*x*(5*d^2 + e^2*x^2))/(8*a^2*c*(a
+ c*x^2)^2) + ((3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c
^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {495, 454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^2)^3} dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{3cd^2+2cexd+ae^2}{(cx^2+a)^2}dx}{4ac} - \frac{(d+ex)(ae-cdx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{(ae^2+3cd^2)\int \frac{1}{cx^2+a}dx}{2a} - \frac{2ade-x(ae^2+3cd^2)}{2a(a+cx^2)}}{4ac} - \frac{(d+ex)(ae-cdx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+3cd^2)}{2a^{3/2}\sqrt{c}} - \frac{2ade-x(ae^2+3cd^2)}{2a(a+cx^2)}}{4ac} - \frac{(d+ex)(ae-cdx)}{4ac(a+cx^2)^2}$$

input
```
Int[(d + e*x)^2/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*e - c*d*x)*(d + e*x))/(a*c*(a + c*x^2)^2) + (-1/2*(2*a*d*e - (3*c
*d^2 + a*e^2)*x)/(a*(a + c*x^2)) + ((3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(2*a^(3/2)*Sqrt[c]))/(4*a*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{\left(a\,e^{2}+3c\,d^{2}\right)x^{3}}{8a^{2}}-\frac{\left(a\,e^{2}-5c\,d^{2}\right)x}{8ac}-\frac{de}{2c}}{\left(c\,x^{2}+a\right)^{2}}+\frac{\left(a\,e^{2}+3c\,d^{2}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^{2}c\sqrt{ac}}$
risch	$\frac{\frac{\left(a\,e^{2}+3c\,d^{2}\right)x^{3}}{8a^{2}}-\frac{\left(a\,e^{2}-5c\,d^{2}\right)x}{8ac}-\frac{de}{2c}}{\left(c\,x^{2}+a\right)^{2}}-\frac{\ln\left(cx+\sqrt{-ac}\right)e^{2}}{16\sqrt{-ac}\,ca}-\frac{3\ln\left(cx+\sqrt{-ac}\right)d^{2}}{16\sqrt{-ac}\,a^{2}}+\frac{\ln\left(-cx+\sqrt{-ac}\right)e^{2}}{16\sqrt{-ac}\,ca}+\frac{3\ln\left(-cx+\sqrt{-ac}\right)d^{2}}{16\sqrt{-ac}\,a^{2}}$

input
```
int((e*x+d)^2/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(a*e^2+3*c*d^2)/a^2*x^3-1/8*(a*e^2-5*c*d^2)/a/c*x-1/2*d*e/c)/(c*x^2+a
)^2+1/8*(a*e^2+3*c*d^2)/a^2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.21

$$\int \frac{(d+ex)^2}{(a+cx^2)^3}\,dx = \left[-\frac{8\,a^3cde - 2\,(3\,ac^3d^2 + a^2c^2e^2)x^3 + (3\,a^2cd^2 + a^3e^2 + (3\,c^3d^2 + ac^2e^2)x^4 + 2\,(3\,ac^2d^2 + a^2ce^2)x^2)\sqrt{-a}}{16\,(a^3c^4x^4 + 2\,a^4c^3x^2 + a^5c^2)} - \frac{4\,a^3cde - (3\,ac^3d^2 + a^2c^2e^2)x^3 - (3\,a^2cd^2 + a^3e^2 + (3\,c^3d^2 + ac^2e^2)x^4 + 2\,(3\,ac^2d^2 + a^2ce^2)x^2)\sqrt{ac}\,\mathrm{arc}}{8\,(a^3c^4x^4 + 2\,a^4c^3x^2 + a^5c^2)}\right.$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*a^3*c*d*e - 2*(3*a*c^3*d^2 + a^2*c^2*e^2)*x^3 + (3*a^2*c*d^2 + a
^3*e^2 + (3*c^3*d^2 + a*c^2*e^2)*x^4 + 2*(3*a*c^2*d^2 + a^2*c*e^2)*x^2)*sq
rt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(5*a^2*c^2*d^2
- a^3*c*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(4*a^3*c*d*e
 - (3*a*c^3*d^2 + a^2*c^2*e^2)*x^3 - (3*a^2*c*d^2 + a^3*e^2 + (3*c^3*d^2 +
 a*c^2*e^2)*x^4 + 2*(3*a*c^2*d^2 + a^2*c*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a
*c)*x/a) - (5*a^2*c^2*d^2 - a^3*c*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a
^5*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^2}{(a+cx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5c^3}}\,(ae^2+3cd^2)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5c^3}}\,(ae^2+3cd^2)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{-4a^2de + x^3(ace^2+3c^2d^2) + x(-a^2e^2+5acd^2)}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

input
```
integrate((e*x+d)**2/(c*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*c**3))*(a*e**2 + 3*c*d**2)*log(-a**3*c*sqrt(-1/(a**5*c**3))
 + x)/16 + sqrt(-1/(a**5*c**3))*(a*e**2 + 3*c*d**2)*log(a**3*c*sqrt(-1/(a*
*5*c**3)) + x)/16 + (-4*a**2*d*e + x**3*(a*c*e**2 + 3*c**2*d**2) + x*(-a**
2*e**2 + 5*a*c*d**2))/(8*a**4*c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(a+cx^2)^3}\,dx = -\frac{4\,a^2de-(3\,c^2d^2+ace^2)x^3-(5\,acd^2-a^2e^2)x}{8\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)} + \frac{(3\,cd^2+ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*a^2*d*e - (3*c^2*d^2 + a*c*e^2)*x^3 - (5*a*c*d^2 - a^2*e^2)*x)/(a^
2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c) + 1/8*(3*c*d^2 + a*e^2)*arctan(c*x/sqrt
(a*c))/(sqrt(a*c)*a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^2}{(a+cx^2)^3}\,dx = \frac{(3\,cd^2+ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c} + \frac{3\,c^2d^2x^3+ace^2x^3+5\,acd^2x-a^2e^2x-4\,a^2de}{8\,(cx^2+a)^2a^2c}$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*c*d^2 + a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(3*c^2
*d^2*x^3 + a*c*e^2*x^3 + 5*a*c*d^2*x - a^2*e^2*x - 4*a^2*d*e)/((c*x^2 + a)
^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2}{(a+cx^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,c\,d^2+a\,e^2)}{8\,a^{5/2}\,c^{3/2}} - \frac{\frac{d\,e}{2\,c}-\frac{x^3\,(3\,c\,d^2+a\,e^2)}{8\,a^2}+\frac{x\,(a\,e^2-5\,c\,d^2)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

input
```
int((d + e*x)^2/(a + c*x^2)^3,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(a*e^2 + 3*c*d^2))/(8*a^(5/2)*c^(3/2)) - ((d*e)
/(2*c) - (x^3*(a*e^2 + 3*c*d^2))/(8*a^2) + (x*(a*e^2 - 5*c*d^2))/(8*a*c))/
(a^2 + c^2*x^4 + 2*a*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.16

$$\int \frac{(d+ex)^2}{(a+cx^2)^3}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3e^2+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2c\,d^2+2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2c\,e^2x^2+6\sqrt{c}\,\sqrt{a}\,atan(}{}$$

input
```
int((e*x+d)^2/(c*x^2+a)^3,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*e**2 + 3*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d**2 + 2*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**2*c*e**2*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a*c**2*d**2*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(
a)))*a*c**2*e**2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c*
*3*d**2*x**4 - 4*a**3*c*d*e - a**3*c*e**2*x + 5*a**2*c**2*d**2*x + a**2*c*
*2*e**2*x**3 + 3*a*c**3*d**2*x**3)/(8*a**3*c**2*(a**2 + 2*a*c*x**2 + c**2*
x**4))
```

3.120 $\int \frac{d+ex}{(a+cx^2)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{d+ex}{(a+cx^2)^3} \, dx = \frac{-ae+cdx}{4ac(a+cx^2)^2} + \frac{3dx}{8a^2(a+cx^2)} + \frac{3d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

output
```
1/4*(c*d*x-a*e)/a/c/(c*x^2+a)^2+3/8*d*x/a^2/(c*x^2+a)+3/8*d*arctan(c^(1/2)
*x/a^(1/2))/a^(5/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{d+ex}{(a+cx^2)^3} \, dx = \frac{\frac{\sqrt{a}(-2a^2e+5acdx+3c^2dx^3)}{(a+cx^2)^2} + 3\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c}$$

input
```
Integrate[(d + e*x)/(a + c*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^2*e + 5*a*c*d*x + 3*c^2*d*x^3))/(a + c*x^2)^2 + 3*Sqrt[c]*
d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {454, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^2)^3} \, dx$$

$$\Big\downarrow \text{454}$$

$$\frac{3d\int \frac{1}{(cx^2+a)^2}dx}{4a}-\frac{ae-cdx}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3d\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a(a+cx^2)}\right)}{4a}-\frac{ae-cdx}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3d\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a(a+cx^2)}\right)}{4a}-\frac{ae-cdx}{4ac(a+cx^2)^2}$$

input
```
Int[(d + e*x)/(a + c*x^2)^3,x]
```

output
```
-1/4*(a*e - c*d*x)/(a*c*(a + c*x^2)^2) + (3*d*(x/(2*a*(a + c*x^2)) + ArcTa
n[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2cdx-2ae}{8ac\left(c\,x^{2}+a\right)^{2}}+\frac{3d\left(\frac{x}{2a\left(c\,x^{2}+a\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2a\sqrt{ac}}\right)}{4a}$	70
risch	$\frac{\frac{3cd\,x^{3}}{8a^{2}}+\frac{5dx}{8a}-\frac{e}{4c}}{\left(c\,x^{2}+a\right)^{2}}-\frac{3d\ln\left(cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^{2}}+\frac{3d\ln\left(-cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^{2}}$	83

input
```
int((e*x+d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*c*d*x-2*a*e)/a/c/(c*x^2+a)^2+3/4*d/a*(1/2*x/a/(c*x^2+a)+1/2/a/(a*c)
^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.83

$$\int \frac{d+ex}{\left(a+cx^2\right)^3}\,dx$$
$$=\left[\frac{6\,ac^2dx^3+10\,a^2cdx-4\,a^3e-3\,\left(c^2dx^4+2\,acdx^2+a^2d\right)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{16\,\left(a^3c^3x^4+2\,a^4c^2x^2+a^5c\right)},\ \frac{3\,ac^2dx^3+5\,a^2cdx}{}\right.$$

input
```
integrate((e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*c^2*d*x^3 + 10*a^2*c*d*x - 4*a^3*e - 3*(c^2*d*x^4 + 2*a*c*d*x^2
 + a^2*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c
^3*x^4 + 2*a^4*c^2*x^2 + a^5*c), 1/8*(3*a*c^2*d*x^3 + 5*a^2*c*d*x - 2*a^3*
e + 3*(c^2*d*x^4 + 2*a*c*d*x^2 + a^2*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(
a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{d+ex}{\left(a+cx^2\right)^3}\,dx = d\left(-\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}\right)+\frac{-2a^2e+5acdx+3c^2dx^3}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

input
```
integrate((e*x+d)/(c*x**2+a)**3,x)
```

output
```
d*(-3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a
**5*c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16) + (-2*a**2*e + 5*a*c*d*x + 3*c
**2*d*x**3)/(8*a**4*c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{(a+cx^2)^3}\,dx = \frac{3\,c^2dx^3+5\,acdx-2\,a^2e}{8\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)} + \frac{3\,d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2}$$

input
```
integrate((e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*c^2*d*x^3 + 5*a*c*d*x - 2*a^2*e)/(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4
*c) + 3/8*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{d+ex}{(a+cx^2)^3}\,dx = \frac{3\,d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2} + \frac{3\,c^2dx^3+5\,acdx-2\,a^2e}{8\,(cx^2+a)^2a^2c}$$

input
```
integrate((e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*c^2*d*x^3 + 5*a*c*d*x
 - 2*a^2*e)/((c*x^2 + a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{d+ex}{(a+cx^2)^3}\,dx = \frac{\frac{5\,d\,x}{8\,a}-\frac{e}{4\,c}+\frac{3\,c\,d\,x^3}{8\,a^2}}{a^2+2\,a\,c\,x^2+c^2\,x^4} + \frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}$$

input
```
int((d + e*x)/(a + c*x^2)^3,x)
```

output
```
((5*d*x)/(8*a) - e/(4*c) + (3*c*d*x^3)/(8*a^2))/(a^2 + c^2*x^4 + 2*a*c*x^2
) + (3*d*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{d+ex}{(a+cx^2)^3}\,dx$$
$$= \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2d+6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)acd\,x^2+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)c^2d\,x^4-2a^3e+5a^2cdx}{8a^3c\,(c^2x^4+2ac\,x^2+a^2)}$$

input
```
int((e*x+d)/(c*x^2+a)^3,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*d + 6*sqrt(c)*sqrt(a
)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*c**2*d*x**4 - 2*a**3*e + 5*a**2*c*d*x + 3*a*c**2*d*x**3
)/(8*a**3*c*(a**2 + 2*a*c*x**2 + c**2*x**4))
```

3.121 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 212

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^3}\,dx=\frac{ae+cdx}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2}+\frac{4a^2e^3+cd(3cd^2+7ae^2)\,x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}+\frac{\sqrt{c}d(3c^2d^4+10acd^2e^2+15a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\left(cd^2+ae^2\right)^3}+\frac{e^5\log(d+ex)}{\left(cd^2+ae^2\right)^3}-\frac{e^5\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^3}$$

output
```
1/4*(c*d*x+a*e)/a/(a*e^2+c*d^2)/(c*x^2+a)^2+1/8*(4*a^2*e^3+c*d*(7*a*e^2+3*
c*d^2)*x)/a^2/(a*e^2+c*d^2)^2/(c*x^2+a)+1/8*c^(1/2)*d*(15*a^2*e^4+10*a*c*d
^2*e^2+3*c^2*d^4)*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^3+e^5*ln
(e*x+d)/(a*e^2+c*d^2)^3-1/2*e^5*ln(c*x^2+a)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex)(a+cx^2)^3}\,dx = \frac{\frac{2(cd^2+ae^2)^2(ae+cdx)}{a(a+cx^2)^2}+\frac{(cd^2+ae^2)(4a^2e^3+3c^2d^3x+7acde^2x)}{a^2(a+cx^2)}+\frac{\sqrt{c}d(3c^2d^4+10acd^2e^2+15a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{5/2}}+8e^5\log(d+ex)}{8\,(cd^2+ae^2)^3}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^2)^3),x]
```

output
```
((2*(c*d^2 + a*e^2)^2*(a*e + c*d*x))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*
(4*a^2*e^3 + 3*c^2*d^3*x + 7*a*c*d*e^2*x))/(a^2*(a + c*x^2)) + (Sqrt[c]*d*
(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(
5/2) + 8*e^5*Log[d + e*x] - 4*e^5*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {496, 25, 686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^3(d+ex)}\,dx$$

$$\Big\downarrow \; 496$$

$$\frac{ae+cdx}{4a(a+cx^2)^2(ae^2+cd^2)}-\frac{\int -\frac{3cd^2+3cexd+4ae^2}{(d+ex)(cx^2+a)^2}dx}{4a(ae^2+cd^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3cd^2+3cexd+4ae^2}{(d+ex)(cx^2+a)^2}dx}{4a(ae^2+cd^2)}+\frac{ae+cdx}{4a(a+cx^2)^2(ae^2+cd^2)}$$

$$\Big\downarrow \; 686$$

$$\frac{\frac{4a^2e^3+cdx(7ae^2+3cd^2)}{2a(a+cx^2)(ae^2+cd^2)}-\frac{\int -\frac{c\left(3c^2d^4+7ace^2d^2+ce\left(3cd^2+7ae^2\right)xd+8a^2e^4\right)}{(d+ex)\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c\left(3c^2d^4+7ace^2d^2+ce\left(3cd^2+7ae^2\right)xd+8a^2e^4\right)}{(d+ex)\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)}+\frac{4a^2e^3+cdx(7ae^2+3cd^2)}{2a(a+cx^2)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3c^2d^4+7ace^2d^2+ce\left(3cd^2+7ae^2\right)xd+8a^2e^4}{(d+ex)\left(cx^2+a\right)}dx}{2a(ae^2+cd^2)}+\frac{4a^2e^3+cdx(7ae^2+3cd^2)}{2a(a+cx^2)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\frac{\int \left(\frac{8a^2e^6}{\left(cd^2+ae^2\right)(d+ex)}+\frac{c\left(3c^2d^5+10ace^2d^3+15a^2e^4d-8a^2e^5x\right)}{\left(cd^2+ae^2\right)\left(cx^2+a\right)}\right)dx}{2a(ae^2+cd^2)}+\frac{4a^2e^3+cdx(7ae^2+3cd^2)}{2a(a+cx^2)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)}{\sqrt{a}\left(ae^2+cd^2\right)}-\frac{4a^2e^5\log\left(a+cx^2\right)}{ae^2+cd^2}+\frac{8a^2e^5\log(d+ex)}{ae^2+cd^2}}{2a(ae^2+cd^2)}+\frac{4a^2e^3+cdx(7ae^2+3cd^2)}{2a(a+cx^2)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x)*(a + c*x^2)^3),x]
```

output
```
(a*e + c*d*x)/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + ((4*a^2*e^3 + c*d*(3*c
*d^2 + 7*a*e^2)*x)/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + ((Sqrt[c]*d*(3*c^2*
d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(
c*d^2 + a*e^2)) + (8*a^2*e^5*Log[d + e*x])/(c*d^2 + a*e^2) - (4*a^2*e^5*Lo
g[a + c*x^2])/(c*d^2 + a*e^2))/(2*a*(c*d^2 + a*e^2)))/(4*a*(c*d^2 + a*e^2)
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

method	result
default	$\frac{e^{5} \ln\left(e x +d \right)}{\left(a \,e^{2}+c \,d^{2}\right)^{3}}+\frac{c \left(\frac{\frac{c d \left(7 a^{2} e^{4}+10 a c \,d^{2} e^{2}+3 c^{2} d^{4}\right) x^{3}}{8 a^{2}}+\left(\frac{1}{2} a \,e^{5}+\frac{1}{2} d^{2} e^{3} c \right) x^{2}+\frac{d \left(9 a^{2} e^{4}+14 a c \,d^{2} e^{2}+5 c^{2} d^{4}\right) x}{8 a}+\frac{e \left(3 a^{2} e^{4}+4 a c \,d^{2} e^{2}+c^{2} d^{4}\right)}{4 c}}{\left(c \,x^{2}+a \right)^{2}}+ \right.}{\left(a \,e^{2}+c \,d^{2}\right)^{3}}$
risch	$\frac{\frac{c^{2} d \left(7 a \,e^{2}+3 c \,d^{2}\right) x^{3}}{8 \left(a^{2} e^{4}+2 a c \,d^{2} e^{2}+c^{2} d^{4}\right) a^{2}}+\frac{c \,e^{3} x^{2}}{2 a^{2} e^{4}+4 a c \,d^{2} e^{2}+2 c^{2} d^{4}}+\frac{d c \left(9 a \,e^{2}+5 c \,d^{2}\right) x}{8 a \left(a^{2} e^{4}+2 a c \,d^{2} e^{2}+c^{2} d^{4}\right)}+\frac{e \left(3 a \,e^{2}+c \,d^{2}\right)}{4 a^{2} e^{4}+8 a c \,d^{2} e^{2}+4 c^{2} d^{4}}}{\left(c \,x^{2}+a \right)^{2}}+\frac{e^{5} \ln\left(e x +d \right)}{e^{6} a^{3}+3 d^{2} e^{4} a^{2} c +3 d^{4} e^{2} a}$

input
```
int(1/(e*x+d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
e^5*ln(e*x+d)/(a*e^2+c*d^2)^3+c/(a*e^2+c*d^2)^3*((1/8*c*d*(7*a^2*e^4+10*a*
c*d^2*e^2+3*c^2*d^4)/a^2*x^3+(1/2*a*e^5+1/2*d^2*e^3*c)*x^2+1/8*d*(9*a^2*e^
4+14*a*c*d^2*e^2+5*c^2*d^4)/a*x+1/4*e*(3*a^2*e^4+4*a*c*d^2*e^2+c^2*d^4)/c)
/(c*x^2+a)^2+1/8/a^2*(-4*a^2*e^5/c*ln(c*x^2+a)+(15*a^2*d*e^4+10*a*c*d^3*e^
2+3*c^2*d^5)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(196) = 392.

Time = 1.90 (sec) , antiderivative size = 1020, normalized size of antiderivative = 4.81

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(4*a^2*c^2*d^4*e + 16*a^3*c*d^2*e^3 + 12*a^4*e^5 + 2*(3*c^4*d^5 + 10
*a*c^3*d^3*e^2 + 7*a^2*c^2*d*e^4)*x^3 + 8*(a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^
2 + (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 + 15*a^4*d*e^4 + (3*c^4*d^5 + 10*a*c
^3*d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*(3*a*c^3*d^5 + 10*a^2*c^2*d^3*e^2 +
 15*a^3*c*d*e^4)*x^2)*sqrt(-c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2
 + a)) + 2*(5*a*c^3*d^5 + 14*a^2*c^2*d^3*e^2 + 9*a^3*c*d*e^4)*x - 8*(a^2*c
^2*e^5*x^4 + 2*a^3*c*e^5*x^2 + a^4*e^5)*log(c*x^2 + a) + 16*(a^2*c^2*e^5*x
^4 + 2*a^3*c*e^5*x^2 + a^4*e^5)*log(e*x + d))/(a^4*c^3*d^6 + 3*a^5*c^2*d^4
*e^2 + 3*a^6*c*d^2*e^4 + a^7*e^6 + (a^2*c^5*d^6 + 3*a^3*c^4*d^4*e^2 + 3*a^
4*c^3*d^2*e^4 + a^5*c^2*e^6)*x^4 + 2*(a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*
a^5*c^2*d^2*e^4 + a^6*c*e^6)*x^2), 1/8*(2*a^2*c^2*d^4*e + 8*a^3*c*d^2*e^3
+ 6*a^4*e^5 + (3*c^4*d^5 + 10*a*c^3*d^3*e^2 + 7*a^2*c^2*d*e^4)*x^3 + 4*(a^
2*c^2*d^2*e^3 + a^3*c*e^5)*x^2 + (3*a^2*c^2*d^5 + 10*a^3*c*d^3*e^2 + 15*a^
4*d*e^4 + (3*c^4*d^5 + 10*a*c^3*d^3*e^2 + 15*a^2*c^2*d*e^4)*x^4 + 2*(3*a*c
^3*d^5 + 10*a^2*c^2*d^3*e^2 + 15*a^3*c*d*e^4)*x^2)*sqrt(c/a)*arctan(x*sqrt
(c/a)) + (5*a*c^3*d^5 + 14*a^2*c^2*d^3*e^2 + 9*a^3*c*d*e^4)*x - 4*(a^2*c^2
*e^5*x^4 + 2*a^3*c*e^5*x^2 + a^4*e^5)*log(c*x^2 + a) + 8*(a^2*c^2*e^5*x^4
+ 2*a^3*c*e^5*x^2 + a^4*e^5)*log(e*x + d))/(a^4*c^3*d^6 + 3*a^5*c^2*d^4*e^
2 + 3*a^6*c*d^2*e^4 + a^7*e^6 + (a^2*c^5*d^6 + 3*a^3*c^4*d^4*e^2 + 3*a^4*c
^3*d^2*e^4 + a^5*c^2*e^6)*x^4 + 2*(a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.82

$$\int \frac{1}{(d+ex)(a+cx^2)^3}\,dx = -\frac{e^5\log\left(cx^2+a\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)} + \frac{e^5\log\left(ex+d\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{\left(3\,c^3d^5+10\,ac^2d^3e^2+15\,a^2cde^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\left(a^2c^3d^6+3\,a^3c^2d^4e^2+3\,a^4cd^2e^4+a^5e^6\right)\sqrt{ac}} + \frac{4\,a^2ce^3x^2+2\,a^2cd^2e+6\,a^3e^3+\left(3\,c^3d^3+7\,ac^2de^2\right)x^3+\left(5\,ac^2d^3+9\,a^2cde^2\right)x}{8\left(a^4c^2d^4+2\,a^5cd^2e^2+a^6e^4+\left(a^2c^4d^4+2\,a^3c^3d^2e^2+a^4c^2e^4\right)x^4+2\left(a^3c^3d^4+2\,a^4c^2d^2e^2+a^5ce^4\right)x^2\right)}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*e^5*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3
*e^6) + e^5*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^
3*e^6) + 1/8*(3*c^3*d^5 + 10*a*c^2*d^3*e^2 + 15*a^2*c*d*e^4)*arctan(c*x/sq
rt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*sq
rt(a*c)) + 1/8*(4*a^2*c*e^3*x^2 + 2*a^2*c*d^2*e + 6*a^3*e^3 + (3*c^3*d^3 +
 7*a*c^2*d*e^2)*x^3 + (5*a*c^2*d^3 + 9*a^2*c*d*e^2)*x)/(a^4*c^2*d^4 + 2*a^
5*c*d^2*e^2 + a^6*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^
4 + 2*(a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.70

$$\int \frac{1}{(d+ex)(a+cx^2)^3}\,dx = \frac{e^6\log\left(|ex+d|\right)}{c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7} - \frac{e^5\log\left(cx^2+a\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)} + \frac{\left(3\,c^3d^5+10\,ac^2d^3e^2+15\,a^2cde^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\left(a^2c^3d^6+3\,a^3c^2d^4e^2+3\,a^4cd^2e^4+a^5e^6\right)\sqrt{ac}} + \frac{2\,a^2c^2d^4e+8\,a^3cd^2e^3+6\,a^4e^5+\left(3\,c^4d^5+10\,ac^3d^3e^2+7\,a^2c^2de^4\right)x^3+4\left(a^2c^2d^2e^3+a^3ce^5\right)x^2+\left(5\,ac^3\right.}{8\left(cd^2+ae^2\right)^3\left(cx^2+a\right)^2a^2}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
e^6*log(abs(e*x + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3
*e^7) - 1/2*e^5*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^
4 + a^3*e^6) + 1/8*(3*c^3*d^5 + 10*a*c^2*d^3*e^2 + 15*a^2*c*d*e^4)*arctan(
c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e
^6)*sqrt(a*c)) + 1/8*(2*a^2*c^2*d^4*e + 8*a^3*c*d^2*e^3 + 6*a^4*e^5 + (3*c
^4*d^5 + 10*a*c^3*d^3*e^2 + 7*a^2*c^2*d*e^4)*x^3 + 4*(a^2*c^2*d^2*e^3 + a^
3*c*e^5)*x^2 + (5*a*c^3*d^5 + 14*a^2*c^2*d^3*e^2 + 9*a^3*c*d*e^4)*x)/((c*d
^2 + a*e^2)^3*(c*x^2 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 987, normalized size of antiderivative = 4.66

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^3*(d + e*x)),x)
```

output
```
((3*a*e^3 + c*d^2*e)/(4*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*c^3
*d^3 + 7*a*c^2*d*e^2))/(8*a^2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5
*c^2*d^3 + 9*a*c*d*e^2))/(8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (c*e^
3*x^2)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2
) + (e^5*log(d + e*x))/(a*e^2 + c*d^2)^3 - (log(9*c^6*d^14*(-a^5*c)^(3/2)
- 576*a^12*e^14*(-a^5*c)^(1/2) - 1326*d^4*e^10*(-a^5*c)^(5/2) + 9*a^7*c^8*
d^14*x + 1377*a^6*d^2*e^12*(-a^5*c)^(3/2) + 576*a^14*c*e^14*x + 319*a^2*c^
4*d^10*e^4*(-a^5*c)^(3/2) + 740*a^3*c^3*d^8*e^6*(-a^5*c)^(3/2) + 1015*a^4*
c^2*d^6*e^8*(-a^5*c)^(3/2) + 78*a^8*c^7*d^12*e^2*x + 319*a^9*c^6*d^10*e^4*
x + 740*a^10*c^5*d^8*e^6*x + 1015*a^11*c^4*d^6*e^8*x + 1326*a^12*c^3*d^4*e
^10*x + 1377*a^13*c^2*d^2*e^12*x + 78*a*c^5*d^12*e^2*(-a^5*c)^(3/2))*(8*a^
5*e^5 + 3*c^2*d^5*(-a^5*c)^(1/2) + 15*a^2*d*e^4*(-a^5*c)^(1/2) + 10*a*c*d^
3*e^2*(-a^5*c)^(1/2)))/(16*(a^8*e^6 + a^5*c^3*d^6 + 3*a^7*c*d^2*e^4 + 3*a^
6*c^2*d^4*e^2)) + (log(576*a^10*e^14*(-a^5*c)^(1/2) + 9*a^5*c^8*d^14*x + 9
*a^3*c^7*d^14*(-a^5*c)^(1/2) - 1377*a^4*d^2*e^12*(-a^5*c)^(3/2) - 319*c^4*
d^10*e^4*(-a^5*c)^(3/2) + 576*a^12*c*e^14*x - 1015*a^2*c^2*d^6*e^8*(-a^5*c
)^(3/2) + 78*a^4*c^6*d^12*e^2*(-a^5*c)^(1/2) + 78*a^6*c^7*d^12*e^2*x + 319
*a^7*c^6*d^10*e^4*x + 740*a^8*c^5*d^8*e^6*x + 1015*a^9*c^4*d^6*e^8*x + 132
6*a^10*c^3*d^4*e^10*x + 1377*a^11*c^2*d^2*e^12*x - 740*a*c^3*d^8*e^6*(-a^5
*c)^(3/2) - 1326*a^3*c*d^4*e^10*(-a^5*c)^(3/2))*(3*c^2*d^5*(-a^5*c)^(1/...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 679, normalized size of antiderivative = 3.20

$$\int \frac{1}{(d+ex)(a+cx^2)^3} dx = \frac{30\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right)a^3cd\,e^4x^2 + 20\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right)a^2c^2d^3e^2x^2 + 15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right)a^2c^2d\,e^4x^4}{}$$

input
```
int(1/(e*x+d)/(c*x^2+a)^3,x)
```

output

```
(15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*d*e**4 + 10*sqrt(c)
*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d**3*e**2 + 30*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d*e**4*x**2 + 3*sqrt(c)*sqrt(a)*at
an((c*x)/(sqrt(c)*sqrt(a)))*a**2*c**2*d**5 + 20*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**2*c**2*d**3*e**2*x**2 + 15*sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a**2*c**2*d*e**4*x**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a*c**3*d**5*x**2 + 10*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a*c**3*d**3*e**2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*c**4*d**5*x**4 - 4*log(a + c*x**2)*a**5*e**5 - 8*log(a + c*x**
2)*a**4*c*e**5*x**2 - 4*log(a + c*x**2)*a**3*c**2*e**5*x**4 + 8*log(d + e*
x)*a**5*e**5 + 16*log(d + e*x)*a**4*c*e**5*x**2 + 8*log(d + e*x)*a**3*c**2
*e**5*x**4 + 4*a**5*e**5 + 6*a**4*c*d**2*e**3 + 9*a**4*c*d*e**4*x + 2*a**3
*c**2*d**4*e + 14*a**3*c**2*d**3*e**2*x + 7*a**3*c**2*d*e**4*x**3 - 2*a**3
*c**2*e**5*x**4 + 5*a**2*c**3*d**5*x + 10*a**2*c**3*d**3*e**2*x**3 - 2*a**
2*c**3*d**2*e**3*x**4 + 3*a*c**4*d**5*x**3)/(8*a**3*(a**5*e**6 + 3*a**4*c*
d**2*e**4 + 2*a**4*c*e**6*x**2 + 3*a**3*c**2*d**4*e**2 + 6*a**3*c**2*d**2*
e**4*x**2 + a**3*c**2*e**6*x**4 + a**2*c**3*d**6 + 6*a**2*c**3*d**4*e**2*x
**2 + 3*a**2*c**3*d**2*e**4*x**4 + 2*a*c**4*d**6*x**2 + 3*a*c**4*d**4*e**2
*x**4 + c**5*d**6*x**4))
```

3.122 $\int \frac{1}{(d+ex)^2(a+cx^2)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 279

$$\int \frac{1}{(d+ex)^2(a+cx^2)^3} dx = -\frac{e^5}{(cd^2+ae^2)^3(d+ex)} + \frac{c(2ade+(cd^2-ae^2)x)}{4a(cd^2+ae^2)^2(a+cx^2)^2} + \frac{c(16a^2de^3+(3c^2d^4+12acd^2e^2-7a^2e^4)x)}{8a^2(cd^2+ae^2)^3(a+cx^2)} + \frac{3\sqrt{c}(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}(cd^2+ae^2)^4} + \frac{6cde^5\log(d+ex)}{(cd^2+ae^2)^4} - \frac{3cde^5\log(a+cx^2)}{(cd^2+ae^2)^4}$$

output
```
-e^5/(a*e^2+c*d^2)^3/(e*x+d)+1/4*c*(2*a*d*e+(-a*e^2+c*d^2)*x)/a/(a*e^2+c*d
^2)^2/(c*x^2+a)^2+1/8*c*(16*a^2*d*e^3+(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4
)*x)/a^2/(a*e^2+c*d^2)^3/(c*x^2+a)+3/8*c^(1/2)*(-5*a^3*e^6+15*a^2*c*d^2*e^
4+5*a*c^2*d^4*e^2+c^3*d^6)*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)
^4+6*c*d*e^5*ln(e*x+d)/(a*e^2+c*d^2)^4-3*c*d*e^5*ln(c*x^2+a)/(a*e^2+c*d^2)
^4
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.86

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^3}\,dx$$
$$=\frac{-\frac{8e^5\left(cd^2+ae^2\right)}{d+ex}+\frac{c\left(cd^2+ae^2\right)\left(3c^2d^4x+12acd^2e^2x+a^2e^3(16d-7ex)\right)}{a^2(a+cx^2)}+\frac{2c\left(cd^2+ae^2\right)^2\left(cd^2x+ae(2d-ex)\right)}{a(a+cx^2)^2}+\frac{3\sqrt{c}\left(c^3d^6+5ac^2d^4e^2+15a^2\cdots\right.}{a\cdots}}{8\left(cd^2+ae^2\right)^4}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^2)^3),x]
```

output
```
((-8*e^5*(c*d^2 + a*e^2))/(d + e*x) + (c*(c*d^2 + a*e^2)*(3*c^2*d^4*x + 12
*a*c*d^2*e^2*x + a^2*e^3*(16*d - 7*e*x)))/(a^2*(a + c*x^2)) + (2*c*(c*d^2
+ a*e^2)^2*(c*d^2*x + a*e*(2*d - e*x)))/(a*(a + c*x^2)^2) + (3*Sqrt[c]*(c^
3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTan[(Sqrt[c]*x)
/Sqrt[a]])/a^(5/2) + 48*c*d*e^5*Log[d + e*x] - 24*c*d*e^5*Log[a + c*x^2])/
(8*(c*d^2 + a*e^2)^4)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {496, 25, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^2\right)^3(d+ex)^2}\,dx$$
$$\Big\downarrow \text{496}$$
$$\frac{ae+cdx}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}-\frac{\int -\frac{3cd^2+4cexd+5ae^2}{(d+ex)^2\left(cx^2+a\right)^2}dx}{4a\left(ae^2+cd^2\right)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3cd^2+4cexd+5ae^2}{(d+ex)^2(cx^2+a)^2}dx}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 686

$$\frac{-\frac{\int -\frac{3c\left(c^2d^4+2ace^2d^2+2ce\left(cd^2+3ae^2\right)xd+5a^2e^4\right)}{(d+ex)^2\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)}-\frac{ae(cd^2-5ae^2)-3cdx(3ae^2+cd^2)}{2a(a+cx^2)(d+ex)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \frac{c^2d^4+2ace^2d^2+2ce\left(cd^2+3ae^2\right)xd+5a^2e^4}{(d+ex)^2\left(cx^2+a\right)}dx}{2a(ae^2+cd^2)}-\frac{ae(cd^2-5ae^2)-3cdx(3ae^2+cd^2)}{2a(a+cx^2)(d+ex)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 657

$$\frac{\frac{3\int \left(\frac{16a^2cde^6}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{\left(-cd^2-5ae^2\right)\left(cd^2-ae^2\right)e^2}{\left(cd^2+ae^2\right)(d+ex)^2}+\frac{c\left(c^3d^6+5ac^2e^2d^4+15a^2ce^4d^2-16a^2ce^5xd-5a^3e^6\right)}{\left(cd^2+ae^2\right)^2\left(cx^2+a\right)}\right)dx}{2a(ae^2+cd^2)}-\frac{ae(cd^2-5ae^2)-3cdx(3ae^2+cd^2)}{2a(a+cx^2)(d+ex)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{3\left(-\frac{8a^2cde^5\log\left(a+cx^2\right)}{\left(ae^2+cd^2\right)^2}+\frac{16a^2cde^5\log(d+ex)}{\left(ae^2+cd^2\right)^2}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(-5a^3e^6+15a^2cd^2e^4+5ac^2d^4e^2+c^3d^6\right)}{\sqrt{a}\left(ae^2+cd^2\right)^2}+\frac{e\left(cd^2-ae^2\right)\left(5ae^2+cd^2\right)}{(d+ex)\left(ae^2+cd^2\right)}\right)}{2a(ae^2+cd^2)}-\frac{ae(cd^2-5ae^2}{2a(a+cx^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^2)^3),x]
```

output
```
(a*e + c*d*x)/(4*a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)^2) + (-1/2*(a*e*(
c*d^2 - 5*a*e^2) - 3*c*d*(c*d^2 + 3*a*e^2)*x)/(a*(c*d^2 + a*e^2)*(d + e*x)
*(a + c*x^2)) + (3*((e*(c*d^2 - a*e^2)*(c*d^2 + 5*a*e^2))/((c*d^2 + a*e^2)
*(d + e*x)) + (Sqrt[c]*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a
^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) + (16*a^2
*c*d*e^5*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (8*a^2*c*d*e^5*Log[a + c*x^2])/
(c*d^2 + a*e^2)^2))/(2*a*(c*d^2 + a*e^2)))/(4*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.11

method	result
default	$-\frac{e^{5}}{\left(a\,e^{2}+c\,d^{2}\right)^{3}\left(ex+d\right)}+\frac{6cd\,e^{5}\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{4}}-\frac{c\left(\frac{\frac{c\left(7e^{6}a^{3}-5d^{2}e^{4}a^{2}c-15d^{4}e^{2}a\,c^{2}-3d^{6}c^{3}\right)x^{3}}{8a^{2}}+\left(-2ad\,e^{5}c-2c^{2}d^{3}e^{3}\right)x^{2}+\frac{\left(9e^{6}a^{3}-3d^{2}e^{4}\right.}{}}{\left(c\,x^{2}+a\right)^{2}}\right.}{}$
risch	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-e^5/(a*e^2+c*d^2)^3/(e*x+d)+6*c*d*e^5*ln(e*x+d)/(a*e^2+c*d^2)^4-c/(a*e^2+
c*d^2)^4*((1/8*c*(7*a^3*e^6-5*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2-3*c^3*d^6)/a^
2*x^3+(-2*a*c*d*e^5-2*c^2*d^3*e^3)*x^2+1/8*(9*a^3*e^6-3*a^2*c*d^2*e^4-17*a
*c^2*d^4*e^2-5*c^3*d^6)/a*x-1/2*d*e*(5*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4))/(c*
x^2+a)^2+3/8/a^2*(8*d*e^5*a^2*ln(c*x^2+a)+(5*a^3*e^6-15*a^2*c*d^2*e^4-5*a*
c^2*d^4*e^2-c^3*d^6)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(265) = 530$.

Time = 5.44 (sec) , antiderivative size = 2322, normalized size of antiderivative = 8.32

$$\int \frac{1}{(d+ex)^2\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*a^2*c^3*d^6*e + 48*a^3*c^2*d^4*e^3 + 24*a^4*c*d^2*e^5 - 16*a^5*e^
7 + 6*(c^5*d^6*e + 5*a*c^4*d^4*e^3 - a^2*c^3*d^2*e^5 - 5*a^3*c^2*e^7)*x^4
+ 6*(c^5*d^7 + 5*a*c^4*d^5*e^2 + 7*a^2*c^3*d^3*e^4 + 3*a^3*c^2*d*e^6)*x^3
+ 2*(5*a*c^4*d^6*e + 33*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 - 25*a^4*c*e^7
)*x^2 - 3*(a^2*c^3*d^7 + 5*a^3*c^2*d^5*e^2 + 15*a^4*c*d^3*e^4 - 5*a^5*d*e^
6 + (c^5*d^6*e + 5*a*c^4*d^4*e^3 + 15*a^2*c^3*d^2*e^5 - 5*a^3*c^2*e^7)*x^5
 + (c^5*d^7 + 5*a*c^4*d^5*e^2 + 15*a^2*c^3*d^3*e^4 - 5*a^3*c^2*d*e^6)*x^4
+ 2*(a*c^4*d^6*e + 5*a^2*c^3*d^4*e^3 + 15*a^3*c^2*d^2*e^5 - 5*a^4*c*e^7)*x
^3 + 2*(a*c^4*d^7 + 5*a^2*c^3*d^5*e^2 + 15*a^3*c^2*d^3*e^4 - 5*a^4*c*d*e^6
)*x^2 + (a^2*c^3*d^6*e + 5*a^3*c^2*d^4*e^3 + 15*a^4*c*d^2*e^5 - 5*a^5*e^7)
*x)*sqrt(-c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(5*a*c^
4*d^7 + 21*a^2*c^3*d^5*e^2 + 27*a^3*c^2*d^3*e^4 + 11*a^4*c*d*e^6)*x - 48*(
a^2*c^3*d*e^6*x^5 + a^2*c^3*d^2*e^5*x^4 + 2*a^3*c^2*d*e^6*x^3 + 2*a^3*c^2*
d^2*e^5*x^2 + a^4*c*d*e^6*x + a^4*c*d^2*e^5)*log(c*x^2 + a) + 96*(a^2*c^3*
d*e^6*x^5 + a^2*c^3*d^2*e^5*x^4 + 2*a^3*c^2*d*e^6*x^3 + 2*a^3*c^2*d^2*e^5*
x^2 + a^4*c*d*e^6*x + a^4*c*d^2*e^5)*log(e*x + d))/(a^4*c^4*d^9 + 4*a^5*c^
3*d^7*e^2 + 6*a^6*c^2*d^5*e^4 + 4*a^7*c*d^3*e^6 + a^8*d*e^8 + (a^2*c^6*d^8
*e + 4*a^3*c^5*d^6*e^3 + 6*a^4*c^4*d^4*e^5 + 4*a^5*c^3*d^2*e^7 + a^6*c^2*e
^9)*x^5 + (a^2*c^6*d^9 + 4*a^3*c^5*d^7*e^2 + 6*a^4*c^4*d^5*e^4 + 4*a^5*c^3
*d^3*e^6 + a^6*c^2*d*e^8)*x^4 + 2*(a^3*c^5*d^8*e + 4*a^4*c^4*d^6*e^3 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\,(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(265) = 530$.

Time = 0.13 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.68

$$\int \frac{1}{(d+ex)^2\,(a+cx^2)^3}\,dx = -\frac{3\,cde^5\log\left(cx^2+a\right)}{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8} + \frac{6\,cde^5\log\left(ex+d\right)}{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8} + \frac{3\left(c^4d^6+5\,ac^3d^4e^2+15\,a^2c^2d^2e^4-5\,a^3ce^6\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\left(a^2c^4d^8+4\,a^3c^3d^6e^2+6\,a^4c^2d^4e^4+4\,a^5cd^2e^6+a^6e^8\right)\sqrt{ac}} + \frac{4\,a^2c^2d^4e+20\,a^3cd^2e^3-8\,a^4e^5+3\left(c^4d^4e+4\,ac^3\right.}{8\left(a^4c^3d^7+3\,a^5c^2d^5e^2+3\,a^6cd^3e^4+a^7de^6+\left(a^2c^5d^6e+3\,a^3c^4d^4e^3+3\,a^4c^3d^2e^5+a^5c^2e^7\right)x^5+\left(a^2c^5d^7\right.\right.}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-3*c*d*e^5*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 +
 4*a^3*c*d^2*e^6 + a^4*e^8) + 6*c*d*e^5*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^
6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + 3/8*(c^4*d^6 + 5*
a*c^3*d^4*e^2 + 15*a^2*c^2*d^2*e^4 - 5*a^3*c*e^6)*arctan(c*x/sqrt(a*c))/((
a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^
6*e^8)*sqrt(a*c)) + 1/8*(4*a^2*c^2*d^4*e + 20*a^3*c*d^2*e^3 - 8*a^4*e^5 +
3*(c^4*d^4*e + 4*a*c^3*d^2*e^3 - 5*a^2*c^2*e^5)*x^4 + 3*(c^4*d^5 + 4*a*c^3
*d^3*e^2 + 3*a^2*c^2*d*e^4)*x^3 + (5*a*c^3*d^4*e + 28*a^2*c^2*d^2*e^3 - 25
*a^3*c*e^5)*x^2 + (5*a*c^3*d^5 + 16*a^2*c^2*d^3*e^2 + 11*a^3*c*d*e^4)*x)/(
a^4*c^3*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e^4 + a^7*d*e^6 + (a^2*c^5*d
^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^5 + (a^2*c^5
*d^7 + 3*a^3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 + a^5*c^2*d*e^6)*x^4 + 2*(a^3
*c^4*d^6*e + 3*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*e^5 + a^6*c*e^7)*x^3 + 2*(a
^3*c^4*d^7 + 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*e^4 + a^6*c*d*e^6)*x^2 + (a
^4*c^3*d^6*e + 3*a^5*c^2*d^4*e^3 + 3*a^6*c*d^2*e^5 + a^7*e^7)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(265) = 530$.

Time = 0.13 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.14

$$\int \frac{1}{(d+ex)^2\,(a+cx^2)^3}\,dx = -\frac{e^{11}}{(c^3d^6e^6+3\,ac^2d^4e^8+3\,a^2cd^2e^{10}+a^3e^{12})(ex+d)} - \frac{3\,cde^5\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8} + \frac{3\,(c^4d^6e^2+5\,ac^3d^4e^4+15\,a^2c^2d^2e^6-5\,a^3ce^8)\arctan\left(\frac{cd-\frac{cd^2}{ex+d}-\frac{ae^2}{ex+d}}{\sqrt{ac}e}\right)}{8\,(a^2c^4d^8+4\,a^3c^3d^6e^2+6\,a^4c^2d^4e^4+4\,a^5cd^2e^6+a^6e^8)\sqrt{ac}e^2} + \frac{3\,c^5d^5e+14\,ac^4d^3e^3-29\,a^2c^3de^5-\frac{9\,c^5d^6e^2+41\,ac^4d^4e^4-121\,a^2c^3d^2e^6+7\,a^3c^2e^8}{(ex+d)e}+\frac{9\,c^5d^7e^3+45\,ac^4d^5e^5-145\,a^2c^3d^3e^7-2}{(ex+d)^2e^2}}{8\,(cd^2+ae^2)^4a^2\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)^2}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-e^11/((c^3*d^6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12)*(e*x
+ d)) - 3*c*d*e^5*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x
 + d)^2)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6
+ a^4*e^8) + 3/8*(c^4*d^6*e^2 + 5*a*c^3*d^4*e^4 + 15*a^2*c^2*d^2*e^6 - 5*a
^3*c*e^8)*arctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/
((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 +
a^6*e^8)*sqrt(a*c)*e^2) + 1/8*(3*c^5*d^5*e + 14*a*c^4*d^3*e^3 - 29*a^2*c^3
*d*e^5 - (9*c^5*d^6*e^2 + 41*a*c^4*d^4*e^4 - 121*a^2*c^3*d^2*e^6 + 7*a^3*c
^2*e^8)/((e*x + d)*e) + (9*c^5*d^7*e^3 + 45*a*c^4*d^5*e^5 - 145*a^2*c^3*d^
3*e^7 - 21*a^3*c^2*d*e^9)/((e*x + d)^2*e^2) - 3*(c^5*d^8*e^4 + 6*a*c^4*d^6
*e^6 - 20*a^2*c^3*d^4*e^8 - 22*a^3*c^2*d^2*e^10 + 3*a^4*c*e^12)/((e*x + d)
^3*e^3))/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 +
 a*e^2/(e*x + d)^2)^2)
```

Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 1296, normalized size of antiderivative = 4.65

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^3*(d + e*x)^2),x)
```

output
```
((c^2*d^4*e - 2*a^2*e^5 + 5*a*c*d^2*e^3)/(2*(a*e^2 + c*d^2)*(a^2*e^4 + c^2
*d^4 + 2*a*c*d^2*e^2)) + (3*x^3*(c^3*d^3 + 3*a*c^2*d*e^2))/(8*a^2*(a^2*e^4
 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*c^2*d^3 + 11*a*c*d*e^2))/(8*a*(a^2*e^
4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (3*x^4*(c^4*d^4*e - 5*a^2*c^2*e^5 + 4*a*c^
3*d^2*e^3))/(8*a^2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)
) + (x^2*(5*c^3*d^4*e - 25*a^2*c*e^5 + 28*a*c^2*d^2*e^3))/(8*a*(a*e^2 + c*
d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2*d + c^2*d*x^4 + c^2*e*x^5
+ a^2*e*x + 2*a*c*d*x^2 + 2*a*c*e*x^3) - (log(c^7*d^16*(-a^5*c)^(3/2) - 25
*a^13*e^16*(-a^5*c)^(1/2) + a^7*c^9*d^16*x - 4508*a*d^4*e^12*(-a^5*c)^(5/2
) - 2644*c*d^6*e^10*(-a^5*c)^(5/2) + 2204*a^7*d^2*e^14*(-a^5*c)^(3/2) + 25
*a^15*c*e^16*x + 76*a^2*c^5*d^12*e^4*(-a^5*c)^(3/2) + 260*a^3*c^4*d^10*e^6
*(-a^5*c)^(3/2) + 510*a^4*c^3*d^8*e^8*(-a^5*c)^(3/2) + 12*a^8*c^8*d^14*e^2
*x + 76*a^9*c^7*d^12*e^4*x + 260*a^10*c^6*d^10*e^6*x + 510*a^11*c^5*d^8*e^
8*x + 2644*a^12*c^4*d^6*e^10*x + 4508*a^13*c^3*d^4*e^12*x + 2204*a^14*c^2*
d^2*e^14*x + 12*a*c^6*d^14*e^2*(-a^5*c)^(3/2))*(c*(3*a^5*d*e^5 + (45*a^2*d
^2*e^4*(-a^5*c)^(1/2))/16) - (15*a^3*e^6*(-a^5*c)^(1/2))/16 + (3*c^3*d^6*(
-a^5*c)^(1/2))/16 + (15*a*c^2*d^4*e^2*(-a^5*c)^(1/2))/16))/(a^9*e^8 + a^5*
c^4*d^8 + 4*a^8*c*d^2*e^6 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4) - (log(
25*a^13*e^16*(-a^5*c)^(1/2) - c^7*d^16*(-a^5*c)^(3/2) + a^7*c^9*d^16*x + 4
508*a*d^4*e^12*(-a^5*c)^(5/2) + 2644*c*d^6*e^10*(-a^5*c)^(5/2) - 2204*a...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1734, normalized size of antiderivative = 6.22

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(c*x^2+a)^3,x)
```

output

```
( - 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*d**2*e**6 - 15*s
qrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*d*e**7*x + 45*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d**4*e**4 + 45*sqrt(c)*sqrt(a)*
atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d**3*e**5*x - 30*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d**2*e**6*x**2 - 30*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d*e**7*x**3 + 15*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**3*c**2*d**6*e**2 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(
sqrt(c)*sqrt(a)))*a**3*c**2*d**5*e**3*x + 90*sqrt(c)*sqrt(a)*atan((c*x)/(s
qrt(c)*sqrt(a)))*a**3*c**2*d**4*e**4*x**2 + 90*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a**3*c**2*d**3*e**5*x**3 - 15*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**3*c**2*d**2*e**6*x**4 - 15*sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*a**3*c**2*d*e**7*x**5 + 3*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**2*c**3*d**8 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*a**2*c**3*d**7*e*x + 30*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sq
rt(a)))*a**2*c**3*d**6*e**2*x**2 + 30*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*a**2*c**3*d**5*e**3*x**3 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*a**2*c**3*d**4*e**4*x**4 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a**2*c**3*d**3*e**5*x**5 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*a*c**4*d**8*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sq
rt(a)))*a*c**4*d**7*e*x**3 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqr...
```

3.123 $\int \frac{(d+ex)^6}{(a+cx^2)^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 290

$$\begin{aligned}\int \frac{(d+ex)^6}{(a+cx^2)^4} dx &= -\frac{2ade(cd^2-3ae^2)(3cd^2-ae^2)-(cd^2-ae^2)(c^2d^4-14acd^2e^2+a^2e^4)x}{6ac^3(a+cx^2)^3} \\ &\quad - \frac{24a^2de^3(5cd^2-3ae^2)-(5c^3d^6+15ac^2d^4e^2-105a^2cd^2e^4+13a^3e^6)x}{24a^2c^3(a+cx^2)^2} \\ &\quad - \frac{48a^3de^5-(5c^3d^6+15ac^2d^4e^2+15a^2cd^2e^4-11a^3e^6)x}{16a^3c^3(a+cx^2)} \\ &\quad + \frac{5(cd^2+ae^2)^3\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}\end{aligned}$$

output
```
-1/6*(2*a*d*e*(-3*a*e^2+c*d^2)*(-a*e^2+3*c*d^2)-(-a*e^2+c*d^2)*(a^2*e^4-14
*a*c*d^2*e^2+c^2*d^4)*x)/a/c^3/(c*x^2+a)^3-1/24*(24*a^2*d*e^3*(-3*a*e^2+5*
c*d^2)-(13*a^3*e^6-105*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+5*c^3*d^6)*x)/a^2/c^
3/(c*x^2+a)^2-1/16*(48*a^3*d*e^5-(-11*a^3*e^6+15*a^2*c*d^2*e^4+15*a*c^2*d^
4*e^2+5*c^3*d^6)*x)/a^3/c^3/(c*x^2+a)+5/16*(a*e^2+c*d^2)^3*arctan(c^(1/2)*
x/a^(1/2))/a^(7/2)/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx = \frac{c^3d^6x - a^3e^5(6d+ex) + 5a^2cd^2e^3(4d+3ex) - 3ac^2d^4e(2d+5ex)}{6ac^3(a+cx^2)^3} + \frac{5c^3d^6x + 15ac^2d^4e^2x + 15a^2cd^2e^4x - a^3e^5(48d+11ex)}{16a^3c^3(a+cx^2)} + \frac{5c^3d^6x + 15ac^2d^4e^2x - 15a^2cd^2e^3(8d+7ex) + a^3e^5(72d+13ex)}{24a^2c^3(a+cx^2)^2} + \frac{5(cd^2+ae^2)^3\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}$$

input
```
Integrate[(d + e*x)^6/(a + c*x^2)^4,x]
```

output
```
(c^3*d^6*x - a^3*e^5*(6*d + e*x) + 5*a^2*c*d^2*e^3*(4*d + 3*e*x) - 3*a*c^2
*d^4*e*(2*d + 5*e*x))/(6*a*c^3*(a + c*x^2)^3) + (5*c^3*d^6*x + 15*a*c^2*d^
4*e^2*x + 15*a^2*c*d^2*e^4*x - a^3*e^5*(48*d + 11*e*x))/(16*a^3*c^3*(a + c
*x^2)) + (5*c^3*d^6*x + 15*a*c^2*d^4*e^2*x - 15*a^2*c*d^2*e^3*(8*d + 7*e*x
) + a^3*e^5*(72*d + 13*e*x))/(24*a^2*c^3*(a + c*x^2)^2) + (5*(c*d^2 + a*e^
2)^3*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(7/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {487, 487, 487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx$$

$$\Big\downarrow \text{ 487}$$

$$\frac{5(ae^2+cd^2)\int \frac{(d+ex)^4}{(cx^2+a)^3}dx}{6ac}-\frac{(d+ex)^5(ae-cdx)}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \ 487$$

$$\frac{5(ae^2+cd^2)\left(\frac{3(ae^2+cd^2)\int \frac{(d+ex)^2}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac(a+cx^2)^2}\right)}{6ac}-\frac{(d+ex)^5(ae-cdx)}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \ 487$$

$$\frac{5(ae^2+cd^2)\left(\frac{3(ae^2+cd^2)\left(\frac{(ae^2+cd^2)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac(a+cx^2)^2}\right)}{6ac}-\frac{(d+ex)^5(ae-cdx)}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \ 218$$

$$\frac{5(ae^2+cd^2)\left(\frac{3(ae^2+cd^2)\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+cd^2)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac(a+cx^2)^2}\right)}{6ac}-\frac{(d+ex)^5(ae-cdx)}{6ac\,(a+cx^2)^3}$$

input
```
Int[(d + e*x)^6/(a + c*x^2)^4,x]
```

output
```
-1/6*((a*e - c*d*x)*(d + e*x)^5)/(a*c*(a + c*x^2)^3) + (5*(c*d^2 + a*e^2)*
(-1/4*((a*e - c*d*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^2) + (3*(c*d^2 + a*e^2)
*(-1/2*((a*e - c*d*x)*(d + e*x))/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))))/(4*a*c)))/(6*a*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.03

method	result
default	$\frac{-\frac{\left(11 e^6 a^3-15 d^2 e^4 a^2 c-15 d^4 e^2 a c^2-5 d^6 c^3\right) x^5}{16 c a^3}-\frac{3 d e^5 x^4}{c}-\frac{5\left(e^6 a^3+3 d^2 e^4 a^2 c-3 d^4 e^2 a c^2-d^6 c^3\right) x^3}{6 a^2 c^2}-\frac{d e^3\left(3 a e^2+5 c d^2\right) x^2}{c^2}-\frac{\left(5 e^6 a^3+15 d^2 e^4\right.}{}}{\left(c x^2+a\right)^3}$
risch	$\frac{-\frac{\left(11 e^6 a^3-15 d^2 e^4 a^2 c-15 d^4 e^2 a c^2-5 d^6 c^3\right) x^5}{16 c a^3}-\frac{3 d e^5 x^4}{c}-\frac{5\left(e^6 a^3+3 d^2 e^4 a^2 c-3 d^4 e^2 a c^2-d^6 c^3\right) x^3}{6 a^2 c^2}-\frac{d e^3\left(3 a e^2+5 c d^2\right) x^2}{c^2}-\frac{\left(5 e^6 a^3+15 d^2 e^4\right.}{}}{\left(c x^2+a\right)^3}$

input
```
int((e*x+d)^6/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/16*(11*a^3*e^6-15*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2-5*c^3*d^6)/c/a^3*x^5-
3*d*e^5/c*x^4-5/6*(a^3*e^6+3*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2-c^3*d^6)/a^2/c^
2*x^3-d*e^3*(3*a*e^2+5*c*d^2)/c^2*x^2-1/16*(5*a^3*e^6+15*a^2*c*d^2*e^4+15*
a*c^2*d^4*e^2-11*c^3*d^6)/a/c^3*x-1/3*d*e*(3*a^2*e^4+5*a*c*d^2*e^2+3*c^2*d
^4)/c^3)/(c*x^2+a)^3+5/16*(a^3*e^6+3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6
)/a^3/c^3/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 1026, normalized size of antiderivative = 3.54

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^6/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(288*a^4*c^3*d*e^5*x^4 + 96*a^4*c^3*d^5*e + 160*a^5*c^2*d^3*e^3 + 9
6*a^6*c*d*e^5 - 6*(5*a*c^6*d^6 + 15*a^2*c^5*d^4*e^2 + 15*a^3*c^4*d^2*e^4 -
 11*a^4*c^3*e^6)*x^5 - 80*(a^2*c^5*d^6 + 3*a^3*c^4*d^4*e^2 - 3*a^4*c^3*d^2
*e^4 - a^5*c^2*e^6)*x^3 + 96*(5*a^4*c^3*d^3*e^3 + 3*a^5*c^2*d*e^5)*x^2 + 1
5*(a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6 + (c^6*d^6
+ 3*a*c^5*d^4*e^2 + 3*a^2*c^4*d^2*e^4 + a^3*c^3*e^6)*x^6 + 3*(a*c^5*d^6 +
3*a^2*c^4*d^4*e^2 + 3*a^3*c^3*d^2*e^4 + a^4*c^2*e^6)*x^4 + 3*(a^2*c^4*d^6
+ 3*a^3*c^3*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*c*e^6)*x^2)*sqrt(-a*c)*log((
c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 6*(11*a^3*c^4*d^6 - 15*a^4*c^3*
d^4*e^2 - 15*a^5*c^2*d^2*e^4 - 5*a^6*c*e^6)*x)/(a^4*c^7*x^6 + 3*a^5*c^6*x^
4 + 3*a^6*c^5*x^2 + a^7*c^4), -1/48*(144*a^4*c^3*d*e^5*x^4 + 48*a^4*c^3*d^
5*e + 80*a^5*c^2*d^3*e^3 + 48*a^6*c*d*e^5 - 3*(5*a*c^6*d^6 + 15*a^2*c^5*d^
4*e^2 + 15*a^3*c^4*d^2*e^4 - 11*a^4*c^3*e^6)*x^5 - 40*(a^2*c^5*d^6 + 3*a^3
*c^4*d^4*e^2 - 3*a^4*c^3*d^2*e^4 - a^5*c^2*e^6)*x^3 + 48*(5*a^4*c^3*d^3*e^
3 + 3*a^5*c^2*d*e^5)*x^2 - 15*(a^3*c^3*d^6 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d
^2*e^4 + a^6*e^6 + (c^6*d^6 + 3*a*c^5*d^4*e^2 + 3*a^2*c^4*d^2*e^4 + a^3*c^
3*e^6)*x^6 + 3*(a*c^5*d^6 + 3*a^2*c^4*d^4*e^2 + 3*a^3*c^3*d^2*e^4 + a^4*c^
2*e^6)*x^4 + 3*(a^2*c^4*d^6 + 3*a^3*c^3*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*
c*e^6)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - 3*(11*a^3*c^4*d^6 - 15*a^4*c
^3*d^4*e^2 - 15*a^5*c^2*d^2*e^4 - 5*a^6*c*e^6)*x)/(a^4*c^7*x^6 + 3*a^5*...
```

Sympy [A] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx = -\frac{5\sqrt{-\frac{1}{a^7c^7}}(ae^2+cd^2)^3\log\left(-\frac{5a^4c^3\sqrt{-\frac{1}{a^7c^7}}(ae^2+cd^2)^3}{5a^3e^6+15a^2cd^2e^4+15ac^2d^4e^2+5c^3d^6}+x\right)}{32} + \frac{5\sqrt{-\frac{1}{a^7c^7}}(ae^2+cd^2)^3\log\left(\frac{5a^4c^3\sqrt{-\frac{1}{a^7c^7}}(ae^2+cd^2)^3}{5a^3e^6+15a^2cd^2e^4+15ac^2d^4e^2+5c^3d^6}+x\right)}{32} + \frac{-48a^5de^5-80a^4cd^3e^3-48a^3c^2d^5e-144a^3c^2de^5x^4+x^5(-33a^3c^2e^6+45a^2c^3d^2e^4+45ac^4d^4e^2+15c^5}{}$$

input
```
integrate((e*x+d)**6/(c*x**2+a)**4,x)
```

output
```
-5*sqrt(-1/(a**7*c**7))*(a*e**2 + c*d**2)**3*log(-5*a**4*c**3*sqrt(-1/(a**
7*c**7))*(a*e**2 + c*d**2)**3/(5*a**3*e**6 + 15*a**2*c*d**2*e**4 + 15*a*c*
*2*d**4*e**2 + 5*c**3*d**6) + x)/32 + 5*sqrt(-1/(a**7*c**7))*(a*e**2 + c*d
**2)**3*log(5*a**4*c**3*sqrt(-1/(a**7*c**7))*(a*e**2 + c*d**2)**3/(5*a**3*
e**6 + 15*a**2*c*d**2*e**4 + 15*a*c**2*d**4*e**2 + 5*c**3*d**6) + x)/32 +
(-48*a**5*d*e**5 - 80*a**4*c*d**3*e**3 - 48*a**3*c**2*d**5*e - 144*a**3*c*
*2*d*e**5*x**4 + x**5*(-33*a**3*c**2*e**6 + 45*a**2*c**3*d**2*e**4 + 45*a*
c**4*d**4*e**2 + 15*c**5*d**6) + x**3*(-40*a**4*c*e**6 - 120*a**3*c**2*d**
2*e**4 + 120*a**2*c**3*d**4*e**2 + 40*a*c**4*d**6) + x**2*(-144*a**4*c*d*e
**5 - 240*a**3*c**2*d**3*e**3) + x*(-15*a**5*e**6 - 45*a**4*c*d**2*e**4 -
45*a**3*c**2*d**4*e**2 + 33*a**2*c**3*d**6))/(48*a**6*c**3 + 144*a**5*c**4
*x**2 + 144*a**4*c**5*x**4 + 48*a**3*c**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx = -\frac{144\,a^3c^2de^5x^4+48\,a^3c^2d^5e+80\,a^4cd^3e^3+48\,a^5de^5-3\,(5\,c^5d^6+15\,ac^4d^4e^2+15\,a^2c^3d^2e^4-11\,a^3c^2e^6}{48} + \frac{5\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^3}$$

input
```
integrate((e*x+d)^6/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(144*a^3*c^2*d*e^5*x^4 + 48*a^3*c^2*d^5*e + 80*a^4*c*d^3*e^3 + 48*a^
5*d*e^5 - 3*(5*c^5*d^6 + 15*a*c^4*d^4*e^2 + 15*a^2*c^3*d^2*e^4 - 11*a^3*c^
2*e^6)*x^5 - 40*(a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 - 3*a^3*c^2*d^2*e^4 - a^4*c
*e^6)*x^3 + 48*(5*a^3*c^2*d^3*e^3 + 3*a^4*c*d*e^5)*x^2 - 3*(11*a^2*c^3*d^6
 - 15*a^3*c^2*d^4*e^2 - 15*a^4*c*d^2*e^4 - 5*a^5*e^6)*x)/(a^3*c^6*x^6 + 3*
a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 5/16*(c^3*d^6 + 3*a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 + a^3*e^6)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx = \frac{5\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^3} + \frac{15\,c^5d^6x^5+45\,ac^4d^4e^2x^5+45\,a^2c^3d^2e^4x^5-33\,a^3c^2e^6x^5-144\,a^3c^2de^5x^4+40\,ac^4d^6x^3+120\,a^2c^3d^4e^2x}{}$$

input
```
integrate((e*x+d)^6/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
5/16*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(15*c^5*d^6*x^5 + 45*a*c^4*d^4*e^2*x^5
 + 45*a^2*c^3*d^2*e^4*x^5 - 33*a^3*c^2*e^6*x^5 - 144*a^3*c^2*d*e^5*x^4 + 4
0*a*c^4*d^6*x^3 + 120*a^2*c^3*d^4*e^2*x^3 - 120*a^3*c^2*d^2*e^4*x^3 - 40*a
^4*c*e^6*x^3 - 240*a^3*c^2*d^3*e^3*x^2 - 144*a^4*c*d*e^5*x^2 + 33*a^2*c^3*
d^6*x - 45*a^3*c^2*d^4*e^2*x - 45*a^4*c*d^2*e^4*x - 15*a^5*e^6*x - 48*a^3*
c^2*d^5*e - 80*a^4*c*d^3*e^3 - 48*a^5*d*e^5)/((c*x^2 + a)^3*a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx = \frac{5\,\mathrm{atan}\left(\frac{\sqrt{c}\,x\,\left(c\,d^2+a\,e^2\right)^3}{\sqrt{a}\,\left(a^3\,e^6+3\,a^2\,c\,d^2\,e^4+3\,a\,c^2\,d^4\,e^2+c^3\,d^6\right)}\right)\,\left(c\,d^2+a\,e^2\right)^3}{16\,a^{7/2}\,c^{7/2}} - \frac{\frac{3\,d\,e^5\,x^4}{c}+\frac{x\,\left(5\,a^3\,e^6+15\,a^2\,c\,d^2\,e^4+15\,a\,c^2\,d^4\,e^2-11\,c^3\,d^6\right)}{16\,a\,c^3}+\frac{d\,e\,\left(3\,a^2\,e^4+5\,a\,c\,d^2\,e^2+3\,c^2\,d^4\right)}{3\,c^3}+\frac{5\,x^3\,\left(a^3\,e^6+3\,a^2\,c\,d^2\,e^4-3\,a\,c^2\,d^4\,e^2-\right.}{6\,a^2\,c^2}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

input
```
int((d + e*x)^6/(a + c*x^2)^4,x)
```

output
```
(5*atan((c^(1/2)*x*(a*e^2 + c*d^2)^3)/(a^(1/2)*(a^3*e^6 + c^3*d^6 + 3*a*c^
2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*(a*e^2 + c*d^2)^3)/(16*a^(7/2)*c^(7/2)) - (
(3*d*e^5*x^4)/c + (x*(5*a^3*e^6 - 11*c^3*d^6 + 15*a*c^2*d^4*e^2 + 15*a^2*c
*d^2*e^4))/(16*a*c^3) + (d*e*(3*a^2*e^4 + 3*c^2*d^4 + 5*a*c*d^2*e^2))/(3*c
^3) + (5*x^3*(a^3*e^6 - c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4))/(6*a
^2*c^2) - (x^5*(5*c^3*d^6 - 11*a^3*e^6 + 15*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e
^4))/(16*a^3*c) + (d*e^3*x^2*(3*a*e^2 + 5*c*d^2))/c^2)/(a^3 + c^3*x^6 + 3*
a^2*c*x^2 + 3*a*c^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.59

$$\int \frac{(d+ex)^6}{(a+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^6/(c*x^2+a)^4,x)
```

output

```
(15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**6*e**6 + 45*sqrt(c)*s
qrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*c*d**2*e**4 + 45*sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*c*e**6*x**2 + 45*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a**4*c**2*d**4*e**2 + 135*sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*a**4*c**2*d**2*e**4*x**2 + 45*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a**4*c**2*e**6*x**4 + 15*sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a**3*c**3*d**6 + 135*sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*a**3*c**3*d**4*e**2*x**2 + 135*sqrt(c)*sqrt(a)*atan((c*x)/(
sqrt(c)*sqrt(a)))*a**3*c**3*d**2*e**4*x**4 + 15*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**3*c**3*e**6*x**6 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(s
qrt(c)*sqrt(a)))*a**2*c**4*d**6*x**2 + 135*sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*a**2*c**4*d**4*e**2*x**4 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(s
qrt(c)*sqrt(a)))*a**2*c**4*d**2*e**4*x**6 + 45*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a*c**5*d**6*x**4 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(
c)*sqrt(a)))*a*c**5*d**4*e**2*x**6 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*c**6*d**6*x**6 - 15*a**6*c*e**6*x - 80*a**5*c**2*d**3*e**3 - 4
5*a**5*c**2*d**2*e**4*x - 40*a**5*c**2*e**6*x**3 - 48*a**4*c**3*d**5*e - 4
5*a**4*c**3*d**4*e**2*x - 240*a**4*c**3*d**3*e**3*x**2 - 120*a**4*c**3*d**
2*e**4*x**3 - 33*a**4*c**3*e**6*x**5 + 33*a**3*c**4*d**6*x + 120*a**3*c**4
*d**4*e**2*x**3 + 45*a**3*c**4*d**2*e**4*x**5 + 48*a**3*c**4*d*e**5*x**...
```

3.124 $\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 240

$$\begin{aligned}\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx &= -\frac{ae(5c^2d^4-10acd^2e^2+a^2e^4)-cd(c^2d^4-10acd^2e^2+5a^2e^4)\,x}{6ac^3\,(a+cx^2)^3} \\ &\quad - \frac{12a^2e^3(5cd^2-ae^2)-5cd(c^2d^4+2acd^2e^2-7a^2e^4)\,x}{24a^2c^3\,(a+cx^2)^2} \\ &\quad - \frac{8a^3e^5-5cd(cd^2+ae^2)^2\,x}{16a^3c^3\,(a+cx^2)} + \frac{5d(cd^2+ae^2)^2\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}\end{aligned}$$

output
```
-1/6*(a*e*(a^2*e^4-10*a*c*d^2*e^2+5*c^2*d^4)-c*d*(5*a^2*e^4-10*a*c*d^2*e^2
+c^2*d^4)*x)/a/c^3/(c*x^2+a)^3-1/24*(12*a^2*e^3*(-a*e^2+5*c*d^2)-5*c*d*(-7
*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*x)/a^2/c^3/(c*x^2+a)^2-1/16*(8*a^3*e^5-5*c
*d*(a*e^2+c*d^2)^2*x)/a^3/c^3/(c*x^2+a)+5/16*d*(a*e^2+c*d^2)^2*arctan(c^(1
/2)*x/a^(1/2))/a^(7/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx$$
$$= \frac{\frac{\sqrt{a}(-8a^5e^5+15c^5d^5x^5+10ac^4d^3x^3(4d^2+3e^2x^2)-a^4ce^3(40d^2+15dex+24e^2x^2)-2a^3c^2e(20d^4+15d^3ex+60d^2e^2x^2+20de^3x^3+12e^4x^4)+a^2c^3d}{(a+cx^2)^3}}{48a^{7/2}c^3}$$

input
```
Integrate[(d + e*x)^5/(a + c*x^2)^4,x]
```

output
```
((Sqrt[a]*(-8*a^5*e^5 + 15*c^5*d^5*x^5 + 10*a*c^4*d^3*x^3*(4*d^2 + 3*e^2*x
^2) - a^4*c*e^3*(40*d^2 + 15*d*e*x + 24*e^2*x^2) - 2*a^3*c^2*e*(20*d^4 + 1
5*d^3*e*x + 60*d^2*e^2*x^2 + 20*d*e^3*x^3 + 12*e^4*x^4) + a^2*c^3*d*x*(33*
d^4 + 80*d^2*e^2*x^2 + 15*e^4*x^4)))/(a + c*x^2)^3 + 15*Sqrt[c]*d*(c*d^2 +
 a*e^2)^2*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {490, 487, 487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx$$
$$\Big\downarrow \; 490$$
$$\frac{5d\int \frac{(d+ex)^4}{(cx^2+a)^3}dx}{6a} + \frac{x(d+ex)^5}{6a(a+cx^2)^3}$$
$$\Big\downarrow \; 487$$

$$\frac{5d\left(\frac{3(ae^2+cd^2)\int \frac{(d+ex)^2}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac(a+cx^2)^2}\right)}{6a}+\frac{x(d+ex)^5}{6a\left(a+cx^2\right)^3}$$

$$\Big\downarrow \text{ 487}$$

$$\frac{5d\left(\frac{3(ae^2+cd^2)\left(\frac{\left(ae^2+cd^2\right)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac\left(a+cx^2\right)}\right)}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac(a+cx^2)^2}\right)}{6a}+\frac{x(d+ex)^5}{6a\left(a+cx^2\right)^3}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{5d\left(\frac{3(ae^2+cd^2)\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ae^2+cd^2\right)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac\left(a+cx^2\right)}\right)}{4ac}-\frac{(d+ex)^3(ae-cdx)}{4ac(a+cx^2)^2}\right)}{6a}+\frac{x(d+ex)^5}{6a\left(a+cx^2\right)^3}$$

input

```
Int[(d + e*x)^5/(a + c*x^2)^4,x]
```

output

```
(x*(d + e*x)^5)/(6*a*(a + c*x^2)^3) + (5*d*(-1/4*((a*e - c*d*x)*(d + e*x)^
3)/(a*c*(a + c*x^2)^2) + (3*(c*d^2 + a*e^2)*(-1/2*((a*e - c*d*x)*(d + e*x)
)/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(
3/2)*c^(3/2))))/(4*a*c)))/(6*a)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 490
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] - Simp[c*(n/(2*a*(
p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{5d\left(a^2e^4+2acd^2e^2+c^2d^4\right)x^5}{16a^3}-\frac{e^5x^4}{2c}-\frac{5d\left(a^2e^4-2acd^2e^2-c^2d^4\right)x^3}{6a^2c}-\frac{e^3\left(ae^2+5cd^2\right)x^2}{2c^2}-\frac{d\left(5a^2e^4+10acd^2e^2-11c^2d^4\right)x}{16ac^2}-\frac{e\left(a^2e^4+5acd^2e^2\right.}{6c^3}}{\left(cx^2+a\right)^3}$
risch	$\frac{\frac{5d\left(a^2e^4+2acd^2e^2+c^2d^4\right)x^5}{16a^3}-\frac{e^5x^4}{2c}-\frac{5d\left(a^2e^4-2acd^2e^2-c^2d^4\right)x^3}{6a^2c}-\frac{e^3\left(ae^2+5cd^2\right)x^2}{2c^2}-\frac{d\left(5a^2e^4+10acd^2e^2-11c^2d^4\right)x}{16ac^2}-\frac{e\left(a^2e^4+5acd^2e^2\right.}{6c^3}}{\left(cx^2+a\right)^3}$

input
```
int((e*x+d)^5/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(5/16*d*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/a^3*x^5-1/2*e^5/c*x^4-5/6*d*(a^2*e
^4-2*a*c*d^2*e^2-c^2*d^4)/a^2/c*x^3-1/2*e^3*(a*e^2+5*c*d^2)/c^2*x^2-1/16*d
*(5*a^2*e^4+10*a*c*d^2*e^2-11*c^2*d^4)/a/c^2*x-1/6*e*(a^2*e^4+5*a*c*d^2*e^
2+5*c^2*d^4)/c^3)/(c*x^2+a)^3+5/16*d*(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/c^2/a
^3/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 822, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(48*a^4*c^2*e^5*x^4 + 80*a^4*c^2*d^4*e + 80*a^5*c*d^2*e^3 + 16*a^6*
e^5 - 30*(a*c^5*d^5 + 2*a^2*c^4*d^3*e^2 + a^3*c^3*d*e^4)*x^5 - 80*(a^2*c^4
*d^5 + 2*a^3*c^3*d^3*e^2 - a^4*c^2*d*e^4)*x^3 + 48*(5*a^4*c^2*d^2*e^3 + a^
5*c*e^5)*x^2 + 15*(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (c^5*d^5 +
2*a*c^4*d^3*e^2 + a^2*c^3*d*e^4)*x^6 + 3*(a*c^4*d^5 + 2*a^2*c^3*d^3*e^2 +
a^3*c^2*d*e^4)*x^4 + 3*(a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2
)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 6*(11*a^3*c^3
*d^5 - 10*a^4*c^2*d^3*e^2 - 5*a^5*c*d*e^4)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4
 + 3*a^6*c^4*x^2 + a^7*c^3), -1/48*(24*a^4*c^2*e^5*x^4 + 40*a^4*c^2*d^4*e
+ 40*a^5*c*d^2*e^3 + 8*a^6*e^5 - 15*(a*c^5*d^5 + 2*a^2*c^4*d^3*e^2 + a^3*c
^3*d*e^4)*x^5 - 40*(a^2*c^4*d^5 + 2*a^3*c^3*d^3*e^2 - a^4*c^2*d*e^4)*x^3 +
 24*(5*a^4*c^2*d^2*e^3 + a^5*c*e^5)*x^2 - 15*(a^3*c^2*d^5 + 2*a^4*c*d^3*e^
2 + a^5*d*e^4 + (c^5*d^5 + 2*a*c^4*d^3*e^2 + a^2*c^3*d*e^4)*x^6 + 3*(a*c^4
*d^5 + 2*a^2*c^3*d^3*e^2 + a^3*c^2*d*e^4)*x^4 + 3*(a^2*c^3*d^5 + 2*a^3*c^2
*d^3*e^2 + a^4*c*d*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - 3*(11*a^3*c
^3*d^5 - 10*a^4*c^2*d^3*e^2 - 5*a^5*c*d*e^4)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x
^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [A] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx = -\frac{5d\sqrt{-\frac{1}{a^7c^5}}\left(ae^2+cd^2\right)^2\log\left(-\frac{5a^4c^2d\sqrt{-\frac{1}{a^7c^5}}\left(ae^2+cd^2\right)^2}{5a^2de^4+10acd^3e^2+5c^2d^5}+x\right)}{32} + \frac{5d\sqrt{-\frac{1}{a^7c^5}}\left(ae^2+cd^2\right)^2\log\left(\frac{5a^4c^2d\sqrt{-\frac{1}{a^7c^5}}\left(ae^2+cd^2\right)^2}{5a^2de^4+10acd^3e^2+5c^2d^5}+x\right)}{32} + \frac{-8a^5e^5-40a^4cd^2e^3-40a^3c^2d^4e-24a^3c^2e^5x^4+x^5\cdot\left(15a^2c^3de^4+30ac^4d^3e^2+15c^5d^5\right)+x^3(-40a^3c^2}{48a^6c^3+144a^5c^4x^2+}$$

input
```
integrate((e*x+d)**5/(c*x**2+a)**4,x)
```

output
```
-5*d*sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)**2*log(-5*a**4*c**2*d*sqrt(-1/
(a**7*c**5))*(a*e**2 + c*d**2)**2/(5*a**2*d*e**4 + 10*a*c*d**3*e**2 + 5*c*
*2*d**5) + x)/32 + 5*d*sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)**2*log(5*a**
4*c**2*d*sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)**2/(5*a**2*d*e**4 + 10*a*c
*d**3*e**2 + 5*c**2*d**5) + x)/32 + (-8*a**5*e**5 - 40*a**4*c*d**2*e**3 -
40*a**3*c**2*d**4*e - 24*a**3*c**2*e**5*x**4 + x**5*(15*a**2*c**3*d*e**4 +
 30*a*c**4*d**3*e**2 + 15*c**5*d**5) + x**3*(-40*a**3*c**2*d*e**4 + 80*a**
2*c**3*d**3*e**2 + 40*a*c**4*d**5) + x**2*(-24*a**4*c*e**5 - 120*a**3*c**2
*d**2*e**3) + x*(-15*a**4*c*d*e**4 - 30*a**3*c**2*d**3*e**2 + 33*a**2*c**3
*d**5))/(48*a**6*c**3 + 144*a**5*c**4*x**2 + 144*a**4*c**5*x**4 + 48*a**3*
c**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx =$$
$$-\frac{24\,a^3c^2e^5x^4+40\,a^3c^2d^4e+40\,a^4cd^2e^3+8\,a^5e^5-15\,(c^5d^5+2\,ac^4d^3e^2+a^2c^3de^4)x^5-40\,(ac^4d^5+2\,a^2 \cdots}{48\,(a^3c^6x^6+3\,a^4c^5x^4+3 \cdots}$$
$$+\frac{5\,(c^2d^5+2\,acd^3e^2+a^2de^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(24*a^3*c^2*e^5*x^4 + 40*a^3*c^2*d^4*e + 40*a^4*c*d^2*e^3 + 8*a^5*e^
5 - 15*(c^5*d^5 + 2*a*c^4*d^3*e^2 + a^2*c^3*d*e^4)*x^5 - 40*(a*c^4*d^5 + 2
*a^2*c^3*d^3*e^2 - a^3*c^2*d*e^4)*x^3 + 24*(5*a^3*c^2*d^2*e^3 + a^4*c*e^5)
*x^2 - 3*(11*a^2*c^3*d^5 - 10*a^3*c^2*d^3*e^2 - 5*a^4*c*d*e^4)*x)/(a^3*c^6
*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 5/16*(c^2*d^5 + 2*a*c*d^
3*e^2 + a^2*d*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx = \frac{5\,(c^2d^5+2\,acd^3e^2+a^2de^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2} + \frac{15\,c^5d^5x^5+30\,ac^4d^3e^2x^5+15\,a^2c^3de^4x^5-24\,a^3c^2e^5x^4+40\,ac^4d^5x^3+80\,a^2c^3d^3e^2x^3-40\,a^3c^2de^4x^3-}{48\,(cx}$$

input
```
integrate((e*x+d)^5/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
5/16*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c
)*a^3*c^2) + 1/48*(15*c^5*d^5*x^5 + 30*a*c^4*d^3*e^2*x^5 + 15*a^2*c^3*d*e^
4*x^5 - 24*a^3*c^2*e^5*x^4 + 40*a*c^4*d^5*x^3 + 80*a^2*c^3*d^3*e^2*x^3 - 4
0*a^3*c^2*d*e^4*x^3 - 120*a^3*c^2*d^2*e^3*x^2 - 24*a^4*c*e^5*x^2 + 33*a^2*
c^3*d^5*x - 30*a^3*c^2*d^3*e^2*x - 15*a^4*c*d*e^4*x - 40*a^3*c^2*d^4*e - 4
0*a^4*c*d^2*e^3 - 8*a^5*e^5)/((c*x^2 + a)^3*a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx = \frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x\,\left(c\,d^2+a\,e^2\right)^2}{\sqrt{a}\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)^2}{16\,a^{7/2}\,c^{5/2}} - \frac{\frac{e\,\left(a^2\,e^4+5\,a\,c\,d^2\,e^2+5\,c^2\,d^4\right)}{6\,c^3}+\frac{e^5\,x^4}{2\,c}+\frac{e^3\,x^2\,\left(5\,c\,d^2+a\,e^2\right)}{2\,c^2}-\frac{5\,d\,x^5\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{16\,a^3}+\frac{d\,x\,\left(5\,a^2\,e^4+10\,a\,c\,d^2\,e^2-11\,c^2\,d^4\right)}{16\,a\,c^2}-}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

input
```
int((d + e*x)^5/(a + c*x^2)^4,x)
```

output
```
(5*d*atan((c^(1/2)*x*(a*e^2 + c*d^2)^2)/(a^(1/2)*(a^2*e^4 + c^2*d^4 + 2*a*
c*d^2*e^2)))*(a*e^2 + c*d^2)^2)/(16*a^(7/2)*c^(5/2)) - ((e*(a^2*e^4 + 5*c^
2*d^4 + 5*a*c*d^2*e^2))/(6*c^3) + (e^5*x^4)/(2*c) + (e^3*x^2*(a*e^2 + 5*c*
d^2))/(2*c^2) - (5*d*x^5*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(16*a^3) + (
d*x*(5*a^2*e^4 - 11*c^2*d^4 + 10*a*c*d^2*e^2))/(16*a*c^2) - (5*d*x^3*(c^2*
d^4 - a^2*e^4 + 2*a*c*d^2*e^2))/(6*a^2*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 +
3*a*c^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.40

$$\int \frac{(d+ex)^5}{(a+cx^2)^4}\,dx$$

$$= \frac{-40a^5c\,d^2e^3 - 40a^4c^2d^4e + 33a^3c^3d^5x + 8a^3c^3e^5x^6 + 40a^2c^4d^5x^3 + 15a\,c^5d^5x^5 + 15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right) c}{}$$

input
```
int((e*x+d)^5/(c*x^2+a)^4,x)
```

output
```
(15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*d*e**4 + 30*sqrt(c)
*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d**3*e**2 + 45*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d*e**4*x**2 + 15*sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c**2*d**5 + 90*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**3*c**2*d**3*e**2*x**2 + 45*sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*a**3*c**2*d*e**4*x**4 + 45*sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a**2*c**3*d**5*x**2 + 90*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a**2*c**3*d**3*e**2*x**4 + 15*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**2*c**3*d*e**4*x**6 + 45*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a*c**4*d**5*x**4 + 30*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a*c**4*d**3*e**2*x**6 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(
c)*sqrt(a)))*c**5*d**5*x**6 - 40*a**5*c*d**2*e**3 - 15*a**5*c*d*e**4*x - 4
0*a**4*c**2*d**4*e - 30*a**4*c**2*d**3*e**2*x - 120*a**4*c**2*d**2*e**3*x*
*2 - 40*a**4*c**2*d*e**4*x**3 + 33*a**3*c**3*d**5*x + 80*a**3*c**3*d**3*e*
*2*x**3 + 15*a**3*c**3*d*e**4*x**5 + 8*a**3*c**3*e**5*x**6 + 40*a**2*c**4*
d**5*x**3 + 30*a**2*c**4*d**3*e**2*x**5 + 15*a*c**5*d**5*x**5)/(48*a**4*c*
*3*(a**3 + 3*a**2*c*x**2 + 3*a*c**2*x**4 + c**3*x**6))
```

3.125 $\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 218

$$\begin{aligned}\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx = &-\frac{4ade(cd^2-ae^2)-(c^2d^4-6acd^2e^2+a^2e^4)\,x}{6ac^2(a+cx^2)^3}\\ &-\frac{24a^2de^3-(5c^2d^4+6acd^2e^2-7a^2e^4)\,x}{24a^2c^2(a+cx^2)^2}\\ &+\frac{(cd^2+ae^2)(5cd^2+ae^2)\,x}{16a^3c^2(a+cx^2)}+\frac{(cd^2+ae^2)(5cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}\end{aligned}$$

output
```
-1/6*(4*a*d*e*(-a*e^2+c*d^2)-(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*x)/a/c^2/(c*x
^2+a)^3-1/24*(24*a^2*d*e^3-(-7*a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)*x)/a^2/c^2
/(c*x^2+a)^2+1/16*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*x/a^3/c^2/(c*x^2+a)+1/16*(
a*e^2+c*d^2)*(a*e^2+5*c*d^2)*arctan(c^(1/2)*x/a^(1/2))/a^(7/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx$$
$$= \frac{15c^4d^4x^5 - a^4e^3(16d+3ex) + 2ac^3d^2x^3(20d^2+9e^2x^2) - 2a^3ce(16d^3+9d^2ex+24de^2x^2+4e^3x^3) + 3a^2}{48a^3c^2\left(a+cx^2\right)^3}$$
$$+ \frac{(5c^2d^4+6acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}$$

input
```
Integrate[(d + e*x)^4/(a + c*x^2)^4,x]
```

output
```
(15*c^4*d^4*x^5 - a^4*e^3*(16*d + 3*e*x) + 2*a*c^3*d^2*x^3*(20*d^2 + 9*e^2
*x^2) - 2*a^3*c*e*(16*d^3 + 9*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3) + 3*a^2*
c^2*x*(11*d^4 + 16*d^2*e^2*x^2 + e^4*x^4))/(48*a^3*c^2*(a + c*x^2)^3) + ((
5*c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7
/2)*c^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {494, 25, 678, 487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx$$
$$\Big\downarrow \; 494$$
$$\frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3} - \frac{\int -\frac{(d+ex)^3(5d+ex)}{(cx^2+a)^3}dx}{6a}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(d+ex)^3(5d+ex)}{(cx^2+a)^3}dx}{6a}+\frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3}$$

$$\Big\downarrow \ 678$$

$$\frac{\frac{3(ae^2+5cd^2)\int \frac{(d+ex)^2}{\left(cx^2+a\right)^2}dx}{4ac}-\frac{(d+ex)^3(ae-5cdx)}{4ac(a+cx^2)^2}}{6a}+\frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3}$$

$$\Big\downarrow \ 487$$

$$\frac{\frac{3(ae^2+5cd^2)\left(\frac{\left(ae^2+cd^2\right)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac\left(a+cx^2\right)}\right)}{4ac}-\frac{(d+ex)^3(ae-5cdx)}{4ac(a+cx^2)^2}}{6a}+\frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{3(ae^2+5cd^2)\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ae^2+cd^2\right)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac\left(a+cx^2\right)}\right)}{4ac}-\frac{(d+ex)^3(ae-5cdx)}{4ac(a+cx^2)^2}}{6a}+\frac{x(d+ex)^4}{6a\left(a+cx^2\right)^3}$$

input
```
Int[(d + e*x)^4/(a + c*x^2)^4,x]
```

output
```
(x*(d + e*x)^4)/(6*a*(a + c*x^2)^3) + (-1/4*((a*e - 5*c*d*x)*(d + e*x)^3)/
(a*c*(a + c*x^2)^2) + (3*(5*c*d^2 + a*e^2)*(-1/2*((a*e - c*d*x)*(d + e*x))
/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3
/2)*c^(3/2))))/(4*a*c))/(6*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 678
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[m*((c*d*f + a*e*g)/(2*a*c*(p + 1)))   Int[(d + e*x)^(
m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && EqQ[S
implify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.91

method	result
default	$\frac{\frac{\left(a^{2} e^{4}+6 a c \,d^{2} e^{2}+5 c^{2} d^{4}\right) x^{5}}{16 a^{3}}-\frac{\left(a^{2} e^{4}-6 a c \,d^{2} e^{2}-5 c^{2} d^{4}\right) x^{3}}{6 a^{2} c}-\frac{d \,e^{3} x^{2}}{c}-\frac{\left(a^{2} e^{4}+6 a c \,d^{2} e^{2}-11 c^{2} d^{4}\right) x}{16 a \,c^{2}}-\frac{d e \left(a \,e^{2}+2 c \,d^{2}\right)}{3 c^{2}}}{\left(c \,x^{2}+a \right)^{3}}+\frac{\left(a^{2} e^{4}+6 a c \,d^{2} e^{2}+5\right.}{16 a^{3}}$
risch	$\frac{\frac{\left(a^{2} e^{4}+6 a c \,d^{2} e^{2}+5 c^{2} d^{4}\right) x^{5}}{16 a^{3}}-\frac{\left(a^{2} e^{4}-6 a c \,d^{2} e^{2}-5 c^{2} d^{4}\right) x^{3}}{6 a^{2} c}-\frac{d \,e^{3} x^{2}}{c}-\frac{\left(a^{2} e^{4}+6 a c \,d^{2} e^{2}-11 c^{2} d^{4}\right) x}{16 a \,c^{2}}-\frac{d e \left(a \,e^{2}+2 c \,d^{2}\right)}{3 c^{2}}}{\left(c \,x^{2}+a \right)^{3}}-\frac{\ln \left(c x +\sqrt{-a c}\right) e^{4}}{32 \sqrt{-a c}\, c^{2} a}-$

input
```
int((e*x+d)^4/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)/a^3*x^5-1/6*(a^2*e^4-6*a*c*d^2*e^2
-5*c^2*d^4)/a^2/c*x^3-d*e^3*x^2/c-1/16*(a^2*e^4+6*a*c*d^2*e^2-11*c^2*d^4)/
a/c^2*x-1/3*d*e*(a*e^2+2*c*d^2)/c^2)/(c*x^2+a)^3+1/16*(a^2*e^4+6*a*c*d^2*e
^2+5*c^2*d^4)/a^3/c^2/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx$$

$$= \left[-\frac{96\,a^4c^2de^3x^2 + 64\,a^4c^2d^3e + 32\,a^5cde^3 - 6\,(5\,ac^5d^4 + 6\,a^2c^4d^2e^2 + a^3c^3e^4)x^5 - 16\,(5\,a^2c^4d^4 + 6\,a^3c^3d}{} \right.$$

$$-\frac{48\,a^4c^2de^3x^2 + 32\,a^4c^2d^3e + 16\,a^5cde^3 - 3\,(5\,ac^5d^4 + 6\,a^2c^4d^2e^2 + a^3c^3e^4)x^5 - 8\,(5\,a^2c^4d^4 + 6\,a^3c^3d^2e}{}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(96*a^4*c^2*d*e^3*x^2 + 64*a^4*c^2*d^3*e + 32*a^5*c*d*e^3 - 6*(5*a*
c^5*d^4 + 6*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)*x^5 - 16*(5*a^2*c^4*d^4 + 6*a^3
*c^3*d^2*e^2 - a^4*c^2*e^4)*x^3 + 3*(5*a^3*c^2*d^4 + 6*a^4*c*d^2*e^2 + a^5
*e^4 + (5*c^5*d^4 + 6*a*c^4*d^2*e^2 + a^2*c^3*e^4)*x^6 + 3*(5*a*c^4*d^4 +
6*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4 + 3*(5*a^2*c^3*d^4 + 6*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a
)) - 6*(11*a^3*c^3*d^4 - 6*a^4*c^2*d^2*e^2 - a^5*c*e^4)*x)/(a^4*c^6*x^6 +
3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3), -1/48*(48*a^4*c^2*d*e^3*x^2 + 32
*a^4*c^2*d^3*e + 16*a^5*c*d*e^3 - 3*(5*a*c^5*d^4 + 6*a^2*c^4*d^2*e^2 + a^3
*c^3*e^4)*x^5 - 8*(5*a^2*c^4*d^4 + 6*a^3*c^3*d^2*e^2 - a^4*c^2*e^4)*x^3 -
3*(5*a^3*c^2*d^4 + 6*a^4*c*d^2*e^2 + a^5*e^4 + (5*c^5*d^4 + 6*a*c^4*d^2*e^
2 + a^2*c^3*e^4)*x^6 + 3*(5*a*c^4*d^4 + 6*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x
^4 + 3*(5*a^2*c^3*d^4 + 6*a^3*c^2*d^2*e^2 + a^4*c*e^4)*x^2)*sqrt(a*c)*arct
an(sqrt(a*c)*x/a) - 3*(11*a^3*c^3*d^4 - 6*a^4*c^2*d^2*e^2 - a^5*c*e^4)*x)/
(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(206) = 412$.

Time = 1.51 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^7c^5}}(ae^2+cd^2)\,(ae^2+5cd^2)\log\left(-\frac{a^4c^2\sqrt{-\frac{1}{a^7c^5}}(ae^2+cd^2)(ae^2+5cd^2)}{a^2e^4+6acd^2e^2+5c^2d^4}+x\right)}{32}$$
$$+\frac{\sqrt{-\frac{1}{a^7c^5}}(ae^2+cd^2)\,(ae^2+5cd^2)\log\left(\frac{a^4c^2\sqrt{-\frac{1}{a^7c^5}}(ae^2+cd^2)(ae^2+5cd^2)}{a^2e^4+6acd^2e^2+5c^2d^4}+x\right)}{32}$$
$$+\frac{-16a^4de^3-32a^3cd^3e-48a^3cde^3x^2+x^5\cdot(3a^2c^2e^4+18ac^3d^2e^2+15c^4d^4)+x^3(-8a^3ce^4+48a^2c^2d^2e^2}{48a^6c^2+144a^5c^3x^2+144a^4c^4x^4+48a^3c^5x^6}$$

input
```
integrate((e*x+d)**4/(c*x**2+a)**4,x)
```

output
```
-sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)*(a*e**2 + 5*c*d**2)*log(-a**4*c**2
*sqrt(-1/(a**7*c**5))*(a*e**2 + c*d**2)*(a*e**2 + 5*c*d**2)/(a**2*e**4 + 6
*a*c*d**2*e**2 + 5*c**2*d**4) + x)/32 + sqrt(-1/(a**7*c**5))*(a*e**2 + c*d
**2)*(a*e**2 + 5*c*d**2)*log(a**4*c**2*sqrt(-1/(a**7*c**5))*(a*e**2 + c*d*
*2)*(a*e**2 + 5*c*d**2)/(a**2*e**4 + 6*a*c*d**2*e**2 + 5*c**2*d**4) + x)/3
2 + (-16*a**4*d*e**3 - 32*a**3*c*d**3*e - 48*a**3*c*d*e**3*x**2 + x**5*(3*
a**2*c**2*e**4 + 18*a*c**3*d**2*e**2 + 15*c**4*d**4) + x**3*(-8*a**3*c*e**
4 + 48*a**2*c**2*d**2*e**2 + 40*a*c**3*d**4) + x*(-3*a**4*e**4 - 18*a**3*c
*d**2*e**2 + 33*a**2*c**2*d**4))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*
a**4*c**4*x**4 + 48*a**3*c**5*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx = -\frac{48\,a^3cde^3x^2+32\,a^3cd^3e+16\,a^4de^3-3\,(5\,c^4d^4+6\,ac^3d^2e^2+a^2c^2e^4)x^5-8\,(5\,ac^3d^4+6\,a^2c^2d^2e^2-a^3c}{48\,(a^3c^5x^6+3\,a^4c^4x^4+3\,a^5c^3x^2+a^6c^2)} + \frac{(5\,c^2d^4+6\,acd^2e^2+a^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(48*a^3*c*d*e^3*x^2 + 32*a^3*c*d^3*e + 16*a^4*d*e^3 - 3*(5*c^4*d^4 +
 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^5 - 8*(5*a*c^3*d^4 + 6*a^2*c^2*d^2*e^2 -
 a^3*c*e^4)*x^3 - 3*(11*a^2*c^2*d^4 - 6*a^3*c*d^2*e^2 - a^4*e^4)*x)/(a^3*c
^5*x^6 + 3*a^4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(5*c^2*d^4 + 6*a*
c*d^2*e^2 + a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx = \frac{(5\,c^2d^4+6\,acd^2e^2+a^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2} + \frac{15\,c^4d^4x^5+18\,ac^3d^2e^2x^5+3\,a^2c^2e^4x^5+40\,ac^3d^4x^3+48\,a^2c^2d^2e^2x^3-8\,a^3ce^4x^3-48\,a^3cde^3x^2+33\,a^2}{48\,(cx^2+a)^3a^3c^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(5*c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c
)*a^3*c^2) + 1/48*(15*c^4*d^4*x^5 + 18*a*c^3*d^2*e^2*x^5 + 3*a^2*c^2*e^4*x
^5 + 40*a*c^3*d^4*x^3 + 48*a^2*c^2*d^2*e^2*x^3 - 8*a^3*c*e^4*x^3 - 48*a^3*
c*d*e^3*x^2 + 33*a^2*c^2*d^4*x - 18*a^3*c*d^2*e^2*x - 3*a^4*e^4*x - 32*a^3
*c*d^3*e - 16*a^4*d*e^3)/((c*x^2 + a)^3*a^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x\,(c\,d^2+a\,e^2)\,(5\,c\,d^2+a\,e^2)}{\sqrt{a}\,(a^2\,e^4+6\,a\,c\,d^2\,e^2+5\,c^2\,d^4)}\right)\,(c\,d^2+a\,e^2)\,(5\,c\,d^2+a\,e^2)}{16\,a^{7/2}\,c^{5/2}} - \frac{\frac{d\,e^3\,x^2}{c} - \frac{x^5\,(a^2\,e^4+6\,a\,c\,d^2\,e^2+5\,c^2\,d^4)}{16\,a^3} + \frac{x\,(a^2\,e^4+6\,a\,c\,d^2\,e^2-11\,c^2\,d^4)}{16\,a\,c^2} + \frac{d\,e\,(2\,c\,d^2+a\,e^2)}{3\,c^2} - \frac{x^3\,(-a^2\,e^4+6\,a\,c\,d^2\,e^2+5\,c^2\,d^4)}{6\,a^2\,c}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

input
```
int((d + e*x)^4/(a + c*x^2)^4,x)
```

output
```
(atan((c^(1/2)*x*(a*e^2 + c*d^2)*(a*e^2 + 5*c*d^2))/(a^(1/2)*(a^2*e^4 + 5*
c^2*d^4 + 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2)*(a*e^2 + 5*c*d^2))/(16*a^(7/2)*
c^(5/2)) - ((d*e^3*x^2)/c - (x^5*(a^2*e^4 + 5*c^2*d^4 + 6*a*c*d^2*e^2))/(1
6*a^3) + (x*(a^2*e^4 - 11*c^2*d^4 + 6*a*c*d^2*e^2))/(16*a*c^2) + (d*e*(a*e
^2 + 2*c*d^2))/(3*c^2) - (x^3*(5*c^2*d^4 - a^2*e^4 + 6*a*c*d^2*e^2))/(6*a^
2*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.53

$$\int \frac{(d+ex)^4}{(a+cx^2)^4}\,dx = \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^5e^4 + 18\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^4c\,d^2e^2 + 9\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^4c\,e^4x^2 + 15\sqrt{c}\,\sqrt{a}}{}$$

input
```
int((e*x+d)^4/(c*x^2+a)^4,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*e**4 + 18*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d**2*e**2 + 9*sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*e**4*x**2 + 15*sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*a**3*c**2*d**4 + 54*sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*a**3*c**2*d**2*e**2*x**2 + 9*sqrt(c)*sqrt(a)*atan((c*x)/(sq
rt(c)*sqrt(a)))*a**3*c**2*e**4*x**4 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(
c)*sqrt(a)))*a**2*c**3*d**4*x**2 + 54*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*a**2*c**3*d**2*e**2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)
*sqrt(a)))*a**2*c**3*e**4*x**6 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sq
rt(a)))*a*c**4*d**4*x**4 + 18*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))
)*a*c**4*d**2*e**2*x**6 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))
*c**5*d**4*x**6 - 16*a**5*c*d*e**3 - 3*a**5*c*e**4*x - 32*a**4*c**2*d**3*e
 - 18*a**4*c**2*d**2*e**2*x - 48*a**4*c**2*d*e**3*x**2 - 8*a**4*c**2*e**4*
x**3 + 33*a**3*c**3*d**4*x + 48*a**3*c**3*d**2*e**2*x**3 + 3*a**3*c**3*e**
4*x**5 + 40*a**2*c**4*d**4*x**3 + 18*a**2*c**4*d**2*e**2*x**5 + 15*a*c**5*
d**4*x**5)/(48*a**4*c**3*(a**3 + 3*a**2*c*x**2 + 3*a*c**2*x**4 + c**3*x**6
))
```

3.126 $\int \frac{(d+ex)^3}{(a+cx^2)^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 174

$$\int \frac{(d+ex)^3}{(a+cx^2)^4} dx = -\frac{ae(3cd^2-ae^2)-cd(cd^2-3ae^2)x}{6ac^2(a+cx^2)^3} - \frac{6a^2e^3-cd(5cd^2+3ae^2)x}{24a^2c^2(a+cx^2)^2} + \frac{d\left(5d^2+\frac{3ae^2}{c}\right)x}{16a^3(a+cx^2)} + \frac{d(5cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

output
```
-1/6*(a*e*(-a*e^2+3*c*d^2)-c*d*(-3*a*e^2+c*d^2)*x)/a/c^2/(c*x^2+a)^3-1/24*
(6*a^2*e^3-c*d*(3*a*e^2+5*c*d^2)*x)/a^2/c^2/(c*x^2+a)^2+1/16*d*(5*d^2+3*a*
e^2/c)*x/a^3/(c*x^2+a)+1/16*d*(3*a*e^2+5*c*d^2)*arctan(c^(1/2)*x/a^(1/2))/
a^(7/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^3}{(a+cx^2)^4} dx = \frac{\frac{\sqrt{a}(-4a^4e^3+15c^4d^3x^5-3a^3ce(8d^2+3dex+4e^2x^2)+3a^2c^2dx(11d^2+8e^2x^2)+ac^3dx^3(40d^2+9e^2x^2))}{(a+cx^2)^3} + 3\sqrt{c}d(5cd^2+3ae^2)\arctan\left(\right.}{48a^{7/2}c^2}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^2)^4,x]
```

output
```
((Sqrt[a]*(-4*a^4*e^3 + 15*c^4*d^3*x^5 - 3*a^3*c*e*(8*d^2 + 3*d*e*x + 4*e^
2*x^2) + 3*a^2*c^2*d*x*(11*d^2 + 8*e^2*x^2) + a*c^3*d*x^3*(40*d^2 + 9*e^2*
x^2)))/(a + c*x^2)^3 + 3*Sqrt[c]*d*(5*c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x)/
Sqrt[a]])/(48*a^(7/2)*c^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {494, 25, 685, 675, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^2)^4}dx$$

$$\Big\downarrow \text{494}$$

$$\frac{x(d+ex)^3}{6a(a+cx^2)^3}-\frac{\int -\frac{(d+ex)^2(5d+2ex)}{(cx^2+a)^3}dx}{6a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)^2(5d+2ex)}{(cx^2+a)^3}dx}{6a}+\frac{x(d+ex)^3}{6a(a+cx^2)^3}$$

$$\Big\downarrow \text{685}$$

$$\frac{\frac{\int \frac{(d+ex)\left(15cd^2+5cexd+4ae^2\right)}{\left(cx^2+a\right)^2}dx}{4ac}-\frac{(d+ex)^2(2ae-5cdx)}{4ac(a+cx^2)^2}}{6a}+\frac{x(d+ex)^3}{6a(a+cx^2)^3}$$

$$\Big\downarrow \text{675}$$

$$\frac{\frac{\frac{3d\left(3ae^2+5cd^2\right)\int \frac{1}{cx^2+a}dx}{2a}-\frac{2e\left(ae^2+5cd^2\right)}{c\left(a+cx^2\right)}+\frac{dx\left(15cd^2-ae^2\right)}{2a\left(a+cx^2\right)}}{4ac}-\frac{(d+ex)^2(2ae-5cdx)}{4ac(a+cx^2)^2}}{6a}+\frac{x(d+ex)^3}{6a(a+cx^2)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{3d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(3ae^2+5cd^2\right)}{2a^{3/2}\sqrt{c}}-\frac{2e\left(ae^2+5cd^2\right)}{c\left(a+cx^2\right)}+\frac{dx\left(15cd^2-ae^2\right)}{2a\left(a+cx^2\right)}}{4ac}-\frac{(d+ex)^2(2ae-5cdx)}{4ac(a+cx^2)^2}}{6a}+\frac{x(d+ex)^3}{6a\left(a+cx^2\right)^3}$$

input

```
Int[(d + e*x)^3/(a + c*x^2)^4,x]
```

output

```
(x*(d + e*x)^3)/(6*a*(a + c*x^2)^3) + (-1/4*((2*a*e - 5*c*d*x)*(d + e*x)^2
)/(a*c*(a + c*x^2)^2) + ((-2*e*(5*c*d^2 + a*e^2))/(c*(a + c*x^2)) + (d*(15
*c*d^2 - a*e^2)*x)/(2*a*(a + c*x^2)) + (3*d*(5*c*d^2 + 3*a*e^2)*ArcTan[(Sq
rt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]))/(4*a*c))/(6*a)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 494

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 675

```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{d(3a e^2+5c d^2)c x^5}{16a^3}+\frac{d(3a e^2+5c d^2)x^3}{6a^2}-\frac{e^3x^2}{4c}-\frac{d(3a e^2-11c d^2)x}{16ac}-\frac{e(a e^2+6c d^2)}{12c^2}}{(c x^2+a)^3}+\frac{d(3a e^2+5c d^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16c a^3\sqrt{ac}}$
risch	$\frac{\frac{d(3a e^2+5c d^2)c x^5}{16a^3}+\frac{d(3a e^2+5c d^2)x^3}{6a^2}-\frac{e^3x^2}{4c}-\frac{d(3a e^2-11c d^2)x}{16ac}-\frac{e(a e^2+6c d^2)}{12c^2}}{(c x^2+a)^3}-\frac{3d\ln(cx+\sqrt{-ac})e^2}{32\sqrt{-ac}\,c a^2}-\frac{5d^3\ln(cx+\sqrt{-ac})}{32\sqrt{-ac}\,a^3}+$

input
```
int((e*x+d)^3/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*d*(3*a*e^2+5*c*d^2)*c/a^3*x^5+1/6/a^2*d*(3*a*e^2+5*c*d^2)*x^3-1/4*e^
3*x^2/c-1/16*d*(3*a*e^2-11*c*d^2)/a/c*x-1/12*e*(a*e^2+6*c*d^2)/c^2)/(c*x^2
+a)^3+1/16*d*(3*a*e^2+5*c*d^2)/c/a^3/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.22

$$\int \frac{(d+ex)^3}{(a+cx^2)^4}\,dx$$
$$= \left[-\frac{24\,a^4ce^3x^2+48\,a^4cd^2e+8\,a^5e^3-6\,(5\,ac^4d^3+3\,a^2c^3de^2)x^5-16\,(5\,a^2c^3d^3+3\,a^3c^2de^2)x^3+3\,(5\,a^3cd\cdots}{\cdots}\right.$$
$$-\frac{12\,a^4ce^3x^2+24\,a^4cd^2e+4\,a^5e^3-3\,(5\,ac^4d^3+3\,a^2c^3de^2)x^5-8\,(5\,a^2c^3d^3+3\,a^3c^2de^2)x^3-3\,(5\,a^3cd^3\cdots}{48\cdots}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(24*a^4*c*e^3*x^2 + 48*a^4*c*d^2*e + 8*a^5*e^3 - 6*(5*a*c^4*d^3 + 3
*a^2*c^3*d*e^2)*x^5 - 16*(5*a^2*c^3*d^3 + 3*a^3*c^2*d*e^2)*x^3 + 3*(5*a^3*
c*d^3 + 3*a^4*d*e^2 + (5*c^4*d^3 + 3*a*c^3*d*e^2)*x^6 + 3*(5*a*c^3*d^3 + 3
*a^2*c^2*d*e^2)*x^4 + 3*(5*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*x^2)*sqrt(-a*c)*lo
g((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 6*(11*a^3*c^2*d^3 - 3*a^4*c*
d*e^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(
12*a^4*c*e^3*x^2 + 24*a^4*c*d^2*e + 4*a^5*e^3 - 3*(5*a*c^4*d^3 + 3*a^2*c^3
*d*e^2)*x^5 - 8*(5*a^2*c^3*d^3 + 3*a^3*c^2*d*e^2)*x^3 - 3*(5*a^3*c*d^3 + 3
*a^4*d*e^2 + (5*c^4*d^3 + 3*a*c^3*d*e^2)*x^6 + 3*(5*a*c^3*d^3 + 3*a^2*c^2*
d*e^2)*x^4 + 3*(5*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(
a*c)*x/a) - 3*(11*a^3*c^2*d^3 - 3*a^4*c*d*e^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4
*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)^3}{(a+cx^2)^4}\,dx = -\frac{d\sqrt{-\frac{1}{a^7c^3}}\cdot(3ae^2+5cd^2)\log\left(-\frac{a^4cd\sqrt{-\frac{1}{a^7c^3}}\cdot(3ae^2+5cd^2)}{3ade^2+5cd^3}+x\right)}{32} + \frac{d\sqrt{-\frac{1}{a^7c^3}}\cdot(3ae^2+5cd^2)\log\left(\frac{a^4cd\sqrt{-\frac{1}{a^7c^3}}\cdot(3ae^2+5cd^2)}{3ade^2+5cd^3}+x\right)}{32} + \frac{-4a^4e^3-24a^3cd^2e-12a^3ce^3x^2+x^5\cdot(9ac^3de^2+15c^4d^3)+x^3\cdot(24a^2c^2de^2+40ac^3d^3)+x(-9a^3cde^2}{48a^6c^2+144a^5c^3x^2+144a^4c^4x^4+48a^3c^5x^6}$$

input
```
integrate((e*x+d)**3/(c*x**2+a)**4,x)
```

output
```
-d*sqrt(-1/(a**7*c**3))*(3*a*e**2 + 5*c*d**2)*log(-a**4*c*d*sqrt(-1/(a**7*
c**3))*(3*a*e**2 + 5*c*d**2)/(3*a*d*e**2 + 5*c*d**3) + x)/32 + d*sqrt(-1/(
a**7*c**3))*(3*a*e**2 + 5*c*d**2)*log(a**4*c*d*sqrt(-1/(a**7*c**3))*(3*a*e
**2 + 5*c*d**2)/(3*a*d*e**2 + 5*c*d**3) + x)/32 + (-4*a**4*e**3 - 24*a**3*
c*d**2*e - 12*a**3*c*e**3*x**2 + x**5*(9*a*c**3*d*e**2 + 15*c**4*d**3) + x
**3*(24*a**2*c**2*d*e**2 + 40*a*c**3*d**3) + x*(-9*a**3*c*d*e**2 + 33*a**2
*c**2*d**3))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*a**4*c**4*x**4 + 48*
a**3*c**5*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3}{(a+cx^2)^4}\,dx = -\frac{12\,a^3ce^3x^2 + 24\,a^3cd^2e + 4\,a^4e^3 - 3\,(5\,c^4d^3 + 3\,ac^3de^2)x^5 - 8\,(5\,ac^3d^3 + 3\,a^2c^2de^2)x^3 - 3\,(11\,a^2c^2d^3 -}{48\,(a^3c^5x^6 + 3\,a^4c^4x^4 + 3\,a^5c^3x^2 + a^6c^2)} + \frac{(5\,cd^3 + 3\,ade^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(12*a^3*c*e^3*x^2 + 24*a^3*c*d^2*e + 4*a^4*e^3 - 3*(5*c^4*d^3 + 3*a*
c^3*d*e^2)*x^5 - 8*(5*a*c^3*d^3 + 3*a^2*c^2*d*e^2)*x^3 - 3*(11*a^2*c^2*d^3
 - 3*a^3*c*d*e^2)*x)/(a^3*c^5*x^6 + 3*a^4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^
2) + 1/16*(5*c*d^3 + 3*a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3}{(a+cx^2)^4}\,dx = \frac{(5\,cd^3 + 3\,ade^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c} + \frac{15\,c^4d^3x^5 + 9\,ac^3de^2x^5 + 40\,ac^3d^3x^3 + 24\,a^2c^2de^2x^3 - 12\,a^3ce^3x^2 + 33\,a^2c^2d^3x - 9\,a^3cde^2x - 24\,a^3cd^2}{48\,(cx^2+a)^3a^3c^2}$$

input
```
integrate((e*x+d)^3/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(5*c*d^3 + 3*a*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*
(15*c^4*d^3*x^5 + 9*a*c^3*d*e^2*x^5 + 40*a*c^3*d^3*x^3 + 24*a^2*c^2*d*e^2*
x^3 - 12*a^3*c*e^3*x^2 + 33*a^2*c^2*d^3*x - 9*a^3*c*d*e^2*x - 24*a^3*c*d^2
*e - 4*a^4*e^3)/((c*x^2 + a)^3*a^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3}{(a+cx^2)^4}\,dx = \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(5\,c\,d^2+3\,a\,e^2\right)}{16\,a^{7/2}\,c^{3/2}} - \frac{\frac{e^3\,x^2}{4\,c}+\frac{e\,\left(6\,c\,d^2+a\,e^2\right)}{12\,c^2}-\frac{d\,x^3\,\left(5\,c\,d^2+3\,a\,e^2\right)}{6\,a^2}+\frac{d\,x\,\left(3\,a\,e^2-11\,c\,d^2\right)}{16\,a\,c}-\frac{c\,d\,x^5\,\left(5\,c\,d^2+3\,a\,e^2\right)}{16\,a^3}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

input
```
int((d + e*x)^3/(a + c*x^2)^4,x)
```

output
```
(d*atan((c^(1/2)*x)/a^(1/2))*(3*a*e^2 + 5*c*d^2))/(16*a^(7/2)*c^(3/2)) - (
(e^3*x^2)/(4*c) + (e*(a*e^2 + 6*c*d^2))/(12*c^2) - (d*x^3*(3*a*e^2 + 5*c*d
^2))/(6*a^2) + (d*x*(3*a*e^2 - 11*c*d^2))/(16*a*c) - (c*d*x^5*(3*a*e^2 + 5
*c*d^2))/(16*a^3))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex)^3}{(a+cx^2)^4}\,dx = \frac{9\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^4d\,e^2+15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3c\,d^3+27\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3cd\,e^2x^2+45\sqrt{c}\,\sqrt{}}{}$$

input
```
int((e*x+d)^3/(c*x^2+a)^4,x)
```

output
```
(9*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*d*e**2 + 15*sqrt(c)*
sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d**3 + 27*sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d*e**2*x**2 + 45*sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*a**2*c**2*d**3*x**2 + 27*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**2*c**2*d*e**2*x**4 + 45*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a*c**3*d**3*x**4 + 9*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*a*c**3*d*e**2*x**6 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sq
rt(a)))*c**4*d**3*x**6 - 4*a**5*e**3 - 24*a**4*c*d**2*e - 9*a**4*c*d*e**2*
x - 12*a**4*c*e**3*x**2 + 33*a**3*c**2*d**3*x + 24*a**3*c**2*d*e**2*x**3 +
 40*a**2*c**3*d**3*x**3 + 9*a**2*c**3*d*e**2*x**5 + 15*a*c**4*d**3*x**5)/(
48*a**4*c**2*(a**3 + 3*a**2*c*x**2 + 3*a*c**2*x**4 + c**3*x**6))
```

3.127 $\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx = -\frac{2ade-(cd^2-ae^2)\,x}{6ac\,(a+cx^2)^3}+\frac{(5cd^2+ae^2)\,x}{24a^2c\,(a+cx^2)^2} + \frac{(5cd^2+ae^2)\,x}{16a^3c\,(a+cx^2)}+\frac{(5cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

output
```
-1/6*(2*a*d*e-(-a*e^2+c*d^2)*x)/a/c/(c*x^2+a)^3+1/24*(a*e^2+5*c*d^2)*x/a^2
/c/(c*x^2+a)^2+1/16*(a*e^2+5*c*d^2)*x/a^3/c/(c*x^2+a)+1/16*(a*e^2+5*c*d^2)
*arctan(c^(1/2)*x/a^(1/2))/a^(7/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx = \frac{15c^3d^2x^5-a^3e(16d+3ex)+ac^2x^3(40d^2+3e^2x^2)+a^2cx(33d^2+8e^2x^2)}{48a^3c\,(a+cx^2)^3} + \frac{(5cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^2)^4,x]
```

output
```
(15*c^3*d^2*x^5 - a^3*e*(16*d + 3*e*x) + a*c^2*x^3*(40*d^2 + 3*e^2*x^2) +
a^2*c*x*(33*d^2 + 8*e^2*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*c*d^2 + a*e^2
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {495, 454, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{5cd^2+4cexd+ae^2}{(cx^2+a)^3}dx}{6ac}-\frac{(d+ex)(ae-cdx)}{6ac(a+cx^2)^3}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{3(ae^2+5cd^2)\int \frac{1}{(cx^2+a)^2}dx}{4a}-\frac{4ade-x(ae^2+5cd^2)}{4a(a+cx^2)^2}}{6ac}-\frac{(d+ex)(ae-cdx)}{6ac(a+cx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3(ae^2+5cd^2)\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a(a+cx^2)}\right)}{4a}-\frac{4ade-x(ae^2+5cd^2)}{4a(a+cx^2)^2}}{6ac}-\frac{(d+ex)(ae-cdx)}{6ac(a+cx^2)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a(a+cx^2)}\right)(ae^2+5cd^2)}{4a}-\frac{4ade-x(ae^2+5cd^2)}{4a(a+cx^2)^2}}{6ac}-\frac{(d+ex)(ae-cdx)}{6ac(a+cx^2)^3}$$

input
```
Int[(d + e*x)^2/(a + c*x^2)^4,x]
```

output
```
-1/6*((a*e - c*d*x)*(d + e*x))/(a*c*(a + c*x^2)^3) + (-1/4*(4*a*d*e - (5*c
*d^2 + a*e^2)*x)/(a*(a + c*x^2)^2) + (3*(5*c*d^2 + a*e^2)*(x/(2*a*(a + c*x
^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])))/(4*a))/(6*a*c)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{\left(a\,e^{2}+5c\,d^{2}\right)c\,x^{5}}{16a^{3}}+\frac{\left(a\,e^{2}+5c\,d^{2}\right)x^{3}}{6a^{2}}-\frac{\left(a\,e^{2}-11c\,d^{2}\right)x}{16ac}-\frac{de}{3c}}{\left(c\,x^{2}+a\right)^{3}}+\frac{\left(a\,e^{2}+5c\,d^{2}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16c\,a^{3}\sqrt{ac}}$
risch	$\frac{\frac{\left(a\,e^{2}+5c\,d^{2}\right)c\,x^{5}}{16a^{3}}+\frac{\left(a\,e^{2}+5c\,d^{2}\right)x^{3}}{6a^{2}}-\frac{\left(a\,e^{2}-11c\,d^{2}\right)x}{16ac}-\frac{de}{3c}}{\left(c\,x^{2}+a\right)^{3}}-\frac{\ln\left(cx+\sqrt{-ac}\right)e^{2}}{32\sqrt{-ac}\,c\,a^{2}}-\frac{5\ln\left(cx+\sqrt{-ac}\right)d^{2}}{32\sqrt{-ac}\,a^{3}}+\frac{\ln\left(-cx+\sqrt{-ac}\right)e^{2}}{32\sqrt{-ac}\,c\,a^{2}}+\frac{5\ln}{}$

input
```
int((e*x+d)^2/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(a*e^2+5*c*d^2)*c/a^3*x^5+1/6/a^2*(a*e^2+5*c*d^2)*x^3-1/16*(a*e^2-11
*c*d^2)/a/c*x-1/3*d*e/c)/(c*x^2+a)^3+1/16*(a*e^2+5*c*d^2)/c/a^3/(a*c)^(1/2
)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.40

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx$$

$$= \left[-\frac{32\,a^4cde-6\,(5\,ac^4d^2+a^2c^3e^2)x^5-16\,(5\,a^2c^3d^2+a^3c^2e^2)x^3+3\,((5\,c^4d^2+ac^3e^2)x^6+5\,a^3cd^2+a^4e^2}{96\,(a^4c^5x^6+3\,a^5c^4}\right.$$

$$-\frac{16\,a^4cde-3\,(5\,ac^4d^2+a^2c^3e^2)x^5-8\,(5\,a^2c^3d^2+a^3c^2e^2)x^3-3\,((5\,c^4d^2+ac^3e^2)x^6+5\,a^3cd^2+a^4e^2+}{48\,(a^4c^5x^6+3\,a^5c^4x^4+}$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(32*a^4*c*d*e - 6*(5*a*c^4*d^2 + a^2*c^3*e^2)*x^5 - 16*(5*a^2*c^3*d
^2 + a^3*c^2*e^2)*x^3 + 3*((5*c^4*d^2 + a*c^3*e^2)*x^6 + 5*a^3*c*d^2 + a^4
*e^2 + 3*(5*a*c^3*d^2 + a^2*c^2*e^2)*x^4 + 3*(5*a^2*c^2*d^2 + a^3*c*e^2)*x
^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 6*(11*a^3*c
^2*d^2 - a^4*c*e^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*
c^2), -1/48*(16*a^4*c*d*e - 3*(5*a*c^4*d^2 + a^2*c^3*e^2)*x^5 - 8*(5*a^2*c
^3*d^2 + a^3*c^2*e^2)*x^3 - 3*((5*c^4*d^2 + a*c^3*e^2)*x^6 + 5*a^3*c*d^2 +
 a^4*e^2 + 3*(5*a*c^3*d^2 + a^2*c^2*e^2)*x^4 + 3*(5*a^2*c^2*d^2 + a^3*c*e^
2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - 3*(11*a^3*c^2*d^2 - a^4*c*e^2)*x
)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{a^7c^3}}(ae^2+5cd^2)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{a^7c^3}}(ae^2+5cd^2)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32} + \frac{-16a^3de+x^5\cdot(3ac^2e^2+15c^3d^2)+x^3\cdot(8a^2ce^2+40ac^2d^2)+x(-3a^3e^2+33a^2cd^2)}{48a^6c+144a^5c^2x^2+144a^4c^3x^4+48a^3c^4x^6}$$

input
```
integrate((e*x+d)**2/(c*x**2+a)**4,x)
```

output
```
-sqrt(-1/(a**7*c**3))*(a*e**2 + 5*c*d**2)*log(-a**4*c*sqrt(-1/(a**7*c**3))
 + x)/32 + sqrt(-1/(a**7*c**3))*(a*e**2 + 5*c*d**2)*log(a**4*c*sqrt(-1/(a*
*7*c**3)) + x)/32 + (-16*a**3*d*e + x**5*(3*a*c**2*e**2 + 15*c**3*d**2) +
x**3*(8*a**2*c*e**2 + 40*a*c**2*d**2) + x*(-3*a**3*e**2 + 33*a**2*c*d**2))
/(48*a**6*c + 144*a**5*c**2*x**2 + 144*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx = \frac{3\,(5\,c^3d^2+ac^2e^2)x^5-16\,a^3de+8\,(5\,ac^2d^2+a^2ce^2)x^3+3\,(11\,a^2cd^2-a^3e^2)x}{48\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)} + \frac{(5\,cd^2+ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(3*(5*c^3*d^2 + a*c^2*e^2)*x^5 - 16*a^3*d*e + 8*(5*a*c^2*d^2 + a^2*c*
e^2)*x^3 + 3*(11*a^2*c*d^2 - a^3*e^2)*x)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*
a^5*c^2*x^2 + a^6*c) + 1/16*(5*c*d^2 + a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(
a*c)*a^3*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx = \frac{(5\,cd^2+ae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c} + \frac{15\,c^3d^2x^5+3\,ac^2e^2x^5+40\,ac^2d^2x^3+8\,a^2ce^2x^3+33\,a^2cd^2x-3\,a^3e^2x-16\,a^3de}{48\,(cx^2+a)^3a^3c}$$

input
```
integrate((e*x+d)^2/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(5*c*d^2 + a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(15*
c^3*d^2*x^5 + 3*a*c^2*e^2*x^5 + 40*a*c^2*d^2*x^3 + 8*a^2*c*e^2*x^3 + 33*a^
2*c*d^2*x - 3*a^3*e^2*x - 16*a^3*d*e)/((c*x^2 + a)^3*a^3*c)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx = \frac{\frac{x^3\,(5\,c\,d^2+a\,e^2)}{6\,a^2}-\frac{d\,e}{3\,c}-\frac{x\,(a\,e^2-11\,c\,d^2)}{16\,a\,c}+\frac{c\,x^5\,(5\,c\,d^2+a\,e^2)}{16\,a^3}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(5\,c\,d^2+a\,e^2)}{16\,a^{7/2}\,c^{3/2}}$$

input
```
int((d + e*x)^2/(a + c*x^2)^4,x)
```

output
```
((x^3*(a*e^2 + 5*c*d^2))/(6*a^2) - (d*e)/(3*c) - (x*(a*e^2 - 11*c*d^2))/(1
6*a*c) + (c*x^5*(a*e^2 + 5*c*d^2))/(16*a^3))/(a^3 + c^3*x^6 + 3*a^2*c*x^2
+ 3*a*c^2*x^4) + (atan((c^(1/2)*x)/a^(1/2))*(a*e^2 + 5*c*d^2))/(16*a^(7/2)
*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.39

$$\int \frac{(d+ex)^2}{(a+cx^2)^4}\,dx = \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^4e^2+15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3c\,d^2+9\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3c\,e^2x^2+45\sqrt{c}\,\sqrt{a}\,at}{}$$

input
```
int((e*x+d)^2/(c*x^2+a)^4,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*e**2 + 15*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d**2 + 9*sqrt(c)*sqrt(a)*atan((
c*x)/(sqrt(c)*sqrt(a)))*a**3*c*e**2*x**2 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(
sqrt(c)*sqrt(a)))*a**2*c**2*d**2*x**2 + 9*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a**2*c**2*e**2*x**4 + 45*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)
*sqrt(a)))*a*c**3*d**2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a
)))*a*c**3*e**2*x**6 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c*
*4*d**2*x**6 - 16*a**4*c*d*e - 3*a**4*c*e**2*x + 33*a**3*c**2*d**2*x + 8*a
**3*c**2*e**2*x**3 + 40*a**2*c**3*d**2*x**3 + 3*a**2*c**3*e**2*x**5 + 15*a
*c**4*d**2*x**5)/(48*a**4*c**2*(a**3 + 3*a**2*c*x**2 + 3*a*c**2*x**4 + c**
3*x**6))
```

3.128 $\int \frac{d+ex}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx=\frac{-ae+cdx}{6ac(a+cx^2)^3}+\frac{5dx}{24a^2(a+cx^2)^2}+\frac{5dx}{16a^3(a+cx^2)}+\frac{5d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{c}}$$

output
```
1/6*(c*d*x-a*e)/a/c/(c*x^2+a)^3+5/24*d*x/a^2/(c*x^2+a)^2+5/16*d*x/a^3/(c*x
^2+a)+5/16*d*arctan(c^(1/2)*x/a^(1/2))/a^(7/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx=\frac{\frac{\sqrt{a}(-8a^3e+33a^2cdx+40ac^2dx^3+15c^3dx^5)}{(a+cx^2)^3}+15\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{48a^{7/2}c}$$

input
```
Integrate[(d + e*x)/(a + c*x^2)^4,x]
```

output
```
((Sqrt[a]*(-8*a^3*e + 33*a^2*c*d*x + 40*a*c^2*d*x^3 + 15*c^3*d*x^5))/(a +
c*x^2)^3 + 15*Sqrt[c]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {454, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^2)^4}dx$$

$$\Big\downarrow \; 454$$

$$\frac{5d\int \frac{1}{(cx^2+a)^3}dx}{6a}-\frac{ae-cdx}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \; 215$$

$$\frac{5d\left(\frac{3\int \frac{1}{\left(cx^2+a\right)^2}dx}{4a}+\frac{x}{4a(a+cx^2)^2}\right)}{6a}-\frac{ae-cdx}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \; 215$$

$$\frac{5d\left(\frac{3\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a\left(a+cx^2\right)}\right)}{4a}+\frac{x}{4a(a+cx^2)^2}\right)}{6a}-\frac{ae-cdx}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{5d\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}\right)}{4a}+\frac{x}{4a(a+cx^2)^2}\right)}{6a}-\frac{ae-cdx}{6ac\,(a+cx^2)^3}$$

input
```
Int[(d + e*x)/(a + c*x^2)^4,x]
```

output
```
-1/6*(a*e - c*d*x)/(a*c*(a + c*x^2)^3) + (5*d*(x/(4*a*(a + c*x^2)^2) + (3*
(x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])))/(
4*a)))/(6*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2cdx-2ae}{12ac(c\,x^{2}+a)^{3}}+\frac{5d\left(\frac{x}{4a\left(c\,x^{2}+a\right)^{2}}+\frac{\frac{3x}{8a\left(c\,x^{2}+a\right)}+\frac{3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a\sqrt{ac}}}{a}\right)}{6a}$	91
risch	$\frac{\frac{5d\,c^{2}x^{5}}{16a^{3}}+\frac{5cd\,x^{3}}{6a^{2}}+\frac{11dx}{16a}-\frac{e}{6c}}{(c\,x^{2}+a)^{3}}-\frac{5d\ln\left(cx+\sqrt{-ac}\right)}{32\sqrt{-ac}\,a^{3}}+\frac{5d\ln\left(-cx+\sqrt{-ac}\right)}{32\sqrt{-ac}\,a^{3}}$	95

input
```
int((e*x+d)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*c*d*x-2*a*e)/a/c/(c*x^2+a)^3+5/6*d/a*(1/4*x/a/(c*x^2+a)^2+3/4/a*(1
/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.03

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx = \left[\frac{30\,ac^3dx^5+80\,a^2c^2dx^3+66\,a^3cdx-16\,a^4e-15\,(c^3dx^6+3\,ac^2dx^4+3\,a^2cdx^2+a^3d)\sqrt{-ac}\log\left(\frac{cx^2-2\ldots}{cx\ldots}\right.}{96\,(a^4c^4x^6+3\,a^5c^3x^4+3\,a^6c^2x^2+a^7c)}\right.$$

input
```
integrate((e*x+d)/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[1/96*(30*a*c^3*d*x^5 + 80*a^2*c^2*d*x^3 + 66*a^3*c*d*x - 16*a^4*e - 15*(c
^3*d*x^6 + 3*a*c^2*d*x^4 + 3*a^2*c*d*x^2 + a^3*d)*sqrt(-a*c)*log((c*x^2 -
2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^4*c^4*x^6 + 3*a^5*c^3*x^4 + 3*a^6*c^2
*x^2 + a^7*c), 1/48*(15*a*c^3*d*x^5 + 40*a^2*c^2*d*x^3 + 33*a^3*c*d*x - 8*
a^4*e + 15*(c^3*d*x^6 + 3*a*c^2*d*x^4 + 3*a^2*c*d*x^2 + a^3*d)*sqrt(a*c)*a
rctan(sqrt(a*c)*x/a))/(a^4*c^4*x^6 + 3*a^5*c^3*x^4 + 3*a^6*c^2*x^2 + a^7*c
)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.61

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx = d\left(-\frac{5\sqrt{-\frac{1}{a^7c}}\log\left(-a^4\sqrt{-\frac{1}{a^7c}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{a^7c}}\log\left(a^4\sqrt{-\frac{1}{a^7c}}+x\right)}{32}\right)+\frac{-8a^3e+33a^2cdx+40ac^2dx^3+15c^3dx^5}{48a^6c+144a^5c^2x^2+144a^4c^3x^4+48a^3c^4x^6}$$

input
```
integrate((e*x+d)/(c*x**2+a)**4,x)
```

output
```
d*(-5*sqrt(-1/(a**7*c))*log(-a**4*sqrt(-1/(a**7*c)) + x)/32 + 5*sqrt(-1/(a
**7*c))*log(a**4*sqrt(-1/(a**7*c)) + x)/32) + (-8*a**3*e + 33*a**2*c*d*x +
 40*a*c**2*d*x**3 + 15*c**3*d*x**5)/(48*a**6*c + 144*a**5*c**2*x**2 + 144*
a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx = \frac{15\,c^3dx^5+40\,ac^2dx^3+33\,a^2cdx-8\,a^3e}{48\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)}+\frac{5\,d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3}$$

input
```
integrate((e*x+d)/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(15*c^3*d*x^5 + 40*a*c^2*d*x^3 + 33*a^2*c*d*x - 8*a^3*e)/(a^3*c^4*x^6
 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a^6*c) + 5/16*d*arctan(c*x/sqrt(a*c))/(
sqrt(a*c)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx = \frac{5\,d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3}+\frac{15\,c^3dx^5+40\,ac^2dx^3+33\,a^2cdx-8\,a^3e}{48\,(cx^2+a)^3a^3c}$$

input
```
integrate((e*x+d)/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
5/16*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3) + 1/48*(15*c^3*d*x^5 + 40*a*c
^2*d*x^3 + 33*a^2*c*d*x - 8*a^3*e)/((c*x^2 + a)^3*a^3*c)
```

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx = \frac{\frac{11\,d\,x}{16\,a}-\frac{e}{6\,c}+\frac{5\,c^2\,d\,x^5}{16\,a^3}+\frac{5\,c\,d\,x^3}{6\,a^2}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}+\frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{c}}$$

input
```
int((d + e*x)/(a + c*x^2)^4,x)
```

output
```
((11*d*x)/(16*a) - e/(6*c) + (5*c^2*d*x^5)/(16*a^3) + (5*c*d*x^3)/(6*a^2))
/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4) + (5*d*atan((c^(1/2)*x)/a^(1/
2)))/(16*a^(7/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.88

$$\int \frac{d+ex}{(a+cx^2)^4}\,dx$$
$$= \frac{15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3d+45\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2cd\,x^2+45\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a\,c^2d\,x^4+15\sqrt{c}\,\sqrt{a}}{48a^4c\,(c^3x^6+3a\,c^2x^4+3a^2c\,x^2+a^3)}$$

input
```
int((e*x+d)/(c*x^2+a)^4,x)
```

output
```
(15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*d + 45*sqrt(c)*sqrt
(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d*x**2 + 45*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d*x**4 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sq
rt(c)*sqrt(a)))*c**3*d*x**6 - 8*a**4*e + 33*a**3*c*d*x + 40*a**2*c**2*d*x*
*3 + 15*a*c**3*d*x**5)/(48*a**4*c*(a**3 + 3*a**2*c*x**2 + 3*a*c**2*x**4 +
c**3*x**6))
```

3.129 $\int \frac{1}{(d+ex)(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 295

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^4}\,dx = \frac{ae+cdx}{6a\left(cd^2+ae^2\right)\left(a+cx^2\right)^3}+\frac{6a^2e^3+cd(5cd^2+11ae^2)\,x}{24a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}+\frac{8a^3e^5+cd(5c^2d^4+16acd^2e^2+19a^2e^4)\,x}{16a^3\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}+\frac{\sqrt{c}d(5c^3d^6+21ac^2d^4e^2+35a^2cd^2e^4+35a^3e^6)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}\left(cd^2+ae^2\right)^4}+\frac{e^7\log(d+ex)}{\left(cd^2+ae^2\right)^4}-\frac{e^7\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^4}$$

output
```
1/6*(c*d*x+a*e)/a/(a*e^2+c*d^2)/(c*x^2+a)^3+1/24*(6*a^2*e^3+c*d*(11*a*e^2+
5*c*d^2)*x)/a^2/(a*e^2+c*d^2)^2/(c*x^2+a)^2+1/16*(8*a^3*e^5+c*d*(19*a^2*e^
4+16*a*c*d^2*e^2+5*c^2*d^4)*x)/a^3/(a*e^2+c*d^2)^3/(c*x^2+a)+1/16*c^(1/2)*
d*(35*a^3*e^6+35*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+5*c^3*d^6)*arctan(c^(1/2)*
x/a^(1/2))/a^(7/2)/(a*e^2+c*d^2)^4+e^7*ln(e*x+d)/(a*e^2+c*d^2)^4-1/2*e^7*l
n(c*x^2+a)/(a*e^2+c*d^2)^4
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)(a+cx^2)^4}\,dx$$
$$= \frac{\frac{8(cd^2+ae^2)^3(ae+cdx)}{a(a+cx^2)^3} + \frac{2(cd^2+ae^2)^2(6a^2e^3+5c^2d^3x+11acde^2x)}{a^2(a+cx^2)^2} + \frac{3(cd^2+ae^2)(8a^3e^5+5c^3d^5x+16ac^2d^3e^2x+19a^2cde^4x)}{a^3(a+cx^2)} + \frac{3\sqrt{c}d(5c^3}{}}{48(cd^2+ae^2)^4}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^2)^4),x]
```

output
```
((8*(c*d^2 + a*e^2)^3*(a*e + c*d*x))/(a*(a + c*x^2)^3) + (2*(c*d^2 + a*e^2
)^2*(6*a^2*e^3 + 5*c^2*d^3*x + 11*a*c*d*e^2*x))/(a^2*(a + c*x^2)^2) + (3*(
c*d^2 + a*e^2)*(8*a^3*e^5 + 5*c^3*d^5*x + 16*a*c^2*d^3*e^2*x + 19*a^2*c*d*
e^4*x))/(a^3*(a + c*x^2)) + (3*Sqrt[c]*d*(5*c^3*d^6 + 21*a*c^2*d^4*e^2 + 3
5*a^2*c*d^2*e^4 + 35*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(7/2) + 48*e^
7*Log[d + e*x] - 24*e^7*Log[a + c*x^2])/(48*(c*d^2 + a*e^2)^4)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {496, 25, 686, 27, 686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^4(d+ex)}\,dx$$
$$\Big\downarrow \; 496$$
$$\frac{ae+cdx}{6a(a+cx^2)^3(ae^2+cd^2)} - \frac{\int -\frac{5cd^2+5cexd+6ae^2}{(d+ex)(cx^2+a)^3}dx}{6a(ae^2+cd^2)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5cd^2+5cexd+6ae^2}{(d+ex)(cx^2+a)^3}dx}{6a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 686

$$\frac{\frac{6a^2e^3+cdx(11ae^2+5cd^2)}{4a(a+cx^2)^2(ae^2+cd^2)}-\frac{\int -\frac{3c\left(5c^2d^4+11ace^2d^2+ce\left(5cd^2+11ae^2\right)xd+8a^2e^4\right)}{(d+ex)\left(cx^2+a\right)^2}dx}{4ac(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \frac{5c^2d^4+11ace^2d^2+ce\left(5cd^2+11ae^2\right)xd+8a^2e^4}{(d+ex)\left(cx^2+a\right)^2}dx}{4a(ae^2+cd^2)}+\frac{6a^2e^3+cdx(11ae^2+5cd^2)}{4a(a+cx^2)^2(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 686

$$\frac{\frac{3\left(\frac{8a^3e^5+cdx\left(19a^2e^4+16acd^2e^2+5c^2d^4\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{\int -\frac{c\left(5c^3d^6+16ac^2e^2d^4+19a^2ce^4d^2+ce\left(5c^2d^4+16ace^2d^2+19a^2e^4\right)xd+16a^3e^6\right)}{(d+ex)\left(cx^2+a\right)}dx}{2ac\left(ae^2+cd^2\right)}\right)}{4a(ae^2+cd^2)}+\frac{6a^2e^3+cdx(11ae^2+5cd^2)}{4a(a+cx^2)^2(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}$$
$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{3\left(\frac{\int \frac{c\left(5c^3d^6+16ac^2e^2d^4+19a^2ce^4d^2+ce\left(5c^2d^4+16ace^2d^2+19a^2e^4\right)xd+16a^3e^6\right)}{(d+ex)\left(cx^2+a\right)}dx}{2ac\left(ae^2+cd^2\right)}+\frac{8a^3e^5+cdx\left(19a^2e^4+16acd^2e^2+5c^2d^4\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}\right)}{4a(ae^2+cd^2)}+\frac{6a^2e^3+cdx(11ae^2+5cd^2)}{4a(a+cx^2)^2(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}$$
$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\left(\frac{\int \frac{5c^3d^6+16ac^2e^2d^4+19a^2ce^4d^2+ce\left(5c^2d^4+16ace^2d^2+19a^2e^4\right)xd+16a^3e^6}{(d+ex)\left(cx^2+a\right)}dx}{2a\left(ae^2+cd^2\right)}+\frac{8a^3e^5+cdx\left(19a^2e^4+16acd^2e^2+5c^2d^4\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}\right)}{4a(ae^2+cd^2)}+\frac{6a^2e^3+cdx(11ae^2+5cd^2)}{4a(a+cx^2)^2(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

$\downarrow$ 657

$$\frac{\frac{3\left(\frac{\int \left(\frac{16a^3e^8}{\left(cd^2+ae^2\right)(d+ex)}+\frac{c\left(5c^3d^7+21ac^2e^2d^5+35a^2ce^4d^3+35a^3e^6d-16a^3e^7x\right)}{\left(cd^2+ae^2\right)\left(cx^2+a\right)}\right)dx}{2a\left(ae^2+cd^2\right)}+\frac{8a^3e^5+cdx\left(19a^2e^4+16acd^2e^2+5c^2d^4\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}\right)}{4a(ae^2+cd^2)}+\frac{6a^2e^3+cdx(11ae^2+\dots}{4a(a+cx^2)^2(ae^2+\dots}}{6a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

$\downarrow$ 2009

$$\frac{\frac{6a^2e^3+cdx(11ae^2+5cd^2)}{4a(a+cx^2)^2(ae^2+cd^2)}+\frac{3\left(\frac{-\frac{8a^3e^7\log\left(a+cx^2\right)}{ae^2+cd^2}+\frac{16a^3e^7\log(d+ex)}{ae^2+cd^2}+\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(35a^3e^6+35a^2cd^2e^4+21ac^2d^4e^2+5c^3d^6\right)}{\sqrt{a}\left(ae^2+cd^2\right)}}{2a\left(ae^2+cd^2\right)}+\frac{8a^3e^5+cdx\left(19a\dots\right.}{2a\left(a+\dots\right.}\dots\right)}{4a(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3\left(ae^2+cd^2\right)}$$

input

```
Int[1/((d + e*x)*(a + c*x^2)^4),x]
```

output

```
(a*e + c*d*x)/(6*a*(c*d^2 + a*e^2)*(a + c*x^2)^3) + ((6*a^2*e^3 + c*d*(5*c
*d^2 + 11*a*e^2)*x)/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + (3*((8*a^3*e^5 +
 c*d*(5*c^2*d^4 + 16*a*c*d^2*e^2 + 19*a^2*e^4)*x)/(2*a*(c*d^2 + a*e^2)*(a
+ c*x^2)) + ((Sqrt[c]*d*(5*c^3*d^6 + 21*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 +
 35*a^3*e^6)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)) + (16*
a^3*e^7*Log[d + e*x])/(c*d^2 + a*e^2) - (8*a^3*e^7*Log[a + c*x^2])/(c*d^2
+ a*e^2))/(2*a*(c*d^2 + a*e^2))))/(4*a*(c*d^2 + a*e^2)))/(6*a*(c*d^2 + a*e
^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.32

method	result
default	$\frac{e^{7} \ln(ex+d)}{(a\,e^{2}+c\,d^{2})^{4}}+\frac{c\left(\frac{c^{2} d\left(19e^{6} a^{3}+35d^{2} e^{4} a^{2} c+21d^{4} e^{2} a\,c^{2}+5d^{6} c^{3}\right) x^{5}}{16a^{3}}+\left(\frac{1}{2} a\,e^{7} c+\frac{1}{2} d^{2} e^{5} c^{2}\right) x^{4}+\frac{cd\left(17e^{6} a^{3}+33d^{2} e^{4} a^{2} c+21d^{4} e^{2} a\,c^{2}+5d^{6} c^{3}\right) x}{6a^{2}}\right.}{}$
risch	Expression too large to display

input
```
int(1/(e*x+d)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
e^7*ln(e*x+d)/(a*e^2+c*d^2)^4+c/(a*e^2+c*d^2)^4*((1/16*c^2*d*(19*a^3*e^6+3
5*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+5*c^3*d^6)/a^3*x^5+(1/2*a*e^7*c+1/2*d^2*e
^5*c^2)*x^4+1/6*c*d*(17*a^3*e^6+33*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+5*c^3*d^
6)/a^2*x^3+(5/4*a^2*e^7+3/2*a*d^2*e^5*c+1/4*c^2*d^4*e^3)*x^2+1/16*d*(29*a^
3*e^6+61*a^2*c*d^2*e^4+43*a*c^2*d^4*e^2+11*c^3*d^6)/a*x+1/12*e*(11*a^3*e^6
+18*a^2*c*d^2*e^4+9*a*c^2*d^4*e^2+2*c^3*d^6)/c)/(c*x^2+a)^3+1/16/a^3*(-8*e
^7*a^3/c*ln(c*x^2+a)+(35*a^3*d*e^6+35*a^2*c*d^3*e^4+21*a*c^2*d^5*e^2+5*c^3
*d^7)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. 2(277) = 554.

Time = 9.91 (sec) , antiderivative size = 1784, normalized size of antiderivative = 6.05

$$\int \frac{1}{(d+ex)\,(a+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[1/96*(16*a^3*c^3*d^6*e + 72*a^4*c^2*d^4*e^3 + 144*a^5*c*d^2*e^5 + 88*a^6*
e^7 + 6*(5*c^6*d^7 + 21*a*c^5*d^5*e^2 + 35*a^2*c^4*d^3*e^4 + 19*a^3*c^3*d*
e^6)*x^5 + 48*(a^3*c^3*d^2*e^5 + a^4*c^2*e^7)*x^4 + 16*(5*a*c^5*d^7 + 21*a
^2*c^4*d^5*e^2 + 33*a^3*c^3*d^3*e^4 + 17*a^4*c^2*d*e^6)*x^3 + 24*(a^3*c^3*
d^4*e^3 + 6*a^4*c^2*d^2*e^5 + 5*a^5*c*e^7)*x^2 + 3*(5*a^3*c^3*d^7 + 21*a^4
*c^2*d^5*e^2 + 35*a^5*c*d^3*e^4 + 35*a^6*d*e^6 + (5*c^6*d^7 + 21*a*c^5*d^5
*e^2 + 35*a^2*c^4*d^3*e^4 + 35*a^3*c^3*d*e^6)*x^6 + 3*(5*a*c^5*d^7 + 21*a^
2*c^4*d^5*e^2 + 35*a^3*c^3*d^3*e^4 + 35*a^4*c^2*d*e^6)*x^4 + 3*(5*a^2*c^4*
d^7 + 21*a^3*c^3*d^5*e^2 + 35*a^4*c^2*d^3*e^4 + 35*a^5*c*d*e^6)*x^2)*sqrt(
-c/a)*log((c*x^2 + 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 6*(11*a^2*c^4*d^7
+ 43*a^3*c^3*d^5*e^2 + 61*a^4*c^2*d^3*e^4 + 29*a^5*c*d*e^6)*x - 48*(a^3*c^
3*e^7*x^6 + 3*a^4*c^2*e^7*x^4 + 3*a^5*c*e^7*x^2 + a^6*e^7)*log(c*x^2 + a)
+ 96*(a^3*c^3*e^7*x^6 + 3*a^4*c^2*e^7*x^4 + 3*a^5*c*e^7*x^2 + a^6*e^7)*log
(e*x + d))/(a^6*c^4*d^8 + 4*a^7*c^3*d^6*e^2 + 6*a^8*c^2*d^4*e^4 + 4*a^9*c*
d^2*e^6 + a^10*e^8 + (a^3*c^7*d^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4
+ 4*a^6*c^4*d^2*e^6 + a^7*c^3*e^8)*x^6 + 3*(a^4*c^6*d^8 + 4*a^5*c^5*d^6*e^
2 + 6*a^6*c^4*d^4*e^4 + 4*a^7*c^3*d^2*e^6 + a^8*c^2*e^8)*x^4 + 3*(a^5*c^5*
d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6 + a^9*c*e^
8)*x^2), 1/48*(8*a^3*c^3*d^6*e + 36*a^4*c^2*d^4*e^3 + 72*a^5*c*d^2*e^5 + 4
4*a^6*e^7 + 3*(5*c^6*d^7 + 21*a*c^5*d^5*e^2 + 35*a^2*c^4*d^3*e^4 + 19*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(a+cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**2+a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(277) = 554$.

Time = 0.14 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.22

$$\int \frac{1}{(d+ex)(a+cx^2)^4}\,dx = -\frac{e^7\log(cx^2+a)}{2(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8)} + \frac{e^7\log(ex+d)}{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8} + \frac{(5\,c^4d^7+21\,ac^3d^5e^2+35\,a^2c^2d^3e^4+35\,a^3cde^6)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,(a^3c^4d^8+4\,a^4c^3d^6e^2+6\,a^5c^2d^4e^4+4\,a^6cd^2e^6+a^7e^8)\sqrt{ac}} + \frac{24\,a^3c^2e^5x^4+8\,a^3c^2d^4e+28\,a^4cd^2e^3+44\,a^5e^5+3\,(5\,c^5d^5+16\,ac^4d^3e^2+19\,a^2c^3de^4)x^5+8\,(5\,ac^4d^5+}{48\,(a^6c^3d^6+3\,a^7c^2d^4e^2+3\,a^8cd^2e^4+a^9e^6+(a^3c^6d^6+3\,a^4c^5d^4e^2+3\,a^5c^4d^2e^4+a^6c^3e^6)x^6+3\,(}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/2*e^7*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4
*a^3*c*d^2*e^6 + a^4*e^8) + e^7*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 +
6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + 1/16*(5*c^4*d^7 + 21*a*c^
3*d^5*e^2 + 35*a^2*c^2*d^3*e^4 + 35*a^3*c*d*e^6)*arctan(c*x/sqrt(a*c))/((a
^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7
*e^8)*sqrt(a*c)) + 1/48*(24*a^3*c^2*e^5*x^4 + 8*a^3*c^2*d^4*e + 28*a^4*c*d
^2*e^3 + 44*a^5*e^5 + 3*(5*c^5*d^5 + 16*a*c^4*d^3*e^2 + 19*a^2*c^3*d*e^4)*
x^5 + 8*(5*a*c^4*d^5 + 16*a^2*c^3*d^3*e^2 + 17*a^3*c^2*d*e^4)*x^3 + 12*(a^
3*c^2*d^2*e^3 + 5*a^4*c*e^5)*x^2 + 3*(11*a^2*c^3*d^5 + 32*a^3*c^2*d^3*e^2
+ 29*a^4*c*d*e^4)*x)/(a^6*c^3*d^6 + 3*a^7*c^2*d^4*e^2 + 3*a^8*c*d^2*e^4 +
a^9*e^6 + (a^3*c^6*d^6 + 3*a^4*c^5*d^4*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c^3*e
^6)*x^6 + 3*(a^4*c^5*d^6 + 3*a^5*c^4*d^4*e^2 + 3*a^6*c^3*d^2*e^4 + a^7*c^2
*e^6)*x^4 + 3*(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c
*e^6)*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(277) = 554$.

Time = 0.12 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.91

$$\int \frac{1}{(d+ex)(a+cx^2)^4}\,dx = \frac{e^8\log(|ex+d|)}{c^4d^8e+4\,ac^3d^6e^3+6\,a^2c^2d^4e^5+4\,a^3cd^2e^7+a^4e^9} - \frac{e^7\log(cx^2+a)}{2\,(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8)} + \frac{(5\,c^4d^7+21\,ac^3d^5e^2+35\,a^2c^2d^3e^4+35\,a^3cde^6)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,(a^3c^4d^8+4\,a^4c^3d^6e^2+6\,a^5c^2d^4e^4+4\,a^6cd^2e^6+a^7e^8)\sqrt{ac}} + \frac{8\,a^3c^3d^6e+36\,a^4c^2d^4e^3+72\,a^5cd^2e^5+44\,a^6e^7+3\,(5\,c^6d^7+21\,ac^5d^5e^2+35\,a^2c^4d^3e^4+19\,a^3c^3de^6)x^5}{}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
e^8*log(abs(e*x + d))/(c^4*d^8*e + 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 + 4
*a^3*c*d^2*e^7 + a^4*e^9) - 1/2*e^7*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*
e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + 1/16*(5*c^4*d^7 + 2
1*a*c^3*d^5*e^2 + 35*a^2*c^2*d^3*e^4 + 35*a^3*c*d*e^6)*arctan(c*x/sqrt(a*c
))/((a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
 + a^7*e^8)*sqrt(a*c)) + 1/48*(8*a^3*c^3*d^6*e + 36*a^4*c^2*d^4*e^3 + 72*a
^5*c*d^2*e^5 + 44*a^6*e^7 + 3*(5*c^6*d^7 + 21*a*c^5*d^5*e^2 + 35*a^2*c^4*d
^3*e^4 + 19*a^3*c^3*d*e^6)*x^5 + 24*(a^3*c^3*d^2*e^5 + a^4*c^2*e^7)*x^4 +
8*(5*a*c^5*d^7 + 21*a^2*c^4*d^5*e^2 + 33*a^3*c^3*d^3*e^4 + 17*a^4*c^2*d*e^
6)*x^3 + 12*(a^3*c^3*d^4*e^3 + 6*a^4*c^2*d^2*e^5 + 5*a^5*c*e^7)*x^2 + 3*(1
1*a^2*c^4*d^7 + 43*a^3*c^3*d^5*e^2 + 61*a^4*c^2*d^3*e^4 + 29*a^5*c*d*e^6)*
x)/((c*d^2 + a*e^2)^4*(c*x^2 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 7.78 (sec) , antiderivative size = 1470, normalized size of antiderivative = 4.98

$$\int \frac{1}{(d+ex)\,(a+cx^2)^4}\,dx = \text{Too large to display}$$

input

```
int(1/((a + c*x^2)^4*(d + e*x)),x)
```

output

```
((11*a^2*e^5 + 2*c^2*d^4*e + 7*a*c*d^2*e^3)/(12*(a^3*e^6 + c^3*d^6 + 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x^2*(c^2*d^2*e^3 + 5*a*c*e^5))/(4*(a^3*e
^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(11*c^3*d^5 + 32*a
*c^2*d^3*e^2 + 29*a^2*c*d*e^4))/(16*a*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4)) + (c^2*e^5*x^4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4)) + (x^3*(5*c^4*d^5 + 16*a*c^3*d^3*e^2 + 17*a^2*c^2*d
*e^4))/(6*a^2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (
x^5*(5*c^5*d^5 + 16*a*c^4*d^3*e^2 + 19*a^2*c^3*d*e^4))/(16*a^3*(a^3*e^6 +
c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))/(a^3 + c^3*x^6 + 3*a^2*c*x^
2 + 3*a*c^2*x^4) + (e^7*log(d + e*x))/(a*e^2 + c*d^2)^4 - (log(25*a^7*c^10
*d^18*x - 2304*a^13*e^18*(-a^7*c)^(1/2) - 25*a^4*c^9*d^18*(-a^7*c)^(1/2) +
 5833*a^5*d^2*e^16*(-a^7*c)^(3/2) + 3612*c^5*d^12*e^6*(-a^7*c)^(3/2) + 230
4*a^16*c*e^18*x + 9660*a^2*c^3*d^8*e^10*(-a^7*c)^(3/2) + 8820*a^3*c^2*d^6*
e^12*(-a^7*c)^(3/2) - 260*a^5*c^8*d^16*e^2*(-a^7*c)^(1/2) - 1236*a^6*c^7*d
^14*e^4*(-a^7*c)^(1/2) + 260*a^8*c^9*d^16*e^2*x + 1236*a^9*c^8*d^14*e^4*x
+ 3612*a^10*c^7*d^12*e^6*x + 7126*a^11*c^6*d^10*e^8*x + 9660*a^12*c^5*d^8*
e^10*x + 8820*a^13*c^4*d^6*e^12*x + 7204*a^14*c^3*d^4*e^14*x + 5833*a^15*c
^2*d^2*e^16*x + 7126*a*c^4*d^10*e^8*(-a^7*c)^(3/2) + 7204*a^4*c*d^4*e^14*(
-a^7*c)^(3/2))*(16*a^7*e^7 + 5*c^3*d^7*(-a^7*c)^(1/2) + 35*a^3*d*e^6*(-a^7
*c)^(1/2) + 21*a*c^2*d^5*e^2*(-a^7*c)^(1/2) + 35*a^2*c*d^3*e^4*(-a^7*c)...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1226, normalized size of antiderivative = 4.16

$$\int \frac{1}{(d+ex)\,(a+cx^2)^4}\,dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)/(c*x^2+a)^4,x)
```

output
```
(105*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**6*d*e**6 + 105*sqrt(
c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*c*d**3*e**4 + 315*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*c*d*e**6*x**2 + 63*sqrt(c)*sqrt(a
)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**2*d**5*e**2 + 315*sqrt(c)*sqrt(a)*
atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**2*d**3*e**4*x**2 + 315*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**2*d*e**6*x**4 + 15*sqrt(c)*sqrt(a
)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c**3*d**7 + 189*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a**3*c**3*d**5*e**2*x**2 + 315*sqrt(c)*sqrt(a)*at
an((c*x)/(sqrt(c)*sqrt(a)))*a**3*c**3*d**3*e**4*x**4 + 105*sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c**3*d*e**6*x**6 + 45*sqrt(c)*sqrt(a)*
atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c**4*d**7*x**2 + 189*sqrt(c)*sqrt(a)*at
an((c*x)/(sqrt(c)*sqrt(a)))*a**2*c**4*d**5*e**2*x**4 + 105*sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c**4*d**3*e**4*x**6 + 45*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**5*d**7*x**4 + 63*sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*a*c**5*d**5*e**2*x**6 + 15*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*c**6*d**7*x**6 - 24*log(a + c*x**2)*a**7*e**7 -
72*log(a + c*x**2)*a**6*c*e**7*x**2 - 72*log(a + c*x**2)*a**5*c**2*e**7*x*
*4 - 24*log(a + c*x**2)*a**4*c**3*e**7*x**6 + 48*log(d + e*x)*a**7*e**7 +
144*log(d + e*x)*a**6*c*e**7*x**2 + 144*log(d + e*x)*a**5*c**2*e**7*x**4 +
 48*log(d + e*x)*a**4*c**3*e**7*x**6 + 36*a**7*e**7 + 64*a**6*c*d**2*e*...
```

3.130 $\int \frac{1}{(d+ex)^2(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 377

$$
\begin{aligned}
\int \frac{1}{(d+ex)^2(a+cx^2)^4}\,dx
&= -\frac{e^7}{(cd^2+ae^2)^4(d+ex)}+\frac{c(2ade+(cd^2-ae^2)x)}{6a(cd^2+ae^2)^2(a+cx^2)^3} \\
&\quad+\frac{c(24a^2de^3+(5c^2d^4+18acd^2e^2-11a^2e^4)x)}{24a^2(cd^2+ae^2)^3(a+cx^2)^2} \\
&\quad+\frac{c(48a^3de^5+(5c^3d^6+23ac^2d^4e^2+47a^2cd^2e^4-19a^3e^6)x)}{16a^3(cd^2+ae^2)^4(a+cx^2)} \\
&\quad+\frac{\sqrt{c}(5c^4d^8+28ac^3d^6e^2+70a^2c^2d^4e^4+140a^3cd^2e^6-35a^4e^8)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}(cd^2+ae^2)^5} \\
&\quad+\frac{8cde^7\log(d+ex)}{(cd^2+ae^2)^5}-\frac{4cde^7\log(a+cx^2)}{(cd^2+ae^2)^5}
\end{aligned}
$$

output
```
-e^7/(a*e^2+c*d^2)^4/(e*x+d)+1/6*c*(2*a*d*e+(-a*e^2+c*d^2)*x)/a/(a*e^2+c*d
^2)^2/(c*x^2+a)^3+1/24*c*(24*a^2*d*e^3+(-11*a^2*e^4+18*a*c*d^2*e^2+5*c^2*d
^4)*x)/a^2/(a*e^2+c*d^2)^3/(c*x^2+a)^2+1/16*c*(48*a^3*d*e^5+(-19*a^3*e^6+4
7*a^2*c*d^2*e^4+23*a*c^2*d^4*e^2+5*c^3*d^6)*x)/a^3/(a*e^2+c*d^2)^4/(c*x^2+
a)+1/16*c^(1/2)*(-35*a^4*e^8+140*a^3*c*d^2*e^6+70*a^2*c^2*d^4*e^4+28*a*c^3
*d^6*e^2+5*c^4*d^8)*arctan(c^(1/2)*x/a^(1/2))/a^(7/2)/(a*e^2+c*d^2)^5+8*c*
d*e^7*ln(e*x+d)/(a*e^2+c*d^2)^5-4*c*d*e^7*ln(c*x^2+a)/(a*e^2+c*d^2)^5
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^4} \, dx$$
$$= \frac{-\frac{48e^7(cd^2+ae^2)}{d+ex} + \frac{3c(cd^2+ae^2)(5c^3d^6x+23ac^2d^4e^2x+47a^2cd^2e^4x+a^3e^5(48d-19ex))}{a^3(a+cx^2)} + \frac{2c(cd^2+ae^2)^2(5c^2d^4x+18acd^2e^2x+a^2e^3(24d-}{a^2(a+cx^2)^2}}{}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^2)^4),x]
```

output
```
((-48*e^7*(c*d^2 + a*e^2))/(d + e*x) + (3*c*(c*d^2 + a*e^2)*(5*c^3*d^6*x +
 23*a*c^2*d^4*e^2*x + 47*a^2*c*d^2*e^4*x + a^3*e^5*(48*d - 19*e*x)))/(a^3*
(a + c*x^2)) + (2*c*(c*d^2 + a*e^2)^2*(5*c^2*d^4*x + 18*a*c*d^2*e^2*x + a^
2*e^3*(24*d - 11*e*x)))/(a^2*(a + c*x^2)^2) + (8*c*(c*d^2 + a*e^2)^3*(c*d^
2*x + a*e*(2*d - e*x)))/(a*(a + c*x^2)^3) + (3*Sqrt[c]*(5*c^4*d^8 + 28*a*c
^3*d^6*e^2 + 70*a^2*c^2*d^4*e^4 + 140*a^3*c*d^2*e^6 - 35*a^4*e^8)*ArcTan[(
Sqrt[c]*x)/Sqrt[a]])/a^(7/2) + 384*c*d*e^7*Log[d + e*x] - 192*c*d*e^7*Log[
a + c*x^2])/(48*(c*d^2 + a*e^2)^5)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {496, 25, 686, 25, 27, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^4 (d+ex)^2}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{ae+cdx}{6a(a+cx^2)^3 (d+ex)(ae^2+cd^2)}-\frac{\int -\frac{5cd^2+6cexd+7ae^2}{(d+ex)^2(cx^2+a)^3}dx}{6a(ae^2+cd^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5cd^2+6cexd+7ae^2}{(d+ex)^2(cx^2+a)^3}dx}{6a(ae^2+cd^2)}+\frac{ae+cdx}{6a(a+cx^2)^3 (d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{686}$$

$$\frac{-\frac{\int -\frac{c\left(15c^2d^4+34ace^2d^2+4ce\left(5cd^2+13ae^2\right)xd+35a^2e^4\right)}{(d+ex)^2\left(cx^2+a\right)^2}dx}{4ac(ae^2+cd^2)}-\frac{ae(cd^2-7ae^2)-cdx(13ae^2+5cd^2)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}}{6a(ae^2+cd^2)}+\frac{ae+cdx}{6a(a+cx^2)^3 (d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{c\left(15c^2d^4+34ace^2d^2+4ce\left(5cd^2+13ae^2\right)xd+35a^2e^4\right)}{(d+ex)^2\left(cx^2+a\right)^2}dx}{4ac(ae^2+cd^2)}-\frac{ae(cd^2-7ae^2)-cdx(13ae^2+5cd^2)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}}{6a(ae^2+cd^2)}+\frac{ae+cdx}{6a(a+cx^2)^3 (d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{15c^2d^4+34ace^2d^2+4ce\left(5cd^2+13ae^2\right)xd+35a^2e^4}{(d+ex)^2\left(cx^2+a\right)^2}dx}{4a(ae^2+cd^2)}-\frac{ae(cd^2-7ae^2)-cdx(13ae^2+5cd^2)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{6a\left(a+cx^2\right)^3(d+ex)\left(ae^2+cd^2\right)}$$

$\downarrow$ 686

$$\frac{-\frac{\frac{\int -\frac{3c\left(5c^3d^6+13ac^2e^2d^4+11a^2ce^4d^2+2ce\left(5c^2d^4+18ace^2d^2+29a^2e^4\right)xd+35a^3e^6\right)}{(d+ex)^2\left(cx^2+a\right)}dx}{2ac\left(ae^2+cd^2\right)}-\frac{ae\left(5cd^2-7ae^2\right)\left(5ae^2+cd^2\right)-3cdx\left(29a^2e^4+18acd^2e^2+5c^2d^4\right)}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}}{4a(ae^2+cd^2)}-}{6a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3(d+ex)\left(ae^2+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{5c^3d^6+13ac^2e^2d^4+11a^2ce^4d^2+2ce\left(5c^2d^4+18ace^2d^2+29a^2e^4\right)xd+35a^3e^6}{(d+ex)^2\left(cx^2+a\right)}dx}{2a\left(ae^2+cd^2\right)}-\frac{ae\left(5cd^2-7ae^2\right)\left(5ae^2+cd^2\right)-3cdx\left(29a^2e^4+18acd^2e^2+5c^2d^4\right)}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}}{4a(ae^2+cd^2)}-\frac{ae(cd}{4a(}}{6a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3(d+ex)\left(ae^2+cd^2\right)}$$

$\downarrow$ 657

$$\frac{\frac{\frac{3\int \left(\frac{128a^3cde^8}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{\left(-5c^3d^6-23ac^2e^2d^4-47a^2ce^4d^2+35a^3e^6\right)e^2}{\left(cd^2+ae^2\right)(d+ex)^2}+\frac{c\left(5c^4d^8+28ac^3e^2d^6+70a^2c^2e^4d^4+140a^3ce^6d^2-128a^3ce^7xd-35a^4e^8\right)}{\left(cd^2+ae^2\right)^2\left(cx^2+a\right)}\right)dx}{2a\left(ae^2+cd^2\right)}-\frac{ae(5}{}}{4a(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3(d+ex)\left(ae^2+cd^2\right)}$$

$\downarrow$ 2009

$$\frac{\frac{\frac{3\left(-\frac{64a^3cde^7\log\left(a+cx^2\right)}{\left(ae^2+cd^2\right)^2}+\frac{128a^3cde^7\log(d+ex)}{\left(ae^2+cd^2\right)^2}+\frac{e\left(-35a^3e^6+47a^2cd^2e^4+23ac^2d^4e^2+5c^3d^6\right)}{(d+ex)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(-35a^4e^8+140a^3cd^2e^6+70a^2c^2d^4e^4+28\right.}{\sqrt{a}\left(ae^2+cd^2\right)^2}\right.}{2a\left(ae^2+cd^2\right)}}{4a(ae^2+cd^2)}}{6a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{6a\left(a+cx^2\right)^3(d+ex)\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^2)^4),x]
```

output
```
(a*e + c*d*x)/(6*a*(c*d^2 + a*e^2)*(d + e*x)*(a + c*x^2)^3) + (-1/4*(a*e*(
c*d^2 - 7*a*e^2) - c*d*(5*c*d^2 + 13*a*e^2)*x)/(a*(c*d^2 + a*e^2)*(d + e*x
)*(a + c*x^2)^2) + (-1/2*(a*e*(5*c*d^2 - 7*a*e^2)*(c*d^2 + 5*a*e^2) - 3*c*
d*(5*c^2*d^4 + 18*a*c*d^2*e^2 + 29*a^2*e^4)*x)/(a*(c*d^2 + a*e^2)*(d + e*x
)*(a + c*x^2)) + (3*((e*(5*c^3*d^6 + 23*a*c^2*d^4*e^2 + 47*a^2*c*d^2*e^4 -
 35*a^3*e^6))/((c*d^2 + a*e^2)*(d + e*x)) + (Sqrt[c]*(5*c^4*d^8 + 28*a*c^3
*d^6*e^2 + 70*a^2*c^2*d^4*e^4 + 140*a^3*c*d^2*e^6 - 35*a^4*e^8)*ArcTan[(Sq
rt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) + (128*a^3*c*d*e^7*Log[d +
e*x])/(c*d^2 + a*e^2)^2 - (64*a^3*c*d*e^7*Log[a + c*x^2])/(c*d^2 + a*e^2)^
2))/(2*a*(c*d^2 + a*e^2)))/(4*a*(c*d^2 + a*e^2)))/(6*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.25

method	result
default	$-\frac{e^{7}}{\left(a\,e^{2}+c\,d^{2}\right)^{4}\left(ex+d\right)}+\frac{8cd\,e^{7}\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{5}}-\frac{c\left(\frac{c^{2}\left(19a^{4}e^{8}-28a^{3}c\,d^{2}e^{6}-70a^{2}c^{2}d^{4}e^{4}-28a\,c^{3}d^{6}e^{2}-5c^{4}d^{8}\right)x^{5}}{16a^{3}}+\left(-3ad\,e^{7}c^{2}-3d^{3}e^{5}c^{3}\right)x\right.}{}$
risch	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-e^7/(a*e^2+c*d^2)^4/(e*x+d)+8*c*d*e^7*ln(e*x+d)/(a*e^2+c*d^2)^5-c/(a*e^2+
c*d^2)^5*((1/16*c^2*(19*a^4*e^8-28*a^3*c*d^2*e^6-70*a^2*c^2*d^4*e^4-28*a*c
^3*d^6*e^2-5*c^4*d^8)/a^3*x^5+(-3*a*c^2*d*e^7-3*c^3*d^3*e^5)*x^4+1/6*c*(17
*a^4*e^8-20*a^3*c*d^2*e^6-60*a^2*c^2*d^4*e^4-28*a*c^3*d^6*e^2-5*c^4*d^8)/a
^2*x^3+(-7*a^2*c*d*e^7-8*a*c^2*d^3*e^5-c^3*d^5*e^3)*x^2+1/16*(29*a^4*e^8-2
0*a^3*c*d^2*e^6-90*a^2*c^2*d^4*e^4-52*a*c^3*d^6*e^2-11*c^4*d^8)/a*x-1/3*d*
e*(13*a^3*e^6+18*a^2*c*d^2*e^4+6*a*c^2*d^4*e^2+c^3*d^6))/(c*x^2+a)^3+1/16/
a^3*(64*a^3*d*e^7*ln(c*x^2+a)+(35*a^4*e^8-140*a^3*c*d^2*e^6-70*a^2*c^2*d^4
*e^4-28*a*c^3*d^6*e^2-5*c^4*d^8)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. 2(361) = 722.

Time = 19.96 (sec) , antiderivative size = 3843, normalized size of antiderivative = 10.19

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^4} dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^4} dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. 2(361) = 722.

Time = 0.16 (sec) , antiderivative size = 1215, normalized size of antiderivative = 3.22

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^4} dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-4*c*d*e^7*log(c*x^2 + a)/(c^5*d^10 + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4
 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e^10) + 8*c*d*e^7*log(e*x +
d)/(c^5*d^10 + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 +
 5*a^4*c*d^2*e^8 + a^5*e^10) + 1/16*(5*c^5*d^8 + 28*a*c^4*d^6*e^2 + 70*a^2
*c^3*d^4*e^4 + 140*a^3*c^2*d^2*e^6 - 35*a^4*c*e^8)*arctan(c*x/sqrt(a*c))/(
(a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e^
6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(a*c)) + 1/48*(16*a^3*c^3*d^6*e + 80*a
^4*c^2*d^4*e^3 + 208*a^5*c*d^2*e^5 - 48*a^6*e^7 + 3*(5*c^6*d^6*e + 23*a*c^
5*d^4*e^3 + 47*a^2*c^4*d^2*e^5 - 35*a^3*c^3*e^7)*x^6 + 3*(5*c^6*d^7 + 23*a
*c^5*d^5*e^2 + 47*a^2*c^4*d^3*e^4 + 29*a^3*c^3*d*e^6)*x^5 + 8*(5*a*c^5*d^6
*e + 23*a^2*c^4*d^4*e^3 + 55*a^3*c^3*d^2*e^5 - 35*a^4*c^2*e^7)*x^4 + 8*(5*
a*c^5*d^7 + 23*a^2*c^4*d^5*e^2 + 43*a^3*c^3*d^3*e^4 + 25*a^4*c^2*d*e^6)*x^
3 + 3*(11*a^2*c^4*d^6*e + 57*a^3*c^3*d^4*e^3 + 161*a^4*c^2*d^2*e^5 - 77*a^
5*c*e^7)*x^2 + (33*a^2*c^4*d^7 + 139*a^3*c^3*d^5*e^2 + 227*a^4*c^2*d^3*e^4
 + 121*a^5*c*d*e^6)*x)/(a^6*c^4*d^9 + 4*a^7*c^3*d^7*e^2 + 6*a^8*c^2*d^5*e^
4 + 4*a^9*c*d^3*e^6 + a^10*d*e^8 + (a^3*c^7*d^8*e + 4*a^4*c^6*d^6*e^3 + 6*
a^5*c^5*d^4*e^5 + 4*a^6*c^4*d^2*e^7 + a^7*c^3*e^9)*x^7 + (a^3*c^7*d^9 + 4*
a^4*c^6*d^7*e^2 + 6*a^5*c^5*d^5*e^4 + 4*a^6*c^4*d^3*e^6 + a^7*c^3*d*e^8)*x
^6 + 3*(a^4*c^6*d^8*e + 4*a^5*c^5*d^6*e^3 + 6*a^6*c^4*d^4*e^5 + 4*a^7*c^3*
d^2*e^7 + a^8*c^2*e^9)*x^5 + 3*(a^4*c^6*d^9 + 4*a^5*c^5*d^7*e^2 + 6*a^6...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(361) = 722$.

Time = 0.13 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.40

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
-e^15/((c^4*d^8*e^8 + 4*a*c^3*d^6*e^10 + 6*a^2*c^2*d^4*e^12 + 4*a^3*c*d^2*
e^14 + a^4*e^16)*(e*x + d)) - 4*c*d*e^7*log(c - 2*c*d/(e*x + d) + c*d^2/(e
*x + d)^2 + a*e^2/(e*x + d)^2)/(c^5*d^10 + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^
6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e^10) + 1/16*(5*c^5*d^8
*e^2 + 28*a*c^4*d^6*e^4 + 70*a^2*c^3*d^4*e^6 + 140*a^3*c^2*d^2*e^8 - 35*a^
4*c*e^10)*arctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/
((a^3*c^5*d^10 + 5*a^4*c^4*d^8*e^2 + 10*a^5*c^3*d^6*e^4 + 10*a^6*c^2*d^4*e
^6 + 5*a^7*c*d^2*e^8 + a^8*e^10)*sqrt(a*c)*e^2) + 1/48*(15*c^7*d^7*e + 79*
a*c^6*d^5*e^3 + 185*a^2*c^5*d^3*e^5 - 295*a^3*c^4*d*e^7 - 3*(25*c^7*d^8*e^
2 + 130*a*c^6*d^6*e^4 + 300*a^2*c^5*d^4*e^6 - 618*a^3*c^4*d^2*e^8 + 19*a^4
*c^3*e^10)/((e*x + d)*e) + 6*(25*c^7*d^9*e^3 + 135*a*c^6*d^7*e^5 + 327*a^2
*c^5*d^5*e^7 - 691*a^3*c^4*d^3*e^9 - 76*a^4*c^3*d*e^11)/((e*x + d)^2*e^2)
- 2*(75*c^7*d^10*e^4 + 440*a*c^6*d^8*e^6 + 1162*a^2*c^5*d^6*e^8 - 2212*a^3
*c^4*d^4*e^10 - 1277*a^4*c^3*d^2*e^12 + 68*a^5*c^2*e^14)/((e*x + d)^3*e^3)
 + 3*(25*c^7*d^11*e^5 + 165*a*c^6*d^9*e^7 + 490*a^2*c^5*d^7*e^9 - 742*a^3*
c^4*d^5*e^11 - 1139*a^4*c^3*d^3*e^13 - 47*a^5*c^2*d*e^15)/((e*x + d)^4*e^4
) - 3*(5*c^7*d^12*e^6 + 38*a*c^6*d^10*e^8 + 131*a^2*c^5*d^8*e^10 - 140*a^3
*c^4*d^6*e^12 - 517*a^4*c^3*d^4*e^14 - 250*a^5*c^2*d^2*e^16 + 29*a^6*c*e^1
8)/((e*x + d)^5*e^5))/((c*d^2 + a*e^2)^5*a^3*(c - 2*c*d/(e*x + d) + c*d^2/
(e*x + d)^2 + a*e^2/(e*x + d)^2)^3)
```

Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 1876, normalized size of antiderivative = 4.98

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^4} dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^4*(d + e*x)^2),x)
```

output
```
((c^3*d^6*e - 3*a^3*e^7 + 5*a*c^2*d^4*e^3 + 13*a^2*c*d^2*e^5)/(3*(a*e^2 +
c*d^2)*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(33*c
^3*d^5 + 106*a*c^2*d^3*e^2 + 121*a^2*c*d*e^4))/(48*a*(a^3*e^6 + c^3*d^6 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x^6*(5*c^6*d^6*e - 35*a^3*c^3*e^7 +
 23*a*c^5*d^4*e^3 + 47*a^2*c^4*d^2*e^5))/(16*a^3*(a^4*e^8 + c^4*d^8 + 4*a*
c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^3*(5*c^4*d^5 + 18
*a*c^3*d^3*e^2 + 25*a^2*c^2*d*e^4))/(6*a^2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4)) + (x^5*(5*c^5*d^5 + 18*a*c^4*d^3*e^2 + 29*a^2*c^
3*d*e^4))/(16*a^3*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4))
 + (x^4*(5*c^5*d^6*e - 35*a^3*c^2*e^7 + 23*a*c^4*d^4*e^3 + 55*a^2*c^3*d^2*
e^5))/(6*a^2*(a*e^2 + c*d^2)*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*
c*d^2*e^4)) + (x^2*(11*c^4*d^6*e - 77*a^3*c*e^7 + 57*a*c^3*d^4*e^3 + 161*a
^2*c^2*d^2*e^5))/(16*a*(a*e^2 + c*d^2)*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4)))/(a^3*d + c^3*d*x^6 + c^3*e*x^7 + a^3*e*x + 3*a^2*c*
d*x^2 + 3*a*c^2*d*x^4 + 3*a^2*c*e*x^3 + 3*a*c^2*e*x^5) - (log(25*c^9*d^20*
(-a^7*c)^(3/2) - 1225*a^17*e^20*(-a^7*c)^(1/2) + 25*a^10*c^11*d^20*x - 291
237*a*d^4*e^16*(-a^7*c)^(5/2) - 184696*c*d^6*e^14*(-a^7*c)^(5/2) + 140106*
a^9*d^2*e^18*(-a^7*c)^(3/2) + 1225*a^20*c*e^20*x + 2069*a^2*c^7*d^16*e^4*(
-a^7*c)^(3/2) + 8568*a^3*c^6*d^14*e^6*(-a^7*c)^(3/2) + 24514*a^4*c^5*d^12*
e^8*(-a^7*c)^(3/2) + 47740*a^5*c^4*d^10*e^10*(-a^7*c)^(3/2) + 62370*a^6...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2894, normalized size of antiderivative = 7.68

$$\int \frac{1}{(d+ex)^2 (a+cx^2)^4} dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(c*x^2+a)^4,x)
```

output
```
( - 105*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**7*d**2*e**8 - 105
*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**7*d*e**9*x + 420*sqrt(c)
*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**6*c*d**4*e**6 + 420*sqrt(c)*sqrt
(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**6*c*d**3*e**7*x - 315*sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a**6*c*d**2*e**8*x**2 - 315*sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a**6*c*d*e**9*x**3 + 210*sqrt(c)*sqrt(a)*at
an((c*x)/(sqrt(c)*sqrt(a)))*a**5*c**2*d**6*e**4 + 210*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a**5*c**2*d**5*e**5*x + 1260*sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*a**5*c**2*d**4*e**6*x**2 + 1260*sqrt(c)*sqrt(a)
*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*c**2*d**3*e**7*x**3 - 315*sqrt(c)*sqrt
(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*c**2*d**2*e**8*x**4 - 315*sqrt(c)*s
qrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*c**2*d*e**9*x**5 + 84*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d**8*e**2 + 84*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d**7*e**3*x + 630*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d**6*e**4*x**2 + 630*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d**5*e**5*x**3 + 1260*sqrt(c
)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d**4*e**6*x**4 + 1260*sq
rt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d**3*e**7*x**5 - 105
*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d**2*e**8*x**6 -
105*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c**3*d*e**9*x**7...
```

3.131 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx = -\frac{3}{26}\arctan\left(\frac{x}{2}\right)+\frac{1}{13}\log(3-x)-\frac{1}{26}\log\left(4+x^2\right)$$

output
```
-3/26*arctan(1/2*x)+1/13*ln(3-x)-1/26*ln(x^2+4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx = -\frac{3}{26}\arctan\left(\frac{x}{2}\right)-\frac{1}{26}\log\left(13+6(-3+x)+(-3+x)^2\right) + \frac{1}{13}\log(-3+x)$$

input
```
Integrate[1/((-3 + x)*(4 + x^2)),x]
```

output
```
(-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/1
3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {479, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-3)(x^2+4)}dx$$

$$\Big\downarrow \; 479$$

$$\frac{1}{13}\int -\frac{x+3}{x^2+4}dx+\frac{1}{13}\log(3-x)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{13}\log(3-x)-\frac{1}{13}\int \frac{x+3}{x^2+4}dx$$

$$\Big\downarrow \; 452$$

$$\frac{1}{13}\left(-3\int \frac{1}{x^2+4}dx-\int \frac{x}{x^2+4}dx\right)+\frac{1}{13}\log(3-x)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{13}\left(-\int \frac{x}{x^2+4}dx-\frac{3}{2}\arctan\left(\frac{x}{2}\right)\right)+\frac{1}{13}\log(3-x)$$

$$\Big\downarrow \; 240$$

$$\frac{1}{13}\left(-\frac{3}{2}\arctan\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(x^2+4\right)\right)+\frac{1}{13}\log(3-x)$$

input
```
Int[1/((-3 + x)*(4 + x^2)),x]
```

output
```
Log[3 - x]/13 + ((-3*ArcTan[x/2])/2 - Log[4 + x^2]/2)/13
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\ln(-3+x)}{13}-\frac{\ln(x^2+4)}{26}-\frac{3\arctan(\frac{x}{2})}{26}$	22
risch	$\frac{\ln(-3+x)}{13}-\frac{\ln(9x^2+36)}{26}-\frac{3\arctan(\frac{x}{2})}{26}$	24
parallelrisch	$\frac{\ln(-3+x)}{13}-\frac{\ln(x-2i)}{26}+\frac{3i\ln(x-2i)}{52}-\frac{\ln(x+2i)}{26}-\frac{3i\ln(x+2i)}{52}$	38

input
```
int(1/(-3+x)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/13*ln(-3+x)-1/26*ln(x^2+4)-3/26*arctan(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{1}{2}x\right) - \frac{1}{26}\log\left(x^2+4\right) + \frac{1}{13}\log\left(x-3\right)$$

input
```
integrate(1/(-3+x)/(x^2+4),x, algorithm="fricas")
```

output
```
-3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = \frac{\log\left(x-3\right)}{13} - \frac{\log\left(x^2+4\right)}{26} - \frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{26}$$

input
```
integrate(1/(-3+x)/(x**2+4),x)
```

output
```
log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{1}{2}x\right) - \frac{1}{26}\log\left(x^2+4\right) + \frac{1}{13}\log\left(x-3\right)$$

input
```
integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")
```

output
```
-3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{1}{2}x\right) - \frac{1}{26}\log\left(x^2+4\right) + \frac{1}{13}\log\left(|x-3|\right)$$

input
```
integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")
```

output
```
-3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = \frac{\ln(x-3)}{13} + \ln(x-2i)\left(-\frac{1}{26}+\frac{3}{52}i\right) + \ln(x+2i)\left(-\frac{1}{26}-\frac{3}{52}i\right)$$

input
```
int(1/((x^2 + 4)*(x - 3)),x)
```

output
```
log(x - 3)/13 - log(x - 2i)*(1/26 - 3i/52) - log(x + 2i)*(1/26 + 3i/52)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3atan\left(\frac{x}{2}\right)}{26} - \frac{\log(x^2+4)}{26} + \frac{\log(x-3)}{13}$$

input
```
int(1/(-3+x)/(x^2+4),x)
```

output
```
( - 3*atan(x/2) - log(x**2 + 4) + 2*log(x - 3))/26
```

3.132 $\int \frac{1}{(2+x)(1+x^2)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{(2+x)(1+x^2)} \, dx = \frac{2\arctan(x)}{5} + \frac{1}{5}\log(2+x) - \frac{1}{10}\log\left(1+x^2\right)$$

output
```
2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)(1+x^2)} \, dx = \frac{2\arctan(x)}{5} + \frac{1}{5}\log(2+x) - \frac{1}{10}\log\left(1+x^2\right)$$

input
```
Integrate[1/((2 + x)*(1 + x^2)),x]
```

output
```
(2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+2)(x^2+1)}dx$$

$$\Big\downarrow \text{479}$$

$$\frac{1}{5}\int \frac{2-x}{x^2+1}dx+\frac{1}{5}\log(x+2)$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{5}\left(2\int \frac{1}{x^2+1}dx-\int \frac{x}{x^2+1}dx\right)+\frac{1}{5}\log(x+2)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5}\left(2\arctan(x)-\int \frac{x}{x^2+1}dx\right)+\frac{1}{5}\log(x+2)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{5}\left(2\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)+\frac{1}{5}\log(x+2)$$

input
```
Int[1/((2 + x)*(1 + x^2)),x]
```

output
```
Log[2 + x]/5 + (2*ArcTan[x] - Log[1 + x^2]/2)/5
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\arctan(x)}{5}+\frac{\ln(2+x)}{5}-\frac{\ln(x^2+1)}{10}$	20
risch	$\frac{2\arctan(x)}{5}+\frac{\ln(2+x)}{5}-\frac{\ln(x^2+1)}{10}$	20
parallelrisch	$\frac{\ln(2+x)}{5}-\frac{\ln(x-i)}{10}-\frac{i\ln(x-i)}{5}-\frac{\ln(x+i)}{10}+\frac{i\ln(x+i)}{5}$	38

input
```
int(1/(2+x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

input
```
integrate(1/(2+x)/(x^2+1),x, algorithm="fricas")
```

output
```
2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{\log(x+2)}{5} - \frac{\log(x^2+1)}{10} + \frac{2\,\mathrm{atan}(x)}{5}$$

input
```
integrate(1/(2+x)/(x**2+1),x)
```

output
```
log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

input
```
integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")
```

output
```
2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(|x+2|)$$

input
```
integrate(1/(2+x)/(x^2+1),x, algorithm="giac")
```

output
```
2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{\ln(x+2)}{5} + \ln(x-\mathrm{i})\left(-\frac{1}{10}-\frac{1}{5}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{10}+\frac{1}{5}\mathrm{i}\right)$$

input
```
int(1/((x^2 + 1)*(x + 2)),x)
```

output
```
log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2atan(x)}{5} - \frac{\log(x^2+1)}{10} + \frac{\log(x+2)}{5}$$

input
```
int(1/(2+x)/(x^2+1),x)
```

output
```
(4*atan(x) - log(x**2 + 1) + 2*log(x + 2))/10
```

3.133 $\int \frac{1}{(1+x)(1+x^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x) - \frac{1}{4}\log(1+x^2)$$

output
```
1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x) - \frac{1}{4}\log(1+x^2)$$

input
```
Integrate[1/((1 + x)*(1 + x^2)),x]
```

output
```
ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)(x^2+1)} \, dx$$

$$\Big\downarrow \ 479$$

$$\frac{1}{2}\int \frac{1-x}{x^2+1}dx+\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \ 452$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx-\int \frac{x}{x^2+1}dx\right)+\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\left(\arctan(x)-\int \frac{x}{x^2+1}dx\right)+\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \ 240$$

$$\frac{1}{2}\left(\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)+\frac{1}{2}\log(x+1)$$

input
```
Int[1/((1 + x)*(1 + x^2)),x]
```

output
```
Log[1 + x]/2 + (ArcTan[x] - Log[1 + x^2]/2)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
parallelrisch	$\frac{\ln(x+1)}{2}-\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

input
```
int(1/(x+1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

input
```
integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(1+x)/(x**2+1),x)
```

output
```
log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

input
```
integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(|x+1|)$$

input
```
integrate(1/(1+x)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/((x^2 + 1)*(x + 1)),x)
```

output
```
log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{atan(x)}{2} - \frac{\log(x^2+1)}{4} + \frac{\log(x+1)}{2}$$

input
```
int(1/(1+x)/(x^2+1),x)
```

output
```
(2*atan(x) - log(x**2 + 1) + 2*log(x + 1))/4
```

3.134 $\int (-3+x) \left(-7+4x^2\right) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int (-3+x) \left(-7+4x^2\right) \, dx = 21x-4x^3+\frac{1}{16}\left(7-4x^2\right)^2$$

output
```
21*x-4*x^3+1/16*(-4*x^2+7)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int (-3+x) \left(-7+4x^2\right) \, dx = 21x-\frac{7x^2}{2}-4x^3+x^4$$

input
```
Integrate[(-3 + x)*(-7 + 4*x^2),x]
```

output
```
21*x - (7*x^2)/2 - 4*x^3 + x^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x-3)\left(4x^2-7\right)\,dx$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{16}\left(7-4x^2\right)^2-3\int \left(4x^2-7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(7-4x^2\right)^2-3\left(\frac{4x^3}{3}-7x\right)$$

input
```
Int[(-3 + x)*(-7 + 4*x^2),x]
```

output
```
(7 - 4*x^2)^2/16 - 3*(-7*x + (4*x^3)/3)
```

Defintions of rubi rules used

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
gosper	$x^4-4x^3-\frac{7}{2}x^2+21x$	18
default	$x^4-4x^3-\frac{7}{2}x^2+21x$	18
norman	$x^4-4x^3-\frac{7}{2}x^2+21x$	18
risch	$x^4-4x^3-\frac{7}{2}x^2+21x$	18
parallelrisch	$x^4-4x^3-\frac{7}{2}x^2+21x$	18
orering	$\frac{x(2x^3-8x^2-7x+42)}{2}$	19

input
```
int((-3+x)*(4*x^2-7),x,method=_RETURNVERBOSE)
```

output
```
x^4-4*x^3-7/2*x^2+21*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx=x^4-4\,x^3-\frac{7}{2}\,x^2+21\,x$$

input
```
integrate((-3+x)*(4*x^2-7),x, algorithm="fricas")
```

output
```
x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx=x^4-4x^3-\frac{7x^2}{2}+21x$$

input
```
integrate((-3+x)*(4*x**2-7),x)
```

output
```
x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx=x^4-4\,x^3-\frac{7}{2}\,x^2+21\,x$$

input
```
integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")
```

output
```
x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx=x^4-4\,x^3-\frac{7}{2}\,x^2+21\,x$$

input
```
integrate((-3+x)*(4*x^2-7),x, algorithm="giac")
```

output
```
x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4\,x^3-\frac{7\,x^2}{2}+21\,x$$

input
```
int((4*x^2 - 7)*(x - 3),x)
```

output
```
21*x - (7*x^2)/2 - 4*x^3 + x^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (-3+x)\left(-7+4x^2\right)\,dx = \frac{x(2x^3-8x^2-7x+42)}{2}$$

input
```
int((-3+x)*(4*x^2-7),x)
```

output
```
(x*(2*x**3 - 8*x**2 - 7*x + 42))/2
```

3.135 $\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = -\frac{4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{7/4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{7/4}}$$

output
```
-4*d*e*(e*x+d)^(1/2)/c-2/3*e*(e*x+d)^(3/2)/c-(c^(1/2)*d-a^(1/2)*e)^(5/2)*a
rctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/c^(7/4)+
(c^(1/2)*d+a^(1/2)*e)^(5/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/
2)*e)^(1/2))/a^(1/2)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = -\frac{2ce\sqrt{d+ex}(7d+ex)+\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}}+\frac{3\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)^3\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d-}}{\sqrt{c}d-\sqrt{a}e}\right.}{\sqrt{a}\sqrt{-cd+\sqrt{a}\sqrt{c}e}}}{3c^2}$$

input
```
Integrate[(d + e*x)^(5/2)/(a - c*x^2),x]
```

output
```
-1/3*(2*c*e*Sqrt[d + e*x]*(7*d + e*x) + (3*(Sqrt[c]*d + Sqrt[a]*e)^2*Sqrt[
-(c*d) - Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[
d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/Sqrt[a] + (3*Sqrt[c]*(Sqrt[c]*d - Sqrt
[a]*e)^3*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*
d - Sqrt[a]*e)])/(Sqrt[a]*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/c^2
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {481, 25, 653, 25, 27, 654, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}dx$$

$$\Big\downarrow \text{481}$$

$$-\frac{\int -\frac{\sqrt{d+ex}(cd^2+2cexd+ae^2)}{a-cx^2}dx}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{d+ex}(cd^2+2cexd+ae^2)}{a-cx^2}dx}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{653}$$

$$\frac{-\frac{\int -\frac{c\left(d\left(cd^2+3ae^2\right)+e\left(3cd^2+ae^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}-4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{c\left(d\left(cd^2+3ae^2\right)+e\left(3cd^2+ae^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}-4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{d(cd^2+3ae^2)+e(3cd^2+ae^2)x}{\sqrt{d+ex}(a-cx^2)}dx-4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 654$$

$$\frac{2\int \frac{e(2d(cd^2-ae^2)-(3cd^2+ae^2)(d+ex))}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}-4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 27$$

$$\frac{2e\int \frac{2d(cd^2-ae^2)-(3cd^2+ae^2)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}-4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 1480$$

$$\frac{2e\left(\frac{(\sqrt{c}d-\sqrt{a}e)^3\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{(\sqrt{a}e+\sqrt{c}d)^3\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}\right)-4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 221$$

$$\frac{2e\left(\frac{(\sqrt{a}e+\sqrt{c}d)^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e}-\frac{(\sqrt{c}d-\sqrt{a}e)^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}e}\right)-4de\sqrt{d+ex}}{c}-\frac{2e(d+ex)^{3/2}}{3c}$$

input
```
Int[(d + e*x)^(5/2)/(a - c*x^2),x]
```

output
```
(-2*e*(d + e*x)^(3/2))/(3*c) + (-4*d*e*Sqrt[d + e*x] + 2*e*(-1/2*((Sqrt[c]
*d - Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqr
t[a]*e]])/(Sqrt[a]*c^(3/4)*e) + ((Sqrt[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^
(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e)))
/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{e\left(\frac{2\sqrt{ex+d}\,(ex+7d)}{3}-\frac{\left(3ad\,e^{2}c+c^{2}d^{3}+\sqrt{ac\,e^{2}}\,a\,e^{2}+3\sqrt{ac\,e^{2}}\,c\,d^{2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(-3ad\,e^{2}c-c^{2}d^{3}+\sqrt{ac\,e^{2}}\,a\right.}{\sqrt{a}}\right.}{c}$
risch	$-\frac{2(ex+7d)\sqrt{ex+d}\,e}{3c}-2e\left(-\frac{\left(3ad\,e^{2}c+c^{2}d^{3}+\sqrt{ac\,e^{2}}\,a\,e^{2}+3\sqrt{ac\,e^{2}}\,c\,d^{2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(-3ac\right.}{}\right.$
derivativedivides	$-2e\left(\frac{\frac{(ex+d)^{\frac{3}{2}}}{3}+2d\sqrt{ex+d}}{c}+\frac{\left(-3ad\,e^{2}c-c^{2}d^{3}+\sqrt{ac\,e^{2}}\,a\,e^{2}+3\sqrt{ac\,e^{2}}\,c\,d^{2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(3ac\right.}{}\right.$
default	$2e\left(-\frac{\frac{(ex+d)^{\frac{3}{2}}}{3}+2d\sqrt{ex+d}}{c}+\frac{\left(3ad\,e^{2}c+c^{2}d^{3}-\sqrt{ac\,e^{2}}\,a\,e^{2}-3\sqrt{ac\,e^{2}}\,c\,d^{2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(-3ac\right.}{}\right.$

input
```
int((e*x+d)^(5/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-e/c*(2/3*(e*x+d)^(1/2)*(e*x+7*d)-(3*a*d*e^2*c+c^2*d^3+(a*c*e^2)^(1/2)*a*e
^2+3*(a*c*e^2)^(1/2)*c*d^2)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2
)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(-3*a*d*e^2*c-c
^2*d^3+(a*c*e^2)^(1/2)*a*e^2+3*(a*c*e^2)^(1/2)*c*d^2)/(a*c*e^2)^(1/2)/((-c
*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2)
)*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(121) = 242$.

Time = 0.14 (sec) , antiderivative size = 1617, normalized size of antiderivative = 9.68

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*c*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt((25*c^
4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a
^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c^2*d^4*e^5 + 8*a^3*
c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) + (10*a*c^4*d^5*e^2 + 20*a^2*c^3*d^3*e^
4 + 2*a^3*c^2*d*e^6 - (a*c^6*d^2 + a^2*c^5*e^2)*sqrt((25*c^4*d^8*e^2 + 100
*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7
)))*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt((25*c^4*d^8*
e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*e^1
0)/(a*c^7)))/(a*c^3))) - 3*c*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4
+ a*c^3*sqrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 2
0*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c
^2*d^4*e^5 + 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) - (10*a*c^4*d^5*e^2
+ 20*a^2*c^3*d^3*e^4 + 2*a^3*c^2*d*e^6 - (a*c^6*d^2 + a^2*c^5*e^2)*sqrt((2
5*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8
 + a^4*e^10)/(a*c^7)))*sqrt((c^2*d^5 + 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^
3*sqrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 + 20*a^3*
c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))) + 3*c*sqrt((c^2*d^5 + 10*a*c*d^3
*e^2 + 5*a^2*d*e^4 - a*c^3*sqrt((25*c^4*d^8*e^2 + 100*a*c^3*d^6*e^4 + 110*
a^2*c^2*d^4*e^6 + 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c
^4*d^8*e - 14*a^2*c^2*d^4*e^5 + 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d...
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = -\int \frac{d^2\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{e^2x^2\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{2dex\sqrt{d+ex}}{-a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(-c*x**2+a),x)
```

output
```
-Integral(d**2*sqrt(d + e*x)/(-a + c*x**2), x) - Integral(e**2*x**2*sqrt(d
 + e*x)/(-a + c*x**2), x) - Integral(2*d*e*x*sqrt(d + e*x)/(-a + c*x**2),
x)
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = \int -\frac{(ex+d)^{\frac{5}{2}}}{cx^2-a}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((e*x + d)^(5/2)/(c*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(121) = 242$.

Time = 0.16 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx =$$

$$-\frac{\left(\sqrt{ac}c^4d^4e+3\sqrt{ac}ac^3d^2e^3-\left(3\sqrt{ac}acd^2e+\sqrt{ac}a^2e^3\right)c^2e^2+2\left(ac^3d^3e-a^2c^2de^3\right)|c||e|\right)\arctan\left(\frac{}{\sqrt{-\frac{c^4d+}{}}}\right.}{\left(ac^4d-\sqrt{ac}ac^3e\right)\sqrt{-c^2d-\sqrt{ac}ce}|e|}$$

$$+\frac{\left(\sqrt{ac}c^4d^4e+3\sqrt{ac}ac^3d^2e^3-\left(3\sqrt{ac}acd^2e+\sqrt{ac}a^2e^3\right)c^2e^2-2\left(ac^3d^3e-a^2c^2de^3\right)|c||e|\right)\arctan\left(\frac{}{\sqrt{-\frac{c^4d-}{}}}\right.}{\left(ac^4d+\sqrt{ac}ac^3e\right)\sqrt{-c^2d+\sqrt{ac}ce}|e|}$$

$$-\frac{2\left((ex+d)^{\frac{3}{2}}c^2e+6\sqrt{ex+d}c^2de\right)}{3c^3}$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-(sqrt(a*c)*c^4*d^4*e + 3*sqrt(a*c)*a*c^3*d^2*e^3 - (3*sqrt(a*c)*a*c*d^2*e
 + sqrt(a*c)*a^2*e^3)*c^2*e^2 + 2*(a*c^3*d^3*e - a^2*c^2*d*e^3)*abs(c)*abs
(e))*arctan(sqrt(e*x + d)/sqrt(-(c^4*d + sqrt(c^8*d^2 - (c^4*d^2 - a*c^3*e
^2)*c^4))/c^4))/((a*c^4*d - sqrt(a*c)*a*c^3*e)*sqrt(-c^2*d - sqrt(a*c)*c*e
)*abs(e)) + (sqrt(a*c)*c^4*d^4*e + 3*sqrt(a*c)*a*c^3*d^2*e^3 - (3*sqrt(a*c
)*a*c*d^2*e + sqrt(a*c)*a^2*e^3)*c^2*e^2 - 2*(a*c^3*d^3*e - a^2*c^2*d*e^3)
*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^4*d - sqrt(c^8*d^2 - (c^4*d^
2 - a*c^3*e^2)*c^4))/c^4))/((a*c^4*d + sqrt(a*c)*a*c^3*e)*sqrt(-c^2*d + sq
rt(a*c)*c*e)*abs(e)) - 2/3*((e*x + d)^(3/2)*c^2*e + 6*sqrt(e*x + d)*c^2*d*
e)/c^3
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 3385, normalized size of antiderivative = 20.27

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(a - c*x^2),x)
```

output
```
- atan((a^3*e^8*(d + e*x)^(1/2)*((e^5*(a^3*c^7)^(1/2))/(4*c^7) + d^5/(4*a*
c) + (5*d^3*e^2)/(2*c^2) + (5*a*d*e^4)/(4*c^3) + (5*d^4*e*(a^3*c^7)^(1/2))
/(4*a^2*c^5) + (5*d^2*e^3*(a^3*c^7)^(1/2))/(2*a*c^6))^(1/2)*32i)/((16*a^4*
e^11)/c^2 + 64*a^2*d^4*e^7 - 80*c^2*d^8*e^3 + (160*a^3*d^2*e^9)/c - 160*a*
c*d^6*e^5 - (160*d^5*e^6*(a^3*c^7)^(1/2))/c^3 + (288*a*d^3*e^8*(a^3*c^7)^(
1/2))/c^4 + (32*a^2*d*e^10*(a^3*c^7)^(1/2))/c^5 - (160*d^7*e^4*(a^3*c^7)^(
1/2))/(a*c^2)) - (d^5*e^3*(a^3*c^7)^(1/2)*(d + e*x)^(1/2)*((e^5*(a^3*c^7)^
(1/2))/(4*c^7) + d^5/(4*a*c) + (5*d^3*e^2)/(2*c^2) + (5*a*d*e^4)/(4*c^3) +
 (5*d^4*e*(a^3*c^7)^(1/2))/(4*a^2*c^5) + (5*d^2*e^3*(a^3*c^7)^(1/2))/(2*a*
c^6))^(1/2)*160i)/((16*a^5*e^11)/c + 160*a^4*d^2*e^9 - 80*a*c^3*d^8*e^3 +
64*a^3*c*d^4*e^7 - 160*a^2*c^2*d^6*e^5 - (160*d^7*e^4*(a^3*c^7)^(1/2))/c -
 (160*a*d^5*e^6*(a^3*c^7)^(1/2))/c^2 + (32*a^3*d*e^10*(a^3*c^7)^(1/2))/c^4
 + (288*a^2*d^3*e^8*(a^3*c^7)^(1/2))/c^3) - (d^3*e^5*(a^3*c^7)^(1/2)*(d +
e*x)^(1/2)*((e^5*(a^3*c^7)^(1/2))/(4*c^7) + d^5/(4*a*c) + (5*d^3*e^2)/(2*c
^2) + (5*a*d*e^4)/(4*c^3) + (5*d^4*e*(a^3*c^7)^(1/2))/(4*a^2*c^5) + (5*d^2
*e^3*(a^3*c^7)^(1/2))/(2*a*c^6))^(1/2)*320i)/(16*a^4*e^11 - 80*c^4*d^8*e^3
 - 160*a*c^3*d^6*e^5 + 160*a^3*c*d^2*e^9 + 64*a^2*c^2*d^4*e^7 - (160*d^7*e
^4*(a^3*c^7)^(1/2))/a - (160*d^5*e^6*(a^3*c^7)^(1/2))/c + (288*a*d^3*e^8*(
a^3*c^7)^(1/2))/c^2 + (32*a^2*d*e^10*(a^3*c^7)^(1/2))/c^3) - (a*d*e^7*(a^3
*c^7)^(1/2)*(d + e*x)^(1/2)*((e^5*(a^3*c^7)^(1/2))/(4*c^7) + d^5/(4*a*c...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.54

$$\int \frac{(d+ex)^{5/2}}{a-cx^2}\,dx = \frac{-6\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)a\,e^2-6\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+}}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{}}}\right.}{}$$

input
```
int((e*x+d)^(5/2)/(-c*x^2+a),x)
```

output
```
( - 6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*e**2 - 6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c
*d**2 + 12*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(s
qrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*d*e - 3*sqrt(a)*sqrt(sqrt(c)*sqrt
(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))
*a*e**2 - 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt
(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d**2 + 3*sqrt(a)*sqrt(sqrt(c)*sqrt
(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*
e**2 + 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e
+ c*d) + sqrt(c)*sqrt(d + e*x))*c*d**2 - 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
+ c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*d*e
 + 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*
d) + sqrt(c)*sqrt(d + e*x))*a*d*e - 28*sqrt(d + e*x)*a*c*d*e - 4*sqrt(d +
e*x)*a*c*e**2*x)/(6*a*c**2)
```

3.136 $\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx = -\frac{2e\sqrt{d+ex}}{c} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}}$$

output
```
-2*e*(e*x+d)^(1/2)/c-(c^(1/2)*d-a^(1/2)*e)^(3/2)*arctanh(c^(1/4)*(e*x+d)^(
1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/c^(5/4)+(c^(1/2)*d+a^(1/2)*e)^(3
/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/c^(
5/4)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx=\frac{-2e\sqrt{d+ex}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\sqrt{-cd-\sqrt{a}\sqrt{c}e}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{a}\sqrt{-cd+\sqrt{a}\sqrt{c}e}}}{c}$$

input
```
Integrate[(d + e*x)^(3/2)/(a - c*x^2),x]
```

output
```
(-2*e*Sqrt[d + e*x] + ((Sqrt[c]*d + Sqrt[a]*e)^2*ArcTan[(Sqrt[-(c*d) - Sqr
t[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/(Sqrt[a]*Sqrt[-(c
*d) - Sqrt[a]*Sqrt[c]*e]) - ((Sqrt[c]*d - Sqrt[a]*e)^2*ArcTan[(Sqrt[-(c*d)
 + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt[a]*Sq
rt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/c
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {481, 25, 654, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}dx$$

$$\Big\downarrow \text{481}$$

$$-\frac{\int -\frac{cd^2+2cexd+ae^2}{\sqrt{d+ex}(a-cx^2)}dx}{c}-\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{cd^2+2cexd+ae^2}{\sqrt{d+ex}(a-cx^2)}dx}{c}-\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{654}$$

$$\frac{2\int \frac{e(cd^2-2c(d+ex)d-ae^2)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e\int \frac{cd^2-2c(d+ex)d-ae^2}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{2e\left(\frac{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)^2\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}(\sqrt{a}e+\sqrt{c}d)^2\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}\right)}{c}-\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2e\left(\frac{(\sqrt{a}e+\sqrt{c}d)^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e}-\frac{(\sqrt{c}d-\sqrt{a}e)^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e}\right)}{c}-\frac{2e\sqrt{d+ex}}{c}$$

input
```
Int[(d + e*x)^(3/2)/(a - c*x^2),x]
```

output
```
(-2*e*Sqrt[d + e*x])/c + (2*e*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]*c^(1/4)*e)
 + ((Sqrt[c]*d + Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqr
t[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e)))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13

method	result
derivativedivides	$-2e\left(\frac{\sqrt{ex+d}}{c}+\frac{\left(-a\,e^2-c\,d^2+2\sqrt{a c\,e^2}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c\,e^2}\right)c}}\right)}{2\sqrt{a c\,e^2}\,\sqrt{\left(-cd+\sqrt{a c\,e^2}\right)c}}-\frac{\left(a\,e^2+c\,d^2+2\sqrt{a c\,e^2}\,d\right)\operatorname{arctanh}\left(\ldots\right.}{2\sqrt{a c\,e^2}\,\sqrt{\left(cd+\sqrt{a c\,e^2}\right)\ldots}}\right.$
pseudoelliptic	$-e\left(\frac{2\sqrt{ex+d}}{c}-\frac{\left(a\,e^2+c\,d^2+2\sqrt{a c\,e^2}\,d\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c\,e^2}\right)c}}\right)}{\sqrt{a c\,e^2}\,\sqrt{\left(cd+\sqrt{a c\,e^2}\right)c}}+\frac{\left(-a\,e^2-c\,d^2+2\sqrt{a c\,e^2}\,d\right)\arctan\left(\ldots\right.}{\sqrt{a c\,e^2}\,\sqrt{\left(-cd+\sqrt{a c\,e^2}\right)c}}\right.$
default	$2e\left(-\frac{\sqrt{ex+d}}{c}+\frac{\left(a\,e^2+c\,d^2-2\sqrt{a c\,e^2}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c\,e^2}\right)c}}\right)}{2\sqrt{a c\,e^2}\,\sqrt{\left(-cd+\sqrt{a c\,e^2}\right)c}}-\frac{\left(-a\,e^2-c\,d^2-2\sqrt{a c\,e^2}\,d\right)\operatorname{arctanh}\left(\ldots\right.}{2\sqrt{a c\,e^2}\,\sqrt{\left(cd+\sqrt{a c\,e^2}\right)c}}\right.$
risch	$-\frac{2e\sqrt{ex+d}}{c}-2e\left(-\frac{\left(a\,e^2+c\,d^2+2\sqrt{a c\,e^2}\,d\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c\,e^2}\right)c}}\right)}{2\sqrt{a c\,e^2}\,\sqrt{\left(cd+\sqrt{a c\,e^2}\right)c}}+\frac{\left(-a\,e^2-c\,d^2+2\sqrt{a c\,e^2}\,d\right)\arctan\left(\ldots\right.}{2\sqrt{a c\,e^2}\,\sqrt{\left(-cd+\ldots\right.}}\right.$

input
```
int((e*x+d)^(3/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*e*(1/c*(e*x+d)^(1/2)+1/2*(-a*e^2-c*d^2+2*(a*c*e^2)^(1/2)*d)/(a*c*e^2)^(
1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e
^2)^(1/2))*c)^(1/2))-1/2*(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)/(a*c*e^2)^(1/2)
/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(
1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(107) = 214$.

Time = 0.12 (sec) , antiderivative size = 974, normalized size of antiderivative = 6.54

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(c*sqrt((c*d^3 + 3*a*d*e^2 + a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4
 + a^2*e^6)/(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5
)*sqrt(e*x + d) + (3*a*c^2*d^2*e^2 + a^2*c*e^4 - a*c^4*d*sqrt((9*c^2*d^4*e
^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt((c*d^3 + 3*a*d*e^2 + a*c^2*sq
rt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) - c*sqrt(
(c*d^3 + 3*a*d*e^2 + a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/
(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x +
 d) - (3*a*c^2*d^2*e^2 + a^2*c*e^4 - a*c^4*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d
^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt((c*d^3 + 3*a*d*e^2 + a*c^2*sqrt((9*c^2*d^
4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) + c*sqrt((c*d^3 + 3*a
*d*e^2 - a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a
*c^2))*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) + (3*a*c
^2*d^2*e^2 + a^2*c*e^4 + a*c^4*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2
*e^6)/(a*c^5)))*sqrt((c*d^3 + 3*a*d*e^2 - a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*
c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) - c*sqrt((c*d^3 + 3*a*d*e^2 - a*c
^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))*log(-
(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) - (3*a*c^2*d^2*e^2 +
 a^2*c*e^4 + a*c^4*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5
)))*sqrt((c*d^3 + 3*a*d*e^2 - a*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 +
a^2*e^6)/(a*c^5)))/(a*c^2))) - 4*sqrt(e*x + d)*e)/c
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx = -\int \frac{d\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{ex\sqrt{d+ex}}{-a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(-c*x**2+a),x)
```

output
```
-Integral(d*sqrt(d + e*x)/(-a + c*x**2), x) - Integral(e*x*sqrt(d + e*x)/(
-a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx = \int -\frac{(ex+d)^{\frac{3}{2}}}{cx^2-a}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((e*x + d)^(3/2)/(c*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(107) = 214$.

Time = 0.17 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx = -\frac{2\sqrt{ex+d}e}{c} - \frac{\left(\sqrt{ac}c^3d^3e - \sqrt{ac}ac^2de^3 + (ac^2d^2e - a^2ce^3)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d+\sqrt{c^4d^2-(c^2d^2-ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d - \sqrt{ac}ac^2e\right)\sqrt{-c^2d - \sqrt{ac}ce}|e|} + \frac{\left(\sqrt{ac}c^3d^3e - \sqrt{ac}ac^2de^3 - (ac^2d^2e - a^2ce^3)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d-\sqrt{c^4d^2-(c^2d^2-ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d + \sqrt{ac}ac^2e\right)\sqrt{-c^2d + \sqrt{ac}ce}|e|}$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-2*sqrt(e*x + d)*e/c - (sqrt(a*c)*c^3*d^3*e - sqrt(a*c)*a*c^2*d*e^3 + (a*c
^2*d^2*e - a^2*c*e^3)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d + s
qrt(c^4*d^2 - (c^2*d^2 - a*c*e^2)*c^2))/c^2))/((a*c^3*d - sqrt(a*c)*a*c^2*
e)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(e)) + (sqrt(a*c)*c^3*d^3*e - sqrt(a*c)
*a*c^2*d*e^3 - (a*c^2*d^2*e - a^2*c*e^3)*abs(c)*abs(e))*arctan(sqrt(e*x +
d)/sqrt(-(c^2*d - sqrt(c^4*d^2 - (c^2*d^2 - a*c*e^2)*c^2))/c^2))/((a*c^3*d
 + sqrt(a*c)*a*c^2*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 1581, normalized size of antiderivative = 10.61

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(a - c*x^2),x)
```

output
```
2*atanh((32*a^2*c*e^6*(d + e*x)^(1/2)*((3*d*e^2)/(4*c^2) + d^3/(4*a*c) - (
e^3*(a^3*c^5)^(1/2))/(4*a*c^5) - (3*d^2*e*(a^3*c^5)^(1/2))/(4*a^2*c^4))^(1
/2))/(16*a^2*d*e^7 - 48*c^2*d^5*e^3 - (16*a*e^8*(a^3*c^5)^(1/2))/c^3 + 32*
a*c*d^3*e^5 - (32*d^2*e^6*(a^3*c^5)^(1/2))/c^2 + (48*d^4*e^4*(a^3*c^5)^(1/
2))/(a*c)) - (32*d*e^5*(a^3*c^5)^(1/2)*(d + e*x)^(1/2)*((3*d*e^2)/(4*c^2)
+ d^3/(4*a*c) - (e^3*(a^3*c^5)^(1/2))/(4*a*c^5) - (3*d^2*e*(a^3*c^5)^(1/2)
)/(4*a^2*c^4))^(1/2))/(48*c^3*d^5*e^3 - 32*a*c^2*d^3*e^5 + (16*a*e^8*(a^3*
c^5)^(1/2))/c^2 - 16*a^2*c*d*e^7 - (48*d^4*e^4*(a^3*c^5)^(1/2))/a + (32*d^
2*e^6*(a^3*c^5)^(1/2))/c) + (96*d^3*e^3*(a^3*c^5)^(1/2)*(d + e*x)^(1/2)*((
3*d*e^2)/(4*c^2) + d^3/(4*a*c) - (e^3*(a^3*c^5)^(1/2))/(4*a*c^5) - (3*d^2*
e*(a^3*c^5)^(1/2))/(4*a^2*c^4))^(1/2))/(16*a^3*d*e^7 - 48*a*c^2*d^5*e^3 +
32*a^2*c*d^3*e^5 - (16*a^2*e^8*(a^3*c^5)^(1/2))/c^3 + (48*d^4*e^4*(a^3*c^5
)^(1/2))/c - (32*a*d^2*e^6*(a^3*c^5)^(1/2))/c^2) + (96*a*c^2*d^2*e^4*(d +
e*x)^(1/2)*((3*d*e^2)/(4*c^2) + d^3/(4*a*c) - (e^3*(a^3*c^5)^(1/2))/(4*a*c
^5) - (3*d^2*e*(a^3*c^5)^(1/2))/(4*a^2*c^4))^(1/2))/(16*a^2*d*e^7 - 48*c^2
*d^5*e^3 - (16*a*e^8*(a^3*c^5)^(1/2))/c^3 + 32*a*c*d^3*e^5 - (32*d^2*e^6*(
a^3*c^5)^(1/2))/c^2 + (48*d^4*e^4*(a^3*c^5)^(1/2))/(a*c)))*((a*c^4*d^3 - a
*e^3*(a^3*c^5)^(1/2) + 3*a^2*c^3*d*e^2 - 3*c*d^2*e*(a^3*c^5)^(1/2))/(4*a^2
*c^5))^(1/2) - 2*atanh((32*d*e^5*(a^3*c^5)^(1/2)*(d + e*x)^(1/2)*((3*d*e^2
)/(4*c^2) + d^3/(4*a*c) + (e^3*(a^3*c^5)^(1/2))/(4*a*c^5) + (3*d^2*e*(a...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex)^{3/2}}{a-cx^2}\,dx = \frac{-2\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)cd+2\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\cdots}}\right.}{}$$

input
```
int((e*x+d)^(3/2)/(-c*x^2+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c*d + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*e
- sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*
d) + sqrt(c)*sqrt(d + e*x))*c*d + sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*lo
g(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d - sqrt(c)*sqr
t(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*
sqrt(d + e*x))*a*e + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c
)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*e - 4*sqrt(d + e*x)*a*c*e)/(
2*a*c**2)
```

3.137 $\int \frac{\sqrt{d+ex}}{a-cx^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 134

$$\int \frac{\sqrt{d+ex}}{a-cx^2} \, dx = -\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{3/4}} + \frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{3/4}}$$

output
```
-(c^(1/2)*d-a^(1/2)*e)^(1/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1
/2)*e)^(1/2))/a^(1/2)/c^(3/4)+(c^(1/2)*d+a^(1/2)*e)^(1/2)*arctanh(c^(1/4)*
(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{a-cx^2} \, dx = \frac{-\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)+\sqrt{-cd+\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{a}c}$$

input
```
Integrate[Sqrt[d + e*x]/(a - c*x^2),x]
```

output
```
(-(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*
e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)]) + Sqrt[-(c*d) + Sqrt[a]*Sqrt[c
]*e]*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d -
Sqrt[a]*e)])/(Sqrt[a]*c)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {483, 25, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{a-cx^2}dx$$

$$\Big\downarrow \text{483}$$

$$2e\int -\frac{d+ex}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{25}$$

$$-2e\int \frac{d+ex}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1450}$$

$$2e\left(-\frac{1}{2}\left(1-\frac{\sqrt{c}d}{\sqrt{a}e}\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}-\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2e}\right)$$

$$\Big\downarrow \text{221}$$

$$2e\left(\frac{\left(1-\frac{\sqrt{c}d}{\sqrt{a}e}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2c^{3/4}e\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)$$

input
```
Int[Sqrt[d + e*x]/(a - c*x^2),x]
```

output
```
2*e*(((1 - (Sqrt[c]*d)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[S
qrt[c]*d - Sqrt[a]*e]])/(2*c^(3/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + (((Sqrt[
c]*d)/Sqrt[a] + e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a
]*e]])/(2*c^(3/4)*e*Sqrt[Sqrt[c]*d + Sqrt[a]*e]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$-\frac{e\left(\frac{\left(-cd+\sqrt{ac\,e^{2}}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(cd+\sqrt{ac\,e^{2}}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}}$	127
derivativedivides	$-2ec\left(\frac{\left(-cd+\sqrt{ac\,e^{2}}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(cd+\sqrt{ac\,e^{2}}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)$	143
default	$-2ec\left(\frac{\left(-cd+\sqrt{ac\,e^{2}}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(cd+\sqrt{ac\,e^{2}}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)$	143

input
```
int((e*x+d)^(1/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-e/(a*c*e^2)^(1/2)*((-c*d+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2
)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-(c*d+(a*c*e^2)^
(1/2))/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*
e^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(94) = 188$.

Time = 0.08 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx = \frac{1}{2}\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\log\left(ac^2\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\sqrt{\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right) - \frac{1}{2}\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\log\left(-ac^2\sqrt{\frac{ac\sqrt{\frac{e^2}{ac^3}}+d}{ac}}\sqrt{\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right) - \frac{1}{2}\sqrt{-\frac{ac\sqrt{\frac{e^2}{ac^3}}-d}{ac}}\log\left(ac^2\sqrt{-\frac{ac\sqrt{\frac{e^2}{ac^3}}-d}{ac}}\sqrt{\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right) + \frac{1}{2}\sqrt{-\frac{ac\sqrt{\frac{e^2}{ac^3}}-d}{ac}}\log\left(-ac^2\sqrt{-\frac{ac\sqrt{\frac{e^2}{ac^3}}-d}{ac}}\sqrt{\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right)$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt((a*c*sqrt(e^2/(a*c^3)) + d)/(a*c))*log(a*c^2*sqrt((a*c*sqrt(e^2/(
a*c^3)) + d)/(a*c))*sqrt(e^2/(a*c^3)) + sqrt(e*x + d)*e) - 1/2*sqrt((a*c*s
qrt(e^2/(a*c^3)) + d)/(a*c))*log(-a*c^2*sqrt((a*c*sqrt(e^2/(a*c^3)) + d)/(
a*c))*sqrt(e^2/(a*c^3)) + sqrt(e*x + d)*e) - 1/2*sqrt(-(a*c*sqrt(e^2/(a*c^
3)) - d)/(a*c))*log(a*c^2*sqrt(-(a*c*sqrt(e^2/(a*c^3)) - d)/(a*c))*sqrt(e^
2/(a*c^3)) + sqrt(e*x + d)*e) + 1/2*sqrt(-(a*c*sqrt(e^2/(a*c^3)) - d)/(a*c
))*log(-a*c^2*sqrt(-(a*c*sqrt(e^2/(a*c^3)) - d)/(a*c))*sqrt(e^2/(a*c^3)) +
 sqrt(e*x + d)*e)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx = -\int \frac{\sqrt{d+ex}}{-a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(-c*x**2+a),x)
```

output
```
-Integral(sqrt(d + e*x)/(-a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx = \int -\frac{\sqrt{ex+d}}{cx^2-a}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x + d)/(c*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(94) = 188$.

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx = \frac{(cd^2e|c| - ae^3|c|)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd+\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{\sqrt{-c^2d-\sqrt{ac}ce}\left(ace-\sqrt{ac}cd\right)|e|} + \frac{(cd^2e|c| - ae^3|c|)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd-\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{\sqrt{-c^2d+\sqrt{ac}ce}\left(ace+\sqrt{ac}cd\right)|e|}$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
(c*d^2*e*abs(c) - a*e^3*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c*d + sqrt(c^2
*d^2 - (c*d^2 - a*e^2)*c))/c))/(sqrt(-c^2*d - sqrt(a*c)*c*e)*(a*c*e - sqrt
(a*c)*c*d)*abs(e)) + (c*d^2*e*abs(c) - a*e^3*abs(c))*arctan(sqrt(e*x + d)/
sqrt(-(c*d - sqrt(c^2*d^2 - (c*d^2 - a*e^2)*c))/c))/(sqrt(-c^2*d + sqrt(a*
c)*c*e)*(a*c*e + sqrt(a*c)*c*d)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\left(16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)\sqrt{d+e\,x}-\frac{16\,c\,d\,e^2\,\left(e\,\sqrt{a^3\,c^3}+a\,c^2\,d\right)\sqrt{d+e\,x}}{a}\right)\sqrt{\frac{e\,\sqrt{a^3\,c^3}+a\,c^2\,d}{4\,a^2\,c^3}}}{16\,c^2\,d^2\,e^3-16\,a\,c\,e^5}\right)\sqrt{\frac{e\,\sqrt{}}{}}$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\left(16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)\sqrt{d+e\,x}+\frac{16\,c\,d\,e^2\,\left(e\,\sqrt{a^3\,c^3}-a\,c^2\,d\right)\sqrt{d+e\,x}}{a}\right)\sqrt{-\frac{e\,\sqrt{a^3\,c^3}-a\,c^2\,d}{4\,a^2\,c^3}}}{16\,c^2\,d^2\,e^3-16\,a\,c\,e^5}\right)\sqrt{-}$$

input
```
int((d + e*x)^(1/2)/(a - c*x^2),x)
```

output
```
- 2*atanh((2*((16*a*c^2*e^4 + 16*c^3*d^2*e^2)*(d + e*x)^(1/2) - (16*c*d*e^
2*(e*(a^3*c^3)^(1/2) + a*c^2*d)*(d + e*x)^(1/2))/a)*((e*(a^3*c^3)^(1/2) +
a*c^2*d)/(4*a^2*c^3))^(1/2))/(16*c^2*d^2*e^3 - 16*a*c*e^5))*((e*(a^3*c^3)^
(1/2) + a*c^2*d)/(4*a^2*c^3))^(1/2) - 2*atanh((2*((16*a*c^2*e^4 + 16*c^3*d
^2*e^2)*(d + e*x)^(1/2) + (16*c*d*e^2*(e*(a^3*c^3)^(1/2) - a*c^2*d)*(d + e
*x)^(1/2))/a)*(-(e*(a^3*c^3)^(1/2) - a*c^2*d)/(4*a^2*c^3))^(1/2))/(16*c^2*
d^2*e^3 - 16*a*c*e^5))*(-(e*(a^3*c^3)^(1/2) - a*c^2*d)/(4*a^2*c^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d+ex}}{a-cx^2}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)-\sqrt{\sqrt{c}\,\sqrt{a}\,e+cd}\log\left(-\sqrt{\sqrt{c}\,\sqrt{a}\,e+cd}+\sqrt{c}\,\sqrt{ex+d}\right)\right.}{2ac}$$

input
```
int((e*x+d)^(1/2)/(-c*x^2+a),x)
```

output
```
(sqrt(a)*( - 2*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))) - sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( -
sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x)) + sqrt(sqrt(c)*sqrt
(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))))/
(2*a*c)
```

3.138 $\int \frac{1}{\sqrt{d+ex}(a-cx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 134

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output
```
-arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/c^(1/4
)/(c^(1/2)*d-a^(1/2)*e)^(1/2)+arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(
1/2)*e)^(1/2))/a^(1/2)/c^(1/4)/(c^(1/2)*d+a^(1/2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}-\frac{\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{-cd+\sqrt{a}\sqrt{c}e}}}{\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a - c*x^2)),x]
```

output
```
(ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt
[a]*e)]/Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e] - ArcTan[(Sqrt[-(c*d) + Sqrt[a]*S
qrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)]/Sqrt[-(c*d) + Sqrt[a]*Sq
rt[c]*e])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {484, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^2)\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 484}$$

$$2e\int \frac{1}{-cd^2+2c(d+ex)d+ae^2-c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{ 1406}$$

$$2e\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{c}(\sqrt{c}d+\sqrt{a}e)-c(d+ex)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}\int \frac{1}{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)-c(d+ex)}d\sqrt{d+ex}}{2\sqrt{a}e}\right)$$

$$\Big\downarrow \text{ 221}$$

$$2e\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)$$

input
```
Int[1/(Sqrt[d + e*x]*(a - c*x^2)),x]
```

output
```
2*e*(-1/2*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]]/(Sq
rt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ArcTanh[(c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]]/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d + Sq
rt[a]*e]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{ce\left(\frac{\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}}$	101
derivativedivides	$-2ec\left(-\frac{\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)$	112
default	$-2ec\left(-\frac{\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)$	112

input
```
int(1/(e*x+d)^(1/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
c*e/(a*c*e^2)^(1/2)*(1/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(
1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+1/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*a
rctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(94) = 188$.

Time = 0.09 (sec) , antiderivative size = 949, normalized size of antiderivative = 7.08

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^
3*c*e^4)) + d)/(a*c*d^2 - a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 - (a*c^2*
d^3 - a^2*c*d*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*
sqrt(((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4)) + d)/(a*c*d^2 - a^2*e^2))) - 1/2*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(e^2/
(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 - a^2*e^2))*log
(sqrt(e*x + d)*e - (a*e^2 - (a*c^2*d^3 - a^2*c*d*e^2)*sqrt(e^2/(a*c^3*d^4
- 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c
^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 - a^2*e^2))) + 1/2*
sqrt(-((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)) - d)/(a*c*d^2 - a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 + (a*c^2*d^3
 - a^2*c*d*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqr
t(-((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^
4)) - d)/(a*c*d^2 - a^2*e^2))) - 1/2*sqrt(-((a*c*d^2 - a^2*e^2)*sqrt(e^2/(
a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 - a^2*e^2))*log(
sqrt(e*x + d)*e - (a*e^2 + (a*c^2*d^3 - a^2*c*d*e^2)*sqrt(e^2/(a*c^3*d^4 -
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(-((a*c*d^2 - a^2*e^2)*sqrt(e^2/(a*c
^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 - a^2*e^2)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx = -\int \frac{1}{-a\sqrt{d+ex}+cx^2\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(-c*x**2+a),x)
```

output
```
-Integral(1/(-a*sqrt(d + e*x) + c*x**2*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \int -\frac{1}{(cx^2-a)\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^2 - a)*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(94) = 188.

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \frac{(ae|c||e|-\sqrt{ac}de|c|)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd+\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{(acd-\sqrt{ac}ae)\sqrt{-c^2d-\sqrt{ac}ce}|e|} + \frac{(ae|c||e|+\sqrt{ac}de|c|)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd-\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{(acd+\sqrt{ac}ae)\sqrt{-c^2d+\sqrt{ac}ce}|e|}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
(a*e*abs(c)*abs(e) - sqrt(a*c)*d*e*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c*d
 + sqrt(c^2*d^2 - (c*d^2 - a*e^2)*c))/c))/((a*c*d - sqrt(a*c)*a*e)*sqrt(-c
^2*d - sqrt(a*c)*c*e)*abs(e)) + (a*e*abs(c)*abs(e) + sqrt(a*c)*d*e*abs(c))
*arctan(sqrt(e*x + d)/sqrt(-(c*d - sqrt(c^2*d^2 - (c*d^2 - a*e^2)*c))/c))/
((a*c*d + sqrt(a*c)*a*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1366, normalized size of antiderivative = 10.19

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
2*atanh((32*a^2*c^5*d^2*e^2*(- (e*(a^3*c)^(1/2))/(4*(a^3*c*e^2 - a^2*c^2*d
^2)) - (a*c*d)/(4*(a^3*c*e^2 - a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))/((16*
a^4*c^6*d^3*e^3)/(a^3*c*e^2 - a^2*c^2*d^2) - (16*a^4*c^4*e^6*(a^3*c)^(1/2)
)/(a^3*c*e^2 - a^2*c^2*d^2) - (16*a^5*c^5*d*e^5)/(a^3*c*e^2 - a^2*c^2*d^2)
 + (16*a^3*c^5*d^2*e^4*(a^3*c)^(1/2))/(a^3*c*e^2 - a^2*c^2*d^2)) - (32*c^3
*e^2*(- (e*(a^3*c)^(1/2))/(4*(a^3*c*e^2 - a^2*c^2*d^2)) - (a*c*d)/(4*(a^3*
c*e^2 - a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))/((16*a^2*c^4*d*e^3)/(a^3*c*e
^2 - a^2*c^2*d^2) + (16*a*c^3*e^4*(a^3*c)^(1/2))/(a^3*c*e^2 - a^2*c^2*d^2)
) + (32*a*c^4*d*e^3*(a^3*c)^(1/2)*(- (e*(a^3*c)^(1/2))/(4*(a^3*c*e^2 - a^2
*c^2*d^2)) - (a*c*d)/(4*(a^3*c*e^2 - a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))
/((16*a^4*c^6*d^3*e^3)/(a^3*c*e^2 - a^2*c^2*d^2) - (16*a^4*c^4*e^6*(a^3*c)
^(1/2))/(a^3*c*e^2 - a^2*c^2*d^2) - (16*a^5*c^5*d*e^5)/(a^3*c*e^2 - a^2*c^
2*d^2) + (16*a^3*c^5*d^2*e^4*(a^3*c)^(1/2))/(a^3*c*e^2 - a^2*c^2*d^2)))*(-
(e*(a^3*c)^(1/2) + a*c*d)/(4*(a^3*c*e^2 - a^2*c^2*d^2)))^(1/2) - 2*atanh((
32*c^3*e^2*((e*(a^3*c)^(1/2))/(4*(a^3*c*e^2 - a^2*c^2*d^2)) - (a*c*d)/(4*(
a^3*c*e^2 - a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))/((16*a^2*c^4*d*e^3)/(a^3
*c*e^2 - a^2*c^2*d^2) - (16*a*c^3*e^4*(a^3*c)^(1/2))/(a^3*c*e^2 - a^2*c^2*
d^2)) - (32*a^2*c^5*d^2*e^2*((e*(a^3*c)^(1/2))/(4*(a^3*c*e^2 - a^2*c^2*d^2
)) - (a*c*d)/(4*(a^3*c*e^2 - a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))/((16*a^
4*c^6*d^3*e^3)/(a^3*c*e^2 - a^2*c^2*d^2) + (16*a^4*c^4*e^6*(a^3*c)^(1/2...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)}\,dx$$
$$= \frac{2\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)cd+2\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)ae+\sqrt{a}\,\sqrt{\sqrt{c}}$$

input
```
int(1/(e*x+d)^(1/2)/(-c*x^2+a),x)
```

output
```
(2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*c*d + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*
d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*e + s
qrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d)
+ sqrt(c)*sqrt(d + e*x))*c*d - sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(s
qrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d - sqrt(c)*sqrt(s
qrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqr
t(d + e*x))*a*e + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*s
qrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*e)/(2*a*c*(a*e**2 - c*d**2))
```

3.139 $\int \frac{1}{(d+ex)^{3/2}(a-cx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 160

$$\int \frac{1}{(d+ex)^{3/2}(a-cx^2)}\,dx = \frac{2e}{(cd^2-ae^2)\sqrt{d+ex}} - \frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} + \frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

output
```
2*e/(-a*e^2+c*d^2)/(e*x+d)^(1/2)-c^(1/4)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^
(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(3/2)+c^(1/4)*arct
anh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/(c^(1/2)*d+
a^(1/2)*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+ex)^{3/2}(a-cx^2)}\,dx = \frac{2e}{(cd^2-ae^2)\sqrt{d+ex}} - \frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\left(\sqrt{c}d+\sqrt{a}e\right)^2} + \frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^2}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a - c*x^2)),x]
```

output
```
(2*e)/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*
ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[
a]*e)])/(Sqrt[a]*(Sqrt[c]*d + Sqrt[a]*e)^2) + (Sqrt[-(c*d) + Sqrt[a]*Sqrt[
c]*e]*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d -
 Sqrt[a]*e)])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)^2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {482, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^2)(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \ 482$$

$$\frac{c\int \frac{d-ex}{\sqrt{d+ex}(a-cx^2)}dx}{cd^2-ae^2} + \frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \ 654$$

$$\frac{2c\int -\frac{e(d-ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}+\frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

$\Big\downarrow$ 25

$$\frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}-\frac{2c\int \frac{e(d-ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}$$

$\Big\downarrow$ 27

$$\frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}-\frac{2ce\int \frac{d-ex}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}$$

$\Big\downarrow$ 1480

$$\frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}-\frac{2ce\left(-\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2e}-\frac{1}{2}\left(1-\frac{\sqrt{c}d}{\sqrt{a}e}\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}\right)}{cd^2-ae^2}$$

$\Big\downarrow$ 221

$$\frac{2e}{\sqrt{d+ex}\,(cd^2-ae^2)}-\frac{2ce\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2c^{3/4}e\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\left(1-\frac{\sqrt{c}d}{\sqrt{a}e}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2c^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{cd^2-ae^2}$$

input
```
Int[1/((d + e*x)^(3/2)*(a - c*x^2)),x]
```

output
```
(2*e)/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*c*e*((((Sqrt[c]*d)/Sqrt[a] + e)
*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*c^(3/4)*
e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ((1 - (Sqrt[c]*d)/(Sqrt[a]*e))*ArcTanh[(c
^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*c^(3/4)*Sqrt[Sqrt[c
]*d + Sqrt[a]*e])))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

method	result
derivativedivides	$-2e\left(-\frac{c^2\left(-\frac{\left(cd-\sqrt{a c e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\, c\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}+\frac{\left(-cd-\sqrt{a c e^2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\, c\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{a e^2-c d^2}+\frac{1}{(a e^2-c d^2)}\right.$
default	$2e\left(\frac{c^2\left(-\frac{\left(cd-\sqrt{a c e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\, c\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}+\frac{\left(-cd-\sqrt{a c e^2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\, c\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{a e^2-c d^2}-\frac{1}{(a e^2-c d^2)\sqrt{ex}}\right.$
pseudoelliptic	$-\frac{2e\left(\frac{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}\, c\left(cd+\sqrt{a c e^2}\right)\sqrt{ex+d}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{2}+\left(\frac{c\sqrt{ex+d}\left(cd-\sqrt{a c e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)}}\right.}{2}\right.\right.}{\sqrt{a c e^2}\,\sqrt{ex+d}\,\sqrt{\left(cd+\sqrt{a c e^2}\right)c}\,\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}\,(a e^2-c d^2}$

input
```
int(1/(e*x+d)^(3/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*e*(-c^2/(a*e^2-c*d^2)*(-1/2*(c*d-(a*c*e^2)^(1/2))/(a*c*e^2)^(1/2)/c/(((c
*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2)
)*c)^(1/2))+1/2*(-c*d-(a*c*e^2)^(1/2))/(a*c*e^2)^(1/2)/c/((-c*d+(a*c*e^2)^
(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)))+
1/(a*e^2-c*d^2)/(e*x+d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2861 vs. $2(118) = 236$.

Time = 0.13 (sec) , antiderivative size = 2861, normalized size of antiderivative = 17.88

$$\int \frac{1}{(d+ex)^{3/2}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*((c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt((c^2*d^3 + 3*a*c*d*e^2 +
 (a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d
^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 1
5*a^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*
e^10 + a^7*e^12)))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*
e^6))*log((3*c^2*d^2*e + a*c*e^3)*sqrt(e*x + d) + (6*a*c^2*d^3*e^2 + 2*a^2
*c*d*e^4 - (a*c^4*d^8 - 2*a^2*c^3*d^6*e^2 + 2*a^4*c*d^2*e^6 - a^5*e^8)*sqr
t((9*c^3*d^4*e^2 + 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^
10*e^2 + 15*a^3*c^4*d^8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*
a^6*c*d^2*e^10 + a^7*e^12)))*sqrt((c^2*d^3 + 3*a*c*d*e^2 + (a*c^3*d^6 - 3*
a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4*e^2 + 6*a*c^2
*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^
4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e^12)
))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6))) - (c*d^3
- a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt((c^2*d^3 + 3*a*c*d*e^2 + (a*c^3*d^6
- 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*sqrt((9*c^3*d^4*e^2 + 6*a
*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 - 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^
8*e^4 - 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 - 6*a^6*c*d^2*e^10 + a^7*e
^12)))/(a*c^3*d^6 - 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6))*log((3
*c^2*d^2*e + a*c*e^3)*sqrt(e*x + d) - (6*a*c^2*d^3*e^2 + 2*a^2*c*d*e^4 ...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\,(a-cx^2)}\,dx = -\int \frac{1}{-ad\sqrt{d+ex} - aex\sqrt{d+ex} + cdx^2\sqrt{d+ex} + cex^3\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(-c*x**2+a),x)
```

output
```
-Integral(1/(-a*d*sqrt(d + e*x) - a*e*x*sqrt(d + e*x) + c*d*x**2*sqrt(d +
e*x) + c*e*x**3*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\,(a-cx^2)}\,dx = \int -\frac{1}{(cx^2-a)(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^2 - a)*(e*x + d)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(118) = 236$.

Time = 0.18 (sec) , antiderivative size = 656, normalized size of antiderivative = 4.10

$$\int \frac{1}{(d+ex)^{3/2}\,(a-cx^2)}\,dx = \frac{2\,e}{(cd^2-ae^2)\sqrt{ex+d}}$$

$$-\frac{\left((cd^2e-ae^3)^2ae|c|-2\left(\sqrt{ac}cd^3e-\sqrt{ac}ade^3\right)|-cd^2e+ae^3||c|+(c^3d^6e-2\,ac^2d^4e^3+a^2cd^2e^5)|c|\right)\text{arcta}}{\left(ac^2d^4e-2\,a^2cd^2e^3+a^3e^5+\sqrt{ac}c^2d^5-2\sqrt{ac}acd^3e^2+\sqrt{ac}a^2de^4\right)\sqrt{-c}}$$

$$-\frac{\left((cd^2e-ae^3)^2ae|c|+2\left(\sqrt{ac}cd^3e-\sqrt{ac}ade^3\right)|-cd^2e+ae^3||c|+(c^3d^6e-2\,ac^2d^4e^3+a^2cd^2e^5)|c|\right)\text{arcta}}{\left(ac^2d^4e-2\,a^2cd^2e^3+a^3e^5-\sqrt{ac}c^2d^5+2\sqrt{ac}acd^3e^2-\sqrt{ac}a^2de^4\right)\sqrt{-c}}$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
2*e/((c*d^2 - a*e^2)*sqrt(e*x + d)) - ((c*d^2*e - a*e^3)^2*a*e*abs(c) - 2*
(sqrt(a*c)*c*d^3*e - sqrt(a*c)*a*d*e^3)*abs(-c*d^2*e + a*e^3)*abs(c) + (c^
3*d^6*e - 2*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*abs(c))*arctan(sqrt(e*x + d)/sq
rt(-(c^2*d^3 - a*c*d*e^2 + sqrt((c^2*d^3 - a*c*d*e^2)^2 - (c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)*(c^2*d^2 - a*c*e^2)))/(c^2*d^2 - a*c*e^2)))/((a*c^2*d^
4*e - 2*a^2*c*d^2*e^3 + a^3*e^5 + sqrt(a*c)*c^2*d^5 - 2*sqrt(a*c)*a*c*d^3*
e^2 + sqrt(a*c)*a^2*d*e^4)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(-c*d^2*e + a*e
^3)) - ((c*d^2*e - a*e^3)^2*a*e*abs(c) + 2*(sqrt(a*c)*c*d^3*e - sqrt(a*c)*
a*d*e^3)*abs(-c*d^2*e + a*e^3)*abs(c) + (c^3*d^6*e - 2*a*c^2*d^4*e^3 + a^2
*c*d^2*e^5)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d^3 - a*c*d*e^2 - sqrt
((c^2*d^3 - a*c*d*e^2)^2 - (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*(c^2*d^2 -
a*c*e^2)))/(c^2*d^2 - a*c*e^2)))/((a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5
 - sqrt(a*c)*c^2*d^5 + 2*sqrt(a*c)*a*c*d^3*e^2 - sqrt(a*c)*a^2*d*e^4)*sqrt
(-c^2*d - sqrt(a*c)*c*e)*abs(-c*d^2*e + a*e^3))
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 4412, normalized size of antiderivative = 27.58

$$\int \frac{1}{(d+ex)^{3/2}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
- atan((((d + e*x)^(1/2)*(16*a^4*c^4*e^10 - 16*c^8*d^8*e^2 + 32*a*c^7*d^6*
e^4 - 32*a^3*c^5*d^2*e^8) + (-(a*c^2*d^3 + a*e^3*(a^3*c)^(1/2) + 3*a^2*c*d
*e^2 + 3*c*d^2*e*(a^3*c)^(1/2))/(4*(a^5*e^6 - a^2*c^3*d^6 - 3*a^4*c*d^2*e^
4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*((d + e*x)^(1/2)*(-(a*c^2*d^3 + a*e^3*(a^3*
c)^(1/2) + 3*a^2*c*d*e^2 + 3*c*d^2*e*(a^3*c)^(1/2))/(4*(a^5*e^6 - a^2*c^3*
d^6 - 3*a^4*c*d^2*e^4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*(64*a*c^9*d^11*e^2 - 64
*a^6*c^4*d*e^12 - 320*a^2*c^8*d^9*e^4 + 640*a^3*c^7*d^7*e^6 - 640*a^4*c^6*
d^5*e^8 + 320*a^5*c^5*d^3*e^10) - 64*a*c^8*d^9*e^3 - 64*a^5*c^4*d*e^11 + 2
56*a^2*c^7*d^7*e^5 - 384*a^3*c^6*d^5*e^7 + 256*a^4*c^5*d^3*e^9))*(-(a*c^2*
d^3 + a*e^3*(a^3*c)^(1/2) + 3*a^2*c*d*e^2 + 3*c*d^2*e*(a^3*c)^(1/2))/(4*(a
^5*e^6 - a^2*c^3*d^6 - 3*a^4*c*d^2*e^4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*1i + (
(d + e*x)^(1/2)*(16*a^4*c^4*e^10 - 16*c^8*d^8*e^2 + 32*a*c^7*d^6*e^4 - 32*
a^3*c^5*d^2*e^8) + (-(a*c^2*d^3 + a*e^3*(a^3*c)^(1/2) + 3*a^2*c*d*e^2 + 3*
c*d^2*e*(a^3*c)^(1/2))/(4*(a^5*e^6 - a^2*c^3*d^6 - 3*a^4*c*d^2*e^4 + 3*a^3
*c^2*d^4*e^2)))^(1/2)*((d + e*x)^(1/2)*(-(a*c^2*d^3 + a*e^3*(a^3*c)^(1/2)
+ 3*a^2*c*d*e^2 + 3*c*d^2*e*(a^3*c)^(1/2))/(4*(a^5*e^6 - a^2*c^3*d^6 - 3*a
^4*c*d^2*e^4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*(64*a*c^9*d^11*e^2 - 64*a^6*c^4*
d*e^12 - 320*a^2*c^8*d^9*e^4 + 640*a^3*c^7*d^7*e^6 - 640*a^4*c^6*d^5*e^8 +
 320*a^5*c^5*d^3*e^10) + 64*a*c^8*d^9*e^3 + 64*a^5*c^4*d*e^11 - 256*a^2*c^
7*d^7*e^5 + 384*a^3*c^6*d^5*e^7 - 256*a^4*c^5*d^3*e^9))*(-(a*c^2*d^3 + ...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.12

$$\int \frac{1}{(d+ex)^{3/2}\,(a-cx^2)}\,dx = \frac{-2\sqrt{a}\,\sqrt{ex+d}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)a\,e^2-2\sqrt{a}\,\sqrt{ex+d}\,\sqrt{}}{}$$

input
```
int(1/(e*x+d)^(3/2)/(-c*x^2+a),x)
```

output

```
( - 2*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e
*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*e**2 - 2*sqrt(a)*sqrt(d
+ e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)))*c*d**2 - 4*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*s
qrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)))*a*d*e - sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - s
qrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*e**2 - sqrt(a)*sqr
t(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c
*d) + sqrt(c)*sqrt(d + e*x))*c*d**2 + sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*s
qrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))
*a*e**2 + sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqr
t(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d**2 + 2*sqrt(c)*sqrt(d +
 e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) +
 sqrt(c)*sqrt(d + e*x))*a*d*e - 2*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(
a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*d
*e - 4*a**2*e**3 + 4*a*c*d**2*e)/(2*sqrt(d + e*x)*a*(a**2*e**4 - 2*a*c*d**
2*e**2 + c**2*d**4))
```

3.140 $\int \frac{1}{(d+ex)^{5/2}(a-cx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 190

$$\int \frac{1}{(d+ex)^{5/2}(a-cx^2)}\,dx = \frac{2e}{3(cd^2-ae^2)(d+ex)^{3/2}}+\frac{4cde}{(cd^2-ae^2)^2\sqrt{d+ex}} - \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}}+\frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}$$

output
```
2/3*e/(-a*e^2+c*d^2)/(e*x+d)^(3/2)+4*c*d*e/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)-
c^(3/4)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)
/(c^(1/2)*d-a^(1/2)*e)^(5/2)+c^(3/4)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2
)*d+a^(1/2)*e)^(1/2))/a^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^{5/2}(a-cx^2)}\,dx = \frac{-2ae^3+2cde(7d+6ex)}{3(cd^2-ae^2)^2(d+ex)^{3/2}} + \frac{c\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}} - \frac{c\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^2\sqrt{-cd+\sqrt{a}\sqrt{c}e}}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(a - c*x^2)),x]
```

output
```
(-2*a*e^3 + 2*c*d*e*(7*d + 6*e*x))/(3*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) +
 (c*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + S
qrt[a]*e)])/(Sqrt[a]*(Sqrt[c]*d + Sqrt[a]*e)^2*Sqrt[-(c*d) - Sqrt[a]*Sqrt[
c]*e]) - (c*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[
c]*d - Sqrt[a]*e)])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)^2*Sqrt[-(c*d) + Sqrt[
a]*Sqrt[c]*e])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {482, 655, 25, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^2)(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{482}$$

$$\frac{c\int \frac{d-ex}{(d+ex)^{3/2}(a-cx^2)}dx}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{655}$$

$$\frac{c\left(\frac{4de}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{\int -\frac{cd^2-2cexd+ae^2}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}\right)}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\left(\frac{\int \frac{cd^2-2cexd+ae^2}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}+\frac{4de}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{654}$$

$$\frac{c\left(\frac{2\int -\frac{e\left(3cd^2-2c(d+ex)d+ae^2\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}+\frac{4de}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{c\left(\frac{4de}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2\int \frac{e\left(3cd^2-2c(d+ex)d+ae^2\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}\right)}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{c\left(\frac{4de}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2e\int \frac{3cd^2-2c(d+ex)d+ae^2}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}\right)}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{c\left(\frac{4de}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2e\left(\frac{\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}\right)}{cd^2-ae^2}\right)}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{c\left(\frac{4de}{\sqrt{d+ex}(cd^2-ae^2)}-\frac{2e\left(\frac{\left(\sqrt{a}e+\sqrt{c}d\right)^2\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{cd^2-ae^2}\right)}{cd^2-ae^2}+\frac{2e}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^(5/2)*(a - c*x^2)),x]
```

output
```
(2*e)/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (c*((4*d*e)/((c*d^2 - a*e^2)*S
qrt[d + e*x]) - (2*e*(((Sqrt[c]*d + Sqrt[a]*e)^2*ArcTanh[(c^(1/4)*Sqrt[d +
 e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d -
 Sqrt[a]*e]) - ((Sqrt[c]*d - Sqrt[a]*e)^2*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/
Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d + Sqrt[a
]*e])))/(c*d^2 - a*e^2)))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.18

method	result
derivativedivides	$-2e\left(\frac{c^2\left(-\frac{\left(a e^2+c d^2-2\sqrt{a c e^2}\, d\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}+\frac{\left(-a e^2-c d^2-2\sqrt{a c e^2}\, d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{\left(a e^2-c d^2\right)^2}\right.$
default	$2e\left(-\frac{c^2\left(-\frac{\left(a e^2+c d^2-2\sqrt{a c e^2}\, d\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}+\frac{\left(-a e^2-c d^2-2\sqrt{a c e^2}\, d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{2\sqrt{a c e^2}\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)}{\left(a e^2-c d^2\right)^2}\right.$
pseudoelliptic	$\frac{e\left(c^2\sqrt{\left(cd+\sqrt{a c e^2}\right)c}\,(ex+d)^{\frac{3}{2}}\left(a e^2+c d^2+2\sqrt{a c e^2}\, d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}}\right)+\sqrt{\left(-cd+\sqrt{a c e^2}\right)c}\left(c^2\left(a e^2+\right.\right.\right.}{\sqrt{a c e^2}\,(ex+d)^{\frac{3}{2}}\sqrt{\left(cd+\sqrt{a c e^2}\right)c}\sqrt{\left(-\right.}}$

input
```
int(1/(e*x+d)^(5/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*e*(c^2/(a*e^2-c*d^2)^2*(-1/2*(a*e^2+c*d^2-2*(a*c*e^2)^(1/2)*d)/(a*c*e^2
)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c
*e^2)^(1/2))*c)^(1/2))+1/2*(-a*e^2-c*d^2-2*(a*c*e^2)^(1/2)*d)/(a*c*e^2)^(1
/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^
2)^(1/2))*c)^(1/2)))+1/3/(a*e^2-c*d^2)/(e*x+d)^(3/2)-2*c*d/(a*e^2-c*d^2)^2
/(e*x+d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5142 vs. $2(144) = 288$.

Time = 0.23 (sec) , antiderivative size = 5142, normalized size of antiderivative = 27.06

$$\int \frac{1}{(d+ex)^{5/2}(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}(a-cx^2)}\,dx = -\int \frac{1}{-ad^2\sqrt{d+ex}-2adex\sqrt{d+ex}-ae^2x^2\sqrt{d+ex}+cd^2x^2\sqrt{d+ex}+2cdex^3\sqrt{d+ex}+ce^2x^4\sqrt{d+ex}}$$

input
```
integrate(1/(e*x+d)**(5/2)/(-c*x**2+a),x)
```

output
```
-Integral(1/(-a*d**2*sqrt(d + e*x) - 2*a*d*e*x*sqrt(d + e*x) - a*e**2*x**2
*sqrt(d + e*x) + c*d**2*x**2*sqrt(d + e*x) + 2*c*d*e*x**3*sqrt(d + e*x) +
c*e**2*x**4*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)}\,dx = \int -\frac{1}{(cx^2-a)(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^2 - a)*(e*x + d)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. 2(144) = 288.

Time = 0.19 (sec) , antiderivative size = 1165, normalized size of antiderivative = 6.13

$$\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
(2*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)^2*sqrt(-c^2*d - sqrt(a*c)*c*e)*sq
rt(a*c)*a*d*e*abs(c) - (3*a*c^3*d^6*e - 5*a^2*c^2*d^4*e^3 + a^3*c*d^2*e^5
+ a^4*e^7)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(c^2*d^4*e - 2*a*c*d^2*e^3 + a^
2*e^5)*abs(c) + (sqrt(a*c)*c^5*d^11*e - 3*sqrt(a*c)*a*c^4*d^9*e^3 + 2*sqrt
(a*c)*a^2*c^3*d^7*e^5 + 2*sqrt(a*c)*a^3*c^2*d^5*e^7 - 3*sqrt(a*c)*a^4*c*d^
3*e^9 + sqrt(a*c)*a^5*d*e^11)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(c))*arctan(
sqrt(e*x + d)/sqrt(-(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4 + sqrt((c^3*d
^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)^2 - (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2
*c*d^2*e^4 - a^3*e^6)*(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)))/(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)))/((a*c^6*d^10 - 5*a^2*c^5*d^8*e^2 + 10*a^3*
c^4*d^6*e^4 - 10*a^4*c^3*d^4*e^6 + 5*a^5*c^2*d^2*e^8 - a^6*c*e^10)*abs(c^2
*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)) - (2*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e
^5)^2*sqrt(-c^2*d + sqrt(a*c)*c*e)*sqrt(a*c)*a*d*e*abs(c) + (3*a*c^3*d^6*e
 - 5*a^2*c^2*d^4*e^3 + a^3*c*d^2*e^5 + a^4*e^7)*sqrt(-c^2*d + sqrt(a*c)*c*
e)*abs(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*abs(c) + (sqrt(a*c)*c^5*d^11*e
 - 3*sqrt(a*c)*a*c^4*d^9*e^3 + 2*sqrt(a*c)*a^2*c^3*d^7*e^5 + 2*sqrt(a*c)*a
^3*c^2*d^5*e^7 - 3*sqrt(a*c)*a^4*c*d^3*e^9 + sqrt(a*c)*a^5*d*e^11)*sqrt(-c
^2*d + sqrt(a*c)*c*e)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c^3*d^5 - 2*a*c^
2*d^3*e^2 + a^2*c*d*e^4 - sqrt((c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)^2
 - (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*(c^3*d^4 - 2...
```

Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 7831, normalized size of antiderivative = 41.22

$$\int \frac{1}{(d+ex)^{5/2}(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - c*x^2)*(d + e*x)^(5/2)),x)
```

output
```
- ((2*e)/(3*(a*e^2 - c*d^2)) - (4*c*d*e*(d + e*x))/(a*e^2 - c*d^2)^2)/(d +
 e*x)^(3/2) - atan((((d + e*x)^(1/2)*(16*a^8*c^5*e^18 + 16*c^13*d^16*e^2 -
 320*a^2*c^11*d^12*e^6 + 1024*a^3*c^10*d^10*e^8 - 1440*a^4*c^9*d^8*e^10 +
1024*a^5*c^8*d^6*e^12 - 320*a^6*c^7*d^4*e^14) + (-(a^2*e^5*(a^3*c^3)^(1/2)
 + a*c^4*d^5 + 5*a^3*c^2*d*e^4 + 10*a^2*c^3*d^3*e^2 + 5*c^2*d^4*e*(a^3*c^3
)^(1/2) + 10*a*c*d^2*e^3*(a^3*c^3)^(1/2))/(4*(a^7*e^10 - a^2*c^5*d^10 - 5*
a^6*c*d^2*e^8 + 5*a^3*c^4*d^8*e^2 - 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^
6)))^(1/2)*(32*a^10*c^4*e^21 - (d + e*x)^(1/2)*(-(a^2*e^5*(a^3*c^3)^(1/2)
+ a*c^4*d^5 + 5*a^3*c^2*d*e^4 + 10*a^2*c^3*d^3*e^2 + 5*c^2*d^4*e*(a^3*c^3)
^(1/2) + 10*a*c*d^2*e^3*(a^3*c^3)^(1/2))/(4*(a^7*e^10 - a^2*c^5*d^10 - 5*a
^6*c*d^2*e^8 + 5*a^3*c^4*d^8*e^2 - 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6
)))^(1/2)*(64*a*c^14*d^21*e^2 + 64*a^11*c^4*d*e^22 - 640*a^2*c^13*d^19*e^4
 + 2880*a^3*c^12*d^17*e^6 - 7680*a^4*c^11*d^15*e^8 + 13440*a^5*c^10*d^13*e
^10 - 16128*a^6*c^9*d^11*e^12 + 13440*a^7*c^8*d^9*e^14 - 7680*a^8*c^7*d^7*
e^16 + 2880*a^9*c^6*d^5*e^18 - 640*a^10*c^5*d^3*e^20) + 96*a*c^13*d^18*e^3
 - 736*a^2*c^12*d^16*e^5 + 2432*a^3*c^11*d^14*e^7 - 4480*a^4*c^10*d^12*e^9
 + 4928*a^5*c^9*d^10*e^11 - 3136*a^6*c^8*d^8*e^13 + 896*a^7*c^7*d^6*e^15 +
 128*a^8*c^6*d^4*e^17 - 160*a^9*c^5*d^2*e^19))*(-(a^2*e^5*(a^3*c^3)^(1/2)
+ a*c^4*d^5 + 5*a^3*c^2*d*e^4 + 10*a^2*c^3*d^3*e^2 + 5*c^2*d^4*e*(a^3*c^3)
^(1/2) + 10*a*c*d^2*e^3*(a^3*c^3)^(1/2))/(4*(a^7*e^10 - a^2*c^5*d^10 - ...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1425, normalized size of antiderivative = 7.50

$$\int \frac{1}{(d+ex)^{5/2}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(5/2)/(-c*x^2+a),x)
```

output

```
(18*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d**2*e**2 + 18*sqrt(a)*s
qrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d*e**3*x + 6*sqrt(a)*sqrt(d + e*x)*sq
rt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*s
qrt(a)*e - c*d)))*c**2*d**4 + 6*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*
c**2*d**3*e*x + 6*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan
((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*d*e**3 +
6*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*
c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*e**4*x + 18*sqrt(c)*sqrt(
d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqr
t(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d**3*e + 18*sqrt(c)*sqrt(d + e*x)*sqrt(sq
rt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a
)*e - c*d)))*a*c*d**2*e**2*x + 9*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a
)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a
*c*d**2*e**2 + 9*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(
- sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*d*e**3*x + 3*
sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sq
rt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c**2*d**4 + 3*sqrt(a)*sqrt(d + ...
```

3.141 $\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 263

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx = \frac{e(cd^2+5ae^2)\sqrt{d+ex}}{2ac^2}+\frac{de(d+ex)^{3/2}}{2ac}+\frac{(ae+cdx)(d+ex)^{5/2}}{2ac(a-cx^2)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\left(2\sqrt{c}d+5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{9/4}}+\frac{\left(2\sqrt{c}d-5\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}c^{9/4}}$$

output
```
1/2*e*(5*a*e^2+c*d^2)*(e*x+d)^(1/2)/a/c^2+1/2*d*e*(e*x+d)^(3/2)/a/c+1/2*(c
*d*x+a*e)*(e*x+d)^(5/2)/a/c/(-c*x^2+a)-1/4*(c^(1/2)*d-a^(1/2)*e)^(5/2)*(2*
c^(1/2)*d+5*a^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)
^(1/2))/a^(3/2)/c^(9/4)+1/4*(2*c^(1/2)*d-5*a^(1/2)*e)*(c^(1/2)*d+a^(1/2)*e
)^(5/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)
/c^(9/4)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^{7/2}}{\left(a-cx^2\right)^2}\,dx = \frac{-\frac{2\sqrt{a}\sqrt{d+ex}\left(5a^2e^3+c^2d^3x+ace\left(3d^2+3dex-4e^2x^2\right)\right)}{-a+cx^2}+\frac{\left(2\sqrt{c}d-5\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^3\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right.}{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}}{4a^{3/2}c^2}$$

input
```
Integrate[(d + e*x)^(7/2)/(a - c*x^2)^2,x]
```

output
```
((-2*Sqrt[a]*Sqrt[d + e*x]*(5*a^2*e^3 + c^2*d^3*x + a*c*e*(3*d^2 + 3*d*e*x
 - 4*e^2*x^2)))/(-a + c*x^2) + ((2*Sqrt[c]*d - 5*Sqrt[a]*e)*(Sqrt[c]*d + S
qrt[a]*e)^3*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[
c]*d + Sqrt[a]*e)])/Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e] - ((Sqrt[c]*d - Sqrt[
a]*e)^3*(2*Sqrt[c]*d + 5*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*
e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*
e])/(4*a^(3/2)*c^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {495, 27, 653, 25, 27, 653, 25, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{\left(a-cx^2\right)^2}\,dx$$

$$\Big\downarrow \; 495$$

$$\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}-\frac{\int -\frac{(d+ex)^{3/2}\left(2cd^2-3cexd-5ae^2\right)}{2\left(a-cx^2\right)}dx}{2ac}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)^{3/2}\left(2cd^2-3cexd-5ae^2\right)}{a-cx^2}dx}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{653}$$

$$\frac{2de(d+ex)^{3/2}-\frac{\int -\frac{c\sqrt{d+ex}\left(2d\left(cd^2-4ae^2\right)-e\left(cd^2+5ae^2\right)x\right)}{a-cx^2}dx}{c}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{c\sqrt{d+ex}\left(2d\left(cd^2-4ae^2\right)-e\left(cd^2+5ae^2\right)x\right)}{a-cx^2}dx}{c}+2de(d+ex)^{3/2}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{d+ex}(2d(cd^2-4ae^2)-e(cd^2+5ae^2)x)}{a-cx^2}dx+2de(d+ex)^{3/2}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{653}$$

$$\frac{-\frac{\int -\frac{\left(cd^2-5ae^2\right)\left(2cd^2+ae^2\right)+cde\left(cd^2-13ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}+\frac{2e\sqrt{d+ex}(5ae^2+cd^2)}{c}+2de(d+ex)^{3/2}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(cd^2-5ae^2\right)\left(2cd^2+ae^2\right)+cde\left(cd^2-13ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}+\frac{2e\sqrt{d+ex}(5ae^2+cd^2)}{c}+2de(d+ex)^{3/2}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{2\int -\frac{e\left(\left(cd^2-ae^2\right)\left(cd^2+5ae^2\right)+cd\left(cd^2-13ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}(5ae^2+cd^2)}{c}+2de(d+ex)^{3/2}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2\int \frac{e\left(\left(cd^2-ae^2\right)\left(cd^2+5ae^2\right)+cd\left(cd^2-13ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}(5ae^2+cd^2)}{c}+2de(d+ex)^{3/2}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{2e\int \frac{(cd^2-ae^2)(cd^2+5ae^2)+cd(cd^2-13ae^2)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}(5ae^2+cd^2)}{c}+2de(d+ex)^{3/2}}{4ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\,(a-cx^2)}$$

$$\Big\downarrow \; 1480$$

$$\frac{-\frac{2e\left(\frac{\sqrt{c}(2\sqrt{c}d-5\sqrt{a}e)(\sqrt{a}e+\sqrt{c}d)^3\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)^3(5\sqrt{a}e+2\sqrt{c}d)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}\right)}{c}+\frac{2e\sqrt{d+e}}{}}{4ac}$$

$$\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\,(a-cx^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{2e\left(\frac{(\sqrt{c}d-\sqrt{a}e)^{5/2}(5\sqrt{a}e+2\sqrt{c}d)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e}-\frac{(2\sqrt{c}d-5\sqrt{a}e)(\sqrt{a}e+\sqrt{c}d)^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e}\right)}{c}+\frac{2e\sqrt{d+ex}(5ae^2+cd^2)}{c}+}{4ac}$$

$$\frac{(d+ex)^{5/2}(ae+cdx)}{2ac\,(a-cx^2)}$$

input
```
Int[(d + e*x)^(7/2)/(a - c*x^2)^2,x]
```

output
```
((a*e + c*d*x)*(d + e*x)^(5/2))/(2*a*c*(a - c*x^2)) + ((2*e*(c*d^2 + 5*a*e
^2)*Sqrt[d + e*x])/c + 2*d*e*(d + e*x)^(3/2) - (2*e*(((Sqrt[c]*d - Sqrt[a]
*e)^(5/2)*(2*Sqrt[c]*d + 5*Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt
[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e) - ((2*Sqrt[c]*d - 5*Sqrt[a
]*e)*(Sqrt[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e)))/c)/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{5\left(e\left(-c\,x^{2}+a\right)c\left(\frac{\left(-13ad\,e^{2}+c\,d^{3}\right)\sqrt{ac\,e^{2}}}{5}+a^{2}e^{4}+\frac{9ac\,d^{2}e^{2}}{5}-\frac{2c^{2}d^{4}}{5}\right)\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)+\cdots}{\cdots}$
derivativedivides	$2e^{3}\left(\frac{\sqrt{ex+d}}{c^{2}}-\frac{\frac{-\frac{cd\left(3a\,e^{2}+c\,d^{2}\right)\left(ex+d\right)^{\frac{3}{2}}}{4a\,e^{2}}-\frac{\left(a^{2}e^{4}-c^{2}d^{4}\right)\sqrt{ex+d}}{4a\,e^{2}}}{-c\left(ex+d\right)^{2}+2cd\left(ex+d\right)+a\,e^{2}-c\,d^{2}}+c\left(\frac{\left(5a^{2}e^{4}+9ac\,d^{2}e^{2}-2c^{2}d^{4}-13\sqrt{ac\,e^{2}}\,ad\,e^{2}+\sqrt{ac\,e^{2}}\cdots\right.}{2\sqrt{ac\,e^{2}}\sqrt{\left(-cd+\sqrt{ac\cdots}\right.}}\cdots\right.}{\cdots}\right.$
default	$2e^{3}\left(\frac{\sqrt{ex+d}}{c^{2}}-\frac{\frac{-\frac{cd\left(3a\,e^{2}+c\,d^{2}\right)\left(ex+d\right)^{\frac{3}{2}}}{4a\,e^{2}}-\frac{\left(a^{2}e^{4}-c^{2}d^{4}\right)\sqrt{ex+d}}{4a\,e^{2}}}{-c\left(ex+d\right)^{2}+2cd\left(ex+d\right)+a\,e^{2}-c\,d^{2}}+c\left(\frac{\left(5a^{2}e^{4}+9ac\,d^{2}e^{2}-2c^{2}d^{4}-13\sqrt{ac\,e^{2}}\,ad\,e^{2}+\sqrt{ac\,e^{2}}\cdots\right.}{2\sqrt{ac\,e^{2}}\sqrt{\left(-cd+\sqrt{ac\cdots}\right.}}\cdots\right.}{\cdots}\right.$
risch	$\frac{2e^{3}\sqrt{ex+d}}{c^{2}}+\frac{2e^{3}\left(\frac{-\frac{cd\left(3a\,e^{2}+c\,d^{2}\right)\left(ex+d\right)^{\frac{3}{2}}}{4a\,e^{2}}-\frac{\left(a^{2}e^{4}-c^{2}d^{4}\right)\sqrt{ex+d}}{4a\,e^{2}}}{c\left(ex+d\right)^{2}-2cd\left(ex+d\right)-a\,e^{2}+c\,d^{2}}+c\left(-\frac{\left(5a^{2}e^{4}+9ac\,d^{2}e^{2}-2c^{2}d^{4}+13\sqrt{ac\,e^{2}}\,ad\,e^{2}-\sqrt{ac\cdots}\right.}{2\sqrt{ac\,e^{2}}\sqrt{\left(cd+\sqrt{\cdots}\right.}}\cdots\right.\right.}{\cdots}$

input
```
int((e*x+d)^(7/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/4/(a*c*e^2)^(1/2)*(e*(-c*x^2+a)*c*(1/5*(-13*a*d*e^2+c*d^3)*(a*c*e^2)^(1
/2)+a^2*e^4+9/5*a*c*d^2*e^2-2/5*c^2*d^4)*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*a
rctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(e*(-c*x^2+a)*c*(1
/5*(13*a*d*e^2-c*d^3)*(a*c*e^2)^(1/2)+a^2*e^4+9/5*a*c*d^2*e^2-2/5*c^2*d^4)
*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))-2*(e*x+d)^(1/2)*
(a*c*e^2)^(1/2)*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(1/5*c^2*d^3*x+3/5*e*a*(-4
/3*e^2*x^2+d*e*x+d^2)*c+a^2*e^3))*((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))/((c*d+
(a*c*e^2)^(1/2))*c)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/c^2/(-c*x^2+a)/
a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2073 vs. $2(200) = 400$.

Time = 0.33 (sec) , antiderivative size = 2073, normalized size of antiderivative = 7.88

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((a*c^3*x^2 - a^2*c^2)*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c*d
^3*e^4 + 105*a^3*d*e^6 + a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*
e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^
9)))/(a^3*c^4))*log((140*c^5*d^10*e^3 - 1771*a*c^4*d^8*e^5 + 6872*a^2*c^3*
d^6*e^7 - 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 + 625*a^5*e^13)*sqrt(
e*x + d) + (35*a^2*c^5*d^6*e^4 + 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d^2*e^8
- 125*a^5*c^2*e^10 - 2*(a^3*c^8*d^3 - 4*a^4*c^7*d*e^2)*sqrt((1225*c^4*d^8*
e^6 - 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 +
 625*a^4*e^14)/(a^3*c^9)))*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^2 + 70*a^2*c*d
^3*e^4 + 105*a^3*d*e^6 + a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*
e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^
9)))/(a^3*c^4))) - (a*c^3*x^2 - a^2*c^2)*sqrt((4*c^3*d^7 - 35*a*c^2*d^5*e^
2 + 70*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^3*c^4*sqrt((1225*c^4*d^8*e^6 - 10
780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^3*c*d^2*e^12 + 625*a^4
*e^14)/(a^3*c^9)))/(a^3*c^4))*log((140*c^5*d^10*e^3 - 1771*a*c^4*d^8*e^5 +
 6872*a^2*c^3*d^6*e^7 - 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 + 625*a
^5*e^13)*sqrt(e*x + d) - (35*a^2*c^5*d^6*e^4 + 21*a^3*c^4*d^4*e^6 - 795*a^
4*c^3*d^2*e^8 - 125*a^5*c^2*e^10 - 2*(a^3*c^8*d^3 - 4*a^4*c^7*d*e^2)*sqrt(
(1225*c^4*d^8*e^6 - 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 + 7700*a^
3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))*sqrt((4*c^3*d^7 - 35*a*c^2*d^5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2-a)^2}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(7/2)/(c*x^2 - a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(200) = 400.

Time = 0.25 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.24

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx = \frac{2\sqrt{ex+d}e^3}{c^2}$$

$$+\frac{\left(\left(\sqrt{ac}cd^3e-13\sqrt{ac}ade^3\right)a^2e^2|c|+\left(ac^2d^4e+4a^2cd^2e^3-5a^3e^5\right)|a||c||e|-\left(2\sqrt{ac}ac^2d^5e-9\sqrt{ac}a^2cd^3e^3\right.}{4\left(a^2c^3d-\sqrt{ac}a^2c^2e\right)\sqrt{-c^2d-\sqrt{ac}ce}|a||e|}$$

$$-\frac{\left((c^2d^3e-13acde^3)a^2e^2|c|-\left(\sqrt{ac}c^2d^4e+4\sqrt{ac}acd^2e^3-5\sqrt{ac}a^2e^5\right)|a||c||e|-(2ac^3d^5e-9a^2c^2d^3e^3-\right.}{4\left(a^2c^3e+\sqrt{ac}ac^3d\right)\sqrt{-c^2d+\sqrt{ac}ce}|a||e|}$$

$$-\frac{(ex+d)^{\frac{3}{2}}c^2d^3e-\sqrt{ex+d}c^2d^4e+3(ex+d)^{\frac{3}{2}}acde^3+\sqrt{ex+d}a^2e^5}{2\left((ex+d)^2c-2(ex+d)cd+cd^2-ae^2\right)ac^2}$$

input
```
integrate((e*x+d)^(7/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*e^3/c^2 + 1/4*((sqrt(a*c)*c*d^3*e - 13*sqrt(a*c)*a*d*e^3)*
a^2*e^2*abs(c) + (a*c^2*d^4*e + 4*a^2*c*d^2*e^3 - 5*a^3*e^5)*abs(a)*abs(c)
*abs(e) - (2*sqrt(a*c)*a*c^2*d^5*e - 9*sqrt(a*c)*a^2*c*d^3*e^3 - 5*sqrt(a*
c)*a^3*d*e^5)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^3*d + sqrt(a^2*c^6*d
^2 - (a*c^3*d^2 - a^2*c^2*e^2)*a*c^3))/(a*c^3)))/((a^2*c^3*d - sqrt(a*c)*a
^2*c^2*e)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(a)*abs(e)) - 1/4*((c^2*d^3*e -
13*a*c*d*e^3)*a^2*e^2*abs(c) - (sqrt(a*c)*c^2*d^4*e + 4*sqrt(a*c)*a*c*d^2*
e^3 - 5*sqrt(a*c)*a^2*e^5)*abs(a)*abs(c)*abs(e) - (2*a*c^3*d^5*e - 9*a^2*c
^2*d^3*e^3 - 5*a^3*c*d*e^5)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^3*d -
sqrt(a^2*c^6*d^2 - (a*c^3*d^2 - a^2*c^2*e^2)*a*c^3))/(a*c^3)))/((a^2*c^3*e
 + sqrt(a*c)*a*c^3*d)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(a)*abs(e)) - 1/2*((
e*x + d)^(3/2)*c^2*d^3*e - sqrt(e*x + d)*c^2*d^4*e + 3*(e*x + d)^(3/2)*a*c
*d*e^3 + sqrt(e*x + d)*a^2*e^5)/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2
- a*e^2)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 7.24 (sec) , antiderivative size = 4090, normalized size of antiderivative = 15.55

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(7/2)/(a - c*x^2)^2,x)
```

output
```
atan((a^2*e^10*(d + e*x)^(1/2)*((105*d*e^6)/(64*c^4) + d^7/(16*a^3*c) + (3
5*d^3*e^4)/(32*a*c^3) - (35*d^5*e^2)/(64*a^2*c^2) + (25*e^7*(a^9*c^9)^(1/2
))/(64*a^4*c^9) + (77*d^2*e^5*(a^9*c^9)^(1/2))/(32*a^5*c^8) - (35*d^4*e^3*
(a^9*c^9)^(1/2))/(64*a^6*c^7))^(1/2)*50i)/((491*a*d^3*e^11)/(2*c) - (885*d
^5*e^9)/2 + (329*c*d^7*e^7)/(2*a) + (50*a^2*d*e^13)/c^2 - (35*c^2*d^9*e^5)
/(2*a^2) + (125*e^14*(a^9*c^9)^(1/2))/(4*a^2*c^7) + (335*d^2*e^12*(a^9*c^9
)^(1/2))/(2*a^3*c^6) - (204*d^4*e^10*(a^9*c^9)^(1/2))/(a^4*c^5) - (7*d^6*e
^8*(a^9*c^9)^(1/2))/(2*a^5*c^4) + (35*d^8*e^6*(a^9*c^9)^(1/2))/(4*a^6*c^3)
) - (d^3*e^7*(a^9*c^9)^(1/2)*(d + e*x)^(1/2)*((105*d*e^6)/(64*c^4) + d^7/(
16*a^3*c) + (35*d^3*e^4)/(32*a*c^3) - (35*d^5*e^2)/(64*a^2*c^2) + (25*e^7*
(a^9*c^9)^(1/2))/(64*a^4*c^9) + (77*d^2*e^5*(a^9*c^9)^(1/2))/(32*a^5*c^8)
- (35*d^4*e^3*(a^9*c^9)^(1/2))/(64*a^6*c^7))^(1/2)*308i)/((329*a^3*c^4*d^7
*e^7)/2 - (35*a^2*c^5*d^9*e^5)/2 - (885*a^4*c^3*d^5*e^9)/2 + (491*a^5*c^2*
d^3*e^11)/2 + 50*a^6*c*d*e^13 + (125*a^2*e^14*(a^9*c^9)^(1/2))/(4*c^4) + (
35*d^8*e^6*(a^9*c^9)^(1/2))/(4*a^2) - (204*d^4*e^10*(a^9*c^9)^(1/2))/c^2 +
 (335*a*d^2*e^12*(a^9*c^9)^(1/2))/(2*c^3) - (7*d^6*e^8*(a^9*c^9)^(1/2))/(2
*a*c)) + (d^5*e^5*(a^9*c^9)^(1/2)*(d + e*x)^(1/2)*((105*d*e^6)/(64*c^4) +
d^7/(16*a^3*c) + (35*d^3*e^4)/(32*a*c^3) - (35*d^5*e^2)/(64*a^2*c^2) + (25
*e^7*(a^9*c^9)^(1/2))/(64*a^4*c^9) + (77*d^2*e^5*(a^9*c^9)^(1/2))/(32*a^5*
c^8) - (35*d^4*e^3*(a^9*c^9)^(1/2))/(64*a^6*c^7))^(1/2)*70i)/(50*a^7*d*...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1267, normalized size of antiderivative = 4.82

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(7/2)/(-c*x^2+a)^2,x)
```

output
```
(16*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d*e**2 - 4*sqrt(a)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d
)))*a*c**2*d**3 - 16*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d +
e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d*e**2*x**2 + 4*sq
rt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)))*c**3*d**3*x**2 - 10*sqrt(c)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*
a**3*e**3 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)
/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d**2*e + 10*sqrt(c)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)))*a**2*c*e**3*x**2 + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*
atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d**
2*e*x**2 + 8*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqr
t(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*c*d*e**2 - 2*sqrt(a)*sqrt(sqrt
(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d
 + e*x))*a*c**2*d**3 - 8*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt
(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c**2*d*e**2*x**2 + 2*
sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d)
 + sqrt(c)*sqrt(d + e*x))*c**3*d**3*x**2 - 8*sqrt(a)*sqrt(sqrt(c)*sqrt(...
```

3.142 $\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 231

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \frac{de\sqrt{d+ex}}{2ac}+\frac{(ae+cdx)(d+ex)^{3/2}}{2ac(a-cx^2)} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(2\sqrt{c}d+3\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{7/4}} + \frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}c^{7/4}}$$

output
```
1/2*d*e*(e*x+d)^(1/2)/a/c+1/2*(c*d*x+a*e)*(e*x+d)^(3/2)/a/c/(-c*x^2+a)-1/4
*(c^(1/2)*d-a^(1/2)*e)^(3/2)*(2*c^(1/2)*d+3*a^(1/2)*e)*arctanh(c^(1/4)*(e*
x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3/2)/c^(7/4)+1/4*(2*c^(1/2)*d-3
*a^(1/2)*e)*(c^(1/2)*d+a^(1/2)*e)^(3/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(
1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/c^(7/4)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \frac{-\frac{2\sqrt{a}c\sqrt{d+ex}(cd^2x+ae(2d+ex))}{-a+cx^2} - \sqrt{-cd-\sqrt{a}\sqrt{c}e}\left(2cd^2-\sqrt{a}\sqrt{c}de-3ae^2\right)\arctan\left(\frac{\sqrt{-cd-}}{\sqrt{}}\right.}{4a^{3/}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a - c*x^2)^2,x]
```

output
```
((-2*Sqrt[a]*c*Sqrt[d + e*x]*(c*d^2*x + a*e*(2*d + e*x)))/(-a + c*x^2) - S
qrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*(2*c*d^2 - Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*
ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[
a]*e)] + Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*(2*c*d^2 + Sqrt[a]*Sqrt[c]*d*e -
 3*a*e^2)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]
*d - Sqrt[a]*e)])/(4*a^(3/2)*c^2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {495, 27, 653, 25, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}dx$$

$$\Big\downarrow \text{ 495}$$

$$\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\,(a-cx^2)} - \frac{\int -\frac{\sqrt{d+ex}(2cd^2-cexd-3ae^2)}{2(a-cx^2)}dx}{2ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{d+ex}(2cd^2-cexd-3ae^2)}{a-cx^2}dx}{4ac} + \frac{(d+ex)^{3/2}(ae+cdx)}{2ac\,(a-cx^2)}$$

$$\Big\downarrow \text{ 653}$$

$$\frac{2de\sqrt{d+ex}-\frac{\int -\frac{c\left(2d\left(cd^2-2ae^2\right)+e\left(cd^2-3ae^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}}{4ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c\left(2d\left(cd^2-2ae^2\right)+e\left(cd^2-3ae^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}+2de\sqrt{d+ex}}{4ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2d\left(cd^2-2ae^2\right)+e\left(cd^2-3ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx+2de\sqrt{d+ex}}{4ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 654$$

$$\frac{2\int -\frac{e\left(d\left(cd^2-ae^2\right)+\left(cd^2-3ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}+2de\sqrt{d+ex}}{4ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2de\sqrt{d+ex}-2\int \frac{e\left(d\left(cd^2-ae^2\right)+\left(cd^2-3ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{4ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2de\sqrt{d+ex}-2e\int \frac{d\left(cd^2-ae^2\right)+\left(cd^2-3ae^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{4ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{2de\sqrt{d+ex}-2e\left(\frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(3\sqrt{a}e+2\sqrt{c}d\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right.}}{2\sqrt{a}e}\right.}{4ac}$$

$$\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{2de\sqrt{d+ex}-2e\left(\frac{(\sqrt{c}d-\sqrt{a}e)^{3/2}(3\sqrt{a}e+2\sqrt{c}d)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}e}-\frac{(2\sqrt{c}d-3\sqrt{a}e)(\sqrt{a}e+\sqrt{c}d)^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e}\right)}{4ac}$$
$$\frac{(d+ex)^{3/2}(ae+cdx)}{2ac\,(a-cx^2)}$$

input
```
Int[(d + e*x)^(5/2)/(a - c*x^2)^2,x]
```

output
```
((a*e + c*d*x)*(d + e*x)^(3/2))/(2*a*c*(a - c*x^2)) + (2*d*e*Sqrt[d + e*x]
 - 2*e*(((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*(2*Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*
e) - ((2*Sqrt[c]*d - 3*Sqrt[a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^(3/2)*ArcTanh[(c
^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e))
)/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{\left(-3 a e^{2}+c d^{2}\right) \sqrt{a c e^{2}}}{4}+c d \left(a e^{2}-\frac{c d^{2}}{2}\right)\right) e \sqrt{\left(c d+\sqrt{a c e^{2}}\right) c}\,\left(-c x^{2}+a\right) \arctan\left(\frac{c \sqrt{e x+d}}{\sqrt{\left(-c d+\sqrt{a c e^{2}}\right) c}}\right)+\sqrt{\left(-c d+\sqrt{a c \cdots}}{\sqrt{a c e^{2}}\, \sqrt{\left(c d+\cdots}$
derivativedivides	$2 e^{3} \left(\frac{\frac{\left(a e^{2}+c d^{2}\right) \left(e x+d\right)^{\frac{3}{2}}}{4 a c e^{2}}+\frac{\left(a e^{2}-c d^{2}\right) d \sqrt{e x+d}}{4 a c e^{2}}}{-c \left(e x+d\right)^{2}+2 c d \left(e x+d\right)+a e^{2}-c d^{2}}+\frac{\left(-4 a d e^{2} c+2 c^{2} d^{3}+3 \sqrt{a c e^{2}}\, a e^{2}-\sqrt{a c e^{2}}\, c d^{2}\right) \arctan\left(\frac{c \sqrt{e x+d}}{\sqrt{\left(-c d+\sqrt{a c e \cdots}}\right)}{2 c \sqrt{a c e^{2}}\, \sqrt{\left(-c d+\sqrt{a c e^{2}}\right) c}} \cdots\right.$
default	$2 e^{3} \left(\frac{\frac{\left(a e^{2}+c d^{2}\right) \left(e x+d\right)^{\frac{3}{2}}}{4 a c e^{2}}+\frac{\left(a e^{2}-c d^{2}\right) d \sqrt{e x+d}}{4 a c e^{2}}}{-c \left(e x+d\right)^{2}+2 c d \left(e x+d\right)+a e^{2}-c d^{2}}+\frac{\left(-4 a d e^{2} c+2 c^{2} d^{3}+3 \sqrt{a c e^{2}}\, a e^{2}-\sqrt{a c e^{2}}\, c d^{2}\right) \arctan\left(\frac{c \sqrt{e x+d}}{\sqrt{\left(-c d+\sqrt{a c e \cdots}}\right)}{2 c \sqrt{a c e^{2}}\, \sqrt{\left(-c d+\sqrt{a c e^{2}}\right) c}} \cdots\right.$

input
```
int((e*x+d)^(5/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(1/4*(-3*a*e^2+c*d^2)*(a*c*e^2)^(1/2)+c*d*(a*e^2-1/2*c*d^2))*e*((c*d+(a*
c*e^2)^(1/2))*c)^(1/2)*(-c*x^2+a)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^
(1/2))*c)^(1/2))+((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-e*(-c*x^2+a)*(1/4*(3*a
*e^2-c*d^2)*(a*c*e^2)^(1/2)+c*d*(a*e^2-1/2*c*d^2))*arctanh(c*(e*x+d)^(1/2)
/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(1/2*c*d^2*x+a*e*(1/2*e*x+d))*((c*d+(a*c
*e^2)^(1/2))*c)^(1/2)*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)))/(a*c*e^2)^(1/2)/((c*
d+(a*c*e^2)^(1/2))*c)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a/c/(-c*x^2+a
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1370 vs. $2(172) = 344$.

Time = 0.14 (sec) , antiderivative size = 1370, normalized size of antiderivative = 5.93

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((a*c^2*x^2 - a^2*c)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4
+ a^3*c^3*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))
/(a^3*c^3))*log(-(20*c^3*d^6*e^3 - 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 -
 81*a^3*e^9)*sqrt(e*x + d) + (5*a^2*c^3*d^3*e^4 - 9*a^3*c^2*d*e^6 - (2*a^3
*c^6*d^2 - 3*a^4*c^5*e^2)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e
^10)/(a^3*c^7)))*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c^3
*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3
))) - (a*c^2*x^2 - a^2*c)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4
+ a^3*c^3*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))
/(a^3*c^3))*log(-(20*c^3*d^6*e^3 - 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 -
 81*a^3*e^9)*sqrt(e*x + d) - (5*a^2*c^3*d^3*e^4 - 9*a^3*c^2*d*e^6 - (2*a^3
*c^6*d^2 - 3*a^4*c^5*e^2)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e
^10)/(a^3*c^7)))*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c^3
*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*c^3
))) + (a*c^2*x^2 - a^2*c)*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4
- a^3*c^3*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))
/(a^3*c^3))*log(-(20*c^3*d^6*e^3 - 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 -
 81*a^3*e^9)*sqrt(e*x + d) + (5*a^2*c^3*d^3*e^4 - 9*a^3*c^2*d*e^6 + (2*a^3
*c^6*d^2 - 3*a^4*c^5*e^2)*sqrt((25*c^2*d^4*e^6 - 90*a*c*d^2*e^8 + 81*a^2*e
^10)/(a^3*c^7)))*sqrt((4*c^2*d^5 - 15*a*c*d^3*e^2 + 15*a^2*d*e^4 - a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2-a)^2}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 - a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(172) = 344.

Time = 0.24 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \frac{\left(2\,ac^4d^4e - 4\,a^2c^3d^2e^3 - (cd^2e - 3\,ae^3)a^2c^2e^2 - \left(\sqrt{ac}c^2d^3e - \sqrt{ac}acde^3\right)|a||c||e|\right)\,\mathrm{arct}}{4\,\left(a^2c^3e - \sqrt{ac}ac^3d\right)\sqrt{-c^2d - \sqrt{ac}ce}|a||e|} + \frac{\left(2\,\sqrt{ac}ac^4d^4e - 4\,\sqrt{ac}a^2c^3d^2e^3 - \left(\sqrt{ac}cd^2e - 3\,\sqrt{ac}ae^3\right)a^2c^2e^2 + (ac^3d^3e - a^2c^2de^3)|a||c||e|\right)\arctan\left(\frac{}{\sqrt{}}\right.}{4\,\left(a^2c^4d + \sqrt{ac}a^2c^3e\right)\sqrt{-c^2d + \sqrt{ac}ce}|a||e|} - \frac{(ex+d)^{\frac{3}{2}}cd^2e - \sqrt{ex+d}cd^3e + (ex+d)^{\frac{3}{2}}ae^3 + \sqrt{ex+d}ade^3}{2\left((ex+d)^2c - 2\,(ex+d)cd + cd^2 - ae^2\right)ac}$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*a*c^4*d^4*e - 4*a^2*c^3*d^2*e^3 - (c*d^2*e - 3*a*e^3)*a^2*c^2*e^2 -
 (sqrt(a*c)*c^2*d^3*e - sqrt(a*c)*a*c*d*e^3)*abs(a)*abs(c)*abs(e))*arctan(
sqrt(e*x + d)/sqrt(-(a*c^2*d + sqrt(a^2*c^4*d^2 - (a*c^2*d^2 - a^2*c*e^2)*
a*c^2))/(a*c^2)))/((a^2*c^3*e - sqrt(a*c)*a*c^3*d)*sqrt(-c^2*d - sqrt(a*c)
*c*e)*abs(a)*abs(e)) + 1/4*(2*sqrt(a*c)*a*c^4*d^4*e - 4*sqrt(a*c)*a^2*c^3*
d^2*e^3 - (sqrt(a*c)*c*d^2*e - 3*sqrt(a*c)*a*e^3)*a^2*c^2*e^2 + (a*c^3*d^3
*e - a^2*c^2*d*e^3)*abs(a)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c^
2*d - sqrt(a^2*c^4*d^2 - (a*c^2*d^2 - a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c
^4*d + sqrt(a*c)*a^2*c^3*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(a)*abs(e)) -
1/2*((e*x + d)^(3/2)*c*d^2*e - sqrt(e*x + d)*c*d^3*e + (e*x + d)^(3/2)*a*e
^3 + sqrt(e*x + d)*a*d*e^3)/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 - a*
e^2)*a*c)
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 1988, normalized size of antiderivative = 8.61

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(a - c*x^2)^2,x)
```

output

```
2*atanh((18*a*e^8*(d + e*x)^(1/2)*(d^5/(16*a^3*c) + (15*d*e^4)/(64*a*c^3)
- (15*d^3*e^2)/(64*a^2*c^2) - (9*e^5*(a^9*c^7)^(1/2))/(64*a^5*c^7) + (5*d^
2*e^3*(a^9*c^7)^(1/2))/(64*a^6*c^6))^(1/2))/((15*d^2*e^9)/c - (43*d^4*e^7)
/(4*a) - (27*a*e^11)/(4*c^2) + (5*c*d^6*e^5)/(2*a^2) + (9*d*e^10*(a^9*c^7)
^(1/2))/(4*a^4*c^5) - (7*d^3*e^8*(a^9*c^7)^(1/2))/(2*a^5*c^4) + (5*d^5*e^6
*(a^9*c^7)^(1/2))/(4*a^6*c^3)) - (10*c*d^2*e^6*(d + e*x)^(1/2)*(d^5/(16*a^
3*c) + (15*d*e^4)/(64*a*c^3) - (15*d^3*e^2)/(64*a^2*c^2) - (9*e^5*(a^9*c^7
)^(1/2))/(64*a^5*c^7) + (5*d^2*e^3*(a^9*c^7)^(1/2))/(64*a^6*c^6))^(1/2))/(
(15*d^2*e^9)/c - (43*d^4*e^7)/(4*a) - (27*a*e^11)/(4*c^2) + (5*c*d^6*e^5)/
(2*a^2) + (9*d*e^10*(a^9*c^7)^(1/2))/(4*a^4*c^5) - (7*d^3*e^8*(a^9*c^7)^(1
/2))/(2*a^5*c^4) + (5*d^5*e^6*(a^9*c^7)^(1/2))/(4*a^6*c^3)) + (18*d*e^7*(a
^9*c^7)^(1/2)*(d + e*x)^(1/2)*(d^5/(16*a^3*c) + (15*d*e^4)/(64*a*c^3) - (1
5*d^3*e^2)/(64*a^2*c^2) - (9*e^5*(a^9*c^7)^(1/2))/(64*a^5*c^7) + (5*d^2*e^
3*(a^9*c^7)^(1/2))/(64*a^6*c^6))^(1/2))/((5*a^2*c^4*d^6*e^5)/2 - (27*a^5*c
*e^11)/4 - (43*a^3*c^3*d^4*e^7)/4 + 15*a^4*c^2*d^2*e^9 + (9*d*e^10*(a^9*c^
7)^(1/2))/(4*c^2) + (5*d^5*e^6*(a^9*c^7)^(1/2))/(4*a^2) - (7*d^3*e^8*(a^9*
c^7)^(1/2))/(2*a*c)) - (10*d^3*e^5*(a^9*c^7)^(1/2)*(d + e*x)^(1/2)*(d^5/(1
6*a^3*c) + (15*d*e^4)/(64*a*c^3) - (15*d^3*e^2)/(64*a^2*c^2) - (9*e^5*(a^9
*c^7)^(1/2))/(64*a^5*c^7) + (5*d^2*e^3*(a^9*c^7)^(1/2))/(64*a^6*c^6))^(1/2
))/(15*a^5*c*d^2*e^9 - (27*a^6*e^11)/4 + (5*a^3*c^3*d^6*e^5)/2 - (43*a^...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 897, normalized size of antiderivative = 3.88

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^(5/2)/(-c*x^2+a)^2,x)
```

output
```
(6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*e**2 - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a
*c*d**2 - 6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(
sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*e**2*x**2 + 4*sqrt(a)*sqrt(sqr
t(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)))*c**2*d**2*x**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan(
(sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*d*e + 2*sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)))*a*c*d*e*x**2 + 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e
+ c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*
e**2 - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)
*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*d**2 - 3*sqrt(a)*sqrt(sqrt(c)*sqrt(
a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*
a*c*e**2*x**2 + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c
)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c**2*d**2*x**2 - 3*sqrt(a)*sqr
t(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqr
t(d + e*x))*a**2*e**2 + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(s
qrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*d**2 + 3*sqrt(a)*sqrt
(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*s...
```

3.143 $\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2} dx = \frac{(ae+cdx)\sqrt{d+ex}}{2ac(a-cx^2)} - \frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\,(2\sqrt{c}d+\sqrt{a}e)\,\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{5/4}} + \frac{(2\sqrt{c}d-\sqrt{a}e)\,\sqrt{\sqrt{c}d+\sqrt{a}e}\,\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}c^{5/4}}$$

output
```
1/2*(c*d*x+a*e)*(e*x+d)^(1/2)/a/c/(-c*x^2+a)-1/4*(c^(1/2)*d-a^(1/2)*e)^(1/
2)*(2*c^(1/2)*d+a^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2
)*e)^(1/2))/a^(3/2)/c^(5/4)+1/4*(2*c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*d+a^(1/2)
*e)^(1/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/
2)/c^(5/4)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}\sqrt{c}(ae+cdx)\sqrt{d+ex}}{a-cx^2} - \left(2\sqrt{c}d-\sqrt{a}e\right)\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right) + (2\cdots}{4a^{3/2}c^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a - c*x^2)^2,x]
```

output
```
((2*Sqrt[a]*Sqrt[c]*(a*e + c*d*x)*Sqrt[d + e*x])/(a - c*x^2) - (2*Sqrt[c]*
d - Sqrt[a]*e)*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - Sqrt
[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)] + (2*Sqrt[c]*d + Sq
rt[a]*e)*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sq
rt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(4*a^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {495, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2}dx$$

$$\Big\downarrow \ 495$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2ac\,(a-cx^2)} - \frac{\int -\frac{2cd^2+cexd-ae^2}{2\sqrt{d+ex}(a-cx^2)}dx}{2ac}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2cd^2+cexd-ae^2}{\sqrt{d+ex}(a-cx^2)}dx}{4ac} + \frac{\sqrt{d+ex}(ae+cdx)}{2ac\,(a-cx^2)}$$

$$\Big\downarrow \ 654$$

$$\frac{\int -\frac{e(cd^2+c(d+ex)d-ae^2)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}+\frac{\sqrt{d+ex}(ae+cdx)}{2ac\,(a-cx^2)}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{d+ex}(ae+cdx)}{2ac\,(a-cx^2)}-\frac{\int \frac{e(cd^2+c(d+ex)d-ae^2)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{d+ex}(ae+cdx)}{2ac\,(a-cx^2)}-\frac{e\int \frac{cd^2+c(d+ex)d-ae^2}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}$$

$\Big\downarrow$ 1480

$$\frac{\sqrt{d+ex}(ae+cdx)}{2ac\,(a-cx^2)}-\frac{e\left(\frac{1}{2}\sqrt{c}\left(\sqrt{c}d-\frac{2cd^2-ae^2}{\sqrt{a}e}\right)\int\frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}+\frac{1}{2}\sqrt{c}\left(\frac{2cd^2-ae^2}{\sqrt{a}e}+\sqrt{c}d\right)\int\frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+}\right.}{2ac}$$

$\Big\downarrow$ 221

$$\frac{\sqrt{d+ex}(ae+cdx)}{2ac\,(a-cx^2)}-\frac{e\left(-\frac{\left(\sqrt{c}d-\frac{2cd^2-ae^2}{\sqrt{a}e}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\left(\frac{2cd^2-ae^2}{\sqrt{a}e}+\sqrt{c}d\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2ac}$$

input
```
Int[(d + e*x)^(3/2)/(a - c*x^2)^2,x]
```

output
```
((a*e + c*d*x)*Sqrt[d + e*x])/(2*a*c*(a - c*x^2)) - (e*(-1/2*((Sqrt[c]*d -
 (2*c*d^2 - a*e^2)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[
c]*d - Sqrt[a]*e]])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) - ((Sqrt[c]*d +
(2*c*d^2 - a*e^2)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c
]*d + Sqrt[a]*e]])/(2*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.20

method	result
derivativedivides	$2e^{3}\left(\frac{\frac{d(ex+d)^{\frac{3}{2}}}{4a\,e^{2}}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\sqrt{ex+d}}{4a\,e^{2}c}}{-c(ex+d)^{2}+2cd(ex+d)+a\,e^{2}-c\,d^{2}}+\frac{\frac{\left(-a\,e^{2}+2c\,d^{2}-\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(a\,e^{2}-2c\,d^{2}-\sqrt{ac\,e\ldots}\right.}{2\sqrt{ac\ldots}}}{4a\,e^{2}}\right.$
default	$2e^{3}\left(\frac{\frac{d(ex+d)^{\frac{3}{2}}}{4a\,e^{2}}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\sqrt{ex+d}}{4a\,e^{2}c}}{-c(ex+d)^{2}+2cd(ex+d)+a\,e^{2}-c\,d^{2}}+\frac{\frac{\left(-a\,e^{2}+2c\,d^{2}-\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(a\,e^{2}-2c\,d^{2}-\sqrt{ac\,e\ldots}\right.}{2\sqrt{ac\ldots}}}{4a\,e^{2}}\right.$
pseudoelliptic	$\frac{-\frac{e\left(-c\,x^{2}+a\right)c\left(a\,e^{2}-2c\,d^{2}+\sqrt{ac\,e^{2}}\,d\right)\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2}+\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}\left(-\frac{ec\left(-c\,x^{2}+a\right)\left(\ldots\right.}{\ldots}\right.}{2\sqrt{ac\,e^{2}}\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}\sqrt{\left(-cd+\sqrt{ac\,e\ldots}\right.}}$

input
```
int((e*x+d)^(3/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*((1/4*d/a/e^2*(e*x+d)^(3/2)+1/4*(a*e^2-c*d^2)/a/e^2/c*(e*x+d)^(1/2))
/(-c*(e*x+d)^2+2*c*d*(e*x+d)+a*e^2-c*d^2)+1/4/a/e^2*(1/2*(-a*e^2+2*c*d^2-(
a*c*e^2)^(1/2)*d)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(
c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-1/2*(a*e^2-2*c*d^2-(a*c*
e^2)^(1/2)*d)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e
*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(154) = 308$.

Time = 0.11 (sec) , antiderivative size = 679, normalized size of antiderivative = 3.25

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = -\frac{(ac^2x^2-a^2c)\sqrt{\frac{a^3c^2\sqrt{\frac{e^6}{a^3c^5}}+4\,cd^3-3\,ade^2}{a^3c^2}}\log\left(-(4\,cd^2e^3-ae^5)\sqrt{ex+d}+\left(2\,a^3c^4d\sqrt{\frac{e^6}{a^3c^5}}+a^2ce^4\right)\sqrt{\frac{a^3c^2\sqrt{\frac{e^6}{a^3c^5}}}{}}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((a*c^2*x^2 - a^2*c)*sqrt((a^3*c^2*sqrt(e^6/(a^3*c^5)) + 4*c*d^3 - 3*
a*d*e^2)/(a^3*c^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*
d*sqrt(e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt((a^3*c^2*sqrt(e^6/(a^3*c^5)) + 4*c
*d^3 - 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^2 - a^2*c)*sqrt((a^3*c^2*sqrt(e^6
/(a^3*c^5)) + 4*c*d^3 - 3*a*d*e^2)/(a^3*c^2))*log(-(4*c*d^2*e^3 - a*e^5)*s
qrt(e*x + d) - (2*a^3*c^4*d*sqrt(e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt((a^3*c^2
*sqrt(e^6/(a^3*c^5)) + 4*c*d^3 - 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^2 - a^2
*c)*sqrt(-(a^3*c^2*sqrt(e^6/(a^3*c^5)) - 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))*l
og(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*d*sqrt(e^6/(a^3*c^5))
 - a^2*c*e^4)*sqrt(-(a^3*c^2*sqrt(e^6/(a^3*c^5)) - 4*c*d^3 + 3*a*d*e^2)/(a
^3*c^2))) + (a*c^2*x^2 - a^2*c)*sqrt(-(a^3*c^2*sqrt(e^6/(a^3*c^5)) - 4*c*d
^3 + 3*a*d*e^2)/(a^3*c^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) - (2*a
^3*c^4*d*sqrt(e^6/(a^3*c^5)) - a^2*c*e^4)*sqrt(-(a^3*c^2*sqrt(e^6/(a^3*c^5
)) - 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))) + 4*(c*d*x + a*e)*sqrt(e*x + d))/(a*
c^2*x^2 - a^2*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\left(a-cx^2\right)^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2-a\right)^2}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 - a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(154) = 308$.

Time = 0.21 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex)^{3/2}}{\left(a-cx^2\right)^2}\,dx = \frac{\left(2\,ac^3d^3e - 2\,a^2c^2de^3 - \left(\sqrt{ac}cd^2e - \sqrt{ac}ae^3\right)|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{ac^2d+\sqrt{a^2c^4d^2-\left(ac^2d^2-a^2}}{ac^2}}}\right)}{4\left(a^2c^2e-\sqrt{ac}ac^2d\right)\sqrt{-c^2d-\sqrt{ac}ce}|a||e|} + \frac{\left(2\,ac^3d^3e - 2\,a^2c^2de^3 + \left(\sqrt{ac}cd^2e - \sqrt{ac}ae^3\right)|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{ac^2d-\sqrt{a^2c^4d^2-\left(ac^2d^2-a^2ce^2\right)ac^2}}{ac^2}}}\right)}{4\left(a^2c^2e+\sqrt{ac}ac^2d\right)\sqrt{-c^2d+\sqrt{ac}ce}|a||e|} - \frac{(ex+d)^{\frac{3}{2}}cde - \sqrt{ex+d}cd^2e + \sqrt{ex+d}ae^3}{2\left((ex+d)^2c - 2\,(ex+d)cd + cd^2 - ae^2\right)ac}$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*a*c^3*d^3*e - 2*a^2*c^2*d*e^3 - (sqrt(a*c)*c*d^2*e - sqrt(a*c)*a*e^
3)*abs(a)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d + sqrt(a^2*c^
4*d^2 - (a*c^2*d^2 - a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c^2*e - sqrt(a*c)*
a*c^2*d)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(a)*abs(e)) + 1/4*(2*a*c^3*d^3*e
- 2*a^2*c^2*d*e^3 + (sqrt(a*c)*c*d^2*e - sqrt(a*c)*a*e^3)*abs(a)*abs(c)*ab
s(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d - sqrt(a^2*c^4*d^2 - (a*c^2*d^2
- a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c^2*e + sqrt(a*c)*a*c^2*d)*sqrt(-c^2*
d + sqrt(a*c)*c*e)*abs(a)*abs(e)) - 1/2*((e*x + d)^(3/2)*c*d*e - sqrt(e*x
+ d)*c*d^2*e + sqrt(e*x + d)*a*e^3)/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*
d^2 - a*e^2)*a*c)
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 704, normalized size of antiderivative = 3.37

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = 2\,\mathrm{atanh}\left(\frac{2\,c\,e^6\,\sqrt{d+e\,x}\,\sqrt{\frac{d^3}{16\,a^3\,c}-\frac{3\,d\,e^2}{64\,a^2\,c^2}-\frac{e^3\,\sqrt{a^9\,c^5}}{64\,a^6\,c^5}}}{\frac{d\,e^7}{2\,a}-\frac{c\,d^3\,e^5}{2\,a^2}+\frac{e^8\,\sqrt{a^9\,c^5}}{4\,a^5\,c^3}-\frac{d^2\,e^6\,\sqrt{a^9\,c^5}}{4\,a^6\,c^2}} + \frac{2\,d\,e^5\,\sqrt{a^9\,c^5}\,\sqrt{d+e\,x}\,\sqrt{\frac{d^3}{16\,a^3\,c}-\frac{3\,d\,e^2}{64\,a^2\,c^2}-\frac{e^3\,\sqrt{a^9\,c^5}}{64\,a^6\,c^5}}}{\frac{e^8\,\sqrt{a^9\,c^5}}{4\,c^2}-\frac{a^3\,c^2\,d^3\,e^5}{2}+\frac{a^4\,c\,d\,e^7}{2}-\frac{d^2\,e^6\,\sqrt{a^9\,c^5}}{4\,a\,c}}\right)\sqrt{-\frac{e^3\,\sqrt{a^9\,c^5}-4\,a^3\,c^4\,d^3+3\,a^4\,c^3\,d\,e^2}{64\,a^6\,c^5}} - \frac{\frac{\left(a\,e^3-c\,d^2\,e\right)\sqrt{d+e\,x}}{2\,a\,c}+\frac{d\,e\,(d+e\,x)^{3/2}}{2\,a}}{c\,(d+e\,x)^2-a\,e^2+c\,d^2-2\,c\,d\,(d+e\,x)} + 2\,\mathrm{atanh}\left(\frac{2\,c\,e^6\,\sqrt{d+e\,x}\,\sqrt{\frac{d^3}{16\,a^3\,c}-\frac{3\,d\,e^2}{64\,a^2\,c^2}+\frac{e^3\,\sqrt{a^9\,c^5}}{64\,a^6\,c^5}}}{\frac{d\,e^7}{2\,a}-\frac{c\,d^3\,e^5}{2\,a^2}-\frac{e^8\,\sqrt{a^9\,c^5}}{4\,a^5\,c^3}+\frac{d^2\,e^6\,\sqrt{a^9\,c^5}}{4\,a^6\,c^2}} + \frac{2\,d\,e^5\,\sqrt{a^9\,c^5}\,\sqrt{d+e\,x}\,\sqrt{\frac{d^3}{16\,a^3\,c}-\frac{3\,d\,e^2}{64\,a^2\,c^2}+\frac{e^3\,\sqrt{a^9\,c^5}}{64\,a^6\,c^5}}}{\frac{e^8\,\sqrt{a^9\,c^5}}{4\,c^2}+\frac{a^3\,c^2\,d^3\,e^5}{2}-\frac{a^4\,c\,d\,e^7}{2}-\frac{d^2\,e^6\,\sqrt{a^9\,c^5}}{4\,a\,c}}\right)$$

input
```
int((d + e*x)^(3/2)/(a - c*x^2)^2,x)
```

output
```
2*atanh((2*c*e^6*(d + e*x)^(1/2)*(d^3/(16*a^3*c) - (3*d*e^2)/(64*a^2*c^2)
- (e^3*(a^9*c^5)^(1/2))/(64*a^6*c^5))^(1/2))/((d*e^7)/(2*a) - (c*d^3*e^5)/
(2*a^2) + (e^8*(a^9*c^5)^(1/2))/(4*a^5*c^3) - (d^2*e^6*(a^9*c^5)^(1/2))/(4
*a^6*c^2)) + (2*d*e^5*(a^9*c^5)^(1/2)*(d + e*x)^(1/2)*(d^3/(16*a^3*c) - (3
*d*e^2)/(64*a^2*c^2) - (e^3*(a^9*c^5)^(1/2))/(64*a^6*c^5))^(1/2))/((e^8*(a
^9*c^5)^(1/2))/(4*c^2) - (a^3*c^2*d^3*e^5)/2 + (a^4*c*d*e^7)/2 - (d^2*e^6*
(a^9*c^5)^(1/2))/(4*a*c)))*(-(e^3*(a^9*c^5)^(1/2) - 4*a^3*c^4*d^3 + 3*a^4*
c^3*d*e^2)/(64*a^6*c^5))^(1/2) - (((a*e^3 - c*d^2*e)*(d + e*x)^(1/2))/(2*a
*c) + (d*e*(d + e*x)^(3/2))/(2*a))/(c*(d + e*x)^2 - a*e^2 + c*d^2 - 2*c*d*
(d + e*x)) + 2*atanh((2*c*e^6*(d + e*x)^(1/2)*(d^3/(16*a^3*c) - (3*d*e^2)/
(64*a^2*c^2) + (e^3*(a^9*c^5)^(1/2))/(64*a^6*c^5))^(1/2))/((d*e^7)/(2*a) -
 (c*d^3*e^5)/(2*a^2) - (e^8*(a^9*c^5)^(1/2))/(4*a^5*c^3) + (d^2*e^6*(a^9*c
^5)^(1/2))/(4*a^6*c^2)) + (2*d*e^5*(a^9*c^5)^(1/2)*(d + e*x)^(1/2)*(d^3/(1
6*a^3*c) - (3*d*e^2)/(64*a^2*c^2) + (e^3*(a^9*c^5)^(1/2))/(64*a^6*c^5))^(1
/2))/((e^8*(a^9*c^5)^(1/2))/(4*c^2) + (a^3*c^2*d^3*e^5)/2 - (a^4*c*d*e^7)/
2 - (d^2*e^6*(a^9*c^5)^(1/2))/(4*a*c)))*((e^3*(a^9*c^5)^(1/2) + 4*a^3*c^4*
d^3 - 3*a^4*c^3*d*e^2)/(64*a^6*c^5))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \frac{-4\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)acd+4\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+\cdots}}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{\cdots}}}\right)\cdots}{\cdots}$$

input
```
int((e*x+d)^(3/2)/(-c*x^2+a)^2,x)
```

output
```
( - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e
 - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c*
*2*d*x**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)
/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*e + 2*sqrt(c)*sqrt(sqrt(c)*
sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
c*d)))*a*c*e*x**2 - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sq
rt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*d + 2*sqrt(a)*sqrt(sqr
t(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(
d + e*x))*c**2*d*x**2 + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(s
qrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*d - 2*sqrt(a)*sqrt(sq
rt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d
+ e*x))*c**2*d*x**2 + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sq
rt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*e - sqrt(c)*sqrt(sqrt
(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d
 + e*x))*a*c*e*x**2 - sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(
c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*e + sqrt(c)*sqrt(sqrt(c)
*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x
))*a*c*e*x**2 + 4*sqrt(d + e*x)*a**2*c*e + 4*sqrt(d + e*x)*a*c**2*d*x)/(8*
a**2*c**2*(a - c*x**2))
```

3.144 $\int \frac{\sqrt{d+ex}}{(a-cx^2)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2} \, dx = \frac{x\sqrt{d+ex}}{2a(a-cx^2)} - \frac{\left(\frac{2\sqrt{c}d}{\sqrt{a}}-e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4ac^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}} + \frac{\left(\frac{2\sqrt{c}d}{\sqrt{a}}+e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4ac^{3/4}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output
```
1/2*x*(e*x+d)^(1/2)/a/(-c*x^2+a)-1/4*(2*c^(1/2)*d/a^(1/2)-e)*arctanh(c^(1/
4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a/c^(3/4)/(c^(1/2)*d-a^(1/2)
*e)^(1/2)+1/4*(2*c^(1/2)*d/a^(1/2)+e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/
2)*d+a^(1/2)*e)^(1/2))/a/c^(3/4)/(c^(1/2)*d+a^(1/2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}x\sqrt{d+ex}}{a-cx^2}+\frac{(2\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{c}\sqrt{-cd-\sqrt{a}\sqrt{c}e}}-\frac{(2\sqrt{c}d-\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{c}\sqrt{-cd+\sqrt{a}\sqrt{c}e}}}{4a^{3/2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a - c*x^2)^2,x]
```

output
```
((2*Sqrt[a]*x*Sqrt[d + e*x])/(a - c*x^2) + ((2*Sqrt[c]*d + Sqrt[a]*e)*ArcT
an[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e
)])/(Sqrt[c]*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]) - ((2*Sqrt[c]*d - Sqrt[a]*e
)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqr
t[a]*e)])/(Sqrt[c]*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {494, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx$$

$$\Big\downarrow \text{494}$$

$$\frac{x\sqrt{d+ex}}{2a\,(a-cx^2)}-\frac{\int -\frac{2d+ex}{2\sqrt{d+ex}(a-cx^2)}dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2d+ex}{\sqrt{d+ex}(a-cx^2)}dx}{4a}+\frac{x\sqrt{d+ex}}{2a\,(a-cx^2)}$$

$$\Big\downarrow \color{blue}{654}$$

$$\frac{\int -\frac{e(2d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}+\frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)}-\frac{\int \frac{e(2d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)}-\frac{e\int \frac{2d+ex}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}$$

$$\Big\downarrow \color{blue}{1480}$$

$$\frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)}-\frac{e\left(\frac{1}{2}\left(1-\frac{2\sqrt{c}d}{\sqrt{a}e}\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}+\frac{1}{2}\left(\frac{2\sqrt{c}d}{\sqrt{a}e}+1\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}\right)}{2a}$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{x\sqrt{d+ex}}{2a\left(a-cx^2\right)}-\frac{e\left(-\frac{\left(1-\frac{2\sqrt{c}d}{\sqrt{a}e}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\left(\frac{2\sqrt{c}d}{\sqrt{a}e}+1\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2c^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2a}$$

input
```
Int[Sqrt[d + e*x]/(a - c*x^2)^2,x]
```

output
```
(x*Sqrt[d + e*x])/(2*a*(a - c*x^2)) - (e*(-1/2*((1 - (2*Sqrt[c]*d)/(Sqrt[a
]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(c^(3/
4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) - ((1 + (2*Sqrt[c]*d)/(Sqrt[a]*e))*ArcTanh
[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*c^(3/4)*Sqrt[Sqr
t[c]*d + Sqrt[a]*e])))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{e c^2 \left(\frac{\frac{\sqrt{a c\, e^2}\,\sqrt{e x+d}}{-c e x+\sqrt{a c\, e^2}} + \frac{\left(\sqrt{a c\, e^2}+2 c d\right) \operatorname{arctanh}\left(\frac{c \sqrt{e x+d}}{\sqrt{\left(c d+\sqrt{a c\, e^2}\right) c}}\right)}{\sqrt{\left(c d+\sqrt{a c\, e^2}\right) c}}}{2 c^2 \sqrt{a c\, e^2}} - \frac{\sqrt{e x+d}}{2 c^2 \left(c e x+\sqrt{a c\, e^2}\right)} + \frac{\arctan\left(\frac{c \sqrt{e x+d}}{\sqrt{\left(-c d+\sqrt{a c\, e^2}\right) c}}\right) d}{\sqrt{\left(-c d+\sqrt{a c\, e^2}\right) c}\, c \sqrt{a c\, e^2}} - \cdots \right.}{2a}$
derivativedivides	$2 e^3 c^2 \left(\frac{\frac{\sqrt{a c\, e^2}\,\sqrt{e x+d}}{2 c^2 \left(-e x+\frac{\sqrt{a c\, e^2}}{c}\right)} + \frac{\left(\sqrt{a c\, e^2}+2 c d\right) \operatorname{arctanh}\left(\frac{c \sqrt{e x+d}}{\sqrt{\left(c d+\sqrt{a c\, e^2}\right) c}}\right)}{2 c \sqrt{\left(c d+\sqrt{a c\, e^2}\right) c}}}{4 c a\, e^2 \sqrt{a c\, e^2}} + \frac{\frac{\sqrt{a c\, e^2}\,\sqrt{e x+d}}{2 c^2 \left(-e x-\frac{\sqrt{a c\, e^2}}{c}\right)} - \frac{\left(\sqrt{a c\, e^2}-2 c d\right) \arctan\left(\cdots\right)}{2 c \sqrt{\left(-c d+\cdots\right)}}}{4 c a\, e^2 \sqrt{a c\, e^2}} \cdots \right.$
default	$2 e^3 c^2 \left(\frac{\frac{\sqrt{a c\, e^2}\,\sqrt{e x+d}}{2 c^2 \left(-e x+\frac{\sqrt{a c\, e^2}}{c}\right)} + \frac{\left(\sqrt{a c\, e^2}+2 c d\right) \operatorname{arctanh}\left(\frac{c \sqrt{e x+d}}{\sqrt{\left(c d+\sqrt{a c\, e^2}\right) c}}\right)}{2 c \sqrt{\left(c d+\sqrt{a c\, e^2}\right) c}}}{4 c a\, e^2 \sqrt{a c\, e^2}} + \frac{\frac{\sqrt{a c\, e^2}\,\sqrt{e x+d}}{2 c^2 \left(-e x-\frac{\sqrt{a c\, e^2}}{c}\right)} - \frac{\left(\sqrt{a c\, e^2}-2 c d\right) \arctan\left(\cdots\right)}{2 c \sqrt{\left(-c d+\cdots\right)}}}{4 c a\, e^2 \sqrt{a c\, e^2}} \cdots \right.$

input

```
int((e*x+d)^(1/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*e*c^2/a*(1/2/c^2/(a*c*e^2)^(1/2)*((a*c*e^2)^(1/2)*(e*x+d)^(1/2)/(-c*e*
x+(a*c*e^2)^(1/2))+((a*c*e^2)^(1/2)+2*c*d)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)
*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)))-1/2/c^2*(e*x+d)
^(1/2)/(c*e*x+(a*c*e^2)^(1/2))+1/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c
*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d/c/(a*c*e^2)^(1/2)-1/2/c
^2/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2
)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. $2(143) = 286$.

Time = 0.12 (sec) , antiderivative size = 1385, normalized size of antiderivative = 7.14

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((a*c*x^2 - a^2)*sqrt((4*c*d^3 - 3*a*d*e^2 + (a^3*c^2*d^2 - a^4*c*e^2
)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2
- a^4*c*e^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x + d) + (a^2*c*d*e^4 - (2
*a^3*c^4*d^4 - 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(e^6/(a^3*c^5*d^4 - 2*
a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt((4*c*d^3 - 3*a*d*e^2 + (a^3*c^2*d^2
- a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a
^3*c^2*d^2 - a^4*c*e^2))) - (a*c*x^2 - a^2)*sqrt((4*c*d^3 - 3*a*d*e^2 + (a
^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^
3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))*log(-(4*c*d^2*e^3 - a*e^5)*sqrt(e*x +
d) - (a^2*c*d*e^4 - (2*a^3*c^4*d^4 - 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt
(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt((4*c*d^3 - 3*a
*d*e^2 + (a^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e
^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))) + (a*c*x^2 - a^2)*sqrt((4*
c*d^3 - 3*a*d*e^2 - (a^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^
4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))*log(-(4*c*d^2*e^
3 - a*e^5)*sqrt(e*x + d) + (a^2*c*d*e^4 + (2*a^3*c^4*d^4 - 3*a^4*c^3*d^2*e
^2 + a^5*c^2*e^4)*sqrt(e^6/(a^3*c^5*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)
))*sqrt((4*c*d^3 - 3*a*d*e^2 - (a^3*c^2*d^2 - a^4*c*e^2)*sqrt(e^6/(a^3*c^5
*d^4 - 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 - a^4*c*e^2))) - (a
*c*x^2 - a^2)*sqrt((4*c*d^3 - 3*a*d*e^2 - (a^3*c^2*d^2 - a^4*c*e^2)*sqr...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2-a)^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 - a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(143) = 286$.

Time = 0.19 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx$$
$$= -\frac{(ex+d)^{\frac{3}{2}}e - \sqrt{ex+d}de}{2\left((ex+d)^2c - 2(ex+d)cd + cd^2 - ae^2\right)a}$$
$$+ \frac{\left(2\,acd^2e|c| - a^2e^3|c| - \sqrt{ac}de|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{acd+\sqrt{a^2c^2d^2-\left(acd^2-a^2e^2\right)ac}}{ac}}}\right)}{4\left(a^2ce - \sqrt{ac}acd\right)\sqrt{-c^2d - \sqrt{ac}ce}|a||e|}$$
$$+ \frac{\left(acde|a||c||e| + 2\sqrt{ac}acd^2e|c| - \sqrt{ac}a^2e^3|c|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{acd-\sqrt{a^2c^2d^2-\left(acd^2-a^2e^2\right)ac}}{ac}}}\right)}{4\left(a^2c^2d + \sqrt{ac}a^2ce\right)\sqrt{-c^2d + \sqrt{ac}ce}|a||e|}$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*((e*x + d)^(3/2)*e - sqrt(e*x + d)*d*e)/((((e*x + d)^2*c - 2*(e*x + d)
*c*d + c*d^2 - a*e^2)*a) + 1/4*(2*a*c*d^2*e*abs(c) - a^2*e^3*abs(c) - sqrt
(a*c)*d*e*abs(a)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c*d + sqrt(a
^2*c^2*d^2 - (a*c*d^2 - a^2*e^2)*a*c))/(a*c)))/((a^2*c*e - sqrt(a*c)*a*c*d
)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(a)*abs(e)) + 1/4*(a*c*d*e*abs(a)*abs(c)
*abs(e) + 2*sqrt(a*c)*a*c*d^2*e*abs(c) - sqrt(a*c)*a^2*e^3*abs(c))*arctan(
sqrt(e*x + d)/sqrt(-(a*c*d - sqrt(a^2*c^2*d^2 - (a*c*d^2 - a^2*e^2)*a*c))/
(a*c)))/((a^2*c^2*d + sqrt(a*c)*a^2*c*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(
a)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 2332, normalized size of antiderivative = 12.02

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(a - c*x^2)^2,x)
```

output
```
- atan((((8*c^3*d*e^3 - 64*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(e^3*(a^9*c^3)^(1
/2) - 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^7*c^3*e^2)))^(
1/2))*(-(e^3*(a^9*c^3)^(1/2) - 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c
^4*d^2 - a^7*c^3*e^2)))^(1/2) + ((a*c^2*e^4 + 4*c^3*d^2*e^2)*(d + e*x)^(1/
2))/a^2)*(-(e^3*(a^9*c^3)^(1/2) - 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^
6*c^4*d^2 - a^7*c^3*e^2)))^(1/2)*1i - ((8*c^3*d*e^3 + 64*a*c^4*d*e^2*(d +
e*x)^(1/2)*(-(e^3*(a^9*c^3)^(1/2) - 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(
a^6*c^4*d^2 - a^7*c^3*e^2)))^(1/2))*(-(e^3*(a^9*c^3)^(1/2) - 4*a^3*c^3*d^3
 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^7*c^3*e^2)))^(1/2) - ((a*c^2*e^4
+ 4*c^3*d^2*e^2)*(d + e*x)^(1/2))/a^2)*(-(e^3*(a^9*c^3)^(1/2) - 4*a^3*c^3*
d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^7*c^3*e^2)))^(1/2)*1i)/(((8*c^
3*d*e^3 - 64*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(e^3*(a^9*c^3)^(1/2) - 4*a^3*c^
3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^7*c^3*e^2)))^(1/2))*(-(e^3*(
a^9*c^3)^(1/2) - 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^7*c
^3*e^2)))^(1/2) + ((a*c^2*e^4 + 4*c^3*d^2*e^2)*(d + e*x)^(1/2))/a^2)*(-(e^
3*(a^9*c^3)^(1/2) - 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^
7*c^3*e^2)))^(1/2) - (4*c^2*d^2*e^3 - a*c*e^5)/(4*a^3) + ((8*c^3*d*e^3 + 6
4*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(e^3*(a^9*c^3)^(1/2) - 4*a^3*c^3*d^3 + 3*a
^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^7*c^3*e^2)))^(1/2))*(-(e^3*(a^9*c^3)^(1
/2) - 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 - a^7*c^3*e^2))...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 908, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)/(-c*x^2+a)^2,x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*e**2 + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*a*c*d**2 + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*e**2*x**2 - 4*sqrt(a)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)))*c**2*d**2*x**2 + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*d*e - 2
*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqr
t(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d*e*x**2 - sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e
 + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2
*e**2 + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a
)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*d**2 + sqrt(a)*sqrt(sqrt(c)*sqrt(a
)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a
*c*e**2*x**2 - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)
*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c**2*d**2*x**2 + sqrt(a)*sqrt(s
qrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d
 + e*x))*a**2*e**2 - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt
(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*d**2 - sqrt(a)*sqrt(sqrt
(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d...
```

3.145 $\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 222

$$\int \frac{1}{\sqrt{d+ex}\left(a-cx^2\right)^2}\,dx = -\frac{(ae-cdx)\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)\left(a-cx^2\right)} - \frac{\left(2\sqrt{c}d-3\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} + \frac{\left(2\sqrt{c}d+3\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

output
```
-1/2*(-c*d*x+a*e)*(e*x+d)^(1/2)/a/(-a*e^2+c*d^2)/(-c*x^2+a)-1/4*(2*c^(1/2)
*d-3*a^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))
/a^(3/2)/c^(1/4)/(c^(1/2)*d-a^(1/2)*e)^(3/2)+1/4*(2*c^(1/2)*d+3*a^(1/2)*e)
*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/c^(1/4
)/(c^(1/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx$$
$$= \frac{-\frac{2\sqrt{a}(-ae+cdx)\sqrt{d+ex}}{(-cd^2+ae^2)(a-cx^2)} + \frac{(2\sqrt{c}d+3\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{(\sqrt{c}d+\sqrt{a}e)\sqrt{-cd-\sqrt{a}\sqrt{c}e}} - \frac{(2\sqrt{c}d-3\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{(\sqrt{c}d-\sqrt{a}e)\sqrt{-cd+\sqrt{a}\sqrt{c}e}}}{4a^{3/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a - c*x^2)^2),x]
```

output
```
((-2*Sqrt[a]*(-(a*e) + c*d*x)*Sqrt[d + e*x])/((-(c*d^2) + a*e^2)*(a - c*x^
2)) + ((2*Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e
]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/((Sqrt[c]*d + Sqrt[a]*e)*Sqrt[-
(c*d) - Sqrt[a]*Sqrt[c]*e]) - ((2*Sqrt[c]*d - 3*Sqrt[a]*e)*ArcTan[(Sqrt[-(
c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/((Sqrt[
c]*d - Sqrt[a]*e)*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {496, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^2)^2\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \ 496$$
$$\frac{\int \frac{2cd^2+cexd-3ae^2}{2\sqrt{d+ex}(a-cx^2)}dx}{2a(cd^2-ae^2)} - \frac{\sqrt{d+ex}(ae-cdx)}{2a(a-cx^2)(cd^2-ae^2)}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2cd^2+cexd-3ae^2}{\sqrt{d+ex}(a-cx^2)}dx}{4a\left(cd^2-ae^2\right)}-\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 654

$$\frac{\int -\frac{e\left(cd^2+c(d+ex)d-3ae^2\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)}-\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 25

$$-\frac{\int \frac{e\left(cd^2+c(d+ex)d-3ae^2\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)}-\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 27

$$-\frac{e\int \frac{cd^2+c(d+ex)d-3ae^2}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)}-\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 1480

$$-\frac{e\left(\frac{1}{2}\sqrt{c}\left(\sqrt{c}d-\frac{2cd^2-3ae^2}{\sqrt{a}e}\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}+\frac{1}{2}\sqrt{c}\left(\frac{2cd^2-3ae^2}{\sqrt{a}e}+\sqrt{c}d\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\surd}{2a\left(cd^2-ae^2\right)}$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 221

$$-\frac{e\left(-\frac{\left(\sqrt{c}d-\frac{2cd^2-3ae^2}{\sqrt{a}e}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\left(\frac{2cd^2-3ae^2}{\sqrt{a}e}+\sqrt{c}d\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2a\left(cd^2-ae^2\right)}-$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

input
```
Int[1/(Sqrt[d + e*x]*(a - c*x^2)^2),x]
```

output
```
-1/2*((a*e - c*d*x)*Sqrt[d + e*x])/(a*(c*d^2 - a*e^2)*(a - c*x^2)) - (e*(-
1/2*((Sqrt[c]*d - (2*c*d^2 - 3*a*e^2)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e
]) - ((Sqrt[c]*d + (2*c*d^2 - 3*a*e^2)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[
d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a
]*e])))/(2*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{e c^2 \left(\frac{\frac{\sqrt{a c e^2}\,\sqrt{ex+d}}{-cex+\sqrt{a c e^2}} + \frac{\left(2cd+3\sqrt{a c e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}\right)}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}}{2c\sqrt{a c e^2}\left(cd+\sqrt{a c e^2}\right)} + \frac{\sqrt{ex+d}}{2c\left(-cd+\sqrt{a c e^2}\right)\left(cex+\sqrt{a c e^2}\right)} - \frac{\arctan\left(\frac{c\sqrt{\cdots}}{\sqrt{\left(-cd+\cdots\right.}}\right.}{\left(-cd+\sqrt{a c e^2}\right)\sqrt{\left(-cd\cdots\right.}} \cdots}{2a}$
derivativedivides	$2e^3c^2\left(\frac{\frac{\sqrt{a c e^2}\,\sqrt{ex+d}}{2c\left(cd+\sqrt{a c e^2}\right)\left(-ex+\frac{\sqrt{a c e^2}}{c}\right)} + \frac{\left(2cd+3\sqrt{a c e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}\right)}{2\left(cd+\sqrt{a c e^2}\right)\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}}{4\sqrt{a c e^2}\,a e^2 c} + \frac{\frac{\sqrt{a c e^2}\,\sqrt{ex+d}}{2c\left(cd-\sqrt{a c e^2}\right)\left(-ex-\frac{\sqrt{a c e^2}}{c}\right.}\cdots}{\cdots} \right.$
default	$2e^3c^2\left(\frac{\frac{\sqrt{a c e^2}\,\sqrt{ex+d}}{2c\left(cd+\sqrt{a c e^2}\right)\left(-ex+\frac{\sqrt{a c e^2}}{c}\right)} + \frac{\left(2cd+3\sqrt{a c e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}\right)}{2\left(cd+\sqrt{a c e^2}\right)\sqrt{\left(cd+\sqrt{a c e^2}\right)c}}}{4\sqrt{a c e^2}\,a e^2 c} + \frac{\frac{\sqrt{a c e^2}\,\sqrt{ex+d}}{2c\left(cd-\sqrt{a c e^2}\right)\left(-ex-\frac{\sqrt{a c e^2}}{c}\right.}\cdots}{\cdots} \right.$

input
```
int(1/(e*x+d)^(1/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*e*c^2/a*(1/2/c/(a*c*e^2)^(1/2)/(c*d+(a*c*e^2)^(1/2))*((a*c*e^2)^(1/2)*
(e*x+d)^(1/2)/(-c*e*x+(a*c*e^2)^(1/2))+(2*c*d+3*(a*c*e^2)^(1/2))/((c*d+(a*
c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(
1/2)))+1/2*(e*x+d)^(1/2)/c/(-c*d+(a*c*e^2)^(1/2))/(c*e*x+(a*c*e^2)^(1/2))-
1/(-c*d+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)
^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))*d/(a*c*e^2)^(1/2)+3/2/(-c*d+(a*c*
e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d
+(a*c*e^2)^(1/2))*c)^(1/2))/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3279 vs. $2(167) = 334$.

Time = 0.36 (sec) , antiderivative size = 3279, normalized size of antiderivative = 14.77

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**(1/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \int \frac{1}{(cx^2-a)^2\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 - a)^2*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(167) = 334$.

Time = 0.19 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.93

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*((a*c*d^2*e - a^2*e^3)^2*c*d*e*abs(c) + (sqrt(a*c)*c^2*d^4*e - 4*sqrt(
a*c)*a*c*d^2*e^3 + 3*sqrt(a*c)*a^2*e^5)*abs(-a*c*d^2*e + a^2*e^3)*abs(c) -
 (2*a*c^4*d^7*e - 7*a^2*c^3*d^5*e^3 + 8*a^3*c^2*d^3*e^5 - 3*a^4*c*d*e^7)*a
bs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d^3 - a^2*c*d*e^2 + sqrt((a*c^2*d
^3 - a^2*c*d*e^2)^2 - (a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)*(a*c^2*d^2 -
 a^2*c*e^2)))/(a*c^2*d^2 - a^2*c*e^2)))/((a^2*c^3*d^4*e - 2*a^3*c^2*d^2*e^
3 + a^4*c*e^5 + sqrt(a*c)*a*c^3*d^5 - 2*sqrt(a*c)*a^2*c^2*d^3*e^2 + sqrt(a
*c)*a^3*c*d*e^4)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(-a*c*d^2*e + a^2*e^3)) -
 1/4*((a*c*d^2*e - a^2*e^3)^2*sqrt(a*c)*d*e*abs(c) - (a*c^2*d^4*e - 4*a^2*
c*d^2*e^3 + 3*a^3*e^5)*abs(-a*c*d^2*e + a^2*e^3)*abs(c) - (2*sqrt(a*c)*a*c
^3*d^7*e - 7*sqrt(a*c)*a^2*c^2*d^5*e^3 + 8*sqrt(a*c)*a^3*c*d^3*e^5 - 3*sqr
t(a*c)*a^4*d*e^7)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d^3 - a^2*c*d*
e^2 - sqrt((a*c^2*d^3 - a^2*c*d*e^2)^2 - (a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^
3*e^4)*(a*c^2*d^2 - a^2*c*e^2)))/(a*c^2*d^2 - a^2*c*e^2)))/((a^2*c^3*d^5 -
 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4 - sqrt(a*c)*a^2*c^2*d^4*e + 2*sqrt(a*c)*a
^3*c*d^2*e^3 - sqrt(a*c)*a^4*e^5)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(-a*c*d^
2*e + a^2*e^3)) - 1/2*((e*x + d)^(3/2)*c*d*e - sqrt(e*x + d)*c*d^2*e - sqr
t(e*x + d)*a*e^3)/((a*c*d^2 - a^2*e^2)*((e*x + d)^2*c - 2*(e*x + d)*c*d +
c*d^2 - a*e^2))
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 5253, normalized size of antiderivative = 23.66

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a - c*x^2)^2*(d + e*x)^(1/2)),x)
```

output
```
atan(((((192*a^5*c^3*e^7 + 64*a^3*c^5*d^4*e^3 - 256*a^4*c^4*d^2*e^5)/(8*(a
^5*e^4 + a^3*c^2*d^4 - 2*a^4*c*d^2*e^2)) + ((d + e*x)^(1/2)*(64*a^5*c^4*d*
e^6 + 64*a^3*c^6*d^5*e^2 - 128*a^4*c^5*d^3*e^4)*(-(4*a^3*c^3*d^5 - 9*a*e^5
*(a^9*c)^(1/2) + 5*c*d^2*e^3*(a^9*c)^(1/2) - 15*a^4*c^2*d^3*e^2 + 15*a^5*c
*d*e^4)/(64*(a^9*c*e^6 - a^6*c^4*d^6 + 3*a^7*c^3*d^4*e^2 - 3*a^8*c^2*d^2*e
^4)))^(1/2))/(a^4*e^4 + a^2*c^2*d^4 - 2*a^3*c*d^2*e^2))*(-(4*a^3*c^3*d^5 -
 9*a*e^5*(a^9*c)^(1/2) + 5*c*d^2*e^3*(a^9*c)^(1/2) - 15*a^4*c^2*d^3*e^2 +
15*a^5*c*d*e^4)/(64*(a^9*c*e^6 - a^6*c^4*d^6 + 3*a^7*c^3*d^4*e^2 - 3*a^8*c
^2*d^2*e^4)))^(1/2) - ((d + e*x)^(1/2)*(9*a^2*c^3*e^6 + 4*c^5*d^4*e^2 - 11
*a*c^4*d^2*e^4))/(a^4*e^4 + a^2*c^2*d^4 - 2*a^3*c*d^2*e^2))*(-(4*a^3*c^3*d
^5 - 9*a*e^5*(a^9*c)^(1/2) + 5*c*d^2*e^3*(a^9*c)^(1/2) - 15*a^4*c^2*d^3*e^
2 + 15*a^5*c*d*e^4)/(64*(a^9*c*e^6 - a^6*c^4*d^6 + 3*a^7*c^3*d^4*e^2 - 3*a
^8*c^2*d^2*e^4)))^(1/2)*1i - (((192*a^5*c^3*e^7 + 64*a^3*c^5*d^4*e^3 - 256
*a^4*c^4*d^2*e^5)/(8*(a^5*e^4 + a^3*c^2*d^4 - 2*a^4*c*d^2*e^2)) - ((d + e*
x)^(1/2)*(64*a^5*c^4*d*e^6 + 64*a^3*c^6*d^5*e^2 - 128*a^4*c^5*d^3*e^4)*(-(
4*a^3*c^3*d^5 - 9*a*e^5*(a^9*c)^(1/2) + 5*c*d^2*e^3*(a^9*c)^(1/2) - 15*a^4
*c^2*d^3*e^2 + 15*a^5*c*d*e^4)/(64*(a^9*c*e^6 - a^6*c^4*d^6 + 3*a^7*c^3*d^
4*e^2 - 3*a^8*c^2*d^2*e^4)))^(1/2))/(a^4*e^4 + a^2*c^2*d^4 - 2*a^3*c*d^2*e
^2))*(-(4*a^3*c^3*d^5 - 9*a*e^5*(a^9*c)^(1/2) + 5*c*d^2*e^3*(a^9*c)^(1/2)
- 15*a^4*c^2*d^3*e^2 + 15*a^5*c*d*e^4)/(64*(a^9*c*e^6 - a^6*c^4*d^6 + 3...
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1305, normalized size of antiderivative = 5.88

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(-c*x^2+a)^2,x)
```

output
```
(8*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d*e**2 - 4*sqrt(a)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)
))*a*c**2*d**3 - 8*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*
x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d*e**2*x**2 + 4*sqrt
(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqr
t(c)*sqrt(a)*e - c*d)))*c**3*d**3*x**2 + 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
- c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**
3*e**3 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(s
qrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d**2*e - 6*sqrt(c)*sqrt(sqrt
(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)))*a**2*c*e**3*x**2 + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan
((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d**2*e*
x**2 + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)
*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*c*d*e**2 - 2*sqrt(a)*sqrt(sqrt(c)*
sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e
*x))*a*c**2*d**3 - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqr
t(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c**2*d*e**2*x**2 + 2*sqrt
(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + s
qrt(c)*sqrt(d + e*x))*c**3*d**3*x**2 - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e...
```

3.146 $\int \frac{1}{(d+ex)^{3/2}(a-cx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 265

$$\int \frac{1}{(d+ex)^{3/2}(a-cx^2)^2}\,dx = -\frac{e(cd^2+5ae^2)}{2a(cd^2-ae^2)^2\sqrt{d+ex}} - \frac{ae-cdx}{2a(cd^2-ae^2)\sqrt{d+ex}(a-cx^2)} - \frac{\sqrt[4]{c}(2\sqrt{c}d-5\sqrt{a}e)\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}} + \frac{\sqrt[4]{c}(2\sqrt{c}d+5\sqrt{a}e)\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}$$

output
```
-1/2*e*(5*a*e^2+c*d^2)/a/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)-1/2*(-c*d*x+a*e)/a
/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x^2+a)-1/4*c^(1/4)*(2*c^(1/2)*d-5*a^(1/2
)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3/2)/(c
^(1/2)*d-a^(1/2)*e)^(5/2)+1/4*c^(1/4)*(2*c^(1/2)*d+5*a^(1/2)*e)*arctanh(c^
(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/(c^(1/2)*d+a^(1/2
)*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx = \frac{\frac{2\sqrt{a}(4a^2e^3-c^2d^2x(d+ex)+ace(2d^2+dex-5e^2x^2))}{(cd^2-ae^2)^2\sqrt{d+ex}(-a+cx^2)} - \frac{(2\sqrt{c}d+5\sqrt{a}e)\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{(\sqrt{c}d+\sqrt{a}e)^3}}{4a^{3/2}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a - c*x^2)^2),x]
```

output
```
((2*Sqrt[a]*(4*a^2*e^3 - c^2*d^2*x*(d + e*x) + a*c*e*(2*d^2 + d*e*x - 5*e^
2*x^2)))/((c*d^2 - a*e^2)^2*Sqrt[d + e*x]*(-a + c*x^2)) - ((2*Sqrt[c]*d +
5*Sqrt[a]*e)*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - Sqrt[a
]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/(Sqrt[c]*d + Sqrt[a]
*e)^3 + ((2*Sqrt[c]*d - 5*Sqrt[a]*e)*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*ArcT
an[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e
)])/(Sqrt[c]*d - Sqrt[a]*e)^3)/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {496, 27, 655, 25, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-cx^2\right)^2(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{\int \frac{2cd^2+3cexd-5ae^2}{2(d+ex)^{3/2}(a-cx^2)}dx}{2a\left(cd^2-ae^2\right)} - \frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2cd^2+3cexd-5ae^2}{(d+ex)^{3/2}(a-cx^2)}dx}{4a\left(cd^2-ae^2\right)} - \frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 655$$

$$\frac{-\frac{\int -\frac{c\left(2d\left(cd^2-4ae^2\right)+e\left(cd^2+5ae^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(5ae^2+cd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c\left(2d\left(cd^2-4ae^2\right)+e\left(cd^2+5ae^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(5ae^2+cd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{c\int \frac{2d\left(cd^2-4ae^2\right)+e\left(cd^2+5ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(5ae^2+cd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 654$$

$$\frac{\frac{2c\int -\frac{e\left(d\left(cd^2-13ae^2\right)+\left(cd^2+5ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2e\left(5ae^2+cd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{2c\int \frac{e\left(d\left(cd^2-13ae^2\right)+\left(cd^2+5ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2e\left(5ae^2+cd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{2ce\int \frac{d\left(cd^2-13ae^2\right)+\left(cd^2+5ae^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2e\left(5ae^2+cd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{-\frac{2ce\left(\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(5\sqrt{a}e+2\sqrt{c}d\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\left(2\sqrt{c}d-5\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}\right)}{cd^2-ae^2}-\frac{2e\left(5ae^2+c\cdots\right.}{\sqrt{d+ex}(cd^2-\cdots}}{4a\left(cd^2-ae^2\right)}$$

$$\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{2ce\left(\frac{(2\sqrt{c}d-5\sqrt{a}e)(\sqrt{a}e+\sqrt{c}d)^2\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{(\sqrt{c}d-\sqrt{a}e)^2(5\sqrt{a}e+2\sqrt{c}d)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{cd^2-ae^2}-\frac{2e(5ae^2+cd^2)}{\sqrt{d+ex}(cd^2-ae^2)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^(3/2)*(a - c*x^2)^2),x]
```

output
```
-1/2*(a*e - c*d*x)/(a*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a - c*x^2)) + ((-2*e*
(c*d^2 + 5*a*e^2))/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*c*e*(((2*Sqrt[c]*d
 - 5*Sqrt[a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^2*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/
Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a
]*e]) - ((Sqrt[c]*d - Sqrt[a]*e)^2*(2*Sqrt[c]*d + 5*Sqrt[a]*e)*ArcTanh[(c^
(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e*Sq
rt[Sqrt[c]*d + Sqrt[a]*e])))/(c*d^2 - a*e^2))/(4*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.31

method	result
derivativedivides	$2e^3 \left(-\frac{c\left(\frac{-\frac{\left(a\,e^2+c\,d^2\right)(ex+d)^{\frac{3}{2}}}{4a\,e^2}+\frac{d\left(3a\,e^2+c\,d^2\right)\sqrt{ex+d}}{4a\,e^2}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{c\left(-\frac{\left(-8ad\,e^2c+2c^2d^3+5\sqrt{ac\,e^2}\,a\,e^2+\sqrt{ac\,e^2}\,c\,d^2\right)\operatorname{arctanh}\left(\right.}{2c\sqrt{ac\,e^2}\,\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right.}{}\right.}{\left(a\,e^2-c\,d^2\right)^2}\right.$
default	$2e^3 \left(-\frac{c\left(\frac{-\frac{\left(a\,e^2+c\,d^2\right)(ex+d)^{\frac{3}{2}}}{4a\,e^2}+\frac{d\left(3a\,e^2+c\,d^2\right)\sqrt{ex+d}}{4a\,e^2}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{c\left(-\frac{\left(-8ad\,e^2c+2c^2d^3+5\sqrt{ac\,e^2}\,a\,e^2+\sqrt{ac\,e^2}\,c\,d^2\right)\operatorname{arctanh}\left(\right.}{2c\sqrt{ac\,e^2}\,\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right.}{}\right.}{\left(a\,e^2-c\,d^2\right)^2}\right.$
pseudoelliptic	$-\frac{2\left(e\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\,\left(-c\,x^2+a\right)\left(\frac{\left(5a\,e^2+c\,d^2\right)\sqrt{ac\,e^2}}{8}+cd\left(a\,e^2-\frac{c\,d^2}{4}\right)\right)c\sqrt{ex+d}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)+\left(e\right.\right.}{}$

input
```
int(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(-c/(a*e^2-c*d^2)^2*((-1/4*(a*e^2+c*d^2)/a/e^2*(e*x+d)^(3/2)+1/4*d*(
3*a*e^2+c*d^2)/a/e^2*(e*x+d)^(1/2))/(-c*(e*x+d)^2+2*c*d*(e*x+d)+a*e^2-c*d^
2)+1/4/a/e^2*c*(-1/2*(-8*a*d*e^2*c+2*c^2*d^3+5*(a*c*e^2)^(1/2)*a*e^2+(a*c*
e^2)^(1/2)*c*d^2)/c/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan
h(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+1/2*(8*a*d*e^2*c-2*c^2*
d^3+5*(a*c*e^2)^(1/2)*a*e^2+(a*c*e^2)^(1/2)*c*d^2)/c/(a*c*e^2)^(1/2)/((-c*
d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))
*c)^(1/2))))-1/(a*e^2-c*d^2)^2/(e*x+d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5703 vs. $2(206) = 412$.

Time = 0.93 (sec) , antiderivative size = 5703, normalized size of antiderivative = 21.52

$$\int \frac{1}{(d+ex)^{3/2}(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**(3/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx = \int \frac{1}{\left(cx^2-a\right)^2(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 - a)^2*(e*x + d)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. 2(206) = 412.

Time = 0.30 (sec) , antiderivative size = 1411, normalized size of antiderivative = 5.32

$$\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*((a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)^2*(c*d^2*e + 5*a*e^3)*abs(
c) + (sqrt(a*c)*c^3*d^7*e - 15*sqrt(a*c)*a*c^2*d^5*e^3 + 27*sqrt(a*c)*a^2*
c*d^3*e^5 - 13*sqrt(a*c)*a^3*d*e^7)*abs(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^
3*e^5)*abs(c) - 2*(a*c^6*d^12*e - 8*a^2*c^5*d^10*e^3 + 22*a^3*c^4*d^8*e^5
- 28*a^4*c^3*d^6*e^7 + 17*a^5*c^2*d^4*e^9 - 4*a^6*c*d^2*e^11)*abs(c))*arct
an(sqrt(e*x + d)/sqrt(-(a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4 + sqrt
((a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)^2 - (a*c^3*d^6 - 3*a^2*c^2*
d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4)))/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/((a^2*c^4*d^8*e - 4
*a^3*c^3*d^6*e^3 + 6*a^4*c^2*d^4*e^5 - 4*a^5*c*d^2*e^7 + a^6*e^9 - sqrt(a*
c)*a*c^4*d^9 + 4*sqrt(a*c)*a^2*c^3*d^7*e^2 - 6*sqrt(a*c)*a^3*c^2*d^5*e^4 +
 4*sqrt(a*c)*a^4*c*d^3*e^6 - sqrt(a*c)*a^5*d*e^8)*sqrt(-c^2*d - sqrt(a*c)*
c*e)*abs(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)) - 1/4*((a*c^2*d^4*e - 2
*a^2*c*d^2*e^3 + a^3*e^5)^2*(sqrt(a*c)*c*d^2*e + 5*sqrt(a*c)*a*e^3)*abs(c)
 - (a*c^4*d^7*e - 15*a^2*c^3*d^5*e^3 + 27*a^3*c^2*d^3*e^5 - 13*a^4*c*d*e^7
)*abs(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)*abs(c) - 2*(sqrt(a*c)*a*c^6
*d^12*e - 8*sqrt(a*c)*a^2*c^5*d^10*e^3 + 22*sqrt(a*c)*a^3*c^4*d^8*e^5 - 28
*sqrt(a*c)*a^4*c^3*d^6*e^7 + 17*sqrt(a*c)*a^5*c^2*d^4*e^9 - 4*sqrt(a*c)*a^
6*c*d^2*e^11)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^3*d^5 - 2*a^2*c^2*d^
3*e^2 + a^3*c*d*e^4 - sqrt((a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4...
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 8700, normalized size of antiderivative = 32.83

$$\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a - c*x^2)^2*(d + e*x)^(3/2)),x)
```

output
```
atan((((-(4*a^3*c^4*d^7 - 25*a^2*e^7*(a^9*c)^(1/2) - 35*a^4*c^3*d^5*e^2 +
70*a^5*c^2*d^3*e^4 + 35*c^2*d^4*e^3*(a^9*c)^(1/2) + 105*a^6*c*d*e^6 - 154*
a*c*d^2*e^5*(a^9*c)^(1/2))/(64*(a^11*e^10 - a^6*c^5*d^10 - 5*a^10*c*d^2*e^
8 + 5*a^7*c^4*d^8*e^2 - 10*a^8*c^3*d^6*e^4 + 10*a^9*c^2*d^4*e^6)))^(1/2)*(
(d + e*x)^(1/2)*(-(4*a^3*c^4*d^7 - 25*a^2*e^7*(a^9*c)^(1/2) - 35*a^4*c^3*d
^5*e^2 + 70*a^5*c^2*d^3*e^4 + 35*c^2*d^4*e^3*(a^9*c)^(1/2) + 105*a^6*c*d*e
^6 - 154*a*c*d^2*e^5*(a^9*c)^(1/2))/(64*(a^11*e^10 - a^6*c^5*d^10 - 5*a^10
*c*d^2*e^8 + 5*a^7*c^4*d^8*e^2 - 10*a^8*c^3*d^6*e^4 + 10*a^9*c^2*d^4*e^6))
)^(1/2)*(2048*a^16*c^4*d*e^22 + 2048*a^6*c^14*d^21*e^2 - 20480*a^7*c^13*d^
19*e^4 + 92160*a^8*c^12*d^17*e^6 - 245760*a^9*c^11*d^15*e^8 + 430080*a^10*
c^10*d^13*e^10 - 516096*a^11*c^9*d^11*e^12 + 430080*a^12*c^8*d^9*e^14 - 24
5760*a^13*c^7*d^7*e^16 + 92160*a^14*c^6*d^5*e^18 - 20480*a^15*c^5*d^3*e^20
) - 3328*a^14*c^4*d*e^21 + 256*a^5*c^13*d^19*e^3 - 5376*a^6*c^12*d^17*e^5
+ 33792*a^7*c^11*d^15*e^7 - 107520*a^8*c^10*d^13*e^9 + 204288*a^9*c^9*d^11
*e^11 - 247296*a^10*c^8*d^9*e^13 + 193536*a^11*c^7*d^7*e^15 - 95232*a^12*c
^6*d^5*e^17 + 26880*a^13*c^5*d^3*e^19) - (d + e*x)^(1/2)*(800*a^12*c^4*e^2
0 + 128*a^3*c^13*d^18*e^2 - 1760*a^4*c^12*d^16*e^4 + 10240*a^5*c^11*d^14*e
^6 - 30848*a^6*c^10*d^12*e^8 + 52480*a^7*c^9*d^10*e^10 - 51008*a^8*c^8*d^8
*e^12 + 25600*a^9*c^7*d^6*e^14 - 3200*a^10*c^6*d^4*e^16 - 2432*a^11*c^5*d^
2*e^18))*(-(4*a^3*c^4*d^7 - 25*a^2*e^7*(a^9*c)^(1/2) - 35*a^4*c^3*d^5*e...
```

Reduce [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 1867, normalized size of antiderivative = 7.05

$$\int \frac{1}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(-c*x^2+a)^2,x)
```

output

```
( - 10*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d +
e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*e**4 - 18*sqrt(a)*sq
rt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d**2*e**2 + 10*sqrt(a)*sqrt(d + e*x
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(
c)*sqrt(a)*e - c*d)))*a**2*c*e**4*x**2 + 4*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt
(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)))*a*c**2*d**4 + 18*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*
*2*d**2*e**2*x**2 - 4*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*
atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**3*d**4*
x**2 - 26*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d
 + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*d*e**3 + 2*sqrt(c
)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt
(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d**3*e + 26*sqrt(c)*sqrt(d + e*
x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt
(c)*sqrt(a)*e - c*d)))*a**2*c*d*e**3*x**2 - 2*sqrt(c)*sqrt(d + e*x)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)))*a*c**2*d**3*e*x**2 - 5*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sq
rt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + ...
```

3.147 $\int \frac{1}{(d+ex)^{5/2}(a-cx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 311

$$\begin{aligned}\int \frac{1}{(d+ex)^{5/2}(a-cx^2)^2}\,dx &= -\frac{e(3cd^2+7ae^2)}{6a(cd^2-ae^2)^2(d+ex)^{3/2}} \\ &- \frac{cde(cd^2+19ae^2)}{2a(cd^2-ae^2)^3\sqrt{d+ex}} - \frac{ae-cdx}{2a(cd^2-ae^2)(d+ex)^{3/2}(a-cx^2)} \\ &- \frac{c^{3/4}(2\sqrt{c}d-7\sqrt{a}e)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)^{7/2}} \\ &+ \frac{c^{3/4}(2\sqrt{c}d+7\sqrt{a}e)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}\left(\sqrt{c}d+\sqrt{a}e\right)^{7/2}}\end{aligned}$$

output
```
-1/6*e*(7*a*e^2+3*c*d^2)/a/(-a*e^2+c*d^2)^2/(e*x+d)^(3/2)-1/2*c*d*e*(19*a*
e^2+c*d^2)/a/(-a*e^2+c*d^2)^3/(e*x+d)^(1/2)-1/2*(-c*d*x+a*e)/a/(-a*e^2+c*d
^2)/(e*x+d)^(3/2)/(-c*x^2+a)-1/4*c^(3/4)*(2*c^(1/2)*d-7*a^(1/2)*e)*arctanh
(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3/2)/(c^(1/2)*d-a^(
1/2)*e)^(7/2)+1/4*c^(3/4)*(2*c^(1/2)*d+7*a^(1/2)*e)*arctanh(c^(1/4)*(e*x+d
)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/(c^(1/2)*d+a^(1/2)*e)^(7/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.15

$$\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)^2}\,dx = \frac{-\frac{2\sqrt{a}\left(4a^3e^5+3c^3d^3x(d+ex)^2-a^2ce^3\left(55d^2+54dex+7e^2x^2\right)+ac^2de\left(-9d^3-9d^2ex+61de^2x^2+57e^3x^3\right)\right)}{\left(cd^2-ae^2\right)^3(d+ex)^{3/2}\left(-a+cx^2\right)}}{12}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(a - c*x^2)^2),x]
```

output
```
((-2*Sqrt[a]*(4*a^3*e^5 + 3*c^3*d^3*x*(d + e*x)^2 - a^2*c*e^3*(55*d^2 + 54
*d*e*x + 7*e^2*x^2) + a*c^2*d*e*(-9*d^3 - 9*d^2*e*x + 61*d*e^2*x^2 + 57*e^
3*x^3)))/((c*d^2 - a*e^2)^3*(d + e*x)^(3/2)*(-a + c*x^2)) + (3*c*(2*Sqrt[c
]*d + 7*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])
/(Sqrt[c]*d + Sqrt[a]*e)])/((Sqrt[c]*d + Sqrt[a]*e)^3*Sqrt[-(c*d) - Sqrt[a
]*Sqrt[c]*e]) - (3*c*(2*Sqrt[c]*d - 7*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d) + Sqr
t[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/((Sqrt[c]*d - Sqr
t[a]*e)^3*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/(12*a^(3/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {496, 27, 655, 25, 27, 655, 25, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-cx^2\right)^2(d+ex)^{5/2}}dx$$

$$\Big\downarrow \; 496$$

$$\frac{\int \frac{2cd^2+5cexd-7ae^2}{2(d+ex)^{5/2}\left(a-cx^2\right)}dx}{2a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2cd^2+5cexd-7ae^2}{(d+ex)^{5/2}(a-cx^2)}dx}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 655$$

$$\frac{-\frac{\int -\frac{c\left(2d\left(cd^2-6ae^2\right)+e\left(3cd^2+7ae^2\right)x\right)}{(d+ex)^{3/2}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c\left(2d\left(cd^2-6ae^2\right)+e\left(3cd^2+7ae^2\right)x\right)}{(d+ex)^{3/2}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{c\int \frac{2d\left(cd^2-6ae^2\right)+e\left(3cd^2+7ae^2\right)x}{(d+ex)^{3/2}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 655$$

$$\frac{\frac{c\left(-\frac{\int -\frac{2c^2d^4-15ace^2d^2+ce\left(cd^2+19ae^2\right)xd-7a^2e^4}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2de\left(19ae^2+cd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{c\left(\frac{\int \frac{2c^2d^4-15ace^2d^2+ce\left(cd^2+19ae^2\right)xd-7a^2e^4}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2de\left(19ae^2+cd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 654$$

$$\frac{\frac{c\left(\frac{2\int -\frac{e\left(c^2d^4-34ace^2d^2+c\left(cd^2+19ae^2\right)(d+ex)d-7a^2e^4\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2de\left(19ae^2+cd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 25$$

$$\frac{\frac{c\left(-\frac{2\int \frac{e\left(c^2d^4-34ace^2d^2+c\left(cd^2+19ae^2\right)(d+ex)d-7a^2e^4\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2de\left(19ae^2+cd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 27$$

$$\frac{\frac{c\left(-\frac{2e\int \frac{c^2d^4-34ace^2d^2+c\left(cd^2+19ae^2\right)(d+ex)d-7a^2e^4}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2de\left(19ae^2+cd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{cd^2-ae^2}-\frac{2e\left(7ae^2+3cd^2\right)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}-\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 1480$$

$$\frac{\frac{c\left(-\frac{2e\left(\frac{\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(7\sqrt{a}e+2\sqrt{c}d\right)\int\frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}\left(2\sqrt{c}d-7\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^3\int\frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}\right)}{cd^2-ae^2}-\frac{2de\left(19a\cdots\right.}{\sqrt{d+ex}\left(c\cdots\right.}\cdots\right.}{cd^2-ae^2}\cdots}{4a\left(cd^2-ae^2\right)}$$

$$\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 221$$

$$\frac{c\left(-\frac{2e\left(\frac{(2\sqrt{c}d-7\sqrt{a}e)(\sqrt{a}e+\sqrt{c}d)^3\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{(\sqrt{c}d-\sqrt{a}e)^3(7\sqrt{a}e+2\sqrt{c}d)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{cd^2-ae^2}-\frac{2de(19ae^2+cd^2)}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{cd^2-ae^2} - \frac{2e}{3(d+}$$

$$4a\left(cd^2-ae^2\right)$$

$$\frac{ae-cdx}{2a\left(a-cx^2\right)(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)^(5/2)*(a - c*x^2)^2),x]
```

output
```
-1/2*(a*e - c*d*x)/(a*(c*d^2 - a*e^2)*(d + e*x)^(3/2)*(a - c*x^2)) + ((-2*
e*(3*c*d^2 + 7*a*e^2))/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + (c*((-2*d*e*(
c*d^2 + 19*a*e^2))/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - (2*e*(((2*Sqrt[c]*d -
 7*Sqrt[a]*e)*(Sqrt[c]*d + Sqrt[a]*e)^3*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sq
rt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*
e]) - ((Sqrt[c]*d - Sqrt[a]*e)^3*(2*Sqrt[c]*d + 7*Sqrt[a]*e)*ArcTanh[(c^(1
/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e*Sqrt
[Sqrt[c]*d + Sqrt[a]*e])))/(c*d^2 - a*e^2)))/(c*d^2 - a*e^2))/(4*a*(c*d^2
 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.29

method	result
derivativedivides	$2e^{3}\left(\frac{c\left(\frac{-\frac{cd\left(3a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{3}{2}}}{4a\,e^{2}}+\frac{\left(a^{2}e^{4}+6ac\,d^{2}e^{2}+c^{2}d^{4}\right)\sqrt{ex+d}}{4a\,e^{2}}}{-c(ex+d)^{2}+2cd(ex+d)+a\,e^{2}-c\,d^{2}}+\frac{c\left(\frac{\left(7a^{2}e^{4}+15ac\,d^{2}e^{2}-2c^{2}d^{4}+19\sqrt{ac\,e^{2}}\,ad\,e^{2}+\sqrt{ac\,e} \cdots\right.}{2\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{a \cdots\right.}}\cdots\right.}{\cdots}\right)}{(c \cdots}\right.$
default	$2e^{3}\left(\frac{c\left(\frac{-\frac{cd\left(3a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{3}{2}}}{4a\,e^{2}}+\frac{\left(a^{2}e^{4}+6ac\,d^{2}e^{2}+c^{2}d^{4}\right)\sqrt{ex+d}}{4a\,e^{2}}}{-c(ex+d)^{2}+2cd(ex+d)+a\,e^{2}-c\,d^{2}}+\frac{c\left(\frac{\left(7a^{2}e^{4}+15ac\,d^{2}e^{2}-2c^{2}d^{4}+19\sqrt{ac\,e^{2}}\,ad\,e^{2}+\sqrt{ac\,e} \cdots\right.}{2\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{a \cdots\right.}}\cdots\right.}{\cdots}\right)}{(c \cdots}\right.$
pseudoelliptic	$\frac{\frac{7e\left(\frac{\left(19ad\,e^{2}+c\,d^{3}\right)\sqrt{ac\,e^{2}}}{7}+a^{2}e^{4}+\frac{15ac\,d^{2}e^{2}}{7}-\frac{2c^{2}d^{4}}{7}\right)\left(-c\,x^{2}+a\right)c^{2}(ex+d)^{\frac{3}{2}}\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{4}+\cdots}{\cdots}$

input
```
int(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(1/(a*e^2-c*d^2)^3*c*((-1/4*c*d*(3*a*e^2+c*d^2)/a/e^2*(e*x+d)^(3/2)+
1/4*(a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)/a/e^2*(e*x+d)^(1/2))/(-c*(e*x+d)^2+2*c
*d*(e*x+d)+a*e^2-c*d^2)+1/4/a/e^2*c*(1/2*(7*a^2*e^4+15*a*c*d^2*e^2-2*c^2*d
^4+19*(a*c*e^2)^(1/2)*a*d*e^2+(a*c*e^2)^(1/2)*c*d^3)/(a*c*e^2)^(1/2)/((-c*
d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))
*c)^(1/2))-1/2*(-7*a^2*e^4-15*a*c*d^2*e^2+2*c^2*d^4+19*(a*c*e^2)^(1/2)*a*d
*e^2+(a*c*e^2)^(1/2)*c*d^3)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2
)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))))-1/3/(a*e^2-c*
d^2)^2/(e*x+d)^(3/2)+4/(a*e^2-c*d^2)^3*c*d/(e*x+d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8308 vs. $2(248) = 496$.

Time = 3.17 (sec) , antiderivative size = 8308, normalized size of antiderivative = 26.71

$$\int \frac{1}{(d+ex)^{5/2}(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2}(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**(5/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)^2}\,dx = \int \frac{1}{\left(cx^2-a\right)^2(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 - a)^2*(e*x + d)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. 2(248) = 496.

Time = 0.34 (sec) , antiderivative size = 1998, normalized size of antiderivative = 6.42

$$\int \frac{1}{(d+ex)^{5/2}\left(a-cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*((a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5 - a^4*e^7)^2*(c^2*
d^3*e + 19*a*c*d*e^3)*abs(c) + (sqrt(a*c)*c^5*d^10*e - 37*sqrt(a*c)*a*c^4*
d^8*e^3 + 98*sqrt(a*c)*a^2*c^3*d^6*e^5 - 82*sqrt(a*c)*a^3*c^2*d^4*e^7 + 13
*sqrt(a*c)*a^4*c*d^2*e^9 + 7*sqrt(a*c)*a^5*e^11)*abs(-a*c^3*d^6*e + 3*a^2*
c^2*d^4*e^3 - 3*a^3*c*d^2*e^5 + a^4*e^7)*abs(c) - (2*a*c^9*d^17*e - 27*a^2
*c^8*d^15*e^3 + 113*a^3*c^7*d^13*e^5 - 223*a^4*c^6*d^11*e^7 + 225*a^5*c^5*
d^9*e^9 - 97*a^6*c^4*d^7*e^11 - 13*a^7*c^3*d^5*e^13 + 27*a^8*c^2*d^3*e^15
- 7*a^9*c*d*e^17)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^4*d^7 - 3*a^2*c^
3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 - a^4*c*d*e^6 + sqrt((a*c^4*d^7 - 3*a^2*c^3*
d^5*e^2 + 3*a^3*c^2*d^3*e^4 - a^4*c*d*e^6)^2 - (a*c^4*d^8 - 4*a^2*c^3*d^6*
e^2 + 6*a^3*c^2*d^4*e^4 - 4*a^4*c*d^2*e^6 + a^5*e^8)*(a*c^4*d^6 - 3*a^2*c^
3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 - a^4*c*e^6)))/(a*c^4*d^6 - 3*a^2*c^3*d^4*e^
2 + 3*a^3*c^2*d^2*e^4 - a^4*c*e^6)))/((a^2*c^6*d^12*e - 6*a^3*c^5*d^10*e^3
 + 15*a^4*c^4*d^8*e^5 - 20*a^5*c^3*d^6*e^7 + 15*a^6*c^2*d^4*e^9 - 6*a^7*c*
d^2*e^11 + a^8*e^13 + sqrt(a*c)*a*c^6*d^13 - 6*sqrt(a*c)*a^2*c^5*d^11*e^2
+ 15*sqrt(a*c)*a^3*c^4*d^9*e^4 - 20*sqrt(a*c)*a^4*c^3*d^7*e^6 + 15*sqrt(a*
c)*a^5*c^2*d^5*e^8 - 6*sqrt(a*c)*a^6*c*d^3*e^10 + sqrt(a*c)*a^7*d*e^12)*sq
rt(-c^2*d + sqrt(a*c)*c*e)*abs(-a*c^3*d^6*e + 3*a^2*c^2*d^4*e^3 - 3*a^3*c*
d^2*e^5 + a^4*e^7)) + 1/4*((a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*
e^5 - a^4*e^7)^2*(c^2*d^3*e + 19*a*c*d*e^3)*abs(c) - (sqrt(a*c)*c^5*d^1...
```

Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 12290, normalized size of antiderivative = 39.52

$$\int \frac{1}{(d+ex)^{5/2}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a - c*x^2)^2*(d + e*x)^(5/2)),x)
```

output

```
- ((2*e^3)/(3*(a*e^2 - c*d^2)) - (20*c*d*e^3*(d + e*x))/(3*(a*e^2 - c*d^2)
^2) - (c*e*(d + e*x)^2*(7*a^2*e^4 + 3*c^2*d^4 + 110*a*c*d^2*e^2))/(6*a*(a*
e^2 - c*d^2)^3) + (c^2*d*e*(19*a*e^2 + c*d^2)*(d + e*x)^3)/(2*a*(a*e^2 - c
*d^2)^3))/((a*e^2 - c*d^2)*(d + e*x)^(3/2) - c*(d + e*x)^(7/2) + 2*c*d*(d
+ e*x)^(5/2)) - atan((((d + e*x)^(1/2)*(1568*a^16*c^5*e^28 - 128*a^3*c^18*
d^26*e^2 + 3040*a^4*c^17*d^24*e^4 - 29120*a^5*c^16*d^22*e^6 + 128128*a^6*c
^15*d^20*e^8 - 282560*a^7*c^14*d^18*e^10 + 242016*a^8*c^13*d^16*e^12 + 282
240*a^9*c^12*d^14*e^14 - 1059840*a^10*c^11*d^12*e^16 + 1403904*a^11*c^10*d
^10*e^18 - 1049440*a^12*c^9*d^8*e^20 + 456512*a^13*c^8*d^6*e^22 - 100480*a
^14*c^7*d^4*e^24 + 4160*a^15*c^6*d^2*e^26) - (-(4*a^3*c^6*d^9 - 49*a^3*e^9
*(a^9*c^3)^(1/2) + 315*a^7*c^2*d*e^8 - 63*a^4*c^5*d^7*e^2 + 189*a^5*c^4*d^
5*e^4 + 1155*a^6*c^3*d^3*e^6 + 105*c^3*d^6*e^3*(a^9*c^3)^(1/2) - 819*a*c^2
*d^4*e^5*(a^9*c^3)^(1/2) - 837*a^2*c*d^2*e^7*(a^9*c^3)^(1/2))/(64*(a^13*e^
14 - a^6*c^7*d^14 - 7*a^12*c*d^2*e^12 + 7*a^7*c^6*d^12*e^2 - 21*a^8*c^5*d^
10*e^4 + 35*a^9*c^4*d^8*e^6 - 35*a^10*c^3*d^6*e^8 + 21*a^11*c^2*d^4*e^10))
)^(1/2)*((d + e*x)^(1/2)*(-(4*a^3*c^6*d^9 - 49*a^3*e^9*(a^9*c^3)^(1/2) + 3
15*a^7*c^2*d*e^8 - 63*a^4*c^5*d^7*e^2 + 189*a^5*c^4*d^5*e^4 + 1155*a^6*c^3
*d^3*e^6 + 105*c^3*d^6*e^3*(a^9*c^3)^(1/2) - 819*a*c^2*d^4*e^5*(a^9*c^3)^(
1/2) - 837*a^2*c*d^2*e^7*(a^9*c^3)^(1/2))/(64*(a^13*e^14 - a^6*c^7*d^14 -
7*a^12*c*d^2*e^12 + 7*a^7*c^6*d^12*e^2 - 21*a^8*c^5*d^10*e^4 + 35*a^9*c...
```

Reduce [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 4569, normalized size of antiderivative = 14.69

$$\int \frac{1}{(d+ex)^{5/2} (a-cx^2)^2} \, dx = \text{Too large to display}$$

input

```
int(1/(e*x+d)^(5/2)/(-c*x^2+a)^2,x)
```

output
```
(156*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*
x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*d**2*e**4 + 156*sqrt
(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*d*e**5*x + 96*sqrt(a)*sqrt(d
+ e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d**4*e**2 + 96*sqrt(a)*sqrt(d + e*x)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)
*sqrt(a)*e - c*d)))*a**2*c**2*d**3*e**3*x - 156*sqrt(a)*sqrt(d + e*x)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)))*a**2*c**2*d**2*e**4*x**2 - 156*sqrt(a)*sqrt(d + e*x)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)))*a**2*c**2*d*e**5*x**3 - 12*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(
c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
 - c*d)))*a*c**3*d**6 - 12*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e -
c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**
3*d**5*e*x - 96*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((
sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**4*e**2
*x**2 - 96*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(
d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**3*e**3*x**3
 + 12*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d ...
```

3.148 $\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 294

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \frac{(ae+cdx)(d+ex)^{5/2}}{4ac(a-cx^2)^2} + \frac{\sqrt{d+ex}(ae(7cd^2-5ae^2)+2cd(3cd^2-2ae^2)x)}{16a^2c^2(a-cx^2)} - \frac{(\sqrt{c}d-\sqrt{a}e)^{3/2}(12cd^2+18\sqrt{a}\sqrt{c}de+5ae^2)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{9/4}} + \frac{(\sqrt{c}d+\sqrt{a}e)^{3/2}(12cd^2-18\sqrt{a}\sqrt{c}de+5ae^2)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{9/4}}$$

output
```
1/4*(c*d*x+a*e)*(e*x+d)^(5/2)/a/c/(-c*x^2+a)^2+1/16*(e*x+d)^(1/2)*(a*e*(-5
*a*e^2+7*c*d^2)+2*c*d*(-2*a*e^2+3*c*d^2)*x)/a^2/c^2/(-c*x^2+a)-1/32*(c^(1/
2)*d-a^(1/2)*e)^(3/2)*(12*c*d^2+18*a^(1/2)*c^(1/2)*d*e+5*a*e^2)*arctanh(c^
(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(5/2)/c^(9/4)+1/32*(c^(
1/2)*d+a^(1/2)*e)^(3/2)*(12*c*d^2-18*a^(1/2)*c^(1/2)*d*e+5*a*e^2)*arctanh(
c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(5/2)/c^(9/4)
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \frac{-\frac{2\sqrt{a}\sqrt{d+ex}(5a^3e^3+6c^3d^3x^3-ac^2dx(10d^2+dex+8e^2x^2)-a^2ce(11d^2+4dex+9e^2x^2))}{(a-cx^2)^2}+\frac{(\sqrt{c}d+\sqrt{a}e)^2(12cd^2-18\sqrt{}}{}}{32}$$

input
```
Integrate[(d + e*x)^(7/2)/(a - c*x^2)^3,x]
```

output
```
((-2*Sqrt[a]*Sqrt[d + e*x]*(5*a^3*e^3 + 6*c^3*d^3*x^3 - a*c^2*d*x*(10*d^2
+ d*e*x + 8*e^2*x^2) - a^2*c*e*(11*d^2 + 4*d*e*x + 9*e^2*x^2)))/(a - c*x^2
)^2 + ((Sqrt[c]*d + Sqrt[a]*e)^2*(12*c*d^2 - 18*Sqrt[a]*Sqrt[c]*d*e + 5*a*
e^2)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d +
Sqrt[a]*e)])/Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e] - ((Sqrt[c]*d - Sqrt[a]*e)^2
*(12*c*d^2 + 18*Sqrt[a]*Sqrt[c]*d*e + 5*a*e^2)*ArcTan[(Sqrt[-(c*d) + Sqrt[
a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/Sqrt[-(c*d) + Sqrt[
a]*Sqrt[c]*e])/(32*a^(5/2)*c^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {495, 27, 684, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{(d+ex)^{5/2}(ae+cdx)}{4ac(a-cx^2)^2}-\frac{\int -\frac{(d+ex)^{3/2}(6cd^2+cexd-5ae^2)}{2(a-cx^2)^2}dx}{4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^{3/2}(6cd^2+cexd-5ae^2)}{(a-cx^2)^2}dx}{8ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\,(a-cx^2)^2}$$

$$\Big\downarrow \text{684}$$

$$\frac{\frac{\sqrt{d+ex}(2cdx(3cd^2-2ae^2)+ae(7cd^2-5ae^2))}{2ac(a-cx^2)}-\frac{\int -\frac{(4cd^2-5ae^2)(3cd^2-ae^2)+2cde(3cd^2-4ae^2)x}{2\sqrt{d+ex}(a-cx^2)}dx}{2ac}}{8ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\,(a-cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(4cd^2-5ae^2)(3cd^2-ae^2)+2cde(3cd^2-4ae^2)x}{\sqrt{d+ex}(a-cx^2)}dx}{4ac}+\frac{\sqrt{d+ex}(2cdx(3cd^2-2ae^2)+ae(7cd^2-5ae^2))}{2ac(a-cx^2)}}{8ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\,(a-cx^2)^2}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{\int -\frac{e((6cd^2-5ae^2)(cd^2-ae^2)+2cd(3cd^2-4ae^2)(d+ex))}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}+\frac{\sqrt{d+ex}(2cdx(3cd^2-2ae^2)+ae(7cd^2-5ae^2))}{2ac(a-cx^2)}}{8ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\,(a-cx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sqrt{d+ex}(2cdx(3cd^2-2ae^2)+ae(7cd^2-5ae^2))}{2ac(a-cx^2)}-\frac{\int \frac{e((6cd^2-5ae^2)(cd^2-ae^2)+2cd(3cd^2-4ae^2)(d+ex))}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}}{8ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\,(a-cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{d+ex}(2cdx(3cd^2-2ae^2)+ae(7cd^2-5ae^2))}{2ac(a-cx^2)}-\frac{e\int \frac{(6cd^2-5ae^2)(cd^2-ae^2)+2cd(3cd^2-4ae^2)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}}{8ac}+\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\,(a-cx^2)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{\sqrt{d+ex}\left(2cdx\left(3cd^2-2ae^2\right)+ae\left(7cd^2-5ae^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{e\left(\frac{\sqrt{c}\left(\sqrt{a}e+\sqrt{c}d\right)^2\left(-18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\int\frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\cdots\right.}{\cdots}\right.}{2ac}}{8ac}$$

$$\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\sqrt{d+ex}\left(2cdx\left(3cd^2-2ae^2\right)+ae\left(7cd^2-5ae^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{e\left(\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}\left(-18\sqrt{a}\sqrt{c}d\cdots\right.}{\cdots}\right.}{2ac}}{8ac}$$

$$\frac{(d+ex)^{5/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

input
```
Int[(d + e*x)^(7/2)/(a - c*x^2)^3,x]
```

output
```
((a*e + c*d*x)*(d + e*x)^(5/2))/(4*a*c*(a - c*x^2)^2) + ((Sqrt[d + e*x]*(a
*e*(7*c*d^2 - 5*a*e^2) + 2*c*d*(3*c*d^2 - 2*a*e^2)*x))/(2*a*c*(a - c*x^2))
 - (e*(((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*(12*c*d^2 + 18*Sqrt[a]*Sqrt[c]*d*e +
 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2
*Sqrt[a]*c^(1/4)*e) - ((Sqrt[c]*d + Sqrt[a]*e)^(3/2)*(12*c*d^2 - 18*Sqrt[a
]*Sqrt[c]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d +
Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e)))/(2*a*c))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 495

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 1480

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$-\frac{5\left(-\frac{\left(\frac{2\left(4ade^{2}-3c\,d^{3}\right)\sqrt{ac\,e^{2}}}{5}+\left(a\,e^{2}-3c\,d^{2}\right)\left(a\,e^{2}-\frac{4c\,d^{2}}{5}\right)\right)e\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}\,\left(-c\,x^{2}+a\right)^{2}c\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2}+\cdots\right.}{\cdots}$
default	$2e^{5}\left(\frac{\frac{d\left(4a\,e^{2}-3c\,d^{2}\right)(ex+d)^{\frac{7}{2}}}{16a^{2}e^{4}}+\frac{\left(9a^{2}e^{4}-23ac\,d^{2}e^{2}+18c^{2}d^{4}\right)(ex+d)^{\frac{5}{2}}}{32a^{2}e^{4}c}-\frac{d\left(7a^{2}e^{4}-16ac\,d^{2}e^{2}+9c^{2}d^{4}\right)(ex+d)^{\frac{3}{2}}}{16a^{2}e^{4}c}-\frac{\left(a\,e^{2}-c\,d^{2}\right)\cdots}{\cdots}}{\left(-c(ex+d)^{2}+2cd(ex+d)+a\,e^{2}-c\,d^{2}\right)^{2}}\cdots\right.$
derivativedivides	$-2e^{5}\left(-\frac{\frac{d\left(4a\,e^{2}-3c\,d^{2}\right)(ex+d)^{\frac{7}{2}}}{16a^{2}e^{4}}+\frac{\left(9a^{2}e^{4}-23ac\,d^{2}e^{2}+18c^{2}d^{4}\right)(ex+d)^{\frac{5}{2}}}{32a^{2}e^{4}c}-\frac{d\left(7a^{2}e^{4}-16ac\,d^{2}e^{2}+9c^{2}d^{4}\right)(ex+d)^{\frac{3}{2}}}{16a^{2}e^{4}c}-\frac{\left(a\,e^{2}-\cdots\right)}{\cdots}}{\left(-c(ex+d)^{2}+2cd(ex+d)+a\,e^{2}-c\,d^{2}\right)^{2}}\cdots\right.$

input
```
int((e*x+d)^(7/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-5/16/(a*c*e^2)^(1/2)*(-1/2*(2/5*(4*a*d*e^2-3*c*d^3)*(a*c*e^2)^(1/2)+(a*e^
2-3*c*d^2)*(a*e^2-4/5*c*d^2))*e*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-c*x^2+a)
^2*c*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((-c*d+(a*c*
e^2)^(1/2))*c)^(1/2)*(-1/2*e*(2/5*(-4*a*d*e^2+3*c*d^3)*(a*c*e^2)^(1/2)+(a*
e^2-3*c*d^2)*(a*e^2-4/5*c*d^2))*(-c*x^2+a)^2*c*arctanh(c*(e*x+d)^(1/2)/((c
*d+(a*c*e^2)^(1/2))*c)^(1/2))+(6/5*d^3*c^3*x^3-2*x*a*d*(4/5*e^2*x^2+1/10*d
*e*x+d^2)*c^2-11/5*e*a^2*(9/11*e^2*x^2+4/11*d*e*x+d^2)*c+e^3*a^3)*((c*d+(a
*c*e^2)^(1/2))*c)^(1/2)*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)))/((c*d+(a*c*e^2)^(1
/2))*c)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a^2/c^2/(-c*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1730 vs. $2(236) = 472$.

Time = 0.21 (sec) , antiderivative size = 1730, normalized size of antiderivative = 5.88

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*((a^2*c^4*x^4 - 2*a^3*c^3*x^2 + a^4*c^2)*sqrt((144*c^3*d^7 - 420*a*c^
2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*sqrt((441*c^2*d^4*
e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((3024*
c^4*d^8*e^5 - 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 - 6250*a^3*c*d^2
*e^11 + 625*a^4*e^13)*sqrt(e*x + d) + (126*a^3*c^4*d^4*e^6 - 255*a^4*c^3*d
^2*e^8 + 125*a^5*c^2*e^10 - (12*a^5*c^8*d^3 - 13*a^6*c^7*d*e^2)*sqrt((441*
c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt((144*c^3
*d^7 - 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*sqr
t((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c
^4))) - (a^2*c^4*x^4 - 2*a^3*c^3*x^2 + a^4*c^2)*sqrt((144*c^3*d^7 - 420*a*
c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*sqrt((441*c^2*d^
4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((302
4*c^4*d^8*e^5 - 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 - 6250*a^3*c*d
^2*e^11 + 625*a^4*e^13)*sqrt(e*x + d) - (126*a^3*c^4*d^4*e^6 - 255*a^4*c^3
*d^2*e^8 + 125*a^5*c^2*e^10 - (12*a^5*c^8*d^3 - 13*a^6*c^7*d*e^2)*sqrt((44
1*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt((144*c
^3*d^7 - 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^5*c^4*s
qrt((441*c^2*d^4*e^10 - 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5
*c^4))) + (a^2*c^4*x^4 - 2*a^3*c^3*x^2 + a^4*c^2)*sqrt((144*c^3*d^7 - 420*
a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 - 105*a^3*d*e^6 - a^5*c^4*sqrt((441*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \int -\frac{(ex+d)^{\frac{7}{2}}}{(cx^2-a)^3}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x + d)^(7/2)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(236) = 472$.

Time $= 0.27$ (sec) , antiderivative size $= 714$, normalized size of antiderivative $= 2.43$

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/32*(2*(3*a*c^2*d^3*e - 4*a^2*c*d*e^3)*e^2*abs(c) + (6*sqrt(a*c)*c^2*d^4
*e - 11*sqrt(a*c)*a*c*d^2*e^3 + 5*sqrt(a*c)*a^2*e^5)*abs(c)*abs(e) - (12*c
^3*d^5*e - 19*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*abs(c))*arctan(sqrt(e*x + d)/
sqrt(-(a^2*c^3*d + sqrt(a^4*c^6*d^2 - (a^2*c^3*d^2 - a^3*c^2*e^2)*a^2*c^3)
)/(a^2*c^3)))/((a^3*c^3*e - sqrt(a*c)*a^2*c^3*d)*sqrt(-c^2*d - sqrt(a*c)*c
*e)*abs(e)) - 1/32*(2*(3*a*c^2*d^3*e - 4*a^2*c*d*e^3)*e^2*abs(c) - (6*sqrt
(a*c)*c^2*d^4*e - 11*sqrt(a*c)*a*c*d^2*e^3 + 5*sqrt(a*c)*a^2*e^5)*abs(c)*a
bs(e) - (12*c^3*d^5*e - 19*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*abs(c))*arctan(s
qrt(e*x + d)/sqrt(-(a^2*c^3*d - sqrt(a^4*c^6*d^2 - (a^2*c^3*d^2 - a^3*c^2*
e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*e + sqrt(a*c)*a^2*c^3*d)*sqrt(-c^2*d
+ sqrt(a*c)*c*e)*abs(e)) - 1/16*(6*(e*x + d)^(7/2)*c^3*d^3*e - 18*(e*x + d
)^(5/2)*c^3*d^4*e + 18*(e*x + d)^(3/2)*c^3*d^5*e - 6*sqrt(e*x + d)*c^3*d^6
*e - 8*(e*x + d)^(7/2)*a*c^2*d*e^3 + 23*(e*x + d)^(5/2)*a*c^2*d^2*e^3 - 32
*(e*x + d)^(3/2)*a*c^2*d^3*e^3 + 17*sqrt(e*x + d)*a*c^2*d^4*e^3 - 9*(e*x +
 d)^(5/2)*a^2*c*e^5 + 14*(e*x + d)^(3/2)*a^2*c*d*e^5 - 16*sqrt(e*x + d)*a^
2*c*d^2*e^5 + 5*sqrt(e*x + d)*a^3*e^7)/(((e*x + d)^2*c - 2*(e*x + d)*c*d +
 c*d^2 - a*e^2)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 2518, normalized size of antiderivative = 8.56

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(7/2)/(a - c*x^2)^3,x)
```

output
```
- ((e*(3*c*d^3 - 4*a*d*e^2)*(d + e*x)^(7/2))/(8*a^2) + ((d + e*x)^(3/2)*(7
*a^2*d*e^5 + 9*c^2*d^5*e - 16*a*c*d^3*e^3))/(8*a^2*c) + ((d + e*x)^(1/2)*(
5*a^3*e^7 - 6*c^3*d^6*e + 17*a*c^2*d^4*e^3 - 16*a^2*c*d^2*e^5))/(16*a^2*c^
2) - (e*(d + e*x)^(5/2)*(9*a^2*e^4 + 18*c^2*d^4 - 23*a*c*d^2*e^2))/(16*a^2
*c))/(c^2*(d + e*x)^4 + a^2*e^4 + c^2*d^4 + (6*c^2*d^2 - 2*a*c*e^2)*(d + e
*x)^2 - (4*c^2*d^3 - 4*a*c*d*e^2)*(d + e*x) - 4*c^2*d*(d + e*x)^3 - 2*a*c*
d^2*e^2) - 2*atanh((25*e^10*(d + e*x)^(1/2)*((9*d^7)/(256*a^5*c) - (105*d*
e^6)/(4096*a^2*c^4) + (385*d^3*e^4)/(4096*a^3*c^3) - (105*d^5*e^2)/(1024*a
^4*c^2) - (25*e^7*(a^15*c^9)^(1/2))/(4096*a^9*c^9) + (21*d^2*e^5*(a^15*c^9
)^(1/2))/(4096*a^10*c^8))^(1/2))/(32*((825*d^5*e^9)/(2048*a^3) + (325*d*e^
13)/(2048*a*c^2) - (63*c*d^7*e^7)/(512*a^4) - (449*d^3*e^11)/(1024*a^2*c)
+ (125*e^14*(a^15*c^9)^(1/2))/(2048*a^8*c^7) - (95*d^2*e^12*(a^15*c^9)^(1/
2))/(512*a^9*c^6) + (381*d^4*e^10*(a^15*c^9)^(1/2))/(2048*a^10*c^5) - (63*
d^6*e^8*(a^15*c^9)^(1/2))/(1024*a^11*c^4))) - (21*d^2*e^8*(d + e*x)^(1/2)*
((9*d^7)/(256*a^5*c) - (105*d*e^6)/(4096*a^2*c^4) + (385*d^3*e^4)/(4096*a^
3*c^3) - (105*d^5*e^2)/(1024*a^4*c^2) - (25*e^7*(a^15*c^9)^(1/2))/(4096*a^
9*c^9) + (21*d^2*e^5*(a^15*c^9)^(1/2))/(4096*a^10*c^8))^(1/2))/(32*((325*d
*e^13)/(2048*c^3) - (63*d^7*e^7)/(512*a^3) - (449*d^3*e^11)/(1024*a*c^2) +
 (825*d^5*e^9)/(2048*a^2*c) + (125*e^14*(a^15*c^9)^(1/2))/(2048*a^7*c^8) -
 (95*d^2*e^12*(a^15*c^9)^(1/2))/(512*a^8*c^7) + (381*d^4*e^10*(a^15*c^9...
```

Reduce [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 1967, normalized size of antiderivative = 6.69

$$\int \frac{(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(7/2)/(-c*x^2+a)^3,x)
```

output
```
(26*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*d*e**2 - 24*sqrt(a)*sqrt(sqrt(c)*sq
rt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*
d)))*a**2*c**2*d**3 - 52*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(
d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d*e**2*x**2
 + 48*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**3*x**2 + 26*sqrt(a)*sqrt(sqrt(
c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
 - c*d)))*a*c**3*d*e**2*x**4 - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**4*d**3*x*
*4 + 10*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt
(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**4*e**3 - 12*sqrt(c)*sqrt(sqrt(c)*sq
rt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*
d)))*a**3*c*d**2*e - 20*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d
 + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*e**3*x**2 + 24*
sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d**2*e*x**2 + 10*sqrt(c)*sqrt(sqrt(c
)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
- c*d)))*a**2*c**2*e**3*x**4 - 12*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d*...
```

3.149 $\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 279

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \frac{(ae+cdx)(d+ex)^{3/2}}{4ac\,(a-cx^2)^2} + \frac{3\sqrt{d+ex}\,(ade+(2cd^2-ae^2)\,x)}{16a^2c\,(a-cx^2)} - \frac{3\sqrt{\sqrt{c}d-\sqrt{a}e}\,(4cd^2+2\sqrt{a}\sqrt{c}de-ae^2)\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{7/4}} + \frac{3\sqrt{\sqrt{c}d+\sqrt{a}e}\,(4cd^2-2\sqrt{a}\sqrt{c}de-ae^2)\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{7/4}}$$

output
```
1/4*(c*d*x+a*e)*(e*x+d)^(3/2)/a/c/(-c*x^2+a)^2+3/16*(e*x+d)^(1/2)*(a*d*e+(
-a*e^2+2*c*d^2)*x)/a^2/c/(-c*x^2+a)-3/32*(c^(1/2)*d-a^(1/2)*e)^(1/2)*(4*c*
d^2+2*a^(1/2)*c^(1/2)*d*e-a*e^2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-
a^(1/2)*e)^(1/2))/a^(5/2)/c^(7/4)+3/32*(c^(1/2)*d+a^(1/2)*e)^(1/2)*(4*c*d^
2-2*a^(1/2)*c^(1/2)*d*e-a*e^2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^
(1/2)*e)^(1/2))/a^(5/2)/c^(7/4)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \frac{-\frac{2\sqrt{a}c\sqrt{d+ex}(6c^2d^2x^3-a^2e(7d+ex)-acx(10d^2+dex+3e^2x^2))}{(a-cx^2)^2} - 3\sqrt{-cd-\sqrt{a}\sqrt{c}e}(4cd^2-2\sqrt{a}\sqrt{c}d}{}$$

input
```
Integrate[(d + e*x)^(5/2)/(a - c*x^2)^3,x]
```

output
```
((-2*Sqrt[a]*c*Sqrt[d + e*x]*(6*c^2*d^2*x^3 - a^2*e*(7*d + e*x) - a*c*x*(1
0*d^2 + d*e*x + 3*e^2*x^2)))/(a - c*x^2)^2 - 3*Sqrt[-(c*d) - Sqrt[a]*Sqrt[
c]*e]*(4*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[(Sqrt[-(c*d) - Sqrt
[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)] + 3*Sqrt[-(c*d) + S
qrt[a]*Sqrt[c]*e]*(4*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[(Sqrt[-
(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(32*a^
(5/2)*c^2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {495, 27, 685, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{(d+ex)^{3/2}(ae+cdx)}{4ac(a-cx^2)^2} - \frac{\int -\frac{3\sqrt{d+ex}(2cd^2+cexd-ae^2)}{2(a-cx^2)^2}dx}{4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\sqrt{d+ex}(2cd^2+cexd-ae^2)}{(a-cx^2)^2}dx}{8ac} + \frac{(d+ex)^{3/2}(ae+cdx)}{4ac(a-cx^2)^2}$$

$$\Big\downarrow \text{685}$$

$$\frac{3\left(\frac{\sqrt{d+ex}\left(x\left(2cd^2-ae^2\right)+ade\right)}{2a\left(a-cx^2\right)}-\frac{\int -\frac{c\left(d\left(4cd^2-3ae^2\right)+e\left(2cd^2-ae^2\right)x\right)}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac}\right)}{8ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{\int \frac{d\left(4cd^2-3ae^2\right)+e\left(2cd^2-ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4a}+\frac{\sqrt{d+ex}\left(x\left(2cd^2-ae^2\right)+ade\right)}{2a\left(a-cx^2\right)}\right)}{8ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{654}$$

$$\frac{3\left(\frac{\int -\frac{e\left(2d\left(cd^2-ae^2\right)+\left(2cd^2-ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}+\frac{\sqrt{d+ex}\left(x\left(2cd^2-ae^2\right)+ade\right)}{2a\left(a-cx^2\right)}\right)}{8ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{\sqrt{d+ex}\left(x\left(2cd^2-ae^2\right)+ade\right)}{2a\left(a-cx^2\right)}-\frac{\int \frac{e\left(2d\left(cd^2-ae^2\right)+\left(2cd^2-ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}\right)}{8ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{\sqrt{d+ex}\left(x\left(2cd^2-ae^2\right)+ade\right)}{2a\left(a-cx^2\right)}-\frac{e\int \frac{2d\left(cd^2-ae^2\right)+\left(2cd^2-ae^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}\right)}{8ac}+\frac{(d+ex)^{3/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{3\left(\frac{\sqrt{d+ex}\left(x\left(2cd^2-ae^2\right)+ade\right)}{2a\left(a-cx^2\right)}-\frac{e\left(\frac{1}{2}\left(-\frac{4c^{3/2}d^3}{\sqrt{a}e}+3\sqrt{a}\sqrt{c}de-ae^2+2cd^2\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\left(-2\sqrt{a}\sqrt{c}de-ae^2+4\right.}{}\right.}{2a}\right.}{8ac}$$

$$\frac{(d+ex)^{3/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{3\left(\frac{\sqrt{d+ex}\left(x\left(2cd^2-ae^2\right)+ade\right)}{2a\left(a-cx^2\right)}-\frac{e\left(-\frac{\sqrt{\sqrt{a}e+\sqrt{c}d}\left(-2\sqrt{a}\sqrt{c}de-ae^2+4cd^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e}-\frac{\left(-\frac{4c^{3/2}d^3}{\sqrt{a}e}+3\sqrt{a}\sqrt{c}de-ae^2+2cd^2\right)\text{arctan}}{2c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}\right.}{2a}\right.}{8ac}$$

$$\frac{(d+ex)^{3/2}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

input
```
Int[(d + e*x)^(5/2)/(a - c*x^2)^3,x]
```

output
```
((a*e + c*d*x)*(d + e*x)^(3/2))/(4*a*c*(a - c*x^2)^2) + (3*((Sqrt[d + e*x]
*(a*d*e + (2*c*d^2 - a*e^2)*x))/(2*a*(a - c*x^2)) - (e*(-1/2*((2*c*d^2 - (
4*c^(3/2)*d^3)/(Sqrt[a]*e) + 3*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(c^(1/
4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(c^(3/4)*Sqrt[Sqrt[c]*d -
Sqrt[a]*e]) - (Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(4*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*
e - a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(
2*Sqrt[a]*c^(3/4)*e)))/(2*a)))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$-\frac{-\frac{9e\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\left(\frac{\left(-a\,e^2+2c\,d^2\right)\sqrt{ac\,e^2}}{3}+cd\left(a\,e^2-\frac{4c\,d^2}{3}\right)\right)\left(-c\,x^2+a\right)^2\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{32}+\frac{7\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)}}{}}{a^2}$
default	$2e^5\left(\frac{\frac{3\left(a\,e^2-2c\,d^2\right)(ex+d)^{\frac{7}{2}}}{32a^2e^4}-\frac{d\left(4a\,e^2-9c\,d^2\right)(ex+d)^{\frac{5}{2}}}{16a^2e^4}+\frac{\left(a^2e^4+17ac\,d^2e^2-18c^2d^4\right)(ex+d)^{\frac{3}{2}}}{32a^2e^4c}+\frac{3\left(a\,e^2-c\,d^2\right)^2d\sqrt{ex+d}}{16a^2e^4c}}{\left(-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\right.$
derivativedivides	$-2e^5\left(-\frac{-\frac{3\left(a\,e^2-2c\,d^2\right)(ex+d)^{\frac{7}{2}}}{32a^2e^4}-\frac{d\left(4a\,e^2-9c\,d^2\right)(ex+d)^{\frac{5}{2}}}{16a^2e^4}+\frac{\left(a^2e^4+17ac\,d^2e^2-18c^2d^4\right)(ex+d)^{\frac{3}{2}}}{32a^2e^4c}+\frac{3\left(a\,e^2-c\,d^2\right)^2d\sqrt{ex+d}}{16a^2e^4c}}{\left(-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2\right)^2}\right.$

input
```
int((e*x+d)^(5/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
7/16/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)/((-c*d+(a*c*e^2)^(1/2
))*c)^(1/2)*(-9/14*e*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(1/3*(-a*e^2+2*c*d^2)
*(a*c*e^2)^(1/2)+c*d*(a*e^2-4/3*c*d^2))*(-c*x^2+a)^2*arctan(c*(e*x+d)^(1/2
)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-9/1
4*e*(1/3*(a*e^2-2*c*d^2)*(a*c*e^2)^(1/2)+c*d*(a*e^2-4/3*c*d^2))*(-c*x^2+a)
^2*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(e*x+d)^(1/2)*
((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(a*c*e^2)^(1/2)*(-6/7*d^2*c^2*x^3+10/7*(3/
10*e^2*x^2+1/10*d*e*x+d^2)*x*a*c+a^2*e*(d+1/7*e*x))))/a^2/c/(-c*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. 2(221) = 442.

Time = 0.13 (sec) , antiderivative size = 1029, normalized size of antiderivative = 3.69

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(e^10/(a^5
*c^7)) + 16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*
c^2*d^4*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d*e^
6 + (4*a^5*c^6*d^2 - a^6*c^5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt((a^5*c^3*sqrt
(e^10/(a^5*c^7)) + 16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3)))
- 3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(e^10/(a^5*c^7
)) + 16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*
d^4*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d*e^6 +
(4*a^5*c^6*d^2 - a^6*c^5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt((a^5*c^3*sqrt(e^1
0/(a^5*c^7)) + 16*c^2*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))) + 3*
(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(e^10/(a^5*c^7))
- 16*c^2*d^5 + 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*d^4
*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d*e^6 - (4*
a^5*c^6*d^2 - a^6*c^5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt(-(a^5*c^3*sqrt(e^10/
(a^5*c^7)) - 16*c^2*d^5 + 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))) - 3*(a
^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(e^10/(a^5*c^7)) -
16*c^2*d^5 + 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(16*c^2*d^4*e
^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d*e^6 - (4*a^
5*c^6*d^2 - a^6*c^5*e^2)*sqrt(e^10/(a^5*c^7)))*sqrt(-(a^5*c^3*sqrt(e^10/(a
^5*c^7)) - 16*c^2*d^5 + 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))) + 4*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \int -\frac{(ex+d)^{\frac{5}{2}}}{(cx^2-a)^3}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x + d)^(5/2)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(221) = 442.

Time = 0.27 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \frac{3\left(4\,c^4d^4e-3\,ac^3d^2e^3-(2\,acd^2e-a^2e^3)c^2e^2-2\left(\sqrt{ac}c^2d^3e-\sqrt{ac}acde^3\right)|c||e|\right)\arctan}{32\left(a^3c^3e-\sqrt{ac}a^2c^3d\right)\sqrt{-c^2d-\sqrt{ac}ce}|e|} + \frac{3\left(4\,c^4d^4e-3\,ac^3d^2e^3-(2\,acd^2e-a^2e^3)c^2e^2+2\left(\sqrt{ac}c^2d^3e-\sqrt{ac}acde^3\right)|c||e|\right)\arctan\left(\frac{\sqrt{e}}{\sqrt{-\frac{a^2c^2d-\sqrt{a^4c^4d^2-}}{}}}\right.}{32\left(a^3c^3e+\sqrt{ac}a^2c^3d\right)\sqrt{-c^2d+\sqrt{ac}ce}|e|} - \frac{6\,(ex+d)^{\frac{7}{2}}c^2d^2e-18\,(ex+d)^{\frac{5}{2}}c^2d^3e+18\,(ex+d)^{\frac{3}{2}}c^2d^4e-6\,\sqrt{ex+d}c^2d^5e-3\,(ex+d)^{\frac{7}{2}}ace^3+8\,(ex}{16\left((ex+d)^2c-2\,(ex+d)cd+\right.}$$

input
```
integrate((e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(4*c^4*d^4*e - 3*a*c^3*d^2*e^3 - (2*a*c*d^2*e - a^2*e^3)*c^2*e^2 - 2*
(sqrt(a*c)*c^2*d^3*e - sqrt(a*c)*a*c*d*e^3)*abs(c)*abs(e))*arctan(sqrt(e*x
 + d)/sqrt(-(a^2*c^2*d + sqrt(a^4*c^4*d^2 - (a^2*c^2*d^2 - a^3*c*e^2)*a^2*
c^2))/(a^2*c^2)))/((a^3*c^3*e - sqrt(a*c)*a^2*c^3*d)*sqrt(-c^2*d - sqrt(a*
c)*c*e)*abs(e)) + 3/32*(4*c^4*d^4*e - 3*a*c^3*d^2*e^3 - (2*a*c*d^2*e - a^2
*e^3)*c^2*e^2 + 2*(sqrt(a*c)*c^2*d^3*e - sqrt(a*c)*a*c*d*e^3)*abs(c)*abs(e
))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^2*d - sqrt(a^4*c^4*d^2 - (a^2*c^2*d^2
 - a^3*c*e^2)*a^2*c^2))/(a^2*c^2)))/((a^3*c^3*e + sqrt(a*c)*a^2*c^3*d)*sqr
t(-c^2*d + sqrt(a*c)*c*e)*abs(e)) - 1/16*(6*(e*x + d)^(7/2)*c^2*d^2*e - 18
*(e*x + d)^(5/2)*c^2*d^3*e + 18*(e*x + d)^(3/2)*c^2*d^4*e - 6*sqrt(e*x + d
)*c^2*d^5*e - 3*(e*x + d)^(7/2)*a*c*e^3 + 8*(e*x + d)^(5/2)*a*c*d*e^3 - 17
*(e*x + d)^(3/2)*a*c*d^2*e^3 + 12*sqrt(e*x + d)*a*c*d^3*e^3 - (e*x + d)^(3
/2)*a^2*e^5 - 6*sqrt(e*x + d)*a^2*d*e^5)/(((e*x + d)^2*c - 2*(e*x + d)*c*d
 + c*d^2 - a*e^2)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 8.05 (sec) , antiderivative size = 1015, normalized size of antiderivative = 3.64

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(a - c*x^2)^3,x)
```

output
```
((3*e*(a*e^2 - 2*c*d^2)*(d + e*x)^(7/2))/(16*a^2) + ((d + e*x)^(3/2)*(a^2*
e^5 - 18*c^2*d^4*e + 17*a*c*d^2*e^3))/(16*a^2*c) - (d*(4*a*e^3 - 9*c*d^2*e
)*(d + e*x)^(5/2))/(8*a^2) + (3*(d + e*x)^(1/2)*(a^2*d*e^5 + c^2*d^5*e - 2
*a*c*d^3*e^3))/(8*a^2*c))/(c^2*(d + e*x)^4 + a^2*e^4 + c^2*d^4 + (6*c^2*d^
2 - 2*a*c*e^2)*(d + e*x)^2 - (4*c^2*d^3 - 4*a*c*d*e^2)*(d + e*x) - 4*c^2*d
*(d + e*x)^3 - 2*a*c*d^2*e^2) - 2*atanh((9*e^8*(d + e*x)^(1/2)*((9*d^5)/(2
56*a^5*c) + (45*d*e^4)/(4096*a^3*c^3) - (45*d^3*e^2)/(1024*a^4*c^2) - (9*e
^5*(a^15*c^7)^(1/2))/(4096*a^10*c^7))^(1/2))/(32*((27*e^11)/(2048*a*c^2) +
 (27*d^4*e^7)/(512*a^3) - (135*d^2*e^9)/(2048*a^2*c) - (27*d*e^10*(a^15*c^
7)^(1/2))/(1024*a^9*c^5) + (27*d^3*e^8*(a^15*c^7)^(1/2))/(1024*a^10*c^4)))
 + (9*d*e^7*(a^15*c^7)^(1/2)*(d + e*x)^(1/2)*((9*d^5)/(256*a^5*c) + (45*d*
e^4)/(4096*a^3*c^3) - (45*d^3*e^2)/(1024*a^4*c^2) - (9*e^5*(a^15*c^7)^(1/2
))/(4096*a^10*c^7))^(1/2))/(32*((27*a^7*c*e^11)/2048 + (27*a^5*c^3*d^4*e^7
)/512 - (135*a^6*c^2*d^2*e^9)/2048 - (27*d*e^10*(a^15*c^7)^(1/2))/(1024*a*
c^2) + (27*d^3*e^8*(a^15*c^7)^(1/2))/(1024*a^2*c))))*(-(9*(e^5*(a^15*c^7)^
(1/2) - 16*a^5*c^6*d^5 - 5*a^7*c^4*d*e^4 + 20*a^6*c^5*d^3*e^2))/(4096*a^10
*c^7))^(1/2) - 2*atanh((9*e^8*(d + e*x)^(1/2)*((9*d^5)/(256*a^5*c) + (45*d
*e^4)/(4096*a^3*c^3) - (45*d^3*e^2)/(1024*a^4*c^2) + (9*e^5*(a^15*c^7)^(1/
2))/(4096*a^10*c^7))^(1/2))/(32*((27*e^11)/(2048*a*c^2) + (27*d^4*e^7)/(51
2*a^3) - (135*d^2*e^9)/(2048*a^2*c) + (27*d*e^10*(a^15*c^7)^(1/2))/(102...
```

Reduce [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 1430, normalized size of antiderivative = 5.13

$$\int \frac{(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(5/2)/(-c*x^2+a)^3,x)
```

output
```
(6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*e**2 - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*
a**2*c*d**2 - 12*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)
*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*e**2*x**2 + 48*sqrt(a)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c
)*sqrt(a)*e - c*d)))*a*c**2*d**2*x**2 + 6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*
*2*e**2*x**4 - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**3*d**2*x**4 - 12*sqrt(c)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)
*sqrt(a)*e - c*d)))*a**3*d*e + 24*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d*e*x
**2 - 12*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d*e*x**4 + 3*sqrt(a)*sqrt(sqrt
(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d
 + e*x))*a**3*e**2 - 12*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(
sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*c*d**2 - 6*sqrt(a)*
sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(
c)*sqrt(d + e*x))*a**2*c*e**2*x**2 + 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e ...
```

3.150 $\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 268

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \frac{(ae+cdx)\sqrt{d+ex}}{4ac(a-cx^2)^2} - \frac{(ae-6cdx)\sqrt{d+ex}}{16a^2c(a-cx^2)} - \frac{3\left(4cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{5/4}\sqrt{\sqrt{c}d-\sqrt{a}e}} + \frac{3\left(4cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{5/4}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output

```
1/4*(c*d*x+a*e)*(e*x+d)^(1/2)/a/c/(-c*x^2+a)^2-1/16*(-6*c*d*x+a*e)*(e*x+d)
^(1/2)/a^2/c/(-c*x^2+a)-3/32*(4*c*d^2-2*a^(1/2)*c^(1/2)*d*e-a*e^2)*arctanh
(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(5/2)/c^(5/4)/(c^(1/
2)*d-a^(1/2)*e)^(1/2)+3/32*(4*c*d^2+2*a^(1/2)*c^(1/2)*d*e-a*e^2)*arctanh(c
^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(5/2)/c^(5/4)/(c^(1/2)
*d+a^(1/2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \frac{-\frac{2\sqrt{a}\sqrt{d+ex}\left(-3a^2e+6c^2dx^3-acx(10d+ex)\right)}{(a-cx^2)^2}+\frac{3\left(4cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}-\frac{3(4cd}{}}{32a^{5/2}c}$$

input
```
Integrate[(d + e*x)^(3/2)/(a - c*x^2)^3,x]
```

output
```
((-2*Sqrt[a]*Sqrt[d + e*x]*(-3*a^2*e + 6*c^2*d*x^3 - a*c*x*(10*d + e*x)))/
(a - c*x^2)^2 + (3*(4*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[(Sqrt[
-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/Sqrt[
-(c*d) - Sqrt[a]*Sqrt[c]*e] - (3*(4*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)
*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt
[a]*e)])/Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e])/(32*a^(5/2)*c)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {495, 27, 686, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}dx$$

$$\Big\downarrow \text{ 495}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4ac\,(a-cx^2)^2}-\frac{\int -\frac{6cd^2+5cexd-ae^2}{2\sqrt{d+ex}(a-cx^2)^2}dx}{4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{6cd^2+5cexd-ae^2}{\sqrt{d+ex}(a-cx^2)^2}dx}{8ac}+\frac{\sqrt{d+ex}(ae+cdx)}{4ac\,(a-cx^2)^2}$$

$$\Big\downarrow \text{686}$$

$$\frac{-\frac{\int -\frac{3c\left(cd^2-ae^2\right)\left(4cd^2+2cexd-ae^2\right)}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-6cdx)}{2a(a-cx^2)}}{8ac}+\frac{\sqrt{d+ex}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{4cd^2+2cexd-ae^2}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4a}-\frac{\sqrt{d+ex}(ae-6cdx)}{2a(a-cx^2)}}{8ac}+\frac{\sqrt{d+ex}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{3\int -\frac{e\left(2cd^2+2c(d+ex)d-ae^2\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}(ae-6cdx)}{2a(a-cx^2)}}{8ac}+\frac{\sqrt{d+ex}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{3\int \frac{e\left(2cd^2+2c(d+ex)d-ae^2\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}(ae-6cdx)}{2a(a-cx^2)}}{8ac}+\frac{\sqrt{d+ex}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3e\int \frac{2cd^2+2c(d+ex)d-ae^2}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}(ae-6cdx)}{2a(a-cx^2)}}{8ac}+\frac{\sqrt{d+ex}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{-\frac{3e\left(\frac{1}{2}\sqrt{c}\left(2\sqrt{c}d-\frac{4cd^2-ae^2}{\sqrt{a}e}\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}+\frac{1}{2}\sqrt{c}\left(\frac{4cd^2-ae^2}{\sqrt{a}e}+2\sqrt{c}d\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}\right)}{2a}-\frac{\sqrt{d+ex}(ae-6}{2a(a-cx^2}}{8ac}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{3e\left(-\frac{\left(2\sqrt{c}d-\frac{4cd^2-ae^2}{\sqrt{a}e}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\left(\frac{4cd^2-ae^2}{\sqrt{a}e}+2\sqrt{c}d\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2a}-\frac{\sqrt{d+ex}(ae-6cdx)}{2a(a-cx^2)}}{8ac}+\frac{\sqrt{d+ex}(ae+cdx)}{4ac\left(a-cx^2\right)^2}$$

input
```
Int[(d + e*x)^(3/2)/(a - c*x^2)^3,x]
```

output
```
((a*e + c*d*x)*Sqrt[d + e*x])/(4*a*c*(a - c*x^2)^2) + (-1/2*((a*e - 6*c*d*
x)*Sqrt[d + e*x])/(a*(a - c*x^2)) - (3*e*(-1/2*((2*Sqrt[c]*d - (4*c*d^2 -
a*e^2)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[
a]*e]])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) - ((2*Sqrt[c]*d + (4*c*d^2 -
 a*e^2)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt
[a]*e]])/(2*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(2*a))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-\frac{3e\sqrt{(cd+\sqrt{ac\,e^2})c}\,(-c\,x^2+a)^2 c\left(a\,e^2-4c\,d^2+2\sqrt{ac\,e^2}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{(-cd+\sqrt{ac\,e^2})c}}\right)}{32}+\frac{3\sqrt{(-cd+\sqrt{ac\,e^2})c}\left(-\frac{ec(-c\,x^2+a)\cdots}{\cdots}\right.}{\cdots}}{a^2c(-c\,x^2+a)^2\sqrt{ac\,e^2}\sqrt{(cd+\cdots}}$
default	$2e^5\left(\frac{-\frac{3cd(ex+d)^{\frac{7}{2}}}{16a^2e^4}+\frac{(a\,e^2+18c\,d^2)(ex+d)^{\frac{5}{2}}}{32a^2e^4}+\frac{d(4a\,e^2-9c\,d^2)(ex+d)^{\frac{3}{2}}}{16a^2e^4}+\frac{3(a\,e^2-c\,d^2)(a\,e^2-2c\,d^2)\sqrt{ex+d}}{32a^2e^4c}}{\left(-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2\right)^2}+\frac{3(-a\,e^2+\cdots}{\cdots}\right.$
derivativedivides	$-2e^5\left(-\frac{-\frac{3cd(ex+d)^{\frac{7}{2}}}{16a^2e^4}+\frac{(a\,e^2+18c\,d^2)(ex+d)^{\frac{5}{2}}}{32a^2e^4}+\frac{d(4a\,e^2-9c\,d^2)(ex+d)^{\frac{3}{2}}}{16a^2e^4}+\frac{3(a\,e^2-c\,d^2)(a\,e^2-2c\,d^2)\sqrt{ex+d}}{32a^2e^4c}}{\left(-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2\right)^2}-\frac{3\left(\cdots\right.}{\cdots}\right.$

input

```
int((e*x+d)^(3/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output

```
3/16*(-1/2*e*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-c*x^2+a)^2*c*(a*e^2-4*c*d^2
+2*(a*c*e^2)^(1/2)*d)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1
/2))+((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-1/2*e*c*(-c*x^2+a)^2*(a*e^2-4*c*d^
2-2*(a*c*e^2)^(1/2)*d)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(
1/2))+(-2*c^2*d*x^3+10/3*x*(1/10*e*x+d)*a*c+a^2*e)*((c*d+(a*c*e^2)^(1/2))*
c)^(1/2)*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)))/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(
1/2))*c)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a^2/c/(-c*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1753 vs. 2(211) = 422.

Time = 0.14 (sec) , antiderivative size = 1753, normalized size of antiderivative = 6.54

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((16*c^2*d^5 - 20*a*c*d^
3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4 -
 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))*log(27*(1
6*c^2*d^4*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d^
2*e^6 - a^4*c*e^8 - (4*a^5*c^6*d^5 - 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4)*
sqrt(e^10/(a^5*c^7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt((16*c^2*d
^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/
(a^5*c^7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e
^2))) - 3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((16*c^2*d^5 - 20*a*c*
d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d^4
 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))*log(27*
(16*c^2*d^4*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*
d^2*e^6 - a^4*c*e^8 - (4*a^5*c^6*d^5 - 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4
)*sqrt(e^10/(a^5*c^7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt((16*c^2
*d^5 - 20*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^1
0/(a^5*c^7*d^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2
*e^2))) + 3*(a^2*c^3*x^4 - 2*a^3*c^2*x^2 + a^4*c)*sqrt((16*c^2*d^5 - 20*a*
c*d^3*e^2 + 5*a^2*d*e^4 - (a^5*c^3*d^2 - a^6*c^2*e^2)*sqrt(e^10/(a^5*c^7*d
^4 - 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 - a^6*c^2*e^2))*log(2
7*(16*c^2*d^4*e^5 - 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \int -\frac{(ex+d)^{\frac{3}{2}}}{(cx^2-a)^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((e*x + d)^(3/2)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(211) = 422.

Time = 0.25 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.87

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \frac{3\left(4\,c^3d^3e-3\,ac^2de^3-\left(2\,\sqrt{ac}cd^2e-\sqrt{ac}ae^3\right)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{a^2c^2d+\sqrt{a^4c^4d^2-\left(a^2c^2d^2-a^3ce^2\right)a^2c^2}}{a^2c^2}}}\right)}{32\left(a^3c^2e-\sqrt{ac}a^2c^2d\right)\sqrt{-c^2d-\sqrt{ac}ce}|e|} + \frac{3\left(4\,c^3d^3e-3\,ac^2de^3+\left(2\,\sqrt{ac}cd^2e-\sqrt{ac}ae^3\right)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{a^2c^2d-\sqrt{a^4c^4d^2-\left(a^2c^2d^2-a^3ce^2\right)a^2c^2}}{a^2c^2}}}\right)}{32\left(a^3c^2e+\sqrt{ac}a^2c^2d\right)\sqrt{-c^2d+\sqrt{ac}ce}|e|} - \frac{6\,(ex+d)^{\frac{7}{2}}c^2de-18\,(ex+d)^{\frac{5}{2}}c^2d^2e+18\,(ex+d)^{\frac{3}{2}}c^2d^3e-6\,\sqrt{ex+d}c^2d^4e-(ex+d)^{\frac{5}{2}}ace^3-8\,(ex+d)^{\frac{3}{2}}acde^3+9\,\sqrt{ex+d}acd^2e^3-3\,\sqrt{ex+d}a^2e^5}{16\left((ex+d)^2c-2\,(ex+d)cd+cd^2-ae^2\right)^2a^2c}$$

input
```
integrate((e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(4*c^3*d^3*e - 3*a*c^2*d*e^3 - (2*sqrt(a*c)*c*d^2*e - sqrt(a*c)*a*e^3
)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^2*d + sqrt(a^4*c^4*d^2
- (a^2*c^2*d^2 - a^3*c*e^2)*a^2*c^2))/(a^2*c^2)))/((a^3*c^2*e - sqrt(a*c)*
a^2*c^2*d)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(e)) + 3/32*(4*c^3*d^3*e - 3*a*
c^2*d*e^3 + (2*sqrt(a*c)*c*d^2*e - sqrt(a*c)*a*e^3)*abs(c)*abs(e))*arctan(
sqrt(e*x + d)/sqrt(-(a^2*c^2*d - sqrt(a^4*c^4*d^2 - (a^2*c^2*d^2 - a^3*c*e
^2)*a^2*c^2))/(a^2*c^2)))/((a^3*c^2*e + sqrt(a*c)*a^2*c^2*d)*sqrt(-c^2*d +
 sqrt(a*c)*c*e)*abs(e)) - 1/16*(6*(e*x + d)^(7/2)*c^2*d*e - 18*(e*x + d)^(
5/2)*c^2*d^2*e + 18*(e*x + d)^(3/2)*c^2*d^3*e - 6*sqrt(e*x + d)*c^2*d^4*e
- (e*x + d)^(5/2)*a*c*e^3 - 8*(e*x + d)^(3/2)*a*c*d*e^3 + 9*sqrt(e*x + d)*
a*c*d^2*e^3 - 3*sqrt(e*x + d)*a^2*e^5)/(((e*x + d)^2*c - 2*(e*x + d)*c*d +
 c*d^2 - a*e^2)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 3191, normalized size of antiderivative = 11.91

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(a - c*x^2)^3,x)
```

output
```
(((4*a*d*e^3 - 9*c*d^3*e)*(d + e*x)^(3/2))/(8*a^2) + (e*(a*e^2 + 18*c*d^2)
*(d + e*x)^(5/2))/(16*a^2) + (3*(d + e*x)^(1/2)*(a^2*e^5 + 2*c^2*d^4*e - 3
*a*c*d^2*e^3))/(16*a^2*c) - (3*c*d*e*(d + e*x)^(7/2))/(8*a^2))/(c^2*(d + e
*x)^4 + a^2*e^4 + c^2*d^4 + (6*c^2*d^2 - 2*a*c*e^2)*(d + e*x)^2 - (4*c^2*d
^3 - 4*a*c*d*e^2)*(d + e*x) - 4*c^2*d*(d + e*x)^3 - 2*a*c*d^2*e^2) + atan(
((((3*(2048*a^6*c^2*e^5 - 4096*a^5*c^3*d^2*e^3))/(2048*a^6) - 64*a*c^4*d*e
^2*(d + e*x)^(1/2)*(-(9*(e^5*(a^15*c^5)^(1/2) - 16*a^5*c^5*d^5 - 5*a^7*c^3
*d*e^4 + 20*a^6*c^4*d^3*e^2))/(4096*(a^10*c^6*d^2 - a^11*c^5*e^2)))^(1/2))
*(-(9*(e^5*(a^15*c^5)^(1/2) - 16*a^5*c^5*d^5 - 5*a^7*c^3*d*e^4 + 20*a^6*c^
4*d^3*e^2))/(4096*(a^10*c^6*d^2 - a^11*c^5*e^2)))^(1/2) + ((d + e*x)^(1/2)
*(9*a^2*c*e^6 + 144*c^3*d^4*e^2 - 36*a*c^2*d^2*e^4))/(64*a^4))*(-(9*(e^5*(
a^15*c^5)^(1/2) - 16*a^5*c^5*d^5 - 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^3*e^2))/
(4096*(a^10*c^6*d^2 - a^11*c^5*e^2)))^(1/2)*1i - (((3*(2048*a^6*c^2*e^5 -
4096*a^5*c^3*d^2*e^3))/(2048*a^6) + 64*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(9*(e
^5*(a^15*c^5)^(1/2) - 16*a^5*c^5*d^5 - 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^3*e^
2))/(4096*(a^10*c^6*d^2 - a^11*c^5*e^2)))^(1/2))*(-(9*(e^5*(a^15*c^5)^(1/2
) - 16*a^5*c^5*d^5 - 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^3*e^2))/(4096*(a^10*c^
6*d^2 - a^11*c^5*e^2)))^(1/2) - ((d + e*x)^(1/2)*(9*a^2*c*e^6 + 144*c^3*d^
4*e^2 - 36*a*c^2*d^2*e^4))/(64*a^4))*(-(9*(e^5*(a^15*c^5)^(1/2) - 16*a^5*c
^5*d^5 - 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^3*e^2))/(4096*(a^10*c^6*d^2 - a...
```

Reduce [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 2012, normalized size of antiderivative = 7.51

$$\int \frac{(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)/(-c*x^2+a)^3,x)
```

output
```
( - 18*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*d*e**2 + 24*sqrt(a)*sqrt(sqrt(c)
*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)))*a**2*c**2*d**3 + 36*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d*e**2*x
**2 - 48*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**3*x**2 - 18*sqrt(a)*sqrt(sq
rt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a
)*e - c*d)))*a*c**3*d*e**2*x**4 + 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)
*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**4*d**3
*x**4 - 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**4*e**3 + 12*sqrt(c)*sqrt(sqrt(c)*
sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
c*d)))*a**3*c*d**2*e + 12*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt
(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*e**3*x**2 - 2
4*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sq
rt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d**2*e*x**2 - 6*sqrt(c)*sqrt(sqrt(
c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
 - c*d)))*a**2*c**2*e**3*x**4 + 12*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*a
tan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d...
```

3.151 $\int \frac{\sqrt{d+ex}}{(a-cx^2)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 281

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3} \, dx = \frac{x\sqrt{d+ex}}{4a(a-cx^2)^2} - \frac{\sqrt{d+ex}(ade-(6cd^2-5ae^2)x)}{16a^2(cd^2-ae^2)(a-cx^2)} - \frac{(12cd^2-18\sqrt{a}\sqrt{c}de+5ae^2)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} + \frac{(12cd^2+18\sqrt{a}\sqrt{c}de+5ae^2)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

output
```
1/4*x*(e*x+d)^(1/2)/a/(-c*x^2+a)^2-1/16*(e*x+d)^(1/2)*(a*d*e-(-5*a*e^2+6*c
*d^2)*x)/a^2/(-a*e^2+c*d^2)/(-c*x^2+a)-1/32*(12*c*d^2-18*a^(1/2)*c^(1/2)*d
*e+5*a*e^2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(
5/2)/c^(3/4)/(c^(1/2)*d-a^(1/2)*e)^(3/2)+1/32*(12*c*d^2+18*a^(1/2)*c^(1/2)
*d*e+5*a*e^2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a
^(5/2)/c^(3/4)/(c^(1/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx$$
$$= \frac{\frac{2\sqrt{a}\sqrt{d+ex}(6c^2d^2x^3+a^2e(d+9ex)-acx(10d^2+dex+5e^2x^2))}{(-cd^2+ae^2)(a-cx^2)^2} - \frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}(12cd^2+18\sqrt{a}\sqrt{c}de+5ae^2)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{c(\sqrt{c}d+\sqrt{a}e)^2} -}{32a^{5/2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a - c*x^2)^3,x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(6*c^2*d^2*x^3 + a^2*e*(d + 9*e*x) - a*c*x*(10*d
^2 + d*e*x + 5*e^2*x^2)))/((-(c*d^2) + a*e^2)*(a - c*x^2)^2) - (Sqrt[-(c*d
) - Sqrt[a]*Sqrt[c]*e]*(12*c*d^2 + 18*Sqrt[a]*Sqrt[c]*d*e + 5*a*e^2)*ArcTa
n[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)
])/(c*(Sqrt[c]*d + Sqrt[a]*e)^2) - ((12*c*d^2 - 18*Sqrt[a]*Sqrt[c]*d*e + 5
*a*e^2)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d
 - Sqrt[a]*e)])/(Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-(c*d) + Sqrt[a]*Sqr
t[c]*e]))/(32*a^(5/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {494, 27, 686, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx$$
$$\Big\downarrow \text{494}$$
$$\frac{x\sqrt{d+ex}}{4a(a-cx^2)^2} - \frac{\int -\frac{6d+5ex}{2\sqrt{d+ex}(a-cx^2)^2}dx}{4a}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{6d+5ex}{\sqrt{d+ex}(a-cx^2)^2}dx}{8a}+\frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2}$$

$$\Big\downarrow \textbf{686}$$

$$\frac{-\frac{\int -\frac{c\left(d\left(12cd^2-13ae^2\right)+e\left(6cd^2-5ae^2\right)x\right)}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ade-x(6cd^2-5ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\int \frac{d\left(12cd^2-13ae^2\right)+e\left(6cd^2-5ae^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ade-x(6cd^2-5ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2}$$

$$\Big\downarrow \textbf{654}$$

$$\frac{\frac{\int -\frac{e\left(2d\left(3cd^2-4ae^2\right)+\left(6cd^2-5ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ade-x(6cd^2-5ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{-\frac{\int \frac{e\left(2d\left(3cd^2-4ae^2\right)+\left(6cd^2-5ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ade-x(6cd^2-5ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{e\int \frac{2d\left(3cd^2-4ae^2\right)+\left(6cd^2-5ae^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ade-x(6cd^2-5ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2}$$

$$\Big\downarrow \textbf{1480}$$

$$-\frac{e\left(\frac{1}{2}\left(-\frac{12c^{3/2}d^3}{\sqrt{a}e}+13\sqrt{a}\sqrt{c}de-5ae^2+6cd^2\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d}{2\sqrt{a}e}\right.}{2a(cd^2-ae^2)}$$

$$8a$$

$$\frac{x\sqrt{d+ex}}{4a\left(a-cx^2\right)^2}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{\frac{e\left(-\frac{(\sqrt{c}d-\sqrt{a}e)\left(18\sqrt{a}\sqrt{c}de+5ae^2+12cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\left(-\frac{12c^{3/2}d^3}{\sqrt{a}e}+13\sqrt{a}\sqrt{c}de-5ae^2+6cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+e}}{2a}}{8a}$$

$$\frac{x\sqrt{d+ex}}{4a\,(a-cx^2)^2}$$

input
```
Int[Sqrt[d + e*x]/(a - c*x^2)^3,x]
```

output
```
(x*Sqrt[d + e*x])/(4*a*(a - c*x^2)^2) + (-1/2*(Sqrt[d + e*x]*(a*d*e - (6*c
*d^2 - 5*a*e^2)*x))/(a*(c*d^2 - a*e^2)*(a - c*x^2)) - (e*(-1/2*((6*c*d^2 -
 (12*c^(3/2)*d^3)/(Sqrt[a]*e) + 13*Sqrt[a]*Sqrt[c]*d*e - 5*a*e^2)*ArcTanh[
(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(c^(3/4)*Sqrt[Sqrt[c
]*d - Sqrt[a]*e]) - ((Sqrt[c]*d - Sqrt[a]*e)*(12*c*d^2 + 18*Sqrt[a]*Sqrt[c
]*d*e + 5*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*
e]])/(2*Sqrt[a]*c^(3/4)*e*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(2*a*(c*d^2 - a*e
^2)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{13e^4\left(\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\,e\left(-c\,x^2+a\right)^2\left(\frac{\left(-5a\,e^2+6c\,d^2\right)\sqrt{ac\,e^2}}{13}+cd\left(a\,e^2-\frac{12c\,d^2}{13}\right)\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)+\left(e(-\cdots\right.\right.}{32\sqrt{ac\,e\cdots}}$
derivativedivides	$-2e^5c^3\left(-\frac{\frac{-\frac{\sqrt{ac\,e^2}\left(6cd+5\sqrt{ac\,e^2}\right)(ex+d)^{\frac{3}{2}}}{4c^2\left(cd+\sqrt{ac\,e^2}\right)}+\frac{\sqrt{ac\,e^2}\left(6cd+7\sqrt{ac\,e^2}\right)\sqrt{ex+d}}{4c^3}}{\left(-ex+\frac{\sqrt{ac\,e^2}}{c}\right)^2}+\frac{\left(5a\,e^2+12c\,d^2+18\sqrt{ac\,e^2}\,d\right)\operatorname{arctanh}\left(\cdots\right)}{4\left(cd+\sqrt{ac\,e^2}\right)\sqrt{\left(cd+\sqrt{ac\,e^2}\cdots\right.}}}{16e^4a^2\sqrt{ac\,e^2}\,c^2}\cdots\right.$
default	$-2e^5c^3\left(-\frac{\frac{-\frac{\sqrt{ac\,e^2}\left(6cd+5\sqrt{ac\,e^2}\right)(ex+d)^{\frac{3}{2}}}{4c^2\left(cd+\sqrt{ac\,e^2}\right)}+\frac{\sqrt{ac\,e^2}\left(6cd+7\sqrt{ac\,e^2}\right)\sqrt{ex+d}}{4c^3}}{\left(-ex+\frac{\sqrt{ac\,e^2}}{c}\right)^2}+\frac{\left(5a\,e^2+12c\,d^2+18\sqrt{ac\,e^2}\,d\right)\operatorname{arctanh}\left(\cdots\right)}{4\left(cd+\sqrt{ac\,e^2}\right)\sqrt{\left(cd+\sqrt{ac\,e^2}\cdots\right.}}}{16e^4a^2\sqrt{ac\,e^2}\,c^2}\cdots\right.$

input
```
int((e*x+d)^(1/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
13/32*e^4*(((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*e*(-c*x^2+a)^2*(1/13*(-5*a*e^2+
6*c*d^2)*(a*c*e^2)^(1/2)+c*d*(a*e^2-12/13*c*d^2))*arctan(c*(e*x+d)^(1/2)/(
(-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(e*(-c*x^2+a)^2*(1/13*(5*a*e^2-6*c*d^2)*(
a*c*e^2)^(1/2)+c*d*(a*e^2-12/13*c*d^2))*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c
*e^2)^(1/2))*c)^(1/2))+2/13*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*((-5*a*c*x^3+9
*a^2*x)*e^2+a*d*(-c*x^2+a)*e-10*x*c*(-3/5*c*x^2+a)*d^2)*(e*x+d)^(1/2)*(a*c
*e^2)^(1/2))*((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))/(a*c*e^2)^(1/2)*c^2/(((c*d+(
a*c*e^2)^(1/2))*c)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a^2/(c*e*x-(a*c*
e^2)^(1/2))^2/(c*e*x+(a*c*e^2)^(1/2))^2/(a*e^2-c*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3787 vs. $2(223) = 446$.

Time = 0.46 (sec) , antiderivative size = 3787, normalized size of antiderivative = 13.48

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \int -\frac{\sqrt{ex+d}}{(cx^2-a)^3}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x + d)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(223) = 446$.

Time = 0.28 (sec) , antiderivative size = 1064, normalized size of antiderivative = 3.79

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/32*((a^2*c*d^2*e - a^3*e^3)^2*(6*c*d^2*e - 5*a*e^3)*abs(c) + 2*(3*sqrt(
a*c)*a*c^2*d^5*e - 7*sqrt(a*c)*a^2*c*d^3*e^3 + 4*sqrt(a*c)*a^3*d*e^5)*abs(
a^2*c*d^2*e - a^3*e^3)*abs(c) - (12*a^3*c^4*d^8*e - 37*a^4*c^3*d^6*e^3 + 3
8*a^5*c^2*d^4*e^5 - 13*a^6*c*d^2*e^7)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(
a^2*c^2*d^3 - a^3*c*d*e^2 + sqrt((a^2*c^2*d^3 - a^3*c*d*e^2)^2 - (a^2*c^2*
d^4 - 2*a^3*c*d^2*e^2 + a^4*e^4)*(a^2*c^2*d^2 - a^3*c*e^2)))/(a^2*c^2*d^2
- a^3*c*e^2)))/((a^4*c^3*d^4*e - 2*a^5*c^2*d^2*e^3 + a^6*c*e^5 - sqrt(a*c)
*a^3*c^3*d^5 + 2*sqrt(a*c)*a^4*c^2*d^3*e^2 - sqrt(a*c)*a^5*c*d*e^4)*sqrt(-
c^2*d - sqrt(a*c)*c*e)*abs(a^2*c*d^2*e - a^3*e^3)) - 1/32*((a^2*c*d^2*e -
a^3*e^3)^2*(6*c*d^2*e - 5*a*e^3)*abs(c) - 2*(3*sqrt(a*c)*a*c^2*d^5*e - 7*s
qrt(a*c)*a^2*c*d^3*e^3 + 4*sqrt(a*c)*a^3*d*e^5)*abs(a^2*c*d^2*e - a^3*e^3)
*abs(c) - (12*a^3*c^4*d^8*e - 37*a^4*c^3*d^6*e^3 + 38*a^5*c^2*d^4*e^5 - 13
*a^6*c*d^2*e^7)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^2*d^3 - a^3*c*d*
e^2 - sqrt((a^2*c^2*d^3 - a^3*c*d*e^2)^2 - (a^2*c^2*d^4 - 2*a^3*c*d^2*e^2
+ a^4*e^4)*(a^2*c^2*d^2 - a^3*c*e^2)))/(a^2*c^2*d^2 - a^3*c*e^2)))/((a^4*c
^3*d^4*e - 2*a^5*c^2*d^2*e^3 + a^6*c*e^5 + sqrt(a*c)*a^3*c^3*d^5 - 2*sqrt(
a*c)*a^4*c^2*d^3*e^2 + sqrt(a*c)*a^5*c*d*e^4)*sqrt(-c^2*d + sqrt(a*c)*c*e)
*abs(a^2*c*d^2*e - a^3*e^3)) - 1/16*(6*(e*x + d)^(7/2)*c^2*d^2*e - 18*(e*x
 + d)^(5/2)*c^2*d^3*e + 18*(e*x + d)^(3/2)*c^2*d^4*e - 6*sqrt(e*x + d)*c^2
*d^5*e - 5*(e*x + d)^(7/2)*a*c*e^3 + 14*(e*x + d)^(5/2)*a*c*d*e^3 - 23*...
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 6163, normalized size of antiderivative = 21.93

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(a - c*x^2)^3,x)
```

output
```
- atan(((((32768*a^7*c^3*d*e^7 + 24576*a^5*c^5*d^5*e^3 - 57344*a^6*c^4*d^3
*e^5)/(4096*(a^8*e^4 + a^6*c^2*d^4 - 2*a^7*c*d^2*e^2)) - ((d + e*x)^(1/2)*
(4096*a^7*c^4*d*e^6 + 4096*a^5*c^6*d^5*e^2 - 8192*a^6*c^5*d^3*e^4)*((144*a
^5*c^5*d^7 - 25*a*e^7*(a^15*c^3)^(1/2) - 105*a^8*c^2*d*e^6 - 420*a^6*c^4*d
^5*e^2 + 385*a^7*c^3*d^3*e^4 + 21*c*d^2*e^5*(a^15*c^3)^(1/2))/(4096*(a^10*
c^6*d^6 - a^13*c^3*e^6 - 3*a^11*c^5*d^4*e^2 + 3*a^12*c^4*d^2*e^4)))^(1/2))
/(64*(a^6*e^4 + a^4*c^2*d^4 - 2*a^5*c*d^2*e^2)))*((144*a^5*c^5*d^7 - 25*a*
e^7*(a^15*c^3)^(1/2) - 105*a^8*c^2*d*e^6 - 420*a^6*c^4*d^5*e^2 + 385*a^7*c
^3*d^3*e^4 + 21*c*d^2*e^5*(a^15*c^3)^(1/2))/(4096*(a^10*c^6*d^6 - a^13*c^3
*e^6 - 3*a^11*c^5*d^4*e^2 + 3*a^12*c^4*d^2*e^4)))^(1/2) + ((d + e*x)^(1/2)
*(25*a^3*c^2*e^8 + 144*c^5*d^6*e^2 - 276*a*c^4*d^4*e^4 + 109*a^2*c^3*d^2*e
^6))/(64*(a^6*e^4 + a^4*c^2*d^4 - 2*a^5*c*d^2*e^2)))*((144*a^5*c^5*d^7 - 2
5*a*e^7*(a^15*c^3)^(1/2) - 105*a^8*c^2*d*e^6 - 420*a^6*c^4*d^5*e^2 + 385*a
^7*c^3*d^3*e^4 + 21*c*d^2*e^5*(a^15*c^3)^(1/2))/(4096*(a^10*c^6*d^6 - a^13
*c^3*e^6 - 3*a^11*c^5*d^4*e^2 + 3*a^12*c^4*d^2*e^4)))^(1/2)*1i - (((32768*
a^7*c^3*d*e^7 + 24576*a^5*c^5*d^5*e^3 - 57344*a^6*c^4*d^3*e^5)/(4096*(a^8*
e^4 + a^6*c^2*d^4 - 2*a^7*c*d^2*e^2)) + ((d + e*x)^(1/2)*(4096*a^7*c^4*d*e
^6 + 4096*a^5*c^6*d^5*e^2 - 8192*a^6*c^5*d^3*e^4)*((144*a^5*c^5*d^7 - 25*a
*e^7*(a^15*c^3)^(1/2) - 105*a^8*c^2*d*e^6 - 420*a^6*c^4*d^5*e^2 + 385*a^7*
c^3*d^3*e^4 + 21*c*d^2*e^5*(a^15*c^3)^(1/2))/(4096*(a^10*c^6*d^6 - a^13...
```

Reduce [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 2568, normalized size of antiderivative = 9.14

$$\int \frac{\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)/(-c*x^2+a)^3,x)
```

output
```
( - 10*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**4*e**4 + 38*sqrt(a)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d
)))*a**3*c*d**2*e**2 + 20*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt
(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*e**4*x**2 - 2
4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sq
rt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d**4 - 76*sqrt(a)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d
)))*a**2*c**2*d**2*e**2*x**2 - 10*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*e*
*4*x**4 + 48*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/
(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**4*x**2 + 38*sqrt(a)*sqr
t(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sq
rt(a)*e - c*d)))*a*c**3*d**2*e**2*x**4 - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e
 - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c*
*4*d**4*x**4 + 16*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**4*d*e**3 - 12*sqrt(c)*sqr
t(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sq
rt(a)*e - c*d)))*a**3*c*d**3*e - 32*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*
atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*...
```

3.152 $\int \frac{1}{\sqrt{d+ex}(a-cx^2)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 315

$$\int \frac{1}{\sqrt{d+ex}(a-cx^2)^3} dx = -\frac{(ae-cdx)\sqrt{d+ex}}{4a(cd^2-ae^2)(a-cx^2)^2} - \frac{\sqrt{d+ex}(ae(cd^2-7ae^2)-6cd(cd^2-2ae^2)x)}{16a^2(cd^2-ae^2)^2(a-cx^2)} - \frac{3(4cd^2-10\sqrt{a}\sqrt{c}de+7ae^2)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)^{5/2}} + \frac{3(4cd^2+10\sqrt{a}\sqrt{c}de+7ae^2)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)^{5/2}}$$

output
```
-1/4*(-c*d*x+a*e)*(e*x+d)^(1/2)/a/(-a*e^2+c*d^2)/(-c*x^2+a)^2-1/16*(e*x+d)
^(1/2)*(a*e*(-7*a*e^2+c*d^2)-6*c*d*(-2*a*e^2+c*d^2)*x)/a^2/(-a*e^2+c*d^2)^
2/(-c*x^2+a)-3/32*(4*c*d^2-10*a^(1/2)*c^(1/2)*d*e+7*a*e^2)*arctanh(c^(1/4)
*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(5/2)/c^(1/4)/(c^(1/2)*d-a^(
1/2)*e)^(5/2)+3/32*(4*c*d^2+10*a^(1/2)*c^(1/2)*d*e+7*a*e^2)*arctanh(c^(1/4
)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(5/2)/c^(1/4)/(c^(1/2)*d+a^
(1/2)*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx$$
$$= \frac{-\frac{2\sqrt{a}\sqrt{d+ex}(-11a^3e^3+6c^3d^3x^3+a^2ce(5d^2+16dex+7e^2x^2)-ac^2dx(10d^2+dex+12e^2x^2))}{(cd^2-ae^2)^2(a-cx^2)^2}+\frac{3(4cd^2+10\sqrt{a}\sqrt{c}de+7ae^2)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}}}{\sqrt{cd-}}\right.}{(\sqrt{c}d+\sqrt{a}e)^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}}}{32a^{5/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a - c*x^2)^3),x]
```

output
```
((-2*Sqrt[a]*Sqrt[d + e*x]*(-11*a^3*e^3 + 6*c^3*d^3*x^3 + a^2*c*e*(5*d^2 +
 16*d*e*x + 7*e^2*x^2) - a*c^2*d*x*(10*d^2 + d*e*x + 12*e^2*x^2)))/((c*d^2
 - a*e^2)^2*(a - c*x^2)^2) + (3*(4*c*d^2 + 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*e^
2)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sq
rt[a]*e)])/((Sqrt[c]*d + Sqrt[a]*e)^2*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]) -
(3*(4*c*d^2 - 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*e^2)*ArcTan[(Sqrt[-(c*d) + Sqrt
[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/((Sqrt[c]*d - Sqrt
[a]*e)^2*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/(32*a^(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {496, 27, 686, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-cx^2)^3\sqrt{d+ex}}dx$$
$$\Big\downarrow \ 496$$
$$\frac{\int \frac{6cd^2+5cexd-7ae^2}{2\sqrt{d+ex}(a-cx^2)^2}dx}{4a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-cdx)}{4a(a-cx^2)^2(cd^2-ae^2)}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{6cd^2+5cexd-7ae^2}{\sqrt{d+ex}(a-cx^2)^2}dx}{8a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-cdx)}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \text{686}$$

$$\frac{-\frac{\int -\frac{3c\left(4c^2d^4-9ace^2d^2+2ce\left(cd^2-2ae^2\right)xd+7a^2e^4\right)}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(cd^2-7ae^2)-6cdx(cd^2-2ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-cdx)}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{4c^2d^4-9ace^2d^2+2ce\left(cd^2-2ae^2\right)xd+7a^2e^4}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(cd^2-7ae^2)-6cdx(cd^2-2ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-cdx)}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{3\int -\frac{e\left(2c^2d^4-5ace^2d^2+2c\left(cd^2-2ae^2\right)(d+ex)d+7a^2e^4\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(cd^2-7ae^2)-6cdx(cd^2-2ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-cdx)}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{3\int \frac{e\left(2c^2d^4-5ace^2d^2+2c\left(cd^2-2ae^2\right)(d+ex)d+7a^2e^4\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(cd^2-7ae^2)-6cdx(cd^2-2ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-cdx)}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3e\int \frac{2c^2d^4-5ace^2d^2+2c\left(cd^2-2ae^2\right)(d+ex)d+7a^2e^4}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(cd^2-7ae^2)-6cdx(cd^2-2ae^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae-cdx)}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{3e\left(\frac{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)^2\left(10\sqrt{a}\sqrt{c}de+7ae^2+4cd^2\right)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}(\sqrt{a}e+\sqrt{c}d)^2\left(-10\sqrt{a}\sqrt{c}de+7ae^2+4cd^2\right)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}}{2\sqrt{a}e}\right.}{2a(cd^2-ae^2)}}{8a\left(cd^2-ae^2\right)}$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{4a\left(a-cx^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{3e\left(\frac{(\sqrt{a}e+\sqrt{c}d)^2\left(-10\sqrt{a}\sqrt{c}de+7ae^2+4cd^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{(\sqrt{c}d-\sqrt{a}e)^2\left(10\sqrt{a}\sqrt{c}de+7ae^2+4cd^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+e}}{}}{8a\left(cd^2-ae^2\right)}$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{4a\left(a-cx^2\right)^2\left(cd^2-ae^2\right)}$$

input
```
Int[1/(Sqrt[d + e*x]*(a - c*x^2)^3),x]
```

output
```
-1/4*((a*e - c*d*x)*Sqrt[d + e*x])/(a*(c*d^2 - a*e^2)*(a - c*x^2)^2) + (-1
/2*(Sqrt[d + e*x]*(a*e*(c*d^2 - 7*a*e^2) - 6*c*d*(c*d^2 - 2*a*e^2)*x))/(a*
(c*d^2 - a*e^2)*(a - c*x^2)) - (3*e*(((Sqrt[c]*d + Sqrt[a]*e)^2*(4*c*d^2 -
 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*e^2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) -
 ((Sqrt[c]*d - Sqrt[a]*e)^2*(4*c*d^2 + 10*Sqrt[a]*Sqrt[c]*d*e + 7*a*e^2)*A
rcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^
(1/4)*e*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(2*a*(c*d^2 - a*e^2)))/(8*a*(c*d^2
- a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.60

method	result
derivativedivides	$-2e^5c^3\left(-\frac{\frac{-\frac{3\sqrt{ac\,e^2}\left(2cd+3\sqrt{ac\,e^2}\right)(ex+d)^{\frac{3}{2}}}{4c^3\left(a\,e^2+c\,d^2+2\sqrt{ac\,e^2}\,d\right)}+\frac{\sqrt{ac\,e^2}\left(6cd+11\sqrt{ac\,e^2}\right)\sqrt{ex+d}}{4c^3\left(cd+\sqrt{ac\,e^2}\right)}}{\left(-ex+\frac{\sqrt{ac\,e^2}}{c}\right)^2}+\frac{3\left(7a\,e^2+4c\,d^2+10\sqrt{ac\,e^2}\,d\right)\operatorname{arctanh}\left(\cdots\right)}{4c\left(a\,e^2+c\,d^2+2\sqrt{ac\,e^2}\,d\right)\sqrt{\left(cd\cdots\right.}}}{16c\,e^4a^2\sqrt{ac\,e^2}}\cdots\right.$
default	$-2e^5c^3\left(-\frac{\frac{-\frac{3\sqrt{ac\,e^2}\left(2cd+3\sqrt{ac\,e^2}\right)(ex+d)^{\frac{3}{2}}}{4c^3\left(a\,e^2+c\,d^2+2\sqrt{ac\,e^2}\,d\right)}+\frac{\sqrt{ac\,e^2}\left(6cd+11\sqrt{ac\,e^2}\right)\sqrt{ex+d}}{4c^3\left(cd+\sqrt{ac\,e^2}\right)}}{\left(-ex+\frac{\sqrt{ac\,e^2}}{c}\right)^2}+\frac{3\left(7a\,e^2+4c\,d^2+10\sqrt{ac\,e^2}\,d\right)\operatorname{arctanh}\left(\cdots\right)}{4c\left(a\,e^2+c\,d^2+2\sqrt{ac\,e^2}\,d\right)\sqrt{\left(cd\cdots\right.}}}{16c\,e^4a^2\sqrt{ac\,e^2}}\cdots\right.$
pseudoelliptic	$-\frac{e^5c^3\left(\frac{\frac{\sqrt{ac\,e^2}\left(\frac{3\left(2cd+3\sqrt{ac\,e^2}\right)(ex+d)^{\frac{3}{2}}}{a\,e^2+c\,d^2+2\sqrt{ac\,e^2}\,d}-\frac{\left(6cd+11\sqrt{ac\,e^2}\right)\sqrt{ex+d}}{cd+\sqrt{ac\,e^2}}\right)}{\left(-cex+\sqrt{ac\,e^2}\right)^2}-\frac{3\left(7a\,e^2+4c\,d^2+10\sqrt{ac\,e^2}\,d\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^2}\right)\cdots}}\right)}{\left(a\,e^2+c\,d^2+2\sqrt{ac\,e^2}\,d\right)\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}}{4c^2e^4a^2\sqrt{ac\,e^2}}\cdots\right.}{\cdots}$

input
```
int(1/(e*x+d)^(1/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*e^5*c^3*(-1/16/c/e^4/a^2/(a*c*e^2)^(1/2)*((-3/4*(a*c*e^2)^(1/2)/c^3*(2*
c*d+3*(a*c*e^2)^(1/2))/(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)*(e*x+d)^(3/2)+1/4
*(a*c*e^2)^(1/2)/c^3*(6*c*d+11*(a*c*e^2)^(1/2))/(c*d+(a*c*e^2)^(1/2))*(e*x
+d)^(1/2))/(-e*x+(a*c*e^2)^(1/2)/c)^2+3/4*(7*a*e^2+4*c*d^2+10*(a*c*e^2)^(1
/2)*d)/c/(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)
*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)))+1/16/c/e^4/a^2/
(a*c*e^2)^(1/2)*((3/4*(a*c*e^2)^(1/2)/c^3*(2*c*d-3*(a*c*e^2)^(1/2))/(a*e^2
+c*d^2-2*(a*c*e^2)^(1/2)*d)*(e*x+d)^(3/2)-1/4*(a*c*e^2)^(1/2)/c^3*(6*c*d-1
1*(a*c*e^2)^(1/2))/(c*d-(a*c*e^2)^(1/2))*(e*x+d)^(1/2))/(-e*x-(a*c*e^2)^(1
/2)/c)^2-3/4*(-7*a*e^2-4*c*d^2+10*(a*c*e^2)^(1/2)*d)/c/(-a*e^2-c*d^2+2*(a*
c*e^2)^(1/2)*d)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((
-c*d+(a*c*e^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5776 vs. 2(258) = 516.

Time = 2.31 (sec) , antiderivative size = 5776, normalized size of antiderivative = 18.34

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**(1/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \int -\frac{1}{(cx^2-a)^3\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^2 - a)^3*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1642 vs. $2(258) = 516$.

Time = 0.27 (sec) , antiderivative size = 1642, normalized size of antiderivative = 5.21

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/32*(2*(a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)^2*(c^2*d^3*e - 2*a*c*
d*e^3)*abs(c) + (2*sqrt(a*c)*a*c^4*d^8*e - 9*sqrt(a*c)*a^2*c^3*d^6*e^3 + 1
9*sqrt(a*c)*a^3*c^2*d^4*e^5 - 19*sqrt(a*c)*a^4*c*d^2*e^7 + 7*sqrt(a*c)*a^5
*e^9)*abs(a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)*abs(c) - (4*a^3*c^7*d
^13*e - 25*a^4*c^6*d^11*e^3 + 67*a^5*c^5*d^9*e^5 - 98*a^6*c^4*d^7*e^7 + 82
*a^7*c^3*d^5*e^9 - 37*a^8*c^2*d^3*e^11 + 7*a^9*c*d*e^13)*abs(c))*arctan(sq
rt(e*x + d)/sqrt(-(a^2*c^3*d^5 - 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4 + sqrt((a
^2*c^3*d^5 - 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)^2 - (a^2*c^3*d^6 - 3*a^3*c^2
*d^4*e^2 + 3*a^4*c*d^2*e^4 - a^5*e^6)*(a^2*c^3*d^4 - 2*a^3*c^2*d^2*e^2 + a
^4*c*e^4)))/(a^2*c^3*d^4 - 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)))/((a^4*c^5*d^8*
e - 4*a^5*c^4*d^6*e^3 + 6*a^6*c^3*d^4*e^5 - 4*a^7*c^2*d^2*e^7 + a^8*c*e^9
- sqrt(a*c)*a^3*c^5*d^9 + 4*sqrt(a*c)*a^4*c^4*d^7*e^2 - 6*sqrt(a*c)*a^5*c^
3*d^5*e^4 + 4*sqrt(a*c)*a^6*c^2*d^3*e^6 - sqrt(a*c)*a^7*c*d*e^8)*sqrt(-c^2
*d - sqrt(a*c)*c*e)*abs(a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)) - 3/32
*(2*(a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)^2*(sqrt(a*c)*c*d^3*e - 2*s
qrt(a*c)*a*d*e^3)*abs(c) - (2*a^2*c^4*d^8*e - 9*a^3*c^3*d^6*e^3 + 19*a^4*c
^2*d^4*e^5 - 19*a^5*c*d^2*e^7 + 7*a^6*e^9)*abs(a^2*c^2*d^4*e - 2*a^3*c*d^2
*e^3 + a^4*e^5)*abs(c) - (4*sqrt(a*c)*a^3*c^6*d^13*e - 25*sqrt(a*c)*a^4*c^
5*d^11*e^3 + 67*sqrt(a*c)*a^5*c^4*d^9*e^5 - 98*sqrt(a*c)*a^6*c^3*d^7*e^7 +
 82*sqrt(a*c)*a^7*c^2*d^5*e^9 - 37*sqrt(a*c)*a^8*c*d^3*e^11 + 7*sqrt(a*...
```

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 8961, normalized size of antiderivative = 28.45

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a - c*x^2)^3*(d + e*x)^(1/2)),x)
```

output
```
- atan(((((3*(14336*a^9*c^3*e^11 + 4096*a^5*c^7*d^8*e^3 - 18432*a^6*c^6*d^
6*e^5 + 38912*a^7*c^5*d^4*e^7 - 38912*a^8*c^4*d^2*e^9))/(2048*(a^10*e^8 +
a^6*c^4*d^8 - 4*a^9*c*d^2*e^6 - 4*a^7*c^3*d^6*e^2 + 6*a^8*c^2*d^4*e^4)) -
((d + e*x)^(1/2)*(-(9*(16*a^5*c^5*d^9 - 49*a^2*e^9*(a^15*c)^(1/2) - 84*a^6
*c^4*d^7*e^2 + 189*a^7*c^3*d^5*e^4 - 210*a^8*c^2*d^3*e^6 - 21*c^2*d^4*e^5*
(a^15*c)^(1/2) + 105*a^9*c*d*e^8 + 54*a*c*d^2*e^7*(a^15*c)^(1/2)))/(4096*(
a^15*c*e^10 - a^10*c^6*d^10 + 5*a^11*c^5*d^8*e^2 - 10*a^12*c^4*d^6*e^4 + 1
0*a^13*c^3*d^4*e^6 - 5*a^14*c^2*d^2*e^8)))^(1/2)*(4096*a^9*c^4*d*e^10 + 40
96*a^5*c^8*d^9*e^2 - 16384*a^6*c^7*d^7*e^4 + 24576*a^7*c^6*d^5*e^6 - 16384
*a^8*c^5*d^3*e^8))/(64*(a^8*e^8 + a^4*c^4*d^8 - 4*a^7*c*d^2*e^6 - 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-(9*(16*a^5*c^5*d^9 - 49*a^2*e^9*(a^15*c
)^(1/2) - 84*a^6*c^4*d^7*e^2 + 189*a^7*c^3*d^5*e^4 - 210*a^8*c^2*d^3*e^6 -
 21*c^2*d^4*e^5*(a^15*c)^(1/2) + 105*a^9*c*d*e^8 + 54*a*c*d^2*e^7*(a^15*c)
^(1/2)))/(4096*(a^15*c*e^10 - a^10*c^6*d^10 + 5*a^11*c^5*d^8*e^2 - 10*a^12
*c^4*d^6*e^4 + 10*a^13*c^3*d^4*e^6 - 5*a^14*c^2*d^2*e^8)))^(1/2) + ((d + e
*x)^(1/2)*(441*a^4*c^3*e^10 + 144*c^7*d^8*e^2 - 612*a*c^6*d^6*e^4 + 1089*a
^2*c^5*d^4*e^6 - 990*a^3*c^4*d^2*e^8))/(64*(a^8*e^8 + a^4*c^4*d^8 - 4*a^7*
c*d^2*e^6 - 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-(9*(16*a^5*c^5*d^9
- 49*a^2*e^9*(a^15*c)^(1/2) - 84*a^6*c^4*d^7*e^2 + 189*a^7*c^3*d^5*e^4 - 2
10*a^8*c^2*d^3*e^6 - 21*c^2*d^4*e^5*(a^15*c)^(1/2) + 105*a^9*c*d*e^8 + ...
```

Reduce [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 3161, normalized size of antiderivative = 10.03

$$\int \frac{1}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(-c*x^2+a)^3,x)
```

output
```
(66*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**4*c*d*e**4 - 66*sqrt(a)*sqrt(sqrt(c)*sq
rt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*
d)))*a**3*c**2*d**3*e**2 - 132*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan(
(sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c**2*d*e**
4*x**2 + 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(
sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**3*d**5 + 132*sqrt(a)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)))*a**2*c**3*d**3*e**2*x**2 + 66*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a
**2*c**3*d*e**4*x**4 - 48*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt
(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**4*d**5*x**2 - 6
6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sq
rt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**4*d**3*e**2*x**4 + 24*sqrt(a)*sqrt(sqrt
(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)))*c**5*d**5*x**4 + 42*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan(
(sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**5*e**5 - 30*
sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)))*a**4*c*d**2*e**3 - 84*sqrt(c)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - ...
```

3.153 $\int \frac{\sqrt{2+3x}}{1-x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{\sqrt{2+3x}}{1-x^2} dx = -\arctan\left(\sqrt{2+3x}\right) + \sqrt{5}\text{arctanh}\left(\frac{\sqrt{2+3x}}{\sqrt{5}}\right)$$

output
```
-arctan((2+3*x)^(1/2))+5^(1/2)*arctanh(1/5*(2+3*x)^(1/2)*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{2+3x}}{1-x^2} dx = -\arctan\left(\sqrt{2+3x}\right) + \sqrt{5}\text{arctanh}\left(\frac{\sqrt{2+3x}}{\sqrt{5}}\right)$$

input
```
Integrate[Sqrt[2 + 3*x]/(1 - x^2),x]
```

output
```
-ArcTan[Sqrt[2 + 3*x]] + Sqrt[5]*ArcTanh[Sqrt[2 + 3*x]/Sqrt[5]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {483, 1450, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}}{1-x^2}dx$$

$$\Big\downarrow \text{483}$$

$$6\int \frac{3x+2}{-(3x+2)^2+4(3x+2)+5}d\sqrt{3x+2}$$

$$\Big\downarrow \text{1450}$$

$$6\left(\frac{1}{6}\int \frac{1}{-3x-3}d\sqrt{3x+2}+\frac{5}{6}\int \frac{1}{3-3x}d\sqrt{3x+2}\right)$$

$$\Big\downarrow \text{217}$$

$$6\left(\frac{5}{6}\int \frac{1}{3-3x}d\sqrt{3x+2}-\frac{1}{6}\arctan\left(\sqrt{3x+2}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$6\left(\frac{1}{6}\sqrt{5}\text{arctanh}\left(\frac{\sqrt{3x+2}}{\sqrt{5}}\right)-\frac{1}{6}\arctan\left(\sqrt{3x+2}\right)\right)$$

input
```
Int[Sqrt[2 + 3*x]/(1 - x^2),x]
```

output
```
6*(-1/6*ArcTan[Sqrt[2 + 3*x]] + (Sqrt[5]*ArcTanh[Sqrt[2 + 3*x]/Sqrt[5]])/6
)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\arctan\left(\sqrt{3x+2}\right)+\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{3x+2}\,\sqrt{5}}{5}\right)$
default	$-\arctan\left(\sqrt{3x+2}\right)+\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{3x+2}\,\sqrt{5}}{5}\right)$
pseudoelliptic	$-\arctan\left(\sqrt{3x+2}\right)+\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{3x+2}\,\sqrt{5}}{5}\right)$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-5\right)x+7\operatorname{RootOf}\left(\_Z^2-5\right)-10\sqrt{3x+2}}{x-1}\right)}{2}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{3\operatorname{RootO}}{}\right.}{}$

input
```
int((3*x+2)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arctan((3*x+2)^(1/2))+5^(1/2)*arctanh(1/5*(3*x+2)^(1/2)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{2+3x}}{1-x^2}\,dx = \frac{1}{2}\sqrt{5}\log\left(\frac{2\sqrt{5}\sqrt{3x+2}+3x+7}{x-1}\right) - \arctan\left(\sqrt{3x+2}\right)$$

input
```
integrate((2+3*x)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(5)*log((2*sqrt(5)*sqrt(3*x + 2) + 3*x + 7)/(x - 1)) - arctan(sqrt
(3*x + 2))
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{2+3x}}{1-x^2}\,dx = -\frac{\sqrt{5}\left(\log\left(\sqrt{3x+2}-\sqrt{5}\right)-\log\left(\sqrt{3x+2}+\sqrt{5}\right)\right)}{2} - \operatorname{atan}\left(\sqrt{3x+2}\right)$$

input
```
integrate((2+3*x)**(1/2)/(-x**2+1),x)
```

output
```
-sqrt(5)*(log(sqrt(3*x + 2) - sqrt(5)) - log(sqrt(3*x + 2) + sqrt(5)))/2 -
 atan(sqrt(3*x + 2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{2+3x}}{1-x^2}\,dx = -\frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{3\,x+2}}{\sqrt{5}+\sqrt{3\,x+2}}\right) - \arctan\left(\sqrt{3\,x+2}\right)$$

input
```
integrate((2+3*x)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(5)*log(-(sqrt(5) - sqrt(3*x + 2))/(sqrt(5) + sqrt(3*x + 2))) - a
rctan(sqrt(3*x + 2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{2+3x}}{1-x^2}\,dx = -\frac{1}{2}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+2\sqrt{3\,x+2}\right|}{2\left(\sqrt{5}+\sqrt{3\,x+2}\right)}\right) - \arctan\left(\sqrt{3\,x+2}\right)$$

input
```
integrate((2+3*x)^(1/2)/(-x^2+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(5)*log(1/2*abs(-2*sqrt(5) + 2*sqrt(3*x + 2))/(sqrt(5) + sqrt(3*x
 + 2))) - arctan(sqrt(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{2+3x}}{1-x^2}\,dx = \sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{3\,x+2}}{5}\right) - \mathrm{atan}\left(\sqrt{3\,x+2}\right)$$

input
```
int(-(3*x + 2)^(1/2)/(x^2 - 1),x)
```

output
```
5^(1/2)*atanh((5^(1/2)*(3*x + 2)^(1/2))/5) - atan((3*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{2+3x}}{1-x^2}\,dx = -atan\left(\sqrt{3x+2}\right) - \frac{\sqrt{5}\log\left(\sqrt{3x+2}-\sqrt{5}\right)}{2} + \frac{\sqrt{5}\log\left(\sqrt{3x+2}+\sqrt{5}\right)}{2}$$

input
```
int((2+3*x)^(1/2)/(-x^2+1),x)
```

output
```
( - 2*atan(sqrt(3*x + 2)) - sqrt(5)*log(sqrt(3*x + 2) - sqrt(5)) + sqrt(5)
*log(sqrt(3*x + 2) + sqrt(5)))/2
```

3.154 $\int \frac{\sqrt{c+dx}}{1-x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt{c+dx}}{1-x^2} \, dx = -\sqrt{c-d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c-d}}\right)+\sqrt{c+d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c+d}}\right)$$

output
```
-(c-d)^(1/2)*arctanh((d*x+c)^(1/2)/(c-d)^(1/2))+(c+d)^(1/2)*arctanh((d*x+c
)^(1/2)/(c+d)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{c+dx}}{1-x^2} \, dx = \sqrt{-c-d}\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c-d}}\right)-\sqrt{-c+d}\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+d}}\right)$$

input
```
Integrate[Sqrt[c + d*x]/(1 - x^2),x]
```

output
```
Sqrt[-c - d]*ArcTan[Sqrt[c + d*x]/Sqrt[-c - d]] - Sqrt[-c + d]*ArcTan[Sqrt
[c + d*x]/Sqrt[-c + d]]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {483, 25, 1450, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{1-x^2}\,dx$$

$$\Big\downarrow \text{483}$$

$$2d\int -\frac{c+dx}{c^2-2(c+dx)c-d^2+(c+dx)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-2d\int \frac{c+dx}{c^2-2(c+dx)c-d^2+(c+dx)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{1450}$$

$$2d\left(-\frac{(c+d)\int \frac{1}{dx-d}d\sqrt{c+dx}}{2d}-\frac{1}{2}\left(1-\frac{c}{d}\right)\int \frac{1}{xd+d}d\sqrt{c+dx}\right)$$

$$\Big\downarrow \text{220}$$

$$2d\left(\frac{\left(1-\frac{c}{d}\right)\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c-d}}\right)}{2\sqrt{c-d}}+\frac{\sqrt{c+d}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c+d}}\right)}{2d}\right)$$

input
```
Int[Sqrt[c + d*x]/(1 - x^2),x]
```

output
```
2*d*(((1 - c/d)*ArcTanh[Sqrt[c + d*x]/Sqrt[c - d]])/(2*Sqrt[c - d]) + (Sqr
t[c + d]*ArcTanh[Sqrt[c + d*x]/Sqrt[c + d]])/(2*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\sqrt{-c+d}\, \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c+d}}\right)+\sqrt{c+d}\, \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c+d}}\right)$	47
derivativedivides	$-2d\left(-\frac{\sqrt{c+d}\, \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c+d}}\right)}{2d}+\frac{\sqrt{-c+d}\, \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c+d}}\right)}{2d}\right)$	57
default	$-2d\left(-\frac{\sqrt{c+d}\, \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c+d}}\right)}{2d}+\frac{\sqrt{-c+d}\, \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c+d}}\right)}{2d}\right)$	57

input
```
int((d*x+c)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-(-c+d)^(1/2)*arctan((d*x+c)^(1/2)/(-c+d)^(1/2))+(c+d)^(1/2)*arctanh((d*x+
c)^(1/2)/(c+d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 4.60

$$\int \frac{\sqrt{c+dx}}{1-x^2}\,dx = \left[\frac{1}{2}\sqrt{c-d}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c-d}+2\,c-d}{x+1}\right) + \frac{1}{2}\sqrt{c+d}\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c+d}+2\,c+d}{x-1}\right), \sqrt{-c+d}\arctan\left(\frac{\sqrt{-c+d}}{\sqrt{dx+c}}\right) + \frac{1}{2}\sqrt{c+d}\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c+d}+2\,c+d}{x-1}\right), -\sqrt{-c-d}\arctan\left(\frac{\sqrt{-c-d}}{\sqrt{dx+c}}\right) + \frac{1}{2}\sqrt{c-d}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c-d}+2\,c-d}{x+1}\right), \sqrt{-c+d}\arctan\left(\frac{\sqrt{-c+d}}{\sqrt{dx+c}}\right) - \sqrt{-c-d}\arctan\left(\frac{\sqrt{-c-d}}{\sqrt{dx+c}}\right)\right]$$

input
```
integrate((d*x+c)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
[1/2*sqrt(c - d)*log((d*x - 2*sqrt(d*x + c)*sqrt(c - d) + 2*c - d)/(x + 1)
) + 1/2*sqrt(c + d)*log((d*x + 2*sqrt(d*x + c)*sqrt(c + d) + 2*c + d)/(x -
 1)), sqrt(-c + d)*arctan(sqrt(-c + d)/sqrt(d*x + c)) + 1/2*sqrt(c + d)*lo
g((d*x + 2*sqrt(d*x + c)*sqrt(c + d) + 2*c + d)/(x - 1)), -sqrt(-c - d)*ar
ctan(sqrt(-c - d)/sqrt(d*x + c)) + 1/2*sqrt(c - d)*log((d*x - 2*sqrt(d*x +
 c)*sqrt(c - d) + 2*c - d)/(x + 1)), sqrt(-c + d)*arctan(sqrt(-c + d)/sqrt
(d*x + c)) - sqrt(-c - d)*arctan(sqrt(-c - d)/sqrt(d*x + c))]
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{c+dx}}{1-x^2}\,dx = \begin{cases} \frac{2\left(\frac{d(c-d)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+d}}\right)}{2\sqrt{-c+d}} - \frac{d(c+d)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c-d}}\right)}{2\sqrt{-c-d}}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**(1/2)/(-x**2+1),x)
```

output
```
Piecewise((2*(d*(c - d)*atan(sqrt(c + d*x)/sqrt(-c + d))/(2*sqrt(-c + d))
- d*(c + d)*atan(sqrt(c + d*x)/sqrt(-c - d))/(2*sqrt(-c - d)))/d, Ne(d, 0)
), (sqrt(c)*(-log(x - 1)/2 + log(x + 1)/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}}{1-x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c-4*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx}}{1-x^2}\,dx = -\sqrt{-c+d}\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c+d}}\right)+\sqrt{-c-d}\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c-d}}\right)$$

input
```
integrate((d*x+c)^(1/2)/(-x^2+1),x, algorithm="giac")
```

output
```
-sqrt(-c + d)*arctan(sqrt(d*x + c)/sqrt(-c + d)) + sqrt(-c - d)*arctan(sqr
t(d*x + c)/sqrt(-c - d))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c+dx}}{1-x^2}\,dx = \operatorname{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c+d}}\right)\,\sqrt{c+d}-\operatorname{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c-d}}\right)\,\sqrt{c-d}$$

input
```
int(-(c + d*x)^(1/2)/(x^2 - 1),x)
```

output
```
atanh((c + d*x)^(1/2)/(c + d)^(1/2))*(c + d)^(1/2) - atanh((c + d*x)^(1/2)
/(c - d)^(1/2))*(c - d)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{c+dx}}{1-x^2}\,dx = \frac{\sqrt{c-d}\log\left(-\sqrt{c-d}+\sqrt{dx+c}\right)}{2}-\frac{\sqrt{c-d}\log\left(\sqrt{c-d}+\sqrt{dx+c}\right)}{2} -\frac{\sqrt{c+d}\log\left(\sqrt{dx+c}-\sqrt{c+d}\right)}{2}+\frac{\sqrt{c+d}\log\left(\sqrt{dx+c}+\sqrt{c+d}\right)}{2}$$

input
```
int((d*x+c)^(1/2)/(-x^2+1),x)
```

output
```
(sqrt(c - d)*log( - sqrt(c - d) + sqrt(c + d*x)) - sqrt(c - d)*log(sqrt(c
- d) + sqrt(c + d*x)) - sqrt(c + d)*log(sqrt(c + d*x) - sqrt(c + d)) + sqr
t(c + d)*log(sqrt(c + d*x) + sqrt(c + d)))/2
```

3.155 $\int \frac{\sqrt{2+3x}}{a-bx^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{\sqrt{2+3x}}{a-bx^2} dx = -\frac{\sqrt{3\sqrt{a}-2\sqrt{b}}\arctan\left(\frac{\sqrt[4]{b}\sqrt{2+3x}}{\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}} + \frac{\sqrt{3\sqrt{a}+2\sqrt{b}}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{2+3x}}{\sqrt{3\sqrt{a}+2\sqrt{b}}}\right)}{\sqrt{a}b^{3/4}}$$

output
```
-(3*a^(1/2)-2*b^(1/2))^(1/2)*arctan(b^(1/4)*(2+3*x)^(1/2)/(3*a^(1/2)-2*b^(
1/2))^(1/2))/a^(1/2)/b^(3/4)+(3*a^(1/2)+2*b^(1/2))^(1/2)*arctanh(b^(1/4)*(
2+3*x)^(1/2)/(3*a^(1/2)+2*b^(1/2))^(1/2))/a^(1/2)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{2+3x}}{a-bx^2} dx$$
$$= \frac{-\sqrt{-3\sqrt{a}\sqrt{b}-2b}\arctan\left(\frac{\sqrt{-3\sqrt{a}\sqrt{b}-2b}\sqrt{2+3x}}{3\sqrt{a}+2\sqrt{b}}\right) - \sqrt{3\sqrt{a}\sqrt{b}-2b}\arctan\left(\frac{\sqrt{3\sqrt{a}\sqrt{b}-2b}\sqrt{2+3x}}{3\sqrt{a}-2\sqrt{b}}\right)}{\sqrt{a}b}$$

input
```
Integrate[Sqrt[2 + 3*x]/(a - b*x^2),x]
```

output
```
(-(Sqrt[-3*Sqrt[a]*Sqrt[b] - 2*b]*ArcTan[(Sqrt[-3*Sqrt[a]*Sqrt[b] - 2*b]*S
qrt[2 + 3*x])/(3*Sqrt[a] + 2*Sqrt[b])]) - Sqrt[3*Sqrt[a]*Sqrt[b] - 2*b]*Ar
cTan[(Sqrt[3*Sqrt[a]*Sqrt[b] - 2*b]*Sqrt[2 + 3*x])/(3*Sqrt[a] - 2*Sqrt[b])
])/(Sqrt[a]*b)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {483, 1450, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}}{a-bx^2}dx$$

$$\Big\downarrow \text{483}$$

$$6\int \frac{3x+2}{-b(3x+2)^2+4b(3x+2)+9a-4b}d\sqrt{3x+2}$$

$$\Big\downarrow \text{1450}$$

$$6\left(\frac{1}{6}\left(3-\frac{2\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{-\sqrt{b}\left(3\sqrt{a}-2\sqrt{b}\right)-b(3x+2)}d\sqrt{3x+2}+\frac{1}{6}\left(\frac{2\sqrt{b}}{\sqrt{a}}+3\right)\int \frac{1}{\left(3\sqrt{a}+2\sqrt{b}\right)\sqrt{b}-b(3x+2)}d\sqrt{3x+2}\right)$$

$$\Big\downarrow \text{218}$$

$$6\left(\frac{1}{6}\left(\frac{2\sqrt{b}}{\sqrt{a}}+3\right)\int \frac{1}{\left(3\sqrt{a}+2\sqrt{b}\right)\sqrt{b}-b(3x+2)}d\sqrt{3x+2}-\frac{\left(3-\frac{2\sqrt{b}}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)}{6b^{3/4}\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)$$

$$\Big\downarrow \text{221}$$

$$6\left(\frac{\left(\frac{2\sqrt{b}}{\sqrt{a}}+3\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}+2\sqrt{b}}}\right)}{6b^{3/4}\sqrt{3\sqrt{a}+2\sqrt{b}}}-\frac{\left(3-\frac{2\sqrt{b}}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt[4]{b}\sqrt{3x+2}}{\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)}{6b^{3/4}\sqrt{3\sqrt{a}-2\sqrt{b}}}\right)$$

input
```
Int[Sqrt[2 + 3*x]/(a - b*x^2),x]
```

output
```
6*(-1/6*((3 - (2*Sqrt[b])/Sqrt[a])*ArcTan[(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*S
qrt[a] - 2*Sqrt[b]]])/(Sqrt[3*Sqrt[a] - 2*Sqrt[b]]*b^(3/4)) + ((3 + (2*Sqr
t[b])/Sqrt[a])*ArcTanh[(b^(1/4)*Sqrt[2 + 3*x])/Sqrt[3*Sqrt[a] + 2*Sqrt[b]]
])/(6*Sqrt[3*Sqrt[a] + 2*Sqrt[b]]*b^(3/4)))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$-\frac{-\frac{\left(3\sqrt{ab}+2b\right)\operatorname{arctanh}\left(\frac{b\sqrt{3x+2}}{\sqrt{\left(3\sqrt{ab}+2b\right)b}}\right)}{\sqrt{\left(3\sqrt{ab}+2b\right)b}}+\frac{\left(3\sqrt{ab}-2b\right)\arctan\left(\frac{b\sqrt{3x+2}}{\sqrt{\left(3\sqrt{ab}-2b\right)b}}\right)}{\sqrt{\left(3\sqrt{ab}-2b\right)b}}}{\sqrt{ab}}$	114
derivativedivides	$-6b\left(-\frac{\left(3\sqrt{ab}+2b\right)\operatorname{arctanh}\left(\frac{b\sqrt{3x+2}}{\sqrt{\left(3\sqrt{ab}+2b\right)b}}\right)}{6b\sqrt{ab}\sqrt{\left(3\sqrt{ab}+2b\right)b}}+\frac{\left(3\sqrt{ab}-2b\right)\arctan\left(\frac{b\sqrt{3x+2}}{\sqrt{\left(3\sqrt{ab}-2b\right)b}}\right)}{6b\sqrt{ab}\sqrt{\left(3\sqrt{ab}-2b\right)b}}\right)$	127
default	$-6b\left(-\frac{\left(3\sqrt{ab}+2b\right)\operatorname{arctanh}\left(\frac{b\sqrt{3x+2}}{\sqrt{\left(3\sqrt{ab}+2b\right)b}}\right)}{6b\sqrt{ab}\sqrt{\left(3\sqrt{ab}+2b\right)b}}+\frac{\left(3\sqrt{ab}-2b\right)\arctan\left(\frac{b\sqrt{3x+2}}{\sqrt{\left(3\sqrt{ab}-2b\right)b}}\right)}{6b\sqrt{ab}\sqrt{\left(3\sqrt{ab}-2b\right)b}}\right)$	127

input
```
int((3*x+2)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*b)^(1/2)*(-(3*(a*b)^(1/2)+2*b)/((3*(a*b)^(1/2)+2*b)*b)^(1/2)*arctanh
(b*(3*x+2)^(1/2)/((3*(a*b)^(1/2)+2*b)*b)^(1/2))+(3*(a*b)^(1/2)-2*b)/((3*(a
*b)^(1/2)-2*b)*b)^(1/2)*arctan(b*(3*x+2)^(1/2)/((3*(a*b)^(1/2)-2*b)*b)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(92) = 184$.

Time = 0.11 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt{2+3x}}{a-bx^2}\,dx = \frac{1}{2}\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\log\left(ab^2\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\sqrt{\frac{1}{ab^3}}+\sqrt{3\,x+2}\right) - \frac{1}{2}\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\log\left(-ab^2\sqrt{\frac{3\,ab\sqrt{\frac{1}{ab^3}}+2}{ab}}\sqrt{\frac{1}{ab^3}}+\sqrt{3\,x+2}\right) - \frac{1}{2}\sqrt{-\frac{3\,ab\sqrt{\frac{1}{ab^3}}-2}{ab}}\log\left(ab^2\sqrt{-\frac{3\,ab\sqrt{\frac{1}{ab^3}}-2}{ab}}\sqrt{\frac{1}{ab^3}}+\sqrt{3\,x+2}\right) + \frac{1}{2}\sqrt{-\frac{3\,ab\sqrt{\frac{1}{ab^3}}-2}{ab}}\log\left(-ab^2\sqrt{-\frac{3\,ab\sqrt{\frac{1}{ab^3}}-2}{ab}}\sqrt{\frac{1}{ab^3}}+\sqrt{3\,x+2}\right)$$

input
```
integrate((2+3*x)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt((3*a*b*sqrt(1/(a*b^3)) + 2)/(a*b))*log(a*b^2*sqrt((3*a*b*sqrt(1/(
a*b^3)) + 2)/(a*b))*sqrt(1/(a*b^3)) + sqrt(3*x + 2)) - 1/2*sqrt((3*a*b*sqr
t(1/(a*b^3)) + 2)/(a*b))*log(-a*b^2*sqrt((3*a*b*sqrt(1/(a*b^3)) + 2)/(a*b)
)*sqrt(1/(a*b^3)) + sqrt(3*x + 2)) - 1/2*sqrt(-(3*a*b*sqrt(1/(a*b^3)) - 2)
/(a*b))*log(a*b^2*sqrt(-(3*a*b*sqrt(1/(a*b^3)) - 2)/(a*b))*sqrt(1/(a*b^3))
 + sqrt(3*x + 2)) + 1/2*sqrt(-(3*a*b*sqrt(1/(a*b^3)) - 2)/(a*b))*log(-a*b^
2*sqrt(-(3*a*b*sqrt(1/(a*b^3)) - 2)/(a*b))*sqrt(1/(a*b^3)) + sqrt(3*x + 2)
)
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}}{a-bx^2}\,dx = -\int \frac{\sqrt{3x+2}}{-a+bx^2}\,dx$$

input
```
integrate((2+3*x)**(1/2)/(-b*x**2+a),x)
```

output
```
-Integral(sqrt(3*x + 2)/(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}}{a-bx^2}\,dx = \int -\frac{\sqrt{3\,x+2}}{bx^2-a}\,dx$$

input
```
integrate((2+3*x)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x + 2)/(b*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(92) = 184.

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{2+3x}}{a-bx^2}\,dx = -\frac{\left(4\,\sqrt{ab}\sqrt{-2\,b^2-3\,\sqrt{ab}b}a+17\,\sqrt{ab}\sqrt{-2\,b^2-3\,\sqrt{ab}b}b\right)|b|\arctan\left(\frac{\sqrt{3\,x+2}}{\sqrt{-\frac{2\,b+\sqrt{(9\,a-4\,b)b+4\,b^2}}{b}}}\right)}{4\,a^2b^3+17\,ab^4} + \frac{\left(4\,\sqrt{ab}\sqrt{-2\,b^2+3\,\sqrt{ab}b}a+17\,\sqrt{ab}\sqrt{-2\,b^2+3\,\sqrt{ab}b}b\right)|b|\arctan\left(\frac{\sqrt{3\,x+2}}{\sqrt{-\frac{2\,b-\sqrt{(9\,a-4\,b)b+4\,b^2}}{b}}}\right)}{4\,a^2b^3+17\,ab^4}$$

input
```
integrate((2+3*x)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
-(4*sqrt(a*b)*sqrt(-2*b^2 - 3*sqrt(a*b)*b)*a + 17*sqrt(a*b)*sqrt(-2*b^2 -
3*sqrt(a*b)*b)*b)*abs(b)*arctan(sqrt(3*x + 2)/sqrt(-(2*b + sqrt((9*a - 4*b
)*b + 4*b^2))/b))/(4*a^2*b^3 + 17*a*b^4) + (4*sqrt(a*b)*sqrt(-2*b^2 + 3*sq
rt(a*b)*b)*a + 17*sqrt(a*b)*sqrt(-2*b^2 + 3*sqrt(a*b)*b)*b)*abs(b)*arctan(
sqrt(3*x + 2)/sqrt(-(2*b - sqrt((9*a - 4*b)*b + 4*b^2))/b))/(4*a^2*b^3 + 1
7*a*b^4)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{2+3x}}{a-bx^2}\,dx$$
$$= 2\,\mathrm{atanh}\left(\frac{2\left(\left(576\,b^3+1296\,a\,b^2\right)\sqrt{3\,x+2}+\frac{288\,b\,\sqrt{3\,x+2}\,\left(3\,\sqrt{a^3\,b^3}-2\,a\,b^2\right)}{a}\right)\sqrt{-\frac{3\,\sqrt{a^3\,b^3}-2\,a\,b^2}{4\,a^2\,b^3}}}{3888\,a\,b-1728\,b^2}\right)\sqrt{-\frac{3\,\sqrt{a^3\,b^3}}{4\,a^2}}$$
$$+2\,\mathrm{atanh}\left(\frac{2\left(\left(576\,b^3+1296\,a\,b^2\right)\sqrt{3\,x+2}-\frac{288\,b\,\sqrt{3\,x+2}\,\left(3\,\sqrt{a^3\,b^3}+2\,a\,b^2\right)}{a}\right)\sqrt{\frac{3\,\sqrt{a^3\,b^3}+2\,a\,b^2}{4\,a^2\,b^3}}}{3888\,a\,b-1728\,b^2}\right)\sqrt{\frac{3\,\sqrt{a^3\,b^3}+}{4\,a^2\,b}}$$

input
```
int((3*x + 2)^(1/2)/(a - b*x^2),x)
```

output
```
2*atanh((2*((1296*a*b^2 + 576*b^3)*(3*x + 2)^(1/2) + (288*b*(3*x + 2)^(1/2
)*(3*(a^3*b^3)^(1/2) - 2*a*b^2))/a)*(-(3*(a^3*b^3)^(1/2) - 2*a*b^2)/(4*a^2
*b^3))^(1/2))/(3888*a*b - 1728*b^2))*(-(3*(a^3*b^3)^(1/2) - 2*a*b^2)/(4*a^
2*b^3))^(1/2) + 2*atanh((2*((1296*a*b^2 + 576*b^3)*(3*x + 2)^(1/2) - (288*
b*(3*x + 2)^(1/2)*(3*(a^3*b^3)^(1/2) + 2*a*b^2))/a)*((3*(a^3*b^3)^(1/2) +
2*a*b^2)/(4*a^2*b^3))^(1/2))/(3888*a*b - 1728*b^2))*((3*(a^3*b^3)^(1/2) +
2*a*b^2)/(4*a^2*b^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{2+3x}}{a-bx^2}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{3\sqrt{b}\,\sqrt{a}-2b}\,atan\left(\frac{\sqrt{3x+2}\,b}{\sqrt{b}\,\sqrt{3\sqrt{b}\,\sqrt{a}-2b}}\right)-\sqrt{3\sqrt{b}\,\sqrt{a}+2b}\log\left(-\sqrt{3\sqrt{b}\,\sqrt{a}+2b}+\sqrt{b}\,\sqrt{3x+2}\right)\right.}{2ab}$$

input
```
int((2+3*x)^(1/2)/(-b*x^2+a),x)
```

output
```
(sqrt(a)*( - 2*sqrt(3*sqrt(b)*sqrt(a) - 2*b)*atan((sqrt(3*x + 2)*b)/(sqrt(
b)*sqrt(3*sqrt(b)*sqrt(a) - 2*b))) - sqrt(3*sqrt(b)*sqrt(a) + 2*b)*log( -
sqrt(3*sqrt(b)*sqrt(a) + 2*b) + sqrt(b)*sqrt(3*x + 2)) + sqrt(3*sqrt(b)*sq
rt(a) + 2*b)*log(sqrt(3*sqrt(b)*sqrt(a) + 2*b) + sqrt(b)*sqrt(3*x + 2))))/
(2*a*b)
```

3.156 $\int (d+ex)^{5/2} (a+cx^2) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int (d+ex)^{5/2} (a+cx^2) \, dx = \frac{2(cd^2+ae^2)(d+ex)^{7/2}}{7e^3} - \frac{4cd(d+ex)^{9/2}}{9e^3} + \frac{2c(d+ex)^{11/2}}{11e^3}$$

output
```
2/7*(a*e^2+c*d^2)*(e*x+d)^(7/2)/e^3-4/9*c*d*(e*x+d)^(9/2)/e^3+2/11*c*(e*x+
d)^(11/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (d+ex)^{5/2} (a+cx^2) \, dx = \frac{2(d+ex)^{7/2}(99ae^2+c(8d^2-28dex+63e^2x^2))}{693e^3}$$

input
```
Integrate[(d + e*x)^(5/2)*(a + c*x^2),x]
```

output
```
(2*(d + e*x)^(7/2)*(99*a*e^2 + c*(8*d^2 - 28*d*e*x + 63*e^2*x^2)))/(693*e^
3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right) (d+ex)^{5/2} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(d+ex)^{5/2} \left(ae^2+cd^2\right)}{e^2}+\frac{c(d+ex)^{9/2}}{e^2}-\frac{2cd(d+ex)^{7/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{7/2} \left(ae^2+cd^2\right)}{7e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}-\frac{4cd(d+ex)^{9/2}}{9e^3}$$

input
```
Int[(d + e*x)^(5/2)*(a + c*x^2),x]
```

output
```
(2*(c*d^2 + a*e^2)*(d + e*x)^(7/2))/(7*e^3) - (4*c*d*(d + e*x)^(9/2))/(9*e
^3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result
gosper	$\frac{2(ex+d)^{\frac{7}{2}}\left(63x^{2}c\,e^{2}-28cdxe+99a\,e^{2}+8c\,d^{2}\right)}{693e^{3}}$
pseudoelliptic	$\frac{2(ex+d)^{\frac{7}{2}}\left(63x^{2}c\,e^{2}-28cdxe+99a\,e^{2}+8c\,d^{2}\right)}{693e^{3}}$
orering	$\frac{2(ex+d)^{\frac{7}{2}}\left(63x^{2}c\,e^{2}-28cdxe+99a\,e^{2}+8c\,d^{2}\right)}{693e^{3}}$
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{11}{2}}}{11}-\frac{4cd(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{7}{2}}}{7}}{e^{3}}$
default	$\frac{\frac{2c(ex+d)^{\frac{11}{2}}}{11}-\frac{4cd(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{7}{2}}}{7}}{e^{3}}$
trager	$\frac{2\left(63c\,e^{5}x^{5}+161d\,e^{4}c\,x^{4}+99a\,e^{5}x^{3}+113c\,d^{2}e^{3}x^{3}+297ad\,e^{4}x^{2}+3d^{3}e^{2}c\,x^{2}+297a\,d^{2}e^{3}x-4c\,d^{4}ex+99a\,d^{3}e^{2}+8c\,d^{5}\right)\sqrt{ex}}{693e^{3}}$
risch	$\frac{2\left(63c\,e^{5}x^{5}+161d\,e^{4}c\,x^{4}+99a\,e^{5}x^{3}+113c\,d^{2}e^{3}x^{3}+297ad\,e^{4}x^{2}+3d^{3}e^{2}c\,x^{2}+297a\,d^{2}e^{3}x-4c\,d^{4}ex+99a\,d^{3}e^{2}+8c\,d^{5}\right)\sqrt{ex}}{693e^{3}}$

input
```
int((e*x+d)^(5/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/693*(e*x+d)^(7/2)*(63*c*e^2*x^2-28*c*d*e*x+99*a*e^2+8*c*d^2)/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(51) = 102.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.71

$$\int (d+ex)^{5/2}\left(a+cx^2\right)\,dx=\frac{2\,(63\,ce^5x^5+161\,cde^4x^4+8\,cd^5+99\,ad^3e^2+(113\,cd^2e^3+99\,ae^5)x^3+3\,(cd^3e^2+99\,ade^4)x^2-}{693\,e^3}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/693*(63*c*e^5*x^5 + 161*c*d*e^4*x^4 + 8*c*d^5 + 99*a*d^3*e^2 + (113*c*d^
2*e^3 + 99*a*e^5)*x^3 + 3*(c*d^3*e^2 + 99*a*d*e^4)*x^2 - (4*c*d^4*e - 297*
a*d^2*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(63) = 126$.

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.46

$$\int (d+ex)^{5/2}\left(a+cx^2\right)\,dx=\begin{cases}\frac{2ad^3\sqrt{d+ex}}{7e}+\frac{6ad^2x\sqrt{d+ex}}{7}+\frac{6adex^2\sqrt{d+ex}}{7}+\frac{2ae^2x^3\sqrt{d+ex}}{7}+\frac{16cd^5\sqrt{d+ex}}{693e^3}-\frac{8cd^4x\sqrt{d+ex}}{693e^2}+\frac{2cd^3x^2\sqrt{d+ex}}{231e}+\\ d^{\frac{5}{2}}\left(ax+\frac{cx^3}{3}\right)\end{cases}$$

input
```
integrate((e*x+d)**(5/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*a*d**3*sqrt(d + e*x)/(7*e) + 6*a*d**2*x*sqrt(d + e*x)/7 + 6*a
*d*e*x**2*sqrt(d + e*x)/7 + 2*a*e**2*x**3*sqrt(d + e*x)/7 + 16*c*d**5*sqrt
(d + e*x)/(693*e**3) - 8*c*d**4*x*sqrt(d + e*x)/(693*e**2) + 2*c*d**3*x**2
*sqrt(d + e*x)/(231*e) + 226*c*d**2*x**3*sqrt(d + e*x)/693 + 46*c*d*e*x**4
*sqrt(d + e*x)/99 + 2*c*e**2*x**5*sqrt(d + e*x)/11, Ne(e, 0)), (d**(5/2)*(
a*x + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int (d+ex)^{5/2}\left(a+cx^2\right)\,dx=\frac{2\left(63\,(ex+d)^{\frac{11}{2}}c-154\,(ex+d)^{\frac{9}{2}}cd+99\,(cd^2+ae^2)(ex+d)^{\frac{7}{2}}\right)}{693\,e^3}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/693*(63*(e*x + d)^(11/2)*c - 154*(e*x + d)^(9/2)*c*d + 99*(c*d^2 + a*e^2
)*(e*x + d)^(7/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 357, normalized size of antiderivative = 5.67

$$\int (d+ex)^{5/2}(a+cx^2)\,dx = \frac{2\left(3465\sqrt{ex+d}ad^3 + 3465\left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+d}d\right)ad^2 + 693\left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}\right.\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(e*x + d)*a*d^3 + 3465*((e*x + d)^(3/2) - 3*sqrt(e*x + d)
*d)*a*d^2 + 693*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*a*d + 231*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x
+ d)*d^2)*c*d^3/e^2 + 99*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e
*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a + 297*(5*(e*x + d)^(7/2) - 21*
(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c*d^2/e
^2 + 33*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*
d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c*d/e^2 + 5*(63*(e*
x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*
x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c/e^2
)/e
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (d+ex)^{5/2}(a+cx^2)\,dx = \frac{2\,{\left(d+e\,x\right)}^{7/2}\,\left(63\,c\,{\left(d+e\,x\right)}^2+99\,a\,e^2+99\,c\,d^2-154\,c\,d\,\left(d+e\,x\right)\right)}{693\,e^3}$$

input
```
int((a + c*x^2)*(d + e*x)^(5/2),x)
```

output
```
(2*(d + e*x)^(7/2)*(63*c*(d + e*x)^2 + 99*a*e^2 + 99*c*d^2 - 154*c*d*(d +
e*x)))/(693*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int (d+ex)^{5/2} \left(a +cx^2\right) dx = \frac{2\sqrt{ex+d}\,(63c\,e^5x^5 + 161cd\,e^4x^4 + 99a\,e^5x^3 + 113c\,d^2e^3x^3 + 297ad\,e^4x^2 + 3c\,d^3e^2x^2 + 297a\,d^2}{693e^3}$$

input
```
int((e*x+d)^(5/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(d + e*x)*(99*a*d**3*e**2 + 297*a*d**2*e**3*x + 297*a*d*e**4*x**2 +
 99*a*e**5*x**3 + 8*c*d**5 - 4*c*d**4*e*x + 3*c*d**3*e**2*x**2 + 113*c*d**
2*e**3*x**3 + 161*c*d*e**4*x**4 + 63*c*e**5*x**5))/(693*e**3)
```

3.157 $\int (d+ex)^{3/2} (a+cx^2) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int (d+ex)^{3/2} (a+cx^2) \, dx = \frac{2(cd^2+ae^2)(d+ex)^{5/2}}{5e^3} - \frac{4cd(d+ex)^{7/2}}{7e^3} + \frac{2c(d+ex)^{9/2}}{9e^3}$$

output
```
2/5*(a*e^2+c*d^2)*(e*x+d)^(5/2)/e^3-4/7*c*d*(e*x+d)^(7/2)/e^3+2/9*c*(e*x+d
)^(9/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (d+ex)^{3/2} (a+cx^2) \, dx = \frac{2(d+ex)^{5/2} (63ae^2 + c(8d^2 - 20dex + 35e^2x^2))}{315e^3}$$

input
```
Integrate[(d + e*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(d + e*x)^(5/2)*(63*a*e^2 + c*(8*d^2 - 20*d*e*x + 35*e^2*x^2)))/(315*e^
3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right) (d+ex)^{3/2} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(d+ex)^{3/2} \left(ae^2+cd^2\right)}{e^2}+\frac{c(d+ex)^{7/2}}{e^2}-\frac{2cd(d+ex)^{5/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{5/2} \left(ae^2+cd^2\right)}{5e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}-\frac{4cd(d+ex)^{7/2}}{7e^3}$$

input
```
Int[(d + e*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^3) - (4*c*d*(d + e*x)^(7/2))/(7*e
^3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2(ex+d)^{\frac{5}{2}}(35x^{2}c\,e^{2}-20cdxe+63a\,e^{2}+8c\,d^{2})}{315e^{3}}$	41
pseudoelliptic	$\frac{2(ex+d)^{\frac{5}{2}}(35x^{2}c\,e^{2}-20cdxe+63a\,e^{2}+8c\,d^{2})}{315e^{3}}$	41
orering	$\frac{2(ex+d)^{\frac{5}{2}}(35x^{2}c\,e^{2}-20cdxe+63a\,e^{2}+8c\,d^{2})}{315e^{3}}$	41
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{9}{2}}}{9}-\frac{4cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{5}{2}}}{5}}{e^{3}}$	48
default	$\frac{\frac{2c(ex+d)^{\frac{9}{2}}}{9}-\frac{4cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{5}{2}}}{5}}{e^{3}}$	48
trager	$\frac{2(35c\,e^{4}x^{4}+50d\,e^{3}c\,x^{3}+63a\,e^{4}x^{2}+3c\,d^{2}e^{2}x^{2}+126ad\,e^{3}x-4c\,d^{3}ex+63a\,d^{2}e^{2}+8c\,d^{4})\sqrt{ex+d}}{315e^{3}}$	85
risch	$\frac{2(35c\,e^{4}x^{4}+50d\,e^{3}c\,x^{3}+63a\,e^{4}x^{2}+3c\,d^{2}e^{2}x^{2}+126ad\,e^{3}x-4c\,d^{3}ex+63a\,d^{2}e^{2}+8c\,d^{4})\sqrt{ex+d}}{315e^{3}}$	85

input
```
int((e*x+d)^(3/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/315*(e*x+d)^(5/2)*(35*c*e^2*x^2-20*c*d*e*x+63*a*e^2+8*c*d^2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

$$\int (d+ex)^{3/2}\left(a+cx^2\right)\,dx=\frac{2\left(35\,ce^4x^4+50\,cde^3x^3+8\,cd^4+63\,ad^2e^2+3\,(cd^2e^2+21\,ae^4)x^2-2\,(2\,cd^3e-63\,ade^3)x\right)\sqrt{ex}}{315\,e^3}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^4*x^4 + 50*c*d*e^3*x^3 + 8*c*d^4 + 63*a*d^2*e^2 + 3*(c*d^2*e
^2 + 21*a*e^4)*x^2 - 2*(2*c*d^3*e - 63*a*d*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int (d+ex)^{3/2} \left(a+cx^2\right)\,dx = \begin{cases} \frac{2\left(-\frac{2cd(d+ex)^{\frac{7}{2}}}{7e^2}+\frac{c(d+ex)^{\frac{9}{2}}}{9e^2}+\frac{(d+ex)^{\frac{5}{2}}\left(ae^2+cd^2\right)}{5e^2}\right)}{e} & \text{for } e \neq 0 \\ d^{\frac{3}{2}}\left(ax+\frac{cx^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(-2*c*d*(d + e*x)**(7/2)/(7*e**2) + c*(d + e*x)**(9/2)/(9*e**
2) + (d + e*x)**(5/2)*(a*e**2 + c*d**2)/(5*e**2))/e, Ne(e, 0)), (d**(3/2)*
(a*x + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int (d+ex)^{3/2} \left(a+cx^2\right)\,dx = \frac{2\left(35\,(ex+d)^{\frac{9}{2}}c-90\,(ex+d)^{\frac{7}{2}}cd+63\,(cd^2+ae^2)(ex+d)^{\frac{5}{2}}\right)}{315\,e^3}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/315*(35*(e*x + d)^(9/2)*c - 90*(e*x + d)^(7/2)*c*d + 63*(c*d^2 + a*e^2)*
(e*x + d)^(5/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(51) = 102$.

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.63

$$\int (d+ex)^{3/2} \left(a+cx^2\right) dx = \frac{2\left(315\sqrt{ex+d}\,ad^2 + 210\left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+d}\,d\right)ad + 21\left(3(ex+d)^{\frac{5}{2}} - 10(ex+d)^{\frac{3}{2}}d + \right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(e*x + d)*a*d^2 + 210*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)
*a*d + 21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2
)*a + 21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)
*c*d^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(
3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*c*d/e^2 + (35*(e*x + d)^(9/2) - 180*(e*x
+ d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqr
t(e*x + d)*d^4)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (d+ex)^{3/2} \left(a+cx^2\right) dx = \frac{2\,{\left(d+e\,x\right)}^{5/2}\,\left(35\,c\,{\left(d+e\,x\right)}^2+63\,a\,e^2+63\,c\,d^2-90\,c\,d\,\left(d+e\,x\right)\right)}{315\,e^3}$$

input
```
int((a + c*x^2)*(d + e*x)^(3/2),x)
```

output
```
(2*(d + e*x)^(5/2)*(35*c*(d + e*x)^2 + 63*a*e^2 + 63*c*d^2 - 90*c*d*(d + e
*x)))/(315*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int (d+ex)^{3/2}\left(a+cx^2\right)\,dx=\frac{2\sqrt{ex+d}\,(35c\,e^4x^4+50cd\,e^3x^3+63a\,e^4x^2+3c\,d^2e^2x^2+126ad\,e^3x-4c\,d^3ex+63a\,d^2e^2+8c$$
$$315e^3$$

input
```
int((e*x+d)^(3/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(d + e*x)*(63*a*d**2*e**2 + 126*a*d*e**3*x + 63*a*e**4*x**2 + 8*c*d
**4 - 4*c*d**3*e*x + 3*c*d**2*e**2*x**2 + 50*c*d*e**3*x**3 + 35*c*e**4*x**
4))/(315*e**3)
```

3.158 $\int \sqrt{d+ex}(a+cx^2)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \sqrt{d+ex}(a+cx^2)\,dx=\frac{2(cd^2+ae^2)\,(d+ex)^{3/2}}{3e^3}-\frac{4cd(d+ex)^{5/2}}{5e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}$$

output
```
2/3*(a*e^2+c*d^2)*(e*x+d)^(3/2)/e^3-4/5*c*d*(e*x+d)^(5/2)/e^3+2/7*c*(e*x+d
)^(7/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \sqrt{d+ex}(a+cx^2)\,dx=\frac{2(d+ex)^{3/2}\,(35ae^2+c(8d^2-12dex+15e^2x^2))}{105e^3}$$

input
```
Integrate[Sqrt[d + e*x]*(a + c*x^2),x]
```

output
```
(2*(d + e*x)^(3/2)*(35*a*e^2 + c*(8*d^2 - 12*d*e*x + 15*e^2*x^2)))/(105*e^
3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{\sqrt{d+ex}(ae^2+cd^2)}{e^2}+\frac{c(d+ex)^{5/2}}{e^2}-\frac{2cd(d+ex)^{3/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)}{3e^3}+\frac{2c(d+ex)^{7/2}}{7e^3}-\frac{4cd(d+ex)^{5/2}}{5e^3}$$

input
```
Int[Sqrt[d + e*x]*(a + c*x^2),x]
```

output
```
(2*(c*d^2 + a*e^2)*(d + e*x)^(3/2))/(3*e^3) - (4*c*d*(d + e*x)^(5/2))/(5*e
^3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{\left(\left(30c\,x^{2}+70a\right)e^{2}-24cdxe+16c\,d^{2}\right)\left(ex+d\right)^{\frac{3}{2}}}{105e^{3}}$	40
gosper	$\frac{2\left(ex+d\right)^{\frac{3}{2}}\left(15x^{2}c\,e^{2}-12cdxe+35a\,e^{2}+8c\,d^{2}\right)}{105e^{3}}$	41
orering	$\frac{2\left(ex+d\right)^{\frac{3}{2}}\left(15x^{2}c\,e^{2}-12cdxe+35a\,e^{2}+8c\,d^{2}\right)}{105e^{3}}$	41
derivativedivides	$\frac{\frac{2c\left(ex+d\right)^{\frac{7}{2}}}{7}-\frac{4cd\left(ex+d\right)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)\left(ex+d\right)^{\frac{3}{2}}}{3}}{e^{3}}$	48
default	$\frac{\frac{2c\left(ex+d\right)^{\frac{7}{2}}}{7}-\frac{4cd\left(ex+d\right)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)\left(ex+d\right)^{\frac{3}{2}}}{3}}{e^{3}}$	48
trager	$\frac{2\left(15c\,e^{3}x^{3}+3cd\,e^{2}x^{2}+35a\,e^{3}x-4c\,d^{2}ex+35ad\,e^{2}+8c\,d^{3}\right)\sqrt{ex+d}}{105e^{3}}$	61
risch	$\frac{2\left(15c\,e^{3}x^{3}+3cd\,e^{2}x^{2}+35a\,e^{3}x-4c\,d^{2}ex+35ad\,e^{2}+8c\,d^{3}\right)\sqrt{ex+d}}{105e^{3}}$	61

input
```
int((e*x+d)^(1/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/105*((30*c*x^2+70*a)*e^2-24*c*d*x*e+16*c*d^2)*(e*x+d)^(3/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \sqrt{d+ex}\left(a+cx^2\right)\,dx = \frac{2\,{\left(15\,ce^{3}x^{3}+3\,cde^{2}x^{2}+8\,cd^{3}+35\,ade^{2}-{\left(4\,cd^{2}e-35\,ae^{3}\right)}x\right)}\sqrt{ex+d}}{105\,e^{3}}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/105*(15*c*e^3*x^3 + 3*c*d*e^2*x^2 + 8*c*d^3 + 35*a*d*e^2 - (4*c*d^2*e -
35*a*e^3)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \sqrt{d+ex}\left(a+cx^2\right)\,dx=\begin{cases}\frac{2\left(-\frac{2cd(d+ex)^{\frac{5}{2}}}{5e^2}+\frac{c(d+ex)^{\frac{7}{2}}}{7e^2}+\frac{(d+ex)^{\frac{3}{2}}\left(ae^2+cd^2\right)}{3e^2}\right)}{e} & \text{for } e\neq 0\\ \sqrt{d}\left(ax+\frac{cx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(-2*c*d*(d + e*x)**(5/2)/(5*e**2) + c*(d + e*x)**(7/2)/(7*e**
2) + (d + e*x)**(3/2)*(a*e**2 + c*d**2)/(3*e**2))/e, Ne(e, 0)), (sqrt(d)*(
a*x + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \sqrt{d+ex}\left(a+cx^2\right)\,dx=\frac{2\left(15\,(ex+d)^{\frac{7}{2}}c-42\,(ex+d)^{\frac{5}{2}}cd+35\,(cd^2+ae^2)(ex+d)^{\frac{3}{2}}\right)}{105\,e^3}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/105*(15*(e*x + d)^(7/2)*c - 42*(e*x + d)^(5/2)*c*d + 35*(c*d^2 + a*e^2)*
(e*x + d)^(3/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.02

$$\int \sqrt{d+ex}\left(a+cx^2\right)\,dx = \frac{2\left(105\sqrt{ex+d}ad + 35\left((ex+d)^{\frac{3}{2}} - 3\sqrt{ex+d}d\right)a + \frac{7\left(3\,(ex+d)^{\frac{5}{2}} - 10\,(ex+d)^{\frac{3}{2}}d + 15\sqrt{ex+d}d^2\right)cd}{e^2} + \frac{3\left(5\,(ex+d)^{\frac{7}{2}} - \cdots\right.}{\cdots}\right.}{105\,e}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(e*x + d)*a*d + 35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*a
+ 7*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d/
e^2 + 3*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2
 - 35*sqrt(e*x + d)*d^3)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \sqrt{d+ex}\left(a+cx^2\right)\,dx = \frac{2\,{\left(d+e\,x\right)}^{3/2}\,\left(15\,c\,{\left(d+e\,x\right)}^2+35\,a\,e^2+35\,c\,d^2-42\,c\,d\,\left(d+e\,x\right)\right)}{105\,e^3}$$

input
```
int((a + c*x^2)*(d + e*x)^(1/2),x)
```

output
```
(2*(d + e*x)^(3/2)*(15*c*(d + e*x)^2 + 35*a*e^2 + 35*c*d^2 - 42*c*d*(d + e
*x)))/(105*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \sqrt{d+ex}\,(a+cx^2)\ dx = \frac{2\sqrt{ex+d}\,(15c\,e^3x^3+3cd\,e^2x^2+35a\,e^3x-4c\,d^2ex+35ad\,e^2+8c\,d^3)}{105e^3}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(d + e*x)*(35*a*d*e**2 + 35*a*e**3*x + 8*c*d**3 - 4*c*d**2*e*x + 3*
c*d*e**2*x**2 + 15*c*e**3*x**3))/(105*e**3)
```

3.159 $\int \frac{a+cx^2}{\sqrt{d+ex}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{a+cx^2}{\sqrt{d+ex}} \, dx = \frac{2(cd^2+ae^2)\sqrt{d+ex}}{e^3} - \frac{4cd(d+ex)^{3/2}}{3e^3} + \frac{2c(d+ex)^{5/2}}{5e^3}$$

output
```
2*(a*e^2+c*d^2)*(e*x+d)^(1/2)/e^3-4/3*c*d*(e*x+d)^(3/2)/e^3+2/5*c*(e*x+d)^
(5/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{d+ex}} \, dx = \frac{2\sqrt{d+ex}(15ae^2+c(8d^2-4dex+3e^2x^2))}{15e^3}$$

input
```
Integrate[(a + c*x^2)/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(15*a*e^2 + c*(8*d^2 - 4*d*e*x + 3*e^2*x^2)))/(15*e^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2\sqrt{d+ex}}+\frac{c(d+ex)^{3/2}}{e^2}-\frac{2cd\sqrt{d+ex}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{d+ex}(ae^2+cd^2)}{e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}-\frac{4cd(d+ex)^{3/2}}{3e^3}$$

input
```
Int[(a + c*x^2)/Sqrt[d + e*x],x]
```

output
```
(2*(c*d^2 + a*e^2)*Sqrt[d + e*x])/e^3 - (4*c*d*(d + e*x)^(3/2))/(3*e^3) +
(2*c*(d + e*x)^(5/2))/(5*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{2\left(3\left(c\,x^{2}+5a\right)e^{2}-4cdxe+8c\,d^{2}\right)\sqrt{ex+d}}{15e^{3}}$	40
gosper	$\frac{2\sqrt{ex+d}\,\left(3x^{2}c\,e^{2}-4cdxe+15a\,e^{2}+8c\,d^{2}\right)}{15e^{3}}$	41
trager	$\frac{2\sqrt{ex+d}\,\left(3x^{2}c\,e^{2}-4cdxe+15a\,e^{2}+8c\,d^{2}\right)}{15e^{3}}$	41
risch	$\frac{2\sqrt{ex+d}\,\left(3x^{2}c\,e^{2}-4cdxe+15a\,e^{2}+8c\,d^{2}\right)}{15e^{3}}$	41
orering	$\frac{2\sqrt{ex+d}\,\left(3x^{2}c\,e^{2}-4cdxe+15a\,e^{2}+8c\,d^{2}\right)}{15e^{3}}$	41
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{5}{2}}}{5}-\frac{4cd(ex+d)^{\frac{3}{2}}}{3}+2a\,e^{2}\sqrt{ex+d}+2c\,d^{2}\sqrt{ex+d}}{e^{3}}$	52
default	$\frac{\frac{2c(ex+d)^{\frac{5}{2}}}{5}-\frac{4cd(ex+d)^{\frac{3}{2}}}{3}+2a\,e^{2}\sqrt{ex+d}+2c\,d^{2}\sqrt{ex+d}}{e^{3}}$	52

input
```
int((c*x^2+a)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*(c*x^2+5*a)*e^2-4*c*d*x*e+8*c*d^2)*(e*x+d)^(1/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{a+cx^2}{\sqrt{d+ex}}\,dx=\frac{2\left(3\,ce^2x^2-4\,cdex+8\,cd^2+15\,ae^2\right)\sqrt{ex+d}}{15\,e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*e^2*x^2 - 4*c*d*e*x + 8*c*d^2 + 15*a*e^2)*sqrt(e*x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{a + cx^2}{\sqrt{d + ex}}\,dx = \begin{cases} \frac{2a\sqrt{d+ex}+\frac{2c\left(d^2\sqrt{d+ex}-\frac{2d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}}{e} & \text{for } e \neq 0 \\ \frac{ax+\frac{cx^3}{3}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)**(1/2),x)
```

output
```
Piecewise(((2*a*sqrt(d + e*x) + 2*c*(d**2*sqrt(d + e*x) - 2*d*(d + e*x)**(
3/2)/3 + (d + e*x)**(5/2)/5)/e**2)/e, Ne(e, 0)), ((a*x + c*x**3/3)/sqrt(d)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a + cx^2}{\sqrt{d + ex}}\,dx = \frac{2\left(15\sqrt{ex+d}a + \frac{\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)c}{e^2}\right)}{15\,e}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(e*x + d)*a + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*
sqrt(e*x + d)*d^2)*c/e^2)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a+cx^2}{\sqrt{d+ex}}\,dx = \frac{2\left(15\sqrt{ex+d}a+\frac{\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)c}{e^2}\right)}{15\,e}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(e*x + d)*a + (3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*
sqrt(e*x + d)*d^2)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+e\,x}\,\left(3\,c\,{(d+e\,x)}^2+15\,a\,e^2+15\,c\,d^2-10\,c\,d\,(d+e\,x)\right)}{15\,e^3}$$

input
```
int((a + c*x^2)/(d + e*x)^(1/2),x)
```

output
```
(2*(d + e*x)^(1/2)*(3*c*(d + e*x)^2 + 15*a*e^2 + 15*c*d^2 - 10*c*d*(d + e*
x)))/(15*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{a+cx^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{ex+d}\,(3c\,e^2x^2-4cdex+15a\,e^2+8c\,d^2)}{15e^3}$$

input
```
int((c*x^2+a)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(15*a*e**2 + 8*c*d**2 - 4*c*d*e*x + 3*c*e**2*x**2))/(15*e
**3)
```

3.160 $\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx=-\frac{2(cd^2+ae^2)}{e^3\sqrt{d+ex}}-\frac{4cd\sqrt{d+ex}}{e^3}+\frac{2c(d+ex)^{3/2}}{3e^3}$$

output
```
(-2*a*e^2-2*c*d^2)/e^3/(e*x+d)^(1/2)-4*c*d*(e*x+d)^(1/2)/e^3+2/3*c*(e*x+d)
^(3/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx=\frac{2(-3ae^2+c(-8d^2-4dex+e^2x^2))}{3e^3\sqrt{d+ex}}$$

input
```
Integrate[(a + c*x^2)/(d + e*x)^(3/2),x]
```

output
```
(2*(-3*a*e^2 + c*(-8*d^2 - 4*d*e*x + e^2*x^2)))/(3*e^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)^{3/2}}+\frac{c\sqrt{d+ex}}{e^2}-\frac{2cd}{e^2\sqrt{d+ex}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(ae^2+cd^2)}{e^3\sqrt{d+ex}}+\frac{2c(d+ex)^{3/2}}{3e^3}-\frac{4cd\sqrt{d+ex}}{e^3}$$

input
```
Int[(a + c*x^2)/(d + e*x)^(3/2),x]
```

output
```
(-2*(c*d^2 + a*e^2))/(e^3*Sqrt[d + e*x]) - (4*c*d*Sqrt[d + e*x])/e^3 + (2*
c*(d + e*x)^(3/2))/(3*e^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\frac{2\left(c\,x^{2}-3a\right)e^{2}}{3}-\frac{8cdxe}{3}-\frac{16c\,d^{2}}{3}}{\sqrt{ex+d}\,e^{3}}$	39
gosper	$-\frac{2\left(-x^{2}c\,e^{2}+4cdxe+3a\,e^{2}+8c\,d^{2}\right)}{3\sqrt{ex+d}\,e^{3}}$	41
trager	$-\frac{2\left(-x^{2}c\,e^{2}+4cdxe+3a\,e^{2}+8c\,d^{2}\right)}{3\sqrt{ex+d}\,e^{3}}$	41
orering	$-\frac{2\left(-x^{2}c\,e^{2}+4cdxe+3a\,e^{2}+8c\,d^{2}\right)}{3\sqrt{ex+d}\,e^{3}}$	41
risch	$-\frac{2c(-ex+5d)\sqrt{ex+d}}{3e^{3}}-\frac{2\left(a\,e^{2}+c\,d^{2}\right)}{e^{3}\sqrt{ex+d}}$	46
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{3}{2}}}{3}-4cd\sqrt{ex+d}-\frac{2\left(a\,e^{2}+c\,d^{2}\right)}{\sqrt{ex+d}}}{e^{3}}$	48
default	$\frac{\frac{2c(ex+d)^{\frac{3}{2}}}{3}-4cd\sqrt{ex+d}-\frac{2\left(a\,e^{2}+c\,d^{2}\right)}{\sqrt{ex+d}}}{e^{3}}$	48

input
```
int((c*x^2+a)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((c*x^2-3*a)*e^2-4*c*d*x*e-8*c*d^2)/(e*x+d)^(1/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx=\frac{2\left(ce^2x^2-4\,cdex-8\,cd^2-3\,ae^2\right)\sqrt{ex+d}}{3\left(e^4x+de^3\right)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(c*e^2*x^2 - 4*c*d*e*x - 8*c*d^2 - 3*a*e^2)*sqrt(e*x + d)/(e^4*x + d*e
^3)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{2cd\sqrt{d+ex}}{e^2}+\frac{c(d+ex)^{\frac{3}{2}}}{3e^2}-\frac{ae^2+cd^2}{e^2\sqrt{d+ex}}\right)}{e} & \text{for } e\neq 0\\ \frac{ax+\frac{cx^3}{3}}{d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(-2*c*d*sqrt(d + e*x)/e**2 + c*(d + e*x)**(3/2)/(3*e**2) - (a
*e**2 + c*d**2)/(e**2*sqrt(d + e*x)))/e, Ne(e, 0)), ((a*x + c*x**3/3)/d**(
3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c-6\sqrt{ex+d}cd}{e^2}-\frac{3(cd^2+ae^2)}{\sqrt{ex+d}e^2}\right)}{3e}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(((e*x + d)^(3/2)*c - 6*sqrt(e*x + d)*c*d)/e^2 - 3*(c*d^2 + a*e^2)/(sq
rt(e*x + d)*e^2))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx = -\frac{2\,(cd^2+ae^2)}{\sqrt{ex+d}e^3} + \frac{2\left((ex+d)^{\frac{3}{2}}ce^6 - 6\sqrt{ex+d}cde^6\right)}{3\,e^9}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*d^2 + a*e^2)/(sqrt(e*x + d)*e^3) + 2/3*((e*x + d)^(3/2)*c*e^6 - 6*sq
rt(e*x + d)*c*d*e^6)/e^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx = -\frac{6\,a\,e^2 - 2\,c\,(d+e\,x)^2 + 6\,c\,d^2 + 12\,c\,d\,(d+e\,x)}{3\,e^3\,\sqrt{d+e\,x}}$$

input
```
int((a + c*x^2)/(d + e*x)^(3/2),x)
```

output
```
-(6*a*e^2 - 2*c*(d + e*x)^2 + 6*c*d^2 + 12*c*d*(d + e*x))/(3*e^3*(d + e*x)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^2}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{3}c\,e^2x^2 - \frac{8}{3}cdex - 2a\,e^2 - \frac{16}{3}c\,d^2}{\sqrt{ex+d}\,e^3}$$

input
```
int((c*x^2+a)/(e*x+d)^(3/2),x)
```

output
```
(2*( - 3*a*e**2 - 8*c*d**2 - 4*c*d*e*x + c*e**2*x**2))/(3*sqrt(d + e*x)*e*
*3)
```

3.161 $\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx=-\frac{2(cd^2+ae^2)}{3e^3(d+ex)^{3/2}}+\frac{4cd}{e^3\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^3}$$

output
```
1/3*(-2*a*e^2-2*c*d^2)/e^3/(e*x+d)^(3/2)+4*c*d/e^3/(e*x+d)^(1/2)+2*c*(e*x+
d)^(1/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx=-\frac{2(cd^2+ae^2-6cd(d+ex)-3c(d+ex)^2)}{3e^3(d+ex)^{3/2}}$$

input
```
Integrate[(a + c*x^2)/(d + e*x)^(5/2),x]
```

output
```
(-2*(c*d^2 + a*e^2 - 6*c*d*(d + e*x) - 3*c*(d + e*x)^2))/(3*e^3*(d + e*x)^
(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)^{5/2}}+\frac{c}{e^2\sqrt{d+ex}}-\frac{2cd}{e^2(d+ex)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(ae^2+cd^2\right)}{3e^3(d+ex)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^3}+\frac{4cd}{e^3\sqrt{d+ex}}$$

input
```
Int[(a + c*x^2)/(d + e*x)^(5/2),x]
```

output
```
(-2*(c*d^2 + a*e^2))/(3*e^3*(d + e*x)^(3/2)) + (4*c*d)/(e^3*Sqrt[d + e*x])
 + (2*c*Sqrt[d + e*x])/e^3
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{2\left(\left(-3c\,x^{2}+a\right)e^{2}-12cdxe-8c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{3}}$	38
gosper	$-\frac{2\left(-3x^{2}c\,e^{2}-12cdxe+a\,e^{2}-8c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{3}}$	40
trager	$-\frac{2\left(-3x^{2}c\,e^{2}-12cdxe+a\,e^{2}-8c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{3}}$	40
orering	$-\frac{2\left(-3x^{2}c\,e^{2}-12cdxe+a\,e^{2}-8c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^{3}}$	40
risch	$\frac{2c\sqrt{ex+d}}{e^{3}}-\frac{2\left(-6cdxe+a\,e^{2}-5c\,d^{2}\right)}{3e^{3}\left(ex+d\right)^{\frac{3}{2}}}$	45
derivativedivides	$\frac{2c\sqrt{ex+d}+\frac{4cd}{\sqrt{ex+d}}-\frac{2\left(a\,e^{2}+c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{3}}$	47
default	$\frac{2c\sqrt{ex+d}+\frac{4cd}{\sqrt{ex+d}}-\frac{2\left(a\,e^{2}+c\,d^{2}\right)}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{3}}$	47

input
```
int((c*x^2+a)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(3/2)*((-3*c*x^2+a)*e^2-12*c*d*x*e-8*c*d^2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx = \frac{2\,(3\,ce^2x^2+12\,cdex+8\,cd^2-ae^2)\sqrt{ex+d}}{3\,(e^5x^2+2\,de^4x+d^2e^3)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*c*e^2*x^2 + 12*c*d*e*x + 8*c*d^2 - a*e^2)*sqrt(e*x + d)/(e^5*x^2 +
2*d*e^4*x + d^2*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(61) = 122$.

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.85

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx = \begin{cases} -\frac{2ae^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{16cd^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{24cdex}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{6ce^2x^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{}} \\ \frac{ax+\frac{cx^3}{3}}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)**(5/2),x)
```

output
```
Piecewise((-2*a*e**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 1
6*c*d**2/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*c*d*e*x/(3
*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*c*e**2*x**2/(3*d*e**3*
sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), ((a*x + c*x**3/3)/d**(
5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{3\sqrt{ex+d}c}{e^2} + \frac{6(ex+d)cd-cd^2-ae^2}{(ex+d)^{\frac{3}{2}}e^2}\right)}{3e}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(e*x + d)*c/e^2 + (6*(e*x + d)*c*d - c*d^2 - a*e^2)/((e*x + d)^
(3/2)*e^2))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{ex+d}c}{e^3} + \frac{2\left(6\left(ex+d\right)cd - cd^2 - ae^2\right)}{3\left(ex+d\right)^{\frac{3}{2}}e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*c/e^3 + 2/3*(6*(e*x + d)*c*d - c*d^2 - a*e^2)/((e*x + d)^(
3/2)*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx = \frac{6\,c\,{\left(d+e\,x\right)}^2 - 2\,a\,e^2 - 2\,c\,d^2 + 12\,c\,d\,\left(d+e\,x\right)}{3\,e^3\,{\left(d+e\,x\right)}^{3/2}}$$

input
```
int((a + c*x^2)/(d + e*x)^(5/2),x)
```

output
```
(6*c*(d + e*x)^2 - 2*a*e^2 - 2*c*d^2 + 12*c*d*(d + e*x))/(3*e^3*(d + e*x)^
(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{a+cx^2}{(d+ex)^{5/2}}\,dx = \frac{2c\,e^2x^2 + 8cdex - \frac{2}{3}a\,e^2 + \frac{16}{3}c\,d^2}{\sqrt{ex+d}\,e^3\,(ex+d)}$$

input
```
int((c*x^2+a)/(e*x+d)^(5/2),x)
```

output
```
(2*( - a*e**2 + 8*c*d**2 + 12*c*d*e*x + 3*c*e**2*x**2))/(3*sqrt(d + e*x)*e
**3*(d + e*x))
```

3.162 $\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx = -\frac{2(cd^2+ae^2)}{5e^3(d+ex)^{5/2}}+\frac{4cd}{3e^3(d+ex)^{3/2}}-\frac{2c}{e^3\sqrt{d+ex}}$$

output
```
1/5*(-2*a*e^2-2*c*d^2)/e^3/(e*x+d)^(5/2)+4/3*c*d/e^3/(e*x+d)^(3/2)-2*c/e^3
/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx = -\frac{2(3ae^2+c(8d^2+20dex+15e^2x^2))}{15e^3(d+ex)^{5/2}}$$

input
```
Integrate[(a + c*x^2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(3*a*e^2 + c*(8*d^2 + 20*d*e*x + 15*e^2*x^2)))/(15*e^3*(d + e*x)^(5/2)
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex)^{7/2}}+\frac{c}{e^2(d+ex)^{3/2}}-\frac{2cd}{e^2(d+ex)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(ae^2+cd^2)}{5e^3(d+ex)^{5/2}}-\frac{2c}{e^3\sqrt{d+ex}}+\frac{4cd}{3e^3(d+ex)^{3/2}}$$

input
```
Int[(a + c*x^2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(c*d^2 + a*e^2))/(5*e^3*(d + e*x)^(5/2)) + (4*c*d)/(3*e^3*(d + e*x)^(3
/2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

method	result	size
pseudoelliptic	$-\frac{2\left(\left(5c\,x^{2}+a\right)e^{2}+\frac{20cdxe}{3}+\frac{8c\,d^{2}}{3}\right)}{5\left(ex+d\right)^{\frac{5}{2}}e^{3}}$	38
gosper	$-\frac{2\left(15x^{2}c\,e^{2}+20cdxe+3a\,e^{2}+8c\,d^{2}\right)}{15\left(ex+d\right)^{\frac{5}{2}}e^{3}}$	41
trager	$-\frac{2\left(15x^{2}c\,e^{2}+20cdxe+3a\,e^{2}+8c\,d^{2}\right)}{15\left(ex+d\right)^{\frac{5}{2}}e^{3}}$	41
orering	$-\frac{2\left(15x^{2}c\,e^{2}+20cdxe+3a\,e^{2}+8c\,d^{2}\right)}{15\left(ex+d\right)^{\frac{5}{2}}e^{3}}$	41
derivativedivides	$\frac{-\frac{2\left(a\,e^{2}+c\,d^{2}\right)}{5\left(ex+d\right)^{\frac{5}{2}}}-\frac{2c}{\sqrt{ex+d}}+\frac{4cd}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{3}}$	48
default	$\frac{-\frac{2\left(a\,e^{2}+c\,d^{2}\right)}{5\left(ex+d\right)^{\frac{5}{2}}}-\frac{2c}{\sqrt{ex+d}}+\frac{4cd}{3\left(ex+d\right)^{\frac{3}{2}}}}{e^{3}}$	48

input
```
int((c*x^2+a)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(e*x+d)^(5/2)*((5*c*x^2+a)*e^2+20/3*c*d*x*e+8/3*c*d^2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx=-\frac{2\,(15\,ce^2x^2+20\,cdex+8\,cd^2+3\,ae^2)\sqrt{ex+d}}{15\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(15*c*e^2*x^2 + 20*c*d*e*x + 8*c*d^2 + 3*a*e^2)*sqrt(e*x + d)/(e^6*x
^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(63) = 126$.

Time = 0.46 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.13

$$\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx = \begin{cases} -\frac{6ae^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{16cd^2}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{\cdots}{15d^2e^3\sqrt{\cdots}} \\ \frac{ax+\frac{cx^3}{3}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-6*a*e**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*
x) + 15*e**5*x**2*sqrt(d + e*x)) - 16*c*d**2/(15*d**2*e**3*sqrt(d + e*x) +
 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 40*c*d*e*x/(15*
d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d
+ e*x)) - 30*c*e**2*x**2/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d
+ e*x) + 15*e**5*x**2*sqrt(d + e*x)), Ne(e, 0)), ((a*x + c*x**3/3)/d**(7/2
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{(d+ex)^{7/2}}\,dx = -\frac{2\left(15\,(ex+d)^2c - 10\,(ex+d)cd + 3\,cd^2 + 3\,ae^2\right)}{15\,(ex+d)^{\frac{5}{2}}e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
-2/15*(15*(e*x + d)^2*c - 10*(e*x + d)*c*d + 3*c*d^2 + 3*a*e^2)/((e*x + d)
^(5/2)*e^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a + cx^2}{(d + ex)^{7/2}}\, dx = -\frac{2\left(15\,(ex + d)^2c - 10\,(ex + d)cd + 3\,cd^2 + 3\,ae^2\right)}{15\,(ex + d)^{\frac{5}{2}}e^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(15*(e*x + d)^2*c - 10*(e*x + d)*c*d + 3*c*d^2 + 3*a*e^2)/((e*x + d)
^(5/2)*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a + cx^2}{(d + ex)^{7/2}}\, dx = -\frac{30\,c\,(d + e\,x)^2 + 6\,a\,e^2 + 6\,c\,d^2 - 20\,c\,d\,(d + e\,x)}{15\,e^3\,(d + e\,x)^{5/2}}$$

input
```
int((a + c*x^2)/(d + e*x)^(7/2),x)
```

output
```
-(30*c*(d + e*x)^2 + 6*a*e^2 + 6*c*d^2 - 20*c*d*(d + e*x))/(15*e^3*(d + e*
x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{a + cx^2}{(d + ex)^{7/2}}\, dx = \frac{-2c\,e^2x^2 - \frac{8}{3}cdex - \frac{2}{5}a\,e^2 - \frac{16}{15}c\,d^2}{\sqrt{ex + d}\,e^3\,(e^2x^2 + 2dex + d^2)}$$

input
```
int((c*x^2+a)/(e*x+d)^(7/2),x)
```

output
```
(2*( - 3*a*e**2 - 8*c*d**2 - 20*c*d*e*x - 15*c*e**2*x**2))/(15*sqrt(d + e*
x)*e**3*(d**2 + 2*d*e*x + e**2*x**2))
```

3.163 $\int (d+ex)^{5/2} \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^2 \, dx = \frac{2(cd^2+ae^2)^2 (d+ex)^{7/2}}{7e^5} - \frac{8cd(cd^2+ae^2)(d+ex)^{9/2}}{9e^5} + \frac{4c(3cd^2+ae^2)(d+ex)^{11/2}}{11e^5} - \frac{8c^2d(d+ex)^{13/2}}{13e^5} + \frac{2c^2(d+ex)^{15/2}}{15e^5}$$

output
```
2/7*(a*e^2+c*d^2)^2*(e*x+d)^(7/2)/e^5-8/9*c*d*(a*e^2+c*d^2)*(e*x+d)^(9/2)/
e^5+4/11*c*(a*e^2+3*c*d^2)*(e*x+d)^(11/2)/e^5-8/13*c^2*d*(e*x+d)^(13/2)/e^
5+2/15*c^2*(e*x+d)^(15/2)/e^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^2 \, dx = \frac{2(d+ex)^{7/2} (6435a^2e^4 + 130ace^2(8d^2-28dex+63e^2x^2) + c^2(128d^4-448d^3ex+1008d^2e^2x}{45045e^5}$$

input
```
Integrate[(d + e*x)^(5/2)*(a + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(7/2)*(6435*a^2*e^4 + 130*a*c*e^2*(8*d^2 - 28*d*e*x + 63*e^2*
x^2) + c^2*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*x^3 + 30
03*e^4*x^4)))/(45045*e^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (d+ex)^{5/2} \, dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{2c(d+ex)^{9/2}\left(ae^2+3cd^2\right)}{e^4}-\frac{4cd(d+ex)^{7/2}\left(ae^2+cd^2\right)}{e^4}+\frac{(d+ex)^{5/2}\left(ae^2+cd^2\right)^2}{e^4}+\frac{c^2(d+ex)^{13/2}}{e^4}-\frac{4c\cdots}{\cdots}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{4c(d+ex)^{11/2}\left(ae^2+3cd^2\right)}{11e^5}-\frac{8cd(d+ex)^{9/2}\left(ae^2+cd^2\right)}{9e^5}+\frac{2(d+ex)^{7/2}\left(ae^2+cd^2\right)^2}{7e^5}+\frac{2c^2(d+ex)^{15/2}}{15e^5}-\frac{8c^2d(d+ex)^{13/2}}{13e^5}$$

input
```
Int[(d + e*x)^(5/2)*(a + c*x^2)^2,x]
```

output
```
(2*(c*d^2 + a*e^2)^2*(d + e*x)^(7/2))/(7*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d
+ e*x)^(9/2))/(9*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(11/2))/(11*e^5)
- (8*c^2*d*(d + e*x)^(13/2))/(13*e^5) + (2*c^2*(d + e*x)^(15/2))/(15*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{7}{2}}\left(\left(\frac{7}{15}c^2x^4+\frac{14}{11}acx^2+a^2\right)e^4-\frac{56xcd\left(\frac{33cx^2}{65}+a\right)e^3}{99}+\frac{16c\left(\frac{63cx^2}{65}+a\right)d^2e^2}{99}-\frac{448xc^2d^3e}{6435}+\frac{128c^2d^4}{6435}\right)}{7e^5}$
gosper	$\frac{2(ex+d)^{\frac{7}{2}}(3003c^2x^4e^4-1848dc^2x^3e^3+8190x^2ace^4+1008x^2c^2d^2e^2-3640xacde^3-448xc^2d^3e+6435a^2e^4+1040acd^2}{45045e^5}$
orering	$\frac{2(ex+d)^{\frac{7}{2}}(3003c^2x^4e^4-1848dc^2x^3e^3+8190x^2ace^4+1008x^2c^2d^2e^2-3640xacde^3-448xc^2d^3e+6435a^2e^4+1040acd^2}{45045e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{15}{2}}}{15}-\frac{8c^2d(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(2\left(ae^2+cd^2\right)c+4d^2c^2\right)(ex+d)^{\frac{11}{2}}}{11}-\frac{8\left(ae^2+cd^2\right)cd(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(ae^2+cd^2\right)^2(ex+d)^{\frac{7}{2}}}{7}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{15}{2}}}{15}-\frac{8c^2d(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(2\left(ae^2+cd^2\right)c+4d^2c^2\right)(ex+d)^{\frac{11}{2}}}{11}-\frac{8\left(ae^2+cd^2\right)cd(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(ae^2+cd^2\right)^2(ex+d)^{\frac{7}{2}}}{7}}{e^5}$
trager	$2(3003c^2e^7x^7+7161c^2de^6x^6+8190ace^7x^5+4473c^2d^2e^5x^5+20930acde^6x^4+35c^2d^3e^4x^4+6435a^2e^7x^3+14690ad^2e^5c$
risch	$2(3003c^2e^7x^7+7161c^2de^6x^6+8190ace^7x^5+4473c^2d^2e^5x^5+20930acde^6x^4+35c^2d^3e^4x^4+6435a^2e^7x^3+14690ad^2e^5c$

input
```
int((e*x+d)^(5/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*(e*x+d)^(7/2)*((7/15*c^2*x^4+14/11*a*c*x^2+a^2)*e^4-56/99*x*c*d*(33/65
*c*x^2+a)*e^3+16/99*c*(63/65*c*x^2+a)*d^2*e^2-448/6435*x*c^2*d^3*e+128/643
5*c^2*d^4)/e^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(107) = 214.

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.72

$$\int (d+ex)^{5/2}\left(a+cx^2\right)^2\,dx=\frac{2\left(3003\,c^2e^7x^7+7161\,c^2de^6x^6+128\,c^2d^7+1040\,acd^5e^2+6435\,a^2d^3e^4+63\left(71\,c^2d^2e^5+130\,a\right.\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3003*c^2*e^7*x^7 + 7161*c^2*d*e^6*x^6 + 128*c^2*d^7 + 1040*a*c*d^
5*e^2 + 6435*a^2*d^3*e^4 + 63*(71*c^2*d^2*e^5 + 130*a*c*e^7)*x^5 + 35*(c^2
*d^3*e^4 + 598*a*c*d*e^6)*x^4 - 5*(8*c^2*d^4*e^3 - 2938*a*c*d^2*e^5 - 1287
*a^2*e^7)*x^3 + 3*(16*c^2*d^5*e^2 + 130*a*c*d^3*e^4 + 6435*a^2*d*e^6)*x^2
- (64*c^2*d^6*e + 520*a*c*d^4*e^3 - 19305*a^2*d^2*e^5)*x)*sqrt(e*x + d)/e^
5
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

$$\int (d+ex)^{5/2}\left(a+cx^2\right)^2\,dx=\begin{cases}\frac{2\left(-\frac{4c^2d(d+ex)^{\frac{13}{2}}}{13e^4}+\frac{c^2(d+ex)^{\frac{15}{2}}}{15e^4}+\frac{(d+ex)^{\frac{11}{2}}\cdot\left(2ace^2+6c^2d^2\right)}{11e^4}+\frac{(d+ex)^{\frac{9}{2}}\left(-4acde^2-4c^2d^3\right)}{9e^4}+\frac{(d+ex)^{\frac{7}{2}}\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{7e^4}\right)}{e} \\ d^{\frac{5}{2}}\left(a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}\right)\end{cases}$$

input
```
integrate((e*x+d)**(5/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(-4*c**2*d*(d + e*x)**(13/2)/(13*e**4) + c**2*(d + e*x)**(15/
2)/(15*e**4) + (d + e*x)**(11/2)*(2*a*c*e**2 + 6*c**2*d**2)/(11*e**4) + (d
 + e*x)**(9/2)*(-4*a*c*d*e**2 - 4*c**2*d**3)/(9*e**4) + (d + e*x)**(7/2)*(
a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(7*e**4))/e, Ne(e, 0)), (d**(5/2)
*(a**2*x + 2*a*c*x**3/3 + c**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^2 \, dx = \frac{2 \left(3003 \left(ex+d\right)^{\frac{15}{2}} c^2 - 13860 \left(ex+d\right)^{\frac{13}{2}} c^2 d + 8190 \left(3 c^2 d^2 + ace^2\right) \left(ex+d\right)^{\frac{11}{2}} - 20020 \left(c^2 d^3 + \right.\right.}{45045 e^5}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/45045*(3003*(e*x + d)^(15/2)*c^2 - 13860*(e*x + d)^(13/2)*c^2*d + 8190*(
3*c^2*d^2 + a*c*e^2)*(e*x + d)^(11/2) - 20020*(c^2*d^3 + a*c*d*e^2)*(e*x +
 d)^(9/2) + 6435*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(7/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(107) = 214.

Time = 0.12 (sec) , antiderivative size = 704, normalized size of antiderivative = 5.54

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(e*x + d)*a^2*d^3 + 45045*((e*x + d)^(3/2) - 3*sqrt(e*x
 + d)*d)*a^2*d^2 + 9009*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqr
t(e*x + d)*d^2)*a^2*d + 6006*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 1
5*sqrt(e*x + d)*d^2)*a*c*d^3/e^2 + 1287*(5*(e*x + d)^(7/2) - 21*(e*x + d)^
(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2 + 7722*(5*(e*
x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x
 + d)*d^3)*a*c*d^2/e^2 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d +
 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4
)*c^2*d^3/e^4 + 858*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x
 + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*c*d/e
^2 + 195*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2
)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x
 + d)*d^5)*c^2*d^2/e^4 + 130*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d
+ 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2
)*d^4 - 693*sqrt(e*x + d)*d^5)*a*c/e^2 + 45*(231*(e*x + d)^(13/2) - 1638*(
e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 +
9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d
^6)*c^2*d/e^4 + 7*(429*(e*x + d)^(15/2) - 3465*(e*x + d)^(13/2)*d + 12285*
(e*x + d)^(11/2)*d^2 - 25025*(e*x + d)^(9/2)*d^3 + 32175*(e*x + d)^(7/2)*d
^4 - 27027*(e*x + d)^(5/2)*d^5 + 15015*(e*x + d)^(3/2)*d^6 - 6435*sqrt(...
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^2 \, dx = \frac{2\,c^2\,{\left(d+e\,x\right)}^{15/2}}{15\,e^5}-\frac{\left(8\,c^2\,d^3+8\,a\,c\,d\,e^2\right)\,{\left(d+e\,x\right)}^{9/2}}{9\,e^5}+\frac{2\,{\left(c\,d^2+a\,e^2\right)}^2\,{\left(d+e\,x\right)}^{7/2}}{7\,e^5}+\frac{\left(12\,c^2\,d^2+4\,a\,c\,e^2\right)\,{\left(d+e\,x\right)}^{11/2}}{11\,e^5}-\frac{8\,c^2\,d\,{\left(d+e\,x\right)}^{13/2}}{13\,e^5}$$

input
```
int((a + c*x^2)^2*(d + e*x)^(5/2),x)
```

output
```
(2*c^2*(d + e*x)^(15/2))/(15*e^5) - ((8*c^2*d^3 + 8*a*c*d*e^2)*(d + e*x)^(
9/2))/(9*e^5) + (2*(a*e^2 + c*d^2)^2*(d + e*x)^(7/2))/(7*e^5) + ((12*c^2*d
^2 + 4*a*c*e^2)*(d + e*x)^(11/2))/(11*e^5) - (8*c^2*d*(d + e*x)^(13/2))/(1
3*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.76

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^2 dx = \frac{2\sqrt{ex+d}\,(3003c^2e^7x^7+7161c^2d\,e^6x^6+8190ac\,e^7x^5+4473c^2d^2e^5x^5+20930acd\,e^6x^4+35c^2d$$

input
```
int((e*x+d)^(5/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(d + e*x)*(6435*a**2*d**3*e**4 + 19305*a**2*d**2*e**5*x + 19305*a**
2*d*e**6*x**2 + 6435*a**2*e**7*x**3 + 1040*a*c*d**5*e**2 - 520*a*c*d**4*e*
*3*x + 390*a*c*d**3*e**4*x**2 + 14690*a*c*d**2*e**5*x**3 + 20930*a*c*d*e**
6*x**4 + 8190*a*c*e**7*x**5 + 128*c**2*d**7 - 64*c**2*d**6*e*x + 48*c**2*d
**5*e**2*x**2 - 40*c**2*d**4*e**3*x**3 + 35*c**2*d**3*e**4*x**4 + 4473*c**
2*d**2*e**5*x**5 + 7161*c**2*d*e**6*x**6 + 3003*c**2*e**7*x**7))/(45045*e*
*5)
```

3.164 $\int (d+ex)^{3/2} \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^2 \, dx = \frac{2(cd^2+ae^2)^2 (d+ex)^{5/2}}{5e^5} - \frac{8cd(cd^2+ae^2)(d+ex)^{7/2}}{7e^5} + \frac{4c(3cd^2+ae^2)(d+ex)^{9/2}}{9e^5} - \frac{8c^2d(d+ex)^{11/2}}{11e^5} + \frac{2c^2(d+ex)^{13/2}}{13e^5}$$

output
```
2/5*(a*e^2+c*d^2)^2*(e*x+d)^(5/2)/e^5-8/7*c*d*(a*e^2+c*d^2)*(e*x+d)^(7/2)/
e^5+4/9*c*(a*e^2+3*c*d^2)*(e*x+d)^(9/2)/e^5-8/11*c^2*d*(e*x+d)^(11/2)/e^5+
2/13*c^2*(e*x+d)^(13/2)/e^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^2 \, dx = \frac{2(d+ex)^{5/2} (9009a^2e^4 + 286ace^2(8d^2-20dex+35e^2x^2) + 3c^2(128d^4 - 320d^3ex + 560d^2e^2x}{45045e^5}$$

input
```
Integrate[(d + e*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(5/2)*(9009*a^2*e^4 + 286*a*c*e^2*(8*d^2 - 20*d*e*x + 35*e^2*
x^2) + 3*c^2*(128*d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e^3*x^3 + 11
55*e^4*x^4)))/(45045*e^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (d+ex)^{3/2} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(d+ex)^{7/2}(ae^2+3cd^2)}{e^4}-\frac{4cd(d+ex)^{5/2}(ae^2+cd^2)}{e^4}+\frac{(d+ex)^{3/2}(ae^2+cd^2)^2}{e^4}+\frac{c^2(d+ex)^{11/2}}{e^4}-4c$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(d+ex)^{9/2}(ae^2+3cd^2)}{9e^5}-\frac{8cd(d+ex)^{7/2}(ae^2+cd^2)}{7e^5}+\frac{2(d+ex)^{5/2}(ae^2+cd^2)^2}{5e^5}+\frac{2c^2(d+ex)^{13/2}}{13e^5}-\frac{8c^2d(d+ex)^{11/2}}{11e^5}$$

input
```
Int[(d + e*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(c*d^2 + a*e^2)^2*(d + e*x)^(5/2))/(5*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d
+ e*x)^(7/2))/(7*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(9/2))/(9*e^5) -
(8*c^2*d*(d + e*x)^(11/2))/(11*e^5) + (2*c^2*(d + e*x)^(13/2))/(13*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{2\left(\left(\frac{5}{13}c^2x^4+\frac{10}{9}ac\,x^2+a^2\right)e^4-\frac{40xcd\left(\frac{63c\,x^2}{143}+a\right)e^3}{63}+\frac{16\left(\frac{105c\,x^2}{143}+a\right)c\,d^2e^2}{63}-\frac{320x\,c^2d^3e}{3003}+\frac{128c^2d^4}{3003}\right)(ex+d)^{\frac{5}{2}}}{5e^5}$
gosper	$\frac{2(ex+d)^{\frac{5}{2}}\left(3465c^2x^4e^4-2520d\,c^2x^3e^3+10010x^2ac\,e^4+1680x^2c^2d^2e^2-5720xacd\,e^3-960x\,c^2d^3e+9009a^2e^4+2288ac\,d\right.}{45045e^5}$
orering	$\frac{2(ex+d)^{\frac{5}{2}}\left(3465c^2x^4e^4-2520d\,c^2x^3e^3+10010x^2ac\,e^4+1680x^2c^2d^2e^2-5720xacd\,e^3-960x\,c^2d^3e+9009a^2e^4+2288ac\,d\right.}{45045e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{13}{2}}}{13}-\frac{8c^2d(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)(ex+d)^{\frac{9}{2}}}{9}-\frac{8\left(a\,e^2+c\,d^2\right)cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^2+c\,d^2\right)^2(ex+d)^{\frac{5}{2}}}{5}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{13}{2}}}{13}-\frac{8c^2d(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)(ex+d)^{\frac{9}{2}}}{9}-\frac{8\left(a\,e^2+c\,d^2\right)cd(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(a\,e^2+c\,d^2\right)^2(ex+d)^{\frac{5}{2}}}{5}}{e^5}$
trager	$\frac{2\left(3465c^2e^6x^6+4410c^2d\,e^5x^5+10010ac\,e^6x^4+105c^2d^2e^4x^4+14300acd\,e^5x^3-120c^2d^3e^3x^3+9009a^2e^6x^2+858ac\,d^2e^4x\right.}{45045e^5}$
risch	$\frac{2\left(3465c^2e^6x^6+4410c^2d\,e^5x^5+10010ac\,e^6x^4+105c^2d^2e^4x^4+14300acd\,e^5x^3-120c^2d^3e^3x^3+9009a^2e^6x^2+858ac\,d^2e^4x\right.}{45045e^5}$

input
```
int((e*x+d)^(3/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*((5/13*c^2*x^4+10/9*a*c*x^2+a^2)*e^4-40/63*x*c*d*(63/143*c*x^2+a)*e^3+
16/63*(105/143*c*x^2+a)*c*d^2*e^2-320/3003*x*c^2*d^3*e+128/3003*c^2*d^4)*(
e*x+d)^(5/2)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.43

$$\int (d+ex)^{3/2} (a+cx^2)^2\, dx = \frac{2\,(3465\,c^2e^6x^6+4410\,c^2de^5x^5+384\,c^2d^6+2288\,acd^4e^2+9009\,a^2d^2e^4+35\,(3\,c^2d^2e^4+286\,ac$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3465*c^2*e^6*x^6 + 4410*c^2*d*e^5*x^5 + 384*c^2*d^6 + 2288*a*c*d^
4*e^2 + 9009*a^2*d^2*e^4 + 35*(3*c^2*d^2*e^4 + 286*a*c*e^6)*x^4 - 20*(6*c^
2*d^3*e^3 - 715*a*c*d*e^5)*x^3 + 3*(48*c^2*d^4*e^2 + 286*a*c*d^2*e^4 + 300
3*a^2*e^6)*x^2 - 2*(96*c^2*d^5*e + 572*a*c*d^3*e^3 - 9009*a^2*d*e^5)*x)*sq
rt(e*x + d)/e^5
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

$$\int (d+ex)^{3/2} (a+cx^2)^2\, dx = \begin{cases} \frac{2\left(-\frac{4c^2d(d+ex)^{\frac{11}{2}}}{11e^4}+\frac{c^2(d+ex)^{\frac{13}{2}}}{13e^4}+\frac{(d+ex)^{\frac{9}{2}}\cdot\left(2ace^2+6c^2d^2\right)}{9e^4}+\frac{(d+ex)^{\frac{7}{2}}\left(-4acde^2-4c^2d^3\right)}{7e^4}+\frac{(d+ex)^{\frac{5}{2}}\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{5e^4}\right)}{e} & \text{f} \\ d^{\frac{3}{2}}\left(a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}\right) & \text{o} \end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(-4*c**2*d*(d + e*x)**(11/2)/(11*e**4) + c**2*(d + e*x)**(13/
2)/(13*e**4) + (d + e*x)**(9/2)*(2*a*c*e**2 + 6*c**2*d**2)/(9*e**4) + (d +
 e*x)**(7/2)*(-4*a*c*d*e**2 - 4*c**2*d**3)/(7*e**4) + (d + e*x)**(5/2)*(a*
*2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(5*e**4))/e, Ne(e, 0)), (d**(3/2)*(
a**2*x + 2*a*c*x**3/3 + c**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^2 dx = \frac{2\left(3465\,(ex+d)^{\frac{13}{2}}c^2 - 16380\,(ex+d)^{\frac{11}{2}}c^2d + 10010\,(3\,c^2d^2+ace^2)(ex+d)^{\frac{9}{2}} - 25740\,(c^2d^3+\right.}{45045\,e^5}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/45045*(3465*(e*x + d)^(13/2)*c^2 - 16380*(e*x + d)^(11/2)*c^2*d + 10010*
(3*c^2*d^2 + a*c*e^2)*(e*x + d)^(9/2) - 25740*(c^2*d^3 + a*c*d*e^2)*(e*x +
 d)^(7/2) + 9009*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(5/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(107) = 214$.

Time = 0.12 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.71

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^2 dx = \frac{2\left(45045\,\sqrt{ex+d}a^2d^2 + 30030\left((ex+d)^{\frac{3}{2}} - 3\,\sqrt{ex+d}d\right)a^2d + 3003\left(3\,(ex+d)^{\frac{5}{2}} - 10\,(ex+\right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(e*x + d)*a^2*d^2 + 30030*((e*x + d)^(3/2) - 3*sqrt(e*x
 + d)*d)*a^2*d + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(
e*x + d)*d^2)*a^2 + 6006*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sq
rt(e*x + d)*d^2)*a*c*d^2/e^2 + 5148*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2
)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*c*d/e^2 + 143*(35*(
e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*
x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^2*d^2/e^4 + 286*(35*(e*x + d)^
(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3
/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*c/e^2 + 130*(63*(e*x + d)^(11/2) - 385*
(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1
155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c^2*d/e^4 + 15*(231*(e*x
+ d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e
*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 +
3003*sqrt(e*x + d)*d^6)*c^2/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^2 \, dx = \frac{2\,c^2\,{\left(d+e\,x\right)}^{13/2}}{13\,e^5}-\frac{\left(8\,c^2\,d^3+8\,a\,c\,d\,e^2\right)\,{\left(d+e\,x\right)}^{7/2}}{7\,e^5}+\frac{2\,{\left(c\,d^2+a\,e^2\right)}^2\,{\left(d+e\,x\right)}^{5/2}}{5\,e^5}+\frac{\left(12\,c^2\,d^2+4\,a\,c\,e^2\right)\,{\left(d+e\,x\right)}^{9/2}}{9\,e^5}-\frac{8\,c^2\,d\,{\left(d+e\,x\right)}^{11/2}}{11\,e^5}$$

input
```
int((a + c*x^2)^2*(d + e*x)^(3/2),x)
```

output
```
(2*c^2*(d + e*x)^(13/2))/(13*e^5) - ((8*c^2*d^3 + 8*a*c*d*e^2)*(d + e*x)^(
7/2))/(7*e^5) + (2*(a*e^2 + c*d^2)^2*(d + e*x)^(5/2))/(5*e^5) + ((12*c^2*d
^2 + 4*a*c*e^2)*(d + e*x)^(9/2))/(9*e^5) - (8*c^2*d*(d + e*x)^(11/2))/(11*
e^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.43

$$\int (d+ex)^{3/2} (a+cx^2)^2 \, dx = \frac{2\sqrt{ex+d}\,(3465c^2e^6x^6+4410c^2d\,e^5x^5+10010ac\,e^6x^4+105c^2d^2e^4x^4+14300acd\,e^5x^3-120c$$

input
```
int((e*x+d)^(3/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(d + e*x)*(9009*a**2*d**2*e**4 + 18018*a**2*d*e**5*x + 9009*a**2*e*
*6*x**2 + 2288*a*c*d**4*e**2 - 1144*a*c*d**3*e**3*x + 858*a*c*d**2*e**4*x*
*2 + 14300*a*c*d*e**5*x**3 + 10010*a*c*e**6*x**4 + 384*c**2*d**6 - 192*c**
2*d**5*e*x + 144*c**2*d**4*e**2*x**2 - 120*c**2*d**3*e**3*x**3 + 105*c**2*
d**2*e**4*x**4 + 4410*c**2*d*e**5*x**5 + 3465*c**2*e**6*x**6))/(45045*e**5
)
```

3.165 $\int \sqrt{d+ex}(a+cx^2)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int \sqrt{d+ex}(a+cx^2)^2\,dx = \frac{2(cd^2+ae^2)^2(d+ex)^{3/2}}{3e^5} - \frac{8cd(cd^2+ae^2)(d+ex)^{5/2}}{5e^5} + \frac{4c(3cd^2+ae^2)(d+ex)^{7/2}}{7e^5} - \frac{8c^2d(d+ex)^{9/2}}{9e^5} + \frac{2c^2(d+ex)^{11/2}}{11e^5}$$

output
```
2/3*(a*e^2+c*d^2)^2*(e*x+d)^(3/2)/e^5-8/5*c*d*(a*e^2+c*d^2)*(e*x+d)^(5/2)/
e^5+4/7*c*(a*e^2+3*c*d^2)*(e*x+d)^(7/2)/e^5-8/9*c^2*d*(e*x+d)^(9/2)/e^5+2/
11*c^2*(e*x+d)^(11/2)/e^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \sqrt{d+ex}(a+cx^2)^2\,dx = \frac{2(d+ex)^{3/2}(1155a^2e^4+66ace^2(8d^2-12dex+15e^2x^2)+c^2(128d^4-192d^3ex+240d^2e^2x^2-280de^3x^3}{3465e^5}$$

input
```
Integrate[Sqrt[d + e*x]*(a + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(3/2)*(1155*a^2*e^4 + 66*a*c*e^2*(8*d^2 - 12*d*e*x + 15*e^2*x
^2) + c^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e
^4*x^4)))/(3465*e^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 \sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(d+ex)^{5/2}\left(ae^2+3cd^2\right)}{e^4}-\frac{4cd(d+ex)^{3/2}\left(ae^2+cd^2\right)}{e^4}+\frac{\sqrt{d+ex}\left(ae^2+cd^2\right)^2}{e^4}+\frac{c^2(d+ex)^{9/2}}{e^4}-\frac{4c^2d(d\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(d+ex)^{7/2}\left(ae^2+3cd^2\right)}{7e^5}-\frac{8cd(d+ex)^{5/2}\left(ae^2+cd^2\right)}{5e^5}+\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)^2}{3e^5}+\frac{2c^2(d+ex)^{11/2}}{11e^5}-\frac{8c^2d(d+ex)^{9/2}}{9e^5}$$

input
```
Int[Sqrt[d + e*x]*(a + c*x^2)^2,x]
```

output
```
(2*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2))/(3*e^5) - (8*c*d*(c*d^2 + a*e^2)*(d
+ e*x)^(5/2))/(5*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(7/2))/(7*e^5) -
(8*c^2*d*(d + e*x)^(9/2))/(9*e^5) + (2*c^2*(d + e*x)^(11/2))/(11*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{3}{2}}\left(\left(\frac{3}{11}c^2x^4+\frac{6}{7}ac\,x^2+a^2\right)e^4-\frac{24xc\left(\frac{35c\,x^2}{99}+a\right)d\,e^3}{35}+\frac{16\left(\frac{5c\,x^2}{11}+a\right)c\,d^2e^2}{35}-\frac{64x\,c^2d^3e}{385}+\frac{128c^2d^4}{1155}\right)}{3e^5}$
gosper	$\frac{2(ex+d)^{\frac{3}{2}}\left(315c^2x^4e^4-280d\,c^2x^3e^3+990x^2ac\,e^4+240x^2c^2d^2e^2-792xacd\,e^3-192x\,c^2d^3e+1155a^2e^4+528ac\,d^2e^2+128\right.}{3465e^5}$
orering	$\frac{2(ex+d)^{\frac{3}{2}}\left(315c^2x^4e^4-280d\,c^2x^3e^3+990x^2ac\,e^4+240x^2c^2d^2e^2-792xacd\,e^3-192x\,c^2d^3e+1155a^2e^4+528ac\,d^2e^2+128\right.}{3465e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{11}{2}}}{11}-\frac{8c^2d(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)(ex+d)^{\frac{7}{2}}}{7}-\frac{8\left(a\,e^2+c\,d^2\right)cd(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^2+c\,d^2\right)^2(ex+d)^{\frac{3}{2}}}{3}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{11}{2}}}{11}-\frac{8c^2d(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)(ex+d)^{\frac{7}{2}}}{7}-\frac{8\left(a\,e^2+c\,d^2\right)cd(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(a\,e^2+c\,d^2\right)^2(ex+d)^{\frac{3}{2}}}{3}}{e^5}$
trager	$\frac{2\left(315c^2e^5x^5+35c^2d\,e^4x^4+990ac\,e^5x^3-40c^2d^2e^3x^3+198acd\,e^4x^2+48c^2d^3e^2x^2+1155a^2e^5x-264ac\,d^2e^3x-64c^2d^4ex\right.}{3465e^5}$
risch	$\frac{2\left(315c^2e^5x^5+35c^2d\,e^4x^4+990ac\,e^5x^3-40c^2d^2e^3x^3+198acd\,e^4x^2+48c^2d^3e^2x^2+1155a^2e^5x-264ac\,d^2e^3x-64c^2d^4ex\right.}{3465e^5}$

input
```
int((e*x+d)^(1/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x+d)^(3/2)*((3/11*c^2*x^4+6/7*a*c*x^2+a^2)*e^4-24/35*x*c*(35/99*c*x
^2+a)*d*e^3+16/35*(5/11*c*x^2+a)*c*d^2*e^2-64/385*x*c^2*d^3*e+128/1155*c^2
*d^4)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \sqrt{d+ex}\left(a+cx^2\right)^2\,dx$$
$$=\frac{2\left(315\,c^2e^5x^5+35\,c^2de^4x^4+128\,c^2d^5+528\,acd^3e^2+1155\,a^2de^4-10\left(4\,c^2d^2e^3-99\,ace^5\right)x^3+6\left(8\,c^2d^3e\right.\right.}{3465\,e^5}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/3465*(315*c^2*e^5*x^5 + 35*c^2*d*e^4*x^4 + 128*c^2*d^5 + 528*a*c*d^3*e^2
 + 1155*a^2*d*e^4 - 10*(4*c^2*d^2*e^3 - 99*a*c*e^5)*x^3 + 6*(8*c^2*d^3*e^2
 + 33*a*c*d*e^4)*x^2 - (64*c^2*d^4*e + 264*a*c*d^2*e^3 - 1155*a^2*e^5)*x)*
sqrt(e*x + d)/e^5
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

$$\int \sqrt{d+ex}\left(a+cx^2\right)^2\,dx$$
$$=\begin{cases}\frac{2\left(-\frac{4c^2d(d+ex)^{\frac{9}{2}}}{9e^4}+\frac{c^2(d+ex)^{\frac{11}{2}}}{11e^4}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(2ace^2+6c^2d^2\right)}{7e^4}+\frac{(d+ex)^{\frac{5}{2}}\left(-4acde^2-4c^2d^3\right)}{5e^4}+\frac{(d+ex)^{\frac{3}{2}}\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{3e^4}\right)}{e} & \text{for } e\neq 0\\ \sqrt{d}\left(a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(-4*c**2*d*(d + e*x)**(9/2)/(9*e**4) + c**2*(d + e*x)**(11/2)
/(11*e**4) + (d + e*x)**(7/2)*(2*a*c*e**2 + 6*c**2*d**2)/(7*e**4) + (d + e
*x)**(5/2)*(-4*a*c*d*e**2 - 4*c**2*d**3)/(5*e**4) + (d + e*x)**(3/2)*(a**2
*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(3*e**4))/e, Ne(e, 0)), (sqrt(d)*(a**
2*x + 2*a*c*x**3/3 + c**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int \sqrt{d+ex}\left(a+cx^2\right)^2\,dx = \frac{2\left(315\,(ex+d)^{\frac{11}{2}}c^2-1540\,(ex+d)^{\frac{9}{2}}c^2d+990\,(3\,c^2d^2+ace^2)(ex+d)^{\frac{7}{2}}-2772\,(c^2d^3+acde^2)(ex+d)^{\frac{5}{2}}\right.}{3465\,e^5}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/3465*(315*(e*x + d)^(11/2)*c^2 - 1540*(e*x + d)^(9/2)*c^2*d + 990*(3*c^2
*d^2 + a*c*e^2)*(e*x + d)^(7/2) - 2772*(c^2*d^3 + a*c*d*e^2)*(e*x + d)^(5/
2) + 1155*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(e*x + d)^(3/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(107) = 214.

Time = 0.12 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.16

$$\int \sqrt{d+ex}\left(a+cx^2\right)^2\,dx = \frac{2\left(3465\,\sqrt{ex+d}a^2d+1155\,\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)a^2+\frac{462\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)acd}{e^2}+\frac{198}{}\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(e*x + d)*a^2*d + 1155*((e*x + d)^(3/2) - 3*sqrt(e*x + d)
*d)*a^2 + 462*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*a*c*d/e^2 + 198*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x
+ d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a*c/e^2 + 11*(35*(e*x + d)^(9/2) -
180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
+ 315*sqrt(e*x + d)*d^4)*c^2*d/e^4 + 5*(63*(e*x + d)^(11/2) - 385*(e*x + d
)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x
 + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c^2/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \sqrt{d+ex}\left(a+cx^2\right)^2\,dx = \frac{2\,c^2\,(d+e\,x)^{11/2}}{11\,e^5}-\frac{\left(8\,c^2\,d^3+8\,a\,c\,d\,e^2\right)\,(d+e\,x)^{5/2}}{5\,e^5}+\frac{2\,\left(c\,d^2+a\,e^2\right)^2\,(d+e\,x)^{3/2}}{3\,e^5}+\frac{\left(12\,c^2\,d^2+4\,a\,c\,e^2\right)\,(d+e\,x)^{7/2}}{7\,e^5}-\frac{8\,c^2\,d\,(d+e\,x)^{9/2}}{9\,e^5}$$

input
```
int((a + c*x^2)^2*(d + e*x)^(1/2),x)
```

output
```
(2*c^2*(d + e*x)^(11/2))/(11*e^5) - ((8*c^2*d^3 + 8*a*c*d*e^2)*(d + e*x)^(
5/2))/(5*e^5) + (2*(a*e^2 + c*d^2)^2*(d + e*x)^(3/2))/(3*e^5) + ((12*c^2*d
^2 + 4*a*c*e^2)*(d + e*x)^(7/2))/(7*e^5) - (8*c^2*d*(d + e*x)^(9/2))/(9*e^
5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.11

$$\int \sqrt{d+ex}\left(a+cx^2\right)^2\,dx = \frac{2\sqrt{ex+d}\,(315c^2e^5x^5+35c^2d\,e^4x^4+990ac\,e^5x^3-40c^2d^2e^3x^3+198acd\,e^4x^2+48c^2d^3e^2x^2+1155a^2e^5x}{3465e^5}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(d + e*x)*(1155*a**2*d*e**4 + 1155*a**2*e**5*x + 528*a*c*d**3*e**2
- 264*a*c*d**2*e**3*x + 198*a*c*d*e**4*x**2 + 990*a*c*e**5*x**3 + 128*c**2
*d**5 - 64*c**2*d**4*e*x + 48*c**2*d**3*e**2*x**2 - 40*c**2*d**2*e**3*x**3
 + 35*c**2*d*e**4*x**4 + 315*c**2*e**5*x**5))/(3465*e**5)
```

3.166 $\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2(cd^2+ae^2)^2\sqrt{d+ex}}{e^5} - \frac{8cd(cd^2+ae^2)(d+ex)^{3/2}}{3e^5} + \frac{4c(3cd^2+ae^2)(d+ex)^{5/2}}{5e^5} - \frac{8c^2d(d+ex)^{7/2}}{7e^5} + \frac{2c^2(d+ex)^{9/2}}{9e^5}$$

output
```
2*(a*e^2+c*d^2)^2*(e*x+d)^(1/2)/e^5-8/3*c*d*(a*e^2+c*d^2)*(e*x+d)^(3/2)/e^
5+4/5*c*(a*e^2+3*c*d^2)*(e*x+d)^(5/2)/e^5-8/7*c^2*d*(e*x+d)^(7/2)/e^5+2/9*
c^2*(e*x+d)^(9/2)/e^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(315a^2e^4+42ace^2(8d^2-4dex+3e^2x^2)+c^2(128d^4-64d^3ex+48d^2e^2x^2-40de^3x^3+35e^4x^4)}{315e^5}$$

input
```
Integrate[(a + c*x^2)^2/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(315*a^2*e^4 + 42*a*c*e^2*(8*d^2 - 4*d*e*x + 3*e^2*x^2) +
 c^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3 + 35*e^4*x^4)))
/(315*e^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 476}$$

$$\int \left(\frac{2c(d+ex)^{3/2}\left(ae^2+3cd^2\right)}{e^4}-\frac{4cd\sqrt{d+ex}\left(ae^2+cd^2\right)}{e^4}+\frac{\left(ae^2+cd^2\right)^2}{e^4\sqrt{d+ex}}+\frac{c^2(d+ex)^{7/2}}{e^4}-\frac{4c^2d(d+ex)^{5/2}}{e^4}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4c(d+ex)^{5/2}\left(ae^2+3cd^2\right)}{5e^5}-\frac{8cd(d+ex)^{3/2}\left(ae^2+cd^2\right)}{3e^5}+\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)^2}{e^5}+\frac{2c^2(d+ex)^{9/2}}{9e^5}-\frac{8c^2d(d+ex)^{7/2}}{7e^5}$$

input
```
Int[(a + c*x^2)^2/Sqrt[d + e*x],x]
```

output
```
(2*(c*d^2 + a*e^2)^2*Sqrt[d + e*x])/e^5 - (8*c*d*(c*d^2 + a*e^2)*(d + e*x)
^(3/2))/(3*e^5) + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^5) - (8*c^2
*d*(d + e*x)^(7/2))/(7*e^5) + (2*c^2*(d + e*x)^(9/2))/(9*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{2\sqrt{ex+d}\left(\left(\frac{1}{9}c^2x^4+\frac{2}{5}ac\,x^2+a^2\right)e^4-\frac{8\left(\frac{5c\,x^2}{21}+a\right)xcd\,e^3}{15}+\frac{16c\,d^2\left(\frac{c\,x^2}{7}+a\right)e^2}{15}-\frac{64x\,c^2d^3e}{315}+\frac{128c^2d^4}{315}\right)}{e^5}$
gosper	$\frac{2\sqrt{ex+d}\,(35c^2x^4e^4-40d\,c^2x^3e^3+126x^2ac\,e^4+48x^2c^2d^2e^2-168xacd\,e^3-64x\,c^2d^3e+315a^2e^4+336ac\,d^2e^2+128c^2d^4)}{315e^5}$
trager	$\frac{2\sqrt{ex+d}\,(35c^2x^4e^4-40d\,c^2x^3e^3+126x^2ac\,e^4+48x^2c^2d^2e^2-168xacd\,e^3-64x\,c^2d^3e+315a^2e^4+336ac\,d^2e^2+128c^2d^4)}{315e^5}$
risch	$\frac{2\sqrt{ex+d}\,(35c^2x^4e^4-40d\,c^2x^3e^3+126x^2ac\,e^4+48x^2c^2d^2e^2-168xacd\,e^3-64x\,c^2d^3e+315a^2e^4+336ac\,d^2e^2+128c^2d^4)}{315e^5}$
orering	$\frac{2\sqrt{ex+d}\,(35c^2x^4e^4-40d\,c^2x^3e^3+126x^2ac\,e^4+48x^2c^2d^2e^2-168xacd\,e^3-64x\,c^2d^3e+315a^2e^4+336ac\,d^2e^2+128c^2d^4)}{315e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{9}{2}}}{9}-\frac{8c^2d(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)(ex+d)^{\frac{5}{2}}}{5}-\frac{8\left(a\,e^2+c\,d^2\right)cd(ex+d)^{\frac{3}{2}}}{3}+2\left(a\,e^2+c\,d^2\right)^2\sqrt{ex+d}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{9}{2}}}{9}-\frac{8c^2d(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)(ex+d)^{\frac{5}{2}}}{5}-\frac{8\left(a\,e^2+c\,d^2\right)cd(ex+d)^{\frac{3}{2}}}{3}+2\left(a\,e^2+c\,d^2\right)^2\sqrt{ex+d}}{e^5}$

input
```
int((c*x^2+a)^2/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*x+d)^(1/2)*((1/9*c^2*x^4+2/5*a*c*x^2+a^2)*e^4-8/15*(5/21*c*x^2+a)*x*c
*d*e^3+16/15*c*d^2*(1/7*c*x^2+a)*e^2-64/315*x*c^2*d^3*e+128/315*c^2*d^4)/e
^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\,(35\,c^2e^4x^4-40\,c^2de^3x^3+128\,c^2d^4+336\,acd^2e^2+315\,a^2e^4+6\,(8\,c^2d^2e^2+21\,ace^4)x^2-8\,(8\,c^2d^3e+21}{315\,e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*c^2*e^4*x^4 - 40*c^2*d*e^3*x^3 + 128*c^2*d^4 + 336*a*c*d^2*e^2 +
 315*a^2*e^4 + 6*(8*c^2*d^2*e^2 + 21*a*c*e^4)*x^2 - 8*(8*c^2*d^3*e + 21*a*
c*d*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \begin{cases} \frac{2\left(-\frac{4c^2d(d+ex)^{\frac{7}{2}}}{7e^4}+\frac{c^2(d+ex)^{\frac{9}{2}}}{9e^4}+\frac{(d+ex)^{\frac{5}{2}}\cdot\left(2ace^2+6c^2d^2\right)}{5e^4}+\frac{(d+ex)^{\frac{3}{2}}\left(-4acde^2-4c^2d^3\right)}{3e^4}+\frac{\sqrt{d+ex}\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{e^4}\right)}{e} & \text{for } e\neq 0 \\ \frac{a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(-4*c**2*d*(d + e*x)**(7/2)/(7*e**4) + c**2*(d + e*x)**(9/2)/
(9*e**4) + (d + e*x)**(5/2)*(2*a*c*e**2 + 6*c**2*d**2)/(5*e**4) + (d + e*x
)**(3/2)*(-4*a*c*d*e**2 - 4*c**2*d**3)/(3*e**4) + sqrt(d + e*x)*(a**2*e**4
 + 2*a*c*d**2*e**2 + c**2*d**4)/e**4)/e, Ne(e, 0)), ((a**2*x + 2*a*c*x**3/
3 + c**2*x**5/5)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\left(315\sqrt{ex+d}a^2+\frac{42\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)ac}{e^2}+\frac{\left(35(ex+d)^{\frac{9}{2}}-180(ex+d)^{\frac{7}{2}}d+378(ex+d)^{\frac{5}{2}}d^2-420(ex+d)^{\frac{3}{2}}\right.}{e^4}\right.}{315\,e}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(315*sqrt(e*x + d)*a^2 + 42*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*
d + 15*sqrt(e*x + d)*d^2)*a*c/e^2 + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7
/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x +
 d)*d^4)*c^2/e^4)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\left(315\sqrt{ex+d}a^2+\frac{42\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)ac}{e^2}+\frac{\left(35(ex+d)^{\frac{9}{2}}-180(ex+d)^{\frac{7}{2}}d+378(ex+d)^{\frac{5}{2}}d^2-420(ex+d)^{\frac{3}{2}}\right.}{e^4}\right.}{315\,e}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(e*x + d)*a^2 + 42*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*
d + 15*sqrt(e*x + d)*d^2)*a*c/e^2 + (35*(e*x + d)^(9/2) - 180*(e*x + d)^(7
/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x +
 d)*d^4)*c^2/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\,c^2\,(d+e\,x)^{9/2}}{9\,e^5} - \frac{(8\,c^2\,d^3+8\,a\,c\,d\,e^2)\,(d+e\,x)^{3/2}}{3\,e^5} + \frac{2\,(c\,d^2+a\,e^2)^2\,\sqrt{d+e\,x}}{e^5} + \frac{(12\,c^2\,d^2+4\,a\,c\,e^2)\,(d+e\,x)^{5/2}}{5\,e^5} - \frac{8\,c^2\,d\,(d+e\,x)^{7/2}}{7\,e^5}$$

input
```
int((a + c*x^2)^2/(d + e*x)^(1/2),x)
```

output
```
(2*c^2*(d + e*x)^(9/2))/(9*e^5) - ((8*c^2*d^3 + 8*a*c*d*e^2)*(d + e*x)^(3/
2))/(3*e^5) + (2*(a*e^2 + c*d^2)^2*(d + e*x)^(1/2))/e^5 + ((12*c^2*d^2 + 4
*a*c*e^2)*(d + e*x)^(5/2))/(5*e^5) - (8*c^2*d*(d + e*x)^(7/2))/(7*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{ex+d}\,(35c^2e^4x^4 - 40c^2d\,e^3x^3 + 126ac\,e^4x^2 + 48c^2d^2e^2x^2 - 168acd\,e^3x - 64c^2d^3ex + 315a^2e^4 + 336a}{315e^5}$$

input
```
int((c*x^2+a)^2/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(315*a**2*e**4 + 336*a*c*d**2*e**2 - 168*a*c*d*e**3*x + 1
26*a*c*e**4*x**2 + 128*c**2*d**4 - 64*c**2*d**3*e*x + 48*c**2*d**2*e**2*x*
*2 - 40*c**2*d*e**3*x**3 + 35*c**2*e**4*x**4))/(315*e**5)
```

3.167 $\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = -\frac{2(cd^2+ae^2)^2}{e^5\sqrt{d+ex}} - \frac{8cd(cd^2+ae^2)\sqrt{d+ex}}{e^5} + \frac{4c(3cd^2+ae^2)(d+ex)^{3/2}}{3e^5} - \frac{8c^2d(d+ex)^{5/2}}{5e^5} + \frac{2c^2(d+ex)^{7/2}}{7e^5}$$

output
```
-2*(a*e^2+c*d^2)^2/e^5/(e*x+d)^(1/2)-8*c*d*(a*e^2+c*d^2)*(e*x+d)^(1/2)/e^5
+4/3*c*(a*e^2+3*c*d^2)*(e*x+d)^(3/2)/e^5-8/5*c^2*d*(e*x+d)^(5/2)/e^5+2/7*c
^2*(e*x+d)^(7/2)/e^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = -\frac{2(105a^2e^4+70ace^2(8d^2+4dex-e^2x^2)+3c^2(128d^4+64d^3ex-16d^2e^2x^2+8de^3x^3-5e^4x^4))}{105e^5\sqrt{d+ex}}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^(3/2),x]
```

output
```
(-2*(105*a^2*e^4 + 70*a*c*e^2*(8*d^2 + 4*d*e*x - e^2*x^2) + 3*c^2*(128*d^4
 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4)))/(105*e^5*Sqrt[
d + e*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c\sqrt{d+ex}(ae^2+3cd^2)}{e^4}-\frac{4cd(ae^2+cd^2)}{e^4\sqrt{d+ex}}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^{3/2}}+\frac{c^2(d+ex)^{5/2}}{e^4}-\frac{4c^2d(d+ex)^{3/2}}{e^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(d+ex)^{3/2}\left(ae^2+3cd^2\right)}{3e^5}-\frac{8cd\sqrt{d+ex}(ae^2+cd^2)}{e^5}-\frac{2\left(ae^2+cd^2\right)^2}{e^5\sqrt{d+ex}}+\frac{2c^2(d+ex)^{7/2}}{7e^5}-\frac{8c^2d(d+ex)^{5/2}}{5e^5}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^(3/2),x]
```

output
```
(-2*(c*d^2 + a*e^2)^2)/(e^5*Sqrt[d + e*x]) - (8*c*d*(c*d^2 + a*e^2)*Sqrt[d
 + e*x])/e^5 + (4*c*(3*c*d^2 + a*e^2)*(d + e*x)^(3/2))/(3*e^5) - (8*c^2*d*
(d + e*x)^(5/2))/(5*e^5) + (2*c^2*(d + e*x)^(7/2))/(7*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{1}{7}c^2x^4-\frac{2}{3}acx^2+a^2\right)e^4+\frac{8xcd\left(\frac{3cx^2}{35}+a\right)e^3}{3}+\frac{16\left(-\frac{3cx^2}{35}+a\right)cd^2e^2}{3}+\frac{64xc^2d^3e}{35}+\frac{128c^2d^4}{35}\right)}{\sqrt{ex+d}\,e^5}$
risch	$-\frac{2c(-15ce^3x^3+39cde^2x^2-70ae^3x-87cd^2ex+350ade^2+279cd^3)\sqrt{ex+d}}{105e^5}-\frac{2(a^2e^4+2acd^2e^2+c^2d^4)}{e^5\sqrt{ex+d}}$
gosper	$-\frac{2(-15c^2x^4e^4+24dc^2x^3e^3-70x^2ace^4-48x^2c^2d^2e^2+280xacde^3+192xc^2d^3e+105a^2e^4+560acd^2e^2+384c^2d^4)}{105\sqrt{ex+d}\,e^5}$
trager	$-\frac{2(-15c^2x^4e^4+24dc^2x^3e^3-70x^2ace^4-48x^2c^2d^2e^2+280xacde^3+192xc^2d^3e+105a^2e^4+560acd^2e^2+384c^2d^4)}{105\sqrt{ex+d}\,e^5}$
orering	$-\frac{2(-15c^2x^4e^4+24dc^2x^3e^3-70x^2ace^4-48x^2c^2d^2e^2+280xacde^3+192xc^2d^3e+105a^2e^4+560acd^2e^2+384c^2d^4)}{105\sqrt{ex+d}\,e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{7}{2}}}{7}-\frac{8c^2d(ex+d)^{\frac{5}{2}}}{5}+\frac{4ace^2(ex+d)^{\frac{3}{2}}}{3}+4c^2d^2(ex+d)^{\frac{3}{2}}-8acde^2\sqrt{ex+d}-8c^2d^3\sqrt{ex+d}-\frac{2\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{\sqrt{ex+d}}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{7}{2}}}{7}-\frac{8c^2d(ex+d)^{\frac{5}{2}}}{5}+\frac{4ace^2(ex+d)^{\frac{3}{2}}}{3}+4c^2d^2(ex+d)^{\frac{3}{2}}-8acde^2\sqrt{ex+d}-8c^2d^3\sqrt{ex+d}-\frac{2\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{\sqrt{ex+d}}}{e^5}$

input
```
int((c*x^2+a)^2/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((-1/7*c^2*x^4-2/3*a*c*x^2+a^2)*e^4+8/3*x*c*d*(3/35*c*x^2+a)*e^3+16/3*(
-3/35*c*x^2+a)*c*d^2*e^2+64/35*x*c^2*d^3*e+128/35*c^2*d^4)/(e*x+d)^(1/2)/e
^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{2\,(15\,c^2e^4x^4 - 24\,c^2de^3x^3 - 384\,c^2d^4 - 560\,acd^2e^2 - 105\,a^2e^4 + 2\,(24\,c^2d^2e^2 + 35\,ace^4)}{105\,(e^6x + de^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*c^2*e^4*x^4 - 24*c^2*d*e^3*x^3 - 384*c^2*d^4 - 560*a*c*d^2*e^2 -
 105*a^2*e^4 + 2*(24*c^2*d^2*e^2 + 35*a*c*e^4)*x^2 - 8*(24*c^2*d^3*e + 35*
a*c*d*e^3)*x)*sqrt(e*x + d)/(e^6*x + d*e^5)
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{4c^2d(d+ex)^{\frac{5}{2}}}{5e^4} + \frac{c^2(d+ex)^{\frac{7}{2}}}{7e^4} + \frac{(d+ex)^{\frac{3}{2}}\cdot(2ace^2+6c^2d^2)}{3e^4} + \frac{\sqrt{d+ex}\left(-4acde^2-4c^2d^3\right)}{e^4} - \frac{\left(ae^2+cd^2\right)^2}{e^4\sqrt{d+ex}}\right)}{e} & \text{for}\ e \neq 0 \\ \frac{a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}}{d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(-4*c**2*d*(d + e*x)**(5/2)/(5*e**4) + c**2*(d + e*x)**(7/2)/
(7*e**4) + (d + e*x)**(3/2)*(2*a*c*e**2 + 6*c**2*d**2)/(3*e**4) + sqrt(d +
 e*x)*(-4*a*c*d*e**2 - 4*c**2*d**3)/e**4 - (a*e**2 + c*d**2)**2/(e**4*sqrt
(d + e*x)))/e, Ne(e, 0)), ((a**2*x + 2*a*c*x**3/3 + c**2*x**5/5)/d**(3/2),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{15\,(ex+d)^{\frac{7}{2}}c^2-84\,(ex+d)^{\frac{5}{2}}c^2d+70\,(3\,c^2d^2+ace^2)(ex+d)^{\frac{3}{2}}-420\,(c^2d^3+acde^2)\sqrt{ex+d}}{e^4}-\frac{105\,(c^2d^4+2\,acd^2e^2+}{\sqrt{ex+d}e^4}\right.}{105\,e}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(e*x + d)^(7/2)*c^2 - 84*(e*x + d)^(5/2)*c^2*d + 70*(3*c^2*d^2
+ a*c*e^2)*(e*x + d)^(3/2) - 420*(c^2*d^3 + a*c*d*e^2)*sqrt(e*x + d))/e^4
- 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)/(sqrt(e*x + d)*e^4))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = -\frac{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)}{\sqrt{ex+d}e^5} + \frac{2\left(15\,(ex+d)^{\frac{7}{2}}c^2e^{30}-84\,(ex+d)^{\frac{5}{2}}c^2de^{30}+210\,(ex+d)^{\frac{3}{2}}c^2d^2e^{30}-420\,\sqrt{ex+d}c^2d^3e^{30}+70\,(ex+d)^{\frac{3}{2}}a\right.}{105\,e^{35}}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
-2*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)/(sqrt(e*x + d)*e^5) + 2/105*(15*(e*
x + d)^(7/2)*c^2*e^30 - 84*(e*x + d)^(5/2)*c^2*d*e^30 + 210*(e*x + d)^(3/2
)*c^2*d^2*e^30 - 420*sqrt(e*x + d)*c^2*d^3*e^30 + 70*(e*x + d)^(3/2)*a*c*e
^32 - 420*sqrt(e*x + d)*a*c*d*e^32)/e^35
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{2\,c^2\,(d+e\,x)^{7/2}}{7\,e^5} - \frac{(8\,c^2\,d^3+8\,a\,c\,d\,e^2)\,\sqrt{d+e\,x}}{e^5} - \frac{2\,a^2\,e^4+4\,a\,c\,d^2\,e^2+2\,c^2\,d^4}{e^5\,\sqrt{d+e\,x}} + \frac{(12\,c^2\,d^2+4\,a\,c\,e^2)\,(d+e\,x)^{3/2}}{3\,e^5} - \frac{8\,c^2\,d\,(d+e\,x)^{5/2}}{5\,e^5}$$

input
```
int((a + c*x^2)^2/(d + e*x)^(3/2),x)
```

output
```
(2*c^2*(d + e*x)^(7/2))/(7*e^5) - ((8*c^2*d^3 + 8*a*c*d*e^2)*(d + e*x)^(1/
2))/e^5 - (2*a^2*e^4 + 2*c^2*d^4 + 4*a*c*d^2*e^2)/(e^5*(d + e*x)^(1/2)) +
((12*c^2*d^2 + 4*a*c*e^2)*(d + e*x)^(3/2))/(3*e^5) - (8*c^2*d*(d + e*x)^(5
/2))/(5*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{7}c^2e^4x^4 - \frac{16}{35}c^2d\,e^3x^3 + \frac{4}{3}ac\,e^4x^2 + \frac{32}{35}c^2d^2e^2x^2 - \frac{16}{3}acd\,e^3x - \frac{128}{35}c^2d^3ex - 2a^2e^4 - \frac{32}{3}ac}{\sqrt{ex+d}\,e^5}$$

input
```
int((c*x^2+a)^2/(e*x+d)^(3/2),x)
```

output
```
(2*( - 105*a**2*e**4 - 560*a*c*d**2*e**2 - 280*a*c*d*e**3*x + 70*a*c*e**4*
x**2 - 384*c**2*d**4 - 192*c**2*d**3*e*x + 48*c**2*d**2*e**2*x**2 - 24*c**
2*d*e**3*x**3 + 15*c**2*e**4*x**4))/(105*sqrt(d + e*x)*e**5)
```

3.168 $\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = -\frac{2(cd^2+ae^2)^2}{3e^5(d+ex)^{3/2}} + \frac{8cd(cd^2+ae^2)}{e^5\sqrt{d+ex}} + \frac{4c(3cd^2+ae^2)\sqrt{d+ex}}{e^5} - \frac{8c^2d(d+ex)^{3/2}}{3e^5} + \frac{2c^2(d+ex)^{5/2}}{5e^5}$$

output
```
-2/3*(a*e^2+c*d^2)^2/e^5/(e*x+d)^(3/2)+8*c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^(1/
2)+4*c*(a*e^2+3*c*d^2)*(e*x+d)^(1/2)/e^5-8/3*c^2*d*(e*x+d)^(3/2)/e^5+2/5*c
^2*(e*x+d)^(5/2)/e^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.78

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2(-5a^2e^4+10ace^2(8d^2+12dex+3e^2x^2)+c^2(128d^4+192d^3ex+48d^2e^2x^2-8de^3x^3}{15e^5(d+ex)^{3/2}}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^(5/2),x]
```

output
```
(2*(-5*a^2*e^4 + 10*a*c*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + c^2*(128*d^4
+ 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4)))/(15*e^5*(d + e
*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4\sqrt{d+ex}}-\frac{4cd(ae^2+cd^2)}{e^4(d+ex)^{3/2}}+\frac{(ae^2+cd^2)^2}{e^4(d+ex)^{5/2}}+\frac{c^2(d+ex)^{3/2}}{e^4}-\frac{4c^2d\sqrt{d+ex}}{e^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c\sqrt{d+ex}(ae^2+3cd^2)}{e^5}+\frac{8cd(ae^2+cd^2)}{e^5\sqrt{d+ex}}-\frac{2(ae^2+cd^2)^2}{3e^5(d+ex)^{3/2}}+\frac{2c^2(d+ex)^{5/2}}{5e^5}-\frac{8c^2d(d+ex)^{3/2}}{3e^5}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^(5/2),x]
```

output
```
(-2*(c*d^2 + a*e^2)^2)/(3*e^5*(d + e*x)^(3/2)) + (8*c*d*(c*d^2 + a*e^2))/(
e^5*Sqrt[d + e*x]) + (4*c*(3*c*d^2 + a*e^2)*Sqrt[d + e*x])/e^5 - (8*c^2*d*
(d + e*x)^(3/2))/(3*e^5) + (2*c^2*(d + e*x)^(5/2))/(5*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{2c(3x^2ce^2-14cdxe+30ae^2+73cd^2)\sqrt{ex+d}}{15e^5}-\frac{2(-12cdxe+ae^2-11cd^2)(ae^2+cd^2)}{3e^5(ex+d)^{\frac{3}{2}}}$	84
pseudoelliptic	$\frac{\frac{2\left(3e^4x^4-8de^3x^3+48d^2e^2x^2+192d^3ex+128d^4\right)c^2}{15}+\frac{32e^2\left(\frac{3}{8}e^2x^2+\frac{3}{2}dex+d^2\right)ac}{3}-\frac{2a^2e^4}{3}}{(ex+d)^{\frac{3}{2}}e^5}$	91
gosper	$-\frac{2(-3c^2x^4e^4+8dc^2x^3e^3-30x^2ace^4-48x^2c^2d^2e^2-120xacde^3-192xc^2d^3e+5a^2e^4-80acd^2e^2-128c^2d^4)}{15(ex+d)^{\frac{3}{2}}e^5}$	106
trager	$-\frac{2(-3c^2x^4e^4+8dc^2x^3e^3-30x^2ace^4-48x^2c^2d^2e^2-120xacde^3-192xc^2d^3e+5a^2e^4-80acd^2e^2-128c^2d^4)}{15(ex+d)^{\frac{3}{2}}e^5}$	106
orering	$-\frac{2(-3c^2x^4e^4+8dc^2x^3e^3-30x^2ace^4-48x^2c^2d^2e^2-120xacde^3-192xc^2d^3e+5a^2e^4-80acd^2e^2-128c^2d^4)}{15(ex+d)^{\frac{3}{2}}e^5}$	106
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{5}{2}}}{5}-\frac{8c^2d(ex+d)^{\frac{3}{2}}}{3}+4ce^2a\sqrt{ex+d}+12c^2d^2\sqrt{ex+d}+\frac{8cd\left(ae^2+cd^2\right)}{\sqrt{ex+d}}-\frac{2\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{3(ex+d)^{\frac{3}{2}}}}{e^5}$	117
default	$\frac{\frac{2c^2(ex+d)^{\frac{5}{2}}}{5}-\frac{8c^2d(ex+d)^{\frac{3}{2}}}{3}+4ce^2a\sqrt{ex+d}+12c^2d^2\sqrt{ex+d}+\frac{8cd\left(ae^2+cd^2\right)}{\sqrt{ex+d}}-\frac{2\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{3(ex+d)^{\frac{3}{2}}}}{e^5}$	117

input
```
int((c*x^2+a)^2/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*c*(3*c*e^2*x^2-14*c*d*e*x+30*a*e^2+73*c*d^2)*(e*x+d)^(1/2)/e^5-2/3*(-
12*c*d*e*x+a*e^2-11*c*d^2)*(a*e^2+c*d^2)/e^5/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\,(3\,c^2e^4x^4 - 8\,c^2de^3x^3 + 128\,c^2d^4 + 80\,acd^2e^2 - 5\,a^2e^4 + 6\,(8\,c^2d^2e^2 + 5\,ace^4)x^2 + 24\,(}{15\,(e^7x^2 + 2\,de^6x + d^2e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*c^2*e^4*x^4 - 8*c^2*d*e^3*x^3 + 128*c^2*d^4 + 80*a*c*d^2*e^2 - 5*a
^2*e^4 + 6*(8*c^2*d^2*e^2 + 5*a*c*e^4)*x^2 + 24*(8*c^2*d^3*e + 5*a*c*d*e^3
)*x)*sqrt(e*x + d)/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.24

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(-\frac{4c^2d(d+ex)^{\frac{3}{2}}}{3e^4} + \frac{c^2(d+ex)^{\frac{5}{2}}}{5e^4} + \frac{4cd\left(ae^2+cd^2\right)}{e^4\sqrt{d+ex}} + \frac{\sqrt{d+ex}\left(2ace^2+6c^2d^2\right)}{e^4} - \frac{\left(ae^2+cd^2\right)^2}{3e^4(d+ex)^{\frac{3}{2}}}\right)}{e} & \text{for } e \neq 0 \\ \frac{a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}}{d^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(-4*c**2*d*(d + e*x)**(3/2)/(3*e**4) + c**2*(d + e*x)**(5/2)/
(5*e**4) + 4*c*d*(a*e**2 + c*d**2)/(e**4*sqrt(d + e*x)) + sqrt(d + e*x)*(2
*a*c*e**2 + 6*c**2*d**2)/e**4 - (a*e**2 + c*d**2)**2/(3*e**4*(d + e*x)**(3
/2)))/e, Ne(e, 0)), ((a**2*x + 2*a*c*x**3/3 + c**2*x**5/5)/d**(5/2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{3\,(ex+d)^{\frac{5}{2}}c^2-20\,(ex+d)^{\frac{3}{2}}c^2d+30\,(3\,c^2d^2+ace^2)\sqrt{ex+d}}{e^4}-\frac{5\,(c^2d^4+2\,acd^2e^2+a^2e^4-12\,(c^2d^3+acde^2)(ex+d))}{(ex+d)^{\frac{3}{2}}e^4}\right)}{15\,e}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/15*((3*(e*x + d)^(5/2)*c^2 - 20*(e*x + d)^(3/2)*c^2*d + 30*(3*c^2*d^2 +
a*c*e^2)*sqrt(e*x + d))/e^4 - 5*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4 - 12*(c
^2*d^3 + a*c*d*e^2)*(e*x + d))/((e*x + d)^(3/2)*e^4))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\,(12\,(ex+d)c^2d^3-c^2d^4+12\,(ex+d)acde^2-2\,acd^2e^2-a^2e^4)}{3\,(ex+d)^{\frac{3}{2}}e^5} + \frac{2\left(3\,(ex+d)^{\frac{5}{2}}c^2e^{20}-20\,(ex+d)^{\frac{3}{2}}c^2de^{20}+90\,\sqrt{ex+d}c^2d^2e^{20}+30\,\sqrt{ex+d}ace^{22}\right)}{15\,e^{25}}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
2/3*(12*(e*x + d)*c^2*d^3 - c^2*d^4 + 12*(e*x + d)*a*c*d*e^2 - 2*a*c*d^2*e
^2 - a^2*e^4)/((e*x + d)^(3/2)*e^5) + 2/15*(3*(e*x + d)^(5/2)*c^2*e^20 - 2
0*(e*x + d)^(3/2)*c^2*d*e^20 + 90*sqrt(e*x + d)*c^2*d^2*e^20 + 30*sqrt(e*x
 + d)*a*c*e^22)/e^25
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\,c^2\,(d+e\,x)^{5/2}}{5\,e^5} - \frac{\frac{2\,a^2\,e^4}{3}+\frac{2\,c^2\,d^4}{3}-(8\,c^2\,d^3+8\,a\,c\,d\,e^2)\,(d+e\,x)+\frac{4\,a\,c\,d^2\,e^2}{3}}{e^5\,(d+e\,x)^{3/2}} + \frac{(12\,c^2\,d^2+4\,a\,c\,e^2)\,\sqrt{d+e\,x}}{e^5} - \frac{8\,c^2\,d\,(d+e\,x)^{3/2}}{3\,e^5}$$

input
```
int((a + c*x^2)^2/(d + e*x)^(5/2),x)
```

output
```
(2*c^2*(d + e*x)^(5/2))/(5*e^5) - ((2*a^2*e^4)/3 + (2*c^2*d^4)/3 - (8*c^2*
d^3 + 8*a*c*d*e^2)*(d + e*x) + (4*a*c*d^2*e^2)/3)/(e^5*(d + e*x)^(3/2)) +
((12*c^2*d^2 + 4*a*c*e^2)*(d + e*x)^(1/2))/e^5 - (8*c^2*d*(d + e*x)^(3/2))
/(3*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{\frac{2}{5}c^2e^4x^4-\frac{16}{15}c^2d\,e^3x^3+4ac\,e^4x^2+\frac{32}{5}c^2d^2e^2x^2+16acd\,e^3x+\frac{128}{5}c^2d^3ex-\frac{2}{3}a^2e^4+\frac{32}{3}a}{\sqrt{ex+d}\,e^5\,(ex+d)}$$

input
```
int((c*x^2+a)^2/(e*x+d)^(5/2),x)
```

output
```
(2*( - 5*a**2*e**4 + 80*a*c*d**2*e**2 + 120*a*c*d*e**3*x + 30*a*c*e**4*x**
2 + 128*c**2*d**4 + 192*c**2*d**3*e*x + 48*c**2*d**2*e**2*x**2 - 8*c**2*d*
e**3*x**3 + 3*c**2*e**4*x**4))/(15*sqrt(d + e*x)*e**5*(d + e*x))
```

3.169 $\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2(cd^2+ae^2)^2}{5e^5(d+ex)^{5/2}} + \frac{8cd(cd^2+ae^2)}{3e^5(d+ex)^{3/2}} - \frac{4c(3cd^2+ae^2)}{e^5\sqrt{d+ex}} - \frac{8c^2d\sqrt{d+ex}}{e^5} + \frac{2c^2(d+ex)^{3/2}}{3e^5}$$

output
```
-2/5*(a*e^2+c*d^2)^2/e^5/(e*x+d)^(5/2)+8/3*c*d*(a*e^2+c*d^2)/e^5/(e*x+d)^(
3/2)-4*c*(a*e^2+3*c*d^2)/e^5/(e*x+d)^(1/2)-8*c^2*d*(e*x+d)^(1/2)/e^5+2/3*c
^2*(e*x+d)^(3/2)/e^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.78

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2(3a^2e^4+2ace^2(8d^2+20dex+15e^2x^2)+c^2(128d^4+320d^3ex+240d^2e^2x^2+40de^3x^3-5e^4x^4))}{15e^5(d+ex)^{5/2}}$$

input
```
Integrate[(a + c*x^2)^2/(d + e*x)^(7/2),x]
```

output
```
(-2*(3*a^2*e^4 + 2*a*c*e^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2) + c^2*(128*d^4
+ 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4)))/(15*e^5*(d +
 e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(ae^2+3cd^2)}{e^4(d+ex)^{3/2}} - \frac{4cd(ae^2+cd^2)}{e^4(d+ex)^{5/2}} + \frac{(ae^2+cd^2)^2}{e^4(d+ex)^{7/2}} + \frac{c^2\sqrt{d+ex}}{e^4} - \frac{4c^2d}{e^4\sqrt{d+ex}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c(ae^2+3cd^2)}{e^5\sqrt{d+ex}} + \frac{8cd(ae^2+cd^2)}{3e^5(d+ex)^{3/2}} - \frac{2(ae^2+cd^2)^2}{5e^5(d+ex)^{5/2}} + \frac{2c^2(d+ex)^{3/2}}{3e^5} - \frac{8c^2d\sqrt{d+ex}}{e^5}$$

input
```
Int[(a + c*x^2)^2/(d + e*x)^(7/2),x]
```

output
```
(-2*(c*d^2 + a*e^2)^2)/(5*e^5*(d + e*x)^(5/2)) + (8*c*d*(c*d^2 + a*e^2))/(
3*e^5*(d + e*x)^(3/2)) - (4*c*(3*c*d^2 + a*e^2))/(e^5*Sqrt[d + e*x]) - (8*
c^2*d*Sqrt[d + e*x])/e^5 + (2*c^2*(d + e*x)^(3/2))/(3*e^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.74

method	result	size
pseudoelliptic	$\frac{\frac{2\left(5e^4x^4-40d\,e^3x^3-240d^2e^2x^2-320d^3ex-128d^4\right)c^2}{15}-\frac{32e^2\left(\frac{15}{8}e^2x^2+\frac{5}{2}dex+d^2\right)ac}{15}-\frac{2a^2e^4}{5}}{(ex+d)^{\frac{5}{2}}e^5}$	91
gosper	$-\frac{2\left(-5c^2x^4e^4+40d\,c^2x^3e^3+30x^2ac\,e^4+240x^2c^2d^2e^2+40xacd\,e^3+320x\,c^2d^3e+3a^2e^4+16ac\,d^2e^2+128c^2d^4\right)}{15(ex+d)^{\frac{5}{2}}e^5}$	106
trager	$-\frac{2\left(-5c^2x^4e^4+40d\,c^2x^3e^3+30x^2ac\,e^4+240x^2c^2d^2e^2+40xacd\,e^3+320x\,c^2d^3e+3a^2e^4+16ac\,d^2e^2+128c^2d^4\right)}{15(ex+d)^{\frac{5}{2}}e^5}$	106
orering	$-\frac{2\left(-5c^2x^4e^4+40d\,c^2x^3e^3+30x^2ac\,e^4+240x^2c^2d^2e^2+40xacd\,e^3+320x\,c^2d^3e+3a^2e^4+16ac\,d^2e^2+128c^2d^4\right)}{15(ex+d)^{\frac{5}{2}}e^5}$	106
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{3}{2}}}{3}-8c^2d\sqrt{ex+d}-\frac{2\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{4c\left(a\,e^2+3c\,d^2\right)}{\sqrt{ex+d}}+\frac{8cd\left(a\,e^2+c\,d^2\right)}{3(ex+d)^{\frac{3}{2}}}}{e^5}$	110
default	$\frac{\frac{2c^2(ex+d)^{\frac{3}{2}}}{3}-8c^2d\sqrt{ex+d}-\frac{2\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{4c\left(a\,e^2+3c\,d^2\right)}{\sqrt{ex+d}}+\frac{8cd\left(a\,e^2+c\,d^2\right)}{3(ex+d)^{\frac{3}{2}}}}{e^5}$	110
risch	$-\frac{2c^2(-ex+11d)\sqrt{ex+d}}{3e^5}-\frac{2\left(30x^2ac\,e^4+90x^2c^2d^2e^2+40xacd\,e^3+160x\,c^2d^3e+3a^2e^4+16ac\,d^2e^2+73c^2d^4\right)}{15e^5\sqrt{ex+d}\,(e^2x^2+2dex+d^2)}$	125

input
```
int((c*x^2+a)^2/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*((5*e^4*x^4-40*d*e^3*x^3-240*d^2*e^2*x^2-320*d^3*e*x-128*d^4)*c^2-16*
e^2*(15/8*e^2*x^2+5/2*d*e*x+d^2)*a*c-3*a^2*e^4)/(e*x+d)^(5/2)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{2\,(5\,c^2e^4x^4 - 40\,c^2de^3x^3 - 128\,c^2d^4 - 16\,acd^2e^2 - 3\,a^2e^4 - 30\,(8\,c^2d^2e^2 + ace^4)x^2 - 40}{15\,(e^8x^3 + 3\,de^7x^2 + 3\,d^2e^6x + d^3e^5)}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*c^2*e^4*x^4 - 40*c^2*d*e^3*x^3 - 128*c^2*d^4 - 16*a*c*d^2*e^2 - 3*
a^2*e^4 - 30*(8*c^2*d^2*e^2 + a*c*e^4)*x^2 - 40*(8*c^2*d^3*e + a*c*d*e^3)*
x)*sqrt(e*x + d)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(119) = 238.

Time = 0.49 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.81

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = \begin{cases} -\frac{6a^2e^4}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}} - \frac{32acd^2e^2}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}} - \frac{}{15d^2e^5\sqrt{}} \\ \frac{a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-6*a**2*e**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d +
 e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 32*a*c*d**2*e**2/(15*d**2*e**5*sqrt(
d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 80*a*
c*d*e**3*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e*
*7*x**2*sqrt(d + e*x)) - 60*a*c*e**4*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30
*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 256*c**2*d**4/(15*
d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d
+ e*x)) - 640*c**2*d**3*e*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt
(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 480*c**2*d**2*e**2*x**2/(15*d**2
*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*
x)) - 80*c**2*d*e**3*x**3/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d
 + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 10*c**2*e**4*x**4/(15*d**2*e**5*sq
rt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)), Ne(
e, 0)), ((a**2*x + 2*a*c*x**3/3 + c**2*x**5/5)/d**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}c^2-12\sqrt{ex+d}c^2d\right)}{e^4} - \frac{3c^2d^4+6acd^2e^2+3a^2e^4+30(3c^2d^2+ace^2)(ex+d)^2-20(c^2d^3+acde^2)(ex+d)}{(ex+d)^{\frac{5}{2}}e^4}\right)}{15e}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
2/15*(5*((e*x + d)^(3/2)*c^2 - 12*sqrt(e*x + d)*c^2*d)/e^4 - (3*c^2*d^4 +
6*a*c*d^2*e^2 + 3*a^2*e^4 + 30*(3*c^2*d^2 + a*c*e^2)*(e*x + d)^2 - 20*(c^2
*d^3 + a*c*d*e^2)*(e*x + d))/((e*x + d)^(5/2)*e^4))/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2\left(90\,(ex+d)^2c^2d^2-20\,(ex+d)c^2d^3+3\,c^2d^4+30\,(ex+d)^2ace^2-20\,(ex+d)acde^2+6\,acd^2e^2+3\,a^2e^4\right)}{15\,(ex+d)^{\frac{5}{2}}e^5} + \frac{2\left((ex+d)^{\frac{3}{2}}c^2e^{10}-12\sqrt{ex+d}c^2de^{10}\right)}{3\,e^{15}}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(90*(e*x + d)^2*c^2*d^2 - 20*(e*x + d)*c^2*d^3 + 3*c^2*d^4 + 30*(e*x
 + d)^2*a*c*e^2 - 20*(e*x + d)*a*c*d*e^2 + 6*a*c*d^2*e^2 + 3*a^2*e^4)/((e*
x + d)^(5/2)*e^5) + 2/3*((e*x + d)^(3/2)*c^2*e^10 - 12*sqrt(e*x + d)*c^2*d
*e^10)/e^15
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2\left(3\,a^2\,e^4+16\,a\,c\,d^2\,e^2+40\,a\,c\,d\,e^3\,x+30\,a\,c\,e^4\,x^2+128\,c^2\,d^4+320\,c^2\,d^3\,e\,x+240\,c^2\,d^2\,e^2\,x^2+40\,c^2\,d\,e^3 \ldots\right)}{15\,e^5\,(d+e\,x)^{5/2}}$$

input
```
int((a + c*x^2)^2/(d + e*x)^(7/2),x)
```

output
```
-(2*(3*a^2*e^4 + 128*c^2*d^4 - 5*c^2*e^4*x^4 + 40*c^2*d*e^3*x^3 + 240*c^2*
d^2*e^2*x^2 + 16*a*c*d^2*e^2 + 30*a*c*e^4*x^2 + 320*c^2*d^3*e*x + 40*a*c*d
*e^3*x))/(15*e^5*(d + e*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

$$\int \frac{(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{\frac{2}{3}c^2e^4x^4 - \frac{16}{3}c^2d\,e^3x^3 - 4ac\,e^4x^2 - 32c^2d^2e^2x^2 - \frac{16}{3}acd\,e^3x - \frac{128}{3}c^2d^3ex - \frac{2}{5}a^2e^4 - \frac{32}{15}a}{\sqrt{ex+d}\,e^5\left(e^2x^2+2dex+d^2\right)}$$

input
```
int((c*x^2+a)^2/(e*x+d)^(7/2),x)
```

output
```
(2*( - 3*a**2*e**4 - 16*a*c*d**2*e**2 - 40*a*c*d*e**3*x - 30*a*c*e**4*x**2
 - 128*c**2*d**4 - 320*c**2*d**3*e*x - 240*c**2*d**2*e**2*x**2 - 40*c**2*d
*e**3*x**3 + 5*c**2*e**4*x**4))/(15*sqrt(d + e*x)*e**5*(d**2 + 2*d*e*x + e
**2*x**2))
```

3.170 $\int (d+ex)^{5/2} \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 204

$$\begin{aligned}\int (d+ex)^{5/2} \left(a+cx^2\right)^3 \, dx &= \frac{2(cd^2+ae^2)^3 (d+ex)^{7/2}}{7e^7} \\ &- \frac{4cd(cd^2+ae^2)^2 (d+ex)^{9/2}}{3e^7} + \frac{6c(cd^2+ae^2)(5cd^2+ae^2)(d+ex)^{11/2}}{11e^7} \\ &- \frac{8c^2d(5cd^2+3ae^2)(d+ex)^{13/2}}{13e^7} + \frac{2c^2(5cd^2+ae^2)(d+ex)^{15/2}}{5e^7} \\ &- \frac{12c^3d(d+ex)^{17/2}}{17e^7} + \frac{2c^3(d+ex)^{19/2}}{19e^7}\end{aligned}$$

output
```
2/7*(a*e^2+c*d^2)^3*(e*x+d)^(7/2)/e^7-4/3*c*d*(a*e^2+c*d^2)^2*(e*x+d)^(9/2
)/e^7+6/11*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^(11/2)/e^7-8/13*c^2*d*(
3*a*e^2+5*c*d^2)*(e*x+d)^(13/2)/e^7+2/5*c^2*(a*e^2+5*c*d^2)*(e*x+d)^(15/2)
/e^7-12/17*c^3*d*(e*x+d)^(17/2)/e^7+2/19*c^3*(e*x+d)^(19/2)/e^7
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$\int (d+ex)^{5/2} (a+cx^2)^3 \, dx = \frac{2(d+ex)^{7/2} (692835a^3e^6 + 20995a^2ce^4(8d^2 - 28dex + 63e^2x^2) + 323ac^2e^2(128d^4 - 448d^3ex$$

input
```
Integrate[(d + e*x)^(5/2)*(a + c*x^2)^3,x]
```

output
```
(2*(d + e*x)^(7/2)*(692835*a^3*e^6 + 20995*a^2*c*e^4*(8*d^2 - 28*d*e*x + 6
3*e^2*x^2) + 323*a*c^2*e^2*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 184
8*d*e^3*x^3 + 3003*e^4*x^4) + 5*c^3*(1024*d^6 - 3584*d^5*e*x + 8064*d^4*e^
2*x^2 - 14784*d^3*e^3*x^3 + 24024*d^2*e^4*x^4 - 36036*d*e^5*x^5 + 51051*e^
6*x^6)))/(4849845*e^7)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^3 (d+ex)^{5/2} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(d+ex)^{13/2} (ae^2+5cd^2)}{e^6} - \frac{4c^2d(d+ex)^{11/2} (3ae^2+5cd^2)}{e^6} + \frac{3c(d+ex)^{9/2} (ae^2+cd^2) (ae^2+5cd^2)}{e^6} -$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c^2(d+ex)^{15/2}\left(ae^2+5cd^2\right)}{5e^7}-\frac{8c^2d(d+ex)^{13/2}\left(3ae^2+5cd^2\right)}{13e^7}+\frac{6c(d+ex)^{11/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{11e^7}-\frac{4cd(d+ex)^{9/2}\left(ae^2+cd^2\right)^2}{3e^7}+\frac{2(d+ex)^{7/2}\left(ae^2+cd^2\right)^3}{7e^7}+\frac{2c^3(d+ex)^{19/2}}{19e^7}-\frac{12c^3d(d+ex)^{17/2}}{17e^7}$$

input
```
Int[(d + e*x)^(5/2)*(a + c*x^2)^3,x]
```

output
```
(2*(c*d^2 + a*e^2)^3*(d + e*x)^(7/2))/(7*e^7) - (4*c*d*(c*d^2 + a*e^2)^2*(
d + e*x)^(9/2))/(3*e^7) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)
^(11/2))/(11*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(13/2))/(13*e^7
) + (2*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(15/2))/(5*e^7) - (12*c^3*d*(d + e*
x)^(17/2))/(17*e^7) + (2*c^3*(d + e*x)^(19/2))/(19*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{7}{2}}\left(\left(\frac{7}{19}e^6x^6-\frac{84}{323}de^5x^5+\frac{56}{323}d^2e^4x^4-\frac{448}{4199}d^3e^3x^3+\frac{2688}{46189}d^4e^2x^2-\frac{3584}{138567}d^5ex+\frac{1024}{138567}d^6\right)c^3+\frac{128e^2\left(\frac{3003}{128}e^4\right.}{}\right.}{7e^7}$
gosper	$\frac{2(ex+d)^{\frac{7}{2}}(255255x^6c^3e^6-180180dc^3x^5e^5+969969x^4ac^2e^6+120120x^4c^3d^2e^4-596904x^3ac^2de^5-73920x^3c^3d^3e^3+1}{}$
orering	$\frac{2(ex+d)^{\frac{7}{2}}(255255x^6c^3e^6-180180dc^3x^5e^5+969969x^4ac^2e^6+120120x^4c^3d^2e^4-596904x^3ac^2de^5-73920x^3c^3d^3e^3+1}{}$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{19}{2}}}{19}-\frac{12c^3d(ex+d)^{\frac{17}{2}}}{17}+\frac{2\left(\left(ae^2+cd^2\right)c^2+8c^3d^2+c\left(2\left(ae^2+cd^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(-8\left(ae^2+cd^2\right)c^2d-2cd\left(2\right.\right.}{}$
default	$\frac{2c^3(ex+d)^{\frac{19}{2}}}{19}-\frac{12c^3d(ex+d)^{\frac{17}{2}}}{17}+\frac{2\left(\left(ae^2+cd^2\right)c^2+8c^3d^2+c\left(2\left(ae^2+cd^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(-8\left(ae^2+cd^2\right)c^2d-2cd\left(2\right.\right.}{}$
trager	$\frac{2(255255c^3e^9x^9+585585c^3de^8x^8+969969ac^2e^9x^7+345345c^3d^2e^7x^7+2313003ac^2de^8x^6+1155c^3d^3e^6x^6+1322685a}{}$
risch	$\frac{2(255255c^3e^9x^9+585585c^3de^8x^8+969969ac^2e^9x^7+345345c^3d^2e^7x^7+2313003ac^2de^8x^6+1155c^3d^3e^6x^6+1322685a}{}$

input
```
int((e*x+d)^(5/2)*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/7*(e*x+d)^(7/2)*((7/19*e^6*x^6-84/323*d*e^5*x^5+56/323*d^2*e^4*x^4-448/4
199*d^3*e^3*x^3+2688/46189*d^4*e^2*x^2-3584/138567*d^5*e*x+1024/138567*d^6
)*c^3+128/2145*e^2*(3003/128*e^4*x^4-231/16*d*e^3*x^3+63/8*d^2*e^2*x^2-7/2
*d^3*e*x+d^4)*a*c^2+8/33*e^4*(63/8*e^2*x^2-7/2*d*e*x+d^2)*a^2*c+e^6*a^3)/e
^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(176) = 352.

Time = 0.09 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.75

$$\int (d+ex)^{5/2}\left(a+cx^2\right)^3\,dx=\frac{2\left(255255\,c^3e^9x^9+585585\,c^3de^8x^8+5120\,c^3d^9+41344\,ac^2d^7e^2+167960\,a^2cd^5e^4+692835\,a^3\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/4849845*(255255*c^3*e^9*x^9 + 585585*c^3*d*e^8*x^8 + 5120*c^3*d^9 + 4134
4*a*c^2*d^7*e^2 + 167960*a^2*c*d^5*e^4 + 692835*a^3*d^3*e^6 + 3003*(115*c^
3*d^2*e^7 + 323*a*c^2*e^9)*x^7 + 231*(5*c^3*d^3*e^6 + 10013*a*c^2*d*e^8)*x
^6 - 63*(20*c^3*d^4*e^5 - 22933*a*c^2*d^2*e^7 - 20995*a^2*c*e^9)*x^5 + 35*
(40*c^3*d^5*e^4 + 323*a*c^2*d^3*e^6 + 96577*a^2*c*d*e^8)*x^4 - 5*(320*c^3*
d^6*e^3 + 2584*a*c^2*d^4*e^5 - 474487*a^2*c*d^2*e^7 - 138567*a^3*e^9)*x^3
+ 3*(640*c^3*d^7*e^2 + 5168*a*c^2*d^5*e^4 + 20995*a^2*c*d^3*e^6 + 692835*a
^3*d*e^8)*x^2 - (2560*c^3*d^8*e + 20672*a*c^2*d^6*e^3 + 83980*a^2*c*d^4*e^
5 - 2078505*a^3*d^2*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.49

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^3 \, dx = \begin{cases} \frac{2\left(-\frac{6c^3d(d+ex)^{\frac{17}{2}}}{17e^6}+\frac{c^3(d+ex)^{\frac{19}{2}}}{19e^6}+\frac{(d+ex)^{\frac{15}{2}}\cdot\left(3ac^2e^2+15c^3d^2\right)}{15e^6}+\frac{(d+ex)^{\frac{13}{2}}\left(-12ac^2de^2-20c^3d^3\right)}{13e^6}+\frac{(d+ex)^{\frac{11}{2}}\cdot\left(3a^2ce^4+18ac^2d^2e^2+\cdots\right)}{11e^6}\cdots\right)}{e} \\ d^{\frac{5}{2}}\left(a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x+d)**(5/2)*(c*x**2+a)**3,x)
```

output
```
Piecewise((2*(-6*c**3*d*(d + e*x)**(17/2)/(17*e**6) + c**3*(d + e*x)**(19/
2)/(19*e**6) + (d + e*x)**(15/2)*(3*a*c**2*e**2 + 15*c**3*d**2)/(15*e**6)
+ (d + e*x)**(13/2)*(-12*a*c**2*d*e**2 - 20*c**3*d**3)/(13*e**6) + (d + e*
x)**(11/2)*(3*a**2*c*e**4 + 18*a*c**2*d**2*e**2 + 15*c**3*d**4)/(11*e**6)
+ (d + e*x)**(9/2)*(-6*a**2*c*d*e**4 - 12*a*c**2*d**3*e**2 - 6*c**3*d**5)/
(9*e**6) + (d + e*x)**(7/2)*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**
4*e**2 + c**3*d**6)/(7*e**6))/e, Ne(e, 0)), (d**(5/2)*(a**3*x + a**2*c*x**
3 + 3*a*c**2*x**5/5 + c**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^3 dx = \frac{2\left(255255\,(ex+d)^{\frac{19}{2}}c^3 - 1711710\,(ex+d)^{\frac{17}{2}}c^3d + 969969\,(5\,c^3d^2+ac^2e^2)(ex+d)^{\frac{15}{2}} - 14922\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/4849845*(255255*(e*x + d)^(19/2)*c^3 - 1711710*(e*x + d)^(17/2)*c^3*d +
969969*(5*c^3*d^2 + a*c^2*e^2)*(e*x + d)^(15/2) - 1492260*(5*c^3*d^3 + 3*a
*c^2*d*e^2)*(e*x + d)^(13/2) + 1322685*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*
c*e^4)*(e*x + d)^(11/2) - 3233230*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4
)*(e*x + d)^(9/2) + 692835*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
a^3*e^6)*(e*x + d)^(7/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1147 vs. 2(176) = 352.

Time = 0.14 (sec) , antiderivative size = 1147, normalized size of antiderivative = 5.62

$$\int (d+ex)^{5/2} \left(a+cx^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
2/4849845*(4849845*sqrt(e*x + d)*a^3*d^3 + 4849845*((e*x + d)^(3/2) - 3*sq
rt(e*x + d)*d)*a^3*d^2 + 969969*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d
+ 15*sqrt(e*x + d)*d^2)*a^3*d + 969969*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(
3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*c*d^3/e^2 + 138567*(5*(e*x + d)^(7/2) -
 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^3
 + 1247103*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*
d^2 - 35*sqrt(e*x + d)*d^3)*a^2*c*d^2/e^2 + 46189*(35*(e*x + d)^(9/2) - 18
0*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 +
315*sqrt(e*x + d)*d^4)*a*c^2*d^3/e^4 + 138567*(35*(e*x + d)^(9/2) - 180*(e
*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*
sqrt(e*x + d)*d^4)*a^2*c*d/e^2 + 62985*(63*(e*x + d)^(11/2) - 385*(e*x + d
)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x
 + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*c^2*d^2/e^4 + 20995*(63*(e*x +
d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x +
d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a^2*c/e^2
 + 1615*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(
9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x
 + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^3*d^3/e^6 + 14535*(231*(e*x +
d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x
 + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 +...
```

Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.92

$$\int (d+ex)^{5/2}\left(a+cx^2\right)^3\,dx=\frac{\left(30\,c^3\,d^2+6\,a\,c^2\,e^2\right)\,{\left(d+e\,x\right)}^{15/2}}{15\,e^7}+\frac{{\left(d+e\,x\right)}^{11/2}\,\left(6\,a^2\,c\,e^4+36\,a\,c^2\,d^2\,e^2+30\,c^3\,d^4\right)}{11\,e^7}+\frac{2\,c^3\,{\left(d+e\,x\right)}^{19/2}}{19\,e^7}+\frac{2\,{\left(c\,d^2+a\,e^2\right)}^3\,{\left(d+e\,x\right)}^{7/2}}{7\,e^7}-\frac{\left(40\,c^3\,d^3+24\,a\,c^2\,d\,e^2\right)\,{\left(d+e\,x\right)}^{13/2}}{13\,e^7}-\frac{12\,c^3\,d\,{\left(d+e\,x\right)}^{17/2}}{17\,e^7}-\frac{4\,c\,d\,{\left(c\,d^2+a\,e^2\right)}^2\,{\left(d+e\,x\right)}^{9/2}}{3\,e^7}$$

input
```
int((a + c*x^2)^3*(d + e*x)^(5/2),x)
```

output
```
((30*c^3*d^2 + 6*a*c^2*e^2)*(d + e*x)^(15/2))/(15*e^7) + ((d + e*x)^(11/2)
*(30*c^3*d^4 + 6*a^2*c*e^4 + 36*a*c^2*d^2*e^2))/(11*e^7) + (2*c^3*(d + e*x
)^(19/2))/(19*e^7) + (2*(a*e^2 + c*d^2)^3*(d + e*x)^(7/2))/(7*e^7) - ((40*
c^3*d^3 + 24*a*c^2*d*e^2)*(d + e*x)^(13/2))/(13*e^7) - (12*c^3*d*(d + e*x)
^(17/2))/(17*e^7) - (4*c*d*(a*e^2 + c*d^2)^2*(d + e*x)^(9/2))/(3*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.83

$$\int (d+ex)^{5/2} (a +cx^2)^3 \, dx = \frac{2\sqrt{ex+d}\,(255255c^3e^9x^9 + 585585c^3d\,e^8x^8 + 969969a\,c^2e^9x^7 + 345345c^3d^2e^7x^7 + 2313003a\,c^2$$

input
```
int((e*x+d)^(5/2)*(c*x^2+a)^3,x)
```

output
```
(2*sqrt(d + e*x)*(692835*a**3*d**3*e**6 + 2078505*a**3*d**2*e**7*x + 20785
05*a**3*d*e**8*x**2 + 692835*a**3*e**9*x**3 + 167960*a**2*c*d**5*e**4 - 83
980*a**2*c*d**4*e**5*x + 62985*a**2*c*d**3*e**6*x**2 + 2372435*a**2*c*d**2
*e**7*x**3 + 3380195*a**2*c*d*e**8*x**4 + 1322685*a**2*c*e**9*x**5 + 41344
*a*c**2*d**7*e**2 - 20672*a*c**2*d**6*e**3*x + 15504*a*c**2*d**5*e**4*x**2
 - 12920*a*c**2*d**4*e**5*x**3 + 11305*a*c**2*d**3*e**6*x**4 + 1444779*a*c
**2*d**2*e**7*x**5 + 2313003*a*c**2*d*e**8*x**6 + 969969*a*c**2*e**9*x**7
 + 5120*c**3*d**9 - 2560*c**3*d**8*e*x + 1920*c**3*d**7*e**2*x**2 - 1600*c*
*3*d**6*e**3*x**3 + 1400*c**3*d**5*e**4*x**4 - 1260*c**3*d**4*e**5*x**5 +
1155*c**3*d**3*e**6*x**6 + 345345*c**3*d**2*e**7*x**7 + 585585*c**3*d*e**8
*x**8 + 255255*c**3*e**9*x**9))/(4849845*e**7)
```

3.171 $\int (d+ex)^{3/2} \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 204

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^3 \, dx = \frac{2(cd^2+ae^2)^3 (d+ex)^{5/2}}{5e^7} - \frac{12cd(cd^2+ae^2)^2 (d+ex)^{7/2}}{7e^7} + \frac{2c(cd^2+ae^2)(5cd^2+ae^2)(d+ex)^{9/2}}{3e^7} - \frac{8c^2d(5cd^2+3ae^2)(d+ex)^{11/2}}{11e^7} + \frac{6c^2(5cd^2+ae^2)(d+ex)^{13/2}}{13e^7} - \frac{4c^3d(d+ex)^{15/2}}{5e^7} + \frac{2c^3(d+ex)^{17/2}}{17e^7}$$

output
```
2/5*(a*e^2+c*d^2)^3*(e*x+d)^(5/2)/e^7-12/7*c*d*(a*e^2+c*d^2)^2*(e*x+d)^(7/
2)/e^7+2/3*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^(9/2)/e^7-8/11*c^2*d*(3
*a*e^2+5*c*d^2)*(e*x+d)^(11/2)/e^7+6/13*c^2*(a*e^2+5*c*d^2)*(e*x+d)^(13/2)
/e^7-4/5*c^3*d*(e*x+d)^(15/2)/e^7+2/17*c^3*(e*x+d)^(17/2)/e^7
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^3 \, dx = \frac{2(d+ex)^{5/2} (51051a^3e^6 + 2431a^2ce^4(8d^2 - 20dex + 35e^2x^2) + 51ac^2e^2(128d^4 - 320d^3ex + 5$$

input
```
Integrate[(d + e*x)^(3/2)*(a + c*x^2)^3,x]
```

output
```
(2*(d + e*x)^(5/2)*(51051*a^3*e^6 + 2431*a^2*c*e^4*(8*d^2 - 20*d*e*x + 35*
e^2*x^2) + 51*a*c^2*e^2*(128*d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e
^3*x^3 + 1155*e^4*x^4) + c^3*(1024*d^6 - 2560*d^5*e*x + 4480*d^4*e^2*x^2 -
 6720*d^3*e^3*x^3 + 9240*d^2*e^4*x^4 - 12012*d*e^5*x^5 + 15015*e^6*x^6)))/
(255255*e^7)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex)^{3/2} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(d+ex)^{11/2} \left(ae^2+5cd^2\right)}{e^6} - \frac{4c^2d(d+ex)^{9/2} \left(3ae^2+5cd^2\right)}{e^6} + \frac{3c(d+ex)^{7/2} \left(ae^2+cd^2\right) \left(ae^2+5cd^2\right)}{e^6} -$$

$$\Big\downarrow \text{2009}$$

$$\frac{6c^2(d+ex)^{13/2}\left(ae^2+5cd^2\right)}{13e^7}-\frac{8c^2d(d+ex)^{11/2}\left(3ae^2+5cd^2\right)}{11e^7}+\frac{2c(d+ex)^{9/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{3e^7}-\frac{12cd(d+ex)^{7/2}\left(ae^2+cd^2\right)^2}{7e^7}+\frac{2(d+ex)^{5/2}\left(ae^2+cd^2\right)^3}{5e^7}+\frac{2c^3(d+ex)^{17/2}}{17e^7}-\frac{4c^3d(d+ex)^{15/2}}{5e^7}$$

input
```
Int[(d + e*x)^(3/2)*(a + c*x^2)^3,x]
```

output
```
(2*(c*d^2 + a*e^2)^3*(d + e*x)^(5/2))/(5*e^7) - (12*c*d*(c*d^2 + a*e^2)^2*
(d + e*x)^(7/2))/(7*e^7) + (2*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x
)^(9/2))/(3*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(11/2))/(11*e^7)
 + (6*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(13/2))/(13*e^7) - (4*c^3*d*(d + e*x
)^(15/2))/(5*e^7) + (2*c^3*(d + e*x)^(17/2))/(17*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{2\left(\frac{\left(5e^6x^6-4d\,e^5x^5+\frac{40}{13}d^2e^4x^4-\frac{320}{143}d^3e^3x^3+\frac{640}{429}d^4e^2x^2-\frac{2560}{3003}d^5ex+\frac{1024}{3003}d^6\right)c^3}{17}+\frac{128e^2\left(\frac{1155}{128}e^4x^4-\frac{105}{16}d\,e^3x^3+\frac{35}{8}d^2e^2x^2-\right.}{1001}\right.}{5e^7}$
gosper	$2(ex+d)^{\frac{5}{2}}\left(15015x^6c^3e^6-12012d\,c^3x^5e^5+58905x^4a\,c^2e^6+9240x^4c^3d^2e^4-42840x^3a\,c^2d\,e^5-6720x^3c^3d^3e^3+85085x^2\right.$
orering	$2(ex+d)^{\frac{5}{2}}\left(15015x^6c^3e^6-12012d\,c^3x^5e^5+58905x^4a\,c^2e^6+9240x^4c^3d^2e^4-42840x^3a\,c^2d\,e^5-6720x^3c^3d^3e^3+85085x^2\right.$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{17}{2}}}{17}-\frac{4c^3d(ex+d)^{\frac{15}{2}}}{5}+\frac{2\left(\left(a\,e^2+c\,d^2\right)c^2+8c^3d^2+c\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(-8\left(a\,e^2+c\,d^2\right)c^2d-2cd\left(2\right.\right.}{}$
default	$\frac{2c^3(ex+d)^{\frac{17}{2}}}{17}-\frac{4c^3d(ex+d)^{\frac{15}{2}}}{5}+\frac{2\left(\left(a\,e^2+c\,d^2\right)c^2+8c^3d^2+c\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(-8\left(a\,e^2+c\,d^2\right)c^2d-2cd\left(2\right.\right.}{}$
trager	$2\left(15015e^8c^3x^8+18018d\,c^3e^7x^7+58905a\,c^2e^8x^6+231c^3d^2e^6x^6+74970a\,c^2d\,e^7x^5-252d^3e^5c^3x^5+85085a^2c\,e^8x^4+1785\right.$
risch	$2\left(15015e^8c^3x^8+18018d\,c^3e^7x^7+58905a\,c^2e^8x^6+231c^3d^2e^6x^6+74970a\,c^2d\,e^7x^5-252d^3e^5c^3x^5+85085a^2c\,e^8x^4+1785\right.$

input
```
int((e*x+d)^(3/2)*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(1/17*(5*e^6*x^6-4*d*e^5*x^5+40/13*d^2*e^4*x^4-320/143*d^3*e^3*x^3+640
/429*d^4*e^2*x^2-2560/3003*d^5*e*x+1024/3003*d^6)*c^3+128/1001*e^2*(1155/1
28*e^4*x^4-105/16*d*e^3*x^3+35/8*d^2*e^2*x^2-5/2*d^3*e*x+d^4)*a*c^2+8/21*e
^4*a^2*(35/8*e^2*x^2-5/2*d*e*x+d^2)*c+e^6*a^3)*(e*x+d)^(5/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.49

$$\int (d+ex)^{3/2}\,(a+cx^2)^3\,dx = \frac{2\,(15015\,c^3e^8x^8+18018\,c^3de^7x^7+1024\,c^3d^8+6528\,ac^2d^6e^2+19448\,a^2cd^4e^4+51051\,a^3d^2e^6}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/255255*(15015*c^3*e^8*x^8 + 18018*c^3*d*e^7*x^7 + 1024*c^3*d^8 + 6528*a*
c^2*d^6*e^2 + 19448*a^2*c*d^4*e^4 + 51051*a^3*d^2*e^6 + 231*(c^3*d^2*e^6 +
 255*a*c^2*e^8)*x^6 - 126*(2*c^3*d^3*e^5 - 595*a*c^2*d*e^7)*x^5 + 35*(8*c^
3*d^4*e^4 + 51*a*c^2*d^2*e^6 + 2431*a^2*c*e^8)*x^4 - 10*(32*c^3*d^5*e^3 +
204*a*c^2*d^3*e^5 - 12155*a^2*c*d*e^7)*x^3 + 3*(128*c^3*d^6*e^2 + 816*a*c^
2*d^4*e^4 + 2431*a^2*c*d^2*e^6 + 17017*a^3*e^8)*x^2 - 2*(256*c^3*d^7*e + 1
632*a*c^2*d^5*e^3 + 4862*a^2*c*d^3*e^5 - 51051*a^3*d*e^7)*x)*sqrt(e*x + d)
/e^7
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.49

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^3 dx = \begin{cases} \frac{2\left(-\frac{2c^3d(d+ex)^{\frac{15}{2}}}{5e^6}+\frac{c^3(d+ex)^{\frac{17}{2}}}{17e^6}+\frac{(d+ex)^{\frac{13}{2}}\cdot\left(3ac^2e^2+15c^3d^2\right)}{13e^6}+\frac{(d+ex)^{\frac{11}{2}}\left(-12ac^2de^2-20c^3d^3\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\cdot\left(3a^2ce^4+18ac^2d^2e^2+\right.}{9e^6}\right.}{e} \\ d^{\frac{3}{2}}\left(a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+a)**3,x)
```

output
```
Piecewise((2*(-2*c**3*d*(d + e*x)**(15/2)/(5*e**6) + c**3*(d + e*x)**(17/2
)/(17*e**6) + (d + e*x)**(13/2)*(3*a*c**2*e**2 + 15*c**3*d**2)/(13*e**6) +
 (d + e*x)**(11/2)*(-12*a*c**2*d*e**2 - 20*c**3*d**3)/(11*e**6) + (d + e*x
)**(9/2)*(3*a**2*c*e**4 + 18*a*c**2*d**2*e**2 + 15*c**3*d**4)/(9*e**6) + (
d + e*x)**(7/2)*(-6*a**2*c*d*e**4 - 12*a*c**2*d**3*e**2 - 6*c**3*d**5)/(7*
e**6) + (d + e*x)**(5/2)*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e
**2 + c**3*d**6)/(5*e**6))/e, Ne(e, 0)), (d**(3/2)*(a**3*x + a**2*c*x**3 +
 3*a*c**2*x**5/5 + c**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^3 \, dx = \frac{2 \left(15015 \, (ex+d)^{\frac{17}{2}} c^3 - 102102 \, (ex+d)^{\frac{15}{2}} c^3 d + 58905 \, (5 \, c^3 d^2 + ac^2 e^2)(ex+d)^{\frac{13}{2}} - 92820 \, (5\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/255255*(15015*(e*x + d)^(17/2)*c^3 - 102102*(e*x + d)^(15/2)*c^3*d + 589
05*(5*c^3*d^2 + a*c^2*e^2)*(e*x + d)^(13/2) - 92820*(5*c^3*d^3 + 3*a*c^2*d
*e^2)*(e*x + d)^(11/2) + 85085*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(
e*x + d)^(9/2) - 218790*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d
)^(7/2) + 51051*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*(e
*x + d)^(5/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(176) = 352$.

Time = 0.13 (sec) , antiderivative size = 784, normalized size of antiderivative = 3.84

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
2/765765*(765765*sqrt(e*x + d)*a^3*d^2 + 510510*((e*x + d)^(3/2) - 3*sqrt(
e*x + d)*d)*a^3*d + 51051*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*s
qrt(e*x + d)*d^2)*a^3 + 153153*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d +
 15*sqrt(e*x + d)*d^2)*a^2*c*d^2/e^2 + 131274*(5*(e*x + d)^(7/2) - 21*(e*x
 + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*c*d/e^2
 + 7293*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*
d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*c^2*d^2/e^4 + 729
3*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 -
420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a^2*c/e^2 + 6630*(63*(e*x
 + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x
 + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*c^2*
d/e^4 + 255*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x +
d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*
(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^3*d^2/e^6 + 765*(231*(e*x
+ d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e
*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 +
3003*sqrt(e*x + d)*d^6)*a*c^2/e^4 + 238*(429*(e*x + d)^(15/2) - 3465*(e*x
+ d)^(13/2)*d + 12285*(e*x + d)^(11/2)*d^2 - 25025*(e*x + d)^(9/2)*d^3 + 3
2175*(e*x + d)^(7/2)*d^4 - 27027*(e*x + d)^(5/2)*d^5 + 15015*(e*x + d)^(3/
2)*d^6 - 6435*sqrt(e*x + d)*d^7)*c^3*d/e^6 + 7*(6435*(e*x + d)^(17/2) -...
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.92

$$\int (d+ex)^{3/2} \left(a+cx^2\right)^3 \, dx = \frac{\left(30\,c^3\,d^2+6\,a\,c^2\,e^2\right)\,{\left(d+e\,x\right)}^{13/2}}{13\,e^7} + \frac{{\left(d+e\,x\right)}^{9/2}\,\left(6\,a^2\,c\,e^4+36\,a\,c^2\,d^2\,e^2+30\,c^3\,d^4\right)}{9\,e^7} + \frac{2\,c^3\,{\left(d+e\,x\right)}^{17/2}}{17\,e^7} + \frac{2\,{\left(c\,d^2+a\,e^2\right)}^3\,{\left(d+e\,x\right)}^{5/2}}{5\,e^7} - \frac{\left(40\,c^3\,d^3+24\,a\,c^2\,d\,e^2\right)\,{\left(d+e\,x\right)}^{11/2}}{11\,e^7} - \frac{4\,c^3\,d\,{\left(d+e\,x\right)}^{15/2}}{5\,e^7} - \frac{12\,c\,d\,\left(c\,d^2+a\,e^2\right)^2\,{\left(d+e\,x\right)}^{7/2}}{7\,e^7}$$

input
```
int((a + c*x^2)^3*(d + e*x)^(3/2),x)
```

output
```
((30*c^3*d^2 + 6*a*c^2*e^2)*(d + e*x)^(13/2))/(13*e^7) + ((d + e*x)^(9/2)*
(30*c^3*d^4 + 6*a^2*c*e^4 + 36*a*c^2*d^2*e^2))/(9*e^7) + (2*c^3*(d + e*x)^
(17/2))/(17*e^7) + (2*(a*e^2 + c*d^2)^3*(d + e*x)^(5/2))/(5*e^7) - ((40*c^
3*d^3 + 24*a*c^2*d*e^2)*(d + e*x)^(11/2))/(11*e^7) - (4*c^3*d*(d + e*x)^(1
5/2))/(5*e^7) - (12*c*d*(a*e^2 + c*d^2)^2*(d + e*x)^(7/2))/(7*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.54

$$\int (d+ex)^{3/2} \left(a +cx^2\right)^3 \, dx = \frac{2\sqrt{ex+d}\,(15015c^3e^8x^8 + 18018c^3d\,e^7x^7 + 58905a\,c^2e^8x^6 + 231c^3d^2e^6x^6 + 74970a\,c^2d\,e^7x^5 -$$

input
```
int((e*x+d)^(3/2)*(c*x^2+a)^3,x)
```

output
```
(2*sqrt(d + e*x)*(51051*a**3*d**2*e**6 + 102102*a**3*d*e**7*x + 51051*a**3
*e**8*x**2 + 19448*a**2*c*d**4*e**4 - 9724*a**2*c*d**3*e**5*x + 7293*a**2*
c*d**2*e**6*x**2 + 121550*a**2*c*d*e**7*x**3 + 85085*a**2*c*e**8*x**4 + 65
28*a*c**2*d**6*e**2 - 3264*a*c**2*d**5*e**3*x + 2448*a*c**2*d**4*e**4*x**2
 - 2040*a*c**2*d**3*e**5*x**3 + 1785*a*c**2*d**2*e**6*x**4 + 74970*a*c**2*
d*e**7*x**5 + 58905*a*c**2*e**8*x**6 + 1024*c**3*d**8 - 512*c**3*d**7*e*x
+ 384*c**3*d**6*e**2*x**2 - 320*c**3*d**5*e**3*x**3 + 280*c**3*d**4*e**4*x
**4 - 252*c**3*d**3*e**5*x**5 + 231*c**3*d**2*e**6*x**6 + 18018*c**3*d*e**
7*x**7 + 15015*c**3*e**8*x**8))/(255255*e**7)
```

3.172 $\int \sqrt{d+ex}\left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 204

$$\begin{aligned}\int \sqrt{d+ex}\left(a+cx^2\right)^3 \, dx =& \frac{2(cd^2+ae^2)^3 (d+ex)^{3/2}}{3e^7} - \frac{12cd(cd^2+ae^2)^2 (d+ex)^{5/2}}{5e^7} \\ &+ \frac{6c(cd^2+ae^2)(5cd^2+ae^2)(d+ex)^{7/2}}{7e^7} \\ &- \frac{8c^2d(5cd^2+3ae^2)(d+ex)^{9/2}}{9e^7} \\ &+ \frac{6c^2(5cd^2+ae^2)(d+ex)^{11/2}}{11e^7} \\ &- \frac{12c^3d(d+ex)^{13/2}}{13e^7} + \frac{2c^3(d+ex)^{15/2}}{15e^7}\end{aligned}$$

output
```
2/3*(a*e^2+c*d^2)^3*(e*x+d)^(3/2)/e^7-12/5*c*d*(a*e^2+c*d^2)^2*(e*x+d)^(5/
2)/e^7+6/7*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^(7/2)/e^7-8/9*c^2*d*(3*
a*e^2+5*c*d^2)*(e*x+d)^(9/2)/e^7+6/11*c^2*(a*e^2+5*c*d^2)*(e*x+d)^(11/2)/e
^7-12/13*c^3*d*(e*x+d)^(13/2)/e^7+2/15*c^3*(e*x+d)^(15/2)/e^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3\,dx$$
$$= \frac{2(d+ex)^{3/2}\left(15015a^3e^6+1287a^2ce^4(8d^2-12dex+15e^2x^2)+39ac^2e^2(128d^4-192d^3ex+240d^2e^2x^2-\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]*(a + c*x^2)^3,x]
```

output
```
(2*(d + e*x)^(3/2)*(15015*a^3*e^6 + 1287*a^2*c*e^4*(8*d^2 - 12*d*e*x + 15*
e^2*x^2) + 39*a*c^2*e^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e
^3*x^3 + 315*e^4*x^4) + c^3*(1024*d^6 - 1536*d^5*e*x + 1920*d^4*e^2*x^2 -
2240*d^3*e^3*x^3 + 2520*d^2*e^4*x^4 - 2772*d*e^5*x^5 + 3003*e^6*x^6)))/(45
045*e^7)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3\sqrt{d+ex}\,dx$$
$$\Big\downarrow \text{476}$$
$$\int \left(\frac{3c^2(d+ex)^{9/2}\left(ae^2+5cd^2\right)}{e^6}-\frac{4c^2d(d+ex)^{7/2}\left(3ae^2+5cd^2\right)}{e^6}+\frac{3c(d+ex)^{5/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^6}-\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{6c^2(d+ex)^{11/2}\left(ae^2+5cd^2\right)}{11e^7}-\frac{8c^2d(d+ex)^{9/2}\left(3ae^2+5cd^2\right)}{9e^7}+\frac{6c(d+ex)^{7/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{7e^7}-\frac{12cd(d+ex)^{5/2}\left(ae^2+cd^2\right)^2}{5e^7}+\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)^3}{3e^7}+\frac{2c^3(d+ex)^{15/2}}{15e^7}-\frac{12c^3d(d+ex)^{13/2}}{13e^7}$$

input
```
Int[Sqrt[d + e*x]*(a + c*x^2)^3,x]
```

output
```
(2*(c*d^2 + a*e^2)^3*(d + e*x)^(3/2))/(3*e^7) - (12*c*d*(c*d^2 + a*e^2)^2*
(d + e*x)^(5/2))/(5*e^7) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x
)^(7/2))/(7*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(9/2))/(9*e^7) +
 (6*c^2*(5*c*d^2 + a*e^2)*(d + e*x)^(11/2))/(11*e^7) - (12*c^3*d*(d + e*x)
^(13/2))/(13*e^7) + (2*c^3*(d + e*x)^(15/2))/(15*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{3}{2}}\left(\left(\frac{1}{5}e^6x^6-\frac{12}{65}d\,e^5x^5+\frac{24}{143}d^2e^4x^4-\frac{64}{429}d^3e^3x^3+\frac{128}{1001}d^4e^2x^2-\frac{512}{5005}d^5ex+\frac{1024}{15015}d^6\right)c^3+\frac{128e^2\left(\frac{315}{128}e^4x^4-\frac{35}{16}d\,e\cdots\right.}{\cdots}\right.}{3e^7}$
gosper	$\frac{2(ex+d)^{\frac{3}{2}}\left(3003x^6c^3e^6-2772d\,c^3x^5e^5+12285x^4a\,c^2e^6+2520x^4c^3d^2e^4-10920x^3a\,c^2d\,e^5-2240x^3c^3d^3e^3+19305x^2a^2c\cdots\right.}{\cdots}$
orering	$\frac{2(ex+d)^{\frac{3}{2}}\left(3003x^6c^3e^6-2772d\,c^3x^5e^5+12285x^4a\,c^2e^6+2520x^4c^3d^2e^4-10920x^3a\,c^2d\,e^5-2240x^3c^3d^3e^3+19305x^2a^2c\cdots\right.}{\cdots}$
trager	$\frac{2\left(3003c^3e^7x^7+231d\,c^3e^6x^6+12285c^2e^7a\,x^5-252c^3d^2e^5x^5+1365d\,e^6c^2a\,x^4+280c^3d^3e^4x^4+19305a^2c\,e^7x^3-1560a\,c^2\cdots\right.}{\cdots}$
risch	$\frac{2\left(3003c^3e^7x^7+231d\,c^3e^6x^6+12285c^2e^7a\,x^5-252c^3d^2e^5x^5+1365d\,e^6c^2a\,x^4+280c^3d^3e^4x^4+19305a^2c\,e^7x^3-1560a\,c^2\cdots\right.}{\cdots}$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{15}{2}}}{15}-\frac{12c^3d(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(\left(a\,e^2+c\,d^2\right)c^2+8c^3d^2+c\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(-8\left(a\,e^2+c\,d^2\right)c^2d-2cd\left(2\cdots\right.\right.}{\cdots}$
default	$\frac{2c^3(ex+d)^{\frac{15}{2}}}{15}-\frac{12c^3d(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(\left(a\,e^2+c\,d^2\right)c^2+8c^3d^2+c\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(-8\left(a\,e^2+c\,d^2\right)c^2d-2cd\left(2\cdots\right.\right.}{\cdots}$

input
```
int((e*x+d)^(1/2)*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x+d)^(3/2)*((1/5*e^6*x^6-12/65*d*e^5*x^5+24/143*d^2*e^4*x^4-64/429*
d^3*e^3*x^3+128/1001*d^4*e^2*x^2-512/5005*d^5*e*x+1024/15015*d^6)*c^3+128/
385*e^2*(315/128*e^4*x^4-35/16*d*e^3*x^3+15/8*d^2*e^2*x^2-3/2*d^3*e*x+d^4)
*a*c^2+24/35*e^4*a^2*(15/8*e^2*x^2-3/2*d*e*x+d^2)*c+e^6*a^3)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.24

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3\,dx$$
$$=\frac{2\left(3003\,c^3e^7x^7+231\,c^3de^6x^6+1024\,c^3d^7+4992\,ac^2d^5e^2+10296\,a^2cd^3e^4+15015\,a^3de^6-63\left(4\,c^3d^2e^5-\cdots\right.\right.}{\cdots}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/45045*(3003*c^3*e^7*x^7 + 231*c^3*d*e^6*x^6 + 1024*c^3*d^7 + 4992*a*c^2*
d^5*e^2 + 10296*a^2*c*d^3*e^4 + 15015*a^3*d*e^6 - 63*(4*c^3*d^2*e^5 - 195*
a*c^2*e^7)*x^5 + 35*(8*c^3*d^3*e^4 + 39*a*c^2*d*e^6)*x^4 - 5*(64*c^3*d^4*e
^3 + 312*a*c^2*d^2*e^5 - 3861*a^2*c*e^7)*x^3 + 3*(128*c^3*d^5*e^2 + 624*a*
c^2*d^3*e^4 + 1287*a^2*c*d*e^6)*x^2 - (512*c^3*d^6*e + 2496*a*c^2*d^4*e^3
+ 5148*a^2*c*d^2*e^5 - 15015*a^3*e^7)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.49

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3\,dx = \begin{cases} \frac{2\left(-\frac{6c^3d(d+ex)^{\frac{13}{2}}}{13e^6}+\frac{c^3(d+ex)^{\frac{15}{2}}}{15e^6}+\frac{(d+ex)^{\frac{11}{2}}\cdot\left(3ac^2e^2+15c^3d^2\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\left(-12ac^2de^2-20c^3d^3\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(3a^2ce^4+18ac^2d^2e^2+15c^3d^4\right)}{7e^6}+\frac{(d+ex)}{} \right.}{e} \\ \sqrt{d}\left(a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+a)**3,x)
```

output
```
Piecewise((2*(-6*c**3*d*(d + e*x)**(13/2)/(13*e**6) + c**3*(d + e*x)**(15/
2)/(15*e**6) + (d + e*x)**(11/2)*(3*a*c**2*e**2 + 15*c**3*d**2)/(11*e**6)
+ (d + e*x)**(9/2)*(-12*a*c**2*d*e**2 - 20*c**3*d**3)/(9*e**6) + (d + e*x)
**(7/2)*(3*a**2*c*e**4 + 18*a*c**2*d**2*e**2 + 15*c**3*d**4)/(7*e**6) + (d
 + e*x)**(5/2)*(-6*a**2*c*d*e**4 - 12*a*c**2*d**3*e**2 - 6*c**3*d**5)/(5*e
**6) + (d + e*x)**(3/2)*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e*
*2 + c**3*d**6)/(3*e**6))/e, Ne(e, 0)), (sqrt(d)*(a**3*x + a**2*c*x**3 + 3
*a*c**2*x**5/5 + c**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3 \, dx$$
$$= \frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^3 - 20790\,(ex+d)^{\frac{13}{2}}c^3d + 12285\,(5\,c^3d^2 + ac^2e^2)(ex+d)^{\frac{11}{2}} - 20020\,(5\,c^3d^3 + 3\,ac^2d\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/45045*(3003*(e*x + d)^(15/2)*c^3 - 20790*(e*x + d)^(13/2)*c^3*d + 12285*
(5*c^3*d^2 + a*c^2*e^2)*(e*x + d)^(11/2) - 20020*(5*c^3*d^3 + 3*a*c^2*d*e^
2)*(e*x + d)^(9/2) + 19305*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(e*x
+ d)^(7/2) - 54054*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d)^(5/
2) + 15015*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*(e*x +
d)^(3/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(176) = 352.

Time = 0.12 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.30

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3 \, dx$$
$$= \frac{2\left(45045\,\sqrt{ex+d}a^3d + 15015\left((ex+d)^{\frac{3}{2}} - 3\,\sqrt{ex+d}d\right)a^3 + \frac{9009\left(3\,(ex+d)^{\frac{5}{2}} - 10\,(ex+d)^{\frac{3}{2}}d + 15\,\sqrt{ex+d}d^2\right)a^2cd}{e^2} + \right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(e*x + d)*a^3*d + 15015*((e*x + d)^(3/2) - 3*sqrt(e*x +
 d)*d)*a^3 + 9009*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x
+ d)*d^2)*a^2*c*d/e^2 + 3861*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 3
5*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*a^2*c/e^2 + 429*(35*(e*x + d
)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^
(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*a*c^2*d/e^4 + 195*(63*(e*x + d)^(11/2)
- 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d
^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*a*c^2/e^4 + 15*(231
*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8
580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*
d^5 + 3003*sqrt(e*x + d)*d^6)*c^3*d/e^6 + 7*(429*(e*x + d)^(15/2) - 3465*(
e*x + d)^(13/2)*d + 12285*(e*x + d)^(11/2)*d^2 - 25025*(e*x + d)^(9/2)*d^3
 + 32175*(e*x + d)^(7/2)*d^4 - 27027*(e*x + d)^(5/2)*d^5 + 15015*(e*x + d)
^(3/2)*d^6 - 6435*sqrt(e*x + d)*d^7)*c^3/e^6)/e
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.92

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3\,dx = \frac{\left(30\,c^3\,d^2+6\,a\,c^2\,e^2\right)\,{\left(d+e\,x\right)}^{11/2}}{11\,e^7} + \frac{{\left(d+e\,x\right)}^{7/2}\,\left(6\,a^2\,c\,e^4+36\,a\,c^2\,d^2\,e^2+30\,c^3\,d^4\right)}{7\,e^7} + \frac{2\,c^3\,{\left(d+e\,x\right)}^{15/2}}{15\,e^7} + \frac{2\,{\left(c\,d^2+a\,e^2\right)}^3\,{\left(d+e\,x\right)}^{3/2}}{3\,e^7} - \frac{\left(40\,c^3\,d^3+24\,a\,c^2\,d\,e^2\right)\,{\left(d+e\,x\right)}^{9/2}}{9\,e^7} - \frac{12\,c^3\,d\,{\left(d+e\,x\right)}^{13/2}}{13\,e^7} - \frac{12\,c\,d\,{\left(c\,d^2+a\,e^2\right)}^2\,{\left(d+e\,x\right)}^{5/2}}{5\,e^7}$$

input
```
int((a + c*x^2)^3*(d + e*x)^(1/2),x)
```

output
```
((30*c^3*d^2 + 6*a*c^2*e^2)*(d + e*x)^(11/2))/(11*e^7) + ((d + e*x)^(7/2)*
(30*c^3*d^4 + 6*a^2*c*e^4 + 36*a*c^2*d^2*e^2))/(7*e^7) + (2*c^3*(d + e*x)^
(15/2))/(15*e^7) + (2*(a*e^2 + c*d^2)^3*(d + e*x)^(3/2))/(3*e^7) - ((40*c^
3*d^3 + 24*a*c^2*d*e^2)*(d + e*x)^(9/2))/(9*e^7) - (12*c^3*d*(d + e*x)^(13
/2))/(13*e^7) - (12*c*d*(a*e^2 + c*d^2)^2*(d + e*x)^(5/2))/(5*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.26

$$\int \sqrt{d+ex}\left(a+cx^2\right)^3 dx$$
$$= \frac{2\sqrt{ex+d}\,(3003c^3e^7x^7+231c^3d\,e^6x^6+12285a\,c^2e^7x^5-252c^3d^2e^5x^5+1365a\,c^2d\,e^6x^4+280c^3d^3e^4x^4+$$

input
```
int((e*x+d)^(1/2)*(c*x^2+a)^3,x)
```

output
```
(2*sqrt(d + e*x)*(15015*a**3*d*e**6 + 15015*a**3*e**7*x + 10296*a**2*c*d**
3*e**4 - 5148*a**2*c*d**2*e**5*x + 3861*a**2*c*d*e**6*x**2 + 19305*a**2*c*
e**7*x**3 + 4992*a*c**2*d**5*e**2 - 2496*a*c**2*d**4*e**3*x + 1872*a*c**2*
d**3*e**4*x**2 - 1560*a*c**2*d**2*e**5*x**3 + 1365*a*c**2*d*e**6*x**4 + 12
285*a*c**2*e**7*x**5 + 1024*c**3*d**7 - 512*c**3*d**6*e*x + 384*c**3*d**5*
e**2*x**2 - 320*c**3*d**4*e**3*x**3 + 280*c**3*d**3*e**4*x**4 - 252*c**3*d
**2*e**5*x**5 + 231*c**3*d*e**6*x**6 + 3003*c**3*e**7*x**7))/(45045*e**7)
```

3.173 $\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\begin{aligned}\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx &= \frac{2(cd^2+ae^2)^3\sqrt{d+ex}}{e^7} - \frac{4cd(cd^2+ae^2)^2(d+ex)^{3/2}}{e^7} \\ &\quad + \frac{6c(cd^2+ae^2)(5cd^2+ae^2)(d+ex)^{5/2}}{5e^7} \\ &\quad - \frac{8c^2d(5cd^2+3ae^2)(d+ex)^{7/2}}{7e^7} + \frac{2c^2(5cd^2+ae^2)(d+ex)^{9/2}}{3e^7} \\ &\quad - \frac{12c^3d(d+ex)^{11/2}}{11e^7} + \frac{2c^3(d+ex)^{13/2}}{13e^7}\end{aligned}$$

output

```
2*(a*e^2+c*d^2)^3*(e*x+d)^(1/2)/e^7-4*c*d*(a*e^2+c*d^2)^2*(e*x+d)^(3/2)/e^
7+6/5*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^(5/2)/e^7-8/7*c^2*d*(3*a*e^2
+5*c*d^2)*(e*x+d)^(7/2)/e^7+2/3*c^2*(a*e^2+5*c*d^2)*(e*x+d)^(9/2)/e^7-12/1
1*c^3*d*(e*x+d)^(11/2)/e^7+2/13*c^3*(e*x+d)^(13/2)/e^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(15015a^3e^6+3003a^2ce^4(8d^2-4dex+3e^2x^2)+143ac^2e^2(128d^4-64d^3ex+48d^2e^2x^2-40de^3}{1501}$$

input
```
Integrate[(a + c*x^2)^3/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(15015*a^3*e^6 + 3003*a^2*c*e^4*(8*d^2 - 4*d*e*x + 3*e^2*
x^2) + 143*a*c^2*e^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3
 + 35*e^4*x^4) + 5*c^3*(1024*d^6 - 512*d^5*e*x + 384*d^4*e^2*x^2 - 320*d^3
*e^3*x^3 + 280*d^2*e^4*x^4 - 252*d*e^5*x^5 + 231*e^6*x^6)))/(15015*e^7)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(d+ex)^{7/2}\,(ae^2+5cd^2)}{e^6}-\frac{4c^2d(d+ex)^{5/2}\,(3ae^2+5cd^2)}{e^6}+\frac{3c(d+ex)^{3/2}\,(ae^2+cd^2)\,(ae^2+5cd^2)}{e^6}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c^2(d+ex)^{9/2}\left(ae^2+5cd^2\right)}{3e^7}-\frac{8c^2d(d+ex)^{7/2}\left(3ae^2+5cd^2\right)}{7e^7}+\frac{6c(d+ex)^{5/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{5e^7}-\frac{4cd(d+ex)^{3/2}\left(ae^2+cd^2\right)^2}{e^7}+\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)^3}{e^7}+\frac{2c^3(d+ex)^{13/2}}{13e^7}-\frac{12c^3d(d+ex)^{11/2}}{11e^7}$$

input
```
Int[(a + c*x^2)^3/Sqrt[d + e*x],x]
```

output
```
(2*(c*d^2 + a*e^2)^3*Sqrt[d + e*x])/e^7 - (4*c*d*(c*d^2 + a*e^2)^2*(d + e*
x)^(3/2))/e^7 + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5
*e^7) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(7/2))/(7*e^7) + (2*c^2*(5*
c*d^2 + a*e^2)*(d + e*x)^(9/2))/(3*e^7) - (12*c^3*d*(d + e*x)^(11/2))/(11*
e^7) + (2*c^3*(d + e*x)^(13/2))/(13*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{2\sqrt{ex+d}\left(\frac{\left(e^6x^6-\frac{12}{11}d\,e^5x^5+\frac{40}{33}d^2e^4x^4-\frac{320}{231}d^3e^3x^3+\frac{128}{77}d^4e^2x^2-\frac{512}{231}d^5ex+\frac{1024}{231}d^6\right)c^3}{13}+\frac{128e^2a\left(\frac{35}{128}e^4x^4-\frac{5}{16}d\,e^3x^3+\frac{3}{8}d^2 \cdots\right.}{105}\right.}{e^7}$
gosper	$\frac{2\sqrt{ex+d}\,(1155x^6c^3e^6-1260d\,c^3x^5e^5+5005x^4a\,c^2e^6+1400x^4c^3d^2e^4-5720x^3a\,c^2d\,e^5-1600x^3c^3d^3e^3+9009x^2a^2c\,e^6+\cdots}{}$
trager	$\frac{2\sqrt{ex+d}\,(1155x^6c^3e^6-1260d\,c^3x^5e^5+5005x^4a\,c^2e^6+1400x^4c^3d^2e^4-5720x^3a\,c^2d\,e^5-1600x^3c^3d^3e^3+9009x^2a^2c\,e^6+\cdots}{}$
risch	$\frac{2\sqrt{ex+d}\,(1155x^6c^3e^6-1260d\,c^3x^5e^5+5005x^4a\,c^2e^6+1400x^4c^3d^2e^4-5720x^3a\,c^2d\,e^5-1600x^3c^3d^3e^3+9009x^2a^2c\,e^6+\cdots}{}$
orering	$\frac{2\sqrt{ex+d}\,(1155x^6c^3e^6-1260d\,c^3x^5e^5+5005x^4a\,c^2e^6+1400x^4c^3d^2e^4-5720x^3a\,c^2d\,e^5-1600x^3c^3d^3e^3+9009x^2a^2c\,e^6+\cdots}{}$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{13}{2}}}{13}-\frac{12c^3d(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(\left(a\,e^2+c\,d^2\right)c^2+8c^3d^2+c\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-8\left(a\,e^2+c\,d^2\right)c^2d-2cd\left(2\left(\cdots\right.\right.\right.}{}$
default	$\frac{2c^3(ex+d)^{\frac{13}{2}}}{13}-\frac{12c^3d(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(\left(a\,e^2+c\,d^2\right)c^2+8c^3d^2+c\left(2\left(a\,e^2+c\,d^2\right)c+4d^2c^2\right)\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-8\left(a\,e^2+c\,d^2\right)c^2d-2cd\left(2\left(\cdots\right.\right.\right.}{}$

input
```
int((c*x^2+a)^3/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*x+d)^(1/2)*(1/13*(e^6*x^6-12/11*d*e^5*x^5+40/33*d^2*e^4*x^4-320/231*d
^3*e^3*x^3+128/77*d^4*e^2*x^2-512/231*d^5*e*x+1024/231*d^6)*c^3+128/105*e^
2*a*(35/128*e^4*x^4-5/16*d*e^3*x^3+3/8*d^2*e^2*x^2-1/2*d^3*e*x+d^4)*c^2+8/
5*(3/8*e^2*x^2-1/2*d*e*x+d^2)*e^4*a^2*c+e^6*a^3)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.01

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\,(1155\,c^3e^6x^6-1260\,c^3de^5x^5+5120\,c^3d^6+18304\,ac^2d^4e^2+24024\,a^2cd^2e^4+15015\,a^3e^6+35\,(40\,c^3d^2e^4 \cdots}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*c^3*e^6*x^6 - 1260*c^3*d*e^5*x^5 + 5120*c^3*d^6 + 18304*a*c^
2*d^4*e^2 + 24024*a^2*c*d^2*e^4 + 15015*a^3*e^6 + 35*(40*c^3*d^2*e^4 + 143
*a*c^2*e^6)*x^4 - 40*(40*c^3*d^3*e^3 + 143*a*c^2*d*e^5)*x^3 + 3*(640*c^3*d
^4*e^2 + 2288*a*c^2*d^2*e^4 + 3003*a^2*c*e^6)*x^2 - 4*(640*c^3*d^5*e + 228
8*a*c^2*d^3*e^3 + 3003*a^2*c*d*e^5)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.52

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{6c^3d(d+ex)^{\frac{11}{2}}}{11e^6}+\frac{c^3(d+ex)^{\frac{13}{2}}}{13e^6}+\frac{(d+ex)^{\frac{9}{2}}\cdot\left(3ac^2e^2+15c^3d^2\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}\left(-12ac^2de^2-20c^3d^3\right)}{7e^6}+\frac{(d+ex)^{\frac{5}{2}}\cdot\left(3a^2ce^4+18ac^2d^2e^2+15c^3d^4\right)}{5e^6}+\frac{(d+ex)^{\frac{3}{2}}\cdots}{}\right.}{e} \\ \frac{a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}}{\sqrt{d}} \end{cases}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(-6*c**3*d*(d + e*x)**(11/2)/(11*e**6) + c**3*(d + e*x)**(13/
2)/(13*e**6) + (d + e*x)**(9/2)*(3*a*c**2*e**2 + 15*c**3*d**2)/(9*e**6) +
(d + e*x)**(7/2)*(-12*a*c**2*d*e**2 - 20*c**3*d**3)/(7*e**6) + (d + e*x)**
(5/2)*(3*a**2*c*e**4 + 18*a*c**2*d**2*e**2 + 15*c**3*d**4)/(5*e**6) + (d +
 e*x)**(3/2)*(-6*a**2*c*d*e**4 - 12*a*c**2*d**3*e**2 - 6*c**3*d**5)/(3*e**
6) + sqrt(d + e*x)*(a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 +
c**3*d**6)/e**6)/e, Ne(e, 0)), ((a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 +
c**3*x**7/7)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{2\left(15015\sqrt{ex+d}a^3 + \frac{3003\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)a^2c}{e^2} + \frac{143\left(35(ex+d)^{\frac{9}{2}}-180(ex+d)^{\frac{7}{2}}d+378(ex+d)^{\frac{5}{2}}d^2-42\right.}{e^4}}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/15015*(15015*sqrt(e*x + d)*a^3 + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^
(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*c/e^2 + 143*(35*(e*x + d)^(9/2) - 180*
(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 31
5*sqrt(e*x + d)*d^4)*a*c^2/e^4 + 5*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^
(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x
 + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^3/e
^6)/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{2\left(15015\sqrt{ex+d}a^3 + \frac{3003\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)a^2c}{e^2} + \frac{143\left(35(ex+d)^{\frac{9}{2}}-180(ex+d)^{\frac{7}{2}}d+378(ex+d)^{\frac{5}{2}}d^2-42\right.}{e^4}}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/15015*(15015*sqrt(e*x + d)*a^3 + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^
(3/2)*d + 15*sqrt(e*x + d)*d^2)*a^2*c/e^2 + 143*(35*(e*x + d)^(9/2) - 180*
(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 31
5*sqrt(e*x + d)*d^4)*a*c^2/e^4 + 5*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^
(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x
 + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^3/e
^6)/e
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{(30\,c^3\,d^2+6\,a\,c^2\,e^2)\,(d+e\,x)^{9/2}}{9\,e^7} + \frac{(d+e\,x)^{5/2}\,(6\,a^2\,c\,e^4+36\,a\,c^2\,d^2\,e^2+30\,c^3\,d^4)}{5\,e^7} + \frac{2\,c^3\,(d+e\,x)^{13/2}}{13\,e^7} + \frac{2\,(c\,d^2+a\,e^2)^3\,\sqrt{d+e\,x}}{e^7} - \frac{(40\,c^3\,d^3+24\,a\,c^2\,d\,e^2)\,(d+e\,x)^{7/2}}{7\,e^7} - \frac{12\,c^3\,d\,(d+e\,x)^{11/2}}{11\,e^7} - \frac{4\,c\,d\,(c\,d^2+a\,e^2)^2\,(d+e\,x)^{3/2}}{e^7}$$

input
```
int((a + c*x^2)^3/(d + e*x)^(1/2),x)
```

output
```
((30*c^3*d^2 + 6*a*c^2*e^2)*(d + e*x)^(9/2))/(9*e^7) + ((d + e*x)^(5/2)*(3
0*c^3*d^4 + 6*a^2*c*e^4 + 36*a*c^2*d^2*e^2))/(5*e^7) + (2*c^3*(d + e*x)^(1
3/2))/(13*e^7) + (2*(a*e^2 + c*d^2)^3*(d + e*x)^(1/2))/e^7 - ((40*c^3*d^3
+ 24*a*c^2*d*e^2)*(d + e*x)^(7/2))/(7*e^7) - (12*c^3*d*(d + e*x)^(11/2))/(
11*e^7) - (4*c*d*(a*e^2 + c*d^2)^2*(d + e*x)^(3/2))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.02

$$\int \frac{(a+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{ex+d}\,(1155c^3e^6x^6 - 1260c^3d\,e^5x^5 + 5005a\,c^2e^6x^4 + 1400c^3d^2e^4x^4 - 5720a\,c^2d\,e^5x^3 - 1600c^3d^3e^3x^3}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(15015*a**3*e**6 + 24024*a**2*c*d**2*e**4 - 12012*a**2*c*
d*e**5*x + 9009*a**2*c*e**6*x**2 + 18304*a*c**2*d**4*e**2 - 9152*a*c**2*d*
*3*e**3*x + 6864*a*c**2*d**2*e**4*x**2 - 5720*a*c**2*d*e**5*x**3 + 5005*a*
c**2*e**6*x**4 + 5120*c**3*d**6 - 2560*c**3*d**5*e*x + 1920*c**3*d**4*e**2
*x**2 - 1600*c**3*d**3*e**3*x**3 + 1400*c**3*d**2*e**4*x**4 - 1260*c**3*d*
e**5*x**5 + 1155*c**3*e**6*x**6))/(15015*e**7)
```

3.174 $\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\begin{aligned}\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx &= -\frac{2(cd^2+ae^2)^3}{e^7\sqrt{d+ex}}-\frac{12cd(cd^2+ae^2)^2\sqrt{d+ex}}{e^7}\\ &+\frac{2c(cd^2+ae^2)(5cd^2+ae^2)(d+ex)^{3/2}}{e^7}-\frac{8c^2d(5cd^2+3ae^2)(d+ex)^{5/2}}{5e^7}\\ &+\frac{6c^2(5cd^2+ae^2)(d+ex)^{7/2}}{7e^7}-\frac{4c^3d(d+ex)^{9/2}}{3e^7}+\frac{2c^3(d+ex)^{11/2}}{11e^7}\end{aligned}$$

output
```
-2*(a*e^2+c*d^2)^3/e^7/(e*x+d)^(1/2)-12*c*d*(a*e^2+c*d^2)^2*(e*x+d)^(1/2)/
e^7+2*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^(3/2)/e^7-8/5*c^2*d*(3*a*e^2
+5*c*d^2)*(e*x+d)^(5/2)/e^7+6/7*c^2*(a*e^2+5*c*d^2)*(e*x+d)^(7/2)/e^7-4/3*
c^3*d*(e*x+d)^(9/2)/e^7+2/11*c^3*(e*x+d)^(11/2)/e^7
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = -\frac{2(1155a^3e^6+1155a^2ce^4(8d^2+4dex-e^2x^2)+99ac^2e^2(128d^4+64d^3ex-16d^2e^2x^2+8de^3x^3-5e^4x^4)+}{1155e^7\sqrt{d+ex}}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^(3/2),x]
```

output
```
(-2*(1155*a^3*e^6 + 1155*a^2*c*e^4*(8*d^2 + 4*d*e*x - e^2*x^2) + 99*a*c^2*
e^2*(128*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4) + 5*
c^3*(1024*d^6 + 512*d^5*e*x - 128*d^4*e^2*x^2 + 64*d^3*e^3*x^3 - 40*d^2*e^
4*x^4 + 28*d*e^5*x^5 - 21*e^6*x^6)))/(1155*e^7*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{3c^2(d+ex)^{5/2}(ae^2+5cd^2)}{e^6}-\frac{4c^2d(d+ex)^{3/2}(3ae^2+5cd^2)}{e^6}+\frac{3c\sqrt{d+ex}(ae^2+cd^2)(ae^2+5cd^2)}{e^6}-\frac{6cd}{e}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{6c^2(d+ex)^{7/2}\left(ae^2+5cd^2\right)}{7e^7}-\frac{8c^2d(d+ex)^{5/2}\left(3ae^2+5cd^2\right)}{5e^7}+\frac{2c(d+ex)^{3/2}\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^7}-\frac{12cd\sqrt{d+ex}\left(ae^2+cd^2\right)^2}{e^7}-\frac{2\left(ae^2+cd^2\right)^3}{e^7\sqrt{d+ex}}+\frac{2c^3(d+ex)^{11/2}}{11e^7}-\frac{4c^3d(d+ex)^{9/2}}{3e^7}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^(3/2),x]
```

output
```
(-2*(c*d^2 + a*e^2)^3)/(e^7*Sqrt[d + e*x]) - (12*c*d*(c*d^2 + a*e^2)^2*Sqr
t[d + e*x])/e^7 + (2*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*(d + e*x)^(3/2))/
e^7 - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(5/2))/(5*e^7) + (6*c^2*(5*c*
d^2 + a*e^2)*(d + e*x)^(7/2))/(7*e^7) - (4*c^3*d*(d + e*x)^(9/2))/(3*e^7)
+ (2*c^3*(d + e*x)^(11/2))/(11*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{(210e^6x^6-280d\,e^5x^5+400d^2e^4x^4-640d^3e^3x^3+1280d^4e^2x^2-5120d^5ex-10240d^6)c^3-25344e^2\left(-\frac{5}{128}e^4x^4+\frac{1}{16}d\,e^3x^3\right.}{1155\sqrt{ex+d}\,e^7}$
risch	$-\frac{2c\left(-105c^2e^5x^5+245c^2d\,e^4x^4-495ac\,e^5x^3-445c^2d^2e^3x^3+1287ac\,e^4d\,x^2+765c^2d^3e^2x^2-1155a^2e^5x-2871ac\,d^2e^3x\right.}{1155e^7}$
gosper	$-\frac{2\left(-105x^6c^3e^6+140d\,c^3x^5e^5-495x^4a\,c^2e^6-200x^4c^3d^2e^4+792x^3a\,c^2d\,e^5+320x^3c^3d^3e^3-1155x^2a^2c\,e^6-1584x^2a\,c^2\right.}{1155}$
trager	$-\frac{2\left(-105x^6c^3e^6+140d\,c^3x^5e^5-495x^4a\,c^2e^6-200x^4c^3d^2e^4+792x^3a\,c^2d\,e^5+320x^3c^3d^3e^3-1155x^2a^2c\,e^6-1584x^2a\,c^2\right.}{1155}$
orering	$-\frac{2\left(-105x^6c^3e^6+140d\,c^3x^5e^5-495x^4a\,c^2e^6-200x^4c^3d^2e^4+792x^3a\,c^2d\,e^5+320x^3c^3d^3e^3-1155x^2a^2c\,e^6-1584x^2a\,c^2\right.}{1155}$
derivativedivides	$\frac{\frac{2c^3(ex+d)^{\frac{11}{2}}}{11}-\frac{4c^3d(ex+d)^{\frac{9}{2}}}{3}+\frac{6a\,c^2e^2(ex+d)^{\frac{7}{2}}}{7}+\frac{30c^3d^2(ex+d)^{\frac{7}{2}}}{7}-\frac{24a\,c^2d\,e^2(ex+d)^{\frac{5}{2}}}{5}-8c^3d^3(ex+d)^{\frac{5}{2}}+2a^2c\,e^4(ex+d)^{\frac{3}{2}}}{}$
default	$\frac{\frac{2c^3(ex+d)^{\frac{11}{2}}}{11}-\frac{4c^3d(ex+d)^{\frac{9}{2}}}{3}+\frac{6a\,c^2e^2(ex+d)^{\frac{7}{2}}}{7}+\frac{30c^3d^2(ex+d)^{\frac{7}{2}}}{7}-\frac{24a\,c^2d\,e^2(ex+d)^{\frac{5}{2}}}{5}-8c^3d^3(ex+d)^{\frac{5}{2}}+2a^2c\,e^4(ex+d)^{\frac{3}{2}}}{}$

input
```
int((c*x^2+a)^3/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1155*((210*e^6*x^6-280*d*e^5*x^5+400*d^2*e^4*x^4-640*d^3*e^3*x^3+1280*d^
4*e^2*x^2-5120*d^5*e*x-10240*d^6)*c^3-25344*e^2*(-5/128*e^4*x^4+1/16*d*e^3
*x^3-1/8*d^2*e^2*x^2+1/2*d^3*e*x+d^4)*a*c^2-18480*e^4*a^2*(-1/8*e^2*x^2+1/
2*d*e*x+d^2)*c-2310*e^6*a^3)/(e*x+d)^(1/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\,(105\,c^3e^6x^6-140\,c^3de^5x^5-5120\,c^3d^6-12672\,ac^2d^4e^2-9240\,a^2cd^2e^4-1155\,a^3e^6+}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(105*c^3*e^6*x^6 - 140*c^3*d*e^5*x^5 - 5120*c^3*d^6 - 12672*a*c^2*d
^4*e^2 - 9240*a^2*c*d^2*e^4 - 1155*a^3*e^6 + 5*(40*c^3*d^2*e^4 + 99*a*c^2*
e^6)*x^4 - 8*(40*c^3*d^3*e^3 + 99*a*c^2*d*e^5)*x^3 + (640*c^3*d^4*e^2 + 15
84*a*c^2*d^2*e^4 + 1155*a^2*c*e^6)*x^2 - 4*(640*c^3*d^5*e + 1584*a*c^2*d^3
*e^3 + 1155*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^8*x + d*e^7)
```

Sympy [A] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{2c^3d(d+ex)^{\frac{9}{2}}}{3e^6}+\frac{c^3(d+ex)^{\frac{11}{2}}}{11e^6}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(3ac^2e^2+15c^3d^2\right)}{7e^6}+\frac{(d+ex)^{\frac{5}{2}}\left(-12ac^2de^2-20c^3d^3\right)}{5e^6}+\frac{(d+ex)^{\frac{3}{2}}\cdot\left(3a^2ce^4+18ac^2\right.}{3e^6}\right.}{e} \\ \frac{a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}}{d^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(-2*c**3*d*(d + e*x)**(9/2)/(3*e**6) + c**3*(d + e*x)**(11/2)
/(11*e**6) + (d + e*x)**(7/2)*(3*a*c**2*e**2 + 15*c**3*d**2)/(7*e**6) + (d
 + e*x)**(5/2)*(-12*a*c**2*d*e**2 - 20*c**3*d**3)/(5*e**6) + (d + e*x)**(3
/2)*(3*a**2*c*e**4 + 18*a*c**2*d**2*e**2 + 15*c**3*d**4)/(3*e**6) + sqrt(d
 + e*x)*(-6*a**2*c*d*e**4 - 12*a*c**2*d**3*e**2 - 6*c**3*d**5)/e**6 - (a*e
**2 + c*d**2)**3/(e**6*sqrt(d + e*x)))/e, Ne(e, 0)), ((a**3*x + a**2*c*x**
3 + 3*a*c**2*x**5/5 + c**3*x**7/7)/d**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{105\,(ex+d)^{\frac{11}{2}}c^3-770\,(ex+d)^{\frac{9}{2}}c^3d+495\,(5\,c^3d^2+ac^2e^2)(ex+d)^{\frac{7}{2}}-924\,(5\,c^3d^3+3\,ac^2de^2)(ex+d)^{\frac{5}{2}}+1155\,(5\,c^3d^4}{e^6}\right.}{115}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/1155*((105*(e*x + d)^(11/2)*c^3 - 770*(e*x + d)^(9/2)*c^3*d + 495*(5*c^3
*d^2 + a*c^2*e^2)*(e*x + d)^(7/2) - 924*(5*c^3*d^3 + 3*a*c^2*d*e^2)*(e*x +
 d)^(5/2) + 1155*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(e*x + d)^(3/2)
 - 6930*(c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*sqrt(e*x + d))/e^6 - 115
5*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)/(sqrt(e*x + d)*e
^6))/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.34

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = -\frac{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)}{\sqrt{ex+d}e^7} + \frac{2\left(105\,(ex+d)^{\frac{11}{2}}c^3e^{70}-770\,(ex+d)^{\frac{9}{2}}c^3de^{70}+2475\,(ex+d)^{\frac{7}{2}}c^3d^2e^{70}-4620\,(ex+d)^{\frac{5}{2}}c^3d^3e^{70}+5775\,(e\right.}{}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
-2*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)/(sqrt(e*x + d)*
e^7) + 2/1155*(105*(e*x + d)^(11/2)*c^3*e^70 - 770*(e*x + d)^(9/2)*c^3*d*e
^70 + 2475*(e*x + d)^(7/2)*c^3*d^2*e^70 - 4620*(e*x + d)^(5/2)*c^3*d^3*e^7
0 + 5775*(e*x + d)^(3/2)*c^3*d^4*e^70 - 6930*sqrt(e*x + d)*c^3*d^5*e^70 +
495*(e*x + d)^(7/2)*a*c^2*e^72 - 2772*(e*x + d)^(5/2)*a*c^2*d*e^72 + 6930*
(e*x + d)^(3/2)*a*c^2*d^2*e^72 - 13860*sqrt(e*x + d)*a*c^2*d^3*e^72 + 1155
*(e*x + d)^(3/2)*a^2*c*e^74 - 6930*sqrt(e*x + d)*a^2*c*d*e^74)/e^77
```

Mupad [B] (verification not implemented)

Time = 6.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{(30\,c^3\,d^2+6\,a\,c^2\,e^2)\,\left(d+e\,x\right)^{7/2}}{7\,e^7} + \frac{\left(d+e\,x\right)^{3/2}\,(6\,a^2\,c\,e^4+36\,a\,c^2\,d^2\,e^2+30\,c^3\,d^4)}{3\,e^7} + \frac{2\,c^3\,\left(d+e\,x\right)^{11/2}}{11\,e^7} - \frac{(40\,c^3\,d^3+24\,a\,c^2\,d\,e^2)\,\left(d+e\,x\right)^{5/2}}{5\,e^7} - \frac{2\,a^3\,e^6+6\,a^2\,c\,d^2\,e^4+6\,a\,c^2\,d^4\,e^2+2\,c^3\,d^6}{e^7\,\sqrt{d+e\,x}} - \frac{4\,c^3\,d\,\left(d+e\,x\right)^{9/2}}{3\,e^7} - \frac{12\,c\,d\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{d+e\,x}}{e^7}$$

input
```
int((a + c*x^2)^3/(d + e*x)^(3/2),x)
```

output
```
((30*c^3*d^2 + 6*a*c^2*e^2)*(d + e*x)^(7/2))/(7*e^7) + ((d + e*x)^(3/2)*(3
0*c^3*d^4 + 6*a^2*c*e^4 + 36*a*c^2*d^2*e^2))/(3*e^7) + (2*c^3*(d + e*x)^(1
1/2))/(11*e^7) - ((40*c^3*d^3 + 24*a*c^2*d*e^2)*(d + e*x)^(5/2))/(5*e^7) -
 (2*a^3*e^6 + 2*c^3*d^6 + 6*a*c^2*d^4*e^2 + 6*a^2*c*d^2*e^4)/(e^7*(d + e*x
)^(1/2)) - (4*c^3*d*(d + e*x)^(9/2))/(3*e^7) - (12*c*d*(a*e^2 + c*d^2)^2*(
d + e*x)^(1/2))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

$$\int \frac{(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{11}c^3e^6x^6 - \frac{8}{33}c^3d\,e^5x^5 + \frac{6}{7}a\,c^2e^6x^4 + \frac{80}{231}c^3d^2e^4x^4 - \frac{48}{35}a\,c^2d\,e^5x^3 - \frac{128}{231}c^3d^3e^3x^3 + 2a^2c\,e}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^(3/2),x)
```

output
```
(2*( - 1155*a**3*e**6 - 9240*a**2*c*d**2*e**4 - 4620*a**2*c*d*e**5*x + 115
5*a**2*c*e**6*x**2 - 12672*a*c**2*d**4*e**2 - 6336*a*c**2*d**3*e**3*x + 15
84*a*c**2*d**2*e**4*x**2 - 792*a*c**2*d*e**5*x**3 + 495*a*c**2*e**6*x**4 -
 5120*c**3*d**6 - 2560*c**3*d**5*e*x + 640*c**3*d**4*e**2*x**2 - 320*c**3*
d**3*e**3*x**3 + 200*c**3*d**2*e**4*x**4 - 140*c**3*d*e**5*x**5 + 105*c**3
*e**6*x**6))/(1155*sqrt(d + e*x)*e**7)
```

3.175 $\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\begin{aligned}\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx &= -\frac{2(cd^2+ae^2)^3}{3e^7(d+ex)^{3/2}}+\frac{12cd(cd^2+ae^2)^2}{e^7\sqrt{d+ex}} \\ &+\frac{6c(cd^2+ae^2)\left(5cd^2+ae^2\right)\sqrt{d+ex}}{e^7}-\frac{8c^2d(5cd^2+3ae^2)\left(d+ex\right)^{3/2}}{3e^7} \\ &+\frac{6c^2(5cd^2+ae^2)\left(d+ex\right)^{5/2}}{5e^7}-\frac{12c^3d(d+ex)^{7/2}}{7e^7}+\frac{2c^3(d+ex)^{9/2}}{9e^7}\end{aligned}$$

output
```
-2/3*(a*e^2+c*d^2)^3/e^7/(e*x+d)^(3/2)+12*c*d*(a*e^2+c*d^2)^2/e^7/(e*x+d)^
(1/2)+6*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)*(e*x+d)^(1/2)/e^7-8/3*c^2*d*(3*a*e
^2+5*c*d^2)*(e*x+d)^(3/2)/e^7+6/5*c^2*(a*e^2+5*c*d^2)*(e*x+d)^(5/2)/e^7-12
/7*c^3*d*(e*x+d)^(7/2)/e^7+2/9*c^3*(e*x+d)^(9/2)/e^7
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2(-105a^3e^6+315a^2ce^4(8d^2+12dex+3e^2x^2)+63ac^2e^2(128d^4+192d^3ex+48d^2e^2x^2$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^(5/2),x]
```

output
```
(2*(-105*a^3*e^6 + 315*a^2*c*e^4*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 63*a*c^2
*e^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4) +
5*c^3*(1024*d^6 + 1536*d^5*e*x + 384*d^4*e^2*x^2 - 64*d^3*e^3*x^3 + 24*d^2
*e^4*x^4 - 12*d*e^5*x^5 + 7*e^6*x^6)))/(315*e^7*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{3c^2(d+ex)^{3/2}\left(ae^2+5cd^2\right)}{e^6}-\frac{4c^2d\sqrt{d+ex}(3ae^2+5cd^2)}{e^6}+\frac{3c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^6\sqrt{d+ex}}-\frac{6cd\left(ae^2+cd^2\right)^2}{e^6(d+ex)^{3/2}}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{6c^2(d+ex)^{5/2}\left(ae^2+5cd^2\right)}{5e^7}-\frac{8c^2d(d+ex)^{3/2}\left(3ae^2+5cd^2\right)}{3e^7}+\frac{6c\sqrt{d+ex}(ae^2+cd^2)\left(ae^2+5cd^2\right)}{e^7}+\frac{12cd\left(ae^2+cd^2\right)^2}{e^7\sqrt{d+ex}}-\frac{2\left(ae^2+cd^2\right)^3}{3e^7(d+ex)^{3/2}}+\frac{2c^3(d+ex)^{9/2}}{9e^7}-\frac{12c^3d(d+ex)^{7/2}}{7e^7}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^(5/2),x]
```

output
```
(-2*(c*d^2 + a*e^2)^3)/(3*e^7*(d + e*x)^(3/2)) + (12*c*d*(c*d^2 + a*e^2)^2
)/(e^7*Sqrt[d + e*x]) + (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2)*Sqrt[d + e*
x])/e^7 - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*(d + e*x)^(3/2))/(3*e^7) + (6*c^2*(
5*c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^7) - (12*c^3*d*(d + e*x)^(7/2))/(7*
e^7) + (2*c^3*(d + e*x)^(9/2))/(9*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

method	result
risch	$\frac{2c\left(35c^2x^4e^4-130d\,c^2x^3e^3+189x^2ac\,e^4+345x^2c^2d^2e^2-882xacd\,e^3-880x\,c^2d^3e+945a^2e^4+4599ac\,d^2e^2+3335c^2d^4\right)\sqrt{\cdots}}{315e^7}$
pseudoelliptic	$\frac{\left(70e^6x^6-120d\,e^5x^5+240d^2e^4x^4-640d^3e^3x^3+3840d^4e^2x^2+15360d^5ex+10240d^6\right)c^3+16128e^2\left(\frac{3}{128}e^4x^4-\frac{1}{16}d\,e^3x^3+\cdots\right.}{315(ex+d)^{\frac{3}{2}}e^7}$
gosper	$-\frac{2\left(-35x^6c^3e^6+60d\,c^3x^5e^5-189x^4a\,c^2e^6-120x^4c^3d^2e^4+504x^3a\,c^2d\,e^5+320x^3c^3d^3e^3-945x^2a^2c\,e^6-3024x^2a\,c^2d^2e\cdots\right.}{315(ex\cdots}$
trager	$-\frac{2\left(-35x^6c^3e^6+60d\,c^3x^5e^5-189x^4a\,c^2e^6-120x^4c^3d^2e^4+504x^3a\,c^2d\,e^5+320x^3c^3d^3e^3-945x^2a^2c\,e^6-3024x^2a\,c^2d^2e\cdots\right.}{315(ex\cdots}$
orering	$-\frac{2\left(-35x^6c^3e^6+60d\,c^3x^5e^5-189x^4a\,c^2e^6-120x^4c^3d^2e^4+504x^3a\,c^2d\,e^5+320x^3c^3d^3e^3-945x^2a^2c\,e^6-3024x^2a\,c^2d^2e\cdots\right.}{315(ex\cdots}$
derivativedivides	$\frac{\frac{2c^3(ex+d)^{\frac{9}{2}}}{9}-\frac{12c^3d(ex+d)^{\frac{7}{2}}}{7}+\frac{6a\,c^2e^2(ex+d)^{\frac{5}{2}}}{5}+6c^3d^2(ex+d)^{\frac{5}{2}}-8a\,c^2d\,e^2(ex+d)^{\frac{3}{2}}-\frac{40c^3d^3(ex+d)^{\frac{3}{2}}}{3}+6a^2c\,e^4\sqrt{ex+d}+\cdots}{e^7}$
default	$\frac{\frac{2c^3(ex+d)^{\frac{9}{2}}}{9}-\frac{12c^3d(ex+d)^{\frac{7}{2}}}{7}+\frac{6a\,c^2e^2(ex+d)^{\frac{5}{2}}}{5}+6c^3d^2(ex+d)^{\frac{5}{2}}-8a\,c^2d\,e^2(ex+d)^{\frac{3}{2}}-\frac{40c^3d^3(ex+d)^{\frac{3}{2}}}{3}+6a^2c\,e^4\sqrt{ex+d}+\cdots}{e^7}$

input
```
int((c*x^2+a)^3/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*c*(35*c^2*e^4*x^4-130*c^2*d*e^3*x^3+189*a*c*e^4*x^2+345*c^2*d^2*e^2*
x^2-882*a*c*d*e^3*x-880*c^2*d^3*e*x+945*a^2*e^4+4599*a*c*d^2*e^2+3335*c^2*
d^4)*(e*x+d)^(1/2)/e^7-2/3*(-18*c*d*e*x+a*e^2-17*c*d^2)*(a^2*e^4+2*a*c*d^2
*e^2+c^2*d^4)/e^7/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\,(35\,c^3e^6x^6 - 60\,c^3de^5x^5 + 5120\,c^3d^6 + 8064\,ac^2d^4e^2 + 2520\,a^2cd^2e^4 - 105\,a^3e^6 + 3\,(4 \cdots}{\cdots}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/315*(35*c^3*e^6*x^6 - 60*c^3*d*e^5*x^5 + 5120*c^3*d^6 + 8064*a*c^2*d^4*e
^2 + 2520*a^2*c*d^2*e^4 - 105*a^3*e^6 + 3*(40*c^3*d^2*e^4 + 63*a*c^2*e^6)*
x^4 - 8*(40*c^3*d^3*e^3 + 63*a*c^2*d*e^5)*x^3 + 3*(640*c^3*d^4*e^2 + 1008*
a*c^2*d^2*e^4 + 315*a^2*c*e^6)*x^2 + 12*(640*c^3*d^5*e + 1008*a*c^2*d^3*e^
3 + 315*a^2*c*d*e^5)*x)*sqrt(e*x + d)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.26

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(-\frac{6c^3d(d+ex)^{\frac{7}{2}}}{7e^6} + \frac{c^3(d+ex)^{\frac{9}{2}}}{9e^6} + \frac{6cd\left(ae^2+cd^2\right)^2}{e^6\sqrt{d+ex}} + \frac{(d+ex)^{\frac{5}{2}}\cdot\left(3ac^2e^2+15c^3d^2\right)}{5e^6} + \frac{(d+ex)^{\frac{3}{2}}\left(-12ac^2de^2-20c^3d^3\right)}{3e^6} + \frac{\sqrt{d+ex}\cdots}{\cdots}\right.}{e} \\ \frac{a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(-6*c**3*d*(d + e*x)**(7/2)/(7*e**6) + c**3*(d + e*x)**(9/2)/
(9*e**6) + 6*c*d*(a*e**2 + c*d**2)**2/(e**6*sqrt(d + e*x)) + (d + e*x)**(5
/2)*(3*a*c**2*e**2 + 15*c**3*d**2)/(5*e**6) + (d + e*x)**(3/2)*(-12*a*c**2
*d*e**2 - 20*c**3*d**3)/(3*e**6) + sqrt(d + e*x)*(3*a**2*c*e**4 + 18*a*c**
2*d**2*e**2 + 15*c**3*d**4)/e**6 - (a*e**2 + c*d**2)**3/(3*e**6*(d + e*x)*
*(3/2)))/e, Ne(e, 0)), ((a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**
7/7)/d**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{35\,(ex+d)^{\frac{9}{2}}c^3-270\,(ex+d)^{\frac{7}{2}}c^3d+189\,(5\,c^3d^2+ac^2e^2)(ex+d)^{\frac{5}{2}}-420\,(5\,c^3d^3+3\,ac^2de^2)(ex+d)^{\frac{3}{2}}+945\,(5\,c^3d^4+6\,a}{e^6}\right.}{315\,e}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/315*((35*(e*x + d)^(9/2)*c^3 - 270*(e*x + d)^(7/2)*c^3*d + 189*(5*c^3*d^
2 + a*c^2*e^2)*(e*x + d)^(5/2) - 420*(5*c^3*d^3 + 3*a*c^2*d*e^2)*(e*x + d)
^(3/2) + 945*(5*c^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(e*x + d))/e^6
- 105*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 - 18*(c^3*d^5
 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(3/2)*e^6))/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.29

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\,(18\,(ex+d)c^3d^5-c^3d^6+36\,(ex+d)ac^2d^3e^2-3\,ac^2d^4e^2+18\,(ex+d)a^2cde^4-3\,a^2}{3\,(ex+d)^{\frac{3}{2}}e^7}$$
$$+\frac{2\left(35\,(ex+d)^{\frac{9}{2}}c^3e^{56}-270\,(ex+d)^{\frac{7}{2}}c^3de^{56}+945\,(ex+d)^{\frac{5}{2}}c^3d^2e^{56}-2100\,(ex+d)^{\frac{3}{2}}c^3d^3e^{56}+4725\,\sqrt{ex}\right.}{31}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
2/3*(18*(e*x + d)*c^3*d^5 - c^3*d^6 + 36*(e*x + d)*a*c^2*d^3*e^2 - 3*a*c^2
*d^4*e^2 + 18*(e*x + d)*a^2*c*d*e^4 - 3*a^2*c*d^2*e^4 - a^3*e^6)/((e*x + d
)^(3/2)*e^7) + 2/315*(35*(e*x + d)^(9/2)*c^3*e^56 - 270*(e*x + d)^(7/2)*c^
3*d*e^56 + 945*(e*x + d)^(5/2)*c^3*d^2*e^56 - 2100*(e*x + d)^(3/2)*c^3*d^3
*e^56 + 4725*sqrt(e*x + d)*c^3*d^4*e^56 + 189*(e*x + d)^(5/2)*a*c^2*e^58 -
 1260*(e*x + d)^(3/2)*a*c^2*d*e^58 + 5670*sqrt(e*x + d)*a*c^2*d^2*e^58 + 9
45*sqrt(e*x + d)*a^2*c*e^60)/e^63
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.12

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{(30\,c^3\,d^2+6\,a\,c^2\,e^2)\,(d+e\,x)^{5/2}}{5\,e^7} + \frac{\sqrt{d+e\,x}\,(6\,a^2\,c\,e^4+36\,a\,c^2\,d^2\,e^2+30\,c^3\,d^4)}{e^7} - \frac{\frac{2\,a^3\,e^6}{3}-(d+e\,x)\,(12\,a^2\,c\,d\,e^4+24\,a\,c^2\,d^3\,e^2+12\,c^3\,d^5)+\frac{2\,c^3\,d^6}{3}+2\,a\,c^2\,d^4\,e^2+2\,a^2\,c\,d^2\,e^4}{e^7\,(d+e\,x)^{3/2}} + \frac{2\,c^3\,(d+e\,x)^{9/2}}{9\,e^7} - \frac{(40\,c^3\,d^3+24\,a\,c^2\,d\,e^2)\,(d+e\,x)^{3/2}}{3\,e^7} - \frac{12\,c^3\,d\,(d+e\,x)^{7/2}}{7\,e^7}$$

input
```
int((a + c*x^2)^3/(d + e*x)^(5/2),x)
```

output
```
((30*c^3*d^2 + 6*a*c^2*e^2)*(d + e*x)^(5/2))/(5*e^7) + ((d + e*x)^(1/2)*(3
0*c^3*d^4 + 6*a^2*c*e^4 + 36*a*c^2*d^2*e^2))/e^7 - ((2*a^3*e^6)/3 - (d + e
*x)*(12*c^3*d^5 + 24*a*c^2*d^3*e^2 + 12*a^2*c*d*e^4) + (2*c^3*d^6)/3 + 2*a
*c^2*d^4*e^2 + 2*a^2*c*d^2*e^4)/(e^7*(d + e*x)^(3/2)) + (2*c^3*(d + e*x)^(
9/2))/(9*e^7) - ((40*c^3*d^3 + 24*a*c^2*d*e^2)*(d + e*x)^(3/2))/(3*e^7) -
(12*c^3*d*(d + e*x)^(7/2))/(7*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{\frac{2}{9}c^3e^6x^6 - \frac{8}{21}c^3d\,e^5x^5 + \frac{6}{5}a\,c^2e^6x^4 + \frac{16}{21}c^3d^2e^4x^4 - \frac{16}{5}a\,c^2d\,e^5x^3 - \frac{128}{63}c^3d^3e^3x^3 + 6a^2c\,e^6}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^(5/2),x)
```

output
```
(2*( - 105*a**3*e**6 + 2520*a**2*c*d**2*e**4 + 3780*a**2*c*d*e**5*x + 945*
a**2*c*e**6*x**2 + 8064*a*c**2*d**4*e**2 + 12096*a*c**2*d**3*e**3*x + 3024
*a*c**2*d**2*e**4*x**2 - 504*a*c**2*d*e**5*x**3 + 189*a*c**2*e**6*x**4 + 5
120*c**3*d**6 + 7680*c**3*d**5*e*x + 1920*c**3*d**4*e**2*x**2 - 320*c**3*d
**3*e**3*x**3 + 120*c**3*d**2*e**4*x**4 - 60*c**3*d*e**5*x**5 + 35*c**3*e*
*6*x**6))/(315*sqrt(d + e*x)*e**7*(d + e*x))
```

3.176 $\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 196

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = -\frac{2(cd^2+ae^2)^3}{5e^7(d+ex)^{5/2}} + \frac{4cd(cd^2+ae^2)^2}{e^7(d+ex)^{3/2}} - \frac{6c(cd^2+ae^2)(5cd^2+ae^2)}{e^7\sqrt{d+ex}} - \frac{8c^2d(5cd^2+3ae^2)\sqrt{d+ex}}{e^7} + \frac{2c^2(5cd^2+ae^2)(d+ex)^{3/2}}{e^7} - \frac{12c^3d(d+ex)^{5/2}}{5e^7} + \frac{2c^3(d+ex)^{7/2}}{7e^7}$$

output
```
-2/5*(a*e^2+c*d^2)^3/e^7/(e*x+d)^(5/2)+4*c*d*(a*e^2+c*d^2)^2/e^7/(e*x+d)^(
3/2)-6*c*(a*e^2+c*d^2)*(a*e^2+5*c*d^2)/e^7/(e*x+d)^(1/2)-8*c^2*d*(3*a*e^2+
5*c*d^2)*(e*x+d)^(1/2)/e^7+2*c^2*(a*e^2+5*c*d^2)*(e*x+d)^(3/2)/e^7-12/5*c^
3*d*(e*x+d)^(5/2)/e^7+2/7*c^3*(e*x+d)^(7/2)/e^7
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx =$$
$$-\frac{2(7a^3e^6+7a^2ce^4(8d^2+20dex+15e^2x^2)+7ac^2e^2(128d^4+320d^3ex+240d^2e^2x^2+40de^3x^3-5e^4x^4)+}{35e^7(d+ex)^{5/2}}$$

input
```
Integrate[(a + c*x^2)^3/(d + e*x)^(7/2),x]
```

output
```
(-2*(7*a^3*e^6 + 7*a^2*c*e^4*(8*d^2 + 20*d*e*x + 15*e^2*x^2) + 7*a*c^2*e^2
*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4) + c^
3*(1024*d^6 + 2560*d^5*e*x + 1920*d^4*e^2*x^2 + 320*d^3*e^3*x^3 - 40*d^2*e
^4*x^4 + 12*d*e^5*x^5 - 5*e^6*x^6)))/(35*e^7*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx$$
$$\Big\downarrow \text{476}$$
$$\int \left(\frac{3c^2\sqrt{d+ex}(ae^2+5cd^2)}{e^6}-\frac{4c^2d(3ae^2+5cd^2)}{e^6\sqrt{d+ex}}+\frac{3c(ae^2+cd^2)(ae^2+5cd^2)}{e^6(d+ex)^{3/2}}-\frac{6cd(ae^2+cd^2)^2}{e^6(d+ex)^{5/2}}+\frac{(ae^2+}{e^6(d+}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{2c^2(d+ex)^{3/2}\left(ae^2+5cd^2\right)}{e^7}-\frac{8c^2d\sqrt{d+ex}\left(3ae^2+5cd^2\right)}{e^7}-\frac{6c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^7\sqrt{d+ex}}+\frac{4cd\left(ae^2+cd^2\right)^2}{e^7(d+ex)^{3/2}}-\frac{2\left(ae^2+cd^2\right)^3}{5e^7(d+ex)^{5/2}}+\frac{2c^3(d+ex)^{7/2}}{7e^7}-\frac{12c^3d(d+ex)^{5/2}}{5e^7}$$

input
```
Int[(a + c*x^2)^3/(d + e*x)^(7/2),x]
```

output
```
(-2*(c*d^2 + a*e^2)^3)/(5*e^7*(d + e*x)^(5/2)) + (4*c*d*(c*d^2 + a*e^2)^2)
/(e^7*(d + e*x)^(3/2)) - (6*c*(c*d^2 + a*e^2)*(5*c*d^2 + a*e^2))/(e^7*Sqrt
[d + e*x]) - (8*c^2*d*(5*c*d^2 + 3*a*e^2)*Sqrt[d + e*x])/e^7 + (2*c^2*(5*c
*d^2 + a*e^2)*(d + e*x)^(3/2))/e^7 - (12*c^3*d*(d + e*x)^(5/2))/(5*e^7) +
(2*c^3*(d + e*x)^(7/2))/(7*e^7)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\frac{2\left(5e^6x^6-12d\,e^5x^5+40d^2e^4x^4-320d^3e^3x^3-1920d^4e^2x^2-2560d^5ex-1024d^6\right)c^3}{35}-\frac{256\left(-\frac{5}{128}e^4x^4+\frac{5}{16}d\,e^3x^3+\frac{15}{8}d^2e^2x^2+\frac{5}{2}d^3e\ldots\right.}{5}}{(ex+d)^{\frac{5}{2}}e^7}$
risch	$-\frac{2c^2\left(-5c\,e^3x^3+27cd\,e^2x^2-35a\,e^3x-106c\,d^2ex+385ad\,e^2+562c\,d^3\right)\sqrt{ex+d}}{35e^7}-\frac{2\left(15x^2ac\,e^4+75x^2c^2d^2e^2+20xacd\,e\ldots\right.}{5e^7\sqrt{\ldots}}$
gosper	$-\frac{2\left(-5x^6c^3e^6+12d\,c^3x^5e^5-35x^4a\,c^2e^6-40x^4c^3d^2e^4+280x^3a\,c^2d\,e^5+320x^3c^3d^3e^3+105x^2a^2c\,e^6+1680x^2a\,c^2d^2e^4+\ldots\right.}{35(ex+d)^{\frac{5}{2}}\ldots}$
trager	$-\frac{2\left(-5x^6c^3e^6+12d\,c^3x^5e^5-35x^4a\,c^2e^6-40x^4c^3d^2e^4+280x^3a\,c^2d\,e^5+320x^3c^3d^3e^3+105x^2a^2c\,e^6+1680x^2a\,c^2d^2e^4+\ldots\right.}{35(ex+d)^{\frac{5}{2}}\ldots}$
orering	$-\frac{2\left(-5x^6c^3e^6+12d\,c^3x^5e^5-35x^4a\,c^2e^6-40x^4c^3d^2e^4+280x^3a\,c^2d\,e^5+320x^3c^3d^3e^3+105x^2a^2c\,e^6+1680x^2a\,c^2d^2e^4+\ldots\right.}{35(ex+d)^{\frac{5}{2}}\ldots}$
derivativedivides	$\frac{\frac{2c^3(ex+d)^{\frac{7}{2}}}{7}-\frac{12c^3d(ex+d)^{\frac{5}{2}}}{5}+2a\,c^2e^2(ex+d)^{\frac{3}{2}}+10c^3d^2(ex+d)^{\frac{3}{2}}-24a\,c^2d\,e^2\sqrt{ex+d}-40c^3d^3\sqrt{ex+d}-\frac{2\left(e^6a^3+3d^2e^4a^2\ldots\right.}{5(e\ldots}}{e^7}$
default	$\frac{\frac{2c^3(ex+d)^{\frac{7}{2}}}{7}-\frac{12c^3d(ex+d)^{\frac{5}{2}}}{5}+2a\,c^2e^2(ex+d)^{\frac{3}{2}}+10c^3d^2(ex+d)^{\frac{3}{2}}-24a\,c^2d\,e^2\sqrt{ex+d}-40c^3d^3\sqrt{ex+d}-\frac{2\left(e^6a^3+3d^2e^4a^2\ldots\right.}{5(e\ldots}}{e^7}$

input
```
int((c*x^2+a)^3/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*((5*e^6*x^6-12*d*e^5*x^5+40*d^2*e^4*x^4-320*d^3*e^3*x^3-1920*d^4*e^2*
x^2-2560*d^5*e*x-1024*d^6)*c^3-896*(-5/128*e^4*x^4+5/16*d*e^3*x^3+15/8*d^2
*e^2*x^2+5/2*d^3*e*x+d^4)*e^2*a*c^2-56*e^4*a^2*(15/8*e^2*x^2+5/2*d*e*x+d^2
)*c-7*e^6*a^3)/(e*x+d)^(5/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\,(5\,c^3e^6x^6-12\,c^3de^5x^5-1024\,c^3d^6-896\,ac^2d^4e^2-56\,a^2cd^2e^4-7\,a^3e^6+5\,(8\,c^3d^2e^4\ldots}{\ldots}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*c^3*e^6*x^6 - 12*c^3*d*e^5*x^5 - 1024*c^3*d^6 - 896*a*c^2*d^4*e^2
- 56*a^2*c*d^2*e^4 - 7*a^3*e^6 + 5*(8*c^3*d^2*e^4 + 7*a*c^2*e^6)*x^4 - 40*
(8*c^3*d^3*e^3 + 7*a*c^2*d*e^5)*x^3 - 15*(128*c^3*d^4*e^2 + 112*a*c^2*d^2*
e^4 + 7*a^2*c*e^6)*x^2 - 20*(128*c^3*d^5*e + 112*a*c^2*d^3*e^3 + 7*a^2*c*d
*e^5)*x)*sqrt(e*x + d)/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.23

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \begin{cases} \frac{2\left(-\frac{6c^3d(d+ex)^{\frac{5}{2}}}{5e^6} + \frac{c^3(d+ex)^{\frac{7}{2}}}{7e^6} + \frac{2cd\left(ae^2+cd^2\right)^2}{e^6(d+ex)^{\frac{3}{2}}} - \frac{3c\left(ae^2+cd^2\right)\left(ae^2+5cd^2\right)}{e^6\sqrt{d+ex}} + \frac{(d+ex)^{\frac{3}{2}}\cdot\left(3ac^2e^2+15c^3d^2\right)}{3e^6} + \frac{\sqrt{d+ex}\left(-12c\right.}{}\right.}{e} \\ \frac{a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((c*x**2+a)**3/(e*x+d)**(7/2),x)
```

output
```
Piecewise((2*(-6*c**3*d*(d + e*x)**(5/2)/(5*e**6) + c**3*(d + e*x)**(7/2)/
(7*e**6) + 2*c*d*(a*e**2 + c*d**2)**2/(e**6*(d + e*x)**(3/2)) - 3*c*(a*e**
2 + c*d**2)*(a*e**2 + 5*c*d**2)/(e**6*sqrt(d + e*x)) + (d + e*x)**(3/2)*(3
*a*c**2*e**2 + 15*c**3*d**2)/(3*e**6) + sqrt(d + e*x)*(-12*a*c**2*d*e**2 -
 20*c**3*d**3)/e**6 - (a*e**2 + c*d**2)**3/(5*e**6*(d + e*x)**(5/2)))/e, N
e(e, 0)), ((a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7)/d**(7/2)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\left(\frac{5\,(ex+d)^{\frac{7}{2}}c^3 - 42\,(ex+d)^{\frac{5}{2}}c^3d + 35\,\left(5\,c^3d^2+ac^2e^2\right)(ex+d)^{\frac{3}{2}} - 140\,\left(5\,c^3d^3+3\,ac^2de^2\right)\sqrt{ex+d}}{e^6} - \frac{7\left(c^3d^6+3\,ac^2d^4e\right.}{}\right.}{35\,e}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
2/35*((5*(e*x + d)^(7/2)*c^3 - 42*(e*x + d)^(5/2)*c^3*d + 35*(5*c^3*d^2 +
a*c^2*e^2)*(e*x + d)^(3/2) - 140*(5*c^3*d^3 + 3*a*c^2*d*e^2)*sqrt(e*x + d)
)/e^6 - 7*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6 + 15*(5*c
^3*d^4 + 6*a*c^2*d^2*e^2 + a^2*c*e^4)*(e*x + d)^2 - 10*(c^3*d^5 + 2*a*c^2*
d^3*e^2 + a^2*c*d*e^4)*(e*x + d))/((e*x + d)^(5/2)*e^6))/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.29

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = -\frac{2\left(75\,(ex+d)^2c^3d^4-10\,(ex+d)c^3d^5+c^3d^6+90\,(ex+d)^2ac^2d^2e^2-20\,(ex+d)ac^2d^3e^2+3\,ac^2d^4e^2+1\right.}{5\,(ex+d)^{\frac{5}{2}}e^7} + \frac{2\left(5\,(ex+d)^{\frac{7}{2}}c^3e^{42}-42\,(ex+d)^{\frac{5}{2}}c^3de^{42}+175\,(ex+d)^{\frac{3}{2}}c^3d^2e^{42}-700\,\sqrt{ex+d}c^3d^3e^{42}+35\,(ex+d)^{\frac{3}{2}}ac\right.}{35\,e^{49}}$$

input
```
integrate((c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-2/5*(75*(e*x + d)^2*c^3*d^4 - 10*(e*x + d)*c^3*d^5 + c^3*d^6 + 90*(e*x +
d)^2*a*c^2*d^2*e^2 - 20*(e*x + d)*a*c^2*d^3*e^2 + 3*a*c^2*d^4*e^2 + 15*(e*
x + d)^2*a^2*c*e^4 - 10*(e*x + d)*a^2*c*d*e^4 + 3*a^2*c*d^2*e^4 + a^3*e^6)
/((e*x + d)^(5/2)*e^7) + 2/35*(5*(e*x + d)^(7/2)*c^3*e^42 - 42*(e*x + d)^(
5/2)*c^3*d*e^42 + 175*(e*x + d)^(3/2)*c^3*d^2*e^42 - 700*sqrt(e*x + d)*c^3
*d^3*e^42 + 35*(e*x + d)^(3/2)*a*c^2*e^44 - 420*sqrt(e*x + d)*a*c^2*d*e^44
)/e^49
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{(30\,c^3\,d^2+6\,a\,c^2\,e^2)\,(d+e\,x)^{3/2}}{3\,e^7}+\frac{2\,c^3\,(d+e\,x)^{7/2}}{7\,e^7} - \frac{(d+e\,x)^2\,(6\,a^2\,c\,e^4+36\,a\,c^2\,d^2\,e^2+30\,c^3\,d^4)-(d+e\,x)\,(4\,a^2\,c\,d\,e^4+8\,a\,c^2\,d^3\,e^2+4\,c^3\,d^5)+\frac{2\,a^3\,e^6}{5}+\frac{2\,c^3}{5}}{e^7\,(d+e\,x)^{5/2}} - \frac{(40\,c^3\,d^3+24\,a\,c^2\,d\,e^2)\,\sqrt{d+e\,x}}{e^7}-\frac{12\,c^3\,d\,(d+e\,x)^{5/2}}{5\,e^7}$$

input
```
int((a + c*x^2)^3/(d + e*x)^(7/2),x)
```

output
```
((30*c^3*d^2 + 6*a*c^2*e^2)*(d + e*x)^(3/2))/(3*e^7) + (2*c^3*(d + e*x)^(7
/2))/(7*e^7) - ((d + e*x)^2*(30*c^3*d^4 + 6*a^2*c*e^4 + 36*a*c^2*d^2*e^2)
- (d + e*x)*(4*c^3*d^5 + 8*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4) + (2*a^3*e^6)/5
+ (2*c^3*d^6)/5 + (6*a*c^2*d^4*e^2)/5 + (6*a^2*c*d^2*e^4)/5)/(e^7*(d + e*x
)^(5/2)) - ((40*c^3*d^3 + 24*a*c^2*d*e^2)*(d + e*x)^(1/2))/e^7 - (12*c^3*d
*(d + e*x)^(5/2))/(5*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{\frac{2}{7}c^3e^6x^6-\frac{24}{35}c^3d\,e^5x^5+2a\,c^2e^6x^4+\frac{16}{7}c^3d^2e^4x^4-16a\,c^2d\,e^5x^3-\frac{128}{7}c^3d^3e^3x^3-6a^2c\,e^6}{}$$

input
```
int((c*x^2+a)^3/(e*x+d)^(7/2),x)
```

output
```
(2*( - 7*a**3*e**6 - 56*a**2*c*d**2*e**4 - 140*a**2*c*d*e**5*x - 105*a**2*
c*e**6*x**2 - 896*a*c**2*d**4*e**2 - 2240*a*c**2*d**3*e**3*x - 1680*a*c**2
*d**2*e**4*x**2 - 280*a*c**2*d*e**5*x**3 + 35*a*c**2*e**6*x**4 - 1024*c**3
*d**6 - 2560*c**3*d**5*e*x - 1920*c**3*d**4*e**2*x**2 - 320*c**3*d**3*e**3
*x**3 + 40*c**3*d**2*e**4*x**4 - 12*c**3*d*e**5*x**5 + 5*c**3*e**6*x**6))/
(35*sqrt(d + e*x)*e**7*(d**2 + 2*d*e*x + e**2*x**2))
```

3.177 $\int \frac{(d+ex)^{5/2}}{a+cx^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 497

$$\int \frac{(d+ex)^{5/2}}{a+cx^2} dx = \frac{4de\sqrt{d+ex}}{c} + \frac{2e(d+ex)^{3/2}}{3c} - \frac{e\left(3cd^2-ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{7/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\left(3cd^2-ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{7/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{e\left(3cd^2-ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}c^{7/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
4*d*e*(e*x+d)^(1/2)/c+2/3*e*(e*x+d)^(3/2)/c-1/2*e*(3*c*d^2-a*e^2-2*c^(1/2)
*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1
/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)
/c^(7/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/2*e*(3*c*d^2-a*e^2-2*c^(
1/2)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+
2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(
1/2)/c^(7/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-1/2*e*(3*c*d^2-a*e^2+2
*c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+
c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2
^(1/2)/c^(7/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx = \frac{2\sqrt{c}e\sqrt{d+ex}(7d+ex)+\frac{3i(\sqrt{c}d+i\sqrt{a}e)^3\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{a}\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}-\frac{3i(\sqrt{c}d-i\sqrt{a}e)^3\arctan\left(\frac{\sqrt{-}}{}\right.}{\sqrt{a}\sqrt{-cd+i\sqrt{}}}}{3c^{3/2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a + c*x^2),x]
```

output
```
(2*Sqrt[c]*e*Sqrt[d + e*x]*(7*d + e*x) + ((3*I)*(Sqrt[c]*d + I*Sqrt[a]*e)^
3*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I
*Sqrt[a]*e)])/(Sqrt[a]*Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]) - ((3*I)*(Sqrt[
c]*d - I*Sqrt[a]*e)^3*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d +
e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/(Sqrt[a]*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]
*e]))/(3*c^(3/2))
```

Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.46, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {481, 653, 27, 654, 25, 27, 1483, 27, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{a+cx^2} \, dx$$

$$\Big\downarrow \text{481}$$

$$\frac{\int \frac{\sqrt{d+ex}(cd^2+2cexd-ae^2)}{cx^2+a}dx}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{653}$$

$$\frac{\frac{\int \frac{c\left(d\left(cd^2-3ae^2\right)+e\left(3cd^2-ae^2\right)x\right)}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{c}+4de\sqrt{d+ex}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d(cd^2-3ae^2)+e(3cd^2-ae^2)x}{\sqrt{d+ex}(cx^2+a)}dx+4de\sqrt{d+ex}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{654}$$

$$\frac{2\int -\frac{e(2d(cd^2+ae^2)-(3cd^2-ae^2)(d+ex))}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}+4de\sqrt{d+ex}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{25}$$

$$\frac{4de\sqrt{d+ex}-2\int \frac{e(2d(cd^2+ae^2)-(3cd^2-ae^2)(d+ex))}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{4de\sqrt{d+ex}-2e\int \frac{2d(cd^2+ae^2)-(3cd^2-ae^2)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{1483}$$

$$\frac{4de\sqrt{d+ex}-2e\left(\frac{\int \frac{\sqrt{cd^2+ae^2}\left(2\sqrt{2}d\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2\sqrt{cd^2+ae^2}d+\frac{3cd^2-ae^2}{\sqrt{c}}\right)\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{cd^2+ae^2}\left(2\sqrt{2}\sqrt{cd\ldots}\right.}{\sqrt[4]{c}\ldots}}{\ldots}\right.}{c}$$

$$\frac{2e(d+ex)^{3/2}}{3c}$$

$\downarrow$ 27

$$\frac{4de\sqrt{d+ex}-2e\left(\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}d\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2\ldots}}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae\ldots}}{\sqrt{c}}\ldots\right.}}{2\sqrt{2}\ldots}\right.}{c}$$

$$\frac{2e(d+ex)^{3/2}}{3c}$$

$\downarrow$ 27

$$\frac{4de\sqrt{d+ex}-2e\left(\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}d\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2\ldots}}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\ldots}}{2\sqrt{2}c\ldots}\right.}{c}$$

$$\frac{2e(d+ex)^{3/2}}{3c}$$

$\downarrow$ 1142

$$\frac{2e(d+ex)^{3/2}}{3c}+$$

$$4de\sqrt{d+ex}-2e\left(\frac{-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{1}{2}\left(3cd^2+2\sqrt{c}\sqrt{\ldots}\right.}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\ldots\right.$$

$\downarrow$ 25

$$\frac{2e(d+ex)^{3/2}}{3c}+$$

$$4de\sqrt{d+ex}-2e\left(\frac{\frac{1}{2}\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int\frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3c\right.}{}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 27

$$\frac{2e(d+ex)^{3/2}}{3c}+$$

$$4de\sqrt{d+ex}-2e\left(\frac{\frac{\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+\right.}{}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 1083

$$\frac{2e(d+ex)^{3/2}}{3c}+$$

$$4de\sqrt{d+ex}-2e\left(\frac{\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int\frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt[4]{c}}+\frac{\left(3cd^2+2\right.}{}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 219

$$\frac{2e(d+ex)^{3/2}}{3c}+$$

$$4de\sqrt{d+ex}-2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}+\frac{\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d\right.}{}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 1103

$$4de\sqrt{d+ex}-2e\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\left(-2\sqrt{c}d\sqrt{ae^2+cd^2}-ae^2+3cd^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\left(2\sqrt{c}d\sqrt{ae^2+cd^2}\right.}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\frac{2e(d+ex)^{3/2}}{3c}$$

input
```
Int[(d + e*x)^(5/2)/(a + c*x^2),x]
```

output
```
(2*e*(d + e*x)^(3/2))/(3*c) + (4*d*e*Sqrt[d + e*x] - 2*e*(((Sqrt[Sqrt[c]*d
 + Sqrt[c*d^2 + a*e^2]]*(3*c*d^2 - a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]
)*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1
/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])
/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]] - ((3*c*d^2 - a*e^2 + 2*Sqrt[c]*d*S
qrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2
]*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + ((Sqrt[Sqrt[c]*d + Sqrt
[c*d^2 + a*e^2]]*(3*c*d^2 - a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTa
nh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*S
qrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt
[c]*d - Sqrt[c*d^2 + a*e^2]] + ((3*c*d^2 - a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2
+ a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[
c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*c^(3/4)*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.49

method	result
pseudoelliptic	$-\frac{\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd}\,\left(\left(a\,e^{2}-2\sqrt{c}\,d\sqrt{a\,e^{2}+c\,d^{2}}-3c\,d^{2}\right)\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-ad\,e^{2}c+3c^{2}d^{3}+2\sqrt{a\,e^{2}+c\,d^{2}}\,c^{\frac{3}{2}}\,d^{2}\right)\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{}}}{4}$
risch	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^(5/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/c^(5/2)*(-1/4*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*((a*e^2-2*c^(1/2)*
d*(a*e^2+c*d^2)^(1/2)-3*c*d^2)*((a*e^2+c*d^2)*c)^(1/2)-a*d*e^2*c+3*c^2*d^3
+2*(a*e^2+c*d^2)^(1/2)*c^(3/2)*d^2)*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e
^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*ln(c^(1/2)*(e*x+d)-(e*x+d)^(1/2)*(2*((a*e^
2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))+1/4*(2*((a*e^2+c*d^2)*
c)^(1/2)+2*c*d)^(1/2)*((a*e^2-2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2)-3*c*d^2)*((a
*e^2+c*d^2)*c)^(1/2)-a*d*e^2*c+3*c^2*d^3+2*(a*e^2+c*d^2)^(1/2)*c^(3/2)*d^2
)*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*ln
(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*
e^2+c*d^2)^(1/2))+e^2*(14/3*c^(3/2)*(e*x+d)^(1/2)*(d+1/7*e*x)*(4*(a*e^2+c*
d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)+(a*c*e^2-3*d^2*c
^2+2*(a*e^2+c*d^2)^(1/2)*c^(3/2)*d)*(arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*(
(a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*
e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e
^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+
c*d^2)*c)^(1/2)-2*c*d)^(1/2))))*a)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^
2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)/a/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1641 vs. $2(400) = 800$.

Time = 0.13 (sec) , antiderivative size = 1641, normalized size of antiderivative = 3.30

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*c*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt(-(25*
c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 +
 a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 14*a^2*c^2*d^4*e^5 - 8*a^
3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) + (10*a*c^4*d^5*e^2 - 20*a^2*c^3*d^3*
e^4 + 2*a^3*c^2*d*e^6 + (a*c^6*d^2 - a^2*c^5*e^2)*sqrt(-(25*c^4*d^8*e^2 -
100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*
c^7)))*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*e^4 + a*c^3*sqrt(-(25*c^4
*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^
4*e^10)/(a*c^7)))/(a*c^3))) - 3*c*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*
d*e^4 + a*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*
e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))*log((5*c^4*d^8*e - 1
4*a^2*c^2*d^4*e^5 - 8*a^3*c*d^2*e^7 + a^4*e^9)*sqrt(e*x + d) - (10*a*c^4*d
^5*e^2 - 20*a^2*c^3*d^3*e^4 + 2*a^3*c^2*d*e^6 + (a*c^6*d^2 - a^2*c^5*e^2)*
sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c
*d^2*e^8 + a^4*e^10)/(a*c^7)))*sqrt(-(c^2*d^5 - 10*a*c*d^3*e^2 + 5*a^2*d*e
^4 + a*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^6*e^4 + 110*a^2*c^2*d^4*e^6
 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^3))) + 3*c*sqrt(-(c^2*d^5 -
 10*a*c*d^3*e^2 + 5*a^2*d*e^4 - a*c^3*sqrt(-(25*c^4*d^8*e^2 - 100*a*c^3*d^
6*e^4 + 110*a^2*c^2*d^4*e^6 - 20*a^3*c*d^2*e^8 + a^4*e^10)/(a*c^7)))/(a*c^
3))*log((5*c^4*d^8*e - 14*a^2*c^2*d^4*e^5 - 8*a^3*c*d^2*e^7 + a^4*e^9)*...
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**(5/2)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx =$$

$$-\frac{\left(\sqrt{-ac}c^4d^4e-3\sqrt{-ac}ac^3d^2e^3+\left(3\sqrt{-ac}acd^2e-\sqrt{-ac}a^2e^3\right)c^2e^2-2\left(ac^3d^3e+a^2c^2de^3\right)|c||e|\right)\arctan\left(\right.}{\left(ac^4d-\sqrt{-ac}ac^3e\right)\sqrt{-c^2d-\sqrt{-ac}ce}|e|}$$

$$+\frac{\left(\sqrt{-ac}c^4d^4e-3\sqrt{-ac}ac^3d^2e^3+\left(3\sqrt{-ac}acd^2e-\sqrt{-ac}a^2e^3\right)c^2e^2+2\left(ac^3d^3e+a^2c^2de^3\right)|c||e|\right)\arctan\left(\right.}{\left(ac^4d+\sqrt{-ac}ac^3e\right)\sqrt{-c^2d+\sqrt{-ac}ce}|e|}$$

$$+\frac{2\left((ex+d)^{\frac{3}{2}}c^2e+6\sqrt{ex+d}c^2de\right)}{3\,c^3}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
-(sqrt(-a*c)*c^4*d^4*e - 3*sqrt(-a*c)*a*c^3*d^2*e^3 + (3*sqrt(-a*c)*a*c*d^
2*e - sqrt(-a*c)*a^2*e^3)*c^2*e^2 - 2*(a*c^3*d^3*e + a^2*c^2*d*e^3)*abs(c)
*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^4*d + sqrt(c^8*d^2 - (c^4*d^2 + a*c
^3*e^2)*c^4))/c^4))/((a*c^4*d - sqrt(-a*c)*a*c^3*e)*sqrt(-c^2*d - sqrt(-a*
c)*c*e)*abs(e)) + (sqrt(-a*c)*c^4*d^4*e - 3*sqrt(-a*c)*a*c^3*d^2*e^3 + (3*
sqrt(-a*c)*a*c*d^2*e - sqrt(-a*c)*a^2*e^3)*c^2*e^2 + 2*(a*c^3*d^3*e + a^2*
c^2*d*e^3)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^4*d - sqrt(c^8*d^2
 - (c^4*d^2 + a*c^3*e^2)*c^4))/c^4))/((a*c^4*d + sqrt(-a*c)*a*c^3*e)*sqrt(
-c^2*d + sqrt(-a*c)*c*e)*abs(e)) + 2/3*((e*x + d)^(3/2)*c^2*e + 6*sqrt(e*x
 + d)*c^2*d*e)/c^3
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 3481, normalized size of antiderivative = 7.00

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(a + c*x^2),x)
```

output
```
(2*e*(d + e*x)^(3/2))/(3*c) - atan((a^3*e^8*(d + e*x)^(1/2)*((e^5*(-a^3*c^
7)^(1/2))/(4*c^7) - d^5/(4*a*c) + (5*d^3*e^2)/(2*c^2) - (5*a*d*e^4)/(4*c^3
) + (5*d^4*e*(-a^3*c^7)^(1/2))/(4*a^2*c^5) - (5*d^2*e^3*(-a^3*c^7)^(1/2))/
(2*a*c^6))^(1/2)*32i)/((16*a^4*e^11)/c^2 + 64*a^2*d^4*e^7 - 80*c^2*d^8*e^3
 - (160*a^3*d^2*e^9)/c + 160*a*c*d^6*e^5 - (160*d^5*e^6*(-a^3*c^7)^(1/2))/
c^3 - (288*a*d^3*e^8*(-a^3*c^7)^(1/2))/c^4 + (32*a^2*d*e^10*(-a^3*c^7)^(1/
2))/c^5 + (160*d^7*e^4*(-a^3*c^7)^(1/2))/(a*c^2)) - (d^5*e^3*(-a^3*c^7)^(1
/2)*(d + e*x)^(1/2)*((e^5*(-a^3*c^7)^(1/2))/(4*c^7) - d^5/(4*a*c) + (5*d^3
*e^2)/(2*c^2) - (5*a*d*e^4)/(4*c^3) + (5*d^4*e*(-a^3*c^7)^(1/2))/(4*a^2*c^
5) - (5*d^2*e^3*(-a^3*c^7)^(1/2))/(2*a*c^6))^(1/2)*160i)/((16*a^5*e^11)/c
- 160*a^4*d^2*e^9 - 80*a*c^3*d^8*e^3 + 64*a^3*c*d^4*e^7 + 160*a^2*c^2*d^6*
e^5 + (160*d^7*e^4*(-a^3*c^7)^(1/2))/c - (160*a*d^5*e^6*(-a^3*c^7)^(1/2))/
c^2 + (32*a^3*d*e^10*(-a^3*c^7)^(1/2))/c^4 - (288*a^2*d^3*e^8*(-a^3*c^7)^(
1/2))/c^3) + (d^3*e^5*(-a^3*c^7)^(1/2)*(d + e*x)^(1/2)*((e^5*(-a^3*c^7)^(1
/2))/(4*c^7) - d^5/(4*a*c) + (5*d^3*e^2)/(2*c^2) - (5*a*d*e^4)/(4*c^3) + (
5*d^4*e*(-a^3*c^7)^(1/2))/(4*a^2*c^5) - (5*d^2*e^3*(-a^3*c^7)^(1/2))/(2*a*
c^6))^(1/2)*320i)/(16*a^4*e^11 - 80*c^4*d^8*e^3 + 160*a*c^3*d^6*e^5 - 160*
a^3*c*d^2*e^9 + 64*a^2*c^2*d^4*e^7 + (160*d^7*e^4*(-a^3*c^7)^(1/2))/a - (1
60*d^5*e^6*(-a^3*c^7)^(1/2))/c - (288*a*d^3*e^8*(-a^3*c^7)^(1/2))/c^2 + (3
2*a^2*d*e^10*(-a^3*c^7)^(1/2))/c^3) - (a*d*e^7*(-a^3*c^7)^(1/2)*(d + e*...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1536, normalized size of antiderivative = 3.09

$$\int \frac{(d+ex)^{5/2}}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+a),x)
```

output
```
(6*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)
*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(
d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e**2 - 6*
sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*at
an((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d +
 e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*d**2 + 18*sq
rt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)
*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqr
t(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*d*e**2 - 6*sqrt(c)*sqrt(sqrt
(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**
2 + c*d**2) - c*d)*sqrt(2)))*c*d**3 - 6*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)
*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d
**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 +
 c*d**2) - c*d)*sqrt(2)))*a*e**2 + 6*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sq
rt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*
d**2) - c*d)*sqrt(2)))*c*d**2 - 18*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d*
*2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2)
 + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*...
```

3.178 $\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 447

$$
\begin{aligned}
\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx &= \frac{2e\sqrt{d+ex}}{c} \\
&- \frac{e\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{5/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&+ \frac{e\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{5/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&- \frac{e\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}c^{5/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}
\end{aligned}
$$

output
```
2*e*(e*x+d)^(1/2)/c-1/2*e*(2*c^(1/2)*d-(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/
2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d
+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/c^(5/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/
2))^(1/2)+1/2*e*(2*c^(1/2)*d-(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^
2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*
d^2)^(1/2))^(1/2))*2^(1/2)/c^(5/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-
1/2*e*(2*c^(1/2)*d+(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d
+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*
x+d)))*2^(1/2)/c^(5/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \frac{2e\sqrt{d+ex}+\frac{i\left(\sqrt{c}d+i\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{a}\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}-\frac{i\left(\sqrt{c}d-i\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{\sqrt{a}\sqrt{-cd+i\sqrt{a}\sqrt{c}e}}}{c}$$

input
```
Integrate[(d + e*x)^(3/2)/(a + c*x^2),x]
```

output
```
(2*e*Sqrt[d + e*x] + (I*(Sqrt[c]*d + I*Sqrt[a]*e)^2*ArcTan[(Sqrt[-(c*d) -
I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/(Sqrt[a]*S
qrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]) - (I*(Sqrt[c]*d - I*Sqrt[a]*e)^2*ArcTan
[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]
*e)])/(Sqrt[a]*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]))/c
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.68, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {481, 654, 25, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx$$

$$\Big\downarrow \; 481$$

$$\frac{\int \frac{cd^2+2cexd-ae^2}{\sqrt{d+ex}(cx^2+a)}dx}{c}+\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \; 654$$

$$\frac{2\int -\frac{e(cd^2-2c(d+ex)d+ae^2)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \; 25$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2\int \frac{e(cd^2-2c(d+ex)d+ae^2)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2e\int \frac{cd^2-2c(d+ex)d+ae^2}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \; 1483$$

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$\frac{2e\left(\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+ae^2\right)+\sqrt[4]{c}\left(cd^2+2\sqrt{c}\sqrt{cd^2+\cdots\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}{\sqrt[4]{c}}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\cdots}}\right.}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$\frac{2e\left(\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+ae^2\right)+\sqrt[4]{c}\left(cd^2+2\sqrt{c}\sqrt{cd^2+\cdots\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{\cdots}}{\sqrt[4]{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\cdots}}\right.}{c}$$

$$\Big\downarrow \; 1142$$

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\left(cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 25

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\left(cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 27

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{\left(cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{\sqrt{}}{d+ex+\sqrt{}}}{}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 1083

$$\frac{2e\sqrt{d+ex}}{c}-2e\left(\frac{\frac{\left(cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\Big\downarrow$ 219

$$\frac{2e\sqrt{d+ex}}{c}-2e\left(\frac{\frac{\left(cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\text{arcta}}{\sqrt{\sqrt{c}d-\sqrt{cd^2+}}}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\Big\downarrow$ 1103

$$\frac{2e\sqrt{d+ex}}{c}-2e\left(\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\left(-2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\sqrt[4]{c}\left(2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right.}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

input

```
Int[(d + e*x)^(3/2)/(a + c*x^2),x]
```

output
```
(2*e*Sqrt[d + e*x])/c - (2*e*((-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]]*(c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*(
-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*
x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sq
rt[c*d^2 + a*e^2]]) - (c^(1/4)*(c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a
*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d
^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqr
t[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-((c^(1/4)*Sqrt
[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2
+ a*e^2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
)/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2
]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (c^(1/4)*(c*d^2 + a*e^2 + 2
*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/
2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$-\frac{-\frac{\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd}\,\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}\,\left(\left(\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}+2cd\right)\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2d^{2}c^{2}-\sqrt{a\,e^{2}+c\,\cdots}\right.}{4}}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((e*x+d)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)
^(1/2)*(-1/4*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(4*(a*e^2+c*d^2)^(1/2
)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(((a*e^2+c*d^2)^(1/2)*c^(
1/2)+2*c*d)*((a*e^2+c*d^2)*c)^(1/2)-2*d^2*c^2-(a*e^2+c*d^2)^(1/2)*c^(3/2)*
d)*ln(c^(1/2)*(e*x+d)-(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2
)+(a*e^2+c*d^2)^(1/2))+1/4*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(4*(a*e
^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(((a*e^2+c*
d^2)^(1/2)*c^(1/2)+2*c*d)*((a*e^2+c*d^2)*c)^(1/2)-2*d^2*c^2-(a*e^2+c*d^2)^
(1/2)*c^(3/2)*d)*ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/
2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))+e^2*(-2*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2
)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*c^(3/2)*(e*x+d)^(1/2)+(-2*c^2*d+(
a*e^2+c*d^2)^(1/2)*c^(3/2))*(arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*
d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2
)*c)^(1/2)-2*c*d)^(1/2))-arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2
)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c
)^(1/2)-2*c*d)^(1/2))))*a)/e/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(354) = 708$.

Time = 0.10 (sec) , antiderivative size = 998, normalized size of antiderivative = 2.23

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(c*sqrt(-(c*d^3 - 3*a*d*e^2 + a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e
^4 + a^2*e^6)/(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e
^5)*sqrt(e*x + d) + (3*a*c^2*d^2*e^2 - a^2*c*e^4 + a*c^4*d*sqrt(-(9*c^2*d^
4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt(-(c*d^3 - 3*a*d*e^2 + a*c^
2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) - c*
sqrt(-(c*d^3 - 3*a*d*e^2 + a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^
2*e^6)/(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*sqr
t(e*x + d) - (3*a*c^2*d^2*e^2 - a^2*c*e^4 + a*c^4*d*sqrt(-(9*c^2*d^4*e^2 -
 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt(-(c*d^3 - 3*a*d*e^2 + a*c^2*sqrt(
-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) + c*sqrt(-(
c*d^3 - 3*a*d*e^2 - a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/
(a*c^5)))/(a*c^2))*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x +
 d) + (3*a*c^2*d^2*e^2 - a^2*c*e^4 - a*c^4*d*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*
d^2*e^4 + a^2*e^6)/(a*c^5)))*sqrt(-(c*d^3 - 3*a*d*e^2 - a*c^2*sqrt(-(9*c^2
*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) - c*sqrt(-(c*d^3 -
 3*a*d*e^2 - a*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)
))/(a*c^2))*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*sqrt(e*x + d) - (
3*a*c^2*d^2*e^2 - a^2*c*e^4 - a*c^4*d*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4
 + a^2*e^6)/(a*c^5)))*sqrt(-(c*d^3 - 3*a*d*e^2 - a*c^2*sqrt(-(9*c^2*d^4*e^
2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^5)))/(a*c^2))) + 4*sqrt(e*x + d)*e)/c
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**(3/2)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \frac{2\sqrt{ex+d}e}{c} - \frac{\left(\sqrt{-ac}c^3d^3e + \sqrt{-ac}ac^2de^3 - (ac^2d^2e + a^2ce^3)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d+\sqrt{c^4d^2-(c^2d^2+ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d - \sqrt{-ac}ac^2e\right)\sqrt{-c^2d - \sqrt{-ac}ce}|e|} + \frac{\left(\sqrt{-ac}c^3d^3e + \sqrt{-ac}ac^2de^3 + (ac^2d^2e + a^2ce^3)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d-\sqrt{c^4d^2-(c^2d^2+ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d + \sqrt{-ac}ac^2e\right)\sqrt{-c^2d + \sqrt{-ac}ce}|e|}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*e/c - (sqrt(-a*c)*c^3*d^3*e + sqrt(-a*c)*a*c^2*d*e^3 - (a*
c^2*d^2*e + a^2*c*e^3)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d +
sqrt(c^4*d^2 - (c^2*d^2 + a*c*e^2)*c^2))/c^2))/((a*c^3*d - sqrt(-a*c)*a*c^
2*e)*sqrt(-c^2*d - sqrt(-a*c)*c*e)*abs(e)) + (sqrt(-a*c)*c^3*d^3*e + sqrt(
-a*c)*a*c^2*d*e^3 + (a*c^2*d^2*e + a^2*c*e^3)*abs(c)*abs(e))*arctan(sqrt(e
*x + d)/sqrt(-(c^2*d - sqrt(c^4*d^2 - (c^2*d^2 + a*c*e^2)*c^2))/c^2))/((a*
c^3*d + sqrt(-a*c)*a*c^2*e)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 1625, normalized size of antiderivative = 3.64

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(a + c*x^2),x)
```

output
```
2*atanh((32*a^2*c*e^6*(d + e*x)^(1/2)*((3*d*e^2)/(4*c^2) - d^3/(4*a*c) + (
e^3*(-a^3*c^5)^(1/2))/(4*a*c^5) - (3*d^2*e*(-a^3*c^5)^(1/2))/(4*a^2*c^4))^
(1/2))/(48*c^2*d^5*e^3 - 16*a^2*d*e^7 - (16*a*e^8*(-a^3*c^5)^(1/2))/c^3 +
32*a*c*d^3*e^5 + (32*d^2*e^6*(-a^3*c^5)^(1/2))/c^2 + (48*d^4*e^4*(-a^3*c^5
)^(1/2))/(a*c)) + (32*d*e^5*(-a^3*c^5)^(1/2)*(d + e*x)^(1/2)*((3*d*e^2)/(4
*c^2) - d^3/(4*a*c) + (e^3*(-a^3*c^5)^(1/2))/(4*a*c^5) - (3*d^2*e*(-a^3*c^
5)^(1/2))/(4*a^2*c^4))^(1/2))/(48*c^3*d^5*e^3 + 32*a*c^2*d^3*e^5 - (16*a*e
^8*(-a^3*c^5)^(1/2))/c^2 - 16*a^2*c*d*e^7 + (48*d^4*e^4*(-a^3*c^5)^(1/2))/
a + (32*d^2*e^6*(-a^3*c^5)^(1/2))/c) - (96*d^3*e^3*(-a^3*c^5)^(1/2)*(d + e
*x)^(1/2)*((3*d*e^2)/(4*c^2) - d^3/(4*a*c) + (e^3*(-a^3*c^5)^(1/2))/(4*a*c
^5) - (3*d^2*e*(-a^3*c^5)^(1/2))/(4*a^2*c^4))^(1/2))/(48*a*c^2*d^5*e^3 - 1
6*a^3*d*e^7 + 32*a^2*c*d^3*e^5 - (16*a^2*e^8*(-a^3*c^5)^(1/2))/c^3 + (48*d
^4*e^4*(-a^3*c^5)^(1/2))/c + (32*a*d^2*e^6*(-a^3*c^5)^(1/2))/c^2) - (96*a*
c^2*d^2*e^4*(d + e*x)^(1/2)*((3*d*e^2)/(4*c^2) - d^3/(4*a*c) + (e^3*(-a^3*
c^5)^(1/2))/(4*a*c^5) - (3*d^2*e*(-a^3*c^5)^(1/2))/(4*a^2*c^4))^(1/2))/(48
*c^2*d^5*e^3 - 16*a^2*d*e^7 - (16*a*e^8*(-a^3*c^5)^(1/2))/c^3 + 32*a*c*d^3
*e^5 + (32*d^2*e^6*(-a^3*c^5)^(1/2))/c^2 + (48*d^4*e^4*(-a^3*c^5)^(1/2))/(
a*c)))*(-(a*c^4*d^3 - a*e^3*(-a^3*c^5)^(1/2) - 3*a^2*c^3*d*e^2 + 3*c*d^2*e
*(-a^3*c^5)^(1/2))/(4*a^2*c^5))^(1/2) - 2*atanh((32*a^2*c*e^6*(d + e*x)^(1
/2)*((3*d*e^2)/(4*c^2) - d^3/(4*a*c) - (e^3*(-a^3*c^5)^(1/2))/(4*a*c^5)...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1113, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^{3/2}}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+a),x)
```

output
```
( - 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*d + 2*
sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(
c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(s
qrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e**2 - 2*sqrt(c)*sqrt(sqrt
(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**
2 + c*d**2) - c*d)*sqrt(2)))*c*d**2 + 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)
*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d
**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 +
 c*d**2) - c*d)*sqrt(2)))*c*d - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**
2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2)
+ 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)))*a*e**2 + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)
*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(
d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*d**2 + sq
rt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2)*log(
 - sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + sqrt(
a*e**2 + c*d**2) + sqrt(c)*d + sqrt(c)*e*x)*c*d - sqrt(a*e**2 + c*d**2)...
```

3.179 $\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 356

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = -\frac{e\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{3/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{3/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{e\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/2*e*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+
d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/c^(3/4)/(-c^(1/2
)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/2*e*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2
))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(
1/2))*2^(1/2)/c^(3/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-1/2*e*arctanh
(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e
^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/c^(3/4)/(c^(1/2)*d+(a*e^2+c*d^2)
^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = \frac{-i\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)+i\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{\sqrt{a}c}$$

input
```
Integrate[Sqrt[d + e*x]/(a + c*x^2),x]
```

output
```
((-I)*Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*S
qrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)] + I*Sqrt[-(c*d) + I*Sq
rt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])
/(Sqrt[c]*d - I*Sqrt[a]*e)])/(Sqrt[a]*c)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {483, 1449, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx$$

$$\Big\downarrow \text{483}$$

$$2e\int \frac{d+ex}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1449}$$

$$2e\left(\frac{\int \frac{\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\frac{\int \frac{\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)$$

$\big\downarrow$ 1142

$$2e\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+e\cdots}}{\sqrt[4]{c}}\cdots\right)}\cdots}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.$$

$\big\downarrow$ 25

$$2e\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}\cdots}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.$$

$\big\downarrow$ 27

$$2e\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.$$

$\big\downarrow$ 1083

$$2e\left(\frac{-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int\frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt[4]{c}}-\frac{\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+e}}{\sqrt[4]{c}}}}{\sqrt{2}\sqrt[4]{c}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$\downarrow$ 219

$$2e\left(\frac{-\frac{\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\right.$$

$\downarrow$ 1103

$$2e\left(\frac{\frac{1}{2}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)-\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\right.}{}\right.}{\sqrt{\sqrt{c}d-\sqrt{ae}}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

input

```
Int[Sqrt[d + e*x]/(a + c*x^2),x]
```

output
```
2*e*((-((Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*ArcTanh[(c^(1/4)*(-((Sqrt[2
]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqr
t[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2
+ a*e^2]]) + Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sq
rt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)]/2)/(2*Sqrt[2]*c^(3/4
)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) - ((Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])
/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]
])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]] + Log[Sqrt[c*d^2 + a*e^2] + Sqr
t[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]
*(d + e*x)]/2)/(2*Sqrt[2]*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$\frac{\left(\sqrt{2\sqrt{(a\,e^2+c\,d^2)c}+2cd}\,\sqrt{4\sqrt{a\,e^2+c\,d^2}\,\sqrt{c}-2\sqrt{(a\,e^2+c\,d^2)c}-2cd}\,\left(\ln\left((-ex-d)\sqrt{c}+\sqrt{ex+d}\,\sqrt{2\sqrt{(a\,e^2+c\,d^2)c}+2c\cdots\right.\right.\right.}{\cdots}$
derivativedivides	$2e\left(\frac{\sqrt{2\sqrt{ac\,e^2+d^2c^2}+2cd}\,\left(-cd+\sqrt{ac\,e^2+d^2c^2}\right)\left(\frac{\ln\left(\sqrt{c}\,(ex+d)-\sqrt{ex+d}\,\sqrt{2\sqrt{\left(a\,e^2+c\,d^2\right)c}+2cd}+\sqrt{a\,e^2+c\,d^2}\right)}{2\sqrt{c}}+\frac{\sqrt{2\sqrt{(\cdots}}}{\cdots}\right.}{4ac\,e^2\cdots}\right.$
default	$2e\left(\frac{\sqrt{2\sqrt{ac\,e^2+d^2c^2}+2cd}\,\left(-cd+\sqrt{ac\,e^2+d^2c^2}\right)\left(\frac{\ln\left(\sqrt{c}\,(ex+d)-\sqrt{ex+d}\,\sqrt{2\sqrt{\left(a\,e^2+c\,d^2\right)c}+2cd}+\sqrt{a\,e^2+c\,d^2}\right)}{2\sqrt{c}}+\frac{\sqrt{2\sqrt{(\cdots}}}{\cdots}\right.}{4ac\,e^2\cdots}\right.$

input
```
int((e*x+d)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)/
c^(3/2)*((2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*c^
(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(ln((-e*x-d)*c^(1/2)+(e*x+d)^
(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))-ln(c^(1
/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c
*d^2)^(1/2)))-4*(c*d+((a*e^2+c*d^2)*c)^(1/2))*(arctan((-2*c^(1/2)*(e*x+d)^
(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1
/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((2*c^(1/2)*(e*x+d)^(1/2
)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-
2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))))*(-c*d+((a*e^2+c*d^2)*c)^(1/2))/e
/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = -\frac{1}{2}\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\log\left(ac^2\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\sqrt{-\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right) + \frac{1}{2}\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\log\left(-ac^2\sqrt{-\frac{ac\sqrt{-\frac{e^2}{ac^3}}+d}{ac}}\sqrt{-\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right) + \frac{1}{2}\sqrt{\frac{ac\sqrt{-\frac{e^2}{ac^3}}-d}{ac}}\log\left(ac^2\sqrt{\frac{ac\sqrt{-\frac{e^2}{ac^3}}-d}{ac}}\sqrt{-\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right) - \frac{1}{2}\sqrt{\frac{ac\sqrt{-\frac{e^2}{ac^3}}-d}{ac}}\log\left(-ac^2\sqrt{\frac{ac\sqrt{-\frac{e^2}{ac^3}}-d}{ac}}\sqrt{-\frac{e^2}{ac^3}}+\sqrt{ex+d}e\right)$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-(a*c*sqrt(-e^2/(a*c^3)) + d)/(a*c))*log(a*c^2*sqrt(-(a*c*sqrt(-
e^2/(a*c^3)) + d)/(a*c))*sqrt(-e^2/(a*c^3)) + sqrt(e*x + d)*e) + 1/2*sqrt(
-(a*c*sqrt(-e^2/(a*c^3)) + d)/(a*c))*log(-a*c^2*sqrt(-(a*c*sqrt(-e^2/(a*c^
3)) + d)/(a*c))*sqrt(-e^2/(a*c^3)) + sqrt(e*x + d)*e) + 1/2*sqrt((a*c*sqrt
(-e^2/(a*c^3)) - d)/(a*c))*log(a*c^2*sqrt((a*c*sqrt(-e^2/(a*c^3)) - d)/(a*
c))*sqrt(-e^2/(a*c^3)) + sqrt(e*x + d)*e) - 1/2*sqrt((a*c*sqrt(-e^2/(a*c^3
)) - d)/(a*c))*log(-a*c^2*sqrt((a*c*sqrt(-e^2/(a*c^3)) - d)/(a*c))*sqrt(-e
^2/(a*c^3)) + sqrt(e*x + d)*e)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = \int \frac{\sqrt{d+ex}}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = \int \frac{\sqrt{ex+d}}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = \frac{(cd^2e|c|+ae^3|c|)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd+\sqrt{c^2d^2-(cd^2+ae^2)c}}{c}}}\right)}{\sqrt{-c^2d-\sqrt{-ac}ce}\,(ace+\sqrt{-ac}cd)|e|} + \frac{(cd^2e|c|+ae^3|c|)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd-\sqrt{c^2d^2-(cd^2+ae^2)c}}{c}}}\right)}{\sqrt{-c^2d+\sqrt{-ac}ce}\,(ace-\sqrt{-ac}cd)|e|}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
(c*d^2*e*abs(c) + a*e^3*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c*d + sqrt(c^2
*d^2 - (c*d^2 + a*e^2)*c))/c))/(sqrt(-c^2*d - sqrt(-a*c)*c*e)*(a*c*e + sqr
t(-a*c)*c*d)*abs(e)) + (c*d^2*e*abs(c) + a*e^3*abs(c))*arctan(sqrt(e*x + d
)/sqrt(-(c*d - sqrt(c^2*d^2 - (c*d^2 + a*e^2)*c))/c))/(sqrt(-c^2*d + sqrt(
-a*c)*c*e)*(a*c*e - sqrt(-a*c)*c*d)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\left(16\,a\,c^2\,e^4-16\,c^3\,d^2\,e^2\right)\sqrt{d+e\,x}+\frac{16\,c\,d\,e^2\left(e\,\sqrt{-a^3\,c^3}+a\,c^2\,d\right)\sqrt{d+e\,x}}{a}\right)\sqrt{-\frac{e\,\sqrt{-a^3\,c^3}+a\,c^2\,d}{4\,a^2\,c^3}}}{16\,c^2\,d^2\,e^3+16\,a\,c\,e^5}\right)\sqrt{\cdots}$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\left(16\,a\,c^2\,e^4-16\,c^3\,d^2\,e^2\right)\sqrt{d+e\,x}-\frac{16\,c\,d\,e^2\left(e\,\sqrt{-a^3\,c^3}-a\,c^2\,d\right)\sqrt{d+e\,x}}{a}\right)\sqrt{\frac{e\,\sqrt{-a^3\,c^3}-a\,c^2\,d}{4\,a^2\,c^3}}}{16\,c^2\,d^2\,e^3+16\,a\,c\,e^5}\right)\sqrt{\frac{e\cdots}{\cdots}}$$

input
```
int((d + e*x)^(1/2)/(a + c*x^2),x)
```

output
```
- 2*atanh((2*((16*a*c^2*e^4 - 16*c^3*d^2*e^2)*(d + e*x)^(1/2) + (16*c*d*e^
2*(e*(-a^3*c^3)^(1/2) + a*c^2*d)*(d + e*x)^(1/2))/a)*(-(e*(-a^3*c^3)^(1/2)
 + a*c^2*d)/(4*a^2*c^3))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5))*(-(e*(-a^3*
c^3)^(1/2) + a*c^2*d)/(4*a^2*c^3))^(1/2) - 2*atanh((2*((16*a*c^2*e^4 - 16*
c^3*d^2*e^2)*(d + e*x)^(1/2) - (16*c*d*e^2*(e*(-a^3*c^3)^(1/2) - a*c^2*d)*
(d + e*x)^(1/2))/a)*((e*(-a^3*c^3)^(1/2) - a*c^2*d)/(4*a^2*c^3))^(1/2))/(1
6*c^2*d^2*e^3 + 16*a*c*e^5))*((e*(-a^3*c^3)^(1/2) - a*c^2*d)/(4*a^2*c^3))^
(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.01

$$\int \frac{\sqrt{d+ex}}{a+cx^2}\,dx = \frac{\sqrt{2}\left(-2\sqrt{a\,e^2+c\,d^2}\,\sqrt{\sqrt{c}\,\sqrt{a\,e^2+c\,d^2}-cd}\,atan\left(\frac{\sqrt{\sqrt{c}\,\sqrt{a\,e^2+c\,d^2}+cd}\,\sqrt{2}-2\sqrt{c}\,\sqrt{ex+d}}{\sqrt{\sqrt{c}\,\sqrt{a\,e^2+c\,d^2}-cd}\,\sqrt{2}}\right)-2\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a\,e^2+ \cdots}}\right.}{}$$

input
```
int((e*x+d)^(1/2)/(c*x^2+a),x)
```

output
```
(sqrt(2)*( - 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) -
c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2))) - 2*sqr
t(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt
(a*e**2 + c*d**2) - c*d)*sqrt(2)))*d + 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c
)*sqrt(a*e**2 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) +
c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) - c*d)*sqrt(2))) + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*a
tan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d
+ e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*d + sqrt(a*e*
*2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*log( - sqrt(d + e*x
)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + sqrt(a*e**2 + c*d**2
) + sqrt(c)*d + sqrt(c)*e*x) - sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) + c*d)*log(sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
+ c*d)*sqrt(2) + sqrt(a*e**2 + c*d**2) + sqrt(c)*d + sqrt(c)*e*x) - sqrt(c
)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*log( - sqrt(d + e*x)*sqrt(sqrt
(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + sqrt(a*e**2 + c*d**2) + sqrt(c)
*d + sqrt(c)*e*x)*d + sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*lo
g(sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + sqr...
```

3.180 $\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 400

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = -\frac{e\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/2*e*arctan((((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+
d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/c^(1/4)/(a*e^2+c
*d^2)^(1/2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/2*e*arctan(((c^(1/2)*
d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a
*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/c^(1/4)/(a*e^2+c*d^2)^(1/2)/(-c^(1/2)*d+
(a*e^2+c*d^2)^(1/2))^(1/2)+1/2*e*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2
+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*
2^(1/2)/c^(1/4)/(a*e^2+c*d^2)^(1/2)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{i\left(\frac{\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}-\frac{\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
(I*(ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d +
 I*Sqrt[a]*e)]/Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e] - ArcTan[(Sqrt[-(c*d) +
I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)]/Sqrt[-(c*d)
 + I*Sqrt[a]*Sqrt[c]*e]))/Sqrt[a]
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {484, 1407, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 484}$$

$$2e\int \frac{1}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{ 1407}$$

$$2e\left(\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt[4]{c}\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d\cdots}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.$$

$\downarrow$ 27

$$2e\left(\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)$$

$\downarrow$ 1142

$$2e\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{\cdots}}{\sqrt[4]{c}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.$$

$\downarrow$ 25

$$2e\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\int \frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+\cdots}}{\sqrt[4]{c}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.$$

$\downarrow$ 27

$$2e\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\Big\downarrow \text{ 1083}$$

$$2e\left(\frac{\frac{\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\int \frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}\right.}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\Big\downarrow \text{ 219}$$

$$2e\left(\frac{\frac{\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt[4]{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\right.$$

$$\Big\downarrow \text{ 1103}$$

$$2e\left(\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\right.}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

input
```
Int[1/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
2*e*((-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*ArcTanh[(c^(1/4)*(-
((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x
]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqr
t[c*d^2 + a*e^2]]) - (c^(1/4)*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sq
rt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)
/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^
2]]) + (-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*ArcTanh[(c^(1/4)*
((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]
))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt
[c*d^2 + a*e^2]]) + (c^(1/4)*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/
(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2
]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd}\,\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}\,\left(\ln\left(\left(-ex-d\right)\sqrt{c}+\sqrt{ex+d}\,\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd-\sqrt{a\,e^{2}+c \cdots}}\cdots\right)\cdots\right)}{4}}{\cdots}$
derivativedivides	$2e\left(\frac{\frac{\left(-\sqrt{2\sqrt{ac\,e^{2}+d^{2}c^{2}}+2cd}\,cd+\sqrt{ac\,e^{2}+d^{2}c^{2}}\,\sqrt{2\sqrt{ac\,e^{2}+d^{2}c^{2}}+2cd}\right)\ln\left(\sqrt{c}\,(ex+d)+\sqrt{ex+d}\,\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd+\sqrt{a\,e^{2} \cdots}}\cdots\right)}{2\sqrt{c}}\cdots}{\cdots}\cdots\right.$
default	$2e\left(\frac{\frac{\left(-\sqrt{2\sqrt{ac\,e^{2}+d^{2}c^{2}}+2cd}\,cd+\sqrt{ac\,e^{2}+d^{2}c^{2}}\,\sqrt{2\sqrt{ac\,e^{2}+d^{2}c^{2}}+2cd}\right)\ln\left(\sqrt{c}\,(ex+d)+\sqrt{ex+d}\,\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd+\sqrt{a\,e^{2} \cdots}}\cdots\right)}{2\sqrt{c}}\cdots}{\cdots}\cdots\right.$

input

```
int(1/(e*x+d)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output

```
1/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)/(a
*e^2+c*d^2)^(1/2)*(-1/4*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(4*(a*e^2+
c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(ln((-e*x-d)*c
^(1/2)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)
^(1/2))-ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)
^(1/2)+(a*e^2+c*d^2)^(1/2)))*(-c*d+((a*e^2+c*d^2)*c)^(1/2))+e^2*(arctan((2
*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+
c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((-2*c^
(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d
^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)))*c*a)/c/a/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(316) = 632$.

Time = 0.09 (sec) , antiderivative size = 941, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 +
a^3*c*e^4)) + d)/(a*c*d^2 + a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 + (a*c^
2*d^3 + a^2*c*d*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
))*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)) + d)/(a*c*d^2 + a^2*e^2))) - 1/2*sqrt(-((a*c*d^2 + a^2*e^2)*sqr
t(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 + a^2*e^
2))*log(sqrt(e*x + d)*e - (a*e^2 + (a*c^2*d^3 + a^2*c*d*e^2)*sqrt(-e^2/(a*
c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(-((a*c*d^2 + a^2*e^2)*sqrt
(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) + d)/(a*c*d^2 + a^2*e^2
))) + 1/2*sqrt(((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e
^2 + a^3*c*e^4)) - d)/(a*c*d^2 + a^2*e^2))*log(sqrt(e*x + d)*e + (a*e^2 -
(a*c^2*d^3 + a^2*c*d*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)))*sqrt(((a*c*d^2 + a^2*e^2)*sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
 + a^3*c*e^4)) - d)/(a*c*d^2 + a^2*e^2))) - 1/2*sqrt(((a*c*d^2 + a^2*e^2)*
sqrt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 + a^2
*e^2))*log(sqrt(e*x + d)*e - (a*e^2 - (a*c^2*d^3 + a^2*c*d*e^2)*sqrt(-e^2/
(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(((a*c*d^2 + a^2*e^2)*sq
rt(-e^2/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)) - d)/(a*c*d^2 + a^2*e
^2)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*sqrt(e*x + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\left(cde|c|-\sqrt{-ac}e|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd+\sqrt{c^2d^2-(cd^2+ae^2)c}}{c}}}\right)}{\sqrt{-c^2d-\sqrt{-ac}ce}\left(ace+\sqrt{-ac}cd\right)|e|} + \frac{\left(cde|c|+\sqrt{-ac}e|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd-\sqrt{c^2d^2-(cd^2+ae^2)c}}{c}}}\right)}{\sqrt{-c^2d+\sqrt{-ac}ce}\left(ace-\sqrt{-ac}cd\right)|e|}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
(c*d*e*abs(c) - sqrt(-a*c)*e*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c*
d + sqrt(c^2*d^2 - (c*d^2 + a*e^2)*c))/c))/(sqrt(-c^2*d - sqrt(-a*c)*c*e)*
(a*c*e + sqrt(-a*c)*c*d)*abs(e)) + (c*d*e*abs(c) + sqrt(-a*c)*e*abs(c)*abs
(e))*arctan(sqrt(e*x + d)/sqrt(-(c*d - sqrt(c^2*d^2 - (c*d^2 + a*e^2)*c))/
c))/(sqrt(-c^2*d + sqrt(-a*c)*c*e)*(a*c*e - sqrt(-a*c)*c*d)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.56 (sec) , antiderivative size = 1366, normalized size of antiderivative = 3.42

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
2*atanh((32*a^2*c^5*d^2*e^2*(- (e*(-a^3*c)^(1/2))/(4*(a^3*c*e^2 + a^2*c^2*
d^2)) - (a*c*d)/(4*(a^3*c*e^2 + a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))/((16
*a^4*c^6*d^3*e^3)/(a^3*c*e^2 + a^2*c^2*d^2) + (16*a^4*c^4*e^6*(-a^3*c)^(1/
2))/(a^3*c*e^2 + a^2*c^2*d^2) + (16*a^5*c^5*d*e^5)/(a^3*c*e^2 + a^2*c^2*d^
2) + (16*a^3*c^5*d^2*e^4*(-a^3*c)^(1/2))/(a^3*c*e^2 + a^2*c^2*d^2)) - (32*
c^3*e^2*(- (e*(-a^3*c)^(1/2))/(4*(a^3*c*e^2 + a^2*c^2*d^2)) - (a*c*d)/(4*(
a^3*c*e^2 + a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))/((16*a^2*c^4*d*e^3)/(a^3
*c*e^2 + a^2*c^2*d^2) + (16*a*c^3*e^4*(-a^3*c)^(1/2))/(a^3*c*e^2 + a^2*c^2
*d^2)) + (32*a*c^4*d*e^3*(-a^3*c)^(1/2)*(- (e*(-a^3*c)^(1/2))/(4*(a^3*c*e^
2 + a^2*c^2*d^2)) - (a*c*d)/(4*(a^3*c*e^2 + a^2*c^2*d^2)))^(1/2)*(d + e*x)
^(1/2))/((16*a^4*c^6*d^3*e^3)/(a^3*c*e^2 + a^2*c^2*d^2) + (16*a^4*c^4*e^6*
(-a^3*c)^(1/2))/(a^3*c*e^2 + a^2*c^2*d^2) + (16*a^5*c^5*d*e^5)/(a^3*c*e^2
+ a^2*c^2*d^2) + (16*a^3*c^5*d^2*e^4*(-a^3*c)^(1/2))/(a^3*c*e^2 + a^2*c^2*
d^2)))*(-(e*(-a^3*c)^(1/2) + a*c*d)/(4*(a^3*c*e^2 + a^2*c^2*d^2)))^(1/2) -
 2*atanh((32*c^3*e^2*((e*(-a^3*c)^(1/2))/(4*(a^3*c*e^2 + a^2*c^2*d^2)) - (
a*c*d)/(4*(a^3*c*e^2 + a^2*c^2*d^2)))^(1/2)*(d + e*x)^(1/2))/((16*a^2*c^4*
d*e^3)/(a^3*c*e^2 + a^2*c^2*d^2) - (16*a*c^3*e^4*(-a^3*c)^(1/2))/(a^3*c*e^
2 + a^2*c^2*d^2)) - (32*a^2*c^5*d^2*e^2*((e*(-a^3*c)^(1/2))/(4*(a^3*c*e^2
+ a^2*c^2*d^2)) - (a*c*d)/(4*(a^3*c*e^2 + a^2*c^2*d^2)))^(1/2)*(d + e*x)^(
1/2))/((16*a^4*c^6*d^3*e^3)/(a^3*c*e^2 + a^2*c^2*d^2) - (16*a^4*c^4*e^6...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1091, normalized size of antiderivative = 2.73

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+a),x)
```

output
```
(sqrt(2)*( - 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) -
c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*d - 2
*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt
(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*
sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqr
t(a*e**2 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*
sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c
*d)*sqrt(2)))*c*d**2 + 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + 2
*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)
))*c*d + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*atan((sqrt(sq
rt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqr
t(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e**2 + 2*sqrt(c)*sqrt(s
qrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**
2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c
*d**2) - c*d)*sqrt(2)))*c*d**2 + sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a
*e**2 + c*d**2) + c*d)*log( - sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d
**2) + c*d)*sqrt(2) + sqrt(a*e**2 + c*d**2) + sqrt(c)*d + sqrt(c)*e*x)*c*d
 - sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*log(...
```

3.181 $\int \frac{1}{(d+ex)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 501

$$\begin{aligned}\int \frac{1}{(d+ex)^{3/2}(a+cx^2)}\,dx = &-\frac{2e}{(cd^2+ae^2)\sqrt{d+ex}} \\ &-\frac{\sqrt[4]{c}e\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\,(cd^2+ae^2)^{3/2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\ &+\frac{\sqrt[4]{c}e\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\,(cd^2+ae^2)^{3/2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\ &+\frac{\sqrt[4]{c}e\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}\,(cd^2+ae^2)^{3/2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\end{aligned}$$

output
```
-2*e/(a*e^2+c*d^2)/(e*x+d)^(1/2)-1/2*c^(1/4)*e*(2*c^(1/2)*d-(a*e^2+c*d^2)^
(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+
d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/(a*e^2+c*d^2)^(3
/2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/2*c^(1/4)*e*(2*c^(1/2)*d-(a*e
^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(
1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/(a*e^2
+c*d^2)^(3/2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/2*c^(1/4)*e*(2*c^(1
/2)*d+(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2
)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2
)/(a*e^2+c*d^2)^(3/2)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.48

$$\int \frac{1}{(d+ex)^{3/2}(a+cx^2)}\,dx = -\frac{2e}{(cd^2+ae^2)\sqrt{d+ex}} + \frac{i\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{a}\left(-i\sqrt{c}d+\sqrt{a}e\right)^2} - \frac{i\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{\sqrt{a}\left(i\sqrt{c}d+\sqrt{a}e\right)^2}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*e)/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (I*Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c
]*e]*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d
+ I*Sqrt[a]*e)]])/(Sqrt[a]*((-I)*Sqrt[c]*d + Sqrt[a]*e)^2) - (I*Sqrt[-(c*d)
 + I*Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d
+ e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)]])/(Sqrt[a]*(I*Sqrt[c]*d + Sqrt[a]*e)^2)
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.43, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {482, 654, 27, 1483, 27, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 482}$$

$$\frac{c\int \frac{d-ex}{\sqrt{d+ex}(cx^2+a)}dx}{ae^2+cd^2}-\frac{2e}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 654}$$

$$\frac{2c\int \frac{e(d-ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}-\frac{2e}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2ce\int \frac{d-ex}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}-\frac{2e}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{2ce\left(\frac{\int \frac{2\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2d+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\sqrt[4]{c}\left(2d+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)}{ae^2+cd^2}-\frac{2e}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2ce\left(\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)}{ae^2+cd^2}-$$

$$\frac{2e}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$\Big\downarrow$ 27

$$\frac{2ce\left(\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}c^{3/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}c^{3/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)}{ae^2+cd^2}-$$

$$\frac{2e}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$\Big\downarrow$ 1142

$$2ce\left(\frac{\frac{\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{1}{2}\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{cd}\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2}}{\sqrt{}}\right.}}{2\sqrt{2}c^{3/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\frac{2e}{(cd^2+ae^2)\sqrt{d+ex}}$$

$\Big\downarrow$ 25

$$\frac{2ce\left(\frac{\frac{\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}+\frac{1}{2}\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\int\frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+\right.}{\sqrt{c}}}}{2\sqrt{2}c^{3/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.}{}$$

$$\frac{2e}{(cd^2+ae^2)\sqrt{d+ex}}$$

$\downarrow$ 27

$$\frac{2ce\left(\frac{\frac{\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}+\frac{\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\sqrt{2}\sqrt{}}}{\sqrt{2}\sqrt[4]{c}}}{2\sqrt{2}c^{3/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.}{}$$

$$\frac{2e}{(cd^2+ae^2)\sqrt{d+ex}}$$

$\downarrow$ 1083

$$\frac{2ce\left(\frac{\frac{\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt{2}\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\int\frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right.}}{\sqrt[4]{c}}}{2\sqrt{2}c^{3/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.}{}$$

$$\frac{2e}{(cd^2+ae^2)\sqrt{d+ex}}$$

$\downarrow$ 219

$$2ce\left(\frac{\frac{\left(2\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{\left(2\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}\right.}{\sqrt{2}\sqrt{}}\right.}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}}{2\sqrt{2}c^{3/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\frac{2e}{(cd^2+ae^2)\sqrt{d+ex}}$$

$$\Big\downarrow \text{1103}$$

$$2ce\left(\frac{-\frac{\left(2\sqrt{c}d-\sqrt{ae^2+cd^2}\right)\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\left(\sqrt{ae^2+cd^2}+2\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\right.}{2\sqrt{2}c^{3/4}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\frac{2e}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

input

```
Int[1/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*e)/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (2*c*e*((-(((2*Sqrt[c]*d - Sqrt[c
*d^2 + a*e^2])*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*ArcTanh[(c^(1/4)*(-((
Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x])
)/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[
c*d^2 + a*e^2]]) - ((2*Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a
*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x
] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*c^(3/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqr
t[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-(((2*Sqrt[c]*d - Sqrt[c*d^2 + a*e^2])*S
qrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[
c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) +
((2*Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(
1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*
x)])/2)/(2*Sqrt[2]*c^(3/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2
 + a*e^2]])))/(c*d^2 + a*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.35

method	result
pseudoelliptic	$\frac{a\sqrt{ex+d}\,e^{2}\left(\sqrt{a\,e^{2}+c\,d^{2}}\,c-2c^{\frac{3}{2}}\,d\right)\arctan\left(\frac{-2\sqrt{c}\,\sqrt{ex+d}+\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd}}{\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}}\right)-a\sqrt{ex+d}\,e^{2}\left(\sqrt{a\,e^{2}+c\,d^{2}}\,c-2c^{\frac{3}{2}}\,d\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^
(1/2)/(a*e^2+c*d^2)^(3/2)/(e*x+d)^(1/2)*(a*(e*x+d)^(1/2)*e^2*((a*e^2+c*d^2
)^(1/2)*c-2*c^(3/2)*d)*arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*
c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^
(1/2)-2*c*d)^(1/2))-a*(e*x+d)^(1/2)*e^2*((a*e^2+c*d^2)^(1/2)*c-2*c^(3/2)*d
)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))
/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-2*
(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(1/8
*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(e*x+d)^(1/2)*((2*c^(1/2)*d+(a*e^
2+c*d^2)^(1/2))*((a*e^2+c*d^2)*c)^(1/2)-c*d*(a*e^2+c*d^2)^(1/2)-2*c^(3/2)*
d^2)*ln((-e*x-d)*c^(1/2)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(
1/2)-(a*e^2+c*d^2)^(1/2))-1/8*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(e*x
+d)^(1/2)*((2*c^(1/2)*d+(a*e^2+c*d^2)^(1/2))*((a*e^2+c*d^2)*c)^(1/2)-c*d*(
a*e^2+c*d^2)^(1/2)-2*c^(3/2)*d^2)*ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*
e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))+(a*e^2+c*d^2)^(1/2)*
c^(1/2)*a*e^2))/e/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2863 vs. $2(403) = 806$.

Time = 0.17 (sec) , antiderivative size = 2863, normalized size of antiderivative = 5.71

$$\int \frac{1}{(d+ex)^{3/2}(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*((c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-(c^2*d^3 - 3*a*c*d*e^2
 + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^
3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2
+ 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d
^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a
^4*e^6))*log(-(3*c^2*d^2*e - a*c*e^3)*sqrt(e*x + d) + (6*a*c^2*d^3*e^2 - 2
*a^2*c*d*e^4 + (a*c^4*d^8 + 2*a^2*c^3*d^6*e^2 - 2*a^4*c*d^2*e^6 - a^5*e^8)
*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c
^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8
 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt(-(c^2*d^3 - 3*a*c*d*e^2 + (a*c^3*d^
6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*d^4*e^2 -
6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4
*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^
7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))) -
(c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-(c^2*d^3 - 3*a*c*d*e^2 + (a*
c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-(9*c^3*d^4*
e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a
^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^1
0 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6
))*log(-(3*c^2*d^2*e - a*c*e^3)*sqrt(e*x + d) - (6*a*c^2*d^3*e^2 - 2*a^...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx=\int \frac{1}{\left(a+cx^2\right)(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.32

$$\int \frac{1}{(d+ex)^{3/2}(a+cx^2)}\,dx = -\frac{2\,e}{(cd^2+ae^2)\sqrt{ex+d}}$$

$$+\frac{\left((cd^2e+ae^3)^2ae|c|+2\left(\sqrt{-ac}cd^3e+\sqrt{-ac}ade^3\right)|-cd^2e-ae^3||c|-(c^3d^6e+2\,ac^2d^4e^3+a^2cd^2e^5)|c|\right)\text{ar}}{\left(ac^2d^4e+2\,a^2cd^2e^3+a^3e^5-\sqrt{-ac}c^2d^5-2\sqrt{-ac}acd^3e^2-\sqrt{-ac}a^2de^4\right)\sqrt{}}$$

$$+\frac{\left((cd^2e+ae^3)^2ae|c|-2\left(\sqrt{-ac}cd^3e+\sqrt{-ac}ade^3\right)|-cd^2e-ae^3||c|-(c^3d^6e+2\,ac^2d^4e^3+a^2cd^2e^5)|c|\right)\text{ar}}{\left(ac^2d^4e+2\,a^2cd^2e^3+a^3e^5+\sqrt{-ac}c^2d^5+2\sqrt{-ac}acd^3e^2+\sqrt{-ac}a^2de^4\right)\sqrt{}}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
-2*e/((c*d^2 + a*e^2)*sqrt(e*x + d)) + ((c*d^2*e + a*e^3)^2*a*e*abs(c) + 2
*(sqrt(-a*c)*c*d^3*e + sqrt(-a*c)*a*d*e^3)*abs(-c*d^2*e - a*e^3)*abs(c) -
(c^3*d^6*e + 2*a*c^2*d^4*e^3 + a^2*c*d^2*e^5)*abs(c))*arctan(sqrt(e*x + d)
/sqrt(-(c^2*d^3 + a*c*d*e^2 + sqrt((c^2*d^3 + a*c*d*e^2)^2 - (c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4)*(c^2*d^2 + a*c*e^2)))/(c^2*d^2 + a*c*e^2)))/((a*c^2
*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5 - sqrt(-a*c)*c^2*d^5 - 2*sqrt(-a*c)*a*c
*d^3*e^2 - sqrt(-a*c)*a^2*d*e^4)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(-c*d^2*
e - a*e^3)) + ((c*d^2*e + a*e^3)^2*a*e*abs(c) - 2*(sqrt(-a*c)*c*d^3*e + sq
rt(-a*c)*a*d*e^3)*abs(-c*d^2*e - a*e^3)*abs(c) - (c^3*d^6*e + 2*a*c^2*d^4*
e^3 + a^2*c*d^2*e^5)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d^3 + a*c*d*e
^2 - sqrt((c^2*d^3 + a*c*d*e^2)^2 - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(c
^2*d^2 + a*c*e^2)))/(c^2*d^2 + a*c*e^2)))/((a*c^2*d^4*e + 2*a^2*c*d^2*e^3
+ a^3*e^5 + sqrt(-a*c)*c^2*d^5 + 2*sqrt(-a*c)*a*c*d^3*e^2 + sqrt(-a*c)*a^2
*d*e^4)*sqrt(-c^2*d - sqrt(-a*c)*c*e)*abs(-c*d^2*e - a*e^3))
```

Mupad [B] (verification not implemented)

Time = 7.40 (sec) , antiderivative size = 4471, normalized size of antiderivative = 8.92

$$\int \frac{1}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
- atan((((d + e*x)^(1/2)*(16*a^4*c^4*e^10 - 16*c^8*d^8*e^2 - 32*a*c^7*d^6*
e^4 + 32*a^3*c^5*d^2*e^8) + (-(a*c^2*d^3 + a*e^3*(-a^3*c)^(1/2) - 3*a^2*c*
d*e^2 - 3*c*d^2*e*(-a^3*c)^(1/2))/(4*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*
e^4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*(64*a*c^8*d^9*e^3 - (d + e*x)^(1/2)*(-(a*
c^2*d^3 + a*e^3*(-a^3*c)^(1/2) - 3*a^2*c*d*e^2 - 3*c*d^2*e*(-a^3*c)^(1/2))
/(4*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*
(64*a*c^9*d^11*e^2 + 64*a^6*c^4*d*e^12 + 320*a^2*c^8*d^9*e^4 + 640*a^3*c^7
*d^7*e^6 + 640*a^4*c^6*d^5*e^8 + 320*a^5*c^5*d^3*e^10) + 64*a^5*c^4*d*e^11
 + 256*a^2*c^7*d^7*e^5 + 384*a^3*c^6*d^5*e^7 + 256*a^4*c^5*d^3*e^9))*(-(a*
c^2*d^3 + a*e^3*(-a^3*c)^(1/2) - 3*a^2*c*d*e^2 - 3*c*d^2*e*(-a^3*c)^(1/2))
/(4*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*
1i + ((d + e*x)^(1/2)*(16*a^4*c^4*e^10 - 16*c^8*d^8*e^2 - 32*a*c^7*d^6*e^4
 + 32*a^3*c^5*d^2*e^8) - (-(a*c^2*d^3 + a*e^3*(-a^3*c)^(1/2) - 3*a^2*c*d*e
^2 - 3*c*d^2*e*(-a^3*c)^(1/2))/(4*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4
 + 3*a^3*c^2*d^4*e^2)))^(1/2)*((d + e*x)^(1/2)*(-(a*c^2*d^3 + a*e^3*(-a^3*
c)^(1/2) - 3*a^2*c*d*e^2 - 3*c*d^2*e*(-a^3*c)^(1/2))/(4*(a^5*e^6 + a^2*c^3
*d^6 + 3*a^4*c*d^2*e^4 + 3*a^3*c^2*d^4*e^2)))^(1/2)*(64*a*c^9*d^11*e^2 + 6
4*a^6*c^4*d*e^12 + 320*a^2*c^8*d^9*e^4 + 640*a^3*c^7*d^7*e^6 + 640*a^4*c^6
*d^5*e^8 + 320*a^5*c^5*d^3*e^10) + 64*a*c^8*d^9*e^3 + 64*a^5*c^4*d*e^11 +
256*a^2*c^7*d^7*e^5 + 384*a^3*c^6*d^5*e^7 + 256*a^4*c^5*d^3*e^9))*(-(a*...
```

Reduce [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 1650, normalized size of antiderivative = 3.29

$$\int \frac{1}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+a),x)
```

output
```
(2*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
- c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2
*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)
))*a*e**2 - 2*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2
 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*
sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c
*d)*sqrt(2)))*c*d**2 - 2*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqr
t(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)
*sqrt(2)))*a*d*e**2 - 2*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c
*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt
(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*
sqrt(2)))*c*d**3 - 2*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt
(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
+ c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d*
*2) - c*d)*sqrt(2)))*a*e**2 + 2*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(s
qrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2
 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*
e**2 + c*d**2) - c*d)*sqrt(2)))*c*d**2 + 2*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt
(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2...
```

3.182 $\int \frac{1}{(d+ex)^{5/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 562

$$
\begin{aligned}
\int \frac{1}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = & -\frac{2e}{3\,(cd^2+ae^2)\,(d+ex)^{3/2}} - \frac{4cde}{(cd^2+ae^2)^2\,\sqrt{d+ex}} \\
& - \frac{c^{3/4}e\left(3cd^2-ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\,(cd^2+ae^2)^{5/2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
& + \frac{c^{3/4}e\left(3cd^2-ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}\,(cd^2+ae^2)^{5/2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
& + \frac{c^{3/4}e\left(3cd^2-ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}\,(cd^2+ae^2)^{5/2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}
\end{aligned}
$$

output
```
-2/3*e/(a*e^2+c*d^2)/(e*x+d)^(3/2)-4*c*d*e/(a*e^2+c*d^2)^2/(e*x+d)^(1/2)-1
/2*c^(3/4)*e*(3*c*d^2-a*e^2-2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1
/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*
d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/(a*e^2+c*d^2)^(5/2)/(-c^(1/2)*d+(a*e
^2+c*d^2)^(1/2))^(1/2)+1/2*c^(3/4)*e*(3*c*d^2-a*e^2-2*c^(1/2)*d*(a*e^2+c*d
^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(
e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/(a*e^2+c*d^2
)^(5/2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/2*c^(3/4)*e*(3*c*d^2-a*e^
2+2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e
^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d))
)*2^(1/2)/(a*e^2+c*d^2)^(5/2)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.47

$$\int \frac{1}{(d+ex)^{5/2}(a+cx^2)}\,dx = -\frac{2e(ae^2+cd(7d+6ex))}{3(cd^2+ae^2)^2(d+ex)^{3/2}} + \frac{\sqrt{c}\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{a}\left(-i\sqrt{c}d+\sqrt{a}e\right)^3} + \frac{\sqrt{c}\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{\sqrt{a}\left(i\sqrt{c}d+\sqrt{a}e\right)^3}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

output
```
(-2*e*(a*e^2 + c*d*(7*d + 6*e*x)))/(3*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2)) +
 (Sqrt[c]*Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - I*Sqrt[
a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/(Sqrt[a]*((-I)*Sq
rt[c]*d + Sqrt[a]*e)^3) + (Sqrt[c]*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*ArcT
an[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[
a]*e)])/(Sqrt[a]*(I*Sqrt[c]*d + Sqrt[a]*e)^3)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.46, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {482, 655, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 482}$$

$$\frac{c\int \frac{d-ex}{(d+ex)^{3/2}(cx^2+a)}dx}{ae^2+cd^2}-\frac{2e}{3(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 655}$$

$$\frac{c\left(\frac{\int \frac{cd^2-2cexd-ae^2}{\sqrt{d+ex}(cx^2+a)}dx}{ae^2+cd^2}-\frac{4de}{\sqrt{d+ex}(ae^2+cd^2)}\right)}{ae^2+cd^2}-\frac{2e}{3(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 654}$$

$$\frac{c\left(\frac{2\int \frac{e(3cd^2-2c(d+ex)d-ae^2)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}-\frac{4de}{\sqrt{d+ex}(ae^2+cd^2)}\right)}{ae^2+cd^2}-\frac{2e}{3(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(\frac{2e\int \frac{3cd^2-2c(d+ex)d-ae^2}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}-\frac{4de}{\sqrt{d+ex}(ae^2+cd^2)}\right)}{ae^2+cd^2}-\frac{2e}{3(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 1483}$$

$$c\left(\frac{2e\left(\frac{\int \frac{\sqrt{2}\left(3cd^2-ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-ae^2\right)+\sqrt[4]{c}\left(3cd^2+2\sqrt{c}\sqrt{c\ldots}\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+\ldots}}}{\sqrt[4]{c}}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}\ldots}}\right.}{ae^2+cd^2}\right.$$

$$\overline{\qquad ae^2+cd^2 \qquad}$$

$$\frac{2e}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)}$$

$\downarrow$ 27

$$c\left(\frac{2e\left(\frac{\int \frac{\sqrt{2}\left(3cd^2-ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-ae^2\right)+\sqrt[4]{c}\left(3cd^2+2\sqrt{c}\sqrt{c\ldots}\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}\ldots}}{\sqrt[4]{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\ldots}}\right.}{ae^2+cd^2}\right.$$

$$\overline{\qquad ae^2+cd^2 \qquad}$$

$$\frac{2e}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)}$$

$\downarrow$ 1142

$$c\left(\frac{2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{}\right)$$

$$\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}$$

$\downarrow$ 25

$$c\left(\frac{2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{}\right)$$

$$\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}$$

$\downarrow$ 27

$$c\left(\frac{2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int \cdots}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right)}{\cdots}\cdots\right)\cdots$$

$$\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}$$

$\Big\downarrow$ 1083

$$c\left(\frac{2e\left(\frac{\frac{\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int \cdots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right)}{\cdots}\cdots\right)\cdots$$

$$\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}$$

$\Big\downarrow$ 219

$$c \left(2e \left(\frac{\frac{\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}} - \frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\mathrm{a}}{\sqrt{\sqrt{c}d-\sqrt{}}}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} \right.\right.$$

$$\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}$$

$\big\downarrow$ 1103

$$c \left(2e \left(\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}} - \frac{1}{2}\sqrt[4]{c}\left(3cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d-ae^2\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} \right.\right.$$

$$\frac{2e}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}$$

input
```
Int[1/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

output
```
(-2*e)/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (c*((-4*d*e)/((c*d^2 + a*e^2)
*Sqrt[d + e*x]) + (2*e*((-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*
(3*c*d^2 - a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*(-((S
qrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))
/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c
*d^2 + a*e^2]]) - (c^(1/4)*(3*c*d^2 - a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e
^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2
 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[
c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-((c^(1/4)*Sqrt[S
qrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(3*c*d^2 - a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2
+ a*e^2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
)/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2
]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (c^(1/4)*(3*c*d^2 - a*e^2 +
 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4
)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)]
)/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
a*e^2]])))/(c*d^2 + a*e^2)))/(c*d^2 + a*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.35

method	result
pseudoelliptic	$-\frac{2\left(-\frac{3a e^{2}(ex+d)^{\frac{3}{2}}\left(ac e^{2}-3d^{2}c^{2}+2\sqrt{a e^{2}+c d^{2}}\, c^{\frac{3}{2}} d\right)\arctan\left(\frac{-2\sqrt{c}\,\sqrt{ex+d}+\sqrt{2\sqrt{\left(a e^{2}+c d^{2}\right)c}+2cd}}{\sqrt{4\sqrt{a e^{2}+c d^{2}}\,\sqrt{c}-2\sqrt{\left(a e^{2}+c d^{2}\right)c}-2cd}}\right)}{2}+\frac{3a e^{2}(ex+d)^{\frac{3}{2}}\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3/2*a*e^2*(e*x+d)^(3/2)*(a*c*e^2-3*d^2*c^2+2*(a*e^2+c*d^2)^(1/2)*c^
(3/2)*d)*arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d
)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(
1/2))+3/2*a*e^2*(e*x+d)^(3/2)*(a*c*e^2-3*d^2*c^2+2*(a*e^2+c*d^2)^(1/2)*c^(
3/2)*d)*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^
(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/
2))+(-3/8*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(e*x+d)^(3/2)*((a*e^2-2*
c^(1/2)*d*(a*e^2+c*d^2)^(1/2)-3*c*d^2)*((a*e^2+c*d^2)*c)^(1/2)-a*d*e^2*c+3
*c^2*d^3+2*(a*e^2+c*d^2)^(1/2)*c^(3/2)*d^2)*ln((-e*x-d)*c^(1/2)+(e*x+d)^(1
/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))+3/8*(2*((
a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(e*x+d)^(3/2)*((a*e^2-2*c^(1/2)*d*(a*e^
2+c*d^2)^(1/2)-3*c*d^2)*((a*e^2+c*d^2)*c)^(1/2)-a*d*e^2*c+3*c^2*d^3+2*(a*e
^2+c*d^2)^(1/2)*c^(3/2)*d^2)*ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c
*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))+a*(a*e^2+c*d^2)^(1/2)*e^2
*(6*c*d*e*x+a*e^2+7*c*d^2))*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2
)*c)^(1/2)-2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c
)^(1/2)-2*c*d)^(1/2)/(e*x+d)^(3/2)/(a*e^2+c*d^2)^(5/2)/e/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5149 vs. $2(460) = 920$.

Time = 0.19 (sec) , antiderivative size = 5149, normalized size of antiderivative = 9.16

$$\int \frac{1}{(d+ex)^{5/2}(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(5/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(460) = 920$.

Time = 0.22 (sec) , antiderivative size = 1215, normalized size of antiderivative = 2.16

$$\int \frac{1}{(d+ex)^{5/2}(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
-(2*(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)^2*a*c*d*e*abs(c) + (3*sqrt(-a*c)
*c^3*d^6*e + 5*sqrt(-a*c)*a*c^2*d^4*e^3 + sqrt(-a*c)*a^2*c*d^2*e^5 - sqrt(
-a*c)*a^3*e^7)*abs(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*abs(c) - (c^6*d^11
*e + 3*a*c^5*d^9*e^3 + 2*a^2*c^4*d^7*e^5 - 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d
^3*e^9 - a^5*c*d*e^11)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c^3*d^5 + 2*a*c
^2*d^3*e^2 + a^2*c*d*e^4 + sqrt((c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)^
2 - (c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*(c^3*d^4 + 2*a
*c^2*d^2*e^2 + a^2*c*e^4)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)))/((a*
c^4*d^8*e + 4*a^2*c^3*d^6*e^3 + 6*a^3*c^2*d^4*e^5 + 4*a^4*c*d^2*e^7 + a^5*
e^9 + sqrt(-a*c)*c^4*d^9 + 4*sqrt(-a*c)*a*c^3*d^7*e^2 + 6*sqrt(-a*c)*a^2*c
^2*d^5*e^4 + 4*sqrt(-a*c)*a^3*c*d^3*e^6 + sqrt(-a*c)*a^4*d*e^8)*sqrt(-c^2*
d - sqrt(-a*c)*c*e)*abs(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)) - (2*(c^2*d^
4*e + 2*a*c*d^2*e^3 + a^2*e^5)^2*a*c*d*e*abs(c) - (3*sqrt(-a*c)*c^3*d^6*e
+ 5*sqrt(-a*c)*a*c^2*d^4*e^3 + sqrt(-a*c)*a^2*c*d^2*e^5 - sqrt(-a*c)*a^3*e
^7)*abs(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*abs(c) - (c^6*d^11*e + 3*a*c^
5*d^9*e^3 + 2*a^2*c^4*d^7*e^5 - 2*a^3*c^3*d^5*e^7 - 3*a^4*c^2*d^3*e^9 - a^
5*c*d*e^11)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c^3*d^5 + 2*a*c^2*d^3*e^2
+ a^2*c*d*e^4 - sqrt((c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)^2 - (c^3*d^
6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*(c^3*d^4 + 2*a*c^2*d^2*e^
2 + a^2*c*e^4)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)))/((a*c^4*d^8*...
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 7908, normalized size of antiderivative = 14.07

$$\int \frac{1}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)*(d + e*x)^(5/2)),x)
```

output
```
- ((2*e)/(3*(a*e^2 + c*d^2)) + (4*c*d*e*(d + e*x))/(a*e^2 + c*d^2)^2)/(d +
 e*x)^(3/2) - atan((((d + e*x)^(1/2)*(320*a^2*c^11*d^12*e^6 - 16*c^13*d^16
*e^2 - 16*a^8*c^5*e^18 + 1024*a^3*c^10*d^10*e^8 + 1440*a^4*c^9*d^8*e^10 +
1024*a^5*c^8*d^6*e^12 + 320*a^6*c^7*d^4*e^14) + (-(a^2*e^5*(-a^3*c^3)^(1/2
) + a*c^4*d^5 + 5*a^3*c^2*d*e^4 - 10*a^2*c^3*d^3*e^2 + 5*c^2*d^4*e*(-a^3*c
^3)^(1/2) - 10*a*c*d^2*e^3*(-a^3*c^3)^(1/2))/(4*(a^7*e^10 + a^2*c^5*d^10 +
 5*a^6*c*d^2*e^8 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4
*e^6)))^(1/2)*(96*a*c^13*d^18*e^3 - (d + e*x)^(1/2)*(-(a^2*e^5*(-a^3*c^3)^
(1/2) + a*c^4*d^5 + 5*a^3*c^2*d*e^4 - 10*a^2*c^3*d^3*e^2 + 5*c^2*d^4*e*(-a
^3*c^3)^(1/2) - 10*a*c*d^2*e^3*(-a^3*c^3)^(1/2))/(4*(a^7*e^10 + a^2*c^5*d^
10 + 5*a^6*c*d^2*e^8 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2
*d^4*e^6)))^(1/2)*(64*a*c^14*d^21*e^2 + 64*a^11*c^4*d*e^22 + 640*a^2*c^13*
d^19*e^4 + 2880*a^3*c^12*d^17*e^6 + 7680*a^4*c^11*d^15*e^8 + 13440*a^5*c^1
0*d^13*e^10 + 16128*a^6*c^9*d^11*e^12 + 13440*a^7*c^8*d^9*e^14 + 7680*a^8*
c^7*d^7*e^16 + 2880*a^9*c^6*d^5*e^18 + 640*a^10*c^5*d^3*e^20) - 32*a^10*c^
4*e^21 + 736*a^2*c^12*d^16*e^5 + 2432*a^3*c^11*d^14*e^7 + 4480*a^4*c^10*d^
12*e^9 + 4928*a^5*c^9*d^10*e^11 + 3136*a^6*c^8*d^8*e^13 + 896*a^7*c^7*d^6*
e^15 - 128*a^8*c^6*d^4*e^17 - 160*a^9*c^5*d^2*e^19))*(-(a^2*e^5*(-a^3*c^3)
^(1/2) + a*c^4*d^5 + 5*a^3*c^2*d*e^4 - 10*a^2*c^3*d^3*e^2 + 5*c^2*d^4*e*(-
a^3*c^3)^(1/2) - 10*a*c*d^2*e^3*(-a^3*c^3)^(1/2))/(4*(a^7*e^10 + a^2*c^...
```

Reduce [B] (verification not implemented)

Time = 23.70 (sec) , antiderivative size = 3434, normalized size of antiderivative = 6.11

$$\int \frac{1}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+a),x)
```

output

```
(18*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
 - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) -
2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2
)))*a*c*d**2*e**2 + 18*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sq
rt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*
d**2) - c*d)*sqrt(2)))*a*c*d*e**3*x - 6*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2
)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqr
t(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)
*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c**2*d**4 - 6*sqrt(d + e*x)*sqrt(a
*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sq
rt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))
/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c**2*d**3*e*x + 6*sq
rt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan
((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e
*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*d*e**4 + 6*
sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*at
an((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d +
 e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*e**5*x -
6*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(...
```

3.183 $\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 607

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2} \, dx = -\frac{e(cd^2-5ae^2)\sqrt{d+ex}}{2ac^2} - \frac{de(d+ex)^{3/2}}{2ac} - \frac{(ae-cdx)(d+ex)^{5/2}}{2ac(a+cx^2)}$$
$$- \frac{e\left((cd^2-5ae^2)\sqrt{cd^2+ae^2}+\sqrt{c}d(cd^2+13ae^2)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{9/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$+ \frac{e\left((cd^2-5ae^2)\sqrt{cd^2+ae^2}+\sqrt{c}d(cd^2+13ae^2)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{9/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$+ \frac{e\left((cd^2-5ae^2)\sqrt{cd^2+ae^2}-\sqrt{c}d(cd^2+13ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{4\sqrt{2}ac^{9/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/2*e*(-5*a*e^2+c*d^2)*(e*x+d)^(1/2)/a/c^2-1/2*d*e*(e*x+d)^(3/2)/a/c-1/2*
(-c*d*x+a*e)*(e*x+d)^(5/2)/a/c/(c*x^2+a)-1/8*e*((-5*a*e^2+c*d^2)*(a*e^2+c*
d^2)^(1/2)+c^(1/2)*d*(13*a*e^2+c*d^2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1
/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))
^(1/2))*2^(1/2)/a/c^(9/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/8*e*((-
5*a*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2)+c^(1/2)*d*(13*a*e^2+c*d^2))*arctan(((c^
(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2
)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(9/4)/(-c^(1/2)*d+(a*e^2+c*d^2
)^(1/2))^(1/2)+1/8*e*((-5*a*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2)-c^(1/2)*d*(13*a
*e^2+c*d^2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a/c^(9/4)/(c
^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}\sqrt{d+ex}(5a^2e^3+c^2d^3x+ace(-3d^2-3dex+4e^2x^2))}{a+cx^2}+\frac{(\sqrt{c}d+i\sqrt{a}e)^3(2i\sqrt{c}d+5\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+e}}{\sqrt{c}d+i\sqrt{a}e}\right.}{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}}{4a^{3/2}c^2}$$

input
```
Integrate[(d + e*x)^(7/2)/(a + c*x^2)^2,x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(5*a^2*e^3 + c^2*d^3*x + a*c*e*(-3*d^2 - 3*d*e*x
 + 4*e^2*x^2)))/(a + c*x^2) + ((Sqrt[c]*d + I*Sqrt[a]*e)^3*((2*I)*Sqrt[c]*
d + 5*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])
/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e] + ((Sqrt[c
]*d - I*Sqrt[a]*e)^3*((-2*I)*Sqrt[c]*d + 5*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d)
+ I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/Sqrt[-(c
*d) + I*Sqrt[a]*Sqrt[c]*e])/(4*a^(3/2)*c^2)
```

Rubi [A] (verified)

Time = 2.82 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {495, 27, 653, 27, 653, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2}\,dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{(d+ex)^{3/2}(2cd^2-3cexd+5ae^2)}{2(cx^2+a)}dx}{2ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^{3/2}(2cd^2-3cexd+5ae^2)}{cx^2+a}dx}{4ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{653}$$

$$\frac{\frac{\int \frac{c\sqrt{d+ex}\left(2d\left(cd^2+4ae^2\right)-e\left(cd^2-5ae^2\right)x\right)}{cx^2+a}dx}{c}-2de(d+ex)^{3/2}}{4ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{d+ex}(2d(cd^2+4ae^2)-e(cd^2-5ae^2)x)}{cx^2+a}dx-2de(d+ex)^{3/2}}{4ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{653}$$

$$\frac{\frac{\int \frac{\left(2cd^2-ae^2\right)\left(cd^2+5ae^2\right)+cde\left(cd^2+13ae^2\right)x}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{c}-\frac{2e\sqrt{d+ex}(cd^2-5ae^2)}{c}-2de(d+ex)^{3/2}}{4ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{2\int \frac{e\left(\left(cd^2-5ae^2\right)\left(cd^2+ae^2\right)+cd\left(cd^2+13ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(cd^2-5ae^2)}{c}-2de(d+ex)^{3/2}}{4ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$\downarrow$ 27

$$\frac{\frac{2e\int \frac{\left(cd^2-5ae^2\right)\left(cd^2+ae^2\right)+cd\left(cd^2+13ae^2\right)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(cd^2-5ae^2)}{c}-2de(d+ex)^{3/2}}{4ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$\downarrow$ 1483

$$\frac{2e\left(\frac{\int \frac{\sqrt{cd^2+ae^2}\left(\sqrt{2}\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}-\sqrt{c}d\left(cd^2+13ae^2\right)\right)\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{cd^2+ae^2}\left(\sqrt{2}\sqrt{cd^2\cdots}\right.}{\cdots}}{\cdots}\right)}{c}\cdots$$

$$4\cdots$$

$$\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$\downarrow$ 27

$$\frac{2e\left(\frac{\int \frac{\sqrt{2}\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}-\sqrt{c}d\left(cd^2+13ae^2\right)\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(\cdots\right.}{d+ex+\cdots}}{\cdots}\right)}{c}\cdots$$

$$4ac$$

$$\frac{(d+ex)^{5/2}(ae-cdx)}{2ac\,(a+cx^2)}$$

$\downarrow$ 1142

$$\frac{-2de(d+ex)^{3/2}-\frac{2e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{c}+\frac{2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}\cdots}}{\cdots}}\cdots}{\sqrt{2}}\cdots}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 25

$$\frac{-2de(d+ex)^{3/2}-\frac{2e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{c}+\frac{2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}\cdots}}{\cdots}}\cdots}{\sqrt{2}}\cdots}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 27

$$\frac{-2de(d+ex)^{3/2}-\frac{2e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{c}+\frac{2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}\cdots}}{\cdots}}\cdots}{\sqrt{2}}}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 1083

$$\frac{-2de(d+ex)^{3/2}-\frac{2e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{c}+\frac{2e\left(\frac{\left(\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}-\sqrt{c}d\left(cd^2+13ae^2\right)\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}}{\sqrt{2}}}{}}{}}{}$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}$$

$\Big\downarrow$ 219

$$\frac{-2de(d+ex)^{3/2}-\frac{2e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{c}+\frac{2e\left(\frac{\left(\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}-\sqrt{c}d\left(cd^2+13ae^2\right)\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}}{\sqrt{2}}}{}}{}}{}$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}$$

$\Big\downarrow$ 1103

$$\frac{-2de(d+ex)^{3/2}-\frac{2e\left(cd^2-5ae^2\right)\sqrt{d+ex}}{c}+\frac{2e\left(-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+}\right.}{\sqrt{2}}\right.}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}}{}}{}}{}$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{2ac\left(cx^2+a\right)}$$

input
```
Int[(d + e*x)^(7/2)/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*e - c*d*x)*(d + e*x)^(5/2))/(a*c*(a + c*x^2)) + ((-2*e*(c*d^2 - 5
*a*e^2)*Sqrt[d + e*x])/c - 2*d*e*(d + e*x)^(3/2) + (2*e*((-((c^(1/4)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 + (c*d^
2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c
]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) -
(c^(1/4)*((c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2] - Sqrt[c]*d*(c*d^2 + 13*a*
e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^
2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt
[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d
^2 + a*e^2]]*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 + (c*d^2 - 5*a*e^2)*Sqrt[c*
d^2 + a*e^2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e
^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a
*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (c^(1/4)*((c*d^2 - 5*a*
e^2)*Sqrt[c*d^2 + a*e^2] - Sqrt[c]*d*(c*d^2 + 13*a*e^2))*Log[Sqrt[c*d^2 +
a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*
x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2
 + a*e^2]])))/c)/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.54

method	result
pseudoelliptic	$\frac{\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}\,\left(-\ln\left(\sqrt{c}\,(ex+d)+\sqrt{ex+d}\,\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd}+\sqrt{a\,e^{2}+c\,d^{2}}\right)+\ln\left(\sqrt{c}\,(ex+d)-\sqrt{ex\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((e*x+d)^(7/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)/
c^(11/2)*(1/4*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c
*d)^(1/2)*(-ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*
c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))+ln(c^(1/2)*(e*x+d)-(e*x+d)^(1/2)*(2*((a*e^
2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2)))*(((-(-5*e^2*x^2+d^2)*
a*c^(7/2)-c^(9/2)*d^2*x^2+5*c^(5/2)*a^2*e^2)*(a*e^2+c*d^2)^(1/2)+13*(c*x^2
+a)*c^3*d*(a*e^2+1/13*c*d^2))*((a*e^2+c*d^2)*c)^(1/2)+((-5*a^2*e^2*c^(7/2)
+c^(9/2)*((c*x^2+a)*d^2-5*a*e^2*x^2))*(a*e^2+c*d^2)^(1/2)-13*(c*x^2+a)*c^4
*d*(a*e^2+1/13*c*d^2))*d)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+e*(e*(-5
*a^2*e^2*c^(7/2)+c^(9/2)*((c*x^2+a)*d^2-5*a*e^2*x^2))*(arctan((2*c^(1/2)*(
e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/
2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((-2*c^(1/2)*(e*x
+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*
c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)))*(a*e^2+c*d^2)^(1/2)+2*(4*
(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(5*a^2*
e^3*c^(7/2)+c^(9/2)*(4*a*e^3*x^2-3*a*d*e^2*x+c*d^3*x-3*a*d^2*e))*(e*x+d)^(
1/2)+13*e*(c*x^2+a)*c^4*d*(arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^
2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*
c)^(1/2)-2*c*d)^(1/2))-arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*
c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2091 vs. $2(497) = 994$.

Time = 0.33 (sec) , antiderivative size = 2091, normalized size of antiderivative = 3.44

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((a*c^3*x^2 + a^2*c^2)*sqrt(-(4*c^3*d^7 + 35*a*c^2*d^5*e^2 + 70*a^2*c
*d^3*e^4 - 105*a^3*d*e^6 + a^3*c^4*sqrt(-(1225*c^4*d^8*e^6 + 10780*a*c^3*d
^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3
*c^9)))/(a^3*c^4))*log(-(140*c^5*d^10*e^3 + 1771*a*c^4*d^8*e^5 + 6872*a^2*
c^3*d^6*e^7 + 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^11 - 625*a^5*e^13)*s
qrt(e*x + d) + (35*a^2*c^5*d^6*e^4 - 21*a^3*c^4*d^4*e^6 - 795*a^4*c^3*d^2*
e^8 + 125*a^5*c^2*e^10 - 2*(a^3*c^8*d^3 + 4*a^4*c^7*d*e^2)*sqrt(-(1225*c^4
*d^8*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e
^12 + 625*a^4*e^14)/(a^3*c^9)))*sqrt(-(4*c^3*d^7 + 35*a*c^2*d^5*e^2 + 70*a
^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^3*c^4*sqrt(-(1225*c^4*d^8*e^6 + 10780*a*c
^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/
(a^3*c^9)))/(a^3*c^4))) - (a*c^3*x^2 + a^2*c^2)*sqrt(-(4*c^3*d^7 + 35*a*c^
2*d^5*e^2 + 70*a^2*c*d^3*e^4 - 105*a^3*d*e^6 + a^3*c^4*sqrt(-(1225*c^4*d^8
*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^10 - 7700*a^3*c*d^2*e^12
+ 625*a^4*e^14)/(a^3*c^9)))/(a^3*c^4))*log(-(140*c^5*d^10*e^3 + 1771*a*c^4
*d^8*e^5 + 6872*a^2*c^3*d^6*e^7 + 8366*a^3*c^2*d^4*e^9 + 2500*a^4*c*d^2*e^
11 - 625*a^5*e^13)*sqrt(e*x + d) - (35*a^2*c^5*d^6*e^4 - 21*a^3*c^4*d^4*e^
6 - 795*a^4*c^3*d^2*e^8 + 125*a^5*c^2*e^10 - 2*(a^3*c^8*d^3 + 4*a^4*c^7*d*
e^2)*sqrt(-(1225*c^4*d^8*e^6 + 10780*a*c^3*d^6*e^8 + 21966*a^2*c^2*d^4*e^1
0 - 7700*a^3*c*d^2*e^12 + 625*a^4*e^14)/(a^3*c^9)))*sqrt(-(4*c^3*d^7 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(7/2)/(c*x^2 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 596, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2}\,dx = \frac{2\sqrt{ex+d}e^3}{c^2}$$

$$+ \frac{\left((c^2d^3e+13\,acde^3)a^2e^2|c| - \left(\sqrt{-ac}c^2d^4e - 4\sqrt{-ac}acd^2e^3 - 5\sqrt{-ac}a^2e^5\right)|a||c||e| + (2\,ac^3d^5e + 9\,a^2c^2d^3}{4\left(a^2c^3e + \sqrt{-ac}ac^3d\right)\sqrt{-c^2d - \sqrt{-ac}ce}|a||e|}$$

$$+ \frac{\left(\left(\sqrt{-ac}cd^3e + 13\sqrt{-ac}ade^3\right)a^2e^2|c| - (ac^2d^4e - 4\,a^2cd^2e^3 - 5\,a^3e^5)|a||c||e| + \left(2\sqrt{-ac}ac^2d^5e + 9\sqrt{-ac}\right.}{4\left(a^2c^3d + \sqrt{-ac}a^2c^2e\right)\sqrt{-c^2d + \sqrt{-ac}c}}$$

$$+ \frac{(ex+d)^{\frac{3}{2}}c^2d^3e - \sqrt{ex+d}c^2d^4e - 3\,(ex+d)^{\frac{3}{2}}acde^3 + \sqrt{ex+d}a^2e^5}{2\left((ex+d)^2c - 2\,(ex+d)cd + cd^2 + ae^2\right)ac^2}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*e^3/c^2 + 1/4*((c^2*d^3*e + 13*a*c*d*e^3)*a^2*e^2*abs(c) -
 (sqrt(-a*c)*c^2*d^4*e - 4*sqrt(-a*c)*a*c*d^2*e^3 - 5*sqrt(-a*c)*a^2*e^5)*
abs(a)*abs(c)*abs(e) + (2*a*c^3*d^5*e + 9*a^2*c^2*d^3*e^3 - 5*a^3*c*d*e^5)
*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^3*d + sqrt(a^2*c^6*d^2 - (a*c^3*d
^2 + a^2*c^2*e^2)*a*c^3))/(a*c^3)))/((a^2*c^3*e + sqrt(-a*c)*a*c^3*d)*sqrt
(-c^2*d - sqrt(-a*c)*c*e)*abs(a)*abs(e)) + 1/4*((sqrt(-a*c)*c*d^3*e + 13*s
qrt(-a*c)*a*d*e^3)*a^2*e^2*abs(c) - (a*c^2*d^4*e - 4*a^2*c*d^2*e^3 - 5*a^3
*e^5)*abs(a)*abs(c)*abs(e) + (2*sqrt(-a*c)*a*c^2*d^5*e + 9*sqrt(-a*c)*a^2*
c*d^3*e^3 - 5*sqrt(-a*c)*a^3*d*e^5)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*
c^3*d - sqrt(a^2*c^6*d^2 - (a*c^3*d^2 + a^2*c^2*e^2)*a*c^3))/(a*c^3)))/((a
^2*c^3*d + sqrt(-a*c)*a^2*c^2*e)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(a)*abs(
e)) + 1/2*((e*x + d)^(3/2)*c^2*d^3*e - sqrt(e*x + d)*c^2*d^4*e - 3*(e*x +
d)^(3/2)*a*c*d*e^3 + sqrt(e*x + d)*a^2*e^5)/(((e*x + d)^2*c - 2*(e*x + d)*
c*d + c*d^2 + a*e^2)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 4192, normalized size of antiderivative = 6.91

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x)^(7/2)/(a + c*x^2)^2,x)
```

output
```
(((a^2*e^5 - c^2*d^4*e)*(d + e*x)^(1/2))/(2*a) + ((c^2*d^3*e - 3*a*c*d*e^3
)*(d + e*x)^(3/2))/(2*a))/(c^3*(d + e*x)^2 + c^3*d^2 + a*c^2*e^2 - 2*c^3*d
*(d + e*x)) - atan((a^2*e^10*(d + e*x)^(1/2)*((105*d*e^6)/(64*c^4) - d^7/(
16*a^3*c) - (35*d^3*e^4)/(32*a*c^3) - (35*d^5*e^2)/(64*a^2*c^2) - (25*e^7*
(-a^9*c^9)^(1/2))/(64*a^4*c^9) + (77*d^2*e^5*(-a^9*c^9)^(1/2))/(32*a^5*c^8
) + (35*d^4*e^3*(-a^9*c^9)^(1/2))/(64*a^6*c^7))^(1/2)*50i)/((885*d^5*e^9)/
2 + (491*a*d^3*e^11)/(2*c) + (329*c*d^7*e^7)/(2*a) - (50*a^2*d*e^13)/c^2 +
 (35*c^2*d^9*e^5)/(2*a^2) + (125*e^14*(-a^9*c^9)^(1/2))/(4*a^2*c^7) - (335
*d^2*e^12*(-a^9*c^9)^(1/2))/(2*a^3*c^6) - (204*d^4*e^10*(-a^9*c^9)^(1/2))/
(a^4*c^5) + (7*d^6*e^8*(-a^9*c^9)^(1/2))/(2*a^5*c^4) + (35*d^8*e^6*(-a^9*c
^9)^(1/2))/(4*a^6*c^3)) + (d^3*e^7*(-a^9*c^9)^(1/2)*(d + e*x)^(1/2)*((105*
d*e^6)/(64*c^4) - d^7/(16*a^3*c) - (35*d^3*e^4)/(32*a*c^3) - (35*d^5*e^2)/
(64*a^2*c^2) - (25*e^7*(-a^9*c^9)^(1/2))/(64*a^4*c^9) + (77*d^2*e^5*(-a^9*
c^9)^(1/2))/(32*a^5*c^8) + (35*d^4*e^3*(-a^9*c^9)^(1/2))/(64*a^6*c^7))^(1/
2)*308i)/((35*a^2*c^5*d^9*e^5)/2 + (329*a^3*c^4*d^7*e^7)/2 + (885*a^4*c^3*
d^5*e^9)/2 + (491*a^5*c^2*d^3*e^11)/2 - 50*a^6*c*d*e^13 + (125*a^2*e^14*(-
a^9*c^9)^(1/2))/(4*c^4) + (35*d^8*e^6*(-a^9*c^9)^(1/2))/(4*a^2) - (204*d^4
*e^10*(-a^9*c^9)^(1/2))/c^2 - (335*a*d^2*e^12*(-a^9*c^9)^(1/2))/(2*c^3) +
(7*d^6*e^8*(-a^9*c^9)^(1/2))/(2*a*c)) + (d^5*e^5*(-a^9*c^9)^(1/2)*(d + e*x
)^(1/2)*((105*d*e^6)/(64*c^4) - d^7/(16*a^3*c) - (35*d^3*e^4)/(32*a*c^3...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4002, normalized size of antiderivative = 6.59

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^(7/2)/(c*x^2+a)^2,x)
```

output
```
( - 16*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c*
d*e**2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)
*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(
c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c
**2*d**3 - 16*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c
*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sq
rt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*
a*c**2*d*e**2*x**2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*
d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(
2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*s
qrt(2)))*c**3*d**3*x**2 + 10*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) -
c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*s
qrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))
*a**3*e**4 - 18*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*
atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d
 + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c*d**2*
e**2 + 10*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((
sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + ...
```

3.184 $\int \frac{(d+ex)^{5/2}}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 533

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^2}\,dx = -\frac{de\sqrt{d+ex}}{2ac} - \frac{(ae-cdx)(d+ex)^{3/2}}{2ac\,(a+cx^2)} - \frac{e\left(cd^2+3ae^2+\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{7/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\left(cd^2+3ae^2+\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{7/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{e\left(cd^2+3ae^2-\sqrt{c}d\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{4\sqrt{2}ac^{7/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/2*d*e*(e*x+d)^(1/2)/a/c-1/2*(-c*d*x+a*e)*(e*x+d)^(3/2)/a/c/(c*x^2+a)-1/
8*e*(c*d^2+3*a*e^2+c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^
2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*
d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(7/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2
)+1/8*e*(c*d^2+3*a*e^2+c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(
a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^
2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(7/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^
(1/2)-1/8*e*(c*d^2+3*a*e^2-c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*
c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)
^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a/c^(7/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}c\sqrt{d+ex}(cd^2x-ae(2d+ex))}{a+cx^2} - i\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\left(2cd^2-i\sqrt{a}\sqrt{c}de+3ae^2\right)\arctan\left(\frac{\sqrt{-cd-}}{}\right.}{4a}$$

input
```
Integrate[(d + e*x)^(5/2)/(a + c*x^2)^2,x]
```

output
```
((2*Sqrt[a]*c*Sqrt[d + e*x]*(c*d^2*x - a*e*(2*d + e*x)))/(a + c*x^2) - I*S
qrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*(2*c*d^2 - I*Sqrt[a]*Sqrt[c]*d*e + 3*a*e
^2)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d +
 I*Sqrt[a]*e)] + I*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*(2*c*d^2 + I*Sqrt[a]
*Sqrt[c]*d*e + 3*a*e^2)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d
+ e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/(4*a^(3/2)*c^2)
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.50, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {495, 27, 653, 27, 654, 27, 1483, 27, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^2} \, dx$$

$$\Big\downarrow \; 495$$

$$\frac{\int \frac{\sqrt{d+ex}(2cd^2-cexd+3ae^2)}{2(cx^2+a)}dx}{2ac} - \frac{(d+ex)^{3/2}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{d+ex}(2cd^2-cexd+3ae^2)}{cx^2+a}dx}{4ac} - \frac{(d+ex)^{3/2}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 653$$

$$\frac{\frac{\int \frac{c\left(2d\left(cd^2+2ae^2\right)+e\left(cd^2+3ae^2\right)x\right)}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{c} - 2de\sqrt{d+ex}}{4ac} - \frac{(d+ex)^{3/2}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2d(cd^2+2ae^2)+e(cd^2+3ae^2)x}{\sqrt{d+ex}(cx^2+a)}dx - 2de\sqrt{d+ex}}{4ac} - \frac{(d+ex)^{3/2}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 654$$

$$\frac{2\int \frac{e(d(cd^2+ae^2)+(cd^2+3ae^2)(d+ex))}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex} - 2de\sqrt{d+ex}}{4ac} - \frac{(d+ex)^{3/2}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{2e\int \frac{d(cd^2+ae^2)+(cd^2+3ae^2)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex} - 2de\sqrt{d+ex}}{4ac} - \frac{(d+ex)^{3/2}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \; 1483$$

$$\frac{2e\left(\frac{\int \frac{\sqrt{cd^2+ae^2}\left(\sqrt{2}d\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(d\sqrt{cd^2+ae^2}-\frac{cd^2+3ae^2}{\sqrt{c}}\right)\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+ae^2\right)+\sqrt[4]{c}\left(c\cdots\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\cdots}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+c\cdots}}\cdots\right.}{4ac}\cdots$$

$$\frac{(d+ex)^{3/2}(ae-cdx)}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2e\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+ae^2\right)+\sqrt[4]{c}\left(cd^3+ae^2d-\cdots\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+\cdots}}}{\sqrt[4]{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+\cdots}}}\cdots\right.}{4ac}\cdots$$

$$\frac{(d+ex)^{3/2}(ae-cdx)}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2e\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+ae^2\right)+\sqrt[4]{c}\left(cd^3+ae^2d-\cdots\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+\cdots}}}{\sqrt[4]{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+\cdots}}}\cdots\right.}{4ac}\cdots$$

$$\frac{(d+ex)^{3/2}(ae-cdx)}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \; 1142$$

$$2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}+\frac{1}{2}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int -\cdots}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}$$

$\Big\downarrow$ 25

$$2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{1}{2}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \cdots}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}$$

$\Big\downarrow$ 27

$$2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{\cdots}{d+ex+\cdots}}{\cdots}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}$$

$\Big\downarrow$ 1083

$$2e\left(\frac{-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int\frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt[4]{c}}-\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\cdots}{\cdots}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}$$

$\downarrow$ 219

$$2e\left(\frac{-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int\frac{\sqrt{\sqrt{c}\cdots}}{d+ex+\frac{\sqrt{cd\cdots}}{\cdots}}}{\sqrt{2}\cdots}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}$$

$\downarrow$ 1103

$$2e\left(\frac{\frac{1}{2}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\log\left(\sqrt{c}(d+ex)-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}+\sqrt{cd^2+ae^2}\right)-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3a\cdots\right.}{\sqrt{\cdots}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{2ac\left(cx^2+a\right)}$$

input
```
Int[(d + e*x)^(5/2)/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*e - c*d*x)*(d + e*x)^(3/2))/(a*c*(a + c*x^2)) + (-2*d*e*Sqrt[d +
e*x] + 2*e*((-((Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c*d^2 + 3*a*e^2 + S
qrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d
+ Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]
*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + ((c*
d^2 + 3*a*e^2 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - S
qrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[
c]*(d + e*x)])/2)/(2*Sqrt[2]*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
) + (-((Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c^(3/2)*d^3 + a*Sqrt[c]*d*e
^2 + Sqrt[c*d^2 + a*e^2]*(c*d^2 + 3*a*e^2))*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*S
qrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d
^2 + a*e^2]])) + (c^(1/4)*(c*d^3 + a*d*e^2 - (Sqrt[c*d^2 + a*e^2]*(c*d^2 +
 3*a*e^2))/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2
]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(4*
a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$-\frac{\frac{e^{2}\left(c\,x^{2}+a\right)\left(c^{\frac{7}{2}} d\sqrt{a\,e^{2}+c\,d^{2}}+3e^{2} a\,c^{3}+c^{4} d^{2}\right) a \arctan\left(\frac{-2\sqrt{c}\,\sqrt{ex+d}+\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd}}{\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}}\right)}{2}+\frac{e^{2}\left(c\,x^{2}+a\right)\left(c^{\frac{7}{2}} d\sqrt{a\,e^{2}}\right.}{}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^(5/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*
(-1/2*e^2*(c*x^2+a)*(c^(7/2)*d*(a*e^2+c*d^2)^(1/2)+3*e^2*a*c^3+c^4*d^2)*a*
arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/
(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))+1/2
*e^2*(c*x^2+a)*(c^(7/2)*d*(a*e^2+c*d^2)^(1/2)+3*e^2*a*c^3+c^4*d^2)*a*arcta
n((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*
e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))+(4*(a*e^2
+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(1/8*(c*x^2+a
)*((-c^(5/2)*d*(a*e^2+c*d^2)^(1/2)+3*c^2*a*e^2+c^3*d^2)*((a*e^2+c*d^2)*c)^
(1/2)+(c^(7/2)*d*(a*e^2+c*d^2)^(1/2)-3*e^2*a*c^3-c^4*d^2)*d)*(2*((a*e^2+c*
d^2)*c)^(1/2)+2*c*d)^(1/2)*ln((-e*x-d)*c^(1/2)+(e*x+d)^(1/2)*(2*((a*e^2+c*
d^2)*c)^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))-1/8*(c*x^2+a)*((-c^(5/2)*d
*(a*e^2+c*d^2)^(1/2)+3*c^2*a*e^2+c^3*d^2)*((a*e^2+c*d^2)*c)^(1/2)+(c^(7/2)
*d*(a*e^2+c*d^2)^(1/2)-3*e^2*a*c^3-c^4*d^2)*d)*(2*((a*e^2+c*d^2)*c)^(1/2)+
2*c*d)^(1/2)*ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2
*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))-(a*e^2*x-c*d^2*x+2*a*d*e)*e*c^(7/2)*a*(e*
x+d)^(1/2)))/e/a^2/c^(9/2)/(c*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(427) = 854$.

Time = 0.20 (sec) , antiderivative size = 1383, normalized size of antiderivative = 2.59

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((a*c^2*x^2 + a^2*c)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4
+ a^3*c^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7))
)/(a^3*c^3))*log((20*c^3*d^6*e^3 + 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e^7 +
 81*a^3*e^9)*sqrt(e*x + d) + (5*a^2*c^3*d^3*e^4 + 9*a^3*c^2*d*e^6 - (2*a^3
*c^6*d^2 + 3*a^4*c^5*e^2)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*
e^10)/(a^3*c^7)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a^3*c
^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(a^3*
c^3))) - (a*c^2*x^2 + a^2*c)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*
e^4 + a^3*c^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c
^7)))/(a^3*c^3))*log((20*c^3*d^6*e^3 + 101*a*c^2*d^4*e^5 + 162*a^2*c*d^2*e
^7 + 81*a^3*e^9)*sqrt(e*x + d) - (5*a^2*c^3*d^3*e^4 + 9*a^3*c^2*d*e^6 - (2
*a^3*c^6*d^2 + 3*a^4*c^5*e^2)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*
a^2*e^10)/(a^3*c^7)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*e^4 + a
^3*c^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a^3*c^7)))/(
a^3*c^3))) + (a*c^2*x^2 + a^2*c)*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^
2*d*e^4 - a^3*c^3*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 + 81*a^2*e^10)/(a
^3*c^7)))/(a^3*c^3))*log((20*c^3*d^6*e^3 + 101*a*c^2*d^4*e^5 + 162*a^2*c*d
^2*e^7 + 81*a^3*e^9)*sqrt(e*x + d) + (5*a^2*c^3*d^3*e^4 + 9*a^3*c^2*d*e^6
+ (2*a^3*c^6*d^2 + 3*a^4*c^5*e^2)*sqrt(-(25*c^2*d^4*e^6 + 90*a*c*d^2*e^8 +
 81*a^2*e^10)/(a^3*c^7)))*sqrt(-(4*c^2*d^5 + 15*a*c*d^3*e^2 + 15*a^2*d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^2}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{\left(cx^2+a\right)^2}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 492, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{5/2}}{\left(a+cx^2\right)^2}\,dx = \frac{\left(2\,ac^4d^4e + 4\,a^2c^3d^2e^3 + (cd^2e + 3\,ae^3)a^2c^2e^2 - \left(\sqrt{-ac}c^2d^3e + \sqrt{-ac}acde^3\right)|a||c||e|\right) a}{4\left(a^2c^3e + \sqrt{-ac}ac^3d\right)\sqrt{-c^2d - \sqrt{-ac}ce}|a||e|}$$
$$+ \frac{\left(2\,ac^4d^4e + 4\,a^2c^3d^2e^3 + (cd^2e + 3\,ae^3)a^2c^2e^2 + \left(\sqrt{-ac}c^2d^3e + \sqrt{-ac}acde^3\right)|a||c||e|\right) \arctan\left(\frac{}{\sqrt{-\frac{ac^2d-\sqrt{a}}{}}}\right.}{4\left(a^2c^3e - \sqrt{-ac}ac^3d\right)\sqrt{-c^2d + \sqrt{-ac}ce}|a||e|}$$
$$+ \frac{(ex+d)^{\frac{3}{2}}cd^2e - \sqrt{ex+d}cd^3e - (ex+d)^{\frac{3}{2}}ae^3 - \sqrt{ex+d}ade^3}{2\left((ex+d)^2c - 2\,(ex+d)cd + cd^2 + ae^2\right)ac}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*a*c^4*d^4*e + 4*a^2*c^3*d^2*e^3 + (c*d^2*e + 3*a*e^3)*a^2*c^2*e^2 -
 (sqrt(-a*c)*c^2*d^3*e + sqrt(-a*c)*a*c*d*e^3)*abs(a)*abs(c)*abs(e))*arcta
n(sqrt(e*x + d)/sqrt(-(a*c^2*d + sqrt(a^2*c^4*d^2 - (a*c^2*d^2 + a^2*c*e^2
)*a*c^2))/(a*c^2)))/((a^2*c^3*e + sqrt(-a*c)*a*c^3*d)*sqrt(-c^2*d - sqrt(-
a*c)*c*e)*abs(a)*abs(e)) + 1/4*(2*a*c^4*d^4*e + 4*a^2*c^3*d^2*e^3 + (c*d^2
*e + 3*a*e^3)*a^2*c^2*e^2 + (sqrt(-a*c)*c^2*d^3*e + sqrt(-a*c)*a*c*d*e^3)*
abs(a)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d - sqrt(a^2*c^4*d
^2 - (a*c^2*d^2 + a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c^3*e - sqrt(-a*c)*a*
c^3*d)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(a)*abs(e)) + 1/2*((e*x + d)^(3/2)
*c*d^2*e - sqrt(e*x + d)*c*d^3*e - (e*x + d)^(3/2)*a*e^3 - sqrt(e*x + d)*a
*d*e^3)/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 + a*e^2)*a*c)
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 2031, normalized size of antiderivative = 3.81

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(a + c*x^2)^2,x)
```

output

```
- (((a*e^3 - c*d^2*e)*(d + e*x)^(3/2))/(2*a*c) + ((a*d*e^3 + c*d^3*e)*(d +
 e*x)^(1/2))/(2*a*c))/(c*(d + e*x)^2 + a*e^2 + c*d^2 - 2*c*d*(d + e*x)) -
2*atanh((18*a*e^8*(d + e*x)^(1/2)*(- d^5/(16*a^3*c) - (15*d*e^4)/(64*a*c^3
) - (15*d^3*e^2)/(64*a^2*c^2) - (9*e^5*(-a^9*c^7)^(1/2))/(64*a^5*c^7) - (5
*d^2*e^3*(-a^9*c^7)^(1/2))/(64*a^6*c^6))^(1/2))/((27*a*e^11)/(4*c^2) + (43
*d^4*e^7)/(4*a) + (15*d^2*e^9)/c + (5*c*d^6*e^5)/(2*a^2) - (9*d*e^10*(-a^9
*c^7)^(1/2))/(4*a^4*c^5) - (7*d^3*e^8*(-a^9*c^7)^(1/2))/(2*a^5*c^4) - (5*d
^5*e^6*(-a^9*c^7)^(1/2))/(4*a^6*c^3)) + (10*c*d^2*e^6*(d + e*x)^(1/2)*(- d
^5/(16*a^3*c) - (15*d*e^4)/(64*a*c^3) - (15*d^3*e^2)/(64*a^2*c^2) - (9*e^5
*(-a^9*c^7)^(1/2))/(64*a^5*c^7) - (5*d^2*e^3*(-a^9*c^7)^(1/2))/(64*a^6*c^6
))^(1/2))/((27*a*e^11)/(4*c^2) + (43*d^4*e^7)/(4*a) + (15*d^2*e^9)/c + (5*
c*d^6*e^5)/(2*a^2) - (9*d*e^10*(-a^9*c^7)^(1/2))/(4*a^4*c^5) - (7*d^3*e^8*
(-a^9*c^7)^(1/2))/(2*a^5*c^4) - (5*d^5*e^6*(-a^9*c^7)^(1/2))/(4*a^6*c^3))
+ (18*d*e^7*(-a^9*c^7)^(1/2)*(d + e*x)^(1/2)*(- d^5/(16*a^3*c) - (15*d*e^4
)/(64*a*c^3) - (15*d^3*e^2)/(64*a^2*c^2) - (9*e^5*(-a^9*c^7)^(1/2))/(64*a^
5*c^7) - (5*d^2*e^3*(-a^9*c^7)^(1/2))/(64*a^6*c^6))^(1/2))/((27*a^5*c*e^11
)/4 + (5*a^2*c^4*d^6*e^5)/2 + (43*a^3*c^3*d^4*e^7)/4 + 15*a^4*c^2*d^2*e^9
- (9*d*e^10*(-a^9*c^7)^(1/2))/(4*c^2) - (5*d^5*e^6*(-a^9*c^7)^(1/2))/(4*a^
2) - (7*d^3*e^8*(-a^9*c^7)^(1/2))/(2*a*c)) + (10*d^3*e^5*(-a^9*c^7)^(1/2)*
(d + e*x)^(1/2)*(- d^5/(16*a^3*c) - (15*d*e^4)/(64*a*c^3) - (15*d^3*e^2...
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 3158, normalized size of antiderivative = 5.92

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^(5/2)/(c*x^2+a)^2,x)
```

output

```
( - 6*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*e**
2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*d**2
 - 6*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(
2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqr
t(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*e**2*
x**2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*s
qrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)
*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c**2*
d**2*x**2 - 8*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*at
an((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d +
 e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*d*e**2 -
4*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqr
t(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt
(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*d**3 - 8*sqrt(c)*sqrt(
sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**
2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqr...
```

3.185 $\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 480

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2} \, dx = -\frac{(ae-cdx)\sqrt{d+ex}}{2ac(a+cx^2)} - \frac{e\left(\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{5/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{e\left(\sqrt{c}d+\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{5/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{e\left(\sqrt{c}d-\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{4\sqrt{2}ac^{5/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/2*(-c*d*x+a*e)*(e*x+d)^(1/2)/a/c/(c*x^2+a)-1/8*e*(c^(1/2)*d+(a*e^2+c*d^
2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e
*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(5/4)/(-c
^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/8*e*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))*
arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2
))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(5/4)/(-c^(1/2)*d+(
a*e^2+c*d^2)^(1/2))^(1/2)-1/8*e*(c^(1/2)*d-(a*e^2+c*d^2)^(1/2))*arctanh(2^
(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+
c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a/c^(5/4)/(c^(1/2)*d+(a*e^2+c*d^2)^
(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}\sqrt{c}(-ae+cdx)\sqrt{d+ex}}{a+cx^2} - i\left(2\sqrt{c}d - i\sqrt{a}e\right)\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{4a^{3/2}c^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a + c*x^2)^2,x]
```

output
```
((2*Sqrt[a]*Sqrt[c]*(-(a*e) + c*d*x)*Sqrt[d + e*x])/(a + c*x^2) - I*(2*Sqr
t[c]*d - I*Sqrt[a]*e)*Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*
d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)] + I*(2
*Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[
-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/(
4*a^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.62, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {495, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}dx$$

$$\Big\downarrow \text{ 495}$$

$$\frac{\int \frac{2cd^2+cexd+ae^2}{2\sqrt{d+ex}(cx^2+a)}dx}{2ac}-\frac{\sqrt{d+ex}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2cd^2+cexd+ae^2}{\sqrt{d+ex}(cx^2+a)}dx}{4ac}-\frac{\sqrt{d+ex}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \text{ 654}$$

$$\frac{\int \frac{e(cd^2+c(d+ex)d+ae^2)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}-\frac{\sqrt{d+ex}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{cd^2+c(d+ex)d+ae^2}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}-\frac{\sqrt{d+ex}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{e\left(\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+ae^2\right)+\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d}}{\sqrt[4]{c}}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}}}\right.}{2ac}$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{2ac(a+cx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$e\left(\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+ae^2\right)+\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}\ldots\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+e\ldots}}{\sqrt[4]{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c\ldots}}}\right.$$

$$\overline{\qquad 2ac \qquad}$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{2ac\left(a+cx^2\right)}$$

$\big\downarrow$ 1142

$$e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int -\ldots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 25

$$e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \ldots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 27

$$e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{\sqrt{\sqrt{c}d \cdots}}{d+ex+\frac{\sqrt{cd^2 \cdots}}{\sqrt{\cdots}}}}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} \cdots\right.$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 1083

$$e\left(\frac{\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{\cdots}{-d+\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} \cdots\right.$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 219

$$e\left(\frac{\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\text{arctanh}\cdots}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}} \cdots\right.$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{2ac\left(cx^2+a\right)}$$

$\big\downarrow$ 1103

$$e\left(\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\left(\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\sqrt[4]{c}\left(-\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+cd^2\right)\log}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{2ac\left(a+cx^2\right)}$$

input

```
Int[(d + e*x)^(3/2)/(a + c*x^2)^2,x]
```

output

```
-1/2*((a*e - c*d*x)*Sqrt[d + e*x])/(a*c*(a + c*x^2)) + (e*((-((c^(1/4)*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c*d^2 + a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 +
 a*e^2])*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]
])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e
^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*(c*d^2 + a*e^2 -
 Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/
2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]]) + (-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c*d^2 + a*e^2
+ Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d
 + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]
*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (c^(
1/4)*(c*d^2 + a*e^2 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^
2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
 Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c
]*d + Sqrt[c*d^2 + a*e^2]])))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.42

method	result
pseudoelliptic	$\dfrac{-\dfrac{e^{2}\left(c\,x^{2}+a\right)\left(c^{\frac{5}{2}}\sqrt{a\,e^{2}+c\,d^{2}}+d\,c^{3}\right)a\arctan\left(\dfrac{-2\sqrt{c}\,\sqrt{ex+d}+\sqrt{2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+2cd}}{\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}}\right)}{2}+\dfrac{e^{2}\left(c\,x^{2}+a\right)\left(c^{\frac{5}{2}}\sqrt{a\,e^{2}+c\,d^{2}}+d\,c^{3}\right)}{}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^(3/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1/2*e^2*(c*x^2+a)*(c^(5/2)*(a*e^2+c*d^2)^(1/2)+d*c^3)*a*arctan((-2*c
^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*
d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))+1/2*e^2*(c*x^2+
a)*(c^(5/2)*(a*e^2+c*d^2)^(1/2)+d*c^3)*a*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(
2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*(
(a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))+(1/8*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d
)^(1/2)*(c*x^2+a)*((-(a*e^2+c*d^2)^(1/2)*c^(3/2)+c^2*d)*((a*e^2+c*d^2)*c)^
(1/2)+d*(c^(5/2)*(a*e^2+c*d^2)^(1/2)-d*c^3))*ln((-e*x-d)*c^(1/2)+(e*x+d)^(
1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))-1/8*(2*(
(a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*(c*x^2+a)*((-(a*e^2+c*d^2)^(1/2)*c^(3/
2)+c^2*d)*((a*e^2+c*d^2)*c)^(1/2)+d*(c^(5/2)*(a*e^2+c*d^2)^(1/2)-d*c^3))*l
n(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a
*e^2+c*d^2)^(1/2))+(c*d*x-a*e)*e*a*(e*x+d)^(1/2)*c^(5/2))*(4*(a*e^2+c*d^2)
^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1
/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)/e/a^2/c^(7/2)/(c*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}\,dx = \frac{(ac^2x^2+a^2c)\sqrt{-\frac{a^3c^2\sqrt{-\frac{e^6}{a^3c^5}}+4\,cd^3+3\,ade^2}{a^3c^2}}\log\left(\left(4\,cd^2e^3+ae^5\right)\sqrt{ex+d}+\left(2\,a^3c^4d\sqrt{-\frac{e}{a^3}}\right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((a*c^2*x^2 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-e^6/(a^3*c^5)) + 4*c*d^3 + 3
*a*d*e^2)/(a^3*c^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*
d*sqrt(-e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt(-(a^3*c^2*sqrt(-e^6/(a^3*c^5)) +
4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^2 + a^2*c)*sqrt(-(a^3*c^2*sqrt
(-e^6/(a^3*c^5)) + 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))*log((4*c*d^2*e^3 + a*e^
5)*sqrt(e*x + d) - (2*a^3*c^4*d*sqrt(-e^6/(a^3*c^5)) + a^2*c*e^4)*sqrt(-(a
^3*c^2*sqrt(-e^6/(a^3*c^5)) + 4*c*d^3 + 3*a*d*e^2)/(a^3*c^2))) - (a*c^2*x^
2 + a^2*c)*sqrt((a^3*c^2*sqrt(-e^6/(a^3*c^5)) - 4*c*d^3 - 3*a*d*e^2)/(a^3*
c^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) + (2*a^3*c^4*d*sqrt(-e^6/(a^
3*c^5)) - a^2*c*e^4)*sqrt((a^3*c^2*sqrt(-e^6/(a^3*c^5)) - 4*c*d^3 - 3*a*d*
e^2)/(a^3*c^2))) + (a*c^2*x^2 + a^2*c)*sqrt((a^3*c^2*sqrt(-e^6/(a^3*c^5))
- 4*c*d^3 - 3*a*d*e^2)/(a^3*c^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d)
- (2*a^3*c^4*d*sqrt(-e^6/(a^3*c^5)) - a^2*c*e^4)*sqrt((a^3*c^2*sqrt(-e^6/(
a^3*c^5)) - 4*c*d^3 - 3*a*d*e^2)/(a^3*c^2))) + 4*(c*d*x - a*e)*sqrt(e*x +
d))/(a*c^2*x^2 + a^2*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}\,dx = \frac{\left(2\,ac^3d^3e+2\,a^2c^2de^3-\left(\sqrt{-ac}cd^2e+\sqrt{-ac}ae^3\right)|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{ac^2d+\sqrt{a^2c^4d^2-\left(ac^2d \cdots}}{ac^2}}}\cdots\right)}{4\left(a^2c^2e+\sqrt{-ac}ac^2d\right)\sqrt{-c^2d-\sqrt{-ac}ce}|a||e|} + \frac{\left(2\,ac^3d^3e+2\,a^2c^2de^3+\left(\sqrt{-ac}cd^2e+\sqrt{-ac}ae^3\right)|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{ac^2d-\sqrt{a^2c^4d^2-\left(ac^2d^2+a^2ce^2\right)ac^2}}{ac^2}}}\right)}{4\left(a^2c^2e-\sqrt{-ac}ac^2d\right)\sqrt{-c^2d+\sqrt{-ac}ce}|a||e|} + \frac{(ex+d)^{\frac{3}{2}}cde-\sqrt{ex+d}cd^2e-\sqrt{ex+d}ae^3}{2\left((ex+d)^2c-2\,(ex+d)cd+cd^2+ae^2\right)ac}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*a*c^3*d^3*e + 2*a^2*c^2*d*e^3 - (sqrt(-a*c)*c*d^2*e + sqrt(-a*c)*a*
e^3)*abs(a)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d + sqrt(a^2*
c^4*d^2 - (a*c^2*d^2 + a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c^2*e + sqrt(-a*
c)*a*c^2*d)*sqrt(-c^2*d - sqrt(-a*c)*c*e)*abs(a)*abs(e)) + 1/4*(2*a*c^3*d^
3*e + 2*a^2*c^2*d*e^3 + (sqrt(-a*c)*c*d^2*e + sqrt(-a*c)*a*e^3)*abs(a)*abs
(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d - sqrt(a^2*c^4*d^2 - (a*c^
2*d^2 + a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c^2*e - sqrt(-a*c)*a*c^2*d)*sqr
t(-c^2*d + sqrt(-a*c)*c*e)*abs(a)*abs(e)) + 1/2*((e*x + d)^(3/2)*c*d*e - s
qrt(e*x + d)*c*d^2*e - sqrt(e*x + d)*a*e^3)/(((e*x + d)^2*c - 2*(e*x + d)*
c*d + c*d^2 + a*e^2)*a*c)
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}\,dx = -\frac{\frac{(c\,d^2\,e+a\,e^3)\,\sqrt{d+e\,x}}{2\,a\,c}-\frac{d\,e\,(d+e\,x)^{3/2}}{2\,a}}{c\,(d+e\,x)^2+a\,e^2+c\,d^2-2\,c\,d\,(d+e\,x)}$$
$$-2\,\mathrm{atanh}\left(\frac{2\,c\,e^6\,\sqrt{d+e\,x}\,\sqrt{-\frac{d^3}{16\,a^3\,c}-\frac{3\,d\,e^2}{64\,a^2\,c^2}-\frac{e^3\,\sqrt{-a^9\,c^5}}{64\,a^6\,c^5}}}{\frac{d\,e^7}{2\,a}+\frac{c\,d^3\,e^5}{2\,a^2}+\frac{e^8\,\sqrt{-a^9\,c^5}}{4\,a^5\,c^3}+\frac{d^2\,e^6\,\sqrt{-a^9\,c^5}}{4\,a^6\,c^2}}-\frac{2\,d\,e^5\,\sqrt{-a^9\,c^5}\,\sqrt{d+e\,x}\,\sqrt{-\frac{d^3}{16\,a^3\,c}-\frac{3\,d\,e^2}{64\,a^2\,c^2}-\frac{e}{}}}{\frac{e^8\,\sqrt{-a^9\,c^5}}{4\,c^2}+\frac{a^3\,c^2\,d^3\,e^5}{2}+\frac{a^4\,c\,d\,e^7}{2}+\frac{d^2\,e^6\,\sqrt{-a^9}}{4\,a\,c}}\right.$$
$$-2\,\mathrm{atanh}\left(\frac{2\,c\,e^6\,\sqrt{d+e\,x}\,\sqrt{\frac{e^3\,\sqrt{-a^9\,c^5}}{64\,a^6\,c^5}-\frac{3\,d\,e^2}{64\,a^2\,c^2}-\frac{d^3}{16\,a^3\,c}}}{\frac{d\,e^7}{2\,a}+\frac{c\,d^3\,e^5}{2\,a^2}-\frac{e^8\,\sqrt{-a^9\,c^5}}{4\,a^5\,c^3}-\frac{d^2\,e^6\,\sqrt{-a^9\,c^5}}{4\,a^6\,c^2}}-\frac{2\,d\,e^5\,\sqrt{-a^9\,c^5}\,\sqrt{d+e\,x}\,\sqrt{\frac{e^3\,\sqrt{-a^9\,c^5}}{64\,a^6\,c^5}-\frac{3\,d\,e^2}{64\,a^2\,c^2}-\frac{}{16}}}{\frac{e^8\,\sqrt{-a^9\,c^5}}{4\,c^2}-\frac{a^3\,c^2\,d^3\,e^5}{2}-\frac{a^4\,c\,d\,e^7}{2}+\frac{d^2\,e^6\,\sqrt{-a^9\,c^5}}{4\,a\,c}}\right.$$

input
```
int((d + e*x)^(3/2)/(a + c*x^2)^2,x)
```

output
```
- (((a*e^3 + c*d^2*e)*(d + e*x)^(1/2))/(2*a*c) - (d*e*(d + e*x)^(3/2))/(2*
a))/(c*(d + e*x)^2 + a*e^2 + c*d^2 - 2*c*d*(d + e*x)) - 2*atanh((2*c*e^6*(
d + e*x)^(1/2)*(- d^3/(16*a^3*c) - (3*d*e^2)/(64*a^2*c^2) - (e^3*(-a^9*c^5
)^(1/2))/(64*a^6*c^5))^(1/2))/((d*e^7)/(2*a) + (c*d^3*e^5)/(2*a^2) + (e^8*
(-a^9*c^5)^(1/2))/(4*a^5*c^3) + (d^2*e^6*(-a^9*c^5)^(1/2))/(4*a^6*c^2)) -
(2*d*e^5*(-a^9*c^5)^(1/2)*(d + e*x)^(1/2)*(- d^3/(16*a^3*c) - (3*d*e^2)/(6
4*a^2*c^2) - (e^3*(-a^9*c^5)^(1/2))/(64*a^6*c^5))^(1/2))/((e^8*(-a^9*c^5)^
(1/2))/(4*c^2) + (a^3*c^2*d^3*e^5)/2 + (a^4*c*d*e^7)/2 + (d^2*e^6*(-a^9*c^
5)^(1/2))/(4*a*c)))*(-(e^3*(-a^9*c^5)^(1/2) + 4*a^3*c^4*d^3 + 3*a^4*c^3*d*
e^2)/(64*a^6*c^5))^(1/2) - 2*atanh((2*c*e^6*(d + e*x)^(1/2)*((e^3*(-a^9*c^
5)^(1/2))/(64*a^6*c^5) - (3*d*e^2)/(64*a^2*c^2) - d^3/(16*a^3*c))^(1/2))/(
(d*e^7)/(2*a) + (c*d^3*e^5)/(2*a^2) - (e^8*(-a^9*c^5)^(1/2))/(4*a^5*c^3) -
 (d^2*e^6*(-a^9*c^5)^(1/2))/(4*a^6*c^2)) - (2*d*e^5*(-a^9*c^5)^(1/2)*(d +
e*x)^(1/2)*((e^3*(-a^9*c^5)^(1/2))/(64*a^6*c^5) - (3*d*e^2)/(64*a^2*c^2) -
 d^3/(16*a^3*c))^(1/2))/((e^8*(-a^9*c^5)^(1/2))/(4*c^2) - (a^3*c^2*d^3*e^5
)/2 - (a^4*c*d*e^7)/2 + (d^2*e^6*(-a^9*c^5)^(1/2))/(4*a*c)))*(-(4*a^3*c^4*
d^3 - e^3*(-a^9*c^5)^(1/2) + 3*a^4*c^3*d*e^2)/(64*a^6*c^5))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 2303, normalized size of antiderivative = 4.80

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+a)^2,x)
```

output
```
( - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*d -
4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*
atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d
 + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c**2*d*x**2
- 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(s
qrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sq
rt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*e**2 - 4*sqrt(c)*sq
rt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*
e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqr
t(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*d**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt
(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
+ c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d*
*2) - c*d)*sqrt(2)))*a*c*e**2*x**2 - 4*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqr
t(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)
*sqrt(2)))*c**2*d**2*x**2 + 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e*
*2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d
)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2...
```

3.186 $\int \frac{\sqrt{d+ex}}{(a+cx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 471

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(a+cx^2)^2} dx &= \frac{x\sqrt{d+ex}}{2a(a+cx^2)} - \frac{e\left(1+\frac{\sqrt{c}d}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{3/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\ &+ \frac{e\left(1+\frac{\sqrt{c}d}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}ac^{3/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\ &- \frac{e\left(1-\frac{\sqrt{c}d}{\sqrt{cd^2+ae^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{4\sqrt{2}ac^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\end{aligned}$$

output
```
1/2*x*(e*x+d)^(1/2)/a/(c*x^2+a)-1/8*e*(1+c^(1/2)*d/(a*e^2+c*d^2)^(1/2))*ar
ctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))
/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(3/4)/(-c^(1/2)*d+(a*
e^2+c*d^2)^(1/2))^(1/2)+1/8*e*(1+c^(1/2)*d/(a*e^2+c*d^2)^(1/2))*arctan(((c
^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/
2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(3/4)/(-c^(1/2)*d+(a*e^2+c*d^
2)^(1/2))^(1/2)-1/8*e*(1-c^(1/2)*d/(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^
(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(
1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a/c^(3/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}x\sqrt{d+ex}}{a+cx^2} + \frac{i(2\sqrt{c}d+i\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{c}\sqrt{-cd-i\sqrt{a}\sqrt{c}e}} - \frac{i(2\sqrt{c}d-i\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{\sqrt{c}\sqrt{-cd+i\sqrt{a}\sqrt{c}e}}}{4a^{3/2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a + c*x^2)^2,x]
```

output
```
((2*Sqrt[a]*x*Sqrt[d + e*x])/(a + c*x^2) + (I*(2*Sqrt[c]*d + I*Sqrt[a]*e)*
ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*S
qrt[a]*e)])/(Sqrt[c]*Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]) - (I*(2*Sqrt[c]*d
 - I*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/
(Sqrt[c]*d - I*Sqrt[a]*e)])/(Sqrt[c]*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]))/
(4*a^(3/2))
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.44, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {494, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^2}dx$$

$$\Big\downarrow \text{494}$$

$$\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}-\frac{\int -\frac{2d+ex}{2\sqrt{d+ex}(cx^2+a)}dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2d+ex}{\sqrt{d+ex}(cx^2+a)}dx}{4a}+\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\int \frac{e(2d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}+\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{2d+ex}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}+\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{1483}$$

$$\frac{e\left(\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\sqrt[4]{c}\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)}{2a}+\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\left(\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\sqrt[4]{c}\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)}{2a}+\frac{x\sqrt{d+ex}}{2a(a+cx^2)}$$

$\downarrow$ 1142

$$\frac{e\left(\frac{\frac{\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)^{3/2}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}-\frac{1}{2}\sqrt[4]{c}\left(d-\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}\right)\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\cdots}}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.}{\cdots}$$

$$\frac{x\sqrt{d+ex}}{2a(a+cx^2)}$$

$\downarrow$ 25

$$\frac{e\left(\frac{\frac{\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)^{3/2}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}+\frac{1}{2}\sqrt[4]{c}\left(d-\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}\right)\int \frac{\sqrt{2}\left(\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\cdots}}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.}{\cdots}$$

$$\frac{x\sqrt{d+ex}}{2a(a+cx^2)}$$

$\downarrow$ 27

$$e\left(\frac{\frac{\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)^{3/2}\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}+\frac{\left(d-\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+\ldots}}{\sqrt[4]{c}}}\ldots}{\sqrt{2}}\ldots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\ldots\right.$$

$$\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{1083}$$

$$e\left(\frac{\frac{\left(d-\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt{2}\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)^{3/2}\int \frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}-\frac{\sqrt{2}\ldots}{}\right.\ldots}{\sqrt{c}}\ldots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\ldots\right.$$

$$\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{219}$$

$$e\left(\frac{\frac{\left(d-\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\ldots}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\ldots\right.}{\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\ldots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\ldots\right.$$

$$\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

$$\Big\downarrow \text{1103}$$

$$e\left(\frac{-\frac{\left(\sqrt{ae^2+cd^2}+\sqrt{c}d\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)}{\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\sqrt[4]{c}\left(d-\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\right.}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\frac{x\sqrt{d+ex}}{2a\left(a+cx^2\right)}$$

input
```
Int[Sqrt[d + e*x]/(a + c*x^2)^2,x]
```

output
```
(x*Sqrt[d + e*x])/(2*a*(a + c*x^2)) + (e*((-(((Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2])^(3/2)*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e
^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 +
a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])) - (c^(1/4)*(d
- Sqrt[c*d^2 + a*e^2]/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*S
qrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2
)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e
^2]]) + (-(((Sqrt[c]*d + Sqrt[c*d^2 + a*e^2])^(3/2)*ArcTanh[(c^(1/4)*((Sqr
t[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(S
qrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d -
Sqrt[c*d^2 + a*e^2]])) + (c^(1/4)*(d - Sqrt[c*d^2 + a*e^2]/Sqrt[c])*Log[Sq
rt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*
Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e
^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex+d}}{\left(c\,x^{2}+a\right)^{2}}dx$$

input
```
int((e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

output
```
int((e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1371 vs. $2(368) = 736$.

Time $= 0.10$ (sec) , antiderivative size $= 1371$, normalized size of antiderivative $= 2.91$

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((a*c*x^2 + a^2)*sqrt(-(4*c*d^3 + 3*a*d*e^2 + (a^3*c^2*d^2 + a^4*c*e^2
)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2
 + a^4*c*e^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*x + d) + (a^2*c*d*e^4 - (2
*a^3*c^4*d^4 + 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(-e^6/(a^3*c^5*d^4 + 2
*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt(-(4*c*d^3 + 3*a*d*e^2 + (a^3*c^2*d^
2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))
/(a^3*c^2*d^2 + a^4*c*e^2))) - (a*c*x^2 + a^2)*sqrt(-(4*c*d^3 + 3*a*d*e^2
+ (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a
^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))*log((4*c*d^2*e^3 + a*e^5)*sqrt(e*
x + d) - (a^2*c*d*e^4 - (2*a^3*c^4*d^4 + 3*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*
sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))*sqrt(-(4*c*d^3
 + 3*a*d*e^2 + (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^
4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))) + (a*c*x^2 + a^2)*s
qrt(-(4*c*d^3 + 3*a*d*e^2 - (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e^6/(a^3*c^5*d
^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*e^2))*log((4*
c*d^2*e^3 + a*e^5)*sqrt(e*x + d) + (a^2*c*d*e^4 + (2*a^3*c^4*d^4 + 3*a^4*c
^3*d^2*e^2 + a^5*c^2*e^4)*sqrt(-e^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5
*c^3*e^4)))*sqrt(-(4*c*d^3 + 3*a*d*e^2 - (a^3*c^2*d^2 + a^4*c*e^2)*sqrt(-e
^6/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4)))/(a^3*c^2*d^2 + a^4*c*
e^2))) - (a*c*x^2 + a^2)*sqrt(-(4*c*d^3 + 3*a*d*e^2 - (a^3*c^2*d^2 + a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^2}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^2}\,dx = \frac{(ex+d)^{\frac{3}{2}}e-\sqrt{ex+d}de}{2\left((ex+d)^2c-2\,(ex+d)cd+cd^2+ae^2\right)a} + \frac{\left(2\,acd^2e|c|+a^2e^3|c|-\sqrt{-ac}de|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{acd+\sqrt{a^2c^2d^2-\left(acd^2+a^2e^2\right)ac}}{ac}}}\right)}{4\left(a^2ce+\sqrt{-ac}acd\right)\sqrt{-c^2d-\sqrt{-ac}ce}|a||e|} + \frac{\left(2\,acd^2e|c|+a^2e^3|c|+\sqrt{-ac}de|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{acd-\sqrt{a^2c^2d^2-\left(acd^2+a^2e^2\right)ac}}{ac}}}\right)}{4\left(a^2ce-\sqrt{-ac}acd\right)\sqrt{-c^2d+\sqrt{-ac}ce}|a||e|}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*((e*x + d)^(3/2)*e - sqrt(e*x + d)*d*e)/(((e*x + d)^2*c - 2*(e*x + d)*
c*d + c*d^2 + a*e^2)*a) + 1/4*(2*a*c*d^2*e*abs(c) + a^2*e^3*abs(c) - sqrt(
-a*c)*d*e*abs(a)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c*d + sqrt(a
^2*c^2*d^2 - (a*c*d^2 + a^2*e^2)*a*c))/(a*c)))/((a^2*c*e + sqrt(-a*c)*a*c*
d)*sqrt(-c^2*d - sqrt(-a*c)*c*e)*abs(a)*abs(e)) + 1/4*(2*a*c*d^2*e*abs(c)
+ a^2*e^3*abs(c) + sqrt(-a*c)*d*e*abs(a)*abs(c)*abs(e))*arctan(sqrt(e*x +
d)/sqrt(-(a*c*d - sqrt(a^2*c^2*d^2 - (a*c*d^2 + a^2*e^2)*a*c))/(a*c)))/((a
^2*c*e - sqrt(-a*c)*a*c*d)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(a)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 2380, normalized size of antiderivative = 5.05

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(a + c*x^2)^2,x)
```

output
```
atan((((8*c^3*d*e^3 - 64*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(e^3*(-a^9*c^3)^(1/
2) + 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 + a^7*c^3*e^2)))^(1
/2))*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c
^4*d^2 + a^7*c^3*e^2)))^(1/2) + ((a*c^2*e^4 - 4*c^3*d^2*e^2)*(d + e*x)^(1/
2))/a^2)*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a
^6*c^4*d^2 + a^7*c^3*e^2)))^(1/2)*1i - ((8*c^3*d*e^3 + 64*a*c^4*d*e^2*(d +
 e*x)^(1/2)*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64
*(a^6*c^4*d^2 + a^7*c^3*e^2)))^(1/2))*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*
d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 + a^7*c^3*e^2)))^(1/2) - ((a*c^2*e
^4 - 4*c^3*d^2*e^2)*(d + e*x)^(1/2))/a^2)*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*
c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 + a^7*c^3*e^2)))^(1/2)*1i)/((4
*c^2*d^2*e^3 + a*c*e^5)/(4*a^3) + ((8*c^3*d*e^3 - 64*a*c^4*d*e^2*(d + e*x)
^(1/2)*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6
*c^4*d^2 + a^7*c^3*e^2)))^(1/2))*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*d^3 +
 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 + a^7*c^3*e^2)))^(1/2) + ((a*c^2*e^4 -
4*c^3*d^2*e^2)*(d + e*x)^(1/2))/a^2)*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*d
^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 + a^7*c^3*e^2)))^(1/2) + ((8*c^3*d*
e^3 + 64*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(e^3*(-a^9*c^3)^(1/2) + 4*a^3*c^3*d
^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 + a^7*c^3*e^2)))^(1/2))*(-(e^3*(-a^
9*c^3)^(1/2) + 4*a^3*c^3*d^3 + 3*a^4*c^2*d*e^2)/(64*(a^6*c^4*d^2 + a^7*...
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 3166, normalized size of antiderivative = 6.72

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^2} dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

output
```
( - 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*e**
2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*d**2
 - 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(
2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqr
t(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*e**2*
x**2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*s
qrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)
*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c**2*
d**2*x**2 - 4*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*at
an((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d +
 e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*d*e**2 -
4*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqr
t(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt
(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*d**3 - 4*sqrt(c)*sqrt(
sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**
2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqr...
```

3.187 $\int \frac{1}{\sqrt{d+ex}(a+cx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 566

$$
\begin{aligned}
\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx
&= \frac{(ae+cdx)\sqrt{d+ex}}{2a\,(cd^2+ae^2)\,(a+cx^2)} \\
&- \frac{e\left(cd^2+3ae^2+\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a\sqrt[4]{c}\,(cd^2+ae^2)^{3/2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&+ \frac{e\left(cd^2+3ae^2+\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a\sqrt[4]{c}\,(cd^2+ae^2)^{3/2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&+ \frac{e\left(cd^2+3ae^2-\sqrt{c}d\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{4\sqrt{2}a\sqrt[4]{c}\,(cd^2+ae^2)^{3/2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}
\end{aligned}
$$

output
```
1/2*(c*d*x+a*e)*(e*x+d)^(1/2)/a/(a*e^2+c*d^2)/(c*x^2+a)-1/8*e*(c*d^2+3*a*e
^2+c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^
(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2
))*2^(1/2)/a/c^(1/4)/(a*e^2+c*d^2)^(3/2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^
(1/2)+1/8*e*(c*d^2+3*a*e^2+c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)
*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(
a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/c^(1/4)/(a*e^2+c*d^2)^(3/2)/(-c^(1/2)
*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/8*e*(c*d^2+3*a*e^2-c^(1/2)*d*(a*e^2+c*d^2)
^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x
+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a/c^(1/4)/(a*e^2+
c*d^2)^(3/2)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}(ae+cdx)\sqrt{d+ex}}{(cd^2+ae^2)(a+cx^2)}+\frac{i(2\sqrt{c}d+3i\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{(\sqrt{c}d+i\sqrt{a}e)\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}-\frac{i(2\sqrt{c}d-3i\sqrt{a}e)\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{(\sqrt{c}d-i\sqrt{a}e)\sqrt{-cd+i\sqrt{a}\sqrt{c}e}}}{4a^{3/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)^2),x]
```

output
```
((2*Sqrt[a]*(a*e + c*d*x)*Sqrt[d + e*x])/((c*d^2 + a*e^2)*(a + c*x^2)) + (
I*(2*Sqrt[c]*d + (3*I)*Sqrt[a]*e)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*
e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/((Sqrt[c]*d + I*Sqrt[a]*e)*S
qrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]) - (I*(2*Sqrt[c]*d - (3*I)*Sqrt[a]*e)*Ar
cTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqr
t[a]*e)])/((Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]))/
(4*a^(3/2))
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {496, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^2\sqrt{d+ex}}dx$$

$$\Big\downarrow \ 496$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)}-\frac{\int -\frac{2cd^2+cexd+3ae^2}{2\sqrt{d+ex}(cx^2+a)}dx}{2a(ae^2+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2cd^2+cexd+3ae^2}{\sqrt{d+ex}(cx^2+a)}dx}{4a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \ 654$$

$$\frac{\int \frac{e(cd^2+c(d+ex)d+3ae^2)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{e\int \frac{cd^2+c(d+ex)d+3ae^2}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \ 1483$$

$$\frac{e\left(\frac{\int \frac{\sqrt{2}\left(cd^2+3ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+3ae^2\right)+\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+}}\right.}{2a(ae^2+cd^2)}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{e\left(\frac{\int \frac{\sqrt{2}\left(cd^2+3ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+3ae^2\right)+\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+\cdots}\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{\cdots}}{\sqrt[4]{c}}}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\cdots}}\cdots\right)}{2a\left(ae^2+cd^2\right)}\cdots$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+$$

$$\frac{e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int -\cdots\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+$$

$$\frac{e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \cdots\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+$$

$$e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{\sqrt{\sqrt{}}}{d+ex+\frac{\sqrt{c}}{}}}{}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+$$

$$e\left(\frac{\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int -}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+$$

$$e\left(\frac{\frac{\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\arctan}{\sqrt{\sqrt{c}d-\sqrt{cd^2+a}}}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+$$

$$e\left(\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{1}{2}\sqrt[4]{c}\left(cd^2-\sqrt{c}\sqrt{cd^2+ae^2}d+3ae^2\right)\text{lo}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

input

```
Int[1/(Sqrt[d + e*x]*(a + c*x^2)^2),x]
```

output

```
((a*e + c*d*x)*Sqrt[d + e*x])/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*((-((
c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c*d^2 + 3*a*e^2 + Sqrt[c]*d
*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c
*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqr
t[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*(c*
d^2 + 3*a*e^2 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - S
qrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[
c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d +
Sqrt[c*d^2 + a*e^2]]) + (-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*
(c*d^2 + 3*a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*((Sqrt[
2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqr
t[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2
+ a*e^2]]) + (c^(1/4)*(c*d^2 + 3*a*e^2 - Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Lo
g[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^
2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 +
 a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(2*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex+d}\,(c\,x^2+a)^2}dx$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

output
```
int(1/(e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3267 vs. 2(458) = 916.

Time = 0.25 (sec) , antiderivative size = 3267, normalized size of antiderivative = 5.77

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx = \int \frac{1}{(cx^2+a)^2\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^2*sqrt(e*x + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*((a*c*d^2*e + a^2*e^3)^2*sqrt(-a*c)*d*e*abs(c) + (a*c^2*d^4*e + 4*a^2
*c*d^2*e^3 + 3*a^3*e^5)*abs(-a*c*d^2*e - a^2*e^3)*abs(c) + (2*sqrt(-a*c)*a
*c^3*d^7*e + 7*sqrt(-a*c)*a^2*c^2*d^5*e^3 + 8*sqrt(-a*c)*a^3*c*d^3*e^5 + 3
*sqrt(-a*c)*a^4*d*e^7)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d^3 + a^2
*c*d*e^2 + sqrt((a*c^2*d^3 + a^2*c*d*e^2)^2 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4)*(a*c^2*d^2 + a^2*c*e^2)))/(a*c^2*d^2 + a^2*c*e^2)))/((a^2*c^3*
d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4 + sqrt(-a*c)*a^2*c^2*d^4*e + 2*sqrt(
-a*c)*a^3*c*d^2*e^3 + sqrt(-a*c)*a^4*e^5)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*ab
s(-a*c*d^2*e - a^2*e^3)) - 1/4*((a*c*d^2*e + a^2*e^3)^2*c*d*e*abs(c) + (sq
rt(-a*c)*c^2*d^4*e + 4*sqrt(-a*c)*a*c*d^2*e^3 + 3*sqrt(-a*c)*a^2*e^5)*abs(
-a*c*d^2*e - a^2*e^3)*abs(c) + (2*a*c^4*d^7*e + 7*a^2*c^3*d^5*e^3 + 8*a^3*
c^2*d^3*e^5 + 3*a^4*c*d*e^7)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d^3
 + a^2*c*d*e^2 - sqrt((a*c^2*d^3 + a^2*c*d*e^2)^2 - (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4)*(a*c^2*d^2 + a^2*c*e^2)))/(a*c^2*d^2 + a^2*c*e^2)))/((a^
2*c^3*d^4*e + 2*a^3*c^2*d^2*e^3 + a^4*c*e^5 + sqrt(-a*c)*a*c^3*d^5 + 2*sqr
t(-a*c)*a^2*c^2*d^3*e^2 + sqrt(-a*c)*a^3*c*d*e^4)*sqrt(-c^2*d - sqrt(-a*c)
*c*e)*abs(-a*c*d^2*e - a^2*e^3)) + 1/2*((e*x + d)^(3/2)*c*d*e - sqrt(e*x +
 d)*c*d^2*e + sqrt(e*x + d)*a*e^3)/((a*c*d^2 + a^2*e^2)*((e*x + d)^2*c - 2
*(e*x + d)*c*d + c*d^2 + a*e^2))
```

Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 5300, normalized size of antiderivative = 9.36

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^2*(d + e*x)^(1/2)),x)
```

output
```
(((a*e^3 - c*d^2*e)*(d + e*x)^(1/2))/(2*a*(a*e^2 + c*d^2)) + (c*d*e*(d + e
*x)^(3/2))/(2*a*(a*e^2 + c*d^2)))/(c*(d + e*x)^2 + a*e^2 + c*d^2 - 2*c*d*(
d + e*x)) - atan(((((192*a^5*c^3*e^7 + 64*a^3*c^5*d^4*e^3 + 256*a^4*c^4*d^
2*e^5)/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) + ((d + e*x)^(1/2)*(6
4*a^5*c^4*d*e^6 + 64*a^3*c^6*d^5*e^2 + 128*a^4*c^5*d^3*e^4)*(-(9*a*e^5*(-a
^9*c)^(1/2) + 4*a^3*c^3*d^5 + 5*c*d^2*e^3*(-a^9*c)^(1/2) + 15*a^4*c^2*d^3*
e^2 + 15*a^5*c*d*e^4)/(64*(a^9*c*e^6 + a^6*c^4*d^6 + 3*a^7*c^3*d^4*e^2 + 3
*a^8*c^2*d^2*e^4)))^(1/2))/(a^4*e^4 + a^2*c^2*d^4 + 2*a^3*c*d^2*e^2))*(-(9
*a*e^5*(-a^9*c)^(1/2) + 4*a^3*c^3*d^5 + 5*c*d^2*e^3*(-a^9*c)^(1/2) + 15*a^
4*c^2*d^3*e^2 + 15*a^5*c*d*e^4)/(64*(a^9*c*e^6 + a^6*c^4*d^6 + 3*a^7*c^3*d
^4*e^2 + 3*a^8*c^2*d^2*e^4)))^(1/2) + ((d + e*x)^(1/2)*(9*a^2*c^3*e^6 + 4*
c^5*d^4*e^2 + 11*a*c^4*d^2*e^4))/(a^4*e^4 + a^2*c^2*d^4 + 2*a^3*c*d^2*e^2)
)*(-(9*a*e^5*(-a^9*c)^(1/2) + 4*a^3*c^3*d^5 + 5*c*d^2*e^3*(-a^9*c)^(1/2) +
 15*a^4*c^2*d^3*e^2 + 15*a^5*c*d*e^4)/(64*(a^9*c*e^6 + a^6*c^4*d^6 + 3*a^7
*c^3*d^4*e^2 + 3*a^8*c^2*d^2*e^4)))^(1/2)*1i - (((192*a^5*c^3*e^7 + 64*a^3
*c^5*d^4*e^3 + 256*a^4*c^4*d^2*e^5)/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)) - ((d + e*x)^(1/2)*(64*a^5*c^4*d*e^6 + 64*a^3*c^6*d^5*e^2 + 128*a^
4*c^5*d^3*e^4)*(-(9*a*e^5*(-a^9*c)^(1/2) + 4*a^3*c^3*d^5 + 5*c*d^2*e^3*(-a
^9*c)^(1/2) + 15*a^4*c^2*d^3*e^2 + 15*a^5*c*d*e^4)/(64*(a^9*c*e^6 + a^6*c^
4*d^6 + 3*a^7*c^3*d^4*e^2 + 3*a^8*c^2*d^2*e^4)))^(1/2))/(a^4*e^4 + a^2*...
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 4039, normalized size of antiderivative = 7.14

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+a)^2,x)
```

output

```
( - 8*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c*d
*e**2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*
sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c
)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c*
*2*d**3 - 8*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d
)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt
(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*
c**2*d*e**2*x**2 - 4*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d*
*2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2)
 - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)))*c**3*d**3*x**2 - 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d
)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt
(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*
*3*e**4 - 10*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*ata
n((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d +
e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c*d**2*e**
2 - 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt
(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x)...
```

3.188 $\int \frac{1}{(d+ex)^{3/2}(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 638

$$
\begin{aligned}
\int \frac{1}{(d+ex)^{3/2}(a+cx^2)^2}\,dx &= \frac{e(cd^2-5ae^2)}{2a(cd^2+ae^2)^2\sqrt{d+ex}} \\
&+ \frac{ae+cdx}{2a(cd^2+ae^2)\sqrt{d+ex}(a+cx^2)} \\
&- \frac{\sqrt[4]{c}e\left(cd^2-5ae^2+\frac{\sqrt{c}d(cd^2+13ae^2)}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a(cd^2+ae^2)^2\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&+ \frac{\sqrt[4]{c}e\left(cd^2-5ae^2+\frac{\sqrt{c}d(cd^2+13ae^2)}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a(cd^2+ae^2)^2\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&- \frac{\sqrt[4]{c}e\left(cd^2-5ae^2-\frac{\sqrt{c}d(cd^2+13ae^2)}{\sqrt{cd^2+ae^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{4\sqrt{2}a(cd^2+ae^2)^2\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}
\end{aligned}
$$

output
```
1/2*e*(-5*a*e^2+c*d^2)/a/(a*e^2+c*d^2)^2/(e*x+d)^(1/2)+1/2*(c*d*x+a*e)/a/(
a*e^2+c*d^2)/(e*x+d)^(1/2)/(c*x^2+a)-1/8*c^(1/4)*e*(c*d^2-5*a*e^2+c^(1/2)*
d*(13*a*e^2+c*d^2)/(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(
1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2)
)^(1/2))*2^(1/2)/a/(a*e^2+c*d^2)^2/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+
1/8*c^(1/4)*e*(c*d^2-5*a*e^2+c^(1/2)*d*(13*a*e^2+c*d^2)/(a*e^2+c*d^2)^(1/2
))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(
1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a/(a*e^2+c*d^2)^2/(-
c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-1/8*c^(1/4)*e*(c*d^2-5*a*e^2-c^(1/2)*
d*(13*a*e^2+c*d^2)/(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d
+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*
x+d)))*2^(1/2)/a/(a*e^2+c*d^2)^2/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.53

$$\int \frac{1}{(d+ex)^{3/2}\,(a+cx^2)^2}\,dx = \frac{\frac{2\sqrt{a}\left(-4a^2e^3+c^2d^2x(d+ex)+ace\left(2d^2+dex-5e^2x^2\right)\right)}{(cd^2+ae^2)^2\sqrt{d+ex}(a+cx^2)}+\frac{i\left(2cd+5i\sqrt{a}\sqrt{c}e\right)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+e}}{\sqrt{c}d+i\sqrt{a}e}\right.}{\left(\sqrt{c}d+i\sqrt{a}e\right)^2\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}}{4a^{3/2}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)^2),x]
```

output
```
((2*Sqrt[a]*(-4*a^2*e^3 + c^2*d^2*x*(d + e*x) + a*c*e*(2*d^2 + d*e*x - 5*e
^2*x^2)))/((c*d^2 + a*e^2)^2*Sqrt[d + e*x]*(a + c*x^2)) + (I*(2*c*d + (5*I
)*Sqrt[a]*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e
*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/((Sqrt[c]*d + I*Sqrt[a]*e)^2*Sqrt[-(c*d)
- I*Sqrt[a]*Sqrt[c]*e]) - (I*(2*c*d - (5*I)*Sqrt[a]*Sqrt[c]*e)*ArcTan[(Sqr
t[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])
/((Sqrt[c]*d - I*Sqrt[a]*e)^2*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]))/(4*a^(3
/2))
```

Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.43, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {496, 27, 655, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^2 (d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 496}$$

$$\frac{ae+cdx}{2a(a+cx^2)\sqrt{d+ex}(ae^2+cd^2)}-\frac{\int -\frac{2cd^2+3cexd+5ae^2}{2(d+ex)^{3/2}(cx^2+a)}dx}{2a(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2cd^2+3cexd+5ae^2}{(d+ex)^{3/2}(cx^2+a)}dx}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)\sqrt{d+ex}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 655}$$

$$\frac{\frac{\int \frac{c\left(2d\left(cd^2+4ae^2\right)+e\left(cd^2-5ae^2\right)x\right)}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{ae^2+cd^2}+\frac{2e\left(cd^2-5ae^2\right)}{\sqrt{d+ex}(ae^2+cd^2)}}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)\sqrt{d+ex}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{c\int \frac{2d\left(cd^2+4ae^2\right)+e\left(cd^2-5ae^2\right)x}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{ae^2+cd^2}+\frac{2e\left(cd^2-5ae^2\right)}{\sqrt{d+ex}(ae^2+cd^2)}}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)\sqrt{d+ex}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 654}$$

$$\frac{\frac{2c\int \frac{e\left(d\left(cd^2+13ae^2\right)+\left(cd^2-5ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}+\frac{2e\left(cd^2-5ae^2\right)}{\sqrt{d+ex}(ae^2+cd^2)}}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)\sqrt{d+ex}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2ce\int \frac{d\left(cd^2+13ae^2\right)+\left(cd^2-5ae^2\right)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}+\frac{2e\left(cd^2-5ae^2\right)}{\sqrt{d+ex}(ae^2+cd^2)}}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)\sqrt{d+ex}(ae^2+cd^2)}$$

$\Big\downarrow$ 1483

$$\frac{\frac{2ce\left(\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+13ae^2\right)+\sqrt[4]{c}\left(\frac{\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}}{\sqrt{c}}-d\left(cd^2+13ae^2\right)\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+13ae^2\right)+\sqrt[4]{c}\ldots}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\ldots\right)}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae\ldots}}\right.}{ae^2+cd^2}\ldots}{4a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{2a\left(a+cx^2\right)\sqrt{d+ex}\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{2ce\left(\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+13ae^2\right)+\sqrt[4]{c}\left(\frac{\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}}{\sqrt{c}}-d\left(cd^2+13ae^2\right)\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cd^2+13ae^2\right)+\sqrt[4]{c}\ldots}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\ldots}}{2\sqrt{2}\sqrt{c}\sqrt{ae\ldots}}\right.}{ae^2+cd^2}\ldots}{4a\left(ae^2+cd^2\right)}$$

$$\frac{ae+cdx}{2a\left(a+cx^2\right)\sqrt{d+ex}\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 1142

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(cx^2+a\right)}+$$

$$\frac{\frac{2e\left(cd^2-5ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2ce\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}+\ldots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\ldots}}\right.}{\ldots}}{\ldots}$$

$\Big\downarrow$ 25

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(cx^2+a\right)}+$$

$$\frac{2e\left(cd^2-5ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2ce\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}-\cdots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{\cdots}}}\cdots\right)}{\cdots}$$

$\downarrow$ 27

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(cx^2+a\right)}+$$

$$\frac{2e\left(cd^2-5ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2ce\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}-\cdots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{c\cdots}}}\cdots\right)}{\cdots}$$

$\downarrow$ 1083

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(cx^2+a\right)}+$$

$$\frac{2e\left(cd^2-5ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2ce\left(\frac{-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\int \frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{c\cdots}}}{\sqrt[4]{c}}\cdots\right)}{\sqrt{c}}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c\cdots}}}\cdots\right)}{\cdots}$$

$\downarrow$ 219

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(cx^2+a\right)}+\frac{\frac{2e\left(cd^2-5ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2ce\left(-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^{3/2}d^3+13a\sqrt{c}e^2d+\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\cdots}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2\cdots}}}\cdots}{\cdots}$$

$\downarrow$ 1103

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)\sqrt{d+ex}\left(cx^2+a\right)}+\frac{\frac{2e\left(cd^2-5ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2ce\left(\frac{\frac{1}{2}\sqrt[4]{c}\left(\frac{\left(cd^2-5ae^2\right)\sqrt{cd^2+ae^2}}{\sqrt{c}}-d\left(cd^2+13ae^2\right)\right)\log\left(\sqrt{c}(d+ex)-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}+\sqrt{cd^2+ae^2}\right)-\frac{\sqrt{\sqrt{c}d+\sqrt{c\cdots}}}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\cdots}}\cdots\right)}{\cdots}}{\cdots}$$

input
```
Int[1/((d + e*x)^(3/2)*(a + c*x^2)^2),x]
```

output
```
(a*e + c*d*x)/(2*a*(c*d^2 + a*e^2)*Sqrt[d + e*x]*(a + c*x^2)) + ((2*e*(c*d
^2 - 5*a*e^2))/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (2*c*e*((-((Sqrt[Sqrt[c]*
d + Sqrt[c*d^2 + a*e^2]]*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 + (c*d^2 - 5*a*
e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sq
rt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d -
 Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]))
+ (c^(1/4)*(((c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2])/Sqrt[c] - d*(c*d^2 + 1
3*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[
c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*
Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-((Sqrt[Sqrt
[c]*d + Sqrt[c*d^2 + a*e^2]]*(c^(3/2)*d^3 + 13*a*Sqrt[c]*d*e^2 + (c*d^2 -
5*a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d +
Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d
- Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]))
 + (c^(1/4)*(c*d^3 + 13*a*d*e^2 - ((c*d^2 - 5*a*e^2)*Sqrt[c*d^2 + a*e^2])/
Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c
*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*S
qrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(c*d^2 + a*e^2
))/(4*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$-\frac{5\left(\sqrt{ex+d}\,\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}\left(\left(\left(\left(c\,x^{2}+a\right)\left(a\,e^{2}-\frac{c\,d^{2}}{5}\right)\sqrt{a\,e^{2}+c\,d^{2}}+\frac{13\left(\frac{a\left(13e^{2}x^{2}+d^{2}\right)c^{\frac{3}{2}}}{13}+\right.}{5}\right.\right.\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/16/c^(1/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c
*d)^(1/2)/(a*e^2+c*d^2)^(5/2)/(e*x+d)^(1/2)*((e*x+d)^(1/2)*(4*(a*e^2+c*d^2
)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(((c*x^2+a)*(a*e^2-
1/5*c*d^2)*(a*e^2+c*d^2)^(1/2)+13/5*(1/13*a*(13*e^2*x^2+d^2)*c^(3/2)+c^(1/
2)*a^2*e^2+1/13*c^(5/2)*d^2*x^2)*d)*((a*e^2+c*d^2)*c)^(1/2)-((c*x^2+a)*c*(
a*e^2-1/5*c*d^2)*(a*e^2+c*d^2)^(1/2)+13/5*(1/13*a*(13*e^2*x^2+d^2)*c^(5/2)
+a^2*e^2*c^(3/2)+1/13*c^(7/2)*d^2*x^2)*d)*d)*(ln((-e*x-d)*c^(1/2)+(e*x+d)^
(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))-ln(c^(1
/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c
*d^2)^(1/2)))*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)-4*e*((e*(c*x^2+a)*c*
(a*e^2-1/5*c*d^2)*(arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(
1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2
)-2*c*d)^(1/2))-arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)
+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*
c*d)^(1/2)))*(e*x+d)^(1/2)-8/5*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*
d^2)*c)^(1/2)-2*c*d)^(1/2)*(-1/2*e*(1/2*d*e*x-5/2*e^2*x^2+d^2)*a*c^(3/2)-1
/4*d^2*x*(e*x+d)*c^(5/2)+a^2*e^3*c^(1/2)))*(a*e^2+c*d^2)^(1/2)-13/5*(e*x+d
)^(1/2)*e*(1/13*a*(13*e^2*x^2+d^2)*c^(5/2)+a^2*e^2*c^(3/2)+1/13*c^(7/2)*d^
2*x^2)*d*(arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*
d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5698 vs. 2(532) = 1064.

Time = 0.91 (sec) , antiderivative size = 5698, normalized size of antiderivative = 8.93

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+cx^2\right)^2(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+a)**2,x)
```

output
```
Integral(1/((a + c*x**2)**2*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{(cx^2+a)^2(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^2*(e*x + d)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(532) = 1064$.

Time = 0.27 (sec) , antiderivative size = 1397, normalized size of antiderivative = 2.19

$$\int \frac{1}{(d+ex)^{3/2} (a+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*((a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)^2*(c*d^2*e - 5*a*e^3)*abs(c
) - (sqrt(-a*c)*c^3*d^7*e + 15*sqrt(-a*c)*a*c^2*d^5*e^3 + 27*sqrt(-a*c)*a^
2*c*d^3*e^5 + 13*sqrt(-a*c)*a^3*d*e^7)*abs(a*c^2*d^4*e + 2*a^2*c*d^2*e^3 +
 a^3*e^5)*abs(c) + 2*(a*c^6*d^12*e + 8*a^2*c^5*d^10*e^3 + 22*a^3*c^4*d^8*e
^5 + 28*a^4*c^3*d^6*e^7 + 17*a^5*c^2*d^4*e^9 + 4*a^6*c*d^2*e^11)*abs(c))*a
rctan(sqrt(e*x + d)/sqrt(-(a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4 + s
qrt((a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)^2 - (a*c^3*d^6 + 3*a^2*c
^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/((a^2*c^4*d^8*e
+ 4*a^3*c^3*d^6*e^3 + 6*a^4*c^2*d^4*e^5 + 4*a^5*c*d^2*e^7 + a^6*e^9 + sqrt
(-a*c)*a*c^4*d^9 + 4*sqrt(-a*c)*a^2*c^3*d^7*e^2 + 6*sqrt(-a*c)*a^3*c^2*d^5
*e^4 + 4*sqrt(-a*c)*a^4*c*d^3*e^6 + sqrt(-a*c)*a^5*d*e^8)*sqrt(-c^2*d - sq
rt(-a*c)*c*e)*abs(a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)) + 1/4*((a*c^2*
d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)^2*(c*d^2*e - 5*a*e^3)*abs(c) + (sqrt(-a
*c)*c^3*d^7*e + 15*sqrt(-a*c)*a*c^2*d^5*e^3 + 27*sqrt(-a*c)*a^2*c*d^3*e^5
+ 13*sqrt(-a*c)*a^3*d*e^7)*abs(a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*ab
s(c) + 2*(a*c^6*d^12*e + 8*a^2*c^5*d^10*e^3 + 22*a^3*c^4*d^8*e^5 + 28*a^4*
c^3*d^6*e^7 + 17*a^5*c^2*d^4*e^9 + 4*a^6*c*d^2*e^11)*abs(c))*arctan(sqrt(e
*x + d)/sqrt(-(a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4 - sqrt((a*c^3*d
^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)^2 - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^...
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 8777, normalized size of antiderivative = 13.76

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^2*(d + e*x)^(3/2)),x)
```

output
```
atan((((-(4*a^3*c^4*d^7 - 25*a^2*e^7*(-a^9*c)^(1/2) + 35*a^4*c^3*d^5*e^2 +
 70*a^5*c^2*d^3*e^4 + 35*c^2*d^4*e^3*(-a^9*c)^(1/2) - 105*a^6*c*d*e^6 + 15
4*a*c*d^2*e^5*(-a^9*c)^(1/2))/(64*(a^11*e^10 + a^6*c^5*d^10 + 5*a^10*c*d^2
*e^8 + 5*a^7*c^4*d^8*e^2 + 10*a^8*c^3*d^6*e^4 + 10*a^9*c^2*d^4*e^6)))^(1/2
)*((d + e*x)^(1/2)*(-(4*a^3*c^4*d^7 - 25*a^2*e^7*(-a^9*c)^(1/2) + 35*a^4*c
^3*d^5*e^2 + 70*a^5*c^2*d^3*e^4 + 35*c^2*d^4*e^3*(-a^9*c)^(1/2) - 105*a^6*
c*d*e^6 + 154*a*c*d^2*e^5*(-a^9*c)^(1/2))/(64*(a^11*e^10 + a^6*c^5*d^10 +
5*a^10*c*d^2*e^8 + 5*a^7*c^4*d^8*e^2 + 10*a^8*c^3*d^6*e^4 + 10*a^9*c^2*d^4
*e^6)))^(1/2)*(2048*a^16*c^4*d*e^22 + 2048*a^6*c^14*d^21*e^2 + 20480*a^7*c
^13*d^19*e^4 + 92160*a^8*c^12*d^17*e^6 + 245760*a^9*c^11*d^15*e^8 + 430080
*a^10*c^10*d^13*e^10 + 516096*a^11*c^9*d^11*e^12 + 430080*a^12*c^8*d^9*e^1
4 + 245760*a^13*c^7*d^7*e^16 + 92160*a^14*c^6*d^5*e^18 + 20480*a^15*c^5*d^
3*e^20) + 3328*a^14*c^4*d*e^21 + 256*a^5*c^13*d^19*e^3 + 5376*a^6*c^12*d^1
7*e^5 + 33792*a^7*c^11*d^15*e^7 + 107520*a^8*c^10*d^13*e^9 + 204288*a^9*c^
9*d^11*e^11 + 247296*a^10*c^8*d^9*e^13 + 193536*a^11*c^7*d^7*e^15 + 95232*
a^12*c^6*d^5*e^17 + 26880*a^13*c^5*d^3*e^19) + (d + e*x)^(1/2)*(128*a^3*c^
13*d^18*e^2 - 800*a^12*c^4*e^20 + 1760*a^4*c^12*d^16*e^4 + 10240*a^5*c^11*
d^14*e^6 + 30848*a^6*c^10*d^12*e^8 + 52480*a^7*c^9*d^10*e^10 + 51008*a^8*c
^8*d^8*e^12 + 25600*a^9*c^7*d^6*e^14 + 3200*a^10*c^6*d^4*e^16 - 2432*a^11*
c^5*d^2*e^18))*(-(4*a^3*c^4*d^7 - 25*a^2*e^7*(-a^9*c)^(1/2) + 35*a^4*c^...
```

Reduce [B] (verification not implemented)

Time = 194.48 (sec) , antiderivative size = 5249, normalized size of antiderivative = 8.23

$$\int \frac{1}{(d+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+a)^2,x)
```

output

```
(10*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
 - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) -
2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2
)))*a**3*e**4 - 18*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a
*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) +
c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) - c*d)*sqrt(2)))*a**2*c*d**2*e**2 + 10*sqrt(d + e*x)*sqrt(a*e**2 + c*d**
2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sq
rt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c
)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c*e**4*x**2 - 4*sqrt(d + e*x
)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*
atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d
 + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*c**2*d**4
- 18*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) -
 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(
2)))*a*c**2*d**2*e**2*x**2 - 4*sqrt(d + e*x)*sqrt(a*e**2 + c*d**2)*sqrt(sq
rt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2
+ c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) - c*d)*sqrt(2)))*c**3*d**4*x**2 - 16*sqrt(c)*sqrt(d + e*x...
```

3.189 $\int \frac{1}{(d+ex)^{5/2}(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 731

$$\int \frac{1}{(d+ex)^{5/2}(a+cx^2)^2}\,dx = \frac{e(3cd^2-7ae^2)}{6a(cd^2+ae^2)^2(d+ex)^{3/2}} + \frac{cde(cd^2-19ae^2)}{2a(cd^2+ae^2)^3\sqrt{d+ex}} + \frac{ae+cdx}{2a(cd^2+ae^2)(d+ex)^{3/2}(a+cx^2)}$$
$$- \frac{c^{3/4}e\left(\sqrt{c}d(cd^2-19ae^2)+\frac{c^2d^4+34acd^2e^2-7a^2e^4}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a(cd^2+ae^2)^3\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$+ \frac{c^{3/4}e\left(\sqrt{c}d(cd^2-19ae^2)+\frac{c^2d^4+34acd^2e^2-7a^2e^4}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{4\sqrt{2}a(cd^2+ae^2)^3\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$- \frac{c^{3/4}e\left(\sqrt{c}d(cd^2-19ae^2)-\frac{c^2d^4+34acd^2e^2-7a^2e^4}{\sqrt{cd^2+ae^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{4\sqrt{2}a(cd^2+ae^2)^3\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
1/6*e*(-7*a*e^2+3*c*d^2)/a/(a*e^2+c*d^2)^2/(e*x+d)^(3/2)+1/2*c*d*e*(-19*a*
e^2+c*d^2)/a/(a*e^2+c*d^2)^3/(e*x+d)^(1/2)+1/2*(c*d*x+a*e)/a/(a*e^2+c*d^2)
/(e*x+d)^(3/2)/(c*x^2+a)-1/8*c^(3/4)*e*(c^(1/2)*d*(-19*a*e^2+c*d^2)+(-7*a^
2*e^4+34*a*c*d^2*e^2+c^2*d^4)/(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e
^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c
*d^2)^(1/2))^(1/2))*2^(1/2)/a/(a*e^2+c*d^2)^3/(-c^(1/2)*d+(a*e^2+c*d^2)^(1
/2))^(1/2)+1/8*c^(3/4)*e*(c^(1/2)*d*(-19*a*e^2+c*d^2)+(-7*a^2*e^4+34*a*c*d
^2*e^2+c^2*d^4)/(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2
))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(
1/2))*2^(1/2)/a/(a*e^2+c*d^2)^3/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-1/8
*c^(3/4)*e*(c^(1/2)*d*(-19*a*e^2+c*d^2)-(-7*a^2*e^4+34*a*c*d^2*e^2+c^2*d^4
)/(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1
/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a/
(a*e^2+c*d^2)^3/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.52

$$\int \frac{1}{(d+ex)^{5/2}\,(a+cx^2)^2}\,dx = \frac{-\frac{2\sqrt{a}\left(4a^3e^5-3c^3d^3x(d+ex)^2+a^2ce^3\left(55d^2+54dex+7e^2x^2\right)+ac^2de\left(-9d^3-9d^2ex+61de^2x^2+57e^3x^3\right)\right.}{(cd^2+ae^2)^3(d+ex)^{3/2}(a+cx^2)}}{}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(a + c*x^2)^2),x]
```

output
```
((-2*Sqrt[a]*(4*a^3*e^5 - 3*c^3*d^3*x*(d + e*x)^2 + a^2*c*e^3*(55*d^2 + 54
*d*e*x + 7*e^2*x^2) + a*c^2*d*e*(-9*d^3 - 9*d^2*e*x + 61*d*e^2*x^2 + 57*e^
3*x^3)))/((c*d^2 + a*e^2)^3*(d + e*x)^(3/2)*(a + c*x^2)) + (3*Sqrt[-(c*d)
- I*Sqrt[a]*Sqrt[c]*e]*((-2*I)*c*d + 7*Sqrt[a]*Sqrt[c]*e)*ArcTan[(Sqrt[-(c
*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/(Sqr
t[c]*d + I*Sqrt[a]*e)^4 + (3*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*((2*I)*c*d
 + 7*Sqrt[a]*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d
+ e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/(Sqrt[c]*d - I*Sqrt[a]*e)^4)/(12*a^(3/
2))
```

Rubi [A] (verified)

Time = 3.58 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.39, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {496, 27, 655, 27, 655, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^2 (d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{ae+cdx}{2a(a+cx^2)(d+ex)^{3/2}(ae^2+cd^2)}-\frac{\int -\frac{2cd^2+5cexd+7ae^2}{2(d+ex)^{5/2}(cx^2+a)}dx}{2a(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2cd^2+5cexd+7ae^2}{(d+ex)^{5/2}(cx^2+a)}dx}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{655}$$

$$\frac{\frac{\int \frac{c\left(2d\left(cd^2+6ae^2\right)+e\left(3cd^2-7ae^2\right)x\right)}{(d+ex)^{3/2}\left(cx^2+a\right)}dx}{ae^2+cd^2}+\frac{2e(3cd^2-7ae^2)}{3(d+ex)^{3/2}(ae^2+cd^2)}}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{c\int \frac{2d\left(cd^2+6ae^2\right)+e\left(3cd^2-7ae^2\right)x}{(d+ex)^{3/2}\left(cx^2+a\right)}dx}{ae^2+cd^2}+\frac{2e(3cd^2-7ae^2)}{3(d+ex)^{3/2}(ae^2+cd^2)}}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{655}$$

$$\frac{\frac{c\left(\frac{\int \frac{2c^2d^4+15ace^2d^2+ce\left(cd^2-19ae^2\right)xd-7a^2e^4}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{ae^2+cd^2}+\frac{2de\left(cd^2-19ae^2\right)}{\sqrt{d+ex}\left(ae^2+cd^2\right)}\right)}{ae^2+cd^2}+\frac{2e(3cd^2-7ae^2)}{3(d+ex)^{3/2}(ae^2+cd^2)}}{4a(ae^2+cd^2)}+\frac{ae+cdx}{2a(a+cx^2)(d+ex)^{3/2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{c\left(\frac{2\int \frac{e\left(c^2d^4+34ace^2d^2+c\left(cd^2-19ae^2\right)(d+ex)d-7a^2e^4\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}+\frac{2de\left(cd^2-19ae^2\right)}{\sqrt{d+ex}\left(ae^2+cd^2\right)}\right)}{ae^2+cd^2}+\frac{2e(3cd^2-7ae^2)}{3(d+ex)^{3/2}(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{2a\left(a+cx^2\right)(d+ex)^{3/2}\left(ae^2+cd^2\right)}$$

$$\Big\downarrow 27$$

$$\frac{\frac{c\left(\frac{2e\int \frac{c^2d^4+34ace^2d^2+c\left(cd^2-19ae^2\right)(d+ex)d-7a^2e^4}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{ae^2+cd^2}+\frac{2de\left(cd^2-19ae^2\right)}{\sqrt{d+ex}\left(ae^2+cd^2\right)}\right)}{ae^2+cd^2}+\frac{2e(3cd^2-7ae^2)}{3(d+ex)^{3/2}(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}+\frac{ae+cdx}{2a\left(a+cx^2\right)(d+ex)^{3/2}\left(ae^2+cd^2\right)}$$

$$\Big\downarrow 1483$$

$$\frac{c\left(\frac{2e\left(\frac{\int \frac{\sqrt{2}\left(c^2d^4+34ace^2d^2-7a^2e^4\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(c^2d^4+34ace^2d^2-\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}d-7a^2e^4\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ \cdots}}}{\cdots}\right)}{ae^2+cd^2}\cdots\right)}{ae^2+cd^2} \over 4a\left(a\cdots\right.}$$

$$\frac{ae+cdx}{2a\left(a+cx^2\right)(d+ex)^{3/2}\left(ae^2+cd^2\right)}$$

$$\Big\downarrow 27$$

$$c\left(\frac{2e\left(\frac{\int \frac{\sqrt{2}\left(c^2d^4+34ace^2d^2-7a^2e^4\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(c^2d^4+34ace^2d^2-\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}d-7a^2e^4\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ \cdots}}}{\cdots}}{\cdots}\right)}{ae^2+cd^2}\cdots\right)$$

$$\overline{\qquad ae^2+cd^2 \cdots}$$

$$4a\,(a \cdots$$

$$\frac{ae+cdx}{2a\left(a+cx^2\right)(d+ex)^{3/2}\left(ae^2+cd^2\right)}$$

$\downarrow$ 1142

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(cx^2+a\right)}+$$

$$\frac{\frac{2e\left(3cd^2-7ae^2\right)}{3(cd^2+ae^2)(d+ex)^{3/2}}+\frac{c\left(\frac{2de\left(cd^2-19ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^2d^4+34ace^2d^2+\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}d-7a^2e^4\right)\int \frac{\cdots}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{\cdots}}{\cdots}}}{\sqrt{2}}\cdots\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(cx^2+a\right)}+$$

$$\frac{2e(3cd^2-7ae^2)}{3(cd^2+ae^2)(d+ex)^{3/2}}+\frac{c\left(\frac{2de\left(cd^2-19ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^2d^4+34ace^2d^2+\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}d-7a^2e^4\right)\int\frac{}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\cdots}}{\sqrt{2}}\cdots\right)}{\cdots}\right)}{\cdots}$$

$\downarrow$ 27

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(cx^2+a\right)}+$$

$$\frac{2e(3cd^2-7ae^2)}{3(cd^2+ae^2)(d+ex)^{3/2}}+\frac{c\left(\frac{2de\left(cd^2-19ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^2d^4+34ace^2d^2+\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}d-7a^2e^4\right)\int\frac{}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\cdots}}{\cdots}}{\sqrt{2}}\cdots\right)}{\cdots}\right)}{\cdots}$$

$\downarrow$ 1083

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(cx^2+a\right)}+$$

$$\frac{\frac{2e\left(3cd^2-7ae^2\right)}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}+\frac{c\left(\frac{2de\left(cd^2-19ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2e\left(\frac{\left(c^2d^4+34ace^2d^2-\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}\,d-7a^2e^4\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+e\cdots}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae\cdots}}}{\sqrt[4]{c}}\cdots}}{\sqrt{2}}\cdots\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

$\downarrow$ 219

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(cx^2+a\right)}+$$

$$\frac{\frac{2e\left(3cd^2-7ae^2\right)}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}+\frac{c\left(\frac{2de\left(cd^2-19ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2e\left(\frac{\left(c^2d^4+34ace^2d^2-\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}\,d-7a^2e^4\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+e\cdots}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae\cdots}}}{\sqrt[4]{c}}\cdots}}{\sqrt{2}}\cdots\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

$\downarrow$ 1103

$$\frac{ae+cdx}{2a\left(cd^2+ae^2\right)(d+ex)^{3/2}\left(cx^2+a\right)}+$$

$$\frac{\frac{2e(3cd^2-7ae^2)}{3(cd^2+ae^2)(d+ex)^{3/2}}+\frac{c\left(\frac{2de\left(cd^2-19ae^2\right)}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{2e\left(-\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(c^2d^4+34ace^2d^2+\sqrt{c}\left(cd^2-19ae^2\right)\sqrt{cd^2+ae^2}\,d-7a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\cdots}{\cdots}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

input
```
Int[1/((d + e*x)^(5/2)*(a + c*x^2)^2),x]
```

output
```
(a*e + c*d*x)/(2*a*(c*d^2 + a*e^2)*(d + e*x)^(3/2)*(a + c*x^2)) + ((2*e*(3
*c*d^2 - 7*a*e^2))/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (c*((2*d*e*(c*d^2
 - 19*a*e^2))/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (2*e*((-((c^(1/4)*Sqrt[Sqr
t[c]*d + Sqrt[c*d^2 + a*e^2]]*(c^2*d^4 + 34*a*c*d^2*e^2 - 7*a^2*e^4 + Sqrt
[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*(-((Sqrt[2]
*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt
[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 +
 a*e^2]]) - (c^(1/4)*(c^2*d^4 + 34*a*c*d^2*e^2 - 7*a^2*e^4 - Sqrt[c]*d*(c*
d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(
1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*
x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2
 + a*e^2]]) + (-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c^2*d^4 +
 34*a*c*d^2*e^2 - 7*a^2*e^4 + Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*
e^2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^
(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])]
)/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (c^(1/4)*(c^2*d^4 + 34*a*c*d^2*
e^2 - 7*a^2*e^4 - Sqrt[c]*d*(c*d^2 - 19*a*e^2)*Sqrt[c*d^2 + a*e^2])*Log[Sq
rt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*
Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e
^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(c*d^2 + a*e^2)))/(c*d^2 +...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 1162, normalized size of antiderivative = 1.59

method	result	size
pseudoelliptic	Expression too large to display	1162
derivativedivides	Expression too large to display	3541
default	Expression too large to display	3541

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
7/4/(a*e^2+c*d^2)^(7/2)*(1/4*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^
2)*c)^(1/2)-2*c*d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*((-19/7*(
-1/19*a*(-19*e^2*x^2+d^2)*c^(3/2)+c^(1/2)*a^2*e^2-1/19*c^(5/2)*d^2*x^2)*d*
(a*e^2+c*d^2)^(1/2)+(a^2*e^4-34/7*a*c*d^2*e^2-1/7*c^2*d^4)*(c*x^2+a))*((a*
e^2+c*d^2)*c)^(1/2)-(-19/7*(-1/19*a*(-19*e^2*x^2+d^2)*c^(5/2)+a^2*e^2*c^(3
/2)-1/19*c^(7/2)*d^2*x^2)*d*(a*e^2+c*d^2)^(1/2)+(a^2*e^4-34/7*a*c*d^2*e^2-
1/7*c^2*d^4)*(c*x^2+a)*c)*d)*(e*x+d)^(3/2)*ln(c^(1/2)*(e*x+d)-(e*x+d)^(1/2
)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))-1/4*(4*(a*e
^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(2*((a*e^2+
c*d^2)*c)^(1/2)+2*c*d)^(1/2)*((-19/7*(-1/19*a*(-19*e^2*x^2+d^2)*c^(3/2)+c^
(1/2)*a^2*e^2-1/19*c^(5/2)*d^2*x^2)*d*(a*e^2+c*d^2)^(1/2)+(a^2*e^4-34/7*a*
c*d^2*e^2-1/7*c^2*d^4)*(c*x^2+a))*((a*e^2+c*d^2)*c)^(1/2)-(-19/7*(-1/19*a*
(-19*e^2*x^2+d^2)*c^(5/2)+a^2*e^2*c^(3/2)-1/19*c^(7/2)*d^2*x^2)*d*(a*e^2+c
*d^2)^(1/2)+(a^2*e^4-34/7*a*c*d^2*e^2-1/7*c^2*d^4)*(c*x^2+a)*c)*d)*(e*x+d)
^(3/2)*ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^
(1/2)+(a*e^2+c*d^2)^(1/2))+e*a*(-8/21*(a*e^2+c*d^2)^(1/2)*(-3/4*d^3*x*(e*x
+d)^2*c^3-9/4*e*(-19/3*e^3*x^3-61/9*d*e^2*x^2+d^2*e*x+d^3)*a*d*c^2+55/4*(7
/55*e^2*x^2+54/55*d*e*x+d^2)*e^3*a^2*c+a^3*e^5)*(4*(a*e^2+c*d^2)^(1/2)*c^(
1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)+e*(arctan((-2*c^(1/2)*(e*x+d)^
(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8281 vs. $2(621) = 1242$.

Time = 3.02 (sec) , antiderivative size = 8281, normalized size of antiderivative = 11.33

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**(5/2)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{\left(cx^2+a\right)^2(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^2*(e*x + d)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2014 vs. $2(621) = 1242$.

Time = 0.35 (sec) , antiderivative size = 2014, normalized size of antiderivative = 2.76

$$\int \frac{1}{(d+ex)^{5/2}\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*((a*c^3*d^6*e + 3*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5 + a^4*e^7)^2*(c^2
*d^3*e - 19*a*c*d*e^3)*abs(c) - (sqrt(-a*c)*c^5*d^10*e + 37*sqrt(-a*c)*a*c
^4*d^8*e^3 + 98*sqrt(-a*c)*a^2*c^3*d^6*e^5 + 82*sqrt(-a*c)*a^3*c^2*d^4*e^7
 + 13*sqrt(-a*c)*a^4*c*d^2*e^9 - 7*sqrt(-a*c)*a^5*e^11)*abs(-a*c^3*d^6*e -
 3*a^2*c^2*d^4*e^3 - 3*a^3*c*d^2*e^5 - a^4*e^7)*abs(c) + (2*a*c^9*d^17*e +
 27*a^2*c^8*d^15*e^3 + 113*a^3*c^7*d^13*e^5 + 223*a^4*c^6*d^11*e^7 + 225*a
^5*c^5*d^9*e^9 + 97*a^6*c^4*d^7*e^11 - 13*a^7*c^3*d^5*e^13 - 27*a^8*c^2*d^
3*e^15 - 7*a^9*c*d*e^17)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^4*d^7 + 3
*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6 + sqrt((a*c^4*d^7 + 3*a
^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)^2 - (a*c^4*d^8 + 4*a^2*c
^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*(a*c^4*d^6 + 3
*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)))/(a*c^4*d^6 + 3*a^2*c^3
*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)))/((a^2*c^6*d^12*e + 6*a^3*c^5*d
^10*e^3 + 15*a^4*c^4*d^8*e^5 + 20*a^5*c^3*d^6*e^7 + 15*a^6*c^2*d^4*e^9 + 6
*a^7*c*d^2*e^11 + a^8*e^13 - sqrt(-a*c)*a*c^6*d^13 - 6*sqrt(-a*c)*a^2*c^5*
d^11*e^2 - 15*sqrt(-a*c)*a^3*c^4*d^9*e^4 - 20*sqrt(-a*c)*a^4*c^3*d^7*e^6 -
 15*sqrt(-a*c)*a^5*c^2*d^5*e^8 - 6*sqrt(-a*c)*a^6*c*d^3*e^10 - sqrt(-a*c)*
a^7*d*e^12)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(-a*c^3*d^6*e - 3*a^2*c^2*d^4
*e^3 - 3*a^3*c*d^2*e^5 - a^4*e^7)) - 1/4*((a*c^3*d^6*e + 3*a^2*c^2*d^4*e^3
 + 3*a^3*c*d^2*e^5 + a^4*e^7)^2*(c^2*d^3*e - 19*a*c*d*e^3)*abs(c) + (sq...
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 12390, normalized size of antiderivative = 16.95

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^2*(d + e*x)^(5/2)),x)
```

output
```
atan((((d + e*x)^(1/2)*(1568*a^16*c^5*e^28 + 128*a^3*c^18*d^26*e^2 + 3040*
a^4*c^17*d^24*e^4 + 29120*a^5*c^16*d^22*e^6 + 128128*a^6*c^15*d^20*e^8 + 2
82560*a^7*c^14*d^18*e^10 + 242016*a^8*c^13*d^16*e^12 - 282240*a^9*c^12*d^1
4*e^14 - 1059840*a^10*c^11*d^12*e^16 - 1403904*a^11*c^10*d^10*e^18 - 10494
40*a^12*c^9*d^8*e^20 - 456512*a^13*c^8*d^6*e^22 - 100480*a^14*c^7*d^4*e^24
 - 4160*a^15*c^6*d^2*e^26) + (-(4*a^3*c^6*d^9 - 49*a^3*e^9*(-a^9*c^3)^(1/2
) + 315*a^7*c^2*d*e^8 + 63*a^4*c^5*d^7*e^2 + 189*a^5*c^4*d^5*e^4 - 1155*a^
6*c^3*d^3*e^6 - 105*c^3*d^6*e^3*(-a^9*c^3)^(1/2) - 819*a*c^2*d^4*e^5*(-a^9
*c^3)^(1/2) + 837*a^2*c*d^2*e^7*(-a^9*c^3)^(1/2))/(64*(a^13*e^14 + a^6*c^7
*d^14 + 7*a^12*c*d^2*e^12 + 7*a^7*c^6*d^12*e^2 + 21*a^8*c^5*d^10*e^4 + 35*
a^9*c^4*d^8*e^6 + 35*a^10*c^3*d^6*e^8 + 21*a^11*c^2*d^4*e^10)))^(1/2)*((d
+ e*x)^(1/2)*(-(4*a^3*c^6*d^9 - 49*a^3*e^9*(-a^9*c^3)^(1/2) + 315*a^7*c^2*
d*e^8 + 63*a^4*c^5*d^7*e^2 + 189*a^5*c^4*d^5*e^4 - 1155*a^6*c^3*d^3*e^6 -
105*c^3*d^6*e^3*(-a^9*c^3)^(1/2) - 819*a*c^2*d^4*e^5*(-a^9*c^3)^(1/2) + 83
7*a^2*c*d^2*e^7*(-a^9*c^3)^(1/2))/(64*(a^13*e^14 + a^6*c^7*d^14 + 7*a^12*c
*d^2*e^12 + 7*a^7*c^6*d^12*e^2 + 21*a^8*c^5*d^10*e^4 + 35*a^9*c^4*d^8*e^6
+ 35*a^10*c^3*d^6*e^8 + 21*a^11*c^2*d^4*e^10)))^(1/2)*(2048*a^21*c^4*d*e^3
2 + 2048*a^6*c^19*d^31*e^2 + 30720*a^7*c^18*d^29*e^4 + 215040*a^8*c^17*d^2
7*e^6 + 931840*a^9*c^16*d^25*e^8 + 2795520*a^10*c^15*d^23*e^10 + 6150144*a
^11*c^14*d^21*e^12 + 10250240*a^12*c^13*d^19*e^14 + 13178880*a^13*c^12*...
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{(ex+d)^{\frac{5}{2}}\left(c\,x^2+a\right)^2}dx$$

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x)
```

output
```
int(1/(e*x+d)^(5/2)/(c*x^2+a)^2,x)
```

3.190 $\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 694

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3} dx = -\frac{de^3\sqrt{d+ex}}{4ac^2} - \frac{(ae-cdx)(d+ex)^{7/2}}{4ac(a+cx^2)^2} - \frac{\sqrt{d+ex}(ade(11cd^2+15ae^2)-(6c^2d^4+3acd^2e^2-7a^2e^4)x)}{16a^2c^2(a+cx^2)} - \frac{3e\left(2c^2d^4+5acd^2e^2+7a^2e^4+2\sqrt{c}d\sqrt{cd^2+ae^2}(cd^2+2ae^2)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{11/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{3e\left(2c^2d^4+5acd^2e^2+7a^2e^4+2\sqrt{c}d\sqrt{cd^2+ae^2}(cd^2+2ae^2)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{11/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{3e\left(2c^2d^4+5acd^2e^2+7a^2e^4-2\sqrt{c}d\sqrt{cd^2+ae^2}(cd^2+2ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{32\sqrt{2}a^2c^{11/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/4*d*e^3*(e*x+d)^(1/2)/a/c^2-1/4*(-c*d*x+a*e)*(e*x+d)^(7/2)/a/c/(c*x^2+a
)^2-1/16*(e*x+d)^(1/2)*(a*d*e*(15*a*e^2+11*c*d^2)-(-7*a^2*e^4+3*a*c*d^2*e^
2+6*c^2*d^4)*x)/a^2/c^2/(c*x^2+a)-3/64*e*(2*c^2*d^4+5*a*c*d^2*e^2+7*a^2*e^
4+2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2)*(2*a*e^2+c*d^2))*arctan(((c^(1/2)*d+(a*e
^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c
*d^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(11/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^
(1/2)+3/64*e*(2*c^2*d^4+5*a*c*d^2*e^2+7*a^2*e^4+2*c^(1/2)*d*(a*e^2+c*d^2)^
(1/2)*(2*a*e^2+c*d^2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/
2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/
a^2/c^(11/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-3/64*e*(2*c^2*d^4+5*a*
c*d^2*e^2+7*a^2*e^4-2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2)*(2*a*e^2+c*d^2))*arcta
nh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a
*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a^2/c^(11/4)/(c^(1/2)*d+(a*e^2
+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3}\,dx = \frac{\frac{2\sqrt{a}\sqrt{c}\sqrt{d+ex}\left(6c^3d^4x^3-a^3e^3(19d+7ex)+ac^2d^2x\left(10d^2+dex+15e^2x^2\right)-a^2ce\left(15d^3+9d^2ex+35de^2x^2+11e^3x^3\right)\right)}{(a+cx^2)^2}+ \cdots}{\cdots}$$

input
```
Integrate[(d + e*x)^(9/2)/(a + c*x^2)^3,x]
```

output
```
((2*Sqrt[a]*Sqrt[c]*Sqrt[d + e*x]*(6*c^3*d^4*x^3 - a^3*e^3*(19*d + 7*e*x)
+ a*c^2*d^2*x*(10*d^2 + d*e*x + 15*e^2*x^2) - a^2*c*e*(15*d^3 + 9*d^2*e*x
+ 35*d*e^2*x^2 + 11*e^3*x^3)))/(a + c*x^2)^2 + (3*(Sqrt[c]*d + I*Sqrt[a]*e
)^3*((4*I)*c*d^2 + 10*Sqrt[a]*Sqrt[c]*d*e - (7*I)*a*e^2)*ArcTan[(Sqrt[-(c*
d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[
-(c*d) - I*Sqrt[a]*Sqrt[c]*e] + (3*(Sqrt[c]*d - I*Sqrt[a]*e)^3*((-4*I)*c*d
^2 + 10*Sqrt[a]*Sqrt[c]*d*e + (7*I)*a*e^2)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]
*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/Sqrt[-(c*d) + I*Sqr
t[a]*Sqrt[c]*e])/(32*a^(5/2)*c^(5/2))
```

Rubi [A] (verified)

Time = 3.50 (sec) , antiderivative size = 968, normalized size of antiderivative = 1.39, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {495, 27, 684, 27, 653, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3}\,dx$$

$$\Big\downarrow \ 495$$

$$\frac{\int \frac{(d+ex)^{5/2}(6cd^2-cexd+7ae^2)}{2(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(d+ex)^{5/2}(6cd^2-cexd+7ae^2)}{(cx^2+a)^2}dx}{8ac}-\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 684$$

$$\frac{\frac{\int \frac{3\sqrt{d+ex}\left(4c^2d^4+9ace^2d^2-2ce\left(cd^2+2ae^2\right)xd+7a^2e^4\right)}{2\left(cx^2+a\right)}dx}{2ac}-\frac{(d+ex)^{3/2}(ae(7ae^2+5cd^2)-2cdx(4ae^2+3cd^2))}{2ac(a+cx^2)}}{8ac}-\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{\sqrt{d+ex}\left(4c^2d^4+9ace^2d^2-2ce\left(cd^2+2ae^2\right)xd+7a^2e^4\right)}{cx^2+a}dx}{4ac}-\frac{(d+ex)^{3/2}(ae(7ae^2+5cd^2)-2cdx(4ae^2+3cd^2))}{2ac(a+cx^2)}}{8ac}-\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 653$$

$$\frac{\frac{3\left(\frac{\int \frac{c\left(d\left(4c^2d^4+11ace^2d^2+11a^2e^4\right)+e\left(2c^2d^4+5ace^2d^2+7a^2e^4\right)x\right)}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{c}-4de\sqrt{d+ex}\left(2ae^2+cd^2\right)\right)}{4ac}-\frac{(d+ex)^{3/2}\left(ae\left(7ae^2+5cd^2\right)-2cdx\left(4ae^2+3cd^2\right)}{2ac(a+cx^2)}}{8ac}$$

$$\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\left(\int \frac{d\left(4c^2d^4+11ace^2d^2+11a^2e^4\right)+e\left(2c^2d^4+5ace^2d^2+7a^2e^4\right)x}{\sqrt{d+ex}\left(cx^2+a\right)}dx-4de\sqrt{d+ex}\left(2ae^2+cd^2\right)\right)}{4ac}-\frac{(d+ex)^{3/2}\left(ae\left(7ae^2+5cd^2\right)-2cdx\left(4ae^2+3cd^2\right)\right)}{2ac(a+cx^2)}}{8ac}-$$

$$\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 654

$$\frac{\frac{3\left(2\int \frac{e\left(2d\left(cd^2+ae^2\right)\left(cd^2+2ae^2\right)+\left(2c^2d^4+5ace^2d^2+7a^2e^4\right)(d+ex)\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}-4de\sqrt{d+ex}\left(2ae^2+cd^2\right)\right)}{4ac}-\frac{(d+ex)^{3/2}\left(ae\left(7ae^2+5cd^2\right)-2cdx\left(4ae\right.\right.}{2ac(a+cx^2)}}{8ac}$$

$$\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\left(2e\int \frac{2d\left(cd^2+ae^2\right)\left(cd^2+2ae^2\right)+\left(2c^2d^4+5ace^2d^2+7a^2e^4\right)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}-4de\sqrt{d+ex}\left(2ae^2+cd^2\right)\right)}{4ac}-\frac{(d+ex)^{3/2}\left(ae\left(7ae^2+5cd^2\right)-2cdx\left(4ae^2+\right.\right.}{2ac(a+cx^2)}}{8ac}$$

$$\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 1483

$$3\left(2e\left(\frac{\int \frac{\sqrt{cd^2+ae^2}\left(2\sqrt{2}d\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2d\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)-\frac{2c^2d^4+5ace^2d^2+7a^2e^4}{\sqrt{c}}\right)\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{cd^2 \cdots}}{\cdots}}{\cdots}\cdots\right.\right.$$

$$\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\downarrow$ 27

$$3\left(2e\left(\frac{\int \frac{2\sqrt{2}d\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2d\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)-\frac{2c^2d^4+5ace^2d^2+7a^2e^4}{\sqrt{c}}\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}d\sqrt{cd^2+ae^2}\sqrt{\cdots}}{\cdots}}{\cdots}\cdots\right.\right. \quad \cdots 4ac \cdots$$

$$\frac{(d+ex)^{7/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\downarrow$ 1142

$$3\left(2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}-\frac{1}{2}\sqrt[4]{c}\left(2d\sqrt{\cdots}\right.\cdots}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\cdots\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{7/2}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 25

$$3\left(2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}+\frac{1}{2}\sqrt[4]{c}\left(2d\sqrt{\right.}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{7/2}}{4ac\left(cx^2+a\right)^2}$$

$\Big\downarrow$ 27

$$3\left(2e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}+\frac{\left(2d\sqrt{cd^2+\right.}{}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{7/2}}{4ac\left(cx^2+a\right)^2}$$

$\Big\downarrow$ 1083

$$3\left(2e\left(\frac{\frac{\left(2d\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)-\frac{2c^2d^4+5ace^2d^2+7a^2e^4}{\sqrt{c}}\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5\right.}{}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{7/2}}{4ac\left(cx^2+a\right)^2}$$

$\Big\downarrow$ 219

$$3\left(2e\left(\frac{\frac{\left(2d\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)-\frac{2c^2d^4+5ace^2d^2+7a^2e^4}{\sqrt{c}}\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace\right.}{}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{7/2}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 1103

$$3\left(2e\left(\frac{-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{1}{2}\sqrt[4]{c}\left(2d\sqrt{cd\right.}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{7/2}}{4ac\left(cx^2+a\right)^2}$$

input
```
Int[(d + e*x)^(9/2)/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*e - c*d*x)*(d + e*x)^(7/2))/(a*c*(a + c*x^2)^2) + (-1/2*((d + e*x
)^(3/2)*(a*e*(5*c*d^2 + 7*a*e^2) - 2*c*d*(3*c*d^2 + 4*a*e^2)*x))/(a*c*(a +
 c*x^2)) + (3*(-4*d*e*(c*d^2 + 2*a*e^2)*Sqrt[d + e*x] + 2*e*((-((Sqrt[Sqrt
[c]*d + Sqrt[c*d^2 + a*e^2]]*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4 + Sqrt
[c]*d*Sqrt[c*d^2 + a*e^2]*(2*c*d^2 + 4*a*e^2))*ArcTanh[(c^(1/4)*(-((Sqrt[2
]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqr
t[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sq
rt[c*d^2 + a*e^2]])) - (c^(1/4)*(2*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2)
 - (2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4)/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2
] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]]) + (-((Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(2*c^2*d^4 + 5*a*c*d^2*
e^2 + 7*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(2*c*d^2 + 4*a*e^2))*ArcTa
nh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*S
qrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*
Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])) + (c^(1/4)*(2*d*Sqrt[c*d^2 + a*e^2
]*(c*d^2 + 2*a*e^2) - (2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4)/Sqrt[c])*Log
[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2
]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[Sqrt[c]*
d + Sqrt[c*d^2 + a*e^2]]))))/(4*a*c))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 991, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$\frac{3\left(c\,x^{2}+a\right)^{2}\left(\left(\left(-c^{\frac{7}{2}}\,d\left(2a\,e^{2}+c\,d^{2}\right)\sqrt{a\,e^{2}+c\,d^{2}}+\frac{7\left(a^{2}e^{4}+\frac{5}{7}ac\,d^{2}e^{2}+\frac{2}{7}c^{2}d^{4}\right)c^{3}}{2}\right)\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+\left(c^{\frac{9}{2}}\,d\left(2a\,e^{2}+c\,d^{2}\right)\sqrt{a\,e^{2}+c\,d^{2}}\right.\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^(9/2)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*((c*x^2+a)^2*((-c^(7/2)*d*(2*a*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2)+7/2*(a^
2*e^4+5/7*a*c*d^2*e^2+2/7*c^2*d^4)*c^3)*((a*e^2+c*d^2)*c)^(1/2)+(c^(9/2)*d
*(2*a*e^2+c*d^2)*(a*e^2+c*d^2)^(1/2)-7/2*(a^2*e^4+5/7*a*c*d^2*e^2+2/7*c^2*
d^4)*c^4)*d)*(ln((-e*x-d)*c^(1/2)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)
+2*c*d)^(1/2)-(a*e^2+c*d^2)^(1/2))-ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a
*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2)))*(4*(a*e^2+c*d^2)^(
1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(
1/2)+2*c*d)^(1/2)-4*e*(e*(arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^
2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*
c)^(1/2)-2*c*d)^(1/2))-arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c
)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(
1/2)-2*c*d)^(1/2)))*(c*x^2+a)^2*c^(9/2)*d*(2*a*e^2+c*d^2)*(a*e^2+c*d^2)^(1
/2)+1/3*(11*a^2*c*e^4*x^3-15*a*c^2*d^2*e^2*x^3-6*c^3*d^4*x^3+35*a^2*c*d*e^
3*x^2-a*c^2*d^3*e*x^2+7*a^3*e^4*x+9*a^2*c*d^2*e^2*x-10*a*c^2*d^4*x+19*a^3*
d*e^3+15*a^2*c*d^3*e)*c^(9/2)*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d
^2)*c)^(1/2)-2*c*d)^(1/2)*(e*x+d)^(1/2)-1/2*e*(arctan((2*c^(1/2)*(e*x+d)^(
1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/
2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((-2*c^(1/2)*(e*x+d)^(1/2
)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-
2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)))*(c*x^2+a)^2*(7*a^2*e^4+5*a*c*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2454 vs. $2(584) = 1168$.

Time = 0.59 (sec) , antiderivative size = 2454, normalized size of antiderivative = 3.54

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(16*c^4*d^9 + 84*a*c
^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 + a^5
*c^5*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14
 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11)))/(a^5*c^5))*log(27*(33
6*c^6*d^12*e^5 + 2460*a*c^5*d^10*e^7 + 8101*a^2*c^4*d^8*e^9 + 14968*a^3*c^
3*d^6*e^11 + 16194*a^4*c^2*d^4*e^13 + 9604*a^5*c*d^2*e^15 + 2401*a^6*e^17)
*sqrt(e*x + d) + 27*(42*a^3*c^6*d^7*e^6 + 192*a^4*c^5*d^5*e^8 + 314*a^5*c^
4*d^3*e^10 + 196*a^6*c^3*d*e^12 - (4*a^5*c^10*d^4 + 9*a^6*c^9*d^2*e^2 + 7*
a^7*c^8*e^4)*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*
d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11)))*sqrt(-(16*c^4
*d^9 + 84*a*c^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^
4*d*e^8 + a^5*c^5*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2
*c^2*d^4*e^14 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11)))/(a^5*c^5
))) - 3*(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(16*c^4*d^9 + 84*a*c
^3*d^7*e^2 + 189*a^2*c^2*d^5*e^4 + 210*a^3*c*d^3*e^6 + 105*a^4*d*e^8 + a^5
*c^5*sqrt(-(441*c^4*d^8*e^10 + 2268*a*c^3*d^6*e^12 + 4974*a^2*c^2*d^4*e^14
 + 5292*a^3*c*d^2*e^16 + 2401*a^4*e^18)/(a^5*c^11)))/(a^5*c^5))*log(27*(33
6*c^6*d^12*e^5 + 2460*a*c^5*d^10*e^7 + 8101*a^2*c^4*d^8*e^9 + 14968*a^3*c^
3*d^6*e^11 + 16194*a^4*c^2*d^4*e^13 + 9604*a^5*c*d^2*e^15 + 2401*a^6*e^17)
*sqrt(e*x + d) - 27*(42*a^3*c^6*d^7*e^6 + 192*a^4*c^5*d^5*e^8 + 314*a^5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3} dx = \int \frac{(ex+d)^{\frac{9}{2}}}{(cx^2+a)^3} dx$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(9/2)/(c*x^2 + a)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*((2*a*c^2*d^4*e + 5*a^2*c*d^2*e^3 + 7*a^3*e^5)*e^2*abs(c) - 2*(sqrt(-
a*c)*c^2*d^5*e + 3*sqrt(-a*c)*a*c*d^3*e^3 + 2*sqrt(-a*c)*a^2*d*e^5)*abs(c)
*abs(e) + (4*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 11*a^2*c*d^2*e^5)*abs(c))*arct
an(sqrt(e*x + d)/sqrt(-(a^2*c^3*d + sqrt(a^4*c^6*d^2 - (a^2*c^3*d^2 + a^3*
c^2*e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*e + sqrt(-a*c)*a^2*c^3*d)*sqrt(-c
^2*d - sqrt(-a*c)*c*e)*abs(e)) + 3/32*((2*a*c^2*d^4*e + 5*a^2*c*d^2*e^3 +
7*a^3*e^5)*e^2*abs(c) + 2*(sqrt(-a*c)*c^2*d^5*e + 3*sqrt(-a*c)*a*c*d^3*e^3
 + 2*sqrt(-a*c)*a^2*d*e^5)*abs(c)*abs(e) + (4*c^3*d^6*e + 11*a*c^2*d^4*e^3
 + 11*a^2*c*d^2*e^5)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d - sqrt(
a^4*c^6*d^2 - (a^2*c^3*d^2 + a^3*c^2*e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*
e - sqrt(-a*c)*a^2*c^3*d)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(e)) + 1/16*(6*
(e*x + d)^(7/2)*c^3*d^4*e - 18*(e*x + d)^(5/2)*c^3*d^5*e + 18*(e*x + d)^(3
/2)*c^3*d^6*e - 6*sqrt(e*x + d)*c^3*d^7*e + 15*(e*x + d)^(7/2)*a*c^2*d^2*e
^3 - 44*(e*x + d)^(5/2)*a*c^2*d^3*e^3 + 53*(e*x + d)^(3/2)*a*c^2*d^4*e^3 -
 24*sqrt(e*x + d)*a*c^2*d^5*e^3 - 11*(e*x + d)^(7/2)*a^2*c*e^5 - 2*(e*x +
d)^(5/2)*a^2*c*d*e^5 + 28*(e*x + d)^(3/2)*a^2*c*d^2*e^5 - 30*sqrt(e*x + d)
*a^2*c*d^3*e^5 - 7*(e*x + d)^(3/2)*a^3*e^7 - 12*sqrt(e*x + d)*a^3*d*e^7)/(
((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 + a*e^2)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 4769, normalized size of antiderivative = 6.87

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(9/2)/(a + c*x^2)^3,x)
```

output
```
- ((3*(d + e*x)^(1/2)*(2*a^3*d*e^7 + c^3*d^7*e + 4*a*c^2*d^5*e^3 + 5*a^2*c
*d^3*e^5))/(8*a^2*c^2) - ((d + e*x)^(3/2)*(18*c^3*d^6*e - 7*a^3*e^7 + 53*a
*c^2*d^4*e^3 + 28*a^2*c*d^2*e^5))/(16*a^2*c^2) - (e*(d + e*x)^(7/2)*(6*c^2
*d^4 - 11*a^2*e^4 + 15*a*c*d^2*e^2))/(16*a^2*c) + (d*(d + e*x)^(5/2)*(a^2*
e^5 + 9*c^2*d^4*e + 22*a*c*d^2*e^3))/(8*a^2*c))/(c^2*(d + e*x)^4 + a^2*e^4
 + c^2*d^4 + (6*c^2*d^2 + 2*a*c*e^2)*(d + e*x)^2 - (4*c^2*d^3 + 4*a*c*d*e^
2)*(d + e*x) - 4*c^2*d*(d + e*x)^3 + 2*a*c*d^2*e^2) - 2*atanh((441*a*e^12*
(d + e*x)^(1/2)*(- (9*d^9)/(256*a^5*c) - (945*d*e^8)/(4096*a*c^5) - (945*d
^3*e^6)/(2048*a^2*c^4) - (1701*d^5*e^4)/(4096*a^3*c^3) - (189*d^7*e^2)/(10
24*a^4*c^2) - (441*e^9*(-a^15*c^11)^(1/2))/(4096*a^8*c^11) - (243*d^2*e^7*
(-a^15*c^11)^(1/2))/(2048*a^9*c^10) - (189*d^4*e^5*(-a^15*c^11)^(1/2))/(40
96*a^10*c^9))^(1/2))/(32*((9261*a*e^17)/(2048*c^3) + (16659*d^6*e^11)/(102
4*a^2) + (15687*d^2*e^15)/(1024*c^2) + (13203*c*d^8*e^9)/(2048*a^3) + (278
1*d^4*e^13)/(128*a*c) + (567*c^2*d^10*e^7)/(512*a^4) - (1323*d*e^16*(-a^15
*c^11)^(1/2))/(512*a^7*c^8) - (6885*d^3*e^14*(-a^15*c^11)^(1/2))/(1024*a^8
*c^7) - (6831*d^5*e^12*(-a^15*c^11)^(1/2))/(1024*a^9*c^6) - (3159*d^7*e^10
*(-a^15*c^11)^(1/2))/(1024*a^10*c^5) - (567*d^9*e^8*(-a^15*c^11)^(1/2))/(1
024*a^11*c^4))) + (243*d^2*e^10*(d + e*x)^(1/2)*(- (9*d^9)/(256*a^5*c) - (
945*d*e^8)/(4096*a*c^5) - (945*d^3*e^6)/(2048*a^2*c^4) - (1701*d^5*e^4)/(4
096*a^3*c^3) - (189*d^7*e^2)/(1024*a^4*c^2) - (441*e^9*(-a^15*c^11)^(1/...
```

Reduce [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 7420, normalized size of antiderivative = 10.69

$$\int \frac{(d+ex)^{9/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(9/2)/(c*x^2+a)^3,x)
```

output
```
( - 42*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**4*e*
*4 - 54*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sq
rt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*
sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**3*c
*d**2*e**2 - 84*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) -
 c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*
sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2))
)*a**3*c*e**4*x**2 - 24*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c
*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt
(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*
sqrt(2)))*a**2*c**2*d**4 - 108*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*
d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
- c*d)*sqrt(2)))*a**2*c**2*d**2*e**2*x**2 - 42*sqrt(a*e**2 + c*d**2)*sqrt(
sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**
2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a
*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c**2*e**4*x**4 - 48*sqrt(a*e**2 + c*
d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt...
```

3.191 $\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 626

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = -\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\,(a+cx^2)^2} - \frac{\sqrt{d+ex}\,(ae(7cd^2+5ae^2)-2cd(3cd^2+2ae^2)\,x)}{16a^2c^2\,(a+cx^2)}$$
$$-\frac{e\left(2\sqrt{c}d(3cd^2+4ae^2)+\sqrt{cd^2+ae^2}(6cd^2+5ae^2)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{9/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$+\frac{e\left(2\sqrt{c}d(3cd^2+4ae^2)+\sqrt{cd^2+ae^2}(6cd^2+5ae^2)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{9/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$-\frac{e\left(2\sqrt{c}d(3cd^2+4ae^2)-\sqrt{cd^2+ae^2}(6cd^2+5ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{32\sqrt{2}a^2c^{9/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/4*(-c*d*x+a*e)*(e*x+d)^(5/2)/a/c/(c*x^2+a)^2-1/16*(e*x+d)^(1/2)*(a*e*(5
*a*e^2+7*c*d^2)-2*c*d*(2*a*e^2+3*c*d^2)*x)/a^2/c^2/(c*x^2+a)-1/64*e*(2*c^(
1/2)*d*(4*a*e^2+3*c*d^2)+(a*e^2+c*d^2)^(1/2)*(5*a*e^2+6*c*d^2))*arctan(((c
^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/
2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(9/4)/(-c^(1/2)*d+(a*e^2+c*
d^2)^(1/2))^(1/2)+1/64*e*(2*c^(1/2)*d*(4*a*e^2+3*c*d^2)+(a*e^2+c*d^2)^(1/2
)*(5*a*e^2+6*c*d^2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)
*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a^
2/c^(9/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-1/64*e*(2*c^(1/2)*d*(4*a*
e^2+3*c*d^2)-(a*e^2+c*d^2)^(1/2)*(5*a*e^2+6*c*d^2))*arctanh(2^(1/2)*c^(1/4
)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)
+c^(1/2)*(e*x+d)))*2^(1/2)/a^2/c^(9/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.13 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = \frac{\frac{2\sqrt{a}\sqrt{d+ex}(-5a^3e^3+6c^3d^3x^3+ac^2dx(10d^2+dex+8e^2x^2)-a^2ce(11d^2+4dex+9e^2x^2))}{(a+cx^2)^2} + \frac{(\sqrt{c}d+i\sqrt{a}e)^2(12icd^2+18}{}}{}$$

input
```
Integrate[(d + e*x)^(7/2)/(a + c*x^2)^3,x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(-5*a^3*e^3 + 6*c^3*d^3*x^3 + a*c^2*d*x*(10*d^2
+ d*e*x + 8*e^2*x^2) - a^2*c*e*(11*d^2 + 4*d*e*x + 9*e^2*x^2)))/(a + c*x^2
)^2 + ((Sqrt[c]*d + I*Sqrt[a]*e)^2*((12*I)*c*d^2 + 18*Sqrt[a]*Sqrt[c]*d*e
- (5*I)*a*e^2)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(
Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e] + ((Sqrt[c]*
d - I*Sqrt[a]*e)^2*((-12*I)*c*d^2 + 18*Sqrt[a]*Sqrt[c]*d*e + (5*I)*a*e^2)*
ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*S
qrt[a]*e)])/Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e])/(32*a^(5/2)*c^2)
```

Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.43, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {495, 27, 684, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx$$

$$\Big\downarrow \; 495$$

$$\frac{\int \frac{(d+ex)^{3/2}(6cd^2+cexd+5ae^2)}{2(cx^2+a)^2}dx}{4ac} - \frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)^{3/2}(6cd^2+cexd+5ae^2)}{(cx^2+a)^2}dx}{8ac} - \frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 684$$

$$\frac{\frac{\int \frac{\left(3cd^2+ae^2\right)\left(4cd^2+5ae^2\right)+2cde\left(3cd^2+4ae^2\right)x}{2\sqrt{d+ex}\left(cx^2+a\right)}dx}{2ac} - \frac{\sqrt{d+ex}\left(ae\left(5ae^2+7cd^2\right)-2cdx\left(2ae^2+3cd^2\right)\right)}{2ac(a+cx^2)}}{8ac} - \frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(3cd^2+ae^2\right)\left(4cd^2+5ae^2\right)+2cde\left(3cd^2+4ae^2\right)x}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{4ac} - \frac{\sqrt{d+ex}\left(ae\left(5ae^2+7cd^2\right)-2cdx\left(2ae^2+3cd^2\right)\right)}{2ac(a+cx^2)}}{8ac} - \frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 654$$

$$\frac{\frac{\int \frac{e\left(\left(cd^2+ae^2\right)\left(6cd^2+5ae^2\right)+2cd\left(3cd^2+4ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac} - \frac{\sqrt{d+ex}\left(ae\left(5ae^2+7cd^2\right)-2cdx\left(2ae^2+3cd^2\right)\right)}{2ac(a+cx^2)}}{8ac} - \frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{e\int \frac{\left(cd^2+ae^2\right)\left(6cd^2+5ae^2\right)+2cd\left(3cd^2+4ae^2\right)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}-\frac{\sqrt{d+ex}\left(ae\left(5ae^2+7cd^2\right)-2cdx\left(2ae^2+3cd^2\right)\right)}{2ac\left(a+cx^2\right)}}{8ac}-\frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 1483$$

$$\frac{e\left(\frac{\int \frac{\sqrt{cd^2+ae^2}\left(\sqrt{2}\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2\sqrt{c}d\left(3cd^2+4ae^2\right)-\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{cd^2+ae^2}\left(\sqrt{2}\sqrt{c\ldots}\right.}{\ldots}}{\ldots}\right.}{2ac}$$

$$\frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{e\left(\frac{\int \frac{\sqrt{2}\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2\sqrt{c}d\left(3cd^2+4ae^2\right)-\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}\ldots}}{d+e\ldots}}{\ldots}\right.}{2ac}$$

$$8ac$$

$$\frac{(d+ex)^{5/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 1142$$

$$e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\left(2\sqrt{c}d\left(3cd^2+4ae^2\cdots\right.\right.}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}$$

$$\Big\downarrow \ 25$$

$$e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\left(2\sqrt{c}d\left(3cd^2+4ae^2\cdots\right.\right.}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}$$

$$\Big\downarrow \ 27$$

$$e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\left(2\sqrt{c}d\left(3cd^2+4ae^2\right)-\sqrt{c}\cdots\right.}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}$$

$$\Big\downarrow \ 1083$$

$$e\left(\frac{-\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\int \frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)-\frac{\left(2\sqrt{c}d\left(3cd^2+\cdots\right.\right.}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}$$

$$\Big\downarrow \ 219$$

$$e\left(\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{\left(2\sqrt{c}d\left(3cd^2+4ae^2\right)-\sqrt{\cdots}\right.}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}$$

$$\Big\downarrow \ 1103$$

$$e\left(\frac{\frac{1}{2}\sqrt[4]{c}\left(2\sqrt{c}d\left(3cd^2+4ae^2\right)-\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\log\left(\sqrt{c}(d+ex)-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}+\sqrt{cd^2+ae^2}\right)-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+\cdots\right.}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.$$

$$\frac{(ae-cdx)(d+ex)^{5/2}}{4ac\left(cx^2+a\right)^2}$$

input
```
Int[(d + e*x)^(7/2)/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*e - c*d*x)*(d + e*x)^(5/2))/(a*c*(a + c*x^2)^2) + (-1/2*(Sqrt[d +
 e*x]*(a*e*(7*c*d^2 + 5*a*e^2) - 2*c*d*(3*c*d^2 + 2*a*e^2)*x))/(a*c*(a + c
*x^2)) + (e*((-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(6*c^(3/2)*
d^3 + 8*a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh
[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*
Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]]) + (c^(1/4)*(2*Sqrt[c]*d*(3*c*d^2 + 4*a*e^2)
 - Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt
[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*
(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) +
 (-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(6*c^(3/2)*d^3 + 8*a*Sq
rt[c]*d*e^2 + Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh[(c^(1/4)*((
Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))
/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c
*d^2 + a*e^2]]) - (c^(1/4)*(2*Sqrt[c]*d*(3*c*d^2 + 4*a*e^2) - Sqrt[c*d^2 +
 a*e^2]*(6*c*d^2 + 5*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqr
t[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/
(2*Sqrt[2]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(2*a*c))/(8*a*
c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(518) = 1036$.

Time = 2.00 (sec) , antiderivative size = 1201, normalized size of antiderivative = 1.92

method	result	size
pseudoelliptic	Expression too large to display	1201
derivativedivides	Expression too large to display	2333
default	Expression too large to display	2333

input
```
int((e*x+d)^(7/2)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*
(-1/2*e^2*((5/6*a^3*e^2*c^(7/2)+c^(9/2)*((5/3*e^2*x^2+d^2)*a^2+2*(5/12*e^2
*x^2+d^2)*x^2*c*a+c^2*d^2*x^4))*(a*e^2+c*d^2)^(1/2)+4/3*(a*e^2+3/4*c*d^2)*
(c*x^2+a)^2*c^4*d)*a*arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)
^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1
/2)-2*c*d)^(1/2))+1/2*e^2*((5/6*a^3*e^2*c^(7/2)+c^(9/2)*((5/3*e^2*x^2+d^2)
*a^2+2*(5/12*e^2*x^2+d^2)*x^2*c*a+c^2*d^2*x^4))*(a*e^2+c*d^2)^(1/2)+4/3*(a
*e^2+3/4*c*d^2)*(c*x^2+a)^2*c^4*d)*a*arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((
a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e
^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))+(1/8*(((-(5/3*e^2*x^2+d^2)*a^2*c^(7/2)-5/
6*a^3*e^2*c^(5/2)-c^(9/2)*x^2*((5/6*e^2*x^2+2*d^2)*a+c*d^2*x^2))*(a*e^2+c*
d^2)^(1/2)+4/3*(a*e^2+3/4*c*d^2)*(c*x^2+a)^2*c^3*d)*((a*e^2+c*d^2)*c)^(1/2
)+((5/6*a^3*e^2*c^(7/2)+c^(9/2)*((5/3*e^2*x^2+d^2)*a^2+2*(5/12*e^2*x^2+d^2
)*x^2*c*a+c^2*d^2*x^4))*(a*e^2+c*d^2)^(1/2)-4/3*(a*e^2+3/4*c*d^2)*(c*x^2+a
)^2*c^4*d)*d)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*ln(c^(1/2)*(e*x+d)-(
e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))-
1/8*(((-(5/3*e^2*x^2+d^2)*a^2*c^(7/2)-5/6*a^3*e^2*c^(5/2)-c^(9/2)*x^2*((5/
6*e^2*x^2+2*d^2)*a+c*d^2*x^2))*(a*e^2+c*d^2)^(1/2)+4/3*(a*e^2+3/4*c*d^2)*(
c*x^2+a)^2*c^3*d)*((a*e^2+c*d^2)*c)^(1/2)+((5/6*a^3*e^2*c^(7/2)+c^(9/2)*((
5/3*e^2*x^2+d^2)*a^2+2*(5/12*e^2*x^2+d^2)*x^2*c*a+c^2*d^2*x^4))*(a*e^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(521) = 1042$.

Time = 0.18 (sec) , antiderivative size = 1751, normalized size of antiderivative = 2.80

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*((a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(144*c^3*d^7 + 420*a*c
^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^5*c^4*sqrt(-(441*c^2*d^
4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*log((302
4*c^4*d^8*e^5 + 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 + 6250*a^3*c*d
^2*e^11 + 625*a^4*e^13)*sqrt(e*x + d) + (126*a^3*c^4*d^4*e^6 + 255*a^4*c^3
*d^2*e^8 + 125*a^5*c^2*e^10 + (12*a^5*c^8*d^3 + 13*a^6*c^7*d*e^2)*sqrt(-(4
41*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqrt(-(144
*c^3*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^5*c^4
*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(
a^5*c^4))) - (a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(144*c^3*d^7 +
420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 + a^5*c^4*sqrt(-(441
*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))/(a^5*c^4))*l
og((3024*c^4*d^8*e^5 + 10908*a*c^3*d^6*e^7 + 13509*a^2*c^2*d^4*e^9 + 6250*
a^3*c*d^2*e^11 + 625*a^4*e^13)*sqrt(e*x + d) - (126*a^3*c^4*d^4*e^6 + 255*
a^4*c^3*d^2*e^8 + 125*a^5*c^2*e^10 + (12*a^5*c^8*d^3 + 13*a^6*c^7*d*e^2)*s
qrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c^9)))*sqr
t(-(144*c^3*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 +
a^5*c^4*sqrt(-(441*c^2*d^4*e^10 + 1050*a*c*d^2*e^12 + 625*a^2*e^14)/(a^5*c
^9)))/(a^5*c^4))) + (a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2)*sqrt(-(144*c^3
*d^7 + 420*a*c^2*d^5*e^2 + 385*a^2*c*d^3*e^4 + 105*a^3*d*e^6 - a^5*c^4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+a)^3}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(7/2)/(c*x^2 + a)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/32*(2*(3*a*c^2*d^3*e + 4*a^2*c*d*e^3)*e^2*abs(c) - (6*sqrt(-a*c)*c^2*d^4
*e + 11*sqrt(-a*c)*a*c*d^2*e^3 + 5*sqrt(-a*c)*a^2*e^5)*abs(c)*abs(e) + (12
*c^3*d^5*e + 19*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*abs(c))*arctan(sqrt(e*x + d
)/sqrt(-(a^2*c^3*d + sqrt(a^4*c^6*d^2 - (a^2*c^3*d^2 + a^3*c^2*e^2)*a^2*c^
3))/(a^2*c^3)))/((a^3*c^3*e + sqrt(-a*c)*a^2*c^3*d)*sqrt(-c^2*d - sqrt(-a*
c)*c*e)*abs(e)) + 1/32*(2*(3*a*c^2*d^3*e + 4*a^2*c*d*e^3)*e^2*abs(c) + (6*
sqrt(-a*c)*c^2*d^4*e + 11*sqrt(-a*c)*a*c*d^2*e^3 + 5*sqrt(-a*c)*a^2*e^5)*a
bs(c)*abs(e) + (12*c^3*d^5*e + 19*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*abs(c))*a
rctan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d - sqrt(a^4*c^6*d^2 - (a^2*c^3*d^2 + a
^3*c^2*e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*e - sqrt(-a*c)*a^2*c^3*d)*sqrt
(-c^2*d + sqrt(-a*c)*c*e)*abs(e)) + 1/16*(6*(e*x + d)^(7/2)*c^3*d^3*e - 18
*(e*x + d)^(5/2)*c^3*d^4*e + 18*(e*x + d)^(3/2)*c^3*d^5*e - 6*sqrt(e*x + d
)*c^3*d^6*e + 8*(e*x + d)^(7/2)*a*c^2*d*e^3 - 23*(e*x + d)^(5/2)*a*c^2*d^2
*e^3 + 32*(e*x + d)^(3/2)*a*c^2*d^3*e^3 - 17*sqrt(e*x + d)*a*c^2*d^4*e^3 -
 9*(e*x + d)^(5/2)*a^2*c*e^5 + 14*(e*x + d)^(3/2)*a^2*c*d*e^5 - 16*sqrt(e*
x + d)*a^2*c*d^2*e^5 - 5*sqrt(e*x + d)*a^3*e^7)/(((e*x + d)^2*c - 2*(e*x +
 d)*c*d + c*d^2 + a*e^2)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 2569, normalized size of antiderivative = 4.10

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(7/2)/(a + c*x^2)^3,x)
```

output
```
((e*(3*c*d^3 + 4*a*d*e^2)*(d + e*x)^(7/2))/(8*a^2) + ((d + e*x)^(3/2)*(7*a
^2*d*e^5 + 9*c^2*d^5*e + 16*a*c*d^3*e^3))/(8*a^2*c) - ((d + e*x)^(1/2)*(5*
a^3*e^7 + 6*c^3*d^6*e + 17*a*c^2*d^4*e^3 + 16*a^2*c*d^2*e^5))/(16*a^2*c^2)
 - (e*(d + e*x)^(5/2)*(9*a^2*e^4 + 18*c^2*d^4 + 23*a*c*d^2*e^2))/(16*a^2*c
))/(c^2*(d + e*x)^4 + a^2*e^4 + c^2*d^4 + (6*c^2*d^2 + 2*a*c*e^2)*(d + e*x
)^2 - (4*c^2*d^3 + 4*a*c*d*e^2)*(d + e*x) - 4*c^2*d*(d + e*x)^3 + 2*a*c*d^
2*e^2) - 2*atanh((25*e^10*(d + e*x)^(1/2)*(- (9*d^7)/(256*a^5*c) - (105*d*
e^6)/(4096*a^2*c^4) - (385*d^3*e^4)/(4096*a^3*c^3) - (105*d^5*e^2)/(1024*a
^4*c^2) - (25*e^7*(-a^15*c^9)^(1/2))/(4096*a^9*c^9) - (21*d^2*e^5*(-a^15*c
^9)^(1/2))/(4096*a^10*c^8))^(1/2))/(32*((825*d^5*e^9)/(2048*a^3) + (325*d*
e^13)/(2048*a*c^2) + (63*c*d^7*e^7)/(512*a^4) + (449*d^3*e^11)/(1024*a^2*c
) + (125*e^14*(-a^15*c^9)^(1/2))/(2048*a^8*c^7) + (95*d^2*e^12*(-a^15*c^9)
^(1/2))/(512*a^9*c^6) + (381*d^4*e^10*(-a^15*c^9)^(1/2))/(2048*a^10*c^5) +
 (63*d^6*e^8*(-a^15*c^9)^(1/2))/(1024*a^11*c^4))) + (21*d^2*e^8*(d + e*x)^
(1/2)*(- (9*d^7)/(256*a^5*c) - (105*d*e^6)/(4096*a^2*c^4) - (385*d^3*e^4)/
(4096*a^3*c^3) - (105*d^5*e^2)/(1024*a^4*c^2) - (25*e^7*(-a^15*c^9)^(1/2))
/(4096*a^9*c^9) - (21*d^2*e^5*(-a^15*c^9)^(1/2))/(4096*a^10*c^8))^(1/2))/(
32*((325*d*e^13)/(2048*c^3) + (63*d^7*e^7)/(512*a^3) + (449*d^3*e^11)/(102
4*a*c^2) + (825*d^5*e^9)/(2048*a^2*c) + (125*e^14*(-a^15*c^9)^(1/2))/(2048
*a^7*c^8) + (95*d^2*e^12*(-a^15*c^9)^(1/2))/(512*a^8*c^7) + (381*d^4*e^...
```

Reduce [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 6099, normalized size of antiderivative = 9.74

$$\int \frac{(d+ex)^{7/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(7/2)/(c*x^2+a)^3,x)
```

output
```
( - 26*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**3*c*
d*e**2 - 24*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d
)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt
(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*
*2*c**2*d**3 - 52*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
 - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) -
2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2
)))*a**2*c**2*d*e**2*x**2 - 48*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*
d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
- c*d)*sqrt(2)))*a*c**3*d**3*x**2 - 26*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*
sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d*
*2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) - c*d)*sqrt(2)))*a*c**3*d*e**2*x**4 - 24*sqrt(a*e**2 + c*d**2)*sqr
t(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt
(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c**4*d**3*x**4 - 10*sqrt(c)*sqrt(sqrt(c
)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 +...
```

3.192 $\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 562

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = -\frac{(ae-cdx)(d+ex)^{3/2}}{4ac(a+cx^2)^2} - \frac{3\sqrt{d+ex}\left(ade-(2cd^2+ae^2)\,x\right)}{16a^2c\,(a+cx^2)}$$
$$-\frac{3e\left(2cd^2+ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{7/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$+\frac{3e\left(2cd^2+ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{7/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$
$$-\frac{3e\left(2cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{32\sqrt{2}a^2c^{7/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/4*(-c*d*x+a*e)*(e*x+d)^(3/2)/a/c/(c*x^2+a)^2-3/16*(e*x+d)^(1/2)*(a*d*e-
(a*e^2+2*c*d^2)*x)/a^2/c/(c*x^2+a)-3/64*e*(2*c*d^2+a*e^2+2*c^(1/2)*d*(a*e^
2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1
/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(
7/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+3/64*e*(2*c*d^2+a*e^2+2*c^(1/2
)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(
1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2
)/a^2/c^(7/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-3/64*e*(2*c*d^2+a*e^2
-2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^
2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))
*2^(1/2)/a^2/c^(7/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = \frac{\frac{2\sqrt{a}c\sqrt{d+ex}\left(6c^2d^2x^3-a^2e(7d+ex)+acx\left(10d^2+dex+3e^2x^2\right)\right)}{(a+cx^2)^2}-3i\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\left(4cd^2-2i\sqrt{a}\sqrt{c}d\right.}{}$$

input
```
Integrate[(d + e*x)^(5/2)/(a + c*x^2)^3,x]
```

output
```
((2*Sqrt[a]*c*Sqrt[d + e*x]*(6*c^2*d^2*x^3 - a^2*e*(7*d + e*x) + a*c*x*(10
*d^2 + d*e*x + 3*e^2*x^2)))/(a + c*x^2)^2 - (3*I)*Sqrt[-(c*d) - I*Sqrt[a]*
Sqrt[c]*e]*(4*c*d^2 - (2*I)*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[(Sqrt[-(c*
d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)] + (3*I
)*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*(4*c*d^2 + (2*I)*Sqrt[a]*Sqrt[c]*d*e
+ a*e^2)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c
]*d - I*Sqrt[a]*e)])/(32*a^(5/2)*c^2)
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.40, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {495, 27, 685, 27, 654, 27, 1483, 27, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}dx$$

$$\Big\downarrow \ 495$$

$$\frac{\int \frac{3\sqrt{d+ex}(2cd^2+cexd+ae^2)}{2(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{3\int \frac{\sqrt{d+ex}(2cd^2+cexd+ae^2)}{(cx^2+a)^2}dx}{8ac}-\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 685$$

$$\frac{3\left(\frac{\int \frac{c\left(d\left(4cd^2+3ae^2\right)+e\left(2cd^2+ae^2\right)x\right)}{2\sqrt{d+ex}\left(cx^2+a\right)}dx}{2ac}-\frac{\sqrt{d+ex}(ade-x(ae^2+2cd^2))}{2a(a+cx^2)}\right)}{8ac}-\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(\frac{\int \frac{d\left(4cd^2+3ae^2\right)+e\left(2cd^2+ae^2\right)x}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{4a}-\frac{\sqrt{d+ex}(ade-x(ae^2+2cd^2))}{2a(a+cx^2)}\right)}{8ac}-\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 654$$

$$\frac{3\left(\frac{\int \frac{e\left(2d\left(cd^2+ae^2\right)+\left(2cd^2+ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}(ade-x(ae^2+2cd^2))}{2a(a+cx^2)}\right)}{8ac}-\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(\frac{e\int \frac{2d\left(cd^2+ae^2\right)+\left(2cd^2+ae^2\right)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}\left(ade-x\left(ae^2+2cd^2\right)\right)}{2a\left(a+cx^2\right)}\right)}{8ac}-\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\downarrow$ 1483

$$\frac{3\left(\frac{e\left(\frac{\int \frac{\sqrt{cd^2+ae^2}\left(2\sqrt{2}d\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2d\sqrt{cd^2+ae^2}-\frac{2cd^2+ae^2}{\sqrt{c}}\right)\sqrt{d+ex}\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{cd^2+ae^2}\left(2\sqrt{2}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+a\cdots}}\cdots\right)}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\cdots\right)}\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2\cdots}}\cdots\right)}{2a}\cdots\right)}{8ac}\cdots$$

$$\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\downarrow$ 27

$$\frac{3\left(\frac{e\left(\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}d\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\left(2cd^2-2\sqrt{c}\sqrt{c\cdots}\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+\cdots}}}{\sqrt[4]{c}}\cdots\right)}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right)}{2a}\cdots\right)}{8ac}\cdots$$

$$\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\downarrow$ 27

$$3\left(\frac{e\left(\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}d\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}\sqrt[4]{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}d+\left(2cd^2-2\sqrt{c}\sqrt{c}\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right)}{2a}\right.$$

$$\overline{8ac}$$

$$\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 1142$$

$$3\left(\frac{e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}+\frac{1}{2}\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int -\cdots}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\cdots}\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(cx^2+a\right)^2}$$

$$\Big\downarrow \; 25$$

$$3\left(e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{1}{2}\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{}{\sqrt[4]{}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 27

$$3\left(e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt[4]{c}}-\frac{\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{}{d+ex}}{}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.\right.$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 1083

$$3\left(\frac{e\left(-\frac{-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{-d+2\left(d-\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)-ex}d\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt[4]{c}}-\frac{\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+a\cdots\right.}{\cdots}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\right)}{\cdots}\right.\cdots$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 219

$$3\left(\frac{e\left(-\frac{-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{\cdots}{d+ex+\cdots}}{\cdots}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\right)}{\cdots}\right.\cdots$$

$$\frac{(ae-cdx)(d+ex)^{3/2}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 1103

$$3\left(e\left(\frac{\frac{1}{2}\left(-2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+2cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}+\sqrt{ae^2+cd^2}+\sqrt{c}(d+ex)\right)-\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\left(2\sqrt{c}d\sqrt{ae^2+cd^2}+ae^2+2cd^2\right)\cdots}{\sqrt{\sqrt{c}\cdots}}}{2\sqrt{2}c^{3/4}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\cdots\right.\right.$$

$$\frac{(d+ex)^{3/2}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

input
```
Int[(d + e*x)^(5/2)/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*e - c*d*x)*(d + e*x)^(3/2))/(a*c*(a + c*x^2)^2) + (3*(-1/2*(Sqrt[
d + e*x]*(a*d*e - (2*c*d^2 + a*e^2)*x))/(a*(a + c*x^2)) + (e*((-((Sqrt[Sqr
t[c]*d + Sqrt[c*d^2 + a*e^2]]*(2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 +
a*e^2])*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]
)/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^
2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + ((2*c*d^2 + a*e^2 - 2*Sqrt
[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[
Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2
*Sqrt[2]*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-((Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]]*(2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^
2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1
/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/
Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - ((2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*S
qrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2
]*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(2*a)))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 902, normalized size of antiderivative = 1.60

method	result
pseudoelliptic	$\frac{3\left(\left(\left(-d\left(c^{\frac{9}{2}} x^{4}+2 a\,x^{2} c^{\frac{7}{2}}+a^{2} c^{\frac{5}{2}}\right)\sqrt{a\,e^{2}+c\,d^{2}}+\frac{c^{2}\left(c\,x^{2}+a\right)^{2}\left(a\,e^{2}+2 c\,d^{2}\right)}{2}\right)\sqrt{\left(a\,e^{2}+c\,d^{2}\right) c}+\left(\left(a^{2} c^{\frac{7}{2}}+c^{\frac{9}{2}} x^{2}\left(c\,x^{2}+2 a\right)\right) d\sqrt{a\,e^{2}\cdots}\right.\right.\right.}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^(5/2)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*(((-d*(c^(9/2)*x^4+2*a*x^2*c^(7/2)+a^2*c^(5/2))*(a*e^2+c*d^2)^(1/2)+1
/2*c^2*(c*x^2+a)^2*(a*e^2+2*c*d^2))*((a*e^2+c*d^2)*c)^(1/2)+((a^2*c^(7/2)+
c^(9/2)*x^2*(c*x^2+2*a))*d*(a*e^2+c*d^2)^(1/2)-1/2*c^3*(c*x^2+a)^2*(a*e^2+
2*c*d^2))*d)*(-ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)
+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))+ln(c^(1/2)*(e*x+d)-(e*x+d)^(1/2)*(2*((a
*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2)))*(4*(a*e^2+c*d^2)^(
1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(
1/2)+2*c*d)^(1/2)+8*e*(1/2*e*(a^2*c^(7/2)+c^(9/2)*x^2*(c*x^2+2*a))*d*(arct
an((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a
*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((
-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^
2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)))*(a*e^2+c*d
^2)^(1/2)+(-7/6*e*(d+1/7*e*x)*a^2*c^(7/2)+c^(9/2)*x*(c*d^2*x^2+5/3*a*(3/10
*e^2*x^2+1/10*d*e*x+d^2)))*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)
*c)^(1/2)-2*c*d)^(1/2)*(e*x+d)^(1/2)+1/4*(arctan((2*c^(1/2)*(e*x+d)^(1/2)+
(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*
((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*
((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a
*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)))*e*(c*x^2+a)^2*c^3*(a*e^2+2*c*d^2))*a)/
c^(9/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(456) = 912$.

Time = 0.10 (sec) , antiderivative size = 1037, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(-e^10/(a
^5*c^7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(1
6*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d*
e^6 - (4*a^5*c^6*d^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt(-(a^5*c^3*
sqrt(-e^10/(a^5*c^7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^
3))) - 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(a^5*c^3*sqrt(-e^10/(
a^5*c^7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c^3))*log(27*(
16*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d
*e^6 - (4*a^5*c^6*d^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt(-(a^5*c^3
*sqrt(-e^10/(a^5*c^7)) + 16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4)/(a^5*c
^3))) + 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(-e^10/(
a^5*c^7)) - 16*c^2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(
16*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*d
*e^6 + (4*a^5*c^6*d^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt((a^5*c^3*
sqrt(-e^10/(a^5*c^7)) - 16*c^2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^
3))) - 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt((a^5*c^3*sqrt(-e^10/(a
^5*c^7)) - 16*c^2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*c^3))*log(27*(1
6*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a^3*c^2*d*
e^6 + (4*a^5*c^6*d^2 + a^6*c^5*e^2)*sqrt(-e^10/(a^5*c^7)))*sqrt((a^5*c^3*s
qrt(-e^10/(a^5*c^7)) - 16*c^2*d^5 - 20*a*c*d^3*e^2 - 5*a^2*d*e^4)/(a^5*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+a)^3}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 + a)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = \frac{3\left(4\,c^4d^4e + 3\,ac^3d^2e^3 + (2\,acd^2e + a^2e^3)c^2e^2 - 2\left(\sqrt{-ac}c^2d^3e + \sqrt{-ac}acde^3\right)|c||e|\right) \mathrm{ar}\cdots}{32\left(a^3c^3e + \sqrt{-ac}a^2c^3d\right)\sqrt{-c^2d - \sqrt{-ac}ce}\,|e|\cdots} + \frac{3\left(4\,c^4d^4e + 3\,ac^3d^2e^3 + (2\,acd^2e + a^2e^3)c^2e^2 + 2\left(\sqrt{-ac}c^2d^3e + \sqrt{-ac}acde^3\right)|c||e|\right)\arctan\left(\frac{\cdots}{\sqrt{-\frac{a^2c^2d-\sqrt{a^4\cdots}}{\cdots}}}\cdots\right.}{32\left(a^3c^3e - \sqrt{-ac}a^2c^3d\right)\sqrt{-c^2d + \sqrt{-ac}ce}\,|e|} + \frac{6\,(ex+d)^{\frac{7}{2}}c^2d^2e - 18\,(ex+d)^{\frac{5}{2}}c^2d^3e + 18\,(ex+d)^{\frac{3}{2}}c^2d^4e - 6\,\sqrt{ex+d}c^2d^5e + 3\,(ex+d)^{\frac{7}{2}}ace^3 - 8\,(ex+\cdots}{16\left((ex+d)^2c - 2\,(ex+d)cd + \cdots\right.}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(4*c^4*d^4*e + 3*a*c^3*d^2*e^3 + (2*a*c*d^2*e + a^2*e^3)*c^2*e^2 - 2*
(sqrt(-a*c)*c^2*d^3*e + sqrt(-a*c)*a*c*d*e^3)*abs(c)*abs(e))*arctan(sqrt(e
*x + d)/sqrt(-(a^2*c^2*d + sqrt(a^4*c^4*d^2 - (a^2*c^2*d^2 + a^3*c*e^2)*a^
2*c^2))/(a^2*c^2)))/((a^3*c^3*e + sqrt(-a*c)*a^2*c^3*d)*sqrt(-c^2*d - sqrt
(-a*c)*c*e)*abs(e)) + 3/32*(4*c^4*d^4*e + 3*a*c^3*d^2*e^3 + (2*a*c*d^2*e +
 a^2*e^3)*c^2*e^2 + 2*(sqrt(-a*c)*c^2*d^3*e + sqrt(-a*c)*a*c*d*e^3)*abs(c)
*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^2*d - sqrt(a^4*c^4*d^2 - (a^2*c
^2*d^2 + a^3*c*e^2)*a^2*c^2))/(a^2*c^2)))/((a^3*c^3*e - sqrt(-a*c)*a^2*c^3
*d)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(e)) + 1/16*(6*(e*x + d)^(7/2)*c^2*d^
2*e - 18*(e*x + d)^(5/2)*c^2*d^3*e + 18*(e*x + d)^(3/2)*c^2*d^4*e - 6*sqrt
(e*x + d)*c^2*d^5*e + 3*(e*x + d)^(7/2)*a*c*e^3 - 8*(e*x + d)^(5/2)*a*c*d*
e^3 + 17*(e*x + d)^(3/2)*a*c*d^2*e^3 - 12*sqrt(e*x + d)*a*c*d^3*e^3 - (e*x
 + d)^(3/2)*a^2*e^5 - 6*sqrt(e*x + d)*a^2*d*e^5)/(((e*x + d)^2*c - 2*(e*x
+ d)*c*d + c*d^2 + a*e^2)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1028, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(a + c*x^2)^3,x)
```

output
```
((3*e*(a*e^2 + 2*c*d^2)*(d + e*x)^(7/2))/(16*a^2) + ((d + e*x)^(3/2)*(18*c
^2*d^4*e - a^2*e^5 + 17*a*c*d^2*e^3))/(16*a^2*c) - (d*(4*a*e^3 + 9*c*d^2*e
)*(d + e*x)^(5/2))/(8*a^2) - (3*(d + e*x)^(1/2)*(a^2*d*e^5 + c^2*d^5*e + 2
*a*c*d^3*e^3))/(8*a^2*c))/(c^2*(d + e*x)^4 + a^2*e^4 + c^2*d^4 + (6*c^2*d^
2 + 2*a*c*e^2)*(d + e*x)^2 - (4*c^2*d^3 + 4*a*c*d*e^2)*(d + e*x) - 4*c^2*d
*(d + e*x)^3 + 2*a*c*d^2*e^2) - 2*atanh((9*e^8*(d + e*x)^(1/2)*(- (9*d^5)/
(256*a^5*c) - (45*d*e^4)/(4096*a^3*c^3) - (45*d^3*e^2)/(1024*a^4*c^2) - (9
*e^5*(-a^15*c^7)^(1/2))/(4096*a^10*c^7))^(1/2))/(32*((27*e^11)/(2048*a*c^2
) + (27*d^4*e^7)/(512*a^3) + (135*d^2*e^9)/(2048*a^2*c) - (27*d*e^10*(-a^1
5*c^7)^(1/2))/(1024*a^9*c^5) - (27*d^3*e^8*(-a^15*c^7)^(1/2))/(1024*a^10*c
^4))) + (9*d*e^7*(-a^15*c^7)^(1/2)*(d + e*x)^(1/2)*(- (9*d^5)/(256*a^5*c)
- (45*d*e^4)/(4096*a^3*c^3) - (45*d^3*e^2)/(1024*a^4*c^2) - (9*e^5*(-a^15*
c^7)^(1/2))/(4096*a^10*c^7))^(1/2))/(32*((27*a^7*c*e^11)/2048 + (27*a^5*c^
3*d^4*e^7)/512 + (135*a^6*c^2*d^2*e^9)/2048 - (27*d*e^10*(-a^15*c^7)^(1/2)
)/(1024*a*c^2) - (27*d^3*e^8*(-a^15*c^7)^(1/2))/(1024*a^2*c))))*(-(9*(e^5*
(-a^15*c^7)^(1/2) + 16*a^5*c^6*d^5 + 5*a^7*c^4*d*e^4 + 20*a^6*c^5*d^3*e^2)
)/(4096*a^10*c^7))^(1/2) - 2*atanh((9*e^8*(d + e*x)^(1/2)*((9*e^5*(-a^15*c
^7)^(1/2))/(4096*a^10*c^7) - (45*d*e^4)/(4096*a^3*c^3) - (45*d^3*e^2)/(102
4*a^4*c^2) - (9*d^5)/(256*a^5*c))^(1/2))/(32*((27*e^11)/(2048*a*c^2) + (27
*d^4*e^7)/(512*a^3) + (135*d^2*e^9)/(2048*a^2*c) + (27*d*e^10*(-a^15*c^...
```

Reduce [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 4855, normalized size of antiderivative = 8.64

$$\int \frac{(d+ex)^{5/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+a)^3,x)
```

output

```
( - 6*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**3*e**
2 - 24*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c*
d**2 - 12*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*
sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c
)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2
*c*e**2*x**2 - 48*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
 - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) -
2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2
)))*a*c**2*d**2*x**2 - 6*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqr
t(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)
*sqrt(2)))*a*c**2*e**2*x**4 - 24*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a
*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) +
c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) - c*d)*sqrt(2)))*c**3*d**2*x**4 - 18*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*...
```

3.193 $\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 596

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx &= -\frac{(ae-cdx)\sqrt{d+ex}}{4ac\,(a+cx^2)^2}+\frac{(ae+6cdx)\sqrt{d+ex}}{16a^2c\,(a+cx^2)} \\ &-\frac{3e\left(2cd^2+ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{5/4}\sqrt{cd^2+ae^2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\ &+\frac{3e\left(2cd^2+ae^2+2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{5/4}\sqrt{cd^2+ae^2}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\ &+\frac{3e\left(2cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{32\sqrt{2}a^2c^{5/4}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\end{aligned}$$

output
```
-1/4*(-c*d*x+a*e)*(e*x+d)^(1/2)/a/c/(c*x^2+a)^2+1/16*(6*c*d*x+a*e)*(e*x+d)
^(1/2)/a^2/c/(c*x^2+a)-3/64*e*(2*c*d^2+a*e^2+2*c^(1/2)*d*(a*e^2+c*d^2)^(1/
2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^
(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(5/4)/(a*e^2+
c*d^2)^(1/2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+3/64*e*(2*c*d^2+a*e^2+
2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(
1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
)*2^(1/2)/a^2/c^(5/4)/(a*e^2+c*d^2)^(1/2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))
^(1/2)+3/64*e*(2*c*d^2+a*e^2-2*c^(1/2)*d*(a*e^2+c*d^2)^(1/2))*arctanh(2^(1
/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*
d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a^2/c^(5/4)/(a*e^2+c*d^2)^(1/2)/(c^(1
/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx = \frac{\frac{2\sqrt{a}\sqrt{d+ex}(-3a^2e+6c^2dx^3+acx(10d+ex))}{(a+cx^2)^2} + \frac{3i(4cd^2+2i\sqrt{a}\sqrt{c}de+ae^2)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}} - \frac{3i(4cd\cdots}{\cdots}}{32a^{5/2}c}$$

input
```
Integrate[(d + e*x)^(3/2)/(a + c*x^2)^3,x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(-3*a^2*e + 6*c^2*d*x^3 + a*c*x*(10*d + e*x)))/(
a + c*x^2)^2 + ((3*I)*(4*c*d^2 + (2*I)*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan
[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]
*e)])/Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e] - ((3*I)*(4*c*d^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d*e + a*e^2)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d +
e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e])/(32*
a^(5/2)*c)
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {495, 27, 686, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{6cd^2+5cexd+ae^2}{2\sqrt{d+ex}(cx^2+a)^2}dx}{4ac}-\frac{\sqrt{d+ex}(ae-cdx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{6cd^2+5cexd+ae^2}{\sqrt{d+ex}(cx^2+a)^2}dx}{8ac}-\frac{\sqrt{d+ex}(ae-cdx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{686}$$

$$\frac{\frac{\sqrt{d+ex}(ae+6cdx)}{2a(a+cx^2)}-\frac{\int -\frac{3c(cd^2+ae^2)(4cd^2+2cexd+ae^2)}{2\sqrt{d+ex}(cx^2+a)}dx}{2ac(ae^2+cd^2)}}{8ac}-\frac{\sqrt{d+ex}(ae-cdx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{4cd^2+2cexd+ae^2}{\sqrt{d+ex}(cx^2+a)}dx}{4a}+\frac{\sqrt{d+ex}(ae+6cdx)}{2a(a+cx^2)}}{8ac}-\frac{\sqrt{d+ex}(ae-cdx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{3\int \frac{e(2cd^2+2c(d+ex)d+ae^2)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}+\frac{\sqrt{d+ex}(ae+6cdx)}{2a(a+cx^2)}}{8ac}-\frac{\sqrt{d+ex}(ae-cdx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3e\int \frac{2cd^2+2c(d+ex)d+ae^2}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}+\frac{\sqrt{d+ex}(ae+6cdx)}{2a(a+cx^2)}}{8ac}-\frac{\sqrt{d+ex}(ae-cdx)}{4ac(a+cx^2)^2}$$

$\downarrow$ 1483

$$\frac{3e\left(\frac{\int \frac{\sqrt{2}\left(2cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+ae^2\right)+\sqrt[4]{c}\left(2cd^2-2\sqrt{c}\sqrt{cd^2+}\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{}}{\sqrt[4]{c}}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}}}\right.}{2a}$$

$$8ac$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\downarrow$ 27

$$\frac{3e\left(\frac{\int \frac{\sqrt{2}\left(2cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+ae^2\right)+\sqrt[4]{c}\left(2cd^2-2\sqrt{c}\sqrt{cd^2+}\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d-}}{\sqrt[4]{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}}}\right.}{2a}$$

$$8ac$$

$$\frac{\sqrt{d+ex}(ae-cdx)}{4ac\left(a+cx^2\right)^2}$$

$\downarrow$ 1142

$$\frac{\frac{\sqrt{d+ex}(ae+6cdx)}{2a(cx^2+a)}+\frac{3e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\left(2cd^2-2\sqrt{}\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.}{}}{}$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 25

$$\frac{\sqrt{d+ex}(ae+6cdx)}{2a(cx^2+a)}+\frac{3e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\left(2cd^2-2\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\cdots\right)}{\cdots}$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 27

$$\frac{\sqrt{d+ex}(ae+6cdx)}{2a(cx^2+a)}+\frac{3e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{\left(2cd^2-2\sqrt{c}\sqrt{cd\cdots}\right.}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right)}{\cdots}$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 1083

$$\frac{\sqrt{d+ex}(ae+6cdx)}{2a(cx^2+a)}+\frac{3e\left(\frac{\frac{\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\cdots}\cdots\right)}{\cdots}$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(cx^2+a\right)^2}$$

$\downarrow$ 219

$$\frac{\frac{\sqrt{d+ex}(ae+6cdx)}{2a(cx^2+a)}+\frac{3e\left(\frac{\frac{\left(2cd^2-2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\cdots\right.}{\cdots}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(cx^2+a\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\sqrt{d+ex}(ae+6cdx)}{2a(cx^2+a)}+\frac{3e\left(\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2cd^2+2\sqrt{c}\sqrt{cd^2+ae^2}d+ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right)}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}-\frac{1}{2}\sqrt[4]{c}\left(2cd^2-\cdots\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(ae-cdx)\sqrt{d+ex}}{4ac\left(cx^2+a\right)^2}$$

input
```
Int[(d + e*x)^(3/2)/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*e - c*d*x)*Sqrt[d + e*x])/(a*c*(a + c*x^2)^2) + (((a*e + 6*c*d*x)
*Sqrt[d + e*x])/(2*a*(a + c*x^2)) + (3*e*((-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqr
t[c*d^2 + a*e^2]]*(2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*ArcT
anh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4)) +
 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[
Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*(2*c*d^2 + a*e^2 - 2*Sqrt[c]*
d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt
[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqr
t[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) +
(-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(2*c*d^2 + a*e^2 + 2*Sqr
t[c]*d*Sqrt[c*d^2 + a*e^2])*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sq
rt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d -
Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) + (c^(1/4)*
(2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2])*Log[Sqrt[c*d^2 + a*e^2
] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]])))/(2*a))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(482) = 964$.

Time = 2.79 (sec) , antiderivative size = 975, normalized size of antiderivative = 1.64

method	result
pseudoelliptic	$-\frac{3\left(\left(\ln\left((-ex-d)\sqrt{c}+\sqrt{ex+d}\,\sqrt{2\sqrt{(a\,e^2+c\,d^2)c}+2cd}-\sqrt{a\,e^2+c\,d^2}\right)-\ln\left(\sqrt{c}\,(ex+d)+\sqrt{ex+d}\,\sqrt{2\sqrt{(a\,e^2+c\,d^2)c}+\right.\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((e*x+d)^(3/2)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output

```
-3/128*((ln((-e*x-d)*c^(1/2)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*
d)^(1/2)-(a*e^2+c*d^2)^(1/2))-ln(c^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+
c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2)))*((-2*c*d*(c*x^2+a)^2*(a
*e^2+c*d^2)^(1/2)+2*a^2*(e^2*x^2+d^2)*c^(3/2)+(e^2*x^4+4*d^2*x^2)*a*c^(5/2
)+2*c^(7/2)*d^2*x^4+a^3*e^2*c^(1/2))*((a*e^2+c*d^2)*c)^(1/2)-(-2*c^2*d*(c*
x^2+a)^2*(a*e^2+c*d^2)^(1/2)+2*a^2*(e^2*x^2+d^2)*c^(5/2)+(e^2*x^4+4*d^2*x^
2)*a*c^(7/2)+2*c^(9/2)*d^2*x^4+a^3*e^2*c^(3/2))*d)*(4*(a*e^2+c*d^2)^(1/2)*
c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+
2*c*d)^(1/2)+4*e*a*(2*((a^2*c^(3/2)*e-10/3*x*(a*(1/10*e*x+d)*c^(5/2)+3/5*c
^(7/2)*d*x^2))*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*
c*d)^(1/2)*(e*x+d)^(1/2)+(arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^
2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*
c)^(1/2)-2*c*d)^(1/2))-arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c
)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(
1/2)-2*c*d)^(1/2)))*e*(c*x^2+a)^2*c^2*d)*(a*e^2+c*d^2)^(1/2)+(arctan((-2*c
^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*
d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((2*c^(1/
2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)
^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)))*e*(2*a^2*(e^2*x^2+
d^2)*c^(5/2)+(e^2*x^4+4*d^2*x^2)*a*c^(7/2)+2*c^(9/2)*d^2*x^4+a^3*e^2*c^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1752 vs. 2(484) = 968.

Time = 0.15 (sec) , antiderivative size = 1752, normalized size of antiderivative = 2.94

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(16*c^2*d^5 + 20*a*c*d
^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5*c^7*d^4
 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2))*log(27*
(16*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) + 27*(2*a^3*c^2*
d^2*e^6 + a^4*c*e^8 + (4*a^5*c^6*d^5 + 7*a^6*c^5*d^3*e^2 + 3*a^7*c^4*d*e^4
)*sqrt(-e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt(-(16*c
^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-
e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*
c^2*e^2))) - 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(16*c^2*d^5 + 2
0*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^10/(a^5*
c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2*e^2))*
log(27*(16*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d) - 27*(2*a
^3*c^2*d^2*e^6 + a^4*c*e^8 + (4*a^5*c^6*d^5 + 7*a^6*c^5*d^3*e^2 + 3*a^7*c^
4*d*e^4)*sqrt(-e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))*sqrt
(-(16*c^2*d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4 + (a^5*c^3*d^2 + a^6*c^2*e^2)
*sqrt(-e^10/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2
 + a^6*c^2*e^2))) + 3*(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)*sqrt(-(16*c^2*
d^5 + 20*a*c*d^3*e^2 + 5*a^2*d*e^4 - (a^5*c^3*d^2 + a^6*c^2*e^2)*sqrt(-e^1
0/(a^5*c^7*d^4 + 2*a^6*c^6*d^2*e^2 + a^7*c^5*e^4)))/(a^5*c^3*d^2 + a^6*c^2
*e^2))*log(27*(16*c^2*d^4*e^5 + 12*a*c*d^2*e^7 + a^2*e^9)*sqrt(e*x + d)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + a)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx = \frac{3\left(4\,c^3d^3e+3\,ac^2de^3-\left(2\,\sqrt{-ac}cd^2e+\sqrt{-ac}ae^3\right)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{a^2c^2d+\sqrt{a^4c^4d^2-\left(a^2c^2\right.}}{a^2c^2}}}\right.}{32\left(a^3c^2e+\sqrt{-ac}a^2c^2d\right)\sqrt{-c^2d-\sqrt{-ac}ce}|e|} + \frac{3\left(4\,c^3d^3e+3\,ac^2de^3+\left(2\,\sqrt{-ac}cd^2e+\sqrt{-ac}ae^3\right)|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{a^2c^2d-\sqrt{a^4c^4d^2-\left(a^2c^2d^2+a^3ce^2\right)a^2c^2}}{a^2c^2}}}\right)}{32\left(a^3c^2e-\sqrt{-ac}a^2c^2d\right)\sqrt{-c^2d+\sqrt{-ac}ce}|e|} + \frac{6\,(ex+d)^{\frac{7}{2}}c^2de-18\,(ex+d)^{\frac{5}{2}}c^2d^2e+18\,(ex+d)^{\frac{3}{2}}c^2d^3e-6\,\sqrt{ex+d}c^2d^4e+(ex+d)^{\frac{5}{2}}ace^3+8\,(ex+d}{16\left((ex+d)^2c-2\,(ex+d)cd+cd^2+ae^2\right)^2a^2c}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(4*c^3*d^3*e + 3*a*c^2*d*e^3 - (2*sqrt(-a*c)*c*d^2*e + sqrt(-a*c)*a*e
^3)*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^2*d + sqrt(a^4*c^4*d^
2 - (a^2*c^2*d^2 + a^3*c*e^2)*a^2*c^2))/(a^2*c^2)))/((a^3*c^2*e + sqrt(-a*
c)*a^2*c^2*d)*sqrt(-c^2*d - sqrt(-a*c)*c*e)*abs(e)) + 3/32*(4*c^3*d^3*e +
3*a*c^2*d*e^3 + (2*sqrt(-a*c)*c*d^2*e + sqrt(-a*c)*a*e^3)*abs(c)*abs(e))*a
rctan(sqrt(e*x + d)/sqrt(-(a^2*c^2*d - sqrt(a^4*c^4*d^2 - (a^2*c^2*d^2 + a
^3*c*e^2)*a^2*c^2))/(a^2*c^2)))/((a^3*c^2*e - sqrt(-a*c)*a^2*c^2*d)*sqrt(-
c^2*d + sqrt(-a*c)*c*e)*abs(e)) + 1/16*(6*(e*x + d)^(7/2)*c^2*d*e - 18*(e*
x + d)^(5/2)*c^2*d^2*e + 18*(e*x + d)^(3/2)*c^2*d^3*e - 6*sqrt(e*x + d)*c^
2*d^4*e + (e*x + d)^(5/2)*a*c*e^3 + 8*(e*x + d)^(3/2)*a*c*d*e^3 - 9*sqrt(e
*x + d)*a*c*d^2*e^3 - 3*sqrt(e*x + d)*a^2*e^5)/(((e*x + d)^2*c - 2*(e*x +
d)*c*d + c*d^2 + a*e^2)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 3204, normalized size of antiderivative = 5.38

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(a + c*x^2)^3,x)
```

output
```
(((4*a*d*e^3 + 9*c*d^3*e)*(d + e*x)^(3/2))/(8*a^2) + (e*(a*e^2 - 18*c*d^2)
*(d + e*x)^(5/2))/(16*a^2) - (3*(d + e*x)^(1/2)*(a^2*e^5 + 2*c^2*d^4*e + 3
*a*c*d^2*e^3))/(16*a^2*c) + (3*c*d*e*(d + e*x)^(7/2))/(8*a^2))/(c^2*(d + e
*x)^4 + a^2*e^4 + c^2*d^4 + (6*c^2*d^2 + 2*a*c*e^2)*(d + e*x)^2 - (4*c^2*d
^3 + 4*a*c*d*e^2)*(d + e*x) - 4*c^2*d*(d + e*x)^3 + 2*a*c*d^2*e^2) + atan(
((((3*(2048*a^6*c^2*e^5 + 4096*a^5*c^3*d^2*e^3))/(2048*a^6) - 64*a*c^4*d*e
^2*(d + e*x)^(1/2)*(-(9*(e^5*(-a^15*c^5)^(1/2) + 16*a^5*c^5*d^5 + 5*a^7*c^
3*d*e^4 + 20*a^6*c^4*d^3*e^2))/(4096*(a^10*c^6*d^2 + a^11*c^5*e^2)))^(1/2)
)*(-(9*(e^5*(-a^15*c^5)^(1/2) + 16*a^5*c^5*d^5 + 5*a^7*c^3*d*e^4 + 20*a^6*
c^4*d^3*e^2))/(4096*(a^10*c^6*d^2 + a^11*c^5*e^2)))^(1/2) - ((d + e*x)^(1/
2)*(9*a^2*c*e^6 + 144*c^3*d^4*e^2 + 36*a*c^2*d^2*e^4))/(64*a^4))*(-(9*(e^5
*(-a^15*c^5)^(1/2) + 16*a^5*c^5*d^5 + 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^3*e^2
))/(4096*(a^10*c^6*d^2 + a^11*c^5*e^2)))^(1/2)*1i - (((3*(2048*a^6*c^2*e^5
 + 4096*a^5*c^3*d^2*e^3))/(2048*a^6) + 64*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(9
*(e^5*(-a^15*c^5)^(1/2) + 16*a^5*c^5*d^5 + 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^
3*e^2))/(4096*(a^10*c^6*d^2 + a^11*c^5*e^2)))^(1/2))*(-(9*(e^5*(-a^15*c^5)
^(1/2) + 16*a^5*c^5*d^5 + 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^3*e^2))/(4096*(a^
10*c^6*d^2 + a^11*c^5*e^2)))^(1/2) + ((d + e*x)^(1/2)*(9*a^2*c*e^6 + 144*c
^3*d^4*e^2 + 36*a*c^2*d^2*e^4))/(64*a^4))*(-(9*(e^5*(-a^15*c^5)^(1/2) + 16
*a^5*c^5*d^5 + 5*a^7*c^3*d*e^4 + 20*a^6*c^4*d^3*e^2))/(4096*(a^10*c^6*d...
```

Reduce [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 6142, normalized size of antiderivative = 10.31

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+a)^3,x)
```

output
```
( - 18*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**3*c*
d*e**2 - 24*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d
)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt
(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*
*2*c**2*d**3 - 36*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
 - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) -
2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2
)))*a**2*c**2*d*e**2*x**2 - 48*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*
d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
- c*d)*sqrt(2)))*a*c**3*d**3*x**2 - 18*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*
sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d*
*2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) - c*d)*sqrt(2)))*a*c**3*d*e**2*x**4 - 24*sqrt(a*e**2 + c*d**2)*sqr
t(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e
**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt
(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c**4*d**3*x**4 - 6*sqrt(c)*sqrt(sqrt(c)
*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + ...
```

3.194 $\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 642

$$\begin{aligned}
&\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx \\
&= \frac{x\sqrt{d+ex}}{4a(a+cx^2)^2} + \frac{\sqrt{d+ex}\left(ade+(6cd^2+5ae^2)x\right)}{16a^2(cd^2+ae^2)(a+cx^2)} \\
&- \frac{e\left(6cd^2+5ae^2+\frac{2\sqrt{c}d(3cd^2+4ae^2)}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{3/4}(cd^2+ae^2)\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&+ \frac{e\left(6cd^2+5ae^2+\frac{2\sqrt{c}d(3cd^2+4ae^2)}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2c^{3/4}(cd^2+ae^2)\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&- \frac{e\left(6cd^2+5ae^2-\frac{2\sqrt{c}d(3cd^2+4ae^2)}{\sqrt{cd^2+ae^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{32\sqrt{2}a^2c^{3/4}(cd^2+ae^2)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}
\end{aligned}$$

output
```
1/4*x*(e*x+d)^(1/2)/a/(c*x^2+a)^2+1/16*(e*x+d)^(1/2)*(a*d*e+(5*a*e^2+6*c*d
^2)*x)/a^2/(a*e^2+c*d^2)/(c*x^2+a)-1/64*e*(6*c*d^2+5*a*e^2+2*c^(1/2)*d*(4*
a*e^2+3*c*d^2)/(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2)
)^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1
/2))*2^(1/2)/a^2/c^(3/4)/(a*e^2+c*d^2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1
/2)+1/64*e*(6*c*d^2+5*a*e^2+2*c^(1/2)*d*(4*a*e^2+3*c*d^2)/(a*e^2+c*d^2)^(1
/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)
^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(3/4)/(a*e^2
+c*d^2)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-1/64*e*(6*c*d^2+5*a*e^2-2*c
^(1/2)*d*(4*a*e^2+3*c*d^2)/(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c
^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(
1/2)*(e*x+d)))*2^(1/2)/a^2/c^(3/4)/(a*e^2+c*d^2)/(c^(1/2)*d+(a*e^2+c*d^2)^
(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx = \frac{\frac{2\sqrt{a}\sqrt{d+ex}(6c^2d^2x^3+a^2e(d+9ex)+acx(10d^2+dex+5e^2x^2))}{(cd^2+ae^2)(a+cx^2)^2}+\frac{i(12cd^2+18i\sqrt{a}\sqrt{c}de-5ae^2)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{c}(\sqrt{c}d+i\sqrt{a}e)\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}-\frac{i(12cd^2-18\cdots}{\cdots}}{32a^{5/2}}$$

input
```
Integrate[Sqrt[d + e*x]/(a + c*x^2)^3,x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(6*c^2*d^2*x^3 + a^2*e*(d + 9*e*x) + a*c*x*(10*d
^2 + d*e*x + 5*e^2*x^2)))/((c*d^2 + a*e^2)*(a + c*x^2)^2) + (I*(12*c*d^2 +
 (18*I)*Sqrt[a]*Sqrt[c]*d*e - 5*a*e^2)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqr
t[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/(Sqrt[c]*(Sqrt[c]*d + I
*Sqrt[a]*e)*Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]) - (I*(12*c*d^2 - (18*I)*Sq
rt[a]*Sqrt[c]*d*e - 5*a*e^2)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sq
rt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/(Sqrt[c]*(Sqrt[c]*d - I*Sqrt[a]*e
)*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]))/(32*a^(5/2))
```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 909, normalized size of antiderivative = 1.42, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {494, 27, 686, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}dx$$

$$\Big\downarrow \ 494$$

$$\frac{x\sqrt{d+ex}}{4a(a+cx^2)^2}-\frac{\int -\frac{6d+5ex}{2\sqrt{d+ex}(cx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{6d+5ex}{\sqrt{d+ex}(cx^2+a)^2}dx}{8a}+\frac{x\sqrt{d+ex}}{4a(a+cx^2)^2}$$

$$\Big\downarrow \ 686$$

$$\frac{\frac{\sqrt{d+ex}(x(5ae^2+6cd^2)+ade)}{2a(a+cx^2)(ae^2+cd^2)}-\frac{\int -\frac{c\left(d\left(12cd^2+13ae^2\right)+e\left(6cd^2+5ae^2\right)x\right)}{2\sqrt{d+ex}\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a(a+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{d\left(12cd^2+13ae^2\right)+e\left(6cd^2+5ae^2\right)x}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{4a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(x(5ae^2+6cd^2)+ade)}{2a(a+cx^2)(ae^2+cd^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a(a+cx^2)^2}$$

$$\Big\downarrow \ 654$$

$$\frac{\frac{\int \frac{e\left(2d\left(3cd^2+4ae^2\right)+\left(6cd^2+5ae^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(x(5ae^2+6cd^2)+ade)}{2a(a+cx^2)(ae^2+cd^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a(a+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{e\int \frac{2d\left(3cd^2+4ae^2\right)+\left(6cd^2+5ae^2\right)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(x(5ae^2+6cd^2)+ade)}{2a(a+cx^2)(ae^2+cd^2)}}{8a}+\frac{x\sqrt{d+ex}}{4a(a+cx^2)^2}$$

$$\Big\downarrow \text{1483}$$

$$\frac{e\left(\frac{\int \frac{2\sqrt{2}d\left(3cd^2+4ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(6cd^3+8ae^2d-\frac{\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)}{\sqrt{c}}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2+4ae^2\right)+\sqrt[4]{c}\Big(\cdots}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2\cdots}}\cdots\right.}{2a(ae^2+cd^2)}$$

$$8a$$

$$\frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\left(\frac{\int \frac{2\sqrt{2}d\left(3cd^2+4ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(6cd^3+8ae^2d-\frac{\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)}{\sqrt{c}}\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{2\sqrt{2}d\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(3cd^2+4ae^2\right)+\sqrt[4]{c}\Big(\cdots}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\cdots}{}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+\cdots}}\cdots\right.}{2a(ae^2+cd^2)}$$

$$8a$$

$$\frac{x\sqrt{d+ex}}{4a\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt{d+ex}x}{4a\left(cx^2+a\right)^2}+$$

$$\frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}\cdots}{\sqrt{2}\sqrt{c}}\cdots}{2\sqrt{2}\sqrt{c}\sqrt{cd\cdots}}\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d+ex}\,x}{4a\left(cx^2+a\right)^2}+\frac{\frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+\frac{e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}}{\sqrt{2}\sqrt{c}}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2}}\right)}{}}{}$$

$\downarrow$ 27

$$\frac{\sqrt{d+ex}\,x}{4a\left(cx^2+a\right)^2}+\frac{\frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+\frac{e\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}}{\sqrt{2}\sqrt{c}}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}}\right)}{}}{}$$

$\downarrow$ 1083

$$\frac{\sqrt{d+ex}\,x}{4a\left(cx^2+a\right)^2}+\frac{\frac{\sqrt{d+ex}\left(ade+\left(6cd^2+5ae^2\right)x\right)}{2a\left(cd^2+ae^2\right)\left(cx^2+a\right)}+\frac{e\left(\frac{\frac{\left(6cd^3+8ae^2d-\frac{\sqrt{cd^2+ae^2}\left(6cd^2+5ae^2\right)}{\sqrt{c}}\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+}}{}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+}}\right)}{}}{}$$

$\downarrow$ 219

$$\frac{\sqrt{d+ex}(ade+(6cd^2+5ae^2)x)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{e\left(\frac{\left(6cd^3+8ae^2d-\frac{\sqrt{cd^2+ae^2}(6cd^2+5ae^2)}{\sqrt{c}}\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt{\sqrt{c}d+\sqrt{c}\cdots}}{\cdots}\right)}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\cdots}}}{\cdots}+\frac{\sqrt{d+ex}x}{4a(cx^2+a)^2}$$

$\downarrow$ 1103

$$\frac{\sqrt{d+ex}(ade+(6cd^2+5ae^2)x)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{e\left(-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(6c^{3/2}d^3+8a\sqrt{c}e^2d+\sqrt{cd^2+ae^2}(6cd^2+5ae^2)\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+a\cdots}}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\cdots\right)}{\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\cdots\right)}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+a\cdots}}}{\cdots}+\frac{\sqrt{d+ex}x}{4a(cx^2+a)^2}$$

input
```
Int[Sqrt[d + e*x]/(a + c*x^2)^3,x]
```

output
```
(x*Sqrt[d + e*x])/(4*a*(a + c*x^2)^2) + ((Sqrt[d + e*x]*(a*d*e + (6*c*d^2
+ 5*a*e^2)*x))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*((-((Sqrt[Sqrt[c]*d
+ Sqrt[c*d^2 + a*e^2]]*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e^2 + Sqrt[c*d^2 + a
*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + S
qrt[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d
- Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]))
 - (c^(1/4)*(6*c*d^3 + 8*a*d*e^2 - (Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2
))/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqr
t[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c
]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-((Sqrt[Sq
rt[c]*d + Sqrt[c*d^2 + a*e^2]]*(6*c^(3/2)*d^3 + 8*a*Sqrt[c]*d*e^2 + Sqrt[c
*d^2 + a*e^2]*(6*c*d^2 + 5*a*e^2))*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]
*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[
c]*d - Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^
2]])) + (c^(1/4)*(6*c*d^3 + 8*a*d*e^2 - (Sqrt[c*d^2 + a*e^2]*(6*c*d^2 + 5*
a*e^2))/Sqrt[c])*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
+ Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*S
qrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/(2*a*(
c*d^2 + a*e^2)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex+d}}{\left(c\,x^{2}+a\right)^{3}}dx$$

input
```
int((e*x+d)^(1/2)/(c*x^2+a)^3,x)
```

output
```
int((e*x+d)^(1/2)/(c*x^2+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3770 vs. $2(536) = 1072$.

Time = 0.42 (sec) , antiderivative size = 3770, normalized size of antiderivative = 5.87

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)^3}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + a)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1068, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/32*((a^2*c*d^2*e + a^3*e^3)^2*(6*c*d^2*e + 5*a*e^3)*abs(c) - 2*(3*sqrt(
-a*c)*a*c^2*d^5*e + 7*sqrt(-a*c)*a^2*c*d^3*e^3 + 4*sqrt(-a*c)*a^3*d*e^5)*a
bs(-a^2*c*d^2*e - a^3*e^3)*abs(c) + (12*a^3*c^4*d^8*e + 37*a^4*c^3*d^6*e^3
 + 38*a^5*c^2*d^4*e^5 + 13*a^6*c*d^2*e^7)*abs(c))*arctan(sqrt(e*x + d)/sqr
t(-(a^2*c^2*d^3 + a^3*c*d*e^2 + sqrt((a^2*c^2*d^3 + a^3*c*d*e^2)^2 - (a^2*
c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(a^2*c^2*d^2 + a^3*c*e^2)))/(a^2*c^2*
d^2 + a^3*c*e^2)))/((a^4*c^3*d^4*e + 2*a^5*c^2*d^2*e^3 + a^6*c*e^5 - sqrt(
-a*c)*a^3*c^3*d^5 - 2*sqrt(-a*c)*a^4*c^2*d^3*e^2 - sqrt(-a*c)*a^5*c*d*e^4)
*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(-a^2*c*d^2*e - a^3*e^3)) - 1/32*((a^2*c
*d^2*e + a^3*e^3)^2*(6*c*d^2*e + 5*a*e^3)*abs(c) + 2*(3*sqrt(-a*c)*a*c^2*d
^5*e + 7*sqrt(-a*c)*a^2*c*d^3*e^3 + 4*sqrt(-a*c)*a^3*d*e^5)*abs(-a^2*c*d^2
*e - a^3*e^3)*abs(c) + (12*a^3*c^4*d^8*e + 37*a^4*c^3*d^6*e^3 + 38*a^5*c^2
*d^4*e^5 + 13*a^6*c*d^2*e^7)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^2*d
^3 + a^3*c*d*e^2 - sqrt((a^2*c^2*d^3 + a^3*c*d*e^2)^2 - (a^2*c^2*d^4 + 2*a
^3*c*d^2*e^2 + a^4*e^4)*(a^2*c^2*d^2 + a^3*c*e^2)))/(a^2*c^2*d^2 + a^3*c*e
^2)))/((a^4*c^3*d^4*e + 2*a^5*c^2*d^2*e^3 + a^6*c*e^5 + sqrt(-a*c)*a^3*c^3
*d^5 + 2*sqrt(-a*c)*a^4*c^2*d^3*e^2 + sqrt(-a*c)*a^5*c*d*e^4)*sqrt(-c^2*d
- sqrt(-a*c)*c*e)*abs(-a^2*c*d^2*e - a^3*e^3)) + 1/16*(6*(e*x + d)^(7/2)*c
^2*d^2*e - 18*(e*x + d)^(5/2)*c^2*d^3*e + 18*(e*x + d)^(3/2)*c^2*d^4*e - 6
*sqrt(e*x + d)*c^2*d^5*e + 5*(e*x + d)^(7/2)*a*c*e^3 - 14*(e*x + d)^(5/...
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 6238, normalized size of antiderivative = 9.72

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(a + c*x^2)^3,x)
```

output
```
atan(((((32768*a^7*c^3*d*e^7 + 24576*a^5*c^5*d^5*e^3 + 57344*a^6*c^4*d^3*e
^5)/(4096*(a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) - ((d + e*x)^(1/2)*(4
096*a^7*c^4*d*e^6 + 4096*a^5*c^6*d^5*e^2 + 8192*a^6*c^5*d^3*e^4)*(-(144*a^
5*c^5*d^7 - 25*a*e^7*(-a^15*c^3)^(1/2) + 105*a^8*c^2*d*e^6 + 420*a^6*c^4*d
^5*e^2 + 385*a^7*c^3*d^3*e^4 - 21*c*d^2*e^5*(-a^15*c^3)^(1/2))/(4096*(a^10
*c^6*d^6 + a^13*c^3*e^6 + 3*a^11*c^5*d^4*e^2 + 3*a^12*c^4*d^2*e^4)))^(1/2)
)/(64*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)))*(-(144*a^5*c^5*d^7 - 25*
a*e^7*(-a^15*c^3)^(1/2) + 105*a^8*c^2*d*e^6 + 420*a^6*c^4*d^5*e^2 + 385*a^
7*c^3*d^3*e^4 - 21*c*d^2*e^5*(-a^15*c^3)^(1/2))/(4096*(a^10*c^6*d^6 + a^13
*c^3*e^6 + 3*a^11*c^5*d^4*e^2 + 3*a^12*c^4*d^2*e^4)))^(1/2) - ((d + e*x)^(
1/2)*(144*c^5*d^6*e^2 - 25*a^3*c^2*e^8 + 276*a*c^4*d^4*e^4 + 109*a^2*c^3*d
^2*e^6))/(64*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)))*(-(144*a^5*c^5*d^
7 - 25*a*e^7*(-a^15*c^3)^(1/2) + 105*a^8*c^2*d*e^6 + 420*a^6*c^4*d^5*e^2 +
 385*a^7*c^3*d^3*e^4 - 21*c*d^2*e^5*(-a^15*c^3)^(1/2))/(4096*(a^10*c^6*d^6
 + a^13*c^3*e^6 + 3*a^11*c^5*d^4*e^2 + 3*a^12*c^4*d^2*e^4)))^(1/2)*1i - ((
(32768*a^7*c^3*d*e^7 + 24576*a^5*c^5*d^5*e^3 + 57344*a^6*c^4*d^3*e^5)/(409
6*(a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) + ((d + e*x)^(1/2)*(4096*a^7*
c^4*d*e^6 + 4096*a^5*c^6*d^5*e^2 + 8192*a^6*c^5*d^3*e^4)*(-(144*a^5*c^5*d^
7 - 25*a*e^7*(-a^15*c^3)^(1/2) + 105*a^8*c^2*d*e^6 + 420*a^6*c^4*d^5*e^2 +
 385*a^7*c^3*d^3*e^4 - 21*c*d^2*e^5*(-a^15*c^3)^(1/2))/(4096*(a^10*c^6*...
```

Reduce [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 7511, normalized size of antiderivative = 11.70

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)/(c*x^2+a)^3,x)
```

output
```
( - 10*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**4*e*
*4 - 38*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sq
rt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*
sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**3*c
*d**2*e**2 - 20*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) -
 c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*
sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2))
)*a**3*c*e**4*x**2 - 24*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c
*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt
(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*
sqrt(2)))*a**2*c**2*d**4 - 76*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e*
*2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d
)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) -
 c*d)*sqrt(2)))*a**2*c**2*d**2*e**2*x**2 - 10*sqrt(a*e**2 + c*d**2)*sqrt(s
qrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2
 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*
e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c**2*e**4*x**4 - 48*sqrt(a*e**2 + c*d
**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(...
```

3.195 $\int \frac{1}{\sqrt{d+ex}(a+cx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 721

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx &= \frac{(ae+cdx)\sqrt{d+ex}}{4a\,(cd^2+ae^2)\,(a+cx^2)^2}+\frac{\sqrt{d+ex}(ae(cd^2+7ae^2)+6cd(cd^2+2ae^2)\,x)}{16a^2\,(cd^2+ae^2)^2\,(a+cx^2)} \\
&\quad -\frac{3e\left(2\sqrt{c}d(cd^2+2ae^2)+\frac{2c^2d^4+5acd^2e^2+7a^2e^4}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2\sqrt[4]{c}\,(cd^2+ae^2)^2\,\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&\quad +\frac{3e\left(2\sqrt{c}d(cd^2+2ae^2)+\frac{2c^2d^4+5acd^2e^2+7a^2e^4}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{32\sqrt{2}a^2\sqrt[4]{c}\,(cd^2+ae^2)^2\,\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} \\
&\quad -\frac{3e\left(2\sqrt{c}d(cd^2+2ae^2)-\frac{2c^2d^4+5acd^2e^2+7a^2e^4}{\sqrt{cd^2+ae^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{32\sqrt{2}a^2\sqrt[4]{c}\,(cd^2+ae^2)^2\,\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}
\end{aligned}$$

output
```
1/4*(c*d*x+a*e)*(e*x+d)^(1/2)/a/(a*e^2+c*d^2)/(c*x^2+a)^2+1/16*(e*x+d)^(1/
2)*(a*e*(7*a*e^2+c*d^2)+6*c*d*(2*a*e^2+c*d^2)*x)/a^2/(a*e^2+c*d^2)^2/(c*x^
2+a)-3/64*e*(2*c^(1/2)*d*(2*a*e^2+c*d^2)+(7*a^2*e^4+5*a*c*d^2*e^2+2*c^2*d^
4)/(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1
/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)
/a^2/c^(1/4)/(a*e^2+c*d^2)^2/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+3/64*e
*(2*c^(1/2)*d*(2*a*e^2+c*d^2)+(7*a^2*e^4+5*a*c*d^2*e^2+2*c^2*d^4)/(a*e^2+c
*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)
*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(1/4
)/(a*e^2+c*d^2)^2/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-3/64*e*(2*c^(1/2)
*d*(2*a*e^2+c*d^2)-(7*a^2*e^4+5*a*c*d^2*e^2+2*c^2*d^4)/(a*e^2+c*d^2)^(1/2)
)*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1
/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/a^2/c^(1/4)/(a*e^2+c*d^
2)^2/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx = \frac{\frac{2\sqrt{a}\sqrt{d+ex}(11a^3e^3+6c^3d^3x^3+a^2ce(5d^2+16dex+7e^2x^2)+ac^2dx(10d^2+dex+12e^2x^2))}{(cd^2+ae^2)^2(a+cx^2)^2}+\frac{3i(4cd^2+10i\sqrt{a}\sqrt{c}de-7ae^2)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{\cdots}}}{\sqrt{c}d+i\sqrt{\cdots}}\right)}{(\sqrt{c}d+i\sqrt{a}e)^2\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}}{32a^{5/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(a + c*x^2)^3),x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(11*a^3*e^3 + 6*c^3*d^3*x^3 + a^2*c*e*(5*d^2 + 1
6*d*e*x + 7*e^2*x^2) + a*c^2*d*x*(10*d^2 + d*e*x + 12*e^2*x^2)))/((c*d^2 +
 a*e^2)^2*(a + c*x^2)^2) + ((3*I)*(4*c*d^2 + (10*I)*Sqrt[a]*Sqrt[c]*d*e -
7*a*e^2)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c
]*d + I*Sqrt[a]*e)])/((Sqrt[c]*d + I*Sqrt[a]*e)^2*Sqrt[-(c*d) - I*Sqrt[a]*
Sqrt[c]*e]) - ((3*I)*(4*c*d^2 - (10*I)*Sqrt[a]*Sqrt[c]*d*e - 7*a*e^2)*ArcT
an[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[
a]*e)])/((Sqrt[c]*d - I*Sqrt[a]*e)^2*Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]))/
(32*a^(5/2))
```

Rubi [A] (verified)

Time = 3.08 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.39, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {496, 27, 686, 27, 654, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^3\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a(a+cx^2)^2(ae^2+cd^2)}-\frac{\int -\frac{6cd^2+5cexd+7ae^2}{2\sqrt{d+ex}(cx^2+a)^2}dx}{4a(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{6cd^2+5cexd+7ae^2}{\sqrt{d+ex}(cx^2+a)^2}dx}{8a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(ae+cdx)}{4a(a+cx^2)^2(ae^2+cd^2)}$$

$$\Big\downarrow \text{686}$$

$$\frac{\frac{\sqrt{d+ex}(6cdx(2ae^2+cd^2)+ae(7ae^2+cd^2))}{2a(a+cx^2)(ae^2+cd^2)}-\frac{\int -\frac{3c\left(4c^2d^4+9ace^2d^2+2ce\left(cd^2+2ae^2\right)xd+7a^2e^4\right)}{2\sqrt{d+ex}\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)}}{8a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(ae+cdx)}{4a(a+cx^2)^2(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{4c^2d^4+9ace^2d^2+2ce\left(cd^2+2ae^2\right)xd+7a^2e^4}{\sqrt{d+ex}\left(cx^2+a\right)}dx}{4a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(6cdx(2ae^2+cd^2)+ae(7ae^2+cd^2))}{2a(a+cx^2)(ae^2+cd^2)}}{8a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(ae+cdx)}{4a(a+cx^2)^2(ae^2+cd^2)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{3\int \frac{e\left(2c^2d^4+5ace^2d^2+2c\left(cd^2+2ae^2\right)(d+ex)d+7a^2e^4\right)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(6cdx(2ae^2+cd^2)+ae(7ae^2+cd^2))}{2a(a+cx^2)(ae^2+cd^2)}}{8a(ae^2+cd^2)}+\frac{\sqrt{d+ex}(ae+cdx)}{4a(a+cx^2)^2(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3e\int \frac{2c^2d^4+5ace^2d^2+2c\left(cd^2+2ae^2\right)(d+ex)d+7a^2e^4}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(ae^2+cd^2)}+\frac{\sqrt{d+ex}\left(6cdx\left(2ae^2+cd^2\right)+ae\left(7ae^2+cd^2\right)\right)}{2a(a+cx^2)(ae^2+cd^2)}}{8a\left(ae^2+cd^2\right)}+\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{1483}$$

$$\frac{3e\left(\frac{\int \frac{\sqrt{2}\left(2c^2d^4+5ace^2d^2+7a^2e^4\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2c^2d^4+5ace^2d^2-2\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)d+7a^2e^4\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae}}\cdots}{\cdots}}{\cdots}\right)}{2a(ae^2+cd^2)} \cdots \frac{\cdots}{8a\left(ae^2\cdots\right.}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3e\left(\frac{\int \frac{\sqrt{2}\left(2c^2d^4+5ace^2d^2+7a^2e^4\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt[4]{c}\left(2c^2d^4+5ace^2d^2-2\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)d+7a^2e^4\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae}}\cdots}{\cdots}}{\cdots}\right)}{2a(ae^2+cd^2)} \cdots \frac{\cdots}{8a\left(ae^2\cdots\right.}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+$$

$$\frac{\frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{3e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\int\frac{}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\sqrt{2}\cdots}}{\sqrt{2}}\cdots\right)}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+$$

$$\frac{\frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{3e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\int\frac{}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\sqrt{2}\cdots}}{\sqrt{2}}\cdots\right)}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+$$

$$\frac{\frac{\sqrt{d+ex}\left(ae\left(cd^2+7ae^2\right)+6cd\left(cd^2+2ae^2\right)x\right)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{3e\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\int\frac{}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\sqrt{2}\cdots}}{\sqrt{2}}\cdots\right)}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+\frac{\frac{\sqrt{d+ex}(ae(cd^2+7ae^2)+6cd(cd^2+2ae^2)x)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{3e\left(\frac{\left(2c^2d^4+5ace^2d^2-2\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)d+7a^2e^4\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}}}{\sqrt{2}}}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 219

$$\frac{\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+\frac{\frac{\sqrt{d+ex}(ae(cd^2+7ae^2)+6cd(cd^2+2ae^2)x)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{3e\left(\frac{\left(2c^2d^4+5ace^2d^2-2\sqrt{c}\sqrt{cd^2+ae^2}\left(cd^2+2ae^2\right)d+7a^2e^4\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}{\sqrt[4]{c}}}}{\sqrt{2}}}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt{d+ex}(ae+cdx)}{4a\left(cd^2+ae^2\right)\left(cx^2+a\right)^2}+\frac{\frac{\sqrt{d+ex}(ae(cd^2+7ae^2)+6cd(cd^2+2ae^2)x)}{2a(cd^2+ae^2)(cx^2+a)}+\frac{3e\left(-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(2c^2d^4+5ace^2d^2+\sqrt{c}\sqrt{cd^2+ae^2}\left(2cd^2+4ae^2\right)d+7a^2e^4\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\left(\cdots\right.}{\cdots}\right.}{\sqrt{\sqrt{c}d-\sqrt{cd^2+ae^2}}}\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[1/(Sqrt[d + e*x]*(a + c*x^2)^3),x]
```

output
```
((a*e + c*d*x)*Sqrt[d + e*x])/(4*a*(c*d^2 + a*e^2)*(a + c*x^2)^2) + ((Sqrt
[d + e*x]*(a*e*(c*d^2 + 7*a*e^2) + 6*c*d*(c*d^2 + 2*a*e^2)*x))/(2*a*(c*d^2
 + a*e^2)*(a + c*x^2)) + (3*e*((-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a
*e^2]]*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a*e
^2]*(2*c*d^2 + 4*a*e^2))*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqr
t[c*d^2 + a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d -
Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*
(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4 - 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(
c*d^2 + 2*a*e^2))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d
 + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*
Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]) + (-((c
^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(2*c^2*d^4 + 5*a*c*d^2*e^2 +
7*a^2*e^4 + Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(2*c*d^2 + 4*a*e^2))*ArcTanh[(c^
(1/4)*((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d
+ e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d
- Sqrt[c*d^2 + a*e^2]]) + (c^(1/4)*(2*c^2*d^4 + 5*a*c*d^2*e^2 + 7*a^2*e^4
- 2*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*(c*d^2 + 2*a*e^2))*Log[Sqrt[c*d^2 + a*e^
2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] +
 Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c
]*d + Sqrt[c*d^2 + a*e^2]])))/(2*a*(c*d^2 + a*e^2)))/(8*a*(c*d^2 + a*e^...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex+d}\,(c\,x^2+a)^3}dx$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+a)^3,x)
```

output
```
int(1/(e*x+d)^(1/2)/(c*x^2+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5779 vs. 2(615) = 1230.

Time = 2.18 (sec) , antiderivative size = 5779, normalized size of antiderivative = 8.02

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx = \int \frac{1}{(cx^2+a)^3\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^3*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1664 vs. $2(615) = 1230$.

Time $= 0.26$ (sec) , antiderivative size $= 1664$, normalized size of antiderivative $= 2.31$

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/32*(2*(a^2*c^2*d^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)^2*(sqrt(-a*c)*c*d^3*e
 + 2*sqrt(-a*c)*a*d*e^3)*abs(c) + (2*a^2*c^4*d^8*e + 9*a^3*c^3*d^6*e^3 + 1
9*a^4*c^2*d^4*e^5 + 19*a^5*c*d^2*e^7 + 7*a^6*e^9)*abs(a^2*c^2*d^4*e + 2*a^
3*c*d^2*e^3 + a^4*e^5)*abs(c) + (4*sqrt(-a*c)*a^3*c^6*d^13*e + 25*sqrt(-a*
c)*a^4*c^5*d^11*e^3 + 67*sqrt(-a*c)*a^5*c^4*d^9*e^5 + 98*sqrt(-a*c)*a^6*c^
3*d^7*e^7 + 82*sqrt(-a*c)*a^7*c^2*d^5*e^9 + 37*sqrt(-a*c)*a^8*c*d^3*e^11 +
 7*sqrt(-a*c)*a^9*d*e^13)*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4 + sqrt((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2
+ a^4*c*d*e^4)^2 - (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^
5*e^6)*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)))/(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4)))/((a^4*c^5*d^9 + 4*a^5*c^4*d^7*e^2 + 6*a^6*c^3
*d^5*e^4 + 4*a^7*c^2*d^3*e^6 + a^8*c*d*e^8 - sqrt(-a*c)*a^4*c^4*d^8*e - 4*
sqrt(-a*c)*a^5*c^3*d^6*e^3 - 6*sqrt(-a*c)*a^6*c^2*d^4*e^5 - 4*sqrt(-a*c)*a
^7*c*d^2*e^7 - sqrt(-a*c)*a^8*e^9)*sqrt(-c^2*d - sqrt(-a*c)*c*e)*abs(a^2*c
^2*d^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)) + 3/32*(2*(a^2*c^2*d^4*e + 2*a^3*c*
d^2*e^3 + a^4*e^5)^2*(c^2*d^3*e + 2*a*c*d*e^3)*abs(c) + (2*sqrt(-a*c)*a*c^
4*d^8*e + 9*sqrt(-a*c)*a^2*c^3*d^6*e^3 + 19*sqrt(-a*c)*a^3*c^2*d^4*e^5 + 1
9*sqrt(-a*c)*a^4*c*d^2*e^7 + 7*sqrt(-a*c)*a^5*e^9)*abs(a^2*c^2*d^4*e + 2*a
^3*c*d^2*e^3 + a^4*e^5)*abs(c) + (4*a^3*c^7*d^13*e + 25*a^4*c^6*d^11*e^3 +
 67*a^5*c^5*d^9*e^5 + 98*a^6*c^4*d^7*e^7 + 82*a^7*c^3*d^5*e^9 + 37*a^8*...
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 9035, normalized size of antiderivative = 12.53

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^3*(d + e*x)^(1/2)),x)
```

output
```
atan(((((3*(14336*a^9*c^3*e^11 + 4096*a^5*c^7*d^8*e^3 + 18432*a^6*c^6*d^6*
e^5 + 38912*a^7*c^5*d^4*e^7 + 38912*a^8*c^4*d^2*e^9))/(2048*(a^10*e^8 + a^
6*c^4*d^8 + 4*a^9*c*d^2*e^6 + 4*a^7*c^3*d^6*e^2 + 6*a^8*c^2*d^4*e^4)) - ((
d + e*x)^(1/2)*(-(9*(16*a^5*c^5*d^9 - 49*a^2*e^9*(-a^15*c)^(1/2) + 84*a^6*
c^4*d^7*e^2 + 189*a^7*c^3*d^5*e^4 + 210*a^8*c^2*d^3*e^6 - 21*c^2*d^4*e^5*(
-a^15*c)^(1/2) + 105*a^9*c*d*e^8 - 54*a*c*d^2*e^7*(-a^15*c)^(1/2)))/(4096*
(a^15*c*e^10 + a^10*c^6*d^10 + 5*a^11*c^5*d^8*e^2 + 10*a^12*c^4*d^6*e^4 +
10*a^13*c^3*d^4*e^6 + 5*a^14*c^2*d^2*e^8)))^(1/2)*(4096*a^9*c^4*d*e^10 + 4
096*a^5*c^8*d^9*e^2 + 16384*a^6*c^7*d^7*e^4 + 24576*a^7*c^6*d^5*e^6 + 1638
4*a^8*c^5*d^3*e^8))/(64*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-(9*(16*a^5*c^5*d^9 - 49*a^2*e^9*(-a^15
*c)^(1/2) + 84*a^6*c^4*d^7*e^2 + 189*a^7*c^3*d^5*e^4 + 210*a^8*c^2*d^3*e^6
 - 21*c^2*d^4*e^5*(-a^15*c)^(1/2) + 105*a^9*c*d*e^8 - 54*a*c*d^2*e^7*(-a^1
5*c)^(1/2)))/(4096*(a^15*c*e^10 + a^10*c^6*d^10 + 5*a^11*c^5*d^8*e^2 + 10*
a^12*c^4*d^6*e^4 + 10*a^13*c^3*d^4*e^6 + 5*a^14*c^2*d^2*e^8)))^(1/2) - ((d
 + e*x)^(1/2)*(441*a^4*c^3*e^10 + 144*c^7*d^8*e^2 + 612*a*c^6*d^6*e^4 + 10
89*a^2*c^5*d^4*e^6 + 990*a^3*c^4*d^2*e^8))/(64*(a^8*e^8 + a^4*c^4*d^8 + 4*
a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-(9*(16*a^5*c^5*
d^9 - 49*a^2*e^9*(-a^15*c)^(1/2) + 84*a^6*c^4*d^7*e^2 + 189*a^7*c^3*d^5*e^
4 + 210*a^8*c^2*d^3*e^6 - 21*c^2*d^4*e^5*(-a^15*c)^(1/2) + 105*a^9*c*d*...
```

Reduce [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 8803, normalized size of antiderivative = 12.21

$$\int \frac{1}{\sqrt{d+ex}\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+a)^3,x)
```

output
```
( - 66*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqr
t(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**4*c*
d*e**4 - 66*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d
)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt
(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*
*3*c**2*d**3*e**2 - 132*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c
*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt
(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*
sqrt(2)))*a**3*c**2*d*e**4*x**2 - 24*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sq
rt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*
d**2) - c*d)*sqrt(2)))*a**2*c**3*d**5 - 132*sqrt(a*e**2 + c*d**2)*sqrt(sqr
t(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 +
 c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e*
*2 + c*d**2) - c*d)*sqrt(2)))*a**2*c**3*d**3*e**2*x**2 - 66*sqrt(a*e**2 +
c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(
c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(s
qrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a**2*c**3*d*e**4*x**4 - 48*s
qrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*...
```

3.196 $\int \frac{\sqrt{2+3x}}{1+x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 162

$$\int \frac{\sqrt{2+3x}}{1+x^2} \, dx = -\frac{3\arctan\left(\frac{\sqrt{2\left(2+\sqrt{13}\right)}-2\sqrt{2+3x}}{\sqrt{2\left(-2+\sqrt{13}\right)}}\right)}{\sqrt{2\left(-2+\sqrt{13}\right)}} + \frac{3\arctan\left(\frac{\sqrt{2\left(2+\sqrt{13}\right)}+2\sqrt{2+3x}}{\sqrt{2\left(-2+\sqrt{13}\right)}}\right)}{\sqrt{2\left(-2+\sqrt{13}\right)}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{2+3x}}{2+\sqrt{13}+3x}\right)}{\sqrt{2\left(2+\sqrt{13}\right)}}$$

output

```
-3*arctan(((4+2*13^(1/2))^(1/2)-2*(2+3*x)^(1/2))/(-4+2*13^(1/2))^(1/2))/(-
4+2*13^(1/2))^(1/2)+3*arctan(((4+2*13^(1/2))^(1/2)+2*(2+3*x)^(1/2))/(-4+2*
13^(1/2))^(1/2))/(-4+2*13^(1/2))^(1/2)-3*arctanh((4+2*13^(1/2))^(1/2)*(2+3
*x)^(1/2)/(2+13^(1/2)+3*x))/(4+2*13^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{2+3x}}{1+x^2}\,dx = \sqrt{2-3i}\arctan\left(\sqrt{-\frac{2}{13}-\frac{3i}{13}}\sqrt{2+3x}\right) + \sqrt{2+3i}\arctan\left(\sqrt{-\frac{2}{13}+\frac{3i}{13}}\sqrt{2+3x}\right)$$

input
```
Integrate[Sqrt[2 + 3*x]/(1 + x^2),x]
```

output
```
Sqrt[2 - 3*I]*ArcTan[Sqrt[-2/13 - (3*I)/13]*Sqrt[2 + 3*x]] + Sqrt[2 + 3*I]
*ArcTan[Sqrt[-2/13 + (3*I)/13]*Sqrt[2 + 3*x]]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.40, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {483, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}}{x^2+1}\,dx$$

$$\Big\downarrow \text{483}$$

$$6\int \frac{3x+2}{(3x+2)^2-4(3x+2)+13}d\sqrt{3x+2}$$

$$\Big\downarrow \text{1447}$$

$$6\left(\frac{1}{2}\int \frac{3x+\sqrt{13}+2}{(3x+2)^2-4(3x+2)+13}d\sqrt{3x+2}-\frac{1}{2}\int \frac{-3x+\sqrt{13}-2}{(3x+2)^2-4(3x+2)+13}d\sqrt{3x+2}\right)$$

$$\Big\downarrow \text{1475}$$

$$6\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{3x-\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2}d\sqrt{3x+2}+\frac{1}{2}\int \frac{1}{3x+\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2}d\sqrt{\right.\right.$$

$\big\downarrow$ 1083

$$6\left(\frac{1}{2}\left(-\int \frac{1}{-3x+2\left(2-\sqrt{13}\right)-2}d\left(2\sqrt{3x+2}-\sqrt{2\left(2+\sqrt{13}\right)}\right)-\int \frac{1}{-3x+2\left(2-\sqrt{13}\right)-2}d\left(2\sqrt{3x+2}+\right.\right.\right.$$

$\big\downarrow$ 217

$$6\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{3x+2}-\sqrt{2\left(2+\sqrt{13}\right)}}{\sqrt{2\left(\sqrt{13}-2\right)}}\right)}{\sqrt{2\left(\sqrt{13}-2\right)}}+\frac{\arctan\left(\frac{2\sqrt{3x+2}+\sqrt{2\left(2+\sqrt{13}\right)}}{\sqrt{2\left(\sqrt{13}-2\right)}}\right)}{\sqrt{2\left(\sqrt{13}-2\right)}}\right)-\frac{1}{2}\int \frac{-3x+\sqrt{13}-2}{(3x+2)^2-4(3x+2)+13}d\sqrt{\right.$$

$\big\downarrow$ 1478

$$6\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2\left(2+\sqrt{13}\right)}-2\sqrt{3x+2}}{3x-\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2}d\sqrt{3x+2}}{2\sqrt{2\left(2+\sqrt{13}\right)}}+\frac{\int -\frac{2\sqrt{3x+2}+\sqrt{2\left(2+\sqrt{13}\right)}}{3x+\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2}d\sqrt{3x+2}}{2\sqrt{2\left(2+\sqrt{13}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2\cdots}{\cdots}\right.}{\sqrt{\cdots}}\right.\right.$$

$\big\downarrow$ 25

$$6\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2\left(2+\sqrt{13}\right)}-2\sqrt{3x+2}}{3x-\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2}d\sqrt{3x+2}}{2\sqrt{2\left(2+\sqrt{13}\right)}}-\frac{\int \frac{2\sqrt{3x+2}+\sqrt{2\left(2+\sqrt{13}\right)}}{3x+\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{3x+2}+\sqrt{13}+2}d\sqrt{3x+2}}{2\sqrt{2\left(2+\sqrt{13}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{3\cdots}}{\cdots}\right.}{\sqrt{2\cdots}}\right.\right.$$

$\big\downarrow$ 1103

$$6\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{3x+2}-\sqrt{2\left(2+\sqrt{13}\right)}}{\sqrt{2\left(\sqrt{13}-2\right)}}\right)}{\sqrt{2\left(\sqrt{13}-2\right)}}+\frac{\arctan\left(\frac{2\sqrt{3x+2}+\sqrt{2\left(2+\sqrt{13}\right)}}{\sqrt{2\left(\sqrt{13}-2\right)}}\right)}{\sqrt{2\left(\sqrt{13}-2\right)}}\right)+\frac{1}{2}\left(\frac{\log\left(3x-\sqrt{2\left(2+\sqrt{13}\right)}\sqrt{3x+\right.}{2\sqrt{2\left(2+\sqrt{13}\right)}}\right.\right.$$

input
```
Int[Sqrt[2 + 3*x]/(1 + x^2),x]
```

output
```
6*((ArcTan[(-Sqrt[2*(2 + Sqrt[13])] + 2*Sqrt[2 + 3*x])/Sqrt[2*(-2 + Sqrt[1
3])]]/Sqrt[2*(-2 + Sqrt[13])] + ArcTan[(Sqrt[2*(2 + Sqrt[13])] + 2*Sqrt[2
+ 3*x])/Sqrt[2*(-2 + Sqrt[13])]]/Sqrt[2*(-2 + Sqrt[13])])/2 + (Log[2 + Sqr
t[13] + 3*x - Sqrt[2*(2 + Sqrt[13])]*Sqrt[2 + 3*x]]/(2*Sqrt[2*(2 + Sqrt[13
])]) - Log[2 + Sqrt[13] + 3*x + Sqrt[2*(2 + Sqrt[13])]*Sqrt[2 + 3*x]]/(2*S
qrt[2*(2 + Sqrt[13])]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

method	result
derivativedivides	$-\frac{\sqrt{4+2\sqrt{13}}\left(-2+\sqrt{13}\right)\left(-\frac{\ln\left(3x+2-\sqrt{3x+2}\,\sqrt{4+2\sqrt{13}}+\sqrt{13}\right)}{2}-\frac{\sqrt{4+2\sqrt{13}}\,\arctan\left(\frac{2\sqrt{3x+2}-\sqrt{4+2\sqrt{13}}}{\sqrt{-4+2\sqrt{13}}}\right)}{\sqrt{-4+2\sqrt{13}}}\right)}{6}-\frac{\sqrt{4+2\sqrt{}}}{}$
default	$-\frac{\sqrt{4+2\sqrt{13}}\left(-2+\sqrt{13}\right)\left(-\frac{\ln\left(3x+2-\sqrt{3x+2}\,\sqrt{4+2\sqrt{13}}+\sqrt{13}\right)}{2}-\frac{\sqrt{4+2\sqrt{13}}\,\arctan\left(\frac{2\sqrt{3x+2}-\sqrt{4+2\sqrt{13}}}{\sqrt{-4+2\sqrt{13}}}\right)}{\sqrt{-4+2\sqrt{13}}}\right)}{6}-\frac{\sqrt{4+2\sqrt{}}}{}$
pseudoelliptic	$\frac{\sqrt{13}\,\ln\left(3x+2-\sqrt{3x+2}\,\sqrt{4+2\sqrt{13}}+\sqrt{13}\right)-\sqrt{13}\,\ln\left(3x+2+\sqrt{3x+2}\,\sqrt{4+2\sqrt{13}}+\sqrt{13}\right)-2\ln\left(3x+2-\sqrt{3x+2}\,\sqrt{4+2\sqrt{}}\right.}{}$
trager	$-\operatorname{RootOf}\left(16\_Z^4+16\_Z^2+13\right)\ln\left(\frac{816\operatorname{RootOf}\left(16\_Z^4+16\_Z^2+13\right)^5x+1848\operatorname{RootOf}\left(16\_Z^4\right.}{}\right.$

input

```
int((3*x+2)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output

```
-1/6*(4+2*13^(1/2))^(1/2)*(-2+13^(1/2))*(-1/2*ln(3*x+2-(3*x+2)^(1/2)*(4+2*
13^(1/2))^(1/2)+13^(1/2))-(4+2*13^(1/2))^(1/2)/(-4+2*13^(1/2))^(1/2)*arcta
n((2*(3*x+2)^(1/2)-(4+2*13^(1/2))^(1/2))/(-4+2*13^(1/2))^(1/2)))-1/6*(4+2*
13^(1/2))^(1/2)*(-2+13^(1/2))*(1/2*ln(3*x+2+(3*x+2)^(1/2)*(4+2*13^(1/2))^(
1/2)+13^(1/2))-(4+2*13^(1/2))^(1/2)/(-4+2*13^(1/2))^(1/2)*arctan(((4+2*13^
(1/2))^(1/2)+2*(3*x+2)^(1/2))/(-4+2*13^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{2+3x}}{1+x^2}\,dx$$
$$= \sqrt{\frac{1}{2}\sqrt{13}+1}\arctan\left(\frac{2}{9}\left(\left(\sqrt{13}+2\right)\sqrt{\frac{1}{2}\sqrt{13}-1}+3\sqrt{3x+2}\right)\sqrt{\frac{1}{2}\sqrt{13}+1}\right)$$
$$-\sqrt{\frac{1}{2}\sqrt{13}+1}\arctan\left(\frac{2}{9}\left(\left(\sqrt{13}+2\right)\sqrt{\frac{1}{2}\sqrt{13}-1}-3\sqrt{3x+2}\right)\sqrt{\frac{1}{2}\sqrt{13}+1}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{13}-1}\log\left(2\sqrt{3x+2}\left(\sqrt{13}+2\right)\sqrt{\frac{1}{2}\sqrt{13}-1}+9x+3\sqrt{13}+6\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{13}-1}\log\left(-2\sqrt{3x+2}\left(\sqrt{13}+2\right)\sqrt{\frac{1}{2}\sqrt{13}-1}+9x+3\sqrt{13}+6\right)$$

input
```
integrate((2+3*x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
sqrt(1/2*sqrt(13) + 1)*arctan(2/9*((sqrt(13) + 2)*sqrt(1/2*sqrt(13) - 1) +
 3*sqrt(3*x + 2))*sqrt(1/2*sqrt(13) + 1)) - sqrt(1/2*sqrt(13) + 1)*arctan(
2/9*((sqrt(13) + 2)*sqrt(1/2*sqrt(13) - 1) - 3*sqrt(3*x + 2))*sqrt(1/2*sqr
t(13) + 1)) - 1/2*sqrt(1/2*sqrt(13) - 1)*log(2*sqrt(3*x + 2)*(sqrt(13) + 2
)*sqrt(1/2*sqrt(13) - 1) + 9*x + 3*sqrt(13) + 6) + 1/2*sqrt(1/2*sqrt(13) -
 1)*log(-2*sqrt(3*x + 2)*(sqrt(13) + 2)*sqrt(1/2*sqrt(13) - 1) + 9*x + 3*s
qrt(13) + 6)
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}}{1+x^2}\,dx = \int \frac{\sqrt{3x+2}}{x^2+1}\,dx$$

input
```
integrate((2+3*x)**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(3*x + 2)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}}{1+x^2}\,dx = \int \frac{\sqrt{3\,x+2}}{x^2+1}\,dx$$

input
```
integrate((2+3*x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x + 2)/(x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{\sqrt{2+3x}}{1+x^2}\,dx &= \frac{1}{2}\sqrt{2\sqrt{13}+4}\arctan\left(\frac{13^{\frac{3}{4}}\left(13^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+\sqrt{3\,x+2}\right)}{13\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)\\
&+\frac{1}{2}\sqrt{2\sqrt{13}+4}\arctan\left(-\frac{13^{\frac{3}{4}}\left(13^{\frac{1}{4}}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}-\sqrt{3\,x+2}\right)}{13\sqrt{-\frac{1}{13}\sqrt{13}+\frac{1}{2}}}\right)\\
&-\frac{1}{4}\sqrt{2\sqrt{13}-4}\log\left(2\cdot 13^{\frac{1}{4}}\sqrt{3\,x+2}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+3\,x+\sqrt{13}+2\right)\\
&+\frac{1}{4}\sqrt{2\sqrt{13}-4}\log\left(-2\cdot 13^{\frac{1}{4}}\sqrt{3\,x+2}\sqrt{\frac{1}{13}\sqrt{13}+\frac{1}{2}}+3\,x+\sqrt{13}+2\right)\end{aligned}$$

input
```
integrate((2+3*x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2*sqrt(13) + 4)*arctan(1/13*13^(3/4)*(13^(1/4)*sqrt(1/13*sqrt(13)
 + 1/2) + sqrt(3*x + 2))/sqrt(-1/13*sqrt(13) + 1/2)) + 1/2*sqrt(2*sqrt(13)
 + 4)*arctan(-1/13*13^(3/4)*(13^(1/4)*sqrt(1/13*sqrt(13) + 1/2) - sqrt(3*x
 + 2))/sqrt(-1/13*sqrt(13) + 1/2)) - 1/4*sqrt(2*sqrt(13) - 4)*log(2*13^(1/
4)*sqrt(3*x + 2)*sqrt(1/13*sqrt(13) + 1/2) + 3*x + sqrt(13) + 2) + 1/4*sqr
t(2*sqrt(13) - 4)*log(-2*13^(1/4)*sqrt(3*x + 2)*sqrt(1/13*sqrt(13) + 1/2)
+ 3*x + sqrt(13) + 2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{2+3x}}{1+x^2}\,dx = -\operatorname{atanh}\left(-\frac{\left(1152\sqrt{3x+2}\left(\sqrt{-\frac{\sqrt{13}}{8}-\frac{1}{4}}-\sqrt{\frac{\sqrt{13}}{8}-\frac{1}{4}}\right)^2-720\sqrt{3x+2}\right)\left(\sqrt{-\frac{\sqrt{13}}{8}-\frac{1}{4}}-\sqrt{\frac{\sqrt{13}}{8}-\right.}{2808}\right.$$

$$\left.-2\sqrt{\frac{\sqrt{13}}{8}-\frac{1}{4}}\right)$$

$$-\operatorname{atanh}\left(\frac{\left(720\sqrt{3x+2}-1152\sqrt{3x+2}\left(\sqrt{-\frac{\sqrt{13}}{8}-\frac{1}{4}}+\sqrt{\frac{\sqrt{13}}{8}-\frac{1}{4}}\right)^2\right)\left(\sqrt{-\frac{\sqrt{13}}{8}-\frac{1}{4}}+\sqrt{\frac{\sqrt{13}}{8}-\frac{1}{4}}\right)}{2808}\right.$$

$$\left.+2\sqrt{\frac{\sqrt{13}}{8}-\frac{1}{4}}\right)$$

input
```
int((3*x + 2)^(1/2)/(x^2 + 1),x)
```

output
```
- atanh(-((1152*(3*x + 2)^(1/2)*((- 13^(1/2)/8 - 1/4)^(1/2) - (13^(1/2)/8
- 1/4)^(1/2))^2 - 720*(3*x + 2)^(1/2))*((- 13^(1/2)/8 - 1/4)^(1/2) - (13^(
1/2)/8 - 1/4)^(1/2)))/2808)*(2*(- 13^(1/2)/8 - 1/4)^(1/2) - 2*(13^(1/2)/8
- 1/4)^(1/2)) - atanh(((720*(3*x + 2)^(1/2) - 1152*(3*x + 2)^(1/2)*((- 13^
(1/2)/8 - 1/4)^(1/2) + (13^(1/2)/8 - 1/4)^(1/2))^2)*((- 13^(1/2)/8 - 1/4)^
(1/2) + (13^(1/2)/8 - 1/4)^(1/2)))/2808)*(2*(- 13^(1/2)/8 - 1/4)^(1/2) + 2
*(13^(1/2)/8 - 1/4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{2+3x}}{1+x^2}\,dx$$
$$= \frac{\sqrt{2}\left(-2\sqrt{\sqrt{13}-2}\,\sqrt{13}\,atan\left(\frac{\sqrt{\sqrt{13}+2}\,\sqrt{2}-2\sqrt{3x+2}}{\sqrt{\sqrt{13}-2}\,\sqrt{2}}\right)-4\sqrt{\sqrt{13}-2}\,atan\left(\frac{\sqrt{\sqrt{13}+2}\,\sqrt{2}-2\sqrt{3x+2}}{\sqrt{\sqrt{13}-2}\,\sqrt{2}}\right)+2\sqrt{\sqrt{13}-}\right.}{}$$

input
```
int((2+3*x)^(1/2)/(x^2+1),x)
```

output
```
(sqrt(2)*( - 2*sqrt(sqrt(13) - 2)*sqrt(13)*atan((sqrt(sqrt(13) + 2)*sqrt(2
) - 2*sqrt(3*x + 2))/(sqrt(sqrt(13) - 2)*sqrt(2))) - 4*sqrt(sqrt(13) - 2)*
atan((sqrt(sqrt(13) + 2)*sqrt(2) - 2*sqrt(3*x + 2))/(sqrt(sqrt(13) - 2)*sq
rt(2))) + 2*sqrt(sqrt(13) - 2)*sqrt(13)*atan((sqrt(sqrt(13) + 2)*sqrt(2) +
 2*sqrt(3*x + 2))/(sqrt(sqrt(13) - 2)*sqrt(2))) + 4*sqrt(sqrt(13) - 2)*ata
n((sqrt(sqrt(13) + 2)*sqrt(2) + 2*sqrt(3*x + 2))/(sqrt(sqrt(13) - 2)*sqrt(
2))) + sqrt(sqrt(13) + 2)*sqrt(13)*log( - sqrt(3*x + 2)*sqrt(sqrt(13) + 2)
*sqrt(2) + sqrt(13) + 3*x + 2) - sqrt(sqrt(13) + 2)*sqrt(13)*log(sqrt(3*x
+ 2)*sqrt(sqrt(13) + 2)*sqrt(2) + sqrt(13) + 3*x + 2) - 2*sqrt(sqrt(13) +
2)*log( - sqrt(3*x + 2)*sqrt(sqrt(13) + 2)*sqrt(2) + sqrt(13) + 3*x + 2) +
 2*sqrt(sqrt(13) + 2)*log(sqrt(3*x + 2)*sqrt(sqrt(13) + 2)*sqrt(2) + sqrt(
13) + 3*x + 2)))/12
```

3.197 $\int \frac{\sqrt{c+dx}}{1+x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 239

$$\int \frac{\sqrt{c+dx}}{1+x^2}\,dx = -\frac{d\arctan\left(\frac{\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}}{\sqrt{-c+\sqrt{c^2+d^2}}}\right)}{\sqrt{2}\sqrt{-c+\sqrt{c^2+d^2}}}+\frac{d\arctan\left(\frac{\sqrt{c+\sqrt{c^2+d^2}}+\sqrt{2}\sqrt{c+dx}}{\sqrt{-c+\sqrt{c^2+d^2}}}\right)}{\sqrt{2}\sqrt{-c+\sqrt{c^2+d^2}}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}{c+\sqrt{c^2+d^2}+dx}\right)}{\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}}$$

output
```
-1/2*d*arctan(((c+(c^2+d^2)^(1/2))^(1/2)-2^(1/2)*(d*x+c)^(1/2))/(-c+(c^2+d
^2)^(1/2))^(1/2))*2^(1/2)/(-c+(c^2+d^2)^(1/2))^(1/2)+1/2*d*arctan(((c+(c^2
+d^2)^(1/2))^(1/2)+2^(1/2)*(d*x+c)^(1/2))/(-c+(c^2+d^2)^(1/2))^(1/2))*2^(1
/2)/(-c+(c^2+d^2)^(1/2))^(1/2)-1/2*d*arctanh(2^(1/2)*(c+(c^2+d^2)^(1/2))^(
1/2)*(d*x+c)^(1/2)/(c+(c^2+d^2)^(1/2)+d*x))*2^(1/2)/(c+(c^2+d^2)^(1/2))^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{c+dx}}{1+x^2} \, dx = i\left(\sqrt{-c-id}\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c-id}}\right)-\sqrt{-c+id}\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+id}}\right)\right)$$

input
```
Integrate[Sqrt[c + d*x]/(1 + x^2),x]
```

output
```
I*(Sqrt[-c - I*d]*ArcTan[Sqrt[c + d*x]/Sqrt[-c - I*d]] - Sqrt[-c + I*d]*Ar
cTan[Sqrt[c + d*x]/Sqrt[-c + I*d]])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.52, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {483, 1449, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{x^2+1} \, dx$$

$$\Big\downarrow \text{483}$$

$$2d\int \frac{c+dx}{c^2-2(c+dx)c+d^2+(c+dx)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{1449}$$

$$2d\left(\frac{\int \frac{\sqrt{c+dx}}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\frac{\int \frac{\sqrt{c+dx}}{c+dx+\sqrt{c^2+d^2}+\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}\right)$$

$$\Big\downarrow \text{1142}$$

$$2d\left(\frac{\frac{\sqrt{\sqrt{c^2+d^2}+c}\int \frac{1}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{\sqrt{2}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}\right)}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\frac{\frac{1}{2}\int -\ldots}{\ldots}\right.$$

$$\Big\downarrow \ 25$$

$$2d\left(\frac{\frac{\sqrt{\sqrt{c^2+d^2}+c}\int \frac{1}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}\right)}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\frac{\frac{1}{2}\int \ldots}{\ldots}\right.$$

$$\Big\downarrow \ 27$$

$$2d\left(\frac{\frac{\sqrt{\sqrt{c^2+d^2}+c}\int \frac{1}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{\sqrt{2}}-\frac{\int \frac{\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{\sqrt{2}}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\frac{\int \frac{\sqrt{c+\sqrt{c^2+\ldots}}}{c+dx+\sqrt{c^2+d^2}+\ldots}}{\ldots}\right.$$

$$\Big\downarrow \ 1083$$

$$2d\left(\frac{-\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}\int \frac{1}{-c+2\left(c-\sqrt{c^2+d^2}\right)-dx}d\left(2\sqrt{c+dx}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\right)-\frac{\int \frac{\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c\ldots}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}\right.$$

$$\Big\downarrow \ 219$$

$$2d\left(\frac{-\frac{\int \frac{\sqrt{c+\sqrt{c^2+d^2}}-\sqrt{2}\sqrt{c+dx}}{c+dx+\sqrt{c^2+d^2}-\sqrt{2}\sqrt{c+\sqrt{c^2+d^2}}\sqrt{c+dx}}d\sqrt{c+dx}}{\sqrt{2}}-\frac{\sqrt{\sqrt{c^2+d^2}+c}\,\text{arctanh}\left(\frac{2\sqrt{c+dx}-\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}{\sqrt{2}\sqrt{c-\sqrt{c^2+d^2}}}\right)}{\sqrt{c-\sqrt{c^2+d^2}}}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\frac{\frac{\int \frac{\sqrt{c+\sqrt{c^2+d^2}}+\sqrt{2}\ldots}{c+dx+\sqrt{c^2+d^2}+\sqrt{2}\sqrt{c+\ldots}}}{\sqrt{2}}}{\ldots}\right.$$

$$\Big\downarrow \ 1103$$

$$2d\left(\frac{\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}\sqrt{c+dx}+\sqrt{c^2+d^2}+c+dx\right)-\frac{\sqrt{\sqrt{c^2+d^2}+c}\,\text{arctanh}\left(\frac{2\sqrt{c+dx}-\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}{\sqrt{2}\sqrt{c-\sqrt{c^2+d^2}}}\right)}{\sqrt{c-\sqrt{c^2+d^2}}}}{2\sqrt{2}\sqrt{\sqrt{c^2+d^2}+c}}-\right.$$

input
```
Int[Sqrt[c + d*x]/(1 + x^2),x]
```

output
```
2*d*((-((Sqrt[c + Sqrt[c^2 + d^2]]*ArcTanh[(-(Sqrt[2]*Sqrt[c + Sqrt[c^2 +
d^2]]) + 2*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[c - Sqrt[c^2 + d^2]])])/Sqrt[c - S
qrt[c^2 + d^2]]) + Log[c + Sqrt[c^2 + d^2] + d*x - Sqrt[2]*Sqrt[c + Sqrt[c
^2 + d^2]]*Sqrt[c + d*x]]/2)/(2*Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]]) - ((Sqr
t[c + Sqrt[c^2 + d^2]]*ArcTanh[(Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]] + 2*Sqrt
[c + d*x])/(Sqrt[2]*Sqrt[c - Sqrt[c^2 + d^2]])])/Sqrt[c - Sqrt[c^2 + d^2]]
 + Log[c + Sqrt[c^2 + d^2] + d*x + Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]]*Sqrt[
c + d*x]]/2)/(2*Sqrt[2]*Sqrt[c + Sqrt[c^2 + d^2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{-\frac{\sqrt{2\sqrt{c^2+d^2}-2c}\,\sqrt{2\sqrt{c^2+d^2}+2c}\,\left(\ln\left(dx+c+\sqrt{dx+c}\,\sqrt{2\sqrt{c^2+d^2}+2c}+\sqrt{c^2+d^2}\right)-\ln\left(dx+c-\sqrt{dx+c}\,\sqrt{2\sqrt{c^2+d^2}+2c}+\sqrt{c^2+d^2}\ldots\right.\right.}{4}}{\sqrt{2\sqrt{c^2+d^2}-2c}\,d\ldots}$
derivativedivides	$2d\left(-\frac{\sqrt{2\sqrt{c^2+d^2}+2c}\,\left(-c+\sqrt{c^2+d^2}\right)\left(\frac{\ln\left(dx+c+\sqrt{dx+c}\,\sqrt{2\sqrt{c^2+d^2}+2c}+\sqrt{c^2+d^2}\right)}{2}-\frac{\sqrt{2\sqrt{c^2+d^2}+2c}\,\arctan\left(\frac{2\sqrt{dx+c}\ldots}{\sqrt{2\ldots}}\right.}{\sqrt{2\sqrt{c^2+d^2}-2c}\ldots}\right.}{4d^2}\ldots\right.$
default	$2d\left(-\frac{\sqrt{2\sqrt{c^2+d^2}+2c}\,\left(-c+\sqrt{c^2+d^2}\right)\left(\frac{\ln\left(dx+c+\sqrt{dx+c}\,\sqrt{2\sqrt{c^2+d^2}+2c}+\sqrt{c^2+d^2}\right)}{2}-\frac{\sqrt{2\sqrt{c^2+d^2}+2c}\,\arctan\left(\frac{2\sqrt{dx+c}\ldots}{\sqrt{2\ldots}}\right.}{\sqrt{2\sqrt{c^2+d^2}-2c}\ldots}\right.}{4d^2}\ldots\right.$

input

```
int((d*x+c)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output

```
-1/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*(-1/4*(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*(ln(d*x+c+(d*x+c)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)+(c^2+d^2)^(1/2))-ln(d*x+c-(d*x+c)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
+(c^2+d^2)^(1/2)))*(-(c^2+d^2)^(1/2)+c)+d^2*(arctan((-2*(d*x+c)^(1/2)+(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))-arctan((2*(d*x+c
)^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx}}{1+x^2} dx = \frac{1}{2}\sqrt{-c+\sqrt{-d^2}}\log\left(\sqrt{dx+c}d+\sqrt{-d^2}\sqrt{-c+\sqrt{-d^2}}\right) - \frac{1}{2}\sqrt{-c+\sqrt{-d^2}}\log\left(\sqrt{dx+c}d-\sqrt{-d^2}\sqrt{-c+\sqrt{-d^2}}\right) - \frac{1}{2}\sqrt{-c-\sqrt{-d^2}}\log\left(\sqrt{dx+c}d+\sqrt{-d^2}\sqrt{-c-\sqrt{-d^2}}\right) + \frac{1}{2}\sqrt{-c-\sqrt{-d^2}}\log\left(\sqrt{dx+c}d-\sqrt{-d^2}\sqrt{-c-\sqrt{-d^2}}\right)$$

input
```
integrate((d*x+c)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(-c + sqrt(-d^2))*log(sqrt(d*x + c)*d + sqrt(-d^2)*sqrt(-c + sqrt(
-d^2))) - 1/2*sqrt(-c + sqrt(-d^2))*log(sqrt(d*x + c)*d - sqrt(-d^2)*sqrt(
-c + sqrt(-d^2))) - 1/2*sqrt(-c - sqrt(-d^2))*log(sqrt(d*x + c)*d + sqrt(-
d^2)*sqrt(-c - sqrt(-d^2))) + 1/2*sqrt(-c - sqrt(-d^2))*log(sqrt(d*x + c)*
d - sqrt(-d^2)*sqrt(-c - sqrt(-d^2)))
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{1+x^2} dx = \int \frac{\sqrt{c+dx}}{x^2+1} dx$$

input
```
integrate((d*x+c)**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(c + d*x)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{1+x^2}\,dx = \int \frac{\sqrt{dx+c}}{x^2+1}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/(x^2 + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{1+x^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{c+dx}}{1+x^2}\,dx = -\operatorname{atan}\left(\frac{2\,c\,\sqrt{-\frac{c}{4}-\frac{d\,1\mathrm{i}}{4}}\,\sqrt{c+d\,x}-d\,\sqrt{-\frac{c}{4}-\frac{d\,1\mathrm{i}}{4}}\,\sqrt{c+d\,x}\,2\mathrm{i}}{c^2+d^2}\right)\sqrt{-\frac{c}{4}-\frac{d\,1\mathrm{i}}{4}}\,2\mathrm{i} + \operatorname{atan}\left(\frac{2\,c\,\sqrt{-\frac{c}{4}+\frac{d\,1\mathrm{i}}{4}}\,\sqrt{c+d\,x}+d\,\sqrt{-\frac{c}{4}+\frac{d\,1\mathrm{i}}{4}}\,\sqrt{c+d\,x}\,2\mathrm{i}}{c^2+d^2}\right)\sqrt{-\frac{c}{4}+\frac{d\,1\mathrm{i}}{4}}\,2\mathrm{i}$$

input
```
int((c + d*x)^(1/2)/(x^2 + 1),x)
```

output
```
atan((2*c*((d*1i)/4 - c/4)^(1/2)*(c + d*x)^(1/2) + d*((d*1i)/4 - c/4)^(1/2
)*(c + d*x)^(1/2)*2i)/(c^2 + d^2))*((d*1i)/4 - c/4)^(1/2)*2i - atan((2*c*(
- c/4 - (d*1i)/4)^(1/2)*(c + d*x)^(1/2) - d*(- c/4 - (d*1i)/4)^(1/2)*(c +
d*x)^(1/2)*2i)/(c^2 + d^2))*(- c/4 - (d*1i)/4)^(1/2)*2i
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{c+dx}}{1+x^2}\,dx$$
$$= \frac{\sqrt{2}\left(-2\sqrt{\sqrt{c^2+d^2}-c}\,\sqrt{c^2+d^2}\,atan\left(\frac{\sqrt{\sqrt{c^2+d^2}+c}\,\sqrt{2}-2\sqrt{dx+c}}{\sqrt{\sqrt{c^2+d^2}-c}\,\sqrt{2}}\right)-2\sqrt{\sqrt{c^2+d^2}-c}\,atan\left(\frac{\sqrt{\sqrt{c^2+d^2}+c}\,\sqrt{2}-2\sqrt{}}{\sqrt{\sqrt{c^2+d^2}-c}\,\sqrt{2}}\right.\right.}{}$$

input
```
int((d*x+c)^(1/2)/(x^2+1),x)
```

output
```
(sqrt(2)*( - 2*sqrt(sqrt(c**2 + d**2) - c)*sqrt(c**2 + d**2)*atan((sqrt(sq
rt(c**2 + d**2) + c)*sqrt(2) - 2*sqrt(c + d*x))/(sqrt(sqrt(c**2 + d**2) -
c)*sqrt(2))) - 2*sqrt(sqrt(c**2 + d**2) - c)*atan((sqrt(sqrt(c**2 + d**2)
+ c)*sqrt(2) - 2*sqrt(c + d*x))/(sqrt(sqrt(c**2 + d**2) - c)*sqrt(2)))*c +
 2*sqrt(sqrt(c**2 + d**2) - c)*sqrt(c**2 + d**2)*atan((sqrt(sqrt(c**2 + d*
*2) + c)*sqrt(2) + 2*sqrt(c + d*x))/(sqrt(sqrt(c**2 + d**2) - c)*sqrt(2)))
 + 2*sqrt(sqrt(c**2 + d**2) - c)*atan((sqrt(sqrt(c**2 + d**2) + c)*sqrt(2)
 + 2*sqrt(c + d*x))/(sqrt(sqrt(c**2 + d**2) - c)*sqrt(2)))*c + sqrt(sqrt(c
**2 + d**2) + c)*sqrt(c**2 + d**2)*log(sqrt(c**2 + d**2) - sqrt(c + d*x)*s
qrt(sqrt(c**2 + d**2) + c)*sqrt(2) + c + d*x) - sqrt(sqrt(c**2 + d**2) + c
)*sqrt(c**2 + d**2)*log(sqrt(c**2 + d**2) + sqrt(c + d*x)*sqrt(sqrt(c**2 +
 d**2) + c)*sqrt(2) + c + d*x) - sqrt(sqrt(c**2 + d**2) + c)*log(sqrt(c**2
 + d**2) - sqrt(c + d*x)*sqrt(sqrt(c**2 + d**2) + c)*sqrt(2) + c + d*x)*c
+ sqrt(sqrt(c**2 + d**2) + c)*log(sqrt(c**2 + d**2) + sqrt(c + d*x)*sqrt(s
qrt(c**2 + d**2) + c)*sqrt(2) + c + d*x)*c))/(4*d)
```

3.198 $\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 314

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx = -\frac{3\arctan\left(\frac{\sqrt{2\sqrt{b}+\sqrt{9a+4b}}-\sqrt{2}\sqrt[4]{b}\sqrt{2+3x}}{\sqrt{-2\sqrt{b}+\sqrt{9a+4b}}}\right)}{\sqrt{2}b^{3/4}\sqrt{-2\sqrt{b}+\sqrt{9a+4b}}} + \frac{3\arctan\left(\frac{\sqrt{2\sqrt{b}+\sqrt{9a+4b}}+\sqrt{2}\sqrt[4]{b}\sqrt{2+3x}}{\sqrt{-2\sqrt{b}+\sqrt{9a+4b}}}\right)}{\sqrt{2}b^{3/4}\sqrt{-2\sqrt{b}+\sqrt{9a+4b}}} - \frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{2+3x}}{\sqrt{9a+4b}+\sqrt{b}(2+3x)}\right)}{\sqrt{2}b^{3/4}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}}$$

output
```
-3/2*arctan(((2*b^(1/2)+(9*a+4*b)^(1/2))^(1/2)-2^(1/2)*b^(1/4)*(2+3*x)^(1/
2))/(-2*b^(1/2)+(9*a+4*b)^(1/2))^(1/2))*2^(1/2)/b^(3/4)/(-2*b^(1/2)+(9*a+4
*b)^(1/2))^(1/2)+3/2*arctan(((2*b^(1/2)+(9*a+4*b)^(1/2))^(1/2)+2^(1/2)*b^(
1/4)*(2+3*x)^(1/2))/(-2*b^(1/2)+(9*a+4*b)^(1/2))^(1/2))*2^(1/2)/b^(3/4)/(-
2*b^(1/2)+(9*a+4*b)^(1/2))^(1/2)-3/2*arctanh(2^(1/2)*b^(1/4)*(2*b^(1/2)+(9
*a+4*b)^(1/2))^(1/2)*(2+3*x)^(1/2)/((9*a+4*b)^(1/2)+b^(1/2)*(2+3*x)))*2^(1
/2)/b^(3/4)/(2*b^(1/2)+(9*a+4*b)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx$$
$$= \frac{\sqrt[4]{-1}\sqrt{3\sqrt{a}\sqrt{b}-2ib}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{3\sqrt{a}\sqrt{b}-2ib}\sqrt{2+3x}}{3\sqrt{a}-2i\sqrt{b}}\right)+(-1)^{3/4}\sqrt{3\sqrt{a}\sqrt{b}+2ib}\arctan\left(\frac{(-1)^{3/4}\sqrt{3\sqrt{a}\sqrt{b}+ ...}}{3\sqrt{a}+2i\sqrt{...}}\right.}{\sqrt{a}b}$$

input
```
Integrate[Sqrt[2 + 3*x]/(a + b*x^2),x]
```

output
```
((-1)^(1/4)*Sqrt[3*Sqrt[a]*Sqrt[b] - (2*I)*b]*ArcTan[((-1)^(1/4)*Sqrt[3*Sq
rt[a]*Sqrt[b] - (2*I)*b]*Sqrt[2 + 3*x])/(3*Sqrt[a] - (2*I)*Sqrt[b])] + (-1
)^(3/4)*Sqrt[3*Sqrt[a]*Sqrt[b] + (2*I)*b]*ArcTan[((-1)^(3/4)*Sqrt[3*Sqrt[a
]*Sqrt[b] + (2*I)*b]*Sqrt[2 + 3*x])/(3*Sqrt[a] + (2*I)*Sqrt[b])])/(Sqrt[a]
*b)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {483, 1449, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 483}$$

$$6\int \frac{3x+2}{b(3x+2)^2-4b(3x+2)+9a+4b}d\sqrt{3x+2}$$

$$\Big\downarrow \text{ 1449}$$

$$6\left(\frac{\int \frac{\sqrt{3x+2}}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}-\frac{\int \frac{\sqrt{3x+2}}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}\right)$$

$\Big\downarrow$ 1142

$$6\left(\frac{\frac{\sqrt{\sqrt{9a+4b}+2\sqrt{b}}\int \frac{1}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{\sqrt{2}\sqrt[4]{b}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{2\sqrt{b}+\sqrt{9a+4b}}-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}\right)}{\sqrt[4]{b}\left(3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2\right)}d\sqrt{3x+}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}\right.$$

$\Big\downarrow$ 25

$$6\left(\frac{\frac{\sqrt{\sqrt{9a+4b}+2\sqrt{b}}\int \frac{1}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{\sqrt{2}\sqrt[4]{b}}-\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{2\sqrt{b}+\sqrt{9a+4b}}-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}\right)}{\sqrt[4]{b}\left(3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2\right)}d\sqrt{3x+2}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}\right.$$

$\Big\downarrow$ 27

$$6\left(\frac{\frac{\sqrt{\sqrt{9a+4b}+2\sqrt{b}}\int \frac{1}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{\sqrt{2\sqrt{b}+\sqrt{9a+4b}}-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}-\frac{\int \frac{\sqrt{2}}{3x+\frac{\sqrt{9a}}{\sqrt{}}}}{}\right.$$

$\Big\downarrow$ 1083

$$6\left(\frac{-\frac{\sqrt{2}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}\int \frac{1}{2\left(2-\frac{\sqrt{9a+4b}}{\sqrt{b}}\right)-3x-2}d\left(2\sqrt{3x+2}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}}{\sqrt[4]{b}}\right)}{\sqrt[4]{b}}-\frac{\int \frac{\sqrt{2\sqrt{b}+\sqrt{9a+4b}}-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}-\right.$$

$\downarrow$ 219

$$6\left(\frac{-\frac{\int \frac{\sqrt{2\sqrt{b}+\sqrt{9a+4b}}-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}}{3x+\frac{\sqrt{9a+4b}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{2\sqrt{b}+\sqrt{9a+4b}}\sqrt{3x+2}}{\sqrt[4]{b}}+2}d\sqrt{3x+2}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt{\sqrt{9a+4b}+2\sqrt{b}}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\left(2\sqrt{3x+2}-\frac{\sqrt{2}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt{2\sqrt{b}-\sqrt{9a+4b}}}\right)}{\sqrt{2\sqrt{b}-\sqrt{9a+4b}}}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}-\right.$$

$\downarrow$ 1103

$$6\left(\frac{\frac{1}{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt{3x+2}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}+\sqrt{9a+4b}+\sqrt{b}(3x+2)\right)-\frac{\sqrt{\sqrt{9a+4b}+2\sqrt{b}}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\left(2\sqrt{3x+2}-\ldots\right)}{\sqrt{2}\sqrt{2\sqrt{b}\ldots}}\right)}{\sqrt{2\sqrt{b}-\sqrt{9a+4b}}}}{2\sqrt{2}b^{3/4}\sqrt{\sqrt{9a+4b}+2\sqrt{b}}}\right.$$

input

```
Int[Sqrt[2 + 3*x]/(a + b*x^2),x]
```

output
```
6*((-((Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*ArcTanh[(b^(1/4)*(-((Sqrt[2]*Sqrt
[2*Sqrt[b] + Sqrt[9*a + 4*b]])/b^(1/4)) + 2*Sqrt[2 + 3*x]))/(Sqrt[2]*Sqrt[
2*Sqrt[b] - Sqrt[9*a + 4*b]])])/Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]]) + Log[S
qrt[9*a + 4*b] - Sqrt[2]*b^(1/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*Sqrt[2
+ 3*x] + Sqrt[b]*(2 + 3*x)]/2)/(2*Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] + Sqrt[9*
a + 4*b]]) - ((Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*ArcTanh[(b^(1/4)*((Sqrt[2
]*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]])/b^(1/4) + 2*Sqrt[2 + 3*x]))/(Sqrt[2]*
Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]])])/Sqrt[2*Sqrt[b] - Sqrt[9*a + 4*b]] + L
og[Sqrt[9*a + 4*b] + Sqrt[2]*b^(1/4)*Sqrt[2*Sqrt[b] + Sqrt[9*a + 4*b]]*Sqr
t[2 + 3*x] + Sqrt[b]*(2 + 3*x)]/2)/(2*Sqrt[2]*b^(3/4)*Sqrt[2*Sqrt[b] + Sqr
t[9*a + 4*b]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{4\left(-\frac{\sqrt{b(9a+4b)}}{2}+b\right)\left(\frac{\sqrt{4\sqrt{9a+4b}\,\sqrt{b}-2\sqrt{b(9a+4b)}-4b}\,\sqrt{2\sqrt{b(9a+4b)}+4b}\,\left(\ln\left(\sqrt{b}\,(3x+2)-\sqrt{3x+2}\,\sqrt{2\sqrt{b(9a+4b)}+4b}+\sqrt{9a+\cdots}\right.\right.}{8}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{\sqrt{2\sqrt{9ab+4b^2}+4b}\,\left(\sqrt{9ab+4b^2}-2b\right)\left(\frac{\ln\left(\sqrt{b}\,(3x+2)+\sqrt{3x+2}\,\sqrt{2\sqrt{b(9a+4b)}+4b}+\sqrt{9a+4b}\right)}{2\sqrt{b}}-\frac{\sqrt{2\sqrt{b(9a+4b)}+4b}\,\arctan\left(\frac{2\cdots}{\sqrt{\cdots}}\right.}{\sqrt{b}\,\sqrt{4\sqrt{9a+4b}\cdots}}\cdots\right.}{6ab}$
default	$-\frac{\sqrt{2\sqrt{9ab+4b^2}+4b}\,\left(\sqrt{9ab+4b^2}-2b\right)\left(\frac{\ln\left(\sqrt{b}\,(3x+2)+\sqrt{3x+2}\,\sqrt{2\sqrt{b(9a+4b)}+4b}+\sqrt{9a+4b}\right)}{2\sqrt{b}}-\frac{\sqrt{2\sqrt{b(9a+4b)}+4b}\,\arctan\left(\frac{2\cdots}{\sqrt{\cdots}}\right.}{\sqrt{b}\,\sqrt{4\sqrt{9a+4b}\cdots}}\cdots\right.}{6ab}$

input
```
int((3*x+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-1/2*(b*(9*a+4*b))^(1/2)+b)/b^(3/2)*(1/8*(4*(9*a+4*b)^(1/2)*b^(1/2)-
2*(b*(9*a+4*b))^(1/2)-4*b)^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)*(ln(b^(
1/2)*(3*x+2)-(3*x+2)^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)+(9*a+4*b)^(1/
2))-ln(b^(1/2)*(3*x+2)+(3*x+2)^(1/2)*(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2)+(9*
a+4*b)^(1/2)))+(b+1/2*(b*(9*a+4*b))^(1/2))*(arctan((2*b^(1/2)*(3*x+2)^(1/2
)-(2*(b*(9*a+4*b))^(1/2)+4*b)^(1/2))/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+
4*b))^(1/2)-4*b)^(1/2))+arctan((2*b^(1/2)*(3*x+2)^(1/2)+(2*(b*(9*a+4*b))^(
1/2)+4*b)^(1/2))/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+4*b))^(1/2)-4*b)^(1/
2))))/(4*(9*a+4*b)^(1/2)*b^(1/2)-2*(b*(9*a+4*b))^(1/2)-4*b)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx = -\frac{1}{2}\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}+2}{ab}}\log\left(ab^2\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}+2}{ab}}\sqrt{-\frac{1}{ab^3}}+\sqrt{3\,x+2}\right)+\frac{1}{2}\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}+2}{ab}}\log\left(-ab^2\sqrt{-\frac{3\,ab\sqrt{-\frac{1}{ab^3}}+2}{ab}}\sqrt{-\frac{1}{ab^3}}+\sqrt{3\,x+2}\right)+\frac{1}{2}\sqrt{\frac{3\,ab\sqrt{-\frac{1}{ab^3}}-2}{ab}}\log\left(ab^2\sqrt{\frac{3\,ab\sqrt{-\frac{1}{ab^3}}-2}{ab}}\sqrt{-\frac{1}{ab^3}}+\sqrt{3\,x+2}\right)-\frac{1}{2}\sqrt{\frac{3\,ab\sqrt{-\frac{1}{ab^3}}-2}{ab}}\log\left(-ab^2\sqrt{\frac{3\,ab\sqrt{-\frac{1}{ab^3}}-2}{ab}}\sqrt{-\frac{1}{ab^3}}+\sqrt{3\,x+2}\right)$$

input
```
integrate((2+3*x)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-(3*a*b*sqrt(-1/(a*b^3)) + 2)/(a*b))*log(a*b^2*sqrt(-(3*a*b*sqrt
(-1/(a*b^3)) + 2)/(a*b))*sqrt(-1/(a*b^3)) + sqrt(3*x + 2)) + 1/2*sqrt(-(3*
a*b*sqrt(-1/(a*b^3)) + 2)/(a*b))*log(-a*b^2*sqrt(-(3*a*b*sqrt(-1/(a*b^3))
+ 2)/(a*b))*sqrt(-1/(a*b^3)) + sqrt(3*x + 2)) + 1/2*sqrt((3*a*b*sqrt(-1/(a
*b^3)) - 2)/(a*b))*log(a*b^2*sqrt((3*a*b*sqrt(-1/(a*b^3)) - 2)/(a*b))*sqrt
(-1/(a*b^3)) + sqrt(3*x + 2)) - 1/2*sqrt((3*a*b*sqrt(-1/(a*b^3)) - 2)/(a*b
))*log(-a*b^2*sqrt((3*a*b*sqrt(-1/(a*b^3)) - 2)/(a*b))*sqrt(-1/(a*b^3)) +
sqrt(3*x + 2))
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx = \int \frac{\sqrt{3x+2}}{a+bx^2}\,dx$$

input
```
integrate((2+3*x)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(3*x + 2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx = \int \frac{\sqrt{3\,x+2}}{bx^2+a}\,dx$$

input
```
integrate((2+3*x)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x + 2)/(b*x^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx = \frac{\left(4\sqrt{-ab}\sqrt{-2\,b^2-3\sqrt{-ab}b}a-17\sqrt{-ab}\sqrt{-2\,b^2-3\sqrt{-ab}b}b\right)|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{3\,x+2}}{\sqrt{-\frac{4\,b+\sqrt{-4\,(9\,a+4\,b)b+16\,b^2}}{b}}}\right)}{4\,a^2b^3-17\,ab^4} + \frac{\left(4\sqrt{-ab}\sqrt{-2\,b^2+3\sqrt{-ab}b}a-17\sqrt{-ab}\sqrt{-2\,b^2+3\sqrt{-ab}b}b\right)|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{3\,x+2}}{\sqrt{-\frac{4\,b-\sqrt{-4\,(9\,a+4\,b)b+16\,b^2}}{b}}}\right)}{4\,a^2b^3-17\,ab^4}$$

input
```
integrate((2+3*x)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
(4*sqrt(-a*b)*sqrt(-2*b^2 - 3*sqrt(-a*b)*b)*a - 17*sqrt(-a*b)*sqrt(-2*b^2
- 3*sqrt(-a*b)*b)*b)*abs(b)*arctan(2*sqrt(1/2)*sqrt(3*x + 2)/sqrt(-(4*b +
sqrt(-4*(9*a + 4*b)*b + 16*b^2))/b))/(4*a^2*b^3 - 17*a*b^4) + (4*sqrt(-a*b
)*sqrt(-2*b^2 + 3*sqrt(-a*b)*b)*a - 17*sqrt(-a*b)*sqrt(-2*b^2 + 3*sqrt(-a*
b)*b)*b)*abs(b)*arctan(2*sqrt(1/2)*sqrt(3*x + 2)/sqrt(-(4*b - sqrt(-4*(9*a
 + 4*b)*b + 16*b^2))/b))/(4*a^2*b^3 - 17*a*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx = -2\,\mathrm{atanh}\left(\frac{2\left(\left(1296\,a\,b^2-576\,b^3\right)\sqrt{3\,x+2}-\frac{288\,b\sqrt{3\,x+2}\left(3\sqrt{-a^3\,b^3}-2\,a\,b^2\right)}{a}\right)\sqrt{\frac{3\sqrt{-a^3\,b^3}-2\,a\,b^2}{4\,a^2\,b^3}}}{1728\,b^2+3888\,a\,b}\right)\sqrt{\frac{3\sqrt{-a}}{4}} -2\,\mathrm{atanh}\left(\frac{2\left(\left(1296\,a\,b^2-576\,b^3\right)\sqrt{3\,x+2}+\frac{288\,b\sqrt{3\,x+2}\left(3\sqrt{-a^3\,b^3}+2\,a\,b^2\right)}{a}\right)\sqrt{-\frac{3\sqrt{-a^3\,b^3}+2\,a\,b^2}{4\,a^2\,b^3}}}{1728\,b^2+3888\,a\,b}\right)\sqrt{-\frac{3\sqrt{}}{}}$$

input
```
int((3*x + 2)^(1/2)/(a + b*x^2),x)
```

output
```
- 2*atanh((2*((1296*a*b^2 - 576*b^3)*(3*x + 2)^(1/2) - (288*b*(3*x + 2)^(1
/2)*(3*(-a^3*b^3)^(1/2) - 2*a*b^2))/a)*((3*(-a^3*b^3)^(1/2) - 2*a*b^2)/(4*
a^2*b^3))^(1/2))/(3888*a*b + 1728*b^2))*((3*(-a^3*b^3)^(1/2) - 2*a*b^2)/(4
*a^2*b^3))^(1/2) - 2*atanh((2*((1296*a*b^2 - 576*b^3)*(3*x + 2)^(1/2) + (2
88*b*(3*x + 2)^(1/2)*(3*(-a^3*b^3)^(1/2) + 2*a*b^2))/a)*(-(3*(-a^3*b^3)^(1
/2) + 2*a*b^2)/(4*a^2*b^3))^(1/2))/(3888*a*b + 1728*b^2))*(-(3*(-a^3*b^3)^
(1/2) + 2*a*b^2)/(4*a^2*b^3))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{2+3x}}{a+bx^2}\,dx$$
$$= \frac{\sqrt{2}\left(-2\sqrt{9a+4b}\,\sqrt{\sqrt{b}\,\sqrt{9a+4b}-2b}\,atan\left(\frac{\sqrt{\sqrt{b}\,\sqrt{9a+4b}+2b}\,\sqrt{2}-2\sqrt{b}\,\sqrt{3x+2}}{\sqrt{\sqrt{b}\,\sqrt{9a+4b}-2b}\,\sqrt{2}}\right)-4\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{9a+4b}-2b}\,ata$$

input
```
int((2+3*x)^(1/2)/(b*x^2+a),x)
```

output
```
(sqrt(2)*( - 2*sqrt(9*a + 4*b)*sqrt(sqrt(b)*sqrt(9*a + 4*b) - 2*b)*atan((s
qrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*sqrt(2) - 2*sqrt(b)*sqrt(3*x + 2))/(sqr
t(sqrt(b)*sqrt(9*a + 4*b) - 2*b)*sqrt(2))) - 4*sqrt(b)*sqrt(sqrt(b)*sqrt(9
*a + 4*b) - 2*b)*atan((sqrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*sqrt(2) - 2*sqr
t(b)*sqrt(3*x + 2))/(sqrt(sqrt(b)*sqrt(9*a + 4*b) - 2*b)*sqrt(2))) + 2*sqr
t(9*a + 4*b)*sqrt(sqrt(b)*sqrt(9*a + 4*b) - 2*b)*atan((sqrt(sqrt(b)*sqrt(9
*a + 4*b) + 2*b)*sqrt(2) + 2*sqrt(b)*sqrt(3*x + 2))/(sqrt(sqrt(b)*sqrt(9*a
 + 4*b) - 2*b)*sqrt(2))) + 4*sqrt(b)*sqrt(sqrt(b)*sqrt(9*a + 4*b) - 2*b)*a
tan((sqrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*sqrt(2) + 2*sqrt(b)*sqrt(3*x + 2)
)/(sqrt(sqrt(b)*sqrt(9*a + 4*b) - 2*b)*sqrt(2))) + sqrt(9*a + 4*b)*sqrt(sq
rt(b)*sqrt(9*a + 4*b) + 2*b)*log( - sqrt(3*x + 2)*sqrt(sqrt(b)*sqrt(9*a +
4*b) + 2*b)*sqrt(2) + sqrt(9*a + 4*b) + 3*sqrt(b)*x + 2*sqrt(b)) - sqrt(9*
a + 4*b)*sqrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*log(sqrt(3*x + 2)*sqrt(sqrt(b
)*sqrt(9*a + 4*b) + 2*b)*sqrt(2) + sqrt(9*a + 4*b) + 3*sqrt(b)*x + 2*sqrt(
b)) - 2*sqrt(b)*sqrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*log( - sqrt(3*x + 2)*s
qrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*sqrt(2) + sqrt(9*a + 4*b) + 3*sqrt(b)*x
 + 2*sqrt(b)) + 2*sqrt(b)*sqrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*log(sqrt(3*x
 + 2)*sqrt(sqrt(b)*sqrt(9*a + 4*b) + 2*b)*sqrt(2) + sqrt(9*a + 4*b) + 3*sq
rt(b)*x + 2*sqrt(b))))/(12*a*b)
```

3.199 $\int \frac{\sqrt{1+x}}{1+x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 157

$$\int \frac{\sqrt{1+x}}{1+x^2} dx = -\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) + \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}}{1+\sqrt{2}+x}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}$$

output
```
-1/2*(2+2*2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)-2*(1+x)^(1/2))/(-2+2*
2^(1/2))^(1/2))+1/2*(2+2*2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)+2*(1+x
)^(1/2))/(-2+2*2^(1/2))^(1/2))-arctanh((2+2*2^(1/2))^(1/2)*(1+x)^(1/2)/(1+
2^(1/2)+x))/(2+2*2^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{1+x}}{1+x^2}\,dx = \sqrt{1-i}\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+x}\right)+\sqrt{1+i}\arctan\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+x}\right)$$

input
```
Integrate[Sqrt[1 + x]/(1 + x^2),x]
```

output
```
Sqrt[1 - I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] + Sqrt[1 + I]*ArcTan[Sqrt
[-1/2 + I/2]*Sqrt[1 + x]]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {483, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{x^2+1}\,dx$$

$$\Big\downarrow \text{483}$$

$$2\int \frac{x+1}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}$$

$$\Big\downarrow \text{1447}$$

$$2\left(\frac{1}{2}\int \frac{x+\sqrt{2}+1}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}-\frac{1}{2}\int \frac{-x+\sqrt{2}-1}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}\right)$$

$$\Big\downarrow \text{1475}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}+\frac{1}{2}\int\frac{1}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}\right)-\right.$$

$\Big\downarrow$ 1083

$$2\left(\frac{1}{2}\left(-\int\frac{1}{-x+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}\right)-\int\frac{1}{-x+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{x+1}+\sqrt{2\left(1\right.}\right.\right.\right.$$

$\Big\downarrow$ 217

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\int\frac{-x+\sqrt{2}-1}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}\right)$$

$\Big\downarrow$ 1478

$$2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\int-\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(\right.}}{\sqrt{2\left(\sqrt{2}-\right.}}\right.}{\sqrt{2\left(\sqrt{2}-1\right)}}\right.\right.$$

$\Big\downarrow$ 25

$$2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\int\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\right.}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right.}{\sqrt{2\left(\sqrt{2}-1\right)}}\right.\right.$$

$\Big\downarrow$ 1103

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}\cdots\right.}{2\sqrt{2\left(1+\sqrt{2}\right)}}\cdots\right.\right.$$

input
```
Int[Sqrt[1 + x]/(1 + x^2),x]
```

output
```
2*((ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]
]/Sqrt[2*(-1 + Sqrt[2])] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/
Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(-1 + Sqrt[2])])/2 + (Log[1 + Sqrt[2] + x -
 Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[2*(1 + Sqrt[2])]) - Log[1 + Sq
rt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[2*(1 + Sqrt[2])]))/
2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{\sqrt{2+2\sqrt{2}}\,\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{2}-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(\right.}{}\right.}{}$
default	$\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{\sqrt{2+2\sqrt{2}}\,\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{2}-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(\right.}{}\right.}{}$
trager	$-\operatorname{RootOf}\left(8\_Z^4+4\_Z^2+1\right)\ln\left(\frac{24\operatorname{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^5 x+26\operatorname{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^3}{}\right.$

input
```
int((x+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(1/2*ln(x+1-(x+1)^(1/2)*(2+2*2^(1/2))^
(1/2)+2^(1/2))+(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1
/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))-1/2*(2+2*2^(1/2))^(1/2)*(2
^(1/2)-1)*(1/2*ln(x+1+(x+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))-(2+2*2^(1/2
))^(1/2)/(-2+2*2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)+2*(x+1)^(1/2))/(
-2+2*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$
\begin{aligned}
\int \frac{\sqrt{1+x}}{1+x^2}\,dx
&= \sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(2\left(\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+\sqrt{x+1}\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right)\\
&\quad-\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(2\left(\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}-\sqrt{x+1}\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right)\\
&\quad-\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(2\sqrt{x+1}\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+x+\sqrt{2}+1\right)\\
&\quad+\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(-2\sqrt{x+1}\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+x+\sqrt{2}+1\right)
\end{aligned}
$$

input
```
integrate((1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
sqrt(1/2*sqrt(2) + 1/2)*arctan(2*((sqrt(2) + 1)*sqrt(1/2*sqrt(2) - 1/2) +
sqrt(x + 1))*sqrt(1/2*sqrt(2) + 1/2)) - sqrt(1/2*sqrt(2) + 1/2)*arctan(2*(
(sqrt(2) + 1)*sqrt(1/2*sqrt(2) - 1/2) - sqrt(x + 1))*sqrt(1/2*sqrt(2) + 1/
2)) - 1/2*sqrt(1/2*sqrt(2) - 1/2)*log(2*sqrt(x + 1)*(sqrt(2) + 1)*sqrt(1/2
*sqrt(2) - 1/2) + x + sqrt(2) + 1) + 1/2*sqrt(1/2*sqrt(2) - 1/2)*log(-2*sq
rt(x + 1)*(sqrt(2) + 1)*sqrt(1/2*sqrt(2) - 1/2) + x + sqrt(2) + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x}}{1+x^2}\,dx = \int \frac{\sqrt{x+1}}{x^2+1}\,dx$$

input
```
integrate((1+x)**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(x + 1)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}}{1+x^2}\,dx = \int \frac{\sqrt{x+1}}{x^2+1}\,dx$$

input
```
integrate((1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 1)/(x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{1+x}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{2}\sqrt{2\sqrt{2}+2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{2\sqrt{2}-2}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) + \frac{1}{4}\sqrt{2\sqrt{2}-2}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)$$

input

```
integrate((1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output

```
1/2*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*
sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + 1/2*sqrt(2*sqrt(2) + 2)*arctan(-1/2*2^(
3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/4
*sqrt(2*sqrt(2) - 2)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(
2) + 1) + 1/4*sqrt(2*sqrt(2) - 2)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) +
2) + x + sqrt(2) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{1+x}}{1+x^2}\,dx = \operatorname{atanh}\left(4\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\sqrt{x+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)+\operatorname{atanh}\left(4\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}-\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\sqrt{x+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}-2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)$$

input
```
int((x + 1)^(1/2)/(x^2 + 1),x)
```

output
```
atanh(4*((- 2^(1/2)/8 - 1/8)^(1/2) + (2^(1/2)/8 - 1/8)^(1/2))^3*(x + 1)^(1
/2))*(2*(- 2^(1/2)/8 - 1/8)^(1/2) + 2*(2^(1/2)/8 - 1/8)^(1/2)) + atanh(4*(
(- 2^(1/2)/8 - 1/8)^(1/2) - (2^(1/2)/8 - 1/8)^(1/2))^3*(x + 1)^(1/2))*(2*(
- 2^(1/2)/8 - 1/8)^(1/2) - 2*(2^(1/2)/8 - 1/8)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{1+x}}{1+x^2}\,dx = -\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} - \sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right) + \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} + \sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right) - \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(-\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{4} + \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{4} + \frac{\sqrt{\sqrt{2}+1}\log\left(-\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{2} - \frac{\sqrt{\sqrt{2}+1}\log\left(\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{2}$$

input
```
int((1+x)^(1/2)/(x^2+1),x)
```

output
```
( - 2*sqrt(sqrt(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) - 2*sqrt(x
 + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) - 4*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(
2) + 1)*sqrt(2) - 2*sqrt(x + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) + 2*sqrt(sqr
t(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) + 2*sqrt(x + 1))/(sqrt(s
qrt(2) - 1)*sqrt(2))) + 4*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) + 1)*sqrt(2
) + 2*sqrt(x + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) - sqrt(sqrt(2) + 1)*sqrt(2
)*log( - sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) + sqrt(2) + x + 1) + sqrt(s
qrt(2) + 1)*sqrt(2)*log(sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) + sqrt(2) +
x + 1) + 2*sqrt(sqrt(2) + 1)*log( - sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2)
+ sqrt(2) + x + 1) - 2*sqrt(sqrt(2) + 1)*log(sqrt(x + 1)*sqrt(sqrt(2) + 1)
*sqrt(2) + sqrt(2) + x + 1))/4
```

3.200 $\int \frac{1}{\sqrt{1+x}(1+x^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 154

$$\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx = -\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}}{1+\sqrt{2}+x}\right)}{2\sqrt{1+\sqrt{2}}}$$

output
```
-1/2*(1+2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)-2*(1+x)^(1/2))/(-2+2*2^
(1/2))^(1/2))+1/2*(1+2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)+2*(1+x)^(1
/2))/(-2+2*2^(1/2))^(1/2))+1/2*arctanh((2+2*2^(1/2))^(1/2)*(1+x)^(1/2)/(1+
2^(1/2)+x))/(1+2^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx = \sqrt{\frac{1}{2}+\frac{i}{2}}\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+x}\right) + \sqrt{\frac{1}{2}-\frac{i}{2}}\arctan\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+x}\right)$$

input
```
Integrate[1/(Sqrt[1 + x]*(1 + x^2)),x]
```

output
```
Sqrt[1/2 + I/2]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] + Sqrt[1/2 - I/2]*Arc
Tan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {484, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}\,(x^2+1)}\,dx$$

$$\Big\downarrow \text{484}$$

$$2\int \frac{1}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}$$

$$\Big\downarrow \text{1407}$$

$$2\left(\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\int \frac{\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}\right)$$

$\downarrow$ 1142

$$2\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}-\frac{1}{2}\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int}{}\right.$$

$\downarrow$ 25

$$2\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}+\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{}{x+}}{}\right.$$

$\downarrow$ 1083

$$2\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-x+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{}{x+}}{}\right.$$

$\downarrow$ 217

$$2\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}+\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{}}{}\right.$$

$\downarrow$ 1103

$$2\left(\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{x+}}{\sqrt{}}\right.}{}\right.$$

input
```
Int[1/(Sqrt[1 + x]*(1 + x^2)),x]
```

output
```
2*((Sqrt[(1 + Sqrt[2])/(-1 + Sqrt[2])]*ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*
Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + S
qrt[2])]*Sqrt[1 + x]]/2)/(4*Sqrt[1 + Sqrt[2]]) + (Sqrt[(1 + Sqrt[2])/(-1 +
 Sqrt[2])]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqr
t[2])]] + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/2)/(4*S
qrt[1 + Sqrt[2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(108) = 216$.

Time = 0.92 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{8}+\frac{\left(2\sqrt{2}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{2}\right)\mathrm{arct}}{2\sqrt{-2+2\sqrt{2}}}$
default	$\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{8}+\frac{\left(2\sqrt{2}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{2}\right)\mathrm{arct}}{2\sqrt{-2+2\sqrt{2}}}$
trager	$-\mathrm{RootOf}\left(32\_Z^4+8\_Z^2+1\right)\ln\left(\frac{64\,\mathrm{RootOf}\left(32\_Z^4+8\_Z^2+1\right)^5 x-16\,\mathrm{RootOf}\left(32\_Z^4+8\_Z^2+\right.}{}\right.$

input
```
int(1/(x+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*ln(x+1+(x+1)^(1/2
)*(2+2*2^(1/2))^(1/2)+2^(1/2))+1/2*(2*2^(1/2)-1/2*(-(2+2*2^(1/2))^(1/2)*2^
(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arc
tan(((2+2*2^(1/2))^(1/2)+2*(x+1)^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(-(2+2*2
^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*ln(x+1-(x+1)^(1/2)*(2+2*2^(1/
2))^(1/2)+2^(1/2))-1/2*(-2*2^(1/2)+1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+
2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1
)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx \\
&= \frac{1}{2}\sqrt{\sqrt{2}+1}\arctan\left(\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+\sqrt{2}\sqrt{x+1}\right)\sqrt{\sqrt{2}+1}\right) \\
&\quad -\frac{1}{2}\sqrt{\sqrt{2}+1}\arctan\left(\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}-\sqrt{2}\sqrt{x+1}\right)\sqrt{\sqrt{2}+1}\right) \\
&\quad +\frac{1}{4}\sqrt{\sqrt{2}-1}\log\left(\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{\sqrt{2}-1}+x+\sqrt{2}+1\right) \\
&\quad -\frac{1}{4}\sqrt{\sqrt{2}-1}\log\left(-\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{\sqrt{2}-1}+x+\sqrt{2}+1\right)
\end{aligned}
$$

input
```
integrate(1/(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(sqrt(2) + 1)*arctan(((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + sqrt(2)*sq
rt(x + 1))*sqrt(sqrt(2) + 1)) - 1/2*sqrt(sqrt(2) + 1)*arctan(((sqrt(2) + 1
)*sqrt(sqrt(2) - 1) - sqrt(2)*sqrt(x + 1))*sqrt(sqrt(2) + 1)) + 1/4*sqrt(s
qrt(2) - 1)*log(sqrt(x + 1)*(sqrt(2) + 2)*sqrt(sqrt(2) - 1) + x + sqrt(2)
+ 1) - 1/4*sqrt(sqrt(2) - 1)*log(-sqrt(x + 1)*(sqrt(2) + 2)*sqrt(sqrt(2) -
 1) + x + sqrt(2) + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx = \int \frac{1}{\sqrt{x+1}\,(x^2+1)}\,dx$$

input
```
integrate(1/(1+x)**(1/2)/(x**2+1),x)
```

output
```
Integral(1/(sqrt(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx = \int \frac{1}{(x^2+1)\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)*sqrt(x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx &= \frac{1}{2}\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &+\frac{1}{2}\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &+\frac{1}{4}\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)\\ &-\frac{1}{4}\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)\end{aligned}$$

input
```
integrate(1/(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sq
rt(x + 1))/sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2) + 1)*arctan(-1/2*2^(3/4)
*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + 1/4*sqr
t(sqrt(2) - 1)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1
) - 1/4*sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x +
 sqrt(2) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx = \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{x+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8} - \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{x+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}} + 2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right) - \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{x+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8} + \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{x+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}} - 2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right)$$

input
```
int(1/((x^2 + 1)*(x + 1)^(1/2)),x)
```

output
```
atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(x + 1)^(1/2))/(128*(2^(1/2)
/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(1/2)/
16 - 1/16)^(1/2)*(x + 1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/
16 - 1/16)^(1/2) - 8))*(2*(- 2^(1/2)/16 - 1/16)^(1/2) + 2*(2^(1/2)/16 - 1/
16)^(1/2)) - atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(x + 1)^(1/2))/
(128*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 8) + (16*2^(1
/2)*(2^(1/2)/16 - 1/16)^(1/2)*(x + 1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2
)*(- 2^(1/2)/16 - 1/16)^(1/2) + 8))*(2*(- 2^(1/2)/16 - 1/16)^(1/2) - 2*(2^
(1/2)/16 - 1/16)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx = -\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} - \frac{\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} + \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} + \frac{\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} - \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(-\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{4} + \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{4} + \frac{\sqrt{\sqrt{2}+1}\log\left(-\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{4} - \frac{\sqrt{\sqrt{2}+1}\log\left(\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{4}$$

input
```
int(1/(1+x)^(1/2)/(x^2+1),x)
```

output
```
( - 2*sqrt(sqrt(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) - 2*sqrt(x
 + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) - 2*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(
2) + 1)*sqrt(2) - 2*sqrt(x + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) + 2*sqrt(sqr
t(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) + 2*sqrt(x + 1))/(sqrt(s
qrt(2) - 1)*sqrt(2))) + 2*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) + 1)*sqrt(2
) + 2*sqrt(x + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) - sqrt(sqrt(2) + 1)*sqrt(2
)*log( - sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) + sqrt(2) + x + 1) + sqrt(s
qrt(2) + 1)*sqrt(2)*log(sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) + sqrt(2) +
x + 1) + sqrt(sqrt(2) + 1)*log( - sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) +
sqrt(2) + x + 1) - sqrt(sqrt(2) + 1)*log(sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqr
t(2) + sqrt(2) + x + 1))/4
```

3.201 $\int \frac{\sqrt{-1+x}}{(1+x^2)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 218

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3} \, dx = \frac{\sqrt{-1+x}x}{4(1+x^2)^2} - \frac{(1-11x)\sqrt{-1+x}}{32(1+x^2)} - \frac{1}{64}\sqrt{\frac{1}{2}\left(-527+373\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2\sqrt{-1+x}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right) + \frac{1}{64}\sqrt{\frac{1}{2}\left(-527+373\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2\sqrt{-1+x}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right) + \frac{1}{64}\sqrt{\frac{1}{2}\left(527+373\sqrt{2}\right)} \operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{-1+x}}{1-\sqrt{2}-x}\right)$$

output

```
1/4*(-1+x)^(1/2)*x/(x^2+1)^2-(1-11*x)*(-1+x)^(1/2)/(32*x^2+32)-1/128*(-105
4+746*2^(1/2))^(1/2)*arctan(((-2+2*2^(1/2))^(1/2)-2*(-1+x)^(1/2))/(2+2*2^(
1/2))^(1/2))+1/128*(-1054+746*2^(1/2))^(1/2)*arctan(((-2+2*2^(1/2))^(1/2)+
2*(-1+x)^(1/2))/(2+2*2^(1/2))^(1/2))+1/128*(1054+746*2^(1/2))^(1/2)*arctan
h((-2+2*2^(1/2))^(1/2)*(-1+x)^(1/2)/(1-2^(1/2)-x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3}\,dx = \frac{1}{64}\left(\frac{2\sqrt{-1+x}(-1+19x-x^2+11x^3)}{(1+x^2)^2} + \sqrt{-527-23i}\arctan\left(\sqrt{\frac{1}{2}-\frac{i}{2}}\sqrt{-1+x}\right) + \sqrt{-527+23i}\arctan\left(\sqrt{\frac{1}{2}+\frac{i}{2}}\sqrt{-1+x}\right)\right)$$

input
```
Integrate[Sqrt[-1 + x]/(1 + x^2)^3,x]
```

output
```
((2*Sqrt[-1 + x]*(-1 + 19*x - x^2 + 11*x^3))/(1 + x^2)^2 + Sqrt[-527 - 23*
I]*ArcTan[Sqrt[1/2 - I/2]*Sqrt[-1 + x]] + Sqrt[-527 + 23*I]*ArcTan[Sqrt[1/
2 + I/2]*Sqrt[-1 + x]])/64
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.39, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {494, 27, 686, 27, 654, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-1}}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{494}$$

$$\frac{\sqrt{x-1}x}{4(x^2+1)^2} - \frac{1}{4}\int \frac{6-5x}{2\sqrt{x-1}(x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}-\frac{1}{8}\int \frac{6-5x}{\sqrt{x-1}\left(x^2+1\right)^2}dx$$

$\Big\downarrow$ 686

$$\frac{1}{8}\left(\frac{1}{4}\int -\frac{25-11x}{2\sqrt{x-1}\left(x^2+1\right)}dx-\frac{(1-11x)\sqrt{x-1}}{4\left(x^2+1\right)}\right)+\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(-\frac{1}{8}\int \frac{25-11x}{\sqrt{x-1}\left(x^2+1\right)}dx-\frac{\sqrt{x-1}(1-11x)}{4\left(x^2+1\right)}\right)+\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 654

$$\frac{1}{8}\left(-\frac{1}{4}\int \frac{14-11(x-1)}{(x-1)^2+2(x-1)+2}d\sqrt{x-1}-\frac{\sqrt{x-1}(1-11x)}{4\left(x^2+1\right)}\right)+\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 1483

$$\frac{1}{8}\left(\frac{1}{4}\left(-\frac{\int \frac{14\sqrt{2\left(-1+\sqrt{2}\right)}-\left(14+11\sqrt{2}\right)\sqrt{x-1}}{x-\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}+\sqrt{2}-1}d\sqrt{x-1}}{4\sqrt{\sqrt{2}-1}}-\frac{\int \frac{\left(14+11\sqrt{2}\right)\sqrt{x-1}+14\sqrt{2\left(-1+\sqrt{2}\right)}}{x+\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}+\sqrt{2}-1}d\sqrt{x-1}}{4\sqrt{\sqrt{2}-1}}\right)-\frac{(1-11x)\sqrt{x-}}{4\left(x^2+1\right)}\right.$$

$$\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{1}{8}\left(\frac{1}{4}\left(-\frac{-\sqrt{373\sqrt{2}-527}\int \frac{1}{x-\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}+\sqrt{2}-1}d\sqrt{x-1}-\frac{1}{2}\left(14+11\sqrt{2}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2\sqrt{x-1}}{x-\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}+\sqrt{2}-1}d\sqrt{}}{4\sqrt{\sqrt{2}-1}}\right.\right.$$

$$\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 25

$$\frac{1}{8}\left(\frac{1}{4}\left(-\frac{\frac{1}{2}(14+11\sqrt{2})\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2\sqrt{x-1}}{x-\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}+\sqrt{2}-1}d\sqrt{x-1}-\sqrt{373\sqrt{2}-527}\int \frac{1}{x-\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}+\sqrt{2}-1}d\sqrt{x-}}{4\sqrt{\sqrt{2}-1}}\right.\right.$$

$$\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\big\downarrow$ 1083

$$\frac{1}{8}\left(\frac{1}{4}\left(-\frac{2\sqrt{373\sqrt{2}-527}\int \frac{1}{-x-2\left(1+\sqrt{2}\right)+1}d\left(2\sqrt{x-1}-\sqrt{2\left(-1+\sqrt{2}\right)}\right)+\frac{1}{2}(14+11\sqrt{2})\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2\sqrt{}}{x-\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}}}{4\sqrt{\sqrt{2}-1}}\right.\right.$$

$$\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\big\downarrow$ 217

$$\frac{1}{8}\left(\frac{1}{4}\left(-\frac{\frac{1}{2}(14+11\sqrt{2})\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2\sqrt{x-1}}{x-\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x-1}+\sqrt{2}-1}d\sqrt{x-1}-\sqrt{\frac{2\left(373\sqrt{2}-527\right)}{1+\sqrt{2}}}\arctan\left(\frac{2\sqrt{x-1}-\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\frac{1}{2}}{}\right.\right.$$

$$\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

$\big\downarrow$ 1103

$$\frac{1}{8}\left(\frac{1}{4}\left(-\frac{-\sqrt{\frac{2\left(373\sqrt{2}-527\right)}{1+\sqrt{2}}}\arctan\left(\frac{2\sqrt{x-1}-\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)-\frac{1}{2}(14+11\sqrt{2})\log\left(x-\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x-1}+\sqrt{2}-\right.}{4\sqrt{\sqrt{2}-1}}\right.\right.$$

$$\frac{\sqrt{x-1}x}{4\left(x^2+1\right)^2}$$

input
```
Int[Sqrt[-1 + x]/(1 + x^2)^3,x]
```

output
```
(Sqrt[-1 + x]*x)/(4*(1 + x^2)^2) + (-1/4*((1 - 11*x)*Sqrt[-1 + x])/(1 + x^
2) + (-1/4*(-(Sqrt[(2*(-527 + 373*Sqrt[2]))/(1 + Sqrt[2])]*ArcTan[(-Sqrt[2
*(-1 + Sqrt[2])] + 2*Sqrt[-1 + x])/Sqrt[2*(1 + Sqrt[2])]]) - ((14 + 11*Sqr
t[2])*Log[-1 + Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*Sqrt[-1 + x] + x])/2)/Sqrt
[-1 + Sqrt[2]] - (-(Sqrt[(2*(-527 + 373*Sqrt[2]))/(1 + Sqrt[2])]*ArcTan[(S
qrt[2*(-1 + Sqrt[2])] + 2*Sqrt[-1 + x])/Sqrt[2*(1 + Sqrt[2])]]) + ((14 + 1
1*Sqrt[2])*Log[-1 + Sqrt[2] + Sqrt[2*(-1 + Sqrt[2])]*Sqrt[-1 + x] + x])/2)
/(4*Sqrt[-1 + Sqrt[2]]))/4)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.20 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.03

method	result
trager	$\frac{(11x^3-x^2+19x-1)\sqrt{x-1}}{32(x^2+1)^2}+\frac{\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4-8432\_Z^2+139129\right)^2-1054\right)\ln\left(\frac{-3008\,\mathrm{RootC}}{}\right.}{}$
risch	$\frac{(11x^3-x^2+19x-1)\sqrt{x-1}}{32(x^2+1)^2}-\frac{9\ln\left(x-1+\sqrt{x-1}\,\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{2}\,\sqrt{-2+2\sqrt{2}}}{128}-\frac{25\ln\left(x-1+\sqrt{x-1}\,\sqrt{-2+2\sqrt{2}}\right.}{256}$
derivativedivides	$-\frac{-\frac{4\left(-759-506\sqrt{2}\right)(x-1)^{\frac{3}{2}}}{23\left(-6-4\sqrt{2}\right)}-\frac{\left(-5336-3588\sqrt{2}\right)\sqrt{-2+2\sqrt{2}}\,(x-1)}{23\left(-6-4\sqrt{2}\right)}-\frac{2\left(-2392\sqrt{2}-3036\right)\sqrt{x-1}}{23\left(-6-4\sqrt{2}\right)}-\frac{\left(-3312\sqrt{2}-4416\right)\sqrt{-2+2\sqrt{2}}}{46\left(-6-4\sqrt{2}\right)}}{128\left(x-1+\sqrt{x-1}\,\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)^2}$
default	$-\frac{-\frac{4\left(-759-506\sqrt{2}\right)(x-1)^{\frac{3}{2}}}{23\left(-6-4\sqrt{2}\right)}-\frac{\left(-5336-3588\sqrt{2}\right)\sqrt{-2+2\sqrt{2}}\,(x-1)}{23\left(-6-4\sqrt{2}\right)}-\frac{2\left(-2392\sqrt{2}-3036\right)\sqrt{x-1}}{23\left(-6-4\sqrt{2}\right)}-\frac{\left(-3312\sqrt{2}-4416\right)\sqrt{-2+2\sqrt{2}}}{46\left(-6-4\sqrt{2}\right)}}{128\left(x-1+\sqrt{x-1}\,\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)^2}$

input
```
int((x-1)^(1/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(11*x^3-x^2+19*x-1)/(x^2+1)^2*(x-1)^(1/2)+1/128*RootOf(_Z^2+16*RootOf
(128*_Z^4-8432*_Z^2+139129)^2-1054)*ln((-3008*RootOf(128*_Z^4-8432*_Z^2+13
9129)^4*x*RootOf(_Z^2+16*RootOf(128*_Z^4-8432*_Z^2+139129)^2-1054)+185916*
RootOf(128*_Z^4-8432*_Z^2+139129)^2*RootOf(_Z^2+16*RootOf(128*_Z^4-8432*_Z
^2+139129)^2-1054)*x+411792*RootOf(128*_Z^4-8432*_Z^2+139129)^2*(x-1)^(1/2
)+21620*RootOf(128*_Z^4-8432*_Z^2+139129)^2*RootOf(_Z^2+16*RootOf(128*_Z^4
-8432*_Z^2+139129)^2-1054)-2870608*RootOf(_Z^2+16*RootOf(128*_Z^4-8432*_Z^
2+139129)^2-1054)*x-14352667*(x-1)^(1/2)-686320*RootOf(_Z^2+16*RootOf(128*
_Z^4-8432*_Z^2+139129)^2-1054))/(16*RootOf(128*_Z^4-8432*_Z^2+139129)^2*x-
527*x+23))+1/32*RootOf(128*_Z^4-8432*_Z^2+139129)*ln(-(12032*RootOf(128*_Z
^4-8432*_Z^2+139129)^5*x-841552*RootOf(128*_Z^4-8432*_Z^2+139129)^3*x+4117
92*RootOf(128*_Z^4-8432*_Z^2+139129)^2*(x-1)^(1/2)+86480*RootOf(128*_Z^4-8
432*_Z^2+139129)^3+14706618*RootOf(128*_Z^4-8432*_Z^2+139129)*x-12774131*(
x-1)^(1/2)-2951590*RootOf(128*_Z^4-8432*_Z^2+139129))/(16*RootOf(128*_Z^4-
8432*_Z^2+139129)^2*x-527*x-23))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3}\,dx = \frac{2\,(x^4+2\,x^2+1)\sqrt{\frac{373}{2}\sqrt{2}-\frac{527}{2}}\arctan\left(\frac{2}{23}\sqrt{x-1}\sqrt{\frac{373}{2}\sqrt{2}-\frac{527}{2}}(7\sqrt{2}+11)+\frac{2}{23}\sqrt{\frac{373}{2}\sqrt{2}+\frac{527}{2}}\sqrt{\frac{373}{2}}\right.}{}$$

input
```
integrate((x-1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/128*(2*(x^4 + 2*x^2 + 1)*sqrt(373/2*sqrt(2) - 527/2)*arctan(2/23*sqrt(x
- 1)*sqrt(373/2*sqrt(2) - 527/2)*(7*sqrt(2) + 11) + 2/23*sqrt(373/2*sqrt(2
) + 527/2)*sqrt(373/2*sqrt(2) - 527/2)*(sqrt(2) - 1)) - 2*(x^4 + 2*x^2 + 1
)*sqrt(373/2*sqrt(2) - 527/2)*arctan(-2/23*sqrt(x - 1)*sqrt(373/2*sqrt(2)
- 527/2)*(7*sqrt(2) + 11) + 2/23*sqrt(373/2*sqrt(2) + 527/2)*sqrt(373/2*sq
rt(2) - 527/2)*(sqrt(2) - 1)) - (x^4 + 2*x^2 + 1)*sqrt(373/2*sqrt(2) + 527
/2)*log(2*sqrt(x - 1)*sqrt(373/2*sqrt(2) + 527/2)*(18*sqrt(2) - 25) + 23*x
 + 23*sqrt(2) - 23) + (x^4 + 2*x^2 + 1)*sqrt(373/2*sqrt(2) + 527/2)*log(-2
*sqrt(x - 1)*sqrt(373/2*sqrt(2) + 527/2)*(18*sqrt(2) - 25) + 23*x + 23*sqr
t(2) - 23) + 4*(11*x^3 - x^2 + 19*x - 1)*sqrt(x - 1))/(x^4 + 2*x^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((x-1)**(1/2)/(x**2+1)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3}\,dx=\int \frac{\sqrt{x-1}}{(x^2+1)^3}\,dx$$

input
```
integrate((x-1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(x - 1)/(x^2 + 1)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{\sqrt{-1+x}}{(1+x^2)^3}\,dx=&\ \frac{1}{128}\sqrt{746\sqrt{2}-1054}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\sqrt{x-1}\right)}{2\sqrt{\sqrt{2}+2}}\right)\\&+\frac{1}{128}\sqrt{746\sqrt{2}-1054}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\sqrt{x-1}\right)}{2\sqrt{\sqrt{2}+2}}\right)\\&-\frac{1}{256}\sqrt{746\sqrt{2}+1054}\log\left(2^{\frac{1}{4}}\sqrt{x-1}\sqrt{-\sqrt{2}+2}+x+\sqrt{2}-1\right)\\&+\frac{1}{256}\sqrt{746\sqrt{2}+1054}\log\left(-2^{\frac{1}{4}}\sqrt{x-1}\sqrt{-\sqrt{2}+2}+x+\sqrt{2}-1\right)\\&+\frac{11\,(x-1)^{\frac{7}{2}}+32\,(x-1)^{\frac{5}{2}}+50\,(x-1)^{\frac{3}{2}}+28\sqrt{x-1}}{32\left((x-1)^2+2\,x\right)^2}\end{aligned}$$

input
```
integrate((x-1)^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(746*sqrt(2) - 1054)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) +
 2) + 2*sqrt(x - 1))/sqrt(sqrt(2) + 2)) + 1/128*sqrt(746*sqrt(2) - 1054)*a
rctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2) - 2*sqrt(x - 1))/sqrt(sqrt(
2) + 2)) - 1/256*sqrt(746*sqrt(2) + 1054)*log(2^(1/4)*sqrt(x - 1)*sqrt(-sq
rt(2) + 2) + x + sqrt(2) - 1) + 1/256*sqrt(746*sqrt(2) + 1054)*log(-2^(1/4
)*sqrt(x - 1)*sqrt(-sqrt(2) + 2) + x + sqrt(2) - 1) + 1/32*(11*(x - 1)^(7/
2) + 32*(x - 1)^(5/2) + 50*(x - 1)^(3/2) + 28*sqrt(x - 1))/((x - 1)^2 + 2*
x)^2
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3} \, dx = \text{Too large to display}$$

input
```
int((x - 1)^(1/2)/(x^2 + 1)^3,x)
```

output
```
atanh((275*(527/32768 - (373*2^(1/2))/32768)^(1/2)*(x - 1)^(1/2))/(64*(28*
(527/32768 - (373*2^(1/2))/32768)^(1/2)*((373*2^(1/2))/32768 + 527/32768)^
(1/2) - 207/4096)) + (275*((373*2^(1/2))/32768 + 527/32768)^(1/2)*(x - 1)^
(1/2))/(64*(28*(527/32768 - (373*2^(1/2))/32768)^(1/2)*((373*2^(1/2))/3276
8 + 527/32768)^(1/2) - 207/4096)) + (373*2^(1/2)*(527/32768 - (373*2^(1/2)
)/32768)^(1/2)*(x - 1)^(1/2))/(128*(28*(527/32768 - (373*2^(1/2))/32768)^(
1/2)*((373*2^(1/2))/32768 + 527/32768)^(1/2) - 207/4096)) - (373*2^(1/2)*(
(373*2^(1/2))/32768 + 527/32768)^(1/2)*(x - 1)^(1/2))/(128*(28*(527/32768
- (373*2^(1/2))/32768)^(1/2)*((373*2^(1/2))/32768 + 527/32768)^(1/2) - 207
/4096)))*(2*(527/32768 - (373*2^(1/2))/32768)^(1/2) + 2*((373*2^(1/2))/327
68 + 527/32768)^(1/2)) - atanh((275*(527/32768 - (373*2^(1/2))/32768)^(1/2
)*(x - 1)^(1/2))/(64*(28*(527/32768 - (373*2^(1/2))/32768)^(1/2)*((373*2^(
1/2))/32768 + 527/32768)^(1/2) + 207/4096)) - (275*((373*2^(1/2))/32768 +
527/32768)^(1/2)*(x - 1)^(1/2))/(64*(28*(527/32768 - (373*2^(1/2))/32768)^
(1/2)*((373*2^(1/2))/32768 + 527/32768)^(1/2) + 207/4096)) + (373*2^(1/2)*
(527/32768 - (373*2^(1/2))/32768)^(1/2)*(x - 1)^(1/2))/(128*(28*(527/32768
 - (373*2^(1/2))/32768)^(1/2)*((373*2^(1/2))/32768 + 527/32768)^(1/2) + 20
7/4096)) + (373*2^(1/2)*((373*2^(1/2))/32768 + 527/32768)^(1/2)*(x - 1)^(1
/2))/(128*(28*(527/32768 - (373*2^(1/2))/32768)^(1/2)*((373*2^(1/2))/32768
 + 527/32768)^(1/2) + 207/4096)))*(2*(527/32768 - (373*2^(1/2))/32768)^...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 845, normalized size of antiderivative = 3.88

$$\int \frac{\sqrt{-1+x}}{(1+x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x-1)^(1/2)/(x^2+1)^3,x)
```

output
```
(50*sqrt(sqrt(2) + 1)*sqrt(2)*atan((sqrt(sqrt(2) - 1)*sqrt(2) - 2*sqrt(x -
 1))/(sqrt(sqrt(2) + 1)*sqrt(2)))*x**4 + 100*sqrt(sqrt(2) + 1)*sqrt(2)*ata
n((sqrt(sqrt(2) - 1)*sqrt(2) - 2*sqrt(x - 1))/(sqrt(sqrt(2) + 1)*sqrt(2)))
*x**2 + 50*sqrt(sqrt(2) + 1)*sqrt(2)*atan((sqrt(sqrt(2) - 1)*sqrt(2) - 2*s
qrt(x - 1))/(sqrt(sqrt(2) + 1)*sqrt(2))) - 72*sqrt(sqrt(2) + 1)*atan((sqrt
(sqrt(2) - 1)*sqrt(2) - 2*sqrt(x - 1))/(sqrt(sqrt(2) + 1)*sqrt(2)))*x**4 -
 144*sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2) - 1)*sqrt(2) - 2*sqrt(x - 1))/(s
qrt(sqrt(2) + 1)*sqrt(2)))*x**2 - 72*sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2)
- 1)*sqrt(2) - 2*sqrt(x - 1))/(sqrt(sqrt(2) + 1)*sqrt(2))) - 50*sqrt(sqrt(
2) + 1)*sqrt(2)*atan((sqrt(sqrt(2) - 1)*sqrt(2) + 2*sqrt(x - 1))/(sqrt(sqr
t(2) + 1)*sqrt(2)))*x**4 - 100*sqrt(sqrt(2) + 1)*sqrt(2)*atan((sqrt(sqrt(2
) - 1)*sqrt(2) + 2*sqrt(x - 1))/(sqrt(sqrt(2) + 1)*sqrt(2)))*x**2 - 50*sqr
t(sqrt(2) + 1)*sqrt(2)*atan((sqrt(sqrt(2) - 1)*sqrt(2) + 2*sqrt(x - 1))/(s
qrt(sqrt(2) + 1)*sqrt(2))) + 72*sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2) - 1)*
sqrt(2) + 2*sqrt(x - 1))/(sqrt(sqrt(2) + 1)*sqrt(2)))*x**4 + 144*sqrt(sqrt
(2) + 1)*atan((sqrt(sqrt(2) - 1)*sqrt(2) + 2*sqrt(x - 1))/(sqrt(sqrt(2) +
1)*sqrt(2)))*x**2 + 72*sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2) - 1)*sqrt(2) +
 2*sqrt(x - 1))/(sqrt(sqrt(2) + 1)*sqrt(2))) + 25*sqrt(sqrt(2) - 1)*sqrt(2
)*log( - sqrt(x - 1)*sqrt(sqrt(2) - 1)*sqrt(2) + sqrt(2) + x - 1)*x**4 + 5
0*sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(x - 1)*sqrt(sqrt(2) - 1)*sqrt(2...
```

3.202 $\int \frac{(c+dx)^{4/3}}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 624

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx = \frac{3d\sqrt[3]{c+dx}}{b} + \frac{\sqrt{3}\left(bc^2-2\sqrt{-a}\sqrt{b}cd-ad^2\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}b^{7/6}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}} - \frac{\sqrt{3}\left(bc^2+2\sqrt{-a}\sqrt{b}cd-ad^2\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}b^{7/6}\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}} - \frac{\left(bc^2+2\sqrt{-a}\sqrt{b}cd-ad^2\right)\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}b^{7/6}\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}} + \frac{\left(bc^2-2\sqrt{-a}\sqrt{b}cd-ad^2\right)\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}b^{7/6}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}} - \frac{3\left(bc^2-2\sqrt{-a}\sqrt{b}cd-ad^2\right)\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{7/6}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}} + \frac{3\left(bc^2+2\sqrt{-a}\sqrt{b}cd-ad^2\right)\log\left(\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{7/6}\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}}$$

output
```
3*d*(d*x+c)^(1/3)/b+1/2*3^(1/2)*(b*c^2-2*(-a)^(1/2)*b^(1/2)*c*d-a*d^2)*arc
tan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3))*3^(1/2)
)/(-a)^(1/2)/b^(7/6)/(b^(1/2)*c-(-a)^(1/2)*d)^(2/3)-1/2*3^(1/2)*(b*c^2+2*(
-a)^(1/2)*b^(1/2)*c*d-a*d^2)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/(b^(1/2
)*c+(-a)^(1/2)*d)^(1/3))*3^(1/2))/(-a)^(1/2)/b^(7/6)/(b^(1/2)*c+(-a)^(1/2)
*d)^(2/3)-1/4*(b*c^2+2*(-a)^(1/2)*b^(1/2)*c*d-a*d^2)*ln((-a)^(1/2)-b^(1/2)
*x)/(-a)^(1/2)/b^(7/6)/(b^(1/2)*c+(-a)^(1/2)*d)^(2/3)+1/4*(b*c^2-2*(-a)^(1
/2)*b^(1/2)*c*d-a*d^2)*ln((-a)^(1/2)+b^(1/2)*x)/(-a)^(1/2)/b^(7/6)/(b^(1/2
)*c-(-a)^(1/2)*d)^(2/3)-3/4*(b*c^2-2*(-a)^(1/2)*b^(1/2)*c*d-a*d^2)*ln((b^(
1/2)*c-(-a)^(1/2)*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(7/6)/(b^(1
/2)*c-(-a)^(1/2)*d)^(2/3)+3/4*(b*c^2+2*(-a)^(1/2)*b^(1/2)*c*d-a*d^2)*ln((b
^(1/2)*c+(-a)^(1/2)*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(7/6)/(b^
(1/2)*c+(-a)^(1/2)*d)^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.21

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx = \frac{d\left(6b\sqrt[3]{c+dx} + \text{RootSum}\left[bc^2+ad^2-2bc\#1^3+b\#1^6\&, \frac{bc^2\log\left(\sqrt[3]{c+dx}-\#1\right)+ad^2\log}{}\right.\right.}{2b^2}$$

input
```
Integrate[(c + d*x)^(4/3)/(a + b*x^2),x]
```

output
```
(d*(6*b*(c + d*x)^(1/3) + RootSum[b*c^2 + a*d^2 - 2*b*c*#1^3 + b*#1^6 & ,
(b*c^2*Log[(c + d*x)^(1/3) - #1] + a*d^2*Log[(c + d*x)^(1/3) - #1] - 2*b*c
*Log[(c + d*x)^(1/3) - #1]*#1^3)/(c*#1^2 - #1^5) & ]))/(2*b^2)
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {481, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx$$

$$\Big\downarrow \ 481$$

$$\frac{\int \frac{bc^2+2bdxc-ad^2}{(c+dx)^{2/3}(bx^2+a)}dx}{b}+\frac{3d\sqrt[3]{c+dx}}{b}$$

$$\Big\downarrow \ 657$$

$$\frac{\int \left(\frac{\sqrt{-a}(bc^2-ad^2)-2a\sqrt{b}cd}{2a\left(\sqrt{-a}-\sqrt{b}x\right)(c+dx)^{2/3}}+\frac{2a\sqrt{b}cd+\sqrt{-a}(bc^2-ad^2)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)(c+dx)^{2/3}}\right)dx}{b}+\frac{3d\sqrt[3]{c+dx}}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sqrt{3}\left(-2\sqrt{-a}\sqrt{b}cd-ad^2+bc^2\right)\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}}-\frac{\sqrt{3}\left(2\sqrt{-a}\sqrt{b}cd-ad^2+bc^2\right)\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[6]{b}\left(\sqrt{-a}d+\sqrt{b}c\right)^{2/3}}-\frac{\left(2\sqrt{-a}\ \cdots\right.}{\cdots}}{\cdots}$$

$$\frac{3d\sqrt[3]{c+dx}}{b}$$

input
```
Int[(c + d*x)^(4/3)/(a + b*x^2),x]
```

output
```
(3*d*(c + d*x)^(1/3))/b + ((Sqrt[3]*(b*c^2 - 2*Sqrt[-a]*Sqrt[b]*c*d - a*d^
2)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3))/(Sqrt[b]*c - Sqrt[-a]*d)^(1/3))
/Sqrt[3]])/(2*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(2/3)) - (Sqrt[3]*
(b*c^2 + 2*Sqrt[-a]*Sqrt[b]*c*d - a*d^2)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^
(1/3))/(Sqrt[b]*c + Sqrt[-a]*d)^(1/3))/Sqrt[3]])/(2*Sqrt[-a]*b^(1/6)*(Sqrt
[b]*c + Sqrt[-a]*d)^(2/3)) - ((b*c^2 + 2*Sqrt[-a]*Sqrt[b]*c*d - a*d^2)*Log
[Sqrt[-a] - Sqrt[b]*x])/(4*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c + Sqrt[-a]*d)^(2/3)
) + ((b*c^2 - 2*Sqrt[-a]*Sqrt[b]*c*d - a*d^2)*Log[Sqrt[-a] + Sqrt[b]*x])/(
4*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(2/3)) - (3*(b*c^2 - 2*Sqrt[-a
]*Sqrt[b]*c*d - a*d^2)*Log[(Sqrt[b]*c - Sqrt[-a]*d)^(1/3) - b^(1/6)*(c + d
*x)^(1/3)])/(4*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(2/3)) + (3*(b*c^
2 + 2*Sqrt[-a]*Sqrt[b]*c*d - a*d^2)*Log[(Sqrt[b]*c + Sqrt[-a]*d)^(1/3) - b
^(1/6)*(c + d*x)^(1/3)])/(4*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c + Sqrt[-a]*d)^(2/3
)))/b
```

Defintions of rubi rules used

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{d\left(6(dx+c)^{\frac{1}{3}}b+\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\left(-2\_R^{3}bc+a\,d^{2}+b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{2}\left(-\_R^{3}+c\right)}\right)\right)}{2b^{2}}$	88
pseudoelliptic	$\frac{d\left(6(dx+c)^{\frac{1}{3}}b+\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\left(-2\_R^{3}bc+a\,d^{2}+b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{2}\left(-\_R^{3}+c\right)}\right)\right)}{2b^{2}}$	88
derivativedivides	$3d\left(\frac{(dx+c)^{\frac{1}{3}}}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\left(2\_R^{3}bc-a\,d^{2}-b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}c}}{6b^{2}}\right)$	93

input
```
int((d*x+c)^(4/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*(6*(d*x+c)^(1/3)*b+sum((-2*_R^3*b*c+a*d^2+b*c^2)*ln((d*x+c)^(1/3)-_R
)/_R^2/(-_R^3+c),_R=RootOf(_Z^6*b-2*_Z^3*b*c+a*d^2+b*c^2)))/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(472) = 944$.

Time = 0.23 (sec) , antiderivative size = 2309, normalized size of antiderivative = 3.70

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(4/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(2*b*(-(4*b*c^3*d - 4*a*c*d^3 + a*b^3*sqrt(-(b^4*c^8 - 12*a*b^3*c^6*d^
2 + 38*a^2*b^2*c^4*d^4 - 12*a^3*b*c^2*d^6 + a^4*d^8)/(a^3*b^7)))/(a*b^3))^
(1/3)*log((b^3*c^6 - 5*a*b^2*c^4*d^2 - 5*a^2*b*c^2*d^4 + a^3*d^6)*(d*x + c
)^(1/3) - (a*b^3*c^4*d - 6*a^2*b^2*c^2*d^3 + a^3*b*d^5 - a^2*b^5*c*sqrt(-(
b^4*c^8 - 12*a*b^3*c^6*d^2 + 38*a^2*b^2*c^4*d^4 - 12*a^3*b*c^2*d^6 + a^4*d
^8)/(a^3*b^7)))*(-(4*b*c^3*d - 4*a*c*d^3 + a*b^3*sqrt(-(b^4*c^8 - 12*a*b^3
*c^6*d^2 + 38*a^2*b^2*c^4*d^4 - 12*a^3*b*c^2*d^6 + a^4*d^8)/(a^3*b^7)))/(a
*b^3))^(1/3)) - (sqrt(-3)*b + b)*(-(4*b*c^3*d - 4*a*c*d^3 + a*b^3*sqrt(-(b
^4*c^8 - 12*a*b^3*c^6*d^2 + 38*a^2*b^2*c^4*d^4 - 12*a^3*b*c^2*d^6 + a^4*d^
8)/(a^3*b^7)))/(a*b^3))^(1/3)*log((b^3*c^6 - 5*a*b^2*c^4*d^2 - 5*a^2*b*c^2
*d^4 + a^3*d^6)*(d*x + c)^(1/3) + 1/2*(a*b^3*c^4*d - 6*a^2*b^2*c^2*d^3 + a
^3*b*d^5 + sqrt(-3)*(a*b^3*c^4*d - 6*a^2*b^2*c^2*d^3 + a^3*b*d^5) - (sqrt(
-3)*a^2*b^5*c + a^2*b^5*c)*sqrt(-(b^4*c^8 - 12*a*b^3*c^6*d^2 + 38*a^2*b^2*
c^4*d^4 - 12*a^3*b*c^2*d^6 + a^4*d^8)/(a^3*b^7)))*(-(4*b*c^3*d - 4*a*c*d^3
 + a*b^3*sqrt(-(b^4*c^8 - 12*a*b^3*c^6*d^2 + 38*a^2*b^2*c^4*d^4 - 12*a^3*b
*c^2*d^6 + a^4*d^8)/(a^3*b^7)))/(a*b^3))^(1/3)) + (sqrt(-3)*b - b)*(-(4*b*
c^3*d - 4*a*c*d^3 + a*b^3*sqrt(-(b^4*c^8 - 12*a*b^3*c^6*d^2 + 38*a^2*b^2*c
^4*d^4 - 12*a^3*b*c^2*d^6 + a^4*d^8)/(a^3*b^7)))/(a*b^3))^(1/3)*log((b^3*c
^6 - 5*a*b^2*c^4*d^2 - 5*a^2*b*c^2*d^4 + a^3*d^6)*(d*x + c)^(1/3) + 1/2*(a
*b^3*c^4*d - 6*a^2*b^2*c^2*d^3 + a^3*b*d^5 - sqrt(-3)*(a*b^3*c^4*d - 6*...
```

Sympy [F]

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx=\int \frac{(c+dx)^{\frac{4}{3}}}{a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(4/3)/(b*x**2+a),x)
```

output
```
Integral((c + d*x)**(4/3)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx = \int \frac{(dx+c)^{\frac{4}{3}}}{bx^2+a}\,dx$$

input
```
integrate((d*x+c)^(4/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(4/3)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx = \int \frac{(dx+c)^{\frac{4}{3}}}{bx^2+a}\,dx$$

input
```
integrate((d*x+c)^(4/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(4/3)/(b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 1906, normalized size of antiderivative = 3.05

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(4/3)/(a + b*x^2),x)
```

output
```
log((486*d^4*(c*(-a^3*b^7)^(1/2) + a^2*b^3*d)*((a^2*d^4*(-a^3*b^7)^(1/2) +
 b^2*c^4*(-a^3*b^7)^(1/2) - 4*a^2*b^5*c^3*d + 4*a^3*b^4*c*d^3 - 6*a*b*c^2*
d^2*(-a^3*b^7)^(1/2))/(a^3*b^7))^(1/3)*(a^3*d^6 + b^3*c^6 - 5*a*b^2*c^4*d^
2 - 5*a^2*b*c^2*d^4))/(a*b) - 486*b*d^4*(a*d^2 + b*c^2)^2*(c + d*x)^(1/3)*
(a^2*d^4 + b^2*c^4 - 6*a*b*c^2*d^2))*((a^2*d^4*(-a^3*b^7)^(1/2) + b^2*c^4*
(-a^3*b^7)^(1/2) - 4*a^2*b^5*c^3*d + 4*a^3*b^4*c*d^3 - 6*a*b*c^2*d^2*(-a^3
*b^7)^(1/2))/(8*a^3*b^7))^(1/3) + log(486*b*d^4*(a*d^2 + b*c^2)^2*(c + d*x
)^(1/3)*(a^2*d^4 + b^2*c^4 - 6*a*b*c^2*d^2) + (486*d^4*(c*(-a^3*b^7)^(1/2)
 - a^2*b^3*d)*(-(a^2*d^4*(-a^3*b^7)^(1/2) + b^2*c^4*(-a^3*b^7)^(1/2) + 4*a
^2*b^5*c^3*d - 4*a^3*b^4*c*d^3 - 6*a*b*c^2*d^2*(-a^3*b^7)^(1/2))/(a^3*b^7)
)^(1/3)*(a^3*d^6 + b^3*c^6 - 5*a*b^2*c^4*d^2 - 5*a^2*b*c^2*d^4))/(a*b))*(-
(a^2*d^4*(-a^3*b^7)^(1/2) + b^2*c^4*(-a^3*b^7)^(1/2) + 4*a^2*b^5*c^3*d - 4
*a^3*b^4*c*d^3 - 6*a*b*c^2*d^2*(-a^3*b^7)^(1/2))/(8*a^3*b^7))^(1/3) + log(
(486*d^4*((3^(1/2)*1i)/2 - 1/2)*(c*(-a^3*b^7)^(1/2) + a^2*b^3*d)*((a^2*d^4
*(-a^3*b^7)^(1/2) + b^2*c^4*(-a^3*b^7)^(1/2) - 4*a^2*b^5*c^3*d + 4*a^3*b^4
*c*d^3 - 6*a*b*c^2*d^2*(-a^3*b^7)^(1/2))/(a^3*b^7))^(1/3)*(a^3*d^6 + b^3*c
^6 - 5*a*b^2*c^4*d^2 - 5*a^2*b*c^2*d^4))/(a*b) - 486*b*d^4*(a*d^2 + b*c^2)
^2*(c + d*x)^(1/3)*(a^2*d^4 + b^2*c^4 - 6*a*b*c^2*d^2))*((3^(1/2)*1i)/2 -
1/2)*((a^2*d^4*(-a^3*b^7)^(1/2) + b^2*c^4*(-a^3*b^7)^(1/2) - 4*a^2*b^5*c^3
*d + 4*a^3*b^4*c*d^3 - 6*a*b*c^2*d^2*(-a^3*b^7)^(1/2))/(8*a^3*b^7))^(1/...
```

Reduce [F]

$$\int \frac{(c+dx)^{4/3}}{a+bx^2}\,dx = \frac{3(dx+c)^{\frac{1}{3}}\,d - \left(\int \frac{(dx+c)^{\frac{1}{3}}}{bd\,x^3+bc\,x^2+adx+ac}dx\right) a\,d^2 + \left(\int \frac{(dx+c)^{\frac{1}{3}}}{bd\,x^3+bc\,x^2+adx+ac}dx\right) b\,c^2 + 2\left(\int \frac{}{bd\,x}\right.}{b}$$

input
```
int((d*x+c)^(4/3)/(b*x^2+a),x)
```

output
```
(3*(c + d*x)**(1/3)*d - int((c + d*x)**(1/3)/(a*c + a*d*x + b*c*x**2 + b*d
*x**3),x)*a*d**2 + int((c + d*x)**(1/3)/(a*c + a*d*x + b*c*x**2 + b*d*x**3
),x)*b*c**2 + 2*int(((c + d*x)**(1/3)*x)/(a*c + a*d*x + b*c*x**2 + b*d*x**
3),x)*b*c*d)/b
```

3.203 $\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 441

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = -\frac{\sqrt{3}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}b^{5/6}} + \frac{\sqrt{3}\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}b^{5/6}} - \frac{\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}b^{5/6}} + \frac{\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}b^{5/6}} - \frac{3\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{5/6}} + \frac{3\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{5/6}}$$

output
```
-1/2*3^(1/2)*(b^(1/2)*c-(-a)^(1/2)*d)^(2/3)*arctan(1/3*(1+2*b^(1/6)*(d*x+c
)^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3))*3^(1/2))/(-a)^(1/2)/b^(5/6)+1/2*3^
(1/2)*(b^(1/2)*c+(-a)^(1/2)*d)^(2/3)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)
/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3))*3^(1/2))/(-a)^(1/2)/b^(5/6)-1/4*(b^(1/2)*
c+(-a)^(1/2)*d)^(2/3)*ln((-a)^(1/2)-b^(1/2)*x)/(-a)^(1/2)/b^(5/6)+1/4*(b^(
1/2)*c-(-a)^(1/2)*d)^(2/3)*ln((-a)^(1/2)+b^(1/2)*x)/(-a)^(1/2)/b^(5/6)-3/4
*(b^(1/2)*c-(-a)^(1/2)*d)^(2/3)*ln((b^(1/2)*c-(-a)^(1/2)*d)^(1/3)-b^(1/6)*
(d*x+c)^(1/3))/(-a)^(1/2)/b^(5/6)+3/4*(b^(1/2)*c+(-a)^(1/2)*d)^(2/3)*ln((b
^(1/2)*c+(-a)^(1/2)*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(5/6)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.15

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = \frac{d\text{RootSum}\left[bc^2+ad^2-2bc\#1^3+b\#1^6\&,\frac{\log\left(\sqrt[3]{c+dx}-\#1\right)\#1^2}{-c+\#1^3}\&\right]}{2b}$$

input
```
Integrate[(c + d*x)^(2/3)/(a + b*x^2),x]
```

output
```
(d*RootSum[b*c^2 + a*d^2 - 2*b*c*#1^3 + b*#1^6 & , (Log[(c + d*x)^(1/3) -
#1]*#1^2)/(-c + #1^3) & ])/(2*b)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 485}$$

$$\int \left(\frac{\sqrt{-a}(c+dx)^{2/3}}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}(c+dx)^{2/3}}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{3}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}b^{5/6}}+\frac{\sqrt{3}\left(\sqrt{-a}d+\sqrt{b}c\right)^{2/3}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}b^{5/6}}-\frac{\left(\sqrt{-a}d+\sqrt{b}c\right)^{2/3}\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}b^{5/6}}+\frac{\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}b^{5/6}}-\frac{3\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{5/6}}+\frac{3\left(\sqrt{-a}d+\sqrt{b}c\right)^{2/3}\log\left(\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{5/6}}$$

input
```
Int[(c + d*x)^(2/3)/(a + b*x^2),x]
```

output
```
-1/2*(Sqrt[3]*(Sqrt[b]*c - Sqrt[-a]*d)^(2/3)*ArcTan[(1 + (2*b^(1/6)*(c + d
*x)^(1/3))/(Sqrt[b]*c - Sqrt[-a]*d)^(1/3))/Sqrt[3]])/(Sqrt[-a]*b^(5/6)) +
(Sqrt[3]*(Sqrt[b]*c + Sqrt[-a]*d)^(2/3)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(
1/3))/(Sqrt[b]*c + Sqrt[-a]*d)^(1/3))/Sqrt[3]])/(2*Sqrt[-a]*b^(5/6)) - ((S
qrt[b]*c + Sqrt[-a]*d)^(2/3)*Log[Sqrt[-a] - Sqrt[b]*x])/(4*Sqrt[-a]*b^(5/6
)) + ((Sqrt[b]*c - Sqrt[-a]*d)^(2/3)*Log[Sqrt[-a] + Sqrt[b]*x])/(4*Sqrt[-a
]*b^(5/6)) - (3*(Sqrt[b]*c - Sqrt[-a]*d)^(2/3)*Log[(Sqrt[b]*c - Sqrt[-a]*d
)^(1/3) - b^(1/6)*(c + d*x)^(1/3)])/(4*Sqrt[-a]*b^(5/6)) + (3*(Sqrt[b]*c +
 Sqrt[-a]*d)^(2/3)*Log[(Sqrt[b]*c + Sqrt[-a]*d)^(1/3) - b^(1/6)*(c + d*x)^
(1/3)])/(4*Sqrt[-a]*b^(5/6))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{2}\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{3}-c}\right)}{2b}$	59
pseudoelliptic	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{2}\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{3}-c}\right)}{2b}$	59
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{4}\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}c}\right)}{2b}$	62

input
```
int((d*x+c)^(2/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*sum(_R^2*ln((d*x+c)^(1/3)-_R)/(_R^3-c),_R=RootOf(_Z^6*b-2*_Z^3*b*c+a
*d^2+b*c^2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(315) = 630$.

Time = 0.12 (sec) , antiderivative size = 1402, normalized size of antiderivative = 3.18

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(2/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(-3) - 1)*(-(a*b^2*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3
*b^5)) + 2*c*d)/(a*b^2))^(1/3)*log(1/2*(a*b^3*c^3 - a^2*b^2*c*d^2 + sqrt(-
3)*(a*b^3*c^3 - a^2*b^2*c*d^2) + (sqrt(-3)*a^3*b^4*d + a^3*b^4*d)*sqrt(-(b
^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^5)))*(-(a*b^2*sqrt(-(b^2*c^4 - 2*
a*b*c^2*d^2 + a^2*d^4)/(a^3*b^5)) + 2*c*d)/(a*b^2))^(2/3) - (b^2*c^4 - a^2
*d^4)*(d*x + c)^(1/3)) - 1/4*(sqrt(-3) + 1)*(-(a*b^2*sqrt(-(b^2*c^4 - 2*a*
b*c^2*d^2 + a^2*d^4)/(a^3*b^5)) + 2*c*d)/(a*b^2))^(1/3)*log(1/2*(a*b^3*c^3
 - a^2*b^2*c*d^2 - sqrt(-3)*(a*b^3*c^3 - a^2*b^2*c*d^2) - (sqrt(-3)*a^3*b^
4*d - a^3*b^4*d)*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^5)))*(-(
a*b^2*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^5)) + 2*c*d)/(a*b^2
))^(2/3) - (b^2*c^4 - a^2*d^4)*(d*x + c)^(1/3)) + 1/4*(sqrt(-3) - 1)*((a*b
^2*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^5)) - 2*c*d)/(a*b^2))^
(1/3)*log(1/2*(a*b^3*c^3 - a^2*b^2*c*d^2 + sqrt(-3)*(a*b^3*c^3 - a^2*b^2*c
*d^2) - (sqrt(-3)*a^3*b^4*d + a^3*b^4*d)*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 +
a^2*d^4)/(a^3*b^5)))*((a*b^2*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^
3*b^5)) - 2*c*d)/(a*b^2))^(2/3) - (b^2*c^4 - a^2*d^4)*(d*x + c)^(1/3)) - 1
/4*(sqrt(-3) + 1)*((a*b^2*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b
^5)) - 2*c*d)/(a*b^2))^(1/3)*log(1/2*(a*b^3*c^3 - a^2*b^2*c*d^2 - sqrt(-3)
*(a*b^3*c^3 - a^2*b^2*c*d^2) + (sqrt(-3)*a^3*b^4*d - a^3*b^4*d)*sqrt(-(b^2
*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^5)))*((a*b^2*sqrt(-(b^2*c^4 - 2*...
```

Sympy [F]

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = \int \frac{(c+dx)^{\frac{2}{3}}}{a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(2/3)/(b*x**2+a),x)
```

output
```
Integral((c + d*x)**(2/3)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = \int \frac{(dx+c)^{\frac{2}{3}}}{bx^2+a}\,dx$$

input
```
integrate((d*x+c)^(2/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(2/3)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = \int \frac{(dx+c)^{\frac{2}{3}}}{bx^2+a}\,dx$$

input
```
integrate((d*x+c)^(2/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2369, normalized size of antiderivative = 5.37

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(2/3)/(a + b*x^2),x)
```

output
```
log((((c + d*x)^(1/3)*(1944*a^3*b^4*d^8 + 1944*a*b^6*c^4*d^4 + 3888*a^2*b^
5*c^2*d^6) + (7776*a^3*b^6*c*d^6 + 7776*a^2*b^7*c^3*d^4)*(-(a*d^2*(-a^3*b^
5)^(1/2) - b*c^2*(-a^3*b^5)^(1/2) + 2*a^2*b^3*c*d)/(8*a^3*b^5))^(2/3))*(-(
a*d^2*(-a^3*b^5)^(1/2) - b*c^2*(-a^3*b^5)^(1/2) + 2*a^2*b^3*c*d)/(8*a^3*b^
5))^(1/3) - 972*a^3*b^3*d^9 + 2916*a*b^5*c^4*d^5 + 1944*a^2*b^4*c^2*d^7)*(
-(a*d^2*(-a^3*b^5)^(1/2) - b*c^2*(-a^3*b^5)^(1/2) + 2*a^2*b^3*c*d)/(8*a^3*
b^5))^(2/3) + (c + d*x)^(1/3)*(486*b^4*c^5*d^5 + 972*a*b^3*c^3*d^7 + 486*a
^2*b^2*c*d^9))*(-(a*d^2*(-a^3*b^5)^(1/2) - b*c^2*(-a^3*b^5)^(1/2) + 2*a^2*
b^3*c*d)/(8*a^3*b^5))^(1/3) + log((((c + d*x)^(1/3)*(1944*a^3*b^4*d^8 + 19
44*a*b^6*c^4*d^4 + 3888*a^2*b^5*c^2*d^6) + (7776*a^3*b^6*c*d^6 + 7776*a^2*
b^7*c^3*d^4)*(-(b*c^2*(-a^3*b^5)^(1/2) - a*d^2*(-a^3*b^5)^(1/2) + 2*a^2*b^
3*c*d)/(8*a^3*b^5))^(2/3))*(-(b*c^2*(-a^3*b^5)^(1/2) - a*d^2*(-a^3*b^5)^(1
/2) + 2*a^2*b^3*c*d)/(8*a^3*b^5))^(1/3) - 972*a^3*b^3*d^9 + 2916*a*b^5*c^4
*d^5 + 1944*a^2*b^4*c^2*d^7)*(-(b*c^2*(-a^3*b^5)^(1/2) - a*d^2*(-a^3*b^5)^
(1/2) + 2*a^2*b^3*c*d)/(8*a^3*b^5))^(2/3) + (c + d*x)^(1/3)*(486*b^4*c^5*d
^5 + 972*a*b^3*c^3*d^7 + 486*a^2*b^2*c*d^9))*(-(b*c^2*(-a^3*b^5)^(1/2) - a
*d^2*(-a^3*b^5)^(1/2) + 2*a^2*b^3*c*d)/(8*a^3*b^5))^(1/3) + log((c + d*x)^
(1/3)*(486*b^4*c^5*d^5 + 972*a*b^3*c^3*d^7 + 486*a^2*b^2*c*d^9) + ((3^(1/2
)*1i)/2 - 1/2)^2*(-(a*d^2*(-a^3*b^5)^(1/2) - b*c^2*(-a^3*b^5)^(1/2) + 2*a^
2*b^3*c*d)/(8*a^3*b^5))^(2/3)*(((c + d*x)^(1/3)*(1944*a^3*b^4*d^8 + 194...
```

Reduce [F]

$$\int \frac{(c+dx)^{2/3}}{a+bx^2}\,dx = \int \frac{(dx+c)^{\frac{2}{3}}}{b\,x^2+a}dx$$

input
```
int((d*x+c)^(2/3)/(b*x^2+a),x)
```

output
```
int((c + d*x)**(2/3)/(a + b*x**2),x)
```

3.204 $\int \frac{\sqrt[3]{c+dx}}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 441

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx = \frac{\sqrt{3}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}b^{2/3}} - \frac{\sqrt{3}\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}b^{2/3}} - \frac{\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}b^{2/3}} + \frac{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}b^{2/3}} - \frac{3\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{2/3}} + \frac{3\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}\log\left(\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{2/3}}$$

output
```
1/2*3^(1/2)*(b^(1/2)*c-(-a)^(1/2)*d)^(1/3)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)
^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3))*3^(1/2))/(-a)^(1/2)/b^(2/3)-1/2*3^(
1/2)*(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/
(b^(1/2)*c+(-a)^(1/2)*d)^(1/3))*3^(1/2))/(-a)^(1/2)/b^(2/3)-1/4*(b^(1/2)*c
+(-a)^(1/2)*d)^(1/3)*ln((-a)^(1/2)-b^(1/2)*x)/(-a)^(1/2)/b^(2/3)+1/4*(b^(1
/2)*c-(-a)^(1/2)*d)^(1/3)*ln((-a)^(1/2)+b^(1/2)*x)/(-a)^(1/2)/b^(2/3)-3/4*
(b^(1/2)*c-(-a)^(1/2)*d)^(1/3)*ln((b^(1/2)*c-(-a)^(1/2)*d)^(1/3)-b^(1/6)*(
d*x+c)^(1/3))/(-a)^(1/2)/b^(2/3)+3/4*(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)*ln((b^
(1/2)*c+(-a)^(1/2)*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx=\frac{d\text{RootSum}\left[bc^2+ad^2-2bc\#1^3+b\#1^6\&,\frac{\log\left(\sqrt[3]{c+dx}-\#1\right)\#1}{-c+\#1^3}\&\right]}{2b}$$

input
```
Integrate[(c + d*x)^(1/3)/(a + b*x^2),x]
```

output
```
(d*RootSum[b*c^2 + a*d^2 - 2*b*c*#1^3 + b*#1^6 & , (Log[(c + d*x)^(1/3) -
#1]*#1)/(-c + #1^3) & ])/(2*b)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{485}$$

$$\int \left(\frac{\sqrt{-a}\sqrt[3]{c+dx}}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sqrt[3]{c+dx}}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}b^{2/3}}-\frac{\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}b^{2/3}}+\frac{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}b^{2/3}}-\frac{3\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{2/3}}+\frac{3\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}\log\left(\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}b^{2/3}}$$

input
```
Int[(c + d*x)^(1/3)/(a + b*x^2),x]
```

output
```
(Sqrt[3]*(Sqrt[b]*c - Sqrt[-a]*d)^(1/3)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(
1/3))/(Sqrt[b]*c - Sqrt[-a]*d)^(1/3))/Sqrt[3]])/(2*Sqrt[-a]*b^(2/3)) - (Sq
rt[3]*(Sqrt[b]*c + Sqrt[-a]*d)^(1/3)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3
))/(Sqrt[b]*c + Sqrt[-a]*d)^(1/3))/Sqrt[3]])/(2*Sqrt[-a]*b^(2/3)) - ((Sqrt
[b]*c + Sqrt[-a]*d)^(1/3)*Log[Sqrt[-a] - Sqrt[b]*x])/(4*Sqrt[-a]*b^(2/3))
 + ((Sqrt[b]*c - Sqrt[-a]*d)^(1/3)*Log[Sqrt[-a] + Sqrt[b]*x])/(4*Sqrt[-a]*b
^(2/3)) - (3*(Sqrt[b]*c - Sqrt[-a]*d)^(1/3)*Log[(Sqrt[b]*c - Sqrt[-a]*d)^(
1/3) - b^(1/6)*(c + d*x)^(1/3)])/(4*Sqrt[-a]*b^(2/3)) + (3*(Sqrt[b]*c + Sq
rt[-a]*d)^(1/3)*Log[(Sqrt[b]*c + Sqrt[-a]*d)^(1/3) - b^(1/6)*(c + d*x)^(1/
3)])/(4*Sqrt[-a]*b^(2/3))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\_R\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{3}-c}\right)}{2b}$	57
pseudoelliptic	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\_R\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{3}-c}\right)}{2b}$	57
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{3}\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}c}\right)}{2b}$	62

input
```
int((d*x+c)^(1/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*sum(_R*ln((d*x+c)^(1/3)-_R)/(_R^3-c),_R=RootOf(_Z^6*b-2*_Z^3*b*c+a*d
^2+b*c^2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(315) = 630$.

Time = 0.08 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-3) + 1)*(-(a*b^2*sqrt(-c^2/(a^3*b^3)) + d)/(a*b^2))^(1/3)*log(
-1/2*(sqrt(-3)*a^2*b^2 + a^2*b^2)*(-(a*b^2*sqrt(-c^2/(a^3*b^3)) + d)/(a*b^
2))^(1/3)*sqrt(-c^2/(a^3*b^3)) + (d*x + c)^(1/3)*c) + 1/4*(sqrt(-3) - 1)*(
-(a*b^2*sqrt(-c^2/(a^3*b^3)) + d)/(a*b^2))^(1/3)*log(1/2*(sqrt(-3)*a^2*b^2
 - a^2*b^2)*(-(a*b^2*sqrt(-c^2/(a^3*b^3)) + d)/(a*b^2))^(1/3)*sqrt(-c^2/(a
^3*b^3)) + (d*x + c)^(1/3)*c) - 1/4*(sqrt(-3) + 1)*((a*b^2*sqrt(-c^2/(a^3*
b^3)) - d)/(a*b^2))^(1/3)*log(1/2*(sqrt(-3)*a^2*b^2 + a^2*b^2)*((a*b^2*sqr
t(-c^2/(a^3*b^3)) - d)/(a*b^2))^(1/3)*sqrt(-c^2/(a^3*b^3)) + (d*x + c)^(1/
3)*c) + 1/4*(sqrt(-3) - 1)*((a*b^2*sqrt(-c^2/(a^3*b^3)) - d)/(a*b^2))^(1/3
)*log(-1/2*(sqrt(-3)*a^2*b^2 - a^2*b^2)*((a*b^2*sqrt(-c^2/(a^3*b^3)) - d)/
(a*b^2))^(1/3)*sqrt(-c^2/(a^3*b^3)) + (d*x + c)^(1/3)*c) + 1/2*(-(a*b^2*sq
rt(-c^2/(a^3*b^3)) + d)/(a*b^2))^(1/3)*log(a^2*b^2*(-(a*b^2*sqrt(-c^2/(a^3
*b^3)) + d)/(a*b^2))^(1/3)*sqrt(-c^2/(a^3*b^3)) + (d*x + c)^(1/3)*c) + 1/2
*((a*b^2*sqrt(-c^2/(a^3*b^3)) - d)/(a*b^2))^(1/3)*log(-a^2*b^2*((a*b^2*sqr
t(-c^2/(a^3*b^3)) - d)/(a*b^2))^(1/3)*sqrt(-c^2/(a^3*b^3)) + (d*x + c)^(1/
3)*c)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx = \int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(1/3)/(b*x**2+a),x)
```

output
```
Integral((c + d*x)**(1/3)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx = \int \frac{(dx+c)^{\frac{1}{3}}}{bx^2+a}\,dx$$

input
```
integrate((d*x+c)^(1/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(1/3)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx = \int \frac{(dx+c)^{\frac{1}{3}}}{bx^2+a}\,dx$$

input
```
integrate((d*x+c)^(1/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1676, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(1/3)/(a + b*x^2),x)
```

output
```
log((486*a^2*b^3*d^8 - 486*b^5*c^4*d^4)*(c + d*x)^(1/3) + (((7776*a^3*b^6*
c*d^6 + 7776*a^2*b^7*c^3*d^4)*(-(c*(-a^3*b^5)^(1/2) + a^2*b^2*d)/(8*a^3*b^
4))^(1/3) + (3888*a^3*b^5*d^7 + 3888*a^2*b^6*c^2*d^5)*(c + d*x)^(1/3))*(-(
c*(-a^3*b^5)^(1/2) + a^2*b^2*d)/(8*a^3*b^4))^(2/3) + 1944*a*b^5*c^3*d^5 +
1944*a^2*b^4*c*d^7)*(-(c*(-a^3*b^5)^(1/2) + a^2*b^2*d)/(8*a^3*b^4))^(1/3))
*(-(c*(-a^3*b^5)^(1/2) + a^2*b^2*d)/(8*a^3*b^4))^(1/3) + log((486*a^2*b^3*
d^8 - 486*b^5*c^4*d^4)*(c + d*x)^(1/3) + (((7776*a^3*b^6*c*d^6 + 7776*a^2*
b^7*c^3*d^4)*((c*(-a^3*b^5)^(1/2) - a^2*b^2*d)/(8*a^3*b^4))^(1/3) + (3888*
a^3*b^5*d^7 + 3888*a^2*b^6*c^2*d^5)*(c + d*x)^(1/3))*((c*(-a^3*b^5)^(1/2)
- a^2*b^2*d)/(8*a^3*b^4))^(2/3) + 1944*a*b^5*c^3*d^5 + 1944*a^2*b^4*c*d^7)
*((c*(-a^3*b^5)^(1/2) - a^2*b^2*d)/(8*a^3*b^4))^(1/3))*((c*(-a^3*b^5)^(1/2
) - a^2*b^2*d)/(8*a^3*b^4))^(1/3) + log((486*a^2*b^3*d^8 - 486*b^5*c^4*d^4
)*(c + d*x)^(1/3) + ((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((388
8*a^3*b^5*d^7 + 3888*a^2*b^6*c^2*d^5)*(c + d*x)^(1/3) + ((3^(1/2)*1i)/2 -
1/2)*(7776*a^3*b^6*c*d^6 + 7776*a^2*b^7*c^3*d^4)*(-(c*(-a^3*b^5)^(1/2) + a
^2*b^2*d)/(8*a^3*b^4))^(1/3))*(-(c*(-a^3*b^5)^(1/2) + a^2*b^2*d)/(8*a^3*b^
4))^(2/3) + 1944*a*b^5*c^3*d^5 + 1944*a^2*b^4*c*d^7)*(-(c*(-a^3*b^5)^(1/2)
 + a^2*b^2*d)/(8*a^3*b^4))^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(-(c*(-a^3*b^5)^(
1/2) + a^2*b^2*d)/(8*a^3*b^4))^(1/3) - log((486*a^2*b^3*d^8 - 486*b^5*c^4*
d^4)*(c + d*x)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 + 1/2)^2...
```

Reduce [F]

$$\int \frac{\sqrt[3]{c+dx}}{a+bx^2}\,dx=\int \frac{(dx+c)^{\frac{1}{3}}}{b\,x^2+a}dx$$

input
```
int((d*x+c)^(1/3)/(b*x^2+a),x)
```

output
```
int((c + d*x)**(1/3)/(a + b*x**2),x)
```

3.205 $\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 441

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}} + \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}} - \frac{\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}} + \frac{\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}} - \frac{3\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}} + \frac{3\log\left(\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}}$$

output

```
-1/2*3^(1/2)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d
)^(1/3))*3^(1/2))/(-a)^(1/2)/b^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3)+1/2*3^
(1/2)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)
)*3^(1/2))/(-a)^(1/2)/b^(1/3)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)-1/4*ln((-a)^(
1/2)-b^(1/2)*x)/(-a)^(1/2)/b^(1/3)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)+1/4*ln((
-a)^(1/2)+b^(1/2)*x)/(-a)^(1/2)/b^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3)-3/4
*ln((b^(1/2)*c-(-a)^(1/2)*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(1/
3)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3)+3/4*ln((b^(1/2)*c+(-a)^(1/2)*d)^(1/3)-b^
(1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(1/3)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.15

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx = -\frac{d\text{RootSum}\left[bc^2+ad^2-2bc\#1^3+b\#1^6\&,\frac{\log\left(\sqrt[3]{c+dx}-\#1\right)}{c\#1-\#1^4}\&\right]}{2b}$$

input
```
Integrate[1/((c + d*x)^(1/3)*(a + b*x^2)),x]
```

output
```
-1/2*(d*RootSum[b*c^2 + a*d^2 - 2*b*c*#1^3 + b*#1^6 & , Log[(c + d*x)^(1/3
) - #1]/(c*#1 - #1^4) & ])/b
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)\sqrt[3]{c+dx}}\,dx$$

$$\Big\downarrow \text{485}$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{b}x\right)\sqrt[3]{c+dx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{b}x\right)\sqrt[3]{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}-\frac{\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+\frac{\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}-\frac{3\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+\frac{3\log\left(\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[3]{b}\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}$$

input
```
Int[1/((c + d*x)^(1/3)*(a + b*x^2)),x]
```

output
```
-1/2*(Sqrt[3]*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3))/(Sqrt[b]*c - Sqrt[-a
]*d)^(1/3))/Sqrt[3]])/(Sqrt[-a]*b^(1/3)*(Sqrt[b]*c - Sqrt[-a]*d)^(1/3)) +
(Sqrt[3]*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3))/(Sqrt[b]*c + Sqrt[-a]*d)^
(1/3))/Sqrt[3]])/(2*Sqrt[-a]*b^(1/3)*(Sqrt[b]*c + Sqrt[-a]*d)^(1/3)) - Log
[Sqrt[-a] - Sqrt[b]*x]/(4*Sqrt[-a]*b^(1/3)*(Sqrt[b]*c + Sqrt[-a]*d)^(1/3))
 + Log[Sqrt[-a] + Sqrt[b]*x]/(4*Sqrt[-a]*b^(1/3)*(Sqrt[b]*c - Sqrt[-a]*d)^
(1/3)) - (3*Log[(Sqrt[b]*c - Sqrt[-a]*d)^(1/3) - b^(1/6)*(c + d*x)^(1/3)])
/(4*Sqrt[-a]*b^(1/3)*(Sqrt[b]*c - Sqrt[-a]*d)^(1/3)) + (3*Log[(Sqrt[b]*c +
 Sqrt[-a]*d)^(1/3) - b^(1/6)*(c + d*x)^(1/3)])/(4*Sqrt[-a]*b^(1/3)*(Sqrt[b
]*c + Sqrt[-a]*d)^(1/3))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R\left(\_R^{3}-c\right)}\right)}{2b}$	59
pseudoelliptic	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R\left(\_R^{3}-c\right)}\right)}{2b}$	59
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\_R\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}c}\right)}{2b}$	60

input
```
int(1/(d*x+c)^(1/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*sum(1/_R*ln((d*x+c)^(1/3)-_R)/(_R^3-c),_R=RootOf(_Z^6*b-2*_Z^3*b*c+a
*d^2+b*c^2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1694 vs. 2(315) = 630.

Time = 0.10 (sec) , antiderivative size = 1694, normalized size of antiderivative = 3.84

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(1/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(-3) - 1)*(((a*b^2*c^2 + a^2*b*d^2)*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^
4*b^2*c^2*d^2 + a^5*b*d^4)) + d)/(a*b^2*c^2 + a^2*b*d^2))^(1/3)*log(-1/2*(
sqrt(-3)*a*b*c^2 + a*b*c^2 + (a^3*b^2*c^2*d + a^4*b*d^3 + sqrt(-3)*(a^3*b^
2*c^2*d + a^4*b*d^3))*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^2 + a^5*b*d
^4)))*(((a*b^2*c^2 + a^2*b*d^2)*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^2
 + a^5*b*d^4)) + d)/(a*b^2*c^2 + a^2*b*d^2))^(2/3) + (d*x + c)^(1/3)*c) -
1/4*(sqrt(-3) + 1)*(((a*b^2*c^2 + a^2*b*d^2)*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^
4*b^2*c^2*d^2 + a^5*b*d^4)) + d)/(a*b^2*c^2 + a^2*b*d^2))^(1/3)*log(1/2*(s
qrt(-3)*a*b*c^2 - a*b*c^2 - (a^3*b^2*c^2*d + a^4*b*d^3 - sqrt(-3)*(a^3*b^2
*c^2*d + a^4*b*d^3))*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^2 + a^5*b*d^
4)))*(((a*b^2*c^2 + a^2*b*d^2)*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^2
+ a^5*b*d^4)) + d)/(a*b^2*c^2 + a^2*b*d^2))^(2/3) + (d*x + c)^(1/3)*c) + 1
/4*(sqrt(-3) - 1)*(-((a*b^2*c^2 + a^2*b*d^2)*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^
4*b^2*c^2*d^2 + a^5*b*d^4)) - d)/(a*b^2*c^2 + a^2*b*d^2))^(1/3)*log(-1/2*(
sqrt(-3)*a*b*c^2 + a*b*c^2 - (a^3*b^2*c^2*d + a^4*b*d^3 + sqrt(-3)*(a^3*b^
2*c^2*d + a^4*b*d^3))*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^2 + a^5*b*d
^4)))*(-((a*b^2*c^2 + a^2*b*d^2)*sqrt(-c^2/(a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^
2 + a^5*b*d^4)) - d)/(a*b^2*c^2 + a^2*b*d^2))^(2/3) + (d*x + c)^(1/3)*c) -
 1/4*(sqrt(-3) + 1)*(-((a*b^2*c^2 + a^2*b*d^2)*sqrt(-c^2/(a^3*b^3*c^4 + 2*
a^4*b^2*c^2*d^2 + a^5*b*d^4)) - d)/(a*b^2*c^2 + a^2*b*d^2))^(1/3)*log(1...
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx=\int \frac{1}{(a+bx^2)\,\sqrt[3]{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/3)/(b*x**2+a),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx = \int \frac{1}{(bx^2+a)(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x + c)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx = \int \frac{1}{(bx^2+a)(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x)^(1/3)),x)
```

output
```
log(243*b^4*d^5*(c + d*x)^(1/3) - ((1944*a*b^5*d^4*(a*d^2 - b*c^2)*(c + d*
x)^(1/3) - 1944*a^2*b^6*c*d^4*(a*d^2 + b*c^2)*((c*(-a^3*b^3)^(1/2) + a^2*b
*d)/(a^3*b^2*(a*d^2 + b*c^2)))^(2/3))*(c*(-a^3*b^3)^(1/2) + a^2*b*d))/(8*a
^3*b^2*(a*d^2 + b*c^2)))*((c*(-a^3*b^3)^(1/2) + a^2*b*d)/(8*(a^3*b^3*c^2 +
 a^4*b^2*d^2)))^(1/3) + log(243*b^4*d^5*(c + d*x)^(1/3) + ((1944*a*b^5*d^4
*(a*d^2 - b*c^2)*(c + d*x)^(1/3) - 1944*a^2*b^6*c*d^4*(a*d^2 + b*c^2)*(-(c
*(-a^3*b^3)^(1/2) - a^2*b*d)/(a^3*b^2*(a*d^2 + b*c^2)))^(2/3))*(c*(-a^3*b^
3)^(1/2) - a^2*b*d))/(8*a^3*b^2*(a*d^2 + b*c^2)))*(-(c*(-a^3*b^3)^(1/2) -
a^2*b*d)/(8*(a^3*b^3*c^2 + a^4*b^2*d^2)))^(1/3) - log(243*b^4*d^5*(c + d*x
)^(1/3) - ((1944*a*b^5*d^4*(a*d^2 - b*c^2)*(c + d*x)^(1/3) - 1944*a^2*b^6*
c*d^4*((3^(1/2)*1i)/2 - 1/2)*(a*d^2 + b*c^2)*((c*(-a^3*b^3)^(1/2) + a^2*b*
d)/(a^3*b^2*(a*d^2 + b*c^2)))^(2/3))*(c*(-a^3*b^3)^(1/2) + a^2*b*d))/(8*a^
3*b^2*(a*d^2 + b*c^2)))*((3^(1/2)*1i)/2 + 1/2)*((c*(-a^3*b^3)^(1/2) + a^2*
b*d)/(8*(a^3*b^3*c^2 + a^4*b^2*d^2)))^(1/3) + log(243*b^4*d^5*(c + d*x)^(1
/3) - ((1944*a*b^5*d^4*(a*d^2 - b*c^2)*(c + d*x)^(1/3) + 1944*a^2*b^6*c*d^
4*((3^(1/2)*1i)/2 + 1/2)*(a*d^2 + b*c^2)*((c*(-a^3*b^3)^(1/2) + a^2*b*d)/(
a^3*b^2*(a*d^2 + b*c^2)))^(2/3))*(c*(-a^3*b^3)^(1/2) + a^2*b*d))/(8*a^3*b^
2*(a*d^2 + b*c^2)))*((3^(1/2)*1i)/2 - 1/2)*((c*(-a^3*b^3)^(1/2) + a^2*b*d)
/(8*(a^3*b^3*c^2 + a^4*b^2*d^2)))^(1/3) - log(243*b^4*d^5*(c + d*x)^(1/3)
+ ((1944*a*b^5*d^4*(a*d^2 - b*c^2)*(c + d*x)^(1/3) - 1944*a^2*b^6*c*d^4...
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(a+bx^2)}\,dx=\int \frac{1}{(dx+c)^{\frac{1}{3}}\,a+(dx+c)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(1/(d*x+c)^(1/3)/(b*x^2+a),x)
```

output
```
int(1/((c + d*x)**(1/3)*a + (c + d*x)**(1/3)*b*x**2),x)
```

3.206 $\int \frac{1}{(c+dx)^{2/3}(a+bx^2)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 441

$$\begin{aligned}\int \frac{1}{(c+dx)^{2/3}(a+bx^2)}\,dx &= \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}} \\ &- \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}} - \frac{\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}} \\ &+ \frac{\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}} - \frac{3\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}} \\ &+ \frac{3\log\left(\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c+\sqrt{-a}d\right)^{2/3}}\end{aligned}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d)
^(1/3))*3^(1/2))/(-a)^(1/2)/b^(1/6)/(b^(1/2)*c-(-a)^(1/2)*d)^(2/3)-1/2*3^(
1/2)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3))
*3^(1/2))/(-a)^(1/2)/b^(1/6)/(b^(1/2)*c+(-a)^(1/2)*d)^(2/3)-1/4*ln((-a)^(1
/2)-b^(1/2)*x)/(-a)^(1/2)/b^(1/6)/(b^(1/2)*c+(-a)^(1/2)*d)^(2/3)+1/4*ln((-
a)^(1/2)+b^(1/2)*x)/(-a)^(1/2)/b^(1/6)/(b^(1/2)*c-(-a)^(1/2)*d)^(2/3)-3/4*
ln((b^(1/2)*c-(-a)^(1/2)*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(1/6
)/(b^(1/2)*c-(-a)^(1/2)*d)^(2/3)+3/4*ln((b^(1/2)*c+(-a)^(1/2)*d)^(1/3)-b^(
1/6)*(d*x+c)^(1/3))/(-a)^(1/2)/b^(1/6)/(b^(1/2)*c+(-a)^(1/2)*d)^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.15

$$\int \frac{1}{(c+dx)^{2/3}(a+bx^2)}\,dx = -\frac{d\text{RootSum}\left[bc^2+ad^2-2bc\#1^3+b\#1^6\&,\frac{\log\left(\sqrt[3]{c+dx}-\#1\right)}{c\#1^2-\#1^5}\&\right]}{2b}$$

input
```
Integrate[1/((c + d*x)^(2/3)*(a + b*x^2)),x]
```

output
```
-1/2*(d*RootSum[b*c^2 + a*d^2 - 2*b*c*#1^3 + b*#1^6 & , Log[(c + d*x)^(1/3
) - #1]/(c*#1^2 - #1^5) & ])/b
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx)^{2/3}}\,dx$$
$$\Big\downarrow \text{485}$$
$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{b}x\right)(c+dx)^{2/3}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{b}x\right)(c+dx)^{2/3}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[6]{b}\left(\sqrt{-a}d+\sqrt{b}c\right)^{2/3}}-\frac{\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{-a}d+\sqrt{b}c\right)^{2/3}}+\frac{\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}}-\frac{3\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{b}c-\sqrt{-a}d\right)^{2/3}}+\frac{3\log\left(\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4\sqrt{-a}\sqrt[6]{b}\left(\sqrt{-a}d+\sqrt{b}c\right)^{2/3}}$$

input
```
Int[1/((c + d*x)^(2/3)*(a + b*x^2)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3))/(Sqrt[b]*c - Sqrt[-a]*d)^
(1/3))/Sqrt[3]])/(2*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(2/3)) - (Sq
rt[3]*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3))/(Sqrt[b]*c + Sqrt[-a]*d)^(1/
3))/Sqrt[3]])/(2*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c + Sqrt[-a]*d)^(2/3)) - Log[Sq
rt[-a] - Sqrt[b]*x]/(4*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c + Sqrt[-a]*d)^(2/3)) +
Log[Sqrt[-a] + Sqrt[b]*x]/(4*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(2/
3)) - (3*Log[(Sqrt[b]*c - Sqrt[-a]*d)^(1/3) - b^(1/6)*(c + d*x)^(1/3)])/(4
*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(2/3)) + (3*Log[(Sqrt[b]*c + Sq
rt[-a]*d)^(1/3) - b^(1/6)*(c + d*x)^(1/3)])/(4*Sqrt[-a]*b^(1/6)*(Sqrt[b]*c
 + Sqrt[-a]*d)^(2/3))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.13

method	result	size
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}c}\right)}{2b}$	59
default	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{2}\left(\_R^{3}-c\right)}\right)}{2b}$	59
pseudoelliptic	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{6}-2bc\_Z^{3}+a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^{2}\left(\_R^{3}-c\right)}\right)}{2b}$	59

input
```
int(1/(d*x+c)^(2/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d/b*sum(1/(_R^5-_R^2*c)*ln((d*x+c)^(1/3)-_R),_R=RootOf(_Z^6*b-2*_Z^3*b
*c+a*d^2+b*c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3194 vs. 2(315) = 630.

Time = 0.22 (sec) , antiderivative size = 3194, normalized size of antiderivative = 7.24

$$\int \frac{1}{(c+dx)^{2/3}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(2/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{2/3}\,(a+bx^2)}\,dx = \int \frac{1}{(a+bx^2)\,(c+dx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(2/3)/(b*x**2+a),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{2/3}\,(a+bx^2)}\,dx = \int \frac{1}{(bx^2+a)(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(2/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x + c)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{2/3}\,(a+bx^2)}\,dx = \int \frac{1}{(bx^2+a)(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(2/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*(d*x + c)^(2/3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 2435, normalized size of antiderivative = 5.52

$$\int \frac{1}{(c+dx)^{2/3}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x)^(2/3)),x)
```

output
```
log((((7776*a^3*b^6*c*d^6 + 7776*a^2*b^7*c^3*d^4)*((a*d^2*(-a^3*b)^(1/2) -
 b*c^2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3*b^3*c^4 + 2*a^4*b
^2*c^2*d^2)))^(1/3) + 7776*a^2*b^6*c*d^5*(c + d*x)^(1/3))*((a*d^2*(-a^3*b)
^(1/2) - b*c^2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3*b^3*c^4 +
 2*a^4*b^2*c^2*d^2)))^(2/3) - 972*a*b^5*d^5)*((a*d^2*(-a^3*b)^(1/2) - b*c^
2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3*b^3*c^4 + 2*a^4*b^2*c^
2*d^2)))^(1/3) - 486*b^5*d^4*(c + d*x)^(1/3))*((a*d^2*(-a^3*b)^(1/2) - b*c
^2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3*b^3*c^4 + 2*a^4*b^2*c
^2*d^2)))^(1/3) + log((((7776*a^3*b^6*c*d^6 + 7776*a^2*b^7*c^3*d^4)*((b*c^
2*(-a^3*b)^(1/2) - a*d^2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3
*b^3*c^4 + 2*a^4*b^2*c^2*d^2)))^(1/3) + 7776*a^2*b^6*c*d^5*(c + d*x)^(1/3)
)*((b*c^2*(-a^3*b)^(1/2) - a*d^2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d
^4 + a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^2)))^(2/3) - 972*a*b^5*d^5)*((b*c^2*(-a
^3*b)^(1/2) - a*d^2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3*b^3*
c^4 + 2*a^4*b^2*c^2*d^2)))^(1/3) - 486*b^5*d^4*(c + d*x)^(1/3))*((b*c^2*(-
a^3*b)^(1/2) - a*d^2*(-a^3*b)^(1/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3*b^3
*c^4 + 2*a^4*b^2*c^2*d^2)))^(1/3) + log(486*b^5*d^4*(c + d*x)^(1/3) + (972
*a*b^5*d^5 - ((3^(1/2)*1i)/2 - 1/2)^2*(((3^(1/2)*1i)/2 - 1/2)*(7776*a^3*b^
6*c*d^6 + 7776*a^2*b^7*c^3*d^4)*((a*d^2*(-a^3*b)^(1/2) - b*c^2*(-a^3*b)^(1
/2) + 2*a^2*b*c*d)/(8*(a^5*b*d^4 + a^3*b^3*c^4 + 2*a^4*b^2*c^2*d^2)))^(...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{2/3}\,(a+bx^2)}\,dx = \int \frac{1}{(dx+c)^{\frac{2}{3}}\,a+(dx+c)^{\frac{2}{3}}\,b\,x^2}dx$$

input
```
int(1/(d*x+c)^(2/3)/(b*x^2+a),x)
```

output
```
int(1/((c + d*x)**(2/3)*a + (c + d*x)**(2/3)*b*x**2),x)
```

3.207 $\int \frac{1}{(c+dx)^{4/3}(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 633

$$\int \frac{1}{(c+dx)^{4/3}(a+bx^2)}\,dx = -\frac{3d}{(bc^2+ad^2)\sqrt[3]{c+dx}} - \frac{\sqrt{3}\sqrt[6]{b}\left(\sqrt{b}c+\sqrt{-a}d\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}}{\sqrt{3}}\right)}{2\sqrt{-a}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\,(bc^2+ad^2)} - \frac{\sqrt{3}\sqrt[6]{b}\left(\sqrt{-a}\sqrt{b}c+ad\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}}}{\sqrt{3}}\right)}{2a\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}\,(bc^2+ad^2)} + \frac{\sqrt[6]{b}\left(\sqrt{-a}\sqrt{b}c+ad\right)\log\left(\sqrt{-a}-\sqrt{b}x\right)}{4a\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}\,(bc^2+ad^2)} - \frac{\sqrt[6]{b}\left(\sqrt{-a}\sqrt{b}c-ad\right)\log\left(\sqrt{-a}+\sqrt{b}x\right)}{4a\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\,(bc^2+ad^2)} + \frac{3\sqrt[6]{b}\left(\sqrt{-a}\sqrt{b}c-ad\right)\log\left(\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4a\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}\,(bc^2+ad^2)} - \frac{3\sqrt[6]{b}\left(\sqrt{-a}\sqrt{b}c+ad\right)\log\left(\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}-\sqrt[6]{b}\sqrt[3]{c+dx}\right)}{4a\sqrt[3]{\sqrt{b}c+\sqrt{-a}d}\,(bc^2+ad^2)}$$

output
```
-3*d/(a*d^2+b*c^2)/(d*x+c)^(1/3)-1/2*3^(1/2)*b^(1/6)*(b^(1/2)*c+(-a)^(1/2)
*d)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3))*
3^(1/2))/(-a)^(1/2)/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3)/(a*d^2+b*c^2)-1/2*3^(1/
2)*b^(1/6)*((-a)^(1/2)*b^(1/2)*c+a*d)*arctan(1/3*(1+2*b^(1/6)*(d*x+c)^(1/3
)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3))*3^(1/2))/a/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3
)/(a*d^2+b*c^2)+1/4*b^(1/6)*((-a)^(1/2)*b^(1/2)*c+a*d)*ln((-a)^(1/2)-b^(1/
2)*x)/a/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)/(a*d^2+b*c^2)-1/4*b^(1/6)*((-a)^(1/
2)*b^(1/2)*c-a*d)*ln((-a)^(1/2)+b^(1/2)*x)/a/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3
)/(a*d^2+b*c^2)+3/4*b^(1/6)*((-a)^(1/2)*b^(1/2)*c-a*d)*ln((b^(1/2)*c-(-a)^
(1/2)*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/a/(b^(1/2)*c-(-a)^(1/2)*d)^(1/3)/(a*
d^2+b*c^2)-3/4*b^(1/6)*((-a)^(1/2)*b^(1/2)*c+a*d)*ln((b^(1/2)*c+(-a)^(1/2)
*d)^(1/3)-b^(1/6)*(d*x+c)^(1/3))/a/(b^(1/2)*c+(-a)^(1/2)*d)^(1/3)/(a*d^2+b
*c^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.18

$$\int \frac{1}{(c+dx)^{4/3}(a+bx^2)}\,dx =$$

$$-\frac{\frac{6d}{\sqrt[3]{c+dx}} - d\text{RootSum}\left[bc^2+ad^2-2bc\#1^3+b\#1^6\&, \frac{-2c\log\left(\sqrt[3]{c+dx}-\#1\right)+\log\left(\sqrt[3]{c+dx}-\#1\right)\#1^3}{c\#1-\#1^4}\&\right]}{2bc^2+2ad^2}$$

input
```
Integrate[1/((c + d*x)^(4/3)*(a + b*x^2)),x]
```

output
```
-(((6*d)/(c + d*x)^(1/3) - d*RootSum[b*c^2 + a*d^2 - 2*b*c*#1^3 + b*#1^6 &
 , (-2*c*Log[(c + d*x)^(1/3) - #1] + Log[(c + d*x)^(1/3) - #1]*#1^3)/(c*#1
 - #1^4) & ])/(2*b*c^2 + 2*a*d^2))
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 571, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {482, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx)^{4/3}}dx$$

$$\Big\downarrow \text{482}$$

$$\frac{b\int \frac{c-dx}{\sqrt[3]{c+dx}(bx^2+a)}dx}{ad^2+bc^2}-\frac{3d}{\sqrt[3]{c+dx}(ad^2+bc^2)}$$

$$\Big\downarrow \text{657}$$

$$\frac{b\int \left(\frac{\sqrt{-a}c-\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{-a}\right)\sqrt[3]{c+dx}}+\frac{\sqrt{-a}c+\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{-a}-\sqrt{b}x\right)\sqrt[3]{c+dx}}\right)dx}{ad^2+bc^2}-\frac{3d}{\sqrt[3]{c+dx}(ad^2+bc^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{\sqrt{3}\left(\sqrt{-a}d+\sqrt{b}c\right)\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}+1}{\sqrt{3}}\right)}{2\sqrt{-a}b^{5/6}\sqrt[3]{\sqrt{b}c-\sqrt{-a}d}}-\frac{\sqrt{3}\left(\sqrt{-a}\sqrt{b}c+ad\right)\arctan\left(\frac{\frac{2\sqrt[6]{b}\sqrt[3]{c+dx}}{\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+1}{\sqrt{3}}\right)}{2ab^{5/6}\sqrt[3]{\sqrt{-a}d+\sqrt{b}c}}+\frac{\left(\sqrt{-a}\sqrt{b}c+ad\right)\log\Big(}{4ab^{5/6}\sqrt[3]{\sqrt{-a}c}}\right.}{}$$

$$\frac{3d}{\sqrt[3]{c+dx}(ad^2+bc^2)}$$

input
```
Int[1/((c + d*x)^(4/3)*(a + b*x^2)),x]
```

output
```
(-3*d)/((b*c^2 + a*d^2)*(c + d*x)^(1/3)) + (b*(-1/2*(Sqrt[3]*(Sqrt[b]*c +
Sqrt[-a]*d)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3))/(Sqrt[b]*c - Sqrt[-a]*
d)^(1/3))/Sqrt[3]])/(Sqrt[-a]*b^(5/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(1/3)) - (S
qrt[3]*(Sqrt[-a]*Sqrt[b]*c + a*d)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/3))/
(Sqrt[b]*c + Sqrt[-a]*d)^(1/3))/Sqrt[3]])/(2*a*b^(5/6)*(Sqrt[b]*c + Sqrt[-
a]*d)^(1/3)) + ((Sqrt[-a]*Sqrt[b]*c + a*d)*Log[Sqrt[-a] - Sqrt[b]*x])/(4*a
*b^(5/6)*(Sqrt[b]*c + Sqrt[-a]*d)^(1/3)) - ((Sqrt[-a]*Sqrt[b]*c - a*d)*Log
[Sqrt[-a] + Sqrt[b]*x])/(4*a*b^(5/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(1/3)) + (3*
(Sqrt[-a]*Sqrt[b]*c - a*d)*Log[(Sqrt[b]*c - Sqrt[-a]*d)^(1/3) - b^(1/6)*(c
 + d*x)^(1/3)])/(4*a*b^(5/6)*(Sqrt[b]*c - Sqrt[-a]*d)^(1/3)) - (3*(Sqrt[-a
]*Sqrt[b]*c + a*d)*Log[(Sqrt[b]*c + Sqrt[-a]*d)^(1/3) - b^(1/6)*(c + d*x)^
(1/3)])/(4*a*b^(5/6)*(Sqrt[b]*c + Sqrt[-a]*d)^(1/3))))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.15

method	result
default	$-\frac{d\left(\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^6-2bc\_Z^3+a\,d^2+b\,c^2\right)}\frac{\left(\_R^3-2c\right)\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R\left(\_R^3-c\right)}\right)(dx+c)^{\frac{1}{3}}+6\right)}{(dx+c)^{\frac{1}{3}}\left(2a\,d^2+2b\,c^2\right)}$
pseudoelliptic	$-\frac{d\left(\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^6-2bc\_Z^3+a\,d^2+b\,c^2\right)}\frac{\left(\_R^3-2c\right)\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R\left(\_R^3-c\right)}\right)(dx+c)^{\frac{1}{3}}+6\right)}{(dx+c)^{\frac{1}{3}}\left(2a\,d^2+2b\,c^2\right)}$
derivativedivides	$3d\left(-\frac{1}{\left(a\,d^2+b\,c^2\right)(dx+c)^{\frac{1}{3}}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^6-2bc\_Z^3+a\,d^2+b\,c^2\right)}\frac{\left(-\_R^4+2\_Rc\right)\ln\left((dx+c)^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2c}}{6a\,d^2+6b\,c^2}\right)$

input
```
int(1/(d*x+c)^(4/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d*(sum(1/_R*(_R^3-2*c)*ln((d*x+c)^(1/3)-_R)/(_R^3-c),_R=RootOf(_Z^6*b-2*_
Z^3*b*c+a*d^2+b*c^2))*(d*x+c)^(1/3)+6)/(d*x+c)^(1/3)/(2*a*d^2+2*b*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6636 vs. 2(491) = 982.

Time = 1.36 (sec) , antiderivative size = 6636, normalized size of antiderivative = 10.48

$$\int \frac{1}{(c+dx)^{4/3}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(4/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{4/3}(a+bx^2)}\,dx = \int \frac{1}{(a+bx^2)(c+dx)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(4/3)/(b*x**2+a),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x)**(4/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{4/3}(a+bx^2)}\,dx = \int \frac{1}{(bx^2+a)(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(4/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x + c)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{4/3}(a+bx^2)}\,dx = \int \frac{1}{(bx^2+a)(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(4/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*(d*x + c)^(4/3)), x)
```

Mupad [B] (verification not implemented)

Time = 6.31 (sec) , antiderivative size = 5055, normalized size of antiderivative = 7.99

$$\int \frac{1}{(c+dx)^{4/3}\left(a+bx^2\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x)^(4/3)),x)
```

output
```
log(((c + d*x)^(1/3)*(38880*a^3*b^13*c^14*d^8 - 1944*a*b^15*c^18*d^4 - 194
4*a^2*b^14*c^16*d^6 - 1944*a^10*b^6*d^22 + 163296*a^4*b^12*c^12*d^10 + 299
376*a^5*b^11*c^10*d^12 + 299376*a^6*b^10*c^8*d^14 + 163296*a^7*b^9*c^6*d^1
6 + 38880*a^8*b^8*c^4*d^18 - 1944*a^9*b^7*c^2*d^20) - (-(a^2*d^4*(-a^3*b)^
(1/2) + b^2*c^4*(-a^3*b)^(1/2) - 4*a^2*b^2*c^3*d + 4*a^3*b*c*d^3 - 6*a*b*c
^2*d^2*(-a^3*b)^(1/2))/(8*(a^7*d^8 + a^3*b^4*c^8 + 4*a^6*b*c^2*d^6 + 4*a^4
*b^3*c^6*d^2 + 6*a^5*b^2*c^4*d^4)))^(2/3)*(7776*a^12*b^6*c*d^24 + 7776*a^2
*b^16*c^21*d^4 + 77760*a^3*b^15*c^19*d^6 + 349920*a^4*b^14*c^17*d^8 + 9331
20*a^5*b^13*c^15*d^10 + 1632960*a^6*b^12*c^13*d^12 + 1959552*a^7*b^11*c^11
*d^14 + 1632960*a^8*b^10*c^9*d^16 + 933120*a^9*b^9*c^7*d^18 + 349920*a^10*
b^8*c^5*d^20 + 77760*a^11*b^7*c^3*d^22))*(-(a^2*d^4*(-a^3*b)^(1/2) + b^2*c
^4*(-a^3*b)^(1/2) - 4*a^2*b^2*c^3*d + 4*a^3*b*c*d^3 - 6*a*b*c^2*d^2*(-a^3*
b)^(1/2))/(8*(a^7*d^8 + a^3*b^4*c^8 + 4*a^6*b*c^2*d^6 + 4*a^4*b^3*c^6*d^2
+ 6*a^5*b^2*c^4*d^4)))^(1/3) - 972*a^9*b^6*d^21 + 2916*a*b^14*c^16*d^5 + 1
9440*a^2*b^13*c^14*d^7 + 54432*a^3*b^12*c^12*d^9 + 81648*a^4*b^11*c^10*d^1
1 + 68040*a^5*b^10*c^8*d^13 + 27216*a^6*b^9*c^6*d^15 - 3888*a^8*b^7*c^2*d^
19)*(-(a^2*d^4*(-a^3*b)^(1/2) + b^2*c^4*(-a^3*b)^(1/2) - 4*a^2*b^2*c^3*d +
 4*a^3*b*c*d^3 - 6*a*b*c^2*d^2*(-a^3*b)^(1/2))/(8*(a^7*d^8 + a^3*b^4*c^8 +
 4*a^6*b*c^2*d^6 + 4*a^4*b^3*c^6*d^2 + 6*a^5*b^2*c^4*d^4)))^(1/3) + log(((
c + d*x)^(1/3)*(38880*a^3*b^13*c^14*d^8 - 1944*a*b^15*c^18*d^4 - 1944*a...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{4/3}\left(a+bx^2\right)}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{3}}\,ac+(dx+c)^{\frac{1}{3}}\,adx+(dx+c)^{\frac{1}{3}}\,bc\,x^2+(dx+c)^{\frac{1}{3}}\,bd\,x^3}dx$$

input
```
int(1/(d*x+c)^(4/3)/(b*x^2+a),x)
```

output

```
int(1/((c + d*x)**(1/3)*a*c + (c + d*x)**(1/3)*a*d*x + (c + d*x)**(1/3)*b*
c*x**2 + (c + d*x)**(1/3)*b*d*x**3),x)
```

3.208 $\int (c+dx)^{3/4} (a-bx^2) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int (c+dx)^{3/4} (a-bx^2) \, dx = -\frac{4(bc^2-ad^2)(c+dx)^{7/4}}{7d^3} + \frac{8bc(c+dx)^{11/4}}{11d^3} - \frac{4b(c+dx)^{15/4}}{15d^3}$$

output
```
-4/7*(-a*d^2+b*c^2)*(d*x+c)^(7/4)/d^3+8/11*b*c*(d*x+c)^(11/4)/d^3-4/15*b*(
d*x+c)^(15/4)/d^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int (c+dx)^{3/4} (a-bx^2) \, dx = -\frac{4(c+dx)^{7/4}(-165ad^2+b(32c^2-56cdx+77d^2x^2))}{1155d^3}$$

input
```
Integrate[(c + d*x)^(3/4)*(a - b*x^2),x]
```

output
```
(-4*(c + d*x)^(7/4)*(-165*a*d^2 + b*(32*c^2 - 56*c*d*x + 77*d^2*x^2)))/(11
55*d^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^2)(c+dx)^{3/4}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(c+dx)^{3/4}(ad^2-bc^2)}{d^2}-\frac{b(c+dx)^{11/4}}{d^2}+\frac{2bc(c+dx)^{7/4}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4(c+dx)^{7/4}(bc^2-ad^2)}{7d^3}-\frac{4b(c+dx)^{15/4}}{15d^3}+\frac{8bc(c+dx)^{11/4}}{11d^3}$$

input
```
Int[(c + d*x)^(3/4)*(a - b*x^2),x]
```

output
```
(-4*(b*c^2 - a*d^2)*(c + d*x)^(7/4))/(7*d^3) + (8*b*c*(c + d*x)^(11/4))/(1
1*d^3) - (4*b*(c + d*x)^(15/4))/(15*d^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{4(dx+c)^{\frac{7}{4}}\left(-77b\,x^{2}d^{2}+56bcdx+165a\,d^{2}-32b\,c^{2}\right)}{1155d^{3}}$	41
pseudoelliptic	$\frac{4(dx+c)^{\frac{7}{4}}\left(-77b\,x^{2}d^{2}+56bcdx+165a\,d^{2}-32b\,c^{2}\right)}{1155d^{3}}$	41
orering	$\frac{4(dx+c)^{\frac{7}{4}}\left(-77b\,x^{2}d^{2}+56bcdx+165a\,d^{2}-32b\,c^{2}\right)}{1155d^{3}}$	41
derivativedivides	$-\frac{4\left(\frac{b(dx+c)^{\frac{15}{4}}}{15}-\frac{2bc(dx+c)^{\frac{11}{4}}}{11}+\frac{\left(-a\,d^{2}+b\,c^{2}\right)(dx+c)^{\frac{7}{4}}}{7}\right)}{d^{3}}$	49
default	$\frac{-\frac{4b(dx+c)^{\frac{15}{4}}}{15}+\frac{8bc(dx+c)^{\frac{11}{4}}}{11}+\frac{4\left(a\,d^{2}-b\,c^{2}\right)(dx+c)^{\frac{7}{4}}}{7}}{d^{3}}$	49
trager	$\frac{4\left(-77b\,d^{3}x^{3}-21bc\,d^{2}x^{2}+165ax\,d^{3}+24b\,c^{2}dx+165a\,d^{2}c-32b\,c^{3}\right)(dx+c)^{\frac{3}{4}}}{1155d^{3}}$	61
risch	$\frac{4\left(-77b\,d^{3}x^{3}-21bc\,d^{2}x^{2}+165ax\,d^{3}+24b\,c^{2}dx+165a\,d^{2}c-32b\,c^{3}\right)(dx+c)^{\frac{3}{4}}}{1155d^{3}}$	61

input
```
int((d*x+c)^(3/4)*(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/1155*(d*x+c)^(7/4)*(-77*b*d^2*x^2+56*b*c*d*x+165*a*d^2-32*b*c^2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int (c+dx)^{3/4}\left(a-bx^2\right)\,dx = -\frac{4\,(77\,bd^3x^3+21\,bcd^2x^2+32\,bc^3-165\,acd^2-3\,(8\,bc^2d+55\,ad^3)x)(dx+c)^{\frac{3}{4}}}{1155\,d^3}$$

input
```
integrate((d*x+c)^(3/4)*(-b*x^2+a),x, algorithm="fricas")
```

output
```
-4/1155*(77*b*d^3*x^3 + 21*b*c*d^2*x^2 + 32*b*c^3 - 165*a*c*d^2 - 3*(8*b*c
^2*d + 55*a*d^3)*x)*(d*x + c)^(3/4)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int (c+dx)^{3/4}\left(a-bx^2\right)\,dx=\begin{cases}\frac{4\cdot\left(\frac{2bc(c+dx)^{\frac{11}{4}}}{11d^2}-\frac{b(c+dx)^{\frac{15}{4}}}{15d^2}-\frac{(c+dx)^{\frac{7}{4}}\left(-ad^2+bc^2\right)}{7d^2}\right)}{d} & \text{for } d\neq 0\\ c^{\frac{3}{4}}\left(ax-\frac{bx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((d*x+c)**(3/4)*(-b*x**2+a),x)
```

output
```
Piecewise((4*(2*b*c*(c + d*x)**(11/4)/(11*d**2) - b*(c + d*x)**(15/4)/(15*
d**2) - (c + d*x)**(7/4)*(-a*d**2 + b*c**2)/(7*d**2))/d, Ne(d, 0)), (c**(3
/4)*(a*x - b*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int (c+dx)^{3/4}\left(a-bx^2\right)\,dx=-\frac{4\left(77\,(dx+c)^{\frac{15}{4}}b-210\,(dx+c)^{\frac{11}{4}}bc+165\,(bc^2-ad^2)(dx+c)^{\frac{7}{4}}\right)}{1155\,d^3}$$

input
```
integrate((d*x+c)^(3/4)*(-b*x^2+a),x, algorithm="maxima")
```

output
```
-4/1155*(77*(d*x + c)^(15/4)*b - 210*(d*x + c)^(11/4)*b*c + 165*(b*c^2 - a
*d^2)*(d*x + c)^(7/4))/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.02

$$\int (c+dx)^{3/4}\left(a-bx^2\right)\,dx = \frac{4\left(385\,(dx+c)^{\frac{3}{4}}ac+55\left(3\,(dx+c)^{\frac{7}{4}}-7\,(dx+c)^{\frac{3}{4}}c\right)a-\frac{5\left(21\,(dx+c)^{\frac{11}{4}}-66\,(dx+c)^{\frac{7}{4}}c+77\,(dx+c)^{\frac{3}{4}}c^2\right)\cdots}{d^2}\cdots\right.}{1155\,d}$$

input
```
integrate((d*x+c)^(3/4)*(-b*x^2+a),x, algorithm="giac")
```

output
```
4/1155*(385*(d*x + c)^(3/4)*a*c + 55*(3*(d*x + c)^(7/4) - 7*(d*x + c)^(3/4
)*c)*a - 5*(21*(d*x + c)^(11/4) - 66*(d*x + c)^(7/4)*c + 77*(d*x + c)^(3/4
)*c^2)*b*c/d^2 - (77*(d*x + c)^(15/4) - 315*(d*x + c)^(11/4)*c + 495*(d*x
+ c)^(7/4)*c^2 - 385*(d*x + c)^(3/4)*c^3)*b/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int (c+dx)^{3/4}\left(a-bx^2\right)\,dx = -\frac{4\,(c+d\,x)^{7/4}\,\left(77\,b\,(c+d\,x)^2-165\,a\,d^2+165\,b\,c^2-210\,b\,c\,(c+d\,x)\right)}{1155\,d^3}$$

input
```
int((a - b*x^2)*(c + d*x)^(3/4),x)
```

output
```
-(4*(c + d*x)^(7/4)*(77*b*(c + d*x)^2 - 165*a*d^2 + 165*b*c^2 - 210*b*c*(c
 + d*x)))/(1155*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int (c+dx)^{3/4} \left(a-bx^2\right) \, dx = \frac{4(dx+c)^{\frac{3}{4}} \left(-77b\, d^3x^3-21bc\, d^2x^2+165a\, d^3x+24b\, c^2dx+165ac\, d^2-32b\, c^3\right)}{1155d^3}$$

input
```
int((d*x+c)^(3/4)*(-b*x^2+a),x)
```

output
```
(4*(c + d*x)**(3/4)*(165*a*c*d**2 + 165*a*d**3*x - 32*b*c**3 + 24*b*c**2*d
*x - 21*b*c*d**2*x**2 - 77*b*d**3*x**3))/(1155*d**3)
```

3.209 $\int \sqrt[4]{c+dx}(a-bx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx=-\frac{4(bc^2-ad^2)\,(c+dx)^{5/4}}{5d^3}+\frac{8bc(c+dx)^{9/4}}{9d^3}-\frac{4b(c+dx)^{13/4}}{13d^3}$$

output
```
-4/5*(-a*d^2+b*c^2)*(d*x+c)^(5/4)/d^3+8/9*b*c*(d*x+c)^(9/4)/d^3-4/13*b*(d*
x+c)^(13/4)/d^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx=-\frac{4(c+dx)^{5/4}\,(-117ad^2+b(32c^2-40cdx+45d^2x^2))}{585d^3}$$

input
```
Integrate[(c + d*x)^(1/4)*(a - b*x^2),x]
```

output
```
(-4*(c + d*x)^(5/4)*(-117*a*d^2 + b*(32*c^2 - 40*c*d*x + 45*d^2*x^2)))/(58
5*d^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right) \sqrt[4]{c+dx} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{\sqrt[4]{c+dx}\left(ad^2-bc^2\right)}{d^2}-\frac{b(c+dx)^{9/4}}{d^2}+\frac{2bc(c+dx)^{5/4}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4(c+dx)^{5/4}\left(bc^2-ad^2\right)}{5d^3}-\frac{4b(c+dx)^{13/4}}{13d^3}+\frac{8bc(c+dx)^{9/4}}{9d^3}$$

input
```
Int[(c + d*x)^(1/4)*(a - b*x^2),x]
```

output
```
(-4*(b*c^2 - a*d^2)*(c + d*x)^(5/4))/(5*d^3) + (8*b*c*(c + d*x)^(9/4))/(9*
d^3) - (4*b*(c + d*x)^(13/4))/(13*d^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{4(dx+c)^{\frac{5}{4}}\left(-45b\,x^{2}d^{2}+40bcdx+117a\,d^{2}-32b\,c^{2}\right)}{585d^{3}}$	41
pseudoelliptic	$\frac{4(dx+c)^{\frac{5}{4}}\left(-45b\,x^{2}d^{2}+40bcdx+117a\,d^{2}-32b\,c^{2}\right)}{585d^{3}}$	41
orering	$\frac{4(dx+c)^{\frac{5}{4}}\left(-45b\,x^{2}d^{2}+40bcdx+117a\,d^{2}-32b\,c^{2}\right)}{585d^{3}}$	41
derivativedivides	$-\frac{4\left(\frac{b(dx+c)^{\frac{13}{4}}}{13}-\frac{2bc(dx+c)^{\frac{9}{4}}}{9}+\frac{\left(-a\,d^{2}+b\,c^{2}\right)(dx+c)^{\frac{5}{4}}}{5}\right)}{d^{3}}$	49
default	$\frac{-\frac{4b(dx+c)^{\frac{13}{4}}}{13}+\frac{8bc(dx+c)^{\frac{9}{4}}}{9}+\frac{4\left(a\,d^{2}-b\,c^{2}\right)(dx+c)^{\frac{5}{4}}}{5}}{d^{3}}$	49
trager	$\frac{4\left(-45b\,d^{3}x^{3}-5bc\,d^{2}x^{2}+117ax\,d^{3}+8b\,c^{2}dx+117a\,d^{2}c-32b\,c^{3}\right)(dx+c)^{\frac{1}{4}}}{585d^{3}}$	61
risch	$\frac{4\left(-45b\,d^{3}x^{3}-5bc\,d^{2}x^{2}+117ax\,d^{3}+8b\,c^{2}dx+117a\,d^{2}c-32b\,c^{3}\right)(dx+c)^{\frac{1}{4}}}{585d^{3}}$	61

input
```
int((d*x+c)^(1/4)*(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/585*(d*x+c)^(5/4)*(-45*b*d^2*x^2+40*b*c*d*x+117*a*d^2-32*b*c^2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx = -\frac{4\,(45\,bd^3x^3+5\,bcd^2x^2+32\,bc^3-117\,acd^2-(8\,bc^2d+117\,ad^3)x)(dx+c)^{\frac{1}{4}}}{585\,d^3}$$

input
```
integrate((d*x+c)^(1/4)*(-b*x^2+a),x, algorithm="fricas")
```

output
```
-4/585*(45*b*d^3*x^3 + 5*b*c*d^2*x^2 + 32*b*c^3 - 117*a*c*d^2 - (8*b*c^2*d
 + 117*a*d^3)*x)*(d*x + c)^(1/4)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx=\begin{cases}\frac{4\cdot\left(\frac{2bc(c+dx)^{\frac{9}{4}}}{9d^2}-\frac{b(c+dx)^{\frac{13}{4}}}{13d^2}-\frac{(c+dx)^{\frac{5}{4}}\left(-ad^2+bc^2\right)}{5d^2}\right)}{d} & \text{for } d\neq 0\\ \sqrt[4]{c}\left(ax-\frac{bx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((d*x+c)**(1/4)*(-b*x**2+a),x)
```

output
```
Piecewise((4*(2*b*c*(c + d*x)**(9/4)/(9*d**2) - b*(c + d*x)**(13/4)/(13*d*
*2) - (c + d*x)**(5/4)*(-a*d**2 + b*c**2)/(5*d**2))/d, Ne(d, 0)), (c**(1/4
)*(a*x - b*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx=-\frac{4\left(45\,(dx+c)^{\frac{13}{4}}b-130\,(dx+c)^{\frac{9}{4}}bc+117\,(bc^2-ad^2)(dx+c)^{\frac{5}{4}}\right)}{585\,d^3}$$

input
```
integrate((d*x+c)^(1/4)*(-b*x^2+a),x, algorithm="maxima")
```

output
```
-4/585*(45*(d*x + c)^(13/4)*b - 130*(d*x + c)^(9/4)*b*c + 117*(b*c^2 - a*d
^2)*(d*x + c)^(5/4))/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(52) = 104$.

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx$$
$$= \frac{4\left(585\,(dx+c)^{\frac{1}{4}}ac + 117\left((dx+c)^{\frac{5}{4}} - 5\,(dx+c)^{\frac{1}{4}}c\right)a - \frac{13\left(5\,(dx+c)^{\frac{9}{4}} - 18\,(dx+c)^{\frac{5}{4}}c + 45\,(dx+c)^{\frac{1}{4}}c^2\right)bc}{d^2} - \frac{3\left(15\,(dx\right.}{}\right.}{585\,d}$$

input
```
integrate((d*x+c)^(1/4)*(-b*x^2+a),x, algorithm="giac")
```

output
```
4/585*(585*(d*x + c)^(1/4)*a*c + 117*((d*x + c)^(5/4) - 5*(d*x + c)^(1/4)*
c)*a - 13*(5*(d*x + c)^(9/4) - 18*(d*x + c)^(5/4)*c + 45*(d*x + c)^(1/4)*c
^2)*b*c/d^2 - 3*(15*(d*x + c)^(13/4) - 65*(d*x + c)^(9/4)*c + 117*(d*x + c
)^(5/4)*c^2 - 195*(d*x + c)^(1/4)*c^3)*b/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx = -\frac{4\,(c+d\,x)^{5/4}\,\left(45\,b\,(c+d\,x)^2 - 117\,a\,d^2 + 117\,b\,c^2 - 130\,b\,c\,(c+d\,x)\right)}{585\,d^3}$$

input
```
int((a - b*x^2)*(c + d*x)^(1/4),x)
```

output
```
-(4*(c + d*x)^(5/4)*(45*b*(c + d*x)^2 - 117*a*d^2 + 117*b*c^2 - 130*b*c*(c
 + d*x)))/(585*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \sqrt[4]{c+dx}\left(a-bx^2\right)\,dx = \frac{4(dx+c)^{\frac{1}{4}}\left(-45b\,d^3x^3-5bc\,d^2x^2+117a\,d^3x+8b\,c^2dx+117ac\,d^2-32b\,c^3\right)}{585d^3}$$

input
```
int((d*x+c)^(1/4)*(-b*x^2+a),x)
```

output
```
(4*(c + d*x)**(1/4)*(117*a*c*d**2 + 117*a*d**3*x - 32*b*c**3 + 8*b*c**2*d*
x - 5*b*c*d**2*x**2 - 45*b*d**3*x**3))/(585*d**3)
```

3.210 $\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx=-\frac{4(bc^2-ad^2)\,(c+dx)^{3/4}}{3d^3}+\frac{8bc(c+dx)^{7/4}}{7d^3}-\frac{4b(c+dx)^{11/4}}{11d^3}$$

output
```
-4/3*(-a*d^2+b*c^2)*(d*x+c)^(3/4)/d^3+8/7*b*c*(d*x+c)^(7/4)/d^3-4/11*b*(d*
x+c)^(11/4)/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx=-\frac{4(c+dx)^{3/4}\,(-77ad^2+b(32c^2-24cdx+21d^2x^2))}{231d^3}$$

input
```
Integrate[(a - b*x^2)/(c + d*x)^(1/4),x]
```

output
```
(-4*(c + d*x)^(3/4)*(-77*a*d^2 + b*(32*c^2 - 24*c*d*x + 21*d^2*x^2)))/(231
*d^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{ad^2-bc^2}{d^2\sqrt[4]{c+dx}}-\frac{b(c+dx)^{7/4}}{d^2}+\frac{2bc(c+dx)^{3/4}}{d^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4(c+dx)^{3/4}\left(bc^2-ad^2\right)}{3d^3}-\frac{4b(c+dx)^{11/4}}{11d^3}+\frac{8bc(c+dx)^{7/4}}{7d^3}$$

input
```
Int[(a - b*x^2)/(c + d*x)^(1/4),x]
```

output
```
(-4*(b*c^2 - a*d^2)*(c + d*x)^(3/4))/(3*d^3) + (8*b*c*(c + d*x)^(7/4))/(7*
d^3) - (4*b*(c + d*x)^(11/4))/(11*d^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{4(dx+c)^{\frac{3}{4}}\left(-21b\,x^{2}d^{2}+24bcdx+77a\,d^{2}-32b\,c^{2}\right)}{231d^{3}}$	41
trager	$\frac{4(dx+c)^{\frac{3}{4}}\left(-21b\,x^{2}d^{2}+24bcdx+77a\,d^{2}-32b\,c^{2}\right)}{231d^{3}}$	41
risch	$\frac{4(dx+c)^{\frac{3}{4}}\left(-21b\,x^{2}d^{2}+24bcdx+77a\,d^{2}-32b\,c^{2}\right)}{231d^{3}}$	41
pseudoelliptic	$\frac{4(dx+c)^{\frac{3}{4}}\left(-21b\,x^{2}d^{2}+24bcdx+77a\,d^{2}-32b\,c^{2}\right)}{231d^{3}}$	41
orering	$\frac{4(dx+c)^{\frac{3}{4}}\left(-21b\,x^{2}d^{2}+24bcdx+77a\,d^{2}-32b\,c^{2}\right)}{231d^{3}}$	41
derivativedivides	$-\frac{4\left(\frac{b(dx+c)^{\frac{11}{4}}}{11}-\frac{2bc(dx+c)^{\frac{7}{4}}}{7}+\frac{\left(-a\,d^{2}+b\,c^{2}\right)(dx+c)^{\frac{3}{4}}}{3}\right)}{d^{3}}$	49
default	$\frac{-\frac{4b(dx+c)^{\frac{11}{4}}}{11}+\frac{8bc(dx+c)^{\frac{7}{4}}}{7}+\frac{4\left(a\,d^{2}-b\,c^{2}\right)(dx+c)^{\frac{3}{4}}}{3}}{d^{3}}$	49

input
```
int((-b*x^2+a)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/231*(d*x+c)^(3/4)*(-21*b*d^2*x^2+24*b*c*d*x+77*a*d^2-32*b*c^2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx = -\frac{4\,(21\,bd^2x^2-24\,bcdx+32\,bc^2-77\,ad^2)(dx+c)^{\frac{3}{4}}}{231\,d^3}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(1/4),x, algorithm="fricas")
```

output
```
-4/231*(21*b*d^2*x^2 - 24*b*c*d*x + 32*b*c^2 - 77*a*d^2)*(d*x + c)^(3/4)/d
^3
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx = \begin{cases} \frac{4\cdot\left(\frac{2bc(c+dx)^{\frac{7}{4}}}{7d^2} - \frac{b(c+dx)^{\frac{11}{4}}}{11d^2} - \frac{(c+dx)^{\frac{3}{4}}\left(-ad^2+bc^2\right)}{3d^2}\right)}{d} & \text{for } d \neq 0 \\ \frac{ax-\frac{bx^3}{3}}{\sqrt[4]{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**2+a)/(d*x+c)**(1/4),x)
```

output
```
Piecewise((4*(2*b*c*(c + d*x)**(7/4)/(7*d**2) - b*(c + d*x)**(11/4)/(11*d*
*2) - (c + d*x)**(3/4)*(-a*d**2 + b*c**2)/(3*d**2))/d, Ne(d, 0)), ((a*x -
b*x**3/3)/c**(1/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx = -\frac{4\left(21\,(dx+c)^{\frac{11}{4}}b - 66\,(dx+c)^{\frac{7}{4}}bc + 77\,(bc^2-ad^2)(dx+c)^{\frac{3}{4}}\right)}{231\,d^3}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(1/4),x, algorithm="maxima")
```

output
```
-4/231*(21*(d*x + c)^(11/4)*b - 66*(d*x + c)^(7/4)*b*c + 77*(b*c^2 - a*d^2
)*(d*x + c)^(3/4))/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx = \frac{4\left(77\,(dx+c)^{\frac{3}{4}}a-\frac{\left(21\,(dx+c)^{\frac{11}{4}}-66\,(dx+c)^{\frac{7}{4}}c+77\,(dx+c)^{\frac{3}{4}}c^2\right)b}{d^2}\right)}{231\,d}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(1/4),x, algorithm="giac")
```

output
```
4/231*(77*(d*x + c)^(3/4)*a - (21*(d*x + c)^(11/4) - 66*(d*x + c)^(7/4)*c
+ 77*(d*x + c)^(3/4)*c^2)*b/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx = -\frac{4\,(c+d\,x)^{3/4}\,\left(21\,b\,(c+d\,x)^2-77\,a\,d^2+77\,b\,c^2-66\,b\,c\,(c+d\,x)\right)}{231\,d^3}$$

input
```
int((a - b*x^2)/(c + d*x)^(1/4),x)
```

output
```
-(4*(c + d*x)^(3/4)*(21*b*(c + d*x)^2 - 77*a*d^2 + 77*b*c^2 - 66*b*c*(c +
d*x)))/(231*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{a-bx^2}{\sqrt[4]{c+dx}}\,dx = \frac{4(dx+c)^{\frac{3}{4}}\left(-21b\,d^2x^2+24bcdx+77a\,d^2-32b\,c^2\right)}{231d^3}$$

input
```
int((-b*x^2+a)/(d*x+c)^(1/4),x)
```

output
```
(4*(c + d*x)**(3/4)*(77*a*d**2 - 32*b*c**2 + 24*b*c*d*x - 21*b*d**2*x**2))
/(231*d**3)
```

3.211 $\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx=-\frac{4(bc^2-ad^2)\sqrt[4]{c+dx}}{d^3}+\frac{8bc(c+dx)^{5/4}}{5d^3}-\frac{4b(c+dx)^{9/4}}{9d^3}$$

output
```
-4*(-a*d^2+b*c^2)*(d*x+c)^(1/4)/d^3+8/5*b*c*(d*x+c)^(5/4)/d^3-4/9*b*(d*x+c
)^(9/4)/d^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx=-\frac{4\sqrt[4]{c+dx}(-45ad^2+b(32c^2-8cdx+5d^2x^2))}{45d^3}$$

input
```
Integrate[(a - b*x^2)/(c + d*x)^(3/4),x]
```

output
```
(-4*(c + d*x)^(1/4)*(-45*a*d^2 + b*(32*c^2 - 8*c*d*x + 5*d^2*x^2)))/(45*d^
3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ad^2-bc^2}{d^2(c+dx)^{3/4}}-\frac{b(c+dx)^{5/4}}{d^2}+\frac{2bc\sqrt[4]{c+dx}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\sqrt[4]{c+dx}\,(bc^2-ad^2)}{d^3}-\frac{4b(c+dx)^{9/4}}{9d^3}+\frac{8bc(c+dx)^{5/4}}{5d^3}$$

input
```
Int[(a - b*x^2)/(c + d*x)^(3/4),x]
```

output
```
(-4*(b*c^2 - a*d^2)*(c + d*x)^(1/4))/d^3 + (8*b*c*(c + d*x)^(5/4))/(5*d^3)
 - (4*b*(c + d*x)^(9/4))/(9*d^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{4(dx+c)^{\frac{1}{4}}\left(-5b\,x^{2}d^{2}+8bcdx+45a\,d^{2}-32b\,c^{2}\right)}{45d^{3}}$	41
trager	$\frac{4(dx+c)^{\frac{1}{4}}\left(-5b\,x^{2}d^{2}+8bcdx+45a\,d^{2}-32b\,c^{2}\right)}{45d^{3}}$	41
risch	$\frac{4(dx+c)^{\frac{1}{4}}\left(-5b\,x^{2}d^{2}+8bcdx+45a\,d^{2}-32b\,c^{2}\right)}{45d^{3}}$	41
pseudoelliptic	$\frac{4(dx+c)^{\frac{1}{4}}\left(-5b\,x^{2}d^{2}+8bcdx+45a\,d^{2}-32b\,c^{2}\right)}{45d^{3}}$	41
orering	$\frac{4(dx+c)^{\frac{1}{4}}\left(-5b\,x^{2}d^{2}+8bcdx+45a\,d^{2}-32b\,c^{2}\right)}{45d^{3}}$	41
derivativedivides	$-\frac{4\left(\frac{b(dx+c)^{\frac{9}{4}}}{9}-\frac{2bc(dx+c)^{\frac{5}{4}}}{5}-a\,d^{2}(dx+c)^{\frac{1}{4}}+b\,c^{2}(dx+c)^{\frac{1}{4}}\right)}{d^{3}}$	53
default	$\frac{-\frac{4b(dx+c)^{\frac{9}{4}}}{9}+\frac{8bc(dx+c)^{\frac{5}{4}}}{5}+4a\,d^{2}(dx+c)^{\frac{1}{4}}-4b\,c^{2}(dx+c)^{\frac{1}{4}}}{d^{3}}$	53

input
```
int((-b*x^2+a)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
4/45*(d*x+c)^(1/4)*(-5*b*d^2*x^2+8*b*c*d*x+45*a*d^2-32*b*c^2)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx=-\frac{4\,(5\,bd^2x^2-8\,bcdx+32\,bc^2-45\,ad^2)(dx+c)^{\frac{1}{4}}}{45\,d^3}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(3/4),x, algorithm="fricas")
```

output
```
-4/45*(5*b*d^2*x^2 - 8*b*c*d*x + 32*b*c^2 - 45*a*d^2)*(d*x + c)^(1/4)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx = \begin{cases} \frac{4a\sqrt[4]{c+dx}-\frac{4b\left(c^2\sqrt[4]{c+dx}-\frac{2c(c+dx)^{\frac{5}{4}}}{5}+\frac{(c+dx)^{\frac{9}{4}}}{9}\right)}{d^2}}{d} & \text{for } d\neq 0 \\ \frac{ax-\frac{bx^3}{3}}{c^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**2+a)/(d*x+c)**(3/4),x)
```

output
```
Piecewise(((4*a*(c + d*x)**(1/4) - 4*b*(c**2*(c + d*x)**(1/4) - 2*c*(c + d
*x)**(5/4)/5 + (c + d*x)**(9/4)/9)/d**2)/d, Ne(d, 0)), ((a*x - b*x**3/3)/c
**(3/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx = \frac{4\left(45\,(dx+c)^{\frac{1}{4}}a-\frac{\left(5\,(dx+c)^{\frac{9}{4}}-18\,(dx+c)^{\frac{5}{4}}c+45\,(dx+c)^{\frac{1}{4}}c^2\right)b}{d^2}\right)}{45\,d}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(3/4),x, algorithm="maxima")
```

output
```
4/45*(45*(d*x + c)^(1/4)*a - (5*(d*x + c)^(9/4) - 18*(d*x + c)^(5/4)*c + 4
5*(d*x + c)^(1/4)*c^2)*b/d^2)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx = \frac{4\left(45\,(dx+c)^{\frac{1}{4}}a - \frac{\left(5\,(dx+c)^{\frac{9}{4}}-18\,(dx+c)^{\frac{5}{4}}c+45\,(dx+c)^{\frac{1}{4}}c^2\right)b}{d^2}\right)}{45\,d}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(3/4),x, algorithm="giac")
```

output
```
4/45*(45*(d*x + c)^(1/4)*a - (5*(d*x + c)^(9/4) - 18*(d*x + c)^(5/4)*c + 4
5*(d*x + c)^(1/4)*c^2)*b/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx = -\frac{4\,(c+d\,x)^{1/4}\,\left(5\,b\,(c+d\,x)^2-45\,a\,d^2+45\,b\,c^2-18\,b\,c\,(c+d\,x)\right)}{45\,d^3}$$

input
```
int((a - b*x^2)/(c + d*x)^(3/4),x)
```

output
```
-(4*(c + d*x)^(1/4)*(5*b*(c + d*x)^2 - 45*a*d^2 + 45*b*c^2 - 18*b*c*(c + d
*x)))/(45*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{a-bx^2}{(c+dx)^{3/4}}\,dx = \frac{4(dx+c)^{\frac{1}{4}}\left(-5b\,d^2x^2+8bcdx+45a\,d^2-32b\,c^2\right)}{45d^3}$$

input
```
int((-b*x^2+a)/(d*x+c)^(3/4),x)
```

output
```
(4*(c + d*x)**(1/4)*(45*a*d**2 - 32*b*c**2 + 8*b*c*d*x - 5*b*d**2*x**2))/(
45*d**3)
```

3.212 $\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx=\frac{4(bc^2-ad^2)}{d^3\sqrt[4]{c+dx}}+\frac{8bc(c+dx)^{3/4}}{3d^3}-\frac{4b(c+dx)^{7/4}}{7d^3}$$

output
```
4*(-a*d^2+b*c^2)/d^3/(d*x+c)^(1/4)+8/3*b*c*(d*x+c)^(3/4)/d^3-4/7*b*(d*x+c)
^(7/4)/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx=-\frac{4(21ad^2+b(-32c^2-8cdx+3d^2x^2))}{21d^3\sqrt[4]{c+dx}}$$

input
```
Integrate[(a - b*x^2)/(c + d*x)^(5/4),x]
```

output
```
(-4*(21*a*d^2 + b*(-32*c^2 - 8*c*d*x + 3*d^2*x^2)))/(21*d^3*(c + d*x)^(1/4
))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^2}{(c+dx)^{5/4}} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ad^2-bc^2}{d^2(c+dx)^{5/4}}-\frac{b(c+dx)^{3/4}}{d^2}+\frac{2bc}{d^2\sqrt[4]{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4(bc^2-ad^2)}{d^3\sqrt[4]{c+dx}}-\frac{4b(c+dx)^{7/4}}{7d^3}+\frac{8bc(c+dx)^{3/4}}{3d^3}$$

input
```
Int[(a - b*x^2)/(c + d*x)^(5/4),x]
```

output
```
(4*(b*c^2 - a*d^2))/(d^3*(c + d*x)^(1/4)) + (8*b*c*(c + d*x)^(3/4))/(3*d^3
) - (4*b*(c + d*x)^(7/4))/(7*d^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$\frac{(-12b\,x^2-84a)\,d^2+32bcdx+128b\,c^2}{21(dx+c)^{\frac{1}{4}}d^3}$	40
gosper	$-\frac{4(3b\,x^2d^2-8bcdx+21a\,d^2-32b\,c^2)}{21(dx+c)^{\frac{1}{4}}d^3}$	41
trager	$-\frac{4(3b\,x^2d^2-8bcdx+21a\,d^2-32b\,c^2)}{21(dx+c)^{\frac{1}{4}}d^3}$	41
orering	$-\frac{4(3b\,x^2d^2-8bcdx+21a\,d^2-32b\,c^2)}{21(dx+c)^{\frac{1}{4}}d^3}$	41
risch	$\frac{4b(-3dx+11c)(dx+c)^{\frac{3}{4}}}{21d^3}-\frac{4(a\,d^2-b\,c^2)}{d^3(dx+c)^{\frac{1}{4}}}$	47
derivativedivides	$-\frac{4\left(\frac{b(dx+c)^{\frac{7}{4}}}{7}-\frac{2bc(dx+c)^{\frac{3}{4}}}{3}-\frac{-a\,d^2+b\,c^2}{(dx+c)^{\frac{1}{4}}}\right)}{d^3}$	49
default	$\frac{-\frac{4b(dx+c)^{\frac{7}{4}}}{7}+\frac{8bc(dx+c)^{\frac{3}{4}}}{3}-\frac{4\left(a\,d^2-b\,c^2\right)}{(dx+c)^{\frac{1}{4}}}}{d^3}$	49

input
```
int((-b*x^2+a)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/21*((-12*b*x^2-84*a)*d^2+32*b*c*d*x+128*b*c^2)/(d*x+c)^(1/4)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx=-\frac{4\,(3\,bd^2x^2-8\,bcdx-32\,bc^2+21\,ad^2)(dx+c)^{\frac{3}{4}}}{21\,(d^4x+cd^3)}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(5/4),x, algorithm="fricas")
```

output
```
-4/21*(3*b*d^2*x^2 - 8*b*c*d*x - 32*b*c^2 + 21*a*d^2)*(d*x + c)^(3/4)/(d^4
*x + c*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx = \begin{cases} \frac{4\cdot\left(\frac{2bc(c+dx)^{\frac{3}{4}}}{3d^2} - \frac{b(c+dx)^{\frac{7}{4}}}{7d^2} - \frac{ad^2-bc^2}{d^2\sqrt[4]{c+dx}}\right)}{d} & \text{for } d\neq 0 \\ \frac{ax-\frac{bx^3}{3}}{c^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**2+a)/(d*x+c)**(5/4),x)
```

output
```
Piecewise((4*(2*b*c*(c + d*x)**(3/4)/(3*d**2) - b*(c + d*x)**(7/4)/(7*d**2
) - (a*d**2 - b*c**2)/(d**2*(c + d*x)**(1/4)))/d, Ne(d, 0)), ((a*x - b*x**
3/3)/c**(5/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx = -\frac{4\left(\frac{3\,(dx+c)^{\frac{7}{4}}b-14\,(dx+c)^{\frac{3}{4}}bc}{d^2} - \frac{21\,(bc^2-ad^2)}{(dx+c)^{\frac{1}{4}}d^2}\right)}{21\,d}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(5/4),x, algorithm="maxima")
```

output
```
-4/21*((3*(d*x + c)^(7/4)*b - 14*(d*x + c)^(3/4)*b*c)/d^2 - 21*(b*c^2 - a*
d^2)/((d*x + c)^(1/4)*d^2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx = \frac{4\,(bc^2-ad^2)}{(dx+c)^{\frac{1}{4}}d^3} - \frac{4\left(3\,(dx+c)^{\frac{7}{4}}bd^{18}-14\,(dx+c)^{\frac{3}{4}}bcd^{18}\right)}{21\,d^{21}}$$

input
```
integrate((-b*x^2+a)/(d*x+c)^(5/4),x, algorithm="giac")
```

output
```
4*(b*c^2 - a*d^2)/((d*x + c)^(1/4)*d^3) - 4/21*(3*(d*x + c)^(7/4)*b*d^18 -
 14*(d*x + c)^(3/4)*b*c*d^18)/d^21
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx = -\frac{12\,b\,(c+d\,x)^2+84\,a\,d^2-84\,b\,c^2-56\,b\,c\,(c+d\,x)}{21\,d^3\,(c+d\,x)^{1/4}}$$

input
```
int((a - b*x^2)/(c + d*x)^(5/4),x)
```

output
```
-(12*b*(c + d*x)^2 + 84*a*d^2 - 84*b*c^2 - 56*b*c*(c + d*x))/(21*d^3*(c +
d*x)^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{a-bx^2}{(c+dx)^{5/4}}\,dx = \frac{-\frac{4}{7}b\,d^2x^2+\frac{32}{21}bcdx-4a\,d^2+\frac{128}{21}b\,c^2}{(dx+c)^{\frac{1}{4}}\,d^3}$$

input
```
int((-b*x^2+a)/(d*x+c)^(5/4),x)
```

output
```
(4*( - 21*a*d**2 + 32*b*c**2 + 8*b*c*d*x - 3*b*d**2*x**2))/(21*(c + d*x)**
(1/4)*d**3)
```

3.213 $\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 300

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx = -\frac{8cd\sqrt[4]{c+dx}}{b}-\frac{4d(c+dx)^{5/4}}{5b} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{13/8}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{13/8}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{13/8}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{13/8}}$$

output
```
-8*c*d*(d*x+c)^(1/4)/b-4/5*d*(d*x+c)^(5/4)/b-(b^(1/2)*c-a^(1/2)*d)^(9/4)*a
rctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(13/8)+
(b^(1/2)*c+a^(1/2)*d)^(9/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2
)*d)^(1/4))/a^(1/2)/b^(13/8)-(b^(1/2)*c-a^(1/2)*d)^(9/4)*arctanh(b^(1/8)*(
d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(13/8)+(b^(1/2)*c+a^(1
/2)*d)^(9/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^
(1/2)/b^(13/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx = \frac{-8bd\sqrt[4]{c+dx}(11c+dx)+5\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-2bc^3d\log\left(\sqrt[4]{c+\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(9/4)/(a - b*x^2),x]
```

output
```
(-8*b*d*(c + d*x)^(1/4)*(11*c + d*x) + 5*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^
4 + b*#1^8 & , (-2*b*c^3*d*Log[(c + d*x)^(1/4) - #1] + 2*a*c*d^3*Log[(c +
d*x)^(1/4) - #1] + 3*b*c^2*d*Log[(c + d*x)^(1/4) - #1]*#1^4 + a*d^3*Log[(c
 + d*x)^(1/4) - #1]*#1^4)/(c*#1^3 - #1^7) & ])/(10*b^2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {481, 25, 653, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx$$

$$\Big\downarrow \text{481}$$

$$-\frac{\int -\frac{\sqrt[4]{c+dx}(bc^2+2bdxc+ad^2)}{a-bx^2}dx}{b}-\frac{4d(c+dx)^{5/4}}{5b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt[4]{c+dx}(bc^2+2bdxc+ad^2)}{a-bx^2}dx}{b}-\frac{4d(c+dx)^{5/4}}{5b}$$

$$\Big\downarrow \text{653}$$

$$\frac{-\frac{\int -\frac{b\left(c\left(bc^2+3ad^2\right)+d\left(3bc^2+ad^2\right)x\right)}{(c+dx)^{3/4}\left(a-bx^2\right)}dx}{b}-8cd\sqrt[4]{c+dx}}{b}-\frac{4d(c+dx)^{5/4}}{5b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\left(c\left(bc^2+3ad^2\right)+d\left(3bc^2+ad^2\right)x\right)}{(c+dx)^{3/4}\left(a-bx^2\right)}dx}{b}-8cd\sqrt[4]{c+dx}}{b}-\frac{4d(c+dx)^{5/4}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{c(bc^2+3ad^2)+d(3bc^2+ad^2)x}{(c+dx)^{3/4}(a-bx^2)}dx-8cd\sqrt[4]{c+dx}}{b}-\frac{4d(c+dx)^{5/4}}{5b}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(\frac{\sqrt{a}c(bc^2+3ad^2)-\frac{ad\left(3bc^2+ad^2\right)}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}+\frac{\frac{ad\left(3bc^2+ad^2\right)}{\sqrt{b}}+\sqrt{a}c(bc^2+3ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx-8cd\sqrt[4]{c+dx}}{b}-\frac{4d(c+dx)^{5/4}}{5b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)^{9/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{5/8}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+}}{\sqrt[4]{\sqrt{b}c-}}\right.}{\sqrt{a}b^{5/8}}}{b}$$

$$\frac{4d(c+dx)^{5/4}}{5b}$$

input
```
Int[(c + d*x)^(9/4)/(a - b*x^2),x]
```

output
```
(-4*d*(c + d*x)^(5/4))/(5*b) + (-8*c*d*(c + d*x)^(1/4) - ((Sqrt[b]*c - Sqr
t[a]*d)^(9/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/
4)])/(Sqrt[a]*b^(5/8)) + ((Sqrt[b]*c + Sqrt[a]*d)^(9/4)*ArcTan[(b^(1/8)*(c
 + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)) - ((Sqrt[
b]*c - Sqrt[a]*d)^(9/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqr
t[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)) + ((Sqrt[b]*c + Sqrt[a]*d)^(9/4)*ArcTanh
[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8
)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.36

method	result
default	$-\frac{d\left(\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\_R^4 a\,d^2+3\_R^4 b\,c^2+2a\,d^2c-2b\,c^3\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+\frac{88\left(\frac{dx}{11}+\right.}{}\right.}{2b^2}$
pseudoelliptic	$-\frac{d\left(\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\_R^4 a\,d^2+3\_R^4 b\,c^2+2a\,d^2c-2b\,c^3\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+\frac{88\left(\frac{dx}{11}+\right.}{}\right.}{2b^2}$
derivativedivides	$-4d\left(\frac{\frac{(dx+c)^{\frac{5}{4}}}{5}+2c(dx+c)^{\frac{1}{4}}}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\left(-a\,d^2-3b\,c^2\right)\_R^4-2a\,d^2c+2b\,c^3\right)\ln\left((dx\right.}{-\_R^7+\_R^3c}}{8b^2}\right.$

input
```
int((d*x+c)^(9/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*(sum((_R^4*a*d^2+3*_R^4*b*c^2+2*a*c*d^2-2*b*c^3)*ln((d*x+c)^(1/4)-_
R)/_R^3/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+88/5*(1/11*d*x+
c)*b*(d*x+c)^(1/4))/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5625 vs. 2(214) = 428.

Time = 1.48 (sec) , antiderivative size = 5625, normalized size of antiderivative = 18.75

$$\int\frac{(c+dx)^{9/4}}{a-bx^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^(9/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx = -\int \frac{c^2\sqrt[4]{c+dx}}{-a+bx^2}\,dx - \int \frac{d^2x^2\sqrt[4]{c+dx}}{-a+bx^2}\,dx - \int \frac{2cdx\sqrt[4]{c+dx}}{-a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(9/4)/(-b*x**2+a),x)
```

output
```
-Integral(c**2*(c + d*x)**(1/4)/(-a + b*x**2), x) - Integral(d**2*x**2*(c
+ d*x)**(1/4)/(-a + b*x**2), x) - Integral(2*c*d*x*(c + d*x)**(1/4)/(-a +
b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{9}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(9/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((d*x + c)^(9/4)/(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{9}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(9/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^(9/4)/(b*x^2 - a), x)
```

Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 12163, normalized size of antiderivative = 40.54

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(9/4)/(a - b*x^2),x)
```

output
```
atan((((((a^4*d^9*(a^5*b^13)^(1/2) + a^2*b^11*c^9 + 9*a^6*b^7*c*d^8 + 36*
a^3*b^10*c^7*d^2 + 126*a^4*b^9*c^5*d^4 + 84*a^5*b^8*c^3*d^6 + 9*b^4*c^8*d*
(a^5*b^13)^(1/2) + 84*a*b^3*c^6*d^3*(a^5*b^13)^(1/2) + 36*a^3*b*c^2*d^7*(a
^5*b^13)^(1/2) + 126*a^2*b^2*c^4*d^5*(a^5*b^13)^(1/2))/(a^4*b^13))^(1/4)*(
(((((a^4*d^9*(a^5*b^13)^(1/2) + a^2*b^11*c^9 + 9*a^6*b^7*c*d^8 + 36*a^3*b^
10*c^7*d^2 + 126*a^4*b^9*c^5*d^4 + 84*a^5*b^8*c^3*d^6 + 9*b^4*c^8*d*(a^5*b
^13)^(1/2) + 84*a*b^3*c^6*d^3*(a^5*b^13)^(1/2) + 36*a^3*b*c^2*d^7*(a^5*b^1
3)^(1/2) + 126*a^2*b^2*c^4*d^5*(a^5*b^13)^(1/2))/(a^4*b^13))^(1/4)*(104857
6*a^6*b^6*d^13 - 1048576*a^3*b^9*c^6*d^7 + 3145728*a^4*b^8*c^4*d^9 - 31457
28*a^5*b^7*c^2*d^11)*1i)/2 - (32768*(c + d*x)^(1/4)*(48*a^6*b^7*c*d^14 - 1
6*a^2*b^11*c^9*d^6 + 96*a^4*b^9*c^5*d^10 - 128*a^5*b^8*c^3*d^12))/b^2)*((a
^4*d^9*(a^5*b^13)^(1/2) + a^2*b^11*c^9 + 9*a^6*b^7*c*d^8 + 36*a^3*b^10*c^7
*d^2 + 126*a^4*b^9*c^5*d^4 + 84*a^5*b^8*c^3*d^6 + 9*b^4*c^8*d*(a^5*b^13)^(
1/2) + 84*a*b^3*c^6*d^3*(a^5*b^13)^(1/2) + 36*a^3*b*c^2*d^7*(a^5*b^13)^(1/
2) + 126*a^2*b^2*c^4*d^5*(a^5*b^13)^(1/2))/(a^4*b^13))^(3/4)*1i)/8 + 52428
8*a^8*c*d^21 + 524288*a*b^7*c^15*d^7 + 1572864*a^7*b*c^3*d^19 + 1572864*a^
2*b^6*c^13*d^9 - 7864320*a^3*b^5*c^11*d^11 + 5767168*a^4*b^4*c^9*d^13 + 57
67168*a^5*b^3*c^7*d^15 - 7864320*a^6*b^2*c^5*d^17)*1i)/2 + (32768*(c + d*x
)^(1/4)*(a^9*d^24 + b^9*c^18*d^6 + 9*a*b^8*c^16*d^8 + 9*a^8*b*c^2*d^22 - 6
0*a^2*b^7*c^14*d^10 + 116*a^3*b^6*c^12*d^12 - 66*a^4*b^5*c^10*d^14 - 66...
```

Reduce [F]

$$\int \frac{(c+dx)^{9/4}}{a-bx^2}\,dx = \int \frac{(dx+c)^{\frac{9}{4}}}{-b\,x^2+a}dx$$

input
```
int((d*x+c)^(9/4)/(-b*x^2+a),x)
```

output
```
int((d*x+c)^(9/4)/(-b*x^2+a),x)
```

3.214 $\int \frac{(c+dx)^{7/4}}{a-bx^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 284

$$\int \frac{(c+dx)^{7/4}}{a-bx^2} dx = -\frac{4d(c+dx)^{3/4}}{3b} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4} \arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{11/8}} - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4} \arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{11/8}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4} \operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{11/8}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4} \operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{11/8}}$$

output
```
-4/3*d*(d*x+c)^(3/4)/b+(b^(1/2)*c-a^(1/2)*d)^(7/4)*arctan(b^(1/8)*(d*x+c)^
(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(11/8)-(b^(1/2)*c+a^(1/2)*d)^
(7/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^
(11/8)-(b^(1/2)*c-a^(1/2)*d)^(7/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*
c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(11/8)+(b^(1/2)*c+a^(1/2)*d)^(7/4)*arctanh(b
^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^(11/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx =$$
$$-\frac{d\left(8b(c+dx)^{3/4}-3\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-bc^2\log\left(\sqrt[4]{c+dx}-\#1\right)+ad^2\log\left(\sqrt[4]{c+dx}-\#\right.}{c\#1-\#1^5}\right.\right.}{6b^2}$$

input
```
Integrate[(c + d*x)^(7/4)/(a - b*x^2),x]
```

output
```
-1/6*(d*(8*b*(c + d*x)^(3/4) - 3*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1
^8 & , (-(b*c^2*Log[(c + d*x)^(1/4) - #1]) + a*d^2*Log[(c + d*x)^(1/4) - #
1] + 2*b*c*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1 - #1^5) & ]))/b^2
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {481, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx$$

$$\Big\downarrow \text{481}$$

$$-\frac{\int -\frac{bc^2+2bdxc+ad^2}{\sqrt[4]{c+dx}(a-bx^2)}dx}{b}-\frac{4d(c+dx)^{3/4}}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bc^2+2bdxc+ad^2}{\sqrt[4]{c+dx}(a-bx^2)}dx}{b}-\frac{4d(c+dx)^{3/4}}{3b}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(\frac{\sqrt{a}(bc^2+ad^2)-2a\sqrt{b}cd}{2a\left(\sqrt{b}x+\sqrt{a}\right)\sqrt[4]{c+dx}}+\frac{2a\sqrt{b}cd+\sqrt{a}(bc^2+ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}\right)dx}{b}-\frac{4d(c+dx)^{3/4}}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)^{7/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{3/8}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+\ldots}}{\sqrt[4]{\sqrt{b}c-\ldots}}\right.}{\sqrt{a}b^{3/8}}}{b}$$

$$\frac{4d(c+dx)^{3/4}}{3b}$$

input

```
Int[(c + d*x)^(7/4)/(a - b*x^2),x]
```

output

```
(-4*d*(c + d*x)^(3/4))/(3*b) + (((Sqrt[b]*c - Sqrt[a]*d)^(7/4)*ArcTan[(b^(
1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)) -
((Sqrt[b]*c + Sqrt[a]*d)^(7/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c
 + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)) - ((Sqrt[b]*c - Sqrt[a]*d)^(7/4)*A
rcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*
b^(3/8)) + ((Sqrt[b]*c + Sqrt[a]*d)^(7/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4)
)/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.32

method	result
default	$-\frac{d\left(8(dx+c)^{\frac{3}{4}}b+3\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(2\_R^{4}bc+a\,d^{2}-b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)\right)}{6b^{2}}$
pseudoelliptic	$-\frac{d\left(8(dx+c)^{\frac{3}{4}}b+3\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(2\_R^{4}bc+a\,d^{2}-b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)\right)}{6b^{2}}$
risch	$-\frac{4d(dx+c)^{\frac{3}{4}}}{3b}-\frac{d\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(2\_R^{4}bc+a\,d^{2}-b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)}{2b^{2}}$
derivativedivides	$-4d\left(\frac{(dx+c)^{\frac{3}{4}}}{3b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(-2c\_R^{6}b+\left(-a\,d^{2}+b\,c^{2}\right)\_R^{2}\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^{7}+\_R^{3}c}}{8b^{2}}\right.$

input
```
int((d*x+c)^(7/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*d*(8*(d*x+c)^(3/4)*b+3*sum(1/_R*(2*_R^4*b*c+a*d^2-b*c^2)*ln((d*x+c)^(
1/4)-_R)/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2)))/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5710 vs. $2(200) = 400$.

Time = 2.91 (sec) , antiderivative size = 5710, normalized size of antiderivative = 20.11

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(7/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx = -\int \frac{c(c+dx)^{\frac{3}{4}}}{-a+bx^2}\,dx - \int \frac{dx(c+dx)^{\frac{3}{4}}}{-a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(7/4)/(-b*x**2+a),x)
```

output
```
-Integral(c*(c + d*x)**(3/4)/(-a + b*x**2), x) - Integral(d*x*(c + d*x)**(
3/4)/(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{7}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(7/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((d*x + c)^(7/4)/(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{7}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(7/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^(7/4)/(b*x^2 - a), x)
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 11459, normalized size of antiderivative = 40.35

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(7/4)/(a - b*x^2),x)
```

output
```
- atan(((((((16384*(64*a^6*b^6*c*d^15 - 64*a^2*b^10*c^9*d^7 + 128*a^3*b^9*
c^7*d^9 - 128*a^5*b^7*c^3*d^13))/b^2 + ((c + d*x)^(1/4)*((a^3*d^7*(a^5*b^1
1)^(1/2) + a^2*b^9*c^7 + 7*a^5*b^6*c*d^6 + 21*a^3*b^8*c^5*d^2 + 35*a^4*b^7
*c^3*d^4 + 7*b^3*c^6*d*(a^5*b^11)^(1/2) + 35*a*b^2*c^4*d^3*(a^5*b^11)^(1/2
) + 21*a^2*b*c^2*d^5*(a^5*b^11)^(1/2))/(a^4*b^11))^(1/4)*(524288*a^6*b^5*d
^14 - 524288*a^2*b^9*c^8*d^6 + 1048576*a^3*b^8*c^6*d^8 - 1048576*a^5*b^6*c
^2*d^12)*1i)/2)*((a^3*d^7*(a^5*b^11)^(1/2) + a^2*b^9*c^7 + 7*a^5*b^6*c*d^6
 + 21*a^3*b^8*c^5*d^2 + 35*a^4*b^7*c^3*d^4 + 7*b^3*c^6*d*(a^5*b^11)^(1/2)
+ 35*a*b^2*c^4*d^3*(a^5*b^11)^(1/2) + 21*a^2*b*c^2*d^5*(a^5*b^11)^(1/2))/(
a^4*b^11))^(3/4)*1i)/8 + (c + d*x)^(1/4)*(163840*a^7*c*d^20 - 32768*b^7*c^
15*d^6 - 163840*a*b^6*c^13*d^8 - 491520*a^6*b*c^3*d^18 + 1146880*a^2*b^5*c
^11*d^10 - 2129920*a^3*b^4*c^9*d^12 + 1474560*a^4*b^3*c^7*d^14 + 32768*a^5
*b^2*c^5*d^16))*((a^3*d^7*(a^5*b^11)^(1/2) + a^2*b^9*c^7 + 7*a^5*b^6*c*d^6
 + 21*a^3*b^8*c^5*d^2 + 35*a^4*b^7*c^3*d^4 + 7*b^3*c^6*d*(a^5*b^11)^(1/2)
+ 35*a*b^2*c^4*d^3*(a^5*b^11)^(1/2) + 21*a^2*b*c^2*d^5*(a^5*b^11)^(1/2))/(
a^4*b^11))^(1/4))/2 - (((((16384*(64*a^6*b^6*c*d^15 - 64*a^2*b^10*c^9*d^7
+ 128*a^3*b^9*c^7*d^9 - 128*a^5*b^7*c^3*d^13))/b^2 - ((c + d*x)^(1/4)*((a^
3*d^7*(a^5*b^11)^(1/2) + a^2*b^9*c^7 + 7*a^5*b^6*c*d^6 + 21*a^3*b^8*c^5*d^
2 + 35*a^4*b^7*c^3*d^4 + 7*b^3*c^6*d*(a^5*b^11)^(1/2) + 35*a*b^2*c^4*d^3*(
a^5*b^11)^(1/2) + 21*a^2*b*c^2*d^5*(a^5*b^11)^(1/2))/(a^4*b^11))^(1/4)*...
```

Reduce [F]

$$\int \frac{(c+dx)^{7/4}}{a-bx^2}\,dx=\int \frac{(dx+c)^{\frac{7}{4}}}{-b\,x^2+a}dx$$

input
```
int((d*x+c)^(7/4)/(-b*x^2+a),x)
```

output
```
int((d*x+c)^(7/4)/(-b*x^2+a),x)
```

3.215 $\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 282

$$\begin{aligned}\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx &= -\frac{4d\sqrt[4]{c+dx}}{b} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{9/8}} \\ &+ \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{9/8}} \\ &- \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{9/8}} \\ &+ \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{9/8}}\end{aligned}$$

output
```
-4*d*(d*x+c)^(1/4)/b-(b^(1/2)*c-a^(1/2)*d)^(5/4)*arctan(b^(1/8)*(d*x+c)^(1
/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(9/8)+(b^(1/2)*c+a^(1/2)*d)^(5/
4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^(9/
8)-(b^(1/2)*c-a^(1/2)*d)^(5/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^
(1/2)*d)^(1/4))/a^(1/2)/b^(9/8)+(b^(1/2)*c+a^(1/2)*d)^(5/4)*arctanh(b^(1/8
)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^(9/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx=-\frac{4d\sqrt[4]{c+dx}}{b}$$
$$+\frac{\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-bc^2d\log\left(\sqrt[4]{c+dx}-\#1\right)+ad^3\log\left(\sqrt[4]{c+dx}-\#1\right)+2bcd\log\left(\sqrt[4]{c+dx}-\right.}{c\#1^3-\#1^7}\right.}{2b^2}$$

input
```
Integrate[(c + d*x)^(5/4)/(a - b*x^2),x]
```

output
```
(-4*d*(c + d*x)^(1/4))/b + RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & ,
 (-(b*c^2*d*Log[(c + d*x)^(1/4) - #1]) + a*d^3*Log[(c + d*x)^(1/4) - #1] +
 2*b*c*d*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1^3 - #1^7) & ]/(2*b^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {481, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx$$

$$\Big\downarrow \; 481$$

$$-\frac{\int -\frac{bc^2+2bdxc+ad^2}{(c+dx)^{3/4}(a-bx^2)}dx}{b}-\frac{4d\sqrt[4]{c+dx}}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{bc^2+2bdxc+ad^2}{(c+dx)^{3/4}(a-bx^2)}dx}{b}-\frac{4d\sqrt[4]{c+dx}}{b}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{\sqrt{a}(bc^2+ad^2)-2a\sqrt{b}cd}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}+\frac{2a\sqrt{b}cd+\sqrt{a}(bc^2+ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx}{b}-\frac{4d\sqrt[4]{c+dx}}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)^{5/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}\sqrt[8]{b}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+}}{\sqrt[4]{\sqrt{b}c-}}\right.}{\sqrt{a}\sqrt[8]{b}}}{b}$$

$$\frac{4d\sqrt[4]{c+dx}}{b}$$

input
```
Int[(c + d*x)^(5/4)/(a - b*x^2),x]
```

output
```
(-4*d*(c + d*x)^(1/4))/b + (-(((Sqrt[b]*c - Sqrt[a]*d)^(5/4)*ArcTan[(b^(1/
8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8))) + (
(Sqrt[b]*c + Sqrt[a]*d)^(5/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c
+ Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)) - ((Sqrt[b]*c - Sqrt[a]*d)^(5/4)*Ar
cTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b
^(1/8)) + ((Sqrt[b]*c + Sqrt[a]*d)^(5/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))
/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 481
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n - 1)/(b*(n - 1))), x] + Simp[1/b   Int[(c + d*x)^(n - 2)*(Simp[b
*c^2 - a*d^2 + 2*b*c*d*x, x]/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}, x]
 && GtQ[n, 1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.32

method	result	size
default	$-\frac{d\left(8(dx+c)^{\frac{1}{4}}b+\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(2\_R^{4}bc+a\,d^{2}-b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)\right)}{2b^{2}}$	90
pseudoelliptic	$-\frac{d\left(8(dx+c)^{\frac{1}{4}}b+\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(2\_R^{4}bc+a\,d^{2}-b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)\right)}{2b^{2}}$	90
derivativedivides	$-4d\left(\frac{(dx+c)^{\frac{1}{4}}}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(-2\_R^{4}bc-a\,d^{2}+b\,c^{2}\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^{7}+\_R^{3}c}}{8b^{2}}\right)$	94

input
```
int((d*x+c)^(5/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*(8*(d*x+c)^(1/4)*b+sum((2*_R^4*b*c+a*d^2-b*c^2)*ln((d*x+c)^(1/4)-_R
)/_R^3/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2)))/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3132 vs. $2(200) = 400$.

Time = 0.18 (sec) , antiderivative size = 3132, normalized size of antiderivative = 11.11

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx = -\int \frac{c\sqrt[4]{c+dx}}{-a+bx^2}\,dx - \int \frac{dx\sqrt[4]{c+dx}}{-a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(5/4)/(-b*x**2+a),x)
```

output
```
-Integral(c*(c + d*x)**(1/4)/(-a + b*x**2), x) - Integral(d*x*(c + d*x)**(
1/4)/(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{5}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(5/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((d*x + c)^(5/4)/(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{5}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(5/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^(5/4)/(b*x^2 - a), x)
```

Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 5862, normalized size of antiderivative = 20.79

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(5/4)/(a - b*x^2),x)
```

output
```
atan((b^3*c^6*(c + d*x)^(1/4)*1i - a^3*d^6*(c + d*x)^(1/4)*1i + a*b^2*c^4*
d^2*(c + d*x)^(1/4)*5i - a^2*b*c^2*d^4*(c + d*x)^(1/4)*5i + (c*(c + d*x)^(
1/4)*(a^2*d^5*(a^5*b^9)^(1/2) - a^2*b^7*c^5 - 5*a^4*b^5*c*d^4 - 10*a^3*b^6
*c^3*d^2 + 5*b^2*c^4*d*(a^5*b^9)^(1/2) + 10*a*b*c^2*d^3*(a^5*b^9)^(1/2))*1
i)/(a^2*b^4))/(a^3*b*d^5*(-(a^2*d^5*(a^5*b^9)^(1/2) - a^2*b^7*c^5 - 5*a^4*
b^5*c*d^4 - 10*a^3*b^6*c^3*d^2 + 5*b^2*c^4*d*(a^5*b^9)^(1/2) + 10*a*b*c^2*
d^3*(a^5*b^9)^(1/2))/(a^4*b^9))^(1/4) + 10*a^2*b^2*c^2*d^3*(-(a^2*d^5*(a^5
*b^9)^(1/2) - a^2*b^7*c^5 - 5*a^4*b^5*c*d^4 - 10*a^3*b^6*c^3*d^2 + 5*b^2*c
^4*d*(a^5*b^9)^(1/2) + 10*a*b*c^2*d^3*(a^5*b^9)^(1/2))/(a^4*b^9))^(1/4) +
5*a*b^3*c^4*d*(-(a^2*d^5*(a^5*b^9)^(1/2) - a^2*b^7*c^5 - 5*a^4*b^5*c*d^4 -
 10*a^3*b^6*c^3*d^2 + 5*b^2*c^4*d*(a^5*b^9)^(1/2) + 10*a*b*c^2*d^3*(a^5*b^
9)^(1/2))/(a^4*b^9))^(1/4)))*(-(a^2*d^5*(a^5*b^9)^(1/2) - a^2*b^7*c^5 - 5*
a^4*b^5*c*d^4 - 10*a^3*b^6*c^3*d^2 + 5*b^2*c^4*d*(a^5*b^9)^(1/2) + 10*a*b*
c^2*d^3*(a^5*b^9)^(1/2))/(16*a^4*b^9))^(1/4)*2i - atan(((((c + d*x)^(1/4)*
(32768*a^6*b*d^18 + 32768*b^7*c^12*d^6 + 65536*a*b^6*c^10*d^8 - 557056*a^2
*b^5*c^8*d^10 + 917504*a^3*b^4*c^6*d^12 - 557056*a^4*b^3*c^4*d^14 + 65536*
a^5*b^2*c^2*d^16) + ((-(a^2*d^5*(a^5*b^9)^(1/2) - a^2*b^7*c^5 - 5*a^4*b^5*
c*d^4 - 10*a^3*b^6*c^3*d^2 + 5*b^2*c^4*d*(a^5*b^9)^(1/2) + 10*a*b*c^2*d^3*
(a^5*b^9)^(1/2))/(a^4*b^9))^(1/4)*(65536*a^6*b^2*d^17 - ((c + d*x)^(1/4)*(
-(a^2*d^5*(a^5*b^9)^(1/2) - a^2*b^7*c^5 - 5*a^4*b^5*c*d^4 - 10*a^3*b^6*...
```

Reduce [F]

$$\int \frac{(c+dx)^{5/4}}{a-bx^2}\,dx = \int \frac{(dx+c)^{\frac{5}{4}}}{-b\,x^2+a}dx$$

input
```
int((d*x+c)^(5/4)/(-b*x^2+a),x)
```

output
```
int((d*x+c)^(5/4)/(-b*x^2+a),x)
```

3.216 $\int \frac{(c+dx)^{3/4}}{a-bx^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$
\begin{aligned}
\int \frac{(c+dx)^{3/4}}{a-bx^2} \, dx = {} & \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4} \arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}} \\
& - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4} \arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}} \\
& - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4} \operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}} \\
& + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4} \operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}}
\end{aligned}
$$

output
```
(b^(1/2)*c-a^(1/2)*d)^(3/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2
)*d)^(1/4))/a^(1/2)/b^(7/8)-(b^(1/2)*c+a^(1/2)*d)^(3/4)*arctan(b^(1/8)*(d*
x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^(7/8)-(b^(1/2)*c-a^(1/2)
*d)^(3/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/
2)/b^(7/8)+(b^(1/2)*c+a^(1/2)*d)^(3/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1
/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^(7/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx=-\frac{d\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{\log\left(\sqrt[4]{c+dx}-\#1\right)\#1^3}{-c+\#1^4}\&\right]}{2b}$$

input
```
Integrate[(c + d*x)^(3/4)/(a - b*x^2),x]
```

output
```
-1/2*(d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (Log[(c + d*x)^(1/
4) - #1]*#1^3)/(-c + #1^4) & ])/b
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx$$

$$\Big\downarrow \text{ 485}$$

$$\int \left(\frac{(c+dx)^{3/4}}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}x\right)}+\frac{(c+dx)^{3/4}}{2\sqrt{a}\left(\sqrt{a}+\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{7/8}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{7/8}}$$

input
```
Int[(c + d*x)^(3/4)/(a - b*x^2),x]
```

output
```
((Sqrt[b]*c - Sqrt[a]*d)^(3/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c
 - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)) - ((Sqrt[b]*c + Sqrt[a]*d)^(3/4)*A
rcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b
^(7/8)) - ((Sqrt[b]*c - Sqrt[a]*d)^(3/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))
/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)) + ((Sqrt[b]*c + Sqrt[a]
*d)^(3/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)]
)/(Sqrt[a]*b^(7/8))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.22

method	result	size
default	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{3}\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{4}-c}\right)}{2b}$	60
pseudoelliptic	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{3}\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{4}-c}\right)}{2b}$	60
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{6}\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^{7}+\_R^{3}c}\right)}{2b}$	64

input
```
int((d*x+c)^(3/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*sum(_R^3*ln((d*x+c)^(1/4)-_R)/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-
a*d^2+b*c^2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2521 vs. 2(187) = 374.

Time = 0.23 (sec) , antiderivative size = 2521, normalized size of antiderivative = 9.44

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(-sqrt((a^2*b^3*sqrt((9*b^2*c^4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^
3*b^7)) + b*c^3 + 3*a*c*d^2)/(a^2*b^3)))*log((6*a^2*b^4*c^3*d^2 + 2*a^3*b^
3*c*d^4 - (a^3*b^7*c^2 + a^4*b^6*d^2)*sqrt((9*b^2*c^4*d^2 + 6*a*b*c^2*d^4
+ a^2*d^6)/(a^3*b^7)))*sqrt(-sqrt((a^2*b^3*sqrt((9*b^2*c^4*d^2 + 6*a*b*c^2
*d^4 + a^2*d^6)/(a^3*b^7)) + b*c^3 + 3*a*c*d^2)/(a^2*b^3)))*sqrt((a^2*b^3*
sqrt((9*b^2*c^4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^3*b^7)) + b*c^3 + 3*a*c*
d^2)/(a^2*b^3)) + (3*b^3*c^6*d - 5*a*b^2*c^4*d^3 + a^2*b*c^2*d^5 + a^3*d^7
)*(d*x + c)^(1/4)) - 1/2*sqrt(-sqrt((a^2*b^3*sqrt((9*b^2*c^4*d^2 + 6*a*b*c
^2*d^4 + a^2*d^6)/(a^3*b^7)) + b*c^3 + 3*a*c*d^2)/(a^2*b^3)))*log(-(6*a^2*
b^4*c^3*d^2 + 2*a^3*b^3*c*d^4 - (a^3*b^7*c^2 + a^4*b^6*d^2)*sqrt((9*b^2*c^
4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^3*b^7)))*sqrt(-sqrt((a^2*b^3*sqrt((9*b
^2*c^4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^3*b^7)) + b*c^3 + 3*a*c*d^2)/(a^2
*b^3)))*sqrt((a^2*b^3*sqrt((9*b^2*c^4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^3*
b^7)) + b*c^3 + 3*a*c*d^2)/(a^2*b^3)) + (3*b^3*c^6*d - 5*a*b^2*c^4*d^3 + a
^2*b*c^2*d^5 + a^3*d^7)*(d*x + c)^(1/4)) + 1/2*sqrt(-sqrt(-(a^2*b^3*sqrt((
9*b^2*c^4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^3*b^7)) - b*c^3 - 3*a*c*d^2)/(
a^2*b^3)))*log((6*a^2*b^4*c^3*d^2 + 2*a^3*b^3*c*d^4 + (a^3*b^7*c^2 + a^4*b
^6*d^2)*sqrt((9*b^2*c^4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^3*b^7)))*sqrt(-s
qrt(-(a^2*b^3*sqrt((9*b^2*c^4*d^2 + 6*a*b*c^2*d^4 + a^2*d^6)/(a^3*b^7)) -
b*c^3 - 3*a*c*d^2)/(a^2*b^3)))*sqrt(-(a^2*b^3*sqrt((9*b^2*c^4*d^2 + 6*a...
```

Sympy [F]

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx = -\int \frac{(c+dx)^{\frac{3}{4}}}{-a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(3/4)/(-b*x**2+a),x)
```

output
```
-Integral((c + d*x)**(3/4)/(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{3}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(3/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((d*x + c)^(3/4)/(b*x^2 - a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(3/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 3910, normalized size of antiderivative = 14.64

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(3/4)/(a - b*x^2),x)
```

output
```
atan((32768*a^2*d^15*(a^5*b^7)^(1/2)*(c + d*x)^(1/4)*(c^3/(a^2*b^2) + (3*c
*d^2)/(a*b^3) + (d^3*(a^5*b^7)^(1/2))/(a^3*b^7) + (3*c^2*d*(a^5*b^7)^(1/2)
)/(a^4*b^6))^(1/4))/(98304*b^7*c^9*d^7 - 262144*a*b^6*c^7*d^9 - 32768*a^4*
b^3*c*d^15 + 196608*a^2*b^5*c^5*d^11 + 196608*b*c^4*d^12*(a^5*b^7)^(1/2) -
 (32768*a^2*d^16*(a^5*b^7)^(1/2))/b - (262144*b^2*c^6*d^10*(a^5*b^7)^(1/2)
)/a + (98304*b^3*c^8*d^8*(a^5*b^7)^(1/2))/a^2) + (262144*b^2*c^6*d^9*(a^5*
b^7)^(1/2)*(c + d*x)^(1/4)*(c^3/(a^2*b^2) + (3*c*d^2)/(a*b^3) + (d^3*(a^5*
b^7)^(1/2))/(a^3*b^7) + (3*c^2*d*(a^5*b^7)^(1/2))/(a^4*b^6))^(1/4))/(98304
*a*b^6*c^9*d^7 - 32768*a^5*b^2*c*d^15 - 262144*a^2*b^5*c^7*d^9 + 196608*a^
3*b^4*c^5*d^11 + 196608*a*c^4*d^12*(a^5*b^7)^(1/2) - 262144*b*c^6*d^10*(a^
5*b^7)^(1/2) - (32768*a^3*d^16*(a^5*b^7)^(1/2))/b^2 + (98304*b^2*c^8*d^8*(
a^5*b^7)^(1/2))/a) - (196608*b*c^4*d^11*(a^5*b^7)^(1/2)*(c + d*x)^(1/4)*(c
^3/(a^2*b^2) + (3*c*d^2)/(a*b^3) + (d^3*(a^5*b^7)^(1/2))/(a^3*b^7) + (3*c^
2*d*(a^5*b^7)^(1/2))/(a^4*b^6))^(1/4))/(196608*c^4*d^12*(a^5*b^7)^(1/2) +
98304*b^6*c^9*d^7 - 262144*a*b^5*c^7*d^9 - 32768*a^4*b^2*c*d^15 + 196608*a
^2*b^4*c^5*d^11 - (32768*a^2*d^16*(a^5*b^7)^(1/2))/b^2 - (262144*b*c^6*d^1
0*(a^5*b^7)^(1/2))/a + (98304*b^2*c^8*d^8*(a^5*b^7)^(1/2))/a^2) + (98304*b
^3*c^8*d^7*(a^5*b^7)^(1/2)*(c + d*x)^(1/4)*(c^3/(a^2*b^2) + (3*c*d^2)/(a*b
^3) + (d^3*(a^5*b^7)^(1/2))/(a^3*b^7) + (3*c^2*d*(a^5*b^7)^(1/2))/(a^4*b^6
))^(1/4))/(32768*a^6*b^2*c*d^15 - 98304*a^2*b^6*c^9*d^7 + 262144*a^3*b^...
```

Reduce [F]

$$\int \frac{(c+dx)^{3/4}}{a-bx^2}\,dx=\int \frac{(dx+c)^{\frac{3}{4}}}{-b\,x^2+a}dx$$

input
```
int((d*x+c)^(3/4)/(-b*x^2+a),x)
```

output
```
int((d*x+c)^(3/4)/(-b*x^2+a),x)
```

3.217 $\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx = -\frac{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}} + \frac{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}} - \frac{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}} + \frac{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}}$$

output
```
-(b^(1/2)*c-a^(1/2)*d)^(1/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/
2)*d)^(1/4))/a^(1/2)/b^(5/8)+(b^(1/2)*c+a^(1/2)*d)^(1/4)*arctan(b^(1/8)*(d
*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^(5/8)-(b^(1/2)*c-a^(1/2
)*d)^(1/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1
/2)/b^(5/8)+(b^(1/2)*c+a^(1/2)*d)^(1/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(
1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/b^(5/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx=-\frac{d\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{\log\left(\sqrt[4]{c+dx}-\#1\right)\#1}{-c+\#1^4}\&\right]}{2b}$$

input
```
Integrate[(c + d*x)^(1/4)/(a - b*x^2),x]
```

output
```
-1/2*(d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (Log[(c + d*x)^(1/
4) - #1]*#1)/(-c + #1^4) & ])/b
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx$$

$$\Big\downarrow \text{ 485}$$

$$\int \left(\frac{\sqrt[4]{c+dx}}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}x\right)}+\frac{\sqrt[4]{c+dx}}{2\sqrt{a}\left(\sqrt{a}+\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}}+\frac{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{5/8}}-\frac{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}}+\frac{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{5/8}}$$

input
```
Int[(c + d*x)^(1/4)/(a - b*x^2),x]
```

output
```
-(((Sqrt[b]*c - Sqrt[a]*d)^(1/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]
*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8))) + ((Sqrt[b]*c + Sqrt[a]*d)^(1/4
)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a
]*b^(5/8)) - ((Sqrt[b]*c - Sqrt[a]*d)^(1/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/
4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)) + ((Sqrt[b]*c + Sqrt
[a]*d)^(1/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/
4)])/(Sqrt[a]*b^(5/8))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.22

method	result	size
default	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)\_R}{\_R^{4}-c}\right)}{2b}$	58
pseudoelliptic	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)\_R}{\_R^{4}-c}\right)}{2b}$	58
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{4}\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^{7}+\_R^{3}c}\right)}{2b}$	64

input
```
int((d*x+c)^(1/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*sum(ln((d*x+c)^(1/4)-_R)*_R/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*
d^2+b*c^2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(187) = 374$.

Time = 0.09 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.99

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(-sqrt((a^2*b^2*sqrt(d^2/(a^3*b^5)) + c)/(a^2*b^2)))*log(a^2*b^3*s
qrt(-sqrt((a^2*b^2*sqrt(d^2/(a^3*b^5)) + c)/(a^2*b^2)))*sqrt(d^2/(a^3*b^5)
) + (d*x + c)^(1/4)*d) - 1/2*sqrt(-sqrt((a^2*b^2*sqrt(d^2/(a^3*b^5)) + c)/
(a^2*b^2)))*log(-a^2*b^3*sqrt(-sqrt((a^2*b^2*sqrt(d^2/(a^3*b^5)) + c)/(a^2
*b^2)))*sqrt(d^2/(a^3*b^5)) + (d*x + c)^(1/4)*d) - 1/2*sqrt(-sqrt(-(a^2*b^
2*sqrt(d^2/(a^3*b^5)) - c)/(a^2*b^2)))*log(a^2*b^3*sqrt(-sqrt(-(a^2*b^2*sq
rt(d^2/(a^3*b^5)) - c)/(a^2*b^2)))*sqrt(d^2/(a^3*b^5)) + (d*x + c)^(1/4)*d
) + 1/2*sqrt(-sqrt(-(a^2*b^2*sqrt(d^2/(a^3*b^5)) - c)/(a^2*b^2)))*log(-a^2
*b^3*sqrt(-sqrt(-(a^2*b^2*sqrt(d^2/(a^3*b^5)) - c)/(a^2*b^2)))*sqrt(d^2/(a
^3*b^5)) + (d*x + c)^(1/4)*d) + 1/2*((a^2*b^2*sqrt(d^2/(a^3*b^5)) + c)/(a^
2*b^2))^(1/4)*log(a^2*b^3*((a^2*b^2*sqrt(d^2/(a^3*b^5)) + c)/(a^2*b^2))^(1
/4)*sqrt(d^2/(a^3*b^5)) + (d*x + c)^(1/4)*d) - 1/2*((a^2*b^2*sqrt(d^2/(a^3
*b^5)) + c)/(a^2*b^2))^(1/4)*log(-a^2*b^3*((a^2*b^2*sqrt(d^2/(a^3*b^5)) +
c)/(a^2*b^2))^(1/4)*sqrt(d^2/(a^3*b^5)) + (d*x + c)^(1/4)*d) - 1/2*(-(a^2*
b^2*sqrt(d^2/(a^3*b^5)) - c)/(a^2*b^2))^(1/4)*log(a^2*b^3*(-(a^2*b^2*sqrt(
d^2/(a^3*b^5)) - c)/(a^2*b^2))^(1/4)*sqrt(d^2/(a^3*b^5)) + (d*x + c)^(1/4)
*d) + 1/2*(-(a^2*b^2*sqrt(d^2/(a^3*b^5)) - c)/(a^2*b^2))^(1/4)*log(-a^2*b^
3*(-(a^2*b^2*sqrt(d^2/(a^3*b^5)) - c)/(a^2*b^2))^(1/4)*sqrt(d^2/(a^3*b^5))
 + (d*x + c)^(1/4)*d)
```

Sympy [F]

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx = -\int \frac{\sqrt[4]{c+dx}}{-a+bx^2}\,dx$$

input
```
integrate((d*x+c)**(1/4)/(-b*x**2+a),x)
```

output
```
-Integral((c + d*x)**(1/4)/(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{1}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(1/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((d*x + c)^(1/4)/(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx = \int -\frac{(dx+c)^{\frac{1}{4}}}{bx^2-a}\,dx$$

input
```
integrate((d*x+c)^(1/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 703, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx = \mathrm{atan}\left(\frac{a\,d^2\,(c+d\,x)^{1/4}+b\,c^2\,(c+d\,x)^{1/4}-\frac{c\,\left(d\,\sqrt{a^5\,b^5}+a^2\,b^3\,c\right)\,(c+d\,x)^{1/4}}{a^2\,b^2}}{a^3\,b^3\,d\,\left(\frac{d\,\sqrt{a^5\,b^5}+a^2\,b^3\,c}{a^4\,b^5}\right)^{5/4}-2\,a\,b\,c\,d\,\left(\frac{d\,\sqrt{a^5\,b^5}+a^2\,b^3\,c}{a^4\,b^5}\right)^{1/4}}\right)\,\left(\frac{d\,\sqrt{a^5\,b^5}+a^2\,b^3\,c}{a^4\,b^5}\right)^{1/4}+\mathrm{atan}\Bigg($$

input
```
int((c + d*x)^(1/4)/(a - b*x^2),x)
```

output
```
atan((a*d^2*(c + d*x)^(1/4) + b*c^2*(c + d*x)^(1/4) - (c*(d*(a^5*b^5)^(1/2
) + a^2*b^3*c)*(c + d*x)^(1/4))/(a^2*b^2))/(a^3*b^3*d*((d*(a^5*b^5)^(1/2)
+ a^2*b^3*c)/(a^4*b^5))^(5/4) - 2*a*b*c*d*((d*(a^5*b^5)^(1/2) + a^2*b^3*c)
/(a^4*b^5))^(1/4)))*((d*(a^5*b^5)^(1/2) + a^2*b^3*c)/(a^4*b^5))^(1/4) + at
an((a*d^2*(c + d*x)^(1/4) + b*c^2*(c + d*x)^(1/4) + (c*(d*(a^5*b^5)^(1/2)
- a^2*b^3*c)*(c + d*x)^(1/4))/(a^2*b^2))/(a^3*b^3*d*(-(d*(a^5*b^5)^(1/2) -
 a^2*b^3*c)/(a^4*b^5))^(5/4) - 2*a*b*c*d*(-(d*(a^5*b^5)^(1/2) - a^2*b^3*c)
/(a^4*b^5))^(1/4)))*(-(d*(a^5*b^5)^(1/2) - a^2*b^3*c)/(a^4*b^5))^(1/4) - a
tan((a*d^2*(c + d*x)^(1/4)*1i + b*c^2*(c + d*x)^(1/4)*1i - (c*(d*(a^5*b^5)
^(1/2) + a^2*b^3*c)*(c + d*x)^(1/4)*1i)/(a^2*b^2))/(a^3*b^3*d*((d*(a^5*b^5
)^(1/2) + a^2*b^3*c)/(a^4*b^5))^(5/4) - 2*a*b*c*d*((d*(a^5*b^5)^(1/2) + a^
2*b^3*c)/(a^4*b^5))^(1/4)))*((d*(a^5*b^5)^(1/2) + a^2*b^3*c)/(16*a^4*b^5))
^(1/4)*2i - atan((a*d^2*(c + d*x)^(1/4)*1i + b*c^2*(c + d*x)^(1/4)*1i + (c
*(d*(a^5*b^5)^(1/2) - a^2*b^3*c)*(c + d*x)^(1/4)*1i)/(a^2*b^2))/(a^3*b^3*d
*(-(d*(a^5*b^5)^(1/2) - a^2*b^3*c)/(a^4*b^5))^(5/4) - 2*a*b*c*d*(-(d*(a^5*
b^5)^(1/2) - a^2*b^3*c)/(a^4*b^5))^(1/4)))*(-(d*(a^5*b^5)^(1/2) - a^2*b^3*
c)/(16*a^4*b^5))^(1/4)*2i
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+dx}}{a-bx^2}\,dx = \int \frac{(dx+c)^{\frac{1}{4}}}{-b\,x^2+a}dx$$

input
```
int((d*x+c)^(1/4)/(-b*x^2+a),x)
```

output
```
int((d*x+c)^(1/4)/(-b*x^2+a),x)
```

3.218 $\int \frac{1}{\sqrt[4]{c+dx}(a-bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}-\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}-\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}$$

output
```
arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(3/8)/
(b^(1/2)*c-a^(1/2)*d)^(1/4)-arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2
)*d)^(1/4))/a^(1/2)/b^(3/8)/(b^(1/2)*c+a^(1/2)*d)^(1/4)-arctanh(b^(1/8)*(d
*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(3/8)/(b^(1/2)*c-a^(1/2
)*d)^(1/4)+arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1
/2)/b^(3/8)/(b^(1/2)*c+a^(1/2)*d)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx = \frac{d\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{\log\left(\sqrt[4]{c+dx}-\#1\right)}{c\#1-\#1^5}\&\right]}{2b}$$

input
```
Integrate[1/((c + d*x)^(1/4)*(a - b*x^2)),x]
```

output
```
(d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , Log[(c + d*x)^(1/4) - #
1]/(c*#1 - #1^5) & ])/(2*b)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)\sqrt[4]{c+dx}}\,dx$$

$$\Big\downarrow \text{485}$$

$$\int \left(\frac{1}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}+\frac{1}{2\sqrt{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt[4]{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}-\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}-\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}$$

input
```
Int[1/((c + d*x)^(1/4)*(a - b*x^2)),x]
```

output
```
ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)]/(Sqrt[a]*b
^(3/8)*(Sqrt[b]*c - Sqrt[a]*d)^(1/4)) - ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(
Sqrt[b]*c + Sqrt[a]*d)^(1/4)]/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c + Sqrt[a]*d)^(1/
4)) - ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)]/(Sq
rt[a]*b^(3/8)*(Sqrt[b]*c - Sqrt[a]*d)^(1/4)) + ArcTanh[(b^(1/8)*(c + d*x)^
(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)]/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c + Sqrt[a
]*d)^(1/4))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.22

method	result	size
default	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)}{2b}$	60
pseudoelliptic	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)}{2b}$	60
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\_R^{2}\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{-\_R^{7}+\_R^{3}c}\right)}{2b}$	64

input
```
int(1/(d*x+c)^(1/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*sum(1/_R*ln((d*x+c)^(1/4)-_R)/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-
a*d^2+b*c^2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(187) = 374$.

Time = 0.12 (sec) , antiderivative size = 2611, normalized size of antiderivative = 9.78

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(1/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-sqrt(((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b
^4*c^2*d^2 + a^5*b^3*d^4)) + c)/(a^2*b^2*c^2 - a^3*b*d^2)))*log((a^2*b*d^2
 - (a^3*b^4*c^3 - a^4*b^3*c*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^2*d^2
 + a^5*b^3*d^4)))*sqrt(-sqrt(((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*
c^4 - 2*a^4*b^4*c^2*d^2 + a^5*b^3*d^4)) + c)/(a^2*b^2*c^2 - a^3*b*d^2)))*s
qrt(((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^2*d^2 +
 a^5*b^3*d^4)) + c)/(a^2*b^2*c^2 - a^3*b*d^2)) + (d*x + c)^(1/4)*d) + 1/2*
sqrt(-sqrt(((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^
2*d^2 + a^5*b^3*d^4)) + c)/(a^2*b^2*c^2 - a^3*b*d^2)))*log(-(a^2*b*d^2 - (
a^3*b^4*c^3 - a^4*b^3*c*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^2*d^2 + a
^5*b^3*d^4)))*sqrt(-sqrt(((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4
- 2*a^4*b^4*c^2*d^2 + a^5*b^3*d^4)) + c)/(a^2*b^2*c^2 - a^3*b*d^2)))*sqrt(
((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^2*d^2 + a^5
*b^3*d^4)) + c)/(a^2*b^2*c^2 - a^3*b*d^2)) + (d*x + c)^(1/4)*d) - 1/2*sqrt
(-sqrt(-((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^2*d
^2 + a^5*b^3*d^4)) - c)/(a^2*b^2*c^2 - a^3*b*d^2)))*log((a^2*b*d^2 + (a^3*
b^4*c^3 - a^4*b^3*c*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^2*d^2 + a^5*b
^3*d^4)))*sqrt(-sqrt(-((a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2
*a^4*b^4*c^2*d^2 + a^5*b^3*d^4)) - c)/(a^2*b^2*c^2 - a^3*b*d^2)))*sqrt(-((
a^2*b^2*c^2 - a^3*b*d^2)*sqrt(d^2/(a^3*b^5*c^4 - 2*a^4*b^4*c^2*d^2 + a^...
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx = -\int \frac{1}{-a\sqrt[4]{c+dx}+bx^2\sqrt[4]{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/4)/(-b*x**2+a),x)
```

output
```
-Integral(1/(-a*(c + d*x)**(1/4) + b*x**2*(c + d*x)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(d*x + c)^(1/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)^(1/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:int()  Bad Argument Typesym2poly/r2
sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
 ValueDon
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 4203, normalized size of antiderivative = 15.74

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)*(c + d*x)^(1/4)),x)
```

output
```
- atan(((((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2))^(1/
4)*((((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2))^(3/4)*(
(((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2))^(1/4)*(5242
88*a^4*b^7*d^10 - 524288*a^2*b^9*c^4*d^6)*(c + d*x)^(1/4)*1i)/2 - 524288*a
^3*b^7*c*d^9 + 524288*a^2*b^8*c^3*d^7)*1i)/8 - (32768*b^7*c^3*d^6 - 32768*
a*b^6*c*d^8)*(c + d*x)^(1/4)))/2 + (((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*
b^4*c^2 - a^5*b^3*d^2))^(1/4)*((((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*
c^2 - a^5*b^3*d^2))^(3/4)*((((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2
- a^5*b^3*d^2))^(1/4)*(524288*a^4*b^7*d^10 - 524288*a^2*b^9*c^4*d^6)*(c +
d*x)^(1/4)*1i)/2 + 524288*a^3*b^7*c*d^9 - 524288*a^2*b^8*c^3*d^7)*1i)/8 -
(32768*b^7*c^3*d^6 - 32768*a*b^6*c*d^8)*(c + d*x)^(1/4)))/2)/((((d*(a^5*b^
3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2))^(1/4)*((((d*(a^5*b^3)^(
1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2))^(3/4)*((((d*(a^5*b^3)^(1/2)
 + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2))^(1/4)*(524288*a^4*b^7*d^10 - 52
4288*a^2*b^9*c^4*d^6)*(c + d*x)^(1/4)*1i)/2 - 524288*a^3*b^7*c*d^9 + 52428
8*a^2*b^8*c^3*d^7)*1i)/8 - (32768*b^7*c^3*d^6 - 32768*a*b^6*c*d^8)*(c + d*
x)^(1/4))*1i)/2 - (((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3
*d^2))^(1/4)*((((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2
))^(3/4)*((((d*(a^5*b^3)^(1/2) + a^2*b^2*c)/(a^4*b^4*c^2 - a^5*b^3*d^2))^(
1/4)*(524288*a^4*b^7*d^10 - 524288*a^2*b^9*c^4*d^6)*(c + d*x)^(1/4)*1i)...
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{4}}\,(-b\,x^2+a)}dx$$

input
```
int(1/(d*x+c)^(1/4)/(-b*x^2+a),x)
```

output
```
int(1/(d*x+c)^(1/4)/(-b*x^2+a),x)
```

3.219 $\int \frac{1}{(c+dx)^{3/4}(a-bx^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)} dx = -\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}} + \frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4}}$$

output
```
-arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(1/8)
/(b^(1/2)*c-a^(1/2)*d)^(3/4)+arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/
2)*d)^(1/4))/a^(1/2)/b^(1/8)/(b^(1/2)*c+a^(1/2)*d)^(3/4)-arctanh(b^(1/8)*(
d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/b^(1/8)/(b^(1/2)*c-a^(1/
2)*d)^(3/4)+arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(
1/2)/b^(1/8)/(b^(1/2)*c+a^(1/2)*d)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.25

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)}\,dx = \frac{d\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{\log\left(\sqrt[4]{c+dx}-\#1\right)}{c\#1^3-\#1^7}\&\right]}{2b}$$

input
```
Integrate[1/((c + d*x)^(3/4)*(a - b*x^2)),x]
```

output
```
(d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , Log[(c + d*x)^(1/4) - #
1]/(c*#1^3 - #1^7) & ])/(2*b)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)(c+dx)^{3/4}}\,dx$$

$$\Big\downarrow \text{485}$$

$$\int \left(\frac{1}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}+\frac{1}{2\sqrt{a}\left(\sqrt{a}+\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}}+\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}}$$

input
```
Int[1/((c + d*x)^(3/4)*(a - b*x^2)),x]
```

output
```
-(ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)]/(Sqrt[a]
*b^(1/8)*(Sqrt[b]*c - Sqrt[a]*d)^(3/4))) + ArcTan[(b^(1/8)*(c + d*x)^(1/4)
)/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)]/(Sqrt[a]*b^(1/8)*(Sqrt[b]*c + Sqrt[a]*d)^
(3/4)) - ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)]/
(Sqrt[a]*b^(1/8)*(Sqrt[b]*c - Sqrt[a]*d)^(3/4)) + ArcTanh[(b^(1/8)*(c + d*
x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)]/(Sqrt[a]*b^(1/8)*(Sqrt[b]*c + Sqr
t[a]*d)^(3/4))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.22

method	result	size
default	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)}{2b}$	60
pseudoelliptic	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)}{2b}$	60
derivativedivides	$\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^{7}+\_R^{3}c}\right)}{2b}$	61

input
```
int(1/(d*x+c)^(3/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*sum(ln((d*x+c)^(1/4)-_R)/_R^3/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-
a*d^2+b*c^2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5497 vs. $2(187) = 374$.

Time = 0.19 (sec) , antiderivative size = 5497, normalized size of antiderivative = 20.59

$$\int \frac{1}{(c+dx)^{3/4}\left(a-bx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(3/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)}\,dx = -\int \frac{1}{-a(c+dx)^{\frac{3}{4}} + bx^2(c+dx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)**(3/4)/(-b*x**2+a),x)
```

output
```
-Integral(1/(-a*(c + d*x)**(3/4) + b*x**2*(c + d*x)**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(d*x + c)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*(d*x + c)^(3/4)), x)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 8501, normalized size of antiderivative = 31.84

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)*(c + d*x)^(3/4)),x)
```

output
```
- atan(((((((((((a*d^3*(a^5*b)^(1/2) - a^2*b^2*c^3 - 3*a^3*b*c*d^2 + 3*b*c
^2*d*(a^5*b)^(1/2))/(a^7*b*d^6 - a^4*b^4*c^6 + 3*a^5*b^3*c^4*d^2 - 3*a^6*b
^2*c^2*d^4))^(1/4)*(2097152*a^4*b^8*c*d^9 - 2097152*a^3*b^9*c^3*d^7)*1i)/2
 + (1572864*a^3*b^8*c*d^8 + 524288*a^2*b^9*c^3*d^6)*(c + d*x)^(1/4))*((a*d
^3*(a^5*b)^(1/2) - a^2*b^2*c^3 - 3*a^3*b*c*d^2 + 3*b*c^2*d*(a^5*b)^(1/2))/
(a^7*b*d^6 - a^4*b^4*c^6 + 3*a^5*b^3*c^4*d^2 - 3*a^6*b^2*c^2*d^4))^(3/4)*1
i)/8 + 65536*a*b^7*d^7)*((a*d^3*(a^5*b)^(1/2) - a^2*b^2*c^3 - 3*a^3*b*c*d^
2 + 3*b*c^2*d*(a^5*b)^(1/2))/(a^7*b*d^6 - a^4*b^4*c^6 + 3*a^5*b^3*c^4*d^2
- 3*a^6*b^2*c^2*d^4))^(1/4)*1i)/2 + 32768*b^7*d^6*(c + d*x)^(1/4))*((a*d^3
*(a^5*b)^(1/2) - a^2*b^2*c^3 - 3*a^3*b*c*d^2 + 3*b*c^2*d*(a^5*b)^(1/2))/(a
^7*b*d^6 - a^4*b^4*c^6 + 3*a^5*b^3*c^4*d^2 - 3*a^6*b^2*c^2*d^4))^(1/4))/2
- ((((((((a*d^3*(a^5*b)^(1/2) - a^2*b^2*c^3 - 3*a^3*b*c*d^2 + 3*b*c^2*d*(
a^5*b)^(1/2))/(a^7*b*d^6 - a^4*b^4*c^6 + 3*a^5*b^3*c^4*d^2 - 3*a^6*b^2*c^2
*d^4))^(1/4)*(2097152*a^4*b^8*c*d^9 - 2097152*a^3*b^9*c^3*d^7)*1i)/2 - (15
72864*a^3*b^8*c*d^8 + 524288*a^2*b^9*c^3*d^6)*(c + d*x)^(1/4))*((a*d^3*(a^
5*b)^(1/2) - a^2*b^2*c^3 - 3*a^3*b*c*d^2 + 3*b*c^2*d*(a^5*b)^(1/2))/(a^7*b
*d^6 - a^4*b^4*c^6 + 3*a^5*b^3*c^4*d^2 - 3*a^6*b^2*c^2*d^4))^(3/4)*1i)/8 +
 65536*a*b^7*d^7)*((a*d^3*(a^5*b)^(1/2) - a^2*b^2*c^3 - 3*a^3*b*c*d^2 + 3*
b*c^2*d*(a^5*b)^(1/2))/(a^7*b*d^6 - a^4*b^4*c^6 + 3*a^5*b^3*c^4*d^2 - 3*a^
6*b^2*c^2*d^4))^(1/4)*1i)/2 - 32768*b^7*d^6*(c + d*x)^(1/4))*((a*d^3*(a...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)}\,dx = \int \frac{1}{(dx+c)^{\frac{3}{4}}(-b\,x^2+a)}dx$$

input
```
int(1/(d*x+c)^(3/4)/(-b*x^2+a),x)
```

output
```
int(1/(d*x+c)^(3/4)/(-b*x^2+a),x)
```

3.220 $\int \frac{1}{(c+dx)^{5/4}(a-bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 293

$$\int \frac{1}{(c+dx)^{5/4}(a-bx^2)}\,dx = \frac{4d}{(bc^2-ad^2)\sqrt[4]{c+dx}} + \frac{\sqrt[8]{b}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}} - \frac{\sqrt[8]{b}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}} - \frac{\sqrt[8]{b}\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}} + \frac{\sqrt[8]{b}\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}}$$

output
```
4*d/(-a*d^2+b*c^2)/(d*x+c)^(1/4)+b^(1/8)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(
1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c-a^(1/2)*d)^(5/4)-b^(1/8)*arcta
n(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c+a^
(1/2)*d)^(5/4)-b^(1/8)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)
^(1/4))/a^(1/2)/(b^(1/2)*c-a^(1/2)*d)^(5/4)+b^(1/8)*arctanh(b^(1/8)*(d*x+c
)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c+a^(1/2)*d)^(5/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.39

$$\int \frac{1}{(c+dx)^{5/4}(a-bx^2)}\,dx = \frac{\frac{8d}{\sqrt[4]{c+dx}} - d\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&, \frac{-2c\log\left(\sqrt[4]{c+dx}-\#1\right)}{c\#1}\right.}{2bc^2-2ad^2}$$

input
```
Integrate[1/((c + d*x)^(5/4)*(a - b*x^2)),x]
```

output
```
((8*d)/(c + d*x)^(1/4) - d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & ,
 (-2*c*Log[(c + d*x)^(1/4) - #1] + Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1 -
 #1^5) & ])/(2*b*c^2 - 2*a*d^2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {482, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)(c+dx)^{5/4}}dx$$

$$\Big\downarrow \text{482}$$

$$\frac{b\int \frac{c-dx}{\sqrt[4]{c+dx}(a-bx^2)}dx}{bc^2-ad^2}+\frac{4d}{\sqrt[4]{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{657}$$

$$\frac{b\int \left(\frac{\sqrt{a}c-\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}+\frac{\sqrt{a}c+\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)\sqrt[4]{c+dx}}\right)dx}{bc^2-ad^2}+\frac{4d}{\sqrt[4]{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right.}{bc^2-ad^2}$$

$$\frac{4d}{\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(5/4)*(a - b*x^2)),x]
```

output
```
(4*d)/((b*c^2 - a*d^2)*(c + d*x)^(1/4)) + (b*(((Sqrt[b]*c + Sqrt[a]*d)*Arc
Tan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(
7/8)*(Sqrt[b]*c - Sqrt[a]*d)^(1/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(
1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sq
rt[b]*c + Sqrt[a]*d)^(1/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c
 + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sqrt[b]*c
 - Sqrt[a]*d)^(1/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c + d*x)
^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sqrt[b]*c + Sqrt
[a]*d)^(1/4))))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.33

method	result
default	$-\frac{d\left(\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\_R^4-2c\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^4-c\right)}\right)(dx+c)^{\frac{1}{4}}+8\right)}{(dx+c)^{\frac{1}{4}}\left(2a\,d^2-2b\,c^2\right)}$
pseudoelliptic	$-\frac{d\left(\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\_R^4-2c\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^4-c\right)}\right)(dx+c)^{\frac{1}{4}}+8\right)}{(dx+c)^{\frac{1}{4}}\left(2a\,d^2-2b\,c^2\right)}$
derivativedivides	$-4d\left(-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\_R^6-2\_R^2c\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^7+\_R^3c}}{8\left(a\,d^2-b\,c^2\right)}+\frac{1}{\left(a\,d^2-b\,c^2\right)(dx+c)^{\frac{1}{4}}}\right)$

input
```
int(1/(d*x+c)^(5/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d*(sum(1/_R*(_R^4-2*c)*ln((d*x+c)^(1/4)-_R)/(_R^4-c),_R=RootOf(_Z^8*b-2*_
Z^4*b*c-a*d^2+b*c^2))*(d*x+c)^(1/4)+8)/(d*x+c)^(1/4)/(2*a*d^2-2*b*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11325 vs. 2(211) = 422.

Time = 0.37 (sec) , antiderivative size = 11325, normalized size of antiderivative = 38.65

$$\int \frac{1}{(c+dx)^{5/4}\,(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(5/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{5/4}\,(a-bx^2)}\,dx = -\int \frac{1}{-ac\sqrt[4]{c+dx} - adx\sqrt[4]{c+dx} + bcx^2\sqrt[4]{c+dx} + bdx^3\sqrt[4]{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(5/4)/(-b*x**2+a),x)
```

output
```
-Integral(1/(-a*c*(c + d*x)**(1/4) - a*d*x*(c + d*x)**(1/4) + b*c*x**2*(c
+ d*x)**(1/4) + b*d*x**3*(c + d*x)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{5/4}\,(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(5/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(d*x + c)^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{5/4}(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(5/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*(d*x + c)^(5/4)), x)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 17873, normalized size of antiderivative = 61.00

$$\int \frac{1}{(c+dx)^{5/4}(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)*(c + d*x)^(5/4)),x)
```

output
```
atan((((((-(a^2*b^3*c^5 + a^2*d^5*(a^5*b)^(1/2) + 5*b^2*c^4*d*(a^5*b)^(1/2
) + 10*a^3*b^2*c^3*d^2 + 5*a^4*b*c*d^4 + 10*a*b*c^2*d^3*(a^5*b)^(1/2))/(a^
9*d^10 - a^4*b^5*c^10 - 5*a^8*b*c^2*d^8 + 5*a^5*b^4*c^8*d^2 - 10*a^6*b^3*c
^6*d^4 + 10*a^7*b^2*c^4*d^6))^(3/4)*(1310720*a^2*b^21*c^26*d^7 - 262144*a^
15*b^8*d^33 - ((-(a^2*b^3*c^5 + a^2*d^5*(a^5*b)^(1/2) + 5*b^2*c^4*d*(a^5*b
)^(1/2) + 10*a^3*b^2*c^3*d^2 + 5*a^4*b*c*d^4 + 10*a*b*c^2*d^3*(a^5*b)^(1/2
))/(a^9*d^10 - a^4*b^5*c^10 - 5*a^8*b*c^2*d^8 + 5*a^5*b^4*c^8*d^2 - 10*a^6
*b^3*c^6*d^4 + 10*a^7*b^2*c^4*d^6))^(1/4)*(c + d*x)^(1/4)*(3145728*a^3*b^2
1*c^26*d^8 - 524288*a^2*b^22*c^28*d^6 - 524288*a^16*b^8*d^34 + 2621440*a^4
*b^20*c^24*d^10 - 85983232*a^5*b^19*c^22*d^12 + 397934592*a^6*b^18*c^20*d^
14 - 1026555904*a^7*b^17*c^18*d^16 + 1747451904*a^8*b^16*c^16*d^18 - 20761
80480*a^9*b^15*c^14*d^20 + 1747451904*a^10*b^14*c^12*d^22 - 1026555904*a^1
1*b^13*c^10*d^24 + 397934592*a^12*b^12*c^8*d^26 - 85983232*a^13*b^11*c^6*d
^28 + 2621440*a^14*b^10*c^4*d^30 + 3145728*a^15*b^9*c^2*d^32)*1i)/2 - 1179
6480*a^3*b^20*c^24*d^9 + 43515904*a^4*b^19*c^22*d^11 - 74973184*a^5*b^18*c
^20*d^13 + 14417920*a^6*b^17*c^18*d^15 + 216268800*a^7*b^16*c^16*d^17 - 51
9045120*a^8*b^15*c^14*d^19 + 657457152*a^9*b^14*c^12*d^21 - 527695872*a^10
*b^13*c^10*d^23 + 273940480*a^11*b^12*c^8*d^25 - 86507520*a^12*b^11*c^6*d^
27 + 13107200*a^13*b^10*c^4*d^29 + 262144*a^14*b^9*c^2*d^31)*1i)/8 + (c +
d*x)^(1/4)*(32768*b^20*c^23*d^6 - 360448*a*b^19*c^21*d^8 - 32768*a^11*b...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{5/4}\,(a-bx^2)}\,dx = \int \frac{1}{(dx+c)^{\frac{5}{4}}\,(-b\,x^2+a)}dx$$

input
```
int(1/(d*x+c)^(5/4)/(-b*x^2+a),x)
```

output
```
int(1/(d*x+c)^(5/4)/(-b*x^2+a),x)
```

3.221 $\int \frac{1}{(c+dx)^{7/4}(a-bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 295

$$\int \frac{1}{(c+dx)^{7/4}(a-bx^2)}\,dx = \frac{4d}{3(bc^2-ad^2)(c+dx)^{3/4}} - \frac{b^{3/8}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}} + \frac{b^{3/8}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4}} - \frac{b^{3/8}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}} + \frac{b^{3/8}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4}}$$

output
```
4/3*d/(-a*d^2+b*c^2)/(d*x+c)^(3/4)-b^(3/8)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b
^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c-a^(1/2)*d)^(7/4)+b^(3/8)*arc
tan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c+
a^(1/2)*d)^(7/4)-b^(3/8)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*
d)^(1/4))/a^(1/2)/(b^(1/2)*c-a^(1/2)*d)^(7/4)+b^(3/8)*arctanh(b^(1/8)*(d*x
+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c+a^(1/2)*d)^(7/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.39

$$\int \frac{1}{(c+dx)^{7/4}\left(a-bx^2\right)}\,dx=\frac{\frac{8d}{(c+dx)^{3/4}}-3d\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-2c\log\left(\sqrt[4]{c+dx}-\#1\right)}{c\#1}\right.}{6bc^2-6ad^2}$$

input
```
Integrate[1/((c + d*x)^(7/4)*(a - b*x^2)),x]
```

output
```
((8*d)/(c + d*x)^(3/4) - 3*d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 &
 , (-2*c*Log[(c + d*x)^(1/4) - #1] + Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1
^3 - #1^7) & ])/(6*b*c^2 - 6*a*d^2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {482, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-bx^2\right)(c+dx)^{7/4}}dx$$

$$\Big\downarrow \text{482}$$

$$\frac{b\int \frac{c-dx}{(c+dx)^{3/4}\left(a-bx^2\right)}dx}{bc^2-ad^2}+\frac{4d}{3(c+dx)^{3/4}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{657}$$

$$\frac{b\int \left(\frac{\sqrt{a}c-\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}+\frac{\sqrt{a}c+\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}\right)dx}{bc^2-ad^2}+\frac{4d}{3(c+dx)^{3/4}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{5/8}\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}}\right.}{bc^2-ad^2}$$

$$\frac{4d}{3(c+dx)^{3/4}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(7/4)*(a - b*x^2)),x]
```

output
```
(4*d)/(3*(b*c^2 - a*d^2)*(c + d*x)^(3/4)) + (b*(-(((Sqrt[b]*c + Sqrt[a]*d)
*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]
*b^(5/8)*(Sqrt[b]*c - Sqrt[a]*d)^(3/4))) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan
[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8
)*(Sqrt[b]*c + Sqrt[a]*d)^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/
8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)*(Sqrt
[b]*c - Sqrt[a]*d)^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c +
 d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)*(Sqrt[b]*c +
 Sqrt[a]*d)^(3/4))))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.33

method	result	siz
default	$-\frac{d\left(3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\_R^{4}-2c\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)(dx+c)^{\frac{3}{4}}+8\right)}{6(a\,d^{2}-b\,c^{2})(dx+c)^{\frac{3}{4}}}$	96
pseudoelliptic	$-\frac{d\left(3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\_R^{4}-2c\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)(dx+c)^{\frac{3}{4}}+8\right)}{6(a\,d^{2}-b\,c^{2})(dx+c)^{\frac{3}{4}}}$	96
derivativedivides	$-4d\left(\frac{1}{3(a\,d^{2}-b\,c^{2})(dx+c)^{\frac{3}{4}}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{8}-2bc\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\_R^{4}-2c\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^{7}+\_R^{3}c}}{8(a\,d^{2}-b\,c^{2})}\right)$	10

input
```
int(1/(d*x+c)^(7/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*d*(3*sum((_R^4-2*c)*ln((d*x+c)^(1/4)-_R)/_R^3/(_R^4-c),_R=RootOf(_Z^8
*b-2*_Z^4*b*c-a*d^2+b*c^2))*(d*x+c)^(3/4)+8)/(a*d^2-b*c^2)/(d*x+c)^(3/4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13123 vs. 2(211) = 422.

Time = 0.67 (sec) , antiderivative size = 13123, normalized size of antiderivative = 44.48

$$\int \frac{1}{(c+dx)^{7/4}\,(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(7/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{7/4}(a-bx^2)}\,dx = -\int \frac{1}{-ac(c+dx)^{\frac{3}{4}} - adx(c+dx)^{\frac{3}{4}} + bcx^2(c+dx)^{\frac{3}{4}} + bdx^3(c+dx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)**(7/4)/(-b*x**2+a),x)
```

output
```
-Integral(1/(-a*c*(c + d*x)**(3/4) - a*d*x*(c + d*x)**(3/4) + b*c*x**2*(c
+ d*x)**(3/4) + b*d*x**3*(c + d*x)**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{7/4}(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(7/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(d*x + c)^(7/4)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{7/4}(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(7/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*(d*x + c)^(7/4)), x)
```

Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 12990, normalized size of antiderivative = 44.03

$$\int \frac{1}{(c+dx)^{7/4}(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)*(c + d*x)^(7/4)),x)
```

output
```
atan((b^3*c^2*(c + d*x)^(1/4) + a*b^2*d^2*(c + d*x)^(1/4) + (a^2*b^5*c^9*(
c + d*x)^(1/4)*(a^3*d^7*(a^5*b^3)^(1/2) + a^2*b^5*c^7 + 7*a^5*b^2*c*d^6 +
21*a^3*b^4*c^5*d^2 + 35*a^4*b^3*c^3*d^4 + 7*b^3*c^6*d*(a^5*b^3)^(1/2) + 35
*a*b^2*c^4*d^3*(a^5*b^3)^(1/2) + 21*a^2*b*c^2*d^5*(a^5*b^3)^(1/2)))/(a^11*
d^14 - a^4*b^7*c^14 - 7*a^10*b*c^2*d^12 + 7*a^5*b^6*c^12*d^2 - 21*a^6*b^5*
c^10*d^4 + 35*a^7*b^4*c^8*d^6 - 35*a^8*b^3*c^6*d^8 + 21*a^9*b^2*c^4*d^10)
+ (8*a^3*b^4*c^7*d^2*(c + d*x)^(1/4)*(a^3*d^7*(a^5*b^3)^(1/2) + a^2*b^5*c^
7 + 7*a^5*b^2*c*d^6 + 21*a^3*b^4*c^5*d^2 + 35*a^4*b^3*c^3*d^4 + 7*b^3*c^6*
d*(a^5*b^3)^(1/2) + 35*a*b^2*c^4*d^3*(a^5*b^3)^(1/2) + 21*a^2*b*c^2*d^5*(a
^5*b^3)^(1/2)))/(a^11*d^14 - a^4*b^7*c^14 - 7*a^10*b*c^2*d^12 + 7*a^5*b^6*
c^12*d^2 - 21*a^6*b^5*c^10*d^4 + 35*a^7*b^4*c^8*d^6 - 35*a^8*b^3*c^6*d^8 +
 21*a^9*b^2*c^4*d^10) - (14*a^4*b^3*c^5*d^4*(c + d*x)^(1/4)*(a^3*d^7*(a^5*
b^3)^(1/2) + a^2*b^5*c^7 + 7*a^5*b^2*c*d^6 + 21*a^3*b^4*c^5*d^2 + 35*a^4*b
^3*c^3*d^4 + 7*b^3*c^6*d*(a^5*b^3)^(1/2) + 35*a*b^2*c^4*d^3*(a^5*b^3)^(1/2
) + 21*a^2*b*c^2*d^5*(a^5*b^3)^(1/2)))/(a^11*d^14 - a^4*b^7*c^14 - 7*a^10*
b*c^2*d^12 + 7*a^5*b^6*c^12*d^2 - 21*a^6*b^5*c^10*d^4 + 35*a^7*b^4*c^8*d^6
 - 35*a^8*b^3*c^6*d^8 + 21*a^9*b^2*c^4*d^10) + (5*a^6*b*c*d^8*(c + d*x)^(1
/4)*(a^3*d^7*(a^5*b^3)^(1/2) + a^2*b^5*c^7 + 7*a^5*b^2*c*d^6 + 21*a^3*b^4*
c^5*d^2 + 35*a^4*b^3*c^3*d^4 + 7*b^3*c^6*d*(a^5*b^3)^(1/2) + 35*a*b^2*c^4*
d^3*(a^5*b^3)^(1/2) + 21*a^2*b*c^2*d^5*(a^5*b^3)^(1/2)))/(a^11*d^14 - a...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{7/4}(a-bx^2)}\,dx = \int \frac{1}{(dx+c)^{\frac{7}{4}}(-b\,x^2+a)}dx$$

input
```
int(1/(d*x+c)^(7/4)/(-b*x^2+a),x)
```

output
```
int(1/(d*x+c)^(7/4)/(-b*x^2+a),x)
```

3.222 $\int \frac{1}{(c+dx)^{9/4}(a-bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 323

$$\int \frac{1}{(c+dx)^{9/4}(a-bx^2)}\,dx = \frac{4d}{5(bc^2-ad^2)(c+dx)^{5/4}} + \frac{8bcd}{(bc^2-ad^2)^2\sqrt[4]{c+dx}} + \frac{b^{5/8}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}} - \frac{b^{5/8}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}} - \frac{b^{5/8}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}} + \frac{b^{5/8}\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{\sqrt{a}\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}}$$

output
```
4/5*d/(-a*d^2+b*c^2)/(d*x+c)^(5/4)+8*b*c*d/(-a*d^2+b*c^2)^2/(d*x+c)^(1/4)+
b^(5/8)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/
(b^(1/2)*c-a^(1/2)*d)^(9/4)-b^(5/8)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*
c+a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c+a^(1/2)*d)^(9/4)-b^(5/8)*arctanh(b^
(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(1/2)/(b^(1/2)*c-a^(1/2
)*d)^(9/4)+b^(5/8)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/
4))/a^(1/2)/(b^(1/2)*c+a^(1/2)*d)^(9/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.52

$$\int \frac{1}{(c+dx)^{9/4}(a-bx^2)}\,dx = \frac{d\left(-8ad^2+8bc(11c+10dx)-5(c+dx)^{5/4}\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+\right.\right.}{10\left(bc^2-\right.}$$

input
```
Integrate[1/((c + d*x)^(9/4)*(a - b*x^2)),x]
```

output
```
(d*(-8*a*d^2 + 8*b*c*(11*c + 10*d*x) - 5*(c + d*x)^(5/4)*RootSum[b*c^2 - a
*d^2 - 2*b*c*#1^4 + b*#1^8 & , (-3*b*c^2*Log[(c + d*x)^(1/4) - #1] - a*d^2
*Log[(c + d*x)^(1/4) - #1] + 2*b*c*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1 -
 #1^5) & ]))/(10*(b*c^2 - a*d^2)^2*(c + d*x)^(5/4))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {482, 655, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)(c+dx)^{9/4}}\,dx$$

$$\Big\downarrow \text{482}$$

$$\frac{b\int \frac{c-dx}{(c+dx)^{5/4}(a-bx^2)}dx}{bc^2-ad^2}+\frac{4d}{5(c+dx)^{5/4}(bc^2-ad^2)}$$

$$\Big\downarrow \text{655}$$

$$\frac{b\left(\frac{8cd}{\sqrt[4]{c+dx}(bc^2-ad^2)}-\frac{\int -\frac{bc^2-2bdxc+ad^2}{\sqrt[4]{c+dx}(a-bx^2)}dx}{bc^2-ad^2}\right)}{bc^2-ad^2}+\frac{4d}{5(c+dx)^{5/4}(bc^2-ad^2)}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\int \frac{bc^2-2bdxc+ad^2}{\sqrt[4]{c+dx}(a-bx^2)}dx}{bc^2-ad^2}+\frac{8cd}{\sqrt[4]{c+dx}(bc^2-ad^2)}\right)}{bc^2-ad^2}+\frac{4d}{5(c+dx)^{5/4}(bc^2-ad^2)}$$

$\Big\downarrow$ 657

$$\frac{b\left(\frac{\int \left(\frac{\sqrt{a}(bc^2+ad^2)-2a\sqrt{b}cd}{2a(\sqrt{a}-\sqrt{b}x)\sqrt[4]{c+dx}}+\frac{2a\sqrt{b}cd+\sqrt{a}(bc^2+ad^2)}{2a(\sqrt{b}x+\sqrt{a})\sqrt[4]{c+dx}}\right)dx}{bc^2-ad^2}+\frac{8cd}{\sqrt[4]{c+dx}(bc^2-ad^2)}\right)}{bc^2-ad^2}+\frac{4d}{5(c+dx)^{5/4}(bc^2-ad^2)}$$

$\Big\downarrow$ 2009

$$\frac{b\left(\frac{-\frac{(\sqrt{b}c-\sqrt{a}d)^2\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}+\frac{(\sqrt{a}d+\sqrt{b}c)^2\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}+\frac{(\sqrt{b}c-\sqrt{a}d)^2\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}-\frac{(\sqrt{a}d}{}}{bc^2-ad^2}\right)}{bc^2-ad^2}$$

$$\frac{4d}{5(c+dx)^{5/4}(bc^2-ad^2)}$$

input
```
Int[1/((c + d*x)^(9/4)*(a - b*x^2)),x]
```

output
```
(4*d)/(5*(b*c^2 - a*d^2)*(c + d*x)^(5/4)) + (b*((8*c*d)/((b*c^2 - a*d^2)*(
c + d*x)^(1/4)) + (((Sqrt[b]*c + Sqrt[a]*d)^2*ArcTan[(b^(1/8)*(c + d*x)^(1
/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c - Sqrt[a]
*d)^(1/4)) - ((Sqrt[b]*c - Sqrt[a]*d)^2*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(
Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c + Sqrt[a]*d)^(1
/4)) - ((Sqrt[b]*c + Sqrt[a]*d)^2*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[
b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c - Sqrt[a]*d)^(1/4))
+ ((Sqrt[b]*c - Sqrt[a]*d)^2*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c
+ Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c + Sqrt[a]*d)^(1/4)))/(b*c
^2 - a*d^2)))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 482
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d,
n}, x] && LtQ[n, -1]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.39

method	result
default	$\frac{d\left(5(dx+c)^{\frac{5}{4}}\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(-2\_R^4bc+a\,d^2+3b\,c^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(-\_R^4+c\right)}\right)+80bcdx-8a\,d^2 \ldots}{10(dx+c)^{\frac{5}{4}}\left(a\,d^2-b\,c^2\right)^2}$
pseudoelliptic	$\frac{d\left(5(dx+c)^{\frac{5}{4}}\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(-2\_R^4bc+a\,d^2+3b\,c^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(-\_R^4+c\right)}\right)+80bcdx-8a\,d^2 \ldots}{10(dx+c)^{\frac{5}{4}}\left(a\,d^2-b\,c^2\right)^2}$
derivativedivides	$-4d\left(\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(2c\_R^6b+\left(-a\,d^2-3b\,c^2\right)\_R^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{-\_R^7+\_R^3c}}{8\left(a\,d^2-b\,c^2\right)^2}+\frac{\ldots}{5\left(a\,d^2-b\,c \ldots\right.}\right.$

input
```
int(1/(d*x+c)^(9/4)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/10*d*(5*(d*x+c)^(5/4)*sum((-2*_R^4*b*c+a*d^2+3*b*c^2)*ln((d*x+c)^(1/4)-_
R)/_R/(-_R^4+c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+80*b*c*d*x-8*a*d
^2+88*b*c^2)/(d*x+c)^(5/4)/(a*d^2-b*c^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20772 vs. 2(237) = 474.

Time = 2.09 (sec) , antiderivative size = 20772, normalized size of antiderivative = 64.31

$$\int \frac{1}{(c+dx)^{9/4}\left(a-bx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(9/4)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{9/4}\,(a-bx^2)}\,dx =$$
$$-\int \frac{1}{-ac^2\sqrt[4]{c+dx}-2acdx\sqrt[4]{c+dx}-ad^2x^2\sqrt[4]{c+dx}+bc^2x^2\sqrt[4]{c+dx}+2bcdx^3\sqrt[4]{c+dx}+bd^2x^4\sqrt[4]{c+dx}}$$

input
```
integrate(1/(d*x+c)**(9/4)/(-b*x**2+a),x)
```

output
```
-Integral(1/(-a*c**2*(c + d*x)**(1/4) - 2*a*c*d*x*(c + d*x)**(1/4) - a*d**
2*x**2*(c + d*x)**(1/4) + b*c**2*x**2*(c + d*x)**(1/4) + 2*b*c*d*x**3*(c +
 d*x)**(1/4) + b*d**2*x**4*(c + d*x)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{9/4}\,(a-bx^2)}\,dx = \int -\frac{1}{(bx^2-a)(dx+c)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(9/4)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(d*x + c)^(9/4)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{9/4}\,(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^(9/4)/(-b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 33153, normalized size of antiderivative = 102.64

$$\int \frac{1}{(c+dx)^{9/4}\,(a-bx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)*(c + d*x)^(9/4)),x)
```

output
```
atan((((((a^4*d^9*(a^5*b^5)^(1/2) - a^2*b^7*c^9 - 9*a^6*b^3*c*d^8 - 36*a^
3*b^6*c^7*d^2 - 126*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^3*d^6 + 9*b^4*c^8*d*(a^
5*b^5)^(1/2) + 84*a*b^3*c^6*d^3*(a^5*b^5)^(1/2) + 36*a^3*b*c^2*d^7*(a^5*b^
5)^(1/2) + 126*a^2*b^2*c^4*d^5*(a^5*b^5)^(1/2))/(a^13*d^18 - a^4*b^9*c^18
- 9*a^12*b*c^2*d^16 + 9*a^5*b^8*c^16*d^2 - 36*a^6*b^7*c^14*d^4 + 84*a^7*b^
6*c^12*d^6 - 126*a^8*b^5*c^10*d^8 + 126*a^9*b^4*c^8*d^10 - 84*a^10*b^3*c^6
*d^12 + 36*a^11*b^2*c^4*d^14))^(3/4)*((((a^4*d^9*(a^5*b^5)^(1/2) - a^2*b^7
*c^9 - 9*a^6*b^3*c*d^8 - 36*a^3*b^6*c^7*d^2 - 126*a^4*b^5*c^5*d^4 - 84*a^5
*b^4*c^3*d^6 + 9*b^4*c^8*d*(a^5*b^5)^(1/2) + 84*a*b^3*c^6*d^3*(a^5*b^5)^(1
/2) + 36*a^3*b*c^2*d^7*(a^5*b^5)^(1/2) + 126*a^2*b^2*c^4*d^5*(a^5*b^5)^(1/
2))/(a^13*d^18 - a^4*b^9*c^18 - 9*a^12*b*c^2*d^16 + 9*a^5*b^8*c^16*d^2 - 3
6*a^6*b^7*c^14*d^4 + 84*a^7*b^6*c^12*d^6 - 126*a^8*b^5*c^10*d^8 + 126*a^9*
b^4*c^8*d^10 - 84*a^10*b^3*c^6*d^12 + 36*a^11*b^2*c^4*d^14))^(1/4)*(c + d*
x)^(1/4)*(524288*a^28*b^9*d^58 - 524288*a^2*b^35*c^52*d^6 + 4194304*a^3*b^
34*c^50*d^8 + 40370176*a^4*b^33*c^48*d^10 - 880803840*a^5*b^32*c^46*d^12 +
 7307526144*a^6*b^31*c^44*d^14 - 38201720832*a^7*b^30*c^42*d^16 + 14299116
3392*a^8*b^29*c^40*d^18 - 405362704384*a^9*b^28*c^38*d^20 + 894689607680*a
^10*b^27*c^36*d^22 - 1552475488256*a^11*b^26*c^34*d^24 + 2099370262528*a^1
2*b^25*c^32*d^26 - 2118844940288*a^13*b^24*c^30*d^28 + 1355437572096*a^14*
b^23*c^28*d^30 - 1355437572096*a^16*b^21*c^24*d^34 + 2118844940288*a^17...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{9/4}\,(a-bx^2)}\,dx = \int \frac{1}{(dx+c)^{\frac{9}{4}}\,(-b\,x^2+a)}dx$$

input
```
int(1/(d*x+c)^(9/4)/(-b*x^2+a),x)
```

output
```
int(1/(d*x+c)^(9/4)/(-b*x^2+a),x)
```

3.223 $\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 435

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx = \frac{d(bc^2+9ad^2)\sqrt[4]{c+dx}}{2ab^2} + \frac{cd(c+dx)^{5/4}}{2ab} + \frac{(ad+bcx)(c+dx)^{9/4}}{2ab(a-bx^2)} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}\left(4\sqrt{b}c+9\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{17/8}} + \frac{\left(4\sqrt{b}c-9\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{17/8}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}\left(4\sqrt{b}c+9\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{17/8}} + \frac{\left(4\sqrt{b}c-9\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{17/8}}$$

output
```
1/2*d*(9*a*d^2+b*c^2)*(d*x+c)^(1/4)/a/b^2+1/2*c*d*(d*x+c)^(5/4)/a/b+1/2*(b
*c*x+a*d)*(d*x+c)^(9/4)/a/b/(-b*x^2+a)-1/8*(b^(1/2)*c-a^(1/2)*d)^(9/4)*(4*
b^(1/2)*c+9*a^(1/2)*d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^
(1/4))/a^(3/2)/b^(17/8)+1/8*(4*b^(1/2)*c-9*a^(1/2)*d)*(b^(1/2)*c+a^(1/2)*d
)^(9/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/
b^(17/8)-1/8*(b^(1/2)*c-a^(1/2)*d)^(9/4)*(4*b^(1/2)*c+9*a^(1/2)*d)*arctanh
(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/b^(17/8)+1/8*(
4*b^(1/2)*c-9*a^(1/2)*d)*(b^(1/2)*c+a^(1/2)*d)^(9/4)*arctanh(b^(1/8)*(d*x+
c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(17/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.01 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx = \frac{-16d^3\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{bc^2\log\left(\sqrt[4]{c+dx}-\#1\right)+ad^2\log\left(\sqrt[4]{c+dx}\right.}{c\#1^3-\#}\right.}{}$$

input
```
Integrate[(c + d*x)^(13/4)/(a - b*x^2)^2,x]
```

output
```
(-16*d^3*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (b*c^2*Log[(c + d
*x)^(1/4) - #1] + a*d^2*Log[(c + d*x)^(1/4) - #1] + 2*b*c*Log[(c + d*x)^(1
/4) - #1]*#1^4)/(c*#1^3 - #1^7) & ] + ((8*b*(c + d*x)^(1/4)*(9*a^2*d^3 + b
^2*c^3*x + a*b*d*(3*c^2 + 3*c*d*x - 8*d^2*x^2)))/(a - b*x^2) + d*RootSum[b
*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (b^2*c^4*Log[(c + d*x)^(1/4) - #1]
+ 24*a*b*c^2*d^2*Log[(c + d*x)^(1/4) - #1] + 7*a^2*d^4*Log[(c + d*x)^(1/4)
 - #1] + 3*b^2*c^3*Log[(c + d*x)^(1/4) - #1]*#1^4 + 9*a*b*c*d^2*Log[(c + d
*x)^(1/4) - #1]*#1^4)/(c*#1^3 - #1^7) & ])/a)/(16*b^3)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {495, 27, 653, 25, 27, 653, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2} \, dx$$

$$\Big\downarrow \ 495$$

$$\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\,(a-bx^2)}-\frac{\int -\frac{(c+dx)^{5/4}(4bc^2-5bdxc-9ad^2)}{4(a-bx^2)}dx}{2ab}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(c+dx)^{5/4}(4bc^2-5bdxc-9ad^2)}{a-bx^2}dx}{8ab}+\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \ 653$$

$$\frac{4cd(c+dx)^{5/4}-\frac{\int -\frac{b\sqrt[4]{c+dx}\left(2c\left(2bc^2-7ad^2\right)-d\left(bc^2+9ad^2\right)x\right)}{a-bx^2}dx}{b}}{8ab}+\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{b\sqrt[4]{c+dx}\left(2c\left(2bc^2-7ad^2\right)-d\left(bc^2+9ad^2\right)x\right)}{a-bx^2}dx}{b}+4cd(c+dx)^{5/4}}{8ab}+\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt[4]{c+dx}\left(2c(2bc^2-7ad^2)-d(bc^2+9ad^2)x\right)}{a-bx^2}dx+4cd(c+dx)^{5/4}}{8ab}+\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \ 653$$

$$\frac{-\frac{\int -\frac{4b^2c^4-15abd^2c^2+bd\left(3bc^2-23ad^2\right)xc-9a^2d^4}{(c+dx)^{3/4}\left(a-bx^2\right)}dx}{b}+\frac{4d\sqrt[4]{c+dx}(9ad^2+bc^2)}{b}+4cd(c+dx)^{5/4}}{8ab}+\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{4b^2c^4-15abd^2c^2+bd\left(3bc^2-23ad^2\right)xc-9a^2d^4}{(c+dx)^{3/4}\left(a-bx^2\right)}dx}{b}+\frac{4d\sqrt[4]{c+dx}\left(9ad^2+bc^2\right)}{b}+4cd(c+dx)^{5/4}}{8ab}+\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\left(a-bx^2\right)}$$

$\Big\downarrow$ 657

$$\frac{\frac{\int \left(\frac{\sqrt{a}\left(4b^2c^4-15abd^2c^2-9a^2d^4\right)-a\sqrt{b}cd\left(3bc^2-23ad^2\right)}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}+\frac{a\sqrt{b}cd\left(3bc^2-23ad^2\right)+\sqrt{a}\left(4b^2c^4-15abd^2c^2-9a^2d^4\right)}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx}{b}+\frac{4d\sqrt[4]{c+dx}\left(9ad^2+bc^2\right)}{b}+4c}{8ab}$$
$$\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\left(a-bx^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\left(9\sqrt{a}d+4\sqrt{b}c\right)\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}}+\frac{\left(4\sqrt{b}c-9\sqrt{a}d\right)\left(\sqrt{a}d+\sqrt{b}c\right)^{9/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}\sqrt[8]{b}}-\frac{\left(9\sqrt{a}d+4\sqrt{b}c\right)\left(\sqrt{b}c-\sqrt{a}d\right)^{9}}{}}{b}$$
$$\frac{(c+dx)^{9/4}(ad+bcx)}{2ab\left(a-bx^2\right)}$$

input
```
Int[(c + d*x)^(13/4)/(a - b*x^2)^2,x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(9/4))/(2*a*b*(a - b*x^2)) + ((4*d*(b*c^2 + 9*a*d
^2)*(c + d*x)^(1/4))/b + 4*c*d*(c + d*x)^(5/4) + (-(((Sqrt[b]*c - Sqrt[a]*
d)^(9/4)*(4*Sqrt[b]*c + 9*Sqrt[a]*d)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqr
t[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8))) + ((4*Sqrt[b]*c - 9*Sqrt[a]
*d)*(Sqrt[b]*c + Sqrt[a]*d)^(9/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b
]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)) - ((Sqrt[b]*c - Sqrt[a]*d)^(9/4
)*(4*Sqrt[b]*c + 9*Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c
 - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)) + ((4*Sqrt[b]*c - 9*Sqrt[a]*d)*(Sq
rt[b]*c + Sqrt[a]*d)^(9/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c +
Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)))/b)/(8*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.41

method	result
default	$\frac{d(-bx^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(-23\_R^4abc\,d^2+3\_R^4b^2c^3-9a^2d^4+8bc^2d^2a+b^2c^4\right)\ln\left((dx+c)\right.}{\_R^3\left(-\_R^4+c\right)}\right.}{16a(-bx^2+a)b^3}$
pseudoelliptic	$\frac{d(-bx^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(-23\_R^4abc\,d^2+3\_R^4b^2c^3-9a^2d^4+8bc^2d^2a+b^2c^4\right)\ln\left((dx+c)\right.}{\_R^3\left(-\_R^4+c\right)}\right.}{16a(-bx^2+a)b^3}$
derivativedivides	$4d^3\left(\frac{(dx+c)^{\frac{1}{4}}}{b^2}-\frac{\frac{-\frac{bc(3a\,d^2+b\,c^2)(dx+c)^{\frac{5}{4}}}{8a\,d^2}-\frac{(a^2d^4-b^2c^4)(dx+c)^{\frac{1}{4}}}{8a\,d^2}}{-b(dx+c)^2+2bc(dx+c)+a\,d^2-b\,c^2}+\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(bc\left(23\right.\right.}{}}{b^2}\right.$

input
```
int((d*x+c)^(13/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(d*(-b*x^2+a)*sum((-23*_R^4*a*b*c*d^2+3*_R^4*b^2*c^3-9*a^2*d^4+8*a*b*
c^2*d^2+b^2*c^4)*ln((d*x+c)^(1/4)-_R)/_R^3/(-_R^4+c),_R=RootOf(_Z^8*b-2*_Z
^4*b*c-a*d^2+b*c^2))+72*b*(d*x+c)^(1/4)*(1/9*c^3*b^2*x+1/3*d*(-8/3*d^2*x^2
+c*d*x+c^2)*a*b+a^2*d^3))/a/(-b*x^2+a)/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7469 vs. 2(320) = 640.

Time = 6.36 (sec) , antiderivative size = 7469, normalized size of antiderivative = 17.17

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(13/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(13/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx=\int \frac{(dx+c)^{\frac{13}{4}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((d*x+c)^(13/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(13/4)/(b*x^2 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)^(13/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 23842, normalized size of antiderivative = 54.81

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(13/4)/(a - b*x^2)^2,x)
```

output
```
(4*d^3*(c + d*x)^(1/4))/b^2 - (atan(((((((((((c + d*x)^(1/4)*(11206656*a^11
*b^9*c*d^16 - 1048576*a^6*b^14*c^11*d^6 + 8847360*a^7*b^13*c^9*d^8 - 31457
280*a^8*b^12*c^7*d^10 + 51773440*a^9*b^11*c^5*d^12 - 39321600*a^10*b^10*c^
3*d^14))/(8*a^6*b^5) - ((-(6561*a^5*d^13*(a^15*b^17)^(1/2) - 256*a^6*b^15*
c^13 - 47385*a^12*b^9*c*d^12 + 3744*a^7*b^14*c^11*d^2 - 20241*a^8*b^13*c^9
*d^4 + 32604*a^9*b^12*c^7*d^6 + 70434*a^10*b^11*c^5*d^8 - 198900*a^11*b^10
*c^3*d^10 - 3120*b^5*c^10*d^3*(a^15*b^17)^(1/2) + 34281*a*b^4*c^8*d^5*(a^1
5*b^17)^(1/2) + 138996*a^4*b*c^2*d^11*(a^15*b^17)^(1/2) - 123084*a^2*b^3*c
^6*d^7*(a^15*b^17)^(1/2) + 106366*a^3*b^2*c^4*d^9*(a^15*b^17)^(1/2))/(a^12
*b^17))^(1/4)*(327680*a^9*b^9*c*d^13 - 327680*a^6*b^12*c^7*d^7 + 983040*a^
7*b^11*c^5*d^9 - 983040*a^8*b^10*c^3*d^11)*1i)/(8*a^4*b^3))*(-(6561*a^5*d^
13*(a^15*b^17)^(1/2) - 256*a^6*b^15*c^13 - 47385*a^12*b^9*c*d^12 + 3744*a^
7*b^14*c^11*d^2 - 20241*a^8*b^13*c^9*d^4 + 32604*a^9*b^12*c^7*d^6 + 70434*
a^10*b^11*c^5*d^8 - 198900*a^11*b^10*c^3*d^10 - 3120*b^5*c^10*d^3*(a^15*b^
17)^(1/2) + 34281*a*b^4*c^8*d^5*(a^15*b^17)^(1/2) + 138996*a^4*b*c^2*d^11*
(a^15*b^17)^(1/2) - 123084*a^2*b^3*c^6*d^7*(a^15*b^17)^(1/2) + 106366*a^3*
b^2*c^4*d^9*(a^15*b^17)^(1/2))/(a^12*b^17))^(3/4)*1i)/4096 + (2*(59049*a^1
0*d^27 + 1280*b^10*c^20*d^7 - 35040*a*b^9*c^18*d^9 + 810648*a^9*b*c^2*d^25
 + 363849*a^2*b^8*c^16*d^11 - 1857992*a^3*b^7*c^14*d^13 + 4903132*a^4*b^6*
c^12*d^15 - 6108024*a^5*b^5*c^10*d^17 + 1739670*a^6*b^4*c^8*d^19 + 3502...
```

Reduce [F]

$$\int \frac{(c+dx)^{13/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{13}{4}}}{(-b\,x^2+a)^2}dx$$

input
```
int((d*x+c)^(13/4)/(-b*x^2+a)^2,x)
```

output
```
int((d*x+c)^(13/4)/(-b*x^2+a)^2,x)
```

3.224 $\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 403

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2} dx = \frac{cd(c+dx)^{3/4}}{2ab} + \frac{(ad+bcx)(c+dx)^{7/4}}{2ab(a-bx^2)} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}\left(4\sqrt{b}c+7\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{15/8}} - \frac{\left(4\sqrt{b}c-7\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{15/8}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}\left(4\sqrt{b}c+7\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{15/8}} + \frac{\left(4\sqrt{b}c-7\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{15/8}}$$

output
```
1/2*c*d*(d*x+c)^(3/4)/a/b+1/2*(b*c*x+a*d)*(d*x+c)^(7/4)/a/b/(-b*x^2+a)+1/8
*(b^(1/2)*c-a^(1/2)*d)^(7/4)*(4*b^(1/2)*c+7*a^(1/2)*d)*arctan(b^(1/8)*(d*x
+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/b^(15/8)-1/8*(4*b^(1/2)*c-7
*a^(1/2)*d)*(b^(1/2)*c+a^(1/2)*d)^(7/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1
/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(15/8)-1/8*(b^(1/2)*c-a^(1/2)*d)^(7/4)*(
4*b^(1/2)*c+7*a^(1/2)*d)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*
d)^(1/4))/a^(3/2)/b^(15/8)+1/8*(4*b^(1/2)*c-7*a^(1/2)*d)*(b^(1/2)*c+a^(1/2
)*d)^(7/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3
/2)/b^(15/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.69

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2}\,dx = \frac{8d^3\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-2c\log\left(\sqrt[4]{c+dx}-\#1\right)-\log\left(\sqrt[4]{c+dx}-\#1\right)}{c\#1-\#1^5}\right.}{}$$

input
```
Integrate[(c + d*x)^(11/4)/(a - b*x^2)^2,x]
```

output
```
(8*d^3*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (-2*c*Log[(c + d*x)
^(1/4) - #1] - Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1 - #1^5) & ] + ((8*b*(
c + d*x)^(3/4)*(b*c^2*x + a*d*(2*c + d*x)))/(a - b*x^2) + d*RootSum[b*c^2
- a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (3*b*c^3*Log[(c + d*x)^(1/4) - #1] + 13*
a*c*d^2*Log[(c + d*x)^(1/4) - #1] + b*c^2*Log[(c + d*x)^(1/4) - #1]*#1^4 +
 a*d^2*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1 - #1^5) & ])/a)/(16*b^2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {495, 27, 653, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2} \, dx$$

$$\Big\downarrow \; 495$$

$$\frac{(c+dx)^{7/4}(ad+bcx)}{2ab(a-bx^2)}-\frac{\int -\frac{(c+dx)^{3/4}(4bc^2-3bdxc-7ad^2)}{4(a-bx^2)}dx}{2ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)^{3/4}(4bc^2-3bdxc-7ad^2)}{a-bx^2}dx}{8ab}+\frac{(c+dx)^{7/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 653$$

$$\frac{4cd(c+dx)^{3/4}-\frac{\int -\frac{b\left(2c\left(2bc^2-5ad^2\right)+d\left(bc^2-7ad^2\right)x\right)}{\sqrt[4]{c+dx}\,(a-bx^2)}dx}{b}}{8ab}+\frac{(c+dx)^{7/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b\left(2c\left(2bc^2-5ad^2\right)+d\left(bc^2-7ad^2\right)x\right)}{\sqrt[4]{c+dx}\,(a-bx^2)}dx}{b}+4cd(c+dx)^{3/4}}{8ab}+\frac{(c+dx)^{7/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2c(2bc^2-5ad^2)+d(bc^2-7ad^2)x}{\sqrt[4]{c+dx}\,(a-bx^2)}dx+4cd(c+dx)^{3/4}}{8ab}+\frac{(c+dx)^{7/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{2\sqrt{a}c(2bc^2-5ad^2)-\frac{ad\left(bc^2-7ad^2\right)}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)\sqrt[4]{c+dx}}+\frac{\frac{ad\left(bc^2-7ad^2\right)}{\sqrt{b}}+2\sqrt{a}c(2bc^2-5ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}\right)dx+4cd(c+dx)^{3/4}}{8ab}+\frac{(c+dx)^{7/4}(ad+bcx)}{2ab\left(a-bx^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\left(7\sqrt{a}d+4\sqrt{b}c\right)\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}}-\frac{\left(4\sqrt{b}c-7\sqrt{a}d\right)\left(\sqrt{a}d+\sqrt{b}c\right)^{7/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{7/8}}-\frac{\left(7\sqrt{a}d+4\sqrt{\cdots}\right.}{}}{8a\cdots}+\frac{(c+dx)^{7/4}(ad+bcx)}{2ab\left(a-bx^2\right)}$$

input
```
Int[(c + d*x)^(11/4)/(a - b*x^2)^2,x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(7/4))/(2*a*b*(a - b*x^2)) + (4*c*d*(c + d*x)^(3/
4) + ((Sqrt[b]*c - Sqrt[a]*d)^(7/4)*(4*Sqrt[b]*c + 7*Sqrt[a]*d)*ArcTan[(b^
(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)) -
 ((4*Sqrt[b]*c - 7*Sqrt[a]*d)*(Sqrt[b]*c + Sqrt[a]*d)^(7/4)*ArcTan[(b^(1/8
)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)) - ((S
qrt[b]*c - Sqrt[a]*d)^(7/4)*(4*Sqrt[b]*c + 7*Sqrt[a]*d)*ArcTanh[(b^(1/8)*(
c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)) + ((4*Sq
rt[b]*c - 7*Sqrt[a]*d)*(Sqrt[b]*c + Sqrt[a]*d)^(7/4)*ArcTanh[(b^(1/8)*(c +
 d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)))/(8*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.35

method	result
default	$\frac{d\left(-b\,x^{2}+a\right)\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(7\,\_R^{4}a\,d^{2}-\_R^{4}b\,c^{2}+3a\,d^{2}c-3b\,c^{3}\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)+1}{16a\,b^{2}\left(-b\,x^{2}+a\right)}$
pseudoelliptic	$\frac{d\left(-b\,x^{2}+a\right)\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(7\,\_R^{4}a\,d^{2}-\_R^{4}b\,c^{2}+3a\,d^{2}c-3b\,c^{3}\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)+1}{16a\,b^{2}\left(-b\,x^{2}+a\right)}$
derivativedivides	$4d^{3}\left(\frac{\frac{\left(a\,d^{2}+b\,c^{2}\right)\left(dx+c\right)^{\frac{7}{4}}}{8ab\,d^{2}}+\frac{c\left(a\,d^{2}-b\,c^{2}\right)\left(dx+c\right)^{\frac{3}{4}}}{8ab\,d^{2}}}{-b\left(dx+c\right)^{2}+2bc\left(dx+c\right)+a\,d^{2}-b\,c^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\left(-7a\,d^{2}+b\,c^{2}\right)\_R^{6}+\right.}{}}{64a\,b^{2}d^{2}}\right.$

input
```
int((d*x+c)^(11/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(d*(-b*x^2+a)*sum(1/_R*(7*_R^4*a*d^2-_R^4*b*c^2+3*a*c*d^2-3*b*c^3)*ln
((d*x+c)^(1/4)-_R)/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+16*(
d*x+c)^(3/4)*b*(1/2*c^2*b*x+a*d*(1/2*d*x+c)))/a/b^2/(-b*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8266 vs. $2(292) = 584$.

Time = 43.82 (sec) , antiderivative size = 8266, normalized size of antiderivative = 20.51

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(11/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(11/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{11}{4}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((d*x+c)^(11/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(11/4)/(b*x^2 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(11/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 16629, normalized size of antiderivative = 41.26

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(11/4)/(a - b*x^2)^2,x)
```

output
```
(atan((((((175616*a^12*b^7*d^19 - 40960*a^6*b^13*c^12*d^7 + 250368*a^7*b^
12*c^10*d^9 - 526848*a^8*b^11*c^8*d^11 + 312320*a^9*b^10*c^6*d^13 + 334848
*a^10*b^9*c^4*d^15 - 505344*a^11*b^8*c^2*d^17)*1i)/(4096*a^7*b^5) - ((c +
d*x)^(1/4)*((2401*a^4*d^11*(a^15*b^15)^(1/2) + 256*a^6*b^13*c^11 + 11319*a
^11*b^8*c*d^10 - 2464*a^7*b^12*c^9*d^2 + 9009*a^8*b^11*c^7*d^4 - 12859*a^9
*b^10*c^5*d^6 - 77*a^10*b^9*c^3*d^8 - 1232*b^4*c^8*d^3*(a^15*b^15)^(1/2) +
 8855*a*b^3*c^6*d^5*(a^15*b^15)^(1/2) + 16709*a^3*b*c^2*d^9*(a^15*b^15)^(1
/2) - 21549*a^2*b^2*c^4*d^7*(a^15*b^15)^(1/2))/(a^12*b^15))^(1/4)*(3211264
*a^11*b^7*d^16 - 1048576*a^6*b^12*c^10*d^6 + 6225920*a^7*b^11*c^8*d^8 - 15
597568*a^8*b^10*c^6*d^10 + 19922944*a^9*b^9*c^4*d^12 - 12713984*a^10*b^8*c
^2*d^14))/(524288*a^6*b^3))*((2401*a^4*d^11*(a^15*b^15)^(1/2) + 256*a^6*b^
13*c^11 + 11319*a^11*b^8*c*d^10 - 2464*a^7*b^12*c^9*d^2 + 9009*a^8*b^11*c^
7*d^4 - 12859*a^9*b^10*c^5*d^6 - 77*a^10*b^9*c^3*d^8 - 1232*b^4*c^8*d^3*(a
^15*b^15)^(1/2) + 8855*a*b^3*c^6*d^5*(a^15*b^15)^(1/2) + 16709*a^3*b*c^2*d
^9*(a^15*b^15)^(1/2) - 21549*a^2*b^2*c^4*d^7*(a^15*b^15)^(1/2))/(a^12*b^15
))^(3/4))/16 + ((c + d*x)^(1/4)*(453789*a^10*c*d^26 - 4096*b^10*c^21*d^6 +
 63744*a*b^9*c^19*d^8 - 2541287*a^9*b*c^3*d^24 - 409584*a^2*b^8*c^17*d^10
+ 1372887*a^3*b^7*c^15*d^12 - 2392653*a^4*b^6*c^13*d^14 + 1305627*a^5*b^5*
c^11*d^16 + 2809911*a^6*b^4*c^9*d^18 - 6420171*a^7*b^3*c^7*d^20 + 5761833*
a^8*b^2*c^5*d^22))/(128*a^6*b^3))*((2401*a^4*d^11*(a^15*b^15)^(1/2) + 2...
```

Reduce [F]

$$\int \frac{(c+dx)^{11/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{11}{4}}}{(-b\,x^2+a)^2}dx$$

input
```
int((d*x+c)^(11/4)/(-b*x^2+a)^2,x)
```

output
```
int((d*x+c)^(11/4)/(-b*x^2+a)^2,x)
```

3.225 $\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 403

$$\begin{aligned}\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx &= \frac{cd\sqrt[4]{c+dx}}{2ab}+\frac{(ad+bcx)(c+dx)^{5/4}}{2ab\,(a-bx^2)} \\ &- \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}\left(4\sqrt{b}c+5\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{13/8}} \\ &+ \frac{\left(4\sqrt{b}c-5\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{13/8}} \\ &- \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}\left(4\sqrt{b}c+5\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{13/8}} \\ &+ \frac{\left(4\sqrt{b}c-5\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{13/8}}\end{aligned}$$

output
```
1/2*c*d*(d*x+c)^(1/4)/a/b+1/2*(b*c*x+a*d)*(d*x+c)^(5/4)/a/b/(-b*x^2+a)-1/8
*(b^(1/2)*c-a^(1/2)*d)^(5/4)*(4*b^(1/2)*c+5*a^(1/2)*d)*arctan(b^(1/8)*(d*x
+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/b^(13/8)+1/8*(4*b^(1/2)*c-5
*a^(1/2)*d)*(b^(1/2)*c+a^(1/2)*d)^(5/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1
/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(13/8)-1/8*(b^(1/2)*c-a^(1/2)*d)^(5/4)*(
4*b^(1/2)*c+5*a^(1/2)*d)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*
d)^(1/4))/a^(3/2)/b^(13/8)+1/8*(4*b^(1/2)*c-5*a^(1/2)*d)*(b^(1/2)*c+a^(1/2
)*d)^(5/4)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3
/2)/b^(13/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx = \frac{8d^3\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-2c\log\left(\sqrt[4]{c+dx}-\#1\right)-\log\left(\sqrt[4]{c+dx}-\#1\right)\cdots}{c\#1^3-\#1^7}\right.}{}$$

input
```
Integrate[(c + d*x)^(9/4)/(a - b*x^2)^2,x]
```

output
```
(8*d^3*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (-2*c*Log[(c + d*x)
^(1/4) - #1] - Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1^3 - #1^7) & ] + ((8*b
*(c + d*x)^(1/4)*(b*c^2*x + a*d*(2*c + d*x)))/(a - b*x^2) + d*RootSum[b*c^
2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (b*c^3*Log[(c + d*x)^(1/4) - #1] + 15*
a*c*d^2*Log[(c + d*x)^(1/4) - #1] + 3*b*c^2*Log[(c + d*x)^(1/4) - #1]*#1^4
 + 3*a*d^2*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1^3 - #1^7) & ])/a)/(16*b^2
)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {495, 27, 653, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2} \, dx$$

$$\Big\downarrow \; 495$$

$$\frac{(c+dx)^{5/4}(ad+bcx)}{2ab(a-bx^2)} - \frac{\int -\frac{\sqrt[4]{c+dx}(4bc^2-bdxc-5ad^2)}{4(a-bx^2)}dx}{2ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt[4]{c+dx}(4bc^2-bdxc-5ad^2)}{a-bx^2}dx}{8ab} + \frac{(c+dx)^{5/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 653$$

$$\frac{4cd\sqrt[4]{c+dx} - \frac{\int -\frac{b\left(2c\left(2bc^2-3ad^2\right)+d\left(3bc^2-5ad^2\right)x\right)}{(c+dx)^{3/4}\left(a-bx^2\right)}dx}{b}}{8ab} + \frac{(c+dx)^{5/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b\left(2c\left(2bc^2-3ad^2\right)+d\left(3bc^2-5ad^2\right)x\right)}{(c+dx)^{3/4}\left(a-bx^2\right)}dx}{b} + 4cd\sqrt[4]{c+dx}}{8ab} + \frac{(c+dx)^{5/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2c(2bc^2-3ad^2)+d(3bc^2-5ad^2)x}{(c+dx)^{3/4}(a-bx^2)}dx + 4cd\sqrt[4]{c+dx}}{8ab} + \frac{(c+dx)^{5/4}(ad+bcx)}{2ab(a-bx^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{2\sqrt{a}c(2bc^2-3ad^2)-\frac{ad\left(3bc^2-5ad^2\right)}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}+\frac{\frac{ad\left(3bc^2-5ad^2\right)}{\sqrt{b}}+2\sqrt{a}c(2bc^2-3ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx+4cd\sqrt[4]{c+dx}}{8ab}+\frac{(c+dx)^{5/4}(ad+bcx)}{2ab\left(a-bx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\left(5\sqrt{a}d+4\sqrt{b}c\right)\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}}+\frac{\left(4\sqrt{b}c-5\sqrt{a}d\right)\left(\sqrt{a}d+\sqrt{b}c\right)^{5/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{5/8}}-\frac{\left(5\sqrt{a}d+4\right.}{}}{8a}$$

$$\frac{(c+dx)^{5/4}(ad+bcx)}{2ab\left(a-bx^2\right)}$$

input
```
Int[(c + d*x)^(9/4)/(a - b*x^2)^2,x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(5/4))/(2*a*b*(a - b*x^2)) + (4*c*d*(c + d*x)^(1/
4) - ((Sqrt[b]*c - Sqrt[a]*d)^(5/4)*(4*Sqrt[b]*c + 5*Sqrt[a]*d)*ArcTan[(b^
(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)) +
 ((4*Sqrt[b]*c - 5*Sqrt[a]*d)*(Sqrt[b]*c + Sqrt[a]*d)^(5/4)*ArcTan[(b^(1/8
)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)) - ((S
qrt[b]*c - Sqrt[a]*d)^(5/4)*(4*Sqrt[b]*c + 5*Sqrt[a]*d)*ArcTanh[(b^(1/8)*(
c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)) + ((4*Sq
rt[b]*c - 5*Sqrt[a]*d)*(Sqrt[b]*c + Sqrt[a]*d)^(5/4)*ArcTanh[(b^(1/8)*(c +
 d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)))/(8*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.35

method	result
default	$\frac{d(-b\,x^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^8-2bc\,\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(5\,\_R^4\,a\,d^2-3\,\_R^4\,b\,c^2+a\,d^2c-b\,c^3\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+16}{16a\,b^2(-b\,x^2+a)}$
pseudoelliptic	$\frac{d(-b\,x^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^8-2bc\,\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(5\,\_R^4\,a\,d^2-3\,\_R^4\,b\,c^2+a\,d^2c-b\,c^3\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+16}{16a\,b^2(-b\,x^2+a)}$
derivativedivides	$4d^3\left(\frac{\frac{\left(a\,d^2+b\,c^2\right)(dx+c)^{\frac{5}{4}}}{8ab\,d^2}+\frac{\left(a\,d^2-b\,c^2\right)c(dx+c)^{\frac{1}{4}}}{8ab\,d^2}}{-b(dx+c)^2+2bc(dx+c)+a\,d^2-b\,c^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^8-2bc\,\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\left(-5a\,d^2+3b\,c^2\right)\_R^4\right.}{-\_}}{64a\,b^2d^2}\right.$

input
```
int((d*x+c)^(9/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(d*(-b*x^2+a)*sum((5*_R^4*a*d^2-3*_R^4*b*c^2+a*c*d^2-b*c^3)*ln((d*x+c
)^(1/4)-_R)/_R^3/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+16*b*(
1/2*c^2*b*x+a*d*(1/2*d*x+c))*(d*x+c)^(1/4))/a/b^2/(-b*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4964 vs. $2(292) = 584$.

Time = 1.31 (sec) , antiderivative size = 4964, normalized size of antiderivative = 12.32

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(9/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(9/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{9}{4}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((d*x+c)^(9/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(9/4)/(b*x^2 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(9/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 17148, normalized size of antiderivative = 42.55

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(9/4)/(a - b*x^2)^2,x)
```

output
```
(atan((((((2*(8125*a^7*c*d^21 - 1280*b^7*c^15*d^7 + 12000*a*b^6*c^13*d^9
- 44925*a^6*b*c^3*d^19 - 47769*a^2*b^5*c^11*d^11 + 104657*a^3*b^4*c^9*d^13
 - 136282*a^4*b^3*c^7*d^15 + 105474*a^5*b^2*c^5*d^17))/a^4 + (((((256*a^6*
b^11*c^9 - 625*a^3*d^9*(a^15*b^13)^(1/2) + 1125*a^10*b^7*c*d^8 - 1440*a^7*
b^10*c^7*d^2 + 3105*a^8*b^9*c^5*d^4 - 3030*a^9*b^8*c^3*d^6 + 240*b^3*c^6*d
^3*(a^15*b^13)^(1/2) - 981*a*b^2*c^4*d^5*(a^15*b^13)^(1/2) + 1350*a^2*b*c^
2*d^7*(a^15*b^13)^(1/2))/(a^12*b^13))^(1/4)*(327680*a^9*b^6*d^13 - 327680*
a^6*b^9*c^6*d^7 + 983040*a^7*b^8*c^4*d^9 - 983040*a^8*b^7*c^2*d^11)*1i)/(8
*a^4) - ((c + d*x)^(1/4)*(983040*a^10*b^7*c*d^14 + 1048576*a^6*b^11*c^9*d^
6 - 4128768*a^7*b^10*c^7*d^8 + 6094848*a^8*b^9*c^5*d^10 - 3997696*a^9*b^8*
c^3*d^12))/(8*a^6*b^2))*((256*a^6*b^11*c^9 - 625*a^3*d^9*(a^15*b^13)^(1/2)
 + 1125*a^10*b^7*c*d^8 - 1440*a^7*b^10*c^7*d^2 + 3105*a^8*b^9*c^5*d^4 - 30
30*a^9*b^8*c^3*d^6 + 240*b^3*c^6*d^3*(a^15*b^13)^(1/2) - 981*a*b^2*c^4*d^5
*(a^15*b^13)^(1/2) + 1350*a^2*b*c^2*d^7*(a^15*b^13)^(1/2))/(a^12*b^13))^(3
/4)*1i)/4096)*((256*a^6*b^11*c^9 - 625*a^3*d^9*(a^15*b^13)^(1/2) + 1125*a^
10*b^7*c*d^8 - 1440*a^7*b^10*c^7*d^2 + 3105*a^8*b^9*c^5*d^4 - 3030*a^9*b^8
*c^3*d^6 + 240*b^3*c^6*d^3*(a^15*b^13)^(1/2) - 981*a*b^2*c^4*d^5*(a^15*b^1
3)^(1/2) + 1350*a^2*b*c^2*d^7*(a^15*b^13)^(1/2))/(a^12*b^13))^(1/4)*1i)/16
 - ((c + d*x)^(1/4)*(15625*a^9*d^24 + 4096*b^9*c^18*d^6 - 39168*a*b^8*c^16
*d^8 - 78750*a^8*b*c^2*d^22 + 158448*a^2*b^7*c^14*d^10 - 346807*a^3*b^6...
```

Reduce [F]

$$\int \frac{(c+dx)^{9/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{9}{4}}}{(-b\,x^2+a)^2}dx$$

input
```
int((d*x+c)^(9/4)/(-b*x^2+a)^2,x)
```

output
```
int((d*x+c)^(9/4)/(-b*x^2+a)^2,x)
```

3.226 $\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 382

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2} dx = \frac{(ad+bcx)(c+dx)^{3/4}}{2ab(a-bx^2)} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}\left(4\sqrt{b}c+3\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{11/8}} - \frac{\left(4\sqrt{b}c-3\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{11/8}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}\left(4\sqrt{b}c+3\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{11/8}} + \frac{\left(4\sqrt{b}c-3\sqrt{a}d\right)\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{11/8}}$$

output
```
1/2*(b*c*x+a*d)*(d*x+c)^(3/4)/a/b/(-b*x^2+a)+1/8*(b^(1/2)*c-a^(1/2)*d)^(3/
4)*(4*b^(1/2)*c+3*a^(1/2)*d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/
2)*d)^(1/4))/a^(3/2)/b^(11/8)-1/8*(4*b^(1/2)*c-3*a^(1/2)*d)*(b^(1/2)*c+a^(
1/2)*d)^(3/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^
(3/2)/b^(11/8)-1/8*(b^(1/2)*c-a^(1/2)*d)^(3/4)*(4*b^(1/2)*c+3*a^(1/2)*d)*a
rctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/b^(11/8)
+1/8*(4*b^(1/2)*c-3*a^(1/2)*d)*(b^(1/2)*c+a^(1/2)*d)^(3/4)*arctanh(b^(1/8)
*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(11/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx = \frac{-8d^3\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{\log\left(\sqrt[4]{c+dx}-\#1\right)}{c\#1-\#1^5}\&\right]+\frac{8b(ad+bcx)(c+dx)^{3/4}}{a-bx^2}}{\cdots}$$

input
```
Integrate[(c + d*x)^(7/4)/(a - b*x^2)^2,x]
```

output
```
(-8*d^3*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , Log[(c + d*x)^(1/4
) - #1]/(c*#1 - #1^5) & ] + ((8*b*(a*d + b*c*x)*(c + d*x)^(3/4))/(a - b*x^
2) + d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (3*b*c^2*Log[(c + d
*x)^(1/4) - #1] + 5*a*d^2*Log[(c + d*x)^(1/4) - #1] + b*c*Log[(c + d*x)^(1
/4) - #1]*#1^4)/(c*#1 - #1^5) & ])/a)/(16*b^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {495, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx$$

$$\Big\downarrow \; 495$$

$$\frac{(c+dx)^{3/4}(ad+bcx)}{2ab\,(a-bx^2)}-\frac{\int -\frac{4bc^2+bdxc-3ad^2}{4\sqrt[4]{c+dx}(a-bx^2)}dx}{2ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4bc^2+bdxc-3ad^2}{\sqrt[4]{c+dx}(a-bx^2)}dx}{8ab}+\frac{(c+dx)^{3/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{\sqrt{a}(4bc^2-3ad^2)-a\sqrt{b}cd}{2a\left(\sqrt{b}x+\sqrt{a}\right)\sqrt[4]{c+dx}}+\frac{a\sqrt{b}cd+\sqrt{a}(4bc^2-3ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}\right)dx}{8ab}+\frac{(c+dx)^{3/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}\left(3\sqrt{a}d+4\sqrt{b}c\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}}-\frac{\left(4\sqrt{b}c-3\sqrt{a}d\right)\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{3/8}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\cdots}{}}{8ab}$$

$$\frac{(c+dx)^{3/4}(ad+bcx)}{2ab\,(a-bx^2)}$$

input
```
Int[(c + d*x)^(7/4)/(a - b*x^2)^2,x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(3/4))/(2*a*b*(a - b*x^2)) + (((Sqrt[b]*c - Sqrt[
a]*d)^(3/4)*(4*Sqrt[b]*c + 3*Sqrt[a]*d)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(
Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)) - ((4*Sqrt[b]*c - 3*Sqrt[
a]*d)*(Sqrt[b]*c + Sqrt[a]*d)^(3/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt
[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)) - ((Sqrt[b]*c - Sqrt[a]*d)^(3
/4)*(4*Sqrt[b]*c + 3*Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]
*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)) + ((4*Sqrt[b]*c - 3*Sqrt[a]*d)*(
Sqrt[b]*c + Sqrt[a]*d)^(3/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c
+ Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)))/(8*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.32

method	result
default	$\frac{-d\left(-b\,x^{2}+a\right)\left(\sum_{\_\_R=\mathrm{RootOf}\left(b\_\_Z^{8}-2bc\_\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\_\_R^{4}bc-3a\,d^{2}+3b\,c^{2}\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_\_R\right)}{\_\_R\left(\_\_R^{4}-c\right)}\right)+8\left(dx+c\right)^{\frac{3}{4}}b(cb}{16a\,b^{2}\left(-b\,x^{2}+a\right)}$
pseudoelliptic	$\frac{-d\left(-b\,x^{2}+a\right)\left(\sum_{\_\_R=\mathrm{RootOf}\left(b\_\_Z^{8}-2bc\_\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\_\_R^{4}bc-3a\,d^{2}+3b\,c^{2}\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_\_R\right)}{\_\_R\left(\_\_R^{4}-c\right)}\right)+8\left(dx+c\right)^{\frac{3}{4}}b(cb}{16a\,b^{2}\left(-b\,x^{2}+a\right)}$
derivativedivides	$4d^{3}\left(\frac{\frac{c\left(dx+c\right)^{\frac{7}{4}}}{8a\,d^{2}}+\frac{\left(a\,d^{2}-b\,c^{2}\right)\left(dx+c\right)^{\frac{3}{4}}}{8a\,d^{2}b}}{-b\left(dx+c\right)^{2}+2bc\left(dx+c\right)+a\,d^{2}-b\,c^{2}}+\frac{\sum_{\_\_R=\mathrm{RootOf}\left(b\_\_Z^{8}-2bc\_\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(c\_\_R^{6}b+3\left(-a\,d^{2}+b\,c^{2}\right)\_\_R^{2}\right.}{-\_\_R^{7}+\_\_R}}{64a\,d^{2}b^{2}}\right.$

input
```
int((d*x+c)^(7/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-d*(-b*x^2+a)*sum(1/_R*(_R^4*b*c-3*a*d^2+3*b*c^2)*ln((d*x+c)^(1/4)-_
R)/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+8*(d*x+c)^(3/4)*b*(b
*c*x+a*d))/a/b^2/(-b*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5011 vs. $2(275) = 550$.

Time = 3.12 (sec) , antiderivative size = 5011, normalized size of antiderivative = 13.12

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(7/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(7/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{7}{4}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((d*x+c)^(7/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(7/4)/(b*x^2 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(7/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 11171, normalized size of antiderivative = 29.24

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(7/4)/(a - b*x^2)^2,x)
```

output
```
(atan((((((41472*a^10*b^6*c*d^15 + 40960*a^6*b^10*c^9*d^7 - 164352*a^7*b^
9*c^7*d^9 + 247296*a^8*b^8*c^5*d^11 - 165376*a^9*b^7*c^3*d^13)*1i)/(4096*a
^7*b^2) - ((c + d*x)^(1/4)*((81*a^2*d^7*(a^15*b^11)^(1/2) + 256*a^6*b^9*c^
7 - 189*a^9*b^6*c*d^6 - 672*a^7*b^8*c^5*d^2 + 609*a^8*b^7*c^3*d^4 + 112*b^
2*c^4*d^3*(a^15*b^11)^(1/2) - 189*a*b*c^2*d^5*(a^15*b^11)^(1/2))/(a^12*b^1
1))^(1/4)*(1048576*a^6*b^9*c^8*d^6 - 589824*a^10*b^5*d^14 - 2555904*a^7*b^
8*c^6*d^8 + 1376256*a^8*b^7*c^4*d^10 + 720896*a^9*b^6*c^2*d^12))/(524288*a
^6))*((81*a^2*d^7*(a^15*b^11)^(1/2) + 256*a^6*b^9*c^7 - 189*a^9*b^6*c*d^6
- 672*a^7*b^8*c^5*d^2 + 609*a^8*b^7*c^3*d^4 + 112*b^2*c^4*d^3*(a^15*b^11)^
(1/2) - 189*a*b*c^2*d^5*(a^15*b^11)^(1/2))/(a^12*b^11))^(3/4))/16 - ((c +
d*x)^(1/4)*(243*a^7*c*d^20 - 4096*b^7*c^15*d^6 + 20736*a*b^6*c^13*d^8 - 28
08*a^6*b*c^3*d^18 - 44016*a^2*b^5*c^11*d^10 + 50671*a^3*b^4*c^9*d^12 - 340
80*a^4*b^3*c^7*d^14 + 13350*a^5*b^2*c^5*d^16))/(128*a^6))*((81*a^2*d^7*(a^
15*b^11)^(1/2) + 256*a^6*b^9*c^7 - 189*a^9*b^6*c*d^6 - 672*a^7*b^8*c^5*d^2
 + 609*a^8*b^7*c^3*d^4 + 112*b^2*c^4*d^3*(a^15*b^11)^(1/2) - 189*a*b*c^2*d
^5*(a^15*b^11)^(1/2))/(a^12*b^11))^(1/4) - (((((41472*a^10*b^6*c*d^15 + 40
960*a^6*b^10*c^9*d^7 - 164352*a^7*b^9*c^7*d^9 + 247296*a^8*b^8*c^5*d^11 -
165376*a^9*b^7*c^3*d^13)*1i)/(4096*a^7*b^2) + ((c + d*x)^(1/4)*((81*a^2*d^
7*(a^15*b^11)^(1/2) + 256*a^6*b^9*c^7 - 189*a^9*b^6*c*d^6 - 672*a^7*b^8*c^
5*d^2 + 609*a^8*b^7*c^3*d^4 + 112*b^2*c^4*d^3*(a^15*b^11)^(1/2) - 189*a...
```

Reduce [F]

$$\int \frac{(c+dx)^{7/4}}{(a-bx^2)^2}\,dx=\int \frac{(dx+c)^{\frac{7}{4}}}{(-b\,x^2+a)^2}dx$$

input
```
int((d*x+c)^(7/4)/(-b*x^2+a)^2,x)
```

output
```
int((d*x+c)^(7/4)/(-b*x^2+a)^2,x)
```

3.227 $\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 380

$$
\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx &= \frac{(ad+bcx)\sqrt[4]{c+dx}}{2ab\,(a-bx^2)} \\
&- \frac{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}\left(4\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{9/8}} \\
&+ \frac{\left(4\sqrt{b}c-\sqrt{a}d\right)\sqrt[4]{\sqrt{b}c+\sqrt{a}d}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{9/8}} \\
&- \frac{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}\left(4\sqrt{b}c+\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{9/8}} \\
&+ \frac{\left(4\sqrt{b}c-\sqrt{a}d\right)\sqrt[4]{\sqrt{b}c+\sqrt{a}d}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{9/8}}
\end{aligned}
$$

output
```
1/2*(b*c*x+a*d)*(d*x+c)^(1/4)/a/b/(-b*x^2+a)-1/8*(b^(1/2)*c-a^(1/2)*d)^(1/
4)*(4*b^(1/2)*c+a^(1/2)*d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)
*d)^(1/4))/a^(3/2)/b^(9/8)+1/8*(4*b^(1/2)*c-a^(1/2)*d)*(b^(1/2)*c+a^(1/2)*
d)^(1/4)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)
/b^(9/8)-1/8*(b^(1/2)*c-a^(1/2)*d)^(1/4)*(4*b^(1/2)*c+a^(1/2)*d)*arctanh(b
^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/b^(9/8)+1/8*(4*b
^(1/2)*c-a^(1/2)*d)*(b^(1/2)*c+a^(1/2)*d)^(1/4)*arctanh(b^(1/8)*(d*x+c)^(1
/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(9/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.58

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx = \frac{-8d^3\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{\log\left(\sqrt[4]{c+dx}-\#1\right)}{c\#1^3-\#1^7}\&\right]+\frac{8b(ad+bcx)\sqrt[4]{c+\cdots}}{a-bx^2}\cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^(5/4)/(a - b*x^2)^2,x]
```

output
```
(-8*d^3*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , Log[(c + d*x)^(1/4
) - #1]/(c*#1^3 - #1^7) & ] + ((8*b*(a*d + b*c*x)*(c + d*x)^(1/4))/(a - b*
x^2) + d*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4 + b*#1^8 & , (b*c^2*Log[(c + d
*x)^(1/4) - #1] + 7*a*d^2*Log[(c + d*x)^(1/4) - #1] + 3*b*c*Log[(c + d*x)^
(1/4) - #1]*#1^4)/(c*#1^3 - #1^7) & ])/a)/(16*b^2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {495, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx$$

$$\Big\downarrow \ 495$$

$$\frac{\sqrt[4]{c+dx}(ad+bcx)}{2ab\,(a-bx^2)}-\frac{\int -\frac{4bc^2+3bdxc-ad^2}{4(c+dx)^{3/4}(a-bx^2)}dx}{2ab}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4bc^2+3bdxc-ad^2}{(c+dx)^{3/4}(a-bx^2)}dx}{8ab}+\frac{\sqrt[4]{c+dx}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \ 657$$

$$\frac{\int \left(\frac{\sqrt{a}(4bc^2-ad^2)-3a\sqrt{b}cd}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}+\frac{3a\sqrt{b}cd+\sqrt{a}(4bc^2-ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx}{8ab}+\frac{\sqrt[4]{c+dx}(ad+bcx)}{2ab\,(a-bx^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}\left(\sqrt{a}d+4\sqrt{b}c\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}}+\frac{\left(4\sqrt{b}c-\sqrt{a}d\right)\sqrt[4]{\sqrt{a}d+\sqrt{b}c}\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}\sqrt[8]{b}}-\frac{\sqrt[4]{\sqrt{b}c \cdots}}{\cdots}}{8ab}$$

$$\frac{\sqrt[4]{c+dx}(ad+bcx)}{2ab\,(a-bx^2)}$$

input
```
Int[(c + d*x)^(5/4)/(a - b*x^2)^2,x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(1/4))/(2*a*b*(a - b*x^2)) + (-(((Sqrt[b]*c - Sqr
t[a]*d)^(1/4)*(4*Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(
Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8))) + ((4*Sqrt[b]*c - Sqrt[a
]*d)*(Sqrt[b]*c + Sqrt[a]*d)^(1/4)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[
b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)) - ((Sqrt[b]*c - Sqrt[a]*d)^(1/
4)*(4*Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c
- Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)) + ((4*Sqrt[b]*c - Sqrt[a]*d)*(Sqrt[
b]*c + Sqrt[a]*d)^(1/4)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqr
t[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)))/(8*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.32

method	result
default	$\frac{-d(-bx^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-ad^2+bc^2\right)}\frac{\left(3\_R^4bc-ad^2+bc^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+8(dx+c)^{\frac{1}{4}}b(cbx}{16ab^2(-bx^2+a)}$
pseudoelliptic	$\frac{-d(-bx^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-ad^2+bc^2\right)}\frac{\left(3\_R^4bc-ad^2+bc^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+8(dx+c)^{\frac{1}{4}}b(cbx}{16ab^2(-bx^2+a)}$
derivativedivides	$4d^3\left(\frac{\frac{c(dx+c)^{\frac{5}{4}}}{8ad^2}+\frac{\left(ad^2-bc^2\right)(dx+c)^{\frac{1}{4}}}{8ad^2b}}{-b(dx+c)^2+2bc(dx+c)+ad^2-bc^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-ad^2+bc^2\right)}\frac{\left(3\_R^4bc-ad^2+bc^2\right)\ln\left((dx+\right.}{-\_R^7+\_R^3c}}{64ad^2b^2}\right.$

input
```
int((d*x+c)^(5/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-d*(-b*x^2+a)*sum((3*_R^4*b*c-a*d^2+b*c^2)*ln((d*x+c)^(1/4)-_R)/_R^3
/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+8*(d*x+c)^(1/4)*b*(b*c
*x+a*d))/a/b^2/(-b*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2425 vs. $2(273) = 546$.

Time = 0.20 (sec) , antiderivative size = 2425, normalized size of antiderivative = 6.38

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/16*((a*b^2*x^2 - a^2*b)*sqrt(-sqrt((a^6*b^4*sqrt((6400*b^2*c^4*d^6 + 16
0*a*b*c^2*d^8 + a^2*d^10)/(a^9*b^9)) + 256*b^2*c^5 - 160*a*b*c^3*d^2 - 15*
a^2*c*d^4)/(a^6*b^4)))*log((4*a^6*b^6*c*sqrt((6400*b^2*c^4*d^6 + 160*a*b*c
^2*d^8 + a^2*d^10)/(a^9*b^9)) + 80*a^2*b^2*c^2*d^4 + a^3*b*d^6)*sqrt(-sqrt
((a^6*b^4*sqrt((6400*b^2*c^4*d^6 + 160*a*b*c^2*d^8 + a^2*d^10)/(a^9*b^9))
+ 256*b^2*c^5 - 160*a*b*c^3*d^2 - 15*a^2*c*d^4)/(a^6*b^4))) - (1280*b^2*c^
4*d^3 - 64*a*b*c^2*d^5 - a^2*d^7)*(d*x + c)^(1/4)) - (a*b^2*x^2 - a^2*b)*s
qrt(-sqrt((a^6*b^4*sqrt((6400*b^2*c^4*d^6 + 160*a*b*c^2*d^8 + a^2*d^10)/(a
^9*b^9)) + 256*b^2*c^5 - 160*a*b*c^3*d^2 - 15*a^2*c*d^4)/(a^6*b^4)))*log(-
(4*a^6*b^6*c*sqrt((6400*b^2*c^4*d^6 + 160*a*b*c^2*d^8 + a^2*d^10)/(a^9*b^9
)) + 80*a^2*b^2*c^2*d^4 + a^3*b*d^6)*sqrt(-sqrt((a^6*b^4*sqrt((6400*b^2*c^
4*d^6 + 160*a*b*c^2*d^8 + a^2*d^10)/(a^9*b^9)) + 256*b^2*c^5 - 160*a*b*c^3
*d^2 - 15*a^2*c*d^4)/(a^6*b^4))) - (1280*b^2*c^4*d^3 - 64*a*b*c^2*d^5 - a^
2*d^7)*(d*x + c)^(1/4)) - (a*b^2*x^2 - a^2*b)*sqrt(-sqrt(-(a^6*b^4*sqrt((6
400*b^2*c^4*d^6 + 160*a*b*c^2*d^8 + a^2*d^10)/(a^9*b^9)) - 256*b^2*c^5 + 1
60*a*b*c^3*d^2 + 15*a^2*c*d^4)/(a^6*b^4)))*log((4*a^6*b^6*c*sqrt((6400*b^2
*c^4*d^6 + 160*a*b*c^2*d^8 + a^2*d^10)/(a^9*b^9)) - 80*a^2*b^2*c^2*d^4 - a
^3*b*d^6)*sqrt(-sqrt(-(a^6*b^4*sqrt((6400*b^2*c^4*d^6 + 160*a*b*c^2*d^8 +
a^2*d^10)/(a^9*b^9)) - 256*b^2*c^5 + 160*a*b*c^3*d^2 + 15*a^2*c*d^4)/(a^6*
b^4))) - (1280*b^2*c^4*d^3 - 64*a*b*c^2*d^5 - a^2*d^7)*(d*x + c)^(1/4))...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{5}{4}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((d*x+c)^(5/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(5/4)/(b*x^2 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(5/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 10441, normalized size of antiderivative = 27.48

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((c + d*x)^(5/4)/(a - b*x^2)^2,x)
```

output
```
- (((a*d^3 - b*c^2*d)*(c + d*x)^(1/4))/(2*a*b) + (c*d*(c + d*x)^(5/4))/(2*
a))/(b*(c + d*x)^2 - a*d^2 + b*c^2 - 2*b*c*(c + d*x)) - (atan((((((((2*(a^5
*b^2*d^17 + 1280*b^7*c^10*d^7 - 3040*a*b^6*c^8*d^9 + 2249*a^2*b^5*c^6*d^11
 - 497*a^3*b^4*c^4*d^13 + 7*a^4*b^3*c^2*d^15))/a^4 - ((((c + d*x)^(1/4)*(5
89824*a^9*b^6*c*d^12 + 1048576*a^6*b^9*c^7*d^6 - 1507328*a^7*b^8*c^5*d^8 -
 131072*a^8*b^7*c^3*d^10))/(8*a^6) - ((327680*a^8*b^7*c*d^11 + 327680*a^6*
b^9*c^5*d^7 - 655360*a^7*b^8*c^3*d^9)*((256*a^6*b^7*c^5 + a*d^5*(a^15*b^9)
^(1/2) - 15*a^8*b^5*c*d^4 - 160*a^7*b^6*c^3*d^2 + 80*b*c^2*d^3*(a^15*b^9)^
(1/2))/(a^12*b^9))^(1/4)*1i)/(8*a^4))*((256*a^6*b^7*c^5 + a*d^5*(a^15*b^9)
^(1/2) - 15*a^8*b^5*c*d^4 - 160*a^7*b^6*c^3*d^2 + 80*b*c^2*d^3*(a^15*b^9)^
(1/2))/(a^12*b^9))^(3/4)*1i)/4096)*((256*a^6*b^7*c^5 + a*d^5*(a^15*b^9)^(1
/2) - 15*a^8*b^5*c*d^4 - 160*a^7*b^6*c^3*d^2 + 80*b*c^2*d^3*(a^15*b^9)^(1/
2))/(a^12*b^9))^(1/4)*1i)/16 - ((c + d*x)^(1/4)*(a^6*b*d^18 + 4096*b^7*c^1
2*d^6 - 8448*a*b^6*c^10*d^8 + 4848*a^2*b^5*c^8*d^10 - 767*a^3*b^4*c^6*d^12
 + 303*a^4*b^3*c^4*d^14 - 33*a^5*b^2*c^2*d^16))/(8*a^6))*((256*a^6*b^7*c^5
 + a*d^5*(a^15*b^9)^(1/2) - 15*a^8*b^5*c*d^4 - 160*a^7*b^6*c^3*d^2 + 80*b*
c^2*d^3*(a^15*b^9)^(1/2))/(a^12*b^9))^(1/4))/16 - (((((2*(a^5*b^2*d^17 + 1
280*b^7*c^10*d^7 - 3040*a*b^6*c^8*d^9 + 2249*a^2*b^5*c^6*d^11 - 497*a^3*b^
4*c^4*d^13 + 7*a^4*b^3*c^2*d^15))/a^4 + ((((c + d*x)^(1/4)*(589824*a^9*b^6
*c*d^12 + 1048576*a^6*b^9*c^7*d^6 - 1507328*a^7*b^8*c^5*d^8 - 131072*a^...
```

Reduce [F]

$$\int \frac{(c+dx)^{5/4}}{(a-bx^2)^2} \, dx = \int \frac{(dx+c)^{\frac{5}{4}}}{(-b\,x^2+a)^2}dx$$

input
```
int((d*x+c)^(5/4)/(-b*x^2+a)^2,x)
```

output
```
int((d*x+c)^(5/4)/(-b*x^2+a)^2,x)
```

3.228 $\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 360

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \frac{x(c+dx)^{3/4}}{2a\,(a-bx^2)} + \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}-d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8ab^{7/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}} - \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}+d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8ab^{7/8}\sqrt[4]{\sqrt{b}c+\sqrt{a}d}} - \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}-d\right)\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8ab^{7/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}} + \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}+d\right)\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8ab^{7/8}\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}$$

output
```
1/2*x*(d*x+c)^(3/4)/a/(-b*x^2+a)+1/8*(4*b^(1/2)*c/a^(1/2)-d)*arctan(b^(1/8
)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a/b^(7/8)/(b^(1/2)*c-a^(1/2)*
d)^(1/4)-1/8*(4*b^(1/2)*c/a^(1/2)+d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)
*c+a^(1/2)*d)^(1/4))/a/b^(7/8)/(b^(1/2)*c+a^(1/2)*d)^(1/4)-1/8*(4*b^(1/2)*
c/a^(1/2)-d)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a/
b^(7/8)/(b^(1/2)*c-a^(1/2)*d)^(1/4)+1/8*(4*b^(1/2)*c/a^(1/2)+d)*arctanh(b^
(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a/b^(7/8)/(b^(1/2)*c+a^(1
/2)*d)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.36

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \frac{8bx(c+dx)^{3/4}+(-ad+bdx^2)\,\text{RootSum}\Big[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-3c\log\left(\sqrt[4]{c+\right.}{}}{}}{16ab\,(a-bx^2)}$$

input
```
Integrate[(c + d*x)^(3/4)/(a - b*x^2)^2,x]
```

output
```
(8*b*x*(c + d*x)^(3/4) + (-(a*d) + b*d*x^2)*RootSum[b*c^2 - a*d^2 - 2*b*c*
#1^4 + b*#1^8 & , (-3*c*Log[(c + d*x)^(1/4) - #1] - Log[(c + d*x)^(1/4) -
#1]*#1^4)/(c*#1 - #1^5) & ])/(16*a*b*(a - b*x^2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {494, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx$$

$$\Big\downarrow \; 494$$

$$\frac{x(c+dx)^{3/4}}{2a\left(a-bx^2\right)}-\frac{\int -\frac{4c+dx}{4\sqrt[4]{c+dx}(a-bx^2)}dx}{2a}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4c+dx}{\sqrt[4]{c+dx}(a-bx^2)}dx}{8a}+\frac{x(c+dx)^{3/4}}{2a\left(a-bx^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{4\sqrt{a}c-\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)\sqrt[4]{c+dx}}+\frac{4\sqrt{a}c+\frac{ad}{\sqrt{b}}}{2a\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}\right)dx}{8a}+\frac{x(c+dx)^{3/4}}{2a\left(a-bx^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\left(4\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}-\frac{\left(\sqrt{a}d+4\sqrt{b}c\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}-\frac{\left(4\sqrt{b}c-\sqrt{a}d\right)\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}}{8a}$$

$$\frac{x(c+dx)^{3/4}}{2a\left(a-bx^2\right)}$$

input
```
Int[(c + d*x)^(3/4)/(a - b*x^2)^2,x]
```

output
```
(x*(c + d*x)^(3/4))/(2*a*(a - b*x^2)) + (((4*Sqrt[b]*c - Sqrt[a]*d)*ArcTan
[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8
)*(Sqrt[b]*c - Sqrt[a]*d)^(1/4)) - ((4*Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1
/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sqr
t[b]*c + Sqrt[a]*d)^(1/4)) - ((4*Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/8)*(
c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sqrt[b]*
c - Sqrt[a]*d)^(1/4)) + ((4*Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c + d
*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sqrt[b]*c + S
qrt[a]*d)^(1/4)))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

method	result
default	$\frac{-d\left(-b\,x^{2}+a\right)\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\_R^{4}+3c\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)+8bx\left(dx+c\right)^{\frac{3}{4}}}{16\left(-b\,x^{2}+a\right)ab}$
pseudoelliptic	$\frac{-d\left(-b\,x^{2}+a\right)\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(\_R^{4}+3c\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^{4}-c\right)}\right)+8bx\left(dx+c\right)^{\frac{3}{4}}}{16\left(-b\,x^{2}+a\right)ab}$
derivativedivides	$4d^{3}\left(\frac{\frac{\left(dx+c\right)^{\frac{7}{4}}}{8a\,d^{2}}-\frac{c\left(dx+c\right)^{\frac{3}{4}}}{8a\,d^{2}}}{-b\left(dx+c\right)^{2}+2bc\left(dx+c\right)+a\,d^{2}-b\,c^{2}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(-\_R^{6}-3\_R^{2}c\right)\ln\left(\left(dx+c\right)\right.}{-\_R^{7}+\_R^{3}c}}{64a\,d^{2}b}\right.$

input
```
int((d*x+c)^(3/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-d*(-b*x^2+a)*sum(1/_R*(_R^4+3*c)*ln((d*x+c)^(1/4)-_R)/(_R^4-c),_R=R
ootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+8*b*x*(d*x+c)^(3/4))/(-b*x^2+a)/a/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4973 vs. 2(261) = 522.

Time = 1.00 (sec) , antiderivative size = 4973, normalized size of antiderivative = 13.81

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(3/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{3}{4}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((d*x+c)^(3/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/4)/(b*x^2 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(3/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 8593, normalized size of antiderivative = 23.87

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(3/4)/(a - b*x^2)^2,x)
```

output

```
atan((((65536*a^9*b^5*d^13 + 5242880*a^6*b^8*c^6*d^7 - 6881280*a^7*b^7*c^
4*d^9 + 1572864*a^8*b^6*c^2*d^11)/(128*a^7) - ((c + d*x)^(1/4)*(-(a*d^5*(a
^15*b^7)^(1/2) - 256*a^6*b^6*c^5 + 15*a^8*b^4*c*d^4 + 160*a^7*b^5*c^3*d^2
+ 80*b*c^2*d^3*(a^15*b^7)^(1/2))/(65536*(a^12*b^8*c^2 - a^13*b^7*d^2)))^(1
/4)*(65536*a^9*b^6*d^12 - 1048576*a^6*b^9*c^6*d^6 + 983040*a^7*b^8*c^4*d^8
))/(8*a^6))*(-(a*d^5*(a^15*b^7)^(1/2) - 256*a^6*b^6*c^5 + 15*a^8*b^4*c*d^4
 + 160*a^7*b^5*c^3*d^2 + 80*b*c^2*d^3*(a^15*b^7)^(1/2))/(65536*(a^12*b^8*c
^2 - a^13*b^7*d^2)))^(3/4) + ((c + d*x)^(1/4)*(2304*a*b^6*c^7*d^8 - 4096*b
^7*c^9*d^6 + 9*a^4*b^3*c*d^14 + 2064*a^2*b^5*c^5*d^10 - 281*a^3*b^4*c^3*d^
12))/(8*a^6))*(-(a*d^5*(a^15*b^7)^(1/2) - 256*a^6*b^6*c^5 + 15*a^8*b^4*c*d
^4 + 160*a^7*b^5*c^3*d^2 + 80*b*c^2*d^3*(a^15*b^7)^(1/2))/(65536*(a^12*b^8
*c^2 - a^13*b^7*d^2)))^(1/4)*1i - (((65536*a^9*b^5*d^13 + 5242880*a^6*b^8*
c^6*d^7 - 6881280*a^7*b^7*c^4*d^9 + 1572864*a^8*b^6*c^2*d^11)/(128*a^7) +
((c + d*x)^(1/4)*(-(a*d^5*(a^15*b^7)^(1/2) - 256*a^6*b^6*c^5 + 15*a^8*b^4*
c*d^4 + 160*a^7*b^5*c^3*d^2 + 80*b*c^2*d^3*(a^15*b^7)^(1/2))/(65536*(a^12*
b^8*c^2 - a^13*b^7*d^2)))^(1/4)*(65536*a^9*b^6*d^12 - 1048576*a^6*b^9*c^6*
d^6 + 983040*a^7*b^8*c^4*d^8))/(8*a^6))*(-(a*d^5*(a^15*b^7)^(1/2) - 256*a^
6*b^6*c^5 + 15*a^8*b^4*c*d^4 + 160*a^7*b^5*c^3*d^2 + 80*b*c^2*d^3*(a^15*b^
7)^(1/2))/(65536*(a^12*b^8*c^2 - a^13*b^7*d^2)))^(3/4) - ((c + d*x)^(1/4)*
(2304*a*b^6*c^7*d^8 - 4096*b^7*c^9*d^6 + 9*a^4*b^3*c*d^14 + 2064*a^2*b^...
```

Reduce [F]

$$\int \frac{(c+dx)^{3/4}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{3}{4}}}{(-b\,x^2+a)^2}dx$$

input

```
int((d*x+c)^(3/4)/(-b*x^2+a)^2,x)
```

output

```
int((d*x+c)^(3/4)/(-b*x^2+a)^2,x)
```

3.229 $\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 364

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx = \frac{x\sqrt[4]{c+dx}}{2a(a-bx^2)} - \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}-3d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8ab^{5/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}} + \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}+3d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8ab^{5/8}\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4}} - \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}-3d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8ab^{5/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}} + \frac{\left(\frac{4\sqrt{b}c}{\sqrt{a}}+3d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8ab^{5/8}\left(\sqrt{b}c+\sqrt{a}d\right)^{3/4}}$$

output
```
1/2*x*(d*x+c)^(1/4)/a/(-b*x^2+a)-1/8*(4*b^(1/2)*c/a^(1/2)-3*d)*arctan(b^(1
/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a/b^(5/8)/(b^(1/2)*c-a^(1/2
)*d)^(3/4)+1/8*(4*b^(1/2)*c/a^(1/2)+3*d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(
1/2)*c+a^(1/2)*d)^(1/4))/a/b^(5/8)/(b^(1/2)*c+a^(1/2)*d)^(3/4)-1/8*(4*b^(1
/2)*c/a^(1/2)-3*d)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/
4))/a/b^(5/8)/(b^(1/2)*c-a^(1/2)*d)^(3/4)+1/8*(4*b^(1/2)*c/a^(1/2)+3*d)*ar
ctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a/b^(5/8)/(b^(1/2
)*c+a^(1/2)*d)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx = \frac{8bx\sqrt[4]{c+dx}+(-ad+bdx^2)\,\text{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{-c\log\left(\sqrt[4]{c+dx}-\#1\right)-3\log\left(\sqrt[4]{c+dx}-\#1\right)\#1^4}{c\#1^3-\#1^7}\&\right]}{16ab\,(a-bx^2)}$$

input
```
Integrate[(c + d*x)^(1/4)/(a - b*x^2)^2,x]
```

output
```
(8*b*x*(c + d*x)^(1/4) + (-(a*d) + b*d*x^2)*RootSum[b*c^2 - a*d^2 - 2*b*c*
#1^4 + b*#1^8 & , (-(c*Log[(c + d*x)^(1/4) - #1]) - 3*Log[(c + d*x)^(1/4)
- #1]*#1^4)/(c*#1^3 - #1^7) & ])/(16*a*b*(a - b*x^2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {494, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx$$

$\Big\downarrow$ 494

$$\frac{x\sqrt[4]{c+dx}}{2a(a-bx^2)}-\frac{\int -\frac{4c+3dx}{4(c+dx)^{3/4}(a-bx^2)}dx}{2a}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{4c+3dx}{(c+dx)^{3/4}(a-bx^2)}dx}{8a}+\frac{x\sqrt[4]{c+dx}}{2a(a-bx^2)}$$

$\Big\downarrow$ 657

$$\frac{\int \left(\frac{4\sqrt{a}c-\frac{3ad}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}+\frac{4\sqrt{a}c+\frac{3ad}{\sqrt{b}}}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx}{8a}+\frac{x\sqrt[4]{c+dx}}{2a(a-bx^2)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\left(4\sqrt{b}c-3\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{5/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}}+\frac{\left(3\sqrt{a}d+4\sqrt{b}c\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{5/8}\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}}-\frac{\left(4\sqrt{b}c-3\sqrt{a}d\right)\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+a}}{\sqrt[4]{\sqrt{b}c-\sqrt{}}}\right.}{\sqrt{a}b^{5/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}}}{8a}$$

$$\frac{x\sqrt[4]{c+dx}}{2a(a-bx^2)}$$

input
```
Int[(c + d*x)^(1/4)/(a - b*x^2)^2,x]
```

output
```
(x*(c + d*x)^(1/4))/(2*a*(a - b*x^2)) + (-(((4*Sqrt[b]*c - 3*Sqrt[a]*d)*Ar
cTan[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^
(5/8)*(Sqrt[b]*c - Sqrt[a]*d)^(3/4))) + ((4*Sqrt[b]*c + 3*Sqrt[a]*d)*ArcTa
n[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/
8)*(Sqrt[b]*c + Sqrt[a]*d)^(3/4)) - ((4*Sqrt[b]*c - 3*Sqrt[a]*d)*ArcTanh[(
b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)*
(Sqrt[b]*c - Sqrt[a]*d)^(3/4)) + ((4*Sqrt[b]*c + 3*Sqrt[a]*d)*ArcTanh[(b^(
1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(5/8)*(Sq
rt[b]*c + Sqrt[a]*d)^(3/4)))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

method	result
default	$\frac{-d\left(-b\,x^{2}+a\right)\left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(3\_R^{4}+c\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)+8bx\left(dx+c\right)^{\frac{1}{4}}}{16\left(-b\,x^{2}+a\right)ab}$
pseudoelliptic	$\frac{-d\left(-b\,x^{2}+a\right)\left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(3\_R^{4}+c\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right)}{\_R^{3}\left(\_R^{4}-c\right)}\right)+8bx\left(dx+c\right)^{\frac{1}{4}}}{16\left(-b\,x^{2}+a\right)ab}$
derivativedivides	$4d^{3}\left(\frac{\frac{\left(dx+c\right)^{\frac{5}{4}}}{8a\,d^{2}}-\frac{c\left(dx+c\right)^{\frac{1}{4}}}{8a\,d^{2}}}{-b\left(dx+c\right)^{2}+2bc\left(dx+c\right)+a\,d^{2}-b\,c^{2}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{8}-2bc\,\_Z^{4}-a\,d^{2}+b\,c^{2}\right)}\frac{\left(3\_R^{4}+c\right)\ln\left(\left(dx+c\right)^{\frac{1}{4}}-\_R\right.}{-\_R^{7}+\_R^{3}c}}{64a\,d^{2}b}\right.$

input
```
int((d*x+c)^(1/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-d*(-b*x^2+a)*sum((3*_R^4+c)*ln((d*x+c)^(1/4)-_R)/_R^3/(_R^4-c),_R=R
ootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+8*b*x*(d*x+c)^(1/4))/(-b*x^2+a)/a/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7535 vs. $2(265) = 530$.

Time = 1.07 (sec) , antiderivative size = 7535, normalized size of antiderivative = 20.70

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx = \int \frac{(dx+c)^{\frac{1}{4}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((d*x+c)^(1/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(1/4)/(b*x^2 - a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 19.45 (sec) , antiderivative size = 14637, normalized size of antiderivative = 40.21

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)^(1/4)/(a - b*x^2)^2,x)
```

output

```
- atan((((((2*(-(81*a^2*d^7*(a^15*b^5)^(1/2) - 256*a^6*b^6*c^7 + 189*a^9*b
^3*c*d^6 + 672*a^7*b^5*c^5*d^2 - 609*a^8*b^4*c^3*d^4 + 112*b^2*c^4*d^3*(a^
15*b^5)^(1/2) - 189*a*b*c^2*d^5*(a^15*b^5)^(1/2))/(65536*(a^12*b^8*c^6 - a
^15*b^5*d^6 - 3*a^13*b^7*c^4*d^2 + 3*a^14*b^6*c^2*d^4)))^(1/4)*(196608*a^8
*b^7*d^11 + 327680*a^6*b^9*c^4*d^7 - 524288*a^7*b^8*c^2*d^9))/a^4 - ((c +
d*x)^(1/4)*(196608*a^8*b^7*c*d^10 + 1048576*a^6*b^9*c^5*d^6 - 983040*a^7*b
^8*c^3*d^8))/(8*a^6))*(-(81*a^2*d^7*(a^15*b^5)^(1/2) - 256*a^6*b^6*c^7 + 1
89*a^9*b^3*c*d^6 + 672*a^7*b^5*c^5*d^2 - 609*a^8*b^4*c^3*d^4 + 112*b^2*c^4
*d^3*(a^15*b^5)^(1/2) - 189*a*b*c^2*d^5*(a^15*b^5)^(1/2))/(65536*(a^12*b^8
*c^6 - a^15*b^5*d^6 - 3*a^13*b^7*c^4*d^2 + 3*a^14*b^6*c^2*d^4)))^(3/4) - (
2*(1280*b^7*c^5*d^7 - 2016*a*b^6*c^3*d^9 + 729*a^2*b^5*c*d^11))/a^4)*(-(81
*a^2*d^7*(a^15*b^5)^(1/2) - 256*a^6*b^6*c^7 + 189*a^9*b^3*c*d^6 + 672*a^7*
b^5*c^5*d^2 - 609*a^8*b^4*c^3*d^4 + 112*b^2*c^4*d^3*(a^15*b^5)^(1/2) - 189
*a*b*c^2*d^5*(a^15*b^5)^(1/2))/(65536*(a^12*b^8*c^6 - a^15*b^5*d^6 - 3*a^1
3*b^7*c^4*d^2 + 3*a^14*b^6*c^2*d^4)))^(1/4) + ((c + d*x)^(1/4)*(729*a^3*b^
4*d^12 + 4096*b^7*c^6*d^6 - 2304*a*b^6*c^4*d^8 - 1296*a^2*b^5*c^2*d^10))/(
8*a^6))*(-(81*a^2*d^7*(a^15*b^5)^(1/2) - 256*a^6*b^6*c^7 + 189*a^9*b^3*c*d
^6 + 672*a^7*b^5*c^5*d^2 - 609*a^8*b^4*c^3*d^4 + 112*b^2*c^4*d^3*(a^15*b^5
)^(1/2) - 189*a*b*c^2*d^5*(a^15*b^5)^(1/2))/(65536*(a^12*b^8*c^6 - a^15*b^
5*d^6 - 3*a^13*b^7*c^4*d^2 + 3*a^14*b^6*c^2*d^4)))^(1/4)*1i - ((((2*(-(...
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+dx}}{(a-bx^2)^2}\,dx=\int \frac{(dx+c)^{\frac{1}{4}}}{(-b\,x^2+a)^2}dx$$

input

```
int((d*x+c)^(1/4)/(-b*x^2+a)^2,x)
```

output

```
int((d*x+c)^(1/4)/(-b*x^2+a)^2,x)
```

3.230 $\int \frac{1}{\sqrt[4]{c+dx}(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 394

$$
\begin{aligned}
\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx = &-\frac{(ad-bcx)(c+dx)^{3/4}}{2a\,(bc^2-ad^2)\,(a-bx^2)} \\
&+\frac{\left(4\sqrt{b}c-5\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{3/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}} \\
&-\frac{\left(4\sqrt{b}c+5\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{3/8}\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}} \\
&-\frac{\left(4\sqrt{b}c-5\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}b^{3/8}\left(\sqrt{b}c-\sqrt{a}d\right)^{5/4}} \\
&+\frac{\left(4\sqrt{b}c+5\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}b^{3/8}\left(\sqrt{b}c+\sqrt{a}d\right)^{5/4}}
\end{aligned}
$$

output
```
-1/2*(-b*c*x+a*d)*(d*x+c)^(3/4)/a/(-a*d^2+b*c^2)/(-b*x^2+a)+1/8*(4*b^(1/2)
*c-5*a^(1/2)*d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/
a^(3/2)/b^(3/8)/(b^(1/2)*c-a^(1/2)*d)^(5/4)-1/8*(4*b^(1/2)*c+5*a^(1/2)*d)*
arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(3/8)/
(b^(1/2)*c+a^(1/2)*d)^(5/4)-1/8*(4*b^(1/2)*c-5*a^(1/2)*d)*arctanh(b^(1/8)*
(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/b^(3/8)/(b^(1/2)*c-a^(1
/2)*d)^(5/4)+1/8*(4*b^(1/2)*c+5*a^(1/2)*d)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(
b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(3/8)/(b^(1/2)*c+a^(1/2)*d)^(5/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx$$

$$= \frac{8b(ad-bcx)(c+dx)^{3/4}+(-ad+bdx^2)\,\mathrm{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1^8\&,\frac{3bc^2\log\left(\sqrt[4]{c+dx}-\#1\right)}{}\right.}{16ab\,(-bc^2+ad^2)\,(a-bx^2)}$$

input
```
Integrate[1/((c + d*x)^(1/4)*(a - b*x^2)^2),x]
```

output
```
(8*b*(a*d - b*c*x)*(c + d*x)^(3/4) + (-(a*d) + b*d*x^2)*RootSum[b*c^2 - a*
d^2 - 2*b*c*#1^4 + b*#1^8 & , (3*b*c^2*Log[(c + d*x)^(1/4) - #1] - 5*a*d^2
*Log[(c + d*x)^(1/4) - #1] + b*c*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1 - #
1^5) & ])/(16*a*b*(-(b*c^2) + a*d^2)*(a - b*x^2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {496, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^2 \sqrt[4]{c+dx}} \, dx$$

$\Big\downarrow$ 496

$$\frac{\int \frac{4bc^2+bdxc-5ad^2}{4\sqrt[4]{c+dx}(a-bx^2)}dx}{2a(bc^2-ad^2)} - \frac{(c+dx)^{3/4}(ad-bcx)}{2a(a-bx^2)(bc^2-ad^2)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{4bc^2+bdxc-5ad^2}{\sqrt[4]{c+dx}(a-bx^2)}dx}{8a(bc^2-ad^2)} - \frac{(c+dx)^{3/4}(ad-bcx)}{2a(a-bx^2)(bc^2-ad^2)}$$

$\Big\downarrow$ 657

$$\frac{\int \left(\frac{\sqrt{a}(4bc^2-5ad^2)-a\sqrt{b}cd}{2a\left(\sqrt{b}x+\sqrt{a}\right)\sqrt[4]{c+dx}} + \frac{a\sqrt{b}cd+\sqrt{a}(4bc^2-5ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}\right)dx}{8a(bc^2-ad^2)} - \frac{(c+dx)^{3/4}(ad-bcx)}{2a(a-bx^2)(bc^2-ad^2)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\left(-\sqrt{a}\sqrt{b}cd-5ad^2+4bc^2\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}} - \frac{\left(\sqrt{a}\sqrt{b}cd-5ad^2+4bc^2\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{3/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}} - \frac{\left(-\sqrt{a}\sqrt{b}cd-5ad^2+4bc^2\right)}{\sqrt{a}b^{3/8}\sqrt[4]{}}}{8a(bc^2-ad^2)}$$

$$\frac{(c+dx)^{3/4}(ad-bcx)}{2a(a-bx^2)(bc^2-ad^2)}$$

input
```
Int[1/((c + d*x)^(1/4)*(a - b*x^2)^2),x]
```

output
```
-1/2*((a*d - b*c*x)*(c + d*x)^(3/4))/(a*(b*c^2 - a*d^2)*(a - b*x^2)) + (((
4*b*c^2 - Sqrt[a]*Sqrt[b]*c*d - 5*a*d^2)*ArcTan[(b^(1/8)*(c + d*x)^(1/4))/
(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c - Sqrt[a]*d)^(
1/4)) - ((4*b*c^2 + Sqrt[a]*Sqrt[b]*c*d - 5*a*d^2)*ArcTan[(b^(1/8)*(c + d*
x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)*(Sqrt[b]*c + Sq
rt[a]*d)^(1/4)) - ((4*b*c^2 - Sqrt[a]*Sqrt[b]*c*d - 5*a*d^2)*ArcTanh[(b^(1
/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(3/8)*(Sqr
t[b]*c - Sqrt[a]*d)^(1/4)) + ((4*b*c^2 + Sqrt[a]*Sqrt[b]*c*d - 5*a*d^2)*Ar
cTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b
^(3/8)*(Sqrt[b]*c + Sqrt[a]*d)^(1/4)))/(8*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.34

method	result
default	$\frac{d(-b\,x^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\_R^4bc-5a\,d^2+3b\,c^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^4-c\right)}\right)+8b(dx+c)^{\frac{3}{4}}(-cb}{16a(a\,d^2-b\,c^2)(-b\,x^2+a)b}$
pseudoelliptic	$\frac{d(-b\,x^2+a)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\_R^4bc-5a\,d^2+3b\,c^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R\left(\_R^4-c\right)}\right)+8b(dx+c)^{\frac{3}{4}}(-cb}{16a(a\,d^2-b\,c^2)(-b\,x^2+a)b}$
derivativedivides	$4d^3\left(\frac{-\frac{bc(dx+c)^{\frac{7}{4}}}{8a\,d^2\left(a\,d^2-b\,c^2\right)}+\frac{\left(a\,d^2+b\,c^2\right)(dx+c)^{\frac{3}{4}}}{8a\,d^2\left(a\,d^2-b\,c^2\right)}}{-b(dx+c)^2+2bc(dx+c)+a\,d^2-b\,c^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(c\_R^6b+\left(-5a\,d^2+3b\,c^2\right)\right.}{\_R}}{64a\,d^2(a\,d^2-b\,c^2)b}\right.$

input
```
int(1/(d*x+c)^(1/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(d*(-b*x^2+a)*sum(1/_R*(_R^4*b*c-5*a*d^2+3*b*c^2)*ln((d*x+c)^(1/4)-_R
)/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+8*b*(d*x+c)^(3/4)*(-b
*c*x+a*d))/a/(a*d^2-b*c^2)/(-b*x^2+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13905 vs. 2(287) = 574.

Time = 28.05 (sec) , antiderivative size = 13905, normalized size of antiderivative = 35.29

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(1/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**(1/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx = \int \frac{1}{(bx^2-a)^2(dx+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^2*(d*x + c)^(1/4)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^(1/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 17149, normalized size of antiderivative = 43.53

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)^2*(c + d*x)^(1/4)),x)
```

output
```
atan(((((21299200*a^9*b^7*c*d^13 - 5242880*a^6*b^10*c^7*d^7 + 25231360*a^7
*b^9*c^5*d^9 - 40239104*a^8*b^8*c^3*d^11)/(128*(a^9*d^4 + a^7*b^2*c^4 - 2*
a^8*b*c^2*d^2)) - ((c + d*x)^(1/4)*((256*a^6*b^6*c^9 - 625*a^3*d^9*(a^15*b
^3)^(1/2) + 1125*a^10*b^2*c*d^8 - 1440*a^7*b^5*c^7*d^2 + 3105*a^8*b^4*c^5*
d^4 - 3030*a^9*b^3*c^3*d^6 + 240*b^3*c^6*d^3*(a^15*b^3)^(1/2) - 981*a*b^2*
c^4*d^5*(a^15*b^3)^(1/2) + 1350*a^2*b*c^2*d^7*(a^15*b^3)^(1/2))/(65536*(a^
12*b^8*c^10 - a^17*b^3*d^10 - 5*a^13*b^7*c^8*d^2 + 10*a^14*b^6*c^6*d^4 - 1
0*a^15*b^5*c^4*d^6 + 5*a^16*b^4*c^2*d^8)))^(1/4)*(1048576*a^6*b^11*c^8*d^6
 - 1638400*a^10*b^7*d^14 - 3604480*a^7*b^10*c^6*d^8 + 2949120*a^8*b^9*c^4*
d^10 + 1245184*a^9*b^8*c^2*d^12))/(8*(a^8*d^4 + a^6*b^2*c^4 - 2*a^7*b*c^2*
d^2)))*((256*a^6*b^6*c^9 - 625*a^3*d^9*(a^15*b^3)^(1/2) + 1125*a^10*b^2*c*
d^8 - 1440*a^7*b^5*c^7*d^2 + 3105*a^8*b^4*c^5*d^4 - 3030*a^9*b^3*c^3*d^6 +
 240*b^3*c^6*d^3*(a^15*b^3)^(1/2) - 981*a*b^2*c^4*d^5*(a^15*b^3)^(1/2) + 1
350*a^2*b*c^2*d^7*(a^15*b^3)^(1/2))/(65536*(a^12*b^8*c^10 - a^17*b^3*d^10
- 5*a^13*b^7*c^8*d^2 + 10*a^14*b^6*c^6*d^4 - 10*a^15*b^5*c^4*d^6 + 5*a^16*
b^4*c^2*d^8)))^(3/4) + ((c + d*x)^(1/4)*(4096*b^9*c^7*d^6 - 16640*a*b^8*c^
5*d^8 - 9375*a^3*b^6*c*d^12 + 22000*a^2*b^7*c^3*d^10))/(8*(a^8*d^4 + a^6*b
^2*c^4 - 2*a^7*b*c^2*d^2)))*((256*a^6*b^6*c^9 - 625*a^3*d^9*(a^15*b^3)^(1/
2) + 1125*a^10*b^2*c*d^8 - 1440*a^7*b^5*c^7*d^2 + 3105*a^8*b^4*c^5*d^4 - 3
030*a^9*b^3*c^3*d^6 + 240*b^3*c^6*d^3*(a^15*b^3)^(1/2) - 981*a*b^2*c^4*...
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{c+dx}\,(a-bx^2)^2}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{4}}\,(-b\,x^2+a)^2}dx$$

input
```
int(1/(d*x+c)^(1/4)/(-b*x^2+a)^2,x)
```

output

```
int(1/(d*x+c)^(1/4)/(-b*x^2+a)^2,x)
```

3.231 $\int \frac{1}{(c+dx)^{3/4}(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 394

$$
\begin{aligned}
\int \frac{1}{(c+dx)^{3/4}(a-bx^2)^2}\,dx &= -\frac{(ad-bcx)\sqrt[4]{c+dx}}{2a(bc^2-ad^2)(a-bx^2)} \\
&- \frac{\left(4\sqrt{b}c-7\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}\sqrt[8]{b}\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}} \\
&+ \frac{\left(4\sqrt{b}c+7\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}\sqrt[8]{b}\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4}} \\
&- \frac{\left(4\sqrt{b}c-7\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}\sqrt[8]{b}\left(\sqrt{b}c-\sqrt{a}d\right)^{7/4}} \\
&+ \frac{\left(4\sqrt{b}c+7\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}\sqrt[8]{b}\left(\sqrt{b}c+\sqrt{a}d\right)^{7/4}}
\end{aligned}
$$

output
```
-1/2*(-b*c*x+a*d)*(d*x+c)^(1/4)/a/(-a*d^2+b*c^2)/(-b*x^2+a)-1/8*(4*b^(1/2)
*c-7*a^(1/2)*d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/
a^(3/2)/b^(1/8)/(b^(1/2)*c-a^(1/2)*d)^(7/4)+1/8*(4*b^(1/2)*c+7*a^(1/2)*d)*
arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(1/8)/
(b^(1/2)*c+a^(1/2)*d)^(7/4)-1/8*(4*b^(1/2)*c-7*a^(1/2)*d)*arctanh(b^(1/8)*
(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/b^(1/8)/(b^(1/2)*c-a^(1
/2)*d)^(7/4)+1/8*(4*b^(1/2)*c+7*a^(1/2)*d)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(
b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/b^(1/8)/(b^(1/2)*c+a^(1/2)*d)^(7/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.45

$$\int \frac{1}{(c+dx)^{3/4}(a-bx^2)^2}\,dx = \frac{8b(ad-bcx)\sqrt[4]{c+dx}+(-ad+bdx^2)\,\mathrm{RootSum}\left[bc^2-ad^2-2bc\#1^4+b\#1\right.}{16ab\left(-bc^2+\right.}$$

input
```
Integrate[1/((c + d*x)^(3/4)*(a - b*x^2)^2),x]
```

output
```
(8*b*(a*d - b*c*x)*(c + d*x)^(1/4) + (-(a*d) + b*d*x^2)*RootSum[b*c^2 - a*
d^2 - 2*b*c*#1^4 + b*#1^8 & , (b*c^2*Log[(c + d*x)^(1/4) - #1] - 7*a*d^2*L
og[(c + d*x)^(1/4) - #1] + 3*b*c*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1^3 -
 #1^7) & ])/(16*a*b*(-(b*c^2) + a*d^2)*(a - b*x^2))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {496, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^2 (c+dx)^{3/4}}\,dx$$

$$\Big\downarrow \; 496$$

$$\frac{\int \frac{4bc^2+3bdxc-7ad^2}{4(c+dx)^{3/4}(a-bx^2)}dx}{2a\,(bc^2-ad^2)}-\frac{\sqrt[4]{c+dx}(ad-bcx)}{2a\,(a-bx^2)\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4bc^2+3bdxc-7ad^2}{(c+dx)^{3/4}(a-bx^2)}dx}{8a\,(bc^2-ad^2)}-\frac{\sqrt[4]{c+dx}(ad-bcx)}{2a\,(a-bx^2)\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{\sqrt{a}(4bc^2-7ad^2)-3a\sqrt{b}cd}{2a\left(\sqrt{b}x+\sqrt{a}\right)(c+dx)^{3/4}}+\frac{3a\sqrt{b}cd+\sqrt{a}(4bc^2-7ad^2)}{2a\left(\sqrt{a}-\sqrt{b}x\right)(c+dx)^{3/4}}\right)dx}{8a\,(bc^2-ad^2)}-\frac{\sqrt[4]{c+dx}(ad-bcx)}{2a\,(a-bx^2)\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\left(-3\sqrt{a}\sqrt{b}cd-7ad^2+4bc^2\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{b}c-\sqrt{a}d\right)^{3/4}}+\frac{\left(3\sqrt{a}\sqrt{b}cd-7ad^2+4bc^2\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}\sqrt[8]{b}\left(\sqrt{a}d+\sqrt{b}c\right)^{3/4}}-\frac{\left(-3\sqrt{a}\sqrt{b}cd-7ad^2+\cdots\right.}{\sqrt{a}\cdots}}{8a\,(bc^2-ad^2)}$$

$$\frac{\sqrt[4]{c+dx}(ad-bcx)}{2a\,(a-bx^2)\,(bc^2-ad^2)}$$

input
```
Int[1/((c + d*x)^(3/4)*(a - b*x^2)^2),x]
```

output
```
-1/2*((a*d - b*c*x)*(c + d*x)^(1/4))/(a*(b*c^2 - a*d^2)*(a - b*x^2)) + (-(
((4*b*c^2 - 3*Sqrt[a]*Sqrt[b]*c*d - 7*a*d^2)*ArcTan[(b^(1/8)*(c + d*x)^(1/
4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)*(Sqrt[b]*c - Sqrt[a]*
d)^(3/4))) + ((4*b*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d - 7*a*d^2)*ArcTan[(b^(1/8)*
(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(1/8)*(Sqrt[b]
*c + Sqrt[a]*d)^(3/4)) - ((4*b*c^2 - 3*Sqrt[a]*Sqrt[b]*c*d - 7*a*d^2)*ArcT
anh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(
1/8)*(Sqrt[b]*c - Sqrt[a]*d)^(3/4)) + ((4*b*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d -
7*a*d^2)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])
/(Sqrt[a]*b^(1/8)*(Sqrt[b]*c + Sqrt[a]*d)^(3/4)))/(8*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.34

method	result
default	$\frac{d(-b x^2+a)\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^8-2bc\_Z^4-a d^2+b c^2\right)}\frac{\left(3\_R^4 bc-7a d^2+b c^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+8b(dx+c)^{\frac{1}{4}}(-cb}{16a(a d^2-b c^2)(-b x^2+a)b}$
pseudoelliptic	$\frac{d(-b x^2+a)\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^8-2bc\_Z^4-a d^2+b c^2\right)}\frac{\left(3\_R^4 bc-7a d^2+b c^2\right)\ln\left((dx+c)^{\frac{1}{4}}-\_R\right)}{\_R^3\left(\_R^4-c\right)}\right)+8b(dx+c)^{\frac{1}{4}}(-cb}{16a(a d^2-b c^2)(-b x^2+a)b}$
derivativedivides	$4d^3\left(\frac{-\frac{bc(dx+c)^{\frac{5}{4}}}{8a d^2\left(a d^2-b c^2\right)}+\frac{\left(a d^2+b c^2\right)(dx+c)^{\frac{1}{4}}}{8a d^2\left(a d^2-b c^2\right)}}{-b(dx+c)^2+2bc(dx+c)+a d^2-b c^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^8-2bc\_Z^4-a d^2+b c^2\right)}\frac{\left(-3\_R^4 bc+7a d^2-b c^2\right)}{-\_R^7+}}{64a d^2(a d^2-b c^2)b}\right.$

input
```
int(1/(d*x+c)^(3/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(d*(-b*x^2+a)*sum((3*_R^4*b*c-7*a*d^2+b*c^2)*ln((d*x+c)^(1/4)-_R)/_R^
3/(_R^4-c),_R=RootOf(_Z^8*b-2*_Z^4*b*c-a*d^2+b*c^2))+8*b*(d*x+c)^(1/4)*(-b
*c*x+a*d))/a/(a*d^2-b*c^2)/(-b*x^2+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14823 vs. $2(287) = 574$.

Time = 13.54 (sec) , antiderivative size = 14823, normalized size of antiderivative = 37.62

$$\int \frac{1}{(c+dx)^{3/4} (a-bx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(3/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{3/4} (a-bx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**(3/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{3/4}\left(a-bx^2\right)^2}\,dx = \int \frac{1}{\left(bx^2-a\right)^2(dx+c)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^2*(d*x + c)^(3/4)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{3/4}\left(a-bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^(3/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 28255, normalized size of antiderivative = 71.71

$$\int \frac{1}{(c+dx)^{3/4}\left(a-bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)^2*(c + d*x)^(3/4)),x)
```

output
```
(atan(((((2401*a^4*d^11*(a^15*b)^(1/2) - 256*a^6*b^6*c^11 + 2464*a^7*b^5*c
^9*d^2 - 9009*a^8*b^4*c^7*d^4 + 12859*a^9*b^3*c^5*d^6 + 77*a^10*b^2*c^3*d^
8 - 1232*b^4*c^8*d^3*(a^15*b)^(1/2) - 11319*a^11*b*c*d^10 - 21549*a^2*b^2*
c^4*d^7*(a^15*b)^(1/2) + 8855*a*b^3*c^6*d^5*(a^15*b)^(1/2) + 16709*a^3*b*c
^2*d^9*(a^15*b)^(1/2))/(a^19*b*d^14 - a^12*b^8*c^14 + 7*a^13*b^7*c^12*d^2
- 21*a^14*b^6*c^10*d^4 + 35*a^15*b^5*c^8*d^6 - 35*a^16*b^4*c^6*d^8 + 21*a^
17*b^3*c^4*d^10 - 7*a^18*b^2*c^2*d^12))^(1/4)*((((2401*a^4*d^11*(a^15*b)^(
1/2) - 256*a^6*b^6*c^11 + 2464*a^7*b^5*c^9*d^2 - 9009*a^8*b^4*c^7*d^4 + 12
859*a^9*b^3*c^5*d^6 + 77*a^10*b^2*c^3*d^8 - 1232*b^4*c^8*d^3*(a^15*b)^(1/2
) - 11319*a^11*b*c*d^10 - 21549*a^2*b^2*c^4*d^7*(a^15*b)^(1/2) + 8855*a*b^
3*c^6*d^5*(a^15*b)^(1/2) + 16709*a^3*b*c^2*d^9*(a^15*b)^(1/2))/(a^19*b*d^1
4 - a^12*b^8*c^14 + 7*a^13*b^7*c^12*d^2 - 21*a^14*b^6*c^10*d^4 + 35*a^15*b
^5*c^8*d^6 - 35*a^16*b^4*c^6*d^8 + 21*a^17*b^3*c^4*d^10 - 7*a^18*b^2*c^2*d
^12))^(1/4)*(((((c + d*x)^(1/4)*(12386304*a^11*b^8*c*d^16 + 1048576*a^6*b^
13*c^11*d^6 - 6750208*a^7*b^12*c^9*d^8 + 11010048*a^8*b^11*c^7*d^10 + 6422
528*a^9*b^10*c^5*d^12 - 24117248*a^10*b^9*c^3*d^14))/(8*(a^10*d^8 + a^6*b^
4*c^8 - 4*a^9*b*c^2*d^6 - 4*a^7*b^3*c^6*d^2 + 6*a^8*b^2*c^4*d^4)) - (((240
1*a^4*d^11*(a^15*b)^(1/2) - 256*a^6*b^6*c^11 + 2464*a^7*b^5*c^9*d^2 - 9009
*a^8*b^4*c^7*d^4 + 12859*a^9*b^3*c^5*d^6 + 77*a^10*b^2*c^3*d^8 - 1232*b^4*
c^8*d^3*(a^15*b)^(1/2) - 11319*a^11*b*c*d^10 - 21549*a^2*b^2*c^4*d^7*(a...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{3/4}\left(a-bx^2\right)^2}\,dx=\int \frac{1}{(dx+c)^{\frac{3}{4}}\left(-b\,x^2+a\right)^2}dx$$

input
```
int(1/(d*x+c)^(3/4)/(-b*x^2+a)^2,x)
```

output
```
int(1/(d*x+c)^(3/4)/(-b*x^2+a)^2,x)
```

3.232 $\int \frac{1}{(c+dx)^{5/4}\left(a-bx^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 437

$$\int \frac{1}{(c+dx)^{5/4}\left(a-bx^2\right)^2}\,dx = -\frac{d(bc^2+9ad^2)}{2a\left(bc^2-ad^2\right)^2\sqrt[4]{c+dx}} - \frac{ad-bcx}{2a\left(bc^2-ad^2\right)\sqrt[4]{c+dx}\left(a-bx^2\right)} + \frac{\sqrt[8]{b}\left(4\sqrt{b}c-9\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}} - \frac{\sqrt[8]{b}\left(4\sqrt{b}c+9\sqrt{a}d\right)\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}} - \frac{\sqrt[8]{b}\left(4\sqrt{b}c-9\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{8a^{3/2}\left(\sqrt{b}c-\sqrt{a}d\right)^{9/4}} + \frac{\sqrt[8]{b}\left(4\sqrt{b}c+9\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c+\sqrt{a}d}}\right)}{8a^{3/2}\left(\sqrt{b}c+\sqrt{a}d\right)^{9/4}}$$

output

```
-1/2*d*(9*a*d^2+b*c^2)/a/(-a*d^2+b*c^2)^2/(d*x+c)^(1/4)-1/2*(-b*c*x+a*d)/a
/(-a*d^2+b*c^2)/(d*x+c)^(1/4)/(-b*x^2+a)+1/8*b^(1/8)*(4*b^(1/2)*c-9*a^(1/2
)*d)*arctan(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/(b^
(1/2)*c-a^(1/2)*d)^(9/4)-1/8*b^(1/8)*(4*b^(1/2)*c+9*a^(1/2)*d)*arctan(b^(1
/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a^(1/2)*d)^(1/4))/a^(3/2)/(b^(1/2)*c+a^(1/2)*
d)^(9/4)-1/8*b^(1/8)*(4*b^(1/2)*c-9*a^(1/2)*d)*arctanh(b^(1/8)*(d*x+c)^(1/
4)/(b^(1/2)*c-a^(1/2)*d)^(1/4))/a^(3/2)/(b^(1/2)*c-a^(1/2)*d)^(9/4)+1/8*b^
(1/8)*(4*b^(1/2)*c+9*a^(1/2)*d)*arctanh(b^(1/8)*(d*x+c)^(1/4)/(b^(1/2)*c+a
^(1/2)*d)^(1/4))/a^(3/2)/(b^(1/2)*c+a^(1/2)*d)^(9/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.80

$$\int \frac{1}{(c+dx)^{5/4}\left(a-bx^2\right)^2}\,dx = \frac{64a^2d^3-8b^2c^2x(c+dx)+8abd(2c^2+cdx-9d^2x^2)+8ad^3\sqrt[4]{c+dx}(-a+}{}$$

input
```
Integrate[1/((c + d*x)^(5/4)*(a - b*x^2)^2),x]
```

output
```
(64*a^2*d^3 - 8*b^2*c^2*x*(c + d*x) + 8*a*b*d*(2*c^2 + c*d*x - 9*d^2*x^2)
+ 8*a*d^3*(c + d*x)^(1/4)*(-a + b*x^2)*RootSum[b*c^2 - a*d^2 - 2*b*c*#1^4
+ b*#1^8 & , (-2*c*Log[(c + d*x)^(1/4) - #1] + Log[(c + d*x)^(1/4) - #1]*#
1^4)/(c*#1 - #1^5) & ] + d*(c + d*x)^(1/4)*(-a + b*x^2)*RootSum[b*c^2 - a*
d^2 - 2*b*c*#1^4 + b*#1^8 & , (3*b*c^3*Log[(c + d*x)^(1/4) - #1] - 7*a*c*d
^2*Log[(c + d*x)^(1/4) - #1] + b*c^2*Log[(c + d*x)^(1/4) - #1]*#1^4 + a*d^
2*Log[(c + d*x)^(1/4) - #1]*#1^4)/(c*#1 - #1^5) & ])/(16*a*(b*c^2 - a*d^2)
^2*(c + d*x)^(1/4)*(-a + b*x^2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {496, 27, 655, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-bx^2\right)^2(c+dx)^{5/4}}dx$$

$$\Big\downarrow \; 496$$

$$\frac{\int \frac{4bc^2+5bdxc-9ad^2}{4(c+dx)^{5/4}(a-bx^2)}dx}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{2a\left(a-bx^2\right)\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4bc^2+5bdxc-9ad^2}{(c+dx)^{5/4}(a-bx^2)}dx}{8a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{2a\left(a-bx^2\right)\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 655$$

$$\frac{-\frac{\int -\frac{b\left(2c\left(2bc^2-7ad^2\right)+d\left(bc^2+9ad^2\right)x\right)}{\sqrt[4]{c+dx}\left(a-bx^2\right)}dx}{bc^2-ad^2}-\frac{4d\left(9ad^2+bc^2\right)}{\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}}{8a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{2a\left(a-bx^2\right)\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b\left(2c\left(2bc^2-7ad^2\right)+d\left(bc^2+9ad^2\right)x\right)}{\sqrt[4]{c+dx}\left(a-bx^2\right)}dx}{bc^2-ad^2}-\frac{4d\left(9ad^2+bc^2\right)}{\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}}{8a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{2a\left(a-bx^2\right)\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{2c\left(2bc^2-7ad^2\right)+d\left(bc^2+9ad^2\right)x}{\sqrt[4]{c+dx}\left(a-bx^2\right)}dx}{bc^2-ad^2}-\frac{4d\left(9ad^2+bc^2\right)}{\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}}{8a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{2a\left(a-bx^2\right)\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\frac{b\int \left(\frac{2\sqrt{a}c\left(2bc^2-7ad^2\right)-\frac{ad\left(bc^2+9ad^2\right)}{\sqrt{b}}}{2a\left(\sqrt{b}x+\sqrt{a}\right)\sqrt[4]{c+dx}}+\frac{2\sqrt{a}c\left(2bc^2-7ad^2\right)+\frac{ad\left(bc^2+9ad^2\right)}{\sqrt{b}}}{2a\left(\sqrt{a}-\sqrt{b}x\right)\sqrt[4]{c+dx}}\right)dx}{bc^2-ad^2}-\frac{4d\left(9ad^2+bc^2\right)}{\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}}{8a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{2a\left(a-bx^2\right)\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{b\left(-\frac{\left(9\sqrt{a}d+4\sqrt{b}c\right)\left(\sqrt{b}c-\sqrt{a}d\right)^2\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{a}d+\sqrt{b}c}}+\frac{\left(4\sqrt{b}c-9\sqrt{a}d\right)\left(\sqrt{a}d+\sqrt{b}c\right)^2\arctan\left(\frac{\sqrt[8]{b}\sqrt[4]{c+dx}}{\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}\right)}{\sqrt{a}b^{7/8}\sqrt[4]{\sqrt{b}c-\sqrt{a}d}}+\frac{\left(9\sqrt{a}d+4\sqrt{b}c\right)\left(\sqrt{b}c-\sqrt{a}d\right)^2 a}{\sqrt{a}b^{7/8}\sqrt[4]{}}\right.}{bc^2-ad^2}}{8a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{2a\left(a-bx^2\right)\sqrt[4]{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(5/4)*(a - b*x^2)^2),x]
```

output
```
-1/2*(a*d - b*c*x)/(a*(b*c^2 - a*d^2)*(c + d*x)^(1/4)*(a - b*x^2)) + ((-4*
d*(b*c^2 + 9*a*d^2))/((b*c^2 - a*d^2)*(c + d*x)^(1/4)) + (b*(((4*Sqrt[b]*c
 - 9*Sqrt[a]*d)*(Sqrt[b]*c + Sqrt[a]*d)^2*ArcTan[(b^(1/8)*(c + d*x)^(1/4))
/(Sqrt[b]*c - Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sqrt[b]*c - Sqrt[a]*d)^
(1/4)) - ((Sqrt[b]*c - Sqrt[a]*d)^2*(4*Sqrt[b]*c + 9*Sqrt[a]*d)*ArcTan[(b^
(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(S
qrt[b]*c + Sqrt[a]*d)^(1/4)) - ((4*Sqrt[b]*c - 9*Sqrt[a]*d)*(Sqrt[b]*c + S
qrt[a]*d)^2*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt[b]*c - Sqrt[a]*d)^(1/4
)])/(Sqrt[a]*b^(7/8)*(Sqrt[b]*c - Sqrt[a]*d)^(1/4)) + ((Sqrt[b]*c - Sqrt[a
]*d)^2*(4*Sqrt[b]*c + 9*Sqrt[a]*d)*ArcTanh[(b^(1/8)*(c + d*x)^(1/4))/(Sqrt
[b]*c + Sqrt[a]*d)^(1/4)])/(Sqrt[a]*b^(7/8)*(Sqrt[b]*c + Sqrt[a]*d)^(1/4))
))/(b*c^2 - a*d^2))/(8*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.42

method	result
default	$-\frac{4\left(\frac{(dx+c)^{\frac{1}{4}} d\left(-b\,x^2+a\right)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(9\_R^4 a\,d^2+\_R^4 b\,c^2-23a\,d^2c+3b\,c^3\right)\ln\left((dx+c)^{\frac{1}{4}}\right.}{\_R\left(\_R^4-c\right)}\right.}{64}\right.}{(dx+c)^{\frac{1}{4}}\left(a\,d^2-b\,c^2\right)^2 a\left(-b\,x^2+a\right)}$
pseudoelliptic	$-\frac{4\left(\frac{(dx+c)^{\frac{1}{4}} d\left(-b\,x^2+a\right)\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(9\_R^4 a\,d^2+\_R^4 b\,c^2-23a\,d^2c+3b\,c^3\right)\ln\left((dx+c)^{\frac{1}{4}}\right.}{\_R\left(\_R^4-c\right)}\right.}{64}\right.}{(dx+c)^{\frac{1}{4}}\left(a\,d^2-b\,c^2\right)^2 a\left(-b\,x^2+a\right)}$
derivativedivides	$4d^3\left(-\frac{b\left(\frac{-\frac{\left(a\,d^2+b\,c^2\right)(dx+c)^{\frac{7}{4}}}{8a\,d^2}+\frac{c\left(3a\,d^2+b\,c^2\right)(dx+c)^{\frac{3}{4}}}{8a\,d^2}}{-b(dx+c)^2+2bc(dx+c)+a\,d^2-b\,c^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-2bc\_Z^4-a\,d^2+b\,c^2\right)}\frac{\left(\left(9a\,d^2+b\,c^2\right)\_R\right.}{}}{64a\,d^2 b}\right.}{\left(a\,d^2-b\,c^2\right)^2}\right.$

input
```
int(1/(d*x+c)^(5/4)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/(d*x+c)^(1/4)*(1/64*(d*x+c)^(1/4)*d*(-b*x^2+a)*sum(1/_R*(9*_R^4*a*d^2+_
R^4*b*c^2-23*a*c*d^2+3*b*c^3)*ln((d*x+c)^(1/4)-_R)/(_R^4-c),_R=RootOf(_Z^8
*b-2*_Z^4*b*c-a*d^2+b*c^2))+a^2*d^3+1/4*(-9/2*d^2*x^2+1/2*c*d*x+c^2)*d*b*a
-1/8*b^2*c^2*x*(d*x+c))/(a*d^2-b*c^2)^2/a/(-b*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23435 vs. $2(326) = 652$.

Time = 133.99 (sec) , antiderivative size = 23435, normalized size of antiderivative = 53.63

$$\int \frac{1}{(c+dx)^{5/4}\left(a-bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(5/4)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{5/4}\left(a-bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**(5/4)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{5/4}\left(a-bx^2\right)^2}\,dx = \int \frac{1}{(bx^2-a)^2(dx+c)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(d*x+c)^(5/4)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^2*(d*x + c)^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{5/4} (a-bx^2)^2} \, dx = \int \frac{1}{(bx^2-a)^2 (dx+c)^{\frac{5}{4}}} \, dx$$

input
```
integrate(1/(d*x+c)^(5/4)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*(d*x + c)^(5/4)), x)
```

Mupad [B] (verification not implemented)

Time = 16.43 (sec) , antiderivative size = 37402, normalized size of antiderivative = 85.59

$$\int \frac{1}{(c+dx)^{5/4} (a-bx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a - b*x^2)^2*(c + d*x)^(5/4)),x)
```

output
```
(atan((((((6561*a^5*d^13*(a^15*b)^(1/2) - 256*a^6*b^7*c^13 + 3744*a^7*b^6
*c^11*d^2 - 20241*a^8*b^5*c^9*d^4 + 32604*a^9*b^4*c^7*d^6 + 70434*a^10*b^3
*c^5*d^8 - 198900*a^11*b^2*c^3*d^10 - 3120*b^5*c^10*d^3*(a^15*b)^(1/2) - 4
7385*a^12*b*c*d^12 - 123084*a^2*b^3*c^6*d^7*(a^15*b)^(1/2) + 106366*a^3*b^
2*c^4*d^9*(a^15*b)^(1/2) + 34281*a*b^4*c^8*d^5*(a^15*b)^(1/2) + 138996*a^4
*b*c^2*d^11*(a^15*b)^(1/2))/(a^21*d^18 - a^12*b^9*c^18 - 9*a^20*b*c^2*d^16
 + 9*a^13*b^8*c^16*d^2 - 36*a^14*b^7*c^14*d^4 + 84*a^15*b^6*c^12*d^6 - 126
*a^16*b^5*c^10*d^8 + 126*a^17*b^4*c^8*d^10 - 84*a^18*b^3*c^6*d^12 + 36*a^1
9*b^2*c^4*d^14))^(3/4)*((((6561*a^5*d^13*(a^15*b)^(1/2) - 256*a^6*b^7*c^13
 + 3744*a^7*b^6*c^11*d^2 - 20241*a^8*b^5*c^9*d^4 + 32604*a^9*b^4*c^7*d^6 +
 70434*a^10*b^3*c^5*d^8 - 198900*a^11*b^2*c^3*d^10 - 3120*b^5*c^10*d^3*(a^
15*b)^(1/2) - 47385*a^12*b*c*d^12 - 123084*a^2*b^3*c^6*d^7*(a^15*b)^(1/2)
+ 106366*a^3*b^2*c^4*d^9*(a^15*b)^(1/2) + 34281*a*b^4*c^8*d^5*(a^15*b)^(1/
2) + 138996*a^4*b*c^2*d^11*(a^15*b)^(1/2))/(a^21*d^18 - a^12*b^9*c^18 - 9*
a^20*b*c^2*d^16 + 9*a^13*b^8*c^16*d^2 - 36*a^14*b^7*c^14*d^4 + 84*a^15*b^6
*c^12*d^6 - 126*a^16*b^5*c^10*d^8 + 126*a^17*b^4*c^8*d^10 - 84*a^18*b^3*c^
6*d^12 + 36*a^19*b^2*c^4*d^14))^(1/4)*(c + d*x)^(1/4)*(5435817984*a^40*b^8
*d^60 - 1073741824*a^13*b^35*c^54*d^6 + 32145145856*a^14*b^34*c^52*d^8 - 4
43992244224*a^15*b^33*c^50*d^10 + 3755747573760*a^16*b^32*c^48*d^12 - 2172
7165808640*a^17*b^31*c^46*d^14 + 90433891860480*a^18*b^30*c^44*d^16 - 2...
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{5/4}\left(a-bx^2\right)^2}\,dx=\int \frac{1}{(dx+c)^{\frac{5}{4}}\left(-b\,x^2+a\right)^2}dx$$

input
```
int(1/(d*x+c)^(5/4)/(-b*x^2+a)^2,x)
```

output
```
int(1/(d*x+c)^(5/4)/(-b*x^2+a)^2,x)
```

3.233 $\int (c+dx)^4\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 207

$$\int (c+dx)^4\sqrt{a+bx^2}\,dx = \frac{(8b^2c^4-12abc^2d^2+a^2d^4)\,x\sqrt{a+bx^2}}{16b^2} + \frac{3cd(c+dx)^2\,(a+bx^2)^{3/2}}{10b} + \frac{d(c+dx)^3\,(a+bx^2)^{3/2}}{6b} + \frac{d(8c(13bc^2-8ad^2)+3d(16bc^2-5ad^2)\,x)\,(a+bx^2)^{3/2}}{120b^2} + \frac{a(8b^2c^4-12abc^2d^2+a^2d^4)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/16*(a^2*d^4-12*a*b*c^2*d^2+8*b^2*c^4)*x*(b*x^2+a)^(1/2)/b^2+3/10*c*d*(d*
x+c)^2*(b*x^2+a)^(3/2)/b+1/6*d*(d*x+c)^3*(b*x^2+a)^(3/2)/b+1/120*d*(8*c*(-
8*a*d^2+13*b*c^2)+3*d*(-5*a*d^2+16*b*c^2)*x)*(b*x^2+a)^(3/2)/b^2+1/16*a*(a
^2*d^4-12*a*b*c^2*d^2+8*b^2*c^4)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2
)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.85

$$\int (c+dx)^4\sqrt{a+bx^2}\,dx$$
$$=\frac{\sqrt{b}\sqrt{a+bx^2}(-a^2d^3(128c+15dx)+2abd(160c^3+90c^2dx+32cd^2x^2+5d^3x^3)+8b^2x(15c^4+40c^3dx+4}{240b^{5/2}}$$

input
```
Integrate[(c + d*x)^4*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-(a^2*d^3*(128*c + 15*d*x)) + 2*a*b*d*(160*c^3 +
 90*c^2*d*x + 32*c*d^2*x^2 + 5*d^3*x^3) + 8*b^2*x*(15*c^4 + 40*c^3*d*x + 4
5*c^2*d^2*x^2 + 24*c*d^3*x^3 + 5*d^4*x^4)) - 15*a*(8*b^2*c^4 - 12*a*b*c^2*
d^2 + a^2*d^4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(240*b^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {497, 27, 687, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^4\,dx$$

$$\Big\downarrow \text{497}$$

$$\frac{\int 3(c+dx)^2\left(2bc^2+3bdxc-ad^2\right)\sqrt{bx^2+a}dx}{6b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c+dx)^2\left(2bc^2+3bdxc-ad^2\right)\sqrt{bx^2+a}dx}{2b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{\int b(c+dx)\left(c\left(10bc^2-11ad^2\right)+d\left(16bc^2-5ad^2\right)x\right)\sqrt{bx^2+a}dx}{5b}+\frac{3}{5}cd\left(a+bx^2\right)^{3/2}(c+dx)^2}{2b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{5}\int (c+dx)\left(c\left(10bc^2-11ad^2\right)+d\left(16bc^2-5ad^2\right)x\right)\sqrt{bx^2+a}dx+\frac{3}{5}cd\left(a+bx^2\right)^{3/2}(c+dx)^2}{2b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\big\downarrow$ 676

$$\frac{\frac{1}{5}\left(\frac{5\left(a^2d^4-12abc^2d^2+8b^2c^4\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{d^2x\left(a+bx^2\right)^{3/2}\left(16bc^2-5ad^2\right)}{4b}+\frac{2cd\left(a+bx^2\right)^{3/2}\left(13bc^2-8ad^2\right)}{3b}\right)+\frac{3}{5}cd\left(a+bx^2\right)^{3/2}(c-}{2b}$$
$$\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\big\downarrow$ 211

$$\frac{\frac{1}{5}\left(\frac{5\left(a^2d^4-12abc^2d^2+8b^2c^4\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{d^2x\left(a+bx^2\right)^{3/2}\left(16bc^2-5ad^2\right)}{4b}+\frac{2cd\left(a+bx^2\right)^{3/2}\left(13bc^2-8ad^2\right)}{3b}\right)+\frac{3}{5}cd}{2b}$$
$$\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{5}\left(\frac{5\left(a^2d^4-12abc^2d^2+8b^2c^4\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{d^2x\left(a+bx^2\right)^{3/2}\left(16bc^2-5ad^2\right)}{4b}+\frac{2cd\left(a+bx^2\right)^{3/2}\left(13bc^2-8ad^2\right)}{3b}\right)}{2b}$$
$$\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{5}\left(\frac{5\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(a^2d^4-12abc^2d^2+8b^2c^4)}{4b}+\frac{d^2x(a+bx^2)^{3/2}(16bc^2-5ad^2)}{4b}+\frac{2cd(a+bx^2)^{3/2}(13bc^2-8ad^2)}{3b}\right)}{2b}$$

$$\frac{d(a+bx^2)^{3/2}(c+dx)^3}{6b}$$

input
```
Int[(c + d*x)^4*Sqrt[a + b*x^2],x]
```

output
```
(d*(c + d*x)^3*(a + b*x^2)^(3/2))/(6*b) + ((3*c*d*(c + d*x)^2*(a + b*x^2)^
(3/2))/5 + ((2*c*d*(13*b*c^2 - 8*a*d^2)*(a + b*x^2)^(3/2))/(3*b) + (d^2*(1
6*b*c^2 - 5*a*d^2)*x*(a + b*x^2)^(3/2))/(4*b) + (5*(8*b^2*c^4 - 12*a*b*c^2
*d^2 + a^2*d^4)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*Sqrt[b])))/(4*b))/5)/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{(-40d^4b^2x^5-192b^2c\,d^3x^4-10a\,d^4b\,x^3-360b^2c^2d^2x^3-64abc\,d^3x^2-320b^2c^3d\,x^2+15a^2d^4x-180a\,c^2d^2xb-120b^2c^4x+128d^3c\,a^2-}{240b^2}$
default	$c^4\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+d^4\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right.$

input
```
int((d*x+c)^4*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-40*b^2*d^4*x^5-192*b^2*c*d^3*x^4-10*a*b*d^4*x^3-360*b^2*c^2*d^2*x
^3-64*a*b*c*d^3*x^2-320*b^2*c^3*d*x^2+15*a^2*d^4*x-180*a*b*c^2*d^2*x-120*b
^2*c^4*x+128*a^2*c*d^3-320*a*b*c^3*d)*(b*x^2+a)^(1/2)/b^2+1/16*a*(a^2*d^4-
12*a*b*c^2*d^2+8*b^2*c^4)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.93

$$\int (c+dx)^4\sqrt{a+bx^2}\,dx$$

$$= \left[\frac{15\,(8\,ab^2c^4-12\,a^2bc^2d^2+a^3d^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(40\,b^3d^4x^5+192\,b^3cd^3x^4+}{}\right.$$

$$-\frac{15\,(8\,ab^2c^4-12\,a^2bc^2d^2+a^3d^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(40\,b^3d^4x^5+192\,b^3cd^3x^4+320\,ab^2c^3d-128}{2}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(15*(8*a*b^2*c^4 - 12*a^2*b*c^2*d^2 + a^3*d^4)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*b^3*d^4*x^5 + 192*b^3*c*d^3*x^
4 + 320*a*b^2*c^3*d - 128*a^2*b*c*d^3 + 10*(36*b^3*c^2*d^2 + a*b^2*d^4)*x^
3 + 64*(5*b^3*c^3*d + a*b^2*c*d^3)*x^2 + 15*(8*b^3*c^4 + 12*a*b^2*c^2*d^2
- a^2*b*d^4)*x)*sqrt(b*x^2 + a))/b^3, -1/240*(15*(8*a*b^2*c^4 - 12*a^2*b*c
^2*d^2 + a^3*d^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (40*b^3*d^
4*x^5 + 192*b^3*c*d^3*x^4 + 320*a*b^2*c^3*d - 128*a^2*b*c*d^3 + 10*(36*b^3
*c^2*d^2 + a*b^2*d^4)*x^3 + 64*(5*b^3*c^3*d + a*b^2*c*d^3)*x^2 + 15*(8*b^3
*c^4 + 12*a*b^2*c^2*d^2 - a^2*b*d^4)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.30

$$\int (c+dx)^4\sqrt{a+bx^2}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{4cd^3x^4}{5}+\frac{d^4x^5}{6}+\frac{x^3\left(\frac{ad^4}{6}+6bc^2d^2\right)}{4b}+\frac{x^2\cdot\left(\frac{4acd^3}{5}+4bc^3d\right)}{3b}+\frac{x\left(6ac^2d^2-\frac{3a\left(\frac{ad^4}{6}+6bc^2d^2\right)}{4b}+bc^4\right)}{2b}+\frac{4ac^3d-\frac{2a\left(\right.}{}}{}\right. \\ \sqrt{a}\left(\begin{cases} c^4x & \text{for } d=0 \\ \frac{(c+dx)^5}{5d} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((d*x+c)**4*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(4*c*d**3*x**4/5 + d**4*x**5/6 + x**3*(a*d**4/
6 + 6*b*c**2*d**2)/(4*b) + x**2*(4*a*c*d**3/5 + 4*b*c**3*d)/(3*b) + x*(6*a
*c**2*d**2 - 3*a*(a*d**4/6 + 6*b*c**2*d**2)/(4*b) + b*c**4)/(2*b) + (4*a*c
**3*d - 2*a*(4*a*c*d**3/5 + 4*b*c**3*d)/(3*b))/b) + (a*c**4 - a*(6*a*c**2*
d**2 - 3*a*(a*d**4/6 + 6*b*c**2*d**2)/(4*b) + b*c**4)/(2*b))*Piecewise((lo
g(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b
*x**2), True)), Ne(b, 0)), (sqrt(a)*Piecewise((c**4*x, Eq(d, 0)), ((c + d*
x)**5/(5*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.15

$$\begin{aligned}\int (c+dx)^4\sqrt{a+bx^2}\,dx &= \frac{(bx^2+a)^{\frac{3}{2}}d^4x^3}{6\,b}+\frac{4\,(bx^2+a)^{\frac{3}{2}}cd^3x^2}{5\,b}+\frac{1}{2}\,\sqrt{bx^2+a}c^4x \\ &+\frac{3\,(bx^2+a)^{\frac{3}{2}}c^2d^2x}{2\,b}-\frac{3\,\sqrt{bx^2+a}ac^2d^2x}{4\,b}-\frac{(bx^2+a)^{\frac{3}{2}}ad^4x}{8\,b^2} \\ &+\frac{\sqrt{bx^2+a}a^2d^4x}{16\,b^2}+\frac{ac^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-\frac{3\,a^2c^2d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\,b^{\frac{3}{2}}} \\ &+\frac{a^3d^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}+\frac{4\,(bx^2+a)^{\frac{3}{2}}c^3d}{3\,b}-\frac{8\,(bx^2+a)^{\frac{3}{2}}acd^3}{15\,b^2}\end{aligned}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*d^4*x^3/b + 4/5*(b*x^2 + a)^(3/2)*c*d^3*x^2/b + 1/2*
sqrt(b*x^2 + a)*c^4*x + 3/2*(b*x^2 + a)^(3/2)*c^2*d^2*x/b - 3/4*sqrt(b*x^2
 + a)*a*c^2*d^2*x/b - 1/8*(b*x^2 + a)^(3/2)*a*d^4*x/b^2 + 1/16*sqrt(b*x^2
+ a)*a^2*d^4*x/b^2 + 1/2*a*c^4*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/4*a^2*c^
2*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/16*a^3*d^4*arcsinh(b*x/sqrt(a*b))
/b^(5/2) + 4/3*(b*x^2 + a)^(3/2)*c^3*d/b - 8/15*(b*x^2 + a)^(3/2)*a*c*d^3/
b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.99

$$\int (c+dx)^4\sqrt{a+bx^2}\,dx$$
$$=\frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5d^4x+24cd^3\right)x+\frac{5\left(36b^4c^2d^2+ab^3d^4\right)}{b^4}\right)x+\frac{32\left(5b^4c^3d+ab^3cd^3\right)}{b^4}\right)x+\frac{15\left(8\right.}{}\right.\right.$$
$$-\frac{(8ab^2c^4-12a^2bc^2d^2+a^3d^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*d^4*x + 24*c*d^3)*x + 5*(36*b^4*c^2*d^2 +
 a*b^3*d^4)/b^4)*x + 32*(5*b^4*c^3*d + a*b^3*c*d^3)/b^4)*x + 15*(8*b^4*c^4
 + 12*a*b^3*c^2*d^2 - a^2*b^2*d^4)/b^4)*x + 64*(5*a*b^3*c^3*d - 2*a^2*b^2*
c*d^3)/b^4) - 1/16*(8*a*b^2*c^4 - 12*a^2*b*c^2*d^2 + a^3*d^4)*log(abs(-sqr
t(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4\sqrt{a+bx^2}\,dx=\int \sqrt{b\,x^2+a}\,(c+d\,x)^4\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^4,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4\sqrt{a+bx^2}\,dx=\int (dx+c)^4\,\sqrt{b\,x^2+a}dx$$

input
```
int((d*x+c)^4*(b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^4*(b*x^2+a)^(1/2),x)
```

3.234 $\int (c+dx)^3\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx=\frac{c(4bc^2-3ad^2)\,x\sqrt{a+bx^2}}{8b}+\frac{d(c+dx)^2\,(a+bx^2)^{3/2}}{5b}+\frac{d(8(6bc^2-ad^2)+21bcdx)\,(a+bx^2)^{3/2}}{60b^2}+\frac{ac(4bc^2-3ad^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*c*(-3*a*d^2+4*b*c^2)*x*(b*x^2+a)^(1/2)/b+1/5*d*(d*x+c)^2*(b*x^2+a)^(3/
2)/b+1/60*d*(21*b*c*d*x-8*a*d^2+48*b*c^2)*(b*x^2+a)^(3/2)/b^2+1/8*a*c*(-3*
a*d^2+4*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx$$
$$=\frac{\sqrt{a+bx^2}(-16a^2d^3+abd(120c^2+45cdx+8d^2x^2)+6b^2x(10c^3+20c^2dx+15cd^2x^2+4d^3x^3))+15a\sqrt{b}}{120b^2}$$

input
```
Integrate[(c + d*x)^3*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*d^3 + a*b*d*(120*c^2 + 45*c*d*x + 8*d^2*x^2) + 6
*b^2*x*(10*c^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3)) + 15*a*Sqrt[b]*c*
(-4*b*c^2 + 3*a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {497, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^3\,dx$$

$$\Big\downarrow \; 497$$

$$\frac{\int (c+dx)\left(5bc^2+7bdxc-2ad^2\right)\sqrt{bx^2+a}dx}{5b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{5}{4}c\left(4bc^2-3ad^2\right)\int \sqrt{bx^2+a}dx+\frac{2d\left(a+bx^2\right)^{3/2}\left(6bc^2-ad^2\right)}{3b}+\frac{7}{4}cd^2x\left(a+bx^2\right)^{3/2}}{5b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{5}{4}c\left(4bc^2-3ad^2\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{2d\left(a+bx^2\right)^{3/2}\left(6bc^2-ad^2\right)}{3b}+\frac{7}{4}cd^2x\left(a+bx^2\right)^{3/2}}{5b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{5}{4}c(4bc^2-3ad^2)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{2d(a+bx^2)^{3/2}(6bc^2-ad^2)}{3b}+\frac{7}{4}cd^2x(a+bx^2)^{3/2}}{5b}+\frac{d(a+bx^2)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{5}{4}c\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc^2-3ad^2)+\frac{2d(a+bx^2)^{3/2}(6bc^2-ad^2)}{3b}+\frac{7}{4}cd^2x(a+bx^2)^{3/2}}{5b}+\frac{d(a+bx^2)^{3/2}(c+dx)^2}{5b}$$

input
```
Int[(c + d*x)^3*Sqrt[a + b*x^2],x]
```

output
```
(d*(c + d*x)^2*(a + b*x^2)^(3/2))/(5*b) + ((2*d*(6*b*c^2 - a*d^2)*(a + b*x
^2)^(3/2))/(3*b) + (7*c*d^2*x*(a + b*x^2)^(3/2))/4 + (5*c*(4*b*c^2 - 3*a*d
^2)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*S
qrt[b])))/4)/(5*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{\left(-24 b^{2} x^{4} d^{3}-90 b^{2} c \,x^{3} d^{2}-8 a b \,d^{3} x^{2}-120 b^{2} c^{2} d \,x^{2}-45 a b c \,d^{2} x -60 c^{3} b^{2} x +16 a^{2} d^{3}-120 a b \,c^{2} d \right) \sqrt{b \,x^{2}+a}}{120 b^{2}}-\frac{a c \left(3 a \,d^{2}-4 b \,c^{2}\right) \ln \left(\sqrt{\right.}}{8 b^{\frac{3}{2}}}$
default	$c^{3} \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)+d^{3} \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{5 b}-\frac{2 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{15 b^{2}}\right)+3 c \,d^{2} \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4 b}-\frac{a \left(\frac{x \sqrt{b \,x^{2}}}{2}\right.}{}\right.$

input
```
int((d*x+c)^3*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*b^2*d^3*x^4-90*b^2*c*d^2*x^3-8*a*b*d^3*x^2-120*b^2*c^2*d*x^2-4
5*a*b*c*d^2*x-60*b^2*c^3*x+16*a^2*d^3-120*a*b*c^2*d)*(b*x^2+a)^(1/2)/b^2-1
/8*a*c/b^(3/2)*(3*a*d^2-4*b*c^2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.99

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx$$
$$=\left[-\frac{15\,(4\,abc^3-3\,a^2cd^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(24\,b^2d^3x^4+90\,b^2cd^2x^3+120\,abc^2d}{240\,b^2}\right.$$
$$-\frac{15\,(4\,abc^3-3\,a^2cd^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(24\,b^2d^3x^4+90\,b^2cd^2x^3+120\,abc^2d-16\,a^2d^3+8\,(15\,b^2}{120\,b^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/240*(15*(4*a*b*c^3 - 3*a^2*c*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - 2*(24*b^2*d^3*x^4 + 90*b^2*c*d^2*x^3 + 120*a*b*c^2*d
 - 16*a^2*d^3 + 8*(15*b^2*c^2*d + a*b*d^3)*x^2 + 15*(4*b^2*c^3 + 3*a*b*c*d
^2)*x)*sqrt(b*x^2 + a))/b^2, -1/120*(15*(4*a*b*c^3 - 3*a^2*c*d^2)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (24*b^2*d^3*x^4 + 90*b^2*c*d^2*x^3 +
 120*a*b*c^2*d - 16*a^2*d^3 + 8*(15*b^2*c^2*d + a*b*d^3)*x^2 + 15*(4*b^2*c
^3 + 3*a*b*c*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.33

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\cdot\left(\frac{3cd^2x^3}{4}+\frac{d^3x^4}{5}+\frac{x^2\left(\frac{ad^3}{5}+3bc^2d\right)}{3b}+\frac{x\left(\frac{3acd^2}{4}+bc^3\right)}{2b}+\frac{3ac^2d-\frac{2a\left(\frac{ad^3}{5}+3bc^2d\right)}{3b}}{b}\right)+\left(ac^3-\frac{a\left(\frac{3acd^2}{4}+bc^3\right)}{2b}\right. \\ \sqrt{a}\left(\begin{cases}c^3x & \text{for } d=0\\ \frac{(c+dx)^4}{4d} & \text{otherwise}\end{cases}\right)\end{cases}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(3*c*d**2*x**3/4 + d**3*x**4/5 + x**2*(a*d**3/
5 + 3*b*c**2*d)/(3*b) + x*(3*a*c*d**2/4 + b*c**3)/(2*b) + (3*a*c**2*d - 2*
a*(a*d**3/5 + 3*b*c**2*d)/(3*b))/b) + (a*c**3 - a*(3*a*c*d**2/4 + b*c**3)/
(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*Piecewise((c**3*x,
 Eq(d, 0)), ((c + d*x)**4/(4*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx=\frac{(bx^2+a)^{\frac{3}{2}}d^3x^2}{5\,b}+\frac{1}{2}\,\sqrt{bx^2+a}c^3x+\frac{3\,(bx^2+a)^{\frac{3}{2}}cd^2x}{4\,b}-\frac{3\,\sqrt{bx^2+a}acd^2x}{8\,b}+\frac{ac^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-\frac{3\,a^2cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{(bx^2+a)^{\frac{3}{2}}c^2d}{b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}ad^3}{15\,b^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*d^3*x^2/b + 1/2*sqrt(b*x^2 + a)*c^3*x + 3/4*(b*x^2 +
 a)^(3/2)*c*d^2*x/b - 3/8*sqrt(b*x^2 + a)*a*c*d^2*x/b + 1/2*a*c^3*arcsinh(
b*x/sqrt(a*b))/sqrt(b) - 3/8*a^2*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + (b
*x^2 + a)^(3/2)*c^2*d/b - 2/15*(b*x^2 + a)^(3/2)*a*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx=\frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,d^3x+15\,cd^2)x+\frac{4\,(15\,b^3c^2d+ab^2d^3)}{b^3}\right)x+\frac{15\,(4\,b^3c^3+3\,ab^2cd^2)}{b^3}\right)x+\frac{8\,(15\,ab\right.-\frac{(4\,abc^3-3\,a^2cd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*d^3*x + 15*c*d^2)*x + 4*(15*b^3*c^2*d + a*
b^2*d^3)/b^3)*x + 15*(4*b^3*c^3 + 3*a*b^2*c*d^2)/b^3)*x + 8*(15*a*b^2*c^2*
d - 2*a^2*b*d^3)/b^3) - 1/8*(4*a*b*c^3 - 3*a^2*c*d^2)*log(abs(-sqrt(b)*x +
 sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx=\int \sqrt{b\,x^2+a}\,(c+d\,x)^3\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.44

$$\int (c+dx)^3\sqrt{a+bx^2}\,dx$$
$$=\frac{-16\sqrt{b\,x^2+a}\,a^2d^3+120\sqrt{b\,x^2+a}\,ab\,c^2d+45\sqrt{b\,x^2+a}\,abc\,d^2x+8\sqrt{b\,x^2+a}\,ab\,d^3x^2+60\sqrt{b\,x^2+a}\,b^2c}{}$$

input
```
int((d*x+c)^3*(b*x^2+a)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x**2)*a**2*d**3 + 120*sqrt(a + b*x**2)*a*b*c**2*d + 45*s
qrt(a + b*x**2)*a*b*c*d**2*x + 8*sqrt(a + b*x**2)*a*b*d**3*x**2 + 60*sqrt(
a + b*x**2)*b**2*c**3*x + 120*sqrt(a + b*x**2)*b**2*c**2*d*x**2 + 90*sqrt(
a + b*x**2)*b**2*c*d**2*x**3 + 24*sqrt(a + b*x**2)*b**2*d**3*x**4 - 45*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*d**2 + 60*sqrt(b)*
log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c**3)/(120*b**2)
```

3.235 $\int (c+dx)^2\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx=\frac{(4bc^2-ad^2)\,x\sqrt{a+bx^2}}{8b}+\frac{d(8c+3dx)\,(a+bx^2)^{3/2}}{12b}+\frac{a(4bc^2-ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(-a*d^2+4*b*c^2)*x*(b*x^2+a)^(1/2)/b+1/12*d*(3*d*x+8*c)*(b*x^2+a)^(3/2
)/b+1/8*a*(-a*d^2+4*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(16acd+12bc^2x+3ad^2x+16bcdx^2+6bd^2x^3)}{24b}+\frac{a(-4bc^2+ad^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[(c + d*x)^2*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(16*a*c*d + 12*b*c^2*x + 3*a*d^2*x + 16*b*c*d*x^2 + 6*b*d
^2*x^3))/(24*b) + (a*(-4*b*c^2 + a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {497, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^2\,dx$$

$$\Big\downarrow \text{497}$$

$$\frac{\int \left(4bc^2+5bdxc-ad^2\right)\sqrt{bx^2+a}dx}{4b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)}{4b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(4bc^2-ad^2\right)\int \sqrt{bx^2+a}dx+\frac{5}{3}cd\left(a+bx^2\right)^{3/2}}{4b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)}{4b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\left(4bc^2-ad^2\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{5}{3}cd\left(a+bx^2\right)^{3/2}}{4b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\left(4bc^2-ad^2\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{5}{3}cd\left(a+bx^2\right)^{3/2}}{4b}+\frac{d\left(a+bx^2\right)^{3/2}(c+dx)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc^2-ad^2)+\frac{5}{3}cd(a+bx^2)^{3/2}}{4b}+\frac{d(a+bx^2)^{3/2}(c+dx)}{4b}$$

input
```
Int[(c + d*x)^2*Sqrt[a + b*x^2],x]
```

output
```
(d*(c + d*x)*(a + b*x^2)^(3/2))/(4*b) + ((5*c*d*(a + b*x^2)^(3/2))/3 + (4*
b*c^2 - a*d^2)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(2*Sqrt[b])))/(4*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\left(6b\,d^{2}x^{3}+16bcd\,x^{2}+3a\,d^{2}x+12c^{2}bx+16acd\right)\sqrt{b\,x^{2}+a}}{24b}-\frac{a\left(a\,d^{2}-4b\,c^{2}\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{3}{2}}}$
default	$c^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+d^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+\frac{2cd\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*b*d^2*x^3+16*b*c*d*x^2+3*a*d^2*x+12*b*c^2*x+16*a*c*d)*(b*x^2+a)^(1
/2)/b-1/8*a*(a*d^2-4*b*c^2)/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.10

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx$$
$$=\left[-\frac{3\,(4\,abc^2-a^2d^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(6\,b^2d^2x^3+16\,b^2cdx^2+16\,abcd+3\,(4\,b^2\right.}{48\,b^2}$$
$$-\frac{3\,(4\,abc^2-a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,b^2d^2x^3+16\,b^2cdx^2+16\,abcd+3\,(4\,b^2c^2+abd^2)x)\sqrt{bx^2+}}{24\,b^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*a*b*c^2 - a^2*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(6*b^2*d^2*x^3 + 16*b^2*c*d*x^2 + 16*a*b*c*d + 3*(4*b^2*
c^2 + a*b*d^2)*x)*sqrt(b*x^2 + a))/b^2, -1/24*(3*(4*a*b*c^2 - a^2*d^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*b^2*d^2*x^3 + 16*b^2*c*d*x^2
 + 16*a*b*c*d + 3*(4*b^2*c^2 + a*b*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{2acd}{3b}+\frac{2cdx^2}{3}+\frac{d^2x^3}{4}+\frac{x\left(\frac{ad^2}{4}+bc^2\right)}{2b}\right)+\left(ac^2-\frac{a\left(\frac{ad^2}{4}+bc^2\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) \\ \sqrt{a}\left(\begin{cases} c^2x & \text{for } d=0 \\ \frac{(c+dx)^3}{3d} & \text{otherwise}\end{cases}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(2*a*c*d/(3*b) + 2*c*d*x**2/3 + d**2*x**3/4 +
x*(a*d**2/4 + b*c**2)/(2*b)) + (a*c**2 - a*(a*d**2/4 + b*c**2)/(2*b))*Piec
ewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(
x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*Piecewise((c**2*x, Eq(d, 0)),
 ((c + d*x)**3/(3*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}c^2x + \frac{(bx^2+a)^{\frac{3}{2}}d^2x}{4\,b} - \frac{\sqrt{bx^2+a}ad^2x}{8\,b} + \frac{ac^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - \frac{a^2d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{2\,(bx^2+a)^{\frac{3}{2}}cd}{3\,b}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*c^2*x + 1/4*(b*x^2 + a)^(3/2)*d^2*x/b - 1/8*sqrt(b*x^2
 + a)*a*d^2*x/b + 1/2*a*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^2*d^2*a
rcsinh(b*x/sqrt(a*b))/b^(3/2) + 2/3*(b*x^2 + a)^(3/2)*c*d/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\frac{16\,acd}{b} + \left(2\left(3\,d^2x+8\,cd\right)x + \frac{3\left(4\,b^2c^2+abd^2\right)}{b^2}\right)x\right) - \frac{\left(4\,abc^2-a^2d^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*(16*a*c*d/b + (2*(3*d^2*x + 8*c*d)*x + 3*(4*b^2*c^2 +
 a*b*d^2)/b^2)*x) - 1/8*(4*a*b*c^2 - a^2*d^2)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx=\int \sqrt{b\,x^2+a}\,(c+d\,x)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.44

$$\int (c+dx)^2\sqrt{a+bx^2}\,dx$$
$$=\frac{16\sqrt{b\,x^2+a}\,abcd+3\sqrt{b\,x^2+a}\,ab\,d^2x+12\sqrt{b\,x^2+a}\,b^2c^2x+16\sqrt{b\,x^2+a}\,b^2cd\,x^2+6\sqrt{b\,x^2+a}\,b^2d^2x^3-}{24b^2}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2),x)
```

output
```
(16*sqrt(a + b*x**2)*a*b*c*d + 3*sqrt(a + b*x**2)*a*b*d**2*x + 12*sqrt(a +
 b*x**2)*b**2*c**2*x + 16*sqrt(a + b*x**2)*b**2*c*d*x**2 + 6*sqrt(a + b*x*
*2)*b**2*d**2*x**3 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*a**2*d**2 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*
*2)/(24*b**2)
```

3.236 $\int (c+dx)\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int (c+dx)\sqrt{a+bx^2}\,dx = \frac{1}{2}cx\sqrt{a+bx^2} + \frac{d(a+bx^2)^{3/2}}{3b} + \frac{a\text{carctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*c*x*(b*x^2+a)^(1/2)+1/3*d*(b*x^2+a)^(3/2)/b+1/2*a*c*arctanh(b^(1/2)*x/
(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (c+dx)\sqrt{a+bx^2}\,dx = \frac{\sqrt{a+bx^2}(2ad+3bcx+2bdx^2)}{6b} - \frac{ac\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[(c + d*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(2*a*d + 3*b*c*x + 2*b*d*x^2))/(6*b) - (a*c*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \sqrt{bx^2+a}dx+\frac{d\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{d\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$c\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{d\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$c\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{d\left(a+bx^2\right)^{3/2}}{3b}$$

input
```
Int[(c + d*x)*Sqrt[a + b*x^2],x]
```

output
```
(d*(a + b*x^2)^(3/2))/(3*b) + c*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$c\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{d\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$	54
risch	$\frac{\left(2bd\,x^{2}+3cbx+2ad\right)\sqrt{b\,x^{2}+a}}{6b}+\frac{ac\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	56

input
```
int((d*x+c)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+1/3*
d*(b*x^2+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int (c+dx)\sqrt{a+bx^2}\,dx = \left[\frac{3\,a\sqrt{b}c\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,bdx^2+3\,bcx+2\,ad)\sqrt{bx^2+a}}{12\,b},\right.$$
$$\left.-\frac{3\,a\sqrt{-b}c\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,bdx^2+3\,bcx+2\,ad)\sqrt{bx^2+a}}{6\,b}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*a*sqrt(b)*c*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*b*d*x^2 + 3*b*c*x + 2*a*d)*sqrt(b*x^2 + a))/b, -1/6*(3*a*sqrt(-b)*c*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*d*x^2 + 3*b*c*x + 2*a*d)*sqrt(b*x^2
+ a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int (c+dx)\sqrt{a+bx^2}\,dx = \begin{cases} \frac{ac\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2}+\sqrt{a+bx^2}\left(\frac{ad}{3b}+\frac{cx}{2}+\frac{dx^2}{3}\right) & \text{for}\ b\neq 0 \\ \sqrt{a}\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((a*c*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + sqrt(a + b*x**2)*(a*d/(3*b)
 + c*x/2 + d*x**2/3), Ne(b, 0)), (sqrt(a)*(c*x + d*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int (c+dx)\sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}cx + \frac{ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{(bx^2+a)^{\frac{3}{2}}d}{3b}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*c*x + 1/2*a*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*
x^2 + a)^(3/2)*d/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int (c+dx)\sqrt{a+bx^2}\,dx = -\frac{ac\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}} + \frac{1}{6}\sqrt{bx^2+a}\left((2\,dx+3\,c)x+\frac{2\,ad}{b}\right)$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*c*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
 a)*((2*d*x + 3*c)*x + 2*a*d/b)
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int (c+dx)\sqrt{a+bx^2}\,dx = \frac{d\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b}+\frac{c\,x\,\sqrt{b\,x^2+a}}{2}+\frac{a\,c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x),x)
```

output
```
(d*(a + b*x^2)^(3/2))/(3*b) + (c*x*(a + b*x^2)^(1/2))/2 + (a*c*log(b^(1/2)
*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (c+dx)\sqrt{a+bx^2}\,dx = \frac{2\sqrt{b\,x^2+a}\,ad+3\sqrt{b\,x^2+a}\,bcx+2\sqrt{b\,x^2+a}\,bd\,x^2+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ac}{6b}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**2)*a*d + 3*sqrt(a + b*x**2)*b*c*x + 2*sqrt(a + b*x**2)*b*
d*x**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c)/(6*b)
```

3.237 $\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx=\frac{\sqrt{a+bx^2}}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc^2+ad^2}\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^2}$$

output
```
(b*x^2+a)^(1/2)/d-b^(1/2)*c*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^2-(a*d^2+
b*c^2)^(1/2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx=\frac{d\sqrt{a+bx^2}+2\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)+\sqrt{b}c\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d^2}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x),x]
```

output
```
(d*Sqrt[a + b*x^2] + 2*Sqrt[-(b*c^2) - a*d^2]*ArcTan[(Sqrt[b]*(c + d*x) -
d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + Sqrt[b]*c*Log[-(Sqrt[b]*x) +
Sqrt[a + b*x^2]])/d^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {493, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{c+dx}dx$$

$$\Big\downarrow \text{493}$$

$$\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\sqrt{a+bx^2}}{d}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{(ad^2+bc^2)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d}+\frac{\sqrt{a+bx^2}}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(ad^2+bc^2)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d}+\frac{\sqrt{a+bx^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(ad^2+bc^2)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{d}+\frac{\sqrt{a+bx^2}}{d}$$

$$\Big\downarrow \text{488}$$

$$\frac{-\frac{(ad^2+bc^2)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{d}+\frac{\sqrt{a+bx^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{\sqrt{ad^2+bc^2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}-\frac{\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{d}+\frac{\sqrt{a+bx^2}}{d}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x),x]
```

output
```
Sqrt[a + b*x^2]/d + (-((Sqrt[b]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d)
 - (Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a
+ b*x^2])])/d)/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(89) = 178$.

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.76

method	result
risch	$\frac{\sqrt{b\,x^2+a}}{d}-\frac{\frac{\sqrt{b}\,c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d}+\frac{\left(a\,d^2+b\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{x+\frac{c}{d}}\right)}{d^2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}}{d}$
default	$\frac{\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}-\frac{\sqrt{b}\,c\ln\left(\frac{-\frac{bc}{d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}\right)}{d}-\frac{\left(a\,d^2+b\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc(}{}}{}\right.}{}}{d}$

input
```
int((b*x^2+a)^(1/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)/d-1/d*(b^(1/2)*c/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+(a*d^2+b*
c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)
+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.57

$$\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx$$

$$=\left[\frac{\sqrt{b}c\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}d+\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2-(2\,b^2c^2+abd^2}{d^2x^2+2\,c}\right.}{2\,d^2}\right.$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b)*c*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b
*x^2 + a)*d + sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 -
 (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^
2 + a))/(d^2*x^2 + 2*c*d*x + c^2)))/d^2, 1/2*(2*sqrt(-b)*c*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) + 2*sqrt(b*x^2 + a)*d + sqrt(b*c^2 + a*d^2)*log((2*a*b
*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 +
a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)))/d^2, 1/2
*(sqrt(b)*c*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*x^2
 + a)*d - 2*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)
*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)))/d^2, (sqr
t(-b)*c*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)*d - sqrt(-b*c
^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b
*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)))/d^2]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx=\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx=-\frac{\sqrt{b}c\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^2}+\frac{\sqrt{a+\frac{bc^2}{d^2}}\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{d}+\frac{\sqrt{bx^2+a}}{d}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c),x, algorithm="maxima")
```

output
```
-sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^2 + sqrt(a + b*c^2/d^2)*arcsinh(b*c*x/
(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d + sqrt(b*x^2 +
a)/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}}{c+dx} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{c+dx} \, dx = \int \frac{\sqrt{b\,x^2+a}}{c+d\,x} \,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x),x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 1048, normalized size of antiderivative = 10.17

$$\int \frac{\sqrt{a+bx^2}}{c+dx}\,dx = \text{Too large to display}$$

input

```
int((b*x^2+a)^(1/2)/(d*x+c),x)
```

output

```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*c - 2*sqrt(2*sqrt(b)*sqrt
(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b
)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*d**2
 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqr
t(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*
d**2 - 2*b*c**2))*b*c**2 - sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c
+ a*d**2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d*
*2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*c
+ sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt
(a*d**2 + b*c**2)*log(sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*
b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*c - sqrt(a*d**2 + b*c**2)*log(
 - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a +
b*x**2)*d + sqrt(b)*d*x)*a*d**2 - sqrt(a*d**2 + b*c**2)*log(sqrt(2*sqrt(b)
*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt(
b)*d*x)*a*d**2 + sqrt(a*d**2 + b*c**2)*log(2*sqrt(b)*sqrt(a*d**2 + b*c**2)
*c + 2*sqrt(b)*sqrt(a + b*x**2)*d**2*x - 2*b*c**2 + 2*b*d**2*x**2)*a*d**2
+ sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*log( - sqrt(
2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**...
```

3.238 $\int \frac{\sqrt{a+bx^2}}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2} \, dx = -\frac{\sqrt{a+bx^2}}{d(c+dx)} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} + \frac{b c\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^2\sqrt{bc^2+ad^2}}$$

output
```
-(b*x^2+a)^(1/2)/d/(d*x+c)+b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^2+
b*c*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^2/(a*d^2+b
*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2} \, dx = -\frac{\frac{d\sqrt{a+bx^2}}{c+dx} - \frac{2bc\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}} + \sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d^2}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x)^2,x]
```

output
```
-(((d*Sqrt[a + b*x^2])/(c + d*x) - (2*b*c*ArcTan[(Sqrt[b]*(c + d*x) - d*Sq
rt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + Sqrt[b]*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {492, 605, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2} \, dx$$

$$\Big\downarrow \ 492$$

$$\frac{b \int \frac{x}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{a+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \ 605$$

$$\frac{b \left(\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{c\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\right)}{d}-\frac{\sqrt{a+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \ 224$$

$$\frac{b \left(\frac{\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{c\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\right)}{d}-\frac{\sqrt{a+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \ 219$$

$$\frac{b \left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}-\frac{c\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\right)}{d}-\frac{\sqrt{a+bx^2}}{d(c+dx)}$$

$$\Big\downarrow \ 488$$

$$\frac{b\left(\frac{c\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}\right)}{d}-\frac{\sqrt{a+bx^2}}{d(c+dx)}$$

$\downarrow$ 219

$$\frac{b\left(\frac{c\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}\right)}{d}-\frac{\sqrt{a+bx^2}}{d(c+dx)}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x)^2,x]
```

output
```
-(Sqrt[a + b*x^2]/(d*(c + d*x))) + (b*(ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
]/(Sqrt[b]*d) + (c*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x
^2])])/(d*Sqrt[b*c^2 + a*d^2])))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(96) = 192$.

Time = 0.27 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.87

method	result
default	$\frac{-\frac{d^2\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\left(\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}-\frac{\sqrt{b}\,c\ln\left(\frac{-\frac{bc}{d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\cdots}\right)}{d}\cdots\right)}{\cdots}}{\cdots}$

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(
a*d^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2
)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)
/d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(97) = 194$.

Time = 0.16 (sec) , antiderivative size = 884, normalized size of antiderivative = 8.04

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/2*((b*c^3 + a*c*d^2 + (b*c^2*d + a*d^3)*x)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + (b*c*d*x + b*c^2)*sqrt(b*c^2 + a*d^2)*log((2
*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^
2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(
b*c^2*d + a*d^3)*sqrt(b*x^2 + a))/(b*c^3*d^2 + a*c*d^4 + (b*c^2*d^3 + a*d^
5)*x), 1/2*(2*(b*c*d*x + b*c^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 -
a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d
^2)*x^2)) + (b*c^3 + a*c*d^2 + (b*c^2*d + a*d^3)*x)*sqrt(b)*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(b*c^2*d + a*d^3)*sqrt(b*x^2 + a))/(
b*c^3*d^2 + a*c*d^4 + (b*c^2*d^3 + a*d^5)*x), -1/2*(2*(b*c^3 + a*c*d^2 + (
b*c^2*d + a*d^3)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c*d*x
 + b*c^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*
b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 +
a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(b*c^2*d + a*d^3)*sqrt(b*x^2 + a))/(b*c
^3*d^2 + a*c*d^4 + (b*c^2*d^3 + a*d^5)*x), ((b*c*d*x + b*c^2)*sqrt(-b*c^2
- a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^
2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (b*c^3 + a*c*d^2 + (b*c^2*d + a*
d^3)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c^2*d + a*d^3)*sq
rt(b*x^2 + a))/(b*c^3*d^2 + a*c*d^4 + (b*c^2*d^3 + a*d^5)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2}\,dx = -\frac{\sqrt{bx^2+a}}{d^2x+cd} + \frac{\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^2} - \frac{bc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}\,d^3}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)/(d^2*x + c*d) + sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^2 - b*c*
arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sq
rt(a + b*c^2/d^2)*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.58

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^2}\,dx$$
$$= \frac{2\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)b\,c^2+2\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}\right.}{}$$

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^2,x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*b*c**2 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*b*c*d*x - 2*sqrt(a*d**2 + b*c**2)*log(c +
d*x)*b*c**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c*d*x - 2*sqrt(a + b*
x**2)*a*d**3 - 2*sqrt(a + b*x**2)*b*c**2*d - sqrt(b)*log(sqrt(a + b*x**2)
- sqrt(b)*x)*a*c*d**2 - sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*d**3*x
 - sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*b*c**3 - sqrt(b)*log(sqrt(a +
 b*x**2) - sqrt(b)*x)*b*c**2*d*x + sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*
x)*a*c*d**2 + sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*x)*a*d**3*x + sqrt(b)
*log(sqrt(a + b*x**2) + sqrt(b)*x)*b*c**3 + sqrt(b)*log(sqrt(a + b*x**2) +
 sqrt(b)*x)*b*c**2*d*x)/(2*d**2*(a*c*d**2 + a*d**3*x + b*c**3 + b*c**2*d*x
))
```

3.239 $\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx=-\frac{(ad-bcx)\sqrt{a+bx^2}}{2\left(bc^2+ad^2\right)(c+dx)^2}-\frac{a b \operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2\left(bc^2+ad^2\right)^{3/2}}$$

output
```
-1/2*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)/(d*x+c)^2-1/2*a*b*arctanh(
(-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx=\frac{(-ad+bcx)\sqrt{a+bx^2}}{2\left(bc^2+ad^2\right)(c+dx)^2}+\frac{ab\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x)^3,x]
```

output
```
((-(a*d) + b*c*x)*Sqrt[a + b*x^2])/(2*(b*c^2 + a*d^2)*(c + d*x)^2) + (a*b*
ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-
(b*c^2) - a*d^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{486}$$

$$\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a b \text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x)^3,x]
```

output
```
-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*
ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 +
a*d^2)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(91) = 182$.

Time = 0.32 (sec) , antiderivative size = 900, normalized size of antiderivative = 8.74

method	result
default	$-\frac{d^2\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{2\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)^2}+\frac{bcd\left(-\frac{d^2\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\left(\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}} \cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(3/2)+1/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*
((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*
c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/
2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*
b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*
d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*d^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2
*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(
a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(
x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))))+1/2*b/(a*d^2+b*c^2)*d
^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((
-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2
-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(92) = 184$.

Time = 0.18 (sec) , antiderivative size = 519, normalized size of antiderivative = 5.04

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx$$

$$= \left[\frac{(abd^2x^2 + 2\,abcdx + abc^2)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx - abc^2 - 2\,a^2d^2 - (2\,b^2c^2 + abd^2)x^2 - 2\,\sqrt{bc^2+ad^2}(bcx-ad)\sqrt{bx^2+a}}{d^2x^2+2\,cdx+c^2}\right) - 2}{4\,(b^2c^6 + 2\,abc^4d^2 + a^2c^2d^4 + (b^2c^4d^2 + 2\,abc^2d^4 + a^2d^6)x^2 + 2\,(b^2c^5d + 2\,ab}\right.$$

$$-\frac{(abd^2x^2 + 2\,abcdx + abc^2)\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{-bc^2-ad^2}(bcx-ad)\sqrt{bx^2+a}}{abc^2+a^2d^2+(b^2c^2+abd^2)x^2}\right) + (abc^2d + a^2d^3 - (b^2c^3 + a}{2\,(b^2c^6 + 2\,abc^4d^2 + a^2c^2d^4 + (b^2c^4d^2 + 2\,abc^2d^4 + a^2d^6)x^2 + 2\,(b^2c^5d + 2\,abc^3d^3 + a^2cd^5}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/4*((a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b
*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 +
a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(a*b*
c^2*d + a^2*d^3 - (b^2*c^3 + a*b*c*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^6 + 2*a
*b*c^4*d^2 + a^2*c^2*d^4 + (b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6)*x^2 + 2
*(b^2*c^5*d + 2*a*b*c^3*d^3 + a^2*c*d^5)*x), -1/2*((a*b*d^2*x^2 + 2*a*b*c*
d*x + a*b*c^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a
*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (a*b*
c^2*d + a^2*d^3 - (b^2*c^3 + a*b*c*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^6 + 2*a
*b*c^4*d^2 + a^2*c^2*d^4 + (b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6)*x^2 + 2
*(b^2*c^5*d + 2*a*b*c^3*d^3 + a^2*c*d^5)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x+c)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(92) = 184$.

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx = -\frac{\sqrt{bx^2+a}bc}{2\,(bc^2d^2x+ad^4x+bc^3d+acd^3)} - \frac{(bx^2+a)^{\frac{3}{2}}}{2\left(bc^2dx^2+ad^3x^2+2\,bc^3x+2\,acd^2x+\frac{bc^4}{d}+ac^2d\right)} + \frac{\sqrt{bx^2+a}b}{2\,(bc^2d+ad^3)} - \frac{b^2c^2\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^5} + \frac{b\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\sqrt{a+\frac{bc^2}{d^2}}d^3}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*sqrt(b*x^2 + a)*b*c/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) - 1/2
*(b*x^2 + a)^(3/2)/(b*c^2*d*x^2 + a*d^3*x^2 + 2*b*c^3*x + 2*a*c*d^2*x + b*
c^4/d + a*c^2*d) + 1/2*sqrt(b*x^2 + a)*b/(b*c^2*d + a*d^3) - 1/2*b^2*c^2*a
rcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a
+ b*c^2/d^2)^(3/2)*d^5) + 1/2*b*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a
*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(92) = 184$.

Time = 0.13 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.07

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx = -\frac{ab\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(bc^2+ad^2)\sqrt{-bc^2-ad^2}} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3b^2c^2d+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3abd^3+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}c^3-\left(\sqrt{b}x-\sqrt{bx^2\right.}}{(bc^2d^2+ad^4)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\left(\sqrt{b}\right.\right.}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-a*b*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*
d^2))/((b*c^2 + a*d^2)*sqrt(-b*c^2 - a*d^2)) + (2*(sqrt(b)*x - sqrt(b*x^2
+ a))^3*b^2*c^2*d + (sqrt(b)*x - sqrt(b*x^2 + a))^3*a*b*d^3 + 2*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*b^(5/2)*c^3 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3
/2)*c*d^2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))*a*b^2*c^2*d + (sqrt(b)*x - sqr
t(b*x^2 + a))*a^2*b*d^3 + a^2*b^(3/2)*c*d^2)/((b*c^2*d^2 + a*d^4)*((sqrt(b
)*x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*c - a
*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx=\int \frac{\sqrt{b\,x^2+a}}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.84

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \frac{\sqrt{a\,d^2+b\,c^2}\,\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)ab\,c^2+2\sqrt{a\,d^2+b\,c^2}\,\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-a\right.}{}$$

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^3,x)
```

output
```
(sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a*b*c**2 + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2
 + b*c**2) - a*d + b*c*x)*a*b*c*d*x + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*d**2*x**2 - sqrt(a*d**2 +
b*c**2)*log(c + d*x)*a*b*c**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c
*d*x - sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*d**2*x**2 - sqrt(a + b*x**2)
*a**2*d**3 - sqrt(a + b*x**2)*a*b*c**2*d + sqrt(a + b*x**2)*a*b*c*d**2*x +
 sqrt(a + b*x**2)*b**2*c**3*x)/(2*(a**2*c**2*d**4 + 2*a**2*c*d**5*x + a**2
*d**6*x**2 + 2*a*b*c**4*d**2 + 4*a*b*c**3*d**3*x + 2*a*b*c**2*d**4*x**2 +
b**2*c**6 + 2*b**2*c**5*d*x + b**2*c**4*d**2*x**2))
```

3.240 $\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx = -\frac{bc(ad-bcx)\sqrt{a+bx^2}}{2\left(bc^2+ad^2\right)^2(c+dx)^2} - \frac{d\left(a+bx^2\right)^{3/2}}{3\left(bc^2+ad^2\right)(c+dx)^3} - \frac{ab^2c\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2\left(bc^2+ad^2\right)^{5/2}}$$

output
```
-1/2*b*c*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2/(d*x+c)^2-1/3*d*(b*x
^2+a)^(3/2)/(a*d^2+b*c^2)/(d*x+c)^3-1/2*a*b^2*c*arctanh((-b*c*x+a*d)/(a*d^
2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx = \frac{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\left(-2a^2d^3+b^2c^2x(3c+dx)-abd\left(5c^2+3cdx+2d^2x^2\right)\right)+3ab^2c(c+dx)^3\log(c+dx}{6\left(bc^2+ad^2\right)^{5/2}(c+dx)^3}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x)^4,x]
```

output
```
(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]*(-2*a^2*d^3 + b^2*c^2*x*(3*c + d*x) -
 a*b*d*(5*c^2 + 3*c*d*x + 2*d^2*x^2)) + 3*a*b^2*c*(c + d*x)^3*Log[c + d*x]
 - 3*a*b^2*c*(c + d*x)^3*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*
x^2]])/(6*(b*c^2 + a*d^2)^(5/2)*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {491, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}dx$$

$$\Big\downarrow \text{491}$$

$$\frac{bc\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{3/2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{486}$$

$$\frac{bc\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{3/2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{bc\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{3/2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{bc\left(-\frac{ab\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2(ad^2+bc^2)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d(a+bx^2)^{3/2}}{3(c+dx)^3(ad^2+bc^2)}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x)^4,x]
```

output
```
-1/3*(d*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^3) + (b*c*(-1/2*((a*
d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(
a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2)^(3
/2))))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(128) = 256$.

Time = 0.37 (sec) , antiderivative size = 988, normalized size of antiderivative = 6.86

method	result
default	$-\frac{d^2\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{3\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)^3}+\frac{bcd\left(-\frac{d^2\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{2\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)^2}+\frac{bcd\left(-\frac{d^2\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b}{d^2}\right.}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}\right.}{}\right.}{}$

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(3/2)+b*c*d/(a*d^2+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+1/2*b*c*d/(a*d^2+b*c
^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^
2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2
)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2
)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*
d^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*
ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d
^2)^(1/2))))+1/2*b/(a*d^2+b*c^2)*d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2
*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/
d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(
1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(129) = 258$.

Time = 0.38 (sec) , antiderivative size = 861, normalized size of antiderivative = 5.98

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(a*b^2*c*d^3*x^3 + 3*a*b^2*c^2*d^2*x^2 + 3*a*b^2*c^3*d*x + a*b^2*
c^4)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c
^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(
d^2*x^2 + 2*c*d*x + c^2)) - 2*(5*a*b^2*c^4*d + 7*a^2*b*c^2*d^3 + 2*a^3*d^5
 - (b^3*c^4*d - a*b^2*c^2*d^3 - 2*a^2*b*d^5)*x^2 - 3*(b^3*c^5 - a^2*b*c*d^
4)*x)*sqrt(b*x^2 + a))/(b^3*c^9 + 3*a*b^2*c^7*d^2 + 3*a^2*b*c^5*d^4 + a^3*
c^3*d^6 + (b^3*c^6*d^3 + 3*a*b^2*c^4*d^5 + 3*a^2*b*c^2*d^7 + a^3*d^9)*x^3
+ 3*(b^3*c^7*d^2 + 3*a*b^2*c^5*d^4 + 3*a^2*b*c^3*d^6 + a^3*c*d^8)*x^2 + 3*
(b^3*c^8*d + 3*a*b^2*c^6*d^3 + 3*a^2*b*c^4*d^5 + a^3*c^2*d^7)*x), -1/6*(3*
(a*b^2*c*d^3*x^3 + 3*a*b^2*c^2*d^2*x^2 + 3*a*b^2*c^3*d*x + a*b^2*c^4)*sqrt
(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)
/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (5*a*b^2*c^4*d + 7*a^2*b
*c^2*d^3 + 2*a^3*d^5 - (b^3*c^4*d - a*b^2*c^2*d^3 - 2*a^2*b*d^5)*x^2 - 3*(
b^3*c^5 - a^2*b*c*d^4)*x)*sqrt(b*x^2 + a))/(b^3*c^9 + 3*a*b^2*c^7*d^2 + 3*
a^2*b*c^5*d^4 + a^3*c^3*d^6 + (b^3*c^6*d^3 + 3*a*b^2*c^4*d^5 + 3*a^2*b*c^2
*d^7 + a^3*d^9)*x^3 + 3*(b^3*c^7*d^2 + 3*a*b^2*c^5*d^4 + 3*a^2*b*c^3*d^6 +
 a^3*c*d^8)*x^2 + 3*(b^3*c^8*d + 3*a*b^2*c^6*d^3 + 3*a^2*b*c^4*d^5 + a^3*c
^2*d^7)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx=\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x+c)**4,x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(129) = 258$.

Time = 0.06 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx = -\frac{\sqrt{bx^2+a}b^2c^2}{2\,(b^2c^4d^2x+2\,abc^2d^4x+a^2d^6x+b^2c^5d+2\,abc^3d^3+a^2cd^5)} - \frac{(bx^2+a)^{\frac{3}{2}}bc}{2\,\left(b^2c^4dx^2+2\,abc^2d^3x^2+a^2d^5x^2+2\,b^2c^5x+4\,abc^3d^2x+2\,a^2cd^4x+\frac{b^2c^6}{d}+2\,abc^4d+a^2c^2d^3\right)} + \frac{\sqrt{bx^2+a}b^2c}{2\,(b^2c^4d+2\,abc^2d^3+a^2d^5)} - \frac{(bx^2+a)^{\frac{3}{2}}}{3\,\left(bc^2d^2x^3+ad^4x^3+3\,bc^3dx^2+3\,acd^3x^2+3\,bc^4x+3\,ac^2d^2x+\frac{bc^5}{d}+ac^3d\right)} - \frac{b^3c^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\,\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^7} + \frac{b^2c\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\,\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^5}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/2*sqrt(b*x^2 + a)*b^2*c^2/(b^2*c^4*d^2*x + 2*a*b*c^2*d^4*x + a^2*d^6*x
+ b^2*c^5*d + 2*a*b*c^3*d^3 + a^2*c*d^5) - 1/2*(b*x^2 + a)^(3/2)*b*c/(b^2*
c^4*d*x^2 + 2*a*b*c^2*d^3*x^2 + a^2*d^5*x^2 + 2*b^2*c^5*x + 4*a*b*c^3*d^2*
x + 2*a^2*c*d^4*x + b^2*c^6/d + 2*a*b*c^4*d + a^2*c^2*d^3) + 1/2*sqrt(b*x^
2 + a)*b^2*c/(b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5) - 1/3*(b*x^2 + a)^(3/2)
/(b*c^2*d^2*x^3 + a*d^4*x^3 + 3*b*c^3*d*x^2 + 3*a*c*d^3*x^2 + 3*b*c^4*x +
3*a*c^2*d^2*x + b*c^5/d + a*c^3*d) - 1/2*b^3*c^3*arcsinh(b*c*x/(sqrt(a*b)*
abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(5/2)*d^7)
+ 1/2*b^2*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*
x + c)))/((a + b*c^2/d^2)^(3/2)*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(129) = 258.

Time = 0.13 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.67

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx = -\frac{ab^2c\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(b^2c^4+2\,abc^2d^2+a^2d^4)\sqrt{-bc^2-ad^2}} - \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5ab^2cd^4-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{7}{2}}c^4d+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4ab^{\frac{5}{2}}c^2d^3-6\left(\sqrt{b}\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-a*b^2*c*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2
 - a*d^2))/((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*sqrt(-b*c^2 - a*d^2)) - 1/3
*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a*b^2*c*d^4 - 6*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*b^(7/2)*c^4*d + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*
d^3 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^5 - 4*(sqrt(b)*x - s
qrt(b*x^2 + a))^3*b^4*c^5 + 14*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a*b^3*c^3*d
^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a^2*b^2*c*d^4 + 6*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a*b^(7/2)*c^4*d - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b
^(5/2)*c^2*d^3 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))*a^2*b^3*c^3*d^2 + 9*(sqrt
(b)*x - sqrt(b*x^2 + a))*a^3*b^2*c*d^4 + a^3*b^(5/2)*c^2*d^3 - 2*a^4*b^(3/
2)*d^5)/((b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6)*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*c - a*d)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x)^4,x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 699, normalized size of antiderivative = 4.85

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^4}\,dx = \frac{3\sqrt{a\,d^2+b\,c^2}\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)a\,b^2c^4+9\sqrt{a\,d^2+b\,c^2}\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-\right.}{}$$

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^4,x)
```

output
```
(3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a*b**2*c**4 + 9*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d*x + 9*sqrt(a*d**2 + b*c**2)*l
og(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**2*
x**2 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a*b**2*c*d**3*x**3 - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a
*b**2*c**4 - 9*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d*x - 9*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**2*x**2 - 3*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*a*b**2*c*d**3*x**3 - 2*sqrt(a + b*x**2)*a**3*d**5 - 7*sq
rt(a + b*x**2)*a**2*b*c**2*d**3 - 3*sqrt(a + b*x**2)*a**2*b*c*d**4*x - 2*s
qrt(a + b*x**2)*a**2*b*d**5*x**2 - 5*sqrt(a + b*x**2)*a*b**2*c**4*d - sqrt
(a + b*x**2)*a*b**2*c**2*d**3*x**2 + 3*sqrt(a + b*x**2)*b**3*c**5*x + sqrt
(a + b*x**2)*b**3*c**4*d*x**2)/(6*(a**3*c**3*d**6 + 3*a**3*c**2*d**7*x + 3
*a**3*c*d**8*x**2 + a**3*d**9*x**3 + 3*a**2*b*c**5*d**4 + 9*a**2*b*c**4*d*
*5*x + 9*a**2*b*c**3*d**6*x**2 + 3*a**2*b*c**2*d**7*x**3 + 3*a*b**2*c**7*d
**2 + 9*a*b**2*c**6*d**3*x + 9*a*b**2*c**5*d**4*x**2 + 3*a*b**2*c**4*d**5*
x**3 + b**3*c**9 + 3*b**3*c**8*d*x + 3*b**3*c**7*d**2*x**2 + b**3*c**6*d**
3*x**3))
```

3.241 $\int \frac{\sqrt{a+bx^2}}{(c+dx)^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 206

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5} \, dx = -\frac{b(4bc^2-ad^2)\,(ad-bcx)\sqrt{a+bx^2}}{8\,(bc^2+ad^2)^3\,(c+dx)^2} - \frac{d(a+bx^2)^{3/2}}{4\,(bc^2+ad^2)\,(c+dx)^4} - \frac{5bcd(a+bx^2)^{3/2}}{12\,(bc^2+ad^2)^2\,(c+dx)^3} - \frac{ab^2(4bc^2-ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{8\,(bc^2+ad^2)^{7/2}}$$

output

```
-1/8*b*(-a*d^2+4*b*c^2)*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^3/(d*x+
c)^2-1/4*d*(b*x^2+a)^(3/2)/(a*d^2+b*c^2)/(d*x+c)^4-5/12*b*c*d*(b*x^2+a)^(3
/2)/(a*d^2+b*c^2)^2/(d*x+c)^3-1/8*a*b^2*(-a*d^2+4*b*c^2)*arctanh((-b*c*x+a
*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 904 vs. $2(206) = 412$.

Time = 11.02 (sec) , antiderivative size = 904, normalized size of antiderivative = 4.39

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5}\,dx$$
$$= \frac{6a^6d^7 + 16b^{11/2}c^6x^4(c+4dx)\left(\sqrt{b}x - \sqrt{a+bx^2}\right) + a^5\left(-24\sqrt{b}d^7x\sqrt{a+bx^2} + bd^5(19c^2 + 4cdx + 57d^2x\right.}{}$$
$$+ \frac{ab^3c^2\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{7/2}} - \frac{a^2b^2d^2\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{4(-bc^2-ad^2)^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c + d*x)^5,x]
```

output
```
(6*a^6*d^7 + 16*b^(11/2)*c^6*x^4*(c + 4*d*x)*(Sqrt[b]*x - Sqrt[a + b*x^2])
 + a^5*(-24*Sqrt[b]*d^7*x*Sqrt[a + b*x^2] + b*d^5*(19*c^2 + 4*c*d*x + 57*d
^2*x^2)) + a^4*(-4*b^(3/2)*d^5*x*Sqrt[a + b*x^2]*(19*c^2 + 4*c*d*x + 21*d^
2*x^2) + b^2*d^3*(28*c^4 + 37*c^3*d*x + 211*c^2*d^2*x^2 + 49*c*d^3*x^3 + 1
23*d^4*x^4)) + 8*a*b^(9/2)*c^2*x^2*(-(Sqrt[b]*x*(-3*c^5 - 12*c^4*d*x + 19*
c^3*d^2*x^2 + 28*c^2*d^3*x^3 + 42*c*d^4*x^4 + 12*d^5*x^5)) + Sqrt[a + b*x^
2]*(-2*c^5 - 8*c^4*d*x + 19*c^3*d^2*x^2 + 28*c^2*d^3*x^3 + 42*c*d^4*x^4 +
12*d^5*x^5)) + a^3*b^(5/2)*d^2*(-(Sqrt[a + b*x^2]*(-13*c^5 + 60*c^4*d*x +
70*c^3*d^2*x^2 + 336*c^2*d^3*x^3 + 87*c*d^4*x^4 + 84*d^5*x^5)) + Sqrt[b]*x
*(-64*c^5 + 36*c^4*d*x + 19*c^3*d^2*x^2 + 456*c^2*d^3*x^3 + 129*c*d^4*x^4
+ 96*d^5*x^5)) - 2*a^2*(-(b^4*x*(4*c^7 + 16*c^6*d*x - 108*c^5*d^2*x^2 - 10
8*c^4*d^3*x^3 - 177*c^3*d^4*x^4 + 84*c^2*d^5*x^5 + 42*c*d^6*x^6 + 12*d^7*x
^7)) + b^(7/2)*Sqrt[a + b*x^2]*(c^7 + 4*c^6*d*x - 70*c^5*d^2*x^2 - 52*c^4*
d^3*x^3 - 93*c^3*d^4*x^4 + 108*c^2*d^5*x^5 + 42*c*d^6*x^6 + 12*d^7*x^7)))/
(24*d^2*(b*c^2 + a*d^2)^3*(c + d*x)^4*(4*a*b*x^2*(3*Sqrt[b]*x - 2*Sqrt[a +
 b*x^2]) + 8*b^2*x^4*(Sqrt[b]*x - Sqrt[a + b*x^2]) - a^2*(-4*Sqrt[b]*x + S
qrt[a + b*x^2]))) + (a*b^3*c^2*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^
2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(7/2) - (a^2*b^2*d^2*ArcTa
n[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(4*(-(b
*c^2) - a*d^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {498, 25, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5} \, dx$$

$$\Big\downarrow \text{ 498}$$

$$-\frac{b \int -\frac{(4c-dx)\sqrt{bx^2+a}}{(c+dx)^4}dx}{4 (ad^2+bc^2)}-\frac{d (a+bx^2)^{3/2}}{4(c+dx)^4 (ad^2+bc^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b \int \frac{(4c-dx)\sqrt{bx^2+a}}{(c+dx)^4}dx}{4 (ad^2+bc^2)}-\frac{d (a+bx^2)^{3/2}}{4(c+dx)^4 (ad^2+bc^2)}$$

$$\Big\downarrow \text{ 679}$$

$$\frac{b \left(\frac{(4bc^2-ad^2) \int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{ad^2+bc^2}-\frac{5cd (a+bx^2)^{3/2}}{3(c+dx)^3(ad^2+bc^2)}\right)}{4 (ad^2+bc^2)}-\frac{d (a+bx^2)^{3/2}}{4(c+dx)^4 (ad^2+bc^2)}$$

$$\Big\downarrow \text{ 486}$$

$$\frac{b \left(\frac{(4bc^2-ad^2) \left(\frac{ab \int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2 (ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2 (ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{5cd (a+bx^2)^{3/2}}{3(c+dx)^3(ad^2+bc^2)}\right)}{4 (ad^2+bc^2)}-\frac{d (a+bx^2)^{3/2}}{4(c+dx)^4 (ad^2+bc^2)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{b\left(\frac{(4bc^2-ad^2)\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{5cd(a+bx^2)^{3/2}}{3(c+dx)^3(ad^2+bc^2)}\right)}{4(ad^2+bc^2)}-\frac{d(a+bx^2)^{3/2}}{4(c+dx)^4(ad^2+bc^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{(4bc^2-ad^2)\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2(ad^2+bc^2)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{5cd(a+bx^2)^{3/2}}{3(c+dx)^3(ad^2+bc^2)}\right)}{4(ad^2+bc^2)}-\frac{d(a+bx^2)^{3/2}}{4(c+dx)^4(ad^2+bc^2)}$$

input
```
Int[Sqrt[a + b*x^2]/(c + d*x)^5,x]
```

output
```
-1/4*(d*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (b*((-5*c*d*(a
+ b*x^2)^(3/2))/(3*(b*c^2 + a*d^2)*(c + d*x)^3) + ((4*b*c^2 - a*d^2)*(-1/2
*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcT
anh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^
2)^(3/2))))/(b*c^2 + a*d^2)))/(4*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1990 vs. $2(186) = 372$.

Time = 0.46 (sec) , antiderivative size = 1991, normalized size of antiderivative = 9.67

method	result	size
default	Expression too large to display	1991

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d^5*(-1/4/(a*d^2+b*c^2)*d^2/(x+c/d)^4*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(3/2)+5/4*b*c*d/(a*d^2+b*c^2)*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d
)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+b*c*d/(a*d^2+b*c
^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(3/2)+1/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b
*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/
d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
)-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*
c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*d^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2*b
*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*
d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))))+1/2*b/(a*d^2+b*c^2)*d^2
*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b
*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1
/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2
*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))))-1/4*b/(a*d^2+b*c^2)*d^2*(-1/2/(a*d^2+b
*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(187) = 374$.

Time = 1.33 (sec) , antiderivative size = 1485, normalized size of antiderivative = 7.21

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*a*b^3*c^6 - a^2*b^2*c^4*d^2 + (4*a*b^3*c^2*d^4 - a^2*b^2*d^6)
*x^4 + 4*(4*a*b^3*c^3*d^3 - a^2*b^2*c*d^5)*x^3 + 6*(4*a*b^3*c^4*d^2 - a^2*
b^2*c^2*d^4)*x^2 + 4*(4*a*b^3*c^5*d - a^2*b^2*c^3*d^3)*x)*sqrt(b*c^2 + a*d
^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2
*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c
^2)) + 2*(28*a*b^3*c^6*d + 47*a^2*b^2*c^4*d^3 + 25*a^3*b*c^2*d^5 + 6*a^4*d
^7 - (2*b^4*c^5*d^2 - 11*a*b^3*c^3*d^4 - 13*a^2*b^2*c*d^6)*x^3 - (8*b^4*c^
6*d - 32*a*b^3*c^4*d^3 - 43*a^2*b^2*c^2*d^5 - 3*a^3*b*d^7)*x^2 - (12*b^4*c
^7 - 25*a*b^3*c^5*d^2 - 41*a^2*b^2*c^3*d^4 - 4*a^3*b*c*d^6)*x)*sqrt(b*x^2
+ a))/(b^4*c^12 + 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^8*d^4 + 4*a^3*b*c^6*d^6 +
 a^4*c^4*d^8 + (b^4*c^8*d^4 + 4*a*b^3*c^6*d^6 + 6*a^2*b^2*c^4*d^8 + 4*a^3*
b*c^2*d^10 + a^4*d^12)*x^4 + 4*(b^4*c^9*d^3 + 4*a*b^3*c^7*d^5 + 6*a^2*b^2*
c^5*d^7 + 4*a^3*b*c^3*d^9 + a^4*c*d^11)*x^3 + 6*(b^4*c^10*d^2 + 4*a*b^3*c^
8*d^4 + 6*a^2*b^2*c^6*d^6 + 4*a^3*b*c^4*d^8 + a^4*c^2*d^10)*x^2 + 4*(b^4*c
^11*d + 4*a*b^3*c^9*d^3 + 6*a^2*b^2*c^7*d^5 + 4*a^3*b*c^5*d^7 + a^4*c^3*d^
9)*x), -1/24*(3*(4*a*b^3*c^6 - a^2*b^2*c^4*d^2 + (4*a*b^3*c^2*d^4 - a^2*b^
2*d^6)*x^4 + 4*(4*a*b^3*c^3*d^3 - a^2*b^2*c*d^5)*x^3 + 6*(4*a*b^3*c^4*d^2
- a^2*b^2*c^2*d^4)*x^2 + 4*(4*a*b^3*c^5*d - a^2*b^2*c^3*d^3)*x)*sqrt(-b*c^
2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*
c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (28*a*b^3*c^6*d + 47*a^2*b^...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx)^5}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x+c)**5,x)
```

output
```
Integral(sqrt(a + b*x**2)/(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(187) = 374$.

Time = 0.09 (sec) , antiderivative size = 977, normalized size of antiderivative = 4.74

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
-5/8*sqrt(b*x^2 + a)*b^3*c^3/(b^3*c^6*d^2*x + 3*a*b^2*c^4*d^4*x + 3*a^2*b*
c^2*d^6*x + a^3*d^8*x + b^3*c^7*d + 3*a*b^2*c^5*d^3 + 3*a^2*b*c^3*d^5 + a^
3*c*d^7) - 5/8*(b*x^2 + a)^(3/2)*b^2*c^2/(b^3*c^6*d*x^2 + 3*a*b^2*c^4*d^3*
x^2 + 3*a^2*b*c^2*d^5*x^2 + a^3*d^7*x^2 + 2*b^3*c^7*x + 6*a*b^2*c^5*d^2*x
+ 6*a^2*b*c^3*d^4*x + 2*a^3*c*d^6*x + b^3*c^8/d + 3*a*b^2*c^6*d + 3*a^2*b*
c^4*d^3 + a^3*c^2*d^5) + 5/8*sqrt(b*x^2 + a)*b^3*c^2/(b^3*c^6*d + 3*a*b^2*
c^4*d^3 + 3*a^2*b*c^2*d^5 + a^3*d^7) - 5/12*(b*x^2 + a)^(3/2)*b*c/(b^2*c^4
*d^2*x^3 + 2*a*b*c^2*d^4*x^3 + a^2*d^6*x^3 + 3*b^2*c^5*d*x^2 + 6*a*b*c^3*d
^3*x^2 + 3*a^2*c*d^5*x^2 + 3*b^2*c^6*x + 6*a*b*c^4*d^2*x + 3*a^2*c^2*d^4*x
 + b^2*c^7/d + 2*a*b*c^5*d + a^2*c^3*d^3) + 1/8*sqrt(b*x^2 + a)*b^2*c/(b^2
*c^4*d^2*x + 2*a*b*c^2*d^4*x + a^2*d^6*x + b^2*c^5*d + 2*a*b*c^3*d^3 + a^2
*c*d^5) + 1/8*(b*x^2 + a)^(3/2)*b/(b^2*c^4*d*x^2 + 2*a*b*c^2*d^3*x^2 + a^2
*d^5*x^2 + 2*b^2*c^5*x + 4*a*b*c^3*d^2*x + 2*a^2*c*d^4*x + b^2*c^6/d + 2*a
*b*c^4*d + a^2*c^2*d^3) - 1/8*sqrt(b*x^2 + a)*b^2/(b^2*c^4*d + 2*a*b*c^2*d
^3 + a^2*d^5) - 1/4*(b*x^2 + a)^(3/2)/(b*c^2*d^3*x^4 + a*d^5*x^4 + 4*b*c^3
*d^2*x^3 + 4*a*c*d^4*x^3 + 6*b*c^4*d*x^2 + 6*a*c^2*d^3*x^2 + 4*b*c^5*x + 4
*a*c^3*d^2*x + b*c^6/d + a*c^4*d) - 5/8*b^4*c^4*arcsinh(b*c*x/(sqrt(a*b)*a
bs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(7/2)*d^9) +
 3/4*b^3*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d
*x + c)))/((a + b*c^2/d^2)^(5/2)*d^7) - 1/8*b^2*arcsinh(b*c*x/(sqrt(a*b...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5} \, dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x+c)^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5} \, dx = \int \frac{\sqrt{b\,x^2+a}}{{\left(c+d\,x\right)}^5} \,d x$$

input
```
int((a + b*x^2)^(1/2)/(c + d*x)^5,x)
```

output
```
int((a + b*x^2)^(1/2)/(c + d*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 1557, normalized size of antiderivative = 7.56

$$\int \frac{\sqrt{a+bx^2}}{(c+dx)^5} \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)/(d*x+c)^5,x)
```

output

```
(3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*b**2*c**4*d**2 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d**3*x + 18*s
qrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*a**2*b**2*c**2*d**4*x**2 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
+ b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x**3 + 3*s
qrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*a**2*b**2*d**6*x**4 - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**6 - 48*sqrt(a*d**2 + b
*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3
*c**5*d*x - 72*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
 b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**2*x**2 - 48*sqrt(a*d**2 + b*c**2)*l
og( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d*
*3*x**3 - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a*b**3*c**2*d**4*x**4 - 3*sqrt(a*d**2 + b*c**2)*log(
c + d*x)*a**2*b**2*c**4*d**2 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*
b**2*c**3*d**3*x - 18*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c**2*d*
*4*x**2 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c*d**5*x**3 - 3*
sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*d**6*x**4 + 12*sqrt(a*d**2 +
b*c**2)*log(c + d*x)*a*b**3*c**6 + 48*sqrt(a*d**2 + b*c**2)*log(c + d*x...
```

3.242 $\int (c+dx)^4 \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 255

$$\int (c+dx)^4 \left(a+bx^2\right)^{3/2} \, dx = \frac{3a(16b^2c^4-16abc^2d^2+a^2d^4)\,x\sqrt{a+bx^2}}{128b^2} + \frac{(16b^2c^4-16abc^2d^2+a^2d^4)\,x(a+bx^2)^{3/2}}{64b^2} + \frac{11cd(c+dx)^2\left(a+bx^2\right)^{5/2}}{56b} + \frac{d(c+dx)^3\left(a+bx^2\right)^{5/2}}{8b} + \frac{d(4c(67bc^2-32ad^2)+5d(26bc^2-7ad^2)\,x)\left(a+bx^2\right)^{5/2}}{560b^2} + \frac{3a^2(16b^2c^4-16abc^2d^2+a^2d^4)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
3/128*a*(a^2*d^4-16*a*b*c^2*d^2+16*b^2*c^4)*x*(b*x^2+a)^(1/2)/b^2+1/64*(a^
2*d^4-16*a*b*c^2*d^2+16*b^2*c^4)*x*(b*x^2+a)^(3/2)/b^2+11/56*c*d*(d*x+c)^2
*(b*x^2+a)^(5/2)/b+1/8*d*(d*x+c)^3*(b*x^2+a)^(5/2)/b+1/560*d*(4*c*(-32*a*d
^2+67*b*c^2)+5*d*(-7*a*d^2+26*b*c^2)*x)*(b*x^2+a)^(5/2)/b^2+3/128*a^2*(a^2
*d^4-16*a*b*c^2*d^2+16*b^2*c^4)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.90

$$\int (c+dx)^4 (a+bx^2)^{3/2}\, dx = \frac{\sqrt{b}\sqrt{a+bx^2}(-a^3d^3(1024c+105dx)+2a^2bd(1792c^3+840c^2dx+256cd^2x^2+35d^3x^3)+16}{}$$

input
```
Integrate[(c + d*x)^4*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-(a^3*d^3*(1024*c + 105*d*x)) + 2*a^2*b*d*(1792*
c^3 + 840*c^2*d*x + 256*c*d^2*x^2 + 35*d^3*x^3) + 16*b^3*x^3*(70*c^4 + 224
*c^3*d*x + 280*c^2*d^2*x^2 + 160*c*d^3*x^3 + 35*d^4*x^4) + 8*a*b^2*x*(350*
c^4 + 896*c^3*d*x + 980*c^2*d^2*x^2 + 512*c*d^3*x^3 + 105*d^4*x^4)) - 105*
a^2*(16*b^2*c^4 - 16*a*b*c^2*d^2 + a^2*d^4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*
x^2]])/(4480*b^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {497, 687, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{3/2} (c+dx)^4 \, dx$$

$$\Big\downarrow \; 497$$

$$\frac{\int (c+dx)^2 (8bc^2+11bdxc-3ad^2)(bx^2+a)^{3/2}dx}{8b} + \frac{d(a+bx^2)^{5/2}(c+dx)^3}{8b}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{\int b(c+dx)\left(c\left(56bc^2-43ad^2\right)+3d\left(26bc^2-7ad^2\right)x\right)\left(bx^2+a\right)^{3/2}dx}{7b}+\frac{11}{7}cd\left(a+bx^2\right)^{5/2}(c+dx)^2}{8b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 27

$$\frac{\frac{1}{7}\int (c+dx)\left(c\left(56bc^2-43ad^2\right)+3d\left(26bc^2-7ad^2\right)x\right)\left(bx^2+a\right)^{3/2}dx+\frac{11}{7}cd\left(a+bx^2\right)^{5/2}(c+dx)^2}{8b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 676

$$\frac{\frac{1}{7}\left(\frac{7\left(a^2d^4-16abc^2d^2+16b^2c^4\right)\int \left(bx^2+a\right)^{3/2}dx}{2b}+\frac{2cd\left(a+bx^2\right)^{5/2}\left(67bc^2-32ad^2\right)}{5b}+\frac{d^2x\left(a+bx^2\right)^{5/2}\left(26bc^2-7ad^2\right)}{2b}\right)+\frac{11}{7}cd\left(a+bx^2\right)^{5/}}{8b}$$
$$\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 211

$$\frac{\frac{1}{7}\left(\frac{7\left(a^2d^4-16abc^2d^2+16b^2c^4\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{2cd\left(a+bx^2\right)^{5/2}\left(67bc^2-32ad^2\right)}{5b}+\frac{d^2x\left(a+bx^2\right)^{5/2}\left(26bc^2-7ad^2\right)}{2b}\right)+}{8b}$$
$$\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 211

$$\frac{\frac{1}{7}\left(\frac{7\left(a^2d^4-16abc^2d^2+16b^2c^4\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{2cd\left(a+bx^2\right)^{5/2}\left(67bc^2-32ad^2\right)}{5b}+\frac{d^2x\left(a+bx^2\right)^{5/2}}{2b}\right.}{8b}$$
$$\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 224

$$\frac{\frac{1}{7}\left(\frac{7\left(a^2d^4-16abc^2d^2+16b^2c^4\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{2cd\left(a+bx^2\right)^{5/2}\left(67bc^2-32ad^2\right)}{5b}+\frac{d^2x(a}{}\right.}{8b}$$

$$\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 219

$$\frac{\frac{1}{7}\left(\frac{7\left(\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(a^2d^4-16abc^2d^2+16b^2c^4\right)}{2b}+\frac{2cd\left(a+bx^2\right)^{5/2}\left(67bc^2-32ad^2\right)}{5b}+\frac{d^2x(a+}{}\right.}{8b}$$

$$\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

input
```
Int[(c + d*x)^4*(a + b*x^2)^(3/2),x]
```

output
```
(d*(c + d*x)^3*(a + b*x^2)^(5/2))/(8*b) + ((11*c*d*(c + d*x)^2*(a + b*x^2)
^(5/2))/7 + ((2*c*d*(67*b*c^2 - 32*a*d^2)*(a + b*x^2)^(5/2))/(5*b) + (d^2*
(26*b*c^2 - 7*a*d^2)*x*(a + b*x^2)^(5/2))/(2*b) + (7*(16*b^2*c^4 - 16*a*b*
c^2*d^2 + a^2*d^4)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2]])/2
+ (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(2*b))/7)/(8*
b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{(-560b^3d^4x^7-2560b^3c\,d^3x^6-840a\,b^2d^4x^5-4480b^3c^2d^2x^5-4096a\,b^2c\,d^3x^4-3584b^3c^3d\,x^4-70a^2d^4b\,x^3-7840b^2c^2d^2a\,x^3-1120b\cdots}{4480b^2}$
default	$c^4\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d^4\left(\frac{x^3(b\,x^2+a)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\cdots}{}\cdots\right)}{\cdots}\right)}{8\cdots}\cdots\right.$

input
```
int((d*x+c)^4*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4480/b^2*(-560*b^3*d^4*x^7-2560*b^3*c*d^3*x^6-840*a*b^2*d^4*x^5-4480*b^
3*c^2*d^2*x^5-4096*a*b^2*c*d^3*x^4-3584*b^3*c^3*d*x^4-70*a^2*b*d^4*x^3-784
0*a*b^2*c^2*d^2*x^3-1120*b^3*c^4*x^3-512*a^2*b*c*d^3*x^2-7168*a*b^2*c^3*d*
x^2+105*a^3*d^4*x-1680*a^2*b*c^2*d^2*x-2800*a*b^2*c^4*x+1024*a^3*c*d^3-358
4*a^2*b*c^3*d)*(b*x^2+a)^(1/2)+3/128*a^2*(a^2*d^4-16*a*b*c^2*d^2+16*b^2*c^
4)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.13

$$\int (c+dx)^4 \left(a+bx^2\right)^{3/2} dx = \left[\frac{105\left(16a^2b^2c^4-16a^3bc^2d^2+a^4d^4\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(560b^4d^4x^7\right.}{} \right.$$

$$-\frac{105\left(16a^2b^2c^4-16a^3bc^2d^2+a^4d^4\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(560b^4d^4x^7+2560b^4cd^3x^6+3584a^2b^2c^3d-\right.}{}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8960*(105*(16*a^2*b^2*c^4 - 16*a^3*b*c^2*d^2 + a^4*d^4)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(560*b^4*d^4*x^7 + 2560*b^4*c
*d^3*x^6 + 3584*a^2*b^2*c^3*d - 1024*a^3*b*c*d^3 + 280*(16*b^4*c^2*d^2 + 3
*a*b^3*d^4)*x^5 + 512*(7*b^4*c^3*d + 8*a*b^3*c*d^3)*x^4 + 70*(16*b^4*c^4 +
 112*a*b^3*c^2*d^2 + a^2*b^2*d^4)*x^3 + 512*(14*a*b^3*c^3*d + a^2*b^2*c*d^
3)*x^2 + 35*(80*a*b^3*c^4 + 48*a^2*b^2*c^2*d^2 - 3*a^3*b*d^4)*x)*sqrt(b*x^
2 + a))/b^3, -1/4480*(105*(16*a^2*b^2*c^4 - 16*a^3*b*c^2*d^2 + a^4*d^4)*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (560*b^4*d^4*x^7 + 2560*b^4*c*
d^3*x^6 + 3584*a^2*b^2*c^3*d - 1024*a^3*b*c*d^3 + 280*(16*b^4*c^2*d^2 + 3*
a*b^3*d^4)*x^5 + 512*(7*b^4*c^3*d + 8*a*b^3*c*d^3)*x^4 + 70*(16*b^4*c^4 +
112*a*b^3*c^2*d^2 + a^2*b^2*d^4)*x^3 + 512*(14*a*b^3*c^3*d + a^2*b^2*c*d^3
)*x^2 + 35*(80*a*b^3*c^4 + 48*a^2*b^2*c^2*d^2 - 3*a^3*b*d^4)*x)*sqrt(b*x^2
 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(248) = 496$.

Time = 0.60 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.03

$$\int (c+dx)^4 \left(a+bx^2\right)^{3/2}\,dx = \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{4bcd^3x^6}{7}+\frac{bd^4x^7}{8}+\frac{x^5\cdot\left(\frac{9abd^4}{8}+6b^2c^2d^2\right)}{6b}+\frac{x^4\cdot\left(\frac{32abcd^3}{7}+4b^2c^3d\right)}{5b}+\frac{x^3\left(a^2d^4+12abc^2d^2-\frac{5a\left(9a\cdots\right.}{}\right.}{4b}\cdots\right. \\ a^{\frac{3}{2}}\left(\begin{cases} c^4x & \text{for } d=0 \\ \frac{(c+dx)^5}{5d} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((d*x+c)**4*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(4*b*c*d**3*x**6/7 + b*d**4*x**7/8 + x**5*(9*a
*b*d**4/8 + 6*b**2*c**2*d**2)/(6*b) + x**4*(32*a*b*c*d**3/7 + 4*b**2*c**3*
d)/(5*b) + x**3*(a**2*d**4 + 12*a*b*c**2*d**2 - 5*a*(9*a*b*d**4/8 + 6*b**2
*c**2*d**2)/(6*b) + b**2*c**4)/(4*b) + x**2*(4*a**2*c*d**3 + 8*a*b*c**3*d
- 4*a*(32*a*b*c*d**3/7 + 4*b**2*c**3*d)/(5*b))/(3*b) + x*(6*a**2*c**2*d**2
 + 2*a*b*c**4 - 3*a*(a**2*d**4 + 12*a*b*c**2*d**2 - 5*a*(9*a*b*d**4/8 + 6*
b**2*c**2*d**2)/(6*b) + b**2*c**4)/(4*b))/(2*b) + (4*a**2*c**3*d - 2*a*(4*
a**2*c*d**3 + 8*a*b*c**3*d - 4*a*(32*a*b*c*d**3/7 + 4*b**2*c**3*d)/(5*b))/
(3*b))/b) + (a**2*c**4 - a*(6*a**2*c**2*d**2 + 2*a*b*c**4 - 3*a*(a**2*d**4
 + 12*a*b*c**2*d**2 - 5*a*(9*a*b*d**4/8 + 6*b**2*c**2*d**2)/(6*b) + b**2*c
**4)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*Piece
wise((c**4*x, Eq(d, 0)), ((c + d*x)**5/(5*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.18

$$\int (c+dx)^4 \left(a+bx^2\right)^{3/2} dx = \frac{(bx^2+a)^{\frac{5}{2}} d^4 x^3}{8\,b} + \frac{4\,(bx^2+a)^{\frac{5}{2}} c d^3 x^2}{7\,b} + \frac{1}{4}\,(bx^2+a)^{\frac{3}{2}} c^4 x + \frac{3}{8}\,\sqrt{bx^2+a}\,a c^4 x + \frac{(bx^2+a)^{\frac{5}{2}} c^2 d^2 x}{b} - \frac{(bx^2+a)^{\frac{3}{2}} a c^2 d^2 x}{4\,b} - \frac{3\,\sqrt{bx^2+a}\,a^2 c^2 d^2 x}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}} a d^4 x}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}} a^2 d^4 x}{64\,b^2} + \frac{3\,\sqrt{bx^2+a}\,a^3 d^4 x}{128\,b^2} + \frac{3\,a^2 c^4 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \frac{3\,a^3 c^2 d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{3\,a^4 d^4 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} + \frac{4\,(bx^2+a)^{\frac{5}{2}} c^3 d}{5\,b} - \frac{8\,(bx^2+a)^{\frac{5}{2}} a c d^3}{35\,b^2}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*d^4*x^3/b + 4/7*(b*x^2 + a)^(5/2)*c*d^3*x^2/b + 1/4*
(b*x^2 + a)^(3/2)*c^4*x + 3/8*sqrt(b*x^2 + a)*a*c^4*x + (b*x^2 + a)^(5/2)*
c^2*d^2*x/b - 1/4*(b*x^2 + a)^(3/2)*a*c^2*d^2*x/b - 3/8*sqrt(b*x^2 + a)*a^
2*c^2*d^2*x/b - 1/16*(b*x^2 + a)^(5/2)*a*d^4*x/b^2 + 1/64*(b*x^2 + a)^(3/2
)*a^2*d^4*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*d^4*x/b^2 + 3/8*a^2*c^4*arcsin
h(b*x/sqrt(a*b))/sqrt(b) - 3/8*a^3*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2)
+ 3/128*a^4*d^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 4/5*(b*x^2 + a)^(5/2)*c^3
*d/b - 8/35*(b*x^2 + a)^(5/2)*a*c*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int (c+dx)^4 \left(a+bx^2\right)^{3/2} dx = \frac{1}{4480}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\left(2\left(7\,bd^4 x + 32\,bcd^3\right)x + \frac{7\,(16\,b^7 c^2 d^2 + 3\,ab^6 d^4)}{b^6}\right)x + \frac{64\,(7\,b^7 c\right.\right.\right.\right.\right.\right. - \frac{3\,(16\,a^2 b^2 c^4 - 16\,a^3 b c^2 d^2 + a^4 d^4)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/4480*sqrt(b*x^2 + a)*((2*((4*(5*(2*(7*b*d^4*x + 32*b*c*d^3)*x + 7*(16*b^
7*c^2*d^2 + 3*a*b^6*d^4)/b^6)*x + 64*(7*b^7*c^3*d + 8*a*b^6*c*d^3)/b^6)*x
+ 35*(16*b^7*c^4 + 112*a*b^6*c^2*d^2 + a^2*b^5*d^4)/b^6)*x + 256*(14*a*b^6
*c^3*d + a^2*b^5*c*d^3)/b^6)*x + 35*(80*a*b^6*c^4 + 48*a^2*b^5*c^2*d^2 - 3
*a^3*b^4*d^4)/b^6)*x + 512*(7*a^2*b^5*c^3*d - 2*a^3*b^4*c*d^3)/b^6) - 3/12
8*(16*a^2*b^2*c^4 - 16*a^3*b*c^2*d^2 + a^4*d^4)*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \left(a+bx^2\right)^{3/2} \, dx = \int \left(b\,x^2+a\right)^{3/2}\,{\left(c+d\,x\right)}^4 \,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)^4,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \left(a+bx^2\right)^{3/2} \, dx = \int (dx+c)^4 \left(b\,x^2+a\right)^{\frac{3}{2}} dx$$

input
```
int((d*x+c)^4*(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^4*(b*x^2+a)^(3/2),x)
```

3.243 $\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 180

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx = \frac{3ac(2bc^2-ad^2)\,x\sqrt{a+bx^2}}{16b} + \frac{c(2bc^2-ad^2)\,x(a+bx^2)^{3/2}}{8b} + \frac{d(c+dx)^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{d(4(8bc^2-ad^2)+15bcdx)\left(a+bx^2\right)^{5/2}}{70b^2} + \frac{3a^2c(2bc^2-ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output

```
3/16*a*c*(-a*d^2+2*b*c^2)*x*(b*x^2+a)^(1/2)/b+1/8*c*(-a*d^2+2*b*c^2)*x*(b*
x^2+a)^(3/2)/b+1/7*d*(d*x+c)^2*(b*x^2+a)^(5/2)/b+1/70*d*(15*b*c*d*x-4*a*d^
2+32*b*c^2)*(b*x^2+a)^(5/2)/b^2+3/16*a^2*c*(-a*d^2+2*b*c^2)*arctanh(b^(1/2
)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96

$$\int (c+dx)^3 (a+bx^2)^{3/2} \, dx = \frac{\sqrt{a+bx^2}(-32a^3d^3 + a^2bd(336c^2 + 105cdx + 16d^2x^2) + 4b^3x^3(35c^3 + 84c^2dx + 70cd^2x^2 + 2$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-32*a^3*d^3 + a^2*b*d*(336*c^2 + 105*c*d*x + 16*d^2*x^2)
 + 4*b^3*x^3*(35*c^3 + 84*c^2*d*x + 70*c*d^2*x^2 + 20*d^3*x^3) + 2*a*b^2*x
*(175*c^3 + 336*c^2*d*x + 245*c*d^2*x^2 + 64*d^3*x^3)) + 105*a^2*Sqrt[b]*c
*(-2*b*c^2 + a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(560*b^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {497, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (c+dx)^3 \, dx$$

$$\Big\downarrow \text{497}$$

$$\frac{\int (c+dx)\left(7bc^2+9bdxc-2ad^2\right)\left(bx^2+a\right)^{3/2}dx}{7b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{7}{2}c\left(2bc^2-ad^2\right)\int \left(bx^2+a\right)^{3/2}dx+\frac{2d\left(a+bx^2\right)^{5/2}\left(8bc^2-ad^2\right)}{5b}+\frac{3}{2}cd^2x\left(a+bx^2\right)^{5/2}}{7b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$\Big\downarrow$ 211

$$\frac{\frac{7}{2}c(2bc^2-ad^2)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{2d(a+bx^2)^{5/2}(8bc^2-ad^2)}{5b}+\frac{3}{2}cd^2x(a+bx^2)^{5/2}}{7b}+\frac{d(a+bx^2)^{5/2}(c+dx)^2}{7b}$$

$\Big\downarrow$ 211

$$\frac{\frac{7}{2}c(2bc^2-ad^2)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{2d(a+bx^2)^{5/2}(8bc^2-ad^2)}{5b}+\frac{3}{2}cd^2x(a+bx}{7b}$$

$$\frac{d(a+bx^2)^{5/2}(c+dx)^2}{7b}$$

$\Big\downarrow$ 224

$$\frac{\frac{7}{2}c(2bc^2-ad^2)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{2d(a+bx^2)^{5/2}(8bc^2-ad^2)}{5b}+\frac{3}{2}cd^2x}{7b}$$

$$\frac{d(a+bx^2)^{5/2}(c+dx)^2}{7b}$$

$\Big\downarrow$ 219

$$\frac{\frac{7}{2}c\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)(2bc^2-ad^2)+\frac{2d(a+bx^2)^{5/2}(8bc^2-ad^2)}{5b}+\frac{3}{2}cd^2x(}{7b}$$

$$\frac{d(a+bx^2)^{5/2}(c+dx)^2}{7b}$$

input
```
Int[(c + d*x)^3*(a + b*x^2)^(3/2),x]
```

output
```
(d*(c + d*x)^2*(a + b*x^2)^(5/2))/(7*b) + ((2*d*(8*b*c^2 - a*d^2)*(a + b*x
^2)^(5/2))/(5*b) + (3*c*d^2*x*(a + b*x^2)^(5/2))/2 + (7*c*(2*b*c^2 - a*d^2
)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/2)/(7*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{\left(-80b^3d^3x^6-280b^3c\,d^2x^5-128a\,b^2d^3x^4-336b^3c^2d\,x^4-490a\,b^2c\,d^2x^3-140b^3c^3x^3-16a^2b\,d^3x^2-672a\,b^2c^2d\,x^2-105a^2bc\,d^2x-35\right.}{560b^2}$
default	$c^3\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d^3\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(b\,x^2+a\right)^{\frac{5}{2}}}{35b^2}\right)+3c\,d^2\left(\frac{x\left(b\,x^2+\right.}{6b}\right.$

input
```
int((d*x+c)^3*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/560/b^2*(-80*b^3*d^3*x^6-280*b^3*c*d^2*x^5-128*a*b^2*d^3*x^4-336*b^3*c^
2*d*x^4-490*a*b^2*c*d^2*x^3-140*b^3*c^3*x^3-16*a^2*b*d^3*x^2-672*a*b^2*c^2
*d*x^2-105*a^2*b*c*d^2*x-350*a*b^2*c^3*x+32*a^3*d^3-336*a^2*b*c^2*d)*(b*x^
2+a)^(1/2)-3/16*a^2*c/b^(3/2)*(a*d^2-2*b*c^2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.23

$$\int (c+dx)^3\left(a+bx^2\right)^{3/2}\,dx=\left[-\frac{105\left(2a^2bc^3-a^3cd^2\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(80b^3d^3x^6+280b^3cd^2x\right.}{}\right.$$

$$-\frac{105\left(2a^2bc^3-a^3cd^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(80b^3d^3x^6+280b^3cd^2x^5+336a^2bc^2d-32a^3d^3+16\left(21b\right.\right.}{5}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/1120*(105*(2*a^2*b*c^3 - a^3*c*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^
2 + a)*sqrt(b)*x - a) - 2*(80*b^3*d^3*x^6 + 280*b^3*c*d^2*x^5 + 336*a^2*b*
c^2*d - 32*a^3*d^3 + 16*(21*b^3*c^2*d + 8*a*b^2*d^3)*x^4 + 70*(2*b^3*c^3 +
 7*a*b^2*c*d^2)*x^3 + 16*(42*a*b^2*c^2*d + a^2*b*d^3)*x^2 + 35*(10*a*b^2*c
^3 + 3*a^2*b*c*d^2)*x)*sqrt(b*x^2 + a))/b^2, -1/560*(105*(2*a^2*b*c^3 - a^
3*c*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (80*b^3*d^3*x^6 + 2
80*b^3*c*d^2*x^5 + 336*a^2*b*c^2*d - 32*a^3*d^3 + 16*(21*b^3*c^2*d + 8*a*b
^2*d^3)*x^4 + 70*(2*b^3*c^3 + 7*a*b^2*c*d^2)*x^3 + 16*(42*a*b^2*c^2*d + a^
2*b*d^3)*x^2 + 35*(10*a*b^2*c^3 + 3*a^2*b*c*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(165) = 330$.

Time = 0.54 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.03

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{bcd^2x^5}{2}+\frac{bd^3x^6}{7}+\frac{x^4\cdot\left(\frac{8abd^3}{7}+3b^2c^2d\right)}{5b}+\frac{x^3\cdot\left(\frac{7abcd^2}{2}+b^2c^3\right)}{4b}+\frac{x^2\left(a^2d^3+6abc^2d-\frac{4a\left(\frac{8abd^3}{7}+3b^2c^2\cdots\right)}{5b}\cdots\right)}{3b}\cdots\right) & \\ a^{\frac{3}{2}}\left(\begin{cases} c^3x & \text{for}\ d=0 \\ \frac{(c+dx)^4}{4d} & \text{otherwise} \end{cases}\right) & \end{cases}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b*c*d**2*x**5/2 + b*d**3*x**6/7 + x**4*(8*a*b
*d**3/7 + 3*b**2*c**2*d)/(5*b) + x**3*(7*a*b*c*d**2/2 + b**2*c**3)/(4*b) +
 x**2*(a**2*d**3 + 6*a*b*c**2*d - 4*a*(8*a*b*d**3/7 + 3*b**2*c**2*d)/(5*b)
)/(3*b) + x*(3*a**2*c*d**2 + 2*a*b*c**3 - 3*a*(7*a*b*c*d**2/2 + b**2*c**3)
/(4*b))/(2*b) + (3*a**2*c**2*d - 2*a*(a**2*d**3 + 6*a*b*c**2*d - 4*a*(8*a*
b*d**3/7 + 3*b**2*c**2*d)/(5*b))/(3*b))/b) + (a**2*c**3 - a*(3*a**2*c*d**2
 + 2*a*b*c**3 - 3*a*(7*a*b*c*d**2/2 + b**2*c**3)/(4*b))/(2*b))*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt
(b*x**2), True)), Ne(b, 0)), (a**(3/2)*Piecewise((c**3*x, Eq(d, 0)), ((c +
 d*x)**4/(4*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.06

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} dx = \frac{(bx^2+a)^{\frac{5}{2}} d^3 x^2}{7\,b} + \frac{1}{4} (bx^2+a)^{\frac{3}{2}} c^3 x + \frac{3}{8} \sqrt{bx^2+a}\, a c^3 x + \frac{(bx^2+a)^{\frac{5}{2}} c d^2 x}{2\,b} - \frac{(bx^2+a)^{\frac{3}{2}} a c d^2 x}{8\,b} - \frac{3\sqrt{bx^2+a}\, a^2 c d^2 x}{16\,b} + \frac{3\,a^2 c^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} - \frac{3\,a^3 c d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,(bx^2+a)^{\frac{5}{2}} c^2 d}{5\,b} - \frac{2\,(bx^2+a)^{\frac{5}{2}} a d^3}{35\,b^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*d^3*x^2/b + 1/4*(b*x^2 + a)^(3/2)*c^3*x + 3/8*sqrt(b
*x^2 + a)*a*c^3*x + 1/2*(b*x^2 + a)^(5/2)*c*d^2*x/b - 1/8*(b*x^2 + a)^(3/2
)*a*c*d^2*x/b - 3/16*sqrt(b*x^2 + a)*a^2*c*d^2*x/b + 3/8*a^2*c^3*arcsinh(b
*x/sqrt(a*b))/sqrt(b) - 3/16*a^3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/
5*(b*x^2 + a)^(5/2)*c^2*d/b - 2/35*(b*x^2 + a)^(5/2)*a*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

$$\int (c+dx)^3 (a+bx^2)^{3/2}\,dx = \frac{1}{560}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\left(2\,bd^3x+7\,bcd^2\right)x+\frac{2\left(21\,b^6c^2d+8\,ab^5d^3\right)}{b^5}\right)x+\frac{35\left(2\,b^6c^3+7\,a\right.}{b^5}\right.\right.\right.\right. - \frac{3\left(2\,a^2bc^3-a^3cd^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/560*sqrt(b*x^2 + a)*((2*((4*(5*(2*b*d^3*x + 7*b*c*d^2)*x + 2*(21*b^6*c^2
*d + 8*a*b^5*d^3)/b^5)*x + 35*(2*b^6*c^3 + 7*a*b^5*c*d^2)/b^5)*x + 8*(42*a
*b^5*c^2*d + a^2*b^4*d^3)/b^5)*x + 35*(10*a*b^5*c^3 + 3*a^2*b^4*c*d^2)/b^5
)*x + 16*(21*a^2*b^4*c^2*d - 2*a^3*b^3*d^3)/b^5) - 3/16*(2*a^2*b*c^3 - a^3
*c*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2}\,dx = \int \left(b\,x^2+a\right)^{3/2}\left(c+d\,x\right)^3\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.65

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} dx = \frac{-32\sqrt{b\,x^2+a}\,a^3d^3+336\sqrt{b\,x^2+a}\,a^2b\,c^2d+105\sqrt{b\,x^2+a}\,a^2bc\,d^2x+16\sqrt{b\,x^2+a}\,a^2b\,d^3x^2}{}$$

input
```
int((d*x+c)^3*(b*x^2+a)^(3/2),x)
```

output
```
( - 32*sqrt(a + b*x**2)*a**3*d**3 + 336*sqrt(a + b*x**2)*a**2*b*c**2*d + 1
05*sqrt(a + b*x**2)*a**2*b*c*d**2*x + 16*sqrt(a + b*x**2)*a**2*b*d**3*x**2
 + 350*sqrt(a + b*x**2)*a*b**2*c**3*x + 672*sqrt(a + b*x**2)*a*b**2*c**2*d
*x**2 + 490*sqrt(a + b*x**2)*a*b**2*c*d**2*x**3 + 128*sqrt(a + b*x**2)*a*b
**2*d**3*x**4 + 140*sqrt(a + b*x**2)*b**3*c**3*x**3 + 336*sqrt(a + b*x**2)
*b**3*c**2*d*x**4 + 280*sqrt(a + b*x**2)*b**3*c*d**2*x**5 + 80*sqrt(a + b*
x**2)*b**3*d**3*x**6 - 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a**3*c*d**2 + 210*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*a**2*b*c**3)/(560*b**2)
```

3.244 $\int (c+dx)^2 (a+bx^2)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int (c+dx)^2 (a+bx^2)^{3/2} dx = \frac{a(6bc^2-ad^2)x\sqrt{a+bx^2}}{16b} + \frac{(6bc^2-ad^2)x(a+bx^2)^{3/2}}{24b} + \frac{d(12c+5dx)(a+bx^2)^{5/2}}{30b} + \frac{a^2(6bc^2-ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(-a*d^2+6*b*c^2)*x*(b*x^2+a)^(1/2)/b+1/24*(-a*d^2+6*b*c^2)*x*(b*x^2
+a)^(3/2)/b+1/30*d*(5*d*x+12*c)*(b*x^2+a)^(5/2)/b+1/16*a^2*(-a*d^2+6*b*c^2
)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int (c+dx)^2 (a+bx^2)^{3/2} dx = \frac{\sqrt{b}\sqrt{a+bx^2}(3a^2d(32c+5dx)+4b^2x^3(15c^2+24cdx+10d^2x^2)+2abx(75c^2+96cdx+35d}{240b^{3/2}}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(3*a^2*d*(32*c + 5*d*x) + 4*b^2*x^3*(15*c^2 + 24*
c*d*x + 10*d^2*x^2) + 2*a*b*x*(75*c^2 + 96*c*d*x + 35*d^2*x^2)) + 15*a^2*(
-6*b*c^2 + a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(240*b^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {497, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (c+dx)^2 \, dx$$

$$\Big\downarrow \text{497}$$

$$\frac{\int \left(6bc^2+7bdxc-ad^2\right)\left(bx^2+a\right)^{3/2}dx}{6b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)}{6b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(6bc^2-ad^2\right)\int \left(bx^2+a\right)^{3/2}dx+\frac{7}{5}cd\left(a+bx^2\right)^{5/2}}{6b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\left(6bc^2-ad^2\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{7}{5}cd\left(a+bx^2\right)^{5/2}}{6b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\left(6bc^2-ad^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{7}{5}cd\left(a+bx^2\right)^{5/2}}{6b}+\frac{d\left(a+bx^2\right)^{5/2}(c+dx)}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6bc^2-ad^2)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{7}{5}cd(a+bx^2)^{5/2}}{6b}+\frac{d(a+bx^2)^{5/2}(c+dx)}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)(6bc^2-ad^2)+\frac{7}{5}cd(a+bx^2)^{5/2}}{6b}+\frac{d(a+bx^2)^{5/2}(c+dx)}{6b}$$

input
```
Int[(c + d*x)^2*(a + b*x^2)^(3/2),x]
```

output
```
(d*(c + d*x)*(a + b*x^2)^(5/2))/(6*b) + ((7*c*d*(a + b*x^2)^(5/2))/5 + (6*
b*c^2 - a*d^2)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

method	result
risch	$\frac{(40b^2d^2x^5+96b^2cd\,x^4+70d^2x^3ab+60b^2c^2x^3+192abcd\,x^2+15a^2d^2x+150ab\,c^2x+96a^2cd)\sqrt{b\,x^2+a}}{240b}-\frac{a^2(a\,d^2-6b\,c^2)\ln\left(\sqrt{b}\,x+\sqrt{\right.}}{16b^{\frac{3}{2}}}$
default	$c^2\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d^2\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{\right.}}{}\right.}{4}\right.}{6b}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/240/b*(40*b^2*d^2*x^5+96*b^2*c*d*x^4+70*a*b*d^2*x^3+60*b^2*c^2*x^3+192*a
*b*c*d*x^2+15*a^2*d^2*x+150*a*b*c^2*x+96*a^2*c*d)*(b*x^2+a)^(1/2)-1/16*a^2
*(a*d^2-6*b*c^2)/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.16

$$\int (c+dx)^2 (a+bx^2)^{3/2}\, dx = \left[-\frac{15\,(6\,a^2bc^2 - a^3d^2)\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) - 2\,(40\,b^3d^2x^5 + 96\,b^3cdx^4 + \ldots}{480\,b^2} - \frac{15\,(6\,a^2bc^2 - a^3d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (40\,b^3d^2x^5 + 96\,b^3cdx^4 + 192\,ab^2cdx^2 + 96\,a^2bcd + 10\,(6\,b^3c^2 \ldots}{240\,b^2}\right.$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/480*(15*(6*a^2*b*c^2 - a^3*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(40*b^3*d^2*x^5 + 96*b^3*c*d*x^4 + 192*a*b^2*c*d*x^2
 + 96*a^2*b*c*d + 10*(6*b^3*c^2 + 7*a*b^2*d^2)*x^3 + 15*(10*a*b^2*c^2 + a^
2*b*d^2)*x)*sqrt(b*x^2 + a))/b^2, -1/240*(15*(6*a^2*b*c^2 - a^3*d^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (40*b^3*d^2*x^5 + 96*b^3*c*d*x^4
+ 192*a*b^2*c*d*x^2 + 96*a^2*b*c*d + 10*(6*b^3*c^2 + 7*a*b^2*d^2)*x^3 + 15
*(10*a*b^2*c^2 + a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.72

$$\int (c+dx)^2 (a+bx^2)^{3/2}\, dx = \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{2a^2cd}{5b} + \frac{4acdx^2}{5} + \frac{2bcdx^4}{5} + \frac{bd^2x^5}{6} + \frac{x^3\cdot\left(\frac{7abd^2}{6} + b^2c^2\right)}{4b} + \frac{x\left(a^2d^2 + 2abc^2 - \frac{3a\left(\frac{7abd^2}{6} + b^2c^2\right)}{4b}\right)}{2b} \ldots\right. \\ a^{\frac{3}{2}}\left(\begin{cases} c^2x & \text{for } d = 0 \\ \frac{(c+dx)^3}{3d} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(2*a**2*c*d/(5*b) + 4*a*c*d*x**2/5 + 2*b*c*d*x
**4/5 + b*d**2*x**5/6 + x**3*(7*a*b*d**2/6 + b**2*c**2)/(4*b) + x*(a**2*d*
*2 + 2*a*b*c**2 - 3*a*(7*a*b*d**2/6 + b**2*c**2)/(4*b))/(2*b)) + (a**2*c**
2 - a*(a**2*d**2 + 2*a*b*c**2 - 3*a*(7*a*b*d**2/6 + b**2*c**2)/(4*b))/(2*b
))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)),
(x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*Piecewise((c**2*x, Eq
(d, 0)), ((c + d*x)**3/(3*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int (c+dx)^2\left(a+bx^2\right)^{3/2}\,dx=\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}c^2x+\frac{3}{8}\sqrt{bx^2+a}ac^2x+\frac{(bx^2+a)^{\frac{5}{2}}d^2x}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}ad^2x}{24\,b}-\frac{\sqrt{bx^2+a}a^2d^2x}{16\,b}+\frac{3\,a^2c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{a^3d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{2\,(bx^2+a)^{\frac{5}{2}}cd}{5\,b}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*c^2*x + 3/8*sqrt(b*x^2 + a)*a*c^2*x + 1/6*(b*x^2 + a
)^(5/2)*d^2*x/b - 1/24*(b*x^2 + a)^(3/2)*a*d^2*x/b - 1/16*sqrt(b*x^2 + a)*
a^2*d^2*x/b + 3/8*a^2*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/16*a^3*d^2*ar
csinh(b*x/sqrt(a*b))/b^(3/2) + 2/5*(b*x^2 + a)^(5/2)*c*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2}\,dx = \frac{1}{240}\left(\frac{96\,a^2cd}{b} + \left(2\left(96\,acd + \left(4\left(5\,bd^2x + 12\,bcd\right)x + \frac{5\left(6\,b^5c^2 + 7\,ab^4d^2\right)}{b^4}\right)x\right)x + \frac{15\,(10\,a\right.\right.$$
$$-\frac{\left(6\,a^2bc^2 - a^3d^2\right)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/240*(96*a^2*c*d/b + (2*(96*a*c*d + (4*(5*b*d^2*x + 12*b*c*d)*x + 5*(6*b^
5*c^2 + 7*a*b^4*d^2)/b^4)*x)*x + 15*(10*a*b^4*c^2 + a^2*b^3*d^2)/b^4)*x)*s
qrt(b*x^2 + a) - 1/16*(6*a^2*b*c^2 - a^3*d^2)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2}\,dx = \int \left(b\,x^2+a\right)^{3/2}\,(c+d\,x)^2\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.55

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} dx = \frac{96\sqrt{b\,x^2+a}\,a^2bcd + 15\sqrt{b\,x^2+a}\,a^2b\,d^2x + 150\sqrt{b\,x^2+a}\,a\,b^2c^2x + 192\sqrt{b\,x^2+a}\,a\,b^2cd\,x^2 +$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2),x)
```

output
```
(96*sqrt(a + b*x**2)*a**2*b*c*d + 15*sqrt(a + b*x**2)*a**2*b*d**2*x + 150*
sqrt(a + b*x**2)*a*b**2*c**2*x + 192*sqrt(a + b*x**2)*a*b**2*c*d*x**2 + 70
*sqrt(a + b*x**2)*a*b**2*d**2*x**3 + 60*sqrt(a + b*x**2)*b**3*c**2*x**3 +
96*sqrt(a + b*x**2)*b**3*c*d*x**4 + 40*sqrt(a + b*x**2)*b**3*d**2*x**5 - 1
5*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d**2 + 90*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c**2)/(240*b**2)
```

3.245 $\int (c+dx) \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int (c+dx) \left(a+bx^2\right)^{3/2} \, dx = \frac{3}{8} acx\sqrt{a+bx^2} + \frac{1}{4} cx\left(a+bx^2\right)^{3/2} + \frac{d\left(a+bx^2\right)^{5/2}}{5b} + \frac{3a^2 c \operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
3/8*a*c*x*(b*x^2+a)^(1/2)+1/4*c*x*(b*x^2+a)^(3/2)+1/5*d*(b*x^2+a)^(5/2)/b+
3/8*a^2*c*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (c+dx) \left(a+bx^2\right)^{3/2} \, dx = \frac{\sqrt{a+bx^2}\left(8a^2d+2b^2x^3(5c+4dx)+abx(25c+16dx)\right)-15a^2\sqrt{b}c\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{40b}$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^2*d + 2*b^2*x^3*(5*c + 4*d*x) + a*b*x*(25*c + 16*d*x
)) - 15*a^2*Sqrt[b]*c*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int (bx^2+a)^{3/2}dx+\frac{d(a+bx^2)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{d(a+bx^2)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{d(a+bx^2)^{5/2}}{5b}$$

$$\Big\downarrow \text{224}$$

$$c\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{d(a+bx^2)^{5/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$c\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{d(a+bx^2)^{5/2}}{5b}$$

input
```
Int[(c + d*x)*(a + b*x^2)^(3/2),x]
```

output
```
(d*(a + b*x^2)^(5/2))/(5*b) + c*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
 + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$c\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{d(b\,x^{2}+a)^{\frac{5}{2}}}{5b}$	70
risch	$\frac{(8b^{2}d\,x^{4}+10b^{2}c\,x^{3}+16abd\,x^{2}+25abcx+8a^{2}d)\sqrt{b\,x^{2}+a}}{40b}+\frac{3a^{2}c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$	80

input
```
int((d*x+c)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+1/5*d*(b*x^2+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{15\,a^2\sqrt{b}c\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^2dx^4+10\,b^2cx^3+16\,abdx^2+25\,abcx\right.}{80\,b}\right.$$
$$\left.-\frac{15\,a^2\sqrt{-b}c\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^2dx^4+10\,b^2cx^3+16\,abdx^2+25\,abcx+8\,a^2d)\sqrt{bx^2+a}}{40\,b}\right]$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/80*(15*a^2*sqrt(b)*c*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(8*b^2*d*x^4 + 10*b^2*c*x^3 + 16*a*b*d*x^2 + 25*a*b*c*x + 8*a^2*d)*sqrt(
b*x^2 + a))/b, -1/40*(15*a^2*sqrt(-b)*c*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (8*b^2*d*x^4 + 10*b^2*c*x^3 + 16*a*b*d*x^2 + 25*a*b*c*x + 8*a^2*d)*sqrt
(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}\frac{3a^2c\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8}+\sqrt{a+bx^2}\left(\frac{a^2d}{5b}+\frac{5acx}{8}+\frac{2adx^2}{5}+\frac{bcx^3}{4}+\frac{bdx^4}{5}\right) & \text{fo}\\ a^{\frac{3}{2}}\left(cx+\frac{dx^2}{2}\right) & \text{ot}\end{cases}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((3*a**2*c*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/8 + sqrt(a + b*x**2)*(a**2
*d/(5*b) + 5*a*c*x/8 + 2*a*d*x**2/5 + b*c*x**3/4 + b*d*x**4/5), Ne(b, 0)),
 (a**(3/2)*(c*x + d*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}cx + \frac{3}{8}\sqrt{bx^2+a}\,acx + \frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}d}{5\,b}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*c*x + 3/8*sqrt(b*x^2 + a)*a*c*x + 3/8*a^2*c*arcsinh(
b*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*d/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\,dx = -\frac{3\,a^2c\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\sqrt{b}} + \frac{1}{40}\sqrt{bx^2+a}\left(\frac{8\,a^2d}{b}+(25\,ac+2\,(8\,ad+(4\,bdx+5\,bc)x)x)x\right)$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3/8*a^2*c*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^
2 + a)*(8*a^2*d/b + (25*a*c + 2*(8*a*d + (4*b*d*x + 5*b*c)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int (c+dx) \left(a+bx^2\right)^{3/2} \, dx = \frac{d\,(b\,x^2+a)^{5/2}}{5\,b} + \frac{c\,x\,(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x),x)
```

output
```
(d*(a + b*x^2)^(5/2))/(5*b) + (c*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2]
, 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int (c+dx) \left(a+bx^2\right)^{3/2} \, dx = \frac{8\sqrt{b\,x^2+a}\,a^2d+25\sqrt{b\,x^2+a}\,abcx+16\sqrt{b\,x^2+a}\,abd\,x^2+10\sqrt{b\,x^2+a}\,b^2c\,x^3+8\sqrt{b\,x^2+a}}{40b}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2),x)
```

output
```
(8*sqrt(a + b*x**2)*a**2*d + 25*sqrt(a + b*x**2)*a*b*c*x + 16*sqrt(a + b*x
**2)*a*b*d*x**2 + 10*sqrt(a + b*x**2)*b**2*c*x**3 + 8*sqrt(a + b*x**2)*b**
2*d*x**4 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c)/
(40*b)
```

3.246 $\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx = \frac{(2(bc^2+ad^2)-bcdx)\sqrt{a+bx^2}}{2d^3}+\frac{(a+bx^2)^{3/2}}{3d} - \frac{\sqrt{b}c(2bc^2+3ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^4} - \frac{(bc^2+ad^2)^{3/2}\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4}$$

output
```
1/2*(-b*c*d*x+2*a*d^2+2*b*c^2)*(b*x^2+a)^(1/2)/d^3+1/3*(b*x^2+a)^(3/2)/d-1
/2*b^(1/2)*c*(3*a*d^2+2*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^4-(a*d
^2+b*c^2)^(3/2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/
d^4
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}(6bc^2+8ad^2-3bcdx+2bd^2x^2)-12(-bc^2-ad^2)^{3/2}\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a-}}{\sqrt{-bc^2-ad^2}}\right.}{6d^4}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x),x]
```

output
```
(d*Sqrt[a + b*x^2]*(6*b*c^2 + 8*a*d^2 - 3*b*c*d*x + 2*b*d^2*x^2) - 12*(-(b
*c^2) - a*d^2)^(3/2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-
(b*c^2) - a*d^2]] + 3*Sqrt[b]*c*(2*b*c^2 + 3*a*d^2)*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/(6*d^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {493, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{c+dx}\,dx$$

$$\Big\downarrow \text{493}$$

$$\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{c+dx}dx}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{\int \frac{b\left(ad\left(bc^2+2ad^2\right)-bc\left(2bc^2+3ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{ad\left(bc^2+2ad^2\right)-bc\left(2bc^2+3ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(3ad^2+2bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(3ad^2+2bc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2+2bc^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{-\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2+2bc^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{-\frac{2\left(ad^2+bc^2\right)^{3/2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2+2bc^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{d}+\frac{\left(a+bx^2\right)^{3/2}}{3d}$$

input

```
Int[(a + b*x^2)^(3/2)/(c + d*x),x]
```

output

```
(a + b*x^2)^(3/2)/(3*d) + (((2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])
/(2*d^2) + (-((Sqrt[b]*c*(2*b*c^2 + 3*a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/d) - (2*(b*c^2 + a*d^2)^(3/2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 +
 a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.50

method	result
risch	$\frac{(2b x^2 d^2-3bcdx+8a d^2+6b c^2)\sqrt{b x^2+a}}{6d^3}-\frac{\frac{2(a^2d^4+2bc^2d^2a+b^2c^4)\ln\left(\frac{\frac{2a d^2+2bc^2}{d^2}-\frac{2bc(x+\frac{c}{d})}{d}+2\sqrt{\frac{a d^2+b c^2}{d^2}}\sqrt{b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}\cdots}}{x+\frac{c}{d}}\right)}{d^2\sqrt{\frac{a d^2+b c^2}{d^2}}}\cdots}{2d^3}$
default	$\frac{\left(b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}+\frac{a d^2+b c^2}{d^2}\right)^{\frac{3}{2}}}{3}-\frac{bc\left(\frac{\left(2b(x+\frac{c}{d})-\frac{2bc}{d}\right)\sqrt{b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}+\frac{a d^2+b c^2}{d^2}}}{4b}+\frac{\left(\frac{4b(a d^2+b c^2)}{d^2}-\frac{4b^2c^2}{d^2}\right)\ln\left(\frac{-\frac{bc}{d}+b\cdots}{\sqrt{\cdots}}\right)\cdots}{\cdots}\right)}{d}$

input
```
int((b*x^2+a)^(3/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*b*d^2*x^2-3*b*c*d*x+8*a*d^2+6*b*c^2)*(b*x^2+a)^(1/2)/d^3-1/2/d^3*(2
*(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^
2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+b^(1/2)*c*(3*a*d^2+2*b*c^2)
/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 774, normalized size of antiderivative = 4.87

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/12*(3*(2*b*c^3 + 3*a*c*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 6*(b*c^2 + a*d^2)^(3/2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*
d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt
(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(2*b*d^3*x^2 - 3*b*c*d^2*x + 6
*b*c^2*d + 8*a*d^3)*sqrt(b*x^2 + a))/d^4, 1/6*(3*(2*b*c^3 + 3*a*c*d^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + 3*(b*c^2 + a*d^2)^(3/2)*log((2*
a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2
 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + (2*b
*d^3*x^2 - 3*b*c*d^2*x + 6*b*c^2*d + 8*a*d^3)*sqrt(b*x^2 + a))/d^4, -1/12*
(12*(b*c^2 + a*d^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*
x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) -
3*(2*b*c^3 + 3*a*c*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(2*b*d^3*x^2 - 3*b*c*d^2*x + 6*b*c^2*d + 8*a*d^3)*sqrt(b*x^2 + a
))/d^4, -1/6*(6*(b*c^2 + a*d^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 -
a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d
^2)*x^2)) - 3*(2*b*c^3 + 3*a*c*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) - (2*b*d^3*x^2 - 3*b*c*d^2*x + 6*b*c^2*d + 8*a*d^3)*sqrt(b*x^2 + a))
/d^4]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx = -\frac{\sqrt{bx^2+a}bcx}{2\,d^2} - \frac{b^{\frac{3}{2}}c^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4} - \frac{3\,a\sqrt{b}c \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,d^2} + \frac{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d} + \frac{\sqrt{bx^2+a}bc^2}{d^3} + \frac{(bx^2+a)^{\frac{3}{2}}}{3\,d} + \frac{\sqrt{bx^2+a}a}{d}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b*x^2 + a)*b*c*x/d^2 - b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 -
3/2*a*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^2 + (a + b*c^2/d^2)^(3/2)*arcsinh
(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d + sqrt(b
*x^2 + a)*b*c^2/d^3 + 1/3*(b*x^2 + a)^(3/2)/d + sqrt(b*x^2 + a)*a/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{c+d\,x}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x),x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1948, normalized size of antiderivative = 12.25

$$\int \frac{(a+bx^2)^{3/2}}{c+dx}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x+c),x)
```

output

```
( - 6*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*c*d**2 - 6*sqrt(b)*sqrt
(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c*
*2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b
*c**2)*c - a*d**2 - 2*b*c**2))*b*c**3 - 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c
**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2
*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*d**4 - 12*sqrt
(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x
**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*
b*c**2))*a*b*c**2*d**2 - 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2
 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a
*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*b**2*c**4 - 3*sqrt(b)*sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log(
 - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a +
b*x**2)*d + sqrt(b)*d*x)*a*c*d**2 - 3*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b
)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt
(b)*d*x)*b*c**3 + 3*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**
2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log(sqrt(2*sqrt(b)*sqrt(a*d**2 + b*...
```

3.247 $\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 175

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = -\frac{2bc\sqrt{a+bx^2}}{d^3} + \frac{bx\sqrt{a+bx^2}}{2d^2} - \frac{(bc^2+ad^2)\sqrt{a+bx^2}}{d^3(c+dx)} + \frac{3\sqrt{b}(2bc^2+ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^4} + \frac{3bc\sqrt{bc^2+ad^2}\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4}$$

output
```
-2*b*c*(b*x^2+a)^(1/2)/d^3+1/2*b*x*(b*x^2+a)^(1/2)/d^2-(a*d^2+b*c^2)*(b*x^
2+a)^(1/2)/d^3/(d*x+c)+3/2*b^(1/2)*(a*d^2+2*b*c^2)*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))/d^4+3*b*c*(a*d^2+b*c^2)^(1/2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^
2)^(1/2)/(b*x^2+a)^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = \frac{\frac{d\sqrt{a+bx^2}(-6bc^2-2ad^2-3bcdx+bd^2x^2)}{c+dx} - 12bc\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right) - 3\sqrt{b}(2}{2d^4}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(-6*b*c^2 - 2*a*d^2 - 3*b*c*d*x + b*d^2*x^2))/(c + d*x
) - 12*b*c*Sqrt[-(b*c^2) - a*d^2]*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b
*x^2])/Sqrt[-(b*c^2) - a*d^2]] - 3*Sqrt[b]*(2*b*c^2 + a*d^2)*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(2*d^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {492, 591, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \ 492$$

$$\frac{3b\int \frac{x\sqrt{bx^2+a}}{c+dx}dx}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow \ 591$$

$$\frac{3b\left(\frac{\int -\frac{acd-\left(2bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}(2c-dx)}{2d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow \ 25$$

$$\frac{3b\left(-\frac{\int \frac{acd-\left(2bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}(2c-dx)}{2d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow \ 719$$

$$\frac{3b\left(-\frac{\frac{2c\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2+2bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}-\frac{\sqrt{a+bx^2}(2c-dx)}{2d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

$$\Big\downarrow \ 224$$

$$\frac{3b\left(-\frac{\frac{2c\left(ad^2+bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2+2bc^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}-\frac{\sqrt{a+bx^2}(2c-dx)}{2d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

$\Big\downarrow$ 219

$$\frac{3b\left(-\frac{\frac{2c\left(ad^2+bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+2bc^2\right)}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2c-dx)}{2d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

$\Big\downarrow$ 488

$$\frac{3b\left(-\frac{-\frac{2c\left(ad^2+bc^2\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+2bc^2\right)}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2c-dx)}{2d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

$\Big\downarrow$ 219

$$\frac{3b\left(-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+2bc^2\right)}{\sqrt{b}d}-\frac{2c\sqrt{ad^2+bc^2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}-\frac{\sqrt{a+bx^2}(2c-dx)}{2d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{d(c+dx)}$$

input

```
Int[(a + b*x^2)^(3/2)/(c + d*x)^2,x]
```

output

```
-((a + b*x^2)^(3/2)/(d*(c + d*x))) + (3*b*(-1/2*((2*c - d*x)*Sqrt[a + b*x^
2])/d^2 - (-(((2*b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqr
t[b]*d)) - (2*c*Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*
d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(153) = 306$.

Time = 0.33 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.55

method	result
risch	$-\frac{(-dx+4c)\sqrt{b\,x^2+a}\,b}{2d^3}+\frac{\frac{2(a^2d^4+2b\,c^2d^2a+b^2c^4)\left(-\frac{d^2\sqrt{b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{(a\,d^2+b\,c^2)(x+\frac{c}{d})}-\frac{bcd\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc(x+\frac{c}{d})}{d}+2\sqrt{a\,d \cdots}}{\cdots}\right)}{(a\,d^2+b\,c \cdots}\right.}{d^3}}{\cdots}$
default	$\frac{-\frac{d^2\left(b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{5}{2}}}{(a\,d^2+b\,c^2)(x+\frac{c}{d})}-\frac{3bcd\left(\frac{\left(b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{3}-\frac{bc\left(\frac{\left(2b(x+\frac{c}{d})-\frac{2bc}{d}\right)\sqrt{b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d} \cdots}{d}}}{4b}\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input

```
int((b*x^2+a)^(3/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2*(-d*x+4*c)*(b*x^2+a)^(1/2)*b/d^3+1/2/d^3*(2*(a^2*d^4+2*a*b*c^2*d^2+b^
2*c^4)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d
^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(
a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+3*b^(1/2)*(a*d^2+2*b*c
^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+8/d^2*b*c*(a*d^2+b*c^2)/((a*d^2+b*c^2)
/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^
(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 888, normalized size of antiderivative = 5.07

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(3*(2*b*c^3 + a*c*d^2 + (2*b*c^2*d + a*d^3)*x)*sqrt(b)*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 6*(b*c*d*x + b*c^2)*sqrt(b*c^2 + a*d^2
)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*s
qrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2
)) + 2*(b*d^3*x^2 - 3*b*c*d^2*x - 6*b*c^2*d - 2*a*d^3)*sqrt(b*x^2 + a))/(d
^5*x + c*d^4), 1/4*(12*(b*c*d*x + b*c^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(
-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^
2 + a*b*d^2)*x^2)) + 3*(2*b*c^3 + a*c*d^2 + (2*b*c^2*d + a*d^3)*x)*sqrt(b)
*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(b*d^3*x^2 - 3*b*c*d^
2*x - 6*b*c^2*d - 2*a*d^3)*sqrt(b*x^2 + a))/(d^5*x + c*d^4), -1/2*(3*(2*b*
c^3 + a*c*d^2 + (2*b*c^2*d + a*d^3)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) - 3*(b*c*d*x + b*c^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*
c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x
 - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - (b*d^3*x^2 - 3*b*c*d
^2*x - 6*b*c^2*d - 2*a*d^3)*sqrt(b*x^2 + a))/(d^5*x + c*d^4), 1/2*(6*(b*c*
d*x + b*c^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d
)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - 3*(2*b*
c^3 + a*c*d^2 + (2*b*c^2*d + a*d^3)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) + (b*d^3*x^2 - 3*b*c*d^2*x - 6*b*c^2*d - 2*a*d^3)*sqrt(b*x^2 + a)
)/(d^5*x + c*d^4)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = -\frac{(bx^2+a)^{\frac{3}{2}}}{d^2x+cd} + \frac{3\sqrt{bx^2+a}bx}{2d^2} + \frac{3b^{\frac{3}{2}}c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4} + \frac{3a\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2d^2} - \frac{3\sqrt{a+\frac{bc^2}{d^2}}bc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^3} - \frac{3\sqrt{bx^2+a}bc}{d^3}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(b*x^2 + a)^(3/2)/(d^2*x + c*d) + 3/2*sqrt(b*x^2 + a)*b*x/d^2 + 3*b^(3/2)
*c^2*arcsinh(b*x/sqrt(a*b))/d^4 + 3/2*a*sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^2
 - 3*sqrt(a + b*c^2/d^2)*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/
(sqrt(a*b)*abs(d*x + c)))/d^3 - 3*sqrt(b*x^2 + a)*b*c/d^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.35

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^2}\,dx = \frac{12\sqrt{a\,d^2+b\,c^2}\,\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)b\,c^2+12\sqrt{a\,d^2+b\,c^2}\,\log\left(-\sqrt{}\right.}{}$$

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^2,x)
```

output
```
(12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*b*c**2 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*b*c*d*x - 12*sqrt(a*d**2 + b*c**2)*log(c
 + d*x)*b*c**2 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c*d*x - 4*sqrt(a
+ b*x**2)*a*d**3 - 12*sqrt(a + b*x**2)*b*c**2*d - 6*sqrt(a + b*x**2)*b*c*d
**2*x + 2*sqrt(a + b*x**2)*b*d**3*x**2 - 3*sqrt(b)*log(sqrt(a + b*x**2) -
sqrt(b)*x)*a*c*d**2 - 3*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*d**3*x
 - 6*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*b*c**3 - 6*sqrt(b)*log(sqrt
(a + b*x**2) - sqrt(b)*x)*b*c**2*d*x + 3*sqrt(b)*log(sqrt(a + b*x**2) + sq
rt(b)*x)*a*c*d**2 + 3*sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*x)*a*d**3*x +
 6*sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*x)*b*c**3 + 6*sqrt(b)*log(sqrt(a
 + b*x**2) + sqrt(b)*x)*b*c**2*d*x)/(4*d**4*(c + d*x))
```

3.248 $\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 182

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3} dx = \frac{b\sqrt{a+bx^2}}{d^3} - \frac{(bc^2+ad^2)\sqrt{a+bx^2}}{2d^3(c+dx)^2} + \frac{5bc\sqrt{a+bx^2}}{2d^3(c+dx)} - \frac{3b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^4} - \frac{3b(2bc^2+ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2d^4\sqrt{bc^2+ad^2}}$$

output
```
b*(b*x^2+a)^(1/2)/d^3-1/2*(a*d^2+b*c^2)*(b*x^2+a)^(1/2)/d^3/(d*x+c)^2+5/2*
b*c*(b*x^2+a)^(1/2)/d^3/(d*x+c)-3*b^(3/2)*c*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/d^4-3/2*b*(a*d^2+2*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b
*x^2+a)^(1/2))/d^4/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3} dx = \frac{\frac{d\sqrt{a+bx^2}(6bc^2-ad^2+9bcdx+2bd^2x^2)}{(c+dx)^2} - \frac{6b(2bc^2+ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}} + 6b^{3/2}c\log\left(-\sqrt{b}x\right.}{2d^4}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x)^3,x]
```

output
```
((d*Sqrt[a + b*x^2]*(6*b*c^2 - a*d^2 + 9*b*c*d*x + 2*b*d^2*x^2))/(c + d*x)
^2 - (6*b*(2*b*c^2 + a*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])
/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + 6*b^(3/2)*c*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/(2*d^4)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {492, 590, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3}dx$$

$$\Big\downarrow \text{ 492}$$

$$\frac{3b\int \frac{x\sqrt{bx^2+a}}{(c+dx)^2}dx}{2d}-\frac{(a+bx^2)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 590}$$

$$\frac{3b\left(\frac{\sqrt{a+bx^2}(2c+dx)}{d^2(c+dx)}-\frac{\int -\frac{ad-2bcx}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}\right)}{2d}-\frac{(a+bx^2)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3b\left(\frac{\int \frac{ad-2bcx}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}+\frac{\sqrt{a+bx^2}(2c+dx)}{d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{3b\left(\frac{\frac{\left(ad^2+2bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2bc\int\frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2}+\frac{\sqrt{a+bx^2}(2c+dx)}{d^2(c+dx)}\right)}{2d}-\frac{\left(a+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{3b\left(\frac{\frac{\left(ad^2+2bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2bc\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d^2}+\frac{\sqrt{a+bx^2}(2c+dx)}{d^2(c+dx)}\right)}{2d}-\frac{\left(a+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{\frac{\left(ad^2+2bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(2c+dx)}{d^2(c+dx)}\right)}{2d}-\frac{\left(a+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{3b\left(\frac{-\frac{\left(ad^2+2bc^2\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{2\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(2c+dx)}{d^2(c+dx)}\right)}{2d}-\frac{\left(a+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{-\frac{\left(ad^2+2bc^2\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}-\frac{2\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(2c+dx)}{d^2(c+dx)}\right)}{2d}-\frac{\left(a+bx^2\right)^{3/2}}{2d(c+dx)^2}$$

input

```
Int[(a + b*x^2)^(3/2)/(c + d*x)^3,x]
```

output
```
-1/2*(a + b*x^2)^(3/2)/(d*(c + d*x)^2) + (3*b*(((2*c + d*x)*Sqrt[a + b*x^2
])/(d^2*(c + d*x)) + ((-2*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
d - ((2*b*c^2 + a*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a +
 b*x^2])])/(d*Sqrt[b*c^2 + a*d^2]))/d^2))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(158) = 316$.

Time = 0.40 (sec) , antiderivative size = 883, normalized size of antiderivative = 4.85

method	result
	$-\frac{\left(a^2d^4+2bc^2d^2a+b^2c^4\right)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{ad^2+bc^2}{d^2}}}{2\left(ad^2+bc^2\right)\left(x+\frac{c}{d}\right)^2}+\frac{3bcd\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{ad^2+bc^2}{d^2}}}{\left(ad^2+bc^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{b\sqrt{bx^2+a}}{d^3}-\frac{\cdots}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b*(b*x^2+a)^(1/2)/d^3-1/d^3*(-(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)/d^4*(-1/2/(a
*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2
*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)
/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^
(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2
*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b
*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d
^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+3*b^(3/2)*c/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
)+2*b/d^2*(a*d^2+3*b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^
2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+4/d^3*b*c*(a*d^2+b*c^2)*(-1/(a*d^2+b*c
^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*
d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*
(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(159) = 318$.

Time = 0.42 (sec) , antiderivative size = 1545, normalized size of antiderivative = 8.49

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/4*(6*(b^2*c^5 + a*b*c^3*d^2 + (b^2*c^3*d^2 + a*b*c*d^4)*x^2 + 2*(b^2*c^
4*d + a*b*c^2*d^3)*x)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 3*(2*b^2*c^4 + a*b*c^2*d^2 + (2*b^2*c^2*d^2 + a*b*d^4)*x^2 + 2*(2*b^
2*c^3*d + a*b*c*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2
*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)
*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(6*b^2*c^4*d + 5*a*b*c^2*
d^3 - a^2*d^5 + 2*(b^2*c^2*d^3 + a*b*d^5)*x^2 + 9*(b^2*c^3*d^2 + a*b*c*d^4
)*x)*sqrt(b*x^2 + a))/(b*c^4*d^4 + a*c^2*d^6 + (b*c^2*d^6 + a*d^8)*x^2 + 2
*(b*c^3*d^5 + a*c*d^7)*x), 1/4*(12*(b^2*c^5 + a*b*c^3*d^2 + (b^2*c^3*d^2 +
 a*b*c*d^4)*x^2 + 2*(b^2*c^4*d + a*b*c^2*d^3)*x)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) + 3*(2*b^2*c^4 + a*b*c^2*d^2 + (2*b^2*c^2*d^2 + a*b*d^4
)*x^2 + 2*(2*b^2*c^3*d + a*b*c*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*
x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2
)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(6*b^2*c^4
*d + 5*a*b*c^2*d^3 - a^2*d^5 + 2*(b^2*c^2*d^3 + a*b*d^5)*x^2 + 9*(b^2*c^3*
d^2 + a*b*c*d^4)*x)*sqrt(b*x^2 + a))/(b*c^4*d^4 + a*c^2*d^6 + (b*c^2*d^6 +
 a*d^8)*x^2 + 2*(b*c^3*d^5 + a*c*d^7)*x), -1/2*(3*(2*b^2*c^4 + a*b*c^2*d^2
 + (2*b^2*c^2*d^2 + a*b*d^4)*x^2 + 2*(2*b^2*c^3*d + a*b*c*d^3)*x)*sqrt(-b*
c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*
b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - 3*(b^2*c^5 + a*b*c^3*d^2 ...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x+c)**3,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(159) = 318$.

Time = 0.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{3\sqrt{bx^2+a}b^2c^2}{2\,(bc^2d^3+ad^5)} - \frac{3\sqrt{bx^2+a}b^2cx}{2\,(bc^2d^2+ad^4)} + \frac{(bx^2+a)^{\frac{3}{2}}bc}{2\,(bc^2d^2x+ad^4x+bc^3d+acd^3)} - \frac{(bx^2+a)^{\frac{5}{2}}}{2\left(bc^2dx^2+ad^3x^2+2\,bc^3x+2\,acd^2x+\frac{bc^4}{d}+ac^2d\right)} + \frac{(bx^2+a)^{\frac{3}{2}}b}{2\,(bc^2d+ad^3)} - \frac{3\,b^{\frac{3}{2}}c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4} + \frac{3\,b^2c^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\sqrt{a+\frac{bc^2}{d^2}}d^5} + \frac{3\sqrt{a+\frac{bc^2}{d^2}}b\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\,d^3} + \frac{3\sqrt{bx^2+a}b}{2\,d^3}$$

input

```
integrate((b*x^2+a)^(3/2)/(d*x+c)^3,x, algorithm="maxima")
```

output

```
3/2*sqrt(b*x^2 + a)*b^2*c^2/(b*c^2*d^3 + a*d^5) - 3/2*sqrt(b*x^2 + a)*b^2*
c*x/(b*c^2*d^2 + a*d^4) + 1/2*(b*x^2 + a)^(3/2)*b*c/(b*c^2*d^2*x + a*d^4*x
 + b*c^3*d + a*c*d^3) - 1/2*(b*x^2 + a)^(5/2)/(b*c^2*d*x^2 + a*d^3*x^2 + 2
*b*c^3*x + 2*a*c*d^2*x + b*c^4/d + a*c^2*d) + 1/2*(b*x^2 + a)^(3/2)*b/(b*c
^2*d + a*d^3) - 3*b^(3/2)*c*arcsinh(b*x/sqrt(a*b))/d^4 + 3/2*b^2*c^2*arcsi
nh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a
+ b*c^2/d^2)*d^5) + 3/2*sqrt(a + b*c^2/d^2)*b*arcsinh(b*c*x/(sqrt(a*b)*abs
(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 + 3/2*sqrt(b*x^2 + a)*b/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(159) = 318$.

Time = 0.15 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3}\,dx = \frac{3\,b^{\frac{3}{2}}c\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{d^4} + \frac{\sqrt{bx^2+a}b}{d^3} + \frac{3\,(2\,b^2c^2+abd^2)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}d^4} + \frac{6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3b^2c^2d+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3abd^3+10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}c^3-5\left(\sqrt{b}x-\sqrt{bx^2}\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\left(\sqrt{b}x-\sqrt{bx}\right.\right.}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
3*b^(3/2)*c*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/d^4 + sqrt(b*x^2 + a)*b
/d^3 + 3*(2*b^2*c^2 + a*b*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d +
sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*d^4) + (6*(sqrt(b)*
x - sqrt(b*x^2 + a))^3*b^2*c^2*d + (sqrt(b)*x - sqrt(b*x^2 + a))^3*a*b*d^3
 + 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^3 - 5*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*b^(3/2)*c*d^2 - 14*(sqrt(b)*x - sqrt(b*x^2 + a))*a*b^2*c^2*d
 + (sqrt(b)*x - sqrt(b*x^2 + a))*a^2*b*d^3 + 5*a^2*b^(3/2)*c*d^2)/(((sqrt(
b)*x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*c -
a*d)^2*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 999, normalized size of antiderivative = 5.49

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^3,x)
```

output
```
(3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a*b*c**2*d**2 + 6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**3*x + 3*sqrt(a*d**2 + b*c**2)*lo
g(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*d**4*x**2 + 6*
sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*b**2*c**4 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d*x + 6*sqrt(a*d**2 + b*c**2)*log(sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**2*d**2*x**2 - 3
*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**2 - 6*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a*b*c*d**3*x - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*d**
4*x**2 - 6*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4 - 12*sqrt(a*d**2 +
 b*c**2)*log(c + d*x)*b**2*c**3*d*x - 6*sqrt(a*d**2 + b*c**2)*log(c + d*x)
*b**2*c**2*d**2*x**2 - sqrt(a + b*x**2)*a**2*d**5 + 5*sqrt(a + b*x**2)*a*b
*c**2*d**3 + 9*sqrt(a + b*x**2)*a*b*c*d**4*x + 2*sqrt(a + b*x**2)*a*b*d**5
*x**2 + 6*sqrt(a + b*x**2)*b**2*c**4*d + 9*sqrt(a + b*x**2)*b**2*c**3*d**2
*x + 2*sqrt(a + b*x**2)*b**2*c**2*d**3*x**2 + 3*sqrt(b)*log(sqrt(a + b*x**
2) - sqrt(b)*x)*a*b*c**3*d**2 + 6*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x
)*a*b*c**2*d**3*x + 3*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*b*c*d**4
*x**2 + 3*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*b**2*c**5 + 6*sqrt(b)*
log(sqrt(a + b*x**2) - sqrt(b)*x)*b**2*c**4*d*x + 3*sqrt(b)*log(sqrt(a ...
```

3.249 $\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 220

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4} \, dx = -\frac{(bc^2+ad^2)\sqrt{a+bx^2}}{3d^3(c+dx)^3} + \frac{7bc\sqrt{a+bx^2}}{6d^3(c+dx)^2} - \frac{b(11bc^2+8ad^2)\sqrt{a+bx^2}}{6d^3(bc^2+ad^2)(c+dx)} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^4} + \frac{b^2c(2bc^2+3ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2d^4(bc^2+ad^2)^{3/2}}$$

output
```
-1/3*(a*d^2+b*c^2)*(b*x^2+a)^(1/2)/d^3/(d*x+c)^3+7/6*b*c*(b*x^2+a)^(1/2)/d
^3/(d*x+c)^2-1/6*b*(8*a*d^2+11*b*c^2)*(b*x^2+a)^(1/2)/d^3/(a*d^2+b*c^2)/(d
*x+c)+b^(3/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^4+1/2*b^2*c*(3*a*d^2+2*
b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^4/(a*d^
2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx =$$

$$-\frac{\frac{d\sqrt{a+bx^2}(2a^2d^4+abd^2(5c^2+9cdx+8d^2x^2)+b^2c^2(6c^2+15cdx+11d^2x^2))}{(bc^2+ad^2)(c+dx)^3}+\frac{6b^2c(2bc^2+3ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}+6b^{3/2}\log\left(\right.}{6d^4}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x)^4,x]
```

output
```
-1/6*((d*Sqrt[a + b*x^2]*(2*a^2*d^4 + a*b*d^2*(5*c^2 + 9*c*d*x + 8*d^2*x^2
) + b^2*c^2*(6*c^2 + 15*c*d*x + 11*d^2*x^2)))/((b*c^2 + a*d^2)*(c + d*x)^3
) + (6*b^2*c*(2*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*
x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) + 6*b^(3/2)*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^4
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {492, 589, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{ 492}$$

$$\frac{b\int \frac{x\sqrt{bx^2+a}}{(c+dx)^3}dx}{d}-\frac{(a+bx^2)^{3/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 589}$$

$$\frac{b\left(\frac{b\int -\frac{2\left(acd-2\left(bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{4d^2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}\left(dx\left(2ad^2+3bc^2\right)+c\left(ad^2+2bc^2\right)\right)}{2d^2(c+dx)^2(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(-\frac{b\int \frac{acd-2\left(bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}\left(dx\left(2ad^2+3bc^2\right)+c\left(ad^2+2bc^2\right)\right)}{2d^2(c+dx)^2(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \; 719$$

$$\frac{b\left(-\frac{b\left(\frac{c\left(3ad^2+2bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2\left(ad^2+bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{2d^2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}\left(dx\left(2ad^2+3bc^2\right)+c\left(ad^2+2bc^2\right)\right)}{2d^2(c+dx)^2(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \; 224$$

$$\frac{b\left(-\frac{b\left(\frac{c\left(3ad^2+2bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2\left(ad^2+bc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{2d^2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}\left(dx\left(2ad^2+3bc^2\right)+c\left(ad^2+2bc^2\right)\right)}{2d^2(c+dx)^2(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(-\frac{b\left(\frac{c\left(3ad^2+2bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d}\right)}{2d^2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}\left(dx\left(2ad^2+3bc^2\right)+c\left(ad^2+2bc^2\right)\right)}{2d^2(c+dx)^2(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \textbf{488}$$

$$\frac{b\left(-\frac{b\left(-\frac{c\left(3ad^2+2bc^2\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d}\right)}{2d^2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(dx(2ad^2+3bc^2)+c(ad^2+2bc^2))}{2d^2(c+dx)^2(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{b\left(-\frac{b\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d}-\frac{c\left(3ad^2+2bc^2\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}\right)}{2d^2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(dx(2ad^2+3bc^2)+c(ad^2+2bc^2))}{2d^2(c+dx)^2(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{3/2}}{3d(c+dx)^3}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x)^4,x]
```

output
```
-1/3*(a + b*x^2)^(3/2)/(d*(c + d*x)^3) + (b*(-1/2*((c*(2*b*c^2 + a*d^2) +
d*(3*b*c^2 + 2*a*d^2)*x)*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^2)*(c + d*x)^2
) - (b*((-2*(b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]
*d) - (c*(2*b*c^2 + 3*a*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sq
rt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*d^2*(b*c^2 + a*d^2))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2445 vs. $2(194) = 388$.

Time = 0.38 (sec) , antiderivative size = 2446, normalized size of antiderivative = 11.12

method	result	size
default	Expression too large to display	2446

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(5/2)+1/3*b*c*d/(a*d^2+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d
)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-1/2*b*c*d/(a*d^2
+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(5/2)-3*b*c*d/(a*d^2+b*c^2)*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2
*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c
^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^
2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^
2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/
d))))+4*b/(a*d^2+b*c^2)*d^2*(1/8*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+3/16*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2
/d^2)/b*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*
c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/
2)))))+3/2*b/(a*d^2+b*c^2)*d^2*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(195) = 390$.

Time = 3.75 (sec) , antiderivative size = 2513, normalized size of antiderivative = 11.42

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
[1/12*(6*(b^3*c^7 + 2*a*b^2*c^5*d^2 + a^2*b*c^3*d^4 + (b^3*c^4*d^3 + 2*a*b
^2*c^2*d^5 + a^2*b*d^7)*x^3 + 3*(b^3*c^5*d^2 + 2*a*b^2*c^3*d^4 + a^2*b*c*d
^6)*x^2 + 3*(b^3*c^6*d + 2*a*b^2*c^4*d^3 + a^2*b*c^2*d^5)*x)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3*(2*b^3*c^6 + 3*a*b^2*c^4*d^
2 + (2*b^3*c^3*d^3 + 3*a*b^2*c*d^5)*x^3 + 3*(2*b^3*c^4*d^2 + 3*a*b^2*c^2*d
^4)*x^2 + 3*(2*b^3*c^5*d + 3*a*b^2*c^3*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*
a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2
 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(6
*b^3*c^6*d + 11*a*b^2*c^4*d^3 + 7*a^2*b*c^2*d^5 + 2*a^3*d^7 + (11*b^3*c^4*
d^3 + 19*a*b^2*c^2*d^5 + 8*a^2*b*d^7)*x^2 + 3*(5*b^3*c^5*d^2 + 8*a*b^2*c^3
*d^4 + 3*a^2*b*c*d^6)*x)*sqrt(b*x^2 + a))/(b^2*c^7*d^4 + 2*a*b*c^5*d^6 + a
^2*c^3*d^8 + (b^2*c^4*d^7 + 2*a*b*c^2*d^9 + a^2*d^11)*x^3 + 3*(b^2*c^5*d^6
 + 2*a*b*c^3*d^8 + a^2*c*d^10)*x^2 + 3*(b^2*c^6*d^5 + 2*a*b*c^4*d^7 + a^2*
c^2*d^9)*x), 1/6*(3*(2*b^3*c^6 + 3*a*b^2*c^4*d^2 + (2*b^3*c^3*d^3 + 3*a*b^
2*c*d^5)*x^3 + 3*(2*b^3*c^4*d^2 + 3*a*b^2*c^2*d^4)*x^2 + 3*(2*b^3*c^5*d +
3*a*b^2*c^3*d^3)*x)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*
x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) +
3*(b^3*c^7 + 2*a*b^2*c^5*d^2 + a^2*b*c^3*d^4 + (b^3*c^4*d^3 + 2*a*b^2*c^2*
d^5 + a^2*b*d^7)*x^3 + 3*(b^3*c^5*d^2 + 2*a*b^2*c^3*d^4 + a^2*b*c*d^6)*x^2
 + 3*(b^3*c^6*d + 2*a*b^2*c^4*d^3 + a^2*b*c^2*d^5)*x)*sqrt(b)*log(-2*b*...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx)^4}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x+c)**4,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(195) = 390$.

Time = 0.08 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.92

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx &= \frac{\sqrt{bx^2+a}\,b^3c^3}{2\,(b^2c^4d^3+2\,abc^2d^5+a^2d^7)} - \frac{\sqrt{bx^2+a}\,b^3c^2x}{2\,(b^2c^4d^2+2\,abc^2d^4+a^2d^6)}\\
&+ \frac{(bx^2+a)^{\frac{3}{2}}b^2c^2}{6\,(b^2c^4d^2x+2\,abc^2d^4x+a^2d^6x+b^2c^5d+2\,abc^3d^3+a^2cd^5)}\\
&- \frac{(bx^2+a)^{\frac{5}{2}}bc}{6\left(b^2c^4dx^2+2\,abc^2d^3x^2+a^2d^5x^2+2\,b^2c^5x+4\,abc^3d^2x+2\,a^2cd^4x+\frac{b^2c^6}{d}+2\,abc^4d+a^2c^2d^3\right)}\\
&+ \frac{(bx^2+a)^{\frac{3}{2}}b^2c}{6\,(b^2c^4d+2\,abc^2d^3+a^2d^5)} - \frac{3\,\sqrt{bx^2+a}\,b^2c}{2\,(bc^2d^3+ad^5)} + \frac{\sqrt{bx^2+a}\,b^2x}{bc^2d^2+ad^4}\\
&- \frac{(bx^2+a)^{\frac{5}{2}}}{3\left(bc^2d^2x^3+ad^4x^3+3\,bc^3dx^2+3\,acd^3x^2+3\,bc^4x+3\,ac^2d^2x+\frac{bc^5}{d}+ac^3d\right)}\\
&- \frac{2\,(bx^2+a)^{\frac{3}{2}}b}{3\,(bc^2d^2x+ad^4x+bc^3d+acd^3)} + \frac{b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4}\\
&+ \frac{b^3c^3\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^7} - \frac{3\,b^2c\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\,\sqrt{a+\frac{bc^2}{d^2}}\,d^5}\end{aligned}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*b^3*c^3/(b^2*c^4*d^3 + 2*a*b*c^2*d^5 + a^2*d^7) - 1/2*
sqrt(b*x^2 + a)*b^3*c^2*x/(b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6) + 1/6*(b
*x^2 + a)^(3/2)*b^2*c^2/(b^2*c^4*d^2*x + 2*a*b*c^2*d^4*x + a^2*d^6*x + b^2
*c^5*d + 2*a*b*c^3*d^3 + a^2*c*d^5) - 1/6*(b*x^2 + a)^(5/2)*b*c/(b^2*c^4*d
*x^2 + 2*a*b*c^2*d^3*x^2 + a^2*d^5*x^2 + 2*b^2*c^5*x + 4*a*b*c^3*d^2*x + 2
*a^2*c*d^4*x + b^2*c^6/d + 2*a*b*c^4*d + a^2*c^2*d^3) + 1/6*(b*x^2 + a)^(3
/2)*b^2*c/(b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5) - 3/2*sqrt(b*x^2 + a)*b^2*
c/(b*c^2*d^3 + a*d^5) + sqrt(b*x^2 + a)*b^2*x/(b*c^2*d^2 + a*d^4) - 1/3*(b
*x^2 + a)^(5/2)/(b*c^2*d^2*x^3 + a*d^4*x^3 + 3*b*c^3*d*x^2 + 3*a*c*d^3*x^2
 + 3*b*c^4*x + 3*a*c^2*d^2*x + b*c^5/d + a*c^3*d) - 2/3*(b*x^2 + a)^(3/2)*
b/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) + b^(3/2)*arcsinh(b*x/sqrt(a
*b))/d^4 + 1/2*b^3*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(
a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^7) - 3/2*b^2*c*arcsinh(b*c*x/
(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d
^2)*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(195) = 390$.

Time = 0.18 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.75

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx = \frac{(2\,b^3c^3+3\,ab^2cd^2)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(bc^2d^4+ad^6)\sqrt{-bc^2-ad^2}} - \frac{b^{\frac{3}{2}}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{d^4} - \frac{18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 b^3c^3d^2+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 ab^2cd^4+54\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{7}{2}}c^4d+27\left(\sqrt{b}\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^4,x, algorithm="giac")
```

output
```
(2*b^3*c^3 + 3*a*b^2*c*d^2)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt
(b)*c)/sqrt(-b*c^2 - a*d^2))/((b*c^2*d^4 + a*d^6)*sqrt(-b*c^2 - a*d^2)) -
b^(3/2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/d^4 - 1/3*(18*(sqrt(b)*x -
sqrt(b*x^2 + a))^5*b^3*c^3*d^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a*b^2*
c*d^4 + 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^4*d + 27*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^3 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^2*b^(3/2)*d^5 + 44*(sqrt(b)*x - sqrt(b*x^2 + a))^3*b^4*c^5 - 34*(sqrt(b
)*x - sqrt(b*x^2 + a))^3*a*b^3*c^3*d^2 - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^
3*a^2*b^2*c*d^4 - 78*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(7/2)*c^4*d - 36*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d^3 + 12*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^3*b^(3/2)*d^5 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))*a^2*b^3*
c^3*d^2 + 33*(sqrt(b)*x - sqrt(b*x^2 + a))*a^3*b^2*c*d^4 - 11*a^3*b^(5/2)*
c^2*d^3 - 8*a^4*b^(3/2)*d^5)/((b*c^2*d^4 + a*d^6)*((sqrt(b)*x - sqrt(b*x^2
 + a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*c - a*d)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x)^4,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1734, normalized size of antiderivative = 7.88

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^4}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^4,x)
```

output
```
(9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b**2*c**4*d**2 + 27*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**3*x + 27*sqrt(a*
d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a*b**2*c**2*d**4*x**2 + 9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**5*x**3 + 6*sqrt(a*d**2 +
b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*
c**6 + 18*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*b**3*c**5*d*x + 18*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
+ b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**4*d**2*x**2 + 6*sqr
t(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*b**3*c**3*d**3*x**3 - 9*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*
*4*d**2 - 27*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d**3*x - 27*sq
rt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**4*x**2 - 9*sqrt(a*d**2 + b
*c**2)*log(c + d*x)*a*b**2*c*d**5*x**3 - 6*sqrt(a*d**2 + b*c**2)*log(c + d
*x)*b**3*c**6 - 18*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**5*d*x - 18*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**4*d**2*x**2 - 6*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*b**3*c**3*d**3*x**3 - 2*sqrt(a + b*x**2)*a**3*d**7 - 7*
sqrt(a + b*x**2)*a**2*b*c**2*d**5 - 9*sqrt(a + b*x**2)*a**2*b*c*d**6*x - 8
*sqrt(a + b*x**2)*a**2*b*d**7*x**2 - 11*sqrt(a + b*x**2)*a*b**2*c**4*d*...
```

3.250 $\int \frac{\left(a+bx^2\right)^{3/2}}{(c+dx)^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{\left(a+bx^2\right)^{3/2}}{(c+dx)^5} \, dx = -\frac{3ab(ad-bcx)\sqrt{a+bx^2}}{8\left(bc^2+ad^2\right)^2(c+dx)^2} - \frac{(ad-bcx)\left(a+bx^2\right)^{3/2}}{4\left(bc^2+ad^2\right)(c+dx)^4} - \frac{3a^2b^2\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{8\left(bc^2+ad^2\right)^{5/2}}$$

output
```
-3/8*a*b*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2/(d*x+c)^2-1/4*(-b*c*
x+a*d)*(b*x^2+a)^(3/2)/(a*d^2+b*c^2)/(d*x+c)^4-3/8*a^2*b^2*arctanh((-b*c*x
+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 723 vs. 2(153) = 306.

Time = 5.87 (sec) , antiderivative size = 723, normalized size of antiderivative = 4.73

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5}\,dx = \frac{1}{8}\left(\frac{16b^5c^4x^3(c^3+4c^2dx+6cd^2x^2+4d^3x^3)\left(\sqrt{b}x-\sqrt{a+bx^2}\right)-2a^5d^7\left(-4\sqrt{b}x+\sqrt{}\right.}{}\right.$$
$$\left.-\frac{6a^2b^2\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{5/2}}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x)^5,x]
```

output
```
((16*b^5*c^4*x^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3)*(Sqrt[b]*x -
Sqrt[a + b*x^2]) - 2*a^5*d^7*(-4*Sqrt[b]*x + Sqrt[a + b*x^2]) + a^4*b*d^5*
(4*Sqrt[b]*x*(5*c^2 + 4*c*d*x + 11*d^2*x^2) - Sqrt[a + b*x^2]*(5*c^2 + 4*c
*d*x + 21*d^2*x^2)) - 8*a*b^4*c^2*x*(Sqrt[a + b*x^2]*(c^5 + 4*c^4*d*x + 11
*c^3*d^2*x^2 + 24*c^2*d^3*x^3 + 24*c*d^4*x^4 + 16*d^5*x^5) - Sqrt[b]*x*(2*
c^5 + 8*c^4*d*x + 17*c^3*d^2*x^2 + 28*c^2*d^3*x^3 + 24*c*d^4*x^4 + 16*d^5*
x^5)) + a^3*(-(b^2*d^4*x*Sqrt[a + b*x^2]*(-5*c^3 + 36*c^2*d*x + 27*c*d^2*x
^2 + 56*d^3*x^3)) + b^(5/2)*d^2*(5*c^5 + 20*c^4*d*x + 10*c^3*d^2*x^2 + 64*
c^2*d^3*x^3 + 33*c*d^4*x^4 + 76*d^5*x^5)) + 2*a^2*(-(b^3*d^2*x*Sqrt[a + b*
x^2]*(10*c^5 + 40*c^4*d*x + 39*c^3*d^2*x^2 + 44*c^2*d^3*x^3 + 6*c*d^4*x^4
+ 20*d^5*x^5)) + b^(7/2)*(c^7 + 4*c^6*d*x + 26*c^5*d^2*x^2 + 84*c^4*d^3*x^
3 + 87*c^3*d^4*x^4 + 76*c^2*d^5*x^5 + 6*c*d^6*x^6 + 20*d^7*x^7)))/(d^4*(b*
c^2 + a*d^2)^2*(c + d*x)^4*(a^2 + 8*a*b*x^2 + 8*b^2*x^4 - 4*a*Sqrt[b]*x*Sq
rt[a + b*x^2] - 8*b^(3/2)*x^3*Sqrt[a + b*x^2])) - (6*a^2*b^2*ArcTan[(Sqrt[
b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d
^2)^(5/2))/8
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5} \, dx$$

$\Big\downarrow$ 486

$$\frac{3ab\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4(ad^2+bc^2)} - \frac{(a+bx^2)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}$$

$\Big\downarrow$ 486

$$\frac{3ab\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)} - \frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{4(ad^2+bc^2)} - \frac{(a+bx^2)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}$$

$\Big\downarrow$ 488

$$\frac{3ab\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2(ad^2+bc^2)} - \frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{4(ad^2+bc^2)} - \frac{(a+bx^2)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}$$

$\Big\downarrow$ 219

$$\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2(ad^2+bc^2)^{3/2}} - \frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{4(ad^2+bc^2)} - \frac{(a+bx^2)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x)^5,x]
```

output
```
-1/4*((a*d - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*
a*b*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) -
(a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c
^2 + a*d^2)^(3/2))))/(4*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4019 vs. $2(137) = 274$.

Time = 0.45 (sec) , antiderivative size = 4020, normalized size of antiderivative = 26.27

method	result	size
default	Expression too large to display	4020

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d^5*(-1/4/(a*d^2+b*c^2)*d^2/(x+c/d)^4*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(5/2)+3/4*b*c*d/(a*d^2+b*c^2)*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d
)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)+1/3*b*c*d/(a*d^2
+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*
d^2+b*c^2)/d^2)^(5/2)-1/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d
)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-3*b*c*d/(a*d^2+b*c
^2)*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*
(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1
/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d)
)/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b
*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c
/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2
)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c
^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(
x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))+4*b/(a*d^2+b*c^2)*d^2*(1/8*(2*
b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)
+3/16*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b*(1/4*(2*b*(x+c/d)-2*b*c/d)/b
*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c
^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))))+3/2*b/(a*d^2+b*c^2)*d^2*(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(138) = 276$.

Time = 1.19 (sec) , antiderivative size = 1123, normalized size of antiderivative = 7.34

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
[1/16*(3*(a^2*b^2*d^4*x^4 + 4*a^2*b^2*c*d^3*x^3 + 6*a^2*b^2*c^2*d^2*x^2 +
4*a^2*b^2*c^3*d*x + a^2*b^2*c^4)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*
b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c
*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(5*a^2*b^2*c^4*d
 + 7*a^3*b*c^2*d^3 + 2*a^4*d^5 - (2*b^4*c^5 + 7*a*b^3*c^3*d^2 + 5*a^2*b^2*
c*d^4)*x^3 - (4*a*b^3*c^4*d - a^2*b^2*c^2*d^3 - 5*a^3*b*d^5)*x^2 - (5*a*b^
3*c^5 + a^2*b^2*c^3*d^2 - 4*a^3*b*c*d^4)*x)*sqrt(b*x^2 + a))/(b^3*c^10 + 3
*a*b^2*c^8*d^2 + 3*a^2*b*c^6*d^4 + a^3*c^4*d^6 + (b^3*c^6*d^4 + 3*a*b^2*c^
4*d^6 + 3*a^2*b*c^2*d^8 + a^3*d^10)*x^4 + 4*(b^3*c^7*d^3 + 3*a*b^2*c^5*d^5
 + 3*a^2*b*c^3*d^7 + a^3*c*d^9)*x^3 + 6*(b^3*c^8*d^2 + 3*a*b^2*c^6*d^4 + 3
*a^2*b*c^4*d^6 + a^3*c^2*d^8)*x^2 + 4*(b^3*c^9*d + 3*a*b^2*c^7*d^3 + 3*a^2
*b*c^5*d^5 + a^3*c^3*d^7)*x), -1/8*(3*(a^2*b^2*d^4*x^4 + 4*a^2*b^2*c*d^3*x
^3 + 6*a^2*b^2*c^2*d^2*x^2 + 4*a^2*b^2*c^3*d*x + a^2*b^2*c^4)*sqrt(-b*c^2
- a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^
2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (5*a^2*b^2*c^4*d + 7*a^3*b*c^2*d
^3 + 2*a^4*d^5 - (2*b^4*c^5 + 7*a*b^3*c^3*d^2 + 5*a^2*b^2*c*d^4)*x^3 - (4*
a*b^3*c^4*d - a^2*b^2*c^2*d^3 - 5*a^3*b*d^5)*x^2 - (5*a*b^3*c^5 + a^2*b^2*
c^3*d^2 - 4*a^3*b*c*d^4)*x)*sqrt(b*x^2 + a))/(b^3*c^10 + 3*a*b^2*c^8*d^2 +
 3*a^2*b*c^6*d^4 + a^3*c^4*d^6 + (b^3*c^6*d^4 + 3*a*b^2*c^4*d^6 + 3*a^2*b*
c^2*d^8 + a^3*d^10)*x^4 + 4*(b^3*c^7*d^3 + 3*a*b^2*c^5*d^5 + 3*a^2*b*c^...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx)^5}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x+c)**5,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1223 vs. $2(138) = 276$.

Time = 0.11 (sec) , antiderivative size = 1223, normalized size of antiderivative = 7.99

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
3/8*sqrt(b*x^2 + a)*b^4*c^4/(b^3*c^6*d^3 + 3*a*b^2*c^4*d^5 + 3*a^2*b*c^2*d
^7 + a^3*d^9) - 3/8*sqrt(b*x^2 + a)*b^4*c^3*x/(b^3*c^6*d^2 + 3*a*b^2*c^4*d
^4 + 3*a^2*b*c^2*d^6 + a^3*d^8) + 1/8*(b*x^2 + a)^(3/2)*b^3*c^3/(b^3*c^6*d
^2*x + 3*a*b^2*c^4*d^4*x + 3*a^2*b*c^2*d^6*x + a^3*d^8*x + b^3*c^7*d + 3*a
*b^2*c^5*d^3 + 3*a^2*b*c^3*d^5 + a^3*c*d^7) - 1/8*(b*x^2 + a)^(5/2)*b^2*c^
2/(b^3*c^6*d*x^2 + 3*a*b^2*c^4*d^3*x^2 + 3*a^2*b*c^2*d^5*x^2 + a^3*d^7*x^2
 + 2*b^3*c^7*x + 6*a*b^2*c^5*d^2*x + 6*a^2*b*c^3*d^4*x + 2*a^3*c*d^6*x + b
^3*c^8/d + 3*a*b^2*c^6*d + 3*a^2*b*c^4*d^3 + a^3*c^2*d^5) + 1/8*(b*x^2 + a
)^(3/2)*b^3*c^2/(b^3*c^6*d + 3*a*b^2*c^4*d^3 + 3*a^2*b*c^2*d^5 + a^3*d^7)
- 3/4*sqrt(b*x^2 + a)*b^3*c^2/(b^2*c^4*d^3 + 2*a*b*c^2*d^5 + a^2*d^7) + 3/
8*sqrt(b*x^2 + a)*b^3*c*x/(b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6) - 1/4*(b
*x^2 + a)^(5/2)*b*c/(b^2*c^4*d^2*x^3 + 2*a*b*c^2*d^4*x^3 + a^2*d^6*x^3 + 3
*b^2*c^5*d*x^2 + 6*a*b*c^3*d^3*x^2 + 3*a^2*c*d^5*x^2 + 3*b^2*c^6*x + 6*a*b
*c^4*d^2*x + 3*a^2*c^2*d^4*x + b^2*c^7/d + 2*a*b*c^5*d + a^2*c^3*d^3) - 3/
8*(b*x^2 + a)^(3/2)*b^2*c/(b^2*c^4*d^2*x + 2*a*b*c^2*d^4*x + a^2*d^6*x + b
^2*c^5*d + 2*a*b*c^3*d^3 + a^2*c*d^5) - 1/8*(b*x^2 + a)^(5/2)*b/(b^2*c^4*d
*x^2 + 2*a*b*c^2*d^3*x^2 + a^2*d^5*x^2 + 2*b^2*c^5*x + 4*a*b*c^3*d^2*x + 2
*a^2*c*d^4*x + b^2*c^6/d + 2*a*b*c^4*d + a^2*c^2*d^3) + 1/8*(b*x^2 + a)^(3
/2)*b^2/(b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5) - 1/4*(b*x^2 + a)^(5/2)/(b*c
^2*d^3*x^4 + a*d^5*x^4 + 4*b*c^3*d^2*x^3 + 4*a*c*d^4*x^3 + 6*b*c^4*d*x^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c+d\,x)^5}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x)^5,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 956, normalized size of antiderivative = 6.25

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^5,x)
```

output

```
(3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*b**2*c**4 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d*x + 18*sqrt(a*d**2 + b
*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2
*c**2*d**2*x**2 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**3*x**3 + 3*sqrt(a*d**2 + b*c**2)
*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*d**4*
x**4 - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c**4 - 12*sqrt(a*d**
2 + b*c**2)*log(c + d*x)*a**2*b**2*c**3*d*x - 18*sqrt(a*d**2 + b*c**2)*log
(c + d*x)*a**2*b**2*c**2*d**2*x**2 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)
*a**2*b**2*c*d**3*x**3 - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*d*
*4*x**4 - 2*sqrt(a + b*x**2)*a**4*d**5 - 7*sqrt(a + b*x**2)*a**3*b*c**2*d*
*3 - 4*sqrt(a + b*x**2)*a**3*b*c*d**4*x - 5*sqrt(a + b*x**2)*a**3*b*d**5*x
**2 - 5*sqrt(a + b*x**2)*a**2*b**2*c**4*d + sqrt(a + b*x**2)*a**2*b**2*c**
3*d**2*x - sqrt(a + b*x**2)*a**2*b**2*c**2*d**3*x**2 + 5*sqrt(a + b*x**2)*
a**2*b**2*c*d**4*x**3 + 5*sqrt(a + b*x**2)*a*b**3*c**5*x + 4*sqrt(a + b*x*
*2)*a*b**3*c**4*d*x**2 + 7*sqrt(a + b*x**2)*a*b**3*c**3*d**2*x**3 + 2*sqrt
(a + b*x**2)*b**4*c**5*x**3)/(8*(a**3*c**4*d**6 + 4*a**3*c**3*d**7*x + 6*a
**3*c**2*d**8*x**2 + 4*a**3*c*d**9*x**3 + a**3*d**10*x**4 + 3*a**2*b*c**6*
d**4 + 12*a**2*b*c**5*d**5*x + 18*a**2*b*c**4*d**6*x**2 + 12*a**2*b*c**...
```

3.251 $\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 195

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6} \, dx = -\frac{3ab^2c(ad-bcx)\sqrt{a+bx^2}}{8(bc^2+ad^2)^3(c+dx)^2} - \frac{bc(ad-bcx)(a+bx^2)^{3/2}}{4(bc^2+ad^2)^2(c+dx)^4} - \frac{d(a+bx^2)^{5/2}}{5(bc^2+ad^2)(c+dx)^5} - \frac{3a^2b^3c\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{8(bc^2+ad^2)^{7/2}}$$

output
```
-3/8*a*b^2*c*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^3/(d*x+c)^2-1/4*b*
c*(-b*c*x+a*d)*(b*x^2+a)^(3/2)/(a*d^2+b*c^2)^2/(d*x+c)^4-1/5*d*(b*x^2+a)^(
5/2)/(a*d^2+b*c^2)/(d*x+c)^5-3/8*a^2*b^3*c*arctanh((-b*c*x+a*d)/(a*d^2+b*c
^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6} \, dx = \frac{\sqrt{a+bx^2}(-8a^4d^5 + 2b^4c^4x^3(5c+dx) - 2a^3bd^3(13c^2+5cdx+8d^2x^2) + ab^3c^2x(25c^3 }{40(bc^2+ad}$$
$$+ \frac{3a^2b^3c\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{4(-bc^2-ad^2)^{7/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x)^6,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^4*d^5 + 2*b^4*c^4*x^3*(5*c + d*x) - 2*a^3*b*d^3*(13
*c^2 + 5*c*d*x + 8*d^2*x^2) + a*b^3*c^2*x*(25*c^3 + 29*c^2*d*x + 45*c*d^2*
x^2 + 9*d^3*x^3) - a^2*b^2*d*(33*c^4 + 45*c^3*d*x + 77*c^2*d^2*x^2 + 25*c*
d^3*x^3 + 8*d^4*x^4)))/(40*(b*c^2 + a*d^2)^3*(c + d*x)^5) + (3*a^2*b^3*c*A
rcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(4*
(-(b*c^2) - a*d^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {491, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6}dx$$

$$\Big\downarrow \text{491}$$

$$\frac{bc\int \frac{(bx^2+a)^{3/2}}{(c+dx)^5}dx}{ad^2+bc^2}-\frac{d(a+bx^2)^{5/2}}{5(c+dx)^5(ad^2+bc^2)}$$

$$\Big\downarrow \text{486}$$

$$\frac{bc\left(\frac{3ab\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4(ad^2+bc^2)}-\frac{(a+bx^2)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d(a+bx^2)^{5/2}}{5(c+dx)^5(ad^2+bc^2)}$$

$$\Big\downarrow \text{486}$$

$$\frac{bc\left(\frac{3ab\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2(ad^2+bc^2)}\right)}{4(ad^2+bc^2)}-\frac{(a+bx^2)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d(a+bx^2)^{5/2}}{5(c+dx)^5(ad^2+bc^2)}$$

$$\Big\downarrow 488$$

$$\frac{bc\left(\frac{3ab\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{5/2}}{5(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 219$$

$$\frac{bc\left(\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{5/2}}{5(c+dx)^5\left(ad^2+bc^2\right)}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x)^6,x]
```

output
```
-1/5*(d*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^5) + (b*c*(-1/4*((a*
d - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*(-1/2
*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcT
anh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^
2)^(3/2))))/(4*(b*c^2 + a*d^2))))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4107 vs. $2(175) = 350$.

Time = 0.59 (sec) , antiderivative size = 4108, normalized size of antiderivative = 21.07

method	result	size
default	Expression too large to display	4108

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d^6*(-1/5/(a*d^2+b*c^2)*d^2/(x+c/d)^5*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(5/2)+b*c*d/(a*d^2+b*c^2)*(-1/4/(a*d^2+b*c^2)*d^2/(x+c/d)^4*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)+3/4*b*c*d/(a*d^2+b*c
^2)*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(5/2)+1/3*b*c*d/(a*d^2+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^
2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-1/2*b*c*d/(a*d^2+b
*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(5/2)-3*b*c*d/(a*d^2+b*c^2)*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2
/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+
b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)
/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)
)))+4*b/(a*d^2+b*c^2)*d^2*(1/8*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+3/16*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d
^2)/b*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(176) = 352$.

Time = 3.30 (sec) , antiderivative size = 1647, normalized size of antiderivative = 8.45

$$\int \frac{(a + bx^2)^{3/2}}{(c + dx)^6} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^6,x, algorithm="fricas")
```

output
```
[1/80*(15*(a^2*b^3*c*d^5*x^5 + 5*a^2*b^3*c^2*d^4*x^4 + 10*a^2*b^3*c^3*d^3*
x^3 + 10*a^2*b^3*c^4*d^2*x^2 + 5*a^2*b^3*c^5*d*x + a^2*b^3*c^6)*sqrt(b*c^2
 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x
^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d
*x + c^2)) - 2*(33*a^2*b^3*c^6*d + 59*a^3*b^2*c^4*d^3 + 34*a^4*b*c^2*d^5 +
 8*a^5*d^7 - (2*b^5*c^6*d + 11*a*b^4*c^4*d^3 + a^2*b^3*c^2*d^5 - 8*a^3*b^2
*d^7)*x^4 - 5*(2*b^5*c^7 + 11*a*b^4*c^5*d^2 + 4*a^2*b^3*c^3*d^4 - 5*a^3*b^
2*c*d^6)*x^3 - (29*a*b^4*c^6*d - 48*a^2*b^3*c^4*d^3 - 93*a^3*b^2*c^2*d^5 -
 16*a^4*b*d^7)*x^2 - 5*(5*a*b^4*c^7 - 4*a^2*b^3*c^5*d^2 - 11*a^3*b^2*c^3*d
^4 - 2*a^4*b*c*d^6)*x)*sqrt(b*x^2 + a))/(b^4*c^13 + 4*a*b^3*c^11*d^2 + 6*a
^2*b^2*c^9*d^4 + 4*a^3*b*c^7*d^6 + a^4*c^5*d^8 + (b^4*c^8*d^5 + 4*a*b^3*c^
6*d^7 + 6*a^2*b^2*c^4*d^9 + 4*a^3*b*c^2*d^11 + a^4*d^13)*x^5 + 5*(b^4*c^9*
d^4 + 4*a*b^3*c^7*d^6 + 6*a^2*b^2*c^5*d^8 + 4*a^3*b*c^3*d^10 + a^4*c*d^12)
*x^4 + 10*(b^4*c^10*d^3 + 4*a*b^3*c^8*d^5 + 6*a^2*b^2*c^6*d^7 + 4*a^3*b*c^
4*d^9 + a^4*c^2*d^11)*x^3 + 10*(b^4*c^11*d^2 + 4*a*b^3*c^9*d^4 + 6*a^2*b^2
*c^7*d^6 + 4*a^3*b*c^5*d^8 + a^4*c^3*d^10)*x^2 + 5*(b^4*c^12*d + 4*a*b^3*c
^10*d^3 + 6*a^2*b^2*c^8*d^5 + 4*a^3*b*c^6*d^7 + a^4*c^4*d^9)*x), -1/40*(15
*(a^2*b^3*c*d^5*x^5 + 5*a^2*b^3*c^2*d^4*x^4 + 10*a^2*b^3*c^3*d^3*x^3 + 10*
a^2*b^3*c^4*d^2*x^2 + 5*a^2*b^3*c^5*d*x + a^2*b^3*c^6)*sqrt(-b*c^2 - a*d^2
)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + ...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx)^6}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x+c)**6,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. $2(176) = 352$.

Time = 0.13 (sec) , antiderivative size = 1762, normalized size of antiderivative = 9.04

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^6,x, algorithm="maxima")
```

output
```
3/8*sqrt(b*x^2 + a)*b^5*c^5/(b^4*c^8*d^3 + 4*a*b^3*c^6*d^5 + 6*a^2*b^2*c^4
*d^7 + 4*a^3*b*c^2*d^9 + a^4*d^11) - 3/8*sqrt(b*x^2 + a)*b^5*c^4*x/(b^4*c^
8*d^2 + 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^4*d^6 + 4*a^3*b*c^2*d^8 + a^4*d^10)
+ 1/8*(b*x^2 + a)^(3/2)*b^4*c^4/(b^4*c^8*d^2*x + 4*a*b^3*c^6*d^4*x + 6*a^2
*b^2*c^4*d^6*x + 4*a^3*b*c^2*d^8*x + a^4*d^10*x + b^4*c^9*d + 4*a*b^3*c^7*
d^3 + 6*a^2*b^2*c^5*d^5 + 4*a^3*b*c^3*d^7 + a^4*c*d^9) - 1/8*(b*x^2 + a)^(
5/2)*b^3*c^3/(b^4*c^8*d*x^2 + 4*a*b^3*c^6*d^3*x^2 + 6*a^2*b^2*c^4*d^5*x^2
+ 4*a^3*b*c^2*d^7*x^2 + a^4*d^9*x^2 + 2*b^4*c^9*x + 8*a*b^3*c^7*d^2*x + 12
*a^2*b^2*c^5*d^4*x + 8*a^3*b*c^3*d^6*x + 2*a^4*c*d^8*x + b^4*c^10/d + 4*a*
b^3*c^8*d + 6*a^2*b^2*c^6*d^3 + 4*a^3*b*c^4*d^5 + a^4*c^2*d^7) + 1/8*(b*x^
2 + a)^(3/2)*b^4*c^3/(b^4*c^8*d + 4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^4*d^5 + 4*
a^3*b*c^2*d^7 + a^4*d^9) - 3/4*sqrt(b*x^2 + a)*b^4*c^3/(b^3*c^6*d^3 + 3*a*
b^2*c^4*d^5 + 3*a^2*b*c^2*d^7 + a^3*d^9) + 3/8*sqrt(b*x^2 + a)*b^4*c^2*x/(
b^3*c^6*d^2 + 3*a*b^2*c^4*d^4 + 3*a^2*b*c^2*d^6 + a^3*d^8) - 1/4*(b*x^2 +
a)^(5/2)*b^2*c^2/(b^3*c^6*d^2*x^3 + 3*a*b^2*c^4*d^4*x^3 + 3*a^2*b*c^2*d^6*
x^3 + a^3*d^8*x^3 + 3*b^3*c^7*d*x^2 + 9*a*b^2*c^5*d^3*x^2 + 9*a^2*b*c^3*d^
5*x^2 + 3*a^3*c*d^7*x^2 + 3*b^3*c^8*x + 9*a*b^2*c^6*d^2*x + 9*a^2*b*c^4*d^
4*x + 3*a^3*c^2*d^6*x + b^3*c^9/d + 3*a*b^2*c^7*d + 3*a^2*b*c^5*d^3 + a^3*
c^3*d^5) - 3/8*(b*x^2 + a)^(3/2)*b^3*c^2/(b^3*c^6*d^2*x + 3*a*b^2*c^4*d^4*
x + 3*a^2*b*c^2*d^6*x + a^3*d^8*x + b^3*c^7*d + 3*a*b^2*c^5*d^3 + 3*a^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(176) = 352$.

Time = 0.17 (sec) , antiderivative size = 1283, normalized size of antiderivative = 6.58

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^6,x, algorithm="giac")
```

output
```
-3/4*a^2*b^3*c*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-
b*c^2 - a*d^2))/((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*s
qrt(-b*c^2 - a*d^2)) - 1/20*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a^2*b^3*c*
d^8 - 40*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(11/2)*c^6*d^3 - 120*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*a*b^(9/2)*c^4*d^5 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))
^8*a^2*b^(7/2)*c^2*d^7 - 40*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(5/2)*d^
9 - 80*(sqrt(b)*x - sqrt(b*x^2 + a))^7*b^6*c^7*d^2 - 240*(sqrt(b)*x - sqrt
(b*x^2 + a))^7*a*b^5*c^5*d^4 + 230*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a^2*b^4
*c^3*d^6 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a^3*b^3*c*d^8 - 80*(sqrt(b)
*x - sqrt(b*x^2 + a))^6*b^(13/2)*c^8*d - 240*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*a*b^(11/2)*c^6*d^3 + 530*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(9/2)*c^
4*d^5 - 570*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(7/2)*c^2*d^7 - 32*(sqrt
(b)*x - sqrt(b*x^2 + a))^5*b^7*c^9 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a*
b^6*c^7*d^2 + 788*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a^2*b^5*c^5*d^4 - 910*(s
qrt(b)*x - sqrt(b*x^2 + a))^5*a^3*b^4*c^3*d^6 + 240*(sqrt(b)*x - sqrt(b*x^
2 + a))^5*a^4*b^3*c*d^8 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(13/2)*c^
8*d + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(11/2)*c^6*d^3 - 1170*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(9/2)*c^4*d^5 + 480*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^4*b^(7/2)*c^2*d^7 - 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b
^(5/2)*d^9 - 80*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a^2*b^6*c^7*d^2 - 400*(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c+d\,x)^6}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x)^6,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 1397, normalized size of antiderivative = 7.16

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^6,x)
```

output
```
(15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**3*c**6 + 75*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**5*d*x + 150*sqrt(a*d**2 +
 b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*
*3*c**4*d**2*x**2 + 150*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**3*d**3*x**3 + 75*sqrt(a*d**2 +
b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**
3*c**2*d**4*x**4 + 15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c*d**5*x**5 - 15*sqrt(a*d**2 + b*c**
2)*log(c + d*x)*a**2*b**3*c**6 - 75*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**
2*b**3*c**5*d*x - 150*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**3*c**4*d*
*2*x**2 - 150*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**3*c**3*d**3*x**3
- 75*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**3*c**2*d**4*x**4 - 15*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**3*c*d**5*x**5 - 8*sqrt(a + b*x**2)*
a**5*d**7 - 34*sqrt(a + b*x**2)*a**4*b*c**2*d**5 - 10*sqrt(a + b*x**2)*a**
4*b*c*d**6*x - 16*sqrt(a + b*x**2)*a**4*b*d**7*x**2 - 59*sqrt(a + b*x**2)*
a**3*b**2*c**4*d**3 - 55*sqrt(a + b*x**2)*a**3*b**2*c**3*d**4*x - 93*sqrt(
a + b*x**2)*a**3*b**2*c**2*d**5*x**2 - 25*sqrt(a + b*x**2)*a**3*b**2*c*d**
6*x**3 - 8*sqrt(a + b*x**2)*a**3*b**2*d**7*x**4 - 33*sqrt(a + b*x**2)*a**2
*b**3*c**6*d - 20*sqrt(a + b*x**2)*a**2*b**3*c**5*d**2*x - 48*sqrt(a + ...
```

3.252 $\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7} dx$

Optimal result

Integrand size = 19, antiderivative size = 269

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7} dx = -\frac{ab^2(6bc^2-ad^2)(ad-bcx)\sqrt{a+bx^2}}{16(bc^2+ad^2)^4(c+dx)^2} - \frac{b(6bc^2-ad^2)(ad-bcx)(a+bx^2)^{3/2}}{24(bc^2+ad^2)^3(c+dx)^4} - \frac{d(a+bx^2)^{5/2}}{6(bc^2+ad^2)(c+dx)^6} - \frac{7bcd(a+bx^2)^{5/2}}{30(bc^2+ad^2)^2(c+dx)^5} - \frac{a^2b^3(6bc^2-ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{16(bc^2+ad^2)^{9/2}}$$

output
```
-1/16*a*b^2*(-a*d^2+6*b*c^2)*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^4/
(d*x+c)^2-1/24*b*(-a*d^2+6*b*c^2)*(-b*c*x+a*d)*(b*x^2+a)^(3/2)/(a*d^2+b*c^
2)^3/(d*x+c)^4-1/6*d*(b*x^2+a)^(5/2)/(a*d^2+b*c^2)/(d*x+c)^6-7/30*b*c*d*(b
*x^2+a)^(5/2)/(a*d^2+b*c^2)^2/(d*x+c)^5-1/16*a^2*b^3*(-a*d^2+6*b*c^2)*arct
anh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(9/2)
```

Mathematica [A] (verified)

Time = 10.50 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7}\,dx = \frac{1}{240}\left(-\frac{\sqrt{a+bx^2}\left(40(bc^2+ad^2)^5-104bc(bc^2+ad^2)^4(c+dx)+2b(bc^2+ad^2)^3(38bc\right.}{}\right.$$
$$+\frac{15a^2b^3(6bc^2-ad^2)\log(c+dx)}{(bc^2+ad^2)^{9/2}}$$
$$\left.+\frac{15a^2b^3(-6bc^2+ad^2)\log\left(ad-bcx+\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\right)}{(bc^2+ad^2)^{9/2}}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c + d*x)^7,x]
```

output
```
(-((Sqrt[a + b*x^2]*(40*(b*c^2 + a*d^2)^5 - 104*b*c*(b*c^2 + a*d^2)^4*(c +
 d*x) + 2*b*(b*c^2 + a*d^2)^3*(38*b*c^2 + 35*a*d^2)*(c + d*x)^2 - 2*b^2*c*
(b*c^2 + a*d^2)^2*(2*b*c^2 + 9*a*d^2)*(c + d*x)^3 - b^2*(b*c^2 + a*d^2)*(4
*b^2*c^4 + 24*a*b*c^2*d^2 - 15*a^2*d^4)*(c + d*x)^4 - b^3*c*(4*b^2*c^4 + 2
8*a*b*c^2*d^2 - 81*a^2*d^4)*(c + d*x)^5))/(d^3*(b*c^2 + a*d^2)^4*(c + d*x)
^6)) + (15*a^2*b^3*(6*b*c^2 - a*d^2)*Log[c + d*x])/(b*c^2 + a*d^2)^(9/2) +
 (15*a^2*b^3*(-6*b*c^2 + a*d^2)*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt
[a + b*x^2]])/(b*c^2 + a*d^2)^(9/2))/240
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {498, 25, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7}\,dx$$

$$\Big\downarrow \; 498$$

$$-\frac{b\int -\frac{(6c-dx)\left(bx^2+a\right)^{3/2}}{(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{5/2}}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\int \frac{(6c-dx)\left(bx^2+a\right)^{3/2}}{(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{5/2}}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 679}$$

$$\frac{b\left(\frac{\left(6bc^2-ad^2\right)\int \frac{\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{ad^2+bc^2}-\frac{7cd\left(a+bx^2\right)^{5/2}}{5(c+dx)^5\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{5/2}}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 486}$$

$$\frac{b\left(\frac{\left(6bc^2-ad^2\right)\left(\frac{3ab\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{7cd\left(a+bx^2\right)^{5/2}}{5(c+dx)^5\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{5/2}}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 486}$$

$$\frac{b\left(\frac{\left(6bc^2-ad^2\right)\left(\frac{3ab\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{7cd\left(a+bx^2\right)^{5/2}}{5(c+dx)^5\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{5/2}}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{b\left(\frac{(6bc^2-ad^2)\left(\frac{3ab\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{7cd(a+bx^2)^{5/2}}{5(c+dx)^5(ad^2+bc^2)}\right)}{6\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{5/2}}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{(6bc^2-ad^2)\left(\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{7cd(a+bx^2)^{5/2}}{5(c+dx)^5(ad^2+bc^2)}\right)}{6\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{5/2}}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

input
```
Int[(a + b*x^2)^(3/2)/(c + d*x)^7,x]
```

output
```
-1/6*(d*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^6) + (b*((-7*c*d*(a
+ b*x^2)^(5/2))/(5*(b*c^2 + a*d^2)*(c + d*x)^5) + ((6*b*c^2 - a*d^2)*(-1/4
*((a*d - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*
(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b
*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 +
 a*d^2)^(3/2))))/(4*(b*c^2 + a*d^2))))/(b*c^2 + a*d^2)))/(6*(b*c^2 + a*d^2
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8230 vs. $2(245) = 490$.

Time = 0.64 (sec) , antiderivative size = 8231, normalized size of antiderivative = 30.60

method	result	size
default	Expression too large to display	8231

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(246) = 492$.

Time = 7.86 (sec) , antiderivative size = 2485, normalized size of antiderivative = 9.24

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^7,x, algorithm="fricas")
```

output
```
[-1/480*(15*(6*a^2*b^4*c^8 - a^3*b^3*c^6*d^2 + (6*a^2*b^4*c^2*d^6 - a^3*b^
3*d^8)*x^6 + 6*(6*a^2*b^4*c^3*d^5 - a^3*b^3*c*d^7)*x^5 + 15*(6*a^2*b^4*c^4
*d^4 - a^3*b^3*c^2*d^6)*x^4 + 20*(6*a^2*b^4*c^5*d^3 - a^3*b^3*c^3*d^5)*x^3
 + 15*(6*a^2*b^4*c^6*d^2 - a^3*b^3*c^4*d^4)*x^2 + 6*(6*a^2*b^4*c^7*d - a^3
*b^3*c^5*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^
2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b
*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(246*a^2*b^4*c^8*d + 513*a^3*b^3
*c^6*d^3 + 433*a^4*b^2*c^4*d^5 + 206*a^5*b*c^2*d^7 + 40*a^6*d^9 - (4*b^6*c
^7*d^2 + 32*a*b^5*c^5*d^4 - 53*a^2*b^4*c^3*d^6 - 81*a^3*b^3*c*d^8)*x^5 - 3
*(8*b^6*c^8*d + 64*a*b^5*c^6*d^3 - 76*a^2*b^4*c^4*d^5 - 137*a^3*b^3*c^2*d^
7 - 5*a^4*b^2*d^9)*x^4 - 2*(30*b^6*c^9 + 239*a*b^5*c^7*d^2 - 158*a^2*b^4*c
^5*d^4 - 388*a^3*b^3*c^3*d^6 - 21*a^4*b^2*c*d^8)*x^3 - 2*(114*a*b^5*c^8*d
- 423*a^2*b^4*c^6*d^3 - 698*a^3*b^3*c^4*d^5 - 196*a^4*b^2*c^2*d^7 - 35*a^5
*b*d^9)*x^2 - 3*(50*a*b^5*c^9 - 117*a^2*b^4*c^7*d^2 - 221*a^3*b^3*c^5*d^4
- 66*a^4*b^2*c^3*d^6 - 12*a^5*b*c*d^8)*x)*sqrt(b*x^2 + a))/(b^5*c^16 + 5*a
*b^4*c^14*d^2 + 10*a^2*b^3*c^12*d^4 + 10*a^3*b^2*c^10*d^6 + 5*a^4*b*c^8*d^
8 + a^5*c^6*d^10 + (b^5*c^10*d^6 + 5*a*b^4*c^8*d^8 + 10*a^2*b^3*c^6*d^10 +
 10*a^3*b^2*c^4*d^12 + 5*a^4*b*c^2*d^14 + a^5*d^16)*x^6 + 6*(b^5*c^11*d^5
+ 5*a*b^4*c^9*d^7 + 10*a^2*b^3*c^7*d^9 + 10*a^3*b^2*c^5*d^11 + 5*a^4*b*c^3
*d^13 + a^5*c*d^15)*x^5 + 15*(b^5*c^12*d^4 + 5*a*b^4*c^10*d^6 + 10*a^2*...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx)^7}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x+c)**7,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(c + d*x)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3297 vs. 2(246) = 492.

Time = 0.20 (sec) , antiderivative size = 3297, normalized size of antiderivative = 12.26

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^7,x, algorithm="maxima")
```

output
```
7/16*sqrt(b*x^2 + a)*b^6*c^6/(b^5*c^10*d^3 + 5*a*b^4*c^8*d^5 + 10*a^2*b^3*
c^6*d^7 + 10*a^3*b^2*c^4*d^9 + 5*a^4*b*c^2*d^11 + a^5*d^13) - 7/16*sqrt(b*
x^2 + a)*b^6*c^5*x/(b^5*c^10*d^2 + 5*a*b^4*c^8*d^4 + 10*a^2*b^3*c^6*d^6 +
10*a^3*b^2*c^4*d^8 + 5*a^4*b*c^2*d^10 + a^5*d^12) + 7/48*(b*x^2 + a)^(3/2)
*b^5*c^5/(b^5*c^10*d^2*x + 5*a*b^4*c^8*d^4*x + 10*a^2*b^3*c^6*d^6*x + 10*a
^3*b^2*c^4*d^8*x + 5*a^4*b*c^2*d^10*x + a^5*d^12*x + b^5*c^11*d + 5*a*b^4*
c^9*d^3 + 10*a^2*b^3*c^7*d^5 + 10*a^3*b^2*c^5*d^7 + 5*a^4*b*c^3*d^9 + a^5*
c*d^11) - 7/48*(b*x^2 + a)^(5/2)*b^4*c^4/(b^5*c^10*d*x^2 + 5*a*b^4*c^8*d^3
*x^2 + 10*a^2*b^3*c^6*d^5*x^2 + 10*a^3*b^2*c^4*d^7*x^2 + 5*a^4*b*c^2*d^9*x
^2 + a^5*d^11*x^2 + 2*b^5*c^11*x + 10*a*b^4*c^9*d^2*x + 20*a^2*b^3*c^7*d^4
*x + 20*a^3*b^2*c^5*d^6*x + 10*a^4*b*c^3*d^8*x + 2*a^5*c*d^10*x + b^5*c^12
/d + 5*a*b^4*c^10*d + 10*a^2*b^3*c^8*d^3 + 10*a^3*b^2*c^6*d^5 + 5*a^4*b*c^
4*d^7 + a^5*c^2*d^9) + 7/48*(b*x^2 + a)^(3/2)*b^5*c^4/(b^5*c^10*d + 5*a*b^
4*c^8*d^3 + 10*a^2*b^3*c^6*d^5 + 10*a^3*b^2*c^4*d^7 + 5*a^4*b*c^2*d^9 + a^
5*d^11) - 15/16*sqrt(b*x^2 + a)*b^5*c^4/(b^4*c^8*d^3 + 4*a*b^3*c^6*d^5 + 6
*a^2*b^2*c^4*d^7 + 4*a^3*b*c^2*d^9 + a^4*d^11) + 1/2*sqrt(b*x^2 + a)*b^5*c
^3*x/(b^4*c^8*d^2 + 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^4*d^6 + 4*a^3*b*c^2*d^8
+ a^4*d^10) - 7/24*(b*x^2 + a)^(5/2)*b^3*c^3/(b^4*c^8*d^2*x^3 + 4*a*b^3*c^
6*d^4*x^3 + 6*a^2*b^2*c^4*d^6*x^3 + 4*a^3*b*c^2*d^8*x^3 + a^4*d^10*x^3 + 3
*b^4*c^9*d*x^2 + 12*a*b^3*c^7*d^3*x^2 + 18*a^2*b^2*c^5*d^5*x^2 + 12*a^3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(246) = 492$.

Time = 0.17 (sec) , antiderivative size = 1875, normalized size of antiderivative = 6.97

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/8*(6*a^2*b^4*c^2 - a^3*b^3*d^2)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d
 + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^4*c^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^
2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8)*sqrt(-b*c^2 - a*d^2)) - 1/120*(90*(
sqrt(b)*x - sqrt(b*x^2 + a))^11*a^2*b^4*c^2*d^9 - 15*(sqrt(b)*x - sqrt(b*x
^2 + a))^11*a^3*b^3*d^11 + 990*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2
)*c^3*d^8 - 165*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c*d^10 - 320*
(sqrt(b)*x - sqrt(b*x^2 + a))^9*b^7*c^8*d^3 - 1280*(sqrt(b)*x - sqrt(b*x^2
 + a))^9*a*b^6*c^6*d^5 + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a^2*b^5*c^4*
d^7 - 2530*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a^3*b^4*c^2*d^9 - 235*(sqrt(b)*
x - sqrt(b*x^2 + a))^9*a^4*b^3*d^11 - 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
b^(15/2)*c^9*d^2 - 1920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(13/2)*c^7*d^4
 + 7380*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(11/2)*c^5*d^6 - 8220*(sqrt(
b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2)*c^3*d^8 + 285*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a^4*b^(7/2)*c*d^10 - 384*(sqrt(b)*x - sqrt(b*x^2 + a))^7*b^8*c^1
0*d - 1728*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a*b^7*c^8*d^3 + 9456*(sqrt(b)*x
 - sqrt(b*x^2 + a))^7*a^2*b^6*c^6*d^5 - 20760*(sqrt(b)*x - sqrt(b*x^2 + a)
)^7*a^3*b^5*c^4*d^7 + 2700*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a^4*b^4*c^2*d^9
 - 390*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a^5*b^3*d^11 - 128*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*b^(17/2)*c^11 + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(15
/2)*c^9*d^2 + 8592*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(13/2)*c^7*d^4...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c+d\,x)^7}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c + d*x)^7,x)
```

output
```
int((a + b*x^2)^(3/2)/(c + d*x)^7, x)
```

Reduce [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 2558, normalized size of antiderivative = 9.51

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x+c)^7,x)
```

output

```
(15*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**3*b**3*c**6*d**2 + 90*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
+ b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c**5*d**3*x + 225
*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**3*b**3*c**4*d**4*x**2 + 300*sqrt(a*d**2 + b*c**2)*log( - sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c**3*d**5*x**3
 + 225*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a**3*b**3*c**2*d**6*x**4 + 90*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c*d**7*x*
*5 + 15*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*a**3*b**3*d**8*x**6 - 90*sqrt(a*d**2 + b*c**2)*log( - sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**4*c**8 - 540*sq
rt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a**2*b**4*c**7*d*x - 1350*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**4*c**6*d**2*x**2 - 1800*
sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*b**4*c**5*d**3*x**3 - 1350*sqrt(a*d**2 + b*c**2)*log( - sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**4*c**4*d**4*x**4
 - 540*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a**2*b**4*c**3*d**5*x**5 - 90*sqrt(a*d**2 + b*c**2)*log...
```

3.253 $\int (c+dx)^4 \left(a+bx^2\right)^{5/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 307

$$\begin{aligned}
\int (c+dx)^4 \left(a+bx^2\right)^{5/2} dx &= \frac{a^2(80b^2c^4-60abc^2d^2+3a^2d^4)\,x\sqrt{a+bx^2}}{256b^2} \\
&+ \frac{a(80b^2c^4-60abc^2d^2+3a^2d^4)\,x(a+bx^2)^{3/2}}{384b^2} \\
&+ \frac{(80b^2c^4-60abc^2d^2+3a^2d^4)\,x(a+bx^2)^{5/2}}{480b^2} \\
&+ \frac{13cd(c+dx)^2\left(a+bx^2\right)^{7/2}}{90b} + \frac{d(c+dx)^3\left(a+bx^2\right)^{7/2}}{10b} \\
&+ \frac{d(16c(103bc^2-40ad^2)+7d(116bc^2-27ad^2)\,x)\left(a+bx^2\right)^{7/2}}{5040b^2} \\
&+ \frac{a^3(80b^2c^4-60abc^2d^2+3a^2d^4)\,\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}
\end{aligned}$$

output
```
1/256*a^2*(3*a^2*d^4-60*a*b*c^2*d^2+80*b^2*c^4)*x*(b*x^2+a)^(1/2)/b^2+1/38
4*a*(3*a^2*d^4-60*a*b*c^2*d^2+80*b^2*c^4)*x*(b*x^2+a)^(3/2)/b^2+1/480*(3*a
^2*d^4-60*a*b*c^2*d^2+80*b^2*c^4)*x*(b*x^2+a)^(5/2)/b^2+13/90*c*d*(d*x+c)^
2*(b*x^2+a)^(7/2)/b+1/10*d*(d*x+c)^3*(b*x^2+a)^(7/2)/b+1/5040*d*(16*c*(-40
*a*d^2+103*b*c^2)+7*d*(-27*a*d^2+116*b*c^2)*x)*(b*x^2+a)^(7/2)/b^2+1/256*a
^3*(3*a^2*d^4-60*a*b*c^2*d^2+80*b^2*c^4)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2)
)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.92

$$\int (c+dx)^4 (a+bx^2)^{5/2} \, dx = \frac{\sqrt{b}\sqrt{a+bx^2}(-5a^4d^3(2048c+189dx)+10a^3bd(4608c^3+1890c^2dx+512cd^2x^2+63d^3x^3)+$$

input
```
Integrate[(c + d*x)^4*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-5*a^4*d^3*(2048*c + 189*d*x) + 10*a^3*b*d*(4608
*c^3 + 1890*c^2*d*x + 512*c*d^2*x^2 + 63*d^3*x^3) + 64*b^4*x^5*(210*c^4 +
720*c^3*d*x + 945*c^2*d^2*x^2 + 560*c*d^3*x^3 + 126*d^4*x^4) + 24*a^2*b^2*
x*(2310*c^4 + 5760*c^3*d*x + 6195*c^2*d^2*x^2 + 3200*c*d^3*x^3 + 651*d^4*x
^4) + 16*a*b^3*x^3*(2730*c^4 + 8640*c^3*d*x + 10710*c^2*d^2*x^2 + 6080*c*d
^3*x^3 + 1323*d^4*x^4)) - 315*a^3*(80*b^2*c^4 - 60*a*b*c^2*d^2 + 3*a^2*d^4
)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(80640*b^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.83, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {497, 687, 27, 676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{5/2} (c+dx)^4 \, dx$$

$$\Big\downarrow \; 497$$

$$\frac{\int (c+dx)^2 (10bc^2+13bdxc-3ad^2) (bx^2+a)^{5/2} dx}{10b} + \frac{d(a+bx^2)^{7/2} (c+dx)^3}{10b}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{\int b(c+dx)\left(c\left(90bc^2-53ad^2\right)+d\left(116bc^2-27ad^2\right)x\right)\left(bx^2+a\right)^{5/2}dx}{9b}+\frac{13}{9}cd\left(a+bx^2\right)^{7/2}(c+dx)^2}{10b}+\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$\downarrow$ 27

$$\frac{\frac{1}{9}\int (c+dx)\left(c\left(90bc^2-53ad^2\right)+d\left(116bc^2-27ad^2\right)x\right)\left(bx^2+a\right)^{5/2}dx+\frac{13}{9}cd\left(a+bx^2\right)^{7/2}(c+dx)^2}{10b}+\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$\downarrow$ 676

$$\frac{\frac{1}{9}\left(\frac{9\left(3a^2d^4-60abc^2d^2+80b^2c^4\right)\int \left(bx^2+a\right)^{5/2}dx}{8b}+\frac{2cd\left(a+bx^2\right)^{7/2}\left(103bc^2-40ad^2\right)}{7b}+\frac{d^2x\left(a+bx^2\right)^{7/2}\left(116bc^2-27ad^2\right)}{8b}\right)+\frac{13}{9}cd(a+bx}{10b}$$
$$\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$\downarrow$ 211

$$\frac{\frac{1}{9}\left(\frac{9\left(3a^2d^4-60abc^2d^2+80b^2c^4\right)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{2cd\left(a+bx^2\right)^{7/2}\left(103bc^2-40ad^2\right)}{7b}+\frac{d^2x\left(a+bx^2\right)^{7/2}\left(116bc^2-27ad^2\right.}{8b}\right.}{10b}$$
$$\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$\downarrow$ 211

$$\frac{\frac{1}{9}\left(\frac{9\left(3a^2d^4-60abc^2d^2+80b^2c^4\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{2cd\left(a+bx^2\right)^{7/2}\left(103bc^2-40ad^2\right)}{7b}+\frac{d^2x(a+bx^2}{}\right.}{10b}$$
$$\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$\downarrow$ 211

$$\frac{\frac{1}{9}\left(\frac{9(3a^2d^4-60abc^2d^2+80b^2c^4)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)}{8b}+\frac{2cd(a+bx^2)^{7/2}(103bc^2-40}{7b}\right.}{10b}$$

$$\frac{d(a+bx^2)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \color{blue}{224}$$

$$\frac{\frac{1}{9}\left(\frac{9(3a^2d^4-60abc^2d^2+80b^2c^4)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)}{8b}+\frac{2cd(a+bx^2)^{7/2}(}{7}\right.}{10b}$$

$$\frac{d(a+bx^2)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{\frac{1}{9}\left(\frac{9\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)(3a^2d^4-60abc^2d^2+80b^2c^4)}{8b}+\frac{2cd(a+bx^2)^{7/2}(1}{7b}\right.}{10b}$$

$$\frac{d(a+bx^2)^{7/2}(c+dx)^3}{10b}$$

input
```
Int[(c + d*x)^4*(a + b*x^2)^(5/2),x]
```

output
```
(d*(c + d*x)^3*(a + b*x^2)^(7/2))/(10*b) + ((13*c*d*(c + d*x)^2*(a + b*x^2
)^(7/2))/9 + ((2*c*d*(103*b*c^2 - 40*a*d^2)*(a + b*x^2)^(7/2))/(7*b) + (d^
2*(116*b*c^2 - 27*a*d^2)*x*(a + b*x^2)^(7/2))/(8*b) + (9*(80*b^2*c^4 - 60*
a*b*c^2*d^2 + 3*a^2*d^4)*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(
3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(2*Sqrt[b])))/4))/6))/(8*b))/9)/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{\left(-8064 b^4 d^4 x^9-35840 b^4 c\, d^3 x^8-21168 a\, b^3 d^4 x^7-60480 b^4 c^2 d^2 x^7-97280 a\, b^3 c\, d^3 x^6-46080 b^4 c^3 d\, x^6-15624 a^2 b^2 d^4 x^5-171360 a\, b^3$
default	$c^4\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d^4\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8b}-\right.}{}\right.$

input
```
int((d*x+c)^4*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/80640/b^2*(-8064*b^4*d^4*x^9-35840*b^4*c*d^3*x^8-21168*a*b^3*d^4*x^7-60
480*b^4*c^2*d^2*x^7-97280*a*b^3*c*d^3*x^6-46080*b^4*c^3*d*x^6-15624*a^2*b^
2*d^4*x^5-171360*a*b^3*c^2*d^2*x^5-13440*b^4*c^4*x^5-76800*a^2*b^2*c*d^3*x
^4-138240*a*b^3*c^3*d*x^4-630*a^3*b*d^4*x^3-148680*a^2*b^2*c^2*d^2*x^3-436
80*a*b^3*c^4*x^3-5120*a^3*b*c*d^3*x^2-138240*a^2*b^2*c^3*d*x^2+945*a^4*d^4
*x-18900*a^3*b*c^2*d^2*x-55440*a^2*b^2*c^4*x+10240*a^4*c*d^3-46080*a^3*b*c
^3*d)*(b*x^2+a)^(1/2)+1/256*a^3*(3*a^2*d^4-60*a*b*c^2*d^2+80*b^2*c^4)/b^(5
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.25

$$\int (c+dx)^4 (a+bx^2)^{5/2}\,dx = \left[\frac{315\,(80\,a^3b^2c^4-60\,a^4bc^2d^2+3\,a^5d^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8064\,b^5d^4 \cdots}{\cdots}\right.$$

$$-\frac{315\,(80\,a^3b^2c^4-60\,a^4bc^2d^2+3\,a^5d^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8064\,b^5d^4x^9+35840\,b^5cd^3x^8+46080\,a^3b^2 \cdots}{\cdots}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/161280*(315*(80*a^3*b^2*c^4 - 60*a^4*b*c^2*d^2 + 3*a^5*d^4)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8064*b^5*d^4*x^9 + 35840
*b^5*c*d^3*x^8 + 46080*a^3*b^2*c^3*d - 10240*a^4*b*c*d^3 + 3024*(20*b^5*c^
2*d^2 + 7*a*b^4*d^4)*x^7 + 5120*(9*b^5*c^3*d + 19*a*b^4*c*d^3)*x^6 + 168*(
80*b^5*c^4 + 1020*a*b^4*c^2*d^2 + 93*a^2*b^3*d^4)*x^5 + 15360*(9*a*b^4*c^3
*d + 5*a^2*b^3*c*d^3)*x^4 + 210*(208*a*b^4*c^4 + 708*a^2*b^3*c^2*d^2 + 3*a
^3*b^2*d^4)*x^3 + 5120*(27*a^2*b^3*c^3*d + a^3*b^2*c*d^3)*x^2 + 315*(176*a
^2*b^3*c^4 + 60*a^3*b^2*c^2*d^2 - 3*a^4*b*d^4)*x)*sqrt(b*x^2 + a))/b^3, -1
/80640*(315*(80*a^3*b^2*c^4 - 60*a^4*b*c^2*d^2 + 3*a^5*d^4)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*b^5*d^4*x^9 + 35840*b^5*c*d^3*x^8 +
46080*a^3*b^2*c^3*d - 10240*a^4*b*c*d^3 + 3024*(20*b^5*c^2*d^2 + 7*a*b^4*d
^4)*x^7 + 5120*(9*b^5*c^3*d + 19*a*b^4*c*d^3)*x^6 + 168*(80*b^5*c^4 + 1020
*a*b^4*c^2*d^2 + 93*a^2*b^3*d^4)*x^5 + 15360*(9*a*b^4*c^3*d + 5*a^2*b^3*c*
d^3)*x^4 + 210*(208*a*b^4*c^4 + 708*a^2*b^3*c^2*d^2 + 3*a^3*b^2*d^4)*x^3 +
 5120*(27*a^2*b^3*c^3*d + a^3*b^2*c*d^3)*x^2 + 315*(176*a^2*b^3*c^4 + 60*a
^3*b^2*c^2*d^2 - 3*a^4*b*d^4)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(298) = 596$.

Time = 0.69 (sec) , antiderivative size = 858, normalized size of antiderivative = 2.79

$$\int (c+dx)^4 \left(a+bx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(4*b**2*c*d**3*x**8/9 + b**2*d**4*x**9/10 + x*
*7*(21*a*b**2*d**4/10 + 6*b**3*c**2*d**2)/(8*b) + x**6*(76*a*b**2*c*d**3/9
 + 4*b**3*c**3*d)/(7*b) + x**5*(3*a**2*b*d**4 + 18*a*b**2*c**2*d**2 - 7*a*
(21*a*b**2*d**4/10 + 6*b**3*c**2*d**2)/(8*b) + b**3*c**4)/(6*b) + x**4*(12
*a**2*b*c*d**3 + 12*a*b**2*c**3*d - 6*a*(76*a*b**2*c*d**3/9 + 4*b**3*c**3*
d)/(7*b))/(5*b) + x**3*(a**3*d**4 + 18*a**2*b*c**2*d**2 + 3*a*b**2*c**4 -
5*a*(3*a**2*b*d**4 + 18*a*b**2*c**2*d**2 - 7*a*(21*a*b**2*d**4/10 + 6*b**3
*c**2*d**2)/(8*b) + b**3*c**4)/(6*b))/(4*b) + x**2*(4*a**3*c*d**3 + 12*a**
2*b*c**3*d - 4*a*(12*a**2*b*c*d**3 + 12*a*b**2*c**3*d - 6*a*(76*a*b**2*c*d
**3/9 + 4*b**3*c**3*d)/(7*b))/(5*b))/(3*b) + x*(6*a**3*c**2*d**2 + 3*a**2*
b*c**4 - 3*a*(a**3*d**4 + 18*a**2*b*c**2*d**2 + 3*a*b**2*c**4 - 5*a*(3*a**
2*b*d**4 + 18*a*b**2*c**2*d**2 - 7*a*(21*a*b**2*d**4/10 + 6*b**3*c**2*d**2
)/(8*b) + b**3*c**4)/(6*b))/(4*b))/(2*b) + (4*a**3*c**3*d - 2*a*(4*a**3*c*
d**3 + 12*a**2*b*c**3*d - 4*a*(12*a**2*b*c*d**3 + 12*a*b**2*c**3*d - 6*a*(
76*a*b**2*c*d**3/9 + 4*b**3*c**3*d)/(7*b))/(5*b))/(3*b))/b) + (a**3*c**4 -
 a*(6*a**3*c**2*d**2 + 3*a**2*b*c**4 - 3*a*(a**3*d**4 + 18*a**2*b*c**2*d**
2 + 3*a*b**2*c**4 - 5*a*(3*a**2*b*d**4 + 18*a*b**2*c**2*d**2 - 7*a*(21*a*b
**2*d**4/10 + 6*b**3*c**2*d**2)/(8*b) + b**3*c**4)/(6*b))/(4*b))/(2*b))*Pi
ecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*lo
g(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*Piecewise((c**4*x, Eq(d...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.19

$$\int (c+dx)^4 \left(a+bx^2\right)^{5/2} dx = \frac{(bx^2+a)^{\frac{7}{2}} d^4 x^3}{10\,b} + \frac{4\,(bx^2+a)^{\frac{7}{2}} c d^3 x^2}{9\,b} + \frac{1}{6}\,(bx^2+a)^{\frac{5}{2}} c^4 x + \frac{5}{24}\,(bx^2+a)^{\frac{3}{2}} a c^4 x + \frac{5}{16}\,\sqrt{bx^2+a} a^2 c^4 x + \frac{3\,(bx^2+a)^{\frac{7}{2}} c^2 d^2 x}{4\,b} - \frac{(bx^2+a)^{\frac{5}{2}} a c^2 d^2 x}{8\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}} a^2 c^2 d^2 x}{32\,b} - \frac{15\,\sqrt{bx^2+a} a^3 c^2 d^2 x}{64\,b} - \frac{3\,(bx^2+a)^{\frac{7}{2}} a d^4 x}{80\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}} a^2 d^4 x}{160\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}} a^3 d^4 x}{128\,b^2} + \frac{3\,\sqrt{bx^2+a} a^4 d^4 x}{256\,b^2} + \frac{5\,a^3 c^4 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} - \frac{15\,a^4 c^2 d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{64\,b^{\frac{3}{2}}} + \frac{3\,a^5 d^4 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}} + \frac{4\,(bx^2+a)^{\frac{7}{2}} c^3 d}{7\,b} - \frac{8\,(bx^2+a)^{\frac{7}{2}} a c d^3}{63\,b^2}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(7/2)*d^4*x^3/b + 4/9*(b*x^2 + a)^(7/2)*c*d^3*x^2/b + 1/6
*(b*x^2 + a)^(5/2)*c^4*x + 5/24*(b*x^2 + a)^(3/2)*a*c^4*x + 5/16*sqrt(b*x^
2 + a)*a^2*c^4*x + 3/4*(b*x^2 + a)^(7/2)*c^2*d^2*x/b - 1/8*(b*x^2 + a)^(5/
2)*a*c^2*d^2*x/b - 5/32*(b*x^2 + a)^(3/2)*a^2*c^2*d^2*x/b - 15/64*sqrt(b*x
^2 + a)*a^3*c^2*d^2*x/b - 3/80*(b*x^2 + a)^(7/2)*a*d^4*x/b^2 + 1/160*(b*x^
2 + a)^(5/2)*a^2*d^4*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*d^4*x/b^2 + 3/256
*sqrt(b*x^2 + a)*a^4*d^4*x/b^2 + 5/16*a^3*c^4*arcsinh(b*x/sqrt(a*b))/sqrt(
b) - 15/64*a^4*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/256*a^5*d^4*arcs
inh(b*x/sqrt(a*b))/b^(5/2) + 4/7*(b*x^2 + a)^(7/2)*c^3*d/b - 8/63*(b*x^2 +
 a)^(7/2)*a*c*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.21

$$\int (c+dx)^4 (a+bx^2)^{5/2}\,dx = \frac{1}{80640}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\left(9\,b^2d^4x+40\,b^2cd^3\right)x+\frac{27\left(20\,b^{10}c^2d^2+7\,ab^9d^4\right)}{b^8}\right)x\right.\right.\right.\right.\right.\right.\right. - \frac{\left(80\,a^3b^2c^4-60\,a^4bc^2d^2+3\,a^5d^4\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^4*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/80640*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*b^2*d^4*x + 40*b^2*c*d^3)*x +
 27*(20*b^10*c^2*d^2 + 7*a*b^9*d^4)/b^8)*x + 320*(9*b^10*c^3*d + 19*a*b^9*
c*d^3)/b^8)*x + 21*(80*b^10*c^4 + 1020*a*b^9*c^2*d^2 + 93*a^2*b^8*d^4)/b^8
)*x + 1920*(9*a*b^9*c^3*d + 5*a^2*b^8*c*d^3)/b^8)*x + 105*(208*a*b^9*c^4 +
 708*a^2*b^8*c^2*d^2 + 3*a^3*b^7*d^4)/b^8)*x + 2560*(27*a^2*b^8*c^3*d + a^
3*b^7*c*d^3)/b^8)*x + 315*(176*a^2*b^8*c^4 + 60*a^3*b^7*c^2*d^2 - 3*a^4*b^
6*d^4)/b^8)*x + 5120*(9*a^3*b^7*c^3*d - 2*a^4*b^6*c*d^3)/b^8) - 1/256*(80*
a^3*b^2*c^4 - 60*a^4*b*c^2*d^2 + 3*a^5*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \left(a+bx^2\right)^{5/2}\,dx = \int \left(b\,x^2+a\right)^{5/2}\,(c+d\,x)^4\,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x)^4,x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \left(a+bx^2\right)^{5/2} dx = \int (dx+c)^4 \left(b\,x^2+a\right)^{\frac{5}{2}} dx$$

input
```
int((d*x+c)^4*(b*x^2+a)^(5/2),x)
```

output
```
int((d*x+c)^4*(b*x^2+a)^(5/2),x)
```

3.254 $\int (c+dx)^3 \left(a+bx^2\right)^{5/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 216

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} dx = \frac{5a^2c(8bc^2-3ad^2)\,x\sqrt{a+bx^2}}{128b} + \frac{5ac(8bc^2-3ad^2)\,x(a+bx^2)^{3/2}}{192b} + \frac{c(8bc^2-3ad^2)\,x(a+bx^2)^{5/2}}{48b} + \frac{d(c+dx)^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{d(16(10bc^2-ad^2)+77bcdx)\left(a+bx^2\right)^{7/2}}{504b^2} + \frac{5a^3c(8bc^2-3ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/128*a^2*c*(-3*a*d^2+8*b*c^2)*x*(b*x^2+a)^(1/2)/b+5/192*a*c*(-3*a*d^2+8*b
*c^2)*x*(b*x^2+a)^(3/2)/b+1/48*c*(-3*a*d^2+8*b*c^2)*x*(b*x^2+a)^(5/2)/b+1/
9*d*(d*x+c)^2*(b*x^2+a)^(7/2)/b+1/504*d*(77*b*c*d*x-16*a*d^2+160*b*c^2)*(b
*x^2+a)^(7/2)/b^2+5/128*a^3*c*(-3*a*d^2+8*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+
a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} dx = \frac{\sqrt{a+bx^2}(-256a^4d^3+a^3bd(3456c^2+945cdx+128d^2x^2)+16b^4x^5(84c^3+216c^2dx+189cd$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-256*a^4*d^3 + a^3*b*d*(3456*c^2 + 945*c*d*x + 128*d^2*x
^2) + 16*b^4*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3*x^3) + 8*a
*b^3*x^3*(546*c^3 + 1296*c^2*d*x + 1071*c*d^2*x^2 + 304*d^3*x^3) + 6*a^2*b
^2*x*(924*c^3 + 1728*c^2*d*x + 1239*c*d^2*x^2 + 320*d^3*x^3)) + 315*a^3*Sq
rt[b]*c*(-8*b*c^2 + 3*a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8064*b^
2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {497, 676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} (c+dx)^3 \, dx$$

$$\Big\downarrow \text{ 497}$$

$$\frac{\int (c+dx) \left(9bc^2+11bdxc-2ad^2\right) \left(bx^2+a\right)^{5/2} dx}{9b} + \frac{d\left(a+bx^2\right)^{7/2} (c+dx)^2}{9b}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{9}{8}c\left(8bc^2-3ad^2\right)\int \left(bx^2+a\right)^{5/2}dx+\frac{2d\left(a+bx^2\right)^{7/2}\left(10bc^2-ad^2\right)}{7b}+\frac{11}{8}cd^2x\left(a+bx^2\right)^{7/2}}{9b}+\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{9}{8}c\left(8bc^2-3ad^2\right)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{2d\left(a+bx^2\right)^{7/2}\left(10bc^2-ad^2\right)}{7b}+\frac{11}{8}cd^2x\left(a+bx^2\right)^{7/2}}{9b}+\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{9}{8}c\left(8bc^2-3ad^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{2d\left(a+bx^2\right)^{7/2}\left(10bc^2-ad^2\right)}{7b}+\frac{11}{8}cd^2\cdots}{9b}$$
$$\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{9}{8}c\left(8bc^2-3ad^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{2d\left(a+bx^2\right)^{7/2}(\cdots}{7b}}{9b}$$
$$\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{9}{8}c\left(8bc^2-3ad^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{2d(a\cdots}{}}{9b}$$
$$\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{9}{8}c\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)\left(8bc^2-3ad^2\right)+\frac{2d(a\cdots}{}}{9b}$$
$$\frac{d\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

input
```
Int[(c + d*x)^3*(a + b*x^2)^(5/2),x]
```

output
```
(d*(c + d*x)^2*(a + b*x^2)^(7/2))/(9*b) + ((2*d*(10*b*c^2 - a*d^2)*(a + b*
x^2)^(7/2))/(7*b) + (11*c*d^2*x*(a + b*x^2)^(7/2))/8 + (9*c*(8*b*c^2 - 3*a
*d^2)*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*
Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))
)/4))/6))/8)/(9*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03

method	result
default	$c^{3} \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)+d^{3} \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{7}{2}}}{9 b}-\frac{2 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}{63 b^{2}}\right) -$
risch	$-\frac{\left(-896 b^{4} d^{3} x^{8}-3024 b^{4} c \,d^{2} x^{7}-2432 a \,b^{3} d^{3} x^{6}-3456 b^{4} c^{2} d \,x^{6}-8568 a \,b^{3} c \,d^{2} x^{5}-1344 b^{4} c^{3} x^{5}-1920 a^{2} b^{2} d^{3} x^{4}-10368 a \,b^{3} c^{2} d \,x^{4}-743}{8064 b}$

input
```
int((d*x+c)^3*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c^3*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^
2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+d^3*(1/9*x^2*(b*
x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+3*c*d^2*(1/8*x*(b*x^2+a)^(7/2)/
b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x
*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+3/7*c^2*d
*(b*x^2+a)^(7/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.40

$$\int (c+dx)^3 (a+bx^2)^{5/2}\, dx = \left[-\frac{315\,(8\,a^3bc^3-3\,a^4cd^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(896\,b^4d^3x^8+3024\,b^4c\cdots}{\cdots}, -\frac{315\,(8\,a^3bc^3-3\,a^4cd^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(896\,b^4d^3x^8+3024\,b^4cd^2x^7+3456\,a^3bc^2d-256\,a^4d^3+1\cdots}{\cdots}\right.$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/16128*(315*(8*a^3*b*c^3 - 3*a^4*c*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) - 2*(896*b^4*d^3*x^8 + 3024*b^4*c*d^2*x^7 + 3456*
a^3*b*c^2*d - 256*a^4*d^3 + 128*(27*b^4*c^2*d + 19*a*b^3*d^3)*x^6 + 168*(8
*b^4*c^3 + 51*a*b^3*c*d^2)*x^5 + 384*(27*a*b^3*c^2*d + 5*a^2*b^2*d^3)*x^4
+ 42*(104*a*b^3*c^3 + 177*a^2*b^2*c*d^2)*x^3 + 128*(81*a^2*b^2*c^2*d + a^3
*b*d^3)*x^2 + 63*(88*a^2*b^2*c^3 + 15*a^3*b*c*d^2)*x)*sqrt(b*x^2 + a))/b^2
, -1/8064*(315*(8*a^3*b*c^3 - 3*a^4*c*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (896*b^4*d^3*x^8 + 3024*b^4*c*d^2*x^7 + 3456*a^3*b*c^2*d -
256*a^4*d^3 + 128*(27*b^4*c^2*d + 19*a*b^3*d^3)*x^6 + 168*(8*b^4*c^3 + 51*
a*b^3*c*d^2)*x^5 + 384*(27*a*b^3*c^2*d + 5*a^2*b^2*d^3)*x^4 + 42*(104*a*b^
3*c^3 + 177*a^2*b^2*c*d^2)*x^3 + 128*(81*a^2*b^2*c^2*d + a^3*b*d^3)*x^2 +
63*(88*a^2*b^2*c^3 + 15*a^3*b*c*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(206) = 412$.

Time = 0.62 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.86

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2}\,dx = \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{3b^2cd^2x^7}{8}+\frac{b^2d^3x^8}{9}+\frac{x^6\cdot\left(\frac{19ab^2d^3}{9}+3b^3c^2d\right)}{7b}+\frac{x^5\cdot\left(\frac{51ab^2cd^2}{8}+b^3c^3\right)}{6b}+\frac{x^4\cdot\left(3a^2bd^3+9ab^2c^2d-\cdots\right)}{5b}\cdots\right. \\ a^{\frac{5}{2}}\left(\begin{cases} c^3x & \text{for } d=0 \\ \frac{(c+dx)^4}{4d} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(3*b**2*c*d**2*x**7/8 + b**2*d**3*x**8/9 + x**
6*(19*a*b**2*d**3/9 + 3*b**3*c**2*d)/(7*b) + x**5*(51*a*b**2*c*d**2/8 + b*
*3*c**3)/(6*b) + x**4*(3*a**2*b*d**3 + 9*a*b**2*c**2*d - 6*a*(19*a*b**2*d*
*3/9 + 3*b**3*c**2*d)/(7*b))/(5*b) + x**3*(9*a**2*b*c*d**2 + 3*a*b**2*c**3
 - 5*a*(51*a*b**2*c*d**2/8 + b**3*c**3)/(6*b))/(4*b) + x**2*(a**3*d**3 + 9
*a**2*b*c**2*d - 4*a*(3*a**2*b*d**3 + 9*a*b**2*c**2*d - 6*a*(19*a*b**2*d**
3/9 + 3*b**3*c**2*d)/(7*b))/(5*b))/(3*b) + x*(3*a**3*c*d**2 + 3*a**2*b*c**
3 - 3*a*(9*a**2*b*c*d**2 + 3*a*b**2*c**3 - 5*a*(51*a*b**2*c*d**2/8 + b**3*
c**3)/(6*b))/(4*b))/(2*b) + (3*a**3*c**2*d - 2*a*(a**3*d**3 + 9*a**2*b*c**
2*d - 4*a*(3*a**2*b*d**3 + 9*a*b**2*c**2*d - 6*a*(19*a*b**2*d**3/9 + 3*b**
3*c**2*d)/(7*b))/(5*b))/(3*b))/b) + (a**3*c**3 - a*(3*a**3*c*d**2 + 3*a**2
*b*c**3 - 3*a*(9*a**2*b*c*d**2 + 3*a*b**2*c**3 - 5*a*(51*a*b**2*c*d**2/8 +
 b**3*c**3)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2)
 + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (
a**(5/2)*Piecewise((c**3*x, Eq(d, 0)), ((c + d*x)**4/(4*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}d^3x^2}{9\,b} + \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}c^3x + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ac^3x + \frac{5}{16}\sqrt{bx^2+a}a^2c^3x + \frac{3\,(bx^2+a)^{\frac{7}{2}}cd^2x}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}acd^2x}{16\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2cd^2x}{64\,b} - \frac{15\,\sqrt{bx^2+a}a^3cd^2x}{128\,b} + \frac{5\,a^3c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} - \frac{15\,a^4cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{3\,(bx^2+a)^{\frac{7}{2}}c^2d}{7\,b} - \frac{2\,(bx^2+a)^{\frac{7}{2}}ad^3}{63\,b^2}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(7/2)*d^3*x^2/b + 1/6*(b*x^2 + a)^(5/2)*c^3*x + 5/24*(b*x^
2 + a)^(3/2)*a*c^3*x + 5/16*sqrt(b*x^2 + a)*a^2*c^3*x + 3/8*(b*x^2 + a)^(7
/2)*c*d^2*x/b - 1/16*(b*x^2 + a)^(5/2)*a*c*d^2*x/b - 5/64*(b*x^2 + a)^(3/2
)*a^2*c*d^2*x/b - 15/128*sqrt(b*x^2 + a)*a^3*c*d^2*x/b + 5/16*a^3*c^3*arcs
inh(b*x/sqrt(a*b))/sqrt(b) - 15/128*a^4*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/
2) + 3/7*(b*x^2 + a)^(7/2)*c^2*d/b - 2/63*(b*x^2 + a)^(7/2)*a*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.32

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2}\,dx = \frac{1}{8064}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\,b^2d^3x+27\,b^2cd^2\right)x+\frac{8\,(27\,b^9c^2d+19\,ab^8d^3)}{b^7}\right)x+\frac{21\,(8 \cdots}{}\right.\right.\right.\right.\right.\right. - \frac{5\,(8\,a^3bc^3-3\,a^4cd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/8064*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*b^2*d^3*x + 27*b^2*c*d^2)*x + 8*(
27*b^9*c^2*d + 19*a*b^8*d^3)/b^7)*x + 21*(8*b^9*c^3 + 51*a*b^8*c*d^2)/b^7)
*x + 48*(27*a*b^8*c^2*d + 5*a^2*b^7*d^3)/b^7)*x + 21*(104*a*b^8*c^3 + 177*
a^2*b^7*c*d^2)/b^7)*x + 64*(81*a^2*b^7*c^2*d + a^3*b^6*d^3)/b^7)*x + 63*(8
8*a^2*b^7*c^3 + 15*a^3*b^6*c*d^2)/b^7)*x + 128*(27*a^3*b^6*c^2*d - 2*a^4*b
^5*d^3)/b^7) - 5/128*(8*a^3*b*c^3 - 3*a^4*c*d^2)*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} dx = \int \left(b\,x^2+a\right)^{5/2} (c+d\,x)^3\, dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.79

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} dx = \frac{-256\sqrt{b\,x^2+a}\,a^4d^3+3456\sqrt{b\,x^2+a}\,a^3b\,c^2d+945\sqrt{b\,x^2+a}\,a^3bc\,d^2x+128\sqrt{b\,x^2+a}\,a^3b\,d}{}$$

input
```
int((d*x+c)^3*(b*x^2+a)^(5/2),x)
```

output
```
( - 256*sqrt(a + b*x**2)*a**4*d**3 + 3456*sqrt(a + b*x**2)*a**3*b*c**2*d +
 945*sqrt(a + b*x**2)*a**3*b*c*d**2*x + 128*sqrt(a + b*x**2)*a**3*b*d**3*x
**2 + 5544*sqrt(a + b*x**2)*a**2*b**2*c**3*x + 10368*sqrt(a + b*x**2)*a**2
*b**2*c**2*d*x**2 + 7434*sqrt(a + b*x**2)*a**2*b**2*c*d**2*x**3 + 1920*sqr
t(a + b*x**2)*a**2*b**2*d**3*x**4 + 4368*sqrt(a + b*x**2)*a*b**3*c**3*x**3
 + 10368*sqrt(a + b*x**2)*a*b**3*c**2*d*x**4 + 8568*sqrt(a + b*x**2)*a*b**
3*c*d**2*x**5 + 2432*sqrt(a + b*x**2)*a*b**3*d**3*x**6 + 1344*sqrt(a + b*x
**2)*b**4*c**3*x**5 + 3456*sqrt(a + b*x**2)*b**4*c**2*d*x**6 + 3024*sqrt(a
 + b*x**2)*b**4*c*d**2*x**7 + 896*sqrt(a + b*x**2)*b**4*d**3*x**8 - 945*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*c*d**2 + 2520*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c**3)/(8064*b**2)
```

3.255 $\int (c+dx)^2 \left(a+bx^2\right)^{5/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 172

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} dx = \frac{5a^2(8bc^2-ad^2)\,x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc^2-ad^2)\,x(a+bx^2)^{3/2}}{192b} + \frac{(8bc^2-ad^2)\,x(a+bx^2)^{5/2}}{48b} + \frac{d(16c+7dx)\,(a+bx^2)^{7/2}}{56b} + \frac{5a^3(8bc^2-ad^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output

```
5/128*a^2*(-a*d^2+8*b*c^2)*x*(b*x^2+a)^(1/2)/b+5/192*a*(-a*d^2+8*b*c^2)*x*
(b*x^2+a)^(3/2)/b+1/48*(-a*d^2+8*b*c^2)*x*(b*x^2+a)^(5/2)/b+1/56*d*(7*d*x+
16*c)*(b*x^2+a)^(7/2)/b+5/128*a^3*(-a*d^2+8*b*c^2)*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int (c+dx)^2 (a+bx^2)^{5/2}\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(3a^3d(256c+35dx)+16b^3x^5(28c^2+48cdx+21d^2x^2)+8ab^2x^3(182c^2+288cd$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(3*a^3*d*(256*c + 35*d*x) + 16*b^3*x^5*(28*c^2 +
48*c*d*x + 21*d^2*x^2) + 8*a*b^2*x^3*(182*c^2 + 288*c*d*x + 119*d^2*x^2) +
 2*a^2*b*x*(924*c^2 + 1152*c*d*x + 413*d^2*x^2)) + 105*a^3*(-8*b*c^2 + a*d
^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2688*b^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {497, 455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} (c+dx)^2 \, dx$$

$$\Big\downarrow \text{497}$$

$$\frac{\int \left(8bc^2+9bdxc-ad^2\right) \left(bx^2+a\right)^{5/2} dx}{8b}+\frac{d\left(a+bx^2\right)^{7/2} (c+dx)}{8b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(8bc^2-ad^2\right) \int \left(bx^2+a\right)^{5/2} dx+\frac{9}{7}cd\left(a+bx^2\right)^{7/2}}{8b}+\frac{d\left(a+bx^2\right)^{7/2} (c+dx)}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8bc^2-ad^2)\left(\frac{5}{6}a\int (bx^2+a)^{3/2}dx+\frac{1}{6}x(a+bx^2)^{5/2}\right)+\frac{9}{7}cd(a+bx^2)^{7/2}}{8b}+\frac{d(a+bx^2)^{7/2}(c+dx)}{8b}$$

$\Big\downarrow$ 211

$$\frac{(8bc^2-ad^2)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)+\frac{9}{7}cd(a+bx^2)^{7/2}}{8b}+\frac{d(a+bx^2)^{7/2}(c+dx)}{8b}$$

$\Big\downarrow$ 211

$$\frac{(8bc^2-ad^2)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)+\frac{9}{7}cd(a+bx^2)^{7/2}}{8b}+\frac{d(a+bx^2)^{7/2}(c+dx)}{8b}$$

$\Big\downarrow$ 224

$$\frac{(8bc^2-ad^2)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)+\frac{9}{7}cd(a-}{8b}+\frac{d(a+bx^2)^{7/2}(c+dx)}{8b}$$

$\Big\downarrow$ 219

$$\frac{\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)(8bc^2-ad^2)+\frac{9}{7}cd(a+}{8b}+\frac{d(a+bx^2)^{7/2}(c+dx)}{8b}$$

input
```
Int[(c + d*x)^2*(a + b*x^2)^(5/2),x]
```

output
```
(d*(c + d*x)*(a + b*x^2)^(7/2))/(8*b) + ((9*c*d*(a + b*x^2)^(7/2))/7 + (8*
b*c^2 - a*d^2)*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 +
(3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*
Sqrt[b])))/4))/6))/(8*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(336b^3d^2x^7+768b^3cd\,x^6+952a\,b^2d^2x^5+448b^3c^2x^5+2304cda\,b^2x^4+826d^2x^3a^2b+1456a\,b^2c^2x^3+2304x^2a^2bcd+105a^3d^2x+1848a^2b}{2688b}$
default	$c^2\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d^2\left(\frac{x(b\,x^2+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\right.}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2688/b*(336*b^3*d^2*x^7+768*b^3*c*d*x^6+952*a*b^2*d^2*x^5+448*b^3*c^2*x^
5+2304*a*b^2*c*d*x^4+826*a^2*b*d^2*x^3+1456*a*b^2*c^2*x^3+2304*a^2*b*c*d*x
^2+105*a^3*d^2*x+1848*a^2*b*c^2*x+768*a^3*c*d)*(b*x^2+a)^(1/2)-5/128*a^3*(
a*d^2-8*b*c^2)/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.21

$$\int (c+dx)^2\left(a+bx^2\right)^{5/2}\,dx=\left[-\frac{105\left(8\,a^3bc^2-a^4d^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(336\,b^4d^2x^7+768\,b^4cdx^6\right.}{}\right.$$

$$-\frac{105\left(8\,a^3bc^2-a^4d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(336\,b^4d^2x^7+768\,b^4cdx^6+2304\,ab^3cdx^4+2304\,a^2b^2cdx^2+\right.}{268}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/5376*(105*(8*a^3*b*c^2 - a^4*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - 2*(336*b^4*d^2*x^7 + 768*b^4*c*d*x^6 + 2304*a*b^3*c*
d*x^4 + 2304*a^2*b^2*c*d*x^2 + 768*a^3*b*c*d + 56*(8*b^4*c^2 + 17*a*b^3*d^
2)*x^5 + 14*(104*a*b^3*c^2 + 59*a^2*b^2*d^2)*x^3 + 21*(88*a^2*b^2*c^2 + 5*
a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^2, -1/2688*(105*(8*a^3*b*c^2 - a^4*d^2)*s
qrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (336*b^4*d^2*x^7 + 768*b^4*c*
d*x^6 + 2304*a*b^3*c*d*x^4 + 2304*a^2*b^2*c*d*x^2 + 768*a^3*b*c*d + 56*(8*
b^4*c^2 + 17*a*b^3*d^2)*x^5 + 14*(104*a*b^3*c^2 + 59*a^2*b^2*d^2)*x^3 + 21
*(88*a^2*b^2*c^2 + 5*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(155) = 310$.

Time = 0.58 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.16

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2}\,dx = \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{2a^3cd}{7b}+\frac{6a^2cdx^2}{7}+\frac{6abcdx^4}{7}+\frac{2b^2cdx^6}{7}+\frac{b^2d^2x^7}{8}+\frac{x^5\cdot\left(\frac{17ab^2d^2}{8}+b^3c^2\right)}{6b}+\frac{x^3\cdot\left(3a^2bd^2+3ab^2c\right.}{} \right. \\ a^{\frac{5}{2}}\left(\begin{cases} c^2x & \text{for } d=0 \\ \frac{(c+dx)^3}{3d} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(2*a**3*c*d/(7*b) + 6*a**2*c*d*x**2/7 + 6*a*b*
c*d*x**4/7 + 2*b**2*c*d*x**6/7 + b**2*d**2*x**7/8 + x**5*(17*a*b**2*d**2/8
 + b**3*c**2)/(6*b) + x**3*(3*a**2*b*d**2 + 3*a*b**2*c**2 - 5*a*(17*a*b**2
*d**2/8 + b**3*c**2)/(6*b))/(4*b) + x*(a**3*d**2 + 3*a**2*b*c**2 - 3*a*(3*
a**2*b*d**2 + 3*a*b**2*c**2 - 5*a*(17*a*b**2*d**2/8 + b**3*c**2)/(6*b))/(4
*b))/(2*b)) + (a**3*c**2 - a*(a**3*d**2 + 3*a**2*b*c**2 - 3*a*(3*a**2*b*d*
*2 + 3*a*b**2*c**2 - 5*a*(17*a*b**2*d**2/8 + b**3*c**2)/(6*b))/(4*b))/(2*b
))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)),
(x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*Piecewise((c**2*x, Eq
(d, 0)), ((c + d*x)**3/(3*d), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} dx = \frac{1}{6} \left(bx^2+a\right)^{\frac{5}{2}} c^2 x + \frac{5}{24} \left(bx^2+a\right)^{\frac{3}{2}} ac^2 x + \frac{5}{16} \sqrt{bx^2+a} a^2 c^2 x + \frac{\left(bx^2+a\right)^{\frac{7}{2}} d^2 x}{8\,b} - \frac{\left(bx^2+a\right)^{\frac{5}{2}} ad^2 x}{48\,b} - \frac{5 \left(bx^2+a\right)^{\frac{3}{2}} a^2 d^2 x}{192\,b} - \frac{5 \sqrt{bx^2+a} a^3 d^2 x}{128\,b} + \frac{5\,a^3 c^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} - \frac{5\,a^4 d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{2 \left(bx^2+a\right)^{\frac{7}{2}} cd}{7\,b}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*c^2*x + 5/24*(b*x^2 + a)^(3/2)*a*c^2*x + 5/16*sqrt(b
*x^2 + a)*a^2*c^2*x + 1/8*(b*x^2 + a)^(7/2)*d^2*x/b - 1/48*(b*x^2 + a)^(5/
2)*a*d^2*x/b - 5/192*(b*x^2 + a)^(3/2)*a^2*d^2*x/b - 5/128*sqrt(b*x^2 + a)
*a^3*d^2*x/b + 5/16*a^3*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*a^4*d^2
*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2/7*(b*x^2 + a)^(7/2)*c*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.11

$$\int (c+dx)^2 (a+bx^2)^{5/2}\, dx = \frac{1}{2688}\left(\frac{768\, a^3cd}{b} + \left(2\left(1152\, a^2cd + \left(4\left(288\, abcd + \left(6\left(7\, b^2d^2x + 16\, b^2cd\right)x + \frac{7\left(8\, b^8c^2 + \right.}{b^6}\right.\right.\right.\right.\right.\right. - \frac{5\left(8\, a^3bc^2 - a^4d^2\right)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{128\, b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/2688*(768*a^3*c*d/b + (2*(1152*a^2*c*d + (4*(288*a*b*c*d + (6*(7*b^2*d^2
*x + 16*b^2*c*d)*x + 7*(8*b^8*c^2 + 17*a*b^7*d^2)/b^6)*x)*x + 7*(104*a*b^7
*c^2 + 59*a^2*b^6*d^2)/b^6)*x)*x + 21*(88*a^2*b^6*c^2 + 5*a^3*b^5*d^2)/b^6
)*x)*sqrt(b*x^2 + a) - 5/128*(8*a^3*b*c^2 - a^4*d^2)*log(abs(-sqrt(b)*x +
sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2}\, dx = \int \left(b\,x^2+a\right)^{5/2}\left(c+d\,x\right)^2 dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.61

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} dx = \frac{768\sqrt{b\,x^2+a}\,a^3bcd + 105\sqrt{b\,x^2+a}\,a^3b\,d^2x + 1848\sqrt{b\,x^2+a}\,a^2b^2c^2x + 2304\sqrt{b\,x^2+a}\,a^2b^2c}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(5/2),x)
```

output
```
(768*sqrt(a + b*x**2)*a**3*b*c*d + 105*sqrt(a + b*x**2)*a**3*b*d**2*x + 18
48*sqrt(a + b*x**2)*a**2*b**2*c**2*x + 2304*sqrt(a + b*x**2)*a**2*b**2*c*d
*x**2 + 826*sqrt(a + b*x**2)*a**2*b**2*d**2*x**3 + 1456*sqrt(a + b*x**2)*a
*b**3*c**2*x**3 + 2304*sqrt(a + b*x**2)*a*b**3*c*d*x**4 + 952*sqrt(a + b*x
**2)*a*b**3*d**2*x**5 + 448*sqrt(a + b*x**2)*b**4*c**2*x**5 + 768*sqrt(a +
 b*x**2)*b**4*c*d*x**6 + 336*sqrt(a + b*x**2)*b**4*d**2*x**7 - 105*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d**2 + 840*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c**2)/(2688*b**2)
```

3.256 $\int (c+dx) \left(a+bx^2\right)^{5/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (c+dx) \left(a+bx^2\right)^{5/2} \, dx = \frac{5}{16}a^2cx\sqrt{a+bx^2} + \frac{5}{24}acx\left(a+bx^2\right)^{3/2} + \frac{1}{6}cx\left(a+bx^2\right)^{5/2} + \frac{d\left(a+bx^2\right)^{7/2}}{7b} + \frac{5a^3c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

output
```
5/16*a^2*c*x*(b*x^2+a)^(1/2)+5/24*a*c*x*(b*x^2+a)^(3/2)+1/6*c*x*(b*x^2+a)^
(5/2)+1/7*d*(b*x^2+a)^(7/2)/b+5/16*a^3*c*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2)
)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int (c+dx) \left(a+bx^2\right)^{5/2} \, dx = \frac{\sqrt{a+bx^2}(48a^3d+8b^3x^5(7c+6dx)+3a^2bx(77c+48dx)+2ab^2x^3(91c+72dx)) - 105a^3\sqrt{\ }}{336b}$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(48*a^3*d + 8*b^3*x^5*(7*c + 6*d*x) + 3*a^2*b*x*(77*c + 4
8*d*x) + 2*a*b^2*x^3*(91*c + 72*d*x)) - 105*a^3*Sqrt[b]*c*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/(336*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} (c+dx) \, dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(bx^2+a\right)^{5/2}dx+\frac{d\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{d\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{d\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{d\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{224}$$

$$c\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{d\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{219}$$

$$c\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{d\left(a+bx^2\right)^{7/2}}{7b}$$

input
```
Int[(c + d*x)*(a + b*x^2)^(5/2),x]
```

output
```
(d*(a + b*x^2)^(7/2))/(7*b) + c*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b
*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*Sqrt[b])))/4))/6)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result	size
default	$c\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{d\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7b}$	86
risch	$\frac{\left(48b^{3}d\,x^{6}+56b^{3}c\,x^{5}+144a\,b^{2}d\,x^{4}+182a\,b^{2}c\,x^{3}+144a^{2}bd\,x^{2}+231a^{2}bcx+48a^{3}d\right)\sqrt{b\,x^{2}+a}}{336b}+\frac{5c\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16\sqrt{b}}$	104

input
```
int((d*x+c)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*d*(b*x^2+a)^(7
/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.09

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{105\,a^3\sqrt{b}c\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(48\,b^3dx^6+56\,b^3cx^5+144\,ab^2dx^4+18}{672\,b}\right.$$

$$-\frac{105\,a^3\sqrt{-b}c\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,b^3dx^6+56\,b^3cx^5+144\,ab^2dx^4+182\,ab^2cx^3+144\,a^2bdx^2+231\,a^2bc}{336\,b}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/672*(105*a^3*sqrt(b)*c*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(48*b^3*d*x^6 + 56*b^3*c*x^5 + 144*a*b^2*d*x^4 + 182*a*b^2*c*x^3 + 144
*a^2*b*d*x^2 + 231*a^2*b*c*x + 48*a^3*d)*sqrt(b*x^2 + a))/b, -1/336*(105*a
^3*sqrt(-b)*c*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*b^3*d*x^6 + 56*b^3*
c*x^5 + 144*a*b^2*d*x^4 + 182*a*b^2*c*x^3 + 144*a^2*b*d*x^2 + 231*a^2*b*c*
x + 48*a^3*d)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\,dx=\begin{cases}\frac{5a^3c\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16}+\sqrt{a+bx^2}\left(\frac{a^3d}{7b}+\frac{11a^2cx}{16}+\frac{3a^2dx^2}{7}+\frac{13abcx^3}{24}+\frac{3abdx}{7}\right. \\ a^{\frac{5}{2}}\left(cx+\frac{dx^2}{2}\right)\end{cases}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise(((5*a**3*c*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/16 + sqrt(a + b*x**2)*(a**
3*d/(7*b) + 11*a**2*c*x/16 + 3*a**2*d*x**2/7 + 13*a*b*c*x**3/24 + 3*a*b*d*
x**4/7 + b**2*c*x**5/6 + b**2*d*x**6/7), Ne(b, 0)), (a**(5/2)*(c*x + d*x**
2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\,dx=\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}cx+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}acx+\frac{5}{16}\sqrt{bx^2+a}a^2cx+\frac{5\,a^3c\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}d}{7\,b}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*c*x + 5/24*(b*x^2 + a)^(3/2)*a*c*x + 5/16*sqrt(b*x^2
 + a)*a^2*c*x + 5/16*a^3*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a
)^(7/2)*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5\,a^3c\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,a^3d}{b}+\left(231\,a^2c+2\left(72\,a^2d+\left(91\,abc+4\left(18\,abd+\left(6\,b^2dx+7\,b^2c\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/16*a^3*c*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*a^3
*d/b + (231*a^2*c + 2*(72*a^2*d + (91*a*b*c + 4*(18*a*b*d + (6*b^2*d*x + 7
*b^2*c)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\,dx = \frac{d\,\left(b\,x^2+a\right)^{7/2}}{7\,b} + \frac{c\,x\,\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x),x)
```

output
```
(d*(a + b*x^2)^(7/2))/(7*b) + (c*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2]
, 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\,dx = \frac{48\sqrt{b\,x^2+a}\,a^3d+231\sqrt{b\,x^2+a}\,a^2bcx+144\sqrt{b\,x^2+a}\,a^2bd\,x^2+182\sqrt{b\,x^2+a}\,a\,b^2c\,x^3+14}{33}$$

input
```
int((d*x+c)*(b*x^2+a)^(5/2),x)
```

output
```
(48*sqrt(a + b*x**2)*a**3*d + 231*sqrt(a + b*x**2)*a**2*b*c*x + 144*sqrt(a
 + b*x**2)*a**2*b*d*x**2 + 182*sqrt(a + b*x**2)*a*b**2*c*x**3 + 144*sqrt(a
 + b*x**2)*a*b**2*d*x**4 + 56*sqrt(a + b*x**2)*b**3*c*x**5 + 48*sqrt(a + b
*x**2)*b**3*d*x**6 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**3*c)/(336*b)
```

3.257 $\int \frac{(a+bx^2)^{5/2}}{c+dx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \frac{(a+bx^2)^{5/2}}{c+dx}\,dx = \frac{\left(8(bc^2+ad^2)^2-bcd(4bc^2+7ad^2)\,x\right)\sqrt{a+bx^2}}{8d^5} + \frac{(4(bc^2+ad^2)-3bcdx)\,(a+bx^2)^{3/2}}{12d^3} + \frac{(a+bx^2)^{5/2}}{5d} - \frac{\sqrt{b}c(8b^2c^4+20abc^2d^2+15a^2d^4)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^6} - \frac{(bc^2+ad^2)^{5/2}\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^6}$$

output
```
1/8*(8*(a*d^2+b*c^2)^2-b*c*d*(7*a*d^2+4*b*c^2)*x)*(b*x^2+a)^(1/2)/d^5+1/12
*(-3*b*c*d*x+4*a*d^2+4*b*c^2)*(b*x^2+a)^(3/2)/d^3+1/5*(b*x^2+a)^(5/2)/d-1/
8*b^(1/2)*c*(15*a^2*d^4+20*a*b*c^2*d^2+8*b^2*c^4)*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/d^6-(a*d^2+b*c^2)^(5/2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2
)/(b*x^2+a)^(1/2))/d^6
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{5/2}}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}(184a^2d^4+abd^2(280c^2-135cdx+88d^2x^2)+2b^2(60c^4-30c^3dx+20c^2d^2\ldots}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x),x]
```

output
```
(d*Sqrt[a + b*x^2]*(184*a^2*d^4 + a*b*d^2*(280*c^2 - 135*c*d*x + 88*d^2*x^
2) + 2*b^2*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4*x
^4)) + 240*(-(b*c^2) - a*d^2)^(5/2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a +
 b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 15*Sqrt[b]*c*(8*b^2*c^4 + 20*a*b*c^2*d^
2 + 15*a^2*d^4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*d^6)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {493, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx}dx$$

$$\Big\downarrow \text{493}$$

$$\frac{\int \frac{(ad-bcx)(bx^2+a)^{3/2}}{c+dx}dx}{d}+\frac{(a+bx^2)^{5/2}}{5d}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{\int \frac{b\left(ad\left(bc^2+4ad^2\right)-bc\left(4bc^2+7ad^2\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{(a+bx^2)^{3/2}(4(ad^2+bc^2)-3bcdx)}{12d^2}}{d}+\frac{(a+bx^2)^{5/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(ad\left(bc^2+4ad^2\right)-bc\left(4bc^2+7ad^2\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}+\frac{(a+bx^2)^{3/2}(4(ad^2+bc^2)-3bcdx)}{12d^2}}{d}+\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

$\downarrow$ 682

$$\frac{\frac{\frac{\int \frac{b\left(ad\left(4b^2c^4+9abd^2c^2+8a^2d^4\right)-bc\left(8b^2c^4+20abd^2c^2+15a^2d^4\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)^2-bcdx\left(7ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}+\frac{(a+bx^2)^{3/2}(4(ad^2+bc^2)-3bcdx)}{12d^2}}{d}+\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{ad\left(4b^2c^4+9abd^2c^2+8a^2d^4\right)-bc\left(8b^2c^4+20abd^2c^2+15a^2d^4\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)^2-bcdx\left(7ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}+\frac{(a+bx^2)^{3/2}(4(ad^2+bc^2)-3bcdx)}{12d^2}}{d}+\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

$\downarrow$ 719

$$\frac{\frac{\frac{\frac{8\left(ad^2+bc^2\right)^3\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)^2-bcdx\left(7ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}+\frac{(a+bx^2)^{3/2}(4(ad^2+bc^2)-3bcdx)}{12d^2}}{d}+\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\frac{8\left(ad^2+bc^2\right)^3\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)^2-bcdx\left(7ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}+\frac{(a+bx^2)^{3/2}(4(ad^2+bc^2)-3bcdx)}{12d^2}}{d}+\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\frac{8\left(ad^2+bc^2\right)^3\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)^2-bcdx\left(7ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}+\frac{(a+bx^2)^{3/}}{}}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

$\Big\downarrow$ 488

$$\frac{\frac{\frac{-\frac{8\left(ad^2+bc^2\right)^3\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)^2-bcdx\left(7ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)}{d}-\frac{8\left(ad^2+bc^2\right)^{5/2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)^2-bcdx\left(7ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x),x]
```

output
```
(a + b*x^2)^(5/2)/(5*d) + (((4*(b*c^2 + a*d^2) - 3*b*c*d*x)*(a + b*x^2)^(3
/2))/(12*d^2) + (((8*(b*c^2 + a*d^2)^2 - b*c*d*(4*b*c^2 + 7*a*d^2)*x)*Sqrt
[a + b*x^2])/(2*d^2) + (-((Sqrt[b]*c*(8*b^2*c^4 + 20*a*b*c^2*d^2 + 15*a^2*
d^4)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d) - (8*(b*c^2 + a*d^2)^(5/2)*A
rcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2))/(
4*d^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.46

method	result
risch	$\frac{\left(24b^2d^4x^4-30b^2c\,d^3x^3+88ab\,d^4x^2+40d^2c^2x^2b^2-135abc\,d^3x-60b^2c^3dx+184a^2d^4+280b\,c^2d^2a+120b^2c^4\right)\sqrt{b\,x^2+a}}{120d^5}-\frac{\frac{8\left(a^3d^6+3a\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{5}{2}}}{5}-\frac{bc\left(\frac{\left(2b\left(x+\frac{c}{d}\right)-\frac{2bc}{d}\right)\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{8b}+\frac{3\left(\frac{4b\left(a\,d^2+b\,c^2\right)}{d^2}-\frac{4b^2c^2}{d^2}\right)\left(\frac{\left(2b\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(5/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*b^2*d^4*x^4-30*b^2*c*d^3*x^3+88*a*b*d^4*x^2+40*b^2*c^2*d^2*x^2-1
35*a*b*c*d^3*x-60*b^2*c^3*d*x+184*a^2*d^4+280*a*b*c^2*d^2+120*b^2*c^4)*(b*
x^2+a)^(1/2)/d^5-1/8/d^5*(8*(a^3*d^6+3*a^2*b*c^2*d^4+3*a*b^2*c^4*d^2+b^3*c
^6)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+
2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2))/(x+c/d))+b^(1/2)*c*(15*a^2*d^4+20*a*b*c^2*d^2+8*b^2*c^4)/d*ln(b^(
1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 29.55 (sec) , antiderivative size = 1176, normalized size of antiderivative = 5.20

$$\int \frac{(a+bx^2)^{5/2}}{c+dx} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/240*(15*(8*b^2*c^5 + 20*a*b*c^3*d^2 + 15*a^2*c*d^4)*sqrt(b)*log(-2*b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 120*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*
d^4)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c
^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(
d^2*x^2 + 2*c*d*x + c^2)) + 2*(24*b^2*d^5*x^4 - 30*b^2*c*d^4*x^3 + 120*b^2
*c^4*d + 280*a*b*c^2*d^3 + 184*a^2*d^5 + 8*(5*b^2*c^2*d^3 + 11*a*b*d^5)*x^
2 - 15*(4*b^2*c^3*d^2 + 9*a*b*c*d^4)*x)*sqrt(b*x^2 + a))/d^6, 1/120*(15*(8
*b^2*c^5 + 20*a*b*c^3*d^2 + 15*a^2*c*d^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) + 60*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*sqrt(b*c^2 + a*d^2)*l
og((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt
(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2))
+ (24*b^2*d^5*x^4 - 30*b^2*c*d^4*x^3 + 120*b^2*c^4*d + 280*a*b*c^2*d^3 + 1
84*a^2*d^5 + 8*(5*b^2*c^2*d^3 + 11*a*b*d^5)*x^2 - 15*(4*b^2*c^3*d^2 + 9*a*
b*c*d^4)*x)*sqrt(b*x^2 + a))/d^6, -1/240*(240*(b^2*c^4 + 2*a*b*c^2*d^2 + a
^2*d^4)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqr
t(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - 15*(8*b^2*c^
5 + 20*a*b*c^3*d^2 + 15*a^2*c*d^4)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(24*b^2*d^5*x^4 - 30*b^2*c*d^4*x^3 + 120*b^2*c^4*d +
280*a*b*c^2*d^3 + 184*a^2*d^5 + 8*(5*b^2*c^2*d^3 + 11*a*b*d^5)*x^2 - 15*(4
*b^2*c^3*d^2 + 9*a*b*c*d^4)*x)*sqrt(b*x^2 + a))/d^6, -1/120*(120*(b^2*c...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{c+dx} dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{c+dx} dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^{5/2}}{c+dx}\,dx = -\frac{\sqrt{bx^2+a}b^2c^3x}{2\,d^4} - \frac{(bx^2+a)^{\frac{3}{2}}bcx}{4\,d^2} - \frac{7\sqrt{bx^2+a}abcx}{8\,d^2} - \frac{b^{\frac{5}{2}}c^5\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^6} - \frac{5\,ab^{\frac{3}{2}}c^3\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,d^4} - \frac{15\,a^2\sqrt{b}c\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,d^2} + \frac{\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}\text{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d} + \frac{\sqrt{bx^2+a}b^2c^4}{d^5} + \frac{(bx^2+a)^{\frac{3}{2}}bc^2}{3\,d^3} + \frac{2\sqrt{bx^2+a}abc^2}{d^3} + \frac{(bx^2+a)^{\frac{5}{2}}}{5\,d} + \frac{(bx^2+a)^{\frac{3}{2}}a}{3\,d} + \frac{\sqrt{bx^2+a}a^2}{d}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b*x^2 + a)*b^2*c^3*x/d^4 - 1/4*(b*x^2 + a)^(3/2)*b*c*x/d^2 - 7/8
*sqrt(b*x^2 + a)*a*b*c*x/d^2 - b^(5/2)*c^5*arcsinh(b*x/sqrt(a*b))/d^6 - 5/
2*a*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 - 15/8*a^2*sqrt(b)*c*arcsinh(b*
x/sqrt(a*b))/d^2 + (a + b*c^2/d^2)^(5/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
+ c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d + sqrt(b*x^2 + a)*b^2*c^4/d^5 + 1/
3*(b*x^2 + a)^(3/2)*b*c^2/d^3 + 2*sqrt(b*x^2 + a)*a*b*c^2/d^3 + 1/5*(b*x^2
 + a)^(5/2)/d + 1/3*(b*x^2 + a)^(3/2)*a/d + sqrt(b*x^2 + a)*a^2/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{c+d\,x}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x),x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{dx+c}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x+c),x)
```

3.258 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 260

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2} dx = -\frac{4bc(bc^2+ad^2)\sqrt{a+bx^2}}{d^5} + \frac{3b(4bc^2+3ad^2)x\sqrt{a+bx^2}}{8d^4} + \frac{b^2x^3\sqrt{a+bx^2}}{4d^2} - \frac{(bc^2+ad^2)^2\sqrt{a+bx^2}}{d^5(c+dx)} - \frac{2bc(a+bx^2)^{3/2}}{3d^3} + \frac{5\sqrt{b}(8b^2c^4+12abc^2d^2+3a^2d^4)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^6} + \frac{5bc(bc^2+ad^2)^{3/2}\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^6}$$

output
```
-4*b*c*(a*d^2+b*c^2)*(b*x^2+a)^(1/2)/d^5+3/8*b*(3*a*d^2+4*b*c^2)*x*(b*x^2+
a)^(1/2)/d^4+1/4*b^2*x^3*(b*x^2+a)^(1/2)/d^2-(a*d^2+b*c^2)^2*(b*x^2+a)^(1/
2)/d^5/(d*x+c)-2/3*b*c*(b*x^2+a)^(3/2)/d^3+5/8*b^(1/2)*(3*a^2*d^4+12*a*b*c
^2*d^2+8*b^2*c^4)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^6+5*b*c*(a*d^2+b*c^
2)^(3/2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^6
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}\,dx = -\frac{\frac{d\sqrt{a+bx^2}(24a^2d^4+abd^2(160c^2+85cdx-27d^2x^2)+2b^2(60c^4+30c^3dx-10c^2d^2x^2+5cd^3x^3-3d^4x^4))}{c+dx} - 240bc(-bc^2-ad^2)^{3/2}\arctan}{24d^6}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^2,x]
```

output
```
-1/24*((d*Sqrt[a + b*x^2]*(24*a^2*d^4 + a*b*d^2*(160*c^2 + 85*c*d*x - 27*d
^2*x^2) + 2*b^2*(60*c^4 + 30*c^3*d*x - 10*c^2*d^2*x^2 + 5*c*d^3*x^3 - 3*d^
4*x^4)))/(c + d*x) - 240*b*c*(-(b*c^2) - a*d^2)^(3/2)*ArcTan[(Sqrt[b]*(c +
 d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 15*Sqrt[b]*(8*b^2*c^4
 + 12*a*b*c^2*d^2 + 3*a^2*d^4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^6
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {492, 591, 25, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{492}$$

$$\frac{5b\int \frac{x(bx^2+a)^{3/2}}{c+dx}dx}{d} - \frac{(a+bx^2)^{5/2}}{d(c+dx)}$$

$$\Big\downarrow \text{591}$$

$$\frac{5b\left(\frac{\int -\frac{\left(acd-\left(4bc^2+3ad^2\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}(4c-3dx)}{12d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{5b\left(-\frac{\int \frac{\left(acd-\left(4bc^2+3ad^2\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}(4c-3dx)}{12d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 682

$$\frac{5b\left(-\frac{\frac{\int \frac{b\left(acd\left(4bc^2+5ad^2\right)-\left(8b^2c^4+12abd^2c^2+3a^2d^4\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(8c\left(ad^2+bc^2\right)-dx\left(3ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}(4c-3dx)}{12d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{5b\left(-\frac{\frac{\int \frac{acd\left(4bc^2+5ad^2\right)-\left(8b^2c^4+12abd^2c^2+3a^2d^4\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(8c\left(ad^2+bc^2\right)-dx\left(3ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}(4c-3dx)}{12d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 719

$$\frac{5b\left(-\frac{\frac{\frac{8c\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8c\left(ad^2+bc^2\right)-dx\left(3ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}(4c-3dx)}{12d^2}\right)}{d}-\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 224

$$5b\left(-\frac{\frac{\frac{8c\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8c\left(ad^2+bc^2\right)-dx\left(3ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}-\frac{(a+bx^2)^{3}}{1}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 219

$$5b\left(-\frac{\frac{\frac{8c\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8c\left(ad^2+bc^2\right)-dx\left(3ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}-\frac{(a+bx^2)^{3}}{1}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 488

$$5b\left(-\frac{\frac{-\frac{8c\left(ad^2+bc^2\right)^2\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8c\left(ad^2+bc^2\right)-dx\left(3ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

$\Big\downarrow$ 219

$$5b\left(-\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)}{\sqrt{b}d}-\frac{8c\left(ad^2+bc^2\right)^{3/2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8c\left(ad^2+bc^2\right)-dx\left(3ad^2+4bc^2\right)\right)}{2d^2}}{4d^2}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{d(c+dx)}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^2,x]
```

output
```
-((a + b*x^2)^(5/2)/(d*(c + d*x))) + (5*b*(-1/12*((4*c - 3*d*x)*(a + b*x^2
)^(3/2))/d^2 - (((8*c*(b*c^2 + a*d^2) - d*(4*b*c^2 + 3*a*d^2)*x)*Sqrt[a +
b*x^2])/(2*d^2) + (-(((8*b^2*c^4 + 12*a*b*c^2*d^2 + 3*a^2*d^4)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (8*c*(b*c^2 + a*d^2)^(3/2)*ArcTa
nh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2))/(4*d^
2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(230) = 460$.

Time = 0.35 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{b(-6b\,d^3x^3+16bc\,d^2x^2-27ax\,d^3-36b\,c^2dx+112a\,d^2c+96b\,c^3)\sqrt{b\,x^2+a}}{24d^5}+\frac{8\left(a^3d^6+3a^2b\,c^2d^4+3a\,b^2c^4d^2+b^3c^6\right)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2 \cdots}}{\left(a\,d^2 \cdots\right.} \cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*b*(-6*b*d^3*x^3+16*b*c*d^2*x^2-27*a*d^3*x-36*b*c^2*d*x+112*a*c*d^2+9
6*b*c^3)*(b*x^2+a)^(1/2)/d^5+1/8/d^5*(8*(a^3*d^6+3*a^2*b*c^2*d^4+3*a*b^2*c
^4*d^2+b^3*c^6)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+
c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2
)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(
x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+5*b^(1/2)*(3*
a^2*d^4+12*a*b*c^2*d^2+8*b^2*c^4)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+48/d^2*b
*c*(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+
b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 1372, normalized size of antiderivative = 5.28

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/48*(15*(8*b^2*c^5 + 12*a*b*c^3*d^2 + 3*a^2*c*d^4 + (8*b^2*c^4*d + 12*a*
b*c^2*d^3 + 3*a^2*d^5)*x)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) + 120*(b^2*c^4 + a*b*c^2*d^2 + (b^2*c^3*d + a*b*c*d^3)*x)*sqrt(b*c
^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)
*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c
*d*x + c^2)) + 2*(6*b^2*d^5*x^4 - 10*b^2*c*d^4*x^3 - 120*b^2*c^4*d - 160*a
*b*c^2*d^3 - 24*a^2*d^5 + (20*b^2*c^2*d^3 + 27*a*b*d^5)*x^2 - 5*(12*b^2*c^
3*d^2 + 17*a*b*c*d^4)*x)*sqrt(b*x^2 + a))/(d^7*x + c*d^6), 1/48*(240*(b^2*
c^4 + a*b*c^2*d^2 + (b^2*c^3*d + a*b*c*d^3)*x)*sqrt(-b*c^2 - a*d^2)*arctan
(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (
b^2*c^2 + a*b*d^2)*x^2)) + 15*(8*b^2*c^5 + 12*a*b*c^3*d^2 + 3*a^2*c*d^4 +
(8*b^2*c^4*d + 12*a*b*c^2*d^3 + 3*a^2*d^5)*x)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*b^2*d^5*x^4 - 10*b^2*c*d^4*x^3 - 120*b^
2*c^4*d - 160*a*b*c^2*d^3 - 24*a^2*d^5 + (20*b^2*c^2*d^3 + 27*a*b*d^5)*x^2
 - 5*(12*b^2*c^3*d^2 + 17*a*b*c*d^4)*x)*sqrt(b*x^2 + a))/(d^7*x + c*d^6),
-1/24*(15*(8*b^2*c^5 + 12*a*b*c^3*d^2 + 3*a^2*c*d^4 + (8*b^2*c^4*d + 12*a*
b*c^2*d^3 + 3*a^2*d^5)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60
*(b^2*c^4 + a*b*c^2*d^2 + (b^2*c^3*d + a*b*c*d^3)*x)*sqrt(b*c^2 + a*d^2)*l
og((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt
(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}\,dx = -\frac{(bx^2+a)^{\frac{5}{2}}}{d^2x+cd} + \frac{5\sqrt{bx^2+a}b^2c^2x}{2\,d^4} + \frac{5\,(bx^2+a)^{\frac{3}{2}}bx}{4\,d^2} + \frac{15\sqrt{bx^2+a}abx}{8\,d^2} + \frac{5\,b^{\frac{5}{2}}c^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^6} + \frac{15\,ab^{\frac{3}{2}}c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,d^4} + \frac{15\,a^2\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,d^2} - \frac{5\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}bc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^3} - \frac{5\sqrt{bx^2+a}b^2c^3}{d^5} - \frac{5\,(bx^2+a)^{\frac{3}{2}}bc}{3\,d^3} - \frac{5\sqrt{bx^2+a}abc}{d^3}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(b*x^2 + a)^(5/2)/(d^2*x + c*d) + 5/2*sqrt(b*x^2 + a)*b^2*c^2*x/d^4 + 5/4
*(b*x^2 + a)^(3/2)*b*x/d^2 + 15/8*sqrt(b*x^2 + a)*a*b*x/d^2 + 5*b^(5/2)*c^
4*arcsinh(b*x/sqrt(a*b))/d^6 + 15/2*a*b^(3/2)*c^2*arcsinh(b*x/sqrt(a*b))/d
^4 + 15/8*a^2*sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^2 - 5*(a + b*c^2/d^2)^(3/2)
*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c))
)/d^3 - 5*sqrt(b*x^2 + a)*b^2*c^3/d^5 - 5/3*(b*x^2 + a)^(3/2)*b*c/d^3 - 5*
sqrt(b*x^2 + a)*a*b*c/d^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}\,dx = \int \frac{\left(b\,x^2+a\right)^{5/2}}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 814, normalized size of antiderivative = 3.13

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^2,x)
```

output

```
(240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a*b*c**2*d**2 + 240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**3*x + 240*sqrt(a*d**2
+ b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**
2*c**4 + 240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*b**2*c**3*d*x - 240*sqrt(a*d**2 + b*c**2)*log(c + d*
x)*a*b*c**2*d**2 - 240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d**3*x - 2
40*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4 - 240*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*b**2*c**3*d*x - 48*sqrt(a + b*x**2)*a**2*d**5 - 320*sqrt(a
+ b*x**2)*a*b*c**2*d**3 - 170*sqrt(a + b*x**2)*a*b*c*d**4*x + 54*sqrt(a +
b*x**2)*a*b*d**5*x**2 - 240*sqrt(a + b*x**2)*b**2*c**4*d - 120*sqrt(a + b*
x**2)*b**2*c**3*d**2*x + 40*sqrt(a + b*x**2)*b**2*c**2*d**3*x**2 - 20*sqrt
(a + b*x**2)*b**2*c*d**4*x**3 + 12*sqrt(a + b*x**2)*b**2*d**5*x**4 - 45*sq
rt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a**2*c*d**4 - 45*sqrt(b)*log(sqrt(
a + b*x**2) - sqrt(b)*x)*a**2*d**5*x - 180*sqrt(b)*log(sqrt(a + b*x**2) -
sqrt(b)*x)*a*b*c**3*d**2 - 180*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a
*b*c**2*d**3*x - 120*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*b**2*c**5 -
 120*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*b**2*c**4*d*x + 45*sqrt(b)*
log(sqrt(a + b*x**2) + sqrt(b)*x)*a**2*c*d**4 + 45*sqrt(b)*log(sqrt(a + b*
x**2) + sqrt(b)*x)*a**2*d**5*x + 180*sqrt(b)*log(sqrt(a + b*x**2) + sqr...
```

3.259 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 265

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \frac{2b(3bc^2+ad^2)\sqrt{a+bx^2}}{d^5} - \frac{3b^2cx\sqrt{a+bx^2}}{2d^4} - \frac{(bc^2+ad^2)^2\sqrt{a+bx^2}}{2d^5(c+dx)^2} + \frac{9bc(bc^2+ad^2)\sqrt{a+bx^2}}{2d^5(c+dx)} + \frac{b(a+bx^2)^{3/2}}{3d^3} - \frac{5b^{3/2}c(4bc^2+3ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^6} - \frac{5b\sqrt{bc^2+ad^2}(4bc^2+ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2d^6}$$

output
```
2*b*(a*d^2+3*b*c^2)*(b*x^2+a)^(1/2)/d^5-3/2*b^2*c*x*(b*x^2+a)^(1/2)/d^4-1/
2*(a*d^2+b*c^2)^2*(b*x^2+a)^(1/2)/d^5/(d*x+c)^2+9/2*b*c*(a*d^2+b*c^2)*(b*x
^2+a)^(1/2)/d^5/(d*x+c)+1/3*b*(b*x^2+a)^(3/2)/d^3-5/2*b^(3/2)*c*(3*a*d^2+4
*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^6-5/2*b*(a*d^2+b*c^2)^(1/2)*(
a*d^2+4*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d
^6
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \frac{-\frac{d\sqrt{a+bx^2}\left(3a^2d^4-abd^2(35c^2+55cdx+14d^2x^2)-b^2(60c^4+90c^3dx+20c^2d^2x^2-5cd^3x^3+2d^4x^4)\right)}{(c+dx)^2}+30b\sqrt{-bc^2}}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^3,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(3*a^2*d^4 - a*b*d^2*(35*c^2 + 55*c*d*x + 14*d^2*x^2
) - b^2*(60*c^4 + 90*c^3*d*x + 20*c^2*d^2*x^2 - 5*c*d^3*x^3 + 2*d^4*x^4)))
/(c + d*x)^2) + 30*b*Sqrt[-(b*c^2) - a*d^2]*(4*b*c^2 + a*d^2)*ArcTan[(Sqrt
[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 15*b^(3/2)*c*
(4*b*c^2 + 3*a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(6*d^6)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {492, 590, 25, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}dx$$

$$\Big\downarrow \text{ 492}$$

$$\frac{5b\int \frac{x(bx^2+a)^{3/2}}{(c+dx)^2}dx}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 590}$$

$$\frac{5b\left(\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}-\frac{\int -\frac{(ad-4bcx)\sqrt{bx^2+a}}{c+dx}dx}{d^2}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5b\left(\frac{\int\frac{(ad-4bcx)\sqrt{bx^2+a}}{c+dx}dx}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 682$$

$$\frac{5b\left(\frac{\frac{\int\frac{2b\left(ad\left(2bc^2+ad^2\right)-bc\left(4bc^2+3ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2-2bcdx\right)}{d^2}}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{5b\left(\frac{\frac{\int\frac{ad\left(2bc^2+ad^2\right)-bc\left(4bc^2+3ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}+\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2-2bcdx\right)}{d^2}}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 719$$

$$\frac{5b\left(\frac{\frac{\frac{\left(ad^2+bc^2\right)\left(ad^2+4bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(3ad^2+4bc^2\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2}+\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2-2bcdx\right)}{d^2}}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 224$$

$$\frac{5b\left(\frac{\frac{\frac{\left(ad^2+bc^2\right)\left(ad^2+4bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(3ad^2+4bc^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d^2}+\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2-2bcdx\right)}{d^2}}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{5b\left(\frac{\frac{\frac{(ad^2+bc^2)(ad^2+4bc^2)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+4bc^2)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(ad^2+4bc^2-2bcdx)}{d^2}}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$\downarrow$ 488

$$\frac{5b\left(\frac{\frac{-\frac{(ad^2+bc^2)(ad^2+4bc^2)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+4bc^2)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(ad^2+4bc^2-2bcdx)}{d^2}}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

$\downarrow$ 219

$$\frac{5b\left(\frac{\frac{-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+4bc^2)}{d}-\frac{\sqrt{ad^2+bc^2}(ad^2+4bc^2)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(ad^2+4bc^2-2bcdx)}{d^2}}{d^2}+\frac{(a+bx^2)^{3/2}(4c+dx)}{3d^2(c+dx)}\right)}{2d}-\frac{(a+bx^2)^{5/2}}{2d(c+dx)^2}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^3,x]
```

output
```
-1/2*(a + b*x^2)^(5/2)/(d*(c + d*x)^2) + (5*b*(((4*c + d*x)*(a + b*x^2)^(3
/2))/(3*d^2*(c + d*x)) + (((4*b*c^2 + a*d^2 - 2*b*c*d*x)*Sqrt[a + b*x^2])/
d^2 + (-((Sqrt[b]*c*(4*b*c^2 + 3*a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/d) - (Sqrt[b*c^2 + a*d^2]*(4*b*c^2 + a*d^2)*ArcTanh[(a*d - b*c*x)/(Sqr
t[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/d^2)/d^2))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(231) = 462$.

Time = 0.43 (sec) , antiderivative size = 967, normalized size of antiderivative = 3.65

method	result
risch	$\frac{b(2bx^2d^2-9bcdx+14ad^2+36bc^2)\sqrt{bx^2+a}}{6d^5}-\frac{2(a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2+b^3c^6)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{ad^2+bc^2}{d^2}}}{2(ad^2+bc^2)\left(x+\frac{c}{d}\right)^2}+\frac{3bcd}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*b*(2*b*d^2*x^2-9*b*c*d*x+14*a*d^2+36*b*c^2)*(b*x^2+a)^(1/2)/d^5-1/2/d^
5*(-2*(a^3*d^6+3*a^2*b*c^2*d^4+3*a*b^2*c^4*d^2+b^3*c^6)/d^4*(-1/2/(a*d^2+b
*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+
3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d
*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^
(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2*b/(a*
d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(
x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2))/(x+c/d)))+6*b/d^2*(a^2*d^4+6*a*b*c^2*d^2+5*b^2*c^4)/((a*d^
2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^
2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/
d))+5*b^(3/2)*c*(3*a*d^2+4*b*c^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+12/d^3*b
*c*(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b
*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/
d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))
))
```

Fricas [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 1553, normalized size of antiderivative = 5.86

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^3,x, algorithm="fricas")
```

output

```
[1/12*(15*(4*b^2*c^5 + 3*a*b*c^3*d^2 + (4*b^2*c^3*d^2 + 3*a*b*c*d^4)*x^2 +
 2*(4*b^2*c^4*d + 3*a*b*c^2*d^3)*x)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 15*(4*b^2*c^4 + a*b*c^2*d^2 + (4*b^2*c^2*d^2 + a*b*d^4
)*x^2 + 2*(4*b^2*c^3*d + a*b*c*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*
x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2
)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(2*b^2*d^5
*x^4 - 5*b^2*c*d^4*x^3 + 60*b^2*c^4*d + 35*a*b*c^2*d^3 - 3*a^2*d^5 + 2*(10
*b^2*c^2*d^3 + 7*a*b*d^5)*x^2 + 5*(18*b^2*c^3*d^2 + 11*a*b*c*d^4)*x)*sqrt(
b*x^2 + a))/(d^8*x^2 + 2*c*d^7*x + c^2*d^6), 1/12*(30*(4*b^2*c^5 + 3*a*b*c
^3*d^2 + (4*b^2*c^3*d^2 + 3*a*b*c*d^4)*x^2 + 2*(4*b^2*c^4*d + 3*a*b*c^2*d^
3)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + 15*(4*b^2*c^4 + a*b*c^
2*d^2 + (4*b^2*c^2*d^2 + a*b*d^4)*x^2 + 2*(4*b^2*c^3*d + a*b*c*d^3)*x)*sqr
t(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b
*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2
+ 2*c*d*x + c^2)) + 2*(2*b^2*d^5*x^4 - 5*b^2*c*d^4*x^3 + 60*b^2*c^4*d + 35
*a*b*c^2*d^3 - 3*a^2*d^5 + 2*(10*b^2*c^2*d^3 + 7*a*b*d^5)*x^2 + 5*(18*b^2*
c^3*d^2 + 11*a*b*c*d^4)*x)*sqrt(b*x^2 + a))/(d^8*x^2 + 2*c*d^7*x + c^2*d^6
), -1/12*(30*(4*b^2*c^4 + a*b*c^2*d^2 + (4*b^2*c^2*d^2 + a*b*d^4)*x^2 + 2*
(4*b^2*c^3*d + a*b*c*d^3)*x)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d
^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx)^3}\,dx$$

input

```
integrate((b*x**2+a)**(5/2)/(d*x+c)**3,x)
```

output

```
Integral((a + b*x**2)**(5/2)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(232) = 464$.

Time = 0.10 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \frac{15\,b^4c^5\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\left(b^{\frac{3}{2}}c^2d^6+a\sqrt{b}d^8\right)} + \frac{15\,ab^3c^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\left(b^{\frac{3}{2}}c^2d^4+a\sqrt{b}d^6\right)} - \frac{15\,\sqrt{bx^2+a}b^3c^3x}{4\,(bc^2d^4+ad^6)} + \frac{5\,(bx^2+a)^{\frac{3}{2}}b^2c^2}{2\,(bc^2d^3+ad^5)} - \frac{5\,(bx^2+a)^{\frac{3}{2}}b^2cx}{2\,(bc^2d^2+ad^4)} - \frac{15\,\sqrt{bx^2+a}ab^2cx}{4\,(bc^2d^2+ad^4)} + \frac{3\,(bx^2+a)^{\frac{5}{2}}bc}{2\,(bc^2d^2x+ad^4x+bc^3d+acd^3)} - \frac{(bx^2+a)^{\frac{7}{2}}}{2\left(bc^2dx^2+ad^3x^2+2\,bc^3x+2\,acd^2x+\frac{bc^4}{d}+ac^2d\right)} + \frac{(bx^2+a)^{\frac{5}{2}}b}{2\,(bc^2d+ad^3)} - \frac{5\,\sqrt{bx^2+a}b^2cx}{4\,d^4} - \frac{55\,b^{\frac{5}{2}}c^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\,d^6} - \frac{15\,ab^{\frac{3}{2}}c\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,d^4} + \frac{15\,\sqrt{a+\frac{bc^2}{d^2}}b^2c^2\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\,d^5} + \frac{5\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}b\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\,d^3} + \frac{10\,\sqrt{bx^2+a}b^2c^2}{d^5} + \frac{5\,(bx^2+a)^{\frac{3}{2}}b}{6\,d^3} + \frac{5\,\sqrt{bx^2+a}ab}{2\,d^3}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
15/4*b^4*c^5*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^6 + a*sqrt(b)*d^8) + 15
/4*a*b^3*c^3*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^4 + a*sqrt(b)*d^6) - 15
/4*sqrt(b*x^2 + a)*b^3*c^3*x/(b*c^2*d^4 + a*d^6) + 5/2*(b*x^2 + a)^(3/2)*b
^2*c^2/(b*c^2*d^3 + a*d^5) - 5/2*(b*x^2 + a)^(3/2)*b^2*c*x/(b*c^2*d^2 + a*
d^4) - 15/4*sqrt(b*x^2 + a)*a*b^2*c*x/(b*c^2*d^2 + a*d^4) + 3/2*(b*x^2 + a
)^(5/2)*b*c/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) - 1/2*(b*x^2 + a)^
(7/2)/(b*c^2*d*x^2 + a*d^3*x^2 + 2*b*c^3*x + 2*a*c*d^2*x + b*c^4/d + a*c^2
*d) + 1/2*(b*x^2 + a)^(5/2)*b/(b*c^2*d + a*d^3) - 5/4*sqrt(b*x^2 + a)*b^2*
c*x/d^4 - 55/4*b^(5/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^6 - 15/2*a*b^(3/2)*c*a
rcsinh(b*x/sqrt(a*b))/d^4 + 15/2*sqrt(a + b*c^2/d^2)*b^2*c^2*arcsinh(b*c*x
/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 + 5/2*(a + b
*c^2/d^2)^(3/2)*b*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*
abs(d*x + c)))/d^3 + 10*sqrt(b*x^2 + a)*b^2*c^2/d^5 + 5/6*(b*x^2 + a)^(3/2
)*b/d^3 + 5/2*sqrt(b*x^2 + a)*a*b/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(232) = 464$.

Time = 0.22 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(x\left(\frac{2\,b^2x}{d^3}-\frac{9\,b^2c}{d^4}\right)+\frac{2\,(18\,b^3c^2d^{13}+7\,ab^2d^{15})}{bd^{18}}\right)$$
$$+\frac{5\left(4\,b^{\frac{5}{2}}c^3+3\,ab^{\frac{3}{2}}cd^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,d^6}$$
$$+\frac{5\,(4\,b^3c^4+5\,ab^2c^2d^2+a^2bd^4)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}\,d^6}$$
$$+\frac{10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3b^3c^4d+11\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3ab^2c^2d^3+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3a^2bd^5+18\left(\sqrt{b}x-\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*(x*(2*b^2*x/d^3 - 9*b^2*c/d^4) + 2*(18*b^3*c^2*d^13 +
7*a*b^2*d^15)/(b*d^18)) + 5/2*(4*b^(5/2)*c^3 + 3*a*b^(3/2)*c*d^2)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/d^6 + 5*(4*b^3*c^4 + 5*a*b^2*c^2*d^2 + a^2*
b*d^4)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 -
 a*d^2))/(sqrt(-b*c^2 - a*d^2)*d^6) + (10*(sqrt(b)*x - sqrt(b*x^2 + a))^3*
b^3*c^4*d + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a*b^2*c^2*d^3 + (sqrt(b)*x
- sqrt(b*x^2 + a))^3*a^2*b*d^5 + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(7/2
)*c^5 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(5/2)*c^3*d^2 - 9*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*c*d^4 - 26*(sqrt(b)*x - sqrt(b*x^2 + a))
*a*b^3*c^4*d - 25*(sqrt(b)*x - sqrt(b*x^2 + a))*a^2*b^2*c^2*d^3 + (sqrt(b)
*x - sqrt(b*x^2 + a))*a^3*b*d^5 + 9*a^2*b^(5/2)*c^3*d^2 + 9*a^3*b^(3/2)*c*
d^4)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))
*sqrt(b)*c - a*d)^2*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1005, normalized size of antiderivative = 3.79

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^3,x)
```

output
```
(30*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a*b*c**2*d**2 + 60*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**3*x + 30*sqrt(a*d**2 + b*c**2)
*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*d**4*x**2 +
 120*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*b**2*c**4 + 240*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d*x + 120*sqrt(a*d**2 + b*c**2)
*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**2*d**2*
x**2 - 30*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**2 - 60*sqrt(a*d**
2 + b*c**2)*log(c + d*x)*a*b*c*d**3*x - 30*sqrt(a*d**2 + b*c**2)*log(c + d
*x)*a*b*d**4*x**2 - 120*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4 - 240
*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**3*d*x - 120*sqrt(a*d**2 + b*c*
*2)*log(c + d*x)*b**2*c**2*d**2*x**2 - 6*sqrt(a + b*x**2)*a**2*d**5 + 70*s
qrt(a + b*x**2)*a*b*c**2*d**3 + 110*sqrt(a + b*x**2)*a*b*c*d**4*x + 28*sqr
t(a + b*x**2)*a*b*d**5*x**2 + 120*sqrt(a + b*x**2)*b**2*c**4*d + 180*sqrt(
a + b*x**2)*b**2*c**3*d**2*x + 40*sqrt(a + b*x**2)*b**2*c**2*d**3*x**2 - 1
0*sqrt(a + b*x**2)*b**2*c*d**4*x**3 + 4*sqrt(a + b*x**2)*b**2*d**5*x**4 +
45*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*b*c**3*d**2 + 90*sqrt(b)*lo
g(sqrt(a + b*x**2) - sqrt(b)*x)*a*b*c**2*d**3*x + 45*sqrt(b)*log(sqrt(a +
b*x**2) - sqrt(b)*x)*a*b*c*d**4*x**2 + 60*sqrt(b)*log(sqrt(a + b*x**2) ...
```

3.260 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 277

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx &= -\frac{4b^2c\sqrt{a+bx^2}}{d^5}+\frac{b^2x\sqrt{a+bx^2}}{2d^4} \\ &\quad -\frac{(bc^2+ad^2)^2\sqrt{a+bx^2}}{3d^5(c+dx)^3}+\frac{13bc(bc^2+ad^2)\sqrt{a+bx^2}}{6d^5(c+dx)^2} \\ &\quad -\frac{b(47bc^2+14ad^2)\sqrt{a+bx^2}}{6d^5(c+dx)}+\frac{5b^{3/2}(4bc^2+ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^6} \\ &\quad +\frac{5b^2c(4bc^2+3ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2d^6\sqrt{bc^2+ad^2}}\end{aligned}$$

output
```
-4*b^2*c*(b*x^2+a)^(1/2)/d^5+1/2*b^2*x*(b*x^2+a)^(1/2)/d^4-1/3*(a*d^2+b*c^
2)^2*(b*x^2+a)^(1/2)/d^5/(d*x+c)^3+13/6*b*c*(a*d^2+b*c^2)*(b*x^2+a)^(1/2)/
d^5/(d*x+c)^2-1/6*b*(14*a*d^2+47*b*c^2)*(b*x^2+a)^(1/2)/d^5/(d*x+c)+5/2*b^
(3/2)*(a*d^2+4*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^6+5/2*b^2*c*(3*
a*d^2+4*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d
^6/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx =$$

$$-\frac{\frac{d\sqrt{a+bx^2}(2a^2d^4+abd^2(5c^2+15cdx+14d^2x^2)+b^2(60c^4+150c^3dx+110c^2d^2x^2+15cd^3x^3-3d^4x^4))}{(c+dx)^3}-\frac{30b^2c(4bc^2+3ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{}}{\sqrt{-bc^2-a}}\right.}{\sqrt{-bc^2-ad^2}}}{6d^6}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^4,x]
```

output
```
-1/6*((d*Sqrt[a + b*x^2]*(2*a^2*d^4 + a*b*d^2*(5*c^2 + 15*c*d*x + 14*d^2*x
^2) + b^2*(60*c^4 + 150*c^3*d*x + 110*c^2*d^2*x^2 + 15*c*d^3*x^3 - 3*d^4*x
^4)))/(c + d*x)^3 - (30*b^2*c*(4*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[b]*(c + d*x
) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + 1
5*b^(3/2)*(4*b*c^2 + a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^6
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {492, 590, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx$$

$$\Big\downarrow \; 492$$

$$\frac{5b\int \frac{x(bx^2+a)^{3/2}}{(c+dx)^3}dx}{3d}-\frac{(a+bx^2)^{5/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \; 590$$

$$\frac{5b\left(\frac{(a+bx^2)^{3/2}(2c+dx)}{2d^2(c+dx)^2}-\frac{3\int -\frac{2(ad-2bcx)\sqrt{bx^2+a}}{(c+dx)^2}dx}{4d^2}\right)}{3d}-\frac{(a+bx^2)^{5/2}}{3d(c+dx)^3}$$

$\Big\downarrow$ 27

$$\frac{5b\left(\frac{3\int \frac{(ad-2bcx)\sqrt{bx^2+a}}{(c+dx)^2}dx}{2d^2}+\frac{(a+bx^2)^{3/2}(2c+dx)}{2d^2(c+dx)^2}\right)}{3d}-\frac{(a+bx^2)^{5/2}}{3d(c+dx)^3}$$

$\Big\downarrow$ 681

$$\frac{5b\left(\frac{3\left(-\frac{\int \frac{2b\left(2acd-\left(4bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2+2bcdx\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{(a+bx^2)^{3/2}(2c+dx)}{2d^2(c+dx)^2}\right)}{3d}-\frac{(a+bx^2)^{5/2}}{3d(c+dx)^3}$$

$\Big\downarrow$ 27

$$\frac{5b\left(\frac{3\left(-\frac{b\int \frac{2acd-\left(4bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2+2bcdx\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{(a+bx^2)^{3/2}(2c+dx)}{2d^2(c+dx)^2}\right)}{3d}-\frac{(a+bx^2)^{5/2}}{3d(c+dx)^3}$$

$\Big\downarrow$ 719

$$\frac{5b\left(\frac{3\left(-\frac{b\left(\frac{c\left(3ad^2+4bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2+4bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2+2bcdx\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{(a+bx^2)^{3/2}(2c+dx)}{2d^2(c+dx)^2}\right)}{3d}-\frac{(a+bx^2)^{5/2}}{3d(c+dx)^3}$$

$\Big\downarrow$ 224

$$\frac{5b\left(\frac{3\left(-\frac{b\left(\frac{c\left(3ad^2+4bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2+4bc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2+2bcdx\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}\right)}{3d}-\frac{\left(a+bx^2\right)^{5/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \ 219$$

$$\frac{5b\left(\frac{3\left(-\frac{b\left(\frac{c\left(3ad^2+4bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+4bc^2\right)}{\sqrt{b}d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2+2bcdx\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}(2c+dx)}{2d^2(c+dx)^2}\right)}{3d}-\frac{\left(a+bx^2\right)^{5/2}}{3d(c+dx)^3}$$

$$\Big\downarrow \ 488$$

$$\frac{5b\left(\frac{3\left(-\frac{b\left(-\frac{c\left(3ad^2+4bc^2\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+4bc^2\right)}{\sqrt{b}d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2+2bcdx\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}}{2d^2(c+}\right)}{3d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{3d(c+dx)^3}$$

$\downarrow$ 219

$$\frac{5b\left(\frac{3\left(-\frac{b\left(-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+4bc^2\right)}{\sqrt{b}d}-\frac{c\left(3ad^2+4bc^2\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+4bc^2+2bcdx\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}(}{2d^2(c+d}\right)}{3d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{3d(c+dx)^3}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^4,x]
```

output
```
-1/3*(a + b*x^2)^(5/2)/(d*(c + d*x)^3) + (5*b*(((2*c + d*x)*(a + b*x^2)^(3
/2))/(2*d^2*(c + d*x)^2) + (3*(-(((4*b*c^2 + a*d^2 + 2*b*c*d*x)*Sqrt[a + b
*x^2])/(d^2*(c + d*x))) - (b*(-(((4*b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(Sqrt[b]*d)) - (c*(4*b*c^2 + 3*a*d^2)*ArcTanh[(a*d - b*c*x)
/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/d^2))/(
2*d^2)))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(243) = 486$.

Time = 0.51 (sec) , antiderivative size = 1726, normalized size of antiderivative = 6.23

method	result	size
risch	Expression too large to display	1726
default	Expression too large to display	3656

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2*(-d*x+8*c)*(b*x^2+a)^(1/2)/d^5+1/2/d^5*(2*(a^3*d^6+3*a^2*b*c^2*d^
4+3*a*b^2*c^4*d^2+b^3*c^6)/d^5*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d)^3*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+5/3*b*c*d/(a*d^2+b*c^2)*(-1/2
/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c
^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^
2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+
1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-
2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-2/3*b/(a*d^2+b*c^2)*d^2*(-1/(a*d^2+b*c^
2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d
/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(
x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2))/(x+c/d))))+5*b^(3/2)*(a*d^2+4*b*c^2)/d*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))+6*b/d^3*(a^2*d^4+6*a*b*c^2*d^2+5*b^2*c^4)*(-1/(a*d^2+b*c^2)*d^2
/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^
2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)
+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(1/2))/(x+c/d)))-12/d^4*b*c*(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)*(-1/2/(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(244) = 488$.

Time = 1.97 (sec) , antiderivative size = 2501, normalized size of antiderivative = 9.03

$$\int \frac{(a + bx^2)^{5/2}}{(c + dx)^4} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
[1/12*(15*(4*b^3*c^7 + 5*a*b^2*c^5*d^2 + a^2*b*c^3*d^4 + (4*b^3*c^4*d^3 +
5*a*b^2*c^2*d^5 + a^2*b*d^7)*x^3 + 3*(4*b^3*c^5*d^2 + 5*a*b^2*c^3*d^4 + a^
2*b*c*d^6)*x^2 + 3*(4*b^3*c^6*d + 5*a*b^2*c^4*d^3 + a^2*b*c^2*d^5)*x)*sqrt
(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 15*(4*b^3*c^6 + 3*a*
b^2*c^4*d^2 + (4*b^3*c^3*d^3 + 3*a*b^2*c*d^5)*x^3 + 3*(4*b^3*c^4*d^2 + 3*a
*b^2*c^2*d^4)*x^2 + 3*(4*b^3*c^5*d + 3*a*b^2*c^3*d^3)*x)*sqrt(b*c^2 + a*d^
2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*
sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^
2)) - 2*(60*b^3*c^6*d + 65*a*b^2*c^4*d^3 + 7*a^2*b*c^2*d^5 + 2*a^3*d^7 - 3
*(b^3*c^2*d^5 + a*b^2*d^7)*x^4 + 15*(b^3*c^3*d^4 + a*b^2*c*d^6)*x^3 + 2*(5
5*b^3*c^4*d^3 + 62*a*b^2*c^2*d^5 + 7*a^2*b*d^7)*x^2 + 15*(10*b^3*c^5*d^2 +
 11*a*b^2*c^3*d^4 + a^2*b*c*d^6)*x)*sqrt(b*x^2 + a))/(b*c^5*d^6 + a*c^3*d^
8 + (b*c^2*d^9 + a*d^11)*x^3 + 3*(b*c^3*d^8 + a*c*d^10)*x^2 + 3*(b*c^4*d^7
 + a*c^2*d^9)*x), 1/12*(30*(4*b^3*c^6 + 3*a*b^2*c^4*d^2 + (4*b^3*c^3*d^3 +
 3*a*b^2*c*d^5)*x^3 + 3*(4*b^3*c^4*d^2 + 3*a*b^2*c^2*d^4)*x^2 + 3*(4*b^3*c
^5*d + 3*a*b^2*c^3*d^3)*x)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2
)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x
^2)) + 15*(4*b^3*c^7 + 5*a*b^2*c^5*d^2 + a^2*b*c^3*d^4 + (4*b^3*c^4*d^3 +
5*a*b^2*c^2*d^5 + a^2*b*d^7)*x^3 + 3*(4*b^3*c^5*d^2 + 5*a*b^2*c^3*d^4 + a^
2*b*c*d^6)*x^2 + 3*(4*b^3*c^6*d + 5*a*b^2*c^4*d^3 + a^2*b*c^2*d^5)*x)*s...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx)^4}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c)**4,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. 2(244) = 488.

Time = 0.12 (sec) , antiderivative size = 1023, normalized size of antiderivative = 3.69

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-5/4*b^5*c^6*arcsinh(b*x/sqrt(a*b))/(b^(5/2)*c^4*d^6 + 2*a*b^(3/2)*c^2*d^8
 + a^2*sqrt(b)*d^10) - 5/4*a*b^4*c^4*arcsinh(b*x/sqrt(a*b))/(b^(5/2)*c^4*d
^4 + 2*a*b^(3/2)*c^2*d^6 + a^2*sqrt(b)*d^8) + 5/4*sqrt(b*x^2 + a)*b^4*c^4*
x/(b^2*c^4*d^4 + 2*a*b*c^2*d^6 + a^2*d^8) - 5/6*(b*x^2 + a)^(3/2)*b^3*c^3/
(b^2*c^4*d^3 + 2*a*b*c^2*d^5 + a^2*d^7) + 5/6*(b*x^2 + a)^(3/2)*b^3*c^2*x/
(b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6) + 5/4*sqrt(b*x^2 + a)*a*b^3*c^2*x/
(b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6) - 5/4*a*b^3*c^2*arcsinh(b*x/sqrt(a
*b))/(b^(3/2)*c^2*d^4 + a*sqrt(b)*d^6) - 1/2*(b*x^2 + a)^(5/2)*b^2*c^2/(b^
2*c^4*d^2*x + 2*a*b*c^2*d^4*x + a^2*d^6*x + b^2*c^5*d + 2*a*b*c^3*d^3 + a^
2*c*d^5) - 5/2*sqrt(b*x^2 + a)*b^3*c^3/(b*c^2*d^5 + a*d^7) + 15/4*sqrt(b*x
^2 + a)*b^3*c^2*x/(b*c^2*d^4 + a*d^6) + 1/6*(b*x^2 + a)^(7/2)*b*c/(b^2*c^4
*d*x^2 + 2*a*b*c^2*d^3*x^2 + a^2*d^5*x^2 + 2*b^2*c^5*x + 4*a*b*c^3*d^2*x +
 2*a^2*c*d^4*x + b^2*c^6/d + 2*a*b*c^4*d + a^2*c^2*d^3) - 1/6*(b*x^2 + a)^
(5/2)*b^2*c/(b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5) - 5/2*(b*x^2 + a)^(3/2)*
b^2*c/(b*c^2*d^3 + a*d^5) + 5/3*(b*x^2 + a)^(3/2)*b^2*x/(b*c^2*d^2 + a*d^4
) + 5/2*sqrt(b*x^2 + a)*a*b^2*x/(b*c^2*d^2 + a*d^4) - 1/3*(b*x^2 + a)^(7/2
)/(b*c^2*d^2*x^3 + a*d^4*x^3 + 3*b*c^3*d*x^2 + 3*a*c*d^3*x^2 + 3*b*c^4*x +
 3*a*c^2*d^2*x + b*c^5/d + a*c^3*d) - 4/3*(b*x^2 + a)^(5/2)*b/(b*c^2*d^2*x
 + a*d^4*x + b*c^3*d + a*c*d^3) + 45/4*b^(5/2)*c^2*arcsinh(b*x/sqrt(a*b))/
d^6 + 5/2*a*b^(3/2)*arcsinh(b*x/sqrt(a*b))/d^4 - 5/2*b^3*c^3*arcsinh(b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(244) = 488$.

Time = 0.27 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx = \frac{1}{2}\sqrt{bx^2+a}\left(\frac{b^2x}{d^4}-\frac{8\,b^2c}{d^5}\right) - \frac{5\left(4\,b^{\frac{5}{2}}c^2+ab^{\frac{3}{2}}d^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,d^6} - \frac{5\,(4\,b^3c^3+3\,ab^2cd^2)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}\,d^6} - \frac{60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 b^3c^3d^2+27\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 ab^2cd^4+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{7}{2}}c^4d+27\left(\sqrt{\right.}\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*(b^2*x/d^4 - 8*b^2*c/d^5) - 5/2*(4*b^(5/2)*c^2 + a*b^(
3/2)*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/d^6 - 5*(4*b^3*c^3 + 3*a*
b^2*c*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c
^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*d^6) - 1/3*(60*(sqrt(b)*x - sqrt(b*x^2
+ a))^5*b^3*c^3*d^2 + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a*b^2*c*d^4 + 210
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^4*d + 27*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a*b^(5/2)*c^2*d^3 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/
2)*d^5 + 188*(sqrt(b)*x - sqrt(b*x^2 + a))^3*b^4*c^5 - 226*(sqrt(b)*x - sq
rt(b*x^2 + a))^3*a*b^3*c^3*d^2 - 84*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a^2*b^
2*c*d^4 - 354*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(7/2)*c^4*d + 24*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*d^5 + 222*(sqrt(b)*x - sqrt(b*x^2 + a
))*a^2*b^3*c^3*d^2 + 57*(sqrt(b)*x - sqrt(b*x^2 + a))*a^3*b^2*c*d^4 - 47*a
^3*b^(5/2)*c^2*d^3 - 14*a^4*b^(3/2)*d^5)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*c - a*d)^3*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx = \int \frac{\left(b\,x^2+a\right)^{5/2}}{\left(c+d\,x\right)^4}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^4,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1756, normalized size of antiderivative = 6.34

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^4,x)
```

output

```
(90*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a*b**2*c**4*d**2 + 270*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**3*x + 270*sqrt
(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*
c*x)*a*b**2*c**2*d**4*x**2 + 90*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**5*x**3 + 120*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*b**3*c**6 + 360*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2
 + b*c**2) - a*d + b*c*x)*b**3*c**5*d*x + 360*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**4*d**2*x**2
 + 120*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*b**3*c**3*d**3*x**3 - 90*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*a*b**2*c**4*d**2 - 270*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d*
*3*x - 270*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**4*x**2 - 90*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**5*x**3 - 120*sqrt(a*d**2 + b
*c**2)*log(c + d*x)*b**3*c**6 - 360*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**
3*c**5*d*x - 360*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**4*d**2*x**2 -
120*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**3*d**3*x**3 - 4*sqrt(a + b*
x**2)*a**3*d**7 - 14*sqrt(a + b*x**2)*a**2*b*c**2*d**5 - 30*sqrt(a + b*x**
2)*a**2*b*c*d**6*x - 28*sqrt(a + b*x**2)*a**2*b*d**7*x**2 - 130*sqrt(a ...
```

3.261 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 308

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5} dx &= \frac{b^2\sqrt{a+bx^2}}{d^5} - \frac{(bc^2+ad^2)^2\sqrt{a+bx^2}}{4d^5(c+dx)^4} \\ &+ \frac{17bc(bc^2+ad^2)\sqrt{a+bx^2}}{12d^5(c+dx)^3} - \frac{b(86bc^2+27ad^2)\sqrt{a+bx^2}}{24d^5(c+dx)^2} \\ &+ \frac{b^2c(154bc^2+139ad^2)\sqrt{a+bx^2}}{24d^5(bc^2+ad^2)(c+dx)} - \frac{5b^{5/2}c\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^6} \\ &- \frac{5b^2(8b^2c^4+12abc^2d^2+3a^2d^4)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{8d^6(bc^2+ad^2)^{3/2}}\end{aligned}$$

output
```
b^2*(b*x^2+a)^(1/2)/d^5-1/4*(a*d^2+b*c^2)^2*(b*x^2+a)^(1/2)/d^5/(d*x+c)^4+
17/12*b*c*(a*d^2+b*c^2)*(b*x^2+a)^(1/2)/d^5/(d*x+c)^3-1/24*b*(27*a*d^2+86*
b*c^2)*(b*x^2+a)^(1/2)/d^5/(d*x+c)^2+1/24*b^2*c*(139*a*d^2+154*b*c^2)*(b*x
^2+a)^(1/2)/d^5/(a*d^2+b*c^2)/(d*x+c)-5*b^(5/2)*c*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/d^6-5/8*b^2*(3*a^2*d^4+12*a*b*c^2*d^2+8*b^2*c^4)*arctanh((-b*c*
x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^6/(a*d^2+b*c^2)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 946 vs. $2(308) = 616$.

Time = 7.67 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.07

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5}\,dx = \frac{ad\left(-6a^5d^6+240b^{9/2}c^3x^3\left(4c^3+15c^2dx+20cd^2x^2+10d^3x^3\right)\left(\sqrt{b}x-\sqrt{a+bx^2}\right)-a^4\left(-30\sqrt{b}d^6x\sqrt{a+bx^2}+bd^4\left(11c^2+20\right.\right.\right.}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^5,x]
```

output
```
((a*d*(-6*a^5*d^6 + 240*b^(9/2)*c^3*x^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2
 + 10*d^3*x^3)*(Sqrt[b]*x - Sqrt[a + b*x^2]) - a^4*(-30*Sqrt[b]*d^6*x*Sqrt
[a + b*x^2] + b*d^4*(11*c^2 + 20*c*d*x + 105*d^2*x^2)) + 5*a^3*(b^(3/2)*d^
4*x*Sqrt[a + b*x^2]*(11*c^2 + 20*c*d*x + 51*d^2*x^2) + b^2*d^2*(20*c^4 + 7
1*c^3*d*x + 61*c^2*d^2*x^2 - 5*c*d^3*x^3 - 99*d^4*x^4)) + 5*a^2*(b^(5/2)*d
*Sqrt[a + b*x^2]*(-3*c^5 - 88*c^4*d*x - 289*c^3*d^2*x^2 - 312*c^2*d^3*x^3
- 108*c*d^4*x^4 + 108*d^5*x^5) + b^3*(24*c^6 + 99*c^5*d*x + 304*c^4*d^2*x^
2 + 643*c^3*d^3*x^3 + 648*c^2*d^4*x^4 + 264*c*d^5*x^5 - 132*d^6*x^6)) + 60
*a*(b^4*x^2*(16*c^6 + 61*c^5*d*x + 96*c^4*d^2*x^2 + 89*c^3*d^3*x^3 + 56*c^
2*d^4*x^4 + 26*c*d^5*x^5 - 4*d^6*x^6) + b^(7/2)*x*Sqrt[a + b*x^2]*(-8*c^6
- 31*c^5*d*x - 56*c^4*d^2*x^2 - 69*c^3*d^3*x^3 - 56*c^2*d^4*x^4 - 26*c*d^5
*x^5 + 4*d^6*x^6))))/((b*c^2 + a*d^2)*(c + d*x)^4*(a^2*(-5*Sqrt[b]*x + Sqr
t[a + b*x^2]) + 16*b^2*x^4*(-(Sqrt[b]*x) + Sqrt[a + b*x^2]) + 4*a*b*x^2*(-
5*Sqrt[b]*x + 3*Sqrt[a + b*x^2]))) + (240*b^4*c^4*ArcTan[(Sqrt[b]*(c + d*x
) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) +
 (360*a*b^3*c^2*d^2*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(
b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) + (90*a^2*b^2*d^4*ArcTan[(Sqrt[
b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d
^2)^(3/2) + (2*b^(5/2)*(77*c^5 + 248*c^4*d*x + 252*c^3*d^2*x^2 + 48*c^2*d^
3*x^3 - 48*c*d^4*x^4 - 12*d^5*x^5 + 60*c*(c + d*x)^4*Log[-(Sqrt[b]*x) +...
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {492, 589, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{(c+dx)^5}dx$$

$$\Big\downarrow \text{492}$$

$$\frac{5b\int \frac{x\left(bx^2+a\right)^{3/2}}{(c+dx)^4}dx}{4d}-\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$$\Big\downarrow \text{589}$$

$$\frac{5b\left(\frac{b\int -\frac{2\left(2acd-\left(4bc^2+3ad^2\right)x\right)\sqrt{bx^2+a}}{(c+dx)^2}dx}{4d^2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(3dx\left(ad^2+2bc^2\right)+c\left(ad^2+4bc^2\right)\right)}{6d^2(c+dx)^3\left(ad^2+bc^2\right)}\right)}{4d}-\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{5b\left(-\frac{b\int \frac{\left(2acd-\left(4bc^2+3ad^2\right)x\right)\sqrt{bx^2+a}}{(c+dx)^2}dx}{2d^2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(3dx\left(ad^2+2bc^2\right)+c\left(ad^2+4bc^2\right)\right)}{6d^2(c+dx)^3\left(ad^2+bc^2\right)}\right)}{4d}-\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$$\Big\downarrow \text{681}$$

$$\frac{5b\left(-\frac{b\left(-\frac{\int \frac{2\left(ad\left(4bc^2+3ad^2\right)-8bc\left(bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}\left(dx\left(3ad^2+4bc^2\right)+8c\left(ad^2+bc^2\right)\right)}{d^2(c+dx)}\right)}{2d^2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(3dx\left(ad^2+2bc^2\right)+c\left(ad^2+4bc^2\right)\right)}{6d^2(c+dx)^3\left(ad^2+bc^2\right)}\right)}{4d}-\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{5b\left(-\frac{b\left(-\frac{\int \frac{ad\left(4bc^2+3ad^2\right)-8bc\left(bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{\sqrt{a+bx^2}\left(dx\left(3ad^2+4bc^2\right)+8c\left(ad^2+bc^2\right)\right)}{d^2(c+dx)}\right)}{2d^2(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}\left(3dx\left(ad^2+2bc^2\right)+c\left(ad^2+4bc^2\right)\right)}{6d^2(c+dx)^3(ad^2+bc^2)}\right)}{4d} - \ldots$$

$$\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$$\Big\downarrow \; 719$$

$$\frac{5b\left(-\frac{b\left(-\frac{\frac{\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{8bc\left(ad^2+bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2}-\frac{\sqrt{a+bx^2}\left(dx\left(3ad^2+4bc^2\right)+8c\left(ad^2+bc^2\right)\right)}{d^2(c+dx)}\right)}{2d^2(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}\ldots}{\ldots}\right)}{4d} \ldots$$

$$\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$$\Big\downarrow \; 224$$

$$\frac{5b\left(-\frac{b\left(-\frac{\frac{\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{8bc\left(ad^2+bc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d^2}-\frac{\sqrt{a+bx^2}\left(dx\left(3ad^2+4bc^2\right)+8c\left(ad^2+bc^2\right)\right)}{d^2(c+dx)}\right)}{2d^2(ad^2+bc^2)}-\frac{(a\ldots}{\ldots}\right)}{4d} \ldots$$

$$\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$$\Big\downarrow \; 219$$

$$\frac{5b\left(-\frac{b\left(-\frac{\frac{\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{8\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{d}}{d^2}-\frac{\sqrt{a+bx^2}\left(dx\left(3ad^2+4bc^2\right)+8c\left(ad^2+bc^2\right)\right)}{d^2(c+dx)}\right)}{2d^2(ad^2+bc^2)}-\frac{(}{}\right.}{4d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$\downarrow$ 488

$$\frac{5b\left(-\frac{b\left(-\frac{-\frac{\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{8\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{d}}{d^2}-\frac{\sqrt{a+bx^2}\left(dx\left(3ad^2+4bc^2\right)+8c\left(ad^2\right.\right.}{d^2(c+dx)}\right.}{2d^2(ad^2+bc^2)}\right.}{4d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

$\downarrow$ 219

$$\frac{5b\left(-\frac{b\left(-\frac{-\frac{\left(3a^2d^4+12abc^2d^2+8b^2c^4\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}-\frac{8\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{d}}{d^2}-\frac{\sqrt{a+bx^2}\left(dx\left(3ad^2+4bc^2\right)+8c\left(ad^2-\right.\right.}{d^2(c+dx)}\right.}{2d^2(ad^2+bc^2)}\right.}{4d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{4d(c+dx)^4}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^5,x]
```

output
```
-1/4*(a + b*x^2)^(5/2)/(d*(c + d*x)^4) + (5*b*(-1/6*((c*(4*b*c^2 + a*d^2)
+ 3*d*(2*b*c^2 + a*d^2)*x)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 + a*d^2)*(c + d*
x)^3) - (b*(-(((8*c*(b*c^2 + a*d^2) + d*(4*b*c^2 + 3*a*d^2)*x)*Sqrt[a + b*
x^2])/(d^2*(c + d*x))) - ((-8*Sqrt[b]*c*(b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/d - ((8*b^2*c^4 + 12*a*b*c^2*d^2 + 3*a^2*d^4)*ArcTanh[
(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^
2]))/d^2))/(2*d^2*(b*c^2 + a*d^2))))/(4*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3029 vs. $2(276) = 552$.

Time = 0.64 (sec), antiderivative size = 3030, normalized size of antiderivative = 9.84

method	result	size
risch	Expression too large to display	3030
default	Expression too large to display	6003

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
b^2*(b*x^2+a)^(1/2)/d^5-1/d^5*(-(a^3*d^6+3*a^2*b*c^2*d^4+3*a*b^2*c^4*d^2+b
^3*c^6)/d^6*(-1/4/(a*d^2+b*c^2)*d^2/(x+c/d)^4*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2)+7/4*b*c*d/(a*d^2+b*c^2)*(-1/3/(a*d^2+b*c^2)*d^2/
(x+c/d)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+5/3*b*c*d/
(a*d^2+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/
(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2
+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+
2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2))/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2
*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-2/3*b/(a*d^2+b*c^2)*
d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^
2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b
*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d
*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))-3/4*b/(a*d^2+b*c^2)*d^2*(-1/
2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d
^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*
c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(277) = 554$.

Time = 7.14 (sec) , antiderivative size = 3733, normalized size of antiderivative = 12.12

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx)^5}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c)**5,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1903 vs. $2(277) = 554$.

Time $= 0.17$ (sec) , antiderivative size $= 1903$, normalized size of antiderivative $= 6.18$

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
-5/16*b^6*c^7*arcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^6 + 3*a*b^(5/2)*c^4*d^
8 + 3*a^2*b^(3/2)*c^2*d^10 + a^3*sqrt(b)*d^12) - 5/16*a*b^5*c^5*arcsinh(b*
x/sqrt(a*b))/(b^(7/2)*c^6*d^4 + 3*a*b^(5/2)*c^4*d^6 + 3*a^2*b^(3/2)*c^2*d^
8 + a^3*sqrt(b)*d^10) + 5/16*sqrt(b*x^2 + a)*b^5*c^5*x/(b^3*c^6*d^4 + 3*a*
b^2*c^4*d^6 + 3*a^2*b*c^2*d^8 + a^3*d^10) + 45/16*b^5*c^5*arcsinh(b*x/sqrt
(a*b))/(b^(5/2)*c^4*d^6 + 2*a*b^(3/2)*c^2*d^8 + a^2*sqrt(b)*d^10) - 5/24*(
b*x^2 + a)^(3/2)*b^4*c^4/(b^3*c^6*d^3 + 3*a*b^2*c^4*d^5 + 3*a^2*b*c^2*d^7
+ a^3*d^9) + 5/24*(b*x^2 + a)^(3/2)*b^4*c^3*x/(b^3*c^6*d^2 + 3*a*b^2*c^4*d
^4 + 3*a^2*b*c^2*d^6 + a^3*d^8) + 5/16*sqrt(b*x^2 + a)*a*b^4*c^3*x/(b^3*c^
6*d^2 + 3*a*b^2*c^4*d^4 + 3*a^2*b*c^2*d^6 + a^3*d^8) + 5/2*a*b^4*c^3*arcsi
nh(b*x/sqrt(a*b))/(b^(5/2)*c^4*d^4 + 2*a*b^(3/2)*c^2*d^6 + a^2*sqrt(b)*d^8
) - 1/8*(b*x^2 + a)^(5/2)*b^3*c^3/(b^3*c^6*d^2*x + 3*a*b^2*c^4*d^4*x + 3*a
^2*b*c^2*d^6*x + a^3*d^8*x + b^3*c^7*d + 3*a*b^2*c^5*d^3 + 3*a^2*b*c^3*d^5
 + a^3*c*d^7) - 5/8*sqrt(b*x^2 + a)*b^4*c^4/(b^2*c^4*d^5 + 2*a*b*c^2*d^7 +
 a^2*d^9) - 15/8*sqrt(b*x^2 + a)*b^4*c^3*x/(b^2*c^4*d^4 + 2*a*b*c^2*d^6 +
a^2*d^8) - 75/16*b^4*c^3*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^6 + a*sqrt(
b)*d^8) + 1/24*(b*x^2 + a)^(7/2)*b^2*c^2/(b^3*c^6*d*x^2 + 3*a*b^2*c^4*d^3*
x^2 + 3*a^2*b*c^2*d^5*x^2 + a^3*d^7*x^2 + 2*b^3*c^7*x + 6*a*b^2*c^5*d^2*x
+ 6*a^2*b*c^3*d^4*x + 2*a^3*c*d^6*x + b^3*c^8/d + 3*a*b^2*c^6*d + 3*a^2*b*
c^4*d^3 + a^3*c^2*d^5) - 1/24*(b*x^2 + a)^(5/2)*b^3*c^2/(b^3*c^6*d + 3*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(c+d\,x)^5}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^5,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2824, normalized size of antiderivative = 9.17

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^5,x)
```

output
```
(45*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**2*c**4*d**4 + 180*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d**5*x + 270*sqrt
(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a**2*b**2*c**2*d**6*x**2 + 180*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**7*x**3 + 45*sqrt(a*d
**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
*2*b**2*d**8*x**4 + 180*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**6*d**2 + 720*sqrt(a*d**2 + b*c**2)
*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**5*d**
3*x + 1080*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*a*b**3*c**4*d**4*x**2 + 720*sqrt(a*d**2 + b*c**2)*log(sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d**5*x**3 +
 180*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a*b**3*c**2*d**6*x**4 + 120*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**8 + 480*sqrt(a*d**2 +
 b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c*
*7*d*x + 720*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*b**4*c**6*d**2*x**2 + 480*sqrt(a*d**2 + b*c**2)*log(sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**5*d**3*x**3 ...
```

3.262 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6} dx$

Optimal result

Integrand size = 19, antiderivative size = 357

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6} dx =& -\frac{(bc^2+ad^2)^2\sqrt{a+bx^2}}{5d^5(c+dx)^5}+\frac{21bc(bc^2+ad^2)\sqrt{a+bx^2}}{20d^5(c+dx)^4} \\ & -\frac{b(137bc^2+44ad^2)\sqrt{a+bx^2}}{60d^5(c+dx)^3}+\frac{b^2c(326bc^2+311ad^2)\sqrt{a+bx^2}}{120d^5(bc^2+ad^2)(c+dx)^2} \\ & -\frac{b^2(274b^2c^4+503abc^2d^2+184a^2d^4)\sqrt{a+bx^2}}{120d^5(bc^2+ad^2)^2(c+dx)}+\frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^6} \\ & +\frac{b^3c(8b^2c^4+20abc^2d^2+15a^2d^4)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{8d^6(bc^2+ad^2)^{5/2}}\end{aligned}$$

output
```
-1/5*(a*d^2+b*c^2)^2*(b*x^2+a)^(1/2)/d^5/(d*x+c)^5+21/20*b*c*(a*d^2+b*c^2)
*(b*x^2+a)^(1/2)/d^5/(d*x+c)^4-1/60*b*(44*a*d^2+137*b*c^2)*(b*x^2+a)^(1/2)
/d^5/(d*x+c)^3+1/120*b^2*c*(311*a*d^2+326*b*c^2)*(b*x^2+a)^(1/2)/d^5/(a*d^
2+b*c^2)/(d*x+c)^2-1/120*b^2*(184*a^2*d^4+503*a*b*c^2*d^2+274*b^2*c^4)*(b*
x^2+a)^(1/2)/d^5/(a*d^2+b*c^2)^2/(d*x+c)+b^(5/2)*arctanh(b^(1/2)*x/(b*x^2+
a)^(1/2))/d^6+1/8*b^3*c*(15*a^2*d^4+20*a*b*c^2*d^2+8*b^2*c^4)*arctanh((-b*
c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^6/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 10.41 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx = \frac{-\frac{d\sqrt{a+bx^2}\left(24(bc^2+ad^2)^4-126bc(bc^2+ad^2)^3(c+dx)+2b(bc^2+ad^2)^2(137bc^2+44ad^2)(c+dx)^2-b^2c(bc^2+ad^2)(326b\right.}{(bc^2+ad^2)^2(c+dx)^5}}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^6,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(24*(b*c^2 + a*d^2)^4 - 126*b*c*(b*c^2 + a*d^2)^3*(c
 + d*x) + 2*b*(b*c^2 + a*d^2)^2*(137*b*c^2 + 44*a*d^2)*(c + d*x)^2 - b^2*c
*(b*c^2 + a*d^2)*(326*b*c^2 + 311*a*d^2)*(c + d*x)^3 + b^2*(274*b^2*c^4 +
503*a*b*c^2*d^2 + 184*a^2*d^4)*(c + d*x)^4))/((b*c^2 + a*d^2)^2*(c + d*x)^
5)) - (15*b^3*c*(8*b^2*c^4 + 20*a*b*c^2*d^2 + 15*a^2*d^4)*Log[c + d*x])/(b
*c^2 + a*d^2)^(5/2) + 120*b^(5/2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]] + (15
*b^3*c*(8*b^2*c^4 + 20*a*b*c^2*d^2 + 15*a^2*d^4)*Log[a*d - b*c*x + Sqrt[b*
c^2 + a*d^2]*Sqrt[a + b*x^2]])/(b*c^2 + a*d^2)^(5/2))/(120*d^6)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {492, 589, 27, 680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx$$

$$\Big\downarrow \text{492}$$

$$\frac{b\int \frac{x(bx^2+a)^{3/2}}{(c+dx)^5}dx}{d}-\frac{(a+bx^2)^{5/2}}{5d(c+dx)^5}$$

$$\Big\downarrow \text{589}$$

$$\frac{b\left(\frac{b\int -\frac{2\left(3acd-4\left(bc^2+ad^2\right)x\right)\sqrt{bx^2+a}}{(c+dx)^3}dx}{8d^2(ad^2+bc^2)}-\frac{(a+bx^2)^{3/2}(dx(4ad^2+7bc^2)+c(ad^2+4bc^2))}{12d^2(c+dx)^4(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\big\downarrow$ 27

$$\frac{b\left(-\frac{b\int \frac{\left(3acd-4\left(bc^2+ad^2\right)x\right)\sqrt{bx^2+a}}{(c+dx)^3}dx}{4d^2(ad^2+bc^2)}-\frac{(a+bx^2)^{3/2}(dx(4ad^2+7bc^2)+c(ad^2+4bc^2))}{12d^2(c+dx)^4(ad^2+bc^2)}\right)}{d}-\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\big\downarrow$ 680

$$\frac{b\left(-\frac{b\left(\frac{\sqrt{a+bx^2}\left(dx\left(8a^2d^4+23abc^2d^2+12b^2c^4\right)+c\left(a^2d^4+12abc^2d^2+8b^2c^4\right)\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{2b\left(acd\left(4bc^2+7ad^2\right)-8\left(bc^2+ad^2\right)^2x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{4d^2\left(ad^2+bc^2\right)}\right)}{4d^2(ad^2+bc^2)}-\frac{(a+bx^2)^{3/2}(dx(4ad \ldots}{12d^2(c+ \ldots}\right.}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\big\downarrow$ 27

$$\frac{b\left(-\frac{b\left(\frac{b\int \frac{acd\left(4bc^2+7ad^2\right)-8\left(bc^2+ad^2\right)^2x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(dx\left(8a^2d^4+23abc^2d^2+12b^2c^4\right)+c\left(a^2d^4+12abc^2d^2+8b^2c^4\right)\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4d^2(ad^2+bc^2)}-\frac{(a+bx^2)^{3/2}(dx(4ad^2+7b \ldots}{12d^2(c+dx)^4(\ldots}\right.}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\big\downarrow$ 719

$$\frac{b\left(-\frac{b\left(\frac{b\left(\frac{c\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{8\left(ad^2+bc^2\right)^2\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{2d^2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(dx\left(8a^2d^4+23abc^2d^2+12b^2c^4\right)+c\left(a^2d^4+12abc^2d^2\right.\right.}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}}{4d^2(ad^2+bc^2)}\right.}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\downarrow$ 224

$$\frac{b\left(-\frac{b\left(\frac{b\left(\frac{c\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{8\left(ad^2+bc^2\right)^2\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{2d^2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(dx\left(8a^2d^4+23abc^2d^2+12b^2c^4\right)+c\left(a^2d^4\right.\right.}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}}{4d^2(ad^2+bc^2)}\right.}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\downarrow$ 219

$$\frac{b\left(-\frac{b\left(\frac{b\left(\frac{c\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{8\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2}{\sqrt{b}d}\right)}{2d^2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(dx\left(8a^2d^4+23abc^2d^2+12b^2c^4\right)+c\left(a^2d^4\cdots\right.\right.}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}\cdots\right)}{4d^2(ad^2+bc^2)}\cdots\right)}{d}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\downarrow$ 488

$$\frac{b\left(-\frac{b\left(\frac{b\left(-\frac{c\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{8\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2}{\sqrt{b}d}\right)}{2d^2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(dx\left(8a^2d^4+23abc^2d^2+12\cdots\right.\right.}{2d^2(c+dx)\cdots}\cdots\right)}{4d^2(ad^2+bc^2)}\cdots\right)}{d}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

$\downarrow$ 219

$$\frac{b\left(-\frac{b\left(\frac{b\left(-\frac{c\left(15a^2d^4+20abc^2d^2+8b^2c^4\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}-\frac{8\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2}{\sqrt{b}d}\right)}{2d^2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(dx\left(8a^2d^4+23abc^2d^2+12\cdots\right.\right.}{2d^2(c+dx)^2\cdots}\right)}{4d^2(ad^2+bc^2)}\cdots\right)}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5d(c+dx)^5}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^6,x]
```

output
```
-1/5*(a + b*x^2)^(5/2)/(d*(c + d*x)^5) + (b*(-1/12*((c*(4*b*c^2 + a*d^2) +
 d*(7*b*c^2 + 4*a*d^2)*x)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 + a*d^2)*(c + d*x
)^4) - (b*(((c*(8*b^2*c^4 + 12*a*b*c^2*d^2 + a^2*d^4) + d*(12*b^2*c^4 + 23
*a*b*c^2*d^2 + 8*a^2*d^4)*x)*Sqrt[a + b*x^2])/(2*d^2*(b*c^2 + a*d^2)*(c +
d*x)^2) + (b*((-8*(b*c^2 + a*d^2)^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(Sqrt[b]*d) - (c*(8*b^2*c^4 + 20*a*b*c^2*d^2 + 15*a^2*d^4)*ArcTanh[(a*d -
b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(
2*d^2*(b*c^2 + a*d^2))))/(4*d^2*(b*c^2 + a*d^2))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9761 vs. 2(323) = 646.

Time = 0.78 (sec) , antiderivative size = 9762, normalized size of antiderivative = 27.34

method	result	size
default	Expression too large to display	9762

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. 2(324) = 648.

Time = 46.13 (sec) , antiderivative size = 5169, normalized size of antiderivative = 14.48

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^6,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx)^6}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c)**6,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3009 vs. $2(324) = 648$.

Time = 0.25 (sec) , antiderivative size = 3009, normalized size of antiderivative = 8.43

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^6,x, algorithm="maxima")
```

output
```
-3/16*b^7*c^8*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^6 + 4*a*b^(7/2)*c^6*d^
8 + 6*a^2*b^(5/2)*c^4*d^10 + 4*a^3*b^(3/2)*c^2*d^12 + a^4*sqrt(b)*d^14) -
3/16*a*b^6*c^6*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^4 + 4*a*b^(7/2)*c^6*d
^6 + 6*a^2*b^(5/2)*c^4*d^8 + 4*a^3*b^(3/2)*c^2*d^10 + a^4*sqrt(b)*d^12) +
3/16*sqrt(b*x^2 + a)*b^6*c^6*x/(b^4*c^8*d^4 + 4*a*b^3*c^6*d^6 + 6*a^2*b^2*
c^4*d^8 + 4*a^3*b*c^2*d^10 + a^4*d^12) + 19/16*b^6*c^6*arcsinh(b*x/sqrt(a*
b))/(b^(7/2)*c^6*d^6 + 3*a*b^(5/2)*c^4*d^8 + 3*a^2*b^(3/2)*c^2*d^10 + a^3*
sqrt(b)*d^12) - 1/8*(b*x^2 + a)^(3/2)*b^5*c^5/(b^4*c^8*d^3 + 4*a*b^3*c^6*d
^5 + 6*a^2*b^2*c^4*d^7 + 4*a^3*b*c^2*d^9 + a^4*d^11) + 1/8*(b*x^2 + a)^(3/
2)*b^5*c^4*x/(b^4*c^8*d^2 + 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^4*d^6 + 4*a^3*b*
c^2*d^8 + a^4*d^10) + 3/16*sqrt(b*x^2 + a)*a*b^5*c^4*x/(b^4*c^8*d^2 + 4*a*
b^3*c^6*d^4 + 6*a^2*b^2*c^4*d^6 + 4*a^3*b*c^2*d^8 + a^4*d^10) + a*b^5*c^4*
arcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^4 + 3*a*b^(5/2)*c^4*d^6 + 3*a^2*b^(3
/2)*c^2*d^8 + a^3*sqrt(b)*d^10) - 3/40*(b*x^2 + a)^(5/2)*b^4*c^4/(b^4*c^8*
d^2*x + 4*a*b^3*c^6*d^4*x + 6*a^2*b^2*c^4*d^6*x + 4*a^3*b*c^2*d^8*x + a^4*
d^10*x + b^4*c^9*d + 4*a*b^3*c^7*d^3 + 6*a^2*b^2*c^5*d^5 + 4*a^3*b*c^3*d^7
 + a^4*c*d^9) - 3/8*sqrt(b*x^2 + a)*b^5*c^5/(b^3*c^6*d^5 + 3*a*b^2*c^4*d^7
 + 3*a^2*b*c^2*d^9 + a^3*d^11) - 5/8*sqrt(b*x^2 + a)*b^5*c^4*x/(b^3*c^6*d^
4 + 3*a*b^2*c^4*d^6 + 3*a^2*b*c^2*d^8 + a^3*d^10) - 45/16*b^5*c^4*arcsinh(
b*x/sqrt(a*b))/(b^(5/2)*c^4*d^6 + 2*a*b^(3/2)*c^2*d^8 + a^2*sqrt(b)*d^1...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1432 vs. $2(324) = 648$.

Time = 0.36 (sec) , antiderivative size = 1432, normalized size of antiderivative = 4.01

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/4*(8*b^5*c^5 + 20*a*b^4*c^3*d^2 + 15*a^2*b^3*c*d^4)*arctan(-((sqrt(b)*x
 - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^2*c^4*d^6 + 2
*a*b*c^2*d^8 + a^2*d^10)*sqrt(-b*c^2 - a*d^2)) - b^(5/2)*log(abs(-sqrt(b)*
x + sqrt(b*x^2 + a)))/d^6 - 1/60*(600*(sqrt(b)*x - sqrt(b*x^2 + a))^9*b^5*
c^5*d^4 + 1140*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a*b^4*c^3*d^6 + 495*(sqrt(b
)*x - sqrt(b*x^2 + a))^9*a^2*b^3*c*d^8 + 3600*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*b^(11/2)*c^6*d^3 + 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(9/2)*c^4*
d^5 + 1935*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(7/2)*c^2*d^7 - 360*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(5/2)*d^9 + 8800*(sqrt(b)*x - sqrt(b*x^2
+ a))^7*b^6*c^7*d^2 + 12200*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a*b^5*c^5*d^4
- 1250*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a^2*b^4*c^3*d^6 - 3030*(sqrt(b)*x -
 sqrt(b*x^2 + a))^7*a^3*b^3*c*d^8 + 10000*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
b^(13/2)*c^8*d + 3800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(11/2)*c^6*d^3 -
 18950*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(9/2)*c^4*d^5 - 8250*(sqrt(b)
*x - sqrt(b*x^2 + a))^6*a^3*b^(7/2)*c^2*d^7 + 720*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^4*b^(5/2)*d^9 + 4384*(sqrt(b)*x - sqrt(b*x^2 + a))^5*b^7*c^9 - 1
3872*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a*b^6*c^7*d^2 - 29076*(sqrt(b)*x - sq
rt(b*x^2 + a))^5*a^2*b^5*c^5*d^4 + 370*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a^3
*b^4*c^3*d^6 + 5520*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a^4*b^3*c*d^8 - 11920*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(13/2)*c^8*d - 3560*(sqrt(b)*x - sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(c+d\,x)^6}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^6,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 3958, normalized size of antiderivative = 11.09

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^6,x)
```

output
```
(225*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a**2*b**3*c**6*d**4 + 1125*sqrt(a*d**2 + b*c**2)*log( - sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**5*d**5*x +
2250*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a**2*b**3*c**4*d**6*x**2 + 2250*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**3*d**7
*x**3 + 1125*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a**2*b**3*c**2*d**8*x**4 + 225*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c*
d**9*x**5 + 300*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2
+ b*c**2) - a*d + b*c*x)*a*b**4*c**8*d**2 + 1500*sqrt(a*d**2 + b*c**2)*log
( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**4*c**7*d**3
*x + 3000*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*a*b**4*c**6*d**4*x**2 + 3000*sqrt(a*d**2 + b*c**2)*log(
 - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**4*c**5*d**5*
x**3 + 1500*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*
c**2) - a*d + b*c*x)*a*b**4*c**4*d**6*x**4 + 300*sqrt(a*d**2 + b*c**2)*log
( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**4*c**3*d**7
*x**5 + 120*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*
c**2) - a*d + b*c*x)*b**5*c**10 + 600*sqrt(a*d**2 + b*c**2)*log( - sqrt...
```

3.263 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7} dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7} dx = -\frac{5a^2b^2(ad-bcx)\sqrt{a+bx^2}}{16(bc^2+ad^2)^3(c+dx)^2} - \frac{5ab(ad-bcx)(a+bx^2)^{3/2}}{24(bc^2+ad^2)^2(c+dx)^4} - \frac{(ad-bcx)(a+bx^2)^{5/2}}{6(bc^2+ad^2)(c+dx)^6} - \frac{5a^3b^3\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{16(bc^2+ad^2)^{7/2}}$$

output
```
-5/16*a^2*b^2*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^3/(d*x+c)^2-5/24*
a*b*(-b*c*x+a*d)*(b*x^2+a)^(3/2)/(a*d^2+b*c^2)^2/(d*x+c)^4-1/6*(-b*c*x+a*d
)*(b*x^2+a)^(5/2)/(a*d^2+b*c^2)/(d*x+c)^6-5/16*a^3*b^3*arctanh((-b*c*x+a*d
)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 10.41 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7} dx = \frac{1}{48}\left(\frac{\sqrt{a+bx^2}(-8a^5d^5+8b^5c^5x^5-2a^4bd^3(13c^2+6cdx+13d^2x^2)+2ab^4c^3x^3(13c^2+\cdots}{\cdots} + \frac{15a^3b^3\log(c+dx)}{(bc^2+ad^2)^{7/2}} - \frac{15a^3b^3\log\left(ad-bcx+\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\right)}{(bc^2+ad^2)^{7/2}}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^7,x]
```

output
```
((Sqrt[a + b*x^2]*(-8*a^5*d^5 + 8*b^5*c^5*x^5 - 2*a^4*b*d^3*(13*c^2 + 6*c*
d*x + 13*d^2*x^2) + 2*a*b^4*c^3*x^3*(13*c^2 + 6*c*d*x + 13*d^2*x^2) - a^3*
b^2*d*(33*c^4 + 54*c^3*d*x + 122*c^2*d^2*x^2 + 54*c*d^3*x^3 + 33*d^4*x^4)
+ a^2*b^3*c*x*(33*c^4 + 54*c^3*d*x + 122*c^2*d^2*x^2 + 54*c*d^3*x^3 + 33*d
^4*x^4)))/((b*c^2 + a*d^2)^3*(c + d*x)^6) + (15*a^3*b^3*Log[c + d*x])/(b*c
^2 + a*d^2)^(7/2) - (15*a^3*b^3*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt
[a + b*x^2]])/(b*c^2 + a*d^2)^(7/2))/48
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{(c+dx)^7}dx$$

$$\Big\downarrow \text{486}$$

$$\frac{5ab\int \frac{\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{486}$$

$$\frac{5ab\left(\frac{3ab\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{486}$$

$$\frac{5ab\left(\frac{3ab\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{5ab\left(\frac{3ab\left(-\frac{ab\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{5ab\left(\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4(ad^2+bc^2)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^7,x]
```

output
```
-1/6*((a*d - b*c*x)*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^6) + (5*
a*b*(-1/4*((a*d - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4)
+ (3*a*b*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^
2) - (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2
*(b*c^2 + a*d^2)^(3/2))))/(4*(b*c^2 + a*d^2))))/(6*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15867 vs. $2(183) = 366$.

Time = 1.02 (sec) , antiderivative size = 15868, normalized size of antiderivative = 78.17

method	result	size
default	Expression too large to display	15868

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(184) = 368$.

Time = 7.81 (sec) , antiderivative size = 1929, normalized size of antiderivative = 9.50

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^7,x, algorithm="fricas")
```

output
```
[1/96*(15*(a^3*b^3*d^6*x^6 + 6*a^3*b^3*c*d^5*x^5 + 15*a^3*b^3*c^2*d^4*x^4
+ 20*a^3*b^3*c^3*d^3*x^3 + 15*a^3*b^3*c^4*d^2*x^2 + 6*a^3*b^3*c^5*d*x + a^
3*b^3*c^6)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2
*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 +
 a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(33*a^3*b^3*c^6*d + 59*a^4*b^2*c^4*d^3
 + 34*a^5*b*c^2*d^5 + 8*a^6*d^7 - (8*b^6*c^7 + 34*a*b^5*c^5*d^2 + 59*a^2*b
^4*c^3*d^4 + 33*a^3*b^3*c*d^6)*x^5 - 3*(4*a*b^5*c^6*d + 22*a^2*b^4*c^4*d^3
 + 7*a^3*b^3*c^2*d^5 - 11*a^4*b^2*d^7)*x^4 - 2*(13*a*b^5*c^7 + 74*a^2*b^4*
c^5*d^2 + 34*a^3*b^3*c^3*d^4 - 27*a^4*b^2*c*d^6)*x^3 - 2*(27*a^2*b^4*c^6*d
 - 34*a^3*b^3*c^4*d^3 - 74*a^4*b^2*c^2*d^5 - 13*a^5*b*d^7)*x^2 - 3*(11*a^2
*b^4*c^7 - 7*a^3*b^3*c^5*d^2 - 22*a^4*b^2*c^3*d^4 - 4*a^5*b*c*d^6)*x)*sqrt
(b*x^2 + a))/(b^4*c^14 + 4*a*b^3*c^12*d^2 + 6*a^2*b^2*c^10*d^4 + 4*a^3*b*c
^8*d^6 + a^4*c^6*d^8 + (b^4*c^8*d^6 + 4*a*b^3*c^6*d^8 + 6*a^2*b^2*c^4*d^10
 + 4*a^3*b*c^2*d^12 + a^4*d^14)*x^6 + 6*(b^4*c^9*d^5 + 4*a*b^3*c^7*d^7 + 6
*a^2*b^2*c^5*d^9 + 4*a^3*b*c^3*d^11 + a^4*c*d^13)*x^5 + 15*(b^4*c^10*d^4 +
 4*a*b^3*c^8*d^6 + 6*a^2*b^2*c^6*d^8 + 4*a^3*b*c^4*d^10 + a^4*c^2*d^12)*x^
4 + 20*(b^4*c^11*d^3 + 4*a*b^3*c^9*d^5 + 6*a^2*b^2*c^7*d^7 + 4*a^3*b*c^5*d
^9 + a^4*c^3*d^11)*x^3 + 15*(b^4*c^12*d^2 + 4*a*b^3*c^10*d^4 + 6*a^2*b^2*c
^8*d^6 + 4*a^3*b*c^6*d^8 + a^4*c^4*d^10)*x^2 + 6*(b^4*c^13*d + 4*a*b^3*c^1
1*d^3 + 6*a^2*b^2*c^9*d^5 + 4*a^3*b*c^7*d^7 + a^4*c^5*d^9)*x), -1/48*(1...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx)^7}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c)**7,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4635 vs. $2(184) = 368$.

Time = 0.32 (sec) , antiderivative size = 4635, normalized size of antiderivative = 22.83

$$\int \frac{(a + bx^2)^{5/2}}{(c + dx)^7} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^7,x, algorithm="maxima")
```

output
```
-5/32*b^8*c^9*arcsinh(b*x/sqrt(a*b))/(b^(11/2)*c^10*d^6 + 5*a*b^(9/2)*c^8*
d^8 + 10*a^2*b^(7/2)*c^6*d^10 + 10*a^3*b^(5/2)*c^4*d^12 + 5*a^4*b^(3/2)*c^
2*d^14 + a^5*sqrt(b)*d^16) - 5/32*a*b^7*c^7*arcsinh(b*x/sqrt(a*b))/(b^(11/
2)*c^10*d^4 + 5*a*b^(9/2)*c^8*d^6 + 10*a^2*b^(7/2)*c^6*d^8 + 10*a^3*b^(5/2
)*c^4*d^10 + 5*a^4*b^(3/2)*c^2*d^12 + a^5*sqrt(b)*d^14) + 5/32*sqrt(b*x^2
+ a)*b^7*c^7*x/(b^5*c^10*d^4 + 5*a*b^4*c^8*d^6 + 10*a^2*b^3*c^6*d^8 + 10*a
^3*b^2*c^4*d^10 + 5*a^4*b*c^2*d^12 + a^5*d^14) + 15/16*b^7*c^7*arcsinh(b*x
/sqrt(a*b))/(b^(9/2)*c^8*d^6 + 4*a*b^(7/2)*c^6*d^8 + 6*a^2*b^(5/2)*c^4*d^1
0 + 4*a^3*b^(3/2)*c^2*d^12 + a^4*sqrt(b)*d^14) - 5/48*(b*x^2 + a)^(3/2)*b^
6*c^6/(b^5*c^10*d^3 + 5*a*b^4*c^8*d^5 + 10*a^2*b^3*c^6*d^7 + 10*a^3*b^2*c^
4*d^9 + 5*a^4*b*c^2*d^11 + a^5*d^13) + 5/48*(b*x^2 + a)^(3/2)*b^6*c^5*x/(b
^5*c^10*d^2 + 5*a*b^4*c^8*d^4 + 10*a^2*b^3*c^6*d^6 + 10*a^3*b^2*c^4*d^8 +
5*a^4*b*c^2*d^10 + a^5*d^12) + 5/32*sqrt(b*x^2 + a)*a*b^6*c^5*x/(b^5*c^10*
d^2 + 5*a*b^4*c^8*d^4 + 10*a^2*b^3*c^6*d^6 + 10*a^3*b^2*c^4*d^8 + 5*a^4*b*
c^2*d^10 + a^5*d^12) + 25/32*a*b^6*c^5*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8
*d^4 + 4*a*b^(7/2)*c^6*d^6 + 6*a^2*b^(5/2)*c^4*d^8 + 4*a^3*b^(3/2)*c^2*d^1
0 + a^4*sqrt(b)*d^12) - 1/16*(b*x^2 + a)^(5/2)*b^5*c^5/(b^5*c^10*d^2*x + 5
*a*b^4*c^8*d^4*x + 10*a^2*b^3*c^6*d^6*x + 10*a^3*b^2*c^4*d^8*x + 5*a^4*b*c
^2*d^10*x + a^5*d^12*x + b^5*c^11*d + 5*a*b^4*c^9*d^3 + 10*a^2*b^3*c^7*d^5
 + 10*a^3*b^2*c^5*d^7 + 5*a^4*b*c^3*d^9 + a^5*c*d^11) - 5/16*sqrt(b*x^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1953 vs. $2(184) = 368$.

Time = 0.20 (sec) , antiderivative size = 1953, normalized size of antiderivative = 9.62

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^7,x, algorithm="giac")
```

output
```
5/8*a^3*b^3*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*
c^2 - a*d^2))/((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*sqr
t(-b*c^2 - a*d^2)) + 1/24*(48*(sqrt(b)*x - sqrt(b*x^2 + a))^11*b^6*c^6*d^5
 + 144*(sqrt(b)*x - sqrt(b*x^2 + a))^11*a*b^5*c^4*d^7 + 144*(sqrt(b)*x - s
qrt(b*x^2 + a))^11*a^2*b^4*c^2*d^9 + 33*(sqrt(b)*x - sqrt(b*x^2 + a))^11*a
^3*b^3*d^11 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(13/2)*c^7*d^4 + 720*
(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(11/2)*c^5*d^6 + 720*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^2*b^(9/2)*c^3*d^8 + 75*(sqrt(b)*x - sqrt(b*x^2 + a))^10
*a^3*b^(7/2)*c*d^10 + 640*(sqrt(b)*x - sqrt(b*x^2 + a))^9*b^7*c^8*d^3 + 18
40*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a*b^6*c^6*d^5 + 1680*(sqrt(b)*x - sqrt(
b*x^2 + a))^9*a^2*b^5*c^4*d^7 - 340*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a^3*b^
4*c^2*d^9 + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a^4*b^3*d^11 + 960*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*b^(15/2)*c^9*d^2 + 2160*(sqrt(b)*x - sqrt(b*x^2 + a
))^8*a*b^(13/2)*c^7*d^4 + 720*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(11/2)
*c^5*d^6 - 2910*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2)*c^3*d^8 + 45*(
sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(7/2)*c*d^10 + 768*(sqrt(b)*x - sqrt(
b*x^2 + a))^7*b^8*c^10*d + 576*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a*b^7*c^8*d
^3 - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a^2*b^6*c^6*d^5 - 5640*(sqrt(b)*
x - sqrt(b*x^2 + a))^7*a^3*b^5*c^4*d^7 + 1800*(sqrt(b)*x - sqrt(b*x^2 + a)
)^7*a^4*b^4*c^2*d^9 + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^7*a^5*b^3*d^11 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(c+d\,x)^7}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^7,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^7, x)
```

Reduce [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 1662, normalized size of antiderivative = 8.19

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^7,x)
```

output
```
(15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**3*b**3*c**6 + 90*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c**5*d*x + 225*sqrt(a*d**2 +
 b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*
*3*c**4*d**2*x**2 + 300*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c**3*d**3*x**3 + 225*sqrt(a*d**2 +
 b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*
*3*c**2*d**4*x**4 + 90*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c*d**5*x**5 + 15*sqrt(a*d**2 + b*c*
*2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*d*
*6*x**6 - 15*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**3*c**6 - 90*sqrt(a
*d**2 + b*c**2)*log(c + d*x)*a**3*b**3*c**5*d*x - 225*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a**3*b**3*c**4*d**2*x**2 - 300*sqrt(a*d**2 + b*c**2)*log(c
+ d*x)*a**3*b**3*c**3*d**3*x**3 - 225*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a
**3*b**3*c**2*d**4*x**4 - 90*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**3*
c*d**5*x**5 - 15*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**3*d**6*x**6 -
8*sqrt(a + b*x**2)*a**6*d**7 - 34*sqrt(a + b*x**2)*a**5*b*c**2*d**5 - 12*s
qrt(a + b*x**2)*a**5*b*c*d**6*x - 26*sqrt(a + b*x**2)*a**5*b*d**7*x**2 - 5
9*sqrt(a + b*x**2)*a**4*b**2*c**4*d**3 - 66*sqrt(a + b*x**2)*a**4*b**2*c**
3*d**4*x - 148*sqrt(a + b*x**2)*a**4*b**2*c**2*d**5*x**2 - 54*sqrt(a + ...
```

3.264 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 246

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx = -\frac{5a^2b^3c(ad-bcx)\sqrt{a+bx^2}}{16\,(bc^2+ad^2)^4\,(c+dx)^2} - \frac{5ab^2c(ad-bcx)\,(a+bx^2)^{3/2}}{24\,(bc^2+ad^2)^3\,(c+dx)^4} - \frac{bc(ad-bcx)\,(a+bx^2)^{5/2}}{6\,(bc^2+ad^2)^2\,(c+dx)^6} - \frac{d(a+bx^2)^{7/2}}{7\,(bc^2+ad^2)\,(c+dx)^7} - \frac{5a^3b^4c\,\mathrm{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{16\,(bc^2+ad^2)^{9/2}}$$

output

```
-5/16*a^2*b^3*c*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^4/(d*x+c)^2-5/2
4*a*b^2*c*(-b*c*x+a*d)*(b*x^2+a)^(3/2)/(a*d^2+b*c^2)^3/(d*x+c)^4-1/6*b*c*(
-b*c*x+a*d)*(b*x^2+a)^(5/2)/(a*d^2+b*c^2)^2/(d*x+c)^6-1/7*d*(b*x^2+a)^(7/2
)/(a*d^2+b*c^2)/(d*x+c)^7-5/16*a^3*b^4*c*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2
)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(9/2)
```

Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx =$$
$$-\frac{\sqrt{a+bx^2}\Big(48(bc^2+ad^2)^6-232bc(bc^2+ad^2)^5(c+dx)+8b(bc^2+ad^2)^4(55bc^2+18ad^2)(c+dx)^2-2b^2}{}$$
$$+\frac{5a^3b^4c\log(c+dx)}{16(bc^2+ad^2)^{9/2}}-\frac{5a^3b^4c\log\left(ad-bcx+\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\right)}{16(bc^2+ad^2)^{9/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^8,x]
```

output
```
-1/336*(Sqrt[a + b*x^2]*(48*(b*c^2 + a*d^2)^6 - 232*b*c*(b*c^2 + a*d^2)^5*
(c + d*x) + 8*b*(b*c^2 + a*d^2)^4*(55*b*c^2 + 18*a*d^2)*(c + d*x)^2 - 2*b^
2*c*(b*c^2 + a*d^2)^3*(200*b*c^2 + 197*a*d^2)*(c + d*x)^3 + 2*b^2*(b*c^2 +
 a*d^2)^2*(80*b^2*c^4 + 159*a*b*c^2*d^2 + 72*a^2*d^4)*(c + d*x)^4 - b^3*c*
(b*c^2 + a*d^2)*(8*b^2*c^4 + 30*a*b*c^2*d^2 + 57*a^2*d^4)*(c + d*x)^5 - b^
3*(8*b^3*c^6 + 38*a*b^2*c^4*d^2 + 87*a^2*b*c^2*d^4 - 48*a^3*d^6)*(c + d*x)
^6))/(d^5*(b*c^2 + a*d^2)^4*(c + d*x)^7) + (5*a^3*b^4*c*Log[c + d*x])/(16*
(b*c^2 + a*d^2)^(9/2)) - (5*a^3*b^4*c*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2
]*Sqrt[a + b*x^2]])/(16*(b*c^2 + a*d^2)^(9/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {491, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx$$
$$\Big\downarrow \; 491$$

$$\frac{bc\int \frac{\left(bx^2+a\right)^{5/2}}{(c+dx)^7}dx}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 486

$$\frac{bc\left(\frac{5ab\int \frac{\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 486

$$\frac{bc\left(\frac{5ab\left(\frac{3ab\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 486

$$\frac{bc\left(\frac{5ab\left(\frac{3ab\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 488

$$\frac{bc\left(\frac{5ab\left(\frac{3ab\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$\frac{bc\left(\frac{5ab\left(\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6(ad^2+bc^2)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6(ad^2+bc^2)}\right)}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^8,x]
```

output
```
-1/7*(d*(a + b*x^2)^(7/2))/((b*c^2 + a*d^2)*(c + d*x)^7) + (b*c*(-1/6*((a*
d - b*c*x)*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^6) + (5*a*b*(-1/4
*((a*d - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*
(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b
*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 +
 a*d^2)^(3/2))))/(4*(b*c^2 + a*d^2))))/(6*(b*c^2 + a*d^2))))/(b*c^2 + a*d^
2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15955 vs. $2(222) = 444$.

Time = 1.52 (sec) , antiderivative size = 15956, normalized size of antiderivative = 64.86

method	result	size
default	Expression too large to display	15956

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(223) = 446$.

Time = 16.85 (sec) , antiderivative size = 2593, normalized size of antiderivative = 10.54

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^8,x, algorithm="fricas")
```

output
```
[1/672*(105*(a^3*b^4*c*d^7*x^7 + 7*a^3*b^4*c^2*d^6*x^6 + 21*a^3*b^4*c^3*d^
5*x^5 + 35*a^3*b^4*c^4*d^4*x^4 + 35*a^3*b^4*c^5*d^3*x^3 + 21*a^3*b^4*c^6*d
^2*x^2 + 7*a^3*b^4*c^7*d*x + a^3*b^4*c^8)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c
*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*
d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(279*a^
3*b^4*c^8*d + 605*a^4*b^3*c^6*d^3 + 526*a^5*b^2*c^4*d^5 + 248*a^6*b*c^2*d^
7 + 48*a^7*d^9 - (8*b^7*c^8*d + 46*a*b^6*c^6*d^3 + 125*a^2*b^5*c^4*d^5 + 3
9*a^3*b^4*c^2*d^7 - 48*a^4*b^3*d^9)*x^6 - 7*(8*b^7*c^9 + 46*a*b^6*c^7*d^2
+ 125*a^2*b^5*c^5*d^4 + 54*a^3*b^4*c^3*d^6 - 33*a^4*b^3*c*d^8)*x^5 - (122*
a*b^6*c^8*d + 922*a^2*b^5*c^6*d^3 - 241*a^3*b^4*c^4*d^5 - 1185*a^4*b^3*c^2
*d^7 - 144*a^5*b^2*d^9)*x^4 - 14*(13*a*b^6*c^9 + 101*a^2*b^5*c^7*d^2 - 101
*a^4*b^3*c^3*d^6 - 13*a^5*b^2*c*d^8)*x^3 - (465*a^2*b^5*c^8*d - 1199*a^3*b
^4*c^6*d^3 - 2362*a^4*b^3*c^4*d^5 - 842*a^5*b^2*c^2*d^7 - 144*a^6*b*d^9)*x
^2 - 7*(33*a^2*b^5*c^9 - 54*a^3*b^4*c^7*d^2 - 125*a^4*b^3*c^5*d^4 - 46*a^5
*b^2*c^3*d^6 - 8*a^6*b*c*d^8)*x)*sqrt(b*x^2 + a))/(b^5*c^17 + 5*a*b^4*c^15
*d^2 + 10*a^2*b^3*c^13*d^4 + 10*a^3*b^2*c^11*d^6 + 5*a^4*b*c^9*d^8 + a^5*c
^7*d^10 + (b^5*c^10*d^7 + 5*a*b^4*c^8*d^9 + 10*a^2*b^3*c^6*d^11 + 10*a^3*b
^2*c^4*d^13 + 5*a^4*b*c^2*d^15 + a^5*d^17)*x^7 + 7*(b^5*c^11*d^6 + 5*a*b^4
*c^9*d^8 + 10*a^2*b^3*c^7*d^10 + 10*a^3*b^2*c^5*d^12 + 5*a^4*b*c^3*d^14 +
a^5*c*d^16)*x^6 + 21*(b^5*c^12*d^5 + 5*a*b^4*c^10*d^7 + 10*a^2*b^3*c^8*...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx=\int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx)^8}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c)**8,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(c + d*x)**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5985 vs. $2(223) = 446$.

Time = 0.34 (sec) , antiderivative size = 5985, normalized size of antiderivative = 24.33

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^8,x, algorithm="maxima")
```

output
```
-5/32*b^9*c^10*arcsinh(b*x/sqrt(a*b))/(b^(13/2)*c^12*d^6 + 6*a*b^(11/2)*c^
10*d^8 + 15*a^2*b^(9/2)*c^8*d^10 + 20*a^3*b^(7/2)*c^6*d^12 + 15*a^4*b^(5/2
)*c^4*d^14 + 6*a^5*b^(3/2)*c^2*d^16 + a^6*sqrt(b)*d^18) - 5/32*a*b^8*c^8*a
rcsinh(b*x/sqrt(a*b))/(b^(13/2)*c^12*d^4 + 6*a*b^(11/2)*c^10*d^6 + 15*a^2*
b^(9/2)*c^8*d^8 + 20*a^3*b^(7/2)*c^6*d^10 + 15*a^4*b^(5/2)*c^4*d^12 + 6*a^
5*b^(3/2)*c^2*d^14 + a^6*sqrt(b)*d^16) + 5/32*sqrt(b*x^2 + a)*b^8*c^8*x/(b
^6*c^12*d^4 + 6*a*b^5*c^10*d^6 + 15*a^2*b^4*c^8*d^8 + 20*a^3*b^3*c^6*d^10
+ 15*a^4*b^2*c^4*d^12 + 6*a^5*b*c^2*d^14 + a^6*d^16) + 15/16*b^8*c^8*arcsi
nh(b*x/sqrt(a*b))/(b^(11/2)*c^10*d^6 + 5*a*b^(9/2)*c^8*d^8 + 10*a^2*b^(7/2
)*c^6*d^10 + 10*a^3*b^(5/2)*c^4*d^12 + 5*a^4*b^(3/2)*c^2*d^14 + a^5*sqrt(b
)*d^16) - 5/48*(b*x^2 + a)^(3/2)*b^7*c^7/(b^6*c^12*d^3 + 6*a*b^5*c^10*d^5
+ 15*a^2*b^4*c^8*d^7 + 20*a^3*b^3*c^6*d^9 + 15*a^4*b^2*c^4*d^11 + 6*a^5*b*
c^2*d^13 + a^6*d^15) + 5/48*(b*x^2 + a)^(3/2)*b^7*c^6*x/(b^6*c^12*d^2 + 6*
a*b^5*c^10*d^4 + 15*a^2*b^4*c^8*d^6 + 20*a^3*b^3*c^6*d^8 + 15*a^4*b^2*c^4*
d^10 + 6*a^5*b*c^2*d^12 + a^6*d^14) + 5/32*sqrt(b*x^2 + a)*a*b^7*c^6*x/(b^
6*c^12*d^2 + 6*a*b^5*c^10*d^4 + 15*a^2*b^4*c^8*d^6 + 20*a^3*b^3*c^6*d^8 +
15*a^4*b^2*c^4*d^10 + 6*a^5*b*c^2*d^12 + a^6*d^14) + 25/32*a*b^7*c^6*arcsi
nh(b*x/sqrt(a*b))/(b^(11/2)*c^10*d^4 + 5*a*b^(9/2)*c^8*d^6 + 10*a^2*b^(7/2
)*c^6*d^8 + 10*a^3*b^(5/2)*c^4*d^10 + 5*a^4*b^(3/2)*c^2*d^12 + a^5*sqrt(b)
*d^14) - 1/16*(b*x^2 + a)^(5/2)*b^6*c^6/(b^6*c^12*d^2*x + 6*a*b^5*c^10*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2428 vs. $2(223) = 446$.

Time = 0.23 (sec) , antiderivative size = 2428, normalized size of antiderivative = 9.87

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^8,x, algorithm="giac")
```

output
```
-5/8*a^3*b^4*c*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-
b*c^2 - a*d^2))/((b^4*c^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*a^3*b*
c^2*d^6 + a^4*d^8)*sqrt(-b*c^2 - a*d^2)) - 1/168*(105*(sqrt(b)*x - sqrt(b*
x^2 + a))^13*a^3*b^4*c*d^12 - 336*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(15/2
)*c^8*d^5 - 1344*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a*b^(13/2)*c^6*d^7 - 201
6*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(11/2)*c^4*d^9 + 21*(sqrt(b)*x -
sqrt(b*x^2 + a))^12*a^3*b^(9/2)*c^2*d^11 - 336*(sqrt(b)*x - sqrt(b*x^2 + a
))^12*a^4*b^(7/2)*d^13 - 1120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*b^8*c^9*d^4
 - 4480*(sqrt(b)*x - sqrt(b*x^2 + a))^11*a*b^7*c^7*d^6 - 6720*(sqrt(b)*x -
 sqrt(b*x^2 + a))^11*a^2*b^6*c^5*d^8 + 3010*(sqrt(b)*x - sqrt(b*x^2 + a))^
11*a^3*b^5*c^3*d^10 - 1820*(sqrt(b)*x - sqrt(b*x^2 + a))^11*a^4*b^4*c*d^12
 - 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(17/2)*c^10*d^3 - 8960*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*a*b^(15/2)*c^8*d^5 - 13440*(sqrt(b)*x - sqrt(b*x^
2 + a))^10*a^2*b^(13/2)*c^6*d^7 + 13370*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
^3*b^(11/2)*c^4*d^9 - 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(9/2)*c^
2*d^11 - 2688*(sqrt(b)*x - sqrt(b*x^2 + a))^9*b^9*c^11*d^2 - 8288*(sqrt(b)
*x - sqrt(b*x^2 + a))^9*a*b^8*c^9*d^4 - 6272*(sqrt(b)*x - sqrt(b*x^2 + a))
^9*a^2*b^7*c^7*d^6 + 42588*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a^3*b^6*c^5*d^8
 - 27370*(sqrt(b)*x - sqrt(b*x^2 + a))^9*a^4*b^5*c^3*d^10 + 4445*(sqrt(b)*
x - sqrt(b*x^2 + a))^9*a^5*b^4*c*d^12 - 1792*(sqrt(b)*x - sqrt(b*x^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(c+d\,x)^8}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^8,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^8, x)
```

Reduce [B] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 2204, normalized size of antiderivative = 8.96

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^8,x)
```

output
```
(105*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a**3*b**4*c**8 + 735*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**4*c**7*d*x + 2205*sqrt(a*d**
2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3
*b**4*c**6*d**2*x**2 + 3675*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**4*c**5*d**3*x**3 + 3675*sqrt(a*d
**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
*3*b**4*c**4*d**4*x**4 + 2205*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**4*c**3*d**5*x**5 + 735*sqrt(a*
d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a
**3*b**4*c**2*d**6*x**6 + 105*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**4*c*d**7*x**7 - 105*sqrt(a*d**
2 + b*c**2)*log(c + d*x)*a**3*b**4*c**8 - 735*sqrt(a*d**2 + b*c**2)*log(c
+ d*x)*a**3*b**4*c**7*d*x - 2205*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b
**4*c**6*d**2*x**2 - 3675*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**4*c**
5*d**3*x**3 - 3675*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**4*c**4*d**4*
x**4 - 2205*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**4*c**3*d**5*x**5 -
735*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**4*c**2*d**6*x**6 - 105*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**4*c*d**7*x**7 - 48*sqrt(a + b*x**2)
*a**7*d**9 - 248*sqrt(a + b*x**2)*a**6*b*c**2*d**7 - 56*sqrt(a + b*x**2...
```

3.265 $\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9} dx$

Optimal result

Integrand size = 19, antiderivative size = 332

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9} dx = -\frac{5a^2b^3(8bc^2-ad^2)(ad-bcx)\sqrt{a+bx^2}}{128(bc^2+ad^2)^5(c+dx)^2} - \frac{5ab^2(8bc^2-ad^2)(ad-bcx)(a+bx^2)^{3/2}}{192(bc^2+ad^2)^4(c+dx)^4} - \frac{b(8bc^2-ad^2)(ad-bcx)(a+bx^2)^{5/2}}{48(bc^2+ad^2)^3(c+dx)^6} - \frac{d(a+bx^2)^{7/2}}{8(bc^2+ad^2)(c+dx)^8} - \frac{9bcd(a+bx^2)^{7/2}}{56(bc^2+ad^2)^2(c+dx)^7} - \frac{5a^3b^4(8bc^2-ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{128(bc^2+ad^2)^{11/2}}$$

output
```
-5/128*a^2*b^3*(-a*d^2+8*b*c^2)*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)
^5/(d*x+c)^2-5/192*a*b^2*(-a*d^2+8*b*c^2)*(-b*c*x+a*d)*(b*x^2+a)^(3/2)/(a*
d^2+b*c^2)^4/(d*x+c)^4-1/48*b*(-a*d^2+8*b*c^2)*(-b*c*x+a*d)*(b*x^2+a)^(5/2
)/(a*d^2+b*c^2)^3/(d*x+c)^6-1/8*d*(b*x^2+a)^(7/2)/(a*d^2+b*c^2)/(d*x+c)^8-
9/56*b*c*d*(b*x^2+a)^(7/2)/(a*d^2+b*c^2)^2/(d*x+c)^7-5/128*a^3*b^4*(-a*d^2
+8*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2
+b*c^2)^(11/2)
```

Mathematica [A] (verified)

Time = 10.78 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9}\,dx = \frac{-\frac{\sqrt{a+bx^2}\left(336(bc^2+ad^2)^7-1584bc(bc^2+ad^2)^6(c+dx)+8b(bc^2+ad^2)^5(362bc^2+119ad^2)(c+dx)^2-8b^2c(bc^2+ad^2)^4(\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(c + d*x)^9,x]
```

output
```
(-((Sqrt[a + b*x^2]*(336*(b*c^2 + a*d^2)^7 - 1584*b*c*(b*c^2 + a*d^2)^6*(c
 + d*x) + 8*b*(b*c^2 + a*d^2)^5*(362*b*c^2 + 119*a*d^2)*(c + d*x)^2 - 8*b^
2*c*(b*c^2 + a*d^2)^4*(310*b*c^2 + 307*a*d^2)*(c + d*x)^3 + 2*b^2*(b*c^2 +
 a*d^2)^3*(440*b^2*c^4 + 880*a*b*c^2*d^2 + 413*a^2*d^4)*(c + d*x)^4 - 2*b^
3*c*(b*c^2 + a*d^2)^2*(8*b^2*c^4 + 32*a*b*c^2*d^2 + 87*a^2*d^4)*(c + d*x)^
5 - b^3*(b*c^2 + a*d^2)*(16*b^3*c^6 + 88*a*b^2*c^4*d^2 + 282*a^2*b*c^2*d^4
 - 105*a^3*d^6)*(c + d*x)^6 - b^4*c*(16*b^3*c^6 + 104*a*b^2*c^4*d^2 + 370*
a^2*b*c^2*d^4 - 663*a^3*d^6)*(c + d*x)^7))/((b*c^2*d + a*d^3)^5*(c + d*x)^
8)) + (105*a^3*b^4*(8*b*c^2 - a*d^2)*Log[c + d*x])/(b*c^2 + a*d^2)^(11/2)
+ (105*a^3*b^4*(-8*b*c^2 + a*d^2)*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sq
rt[a + b*x^2]])/(b*c^2 + a*d^2)^(11/2))/2688
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {498, 25, 679, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9}\,dx$$

$$\Big\downarrow \text{ 498}$$

$$-\frac{b\int -\frac{(8c-dx)(bx^2+a)^{5/2}}{(c+dx)^8}dx}{8(ad^2+bc^2)}-\frac{d(a+bx^2)^{7/2}}{8(c+dx)^8(ad^2+bc^2)}$$

$$\Big\downarrow 25$$

$$\frac{b\int \frac{(8c-dx)\left(bx^2+a\right)^{5/2}}{(c+dx)^8}dx}{8\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{7/2}}{8(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 679$$

$$\frac{b\left(\frac{\left(8bc^2-ad^2\right)\int \frac{\left(bx^2+a\right)^{5/2}}{(c+dx)^7}dx}{ad^2+bc^2}-\frac{9cd\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}\right)}{8\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{7/2}}{8(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 486$$

$$\frac{b\left(\frac{\left(8bc^2-ad^2\right)\left(\frac{5ab\int \frac{\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{9cd\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}\right)}{8\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{7/2}}{8(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 486$$

$$\frac{b\left(\frac{\left(8bc^2-ad^2\right)\left(\frac{5ab\left(\frac{3ab\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{9cd\left(a+bx^2\right)^{7/2}}{7(c+dx)^7\left(ad^2+bc^2\right)}\right)}{8\left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{7/2}}{8(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 486$$

$$\frac{b\left(\frac{(8bc^2-ad^2)\left(\frac{5ab\left(\frac{3ab\left(\frac{ab\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{9cd\left(a+bx^2\right)^{7/2}}{7(c+dx)^7(ad^2+bc^2)}\right.}{8\left(ad^2+bc^2\right)}$$

$$\frac{d\left(a+bx^2\right)^{7/2}}{8(c+dx)^8\left(ad^2+bc^2\right)}$$

$\downarrow$ 488

$$\frac{b\left(\frac{(8bc^2-ad^2)\left(\frac{5ab\left(\frac{3ab\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{9a \cdots}{7(c \cdots}\right)}{8\left(ad^2+bc^2\right)}$$

$$\frac{d\left(a+bx^2\right)^{7/2}}{8(c+dx)^8\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 219

$$\frac{b\left(\frac{(8bc^2-ad^2)\left(\frac{5ab\left(\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{9c}{7(c+}\right.}{8\left(ad^2+bc^2\right)}$$

$$\frac{d\left(a+bx^2\right)^{7/2}}{8(c+dx)^8\left(ad^2+bc^2\right)}$$

input
```
Int[(a + b*x^2)^(5/2)/(c + d*x)^9,x]
```

output
```
-1/8*(d*(a + b*x^2)^(7/2))/((b*c^2 + a*d^2)*(c + d*x)^8) + (b*((-9*c*d*(a
+ b*x^2)^(7/2))/(7*(b*c^2 + a*d^2)*(c + d*x)^7) + ((8*b*c^2 - a*d^2)*(-1/6
*((a*d - b*c*x)*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^6) + (5*a*b*
(-1/4*((a*d - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3
*a*b*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) -
 (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*
c^2 + a*d^2)^(3/2))))/(4*(b*c^2 + a*d^2))))/(6*(b*c^2 + a*d^2))))/(b*c^2 +
 a*d^2)))/(8*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_^2)]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31926 vs. 2(304) = 608.

Time = 2.26 (sec) , antiderivative size = 31927, normalized size of antiderivative = 96.17

method	result	size
default	Expression too large to display	31927

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. 2(305) = 610.

Time = 31.79 (sec) , antiderivative size = 3693, normalized size of antiderivative = 11.12

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^9,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x+c)**9,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9514 vs. 2(305) = 610.

Time = 0.45 (sec) , antiderivative size = 9514, normalized size of antiderivative = 28.66

$$\int \frac{(a + bx^2)^{5/2}}{(c + dx)^9} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^9,x, algorithm="maxima")
```

output
```
-45/256*b^10*c^11*arcsinh(b*x/sqrt(a*b))/(b^(15/2)*c^14*d^6 + 7*a*b^(13/2)
*c^12*d^8 + 21*a^2*b^(11/2)*c^10*d^10 + 35*a^3*b^(9/2)*c^8*d^12 + 35*a^4*b
^(7/2)*c^6*d^14 + 21*a^5*b^(5/2)*c^4*d^16 + 7*a^6*b^(3/2)*c^2*d^18 + a^7*s
qrt(b)*d^20) - 45/256*a*b^9*c^9*arcsinh(b*x/sqrt(a*b))/(b^(15/2)*c^14*d^4
+ 7*a*b^(13/2)*c^12*d^6 + 21*a^2*b^(11/2)*c^10*d^8 + 35*a^3*b^(9/2)*c^8*d^
10 + 35*a^4*b^(7/2)*c^6*d^12 + 21*a^5*b^(5/2)*c^4*d^14 + 7*a^6*b^(3/2)*c^2
*d^16 + a^7*sqrt(b)*d^18) + 45/256*sqrt(b*x^2 + a)*b^9*c^9*x/(b^7*c^14*d^4
 + 7*a*b^6*c^12*d^6 + 21*a^2*b^5*c^10*d^8 + 35*a^3*b^4*c^8*d^10 + 35*a^4*b
^3*c^6*d^12 + 21*a^5*b^2*c^4*d^14 + 7*a^6*b*c^2*d^16 + a^7*d^18) + 275/256
*b^9*c^9*arcsinh(b*x/sqrt(a*b))/(b^(13/2)*c^12*d^6 + 6*a*b^(11/2)*c^10*d^8
 + 15*a^2*b^(9/2)*c^8*d^10 + 20*a^3*b^(7/2)*c^6*d^12 + 15*a^4*b^(5/2)*c^4*
d^14 + 6*a^5*b^(3/2)*c^2*d^16 + a^6*sqrt(b)*d^18) - 15/128*(b*x^2 + a)^(3/
2)*b^8*c^8/(b^7*c^14*d^3 + 7*a*b^6*c^12*d^5 + 21*a^2*b^5*c^10*d^7 + 35*a^3
*b^4*c^8*d^9 + 35*a^4*b^3*c^6*d^11 + 21*a^5*b^2*c^4*d^13 + 7*a^6*b*c^2*d^1
5 + a^7*d^17) + 15/128*(b*x^2 + a)^(3/2)*b^8*c^7*x/(b^7*c^14*d^2 + 7*a*b^6
*c^12*d^4 + 21*a^2*b^5*c^10*d^6 + 35*a^3*b^4*c^8*d^8 + 35*a^4*b^3*c^6*d^10
 + 21*a^5*b^2*c^4*d^12 + 7*a^6*b*c^2*d^14 + a^7*d^16) + 45/256*sqrt(b*x^2
+ a)*a*b^8*c^7*x/(b^7*c^14*d^2 + 7*a*b^6*c^12*d^4 + 21*a^2*b^5*c^10*d^6 +
35*a^3*b^4*c^8*d^8 + 35*a^4*b^3*c^6*d^10 + 21*a^5*b^2*c^4*d^12 + 7*a^6*b*c
^2*d^14 + a^7*d^16) + 115/128*a*b^8*c^7*arcsinh(b*x/sqrt(a*b))/(b^(13/2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3216 vs. $2(305) = 610$.

Time = 0.25 (sec) , antiderivative size = 3216, normalized size of antiderivative = 9.69

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x+c)^9,x, algorithm="giac")
```

output
```
5/64*(8*a^3*b^5*c^2 - a^4*b^4*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*
d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^5*c^10 + 5*a*b^4*c^8*d^2 + 10*a^2
*b^3*c^6*d^4 + 10*a^3*b^2*c^4*d^6 + 5*a^4*b*c^2*d^8 + a^5*d^10)*sqrt(-b*c^
2 - a*d^2)) - 1/1344*(840*(sqrt(b)*x - sqrt(b*x^2 + a))^15*a^3*b^5*c^2*d^1
3 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^15*a^4*b^4*d^15 + 12600*(sqrt(b)*x -
 sqrt(b*x^2 + a))^14*a^3*b^(11/2)*c^3*d^12 - 1575*(sqrt(b)*x - sqrt(b*x^2
+ a))^14*a^4*b^(9/2)*c*d^14 - 3584*(sqrt(b)*x - sqrt(b*x^2 + a))^13*b^9*c^
10*d^5 - 17920*(sqrt(b)*x - sqrt(b*x^2 + a))^13*a*b^8*c^8*d^7 - 35840*(sqr
t(b)*x - sqrt(b*x^2 + a))^13*a^2*b^7*c^6*d^9 + 45920*(sqrt(b)*x - sqrt(b*x
^2 + a))^13*a^3*b^6*c^4*d^11 - 34580*(sqrt(b)*x - sqrt(b*x^2 + a))^13*a^4*
b^5*c^2*d^13 - 2779*(sqrt(b)*x - sqrt(b*x^2 + a))^13*a^5*b^4*d^15 - 8960*(
sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(19/2)*c^11*d^4 - 44800*(sqrt(b)*x - sqr
t(b*x^2 + a))^12*a*b^(17/2)*c^9*d^6 - 89600*(sqrt(b)*x - sqrt(b*x^2 + a))^
12*a^2*b^(15/2)*c^7*d^8 + 208880*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(1
3/2)*c^5*d^10 - 165830*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(11/2)*c^3*d
^12 + 1505*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*b^(9/2)*c*d^14 - 14336*(sq
rt(b)*x - sqrt(b*x^2 + a))^11*b^10*c^12*d^3 - 75264*(sqrt(b)*x - sqrt(b*x^
2 + a))^11*a*b^9*c^10*d^5 - 161280*(sqrt(b)*x - sqrt(b*x^2 + a))^11*a^2*b^
8*c^8*d^7 + 486528*(sqrt(b)*x - sqrt(b*x^2 + a))^11*a^3*b^7*c^6*d^9 - 6501
60*(sqrt(b)*x - sqrt(b*x^2 + a))^11*a^4*b^6*c^4*d^11 + 46620*(sqrt(b)*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(c+d\,x)^9}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(c + d*x)^9,x)
```

output
```
int((a + b*x^2)^(5/2)/(c + d*x)^9, x)
```

Reduce [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 3709, normalized size of antiderivative = 11.17

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x+c)^9,x)
```

output

```
(105*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a**4*b**4*c**8*d**2 + 840*sqrt(a*d**2 + b*c**2)*log( - sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b**4*c**7*d**3*x + 2
940*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**4*b**4*c**6*d**4*x**2 + 5880*sqrt(a*d**2 + b*c**2)*log( -
sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b**4*c**5*d**5*
x**3 + 7350*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*
c**2) - a*d + b*c*x)*a**4*b**4*c**4*d**6*x**4 + 5880*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b**4*c*
*3*d**7*x**5 + 2940*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*a**4*b**4*c**2*d**8*x**6 + 840*sqrt(a*d**2 +
b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*
b**4*c*d**9*x**7 + 105*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b**4*d**10*x**8 - 840*sqrt(a*d**2 + b
*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b
**5*c**10 - 6720*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2
 + b*c**2) - a*d + b*c*x)*a**3*b**5*c**9*d*x - 23520*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**5*c*
*8*d**2*x**2 - 47040*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a**3*b**5*c**7*d**3*x**3 - 58800*sqrt(a*d...
```

3.266 $\int \frac{\sqrt{2+x^2}}{1+4x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx=\frac{\sqrt{2+x^2}}{4}-\frac{1}{16}\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{16}\sqrt{33}\text{arctanh}\left(\frac{8-x}{\sqrt{33}\sqrt{2+x^2}}\right)$$

output
```
1/4*(x^2+2)^(1/2)-1/16*arcsinh(1/2*x*2^(1/2))-1/16*33^(1/2)*arctanh(1/33*(
8-x)*33^(1/2)/(x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx=\frac{1}{16}\left(4\sqrt{2+x^2}+2\sqrt{33}\text{arctanh}\left(\frac{1+4x-4\sqrt{2+x^2}}{\sqrt{33}}\right)+\log\left(-x+\sqrt{2+x^2}\right)\right)$$

input
```
Integrate[Sqrt[2 + x^2]/(1 + 4*x),x]
```

output
```
(4*Sqrt[2 + x^2] + 2*Sqrt[33]*ArcTanh[(1 + 4*x - 4*Sqrt[2 + x^2])/Sqrt[33]
] + Log[-x + Sqrt[2 + x^2]])/16
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {493, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2}}{4x+1}dx$$

$$\Big\downarrow \text{493}$$

$$\frac{1}{4}\int \frac{8-x}{(4x+1)\sqrt{x^2+2}}dx+\frac{\sqrt{x^2+2}}{4}$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{4}\left(\frac{33}{4}\int \frac{1}{(4x+1)\sqrt{x^2+2}}dx-\frac{1}{4}\int \frac{1}{\sqrt{x^2+2}}dx\right)+\frac{\sqrt{x^2+2}}{4}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\frac{33}{4}\int \frac{1}{(4x+1)\sqrt{x^2+2}}dx-\frac{1}{4}\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\right)+\frac{\sqrt{x^2+2}}{4}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{4}\left(-\frac{33}{4}\int \frac{1}{33-\frac{(8-x)^2}{x^2+2}}d\frac{8-x}{\sqrt{x^2+2}}-\frac{1}{4}\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\right)+\frac{\sqrt{x^2+2}}{4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-\frac{1}{4}\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{4}\sqrt{33}\text{arctanh}\left(\frac{8-x}{\sqrt{33}\sqrt{x^2+2}}\right)\right)+\frac{\sqrt{x^2+2}}{4}$$

input
```
Int[Sqrt[2 + x^2]/(1 + 4*x),x]
```

output
```
Sqrt[2 + x^2]/4 + (-1/4*ArcSinh[x/Sqrt[2]] - (Sqrt[33]*ArcTanh[(8 - x)/(Sq
rt[33]*Sqrt[2 + x^2])])/4)/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{x^{2}+2}}{4}-\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{2}}{2}\right)}{16}-\frac{\sqrt{33}\, \operatorname{arctanh}\left(\frac{8\left(4-\frac{x}{2}\right)\sqrt{33}}{33\sqrt{16\left(x+\frac{1}{4}\right)^{2}-8x+31}}\right)}{16}$	50
default	$\frac{\sqrt{16\left(x+\frac{1}{4}\right)^{2}-8x+31}}{16}-\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{2}}{2}\right)}{16}-\frac{\sqrt{33}\, \operatorname{arctanh}\left(\frac{8\left(4-\frac{x}{2}\right)\sqrt{33}}{33\sqrt{16\left(x+\frac{1}{4}\right)^{2}-8x+31}}\right)}{16}$	57
trager	$\frac{\sqrt{x^{2}+2}}{4}-\frac{\ln\left(x+\sqrt{x^{2}+2}\right)}{16}+\frac{\operatorname{RootOf}\left(\_Z^{2}-33\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-33\right)x+33\sqrt{x^{2}+2}-8\operatorname{RootOf}\left(\_Z^{2}-33\right)}{1+4x}\right)}{16}$	66

input
```
int((x^2+2)^(1/2)/(1+4*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2+2)^(1/2)-1/16*arcsinh(1/2*x*2^(1/2))-1/16*33^(1/2)*arctanh(8/33*(
4-1/2*x)*33^(1/2)/(16*(x+1/4)^2-8*x+31)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx = \frac{1}{16}\sqrt{33}\log\left(-\frac{\sqrt{33}(x-8)+\sqrt{x^2+2}\left(\sqrt{33}+33\right)+x-8}{4\,x+1}\right) + \frac{1}{4}\sqrt{x^2+2}+\frac{1}{16}\log\left(-x+\sqrt{x^2+2}\right)$$

input
```
integrate((x^2+2)^(1/2)/(1+4*x),x, algorithm="fricas")
```

output
```
1/16*sqrt(33)*log(-(sqrt(33)*(x - 8) + sqrt(x^2 + 2)*(sqrt(33) + 33) + x -
 8)/(4*x + 1)) + 1/4*sqrt(x^2 + 2) + 1/16*log(-x + sqrt(x^2 + 2))
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx = \int \frac{\sqrt{x^2+2}}{4x+1}\,dx$$

input
```
integrate((x**2+2)**(1/2)/(1+4*x),x)
```

output
```
Integral(sqrt(x**2 + 2)/(4*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx = \frac{1}{16}\sqrt{33}\,\mathrm{arsinh}\left(\frac{\sqrt{2}x}{2\,|4\,x+1|} - \frac{4\,\sqrt{2}}{|4\,x+1|}\right) + \frac{1}{4}\,\sqrt{x^2+2} - \frac{1}{16}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{2}x\right)$$

input
```
integrate((x^2+2)^(1/2)/(1+4*x),x, algorithm="maxima")
```

output
```
1/16*sqrt(33)*arcsinh(1/2*sqrt(2)*x/abs(4*x + 1) - 4*sqrt(2)/abs(4*x + 1))
 + 1/4*sqrt(x^2 + 2) - 1/16*arcsinh(1/2*sqrt(2)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx = \frac{1}{16}\sqrt{33}\log\left(\frac{\left|-4\,x-\sqrt{33}+4\,\sqrt{x^2+2}-1\right|}{\left|-4\,x+\sqrt{33}+4\,\sqrt{x^2+2}-1\right|}\right) + \frac{1}{4}\,\sqrt{x^2+2} + \frac{1}{16}\,\log\left(-x+\sqrt{x^2+2}\right)$$

input
```
integrate((x^2+2)^(1/2)/(1+4*x),x, algorithm="giac")
```

output
```
1/16*sqrt(33)*log(abs(-4*x - sqrt(33) + 4*sqrt(x^2 + 2) - 1)/abs(-4*x + sq
rt(33) + 4*sqrt(x^2 + 2) - 1)) + 1/4*sqrt(x^2 + 2) + 1/16*log(-x + sqrt(x^
2 + 2))
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx = \frac{\sqrt{x^2+2}}{4} - \frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,x}{2}\right)}{16} + \frac{\sqrt{33}\,\left(132\,\ln\left(x+\frac{1}{4}\right) - 132\,\ln\left(x-\sqrt{33}\,\sqrt{x^2+2}-8\right)\right)}{2112}$$

input
```
int((x^2 + 2)^(1/2)/(4*x + 1),x)
```

output
```
(x^2 + 2)^(1/2)/4 - asinh((2^(1/2)*x)/2)/16 + (33^(1/2)*(132*log(x + 1/4)
- 132*log(x - 33^(1/2)*(x^2 + 2)^(1/2) - 8)))/2112
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{2+x^2}}{1+4x}\,dx = \frac{\sqrt{33}\,atan\left(\frac{4\sqrt{x^2+2}\,i+4ix}{\sqrt{33}-1}\right)i}{16} + \frac{\sqrt{x^2+2}}{4} + \frac{\sqrt{33}\log\left(16\sqrt{x^2+2}\,x+\sqrt{33}+16x^2-1\right)}{32} - \frac{\sqrt{33}\log\left(\frac{4\sqrt{x^2+2}+\sqrt{33}+4x+1}{\sqrt{2}}\right)}{16} - \frac{\log\left(\frac{\sqrt{x^2+2}+x}{\sqrt{2}}\right)}{16}$$

input
```
int((x^2+2)^(1/2)/(1+4*x),x)
```

output
```
(2*sqrt(33)*atan((4*sqrt(x**2 + 2)*i + 4*i*x)/(sqrt(33) - 1))*i + 8*sqrt(x
**2 + 2) + sqrt(33)*log(16*sqrt(x**2 + 2)*x + sqrt(33) + 16*x**2 - 1) - 2*
sqrt(33)*log((4*sqrt(x**2 + 2) + sqrt(33) + 4*x + 1)/sqrt(2)) - 2*log((sqr
t(x**2 + 2) + x)/sqrt(2)))/32
```

3.267 $\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx=\frac{\sqrt{1+2x^2}}{2\sqrt{2}}-\frac{5}{8}\text{arcsinh}\left(\sqrt{2}x\right)-\frac{1}{8}\sqrt{33}\text{arctanh}\left(\frac{\sqrt{\frac{2}{33}}(2-5x)}{\sqrt{1+2x^2}}\right)$$

output
```
1/4*(2*x^2+1)^(1/2)*2^(1/2)-5/8*arcsinh(x*2^(1/2))-1/8*33^(1/2)*arctanh(1/
33*66^(1/2)*(2-5*x)/(2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx=\frac{1}{8}\left(2\sqrt{2+4x^2}+2\sqrt{33}\text{arctanh}\left(\frac{5+4x-2\sqrt{2+4x^2}}{\sqrt{33}}\right)+5\log\left(-2x+\sqrt{2+4x^2}\right)\right)$$

input
```
Integrate[Sqrt[2 + 4*x^2]/(5 + 4*x),x]
```

output
```
(2*Sqrt[2 + 4*x^2] + 2*Sqrt[33]*ArcTanh[(5 + 4*x - 2*Sqrt[2 + 4*x^2])/Sqrt
[33]] + 5*Log[-2*x + Sqrt[2 + 4*x^2]])/8
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {493, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+2}}{4x+5}dx$$

$$\Big\downarrow \text{ 493}$$

$$\frac{1}{4}\int \frac{2\sqrt{2}(2-5x)}{(4x+5)\sqrt{2x^2+1}}dx+\frac{\sqrt{2x^2+1}}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2-5x}{(4x+5)\sqrt{2x^2+1}}dx}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\frac{33}{4}\int \frac{1}{(4x+5)\sqrt{2x^2+1}}dx-\frac{5}{4}\int \frac{1}{\sqrt{2x^2+1}}dx}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\frac{33}{4}\int \frac{1}{(4x+5)\sqrt{2x^2+1}}dx-\frac{5\text{arcsinh}\left(\sqrt{2}x\right)}{4\sqrt{2}}}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{-\frac{33}{4}\int \frac{1}{66-\frac{(4-10x)^2}{2x^2+1}}d\frac{4-10x}{\sqrt{2x^2+1}}-\frac{5\text{arcsinh}\left(\sqrt{2}x\right)}{4\sqrt{2}}}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{-\frac{5\operatorname{arcsinh}\left(\sqrt{2}x\right)}{4\sqrt{2}}-\frac{1}{4}\sqrt{\frac{33}{2}}\operatorname{arctanh}\left(\frac{4-10x}{\sqrt{66}\sqrt{2x^2+1}}\right)}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2\sqrt{2}}$$

input
```
Int[Sqrt[2 + 4*x^2]/(5 + 4*x),x]
```

output
```
Sqrt[1 + 2*x^2]/(2*Sqrt[2]) + ((-5*ArcSinh[Sqrt[2]*x])/(4*Sqrt[2]) - (Sqrt
[33/2]*ArcTanh[(4 - 10*x)/(Sqrt[66]*Sqrt[1 + 2*x^2])])/4)/Sqrt[2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\sqrt{16\left(x+\frac{5}{4}\right)^{2}-40x-17}}{8}-\frac{5\,\operatorname{arcsinh}\left(x\sqrt{2}\right)}{8}-\frac{\sqrt{33}\,\operatorname{arctanh}\left(\frac{2(4-10x)\sqrt{33}}{33\sqrt{16\left(x+\frac{5}{4}\right)^{2}-40x-17}}\right)}{8}$	56
risch	$\frac{2x^{2}+1}{2\sqrt{4x^{2}+2}}-\frac{5\,\operatorname{arcsinh}\left(x\sqrt{2}\right)}{8}-\frac{\sqrt{33}\,\operatorname{arctanh}\left(\frac{2(4-10x)\sqrt{33}}{33\sqrt{16\left(x+\frac{5}{4}\right)^{2}-40x-17}}\right)}{8}$	58
trager	$\frac{\sqrt{4x^{2}+2}}{4}-\frac{\operatorname{RootOf}\left(\_Z^{2}-33\right)\ln\left(\frac{-10\operatorname{RootOf}\left(\_Z^{2}-33\right)x+33\sqrt{4x^{2}+2}+4\operatorname{RootOf}\left(\_Z^{2}-33\right)}{5+4x}\right)}{8}+\frac{5\ln\left(2x-\sqrt{4x^{2}+2}\right)}{8}$	77

input
```
int((4*x^2+2)^(1/2)/(5+4*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*(16*(x+5/4)^2-40*x-17)^(1/2)-5/8*arcsinh(x*2^(1/2))-1/8*33^(1/2)*arcta
nh(2/33*(4-10*x)*33^(1/2)/(16*(x+5/4)^2-40*x-17)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int\frac{\sqrt{2+4x^2}}{5+4x}\,dx=\frac{1}{8}\sqrt{33}\log\left(-\frac{2\sqrt{33}(5x-2)+\sqrt{4x^2+2}\,(5\sqrt{33}+33)+50x-20}{4x+5}\right)+\frac{1}{4}\sqrt{4x^2+2}+\frac{5}{8}\log\left(-2x+\sqrt{4x^2+2}\right)$$

input
```
integrate((4*x^2+2)^(1/2)/(5+4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(33)*log(-(2*sqrt(33)*(5*x - 2) + sqrt(4*x^2 + 2)*(5*sqrt(33) + 33
) + 50*x - 20)/(4*x + 5)) + 1/4*sqrt(4*x^2 + 2) + 5/8*log(-2*x + sqrt(4*x^
2 + 2))
```

Sympy [F]

$$\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx = \sqrt{2}\int \frac{\sqrt{2x^2+1}}{4x+5}\,dx$$

input
```
integrate((4*x**2+2)**(1/2)/(5+4*x),x)
```

output
```
sqrt(2)*Integral(sqrt(2*x**2 + 1)/(4*x + 5), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx = \frac{1}{8}\sqrt{33}\,\operatorname{arsinh}\left(\frac{5\sqrt{2}x}{|4x+5|}-\frac{2\sqrt{2}}{|4x+5|}\right)+\frac{1}{4}\sqrt{4x^2+2}-\frac{5}{8}\operatorname{arsinh}\left(\sqrt{2}x\right)$$

input
```
integrate((4*x^2+2)^(1/2)/(5+4*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(33)*arcsinh(5*sqrt(2)*x/abs(4*x + 5) - 2*sqrt(2)/abs(4*x + 5)) +
1/4*sqrt(4*x^2 + 2) - 5/8*arcsinh(sqrt(2)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(48) = 96$.

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx$$
$$= \frac{1}{16}\sqrt{2}\left(5\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)+\sqrt{66}\log\left(-\frac{\left|-4\sqrt{2}x-\sqrt{66}-5\sqrt{2}+4\sqrt{2\,x^2+1}\right|}{4\sqrt{2}x-\sqrt{66}+5\sqrt{2}-4\sqrt{2\,x^2+1}}\right)+4\sqrt{2\,x^2+1}\right)$$

input
```
integrate((4*x^2+2)^(1/2)/(5+4*x),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(5*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + sqrt(66)*log(-
abs(-4*sqrt(2)*x - sqrt(66) - 5*sqrt(2) + 4*sqrt(2*x^2 + 1))/(4*sqrt(2)*x
- sqrt(66) + 5*sqrt(2) - 4*sqrt(2*x^2 + 1))) + 4*sqrt(2*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx = \frac{\sqrt{x^2+\frac{1}{2}}}{2} - \frac{5\,\mathrm{asinh}\left(\sqrt{2}\,x\right)}{8} + \frac{\sqrt{33}\left(132\,\ln\left(x+\frac{5}{4}\right)-132\,\ln\left(x-\frac{\sqrt{33}\sqrt{x^2+\frac{1}{2}}}{5}-\frac{2}{5}\right)\right)}{1056}$$

input
```
int((4*x^2 + 2)^(1/2)/(4*x + 5),x)
```

output
```
(x^2 + 1/2)^(1/2)/2 - (5*asinh(2^(1/2)*x))/8 + (33^(1/2)*(132*log(x + 5/4)
 - 132*log(x - (33^(1/2)*(x^2 + 1/2)^(1/2))/5 - 2/5)))/1056
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{2+4x^2}}{5+4x}\,dx = \frac{\sqrt{33}\,atan\left(\frac{4\sqrt{2x^2+1}\,i+4\sqrt{2}\,ix}{\sqrt{66}-5\sqrt{2}}\right)i}{8} + \frac{\sqrt{2x^2+1}\,\sqrt{2}}{4} + \frac{\sqrt{33}\log\left(8\sqrt{2x^2+1}\,\sqrt{2}\,x+5\sqrt{33}+16x^2-25\right)}{16} - \frac{\sqrt{33}\log\left(2\sqrt{2x^2+1}+\frac{\sqrt{66}}{2}+2\sqrt{2}\,x+\frac{5\sqrt{2}}{2}\right)}{8} - \frac{5\log\left(\sqrt{2x^2+1}+\sqrt{2}\,x\right)}{8}$$

input

```
int((4*x^2+2)^(1/2)/(5+4*x),x)
```

output

```
(2*sqrt(33)*atan((4*sqrt(2*x**2 + 1)*i + 4*sqrt(2)*i*x)/(sqrt(66) - 5*sqrt
(2)))*i + 4*sqrt(2*x**2 + 1)*sqrt(2) + sqrt(33)*log(8*sqrt(2*x**2 + 1)*sqr
t(2)*x + 5*sqrt(33) + 16*x**2 - 25) - 2*sqrt(33)*log((4*sqrt(2*x**2 + 1) +
 sqrt(66) + 4*sqrt(2)*x + 5*sqrt(2))/2) - 10*log(sqrt(2*x**2 + 1) + sqrt(2
)*x))/16
```

3.268 $\int (2+3x)\sqrt{-5+7x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = x\sqrt{-5+7x^2}+\frac{1}{7}\left(-5+7x^2\right)^{3/2}-\frac{5\text{arctanh}\left(\frac{\sqrt{7}x}{\sqrt{-5+7x^2}}\right)}{\sqrt{7}}$$

output
```
x*(7*x^2-5)^(1/2)+1/7*(7*x^2-5)^(3/2)-5/7*arctanh(7^(1/2)*x/(7*x^2-5)^(1/2
))*7^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = \frac{1}{7}\sqrt{-5+7x^2}\left(-5+7x+7x^2\right)-\frac{10\text{arctanh}\left(\frac{\sqrt{-5+7x^2}}{\sqrt{5}+\sqrt{7}x}\right)}{\sqrt{7}}$$

input
```
Integrate[(2 + 3*x)*Sqrt[-5 + 7*x^2],x]
```

output
```
(Sqrt[-5 + 7*x^2]*(-5 + 7*x + 7*x^2))/7 - (10*ArcTanh[Sqrt[-5 + 7*x^2]/(Sq
rt[5] + Sqrt[7]*x)])/Sqrt[7]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)\sqrt{7x^2-5}\,dx$$

$$\Big\downarrow \text{455}$$

$$2\int \sqrt{7x^2-5}dx+\frac{1}{7}\left(7x^2-5\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$2\left(\frac{1}{2}x\sqrt{7x^2-5}-\frac{5}{2}\int \frac{1}{\sqrt{7x^2-5}}dx\right)+\frac{1}{7}\left(7x^2-5\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$2\left(\frac{1}{2}x\sqrt{7x^2-5}-\frac{5}{2}\int \frac{1}{1-\frac{7x^2}{7x^2-5}}d\frac{x}{\sqrt{7x^2-5}}\right)+\frac{1}{7}\left(7x^2-5\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}x\sqrt{7x^2-5}-\frac{5\text{arctanh}\left(\frac{\sqrt{7}x}{\sqrt{7x^2-5}}\right)}{2\sqrt{7}}\right)+\frac{1}{7}\left(7x^2-5\right)^{3/2}$$

input
```
Int[(2 + 3*x)*Sqrt[-5 + 7*x^2],x]
```

output
```
(-5 + 7*x^2)^(3/2)/7 + 2*((x*Sqrt[-5 + 7*x^2])/2 - (5*ArcTanh[(Sqrt[7]*x)/
Sqrt[-5 + 7*x^2]])/(2*Sqrt[7]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result
risch	$\frac{(7x^2+7x-5)\sqrt{7x^2-5}}{7}-\frac{5\ln\left(x\sqrt{7}+\sqrt{7x^2-5}\right)\sqrt{7}}{7}$
default	$x\sqrt{7x^2-5}-\frac{5\ln\left(x\sqrt{7}+\sqrt{7x^2-5}\right)\sqrt{7}}{7}+\frac{(7x^2-5)^{\frac{3}{2}}}{7}$
trager	$\left(x^2+x-\frac{5}{7}\right)\sqrt{7x^2-5}+\frac{5\,\mathrm{RootOf}\left(\_Z^2-7\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-7\right)\sqrt{7x^2-5}+7x\right)}{7}$
meijerg	$\frac{5i\sqrt{\mathrm{signum}\left(-1+\frac{7x^2}{5}\right)}\sqrt{7}\left(-\frac{2i\sqrt{35}\,\sqrt{\pi}\,x\sqrt{-\frac{7x^2}{5}+1}}{5}-2i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{7}\,\sqrt{5}}{5}\right)\right)}{14\sqrt{\pi}\,\sqrt{-\mathrm{signum}\left(-1+\frac{7x^2}{5}\right)}}+\frac{15\sqrt{5}\,\sqrt{\mathrm{signum}\left(-1+\frac{7x^2}{5}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2-\frac{14}{}\right.}{}\right.}{28\sqrt{\pi}\,\sqrt{-\mathrm{signum}\left(-1+\frac{7x^2}{5}\right)}}$

input
```
int((3*x+2)*(7*x^2-5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*(7*x^2+7*x-5)*(7*x^2-5)^(1/2)-5/7*ln(x*7^(1/2)+(7*x^2-5)^(1/2))*7^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = \frac{1}{7}\left(7\,x^2+7\,x-5\right)\sqrt{7\,x^2-5} + \frac{5}{14}\,\sqrt{7}\log\left(-2\,\sqrt{7}\sqrt{7\,x^2-5}x+14\,x^2-5\right)$$

input
```
integrate((2+3*x)*(7*x^2-5)^(1/2),x, algorithm="fricas")
```

output
```
1/7*(7*x^2 + 7*x - 5)*sqrt(7*x^2 - 5) + 5/14*sqrt(7)*log(-2*sqrt(7)*sqrt(7
*x^2 - 5)*x + 14*x^2 - 5)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = x^2\sqrt{7x^2-5}+x\sqrt{7x^2-5}-\frac{5\sqrt{7x^2-5}}{7} - \frac{5\sqrt{7}\log\left(7x+\sqrt{7}\sqrt{7x^2-5}\right)}{7}$$

input
```
integrate((2+3*x)*(7*x**2-5)**(1/2),x)
```

output
```
x**2*sqrt(7*x**2 - 5) + x*sqrt(7*x**2 - 5) - 5*sqrt(7*x**2 - 5)/7 - 5*sqrt
(7)*log(7*x + sqrt(7)*sqrt(7*x**2 - 5))/7
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = \frac{1}{7}\left(7\,x^2-5\right)^{\frac{3}{2}} + \sqrt{7\,x^2-5}x - \frac{5}{7}\,\sqrt{7}\log\left(2\,\sqrt{7}\sqrt{7\,x^2-5}+14\,x\right)$$

input
```
integrate((2+3*x)*(7*x^2-5)^(1/2),x, algorithm="maxima")
```

output
```
1/7*(7*x^2 - 5)^(3/2) + sqrt(7*x^2 - 5)*x - 5/7*sqrt(7)*log(2*sqrt(7)*sqrt
(7*x^2 - 5) + 14*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = \frac{1}{7}\left(7\,(x+1)x-5\right)\sqrt{7\,x^2-5} + \frac{5}{7}\,\sqrt{7}\log\left(\left|-\sqrt{7}x+\sqrt{7\,x^2-5}\right|\right)$$

input
```
integrate((2+3*x)*(7*x^2-5)^(1/2),x, algorithm="giac")
```

output
```
1/7*(7*(x + 1)*x - 5)*sqrt(7*x^2 - 5) + 5/7*sqrt(7)*log(abs(-sqrt(7)*x + s
qrt(7*x^2 - 5)))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = x\,\sqrt{7\,x^2-5} - \frac{5\,\sqrt{7}\,\ln\left(\sqrt{7}\,x+\sqrt{7\,x^2-5}\right)}{7} + \frac{\left(7\,x^2-5\right)^{3/2}}{7}$$

input
```
int((3*x + 2)*(7*x^2 - 5)^(1/2),x)
```

output
```
x*(7*x^2 - 5)^(1/2) - (5*7^(1/2)*log(7^(1/2)*x + (7*x^2 - 5)^(1/2)))/7 + (
7*x^2 - 5)^(3/2)/7
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int (2+3x)\sqrt{-5+7x^2}\,dx = \sqrt{7x^2-5}\,x^2+\sqrt{7x^2-5}\,x - \frac{5\sqrt{7x^2-5}}{7} - \frac{5\sqrt{7}\log\left(\frac{\sqrt{7x^2-5}+\sqrt{7}\,x}{\sqrt{5}}\right)}{7}$$

input
```
int((2+3*x)*(7*x^2-5)^(1/2),x)
```

output
```
(7*sqrt(7*x**2 - 5)*x**2 + 7*sqrt(7*x**2 - 5)*x - 5*sqrt(7*x**2 - 5) - 5*s
qrt(7)*log((sqrt(7*x**2 - 5) + sqrt(7)*x)/sqrt(5)))/7
```

3.269 $\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \frac{7cd(c+dx)^2\sqrt{a+bx^2}}{12b} + \frac{d(c+dx)^3\sqrt{a+bx^2}}{4b} + \frac{d\left(4c(19bc^2-16ad^2)+d(26bc^2-9ad^2)\,x\right)\sqrt{a+bx^2}}{24b^2} + \frac{(8b^2c^4-24abc^2d^2+3a^2d^4)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
7/12*c*d*(d*x+c)^2*(b*x^2+a)^(1/2)/b+1/4*d*(d*x+c)^3*(b*x^2+a)^(1/2)/b+1/2
4*d*(4*c*(-16*a*d^2+19*b*c^2)+d*(-9*a*d^2+26*b*c^2)*x)*(b*x^2+a)^(1/2)/b^2
+1/8*(3*a^2*d^4-24*a*b*c^2*d^2+8*b^2*c^4)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2
))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}\left(96bc^3d-64acd^3+72bc^2d^2x-9ad^4x+32bcd^3x^2+6bd^4x^3\right)}{24b^2} + \frac{\left(-8b^2c^4+24abc^2d^2-3a^2d^4\right)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[(c + d*x)^4/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(96*b*c^3*d - 64*a*c*d^3 + 72*b*c^2*d^2*x - 9*a*d^4*x + 3
2*b*c*d^3*x^2 + 6*b*d^4*x^3))/(24*b^2) + ((-8*b^2*c^4 + 24*a*b*c^2*d^2 - 3
*a^2*d^4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {497, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 497$$

$$\frac{\int \frac{(c+dx)^2\left(4bc^2+7bdxc-3ad^2\right)}{\sqrt{bx^2+a}}dx}{4b} + \frac{d\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{\int \frac{b(c+dx)\left(c\left(12bc^2-23ad^2\right)+d\left(26bc^2-9ad^2\right)x\right)}{\sqrt{bx^2+a}}dx}{3b} + \frac{7}{3}cd\sqrt{a+bx^2}(c+dx)^2}{4b} + \frac{d\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}\int \frac{(c+dx)\left(c\left(12bc^2-23ad^2\right)+d\left(26bc^2-9ad^2\right)x\right)}{\sqrt{bx^2+a}}dx+\frac{7}{3}cd\sqrt{a+bx^2}(c+dx)^2}{4b}+\frac{d\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{1}{3}\left(\frac{3\left(3a^2d^4-24abc^2d^2+8b^2c^4\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{d^2x\sqrt{a+bx^2}\left(26bc^2-9ad^2\right)}{2b}+\frac{2cd\sqrt{a+bx^2}\left(19bc^2-16ad^2\right)}{b}\right)+\frac{7}{3}cd\sqrt{a+bx^2}(c+dx)^2}{4b}+\frac{d\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{3}\left(\frac{3\left(3a^2d^4-24abc^2d^2+8b^2c^4\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{d^2x\sqrt{a+bx^2}\left(26bc^2-9ad^2\right)}{2b}+\frac{2cd\sqrt{a+bx^2}\left(19bc^2-16ad^2\right)}{b}\right)+\frac{7}{3}cd\sqrt{a+bx^2}(c+dx)^2}{4b}+\frac{d\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{3}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4-24abc^2d^2+8b^2c^4\right)}{2b^{3/2}}+\frac{d^2x\sqrt{a+bx^2}\left(26bc^2-9ad^2\right)}{2b}+\frac{2cd\sqrt{a+bx^2}\left(19bc^2-16ad^2\right)}{b}\right)+\frac{7}{3}cd\sqrt{a+bx^2}(c+dx)^2}{4b}+\frac{d\sqrt{a+bx^2}(c+dx)^3}{4b}$$

input

```
Int[(c + d*x)^4/Sqrt[a + b*x^2],x]
```

output

```
(d*(c + d*x)^3*Sqrt[a + b*x^2])/(4*b) + ((7*c*d*(c + d*x)^2*Sqrt[a + b*x^2
])/3 + ((2*c*d*(19*b*c^2 - 16*a*d^2)*Sqrt[a + b*x^2])/b + (d^2*(26*b*c^2 -
 9*a*d^2)*x*Sqrt[a + b*x^2])/(2*b) + (3*(8*b^2*c^4 - 24*a*b*c^2*d^2 + 3*a^
2*d^4)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/3)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{d(-6b\,d^3x^3-32bc\,d^2x^2+9ax\,d^3-72b\,c^2dx+64a\,d^2c-96b\,c^3)\sqrt{b\,x^2+a}}{24b^2}+\frac{(3a^2d^4-24b\,c^2d^2a+8b^2c^4)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$
default	$\frac{c^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d^4\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+\frac{4d\,c^3\sqrt{b\,x^2+a}}{b}+6c^2d^2\left(\frac{x\sqrt{b\,x^2}}{2b}\right.$

input
```
int((d*x+c)^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*d*(-6*b*d^3*x^3-32*b*c*d^2*x^2+9*a*d^3*x-72*b*c^2*d*x+64*a*c*d^2-96*
b*c^3)*(b*x^2+a)^(1/2)/b^2+1/8*(3*a^2*d^4-24*a*b*c^2*d^2+8*b^2*c^4)/b^(5/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{3\,(8\,b^2c^4-24\,abc^2d^2+3\,a^2d^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(6\,b^2d^4x^3+32\,b^2cd^3x^2+96\,b}{48\,b^3}\right.$$
$$-\frac{3\,(8\,b^2c^4-24\,abc^2d^2+3\,a^2d^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,b^2d^4x^3+32\,b^2cd^3x^2+96\,b^2c^3d-64\,abcd^3+}{24\,b^3}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*b^2*c^4 - 24*a*b*c^2*d^2 + 3*a^2*d^4)*sqrt(b)*log(-2*b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*b^2*d^4*x^3 + 32*b^2*c*d^3*x^2 + 96
*b^2*c^3*d - 64*a*b*c*d^3 + 9*(8*b^2*c^2*d^2 - a*b*d^4)*x)*sqrt(b*x^2 + a)
)/b^3, -1/24*(3*(8*b^2*c^4 - 24*a*b*c^2*d^2 + 3*a^2*d^4)*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (6*b^2*d^4*x^3 + 32*b^2*c*d^3*x^2 + 96*b^2*c^
3*d - 64*a*b*c*d^3 + 9*(8*b^2*c^2*d^2 - a*b*d^4)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{4cd^3x^2}{3b}+\frac{d^4x^3}{4b}+\frac{x\left(-\frac{3ad^4}{4b}+6c^2d^2\right)}{2b}+\frac{-\frac{8acd^3}{3b}+4c^3d}{b}\right)+\left(-\frac{a\left(-\frac{3ad^4}{4b}+6c^2d^2\right)}{2b}+c^4\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}\right.}{\sqrt{b}} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right. \\ \frac{\begin{cases} c^4x & \text{for } d=0 \\ \frac{(c+dx)^5}{5d} & \text{otherwise}\end{cases}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x+c)**4/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(4*c*d**3*x**2/(3*b) + d**4*x**3/(4*b) + x*(-3
*a*d**4/(4*b) + 6*c**2*d**2)/(2*b) + (-8*a*c*d**3/(3*b) + 4*c**3*d)/b) + (
-a*(-3*a*d**4/(4*b) + 6*c**2*d**2)/(2*b) + c**4)*Piecewise((log(2*sqrt(b)*
sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True
)), Ne(b, 0)), (Piecewise((c**4*x, Eq(d, 0)), ((c + d*x)**5/(5*d), True))/
sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^4x^3}{4\,b} + \frac{4\,\sqrt{bx^2+a}cd^3x^2}{3\,b} + \frac{3\,\sqrt{bx^2+a}c^2d^2x}{b} - \frac{3\,\sqrt{bx^2+a}ad^4x}{8\,b^2} + \frac{c^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{3\,ac^2d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{3\,a^2d^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{4\,\sqrt{bx^2+a}c^3d}{b} - \frac{8\,\sqrt{bx^2+a}acd^3}{3\,b^2}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*d^4*x^3/b + 4/3*sqrt(b*x^2 + a)*c*d^3*x^2/b + 3*sqrt(b
*x^2 + a)*c^2*d^2*x/b - 3/8*sqrt(b*x^2 + a)*a*d^4*x/b^2 + c^4*arcsinh(b*x/
sqrt(a*b))/sqrt(b) - 3*a*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*a^2*
d^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 4*sqrt(b*x^2 + a)*c^3*d/b - 8/3*sqrt(
b*x^2 + a)*a*c*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \frac{1}{24}\,\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,d^4x}{b} + \frac{16\,cd^3}{b}\right)x + \frac{9\,(8\,b^3c^2d^2 - ab^2d^4)}{b^4}\right)x + \frac{32\,(3\,b^3c^3d - 2\,ab^2cd^3)}{b^4}\right) - \frac{(8\,b^2c^4 - 24\,abc^2d^2 + 3\,a^2d^4)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*d^4*x/b + 16*c*d^3/b)*x + 9*(8*b^3*c^2*d^2 - a
*b^2*d^4)/b^4)*x + 32*(3*b^3*c^3*d - 2*a*b^2*c*d^3)/b^4) - 1/8*(8*b^2*c^4
- 24*a*b*c^2*d^2 + 3*a^2*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/
2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^4}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x)^4/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^4/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx+c)^4}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x+c)^4/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^4/(b*x^2+a)^(1/2),x)
```

3.270 $\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx=\frac{d(c+dx)^2\sqrt{a+bx^2}}{3b}+\frac{d(4(4bc^2-ad^2)+5bcdx)\sqrt{a+bx^2}}{6b^2}+\frac{c(2bc^2-3ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/3*d*(d*x+c)^2*(b*x^2+a)^(1/2)/b+1/6*d*(5*b*c*d*x-4*a*d^2+16*b*c^2)*(b*x^
2+a)^(1/2)/b^2+1/2*c*(-3*a*d^2+2*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx=\frac{d\sqrt{a+bx^2}(-4ad^2+b(18c^2+9cdx+2d^2x^2))+3\sqrt{b}c(-2bc^2+3ad^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{6b^2}$$

input
```
Integrate[(c + d*x)^3/Sqrt[a + b*x^2],x]
```

output
```
(d*Sqrt[a + b*x^2]*(-4*a*d^2 + b*(18*c^2 + 9*c*d*x + 2*d^2*x^2)) + 3*Sqrt[
b]*c*(-2*b*c^2 + 3*a*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(6*b^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {497, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 497$$

$$\frac{\int \frac{(c+dx)(3bc^2+5bdxc-2ad^2)}{\sqrt{bx^2+a}}dx}{3b}+\frac{d\sqrt{a+bx^2}(c+dx)^2}{3b}$$

$$\Big\downarrow \ 676$$

$$\frac{\frac{3}{2}c(2bc^2-3ad^2)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2d\sqrt{a+bx^2}(4bc^2-ad^2)}{b}+\frac{5}{2}cd^2x\sqrt{a+bx^2}}{3b}+\frac{d\sqrt{a+bx^2}(c+dx)^2}{3b}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{3}{2}c(2bc^2-3ad^2)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2d\sqrt{a+bx^2}(4bc^2-ad^2)}{b}+\frac{5}{2}cd^2x\sqrt{a+bx^2}}{3b}+\frac{d\sqrt{a+bx^2}(c+dx)^2}{3b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{3c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc^2-3ad^2)}{2\sqrt{b}}+\frac{2d\sqrt{a+bx^2}(4bc^2-ad^2)}{b}+\frac{5}{2}cd^2x\sqrt{a+bx^2}}{3b}+\frac{d\sqrt{a+bx^2}(c+dx)^2}{3b}$$

input
```
Int[(c + d*x)^3/Sqrt[a + b*x^2],x]
```

output
```
(d*(c + d*x)^2*Sqrt[a + b*x^2])/(3*b) + ((2*d*(4*b*c^2 - a*d^2)*Sqrt[a + b
*x^2])/b + (5*c*d^2*x*Sqrt[a + b*x^2])/2 + (3*c*(2*b*c^2 - 3*a*d^2)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))/(3*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{d\left(-2b\,x^2d^2-9bcdx+4a\,d^2-18b\,c^2\right)\sqrt{b\,x^2+a}}{6b^2}-\frac{c\left(3a\,d^2-2b\,c^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$
default	$\frac{c^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d^3\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+3c\,d^2\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{3c^2d\sqrt{b\,x^2}}{b}$

input
```
int((d*x+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*d*(-2*b*d^2*x^2-9*b*c*d*x+4*a*d^2-18*b*c^2)*(b*x^2+a)^(1/2)/b^2-1/2*c
/b^(3/2)*(3*a*d^2-2*b*c^2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.64

$$\int\frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\,(2\,bc^3-3\,acd^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,bd^3x^2+9\,bcd^2x+18\,bc^2d-4\,ad^3)\sqrt{b}}{12\,b^2}\right.$$
$$\left.-\frac{3\,(2\,bc^3-3\,acd^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,bd^3x^2+9\,bcd^2x+18\,bc^2d-4\,ad^3)\sqrt{bx^2+a}}{6\,b^2}\right]$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*b*c^3 - 3*a*c*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(2*b*d^3*x^2 + 9*b*c*d^2*x + 18*b*c^2*d - 4*a*d^3)*sqrt(
b*x^2 + a))/b^2, -1/6*(3*(2*b*c^3 - 3*a*c*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)) - (2*b*d^3*x^2 + 9*b*c*d^2*x + 18*b*c^2*d - 4*a*d^3)*sqrt
(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{3cd^2x}{2b}+\frac{d^3x^2}{3b}+\frac{-\frac{2ad^3}{3b}+3c^2d}{b}\right)+\left(-\frac{3acd^2}{2b}+c^3\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for } b \neq 0 \\ \frac{\begin{cases}c^3x & \text{for } d=0\\ \frac{(c+dx)^4}{4d} & \text{otherwise}\end{cases}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**3/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(3*c*d**2*x/(2*b) + d**3*x**2/(3*b) + (-2*a*d*
*3/(3*b) + 3*c**2*d)/b) + (-3*a*c*d**2/(2*b) + c**3)*Piecewise((log(2*sqrt
(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
True)), Ne(b, 0)), (Piecewise((c**3*x, Eq(d, 0)), ((c + d*x)**4/(4*d), Tru
e))/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^3x^2}{3\,b}+\frac{3\,\sqrt{bx^2+a}cd^2x}{2\,b}+\frac{c^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{3\,acd^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,\sqrt{bx^2+a}c^2d}{b}-\frac{2\,\sqrt{bx^2+a}ad^3}{3\,b^2}$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*d^3*x^2/b + 3/2*sqrt(b*x^2 + a)*c*d^2*x/b + c^3*arcsin
h(b*x/sqrt(a*b))/sqrt(b) - 3/2*a*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3*
sqrt(b*x^2 + a)*c^2*d/b - 2/3*sqrt(b*x^2 + a)*a*d^3/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,d^3x}{b}+\frac{9\,cd^2}{b}\right)x+\frac{2\,(9\,b^2c^2d-2\,abd^3)}{b^3}\right) - \frac{(2\,bc^3-3\,acd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*d^3*x/b + 9*c*d^2/b)*x + 2*(9*b^2*c^2*d - 2*a*b*d^
3)/b^3) - 1/2*(2*b*c^3 - 3*a*c*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))
/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^3}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x)^3/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^3/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-4\sqrt{b\,x^2+a}\,a\,d^3+18\sqrt{b\,x^2+a}\,b\,c^2d+9\sqrt{b\,x^2+a}\,bc\,d^2x+2\sqrt{b\,x^2+a}\,b\,d^3x^2-9\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{6b^2}$$

input
```
int((d*x+c)^3/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**2)*a*d**3 + 18*sqrt(a + b*x**2)*b*c**2*d + 9*sqrt(a +
b*x**2)*b*c*d**2*x + 2*sqrt(a + b*x**2)*b*d**3*x**2 - 9*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c*d**2 + 6*sqrt(b)*log((sqrt(a + b*x**
2) + sqrt(b)*x)/sqrt(a))*b*c**3)/(6*b**2)
```

3.271 $\int \frac{(c+dx)^2}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}} \, dx = \frac{d(4c+dx)\sqrt{a+bx^2}}{2b} + \frac{(2bc^2-ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*d*(d*x+4*c)*(b*x^2+a)^(1/2)/b+1/2*(-a*d^2+2*b*c^2)*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}} \, dx = \frac{d(4c+dx)\sqrt{a+bx^2}}{2b} + \frac{(-2bc^2+ad^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[(c + d*x)^2/Sqrt[a + b*x^2],x]
```

output
```
(d*(4*c + d*x)*Sqrt[a + b*x^2])/(2*b) + ((-2*b*c^2 + a*d^2)*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {497, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{497}$$

$$\frac{\int \frac{2bc^2+3bdxc-ad^2}{\sqrt{bx^2+a}}dx}{2b}+\frac{d\sqrt{a+bx^2}(c+dx)}{2b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(2bc^2-ad^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx+3cd\sqrt{a+bx^2}}{2b}+\frac{d\sqrt{a+bx^2}(c+dx)}{2b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\left(2bc^2-ad^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+3cd\sqrt{a+bx^2}}{2b}+\frac{d\sqrt{a+bx^2}(c+dx)}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2bc^2-ad^2\right)}{\sqrt{b}}+3cd\sqrt{a+bx^2}}{2b}+\frac{d\sqrt{a+bx^2}(c+dx)}{2b}$$

input
```
Int[(c + d*x)^2/Sqrt[a + b*x^2],x]
```

output
```
(d*(c + d*x)*Sqrt[a + b*x^2])/(2*b) + (3*c*d*Sqrt[a + b*x^2] + ((2*b*c^2 -
 a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{d(dx+4c)\sqrt{b\,x^{2}+a}}{2b}-\frac{\left(a\,d^{2}-2b\,c^{2}\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	57
default	$\frac{c^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+d^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{2cd\sqrt{b\,x^{2}+a}}{b}$	83

input
```
int((d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*(d*x+4*c)*(b*x^2+a)^(1/2)/b-1/2*(a*d^2-2*b*c^2)/b^(3/2)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.03

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \left[-\frac{(2\,bc^2-ad^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(bd^2x+4\,bcd)\sqrt{bx^2+a}}{4\,b^2},\right.$$
$$\left.-\frac{(2\,bc^2-ad^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(bd^2x+4\,bcd)\sqrt{bx^2+a}}{2\,b^2}\right]$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*b*c^2 - a*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(b*d^2*x + 4*b*c*d)*sqrt(b*x^2 + a))/b^2, -1/2*((2*b*c^2 - a*d^
2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*d^2*x + 4*b*c*d)*sqrt(
b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\cdot\left(\frac{2cd}{b}+\frac{d^2x}{2b}\right)+\left(-\frac{ad^2}{2b}+c^2\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for}\ b\neq 0\\ \frac{\begin{cases}c^2x & \text{for}\ d=0\\ \frac{(c+dx)^3}{3d} & \text{otherwise}\end{cases}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

input
```
integrate((d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(2*c*d/b + d**2*x/(2*b)) + (-a*d**2/(2*b) + c*
*2)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)),
 (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (Piecewise((c**2*x, Eq(d, 0)),
 ((c + d*x)**3/(3*d), True))/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^2x}{2\,b} + \frac{c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{ad^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{2\,\sqrt{bx^2+a}cd}{b}$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*d^2*x/b + c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*d
^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*sqrt(b*x^2 + a)*c*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\sqrt{bx^2+a}\left(\frac{d^2x}{b} + \frac{4\,cd}{b}\right) - \frac{(2\,bc^2-ad^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*(d^2*x/b + 4*c*d/b) - 1/2*(2*b*c^2 - a*d^2)*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x)^2/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^2/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \frac{4\sqrt{b\,x^2+a}\,bcd+\sqrt{b\,x^2+a}\,b\,d^2x-\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a\,d^2+2\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,c^2}{2b^2}$$

input
```
int((d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(4*sqrt(a + b*x**2)*b*c*d + sqrt(a + b*x**2)*b*d**2*x - sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*d**2 + 2*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*b*c**2)/(2*b**2)
```

3.272 $\int \frac{c+dx}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{c+dx}{\sqrt{a+bx^2}} \, dx = \frac{d\sqrt{a+bx^2}}{b} + \frac{c\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
d*(b*x^2+a)^(1/2)/b+c*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{c+dx}{\sqrt{a+bx^2}} \, dx = \frac{d\sqrt{a+bx^2}}{b} - \frac{c\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(c + d*x)/Sqrt[a + b*x^2],x]
```

output
```
(d*Sqrt[a + b*x^2])/b - (c*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 455$$

$$c\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{d\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \; 224$$

$$c\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{d\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{d\sqrt{a+bx^2}}{b}$$

input
```
Int[(c + d*x)/Sqrt[a + b*x^2],x]
```

output
```
(d*Sqrt[a + b*x^2])/b + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{d\sqrt{b\,x^{2}+a}}{b}$	37
risch	$\frac{c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{d\sqrt{b\,x^{2}+a}}{b}$	37

input
```
int((d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+d*(b*x^2+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{c+dx}{\sqrt{a+bx^2}}\,dx = \left[\frac{\sqrt{b}c\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}d}{2\,b},\right. \\ \left. -\frac{\sqrt{-b}c\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}d}{b}\right]$$

input
```
integrate((d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b)*c*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b
*x^2 + a)*d)/b, -(sqrt(-b)*c*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x
^2 + a)*d)/b]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{c+dx}{\sqrt{a+bx^2}}\,dx = \begin{cases} c\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) + \frac{d\sqrt{a+bx^2}}{b} & \text{for } b\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((c*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + d*sqrt(a + b*x**2)/b, Ne(b, 0))
, ((c*x + d*x**2/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{c+dx}{\sqrt{a+bx^2}}\,dx = \frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}\,d}{b}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{\sqrt{a+bx^2}}\,dx = -\frac{c\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}d}{b}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-c*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*d/b
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{\sqrt{a+bx^2}}\,dx = \frac{d\sqrt{b\,x^2+a}}{b}+\frac{c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int((c + d*x)/(a + b*x^2)^(1/2),x)
```

output
```
(d*(a + b*x^2)^(1/2))/b + (c*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,d+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)c}{b}$$

input
```
int((d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*d + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
c)/b
```

3.273 $\int \frac{1}{(c+dx)\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{\sqrt{bc^2+ad^2}}$$

output
```
-arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}} \, dx = -\frac{2\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}$$

input
```
Integrate[1/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]]
)/Sqrt[-(b*c^2) - a*d^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}(c+dx)} \, dx$$

$$\Big\downarrow \ 488$$

$$-\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\sqrt{ad^2+bc^2}}$$

input
```
Int[1/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
-(ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])]/Sqrt[b*c^2
+ a*d^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(48) = 96$.

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.35

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{x+\frac{c}{d}}\right)}{d\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}$	127

input
```
int(1/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*(
(a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
1/2))/(x+c/d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.91

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}}\,dx = \left[\frac{\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2-(2\,b^2c^2+abd^2)x^2-2\,\sqrt{bc^2+ad^2}(bcx-ad)\sqrt{bx^2+a}}{d^2x^2+2\,cdx+c^2}\right)}{2\,\sqrt{bc^2+ad^2}},\right.$$
$$\left.-\frac{\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{-bc^2-ad^2}(bcx-ad)\sqrt{bx^2+a}}{abc^2+a^2d^2+(b^2c^2+abd^2)x^2}\right)}{bc^2+ad^2}\right]$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 -
2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x +
c^2))/sqrt(b*c^2 + a*d^2), -sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^
2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*
x^2))/(b*c^2 + a*d^2)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sq
rt(a + b*c^2/d^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^
2))/sqrt(-b*c^2 - a*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1019, normalized size of antiderivative = 18.87

$$\int \frac{1}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*c - 2*sqrt(2*sqrt(b)*sqrt
(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b
)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*d**2
 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqr
t(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*
d**2 - 2*b*c**2))*b*c**2 - sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c
+ a*d**2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d*
*2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*c
+ sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt
(a*d**2 + b*c**2)*log(sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*
b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*c - sqrt(a*d**2 + b*c**2)*log(
 - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a +
b*x**2)*d + sqrt(b)*d*x)*a*d**2 - sqrt(a*d**2 + b*c**2)*log(sqrt(2*sqrt(b)
*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt(
b)*d*x)*a*d**2 + sqrt(a*d**2 + b*c**2)*log(2*sqrt(b)*sqrt(a*d**2 + b*c**2)
*c + 2*sqrt(b)*sqrt(a + b*x**2)*d**2*x - 2*b*c**2 + 2*b*d**2*x**2)*a*d**2
+ sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*log( - sqrt(
2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**...
```

3.274 $\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx = -\frac{d\sqrt{a+bx^2}}{(bc^2+ad^2)(c+dx)} - \frac{b c \operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{3/2}}$$

output
```
-d*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)/(d*x+c)-b*c*arctanh((-b*c*x+a*d)/(a*d^2+b
*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx = -\frac{d\sqrt{a+bx^2}}{(bc^2+ad^2)(c+dx)} + \frac{2bc\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}$$

input
```
Integrate[1/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-((d*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x))) + (2*b*c*ArcTan[(Sqrt[b
]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^
2)^(3/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}(c+dx)^2} \, dx$$

$$\Big\downarrow \text{491}$$

$$\frac{bc\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{d\sqrt{a+bx^2}}{(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{bc\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{d\sqrt{a+bx^2}}{(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b c \,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{d\sqrt{a+bx^2}}{(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[1/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-((d*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x))) - (b*c*ArcTanh[(a*d - b
*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.36

method	result	size
default	$\frac{-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\,\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{x+\frac{c}{d}}\right)}{\left(a\,d^2+b\,c^2\right)\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}}{d^2}$	215

input
```
int(1/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2
+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c
/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.19

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{(bcdx+bc^2)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2-(2\,b^2c^2+abd^2)x^2-2\,\sqrt{bc^2+ad^2}(bcx-ad)\sqrt{bx^2+a}}{d^2x^2+2\,cdx+c^2}\right)-2\,(bc^2d+ad^3)\sqrt{}}{2\,(b^2c^5+2\,abc^3d^2+a^2cd^4+(b^2c^4d+2\,abc^2d^3+a^2d^5)x)}\right.$$
$$\left.-\frac{(bcdx+bc^2)\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{-bc^2-ad^2}(bcx-ad)\sqrt{bx^2+a}}{abc^2+a^2d^2+(b^2c^2+abd^2)x^2}\right)+(bc^2d+ad^3)\sqrt{bx^2+a}}{b^2c^5+2\,abc^3d^2+a^2cd^4+(b^2c^4d+2\,abc^2d^3+a^2d^5)x}\right]$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((b*c*d*x + b*c^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2
*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)
*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(b*c^2*d + a*d^3)*sqrt(b*
x^2 + a))/(b^2*c^5 + 2*a*b*c^3*d^2 + a^2*c*d^4 + (b^2*c^4*d + 2*a*b*c^2*d^
3 + a^2*d^5)*x), -((b*c*d*x + b*c^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c
^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 +
a*b*d^2)*x^2)) + (b*c^2*d + a*d^3)*sqrt(b*x^2 + a))/(b^2*c^5 + 2*a*b*c^3*d
^2 + a^2*c*d^4 + (b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5)*x)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bx^2+a}}{bc^2x+ad^2x+\frac{bc^3}{d}+acd}+\frac{bc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)/(b*c^2*x + a*d^2*x + b*c^3/d + a*c*d) + b*c*arcsinh(b*c*x
/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)
^(3/2)*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.54

$$\int \frac{1}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a\,d^2+b\,c^2}\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)b\,c^2+\sqrt{a\,d^2+b\,c^2}\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad\right.}{a^2d^5x+2ab\,c^2d^3x+b^2c}$$

input
```
int(1/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*b*c**2 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*b*c*d*x - sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c**2
- sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c*d*x - sqrt(a + b*x**2)*a*d**3 - s
qrt(a + b*x**2)*b*c**2*d)/(a**2*c*d**4 + a**2*d**5*x + 2*a*b*c**3*d**2 + 2
*a*b*c**2*d**3*x + b**2*c**5 + b**2*c**4*d*x)
```

3.275 $\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx = -\frac{d\sqrt{a+bx^2}}{2\left(bc^2+ad^2\right)(c+dx)^2} - \frac{3bcd\sqrt{a+bx^2}}{2\left(bc^2+ad^2\right)^2(c+dx)} - \frac{b(2bc^2-ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2\left(bc^2+ad^2\right)^{5/2}}$$

output
```
-1/2*d*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)/(d*x+c)^2-3/2*b*c*d*(b*x^2+a)^(1/2)/(
a*d^2+b*c^2)^2/(d*x+c)-1/2*b*(-a*d^2+2*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+
b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx = -\frac{d\sqrt{a+bx^2}(ad^2+bc(4c+3dx))}{2\left(bc^2+ad^2\right)^2(c+dx)^2} - \frac{b(2bc^2-ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[1/((c + d*x)^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(d*Sqrt[a + b*x^2]*(a*d^2 + b*c*(4*c + 3*d*x)))/((b*c^2 + a*d^2)^2*(c
 + d*x)^2) - (b*(2*b*c^2 - a*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b
*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {498, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}(c+dx)^3}dx$$

$$\Big\downarrow \text{ 498}$$

$$-\frac{b\int -\frac{2c-dx}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\int \frac{2c-dx}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 679}$$

$$\frac{b\left(\frac{\left(2bc^2-ad^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{3cd\sqrt{a+bx^2}}{(c+dx)\left(ad^2+bc^2\right)}\right)}{2\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{b\left(-\frac{\left(2bc^2-ad^2\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{3cd\sqrt{a+bx^2}}{(c+dx)\left(ad^2+bc^2\right)}\right)}{2\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b\left(-\frac{(2bc^2-ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}-\frac{3cd\sqrt{a+bx^2}}{(c+dx)(ad^2+bc^2)}\right)}{2\,(ad^2+bc^2)}-\frac{d\sqrt{a+bx^2}}{2(c+dx)^2\,(ad^2+bc^2)}$$

input
```
Int[1/((c + d*x)^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(d*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) + (b*((-3*c*d*Sqrt[
a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) - ((2*b*c^2 - a*d^2)*ArcTanh[(a*d
- b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)))/(
2*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(129) = 258$.

Time = 0.35 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.06

method	result
default	$\frac{-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}}}{2\left(a d^2+b c^2\right)\left(x+\frac{c}{d}\right)^2}+\frac{3bcd\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}}}{\left(a d^2+b c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{\frac{2a d^2+2b c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a d^2+b c^2}{d^2}}}{x+\frac{c}{d}}\right.}{\left(a d^2+b c^2\right)\sqrt{\frac{a d}{}}}\right.}{2\left(a d^2+b c^2\right)}}{d^3}$

input
```
int(1/(d*x+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/
((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^
2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+
b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(130) = 260$.

Time = 0.26 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.92

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{(2\,b^2c^4 - abc^2d^2 + (2\,b^2c^2d^2 - abd^4)x^2 + 2\,(2\,b^2c^3d - abcd^3)x)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx - abc^2 - 2\,a^2d^2 - (2\,b^2c^2}{d}\right.}{4\,(b^3c^8 + 3\,ab^2c^6d^2 + 3\,a^2bc^4d^4 + a^3c^2d^6 + (b^3c^6d^2 + 3\,ab^2c^4d^4 + 3\,a}\right.$$
$$-\frac{(2\,b^2c^4 - abc^2d^2 + (2\,b^2c^2d^2 - abd^4)x^2 + 2\,(2\,b^2c^3d - abcd^3)x)\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{-bc^2-ad^2}(bcx-ad)\sqrt{}}{abc^2+a^2d^2+(b^2c^2+ab}\right.}{2\,(b^3c^8 + 3\,ab^2c^6d^2 + 3\,a^2bc^4d^4 + a^3c^2d^6 + (b^3c^6d^2 + 3\,ab^2c^4d^4 + 3\,a^2bc^2d^6 + a^3d^8)}$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*b^2*c^4 - a*b*c^2*d^2 + (2*b^2*c^2*d^2 - a*b*d^4)*x^2 + 2*(2*b^2
*c^3*d - a*b*c*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*
a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*
sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(4*b^2*c^4*d + 5*a*b*c^2*d
^3 + a^2*d^5 + 3*(b^2*c^3*d^2 + a*b*c*d^4)*x)*sqrt(b*x^2 + a))/(b^3*c^8 +
3*a*b^2*c^6*d^2 + 3*a^2*b*c^4*d^4 + a^3*c^2*d^6 + (b^3*c^6*d^2 + 3*a*b^2*c
^4*d^4 + 3*a^2*b*c^2*d^6 + a^3*d^8)*x^2 + 2*(b^3*c^7*d + 3*a*b^2*c^5*d^3 +
 3*a^2*b*c^3*d^5 + a^3*c*d^7)*x), -1/2*((2*b^2*c^4 - a*b*c^2*d^2 + (2*b^2*
c^2*d^2 - a*b*d^4)*x^2 + 2*(2*b^2*c^3*d - a*b*c*d^3)*x)*sqrt(-b*c^2 - a*d^
2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^
2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (4*b^2*c^4*d + 5*a*b*c^2*d^3 + a^2*d^5
 + 3*(b^2*c^3*d^2 + a*b*c*d^4)*x)*sqrt(b*x^2 + a))/(b^3*c^8 + 3*a*b^2*c^6*
d^2 + 3*a^2*b*c^4*d^4 + a^3*c^2*d^6 + (b^3*c^6*d^2 + 3*a*b^2*c^4*d^4 + 3*a
^2*b*c^2*d^6 + a^3*d^8)*x^2 + 2*(b^3*c^7*d + 3*a*b^2*c^5*d^3 + 3*a^2*b*c^3
*d^5 + a^3*c*d^7)*x)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.68

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx = -\frac{3\sqrt{bx^2+a}\,abc}{2\left(b^2c^4x+2\,abc^2d^2x+a^2d^4x+\frac{b^2c^5}{d}+2\,abc^3d+a^2cd^3\right)} - \frac{\sqrt{bx^2+a}}{2\left(bc^2dx^2+ad^3x^2+2\,bc^3x+2\,acd^2x+\frac{bc^4}{d}+ac^2d\right)} + \frac{3\,b^2c^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^5} - \frac{b\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3}$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*sqrt(b*x^2 + a)*b*c/(b^2*c^4*x + 2*a*b*c^2*d^2*x + a^2*d^4*x + b^2*c^
5/d + 2*a*b*c^3*d + a^2*c*d^3) - 1/2*sqrt(b*x^2 + a)/(b*c^2*d*x^2 + a*d^3*
x^2 + 2*b*c^3*x + 2*a*c*d^2*x + b*c^4/d + a*c^2*d) + 3/2*b^2*c^2*arcsinh(b
*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/
d^2)^(5/2)*d^5) - 1/2*b*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt
(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(130) = 260$.

Time = 0.13 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.42

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx = -b\left(\frac{(2\,bc^2-ad^2)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(b^2c^4+2\,abc^2d^2+a^2d^4)\sqrt{-bc^2-ad^2}} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3bc^2d-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3ad}{(b}\right.$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-b*((2*b*c^2 - a*d^2)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)
/sqrt(-b*c^2 - a*d^2))/((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*sqrt(-b*c^2 -
a*d^2)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^3*b*c^2*d - (sqrt(b)*x - sqrt(b
*x^2 + a))^3*a*d^3 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c^3 - 3*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c*d^2 - 10*(sqrt(b)*x - sqrt(b*x^2
+ a))*a*b*c^2*d - (sqrt(b)*x - sqrt(b*x^2 + a))*a^2*d^3 + 3*a^2*sqrt(b)*c*
d^2)/((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^2
*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*c - a*d)^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(c+d\,x)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 729, normalized size of antiderivative = 5.03

$$\int \frac{1}{(c+dx)^3\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)ab\,c^2d^2+2\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b}\right.}{}$$

input
```
int(1/(d*x+c)^3/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a*b*c**2*d**2 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**3*x + sqrt(a*d**2 + b*c**2)*
log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*d**4*x**2
 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*b**2*c**4 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d*x - 2*sqrt(a*d**2 + b*c**
2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**2*
d**2*x**2 - sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**2 - 2*sqrt(a*d*
*2 + b*c**2)*log(c + d*x)*a*b*c*d**3*x - sqrt(a*d**2 + b*c**2)*log(c + d*x
)*a*b*d**4*x**2 + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4 + 4*sqrt(
a*d**2 + b*c**2)*log(c + d*x)*b**2*c**3*d*x + 2*sqrt(a*d**2 + b*c**2)*log(
c + d*x)*b**2*c**2*d**2*x**2 - sqrt(a + b*x**2)*a**2*d**5 - 5*sqrt(a + b*x
**2)*a*b*c**2*d**3 - 3*sqrt(a + b*x**2)*a*b*c*d**4*x - 4*sqrt(a + b*x**2)*
b**2*c**4*d - 3*sqrt(a + b*x**2)*b**2*c**3*d**2*x)/(2*(a**3*c**2*d**6 + 2*
a**3*c*d**7*x + a**3*d**8*x**2 + 3*a**2*b*c**4*d**4 + 6*a**2*b*c**3*d**5*x
 + 3*a**2*b*c**2*d**6*x**2 + 3*a*b**2*c**6*d**2 + 6*a*b**2*c**5*d**3*x + 3
*a*b**2*c**4*d**4*x**2 + b**3*c**8 + 2*b**3*c**7*d*x + b**3*c**6*d**2*x**2
))
```

3.276 $\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx = -\frac{d\sqrt{a+bx^2}}{3(bc^2+ad^2)(c+dx)^3} - \frac{5bcd\sqrt{a+bx^2}}{6(bc^2+ad^2)^2(c+dx)^2} - \frac{bd(11bc^2-4ad^2)\sqrt{a+bx^2}}{6(bc^2+ad^2)^3(c+dx)} - \frac{b^2c(2bc^2-3ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2(bc^2+ad^2)^{7/2}}$$

output
```
-1/3*d*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)/(d*x+c)^3-5/6*b*c*d*(b*x^2+a)^(1/2)/(
a*d^2+b*c^2)^2/(d*x+c)^2-1/6*b*d*(-4*a*d^2+11*b*c^2)*(b*x^2+a)^(1/2)/(a*d^
2+b*c^2)^3/(d*x+c)-1/2*b^2*c*(-3*a*d^2+2*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^
2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.06

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx$$
$$= \frac{-d\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\left(2(bc^2+ad^2)^2+5bc(bc^2+ad^2)(c+dx)+b(11bc^2-4ad^2)(c+dx)^2\right)+3b^2c(2\ldots}{6(bc^2+ad^2)^7\ldots}$$

input
```
Integrate[1/((c + d*x)^4*Sqrt[a + b*x^2]),x]
```

output
```
(-(d*Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]*(2*(b*c^2 + a*d^2)^2 + 5*b*c*(b*c
^2 + a*d^2)*(c + d*x) + b*(11*b*c^2 - 4*a*d^2)*(c + d*x)^2)) + 3*b^2*c*(2*
b*c^2 - 3*a*d^2)*(c + d*x)^3*Log[c + d*x] - 3*b^2*c*(2*b*c^2 - 3*a*d^2)*(c
 + d*x)^3*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/(6*(b*c^
2 + a*d^2)^(7/2)*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {498, 25, 688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}(c+dx)^4}dx$$

$$\Big\downarrow \; 498$$

$$-\frac{b\int -\frac{3c-2dx}{(c+dx)^3\sqrt{bx^2+a}}dx}{3(ad^2+bc^2)}-\frac{d\sqrt{a+bx^2}}{3(c+dx)^3(ad^2+bc^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{b\int \frac{3c-2dx}{(c+dx)^3\sqrt{bx^2+a}}dx}{3(ad^2+bc^2)}-\frac{d\sqrt{a+bx^2}}{3(c+dx)^3(ad^2+bc^2)}$$

$$\Big\downarrow \text{688}$$

$$\frac{b\left(-\frac{\int -\frac{2\left(3bc^2-2ad^2\right)-5bcdx}{(c+dx)^2\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{5cd\sqrt{a+bx^2}}{2(c+dx)^2(ad^2+bc^2)}\right)}{3\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{2\left(3bc^2-2ad^2\right)-5bcdx}{(c+dx)^2\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{5cd\sqrt{a+bx^2}}{2(c+dx)^2(ad^2+bc^2)}\right)}{3\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{679}$$

$$\frac{b\left(\frac{\frac{3bc\left(2bc^2-3ad^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{d\sqrt{a+bx^2}\left(11bc^2-4ad^2\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2(ad^2+bc^2)}-\frac{5cd\sqrt{a+bx^2}}{2(c+dx)^2(ad^2+bc^2)}\right)}{3\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{b\left(\frac{-\frac{3bc\left(2bc^2-3ad^2\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{d\sqrt{a+bx^2}\left(11bc^2-4ad^2\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2(ad^2+bc^2)}-\frac{5cd\sqrt{a+bx^2}}{2(c+dx)^2(ad^2+bc^2)}\right)}{3\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{-\frac{3bc\left(2bc^2-3ad^2\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{d\sqrt{a+bx^2}\left(11bc^2-4ad^2\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2(ad^2+bc^2)}-\frac{5cd\sqrt{a+bx^2}}{2(c+dx)^2(ad^2+bc^2)}\right)}{3\left(ad^2+bc^2\right)}-\frac{d\sqrt{a+bx^2}}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

input
```
Int[1/((c + d*x)^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(d*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^3) + (b*((-5*c*d*Sqrt[
a + b*x^2])/(2*(b*c^2 + a*d^2)*(c + d*x)^2) + (-((d*(11*b*c^2 - 4*a*d^2)*S
qrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x))) - (3*b*c*(2*b*c^2 - 3*a*d^2)*
ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d
^2)^(3/2))/(2*(b*c^2 + a*d^2))))/(3*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(178) = 356$.

Time = 0.36 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.85

method	result
default	$\frac{-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{3\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)^3}+\frac{5bcd\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{2\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)^2}+\frac{3bcd\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)} -\right.}{}\right.}{}}{}$

input
```
int(1/(d*x+c)^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2)+5/3*b*c*d/(a*d^2+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d
)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2
+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d
^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/(
(a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2
+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/
(x+c/d)))-2/3*b/(a*d^2+b*c^2)*d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*
c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d
^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(179) = 358$.

Time = 0.44 (sec) , antiderivative size = 1139, normalized size of antiderivative = 5.75

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*b^3*c^6 - 3*a*b^2*c^4*d^2 + (2*b^3*c^3*d^3 - 3*a*b^2*c*d^5)*x
^3 + 3*(2*b^3*c^4*d^2 - 3*a*b^2*c^2*d^4)*x^2 + 3*(2*b^3*c^5*d - 3*a*b^2*c^
3*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*
b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 +
a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(18*b^3*c^6*d + 23*a*b^2*c^4*d^3 + 7*a^
2*b*c^2*d^5 + 2*a^3*d^7 + (11*b^3*c^4*d^3 + 7*a*b^2*c^2*d^5 - 4*a^2*b*d^7)
*x^2 + 3*(9*b^3*c^5*d^2 + 8*a*b^2*c^3*d^4 - a^2*b*c*d^6)*x)*sqrt(b*x^2 + a
))/(b^4*c^11 + 4*a*b^3*c^9*d^2 + 6*a^2*b^2*c^7*d^4 + 4*a^3*b*c^5*d^6 + a^4
*c^3*d^8 + (b^4*c^8*d^3 + 4*a*b^3*c^6*d^5 + 6*a^2*b^2*c^4*d^7 + 4*a^3*b*c^
2*d^9 + a^4*d^11)*x^3 + 3*(b^4*c^9*d^2 + 4*a*b^3*c^7*d^4 + 6*a^2*b^2*c^5*d
^6 + 4*a^3*b*c^3*d^8 + a^4*c*d^10)*x^2 + 3*(b^4*c^10*d + 4*a*b^3*c^8*d^3 +
 6*a^2*b^2*c^6*d^5 + 4*a^3*b*c^4*d^7 + a^4*c^2*d^9)*x), -1/6*(3*(2*b^3*c^6
 - 3*a*b^2*c^4*d^2 + (2*b^3*c^3*d^3 - 3*a*b^2*c*d^5)*x^3 + 3*(2*b^3*c^4*d^
2 - 3*a*b^2*c^2*d^4)*x^2 + 3*(2*b^3*c^5*d - 3*a*b^2*c^3*d^3)*x)*sqrt(-b*c^
2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*
c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (18*b^3*c^6*d + 23*a*b^2*c^4*d
^3 + 7*a^2*b*c^2*d^5 + 2*a^3*d^7 + (11*b^3*c^4*d^3 + 7*a*b^2*c^2*d^5 - 4*a
^2*b*d^7)*x^2 + 3*(9*b^3*c^5*d^2 + 8*a*b^2*c^3*d^4 - a^2*b*c*d^6)*x)*sqrt(
b*x^2 + a))/(b^4*c^11 + 4*a*b^3*c^9*d^2 + 6*a^2*b^2*c^7*d^4 + 4*a^3*b*c^5*
d^6 + a^4*c^3*d^8 + (b^4*c^8*d^3 + 4*a*b^3*c^6*d^5 + 6*a^2*b^2*c^4*d^7 ...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx)^4}\,dx$$

input
```
integrate(1/(d*x+c)**4/(b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(179) = 358$.

Time = 0.06 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.42

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx$$
$$= -\frac{5\sqrt{bx^2+a}b^2c^2}{2\left(b^3c^6x+3\,ab^2c^4d^2x+3\,a^2bc^2d^4x+a^3d^6x+\frac{b^3c^7}{d}+3\,ab^2c^5d+3\,a^2bc^3d^3+a^3cd^5\right)}$$
$$-\frac{5\sqrt{bx^2+a}bc}{6\left(b^2c^4dx^2+2\,abc^2d^3x^2+a^2d^5x^2+2\,b^2c^5x+4\,abc^3d^2x+2\,a^2cd^4x+\frac{b^2c^6}{d}+2\,abc^4d+a^2c^2d^3\right)}$$
$$+\frac{2\sqrt{bx^2+a}b}{3\left(b^2c^4x+2\,abc^2d^2x+a^2d^4x+\frac{b^2c^5}{d}+2\,abc^3d+a^2cd^3\right)}$$
$$-\frac{\sqrt{bx^2+a}}{3\left(bc^2d^2x^3+ad^4x^3+3\,bc^3dx^2+3\,acd^3x^2+3\,bc^4x+3\,ac^2d^2x+\frac{bc^5}{d}+ac^3d\right)}$$
$$+\frac{5\,b^3c^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{7}{2}}d^7}-\frac{3\,b^2c\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^5}$$

input
```
integrate(1/(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-5/2*sqrt(b*x^2 + a)*b^2*c^2/(b^3*c^6*x + 3*a*b^2*c^4*d^2*x + 3*a^2*b*c^2*
d^4*x + a^3*d^6*x + b^3*c^7/d + 3*a*b^2*c^5*d + 3*a^2*b*c^3*d^3 + a^3*c*d^
5) - 5/6*sqrt(b*x^2 + a)*b*c/(b^2*c^4*d*x^2 + 2*a*b*c^2*d^3*x^2 + a^2*d^5*
x^2 + 2*b^2*c^5*x + 4*a*b*c^3*d^2*x + 2*a^2*c*d^4*x + b^2*c^6/d + 2*a*b*c^
4*d + a^2*c^2*d^3) + 2/3*sqrt(b*x^2 + a)*b/(b^2*c^4*x + 2*a*b*c^2*d^2*x +
a^2*d^4*x + b^2*c^5/d + 2*a*b*c^3*d + a^2*c*d^3) - 1/3*sqrt(b*x^2 + a)/(b*
c^2*d^2*x^3 + a*d^4*x^3 + 3*b*c^3*d*x^2 + 3*a*c*d^3*x^2 + 3*b*c^4*x + 3*a*
c^2*d^2*x + b*c^5/d + a*c^3*d) + 5/2*b^3*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(
d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(7/2)*d^7) - 3/
2*b^2*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x +
c)))/((a + b*c^2/d^2)^(5/2)*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(179) = 358$.

Time = 0.14 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.99

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{3}b^{\frac{3}{2}}\left(\frac{3\left(2\,b^{\frac{3}{2}}c^3-3\,a\sqrt{b}cd^2\right)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\left(b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6\right)\sqrt{-bc^2-ad^2}} - \frac{6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5b^{\frac{3}{2}}c^3d^2-9\left(\sqrt{b}x-\right.}{}\right.$$

input
```
integrate(1/(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*b^(3/2)*(3*(2*b^(3/2)*c^3 - 3*a*sqrt(b)*c*d^2)*arctan(-((sqrt(b)*x - s
qrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^3*c^6 + 3*a*b^2*c
^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*sqrt(-b*c^2 - a*d^2)) - (6*(sqrt(b)*x
- sqrt(b*x^2 + a))^5*b^(3/2)*c^3*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^5*a
*sqrt(b)*c*d^4 + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^2*c^4*d - 45*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a*b*c^2*d^3 + 44*(sqrt(b)*x - sqrt(b*x^2 + a))^3*
b^(5/2)*c^5 - 82*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a*b^(3/2)*c^3*d^2 + 24*(s
qrt(b)*x - sqrt(b*x^2 + a))^3*a^2*sqrt(b)*c*d^4 - 102*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*b^2*c^4*d + 36*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b*c^2*d^3
 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*d^5 + 60*(sqrt(b)*x - sqrt(b*x^2
 + a))*a^2*b^(3/2)*c^3*d^2 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))*a^3*sqrt(b)*
c*d^4 - 11*a^3*b*c^2*d^3 + 4*a^4*d^5)/((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*
b*c^2*d^4 + a^3*d^6)*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x - s
qrt(b*x^2 + a))*sqrt(b)*c - a*d)^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^4}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)^4),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1162, normalized size of antiderivative = 5.87

$$\int \frac{1}{(c+dx)^4\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^4/(b*x^2+a)^(1/2),x)
```

output
```
(9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b**2*c**4*d**2 + 27*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**3*x + 27*sqrt(a*
d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a*b**2*c**2*d**4*x**2 + 9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**5*x**3 - 6*sqrt(a*d**2 +
b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*
c**6 - 18*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*b**3*c**5*d*x - 18*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
+ b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**4*d**2*x**2 - 6*sqr
t(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*b**3*c**3*d**3*x**3 - 9*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*
*4*d**2 - 27*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d**3*x - 27*sq
rt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**4*x**2 - 9*sqrt(a*d**2 + b
*c**2)*log(c + d*x)*a*b**2*c*d**5*x**3 + 6*sqrt(a*d**2 + b*c**2)*log(c + d
*x)*b**3*c**6 + 18*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**5*d*x + 18*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**4*d**2*x**2 + 6*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*b**3*c**3*d**3*x**3 - 2*sqrt(a + b*x**2)*a**3*d**7 - 7*
sqrt(a + b*x**2)*a**2*b*c**2*d**5 + 3*sqrt(a + b*x**2)*a**2*b*c*d**6*x + 4
*sqrt(a + b*x**2)*a**2*b*d**7*x**2 - 23*sqrt(a + b*x**2)*a*b**2*c**4*d*...
```

3.277 $\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = -\frac{4acd(bc^2-ad^2)-(b^2c^4-6abc^2d^2+a^2d^4)\,x}{ab^2\sqrt{a+bx^2}} + \frac{4cd^3\sqrt{a+bx^2}}{b^2} + \frac{d^4x\sqrt{a+bx^2}}{2b^2} + \frac{3d^2(4bc^2-ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
-(4*a*c*d*(-a*d^2+b*c^2)-(a^2*d^4-6*a*b*c^2*d^2+b^2*c^4)*x)/a/b^2/(b*x^2+a
)^(1/2)+4*c*d^3*(b*x^2+a)^(1/2)/b^2+1/2*d^4*x*(b*x^2+a)^(1/2)/b^2+3/2*d^2*
(-a*d^2+4*b*c^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \frac{2b^2c^4x+a^2d^3(16c+3dx)+abd(-8c^3-12c^2dx+8cd^2x^2+d^3x^3)}{2ab^2\sqrt{a+bx^2}} - \frac{3(4bc^2d^2-ad^4)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[(c + d*x)^4/(a + b*x^2)^(3/2),x]
```

output
```
(2*b^2*c^4*x + a^2*d^3*(16*c + 3*d*x) + a*b*d*(-8*c^3 - 12*c^2*d*x + 8*c*d
^2*x^2 + d^3*x^3))/(2*a*b^2*Sqrt[a + b*x^2]) - (3*(4*b*c^2*d^2 - a*d^4)*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {495, 27, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{3d(ad-bcx)(c+dx)^2}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^3(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3d\int \frac{(ad-bcx)(c+dx)^2}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^3(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{687}$$

$$\frac{3d\left(\frac{\int \frac{b(c+dx)\left(5acd-\left(2bc^2-3ad^2\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}-\frac{1}{3}c\sqrt{a+bx^2}(c+dx)^2\right)}{ab}-\frac{(c+dx)^3(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3d\left(\frac{1}{3}\int \frac{(c+dx)(5acd-(2bc^2-3ad^2)x)}{\sqrt{bx^2+a}}dx-\frac{1}{3}c\sqrt{a+bx^2}(c+dx)^2\right)}{ab}-\frac{(c+dx)^3(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{676}$$

$$\frac{3d\left(\frac{1}{3}\left(\frac{3ad(4bc^2-ad^2)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{2c\sqrt{a+bx^2}(bc^2-4ad^2)}{b}-\frac{dx\sqrt{a+bx^2}(2bc^2-3ad^2)}{2b}\right)-\frac{1}{3}c\sqrt{a+bx^2}(c+dx)^2\right)}{ab}-\frac{(c+dx)^3(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{3d\left(\frac{1}{3}\left(\frac{3ad(4bc^2-ad^2)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{2c\sqrt{a+bx^2}(bc^2-4ad^2)}{b}-\frac{dx\sqrt{a+bx^2}(2bc^2-3ad^2)}{2b}\right)-\frac{1}{3}c\sqrt{a+bx^2}(c+dx)^2\right)}{ab}-\frac{(c+dx)^3(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3d\left(\frac{1}{3}\left(\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc^2-ad^2)}{2b^{3/2}}-\frac{2c\sqrt{a+bx^2}(bc^2-4ad^2)}{b}-\frac{dx\sqrt{a+bx^2}(2bc^2-3ad^2)}{2b}\right)-\frac{1}{3}c\sqrt{a+bx^2}(c+dx)^2\right)}{ab}-\frac{(c+dx)^3(ad-bcx)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x)^4/(a + b*x^2)^(3/2),x]
```

output
```
-(((a*d - b*c*x)*(c + d*x)^3)/(a*b*Sqrt[a + b*x^2])) + (3*d*(-1/3*(c*(c +
d*x)^2*Sqrt[a + b*x^2]) + ((-2*c*(b*c^2 - 4*a*d^2)*Sqrt[a + b*x^2])/b - (d
*(2*b*c^2 - 3*a*d^2)*x*Sqrt[a + b*x^2])/(2*b) + (3*a*d*(4*b*c^2 - a*d^2)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/3))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

method	result
risch	$\frac{d^{3}(dx+8c)\sqrt{b\,x^{2}+a}}{2b^{2}}-\frac{\frac{a\,d^{4}x}{\sqrt{b\,x^{2}+a}}+3b\,d^{2}\left(a\,d^{2}-4b\,c^{2}\right)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{8dc\left(a\,d^{2}-b\,c^{2}\right)}{\sqrt{b\,x^{2}+a}}-\frac{2b^{2}c^{4}x}{a\sqrt{b\,x^{2}+a}}}{2b^{2}}$
default	$\frac{c^{4}x}{\sqrt{b\,x^{2}+a}\,a}+d^{4}\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)-\frac{4d\,c^{3}}{b\sqrt{b\,x^{2}+a}}+6c^{2}d^{2}\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln(\sqrt{}}{}\right.$

input
```
int((d*x+c)^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d^3*(d*x+8*c)*(b*x^2+a)^(1/2)/b^2-1/2/b^2*(a*d^4*x/(b*x^2+a)^(1/2)+3*b
*d^2*(a*d^2-4*b*c^2)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2)))-8*d*c*(a*d^2-b*c^2)/(b*x^2+a)^(1/2)-2*b^2*c^4*x/a/(b*x^2+a)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.42

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(4\,a^2bc^2d^2-a^3d^4+(4\,ab^2c^2d^2-a^2bd^4)x^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right.}{4}\right.$$

$$-\frac{3\,(4\,a^2bc^2d^2-a^3d^4+(4\,ab^2c^2d^2-a^2bd^4)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(ab^2d^4x^3+8\,ab^2cd^3x^2-8\,ab^2c^3d+}{2\,(ab^4x^2+a^2b^3)}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(3*(4*a^2*b*c^2*d^2 - a^3*d^4 + (4*a*b^2*c^2*d^2 - a^2*b*d^4)*x^2)*s
qrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(a*b^2*d^4*x^3
+ 8*a*b^2*c*d^3*x^2 - 8*a*b^2*c^3*d + 16*a^2*b*c*d^3 + (2*b^3*c^4 - 12*a*b
^2*c^2*d^2 + 3*a^2*b*d^4)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), -1/2*
(3*(4*a^2*b*c^2*d^2 - a^3*d^4 + (4*a*b^2*c^2*d^2 - a^2*b*d^4)*x^2)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b^2*d^4*x^3 + 8*a*b^2*c*d^3*x^2
- 8*a*b^2*c^3*d + 16*a^2*b*c*d^3 + (2*b^3*c^4 - 12*a*b^2*c^2*d^2 + 3*a^2*b
*d^4)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^4}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**4/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**4/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx &= \frac{d^4x^3}{2\sqrt{bx^2+a}b} + \frac{4\,cd^3x^2}{\sqrt{bx^2+a}b} + \frac{c^4x}{\sqrt{bx^2+a}a} \\ &\quad - \frac{6\,c^2d^2x}{\sqrt{bx^2+a}b} + \frac{3\,ad^4x}{2\sqrt{bx^2+a}b^2} + \frac{6\,c^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} \\ &\quad - \frac{3\,ad^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} - \frac{4\,c^3d}{\sqrt{bx^2+a}b} + \frac{8\,acd^3}{\sqrt{bx^2+a}b^2}\end{aligned}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*d^4*x^3/(sqrt(b*x^2 + a)*b) + 4*c*d^3*x^2/(sqrt(b*x^2 + a)*b) + c^4*x/
(sqrt(b*x^2 + a)*a) - 6*c^2*d^2*x/(sqrt(b*x^2 + a)*b) + 3/2*a*d^4*x/(sqrt(
b*x^2 + a)*b^2) + 6*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 3/2*a*d^4*arc
sinh(b*x/sqrt(a*b))/b^(5/2) - 4*c^3*d/(sqrt(b*x^2 + a)*b) + 8*a*c*d^3/(sqr
t(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{d^4x}{b}+\frac{8\,cd^3}{b}\right)x+\frac{2\,b^4c^4-12\,ab^3c^2d^2+3\,a^2b^2d^4}{ab^4}\right)x-\frac{8\,(ab^3c^3d-2\,a^2b^2cd^3)}{ab^4}}{2\,\sqrt{bx^2+a}} - \frac{3\,(4\,bc^2d^2-ad^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((d^4*x/b + 8*c*d^3/b)*x + (2*b^4*c^4 - 12*a*b^3*c^2*d^2 + 3*a^2*b^2*
d^4)/(a*b^4))*x - 8*(a*b^3*c^3*d - 2*a^2*b^2*c*d^3)/(a*b^4))/sqrt(b*x^2 +
a) - 3/2*(4*b*c^2*d^2 - a*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5
/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{{\left(c+d\,x\right)}^4}{{\left(b\,x^2+a\right)}^{3/2}} \,dx$$

input
```
int((c + d*x)^4/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^4/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^4}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^4/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^4/(b*x^2+a)^(3/2),x)
```

3.278 $\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = -\frac{ad(3bc^2-ad^2)-bc(bc^2-3ad^2)\,x}{ab^2\sqrt{a+bx^2}} + \frac{d^3\sqrt{a+bx^2}}{b^2} + \frac{3cd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(a*d*(-a*d^2+3*b*c^2)-b*c*(-3*a*d^2+b*c^2)*x)/a/b^2/(b*x^2+a)^(1/2)+d^3*(
b*x^2+a)^(1/2)/b^2+3*c*d^2*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \frac{2a^2d^3+b^2c^3x+abd(-3c^2-3cdx+d^2x^2)}{ab^2\sqrt{a+bx^2}} - \frac{3cd^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(c + d*x)^3/(a + b*x^2)^(3/2),x]
```

output
```
(2*a^2*d^3 + b^2*c^3*x + a*b*d*(-3*c^2 - 3*c*d*x + d^2*x^2))/(a*b^2*Sqrt[a
 + b*x^2]) - (3*c*d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {495, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{2d(ad-bcx)(c+dx)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\int \frac{(ad-bcx)(c+dx)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{676}$$

$$\frac{2d\left(\frac{3}{2}acd\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(bc^2-ad^2)}{b}-\frac{1}{2}cdx\sqrt{a+bx^2}\right)}{ab}-\frac{(c+dx)^2(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{2d\left(\frac{3}{2}acd\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(bc^2-ad^2)}{b}-\frac{1}{2}cdx\sqrt{a+bx^2}\right)}{ab}-\frac{(c+dx)^2(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2d\left(\frac{3acd\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{\sqrt{a+bx^2}(bc^2-ad^2)}{b}-\frac{1}{2}cdx\sqrt{a+bx^2}\right)}{ab}-\frac{(c+dx)^2(ad-bcx)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x)^3/(a + b*x^2)^(3/2),x]
```

output
```
-(((a*d - b*c*x)*(c + d*x)^2)/(a*b*Sqrt[a + b*x^2])) + (2*d*(-(((b*c^2 - a
*d^2)*Sqrt[a + b*x^2])/b) - (c*d*x*Sqrt[a + b*x^2])/2 + (3*a*c*d*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{d^{3}\sqrt{b\,x^{2}+a}}{b^{2}}+\frac{\frac{b\,c^{3}x}{a\sqrt{b\,x^{2}+a}}+\frac{d\left(a\,d^{2}-3b\,c^{2}\right)}{b\sqrt{b\,x^{2}+a}}+3bc\,d^{2}\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{b}$	110
default	$\frac{c^{3}x}{\sqrt{b\,x^{2}+a}\,a}+d^{3}\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)+3c\,d^{2}\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{3c^{2}d}{b\sqrt{b\,x^{2}+a}}$	115

input
```
int((d*x+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d^3*(b*x^2+a)^(1/2)/b^2+1/b*(b*c^3*x/a/(b*x^2+a)^(1/2)+d*(a*d^2-3*b*c^2)/b
/(b*x^2+a)^(1/2)+3*b*c*d^2*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b
*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.41

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \left[\frac{3\left(abcd^2x^2+a^2cd^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(abd^3x^2-3\,abc^2d+\right.}{2\left(ab^3x^2+a^2b^2\right)}\right.$$

$$\left.-\frac{3\left(abcd^2x^2+a^2cd^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(abd^3x^2-3\,abc^2d+2\,a^2d^3+\left(b^2c^3-3\,abcd^2\right)x\right)\sqrt{bx^2+a}}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*c*d^2*x^2 + a^2*c*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 2*(a*b*d^3*x^2 - 3*a*b*c^2*d + 2*a^2*d^3 + (b^2*c^3 -
3*a*b*c*d^2)*x)*sqrt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2), -(3*(a*b*c*d^2*x^2
 + a^2*c*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b*d^3*x^2 -
 3*a*b*c^2*d + 2*a^2*d^3 + (b^2*c^3 - 3*a*b*c*d^2)*x)*sqrt(b*x^2 + a))/(a*
b^3*x^2 + a^2*b^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^3}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**3/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \frac{d^3x^2}{\sqrt{bx^2+a}b} + \frac{c^3x}{\sqrt{bx^2+a}a} - \frac{3\,cd^2x}{\sqrt{bx^2+a}b} + \frac{3\,cd^2\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{3\,c^2d}{\sqrt{bx^2+a}b} + \frac{2\,ad^3}{\sqrt{bx^2+a}b^2}$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
d^3*x^2/(sqrt(b*x^2 + a)*b) + c^3*x/(sqrt(b*x^2 + a)*a) - 3*c*d^2*x/(sqrt(
b*x^2 + a)*b) + 3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 3*c^2*d/(sqrt(b*x
^2 + a)*b) + 2*a*d^3/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = -\frac{3\,cd^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{\left(\frac{d^3x}{b} + \frac{b^3c^3-3\,ab^2cd^2}{ab^3}\right)x - \frac{3\,ab^2c^2d-2\,a^2bd^3}{ab^3}}{\sqrt{bx^2+a}}$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3*c*d^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + ((d^3*x/b + (b^3
*c^3 - 3*a*b^2*c*d^2)/(a*b^3))*x - (3*a*b^2*c^2*d - 2*a^2*b*d^3)/(a*b^3))/
sqrt(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)^3}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c + d*x)^3/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^3/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b\,x^2+a}\,a^2d^3 - 3\sqrt{b\,x^2+a}\,ab\,c^2d - 3\sqrt{b\,x^2+a}\,abc\,d^2x + \sqrt{b\,x^2+a}\,ab\,d^3x^2 + \sqrt{b\,x}$$

input
```
int((d*x+c)^3/(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**2)*a**2*d**3 - 3*sqrt(a + b*x**2)*a*b*c**2*d - 3*sqrt(a +
 b*x**2)*a*b*c*d**2*x + sqrt(a + b*x**2)*a*b*d**3*x**2 + sqrt(a + b*x**2)*
b**2*c**3*x + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c
*d**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*d**2*x
**2 - 3*sqrt(b)*a**2*c*d**2 + sqrt(b)*a*b*c**3 - 3*sqrt(b)*a*b*c*d**2*x**2
 + sqrt(b)*b**2*c**3*x**2)/(a*b**2*(a + b*x**2))
```

3.279 $\int \frac{(c+dx)^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{(c+dx)^2}{\left(a+bx^2\right)^{3/2}}\,dx=-\frac{2acd-\left(bc^2-ad^2\right)x}{ab\sqrt{a+bx^2}}+\frac{d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(2*a*c*d-(-a*d^2+b*c^2)*x)/a/b/(b*x^2+a)^(1/2)+d^2*arctanh(b^(1/2)*x/(b*x
^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{-2acd+bc^2x-ad^2x}{ab\sqrt{a+bx^2}}-\frac{d^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(c + d*x)^2/(a + b*x^2)^(3/2),x]
```

output
```
(-2*a*c*d + b*c^2*x - a*d^2*x)/(a*b*Sqrt[a + b*x^2]) - (d^2*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {495, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{d(ad-bcx)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{ad-bcx}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{d\left(ad\int \frac{1}{\sqrt{bx^2+a}}dx-c\sqrt{a+bx^2}\right)}{ab}-\frac{(c+dx)(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(ad\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-c\sqrt{a+bx^2}\right)}{ab}-\frac{(c+dx)(ad-bcx)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{ad\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-c\sqrt{a+bx^2}\right)}{ab}-\frac{(c+dx)(ad-bcx)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x)^2/(a + b*x^2)^(3/2),x]
```

output
```
-(((a*d - b*c*x)*(c + d*x))/(a*b*Sqrt[a + b*x^2])) + (d*(-(c*Sqrt[a + b*x^
2]) + (a*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{c^2 x}{\sqrt{b\,x^2+a}\,a}+d^2\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{2cd}{b\sqrt{b\,x^2+a}}$	75

input
```
int((d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c^2/(b*x^2+a)^(1/2)/a*x+d^2*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2)))-2*c*d/b/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.91

$$\int \frac{(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(abd^2x^2+a^2d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,abcd-(b^2c^2-abd^2)x}{2\,(ab^3x^2+a^2b^2)}\right.$$

$$\left.-\frac{(abd^2x^2+a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(2\,abcd-(b^2c^2-abd^2)x)\sqrt{bx^2+a}}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*b*d^2*x^2 + a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(2*a*b*c*d - (b^2*c^2 - a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^
3*x^2 + a^2*b^2), -((a*b*d^2*x^2 + a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) + (2*a*b*c*d - (b^2*c^2 - a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^
3*x^2 + a^2*b^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \frac{c^2x}{\sqrt{bx^2+a}a} - \frac{d^2x}{\sqrt{bx^2+a}b} + \frac{d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{2\,cd}{\sqrt{bx^2+a}b}$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
c^2*x/(sqrt(b*x^2 + a)*a) - d^2*x/(sqrt(b*x^2 + a)*b) + d^2*arcsinh(b*x/sq
rt(a*b))/b^(3/2) - 2*c*d/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = -\frac{d^2 \log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}} - \frac{\frac{2\,cd}{b} - \frac{(b^2c^2 - abd^2)x}{ab^2}}{\sqrt{bx^2+a}}$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-d^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - (2*c*d/b - (b^2*c^2
- a*b*d^2)*x/(a*b^2))/sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \frac{d^2\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}+\frac{c^2\,x}{a\,\sqrt{b\,x^2+a}}-\frac{d^2\,x}{b\,\sqrt{b\,x^2+a}}-\frac{2\,c\,d}{b\,\sqrt{b\,x^2+a}}$$

input
```
int((c + d*x)^2/(a + b*x^2)^(3/2),x)
```

output
```
(d^2*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (c^2*x)/(a*(a + b*x^2)^
(1/2)) - (d^2*x)/(b*(a + b*x^2)^(1/2)) - (2*c*d)/(b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.36

$$\int \frac{(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,abcd-\sqrt{b\,x^2+a}\,ab\,d^2x+\sqrt{b\,x^2+a}\,b^2c^2x+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2d^2}{a\,b^2\,(b\,x^2+}$$

input
```
int((d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a*b*c*d - sqrt(a + b*x**2)*a*b*d**2*x + sqrt(a + b*
x**2)*b**2*c**2*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
*2*d**2 + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**
2 - sqrt(b)*a**2*d**2 + sqrt(b)*a*b*c**2 - sqrt(b)*a*b*d**2*x**2 + sqrt(b)
*b**2*c**2*x**2)/(a*b**2*(a + b*x**2))
```

3.280 $\int \frac{c+dx}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{c+dx}{\left(a+bx^2\right)^{3/2}} \, dx = -\frac{ad-bcx}{ab\sqrt{a+bx^2}}$$

output
```
-(-b*c*x+a*d)/a/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{-ad+bcx}{ab\sqrt{a+bx^2}}$$

input
```
Integrate[(c + d*x)/(a + b*x^2)^(3/2),x]
```

output
```
(-(a*d) + b*c*x)/(a*b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 453}$$

$$-\frac{ad-bcx}{ab\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*d - b*c*x)/(a*b*Sqrt[a + b*x^2]))
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{-cbx+ad}{ab\sqrt{b\,x^{2}+a}}$	27
trager	$-\frac{-cbx+ad}{ab\sqrt{b\,x^{2}+a}}$	27
orering	$-\frac{-cbx+ad}{ab\sqrt{b\,x^{2}+a}}$	27
default	$\frac{cx}{\sqrt{b\,x^{2}+a}\,a}-\frac{d}{b\sqrt{b\,x^{2}+a}}$	32

input
```
int((d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-b*c*x+a*d)/a/b/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{c+dx}{(a+bx^2)^{3/2}}\,dx = \frac{(bcx-ad)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{c+dx}{(a+bx^2)^{3/2}}\,dx = d\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
d*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)
) + c*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{c+dx}{(a+bx^2)^{3/2}}\,dx = \frac{cx}{\sqrt{bx^2+a}a} - \frac{d}{\sqrt{bx^2+a}b}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
c*x/(sqrt(b*x^2 + a)*a) - d/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{c+dx}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{cx}{a} - \frac{d}{b}}{\sqrt{bx^2+a}}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(c*x/a - d/b)/sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 6.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{d}{b} - \frac{c\,x}{a}}{\sqrt{b\,x^2+a}}$$

input
```
int((c + d*x)/(a + b*x^2)^(3/2),x)
```

output
```
-(d/b - (c*x)/a)/(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{c+dx}{(a+bx^2)^{3/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,ad+\sqrt{b\,x^2+a}\,bcx+\sqrt{b}\,ac+\sqrt{b}\,bc\,x^2}{ab\,(b\,x^2+a)}$$

input
```
int((d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a*d + sqrt(a + b*x**2)*b*c*x + sqrt(b)*a*c + sqrt(b)*
b*c*x**2)/(a*b*(a + b*x**2))
```

3.281 $\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\frac{ad+bcx}{a\left(bc^2+ad^2\right)\sqrt{a+bx^2}}-\frac{d^2\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{\left(bc^2+ad^2\right)^{3/2}}$$

output
```
(b*c*x+a*d)/a/(a*d^2+b*c^2)/(b*x^2+a)^(1/2)-d^2*arctanh((-b*c*x+a*d)/(a*d^
2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\frac{ad+bcx}{a\left(bc^2+ad^2\right)\sqrt{a+bx^2}}+\frac{2d^2\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{3/2}}$$

input
```
Integrate[1/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(a*d + b*c*x)/(a*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) + (2*d^2*ArcTan[(Sqrt[b]
*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2
)^(3/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {496, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(ad^2+bc^2)}-\frac{\int -\frac{ad^2}{(c+dx)\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ad^2}{(c+dx)\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{ad+bcx}{a\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{488}$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(ad^2+bc^2)}-\frac{d^2\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(ad^2+bc^2)}-\frac{d^2\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}$$

input
```
Int[1/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(a*d + b*c*x)/(a*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) - (d^2*ArcTanh[(a*d - b*
c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.35

method	result
default	$\frac{\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{2bcd\left(2b\left(x+\frac{c}{d}\right)-\frac{2bc}{d}\right)}{\left(a\,d^2+b\,c^2\right)\left(\frac{4b\left(a\,d^2+b\,c^2\right)}{d^2}-\frac{4b^2c^2}{d^2}\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}-\frac{d^2\ln\left(\frac{2a\,d^2 \cdots}{d}\cdots\right.}{\cdots}}{d}$

input
```
int(1/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*
b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^
2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+
c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2
)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(87) = 174.

Time = 0.14 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.85

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{\left(abd^2x^2+a^2d^2\right)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2-\left(2\,b^2c^2+abd^2\right)x^2-2\,\sqrt{bc^2+ad^2}\,(bc\cdots}{d^2x^2+2\,cdx+c^2}\cdots\right)\cdots}{2\left(a^2b^2c^4+2\,a^3bc^2d^2+a^4d^4+\left(ab^3c^4+\cdots\right.\right.}\cdots\right.$$

$$-\frac{\left(abd^2x^2+a^2d^2\right)\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{-bc^2-ad^2}\,(bcx-ad)\sqrt{bx^2+a}}{abc^2+a^2d^2+\left(b^2c^2+abd^2\right)x^2}\right)-\left(abc^2d+a^2d^3+\left(b^2c^3+abcd^2\right)x\right)\sqrt{bx^2+\cdots}}{a^2b^2c^4+2\,a^3bc^2d^2+a^4d^4+\left(ab^3c^4+2\,a^2b^2c^2d^2+a^3bd^4\right)x^2}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*b*d^2*x^2 + a^2*d^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c
^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x
- a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(a*b*c^2*d + a^2*d^
3 + (b^2*c^3 + a*b*c*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^2*c^4 + 2*a^3*b*c^2*d
^2 + a^4*d^4 + (a*b^3*c^4 + 2*a^2*b^2*c^2*d^2 + a^3*b*d^4)*x^2), -((a*b*d^
2*x^2 + a^2*d^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x -
 a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (a*
b*c^2*d + a^2*d^3 + (b^2*c^3 + a*b*c*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^2*c^4
 + 2*a^3*b*c^2*d^2 + a^4*d^4 + (a*b^3*c^4 + 2*a^2*b^2*c^2*d^2 + a^3*b*d^4)
*x^2)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.31

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\frac{bcx}{\sqrt{bx^2+a}abc^2+\sqrt{bx^2+a}a^2d^2}+\frac{1}{\frac{\sqrt{bx^2+a}bc^2}{d}+\sqrt{bx^2+a}ad}+\frac{\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
b*c*x/(sqrt(b*x^2 + a)*a*b*c^2 + sqrt(b*x^2 + a)*a^2*d^2) + 1/(sqrt(b*x^2
+ a)*b*c^2/d + sqrt(b*x^2 + a)*a*d) + arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c
)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(bc^2+ad^2)\sqrt{-bc^2-ad^2}} + \frac{\frac{(b^2c^3+abcd^2)x}{ab^2c^4+2\,a^2bc^2d^2+a^3d^4}+\frac{abc^2d+a^2d^3}{ab^2c^4+2\,a^2bc^2d^2+a^3d^4}}{\sqrt{bx^2+a}}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-2*d^2*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 -
a*d^2))/((b*c^2 + a*d^2)*sqrt(-b*c^2 - a*d^2)) + ((b^2*c^3 + a*b*c*d^2)*x/
(a*b^2*c^4 + 2*a^2*b*c^2*d^2 + a^3*d^4) + (a*b*c^2*d + a^2*d^3)/(a*b^2*c^4
 + 2*a^2*b*c^2*d^2 + a^3*d^4))/sqrt(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2247, normalized size of antiderivative = 23.90

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*c - 2*sqrt(b)*sqrt(2*sq
rt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*a
tan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2
)*c - a*d**2 - 2*b*c**2))*b*c*x**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2
)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sq
rt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*d**2 - 2*sqrt(2*s
qrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)
*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c*
*2))*a*b*c**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c*
*2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b
*c**2)*c - a*d**2 - 2*b*c**2))*a*b*d**2*x**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**
2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)
/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*b**2*c**2*x*
*2 - sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*s
qrt(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**
2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*a*c - sqrt(b)*sqrt(2*sqr
t(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*lo
g( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt...
```

3.282 $\int \frac{1}{(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \frac{1}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = -\frac{d}{(bc^2+ad^2)(c+dx)\sqrt{a+bx^2}} + \frac{b(3acd+(bc^2-2ad^2)x)}{a(bc^2+ad^2)^2\sqrt{a+bx^2}} - \frac{3bcd^2\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{5/2}}$$

output
```
-d/(a*d^2+b*c^2)/(d*x+c)/(b*x^2+a)^(1/2)+b*(3*a*c*d+(-2*a*d^2+b*c^2)*x)/a/
(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)-3*b*c*d^2*arctanh((-b*c*x+a*d)/(a*d^2+b*c^
2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \frac{-a^2d^3+b^2c^2x(c+dx)+abd(2c^2+cdx-2d^2x^2)}{a(bc^2+ad^2)^2(c+dx)\sqrt{a+bx^2}} - \frac{6bcd^2\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{5/2}}$$

input
```
Integrate[1/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a^2*d^3) + b^2*c^2*x*(c + d*x) + a*b*d*(2*c^2 + c*d*x - 2*d^2*x^2))/(a*
(b*c^2 + a*d^2)^2*(c + d*x)*Sqrt[a + b*x^2]) - (6*b*c*d^2*ArcTan[(Sqrt[b]*
(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)
^(5/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {496, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \; 496$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}-\frac{\int -\frac{d(2ad+bcx)}{(c+dx)^2\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{d(2ad+bcx)}{(c+dx)^2\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{2ad+bcx}{(c+dx)^2\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \; 679$$

$$\frac{d\left(\frac{3abcd\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}(bc^2-2ad^2)}{(c+dx)(ad^2+bc^2)}\right)}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \; 488$$

$$\frac{d\left(\frac{\sqrt{a+bx^2}(bc^2-2ad^2)}{(c+dx)(ad^2+bc^2)}-\frac{3abcd\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\sqrt{a+bx^2}(bc^2-2ad^2)}{(c+dx)(ad^2+bc^2)}-\frac{3abcd\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[1/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(a*d + b*c*x)/(a*(b*c^2 + a*d^2)*(c + d*x)*Sqrt[a + b*x^2]) + (d*(((b*c^2
- 2*a*d^2)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) - (3*a*b*c*d*ArcTa
nh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(
3/2)))/(a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(133) = 266$.

Time = 0.29 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.60

method	result
default	$-\frac{d^2}{\left(a d^2+b c^2\right)\left(x+\frac{c}{d}\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}}}+\frac{3bcd\left(\frac{d^2}{\left(a d^2+b c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}}}+\frac{\cdots}{\left(a d^2+b c^2\right)\left(\frac{4b\left(a d^2+b\cdots\right.}{d^2}\cdots\right.}\cdots\right.}{\cdots}$

input
```
int(1/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-
2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*l
n((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c^2
)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/
d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(134) = 268$.

Time = 0.19 (sec) , antiderivative size = 900, normalized size of antiderivative = 6.29

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b^2*c*d^3*x^3 + a*b^2*c^2*d^2*x^2 + a^2*b*c*d^3*x + a^2*b*c^2*d
^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^
2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d
^2*x^2 + 2*c*d*x + c^2)) + 2*(2*a*b^2*c^4*d + a^2*b*c^2*d^3 - a^3*d^5 + (b
^3*c^4*d - a*b^2*c^2*d^3 - 2*a^2*b*d^5)*x^2 + (b^3*c^5 + 2*a*b^2*c^3*d^2 +
 a^2*b*c*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^3*c^7 + 3*a^3*b^2*c^5*d^2 + 3*a^4
*b*c^3*d^4 + a^5*c*d^6 + (a*b^4*c^6*d + 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^2*
d^5 + a^4*b*d^7)*x^3 + (a*b^4*c^7 + 3*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^3*d^4
+ a^4*b*c*d^6)*x^2 + (a^2*b^3*c^6*d + 3*a^3*b^2*c^4*d^3 + 3*a^4*b*c^2*d^5
+ a^5*d^7)*x), -(3*(a*b^2*c*d^3*x^3 + a*b^2*c^2*d^2*x^2 + a^2*b*c*d^3*x +
a^2*b*c^2*d^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a
*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (2*a*
b^2*c^4*d + a^2*b*c^2*d^3 - a^3*d^5 + (b^3*c^4*d - a*b^2*c^2*d^3 - 2*a^2*b
*d^5)*x^2 + (b^3*c^5 + 2*a*b^2*c^3*d^2 + a^2*b*c*d^4)*x)*sqrt(b*x^2 + a))/
(a^2*b^3*c^7 + 3*a^3*b^2*c^5*d^2 + 3*a^4*b*c^3*d^4 + a^5*c*d^6 + (a*b^4*c^
6*d + 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^2*d^5 + a^4*b*d^7)*x^3 + (a*b^4*c^7
+ 3*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^3*d^4 + a^4*b*c*d^6)*x^2 + (a^2*b^3*c^6*
d + 3*a^3*b^2*c^4*d^3 + 3*a^4*b*c^2*d^5 + a^5*d^7)*x)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(134) = 268$.

Time = 0.06 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.99

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \frac{3\,b^2c^2x}{\sqrt{bx^2+a}ab^2c^4 + 2\sqrt{bx^2+a}a^2bc^2d^2 + \sqrt{bx^2+a}a^3d^4} + \frac{3\,bc}{\frac{\sqrt{bx^2+a}b^2c^4}{d} + 2\sqrt{bx^2+a}abc^2d + \sqrt{bx^2+a}a^2d^3} - \frac{2\,bx}{\sqrt{bx^2+a}abc^2 + \sqrt{bx^2+a}a^2d^2} - \frac{1}{\sqrt{bx^2+a}bc^2x + \sqrt{bx^2+a}ad^2x + \frac{\sqrt{bx^2+a}bc^3}{d} + \sqrt{bx^2+a}acd} + \frac{3\,bc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^3}$$

input

```
integrate(1/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output

```
3*b^2*c^2*x/(sqrt(b*x^2 + a)*a*b^2*c^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^2 +
 sqrt(b*x^2 + a)*a^3*d^4) + 3*b*c/(sqrt(b*x^2 + a)*b^2*c^4/d + 2*sqrt(b*x^
2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) - 2*b*x/(sqrt(b*x^2 + a)*a*b*c
^2 + sqrt(b*x^2 + a)*a^2*d^2) - 1/(sqrt(b*x^2 + a)*b*c^2*x + sqrt(b*x^2 +
a)*a*d^2*x + sqrt(b*x^2 + a)*b*c^3/d + sqrt(b*x^2 + a)*a*c*d) + 3*b*c*arcs
inh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b
*c^2/d^2)^(5/2)*d^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 720, normalized size of antiderivative = 5.03

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \frac{3\sqrt{a\,d^2+b\,c^2}\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)a^2b\,c^2d^2+3\sqrt{a\,d^2+b}$$

input
```
int(1/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output

```
(3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*b*c**2*d**2 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**3*x + 3*sqrt(a*d**2 + b*c*
*2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*
d**2*x**2 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c
**2) - a*d + b*c*x)*a*b**2*c*d**3*x**3 - 3*sqrt(a*d**2 + b*c**2)*log(c + d
*x)*a**2*b*c**2*d**2 - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**3*
x - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**2*x**2 - 3*sqrt(a*
d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**3*x**3 - sqrt(a + b*x**2)*a**3*d**
5 + sqrt(a + b*x**2)*a**2*b*c**2*d**3 + sqrt(a + b*x**2)*a**2*b*c*d**4*x -
 2*sqrt(a + b*x**2)*a**2*b*d**5*x**2 + 2*sqrt(a + b*x**2)*a*b**2*c**4*d +
2*sqrt(a + b*x**2)*a*b**2*c**3*d**2*x - sqrt(a + b*x**2)*a*b**2*c**2*d**3*
x**2 + sqrt(a + b*x**2)*b**3*c**5*x + sqrt(a + b*x**2)*b**3*c**4*d*x**2)/(
a*(a**4*c*d**6 + a**4*d**7*x + 3*a**3*b*c**3*d**4 + 3*a**3*b*c**2*d**5*x +
 a**3*b*c*d**6*x**2 + a**3*b*d**7*x**3 + 3*a**2*b**2*c**5*d**2 + 3*a**2*b*
*2*c**4*d**3*x + 3*a**2*b**2*c**3*d**4*x**2 + 3*a**2*b**2*c**2*d**5*x**3 +
 a*b**3*c**7 + a*b**3*c**6*d*x + 3*a*b**3*c**5*d**2*x**2 + 3*a*b**3*c**4*d
**3*x**3 + b**4*c**7*x**2 + b**4*c**6*d*x**3))
```

3.283 $\int \frac{1}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 215

$$\begin{aligned}\int \frac{1}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx = &-\frac{d}{2\left(bc^2+ad^2\right)(c+dx)^2\sqrt{a+bx^2}}\\ &-\frac{5bcd}{2\left(bc^2+ad^2\right)^2(c+dx)\sqrt{a+bx^2}}+\frac{b(3ad(4bc^2-ad^2)+bc(2bc^2-13ad^2)\,x)}{2a\left(bc^2+ad^2\right)^3\sqrt{a+bx^2}}\\ &-\frac{3bd^2(4bc^2-ad^2)\,\mathrm{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2\left(bc^2+ad^2\right)^{7/2}}\end{aligned}$$

output
```
-1/2*d/(a*d^2+b*c^2)/(d*x+c)^2/(b*x^2+a)^(1/2)-5/2*b*c*d/(a*d^2+b*c^2)^2/(
d*x+c)/(b*x^2+a)^(1/2)+1/2*b*(3*a*d*(-a*d^2+4*b*c^2)+b*c*(-13*a*d^2+2*b*c^
2)*x)/a/(a*d^2+b*c^2)^3/(b*x^2+a)^(1/2)-3/2*b*d^2*(-a*d^2+4*b*c^2)*arctanh
((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 10.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{-a^3d^5+2b^3c^3x(c+dx)^2-a^2bd^3(10c^2+11cdx+3d^2x^2)+ab^2cd(6c^3+}{a(bc^2+ad^2)^3(c+dx)^2\sqrt{a+bx^2}}\right.$$
$$+\frac{3bd^2(4bc^2-ad^2)\log(c+dx)}{(bc^2+ad^2)^{7/2}}$$
$$\left.+\frac{3bd^2(-4bc^2+ad^2)\log\left(ad-bcx+\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\right)}{(bc^2+ad^2)^{7/2}}\right)$$

input
```
Integrate[1/((c + d*x)^3*(a + b*x^2)^(3/2)),x]
```

output
```
((-(a^3*d^5) + 2*b^3*c^3*x*(c + d*x)^2 - a^2*b*d^3*(10*c^2 + 11*c*d*x + 3*
d^2*x^2) + a*b^2*c*d*(6*c^3 + 6*c^2*d*x - 14*c*d^2*x^2 - 13*d^3*x^3))/(a*(
b*c^2 + a*d^2)^3*(c + d*x)^2*Sqrt[a + b*x^2]) + (3*b*d^2*(4*b*c^2 - a*d^2)
*Log[c + d*x])/(b*c^2 + a*d^2)^(7/2) + (3*b*d^2*(-4*b*c^2 + a*d^2)*Log[a*d
 - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/(b*c^2 + a*d^2)^(7/2))/2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {496, 25, 27, 688, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx)^3}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}-\frac{\int -\frac{d(3ad+2bcx)}{(c+dx)^3\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(3ad+2bcx)}{(c+dx)^3\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{d\int \frac{3ad+2bcx}{(c+dx)^3\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 688$$

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(2bc^2-3ad^2\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{b\left(10acd+\left(2bc^2-3ad^2\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{d\left(\frac{\int \frac{b\left(10acd+\left(2bc^2-3ad^2\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(2bc^2-3ad^2\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{d\left(\frac{b\int \frac{10acd+\left(2bc^2-3ad^2\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(2bc^2-3ad^2\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 679$$

$$\frac{d\left(\frac{b\left(\frac{3ad\left(4bc^2-ad^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{c\sqrt{a+bx^2}\left(2bc^2-13ad^2\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(2bc^2-3ad^2\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+$$
$$\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 488$$

$$\frac{d\left(\frac{b\left(\frac{c\sqrt{a+bx^2}(2bc^2-13ad^2)}{(c+dx)(ad^2+bc^2)}-\frac{3ad(4bc^2-ad^2)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}\right)}{2(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(2bc^2-3ad^2)}{2(c+dx)^2(ad^2+bc^2)}\right)}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}$$

$\downarrow$ 219

$$\frac{d\left(\frac{b\left(\frac{c\sqrt{a+bx^2}(2bc^2-13ad^2)}{(c+dx)(ad^2+bc^2)}-\frac{3ad(4bc^2-ad^2)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}\right)}{2(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(2bc^2-3ad^2)}{2(c+dx)^2(ad^2+bc^2)}\right)}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}$$

input
```
Int[1/((c + d*x)^3*(a + b*x^2)^(3/2)),x]
```

output
```
(a*d + b*c*x)/(a*(b*c^2 + a*d^2)*(c + d*x)^2*Sqrt[a + b*x^2]) + (d*(((2*b*
c^2 - 3*a*d^2)*Sqrt[a + b*x^2])/(2*(b*c^2 + a*d^2)*(c + d*x)^2) + (b*((c*(
2*b*c^2 - 13*a*d^2)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) - (3*a*d*
(4*b*c^2 - a*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^
2])])/(b*c^2 + a*d^2)^(3/2)))/(2*(b*c^2 + a*d^2))))/(a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(195) = 390$.

Time = 0.35 (sec) , antiderivative size = 933, normalized size of antiderivative = 4.34

method	result
default	$-\frac{d^{2}}{2\left(a\,d^{2}+b\,c^{2}\right)\left(x+\frac{c}{d}\right)^{2}\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}+\frac{5bcd\left(-\frac{d^{2}}{\left(a\,d^{2}+b\,c^{2}\right)\left(x+\frac{c}{d}\right)\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}+\frac{3bcd\left(\ldots\right.}{\left(a\,d^{2}\ldots\right.}\right.}{\ldots}$

input
```
int(1/(d*x+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2)+5/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2
)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/
2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^
2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+
b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/
d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/
d^2)^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*
d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d
^2)^(1/2))-3/2*b/(a*d^2+b*c^2)*d^2*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c
/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b
*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((
2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(196) = 392$.

Time = 0.46 (sec) , antiderivative size = 1558, normalized size of antiderivative = 7.25

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(3*(4*a^2*b^2*c^4*d^2 - a^3*b*c^2*d^4 + (4*a*b^3*c^2*d^4 - a^2*b^2*d
^6)*x^4 + 2*(4*a*b^3*c^3*d^3 - a^2*b^2*c*d^5)*x^3 + (4*a*b^3*c^4*d^2 + 3*a
^2*b^2*c^2*d^4 - a^3*b*d^6)*x^2 + 2*(4*a^2*b^2*c^3*d^3 - a^3*b*c*d^5)*x)*s
qrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a
*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^
2 + 2*c*d*x + c^2)) - 2*(6*a*b^3*c^6*d - 4*a^2*b^2*c^4*d^3 - 11*a^3*b*c^2*
d^5 - a^4*d^7 + (2*b^4*c^5*d^2 - 11*a*b^3*c^3*d^4 - 13*a^2*b^2*c*d^6)*x^3
+ (4*b^4*c^6*d - 10*a*b^3*c^4*d^3 - 17*a^2*b^2*c^2*d^5 - 3*a^3*b*d^7)*x^2
+ (2*b^4*c^7 + 8*a*b^3*c^5*d^2 - 5*a^2*b^2*c^3*d^4 - 11*a^3*b*c*d^6)*x)*sq
rt(b*x^2 + a))/(a^2*b^4*c^10 + 4*a^3*b^3*c^8*d^2 + 6*a^4*b^2*c^6*d^4 + 4*a
^5*b*c^4*d^6 + a^6*c^2*d^8 + (a*b^5*c^8*d^2 + 4*a^2*b^4*c^6*d^4 + 6*a^3*b^
3*c^4*d^6 + 4*a^4*b^2*c^2*d^8 + a^5*b*d^10)*x^4 + 2*(a*b^5*c^9*d + 4*a^2*b
^4*c^7*d^3 + 6*a^3*b^3*c^5*d^5 + 4*a^4*b^2*c^3*d^7 + a^5*b*c*d^9)*x^3 + (a
*b^5*c^10 + 5*a^2*b^4*c^8*d^2 + 10*a^3*b^3*c^6*d^4 + 10*a^4*b^2*c^4*d^6 +
5*a^5*b*c^2*d^8 + a^6*d^10)*x^2 + 2*(a^2*b^4*c^9*d + 4*a^3*b^3*c^7*d^3 + 6
*a^4*b^2*c^5*d^5 + 4*a^5*b*c^3*d^7 + a^6*c*d^9)*x), -1/2*(3*(4*a^2*b^2*c^4
*d^2 - a^3*b*c^2*d^4 + (4*a*b^3*c^2*d^4 - a^2*b^2*d^6)*x^4 + 2*(4*a*b^3*c^
3*d^3 - a^2*b^2*c*d^5)*x^3 + (4*a*b^3*c^4*d^2 + 3*a^2*b^2*c^2*d^4 - a^3*b*
d^6)*x^2 + 2*(4*a^2*b^2*c^3*d^3 - a^3*b*c*d^5)*x)*sqrt(-b*c^2 - a*d^2)*arc
tan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(196) = 392.

Time = 0.08 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.93

$$\int \frac{1}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{15\,b^3c^3x}{2\left(\sqrt{bx^2+a}ab^3c^6+3\sqrt{bx^2+a}a^2b^2c^4d^2+3\sqrt{bx^2+a}a^3bc^2d^4+\sqrt{bx^2+a}a^4\right.}$$
$$+\frac{15\,b^2c^2}{2\left(\frac{\sqrt{bx^2+a}b^3c^6}{d}+3\sqrt{bx^2+a}ab^2c^4d+3\sqrt{bx^2+a}a^2bc^2d^3+\sqrt{bx^2+a}a^3d^5\right)}$$
$$-\frac{13\,b^2cx}{2\left(\sqrt{bx^2+a}ab^2c^4+2\sqrt{bx^2+a}a^2bc^2d^2+\sqrt{bx^2+a}a^3d^4\right)}$$
$$-\frac{5\,bc}{2\left(\sqrt{bx^2+a}b^2c^4x+2\sqrt{bx^2+a}abc^2d^2x+\sqrt{bx^2+a}a^2d^4x+\frac{\sqrt{bx^2+a}b^2c^5}{d}+2\sqrt{bx^2+a}abc^3d+\sqrt{bx^2+a}a^2\right.}$$
$$-\frac{3\,b}{2\left(\frac{\sqrt{bx^2+a}b^2c^4}{d}+2\sqrt{bx^2+a}abc^2d+\sqrt{bx^2+a}a^2d^3\right)}$$
$$-\frac{1}{2\left(\sqrt{bx^2+a}bc^2dx^2+\sqrt{bx^2+a}ad^3x^2+2\sqrt{bx^2+a}bc^3x+2\sqrt{bx^2+a}acd^2x+\frac{\sqrt{bx^2+a}bc^4}{d}+\sqrt{bx^2+a}ac^2d\right.}$$
$$+\frac{15\,b^2c^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{7}{2}}d^5}-\frac{3\,b\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^3}$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
15/2*b^3*c^3*x/(sqrt(b*x^2 + a)*a*b^3*c^6 + 3*sqrt(b*x^2 + a)*a^2*b^2*c^4*
d^2 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^4 + sqrt(b*x^2 + a)*a^4*d^6) + 15/2*b^
2*c^2/(sqrt(b*x^2 + a)*b^3*c^6/d + 3*sqrt(b*x^2 + a)*a*b^2*c^4*d + 3*sqrt(
b*x^2 + a)*a^2*b*c^2*d^3 + sqrt(b*x^2 + a)*a^3*d^5) - 13/2*b^2*c*x/(sqrt(b
*x^2 + a)*a*b^2*c^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^2 + sqrt(b*x^2 + a)*a^
3*d^4) - 5/2*b*c/(sqrt(b*x^2 + a)*b^2*c^4*x + 2*sqrt(b*x^2 + a)*a*b*c^2*d^
2*x + sqrt(b*x^2 + a)*a^2*d^4*x + sqrt(b*x^2 + a)*b^2*c^5/d + 2*sqrt(b*x^2
 + a)*a*b*c^3*d + sqrt(b*x^2 + a)*a^2*c*d^3) - 3/2*b/(sqrt(b*x^2 + a)*b^2*
c^4/d + 2*sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) - 1/2/(sqrt
(b*x^2 + a)*b*c^2*d*x^2 + sqrt(b*x^2 + a)*a*d^3*x^2 + 2*sqrt(b*x^2 + a)*b*
c^3*x + 2*sqrt(b*x^2 + a)*a*c*d^2*x + sqrt(b*x^2 + a)*b*c^4/d + sqrt(b*x^2
 + a)*a*c^2*d) + 15/2*b^2*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d
/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(7/2)*d^5) - 3/2*b*arcsinh(b*c
*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^
2)^(5/2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(196) = 392$.

Time = 0.15 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.16

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \frac{\frac{(b^6c^9-6a^2b^4c^5d^4-8a^3b^3c^3d^6-3a^4b^2cd^8)x}{ab^6c^{12}+6a^2b^5c^{10}d^2+15a^3b^4c^8d^4+20a^4b^3c^6d^6+15a^5b^2c^4d^8+6a^6bc^2d^{10}+a^7d^{12}} + \frac{}{ab^6c^{12}+6a^2b^5c^{10}}}{\sqrt{bx^2+a}}$$
$$+ \frac{3\,(4\,b^2c^2d^2-abd^4)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6)\sqrt{-bc^2-ad^2}}$$
$$- \frac{6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 b^2c^2d^3 - \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 abd^5 + 14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{5}{2}}c^3d^2 - 7\left(\sqrt{b}x-\sqrt{\right.}\right.}{(b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6)\left(\left(\sqrt{b}x-\sqrt{bx^2+}\right.\right.}$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((b^6*c^9 - 6*a^2*b^4*c^5*d^4 - 8*a^3*b^3*c^3*d^6 - 3*a^4*b^2*c*d^8)*x/(a*
b^6*c^12 + 6*a^2*b^5*c^10*d^2 + 15*a^3*b^4*c^8*d^4 + 20*a^4*b^3*c^6*d^6 +
15*a^5*b^2*c^4*d^8 + 6*a^6*b*c^2*d^10 + a^7*d^12) + (3*a*b^5*c^8*d + 8*a^2
*b^4*c^6*d^3 + 6*a^3*b^3*c^4*d^5 - a^5*b*d^9)/(a*b^6*c^12 + 6*a^2*b^5*c^10
*d^2 + 15*a^3*b^4*c^8*d^4 + 20*a^4*b^3*c^6*d^6 + 15*a^5*b^2*c^4*d^8 + 6*a^
6*b*c^2*d^10 + a^7*d^12))/sqrt(b*x^2 + a) + 3*(4*b^2*c^2*d^2 - a*b*d^4)*ar
ctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/
((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*sqrt(-b*c^2 - a*d
^2)) - (6*(sqrt(b)*x - sqrt(b*x^2 + a))^3*b^2*c^2*d^3 - (sqrt(b)*x - sqrt(
b*x^2 + a))^3*a*b*d^5 + 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^3*d^2
 - 7*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2)*c*d^4 - 22*(sqrt(b)*x - sqr
t(b*x^2 + a))*a*b^2*c^2*d^3 - (sqrt(b)*x - sqrt(b*x^2 + a))*a^2*b*d^5 + 7*
a^2*b^(3/2)*c*d^4)/((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6
)*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqr
t(b)*c - a*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(c+d\,x)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1666, normalized size of antiderivative = 7.75

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^3/(b*x^2+a)^(3/2),x)
```

output

```
(3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**3*b*c**2*d**4 + 6*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*c*d**5*x + 3*sqrt(a*d**2
+ b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**
3*b*d**6*x**2 - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**4*d**2 - 24*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*
*3*d**3*x - 9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**4*x**2 + 6*sqrt(a*d**2 + b*c**2)*
log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d
**5*x**3 + 3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a**2*b**2*d**6*x**4 - 12*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**2*x
**2 - 24*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*a*b**3*c**3*d**3*x**3 - 12*sqrt(a*d**2 + b*c**2)*log( -
sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**2*d**4*x**
4 - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b*c**2*d**4 - 6*sqrt(a*d**2
+ b*c**2)*log(c + d*x)*a**3*b*c*d**5*x - 3*sqrt(a*d**2 + b*c**2)*log(c + d
*x)*a**3*b*d**6*x**2 + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c**
4*d**2 + 24*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c**3*d**3*x + ...
```

3.284 $\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 283

$$\begin{aligned}\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}} dx =& -\frac{d}{3(bc^2+ad^2)(c+dx)^3\sqrt{a+bx^2}} \\ &- \frac{7bcd}{6(bc^2+ad^2)^2(c+dx)^2\sqrt{a+bx^2}} - \frac{bd(27bc^2-8ad^2)}{6(bc^2+ad^2)^3(c+dx)\sqrt{a+bx^2}} \\ &+ \frac{b^2(15acd(4bc^2-3ad^2)+(6b^2c^4-83abc^2d^2+16a^2d^4)x)}{6a(bc^2+ad^2)^4\sqrt{a+bx^2}} \\ &- \frac{5b^2cd^2(4bc^2-3ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2(bc^2+ad^2)^{9/2}}\end{aligned}$$

output
```
-1/3*d/(a*d^2+b*c^2)/(d*x+c)^3/(b*x^2+a)^(1/2)-7/6*b*c*d/(a*d^2+b*c^2)^2/(
d*x+c)^2/(b*x^2+a)^(1/2)-1/6*b*d*(-8*a*d^2+27*b*c^2)/(a*d^2+b*c^2)^3/(d*x+
c)/(b*x^2+a)^(1/2)+1/6*b^2*(15*a*c*d*(-3*a*d^2+4*b*c^2)+(16*a^2*d^4-83*a*b
*c^2*d^2+6*b^2*c^4)*x)/a/(a*d^2+b*c^2)^4/(b*x^2+a)^(1/2)-5/2*b^2*c*d^2*(-3
*a*d^2+4*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/
(a*d^2+b*c^2)^(9/2)
```

Mathematica [A] (verified)

Time = 10.57 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.99

$$\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}} dx = \frac{1}{6} \left(\frac{\sqrt{a+bx^2}\left(-\frac{2d^3(bc^2+ad^2)^2}{(c+dx)^3} - \frac{11bcd^3(bc^2+ad^2)}{(c+dx)^2} + \frac{bd^3(-47bc^2+10ad^2)}{c+dx} + \frac{6b^2(b^2c^4x+2\right.}{}}{(bc^2+ad^2)^4} \right.$$
$$+ \frac{15b^2cd^2(4bc^2-3ad^2)\log(c+dx)}{(bc^2+ad^2)^{9/2}}$$
$$\left. - \frac{15b^2cd^2(4bc^2-3ad^2)\log\left(ad-bcx+\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\right)}{(bc^2+ad^2)^{9/2}} \right)$$

input
```
Integrate[1/((c + d*x)^4*(a + b*x^2)^(3/2)),x]
```

output
```
((Sqrt[a + b*x^2]*((-2*d^3*(b*c^2 + a*d^2)^2)/(c + d*x)^3 - (11*b*c*d^3*(b
*c^2 + a*d^2))/(c + d*x)^2 + (b*d^3*(-47*b*c^2 + 10*a*d^2))/(c + d*x) + (6
*b^2*(b^2*c^4*x + 2*a*b*c^2*d*(2*c - 3*d*x) + a^2*d^3*(-4*c + d*x)))/(a*(a
 + b*x^2))))/(b*c^2 + a*d^2)^4 + (15*b^2*c*d^2*(4*b*c^2 - 3*a*d^2)*Log[c +
 d*x])/(b*c^2 + a*d^2)^(9/2) - (15*b^2*c*d^2*(4*b*c^2 - 3*a*d^2)*Log[a*d -
 b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/(b*c^2 + a*d^2)^(9/2))/6
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {496, 25, 27, 688, 25, 27, 688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2} (c+dx)^4} dx$$

$$\Big\downarrow \; 496$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}-\frac{\int -\frac{d(4ad+3bcx)}{(c+dx)^4\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(4ad+3bcx)}{(c+dx)^4\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{4ad+3bcx}{(c+dx)^4\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{688}$$

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(3bc^2-4ad^2\right)}{3(c+dx)^3\left(ad^2+bc^2\right)}-\frac{\int -\frac{b\left(21acd+2\left(3bc^2-4ad^2\right)x\right)}{(c+dx)^3\sqrt{bx^2+a}}dx}{3\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{\int \frac{b\left(21acd+2\left(3bc^2-4ad^2\right)x\right)}{(c+dx)^3\sqrt{bx^2+a}}dx}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(3bc^2-4ad^2\right)}{3(c+dx)^3\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{b\int \frac{21acd+2\left(3bc^2-4ad^2\right)x}{(c+dx)^3\sqrt{bx^2+a}}dx}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(3bc^2-4ad^2\right)}{3(c+dx)^3\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{688}$$

$$\frac{d\left(\frac{b\left(\frac{c\sqrt{a+bx^2}\left(6bc^2-29ad^2\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{2ad\left(27bc^2-8ad^2\right)+bc\left(6bc^2-29ad^2\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}\right)}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(3bc^2-4ad^2\right)}{3(c+dx)^3\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{b\left(\frac{\int \frac{2ad(27bc^2-8ad^2)+bc(6bc^2-29ad^2)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}+\frac{c\sqrt{a+bx^2}(6bc^2-29ad^2)}{2(c+dx)^2(ad^2+bc^2)}\right)}{3(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(3bc^2-4ad^2)}{3(c+dx)^3(ad^2+bc^2)}\right)}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3(ad^2+bc^2)}$$

$\big\downarrow$ 679

$$\frac{d\left(\frac{b\left(\frac{\frac{15abcd(4bc^2-3ad^2)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}(16a^2d^4-83abc^2d^2+6b^2c^4)}{(c+dx)(ad^2+bc^2)}}{2(ad^2+bc^2)}+\frac{c\sqrt{a+bx^2}(6bc^2-29ad^2)}{2(c+dx)^2(ad^2+bc^2)}\right)}{3(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(3bc^2-4ad^2)}{3(c+dx)^3(ad^2+bc^2)}\right)}{a(ad^2+bc^2)}+\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3(ad^2+bc^2)}$$

$\big\downarrow$ 488

$$\frac{d\left(\frac{b\left(\frac{\frac{\sqrt{a+bx^2}(16a^2d^4-83abc^2d^2+6b^2c^4)}{(c+dx)(ad^2+bc^2)}-\frac{15abcd(4bc^2-3ad^2)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}}{2(ad^2+bc^2)}+\frac{c\sqrt{a+bx^2}(6bc^2-29ad^2)}{2(c+dx)^2(ad^2+bc^2)}\right)}{3(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(3bc^2-4ad}{3(c+dx)^3(ad^2+bc}\right)}{a(ad^2+bc^2)}$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3(ad^2+bc^2)}$$

$\big\downarrow$ 219

$$\frac{d\left(\frac{b\left(\frac{\frac{\sqrt{a+bx^2}\left(16a^2d^4-83abc^2d^2+6b^2c^4\right)}{(c+dx)\left(ad^2+bc^2\right)}-\frac{15abcd\left(4bc^2-3ad^2\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}}{2\left(ad^2+bc^2\right)}+\frac{c\sqrt{a+bx^2}\left(6bc^2-29ad^2\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{3(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(3bc^2-4ad^2)}{3(c+dx)^3(ad^2+bc^2)}\right)}{a\left(ad^2+bc^2\right)}$$

$$\frac{ad+bcx}{a\sqrt{a+bx^2}(c+dx)^3\left(ad^2+bc^2\right)}$$

input
```
Int[1/((c + d*x)^4*(a + b*x^2)^(3/2)),x]
```

output
```
(a*d + b*c*x)/(a*(b*c^2 + a*d^2)*(c + d*x)^3*Sqrt[a + b*x^2]) + (d*(((3*b*
c^2 - 4*a*d^2)*Sqrt[a + b*x^2])/(3*(b*c^2 + a*d^2)*(c + d*x)^3) + (b*((c*(
6*b*c^2 - 29*a*d^2)*Sqrt[a + b*x^2])/(2*(b*c^2 + a*d^2)*(c + d*x)^2) + (((
6*b^2*c^4 - 83*a*b*c^2*d^2 + 16*a^2*d^4)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)
*(c + d*x)) - (15*a*b*c*d*(4*b*c^2 - 3*a*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[
b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2))/(2*(b*c^2 + a*d^2
))))/(3*(b*c^2 + a*d^2))))/(a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1550 vs. $2(259) = 518$.

Time = 0.41 (sec) , antiderivative size = 1551, normalized size of antiderivative = 5.48

method	result	size
default	Expression too large to display	1551

input
```
int(1/(d*x+c)^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(-1/3/(a*d^2+b*c^2)*d^2/(x+c/d)^3/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2)+7/3*b*c*d/(a*d^2+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d
)^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+5/2*b*c*d/(a*d^2
+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d
)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c
/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)
*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c
^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-3/2*b/(a*d^2+b*c^2)*d^2*(
1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+
2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c
^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c
^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+
2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2))/(x+c/d))))-4/3*b/(a*d^2+b*c^2)*d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2
)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(260) = 520$.

Time = 0.95 (sec) , antiderivative size = 2226, normalized size of antiderivative = 7.87

$$\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(15*(4*a^2*b^3*c^6*d^2 - 3*a^3*b^2*c^4*d^4 + (4*a*b^4*c^3*d^5 - 3*a
^2*b^3*c*d^7)*x^5 + 3*(4*a*b^4*c^4*d^4 - 3*a^2*b^3*c^2*d^6)*x^4 + (12*a*b^
4*c^5*d^3 - 5*a^2*b^3*c^3*d^5 - 3*a^3*b^2*c*d^7)*x^3 + (4*a*b^4*c^6*d^2 +
9*a^2*b^3*c^4*d^4 - 9*a^3*b^2*c^2*d^6)*x^2 + 3*(4*a^2*b^3*c^5*d^3 - 3*a^3*
b^2*c^3*d^5)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2
 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*
x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(24*a*b^4*c^8*d - 60*a^2*b^3*c^6*
d^3 - 89*a^3*b^2*c^4*d^5 - 7*a^4*b*c^2*d^7 - 2*a^5*d^9 + (6*b^5*c^6*d^3 -
77*a*b^4*c^4*d^5 - 67*a^2*b^3*c^2*d^7 + 16*a^3*b^2*d^9)*x^4 + 3*(6*b^5*c^7
*d^2 - 57*a*b^4*c^5*d^4 - 62*a^2*b^3*c^3*d^6 + a^3*b^2*c*d^8)*x^3 + 2*(9*b
^5*c^8*d - 39*a*b^4*c^6*d^3 - 101*a^2*b^3*c^4*d^5 - 49*a^3*b^2*c^2*d^7 + 4
*a^4*b*d^9)*x^2 + 3*(2*b^5*c^9 + 14*a*b^4*c^7*d^2 - 45*a^2*b^3*c^5*d^4 - 5
4*a^3*b^2*c^3*d^6 + 3*a^4*b*c*d^8)*x)*sqrt(b*x^2 + a))/(a^2*b^5*c^13 + 5*a
^3*b^4*c^11*d^2 + 10*a^4*b^3*c^9*d^4 + 10*a^5*b^2*c^7*d^6 + 5*a^6*b*c^5*d^
8 + a^7*c^3*d^10 + (a*b^6*c^10*d^3 + 5*a^2*b^5*c^8*d^5 + 10*a^3*b^4*c^6*d^
7 + 10*a^4*b^3*c^4*d^9 + 5*a^5*b^2*c^2*d^11 + a^6*b*d^13)*x^5 + 3*(a*b^6*c
^11*d^2 + 5*a^2*b^5*c^9*d^4 + 10*a^3*b^4*c^7*d^6 + 10*a^4*b^3*c^5*d^8 + 5*
a^5*b^2*c^3*d^10 + a^6*b*c*d^12)*x^4 + (3*a*b^6*c^12*d + 16*a^2*b^5*c^10*d
^3 + 35*a^3*b^4*c^8*d^5 + 40*a^4*b^3*c^6*d^7 + 25*a^5*b^2*c^4*d^9 + 8*a^6*
b*c^2*d^11 + a^7*d^13)*x^3 + (a*b^6*c^13 + 8*a^2*b^5*c^11*d^2 + 25*a^3*...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}} (c+dx)^4}\,dx$$

input
```
integrate(1/(d*x+c)**4/(b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(260) = 520$.

Time = 0.10 (sec) , antiderivative size = 1179, normalized size of antiderivative = 4.17

$$\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
35/2*b^4*c^4*x/(sqrt(b*x^2 + a)*a*b^4*c^8 + 4*sqrt(b*x^2 + a)*a^2*b^3*c^6*
d^2 + 6*sqrt(b*x^2 + a)*a^3*b^2*c^4*d^4 + 4*sqrt(b*x^2 + a)*a^4*b*c^2*d^6
+ sqrt(b*x^2 + a)*a^5*d^8) + 35/2*b^3*c^3/(sqrt(b*x^2 + a)*b^4*c^8/d + 4*s
qrt(b*x^2 + a)*a*b^3*c^6*d + 6*sqrt(b*x^2 + a)*a^2*b^2*c^4*d^3 + 4*sqrt(b*
x^2 + a)*a^3*b*c^2*d^5 + sqrt(b*x^2 + a)*a^4*d^7) - 115/6*b^3*c^2*x/(sqrt(
b*x^2 + a)*a*b^3*c^6 + 3*sqrt(b*x^2 + a)*a^2*b^2*c^4*d^2 + 3*sqrt(b*x^2 +
a)*a^3*b*c^2*d^4 + sqrt(b*x^2 + a)*a^4*d^6) - 35/6*b^2*c^2/(sqrt(b*x^2 + a
)*b^3*c^6*x + 3*sqrt(b*x^2 + a)*a*b^2*c^4*d^2*x + 3*sqrt(b*x^2 + a)*a^2*b*
c^2*d^4*x + sqrt(b*x^2 + a)*a^3*d^6*x + sqrt(b*x^2 + a)*b^3*c^7/d + 3*sqrt
(b*x^2 + a)*a*b^2*c^5*d + 3*sqrt(b*x^2 + a)*a^2*b*c^3*d^3 + sqrt(b*x^2 + a
)*a^3*c*d^5) - 15/2*b^2*c/(sqrt(b*x^2 + a)*b^3*c^6/d + 3*sqrt(b*x^2 + a)*a
*b^2*c^4*d + 3*sqrt(b*x^2 + a)*a^2*b*c^2*d^3 + sqrt(b*x^2 + a)*a^3*d^5) +
8/3*b^2*x/(sqrt(b*x^2 + a)*a*b^2*c^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^2 + s
qrt(b*x^2 + a)*a^3*d^4) - 7/6*b*c/(sqrt(b*x^2 + a)*b^2*c^4*d*x^2 + 2*sqrt(
b*x^2 + a)*a*b*c^2*d^3*x^2 + sqrt(b*x^2 + a)*a^2*d^5*x^2 + 2*sqrt(b*x^2 +
a)*b^2*c^5*x + 4*sqrt(b*x^2 + a)*a*b*c^3*d^2*x + 2*sqrt(b*x^2 + a)*a^2*c*d
^4*x + sqrt(b*x^2 + a)*b^2*c^6/d + 2*sqrt(b*x^2 + a)*a*b*c^4*d + sqrt(b*x^
2 + a)*a^2*c^2*d^3) + 4/3*b/(sqrt(b*x^2 + a)*b^2*c^4*x + 2*sqrt(b*x^2 + a)
*a*b*c^2*d^2*x + sqrt(b*x^2 + a)*a^2*d^4*x + sqrt(b*x^2 + a)*b^2*c^5/d + 2
*sqrt(b*x^2 + a)*a*b*c^3*d + sqrt(b*x^2 + a)*a^2*c*d^3) - 1/3/(sqrt(b*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(260) = 520$.

Time = 0.20 (sec) , antiderivative size = 1075, normalized size of antiderivative = 3.80

$$\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((b^8*c^12 - 2*a*b^7*c^10*d^2 - 17*a^2*b^6*c^8*d^4 - 28*a^3*b^5*c^6*d^6 -
17*a^4*b^4*c^4*d^8 - 2*a^5*b^3*c^2*d^10 + a^6*b^2*d^12)*x/(a*b^8*c^16 + 8*
a^2*b^7*c^14*d^2 + 28*a^3*b^6*c^12*d^4 + 56*a^4*b^5*c^10*d^6 + 70*a^5*b^4*
c^8*d^8 + 56*a^6*b^3*c^6*d^10 + 28*a^7*b^2*c^4*d^12 + 8*a^8*b*c^2*d^14 + a
^9*d^16) + 4*(a*b^7*c^11*d + 3*a^2*b^6*c^9*d^3 + 2*a^3*b^5*c^7*d^5 - 2*a^4
*b^4*c^5*d^7 - 3*a^5*b^3*c^3*d^9 - a^6*b^2*c*d^11)/(a*b^8*c^16 + 8*a^2*b^7
*c^14*d^2 + 28*a^3*b^6*c^12*d^4 + 56*a^4*b^5*c^10*d^6 + 70*a^5*b^4*c^8*d^8
 + 56*a^6*b^3*c^6*d^10 + 28*a^7*b^2*c^4*d^12 + 8*a^8*b*c^2*d^14 + a^9*d^16
))/sqrt(b*x^2 + a) + 5*(4*b^3*c^3*d^2 - 3*a*b^2*c*d^4)*arctan(-((sqrt(b)*x
 - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^4*c^8 + 4*a*b
^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8)*sqrt(-b*c^2 -
a*d^2)) - 1/3*(36*(sqrt(b)*x - sqrt(b*x^2 + a))^5*b^3*c^3*d^4 - 21*(sqrt(b
)*x - sqrt(b*x^2 + a))^5*a*b^2*c*d^6 + 162*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*b^(7/2)*c^4*d^3 - 117*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^5 +
 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^7 + 188*(sqrt(b)*x - sqrt
(b*x^2 + a))^3*b^4*c^5*d^2 - 322*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a*b^3*c^3
*d^4 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^3*a^2*b^2*c*d^6 - 402*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*a*b^(7/2)*c^4*d^3 + 144*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^2*b^(5/2)*c^2*d^5 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*d^7
 + 246*(sqrt(b)*x - sqrt(b*x^2 + a))*a^2*b^3*c^3*d^4 - 39*(sqrt(b)*x - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^4}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)^4),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2350, normalized size of antiderivative = 8.30

$$\int \frac{1}{(c+dx)^4 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^4/(b*x^2+a)^(3/2),x)
```

output
```
(45*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**3*b**2*c**4*d**4 + 135*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
 + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**2*c**3*d**5*x + 13
5*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a**3*b**2*c**2*d**6*x**2 + 45*sqrt(a*d**2 + b*c**2)*log( - sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**2*c*d**7*x**3 -
60*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*b**3*c**6*d**2 - 180*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
+ b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**5*d**3*x - 135
*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**3*c**4*d**4*x**2 + 75*sqrt(a*d**2 + b*c**2)*log( - sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**3*d**5*x**3
+ 135*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a**2*b**3*c**2*d**6*x**4 + 45*sqrt(a*d**2 + b*c**2)*log( -
sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c*d**7*x**
5 - 60*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a*b**4*c**6*d**2*x**2 - 180*sqrt(a*d**2 + b*c**2)*log( - s
qrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**4*c**5*d**3*x**3
 - 180*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a*b**4*c**4*d**4*x**4 - 60*sqrt(a*d**2 + b*c**2)*log( -...
```

3.285 $\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 205

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = -\frac{ad(5b^2c^4-10abc^2d^2+a^2d^4)-bc(b^2c^4-10abc^2d^2+5a^2d^4)\,x}{3ab^3\,(a+bx^2)^{3/2}} - \frac{2(3a^2d^3(5bc^2-ad^2)-bc(b^2c^4+5abc^2d^2-10a^2d^4)\,x)}{3a^2b^3\sqrt{a+bx^2}} + \frac{d^5\sqrt{a+bx^2}}{b^3} + \frac{5cd^4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output

```
-1/3*(a*d*(a^2*d^4-10*a*b*c^2*d^2+5*b^2*c^4)-b*c*(5*a^2*d^4-10*a*b*c^2*d^2
+b^2*c^4)*x)/a/b^3/(b*x^2+a)^(3/2)-2/3*(3*a^2*d^3*(-a*d^2+5*b*c^2)-b*c*(-1
0*a^2*d^4+5*a*b*c^2*d^2+b^2*c^4)*x)/a^2/b^3/(b*x^2+a)^(1/2)+d^5*(b*x^2+a)^
(1/2)/b^3+5*c*d^4*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \frac{8a^4d^5 + 2b^4c^5x^3 + ab^3c^3x(3c^2 + 10d^2x^2) + a^3bd^3(-20c^2 - 15cdx + 12d^2x^2) + a^2b^2d(-}{3a^2b^3\,(a}$$

input
```
Integrate[(c + d*x)^5/(a + b*x^2)^(5/2),x]
```

output
```
(8*a^4*d^5 + 2*b^4*c^5*x^3 + a*b^3*c^3*x*(3*c^2 + 10*d^2*x^2) + a^3*b*d^3*
(-20*c^2 - 15*c*d*x + 12*d^2*x^2) + a^2*b^2*d*(-5*c^4 - 30*c^2*d^2*x^2 - 2
0*c*d^3*x^3 + 3*d^4*x^4) - 15*a^2*Sqrt[b]*c*d^4*(a + b*x^2)^(3/2)*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(3*a^2*b^3*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {495, 27, 684, 25, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 495$$

$$\frac{\int \frac{2(c+dx)^3(bc^2-bdxc+2ad^2)}{(bx^2+a)^{3/2}}dx}{3ab} - \frac{(c+dx)^4(ad-bcx)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{(c+dx)^3(bc^2-bdxc+2ad^2)}{(bx^2+a)^{3/2}}dx}{3ab} - \frac{(c+dx)^4(ad-bcx)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 684$$

$$\frac{2\left(\frac{\int -\frac{d(c+dx)\left(ad\left(bc^2-4ad^2\right)+bc\left(2bc^2+7ad^2\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(2a^2d^3-bcx\left(3ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}\right)}{3ab}-\frac{(c+dx)^4(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 25

$$\frac{2\left(-\frac{\int \frac{d(c+dx)\left(ad\left(bc^2-4ad^2\right)+bc\left(2bc^2+7ad^2\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(2a^2d^3-bcx\left(3ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}\right)}{3ab}-\frac{(c+dx)^4(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{2\left(-\frac{d\int \frac{(c+dx)\left(ad\left(bc^2-4ad^2\right)+bc\left(2bc^2+7ad^2\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(2a^2d^3-bcx\left(3ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}\right)}{3ab}-\frac{(c+dx)^4(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 676

$$\frac{2\left(-\frac{d\left(-\frac{15}{2}a^2cd^3\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}\left(-2a^2d^4+4abc^2d^2+b^2c^4\right)}{b}+\frac{1}{2}cdx\sqrt{a+bx^2}\left(7ad^2+2bc^2\right)\right)}{ab}-\frac{(c+dx)^2\left(2a^2d^3-bcx\left(3ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}\right)}{3ab}-\frac{(c+dx)^4(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 224

$$\frac{2\left(-\frac{d\left(-\frac{15}{2}a^2cd^3\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}\left(-2a^2d^4+4abc^2d^2+b^2c^4\right)}{b}+\frac{1}{2}cdx\sqrt{a+bx^2}\left(7ad^2+2bc^2\right)\right)}{ab}-\frac{(c+dx)^2\left(2a^2d^3-bcx\left(3ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}\right)}{3ab}-\frac{(c+dx)^4(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 219

$$\frac{2\left(-\frac{d\left(-\frac{15a^2cd^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{2\sqrt{a+bx^2}\left(-2a^2d^4+4abc^2d^2+b^2c^4\right)}{b}+\frac{1}{2}cdx\sqrt{a+bx^2}\left(7ad^2+2bc^2\right)\right)}{ab}-\frac{(c+dx)^2\left(2a^2d^3-bcx\left(3ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}\right)}{3ab}$$
$$\frac{(c+dx)^4(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x)^5/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((a*d - b*c*x)*(c + d*x)^4)/(a*b*(a + b*x^2)^(3/2)) + (2*(-(((c + d*x
)^2*(2*a^2*d^3 - b*c*(b*c^2 + 3*a*d^2)*x))/(a*b*Sqrt[a + b*x^2])) - (d*((2
*(b^2*c^4 + 4*a*b*c^2*d^2 - 2*a^2*d^4)*Sqrt[a + b*x^2])/b + (c*d*(2*b*c^2
+ 7*a*d^2)*x*Sqrt[a + b*x^2])/2 - (15*a^2*c*d^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*Sqrt[b])))/(a*b)))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.37

method	result
default	$c^{5}\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)+d^{5}\left(\frac{x^{4}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)}{b}\right)+5c\,d^{4}\left(-\frac{x^{3}}{3b(b\,x^{2}+\cdots}\right.$
risch	$\frac{d^{5}\sqrt{b\,x^{2}+a}}{b^{3}}+\frac{\frac{5c\,d^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{\left(\sqrt{-ab}\,a^{2}d^{5}-10\sqrt{-ab}\,ab\,c^{2}d^{3}+5\sqrt{-ab}\,b^{2}c^{4}d+5a^{2}bc\,d^{4}-10a\,b^{2}c^{3}d^{2}+b^{3}c^{5}\right)\left(-\frac{\sqrt{b\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\cdots}}{3\sqrt{-ab}\left(\cdots\right.}\right.}{4a\,b^{2}}}{\cdots}$

input
```
int((d*x+c)^5/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c^5*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x)+d^5*(x^4/b/(b*x^2+
a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2)))+5*c*d^4
*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))))+10*c^2*d^3*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2
+a)^(3/2))+10*c^3*d^2*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)
^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x))-5/3*c^4*d/b/(b*x^2+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.38

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \left[\frac{15\left(a^2b^2cd^4x^4+2\,a^3bcd^4x^2+a^4cd^4\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(3\,a^2\cdots}{\cdots}\right.$$

$$-\frac{15\left(a^2b^2cd^4x^4+2\,a^3bcd^4x^2+a^4cd^4\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(3\,a^2b^2d^5x^4-5\,a^2b^2c^4d-20\,a^3bc^2d^3+8\,a^4d\cdots\right.}{3\left(a^2b^5x^4+2\,a^3\cdots\right.}$$

input
```
integrate((d*x+c)^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a^2*b^2*c*d^4*x^4 + 2*a^3*b*c*d^4*x^2 + a^4*c*d^4)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(3*a^2*b^2*d^5*x^4 - 5*a^2*
b^2*c^4*d - 20*a^3*b*c^2*d^3 + 8*a^4*d^5 + 2*(b^4*c^5 + 5*a*b^3*c^3*d^2 -
10*a^2*b^2*c*d^4)*x^3 - 6*(5*a^2*b^2*c^2*d^3 - 2*a^3*b*d^5)*x^2 + 3*(a*b^3
*c^5 - 5*a^3*b*c*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3), -1/3*(15*(a^2*b^2*c*d^4*x^4 + 2*a^3*b*c*d^4*x^2 + a^4*c*d^4)*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b^2*d^5*x^4 - 5*a^2*b^2*c
^4*d - 20*a^3*b*c^2*d^3 + 8*a^4*d^5 + 2*(b^4*c^5 + 5*a*b^3*c^3*d^2 - 10*a^
2*b^2*c*d^4)*x^3 - 6*(5*a^2*b^2*c^2*d^3 - 2*a^3*b*d^5)*x^2 + 3*(a*b^3*c^5
- 5*a^3*b*c*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^
3)]
```

Sympy [F]

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^5}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**5/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x)**5/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = & -\frac{5}{3}\,cd^4x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) \\ & +\frac{d^5x^4}{(bx^2+a)^{\frac{3}{2}}b}-\frac{10\,c^2d^3x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{4\,ad^5x^2}{(bx^2+a)^{\frac{3}{2}}b^2}+\frac{2\,c^5x}{3\,\sqrt{bx^2+a}a^2} \\ & +\frac{c^5x}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{10\,c^3d^2x}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{10\,c^3d^2x}{3\,\sqrt{bx^2+a}ab}-\frac{5\,cd^4x}{3\,\sqrt{bx^2+a}b^2} \\ & +\frac{5\,cd^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{5\,c^4d}{3\,(bx^2+a)^{\frac{3}{2}}b}-\frac{20\,ac^2d^3}{3\,(bx^2+a)^{\frac{3}{2}}b^2}+\frac{8\,a^2d^5}{3\,(bx^2+a)^{\frac{3}{2}}b^3}\end{aligned}$$

input
```
integrate((d*x+c)^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-5/3*c*d^4*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) +
 d^5*x^4/((b*x^2 + a)^(3/2)*b) - 10*c^2*d^3*x^2/((b*x^2 + a)^(3/2)*b) + 4*
a*d^5*x^2/((b*x^2 + a)^(3/2)*b^2) + 2/3*c^5*x/(sqrt(b*x^2 + a)*a^2) + 1/3*
c^5*x/((b*x^2 + a)^(3/2)*a) - 10/3*c^3*d^2*x/((b*x^2 + a)^(3/2)*b) + 10/3*
c^3*d^2*x/(sqrt(b*x^2 + a)*a*b) - 5/3*c*d^4*x/(sqrt(b*x^2 + a)*b^2) + 5*c*
d^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/3*c^4*d/((b*x^2 + a)^(3/2)*b) - 20/
3*a*c^2*d^3/((b*x^2 + a)^(3/2)*b^2) + 8/3*a^2*d^5/((b*x^2 + a)^(3/2)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = -\frac{5\,cd^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}} + \frac{\left(\left(\left(\frac{3\,d^5x}{b}+\frac{2\,(b^6c^5+5\,ab^5c^3d^2-10\,a^2b^4cd^4)}{a^2b^5}\right)x-\frac{6\,(5\,a^2b^4c^2d^3-2\,a^3b^3d^5)}{a^2b^5}\right)x+\frac{3\,(ab^5c^5-5\,a^3b^3cd^4)}{a^2b^5}\right)x-\frac{5\,a^2b^4c^4d+20\,a^3b^3c^2d^3-}{a^2b^5}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5*c*d^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/3*((((3*d^5*x/
b + 2*(b^6*c^5 + 5*a*b^5*c^3*d^2 - 10*a^2*b^4*c*d^4)/(a^2*b^5))*x - 6*(5*a
^2*b^4*c^2*d^3 - 2*a^3*b^3*d^5)/(a^2*b^5))*x + 3*(a*b^5*c^5 - 5*a^3*b^3*c*
d^4)/(a^2*b^5))*x - (5*a^2*b^4*c^4*d + 20*a^3*b^3*c^2*d^3 - 8*a^4*b^2*d^5)
/(a^2*b^5))/(b*x^2 + a)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+d\,x)^5}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c + d*x)^5/(a + b*x^2)^(5/2),x)
```

output
```
int((c + d*x)^5/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^5}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^5/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x+c)^5/(b*x^2+a)^(5/2),x)
```

3.286 $\int \frac{(c+dx)^4}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 156

$$\int \frac{(c+dx)^4}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{4acd(bc^2-ad^2)-\left(b^2c^4-6abc^2d^2+a^2d^4\right)x}{3ab^2\left(a+bx^2\right)^{3/2}} - \frac{2(6a^2cd^3-\left(b^2c^4+3abc^2d^2-2a^2d^4\right)x)}{3a^2b^2\sqrt{a+bx^2}} + \frac{d^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
-1/3*(4*a*c*d*(-a*d^2+b*c^2)-(a^2*d^4-6*a*b*c^2*d^2+b^2*c^4)*x)/a/b^2/(b*x
^2+a)^(3/2)-2/3*(6*a^2*c*d^3-(-2*a^2*d^4+3*a*b*c^2*d^2+b^2*c^4)*x)/a^2/b^2
/(b*x^2+a)^(1/2)+d^4*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^4}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{2b^3c^4x^3-a^3d^3(8c+3dx)+3ab^2c^2x(c^2+2d^2x^2)-4a^2bd(c^3+3cd^2x^2+d^3x^3)}{3a^2b^2\left(a+bx^2\right)^{3/2}} - \frac{d^4\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[(c + d*x)^4/(a + b*x^2)^(5/2),x]
```

output
```
(2*b^3*c^4*x^3 - a^3*d^3*(8*c + 3*d*x) + 3*a*b^2*c^2*x*(c^2 + 2*d^2*x^2) -
 4*a^2*b*d*(c^3 + 3*c*d^2*x^2 + d^3*x^3))/(3*a^2*b^2*(a + b*x^2)^(3/2)) -
(d^4*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {495, 684, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{(c+dx)^2(2bc^2-bdxc+3ad^2)}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{(c+dx)^3(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{684}$$

$$\frac{\frac{\int \frac{d\left(3a^2d^3-bc\left(2bc^2+5ad^2\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)\left(ad\left(3ad^2+bc^2\right)-2bcx\left(2ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}}{3ab}-\frac{(c+dx)^3(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{3a^2d^3-bc\left(2bc^2+5ad^2\right)x}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)\left(ad\left(3ad^2+bc^2\right)-2bcx\left(2ad^2+bc^2\right)\right)}{ab\sqrt{a+bx^2}}}{3ab}-\frac{(c+dx)^3(ad-bcx)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{d\left(3a^2d^3\int\frac{1}{\sqrt{bx^2+a}}dx-c\sqrt{a+bx^2}(5ad^2+2bc^2)\right)}{ab}-\frac{(c+dx)(ad(3ad^2+bc^2)-2bcx(2ad^2+bc^2))}{ab\sqrt{a+bx^2}}}{3ab}-\frac{(c+dx)^3(ad-bcx)}{3ab(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 224

$$\frac{\frac{d\left(3a^2d^3\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-c\sqrt{a+bx^2}(5ad^2+2bc^2)\right)}{ab}-\frac{(c+dx)(ad(3ad^2+bc^2)-2bcx(2ad^2+bc^2))}{ab\sqrt{a+bx^2}}}{3ab}-\frac{(c+dx)^3(ad-bcx)}{3ab(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{d\left(\frac{3a^2d^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-c\sqrt{a+bx^2}(5ad^2+2bc^2)\right)}{ab}-\frac{(c+dx)(ad(3ad^2+bc^2)-2bcx(2ad^2+bc^2))}{ab\sqrt{a+bx^2}}}{3ab}-\frac{(c+dx)^3(ad-bcx)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[(c + d*x)^4/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((a*d - b*c*x)*(c + d*x)^3)/(a*b*(a + b*x^2)^(3/2)) + (-(((c + d*x)*(
a*d*(b*c^2 + 3*a*d^2) - 2*b*c*(b*c^2 + 2*a*d^2)*x))/(a*b*Sqrt[a + b*x^2]))
 + (d*(-(c*(2*b*c^2 + 5*a*d^2)*Sqrt[a + b*x^2]) + (3*a^2*d^3*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.39

method	result
default	$c^4\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d^4\left(-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)-\frac{4d\,c^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+6c^2$

input
```
int((d*x+c)^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x)+d^4*(-1/3*x^3/b/(b
*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))))-4/3*d*c^3/b/(b*x^2+a)^(3/2)+6*c^2*d^2*(-1/2*x/b/(b*x^2+a)^(3/2)+1/
2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x))+4*c*d^3*(-x^2/b
/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.57

$$\int \frac{(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(a^2b^2d^4x^4+2\,a^3bd^4x^2+a^4d^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(12\,a^2b^2c}{6\,(a^2b^5x^4}\right.$$

$$-\frac{3\,(a^2b^2d^4x^4+2\,a^3bd^4x^2+a^4d^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(12\,a^2b^2cd^3x^2+4\,a^2b^2c^3d+8\,a^3bcd^3-2\,(b^4c^4+}{3\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*b^2*d^4*x^4 + 2*a^3*b*d^4*x^2 + a^4*d^4)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(12*a^2*b^2*c*d^3*x^2 + 4*a^2*b^2*
c^3*d + 8*a^3*b*c*d^3 - 2*(b^4*c^4 + 3*a*b^3*c^2*d^2 - 2*a^2*b^2*d^4)*x^3
- 3*(a*b^3*c^4 - a^3*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x
^2 + a^4*b^3), -1/3*(3*(a^2*b^2*d^4*x^4 + 2*a^3*b*d^4*x^2 + a^4*d^4)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (12*a^2*b^2*c*d^3*x^2 + 4*a^2*b^2
*c^3*d + 8*a^3*b*c*d^3 - 2*(b^4*c^4 + 3*a*b^3*c^2*d^2 - 2*a^2*b^2*d^4)*x^3
 - 3*(a*b^3*c^4 - a^3*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*
x^2 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^4}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**4/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x)**4/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}d^4x\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) - \frac{4cd^3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2c^4x}{3\sqrt{bx^2+a}a^2} + \frac{c^4x}{3(bx^2+a)^{\frac{3}{2}}a} - \frac{2c^2d^2x}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2c^2d^2x}{\sqrt{bx^2+a}ab} - \frac{d^4x}{3\sqrt{bx^2+a}b^2} + \frac{d^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{4c^3d}{3(bx^2+a)^{\frac{3}{2}}b} - \frac{8acd^3}{3(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*d^4*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 4
*c*d^3*x^2/((b*x^2 + a)^(3/2)*b) + 2/3*c^4*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c
^4*x/((b*x^2 + a)^(3/2)*a) - 2*c^2*d^2*x/((b*x^2 + a)^(3/2)*b) + 2*c^2*d^2
*x/(sqrt(b*x^2 + a)*a*b) - 1/3*d^4*x/(sqrt(b*x^2 + a)*b^2) + d^4*arcsinh(b
*x/sqrt(a*b))/b^(5/2) - 4/3*c^3*d/((b*x^2 + a)^(3/2)*b) - 8/3*a*c*d^3/((b*
x^2 + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = -\frac{d^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}} - \frac{\left(2\left(\frac{6\,cd^3}{b}-\frac{(b^5c^4+3\,ab^4c^2d^2-2\,a^2b^3d^4)x}{a^2b^4}\right)x-\frac{3\,(ab^4c^4-a^3b^2d^4)}{a^2b^4}\right)x+\frac{4\,(a^2b^3c^3d+2\,a^3b^2cd^3)}{a^2b^4}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-d^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/3*((2*(6*c*d^3/b -
 (b^5*c^4 + 3*a*b^4*c^2*d^2 - 2*a^2*b^3*d^4)*x/(a^2*b^4))*x - 3*(a*b^4*c^4
 - a^3*b^2*d^4)/(a^2*b^4))*x + 4*(a^2*b^3*c^3*d + 2*a^3*b^2*c*d^3)/(a^2*b^
4))/(b*x^2 + a)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+d\,x)^4}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c + d*x)^4/(a + b*x^2)^(5/2),x)
```

output
```
int((c + d*x)^4/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.42

$$\int \frac{(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^3bc\,d^3-3\sqrt{b\,x^2+a}\,a^3b\,d^4x-4\sqrt{b\,x^2+a}\,a^2b^2c^3d-12\sqrt{b\,x^2+a}\,a^2b^2c\,d}{}$$

input
```
int((d*x+c)^4/(b*x^2+a)^(5/2),x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**3*b*c*d**3 - 3*sqrt(a + b*x**2)*a**3*b*d**4*x -
4*sqrt(a + b*x**2)*a**2*b**2*c**3*d - 12*sqrt(a + b*x**2)*a**2*b**2*c*d**3
*x**2 - 4*sqrt(a + b*x**2)*a**2*b**2*d**4*x**3 + 3*sqrt(a + b*x**2)*a*b**3
*c**4*x + 6*sqrt(a + b*x**2)*a*b**3*c**2*d**2*x**3 + 2*sqrt(a + b*x**2)*b*
*4*c**4*x**3 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*
d**4 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d**4*x
**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d**4
*x**4 + 6*sqrt(b)*a**3*b*c**2*d**2 - 2*sqrt(b)*a**2*b**2*c**4 + 12*sqrt(b)
*a**2*b**2*c**2*d**2*x**2 - 4*sqrt(b)*a*b**3*c**4*x**2 + 6*sqrt(b)*a*b**3*
c**2*d**2*x**4 - 2*sqrt(b)*b**4*c**4*x**4)/(3*a**2*b**3*(a**2 + 2*a*b*x**2
 + b**2*x**4))
```

3.287 $\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx=-\frac{ad(3bc^2-ad^2)-bc(bc^2-3ad^2)\,x}{3ab^2\,(a+bx^2)^{3/2}}-\frac{3a^2d^3-bc(2bc^2+3ad^2)\,x}{3a^2b^2\sqrt{a+bx^2}}$$

output
```
-1/3*(a*d*(-a*d^2+3*b*c^2)-b*c*(-3*a*d^2+b*c^2)*x)/a/b^2/(b*x^2+a)^(3/2)-1
/3*(3*a^2*d^3-b*c*(3*a*d^2+2*b*c^2)*x)/a^2/b^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx=\frac{-2a^3d^3+2b^3c^3x^3-3a^2bd(c^2+d^2x^2)+3ab^2cx(c^2+d^2x^2)}{3a^2b^2\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(c + d*x)^3/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a^3*d^3 + 2*b^3*c^3*x^3 - 3*a^2*b*d*(c^2 + d^2*x^2) + 3*a*b^2*c*x*(c^2
 + d^2*x^2))/(3*a^2*b^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {487, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 487}$$
$$\frac{2(ad^2+bc^2)\int \frac{c+dx}{(bx^2+a)^{3/2}}dx}{3ab} - \frac{(c+dx)^2(ad-bcx)}{3ab(a+bx^2)^{3/2}}$$
$$\Big\downarrow \text{ 453}$$
$$-\frac{2(ad^2+bc^2)(ad-bcx)}{3a^2b^2\sqrt{a+bx^2}} - \frac{(c+dx)^2(ad-bcx)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[(c + d*x)^3/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((a*d - b*c*x)*(c + d*x)^2)/(a*b*(a + b*x^2)^(3/2)) - (2*(b*c^2 + a*d
^2)*(a*d - b*c*x))/(3*a^2*b^2*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

method	result
gosper	$-\frac{-3a\,b^{2}c\,d^{2}x^{3}-2b^{3}c^{3}x^{3}+3a^{2}b\,d^{3}x^{2}-3a\,b^{2}c^{3}x+2a^{3}d^{3}+3a^{2}b\,c^{2}d}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
trager	$-\frac{-3a\,b^{2}c\,d^{2}x^{3}-2b^{3}c^{3}x^{3}+3a^{2}b\,d^{3}x^{2}-3a\,b^{2}c^{3}x+2a^{3}d^{3}+3a^{2}b\,c^{2}d}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
orering	$-\frac{-3a\,b^{2}c\,d^{2}x^{3}-2b^{3}c^{3}x^{3}+3a^{2}b\,d^{3}x^{2}-3a\,b^{2}c^{3}x+2a^{3}d^{3}+3a^{2}b\,c^{2}d}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
default	$c^{3}\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)+d^{3}\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)+3c\,d^{2}\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+\ldots\right.}\right.}{\ldots}\right.$

input
```
int((d*x+c)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*a*b^2*c*d^2*x^3-2*b^3*c^3*x^3+3*a^2*b*d^3*x^2-3*a*b^2*c^3*x+2*a^3
*d^3+3*a^2*b*c^2*d)/(b*x^2+a)^(3/2)/a^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(3\,a^2bd^3x^2-3\,ab^2c^3x+3\,a^2bc^2d+2\,a^3d^3-(2\,b^3c^3+3\,ab^2cd^2)x^3\right)\sqrt{bx^2+a}}{3\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*b*d^3*x^2 - 3*a*b^2*c^3*x + 3*a^2*b*c^2*d + 2*a^3*d^3 - (2*b^3
*c^3 + 3*a*b^2*c*d^2)*x^3)*sqrt(b*x^2 + a)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^3}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**3/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x)**3/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = -\frac{d^3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,c^3x}{3\,\sqrt{bx^2+a}a^2} + \frac{c^3x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{cd^2x}{(bx^2+a)^{\frac{3}{2}}b} + \frac{cd^2x}{\sqrt{bx^2+a}ab} - \frac{c^2d}{(bx^2+a)^{\frac{3}{2}}b} - \frac{2\,ad^3}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-d^3*x^2/((b*x^2 + a)^(3/2)*b) + 2/3*c^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^3
*x/((b*x^2 + a)^(3/2)*a) - c*d^2*x/((b*x^2 + a)^(3/2)*b) + c*d^2*x/(sqrt(b
*x^2 + a)*a*b) - c^2*d/((b*x^2 + a)^(3/2)*b) - 2/3*a*d^3/((b*x^2 + a)^(3/2
)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{3\,c^3}{a} - \left(\frac{3\,d^3}{b} - \frac{(2\,b^3c^3+3\,ab^2cd^2)x}{a^2b^2}\right)x\right)x - \frac{3\,a^2bc^2d+2\,a^3d^3}{a^2b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((3*c^3/a - (3*d^3/b - (2*b^3*c^3 + 3*a*b^2*c*d^2)*x/(a^2*b^2))*x)*x -
 (3*a^2*b*c^2*d + 2*a^3*d^3)/(a^2*b^2))/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = -\frac{2\,a^3\,d^3+3\,a^2\,b\,c^2\,d+3\,a^2\,b\,d^3\,x^2-3\,a\,b^2\,c^3\,x-3\,a\,b^2\,c\,d^2\,x^3-2\,b^3\,c^3\,x^3}{3\,a^2\,b^2\,\left(b\,x^2+a\right)^{3/2}}$$

input
```
int((c + d*x)^3/(a + b*x^2)^(5/2),x)
```

output
```
-(2*a^3*d^3 - 2*b^3*c^3*x^3 + 3*a^2*b*d^3*x^2 + 3*a^2*b*c^2*d - 3*a*b^2*c^
3*x - 3*a*b^2*c*d^2*x^3)/(3*a^2*b^2*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.12

$$\int \frac{(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^3d^3-3\sqrt{b\,x^2+a}\,a^2b\,c^2d-3\sqrt{b\,x^2+a}\,a^2b\,d^3x^2+3\sqrt{b\,x^2+a}\,a\,b^2c^3x+}{}$$

input
```
int((d*x+c)^3/(b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**3*d**3 - 3*sqrt(a + b*x**2)*a**2*b*c**2*d - 3*sq
rt(a + b*x**2)*a**2*b*d**3*x**2 + 3*sqrt(a + b*x**2)*a*b**2*c**3*x + 3*sqr
t(a + b*x**2)*a*b**2*c*d**2*x**3 + 2*sqrt(a + b*x**2)*b**3*c**3*x**3 + 3*s
qrt(b)*a**3*c*d**2 - 2*sqrt(b)*a**2*b*c**3 + 6*sqrt(b)*a**2*b*c*d**2*x**2
- 4*sqrt(b)*a*b**2*c**3*x**2 + 3*sqrt(b)*a*b**2*c*d**2*x**4 - 2*sqrt(b)*b*
*3*c**3*x**4)/(3*a**2*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.288 $\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}} \, dx = -\frac{2acd-(bc^2-ad^2)\,x}{3ab\,(a+bx^2)^{3/2}}+\frac{(2bc^2+ad^2)\,x}{3a^2b\sqrt{a+bx^2}}$$

output
```
-1/3*(2*a*c*d-(-a*d^2+b*c^2)*x)/a/b/(b*x^2+a)^(3/2)+1/3*(a*d^2+2*b*c^2)*x/
a^2/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}} \, dx = \frac{-2a^2cd+3abc^2x+2b^2c^2x^3+abd^2x^3}{3a^2b\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(c + d*x)^2/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a^2*c*d + 3*a*b*c^2*x + 2*b^2*c^2*x^3 + a*b*d^2*x^3)/(3*a^2*b*(a + b*x
^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {490, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 490}$$

$$\frac{2c\int \frac{c+dx}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x(c+dx)^2}{3a(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 453}$$

$$\frac{x(c+dx)^2}{3a(a+bx^2)^{3/2}}-\frac{2c(ad-bcx)}{3a^2b\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x)^2/(a + b*x^2)^(5/2),x]
```

output
```
(x*(c + d*x)^2)/(3*a*(a + b*x^2)^(3/2)) - (2*c*(a*d - b*c*x))/(3*a^2*b*Sqr
t[a + b*x^2])
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 490
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] - Simp[c*(n/(2*a*(
p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{-d^{2} x^{3} a b -2 b^{2} c^{2} x^{3}-3 a b \,c^{2} x +2 a^{2} c d}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2} b}$	55
trager	$-\frac{-d^{2} x^{3} a b -2 b^{2} c^{2} x^{3}-3 a b \,c^{2} x +2 a^{2} c d}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2} b}$	55
orering	$-\frac{-d^{2} x^{3} a b -2 b^{2} c^{2} x^{3}-3 a b \,c^{2} x +2 a^{2} c d}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2} b}$	55
default	$c^{2} \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)+d^{2} \left(-\frac{x}{2 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{a \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)}{2 b}\right)-\frac{2 c d}{3 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}$	110

input
```
int((d*x+c)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-a*b*d^2*x^3-2*b^2*c^2*x^3-3*a*b*c^2*x+2*a^2*c*d)/(b*x^2+a)^(3/2)/a^
2/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(3\,abc^2x - 2\,a^2cd + (2\,b^2c^2 + abd^2)x^3)\sqrt{bx^2+a}}{3\,(a^2b^3x^4 + 2\,a^3b^2x^2 + a^4b)}$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*a*b*c^2*x - 2*a^2*c*d + (2*b^2*c^2 + a*b*d^2)*x^3)*sqrt(b*x^2 + a)/
(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x)**2/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \frac{2\,c^2x}{3\,\sqrt{bx^2+a}a^2} + \frac{c^2x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{d^2x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{d^2x}{3\,\sqrt{bx^2+a}ab} - \frac{2\,cd}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*c^2*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^2*x/((b*x^2 + a)^(3/2)*a) - 1/3*d^
2*x/((b*x^2 + a)^(3/2)*b) + 1/3*d^2*x/(sqrt(b*x^2 + a)*a*b) - 2/3*c*d/((b*
x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{3\,c^2}{a} + \frac{(2\,b^2c^2+abd^2)x^2}{a^2b}\right)x - \frac{2\,cd}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((3*c^2/a + (2*b^2*c^2 + a*b*d^2)*x^2/(a^2*b))*x - 2*c*d/b)/(b*x^2 + a
)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \frac{a\,d^2\,x\,(b\,x^2+a)-2\,a^2\,c\,d-a^2\,d^2\,x+2\,b\,c^2\,x\,(b\,x^2+a)+a\,b\,c^2\,x}{3\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((c + d*x)^2/(a + b*x^2)^(5/2),x)
```

output
```
(a*d^2*x*(a + b*x^2) - 2*a^2*c*d - a^2*d^2*x + 2*b*c^2*x*(a + b*x^2) + a*b
*c^2*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.27

$$\int \frac{(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2bcd+3\sqrt{b\,x^2+a}\,a\,b^2c^2x+\sqrt{b\,x^2+a}\,a\,b^2d^2x^3+2\sqrt{b\,x^2+a}\,b^3c^2x^3+}{3a^2b^2\,(b^2x^4+2a}$$

input
```
int((d*x+c)^2/(b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*b*c*d + 3*sqrt(a + b*x**2)*a*b**2*c**2*x + sqr
t(a + b*x**2)*a*b**2*d**2*x**3 + 2*sqrt(a + b*x**2)*b**3*c**2*x**3 + sqrt(
b)*a**3*d**2 - 2*sqrt(b)*a**2*b*c**2 + 2*sqrt(b)*a**2*b*d**2*x**2 - 4*sqrt
(b)*a*b**2*c**2*x**2 + sqrt(b)*a*b**2*d**2*x**4 - 2*sqrt(b)*b**3*c**2*x**4
)/(3*a**2*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.289 $\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx = \frac{-ad+bcx}{3ab\,(a+bx^2)^{3/2}} + \frac{2cx}{3a^2\sqrt{a+bx^2}}$$

output
```
1/3*(b*c*x-a*d)/a/b/(b*x^2+a)^(3/2)+2/3*c*x/a^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx = \frac{-a^2d+3abcx+2b^2cx^3}{3a^2b\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(c + d*x)/(a + b*x^2)^(5/2),x]
```

output
```
(-(a^2*d) + 3*a*b*c*x + 2*b^2*c*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 454}$$

$$\frac{2c\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}-\frac{ad-bcx}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}-\frac{ad-bcx}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Int[(c + d*x)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*d - b*c*x)/(a*b*(a + b*x^2)^(3/2)) + (2*c*x)/(3*a^2*Sqrt[a + b*x^2
])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{-2b^{2}c\,x^{3}-3abcx+a^{2}d}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	39
trager	$-\frac{-2b^{2}c\,x^{3}-3abcx+a^{2}d}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	39
orering	$-\frac{-2b^{2}c\,x^{3}-3abcx+a^{2}d}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	39
default	$c\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)-\frac{d}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	50

input
```
int((d*x+c)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-2*b^2*c*x^3-3*a*b*c*x+a^2*d)/(b*x^2+a)^(3/2)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{c+dx}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{\left(2\,b^2cx^3+3\,abcx-a^2d\right)\sqrt{bx^2+a}}{3\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b^2*c*x^3 + 3*a*b*c*x - a^2*d)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3
*b^2*x^2 + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.

Time = 3.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.86

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx = c\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + d\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)/(b*x**2+a)**(5/2),x)
```

output
```
c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + d*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,cx}{3\,\sqrt{bx^2+a}a^2} + \frac{cx}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{d}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*x/((b*x^2 + a)^(3/2)*a) - 1/3*d/((b*
x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{2\,bcx^2}{a^2}+\frac{3\,c}{a}\right)x-\frac{d}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((2*b*c*x^2/a^2 + 3*c/a)*x - d/b)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,b\,c\,x\,(b\,x^2+a)-a^2\,d+a\,b\,c\,x}{3\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((c + d*x)/(a + b*x^2)^(5/2),x)
```

output
```
(2*b*c*x*(a + b*x^2) - a^2*d + a*b*c*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.02

$$\int \frac{c+dx}{(a+bx^2)^{5/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^2d+3\sqrt{b\,x^2+a}\,abcx+2\sqrt{b\,x^2+a}\,b^2c\,x^3-2\sqrt{b}\,a^2c-4\sqrt{b}\,abc\,x^2-2\sqrt{}}{3a^2b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((d*x+c)/(b*x^2+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*d + 3*sqrt(a + b*x**2)*a*b*c*x + 2*sqrt(a + b*x*
*2)*b**2*c*x**3 - 2*sqrt(b)*a**2*c - 4*sqrt(b)*a*b*c*x**2 - 2*sqrt(b)*b**2
*c*x**4)/(3*a**2*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.290 $\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx = \frac{ad+bcx}{3a\left(bc^2+ad^2\right)\left(a+bx^2\right)^{3/2}} + \frac{3a^2d^3+bc(2bc^2+5ad^2)\,x}{3a^2\left(bc^2+ad^2\right)^2\sqrt{a+bx^2}} - \frac{d^4\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{\left(bc^2+ad^2\right)^{5/2}}$$

output
```
1/3*(b*c*x+a*d)/a/(a*d^2+b*c^2)/(b*x^2+a)^(3/2)+1/3*(3*a^2*d^3+b*c*(5*a*d^
2+2*b*c^2)*x)/a^2/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)-d^4*arctanh((-b*c*x+a*d)
/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx = \frac{4a^3d^3+2b^3c^3x^3+a^2bd(c^2+6cdx+3d^2x^2)+ab^2cx(3c^2+5d^2x^2)}{3a^2\left(bc^2+ad^2\right)^2\left(a+bx^2\right)^{3/2}} - \frac{2d^4\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[1/((c + d*x)*(a + b*x^2)^(5/2)),x]
```

output
```
(4*a^3*d^3 + 2*b^3*c^3*x^3 + a^2*b*d*(c^2 + 6*c*d*x + 3*d^2*x^2) + a*b^2*c
*x*(3*c^2 + 5*d^2*x^2))/(3*a^2*(b*c^2 + a*d^2)^2*(a + b*x^2)^(3/2)) - (2*d
^4*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])
/(-(b*c^2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {496, 25, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx)}dx$$

$$\Big\downarrow \ 496$$

$$\frac{ad+bcx}{3a(a+bx^2)^{3/2}(ad^2+bc^2)}-\frac{\int -\frac{2bc^2+2bdxc+3ad^2}{(c+dx)(bx^2+a)^{3/2}}dx}{3a(ad^2+bc^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2bc^2+2bdxc+3ad^2}{(c+dx)(bx^2+a)^{3/2}}dx}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(ad^2+bc^2)}$$

$$\Big\downarrow \ 686$$

$$\frac{\frac{3a^2d^3+bcx(5ad^2+2bc^2)}{a\sqrt{a+bx^2}(ad^2+bc^2)}-\frac{\int -\frac{3a^2bd^4}{(c+dx)\sqrt{bx^2+a}}dx}{ab(ad^2+bc^2)}}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(ad^2+bc^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3ad^4\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{3a^2d^3+bcx(5ad^2+2bc^2)}{a\sqrt{a+bx^2}(ad^2+bc^2)}}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{3a^2d^3+bcx(5ad^2+2bc^2)}{a\sqrt{a+bx^2}(ad^2+bc^2)}-\frac{3ad^4\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3a^2d^3+bcx(5ad^2+2bc^2)}{a\sqrt{a+bx^2}(ad^2+bc^2)}-\frac{3ad^4\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

input
```
Int[1/((c + d*x)*(a + b*x^2)^(5/2)),x]
```

output
```
(a*d + b*c*x)/(3*a*(b*c^2 + a*d^2)*(a + b*x^2)^(3/2)) + ((3*a^2*d^3 + b*c*
(2*b*c^2 + 5*a*d^2)*x)/(a*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) - (3*a*d^4*ArcT
anh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^
(3/2))/(3*a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(140) = 280$.

Time = 0.27 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.90

method	result
default	$\frac{\frac{d^2}{3\left(a d^2+b c^2\right)\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}\right)^{\frac{3}{2}}}+\frac{bcd\left(\frac{\frac{4b\left(x+\frac{c}{d}\right)}{3}-\frac{4bc}{3d}}{\left(\frac{4b\left(a d^2+b c^2\right)}{d^2}-\frac{4b^2c^2}{d^2}\right)\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}\right)^{\frac{3}{2}}}+\frac{\cdots}{3\left(\frac{4b\left(a d^2\cdots\right.}{d^2}\cdots\right.}\cdots\right.}{a d^2+b c^2}\cdots}{\cdots}$

input
```
int(1/(d*x+c)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(3/2)+b*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d
^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16
/3*b/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))+1/(a*d^2+b*c^2)*d^2*(1/(a*
d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c
*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^
2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d
^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a
*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/
2))/(x+c/d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(141) = 282$.

Time = 0.17 (sec) , antiderivative size = 858, normalized size of antiderivative = 5.57

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*b^2*d^4*x^4 + 2*a^3*b*d^4*x^2 + a^4*d^4)*sqrt(b*c^2 + a*d^2)*
log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqr
t(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2))
 + 2*(a^2*b^2*c^4*d + 5*a^3*b*c^2*d^3 + 4*a^4*d^5 + (2*b^4*c^5 + 7*a*b^3*c
^3*d^2 + 5*a^2*b^2*c*d^4)*x^3 + 3*(a^2*b^2*c^2*d^3 + a^3*b*d^5)*x^2 + 3*(a
*b^3*c^5 + 3*a^2*b^2*c^3*d^2 + 2*a^3*b*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b^3
*c^6 + 3*a^5*b^2*c^4*d^2 + 3*a^6*b*c^2*d^4 + a^7*d^6 + (a^2*b^5*c^6 + 3*a^
3*b^4*c^4*d^2 + 3*a^4*b^3*c^2*d^4 + a^5*b^2*d^6)*x^4 + 2*(a^3*b^4*c^6 + 3*
a^4*b^3*c^4*d^2 + 3*a^5*b^2*c^2*d^4 + a^6*b*d^6)*x^2), -1/3*(3*(a^2*b^2*d^
4*x^4 + 2*a^3*b*d^4*x^2 + a^4*d^4)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2
 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*
b*d^2)*x^2)) - (a^2*b^2*c^4*d + 5*a^3*b*c^2*d^3 + 4*a^4*d^5 + (2*b^4*c^5 +
 7*a*b^3*c^3*d^2 + 5*a^2*b^2*c*d^4)*x^3 + 3*(a^2*b^2*c^2*d^3 + a^3*b*d^5)*
x^2 + 3*(a*b^3*c^5 + 3*a^2*b^2*c^3*d^2 + 2*a^3*b*c*d^4)*x)*sqrt(b*x^2 + a)
)/(a^4*b^3*c^6 + 3*a^5*b^2*c^4*d^2 + 3*a^6*b*c^2*d^4 + a^7*d^6 + (a^2*b^5*
c^6 + 3*a^3*b^4*c^4*d^2 + 3*a^4*b^3*c^2*d^4 + a^5*b^2*d^6)*x^4 + 2*(a^3*b^
4*c^6 + 3*a^4*b^3*c^4*d^2 + 3*a^5*b^2*c^2*d^4 + a^6*b*d^6)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**2+a)**(5/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(141) = 282$.

Time = 0.06 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.84

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx = \frac{bcx}{3\left((bx^2+a)^{\frac{3}{2}}abc^2+(bx^2+a)^{\frac{3}{2}}a^2d^2\right)} + \frac{bcx}{2\sqrt{bx^2+a}a^2bc^2+\frac{\sqrt{bx^2+a}ab^2c^4}{d^2}+\sqrt{bx^2+a}a^3d^2} + \frac{2\,bcx}{3\left(\sqrt{bx^2+a}a^2bc^2+\sqrt{bx^2+a}a^3d^2\right)} + \frac{1}{\frac{\sqrt{bx^2+a}b^2c^4}{d^3}+\frac{2\sqrt{bx^2+a}abc^2}{d}+\sqrt{bx^2+a}a^2d} + \frac{1}{3\left(\frac{(bx^2+a)^{\frac{3}{2}}bc^2}{d}+(bx^2+a)^{\frac{3}{2}}ad\right)} + \frac{\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*b*c*x/((b*x^2 + a)^(3/2)*a*b*c^2 + (b*x^2 + a)^(3/2)*a^2*d^2) + b*c*x/
(2*sqrt(b*x^2 + a)*a^2*b*c^2 + sqrt(b*x^2 + a)*a*b^2*c^4/d^2 + sqrt(b*x^2
+ a)*a^3*d^2) + 2/3*b*c*x/(sqrt(b*x^2 + a)*a^2*b*c^2 + sqrt(b*x^2 + a)*a^3
*d^2) + 1/(sqrt(b*x^2 + a)*b^2*c^4/d^3 + 2*sqrt(b*x^2 + a)*a*b*c^2/d + sqr
t(b*x^2 + a)*a^2*d) + 1/3/((b*x^2 + a)^(3/2)*b*c^2/d + (b*x^2 + a)^(3/2)*a
*d) + arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)
))/((a + b*c^2/d^2)^(5/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. 2(141) = 282.

Time = 0.14 (sec) , antiderivative size = 998, normalized size of antiderivative = 6.48

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-2*d^4*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 -
a*d^2))/((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*sqrt(-b*c^2 - a*d^2)) + 1/3*(
(((2*b^10*c^15 + 17*a*b^9*c^13*d^2 + 60*a^2*b^8*c^11*d^4 + 115*a^3*b^7*c^9
*d^6 + 130*a^4*b^6*c^7*d^8 + 87*a^5*b^5*c^5*d^10 + 32*a^6*b^4*c^3*d^12 + 5
*a^7*b^3*c*d^14)*x/(a^2*b^9*c^16 + 8*a^3*b^8*c^14*d^2 + 28*a^4*b^7*c^12*d^
4 + 56*a^5*b^6*c^10*d^6 + 70*a^6*b^5*c^8*d^8 + 56*a^7*b^4*c^6*d^10 + 28*a^
8*b^3*c^4*d^12 + 8*a^9*b^2*c^2*d^14 + a^10*b*d^16) + 3*(a^2*b^8*c^12*d^3 +
 6*a^3*b^7*c^10*d^5 + 15*a^4*b^6*c^8*d^7 + 20*a^5*b^5*c^6*d^9 + 15*a^6*b^4
*c^4*d^11 + 6*a^7*b^3*c^2*d^13 + a^8*b^2*d^15)/(a^2*b^9*c^16 + 8*a^3*b^8*c
^14*d^2 + 28*a^4*b^7*c^12*d^4 + 56*a^5*b^6*c^10*d^6 + 70*a^6*b^5*c^8*d^8 +
 56*a^7*b^4*c^6*d^10 + 28*a^8*b^3*c^4*d^12 + 8*a^9*b^2*c^2*d^14 + a^10*b*d
^16))*x + 3*(a*b^9*c^15 + 8*a^2*b^8*c^13*d^2 + 27*a^3*b^7*c^11*d^4 + 50*a^
4*b^6*c^9*d^6 + 55*a^5*b^5*c^7*d^8 + 36*a^6*b^4*c^5*d^10 + 13*a^7*b^3*c^3*
d^12 + 2*a^8*b^2*c*d^14)/(a^2*b^9*c^16 + 8*a^3*b^8*c^14*d^2 + 28*a^4*b^7*c
^12*d^4 + 56*a^5*b^6*c^10*d^6 + 70*a^6*b^5*c^8*d^8 + 56*a^7*b^4*c^6*d^10 +
 28*a^8*b^3*c^4*d^12 + 8*a^9*b^2*c^2*d^14 + a^10*b*d^16))*x + (a^2*b^8*c^1
4*d + 10*a^3*b^7*c^12*d^3 + 39*a^4*b^6*c^10*d^5 + 80*a^5*b^5*c^8*d^7 + 95*
a^6*b^4*c^6*d^9 + 66*a^7*b^3*c^4*d^11 + 25*a^8*b^2*c^2*d^13 + 4*a^9*b*d^15
)/(a^2*b^9*c^16 + 8*a^3*b^8*c^14*d^2 + 28*a^4*b^7*c^12*d^4 + 56*a^5*b^6*c^
10*d^6 + 70*a^6*b^5*c^8*d^8 + 56*a^7*b^4*c^6*d^10 + 28*a^8*b^3*c^4*d^12...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx=\int \frac{1}{{\left(b\,x^2+a\right)}^{5/2}\,\left(c+d\,x\right)} \,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 3779, normalized size of antiderivative = 24.54

$$\int \frac{1}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)/(b*x^2+a)^(5/2),x)
```

output
```
( - 6*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**3*c*d**2 - 12*sqrt(b)*
sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 +
b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2
 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b*c*d**2*x**2 - 6*sqrt(b)*sqrt(2*s
qrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*
atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**
2)*c - a*d**2 - 2*b*c**2))*a*b**2*c*d**2*x**4 - 6*sqrt(2*sqrt(b)*sqrt(a*d*
*2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x
)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**4*d**4 -
 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(
a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d*
*2 - 2*b*c**2))*a**3*b*c**2*d**2 - 12*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)
*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqr
t(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**3*b*d**4*x**2 - 12*s
qrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a +
b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 -
 2*b*c**2))*a**2*b**2*c**2*d**2*x**2 - 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c*
*2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt...
```

3.291 $\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 217

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}} \, dx = -\frac{d}{(bc^2+ad^2)(c+dx)(a+bx^2)^{3/2}} + \frac{b(5acd+(bc^2-4ad^2)x)}{3a(bc^2+ad^2)^2(a+bx^2)^{3/2}} + \frac{b(15a^2cd^3+(2b^2c^4+9abc^2d^2-8a^2d^4)x)}{3a^2(bc^2+ad^2)^3\sqrt{a+bx^2}} - \frac{5bcd^4\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{7/2}}$$

output
```
-d/(a*d^2+b*c^2)/(d*x+c)/(b*x^2+a)^(3/2)+1/3*b*(5*a*c*d+(-4*a*d^2+b*c^2)*x
)/a/(a*d^2+b*c^2)^2/(b*x^2+a)^(3/2)+1/3*b*(15*a^2*c*d^3+(-8*a^2*d^4+9*a*b*
c^2*d^2+2*b^2*c^4)*x)/a^2/(a*d^2+b*c^2)^3/(b*x^2+a)^(1/2)-5*b*c*d^4*arctan
h((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.14

$$\int \frac{1}{(c+dx)^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{-3a^4d^5+2b^4c^4x^3(c+dx)+2a^3bd^3(7c^2+4cdx-6d^2x^2)+3ab^3c^2x(c^3+c^2}{3a^2\left(bc^2+ad^2\right)^3(c} + \frac{10bcd^4\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{7/2}}$$

input
```
Integrate[1/((c + d*x)^2*(a + b*x^2)^(5/2)),x]
```

output
```
(-3*a^4*d^5 + 2*b^4*c^4*x^3*(c + d*x) + 2*a^3*b*d^3*(7*c^2 + 4*c*d*x - 6*d
^2*x^2) + 3*a*b^3*c^2*x*(c^3 + c^2*d*x + 3*c*d^2*x^2 + 3*d^3*x^3) + a^2*b^
2*d*(2*c^4 + 11*c^3*d*x + 21*c^2*d^2*x^2 + 7*c*d^3*x^3 - 8*d^4*x^4))/(3*a^
2*(b*c^2 + a*d^2)^3*(c + d*x)*(a + b*x^2)^(3/2)) + (10*b*c*d^4*ArcTan[(Sqr
t[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a
*d^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {496, 25, 686, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)} - \frac{\int -\frac{2\left(bc^2+2ad^2\right)+3bcdx}{(c+dx)^2\left(bx^2+a\right)^{3/2}}dx}{3a\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(bc^2+2ad^2)+3bcdx}{(c+dx)^2(bx^2+a)^{3/2}}dx}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)}$$

$\Big\downarrow$ 686

$$\frac{-\frac{\int \frac{bd\left(2ad\left(bc^2-4ad^2\right)-bc\left(2bc^2+7ad^2\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab(ad^2+bc^2)}-\frac{ad(bc^2-4ad^2)-bcx(7ad^2+2bc^2)}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{d\int \frac{2ad\left(bc^2-4ad^2\right)-bc\left(2bc^2+7ad^2\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}-\frac{ad(bc^2-4ad^2)-bcx(7ad^2+2bc^2)}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)}$$

$\Big\downarrow$ 679

$$\frac{-\frac{d\left(-\frac{15a^2bcd^3\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(-8a^2d^4+9abc^2d^2+2b^2c^4\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\frac{ad(bc^2-4ad^2)-bcx(7ad^2+2bc^2)}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)}$$

$\Big\downarrow$ 488

$$\frac{-\frac{d\left(\frac{15a^2bcd^3\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(-8a^2d^4+9abc^2d^2+2b^2c^4\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\frac{ad(bc^2-4ad^2)-bcx(7ad^2+2bc^2)}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}}{3a(ad^2+bc^2)}+\frac{ad+bcx}{3a(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)}$$

$\Big\downarrow$ 219

$$\frac{-\frac{d\left(\frac{15a^2bcd^3\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}\left(-8a^2d^4+9abc^2d^2+2b^2c^4\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\frac{ad(bc^2-4ad^2)-bcx(7ad^2+2bc^2)}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[1/((c + d*x)^2*(a + b*x^2)^(5/2)),x]
```

output
```
(a*d + b*c*x)/(3*a*(b*c^2 + a*d^2)*(c + d*x)*(a + b*x^2)^(3/2)) + (-((a*d*
(b*c^2 - 4*a*d^2) - b*c*(2*b*c^2 + 7*a*d^2)*x)/(a*(b*c^2 + a*d^2)*(c + d*x
)*Sqrt[a + b*x^2])) - (d*(-(((2*b^2*c^4 + 9*a*b*c^2*d^2 - 8*a^2*d^4)*Sqrt[
a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x))) + (15*a^2*b*c*d^3*ArcTanh[(a*d -
b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)))/(a*
(b*c^2 + a*d^2)))/(3*a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(201) = 402$.

Time = 0.29 (sec) , antiderivative size = 899, normalized size of antiderivative = 4.14

method	result
default	$-\frac{d^2}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}+\frac{5bcd\left(\frac{d^2}{3\left(a\,d^2+b\,c^2\right)\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}+\frac{bcd\left(\frac{\ldots}{\left(\frac{4b\left(a\,d^2\ldots\right)}{d^2}\ldots\right)}\ldots\right)}{\ldots}\right)}{\ldots}$

input

```
int(1/(d*x+c)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(3/2)+5*b*c*d/(a*d^2+b*c^2)*(1/3/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+b*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+
c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(
x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16/3*b/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^
2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2))+1/(a*d^2+b*c^2)*d^2*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c
/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(
4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^
2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))-4*b/(a*d^2+b*c^2)*d^2*(2/
3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2
-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16/3*b/(4*b*(a*d^2+b*c^2)/d^2-4*
b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(202) = 404$.

Time = 0.36 (sec) , antiderivative size = 1678, normalized size of antiderivative = 7.73

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a^2*b^3*c*d^5*x^5 + a^2*b^3*c^2*d^4*x^4 + 2*a^3*b^2*c*d^5*x^3 +
2*a^3*b^2*c^2*d^4*x^2 + a^4*b*c*d^5*x + a^4*b*c^2*d^4)*sqrt(b*c^2 + a*d^2)
*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sq
rt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*(2*a^2*b^3*c^6*d + 16*a^3*b^2*c^4*d^3 + 11*a^4*b*c^2*d^5 - 3*a^5*d^7
 + (2*b^5*c^6*d + 11*a*b^4*c^4*d^3 + a^2*b^3*c^2*d^5 - 8*a^3*b^2*d^7)*x^4
+ (2*b^5*c^7 + 11*a*b^4*c^5*d^2 + 16*a^2*b^3*c^3*d^4 + 7*a^3*b^2*c*d^6)*x^
3 + 3*(a*b^4*c^6*d + 8*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^2*d^5 - 4*a^4*b*d^7)*
x^2 + (3*a*b^4*c^7 + 14*a^2*b^3*c^5*d^2 + 19*a^3*b^2*c^3*d^4 + 8*a^4*b*c*d
^6)*x)*sqrt(b*x^2 + a))/(a^4*b^4*c^9 + 4*a^5*b^3*c^7*d^2 + 6*a^6*b^2*c^5*d
^4 + 4*a^7*b*c^3*d^6 + a^8*c*d^8 + (a^2*b^6*c^8*d + 4*a^3*b^5*c^6*d^3 + 6*
a^4*b^4*c^4*d^5 + 4*a^5*b^3*c^2*d^7 + a^6*b^2*d^9)*x^5 + (a^2*b^6*c^9 + 4*
a^3*b^5*c^7*d^2 + 6*a^4*b^4*c^5*d^4 + 4*a^5*b^3*c^3*d^6 + a^6*b^2*c*d^8)*x
^4 + 2*(a^3*b^5*c^8*d + 4*a^4*b^4*c^6*d^3 + 6*a^5*b^3*c^4*d^5 + 4*a^6*b^2*
c^2*d^7 + a^7*b*d^9)*x^3 + 2*(a^3*b^5*c^9 + 4*a^4*b^4*c^7*d^2 + 6*a^5*b^3*
c^5*d^4 + 4*a^6*b^2*c^3*d^6 + a^7*b*c*d^8)*x^2 + (a^4*b^4*c^8*d + 4*a^5*b^
3*c^6*d^3 + 6*a^6*b^2*c^4*d^5 + 4*a^7*b*c^2*d^7 + a^8*d^9)*x), -1/3*(15*(a
^2*b^3*c*d^5*x^5 + a^2*b^3*c^2*d^4*x^4 + 2*a^3*b^2*c*d^5*x^3 + 2*a^3*b^2*c
^2*d^4*x^2 + a^4*b*c*d^5*x + a^4*b*c^2*d^4)*sqrt(-b*c^2 - a*d^2)*arctan(sq
rt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}} \, dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}} (c+dx)^2} \, dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(202) = 404.

Time = 0.09 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.59

$$\begin{aligned}
\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}} \, dx = & \frac{5 b^2 c^2 x}{3 \left((bx^2+a)^{\frac{3}{2}} a b^2 c^4 + 2 (bx^2+a)^{\frac{3}{2}} a^2 b c^2 d^2 + (bx^2+a)^{\frac{3}{2}} a^3 d^4 \right)} \\
& + \frac{5 b^2 c^2 x}{3 \sqrt{bx^2+a} a^2 b^2 c^4 + \frac{\sqrt{bx^2+a} a b^3 c^6}{d^2} + 3 \sqrt{bx^2+a} a^3 b c^2 d^2 + \sqrt{bx^2+a} a^4 d^4} \\
& + \frac{10 b^2 c^2 x}{3 \left(\sqrt{bx^2+a} a^2 b^2 c^4 + 2 \sqrt{bx^2+a} a^3 b c^2 d^2 + \sqrt{bx^2+a} a^4 d^4 \right)} \\
& + \frac{5 bc}{\frac{\sqrt{bx^2+a} b^3 c^6}{d^3} + \frac{3 \sqrt{bx^2+a} a b^2 c^4}{d} + 3 \sqrt{bx^2+a} a^2 b c^2 d + \sqrt{bx^2+a} a^3 d^3} \\
& + \frac{5 bc}{3 \left(\frac{(bx^2+a)^{\frac{3}{2}} b^2 c^4}{d} + 2 (bx^2+a)^{\frac{3}{2}} a b c^2 d + (bx^2+a)^{\frac{3}{2}} a^2 d^3 \right)} \\
& - \frac{4 bx}{3 \left((bx^2+a)^{\frac{3}{2}} a b c^2 + (bx^2+a)^{\frac{3}{2}} a^2 d^2 \right)} - \frac{8 bx}{3 \left(\sqrt{bx^2+a} a^2 b c^2 + \sqrt{bx^2+a} a^3 d^2 \right)} \\
& - \frac{1}{(bx^2+a)^{\frac{3}{2}} b c^2 x + (bx^2+a)^{\frac{3}{2}} a d^2 x + \frac{(bx^2+a)^{\frac{3}{2}} b c^3}{d} + (bx^2+a)^{\frac{3}{2}} a c d} \\
& + \frac{5 bc \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|} \right)}{\left(a + \frac{bc^2}{d^2} \right)^{\frac{7}{2}} d^3}
\end{aligned}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
5/3*b^2*c^2*x/((b*x^2 + a)^(3/2)*a*b^2*c^4 + 2*(b*x^2 + a)^(3/2)*a^2*b*c^2
*d^2 + (b*x^2 + a)^(3/2)*a^3*d^4) + 5*b^2*c^2*x/(3*sqrt(b*x^2 + a)*a^2*b^2
*c^4 + sqrt(b*x^2 + a)*a*b^3*c^6/d^2 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^2 + s
qrt(b*x^2 + a)*a^4*d^4) + 10/3*b^2*c^2*x/(sqrt(b*x^2 + a)*a^2*b^2*c^4 + 2*
sqrt(b*x^2 + a)*a^3*b*c^2*d^2 + sqrt(b*x^2 + a)*a^4*d^4) + 5*b*c/(sqrt(b*x
^2 + a)*b^3*c^6/d^3 + 3*sqrt(b*x^2 + a)*a*b^2*c^4/d + 3*sqrt(b*x^2 + a)*a^
2*b*c^2*d + sqrt(b*x^2 + a)*a^3*d^3) + 5/3*b*c/((b*x^2 + a)^(3/2)*b^2*c^4/
d + 2*(b*x^2 + a)^(3/2)*a*b*c^2*d + (b*x^2 + a)^(3/2)*a^2*d^3) - 4/3*b*x/(
(b*x^2 + a)^(3/2)*a*b*c^2 + (b*x^2 + a)^(3/2)*a^2*d^2) - 8/3*b*x/(sqrt(b*x
^2 + a)*a^2*b*c^2 + sqrt(b*x^2 + a)*a^3*d^2) - 1/((b*x^2 + a)^(3/2)*b*c^2*
x + (b*x^2 + a)^(3/2)*a*d^2*x + (b*x^2 + a)^(3/2)*b*c^3/d + (b*x^2 + a)^(3
/2)*a*c*d) + 5*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)
*abs(d*x + c)))/((a + b*c^2/d^2)^(7/2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. 2(202) = 404.

Time = 0.25 (sec) , antiderivative size = 1180, normalized size of antiderivative = 5.44

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(15*b*c*d^7*log(abs(-b*c*d + sqrt(b*c^2 + a*d^2)*(sqrt(b - 2*b*c/(d*x
 + c) + b*c^2/(d*x + c)^2 + a*d^2/(d*x + c)^2) + sqrt(b*c^2*d^2 + a*d^4)/(
(d*x + c)*d))*abs(d)))/((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3
*d^6)*sqrt(b*c^2 + a*d^2)*abs(d)*sgn(1/(d*x + c))*sgn(d)) - (15*a^2*b^(3/2
)*c*d^7*log(abs(-b*c*d + sqrt(b*c^2 + a*d^2)*sqrt(b)*abs(d))) - 2*sqrt(b*c
^2 + a*d^2)*b^3*c^4*d^2*abs(d) - 9*sqrt(b*c^2 + a*d^2)*a*b^2*c^2*d^4*abs(d
) + 8*sqrt(b*c^2 + a*d^2)*a^2*b*d^6*abs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(
b*c^2 + a*d^2)*a^2*b^(7/2)*c^6*abs(d) + 3*sqrt(b*c^2 + a*d^2)*a^3*b^(5/2)*
c^4*d^2*abs(d) + 3*sqrt(b*c^2 + a*d^2)*a^4*b^(3/2)*c^2*d^4*abs(d) + sqrt(b
*c^2 + a*d^2)*a^5*sqrt(b)*d^6*abs(d)) - ((2*b^5*c^4*d^13 + 9*a*b^4*c^2*d^1
5 - 8*a^2*b^3*d^17)/(a^2*b^4*c^6*d^11*sgn(1/(d*x + c))*sgn(d) + 3*a^3*b^3*
c^4*d^13*sgn(1/(d*x + c))*sgn(d) + 3*a^4*b^2*c^2*d^15*sgn(1/(d*x + c))*sgn
(d) + a^5*b*d^17*sgn(1/(d*x + c))*sgn(d)) - (3*(2*b^5*c^5*d^14 + 9*a*b^4*c
^3*d^16 - 13*a^2*b^3*c*d^18)/(a^2*b^4*c^6*d^11*sgn(1/(d*x + c))*sgn(d) + 3
*a^3*b^3*c^4*d^13*sgn(1/(d*x + c))*sgn(d) + 3*a^4*b^2*c^2*d^15*sgn(1/(d*x
+ c))*sgn(d) + a^5*b*d^17*sgn(1/(d*x + c))*sgn(d)) - (6*(b^5*c^6*d^15 + 5*
a*b^4*c^4*d^17 - 8*a^2*b^3*c^2*d^19 - 2*a^3*b^2*d^21)/(a^2*b^4*c^6*d^11*sg
n(1/(d*x + c))*sgn(d) + 3*a^3*b^3*c^4*d^13*sgn(1/(d*x + c))*sgn(d) + 3*a^4
*b^2*c^2*d^15*sgn(1/(d*x + c))*sgn(d) + a^5*b*d^17*sgn(1/(d*x + c))*sgn(d)
) - (2*(b^5*c^7*d^16 + 6*a*b^4*c^5*d^18 - 11*a^2*b^3*c^3*d^20 - 16*a^3*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}} dx = \int \frac{1}{(b x^2+a)^{5/2} (c+d x)^2} dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1399, normalized size of antiderivative = 6.45

$$\int \frac{1}{(c+dx)^2 (a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(b*x^2+a)^(5/2),x)
```

output
```
(15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**4*b*c**2*d**4 + 15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b*c*d**5*x + 30*sqrt(a*d**2 + b
*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**2
*c**2*d**4*x**2 + 30*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*a**3*b**2*c*d**5*x**3 + 15*sqrt(a*d**2 + b*c**2
)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**2
*d**4*x**4 + 15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a**2*b**3*c*d**5*x**5 - 15*sqrt(a*d**2 + b*c**2)*log
(c + d*x)*a**4*b*c**2*d**4 - 15*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**4*b*
c*d**5*x - 30*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**2*c**2*d**4*x**2
- 30*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b**2*c*d**5*x**3 - 15*sqrt(a*
d**2 + b*c**2)*log(c + d*x)*a**2*b**3*c**2*d**4*x**4 - 15*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*a**2*b**3*c*d**5*x**5 - 3*sqrt(a + b*x**2)*a**5*d**7 +
11*sqrt(a + b*x**2)*a**4*b*c**2*d**5 + 8*sqrt(a + b*x**2)*a**4*b*c*d**6*x
- 12*sqrt(a + b*x**2)*a**4*b*d**7*x**2 + 16*sqrt(a + b*x**2)*a**3*b**2*c**
4*d**3 + 19*sqrt(a + b*x**2)*a**3*b**2*c**3*d**4*x + 9*sqrt(a + b*x**2)*a*
*3*b**2*c**2*d**5*x**2 + 7*sqrt(a + b*x**2)*a**3*b**2*c*d**6*x**3 - 8*sqrt
(a + b*x**2)*a**3*b**2*d**7*x**4 + 2*sqrt(a + b*x**2)*a**2*b**3*c**6*d + 1
4*sqrt(a + b*x**2)*a**2*b**3*c**5*d**2*x + 24*sqrt(a + b*x**2)*a**2*b**...
```

3.292 $\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 300

$$\begin{aligned}\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}} \, dx &= -\frac{d}{2 (bc^2+ad^2) (c+dx)^2 (a+bx^2)^{3/2}} \\ &- \frac{7bcd}{2 (bc^2+ad^2)^2 (c+dx) (a+bx^2)^{3/2}} + \frac{b(5ad(6bc^2-ad^2)+bc(2bc^2-33ad^2) x)}{6a (bc^2+ad^2)^3 (a+bx^2)^{3/2}} \\ &+ \frac{b(15a^2d^3(6bc^2-ad^2)+bc(4b^2c^4+28abc^2d^2-81a^2d^4) x)}{6a^2 (bc^2+ad^2)^4 \sqrt{a+bx^2}} \\ &- \frac{5bd^4(6bc^2-ad^2) \operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2 (bc^2+ad^2)^{9/2}}\end{aligned}$$

output
```
-1/2*d/(a*d^2+b*c^2)/(d*x+c)^2/(b*x^2+a)^(3/2)-7/2*b*c*d/(a*d^2+b*c^2)^2/(
d*x+c)/(b*x^2+a)^(3/2)+1/6*b*(5*a*d*(-a*d^2+6*b*c^2)+b*c*(-33*a*d^2+2*b*c^
2)*x)/a/(a*d^2+b*c^2)^3/(b*x^2+a)^(3/2)+1/6*b*(15*a^2*d^3*(-a*d^2+6*b*c^2)
+b*c*(-81*a^2*d^4+28*a*b*c^2*d^2+4*b^2*c^4)*x)/a^2/(a*d^2+b*c^2)^4/(b*x^2+
a)^(1/2)-5/2*b*d^4*(-a*d^2+6*b*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/
2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(9/2)
```

Mathematica [A] (verified)

Time = 10.80 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.99

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}}\,dx = \frac{1}{6}\Bigg(\frac{\sqrt{a+bx^2}\Big(-\frac{3d^5(bc^2+ad^2)}{(c+dx)^2}-\frac{33bcd^5}{c+dx}+\frac{4b(-3a^3d^5+b^3c^5x+7ab^2c^3d^2x+3a^2bcd^3(5c-4dx)}{a^2(a+bx^2)}}{(bc^2+ad^2)^4}$$
$$+\frac{15bd^4(6bc^2-ad^2)\log(c+dx)}{(bc^2+ad^2)^{9/2}}$$
$$+\frac{15bd^4(-6bc^2+ad^2)\log\left(ad-bcx+\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\right)}{(bc^2+ad^2)^{9/2}}\Bigg)$$

input
```
Integrate[1/((c + d*x)^3*(a + b*x^2)^(5/2)),x]
```

output
```
((Sqrt[a + b*x^2]*((-3*d^5*(b*c^2 + a*d^2))/(c + d*x)^2 - (33*b*c*d^5)/(c
+ d*x) + (4*b*(-3*a^3*d^5 + b^3*c^5*x + 7*a*b^2*c^3*d^2*x + 3*a^2*b*c*d^3*
(5*c - 4*d*x)))/(a^2*(a + b*x^2)) + (2*b*(b*c^2 + a*d^2)*(-(a^2*d^3) + b^2
*c^3*x + 3*a*b*c*d*(c - d*x)))/(a*(a + b*x^2)^2)))/(b*c^2 + a*d^2)^4 + (15
*b*d^4*(6*b*c^2 - a*d^2)*Log[c + d*x])/(b*c^2 + a*d^2)^(9/2) + (15*b*d^4*(
-6*b*c^2 + a*d^2)*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/
(b*c^2 + a*d^2)^(9/2))/6
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {496, 25, 686, 27, 688, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx)^3}\,dx$$

$$\Big\downarrow \; 496$$

$$\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{2bc^2+4bdxc+5ad^2}{(c+dx)^3\left(bx^2+a\right)^{3/2}}dx}{3a\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2bc^2+4bdxc+5ad^2}{(c+dx)^3\left(bx^2+a\right)^{3/2}}dx}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 686}$$

$$\frac{-\frac{\int \frac{bd\left(3ad\left(2bc^2-5ad^2\right)-2bc\left(2bc^2+9ad^2\right)x\right)}{(c+dx)^3\sqrt{bx^2+a}}dx}{ab\left(ad^2+bc^2\right)}-\frac{ad\left(2bc^2-5ad^2\right)-bcx\left(9ad^2+2bc^2\right)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{d\int \frac{3ad\left(2bc^2-5ad^2\right)-2bc\left(2bc^2+9ad^2\right)x}{(c+dx)^3\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}-\frac{ad\left(2bc^2-5ad^2\right)-bcx\left(9ad^2+2bc^2\right)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 688}$$

$$\frac{-\frac{d\left(-\frac{\int -\frac{b\left(2acd\left(2bc^2-33ad^2\right)-\left(4b^2c^4+24abd^2c^2-15a^2d^4\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(-15a^2d^4+24abc^2d^2+4b^2c^4\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}-\frac{ad\left(2bc^2-5ad^2\right)-bcx\left(9ad^2+2bc^2\right)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{d\left(\frac{\int \frac{b\left(2acd\left(2bc^2-33ad^2\right)-\left(4b^2c^4+24abd^2c^2-15a^2d^4\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(-15a^2d^4+24abc^2d^2+4b^2c^4\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}-\frac{ad\left(2bc^2-5ad^2\right)-bcx\left(9ad^2+2bc^2\right)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{d\left(\frac{b\int \frac{2acd\left(2bc^2-33ad^2\right)-\left(4b^2c^4+24abd^2c^2-15a^2d^4\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(-15a^2d^4+24abc^2d^2+4b^2c^4\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\frac{ad(2bc^2-5ad^2)-bcx(9ad^2+2bc^2)}{a\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}+\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 679

$$\frac{-\frac{d\left(\frac{b\left(-\frac{15a^2d^3\left(6bc^2-ad^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{c\sqrt{a+bx^2}\left(-81a^2d^4+28abc^2d^2+4b^2c^4\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(-15a^2d^4+24abc^2d^2+4b^2c^4\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\frac{ad(2bc^2-5a\cdots}{a\sqrt{a+bx^2}\cdots}}{3a\left(ad^2+bc^2\right)}\cdots$$

$$\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 488

$$\frac{-\frac{d\left(\frac{b\left(\frac{15a^2d^3\left(6bc^2-ad^2\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{c\sqrt{a+bx^2}\left(-81a^2d^4+28abc^2d^2+4b^2c^4\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(-15a^2d^4+24abc^2d^2+4b^2c^4\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\cdots}{3a\left(ad^2+bc^2\right)}\cdots$$

$$\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$\frac{-\frac{d\left(\frac{b\left(\frac{15a^2d^3\left(6bc^2-ad^2\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{c\sqrt{a+bx^2}\left(-81a^2d^4+28abc^2d^2+4b^2c^4\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(-15a^2d^4+24abc^2d^2+4b^2c^4\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\cdots}{3a\left(ad^2+bc^2\right)}\cdots$$

$$\frac{ad+bcx}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

input
```
Int[1/((c + d*x)^3*(a + b*x^2)^(5/2)),x]
```

output
```
(a*d + b*c*x)/(3*a*(b*c^2 + a*d^2)*(c + d*x)^2*(a + b*x^2)^(3/2)) + (-((a*
d*(2*b*c^2 - 5*a*d^2) - b*c*(2*b*c^2 + 9*a*d^2)*x)/(a*(b*c^2 + a*d^2)*(c +
 d*x)^2*Sqrt[a + b*x^2])) - (d*(-1/2*((4*b^2*c^4 + 24*a*b*c^2*d^2 - 15*a^2
*d^4)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) + (b*(-((c*(4*b^2*c^4
 + 28*a*b*c^2*d^2 - 81*a^2*d^4)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x
))) + (15*a^2*d^3*(6*b*c^2 - a*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*
d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)))/(2*(b*c^2 + a*d^2))))/(a*(
b*c^2 + a*d^2)))/(3*a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1602 vs. $2(276) = 552$.

Time = 0.36 (sec) , antiderivative size = 1603, normalized size of antiderivative = 5.34

method	result	size
default	Expression too large to display	1603

input
```
int(1/(d*x+c)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(3/2)+7/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+5*b*c*d/(a*d^2+b*c^2
)*(1/3/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
3/2)+b*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2
-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16/3
*b/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))+1/(a*d^2+b*c^2)*d^2*(1/(a*d^
2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d
/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)
/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2
/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d
^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
)/(x+c/d))))-4*b/(a*d^2+b*c^2)*d^2*(2/3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+
b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(3/2)+16/3*b/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)
/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))-5/2*b/(a*d^2+b*c^
2)*d^2*(1/3/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d
^2)^(3/2)+b*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2
)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1301 vs. $2(277) = 554$.

Time = 1.03 (sec) , antiderivative size = 2628, normalized size of antiderivative = 8.76

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(15*(6*a^4*b^2*c^4*d^4 - a^5*b*c^2*d^6 + (6*a^2*b^4*c^2*d^6 - a^3*b
^3*d^8)*x^6 + 2*(6*a^2*b^4*c^3*d^5 - a^3*b^3*c*d^7)*x^5 + (6*a^2*b^4*c^4*d
^4 + 11*a^3*b^3*c^2*d^6 - 2*a^4*b^2*d^8)*x^4 + 4*(6*a^3*b^3*c^3*d^5 - a^4*
b^2*c*d^7)*x^3 + (12*a^3*b^3*c^4*d^4 + 4*a^4*b^2*c^2*d^6 - a^5*b*d^8)*x^2
+ 2*(6*a^4*b^2*c^3*d^5 - a^5*b*c*d^7)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*
d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d
^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(6*a^2*b
^4*c^8*d + 70*a^3*b^3*c^6*d^3 + 14*a^4*b^2*c^4*d^5 - 53*a^5*b*c^2*d^7 - 3*
a^6*d^9 + (4*b^6*c^7*d^2 + 32*a*b^5*c^5*d^4 - 53*a^2*b^4*c^3*d^6 - 81*a^3*
b^3*c*d^8)*x^5 + (8*b^6*c^8*d + 64*a*b^5*c^6*d^3 - 16*a^2*b^4*c^4*d^5 - 87
*a^3*b^3*c^2*d^7 - 15*a^4*b^2*d^9)*x^4 + 2*(2*b^6*c^9 + 19*a*b^5*c^7*d^2 +
 65*a^2*b^4*c^5*d^4 - 24*a^3*b^3*c^3*d^6 - 72*a^4*b^2*c*d^8)*x^3 + 2*(6*a*
b^5*c^8*d + 63*a^2*b^4*c^6*d^3 - 7*a^3*b^3*c^4*d^5 - 74*a^4*b^2*c^2*d^7 -
10*a^5*b*d^9)*x^2 + (6*a*b^5*c^9 + 42*a^2*b^4*c^7*d^2 + 110*a^3*b^3*c^5*d^
4 + 13*a^4*b^2*c^3*d^6 - 61*a^5*b*c*d^8)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^12
 + 5*a^5*b^4*c^10*d^2 + 10*a^6*b^3*c^8*d^4 + 10*a^7*b^2*c^6*d^6 + 5*a^8*b*
c^4*d^8 + a^9*c^2*d^10 + (a^2*b^7*c^10*d^2 + 5*a^3*b^6*c^8*d^4 + 10*a^4*b^
5*c^6*d^6 + 10*a^5*b^4*c^4*d^8 + 5*a^6*b^3*c^2*d^10 + a^7*b^2*d^12)*x^6 +
2*(a^2*b^7*c^11*d + 5*a^3*b^6*c^9*d^3 + 10*a^4*b^5*c^7*d^5 + 10*a^5*b^4*c^
5*d^7 + 5*a^6*b^3*c^3*d^9 + a^7*b^2*c*d^11)*x^5 + (a^2*b^7*c^12 + 7*a^3...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3 \left(a+bx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx\right)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(b*x**2+a)**(5/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(277) = 554$.

Time = 0.12 (sec) , antiderivative size = 1176, normalized size of antiderivative = 3.92

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
35/6*b^3*c^3*x/((b*x^2 + a)^(3/2)*a*b^3*c^6 + 3*(b*x^2 + a)^(3/2)*a^2*b^2*
c^4*d^2 + 3*(b*x^2 + a)^(3/2)*a^3*b*c^2*d^4 + (b*x^2 + a)^(3/2)*a^4*d^6) +
 35/2*b^3*c^3*x/(4*sqrt(b*x^2 + a)*a^2*b^3*c^6 + sqrt(b*x^2 + a)*a*b^4*c^8
/d^2 + 6*sqrt(b*x^2 + a)*a^3*b^2*c^4*d^2 + 4*sqrt(b*x^2 + a)*a^4*b*c^2*d^4
 + sqrt(b*x^2 + a)*a^5*d^6) + 35/3*b^3*c^3*x/(sqrt(b*x^2 + a)*a^2*b^3*c^6
+ 3*sqrt(b*x^2 + a)*a^3*b^2*c^4*d^2 + 3*sqrt(b*x^2 + a)*a^4*b*c^2*d^4 + sq
rt(b*x^2 + a)*a^5*d^6) + 35/2*b^2*c^2/(sqrt(b*x^2 + a)*b^4*c^8/d^3 + 4*sqr
t(b*x^2 + a)*a*b^3*c^6/d + 6*sqrt(b*x^2 + a)*a^2*b^2*c^4*d + 4*sqrt(b*x^2
+ a)*a^3*b*c^2*d^3 + sqrt(b*x^2 + a)*a^4*d^5) + 35/6*b^2*c^2/((b*x^2 + a)^
(3/2)*b^3*c^6/d + 3*(b*x^2 + a)^(3/2)*a*b^2*c^4*d + 3*(b*x^2 + a)^(3/2)*a^
2*b*c^2*d^3 + (b*x^2 + a)^(3/2)*a^3*d^5) - 11/2*b^2*c*x/((b*x^2 + a)^(3/2)
*a*b^2*c^4 + 2*(b*x^2 + a)^(3/2)*a^2*b*c^2*d^2 + (b*x^2 + a)^(3/2)*a^3*d^4
) - 5/2*b^2*c*x/(3*sqrt(b*x^2 + a)*a^2*b^2*c^4 + sqrt(b*x^2 + a)*a*b^3*c^6
/d^2 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^2 + sqrt(b*x^2 + a)*a^4*d^4) - 11*b^2
*c*x/(sqrt(b*x^2 + a)*a^2*b^2*c^4 + 2*sqrt(b*x^2 + a)*a^3*b*c^2*d^2 + sqrt
(b*x^2 + a)*a^4*d^4) - 7/2*b*c/((b*x^2 + a)^(3/2)*b^2*c^4*x + 2*(b*x^2 + a
)^(3/2)*a*b*c^2*d^2*x + (b*x^2 + a)^(3/2)*a^2*d^4*x + (b*x^2 + a)^(3/2)*b^
2*c^5/d + 2*(b*x^2 + a)^(3/2)*a*b*c^3*d + (b*x^2 + a)^(3/2)*a^2*c*d^3) - 5
/2*b/(sqrt(b*x^2 + a)*b^3*c^6/d^3 + 3*sqrt(b*x^2 + a)*a*b^2*c^4/d + 3*sqrt
(b*x^2 + a)*a^2*b*c^2*d + sqrt(b*x^2 + a)*a^3*d^3) - 5/6*b/((b*x^2 + a)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2179 vs. $2(277) = 554$.

Time = 0.28 (sec) , antiderivative size = 2179, normalized size of antiderivative = 7.26

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input

```
integrate(1/(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output

```
5*(6*b^2*c^2*d^4 - a*b*d^6)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqr
t(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^4*c^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*
d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8)*sqrt(-b*c^2 - a*d^2)) + 1/3*((2*((b^18*c^
29 + 19*a*b^17*c^27*d^2 + 138*a^2*b^16*c^25*d^4 + 538*a^3*b^15*c^23*d^6 +
1243*a^4*b^14*c^21*d^8 + 1617*a^5*b^13*c^19*d^10 + 528*a^6*b^12*c^17*d^12
- 2244*a^7*b^11*c^15*d^14 - 5049*a^8*b^10*c^13*d^16 - 5819*a^9*b^9*c^11*d^
18 - 4334*a^10*b^8*c^9*d^20 - 2166*a^11*b^7*c^7*d^22 - 707*a^12*b^6*c^5*d^
24 - 137*a^13*b^5*c^3*d^26 - 12*a^14*b^4*c*d^28)*x/(a^2*b^17*c^32 + 16*a^3
*b^16*c^30*d^2 + 120*a^4*b^15*c^28*d^4 + 560*a^5*b^14*c^26*d^6 + 1820*a^6*
b^13*c^24*d^8 + 4368*a^7*b^12*c^22*d^10 + 8008*a^8*b^11*c^20*d^12 + 11440*
a^9*b^10*c^18*d^14 + 12870*a^10*b^9*c^16*d^16 + 11440*a^11*b^8*c^14*d^18 +
 8008*a^12*b^7*c^12*d^20 + 4368*a^13*b^6*c^10*d^22 + 1820*a^14*b^5*c^8*d^2
4 + 560*a^15*b^4*c^6*d^26 + 120*a^16*b^3*c^4*d^28 + 16*a^17*b^2*c^2*d^30 +
 a^18*b*d^32) + 3*(5*a^2*b^16*c^26*d^3 + 59*a^3*b^15*c^24*d^5 + 318*a^4*b^
14*c^22*d^7 + 1034*a^5*b^13*c^20*d^9 + 2255*a^6*b^12*c^18*d^11 + 3465*a^7*
b^11*c^16*d^13 + 3828*a^8*b^10*c^14*d^15 + 3036*a^9*b^9*c^12*d^17 + 1683*a
^10*b^8*c^10*d^19 + 605*a^11*b^7*c^8*d^21 + 110*a^12*b^6*c^6*d^23 - 6*a^13
*b^5*c^4*d^25 - 7*a^14*b^4*c^2*d^27 - a^15*b^3*d^29)/(a^2*b^17*c^32 + 16*a
^3*b^16*c^30*d^2 + 120*a^4*b^15*c^28*d^4 + 560*a^5*b^14*c^26*d^6 + 1820*a^
6*b^13*c^24*d^8 + 4368*a^7*b^12*c^22*d^10 + 8008*a^8*b^11*c^20*d^12 + 1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(c+d\,x)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x)^3),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 2772, normalized size of antiderivative = 9.24

$$\int \frac{1}{(c+dx)^3 (a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^3/(b*x^2+a)^(5/2),x)
```

output
```
(15*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**5*b*c**2*d**6 + 30*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**5*b*c*d**7*x + 15*sqrt(a*d*
*2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*
a**5*b*d**8*x**2 - 90*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a**4*b**2*c**4*d**4 - 180*sqrt(a*d**2 + b*c
**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b**
2*c**3*d**5*x - 60*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*a**4*b**2*c**2*d**6*x**2 + 60*sqrt(a*d**2 + b*
c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b*
*2*c*d**7*x**3 + 30*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*a**4*b**2*d**8*x**4 - 180*sqrt(a*d**2 + b*c**
2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*
c**4*d**4*x**2 - 360*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c**3*d**5*x**3 - 165*sqrt(a*d**2 +
 b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3
*b**3*c**2*d**6*x**4 + 30*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**3*c*d**7*x**5 + 15*sqrt(a*d**2
+ b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**
3*b**3*d**8*x**6 - 90*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr...
```

3.293 $\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = -\frac{2c(c+dx)}{d\sqrt{c^2+d^2x^2}} + \frac{\sqrt{c^2+d^2x^2}}{d} + \frac{3c\operatorname{arctanh}\left(\frac{dx}{\sqrt{c^2+d^2x^2}}\right)}{d}$$

output
```
-2*c*(d*x+c)/d/(d^2*x^2+c^2)^(1/2)+(d^2*x^2+c^2)^(1/2)/d+3*c*arctanh(d*x/(
d^2*x^2+c^2)^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = \frac{\frac{-c^2-2cdx+d^2x^2}{\sqrt{c^2+d^2x^2}} + 3c\log\left(\sqrt{c^2}-dx-\sqrt{c^2+d^2x^2}\right) - 3c\log\left(d\left(\sqrt{c^2}+dx-\sqrt{c^2+d^2\right.\right.}{d}$$

input
```
Integrate[(c + d*x)^3/(c^2 + d^2*x^2)^(3/2),x]
```

output
```
((-c^2 - 2*c*d*x + d^2*x^2)/Sqrt[c^2 + d^2*x^2] + 3*c*Log[Sqrt[c^2] - d*x
- Sqrt[c^2 + d^2*x^2]] - 3*c*Log[d*(Sqrt[c^2] + d*x - Sqrt[c^2 + d^2*x^2])
])/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {495, 27, 643, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\int \frac{2cd^2(c-dx)(c+dx)}{\sqrt{c^2+d^2x^2}}dx}{c^2d^2}-\frac{(c-dx)(c+dx)^2}{cd\sqrt{c^2+d^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(c-dx)(c+dx)}{\sqrt{c^2+d^2x^2}}dx}{c}-\frac{(c-dx)(c+dx)^2}{cd\sqrt{c^2+d^2x^2}}$$

$$\Big\downarrow \text{643}$$

$$\frac{2\int \frac{c^2-d^2x^2}{\sqrt{c^2+d^2x^2}}dx}{c}-\frac{(c-dx)(c+dx)^2}{cd\sqrt{c^2+d^2x^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{2\left(\frac{3}{2}c^2\int \frac{1}{\sqrt{c^2+d^2x^2}}dx-\frac{1}{2}x\sqrt{c^2+d^2x^2}\right)}{c}-\frac{(c-dx)(c+dx)^2}{cd\sqrt{c^2+d^2x^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{2\left(\frac{3}{2}c^2\int \frac{1}{1-\frac{d^2x^2}{c^2+d^2x^2}}d\frac{x}{\sqrt{c^2+d^2x^2}}-\frac{1}{2}x\sqrt{c^2+d^2x^2}\right)}{c}-\frac{(c-dx)(c+dx)^2}{cd\sqrt{c^2+d^2x^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{3c^2\text{arctanh}\left(\frac{dx}{\sqrt{c^2+d^2x^2}}\right)}{2d}-\frac{1}{2}x\sqrt{c^2+d^2x^2}\right)}{c}-\frac{(c-dx)(c+dx)^2}{cd\sqrt{c^2+d^2x^2}}$$

input
```
Int[(c + d*x)^3/(c^2 + d^2*x^2)^(3/2),x]
```

output
```
-(((c - d*x)*(c + d*x)^2)/(c*d*Sqrt[c^2 + d^2*x^2])) + (2*(-1/2*(x*Sqrt[c^
2 + d^2*x^2]) + (3*c^2*ArcTanh[(d*x)/Sqrt[c^2 + d^2*x^2]])/(2*d)))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

method	result
risch	$\frac{\sqrt{d^2x^2+c^2}}{d}-\frac{2cx}{\sqrt{d^2x^2+c^2}}+\frac{3c\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2+c^2}\right)}{\sqrt{d^2}}-\frac{2c^2}{d\sqrt{d^2x^2+c^2}}$
default	$\frac{cx}{\sqrt{d^2x^2+c^2}}+d^3\left(\frac{x^2}{d^2\sqrt{d^2x^2+c^2}}+\frac{2c^2}{d^4\sqrt{d^2x^2+c^2}}\right)+3c\,d^2\left(-\frac{x}{d^2\sqrt{d^2x^2+c^2}}+\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2+c^2}\right)}{d^2\sqrt{d^2}}\right)-\frac{3c^2}{d\sqrt{d^2x^2+c^2}}$

input
```
int((d*x+c)^3/(d^2*x^2+c^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(d^2*x^2+c^2)^(1/2)/d-2*c*x/(d^2*x^2+c^2)^(1/2)+3*c*ln(d^2*x/(d^2)^(1/2)+(
d^2*x^2+c^2)^(1/2))/(d^2)^(1/2)-2*c^2/d/(d^2*x^2+c^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = -\frac{2\,cd^2x^2+2\,c^3+3\,(cd^2x^2+c^3)\log\left(-dx+\sqrt{d^2x^2+c^2}\right)-(d^2x^2-2\,cdx-c^2)\sqrt{d^2x^2+c^2}}{d^3x^2+c^2d}$$

input
```
integrate((d*x+c)^3/(d^2*x^2+c^2)^(3/2),x, algorithm="fricas")
```

output
```
-(2*c*d^2*x^2 + 2*c^3 + 3*(c*d^2*x^2 + c^3)*log(-d*x + sqrt(d^2*x^2 + c^2)
) - (d^2*x^2 - 2*c*d*x - c^2)*sqrt(d^2*x^2 + c^2))/(d^3*x^2 + c^2*d)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = \int \frac{(c+dx)^3}{(c^2+d^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**3/(d**2*x**2+c**2)**(3/2),x)
```

output
```
Integral((c + d*x)**3/(c**2 + d**2*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = \frac{dx^2}{\sqrt{d^2x^2+c^2}} - \frac{2\,cx}{\sqrt{d^2x^2+c^2}} + \frac{3\,c\,\mathrm{arsinh}\left(\frac{dx}{c}\right)}{d} - \frac{c^2}{\sqrt{d^2x^2+c^2}d}$$

input
```
integrate((d*x+c)^3/(d^2*x^2+c^2)^(3/2),x, algorithm="maxima")
```

output
```
d*x^2/sqrt(d^2*x^2 + c^2) - 2*c*x/sqrt(d^2*x^2 + c^2) + 3*c*arcsinh(d*x/c)
/d - c^2/(sqrt(d^2*x^2 + c^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = -\frac{3\,c\log\left(-x|d|+\sqrt{d^2x^2+c^2}\right)}{|d|} + \frac{(dx-2\,c)x-\frac{c^2}{d}}{\sqrt{d^2x^2+c^2}}$$

input
```
integrate((d*x+c)^3/(d^2*x^2+c^2)^(3/2),x, algorithm="giac")
```

output
```
-3*c*log(-x*abs(d) + sqrt(d^2*x^2 + c^2))/abs(d) + ((d*x - 2*c)*x - c^2/d)
/sqrt(d^2*x^2 + c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = \int \frac{(c+d\,x)^3}{(c^2+d^2\,x^2)^{3/2}}\,dx$$

input
```
int((c + d*x)^3/(c^2 + d^2*x^2)^(3/2),x)
```

output
```
int((c + d*x)^3/(c^2 + d^2*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)^3}{(c^2+d^2x^2)^{3/2}}\,dx = \frac{-\sqrt{d^2x^2+c^2}\,c^2-2\sqrt{d^2x^2+c^2}\,cdx+\sqrt{d^2x^2+c^2}\,d^2x^2+3\log\left(\frac{\sqrt{d^2x^2+c^2}+dx}{c}\right)c^3+3}{d\,(d^2x^2+c^2)}$$

input
```
int((d*x+c)^3/(d^2*x^2+c^2)^(3/2),x)
```

output
```
( - sqrt(c**2 + d**2*x**2)*c**2 - 2*sqrt(c**2 + d**2*x**2)*c*d*x + sqrt(c*
*2 + d**2*x**2)*d**2*x**2 + 3*log((sqrt(c**2 + d**2*x**2) + d*x)/c)*c**3 +
 3*log((sqrt(c**2 + d**2*x**2) + d*x)/c)*c*d**2*x**2 - 2*c**3 - 2*c*d**2*x
**2)/(d*(c**2 + d**2*x**2))
```

3.294 $\int \frac{3+x}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx=-\sqrt{1-x^2}+3\arcsin(x)$$

output
```
-(-x^2+1)^(1/2)+3*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx=-\sqrt{1-x^2}-6\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(3 + x)/Sqrt[1 - x^2],x]
```

output
```
-Sqrt[1 - x^2] - 6*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+3}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{455}$$

$$3\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}$$

$$\Big\downarrow \text{223}$$

$$3\arcsin(x)-\sqrt{1-x^2}$$

input
```
Int[(3 + x)/Sqrt[1 - x^2],x]
```

output
```
-Sqrt[1 - x^2] + 3*ArcSin[x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\sqrt{-x^2+1}+3\arcsin\left(x\right)$	17
risch	$\frac{x^2-1}{\sqrt{-x^2+1}}+3\arcsin\left(x\right)$	21
meijerg	$3\arcsin\left(x\right)-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}$	31
trager	$-\sqrt{-x^2+1}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	40

input
```
int((3+x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)+3*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx=-\sqrt{-x^2+1}-6\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((3+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 1) - 6*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx=-\sqrt{1-x^2}+3\,\mathrm{asin}\left(x\right)$$

input
```
integrate((3+x)/(-x**2+1)**(1/2),x)
```

output
```
-sqrt(1 - x**2) + 3*asin(x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + 3\,\arcsin(x)$$

input
```
integrate((3+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1) + 3*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + 3\,\arcsin(x)$$

input
```
integrate((3+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1) + 3*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx = 3\,\mathrm{asin}(x) - \sqrt{1-x^2}$$

input
```
int((x + 3)/(1 - x^2)^(1/2),x)
```

output
```
3*asin(x) - (1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{3+x}{\sqrt{1-x^2}}\,dx = 3asin(x) - \sqrt{-x^2+1}$$

input
```
int((3+x)/(-x^2+1)^(1/2),x)
```

output
```
3*asin(x) - sqrt( - x**2 + 1)
```

3.295 $\int \frac{1+x}{\sqrt{4-x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1+x}{\sqrt{4-x^2}} \, dx = -\sqrt{4-x^2} + \arcsin\left(\frac{x}{2}\right)$$

output
```
-(-x^2+4)^(1/2)+arcsin(1/2*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{1+x}{\sqrt{4-x^2}} \, dx = -\sqrt{4-x^2} - 2\arctan\left(\frac{\sqrt{4-x^2}}{2+x}\right)$$

input
```
Integrate[(1 + x)/Sqrt[4 - x^2],x]
```

output
```
-Sqrt[4 - x^2] - 2*ArcTan[Sqrt[4 - x^2]/(2 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt{4-x^2}}\,dx$$

$$\Big\downarrow \ 455$$

$$\int \frac{1}{\sqrt{4-x^2}}dx-\sqrt{4-x^2}$$

$$\Big\downarrow \ 223$$

$$\arcsin\left(\frac{x}{2}\right)-\sqrt{4-x^2}$$

input
```
Int[(1 + x)/Sqrt[4 - x^2],x]
```

output
```
-Sqrt[4 - x^2] + ArcSin[x/2]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$-\sqrt{-x^{2}+4}+\arcsin\left(\frac{x}{2}\right)$	17
risch	$\frac{x^{2}-4}{\sqrt{-x^{2}+4}}+\arcsin\left(\frac{x}{2}\right)$	21
meijerg	$\arcsin\left(\frac{x}{2}\right)-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-\frac{x^{2}}{4}+1}}{\sqrt{\pi}}$	31
trager	$-\sqrt{-x^{2}+4}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+4}+x\right)$	39

input
```
int((x+1)/(-x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+4)^(1/2)+arcsin(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{1+x}{\sqrt{4-x^2}}\,dx=-\sqrt{-x^{2}+4}-2\,\arctan\left(\frac{\sqrt{-x^{2}+4}-2}{x}\right)$$

input
```
integrate((1+x)/(-x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 4) - 2*arctan((sqrt(-x^2 + 4) - 2)/x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1+x}{\sqrt{4-x^2}}\,dx = -\sqrt{4-x^2} + \operatorname{asin}\left(\frac{x}{2}\right)$$

input
```
integrate((1+x)/(-x**2+4)**(1/2),x)
```

output
```
-sqrt(4 - x**2) + asin(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1+x}{\sqrt{4-x^2}}\,dx = -\sqrt{-x^2+4} + \arcsin\left(\frac{1}{2}\,x\right)$$

input
```
integrate((1+x)/(-x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 4) + arcsin(1/2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1+x}{\sqrt{4-x^2}}\,dx = -\sqrt{-x^2+4} + \arcsin\left(\frac{1}{2}\,x\right)$$

input
```
integrate((1+x)/(-x^2+4)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 4) + arcsin(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1+x}{\sqrt{4-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{2}\right) - \sqrt{4-x^2}$$

input
```
int((x + 1)/(4 - x^2)^(1/2),x)
```

output
```
asin(x/2) - (4 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1+x}{\sqrt{4-x^2}}\,dx = asin\left(\frac{x}{2}\right) - \sqrt{-x^2+4}$$

input
```
int((1+x)/(-x^2+4)^(1/2),x)
```

output
```
asin(x/2) - sqrt( - x**2 + 4)
```

3.296 $\int \frac{2+x}{\sqrt{9+x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{2+x}{\sqrt{9+x^2}} \, dx = \sqrt{9+x^2} + 2\text{arcsinh}\left(\frac{x}{3}\right)$$

output
```
(x^2+9)^(1/2)+2*arcsinh(1/3*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{2+x}{\sqrt{9+x^2}} \, dx = \sqrt{9+x^2} - 2\log\left(-x+\sqrt{9+x^2}\right)$$

input
```
Integrate[(2 + x)/Sqrt[9 + x^2],x]
```

output
```
Sqrt[9 + x^2] - 2*Log[-x + Sqrt[9 + x^2]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{\sqrt{x^2+9}} dx$$
$$\Big\downarrow \text{455}$$
$$2\int \frac{1}{\sqrt{x^2+9}}dx+\sqrt{x^2+9}$$
$$\Big\downarrow \text{222}$$
$$2\text{arcsinh}\left(\frac{x}{3}\right)+\sqrt{x^2+9}$$

input
```
Int[(2 + x)/Sqrt[9 + x^2],x]
```

output
```
Sqrt[9 + x^2] + 2*ArcSinh[x/3]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{x^2+9}+2\,\mathrm{arcsinh}\left(\frac{x}{3}\right)$	15
risch	$\sqrt{x^2+9}+2\,\mathrm{arcsinh}\left(\frac{x}{3}\right)$	15
trager	$\sqrt{x^2+9}+2\ln\left(x+\sqrt{x^2+9}\right)$	21
meijerg	$2\,\mathrm{arcsinh}\left(\frac{x}{3}\right)+\frac{-3\sqrt{\pi}+3\sqrt{\pi}\,\sqrt{\frac{x^2}{9}+1}}{\sqrt{\pi}}$	33

input
```
int((2+x)/(x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2+9)^(1/2)+2*arcsinh(1/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{2+x}{\sqrt{9+x^2}}\,dx=\sqrt{x^2+9}-2\,\log\left(-x+\sqrt{x^2+9}\right)$$

input
```
integrate((2+x)/(x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + 9) - 2*log(-x + sqrt(x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{\sqrt{9+x^2}}\,dx=\sqrt{x^2+9}+2\,\mathrm{asinh}\left(\frac{x}{3}\right)$$

input
```
integrate((2+x)/(x**2+9)**(1/2),x)
```

output
```
sqrt(x**2 + 9) + 2*asinh(x/3)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{\sqrt{9+x^2}}\,dx = \sqrt{x^2+9} + 2\,\operatorname{arsinh}\left(\frac{1}{3}\,x\right)$$

input
```
integrate((2+x)/(x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 9) + 2*arcsinh(1/3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{2+x}{\sqrt{9+x^2}}\,dx = \sqrt{x^2+9} - 2\,\log\left(-x+\sqrt{x^2+9}\right)$$

input
```
integrate((2+x)/(x^2+9)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 9) - 2*log(-x + sqrt(x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{\sqrt{9+x^2}}\,dx = 2\,\operatorname{asinh}\left(\frac{x}{3}\right) + \sqrt{x^2+9}$$

input
```
int((x + 2)/(x^2 + 9)^(1/2),x)
```

output
```
2*asinh(x/3) + (x^2 + 9)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{2+x}{\sqrt{9+x^2}}\,dx = \sqrt{x^2+9} + 2\log\left(\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right)$$

input
```
int((2+x)/(x^2+9)^(1/2),x)
```

output
```
sqrt(x**2 + 9) + 2*log((sqrt(x**2 + 9) + x)/3)
```

3.297 $\int \frac{(a+bx)^2}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}} \, dx = -\frac{1}{2}b(4a+bx)\sqrt{1-x^2}+\frac{1}{2}\left(2a^2+b^2\right)\arcsin(x)$$

output
```
-1/2*b*(b*x+4*a)*(-x^2+1)^(1/2)+1/2*(2*a^2+b^2)*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}} \, dx = -\frac{1}{2}b(4a+bx)\sqrt{1-x^2}+\left(2a^2+b^2\right)\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[(a + b*x)^2/Sqrt[1 - x^2],x]
```

output
```
-1/2*(b*(4*a + b*x)*Sqrt[1 - x^2]) + (2*a^2 + b^2)*ArcTan[x/(-1 + Sqrt[1 -
 x^2])]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {497, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \text{ 497}$$

$$-\frac{1}{2}\int -\frac{2a^2+3bxa+b^2}{\sqrt{1-x^2}}dx-\frac{1}{2}b\sqrt{1-x^2}(a+bx)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int \frac{2a^2+3bxa+b^2}{\sqrt{1-x^2}}dx-\frac{1}{2}b\sqrt{1-x^2}(a+bx)$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{1}{\sqrt{1-x^2}}dx-3ab\sqrt{1-x^2}\right)-\frac{1}{2}b\sqrt{1-x^2}(a+bx)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\arcsin(x)-3ab\sqrt{1-x^2}\right)-\frac{1}{2}b\sqrt{1-x^2}(a+bx)$$

input
```
Int[(a + b*x)^2/Sqrt[1 - x^2],x]
```

output
```
-1/2*(b*(a + b*x)*Sqrt[1 - x^2]) + (-3*a*b*Sqrt[1 - x^2] + (2*a^2 + b^2)*A
rcSin[x])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{b(bx+4a)(x^2-1)}{2\sqrt{-x^2+1}}+\left(a^2+\frac{b^2}{2}\right)\arcsin(x)$	38
default	$a^2\arcsin(x)+b^2\left(-\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}\right)-2ab\sqrt{-x^2+1}$	42
trager	$-\frac{b(bx+4a)\sqrt{-x^2+1}}{2}+\frac{(2a^2+b^2)\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	57
meijerg	$\frac{ib^2\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\arcsin(x)\right)}{2\sqrt{\pi}}-\frac{ab\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{-x^2+1}\right)}{\sqrt{\pi}}+a^2\arcsin(x)$	69

input
```
int((b*x+a)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*(b*x+4*a)*(x^2-1)/(-x^2+1)^(1/2)+(a^2+1/2*b^2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx = -\left(2\,a^2+b^2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\frac{1}{2}\left(b^2x+4\,ab\right)\sqrt{-x^2+1}$$

input
```
integrate((b*x+a)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(2*a^2 + b^2)*arctan((sqrt(-x^2 + 1) - 1)/x) - 1/2*(b^2*x + 4*a*b)*sqrt(-
x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx = a^2\,\mathrm{asin}\,(x)-2ab\sqrt{1-x^2}-\frac{b^2x\sqrt{1-x^2}}{2}+\frac{b^2\,\mathrm{asin}\,(x)}{2}$$

input
```
integrate((b*x+a)**2/(-x**2+1)**(1/2),x)
```

output
```
a**2*asin(x) - 2*a*b*sqrt(1 - x**2) - b**2*x*sqrt(1 - x**2)/2 + b**2*asin(
x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}b^2x+a^2\arcsin\left(x\right)+\frac{1}{2}\,b^2\arcsin\left(x\right)-2\sqrt{-x^2+1}ab$$

input
```
integrate((b*x+a)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2 + 1)*b^2*x + a^2*arcsin(x) + 1/2*b^2*arcsin(x) - 2*sqrt(-x^
2 + 1)*a*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\left(2\,a^2+b^2\right)\arcsin\left(x\right)-\frac{1}{2}\left(b^2x+4\,ab\right)\sqrt{-x^2+1}$$

input
```
integrate((b*x+a)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*a^2 + b^2)*arcsin(x) - 1/2*(b^2*x + 4*a*b)*sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx = \mathrm{asin}(x)\,\left(a^2+\frac{b^2}{2}\right)-\left(\frac{x\,b^2}{2}+2\,a\,b\right)\,\sqrt{1-x^2}$$

input
```
int((a + b*x)^2/(1 - x^2)^(1/2),x)
```

output
```
asin(x)*(a^2 + b^2/2) - (2*a*b + (b^2*x)/2)*(1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^2}{\sqrt{1-x^2}}\,dx = asin(x)\,a^2+\frac{asin(x)\,b^2}{2}-2\sqrt{-x^2+1}\,ab-\frac{\sqrt{-x^2+1}\,b^2x}{2}+2ab$$

input
```
int((b*x+a)^2/(-x^2+1)^(1/2),x)
```

output
```
(2*asin(x)*a**2 + asin(x)*b**2 - 4*sqrt( - x**2 + 1)*a*b - sqrt( - x**2 +
1)*b**2*x + 4*a*b)/2
```

3.298 $\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx = \frac{1}{2}b(4a+bx)\sqrt{1+x^2}+\frac{1}{2}\left(2a^2-b^2\right)\operatorname{arcsinh}(x)$$

output
```
1/2*b*(b*x+4*a)*(x^2+1)^(1/2)+1/2*(2*a^2-b^2)*arcsinh(x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx = \frac{1}{2}b(4a+bx)\sqrt{1+x^2}+\frac{1}{2}\left(-2a^2+b^2\right)\log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[(a + b*x)^2/Sqrt[1 + x^2],x]
```

output
```
(b*(4*a + b*x)*Sqrt[1 + x^2])/2 + ((-2*a^2 + b^2)*Log[-x + Sqrt[1 + x^2]])
/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {497, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \ 497$$

$$\frac{1}{2}\int \frac{2a^2+3bxa-b^2}{\sqrt{x^2+1}}dx+\frac{1}{2}b\sqrt{x^2+1}(a+bx)$$

$$\Big\downarrow \ 455$$

$$\frac{1}{2}\left(\left(2a^2-b^2\right)\int \frac{1}{\sqrt{x^2+1}}dx+3ab\sqrt{x^2+1}\right)+\frac{1}{2}b\sqrt{x^2+1}(a+bx)$$

$$\Big\downarrow \ 222$$

$$\frac{1}{2}\left(\left(2a^2-b^2\right)\operatorname{arcsinh}(x)+3ab\sqrt{x^2+1}\right)+\frac{1}{2}b\sqrt{x^2+1}(a+bx)$$

input
```
Int[(a + b*x)^2/Sqrt[1 + x^2],x]
```

output
```
(b*(a + b*x)*Sqrt[1 + x^2])/2 + (3*a*b*Sqrt[1 + x^2] + (2*a^2 - b^2)*ArcSi
nh[x])/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{b(bx+4a)\sqrt{x^2+1}}{2}+\left(a^2-\frac{b^2}{2}\right)\operatorname{arcsinh}\left(x\right)$	31
default	$a^2\operatorname{arcsinh}\left(x\right)+b^2\left(\frac{x\sqrt{x^2+1}}{2}-\frac{\operatorname{arcsinh}(x)}{2}\right)+2ab\sqrt{x^2+1}$	38
trager	$\frac{b(bx+4a)\sqrt{x^2+1}}{2}+\frac{(2a^2-b^2)\ln\left(\sqrt{x^2+1}+x\right)}{2}$	42
meijerg	$\frac{b^2\left(\sqrt{\pi}\,x\sqrt{x^2+1}-\sqrt{\pi}\,\operatorname{arcsinh}(x)\right)}{2\sqrt{\pi}}+\frac{ab\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^2+1}\right)}{\sqrt{\pi}}+a^2\operatorname{arcsinh}\left(x\right)$	60

input
```
int((b*x+a)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*(b*x+4*a)*(x^2+1)^(1/2)+(a^2-1/2*b^2)*arcsinh(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx=-\frac{1}{2}\left(2\,a^2-b^2\right)\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\left(b^2x+4\,ab\right)\sqrt{x^2+1}$$

input
```
integrate((b*x+a)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*a^2 - b^2)*log(-x + sqrt(x^2 + 1)) + 1/2*(b^2*x + 4*a*b)*sqrt(x^2
+ 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx = a^2\operatorname{asinh}(x) + 2ab\sqrt{x^2+1} + \frac{b^2x\sqrt{x^2+1}}{2} - \frac{b^2\operatorname{asinh}(x)}{2}$$

input
```
integrate((b*x+a)**2/(x**2+1)**(1/2),x)
```

output
```
a**2*asinh(x) + 2*a*b*sqrt(x**2 + 1) + b**2*x*sqrt(x**2 + 1)/2 - b**2*asin
h(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}b^2x + a^2\operatorname{arsinh}(x) - \frac{1}{2}b^2\operatorname{arsinh}(x) + 2\sqrt{x^2+1}ab$$

input
```
integrate((b*x+a)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 + 1)*b^2*x + a^2*arcsinh(x) - 1/2*b^2*arcsinh(x) + 2*sqrt(x^2
 + 1)*a*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx = -\frac{1}{2}\left(2a^2-b^2\right)\log\left(-x+\sqrt{x^2+1}\right) + \frac{1}{2}\left(b^2x+4ab\right)\sqrt{x^2+1}$$

input
```
integrate((b*x+a)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(2*a^2 - b^2)*log(-x + sqrt(x^2 + 1)) + 1/2*(b^2*x + 4*a*b)*sqrt(x^2
+ 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx = \left(\frac{x\,b^2}{2}+2\,a\,b\right)\,\sqrt{x^2+1}+\operatorname{asinh}(x)\,\left(a^2-\frac{b^2}{2}\right)$$

input
```
int((a + b*x)^2/(x^2 + 1)^(1/2),x)
```

output
```
(2*a*b + (b^2*x)/2)*(x^2 + 1)^(1/2) + asinh(x)*(a^2 - b^2/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)^2}{\sqrt{1+x^2}}\,dx = 2\sqrt{x^2+1}\,ab+\frac{\sqrt{x^2+1}\,b^2x}{2} + \log\left(\sqrt{x^2+1}+x\right)a^2-\frac{\log\left(\sqrt{x^2+1}+x\right)b^2}{2}$$

input
```
int((b*x+a)^2/(x^2+1)^(1/2),x)
```

output
```
(4*sqrt(x**2 + 1)*a*b + sqrt(x**2 + 1)*b**2*x + 2*log(sqrt(x**2 + 1) + x)*
a**2 - log(sqrt(x**2 + 1) + x)*b**2)/2
```

3.299 $\int \frac{2+3x}{(4+x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{2+3x}{(4+x^2)^{3/2}} \, dx = -\frac{6-x}{2\sqrt{4+x^2}}$$

output
```
-1/2*(6-x)/(x^2+4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{2+3x}{(4+x^2)^{3/2}} \, dx = \frac{-6+x}{2\sqrt{4+x^2}}$$

input
```
Integrate[(2 + 3*x)/(4 + x^2)^(3/2),x]
```

output
```
(-6 + x)/(2*Sqrt[4 + x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{(x^2+4)^{3/2}}\,dx$$

$$\Big\downarrow \; 453$$

$$-\frac{6-x}{2\sqrt{x^2+4}}$$

input
```
Int[(2 + 3*x)/(4 + x^2)^(3/2),x]
```

output
```
-1/2*(6 - x)/Sqrt[4 + x^2]
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x-6}{2\sqrt{x^2+4}}$	13
trager	$\frac{x-6}{2\sqrt{x^2+4}}$	13
risch	$\frac{x-6}{2\sqrt{x^2+4}}$	13
orering	$\frac{x-6}{2\sqrt{x^2+4}}$	13
default	$\frac{x}{2\sqrt{x^2+4}}-\frac{3}{\sqrt{x^2+4}}$	21
meijerg	$\frac{x}{4\sqrt{1+\frac{x^2}{4}}}+\frac{\frac{3\sqrt{\pi}}{2}-\frac{3\sqrt{\pi}}{2\sqrt{1+\frac{x^2}{4}}}}{\sqrt{\pi}}$	37

input
```
int((3*x+2)/(x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x-6)/(x^2+4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{2+3x}{(4+x^2)^{3/2}}\,dx = \frac{x^2+\sqrt{x^2+4}(x-6)+4}{2\,(x^2+4)}$$

input
```
integrate((2+3*x)/(x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(x^2 + sqrt(x^2 + 4)*(x - 6) + 4)/(x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{2+3x}{(4+x^2)^{3/2}}\,dx = \frac{x}{2\sqrt{x^2+4}} - \frac{3}{\sqrt{x^2+4}}$$

input
```
integrate((2+3*x)/(x**2+4)**(3/2),x)
```

output
```
x/(2*sqrt(x**2 + 4)) - 3/sqrt(x**2 + 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{2+3x}{(4+x^2)^{3/2}}\,dx = \frac{x}{2\,\sqrt{x^2+4}} - \frac{3}{\sqrt{x^2+4}}$$

input
```
integrate((2+3*x)/(x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x/sqrt(x^2 + 4) - 3/sqrt(x^2 + 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{2+3x}{(4+x^2)^{3/2}}\,dx = \frac{x-6}{2\,\sqrt{x^2+4}}$$

input
```
integrate((2+3*x)/(x^2+4)^(3/2),x, algorithm="giac")
```

output
```
1/2*(x - 6)/sqrt(x^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{2+3x}{(4+x^2)^{3/2}}\,dx = \frac{x-6}{2\sqrt{x^2+4}}$$

input
```
int((3*x + 2)/(x^2 + 4)^(3/2),x)
```

output
```
(x - 6)/(2*(x^2 + 4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{2+3x}{(4+x^2)^{3/2}}\,dx = \frac{\sqrt{x^2+4}\,x-6\sqrt{x^2+4}+x^2+4}{2x^2+8}$$

input
```
int((2+3*x)/(x^2+4)^(3/2),x)
```

output
```
(sqrt(x**2 + 4)*x - 6*sqrt(x**2 + 4) + x**2 + 4)/(2*(x**2 + 4))
```

3.300 $\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 414

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = -\frac{4}{105}\left(\frac{3c^2}{d}+\frac{5ad}{b}\right)\sqrt{c+dx}\sqrt{a-bx^2}-\frac{4c(c+dx)^{3/2}\sqrt{a-bx^2}}{35d}+\frac{2(c+dx)^{5/2}\sqrt{a-bx^2}}{7d}$$
$$-\frac{4\sqrt{a}c(3bc^2+29ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{105\sqrt{b}d^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$+\frac{4\sqrt{a}(3b^2c^4+2abc^2d^2-5a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{105b^{3/2}d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-4/105*(3*c^2/d+5*a*d/b)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)-4/35*c*(d*x+c)^(3/
2)*(-b*x^2+a)^(1/2)/d+2/7*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/d-4/105*a^(1/2)*c
*(29*a*d^2+3*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/b^(1/2)/d^2/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1
/2)+4/105*a^(1/2)*(-5*a^2*d^4+2*a*b*c^2*d^2+3*b^2*c^4)*(b^(1/2)*(d*x+c)/(b
^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^
(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3
/2)/d^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.45 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.27

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = \frac{2\sqrt{a-bx^2}\left((c+dx)\left(-10ad^2+3b(c^2+8cdx+5d^2x^2)\right)-\frac{2\left(cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(3bc^2+29ad^2)\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Sqrt[a - b*x^2],x]
```

output
```
(2*Sqrt[a - b*x^2]*((c + d*x)*(-10*a*d^2 + 3*b*(c^2 + 8*c*d*x + 5*d^2*x^2)
) - (2*(c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*b*c^2 + 29*a*d^2)*(a - b*x
^2) + I*Sqrt[b]*c*(3*b^(3/2)*c^3 - 3*Sqrt[a]*b*c^2*d + 29*a*Sqrt[b]*c*d^2
- 29*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[
a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]
*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(3*b^(3/2)*c^3 - 27*Sqrt[a]*b*c^2*d + 29*a*
Sqrt[b]*c*d^2 - 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*S
qrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*
ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a
]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b
*x^2))))/(105*b*d*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {497, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-bx^2}(c+dx)^{3/2}\,dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{2\int -\frac{(7bc^2+8bdxc+ad^2)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{7b}-\frac{2d(a-bx^2)^{3/2}\sqrt{c+dx}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(7bc^2+8bdxc+ad^2)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{7b}-\frac{2d(a-bx^2)^{3/2}\sqrt{c+dx}}{7b}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(5ad^2+3bc^2+24bcdx)}{15d}-\frac{4\int -\frac{bd\left(ad\left(27bc^2+5ad^2\right)+bc\left(3bc^2+29ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}}{7b}-\frac{2d(a-bx^2)^{3/2}\sqrt{c+dx}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{ad\left(27bc^2+5ad^2\right)+bc\left(3bc^2+29ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(5ad^2+3bc^2+24bcdx)}{15d}}{7b}-\frac{2d(a-bx^2)^{3/2}\sqrt{c+dx}}{7b}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{2\left(\frac{bc\left(29ad^2+3bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(5ad^2+3bc^2+24bcdx)}{15d}}{7b}-\frac{2d(a-bx^2)^{3/2}\sqrt{c+dx}}{7b}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{2\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\left(29ad^2+3bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(5ad^2+3bc^2+24bcdx\right)}{15d}}{7b}-\frac{2d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7b}$$

$\downarrow$ 508

$$\frac{\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(5ad^2}{15d}}{7b}$$

$$\frac{2d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7b}$$

$\downarrow$ 327

$$\frac{\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(5a}{15d}}{7b}$$

$$\frac{2d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7b}$$

$\downarrow$ 512

$$\frac{\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d}+2\sqrt{a-bx^2}\sqrt{c}}{7b}$$

$$\frac{2d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7b}$$

$\Big\downarrow$ 511

$$\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{15d}}{7b}$$

$$\frac{2d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7b}$$

$\Big\downarrow$ 321

$$\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{15d}}{7b}$$

$$\frac{2d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7b}$$

input
```
Int[(c + d*x)^(3/2)*Sqrt[a - b*x^2],x]
```

output
```
(-2*d*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/(7*b) + ((2*Sqrt[c + d*x]*(3*b*c^2
+ 5*a*d^2 + 24*b*c*d*x)*Sqrt[a - b*x^2])/(15*d) + (2*((-2*Sqrt[a]*Sqrt[b]*
c*(3*b*c^2 + 29*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[
Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/
(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2
*Sqrt[a]*(b*c^2 - a*d^2)*(3*b*c^2 + 5*a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqr
t[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b
]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[
c + d*x]*Sqrt[a - b*x^2])))/(15*d))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(338) = 676$.

Time = 2.76 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{2(-15b\,x^2d^2-24bcdx+10a\,d^2-3b\,c^2)\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{105bd}+\frac{2\left(\frac{c\left(29a\,d^2+3b\,c^2\right)\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}}{}\right.}{}$
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{2d\,x^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7}+\frac{16cx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{35}-\frac{2\left(\frac{2a\,d^2}{7}-\frac{3b\,c^2}{35}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}+\frac{2\left(\frac{19a}{35}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(3/2)*(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(-15*b*d^2*x^2-24*b*c*d*x+10*a*d^2-3*b*c^2)*(d*x+c)^(1/2)/b/d*(-b*x
^2+a)^(1/2)+2/105/d/b*(c*(29*a*d^2+3*b*c^2)*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a
*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*
(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2
)*((c/d-1/b*(a*b)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b
)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*Ellipti
cF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)))+5*a^2*d^3/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a
*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*
(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2
)*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a
*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))+27*a*c^2*d*(a*b)^(1/2)*2^(1/2)*((x
+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*
c)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(
-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))*((d*x+c)*(-b*x^2+a))^(1/2)
/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.67

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = \frac{2\left(2\left(3\,b^2c^4-52\,abc^2d^2-15\,a^2d^4\right)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,a}{27\,bd^3}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(2*(3*b^2*c^4 - 52*a*b*c^2*d^2 - 15*a^2*d^4)*sqrt(-b*d)*weierstrassP
Inverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
1/3*(3*d*x + c)/d) + 6*(3*b^2*c^3*d + 29*a*b*c*d^3)*sqrt(-b*d)*weierstrass
Zeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), wei
erstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/
(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(15*b^2*d^4*x^2 + 24*b^2*c*d^3*x + 3*b^2*
c^2*d^2 - 10*a*b*d^4)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^2*d^3)
```

Sympy [F]

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = \int \sqrt{a-bx^2}(c+dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = \int \sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)*(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = \int \sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)*(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = \int \sqrt{a-b\,x^2}\,(c+d\,x)^{3/2}\,dx$$

input
```
int((a - b*x^2)^(1/2)*(c + d*x)^(3/2),x)
```

output
```
int((a - b*x^2)^(1/2)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\sqrt{a-bx^2}\,dx = \frac{-26\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ad+16\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bcx+10\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{}$$

input
```
int((d*x+c)^(3/2)*(-b*x^2+a)^(1/2),x)
```

output
```
( - 26*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d + 16*sqrt(c + d*x)*sqrt(a - b*x*
*2)*b*c*x + 10*sqrt(c + d*x)*sqrt(a - b*x**2)*b*d*x**2 - 29*int((sqrt(c +
d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*d**
2 - 3*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 -
b*d*x**3),x)*b**2*c**2 + 13*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*
d*x - b*c*x**2 - b*d*x**3),x)*a**2*d**2 + 19*int((sqrt(c + d*x)*sqrt(a - b
*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**2)/(35*b)
```

3.301 $\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 357

$$\begin{aligned}\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx &= -\frac{4c\sqrt{c+dx}\sqrt{a-bx^2}}{15d}+\frac{2(c+dx)^{3/2}\sqrt{a-bx^2}}{5d}\\ &-\frac{4\sqrt{a}(bc^2+3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15\sqrt{b}d^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\\ &+\frac{4\sqrt{a}c(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15\sqrt{b}d^2\sqrt{c+dx}\sqrt{a-bx^2}}\end{aligned}$$

output
```
-4/15*c*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d+2/5*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2
)/d-4/15*a^(1/2)*(3*a*d^2+b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE
(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(
1/2)*d))^(1/2))/b^(1/2)/d^2/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/
(-b*x^2+a)^(1/2)+4/15*a^(1/2)*c*(-a*d^2+b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c
+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(
1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^2/
(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.23 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.37

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx$$

$$= \frac{\sqrt{a-bx^2}\left(\frac{2(c+dx)(c+3dx)}{d}+\frac{4\left(d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\,(bc^2+3ad^2)\,(a-bx^2)+i\sqrt{b}\left(b^{3/2}c^3-\sqrt{a}bc^2d+3a\sqrt{b}cd^2-3a^{3/2}d^3\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}\cdots}{c+\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[c + d*x]*Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[a - b*x^2]*((2*(c + d*x)*(c + 3*d*x))/d + (4*(d^2*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*(b*c^2 + 3*a*d^2)*(a - b*x^2) + I*Sqrt[b]*(b^(3/2)*c^3 - Sqrt
[a]*b*c^2*d + 3*a*Sqrt[b]*c*d^2 - 3*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b]
+ x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^
(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (
Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*(b*c
^2 - 4*Sqrt[a]*Sqrt[b]*c*d + 3*a*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c +
 Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(b*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*(-a + b*x^2))))/(15*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {493, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-bx^2}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{ 493}$$

$$\frac{2\int \frac{(ad+bcx)\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{5d}+\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{2\left(-\frac{2\int -\frac{b\left(4acd+\left(bc^2+3ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{5d}+\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(\frac{1}{3}\int \frac{4acd+\left(bc^2+3ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{5d}+\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{2\left(\frac{1}{3}\left(\frac{\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{5d}+\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 509}$$

$$\frac{2\left(\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{5d}+\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$$\Big\downarrow \text{ 508}$$

$$\frac{2\left(\frac{1}{3}\left(-\frac{c(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3ad^2+bc^2)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}}{5d}$$

$$\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$\downarrow$ 327

$$\frac{2\left(\frac{1}{3}\left(-\frac{c(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3ad^2+bc^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+d}}{5d}$$

$$\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$\downarrow$ 512

$$\frac{2\left(\frac{1}{3}\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3ad^2+bc^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{2}{3}c\sqrt{a-bx}}{5d}$$

$$\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$\downarrow$ 511

$$\frac{2\left(\frac{1}{3}\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3ad^2+bc^2)E\left(\arcsin\left(\sqrt{\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{5d}$$

$$\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

$$\Big\downarrow \; 321$$

$$\frac{2\left(\frac{1}{3}\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3ad^2+bc^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.\right.}{5d}$$

$$\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}}{5d}$$

input
```
Int[Sqrt[c + d*x]*Sqrt[a - b*x^2],x]
```

output
```
(2*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*d) + (2*((-2*c*Sqrt[c + d*x]*Sqrt[a
 - b*x^2])/3 + ((-2*Sqrt[a]*(b*c^2 + 3*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqr
t[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sq
rt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c
 + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqr
t[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sq
rt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/3))/(5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.37

method	result
risch	$\frac{2(3dx+c)\sqrt{dx+c}\,\sqrt{-b\,x^{2}+a}}{15d}+\frac{2\left(\frac{\left(3a\,d^{2}+b\,c^{2}\right)\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\left(\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{\left(x+\sqrt{\ldots}\right)}{\sqrt{\ldots}}}}{2}\ldots\right.\right.}{b\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+\ldots}}\ldots\right)}{\ldots}$
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^{2}+a)}\left(\frac{2x\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{5}+\frac{2c\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{15d}+\frac{16ac\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\mathrm{El}\ldots}{15\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+\ldots}}\ldots\right)}{\ldots}$
default	Expression too large to display

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*d*x+c)/d*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)+2/15/d*((3*a*d^2+b*c^2)/b*
(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/
d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d-1/b*(a*b)^(1/2))*EllipticE(1/2*2^(1/
2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a
*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^
(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))+4*a*c*d/b*(a
*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b
/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))*((d*x
+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.65

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx$$
$$= \frac{2\left(2\,(bc^3-9\,acd^2)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\frac{3\,dx+c}{3\,d}\right)+6\,(bc^2d+3\,ad^3)\sqrt{-bd}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(2*(b*c^3 - 9*a*c*d^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*
a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*
(b*c^2*d + 3*a*d^3)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^
2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*
a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3
*(3*b*d^3*x + b*c*d^2)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b*d^3)
```

Sympy [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx=\int \sqrt{a-bx^2}\sqrt{c+dx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*sqrt(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx=\int \sqrt{-bx^2+a}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)*sqrt(d*x + c), x)
```

Giac [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx=\int \sqrt{-bx^2+a}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)*sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx = \int \sqrt{a-b\,x^2}\,\sqrt{c+d\,x}\,dx$$

input
```
int((a - b*x^2)^(1/2)*(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/2)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\,dx = \frac{-2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ad+2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bcx-3\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)ab\,d^2-\left(\int \frac{\sqrt{dx+c}\,\sqrt{}}{-bd\,x^3-bc}\right.}{5bc}$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d + 2*sqrt(c + d*x)*sqrt(a - b*x**2
)*b*c*x - 3*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x
**2 - b*d*x**3),x)*a*b*d**2 - int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a
*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**2*c**2 + int((sqrt(c + d*x)*sqrt(a
 - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*d**2 + 3*int((sqrt
(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**
2)/(5*b*c)
```

3.302 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 316

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}\sqrt{a-bx^2}}{3d} - \frac{4\sqrt{a}\sqrt{b}c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3d^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{4\sqrt{a}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{b}d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d-4/3*a^(1/2)*b^(1/2)*c*(d*x+c)^(1/2)*(
1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^2/(b^(1/2)*(d*x+c)/(b^(1/2)*c+
a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+4/3*a^(1/2)*(-a*d^2+b*c^2)*(b^(1/2)*(d*
x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/b^(1/2)/d^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.66 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{a-bx^2}\left(-c+dx+\frac{2i\sqrt{b}c\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}}{\sqrt{c+dx}}\right)\middle|\frac{\sqrt{b}c+\sqrt{a}d}{\sqrt{b}c-\sqrt{a}d}\right)}{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(-a+bx^2)}+\frac{2i\sqrt{a}\left(\cdots\right.}{\cdots}\right.}{3d\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[a - b*x^2]/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-c + d*x + ((2*I)*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*Sq
rt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)
/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) + ((2*I)*Sqrt[a]*(Sqrt[b]*
c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*
d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c
- Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*d*Sqrt
[c + d*x])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {493, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{493}$$

$$\frac{2\int \frac{ad+bcx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

$$\Big\downarrow \text{600}$$

$$\frac{2\left(\frac{bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

$$\Big\downarrow \text{509}$$

$$\frac{2\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

$$\Big\downarrow \text{508}$$

$$\frac{2\left(-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

$$\Big\downarrow \text{327}$$

$$\frac{2\left(-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

$$\Big\downarrow \text{512}$$

$$\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

$\downarrow$ 511

$$\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2}{\frac{\sqrt{bc}}{\sqrt{a}}}\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3d}$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

$\downarrow$ 321

$$\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3d}$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{3d}$$

input
```
Int[Sqrt[a - b*x^2]/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*d) + (2*((-2*Sqrt[a]*Sqrt[b]*c*Sqrt[c
 + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*Sqrt
[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Elliptic
F[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a
] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*d)
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.48

method	result
risch	$\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^{2}+a}}{3d}+\frac{2\left(\frac{ad\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}}{2},\sqrt{-\frac{2\sqrt{ab}}{b\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)}}\right)}{b\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}+\frac{c\sqrt{ab}\cdots}{\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^{2}+a)}\left(\frac{2\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{3d}+\frac{4a\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\cdots\right)}{3\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}\cdots\right)}{\sqrt{\cdots}}$
default	$-\frac{2\sqrt{-b\,x^{2}+a}\,\sqrt{dx+c}\left(2\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\,\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\,\sqrt{\frac{\left(bx+\sqrt{ab}\right)d}{d\sqrt{ab}-bc}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)\sqrt{ab}\,a\,d^{3}-2\sqrt{-\frac{(dx+\cdots}{d\sqrt{ab}\cdots}}\cdots\right)}{\cdots}$

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d+2/3/d*(a*d/b*(a*b)^(1/2)*2^(1/2)*((x+
1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2
)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c
)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-
2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))+c*(a*b)^(1/2)*2^(1/2)*((x+1/
b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*
(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*((c/d-1/b*(a*b)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/
(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*Ell
ipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(
1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))))*((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1
/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx = \frac{2\left(6\sqrt{-bd}bcd\text{weierstrassZeta}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3}\right.\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(6*sqrt(-b*d)*b*c*d*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/
27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/
(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*sqrt(-
b*x^2 + a)*sqrt(d*x + c)*b*d^2 + 2*(b*c^2 - 3*a*d^2)*sqrt(-b*d)*weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d))/(b*d^3)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx=\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx=\int \frac{\sqrt{-bx^2+a}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx=\int \frac{\sqrt{-bx^2+a}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx=\int \frac{\sqrt{a-b\,x^2}}{\sqrt{c+d\,x}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx}}\,dx=\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{dx+c}dx$$

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(c + d*x),x)
```

3.303 $\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 298

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx = -\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}} + \frac{4\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{d^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{4\sqrt{a}\sqrt{b}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*(-b*x^2+a)^(1/2)/d/(d*x+c)^(1/2)+4*a^(1/2)*b^(1/2)*(d*x+c)^(1/2)*(1-b*x
^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^2/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/
2)*d))^(1/2)/(-b*x^2+a)^(1/2)-4*a^(1/2)*b^(1/2)*c*(b^(1/2)*(d*x+c)/(b^(1/2
)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^2/(d*x+
c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.31 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-1-\frac{2\left(d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\,(a-bx^2)+i\sqrt{b}\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}\,(c+dx)^{3/2}E\left(i\operatorname{arcsi}\right.\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[a - b*x^2]/(c + d*x)^(3/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-1 - (2*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2
) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellip
ticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c +
 Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*Sqrt[(d*(Sqrt[a
]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*
(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
+ d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(d*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {492, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 492}$$

$$-\frac{2b\int \frac{x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

$\downarrow$ 600

$$-\frac{2b\left(\frac{\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

$\downarrow$ 509

$$-\frac{2b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

$\downarrow$ 508

$$-\frac{2b\left(-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

$\downarrow$ 327

$$-\frac{2b\left(-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

$\downarrow$ 512

$$-\frac{2b\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

$\downarrow$ 511

$$-\frac{2b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

$\downarrow$ 321

$$-\frac{2b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{d}-\frac{2\sqrt{a-bx^2}}{d\sqrt{c+dx}}$$

input
```
Int[Sqrt[a - b*x^2]/(c + d*x)^(3/2),x]
```

output
```
(-2*Sqrt[a - b*x^2])/(d*Sqrt[c + d*x]) - (2*b*((-2*Sqrt[a]*Sqrt[c + d*x]*S
qrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]]
, (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(S
qrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*Sqrt[(Sqrt[b]*(c +
d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[
b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/d
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(dx+c)(-bx^2+a)}\left(-\frac{2(-bd\,x^2+ad)}{d^2\sqrt{\left(x+\frac{c}{d}\right)(-bd\,x^2+ad)}}-\frac{4b\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\left(\left(-\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\operatorname{EllipticE}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{\cdots}}{b}}}\cdots\right.\right.}{d\sqrt{-bd\,x^3-bc\,x^2+adx+\cdots}}\cdots\right)}{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}\cdots$
default	$\frac{2\sqrt{-b\,x^2+a}\,\sqrt{dx+c}\left(2\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\sqrt{\frac{(-bx+\sqrt{ab})d}{d\sqrt{ab}+bc}}\sqrt{\frac{(bx+\sqrt{ab})d}{d\sqrt{ab}-bc}}\operatorname{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)a\,d^2-2\sqrt{ab}\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-b\cdots}}\cdots\right.}{\cdots}$

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2*(-b*d*x^2+a*
d)/d^2/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-4*b/d*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/
b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx =$$
$$-\frac{2\left(3\sqrt{-bx^2+a}\sqrt{dx+c}d^2+2\left(cdx+c^2\right)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4\left(bc^2+3\,ad^2\right)}{3\,bd^2},-\frac{8\left(bc^3-9\,acd^2\right)}{27\,bd^3},\frac{3\,dx+c}{3\,d}\right)+\right.}{}$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(-b*x^2 + a)*sqrt(d*x + c)*d^2 + 2*(c*d*x + c^2)*sqrt(-b*d)*we
ierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)
/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(d^2*x + c*d)*sqrt(-b*d)*weierstrassZeta(
4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstr
assPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^
3), 1/3*(3*d*x + c)/d)))/(d^4*x + c*d^3)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{a-bx^2}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(c + d*x)^(3/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{d^2x^2+2cdx+c^2}dx$$

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.304 $\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 362

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx=-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}+\frac{4bc\sqrt{a-bx^2}}{3d\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{4\sqrt{a}b^{3/2}c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3d^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{4\sqrt{a}\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*(-b*x^2+a)^(1/2)/d/(d*x+c)^(3/2)+4/3*b*c*(-b*x^2+a)^(1/2)/d/(-a*d^2+b
*c^2)/(d*x+c)^(1/2)-4/3*a^(1/2)*b^(3/2)*c*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*
EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1
/2)*c+a^(1/2)*d))^(1/2))/d^2/(-a*d^2+b*c^2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^
(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+4/3*a^(1/2)*b^(1/2)*(b^(1/2)*(d*x+c)/(b^(
1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^2/(d
*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.82 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-\frac{bc^2}{c+dx}+\frac{ad^2}{c+dx}+\frac{2ib^{3/2}c\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\cdots}}{\sqrt{\cdots}}\right)\cdots\right)}{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(-a+bx^2)}\cdots\right)}{3\left(bc^2\cdots\right)}$$

input
```
Integrate[Sqrt[a - b*x^2]/(c + d*x)^(5/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-((b*c^2)/(c + d*x)) + (a*d^2)/(c + d*x) + ((2*I)*b^(3
/2)*c*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sq
rt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*A
rcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]
*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*
x^2)) + ((2*I)*Sqrt[a]*b*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b]
+ x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (
Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]]*(-a + b*x^2))))/(3*(b*c^2*d - a*d^3)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {492, 594, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 492}$$

$$-\frac{2b\int \frac{x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3d}-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{594}$$

$$-\frac{2b\left(\frac{2\int -\frac{ad+bcx}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{3d}-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\left(-\frac{\int \frac{ad+bcx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{3d}-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{600}$$

$$-\frac{2b\left(-\frac{\frac{bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{3d}-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{509}$$

$$-\frac{2b\left(-\frac{\frac{bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{3d}-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{508}$$

$$-\frac{2b\left(-\frac{-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{3d}-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 327

$$-\frac{2b\left(-\frac{-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{3d}-\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 512

$$-\frac{2b\left(-\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{3d}-\ldots$$

$\Big\downarrow$ 511

$$-\frac{2b\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}\right)}{3d}$$

$$\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 321

$$-\frac{2b\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\cdots\right)}{3d}$$

$$\frac{2\sqrt{a-bx^2}}{3d(c+dx)^{3/2}}$$

input
```
Int[Sqrt[a - b*x^2]/(c + d*x)^(5/2),x]
```

output
```
(-2*Sqrt[a - b*x^2])/(3*d*(c + d*x)^(3/2)) - (2*b*((-2*c*Sqrt[a - b*x^2])/
((b*c^2 - a*d^2)*Sqrt[c + d*x]) - ((-2*Sqrt[a]*Sqrt[b]*c*Sqrt[c + d*x]*Sqr
t[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
(2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
 Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(
c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(S
qrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2)))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(292) = 584$.

Time = 1.12 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.88

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3d^3\left(x+\frac{c}{d}\right)^2}-\frac{4\left(-bd\,x^2+ad\right)bc}{3\left(a\,d^2-b\,c^2\right)d^2\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}+\frac{2\left(-\frac{2b}{3d^2}-\frac{2b^2c^2}{3d^2\left(a\,d^2-b\,c^2\right)}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/3/d^3*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2-4/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)/d^
2*b*c/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(-2/3*b/d^2-2/3*b^2*c^2/d^2/(a*d^2-
b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/
b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2))-4/3*b^2/d*c/(a*d^2-b*c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)
*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b
)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(
1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx = \frac{2\left(2\left(bc^4-3\,ac^2d^2+\left(bc^2d^2-3\,ad^4\right)x^2+2\left(bc^3d-3\,acd^3\right)x\right)\sqrt{-bd}\text{weierstrassPInverse}\right.}{}$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(2*(b*c^4 - 3*a*c^2*d^2 + (b*c^2*d^2 - 3*a*d^4)*x^2 + 2*(b*c^3*d - 3*a
*c*d^3)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -
8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(b*c*d^3*x^2 + 2*
b*c^2*d^2*x + b*c^3*d)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 +
 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d))
+ 3*(2*b*c*d^3*x + b*c^2*d^2 + a*d^4)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b*c
^4*d^3 - a*c^2*d^5 + (b*c^2*d^5 - a*d^7)*x^2 + 2*(b*c^3*d^4 - a*c*d^6)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{a-bx^2}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx=\int \frac{\sqrt{-bx^2+a}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx=\int \frac{\sqrt{-bx^2+a}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx=\int \frac{\sqrt{a-b\,x^2}}{{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(c + d*x)^(5/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(5/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3),x)
```

3.305 $\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 443

$$\begin{aligned}
\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}} \, dx = & -\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}} \\
& + \frac{4bc\sqrt{a-bx^2}}{15d\,(bc^2-ad^2)\,(c+dx)^{3/2}} + \frac{4b(bc^2+3ad^2)\,\sqrt{a-bx^2}}{15d\,(bc^2-ad^2)^2\,\sqrt{c+dx}} \\
& - \frac{4\sqrt{a}b^{3/2}(bc^2+3ad^2)\,\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15d^2\,(bc^2-ad^2)^2\,\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \\
& + \frac{4\sqrt{a}b^{3/2}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15d^2\,(bc^2-ad^2)\,\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-2/5*(-b*x^2+a)^(1/2)/d/(d*x+c)^(5/2)+4/15*b*c*(-b*x^2+a)^(1/2)/d/(-a*d^2+
b*c^2)/(d*x+c)^(3/2)+4/15*b*(3*a*d^2+b*c^2)*(-b*x^2+a)^(1/2)/d/(-a*d^2+b*c
^2)^2/(d*x+c)^(1/2)-4/15*a^(1/2)*b^(3/2)*(3*a*d^2+b*c^2)*(d*x+c)^(1/2)*(1-
b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^2/(-a*d^2+b*c^2)^2/(b^(1/2)*(d*x
+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+4/15*a^(1/2)*b^(3/2)*c*(
b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1
/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/d^2/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.53 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-\frac{3(bc^2-ad^2)^2}{(c+dx)^2}+\frac{2bc(bc^2-ad^2)}{c+dx}-\frac{2ib^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(bc^2+3ad^2)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)}{d^2(-a+bx^2)}\right.}{}$$

input
```
Integrate[Sqrt[a - b*x^2]/(c + d*x)^(7/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*((-3*(b*c^2 - a*d^2)^2)/(c + d*x)^2 + (2*b*c*(b*c^2 - a
*d^2))/(c + d*x) - ((2*I)*b^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 + 3*a*
d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
 - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*
d)])/(d^2*(-a + b*x^2)) + ((2*I)*Sqrt[a]*b^(3/2)*(b*c^2 - 4*Sqrt[a]*Sqrt[b
]*c*d + 3*a*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a
]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*
c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(15*d*(
b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {492, 594, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx$$

$$\Big\downarrow \text{492}$$

$$-\frac{2b\int \frac{x}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{5d}-\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{594}$$

$$-\frac{2b\left(\frac{2\int -\frac{3ad-bcx}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}-\frac{2c\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\left(-\frac{\int \frac{3ad-bcx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}-\frac{2c\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{688}$$

$$-\frac{2b\left(-\frac{\frac{2\int \frac{b\left(4acd+\left(bc^2+3ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}-\frac{2c\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\left(-\frac{\frac{b\int \frac{4acd+\left(bc^2+3ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}-\frac{2c\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 600$$

$$-\frac{2b\left(-\frac{\frac{b\left(\frac{\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3\left(bc^2-ad^2\right)}-\frac{2c\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}\right)}{5d}-\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 509$$

$$-\frac{2b\left(-\frac{\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3\left(bc^2-ad^2\right)}-\frac{2c\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}\right)}{5d}-\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 508$$

$$-\frac{2b\left(-\frac{\frac{b\left(-\frac{c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}-\frac{2c}{3(c+dx)}\right)}{5d}$$

$$\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 327

$$-\frac{2b\left(-\frac{\frac{b\left(-\frac{c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}-\frac{2}{3(c+d}\right)}{5d}$$

$$\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 512

$$-\frac{2b\left(-\frac{\frac{b\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)} \cdots\right)}{5d}$$

$$\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 511

$$-\frac{2b\left(-\frac{\frac{b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}} \cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\cdots\right)}{bc^2-ad^2}\cdots}{3(bc^2-ad^2)}\cdots\right)}{5d}$$

$$\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 321

$$-\frac{2b\left(-\frac{\frac{b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\cdots}{\frac{\sqrt{b}c}{\sqrt{a}}\cdots}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{bc^2-ad^2}\cdots}{3(bc^2-ad^2)}\cdots\right.}{5d}\cdots$$

$$\frac{2\sqrt{a-bx^2}}{5d(c+dx)^{5/2}}$$

input
```
Int[Sqrt[a - b*x^2]/(c + d*x)^(7/2),x]
```

output
```
(-2*Sqrt[a - b*x^2])/(5*d*(c + d*x)^(5/2)) - (2*b*((-2*c*Sqrt[a - b*x^2])/
(3*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) - ((2*(b*c^2 + 3*a*d^2)*Sqrt[a - b*x^2
])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + (b*((-2*Sqrt[a]*(b*c^2 + 3*a*d^2)*Sqr
t[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[
a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(
c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*(b*c^2
- a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2
)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt
[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(b*c^2 -
 a*d^2))/(3*(b*c^2 - a*d^2))))/(5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(367) = 734$.

Time = 1.34 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.77

method	result
elliptic	$\frac{\sqrt{(dx+c)(-bx^2+a)}\left(-\frac{2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d^4\left(x+\frac{c}{d}\right)^3}-\frac{4bc\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{15d^3\left(a\,d^2-b\,c^2\right)\left(x+\frac{c}{d}\right)^2}+\frac{4\left(-bd\,x^2+ad\right)b\left(3a\,d^2+b\,c^2\right)}{15\left(a\,d^2-b\,c^2\right)^2d^2\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}+\frac{2\left(\frac{2b^2}{15d^2\left(a\,d\right.}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/5/d^4*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3-4/15*b/d^3/(a*d^2-b*c^2)*c*(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+4/15*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2/d
^2*b*(3*a*d^2+b*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(2/15*b^2/d^2*c/(a*d
^2-b*c^2)+2/15*b^2/d^2*c*(3*a*d^2+b*c^2)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+4/15*b^2/d*(
3*a*d^2+b*c^2)/(a*d^2-b*c^2)^2*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)
*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx = \frac{2\left(2\left(b^2c^6-9\,abc^4d^2+(b^2c^3d^3-9\,abcd^5)x^3+3\,(b^2c^4d^2-9\,abc^2d^4)x^2+3\,(b^2c^5d-9\,ab\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/45*(2*(b^2*c^6 - 9*a*b*c^4*d^2 + (b^2*c^3*d^3 - 9*a*b*c*d^5)*x^3 + 3*(b^
2*c^4*d^2 - 9*a*b*c^2*d^4)*x^2 + 3*(b^2*c^5*d - 9*a*b*c^3*d^3)*x)*sqrt(-b*
d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c
*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(b^2*c^5*d + 3*a*b*c^3*d^3 + (b^2*c^
2*d^4 + 3*a*b*d^6)*x^3 + 3*(b^2*c^3*d^3 + 3*a*b*c*d^5)*x^2 + 3*(b^2*c^4*d^
2 + 3*a*b*c^2*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*
d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 +
3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) +
 3*(b^2*c^4*d^2 + 10*a*b*c^2*d^4 - 3*a^2*d^6 + 2*(b^2*c^2*d^4 + 3*a*b*d^6)
*x^2 + 2*(3*b^2*c^3*d^3 + 5*a*b*c*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/
(b^2*c^7*d^3 - 2*a*b*c^5*d^5 + a^2*c^3*d^7 + (b^2*c^4*d^6 - 2*a*b*c^2*d^8
+ a^2*d^10)*x^3 + 3*(b^2*c^5*d^5 - 2*a*b*c^3*d^7 + a^2*c*d^9)*x^2 + 3*(b^2
*c^6*d^4 - 2*a*b*c^4*d^6 + a^2*c^2*d^8)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{a-bx^2}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(d*x+c)**(7/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{(c+d\,x)^{7/2}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(c + d*x)^(7/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx$$

input
```
int((-b*x^2+a)^(1/2)/(d*x+c)^(7/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2
 + 4*c*d**3*x**3 + d**4*x**4),x)
```

3.306 $\int (c+dx)^{3/2} \left(a-bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 512

$$
\begin{aligned}
\int (c+dx)^{3/2} \left(a-bx^2\right)^{3/2} \, dx = & -\frac{8\left(21ac^2-\frac{4bc^4}{d^2}+\frac{15a^2d^2}{b}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{1155d} \\
& +\frac{32c(bc^2-4ad^2)\,(c+dx)^{3/2}\sqrt{a-bx^2}}{1155d^3} \\
& -\frac{4(c+dx)^{5/2}\,(4bc^2-9ad^2-7bcdx)\sqrt{a-bx^2}}{231d^3}+\frac{2(c+dx)^{5/2}\,(a-bx^2)^{3/2}}{11d} \\
& +\frac{32\sqrt{a}c(bc^2-9ad^2)\,(bc^2+3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{1155\sqrt{b}d^4\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \\
& -\frac{8\sqrt{a}(4b^3c^6-25ab^2c^4d^2+6a^2bc^2d^4+15a^3d^6)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}\cdots}\right.}{1155b^{3/2}d^4\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}
$$

output
```
-8/1155*(21*a*c^2-4*b*c^4/d^2+15*a^2*d^2/b)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)
/d+32/1155*c*(-4*a*d^2+b*c^2)*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^3-4/231*(d*
x+c)^(5/2)*(-7*b*c*d*x-9*a*d^2+4*b*c^2)*(-b*x^2+a)^(1/2)/d^3+2/11*(d*x+c)^
(5/2)*(-b*x^2+a)^(3/2)/d+32/1155*a^(1/2)*c*(-9*a*d^2+b*c^2)*(3*a*d^2+b*c^2
)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2
)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^4/(b^
(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-8/1155*a^(1/2)
*(15*a^3*d^6+6*a^2*b*c^2*d^4-25*a*b^2*c^4*d^2+4*b^3*c^6)*(b^(1/2)*(d*x+c)/
(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^
(3/2)/d^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.65 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.29

$$\int (c+dx)^{3/2} \left(a -bx^2\right)^{3/2} dx = \frac{2\sqrt{a-bx^2}\left(-((c+dx)\left(60a^2d^4-abd^2(47c^2+326cdx+195d^2x^2)+b^2(8c^4-6c^3dx+5c^2d\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*(a - b*x^2)^(3/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-((c + d*x)*(60*a^2*d^4 - a*b*d^2*(47*c^2 + 326*c*d*x
+ 195*d^2*x^2) + b^2*(8*c^4 - 6*c^3*d*x + 5*c^2*d^2*x^2 + 140*c*d^3*x^3 +
105*d^4*x^4))) + (4*(4*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b^2*c^4 - 6*a
*b*c^2*d^2 - 27*a^2*d^4)*(a - b*x^2) + (4*I)*Sqrt[b]*c*(b^(5/2)*c^5 - Sqrt
[a]*b^2*c^4*d - 6*a*b^(3/2)*c^3*d^2 + 6*a^(3/2)*b*c^2*d^3 - 27*a^2*Sqrt[b]
*c*d^4 + 27*a^(5/2)*d^5)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(
((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSin
h[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(
Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(4*b^(5/2)*c^5 - Sqrt[a]*b^2*c^4*d -
 24*a*b^(3/2)*c^3*d^2 + 114*a^(3/2)*b*c^2*d^3 - 108*a^2*Sqrt[b]*c*d^4 + 15
*a^(5/2)*d^5)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d
)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c -
 Sqrt[a]*d)]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(1155*b*
d^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {497, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{3/2} (c+dx)^{3/2} \, dx$$

$$\Big\downarrow \ 497$$

$$-\frac{2\int -\frac{\left(11bc^2+12bdxc+ad^2\right)\left(a-bx^2\right)^{3/2}}{2\sqrt{c+dx}}dx}{11b}-\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(11bc^2+12bdxc+ad^2\right)\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx}{11b}-\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

$$\Big\downarrow \ 682$$

$$\frac{\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}(3ad^2+bc^2+28bcdx)}{21d}-\frac{4\int -\frac{3bd\left(ad\left(29bc^2+3ad^2\right)+bc\left(bc^2+31ad^2\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{21bd^2}}{11b}-\frac{2d(a-bx^2)^{5/2}\sqrt{c+dx}}{11b}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\int \frac{\left(ad\left(29bc^2+3ad^2\right)+bc\left(bc^2+31ad^2\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{7d}+\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}(3ad^2+bc^2+28bcdx)}{21d}}{11b}-\frac{2d(a-bx^2)^{5/2}\sqrt{c+dx}}{11b}$$

$\Big\downarrow$ 682

$$\frac{\frac{2\left(-\frac{4\int \frac{b\left(ad\left(b^2c^4-114abd^2c^2-15a^2d^4\right)+4bc\left(bc^2-9ad^2\right)\left(bc^2+3ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-15a^2d^4-3bcdx\left(31ad^2+bc^2\right)-21abc^2d^2+4b^2c^4\right)}{15d^2}\right)}{7d}+\frac{2(a\cdots}{\cdots}}{11b}-\frac{2d(a-bx^2)^{5/2}\sqrt{c+dx}}{11b}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\left(-\frac{2\int \frac{ad\left(b^2c^4-114abd^2c^2-15a^2d^4\right)+4bc\left(bc^2-9ad^2\right)\left(bc^2+3ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-15a^2d^4-3bcdx\left(31ad^2+bc^2\right)-21abc^2d^2+4b^2c^4\right)}{15d^2}\right)}{7d}+\frac{2(a-b\cdots}{\cdots}}{11b}-\frac{2d(a-bx^2)^{5/2}\sqrt{c+dx}}{11b}$$

$\Big\downarrow$ 600

$$\frac{\frac{2\left(-\frac{2\left(\frac{4bc\left(bc^2-9ad^2\right)\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(-15a^2d^4-21abc^2d^2+4b^2c^4\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-15a^2d^4-3bcdx\left(31\cdots\right.\right.}{15d^2}\cdots\right)}{7d}\cdots}{11b}-\frac{2d(a-bx^2)^{5/2}\sqrt{c+dx}}{11b}$$

$\Big\downarrow$ 509

$$\frac{\dfrac{2\left(-\dfrac{2\left(\dfrac{4bc\sqrt{1-\frac{bx^2}{a}}\left(bc^2-9ad^2\right)\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\dfrac{\left(bc^2-ad^2\right)\left(-15a^2d^4-21abc^2d^2+4b^2c^4\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\dfrac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-15a^2d^4-\right.}{}\right.}{7d}}{11b}$$

$$\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

$$\Big\downarrow \; 508$$

$$\frac{\dfrac{2\left(-\dfrac{2\left(-\dfrac{\left(bc^2-ad^2\right)\left(-15a^2d^4-21abc^2d^2+4b^2c^4\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\dfrac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-9ad^2\right)\left(3ad^2+bc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}\right.}{7d}}{11b}$$

$$\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

$$\Big\downarrow \; 327$$

$$2\left(-\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(-15a^2d^4-21abc^2d^2+4b^2c^4\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-9ad^2\right)\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots}{15d^2}\cdots}{7d}\cdots}{11b}\cdots$$

$$\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

$\Big\downarrow$ 512

$$2\left(-\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(-15a^2d^4-21abc^2d^2+4b^2c^4\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-9ad^2\right)\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\cdots\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots}{15d^2}\cdots}{7d}\cdots}{11b}\cdots$$

$$\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

$\Big\downarrow$ 511

$$2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(-15a^2d^4-21abc^2d^2+4b^2c^4\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-\right.}{}\right)}{15d^2}\right.}{7d}$$

$$\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

$\big\downarrow$ 321

$$2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(-15a^2d^4-21abc^2d^2+4b^2c^4\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-9ad^2\right)\left(\right.}{d\sqrt{a-b}}\right)}{15d^2}\right.}{7d}$$

$$\frac{2d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11b}$$

input
```
Int[(c + d*x)^(3/2)*(a - b*x^2)^(3/2),x]
```

output
```
(-2*d*Sqrt[c + d*x]*(a - b*x^2)^(5/2))/(11*b) + ((2*Sqrt[c + d*x]*(b*c^2 +
 3*a*d^2 + 28*b*c*d*x)*(a - b*x^2)^(3/2))/(21*d) + (2*((-2*Sqrt[c + d*x]*(
4*b^2*c^4 - 21*a*b*c^2*d^2 - 15*a^2*d^4 - 3*b*c*d*(b*c^2 + 31*a*d^2)*x)*Sq
rt[a - b*x^2])/(15*d^2) - (2*((-8*Sqrt[a]*Sqrt[b]*c*(b*c^2 - 9*a*d^2)*(b*c
^2 + 3*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(
Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*
(b*c^2 - a*d^2)*(4*b^2*c^4 - 21*a*b*c^2*d^2 - 15*a^2*d^4)*Sqrt[(Sqrt[b]*(c
 + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqr
t[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sq
rt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(7*d))/(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(430) = 860$.

Time = 4.42 (sec) , antiderivative size = 934, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{2\left(105b^{2}d^{4}x^{4}+140b^{2}c\,d^{3}x^{3}-195ab\,d^{4}x^{2}+5d^{2}c^{2}x^{2}b^{2}-326abc\,d^{3}x-6b^{2}c^{3}dx+60a^{2}d^{4}-47b\,c^{2}d^{2}a+8b^{2}c^{4}\right)\sqrt{dx+c}\,\sqrt{-b\,x^{2}+a}}{1155b\,d^{3}}+ -$ ... 4
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^(3/2)*(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155/b*(105*b^2*d^4*x^4+140*b^2*c*d^3*x^3-195*a*b*d^4*x^2+5*b^2*c^2*d^2
*x^2-326*a*b*c*d^3*x-6*b^2*c^3*d*x+60*a^2*d^4-47*a*b*c^2*d^2+8*b^2*c^4)*(d
*x+c)^(1/2)/d^3*(-b*x^2+a)^(1/2)+4/1155/b/d^3*(4*c*(27*a^2*d^4+6*a*b*c^2*d
^2-b^2*c^4)*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2
))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d-1/b*(a*b)^(1/2))*Ellipti
cE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)
/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/
2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))+
15*a^3*d^5/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)
*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/
2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b
*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2))
)^(1/2))-a*b*c^4*d*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))
^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*
b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*(
(x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^
(1/2)))^(1/2))+114*a^2*c^2*d^3*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/
(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^
(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elliptic...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.72

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^{3/2} \, dx = -\frac{2 \left(4 \left(4 b^3c^6-27 ab^2c^4d^2+234 a^2bc^2d^4+45 a^3d^6\right)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4 \left(bc^2+3 ad^2\right)}{3 bd^2}, -\frac{8 \left(bc^3-9 acd^2\right)}{27 bd^3}, 3\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3465*(4*(4*b^3*c^6 - 27*a*b^2*c^4*d^2 + 234*a^2*b*c^2*d^4 + 45*a^3*d^6)
*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^
3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 48*(b^3*c^5*d - 6*a*b^2*c^3*d
^3 - 27*a^2*b*c*d^5)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d
^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3
*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) +
3*(105*b^3*d^6*x^4 + 140*b^3*c*d^5*x^3 + 8*b^3*c^4*d^2 - 47*a*b^2*c^2*d^4
+ 60*a^2*b*d^6 + 5*(b^3*c^2*d^4 - 39*a*b^2*d^6)*x^2 - 2*(3*b^3*c^3*d^3 + 1
63*a*b^2*c*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^2*d^5)
```

Sympy [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^{3/2} \, dx = \int \left(a-bx^2\right)^{\frac{3}{2}} (c+dx)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x+c)**(3/2)*(-b*x**2+a)**(3/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^{3/2} \, dx = \int \left(-bx^2+a\right)^{\frac{3}{2}} (dx+c)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^{3/2} \, dx = \int \left(-bx^2+a\right)^{\frac{3}{2}} (dx+c)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^{3/2} \, dx = \int \left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2} \,d x$$

input
```
int((a - b*x^2)^(3/2)*(c + d*x)^(3/2),x)
```

output
```
int((a - b*x^2)^(3/2)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2} \left(a -bx^2\right)^{3/2} \, dx = \frac{-\frac{184\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,a^2 d^3}{385}-\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ab\,c^2 d}{1155}+\frac{652\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,abc\,d^2 x}{1155}+\frac{26\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ab\,d}{77}}{}$$

input
```
int((d*x+c)^(3/2)*(-b*x^2+a)^(3/2),x)
```

output
```
(2*( - 276*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 - sqrt(c + d*x)*sqrt(a
 - b*x**2)*a*b*c**2*d + 326*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x +
195*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d**3*x**2 + 6*sqrt(c + d*x)*sqrt(a
- b*x**2)*b**2*c**3*x - 5*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x**2
- 140*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 - 105*sqrt(c + d*x)*
sqrt(a - b*x**2)*b**2*d**3*x**4 - 324*int((sqrt(c + d*x)*sqrt(a - b*x**2)*
x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*d**4 - 72*int((sqrt(c
+ d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**
2*c**2*d**2 + 12*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x -
b*c*x**2 - b*d*x**3),x)*b**3*c**4 + 138*int((sqrt(c + d*x)*sqrt(a - b*x**2
))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*d**4 + 252*int((sqrt(c + d*
x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*c**2*d*
*2 - 6*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*
x**3),x)*a*b**2*c**4))/(1155*b*d**2)
```

3.307 $\int \sqrt{c+dx}(a-bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 445

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx=\frac{32c(bc^2-3ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}{315d^3}$$
$$-\frac{4(c+dx)^{3/2}\left(4bc^2-7ad^2-5bcdx\right)\sqrt{a-bx^2}}{105d^3}+\frac{2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}{9d}$$
$$+\frac{8\sqrt{a}(4b^2c^4-15abc^2d^2-21a^2d^4)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{315\sqrt{b}d^4\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$-\frac{32\sqrt{a}c(b^2c^4-4abc^2d^2+3a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{315\sqrt{b}d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
32/315*c*(-3*a*d^2+b*c^2)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^3-4/105*(d*x+c)
^(3/2)*(-5*b*c*d*x-7*a*d^2+4*b*c^2)*(-b*x^2+a)^(1/2)/d^3+2/9*(d*x+c)^(3/2)
*(-b*x^2+a)^(3/2)/d+8/315*a^(1/2)*(-21*a^2*d^4-15*a*b*c^2*d^2+4*b^2*c^4)*(
d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2
^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^4/(b^(1/
2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-32/315*a^(1/2)*c*
(3*a^2*d^4-4*a*b*c^2*d^2+b^2*c^4)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^
(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),
2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^4/(d*x+c)^(1/2)
/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.46 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.37

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx=\frac{\sqrt{a-bx^2}\left(-\frac{2(c+dx)\left(-ad^2(29c+77dx)+b\left(8c^3-6c^2dx+5cd^2x^2+35d^3x^3\right)\right)}{d^3}+\frac{8\left(d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(4b^2c^4-15abc^2d^2-21a\right.\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]*(a - b*x^2)^(3/2),x]
```

output
```
(Sqrt[a - b*x^2]*((-2*(c + d*x)*(-(a*d^2*(29*c + 77*d*x)) + b*(8*c^3 - 6*c
^2*d*x + 5*c*d^2*x^2 + 35*d^3*x^3)))/d^3 + (8*(d^2*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]*(4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a^2*d^4)*(a - b*x^2) + I*Sqrt[b]*
(4*b^(5/2)*c^5 - 4*Sqrt[a]*b^2*c^4*d - 15*a*b^(3/2)*c^3*d^2 + 15*a^(3/2)*b
*c^2*d^3 - 21*a^2*Sqrt[b]*c*d^4 + 21*a^(5/2)*d^5)*Sqrt[(d*(Sqrt[a]/Sqrt[b]
 + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)
^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]],
(Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*(4*
b^2*c^4 - Sqrt[a]*b^(3/2)*c^3*d - 15*a*b*c^2*d^2 + 33*a^(3/2)*Sqrt[b]*c*d^
3 - 21*a^2*d^4)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]
*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c
 - Sqrt[a]*d)]))/(b*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(315
*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {493, 687, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{3/2}\sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{ 493}$$

$$\frac{2\int (ad+bcx)\sqrt{c+dx}\sqrt{a-bx^2}dx}{3d}+\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{2\left(-\frac{2\int -\frac{b\left(8acd+\left(bc^2+7ad^2\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{7b}-\frac{2}{7}c\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\right)}{3d}+\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(\frac{1}{7}\int \frac{\left(8acd+\left(bc^2+7ad^2\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx-\frac{2}{7}c\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\right)}{3d}+\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\downarrow \text{682}$$

$$\frac{2\left(\frac{1}{7}\left(-\frac{4\int \frac{b\left(acd\left(bc^2-33ad^2\right)+\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4c\left(bc^2-3ad^2\right)-3dx\left(7ad^2+bc^2\right)\right)}{15d^2}\right)-\frac{2}{7}c\left(a-bx^2\right)^{3/}\right.}{3d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{7}\left(-\frac{2\int \frac{acd\left(bc^2-33ad^2\right)+\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4c\left(bc^2-3ad^2\right)-3dx\left(7ad^2+bc^2\right)\right)}{15d^2}\right)-\frac{2}{7}c\left(a-bx^2\right)^{3/2}\right.}{3d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\downarrow \text{600}$$

$$\frac{2\left(\frac{1}{7}\left(-\frac{2\left(\frac{\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4c\left(bc^2-3ad^2\right)-3dx\right.}{15d^2}\right.\right.}{3d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\downarrow \text{509}$$

$$\frac{2\left(\frac{1}{7}\left(-\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4c\left(bc^2-3a\right.\right.}{15d^2}\right.\right.}{3d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\downarrow \text{508}$$

$$2\left(\frac{1}{7}\left(-\frac{2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2}{}\right.\right.$$

$$\overline{\qquad\qquad\qquad\qquad 3d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \; 327$$

$$2\left(\frac{1}{7}\left(-\frac{2\left(-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\right.\right.$$

$$\overline{\qquad\qquad\qquad\qquad 3d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \; 512$$

$$2\left(\frac{1}{7}\left(-\frac{2\left(-\frac{4c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+\cdots}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{15d^2}\cdots\right)\cdots\right)$$

$$\cdots 3d$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$\big\downarrow$ 511

$$2\left(\frac{1}{7}\left(-\frac{2\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2\cdots\right.}{\sqrt{b}d\sqrt{a-\cdots}}\cdots\right)}{15d^2}\cdots\right)\cdots\right)$$

$$\cdots 3d$$

$$\frac{2\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9d}$$

$\big\downarrow$ 321

$$\frac{2\left(\frac{1}{7}\left(-\frac{2\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}}\right)}{15d^2}\cdots\right.\right.}{3d\cdots}$$

$$\frac{2(a-bx^2)^{3/2}(c+dx)^{3/2}}{9d}$$

input
```
Int[Sqrt[c + d*x]*(a - b*x^2)^(3/2),x]
```

output
```
(2*(c + d*x)^(3/2)*(a - b*x^2)^(3/2))/(9*d) + (2*((-2*c*Sqrt[c + d*x]*(a -
 b*x^2)^(3/2))/7 + ((-2*Sqrt[c + d*x]*(4*c*(b*c^2 - 3*a*d^2) - 3*d*(b*c^2
+ 7*a*d^2)*x)*Sqrt[a - b*x^2])/(15*d^2) - (2*((-2*Sqrt[a]*(4*b^2*c^4 - 15*
a*b*c^2*d^2 - 21*a^2*d^4)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcS
in[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)
])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b
*x^2]) + (8*Sqrt[a]*c*(b*c^2 - 3*a*d^2)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c +
 d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[
1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt
[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2))/7))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.61

method	result
risch	$\frac{2\left(-35 b \,d^{3} x^{3}-5 b c \,d^{2} x^{2}+77 a x \,d^{3}+6 b \,c^{2} d x +29 a \,d^{2} c -8 b \,c^{3}\right) \sqrt{d x +c}\, \sqrt{-b \,x^{2}+a}}{315 d^{3}}+\frac{4\left(\frac{\left(21 a^{2} d^{4}+15 b \,c^{2} d^{2} a -4 b^{2} c^{4}\right) \sqrt{a b}\, \sqrt{2}\, \sqrt{\frac{\left(x +\frac{\sqrt{a b}}{b}\right)}{\sqrt{a b}}}}{}\right.}{}$
elliptic	$\frac{\sqrt{\left(d x +c \right) \left(-b \,x^{2}+a \right)}\, \left(-\frac{2 b \,x^{3} \sqrt{-b d \,x^{3}-b c \,x^{2}+a d x +a c}}{9}-\frac{2 b c \,x^{2} \sqrt{-b d \,x^{3}-b c \,x^{2}+a d x +a c}}{63 d}-\frac{2 \left(-\frac{11 d a b}{9}-\frac{2 b^{2} c^{2}}{21 d}\right) x \sqrt{-b d \,x^{3}-b c \,x^{2}+a d x +a c}}{5 b d}-\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(-35*b*d^3*x^3-5*b*c*d^2*x^2+77*a*d^3*x+6*b*c^2*d*x+29*a*c*d^2-8*b*c
^3)/d^3*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)+4/315/d^3*((21*a^2*d^4+15*a*b*c^2*d
^2-4*b^2*c^4)/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1
/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^
(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d-1/b*(a*b)^(1/2))*Ell
ipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(
1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)
^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2
)))+33*d^3*c*a^2/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))
^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*
b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*(
(x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^
(1/2)))^(1/2))-a*c^3*d*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1
/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b
/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/
2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a
*b)^(1/2)))^(1/2)))*((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.70

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx = -\frac{2\left(8\left(2b^2c^5-9abc^3d^2+39a^2cd^4\right)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\left(bc^2+3ad^2\right)}{3bd^2},-\frac{8\left(bc^3-9acd^2\right)}{27bd^3},\frac{3dx+c}{3d}\right)+12\left(4b^2\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/945*(8*(2*b^2*c^5 - 9*a*b*c^3*d^2 + 39*a^2*c*d^4)*sqrt(-b*d)*weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d) + 12*(4*b^2*c^4*d - 15*a*b*c^2*d^3 - 21*a^2*d^5)*sqrt
(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c
*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(35*b^2*d^5*x^3 + 5*b^2
*c*d^4*x^2 + 8*b^2*c^3*d^2 - 29*a*b*c*d^4 - (6*b^2*c^2*d^3 + 77*a*b*d^5)*x
)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b*d^5)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx=\int \left(a-bx^2\right)^{\frac{3}{2}}\sqrt{c+dx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-b*x**2+a)**(3/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*sqrt(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx=\int \left(-bx^2+a\right)^{\frac{3}{2}}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)*sqrt(d*x + c), x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx=\int \left(-bx^2+a\right)^{\frac{3}{2}}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)*sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx=\int \left(a-b\,x^2\right)^{3/2}\,\sqrt{c+d\,x}\,dx$$

input
```
int((a - b*x^2)^(3/2)*(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^(3/2)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\,dx=\frac{-\frac{4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,a^2d^3}{15}-\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ab\,c^2d}{315}+\frac{22\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,abc\,d^2x}{45}+\frac{4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,b^2c^3x}{105}-}{}$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(3/2),x)
```

output
```
(2*( - 42*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 - sqrt(c + d*x)*sqrt(a
- b*x**2)*a*b*c**2*d + 77*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x + 6*
sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**3*x - 5*sqrt(c + d*x)*sqrt(a - b*x*
*2)*b**2*c**2*d*x**2 - 35*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3
- 63*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b
*d*x**3),x)*a**2*b*d**4 - 45*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*
c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c**2*d**2 + 12*int((sqrt(c + d*
x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**3*c**4
 + 21*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x
**3),x)*a**3*d**4 + 81*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x -
 b*c*x**2 - b*d*x**3),x)*a**2*b*c**2*d**2 - 6*int((sqrt(c + d*x)*sqrt(a -
b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c**4))/(315*b*c*d**
2)
```

3.308 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 390

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = -\frac{4\sqrt{c+dx}(4bc^2-5ad^2-3bcdx)\sqrt{a-bx^2}}{35d^3} + \frac{2\sqrt{c+dx}(a-bx^2)^{3/2}}{7d} + \frac{32\sqrt{a}\sqrt{b}c(bc^2-2ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{35d^4\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{8\sqrt{a}(4b^2c^4-9abc^2d^2+5a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{35\sqrt{b}d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-4/35*(d*x+c)^(1/2)*(-3*b*c*d*x-5*a*d^2+4*b*c^2)*(-b*x^2+a)^(1/2)/d^3+2/7*
(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/d+32/35*a^(1/2)*b^(1/2)*c*(-2*a*d^2+b*c^2)*
(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^4/(b^(1/2)*(d*x
+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-8/35*a^(1/2)*(5*a^2*d^4-
9*a*b*c^2*d^2+4*b^2*c^4)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-
b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^4/(d*x+c)^(1/2)/(-b*x^2+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.53 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.35

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{2(c+dx)\left(-15ad^2+b\left(8c^2-6cdx+5d^2x^2\right)\right)}{d^3}+\frac{8\left(4cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(bc^2-2ad^2\right)\left(a-bx^2\right)+4i\sqrt{b}c\left(b^{3/\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a - b*x^2)^(3/2)/Sqrt[c + d*x],x]
```

output
```
(Sqrt[a - b*x^2]*((-2*(c + d*x)*(-15*a*d^2 + b*(8*c^2 - 6*c*d*x + 5*d^2*x^
2)))/d^3 + (8*(4*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - 2*a*d^2)*(a
 - b*x^2) + (4*I)*Sqrt[b]*c*(b^(3/2)*c^3 - Sqrt[a]*b*c^2*d - 2*a*Sqrt[b]*c
*d^2 + 2*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((S
qrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqr
t[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(4*b^(3/2)*c^3 - Sqrt[a]*b*c^2*d - 8*a*
Sqrt[b]*c*d^2 + 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*S
qrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*
ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a
]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b
*x^2))))/(35*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {493, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \ 493$$

$$\frac{6\int \frac{(ad+bcx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{7d}+\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}}{7d}$$

$$\Big\downarrow \ 682$$

$$\frac{6\left(-\frac{4\int \frac{b\left(ad\left(bc^2-5ad^2\right)+4bc\left(bc^2-2ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5ad^2+4bc^2-3bcdx)}{15d^2}\right)}{7d}+\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}}{7d}$$

$$\Big\downarrow \ 27$$

$$\frac{6\left(-\frac{2\int \frac{ad\left(bc^2-5ad^2\right)+4bc\left(bc^2-2ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5ad^2+4bc^2-3bcdx)}{15d^2}\right)}{7d}+\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}}{7d}$$

$$\Big\downarrow \ 600$$

$$\frac{6\left(-\frac{2\left(\frac{4bc\left(bc^2-2ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5ad^2+4bc^2-3bcdx)}{15d^2}\right)}{7d}+\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}}{7d}$$

$$\Big\downarrow \text{509}$$

$$\frac{6\left(-\frac{2\left(\frac{4bc\sqrt{1-\frac{bx^2}{a}}\left(bc^2-2ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-5ad^2+4bc^2-3bcdx\right)}{15d^2}\right)}{7d}+\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7d}$$

$$\Big\downarrow \text{508}$$

$$\frac{6\left(-\frac{2\left(-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(}{1}\right.}{7d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7d}$$

$$\Big\downarrow \text{327}$$

$$6\left(-\frac{2\left(-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\cdots}{\cdots}\right. \over 7d$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7d}$$

$\downarrow$ 512

$$6\left(-\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-b\cdots}}{\cdots}\right. \over 7d$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7d}$$

$\downarrow$ 511

$$\frac{6\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\text{arcs}\ldots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+d\ldots}{\sqrt{a}d+\sqrt{\ldots}}}}\ldots\right.}{15d^2}\ldots\right.}{7d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7d}$$

$\Big\downarrow$ 321

$$\frac{6\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\ldots}}{\ldots}\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\ldots\right.}{15d^2}\ldots\right.}{7d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7d}$$

input
```
Int[(a - b*x^2)^(3/2)/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/(7*d) + (6*((-2*Sqrt[c + d*x]*(4*b*c^2
 - 5*a*d^2 - 3*b*c*d*x)*Sqrt[a - b*x^2])/(15*d^2) - (2*((-8*Sqrt[a]*Sqrt[b
]*c*(b*c^2 - 2*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[S
qrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(
d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*
Sqrt[a]*(4*b*c^2 - 5*a*d^2)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt
[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]
*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c
 + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(320) = 640$.

Time = 3.58 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.74

method	result
risch	$\frac{2(-5b\,x^2d^2+6bcdx+15a\,d^2-8b\,c^2)\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{35d^3}+\frac{4\left(\frac{4c\left(2a\,d^2-b\,c^2\right)\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\left(\frac{c}{d}-\ldots\right.}{\ldots}\ldots\right)}{\ldots}$
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2b\,x^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7d}+\frac{12bcx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{35d^2}-\frac{2\left(-\frac{9ab}{7}+\frac{24b^2c^2}{35d^2}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}+\frac{2(\ldots}{\ldots}\ldots\right.}{\ldots}$
default	Expression too large to display

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(-5*b*d^2*x^2+6*b*c*d*x+15*a*d^2-8*b*c^2)/d^3*(d*x+c)^(1/2)*(-b*x^2+a
)^(1/2)+4/35/d^3*(4*c*(2*a*d^2-b*c^2)*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1
/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b
*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/
d-1/b*(a*b)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2
))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2
*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-
1/b*(a*b)^(1/2)))^(1/2)))+5*a^2*d^3/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1
/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b
*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elli
pticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1
/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-a*c^2*d*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b
)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x
-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*
EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b
)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))*((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^
(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.71

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = -\frac{2\left(4\left(4\,b^2c^4-11\,abc^2d^2+15\,a^2d^4\right)\sqrt{-bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\frac{3\,dx+c}{3\,d}\right)+48\,(b^2c\right.}{}$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(4*(4*b^2*c^4 - 11*a*b*c^2*d^2 + 15*a^2*d^4)*sqrt(-b*d)*weierstrass
PInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
 1/3*(3*d*x + c)/d) + 48*(b^2*c^3*d - 2*a*b*c*d^3)*sqrt(-b*d)*weierstrassZ
eta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weie
rstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), 1/3*(3*d*x + c)/d)) + 3*(5*b^2*d^4*x^2 - 6*b^2*c*d^3*x + 8*b^2*c^2
*d^2 - 15*a*b*d^4)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b*d^5)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = \int \frac{(a-b\,x^2)^{3/2}}{\sqrt{c+d\,x}}\,dx$$

input
```
int((a - b*x^2)^(3/2)/(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^(3/2)/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx}}\,dx = \frac{-\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ad}{35}+\frac{12\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bcx}{35}-\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bd\,x^2}{7}-\frac{48\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^3-bc\,x^2+adx+ac}\right.}{35}}{}$$

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(1/2),x)
```

output
```
(2*( - sqrt(c + d*x)*sqrt(a - b*x**2)*a*d + 6*sqrt(c + d*x)*sqrt(a - b*x**
2)*b*c*x - 5*sqrt(c + d*x)*sqrt(a - b*x**2)*b*d*x**2 - 24*int((sqrt(c + d*
x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*d**2
+ 12*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b
*d*x**3),x)*b**2*c**2 + 18*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d
*x - b*c*x**2 - b*d*x**3),x)*a**2*d**2 - 6*int((sqrt(c + d*x)*sqrt(a - b*x
**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**2))/(35*d**2)
```

3.309 $\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 386

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \frac{28bc\sqrt{c+dx}\sqrt{a-bx^2}}{5d^3} - \frac{12b(c+dx)^{3/2}\sqrt{a-bx^2}}{5d^3} - \frac{2(a-bx^2)^{3/2}}{d\sqrt{c+dx}}$$
$$- \frac{8\sqrt{a}\sqrt{b}(4bc^2-3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{5d^4\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$+ \frac{32\sqrt{a}\sqrt{b}c(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{5d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
28/5*b*c*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^3-12/5*b*(d*x+c)^(3/2)*(-b*x^2+a
)^(1/2)/d^3-2*(-b*x^2+a)^(3/2)/d/(d*x+c)^(1/2)-8/5*a^(1/2)*b^(1/2)*(-3*a*d
^2+4*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(
1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^4/(
b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+32/5*a^(1/2)
*b^(1/2)*c*(-a*d^2+b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1
-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*
(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.17 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.31

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(5bc^2-5ad^2+3bc(c+dx)-bdx(c+dx)-\frac{4\left(d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\,(4bc^2-3ad^2)\,(a-bx^2)\right.}{}\right.}{}$$

input
```
Integrate[(a - b*x^2)^(3/2)/(c + d*x)^(3/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(5*b*c^2 - 5*a*d^2 + 3*b*c*(c + d*x) - b*d*x*(c + d*x)
- (4*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(4*b*c^2 - 3*a*d^2)*(a - b*x^2) +
 I*Sqrt[b]*(4*b^(3/2)*c^3 - 4*Sqrt[a]*b*c^2*d - 3*a*Sqrt[b]*c*d^2 + 3*a^(3
/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqr
t[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt
[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*(4*b*c^2 - Sqrt[a]*Sqrt[b]*c*d - 3*a*d^2)*Sq
rt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)
/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(
d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(5*d^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {492, 591, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{492}$$

$$-\frac{6b\int \frac{x\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{d}-\frac{2(a-bx^2)^{3/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{591}$$

$$-\frac{6b\left(\frac{4\int -\frac{acd+(4bc^2-3ad^2)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}(4c-3dx)\sqrt{c+dx}}{15d^2}\right)}{d}-\frac{2(a-bx^2)^{3/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{6b\left(-\frac{2\int \frac{acd+(4bc^2-3ad^2)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(4c-3dx)}{15d^2}\right)}{d}-\frac{2(a-bx^2)^{3/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{600}$$

$$-\frac{6b\left(-\frac{2\left(\frac{(4bc^2-3ad^2)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{4c(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(4c-3dx)}{15d^2}\right)}{d}-\frac{2(a-bx^2)^{3/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{509}$$

$$-\frac{6b\left(-\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-3ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(4c-3dx)}{15d^2}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 508

$$-\frac{6b\left(-\frac{2\left(-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(4c-3dx)}{15d^2}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 327

$$-\frac{6b\left(-\frac{2\left(-\frac{4c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(4c-3dx}{15d^2}\right.}{d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 512

$$-\frac{6b\left(-\frac{2\left(-\frac{4c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{15d^2}\right.}{d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 511

$$-\frac{6b\left(-\frac{2\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\ldots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}\ldots\right)}{d}\ldots$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{d\sqrt{c+dx}}$$

$\big\downarrow$ 321

$$-\frac{6b\left(-\frac{2\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\ldots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}\ldots\right)}{d}\ldots$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{d\sqrt{c+dx}}$$

input
```
Int[(a - b*x^2)^(3/2)/(c + d*x)^(3/2),x]
```

output
```
(-2*(a - b*x^2)^(3/2))/(d*Sqrt[c + d*x]) - (6*b*((-2*(4*c - 3*d*x)*Sqrt[c
+ d*x]*Sqrt[a - b*x^2])/(15*d^2) - (2*((-2*Sqrt[a]*(4*b*c^2 - 3*a*d^2)*Sqr
t[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[
a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(
c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (8*Sqrt[a]*c*(b*c^2
- a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2
)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt
[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)
))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. 2(312) = 624.

Time = 6.98 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.95

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2(-bd\,x^2+ad)(a\,d^2-b\,c^2)}{d^4\sqrt{(x+\frac{c}{d})(-bd\,x^2+ad)}}-\frac{2bx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d^2}+\frac{6bc\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d^3}+\frac{2\left(\frac{bc(2a\,d^2-b\,c^2)}{d^4}-\frac{(a}{}\right.}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2*(-b*d*x^2+a*
d)*(a*d^2-b*c^2)/d^4/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2/5*b/d^2*x*(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)+6/5*b/d^3*c*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*
(b*c*(2*a*d^2-b*c^2)/d^4-(a*d^2-b*c^2)*b/d^4*c-1/5*c/d^2*a*b)*(c/d-1/b*(a*
b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2
)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-b/d^
3*(2*a*d^2-b*c^2)-(a*d^2-b*c^2)*b/d^3+3/5*a*b/d+6/5*b^2/d^3*c^2)*(c/d-1/b*
(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(
1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.77

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \frac{2\left(8\left(2\,bc^4-3\,ac^2d^2+(2\,bc^3d-3\,acd^3)x\right)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(8*(2*b*c^4 - 3*a*c^2*d^2 + (2*b*c^3*d - 3*a*c*d^3)*x)*sqrt(-b*d)*wei
erstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/
(b*d^3), 1/3*(3*d*x + c)/d) + 12*(4*b*c^3*d - 3*a*c*d^3 + (4*b*c^2*d^2 - 3
*a*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(b*d^4*x
^2 - 2*b*c*d^3*x - 8*b*c^2*d^2 + 5*a*d^4)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/
(d^6*x + c*d^5)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(a-b\,x^2)^{3/2}}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((a - b*x^2)^(3/2)/(c + d*x)^(3/2),x)
```

output
```
int((a - b*x^2)^(3/2)/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{3/2}}\,dx = \frac{-\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ad}{5} + \frac{4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bcx}{5} - \frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bd\,x^2}{5} - \frac{8\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x}}{-b\,d^2x^4-2bcd\,x^3+a\,d^2x^2 \cdots}\right.}{\cdots}}{\cdots}$$

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(3/2),x)
```

output
```
(2*( - sqrt(c + d*x)*sqrt(a - b*x**2)*a*d + 2*sqrt(c + d*x)*sqrt(a - b*x**
2)*b*c*x - sqrt(c + d*x)*sqrt(a - b*x**2)*b*d*x**2 - 4*int((sqrt(c + d*x)*
sqrt(a - b*x**2)*x**2)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2
*b*c*d*x**3 - b*d**2*x**4),x)*a*b*c*d**2 - 4*int((sqrt(c + d*x)*sqrt(a - b
*x**2)*x**2)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**
3 - b*d**2*x**4),x)*a*b*d**3*x + 4*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**
2)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2
*x**4),x)*b**2*c**3 + 4*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**2
+ 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*b
**2*c**2*d*x + 2*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*a**2*c*d**2 +
 2*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d*x + a*d**2*x**2
- b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*a**2*d**3*x - 2*int((sqrt(c
 + d*x)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2
- 2*b*c*d*x**3 - b*d**2*x**4),x)*a*b*c**3 - 2*int((sqrt(c + d*x)*sqrt(a -
b*x**2))/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 -
b*d**2*x**4),x)*a*b*c**2*d*x))/(5*d**2*(c + d*x))
```

3.310 $\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 374

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx = -\frac{4bc\sqrt{a-bx^2}}{d^3\sqrt{c+dx}} - \frac{4b\sqrt{c+dx}\sqrt{a-bx^2}}{3d^3} - \frac{2(a-bx^2)^{3/2}}{3d(c+dx)^{3/2}} + \frac{32\sqrt{a}b^{3/2}c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3d^4\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{8\sqrt{a}\sqrt{b}(4bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-4*b*c*(-b*x^2+a)^(1/2)/d^3/(d*x+c)^(1/2)-4/3*b*(d*x+c)^(1/2)*(-b*x^2+a)^(
1/2)/d^3-2/3*(-b*x^2+a)^(3/2)/d/(d*x+c)^(3/2)+32/3*a^(1/2)*b^(3/2)*c*(d*x+
c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/
2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^4/(b^(1/2)*(d*x+c)/(
b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-8/3*a^(1/2)*b^(1/2)*(-a*d^2+4
*b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*El
lipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2
)*c+a^(1/2)*d))^(1/2))/d^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.90 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.22

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(8bc+\frac{bc^2}{c+dx}-\frac{ad^2}{c+dx}-b(c+dx)-\frac{16ib^{3/2}c\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx \ldots}{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(\ldots}\ldots\right.}{\ldots}$$

input
```
Integrate[(a - b*x^2)^(3/2)/(c + d*x)^(5/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(8*b*c + (b*c^2)/(c + d*x) - (a*d^2)/(c + d*x) - b*(c +
 d*x) - ((16*I)*b^(3/2)*c*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b]
 + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)
^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]],
(Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]*(-a + b*x^2)) - ((4*I)*Sqrt[a]*b*(4*Sqrt[b]*c - Sqrt[a]*d)*Sqr
t[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/
(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*d^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {492, 590, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \ 492$$

$$-\frac{2b\int \frac{x\sqrt{a-bx^2}}{(c+dx)^{3/2}}dx}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 590$$

$$-\frac{2b\left(\frac{2\sqrt{a-bx^2}(4c+dx)}{3d^2\sqrt{c+dx}}-\frac{4\int -\frac{ad+4bcx}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\left(\frac{2\int \frac{ad+4bcx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2}+\frac{2\sqrt{a-bx^2}(4c+dx)}{3d^2\sqrt{c+dx}}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 600$$

$$-\frac{2b\left(\frac{2\left(\frac{4bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(4bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}(4c+dx)}{3d^2\sqrt{c+dx}}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 509$$

$$-\frac{2b\left(\frac{2\left(\frac{4bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(4bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}(4c+dx)}{3d^2\sqrt{c+dx}}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 508$$

$$-\frac{2b\left(\frac{2\left(-\frac{\left(4bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}(4c+dx)}{3d^2\sqrt{c+dx}}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 327

$$-\frac{2b\left(\frac{2\left(-\frac{\left(4bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}(4c+dx)}{3d^2\sqrt{c+dx}}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$\Big\downarrow$ 512

$$-\frac{2b\left(\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}(4c+dx)}{3d^2\sqrt{c+dx}}\right)}{d}-\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 511}$$

$$-\frac{2b\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+\cdots}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}\cdots\right)}{d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{2b\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}-\right.}{d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{3d(c+dx)^{3/2}}$$

input
```
Int[(a - b*x^2)^(3/2)/(c + d*x)^(5/2),x]
```

output
```
(-2*(a - b*x^2)^(3/2))/(3*d*(c + d*x)^(3/2)) - (2*b*((2*(4*c + d*x)*Sqrt[a
 - b*x^2])/(3*d^2*Sqrt[c + d*x]) + (2*((-8*Sqrt[a]*Sqrt[b]*c*Sqrt[c + d*x]
*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2
]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]
*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(4*b*c^2 - a*d^2)*Sqrt[(Sqr
t[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d
)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*d^2)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(300) = 600$.

Time = 8.14 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\left(a\,d^2-b\,c^2\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3d^5\left(x+\frac{c}{d}\right)^2}-\frac{16\left(-bd\,x^2+ad\right)bc}{3d^4\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}-\frac{2b\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3d^3}+\frac{2\left(-\frac{b\left(2a\,d^2-3\right.}{d^4}\right.}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/3*(a*d^2-b*c
^2)/d^5*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2-16/3*(-b*d*x^2+a*d)/d
^4*b*c/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2/3*b/d^3*(-b*d*x^3-b*c*x^2+a*d*x+a*
c)^(1/2)+2*(-b*(2*a*d^2-3*b*c^2)/d^4+1/3*(a*d^2-b*c^2)*b/d^4-8/3*b^2/d^4*c
^2+1/3*a*b/d^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/
(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF
(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2))-32/3*b^2/d^3*c*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2))
)^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(
1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2
))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.86

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx = -\frac{2\left(4\left(4\,bc^4-3\,ac^2d^2+(4\,bc^2d^2-3\,ad^4)x^2+2\,(4\,bc^3d-3\,acd^3)x\right)\sqrt{-bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(4*(4*b*c^4 - 3*a*c^2*d^2 + (4*b*c^2*d^2 - 3*a*d^4)*x^2 + 2*(4*b*c^3*
d - 3*a*c*d^3)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*
d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 48*(b*c*d^3*
x^2 + 2*b*c^2*d^2*x + b*c^3*d)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a
*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*
(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x +
 c)/d)) + 3*(b*d^4*x^2 + 10*b*c*d^3*x + 8*b*c^2*d^2 + a*d^4)*sqrt(-b*x^2 +
 a)*sqrt(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(d*x+c)**(5/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)/(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx=\int \frac{{\left(a-b\,x^2\right)}^{3/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

input
```
int((a - b*x^2)^(3/2)/(c + d*x)^(5/2),x)
```

output
```
int((a - b*x^2)^(3/2)/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(5/2),x)
```

output
```
(2*( - 5*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d + 6*sqrt(c + d*x)*sqrt(a - b*x
**2)*b*c*x - sqrt(c + d*x)*sqrt(a - b*x**2)*b*d*x**2 + 12*int((sqrt(c + d*
x)*sqrt(a - b*x**2)*x**2)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**
3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)
*b**2*c**4 + 24*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**3 + 3*a*c*
*2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3
*b*c*d**2*x**4 - b*d**3*x**5),x)*b**2*c**3*d*x + 12*int((sqrt(c + d*x)*sqr
t(a - b*x**2)*x**2)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3
 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)*b**2*
c**2*d**2*x**2 - 6*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**3 + 3*a*c**2
*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b
*c*d**2*x**4 - b*d**3*x**5),x)*a**2*c**2*d**2 - 12*int((sqrt(c + d*x)*sqrt
(a - b*x**2))/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c
**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)*a**2*c*d**3
*x - 6*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**3 + 3*a*c**2*d*x + 3*a*c
*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4
 - b*d**3*x**5),x)*a**2*d**4*x**2 - 6*int((sqrt(c + d*x)*sqrt(a - b*x**2))
/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*
b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)*a*b*c**4 - 12*int((sqrt(
c + d*x)*sqrt(a - b*x**2))/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a...
```

3.311 $\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 417

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx = -\frac{4bc\sqrt{a-bx^2}}{5d^3(c+dx)^{3/2}}+\frac{4b(5bc^2-3ad^2)\sqrt{a-bx^2}}{5d^3(bc^2-ad^2)\sqrt{c+dx}}-\frac{2(a-bx^2)^{3/2}}{5d(c+dx)^{5/2}}$$
$$-\frac{8\sqrt{a}b^{3/2}(4bc^2-3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{5d^4(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$+\frac{32\sqrt{a}b^{3/2}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{5d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output

```
-4/5*b*c*(-b*x^2+a)^(1/2)/d^3/(d*x+c)^(3/2)+4/5*b*(-3*a*d^2+5*b*c^2)*(-b*x
^2+a)^(1/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(1/2)-2/5*(-b*x^2+a)^(3/2)/d/(d*x+c
)^(5/2)-8/5*a^(1/2)*b^(3/2)*(-3*a*d^2+4*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(
1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/
(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^4/(-a*d^2+b*c^2)/(b^(1/2)*(d*x+c)/(b^(1/2)
*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+32/5*a^(1/2)*b^(3/2)*c*(b^(1/2)*(d*x
+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2)
)/d^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.73 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.24

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(b(11bc^2-7ad^2)-4b(4bc^2-3ad^2)+\frac{(bc^2-ad^2)^2}{(c+dx)^2}-\frac{4bc(bc^2-ad^2)}{c+dx}-\frac{4ib^2\sqrt{-c+\cdots}}{\cdots}\cdots\right.}{\cdots}$$

input

```
Integrate[(a - b*x^2)^(3/2)/(c + d*x)^(7/2),x]
```

output

```
(2*Sqrt[a - b*x^2]*(b*(11*b*c^2 - 7*a*d^2) - 4*b*(4*b*c^2 - 3*a*d^2) + (b*
c^2 - a*d^2)^2/(c + d*x)^2 - (4*b*c*(b*c^2 - a*d^2))/(c + d*x) - ((4*I)*b^
2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(4*b*c^2 - 3*a*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt
[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d
*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]
], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*(-a + b*x^2)) +
((4*I)*Sqrt[a]*b^(3/2)*(4*b*c^2 - Sqrt[a]*Sqrt[b]*c*d - 3*a*d^2)*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(5*d^3*(b*c^2 - a*d^2)*Sqrt[c + d
*x])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {492, 589, 25, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \ 492$$

$$-\frac{6b\int \frac{x\sqrt{a-bx^2}}{(c+dx)^{5/2}}dx}{5d}-\frac{2(a-bx^2)^{3/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 589$$

$$-\frac{6b\left(\frac{2b\int -\frac{acd+(4bc^2-3ad^2)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}(dx(5bc^2-3ad^2)+2c(2bc^2-ad^2))}{3d^2(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2(a-bx^2)^{3/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{6b\left(-\frac{2b\int \frac{acd+(4bc^2-3ad^2)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}(dx(5bc^2-3ad^2)+2c(2bc^2-ad^2))}{3d^2(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2(a-bx^2)^{3/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 600$$

$$-\frac{6b\left(-\frac{2b\left(\frac{(4bc^2-3ad^2)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{4c(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}(dx(5bc^2-3ad^2)+2c(2bc^2-ad^2))}{3d^2(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2(a-bx^2)^{3/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 509$$

$$-\frac{6b\left(-\frac{2b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-3ad^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{4c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(dx\left(5bc^2-3ad^2\right)+2c\left(2bc^2-ad^2\right)\right)}{3d^2(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5d}-\frac{2\left(a-bx^2\right)^{3/2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 508

$$-\frac{6b\left(-\frac{2b\left(-\frac{4c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(dx\left(5bc^2-3ad^2\right)\cdots\right)}{3d^2(c+dx)^{3/2}(bc\cdots)}\right)}{5d}-\frac{2\left(a-bx^2\right)^{3/2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 327

$$-\frac{6b\left(-\frac{2b\left(-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(dx\left(5bc^2-3ad^2\cdots\right.\right.}{3d^2(c+dx)^{3/2}(\cdots}\cdots\right)}{5d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \; 512$$

$$-\frac{6b\left(-\frac{2b\left(-\frac{4c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(dx\left(5\cdots\right.\right.}{3d^2(c\cdots}\cdots\right)}{5d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \; 511$$

$$-\frac{6b\left(-\frac{2b\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2(bc^2-ad^2)}\right)}{5d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 321

$$-\frac{6b\left(-\frac{2b\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2(bc^2-ad^2)}\right)}{5d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{5d(c+dx)^{5/2}}$$

input
```
Int[(a - b*x^2)^(3/2)/(c + d*x)^(7/2),x]
```

output
```
(-2*(a - b*x^2)^(3/2))/(5*d*(c + d*x)^(5/2)) - (6*b*((-2*(2*c*(2*b*c^2 - a
*d^2) + d*(5*b*c^2 - 3*a*d^2)*x)*Sqrt[a - b*x^2])/(3*d^2*(b*c^2 - a*d^2)*(
c + d*x)^(3/2)) - (2*b*((-2*Sqrt[a]*(4*b*c^2 - 3*a*d^2)*Sqrt[c + d*x]*Sqrt
[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt
[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (8*Sqrt[a]*c*(b*c^2 - a*d^2)*Sqrt[(
Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[
ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
+ d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*d^2*(b*c^2 - a*d^2))
))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(341) = 682$.

Time = 3.98 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.87

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\left(a\,d^2-b\,c^2\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d^6\left(x+\frac{c}{d}\right)^3}-\frac{8bc\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d^5\left(x+\frac{c}{d}\right)^2}+\frac{2\left(-bd\,x^2+ad\right)b\left(7a\,d^2-11b\,c^2\right)}{5\left(a\,d^2-b\,c^2\right)d^4\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}+\frac{2(}{}\right.}{}$
default	Expression too large to display

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/5*(a*d^2-b*c
^2)/d^6*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3-8/5*b/d^5*c*(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+2/5*(-b*d*x^2+a*d)/(a*d^2-b*c^2)/d^4*b
*(7*a*d^2-11*b*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(-11/5*b^2*c/d^4+1/5*
b^2*c/d^4*(7*a*d^2-11*b*c^2)/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+
1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(b^2/d^3+1/5*b^2/d^3*(7*
a*d^2-11*b*c^2)/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)
*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.27

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx = \frac{2\left(8\left(2\,b^2c^6-3\,abc^4d^2+\left(2\,b^2c^3d^3-3\,abcd^5\right)x^3+3\left(2\,b^2c^4d^2-3\,abc^2d^4\right)x^2+3\left(2\,b^2c^5\right.\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(8*(2*b^2*c^6 - 3*a*b*c^4*d^2 + (2*b^2*c^3*d^3 - 3*a*b*c*d^5)*x^3 + 3
*(2*b^2*c^4*d^2 - 3*a*b*c^2*d^4)*x^2 + 3*(2*b^2*c^5*d - 3*a*b*c^3*d^3)*x)*
sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3
 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(4*b^2*c^5*d - 3*a*b*c^3*d^
3 + (4*b^2*c^2*d^4 - 3*a*b*d^6)*x^3 + 3*(4*b^2*c^3*d^3 - 3*a*b*c*d^5)*x^2
+ 3*(4*b^2*c^4*d^2 - 3*a*b*c^2*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c
^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInve
rse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*
(3*d*x + c)/d)) + 3*(8*b^2*c^4*d^2 - 5*a*b*c^2*d^4 + a^2*d^6 + (11*b^2*c^2
*d^4 - 7*a*b*d^6)*x^2 + 2*(9*b^2*c^3*d^3 - 5*a*b*c*d^5)*x)*sqrt(-b*x^2 + a
)*sqrt(d*x + c))/(b*c^5*d^5 - a*c^3*d^7 + (b*c^2*d^8 - a*d^10)*x^3 + 3*(b*
c^3*d^7 - a*c*d^9)*x^2 + 3*(b*c^4*d^6 - a*c^2*d^8)*x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(d*x+c)**(7/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)/(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx=\int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx=\int \frac{(a-b\,x^2)^{3/2}}{(c+d\,x)^{7/2}}\,dx$$

input
```
int((a - b*x^2)^(3/2)/(c + d*x)^(7/2),x)
```

output
```
int((a - b*x^2)^(3/2)/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{7/2}}\,dx=\text{too large to display}$$

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(7/2),x)
```

output

```
(2*(7*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d - 6*sqrt(c + d*x)*sqrt(a - b*x**2
)*b*c*x - sqrt(c + d*x)*sqrt(a - b*x**2)*b*d*x**2 - 12*int((sqrt(c + d*x)*
sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4*a*c
*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2*d**2*x
**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*c**3*d**2 - 36*int((sqrt(c + d
*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4
*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2*d*
*2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*c**2*d**3*x - 36*int((sqrt
(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x*
*2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c
**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*c*d**4*x**2 - 12*int
((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d
**2*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 -
 6*b*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*d**5*x**3 - 12
*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c*
*2*d**2*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x*
*3 - 6*b*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*b**2*c**5 - 36
*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c*
*2*d**2*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x*
*3 - 6*b*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*b**2*c**4*d...
```

3.312 $\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 501

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}} dx = -\frac{12bc\sqrt{a-bx^2}}{35d^3(c+dx)^{5/2}} + \frac{4b(7bc^2-5ad^2)\sqrt{a-bx^2}}{35d^3(bc^2-ad^2)(c+dx)^{3/2}} - \frac{32b^2c(bc^2-2ad^2)\sqrt{a-bx^2}}{35d^3(bc^2-ad^2)^2\sqrt{c+dx}} - \frac{2(a-bx^2)^{3/2}}{7d(c+dx)^{7/2}} + \frac{32\sqrt{a}b^{5/2}c(bc^2-2ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{35d^4(bc^2-ad^2)^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{8\sqrt{a}b^{3/2}(4bc^2-5ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{35d^4(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-12/35*b*c*(-b*x^2+a)^(1/2)/d^3/(d*x+c)^(5/2)+4/35*b*(-5*a*d^2+7*b*c^2)*(-
b*x^2+a)^(1/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(3/2)-32/35*b^2*c*(-2*a*d^2+b*c^
2)*(-b*x^2+a)^(1/2)/d^3/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-2/7*(-b*x^2+a)^(3/2
)/d/(d*x+c)^(7/2)+32/35*a^(1/2)*b^(5/2)*c*(-2*a*d^2+b*c^2)*(d*x+c)^(1/2)*(
1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^4/(-a*d^2+b*c^2)^2/(b^(1/2)*(d
*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-8/35*a^(1/2)*b^(3/2)*(
-5*a*d^2+4*b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a
)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)
*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^4/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.61 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.08

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(\frac{5(bc^2-ad^2)^3}{(c+dx)^3} - \frac{16bc(bc^2-ad^2)^2}{(c+dx)^2} + \frac{b(19bc^2-15ad^2)(bc^2-ad^2)}{c+dx} + \frac{16ib^3c\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(bc^2-2ad^2}{}\right.}{}$$

input
```
Integrate[(a - b*x^2)^(3/2)/(c + d*x)^(9/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*((5*(b*c^2 - a*d^2)^3)/(c + d*x)^3 - (16*b*c*(b*c^2 - a
*d^2)^2)/(c + d*x)^2 + (b*(19*b*c^2 - 15*a*d^2)*(b*c^2 - a*d^2))/(c + d*x)
 + ((16*I)*b^3*c*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - 2*a*d^2)*Sqrt[(d*
(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c +
d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/
Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*(-a
 + b*x^2)) - ((4*I)*Sqrt[a]*b^2*(4*b^(3/2)*c^3 - Sqrt[a]*b*c^2*d - 8*a*Sqr
t[b]*c*d^2 + 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt
[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*Arc
Sinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d
)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)
)))/(35*d^3*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {492, 589, 25, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{492}$$

$$-\frac{6b\int \frac{x\sqrt{a-bx^2}}{(c+dx)^{7/2}}dx}{7d}-\frac{2(a-bx^2)^{3/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{589}$$

$$-\frac{6b\left(\frac{2b\int -\frac{3acd+(4bc^2-5ad^2)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{15d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}(dx(7bc^2-5ad^2)+2c(2bc^2-ad^2))}{15d^2(c+dx)^{5/2}(bc^2-ad^2)}\right)}{7d}-\frac{2(a-bx^2)^{3/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{6b\left(-\frac{2b\int \frac{3acd+(4bc^2-5ad^2)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{15d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}(dx(7bc^2-5ad^2)+2c(2bc^2-ad^2))}{15d^2(c+dx)^{5/2}(bc^2-ad^2)}\right)}{7d}-\frac{2(a-bx^2)^{3/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{688}$$

$$-\frac{6b\left(-\frac{2b\left(\frac{2\int -\frac{ad(bc^2-5ad^2)+4bc(bc^2-2ad^2)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{8c\sqrt{a-bx^2}(bc^2-2ad^2)}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{15d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}(dx(7bc^2-5ad^2)+2c(2bc^2-ad^2))}{15d^2(c+dx)^{5/2}(bc^2-ad^2)}\right)}{7d}-\frac{2(a-bx^2)^{3/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{6b\left(-\frac{2b\left(-\frac{\int \frac{ad\left(bc^2-5ad^2\right)+4bc\left(bc^2-2ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{8c\sqrt{a-bx^2}\left(bc^2-2ad^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{15d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(dx\left(7bc^2-5ad^2\right)+2c\left(2bc^2-ad^2\right)\right)}{15d^2(c+dx)^{5/2}(bc^2-ad^2)}\right)}{7d}-\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

$\big\downarrow$ 600

$$-\frac{6b\left(-\frac{2b\left(-\frac{\frac{4bc\left(bc^2-2ad^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}-\frac{8c\sqrt{a-bx^2}\left(bc^2-2ad^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{15d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(dx\left(7bc^2-5ad^2\right.\right.}{15d^2(c+dx)^{5/2}(\ldots}\right)}{7d}-\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

$\big\downarrow$ 509

$$-\frac{6b\left(-\frac{2b\left(-\frac{\frac{4bc\sqrt{1-\frac{bx^2}{a}}\left(bc^2-2ad^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}-\frac{8c\sqrt{a-bx^2}\left(bc^2-2ad^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{15d^2(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(dx\left(7b\ldots\right.\right.}{15d^2(c+\ldots}\right)}{7d}-\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

$\big\downarrow$ 508

$$-\frac{6b\left(-\frac{2b\left(-\frac{-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{8c\sqrt{a-bx^2}\left(bc^2-2\cdots\right.}{\sqrt{c+dx}\left(bc^2-ad^2\cdots\right.}\cdots\right)}{15d^2(bc^2-ad^2)}\cdots\right)}{7d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \; 327$$

$$-\frac{6b\left(-\frac{2b\left(-\frac{-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{8c\sqrt{a-bx^2}\left(bc^2 \cdots\right.}{\sqrt{c+dx}\left(bc^2-\cdots\right.}\right.}{15d^2(bc^2-ad^2)}\cdots\right.}{7d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \; 512$$

$$-\frac{6b\left(-\frac{2b\left(-\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{8c\sqrt{a\cdots}}{\sqrt{c\cdots}}\right.}{15d^2(bc^2-ad^2)}\cdots\right.}{7d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \; 511$$

$$-\frac{6b\left(-\frac{2b\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)}{d\sqrt{a-bx^2}\sqrt{}}}{bc^2-ad^2}}{15d^2(bc^2-ad^2)}}{7d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

$\downarrow$ 321

$$-\frac{6b\left(-\frac{2b\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\text{arcsi}\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx}{\sqrt{a}d+\sqrt{b}}}}}{bc^2-ad^2}}{15d^2(bc^2-ad^2)}}{7d}$$

$$\frac{2\left(a-bx^2\right)^{3/2}}{7d(c+dx)^{7/2}}$$

input
```
Int[(a - b*x^2)^(3/2)/(c + d*x)^(9/2),x]
```

output
```
(-2*(a - b*x^2)^(3/2))/(7*d*(c + d*x)^(7/2)) - (6*b*((-2*(2*c*(2*b*c^2 - a
*d^2) + d*(7*b*c^2 - 5*a*d^2)*x)*Sqrt[a - b*x^2])/(15*d^2*(b*c^2 - a*d^2)*
(c + d*x)^(5/2)) - (2*b*((-8*c*(b*c^2 - 2*a*d^2)*Sqrt[a - b*x^2])/((b*c^2
- a*d^2)*Sqrt[c + d*x]) - ((-8*Sqrt[a]*Sqrt[b]*c*(b*c^2 - 2*a*d^2)*Sqrt[c
+ d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/
Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(S
qrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(4*b*c^2 - 5*a*d^2)*(
b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 -
(b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/
((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*
c^2 - a*d^2)))/(15*d^2*(b*c^2 - a*d^2))))/(7*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(419) = 838$.

Time = 5.82 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\left(a\,d^2-b\,c^2\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7d^7\left(x+\frac{c}{d}\right)^4}-\frac{32bc\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{35d^6\left(x+\frac{c}{d}\right)^3}+\frac{2b\left(15a\,d^2-19b\,c^2\right)\sqrt{-bd\,x^3-bc\,x^2+adx+a}}{35\left(a\,d^2-b\,c^2\right)d^5\left(x+\frac{c}{d}\right)^2}\right.}{}$
default	Expression too large to display

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/7*(a*d^2-b*c
^2)/d^7*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^4-32/35*b/d^6*c*(-b*d*x
^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3+2/35*b*(15*a*d^2-19*b*c^2)/(a*d^2-b*
c^2)/d^5*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+32/35*(-b*d*x^2+a*d)
/(a*d^2-b*c^2)^2/d^4*b^2*c*(2*a*d^2-b*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+
2*(b^2/d^4-1/35*b^2*(15*a*d^2-19*b*c^2)/d^4/(a*d^2-b*c^2)+16/35*b^3*c^2/d^
4*(2*a*d^2-b*c^2)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+32/35*b^3/d^3*c*(2*a*d^2-b*c^2)/(a*
d^2-b*c^2)^2*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(
(x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c
/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(
a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*
b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.62

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(4*(4*b^3*c^8 - 11*a*b^2*c^6*d^2 + 15*a^2*b*c^4*d^4 + (4*b^3*c^4*d^
4 - 11*a*b^2*c^2*d^6 + 15*a^2*b*d^8)*x^4 + 4*(4*b^3*c^5*d^3 - 11*a*b^2*c^3
*d^5 + 15*a^2*b*c*d^7)*x^3 + 6*(4*b^3*c^6*d^2 - 11*a*b^2*c^4*d^4 + 15*a^2*
b*c^2*d^6)*x^2 + 4*(4*b^3*c^7*d - 11*a*b^2*c^5*d^3 + 15*a^2*b*c^3*d^5)*x)*
sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3
 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 48*(b^3*c^7*d - 2*a*b^2*c^5*d^
3 + (b^3*c^3*d^5 - 2*a*b^2*c*d^7)*x^4 + 4*(b^3*c^4*d^4 - 2*a*b^2*c^2*d^6)*
x^3 + 6*(b^3*c^5*d^3 - 2*a*b^2*c^3*d^5)*x^2 + 4*(b^3*c^6*d^2 - 2*a*b^2*c^4
*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(
b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d
^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(8*b^3*c^6
*d^2 - 15*a*b^2*c^4*d^4 - 14*a^2*b*c^2*d^6 + 5*a^3*d^8 + 16*(b^3*c^3*d^5 -
 2*a*b^2*c*d^7)*x^3 + (29*b^3*c^4*d^4 - 62*a*b^2*c^2*d^6 - 15*a^2*b*d^8)*x
^2 + 2*(13*b^3*c^5*d^3 - 30*a*b^2*c^3*d^5 - 7*a^2*b*c*d^7)*x)*sqrt(-b*x^2
+ a)*sqrt(d*x + c))/(b^2*c^8*d^5 - 2*a*b*c^6*d^7 + a^2*c^4*d^9 + (b^2*c^4*
d^9 - 2*a*b*c^2*d^11 + a^2*d^13)*x^4 + 4*(b^2*c^5*d^8 - 2*a*b*c^3*d^10 + a
^2*c*d^12)*x^3 + 6*(b^2*c^6*d^7 - 2*a*b*c^4*d^9 + a^2*c^2*d^11)*x^2 + 4*(b
^2*c^7*d^6 - 2*a*b*c^5*d^8 + a^2*c^3*d^10)*x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{(c+dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(d*x+c)**(9/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)/(c + d*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(a-b\,x^2)^{3/2}}{(c+d\,x)^{9/2}}\,dx$$

input
```
int((a - b*x^2)^(3/2)/(c + d*x)^(9/2),x)
```

output
```
int((a - b*x^2)^(3/2)/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}}{(c+dx)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((-b*x^2+a)^(3/2)/(d*x+c)^(9/2),x)
```

output
```
(2*( - 3*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d + 2*sqrt(c + d*x)*sqrt(a - b*x
**2)*b*c*x + sqrt(c + d*x)*sqrt(a - b*x**2)*b*d*x**2 + 8*int((sqrt(c + d*x
)*sqrt(a - b*x**2)*x**2)/(a*c**5 + 5*a*c**4*d*x + 10*a*c**3*d**2*x**2 + 10
*a*c**2*d**3*x**3 + 5*a*c*d**4*x**4 + a*d**5*x**5 - b*c**5*x**2 - 5*b*c**4
*d*x**3 - 10*b*c**3*d**2*x**4 - 10*b*c**2*d**3*x**5 - 5*b*c*d**4*x**6 - b*
d**5*x**7),x)*a*b*c**4*d**2 + 32*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)
/(a*c**5 + 5*a*c**4*d*x + 10*a*c**3*d**2*x**2 + 10*a*c**2*d**3*x**3 + 5*a*
c*d**4*x**4 + a*d**5*x**5 - b*c**5*x**2 - 5*b*c**4*d*x**3 - 10*b*c**3*d**2
*x**4 - 10*b*c**2*d**3*x**5 - 5*b*c*d**4*x**6 - b*d**5*x**7),x)*a*b*c**3*d
**3*x + 48*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**5 + 5*a*c**4*d*
x + 10*a*c**3*d**2*x**2 + 10*a*c**2*d**3*x**3 + 5*a*c*d**4*x**4 + a*d**5*x
**5 - b*c**5*x**2 - 5*b*c**4*d*x**3 - 10*b*c**3*d**2*x**4 - 10*b*c**2*d**3
*x**5 - 5*b*c*d**4*x**6 - b*d**5*x**7),x)*a*b*c**2*d**4*x**2 + 32*int((sqr
t(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**5 + 5*a*c**4*d*x + 10*a*c**3*d**2*
x**2 + 10*a*c**2*d**3*x**3 + 5*a*c*d**4*x**4 + a*d**5*x**5 - b*c**5*x**2 -
 5*b*c**4*d*x**3 - 10*b*c**3*d**2*x**4 - 10*b*c**2*d**3*x**5 - 5*b*c*d**4*
x**6 - b*d**5*x**7),x)*a*b*c*d**5*x**3 + 8*int((sqrt(c + d*x)*sqrt(a - b*x
**2)*x**2)/(a*c**5 + 5*a*c**4*d*x + 10*a*c**3*d**2*x**2 + 10*a*c**2*d**3*x
**3 + 5*a*c*d**4*x**4 + a*d**5*x**5 - b*c**5*x**2 - 5*b*c**4*d*x**3 - 10*b
*c**3*d**2*x**4 - 10*b*c**2*d**3*x**5 - 5*b*c*d**4*x**6 - b*d**5*x**7),...
```

3.313 $\int \sqrt{c+dx}(a-bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 563

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{5/2}\,dx = -\frac{16c(32b^2c^4-113abc^2d^2+177a^2d^4)\sqrt{c+dx}\sqrt{a-bx^2}}{9009d^5}$$
$$+\frac{8(c+dx)^{3/2}\left(32b^2c^4-73abc^2d^2+77a^2d^4-20bcd(2bc^2-5ad^2)x\right)\sqrt{a-bx^2}}{3003d^5}$$
$$-\frac{20(c+dx)^{3/2}\left(8bc^2-11ad^2-9bcdx\right)\left(a-bx^2\right)^{3/2}}{1287d^3}+\frac{2(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}{13d}$$
$$-\frac{16\sqrt{a}(32b^3c^6-137ab^2c^4d^2+258a^2bc^2d^4+231a^3d^6)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{9009\sqrt{b}d^6\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$+\frac{16\sqrt{a}c(32b^3c^6-145ab^2c^4d^2+290a^2bc^2d^4-177a^3d^6)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{9009\sqrt{b}d^6\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-16/9009*c*(177*a^2*d^4-113*a*b*c^2*d^2+32*b^2*c^4)*(d*x+c)^(1/2)*(-b*x^2+
a)^(1/2)/d^5+8/3003*(d*x+c)^(3/2)*(32*b^2*c^4-73*a*b*c^2*d^2+77*a^2*d^4-20
*b*c*d*(-5*a*d^2+2*b*c^2)*x)*(-b*x^2+a)^(1/2)/d^5-20/1287*(d*x+c)^(3/2)*(-
9*b*c*d*x-11*a*d^2+8*b*c^2)*(-b*x^2+a)^(3/2)/d^3+2/13*(d*x+c)^(3/2)*(-b*x^
2+a)^(5/2)/d-16/9009*a^(1/2)*(231*a^3*d^6+258*a^2*b*c^2*d^4-137*a*b^2*c^4*
d^2+32*b^3*c^6)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x
/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b
^(1/2)/d^6/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+
16/9009*a^(1/2)*c*(-177*a^3*d^6+290*a^2*b*c^2*d^4-145*a*b^2*c^4*d^2+32*b^3
*c^6)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*Elli
pticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/b^(1/2)/d^6/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.69 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.33

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{5/2}\,dx = \frac{2\sqrt{a-bx^2}\left((c+dx)\left(a^2d^4(971c+2387dx)-2abd^2(266c^3-197c^2dx+163cd^2x^2+1078d^3\right.\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]*(a - b*x^2)^(5/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*((c + d*x)*(a^2*d^4*(971*c + 2387*d*x) - 2*a*b*d^2*(266
*c^3 - 197*c^2*d*x + 163*c*d^2*x^2 + 1078*d^3*x^3) + b^2*(128*c^5 - 96*c^4
*d*x + 80*c^3*d^2*x^2 - 70*c^2*d^3*x^3 + 63*c*d^4*x^4 + 693*d^5*x^5)) - (8
*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(32*b^3*c^6 - 137*a*b^2*c^4*d^2 + 258
*a^2*b*c^2*d^4 + 231*a^3*d^6)*(a - b*x^2) + I*Sqrt[b]*(32*b^(7/2)*c^7 - 32
*Sqrt[a]*b^3*c^6*d - 137*a*b^(5/2)*c^5*d^2 + 137*a^(3/2)*b^2*c^4*d^3 + 258
*a^2*b^(3/2)*c^3*d^4 - 258*a^(5/2)*b*c^2*d^5 + 231*a^3*Sqrt[b]*c*d^6 - 231
*a^(7/2)*d^7)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d
)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c -
 Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*(32*b^3*c^6 - 8*Sqrt[a]*b^(5/2)*c^5*d -
 137*a*b^2*c^4*d^2 + 32*a^(3/2)*b^(3/2)*c^3*d^3 + 258*a^2*b*c^2*d^4 - 408*
a^(5/2)*Sqrt[b]*c*d^5 + 231*a^3*d^6)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d
*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellipt
icF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c +
Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(b*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]*(a - b*x^2))))/(9009*d^5*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {493, 687, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{5/2} \sqrt{c+dx} \, dx$$

$$\Big\downarrow \text{ 493}$$

$$\frac{10 \int (ad+bcx)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}dx}{13d}+\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{10\left(-\frac{2\int -\frac{b\left(12acd+\left(bc^2+11ad^2\right)x\right)\left(a-bx^2\right)^{3/2}}{2\sqrt{c+dx}}dx}{11b}-\frac{2}{11}c\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\right)}{13d}+\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 27

$$\frac{10\left(\frac{1}{11}\int \frac{\left(12acd+\left(bc^2+11ad^2\right)x\right)\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx-\frac{2}{11}c\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\right)}{13d}+\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 682

$$\frac{10\left(\frac{1}{11}\left(-\frac{4\int \frac{b\left(acd\left(bc^2-97ad^2\right)+\left(8b^2c^4-27abd^2c^2-77a^2d^4\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{21bd^2}-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(4c\left(2bc^2-5ad^2\right)-7dx\left(11ad^2+bc^2\right)\right)}{63d^2}\right)-\ldots}{13d}\quad\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 27

$$\frac{10\left(\frac{1}{11}\left(-\frac{2\int \frac{\left(acd\left(bc^2-97ad^2\right)+\left(8b^2c^4-27abd^2c^2-77a^2d^4\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{21d^2}-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(4c\left(2bc^2-5ad^2\right)-7dx\left(11ad^2+bc^2\right)\right)}{63d^2}\right)-\ldots}{13d}\quad\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 682

$$\frac{10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{4\int \frac{b\left(8acd\left(b^2c^4-4abd^2c^2+51a^2d^4\right)+\left(32b^3c^6-137ab^2d^2c^4+258a^2bd^4c^2+231a^3d^6\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(c\left(177a^2d^4-113abc^2d^2+32\ldots\right.\right.}{15d^2\ldots}\right.}{21d^2}\ldots\right.\right.}{13\ldots}\quad\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 27

$$10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{2\int \frac{8acd\left(b^2c^4-4abd^2c^2+51a^2d^4\right)+\left(32b^3c^6-137ab^2d^2c^4+258a^2bd^4c^2+231a^3d^6\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(c\left(177a^2d^4-113abc^2d^2+32b^2c^4\right.\right.}{15d^2}\cdots\right)}{21d^2}\cdots\right)\cdots\right.$$

$$\cdots\overline{\qquad 13d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\Big\downarrow$ 600

$$10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{2\left(\frac{\left(231a^3d^6+258a^2bc^2d^4-137ab^2c^4d^2+32b^3c^6\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(bc^2-ad^2\right)\left(177a^2d^4-113abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\cdots}{\cdots}\right)}{21d^2}\cdots\right)\cdots\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\Big\downarrow$ 509

$$10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(231a^3d^6+258a^2bc^2d^4-137ab^2c^4d^2+32b^3c^6\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(bc^2-ad^2\right)\left(177a^2d^4-113abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{d}\cdots\right)}{15d^2}\cdots\right)}{21d^2}\cdots\right)\cdots\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 508

$$10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{2\left(-\frac{c\left(bc^2-ad^2\right)\left(177a^2d^4-113abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(231a^3d^6+258a^2bc^2d^4-137ab^2c^4d^2+32b^3c^6\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{15d^2}\right.}{21d^2}\right.\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 327

$$10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{2\left(-\frac{c\left(bc^2-ad^2\right)\left(177a^2d^4-113abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(231a^3d^6+258a^2bc^2d^4-137ab^2c^4d^2+32b^3c^6\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{15d^2}\right.}{21d^2}\right.\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 512

$$10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{2\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(177a^2d^4-113abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(231a^3d^6+258a^2bc^2d^4-137ab^2c^4d\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right.}{15d^2}\right.}{21d}\right.\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 511

$$10\left(\frac{1}{11}\left(-\frac{2\left(2-\frac{2\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(177a^2d^4-113abc^2d^2+32b^2c^4\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx}{a}}\cdots}{\cdots}\right)}{15d^2}\right)}{\cdots}\right)\right)$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

$\downarrow$ 321

$$10\left(\frac{1}{11}\left(-\frac{2\left(-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(c\left(177a^2d^4-113abc^2d^2+32b^2c^4\right)-3dx\left(-77a^2d^4-27abc^2d^2+8b^2c^4\right)\right)}{15d^2}-\frac{2\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(177a^2d^4-113abc^2\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13d}$$

input
```
Int[Sqrt[c + d*x]*(a - b*x^2)^(5/2),x]
```

output
```
(2*(c + d*x)^(3/2)*(a - b*x^2)^(5/2))/(13*d) + (10*((-2*c*Sqrt[c + d*x]*(a
 - b*x^2)^(5/2))/11 + ((-2*Sqrt[c + d*x]*(4*c*(2*b*c^2 - 5*a*d^2) - 7*d*(b
*c^2 + 11*a*d^2)*x)*(a - b*x^2)^(3/2))/(63*d^2) - (2*((-2*Sqrt[c + d*x]*(c
*(32*b^2*c^4 - 113*a*b*c^2*d^2 + 177*a^2*d^4) - 3*d*(8*b^2*c^4 - 27*a*b*c^
2*d^2 - 77*a^2*d^4)*x)*Sqrt[a - b*x^2])/(15*d^2) - (2*((-2*Sqrt[a]*(32*b^3
*c^6 - 137*a*b^2*c^4*d^2 + 258*a^2*b*c^2*d^4 + 231*a^3*d^6)*Sqrt[c + d*x]*
Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]
], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*(b*c^2 - a*d^2)*(3
2*b^2*c^4 - 113*a*b*c^2*d^2 + 177*a^2*d^4)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[
b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*
x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c
+ d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(21*d^2))/11))/(13*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. 2(481) = 962.

Time = 2.96 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.74

method	result
risch	$\frac{2(693b^2x^5d^5+63b^2c\,x^4d^4-2156ab\,d^5x^3-70c^2d^3x^3b^2-326abc\,d^4x^2+80b^2c^3d^2x^2+2387a^2x\,d^5+394ab\,c^2d^3x-96b^2c^4dx+971a^2c\,d^4}{9009d^5}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9009*(693*b^2*d^5*x^5+63*b^2*c*d^4*x^4-2156*a*b*d^5*x^3-70*b^2*c^2*d^3*x
^3-326*a*b*c*d^4*x^2+80*b^2*c^3*d^2*x^2+2387*a^2*d^5*x+394*a*b*c^2*d^3*x-9
6*b^2*c^4*d*x+971*a^2*c*d^4-532*a*b*c^3*d^2+128*b^2*c^5)*(d*x+c)^(1/2)/d^5
*(-b*x^2+a)^(1/2)+8/9009/d^5*((231*a^3*d^6+258*a^2*b*c^2*d^4-137*a*b^2*c^4
*d^2+32*b^3*c^6)/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))
^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*
b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d-1/b*(a*b)^(1/2))*
EllipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b
)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*2^(1/2)*((x+1/b*(a
*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(
1/2)))+408*a^3*c*d^5/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1
/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b
/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/
2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a
*b)^(1/2)))^(1/2))-32*c^3*d^3*a^2*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))
*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*
b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elliptic
F(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/
(c/d-1/b*(a*b)^(1/2)))^(1/2))+8*b*c^5*d*a*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b
)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.75

$$\int \sqrt{c+dx}(a-bx^2)^{5/2}\,dx = \frac{2\left(8\left(32\,b^3c^7-161\,ab^2c^5d^2+354\,a^2bc^3d^4-993\,a^3cd^6\right)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2}{3\,bd^2}\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27027*(8*(32*b^3*c^7 - 161*a*b^2*c^5*d^2 + 354*a^2*b*c^3*d^4 - 993*a^3*c
*d^6)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*
(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 24*(32*b^3*c^6*d - 137*a
*b^2*c^4*d^3 + 258*a^2*b*c^2*d^5 + 231*a^3*d^7)*sqrt(-b*d)*weierstrassZeta
(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierst
rassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d
^3), 1/3*(3*d*x + c)/d)) + 3*(693*b^3*d^7*x^5 + 63*b^3*c*d^6*x^4 + 128*b^3
*c^5*d^2 - 532*a*b^2*c^3*d^4 + 971*a^2*b*c*d^6 - 14*(5*b^3*c^2*d^5 + 154*a
*b^2*d^7)*x^3 + 2*(40*b^3*c^3*d^4 - 163*a*b^2*c*d^6)*x^2 - (96*b^3*c^4*d^3
 - 394*a*b^2*c^2*d^5 - 2387*a^2*b*d^7)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/
(b*d^7)
```

Sympy [F]

$$\int \sqrt{c+dx}\,(a-bx^2)^{5/2}\,dx = \int (a-bx^2)^{\frac{5}{2}}\sqrt{c+dx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-b*x**2+a)**(5/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)*sqrt(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\,(a-bx^2)^{5/2}\,dx = \int (-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)*sqrt(d*x + c), x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{5/2} dx = \int \left(-bx^2+a\right)^{\frac{5}{2}}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)*sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{5/2} dx = \int \left(a-b\,x^2\right)^{5/2}\sqrt{c+d\,x}\,dx$$

input
```
int((a - b*x^2)^(5/2)*(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^(5/2)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(5/2),x)
```

output

```
(2*( - 924*sqrt(c + d*x)*sqrt(a - b*x**2)*a**3*d**5 - 61*sqrt(c + d*x)*sqr
t(a - b*x**2)*a**2*b*c**2*d**3 + 2387*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*
b*c*d**4*x + 16*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c**4*d + 394*sqrt(c
+ d*x)*sqrt(a - b*x**2)*a*b**2*c**3*d**2*x - 326*sqrt(c + d*x)*sqrt(a - b*
x**2)*a*b**2*c**2*d**3*x**2 - 2156*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c
*d**4*x**3 - 96*sqrt(c + d*x)*sqrt(a - b*x**2)*b**3*c**5*x + 80*sqrt(c + d
*x)*sqrt(a - b*x**2)*b**3*c**4*d*x**2 - 70*sqrt(c + d*x)*sqrt(a - b*x**2)*
b**3*c**3*d**2*x**3 + 63*sqrt(c + d*x)*sqrt(a - b*x**2)*b**3*c**2*d**3*x**
4 + 693*sqrt(c + d*x)*sqrt(a - b*x**2)*b**3*c*d**4*x**5 - 1386*int((sqrt(c
 + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3
*b*d**6 - 1548*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*
c*x**2 - b*d*x**3),x)*a**2*b**2*c**2*d**4 + 822*int((sqrt(c + d*x)*sqrt(a
- b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*c**4*d**2 -
192*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*
d*x**3),x)*b**4*c**6 + 462*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d
*x - b*c*x**2 - b*d*x**3),x)*a**4*d**6 + 2148*int((sqrt(c + d*x)*sqrt(a -
b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*b*c**2*d**4 - 402*int
((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a
**2*b**2*c**4*d**2 + 96*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x
- b*c*x**2 - b*d*x**3),x)*a*b**3*c**6))/(9009*b*c*d**4)
```

3.314 $\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 492

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{8\sqrt{c+dx}\left(32b^2c^4-69abc^2d^2+45a^2d^4-24bcd(bc^2-2ad^2)\,x\right)\sqrt{a-bx^2}}{693d^5}$$
$$-\frac{20\sqrt{c+dx}\,(8bc^2-9ad^2-7bcdx)\,(a-bx^2)^{3/2}}{693d^3}+\frac{2\sqrt{c+dx}\,(a-bx^2)^{5/2}}{11d}$$
$$-\frac{16\sqrt{a}\sqrt{b}c(32b^2c^4-93abc^2d^2+93a^2d^4)\,\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{693d^6\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$+\frac{16\sqrt{a}(32b^3c^6-101ab^2c^4d^2+114a^2bc^2d^4-45a^3d^6)\,\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),-\ldots\right)}{693\sqrt{b}d^6\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
8/693*(d*x+c)^(1/2)*(32*b^2*c^4-69*a*b*c^2*d^2+45*a^2*d^4-24*b*c*d*(-2*a*d
^2+b*c^2)*x)*(-b*x^2+a)^(1/2)/d^5-20/693*(d*x+c)^(1/2)*(-7*b*c*d*x-9*a*d^2
+8*b*c^2)*(-b*x^2+a)^(3/2)/d^3+2/11*(d*x+c)^(1/2)*(-b*x^2+a)^(5/2)/d-16/69
3*a^(1/2)*b^(1/2)*c*(93*a^2*d^4-93*a*b*c^2*d^2+32*b^2*c^4)*(d*x+c)^(1/2)*(
1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(b^(1/2)*(d*x+c)/(b^(1/2)*c+
a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+16/693*a^(1/2)*(-45*a^3*d^6+114*a^2*b*c
^2*d^4-101*a*b^2*c^4*d^2+32*b^3*c^6)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d
))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/
2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^6/(d*x+c)^(1
/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.32 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.33

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{a-bx^2}\Bigg((c+dx)\,(333a^2d^4-2abd^2(178c^2-131cdx+108d^2x^2)+b^2(128c^4-96c}{}$$

input
```
Integrate[(a - b*x^2)^(5/2)/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*((c + d*x)*(333*a^2*d^4 - 2*a*b*d^2*(178*c^2 - 131*c*d*
x + 108*d^2*x^2) + b^2*(128*c^4 - 96*c^3*d*x + 80*c^2*d^2*x^2 - 70*c*d^3*x
^3 + 63*d^4*x^4)) - (8*(c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(32*b^2*c^4 -
 93*a*b*c^2*d^2 + 93*a^2*d^4)*(a - b*x^2) + I*Sqrt[b]*c*(32*b^(5/2)*c^5 -
32*Sqrt[a]*b^2*c^4*d - 93*a*b^(3/2)*c^3*d^2 + 93*a^(3/2)*b*c^2*d^3 + 93*a^
2*Sqrt[b]*c*d^4 - 93*a^(5/2)*d^5)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)
]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE
[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqr
t[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(32*b^(5/2)*c^5 - 8*Sqrt[a]
*b^2*c^4*d - 93*a*b^(3/2)*c^3*d^2 + 21*a^(3/2)*b*c^2*d^3 + 93*a^2*Sqrt[b]*
c*d^4 - 45*a^(5/2)*d^5)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-((
(Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh
[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(S
qrt[b]*c - Sqrt[a]*d)]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2)))
)/(693*d^5*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {493, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 493}$$

$$\frac{10\int \frac{(ad+bcx)\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx}{11d}+\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$$\Big\downarrow \text{ 682}$$

$$\frac{10\left(-\frac{4\int \frac{b\left(ad\left(bc^2-9ad^2\right)+8bc\left(bc^2-2ad^2\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{21bd^2}-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(-9ad^2+8bc^2-7bcdx\right)}{63d^2}\right)}{11d}+\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$$\Big\downarrow \; 27$$

$$\frac{10\left(-\frac{2\int \frac{\left(ad\left(bc^2-9ad^2\right)+8bc\left(bc^2-2ad^2\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{21d^2}-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(-9ad^2+8bc^2-7bcdx\right)}{63d^2}\right)}{11d}+\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$$\Big\downarrow \; 682$$

$$\frac{10\left(-\frac{2\left(-\frac{4\int \frac{b\left(ad\left(8b^2c^4-21abd^2c^2+45a^2d^4\right)+bc\left(32b^2c^4-93abd^2c^2+93a^2d^4\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(45a^2d^4-24bcdx\left(bc^2-2ad^2\right)-69abc^2d^2+32b^2c^4\right)}{15d^2}\right)}{21d^2}\right.}{11d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$$\Big\downarrow \; 27$$

$$\frac{10\left(-\frac{2\left(-\frac{2\int \frac{ad\left(8b^2c^4-21abd^2c^2+45a^2d^4\right)+bc\left(32b^2c^4-93abd^2c^2+93a^2d^4\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(45a^2d^4-24bcdx\left(bc^2-2ad^2\right)-69abc^2d^2+32b^2c^4\right)}{15d^2}\right)}{21d^2}\right.}{11d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$$\Big\downarrow \; 600$$

$$\frac{10\left(-\frac{2\left(-\frac{2\left(\frac{bc\left(93a^2d^4-93abc^2d^2+32b^2c^4\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(45a^2d^4-69abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(45a^2d^4-\cdots\right.}{\cdots}\right)}{21d^2}\cdots\right)}{11d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$\downarrow$ 509

$$\frac{10\left(-\frac{2\left(-\frac{2\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\left(93a^2d^4-93abc^2d^2+32b^2c^4\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(45a^2d^4-69abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\cdots}{\cdots}\right)}{21d^2}\cdots\right)}{11d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$\downarrow$ 508

$$\frac{10\left(-\frac{2\left(2-\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(45a^2d^4-69abc^2d^2+32b^2c^4\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(93a^2d^4-93abc^2d^2+32b^2c^4\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}\cdots}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{15d^2}\cdots\right)}{21d^2}\cdots\right)}{11d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$\downarrow$ 327

$$\frac{10\left(-\frac{2\left(-\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(45a^2d^4-69abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(93a^2d^4-93abc^2d^2+32b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{15d^2}}{21d^2}}{11d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$$\Big\downarrow \; 512$$

$$\frac{10\left(-\frac{2\left(-\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(45a^2d^4-69abc^2d^2+32b^2c^4\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(93a^2d^4-93abc^2d^2+32b^2c^4\right)E\left(\arcsin\left(\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{15d^2}}{21d^2}}{11d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$\downarrow$ 511

$$10\left(-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(45a^2d^4-69abc^2d^2+32b^2c^4\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+\cdots}}{\cdots}\right)}{15d^2}\cdots\right)}{21d^2}\cdots\right)$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

$\downarrow$ 321

$$10\left(-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(45a^2d^4-69abc^2d^2+32b^2c^4\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(93a^2\cdots\right.}{\cdots}\right)}{15d^2}\cdots}{21d^2}\cdots\right)$$

$$\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11d}$$

input
```
Int[(a - b*x^2)^(5/2)/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(a - b*x^2)^(5/2))/(11*d) + (10*((-2*Sqrt[c + d*x]*(8*b*c
^2 - 9*a*d^2 - 7*b*c*d*x)*(a - b*x^2)^(3/2))/(63*d^2) - (2*((-2*Sqrt[c + d
*x]*(32*b^2*c^4 - 69*a*b*c^2*d^2 + 45*a^2*d^4 - 24*b*c*d*(b*c^2 - 2*a*d^2)
*x)*Sqrt[a - b*x^2])/(15*d^2) - (2*((-2*Sqrt[a]*Sqrt[b]*c*(32*b^2*c^4 - 93
*a*b*c^2*d^2 + 93*a^2*d^4)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[Arc
Sin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d
)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
 + (2*Sqrt[a]*(b*c^2 - a*d^2)*(32*b^2*c^4 - 69*a*b*c^2*d^2 + 45*a^2*d^4)*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt
[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(21*d^2)
))/(11*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 926 vs. $2(416) = 832$.

Time = 4.23 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.88

method	result
risch	$\frac{2\left(63b^2d^4x^4-70b^2c\,d^3x^3-216ab\,d^4x^2+80d^2c^2x^2b^2+262abc\,d^3x-96b^2c^3dx+333a^2d^4-356b\,c^2d^2a+128b^2c^4\right)\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{693d^5}+$
elliptic	Expression too large to display
default	Expression too large to display

input

```
int((-b*x^2+a)^(5/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/693*(63*b^2*d^4*x^4-70*b^2*c*d^3*x^3-216*a*b*d^4*x^2+80*b^2*c^2*d^2*x^2+
262*a*b*c*d^3*x-96*b^2*c^3*d*x+333*a^2*d^4-356*a*b*c^2*d^2+128*b^2*c^4)/d^
5*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)+8/693/d^5*(c*(93*a^2*d^4-93*a*b*c^2*d^2+3
2*b^2*c^4)*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2)
)^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d-1/b*(a*b)^(1/2))*Elliptic
E(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/
(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2
))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))+4
5*a^3*d^5/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2
))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b*
(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))
^(1/2))+8*a*b*c^4*d*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2)
)^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a
*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*
((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)
^(1/2)))^(1/2))-21*a^2*c^2*d^3*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/
(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^
(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elliptic...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.76

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{2\left(8\left(32\,b^3c^6-117\,ab^2c^4d^2+156\,a^2bc^2d^4-135\,a^3d^6\right)\sqrt{-bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(bc^2\right.}{3}\right.}{}$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/2079*(8*(32*b^3*c^6 - 117*a*b^2*c^4*d^2 + 156*a^2*b*c^2*d^4 - 135*a^3*d^
6)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*
c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 24*(32*b^3*c^5*d - 93*a*b^2
*c^3*d^3 + 93*a^2*b*c*d^5)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2
)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c
^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/
d)) + 3*(63*b^3*d^6*x^4 - 70*b^3*c*d^5*x^3 + 128*b^3*c^4*d^2 - 356*a*b^2*c
^2*d^4 + 333*a^2*b*d^6 + 8*(10*b^3*c^2*d^4 - 27*a*b^2*d^6)*x^2 - 2*(48*b^3
*c^3*d^3 - 131*a*b^2*c*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b*d^7)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{5}{2}}}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(5/2)/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \int \frac{(a-b\,x^2)^{5/2}}{\sqrt{c+d\,x}}\,dx$$

input
```
int((a - b*x^2)^(5/2)/(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^(5/2)/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{-\frac{26\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,a^2d^3}{231}+\frac{32\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,ab\,c^2d}{693}+\frac{524\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,abc\,d^2x}{693}-\frac{48\sqrt{dx+c}\,\sqrt{-b\,x^{2}}}{77}}{}$$

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(1/2),x)
```

output

```
(2*( - 39*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 + 16*sqrt(c + d*x)*sqrt
(a - b*x**2)*a*b*c**2*d + 262*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x
- 216*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d**3*x**2 - 96*sqrt(c + d*x)*sqrt
(a - b*x**2)*b**2*c**3*x + 80*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x
**2 - 70*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 + 63*sqrt(c + d*x
)*sqrt(a - b*x**2)*b**2*d**3*x**4 - 558*int((sqrt(c + d*x)*sqrt(a - b*x**2
)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*d**4 + 558*int((sqrt
(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*
b**2*c**2*d**2 - 192*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*
x - b*c*x**2 - b*d*x**3),x)*b**3*c**4 + 366*int((sqrt(c + d*x)*sqrt(a - b*
x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*d**4 - 270*int((sqrt(c
+ d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*c**
2*d**2 + 96*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 -
 b*d*x**3),x)*a*b**2*c**4))/(693*d**4)
```

3.315 $\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 477

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = -\frac{8b\sqrt{c+dx}(c(32bc^2-33ad^2)-3d(8bc^2-7ad^2)\,x)\sqrt{a-bx^2}}{63d^5} + \frac{100bc\sqrt{c+dx}(a-bx^2)^{3/2}}{21d^3} - \frac{20b(c+dx)^{3/2}\,(a-bx^2)^{3/2}}{9d^3} - \frac{2(a-bx^2)^{5/2}}{d\sqrt{c+dx}} + \frac{16\sqrt{a}\sqrt{b}(32b^2c^4-57abc^2d^2+21a^2d^4)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{63d^6\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{16\sqrt{a}\sqrt{b}c(32b^2c^4-65abc^2d^2+33a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{63d^6\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-8/63*b*(d*x+c)^(1/2)*(c*(-33*a*d^2+32*b*c^2)-3*d*(-7*a*d^2+8*b*c^2)*x)*(-
b*x^2+a)^(1/2)/d^5+100/21*b*c*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/d^3-20/9*b*(d
*x+c)^(3/2)*(-b*x^2+a)^(3/2)/d^3-2*(-b*x^2+a)^(5/2)/d/(d*x+c)^(1/2)+16/63*
a^(1/2)*b^(1/2)*(21*a^2*d^4-57*a*b*c^2*d^2+32*b^2*c^4)*(d*x+c)^(1/2)*(1-b*
x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^
(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1
/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-16/63*a^(1/2)*b^(1/2)*c*(33*a^2*d^4-65*a*b*
c^2*d^2+32*b^2*c^4)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2
/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/
2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.89 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.36

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-63(bc^2-ad^2)^2-bc(65bc^2-86ad^2)(c+dx)+bd(33bc^2-28ad^2)x(c+\right.}{}$$

input
```
Integrate[(a - b*x^2)^(5/2)/(c + d*x)^(3/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-63*(b*c^2 - a*d^2)^2 - b*c*(65*b*c^2 - 86*a*d^2)*(c +
 d*x) + b*d*(33*b*c^2 - 28*a*d^2)*x*(c + d*x) - 17*b^2*c*d^2*x^2*(c + d*x)
 + 7*b^2*d^3*x^3*(c + d*x) + (8*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(32*b^
2*c^4 - 57*a*b*c^2*d^2 + 21*a^2*d^4)*(a - b*x^2) + I*Sqrt[b]*(32*b^(5/2)*c
^5 - 32*Sqrt[a]*b^2*c^4*d - 57*a*b^(3/2)*c^3*d^2 + 57*a^(3/2)*b*c^2*d^3 +
21*a^2*Sqrt[b]*c*d^4 - 21*a^(5/2)*d^5)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
 d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Elli
pticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c
+ Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*(32*b^2*c^4 -
8*Sqrt[a]*b^(3/2)*c^3*d - 57*a*b*c^2*d^2 + 12*a^(3/2)*Sqrt[b]*c*d^3 + 21*a
^2*d^4)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt
[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[
a]*d)]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(63*d^5*Sqrt[c
 + d*x])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {492, 591, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/2}}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{492}$$

$$-\frac{10b\int \frac{x\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx}{d}-\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{591}$$

$$-\frac{10b\left(\frac{4\int -\frac{\left(acd+\left(8bc^2-7ad^2\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{21d^2}-\frac{2\left(a-bx^2\right)^{3/2}(8c-7dx)\sqrt{c+dx}}{63d^2}\right)}{d}-\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10b\left(-\frac{2\int \frac{\left(acd+\left(8bc^2-7ad^2\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{21d^2}-\frac{2\left(a-bx^2\right)^{3/2}(8c-7dx)\sqrt{c+dx}}{63d^2}\right)}{d}-\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 682

$$-\frac{10b\left(-\frac{2\left(-\frac{4\int \frac{b\left(4acd\left(2bc^2-3ad^2\right)+\left(32b^2c^4-57abd^2c^2+21a^2d^4\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(c\left(32bc^2-33ad^2\right)-3dx\left(8bc^2-7ad^2\right)\right)}{15d^2}\right)}{21d^2}-\frac{2\left(a-bx^2\right)^{3/2}\cdots}{\cdots}\right)}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 27

$$-\frac{10b\left(-\frac{2\left(-\frac{2\int \frac{4acd\left(2bc^2-3ad^2\right)+\left(32b^2c^4-57abd^2c^2+21a^2d^4\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(c\left(32bc^2-33ad^2\right)-3dx\left(8bc^2-7ad^2\right)\right)}{15d^2}\right)}{21d^2}-\frac{2\left(a-bx^2\right)^{3/2}(8c\cdots}{63d\cdots}\right)}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 600

$$-\frac{10b\left(-\frac{2\left(-\frac{2\left(\frac{\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(c\left(32bc^2-33ad^2\right)-\cdots\right.}{15d^2}\cdots\right)}{21d^2}\cdots\right)}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 509

$$-\frac{10b\left(-\frac{2\left(-\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(c\left(32bc^2-\right.\right.}{15}\right.}{21d^2}}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$$\downarrow 508$$

$$-\frac{10b\left(-\frac{2\left(-\frac{2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\cdots\right)}{15d^2}\cdots\right)}{21d^2}\cdots\right)}{d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 327$$

$$-\frac{10b\left(-\frac{2\left(-\frac{2\left(-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}}{21d^2}\right)}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$\downarrow$ 512

$$-\frac{10b\left(-\frac{2\left(-\frac{2\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}}{21d^2}\right)}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 511$$

$$-\frac{10b\left(-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-\cdots\right.}{\cdots}\right.}{15d^2}\cdots}{21d^2}\cdots}{}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 321$$

$$-\frac{10b\left(-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc^2d\right.}{\sqrt{b}d\sqrt{a-}}\right)}{15d^2}}{21d^2}}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{d\sqrt{c+dx}}$$

input
```
Int[(a - b*x^2)^(5/2)/(c + d*x)^(3/2),x]
```

output
```
(-2*(a - b*x^2)^(5/2))/(d*Sqrt[c + d*x]) - (10*b*((-2*(8*c - 7*d*x)*Sqrt[c
 + d*x]*(a - b*x^2)^(3/2))/(63*d^2) - (2*((-2*Sqrt[c + d*x]*(c*(32*b*c^2 -
 33*a*d^2) - 3*d*(8*b*c^2 - 7*a*d^2)*x)*Sqrt[a - b*x^2])/(15*d^2) - (2*((-
2*Sqrt[a]*(32*b^2*c^4 - 57*a*b*c^2*d^2 + 21*a^2*d^4)*Sqrt[c + d*x]*Sqrt[1
- (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d
)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]
*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*(32*b*c^2 - 33*a*d^2)*(b*
c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((
Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*
d^2)))/(21*d^2)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(397) = 794.

Time = 6.69 (sec) , antiderivative size = 1267, normalized size of antiderivative = 2.66

method	result	size
elliptic	Expression too large to display	1267
risch	Expression too large to display	1413
default	Expression too large to display	1656

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2*(-b*d*x^2+a*
d)*(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/d^6/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2/9*
b^2/d^2*x^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-34/63*b^2/d^3*c*x^2*(-b*d*x
^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(b^2/d^3*(3*a*d^2-b*c^2)-7/9*b^2/d*a-34/21
*b^3/d^3*c^2)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-b^2*c/d^4*(3*
a*d^2-b*c^2)+43/63*b^2/d^2*a*c-4/5*(b^2/d^3*(3*a*d^2-b*c^2)-7/9*b^2/d*a-34
/21*b^3/d^3*c^2)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(b*c*(3*a^2
*d^4-3*a*b*c^2*d^2+b^2*c^4)/d^6-(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)*b/d^6*c+2/
5*(b^2/d^3*(3*a*d^2-b*c^2)-7/9*b^2/d*a-34/21*b^3/d^3*c^2)/b/d*a*c+1/3*(-b^
2*c/d^4*(3*a*d^2-b*c^2)+43/63*b^2/d^2*a*c-4/5*(b^2/d^3*(3*a*d^2-b*c^2)-7/9
*b^2/d*a-34/21*b^3/d^3*c^2)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)
*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a
*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-b/d^5*(3*a^2*d^4-3*a*b*c^2*d
^2+b^2*c^4)-(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)*b/d^5+68/63*b^2/d^3*c^2*a+3/5*
(b^2/d^3*(3*a*d^2-b*c^2)-7/9*b^2/d*a-34/21*b^3/d^3*c^2)/b*a-2/3*(-b^2*c/d^
4*(3*a*d^2-b*c^2)+43/63*b^2/d^2*a*c-4/5*(b^2/d^3*(3*a*d^2-b*c^2)-7/9*b^2/d
*a-34/21*b^3/d^3*c^2)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a
*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.88

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = -\frac{2\left(8\left(32\,b^2c^6-81\,abc^4d^2+57\,a^2c^2d^4+\left(32\,b^2c^5d-81\,abc^3d^3+57\,a^2cd^5\right)x\right)\sqrt{-bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/189*(8*(32*b^2*c^6 - 81*a*b*c^4*d^2 + 57*a^2*c^2*d^4 + (32*b^2*c^5*d -
81*a*b*c^3*d^3 + 57*a^2*c*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^
2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d
) + 24*(32*b^2*c^5*d - 57*a*b*c^3*d^3 + 21*a^2*c*d^5 + (32*b^2*c^4*d^2 - 5
7*a*b*c^2*d^4 + 21*a^2*d^6)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a
*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*
(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x +
 c)/d)) - 3*(7*b^2*d^6*x^4 - 10*b^2*c*d^5*x^3 - 128*b^2*c^4*d^2 + 212*a*b*
c^2*d^4 - 63*a^2*d^6 + 4*(4*b^2*c^2*d^4 - 7*a*b*d^6)*x^2 - 2*(16*b^2*c^3*d
^3 - 29*a*b*c*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(d^8*x + c*d^7)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(a-bx^2)^{\frac{5}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(5/2)/(d*x+c)**(3/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = \int \frac{(a-b\,x^2)^{5/2}}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((a - b*x^2)^(5/2)/(c + d*x)^(3/2),x)
```

output
```
int((a - b*x^2)^(5/2)/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(3/2),x)
```

output
```
(2*( - 27*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 + 16*sqrt(c + d*x)*sqrt
(a - b*x**2)*a*b*c**2*d + 58*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x -
 28*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d**3*x**2 - 32*sqrt(c + d*x)*sqrt(a
 - b*x**2)*b**2*c**3*x + 16*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x**
2 - 10*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 + 7*sqrt(c + d*x)*s
qrt(a - b*x**2)*b**2*d**3*x**4 - 66*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x*
*2)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**
2*x**4),x)*a**2*b*c*d**4 - 66*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a
*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4
),x)*a**2*b*d**5*x + 130*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**2
 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*
a*b**2*c**3*d**2 + 130*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**2 +
 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*a*
b**2*c**2*d**3*x - 64*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**2 +
2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*b**
3*c**5 - 64*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*b**3*c**4*d*x
 + 18*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*a**3*c*d**4 + 18*int((sq
rt(c + d*x)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**...
```

3.316 $\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 451

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx = \frac{8b\sqrt{c+dx}(32bc^2-5ad^2-24bcdx)\sqrt{a-bx^2}}{21d^5} - \frac{20bc(a-bx^2)^{3/2}}{3d^3\sqrt{c+dx}} - \frac{20b\sqrt{c+dx}(a-bx^2)^{3/2}}{21d^3} - \frac{2(a-bx^2)^{5/2}}{3d(c+dx)^{3/2}} - \frac{16\sqrt{a}b^{3/2}c(32bc^2-29ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{21d^6\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{16\sqrt{a}\sqrt{b}(32b^2c^4-37abc^2d^2+5a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{21d^6\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
8/21*b*(d*x+c)^(1/2)*(-24*b*c*d*x-5*a*d^2+32*b*c^2)*(-b*x^2+a)^(1/2)/d^5-2
0/3*b*c*(-b*x^2+a)^(3/2)/d^3/(d*x+c)^(1/2)-20/21*b*(d*x+c)^(1/2)*(-b*x^2+a
)^(3/2)/d^3-2/3*(-b*x^2+a)^(5/2)/d/(d*x+c)^(3/2)-16/21*a^(1/2)*b^(3/2)*c*(
-29*a*d^2+32*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/d^6/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+16/2
1*a^(1/2)*b^(1/2)*(5*a^2*d^4-37*a*b*c^2*d^2+32*b^2*c^4)*(b^(1/2)*(d*x+c)/(
b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a
^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6
/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.71 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.29

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{-14a^2d^4-4abd^2(50c^2+65cdx+8d^2x^2)+2b^2(128c^4+160c^3dx+16c^2d^2x^2-6cd^3x^3+3d^4x^4)}{d^5(c+dx)} - \frac{16b\left(c \cdots\right.}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(a - b*x^2)^(5/2)/(c + d*x)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*((-14*a^2*d^4 - 4*a*b*d^2*(50*c^2 + 65*c*d*x + 8*d^2*x^2)
 + 2*b^2*(128*c^4 + 160*c^3*d*x + 16*c^2*d^2*x^2 - 6*c*d^3*x^3 + 3*d^4*x^4
))/(d^5*(c + d*x)) - (16*b*(c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(32*b*c^2
 - 29*a*d^2)*(-a + b*x^2) - I*Sqrt[b]*c*(32*b^(3/2)*c^3 - 32*Sqrt[a]*b*c^2
*d - 29*a*Sqrt[b]*c*d^2 + 29*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(
c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*E
llipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]
*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*d*(32*b^(3/2)*c^3 - 8
*Sqrt[a]*b*c^2*d - 29*a*Sqrt[b]*c*d^2 + 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sq
rt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
 d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*
x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^7*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(21*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {492, 590, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/2}}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 492}$$

$$-\frac{10b\int \frac{x\left(a-bx^2\right)^{3/2}}{(c+dx)^{3/2}}dx}{3d}-\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 590}$$

$$-\frac{10b\left(\frac{2\left(a-bx^2\right)^{3/2}(8c+dx)}{7d^2\sqrt{c+dx}}-\frac{12\int -\frac{(ad+8bcx)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{7d^2}\right)}{3d}-\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{10b\left(\frac{6\int \frac{(ad+8bcx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{7d^2}+\frac{2\left(a-bx^2\right)^{3/2}(8c+dx)}{7d^2\sqrt{c+dx}}\right)}{3d}-\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 682}$$

$$-\frac{10b\left(\frac{6\left(-\frac{4\int \frac{b\left(ad\left(8bc^2-5ad^2\right)+bc\left(32bc^2-29ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-5ad^2+32bc^2-24bcdx\right)}{15d^2}\right)}{7d^2}+\frac{2\left(a-bx^2\right)^{3/2}(8c+dx)}{7d^2\sqrt{c+dx}}\right)}{3d}-\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{10b\left(\frac{6\left(-\frac{2\int \frac{ad\left(8bc^2-5ad^2\right)+bc\left(32bc^2-29ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-5ad^2+32bc^2-24bcdx\right)}{15d^2}\right)}{7d^2}+\frac{2\left(a-bx^2\right)^{3/2}(8c+dx)}{7d^2\sqrt{c+dx}}\right)}{3d}-\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$\downarrow$ 600

$$-\frac{10b\left(\frac{6\left(-\frac{2\left(\frac{bc\left(32bc^2-29ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-5ad^2+32bc^2-24bcdx\right)}{15d^2}\right)}{7d^2}+2(a \cdots\right)}{3d} \cdots -\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$\downarrow$ 509

$$-\frac{10b\left(\frac{6\left(-\frac{2\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-29ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-5ad^2+32bc^2-24bcdx\right)}{15d^2}\right)}{7d^2} \cdots\right)}{3d} \cdots -\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$\downarrow$ 508

$$-\frac{10b\left(\frac{6\left(-\frac{2\left(-\frac{\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-29ad^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-bx^2}\cdots}{\cdots}\right)}{7d^2}\cdots\right)}{3d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$\downarrow$ 327

$$-\frac{10b\left(\frac{6\left(-\frac{2\left(-\frac{\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-29ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\frac{2\sqrt{a-bx}\cdots}{\cdots}\right)}{7d^2}\cdots\right)}{3d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$\big\downarrow$ 512

$$-\frac{10b\left(\frac{6\left(-\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-29ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{15d^2}-\cdots\right)}{7d^2}\cdots\right)}{3d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 511$$

$$-\frac{10b\left(\frac{6\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-29\right.}{d\sqrt{a-bx}}\right.}{15d^2}\right.}{7d^2}\right.}{3d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 321$$

$$-\frac{10b\left(\frac{6\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-29ad^2\right)E\left(\ldots\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c\ldots}{\sqrt{a}d\ldots}}}\right)}{15d^2}\ldots\right)}{7d^2}\ldots\right)}{3d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{3d(c+dx)^{3/2}}$$

input
```
Int[(a - b*x^2)^(5/2)/(c + d*x)^(5/2),x]
```

output
```
(-2*(a - b*x^2)^(5/2))/(3*d*(c + d*x)^(3/2)) - (10*b*((2*(8*c + d*x)*(a -
b*x^2)^(3/2))/(7*d^2*Sqrt[c + d*x]) + (6*((-2*Sqrt[c + d*x]*(32*b*c^2 - 5*
a*d^2 - 24*b*c*d*x)*Sqrt[a - b*x^2])/(15*d^2) - (2*((-2*Sqrt[a]*Sqrt[b]*c*
(32*b*c^2 - 29*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[S
qrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(
d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*
Sqrt[a]*(32*b*c^2 - 5*a*d^2)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqr
t[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b
]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[
c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(7*d^2)))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(369) = 738$.

Time = 8.45 (sec) , antiderivative size = 1041, normalized size of antiderivative = 2.31

method	result	size
elliptic	Expression too large to display	1041
risch	Expression too large to display	1971
default	Expression too large to display	2513

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/3*(a^2*d^4-2
*a*b*c^2*d^2+b^2*c^4)/d^7*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2-28/
3*(-b*d*x^2+a*d)*(a*d^2-b*c^2)*b*c/d^6/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2/7*
b^2/d^3*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-8/7*b^2*c/d^4*x*(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)-2/3*(3*b^2/d^4*(a*d^2-b*c^2)-5/7*b^2/d^2*a-16/7*b^
3*c^2/d^4)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-b*(3*a^2*d^4-9*a*b*c
^2*d^2+5*b^2*c^4)/d^6+1/3*(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)*b/d^6-14/3*c^2*(
a*d^2-b*c^2)*b^2/d^6+8/7*b^2*c^2/d^4*a+1/3*(3*b^2/d^4*(a*d^2-b*c^2)-5/7*b^
2/d^2*a-16/7*b^3*c^2/d^4)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-2*b^2*c/d^5*(3*a*d^2-2*b*c^2)-14/3*
c*(a*d^2-b*c^2)/d^5*b^2+8/7*b^2/d^3*a*c-2/3*(3*b^2/d^4*(a*d^2-b*c^2)-5/7*b
^2/d^2*a-16/7*b^3*c^2/d^4)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a
*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1
/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)
^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2
)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.05

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx = \frac{2\left(8\,(32\,b^2c^6 - 53\,abc^4d^2 + 15\,a^2c^2d^4 + (32\,b^2c^4d^2 - 53\,abc^2d^4 + 15\,a^2d^6)x^2 + 2\,(32\,b^2c\right.}{}$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/63*(8*(32*b^2*c^6 - 53*a*b*c^4*d^2 + 15*a^2*c^2*d^4 + (32*b^2*c^4*d^2 -
53*a*b*c^2*d^4 + 15*a^2*d^6)*x^2 + 2*(32*b^2*c^5*d - 53*a*b*c^3*d^3 + 15*a
^2*c*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
 -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 24*(32*b^2*c^5*d
- 29*a*b*c^3*d^3 + (32*b^2*c^3*d^3 - 29*a*b*c*d^5)*x^2 + 2*(32*b^2*c^4*d^2
 - 29*a*b*c^2*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*
d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 +
3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) +
 3*(3*b^2*d^6*x^4 - 6*b^2*c*d^5*x^3 + 128*b^2*c^4*d^2 - 100*a*b*c^2*d^4 -
7*a^2*d^6 + 16*(b^2*c^2*d^4 - a*b*d^6)*x^2 + 10*(16*b^2*c^3*d^3 - 13*a*b*c
*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx=\int \frac{(a-bx^2)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(5/2)/(d*x+c)**(5/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)/(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx=\int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx = \int \frac{(a-b\,x^2)^{5/2}}{(c+d\,x)^{5/2}}\,dx$$

input
```
int((a - b*x^2)^(5/2)/(c + d*x)^(5/2),x)
```

output
```
int((a - b*x^2)^(5/2)/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(5/2),x)
```

output
```
(2*( - 83*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 + 80*sqrt(c + d*x)*sqrt
(a - b*x**2)*a*b*c**2*d + 102*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x
- 16*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d**3*x**2 - 96*sqrt(c + d*x)*sqrt(
a - b*x**2)*b**2*c**3*x + 16*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x*
*2 - 6*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 + 3*sqrt(c + d*x)*s
qrt(a - b*x**2)*b**2*d**3*x**4 + 18*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x*
*2)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 -
 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)*a**2*b*c**2*d**4 + 36
*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*
d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4
- b*d**3*x**5),x)*a**2*b*c*d**5*x + 18*int((sqrt(c + d*x)*sqrt(a - b*x**2)
*x**2)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**
2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)*a**2*b*d**6*x**2 +
 174*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**3 + 3*a*c**2*d*x + 3*
a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x
**4 - b*d**3*x**5),x)*a*b**2*c**4*d**2 + 348*int((sqrt(c + d*x)*sqrt(a - b
*x**2)*x**2)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c*
*3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)*a*b**2*c**3*
d**3*x + 174*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3...
```

3.317 $\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 492

$$\begin{aligned}
\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx &= -\frac{8b^2\sqrt{c+dx}(c(32bc^2-17ad^2)-3d(8bc^2-3ad^2)\,x)\sqrt{a-bx^2}}{15d^5\,(bc^2-ad^2)} \\
&- \frac{4bc(a-bx^2)^{3/2}}{3d^3(c+dx)^{3/2}} + \frac{4b(3bc^2-ad^2)\,(a-bx^2)^{3/2}}{d^3\,(bc^2-ad^2)\sqrt{c+dx}} - \frac{2(a-bx^2)^{5/2}}{5d(c+dx)^{5/2}} \\
&+ \frac{16\sqrt{a}b^{3/2}(32bc^2-9ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15d^6\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \\
&- \frac{16\sqrt{a}b^{3/2}c(32bc^2-17ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15d^6\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-8/15*b^2*(d*x+c)^(1/2)*(c*(-17*a*d^2+32*b*c^2)-3*d*(-3*a*d^2+8*b*c^2)*x)*
(-b*x^2+a)^(1/2)/d^5/(-a*d^2+b*c^2)-4/3*b*c*(-b*x^2+a)^(3/2)/d^3/(d*x+c)^(
3/2)+4*b*(-a*d^2+3*b*c^2)*(-b*x^2+a)^(3/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(1/2
)-2/5*(-b*x^2+a)^(5/2)/d/(d*x+c)^(5/2)+16/15*a^(1/2)*b^(3/2)*(-9*a*d^2+32*
b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))
^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(b^(1/
2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-16/15*a^(1/2)*b^(
3/2)*c*(-17*a*d^2+32*b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*
(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(d*x+c)^(1/2)/(-b*x^2+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.35 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.04

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(4b(32bc^2-9ad^2)-\frac{3(bc^2-ad^2)^2}{(c+dx)^2}+\frac{22bc(bc^2-ad^2)}{c+dx}-19b^2c(c+dx)+3b^2dx(c\right.}{}$$

input
```
Integrate[(a - b*x^2)^(5/2)/(c + d*x)^(7/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(4*b*(32*b*c^2 - 9*a*d^2) - (3*(b*c^2 - a*d^2)^2)/(c +
d*x)^2 + (22*b*c*(b*c^2 - a*d^2))/(c + d*x) - 19*b^2*c*(c + d*x) + 3*b^2*d
*x*(c + d*x) + ((8*I)*b^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(32*b*c^2 - 9*a*d
^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
- d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d
)])/(d^2*(-a + b*x^2)) - ((8*I)*Sqrt[a]*b^(3/2)*(32*b*c^2 - 8*Sqrt[a]*Sqrt
[b]*c*d - 9*a*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt
[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b
]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(15*d
^5*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {492, 590, 27, 681, 25, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx$$

$$\Big\downarrow \ 492$$

$$-\frac{2b\int \frac{x(a-bx^2)^{3/2}}{(c+dx)^{5/2}}dx}{d}-\frac{2(a-bx^2)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 590$$

$$-\frac{2b\left(\frac{2(a-bx^2)^{3/2}(8c+3dx)}{15d^2(c+dx)^{3/2}}-\frac{4\int -\frac{(3ad+8bcx)\sqrt{a-bx^2}}{2(c+dx)^{3/2}}dx}{5d^2}\right)}{d}-\frac{2(a-bx^2)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{2b\left(\frac{2\int \frac{(3ad+8bcx)\sqrt{a-bx^2}}{(c+dx)^{3/2}}dx}{5d^2}+\frac{2(a-bx^2)^{3/2}(8c+3dx)}{15d^2(c+dx)^{3/2}}\right)}{d}-\frac{2(a-bx^2)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 681$$

$$-\frac{2b\left(\frac{2\left(\frac{2\sqrt{a-bx^2}(-9ad^2+32bc^2+8bcdx)}{3d^2\sqrt{c+dx}}-\frac{2\int -\frac{b(8acd+(32bc^2-9ad^2)x)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2}\right)}{5d^2}+\frac{2(a-bx^2)^{3/2}(8c+3dx)}{15d^2(c+dx)^{3/2}}\right)}{d}-\frac{2(a-bx^2)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{2b\left(\frac{2\left(\frac{2\int \frac{b\left(8acd+\left(32bc^2-9ad^2\right)x\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2}+\frac{2\sqrt{a-bx^2}\left(-9ad^2+32bc^2+8bcdx\right)}{3d^2\sqrt{c+dx}}\right)}{5d^2}+\frac{2\left(a-bx^2\right)^{3/2}(8c+3dx)}{15d^2(c+dx)^{3/2}}\right)}{d}-\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2b\left(\frac{2\left(\frac{2b\int \frac{8acd+\left(32bc^2-9ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2}+\frac{2\sqrt{a-bx^2}\left(-9ad^2+32bc^2+8bcdx\right)}{3d^2\sqrt{c+dx}}\right)}{5d^2}+\frac{2\left(a-bx^2\right)^{3/2}(8c+3dx)}{15d^2(c+dx)^{3/2}}\right)}{d}-\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 600}$$

$$-\frac{2b\left(\frac{2\left(\frac{2b\left(\frac{\left(32bc^2-9ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(32bc^2-17ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}\left(-9ad^2+32bc^2+8bcdx\right)}{3d^2\sqrt{c+dx}}\right)}{5d^2}+\frac{2\left(a-bx^2\right)^{3/2}(8c+3dx)}{15d^2(c+dx)^{3/2}}\right)}{d}-\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 509}$$

$$-\frac{2b\left(\frac{2\left(\frac{2b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-9ad^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(32bc^2-17ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}\left(-9ad^2+32bc^2+8bcdx\right)}{3d^2\sqrt{c+dx}}\right)}{5d^2}+\frac{2\left(a-bx^2\right)^{3/2}(8c}{15d^2(c+dx)}\right.}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$\Big\downarrow$ 508

$$-\frac{2b\left(\frac{2\left(\frac{2b\left(-\frac{c\left(32bc^2-17ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-9ad^2\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}\left(-9ad^2+32bc^2+8\right.}{3d^2\sqrt{c+dx}}\right)}{5d^2}\right.}{d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$\downarrow$ 327

$$-\frac{2b\left(\frac{2\left(\frac{2b\left(-\frac{c\left(32bc^2-17ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-9ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}\left(-9ad^2+32bc^2\cdots\right)}{3d^2\sqrt{c+dx}}\cdots\right)}{5d^2}\cdots\right)}{d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \; 512$$

$$-\frac{2b\left(\frac{2\left(\frac{2b\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-17ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-9ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}\left(-9a\cdots\right)}{3d^2\cdots}\cdots\right)}{5d^2}\cdots\right)}{d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \; 511$$

$$-\frac{2b\left(\frac{2\left(\frac{2b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-17ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-9ad^2\right)E\left(\arcsin\left(\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}\cdots\right)}{3d^2}\cdots\right)}{5d^2}\cdots\right)}{d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \; 321$$

$$-\frac{2b\left(\frac{2\left(\frac{2b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-17ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-9ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}}}{\sqrt{2}}\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{3d^2}\cdots\right)}{5d^2}\cdots\right)}{d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{5d(c+dx)^{5/2}}$$

input
```
Int[(a - b*x^2)^(5/2)/(c + d*x)^(7/2),x]
```

output
```
(-2*(a - b*x^2)^(5/2))/(5*d*(c + d*x)^(5/2)) - (2*b*((2*(8*c + 3*d*x)*(a -
 b*x^2)^(3/2))/(15*d^2*(c + d*x)^(3/2)) + (2*((2*(32*b*c^2 - 9*a*d^2 + 8*b
*c*d*x)*Sqrt[a - b*x^2])/(3*d^2*Sqrt[c + d*x]) + (2*b*((-2*Sqrt[a]*(32*b*c
^2 - 9*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*
d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*
Sqrt[a]*c*(32*b*c^2 - 17*a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt
[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt
[a - b*x^2])))/(3*d^2)))/(5*d^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(412) = 824$.

Time = 10.86 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\left(a^2d^4-2b\,c^2d^2a+b^2c^4\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d^8\left(x+\frac{c}{d}\right)^3}-\frac{44b\left(a\,d^2-b\,c^2\right)c\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{15d^7\left(x+\frac{c}{d}\right)^2}+\frac{8\left(-bd\,x^2+ad\right)\left(9a\right.}{15d^6\sqrt{\left(x+\frac{c}{d}\right)\left(-\right.}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/5*(a^2*d^4-2
*a*b*c^2*d^2+b^2*c^4)/d^8*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3-44/
15*b/d^7*(a*d^2-b*c^2)*c*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+8/15
*(-b*d*x^2+a*d)*(9*a*d^2-32*b*c^2)*b/d^6/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2/
5*b^2/d^4*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-38/15*b^2*c/d^5*(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)+2*(-b^2*c*(9*a*d^2-10*b*c^2)/d^6+22/15*c*(a*d^2-b*
c^2)/d^6*b^2+4/15*(9*a*d^2-32*b*c^2)*b^2*c/d^6+13/15*b^2/d^4*a*c)*(c/d-1/b
*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^
(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)
^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(3
*b^2/d^5*(a*d^2-2*b*c^2)+4/15*(9*a*d^2-32*b*c^2)*b^2/d^5-3/5*b^2/d^3*a-38/
15*b^3*c^2/d^5)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/
(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/
b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*
(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+
c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.03

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx = -\frac{2\left(8\left(32\,b^2c^6-33\,abc^4d^2+(32\,b^2c^3d^3-33\,abcd^5)x^3+3\,(32\,b^2c^4d^2-33\,abc^2d^4)x^2+3\,(32\,b^2c^5d-33\,abc^3\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/45*(8*(32*b^2*c^6 - 33*a*b*c^4*d^2 + (32*b^2*c^3*d^3 - 33*a*b*c*d^5)*x^
3 + 3*(32*b^2*c^4*d^2 - 33*a*b*c^2*d^4)*x^2 + 3*(32*b^2*c^5*d - 33*a*b*c^3
*d^3)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/
27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 24*(32*b^2*c^5*d - 9*
a*b*c^3*d^3 + (32*b^2*c^2*d^4 - 9*a*b*d^6)*x^3 + 3*(32*b^2*c^3*d^3 - 9*a*b
*c*d^5)*x^2 + 3*(32*b^2*c^4*d^2 - 9*a*b*c^2*d^4)*x)*sqrt(-b*d)*weierstrass
Zeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), wei
erstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/
(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(3*b^2*d^6*x^4 - 10*b^2*c*d^5*x^3 - 128*b
^2*c^4*d^2 + 20*a*b*c^2*d^4 - 3*a^2*d^6 - 4*(44*b^2*c^2*d^4 - 9*a*b*d^6)*x
^2 - 2*(144*b^2*c^3*d^3 - 25*a*b*c*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))
/(d^10*x^3 + 3*c*d^9*x^2 + 3*c^2*d^8*x + c^3*d^7)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx = \int \frac{(a-bx^2)^{\frac{5}{2}}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(5/2)/(d*x+c)**(7/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)/(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx = \int \frac{(a-b\,x^2)^{5/2}}{(c+d\,x)^{7/2}}\,dx$$

input
```
int((a - b*x^2)^(5/2)/(c + d*x)^(7/2),x)
```

output
```
int((a - b*x^2)^(5/2)/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(7/2),x)
```

output
```
(2*(273*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 - 560*sqrt(c + d*x)*sqrt(
a - b*x**2)*a*b*c**2*d - 230*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x -
 36*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d**3*x**2 + 480*sqrt(c + d*x)*sqrt(
a - b*x**2)*b**2*c**3*x + 80*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x*
*2 - 10*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 + 3*sqrt(c + d*x)*
sqrt(a - b*x**2)*b**2*d**3*x**4 - 450*int((sqrt(c + d*x)*sqrt(a - b*x**2)*
x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4*a*c*d**3*x**3 + a*d*
*4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2*d**2*x**4 - 4*b*c*d**3*
x**5 - b*d**4*x**6),x)*a**2*b*c**3*d**4 - 1350*int((sqrt(c + d*x)*sqrt(a -
 b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4*a*c*d**3*x*
*3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2*d**2*x**4 - 4*
b*c*d**3*x**5 - b*d**4*x**6),x)*a**2*b*c**2*d**5*x - 1350*int((sqrt(c + d*
x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4*
a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2*d**
2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a**2*b*c*d**6*x**2 - 450*int((s
qrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2
*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*
b*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a**2*b*d**7*x**3 + 45
0*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c
**2*d**2*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*...
```

3.318 $\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 507

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \frac{8b^2(c(32bc^2-29ad^2)+d(8bc^2-5ad^2)\,x)\sqrt{a-bx^2}}{21d^5\,(bc^2-ad^2)\sqrt{c+dx}} - \frac{4bc(a-bx^2)^{3/2}}{7d^3(c+dx)^{5/2}} + \frac{4b(11bc^2-5ad^2)\,(a-bx^2)^{3/2}}{21d^3\,(bc^2-ad^2)\,(c+dx)^{3/2}} - \frac{2(a-bx^2)^{5/2}}{7d(c+dx)^{7/2}} - \frac{16\sqrt{a}b^{5/2}c(32bc^2-29ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{21d^6\,(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{16\sqrt{a}b^{3/2}(32bc^2-5ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{21d^6\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
8/21*b^2*(c*(-29*a*d^2+32*b*c^2)+d*(-5*a*d^2+8*b*c^2)*x)*(-b*x^2+a)^(1/2)/
d^5/(-a*d^2+b*c^2)/(d*x+c)^(1/2)-4/7*b*c*(-b*x^2+a)^(3/2)/d^3/(d*x+c)^(5/2
)+4/21*b*(-5*a*d^2+11*b*c^2)*(-b*x^2+a)^(3/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(
3/2)-2/7*(-b*x^2+a)^(5/2)/d/(d*x+c)^(7/2)-16/21*a^(1/2)*b^(5/2)*c*(-29*a*d
^2+32*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^
(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/
(-a*d^2+b*c^2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1
/2)+16/21*a^(1/2)*b^(3/2)*(-5*a*d^2+32*b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+
a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(d*x+c)^(
1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.19 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.19

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(2b^2c(79bc^2-67ad^2)-8b^2c(32bc^2-29ad^2)-\frac{3(bc^2-ad^2)^3}{(c+dx)^3}+\frac{18bc(bc^2-ad^2)^2}{(c+dx)^2}\right.}{}$$

input
```
Integrate[(a - b*x^2)^(5/2)/(c + d*x)^(9/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(2*b^2*c*(79*b*c^2 - 67*a*d^2) - 8*b^2*c*(32*b*c^2 - 29
*a*d^2) - (3*(b*c^2 - a*d^2)^3)/(c + d*x)^3 + (18*b*c*(b*c^2 - a*d^2)^2)/(
c + d*x)^2 - (4*b*(13*b*c^2 - 4*a*d^2)*(b*c^2 - a*d^2))/(c + d*x) + 7*b^2*
(b*c^2 - a*d^2)*(c + d*x) - ((8*I)*b^3*c*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3
2*b*c^2 - 29*a*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqr
t[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[
b]*c - Sqrt[a]*d)])/(d^2*(-a + b*x^2)) + ((8*I)*Sqrt[a]*b^2*(32*b^(3/2)*c^
3 - 8*Sqrt[a]*b*c^2*d - 29*a*Sqrt[b]*c*d^2 + 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[
a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]
*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(21*d^5*(b*c^2 - a*d^2)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {492, 589, 25, 681, 25, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/2}}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{ 492}$$

$$-\frac{10b\int \frac{x\left(a-bx^2\right)^{3/2}}{(c+dx)^{7/2}}dx}{7d}-\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{ 589}$$

$$-\frac{10b\left(\frac{2b\int -\frac{\left(3acd+\left(8bc^2-5ad^2\right)x\right)\sqrt{a-bx^2}}{(c+dx)^{3/2}}dx}{5d^2\left(bc^2-ad^2\right)}-\frac{2\left(a-bx^2\right)^{3/2}\left(dx\left(11bc^2-5ad^2\right)+2c\left(4bc^2-ad^2\right)\right)}{15d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}\right)}{7d}-\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{10b\left(-\frac{2b\int\frac{\left(3acd+\left(8bc^2-5ad^2\right)x\right)\sqrt{a-bx^2}}{(c+dx)^{3/2}}dx}{5d^2(bc^2-ad^2)}-\frac{2(a-bx^2)^{3/2}(dx(11bc^2-5ad^2)+2c(4bc^2-ad^2))}{15d^2(c+dx)^{5/2}(bc^2-ad^2)}\right)}{7d}-\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$\Big\downarrow$ 681

$$-\frac{10b\left(-\frac{2b\left(\frac{2\sqrt{a-bx^2}\left(dx\left(8bc^2-5ad^2\right)+c\left(32bc^2-29ad^2\right)\right)}{3d^2\sqrt{c+dx}}-\frac{2\int-\frac{ad\left(8bc^2-5ad^2\right)+bc\left(32bc^2-29ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2}\right)}{5d^2(bc^2-ad^2)}-\frac{2(a-bx^2)^{3/2}(dx(11bc^2-5ad^2)+2c(4bc^2-ad^2))}{15d^2(c+dx)^{5/2}(bc^2-ad^2)}\right)}{7d}-\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{10b\left(-\frac{2b\left(\frac{2\int\frac{ad\left(8bc^2-5ad^2\right)+bc\left(32bc^2-29ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2}+\frac{2\sqrt{a-bx^2}\left(dx\left(8bc^2-5ad^2\right)+c\left(32bc^2-29ad^2\right)\right)}{3d^2\sqrt{c+dx}}\right)}{5d^2(bc^2-ad^2)}-\frac{2(a-bx^2)^{3/2}(dx(11bc^2-5ad^2)+2c(4bc^2-ad^2))}{15d^2(c+dx)^{5/2}(bc^2-ad^2)}\right)}{7d}-\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$\Big\downarrow$ 600

$$-\frac{10b\left(-\frac{2b\left(\frac{2\left(\frac{bc\left(32bc^2-29ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}\left(dx\left(8bc^2-5ad^2\right)+c\left(32bc^2-29ad^2\right)\right)}{3d^2\sqrt{c+dx}}\right)}{5d^2(bc^2-ad^2)}\cdots\right)}{7d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$\Big\downarrow$ 509

$$-\frac{10b\left(-\frac{2b\left(\frac{2\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-29ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2}+\frac{2\sqrt{a-bx^2}\left(dx\left(8bc^2-5ad^2\right)+c\left(32bc^2-29a\cdots\right.\right.}{3d^2\sqrt{c+dx}}\cdots}{5d^2(bc^2-ad^2)}\cdots\right)}{7d}\cdots$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$\Big\downarrow$ 508

$$-\frac{10b\left(-\frac{2b\left(\frac{2\left(-\frac{\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-29ad^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-bx\cdots}}{\cdots}\right)}{5d^2(bc^2-ad^2)}\cdots\right)}{7d\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$\downarrow$ 327

$$-\frac{10b\left(-\frac{2b\left(\frac{2\left(-\frac{(32bc^2-5ad^2)(bc^2-ad^2)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(32bc^2-29ad^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}+\frac{2\sqrt{a-b}\cdots}{\cdots}\right)}{5d^2(bc^2-ad^2)}\cdots\right)}{7d}\cdots$$

$$\frac{2(a-bx^2)^{5/2}}{7d(c+dx)^{7/2}}$$

$\downarrow$ 512

$$-\frac{10b\left(-\frac{2b\left(\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}(32bc^2-5ad^2)(bc^2-ad^2)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(32bc^2-29ad^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{3d^2}\cdots\right)}{5d^2(bc^2-ad^2)}\cdots\right)}{7d}\cdots$$

$$\frac{2(a-bx^2)^{5/2}}{7d(c+dx)^{7/2}}$$

$\downarrow$ 511

$$-\frac{10b\left(-\frac{2b\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-\cdots\right.}{d\sqrt{a-b\cdots}}\cdots\right)}{3d^2}\cdots\right)}{5d^2\left(bc^2-ad^2\right)}\cdots\right)}{7\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

$\downarrow$ 321

$$-\frac{10b\left(-\frac{2b\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(32bc^2-29ad^2\right)E\cdots}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}}\cdots\right)}{3d^2}\cdots\right)}{5d^2(bc^2-ad^2)}\cdots\right)}{7d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{7d(c+dx)^{7/2}}$$

input
```
Int[(a - b*x^2)^(5/2)/(c + d*x)^(9/2),x]
```

output
```
(-2*(a - b*x^2)^(5/2))/(7*d*(c + d*x)^(7/2)) - (10*b*((-2*(2*c*(4*b*c^2 -
a*d^2) + d*(11*b*c^2 - 5*a*d^2)*x)*(a - b*x^2)^(3/2))/(15*d^2*(b*c^2 - a*d
^2)*(c + d*x)^(5/2)) - (2*b*((2*(c*(32*b*c^2 - 29*a*d^2) + d*(8*b*c^2 - 5*
a*d^2)*x)*Sqrt[a - b*x^2])/(3*d^2*Sqrt[c + d*x]) + (2*((-2*Sqrt[a]*Sqrt[b]
*c*(32*b*c^2 - 29*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)]
)/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) +
(2*Sqrt[a]*(32*b*c^2 - 5*a*d^2)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sq
rt[c + d*x]*Sqrt[a - b*x^2])))/(3*d^2)))/(5*d^2*(b*c^2 - a*d^2))))/(7*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(425) = 850$.

Time = 14.48 (sec) , antiderivative size = 939, normalized size of antiderivative = 1.85

method	result
elliptic	$\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\left(a^2d^4-2b\,c^2d^2a+b^2c^4\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7d^9\left(x+\frac{c}{d}\right)^4}-\frac{12\left(a\,d^2-b\,c^2\right)bc\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7d^8\left(x+\frac{c}{d}\right)^3}+\frac{8\left(4a\,d^2-13b\,c^2\right)b\sqrt{}}{21d^7}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/7*(a^2*d^4-2
*a*b*c^2*d^2+b^2*c^4)/d^9*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^4-12/
7/d^8*(a*d^2-b*c^2)*b*c*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3+8/21*
(4*a*d^2-13*b*c^2)*b/d^7*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+4/21
*(-b*d*x^2+a*d)/(a*d^2-b*c^2)/d^6*b^2*c*(67*a*d^2-79*b*c^2)/((x+c/d)*(-b*d
*x^2+a*d))^(1/2)+2/3*b^2/d^5*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(b^2*(3*
a*d^2-10*b*c^2)/d^6-4/21*b^2*(4*a*d^2-13*b*c^2)/d^6+2/21*b^3*c^2/d^6*(67*a
*d^2-79*b*c^2)/(a*d^2-b*c^2)-1/3*b^2/d^4*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+
1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(14/3*b^3*c/d^5+2/21*b^3
*c/d^5*(67*a*d^2-79*b*c^2)/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)
^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*
(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*
b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.54

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/63*(8*(32*b^3*c^8 - 53*a*b^2*c^6*d^2 + 15*a^2*b*c^4*d^4 + (32*b^3*c^4*d^
4 - 53*a*b^2*c^2*d^6 + 15*a^2*b*d^8)*x^4 + 4*(32*b^3*c^5*d^3 - 53*a*b^2*c^
3*d^5 + 15*a^2*b*c*d^7)*x^3 + 6*(32*b^3*c^6*d^2 - 53*a*b^2*c^4*d^4 + 15*a^
2*b*c^2*d^6)*x^2 + 4*(32*b^3*c^7*d - 53*a*b^2*c^5*d^3 + 15*a^2*b*c^3*d^5)*
x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*
c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 24*(32*b^3*c^7*d - 29*a*b^2
*c^5*d^3 + (32*b^3*c^3*d^5 - 29*a*b^2*c*d^7)*x^4 + 4*(32*b^3*c^4*d^4 - 29*
a*b^2*c^2*d^6)*x^3 + 6*(32*b^3*c^5*d^3 - 29*a*b^2*c^3*d^5)*x^2 + 4*(32*b^3
*c^6*d^2 - 29*a*b^2*c^4*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*
a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3
*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x
+ c)/d)) + 3*(128*b^3*c^6*d^2 - 100*a*b^2*c^4*d^4 - 7*a^2*b*c^2*d^6 + 3*a^
3*d^8 + 7*(b^3*c^2*d^6 - a*b^2*d^8)*x^4 + 6*(31*b^3*c^3*d^5 - 27*a*b^2*c*d
^7)*x^3 + 8*(58*b^3*c^4*d^4 - 47*a*b^2*c^2*d^6 - 2*a^2*b*d^8)*x^2 + 2*(208
*b^3*c^5*d^3 - 165*a*b^2*c^3*d^5 - 7*a^2*b*c*d^7)*x)*sqrt(-b*x^2 + a)*sqrt
(d*x + c))/(b*c^6*d^7 - a*c^4*d^9 + (b*c^2*d^11 - a*d^13)*x^4 + 4*(b*c^3*d
^10 - a*c*d^12)*x^3 + 6*(b*c^4*d^9 - a*c^2*d^11)*x^2 + 4*(b*c^5*d^8 - a*c^
3*d^10)*x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(a-bx^2)^{\frac{5}{2}}}{(c+dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(5/2)/(d*x+c)**(9/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)/(c + d*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \int \frac{(a-b\,x^2)^{5/2}}{(c+d\,x)^{9/2}}\,dx$$

input
```
int((a - b*x^2)^(5/2)/(c + d*x)^(9/2),x)
```

output
```
int((a - b*x^2)^(5/2)/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(9/2),x)
```

output
```
(2*( - 9*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 + 240*sqrt(c + d*x)*sqrt
(a - b*x**2)*a*b*c**2*d + 10*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x +
 8*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d**3*x**2 - 160*sqrt(c + d*x)*sqrt(a
 - b*x**2)*b**2*c**3*x - 80*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x**
2 - 10*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 + sqrt(c + d*x)*sqr
t(a - b*x**2)*b**2*d**3*x**4 + 30*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2
)/(a*c**5 + 5*a*c**4*d*x + 10*a*c**3*d**2*x**2 + 10*a*c**2*d**3*x**3 + 5*a
*c*d**4*x**4 + a*d**5*x**5 - b*c**5*x**2 - 5*b*c**4*d*x**3 - 10*b*c**3*d**
2*x**4 - 10*b*c**2*d**3*x**5 - 5*b*c*d**4*x**6 - b*d**5*x**7),x)*a**2*b*c*
*4*d**4 + 120*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**5 + 5*a*c**4
*d*x + 10*a*c**3*d**2*x**2 + 10*a*c**2*d**3*x**3 + 5*a*c*d**4*x**4 + a*d**
5*x**5 - b*c**5*x**2 - 5*b*c**4*d*x**3 - 10*b*c**3*d**2*x**4 - 10*b*c**2*d
**3*x**5 - 5*b*c*d**4*x**6 - b*d**5*x**7),x)*a**2*b*c**3*d**5*x + 180*int(
(sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**5 + 5*a*c**4*d*x + 10*a*c**3*d
**2*x**2 + 10*a*c**2*d**3*x**3 + 5*a*c*d**4*x**4 + a*d**5*x**5 - b*c**5*x*
*2 - 5*b*c**4*d*x**3 - 10*b*c**3*d**2*x**4 - 10*b*c**2*d**3*x**5 - 5*b*c*d
**4*x**6 - b*d**5*x**7),x)*a**2*b*c**2*d**6*x**2 + 120*int((sqrt(c + d*x)*
sqrt(a - b*x**2)*x**2)/(a*c**5 + 5*a*c**4*d*x + 10*a*c**3*d**2*x**2 + 10*a
*c**2*d**3*x**3 + 5*a*c*d**4*x**4 + a*d**5*x**5 - b*c**5*x**2 - 5*b*c**4*d
*x**3 - 10*b*c**3*d**2*x**4 - 10*b*c**2*d**3*x**5 - 5*b*c*d**4*x**6 - b...
```

3.319 $\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 608

$$\begin{aligned}
\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx &= -\frac{16b^2(32b^2c^4-57abc^2d^2+21a^2d^4)\sqrt{a-bx^2}}{63d^5(bc^2-ad^2)^2\sqrt{c+dx}} \\
&+ \frac{8b^2(c(32bc^2-33ad^2)+3d(8bc^2-7ad^2)x)\sqrt{a-bx^2}}{63d^5(bc^2-ad^2)(c+dx)^{3/2}} \\
&- \frac{20bc(a-bx^2)^{3/2}}{63d^3(c+dx)^{7/2}} + \frac{4b(13bc^2-7ad^2)(a-bx^2)^{3/2}}{63d^3(bc^2-ad^2)(c+dx)^{5/2}} - \frac{2(a-bx^2)^{5/2}}{9d(c+dx)^{9/2}} \\
&+ \frac{16\sqrt{a}b^{5/2}(32b^2c^4-57abc^2d^2+21a^2d^4)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{63d^6(bc^2-ad^2)^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \\
&- \frac{16\sqrt{a}b^{5/2}c(32bc^2-33ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{63d^6(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-16/63*b^2*(21*a^2*d^4-57*a*b*c^2*d^2+32*b^2*c^4)*(-b*x^2+a)^(1/2)/d^5/(-a
*d^2+b*c^2)^2/(d*x+c)^(1/2)+8/63*b^2*(c*(-33*a*d^2+32*b*c^2)+3*d*(-7*a*d^2
+8*b*c^2)*x)*(-b*x^2+a)^(1/2)/d^5/(-a*d^2+b*c^2)/(d*x+c)^(3/2)-20/63*b*c*(
-b*x^2+a)^(3/2)/d^3/(d*x+c)^(7/2)+4/63*b*(-7*a*d^2+13*b*c^2)*(-b*x^2+a)^(3
/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(5/2)-2/9*(-b*x^2+a)^(5/2)/d/(d*x+c)^(9/2)+
16/63*a^(1/2)*b^(5/2)*(21*a^2*d^4-57*a*b*c^2*d^2+32*b^2*c^4)*(d*x+c)^(1/2)
*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/
2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(-a*d^2+b*c^2)^2/(b^(1/2)*
(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-16/63*a^(1/2)*b^(5/2
)*c*(-33*a*d^2+32*b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-
b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^6/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(
-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.23 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.11

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(8b^2(32b^2c^4-57abc^2d^2+21a^2d^4)-b^2(193b^2c^4-330abc^2d^2+105a^2d^4)-\right.}{}$$

input
```
Integrate[(a - b*x^2)^(5/2)/(c + d*x)^(11/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(8*b^2*(32*b^2*c^4 - 57*a*b*c^2*d^2 + 21*a^2*d^4) - b^2
*(193*b^2*c^4 - 330*a*b*c^2*d^2 + 105*a^2*d^4) - (7*(b*c^2 - a*d^2)^4)/(c
+ d*x)^4 + (38*b*c*(b*c^2 - a*d^2)^3)/(c + d*x)^3 - (4*b*(22*b*c^2 - 7*a*d
^2)*(b*c^2 - a*d^2)^2)/(c + d*x)^2 + (2*b^2*c*(61*b*c^2 - 57*a*d^2)*(b*c^2
 - a*d^2))/(c + d*x) + ((8*I)*b^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(32*b^2*c
^4 - 57*a*b*c^2*d^2 + 21*a^2*d^4)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)
]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE
[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqr
t[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*(-a + b*x^2)) - ((8*I)*Sqrt[a]*b^(5
/2)*(32*b^2*c^4 - 8*Sqrt[a]*b^(3/2)*c^3*d - 57*a*b*c^2*d^2 + 12*a^(3/2)*Sq
rt[b]*c*d^3 + 21*a^2*d^4)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-
(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSi
nh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/
(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)))
)/(63*d^5*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {492, 589, 25, 680, 25, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/2}}{(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{492}$$

$$-\frac{10b\int \frac{x\left(a-bx^2\right)^{3/2}}{(c+dx)^{9/2}}dx}{9d}-\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\Big\downarrow \text{589}$$

$$-\frac{10b\left(\frac{6b\int -\frac{\left(5acd+\left(8bc^2-7ad^2\right)x\right)\sqrt{a-bx^2}}{(c+dx)^{5/2}}dx}{35d^2\left(bc^2-ad^2\right)}-\frac{2\left(a-bx^2\right)^{3/2}\left(dx\left(13bc^2-7ad^2\right)+2c\left(4bc^2-ad^2\right)\right)}{35d^2(c+dx)^{7/2}\left(bc^2-ad^2\right)}\right)}{9d}-\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{10b\left(-\frac{6b\int \frac{\left(5acd+\left(8bc^2-7ad^2\right)x\right)\sqrt{a-bx^2}}{(c+dx)^{5/2}}dx}{35d^2(bc^2-ad^2)}-\frac{2(a-bx^2)^{3/2}(dx(13bc^2-7ad^2)+2c(4bc^2-ad^2))}{35d^2(c+dx)^{7/2}(bc^2-ad^2)}\right)}{9d}-\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\Big\downarrow \; 680$$

$$-\frac{10b\left(-\frac{6b\left(\frac{2\int -\frac{b\left(4acd\left(2bc^2-3ad^2\right)+\left(32b^2c^4-57abd^2c^2+21a^2d^4\right)x\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}\left(dx\left(21a^2d^4-69abc^2d^2+40b^2c^4\right)+c\left(9a^2d^4-49abc^2d^2+32b^2c^4\right)\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}\right)}{35d^2(bc^2-ad^2)}\right.}{9d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2\int \frac{b\left(4acd\left(2bc^2-3ad^2\right)+\left(32b^2c^4-57abd^2c^2+21a^2d^4\right)x\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}\left(dx\left(21a^2d^4-69abc^2d^2+40b^2c^4\right)+c\left(9a^2d^4-49abc^2d^2+32b^2c^4\right)\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}\right)}{35d^2(bc^2-ad^2)}\right.}{9d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{4acd\left(2bc^2-3ad^2\right)+\left(32b^2c^4-57abd^2c^2+21a^2d^4\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3d^2\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}\left(dx\left(21a^2d^4-69abc^2d^2+40b^2c^4\right)+c\left(9a^2d^4-49abc^2d^2+32b^2c^4\right)\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}\right)}{35d^2(bc^2-ad^2)}\right.}{9d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$\downarrow$ 600

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\left(\frac{\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}\left(dx\left(21a^2d^4-69abc^2d^2+\cdots\right.\right.}{3d^2(c+dx\cdots}\right)}{35d^2(bc^2-ad^2)}\cdots}{9d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$\downarrow$ 509

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{3d^2\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}\left(dx\left(21a^2d^4-6\cdots\right.\right.}{\cdots}\right)}{35d^2(bc^2-ad^2)}\cdots}{9d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$\downarrow$ 508

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}d}{d}\right.}{3d^2\left(bc^2-ad^2\right)}\right.}{35d^2(bc^2-ad^2)}\right.}{9d}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$\downarrow$ 327

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\left(-\frac{c\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+\cdots}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{3d^2\left(bc^2-ad^2\right)}\cdots\right)}{35d^2(bc^2-ad^2)}\cdots\right)}{9d\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\Big\downarrow \; 512$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc^2d^2+32b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\cdots\right)\cdots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{3d^2\left(bc^2-ad^2\right)}\cdots\right)}{35d^2(bc^2-ad^2)}\cdots\right)}{9\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\downarrow 511$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d \cdots}{\cdots}\cdots\right)}{3d^2\left(bc^2-ad^2\right)}\cdots\right)}{35d^2(bc^2- \cdots}\cdots\right)}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

$$\downarrow 321$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(32bc^2-33ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2d^4-57abc\ldots\right.}{\sqrt{b}d\sqrt{a\ldots}}\right)}{3d^2\left(bc^2-ad^2\right)}\ldots\right)}{35d^2(bc^2-ad^2\ldots}\ldots\right)}{\ldots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}}{9d(c+dx)^{9/2}}$$

input
```
Int[(a - b*x^2)^(5/2)/(c + d*x)^(11/2),x]
```

output
```
(-2*(a - b*x^2)^(5/2))/(9*d*(c + d*x)^(9/2)) - (10*b*((-2*(2*c*(4*b*c^2 -
a*d^2) + d*(13*b*c^2 - 7*a*d^2)*x)*(a - b*x^2)^(3/2))/(35*d^2*(b*c^2 - a*d
^2)*(c + d*x)^(7/2)) - (6*b*((-2*(c*(32*b^2*c^4 - 49*a*b*c^2*d^2 + 9*a^2*d
^4) + d*(40*b^2*c^4 - 69*a*b*c^2*d^2 + 21*a^2*d^4)*x)*Sqrt[a - b*x^2])/(3*
d^2*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) - (2*b*((-2*Sqrt[a]*(32*b^2*c^4 - 57*
a*b*c^2*d^2 + 21*a^2*d^4)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcS
in[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)
])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b
*x^2]) + (2*Sqrt[a]*c*(32*b*c^2 - 33*a*d^2)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*
(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[S
qrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(
Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*d^2*(b*c^2 - a*d^2))))/(35*d
^2*(b*c^2 - a*d^2))))/(9*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

Maple [A] (verified)

Time = 9.16 (sec) , antiderivative size = 1008, normalized size of antiderivative = 1.66

method	result	size
elliptic	Expression too large to display	1008
default	Expression too large to display	7831

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/9*(a^2*d^4-2
*a*b*c^2*d^2+b^2*c^4)/d^10*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^5-76
/63/d^9*(a*d^2-b*c^2)*b*c*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^4+8/6
3*(7*a*d^2-22*b*c^2)*b/d^8*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3+4/
63*b^2*c*(57*a*d^2-61*b*c^2)/d^7/(a*d^2-b*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c
)^(1/2)/(x+c/d)^2-2/63*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2/d^6*b^2*(105*a^2*d^4
-330*a*b*c^2*d^2+193*b^2*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(5*b^3*c/d^
6-2/63*b^3*c*(57*a*d^2-61*b*c^2)/d^6/(a*d^2-b*c^2)-1/63*b^3*c/d^6*(105*a^2
*d^4-330*a*b*c^2*d^2+193*b^2*c^4)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*(
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),
((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-b^3/d^5-1/63*b^
3/d^5*(105*a^2*d^4-330*a*b*c^2*d^2+193*b^2*c^4)/(a*d^2-b*c^2)^2)*(c/d-1/b*
(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(
1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(520) = 1040$.

Time = 0.18 (sec) , antiderivative size = 1118, normalized size of antiderivative = 1.84

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
-2/189*(8*(32*b^4*c^10 - 81*a*b^3*c^8*d^2 + 57*a^2*b^2*c^6*d^4 + (32*b^4*c
^5*d^5 - 81*a*b^3*c^3*d^7 + 57*a^2*b^2*c*d^9)*x^5 + 5*(32*b^4*c^6*d^4 - 81
*a*b^3*c^4*d^6 + 57*a^2*b^2*c^2*d^8)*x^4 + 10*(32*b^4*c^7*d^3 - 81*a*b^3*c
^5*d^5 + 57*a^2*b^2*c^3*d^7)*x^3 + 10*(32*b^4*c^8*d^2 - 81*a*b^3*c^6*d^4 +
 57*a^2*b^2*c^4*d^6)*x^2 + 5*(32*b^4*c^9*d - 81*a*b^3*c^7*d^3 + 57*a^2*b^2
*c^5*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
 -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 24*(32*b^4*c^9*d
- 57*a*b^3*c^7*d^3 + 21*a^2*b^2*c^5*d^5 + (32*b^4*c^4*d^6 - 57*a*b^3*c^2*d
^8 + 21*a^2*b^2*d^10)*x^5 + 5*(32*b^4*c^5*d^5 - 57*a*b^3*c^3*d^7 + 21*a^2*
b^2*c*d^9)*x^4 + 10*(32*b^4*c^6*d^4 - 57*a*b^3*c^4*d^6 + 21*a^2*b^2*c^2*d^
8)*x^3 + 10*(32*b^4*c^7*d^3 - 57*a*b^3*c^5*d^5 + 21*a^2*b^2*c^3*d^7)*x^2 +
 5*(32*b^4*c^8*d^2 - 57*a*b^3*c^6*d^4 + 21*a^2*b^2*c^4*d^6)*x)*sqrt(-b*d)*
weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 -
9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(128*b^4*c^8*d^2 - 212*a*b^3*c
^6*d^4 + 63*a^2*b^2*c^4*d^6 - 18*a^3*b*c^2*d^8 + 7*a^4*d^10 + (193*b^4*c^4
*d^6 - 330*a*b^3*c^2*d^8 + 105*a^2*b^2*d^10)*x^4 + 2*(325*b^4*c^5*d^5 - 54
2*a*b^3*c^3*d^7 + 153*a^2*b^2*c*d^9)*x^3 + 4*(220*b^4*c^6*d^4 - 369*a*b^3*
c^4*d^6 + 108*a^2*b^2*c^2*d^8 - 7*a^3*b*d^10)*x^2 + 2*(272*b^4*c^7*d^3 - 4
53*a*b^3*c^5*d^5 + 126*a^2*b^2*c^3*d^7 - 9*a^3*b*c*d^9)*x)*sqrt(-b*x^2 ...
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx = \int \frac{(a-bx^2)^{\frac{5}{2}}}{(c+dx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(5/2)/(d*x+c)**(11/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)/(c + d*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(11/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)/(d*x + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx=\int \frac{(a-b\,x^2)^{5/2}}{(c+d\,x)^{11/2}}\,dx$$

input
```
int((a - b*x^2)^(5/2)/(c + d*x)^(11/2),x)
```

output
```
int((a - b*x^2)^(5/2)/(c + d*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/2}}{(c+dx)^{11/2}}\,dx=\text{too large to display}$$

input
```
int((-b*x^2+a)^(5/2)/(d*x+c)^(11/2),x)
```

output

```
(2*( - 55*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 - 176*sqrt(c + d*x)*sqr
t(a - b*x**2)*a*b*c**2*d + 18*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x
+ 4*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d**3*x**2 + 96*sqrt(c + d*x)*sqrt(a
 - b*x**2)*b**2*c**3*x + 80*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x**
2 + 30*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 + 3*sqrt(c + d*x)*s
qrt(a - b*x**2)*b**2*d**3*x**4 + 198*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x
**2)/(a*c**6 + 6*a*c**5*d*x + 15*a*c**4*d**2*x**2 + 20*a*c**3*d**3*x**3 +
15*a*c**2*d**4*x**4 + 6*a*c*d**5*x**5 + a*d**6*x**6 - b*c**6*x**2 - 6*b*c*
*5*d*x**3 - 15*b*c**4*d**2*x**4 - 20*b*c**3*d**3*x**5 - 15*b*c**2*d**4*x**
6 - 6*b*c*d**5*x**7 - b*d**6*x**8),x)*a**2*b*c**5*d**4 + 990*int((sqrt(c +
 d*x)*sqrt(a - b*x**2)*x**2)/(a*c**6 + 6*a*c**5*d*x + 15*a*c**4*d**2*x**2
+ 20*a*c**3*d**3*x**3 + 15*a*c**2*d**4*x**4 + 6*a*c*d**5*x**5 + a*d**6*x**
6 - b*c**6*x**2 - 6*b*c**5*d*x**3 - 15*b*c**4*d**2*x**4 - 20*b*c**3*d**3*x
**5 - 15*b*c**2*d**4*x**6 - 6*b*c*d**5*x**7 - b*d**6*x**8),x)*a**2*b*c**4*
d**5*x + 1980*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**6 + 6*a*c**5
*d*x + 15*a*c**4*d**2*x**2 + 20*a*c**3*d**3*x**3 + 15*a*c**2*d**4*x**4 + 6
*a*c*d**5*x**5 + a*d**6*x**6 - b*c**6*x**2 - 6*b*c**5*d*x**3 - 15*b*c**4*d
**2*x**4 - 20*b*c**3*d**3*x**5 - 15*b*c**2*d**4*x**6 - 6*b*c*d**5*x**7 - b
*d**6*x**8),x)*a**2*b*c**3*d**6*x**2 + 1980*int((sqrt(c + d*x)*sqrt(a - b*
x**2)*x**2)/(a*c**6 + 6*a*c**5*d*x + 15*a*c**4*d**2*x**2 + 20*a*c**3*d*...
```

3.320 $\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 411

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx = -\frac{2d(71bc^2+25ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}{105b^2} - \frac{24cd(c+dx)^{3/2}\sqrt{a-bx^2}}{35b} - \frac{2d(c+dx)^{5/2}\sqrt{a-bx^2}}{7b} - \frac{32\sqrt{a}c(11bc^2+13ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{105b^{3/2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(71b^2c^4-46abc^2d^2-25a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{105b^{5/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/105*d*(25*a*d^2+71*b*c^2)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2-24/35*c*d*
(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b-2/7*d*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b-32
/105*a^(1/2)*c*(13*a*d^2+11*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*Ellipti
cE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a
^(1/2)*d))^(1/2))/b^(3/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-
b*x^2+a)^(1/2)+2/105*a^(1/2)*(-25*a^2*d^4-46*a*b*c^2*d^2+71*b^2*c^4)*(b^(1
/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(
1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d
))^(1/2))/b^(5/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.58 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-((c+dx)\left(25ad^3+bd(122c^2+66cdx+15d^2x^2)\right)) - \frac{16cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(11bc^2+\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(7/2)/Sqrt[a - b*x^2],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-((c + d*x)*(25*a*d^3 + b*d*(122*c^2 + 66*c*d*x + 15*d
^2*x^2))) - (16*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(11*b*c^2 + 13*a*d^2)
*(a - b*x^2) + (16*I)*Sqrt[b]*c*(11*b^(3/2)*c^3 - 11*Sqrt[a]*b*c^2*d + 13*
a*Sqrt[b]*c*d^2 - 13*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)
]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE
[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqr
t[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*(105*b^2*c^4 - 176*Sqrt[a]*b^(3/2)*c^
3*d + 254*a*b*c^2*d^2 - 208*a^(3/2)*Sqrt[b]*c*d^3 + 25*a^2*d^4)*Sqrt[(d*(S
qrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*
x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sq
rt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(105*b^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {497, 27, 687, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{2\int -\frac{(c+dx)^{3/2}(7bc^2+12bdxc+5ad^2)}{2\sqrt{a-bx^2}}dx}{7b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^{3/2}(7bc^2+12bdxc+5ad^2)}{\sqrt{a-bx^2}}dx}{7b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$$\Big\downarrow \text{687}$$

$$\frac{-\frac{2\int -\frac{b\sqrt{c+dx}\left(c\left(35bc^2+61ad^2\right)+d\left(71bc^2+25ad^2\right)x\right)}{2\sqrt{a-bx^2}}dx}{5b}-\frac{24}{5}cd\sqrt{a-bx^2}(c+dx)^{3/2}}{7b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\int \frac{\sqrt{c+dx}(c(35bc^2+61ad^2)+d(71bc^2+25ad^2)x)}{\sqrt{a-bx^2}}dx-\frac{24}{5}cd\sqrt{a-bx^2}(c+dx)^{3/2}}{7b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{1}{5}\left(-\frac{2\int -\frac{105b^2c^4+254abd^2c^2+16bd\left(11bc^2+13ad^2\right)xc+25a^2d^4}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}(25ad^2+71bc^2)}{3b}\right)-\frac{24}{5}cd\sqrt{a-bx^2}(c+dx)^{3/2}}{7b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\downarrow$ 27

$$\frac{\frac{1}{5}\left(\frac{\int \frac{105b^2c^4+254abd^2c^2+16bd\left(11bc^2+13ad^2\right)xc+25a^2d^4}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}\left(25ad^2+71bc^2\right)}{3b}\right)-\frac{24}{5}cd\sqrt{a-bx^2}(c+dx)^{3/2}}{7b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\downarrow$ 600

$$\frac{\frac{1}{5}\left(\frac{16bc\left(13ad^2+11bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-\left(bc^2-ad^2\right)\left(25ad^2+71bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}\left(25ad^2+71bc^2\right)}{3b}\right)-\frac{24}{5}cd\sqrt{a-}}{7b}$$
$$\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\downarrow$ 509

$$\frac{\frac{1}{5}\left(\frac{\frac{16bc\sqrt{1-\frac{bx^2}{a}}\left(13ad^2+11bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\left(bc^2-ad^2\right)\left(25ad^2+71bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}\left(25ad^2+71bc^2\right)}{3b}\right)-\frac{24}{5}cd}{7b}$$
$$\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\downarrow$ 508

$$\frac{\frac{1}{5}\left(\frac{-\left(\left(bc^2-ad^2\right)\left(25ad^2+71bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{32\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(13ad^2+11bc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c-}}{}\right.}{7b}$$
$$\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\big\downarrow$ 327

$$\frac{\frac{1}{5}\left(\frac{-\left(bc^2-ad^2\right)\left(25ad^2+71bc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{32\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(13ad^2+11bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+\cdots}}{3\cdots}\right)}{7b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\big\downarrow$ 512

$$\frac{\frac{1}{5}\left(\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(25ad^2+71bc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{32\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(13ad^2+11bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2d\sqrt{a-b\cdots}}{\cdots}\right)}{7b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\big\downarrow$ 511

$$\frac{\frac{1}{5}\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(25ad^2+71bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{32\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(13ad^2+11bc^2\right)E\left(\text{arc}\cdots\right.}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}\cdots\right)}{7b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

$\big\downarrow$ 321

$$\frac{\frac{1}{5}\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(25ad^2+71bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{32\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(13ad^2+11bc^2\right)E\left(\arcsin\left(\sqrt{1\right.\right.}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}\right.}{7b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}$$

input
```
Int[(c + d*x)^(7/2)/Sqrt[a - b*x^2],x]
```

output
```
(-2*d*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/(7*b) + ((-24*c*d*(c + d*x)^(3/2)*S
qrt[a - b*x^2])/5 + ((-2*d*(71*b*c^2 + 25*a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*
x^2])/(3*b) + ((-32*Sqrt[a]*Sqrt[b]*c*(11*b*c^2 + 13*a*d^2)*Sqrt[c + d*x]*
Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]
], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
+ Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(71*b*c^2 + 25
*a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)
/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[
b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b))/5)/(7
*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(335) = 670$.

Time = 2.97 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.79

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2d^3x^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7b}-\frac{44c\,d^2x\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{35b}-\frac{2\left(\frac{122c^2d^2}{35}+\frac{5a\,d^4}{7b}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}\right.}{}$
risch	$-\frac{2d(15b\,x^2d^2+66bcdx+25a\,d^2+122b\,c^2)\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{105b^2}+\frac{\left(16dc\left(13a\,d^2+11b\,c^2\right)\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{a}}{b}\right.}{\sqrt{ab}}}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/7*d^3/b*x^2*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-44/35*c*d^2/b*x*(-b*d*x^3-b*c*x^2+a*d*x
+a*c)^(1/2)-2/3*(122/35*c^2*d^2+5/7*a/b*d^4)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)+2*(c^4+44/35*a/b*c^2*d^2+1/3*a/b*(122/35*c^2*d^2+5/7*a/b*d^4))*(
c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(
1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(
1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/
b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
))+2*(4*d*c^3+86/35*a/b*c*d^3-2/3*c/d*(122/35*c^2*d^2+5/7*a/b*d^4))*(c/d-1
/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2))
)^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*Ellipti
cE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{2\left((139\,b^2c^4+554\,abc^2d^2+75\,a^2d^4)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\frac{3\,dx+c}{3\,d}\right)-48\,(1$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((139*b^2*c^4 + 554*a*b*c^2*d^2 + 75*a^2*d^4)*sqrt(-b*d)*weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d) - 48*(11*b^2*c^3*d + 13*a*b*c*d^3)*sqrt(-b*d)*weierst
rassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
 weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d
^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(15*b^2*d^4*x^2 + 66*b^2*c*d^3*x + 12
2*b^2*c^2*d^2 + 25*a*b*d^4)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(c+dx)^{\frac{7}{2}}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x+c)**(7/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x)**(7/2)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(dx+c)^{\frac{7}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(7/2)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(dx+c)^{\frac{7}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(7/2)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(c+d\,x)^{7/2}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((c + d*x)^(7/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^(7/2)/(a - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{7/2}}{\sqrt{a-bx^2}}\,dx = \frac{-86\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,a\,d^3 - 140\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,b\,c^2d - 44\sqrt{dx+c}\,\sqrt{-b\,x^2+}}{}$$

input
```
int((d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 86*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d**3 - 140*sqrt(c + d*x)*sqrt(a -
b*x**2)*b*c**2*d - 44*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*d**2*x - 10*sqrt(
c + d*x)*sqrt(a - b*x**2)*b*d**3*x**2 - 104*int((sqrt(c + d*x)*sqrt(a - b*
x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*d**4 - 88*int((sqrt
(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b*
*2*c**2*d**2 + 43*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*
x**2 - b*d*x**3),x)*a**2*d**4 + 114*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(
a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**2*d**2 + 35*int((sqrt(c + d*x
)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**2*c**4)/(35*
b**2)
```

3.321 $\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 354

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx = -\frac{16cd\sqrt{c+dx}\sqrt{a-bx^2}}{15b}-\frac{2d(c+dx)^{3/2}\sqrt{a-bx^2}}{5b} - \frac{2\sqrt{a}(23bc^2+9ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15b^{3/2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{16\sqrt{a}c(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15b^{3/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-16/15*c*d*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b-2/5*d*(d*x+c)^(3/2)*(-b*x^2+a)
^(1/2)/b-2/15*a^(1/2)*(9*a*d^2+23*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*E
llipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/
2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1
/2)/(-b*x^2+a)^(1/2)+16/15*a^(1/2)*c*(-a*d^2+b*c^2)*(b^(1/2)*(d*x+c)/(b^(1
/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/
2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)
/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.46 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{2d(c+dx)(11c+3dx)}{b}+\frac{2\left(d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\,(23bc^2+9ad^2)\,(a-bx^2)+i\sqrt{b}\left(23b^{3/2}c^3-23\sqrt{a}bc^2d+9a\sqrt{b}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^(5/2)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[a - b*x^2]*((-2*d*(c + d*x)*(11*c + 3*d*x))/b + (2*(d^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]*(23*b*c^2 + 9*a*d^2)*(a - b*x^2) + I*Sqrt[b]*(23*b^(3/2
)*c^3 - 23*Sqrt[a]*b*c^2*d + 9*a*Sqrt[b]*c*d^2 - 9*a^(3/2)*d^3)*Sqrt[(d*(S
qrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*
x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sq
rt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[b]
*(15*b^(3/2)*c^3 - 23*Sqrt[a]*b*c^2*d + 17*a*Sqrt[b]*c*d^2 - 9*a^(3/2)*d^3
)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
))/(b^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(15*Sqrt[c + d*x]
)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {497, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx$$

$$\Big\downarrow \; 497$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(5bc^2+8bdxc+3ad^2\right)}{2\sqrt{a-bx^2}}dx}{5b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(5bc^2+8bdxc+3ad^2\right)}{\sqrt{a-bx^2}}dx}{5b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 687$$

$$\frac{-\frac{2\int -\frac{b\left(c\left(15bc^2+17ad^2\right)+d\left(23bc^2+9ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{16}{3}cd\sqrt{a-bx^2}\sqrt{c+dx}}{5b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}\int \frac{c\left(15bc^2+17ad^2\right)+d\left(23bc^2+9ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{16}{3}cd\sqrt{a-bx^2}\sqrt{c+dx}}{5b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{1}{3}\left(\left(9ad^2+23bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-8c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{16}{3}cd\sqrt{a-bx^2}\sqrt{c+dx}}{5b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 509$$

$$\frac{\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(9ad^2+23bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-8c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{16}{3}cd\sqrt{a-bx^2}\sqrt{c+dx}}{5b}-\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 508$$

$$\frac{\frac{1}{3}\left(-8c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{16}{3}cd\sqrt{a-bx^2}\sqrt{\cdots}}{5b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{1}{3}\left(-8c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{16}{3}cd\sqrt{a-bx\cdots}}{5b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{1}{3}\left(-\frac{8c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{16}{3}cd\sqrt{a-\cdots}}{5b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{1}{3}\left(\frac{16\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)E\left(\arcsin\left(\sqrt{1\cdots}\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right.}{5b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

$\downarrow$ 321

$$\frac{\frac{1}{3}\left(\frac{16\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(9ad^2+23bc^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}}{\sqrt{}}}}{\sqrt{2}}\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5b}$$

$$\frac{2d\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}$$

input
```
Int[(c + d*x)^(5/2)/Sqrt[a - b*x^2],x]
```

output
```
(-2*d*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b) + ((-16*c*d*Sqrt[c + d*x]*Sqr
t[a - b*x^2])/3 + ((-2*Sqrt[a]*(23*b*c^2 + 9*a*d^2)*Sqrt[c + d*x]*Sqrt[1 -
 (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)
/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
+ Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (16*Sqrt[a]*c*(b*c^2 - a*d^2)*Sqrt[(Sqrt[
b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)]
)/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/3)/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(284) = 568$.

Time = 2.03 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.78

method	result
elliptic	$\sqrt{(dx+c)(-bx^2+a)}\left(-\frac{2d^2x\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5b}-\frac{22cd\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{15b}+\frac{2\left(c^3+\frac{17ca\,d^2}{15b}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{a \ldots}}{b}}}}{\sqrt{-bd\,x^3-\ldots}}\ldots\right.$ …
risch	$-\frac{2(3dx+11c)\sqrt{dx+c}\,d\sqrt{-b\,x^2+a}}{15b}+\frac{\left(\frac{d\left(9a\,d^2+23b\,c^2\right)\sqrt{ab}\,\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\left(\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\mathrm{EllipticE}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.\right.\ldots}{b\sqrt{-bd\,x^3-bc\,x^2\ldots}}\ldots\right.}{\ldots}$ …
default	Expression too large to display

input
```
int((d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/5*d^2/b*x*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-22/15*c*d/b*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)+2*(c^3+17/15/b*c*a*d^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b
)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b
*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(
1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2
))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(23/15*c^2*d+3/5*a/b*d^3)*(c/d-1/b*(a*
b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2
)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^
(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{2\left(2\left(11\,bc^3+21\,acd^2\right)\sqrt{-bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\frac{3\,dx+c}{3\,d}\right)-3\left(23\,bc^2d+9\,ad^3\right)\right.}{}$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(2*(11*b*c^3 + 21*a*c*d^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2
 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)
 - 3*(23*b*c^2*d + 9*a*d^3)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^
2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*
c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)
/d)) + 3*(3*b*d^3*x + 11*b*c*d^2)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(c+dx)^{\frac{5}{2}}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x)**(5/2)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(5/2)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx=\int \frac{(c+d\,x)^{5/2}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((c + d*x)^(5/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^(5/2)/(a - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a-bx^2}}\,dx=\frac{-6\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,a\,d^3-30\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,b\,c^2d-4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bc}{}$$

input
```
int((d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d**3 - 30*sqrt(c + d*x)*sqrt(a - b*
x**2)*b*c**2*d - 4*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*d**2*x - 9*int((sqrt
(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*
b*d**4 - 23*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x
**2 - b*d*x**3),x)*b**2*c**2*d**2 + 3*int((sqrt(c + d*x)*sqrt(a - b*x**2))
/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*d**4 + 19*int((sqrt(c + d*x)*
sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**2*d**2 + 1
0*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3)
,x)*b**2*c**4)/(10*b**2*c)
```

3.322 $\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 311

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx = -\frac{2d\sqrt{c+dx}\sqrt{a-bx^2}}{3b} - \frac{8\sqrt{a}c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3b^{3/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*d*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b-8/3*a^(1/2)*c*(d*x+c)^(1/2)*(1-b*x
^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a
^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+2/3*a^(1/2)*(-a*d^2+b*c^2)*(b^(1/2)*(d*x
+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2)
)/b^(3/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.32 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx = \frac{2d\sqrt{a-bx^2}\left(-5c-dx+\frac{4i\sqrt{b}c\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}}{\sqrt{c+dx}}\right) \cdots\right.}{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(-a+bx^2\right)} \cdots\right)}{\cdots}$$

input
```
Integrate[(c + d*x)^(3/2)/Sqrt[a - b*x^2],x]
```

output
```
(2*d*Sqrt[a - b*x^2]*(-5*c - d*x + ((4*I)*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d
)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
)/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*(3*b*c^2 - 4*Sqrt
[a]*Sqrt[b]*c*d + a*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(
((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSin
h[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(
Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))
))/(3*b*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {497, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx$$

$$\Big\downarrow \; 497$$

$$-\frac{2\int -\frac{3bc^2+4bdxc+ad^2}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3bc^2+4bdxc+ad^2}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}$$

$$\Big\downarrow \ 600$$

$$\frac{4bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}$$

$$\Big\downarrow \ 509$$

$$\frac{\frac{4bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}$$

$$\Big\downarrow \ 508$$

$$\frac{-\left(\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}$$

$$\Big\downarrow \ 327$$

$$\frac{-\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}$$

$$\Big\downarrow \ 512$$

$$\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}$$

$$\Big\downarrow \text{511}$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}}{\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-}{\frac{2d\sqrt{a-bx^2}\sqrt{c+dx}}{3b}}$$

input
```
Int[(c + d*x)^(3/2)/Sqrt[a - b*x^2],x]
```

output
```
(-2*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b) + ((-8*Sqrt[a]*Sqrt[b]*c*Sqrt[c
 + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/(Sq
rt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*Sqrt[(
Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[
ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
+ d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(247) = 494$.

Time = 1.15 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.87

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2d\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3b}+\frac{2\left(c^2+\frac{d^2a}{3b}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\right.}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
risch	$-\frac{2d\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{3b}+\frac{\left(\frac{a\,d^2\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}}{2},\sqrt{-\frac{2\sqrt{ab}}{b\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)}}\right)}{b\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}+\right.}{}$
default	$\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\left(4\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\,\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\,\sqrt{\frac{\left(bx+\sqrt{ab}\right)d}{d\sqrt{ab}-bc}}\,\operatorname{EllipticE}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)abc\,d^2-4\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\right.}{}$

input
```
int((d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/3*d/b*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(c^2+1/3*d^2/b*a)*(c/d-1/b*(a*b)^(1/2))*((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1
/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((
-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+8/3*c*d*(c/d-1/b*(a*b
)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)
/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+
c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx = \frac{2\left(12\sqrt{-bd}bcd\text{weierstrassZeta}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3}{3\,bd}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(12*sqrt(-b*d)*b*c*d*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8
/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)
/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*sqrt(
-b*x^2 + a)*sqrt(d*x + c)*b*d^2 - (5*b*c^2 + 3*a*d^2)*sqrt(-b*d)*weierstra
ssPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3
), 1/3*(3*d*x + c)/d))/(b^2*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x)**(3/2)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx=\int \frac{{\left(c+d\,x\right)}^{3/2}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((c + d*x)^(3/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^(3/2)/(a - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a-bx^2}}\,dx=\frac{-2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,d-2\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)b\,d^2+\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)}{b}$$

input
```
int((d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a - b*x**2)*d - 2*int((sqrt(c + d*x)*sqrt(a - b*x
**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b*d**2 + int((sqrt(c + d
*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*d**2 + int(
(sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b*
c**2)/b
```

3.323 $\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx = -\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$

output
```
-2*a^(1/2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2
)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.39 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx$$
$$= \frac{2i\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\sqrt{a}+\sqrt{b}x\right)}{-\sqrt{b}c+\sqrt{a}d}}\sqrt{c+dx}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\right)|\frac{\sqrt{b}c+\sqrt{a}d}{\sqrt{b}c-\sqrt{a}d}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\right.\right.\right.}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{d\left(-\sqrt{a}+\sqrt{b}x\right)}}\sqrt{a-bx^2}}$$

input
```
Integrate[Sqrt[c + d*x]/Sqrt[a - b*x^2],x]
```

output
```
((2*I)*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a] + Sqrt[b]*x))/(-(Sqrt[b]*c
) + Sqrt[a]*d)]*Sqrt[c + d*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[b]*(c + d*
x))/(Sqrt[b]*c + Sqrt[a]*d))]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[
a]*d)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[
a]*d))]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(Sqrt[b]*d*Sq
rt[(Sqrt[b]*(c + d*x))/(d*(-Sqrt[a] + Sqrt[b]*x))]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {509, 508, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{509}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{508}$$

$$-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

input
```
Int[Sqrt[c + d*x]/Sqrt[a - b*x^2],x]
```

output
```
(-2*Sqrt[a]*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(105) = 210$.

Time = 0.37 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.87

method	result
default	$\frac{2\sqrt{dx+c}\,\sqrt{-b\,x^{2}+a}\,\left(bc-d\sqrt{ab}\right)\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\,\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\,\sqrt{\frac{\left(bx+\sqrt{ab}\right)d}{d\sqrt{ab}-bc}}\,\left(\sqrt{ab}\ \mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)d+c\,\mathrm{Ellip}\right.}{d(-bd\,x^{3}-bc\,x^{2}}$
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^{2}+a)}\left(\frac{2c\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\right)}{\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}+\frac{2d\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}}{}\right.}{\sqrt{dx+c}\,\sqrt{-b\,x^{2}}}$

input
```
int((d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)*(b*c-d*(a*b)^(1/2))*(-(d*x+c)*b/(d*(a*b)^
(1/2)-b*c))^(1/2)*((-b*x+(a*b)^(1/2))*d/(d*(a*b)^(1/2)+b*c))^(1/2)*((b*x+(
a*b)^(1/2))*d/(d*(a*b)^(1/2)-b*c))^(1/2)*((a*b)^(1/2)*EllipticF((-(d*x+c)*
b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(d*(a*b)^(1/2)-b*c)/(d*(a*b)^(1/2)+b*c))^(1
/2))*d+c*EllipticF((-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(d*(a*b)^(1/2)
-b*c)/(d*(a*b)^(1/2)+b*c))^(1/2))*b-(a*b)^(1/2)*EllipticE((-(d*x+c)*b/(d*(
a*b)^(1/2)-b*c))^(1/2),(-(d*(a*b)^(1/2)-b*c)/(d*(a*b)^(1/2)+b*c))^(1/2))*d
-EllipticE((-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(d*(a*b)^(1/2)-b*c)/(d
*(a*b)^(1/2)+b*c))^(1/2))*b*c)/d/(-b*d*x^3-b*c*x^2+a*d*x+a*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{2\left(2\sqrt{-bd}c\,\mathrm{weierstrassPInverse}\left(\frac{4\left(bc^{2}+3\,ad^{2}\right)}{3\,bd^{2}},-\frac{8\left(bc^{3}-9\,acd^{2}\right)}{27\,bd^{3}},\frac{3\,dx+c}{3\,d}\right)-3\sqrt{-bd}d\,\mathrm{weierstrassZeta}\left(\frac{4\left(bc^{2}+3\right.}{3\,ba}\right.\right.}{3\,bd}$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*sqrt(-b*d)*c*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8
/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) - 3*sqrt(-b*d)*d*weier
strassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3
), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c
*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)))/(b*d)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{dx+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{c+d\,x}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((c + d*x)^(1/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^(1/2)/(a - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b\,x^2+a}dx$$

input
```
int((d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a - b*x**2),x)
```

3.324 $\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2\sqrt{a}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*a^(1/2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)
*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(
1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$= \frac{2i\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}}{\sqrt{c+dx}}\right),\frac{\sqrt{b}c+\sqrt{a}d}{\sqrt{b}c-\sqrt{a}d}\right)}{d\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\sqrt{a-bx^2}}$$

input
```
Integrate[1/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
((2*I)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[
b] - d*x)/(c + d*x))]*(c + d*x)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])
/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{512}$$
$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}$$
$$\Big\downarrow \text{511}$$

$$-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}$$

$$\Big\downarrow \text{321}$$

$$-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}$$

input
```
Int[1/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*Sqrt[a]*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((S
qrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{2\left(bc-d\sqrt{ab}\right)\operatorname{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)\sqrt{\frac{\left(bx+\sqrt{ab}\right)d}{d\sqrt{ab}-bc}}\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\sqrt{-b\,x^{2}+a}\,\sqrt{dx+c}}{bd\left(-bd\,x^{3}-bc\,x^{2}+adx+ac\right)}$	194
elliptic	$\frac{2\sqrt{(dx+c)\left(-b\,x^{2}+a\right)}\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\right)}{\sqrt{dx+c}\,\sqrt{-b\,x^{2}+a}\,\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}$	237

input
```
int(1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*c-d*(a*b)^(1/2))*EllipticF((-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(
d*(a*b)^(1/2)-b*c)/(d*(a*b)^(1/2)+b*c))^(1/2))*((b*x+(a*b)^(1/2))*d/(d*(a*
b)^(1/2)-b*c))^(1/2)*((-b*x+(a*b)^(1/2))*d/(d*(a*b)^(1/2)+b*c))^(1/2)*(-(d
*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2)*(-b*x^2+a)^(1/2)*(d*x+c)^(1/2)/b/d/(-b*
d*x^3-b*c*x^2+a*d*x+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\frac{3\,dx+c}{3\,d}\right)}{bd}$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*
c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)/(b*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^3-bc\,x^2+adx+ac}dx$$

input
```
int(1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x
)
```

3.325 $\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 185

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2d\sqrt{a-bx^2}}{(bc^2-ad^2)\sqrt{c+dx}} - \frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$

output
```
2*d*(-b*x^2+a)^(1/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)-2*a^(1/2)*b^(1/2)*(d*x+c
)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2
),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/(-a*d^2+b*c^2)/(b^(1/2)
*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.18 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.69

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2\left(\frac{d^2(a-bx^2)}{bc^2-ad^2} - i\sqrt{\frac{d(\sqrt{a}-\sqrt{b}x)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{\frac{d(\sqrt{a}+\sqrt{b}x)}{-\sqrt{b}c+\sqrt{a}d}}\sqrt{-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{b}(c+}{\sqrt{b}c+}}\right.\right.\right.\right.}{d\sqrt{c+dx}\sqrt{a-}}$$

input
```
Integrate[1/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*((d^2*(a - b*x^2))/(b*c^2 - a*d^2) - I*Sqrt[(d*(Sqrt[a] - Sqrt[b]*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[(d*(Sqrt[a] + Sqrt[b]*x))/(-(Sqrt[b]*c) + Sqr
t[a]*d)]*Sqrt[-((Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d))]*(EllipticE[I
*ArcSinh[Sqrt[-((Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d))]], (Sqrt[b]*c
 + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[
b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d))]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]
*c - Sqrt[a]*d)])))/(d*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {498, 27, 509, 508, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{498}$$

$$\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}\,(bc^2-ad^2)} - \frac{2b\int -\frac{\sqrt{c+dx}}{2\sqrt{a-bx^2}}dx}{bc^2-ad^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \textbf{509}$$

$$\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\,(bc^2-ad^2)}+\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \textbf{508}$$

$$\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\,(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \textbf{327}$$

$$\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\,(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

input
```
Int[1/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*d*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) - (2*Sqrt[a]*Sqrt[b]
*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/S
qrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/((b*c^2 - a*d^2)*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(154) = 308$.

Time = 2.70 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.34

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2(-bd\,x^2+ad)}{(a\,d^2-b\,c^2)\sqrt{(x+\frac{c}{d})(-bd\,x^2+ad)}}-\frac{2bc\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\right.}{(a\,d^2-b\,c^2)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
default	$-\frac{2\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\sqrt{\frac{(-bx+\sqrt{ab})d}{d\sqrt{ab}+bc}}\sqrt{\frac{(bx+\sqrt{ab})d}{d\sqrt{ab}-bc}}\,\mathrm{EllipticE}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)a\,d^2-\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\sqrt{\frac{(-bx+\sqrt{ab})d}{d\sqrt{ab}+bc}}\right.}{}$

input
```
int(1/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2*(-b*d*x^2+a*
d)/(a*d^2-b*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2*b*c/(a*d^2-b*c^2)*(c/d-1
/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2))
)^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-2*
b*d/(a*d^2-b*c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2)
)/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-
1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((
x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b
)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.29

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2\left(3\sqrt{-bx^2+a}\sqrt{dx+c}\,cd^2-2\left(cdx+c^2\right)\sqrt{-bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(bc^2+3a\right.}{3\,bd^2}\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(-b*x^2 + a)*sqrt(d*x + c)*d^2 - 2*(c*d*x + c^2)*sqrt(-b*d)*wei
erstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/
(b*d^3), 1/3*(3*d*x + c)/d) + 3*(d^2*x + c*d)*sqrt(-b*d)*weierstrassZeta(4
/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstra
ssPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3
), 1/3*(3*d*x + c)/d)))/(b*c^3*d - a*c*d^3 + (b*c^2*d^2 - a*d^4)*x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{1}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{1}{\sqrt{a-b\,x^2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b\,d^2x^4-2bcd\,x^3+a\,d^2x^2-b\,c^2x^2+2acdx+a\,c^2}dx$$

input
```
int(1/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b
*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)
```

3.326 $\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 380

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2d\sqrt{a-bx^2}}{3(bc^2-ad^2)(c+dx)^{3/2}} + \frac{8bcd\sqrt{a-bx^2}}{3(bc^2-ad^2)^2\sqrt{c+dx}} - \frac{8\sqrt{a}b^{3/2}c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3(bc^2-ad^2)^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3*d*(-b*x^2+a)^(1/2)/(-a*d^2+b*c^2)/(d*x+c)^(3/2)+8/3*b*c*d*(-b*x^2+a)^(
1/2)/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-8/3*a^(1/2)*b^(3/2)*c*(d*x+c)^(1/2)*(1
-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*
(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/(-a*d^2+b*c^2)^2/(b^(1/2)*(d*x+c)
/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+2/3*a^(1/2)*b^(1/2)*(b^(1/2
)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-
b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))
^(1/2))/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.38 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.21

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2d\sqrt{a-bx^2}\left(\frac{bc^2}{c+dx}-\frac{ad^2}{c+dx}+\frac{4ib^{3/2}c\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\mathrm{arc}\right.}{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(-a+bx^2)}\right.}{}$$

input
```
Integrate[1/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*d*Sqrt[a - b*x^2]*((b*c^2)/(c + d*x) - (a*d^2)/(c + d*x) + ((4*I)*b^(3/
2)*c*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqr
t[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*Ar
cSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*
d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x
^2)) - (I*b*(3*b*c^2 - 4*Sqrt[a]*Sqrt[b]*c*d + a*d^2)*Sqrt[(d*(Sqrt[a]/Sqr
t[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x
]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {498, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \; 498$$

$$\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{2b\int -\frac{3c-dx}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{3c-dx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 688$$

$$\frac{b\left(\frac{2\int \frac{3bc^2+4bdxc+ad^2}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{8cd\sqrt{a-bx^2}}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{\int \frac{3bc^2+4bdxc+ad^2}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{8cd\sqrt{a-bx^2}}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 600$$

$$\frac{b\left(\frac{4bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{8cd\sqrt{a-bx^2}}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 509$$

$$\frac{b\left(\frac{\frac{4bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{8cd\sqrt{a-bx^2}}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 508$$

$$\frac{b\left(\frac{-\left(\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\frac{8cd\sqrt{a-bx^2}}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{b\left(\frac{-\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\frac{8cd\sqrt{a-bx^2}}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{512}$$

$$\frac{b\left(\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\frac{8cd\sqrt{a-bx^2}}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{511}$$

$$\frac{b\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\right.}{3\left(bc^2-ad^2\right)}$$

$$\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 321

$$\frac{b\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\frac{8cd\sqrt{}}{\sqrt{c+dx}}\right.}{3\left(bc^2-ad^2\right)}$$

$$\frac{2d\sqrt{a-bx^2}}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*d*Sqrt[a - b*x^2])/(3*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) + (b*((8*c*d*Sqr
t[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((-8*Sqrt[a]*Sqrt[b]*c*Sqr
t[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[
a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/
(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*Sqr
t[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[
a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2)))/(3*(b
*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(310) = 620$.

Time = 5.38 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3\left(a\,d^2-b\,c^2\right)d\left(x+\frac{c}{d}\right)^2}+\frac{8\left(-bd\,x^2+ad\right)bc}{3\left(a\,d^2-b\,c^2\right)^2\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}+\frac{2\left(\frac{b}{3a\,d^2-3b\,c^2}+\frac{4b^2c^2}{3\left(a\,d^2-b\,c^2\right)^2}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{}}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/3/(a*d^2-b*c
^2)/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+8/3*(-b*d*x^2+a*d)/(a*d
^2-b*c^2)^2*b*c/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(1/3/(a*d^2-b*c^2)*b+4/3*
b^2*c^2/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b
)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*
EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))+8/3*b^2*c*d/(a*d^2-b*c^2)^2*(c/d-1/b*(a*b)^(1
/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2
)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.01

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = -\frac{2\left(\left(5\,bc^4+3\,ac^2d^2+\left(5\,bc^2d^2+3\,ad^4\right)x^2+2\left(5\,bc^3d+3\,acd^3\right)x\right)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\left(bc^2+3\,ad^2\right)}{3\,bd^2},-\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*((5*b*c^4 + 3*a*c^2*d^2 + (5*b*c^2*d^2 + 3*a*d^4)*x^2 + 2*(5*b*c^3*d
+ 3*a*c*d^3)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^
2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) - 12*(b*c*d^3*x^
2 + 2*b*c^2*d^2*x + b*c^3*d)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d
^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b
*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c
)/d)) - 3*(4*b*c*d^3*x + 5*b*c^2*d^2 - a*d^4)*sqrt(-b*x^2 + a)*sqrt(d*x +
c))/(b^2*c^6*d - 2*a*b*c^4*d^3 + a^2*c^2*d^5 + (b^2*c^4*d^3 - 2*a*b*c^2*d^
5 + a^2*d^7)*x^2 + 2*(b^2*c^5*d^2 - 2*a*b*c^3*d^4 + a^2*c*d^6)*x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{a-b\,x^2}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b\,d^3x^5-3bc\,d^2x^4+a\,d^3x^3-3b\,c^2d\,x^3+3ac\,d^2x^2-b\,c^3x^2+3a\,c^2dx+a}$$

input
```
int(1/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x
**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d*
*3*x**5),x)
```

3.327 $\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 447

$$\begin{aligned}\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx &= \frac{2d\sqrt{a-bx^2}}{5(bc^2-ad^2)(c+dx)^{5/2}} \\ &+ \frac{16bcd\sqrt{a-bx^2}}{15(bc^2-ad^2)^2(c+dx)^{3/2}} + \frac{2bd(23bc^2+9ad^2)\sqrt{a-bx^2}}{15(bc^2-ad^2)^3\sqrt{c+dx}} \\ &- \frac{2\sqrt{a}b^{3/2}(23bc^2+9ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15(bc^2-ad^2)^3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \\ &+ \frac{16\sqrt{a}b^{3/2}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}\end{aligned}$$

output
```
2/5*d*(-b*x^2+a)^(1/2)/(-a*d^2+b*c^2)/(d*x+c)^(5/2)+16/15*b*c*d*(-b*x^2+a)
^(1/2)/(-a*d^2+b*c^2)^2/(d*x+c)^(3/2)+2/15*b*d*(9*a*d^2+23*b*c^2)*(-b*x^2+
a)^(1/2)/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)-2/15*a^(1/2)*b^(3/2)*(9*a*d^2+23*b
*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/(-a*d^2+b*c
^2)^3/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+16/15
*a^(1/2)*b^(3/2)*c*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/
a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2
)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.40 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.12

$$\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \frac{2d\sqrt{a-bx^2}\left(\frac{3(bc^2-ad^2)^2}{(c+dx)^2}+\frac{8bc(bc^2-ad^2)}{c+dx}-\frac{ib^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(23bc^2+9ad^2)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}\cdots}{c\cdots}}\cdots}{d^2\cdots}\cdots\right)}{\cdots}$$

input
```
Integrate[1/((c + d*x)^(7/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*d*Sqrt[a - b*x^2]*((3*(b*c^2 - a*d^2)^2)/(c + d*x)^2 + (8*b*c*(b*c^2 -
a*d^2))/(c + d*x) - (I*b^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(23*b*c^2 + 9*a*
d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
 - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*
d)])/(d^2*(-a + b*x^2)) - (I*b^(3/2)*(15*b^(3/2)*c^3 - 23*Sqrt[a]*b*c^2*d
+ 17*a*Sqrt[b]*c*d^2 - 9*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c +
 Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*
(-a + b*x^2))))/(15*(b*c^2 - a*d^2)^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {498, 27, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 498}$$

$$\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}-\frac{2b\int -\frac{5c-3dx}{2(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{5\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{5c-3dx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{5\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 688}$$

$$\frac{b\left(\frac{2\int \frac{3\left(5bc^2+3ad^2\right)-8bcdx}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{16cd\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\left(\frac{\int \frac{3\left(5bc^2+3ad^2\right)-8bcdx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{16cd\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 688}$$

$$\frac{b\left(\frac{\frac{2\int \frac{b\left(c\left(15bc^2+17ad^2\right)+d\left(23bc^2+9ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\left(9ad^2+23bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}+\frac{16cd\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\frac{b\int\frac{c\left(15bc^2+17ad^2\right)+d\left(23bc^2+9ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\left(9ad^2+23bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}+\frac{16cd\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{600}$$

$$\frac{b\left(\frac{\frac{b\left(\left(9ad^2+23bc^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-8c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\left(9ad^2+23bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}+\frac{16cd\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{509}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(9ad^2+23bc^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-8c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\left(9ad^2+23bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}+\frac{16cd\sqrt{a-bx^2}}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{5\left(bc^2-ad^2\right)}+\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{508}$$

$$\frac{b\left(\frac{\frac{b\left(-8c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\left(9ad^2+23bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}+\frac{16cd\ldots}{3(c+dx)^{\ldots}}\right)}{5\left(bc^2-ad^2\right)}$$

$$\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 327

$$\frac{b\left(\frac{\frac{b\left(-8c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\left(9ad^2+23bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}+\frac{16\ldots}{3(c+dx\ldots}\right)}{5\left(bc^2-ad^2\right)}$$

$$\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 512

$$b\left(\frac{\frac{b\left(-\frac{8c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\left(9ad^2+23bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}\right.$$

$$\overline{\qquad 5\left(bc^2-ad^2\right)}$$

$$\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{511}$$

$$b\left(\frac{\frac{b\left(\frac{16\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.$$

$$\overline{\qquad 5\left(bc^2-ad^2\right)}$$

$$\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{321}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{16\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2+23bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}\right.}{5\left(bc^2-ad^2\right)}$$

$$\frac{2d\sqrt{a-bx^2}}{5(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(7/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*d*Sqrt[a - b*x^2])/(5*(b*c^2 - a*d^2)*(c + d*x)^(5/2)) + (b*((16*c*d*Sq
rt[a - b*x^2])/(3*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) + ((2*d*(23*b*c^2 + 9*a
*d^2)*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + (b*((-2*Sqrt[a]*(
23*b*c^2 + 9*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqr
t[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sq
rt[b]*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) +
 (16*Sqrt[a]*c*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[
a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/
Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a
- b*x^2])))/(b*c^2 - a*d^2))/(3*(b*c^2 - a*d^2))))/(5*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(371) = 742$.

Time = 7.75 (sec) , antiderivative size = 790, normalized size of antiderivative = 1.77

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5\left(a\,d^2-b\,c^2\right)d^2\left(x+\frac{c}{d}\right)^3}+\frac{16bc\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{15\left(a\,d^2-b\,c^2\right)^2 d\left(x+\frac{c}{d}\right)^2}-\frac{2\left(-bd\,x^2+ad\right)b\left(9a\,d^2+23b\,c^2\right)}{15\left(a\,d^2-b\,c^2\right)^3\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}+\frac{2\left(-\frac{8b^2}{15\left(a\,d^2-\cdots\right.}\right.}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/5/(a*d^2-b*c
^2)/d^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3+16/15*b/(a*d^2-b*c^2)
^2*c/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2-2/15*(-b*d*x^2+a*d)/(a
*d^2-b*c^2)^3*b*(9*a*d^2+23*b*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(-8/15
*b^2*c/(a*d^2-b*c^2)^2-1/15*b^2*c*(9*a*d^2+23*b*c^2)/(a*d^2-b*c^2)^3)*(c/d
-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2
))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2
)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(
a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-
2/15*b^2*d*(9*a*d^2+23*b*c^2)/(a*d^2-b*c^2)^3*(c/d-1/b*(a*b)^(1/2))*((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(
a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+
1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx =$$

$$-\frac{2\left(2\left(11\,b^2c^6+21\,abc^4d^2+\left(11\,b^2c^3d^3+21\,abcd^5\right)x^3+3\left(11\,b^2c^4d^2+21\,abc^2d^4\right)x^2+3\left(11\,b^2c^5d+21\,abc^3\right.\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(2*(11*b^2*c^6 + 21*a*b*c^4*d^2 + (11*b^2*c^3*d^3 + 21*a*b*c*d^5)*x^
3 + 3*(11*b^2*c^4*d^2 + 21*a*b*c^2*d^4)*x^2 + 3*(11*b^2*c^5*d + 21*a*b*c^3
*d^3)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/
27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) - 3*(23*b^2*c^5*d + 9*a
*b*c^3*d^3 + (23*b^2*c^2*d^4 + 9*a*b*d^6)*x^3 + 3*(23*b^2*c^3*d^3 + 9*a*b*
c*d^5)*x^2 + 3*(23*b^2*c^4*d^2 + 9*a*b*c^2*d^4)*x)*sqrt(-b*d)*weierstrassZ
eta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weie
rstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), 1/3*(3*d*x + c)/d)) - 3*(34*b^2*c^4*d^2 - 5*a*b*c^2*d^4 + 3*a^2*d^
6 + (23*b^2*c^2*d^4 + 9*a*b*d^6)*x^2 + 2*(27*b^2*c^3*d^3 + 5*a*b*c*d^5)*x)
*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^3*c^9*d - 3*a*b^2*c^7*d^3 + 3*a^2*b*c^
5*d^5 - a^3*c^3*d^7 + (b^3*c^6*d^4 - 3*a*b^2*c^4*d^6 + 3*a^2*b*c^2*d^8 - a
^3*d^10)*x^3 + 3*(b^3*c^7*d^3 - 3*a*b^2*c^5*d^5 + 3*a^2*b*c^3*d^7 - a^3*c*
d^9)*x^2 + 3*(b^3*c^8*d^2 - 3*a*b^2*c^6*d^4 + 3*a^2*b*c^4*d^6 - a^3*c^2*d^
8)*x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\,(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)**(7/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*(c + d*x)**(7/2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*(d*x + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \int \frac{1}{\sqrt{a-b\,x^2}\,(c+d\,x)^{7/2}}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(7/2)),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c + d*x)^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b\,d^4x^6-4bc\,d^3x^5+a\,d^4x^4-6b\,c^2d^2x^4+4ac\,d^3x^3-4b\,c^3d\,x^3+6a\,c^2d^2x}$$

input
```
int(1/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**
2*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6
*b*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)
```

3.328 $\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 421

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx = \frac{(ad+bcx)(c+dx)^{5/2}}{ab\sqrt{a-bx^2}} + \frac{d(3bc^2+5ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}{3ab^2} + \frac{cd(c+dx)^{3/2}\sqrt{a-bx^2}}{ab} + \frac{c(3bc^2+29ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}b^{3/2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{(3b^2c^4+2abc^2d^2-5a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}b^{5/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(b*c*x+a*d)*(d*x+c)^(5/2)/a/b/(-b*x^2+a)^(1/2)+1/3*d*(5*a*d^2+3*b*c^2)*(d*
x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/b^2+c*d*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/a/b+1
/3*c*(29*a*d^2+3*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b
^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^
(1/2))/a^(1/2)/b^(3/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x
^2+a)^(1/2)-1/3*(-5*a^2*d^4+2*a*b*c^2*d^2+3*b^2*c^4)*(b^(1/2)*(d*x+c)/(b^(
1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2
)/b^(5/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.20 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx = \frac{cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(3bc^2+29ad^2)\,(a-bx^2)+d\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(c+dx)\,(5a^2d^3+3b^2c^3x+ab}{}$$

input
```
Integrate[(c + d*x)^(7/2)/(a - b*x^2)^(3/2),x]
```

output
```
(c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*b*c^2 + 29*a*d^2)*(a - b*x^2) + d
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)*(5*a^2*d^3 + 3*b^2*c^3*x + a*b*d
*(9*c^2 + 9*c*d*x - 2*d^2*x^2)) + I*Sqrt[b]*c*(3*b^(3/2)*c^3 - 3*Sqrt[a]*b
*c^2*d + 29*a*Sqrt[b]*c*d^2 - 29*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x
))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/
2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqr
t[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(3*b^(3/2)*c^3
- 27*Sqrt[a]*b*c^2*d + 29*a*Sqrt[b]*c*d^2 - 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a
]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*
(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
+ d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(3*a*b^2*d*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {495, 27, 687, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 495}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{\int \frac{5d(ad+bcx)(c+dx)^{3/2}}{2\sqrt{a-bx^2}}dx}{ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\int \frac{(ad+bcx)(c+dx)^{3/2}}{\sqrt{a-bx^2}}dx}{2ab}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(-\frac{2\int -\frac{b\sqrt{c+dx}\left(8acd+\left(3bc^2+5ad^2\right)x\right)}{2\sqrt{a-bx^2}}dx}{5b}-\frac{2}{5}c\sqrt{a-bx^2}(c+dx)^{3/2}\right)}{2ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\int \frac{\sqrt{c+dx}(8acd+(3bc^2+5ad^2)x)}{\sqrt{a-bx^2}}dx-\frac{2}{5}c\sqrt{a-bx^2}(c+dx)^{3/2}\right)}{2ab}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-$$
$$\frac{5d\left(\frac{1}{5}\left(-\frac{2\int -\frac{ad\left(27bc^2+5ad^2\right)+bc\left(3bc^2+29ad^2\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(5ad^2+3bc^2)}{3b}\right)-\frac{2}{5}c\sqrt{a-bx^2}(c+dx)^{3/2}\right)}{2ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\left(\frac{\int \frac{ad\left(27bc^2+5ad^2\right)+bc\left(3bc^2+29ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(5ad^2+3bc^2\right)}{3b}\right)-\frac{2}{5}c\sqrt{a-bx^2}(c+dx)^{3/2}\right)}{2ab}$$

$$\Big\downarrow \; 600$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\left(\frac{\frac{bc\left(29ad^2+3bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(5ad^2+3bc^2\right)}{3b}\right)-\frac{2}{5}c\sqrt{a-bx^2}(c+\right.}{2ab}$$

$$\Big\downarrow \; 509$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\left(\frac{\frac{bc\sqrt{1-\frac{bx^2}{a}}\left(29ad^2+3bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(5ad^2+3bc^2\right)}{3b}\right)-\frac{2}{5}c\sqrt{a-\right.}{2ab}$$

$$\Big\downarrow \; 508$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\left(\frac{-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(\right.}{3b}\right.\right.}{2ab}$$

$$\Big\downarrow \; 327$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\left(\frac{-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\cdots}{3b}\cdots\right.\right.}{2ab}$$

$$\Big\downarrow \; 512$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\left(\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{2\sqrt{a-b\cdots}}{\cdots}\cdots\right.\right.}{2ab}$$

$$\Big\downarrow \; 511$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{5d\left(\frac{1}{5}\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+\cdots}{\sqrt{a}d+\cdots}}}}{3b}\cdots\right.\right.}{2ab}$$

$$\Big\downarrow \; 321$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-$$

$$\frac{5d\left(\frac{1}{5}\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\sqrt{\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}\right.\right.}{2ab}$$

input
```
Int[(c + d*x)^(7/2)/(a - b*x^2)^(3/2),x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(5/2))/(a*b*Sqrt[a - b*x^2]) - (5*d*((-2*c*(c + d
*x)^(3/2)*Sqrt[a - b*x^2])/5 + ((-2*(3*b*c^2 + 5*a*d^2)*Sqrt[c + d*x]*Sqrt
[a - b*x^2])/(3*b) + ((-2*Sqrt[a]*Sqrt[b]*c*(3*b*c^2 + 29*a*d^2)*Sqrt[c +
d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sq
rt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqr
t[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*b*c^
2 + 5*a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((
Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b)
)/5))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(349) = 698$.

Time = 5.48 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(dx+c)(-bx^2+a)}\left(-\frac{2(-bdx-bc)\left(\frac{(3a d^2+b c^2)cx}{2b^2 a}+\frac{d(a d^2+3b c^2)}{2b^3}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2d^3\sqrt{-bd x^3-bc x^2+adx+ac}}{3b^2}+\frac{2\left(-\frac{d^2(a d^2+6b c^2)}{b^2}+\frac{a^2d^4+6b c}{b}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^(7/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*(3*a*d^2+b*c^2)/b^2*c/a*x+1/2*d*(a*d^2+3*b*c^2)/b^3)/((x^2-a/b)*(-b*
d*x-b*c))^(1/2)+2/3*d^3/b^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-d^2*(a*
d^2+6*b*c^2)/b^2+1/b^2*(a^2*d^4+6*a*b*c^2*d^2+b^2*c^4)/a-1/2/b^2*d^2*(a*d^
2+3*b*c^2)-1/b*c^2*(3*a*d^2+b*c^2)/a-1/3*a/b^2*d^4)*(c/d-1/b*(a*b)^(1/2))*
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-10/3*c*d^3/b-1
/2*(3*a*d^2+b*c^2)*c*d/a/b)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(
a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx = \frac{(3\,ab^2c^4 - 52\,a^2bc^2d^2 - 15\,a^3d^4 - (3\,b^3c^4 - 52\,ab^2c^2d^2 - 15\,a^2bd^4)x^2)\sqrt{-bd}\text{weierstras}}{}$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/9*((3*a*b^2*c^4 - 52*a^2*b*c^2*d^2 - 15*a^3*d^4 - (3*b^3*c^4 - 52*a*b^2*
c^2*d^2 - 15*a^2*b*d^4)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3
*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3
*(3*a*b^2*c^3*d + 29*a^2*b*c*d^3 - (3*b^3*c^3*d + 29*a*b^2*c*d^3)*x^2)*sqr
t(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*
c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(
b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(2*a*b^2*d^4*x^2 - 9*a
*b^2*c^2*d^2 - 5*a^2*b*d^4 - 3*(b^3*c^3*d + 3*a*b^2*c*d^3)*x)*sqrt(-b*x^2
+ a)*sqrt(d*x + c))/(a*b^4*d*x^2 - a^2*b^3*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^{\frac{7}{2}}}{(a-bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(7/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**(7/2)/(a - b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{7}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(7/2)/(-b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{7}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(7/2)/(-b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx=\int \frac{(c+d\,x)^{7/2}}{(a-b\,x^2)^{3/2}}\,dx$$

input
```
int((c + d*x)^(7/2)/(a - b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^(7/2)/(a - b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(7/2)/(-b*x^2+a)^(3/2),x)
```

output
```
(34*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d**3 + 12*sqrt(c + d*x)*sqrt(a - b*x*
*2)*b*c**2*d - 20*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*d**2*x - 2*sqrt(c + d
*x)*sqrt(a - b*x**2)*b*d**3*x**2 - 17*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*
a*c**2 - sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqr
t(a - b*x**2)*b*d**2*x**4),x)*a**3*c*d**4 + 14*int(sqrt(c + d*x)/(sqrt(a -
 b*x**2)*a*c**2 - sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x
**2 + sqrt(a - b*x**2)*b*d**2*x**4),x)*a**2*b*c**3*d**2 + 17*int(sqrt(c +
d*x)/(sqrt(a - b*x**2)*a*c**2 - sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*
x**2)*b*c**2*x**2 + sqrt(a - b*x**2)*b*d**2*x**4),x)*a**2*b*c*d**4*x**2 +
3*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a*c**2 - sqrt(a - b*x**2)*a*d**2*x**
2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqrt(a - b*x**2)*b*d**2*x**4),x)*a*b**2
*c**5 - 14*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a*c**2 - sqrt(a - b*x**2)*a
*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqrt(a - b*x**2)*b*d**2*x**4),
x)*a*b**2*c**3*d**2*x**2 - 3*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a*c**2 -
sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqrt(a - b*x
**2)*b*d**2*x**4),x)*b**3*c**5*x**2 - 12*int((sqrt(c + d*x)*sqrt(a - b*x**
2)*x**2)/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 +
b**2*d*x**5),x)*a**2*b*d**4 + 12*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)
/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x
**5),x)*a*b**2*c**2*d**2 + 12*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)...
```

3.329 $\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 355

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \frac{(ad+bcx)(c+dx)^{3/2}}{ab\sqrt{a-bx^2}}+\frac{cd\sqrt{c+dx}\sqrt{a-bx^2}}{ab} + \frac{(bc^2+3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}b^{3/2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{c(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}b^{3/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(b*c*x+a*d)*(d*x+c)^(3/2)/a/b/(-b*x^2+a)^(1/2)+c*d*(d*x+c)^(1/2)*(-b*x^2+a
)^(1/2)/a/b+(3*a*d^2+b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*
(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*
d))^(1/2))/a^(1/2)/b^(3/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(
-b*x^2+a)^(1/2)-c*(-a*d^2+b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(
1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2
^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(3/2)/(d*x+c)^(1
/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.04 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{b(c+dx)(bc^2x+ad(2c+dx))}{-a+bx^2} - \frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(bc^2+3ad^2)(a-bx^2)+i\sqrt{b}\left(b^{3/2}c^3-\sqrt{a}bc^2d+3a\sqrt{b}c\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(5/2)/(a - b*x^2)^(3/2),x]
```

output
```
(Sqrt[a - b*x^2]*(-((b*(c + d*x)*(b*c^2*x + a*d*(2*c + d*x)))/(-a + b*x^2)
) - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 + 3*a*d^2)*(a - b*x^2) + I*
Sqrt[b]*(b^(3/2)*c^3 - Sqrt[a]*b*c^2*d + 3*a*Sqrt[b]*c*d^2 - 3*a^(3/2)*d^3
)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
 + I*Sqrt[a]*Sqrt[b]*d*(b*c^2 - 4*Sqrt[a]*Sqrt[b]*c*d + 3*a*d^2)*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(a*b^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {495, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{495}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{\int \frac{3d(ad+bcx)\sqrt{c+dx}}{2\sqrt{a-bx^2}}dx}{ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\int \frac{(ad+bcx)\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{2ab}$$

$$\Big\downarrow \text{687}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(-\frac{2\int -\frac{b\left(4acd+\left(bc^2+3ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{2ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(\frac{1}{3}\int \frac{4acd+(bc^2+3ad^2)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{2ab}$$

$$\Big\downarrow \text{600}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(\frac{1}{3}\left(\frac{(3ad^2+bc^2)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{2ab}$$

$$\Big\downarrow \text{509}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}(3ad^2+bc^2)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{2ab}$$

$$\Big\downarrow \text{508}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(\frac{1}{3}\left(-\frac{c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\right)}{2ab}$$

$\downarrow$ 327

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(\frac{1}{3}\left(-\frac{c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{2}{3}c\sqrt{a-bx^2}\sqrt{c+\right)}{2ab}$$

$\downarrow$ 512

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(\frac{1}{3}\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)-\frac{2}{3}c\sqrt{a-b}\right)}{2ab}$$

$\downarrow$ 511

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{3d\left(\frac{1}{3}\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)\right)}{2ab}$$

$\downarrow$ 321

$$\frac{(c+dx)^{3/2}(ad+bcx)}{ab\sqrt{a-bx^2}} - \frac{3d\left(\frac{1}{3}\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}} - \frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3ad^2+bc^2)E\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.\right.}{2ab}$$

input
```
Int[(c + d*x)^(5/2)/(a - b*x^2)^(3/2),x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(3/2))/(a*b*Sqrt[a - b*x^2]) - (3*d*((-2*c*Sqrt[c
 + d*x]*Sqrt[a - b*x^2])/3 + ((-2*Sqrt[a]*(b*c^2 + 3*a*d^2)*Sqrt[c + d*x]*
Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]
], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*(b*c^2 - a*d^2)*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt
[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/3))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(293) = 586.

Time = 2.53 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.91

method	result
elliptic	$\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2(-bdx-bc)\left(\frac{\left(a\,d^2+b\,c^2\right)x}{2a\,b^2}+\frac{cd}{b^2}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(-\frac{4c\,d^2}{b}+\frac{c\left(3a\,d^2+b\,c^2\right)}{ab}-\frac{c\left(a\,d^2+b\,c^2\right)}{ba}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{a}}{b}}}}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.$
default	Expression too large to display

input
```
int((d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*(a*d^2+b*c^2)/a/b^2*x+c*d/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(-4*
c*d^2/b+c*(3*a*d^2+b*c^2)/a/b-1/b*c*(a*d^2+b*c^2)/a)*(c/d-1/b*(a*b)^(1/2))
*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b
)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x
^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2
),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-d^3/b-1/2*(a*
d^2+b*c^2)*d/a/b)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2)
)/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-
1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((
x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b
)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \frac{(abc^3-9\,a^2cd^2-(b^2c^3-9\,abcd^2)x^2)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9}{27\,bd}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((a*b*c^3 - 9*a^2*c*d^2 - (b^2*c^3 - 9*a*b*c*d^2)*x^2)*sqrt(-b*d)*weie
rstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), 1/3*(3*d*x + c)/d) + 3*(a*b*c^2*d + 3*a^2*d^3 - (b^2*c^2*d + 3*a*b
*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(2*a*b*c
*d^2 + (b^2*c^2*d + a*b*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a*b^3*d*x
^2 - a^2*b^2*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^{\frac{5}{2}}}{(a-bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**(5/2)/(a - b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(5/2)/(-b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)/(-b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)^{5/2}}{(a-b\,x^2)^{3/2}}\,dx$$

input
```
int((c + d*x)^(5/2)/(a - b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^(5/2)/(a - b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
( - 3*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a*c**2 - sqrt(a
 - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqrt(a - b*x**2)*b
*d**2*x**4),x)*a**3*c*d**4 + 2*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a
- b*x**2)*a*c**2 - sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*
x**2 + sqrt(a - b*x**2)*b*d**2*x**4),x)*a**2*b*c**3*d**2 + sqrt(a - b*x**2
)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a*c**2 - sqrt(a - b*x**2)*a*d**2*x**
2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqrt(a - b*x**2)*b*d**2*x**4),x)*a*b**2
*c**5 - sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)/(a**2*c
 + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)*
a*b**2*c*d**3 + sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)
/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x
**5),x)*b**3*c**3*d - 3*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**
2)*x**2)/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 +
b**2*d*x**5),x)*a**2*b*d**4 + 3*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a
 - b*x**2)*x**2)/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c
*x**4 + b**2*d*x**5),x)*a*b**2*c**2*d**2 + 2*sqrt(a - b*x**2)*int((sqrt(c
+ d*x)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3
+ b**2*c*x**4 + b**2*d*x**5),x)*a**2*b*c**2*d**2 - 2*sqrt(a - b*x**2)*int(
(sqrt(c + d*x)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b
*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)*a*b**2*c**4 + 3*sqrt(a - b*x**2...
```

3.330 $\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 313

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx = \frac{(ad+bcx)\sqrt{c+dx}}{ab\sqrt{a-bx^2}} + \frac{c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}b^{3/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(b*c*x+a*d)*(d*x+c)^(1/2)/a/b/(-b*x^2+a)^(1/2)+c*(d*x+c)^(1/2)*(1-b*x^2/a)
^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*
c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-(-a*d^2+b*c^2)*(b^(1/2)*(d*x+c)/(b^(1
/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/
2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)
/b^(3/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.69 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.41

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(cd+\frac{(ad+bcx)(c+dx)}{a-bx^2}-\frac{i\sqrt{b}c\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\text{arcsinh}\left(\cdots\right.\right.}{d\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(-a+bx^2)}\cdots\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^(3/2)/(a - b*x^2)^(3/2),x]
```

output
```
(Sqrt[a - b*x^2]*(c*d + ((a*d + b*c*x)*(c + d*x))/(a - b*x^2) - (I*Sqrt[b]
*c*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[
-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcS
inh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)
/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))
 + (I*Sqrt[a]*(-(Sqrt[b]*c) + Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c
 + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*El
lipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*
c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(
-a + b*x^2))))/(a*b*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {495, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 495$$

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{\int \frac{d(ad+bcx)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\int \frac{ad+bcx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab}$$

$\downarrow$ 600

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ab}$$

$\downarrow$ 509

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ab}$$

$\downarrow$ 508

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2ab}$$

$\downarrow$ 327

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2ab}$$

$\downarrow$ 512

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2ab}$$

$\Big\downarrow$ 511

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2}{\frac{\sqrt{b}c}{\sqrt{a}}}\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2ab}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{c+dx}(ad+bcx)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2ab}$$

input
```
Int[(c + d*x)^(3/2)/(a - b*x^2)^(3/2),x]
```

output
```
((a*d + b*c*x)*Sqrt[c + d*x])/(a*b*Sqrt[a - b*x^2]) - (d*((-2*Sqrt[a]*Sqrt
[b]*c*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]
*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*
(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 -
 a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)
/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[
b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(255) = 510$.

Time = 0.75 (sec) , antiderivative size = 629, normalized size of antiderivative = 2.01

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2(-bdx-bc)\left(\frac{cx}{2ab}+\frac{d}{2b^2}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(-\frac{3d^2}{2b}+\frac{a\,d^2+b\,c^2}{ba}-\frac{c^2}{a}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\mathrm{EllipticF}\left(\right.}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
default	$\frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\,\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\,\sqrt{\frac{\left(bx+\sqrt{ab}\right)d}{d\sqrt{ab}-bc}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)\sqrt{ab}\,a\,d^3-\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\,\sqrt{\right.}{}$

input
```
int((d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*c/a/b*x+1/2*d/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(-3/2*d^2/b+(a*d
^2+b*c^2)/b/a-c^2/a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1
/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elli
pticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2))-c*d/a*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)
*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx = \frac{(abc^2 - 3\,a^2d^2 - (b^2c^2 - 3\,abd^2)x^2)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2}, -\frac{8\,(bc^3-9\,ac\cdots}{27\,bd^3}\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((a*b*c^2 - 3*a^2*d^2 - (b^2*c^2 - 3*a*b*d^2)*x^2)*sqrt(-b*d)*weierstr
assPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^
3), 1/3*(3*d*x + c)/d) - 3*(b^2*c*d*x^2 - a*b*c*d)*sqrt(-b*d)*weierstrassZ
eta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weie
rstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), 1/3*(3*d*x + c)/d)) - 3*(b^2*c*d*x + a*b*d^2)*sqrt(-b*x^2 + a)*sqr
t(d*x + c))/(a*b^3*d*x^2 - a^2*b^2*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}}{(a-bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**(3/2)/(a - b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)/(-b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)/(-b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx=\int \frac{(c+d\,x)^{3/2}}{(a-b\,x^2)^{3/2}}\,dx$$

input
```
int((c + d*x)^(3/2)/(a - b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^(3/2)/(a - b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)/(a**2*c + a*
*2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)*a**2*
b*c*d**3 - 2*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)/(a
**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5
),x)*a*b**2*c**3*d - 2*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2
)*x**3)/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b
**2*d*x**5),x)*a*b**2*c*d**3*x**2 + 2*sqrt(a - b*x**2)*int((sqrt(c + d*x)*
sqrt(a - b*x**2)*x**3)/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 +
b**2*c*x**4 + b**2*d*x**5),x)*b**3*c**3*d*x**2 + 3*sqrt(a - b*x**2)*int((s
qrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*
a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)*a**3*d**4 - 3*sqrt(a - b*x**2)*
int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x - 2*a*b*c*x**
2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)*a**2*b*c**2*d**2 - 3*sqrt
(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x
- 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)*a**2*b*d**4*
x**2 + 3*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a**2*
c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)
*a*b**2*c**2*d**2*x**2 - 4*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*
x**2))/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b*
*2*d*x**5),x)*a**3*c**2*d**2 + 4*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sq...
```

3.331 $\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 291

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}} dx = \frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}} + \frac{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
x*(d*x+c)^(1/2)/a/(-b*x^2+a)^(1/2)+(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*Ellipti
cE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a
^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^
(1/2)/(-b*x^2+a)^(1/2)-c*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-
b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(d*x+c)^(1/2)/(-b*
x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.73 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{x(c+dx)}{a-bx^2} - \frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(a-bx^2)+i\sqrt{b}\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\operatorname{arcs}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]/(a - b*x^2)^(3/2),x]
```

output
```
(Sqrt[a - b*x^2]*((x*(c + d*x))/(a - b*x^2) - (d^2*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/S
qrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
+ d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d
*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]
*d*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
 d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)
])/(b*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(a*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {494, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{494}$$

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{\int \frac{dx}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a}$$

$\Big\downarrow$ 27

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\int \frac{x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a}$$

$\Big\downarrow$ 600

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a}$$

$\Big\downarrow$ 509

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a}$$

$\Big\downarrow$ 508

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\left(-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}$$

$\Big\downarrow$ 327

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\left(-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}$$

$\Big\downarrow$ 512

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}$$

$$\Big\downarrow \text{ 511}$$

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{x\sqrt{c+dx}}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}$$

input
```
Int[Sqrt[c + d*x]/(a - b*x^2)^(3/2),x]
```

output
```
(x*Sqrt[c + d*x])/(a*Sqrt[a - b*x^2]) - (d*((-2*Sqrt[a]*Sqrt[c + d*x]*Sqrt
[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt
[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*Sqrt[(Sqrt[b]*(c + d*x
))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*
d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{(-bdx-bc)x}{ba\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}-\frac{d\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\left(\left(-\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\right.\right.}{a\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}$
default	$\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\sqrt{\frac{\left(bx+\sqrt{ab}\right)d}{d\sqrt{ab}-bc}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)a\,d^2-\sqrt{ab}\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\right.$

input
```
int((d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-(-b*d*x-b*c)/b
/a*x/((x^2-a/b)*(-b*d*x-b*c))^(1/2)-d/a*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/
d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/
2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*
d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(
1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a
*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx =$$

$$-\frac{3\sqrt{-bx^2+a}\sqrt{dx+c}bdx+(bcx^2-ac)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4(bc^2+3ad^2)}{3bd^2},-\frac{8(bc^3-9acd^2)}{27bd^3},\frac{3dx+c}{3d}\right)+3}{3(}$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(3*sqrt(-b*x^2 + a)*sqrt(d*x + c)*b*d*x + (b*c*x^2 - a*c)*sqrt(-b*d)*
weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^
2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(b*d*x^2 - a*d)*sqrt(-b*d)*weierstrassZ
eta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weie
rstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), 1/3*(3*d*x + c)/d)))/(a*b^2*d*x^2 - a^2*b*d)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx}}{(a-bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)/(a - b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/(-b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/(-b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+d\,x}}{(a-b\,x^2)^{3/2}}\,dx$$

input
```
int((c + d*x)^(1/2)/(a - b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^(1/2)/(a - b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{3/2}}\,dx = \frac{2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,x+\left(\int \frac{\sqrt{dx+c}}{\sqrt{-b\,x^2+a}\,a\,c^2-\sqrt{-b\,x^2+a}\,a\,d^2x^2-\sqrt{-b\,x^2+a}\,b\,c^2x^2+\sqrt{-b\,x^2+a}\,b\,d^2x^4}dx\right.}{}$$

input
```
int((d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(c + d*x)*sqrt(a - b*x**2)*x + int(sqrt(c + d*x)/(sqrt(a - b*x**2)*
a*c**2 - sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqr
t(a - b*x**2)*b*d**2*x**4),x)*a**2*c**2 - int(sqrt(c + d*x)/(sqrt(a - b*x*
*2)*a*c**2 - sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 +
 sqrt(a - b*x**2)*b*d**2*x**4),x)*a*b*c**2*x**2 + int((sqrt(c + d*x)*sqrt(
a - b*x**2)*x**3)/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*
c*x**4 + b**2*d*x**5),x)*a*b*d - int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)
/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x
**5),x)*b**2*d*x**2 - int((sqrt(c + d*x)*x)/(sqrt(a - b*x**2)*a*c**2 - sqr
t(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqrt(a - b*x**2
)*b*d**2*x**4),x)*a**2*c*d + int((sqrt(c + d*x)*x)/(sqrt(a - b*x**2)*a*c**
2 - sqrt(a - b*x**2)*a*d**2*x**2 - sqrt(a - b*x**2)*b*c**2*x**2 + sqrt(a -
 b*x**2)*b*d**2*x**4),x)*a*b*c*d*x**2)/(3*a*(a - b*x**2))
```

3.332 $\int \frac{1}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 328

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = -\frac{(ad-bcx)\sqrt{c+dx}}{a\,(bc^2-ad^2)\sqrt{a-bx^2}} + \frac{\sqrt{b}c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\,(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-(-b*c*x+a*d)*(d*x+c)^(1/2)/a/(-a*d^2+b*c^2)/(-b*x^2+a)^(1/2)+b^(1/2)*c*(d
*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^
(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a*d^2+b*c
^2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-(b^(1/2
)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-
b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))
^(1/2))/a^(1/2)/b^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.04 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx = \frac{i\left(-id\left(\sqrt{b}c+\sqrt{a}d\right)\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}x\sqrt{c+dx}+\sqrt{b}c\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+\right.}{}$$

input
```
Integrate[1/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(I*((-I)*d*(Sqrt[b]*c + Sqrt[a]*d)*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*x*Sqrt[c
 + d*x] + Sqrt[b]*c*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqr
t[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^2*EllipticE[I*ArcSinh[Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c
 - Sqrt[a]*d)] + Sqrt[a]*d*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[
-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^2*EllipticF[I*ArcSinh[
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sq
rt[b]*c - Sqrt[a]*d)]))/(a*d*(Sqrt[b]*c + Sqrt[a]*d)*Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]*(c + d*x)*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {496, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 496$$

$$\frac{\int -\frac{d(ad+bcx)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{d\int \frac{ad+bcx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{600}$$

$$-\frac{d\left(\frac{bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{509}$$

$$-\frac{d\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{508}$$

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{327}$$

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{512}$$

$$-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

input
```
Int[1/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
-(((a*d - b*c*x)*Sqrt[c + d*x])/(a*(b*c^2 - a*d^2)*Sqrt[a - b*x^2])) - (d*
((-2*Sqrt[a]*Sqrt[b]*c*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[
Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/
(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2
*Sqrt[a]*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]
*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2
]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*
x^2])))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(270) = 540$.

Time = 3.34 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.04

method	result
default	$\frac{\left(-\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\sqrt{\frac{\left(bx+\sqrt{ab}\right)d}{d\sqrt{ab}-bc}}\,\text{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}},\sqrt{-\frac{d\sqrt{ab}-bc}{d\sqrt{ab}+bc}}\right)\sqrt{ab}\,a\,d^3+\sqrt{-\frac{(dx+c)b}{d\sqrt{ab}-bc}}\sqrt{\frac{\left(-bx+\sqrt{ab}\right)d}{d\sqrt{ab}+bc}}\sqrt{\left(\right.}}{}$
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2(-bdx-bc)\left(-\frac{cx}{2\left(a\,d^2-b\,c^2\right)a}+\frac{d}{2\left(a\,d^2-b\,c^2\right)b}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(\frac{1}{a}-\frac{d^2}{2\left(a\,d^2-b\,c^2\right)}+\frac{b\,c^2}{\left(a\,d^2-b\,c^2\right)a}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{}}{\sqrt{-bd\,x^3-bc}}\right.}{}$

input
```
int(1/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2)*((-b*x+(a*b)^(1/2))*d/(d*(a*b)^(1
/2)+b*c))^(1/2)*((b*x+(a*b)^(1/2))*d/(d*(a*b)^(1/2)-b*c))^(1/2)*EllipticF(
(-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(d*(a*b)^(1/2)-b*c)/(d*(a*b)^(1/2
)+b*c))^(1/2))*(a*b)^(1/2)*a*d^3+(-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2)*((
-b*x+(a*b)^(1/2))*d/(d*(a*b)^(1/2)+b*c))^(1/2)*((b*x+(a*b)^(1/2))*d/(d*(a*
b)^(1/2)-b*c))^(1/2)*EllipticF((-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(d
*(a*b)^(1/2)-b*c)/(d*(a*b)^(1/2)+b*c))^(1/2))*(a*b)^(1/2)*b*c^2*d+(-(d*x+c
)*b/(d*(a*b)^(1/2)-b*c))^(1/2)*((-b*x+(a*b)^(1/2))*d/(d*(a*b)^(1/2)+b*c))^
(1/2)*((b*x+(a*b)^(1/2))*d/(d*(a*b)^(1/2)-b*c))^(1/2)*EllipticE((-(d*x+c)*
b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(d*(a*b)^(1/2)-b*c)/(d*(a*b)^(1/2)+b*c))^(1
/2))*a*b*c*d^2-(-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2)*((-b*x+(a*b)^(1/2))*
d/(d*(a*b)^(1/2)+b*c))^(1/2)*((b*x+(a*b)^(1/2))*d/(d*(a*b)^(1/2)-b*c))^(1/
2)*EllipticE((-(d*x+c)*b/(d*(a*b)^(1/2)-b*c))^(1/2),(-(d*(a*b)^(1/2)-b*c)/
(d*(a*b)^(1/2)+b*c))^(1/2))*b^2*c^3-b^2*c*d^2*x^2+a*b*d^3*x-b^2*c^2*d*x+a*
b*c*d^2)*(-b*x^2+a)^(1/2)*(d*x+c)^(1/2)/d/b/a/(a*d^2-b*c^2)/(-b*d*x^3-b*c*
x^2+a*d*x+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = -\frac{(abc^2-3\,a^2d^2-(b^2c^2-3\,abd^2)x^2)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\frac{3\,dx+c}{3\,d}\right)-3\,(b^2c}{}$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((a*b*c^2 - 3*a^2*d^2 - (b^2*c^2 - 3*a*b*d^2)*x^2)*sqrt(-b*d)*weierst
rassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d
^3), 1/3*(3*d*x + c)/d) - 3*(b^2*c*d*x^2 - a*b*c*d)*sqrt(-b*d)*weierstrass
Zeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), wei
erstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/
(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(b^2*c*d*x - a*b*d^2)*sqrt(-b*x^2 + a)*sq
rt(d*x + c))/(a^2*b^2*c^2*d - a^3*b*d^3 - (a*b^3*c^2*d - a^2*b^2*d^3)*x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{1}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a - b*x**2)**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{1}{(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{b^2d\,x^5+b^2c\,x^4-2abd\,x^3-2abc\,x^2+a^2dx+a^2c}dx$$

input
```
int(1/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2
*a*b*d*x**3 + b**2*c*x**4 + b**2*d*x**5),x)
```

3.333 $\int \frac{1}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 405

$$\int \frac{1}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx = \frac{2d}{(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{b\sqrt{c+dx}(4acd-(bc^2+3ad^2)x)}{a(bc^2-ad^2)^2\sqrt{a-bx^2}} + \frac{\sqrt{b}(bc^2+3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}(bc^2-ad^2)^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{\sqrt{b}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2*d/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-b*(d*x+c)^(1/2)*(4*a*c*d
-(3*a*d^2+b*c^2)*x)/a/(-a*d^2+b*c^2)^2/(-b*x^2+a)^(1/2)+b^(1/2)*(3*a*d^2+b
*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a
*d^2+b*c^2)^2/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/
2)-b^(1/2)*c*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/
2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b
^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.13 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.18

$$\int \frac{1}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\frac{\sqrt{a-bx^2}\left(bc^2d+ad^3-\frac{b(c+dx)\left(bc^2x+ad(-2c+dx)\right)}{-a+bx^2}+\frac{ib\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(bc^2+3ad^2\right)\sqrt{\frac{d\left(\cdots\right)}{c+\cdots}}\cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[1/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(b*c^2*d + a*d^3 - (b*(c + d*x)*(b*c^2*x + a*d*(-2*c + d*
x)))/(-a + b*x^2) + (I*b*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 + 3*a*d^2)*
Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*
x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/
(d*(-a + b*x^2)) - (I*Sqrt[a]*Sqrt[b]*(b*c^2 - 4*Sqrt[a]*Sqrt[b]*c*d + 3*a
*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b
] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]
*d)])/(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(a*(b*c^2 - a*d^2)^2
*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {496, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{\int -\frac{d(3ad-bcx)}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{3ad-bcx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{688}$$

$$-\frac{d\left(\frac{2\int \frac{b\left(4acd+\left(bc^2+3ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}(3ad^2+bc^2)}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{2a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\left(\frac{b\int \frac{4acd+\left(bc^2+3ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}(3ad^2+bc^2)}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{2a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{600}$$

$$-\frac{d\left(\frac{b\left(\frac{\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}(3ad^2+bc^2)}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{2a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{509}$$

$$-\frac{d\left(\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(3ad^2+bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{508}$$

$$-\frac{d\left(\frac{b\left(-\frac{c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$-\frac{d\left(\frac{b\left(-\frac{c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{512}$$

$$-\frac{d\left(\frac{b\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2+bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{511}$$

$$-\frac{d\left(\frac{b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 321

$$-\frac{d\left(\frac{b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2+bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
-((a*d - b*c*x)/(a*(b*c^2 - a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])) - (d*((
2*(b*c^2 + 3*a*d^2)*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + (b*
((-2*Sqrt[a]*(b*c^2 + 3*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE
[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
 + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[
a - b*x^2]) + (2*Sqrt[a]*c*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[
b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*
x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c
+ d*x]*Sqrt[a - b*x^2])))/(b*c^2 - a*d^2)))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(343) = 686$.

Time = 5.01 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.98

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{2bd\left(\frac{(3a\,d^2+b\,c^2)x^2}{2a(a^2d^4-2b\,c^2d^2a+b^2c^4)}-\frac{cx}{2(a\,d^2-b\,c^2)ad}-\frac{a\,d^2+b\,c^2}{(a^2d^4-2b\,c^2d^2a+b^2c^4)b}\right)}{\sqrt{-\left(x^3+\frac{c\,x^2}{d}-\frac{ax}{b}-\frac{ac}{bd}\right)bd}}+\frac{2\left(-\frac{bc(3a\,d^2-b\,c^2)}{a(a^2d^4-2b\,c^2d^2a+b^2c^4)}+\frac{}{(a}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(2*b*d*(1/2/a/(a
^2*d^4-2*a*b*c^2*d^2+b^2*c^4)*(3*a*d^2+b*c^2)*x^2-1/2/(a*d^2-b*c^2)*c/a/d*
x-(a*d^2+b*c^2)/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/b)/(-(x^3+c/d*x^2-a*x/b-a/
b*c/d)*b*d)^(1/2)+2*(-b*c*(3*a*d^2-b*c^2)/a/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4
)+1/(a*d^2-b*c^2)*b*c/a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b
)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*
EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))-1/a/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)*(3*a*d^2+
b*c^2)*b*d*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d
+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*
b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.38

$$\int \frac{1}{(c+dx)^{3/2} (a-bx^2)^{3/2}} \, dx = -\frac{(abc^4 - 9\,a^2c^2d^2 - (b^2c^3d - 9\,abcd^3)x^3 - (b^2c^4 - 9\,abc^2d^2)x^2 + (abc^3d - 9\,a^2cd^3)x)\sqrt{-bd}\text{weierstrassPInv}}{}$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((a*b*c^4 - 9*a^2*c^2*d^2 - (b^2*c^3*d - 9*a*b*c*d^3)*x^3 - (b^2*c^4
- 9*a*b*c^2*d^2)*x^2 + (a*b*c^3*d - 9*a^2*c*d^3)*x)*sqrt(-b*d)*weierstrass
PInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
 1/3*(3*d*x + c)/d) + 3*(a*b*c^3*d + 3*a^2*c*d^3 - (b^2*c^2*d^2 + 3*a*b*d^
4)*x^3 - (b^2*c^3*d + 3*a*b*c*d^3)*x^2 + (a*b*c^2*d^2 + 3*a^2*d^4)*x)*sqrt
(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c
*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(2*a*b*c^2*d^2 + 2*a^2*
d^4 - (b^2*c^2*d^2 + 3*a*b*d^4)*x^2 - (b^2*c^3*d - a*b*c*d^3)*x)*sqrt(-b*x
^2 + a)*sqrt(d*x + c))/(a^2*b^2*c^5*d - 2*a^3*b*c^3*d^3 + a^4*c*d^5 - (a*b
^3*c^4*d^2 - 2*a^2*b^2*c^2*d^4 + a^3*b*d^6)*x^3 - (a*b^3*c^5*d - 2*a^2*b^2
*c^3*d^3 + a^3*b*c*d^5)*x^2 + (a^2*b^2*c^4*d^2 - 2*a^3*b*c^2*d^4 + a^4*d^6
)*x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{3/2} (a-bx^2)^{3/2}} \, dx = \int \frac{1}{(a-bx^2)^{\frac{3}{2}} (c+dx)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a - b*x**2)**(3/2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \left(\int \frac{\sqrt{dx+c}}{\sqrt{-b\,x^2+a}\,a\,c^4-2\sqrt{-b\,x^2+a}\,a\,c^2d^2x^2+\sqrt{-b\,x^2+a}\,a\,d^4x^4-\sqrt{-b}\right.$$
$$+\left(\int \frac{\sqrt{dx+c}\,x^2}{\sqrt{-b\,x^2+a}\,a\,c^4-2\sqrt{-b\,x^2+a}\,a\,c^2d^2x^2+\sqrt{-b\,x^2+a}\,a\,d^4x^4-\sqrt{-b\,x^2+a}\,b\,c^4x^2+2\sqrt{-b\,x^2+a}\,b}\right.$$
$$-2\left(\int \frac{\sqrt{dx+c}\,x}{\sqrt{-b\,x^2+a}\,a\,c^4-2\sqrt{-b\,x^2+a}\,a\,c^2d^2x^2+\sqrt{-b\,x^2+a}\,a\,d^4x^4-\sqrt{-b\,x^2+a}\,b\,c^4x^2+2\sqrt{-b\,x^2+a}}\right.$$

input
```
int(1/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a*c**4 - 2*sqrt(a - b*x**2)*a*c**2*d**
2*x**2 + sqrt(a - b*x**2)*a*d**4*x**4 - sqrt(a - b*x**2)*b*c**4*x**2 + 2*s
qrt(a - b*x**2)*b*c**2*d**2*x**4 - sqrt(a - b*x**2)*b*d**4*x**6),x)*c**2 +
 int((sqrt(c + d*x)*x**2)/(sqrt(a - b*x**2)*a*c**4 - 2*sqrt(a - b*x**2)*a*
c**2*d**2*x**2 + sqrt(a - b*x**2)*a*d**4*x**4 - sqrt(a - b*x**2)*b*c**4*x*
*2 + 2*sqrt(a - b*x**2)*b*c**2*d**2*x**4 - sqrt(a - b*x**2)*b*d**4*x**6),x
)*d**2 - 2*int((sqrt(c + d*x)*x)/(sqrt(a - b*x**2)*a*c**4 - 2*sqrt(a - b*x
**2)*a*c**2*d**2*x**2 + sqrt(a - b*x**2)*a*d**4*x**4 - sqrt(a - b*x**2)*b*
c**4*x**2 + 2*sqrt(a - b*x**2)*b*c**2*d**2*x**4 - sqrt(a - b*x**2)*b*d**4*
x**6),x)*c*d
```

3.334 $\int \frac{1}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 484

$$\int \frac{1}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \frac{2d}{3(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}} + \frac{16bcd}{3(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{b\sqrt{c+dx}(ad(27bc^2+5ad^2)-bc(3bc^2+29ad^2)x)}{3a(bc^2-ad^2)^3\sqrt{a-bx^2}} + \frac{b^{3/2}c(3bc^2+29ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}(bc^2-ad^2)^3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{\sqrt{b}(3bc^2+5ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3*d/(-a*d^2+b*c^2)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2)+16/3*b*c*d/(-a*d^2+b*c
^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/3*b*(d*x+c)^(1/2)*(a*d*(5*a*d^2+27*
b*c^2)-b*c*(29*a*d^2+3*b*c^2)*x)/a/(-a*d^2+b*c^2)^3/(-b*x^2+a)^(1/2)+1/3*b
^(3/2)*c*(29*a*d^2+3*b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*
(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*
d))^(1/2))/a^(1/2)/(-a*d^2+b*c^2)^3/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d)
)^(1/2)/(-b*x^2+a)^(1/2)-1/3*b^(1/2)*(5*a*d^2+3*b*c^2)*(b^(1/2)*(d*x+c)/(b
^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^
(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1
/2)/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.86 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.26

$$\int \frac{1}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \frac{d\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(2a^3d^5+3b^3c^3x(c+dx)^2-a^2bd^3\left(25c^2+26cdx+5d^2x^2\right)+c\right.}{}$$

input
```
Integrate[1/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(2*a^3*d^5 + 3*b^3*c^3*x*(c + d*x)^2 - a
^2*b*d^3*(25*c^2 + 26*c*d*x + 5*d^2*x^2) + a*b^2*c*d*(-9*c^3 - 9*c^2*d*x +
 31*c*d^2*x^2 + 29*d^3*x^3)) + b*(c + d*x)*(c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]]*(3*b*c^2 + 29*a*d^2)*(a - b*x^2) + I*Sqrt[b]*c*(3*b^(3/2)*c^3 - 3*S
qrt[a]*b*c^2*d + 29*a*Sqrt[b]*c*d^2 - 29*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqr
t[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x
]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(3*b^(3
/2)*c^3 - 27*Sqrt[a]*b*c^2*d + 29*a*Sqrt[b]*c*d^2 - 5*a^(3/2)*d^3)*Sqrt[(d
*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c +
 d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(3*a*d
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)^3*(c + d*x)^(3/2)*Sqrt[a -
 b*x^2])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {496, 27, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{\int -\frac{d(5ad-3bcx)}{2(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{5ad-3bcx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{688}$$

$$-\frac{d\left(\frac{2\int \frac{b\left(24acd-\left(3bc^2+5ad^2\right)x\right)}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}(5ad^2+3bc^2)}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{2a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\left(\frac{b\int \frac{24acd-\left(3bc^2+5ad^2\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}(5ad^2+3bc^2)}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{2a(bc^2-ad^2)}-\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{688}$$

$$-\frac{d\left(\frac{b\left(\frac{2\int \frac{ad\left(27bc^2+5ad^2\right)+bc\left(3bc^2+29ad^2\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2c\sqrt{a-bx^2}\left(29ad^2+3bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}(5ad^2+3bc^2)}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{d\left(\frac{b\left(\frac{\int \frac{ad\left(27bc^2+5ad^2\right)+bc\left(3bc^2+29ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2c\sqrt{a-bx^2}\left(29ad^2+3bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}(5ad^2+3bc^2)}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 600$$

$$-\frac{d\left(\frac{b\left(\frac{\frac{bc\left(29ad^2+3bc^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}+\frac{2c\sqrt{a-bx^2}\left(29ad^2+3bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}(5ad^2+3bc^2)}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{2a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 509$$

$$-\frac{d\left(\frac{b\left(\frac{\frac{bc\sqrt{1-\frac{bx^2}{a}}\left(29ad^2+3bc^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}+\frac{2c\sqrt{a-bx^2}\left(29ad^2+3bc^2\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}\left(5ad^2+3bc\right.}{3(c+dx)^{3/2}(bc^2-ad\ldots}\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$-\frac{d\left(\frac{b\left(\frac{-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\frac{2c\sqrt{a-bx^2}\left(29ad^2+3bc\ldots\right.}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{3(bc^2-ad^2)}\ldots\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 327

$$-\frac{d\left(\frac{b\left(\frac{-\frac{\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}}{bc^2-ad^2}+\frac{2c\sqrt{a-bx^2}\left(29ad^2+3\cdots\right.}{\sqrt{c+dx}\left(bc^2-ad^2\cdots\right.}\cdots\right)}{3(bc^2-ad^2)}\cdots\right)}{2a\left(bc^2-ad^2\right)\cdots}$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 512

$$-\frac{d\left(\frac{b\left(\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}}{bc^2-ad^2}+\frac{2c\sqrt{a-bx^2}\cdots}{\sqrt{c+dx}\cdots}\cdots\right)}{3(bc^2-ad^2)}\cdots\right)}{2a\left(bc^2-ad^2\right)\cdots}$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 511

$$-\frac{d\left(\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+\cdots}{\sqrt{a}d+\cdots}}}\cdots\right)}{bc^2-ad^2}\cdots}{3(bc^2-ad^2)}\cdots\right)}{2a\left(bc^2-ad^2\right)}\cdots$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 321

$$-\frac{d\left(\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5ad^2+3bc^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(29ad^2+3bc^2\right)E\left(\arcsin\left(\cdots\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{bc^2-ad^2}\cdots}{3(bc^2-ad^2)}\cdots\right)}{2a\left(bc^2-ad^2\right)}\cdots$$

$$\frac{ad-bcx}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
-((a*d - b*c*x)/(a*(b*c^2 - a*d^2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2])) - (d*
((2*(3*b*c^2 + 5*a*d^2)*Sqrt[a - b*x^2])/(3*(b*c^2 - a*d^2)*(c + d*x)^(3/2
)) + (b*((2*c*(3*b*c^2 + 29*a*d^2)*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[
c + d*x]) + ((-2*Sqrt[a]*Sqrt[b]*c*(3*b*c^2 + 29*a*d^2)*Sqrt[c + d*x]*Sqrt
[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*b*c^2 + 5*a*d
^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*
EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c
)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2
)))/(3*(b*c^2 - a*d^2))))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(408) = 816$.

Time = 7.59 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.95

method	result
elliptic	$\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2d\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3\left(a\,d^2-b\,c^2\right)^2\left(x+\frac{c}{d}\right)^2}+\frac{20\left(-bd\,x^2+ad\right)d^2bc}{3\left(a\,d^2-b\,c^2\right)^3\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}-\frac{2(-bdx-bc)\left(-\frac{bc\left(3a\,d^2+b\,c^2\right)x}{2a\left(a\,d^2-b\,c^2\right)^3}+\frac{d\left(a\,d^2+3b\right.}{2\left(a\,d^2-b\,c\right.}\right.}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}\right.$
default	Expression too large to display

input
```
int(1/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(-2/3*d/(a*d^2-b
*c^2)^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+20/3*(-b*d*x^2+a*d)*d
^2/(a*d^2-b*c^2)^3*b*c/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2*(-b*d*x-b*c)*(-1/2
*b*c*(3*a*d^2+b*c^2)/a/(a*d^2-b*c^2)^3*x+1/2*d*(a*d^2+3*b*c^2)/(a*d^2-b*c^
2)^3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(1/3*b*d^2/(a*d^2-b*c^2)^2+10/3*b^2
*c^2*d^2/(a*d^2-b*c^2)^3+b/(a*d^2-b*c^2)^2*(a*d^2+b*c^2)/a-1/2*b*d^2*(a*d^
2+3*b*c^2)/(a*d^2-b*c^2)^3+b^2*c^2*(3*a*d^2+b*c^2)/a/(a*d^2-b*c^2)^3)*(c/d
-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2
))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2
)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(
a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+
2*(10/3*b^2*c*d^3/(a*d^2-b*c^2)^3+1/2*b^2*c*d*(3*a*d^2+b*c^2)/a/(a*d^2-b*c
^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b
*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(
1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(408) = 816$.

Time = 0.17 (sec) , antiderivative size = 925, normalized size of antiderivative = 1.91

$$\int \frac{1}{(c+dx)^{5/2} (a-bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/9*((3*a*b^2*c^6 - 52*a^2*b*c^4*d^2 - 15*a^3*c^2*d^4 - (3*b^3*c^4*d^2 -
52*a*b^2*c^2*d^4 - 15*a^2*b*d^6)*x^4 - 2*(3*b^3*c^5*d - 52*a*b^2*c^3*d^3 -
 15*a^2*b*c*d^5)*x^3 - (3*b^3*c^6 - 55*a*b^2*c^4*d^2 + 37*a^2*b*c^2*d^4 +
15*a^3*d^6)*x^2 + 2*(3*a*b^2*c^5*d - 52*a^2*b*c^3*d^3 - 15*a^3*c*d^5)*x)*s
qrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3
- 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(3*a*b^2*c^5*d + 29*a^2*b*c^3
*d^3 - (3*b^3*c^3*d^3 + 29*a*b^2*c*d^5)*x^4 - 2*(3*b^3*c^4*d^2 + 29*a*b^2*
c^2*d^4)*x^3 - (3*b^3*c^5*d + 26*a*b^2*c^3*d^3 - 29*a^2*b*c*d^5)*x^2 + 2*(
3*a*b^2*c^4*d^2 + 29*a^2*b*c^2*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c
^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInve
rse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*
(3*d*x + c)/d)) + 3*(9*a*b^2*c^4*d^2 + 25*a^2*b*c^2*d^4 - 2*a^3*d^6 - (3*b
^3*c^3*d^3 + 29*a*b^2*c*d^5)*x^3 - (6*b^3*c^4*d^2 + 31*a*b^2*c^2*d^4 - 5*a
^2*b*d^6)*x^2 - (3*b^3*c^5*d - 9*a*b^2*c^3*d^3 - 26*a^2*b*c*d^5)*x)*sqrt(-
b*x^2 + a)*sqrt(d*x + c))/(a^2*b^3*c^8*d - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^4
*d^5 - a^5*c^2*d^7 - (a*b^4*c^6*d^3 - 3*a^2*b^3*c^4*d^5 + 3*a^3*b^2*c^2*d^
7 - a^4*b*d^9)*x^4 - 2*(a*b^4*c^7*d^2 - 3*a^2*b^3*c^5*d^4 + 3*a^3*b^2*c^3*
d^6 - a^4*b*c*d^8)*x^3 - (a*b^4*c^8*d - 4*a^2*b^3*c^6*d^3 + 6*a^3*b^2*c^4*
d^5 - 4*a^4*b*c^2*d^7 + a^5*d^9)*x^2 + 2*(a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^5*
d^4 + 3*a^4*b*c^3*d^6 - a^5*c*d^8)*x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(a-bx^2\right)^{\frac{3}{2}}(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a - b*x**2)**(3/2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2 -}}{b^2d^3x^7+3b^2c\,d^2x^6-2ab\,d^3x^5+3b^2c^2d\,x^5-6abc\,d^2x^4+b^2c^3x^4+a^2d^3}$$

input
```
int(1/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a**2*c**3 + 3*a**2*c**2*d*x + 3*a**2
*c*d**2*x**2 + a**2*d**3*x**3 - 2*a*b*c**3*x**2 - 6*a*b*c**2*d*x**3 - 6*a*
b*c*d**2*x**4 - 2*a*b*d**3*x**5 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**5 + 3*
b**2*c*d**2*x**6 + b**2*d**3*x**7),x)
```

3.335 $\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 472

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = \frac{(ad+bcx)(c+dx)^{7/2}}{3ab\,(a-bx^2)^{3/2}} + \frac{(c+dx)^{3/2}\,(ad(bc^2-7ad^2)+2bc(2bc^2-5ad^2)\,x)}{6a^2b^2\sqrt{a-bx^2}} + \frac{2cd(bc^2-3ad^2)\,\sqrt{c+dx}\sqrt{a-bx^2}}{3a^2b^2} + \frac{(4b^2c^4-15abc^2d^2-21a^2d^4)\,\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{5/2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{2c(b^2c^4-4abc^2d^2+3a^2d^4)\,\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3a^{3/2}b^{5/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(b*c*x+a*d)*(d*x+c)^(7/2)/a/b/(-b*x^2+a)^(3/2)+1/6*(d*x+c)^(3/2)*(a*d*
(-7*a*d^2+b*c^2)+2*b*c*(-5*a*d^2+2*b*c^2)*x)/a^2/b^2/(-b*x^2+a)^(1/2)+2/3*
c*d*(-3*a*d^2+b*c^2)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/b^2+1/6*(-21*a^2*d
^4-15*a*b*c^2*d^2+4*b^2*c^4)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)
*d))^(1/2))/a^(3/2)/b^(5/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/
(-b*x^2+a)^(1/2)-2/3*c*(3*a^2*d^4-4*a*b*c^2*d^2+b^2*c^4)*(b^(1/2)*(d*x+c)/
(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^
(3/2)/b^(5/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.30 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{(c+dx)\left(4b^3c^4x^3+a^3d^3(19c+7dx)-ab^2c^2x\left(6c^2+cdx+15d^2x^2\right)-a^2bd\left(7c^3-3c^2dx+27cd^2x^2+9d^3x^3\cdots\right.\right.}{a^2b^2(a-bx^2)^2}\cdots\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^(9/2)/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(-(((c + d*x)*(4*b^3*c^4*x^3 + a^3*d^3*(19*c + 7*d*x) - a
*b^2*c^2*x*(6*c^2 + c*d*x + 15*d^2*x^2) - a^2*b*d*(7*c^3 - 3*c^2*d*x + 27*
c*d^2*x^2 + 9*d^3*x^3)))/(a^2*b^2*(a - b*x^2)^2)) + (d^2*Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]]*(4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a^2*d^4)*(-a + b*x^2) - I*S
qrt[b]*(4*b^(5/2)*c^5 - 4*Sqrt[a]*b^2*c^4*d - 15*a*b^(3/2)*c^3*d^2 + 15*a^
(3/2)*b*c^2*d^3 - 21*a^2*Sqrt[b]*c*d^4 + 21*a^(5/2)*d^5)*Sqrt[(d*(Sqrt[a]/
Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
 + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c +
d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[b
]*d*(4*b^2*c^4 - Sqrt[a]*b^(3/2)*c^3*d - 15*a*b*c^2*d^2 + 33*a^(3/2)*Sqrt[
b]*c*d^3 - 21*a^2*d^4)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((
Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sq
rt[b]*c - Sqrt[a]*d)])/(a^2*b^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x
^2))))/(6*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {495, 27, 684, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{495}$$

$$\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}-\frac{\int -\frac{(c+dx)^{5/2}(4bc^2-3bdxc-7ad^2)}{2(a-bx^2)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^{5/2}(4bc^2-3bdxc-7ad^2)}{(a-bx^2)^{3/2}}dx}{6ab}+\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \text{684}$$

$$\frac{\frac{(c+dx)^{3/2}\left(2bcx\left(2bc^2-5ad^2\right)+ad\left(bc^2-7ad^2\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\int -\frac{3d\sqrt{c+dx}\left(ad\left(bc^2+7ad^2\right)-4bc\left(bc^2-3ad^2\right)x\right)}{2\sqrt{a-bx^2}}dx}{ab}}{6ab}+\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3d\int \frac{\sqrt{c+dx}\left(ad\left(bc^2+7ad^2\right)-4bc\left(bc^2-3ad^2\right)x\right)}{\sqrt{a-bx^2}}dx}{2ab}+\frac{(c+dx)^{3/2}\left(2bcx\left(2bc^2-5ad^2\right)+ad\left(bc^2-7ad^2\right)\right)}{ab\sqrt{a-bx^2}}}{6ab}+\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{3d\left(\frac{8}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-3ad^2\right)-\frac{2\int \frac{b\left(acd\left(bc^2-33ad^2\right)+\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}\right)}{2ab}+\frac{(c+dx)^{3/2}\left(2bcx\left(2bc^2-5ad^2\right)+ad\left(bc^2-7ad^2\right)\right)}{ab\sqrt{a-bx^2}}}{6ab}+\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3d\left(\frac{8}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-3ad^2\right)-\frac{1}{3}\int \frac{acd\left(bc^2-33ad^2\right)+\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{2ab}+\frac{(c+dx)^{3/2}\left(2bcx\left(2bc^2-5ad^2\right)+ad\left(bc^2-7ad^2\right)\right)}{ab\sqrt{a-bx^2}}}{6ab}+\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{3d\left(\frac{1}{3}\left(\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)+\frac{8}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-3ad^2\right)\right)}{2ab}+\frac{(c+dx)^{3/}}{}}{6ab}$$

$$\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 509$$

$$\frac{\frac{3d\left(\frac{1}{3}\left(\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)+\frac{8}{3}c\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-3ad^2\right)\right)}{2ab}+\cdots}{6ab}$$

$$\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 508

$$\frac{\frac{3d\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)+\frac{8}{3}c\sqrt{a-bx^2}\cdots\right)}{2ab}\cdots}{6ab}$$

$$\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 327

$$\frac{\frac{3d\left(\frac{1}{3}\left(\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)+\frac{8}{3}c\sqrt{a-b\cdots}\right)}{2ab}\cdots}{6ab}$$

$$\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 512

$$\frac{\frac{3d\left(\frac{1}{3}\left(\frac{4c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)+\cdots\right)}{2ab}\cdots}{6ab}$$

$$\frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 511

$$\frac{3d\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \cdots}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\cdots\right)\cdots\right)}{2ab} \cdots \Big/ 6ab + \frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 321

$$\frac{3d\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\cdots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\cdots\right)\cdots\right)}{2ab} \cdots \Big/ 6ab + \frac{(c+dx)^{7/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

input

```
Int[(c + d*x)^(9/2)/(a - b*x^2)^(5/2),x]
```

output

```
((a*d + b*c*x)*(c + d*x)^(7/2))/(3*a*b*(a - b*x^2)^(3/2)) + (((c + d*x)^(3
/2)*(a*d*(b*c^2 - 7*a*d^2) + 2*b*c*(2*b*c^2 - 5*a*d^2)*x))/(a*b*Sqrt[a - b
*x^2]) + (3*d*((8*c*(b*c^2 - 3*a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + (
(2*Sqrt[a]*(4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a^2*d^4)*Sqrt[c + d*x]*Sqrt[1
- (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d
)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]
*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (8*Sqrt[a]*c*(b*c^2 - 3*a*d^2)*(b*c^2
- a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2
)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt
[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/3))/(2*a*
b))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. 2(396) = 792.

Time = 5.34 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.85

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{\left(\frac{\left(a^2d^4+6b\,c^2d^2a+b^2c^4\right)x}{3a\,b^4}+\frac{4cd\left(a\,d^2+b\,c^2\right)}{3b^4}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{\left(x^2-\frac{a}{b}\right)^2}-\frac{2(-bdx-bc)\left(-\frac{\left(9a^2d^4+15b\,c^2d^2a-4b^2c^4\right)}{12a^2b^3}\right.}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc}}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(9/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((1/3*(a^2*d^4+6
*a*b*c^2*d^2+b^2*c^4)/a/b^4*x+4/3*c*d*(a*d^2+b*c^2)/b^4)*(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/12*(9*a^2*d^4+15*a*b*c^2*
d^2-4*b^2*c^4)/a^2/b^3*x-1/12*(27*a*d^2+b*c^2)*c*d/a/b^3)/((x^2-a/b)*(-b*d
*x-b*c))^(1/2)+2*(5*c*d^4/b^2-2/3*c*(9*a^2*d^4+4*a*b*c^2*d^2-b^2*c^4)/a^2/
b^2+1/12/b^2*d^2*(27*a*d^2+b*c^2)*c/a+1/6/b^2*c*(9*a^2*d^4+15*a*b*c^2*d^2-
4*b^2*c^4)/a^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/
(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF
(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2))+2*(d^5/b^2+1/12*(9*a^2*d^4+15*a*b*c^2*d^2-4*b^2*c^4)*d
/a^2/b^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-
1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+
1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b
)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2))
)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = -\frac{2\,(2\,a^2b^2c^5-9\,a^3bc^3d^2+39\,a^4cd^4+(2\,b^4c^5-9\,ab^3c^3d^2+39\,a^2b^2cd^4)x^4-2\,(2\,ab^3c^5-9\,a^2b^2c^3d^2+39\,a^3b}{}$$

input
```
integrate((d*x+c)^(9/2)/(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*(2*(2*a^2*b^2*c^5 - 9*a^3*b*c^3*d^2 + 39*a^4*c*d^4 + (2*b^4*c^5 - 9*
a*b^3*c^3*d^2 + 39*a^2*b^2*c*d^4)*x^4 - 2*(2*a*b^3*c^5 - 9*a^2*b^2*c^3*d^2
 + 39*a^3*b*c*d^4)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^
2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(4*a
^2*b^2*c^4*d - 15*a^3*b*c^2*d^3 - 21*a^4*d^5 + (4*b^4*c^4*d - 15*a*b^3*c^2
*d^3 - 21*a^2*b^2*d^5)*x^4 - 2*(4*a*b^3*c^4*d - 15*a^2*b^2*c^2*d^3 - 21*a^
3*b*d^5)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8
/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)
/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(7*a^
2*b^2*c^3*d^2 - 19*a^3*b*c*d^4 - (4*b^4*c^4*d - 15*a*b^3*c^2*d^3 - 9*a^2*b
^2*d^5)*x^3 + (a*b^3*c^3*d^2 + 27*a^2*b^2*c*d^4)*x^2 + (6*a*b^3*c^4*d - 3*
a^2*b^2*c^2*d^3 - 7*a^3*b*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a^2*b^5
*d*x^4 - 2*a^3*b^4*d*x^2 + a^4*b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(9/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{9}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(9/2)/(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(9/2)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{9}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(9/2)/(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(9/2)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(c+d\,x)^{9/2}}{(a-b\,x^2)^{5/2}}\,dx$$

input
```
int((c + d*x)^(9/2)/(a - b*x^2)^(5/2),x)
```

output
```
int((c + d*x)^(9/2)/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{9/2}}{(a-bx^2)^{5/2}}\,dx = \text{too large to display}$$

input

```
int((d*x+c)^(9/2)/(-b*x^2+a)^(5/2),x)
```

output

```
(21*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(
a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a -
 b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2
)*b**2*d**2*x**6),x)*a**5*c*d**6 - 39*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(
sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a -
b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2
)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*a**4*b*c**3*d**4 -
42*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a
 - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a -
b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)
*b**2*d**2*x**6),x)*a**4*b*c*d**6*x**2 + 15*sqrt(a - b*x**2)*int(sqrt(c +
d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqr
t(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a -
b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*a**3*b**2*c**
5*d**2 + 78*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2
 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*
sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a
- b*x**2)*b**2*d**2*x**6),x)*a**3*b**2*c**3*d**4*x**2 + 21*sqrt(a - b*x**2
)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d*
*2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**...
```

3.336 $\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 414

$$\begin{aligned}\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx &= \frac{(ad+bcx)(c+dx)^{5/2}}{3ab\,(a-bx^2)^{3/2}}\\
&+\frac{\sqrt{c+dx}\,(ad(3bc^2-5ad^2)+2bc(2bc^2-3ad^2)\,x)}{6a^2b^2\sqrt{a-bx^2}}\\
&+\frac{2c(bc^2-2ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3a^{3/2}b^{3/2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\\
&-\frac{(4b^2c^4-9abc^2d^2+5a^2d^4)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{5/2}\sqrt{c+dx}\sqrt{a-bx^2}}\end{aligned}$$

output
```
1/3*(b*c*x+a*d)*(d*x+c)^(5/2)/a/b/(-b*x^2+a)^(3/2)+1/6*(d*x+c)^(1/2)*(a*d*
(-5*a*d^2+3*b*c^2)+2*b*c*(-3*a*d^2+2*b*c^2)*x)/a^2/b^2/(-b*x^2+a)^(1/2)+2/
3*c*(-2*a*d^2+b*c^2)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1
/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/
2))/a^(3/2)/b^(3/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+
a)^(1/2)-1/6*(5*a^2*d^4-9*a*b*c^2*d^2+4*b^2*c^4)*(b^(1/2)*(d*x+c)/(b^(1/2)
*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))
^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^
(5/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.48 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{(c+dx)\left(5a^3d^3+4b^3c^3x^3+a^2bd(-5c^2+2cdx-7d^2x^2)-ab^2cx(6c^2+cdx+8d^2x^2)\right)}{a^2b^2(a-bx^2)^2}+\frac{4cd^2\sqrt{-c+\frac{\sqrt{}}{\sqrt{}}}}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(7/2)/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(-(((c + d*x)*(5*a^3*d^3 + 4*b^3*c^3*x^3 + a^2*b*d*(-5*c^
2 + 2*c*d*x - 7*d^2*x^2) - a*b^2*c*x*(6*c^2 + c*d*x + 8*d^2*x^2)))/(a^2*b^
2*(a - b*x^2)^2)) + (4*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - 2*a*d
^2)*(-a + b*x^2) - (4*I)*Sqrt[b]*c*(b^(3/2)*c^3 - Sqrt[a]*b*c^2*d - 2*a*Sq
rt[b]*c*d^2 + 2*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqr
t[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*Ar
cSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*
d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*d*(4*b^(3/2)*c^3 - Sqrt[a]*b*c^2*d
 - 8*a*Sqrt[b]*c*d^2 + 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c +
 Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a^2*b^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]*(-a + b*x^2))))/(6*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {495, 27, 684, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}dx$$

$$\Big\downarrow \ 495$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{3ab(a-bx^2)^{3/2}}-\frac{\int -\frac{(c+dx)^{3/2}(4bc^2-bdxc-5ad^2)}{2(a-bx^2)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(c+dx)^{3/2}(4bc^2-bdxc-5ad^2)}{(a-bx^2)^{3/2}}dx}{6ab}+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab(a-bx^2)^{3/2}}$$

$$\Big\downarrow \ 684$$

$$\frac{\frac{\sqrt{c+dx}(2bcx(2bc^2-3ad^2)+ad(3bc^2-5ad^2))}{ab\sqrt{a-bx^2}}-\frac{\int \frac{d\left(ad\left(bc^2-5ad^2\right)+4bc\left(bc^2-2ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab}}{6ab}+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab(a-bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sqrt{c+dx}(2bcx(2bc^2-3ad^2)+ad(3bc^2-5ad^2))}{ab\sqrt{a-bx^2}}-\frac{d\int \frac{ad\left(bc^2-5ad^2\right)+4bc\left(bc^2-2ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab}}{6ab}+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab(a-bx^2)^{3/2}}$$

$$\Big\downarrow \ 600$$

$$\frac{\frac{\sqrt{c+dx}(2bcx(2bc^2-3ad^2)+ad(3bc^2-5ad^2))}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{4bc\left(bc^2-2ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ab}}{6ab}+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab(a-bx^2)^{3/2}}$$

$$\Big\downarrow \ 509$$

$$\frac{\frac{\sqrt{c+dx}\left(2bcx\left(2bc^2-3ad^2\right)+ad\left(3bc^2-5ad^2\right)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{4bc\sqrt{1-\frac{bx^2}{a}}\left(bc^2-2ad^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ab}}{6ab}+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 508

$$\frac{\frac{\sqrt{c+dx}\left(2bcx\left(2bc^2-3ad^2\right)+ad\left(3bc^2-5ad^2\right)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)\int\frac{\sqrt{1-\frac{d\left(1-\cdots\right)}{\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}\cdots\right.}}\cdots}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}\cdots\right.}{2ab}\cdots}{6ab}\cdots+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 327

$$\frac{\frac{\sqrt{c+dx}\left(2bcx\left(2bc^2-3ad^2\right)+ad\left(3bc^2-5ad^2\right)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.\cdots}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}\cdots\right.}{2ab}\cdots}{6ab}\cdots+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 512

$$\frac{\frac{\sqrt{c+dx}\left(2bcx\left(2bc^2-3ad^2\right)+ad\left(3bc^2-5ad^2\right)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(\cdots}{\sqrt{a}\cdots}}\cdots}\cdots\right.}{2ab}\cdots}{6ab}\cdots+\frac{(c+dx)^{5/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 511

$$\frac{\frac{\sqrt{c+dx}\left(2bcx\left(2bc^2-3ad^2\right)+ad\left(3bc^2-5ad^2\right)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{2ab}}{6ab}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 321

$$\frac{\frac{\sqrt{c+dx}\left(2bcx\left(2bc^2-3ad^2\right)+ad\left(3bc^2-5ad^2\right)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{\cdots}}{\cdots}\right.}{2ab}}{6ab}$$

$$\frac{(c+dx)^{5/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

input

```
Int[(c + d*x)^(7/2)/(a - b*x^2)^(5/2),x]
```

output

```
((a*d + b*c*x)*(c + d*x)^(5/2))/(3*a*b*(a - b*x^2)^(3/2)) + ((Sqrt[c + d*x
]*(a*d*(3*b*c^2 - 5*a*d^2) + 2*b*c*(2*b*c^2 - 3*a*d^2)*x))/(a*b*Sqrt[a - b
*x^2]) - (d*((-8*Sqrt[a]*Sqrt[b]*c*(b*c^2 - 2*a*d^2)*Sqrt[c + d*x]*Sqrt[1
- (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d
)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqr
t[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(4*b*c^2 - 5*a*d^2)*(b*c^2 - a*d^2)
*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/S
qrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(2*a*b))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(344) = 688$.

Time = 4.67 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.95

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{\left(\frac{\left(3a\,d^2+b\,c^2\right)cx}{3b^3a}+\frac{d\left(a\,d^2+3b\,c^2\right)}{3b^4}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{\left(x^2-\frac{a}{b}\right)^2}-\frac{2(-bdx-bc)\left(-\frac{\left(2a\,d^2-b\,c^2\right)cx}{3b^2a^2}-\frac{\left(7a\,d^2+b\,c^2\right)d}{12a\,b^3}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(7/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((1/3*(3*a*d^2+b
*c^2)/b^3*c/a*x+1/3*d*(a*d^2+3*b*c^2)/b^4)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1
/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/3*(2*a*d^2-b*c^2)/b^2*c/a^2*x-1/12*(7*a
*d^2+b*c^2)*d/a/b^3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(d^4/b^2-1/6/b^2*(7*
a^2*d^4+9*a*b*c^2*d^2-4*b^2*c^4)/a^2+1/12/b^2*d^2*(7*a*d^2+b*c^2)/a+2/3/b*
c^2*(2*a*d^2-b*c^2)/a^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b
)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*
EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))+2/3*(2*a*d^2-b*c^2)*c*d/a^2/b*(c/d-1/b*(a*b)^
(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1
/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx =$$

$$-\frac{(4\,a^2b^2c^4-11\,a^3bc^2d^2+15\,a^4d^4+(4\,b^4c^4-11\,ab^3c^2d^2+15\,a^2b^2d^4)x^4-2\,(4\,ab^3c^4-11\,a^2b^2c^2d^2+15\,a^3b}{}$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*a^2*b^2*c^4 - 11*a^3*b*c^2*d^2 + 15*a^4*d^4 + (4*b^4*c^4 - 11*a*
b^3*c^2*d^2 + 15*a^2*b^2*d^4)*x^4 - 2*(4*a*b^3*c^4 - 11*a^2*b^2*c^2*d^2 +
15*a^3*b*d^4)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(a^2*b^2
*c^3*d - 2*a^3*b*c*d^3 + (b^4*c^3*d - 2*a*b^3*c*d^3)*x^4 - 2*(a*b^3*c^3*d
- 2*a^2*b^2*c*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2
+ 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d))
 - 3*(5*a^2*b^2*c^2*d^2 - 5*a^3*b*d^4 - 4*(b^4*c^3*d - 2*a*b^3*c*d^3)*x^3
+ (a*b^3*c^2*d^2 + 7*a^2*b^2*d^4)*x^2 + 2*(3*a*b^3*c^3*d - a^2*b^2*c*d^3)*
x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a^2*b^5*d*x^4 - 2*a^3*b^4*d*x^2 + a^4*
b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(7/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{7}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(7/2)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx=\int \frac{(dx+c)^{\frac{7}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(7/2)/(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(7/2)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx=\int \frac{(c+d\,x)^{7/2}}{(a-b\,x^2)^{5/2}}\,dx$$

input
```
int((c + d*x)^(7/2)/(a - b*x^2)^(5/2),x)
```

output
```
int((c + d*x)^(7/2)/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{7/2}}{(a-bx^2)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((d*x+c)^(7/2)/(-b*x^2+a)^(5/2),x)
```

output
```
(3*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a
 - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a -
b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)
*b**2*d**2*x**6),x)*a**6*c*d**8 - 9*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sq
rt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*
x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*
b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*a**5*b*c**3*d**6 - 6*
sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a -
b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x
**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b*
*2*d**2*x**6),x)*a**5*b*c*d**8*x**2 - 81*sqrt(a - b*x**2)*int(sqrt(c + d*x
)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a
 - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x
**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*a**4*b**2*c**5*d
**4 + 18*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 -
sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqr
t(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b
*x**2)*b**2*d**2*x**6),x)*a**4*b**2*c**3*d**6*x**2 + 3*sqrt(a - b*x**2)*in
t(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x
**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x*...
```

3.337 $\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 385

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx = \frac{(ad+bcx)(c+dx)^{3/2}}{3ab\,(a-bx^2)^{3/2}}+\frac{\sqrt{c+dx}\,(acd+(4bc^2-3ad^2)\,x)}{6a^2b\sqrt{a-bx^2}}$$
$$+\frac{(4bc^2-3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{3/2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$-\frac{2c(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3a^{3/2}b^{3/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(b*c*x+a*d)*(d*x+c)^(3/2)/a/b/(-b*x^2+a)^(3/2)+1/6*(d*x+c)^(1/2)*(a*c*
d+(-3*a*d^2+4*b*c^2)*x)/a^2/b/(-b*x^2+a)^(1/2)+1/6*(-3*a*d^2+4*b*c^2)*(d*x
+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1
/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(3/2)/(b^(1
/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-2/3*c*(-a*d^2+b*
c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*Ellip
ticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c
+a^(1/2)*d))^(1/2))/a^(3/2)/b^(3/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.21 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(-4b^2c^2x^3+a^2d(3c-dx)+abx\left(6c^2+cdx+3d^2x^2\right)\right)}{a^2b\left(a-bx^2\right)^2}+\frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(4bc^2-3ad^2\right)\left(-a+bx^2\right)-}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(5/2)/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(-4*b^2*c^2*x^3 + a^2*d*(3*c - d*x) + a*b*x*(
6*c^2 + c*d*x + 3*d^2*x^2)))/(a^2*b*(a - b*x^2)^2) + (d^2*Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]]*(4*b*c^2 - 3*a*d^2)*(-a + b*x^2) - I*Sqrt[b]*(4*b^(3/2)*c^3
 - 4*Sqrt[a]*b*c^2*d - 3*a*Sqrt[b]*c*d^2 + 3*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]
/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(
c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c +
 d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[
b]*d*(4*b*c^2 - Sqrt[a]*Sqrt[b]*c*d - 3*a*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] +
x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3
/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sq
rt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a^2*b^2*d*Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]]*(-a + b*x^2))))/(6*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {495, 27, 685, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 495$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}-\frac{\int -\frac{\sqrt{c+dx}\,(4bc^2+bdxc-3ad^2)}{2(a-bx^2)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\,(4bc^2+bdxc-3ad^2)}{(a-bx^2)^{3/2}}dx}{6ab}+\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \; 685$$

$$\frac{\frac{\sqrt{c+dx}\,(x\,(4bc^2-3ad^2)+acd)}{a\sqrt{a-bx^2}}-\frac{\int \frac{bd\left(acd+\left(4bc^2-3ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab}}{6ab}+\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{c+dx}\,(x\,(4bc^2-3ad^2)+acd)}{a\sqrt{a-bx^2}}-\frac{d\int \frac{acd+\left(4bc^2-3ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a}}{6ab}+\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{\sqrt{c+dx}\,(x\,(4bc^2-3ad^2)+acd)}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{\left(4bc^2-3ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a}}{6ab}+\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \; 509$$

$$\frac{\frac{\sqrt{c+dx}\,(x\,(4bc^2-3ad^2)+acd)}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-3ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a}}{6ab}+\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \; 508$$

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4bc^2-3ad^2\right)+acd\right)}{a\sqrt{a-bx^2}}-\frac{d\left(-\frac{4c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}}{6ab}+\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 327

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4bc^2-3ad^2\right)+acd\right)}{a\sqrt{a-bx^2}}-\frac{d\left(-\frac{4c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}}{6ab}+\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 512

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4bc^2-3ad^2\right)+acd\right)}{a\sqrt{a-bx^2}}-\frac{d\left(-\frac{4c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}}{6ab}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 511

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4bc^2-3ad^2\right)+acd\right)}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2\cdots\right.}{\sqrt{b}d\cdots}\right.}{2a}}{6ab}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 321$$

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4bc^2-3ad^2\right)+acd\right)}{a\sqrt{a-bx^2}}-\frac{d\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)\cdots}{\sqrt{b}d\sqrt{a-bx^2}\cdots}\right.}{2a}}{6ab}$$

$$\frac{(c+dx)^{3/2}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x)^(5/2)/(a - b*x^2)^(5/2),x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(3/2))/(3*a*b*(a - b*x^2)^(3/2)) + ((Sqrt[c + d*x
]*(a*c*d + (4*b*c^2 - 3*a*d^2)*x))/(a*Sqrt[a - b*x^2]) - (d*((-2*Sqrt[a]*(
4*b*c^2 - 3*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt
[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqr
t[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
+ (8*Sqrt[a]*c*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[
a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/
Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[
a - b*x^2])))/(2*a))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(315) = 630$.

Time = 4.57 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.94

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{\left(\frac{(a\,d^2+b\,c^2)x}{3a\,b^3}+\frac{2cd}{3b^3}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{\left(x^2-\frac{a}{b}\right)^2}-\frac{2(-bdx-bc)\left(-\frac{(3a\,d^2-4b\,c^2)x}{12a^2b^2}-\frac{cd}{12a\,b^2}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(-\frac{2c\left(a\,d^2-b\,c^2\right)}{3b\,a^2}+\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(5/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((1/3*(a*d^2+b*c
^2)/a/b^3*x+2/3*c*d/b^3)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*
(-b*d*x-b*c)*(-1/12*(3*a*d^2-4*b*c^2)/a^2/b^2*x-1/12*c*d/a/b^2)/((x^2-a/b)
*(-b*d*x-b*c))^(1/2)+2*(-2/3/b*c*(a*d^2-b*c^2)/a^2+1/12/b*d^2*c/a+1/6/b*c*
(3*a*d^2-4*b*c^2)/a^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^
(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))+1/6*d*(3*a*d^2-4*b*c^2)/a^2/b*(c/d-1/b*(a*b)^(1
/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2
)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx =$$

$$-\frac{2\left(2\,a^2bc^3-3\,a^3cd^2+\left(2\,b^3c^3-3\,ab^2cd^2\right)x^4-2\left(2\,ab^2c^3-3\,a^2bcd^2\right)x^2\right)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\left(bc^2+3\right.}{3\,bd}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*(2*(2*a^2*b*c^3 - 3*a^3*c*d^2 + (2*b^3*c^3 - 3*a*b^2*c*d^2)*x^4 - 2*
(2*a*b^2*c^3 - 3*a^2*b*c*d^2)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c
^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/
d) + 3*(4*a^2*b*c^2*d - 3*a^3*d^3 + (4*b^3*c^2*d - 3*a*b^2*d^3)*x^4 - 2*(4
*a*b^2*c^2*d - 3*a^2*b*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3
*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/
3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x
 + c)/d)) - 3*(a*b^2*c*d^2*x^2 + 3*a^2*b*c*d^2 - (4*b^3*c^2*d - 3*a*b^2*d^
3)*x^3 + (6*a*b^2*c^2*d - a^2*b*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a
^2*b^4*d*x^4 - 2*a^3*b^3*d*x^2 + a^4*b^2*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^{\frac{5}{2}}}{(a-bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(5/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x)**(5/2)/(a - b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(5/2)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(c+d\,x)^{5/2}}{(a-b\,x^2)^{5/2}}\,dx$$

input
```
int((c + d*x)^(5/2)/(a - b*x^2)^(5/2),x)
```

output
```
int((c + d*x)^(5/2)/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}}{(a-bx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^(5/2)/(-b*x^2+a)^(5/2),x)
```

output

```
(sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a -
 b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*
x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b
**2*d**2*x**6),x)*a**3*c*d**4 - 7*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt
(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x*
*2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b*
*2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*a**2*b*c**3*d**2 - sqrt
(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x*
*2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)
*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d
**2*x**6),x)*a**2*b*c*d**4*x**2 + 6*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sq
rt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*
x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*
b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*a*b**2*c**5 + 7*sqrt(
a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**
2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*
a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d*
*2*x**6),x)*a*b**2*c**3*d**2*x**2 - 6*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(
sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a -
b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x...
```

3.338 $\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 362

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx = \frac{(ad+bcx)\sqrt{c+dx}}{3ab(a-bx^2)^{3/2}} - \frac{(ad-4bcx)\sqrt{c+dx}}{6a^2b\sqrt{a-bx^2}} + \frac{2c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3a^{3/2}\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{(4bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{3/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(b*c*x+a*d)*(d*x+c)^(1/2)/a/b/(-b*x^2+a)^(3/2)-1/6*(-4*b*c*x+a*d)*(d*x
+c)^(1/2)/a^2/b/(-b*x^2+a)^(1/2)+2/3*c*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*Ell
ipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)
*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(1/2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*
d))^(1/2)/(-b*x^2+a)^(1/2)-1/6*(-a*d^2+4*b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*
c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(
3/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.67 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)(a^2d-4b^2cx^3+abx(6c+dx))}{a^2b(a-bx^2)^2} + \frac{4cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\,(-a+bx^2)-4i\sqrt{b}c\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(a^2*d - 4*b^2*c*x^3 + a*b*x*(6*c + d*x)))/(a
^2*b*(a - b*x^2)^2) + (4*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)
 - (4*I)*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/
(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*
EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b
]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(-4*Sqrt[b]*c + Sq
rt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqr
t[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt
[a]*d)])/(a^2*b*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(6*Sqrt[c
 + d*x])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {495, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{495}$$

$$\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}-\frac{\int -\frac{4bc^2+3bdxc-ad^2}{2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4bc^2+3bdxc-ad^2}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{-\frac{\int \frac{bd\left(bc^2-ad^2\right)(ad+4bcx)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad-4bcx)}{a\sqrt{a-bx^2}}}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{d\int \frac{ad+4bcx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a}-\frac{\sqrt{c+dx}(ad-4bcx)}{a\sqrt{a-bx^2}}}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{600}$$

$$\frac{-\frac{d\left(\frac{4bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(4bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a}-\frac{\sqrt{c+dx}(ad-4bcx)}{a\sqrt{a-bx^2}}}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{509}$$

$$\frac{-\frac{d\left(\frac{4bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(4bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a}-\frac{\sqrt{c+dx}(ad-4bcx)}{a\sqrt{a-bx^2}}}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{508}$$

$$-\frac{\frac{d\left(-\frac{\left(4bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}-\frac{\sqrt{c+dx}(ad-4bcx)}{a\sqrt{a-bx^2}}}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{327}$$

$$-\frac{\frac{d\left(-\frac{\left(4bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}-\frac{\sqrt{c+dx}(ad-4bcx)}{a\sqrt{a-bx^2}}}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{512}$$

$$-\frac{\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}-\frac{\sqrt{c+dx}(ad-4bcx)}{a\sqrt{a-bx^2}}}{6ab}+\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{511}$$

$$-\frac{-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}}{6ab}$$

$$\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\Big\downarrow$ 321

$$-\frac{-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a}-\frac{\sqrt{c}}{}}{6ab}$$

$$\frac{\sqrt{c+dx}(ad+bcx)}{3ab\left(a-bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x)^(3/2)/(a - b*x^2)^(5/2),x]
```

output
```
((a*d + b*c*x)*Sqrt[c + d*x])/(3*a*b*(a - b*x^2)^(3/2)) + (-(((a*d - 4*b*c
*x)*Sqrt[c + d*x])/(a*Sqrt[a - b*x^2])) - (d*((-8*Sqrt[a]*Sqrt[b]*c*Sqrt[c
 + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(4*b*c^2 - a*d^2)*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt
[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(2*a))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(292) = 584$.

Time = 1.24 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.91

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{\left(\frac{cx}{3a\,b^2}+\frac{d}{3b^3}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{\left(x^2-\frac{a}{b}\right)^2}-\frac{2(-bdx-bc)\left(\frac{cx}{3b\,a^2}-\frac{d}{12a\,b^2}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(-\frac{a\,d^2-4b\,c^2}{6b\,a^2}+\frac{d^2}{12ba}-\frac{2c^2}{3a^2}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((1/3*c/a/b^2*x+
1/3*d/b^3)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(
1/3/b/a^2*c*x-1/12*d/a/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(-1/6/b*(a*d^
2-4*b*c^2)/a^2+1/12/b*d^2/a-2/3*c^2/a^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c
/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a
*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-2/3*c*d/a^2*(c/d-1/b*(a*b)^(1
/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2
)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx =$$

$$-\frac{(4\,a^2bc^2-3\,a^3d^2+(4\,b^3c^2-3\,ab^2d^2)x^4-2\,(4\,ab^2c^2-3\,a^2bd^2)x^2)\sqrt{-bd}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*a^2*b*c^2 - 3*a^3*d^2 + (4*b^3*c^2 - 3*a*b^2*d^2)*x^4 - 2*(4*a*b
^2*c^2 - 3*a^2*b*d^2)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a
*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*
(b^3*c*d*x^4 - 2*a*b^2*c*d*x^2 + a^2*b*c*d)*sqrt(-b*d)*weierstrassZeta(4/3
*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrass
PInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
 1/3*(3*d*x + c)/d)) + 3*(4*b^3*c*d*x^3 - a*b^2*d^2*x^2 - 6*a*b^2*c*d*x -
a^2*b*d^2)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a^2*b^4*d*x^4 - 2*a^3*b^3*d*x^
2 + a^4*b^2*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}}{(a-bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x)**(3/2)/(a - b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx = \int \frac{(c+d\,x)^{3/2}}{(a-b\,x^2)^{5/2}}\,dx$$

input
```
int((c + d*x)^(3/2)/(a - b*x^2)^(5/2),x)
```

output
```
int((c + d*x)^(3/2)/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}}{(a-bx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x)
```

output
```
( - sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(
a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a -
 b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2
)*b**2*d**2*x**6),x)*a**2*c*d**2 + 3*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(s
qrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b
*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)
*b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*a*b*c**3 + sqrt(a -
b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a
**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*
d**2*x**4 + sqrt(a - b*x**2)*b**2*c**2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x
**6),x)*a*b*c*d**2*x**2 - 3*sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b
*x**2)*a**2*c**2 - sqrt(a - b*x**2)*a**2*d**2*x**2 - 2*sqrt(a - b*x**2)*a*
b*c**2*x**2 + 2*sqrt(a - b*x**2)*a*b*d**2*x**4 + sqrt(a - b*x**2)*b**2*c**
2*x**4 - sqrt(a - b*x**2)*b**2*d**2*x**6),x)*b**2*c**3*x**2 - 2*sqrt(a - b
*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c + sqrt(a - b*x**2)*a**2*
d*x - 2*sqrt(a - b*x**2)*a*b*c*x**2 - 2*sqrt(a - b*x**2)*a*b*d*x**3 + sqrt
(a - b*x**2)*b**2*c*x**4 + sqrt(a - b*x**2)*b**2*d*x**5),x)*a**2*d**2 + 2*
sqrt(a - b*x**2)*int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c + sqrt(a - b*x
**2)*a**2*d*x - 2*sqrt(a - b*x**2)*a*b*c*x**2 - 2*sqrt(a - b*x**2)*a*b*d*x
**3 + sqrt(a - b*x**2)*b**2*c*x**4 + sqrt(a - b*x**2)*b**2*d*x**5),x)*a...
```

3.339 $\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 389

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx = \frac{x\sqrt{c+dx}}{3a(a-bx^2)^{3/2}} - \frac{\sqrt{c+dx}\,(acd-(4bc^2-3ad^2)\,x)}{6a^2(bc^2-ad^2)\sqrt{a-bx^2}} + \frac{(4bc^2-3ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}\sqrt{b}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{2c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3a^{3/2}\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output

```
1/3*x*(d*x+c)^(1/2)/a/(-b*x^2+a)^(3/2)-1/6*(d*x+c)^(1/2)*(a*c*d-(-3*a*d^2+
4*b*c^2)*x)/a^2/(-a*d^2+b*c^2)/(-b*x^2+a)^(1/2)+1/6*(-3*a*d^2+4*b*c^2)*(d*
x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(
1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(1/2)/(-a*
d^2+b*c^2)/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-
2/3*c*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*Elli
pticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.27 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)(4b^2c^2x^3+a^2d(c+5dx)-abx(6c^2+cdx+3d^2x^2))}{a^2(-bc^2+ad^2)(a-bx^2)^2}+\frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(4bc^2-3ad^2)(-a+bx^2)-i\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[c + d*x]/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(4*b^2*c^2*x^3 + a^2*d*(c + 5*d*x) - a*b*x*(6
*c^2 + c*d*x + 3*d^2*x^2)))/(a^2*(-(b*c^2) + a*d^2)*(a - b*x^2)^2) + (d^2*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(4*b*c^2 - 3*a*d^2)*(-a + b*x^2) - I*Sqrt[b
]*(4*b^(3/2)*c^3 - 4*Sqrt[a]*b*c^2*d - 3*a*Sqrt[b]*c*d^2 + 3*a^(3/2)*d^3)*
Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*
x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] -
 I*Sqrt[a]*Sqrt[b]*d*(4*b*c^2 - Sqrt[a]*Sqrt[b]*c*d - 3*a*d^2)*Sqrt[(d*(Sq
rt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x
))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqr
t[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a^2*b*d*Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)*(-a + b*x^2))))/(6*Sqrt[c + d
*x])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {494, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{\left(a-bx^2\right)^{5/2}}dx$$

$\big\downarrow$ 494

$$\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}-\frac{\int -\frac{4c+3dx}{2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{3a}$$

$\big\downarrow$ 27

$$\frac{\int \frac{4c+3dx}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{6a}+\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 686

$$\frac{-\frac{\int \frac{bd\left(acd+\left(4bc^2-3ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}\left(acd-x\left(4bc^2-3ad^2\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}}{6a}+\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{-\frac{d\int \frac{acd+\left(4bc^2-3ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}\left(acd-x\left(4bc^2-3ad^2\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}}{6a}+\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 600

$$\frac{-\frac{d\left(\frac{\left(4bc^2-3ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}\left(acd-x\left(4bc^2-3ad^2\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}}{6a}+\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 509

$$\frac{-\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-3ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}\left(acd-x\left(4bc^2-3ad^2\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}}{6a}+\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\big\downarrow$ 508

$$\frac{-\frac{d\left(-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(acd-x\left(4bc^2-3ad^2\right)\right)}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a}+\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\downarrow$ 327

$$\frac{-\frac{d\left(-\frac{4c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(acd-x\left(4bc^2-3ad^2\right)\right)}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a}+\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\downarrow$ 512

$$\frac{-\frac{d\left(-\frac{4c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(acd-x\left(4bc^2-3a\right.\right.}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a}$$

$$\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$\downarrow$ 511

$$\frac{-\frac{d\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a(bc^2-ad^2)}}{6a}$$

$$\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 321$$

$$\frac{-\frac{d\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2-3ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a(bc^2-ad^2)}}{6a}$$

$$\frac{x\sqrt{c+dx}}{3a\left(a-bx^2\right)^{3/2}}$$

input

```
Int[Sqrt[c + d*x]/(a - b*x^2)^(5/2),x]
```

output

```
(x*Sqrt[c + d*x])/(3*a*(a - b*x^2)^(3/2)) + (-((Sqrt[c + d*x]*(a*c*d - (4*
b*c^2 - 3*a*d^2)*x))/(a*(b*c^2 - a*d^2)*Sqrt[a - b*x^2])) - (d*((-2*Sqrt[a
]*(4*b*c^2 - 3*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[S
qrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(
Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2
]) + (8*Sqrt[a]*c*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sq
rt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a
]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sq
rt[a - b*x^2])))/(2*a*(b*c^2 - a*d^2)))/(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(319) = 638$.

Time = 1.18 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.98

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{x\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3b^2a\left(x^2-\frac{a}{b}\right)^2}-\frac{2(-bdx-bc)\left(\frac{\left(3a\,d^2-4b\,c^2\right)x}{12a^2\left(a\,d^2-b\,c^2\right)b}+\frac{dc}{12a\left(a\,d^2-b\,c^2\right)b}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(\frac{2c}{3a^2}-\frac{d^2c}{12a\left(a\,d^2-b\,c^2\right)}-\frac{c\left(3a\right.}{6a^2\left(\right.}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*(1/3/b^2/a*x*(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(1/12*(3*a*d^2-
4*b*c^2)/a^2/(a*d^2-b*c^2)/b*x+1/12*d*c/a/(a*d^2-b*c^2)/b)/((x^2-a/b)*(-b*
d*x-b*c))^(1/2)+2*(2/3*c/a^2-1/12*d^2*c/a/(a*d^2-b*c^2)-1/6*c*(3*a*d^2-4*b
*c^2)/a^2/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b
)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*
EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))-1/6*d*(3*a*d^2-4*b*c^2)/(a*d^2-b*c^2)/a^2*(c/
d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/
2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*Elli
pticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)
^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx =$$

$$-\frac{2\left(2\,a^2bc^3-3\,a^3cd^2+(2\,b^3c^3-3\,ab^2cd^2)x^4-2\,(2\,ab^2c^3-3\,a^2bcd^2)x^2\right)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\right.}{3\,bd^{\cdots}}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*(2*(2*a^2*b*c^3 - 3*a^3*c*d^2 + (2*b^3*c^3 - 3*a*b^2*c*d^2)*x^4 - 2*
(2*a*b^2*c^3 - 3*a^2*b*c*d^2)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c
^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/
d) + 3*(4*a^2*b*c^2*d - 3*a^3*d^3 + (4*b^3*c^2*d - 3*a*b^2*d^3)*x^4 - 2*(4
*a*b^2*c^2*d - 3*a^2*b*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3
*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/
3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x
 + c)/d)) - 3*(a*b^2*c*d^2*x^2 - a^2*b*c*d^2 - (4*b^3*c^2*d - 3*a*b^2*d^3)
*x^3 + (6*a*b^2*c^2*d - 5*a^2*b*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a
^4*b^2*c^2*d - a^5*b*d^3 + (a^2*b^4*c^2*d - a^3*b^3*d^3)*x^4 - 2*(a^3*b^3*
c^2*d - a^4*b^2*d^3)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx}}{(a-bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x)/(a - b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+d\,x}}{(a-b\,x^2)^{5/2}}\,dx$$

input
```
int((c + d*x)^(1/2)/(a - b*x^2)^(5/2),x)
```

output
```
int((c + d*x)^(1/2)/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b^3x^6+3a\,b^2x^4-3a^2b\,x^2+a^3}dx$$

input
```
int((d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a**3 - 3*a**2*b*x**2 + 3*a*b**2*x**4
 - b**3*x**6),x)
```

3.340 $\int \frac{1}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 449

$$
\begin{aligned}
\int \frac{1}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx &= -\frac{(ad-bcx)\sqrt{c+dx}}{3a\left(bc^2-ad^2\right)\left(a-bx^2\right)^{3/2}} \\
&- \frac{\sqrt{c+dx}(ad(bc^2-5ad^2)-4bc(bc^2-2ad^2)\,x)}{6a^2\left(bc^2-ad^2\right)^2\sqrt{a-bx^2}} \\
&+ \frac{2\sqrt{b}c(bc^2-2ad^2)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3a^{3/2}\left(bc^2-ad^2\right)^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \\
&- \frac{(4bc^2-5ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}\sqrt{b}\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}
$$

output
```
-1/3*(-b*c*x+a*d)*(d*x+c)^(1/2)/a/(-a*d^2+b*c^2)/(-b*x^2+a)^(3/2)-1/6*(d*x
+c)^(1/2)*(a*d*(-5*a*d^2+b*c^2)-4*b*c*(-2*a*d^2+b*c^2)*x)/a^2/(-a*d^2+b*c^
2)^2/(-b*x^2+a)^(1/2)+2/3*b^(1/2)*c*(-2*a*d^2+b*c^2)*(d*x+c)^(1/2)*(1-b*x^
2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1
/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/(-a*d^2+b*c^2)^2/(b^(1/2)*(d*x
+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-1/6*(-5*a*d^2+4*b*c^2)*(
b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1
/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/a^(3/2)/b^(1/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.85 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{(c+dx)\left(2a(bc^2-ad^2)(ad-bcx)+(a-bx^2)\left(-5a^2d^3-4b^2c^3x+abcd(c+8dx)\right)\right)}{(a-bx^2)^2}+\frac{4cd^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(-(((c + d*x)*(2*a*(b*c^2 - a*d^2)*(a*d - b*c*x) + (a - b
*x^2)*(-5*a^2*d^3 - 4*b^2*c^3*x + a*b*c*d*(c + 8*d*x))))/(a - b*x^2)^2) +
(4*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - 2*a*d^2)*(a - b*x^2) + (4
*I)*Sqrt[b]*c*(b^(3/2)*c^3 - Sqrt[a]*b*c^2*d - 2*a*Sqrt[b]*c*d^2 + 2*a^(3/
2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt
[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[
a]*d)] + I*Sqrt[a]*d*(4*b^(3/2)*c^3 - Sqrt[a]*b*c^2*d - 8*a*Sqrt[b]*c*d^2
+ 5*a^(3/2)*d^3)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a
]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*
c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(6*a^2*(
b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {496, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 496}$$

$$\frac{\int \frac{4bc^2+3bdxc-5ad^2}{2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{3a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a(a-bx^2)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4bc^2+3bdxc-5ad^2}{\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{6a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a(a-bx^2)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 686}$$

$$\frac{-\frac{\int \frac{bd\left(ad\left(bc^2-5ad^2\right)+4bc\left(bc^2-2ad^2\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(bc^2-5ad^2)-4bcx(bc^2-2ad^2))}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a(a-bx^2)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{d\int \frac{ad\left(bc^2-5ad^2\right)+4bc\left(bc^2-2ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(bc^2-5ad^2)-4bcx(bc^2-2ad^2))}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a(a-bx^2)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{-\frac{d\left(\frac{4bc\left(bc^2-2ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(bc^2-5ad^2)-4bcx(bc^2-2ad^2))}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 509}$$

$$\frac{-\frac{d\left(\frac{4bc\sqrt{1-\frac{bx^2}{a}}\left(bc^2-2ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(bc^2-5ad^2)-4bcx(bc^2-2ad^2))}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 508}$$

$$\frac{-\frac{d\left(-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(bc^2-5ad^2)-4bcx(bc^2-2ad^2))}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{-\frac{d\left(-\frac{\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(bc^2-5ad^2)-4bcx(bc^2-2ad^2))}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 512$$

$$-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(}{a}$$

$$6a\left(bc^2-ad^2\right)$$

$$\frac{\sqrt{c+dx}(ad-bcx)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a(bc^2-ad^2)}$$

$$6a\left(bc^2-ad^2\right)$$

$$\frac{\sqrt{c+dx}(ad-bcx)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-5ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{8\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-2ad^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a(bc^2-ad^2)}$$

$$6a\left(bc^2-ad^2\right)$$

$$\frac{\sqrt{c+dx}(ad-bcx)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[1/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
-1/3*((a*d - b*c*x)*Sqrt[c + d*x])/(a*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) +
 (-((Sqrt[c + d*x]*(a*d*(b*c^2 - 5*a*d^2) - 4*b*c*(b*c^2 - 2*a*d^2)*x))/(a
*(b*c^2 - a*d^2)*Sqrt[a - b*x^2])) - (d*((-8*Sqrt[a]*Sqrt[b]*c*(b*c^2 - 2*
a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b
]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]
*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(4*b*c^
2 - 5*a*d^2)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]
*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sq
rt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a
- b*x^2])))/(2*a*(b*c^2 - a*d^2)))/(6*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(379) = 758.

Time = 6.35 (sec) , antiderivative size = 862, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(\frac{\left(-\frac{cx}{3\left(a\,d^2-b\,c^2\right)ba}+\frac{d}{3\left(a\,d^2-b\,c^2\right)b^2}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{\left(x^2-\frac{a}{b}\right)^2}-\frac{2(-bdx-bc)\left(-\frac{c\left(2a\,d^2-b\,c^2\right)x}{3a^2\left(a\,d^2-b\,c^2\right)^2}+\frac{d\left(5a\,d^2-b\,c^2\right)}{12\left(a\,d^2-b\,c^2\right)^2}\right.}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}\right.}{}$
default	Expression too large to display

input
```
int(1/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((-1/3/(a*d^2-b*
c^2)/b*c/a*x+1/3*d/(a*d^2-b*c^2)/b^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(
x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/3*c*(2*a*d^2-b*c^2)/a^2/(a*d^2-b*c^2)^2*x+1/
12*d*(5*a*d^2-b*c^2)/(a*d^2-b*c^2)^2/b/a)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2
*(1/6/(a*d^2-b*c^2)*(5*a*d^2-4*b*c^2)/a^2-1/12*d^2*(5*a*d^2-b*c^2)/(a*d^2-
b*c^2)^2/a+2/3*b*c^2*(2*a*d^2-b*c^2)/a^2/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2/3*d*b*c*(2
*a*d^2-b*c^2)/(a*d^2-b*c^2)^2/a^2*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*
(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*
c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1
/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2)
)/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx =$$

$$-\frac{(4\,a^2b^2c^4-11\,a^3bc^2d^2+15\,a^4d^4+(4\,b^4c^4-11\,ab^3c^2d^2+15\,a^2b^2d^4)x^4-2\,(4\,ab^3c^4-11\,a^2b^2c^2d^2+15\,a^3b}{}$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*a^2*b^2*c^4 - 11*a^3*b*c^2*d^2 + 15*a^4*d^4 + (4*b^4*c^4 - 11*a*
b^3*c^2*d^2 + 15*a^2*b^2*d^4)*x^4 - 2*(4*a*b^3*c^4 - 11*a^2*b^2*c^2*d^2 +
15*a^3*b*d^4)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(a^2*b^2
*c^3*d - 2*a^3*b*c*d^3 + (b^4*c^3*d - 2*a*b^3*c*d^3)*x^4 - 2*(a*b^3*c^3*d
- 2*a^2*b^2*c*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2
+ 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d))
 + 3*(3*a^2*b^2*c^2*d^2 - 7*a^3*b*d^4 + 4*(b^4*c^3*d - 2*a*b^3*c*d^3)*x^3
- (a*b^3*c^2*d^2 - 5*a^2*b^2*d^4)*x^2 - 2*(3*a*b^3*c^3*d - 5*a^2*b^2*c*d^3
)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a^4*b^3*c^4*d - 2*a^5*b^2*c^2*d^3 +
a^6*b*d^5 + (a^2*b^5*c^4*d - 2*a^3*b^4*c^2*d^3 + a^4*b^3*d^5)*x^4 - 2*(a^3
*b^4*c^4*d - 2*a^4*b^3*c^2*d^3 + a^5*b^2*d^5)*x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{1}{(a-bx^2)^{\frac{5}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Integral(1/((a - b*x**2)**(5/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{1}{(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b^3d\,x^7-b^3c\,x^6+3a\,b^2d\,x^5+3a\,b^2c\,x^4-3a^2bd\,x^3-3a^2bc\,x^2+a^3dx+a}$$

input
```
int(1/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a**3*c + a**3*d*x - 3*a**2*b*c*x**2
- 3*a**2*b*d*x**3 + 3*a*b**2*c*x**4 + 3*a*b**2*d*x**5 - b**3*c*x**6 - b**3
*d*x**7),x)
```

3.341 $\int \frac{1}{(c+dx)^{3/2}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 529

$$\begin{aligned}
\int \frac{1}{(c+dx)^{3/2}(a-bx^2)^{5/2}}\,dx &= \frac{2d}{(bc^2-ad^2)\sqrt{c+dx}\,(a-bx^2)^{3/2}} \\
&- \frac{b\sqrt{c+dx}(8acd-(bc^2+7ad^2)\,x)}{3a\,(bc^2-ad^2)^2\,(a-bx^2)^{3/2}} \\
&- \frac{b\sqrt{c+dx}(acd(bc^2-33ad^2)-(4b^2c^4-15abc^2d^2-21a^2d^4)\,x)}{6a^2\,(bc^2-ad^2)^3\,\sqrt{a-bx^2}} \\
&+ \frac{\sqrt{b}(4b^2c^4-15abc^2d^2-21a^2d^4)\,\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}\,(bc^2-ad^2)^3\,\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \\
&- \frac{2\sqrt{b}c(bc^2-3ad^2)\,\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3a^{3/2}\,(bc^2-ad^2)^2\,\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
2*d/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2)-1/3*b*(d*x+c)^(1/2)*(8*a
*c*d-(7*a*d^2+b*c^2)*x)/a/(-a*d^2+b*c^2)^2/(-b*x^2+a)^(3/2)-1/6*b*(d*x+c)^
(1/2)*(a*c*d*(-33*a*d^2+b*c^2)-(-21*a^2*d^4-15*a*b*c^2*d^2+4*b^2*c^4)*x)/a
^2/(-a*d^2+b*c^2)^3/(-b*x^2+a)^(1/2)+1/6*b^(1/2)*(-21*a^2*d^4-15*a*b*c^2*d
^2+4*b^2*c^4)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a
^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(
3/2)/(-a*d^2+b*c^2)^3/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^
2+a)^(1/2)-2/3*b^(1/2)*c*(-3*a*d^2+b*c^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1
/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/(-a*d^2+b
*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.18 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.16

$$\int \frac{1}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx =$$

$$-\frac{\sqrt{a-bx^2}\left(-4b^2c^4d+15abc^2d^3+9a^2d^5+\frac{2ab(-bc^2+ad^2)(c+dx)(bc^2x+ad(-2c+dx))}{(a-bx^2)^2}+\frac{b(c+dx)(4b^2c^4x+3a^2d^3(7c-3dx)-}{-a+bx^2}\right.}{}$$

input
```
Integrate[1/((c + d*x)^(3/2)*(a - b*x^2)^(5/2)),x]
```

output
```
-1/6*(Sqrt[a - b*x^2]*(-4*b^2*c^4*d + 15*a*b*c^2*d^3 + 9*a^2*d^5 + (2*a*b*
(-(b*c^2) + a*d^2)*(c + d*x)*(b*c^2*x + a*d*(-2*c + d*x)))/(a - b*x^2)^2 +
 (b*(c + d*x)*(4*b^2*c^4*x + 3*a^2*d^3*(7*c - 3*d*x) - a*b*c^2*d*(c + 15*d
*x)))/(-a + b*x^2) - (I*b*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(4*b^2*c^4 - 15*a
*b*c^2*d^2 - 21*a^2*d^4)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(
((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSin
h[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(
Sqrt[b]*c - Sqrt[a]*d)])/(d*(-a + b*x^2)) + (I*Sqrt[a]*Sqrt[b]*(4*b^2*c^4
- Sqrt[a]*b^(3/2)*c^3*d - 15*a*b*c^2*d^2 + 33*a^(3/2)*Sqrt[b]*c*d^3 - 21*a
^2*d^4)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt
[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[
a]*d)])/(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(a^2*(b*c^2 - a*d^
2)^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {496, 27, 686, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{5/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 496}$$

$$\frac{\int \frac{4bc^2+5bdxc-7ad^2}{2(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{3a(bc^2-ad^2)}-\frac{ad-bcx}{3a(a-bx^2)^{3/2}\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4bc^2+5bdxc-7ad^2}{(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{6a(bc^2-ad^2)}-\frac{ad-bcx}{3a(a-bx^2)^{3/2}\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 686}$$

$$\frac{\frac{4bcx(bc^2-3ad^2)+ad(7ad^2+bc^2)}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}-\frac{\int -\frac{bd\left(3ad\left(bc^2+7ad^2\right)+4bc\left(bc^2-3ad^2\right)x\right)}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{ab(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{d\int \frac{3ad\left(bc^2+7ad^2\right)+4bc\left(bc^2-3ad^2\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}+\frac{4bcx(bc^2-3ad^2)+ad(7ad^2+bc^2)}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 688$$

$$\frac{\frac{d\left(\frac{2\int -\frac{b\left(acd\left(bc^2-33ad^2\right)+\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a(bc^2-ad^2)}+\frac{4bcx(bc^2-3ad^2)+ad(7ad^2+bc^2)}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{d\left(-\frac{b\int \frac{acd\left(bc^2-33ad^2\right)+\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a(bc^2-ad^2)}+\frac{4bcx(bc^2-3ad^2)+ad(7ad^2+bc^2)}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}-\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{d\left(-\frac{b\left(\frac{\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a(bc^2-ad^2)}+\frac{4bcx\cdots}{a\cdots}}{6a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$\frac{\frac{d\left(-\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$\frac{\frac{d\left(-\frac{b\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(\cdots\right.}{\cdots}\right)}{2a(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 327

$$\frac{d\left(-\frac{b\left(-\frac{4c\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\cdots}{\cdots}\right.}{2a(bc^2-ad^2)}$$

$$6a\left(bc^2-ad^2\right)$$

$$\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 512

$$\frac{d\left(-\frac{b\left(-\frac{4c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{bc^2-ad^2}-\cdots\right.}{2a(bc^2-ad^2)}$$

$$6a\left(bc^2-ad^2\right)$$

$$\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 511

$$d\left(-\frac{b\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\cdots}}\right.}{bc^2-ad^2}\right.$$

$$\frac{\cdots}{2a(bc^2-ad^2)}$$

$$\frac{\cdots}{6a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 321

$$d\left(-\frac{b\left(\frac{8\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\left(bc^2-3ad^2\right)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)E\left(\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{bc^2-ad^2}\right.$$

$$\frac{\cdots}{2a(bc^2-ad^2)}$$

$$\frac{\cdots}{6a\left(bc^2-ad^2\right)}$$

$$\frac{ad-bcx}{3a\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[1/((c + d*x)^(3/2)*(a - b*x^2)^(5/2)),x]
```

output
```
-1/3*(a*d - b*c*x)/(a*(b*c^2 - a*d^2)*Sqrt[c + d*x]*(a - b*x^2)^(3/2)) + (
(a*d*(b*c^2 + 7*a*d^2) + 4*b*c*(b*c^2 - 3*a*d^2)*x)/(a*(b*c^2 - a*d^2)*Sqr
t[c + d*x]*Sqrt[a - b*x^2]) + (d*((-2*(4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a^2
*d^4)*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) - (b*((-2*Sqrt[a]*(
4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a^2*d^4)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]
*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*
c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]
*d)]*Sqrt[a - b*x^2]) + (8*Sqrt[a]*c*(b*c^2 - 3*a*d^2)*(b*c^2 - a*d^2)*Sqr
t[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[
a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(b*c^2 - a*d^2)))/(2
*a*(b*c^2 - a*d^2)))/(6*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. $2(455) = 910$.

Time = 8.01 (sec) , antiderivative size = 1008, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1008
default	Expression too large to display	3158

input
```
int(1/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(-b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((1/3*(a*d^2+b*c
^2)/a/(a*d^2-b*c^2)^2/b*x-2/3*c*d/(a*d^2-b*c^2)^2/b)*(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(1/12*(9*a^2*d^4+15*a*b*c^2*d^2-4
*b^2*c^4)/a^2/(a*d^2-b*c^2)^3*x-1/12*c*d*(21*a*d^2-b*c^2)/a/(a*d^2-b*c^2)^
3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)-2*(-b*d*x^2+a*d)*d^4/(a*d^2-b*c^2)^3/((x
+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(-2/3*b*c*(3*a*d^2-b*c^2)/(a*d^2-b*c^2)^2/a^
2+1/12*b*d^2*c*(21*a*d^2-b*c^2)/a/(a*d^2-b*c^2)^3-1/6*b*c*(9*a^2*d^4+15*a*
b*c^2*d^2-4*b^2*c^4)/a^2/(a*d^2-b*c^2)^3-b*c*d^4/(a*d^2-b*c^2)^3)*(c/d-1/b
*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^
(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)
^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-
1/12*b*d*(9*a^2*d^4+15*a*b*c^2*d^2-4*b^2*c^4)/a^2/(a*d^2-b*c^2)^3-d^5*b/(a
*d^2-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(
-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b
*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c
/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(
1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(455) = 910$.

Time = 0.15 (sec) , antiderivative size = 1158, normalized size of antiderivative = 2.19

$$\int \frac{1}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*(2*(2*a^2*b^2*c^6 - 9*a^3*b*c^4*d^2 + 39*a^4*c^2*d^4 + (2*b^4*c^5*d
 - 9*a*b^3*c^3*d^3 + 39*a^2*b^2*c*d^5)*x^5 + (2*b^4*c^6 - 9*a*b^3*c^4*d^2 +
 39*a^2*b^2*c^2*d^4)*x^4 - 2*(2*a*b^3*c^5*d - 9*a^2*b^2*c^3*d^3 + 39*a^3*b
*c*d^5)*x^3 - 2*(2*a*b^3*c^6 - 9*a^2*b^2*c^4*d^2 + 39*a^3*b*c^2*d^4)*x^2 +
 (2*a^2*b^2*c^5*d - 9*a^3*b*c^3*d^3 + 39*a^4*c*d^5)*x)*sqrt(-b*d)*weierstr
assPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^
3), 1/3*(3*d*x + c)/d) + 3*(4*a^2*b^2*c^5*d - 15*a^3*b*c^3*d^3 - 21*a^4*c*
d^5 + (4*b^4*c^4*d^2 - 15*a*b^3*c^2*d^4 - 21*a^2*b^2*d^6)*x^5 + (4*b^4*c^5
*d - 15*a*b^3*c^3*d^3 - 21*a^2*b^2*c*d^5)*x^4 - 2*(4*a*b^3*c^4*d^2 - 15*a^
2*b^2*c^2*d^4 - 21*a^3*b*d^6)*x^3 - 2*(4*a*b^3*c^5*d - 15*a^2*b^2*c^3*d^3
 - 21*a^3*b*c*d^5)*x^2 + (4*a^2*b^2*c^4*d^2 - 15*a^3*b*c^2*d^4 - 21*a^4*d^6
)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^
3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
 -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(5*a^2*b^2*c^4
*d^2 - 25*a^3*b*c^2*d^4 - 12*a^4*d^6 + (4*b^4*c^4*d^2 - 15*a*b^3*c^2*d^4 -
 21*a^2*b^2*d^6)*x^4 + 4*(b^4*c^5*d - 4*a*b^3*c^3*d^3 + 3*a^2*b^2*c*d^5)*x
^3 - (7*a*b^3*c^4*d^2 - 36*a^2*b^2*c^2*d^4 - 35*a^3*b*d^6)*x^2 - 2*(3*a*b^
3*c^5*d - 10*a^2*b^2*c^3*d^3 + 7*a^3*b*c*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x
 + c))/(a^4*b^3*c^7*d - 3*a^5*b^2*c^5*d^3 + 3*a^6*b*c^3*d^5 - a^7*c*d^7 +
(a^2*b^5*c^6*d^2 - 3*a^3*b^4*c^4*d^4 + 3*a^4*b^3*c^2*d^6 - a^5*b^2*d^8)...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{3/2}\,(a-bx^2)^{5/2}}\,dx = \int \frac{1}{(a-bx^2)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)**(3/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Integral(1/((a - b*x**2)**(5/2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{3/2}(a-bx^2)^{5/2}}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(5/2)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^{3/2}(a-bx^2)^{5/2}}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(5/2)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{3/2}(a-bx^2)^{5/2}}\,dx=\int \frac{1}{(a-b\,x^2)^{5/2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int(1/((a - b*x^2)^(5/2)*(c + d*x)^(3/2)),x)
```

output
```
int(1/((a - b*x^2)^(5/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx = \left(\int \frac{}{\sqrt{-b\,x^2+a}\,a^2c^4-2\sqrt{-b\,x^2+a}\,a^2c^2d^2x^2+\sqrt{-b\,x^2+a}\,a^2d^4x^4-2\sqrt{}\right.$$
$$+\left(\int \frac{\sqrt{dx}}{\sqrt{-b\,x^2+a}\,a^2c^4-2\sqrt{-b\,x^2+a}\,a^2c^2d^2x^2+\sqrt{-b\,x^2+a}\,a^2d^4x^4-2\sqrt{-b\,x^2+a}\,ab\,c^4x^2+4\sqrt{-b\,x^2+}}\right.$$
$$-2\left(\int \frac{\sqrt{}}{\sqrt{-b\,x^2+a}\,a^2c^4-2\sqrt{-b\,x^2+a}\,a^2c^2d^2x^2+\sqrt{-b\,x^2+a}\,a^2d^4x^4-2\sqrt{-b\,x^2+a}\,ab\,c^4x^2+4\sqrt{-b\,x^2}}\right.$$

input

```
int(1/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x)
```

output

```
int(sqrt(c + d*x)/(sqrt(a - b*x**2)*a**2*c**4 - 2*sqrt(a - b*x**2)*a**2*c*
*2*d**2*x**2 + sqrt(a - b*x**2)*a**2*d**4*x**4 - 2*sqrt(a - b*x**2)*a*b*c*
*4*x**2 + 4*sqrt(a - b*x**2)*a*b*c**2*d**2*x**4 - 2*sqrt(a - b*x**2)*a*b*d
**4*x**6 + sqrt(a - b*x**2)*b**2*c**4*x**4 - 2*sqrt(a - b*x**2)*b**2*c**2*
d**2*x**6 + sqrt(a - b*x**2)*b**2*d**4*x**8),x)*c**2 + int((sqrt(c + d*x)*
x**2)/(sqrt(a - b*x**2)*a**2*c**4 - 2*sqrt(a - b*x**2)*a**2*c**2*d**2*x**2
 + sqrt(a - b*x**2)*a**2*d**4*x**4 - 2*sqrt(a - b*x**2)*a*b*c**4*x**2 + 4*
sqrt(a - b*x**2)*a*b*c**2*d**2*x**4 - 2*sqrt(a - b*x**2)*a*b*d**4*x**6 + s
qrt(a - b*x**2)*b**2*c**4*x**4 - 2*sqrt(a - b*x**2)*b**2*c**2*d**2*x**6 +
sqrt(a - b*x**2)*b**2*d**4*x**8),x)*d**2 - 2*int((sqrt(c + d*x)*x)/(sqrt(a
 - b*x**2)*a**2*c**4 - 2*sqrt(a - b*x**2)*a**2*c**2*d**2*x**2 + sqrt(a - b
*x**2)*a**2*d**4*x**4 - 2*sqrt(a - b*x**2)*a*b*c**4*x**2 + 4*sqrt(a - b*x*
*2)*a*b*c**2*d**2*x**4 - 2*sqrt(a - b*x**2)*a*b*d**4*x**6 + sqrt(a - b*x**
2)*b**2*c**4*x**4 - 2*sqrt(a - b*x**2)*b**2*c**2*d**2*x**6 + sqrt(a - b*x*
*2)*b**2*d**4*x**8),x)*c*d
```

3.342 $\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(d-ex)}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}}\right)}{2^{2/3}\sqrt{3}d^{2/3}e}-\frac{\log(d+ex)}{2\ 2^{2/3}d^{2/3}e}+\frac{\log\left(3\ 2^{2/3}d^{2/3}e^2(d-ex)-6de^2\sqrt[3]{d^2+3e^2x^2}\right)}{2\ 2^{2/3}d^{2/3}e}$$

output
```
-1/6*arctan(1/3*3^(1/2)+1/3*2^(2/3)*(-e*x+d)*3^(1/2)/d^(1/3)/(3*e^2*x^2+d^
2)^(1/3))*2^(1/3)*3^(1/2)/d^(2/3)/e-1/4*ln(e*x+d)*2^(1/3)/d^(2/3)/e+1/4*ln
(3*2^(2/3)*d^(2/3)*e^2*(-e*x+d)-6*d*e^2*(3*e^2*x^2+d^2)^(1/3))*2^(1/3)/d^(
2/3)/e
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.72

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx$$

$$=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}}{2^{2/3}d-2^{2/3}ex+\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}}\right)+2\log\left(\sqrt{e}\left(-2^{2/3}d+2^{2/3}ex+2\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}\right)\right)-\log}{6\ 2^{2/}}$$

input
```
Integrate[1/((d + e*x)*(d^2 + 3*e^2*x^2)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(d^2 + 3*e^2*x^2)^(1/3))/(2^(2/3)*d - 2
^(2/3)*e*x + d^(1/3)*(d^2 + 3*e^2*x^2)^(1/3))] + 2*Log[Sqrt[e]*(-(2^(2/3)*
d) + 2^(2/3)*e*x + 2*d^(1/3)*(d^2 + 3*e^2*x^2)^(1/3))] - Log[e*(2^(1/3)*d^
2 - 2*2^(1/3)*d*e*x + 2^(1/3)*e^2*x^2 + 2^(2/3)*d^(4/3)*(d^2 + 3*e^2*x^2)^
(1/3) - 2^(2/3)*d^(1/3)*e*x*(d^2 + 3*e^2*x^2)^(1/3) + 2*d^(2/3)*(d^2 + 3*e
^2*x^2)^(2/3))])/(6*2^(2/3)*d^(2/3)*e)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{501\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx$$

$$\Big\downarrow \; 501$$

$$-\frac{\arctan\left(\frac{2^{2/3}(d-ex)}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{d^2+3e^2x^2}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}e}-\frac{\log(d+ex)}{2\ 2^{2/3}d^{2/3}e}+\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{d}e^2\sqrt[3]{d^2+3e^2x^2}+3de^2-3e^3x\right)}{2\ 2^{2/3}d^{2/3}e}$$

input
```
Int[1/((d + e*x)*(d^2 + 3*e^2*x^2)^(1/3)),x]
```

output
```
-(ArcTan[1/Sqrt[3] + (2^(2/3)*(d - e*x))/(Sqrt[3]*d^(1/3)*(d^2 + 3*e^2*x^2
)^(1/3))]/(2^(2/3)*Sqrt[3]*d^(2/3)*e)) - Log[d + e*x]/(2*2^(2/3)*d^(2/3)*e
) + Log[3*d*e^2 - 3*e^3*x - 3*2^(1/3)*d^(1/3)*e^2*(d^2 + 3*e^2*x^2)^(1/3)]
/(2*2^(2/3)*d^(2/3)*e)
```

Defintions of rubi rules used

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

Maple [F]

$$\int \frac{1}{(ex+d)\left(3e^2x^2+d^2\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x)
```

output
```
int(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(122) = 244$.

Time = 22.92 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.23

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx = -\frac{4\sqrt{3}d\sqrt{4^{\frac{1}{3}}\left(d^2\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}\left(3e^2x^2+d^2\right)^{\frac{2}{3}}\left(d^2\right)^{\frac{2}{3}}(ex-d)+4^{\frac{1}{3}}\left(e^3x^3+3de^2x^2+3d^2ex+d^3\right)\left(d^2\right)^{\frac{1}{3}}+4\left(de^2x^2-2d^2ex+d^3\right)(3\right.}{6\left(de^3x^3-9d^2e^2x^2+3d^3ex-3d^4\right)}$$

input
```
integrate(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x, algorithm="fricas")
```

output
```
-1/24*(4*sqrt(3)*d*sqrt(4^(1/3)*(d^2)^(1/3))*arctan(1/6*sqrt(3)*(2*4^(2/3)
*(3*e^2*x^2 + d^2)^(2/3)*(d^2)^(2/3)*(e*x - d) + 4^(1/3)*(e^3*x^3 + 3*d*e^
2*x^2 + 3*d^2*e*x + d^3)*(d^2)^(1/3) + 4*(d*e^2*x^2 - 2*d^2*e*x + d^3)*(3*
e^2*x^2 + d^2)^(1/3))*sqrt(4^(1/3)*(d^2)^(1/3))/(d*e^3*x^3 - 9*d^2*e^2*x^2
 + 3*d^3*e*x - 3*d^4)) + 4^(2/3)*(d^2)^(2/3)*log((4^(2/3)*(3*e^2*x^2 + d^2
)^(2/3)*(d^2)^(2/3) + 4^(1/3)*(e^2*x^2 - 2*d*e*x + d^2)*(d^2)^(1/3) - 2*(3
*e^2*x^2 + d^2)^(1/3)*(d*e*x - d^2))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*4^(2/3
)*(d^2)^(2/3)*log((4^(1/3)*(d^2)^(1/3)*(e*x - d) + 2*(3*e^2*x^2 + d^2)^(1/
3)*d)/(e*x + d)))/(d^2*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx = \int \frac{1}{(d+ex)\,\sqrt[3]{d^2+3e^2x^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(3*e**2*x**2+d**2)**(1/3),x)
```

output
```
Integral(1/((d + e*x)*(d**2 + 3*e**2*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx = \int \frac{1}{(3\,e^2x^2+d^2)^{\frac{1}{3}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((3*e^2*x^2 + d^2)^(1/3)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx = \int \frac{1}{(3\,e^2x^2+d^2)^{\frac{1}{3}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((3*e^2*x^2 + d^2)^(1/3)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx = \int \frac{1}{\left(d^2+3\,e^2\,x^2\right)^{1/3}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((d^2 + 3*e^2*x^2)^(1/3)*(d + e*x)),x)
```

output
```
int(1/((d^2 + 3*e^2*x^2)^(1/3)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\sqrt[3]{d^2+3e^2x^2}}\,dx = \int \frac{1}{(3e^2x^2+d^2)^{\frac{1}{3}}\,d+(3e^2x^2+d^2)^{\frac{1}{3}}\,ex}dx$$

input
```
int(1/(e*x+d)/(3*e^2*x^2+d^2)^(1/3),x)
```

output
```
int(1/((d**2 + 3*e**2*x**2)**(1/3)*d + (d**2 + 3*e**2*x**2)**(1/3)*e*x),x)
```

3.343 $\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 558

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx = \frac{1}{30}(2+3x)^2\left(4+27x^2\right)^{2/3}+\frac{4}{35}(7+4x)\left(4+27x^2\right)^{2/3}-\frac{96x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)}$$

$$+\frac{16\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+2}}\right.\right.}{21\ 3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$

$$-\frac{32\ 2^{5/6}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+\left(4+27x^2\right)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+27}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27}}\right.\right.}{63\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$

output
```
1/30*(2+3*x)^2*(27*x^2+4)^(2/3)+4/35*(7+4*x)*(27*x^2+4)^(2/3)-96*x/(7*2^(2
/3)*(1-3^(1/2))-7*(27*x^2+4)^(1/3))+16/63*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2)
)*(2^(2/3)-(27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(27*x^2+4)^(1/3)+(27*x^2+
4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)*EllipticE((2^(2/
3)*(1+3^(1/2))-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)),2*
I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(
27*x^2+4)^(1/3))^2)^(1/2)-32/189*2^(5/6)*(2^(2/3)-(27*x^2+4)^(1/3))*((2*2^
(1/3)+2^(2/3)*(27*x^2+4)^(1/3)+(27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(27*
x^2+4)^(1/3))^2)^(1/2)*EllipticF((2^(2/3)*(1+3^(1/2))-(27*x^2+4)^(1/3))/(2
^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(-(2^(2/3)-(
27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.09

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx = \frac{1}{210}\left(4+27x^2\right)^{2/3}\left(196+180x+63x^2\right) + \frac{16}{7}\sqrt[3]{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{27x^2}{4}\right)$$

input
```
Integrate[(2 + 3*x)^3/(4 + 27*x^2)^(1/3),x]
```

output
```
((4 + 27*x^2)^(2/3)*(196 + 180*x + 63*x^2))/210 + (16*2^(1/3)*x*Hypergeome
tric2F1[1/3, 1/2, 3/2, (-27*x^2)/4])/7
```

Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {497, 27, 676, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^3}{\sqrt[3]{27x^2+4}}dx$$

$$\Big\downarrow \text{497}$$

$$\frac{1}{90}\int \frac{288(3x+1)(3x+2)}{\sqrt[3]{27x^2+4}}dx+\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{16}{5}\int \frac{(3x+1)(3x+2)}{\sqrt[3]{27x^2+4}}dx+\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2$$

$$\Big\downarrow \text{676}$$

$$\frac{16}{5}\left(\frac{10}{7}\int \frac{1}{\sqrt[3]{27x^2+4}}dx+\frac{1}{7}\left(27x^2+4\right)^{2/3}x+\frac{1}{4}\left(27x^2+4\right)^{2/3}\right)+\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2$$

$$\Big\downarrow \text{233}$$

$$\frac{16}{5}\left(\frac{5\sqrt{x^2}\int \frac{\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}}{7\sqrt{3}x}+\frac{1}{7}\left(27x^2+4\right)^{2/3}x+\frac{1}{4}\left(27x^2+4\right)^{2/3}\right)+$$
$$\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2$$

$$\Big\downarrow \text{833}$$

$$\frac{16}{5}\left(\frac{5\sqrt{x^2}\left(2^{2/3}\left(1+\sqrt{3}\right)\int \frac{1}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}\right)}{7\sqrt{3}x}+\frac{1}{7}\left(27x^2+4\right)^{2/3}x\right.$$
$$\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2$$

$$\Big\downarrow \text{760}$$

$$\frac{16}{5}\left(\frac{5\sqrt{x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{\cdots}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{\cdots}\right.}}}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{\cdots}{\left(2^{2/3}\cdots\right.}}}\right.}{7\sqrt{3}x}\right.$$

$$\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2$$

$\big\downarrow$ 2418

$$\frac{16}{5}\left(\frac{5\sqrt{x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{\cdots}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{\cdots}}\right.\right.}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}}\right.}{\cdots}\right.$$

$$\frac{1}{30}\left(27x^2+4\right)^{2/3}(3x+2)^2$$

input
```
Int[(2 + 3*x)^3/(4 + 27*x^2)^(1/3),x]
```

output
```
((2 + 3*x)^2*(4 + 27*x^2)^(2/3))/30 + (16*((4 + 27*x^2)^(2/3)/4 + (x*(4 +
27*x^2)^(2/3))/7 + (5*Sqrt[x^2]*((-6*Sqrt[3]*Sqrt[x^2])/(2^(2/3)*(1 - Sqrt
[3]) - (4 + 27*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27
*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(
2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^
(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) - (4
+ 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - (2*2^
(1/3)*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[
(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1
- Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]
) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3*3^(3/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(
2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)])))/(7*Sqrt[3]*x)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

method	result
risch	$\frac{(63x^2+180x+196)(27x^2+4)^{\frac{2}{3}}}{210}+\frac{16\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{27x^2}{4}\right)}{7}$
meijerg	$4\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{27x^2}{4}\right)+\frac{27\,2^{\frac{1}{3}}x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{3},2\right],[3],-\frac{27x^2}{4}\right)}{8}+9\,2^{\frac{1}{3}}x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{3}\right.\right.$

input
```
int((3*x+2)^3/(27*x^2+4)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/210*(63*x^2+180*x+196)*(27*x^2+4)^(2/3)+16/7*2^(1/3)*x*hypergeom([1/3,1/
2],[3/2],-27/4*x^2)
```

Fricas [F]

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx=\int \frac{(3\,x+2)^3}{(27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
integral((27*x^3 + 54*x^2 + 36*x + 8)/(27*x^2 + 4)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.15

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx=9\cdot\sqrt[3]{2}x^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{27x^2e^{i\pi}}{4}\right)+\frac{3x^2(27x^2+4)^{\frac{2}{3}}}{10}+4\cdot\sqrt[3]{2}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{i\pi}}{4}\right)+\frac{14(27x^2+4)^{\frac{2}{3}}}{15}$$

input
```
integrate((2+3*x)**3/(27*x**2+4)**(1/3),x)
```

output
```
9*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(I*pi)/4) + 3*x
**2*(27*x**2 + 4)**(2/3)/10 + 4*2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x*
*2*exp_polar(I*pi)/4) + 14*(27*x**2 + 4)**(2/3)/15
```

Maxima [F]

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx = \int \frac{(3\,x+2)^3}{(27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)^3/(27*x^2 + 4)^(1/3), x)
```

Giac [F]

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx = \int \frac{(3\,x+2)^3}{(27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate((3*x + 2)^3/(27*x^2 + 4)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx=\int \frac{{\left(3\,x+2\right)}^3}{{\left(27\,x^2+4\right)}^{1/3}} \,d x$$

input
```
int((3*x + 2)^3/(27*x^2 + 4)^(1/3),x)
```

output
```
int((3*x + 2)^3/(27*x^2 + 4)^(1/3), x)
```

Reduce [F]

$$\int \frac{(2+3x)^3}{\sqrt[3]{4+27x^2}}\,dx=27\left(\int \frac{x^{3}}{\left(27x^{2}+4\right)^{\frac{1}{3}}}d x \right)+54\left(\int \frac{x^{2}}{\left(27x^{2}+4\right)^{\frac{1}{3}}}d x \right)+36\left(\int \frac{x}{\left(27x^{2}+4\right)^{\frac{1}{3}}}d x \right)+8\left(\int \frac{1}{\left(27x^{2}+4\right)^{\frac{1}{3}}}d x \right)$$

input
```
int((2+3*x)^3/(27*x^2+4)^(1/3),x)
```

output
```
27*int(x**3/(27*x**2 + 4)**(1/3),x) + 54*int(x**2/(27*x**2 + 4)**(1/3),x)
+ 36*int(x/(27*x**2 + 4)**(1/3),x) + 8*int(1/(27*x**2 + 4)**(1/3),x)
```

3.344 $\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 536

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \frac{1}{21}(7+3x)\left(4+27x^2\right)^{2/3} - \frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)}$$
$$+\frac{4\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}\right.\right.}{7\,3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$
$$-\frac{8\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}\right.\right.}{21\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$

output
```
1/21*(7+3*x)*(27*x^2+4)^(2/3)-72*x/(7*2^(2/3)*(1-3^(1/2))-7*(27*x^2+4)^(1/
3))+4/21*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(2/3)-(27*x^2+4)^(1/3))*((2*
2^(1/3)+2^(2/3)*(27*x^2+4)^(1/3)+(27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(2
7*x^2+4)^(1/3))^2)^(1/2)*EllipticE((2^(2/3)*(1+3^(1/2))-(27*x^2+4)^(1/3))/
(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)
-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)-8/63*2^
(5/6)*(2^(2/3)-(27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(27*x^2+4)^(1/3)+(27*
x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)*EllipticF((2
^(2/3)*(1+3^(1/2))-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)
),2*I-I*3^(1/2))*3^(3/4)/x/(-(2^(2/3)-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2
))-(27*x^2+4)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \frac{1}{21}\left((7+3x)\left(4+27x^2\right)^{2/3} + 36\sqrt[3]{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{27x^2}{4}\right)\right)$$

input
```
Integrate[(2 + 3*x)^2/(4 + 27*x^2)^(1/3),x]
```

output
```
((7 + 3*x)*(4 + 27*x^2)^(2/3) + 36*2^(1/3)*x*Hypergeometric2F1[1/3, 1/2, 3
/2, (-27*x^2)/4])/21
```

Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {497, 27, 455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^2}{\sqrt[3]{27x^2+4}}\,dx$$

$$\Big\downarrow \text{497}$$

$$\frac{1}{63}\int \frac{108(5x+2)}{\sqrt[3]{27x^2+4}}dx+\frac{1}{21}\left(27x^2+4\right)^{2/3}(3x+2)$$

$$\Big\downarrow \text{27}$$

$$\frac{12}{7}\int \frac{5x+2}{\sqrt[3]{27x^2+4}}dx+\frac{1}{21}\left(27x^2+4\right)^{2/3}(3x+2)$$

$$\Big\downarrow \text{455}$$

$$\frac{12}{7}\left(2\int \frac{1}{\sqrt[3]{27x^2+4}}dx+\frac{5}{36}\left(27x^2+4\right)^{2/3}\right)+\frac{1}{21}\left(27x^2+4\right)^{2/3}(3x+2)$$

$$\Big\downarrow \text{233}$$

$$\frac{12}{7}\left(\frac{\sqrt{x^2}\int \frac{\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}}{\sqrt{3}x}+\frac{5}{36}\left(27x^2+4\right)^{2/3}\right)+\frac{1}{21}\left(27x^2+4\right)^{2/3}(3x+2)$$

$$\Big\downarrow \text{833}$$

$$\frac{12}{7}\left(\frac{\sqrt{x^2}\left(2^{2/3}\left(1+\sqrt{3}\right)\int \frac{1}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}\right)}{\sqrt{3}x}+\frac{5}{36}\left(27x^2+4\right)^{2/3}\right)+$$

$$\frac{1}{21}\left(27x^2+4\right)^{2/3}(3x+2)$$

$$\Big\downarrow \text{760}$$

$$\frac{12}{7}\left(\frac{\sqrt{x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{2}\right.}}}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{}{\left(2^{2/3}\right.}}}\right.}{\sqrt{3}x}\right.$$

$$\frac{1}{21}\left(27x^2+4\right)^{2/3}(3x+2)$$

$\downarrow$ 2418

$$\frac{12}{7}\left(\frac{\sqrt{x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{}}\right.\right.}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}}\right.}{}\right.$$

$$\frac{1}{21}\left(27x^2+4\right)^{2/3}(3x+2)$$

input
```
Int[(2 + 3*x)^2/(4 + 27*x^2)^(1/3),x]
```

output
```
((2 + 3*x)*(4 + 27*x^2)^(2/3))/21 + (12*((5*(4 + 27*x^2)^(2/3))/36 + (Sqrt
[x^2]*((-6*Sqrt[3]*Sqrt[x^2])/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))
 + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/
3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3
]) - (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 +
 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(3*3^(1/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*
(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1
 + Sqrt[3])*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 +
27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^
(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2
^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*S
qrt[x^2]*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4
 + 27*x^2)^(1/3))^2)])))/(Sqrt[3]*x)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.07

method	result
risch	$\frac{(7+3x)\left(27x^2+4\right)^{\frac{2}{3}}}{21}+\frac{12\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{27x^2}{4}\right)}{7}$
meijerg	$2\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{27x^2}{4}\right)+\frac{3\,2^{\frac{1}{3}}x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{3}{2}\right],\left[\frac{5}{2}\right],-\frac{27x^2}{4}\right)}{2}+3\,2^{\frac{1}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3}\right.\right.$

input
```
int((3*x+2)^2/(27*x^2+4)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/21*(7+3*x)*(27*x^2+4)^(2/3)+12/7*2^(1/3)*x*hypergeom([1/3,1/2],[3/2],-27
/4*x^2)
```

Fricas [F]

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \int \frac{(3\,x+2)^2}{(27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
integral((9*x^2 + 12*x + 4)/(27*x^2 + 4)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \frac{3\cdot\sqrt[3]{2}x^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{27x^2e^{i\pi}}{4}\right)}{2} + 2\cdot\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{i\pi}}{4}\right) + \frac{(27x^2+4)^{\frac{2}{3}}}{3}$$

input
```
integrate((2+3*x)**2/(27*x**2+4)**(1/3),x)
```

output
```
3*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(I*pi)/4)/2 + 2
*2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(I*pi)/4) + (27*x**
2 + 4)**(2/3)/3
```

Maxima [F]

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \int \frac{(3\,x+2)^2}{(27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)^2/(27*x^2 + 4)^(1/3), x)
```

Giac [F]

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \int \frac{(3\,x+2)^2}{(27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate((3*x + 2)^2/(27*x^2 + 4)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = \int \frac{(3\,x+2)^2}{(27\,x^2+4)^{1/3}}\,dx$$

input
```
int((3*x + 2)^2/(27*x^2 + 4)^(1/3),x)
```

output
```
int((3*x + 2)^2/(27*x^2 + 4)^(1/3), x)
```

Reduce [F]

$$\int \frac{(2+3x)^2}{\sqrt[3]{4+27x^2}}\,dx = 9\left(\int \frac{x^2}{(27x^2+4)^{\frac{1}{3}}}dx\right) + 12\left(\int \frac{x}{(27x^2+4)^{\frac{1}{3}}}dx\right) + 4\left(\int \frac{1}{(27x^2+4)^{\frac{1}{3}}}dx\right)$$

input
```
int((2+3*x)^2/(27*x^2+4)^(1/3),x)
```

output
```
9*int(x**2/(27*x**2 + 4)**(1/3),x) + 12*int(x/(27*x**2 + 4)**(1/3),x) + 4*
int(1/(27*x**2 + 4)**(1/3),x)
```

3.345 $\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 529

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx = \frac{1}{12}\left(4+27x^2\right)^{2/3} - \frac{6x}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}$$

$$+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4+27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{4+27x^2}}{2^{2/3}(1-\sqrt{3})-\sqrt[3]{4+27x^2}}\right.\right.}{3\,3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4+27x^2}\right)^2}}}$$

$$-\frac{2\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4+27x^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{4+27x^2}}{2^{2/3}(1-\sqrt{3})-\sqrt[3]{4+27x^2}}\right.\right.}{9\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}(1-\sqrt{3})-\sqrt[3]{4+27x^2}\right)^2}}}$$

output
```
1/12*(27*x^2+4)^(2/3)-6*x/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))+1/9*2^(1/
3)*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(2/3)-(27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3
)*(27*x^2+4)^(1/3)+(27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)
)^2)^(1/2)*EllipticE((2^(2/3)*(1+3^(1/2))-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^
(1/2))-(27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)-(27*x^2+4)^(1
/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)-2/27*2^(5/6)*(2^(2/3)
-(27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(27*x^2+4)^(1/3)+(27*x^2+4)^(2/3))/
(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)*EllipticF((2^(2/3)*(1+3^(1
/2))-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)),2*I-I*3^(1/2
))*3^(3/4)/x/(-(2^(2/3)-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^
(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.08

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx=\frac{1}{12}\left(4+27x^2\right)^{2/3}+\sqrt[3]{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{27x^2}{4}\right)$$

input
```
Integrate[(2 + 3*x)/(4 + 27*x^2)^(1/3),x]
```

output
```
(4 + 27*x^2)^(2/3)/12 + 2^(1/3)*x*Hypergeometric2F1[1/3, 1/2, 3/2, (-27*x^
2)/4]
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{\sqrt[3]{27x^2+4}}\,dx$$

$$\Big\downarrow \text{ 455}$$

$$2\int \frac{1}{\sqrt[3]{27x^2+4}}dx+\frac{1}{12}\left(27x^2+4\right)^{2/3}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{\sqrt{x^2}\int \frac{\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}}{\sqrt{3}x}+\frac{1}{12}\left(27x^2+4\right)^{2/3}$$

$$\Big\downarrow \text{ 833}$$

$$\frac{\sqrt{x^2}\left(2^{2/3}\left(1+\sqrt{3}\right)\int \frac{1}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}\right)}{\sqrt{3}x}+\frac{1}{12}\left(27x^2+4\right)^{2/3}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\sqrt{x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2 - \cdots}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2 \cdots}\right.}}}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{2^{2/3}- \cdots}{\left(2^{2/3}\left(1-\sqrt{3}\right.\right.\cdots}}} \cdots\right)}{\sqrt{3}x} \cdots$$

$$\frac{1}{12}\left(27x^2+4\right)^{2/3}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{\sqrt{x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2 \cdots}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2 \cdots}}\right.\right.\cdots}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}} \cdots\right)}{\cdots}$$

$$\frac{1}{12}\left(27x^2+4\right)^{2/3}$$

input
```
Int[(2 + 3*x)/(4 + 27*x^2)^(1/3),x]
```

output
```
(4 + 27*x^2)^(2/3)/12 + (Sqrt[x^2]*((-6*Sqrt[3]*Sqrt[x^2])/(2^(2/3)*(1 - S
qrt[3]) - (4 + 27*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 +
 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2
)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[
(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 +
 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) -
(4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - (2
*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sq
rt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*
(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt
[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(3*3^(3/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3)
)/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)])))/(Sqrt[3]*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

method	result	size
risch	$\frac{(27x^2+4)^{\frac{2}{3}}}{12}+2^{\frac{1}{3}}x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{27x^2}{4}\right)$	29
meijerg	$2^{\frac{1}{3}}x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{27x^2}{4}\right)+\frac{3\,2^{\frac{1}{3}}x^2\,\text{hypergeom}\left(\left[\frac{1}{3},1\right],[2],-\frac{27x^2}{4}\right)}{4}$	37

input
```
int((3*x+2)/(27*x^2+4)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/12*(27*x^2+4)^(2/3)+2^(1/3)*x*hypergeom([1/3,1/2],[3/2],-27/4*x^2)
```

Fricas [F]

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx=\int \frac{3\,x+2}{\left(27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
integral((3*x + 2)/(27*x^2 + 4)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.07

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx=\sqrt[3]{2}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{i\pi}}{4}\right)+\frac{(27x^2+4)^{\frac{2}{3}}}{12}$$

input
```
integrate((2+3*x)/(27*x**2+4)**(1/3),x)
```

output
```
2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(I*pi)/4) + (27*x**2
 + 4)**(2/3)/12
```

Maxima [F]

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx=\int \frac{3\,x+2}{\left(27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)/(27*x^2 + 4)^(1/3), x)
```

Giac [F]

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx = \int \frac{3\,x+2}{\left(27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*x)/(27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate((3*x + 2)/(27*x^2 + 4)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx = \frac{\left(27\,x^2+4\right)^{2/3}}{12} + 2^{1/3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{27\,x^2}{4}\right)$$

input
```
int((3*x + 2)/(27*x^2 + 4)^(1/3),x)
```

output
```
(27*x^2 + 4)^(2/3)/12 + 2^(1/3)*x*hypergeom([1/3, 1/2], 3/2, -(27*x^2)/4)
```

Reduce [F]

$$\int \frac{2+3x}{\sqrt[3]{4+27x^2}}\,dx = 3\left(\int \frac{x}{\left(27x^2+4\right)^{\frac{1}{3}}}dx\right) + 2\left(\int \frac{1}{\left(27x^2+4\right)^{\frac{1}{3}}}dx\right)$$

input
```
int((2+3*x)/(27*x^2+4)^(1/3),x)
```

output
```
3*int(x/(27*x**2 + 4)**(1/3),x) + 2*int(1/(27*x**2 + 4)**(1/3),x)
```

3.346 $\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}} - \frac{\log(2+3x)}{12\sqrt[3]{2}} + \frac{\log\left(54\sqrt[3]{2}(2-3x)-108\sqrt[3]{4+27x^2}\right)}{12\sqrt[3]{2}}$$

output
```
1/36*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-3*x)*3^(1/2)/(27*x^2+4)^(1/3))*2^(
2/3)*3^(1/2)-1/24*ln(2+3*x)*2^(2/3)+1/24*ln(54*2^(1/3)*(2-3*x)-108*(27*x^2
+4)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.52

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt[3]{2}-3\sqrt[3]{2}x+\sqrt[3]{4+27x^2}}{\sqrt{3}\sqrt[3]{4+27x^2}}\right) - 2\log\left(-2\sqrt[3]{2}+3\sqrt[3]{2}x+2\sqrt[3]{4+27x^2}\right) + \log\left(4\ 2^{2/3} - 12\ 2^{\cdots}\right.}{36\sqrt[3]{2}}$$

input
```
Integrate[1/((2 + 3*x)*(4 + 27*x^2)^(1/3)),x]
```

output
```
-1/36*(2*Sqrt[3]*ArcTan[(2*2^(1/3) - 3*2^(1/3)*x + (4 + 27*x^2)^(1/3))/(Sq
rt[3]*(4 + 27*x^2)^(1/3))] - 2*Log[-2*2^(1/3) + 3*2^(1/3)*x + 2*(4 + 27*x^
2)^(1/3)] + Log[4*2^(2/3) - 12*2^(2/3)*x + 9*2^(2/3)*x^2 + 4*(4 + 27*x^2)^
(2/3) + 2*(2 - 3*x)*(8 + 54*x^2)^(1/3)])/2^(1/3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {501}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)\sqrt[3]{27x^2+4}}dx$$

$$\Big\downarrow \; 501$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{27x^2+4}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(-27\ 2^{2/3}\sqrt[3]{27x^2+4}-81x+54\right)}{12\sqrt[3]{2}}-\frac{\log(3x+2)}{12\sqrt[3]{2}}$$

input
```
Int[1/((2 + 3*x)*(4 + 27*x^2)^(1/3)),x]
```

output
```
-1/6*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27*x^2)^(1/3))]/
(2^(1/3)*Sqrt[3]) - Log[2 + 3*x]/(12*2^(1/3)) + Log[54 - 81*x - 27*2^(2/3)
*(4 + 27*x^2)^(1/3)]/(12*2^(1/3))
```

Defintions of rubi rules used

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

Maple [F]

$$\int \frac{1}{(3x+2)\,(27x^2+4)^{\frac{1}{3}}}dx$$

input
```
int(1/(3*x+2)/(27*x^2+4)^(1/3),x)
```

output
```
int(1/(3*x+2)/(27*x^2+4)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(77) = 154$.

Time = 1.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.94

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = -\frac{1}{6} \cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{1}{6}}\left(4\cdot 2^{\frac{2}{3}}(27\,x^2+4)^{\frac{2}{3}}(3\,x-2)+2^{\frac{1}{3}}(27\,x^3+54\,x^2+36\,x+8)+4\,(27\,x^2+4)^{\frac{1}{3}}(9\,x\right.}{3\,(9\,x^3-54\,x^2+12\,x-8)}\right) - \frac{1}{72}\cdot 2^{\frac{2}{3}}\log\left(\frac{2\cdot 2^{\frac{2}{3}}(27\,x^2+4)^{\frac{2}{3}}+2^{\frac{1}{3}}(9\,x^2-12\,x+4)-2\,(27\,x^2+4)^{\frac{1}{3}}(3\,x-2)}{9\,x^2+12\,x+4}\right) + \frac{1}{36}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}(3\,x-2)+2\,(27\,x^2+4)^{\frac{1}{3}}}{3\,x+2}\right)$$

input
```
integrate(1/(2+3*x)/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
-1/6*2^(1/6)*sqrt(1/6)*arctan(1/3*2^(1/6)*sqrt(1/6)*(4*2^(2/3)*(27*x^2 + 4
)^(2/3)*(3*x - 2) + 2^(1/3)*(27*x^3 + 54*x^2 + 36*x + 8) + 4*(27*x^2 + 4)^
(1/3)*(9*x^2 - 12*x + 4))/(9*x^3 - 54*x^2 + 12*x - 8)) - 1/72*2^(2/3)*log(
(2*2^(2/3)*(27*x^2 + 4)^(2/3) + 2^(1/3)*(9*x^2 - 12*x + 4) - 2*(27*x^2 + 4
)^(1/3)*(3*x - 2))/(9*x^2 + 12*x + 4)) + 1/36*2^(2/3)*log((2^(1/3)*(3*x -
2) + 2*(27*x^2 + 4)^(1/3))/(3*x + 2))
```

Sympy [F]

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(3x+2)\sqrt[3]{27x^2+4}}\,dx$$

input
```
integrate(1/(2+3*x)/(27*x**2+4)**(1/3),x)
```

output
```
Integral(1/((3*x + 2)*(27*x**2 + 4)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(27\,x^2+4)^{\frac{1}{3}}(3\,x+2)}\,dx$$

input
```
integrate(1/(2+3*x)/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(27\,x^2+4)^{\frac{1}{3}}(3\,x+2)}\,dx$$

input
```
integrate(1/(2+3*x)/(27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(3\,x+2)\,\left(27\,x^2+4\right)^{1/3}}\,dx$$

input
```
int(1/((3*x + 2)*(27*x^2 + 4)^(1/3)),x)
```

output
```
int(1/((3*x + 2)*(27*x^2 + 4)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(2+3x)\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{3\,(27x^2+4)^{\frac{1}{3}}\,x+2\,(27x^2+4)^{\frac{1}{3}}}dx$$

input
```
int(1/(2+3*x)/(27*x^2+4)^(1/3),x)
```

output
```
int(1/(3*(27*x**2 + 4)**(1/3)*x + 2*(27*x**2 + 4)**(1/3)),x)
```

3.347 $\int \frac{1}{(2+3x)^2 \sqrt[3]{4+27x^2}}\, dx$

Optimal result

Integrand size = 19, antiderivative size = 637

$$\int \frac{1}{(2+3x)^2 \sqrt[3]{4+27x^2}}\, dx$$

$$= -\frac{(4+27x^2)^{2/3}}{48(2+3x)} - \frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)} - \frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}}$$

$$+ \frac{\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}\, E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}\right)\right.}{48\ 2^{2/3}3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$

$$- \frac{\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}\right), -\right.}{72\sqrt[6]{2}\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$

$$- \frac{\log(2+3x)}{48\sqrt[3]{2}} + \frac{\log\left(54\sqrt[3]{2}(2-3x)-108\sqrt[3]{4+27x^2}\right)}{48\sqrt[3]{2}}$$

output
```
-1/48*(27*x^2+4)^(2/3)/(2+3*x)-3*x/(16*2^(2/3)*(1-3^(1/2))-16*(27*x^2+4)^(
1/3))+1/144*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-3*x)*3^(1/2)/(27*x^2+4)^(1/
3))*2^(2/3)*3^(1/2)+1/288*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(2/3)-(27*x
^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(27*x^2+4)^(1/3)+(27*x^2+4)^(2/3))/(2^(2/
3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)*EllipticE((2^(2/3)*(1+3^(1/2))-(
27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(
1/4)/x/(-(2^(2/3)-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))
^2)^(1/2)-1/432*2^(5/6)*(2^(2/3)-(27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(27
*x^2+4)^(1/3)+(27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^
(1/2)*EllipticF((2^(2/3)*(1+3^(1/2))-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2)
)-(27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(-(2^(2/3)-(27*x^2+4)^(1/3))/
(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)-1/96*ln(2+3*x)*2^(2/3)+1/9
6*ln(54*2^(1/3)*(2-3*x)-108*(27*x^2+4)^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.69 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.33

$$\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx$$
$$= \frac{-4(4+27x^2)-8\sqrt[3]{3}(2+3x)\sqrt[3]{\frac{-2i\sqrt{3}+9x}{2+3x}}\sqrt[3]{\frac{2i\sqrt{3}+9x}{2+3x}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6+2i\sqrt{3}}{6+9x}\right)+\sqrt[3]{6}\sqrt[3]{\cdots}}{192(2+3x)\sqrt[3]{4+27x^2}}$$

input
```
Integrate[1/((2 + 3*x)^2*(4 + 27*x^2)^(1/3)),x]
```

output
```
(-4*(4 + 27*x^2) - 8*3^(1/3)*(2 + 3*x)*(((-2*I)*Sqrt[3] + 9*x)/(2 + 3*x))^
(1/3)*(((2*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3,
 (6 - (2*I)*Sqrt[3])/(6 + 9*x), (6 + (2*I)*Sqrt[3])/(6 + 9*x)] + 6^(1/3)*(
2*Sqrt[3] - (9*I)*x)^(1/3)*(2 + 3*x)*(-2*I + 3*Sqrt[3]*x)*Hypergeometric2F
1[1/3, 2/3, 5/3, 1/2 + ((3*I)/4)*Sqrt[3]*x])/(192*(2 + 3*x)*(4 + 27*x^2)^(
1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {498, 25, 719, 233, 501, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)^2 \sqrt[3]{27x^2+4}}dx$$

$$\Big\downarrow \text{498}$$

$$-\frac{3}{16}\int -\frac{x+2}{(3x+2)\sqrt[3]{27x^2+4}}dx-\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{16}\int \frac{x+2}{(3x+2)\sqrt[3]{27x^2+4}}dx-\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

$$\Big\downarrow \text{719}$$

$$\frac{3}{16}\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{27x^2+4}}dx+\frac{4}{3}\int \frac{1}{(3x+2)\sqrt[3]{27x^2+4}}dx\right)-\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

$$\Big\downarrow \text{233}$$

$$\frac{3}{16}\left(\frac{4}{3}\int \frac{1}{(3x+2)\sqrt[3]{27x^2+4}}dx+\frac{\sqrt{x^2}\int \frac{\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}}{6\sqrt{3}x}\right)-\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

$$\Big\downarrow \text{501}$$

$$\frac{3}{16}\left(\frac{\sqrt{x^2}\int \frac{\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}}{6\sqrt{3}x}+\frac{4}{3}\left(-\frac{\arctan\left(\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{27x^2+4}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(-27\ 2^{2/3}\sqrt[3]{27x^2+4}-81x -\right.}{12\sqrt[3]{2}}\right.\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

$$\Big\downarrow \text{833}$$

$$\frac{3}{16}\left(\frac{\sqrt{x^2}\left(2^{2/3}(1+\sqrt{3})\int\frac{1}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\int\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}\right)}{6\sqrt{3}x}+\frac{4}{3}\left(-\frac{\arctan\left(\frac{\sqrt[3]{\cdots}}{\sqrt{3}\sqrt[3]{\cdots}}\right)}{6\cdots}\right.\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

$\big\downarrow$ 760

$$\frac{3}{16}\left(\frac{\sqrt{x^2}\left(-\int\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{2\cdots}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{2\cdots}\right.}}}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{\cdots}{\left(2^{2/3}\cdots\right.}}}\right.}{6\sqrt{3}x}\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

$\big\downarrow$ 2418

$$\frac{3}{16}\left(\frac{\sqrt{x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{\cdots}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{\cdots}}\right.\right.}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}}\cdots\right.}{\cdots}\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{48(3x+2)}$$

input
```
Int[1/((2 + 3*x)^2*(4 + 27*x^2)^(1/3)),x]
```

output
```
-1/48*(4 + 27*x^2)^(2/3)/(2 + 3*x) + (3*((Sqrt[x^2]*((-6*Sqrt[3]*Sqrt[x^2]
)/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]
]*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1
/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*
EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1
- Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[x^2]*S
qrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)
^(1/3))^2)]) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 +
27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)
^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(
2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 +
27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) - (
4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)])))/(6*
Sqrt[3]*x) + (4*(-1/6*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 +
 27*x^2)^(1/3))]/(2^(1/3)*Sqrt[3]) - Log[2 + 3*x]/(12*2^(1/3)) + Log[54 -
81*x - 27*2^(2/3)*(4 + 27*x^2)^(1/3)]/(12*2^(1/3))))/3))/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(3x+2)^2 (27x^2+4)^{\frac{1}{3}}} dx$$

input
```
int(1/(3*x+2)^2/(27*x^2+4)^(1/3),x)
```

output
```
int(1/(3*x+2)^2/(27*x^2+4)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}} dx = \int \frac{1}{(27 x^2+4)^{\frac{1}{3}}(3 x+2)^2} dx$$

input
```
integrate(1/(2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
integral((27*x^2 + 4)^(2/3)/(243*x^4 + 324*x^3 + 144*x^2 + 48*x + 16), x)
```

Sympy [F]

$$\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(3x+2)^2\sqrt[3]{27x^2+4}}\,dx$$

input
```
integrate(1/(2+3*x)**2/(27*x**2+4)**(1/3),x)
```

output
```
Integral(1/((3*x + 2)**2*(27*x**2 + 4)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(27\,x^2+4)^{\frac{1}{3}}(3\,x+2)^2}\,dx$$

input
```
integrate(1/(2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^2), x)
```

Giac [F]

$$\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(27\,x^2+4)^{\frac{1}{3}}(3\,x+2)^2}\,dx$$

input
```
integrate(1/(2+3*x)^2/(27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(3\,x+2)^2\,(27\,x^2+4)^{1/3}}\,dx$$

input
```
int(1/((3*x + 2)^2*(27*x^2 + 4)^(1/3)),x)
```

output
```
int(1/((3*x + 2)^2*(27*x^2 + 4)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(2+3x)^2\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{9\,(27x^2+4)^{\frac{1}{3}}\,x^2+12\,(27x^2+4)^{\frac{1}{3}}\,x+4\,(27x^2+4)^{\frac{1}{3}}}dx$$

input
```
int(1/(2+3*x)^2/(27*x^2+4)^(1/3),x)
```

output
```
int(1/(9*(27*x**2 + 4)**(1/3)*x**2 + 12*(27*x**2 + 4)**(1/3)*x + 4*(27*x**
2 + 4)**(1/3)),x)
```

3.348 $\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 659

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx = -\frac{(4+27x^2)^{2/3}}{96(2+3x)^2} - \frac{(4+27x^2)^{2/3}}{96(2+3x)}$$
$$-\frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)} - \frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{4+27x^2}}\right)}{96\sqrt[3]{2}\sqrt{3}}$$
$$+\frac{\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}\right)\right.}{96\,2^{2/3}3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$
$$-\frac{\left(2^{2/3}-\sqrt[3]{4+27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4+27x^2}+(4+27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4+27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}}\right),-\right.}{144\sqrt[6]{2}\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4+27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4+27x^2}\right)^2}}}$$
$$-\frac{\log(2+3x)}{192\sqrt[3]{2}} + \frac{\log\left(54\sqrt[3]{2}(2-3x)-108\sqrt[3]{4+27x^2}\right)}{192\sqrt[3]{2}}$$

output
```
-1/96*(27*x^2+4)^(2/3)/(2+3*x)^2-(27*x^2+4)^(2/3)/(192+288*x)-3*x/(32*2^(2
/3)*(1-3^(1/2))-32*(27*x^2+4)^(1/3))+1/576*arctan(-1/3*3^(1/2)-1/3*2^(1/3)
*(2-3*x)*3^(1/2)/(27*x^2+4)^(1/3))*2^(2/3)*3^(1/2)+1/576*2^(1/3)*(1/2*6^(1
/2)+1/2*2^(1/2))*(2^(2/3)-(27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(27*x^2+4)
^(1/3)+(27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)*E
llipticE((2^(2/3)*(1+3^(1/2))-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x
^2+4)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)-(27*x^2+4)^(1/3))/(2^(2/3
)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)-1/864*2^(5/6)*(2^(2/3)-(27*x^2+4)
^(1/3))*((2*2^(1/3)+2^(2/3)*(27*x^2+4)^(1/3)+(27*x^2+4)^(2/3))/(2^(2/3)*(1
-3^(1/2))-(27*x^2+4)^(1/3))^2)^(1/2)*EllipticF((2^(2/3)*(1+3^(1/2))-(27*x^
2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/
x/(-(2^(2/3)-(27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(27*x^2+4)^(1/3))^2)^(
1/2)-1/384*ln(2+3*x)*2^(2/3)+1/384*ln(54*2^(1/3)*(2-3*x)-108*(27*x^2+4)^(1
/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.67 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.34

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx$$

$$= \frac{-12(4+4x+27x^2+27x^3)-4\sqrt[3]{3}(2+3x)^2\sqrt[3]{\frac{-2i\sqrt{3}+9x}{2+3x}}\sqrt[3]{\frac{2i\sqrt{3}+9x}{2+3x}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{6-2i\sqrt{3}}{6+9x},\frac{6}{}\right.}{384(2+3x)^2\sqrt[3]{4+}}$$

input
```
Integrate[1/((2 + 3*x)^3*(4 + 27*x^2)^(1/3)),x]
```

output
```
(-12*(4 + 4*x + 27*x^2 + 27*x^3) - 4*3^(1/3)*(2 + 3*x)^2*(((-2*I)*Sqrt[3]
+ 9*x)/(2 + 3*x))^(1/3)*(((2*I)*Sqrt[3] + 9*x)/(2 + 3*x))^(1/3)*AppellF1[2
/3, 1/3, 1/3, 5/3, (6 - (2*I)*Sqrt[3])/(6 + 9*x), (6 + (2*I)*Sqrt[3])/(6 +
 9*x)] + 6^(1/3)*(2*Sqrt[3] - (9*I)*x)^(1/3)*(2 + 3*x)^2*(-2*I + 3*Sqrt[3]
*x)*Hypergeometric2F1[1/3, 2/3, 5/3, 1/2 + ((3*I)/4)*Sqrt[3]*x])/(384*(2 +
 3*x)^2*(4 + 27*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {498, 27, 688, 27, 719, 233, 501, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)^3 \sqrt[3]{27x^2+4}}dx$$

$$\Big\downarrow \text{498}$$

$$-\frac{3}{32}\int -\frac{2(2-x)}{(3x+2)^2 \sqrt[3]{27x^2+4}}dx-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{16}\int \frac{2-x}{(3x+2)^2 \sqrt[3]{27x^2+4}}dx-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{688}$$

$$\frac{3}{16}\left(-\frac{1}{144}\int -\frac{24(3x+4)}{(3x+2)\sqrt[3]{27x^2+4}}dx-\frac{\left(27x^2+4\right)^{2/3}}{18(3x+2)}\right)-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{16}\left(\frac{1}{6}\int \frac{3x+4}{(3x+2)\sqrt[3]{27x^2+4}}dx-\frac{\left(27x^2+4\right)^{2/3}}{18(3x+2)}\right)-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{719}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(\int \frac{1}{\sqrt[3]{27x^2+4}}dx+2\int \frac{1}{(3x+2)\sqrt[3]{27x^2+4}}dx\right)-\frac{\left(27x^2+4\right)^{2/3}}{18(3x+2)}\right)-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{233}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(2\int \frac{1}{(3x+2)\sqrt[3]{27x^2+4}}dx+\frac{\sqrt{x^2}\int \frac{\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}}{2\sqrt{3}x}\right)-\frac{\left(27x^2+4\right)^{2/3}}{18(3x+2)}\right)-\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{501}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(\frac{\sqrt{x^2}\int \frac{\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}}{2\sqrt{3}x}+2\left(-\frac{\arctan\left(\frac{\sqrt[3]{2}(2-3x)}{\sqrt{3}\sqrt[3]{27x^2+4}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(-27\ 2^{2/3}\sqrt[3]{27x^2+4}-8\right.}{12\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{833}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(\frac{\sqrt{x^2}\left(2^{2/3}(1+\sqrt{3})\int \frac{1}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}\right)}{2\sqrt{3}x}+2\left(-\frac{\arctan\left(-\right.}{}\right.\right.\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{760}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(\frac{\sqrt{x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{27x^2+4}}{3\sqrt{3}\sqrt{x^2}}d\sqrt[3]{27x^2+4}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\ldots}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\ldots\right.}}}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\ldots}}\right.}{2\sqrt{3}x}\right.\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

$$\Big\downarrow \text{2418}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(\frac{\sqrt{x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{27x^2+4}\right)\sqrt{\frac{\left(27x^2+4\right)^{2/3}+2^{2/3}\sqrt[3]{27x^2+4}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)}{2^{2/3}\left(1-\sqrt{3}\right)}\right.\right.}{3\ 3^{3/4}\sqrt{x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{27x^2+4}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{27x^2+4}\right)^2}}}\right.}{}\right.\right.$$

$$\frac{\left(27x^2+4\right)^{2/3}}{96(3x+2)^2}$$

input
```
Int[1/((2 + 3*x)^3*(4 + 27*x^2)^(1/3)),x]
```

output
```
-1/96*(4 + 27*x^2)^(2/3)/(2 + 3*x)^2 + (3*(-1/18*(4 + 27*x^2)^(2/3)/(2 + 3
*x) + ((Sqrt[x^2]*((-6*Sqrt[3]*Sqrt[x^2])/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27
*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 + 27*x^2)^(1/3))*S
qrt[(2*2^(1/3) + 2^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)
*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqr
t[3]) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3
))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))^2)]) - (2*2^(1/3)*Sqrt[2 -
 Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 + 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2
^(2/3)*(4 + 27*x^2)^(1/3) + (4 + 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (
4 + 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 + 27*x^
2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 + 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(3*3^(3/4)*Sqrt[x^2]*Sqrt[-((2^(2/3) - (4 + 27*x^2)^(1/3))/(2^(2/3)*(1 - S
qrt[3]) - (4 + 27*x^2)^(1/3))^2)])))/(2*Sqrt[3]*x) + 2*(-1/6*ArcTan[1/Sqrt
[3] + (2^(1/3)*(2 - 3*x))/(Sqrt[3]*(4 + 27*x^2)^(1/3))]/(2^(1/3)*Sqrt[3])
- Log[2 + 3*x]/(12*2^(1/3)) + Log[54 - 81*x - 27*2^(2/3)*(4 + 27*x^2)^(1/3
)]/(12*2^(1/3)))/6))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(3x+2)^3 (27x^2+4)^{\frac{1}{3}}}dx$$

input
```
int(1/(3*x+2)^3/(27*x^2+4)^(1/3),x)
```

output
```
int(1/(3*x+2)^3/(27*x^2+4)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(27\,x^2+4)^{\frac{1}{3}}(3\,x+2)^3}\,dx$$

input
```
integrate(1/(2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
integral((27*x^2 + 4)^(2/3)/(729*x^5 + 1458*x^4 + 1080*x^3 + 432*x^2 + 144
*x + 32), x)
```

Sympy [F]

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(3x+2)^3\,\sqrt[3]{27x^2+4}}\,dx$$

input
```
integrate(1/(2+3*x)**3/(27*x**2+4)**(1/3),x)
```

output
```
Integral(1/((3*x + 2)**3*(27*x**2 + 4)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(27\,x^2+4)^{\frac{1}{3}}(3\,x+2)^3}\,dx$$

input
```
integrate(1/(2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^3), x)
```

Giac [F]

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(27\,x^2+4)^{\frac{1}{3}}(3\,x+2)^3}\,dx$$

input
```
integrate(1/(2+3*x)^3/(27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((27*x^2 + 4)^(1/3)*(3*x + 2)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx = \int \frac{1}{(3\,x+2)^3\,(27\,x^2+4)^{1/3}}\,dx$$

input
```
int(1/((3*x + 2)^3*(27*x^2 + 4)^(1/3)),x)
```

output
```
int(1/((3*x + 2)^3*(27*x^2 + 4)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(2+3x)^3\sqrt[3]{4+27x^2}}\,dx$$
$$= \int \frac{1}{27\,(27x^2+4)^{\frac{1}{3}}\,x^3+54\,(27x^2+4)^{\frac{1}{3}}\,x^2+36\,(27x^2+4)^{\frac{1}{3}}\,x+8\,(27x^2+4)^{\frac{1}{3}}}dx$$

input
```
int(1/(2+3*x)^3/(27*x^2+4)^(1/3),x)
```

output
```
int(1/(27*(27*x**2 + 4)**(1/3)*x**3 + 54*(27*x**2 + 4)**(1/3)*x**2 + 36*(2
7*x**2 + 4)**(1/3)*x + 8*(27*x**2 + 4)**(1/3)),x)
```

3.349 $\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 564

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = -\frac{4}{35}(7i-4x)\left(4-27x^2\right)^{2/3} - \frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3} - \frac{96x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)} - \frac{16\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-2}}\right.\right.}{21\,3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}} + \frac{32\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-2}}\right.\right.}{63\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}$$

output
```
-4/35*(7*I-4*x)*(-27*x^2+4)^(2/3)-1/30*I*(2+3*I*x)^2*(-27*x^2+4)^(2/3)-96*
x/(7*2^(2/3)*(1-3^(1/2))-7*(-27*x^2+4)^(1/3))-16/63*2^(1/3)*(1/2*6^(1/2)+1
/2*2^(1/2))*(2^(2/3)-(-27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(-27*x^2+4)^(1
/3)+(-27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)*El
lipticE((2^(2/3)*(1+3^(1/2))-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*
x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)-(-27*x^2+4)^(1/3))/(2^(2
/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)+32/189*2^(5/6)*(2^(2/3)-(-27*x
^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(-27*x^2+4)^(1/3)+(-27*x^2+4)^(2/3))/(2^(
2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)*EllipticF((2^(2/3)*(1+3^(1/2)
)-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3)),2*I-I*3^(1/2)
)*3^(3/4)/x/(-(2^(2/3)-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)
^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.11

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = \left(4-27x^2\right)^{2/3}\left(-\frac{14i}{15}+\frac{6x}{7}+\frac{3ix^2}{10}\right) + \frac{16}{7}\sqrt[3]{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{27x^2}{4}\right)$$

input
```
Integrate[(2 + (3*I)*x)^3/(4 - 27*x^2)^(1/3),x]
```

output
```
(4 - 27*x^2)^(2/3)*((-14*I)/15 + (6*x)/7 + ((3*I)/10)*x^2) + (16*2^(1/3)*x
*Hypergeometric2F1[1/3, 1/2, 3/2, (27*x^2)/4])/7
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {497, 27, 676, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{1}{90}\int -\frac{288(3ix+1)(3ix+2)}{\sqrt[3]{4-27x^2}}dx-\frac{1}{30}i\left(4-27x^2\right)^{2/3}(2+3ix)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{16}{5}\int \frac{(3ix+1)(3ix+2)}{\sqrt[3]{4-27x^2}}dx-\frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{676}$$

$$\frac{16}{5}\left(\frac{10}{7}\int \frac{1}{\sqrt[3]{4-27x^2}}dx+\frac{1}{7}\left(4-27x^2\right)^{2/3}x-\frac{1}{4}i\left(4-27x^2\right)^{2/3}\right)-\frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{233}$$

$$\frac{16}{5}\left(-\frac{5\sqrt{-x^2}\int \frac{\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}}{7\sqrt{3}x}+\frac{1}{7}\left(4-27x^2\right)^{2/3}x-\frac{1}{4}i\left(4-27x^2\right)^{2/3}\right)-\frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{833}$$

$$\frac{16}{5}\left(-\frac{5\sqrt{-x^2}\left(2^{2/3}(1+\sqrt{3})\int \frac{1}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\int \frac{2^{2/3}(1+\sqrt{3})-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}\right)}{7\sqrt{3}x}+\frac{1}{7}\left(4-27x^2\right)^{2}\right.$$

$$\frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3}$$

$\downarrow$ 760

$$\frac{16}{5}\left(-\frac{5\sqrt{-x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/}}{\left(2^{2/3}\left(1-\sqrt{3}\right)\right.}}}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-}}\right.}{7\sqrt{3}x}\right.$$

$$\frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3}$$

$\downarrow$ 2418

$$\frac{16}{5}\left(-\frac{5\sqrt{-x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{\right.}}{2^{2/3}\left(1-\sqrt{\right.}}\right.\right.}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}\right.}{}\right.$$

$$\frac{1}{30}i(2+3ix)^2\left(4-27x^2\right)^{2/3}$$

input
```
Int[(2 + (3*I)*x)^3/(4 - 27*x^2)^(1/3),x]
```

output
```
(-1/30*I)*(2 + (3*I)*x)^2*(4 - 27*x^2)^(2/3) + (16*((-1/4*I)*(4 - 27*x^2)^
(2/3) + (x*(4 - 27*x^2)^(2/3))/7 - (5*Sqrt[-x^2]*((-6*Sqrt[3]*Sqrt[-x^2])/
(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*
(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3
) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*El
lipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 -
Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-x^2]*Sq
rt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^
(1/3))^2)]) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 - 2
7*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^
(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2
^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 2
7*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*Sqrt[-x^2]*Sqrt[-((2^(2/3) - (
4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)])))/(7*
Sqrt[3]*x)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 760

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.09

method	result
risch	$-\frac{i(63x^2-180ix-196)(27x^2-4)}{210(-27x^2+4)^{\frac{1}{3}}}+\frac{16\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{27x^2}{4}\right)}{7}$
meijerg	$4\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{27x^2}{4}\right)+9i2^{\frac{1}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1\right],[2],\frac{27x^2}{4}\right)-9\,2^{\frac{1}{3}}x^3\,\mathrm{hyperge}$

input
```
int((2+3*I*x)^3/(-27*x^2+4)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/210*I*(-180*I*x+63*x^2-196)*(27*x^2-4)/(-27*x^2+4)^(1/3)+16/7*2^(1/3)*x
*hypergeom([1/3,1/2],[3/2],27/4*x^2)
```

Fricas [F]

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx=\int \frac{(3i\,x+2)^3}{(-27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
1/630*(630*x*integral(128/63*(-27*x^2 + 4)^(2/3)/(27*x^4 - 4*x^2), x) + (1
89*I*x^3 + 540*x^2 - 588*I*x - 320)*(-27*x^2 + 4)^(2/3))/x
```

Sympy [A] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.26

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = -9\cdot\sqrt[3]{2}x^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right)+4\cdot\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right)-i(4-27x^2)^{\frac{2}{3}}-27i\left(\begin{cases}\frac{x^2(27x^2-4)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}}{90}+\frac{(27x^2-4)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}}{405} & \text{for } |x^2|>\frac{4}{27}\\ -\frac{x^2(4-27x^2)^{\frac{2}{3}}}{90}-\frac{(4-27x^2)^{\frac{2}{3}}}{405} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((2+3*I*x)**3/(-27*x**2+4)**(1/3),x)
```

output
```
-9*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(2*I*pi)/4) +
4*2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(2*I*pi)/4) - I*(4
 - 27*x**2)**(2/3) - 27*I*Piecewise((x**2*(27*x**2 - 4)**(2/3)*exp(-I*pi/3
)/90 + (27*x**2 - 4)**(2/3)*exp(-I*pi/3)/405, Abs(x**2) > 4/27), (-x**2*(4
 - 27*x**2)**(2/3)/90 - (4 - 27*x**2)**(2/3)/405, True))
```

Maxima [F]

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{(3i\,x+2)^3}{(-27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((3*I*x + 2)^3/(-27*x^2 + 4)^(1/3), x)
```

Giac [F]

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{(3i\,x+2)^3}{(-27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate((3*I*x + 2)^3/(-27*x^2 + 4)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{(2+x\,3\mathrm{i})^3}{(4-27\,x^2)^{1/3}}\,dx$$

input
```
int((x*3i + 2)^3/(4 - 27*x^2)^(1/3),x)
```

output
```
int((x*3i + 2)^3/(4 - 27*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(2+3ix)^3}{\sqrt[3]{4-27x^2}}\,dx = -27\left(\int \frac{x^3}{(-27x^2+4)^{\frac{1}{3}}}dx\right)i-54\left(\int \frac{x^2}{(-27x^2+4)^{\frac{1}{3}}}dx\right) +36\left(\int \frac{x}{(-27x^2+4)^{\frac{1}{3}}}dx\right)i+8\left(\int \frac{1}{(-27x^2+4)^{\frac{1}{3}}}dx\right)$$

input
```
int((2+3*I*x)^3/(-27*x^2+4)^(1/3),x)
```

output
```
- 27*int(x**3/( - 27*x**2 + 4)**(1/3),x)*i - 54*int(x**2/( - 27*x**2 + 4)
**(1/3),x) + 36*int(x/( - 27*x**2 + 4)**(1/3),x)*i + 8*int(1/( - 27*x**2 +
 4)**(1/3),x)
```

3.350 $\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 540

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = -\frac{1}{21}i(7+3ix)\left(4-27x^2\right)^{2/3} - \frac{72x}{7\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)}$$

$$- \frac{4\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27\cdots}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27\cdots}}\cdots\right.\right.}{7\ 3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}$$

$$+ \frac{8\ 2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x\cdots}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x\cdots}}\cdots\right.\right.}{21\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}$$

output
```
-1/21*I*(7+3*I*x)*(-27*x^2+4)^(2/3)-72*x/(7*2^(2/3)*(1-3^(1/2))-7*(-27*x^2
+4)^(1/3))-4/21*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(2/3)-(-27*x^2+4)^(1/
3))*((2*2^(1/3)+2^(2/3)*(-27*x^2+4)^(1/3)+(-27*x^2+4)^(2/3))/(2^(2/3)*(1-3
^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)*EllipticE((2^(2/3)*(1+3^(1/2))-(-27*x^
2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(1/4)
/x/(-(2^(2/3)-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2
)^(1/2)+8/63*2^(5/6)*(2^(2/3)-(-27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(-27*
x^2+4)^(1/3)+(-27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)
^(1/2)*EllipticF((2^(2/3)*(1+3^(1/2))-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/
2))-(-27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(-(2^(2/3)-(-27*x^2+4)^(1/
3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = \left(-\frac{i}{3}+\frac{x}{7}\right)\left(4-27x^2\right)^{2/3} + \frac{12}{7}\sqrt[3]{2}x \,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{27x^2}{4}\right)$$

input
```
Integrate[(2 + (3*I)*x)^2/(4 - 27*x^2)^(1/3),x]
```

output
```
(-1/3*I + x/7)*(4 - 27*x^2)^(2/3) + (12*2^(1/3)*x*Hypergeometric2F1[1/3, 1
/2, 3/2, (27*x^2)/4])/7
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {497, 27, 455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx$$

$$\Big\downarrow \text{ 497}$$

$$-\frac{1}{63}\int -\frac{108(5ix+2)}{\sqrt[3]{4-27x^2}}dx-\frac{1}{21}i\left(4-27x^2\right)^{2/3}(2+3ix)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{12}{7}\int \frac{5ix+2}{\sqrt[3]{4-27x^2}}dx-\frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{12}{7}\left(2\int \frac{1}{\sqrt[3]{4-27x^2}}dx-\frac{5}{36}i\left(4-27x^2\right)^{2/3}\right)-\frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{12}{7}\left(-\frac{\sqrt{-x^2}\int \frac{\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}}{\sqrt{3}x}-\frac{5}{36}i\left(4-27x^2\right)^{2/3}\right)-\frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{ 833}$$

$$\frac{12}{7}\left(-\frac{\sqrt{-x^2}\left(2^{2/3}\left(1+\sqrt{3}\right)\int \frac{1}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}\right)}{\sqrt{3}x}-\frac{5}{36}i\left(4-27x^2\right)\right.$$

$$\frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{12}{7}\left(-\frac{\sqrt{-x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\cdots}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\cdots\right.}}}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-\cdots}}\cdots\right.}{\sqrt{3}x}\cdots\right.$$

$$\frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3}$$

$\downarrow$ 2418

$$\frac{12}{7}\left(-\frac{\sqrt{-x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)\cdots}{2^{2/3}\left(1-\sqrt{3}\right)\cdots}\right.\right.}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{1}{21}i(2+3ix)\left(4-27x^2\right)^{2/3}$$

input
```
Int[(2 + (3*I)*x)^2/(4 - 27*x^2)^(1/3),x]
```

output
```
(-1/21*I)*(2 + (3*I)*x)*(4 - 27*x^2)^(2/3) + (12*(((-5*I)/36)*(4 - 27*x^2)
^(2/3) - (Sqrt[-x^2]*((-6*Sqrt[3]*Sqrt[-x^2])/(2^(2/3)*(1 - Sqrt[3]) - (4
- 27*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3
))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(
2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 +
 Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-x^2]*Sqrt[-((2^(2/3) - (4 - 27*x^2)
^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)]) - (2*2^(1/3)*Sqr
t[2 - Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3
) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]
) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 -
27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(3*3^(3/4)*Sqrt[-x^2]*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*
(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)])))/(Sqrt[3]*x)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

method	result
risch	$-\frac{(-7i+3x)(27x^2-4)}{21(-27x^2+4)^{\frac{1}{3}}}+\frac{12\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{27x^2}{4}\right)}{7}$
meijerg	$2\,2^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{27x^2}{4}\right)+3i2^{\frac{1}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1\right],[2],\frac{27x^2}{4}\right)-\frac{3\,2^{\frac{1}{3}}x^3\,\mathrm{hypergeom}\left(\left[\right.\right.}{2}$

input
```
int((2+3*I*x)^2/(-27*x^2+4)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-7*I+3*x)*(27*x^2-4)/(-27*x^2+4)^(1/3)+12/7*2^(1/3)*x*hypergeom([1/
3,1/2],[3/2],27/4*x^2)
```

Fricas [F]

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{(3i\,x+2)^2}{(-27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
1/21*(21*x*integral(32/21*(-27*x^2 + 4)^(2/3)/(27*x^4 - 4*x^2), x) + (3*x^
2 - 7*I*x - 8)*(-27*x^2 + 4)^(2/3))/x
```

Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.14

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = -\frac{3\cdot\sqrt[3]{2}x^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right)}{2} + 2\cdot\sqrt[3]{2}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right) - \frac{i(4-27x^2)^{\frac{2}{3}}}{3}$$

input
```
integrate((2+3*I*x)**2/(-27*x**2+4)**(1/3),x)
```

output
```
-3*2**(1/3)*x**3*hyper((1/3, 3/2), (5/2,), 27*x**2*exp_polar(2*I*pi)/4)/2
+ 2*2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(2*I*pi)/4) - I*
(4 - 27*x**2)**(2/3)/3
```

Maxima [F]

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{(3i\,x+2)^2}{(-27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((3*I*x + 2)^2/(-27*x^2 + 4)^(1/3), x)
```

Giac [F]

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{(3i\,x+2)^2}{(-27\,x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate((3*I*x + 2)^2/(-27*x^2 + 4)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{(2+x\,3\mathrm{i})^2}{(4-27\,x^2)^{1/3}}\,dx$$

input
```
int((x*3i + 2)^2/(4 - 27*x^2)^(1/3),x)
```

output
```
int((x*3i + 2)^2/(4 - 27*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(2+3ix)^2}{\sqrt[3]{4-27x^2}}\,dx = -9\left(\int \frac{x^2}{(-27x^2+4)^{\frac{1}{3}}}dx\right)+12\left(\int \frac{x}{(-27x^2+4)^{\frac{1}{3}}}dx\right)i + 4\left(\int \frac{1}{(-27x^2+4)^{\frac{1}{3}}}dx\right)$$

input
```
int((2+3*I*x)^2/(-27*x^2+4)^(1/3),x)
```

output
```
 - 9*int(x**2/( - 27*x**2 + 4)**(1/3),x) + 12*int(x/( - 27*x**2 + 4)**(1/3
),x)*i + 4*int(1/( - 27*x**2 + 4)**(1/3),x)
```

3.351 $\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 531

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx = -\frac{1}{12}i\left(4-27x^2\right)^{2/3} - \frac{6x}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}$$
$$-\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right.\right.}{3\,3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}$$
$$+\frac{2\,2^{5/6}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right.\right.}{9\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}$$

output
```
-1/12*I*(-27*x^2+4)^(2/3)-6*x/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))-1/9*
2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(2/3)-(-27*x^2+4)^(1/3))*((2*2^(1/3)+
2^(2/3)*(-27*x^2+4)^(1/3)+(-27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2
+4)^(1/3))^2)^(1/2)*EllipticE((2^(2/3)*(1+3^(1/2))-(-27*x^2+4)^(1/3))/(2^(
2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)-(-
27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)+2/27*2^(
5/6)*(2^(2/3)-(-27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(-27*x^2+4)^(1/3)+(-2
7*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)*EllipticF
((2^(2/3)*(1+3^(1/2))-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^
(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(-(2^(2/3)-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-
3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.87 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.08

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx = -\frac{1}{12}i\left(4-27x^2\right)^{2/3}+\sqrt[3]{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{27x^2}{4}\right)$$

input
```
Integrate[(2 + (3*I)*x)/(4 - 27*x^2)^(1/3),x]
```

output
```
(-1/12*I)*(4 - 27*x^2)^(2/3) + 2^(1/3)*x*Hypergeometric2F1[1/3, 1/2, 3/2,
(27*x^2)/4]
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx$$

$$\Big\downarrow \text{455}$$

$$2\int \frac{1}{\sqrt[3]{4-27x^2}}dx-\frac{1}{12}i\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{233}$$

$$-\frac{\sqrt{-x^2}\int \frac{\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}}{\sqrt{3}x}-\frac{1}{12}i\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{833}$$

$$-\frac{\sqrt{-x^2}\left(2^{2/3}\left(1+\sqrt{3}\right)\int \frac{1}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}\right)}{\sqrt{3}x}-\frac{1}{12}i\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{760}$$

$$-\frac{\sqrt{-x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-\cdots}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4\cdots}\right.}}}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-\frac{\cdots}{\left(2^{2/3}\cdots\right.}}}\cdots\right.}{\sqrt{3}x}\cdots$$

$$\frac{1}{12}i\left(4-27x^2\right)^{2/3}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{\sqrt{-x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4\cdots}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4\cdots}}\right.\right.\cdots}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}\cdots\right.}{\cdots}$$

$$\frac{1}{12}i\left(4-27x^2\right)^{2/3}$$

input
```
Int[(2 + (3*I)*x)/(4 - 27*x^2)^(1/3),x]
```

output
```
(-1/12*I)*(4 - 27*x^2)^(2/3) - (Sqrt[-x^2]*((-6*Sqrt[3]*Sqrt[-x^2])/(2^(2/
3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/
3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4
 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*Elliptic
E[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3
]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-x^2]*Sqrt[-((
2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))
^2)]) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 - 27*x^2)
^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))
/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)
*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*Sqrt[-x^2]*Sqrt[-((2^(2/3) - (4 - 27
*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)])))/(Sqrt[3]*
x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

method	result	size
risch	$\frac{i(27x^2-4)}{12(-27x^2+4)^{\frac{1}{3}}}+2^{\frac{1}{3}}x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{27x^2}{4}\right)$	37
meijerg	$2^{\frac{1}{3}}x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{27x^2}{4}\right)+\frac{3i2^{\frac{1}{3}}x^2\,\text{hypergeom}\left(\left[\frac{1}{3},1\right],[2],\frac{27x^2}{4}\right)}{4}$	38

input
```
int((2+3*I*x)/(-27*x^2+4)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*(27*x^2-4)/(-27*x^2+4)^(1/3)+2^(1/3)*x*hypergeom([1/3,1/2],[3/2],27
/4*x^2)
```

Fricas [F]

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}} \, dx = \int \frac{3i\,x+2}{\left(-27\,x^2+4\right)^{\frac{1}{3}}} \, dx$$

input
```
integrate((2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
1/36*(36*x*integral(8/9*(-27*x^2 + 4)^(2/3)/(27*x^4 - 4*x^2), x) + (-27*x^
2 + 4)^(2/3)*(-3*I*x - 8))/x
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}} \, dx = \sqrt[3]{2}x_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{27x^2e^{2i\pi}}{4}\right) - \frac{i(4-27x^2)^{\frac{2}{3}}}{12}$$

input
```
integrate((2+3*I*x)/(-27*x**2+4)**(1/3),x)
```

output
```
2**(1/3)*x*hyper((1/3, 1/2), (3/2,), 27*x**2*exp_polar(2*I*pi)/4) - I*(4 -
 27*x**2)**(2/3)/12
```

Maxima [F]

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}} \, dx = \int \frac{3i\,x+2}{\left(-27\,x^2+4\right)^{\frac{1}{3}}} \, dx$$

input
```
integrate((2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((3*I*x + 2)/(-27*x^2 + 4)^(1/3), x)
```

Giac [F]

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx = \int \frac{3i\,x+2}{\left(-27\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate((3*I*x + 2)/(-27*x^2 + 4)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.05

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx = 2^{1/3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ \frac{27\,x^2}{4}\right) - \frac{\left(4-27\,x^2\right)^{2/3}\,1\mathrm{i}}{12}$$

input
```
int((x*3i + 2)/(4 - 27*x^2)^(1/3),x)
```

output
```
2^(1/3)*x*hypergeom([1/3, 1/2], 3/2, (27*x^2)/4) - ((4 - 27*x^2)^(2/3)*1i)
/12
```

Reduce [F]

$$\int \frac{2+3ix}{\sqrt[3]{4-27x^2}}\,dx = 3\left(\int \frac{x}{\left(-27x^2+4\right)^{\frac{1}{3}}}dx\right)i + 2\left(\int \frac{1}{\left(-27x^2+4\right)^{\frac{1}{3}}}dx\right)$$

input
```
int((2+3*I*x)/(-27*x^2+4)^(1/3),x)
```

output
```
3*int(x/( - 27*x**2 + 4)**(1/3),x)*i + 2*int(1/( - 27*x**2 + 4)**(1/3),x)
```

3.352 $\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx=\frac{i\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}+\frac{i\log(2+3ix)}{12\sqrt[3]{2}}-\frac{i\log\left(-54\sqrt[3]{2}(2-3ix)+108\sqrt[3]{4-27x^2}\right)}{12\sqrt[3]{2}}$$

output
```
-1/36*I*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-3*I*x)*3^(1/2)/(-27*x^2+4)^(1/3
))*2^(2/3)*3^(1/2)+1/24*I*ln(2+3*I*x)*2^(2/3)-1/24*I*ln(-54*2^(1/3)*(2-3*I
*x)+108*(-27*x^2+4)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.48

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx = -\frac{i\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{4-27x^2}}{2\sqrt[3]{2}-3i\sqrt[3]{2}x+\sqrt[3]{4-27x^2}}\right)+2\log\left(-2\sqrt[3]{2}+3i\sqrt[3]{2}x+2\sqrt[3]{4-27x^2}\right)-\log\left(-4 2^{2/3}+\right.\right.}{36\sqrt[3]{2}}$$

input
```
Integrate[1/((2 + (3*I)*x)*(4 - 27*x^2)^(1/3)),x]
```

output
```
((-1/36*I)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(4 - 27*x^2)^(1/3))/(2*2^(1/3) - (3*
I)*2^(1/3)*x + (4 - 27*x^2)^(1/3))] + 2*Log[-2*2^(1/3) + (3*I)*2^(1/3)*x +
 2*(4 - 27*x^2)^(1/3)] - Log[-4*2^(2/3) + (12*I)*2^(2/3)*x + 9*2^(2/3)*x^2
 + 2*(-2 + (3*I)*x)*(8 - 54*x^2)^(1/3) - 4*(4 - 27*x^2)^(2/3)]))/2^(1/3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {501}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}dx$$

$$\Big\downarrow \text{501}$$

$$\frac{i\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}-\frac{i\log\left(27\ 2^{2/3}\sqrt[3]{4-27x^2}+81ix-54\right)}{12\sqrt[3]{2}}+\frac{i\log(2+3ix)}{12\sqrt[3]{2}}$$

input
```
Int[1/((2 + (3*I)*x)*(4 - 27*x^2)^(1/3)),x]
```

output
```
((I/6)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x))/(Sqrt[3]*(4 - 27*x^2)^(1
/3))])/(2^(1/3)*Sqrt[3]) + ((I/12)*Log[2 + (3*I)*x])/2^(1/3) - ((I/12)*Log
[-54 + (81*I)*x + 27*2^(2/3)*(4 - 27*x^2)^(1/3)])/2^(1/3)
```

Defintions of rubi rules used

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

Maple [F]

$$\int \frac{1}{(3ix+2)\,(-27x^2+4)^{\frac{1}{3}}}dx$$

input
```
int(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x)
```

output
```
int(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx = -i\int \frac{1}{3x\sqrt[3]{4-27x^2}-2i\sqrt[3]{4-27x^2}}\,dx$$

input
```
integrate(1/(2+3*I*x)/(-27*x**2+4)**(1/3),x)
```

output
```
-I*Integral(1/(3*x*(4 - 27*x**2)**(1/3) - 2*I*(4 - 27*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(-27\,x^2+4)^{\frac{1}{3}}(3i\,x+2)}\,dx$$

input
```
integrate(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(-27\,x^2+4)^{\frac{1}{3}}(3i\,x+2)}\,dx$$

input
```
integrate(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(2+x\,3\mathrm{i})\,\left(4-27\,x^2\right)^{1/3}}\,dx$$

input
```
int(1/((x*3i + 2)*(4 - 27*x^2)^(1/3)),x)
```

output
```
int(1/((x*3i + 2)*(4 - 27*x^2)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(2+3ix)\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{3\left(-27x^2+4\right)^{\frac{1}{3}} ix + 2\left(-27x^2+4\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(2+3*I*x)/(-27*x^2+4)^(1/3),x)
```

output
```
int(1/(3*( - 27*x**2 + 4)**(1/3)*i*x + 2*( - 27*x**2 + 4)**(1/3)),x)
```

3.353 $\int \frac{1}{(2+3ix)^2 \sqrt[3]{4-27x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 653

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx$$

$$= \frac{i(4-27x^2)^{2/3}}{48(2+3ix)} - \frac{3x}{16\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)} + \frac{i\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{24\sqrt[3]{2}\sqrt{3}}$$

$$- \frac{\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right)\right.}{48\ 2^{2/3}3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}$$

$$+ \frac{\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right),\right.}{72\sqrt[6]{2}\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}$$

$$+ \frac{i\log(2+3ix)}{48\sqrt[3]{2}} - \frac{i\log\left(-54\sqrt[3]{2}(2-3ix)+108\sqrt[3]{4-27x^2}\right)}{48\sqrt[3]{2}}$$

output
```
1/48*I*(-27*x^2+4)^(2/3)/(2+3*I*x)-3*x/(16*2^(2/3)*(1-3^(1/2))-16*(-27*x^2
+4)^(1/3))-1/144*I*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-3*I*x)*3^(1/2)/(-27*
x^2+4)^(1/3))*2^(2/3)*3^(1/2)-1/288*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(
2/3)-(-27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(-27*x^2+4)^(1/3)+(-27*x^2+4)^
(2/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)*EllipticE((2^(2/3)
*(1+3^(1/2))-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3)),2*
I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-
(-27*x^2+4)^(1/3))^2)^(1/2)+1/432*2^(5/6)*(2^(2/3)-(-27*x^2+4)^(1/3))*((2*
2^(1/3)+2^(2/3)*(-27*x^2+4)^(1/3)+(-27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-
(-27*x^2+4)^(1/3))^2)^(1/2)*EllipticF((2^(2/3)*(1+3^(1/2))-(-27*x^2+4)^(1/
3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(-(2^
(2/3)-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)+
1/96*I*ln(2+3*I*x)*2^(2/3)-1/96*I*ln(-54*2^(1/3)*(2-3*I*x)+108*(-27*x^2+4)
^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.20

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx = \frac{\sqrt[3]{\frac{2\sqrt{3}-9x}{2i-3x}}\sqrt[3]{\frac{2\sqrt{3}+9x}{-2i+3x}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},\frac{1}{3},\frac{8}{3},\frac{2\left(3i+\sqrt{3}\right)}{6i-9x},\frac{2\left(-3i+\sqrt{3}\right)}{-6i+9x}\right)}{5\ 3^{2/3}(-2i+3x)\sqrt[3]{4-27x^2}}$$

input
```
Integrate[1/((2 + (3*I)*x)^2*(4 - 27*x^2)^(1/3)),x]
```

output
```
(((2*Sqrt[3] - 9*x)/(2*I - 3*x))^(1/3)*((2*Sqrt[3] + 9*x)/(-2*I + 3*x))^(1
/3)*AppellF1[5/3, 1/3, 1/3, 8/3, (2*(3*I + Sqrt[3]))/(6*I - 9*x), (2*(-3*I
 + Sqrt[3]))/(-6*I + 9*x)])/(5*3^(2/3)*(-2*I + 3*x)*(4 - 27*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {498, 25, 719, 233, 501, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}dx$$

$$\Big\downarrow \text{ 498}$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}-\frac{3}{16}\int -\frac{ix+2}{(3ix+2)\sqrt[3]{4-27x^2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3}{16}\int \frac{ix+2}{(3ix+2)\sqrt[3]{4-27x^2}}dx+\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{3}{16}\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{4-27x^2}}dx+\frac{4}{3}\int \frac{1}{(3ix+2)\sqrt[3]{4-27x^2}}dx\right)+\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{3}{16}\left(-\frac{\sqrt{-x^2}\int \frac{\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}}{6\sqrt{3}x}+\frac{4}{3}\int \frac{1}{(3ix+2)\sqrt[3]{4-27x^2}}dx\right)+\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}$$

$$\Big\downarrow \text{ 501}$$

$$\frac{3}{16}\left(-\frac{\sqrt{-x^2}\int \frac{\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}}{6\sqrt{3}x}+\frac{4}{3}\left(\frac{i\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}-\frac{i\log\left(27\ 2^{2/3}\sqrt[3]{4-27x^2}+81\right.}{12\sqrt[3]{2}}\right.\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}$$

$$\Big\downarrow \text{ 833}$$

$$\frac{3}{16}\left(-\frac{\sqrt{-x^2}\left(2^{2/3}(1+\sqrt{3})\int \frac{1}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}\right)}{6\sqrt{3}x}+\frac{4}{3}\left(\frac{i\arctan\left(\right.}{}\right.\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}$$

$$\Big\downarrow \; 760$$

$$\frac{3}{16}\left(-\frac{\sqrt{-x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\right.}}}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-}}\right.}{6\sqrt{3}x}\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}$$

$$\Big\downarrow \; 2418$$

$$\frac{3}{16}\left(-\frac{\sqrt{-x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)}{2^{2/3}\left(1-\sqrt{3}\right)}\right.\right.}{3\,3^{3/4}\sqrt{-x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}\right.}{}\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{48(2+3ix)}$$

input
```
Int[1/((2 + (3*I)*x)^2*(4 - 27*x^2)^(1/3)),x]
```

output
```
((I/48)*(4 - 27*x^2)^(2/3))/(2 + (3*I)*x) + (3*(-1/6*(Sqrt[-x^2]*((-6*Sqrt
[3]*Sqrt[-x^2])/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3)) + (2^(1/3)*Sq
rt[2 + Sqrt[3]]*(2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(
4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x
^2)^(1/3))^2]*EllipticE[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3)
)/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/
4)*Sqrt[-x^2]*Sqrt[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3])
 - (4 - 27*x^2)^(1/3))^2)]) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(
2^(2/3) - (4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3)
 + (4 - 27*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - S
qrt[3]) - (4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*Sqrt[-x^2]*Sqr
t[-((2^(2/3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(
1/3))^2)])))/(Sqrt[3]*x) + (4*(((I/6)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*
I)*x))/(Sqrt[3]*(4 - 27*x^2)^(1/3))])/(2^(1/3)*Sqrt[3]) + ((I/12)*Log[2 +
(3*I)*x])/2^(1/3) - ((I/12)*Log[-54 + (81*I)*x + 27*2^(2/3)*(4 - 27*x^2)^(
1/3)])/2^(1/3)))/3))/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(3ix+2)^2 (-27x^2+4)^{\frac{1}{3}}} dx$$

input
```
int(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x)
```

output
```
int(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(2+3ix)^2 \sqrt[3]{4-27x^2}} dx = \text{Timed out}$$

input
```
integrate(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx = -\int \frac{1}{9x^2\sqrt[3]{4-27x^2}-12ix\sqrt[3]{4-27x^2}-4\sqrt[3]{4-27x^2}}\,dx$$

input
```
integrate(1/(2+3*I*x)**2/(-27*x**2+4)**(1/3),x)
```

output
```
-Integral(1/(9*x**2*(4 - 27*x**2)**(1/3) - 12*I*x*(4 - 27*x**2)**(1/3) - 4
*(4 - 27*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(-27\,x^2+4)^{\frac{1}{3}}(3i\,x+2)^2}\,dx$$

input
```
integrate(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^2), x)
```

Giac [F]

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(-27\,x^2+4)^{\frac{1}{3}}(3i\,x+2)^2}\,dx$$

input
```
integrate(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(2+x\,3\mathrm{i})^2\,(4-27\,x^2)^{1/3}}\,dx$$

input
```
int(1/((x*3i + 2)^2*(4 - 27*x^2)^(1/3)),x)
```

output
```
int(1/((x*3i + 2)^2*(4 - 27*x^2)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(2+3ix)^2\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{12\,(-27x^2+4)^{\frac{1}{3}}\,ix-9\,(-27x^2+4)^{\frac{1}{3}}\,x^2+4\,(-27x^2+4)^{\frac{1}{3}}}dx$$

input
```
int(1/(2+3*I*x)^2/(-27*x^2+4)^(1/3),x)
```

output
```
int(1/(12*( - 27*x**2 + 4)**(1/3)*i*x - 9*( - 27*x**2 + 4)**(1/3)*x**2 + 4
*( - 27*x**2 + 4)**(1/3)),x)
```

3.354 $\int \frac{1}{(2+3ix)^3 \sqrt[3]{4-27x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 679

$$
\begin{aligned}
\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx &= \frac{i(4-27x^2)^{2/3}}{96(2+3ix)^2}+\frac{i(4-27x^2)^{2/3}}{96(2+3ix)} \\
&- \frac{3x}{32\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)}+\frac{i\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{96\sqrt[3]{2}\sqrt{3}} \\
&- \frac{\sqrt{2+\sqrt{3}}\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,E\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right)\right.}{96\ 2^{2/3}3^{3/4}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}} \\
&+ \frac{\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{4-27x^2}+(4-27x^2)^{2/3}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}}\right),\right.}{144\sqrt[6]{2}\sqrt[4]{3}x\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}} \\
&+ \frac{i\log(2+3ix)}{192\sqrt[3]{2}}-\frac{i\log\left(-54\sqrt[3]{2}(2-3ix)+108\sqrt[3]{4-27x^2}\right)}{192\sqrt[3]{2}}
\end{aligned}
$$

output
```
1/96*I*(-27*x^2+4)^(2/3)/(2+3*I*x)^2+1/96*I*(-27*x^2+4)^(2/3)/(2+3*I*x)-3*
x/(32*2^(2/3)*(1-3^(1/2))-32*(-27*x^2+4)^(1/3))-1/576*I*arctan(-1/3*3^(1/2
)-1/3*2^(1/3)*(2-3*I*x)*3^(1/2)/(-27*x^2+4)^(1/3))*2^(2/3)*3^(1/2)-1/576*2
^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(2/3)-(-27*x^2+4)^(1/3))*((2*2^(1/3)+2
^(2/3)*(-27*x^2+4)^(1/3)+(-27*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+
4)^(1/3))^2)^(1/2)*EllipticE((2^(2/3)*(1+3^(1/2))-(-27*x^2+4)^(1/3))/(2^(2
/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/x/(-(2^(2/3)-(-2
7*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)+1/864*2^(
5/6)*(2^(2/3)-(-27*x^2+4)^(1/3))*((2*2^(1/3)+2^(2/3)*(-27*x^2+4)^(1/3)+(-2
7*x^2+4)^(2/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)*EllipticF
((2^(2/3)*(1+3^(1/2))-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-3^(1/2))-(-27*x^2+4)^
(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(-(2^(2/3)-(-27*x^2+4)^(1/3))/(2^(2/3)*(1-
3^(1/2))-(-27*x^2+4)^(1/3))^2)^(1/2)+1/384*I*ln(2+3*I*x)*2^(2/3)-1/384*I*l
n(-54*2^(1/3)*(2-3*I*x)+108*(-27*x^2+4)^(1/3))*2^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.53 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.20

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = -\frac{i\sqrt[3]{\frac{2\sqrt{3}-9x}{2i-3x}}\sqrt[3]{\frac{2\sqrt{3}+9x}{-2i+3x}}\,\text{AppellF1}\left(\frac{8}{3},\frac{1}{3},\frac{1}{3},\frac{11}{3},\frac{2(3i+\sqrt{3})}{6i-9x},\frac{2(-3i+\sqrt{3})}{-6i+9x}\right)}{8\,3^{2/3}(2i-3x)^2\sqrt[3]{4-27x^2}}$$

input
```
Integrate[1/((2 + (3*I)*x)^3*(4 - 27*x^2)^(1/3)),x]
```

output
```
((-1/8*I)*((2*Sqrt[3] - 9*x)/(2*I - 3*x))^(1/3)*((2*Sqrt[3] + 9*x)/(-2*I +
 3*x))^(1/3)*AppellF1[8/3, 1/3, 1/3, 11/3, (2*(3*I + Sqrt[3]))/(6*I - 9*x)
, (2*(-3*I + Sqrt[3]))/(-6*I + 9*x)])/(3^(2/3)*(2*I - 3*x)^2*(4 - 27*x^2)^
(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {498, 27, 688, 27, 719, 233, 501, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}dx$$

$$\Big\downarrow \text{498}$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}-\frac{3}{32}\int -\frac{2(2-ix)}{(3ix+2)^2\sqrt[3]{4-27x^2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{16}\int \frac{2-ix}{(3ix+2)^2\sqrt[3]{4-27x^2}}dx+\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$$\Big\downarrow \text{688}$$

$$\frac{3}{16}\left(\frac{1}{144}\int \frac{24(3ix+4)}{(3ix+2)\sqrt[3]{4-27x^2}}dx+\frac{i\left(4-27x^2\right)^{2/3}}{18(2+3ix)}\right)+\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{16}\left(\frac{1}{6}\int \frac{3ix+4}{(3ix+2)\sqrt[3]{4-27x^2}}dx+\frac{i\left(4-27x^2\right)^{2/3}}{18(2+3ix)}\right)+\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$$\Big\downarrow \text{719}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(\int \frac{1}{\sqrt[3]{4-27x^2}}dx+2\int \frac{1}{(3ix+2)\sqrt[3]{4-27x^2}}dx\right)+\frac{i\left(4-27x^2\right)^{2/3}}{18(2+3ix)}\right)+\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$$\Big\downarrow \text{233}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(-\frac{\sqrt{-x^2}\int \frac{\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}}{2\sqrt{3}x}+2\int \frac{1}{(3ix+2)\sqrt[3]{4-27x^2}}dx\right)+\frac{i\left(4-27x^2\right)^{2/3}}{18(2+3ix)}\right)+$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$$\Big\downarrow \text{501}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(-\frac{\sqrt{-x^2}\int \frac{\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}}{2\sqrt{3}x}+2\left(\frac{i\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-3ix)}{\sqrt{3}\sqrt[3]{4-27x^2}}\right)}{6\sqrt[3]{2}\sqrt{3}}-\frac{i\log\left(27\ 2^{2/3}\sqrt[3]{4-27x^2}+\right.}{12\sqrt[3]{2}}\right.\right.\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$$\Big\downarrow \text{833}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(-\frac{\sqrt{-x^2}\left(2^{2/3}\left(1+\sqrt{3}\right)\int \frac{1}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}\right)}{2\sqrt{3}x}+2\left(\frac{i\arctan}{}\right.\right.\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$$\Big\downarrow \text{760}$$

$$\frac{3}{16}\left(\frac{1}{6}\left(-\frac{\sqrt{-x^2}\left(-\int \frac{2^{2/3}\left(1+\sqrt{3}\right)-\sqrt[3]{4-27x^2}}{3\sqrt{3}\sqrt{-x^2}}d\sqrt[3]{4-27x^2}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+\cdots}{\left(2^{2/3}\left(1-\cdots\right.\right.}}}{3\ 3^{3/4}\sqrt{-x^2}\cdots}\right.}{2\sqrt{3}x}\right.\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

$\downarrow$ 2418

$$\frac{3}{16}\left(\frac{1}{6}\left(-\frac{\sqrt{-x^2}\left(-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(2^{2/3}-\sqrt[3]{4-27x^2}\right)\sqrt{\frac{\left(4-27x^2\right)^{2/3}+2^{2/3}\sqrt[3]{4-27x^2}+2\sqrt[3]{2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1\cdots\right.}{2^{2/3}\left(1\cdots\right.}\right.\right.}{3\ 3^{3/4}\sqrt{-x^2}\sqrt{-\frac{2^{2/3}-\sqrt[3]{4-27x^2}}{\left(2^{2/3}\left(1-\sqrt{3}\right)-\sqrt[3]{4-27x^2}\right)^2}}}\right.}{\cdots}\right.\right.$$

$$\frac{i\left(4-27x^2\right)^{2/3}}{96(2+3ix)^2}$$

input
```
Int[1/((2 + (3*I)*x)^3*(4 - 27*x^2)^(1/3)),x]
```

output
```
((I/96)*(4 - 27*x^2)^(2/3))/(2 + (3*I)*x)^2 + (3*(((I/18)*(4 - 27*x^2)^(2/
3))/(2 + (3*I)*x) + (-1/2*(Sqrt[-x^2]*((-6*Sqrt[3]*Sqrt[-x^2])/(2^(2/3)*(1
 - Sqrt[3]) - (4 - 27*x^2)^(1/3)) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) -
(4 - 27*x^2)^(1/3))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27
*x^2)^(2/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticE[Arc
Sin[(2^(2/3)*(1 + Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) -
(4 - 27*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-x^2]*Sqrt[-((2^(2/
3) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)])
 - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(2^(2/3) - (4 - 27*x^2)^(1/3
))*Sqrt[(2*2^(1/3) + 2^(2/3)*(4 - 27*x^2)^(1/3) + (4 - 27*x^2)^(2/3))/(2^(
2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2]*EllipticF[ArcSin[(2^(2/3)*(1 +
 Sqrt[3]) - (4 - 27*x^2)^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*Sqrt[-x^2]*Sqrt[-((2^(2/3) - (4 - 27*x^2)
^(1/3))/(2^(2/3)*(1 - Sqrt[3]) - (4 - 27*x^2)^(1/3))^2)])))/(Sqrt[3]*x) +
2*(((I/6)*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - (3*I)*x))/(Sqrt[3]*(4 - 27*x^2)
^(1/3))])/(2^(1/3)*Sqrt[3]) + ((I/12)*Log[2 + (3*I)*x])/2^(1/3) - ((I/12)*
Log[-54 + (81*I)*x + 27*2^(2/3)*(4 - 27*x^2)^(1/3)])/2^(1/3)))/6))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(3ix+2)^3 (-27x^2+4)^{\frac{1}{3}}}\,dx$$

input
```
int(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x)
```

output
```
int(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = i\int \frac{1}{27x^3\sqrt[3]{4-27x^2}-54ix^2\sqrt[3]{4-27x^2}-36x\sqrt[3]{4-27x^2}+8i\sqrt[3]{4-27x^2}}\,dx$$

input
```
integrate(1/(2+3*I*x)**3/(-27*x**2+4)**(1/3),x)
```

output
```
I*Integral(1/(27*x**3*(4 - 27*x**2)**(1/3) - 54*I*x**2*(4 - 27*x**2)**(1/3
) - 36*x*(4 - 27*x**2)**(1/3) + 8*I*(4 - 27*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(-27\,x^2+4)^{\frac{1}{3}}(3i\,x+2)^3}\,dx$$

input
```
integrate(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^3), x)
```

Giac [F]

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(-27\,x^2+4)^{\frac{1}{3}}(3i\,x+2)^3}\,dx$$

input
```
integrate(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-27*x^2 + 4)^(1/3)*(3*I*x + 2)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = \int \frac{1}{(2+x\,3\mathrm{i})^3\,(4-27\,x^2)^{1/3}}\,dx$$

input
```
int(1/((x*3i + 2)^3*(4 - 27*x^2)^(1/3)),x)
```

output
```
int(1/((x*3i + 2)^3*(4 - 27*x^2)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{(2+3ix)^3\sqrt[3]{4-27x^2}}\,dx = -\left(\int \frac{1}{27\,(-27x^2+4)^{\frac{1}{3}}\,i\,x^3-36\,(-27x^2+4)^{\frac{1}{3}}\,ix+54\,(-27x^2+4)^{\frac{1}{3}}\,x^2-8\,(-27x^2+4)^{\frac{1}{3}}}dx\right)$$

input
```
int(1/(2+3*I*x)^3/(-27*x^2+4)^(1/3),x)
```

output
```
 - int(1/(27*( - 27*x**2 + 4)**(1/3)*i*x**3 - 36*( - 27*x**2 + 4)**(1/3)*i
*x + 54*( - 27*x**2 + 4)**(1/3)*x**2 - 8*( - 27*x**2 + 4)**(1/3)),x)
```

3.355 $\int \frac{1}{(\sqrt{3}+x)\sqrt[3]{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{1}{(\sqrt{3}+x)\sqrt[3]{1+x^2}}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(\sqrt{3}-x)}{3\sqrt[3]{1+x^2}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(\sqrt{3}+x)}{2\,2^{2/3}} + \frac{\log\left(2^{2/3}(\sqrt{3}-x)-2\sqrt{3}\sqrt[3]{1+x^2}\right)}{2\,2^{2/3}}$$

output
```
1/6*arctan(-1/3*3^(1/2)-1/3*2^(2/3)*(3^(1/2)-x)/(x^2+1)^(1/3))*2^(1/3)*3^(
1/2)-1/4*ln(x+3^(1/2))*2^(1/3)+1/4*ln(2^(2/3)*(3^(1/2)-x)-2*3^(1/2)*(x^2+1
)^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.62

$$\int \frac{1}{(\sqrt{3}+x)\sqrt[3]{1+x^2}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{2^{2/3}x-\sqrt{3}\left(2^{2/3}+\sqrt[3]{1+x^2}\right)}{3\sqrt[3]{1+x^2}}\right)+2\log\left(-32^{2/3}+2^{2/3}\sqrt{3}x+6\sqrt[3]{1+x^2}\right)-\log\left(-\sqrt[3]{2}x^2+\sqrt[3]{2}\sqrt{}\right.}{6\,2^{2/3}}$$

input
```
Integrate[1/((Sqrt[3] + x)*(1 + x^2)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(2^(2/3)*x - Sqrt[3]*(2^(2/3) + (1 + x^2)^(1/3)))/(3*(1
+ x^2)^(1/3))] + 2*Log[-3*2^(2/3) + 2^(2/3)*Sqrt[3]*x + 6*(1 + x^2)^(1/3)]
 - Log[-(2^(1/3)*x^2) + 2^(1/3)*Sqrt[3]*x*(2 + 2^(1/3)*(1 + x^2)^(1/3)) -
3*(2^(1/3) + 2^(2/3)*(1 + x^2)^(1/3) + 2*(1 + x^2)^(2/3))])/(6*2^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {501}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+\sqrt{3}\right)\sqrt[3]{x^2+1}}dx$$

$$\Big\downarrow \text{501}$$

$$-\frac{\arctan\left(\frac{2^{2/3}\left(\sqrt{3}-x\right)}{3\sqrt[3]{x^2+1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}-x+\sqrt{3}\right)}{2\ 2^{2/3}}-\frac{\log\left(x+\sqrt{3}\right)}{2\ 2^{2/3}}$$

input
```
Int[1/((Sqrt[3] + x)*(1 + x^2)^(1/3)),x]
```

output
```
-(ArcTan[1/Sqrt[3] + (2^(2/3)*(Sqrt[3] - x))/(3*(1 + x^2)^(1/3))]/(2^(2/3)
*Sqrt[3])) - Log[Sqrt[3] + x]/(2*2^(2/3)) + Log[Sqrt[3] - x - 2^(1/3)*Sqrt
[3]*(1 + x^2)^(1/3)]/(2*2^(2/3))
```

Defintions of rubi rules used

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 26.01 (sec) , antiderivative size = 2447, normalized size of antiderivative = 23.08

method	result	size
trager	Expression too large to display	2447

input
```
int(1/(3^(1/2)+x)/(x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/18*3^(1/2)*(6*ln(-(60*RootOf(_Z^3-6*3^(1/2))^2*RootOf(RootOf(_Z^3-6*3^(
1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)^2*x^3+2*RootOf(_Z^3-6*3^(1/2)
)^3*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*x
^3-144*RootOf(_Z^3-6*3^(1/2))^2*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_
Z^3-6*3^(1/2))*_Z+36*_Z^2)*3^(1/2)*(x^2+1)^(2/3)*x-864*RootOf(_Z^3-6*3^(1/
2))*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*(
x^2+1)^(1/3)*x+144*RootOf(_Z^3-6*3^(1/2))*RootOf(RootOf(_Z^3-6*3^(1/2))^2+
6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*3^(1/2)*(x^2+1)^(1/3)*x^2-54*RootOf(_
Z^3-6*3^(1/2))^2*(x^2+1)^(1/3)*x+9*RootOf(_Z^3-6*3^(1/2))^2*3^(1/2)*(x^2+1
)^(1/3)*x^2+540*RootOf(_Z^3-6*3^(1/2))^2*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6
*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)^2*x+18*RootOf(_Z^3-6*3^(1/2))^3*RootOf
(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*x+432*RootO
f(_Z^3-6*3^(1/2))^2*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2
))*_Z+36*_Z^2)*(x^2+1)^(2/3)-30*3^(1/2)*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*
RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*x^3-RootOf(_Z^3-6*3^(1/2))*3^(1/2)*x^3-
162*(x^2+1)^(2/3)*x+432*RootOf(_Z^3-6*3^(1/2))*RootOf(RootOf(_Z^3-6*3^(1/2
))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*3^(1/2)*(x^2+1)^(1/3)+27*RootOf(
_Z^3-6*3^(1/2))^2*3^(1/2)*(x^2+1)^(1/3)-270*RootOf(RootOf(_Z^3-6*3^(1/2))^
2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*3^(1/2)*x+1890*RootOf(RootOf(_Z^3-6
*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*x^2-9*RootOf(_Z^3-6*3^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(77) = 154$.

Time = 2.92 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.66

$$\int \frac{1}{(\sqrt{3}+x)\sqrt[3]{1+x^2}}\,dx = -\frac{1}{6}$$

$$\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}\left(x^4+8\sqrt{3}x^3-18x^2-27\right)(x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^6+99x^4+243x^2+12\sqrt{3}(x^5\right.\right.}{6\left(x^6-225x^4-\right.}\right.$$

$$-\frac{1}{24}$$

$$\cdot 4^{\frac{2}{3}}\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(x^2-2\sqrt{3}x+3\right)(x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^4+18x^2-4\sqrt{3}(x^3+3x)+9\right)+2\left(9x^2-\sqrt{3}(x^3+\right.}{x^4-6x^2+9}\right.$$

$$+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}\left(x^2-2\sqrt{3}x+3\right)+2(x^2+1)^{\frac{1}{3}}\left(\sqrt{3}x-3\right)}{x^2-3}\right)$$

input
```
integrate(1/(3^(1/2)+x)/(x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(x^4 + 8*sqrt(3
)*x^3 - 18*x^2 - 27)*(x^2 + 1)^(2/3) + 4^(1/3)*(x^6 + 99*x^4 + 243*x^2 + 1
2*sqrt(3)*(x^5 + 10*x^3 + 9*x) + 81) + 4*(21*x^4 + 54*x^2 + sqrt(3)*(x^5 -
 42*x^3 - 27*x) + 81)*(x^2 + 1)^(1/3))/(x^6 - 225*x^4 - 405*x^2 - 243)) -
1/24*4^(2/3)*log((3*4^(2/3)*(x^2 - 2*sqrt(3)*x + 3)*(x^2 + 1)^(2/3) + 4^(1
/3)*(x^4 + 18*x^2 - 4*sqrt(3)*(x^3 + 3*x) + 9) + 2*(9*x^2 - sqrt(3)*(x^3 +
 9*x) + 9)*(x^2 + 1)^(1/3))/(x^4 - 6*x^2 + 9)) + 1/12*4^(2/3)*log((4^(1/3)
*(x^2 - 2*sqrt(3)*x + 3) + 2*(x^2 + 1)^(1/3)*(sqrt(3)*x - 3))/(x^2 - 3))
```

Sympy [F]

$$\int \frac{1}{\left(\sqrt{3}+x\right)\sqrt[3]{1+x^2}}\,dx=\int \frac{1}{\left(x+\sqrt{3}\right)\sqrt[3]{x^2+1}}\,dx$$

input
```
integrate(1/(3**(1/2)+x)/(x**2+1)**(1/3),x)
```

output
```
Integral(1/((x + sqrt(3))*(x**2 + 1)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{3}+x\right)\sqrt[3]{1+x^2}}\,dx=\int \frac{1}{(x^2+1)^{\frac{1}{3}}\left(x+\sqrt{3}\right)}\,dx$$

input
```
integrate(1/(3^(1/2)+x)/(x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)^(1/3)*(x + sqrt(3))), x)
```

Giac [F]

$$\int \frac{1}{\left(\sqrt{3}+x\right)\sqrt[3]{1+x^2}}\,dx=\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x+\sqrt{3}\right)}\,dx$$

input
```
integrate(1/(3^(1/2)+x)/(x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 1)^(1/3)*(x + sqrt(3))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{3}+x\right)\sqrt[3]{1+x^2}}\,dx=\int \frac{1}{\left(x^2+1\right)^{1/3}\,\left(x+\sqrt{3}\right)}\,dx$$

input
```
int(1/((x^2 + 1)^(1/3)*(x + 3^(1/2))),x)
```

output
```
int(1/((x^2 + 1)^(1/3)*(x + 3^(1/2))), x)
```

Reduce [F]

$$\int \frac{1}{\left(\sqrt{3}+x\right)\sqrt[3]{1+x^2}}\,dx=-\sqrt{3}\left(\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}x^2-3\left(x^2+1\right)^{\frac{1}{3}}}dx\right)+\int \frac{x}{\left(x^2+1\right)^{\frac{1}{3}}x^2-3\left(x^2+1\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(3^(1/2)+x)/(x^2+1)^(1/3),x)
```

output
```
- sqrt(3)*int(1/((x**2 + 1)**(1/3)*x**2 - 3*(x**2 + 1)**(1/3)),x) + int(x
/((x**2 + 1)**(1/3)*x**2 - 3*(x**2 + 1)**(1/3)),x)
```

3.356 $\int \frac{1}{(\sqrt{3}-x)\sqrt[3]{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{1+x^2}}\,dx=\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}\left(\sqrt{3}+x\right)}{3\sqrt[3]{1+x^2}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt{3}-x\right)}{2\ 2^{2/3}}-\frac{\log\left(2^{2/3}\left(\sqrt{3}+x\right)-2\sqrt{3}\sqrt[3]{1+x^2}\right)}{2\ 2^{2/3}}$$

output
```
1/6*arctan(1/3*3^(1/2)+1/3*2^(2/3)*(x+3^(1/2))/(x^2+1)^(1/3))*2^(1/3)*3^(1
/2)+1/4*ln(3^(1/2)-x)*2^(1/3)-1/4*ln(2^(2/3)*(x+3^(1/2))-2*3^(1/2)*(x^2+1)
^(1/3))*2^(1/3)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{1+x^2}}\,dx$$

$$=\frac{2\sqrt{3}\arctan\left(\frac{2^{2/3}x+\sqrt{3}\left(2^{2/3}+\sqrt[3]{1+x^2}\right)}{3\sqrt[3]{1+x^2}}\right)-2\log\left(3\ 2^{2/3}+2^{2/3}\sqrt{3}x-6\sqrt[3]{1+x^2}\right)+\log\left(\sqrt[3]{2}x^2+\sqrt[3]{2}\sqrt{3}x\right.}{6\ 2^{2/3}}$$

input
```
Integrate[1/((Sqrt[3] - x)*(1 + x^2)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(2^(2/3)*x + Sqrt[3]*(2^(2/3) + (1 + x^2)^(1/3)))/(3*(1
+ x^2)^(1/3))] - 2*Log[3*2^(2/3) + 2^(2/3)*Sqrt[3]*x - 6*(1 + x^2)^(1/3)]
+ Log[2^(1/3)*x^2 + 2^(1/3)*Sqrt[3]*x*(2 + 2^(1/3)*(1 + x^2)^(1/3)) + 3*(2
^(1/3) + 2^(2/3)*(1 + x^2)^(1/3) + 2*(1 + x^2)^(2/3))])/(6*2^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{501\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{x^2+1}}dx$$

$$\Big\downarrow \text{501}$$

$$\frac{\arctan\left(\frac{2^{2/3}\left(x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+x+\sqrt{3}\right)}{2\ 2^{2/3}}+\frac{\log\left(\sqrt{3}-x\right)}{2\ 2^{2/3}}$$

input
```
Int[1/((Sqrt[3] - x)*(1 + x^2)^(1/3)),x]
```

output
```
ArcTan[1/Sqrt[3] + (2^(2/3)*(Sqrt[3] + x))/(3*(1 + x^2)^(1/3))]/(2^(2/3)*S
qrt[3]) + Log[Sqrt[3] - x]/(2*2^(2/3)) - Log[Sqrt[3] + x - 2^(1/3)*Sqrt[3]
*(1 + x^2)^(1/3)]/(2*2^(2/3))
```

Defintions of rubi rules used

rule 501
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> With[
{q = Rt[6*b^2*(d^2/c^2), 3]}, Simp[(-Sqrt[3])*b*d*(ArcTan[1/Sqrt[3] + 2*b*(
(c - d*x)/(Sqrt[3]*c*q*(a + b*x^2)^(1/3)))]/(c^2*q^2)), x] + (-Simp[3*b*d*(
Log[c + d*x]/(2*c^2*q^2)), x] + Simp[3*b*d*(Log[b*c - b*d*x - c*q*(a + b*x^
2)^(1/3)]/(2*c^2*q^2)), x])] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 - 3*a*d
^2, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 26.01 (sec) , antiderivative size = 1625, normalized size of antiderivative = 15.78

method	result	size
trager	Expression too large to display	1625

input
```
int(1/(3^(1/2)-x)/(x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/18*3^(1/2)*(6*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*
_Z+36*_Z^2)*ln(-(-72*RootOf(_Z^3-6*3^(1/2))^2*RootOf(RootOf(_Z^3-6*3^(1/2)
)^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*3^(1/2)*(x^2+1)^(2/3)*x-6*RootOf(
_Z^3-6*3^(1/2))^2*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))
*_Z+36*_Z^2)^2*x^3+4*RootOf(_Z^3-6*3^(1/2))^3*RootOf(RootOf(_Z^3-6*3^(1/2)
)^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*x^3+270*RootOf(_Z^3-6*3^(1/2))*Ro
otOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*(x^2+1)
^(1/3)*x+72*RootOf(_Z^3-6*3^(1/2))^2*(x^2+1)^(1/3)*x+45*RootOf(_Z^3-6*3^(1
/2))*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*
3^(1/2)*(x^2+1)^(1/3)*x^2+12*RootOf(_Z^3-6*3^(1/2))^2*3^(1/2)*(x^2+1)^(1/3
)*x^2-81*(x^2+1)^(2/3)*x+3*3^(1/2)*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootO
f(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*x^3-2*RootOf(_Z^3-6*3^(1/2))*3^(1/2)*x^3-54*
RootOf(_Z^3-6*3^(1/2))^2*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3
^(1/2))*_Z+36*_Z^2)^2*x+36*RootOf(_Z^3-6*3^(1/2))^3*RootOf(RootOf(_Z^3-6*3
^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*x-216*RootOf(_Z^3-6*3^(1/2)
)^2*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*(
x^2+1)^(2/3)+135*RootOf(_Z^3-6*3^(1/2))*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*
RootOf(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*3^(1/2)*(x^2+1)^(1/3)+36*RootOf(_Z^3-6*
3^(1/2))^2*3^(1/2)*(x^2+1)^(1/3)+27*RootOf(RootOf(_Z^3-6*3^(1/2))^2+6*Root
Of(_Z^3-6*3^(1/2))*_Z+36*_Z^2)*3^(1/2)*x-18*RootOf(_Z^3-6*3^(1/2))*3^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(74) = 148$.

Time = 2.96 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.74

$$\int \frac{1}{(\sqrt{3}-x)\sqrt[3]{1+x^2}}\,dx = -\frac{1}{6} \cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}\left(x^4-8\sqrt{3}x^3-18x^2-27\right)\left(x^2+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^6+99x^4+243x^2-12\sqrt{3}(\right.\right.}{6\left(x^6-225x^4-\right.}\right.$$
$$+\frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(x^2+2\sqrt{3}x+3\right)\left(x^2+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^4+18x^2+4\sqrt{3}(x^3+3x)+9\right)+2\left(9x^2+\sqrt{3}(x^3+\right.}{x^4-6x^2+9}\right.$$
$$-\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}\left(x^2+2\sqrt{3}x+3\right)-2\left(x^2+1\right)^{\frac{1}{3}}\left(\sqrt{3}x+3\right)}{x^2-3}\right)$$

input
```
integrate(1/(3^(1/2)-x)/(x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(x^4 - 8*sqrt(
3)*x^3 - 18*x^2 - 27)*(x^2 + 1)^(2/3) + 4^(1/3)*(x^6 + 99*x^4 + 243*x^2 -
12*sqrt(3)*(x^5 + 10*x^3 + 9*x) + 81) + 4*(21*x^4 + 54*x^2 - sqrt(3)*(x^5
- 42*x^3 - 27*x) + 81)*(x^2 + 1)^(1/3))/(x^6 - 225*x^4 - 405*x^2 - 243)) +
 1/24*4^(2/3)*log((3*4^(2/3)*(x^2 + 2*sqrt(3)*x + 3)*(x^2 + 1)^(2/3) + 4^(
1/3)*(x^4 + 18*x^2 + 4*sqrt(3)*(x^3 + 3*x) + 9) + 2*(9*x^2 + sqrt(3)*(x^3
+ 9*x) + 9)*(x^2 + 1)^(1/3))/(x^4 - 6*x^2 + 9)) - 1/12*4^(2/3)*log((4^(1/3
)*(x^2 + 2*sqrt(3)*x + 3) - 2*(x^2 + 1)^(1/3)*(sqrt(3)*x + 3))/(x^2 - 3))
```

Sympy [F]

$$\int \frac{1}{(\sqrt{3}-x)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{x\sqrt[3]{x^2+1}-\sqrt{3}\sqrt[3]{x^2+1}}\,dx$$

input
```
integrate(1/(3**(1/2)-x)/(x**2+1)**(1/3),x)
```

output
```
-Integral(1/(x*(x**2 + 1)**(1/3) - sqrt(3)*(x**2 + 1)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(\sqrt{3}-x)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x-\sqrt{3})}\,dx$$

input
```
integrate(1/(3^(1/2)-x)/(x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((x^2 + 1)^(1/3)*(x - sqrt(3))), x)
```

Giac [F]

$$\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{1+x^2}}\,dx=\int -\frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x-\sqrt{3}\right)}\,dx$$

input
```
integrate(1/(3^(1/2)-x)/(x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((x^2 + 1)^(1/3)*(x - sqrt(3))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{1+x^2}}\,dx=-\int \frac{1}{\left(x^2+1\right)^{1/3}\,\left(x-\sqrt{3}\right)}\,dx$$

input
```
int(-1/((x^2 + 1)^(1/3)*(x - 3^(1/2))),x)
```

output
```
-int(1/((x^2 + 1)^(1/3)*(x - 3^(1/2))), x)
```

Reduce [F]

$$\int \frac{1}{\left(\sqrt{3}-x\right)\sqrt[3]{1+x^2}}\,dx=-\sqrt{3}\left(\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}x^2-3\left(x^2+1\right)^{\frac{1}{3}}}dx\right)-\left(\int \frac{x}{\left(x^2+1\right)^{\frac{1}{3}}x^2-3\left(x^2+1\right)^{\frac{1}{3}}}dx\right)$$

input
```
int(1/(3^(1/2)-x)/(x^2+1)^(1/3),x)
```

output
```
- (sqrt(3)*int(1/((x**2 + 1)**(1/3)*x**2 - 3*(x**2 + 1)**(1/3)),x) + int(
x/((x**2 + 1)**(1/3)*x**2 - 3*(x**2 + 1)**(1/3)),x))
```

3.357 $\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx = -\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{1+x}{\sqrt{3}\sqrt[3]{1-x^2}}\right)-\frac{1}{4}\log(3-x)-\frac{1}{8}\log(3+3x)+\frac{3}{8}\log\left(3+3x+6\sqrt[3]{1-x^2}\right)$$

output
```
1/4*3^(1/2)*arctan(-1/3*3^(1/2)+1/3*(1+x)*3^(1/2)/(-x^2+1)^(1/3))-1/4*ln(3
-x)-1/8*ln(3+3*x)+3/8*ln(3+3*x+6*(-x^2+1)^(1/3))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx = \frac{1}{8}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{1+x-\sqrt[3]{1-x^2}}\right)+2\log\left(1+x+2\sqrt[3]{1-x^2}\right)-\log\left(1+2x+x^2-2(1+x)\sqrt[3]{1-x^2}+4\left(1-x^2\right)^{2/3}\right)\right)$$

input
```
Integrate[1/((3 - x)*(1 - x^2)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(1 + x - (1 - x^2)^(1/3))] +
2*Log[1 + x + 2*(1 - x^2)^(1/3)] - Log[1 + 2*x + x^2 - 2*(1 + x)*(1 - x^2)
^(1/3) + 4*(1 - x^2)^(2/3)])/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {502, 133}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}dx$$

$$\Big\downarrow \text{502}$$

$$\int \frac{1}{\sqrt[3]{1-x}(3-x)\sqrt[3]{x+1}}dx$$

$$\Big\downarrow \text{133}$$

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)-\frac{1}{4}\log(3-x)+\frac{3}{8}\log\left(-\frac{1}{2}(x+1)^{2/3}-\sqrt[3]{1-x}\right)$$

input
```
Int[1/((3 - x)*(1 - x^2)^(1/3)),x]
```

output
```
-1/4*(Sqrt[3]*ArcTan[1/Sqrt[3] - (1 + x)^(2/3)/(Sqrt[3]*(1 - x)^(1/3))]) -
 Log[3 - x]/4 + (3*Log[-(1 - x)^(1/3) - (1 + x)^(2/3)/2])/8
```

Defintions of rubi rules used

rule 133
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_] :> With[{q = Rt[b*((b*e - a*f)/(b*c - a*d)^2), 3]}, Simp[-Log[
a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[Sqrt[3]*(ArcTan[1/Sqrt[3] + 2*q*((c
 + d*x)^(2/3)/(Sqrt[3]*(e + f*x)^(1/3)))]/(2*q*(b*c - a*d))), x] + Simp[3*(
Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)]/(4*q*(b*c - a*d))), x])] /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

rule 502
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> Simp[
a^(1/3)   Int[1/((c + d*x)*(1 - 3*d*(x/c))^(1/3)*(1 + 3*d*(x/c))^(1/3)), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 9*a*d^2, 0] && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.24 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.62

method	result
trager	$-\frac{\ln\left(-\frac{96\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2 x^2+864\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(-x^2+1\right)^{\frac{2}{3}}-432\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(-x^2+1\right)^{\frac{1}{3}}x+288\,\mathrm{I}}{}\right.}{}$

input
```
int(1/(3-x)/(-x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(-(96*RootOf(4*_Z^2+2*_Z+1)^2*x^2+864*RootOf(4*_Z^2+2*_Z+1)*(-x^2+1
)^(2/3)-432*RootOf(4*_Z^2+2*_Z+1)*(-x^2+1)^(1/3)*x+288*RootOf(4*_Z^2+2*_Z+
1)^2*x+278*RootOf(4*_Z^2+2*_Z+1)*x^2-516*(-x^2+1)^(2/3)-432*RootOf(4*_Z^2+
2*_Z+1)*(-x^2+1)^(1/3)+258*(-x^2+1)^(1/3)*x+492*RootOf(4*_Z^2+2*_Z+1)*x+17
*x^2+258*(-x^2+1)^(1/3)+342*RootOf(4*_Z^2+2*_Z+1)-918*x+969)/(-3+x)^2)-1/2
*ln(-(96*RootOf(4*_Z^2+2*_Z+1)^2*x^2+864*RootOf(4*_Z^2+2*_Z+1)*(-x^2+1)^(2
/3)-432*RootOf(4*_Z^2+2*_Z+1)*(-x^2+1)^(1/3)*x+288*RootOf(4*_Z^2+2*_Z+1)^2
*x+278*RootOf(4*_Z^2+2*_Z+1)*x^2-516*(-x^2+1)^(2/3)-432*RootOf(4*_Z^2+2*_Z
+1)*(-x^2+1)^(1/3)+258*(-x^2+1)^(1/3)*x+492*RootOf(4*_Z^2+2*_Z+1)*x+17*x^2
+258*(-x^2+1)^(1/3)+342*RootOf(4*_Z^2+2*_Z+1)-918*x+969)/(-3+x)^2)*RootOf(
4*_Z^2+2*_Z+1)+1/2*RootOf(4*_Z^2+2*_Z+1)*ln((-48*RootOf(4*_Z^2+2*_Z+1)^2*x
^2+432*RootOf(4*_Z^2+2*_Z+1)*(-x^2+1)^(2/3)-216*RootOf(4*_Z^2+2*_Z+1)*(-x^
2+1)^(1/3)*x-144*RootOf(4*_Z^2+2*_Z+1)^2*x+91*RootOf(4*_Z^2+2*_Z+1)*x^2+47
4*(-x^2+1)^(2/3)-216*RootOf(4*_Z^2+2*_Z+1)*(-x^2+1)^(1/3)-237*(-x^2+1)^(1/
3)*x+102*RootOf(4*_Z^2+2*_Z+1)*x+49*x^2-237*(-x^2+1)^(1/3)+171*RootOf(4*_Z
^2+2*_Z+1)+546*x-399)/(-3+x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx =$$
$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{18031\sqrt{3}(-x^2+1)^{\frac{1}{3}}(x+1)+\sqrt{3}(5054\,x^2-8497\,x+23659)+57889\sqrt{3}(-x^2+1)^{\frac{2}{3}}}{6859\,x^2+240699\,x-220122}\right)$$
$$+\frac{1}{8}\log\left(\frac{x^2+6\,(-x^2+1)^{\frac{1}{3}}(x+1)-6\,x+12\,(-x^2+1)^{\frac{2}{3}}+9}{x^2-6\,x+9}\right)$$

input
```
integrate(1/(3-x)/(-x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*sqrt(3)*arctan((18031*sqrt(3)*(-x^2 + 1)^(1/3)*(x + 1) + sqrt(3)*(505
4*x^2 - 8497*x + 23659) + 57889*sqrt(3)*(-x^2 + 1)^(2/3))/(6859*x^2 + 2406
99*x - 220122)) + 1/8*log((x^2 + 6*(-x^2 + 1)^(1/3)*(x + 1) - 6*x + 12*(-x
^2 + 1)^(2/3) + 9)/(x^2 - 6*x + 9))
```

Sympy [F]

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx = -\int \frac{1}{x\sqrt[3]{1-x^2}-3\sqrt[3]{1-x^2}}\,dx$$

input
```
integrate(1/(3-x)/(-x**2+1)**(1/3),x)
```

output
```
-Integral(1/(x*(1 - x**2)**(1/3) - 3*(1 - x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx = \int -\frac{1}{(-x^2+1)^{\frac{1}{3}}(x-3)}\,dx$$

input
```
integrate(1/(3-x)/(-x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((-x^2 + 1)^(1/3)*(x - 3)), x)
```

Giac [F]

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx = \int -\frac{1}{(-x^2+1)^{\frac{1}{3}}(x-3)}\,dx$$

input
```
integrate(1/(3-x)/(-x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((-x^2 + 1)^(1/3)*(x - 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx = -\int \frac{1}{\left(1-x^2\right)^{1/3}\,(x-3)}\,dx$$

input
```
int(-1/((1 - x^2)^(1/3)*(x - 3)),x)
```

output
```
-int(1/((1 - x^2)^(1/3)*(x - 3)), x)
```

Reduce [F]

$$\int \frac{1}{(3-x)\sqrt[3]{1-x^2}}\,dx = -\left(\int \frac{1}{\left(-x^2+1\right)^{\frac{1}{3}}x-3\left(-x^2+1\right)^{\frac{1}{3}}}dx\right)$$

input
```
int(1/(3-x)/(-x^2+1)^(1/3),x)
```

output
```
- int(1/(( - x**2 + 1)**(1/3)*x - 3*( - x**2 + 1)**(1/3)),x)
```

3.358 $\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx = \frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{1-x}{\sqrt{3}\sqrt[3]{1-x^2}}\right)+\frac{1}{8}\log(3-3x)+\frac{1}{4}\log(3+x)-\frac{3}{8}\log\left(3-3x+6\sqrt[3]{1-x^2}\right)$$

output
```
1/4*3^(1/2)*arctan(1/3*3^(1/2)-1/3*(1-x)*3^(1/2)/(-x^2+1)^(1/3))+1/8*ln(3-
3*x)+1/4*ln(3+x)-3/8*ln(3-3*x+6*(-x^2+1)^(1/3))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx = \frac{1}{8}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{-1+x+\sqrt[3]{1-x^2}}\right)-2\log\left(1-x+2\sqrt[3]{1-x^2}\right)+\log\left(1-2x+x^2+2(-1+x)\sqrt[3]{1-x^2}+4\left(1-x^2\right)^{2/3}\right)\right)$$

input
```
Integrate[1/((3 + x)*(1 - x^2)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-1 + x + (1 - x^2)^(1/3))] -
 2*Log[1 - x + 2*(1 - x^2)^(1/3)] + Log[1 - 2*x + x^2 + 2*(-1 + x)*(1 - x^
2)^(1/3) + 4*(1 - x^2)^(2/3)])/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {502, 133}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt[3]{1-x^2}}dx$$

$$\Big\downarrow \text{ 502}$$

$$\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x+1}(x+3)}dx$$

$$\Big\downarrow \text{ 133}$$

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)+\frac{1}{4}\log(x+3)-\frac{3}{8}\log\left(-\frac{1}{2}(1-x)^{2/3}-\sqrt[3]{x+1}\right)$$

input
```
Int[1/((3 + x)*(1 - x^2)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[1/Sqrt[3] - (1 - x)^(2/3)/(Sqrt[3]*(1 + x)^(1/3))])/4 + Lo
g[3 + x]/4 - (3*Log[-1/2*(1 - x)^(2/3) - (1 + x)^(1/3)])/8
```

Defintions of rubi rules used

rule 133
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_] :> With[{q = Rt[b*((b*e - a*f)/(b*c - a*d)^2), 3]}, Simp[-Log[
a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[Sqrt[3]*(ArcTan[1/Sqrt[3] + 2*q*((c
 + d*x)^(2/3)/(Sqrt[3]*(e + f*x)^(1/3)))]/(2*q*(b*c - a*d))), x] + Simp[3*(
Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)]/(4*q*(b*c - a*d))), x])] /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

rule 502
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> Simp[
a^(1/3)   Int[1/((c + d*x)*(1 - 3*d*(x/c))^(1/3)*(1 + 3*d*(x/c))^(1/3)), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 9*a*d^2, 0] && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.25 (sec) , antiderivative size = 616, normalized size of antiderivative = 7.60

method	result
trager	$\frac{\operatorname{RootOf}\left(4\_Z^2-2\_Z+1\right)\ln\left(-\frac{48\operatorname{RootOf}\left(4\_Z^2-2\_Z+1\right)^2x^2+432\operatorname{RootOf}\left(4\_Z^2-2\_Z+1\right)\left(-x^2+1\right)^{\frac{2}{3}}+216\operatorname{RootOf}\left(4\_Z^2-2\right.}{}\right.}{}$

input
```
int(1/(3+x)/(-x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*RootOf(4*_Z^2-2*_Z+1)*ln(-(48*RootOf(4*_Z^2-2*_Z+1)^2*x^2+432*RootOf(4
*_Z^2-2*_Z+1)*(-x^2+1)^(2/3)+216*RootOf(4*_Z^2-2*_Z+1)*(-x^2+1)^(1/3)*x-14
4*RootOf(4*_Z^2-2*_Z+1)^2*x+91*RootOf(4*_Z^2-2*_Z+1)*x^2-474*(-x^2+1)^(2/3
)-216*RootOf(4*_Z^2-2*_Z+1)*(-x^2+1)^(1/3)-237*(-x^2+1)^(1/3)*x-102*RootOf
(4*_Z^2-2*_Z+1)*x-49*x^2+237*(-x^2+1)^(1/3)+171*RootOf(4*_Z^2-2*_Z+1)+546*
x+399)/(3+x)^2)+1/4*ln((-96*RootOf(4*_Z^2-2*_Z+1)^2*x^2+864*RootOf(4*_Z^2-
2*_Z+1)*(-x^2+1)^(2/3)+432*RootOf(4*_Z^2-2*_Z+1)*(-x^2+1)^(1/3)*x+288*Root
Of(4*_Z^2-2*_Z+1)^2*x+278*RootOf(4*_Z^2-2*_Z+1)*x^2+516*(-x^2+1)^(2/3)-432
*RootOf(4*_Z^2-2*_Z+1)*(-x^2+1)^(1/3)+258*(-x^2+1)^(1/3)*x-492*RootOf(4*_Z
^2-2*_Z+1)*x-17*x^2-258*(-x^2+1)^(1/3)+342*RootOf(4*_Z^2-2*_Z+1)-918*x-969
)/(3+x)^2)-1/2*ln((-96*RootOf(4*_Z^2-2*_Z+1)^2*x^2+864*RootOf(4*_Z^2-2*_Z+
1)*(-x^2+1)^(2/3)+432*RootOf(4*_Z^2-2*_Z+1)*(-x^2+1)^(1/3)*x+288*RootOf(4*
_Z^2-2*_Z+1)^2*x+278*RootOf(4*_Z^2-2*_Z+1)*x^2+516*(-x^2+1)^(2/3)-432*Root
Of(4*_Z^2-2*_Z+1)*(-x^2+1)^(1/3)+258*(-x^2+1)^(1/3)*x-492*RootOf(4*_Z^2-2*
_Z+1)*x-17*x^2-258*(-x^2+1)^(1/3)+342*RootOf(4*_Z^2-2*_Z+1)-918*x-969)/(3+
x)^2)*RootOf(4*_Z^2-2*_Z+1)
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.42

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx$$
$$= \frac{1}{4}\sqrt{3}\arctan\left(-\frac{18031\sqrt{3}(-x^2+1)^{\frac{1}{3}}(x-1)-\sqrt{3}(5054\,x^2+8497\,x+23659)-57889\sqrt{3}(-x^2+1)^{\frac{2}{3}}}{6859\,x^2-240699\,x-220122}\right)$$
$$-\frac{1}{8}\log\left(\frac{x^2-6\,(-x^2+1)^{\frac{1}{3}}(x-1)+6\,x+12\,(-x^2+1)^{\frac{2}{3}}+9}{x^2+6\,x+9}\right)$$

input
```
integrate(1/(3+x)/(-x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
1/4*sqrt(3)*arctan(-(18031*sqrt(3)*(-x^2 + 1)^(1/3)*(x - 1) - sqrt(3)*(505
4*x^2 + 8497*x + 23659) - 57889*sqrt(3)*(-x^2 + 1)^(2/3))/(6859*x^2 - 2406
99*x - 220122)) - 1/8*log((x^2 - 6*(-x^2 + 1)^(1/3)*(x - 1) + 6*x + 12*(-x
^2 + 1)^(2/3) + 9)/(x^2 + 6*x + 9))
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)(x+1)}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x**2+1)**(1/3),x)
```

output
```
Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx = \int \frac{1}{(-x^2+1)^{\frac{1}{3}}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-x^2 + 1)^(1/3)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx = \int \frac{1}{(-x^2+1)^{\frac{1}{3}}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-x^2 + 1)^(1/3)*(x + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx = \int \frac{1}{\left(1-x^2\right)^{1/3}\,(x+3)}\,dx$$

input
```
int(1/((1 - x^2)^(1/3)*(x + 3)),x)
```

output
```
int(1/((1 - x^2)^(1/3)*(x + 3)), x)
```

Reduce [F]

$$\int \frac{1}{(3+x)\sqrt[3]{1-x^2}}\,dx = \int \frac{1}{\left(-x^2+1\right)^{\frac{1}{3}} x+3\left(-x^2+1\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(3+x)/(-x^2+1)^(1/3),x)
```

output
```
int(1/(( - x**2 + 1)**(1/3)*x + 3*( - x**2 + 1)**(1/3)),x)
```

3.359 $\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{c-3dx}{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c^2-9d^2x^2}}\right)}{4c^{2/3}d}+\frac{\log(c-3dx)}{8c^{2/3}d}+\frac{\log(c+dx)}{4c^{2/3}d}-\frac{3\log\left(c^{2/3}(c-3dx)+2c\sqrt[3]{c^2-9d^2x^2}\right)}{8c^{2/3}d}$$

output
```
-1/4*3^(1/2)*arctan(-1/3*3^(1/2)+1/3*(-3*d*x+c)*3^(1/2)/c^(1/3)/(-9*d^2*x^
2+c^2)^(1/3))/c^(2/3)/d+1/8*ln(-3*d*x+c)/c^(2/3)/d+1/4*ln(d*x+c)/c^(2/3)/d
-3/8*ln(c^(2/3)*(-3*d*x+c)+2*c*(-9*d^2*x^2+c^2)^(1/3))/c^(2/3)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.92

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx$$
$$= \frac{2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{2\sqrt{3}\sqrt[3]{c}\sqrt[3]{c^2-9d^2x^2}}{c-i\sqrt{3}c+3i\left(i+\sqrt{3}\right)dx+2\sqrt[3]{c}\sqrt[3]{c^2-9d^2x^2}}\right)+\left(1+i\sqrt{3}\right)\left(2\log\left(\sqrt{d}\left(-c+i\sqrt{3}c+3d\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)*(c^2 - 9*d^2*x^2)^(1/3)),x]
```

output
```
(2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(2*Sqrt[3]*c^(1/3)*(c^2 - 9*d^2*x^2)^(1
/3))/(c - I*Sqrt[3]*c + (3*I)*(I + Sqrt[3])*d*x + 2*c^(1/3)*(c^2 - 9*d^2*x
^2)^(1/3))] + (1 + I*Sqrt[3])*(2*Log[Sqrt[d]*(-c + I*Sqrt[3]*c + 3*d*x - (
3*I)*Sqrt[3]*d*x + 4*c^(1/3)*(c^2 - 9*d^2*x^2)^(1/3))] - Log[d*(c - 3*d*x
+ 2*c^(1/3)*(c^2 - 9*d^2*x^2)^(1/3))*(-c - I*Sqrt[3]*c + 3*d*x + (3*I)*Sqr
t[3]*d*x + 4*c^(1/3)*(c^2 - 9*d^2*x^2)^(1/3))]))/(16*c^(2/3)*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {503, 502, 133}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx$$
$$\Big\downarrow \text{503}$$
$$\frac{\sqrt[3]{1-\frac{9d^2x^2}{c^2}}\int \frac{1}{(c+dx)\sqrt[3]{1-\frac{9d^2x^2}{c^2}}}dx}{\sqrt[3]{c^2-9d^2x^2}}$$
$$\Big\downarrow \text{502}$$

$$\frac{\sqrt[3]{1-\frac{9d^2x^2}{c^2}}\int \frac{1}{(c+dx)\sqrt[3]{1-\frac{3dx}{c}}\sqrt[3]{\frac{3dx}{c}+1}}dx}{\sqrt[3]{c^2-9d^2x^2}}$$

$\Big\downarrow$ 133

$$\frac{\sqrt[3]{1-\frac{9d^2x^2}{c^2}}\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{\left(1-\frac{3dx}{c}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3dx}{c}+1}}\right)}{4d}+\frac{\log(c+dx)}{4d}-\frac{3\log\left(-\frac{1}{2}\left(1-\frac{3dx}{c}\right)^{2/3}-\sqrt[3]{\frac{3dx}{c}+1}\right)}{8d}\right)}{\sqrt[3]{c^2-9d^2x^2}}$$

input
```
Int[1/((c + d*x)*(c^2 - 9*d^2*x^2)^(1/3)),x]
```

output
```
((1 - (9*d^2*x^2)/c^2)^(1/3)*((Sqrt[3]*ArcTan[1/Sqrt[3] - (1 - (3*d*x)/c)^
(2/3)/(Sqrt[3]*(1 + (3*d*x)/c)^(1/3))])/(4*d) + Log[c + d*x]/(4*d) - (3*Lo
g[-1/2*(1 - (3*d*x)/c)^(2/3) - (1 + (3*d*x)/c)^(1/3)])/(8*d)))/(c^2 - 9*d^
2*x^2)^(1/3)
```

Defintions of rubi rules used

rule 133
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_] :> With[{q = Rt[b*((b*e - a*f)/(b*c - a*d)^2), 3]}, Simp[-Log[
a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[Sqrt[3]*(ArcTan[1/Sqrt[3] + 2*q*((c
 + d*x)^(2/3)/(Sqrt[3]*(e + f*x)^(1/3)))]/(2*q*(b*c - a*d))), x] + Simp[3*(
Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)]/(4*q*(b*c - a*d))), x])] /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

rule 502
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> Simp[
a^(1/3)   Int[1/((c + d*x)*(1 - 3*d*(x/c))^(1/3)*(1 + 3*d*(x/c))^(1/3)), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 9*a*d^2, 0] && GtQ[a, 0]
```

rule 503
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/3)), x_Symbol] :> Simp[
(1 + b*(x^2/a))^(1/3)/(a + b*x^2)^(1/3)   Int[1/((c + d*x)*(1 + b*(x^2/a))^
(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 9*a*d^2, 0] &&  !Gt
Q[a, 0]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(-9d^2x^2+c^2\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(-9*d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/(d*x+c)/(-9*d^2*x^2+c^2)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(-9*d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-\left(-c+3dx\right)\left(c+3dx\right)}\left(c+dx\right)}\,dx$$

input
```
integrate(1/(d*x+c)/(-9*d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + 3*d*x)*(c + 3*d*x))**(1/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx = \int \frac{1}{\left(-9\,d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-9*d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-9*d^2*x^2 + c^2)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx = \int \frac{1}{\left(-9\,d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-9*d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-9*d^2*x^2 + c^2)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx = \int \frac{1}{\left(c^2-9\,d^2\,x^2\right)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((c^2 - 9*d^2*x^2)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((c^2 - 9*d^2*x^2)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-9d^2x^2}}\,dx = \int \frac{1}{(-9d^2x^2+c^2)^{\frac{1}{3}}\,c+(-9d^2x^2+c^2)^{\frac{1}{3}}\,dx}dx$$

input
```
int(1/(d*x+c)/(-9*d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c**2 - 9*d**2*x**2)**(1/3)*c + (c**2 - 9*d**2*x**2)**(1/3)*d*x),x)
```

3.360 $\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}} \, dx = -\frac{1}{7}(1+x)^2 \left(1-2x^2\right)^{3/4} - \frac{1}{105}(100+33x) \left(1-2x^2\right)^{3/4} + \frac{8}{5}\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)$$

output
```
-1/7*(1+x)^2*(-2*x^2+1)^(3/4)-1/105*(100+33*x)*(-2*x^2+1)^(3/4)+8/5*2^(1/2
)*EllipticE(sin(1/2*arcsin(x*2^(1/2))),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 17.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}} \, dx = -\frac{1}{105}\left(1-2x^2\right)^{3/4} \left(115+63x+15x^2\right) + \frac{8}{5}x \operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)$$

input
```
Integrate[(1 + x)^3/(1 - 2*x^2)^(1/4),x]
```

output
```
-1/105*((1 - 2*x^2)^(3/4)*(115 + 63*x + 15*x^2)) + (8*x*Hypergeometric2F1[
1/4, 1/2, 3/2, 2*x^2])/5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {497, 25, 676, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^3}{\sqrt[4]{1-2x^2}}dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{1}{7}\int -\frac{(x+1)(11x+9)}{\sqrt[4]{1-2x^2}}dx-\frac{1}{7}\left(1-2x^2\right)^{3/4}(x+1)^2$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}\int \frac{(x+1)(11x+9)}{\sqrt[4]{1-2x^2}}dx-\frac{1}{7}(x+1)^2\left(1-2x^2\right)^{3/4}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{7}\left(\frac{56}{5}\int \frac{1}{\sqrt[4]{1-2x^2}}dx-\frac{11}{5}\left(1-2x^2\right)^{3/4}x-\frac{20}{3}\left(1-2x^2\right)^{3/4}\right)-\frac{1}{7}(x+1)^2\left(1-2x^2\right)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{1}{7}\left(\frac{56}{5}\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)-\frac{11}{5}\left(1-2x^2\right)^{3/4}x-\frac{20}{3}\left(1-2x^2\right)^{3/4}\right)-\frac{1}{7}(x+1)^2\left(1-2x^2\right)^{3/4}$$

input
```
Int[(1 + x)^3/(1 - 2*x^2)^(1/4),x]
```

output
```
-1/7*((1 + x)^2*(1 - 2*x^2)^(3/4)) + ((-20*(1 - 2*x^2)^(3/4))/3 - (11*x*(1
 - 2*x^2)^(3/4))/5 + (56*Sqrt[2]*EllipticE[ArcSin[Sqrt[2]*x]/2, 2])/5)/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

method	result
risch	$\frac{(15x^2+63x+115)(2x^2-1)}{105(-2x^2+1)^{\frac{1}{4}}}+\frac{8x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],2x^2\right)}{5}$
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],2x^2\right)+\frac{x^4\,\text{hypergeom}\left(\left[\frac{1}{4},2\right],[3],2x^2\right)}{4}+x^3\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],2x^2\right)+\frac{3}{}$

input
```
int((x+1)^3/(-2*x^2+1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/105*(15*x^2+63*x+115)*(2*x^2-1)/(-2*x^2+1)^(1/4)+8/5*x*hypergeom([1/4,1/
2],[3/2],2*x^2)
```

Fricas [F]

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^3}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^3/(-2*x^2+1)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(x^3 + 3*x^2 + 3*x + 1)*(-2*x^2 + 1)^(3/4)/(2*x^2 - 1), x)
```

Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.02

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}}\,dx = x^3\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|2x^2e^{2i\pi}\right) + x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|2x^2e^{2i\pi}\right) - (1-2x^2)^{\frac{3}{4}} + \begin{cases} \frac{x^2(2x^2-1)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}}{7} + \frac{2(2x^2-1)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}}{21} & \text{for } |x^2| > \frac{1}{2} \\ -\frac{x^2(1-2x^2)^{\frac{3}{4}}}{7} - \frac{2(1-2x^2)^{\frac{3}{4}}}{21} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**3/(-2*x**2+1)**(1/4),x)
```

output
```
x**3*hyper((1/4, 3/2), (5/2,), 2*x**2*exp_polar(2*I*pi)) + x*hyper((1/4, 1
/2), (3/2,), 2*x**2*exp_polar(2*I*pi)) - (1 - 2*x**2)**(3/4) + Piecewise((
x**2*(2*x**2 - 1)**(3/4)*exp(-I*pi/4)/7 + 2*(2*x**2 - 1)**(3/4)*exp(-I*pi/
4)/21, Abs(x**2) > 1/2), (-x**2*(1 - 2*x**2)**(3/4)/7 - 2*(1 - 2*x**2)**(3
/4)/21, True))
```

Maxima [F]

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^3}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^3/(-2*x^2+1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x + 1)^3/(-2*x^2 + 1)^(1/4), x)
```

Giac [F]

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^3}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^3/(-2*x^2+1)^(1/4),x, algorithm="giac")
```

output
```
integrate((x + 1)^3/(-2*x^2 + 1)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^3}{(1-2\,x^2)^{1/4}}\,dx$$

input
```
int((x + 1)^3/(1 - 2*x^2)^(1/4),x)
```

output
```
int((x + 1)^3/(1 - 2*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(1+x)^3}{\sqrt[4]{1-2x^2}}\,dx = -\frac{(-2x^2+1)^{\frac{3}{4}}\,x^2}{7} - \frac{23(-2x^2+1)^{\frac{3}{4}}}{21} + 3\left(\int \frac{x^2}{(-2x^2+1)^{\frac{1}{4}}}dx\right) + \int \frac{1}{(-2x^2+1)^{\frac{1}{4}}}dx$$

input
```
int((1+x)^3/(-2*x^2+1)^(1/4),x)
```

output
```
( - 3*( - 2*x**2 + 1)**(3/4)*x**2 - 23*( - 2*x**2 + 1)**(3/4) + 63*int(x**
2/( - 2*x**2 + 1)**(1/4),x) + 21*int(1/( - 2*x**2 + 1)**(1/4),x))/21
```

3.361 $\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx=-\frac{1}{15}(10+3x)\left(1-2x^2\right)^{3/4}+\frac{6}{5}\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)$$

output
```
-1/15*(10+3*x)*(-2*x^2+1)^(3/4)+6/5*2^(1/2)*EllipticE(sin(1/2*arcsin(x*2^(
1/2))),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.99 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx=-\frac{1}{15}(10+3x)\left(1-2x^2\right)^{3/4}+\frac{6}{5}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)$$

input
```
Integrate[(1 + x)^2/(1 - 2*x^2)^(1/4),x]
```

output
```
-1/15*((10 + 3*x)*(1 - 2*x^2)^(3/4)) + (6*x*Hypergeometric2F1[1/4, 1/2, 3/
2, 2*x^2])/5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {497, 25, 455, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{\sqrt[4]{1-2x^2}}dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{1}{5}\int -\frac{7x+6}{\sqrt[4]{1-2x^2}}dx-\frac{1}{5}\left(1-2x^2\right)^{3/4}(x+1)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\int \frac{7x+6}{\sqrt[4]{1-2x^2}}dx-\frac{1}{5}(x+1)\left(1-2x^2\right)^{3/4}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{5}\left(6\int \frac{1}{\sqrt[4]{1-2x^2}}dx-\frac{7}{3}\left(1-2x^2\right)^{3/4}\right)-\frac{1}{5}(x+1)\left(1-2x^2\right)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{1}{5}\left(6\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\bigg|2\right)-\frac{7}{3}\left(1-2x^2\right)^{3/4}\right)-\frac{1}{5}(x+1)\left(1-2x^2\right)^{3/4}$$

input
```
Int[(1 + x)^2/(1 - 2*x^2)^(1/4),x]
```

output
```
-1/5*((1 + x)*(1 - 2*x^2)^(3/4)) + ((-7*(1 - 2*x^2)^(3/4))/3 + 6*Sqrt[2]*E
llipticE[ArcSin[Sqrt[2]*x]/2, 2])/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	si
risch	$\frac{(10+3x)\left(2x^2-1\right)}{15\left(-2x^2+1\right)^{\frac{1}{4}}}+\frac{6x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],2x^2\right)}{5}$	3
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],2x^2\right)+\frac{x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],2x^2\right)}{3}+x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{4},1\right],[2],2x^2\right)$	4

input
```
int((x+1)^2/(-2*x^2+1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/15*(10+3*x)*(2*x^2-1)/(-2*x^2+1)^(1/4)+6/5*x*hypergeom([1/4,1/2],[3/2],2
*x^2)
```

Fricas [F]

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^2}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^2/(-2*x^2+1)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(x^2 + 2*x + 1)*(-2*x^2 + 1)^(3/4)/(2*x^2 - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx = \frac{x^3{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|2x^2e^{2i\pi}\right)}{3} + x{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|2x^2e^{2i\pi}\right) - \frac{2(1-2x^2)^{\frac{3}{4}}}{3}$$

input
```
integrate((1+x)**2/(-2*x**2+1)**(1/4),x)
```

output
```
x**3*hyper((1/4, 3/2), (5/2,), 2*x**2*exp_polar(2*I*pi))/3 + x*hyper((1/4,
 1/2), (3/2,), 2*x**2*exp_polar(2*I*pi)) - 2*(1 - 2*x**2)**(3/4)/3
```

Maxima [F]

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^2}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^2/(-2*x^2+1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x + 1)^2/(-2*x^2 + 1)^(1/4), x)
```

Giac [F]

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^2}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^2/(-2*x^2+1)^(1/4),x, algorithm="giac")
```

output
```
integrate((x + 1)^2/(-2*x^2 + 1)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{(x+1)^2}{(1-2\,x^2)^{1/4}}\,dx$$

input
```
int((x + 1)^2/(1 - 2*x^2)^(1/4),x)
```

output
```
int((x + 1)^2/(1 - 2*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(1+x)^2}{\sqrt[4]{1-2x^2}}\,dx = -\frac{2(-2x^2+1)^{\frac{3}{4}}}{3} + \int \frac{x^2}{(-2x^2+1)^{\frac{1}{4}}}dx + \int \frac{1}{(-2x^2+1)^{\frac{1}{4}}}dx$$

input
```
int((1+x)^2/(-2*x^2+1)^(1/4),x)
```

output
```
( - 2*( - 2*x**2 + 1)**(3/4) + 3*int(x**2/( - 2*x**2 + 1)**(1/4),x) + 3*in
t(1/( - 2*x**2 + 1)**(1/4),x))/3
```

3.362 $\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx = -\frac{1}{3}\left(1-2x^2\right)^{3/4}+\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)$$

output
```
-1/3*(-2*x^2+1)^(3/4)+2^(1/2)*EllipticE(sin(1/2*arcsin(x*2^(1/2))),2^(1/2)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx = -\frac{1}{3}\left(1-2x^2\right)^{3/4}+x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)$$

input
```
Integrate[(1 + x)/(1 - 2*x^2)^(1/4),x]
```

output
```
-1/3*(1 - 2*x^2)^(3/4) + x*Hypergeometric2F1[1/4, 1/2, 3/2, 2*x^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {455, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt[4]{1-2x^2}}\,dx$$

$$\Big\downarrow \ 455$$

$$\int \frac{1}{\sqrt[4]{1-2x^2}}dx-\frac{1}{3}\left(1-2x^2\right)^{3/4}$$

$$\Big\downarrow \ 226$$

$$\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)-\frac{1}{3}\left(1-2x^2\right)^{3/4}$$

input
```
Int[(1 + x)/(1 - 2*x^2)^(1/4),x]
```

output
```
-1/3*(1 - 2*x^2)^(3/4) + Sqrt[2]*EllipticE[ArcSin[Sqrt[2]*x]/2, 2]
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],2x^{2}\right)+\frac{x^{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},1\right],[2],2x^{2}\right)}{2}$	31
risch	$\frac{2x^{2}-1}{3\left(-2x^{2}+1\right)^{\frac{1}{4}}}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],2x^{2}\right)$	33

input
```
int((x+1)/(-2*x^2+1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,1/2],[3/2],2*x^2)+1/2*x^2*hypergeom([1/4,1],[2],2*x^2)
```

Fricas [F]

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx=\int \frac{x+1}{\left(-2\,x^{2}+1\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)/(-2*x^2+1)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-2*x^2 + 1)^(3/4)*(x + 1)/(2*x^2 - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx=x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,2x^{2}e^{2i\pi}\right)-\frac{\left(1-2x^{2}\right)^{\frac{3}{4}}}{3}$$

input
```
integrate((1+x)/(-2*x**2+1)**(1/4),x)
```

output
```
x*hyper((1/4, 1/2), (3/2,), 2*x**2*exp_polar(2*I*pi)) - (1 - 2*x**2)**(3/4
)/3
```

Maxima [F]

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{x+1}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)/(-2*x^2+1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(-2*x^2 + 1)^(1/4), x)
```

Giac [F]

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{x+1}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)/(-2*x^2+1)^(1/4),x, algorithm="giac")
```

output
```
integrate((x + 1)/(-2*x^2 + 1)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ 2\,x^2\right) - \frac{\left(1-2\,x^2\right)^{3/4}}{3}$$

input
```
int((x + 1)/(1 - 2*x^2)^(1/4),x)
```

output
```
x*hypergeom([1/4, 1/2], 3/2, 2*x^2) - (1 - 2*x^2)^(3/4)/3
```

Reduce [F]

$$\int \frac{1+x}{\sqrt[4]{1-2x^2}}\,dx = -\frac{(-2x^2+1)^{\frac{3}{4}}}{3} + \int \frac{1}{(-2x^2+1)^{\frac{1}{4}}}dx$$

input
```
int((1+x)/(-2*x^2+1)^(1/4),x)
```

output
```
( - ( - 2*x**2 + 1)**(3/4) + 3*int(1/( - 2*x**2 + 1)**(1/4),x))/3
```

3.363 $\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = \sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)$$

output
```
2^(1/2)*EllipticE(sin(1/2*arcsin(x*2^(1/2))),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)$$

input
```
Integrate[(1 - 2*x^2)^(-1/4),x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 3/2, 2*x^2]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx$$

$$\Big\downarrow \ 226$$

$$\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)$$

input
```
Int[(1 - 2*x^2)^(-1/4),x]
```

output
```
Sqrt[2]*EllipticE[ArcSin[Sqrt[2]*x]/2, 2]
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],2x^2\right)$	14

input
```
int(1/(-2*x^2+1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,1/2],[3/2],2*x^2)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-2*x^2+1)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-2*x^2 + 1)^(3/4)/(2*x^2 - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = x\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| 2x^2e^{2i\pi}\right)$$

input
```
integrate(1/(-2*x**2+1)**(1/4),x)
```

output
```
x*hyper((1/4, 1/2), (3/2,), 2*x**2*exp_polar(2*I*pi))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-2*x^2+1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-2*x^2 + 1)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-2*x^2+1)^(1/4),x, algorithm="giac")
```

output
```
integrate((-2*x^2 + 1)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = x\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{3}{2};\;2\,x^2\right)$$

input
```
int(1/(1 - 2*x^2)^(1/4),x)
```

output
```
x*hypergeom([1/4, 1/2], 3/2, 2*x^2)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2x^2+1)^{\frac{1}{4}}}dx$$

input
```
int(1/(-2*x^2+1)^(1/4),x)
```

output
```
int(1/( - 2*x**2 + 1)**(1/4),x)
```

3.364 $\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx = \frac{\sqrt[4]{-1+2x^2}\arctan\left(\frac{\sqrt[4]{-1+2x^2}}{1+x-\sqrt{-1+2x^2}}\right)}{2\sqrt[4]{1-2x^2}} - \frac{\sqrt[4]{-1+2x^2}\operatorname{arctanh}\left(\frac{\sqrt[4]{-1+2x^2}}{1+x+\sqrt{-1+2x^2}}\right)}{2\sqrt[4]{1-2x^2}}$$

output
```
1/2*(2*x^2-1)^(1/4)*arctan((2*x^2-1)^(1/4)/(1+x-(2*x^2-1)^(1/2)))/(-2*x^2+
1)^(1/4)-1/2*(2*x^2-1)^(1/4)*arctanh((2*x^2-1)^(1/4)/(1+x+(2*x^2-1)^(1/2))
)/(-2*x^2+1)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 13.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{-\frac{\sqrt{2}-2x}{1+x}}\sqrt[4]{\frac{\sqrt{2}+2x}{1+x}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2+\sqrt{2}}{2+2x},\frac{2-\sqrt{2}}{2+2x}\right)}{\sqrt[4]{1-2x^2}}$$

input
```
Integrate[1/((1 + x)*(1 - 2*x^2)^(1/4)),x]
```

output
```
-((Sqrt[2]*(-((Sqrt[2] - 2*x)/(1 + x)))^(1/4)*((Sqrt[2] + 2*x)/(1 + x))^(1
/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (2 + Sqrt[2])/(2 + 2*x), (2 - Sqrt[2])/(2
 + 2*x)])/(1 - 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {500, 499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt[4]{1-2x^2}}\,dx$$

$$\Big\downarrow \text{500}$$

$$\frac{\sqrt[4]{2x^2-1}\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx}{\sqrt[4]{1-2x^2}}$$

$$\Big\downarrow \text{499}$$

$$\frac{\sqrt[4]{2x^2-1}\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)\right)}{\sqrt[4]{1-2x^2}}$$

input
```
Int[1/((1 + x)*(1 - 2*x^2)^(1/4)),x]
```

output
```
((-1 + 2*x^2)^(1/4)*(ArcTan[(-1 + 2*x^2)^(1/4)/(1 + x - Sqrt[-1 + 2*x^2])]
/2 - ArcTanh[(-1 + 2*x^2)^(1/4)/(1 + x + Sqrt[-1 + 2*x^2])]/2))/(1 - 2*x^2
)^(1/4)
```

Defintions of rubi rules used

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 500
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[
(-a - b*x^2)^(1/4)/(a + b*x^2)^(1/4)   Int[1/((c + d*x)*(-a - b*x^2)^(1/4))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 2*a*d^2, 0] && PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.87

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{2\left(-2x^2+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^3 x+\left(-2x^2+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)+\left(-2x^2+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^3-\sqrt{-}}{(x+1)^2}\right.}{2}$

input
```
int(1/(x+1)/(-2*x^2+1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*RootOf(_Z^4+1)*ln(-(2*(-2*x^2+1)^(1/4)*RootOf(_Z^4+1)^3*x+(-2*x^2+1)^
(3/4)*RootOf(_Z^4+1)+(-2*x^2+1)^(1/4)*RootOf(_Z^4+1)^3-(-2*x^2+1)^(1/2)*x+
3*x*RootOf(_Z^4+1)^2+2*RootOf(_Z^4+1)^2)/(x+1)^2)+1/2*RootOf(_Z^4+1)^3*ln(
(-(-2*x^2+1)^(1/2)*RootOf(_Z^4+1)*x-2*(-2*x^2+1)^(1/4)*RootOf(_Z^4+1)^2*x-
3*RootOf(_Z^4+1)^3*x+(-2*x^2+1)^(3/4)-(-2*x^2+1)^(1/4)*RootOf(_Z^4+1)^2-2*
RootOf(_Z^4+1)^3)/(x+1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(93) = 186$.

Time = 1.21 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.10

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx$$
$$= \frac{1}{4}\sqrt{2}\arctan\left(\frac{x^4+\sqrt{2}(x^2-x-1)(-2x^2+1)^{\frac{3}{4}}-5x^2-\sqrt{2}(x^3-3x^2-4x-1)(-2x^2+1)^{\frac{1}{4}}-(x^2+}{x^4-4x^3+2x^2+8x+3}\right.$$
$$+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^4-\sqrt{2}(x^2-x-1)(-2x^2+1)^{\frac{3}{4}}-5x^2+\sqrt{2}(x^3-3x^2-4x-1)(-2x^2+1)^{\frac{1}{4}}-(}{x^4-4x^3+2x^2+8x+3}\right.$$
$$+\frac{1}{8}\sqrt{2}\log\left(-\frac{x^2+\sqrt{2}(-2x^2+1)^{\frac{1}{4}}(x+1)+\sqrt{2}(-2x^2+1)^{\frac{3}{4}}-2x-\sqrt{-2x^2+1}-2}{x^2+2x+1}\right)$$
$$-\frac{1}{8}\sqrt{2}\log\left(-\frac{x^2-\sqrt{2}(-2x^2+1)^{\frac{1}{4}}(x+1)-\sqrt{2}(-2x^2+1)^{\frac{3}{4}}-2x-\sqrt{-2x^2+1}-2}{x^2+2x+1}\right)$$

input
```
integrate(1/(1+x)/(-2*x^2+1)^(1/4),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan((x^4 + sqrt(2)*(x^2 - x - 1)*(-2*x^2 + 1)^(3/4) - 5*x^2
 - sqrt(2)*(x^3 - 3*x^2 - 4*x - 1)*(-2*x^2 + 1)^(1/4) - (x^2 + 2*x + 1)*sq
rt(-2*x^2 + 1) - 6*x - 2)/(x^4 - 4*x^3 + 2*x^2 + 8*x + 3)) + 1/4*sqrt(2)*a
rctan(-(x^4 - sqrt(2)*(x^2 - x - 1)*(-2*x^2 + 1)^(3/4) - 5*x^2 + sqrt(2)*(
x^3 - 3*x^2 - 4*x - 1)*(-2*x^2 + 1)^(1/4) - (x^2 + 2*x + 1)*sqrt(-2*x^2 +
1) - 6*x - 2)/(x^4 - 4*x^3 + 2*x^2 + 8*x + 3)) + 1/8*sqrt(2)*log(-(x^2 + s
qrt(2)*(-2*x^2 + 1)^(1/4)*(x + 1) + sqrt(2)*(-2*x^2 + 1)^(3/4) - 2*x - sqr
t(-2*x^2 + 1) - 2)/(x^2 + 2*x + 1)) - 1/8*sqrt(2)*log(-(x^2 - sqrt(2)*(-2*
x^2 + 1)^(1/4)*(x + 1) - sqrt(2)*(-2*x^2 + 1)^(3/4) - 2*x - sqrt(-2*x^2 +
1) - 2)/(x^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\sqrt[4]{1-2x^2}\,(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-2*x**2+1)**(1/4),x)
```

output
```
Integral(1/((1 - 2*x**2)**(1/4)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-2*x^2+1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-2*x^2 + 1)^(1/4)*(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-2*x^2+1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-2*x^2 + 1)^(1/4)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\left(1-2\,x^2\right)^{1/4}\,(x+1)}\,dx$$

input
```
int(1/((1 - 2*x^2)^(1/4)*(x + 1)),x)
```

output
```
int(1/((1 - 2*x^2)^(1/4)*(x + 1)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2x^2+1)^{\frac{1}{4}}\,x+(-2x^2+1)^{\frac{1}{4}}}dx$$

input
```
int(1/(1+x)/(-2*x^2+1)^(1/4),x)
```

output
```
int(1/(( - 2*x**2 + 1)**(1/4)*x + ( - 2*x**2 + 1)**(1/4)),x)
```

3.365 $\int \frac{1}{(1+x)^2 \sqrt[4]{1-2x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{1}{(1+x)^2\sqrt[4]{1-2x^2}}\,dx = \frac{(1-2x^2)^{3/4}}{1+x} + \frac{\sqrt[4]{-1+2x^2}\arctan\left(\frac{\sqrt[4]{-1+2x^2}}{1+x-\sqrt{-1+2x^2}}\right)}{2\sqrt[4]{1-2x^2}} - \frac{\sqrt[4]{-1+2x^2}\operatorname{arctanh}\left(\frac{\sqrt[4]{-1+2x^2}}{1+x+\sqrt{-1+2x^2}}\right)}{2\sqrt[4]{1-2x^2}} + \sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)$$

output
```
(-2*x^2+1)^(3/4)/(1+x)+1/2*(2*x^2-1)^(1/4)*arctan((2*x^2-1)^(1/4)/(1+x-(2*
x^2-1)^(1/2)))/(-2*x^2+1)^(1/4)-1/2*(2*x^2-1)^(1/4)*arctanh((2*x^2-1)^(1/4
)/(1+x+(2*x^2-1)^(1/2)))/(-2*x^2+1)^(1/4)+2^(1/2)*EllipticE(sin(1/2*arcsin
(x*2^(1/2))),2^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.86 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \frac{1}{(1+x)^2\sqrt[4]{1-2x^2}}\,dx$$
$$= \frac{3-6x^2-3\sqrt{2}\sqrt[4]{-\frac{\sqrt{2}-2x}{1+x}}(1+x)\sqrt[4]{\frac{\sqrt{2}+2x}{1+x}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2+\sqrt{2}}{2+2x},\frac{2-\sqrt{2}}{2+2x}\right)+2\sqrt[8]{2}(1+x)\sqrt[4]{\sqrt{2}+}}{3(1+x)\sqrt[4]{1-2x^2}}$$

input
```
Integrate[1/((1 + x)^2*(1 - 2*x^2)^(1/4)),x]
```

output
```
(3 - 6*x^2 - 3*Sqrt[2]*(-((Sqrt[2] - 2*x)/(1 + x)))^(1/4)*(1 + x)*((Sqrt[2
] + 2*x)/(1 + x))^(1/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (2 + Sqrt[2])/(2 + 2*
x), (2 - Sqrt[2])/(2 + 2*x)] + 2*2^(1/8)*(1 + x)*(Sqrt[2] + 2*x)^(1/4)*(-1
 + Sqrt[2]*x)*Hypergeometric2F1[1/4, 3/4, 7/4, 1/2 - x/Sqrt[2]])/(3*(1 + x
)*(1 - 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {498, 27, 719, 226, 500, 499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^2\sqrt[4]{1-2x^2}}\,dx$$
$$\Big\downarrow \text{498}$$
$$\frac{\left(1-2x^2\right)^{3/4}}{x+1}-2\int -\frac{x+2}{2(x+1)\sqrt[4]{1-2x^2}}dx$$
$$\Big\downarrow \text{27}$$

$$\int \frac{x+2}{(x+1)\sqrt[4]{1-2x^2}}dx + \frac{(1-2x^2)^{3/4}}{x+1}$$

$$\Big\downarrow \text{719}$$

$$\int \frac{1}{\sqrt[4]{1-2x^2}}dx + \int \frac{1}{(x+1)\sqrt[4]{1-2x^2}}dx + \frac{(1-2x^2)^{3/4}}{x+1}$$

$$\Big\downarrow \text{226}$$

$$\int \frac{1}{(x+1)\sqrt[4]{1-2x^2}}dx + \sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right) + \frac{(1-2x^2)^{3/4}}{x+1}$$

$$\Big\downarrow \text{500}$$

$$\frac{\sqrt[4]{2x^2-1}\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx}{\sqrt[4]{1-2x^2}} + \sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right) + \frac{(1-2x^2)^{3/4}}{x+1}$$

$$\Big\downarrow \text{499}$$

$$\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right) + \frac{\sqrt[4]{2x^2-1}\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right) - \frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)\right)}{\sqrt[4]{1-2x^2}} + \frac{(1-2x^2)^{3/4}}{x+1}$$

input
```
Int[1/((1 + x)^2*(1 - 2*x^2)^(1/4)),x]
```

output
```
(1 - 2*x^2)^(3/4)/(1 + x) + ((-1 + 2*x^2)^(1/4)*(ArcTan[(-1 + 2*x^2)^(1/4)
/(1 + x - Sqrt[-1 + 2*x^2])]/2 - ArcTanh[(-1 + 2*x^2)^(1/4)/(1 + x + Sqrt[
-1 + 2*x^2])]/2))/(1 - 2*x^2)^(1/4) + Sqrt[2]*EllipticE[ArcSin[Sqrt[2]*x]/
2, 2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 500
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[
(-a - b*x^2)^(1/4)/(a + b*x^2)^(1/4)   Int[1/((c + d*x)*(-a - b*x^2)^(1/4))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 2*a*d^2, 0] && PosQ[a]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{1}{\left(x+1\right)^{2} \left(-2x^{2}+1\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(x+1)^2/(-2*x^2+1)^(1/4),x)
```

output
```
int(1/(x+1)^2/(-2*x^2+1)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\left(-2\,x^{2}+1\right)^{\frac{1}{4}}\left(x+1\right)^{2}}\,dx$$

input
```
integrate(1/(1+x)^2/(-2*x^2+1)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-2*x^2 + 1)^(3/4)/(2*x^4 + 4*x^3 + x^2 - 2*x - 1), x)
```

Sympy [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\sqrt[4]{1-2x^{2}}\left(x+1\right)^{2}}\,dx$$

input
```
integrate(1/(1+x)**2/(-2*x**2+1)**(1/4),x)
```

output
```
Integral(1/((1 - 2*x**2)**(1/4)*(x + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}(x+1)^2}\,dx$$

input
```
integrate(1/(1+x)^2/(-2*x^2+1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-2*x^2 + 1)^(1/4)*(x + 1)^2), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}(x+1)^2}\,dx$$

input
```
integrate(1/(1+x)^2/(-2*x^2+1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-2*x^2 + 1)^(1/4)*(x + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^2\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(1-2\,x^2)^{1/4}\,(x+1)^2}\,dx$$

input
```
int(1/((1 - 2*x^2)^(1/4)*(x + 1)^2),x)
```

output
```
int(1/((1 - 2*x^2)^(1/4)*(x + 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)^2 \sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\left(-2x^2+1\right)^{\frac{1}{4}} x^2 + 2\left(-2x^2+1\right)^{\frac{1}{4}} x + \left(-2x^2+1\right)^{\frac{1}{4}}}\,dx$$

input
```
int(1/(1+x)^2/(-2*x^2+1)^(1/4),x)
```

output
```
int(1/(( - 2*x**2 + 1)**(1/4)*x**2 + 2*( - 2*x**2 + 1)**(1/4)*x + ( - 2*x*
*2 + 1)**(1/4)),x)
```

3.366 $\int \frac{1}{(1+x)^3 \sqrt[4]{1-2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{1}{(1+x)^3\sqrt[4]{1-2x^2}}\,dx = \frac{(1-2x^2)^{3/4}}{2(1+x)^2}+\frac{5(1-2x^2)^{3/4}}{2(1+x)} + \frac{\sqrt[4]{-1+2x^2}\arctan\left(\frac{\sqrt[4]{-1+2x^2}}{1+x-\sqrt{-1+2x^2}}\right)}{\sqrt[4]{1-2x^2}} - \frac{\sqrt[4]{-1+2x^2}\operatorname{arctanh}\left(\frac{\sqrt[4]{-1+2x^2}}{1+x+\sqrt{-1+2x^2}}\right)}{\sqrt[4]{1-2x^2}} + \frac{5E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)}{\sqrt{2}}$$

output

```
1/2*(-2*x^2+1)^(3/4)/(1+x)^2+5*(-2*x^2+1)^(3/4)/(2+2*x)+(2*x^2-1)^(1/4)*ar
ctan((2*x^2-1)^(1/4)/(1+x-(2*x^2-1)^(1/2)))/(-2*x^2+1)^(1/4)-(2*x^2-1)^(1/
4)*arctanh((2*x^2-1)^(1/4)/(1+x+(2*x^2-1)^(1/2)))/(-2*x^2+1)^(1/4)+5/2*2^(
1/2)*EllipticE(sin(1/2*arcsin(x*2^(1/2))),2^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 19.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.29

$$\int \frac{1}{(1+x)^3\sqrt[4]{1-2x^2}}\,dx$$
$$= \frac{\frac{15-30x^2}{1+x}+\frac{3-6x^2}{(1+x)^2}-12\sqrt{2}\sqrt[4]{-\frac{\sqrt{2}-2x}{1+x}}\sqrt[4]{\frac{\sqrt{2}+2x}{1+x}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2+\sqrt{2}}{2+2x},\frac{2-\sqrt{2}}{2+2x}\right)-10\sqrt[8]{2}\sqrt[4]{\sqrt{2}+2x}\,\text{I}}{6\sqrt[4]{1-2}}$$

input
```
Integrate[1/((1 + x)^3*(1 - 2*x^2)^(1/4)),x]
```

output
```
((15 - 30*x^2)/(1 + x) + (3 - 6*x^2)/(1 + x)^2 - 12*Sqrt[2]*(-((Sqrt[2] -
2*x)/(1 + x)))^(1/4)*((Sqrt[2] + 2*x)/(1 + x))^(1/4)*AppellF1[1/2, 1/4, 1/
4, 3/2, (2 + Sqrt[2])/(2 + 2*x), (2 - Sqrt[2])/(2 + 2*x)] - 10*2^(1/8)*(Sq
rt[2] + 2*x)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, 1/2 - x/Sqrt[2]] + 10*
2^(5/8)*x*(Sqrt[2] + 2*x)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, 1/2 - x/S
qrt[2]])/(6*(1 - 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {498, 27, 688, 719, 226, 500, 499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^3\sqrt[4]{1-2x^2}}dx$$
$$\Big\downarrow \text{498}$$
$$\frac{\left(1-2x^2\right)^{3/4}}{2(x+1)^2}-\int -\frac{4-x}{2(x+1)^2\sqrt[4]{1-2x^2}}dx$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\int \frac{4-x}{(x+1)^2\sqrt[4]{1-2x^2}}dx+\frac{\left(1-2x^2\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow 688$$

$$\frac{1}{2}\left(\int \frac{5x+9}{(x+1)\sqrt[4]{1-2x^2}}dx+\frac{5\left(1-2x^2\right)^{3/4}}{x+1}\right)+\frac{\left(1-2x^2\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow 719$$

$$\frac{1}{2}\left(5\int \frac{1}{\sqrt[4]{1-2x^2}}dx+4\int \frac{1}{(x+1)\sqrt[4]{1-2x^2}}dx+\frac{5\left(1-2x^2\right)^{3/4}}{x+1}\right)+\frac{\left(1-2x^2\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow 226$$

$$\frac{1}{2}\left(4\int \frac{1}{(x+1)\sqrt[4]{1-2x^2}}dx+5\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)+\frac{5\left(1-2x^2\right)^{3/4}}{x+1}\right)+\frac{\left(1-2x^2\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow 500$$

$$\frac{1}{2}\left(\frac{4\sqrt[4]{2x^2-1}\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx}{\sqrt[4]{1-2x^2}}+5\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)+\frac{5\left(1-2x^2\right)^{3/4}}{x+1}\right)+$$
$$\frac{\left(1-2x^2\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow 499$$

$$\frac{1}{2}\left(5\sqrt{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}x\right)\middle|2\right)+\frac{4\sqrt[4]{2x^2-1}\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)\right)}{\sqrt[4]{1-2x^2}}+\frac{5(1-}{x}\right.$$
$$\frac{\left(1-2x^2\right)^{3/4}}{2(x+1)^2}$$

input
```
Int[1/((1 + x)^3*(1 - 2*x^2)^(1/4)),x]
```

output
```
(1 - 2*x^2)^(3/4)/(2*(1 + x)^2) + ((5*(1 - 2*x^2)^(3/4))/(1 + x) + (4*(-1
+ 2*x^2)^(1/4)*(ArcTan[(-1 + 2*x^2)^(1/4)/(1 + x - Sqrt[-1 + 2*x^2])]/2 -
ArcTanh[(-1 + 2*x^2)^(1/4)/(1 + x + Sqrt[-1 + 2*x^2])]/2))/(1 - 2*x^2)^(1/
4) + 5*Sqrt[2]*EllipticE[ArcSin[Sqrt[2]*x]/2, 2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 500
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[
(-a - b*x^2)^(1/4)/(a + b*x^2)^(1/4)   Int[1/((c + d*x)*(-a - b*x^2)^(1/4))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 2*a*d^2, 0] && PosQ[a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{1}{\left(x+1\right)^{3} \left(-2x^{2}+1\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(x+1)^3/(-2*x^2+1)^(1/4),x)
```

output
```
int(1/(x+1)^3/(-2*x^2+1)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{1-2x^2}}\,dx=\int \frac{1}{{\left(-2\,x^{2}+1\right)}^{\frac{1}{4}}{\left(x+1\right)}^{3}}\,dx$$

input
```
integrate(1/(1+x)^3/(-2*x^2+1)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-2*x^2 + 1)^(3/4)/(2*x^5 + 6*x^4 + 5*x^3 - x^2 - 3*x - 1), x)
```

Sympy [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\sqrt[4]{1-2x^2}\,(x+1)^3}\,dx$$

input
```
integrate(1/(1+x)**3/(-2*x**2+1)**(1/4),x)
```

output
```
Integral(1/((1 - 2*x**2)**(1/4)*(x + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}(x+1)^3}\,dx$$

input
```
integrate(1/(1+x)^3/(-2*x^2+1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-2*x^2 + 1)^(1/4)*(x + 1)^3), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{(-2\,x^2+1)^{\frac{1}{4}}(x+1)^3}\,dx$$

input
```
integrate(1/(1+x)^3/(-2*x^2+1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-2*x^2 + 1)^(1/4)*(x + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^3 \sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\left(1-2\,x^2\right)^{1/4}\,(x+1)^3}\,dx$$

input
```
int(1/((1 - 2*x^2)^(1/4)*(x + 1)^3),x)
```

output
```
int(1/((1 - 2*x^2)^(1/4)*(x + 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)^3 \sqrt[4]{1-2x^2}}\,dx = \int \frac{1}{\left(-2x^2+1\right)^{\frac{1}{4}} x^3+3\left(-2x^2+1\right)^{\frac{1}{4}} x^2+3\left(-2x^2+1\right)^{\frac{1}{4}} x+\left(-2x^2+1\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(1+x)^3/(-2*x^2+1)^(1/4),x)
```

output
```
int(1/(( - 2*x**2 + 1)**(1/4)*x**3 + 3*( - 2*x**2 + 1)**(1/4)*x**2 + 3*( -
 2*x**2 + 1)**(1/4)*x + ( - 2*x**2 + 1)**(1/4)),x)
```

3.367 $\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = -\frac{(c+dx)^2\left(ac^2-2ad^2x^2\right)^{3/4}}{7ad} - \frac{c(100c+33dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{105ad} + \frac{8\sqrt{2}c^4\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{5d\sqrt[4]{ac^2-2ad^2x^2}}$$

output
```
-1/7*(d*x+c)^2*(-2*a*d^2*x^2+a*c^2)^(3/4)/a/d-1/105*c*(33*d*x+100*c)*(-2*a
*d^2*x^2+a*c^2)^(3/4)/a/d+8/5*2^(1/2)*c^4*(1-2*d^2*x^2/c^2)^(1/4)*Elliptic
E(sin(1/2*arcsin(2^(1/2)*d*x/c)),2^(1/2))/d/(-2*a*d^2*x^2+a*c^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx$$
$$= \frac{-115c^4-63c^3dx+215c^2d^2x^2+126cd^3x^3+30d^4x^4+168c^3dx\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\ \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\right.}{105d\sqrt[4]{a(c^2-2d^2x^2)}}$$

input
```
Integrate[(c + d*x)^3/(a*c^2 - 2*a*d^2*x^2)^(1/4),x]
```

output
```
(-115*c^4 - 63*c^3*d*x + 215*c^2*d^2*x^2 + 126*c*d^3*x^3 + 30*d^4*x^4 + 16
8*c^3*d*x*(1 - (2*d^2*x^2)/c^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (2*
d^2*x^2)/c^2])/(105*d*(a*(c^2 - 2*d^2*x^2))^(1/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {497, 25, 27, 676, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}dx$$

$$\Big\downarrow \ 497$$

$$-\frac{\int -\frac{acd^2(c+dx)(9c+11dx)}{\sqrt[4]{ac^2-2ad^2x^2}}dx}{7ad^2}-\frac{(c+dx)^2\left(ac^2-2ad^2x^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{acd^2(c+dx)(9c+11dx)}{\sqrt[4]{ac^2-2ad^2x^2}}dx}{7ad^2}-\frac{(c+dx)^2\left(ac^2-2ad^2x^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow 27$$

$$\frac{1}{7}c\int \frac{(c+dx)(9c+11dx)}{\sqrt[4]{ac^2-2ad^2x^2}}dx-\frac{(c+dx)^2\left(ac^2-2ad^2x^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow 676$$

$$\frac{1}{7}c\left(\frac{56}{5}c^2\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}dx-\frac{20c\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}-\frac{11x\left(ac^2-2ad^2x^2\right)^{3/4}}{5a}\right)-\frac{(c+dx)^2\left(ac^2-2ad^2x^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow 227$$

$$\frac{1}{7}c\left(\frac{56c^2\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\int \frac{1}{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}}dx}{5\sqrt[4]{ac^2-2ad^2x^2}}-\frac{20c\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}-\frac{11x\left(ac^2-2ad^2x^2\right)^{3/4}}{5a}\right)-\frac{(c+dx)^2\left(ac^2-2ad^2x^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow 226$$

$$\frac{1}{7}c\left(\frac{56\sqrt{2}c^3\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{5d\sqrt[4]{ac^2-2ad^2x^2}}-\frac{20c\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}-\frac{11x\left(ac^2-2ad^2x^2\right)^{3/4}}{5a}\right)-\frac{(c+dx)^2\left(ac^2-2ad^2x^2\right)^{3/4}}{7ad}$$

input
```
Int[(c + d*x)^3/(a*c^2 - 2*a*d^2*x^2)^(1/4),x]
```

output
```
-1/7*((c + d*x)^2*(a*c^2 - 2*a*d^2*x^2)^(3/4))/(a*d) + (c*((-20*c*(a*c^2 -
 2*a*d^2*x^2)^(3/4))/(3*a*d) - (11*x*(a*c^2 - 2*a*d^2*x^2)^(3/4))/(5*a) +
(56*Sqrt[2]*c^3*(1 - (2*d^2*x^2)/c^2)^(1/4)*EllipticE[ArcSin[(Sqrt[2]*d*x)
/c]/2, 2])/(5*d*(a*c^2 - 2*a*d^2*x^2)^(1/4))))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \frac{(dx+c)^3}{(-2a\,d^2x^2+a\,c^2)^{\frac{1}{4}}}dx$$

input
```
int((d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int((d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{(dx+c)^3}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-2*a*d^2*x^2 + a*c^2)
^(3/4)/(2*a*d^2*x^2 - a*c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 700, normalized size of antiderivative = 4.86

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3/(-2*a*d**2*x**2+a*c**2)**(1/4),x)
```

output
```
3*c**2*d*Piecewise((zoo*x**2, Eq(a, 0)), (x**2/(2*(a*c**2)**(1/4)), Eq(d**
2, 0)), (-(a*c**2 - 2*a*d**2*x**2)**(3/4)/(3*a*d**2), True)) + d**3*Piecew
ise((-2*c**(15/2)*(-1 + 2*d**2*x**2/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4*e
xp(I*pi/4) + 42*a**(1/4)*c**2*d**6*x**2*exp(I*pi/4)) - 2*c**(15/2)/(-21*a*
*(1/4)*c**4*d**4 + 42*a**(1/4)*c**2*d**6*x**2) + c**(11/2)*d**2*x**2*(-1 +
 2*d**2*x**2/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(1/4
)*c**2*d**6*x**2*exp(I*pi/4)) + 4*c**(11/2)*d**2*x**2/(-21*a**(1/4)*c**4*d
**4 + 42*a**(1/4)*c**2*d**6*x**2) + 6*c**(7/2)*d**4*x**4*(-1 + 2*d**2*x**2
/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(1/4)*c**2*d**6*
x**2*exp(I*pi/4)), Abs(d**2*x**2/c**2) > 1/2), (2*c**(15/2)*(1 - 2*d**2*x*
*2/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4 + 42*a**(1/4)*c**2*d**6*x**2) - 2*
c**(15/2)/(-21*a**(1/4)*c**4*d**4 + 42*a**(1/4)*c**2*d**6*x**2) - c**(11/2
)*d**2*x**2*(1 - 2*d**2*x**2/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4 + 42*a**
(1/4)*c**2*d**6*x**2) + 4*c**(11/2)*d**2*x**2/(-21*a**(1/4)*c**4*d**4 + 42
*a**(1/4)*c**2*d**6*x**2) - 6*c**(7/2)*d**4*x**4*(1 - 2*d**2*x**2/c**2)**(
3/4)/(-21*a**(1/4)*c**4*d**4 + 42*a**(1/4)*c**2*d**6*x**2), True)) + c**(5
/2)*x*hyper((1/4, 1/2), (3/2,), 2*d**2*x**2*exp_polar(2*I*pi)/c**2)/a**(1/
4) + sqrt(c)*d**2*x**3*hyper((1/4, 3/2), (5/2,), 2*d**2*x**2*exp_polar(2*I
*pi)/c**2)/a**(1/4)
```

Maxima [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{(dx+c)^3}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3/(-2*a*d^2*x^2 + a*c^2)^(1/4), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{(dx+c)^3}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(-2*a*d^2*x^2 + a*c^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{(c+d\,x)^3}{(a\,c^2-2\,a\,d^2\,x^2)^{1/4}}\,dx$$

input
```
int((c + d*x)^3/(a*c^2 - 2*a*d^2*x^2)^(1/4),x)
```

output
```
int((c + d*x)^3/(a*c^2 - 2*a*d^2*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{-23(-2d^2x^2+c^2)^{\frac{3}{4}}\,c^2-3(-2d^2x^2+c^2)^{\frac{3}{4}}\,d^2x^2+63\left(\int \frac{x^2}{(-2d^2x^2+c^2)^{\frac{1}{4}}}dx\right)c\,d^3+21\left(\int \frac{1}{(-2d^2x^2+c^2)^{\frac{1}{4}}}dx\right)c^3}{21a^{\frac{1}{4}}d}$$

input
```
int((d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
( - 23*(c**2 - 2*d**2*x**2)**(3/4)*c**2 - 3*(c**2 - 2*d**2*x**2)**(3/4)*d*
*2*x**2 + 63*int(x**2/(c**2 - 2*d**2*x**2)**(1/4),x)*c*d**3 + 21*int(1/(c*
*2 - 2*d**2*x**2)**(1/4),x)*c**3*d)/(21*a**(1/4)*d)
```

3.368 $\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=-\frac{(10c+3dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{15ad}+\frac{6\sqrt{2}c^3\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{5d\sqrt[4]{ac^2-2ad^2x^2}}$$

output
```
-1/15*(3*d*x+10*c)*(-2*a*d^2*x^2+a*c^2)^(3/4)/a/d+6/5*2^(1/2)*c^3*(1-2*d^2
*x^2/c^2)^(1/4)*EllipticE(sin(1/2*arcsin(2^(1/2)*d*x/c)),2^(1/2))/d/(-2*a*
d^2*x^2+a*c^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\frac{-10c^3-3c^2dx+20cd^2x^2+6d^3x^3+18c^2dx\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{2d^2x^2}{c^2}\right)}{15d\sqrt[4]{a\left(c^2-2d^2x^2\right)}}$$

input
```
Integrate[(c + d*x)^2/(a*c^2 - 2*a*d^2*x^2)^(1/4),x]
```

output
```
(-10*c^3 - 3*c^2*d*x + 20*c*d^2*x^2 + 6*d^3*x^3 + 18*c^2*d*x*(1 - (2*d^2*x
^2)/c^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (2*d^2*x^2)/c^2])/(15*d*(a
*(c^2 - 2*d^2*x^2))^(1/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {497, 25, 27, 455, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{\int -\frac{acd^2(6c+7dx)}{\sqrt[4]{ac^2-2ad^2x^2}}dx}{5ad^2}-\frac{(c+dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{acd^2(6c+7dx)}{\sqrt[4]{ac^2-2ad^2x^2}}dx}{5ad^2}-\frac{(c+dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}c\int \frac{6c+7dx}{\sqrt[4]{ac^2-2ad^2x^2}}dx-\frac{(c+dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{5}c\left(6c\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}dx-\frac{7\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}\right)-\frac{(c+dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{5}c\left(\frac{6c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\int\frac{1}{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}-\frac{7\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}\right)-\frac{(c+dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{5ad}$$

$\downarrow$ 226

$$\frac{1}{5}c\left(\frac{6\sqrt{2}c^2\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}-\frac{7\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}\right)-\frac{(c+dx)\left(ac^2-2ad^2x^2\right)^{3/4}}{5ad}$$

input
```
Int[(c + d*x)^2/(a*c^2 - 2*a*d^2*x^2)^(1/4),x]
```

output
```
-1/5*((c + d*x)*(a*c^2 - 2*a*d^2*x^2)^(3/4))/(a*d) + (c*((-7*(a*c^2 - 2*a*
d^2*x^2)^(3/4))/(3*a*d) + (6*Sqrt[2]*c^2*(1 - (2*d^2*x^2)/c^2)^(1/4)*Ellip
ticE[ArcSin[(Sqrt[2]*d*x)/c]/2, 2])/(d*(a*c^2 - 2*a*d^2*x^2)^(1/4))))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(-2a\,d^2x^2+a\,c^2)^{\frac{1}{4}}}dx$$

input
```
int((d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int((d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{(dx+c)^2}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-2*a*d^2*x^2 + a*c^2)^(3/4)*(d^2*x^2 + 2*c*d*x + c^2)/(2*a*d^2*
x^2 - a*c^2), x)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = 2cd\left(\begin{cases} \tilde{\infty}x^2 & \text{for } a=0 \\ \frac{x^2}{2\sqrt[4]{ac^2}} & \text{for } d^2=0 \\ -\frac{(ac^2-2ad^2x^2)^{\frac{3}{4}}}{3ad^2} & \text{otherwise} \end{cases}\right) + \frac{c^{\frac{3}{2}}x\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{2d^2x^2e^{2i\pi}}{c^2}\right)}{\sqrt[4]{a}} + \frac{d^2x^3\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{2d^2x^2e^{2i\pi}}{c^2}\right)}{3\sqrt[4]{a}\sqrt{c}}$$

input
```
integrate((d*x+c)**2/(-2*a*d**2*x**2+a*c**2)**(1/4),x)
```

output
```
2*c*d*Piecewise((zoo*x**2, Eq(a, 0)), (x**2/(2*(a*c**2)**(1/4)), Eq(d**2,
0)), (-(a*c**2 - 2*a*d**2*x**2)**(3/4)/(3*a*d**2), True)) + c**(3/2)*x*hyp
er((1/4, 1/2), (3/2,), 2*d**2*x**2*exp_polar(2*I*pi)/c**2)/a**(1/4) + d**2
*x**3*hyper((1/4, 3/2), (5/2,), 2*d**2*x**2*exp_polar(2*I*pi)/c**2)/(3*a**
(1/4)*sqrt(c))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{(dx+c)^2}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(-2*a*d^2*x^2 + a*c^2)^(1/4), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{(dx+c)^2}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(-2*a*d^2*x^2 + a*c^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{(c+d\,x)^2}{(a\,c^2-2\,a\,d^2\,x^2)^{1/4}}\,dx$$

input
```
int((c + d*x)^2/(a*c^2 - 2*a*d^2*x^2)^(1/4),x)
```

output
```
int((c + d*x)^2/(a*c^2 - 2*a*d^2*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{-2(-2d^2x^2+c^2)^{\frac{3}{4}}c+3\left(\int \frac{x^2}{(-2d^2x^2+c^2)^{\frac{1}{4}}}dx\right)d^3+3\left(\int \frac{1}{(-2d^2x^2+c^2)^{\frac{1}{4}}}dx\right)c^2d}{3a^{\frac{1}{4}}d}$$

input
```
int((d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
( - 2*(c**2 - 2*d**2*x**2)**(3/4)*c + 3*int(x**2/(c**2 - 2*d**2*x**2)**(1/
4),x)*d**3 + 3*int(1/(c**2 - 2*d**2*x**2)**(1/4),x)*c**2*d)/(3*a**(1/4)*d)
```

3.369 $\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=-\frac{(ac^2-2ad^2x^2)^{3/4}}{3ad}+\frac{\sqrt{2}c^2\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}$$

output
```
-1/3*(-2*a*d^2*x^2+a*c^2)^(3/4)/a/d+2^(1/2)*c^2*(1-2*d^2*x^2/c^2)^(1/4)*El
lipticE(sin(1/2*arcsin(2^(1/2)*d*x/c)),2^(1/2))/d/(-2*a*d^2*x^2+a*c^2)^(1/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\frac{-c^2+2d^2x^2+3cdx\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{2d^2x^2}{c^2}\right)}{3d\sqrt[4]{a(c^2-2d^2x^2)}}$$

input
```
Integrate[(c + d*x)/(a*c^2 - 2*a*d^2*x^2)^(1/4),x]
```

output
```
(-c^2 + 2*d^2*x^2 + 3*c*d*x*(1 - (2*d^2*x^2)/c^2)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, (2*d^2*x^2)/c^2])/(3*d*(a*(c^2 - 2*d^2*x^2))^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {455, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}dx-\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}$$

$$\Big\downarrow \text{227}$$

$$\frac{c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\int \frac{1}{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}-\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}$$

$$\Big\downarrow \text{226}$$

$$\frac{\sqrt{2}c^2\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}-\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{3ad}$$

input
```
Int[(c + d*x)/(a*c^2 - 2*a*d^2*x^2)^(1/4),x]
```

output
```
-1/3*(a*c^2 - 2*a*d^2*x^2)^(3/4)/(a*d) + (Sqrt[2]*c^2*(1 - (2*d^2*x^2)/c^2
)^(1/4)*EllipticE[ArcSin[(Sqrt[2]*d*x)/c]/2, 2])/(d*(a*c^2 - 2*a*d^2*x^2)^
(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int \frac{dx + c}{\left(-2a\,d^{2}x^{2} + a\,c^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int((d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int((d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{dx+c}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-2*a*d^2*x^2 + a*c^2)^(3/4)*(d*x + c)/(2*a*d^2*x^2 - a*c^2), x)
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=d\left(\begin{cases}\tilde{\infty}x^2 & \text{for } a=0\\ \frac{x^2}{2\sqrt[4]{ac^2}} & \text{for } d^2=0\\ -\frac{\left(ac^2-2ad^2x^2\right)^{\frac{3}{4}}}{3ad^2} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{c}x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{2d^2x^2e^{2i\pi}}{c^2}\right)}{\sqrt[4]{a}}$$

input
```
integrate((d*x+c)/(-2*a*d**2*x**2+a*c**2)**(1/4),x)
```

output
```
d*Piecewise((zoo*x**2, Eq(a, 0)), (x**2/(2*(a*c**2)**(1/4)), Eq(d**2, 0)),
 (-(a*c**2 - 2*a*d**2*x**2)**(3/4)/(3*a*d**2), True)) + sqrt(c)*x*hyper((1
/4, 1/2), (3/2,), 2*d**2*x**2*exp_polar(2*I*pi)/c**2)/a**(1/4)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{dx+c}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(-2*a*d^2*x^2 + a*c^2)^(1/4), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{dx+c}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(-2*a*d^2*x^2 + a*c^2)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{c\,x\,\left(1-\frac{2\,d^2\,x^2}{c^2}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{2\,d^2\,x^2}{c^2}\right)}{\left(a\,c^2-2\,a\,d^2\,x^2\right)^{1/4}} - \frac{\left(a\,\left(c^2-2\,d^2\,x^2\right)\right)^{3/4}}{3\,a\,d}$$

input
```
int((c + d*x)/(a*c^2 - 2*a*d^2*x^2)^(1/4),x)
```

output
```
(c*x*(1 - (2*d^2*x^2)/c^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, (2*d^2*x^2)/c^
2))/(a*c^2 - 2*a*d^2*x^2)^(1/4) - (a*(c^2 - 2*d^2*x^2))^(3/4)/(3*a*d)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{-\left(-2d^2x^2+c^2\right)^{\frac{3}{4}}+3\left(\int \frac{1}{\left(-2d^2x^2+c^2\right)^{\frac{1}{4}}}dx\right)cd}{3a^{\frac{1}{4}}d}$$

input
```
int((d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
( - (c**2 - 2*d**2*x**2)**(3/4) + 3*int(1/(c**2 - 2*d**2*x**2)**(1/4),x)*c
*d)/(3*a**(1/4)*d)
```

3.370 $\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\frac{\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}$$

output
```
2^(1/2)*c*(1-2*d^2*x^2/c^2)^(1/4)*EllipticE(sin(1/2*arcsin(2^(1/2)*d*x/c))
,2^(1/2))/d/(-2*a*d^2*x^2+a*c^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.85 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\frac{x\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{2d^2x^2}{c^2}\right)}{\sqrt[4]{a\left(c^2-2d^2x^2\right)}}$$

input
```
Integrate[(a*c^2 - 2*a*d^2*x^2)^(-1/4),x]
```

output
```
(x*(1 - (2*d^2*x^2)/c^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (2*d^2*x^2
)/c^2])/(a*(c^2 - 2*d^2*x^2))^(1/4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}dx$$

$$\Big\downarrow \text{ 227}$$

$$\frac{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\int \frac{1}{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}$$

$$\Big\downarrow \text{ 226}$$

$$\frac{\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}$$

input
```
Int[(a*c^2 - 2*a*d^2*x^2)^(-1/4),x]
```

output
```
(Sqrt[2]*c*(1 - (2*d^2*x^2)/c^2)^(1/4)*EllipticE[ArcSin[(Sqrt[2]*d*x)/c]/2
, 2])/(d*(a*c^2 - 2*a*d^2*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-2a\,d^{2}x^{2}+a\,c^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{1}{\left(-2\,ad^{2}x^{2}+ac^{2}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-2*a*d^2*x^2 + a*c^2)^(3/4)/(2*a*d^2*x^2 - a*c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{2d^2x^2e^{2i\pi}}{c^2}\right)}{\sqrt[4]{a}\sqrt{c}}$$

input
```
integrate(1/(-2*a*d**2*x**2+a*c**2)**(1/4),x)
```

output
```
x*hyper((1/4, 1/2), (3/2,), 2*d**2*x**2*exp_polar(2*I*pi)/c**2)/(a**(1/4)*
sqrt(c))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-2*a*d^2*x^2 + a*c^2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{(-2\,ad^2x^2+ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((-2*a*d^2*x^2 + a*c^2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\frac{x\,\left(1-\frac{2\,d^2\,x^2}{c^2}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{2\,d^2\,x^2}{c^2}\right)}{\left(a\,c^2-2\,a\,d^2\,x^2\right)^{1/4}}$$

input
```
int(1/(a*c^2 - 2*a*d^2*x^2)^(1/4),x)
```

output
```
(x*(1 - (2*d^2*x^2)/c^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, (2*d^2*x^2)/c^2)
)/(a*c^2 - 2*a*d^2*x^2)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\frac{\int \frac{1}{(-2d^2x^2+c^2)^{\frac{1}{4}}}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/(c**2 - 2*d**2*x**2)**(1/4),x)/a**(1/4)
```

3.371 $\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 252

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{\sqrt[4]{-ac^2+2ad^2x^2}\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)-c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d\sqrt[4]{ac^2-2ad^2x^2}} - \frac{\sqrt[4]{-ac^2+2ad^2x^2}\,\text{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)+c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d\sqrt[4]{ac^2-2ad^2x^2}}$$

output

```
1/2*(2*a*d^2*x^2-a*c^2)^(1/4)*arctan(a^(1/4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(
1/4)/(a^(1/2)*c*(d*x+c)-c*(2*a*d^2*x^2-a*c^2)^(1/2)))/a^(1/4)/c^(1/2)/d/(-
2*a*d^2*x^2+a*c^2)^(1/4)-1/2*(2*a*d^2*x^2-a*c^2)^(1/4)*arctanh(a^(1/4)*c^(
3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)+c*(2*a*d^2*x^2-a*c^2)^(1
/2)))/a^(1/4)/c^(1/2)/d/(-2*a*d^2*x^2+a*c^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 7.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.70

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{\frac{d\left(-\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\sqrt[4]{\frac{d\left(\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2c-\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx},\frac{2c+\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx}\right)}{d\sqrt[4]{a(c^2-2d^2x^2)}}$$

input
```
Integrate[1/((c + d*x)*(a*c^2 - 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-((Sqrt[2]*((d*(-(Sqrt[2]*Sqrt[c^2/d^2]) + 2*x))/(c + d*x))^(1/4)*((d*(Sqr
t[2]*Sqrt[c^2/d^2] + 2*x))/(c + d*x))^(1/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (
2*c - Sqrt[2]*Sqrt[c^2/d^2]*d)/(2*c + 2*d*x), (2*c + Sqrt[2]*Sqrt[c^2/d^2]
*d)/(2*c + 2*d*x)])/(d*(a*(c^2 - 2*d^2*x^2))^(1/4)))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.85, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {500, 499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx$$

$$\Big\downarrow \text{500}$$

$$\frac{\sqrt[4]{2ad^2x^2-ac^2}\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}$$

$$\Big\downarrow \text{499}$$

$$\frac{\sqrt[4]{2ad^2x^2-ac^2}\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{c\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c(c+dx)}\right)}{2\sqrt[4]{a}\sqrt{c}d}\right)}{\sqrt[4]{ac^2-2ad^2x^2}}$$

input
```
Int[1/((c + d*x)*(a*c^2 - 2*a*d^2*x^2)^(1/4)),x]
```

output
```
((-(a*c^2) + 2*a*d^2*x^2)^(1/4)*(ArcTan[(a^(1/4)*c^(3/2)*(-(a*c^2) + 2*a*d
^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x) - c*Sqrt[-(a*c^2) + 2*a*d^2*x^2])]/(2*
a^(1/4)*Sqrt[c]*d) - ArcTanh[(a^(1/4)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/
4))/(Sqrt[a]*c*(c + d*x) + c*Sqrt[-(a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqr
t[c]*d)))/(a*c^2 - 2*a*d^2*x^2)^(1/4)
```

Defintions of rubi rules used

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 500
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[
(-a - b*x^2)^(1/4)/(a + b*x^2)^(1/4)   Int[1/((c + d*x)*(-a - b*x^2)^(1/4))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 2*a*d^2, 0] && PosQ[a]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(-2a\,d^2x^2+a\,c^2\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/(d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{1}{\sqrt[4]{-a\left(-c^2+2d^2x^2\right)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(-2*a*d**2*x**2+a*c**2)**(1/4),x)
```

output
```
Integral(1/((-a*(-c**2 + 2*d**2*x**2))**(1/4)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{1}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-2*a*d^2*x^2 + a*c^2)^(1/4)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{1}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-2*a*d^2*x^2 + a*c^2)^(1/4)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{1}{\left(a\,c^2-2\,a\,d^2\,x^2\right)^{1/4}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a*c^2 - 2*a*d^2*x^2)^(1/4)*(c + d*x)),x)
```

output
```
int(1/((a*c^2 - 2*a*d^2*x^2)^(1/4)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\frac{\int \frac{1}{(-2d^2x^2+c^2)^{\frac{1}{4}}c+(-2d^2x^2+c^2)^{\frac{1}{4}}dx}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(d*x+c)/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/((c**2 - 2*d**2*x**2)**(1/4)*c + (c**2 - 2*d**2*x**2)**(1/4)*d*x),x)
/a**(1/4)
```

3.372 $\int \frac{1}{(c+dx)^2 \sqrt[4]{ac^2-2ad^2x^2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 354

$$\int \frac{1}{(c+dx)^2 \sqrt[4]{ac^2-2ad^2x^2}} \, dx = \frac{(ac^2-2ad^2x^2)^{3/4}}{ac^2d(c+dx)} + \frac{\sqrt[4]{-ac^2+2ad^2x^2} \arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)-c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}c^{3/2}d\sqrt[4]{ac^2-2ad^2x^2}} - \frac{\sqrt[4]{-ac^2+2ad^2x^2}\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)+c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}c^{3/2}d\sqrt[4]{ac^2-2ad^2x^2}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{cd\sqrt[4]{ac^2-2ad^2x^2}}$$

output
```
(-2*a*d^2*x^2+a*c^2)^(3/4)/a/c^2/d/(d*x+c)+1/2*(2*a*d^2*x^2-a*c^2)^(1/4)*a
rctan(a^(1/4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)-c*(2*a*
d^2*x^2-a*c^2)^(1/2)))/a^(1/4)/c^(3/2)/d/(-2*a*d^2*x^2+a*c^2)^(1/4)-1/2*(2
*a*d^2*x^2-a*c^2)^(1/4)*arctanh(a^(1/4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/
(a^(1/2)*c*(d*x+c)+c*(2*a*d^2*x^2-a*c^2)^(1/2)))/a^(1/4)/c^(3/2)/d/(-2*a*d
^2*x^2+a*c^2)^(1/4)+2^(1/2)*(1-2*d^2*x^2/c^2)^(1/4)*EllipticE(sin(1/2*arcs
in(2^(1/2)*d*x/c)),2^(1/2))/c/d/(-2*a*d^2*x^2+a*c^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.53

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{\frac{d\left(-\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\sqrt[4]{\frac{d\left(\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{4},\frac{1}{4},\frac{5}{2},\frac{2c-\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx},\frac{2c+\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx}\right)}{3d(c+dx)\sqrt[4]{a(c^2-2d^2x^2)}}$$

input
```
Integrate[1/((c + d*x)^2*(a*c^2 - 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-1/3*(Sqrt[2]*((d*(-(Sqrt[2]*Sqrt[c^2/d^2]) + 2*x))/(c + d*x))^(1/4)*((d*(
Sqrt[2]*Sqrt[c^2/d^2] + 2*x))/(c + d*x))^(1/4)*AppellF1[3/2, 1/4, 1/4, 5/2
, (2*c - Sqrt[2]*Sqrt[c^2/d^2]*d)/(2*c + 2*d*x), (2*c + Sqrt[2]*Sqrt[c^2/d
^2]*d)/(2*c + 2*d*x)])/(d*(c + d*x)*(a*(c^2 - 2*d^2*x^2))^(1/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {498, 27, 719, 227, 226, 500, 499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}dx$$

$$\Big\downarrow \text{498}$$

$$\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}-\frac{2\int -\frac{2c+dx}{2(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx}{c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2c+dx}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx}{c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\downarrow$ 719

$$\frac{\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}dx+c\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx}{c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\downarrow$ 227

$$\frac{c\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx+\frac{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\int \frac{1}{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}}{c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\downarrow$ 226

$$\frac{c\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx+\frac{\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}}{c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\downarrow$ 500

$$\frac{\frac{c\sqrt[4]{2ad^2x^2-ac^2}\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}+\frac{\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}}{c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\downarrow$ 499

$$\frac{\frac{\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}+\frac{c\sqrt[4]{2ad^2x^2-ac^2}\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2}\ldots}{c\sqrt{2ad^2x^2-ac^2}+\ldots}\right)}{2\sqrt[4]{a}\sqrt{c}d}\ldots\right)}{\sqrt[4]{ac^2-2ad^2x^2}}}{c^2}$$

$$\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}$$

input
```
Int[1/((c + d*x)^2*(a*c^2 - 2*a*d^2*x^2)^(1/4)),x]
```

output
```
(a*c^2 - 2*a*d^2*x^2)^(3/4)/(a*c^2*d*(c + d*x)) + ((c*(-(a*c^2) + 2*a*d^2*
x^2)^(1/4)*(ArcTan[(a^(1/4)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[
a]*c*(c + d*x) - c*Sqrt[-(a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d) -
ArcTanh[(a^(1/4)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d
*x) + c*Sqrt[-(a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d)))/(a*c^2 - 2*
a*d^2*x^2)^(1/4) + (Sqrt[2]*c*(1 - (2*d^2*x^2)/c^2)^(1/4)*EllipticE[ArcSin
[(Sqrt[2]*d*x)/c]/2, 2])/(d*(a*c^2 - 2*a*d^2*x^2)^(1/4)))/c^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 500
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[
(-a - b*x^2)^(1/4)/(a + b*x^2)^(1/4)   Int[1/((c + d*x)*(-a - b*x^2)^(1/4))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 2*a*d^2, 0] && PosQ[a]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{2} \left(-2a\,d^{2}x^{2}+a\,c^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/(d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{\sqrt[4]{-a\left(-c^2+2d^2x^2\right)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(-2*a*d**2*x**2+a*c**2)**(1/4),x)
```

output
```
Integral(1/((-a*(-c**2 + 2*d**2*x**2))**(1/4)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-2*a*d^2*x^2 + a*c^2)^(1/4)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-2*a*d^2*x^2 + a*c^2)^(1/4)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{\left(a\,c^2-2\,a\,d^2\,x^2\right)^{1/4}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a*c^2 - 2*a*d^2*x^2)^(1/4)*(c + d*x)^2),x)
```

output
```
int(1/((a*c^2 - 2*a*d^2*x^2)^(1/4)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{\int \frac{1}{(-2d^2x^2+c^2)^{\frac{1}{4}}c^2+2(-2d^2x^2+c^2)^{\frac{1}{4}}cdx+(-2d^2x^2+c^2)^{\frac{1}{4}}d^2x^2}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(d*x+c)^2/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/((c**2 - 2*d**2*x**2)**(1/4)*c**2 + 2*(c**2 - 2*d**2*x**2)**(1/4)*c*
d*x + (c**2 - 2*d**2*x**2)**(1/4)*d**2*x**2),x)/a**(1/4)
```

3.373 $\int \frac{1}{(c+dx)^3 \sqrt[4]{ac^2-2ad^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 392

$$\begin{aligned}\int \frac{1}{(c+dx)^3\sqrt[4]{ac^2-2ad^2x^2}}\,dx &= \frac{(ac^2-2ad^2x^2)^{3/4}}{2ac^2d(c+dx)^2}+\frac{5(ac^2-2ad^2x^2)^{3/4}}{2ac^3d(c+dx)} \\ &+ \frac{\sqrt[4]{-ac^2+2ad^2x^2}\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)-c\sqrt{-ac^2+2ad^2x^2}}\right)}{\sqrt[4]{a}c^{5/2}d\sqrt[4]{ac^2-2ad^2x^2}} \\ &- \frac{\sqrt[4]{-ac^2+2ad^2x^2}\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)+c\sqrt{-ac^2+2ad^2x^2}}\right)}{\sqrt[4]{a}c^{5/2}d\sqrt[4]{ac^2-2ad^2x^2}} \\ &+ \frac{5\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{\sqrt{2}c^2d\sqrt[4]{ac^2-2ad^2x^2}}\end{aligned}$$

output
```
1/2*(-2*a*d^2*x^2+a*c^2)^(3/4)/a/c^2/d/(d*x+c)^2+5/2*(-2*a*d^2*x^2+a*c^2)^
(3/4)/a/c^3/d/(d*x+c)+(2*a*d^2*x^2-a*c^2)^(1/4)*arctan(a^(1/4)*c^(3/2)*(2*
a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)-c*(2*a*d^2*x^2-a*c^2)^(1/2)))/a^
(1/4)/c^(5/2)/d/(-2*a*d^2*x^2+a*c^2)^(1/4)-(2*a*d^2*x^2-a*c^2)^(1/4)*arcta
nh(a^(1/4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)+c*(2*a*d^2
*x^2-a*c^2)^(1/2)))/a^(1/4)/c^(5/2)/d/(-2*a*d^2*x^2+a*c^2)^(1/4)+5/2*(1-2*
d^2*x^2/c^2)^(1/4)*EllipticE(sin(1/2*arcsin(2^(1/2)*d*x/c)),2^(1/2))*2^(1/
2)/c^2/d/(-2*a*d^2*x^2+a*c^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.47

$$\int \frac{1}{(c+dx)^3\sqrt[4]{ac^2-2ad^2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{\frac{d\left(-\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\sqrt[4]{\frac{d\left(\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\,\text{AppellF1}\left(\frac{5}{2},\frac{1}{4},\frac{1}{4},\frac{7}{2},\frac{2c-\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx},\frac{2c+\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx}\right)}{5d(c+dx)^2\sqrt[4]{a(c^2-2d^2x^2)}}$$

input
```
Integrate[1/((c + d*x)^3*(a*c^2 - 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-1/5*(Sqrt[2]*((d*(-(Sqrt[2]*Sqrt[c^2/d^2]) + 2*x))/(c + d*x))^(1/4)*((d*(
Sqrt[2]*Sqrt[c^2/d^2] + 2*x))/(c + d*x))^(1/4)*AppellF1[5/2, 1/4, 1/4, 7/2
, (2*c - Sqrt[2]*Sqrt[c^2/d^2]*d)/(2*c + 2*d*x), (2*c + Sqrt[2]*Sqrt[c^2/d
^2]*d)/(2*c + 2*d*x)])/(d*(c + d*x)^2*(a*(c^2 - 2*d^2*x^2))^(1/4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {498, 27, 688, 27, 719, 227, 226, 500, 499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3\sqrt[4]{ac^2-2ad^2x^2}}\,dx$$

$$\Big\downarrow \text{498}$$

$$\frac{(ac^2-2ad^2x^2)^{3/4}}{2ac^2d(c+dx)^2}-\frac{\int -\frac{4c-dx}{2(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}dx}{c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4c-dx}{(c+dx)^2\sqrt[4]{ac^2-2ad^2x^2}}dx}{2c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\Big\downarrow$ 688

$$\frac{\frac{\int \frac{acd^2(9c+5dx)}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx}{ac^2d^2}+\frac{5\left(ac^2-2ad^2x^2\right)^{3/4}}{acd(c+dx)}}{2c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{9c+5dx}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx}{c}+\frac{5\left(ac^2-2ad^2x^2\right)^{3/4}}{acd(c+dx)}}{2c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\Big\downarrow$ 719

$$\frac{\frac{5\int \frac{1}{\sqrt[4]{ac^2-2ad^2x^2}}dx+4c\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx}{c}+\frac{5\left(ac^2-2ad^2x^2\right)^{3/4}}{acd(c+dx)}}{2c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\Big\downarrow$ 227

$$\frac{\frac{4c\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx+\frac{5\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\int \frac{1}{\sqrt[4]{1-\frac{2d^2x^2}{c^2}}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}}{c}+\frac{5\left(ac^2-2ad^2x^2\right)^{3/4}}{acd(c+dx)}}{2c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\Big\downarrow$ 226

$$\frac{\frac{4c\int \frac{1}{(c+dx)\sqrt[4]{ac^2-2ad^2x^2}}dx+\frac{5\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}}{c}+\frac{5\left(ac^2-2ad^2x^2\right)^{3/4}}{acd(c+dx)}}{2c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\Big\downarrow$ 500

$$\frac{\frac{\frac{4c\sqrt[4]{2ad^2x^2-ac^2}\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{\sqrt[4]{ac^2-2ad^2x^2}}+\frac{5\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}}{c}+\frac{5\left(ac^2-2ad^2x^2\right)^{3/4}}{acd(c+dx)}}{2c^2}+\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\Big\downarrow$ 499

$$\frac{\frac{\frac{5\sqrt{2}c\sqrt[4]{1-\frac{2d^2x^2}{c^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}dx}{c}\right)\middle|2\right)}{d\sqrt[4]{ac^2-2ad^2x^2}}+\frac{4c\sqrt[4]{2ad^2x^2-ac^2}\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x}}{c\sqrt{2ad^2x^2-ac^2}+\sqrt{a}}\right.}{2\sqrt[4]{a}\sqrt{c}d}\right.}{\sqrt[4]{ac^2-2ad^2x^2}}}{c}}{2c^2}$$

$$\frac{\left(ac^2-2ad^2x^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

input

```
Int[1/((c + d*x)^3*(a*c^2 - 2*a*d^2*x^2)^(1/4)),x]
```

output

```
(a*c^2 - 2*a*d^2*x^2)^(3/4)/(2*a*c^2*d*(c + d*x)^2) + ((5*(a*c^2 - 2*a*d^2
*x^2)^(3/4))/(a*c*d*(c + d*x)) + ((4*c*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)*(Arc
Tan[(a^(1/4)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x)
- c*Sqrt[-(a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d) - ArcTanh[(a^(1/4
)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x) + c*Sqrt[-(
a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d)))/(a*c^2 - 2*a*d^2*x^2)^(1/4
) + (5*Sqrt[2]*c*(1 - (2*d^2*x^2)/c^2)^(1/4)*EllipticE[ArcSin[(Sqrt[2]*d*x
)/c]/2, 2])/(d*(a*c^2 - 2*a*d^2*x^2)^(1/4)))/c)/(2*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 500
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[
(-a - b*x^2)^(1/4)/(a + b*x^2)^(1/4)   Int[1/((c + d*x)*(-a - b*x^2)^(1/4))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + 2*a*d^2, 0] && PosQ[a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{1}{(dx+c)^{3} \left(-2a\,d^{2}x^{2}+a\,c^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/(d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3 \sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{\sqrt[4]{-a\left(-c^2+2d^2x^2\right)}\,(c+dx)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(-2*a*d**2*x**2+a*c**2)**(1/4),x)
```

output
```
Integral(1/((-a*(-c**2 + 2*d**2*x**2))**(1/4)*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3 \sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-2*a*d^2*x^2 + a*c^2)^(1/4)*(d*x + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^3 \sqrt[4]{ac^2-2ad^2x^2}}\,dx = \int \frac{1}{\left(-2\,ad^2x^2+ac^2\right)^{\frac{1}{4}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-2*a*d^2*x^2 + a*c^2)^(1/4)*(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[4]{ac^2-2ad^2x^2}}\,dx=\int \frac{1}{{\left(a\,c^2-2\,a\,d^2\,x^2\right)}^{1/4}\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(1/((a*c^2 - 2*a*d^2*x^2)^(1/4)*(c + d*x)^3),x)
```

output
```
int(1/((a*c^2 - 2*a*d^2*x^2)^(1/4)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^3\sqrt[4]{ac^2-2ad^2x^2}}\,dx = \frac{\int \frac{1}{(-2d^2x^2+c^2)^{\frac{1}{4}}c^3+3(-2d^2x^2+c^2)^{\frac{1}{4}}c^2dx+3(-2d^2x^2+c^2)^{\frac{1}{4}}c\,d^2x^2+(-2d^2x^2+c^2)^{\frac{1}{4}}d^3x^3}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(d*x+c)^3/(-2*a*d^2*x^2+a*c^2)^(1/4),x)
```

output
```
int(1/((c**2 - 2*d**2*x**2)**(1/4)*c**3 + 3*(c**2 - 2*d**2*x**2)**(1/4)*c*
*2*d*x + 3*(c**2 - 2*d**2*x**2)**(1/4)*c*d**2*x**2 + (c**2 - 2*d**2*x**2)*
*(1/4)*d**3*x**3),x)/a**(1/4)
```

3.374 $\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 204

$$\begin{aligned}\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx &= \frac{1}{7}(1+x)^2\left(-1+2x^2\right)^{3/4}+\frac{1}{105}(100+33x)\left(-1+2x^2\right)^{3/4}+\frac{16x\sqrt[4]{-1+2x^2}}{5\left(1+\sqrt{-1+2x^2}\right)}\\ &\quad-\frac{8\sqrt{2}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right)|\frac{1}{2}\right)}{5x}\\ &\quad+\frac{4\sqrt{2}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right),\frac{1}{2}\right)}{5x}\end{aligned}$$

output
```
1/7*(1+x)^2*(2*x^2-1)^(3/4)+1/105*(100+33*x)*(2*x^2-1)^(3/4)+16*x*(2*x^2-1
)^(1/4)/(5+5*(2*x^2-1)^(1/2))-8/5*2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2)^(1/2
)*(1+(2*x^2-1)^(1/2))*EllipticE(sin(2*arctan((2*x^2-1)^(1/4))),1/2*2^(1/2)
)/x+4/5*2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(1/2))*Inve
rseJacobiAM(2*arctan((2*x^2-1)^(1/4)),1/2*2^(1/2))/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.91 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.31

$$\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx = \frac{-115-63x+215x^2+126x^3+30x^4+168x\sqrt[4]{1-2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)}{105\sqrt[4]{-1+2x^2}}$$

input
```
Integrate[(1 + x)^3/(-1 + 2*x^2)^(1/4),x]
```

output
```
(-115 - 63*x + 215*x^2 + 126*x^3 + 30*x^4 + 168*x*(1 - 2*x^2)^(1/4)*Hyperg
eometric2F1[1/4, 1/2, 3/2, 2*x^2])/(105*(-1 + 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {497, 676, 228, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^3}{\sqrt[4]{2x^2-1}}dx$$

$$\Big\downarrow \text{ 497}$$

$$\frac{1}{7}\int \frac{(x+1)(11x+9)}{\sqrt[4]{2x^2-1}}dx+\frac{1}{7}\left(2x^2-1\right)^{3/4}(x+1)^2$$

$$\Big\downarrow \text{ 676}$$

$$\frac{1}{7}\left(\frac{56}{5}\int \frac{1}{\sqrt[4]{2x^2-1}}dx+\frac{11}{5}\left(2x^2-1\right)^{3/4}x+\frac{20}{3}\left(2x^2-1\right)^{3/4}\right)+\frac{1}{7}\left(2x^2-1\right)^{3/4}(x+1)^2$$

$$\Big\downarrow \text{ 228}$$

$$\frac{1}{7}\left(\frac{56\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{5x}+\frac{11}{5}\left(2x^2-1\right)^{3/4}x+\frac{20}{3}\left(2x^2-1\right)^{3/4}\right)+\frac{1}{7}\left(2x^2-1\right)^{3/4}(x+1)^2$$

$$\Big\downarrow \; 834$$

$$\frac{1}{7}\left(\frac{56\sqrt{2}\sqrt{x^2}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{5x}+\frac{11}{5}\left(2x^2-1\right)^{3/4}x+\frac{20}{3}\left(2x^2-1\right)^{3/4}\right)+\frac{1}{7}\left(2x^2-1\right)^{3/4}(x+1)^2$$

$$\Big\downarrow \; 761$$

$$\frac{1}{7}\left(\frac{56\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{5x}+\frac{11}{5}\left(2x^2-\right.\right.$$

$$\frac{1}{7}\left(2x^2-1\right)^{3/4}(x+1)^2$$

$$\Big\downarrow \; 1510$$

$$\frac{1}{7}\left(\frac{56\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{\right.\right.}{\sqrt{x^2}}\right.}{5x}\right.$$

$$\frac{1}{7}\left(2x^2-1\right)^{3/4}(x+1)^2$$

input
```
Int[(1 + x)^3/(-1 + 2*x^2)^(1/4),x]
```

output
```
((1 + x)^2*(-1 + 2*x^2)^(3/4))/7 + ((20*(-1 + 2*x^2)^(3/4))/3 + (11*x*(-1
+ 2*x^2)^(3/4))/5 + (56*Sqrt[2]*Sqrt[x^2]*((Sqrt[2]*Sqrt[x^2]*(-1 + 2*x^2)
^(1/4))/(1 + Sqrt[-1 + 2*x^2]) - (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 +
Sqrt[-1 + 2*x^2])*EllipticE[2*ArcTan[(-1 + 2*x^2)^(1/4)], 1/2])/Sqrt[x^2]
+ (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*EllipticF[2*A
rcTan[(-1 + 2*x^2)^(1/4)], 1/2])/(2*Sqrt[x^2])))/(5*x))/7
```

Defintions of rubi rules used

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.29

method	result
risch	$\frac{(15x^2+63x+115)(2x^2-1)^{\frac{3}{4}}}{105}+\frac{8(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}}x\operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],2x^2)}{5\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}$
meijerg	$\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}}x\operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],2x^2)}{\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}+\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}}x^4\operatorname{hypergeom}([\frac{1}{4},2],[3],2x^2)}{4\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}+\frac{(-\operatorname{signum}(2x^2-\cdots}{}$

input
```
int((x+1)^3/(2*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/105*(15*x^2+63*x+115)*(2*x^2-1)^(3/4)+8/5/signum(2*x^2-1)^(1/4)*(-signum
(2*x^2-1))^(1/4)*x*hypergeom([1/4,1/2],[3/2],2*x^2)
```

Fricas [F]

$$\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx=\int \frac{(x+1)^3}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^3/(2*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((x^3 + 3*x^2 + 3*x + 1)/(2*x^2 - 1)^(1/4), x)
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.63

$$\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx = x^3e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|2x^2\right)+xe^{-\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|2x^2\right)+\left(2x^2-1\right)^{\frac{3}{4}} + \begin{cases}\frac{x^2(2x^2-1)^{\frac{3}{4}}}{7}+\frac{2(2x^2-1)^{\frac{3}{4}}}{21} & \text{for } |x^2|>\frac{1}{2}\\ -\frac{x^2(1-2x^2)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}}{7}-\frac{2(1-2x^2)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}}{21} & \text{otherwise}\end{cases}$$

input
```
integrate((1+x)**3/(2*x**2-1)**(1/4),x)
```

output
```
x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 2*x**2) + x*exp(-I*pi/4)*hyper
((1/4, 1/2), (3/2,), 2*x**2) + (2*x**2 - 1)**(3/4) + Piecewise((x**2*(2*x*
*2 - 1)**(3/4)/7 + 2*(2*x**2 - 1)**(3/4)/21, Abs(x**2) > 1/2), (-x**2*(1 -
 2*x**2)**(3/4)*exp(-I*pi/4)/7 - 2*(1 - 2*x**2)**(3/4)*exp(-I*pi/4)/21, Tr
ue))
```

Maxima [F]

$$\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{(x+1)^3}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^3/(2*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x + 1)^3/(2*x^2 - 1)^(1/4), x)
```

Giac [F]

$$\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{(x+1)^3}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^3/(2*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate((x + 1)^3/(2*x^2 - 1)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{(x+1)^3}{(2\,x^2-1)^{1/4}}\,dx$$

input
```
int((x + 1)^3/(2*x^2 - 1)^(1/4),x)
```

output
```
int((x + 1)^3/(2*x^2 - 1)^(1/4), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(1+x)^3}{\sqrt[4]{-1+2x^2}}\,dx &= \frac{\sqrt{2x^2-1}\,\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,\sqrt{2}\,x^3}{7}\\ &+\frac{23\sqrt{2x^2-1}\,\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,\sqrt{2}\,x}{21}\\ &-\frac{2\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,x^4}{7}\\ &-\frac{43\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,x^2}{21}\\ &+\frac{23\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}}{21}\\ &+3\left(\int \frac{x^2}{(2x^2-1)^{\frac{1}{4}}}dx\right)+\int \frac{1}{(2x^2-1)^{\frac{1}{4}}}dx\end{aligned}$$

input
```
int((1+x)^3/(2*x^2-1)^(1/4),x)
```

output
```
(3*sqrt(2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x + 2*x**2 - 1)*sqrt(sqr
t(2*x**2 - 1) + sqrt(2)*x)*sqrt(2)*x**3 + 23*sqrt(2*x**2 - 1)*sqrt(sqrt(2*
x**2 - 1)*sqrt(2)*x + 2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1) + sqrt(2)*x)*sqrt(
2)*x - 6*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x + 2*x**2 - 1)*sqrt(sqrt(2*x**2 -
1) + sqrt(2)*x)*x**4 - 43*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x + 2*x**2 - 1)*sq
rt(sqrt(2*x**2 - 1) + sqrt(2)*x)*x**2 + 23*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x
 + 2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1) + sqrt(2)*x) + 63*int(x**2/(2*x**2 -
1)**(1/4),x) + 21*int(1/(2*x**2 - 1)**(1/4),x))/21
```

3.375 $\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 184

$$\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx = \frac{1}{15}(10+3x)\left(-1+2x^2\right)^{3/4}+\frac{12x\sqrt[4]{-1+2x^2}}{5\left(1+\sqrt{-1+2x^2}\right)} - \frac{6\sqrt{2}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right)|\frac{1}{2}\right)}{5x} + \frac{3\sqrt{2}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right),\frac{1}{2}\right)}{5x}$$

output
```
1/15*(10+3*x)*(2*x^2-1)^(3/4)+12*x*(2*x^2-1)^(1/4)/(5+5*(2*x^2-1)^(1/2))-6
/5*2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(1/2))*EllipticE
(sin(2*arctan((2*x^2-1)^(1/4))),1/2*2^(1/2))/x+3/5*2^(1/2)*(x^2/(1+(2*x^2-
1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(1/2))*InverseJacobiAM(2*arctan((2*x^2-1)^
(1/4)),1/2*2^(1/2))/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

$$\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx = \frac{-10-3x+20x^2+6x^3+18x\sqrt[4]{1-2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)}{15\sqrt[4]{-1+2x^2}}$$

input
```
Integrate[(1 + x)^2/(-1 + 2*x^2)^(1/4),x]
```

output
```
(-10 - 3*x + 20*x^2 + 6*x^3 + 18*x*(1 - 2*x^2)^(1/4)*Hypergeometric2F1[1/4
, 1/2, 3/2, 2*x^2])/(15*(-1 + 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {497, 455, 228, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{\sqrt[4]{2x^2-1}}\,dx$$

$$\Big\downarrow \text{497}$$

$$\frac{1}{5}\int \frac{7x+6}{\sqrt[4]{2x^2-1}}dx+\frac{1}{5}\left(2x^2-1\right)^{3/4}(x+1)$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{5}\left(6\int \frac{1}{\sqrt[4]{2x^2-1}}dx+\frac{7}{3}\left(2x^2-1\right)^{3/4}\right)+\frac{1}{5}\left(2x^2-1\right)^{3/4}(x+1)$$

$$\Big\downarrow \text{228}$$

$$\frac{1}{5}\left(\frac{6\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{x}+\frac{7}{3}\left(2x^2-1\right)^{3/4}\right)+\frac{1}{5}\left(2x^2-1\right)^{3/4}(x+1)$$

$$\Big\downarrow \text{834}$$

$$\frac{1}{5}\left(\frac{6\sqrt{2}\sqrt{x^2}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+\frac{7}{3}\left(2x^2-1\right)^{3/4}\right)+$$
$$\frac{1}{5}\left(2x^2-1\right)^{3/4}(x+1)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{5}\left(\frac{6\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+\frac{7}{3}\left(2x^2-1\right)\right.$$
$$\frac{1}{5}\left(2x^2-1\right)^{3/4}(x+1)$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{5}\left(\frac{6\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{2\right.\right.}{\sqrt{x^2}}\right.}{x}\right.$$
$$\frac{1}{5}\left(2x^2-1\right)^{3/4}(x+1)$$

input
```
Int[(1 + x)^2/(-1 + 2*x^2)^(1/4),x]
```

output
```
((1 + x)*(-1 + 2*x^2)^(3/4))/5 + ((7*(-1 + 2*x^2)^(3/4))/3 + (6*Sqrt[2]*Sq
rt[x^2]*((Sqrt[2]*Sqrt[x^2]*(-1 + 2*x^2)^(1/4))/(1 + Sqrt[-1 + 2*x^2]) - (
Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*EllipticE[2*ArcT
an[(-1 + 2*x^2)^(1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])
^2]*(1 + Sqrt[-1 + 2*x^2])*EllipticF[2*ArcTan[(-1 + 2*x^2)^(1/4)], 1/2])/(
2*Sqrt[x^2])))/x)/5
```

Defintions of rubi rules used

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.29

method	result
risch	$\frac{(10+3x)(2x^2-1)^{\frac{3}{4}}}{15}+\frac{6(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}}x\operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],2x^2)}{5\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}$
meijerg	$\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}}x\operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],2x^2)}{\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}+\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}}x^3\operatorname{hypergeom}([\frac{1}{4},\frac{3}{2}],[\frac{5}{2}],2x^2)}{3\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}+\frac{(-\operatorname{signum}(2x^2$

input
```
int((x+1)^2/(2*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/15*(10+3*x)*(2*x^2-1)^(3/4)+6/5/signum(2*x^2-1)^(1/4)*(-signum(2*x^2-1))
^(1/4)*x*hypergeom([1/4,1/2],[3/2],2*x^2)
```

Fricas [F]

$$\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx=\int \frac{(x+1)^2}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^2/(2*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((x^2 + 2*x + 1)/(2*x^2 - 1)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.30

$$\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx = \frac{x^3 e^{-\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| 2x^2\right)}{3} + x e^{-\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| 2x^2\right) + \frac{2(2x^2-1)^{\frac{3}{4}}}{3}$$

input
```
integrate((1+x)**2/(2*x**2-1)**(1/4),x)
```

output
```
x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 2*x**2)/3 + x*exp(-I*pi/4)*hyp
er((1/4, 1/2), (3/2,), 2*x**2) + 2*(2*x**2 - 1)**(3/4)/3
```

Maxima [F]

$$\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{(x+1)^2}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^2/(2*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x + 1)^2/(2*x^2 - 1)^(1/4), x)
```

Giac [F]

$$\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{(x+1)^2}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)^2/(2*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate((x + 1)^2/(2*x^2 - 1)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{(x+1)^2}{(2\,x^2-1)^{1/4}}\,dx$$

input
```
int((x + 1)^2/(2*x^2 - 1)^(1/4),x)
```

output
```
int((x + 1)^2/(2*x^2 - 1)^(1/4), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(1+x)^2}{\sqrt[4]{-1+2x^2}}\,dx &= \frac{2\sqrt{2x^2-1}\,\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,\sqrt{2}\,x}{3} \\ &\quad - \frac{4\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,x^2}{3} \\ &\quad + \frac{2\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}}{3} \\ &\quad + \int \frac{x^2}{(2x^2-1)^{\frac{1}{4}}}dx + \int \frac{1}{(2x^2-1)^{\frac{1}{4}}}dx\end{aligned}$$

input
```
int((1+x)^2/(2*x^2-1)^(1/4),x)
```

output
```
(2*sqrt(2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x + 2*x**2 - 1)*sqrt(sqr
t(2*x**2 - 1) + sqrt(2)*x)*sqrt(2)*x - 4*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x +
 2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1) + sqrt(2)*x)*x**2 + 2*sqrt(sqrt(2*x**2
- 1)*sqrt(2)*x + 2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1) + sqrt(2)*x) + 3*int(x*
*2/(2*x**2 - 1)**(1/4),x) + 3*int(1/(2*x**2 - 1)**(1/4),x))/3
```

3.376 $\int \frac{1+x}{\sqrt[4]{-1+2x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 172

$$\begin{aligned}\int \frac{1+x}{\sqrt[4]{-1+2x^2}} \, dx &= \frac{1}{3}\left(-1+2x^2\right)^{3/4} + \frac{2x\sqrt[4]{-1+2x^2}}{1+\sqrt{-1+2x^2}} \\ &- \frac{\sqrt{2}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right) E\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right)|\frac{1}{2}\right)}{x} \\ &+ \frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right) \text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right),\frac{1}{2}\right)}{\sqrt{2}x}\end{aligned}$$

output
```
1/3*(2*x^2-1)^(3/4)+2*x*(2*x^2-1)^(1/4)/(1+(2*x^2-1)^(1/2))-2^(1/2)*(x^2/(
1+(2*x^2-1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(1/2))*EllipticE(sin(2*arctan((2*
x^2-1)^(1/4))),1/2*2^(1/2))/x+1/2*2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2)^(1/2
)*(1+(2*x^2-1)^(1/2))*InverseJacobiAM(2*arctan((2*x^2-1)^(1/4)),1/2*2^(1/2
))/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.30

$$\int \frac{1+x}{\sqrt[4]{-1+2x^2}}\,dx = \frac{-1+2x^2+3x\sqrt[4]{1-2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)}{3\sqrt[4]{-1+2x^2}}$$

input
```
Integrate[(1 + x)/(-1 + 2*x^2)^(1/4),x]
```

output
```
(-1 + 2*x^2 + 3*x*(1 - 2*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, 2*x^2
])/(3*(-1 + 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {455, 228, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt[4]{2x^2-1}}dx$$

$$\Big\downarrow \text{455}$$

$$\int \frac{1}{\sqrt[4]{2x^2-1}}dx+\frac{1}{3}\left(2x^2-1\right)^{3/4}$$

$$\Big\downarrow \text{228}$$

$$\frac{\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{x}+\frac{1}{3}\left(2x^2-1\right)^{3/4}$$

$$\Big\downarrow \text{834}$$

$$\frac{\sqrt{2}\sqrt{x^2}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+\frac{1}{3}\left(2x^2-1\right)^{3/4}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\int\frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+\frac{1}{3}\left(2x^2-1\right)^{3/4}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{2x^2-\right.\right.}{\sqrt{x^2}}\right.}{x}$$

$$\frac{1}{3}\left(2x^2-1\right)^{3/4}$$

input
```
Int[(1 + x)/(-1 + 2*x^2)^(1/4),x]
```

output
```
(-1 + 2*x^2)^(3/4)/3 + (Sqrt[2]*Sqrt[x^2]*((Sqrt[2]*Sqrt[x^2]*(-1 + 2*x^2)
^(1/4))/(1 + Sqrt[-1 + 2*x^2]) - (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 +
Sqrt[-1 + 2*x^2])*EllipticE[2*ArcTan[(-1 + 2*x^2)^(1/4)], 1/2])/Sqrt[x^2]
+ (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*EllipticF[2*A
rcTan[(-1 + 2*x^2)^(1/4)], 1/2])/(2*Sqrt[x^2])))/x
```

Defintions of rubi rules used

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.28

method	result	size
risch	$\frac{(2x^2-1)^{\frac{3}{4}}}{3}+\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}} x \operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],2x^2)}{\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}$	48
meijerg	$\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}} x \operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],2x^2)}{\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}+\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}} x^2 \operatorname{hypergeom}([\frac{1}{4},1],[2],2x^2)}{2 \operatorname{signum}(2x^2-1)^{\frac{1}{4}}}$	75

input
```
int((x+1)/(2*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*x^2-1)^(3/4)+1/signum(2*x^2-1)^(1/4)*(-signum(2*x^2-1))^(1/4)*x*hyp
ergeom([1/4,1/2],[3/2],2*x^2)
```

Fricas [F]

$$\int \frac{1+x}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{x+1}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)/(2*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((x + 1)/(2*x^2 - 1)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.18

$$\int \frac{1+x}{\sqrt[4]{-1+2x^2}}\,dx = xe^{-\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|2x^2\right)+\frac{(2x^2-1)^{\frac{3}{4}}}{3}$$

input
```
integrate((1+x)/(2*x**2-1)**(1/4),x)
```

output
```
x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 2*x**2) + (2*x**2 - 1)**(3/4)/3
```

Maxima [F]

$$\int \frac{1+x}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{x+1}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)/(2*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(2*x^2 - 1)^(1/4), x)
```

Giac [F]

$$\int \frac{1+x}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{x+1}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((1+x)/(2*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate((x + 1)/(2*x^2 - 1)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.24

$$\int \frac{1+x}{\sqrt[4]{-1+2x^2}}\,dx = \frac{(2\,x^2-1)^{3/4}}{3} + \frac{x\,(1-2\,x^2)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ 2\,x^2\right)}{(2\,x^2-1)^{1/4}}$$

input
```
int((x + 1)/(2*x^2 - 1)^(1/4),x)
```

output
```
(2*x^2 - 1)^(3/4)/3 + (x*(1 - 2*x^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, 2*x^
2))/(2*x^2 - 1)^(1/4)
```

Reduce [F]

$$\begin{aligned}\int \frac{1+x}{\sqrt[4]{-1+2x^2}}\,dx &= \frac{\sqrt{2x^2-1}\,\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,\sqrt{2}\,x}{3} \\ &\quad - \frac{2\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}\,x^2}{3} \\ &\quad + \frac{\sqrt{\sqrt{2x^2-1}\,\sqrt{2}\,x+2x^2-1}\,\sqrt{\sqrt{2x^2-1}+\sqrt{2}\,x}}{3} \\ &\quad + \int \frac{1}{(2x^2-1)^{\frac{1}{4}}}dx\end{aligned}$$

input

```
int((1+x)/(2*x^2-1)^(1/4),x)
```

output

```
(sqrt(2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x + 2*x**2 - 1)*sqrt(sqrt(
2*x**2 - 1) + sqrt(2)*x)*sqrt(2)*x - 2*sqrt(sqrt(2*x**2 - 1)*sqrt(2)*x + 2
*x**2 - 1)*sqrt(sqrt(2*x**2 - 1) + sqrt(2)*x)*x**2 + sqrt(sqrt(2*x**2 - 1)
*sqrt(2)*x + 2*x**2 - 1)*sqrt(sqrt(2*x**2 - 1) + sqrt(2)*x) + 3*int(1/(2*x
**2 - 1)**(1/4),x))/3
```

3.377 $\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 157

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = \frac{2x\sqrt[4]{-1+2x^2}}{1+\sqrt{-1+2x^2}} - \frac{\sqrt{2}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right)|\frac{1}{2}\right)}{x} + \frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right),\frac{1}{2}\right)}{\sqrt{2}x}$$

output
```
2*x*(2*x^2-1)^(1/4)/(1+(2*x^2-1)^(1/2))-2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2
)^(1/2)*(1+(2*x^2-1)^(1/2))*EllipticE(sin(2*arctan((2*x^2-1)^(1/4))),1/2*2
^(1/2))/x+1/2*2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(1/2)
)*InverseJacobiAM(2*arctan((2*x^2-1)^(1/4)),1/2*2^(1/2))/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = \frac{x\sqrt[4]{1-2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},2x^2\right)}{\sqrt[4]{-1+2x^2}}$$

input
```
Integrate[(-1 + 2*x^2)^(-1/4),x]
```

output
```
(x*(1 - 2*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, 2*x^2])/(-1 + 2*x^2)
^(1/4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {228, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2x^2-1}}dx$$

$$\Big\downarrow \text{228}$$

$$\frac{\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{x}$$

$$\Big\downarrow \text{834}$$

$$\frac{\sqrt{2}\sqrt{x^2}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\int\frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{2x^2-\right.\right.}{\sqrt{x^2}}}{x}$$

input
```
Int[(-1 + 2*x^2)^(-1/4),x]
```

output
```
(Sqrt[2]*Sqrt[x^2]*((Sqrt[2]*Sqrt[x^2]*(-1 + 2*x^2)^(1/4))/(1 + Sqrt[-1 +
2*x^2]) - (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*Ellip
ticE[2*ArcTan[(-1 + 2*x^2)^(1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(1 + Sqrt[-
1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*EllipticF[2*ArcTan[(-1 + 2*x^2)^(1/4
)], 1/2])/(2*Sqrt[x^2])))/x
```

Defintions of rubi rules used

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

method	result	size
meijerg	$\frac{(-\operatorname{signum}(2x^2-1))^{\frac{1}{4}} x \operatorname{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],2x^2)}{\operatorname{signum}(2x^2-1)^{\frac{1}{4}}}$	36

input
```
int(1/(2*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/signum(2*x^2-1)^(1/4)*(-signum(2*x^2-1))^(1/4)*x*hypergeom([1/4,1/2],[3/
2],2*x^2)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((2*x^2 - 1)^(-1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = xe^{-\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|2x^2\right)$$

input
```
integrate(1/(2*x**2-1)**(1/4),x)
```

output
```
x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 2*x**2)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate((2*x^2 - 1)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate((2*x^2 - 1)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = \frac{x\,(1-2\,x^2)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ 2\,x^2\right)}{(2\,x^2-1)^{1/4}}$$

input
```
int(1/(2*x^2 - 1)^(1/4),x)
```

output
```
(x*(1 - 2*x^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, 2*x^2))/(2*x^2 - 1)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2x^2-1)^{\frac{1}{4}}}dx$$

input
```
int(1/(2*x^2-1)^(1/4),x)
```

output
```
int(1/(2*x**2 - 1)**(1/4),x)
```

3.378 $\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt[4]{-1+2x^2}}{1+x-\sqrt{-1+2x^2}}\right) - \frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{-1+2x^2}}{1+x+\sqrt{-1+2x^2}}\right)$$

output
```
1/2*arctan((2*x^2-1)^(1/4)/(1+x-(2*x^2-1)^(1/2)))-1/2*arctanh((2*x^2-1)^(1
/4)/(1+x+(2*x^2-1)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 12.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{-\frac{\sqrt{2}-2x}{1+x}}\sqrt[4]{\frac{\sqrt{2}+2x}{1+x}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2-\sqrt{2}}{2+2x},\frac{2+\sqrt{2}}{2+2x}\right)}{\sqrt[4]{-1+2x^2}}$$

input
```
Integrate[1/((1 + x)*(-1 + 2*x^2)^(1/4)),x]
```

output
```
-((Sqrt[2]*(-((Sqrt[2] - 2*x)/(1 + x)))^(1/4)*((Sqrt[2] + 2*x)/(1 + x))^(1
/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (2 - Sqrt[2])/(2 + 2*x), (2 + Sqrt[2])/(2
 + 2*x)])/(-1 + 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{499\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx$$

$$\Big\downarrow \text{499}$$

$$\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)$$

input
```
Int[1/((1 + x)*(-1 + 2*x^2)^(1/4)),x]
```

output
```
ArcTan[(-1 + 2*x^2)^(1/4)/(1 + x - Sqrt[-1 + 2*x^2])]/2 - ArcTanh[(-1 + 2*
x^2)^(1/4)/(1 + x + Sqrt[-1 + 2*x^2])]/2
```

Defintions of rubi rules used

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.10

method	result
trager	$-\frac{\ln\left(\frac{\left(2x^2-1\right)^{\frac{3}{4}}-x\sqrt{2x^2-1}-2\left(2x^2-1\right)^{\frac{1}{4}}x-\left(2x^2-1\right)^{\frac{1}{4}}-3x-2}{(x+1)^2}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\sqrt{2x^2-1}\operatorname{RootOf}\left(\_Z^2+1\right)x-\left(2x^2-1\right)}{}\right.}{}$

input
```
int(1/(x+1)/(2*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(((2*x^2-1)^(3/4)-x*(2*x^2-1)^(1/2)-2*(2*x^2-1)^(1/4)*x-(2*x^2-1)^(
1/4)-3*x-2)/(x+1)^2)+1/2*RootOf(_Z^2+1)*ln(-((2*x^2-1)^(1/2)*RootOf(_Z^2+1
)*x-(2*x^2-1)^(3/4)-3*RootOf(_Z^2+1)*x-2*(2*x^2-1)^(1/4)*x-2*RootOf(_Z^2+1
)-(2*x^2-1)^(1/4))/(x+1)^2)
```

Fricas [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.35

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = -\frac{1}{2}\arctan\left(\frac{(2x^2-1)^{\frac{1}{4}}(x+1)+(2x^2-1)^{\frac{3}{4}}}{x^2-2x-2}\right) + \frac{1}{2}\log\left(\frac{\sqrt{2x^2-1}x-(2x^2-1)^{\frac{1}{4}}(2x+1)+3x+(2x^2-1)^{\frac{3}{4}}+2}{x^2+2x+1}\right)$$

input
```
integrate(1/(1+x)/(2*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
-1/2*arctan(((2*x^2 - 1)^(1/4)*(x + 1) + (2*x^2 - 1)^(3/4))/(x^2 - 2*x - 2
)) + 1/2*log((sqrt(2*x^2 - 1)*x - (2*x^2 - 1)^(1/4)*(2*x + 1) + 3*x + (2*x
^2 - 1)^(3/4) + 2)/(x^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}\,dx$$

input
```
integrate(1/(1+x)/(2*x**2-1)**(1/4),x)
```

output
```
Integral(1/((x + 1)*(2*x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2x^2-1)^{\frac{1}{4}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(2*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 - 1)^(1/4)*(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(2*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((2*x^2 - 1)^(1/4)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{1/4}\,(x+1)}\,dx$$

input
```
int(1/((2*x^2 - 1)^(1/4)*(x + 1)),x)
```

output
```
int(1/((2*x^2 - 1)^(1/4)*(x + 1)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2x^2-1)^{\frac{1}{4}}\,x+(2x^2-1)^{\frac{1}{4}}}dx$$

input
```
int(1/(1+x)/(2*x^2-1)^(1/4),x)
```

output
```
int(1/((2*x**2 - 1)**(1/4)*x + (2*x**2 - 1)**(1/4)),x)
```

3.379 $\int \frac{1}{(1+x)^2 \sqrt[4]{-1+2x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 243

$$\begin{aligned}\int \frac{1}{(1+x)^2 \sqrt[4]{-1+2x^2}} \, dx &= -\frac{(-1+2x^2)^{3/4}}{1+x} + \frac{2x\sqrt[4]{-1+2x^2}}{1+\sqrt{-1+2x^2}} \\ &+ \frac{1}{2}\arctan\left(\frac{\sqrt[4]{-1+2x^2}}{1+x-\sqrt{-1+2x^2}}\right) - \frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{-1+2x^2}}{1+x+\sqrt{-1+2x^2}}\right) \\ &- \frac{\sqrt{2}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right) E\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right)|\frac{1}{2}\right)}{x} \\ &+ \frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right),\frac{1}{2}\right)}{\sqrt{2}x}\end{aligned}$$

output
```
-(2*x^2-1)^(3/4)/(1+x)+2*x*(2*x^2-1)^(1/4)/(1+(2*x^2-1)^(1/2))+1/2*arctan(
(2*x^2-1)^(1/4)/(1+x-(2*x^2-1)^(1/2)))-1/2*arctanh((2*x^2-1)^(1/4)/(1+x+(2
*x^2-1)^(1/2)))-2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(1/
2))*EllipticE(sin(2*arctan((2*x^2-1)^(1/4))),1/2*2^(1/2))/x+1/2*2^(1/2)*(x
^2/(1+(2*x^2-1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(1/2))*InverseJacobiAM(2*arct
an((2*x^2-1)^(1/4)),1/2*2^(1/2))/x
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1+x)^2\sqrt[4]{-1+2x^2}}\,dx = \frac{3-6x^2-3\sqrt{2}\sqrt[4]{-\frac{\sqrt{2}-2x}{1+x}}(1+x)\sqrt[4]{\frac{\sqrt{2}+2x}{1+x}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2-\sqrt{2}}{2+2x},\frac{2+\sqrt{2}}{2+2x}\right)+2\sqrt[8]{2}(1+x)\sqrt[4]{\sqrt{2}+}}{3(1+x)\sqrt[4]{-1+2x^2}}$$

input
```
Integrate[1/((1 + x)^2*(-1 + 2*x^2)^(1/4)),x]
```

output
```
(3 - 6*x^2 - 3*Sqrt[2]*(-((Sqrt[2] - 2*x)/(1 + x)))^(1/4)*(1 + x)*((Sqrt[2
] + 2*x)/(1 + x))^(1/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (2 - Sqrt[2])/(2 + 2*
x), (2 + Sqrt[2])/(2 + 2*x)] + 2*2^(1/8)*(1 + x)*(Sqrt[2] + 2*x)^(1/4)*(-1
 + Sqrt[2]*x)*Hypergeometric2F1[1/4, 3/4, 7/4, 1/2 - x/Sqrt[2]])/(3*(1 + x
)*(-1 + 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {498, 27, 719, 228, 499, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^2\sqrt[4]{2x^2-1}}\,dx$$

$$\Big\downarrow \text{498}$$

$$-2\int -\frac{x+2}{2(x+1)\sqrt[4]{2x^2-1}}dx-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+2}{(x+1)\sqrt[4]{2x^2-1}}dx-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

$\Big\downarrow$ 719

$$\int \frac{1}{\sqrt[4]{2x^2-1}}dx+\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

$\Big\downarrow$ 228

$$\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx+\frac{\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{x}-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

$\Big\downarrow$ 499

$$\frac{\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{x}+\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

$\Big\downarrow$ 834

$$\frac{\sqrt{2}\sqrt{x^2}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

$\Big\downarrow$ 761

$$\frac{\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

$\Big\downarrow$ 1510

$$\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)+$$

$$\frac{\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{2x^2-\cdots\right.\right.}{\sqrt{x^2}}\cdots\right.}{x}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{x+1}$$

input
```
Int[1/((1 + x)^2*(-1 + 2*x^2)^(1/4)),x]
```

output
```
-((-1 + 2*x^2)^(3/4)/(1 + x)) + ArcTan[(-1 + 2*x^2)^(1/4)/(1 + x - Sqrt[-1
 + 2*x^2])]/2 - ArcTanh[(-1 + 2*x^2)^(1/4)/(1 + x + Sqrt[-1 + 2*x^2])]/2 +
 (Sqrt[2]*Sqrt[x^2]*((Sqrt[2]*Sqrt[x^2]*(-1 + 2*x^2)^(1/4))/(1 + Sqrt[-1 +
 2*x^2]) - (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*Elli
pticE[2*ArcTan[(-1 + 2*x^2)^(1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(1 + Sqrt[
-1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*EllipticF[2*ArcTan[(-1 + 2*x^2)^(1/
4)], 1/2])/(2*Sqrt[x^2])))/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(x+1)^2 (2x^2-1)^{\frac{1}{4}}} dx$$

input
```
int(1/(x+1)^2/(2*x^2-1)^(1/4),x)
```

output
```
int(1/(x+1)^2/(2*x^2-1)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{\left(2\,x^2-1\right)^{\frac{1}{4}}(x+1)^2}\,dx$$

input
```
integrate(1/(1+x)^2/(2*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((2*x^2 - 1)^(3/4)/(2*x^4 + 4*x^3 + x^2 - 2*x - 1), x)
```

Sympy [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(x+1)^2\,\sqrt[4]{2x^2-1}}\,dx$$

input
```
integrate(1/(1+x)**2/(2*x**2-1)**(1/4),x)
```

output
```
Integral(1/((x + 1)**2*(2*x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{\left(2\,x^2-1\right)^{\frac{1}{4}}(x+1)^2}\,dx$$

input
```
integrate(1/(1+x)^2/(2*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 - 1)^(1/4)*(x + 1)^2), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}(x+1)^2}\,dx$$

input
```
integrate(1/(1+x)^2/(2*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((2*x^2 - 1)^(1/4)*(x + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^2\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{1/4}\,(x+1)^2}\,dx$$

input
```
int(1/((2*x^2 - 1)^(1/4)*(x + 1)^2),x)
```

output
```
int(1/((2*x^2 - 1)^(1/4)*(x + 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)^2\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2x^2-1)^{\frac{1}{4}}\,x^2+2\,(2x^2-1)^{\frac{1}{4}}\,x+(2x^2-1)^{\frac{1}{4}}}dx$$

input
```
int(1/(1+x)^2/(2*x^2-1)^(1/4),x)
```

output
```
int(1/((2*x**2 - 1)**(1/4)*x**2 + 2*(2*x**2 - 1)**(1/4)*x + (2*x**2 - 1)**
(1/4)),x)
```

3.380 $\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 262

$$\begin{aligned}\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx &= -\frac{(-1+2x^2)^{3/4}}{2(1+x)^2}-\frac{5(-1+2x^2)^{3/4}}{2(1+x)}+\frac{5x\sqrt[4]{-1+2x^2}}{1+\sqrt{-1+2x^2}}\\&\quad+\arctan\left(\frac{\sqrt[4]{-1+2x^2}}{1+x-\sqrt{-1+2x^2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt[4]{-1+2x^2}}{1+x+\sqrt{-1+2x^2}}\right)\\&\quad-\frac{5\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right)|\frac{1}{2}\right)}{\sqrt{2}x}\\&\quad+\frac{5\sqrt{\frac{x^2}{\left(1+\sqrt{-1+2x^2}\right)^2}}\left(1+\sqrt{-1+2x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+2x^2}\right),\frac{1}{2}\right)}{2\sqrt{2}x}\end{aligned}$$

output
```
-1/2*(2*x^2-1)^(3/4)/(1+x)^2-5*(2*x^2-1)^(3/4)/(2+2*x)+5*x*(2*x^2-1)^(1/4)
/(1+(2*x^2-1)^(1/2))+arctan((2*x^2-1)^(1/4)/(1+x-(2*x^2-1)^(1/2)))-arctanh
((2*x^2-1)^(1/4)/(1+x+(2*x^2-1)^(1/2)))-5/2*2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2
))^2)^(1/2)*(1+(2*x^2-1)^(1/2))*EllipticE(sin(2*arctan((2*x^2-1)^(1/4))),1
/2*2^(1/2))/x+5/4*2^(1/2)*(x^2/(1+(2*x^2-1)^(1/2))^2)^(1/2)*(1+(2*x^2-1)^(
1/2))*InverseJacobiAM(2*arctan((2*x^2-1)^(1/4)),1/2*2^(1/2))/x
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 19.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.83

$$\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx = \frac{\frac{15-30x^2}{1+x}+\frac{3-6x^2}{(1+x)^2}-12\sqrt{2}\sqrt[4]{-\frac{\sqrt{2}-2x}{1+x}}\sqrt[4]{\frac{\sqrt{2}+2x}{1+x}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2-\sqrt{2}}{2+2x},\frac{2+\sqrt{2}}{2+2x}\right)-10\sqrt[8]{2}\sqrt[4]{\sqrt{2}+2x}\,\text{I}}{6\sqrt[4]{-1+}}$$

input
```
Integrate[1/((1 + x)^3*(-1 + 2*x^2)^(1/4)),x]
```

output
```
((15 - 30*x^2)/(1 + x) + (3 - 6*x^2)/(1 + x)^2 - 12*Sqrt[2]*(-((Sqrt[2] -
2*x)/(1 + x)))^(1/4)*((Sqrt[2] + 2*x)/(1 + x))^(1/4)*AppellF1[1/2, 1/4, 1/
4, 3/2, (2 - Sqrt[2])/(2 + 2*x), (2 + Sqrt[2])/(2 + 2*x)] - 10*2^(1/8)*(Sq
rt[2] + 2*x)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, 1/2 - x/Sqrt[2]] + 10*
2^(5/8)*x*(Sqrt[2] + 2*x)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, 1/2 - x/S
qrt[2]])/(6*(-1 + 2*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {498, 27, 688, 25, 719, 228, 499, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^3\sqrt[4]{2x^2-1}}\,dx$$

$$\Big\downarrow \text{ 498}$$

$$-\int -\frac{4-x}{2(x+1)^2\sqrt[4]{2x^2-1}}dx-\frac{(2x^2-1)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \frac{4-x}{(x+1)^2\sqrt[4]{2x^2-1}}dx-\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 688$$

$$\frac{1}{2}\left(-\int -\frac{5x+9}{(x+1)\sqrt[4]{2x^2-1}}dx-\frac{5\left(2x^2-1\right)^{3/4}}{x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\int \frac{5x+9}{(x+1)\sqrt[4]{2x^2-1}}dx-\frac{5\left(2x^2-1\right)^{3/4}}{x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 719$$

$$\frac{1}{2}\left(5\int \frac{1}{\sqrt[4]{2x^2-1}}dx+4\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx-\frac{5\left(2x^2-1\right)^{3/4}}{x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 228$$

$$\frac{1}{2}\left(4\int \frac{1}{(x+1)\sqrt[4]{2x^2-1}}dx+\frac{5\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{x}-\frac{5\left(2x^2-1\right)^{3/4}}{x+1}\right)-\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 499$$

$$\frac{1}{2}\left(\frac{5\sqrt{2}\sqrt{x^2}\int \frac{\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}}{x}+4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)\right)-\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 834$$

$$\frac{1}{2}\left(\frac{5\sqrt{2}\sqrt{x^2}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}-\int \frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\right.\right.$$
$$\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$$\Big\downarrow \ 761$$

$$\frac{1}{2}\left(\frac{5\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{x^2}}-\int\frac{1-\sqrt{2x^2-1}}{\sqrt{2}\sqrt{x^2}}d\sqrt[4]{2x^2-1}\right)}{x}+4\left(\frac{1}{2}\text{arcta}\right.\right.$$

$$\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

$\downarrow$ 1510

$$\frac{1}{2}\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{2x^2-1}}{-\sqrt{2x^2-1}+x+1}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{2x^2-1}}{\sqrt{2x^2-1}+x+1}\right)\right)+\frac{5\sqrt{2}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{2x^2-1}+1\right)^2}}\left(\sqrt{2x^2-1}+\right.}{}\right.}{}\right.$$

$$\frac{\left(2x^2-1\right)^{3/4}}{2(x+1)^2}$$

input
```
Int[1/((1 + x)^3*(-1 + 2*x^2)^(1/4)),x]
```

output
```
-1/2*(-1 + 2*x^2)^(3/4)/(1 + x)^2 + ((-5*(-1 + 2*x^2)^(3/4))/(1 + x) + 4*(
ArcTan[(-1 + 2*x^2)^(1/4)/(1 + x - Sqrt[-1 + 2*x^2])]/2 - ArcTanh[(-1 + 2*
x^2)^(1/4)/(1 + x + Sqrt[-1 + 2*x^2])]/2) + (5*Sqrt[2]*Sqrt[x^2]*((Sqrt[2]
*Sqrt[x^2]*(-1 + 2*x^2)^(1/4))/(1 + Sqrt[-1 + 2*x^2]) - (Sqrt[x^2/(1 + Sqr
t[-1 + 2*x^2])^2]*(1 + Sqrt[-1 + 2*x^2])*EllipticE[2*ArcTan[(-1 + 2*x^2)^(
1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(1 + Sqrt[-1 + 2*x^2])^2]*(1 + Sqrt[-1
+ 2*x^2])*EllipticF[2*ArcTan[(-1 + 2*x^2)^(1/4)], 1/2])/(2*Sqrt[x^2])))/x)
/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(x+1)^3 (2x^2-1)^{\frac{1}{4}}} dx$$

input
```
int(1/(x+1)^3/(2*x^2-1)^(1/4),x)
```

output
```
int(1/(x+1)^3/(2*x^2-1)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}(x+1)^3}\,dx$$

input
```
integrate(1/(1+x)^3/(2*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((2*x^2 - 1)^(3/4)/(2*x^5 + 6*x^4 + 5*x^3 - x^2 - 3*x - 1), x)
```

Sympy [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(x+1)^3\sqrt[4]{2x^2-1}}\,dx$$

input
```
integrate(1/(1+x)**3/(2*x**2-1)**(1/4),x)
```

output
```
Integral(1/((x + 1)**3*(2*x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}(x+1)^3}\,dx$$

input
```
integrate(1/(1+x)^3/(2*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 - 1)^(1/4)*(x + 1)^3), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{\frac{1}{4}}(x+1)^3}\,dx$$

input
```
integrate(1/(1+x)^3/(2*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((2*x^2 - 1)^(1/4)*(x + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2\,x^2-1)^{1/4}\,(x+1)^3}\,dx$$

input
```
int(1/((2*x^2 - 1)^(1/4)*(x + 1)^3),x)
```

output
```
int(1/((2*x^2 - 1)^(1/4)*(x + 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)^3\sqrt[4]{-1+2x^2}}\,dx = \int \frac{1}{(2x^2-1)^{\frac{1}{4}}\,x^3+3\,(2x^2-1)^{\frac{1}{4}}\,x^2+3\,(2x^2-1)^{\frac{1}{4}}\,x+(2x^2-1)^{\frac{1}{4}}}dx$$

input
```
int(1/(1+x)^3/(2*x^2-1)^(1/4),x)
```

output
```
int(1/((2*x**2 - 1)**(1/4)*x**3 + 3*(2*x**2 - 1)**(1/4)*x**2 + 3*(2*x**2 -
 1)**(1/4)*x + (2*x**2 - 1)**(1/4)),x)
```

3.381 $\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 401

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\frac{(c+dx)^2\left(-ac^2+2ad^2x^2\right)^{3/4}}{7ad}+\frac{c(100c+33dx)\left(-ac^2+2ad^2x^2\right)^{3/4}}{105ad}+\frac{16c^3x\sqrt[4]{-ac^2+2ad^2x^2}}{5\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)}-\frac{8\sqrt{2}c^{7/2}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}d^2x}+\frac{4\sqrt{2}c^{7/2}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{3/4}d^2x}$$

output
```
1/7*(d*x+c)^2*(2*a*d^2*x^2-a*c^2)^(3/4)/a/d+1/105*c*(33*d*x+100*c)*(2*a*d^
2*x^2-a*c^2)^(3/4)/a/d+16*c^3*x*(2*a*d^2*x^2-a*c^2)^(1/4)/(5*a^(1/2)*c+5*(
2*a*d^2*x^2-a*c^2)^(1/2))-8/5*2^(1/2)*c^(7/2)*(a*d^2*x^2/(a^(1/2)*c+(2*a*d
^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*Ellipt
icE(sin(2*arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/
a^(3/4)/d^2/x+4/5*2^(1/2)*c^(7/2)*(a*d^2*x^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2
)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*InverseJacobiAM(2*
arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(3/4)/d^2
/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.28

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{-115c^4-63c^3dx+215c^2d^2x^2+126cd^3x^3+30d^4x^4+168c^3dx\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2}\right.}{105d\sqrt[4]{-a\,(c^2-2d^2x^2)}}$$

input
```
Integrate[(c + d*x)^3/(-(a*c^2) + 2*a*d^2*x^2)^(1/4),x]
```

output
```
(-115*c^4 - 63*c^3*d*x + 215*c^2*d^2*x^2 + 126*c*d^3*x^3 + 30*d^4*x^4 + 16
8*c^3*d*x*(1 - (2*d^2*x^2)/c^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (2*
d^2*x^2)/c^2])/(105*d*(-(a*(c^2 - 2*d^2*x^2)))^(1/4))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {497, 27, 676, 228, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\sqrt[4]{2ad^2x^2-ac^2}}dx$$

$$\Big\downarrow \text{ 497}$$

$$\frac{\int \frac{acd^2(c+dx)(9c+11dx)}{\sqrt[4]{2ad^2x^2-ac^2}}dx}{7ad^2}+\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}c\int \frac{(c+dx)(9c+11dx)}{\sqrt[4]{2ad^2x^2-ac^2}}dx+\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{1}{7}c\left(\frac{56}{5}c^2\int \frac{1}{\sqrt[4]{2ad^2x^2-ac^2}}dx+\frac{20c\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}+\frac{11x\left(2ad^2x^2-ac^2\right)^{3/4}}{5a}\right)+\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow \text{ 228}$$

$$\frac{1}{7}c\left(\frac{56\sqrt{2}c^2\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{5ad^2x}+\frac{20c\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}+\frac{11x\left(2ad^2x^2-ac^2\right)^{3/4}}{5a}\right)+\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{1}{7}c\left(\frac{56\sqrt{2}c^2\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{a}c\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{5ad^2x}+\frac{20c(\ldots\right.$$

$$\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}c\left(\frac{56\sqrt{2}c^2\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int\frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\int\frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{5ad^2x}+\frac{20c(2ad^2x\cdots}{3\cdots}\right.$$

$$\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

$\Big\downarrow$ 761

$$\frac{1}{7}c\left(\frac{56\sqrt{2}c^2\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\int\frac{\sqrt{a}c\cdots}{\sqrt{\cdots}}\cdots\right.}{5ad^2x}\cdots\right.$$

$$\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

$\Big\downarrow$ 1510

$$\frac{1}{7}c\left(\frac{56\sqrt{2}c^2\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right.}{5\cdots}\cdots\right.$$

$$\frac{(c+dx)^2\left(2ad^2x^2-ac^2\right)^{3/4}}{7ad}$$

input
```
Int[(c + d*x)^3/(-(a*c^2) + 2*a*d^2*x^2)^(1/4),x]
```

output
```
((c + d*x)^2*(-(a*c^2) + 2*a*d^2*x^2)^(3/4))/(7*a*d) + (c*((20*c*(-(a*c^2)
 + 2*a*d^2*x^2)^(3/4))/(3*a*d) + (11*x*(-(a*c^2) + 2*a*d^2*x^2)^(3/4))/(5*
a) + (56*Sqrt[2]*c^2*Sqrt[(d^2*x^2)/c^2]*((a*c^2*(-(a*c^2) + 2*a*d^2*x^2)^
(1/4)*Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)])/(Sqrt[a]*c + Sqrt[-(a*c^
2) + 2*a*d^2*x^2]) - (Sqrt[2]*a^(1/4)*Sqrt[c]*Sqrt[(a*d^2*x^2)/(Sqrt[a]*c
+ Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^
2])*EllipticE[2*ArcTan[(-(a*c^2) + 2*a*d^2*x^2)^(1/4)/(a^(1/4)*Sqrt[c])],
1/2])/Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)] + (a^(1/4)*Sqrt[c]*Sqrt[(
a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]*c + Sqrt
[-(a*c^2) + 2*a*d^2*x^2])*EllipticF[2*ArcTan[(-(a*c^2) + 2*a*d^2*x^2)^(1/4
)/(a^(1/4)*Sqrt[c])], 1/2])/(Sqrt[2]*Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*
c^2)])))/(5*a*d^2*x)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{(dx+c)^3}{(2a\,d^2x^2-a\,c^2)^{\frac{1}{4}}}dx$$

input
```
int((d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int((d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(dx+c)^3}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/(2*a*d^2*x^2 - a*c^2)^(
1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.03

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3/(2*a*d**2*x**2-a*c**2)**(1/4),x)
```

output
```
3*c**2*d*Piecewise((zoo*x**2, Eq(a, 0)), (x**2/(2*(-a*c**2)**(1/4)), Eq(d*
*2, 0)), ((-a*c**2 + 2*a*d**2*x**2)**(3/4)/(3*a*d**2), True)) + d**3*Piece
wise((-2*c**(15/2)*(-1 + 2*d**2*x**2/c**2)**(3/4)*exp(I*pi/4)/(-21*a**(1/4
)*c**4*d**4*exp(I*pi/4) + 42*a**(1/4)*c**2*d**6*x**2*exp(I*pi/4)) - 2*c**(
15/2)/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(1/4)*c**2*d**6*x**2*exp
(I*pi/4)) + c**(11/2)*d**2*x**2*(-1 + 2*d**2*x**2/c**2)**(3/4)*exp(I*pi/4)
/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(1/4)*c**2*d**6*x**2*exp(I*pi
/4)) + 4*c**(11/2)*d**2*x**2/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(
1/4)*c**2*d**6*x**2*exp(I*pi/4)) + 6*c**(7/2)*d**4*x**4*(-1 + 2*d**2*x**2/
c**2)**(3/4)*exp(I*pi/4)/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(1/4)
*c**2*d**6*x**2*exp(I*pi/4)), Abs(d**2*x**2/c**2) > 1/2), (2*c**(15/2)*(1
- 2*d**2*x**2/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(1/
4)*c**2*d**6*x**2*exp(I*pi/4)) - 2*c**(15/2)/(-21*a**(1/4)*c**4*d**4*exp(I
*pi/4) + 42*a**(1/4)*c**2*d**6*x**2*exp(I*pi/4)) - c**(11/2)*d**2*x**2*(1
- 2*d**2*x**2/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4) + 42*a**(1/
4)*c**2*d**6*x**2*exp(I*pi/4)) + 4*c**(11/2)*d**2*x**2/(-21*a**(1/4)*c**4*
d**4*exp(I*pi/4) + 42*a**(1/4)*c**2*d**6*x**2*exp(I*pi/4)) - 6*c**(7/2)*d*
*4*x**4*(1 - 2*d**2*x**2/c**2)**(3/4)/(-21*a**(1/4)*c**4*d**4*exp(I*pi/4)
+ 42*a**(1/4)*c**2*d**6*x**2*exp(I*pi/4)), True)) + c**(5/2)*x*exp(-I*pi/4
)*hyper((1/4, 1/2), (3/2,), 2*d**2*x**2/c**2)/a**(1/4) + sqrt(c)*d**2*x...
```

Maxima [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(dx+c)^3}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(dx+c)^3}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(c+d\,x)^3}{(2\,a\,d^2\,x^2-a\,c^2)^{1/4}}\,dx$$

input
```
int((c + d*x)^3/(2*a*d^2*x^2 - a*c^2)^(1/4),x)
```

output
```
int((c + d*x)^3/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$$

$$= \frac{23\sqrt{\sqrt{2d^2x^2-c^2}\,\sqrt{2}\,dx-c^2+2d^2x^2}\,\sqrt{\sqrt{2d^2x^2-c^2}+\sqrt{2}\,dx}\,\sqrt{2d^2x^2-c^2}\,\sqrt{2}\,c^2dx+3\sqrt{\sqrt{2d^2x^2-c^2}}}{}$$

input
```
int((d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
(23*sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2*x**2)*sqr
t(sqrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*sqrt( - c**2 + 2*d**2*x**2)*s
qrt(2)*c**2*d*x + 3*sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 +
2*d**2*x**2)*sqrt(sqrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*sqrt( - c**2
+ 2*d**2*x**2)*sqrt(2)*d**3*x**3 + 23*sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqr
t(2)*d*x - c**2 + 2*d**2*x**2)*sqrt(sqrt( - c**2 + 2*d**2*x**2) + sqrt(2)*
d*x)*c**4 - 43*sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**
2*x**2)*sqrt(sqrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*c**2*d**2*x**2 - 6
*sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2*x**2)*sqrt(s
qrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*d**4*x**4 + 63*int(x**2/( - c**2
 + 2*d**2*x**2)**(1/4),x)*c**3*d**3 + 21*int(1/( - c**2 + 2*d**2*x**2)**(1
/4),x)*c**5*d)/(21*a**(1/4)*c**2*d)
```

3.382 $\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 363

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{(10c+3dx)\left(-ac^2+2ad^2x^2\right)^{3/4}}{15ad}+\frac{12c^2x\sqrt[4]{-ac^2+2ad^2x^2}}{5\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)} - \frac{6\sqrt{2}c^{5/2}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}d^2x} + \frac{3\sqrt{2}c^{5/2}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{3/4}d^2x}$$

output
```
1/15*(3*d*x+10*c)*(2*a*d^2*x^2-a*c^2)^(3/4)/a/d+12*c^2*x*(2*a*d^2*x^2-a*c^
2)^(1/4)/(5*a^(1/2)*c+5*(2*a*d^2*x^2-a*c^2)^(1/2))-6/5*2^(1/2)*c^(5/2)*(a*
d^2*x^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2
*x^2-a*c^2)^(1/2))*EllipticE(sin(2*arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4
)/c^(1/2))),1/2*2^(1/2))/a^(3/4)/d^2/x+3/5*2^(1/2)*c^(5/2)*(a*d^2*x^2/(a^(
1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^
(1/2))*InverseJacobiAM(2*arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2))
,1/2*2^(1/2))/a^(3/4)/d^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.28

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$$
$$= \frac{-10c^3-3c^2dx+20cd^2x^2+6d^3x^3+18c^2dx\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{2d^2x^2}{c^2}\right)}{15d\sqrt[4]{-a\left(c^2-2d^2x^2\right)}}$$

input
```
Integrate[(c + d*x)^2/(-(a*c^2) + 2*a*d^2*x^2)^(1/4),x]
```

output
```
(-10*c^3 - 3*c^2*d*x + 20*c*d^2*x^2 + 6*d^3*x^3 + 18*c^2*d*x*(1 - (2*d^2*x
^2)/c^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (2*d^2*x^2)/c^2])/(15*d*(-
(a*(c^2 - 2*d^2*x^2)))^(1/4))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {497, 27, 455, 228, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt[4]{2ad^2x^2-ac^2}}\,dx$$
$$\Big\downarrow \text{497}$$
$$\frac{\int \frac{acd^2(6c+7dx)}{\sqrt[4]{2ad^2x^2-ac^2}}dx}{5ad^2}+\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{5}c\int \frac{6c+7dx}{\sqrt[4]{2ad^2x^2-ac^2}}dx+\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow 455$$

$$\frac{1}{5}c\left(6c\int \frac{1}{\sqrt[4]{2ad^2x^2-ac^2}}dx+\frac{7\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}\right)+\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow 228$$

$$\frac{1}{5}c\left(\frac{6\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{ad^2x}+\frac{7\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}\right)+$$

$$\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow 834$$

$$\frac{1}{5}c\left(\frac{6\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{a}c\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+\frac{7(2ad^2\right.$$

$$\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}c\left(\frac{6\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+\frac{7(2ad^2x^2-}{3ad}\right.$$

$$\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow 761$$

$$\frac{1}{5}c\left(\frac{6\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\int\frac{\sqrt{a}c-\sqrt{\cdots}}{\sqrt{\frac{2ad\cdots}{}}}\cdots\right)}{ad^2x}\cdots\right)$$

$$\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$

$$\Big\downarrow \; 1510$$

$$\frac{1}{5}c\left(\frac{6\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\cdots}{\cdots}\right)}{ac\cdots}\cdots\right)$$

$$\frac{(c+dx)\left(2ad^2x^2-ac^2\right)^{3/4}}{5ad}$$

input
```
Int[(c + d*x)^2/(-(a*c^2) + 2*a*d^2*x^2)^(1/4),x]
```

output
```
((c + d*x)*(-(a*c^2) + 2*a*d^2*x^2)^(3/4))/(5*a*d) + (c*((7*(-(a*c^2) + 2*
a*d^2*x^2)^(3/4))/(3*a*d) + (6*Sqrt[2]*c*Sqrt[(d^2*x^2)/c^2]*((a*c^2*(-(a*
c^2) + 2*a*d^2*x^2)^(1/4)*Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)])/(Sqr
t[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2]) - (Sqrt[2]*a^(1/4)*Sqrt[c]*Sqrt[(a*
d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]*c + Sqrt[-
(a*c^2) + 2*a*d^2*x^2])*EllipticE[2*ArcTan[(-(a*c^2) + 2*a*d^2*x^2)^(1/4)/
(a^(1/4)*Sqrt[c])], 1/2])/Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)] + (a^
(1/4)*Sqrt[c]*Sqrt[(a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^
2]*(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])*EllipticF[2*ArcTan[(-(a*c^2)
 + 2*a*d^2*x^2)^(1/4)/(a^(1/4)*Sqrt[c])], 1/2])/(Sqrt[2]*Sqrt[1 + (-(a*c^2
) + 2*a*d^2*x^2)/(a*c^2)])))/(a*d^2*x)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(2a\,d^2x^2-a\,c^2)^{\frac{1}{4}}}dx$$

input
```
int((d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int((d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(dx+c)^2}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.36

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = 2cd\left(\begin{cases}\tilde{\infty}x^2 & \text{for } a=0\\ \frac{x^2}{2\sqrt[4]{-ac^2}} & \text{for } d^2=0\\ \frac{(-ac^2+2ad^2x^2)^{\frac{3}{4}}}{3ad^2} & \text{otherwise}\end{cases}\right) + \frac{c^{\frac{3}{2}}xe^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{2d^2x^2}{c^2}\right)}{\sqrt[4]{a}} + \frac{d^2x^3e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{2d^2x^2}{c^2}\right)}{3\sqrt[4]{a}\sqrt{c}}$$

input
```
integrate((d*x+c)**2/(2*a*d**2*x**2-a*c**2)**(1/4),x)
```

output
```
2*c*d*Piecewise((zoo*x**2, Eq(a, 0)), (x**2/(2*(-a*c**2)**(1/4)), Eq(d**2,
 0)), ((-a*c**2 + 2*a*d**2*x**2)**(3/4)/(3*a*d**2), True)) + c**(3/2)*x*ex
p(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 2*d**2*x**2/c**2)/a**(1/4) + d**2*x**
3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 2*d**2*x**2/c**2)/(3*a**(1/4)*sqr
t(c))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(dx+c)^2}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(dx+c)^2}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{(c+d\,x)^2}{(2\,a\,d^2\,x^2-a\,c^2)^{1/4}}\,dx$$

input
```
int((c + d*x)^2/(2*a*d^2*x^2 - a*c^2)^(1/4),x)
```

output
```
int((c + d*x)^2/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$$

$$= \frac{2\sqrt{\sqrt{2d^2x^2-c^2}\,\sqrt{2}\,dx-c^2+2d^2x^2}\,\sqrt{\sqrt{2d^2x^2-c^2}+\sqrt{2}\,dx}\,\sqrt{2d^2x^2-c^2}\,\sqrt{2}\,dx+2\sqrt{\sqrt{2d^2x^2-c^2}\,\sqrt{2}\cdots}}{\cdots}$$

input
```
int((d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
(2*sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2*x**2)*sqrt
(sqrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*sqrt( - c**2 + 2*d**2*x**2)*sq
rt(2)*d*x + 2*sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2
*x**2)*sqrt(sqrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*c**2 - 4*sqrt(sqrt(
 - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2*x**2)*sqrt(sqrt( - c**2
 + 2*d**2*x**2) + sqrt(2)*d*x)*d**2*x**2 + 3*int(x**2/( - c**2 + 2*d**2*x*
*2)**(1/4),x)*c*d**3 + 3*int(1/( - c**2 + 2*d**2*x**2)**(1/4),x)*c**3*d)/(
3*a**(1/4)*c*d)
```

3.383 $\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 346

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{\left(-ac^2+2ad^2x^2\right)^{3/4}}{3ad}+\frac{2cx\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}}$$
$$-\frac{\sqrt{2}c^{3/2}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}d^2x}$$
$$+\frac{c^{3/2}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}a^{3/4}d^2x}$$

output
```
1/3*(2*a*d^2*x^2-a*c^2)^(3/4)/a/d+2*c*x*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)
*c+(2*a*d^2*x^2-a*c^2)^(1/2))-2^(1/2)*c^(3/2)*(a*d^2*x^2/(a^(1/2)*c+(2*a*d
^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*Ellipt
icE(sin(2*arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/
a^(3/4)/d^2/x+1/2*c^(3/2)*(a*d^2*x^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))
^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*InverseJacobiAM(2*arctan((
2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2)),1/2*2^(1/2))*2^(1/2)/a^(3/4)/d^2
/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{-c^2+2d^2x^2+3cdx\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{2d^2x^2}{c^2}\right)}{3d\sqrt[4]{-a\left(c^2-2d^2x^2\right)}}$$

input
```
Integrate[(c + d*x)/(-(a*c^2) + 2*a*d^2*x^2)^(1/4),x]
```

output
```
(-c^2 + 2*d^2*x^2 + 3*c*d*x*(1 - (2*d^2*x^2)/c^2)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, (2*d^2*x^2)/c^2])/(3*d*(-(a*(c^2 - 2*d^2*x^2)))^(1/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {455, 228, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt[4]{2ad^2x^2-ac^2}}dx$$

$$\Big\downarrow \text{455}$$

$$c\int \frac{1}{\sqrt[4]{2ad^2x^2-ac^2}}dx+\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}$$

$$\Big\downarrow \text{228}$$

$$\frac{\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{ad^2x}+\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}$$

$$\Big\downarrow \ 834$$

$$\frac{\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{a}c\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}$$

$$\Big\downarrow \ 761$$

$$\frac{\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}}}\right.}{ad^2x}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}$$

$$\Big\downarrow \ 1510$$

$$\frac{\sqrt{2}c\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\frac{}{(\sqrt{2a}}}}{}\right.}{ad^2x}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{3ad}$$

input
```
Int[(c + d*x)/(-(a*c^2) + 2*a*d^2*x^2)^(1/4),x]
```

output
```
(-(a*c^2) + 2*a*d^2*x^2)^(3/4)/(3*a*d) + (Sqrt[2]*c*Sqrt[(d^2*x^2)/c^2]*((
a*c^2*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)*Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*
c^2)])/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2]) - (Sqrt[2]*a^(1/4)*Sqrt[
c]*Sqrt[(a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]
*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])*EllipticE[2*ArcTan[(-(a*c^2) + 2*a*d^2*
x^2)^(1/4)/(a^(1/4)*Sqrt[c])], 1/2])/Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*
c^2)] + (a^(1/4)*Sqrt[c]*Sqrt[(a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a
*d^2*x^2])^2]*(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])*EllipticF[2*ArcTa
n[(-(a*c^2) + 2*a*d^2*x^2)^(1/4)/(a^(1/4)*Sqrt[c])], 1/2])/(Sqrt[2]*Sqrt[1
 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)])))/(a*d^2*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{dx+c}{\left(2a\,d^{2}x^{2}-a\,c^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int((d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int((d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{dx+c}{\left(2\,ad^{2}x^{2}-ac^{2}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((d*x + c)/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=d\left(\begin{cases}\tilde{\infty}x^2 & \text{for } a=0\\ \frac{x^2}{2\sqrt[4]{-ac^2}} & \text{for } d^2=0\\ \frac{\left(-ac^2+2ad^2x^2\right)^{\frac{3}{4}}}{3ad^2} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{c}xe^{-\frac{i\pi}{4}}{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{2d^2x^2}{c^2}\right)}{\sqrt[4]{a}}$$

input
```
integrate((d*x+c)/(2*a*d**2*x**2-a*c**2)**(1/4),x)
```

output
```
d*Piecewise((zoo*x**2, Eq(a, 0)), (x**2/(2*(-a*c**2)**(1/4)), Eq(d**2, 0))
, ((-a*c**2 + 2*a*d**2*x**2)**(3/4)/(3*a*d**2), True)) + sqrt(c)*x*exp(-I*
pi/4)*hyper((1/4, 1/2), (3/2,), 2*d**2*x**2/c**2)/a**(1/4)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{dx+c}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{dx+c}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(2*a*d^2*x^2 - a*c^2)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.23

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\frac{\left(-a\left(c^2-2\,d^2\,x^2\right)\right)^{3/4}}{3\,a\,d}+\frac{c\,x\,\left(1-\frac{2\,d^2\,x^2}{c^2}\right)^{1/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{2\,d^2\,x^2}{c^2}\right)}{\left(2\,a\,d^2\,x^2-a\,c^2\right)^{1/4}}$$

input
```
int((c + d*x)/(2*a*d^2*x^2 - a*c^2)^(1/4),x)
```

output
```
(-a*(c^2 - 2*d^2*x^2))^(3/4)/(3*a*d) + (c*x*(1 - (2*d^2*x^2)/c^2)^(1/4)*hy
pergeom([1/4, 1/2], 3/2, (2*d^2*x^2)/c^2))/(2*a*d^2*x^2 - a*c^2)^(1/4)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$$
$$=\frac{\sqrt{\sqrt{2d^2x^2-c^2}\,\sqrt{2}\,dx-c^2+2d^2x^2}\,\sqrt{\sqrt{2d^2x^2-c^2}+\sqrt{2}\,dx}\,\sqrt{2d^2x^2-c^2}\,\sqrt{2}\,dx+\sqrt{\sqrt{2d^2x^2-c^2}\,\sqrt{2}\,dx}}{}$$

input
```
int((d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
(sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2*x**2)*sqrt(s
qrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*sqrt( - c**2 + 2*d**2*x**2)*sqrt
(2)*d*x + sqrt(sqrt( - c**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2*x**
2)*sqrt(sqrt( - c**2 + 2*d**2*x**2) + sqrt(2)*d*x)*c**2 - 2*sqrt(sqrt( - c
**2 + 2*d**2*x**2)*sqrt(2)*d*x - c**2 + 2*d**2*x**2)*sqrt(sqrt( - c**2 + 2
*d**2*x**2) + sqrt(2)*d*x)*d**2*x**2 + 3*int(1/( - c**2 + 2*d**2*x**2)**(1
/4),x)*c**3*d)/(3*a**(1/4)*c**2*d)
```

3.384 $\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 315

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{2x\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}} - \frac{\sqrt{2}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}d^2x} + \frac{\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}a^{3/4}d^2x}$$

output

```
2*x*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))-2^(1/2
)*c^(1/2)*(a*d^2*x^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/
2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*EllipticE(sin(2*arctan((2*a*d^2*x^2-a*c^2)
^(1/4)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(3/4)/d^2/x+1/2*c^(1/2)*(a*d^2*x^2
/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*
c^2)^(1/2))*InverseJacobiAM(2*arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(
1/2)),1/2*2^(1/2))*2^(1/2)/a^(3/4)/d^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\frac{x\sqrt[4]{1-\frac{2d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{2d^2x^2}{c^2}\right)}{\sqrt[4]{-a(c^2-2d^2x^2)}}$$

input
```
Integrate[(-(a*c^2) + 2*a*d^2*x^2)^(-1/4),x]
```

output
```
(x*(1 - (2*d^2*x^2)/c^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (2*d^2*x^2
)/c^2])/(-(a*(c^2 - 2*d^2*x^2)))^(1/4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {228, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2ad^2x^2-ac^2}}\,dx$$

$$\Big\downarrow \text{228}$$

$$\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{ad^2x}$$

$$\Big\downarrow \text{834}$$

$$\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{a}c\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int\frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\int\frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}$$

$\downarrow$ 761

$$\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\int\frac{\sqrt{a}c-\sqrt{2ad^2x^2-}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}}}\right.}{ad^2x}$$

$\downarrow$ 1510

$$\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\frac{}{(\sqrt{2ac}}}}{}\right.}{ad^2x}$$

input
```
Int[(-(a*c^2) + 2*a*d^2*x^2)^(-1/4),x]
```

output
```
(Sqrt[2]*Sqrt[(d^2*x^2)/c^2]*((a*c^2*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)*Sqrt[1
 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)])/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2
*x^2]) - (Sqrt[2]*a^(1/4)*Sqrt[c]*Sqrt[(a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c
^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])*Elliptic
E[2*ArcTan[(-(a*c^2) + 2*a*d^2*x^2)^(1/4)/(a^(1/4)*Sqrt[c])], 1/2])/Sqrt[1
 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)] + (a^(1/4)*Sqrt[c]*Sqrt[(a*d^2*x^2)/(
Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]*c + Sqrt[-(a*c^2) +
2*a*d^2*x^2])*EllipticF[2*ArcTan[(-(a*c^2) + 2*a*d^2*x^2)^(1/4)/(a^(1/4)*S
qrt[c])], 1/2])/(Sqrt[2]*Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)])))/(a*
d^2*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(2a\,d^2x^2 - a\,c^2)^{\frac{1}{4}}}dx$$

input
```
int(1/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{\left(2\,ad^2x^2-ac^2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((2*a*d^2*x^2 - a*c^2)^(-1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{xe^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{2d^2x^2}{c^2}\right)}{\sqrt[4]{a}\sqrt{c}}$$

input
```
integrate(1/(2*a*d**2*x**2-a*c**2)**(1/4),x)
```

output
```
x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 2*d**2*x**2/c**2)/(a**(1/4)*sqrt(
c))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{\left(2\,ad^2x^2-ac^2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((2*a*d^2*x^2 - a*c^2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{1}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((2*a*d^2*x^2 - a*c^2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.16

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\frac{x\left(1-\frac{2\,d^2\,x^2}{c^2}\right)^{1/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{2\,d^2\,x^2}{c^2}\right)}{(2\,a\,d^2\,x^2-a\,c^2)^{1/4}}$$

input
```
int(1/(2*a*d^2*x^2 - a*c^2)^(1/4),x)
```

output
```
(x*(1 - (2*d^2*x^2)/c^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, (2*d^2*x^2)/c^2)
)/(2*a*d^2*x^2 - a*c^2)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\frac{\int \frac{1}{(2d^2x^2-c^2)^{\frac{1}{4}}}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/( - c**2 + 2*d**2*x**2)**(1/4),x)/a**(1/4)
```

3.385 $\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)-c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)+c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}$$

output
```
1/2*arctan(a^(1/4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)-c*
(2*a*d^2*x^2-a*c^2)^(1/2)))/a^(1/4)/c^(1/2)/d-1/2*arctanh(a^(1/4)*c^(3/2)*
(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)+c*(2*a*d^2*x^2-a*c^2)^(1/2)))
/a^(1/4)/c^(1/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{\frac{d\left(-\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\sqrt[4]{\frac{d\left(\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{2c-\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx},\frac{2c+\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx}\right)}{d\sqrt[4]{-a\left(c^2-2d^2x^2\right)}}$$

input
```
Integrate[1/((c + d*x)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-((Sqrt[2]*((d*(-(Sqrt[2]*Sqrt[c^2/d^2]) + 2*x))/(c + d*x))^(1/4)*((d*(Sqr
t[2]*Sqrt[c^2/d^2] + 2*x))/(c + d*x))^(1/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (
2*c - Sqrt[2]*Sqrt[c^2/d^2]*d)/(2*c + 2*d*x), (2*c + Sqrt[2]*Sqrt[c^2/d^2]
*d)/(2*c + 2*d*x)])/(d*(-(a*(c^2 - 2*d^2*x^2)))^(1/4)))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{499\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}\,dx$$

$$\Big\downarrow \; 499$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{c\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c(c+dx)}\right)}{2\sqrt[4]{a}\sqrt{c}d}$$

input
```
Int[1/((c + d*x)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)),x]
```

output
```
ArcTan[(a^(1/4)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*
x) - c*Sqrt[-(a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d) - ArcTanh[(a^(
1/4)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x) + c*Sqrt
[-(a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d)
```

Defintions of rubi rules used

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(2a\,d^2x^2-a\,c^2\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/(d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{\sqrt[4]{a\,(-c^2+2d^2x^2)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(2*a*d**2*x**2-a*c**2)**(1/4),x)
```

output
```
Integral(1/((a*(-c**2 + 2*d**2*x**2))**(1/4)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{(2\,a\,d^2\,x^2-a\,c^2)^{1/4}\,(c+d\,x)}\,dx$$

input
```
int(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(c + d*x)),x)
```

output
```
int(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{\int \frac{1}{(2d^2x^2-c^2)^{\frac{1}{4}}c+(2d^2x^2-c^2)^{\frac{1}{4}}dx}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(d*x+c)/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/(( - c**2 + 2*d**2*x**2)**(1/4)*c + ( - c**2 + 2*d**2*x**2)**(1/4)*d
*x),x)/a**(1/4)
```

3.386 $\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 529

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = -\frac{\left(-ac^2+2ad^2x^2\right)^{3/4}}{ac^2d(c+dx)}+\frac{2x\sqrt[4]{-ac^2+2ad^2x^2}}{c^2\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)}$$
$$+\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)-c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}c^{3/2}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)+c\sqrt{-ac^2+2ad^2x^2}}\right)}{2\sqrt[4]{a}c^{3/2}d}$$
$$-\frac{\sqrt{2}\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}d^2x}$$
$$+\frac{\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}a^{3/4}c^{3/2}d^2x}$$

output
```
-(2*a*d^2*x^2-a*c^2)^(3/4)/a/c^2/d/(d*x+c)+2*x*(2*a*d^2*x^2-a*c^2)^(1/4)/c
^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))+1/2*arctan(a^(1/4)*c^(3/2)*(2*a*d
^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)-c*(2*a*d^2*x^2-a*c^2)^(1/2)))/a^(1/
4)/c^(3/2)/d-1/2*arctanh(a^(1/4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2
)*c*(d*x+c)+c*(2*a*d^2*x^2-a*c^2)^(1/2)))/a^(1/4)/c^(3/2)/d-2^(1/2)*(a*d^2
*x^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^
2-a*c^2)^(1/2))*EllipticE(sin(2*arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c
^(1/2))),1/2*2^(1/2))/a^(3/4)/c^(3/2)/d^2/x+1/2*(a*d^2*x^2/(a^(1/2)*c+(2*a
*d^2*x^2-a*c^2)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*Inve
rseJacobiAM(2*arctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2)),1/2*2^(1/2
))*2^(1/2)/a^(3/4)/c^(3/2)/d^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.35

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{\frac{d\left(-\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\sqrt[4]{\frac{d\left(\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{4},\frac{1}{4},\frac{5}{2},\frac{2c-\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx},\frac{2c+\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx}\right)}{3d(c+dx)\sqrt[4]{-a\left(c^2-2d^2x^2\right)}}$$

input
```
Integrate[1/((c + d*x)^2*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-1/3*(Sqrt[2]*((d*(-(Sqrt[2]*Sqrt[c^2/d^2]) + 2*x))/(c + d*x))^(1/4)*((d*(
Sqrt[2]*Sqrt[c^2/d^2] + 2*x))/(c + d*x))^(1/4)*AppellF1[3/2, 1/4, 1/4, 5/2
, (2*c - Sqrt[2]*Sqrt[c^2/d^2]*d)/(2*c + 2*d*x), (2*c + Sqrt[2]*Sqrt[c^2/d
^2]*d)/(2*c + 2*d*x)])/(d*(c + d*x)*(-(a*(c^2 - 2*d^2*x^2)))^(1/4))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {498, 27, 719, 228, 499, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2 \sqrt[4]{2ad^2x^2-ac^2}} \, dx$$

$$\Big\downarrow \ 498$$

$$-\frac{2\int -\frac{2c+dx}{2(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2c+dx}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$$\Big\downarrow \ 719$$

$$\frac{\int \frac{1}{\sqrt[4]{2ad^2x^2-ac^2}}dx+c\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$$\Big\downarrow \ 228$$

$$\frac{c\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx+\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{ad^2x}}{c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$$\Big\downarrow \ 499$$

$$\frac{\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{ad^2x}+c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-a}}{c\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c(c+dx}\right.}{2\sqrt[4]{a}\sqrt{c}d}\right.}{c^2}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\Big\downarrow$ 834

$$\frac{\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int\frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{a}c\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2a}}{\sqrt{a}c(c+dx)-c\sqrt{}}\right.}{2\sqrt[4]{a}\sqrt{c}}\right.}{c^2}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int\frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\int\frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x}}\right.}{2\sqrt[4]{a}\sqrt{c}d}\right.}{c^2}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\Big\downarrow$ 761

$$\frac{\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\int\frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad}\right.}{ad^2x}}{c^2}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

$\Big\downarrow$ 1510

$$\frac{c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{c\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c(c+dx)}\right)}{2\sqrt[4]{a}\sqrt{c}d}\right)+\frac{\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}}{}\right.}{}}{}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{ac^2d(c+dx)}$$

input
```
Int[1/((c + d*x)^2*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-((-(a*c^2) + 2*a*d^2*x^2)^(3/4)/(a*c^2*d*(c + d*x))) + (c*(ArcTan[(a^(1/4
)*c^(3/2)*(-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x) - c*Sqrt[-(
a*c^2) + 2*a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d) - ArcTanh[(a^(1/4)*c^(3/2)*(
-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x) + c*Sqrt[-(a*c^2) + 2*
a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d)) + (Sqrt[2]*Sqrt[(d^2*x^2)/c^2]*((a*c^2
*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)*Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)]
)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2]) - (Sqrt[2]*a^(1/4)*Sqrt[c]*Sq
rt[(a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]*c +
Sqrt[-(a*c^2) + 2*a*d^2*x^2])*EllipticE[2*ArcTan[(-(a*c^2) + 2*a*d^2*x^2)^
(1/4)/(a^(1/4)*Sqrt[c])], 1/2])/Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)]
 + (a^(1/4)*Sqrt[c]*Sqrt[(a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*
x^2])^2]*(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])*EllipticF[2*ArcTan[(-(
a*c^2) + 2*a*d^2*x^2)^(1/4)/(a^(1/4)*Sqrt[c])], 1/2])/(Sqrt[2]*Sqrt[1 + (-
(a*c^2) + 2*a*d^2*x^2)/(a*c^2)])))/(a*d^2*x))/c^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(dx+c)^2 (2a\,d^2x^2-a\,c^2)^{\frac{1}{4}}}dx$$

input
```
int(1/(d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/(d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{\sqrt[4]{a\left(-c^2+2d^2x^2\right)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(2*a*d**2*x**2-a*c**2)**(1/4),x)
```

output
```
Integral(1/((a*(-c**2 + 2*d**2*x**2))**(1/4)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{1}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{1}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{1}{{\left(2\,a\,d^2\,x^2-a\,c^2\right)}^{1/4}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(c + d*x)^2),x)
```

output
```
int(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \frac{\int \frac{1}{(2d^2x^2-c^2)^{\frac{1}{4}}c^2+2(2d^2x^2-c^2)^{\frac{1}{4}}cdx+(2d^2x^2-c^2)^{\frac{1}{4}}d^2x^2}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(d*x+c)^2/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/(( - c**2 + 2*d**2*x**2)**(1/4)*c**2 + 2*( - c**2 + 2*d**2*x**2)**(1
/4)*c*d*x + ( - c**2 + 2*d**2*x**2)**(1/4)*d**2*x**2),x)/a**(1/4)
```

3.387 $\int \frac{1}{(c+dx)^3 \sqrt[4]{-ac^2+2ad^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 569

$$
\begin{aligned}
&\int \frac{1}{(c+dx)^3 \sqrt[4]{-ac^2+2ad^2x^2}}\,dx \\
&= -\frac{(-ac^2+2ad^2x^2)^{3/4}}{2ac^2d(c+dx)^2} - \frac{5(-ac^2+2ad^2x^2)^{3/4}}{2ac^3d(c+dx)} + \frac{5x\sqrt[4]{-ac^2+2ad^2x^2}}{c^3\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)} \\
&+ \frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)-c\sqrt{-ac^2+2ad^2x^2}}\right)}{\sqrt[4]{a}c^{5/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt{a}c(c+dx)+c\sqrt{-ac^2+2ad^2x^2}}\right)}{\sqrt[4]{a}c^{5/2}d} \\
&- \frac{5\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt{2}a^{3/4}c^{5/2}d^2x} \\
&+ \frac{5\sqrt{\frac{ad^2x^2}{\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)^2}}\left(\sqrt{a}c+\sqrt{-ac^2+2ad^2x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-ac^2+2ad^2x^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2\sqrt{2}a^{3/4}c^{5/2}d^2x}
\end{aligned}
$$

output
```
-1/2*(2*a*d^2*x^2-a*c^2)^(3/4)/a/c^2/d/(d*x+c)^2-5/2*(2*a*d^2*x^2-a*c^2)^(
3/4)/a/c^3/d/(d*x+c)+5*x*(2*a*d^2*x^2-a*c^2)^(1/4)/c^3/(a^(1/2)*c+(2*a*d^2
*x^2-a*c^2)^(1/2))+arctan(a^(1/4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/
2)*c*(d*x+c)-c*(2*a*d^2*x^2-a*c^2)^(1/2)))/a^(1/4)/c^(5/2)/d-arctanh(a^(1/
4)*c^(3/2)*(2*a*d^2*x^2-a*c^2)^(1/4)/(a^(1/2)*c*(d*x+c)+c*(2*a*d^2*x^2-a*c
^2)^(1/2)))/a^(1/4)/c^(5/2)/d-5/2*(a*d^2*x^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2
)^(1/2))^2)^(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*EllipticE(sin(2*ar
ctan((2*a*d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*2^(1/2)/a^(3
/4)/c^(5/2)/d^2/x+5/4*(a*d^2*x^2/(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))^2)^
(1/2)*(a^(1/2)*c+(2*a*d^2*x^2-a*c^2)^(1/2))*InverseJacobiAM(2*arctan((2*a*
d^2*x^2-a*c^2)^(1/4)/a^(1/4)/c^(1/2)),1/2*2^(1/2))*2^(1/2)/a^(3/4)/c^(5/2)
/d^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.33

$$\int \frac{1}{(c+dx)^3\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = -\frac{\sqrt{2}\sqrt[4]{\frac{d\left(-\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\sqrt[4]{\frac{d\left(\sqrt{2}\sqrt{\frac{c^2}{d^2}}+2x\right)}{c+dx}}\operatorname{AppellF1}\left(\frac{5}{2},\frac{1}{4},\frac{1}{4},\frac{7}{2},\frac{2c-\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx},\frac{2c+\sqrt{2}\sqrt{\frac{c^2}{d^2}}d}{2c+2dx}\right)}{5d(c+dx)^2\sqrt[4]{-a\left(c^2-2d^2x^2\right)}}$$

input
```
Integrate[1/((c + d*x)^3*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-1/5*(Sqrt[2]*((d*(-(Sqrt[2]*Sqrt[c^2/d^2]) + 2*x))/(c + d*x))^(1/4)*((d*(
Sqrt[2]*Sqrt[c^2/d^2] + 2*x))/(c + d*x))^(1/4)*AppellF1[5/2, 1/4, 1/4, 7/2
, (2*c - Sqrt[2]*Sqrt[c^2/d^2]*d)/(2*c + 2*d*x), (2*c + Sqrt[2]*Sqrt[c^2/d
^2]*d)/(2*c + 2*d*x)])/(d*(c + d*x)^2*(-(a*(c^2 - 2*d^2*x^2)))^(1/4))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {498, 27, 688, 25, 27, 719, 228, 499, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3\sqrt[4]{2ad^2x^2-ac^2}}dx$$

$$\Big\downarrow \text{498}$$

$$-\frac{\int -\frac{4c-dx}{2(c+dx)^2\sqrt[4]{2ad^2x^2-ac^2}}dx}{c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4c-dx}{(c+dx)^2\sqrt[4]{2ad^2x^2-ac^2}}dx}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \text{688}$$

$$\frac{-\frac{\int -\frac{acd^2(9c+5dx)}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{ac^2d^2}-\frac{5\left(2ad^2x^2-ac^2\right)^{3/4}}{acd(c+dx)}}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{acd^2(9c+5dx)}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{ac^2d^2}-\frac{5\left(2ad^2x^2-ac^2\right)^{3/4}}{acd(c+dx)}}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{9c+5dx}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{c}-\frac{5\left(2ad^2x^2-ac^2\right)^{3/4}}{acd(c+dx)}}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{5\int \frac{1}{\sqrt[4]{2ad^2x^2-ac^2}}dx+4c\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx}{c}-\frac{5\left(2ad^2x^2-ac^2\right)^{3/4}}{acd(c+dx)}}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \; 228$$

$$\frac{\frac{4c\int \frac{1}{(c+dx)\sqrt[4]{2ad^2x^2-ac^2}}dx+\frac{5\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{ad^2x}}{c}-\frac{5\left(2ad^2x^2-ac^2\right)^{3/4}}{acd(c+dx)}}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \; 499$$

$$\frac{\frac{\frac{5\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\int \frac{\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}}{ad^2x}+4c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{c\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c(c+dx)}\right)}{2\sqrt[4]{a}\sqrt{c}d}\cdots}{c}\cdots}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \; 834$$

$$\frac{\frac{\frac{5\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{a}c\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+4c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2\cdots}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2\cdots}}\cdots\right.}{2\sqrt[4]{a}\sqrt{c}d}\cdots}{c}\cdots}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{5\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt{a}c\int \frac{1}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}-\int \frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2x^2-ac^2}\right)}{ad^2x}+4c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-a\cdots}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac\cdots}}\cdots\right.}{2\sqrt[4]{a}\sqrt{c}d}\cdots}{c}\cdots}{2c^2}-\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{5\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\frac{\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}-\int\frac{\sqrt{a}c-\sqrt{2ad^2x^2-ac^2}}{\sqrt{\frac{2ad^2x^2-ac^2}{ac^2}+1}}d\sqrt[4]{2ad^2}\right.}{ad^2x}}{c}}{2c^2}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

$\downarrow$ 1510

$$\frac{4c\left(\frac{\arctan\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{\sqrt{a}c(c+dx)-c\sqrt{2ad^2x^2-ac^2}}\right)}{2\sqrt[4]{a}\sqrt{c}d}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}c^{3/2}\sqrt[4]{2ad^2x^2-ac^2}}{c\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c(c+dx)}\right)}{2\sqrt[4]{a}\sqrt{c}d}\right)+\frac{5\sqrt{2}\sqrt{\frac{d^2x^2}{c^2}}\left(\sqrt[4]{a}\sqrt{c}\sqrt{\frac{ad^2x^2}{\left(\sqrt{2ad^2x^2-ac^2}+\sqrt{a}c\right)^2}}\left(\sqrt{2a\right.\right.}{}}{}}{}$$

$$\frac{\left(2ad^2x^2-ac^2\right)^{3/4}}{2ac^2d(c+dx)^2}$$

input
```
Int[1/((c + d*x)^3*(-(a*c^2) + 2*a*d^2*x^2)^(1/4)),x]
```

output
```
-1/2*(-(a*c^2) + 2*a*d^2*x^2)^(3/4)/(a*c^2*d*(c + d*x)^2) + ((-5*(-(a*c^2)
 + 2*a*d^2*x^2)^(3/4))/(a*c*d*(c + d*x)) + (4*c*(ArcTan[(a^(1/4)*c^(3/2)*(
-(a*c^2) + 2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x) - c*Sqrt[-(a*c^2) + 2*
a*d^2*x^2])]/(2*a^(1/4)*Sqrt[c]*d) - ArcTanh[(a^(1/4)*c^(3/2)*(-(a*c^2) +
2*a*d^2*x^2)^(1/4))/(Sqrt[a]*c*(c + d*x) + c*Sqrt[-(a*c^2) + 2*a*d^2*x^2])
]/(2*a^(1/4)*Sqrt[c]*d)) + (5*Sqrt[2]*Sqrt[(d^2*x^2)/c^2]*((a*c^2*(-(a*c^2
) + 2*a*d^2*x^2)^(1/4)*Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)])/(Sqrt[a
]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2]) - (Sqrt[2]*a^(1/4)*Sqrt[c]*Sqrt[(a*d^2
*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*(Sqrt[a]*c + Sqrt[-(a*
c^2) + 2*a*d^2*x^2])*EllipticE[2*ArcTan[(-(a*c^2) + 2*a*d^2*x^2)^(1/4)/(a^
(1/4)*Sqrt[c])], 1/2])/Sqrt[1 + (-(a*c^2) + 2*a*d^2*x^2)/(a*c^2)] + (a^(1/
4)*Sqrt[c]*Sqrt[(a*d^2*x^2)/(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])^2]*
(Sqrt[a]*c + Sqrt[-(a*c^2) + 2*a*d^2*x^2])*EllipticF[2*ArcTan[(-(a*c^2) +
2*a*d^2*x^2)^(1/4)/(a^(1/4)*Sqrt[c])], 1/2])/(Sqrt[2]*Sqrt[1 + (-(a*c^2) +
 2*a*d^2*x^2)/(a*c^2)])))/(a*d^2*x))/c)/(2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 499
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> With[
{q = Rt[-a, 4]}, Simp[(1/(2*d*q))*ArcTan[c*q*((a + b*x^2)^(1/4)/(q^2*(c + d
*x) - c*Sqrt[a + b*x^2]))], x] - Simp[(1/(2*d*q))*ArcTanh[c*q*((a + b*x^2)^
(1/4)/(q^2*(c + d*x) + c*Sqrt[a + b*x^2]))], x]] /; FreeQ[{a, b, c, d}, x]
&& EqQ[b*c^2 + 2*a*d^2, 0] && NegQ[a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(dx+c)^{3}\left(2a\,d^{2}x^{2}-a\,c^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/(d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3\sqrt[4]{-ac^2+2ad^2x^2}}\,dx = \int \frac{1}{\sqrt[4]{a\left(-c^{2}+2d^{2}x^{2}\right)}\left(c+dx\right)^{3}}\,dx$$

input
```
integrate(1/(d*x+c)**3/(2*a*d**2*x**2-a*c**2)**(1/4),x)
```

output
```
Integral(1/((a*(-c**2 + 2*d**2*x**2))**(1/4)*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{1}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(d*x + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^3\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{1}{(2\,ad^2x^2-ac^2)^{\frac{1}{4}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[4]{-ac^2+2ad^2x^2}}\,dx=\int \frac{1}{\left(2\,a\,d^2\,x^2-a\,c^2\right)^{1/4}\,\left(c+d\,x\right)^3}\,dx$$

input
```
int(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(c + d*x)^3),x)
```

output
```
int(1/((2*a*d^2*x^2 - a*c^2)^(1/4)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^3 \sqrt[4]{-ac^2+2ad^2x^2}}\,dx$$
$$= \frac{\int \frac{1}{(2d^2x^2-c^2)^{\frac{1}{4}}c^3+3(2d^2x^2-c^2)^{\frac{1}{4}}c^2dx+3(2d^2x^2-c^2)^{\frac{1}{4}}c\,d^2x^2+(2d^2x^2-c^2)^{\frac{1}{4}}d^3x^3}dx}{a^{\frac{1}{4}}}$$

input
```
int(1/(d*x+c)^3/(2*a*d^2*x^2-a*c^2)^(1/4),x)
```

output
```
int(1/(( - c**2 + 2*d**2*x**2)**(1/4)*c**3 + 3*( - c**2 + 2*d**2*x**2)**(1
/4)*c**2*d*x + 3*( - c**2 + 2*d**2*x**2)**(1/4)*c*d**2*x**2 + ( - c**2 + 2
*d**2*x**2)**(1/4)*d**3*x**3),x)/a**(1/4)
```

3.388 $\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 278

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{\sqrt{b}\sqrt[4]{b^2c+a^2d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{\sqrt{b}\sqrt[4]{b^2c+a^2d}} - \frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}},\arcsin\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right),-1\right)}{b\sqrt{b^2c+a^2d}\,x} + \frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}},\arcsin\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right),-1\right)}{b\sqrt{b^2c+a^2d}\,x}$$

output
```
arctan(b^(1/2)*(d*x^2+c)^(1/4)/(a^2*d+b^2*c)^(1/4))/b^(1/2)/(a^2*d+b^2*c)^
(1/4)-arctanh(b^(1/2)*(d*x^2+c)^(1/4)/(a^2*d+b^2*c)^(1/4))/b^(1/2)/(a^2*d+
b^2*c)^(1/4)-a*c^(1/4)*(-d*x^2/c)^(1/2)*EllipticPi((d*x^2+c)^(1/4)/c^(1/4)
,-b*c^(1/2)/(a^2*d+b^2*c)^(1/2),I)/b/(a^2*d+b^2*c)^(1/2)/x+a*c^(1/4)*(-d*x
^2/c)^(1/2)*EllipticPi((d*x^2+c)^(1/4)/c^(1/4),b*c^(1/2)/(a^2*d+b^2*c)^(1/
2),I)/b/(a^2*d+b^2*c)^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.73 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx$$
$$= -\frac{2\sqrt[4]{\frac{b\left(-\sqrt{-\frac{c}{d}}+x\right)}{a+bx}}\sqrt[4]{\frac{b\left(\sqrt{-\frac{c}{d}}+x\right)}{a+bx}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{a-b\sqrt{-\frac{c}{d}}}{a+bx},\frac{a+b\sqrt{-\frac{c}{d}}}{a+bx}\right)}{b\sqrt[4]{c+dx^2}}$$

input
```
Integrate[1/((a + b*x)*(c + d*x^2)^(1/4)),x]
```

output
```
(-2*((b*(-Sqrt[-(c/d)] + x))/(a + b*x))^(1/4)*((b*(Sqrt[-(c/d)] + x))/(a +
 b*x))^(1/4)*AppellF1[1/2, 1/4, 1/4, 3/2, (a - b*Sqrt[-(c/d)])/(a + b*x),
(a + b*Sqrt[-(c/d)])/(a + b*x)])/(b*(c + d*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {504, 310, 353, 73, 27, 827, 218, 221, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx$$
$$\Big\downarrow \text{ 504}$$
$$a\int \frac{1}{(a^2-b^2x^2)\sqrt[4]{dx^2+c}}dx-b\int \frac{x}{(a^2-b^2x^2)\sqrt[4]{dx^2+c}}dx$$
$$\Big\downarrow \text{ 310}$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{\sqrt{dx^2+c}}{\left(da^2+b^2c-b^2\left(dx^2+c\right)\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{x}-b\int \frac{x}{\left(a^2-b^2x^2\right)\sqrt[4]{dx^2+c}}dx$$

$$\Big\downarrow \; 353$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{\sqrt{dx^2+c}}{\left(da^2+b^2c-b^2\left(dx^2+c\right)\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{x}-\frac{1}{2}b\int \frac{1}{\left(a^2-b^2x^2\right)\sqrt[4]{dx^2+c}}dx^2$$

$$\Big\downarrow \; 73$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{\sqrt{dx^2+c}}{\left(da^2+b^2c-b^2\left(dx^2+c\right)\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{x}-\frac{2b\int \frac{dx^4}{\left(a^2+\frac{b^2c}{d}\right)d-b^2x^8}d\sqrt[4]{dx^2+c}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{\sqrt{dx^2+c}}{\left(da^2+b^2c-b^2\left(dx^2+c\right)\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{x}-2b\int \frac{x^4}{-b^2x^8+b^2c+a^2d}d\sqrt[4]{dx^2+c}$$

$$\Big\downarrow \; 827$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{\sqrt{dx^2+c}}{\left(da^2+b^2c-b^2\left(dx^2+c\right)\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{x}-2b\left(\frac{\int \frac{1}{\sqrt{da^2+b^2c}-bx^4}d\sqrt[4]{dx^2+c}}{2b}-\frac{\int \frac{1}{bx^4+\sqrt{da^2+b^2c}}d\sqrt[4]{dx^2+c}}{2b}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{\sqrt{dx^2+c}}{\left(da^2+b^2c-b^2\left(dx^2+c\right)\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{x}-2b\left(\frac{\int \frac{1}{\sqrt{da^2+b^2c}-bx^4}d\sqrt[4]{dx^2+c}}{2b}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2b^{3/2}\sqrt[4]{a^2d+b^2c}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{\sqrt{dx^2+c}}{(da^2+b^2c-b^2(dx^2+c))\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{x}-2b\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2b^{3/2}\sqrt[4]{a^2d+b^2c}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2b^{3/2}\sqrt[4]{a^2d+b^2c}}\right)$$

$\Big\downarrow$ 993

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\left(\frac{\int \frac{1}{\left(\sqrt{da^2+b^2c}-b\sqrt{dx^2+c}\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{2b}-\frac{\int \frac{1}{\left(\sqrt{dx^2+c}b+\sqrt{da^2+b^2c}\right)\sqrt{1-\frac{dx^2+c}{c}}}d\sqrt[4]{dx^2+c}}{2b}\right)}{x}-2b\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2b^{3/2}\sqrt[4]{a^2d+b^2c}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2b^{3/2}\sqrt[4]{a^2d+b^2c}}\right)$$

$\Big\downarrow$ 1542

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\left(\frac{\sqrt[4]{c}\,\text{EllipticPi}\left(\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}},\arcsin\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right),-1\right)}{2b\sqrt{a^2d+b^2c}}-\frac{\sqrt[4]{c}\,\text{EllipticPi}\left(-\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}},\arcsin\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right),-1\right)}{2b\sqrt{a^2d+b^2c}}\right)}{x}-2b\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2b^{3/2}\sqrt[4]{a^2d+b^2c}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2b^{3/2}\sqrt[4]{a^2d+b^2c}}\right)$$

input
```
Int[1/((a + b*x)*(c + d*x^2)^(1/4)),x]
```

output
```
-2*b*(-1/2*ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)]/(b^(3
/2)*(b^2*c + a^2*d)^(1/4)) + ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c +
a^2*d)^(1/4)]/(2*b^(3/2)*(b^2*c + a^2*d)^(1/4))) + (2*a*Sqrt[-((d*x^2)/c)]
*(-1/2*(c^(1/4)*EllipticPi[-((b*Sqrt[c])/Sqrt[b^2*c + a^2*d]), ArcSin[(c +
 d*x^2)^(1/4)/c^(1/4)], -1])/(b*Sqrt[b^2*c + a^2*d]) + (c^(1/4)*EllipticPi
[(b*Sqrt[c])/Sqrt[b^2*c + a^2*d], ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1])/
(2*b*Sqrt[b^2*c + a^2*d])))/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(bx+a)\,(d\,x^2+c)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x+a)/(d*x^2+c)^(1/4),x)
```

output
```
int(1/(b*x+a)/(d*x^2+c)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = \int \frac{1}{(a+bx)\,\sqrt[4]{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x**2+c)**(1/4),x)
```

output
```
Integral(1/((a + b*x)*(c + d*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = \int \frac{1}{(dx^2+c)^{\frac{1}{4}}(bx+a)}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 + c)^(1/4)*(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = \int \frac{1}{(dx^2+c)^{\frac{1}{4}}(bx+a)}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 + c)^(1/4)*(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = \int \frac{1}{(d\,x^2+c)^{1/4}\,(a+b\,x)}\,dx$$

input
```
int(1/((c + d*x^2)^(1/4)*(a + b*x)),x)
```

output
```
int(1/((c + d*x^2)^(1/4)*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)\sqrt[4]{c+dx^2}}\,dx = \int \frac{1}{(d\,x^2+c)^{\frac{1}{4}}\,a+(d\,x^2+c)^{\frac{1}{4}}\,bx}dx$$

input
```
int(1/(b*x+a)/(d*x^2+c)^(1/4),x)
```

output
```
int(1/((c + d*x**2)**(1/4)*a + (c + d*x**2)**(1/4)*b*x),x)
```

3.389 $\int \frac{1}{(a+bx)(c+dx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \frac{1}{(a+bx)(c+dx^2)^{3/4}}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{(b^2c+a^2d)^{3/4}} - \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)}{(b^2c+a^2d)^{3/4}} + \frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}},\arcsin\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right),-1\right)}{(b^2c+a^2d)\,x} + \frac{a\sqrt[4]{c}\sqrt{-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{b\sqrt{c}}{\sqrt{b^2c+a^2d}},\arcsin\left(\frac{\sqrt[4]{c+dx^2}}{\sqrt[4]{c}}\right),-1\right)}{(b^2c+a^2d)\,x}$$

output
```
-b^(1/2)*arctan(b^(1/2)*(d*x^2+c)^(1/4)/(a^2*d+b^2*c)^(1/4))/(a^2*d+b^2*c)
^(3/4)-b^(1/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/4)/(a^2*d+b^2*c)^(1/4))/(a^2*d
+b^2*c)^(3/4)+a*c^(1/4)*(-d*x^2/c)^(1/2)*EllipticPi((d*x^2+c)^(1/4)/c^(1/4
),-b*c^(1/2)/(a^2*d+b^2*c)^(1/2),I)/(a^2*d+b^2*c)/x+a*c^(1/4)*(-d*x^2/c)^(
1/2)*EllipticPi((d*x^2+c)^(1/4)/c^(1/4),b*c^(1/2)/(a^2*d+b^2*c)^(1/2),I)/(
a^2*d+b^2*c)/x
```

Mathematica [A] (verified)

Time = 7.74 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+bx)\,(c+dx^2)^{3/4}}\,dx = \frac{-\sqrt{b}\sqrt[4]{b^2c+a^2d}\,x\left(\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{b^2c+a^2d}}\right)\right)+a\sqrt[4]{}\cdots}{\cdots}$$

input
```
Integrate[1/((a + b*x)*(c + d*x^2)^(3/4)),x]
```

output
```
(-(Sqrt[b]*(b^2*c + a^2*d)^(1/4)*x*(ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^
2*c + a^2*d)^(1/4)] + ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^
(1/4)])) + a*c^(1/4)*Sqrt[-((d*x^2)/c)]*EllipticPi[-((b*Sqrt[c])/Sqrt[b^2*
c + a^2*d]), ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1] + a*c^(1/4)*Sqrt[-((d*
x^2)/c)]*EllipticPi[(b*Sqrt[c])/Sqrt[b^2*c + a^2*d], ArcSin[(c + d*x^2)^(1
/4)/c^(1/4)], -1])/((b^2*c + a^2*d)*x)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\,(c+dx^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 504$$

$$a\int \frac{1}{(a^2-b^2x^2)\,(dx^2+c)^{3/4}}dx - b\int \frac{x}{(a^2-b^2x^2)\,(dx^2+c)^{3/4}}dx$$

$$\Big\downarrow \; 312$$

$$\frac{a\sqrt{-\frac{dx^2}{c}}\int \frac{1}{\sqrt{-\frac{dx^2}{c}}(a^2-b^2x^2)(dx^2+c)^{3/4}}dx^2}{2x} - b\int \frac{x}{(a^2-b^2x^2)\,(dx^2+c)^{3/4}}dx$$

$$\Big\downarrow \text{118}$$

$$-b\int \frac{x}{(a^2-b^2x^2)(dx^2+c)^{3/4}}dx-\frac{2a\sqrt{-\frac{dx^2}{c}}\int -\frac{1}{(-b^2x^8+b^2c+a^2d)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{1}{(-b^2x^8+b^2c+a^2d)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{x}-b\int \frac{x}{(a^2-b^2x^2)(dx^2+c)^{3/4}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{1}{(-b^2x^8+b^2c+a^2d)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{x}-\frac{1}{2}b\int \frac{1}{(a^2-b^2x^2)(dx^2+c)^{3/4}}dx^2$$

$$\Big\downarrow \text{73}$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{1}{(-b^2x^8+b^2c+a^2d)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{x}-\frac{2b\int \frac{1}{-\frac{b^2x^8}{d}+a^2+\frac{b^2c}{d}}d\sqrt[4]{dx^2+c}}{d}$$

$$\Big\downarrow \text{756}$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{1}{(-b^2x^8+b^2c+a^2d)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{x}-\frac{2b\left(\frac{d\int \frac{1}{\sqrt{da^2+b^2c}-bx^4}d\sqrt[4]{dx^2+c}}{2\sqrt{a^2d+b^2c}}+\frac{d\int \frac{1}{bx^4+\sqrt{da^2+b^2c}}d\sqrt[4]{dx^2+c}}{2\sqrt{a^2d+b^2c}}\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{1}{(-b^2x^8+b^2c+a^2d)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{x}-\frac{2b\left(\frac{d\int \frac{1}{\sqrt{da^2+b^2c}-bx^4}d\sqrt[4]{dx^2+c}}{2\sqrt{a^2d+b^2c}}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2\sqrt{b}(a^2d+b^2c)^{3/4}}\right)}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2a\sqrt{-\frac{dx^2}{c}}\int \frac{1}{(-b^2x^8+b^2c+a^2d)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{x}-\frac{2b\left(\frac{d\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2\sqrt{b}(a^2d+b^2c)^{3/4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2\sqrt{b}(a^2d+b^2c)^{3/4}}\right)}{d}$$

$$\Big\downarrow \; 925$$

$$-\frac{2a\sqrt{-\frac{dx^2}{c}}\left(-\frac{\int \frac{1}{\left(1-\frac{bx^4}{\sqrt{da^2+b^2c}}\right)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{2(a^2d+b^2c)}-\frac{\int \frac{1}{\left(\frac{bx^4}{\sqrt{da^2+b^2c}}+1\right)\sqrt{1-\frac{x^8}{c}}}d\sqrt[4]{dx^2+c}}{2(a^2d+b^2c)}\right)}{x}-\frac{2b\left(\frac{d\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2\sqrt{b}(a^2d+b^2c)^{3/4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2\sqrt{b}(a^2d+b^2c)^{3/4}}\right)}{d}$$

$$\Big\downarrow \; 1542$$

$$-\frac{2a\sqrt{-\frac{dx^2}{c}}\left(-\frac{\sqrt[4]{c}\,\text{EllipticPi}\left(-\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}},\arcsin\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right),-1\right)}{2(a^2d+b^2c)}-\frac{\sqrt[4]{c}\,\text{EllipticPi}\left(\frac{b\sqrt{c}}{\sqrt{da^2+b^2c}},\arcsin\left(\frac{\sqrt[4]{dx^2+c}}{\sqrt[4]{c}}\right),-1\right)}{2(a^2d+b^2c)}\right)}{x}-\frac{2b\left(\frac{d\arctan\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2\sqrt{b}(a^2d+b^2c)^{3/4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{c+dx^2}}{\sqrt[4]{a^2d+b^2c}}\right)}{2\sqrt{b}(a^2d+b^2c)^{3/4}}\right)}{d}$$

input
```
Int[1/((a + b*x)*(c + d*x^2)^(3/4)),x]
```

output
```
(-2*b*((d*ArcTan[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2*c + a^2*d)^(1/4)])/(2*Sq
rt[b]*(b^2*c + a^2*d)^(3/4)) + (d*ArcTanh[(Sqrt[b]*(c + d*x^2)^(1/4))/(b^2
*c + a^2*d)^(1/4)])/(2*Sqrt[b]*(b^2*c + a^2*d)^(3/4))))/d - (2*a*Sqrt[-((d
*x^2)/c)]*(-1/2*(c^(1/4)*EllipticPi[-((b*Sqrt[c])/Sqrt[b^2*c + a^2*d]), Ar
cSin[(c + d*x^2)^(1/4)/c^(1/4)], -1])/(b^2*c + a^2*d) - (c^(1/4)*EllipticP
i[(b*Sqrt[c])/Sqrt[b^2*c + a^2*d], ArcSin[(c + d*x^2)^(1/4)/c^(1/4)], -1])
/(2*(b^2*c + a^2*d))))/x
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(bx+a)\,(d\,x^2+c)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x+a)/(d*x^2+c)^(3/4),x)
```

output
```
int(1/(b*x+a)/(d*x^2+c)^(3/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\left(c+dx^2\right)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\left(c+dx^2\right)^{3/4}}\,dx = \int \frac{1}{(a+bx)\left(c+dx^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x**2+c)**(3/4),x)
```

output
```
Integral(1/((a + b*x)*(c + d*x**2)**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\left(c+dx^2\right)^{3/4}}\,dx = \int \frac{1}{(dx^2+c)^{\frac{3}{4}}(bx+a)}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 + c)^(3/4)*(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\,(c+dx^2)^{3/4}}\,dx=\int \frac{1}{(dx^2+c)^{\frac{3}{4}}(bx+a)}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 + c)^(3/4)*(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\,(c+dx^2)^{3/4}}\,dx=\int \frac{1}{(d\,x^2+c)^{3/4}\,(a+b\,x)}\,dx$$

input
```
int(1/((c + d*x^2)^(3/4)*(a + b*x)),x)
```

output
```
int(1/((c + d*x^2)^(3/4)*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)\,(c+dx^2)^{3/4}}\,dx=\int \frac{1}{(d\,x^2+c)^{\frac{3}{4}}\,a+(d\,x^2+c)^{\frac{3}{4}}\,bx}dx$$

input
```
int(1/(b*x+a)/(d*x^2+c)^(3/4),x)
```

output
```
int(1/((c + d*x**2)**(3/4)*a + (c + d*x**2)**(3/4)*b*x),x)
```

3.390 $\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx = -\frac{2\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt[4]{-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right.}{\sqrt{c}\left(d+\frac{\sqrt{-a}e}{\sqrt{c}}\right)\sqrt{d+ex}\sqrt[4]{a+cx^2}}$$

output

```
-2*((-a)^(1/2)-c^(1/2)*x)*(-(c^(1/2)*d+(-a)^(1/2)*e)*((-a)^(1/2)+c^(1/2)*x
)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)^(1/2)-c^(1/2)*x))^(1/4)*hypergeom([-1/2,
1/4],[1/2],2*(-a)^(1/2)*c^(1/2)*(e*x+d)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)^(1/
2)-c^(1/2)*x))/c^(1/2)/(d+(-a)^(1/2)*e/c^(1/2))/(e*x+d)^(1/2)/(c*x^2+a)^(1
/4)
```

Mathematica [A] (verified)

Time = 20.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.53

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx = \frac{(-ae+cdx)\,(a+cx^2)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{(ae-cdx)^2}{ac(d+ex)^2}\right)}{ac(d+ex)^{5/2}\left(\frac{(cd^2+ae^2)(a+cx^2)}{ac(d+ex)^2}\right)^{3/4}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(a + c*x^2)^(1/4)),x]
```

output
```
((-(a*e) + c*d*x)*(a + c*x^2)^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((a*
e - c*d*x)^2/(a*c*(d + e*x)^2))])/(a*c*(d + e*x)^(5/2)*(((c*d^2 + a*e^2)*(
a + c*x^2))/(a*c*(d + e*x)^2))^(3/4))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+cx^2}(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \; 489$$

$$-\frac{2\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt[4]{-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\frac{2\sqrt{-a}\sqrt{c}(d+ex)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)}{\sqrt[4]{a+cx^2}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

input
```
Int[1/((d + e*x)^(3/2)*(a + c*x^2)^(1/4)),x]
```

output
```
(-2*(Sqrt[-a] - Sqrt[c]*x)*(-(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] + Sqrt[c
]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))))^(1/4)*Hypergeome
tric2F1[-1/2, 1/4, 1/2, (2*Sqrt[-a]*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[
-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d + Sqrt[-a]*e)*Sqrt[d + e*x]*(
a + c*x^2)^(1/4))
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

Maple [F]

$$\int \frac{1}{(ex+d)^{\frac{3}{2}} (c\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x)
```

output
```
int(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx = \int \frac{1}{(cx^2+a)^{\frac{1}{4}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^(3/4)*sqrt(e*x + d)/(c*e^2*x^4 + 2*c*d*e*x^3 + 2*a*d*
e*x + a*d^2 + (c*d^2 + a*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx = \int \frac{1}{\sqrt[4]{a+cx^2}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+a)**(1/4),x)
```

output
```
Integral(1/((a + c*x**2)**(1/4)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx = \int \frac{1}{(cx^2+a)^{\frac{1}{4}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^(1/4)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx = \int \frac{1}{(cx^2+a)^{\frac{1}{4}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + a)^(1/4)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx=\int \frac{1}{{\left(c\,x^2+a\right)}^{1/4}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((a + c*x^2)^(1/4)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((a + c*x^2)^(1/4)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt[4]{a+cx^2}}\,dx=\frac{-2\sqrt{ex+d}\,(c\,x^2+a)^{\frac{1}{4}}\,ae+2\sqrt{ex+d}\,(c\,x^2+a)^{\frac{1}{4}}\,cdx-2\sqrt{c\,x^2+a}\left(\int \frac{}{c^2e^2x}\right.}{}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+a)^(1/4),x)
```

output

```
( - 2*sqrt(d + e*x)*(a + c*x**2)**(1/4)*a*e + 2*sqrt(d + e*x)*(a + c*x**2)
**(1/4)*c*d*x - 2*sqrt(a + c*x**2)*int((sqrt(d + e*x)*(a + c*x**2)**(3/4)*
x**3)/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*c*d**2*x**2 + 4*a*c
*d*e*x**3 + 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c**2*e**2
*x**6),x)*c**2*d**2*e - 2*sqrt(a + c*x**2)*int((sqrt(d + e*x)*(a + c*x**2)
**(3/4)*x**3)/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*c*d**2*x**2
 + 4*a*c*d*e*x**3 + 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c
**2*e**2*x**6),x)*c**2*d*e**2*x - sqrt(a + c*x**2)*int((sqrt(d + e*x)*(a +
 c*x**2)**(3/4))/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*c*d**2*x
**2 + 4*a*c*d*e*x**3 + 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5
+ c**2*e**2*x**6),x)*a**2*d*e**2 - sqrt(a + c*x**2)*int((sqrt(d + e*x)*(a
+ c*x**2)**(3/4))/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*c*d**2*
x**2 + 4*a*c*d*e*x**3 + 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5
 + c**2*e**2*x**6),x)*a**2*e**3*x + sqrt(a + c*x**2)*int((sqrt(d + e*x)*(a
 + c*x**2)**(3/4))/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*c*d**2
*x**2 + 4*a*c*d*e*x**3 + 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**
5 + c**2*e**2*x**6),x)*a*c*d**3 + sqrt(a + c*x**2)*int((sqrt(d + e*x)*(a +
 c*x**2)**(3/4))/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*c*d**2*x
**2 + 4*a*c*d*e*x**3 + 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5
+ c**2*e**2*x**6),x)*a*c*d**2*e*x)/(3*sqrt(a + c*x**2)*c*d**2*(d + e*x)...
```

3.391 $\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 164

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right) - \frac{\sqrt{3}\arctan\left(\frac{1-2^{5/6}\sqrt[6]{1+x^2}}{\sqrt{3}}\right)}{2\sqrt[6]{2}} + \frac{\sqrt{3}\arctan\left(\frac{1+2^{5/6}\sqrt[6]{1+x^2}}{\sqrt{3}}\right)}{2\sqrt[6]{2}} - \frac{\text{arctanh}\left(\frac{\sqrt[6]{1+x^2}}{\sqrt[6]{2}}\right)}{\sqrt[6]{2}} - \frac{\text{arctanh}\left(\frac{\sqrt[6]{2}\sqrt[6]{1+x^2}}{\sqrt[3]{2}+\sqrt[3]{1+x^2}}\right)}{2\sqrt[6]{2}}$$

output
```
x*AppellF1(1/2,1,1/6,3/2,x^2,-x^2)-1/4*3^(1/2)*arctan(1/3*(1-2^(5/6)*(x^2+
1)^(1/6))*3^(1/2))*2^(5/6)+1/4*3^(1/2)*arctan(1/3*(1+2^(5/6)*(x^2+1)^(1/6)
)*3^(1/2))*2^(5/6)-1/2*arctanh(1/2*2^(5/6)*(x^2+1)^(1/6))*2^(5/6)-1/4*arct
anh(2^(1/6)*(x^2+1)^(1/6)/(2^(1/3)+(x^2+1)^(1/3)))*2^(5/6)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.70 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = -\frac{3\sqrt[6]{\frac{-i+x}{1+x}}\sqrt[6]{\frac{i+x}{1+x}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{6},\frac{1}{6},\frac{4}{3},\frac{1-i}{1+x},\frac{1+i}{1+x}\right)}{\sqrt[6]{1+x^2}}$$

input
```
Integrate[1/((1 + x)*(1 + x^2)^(1/6)),x]
```

output
```
(-3*((-I + x)/(1 + x))^(1/6)*((I + x)/(1 + x))^(1/6)*AppellF1[1/3, 1/6, 1/
6, 4/3, (1 - I)/(1 + x), (1 + I)/(1 + x)])/(1 + x^2)^(1/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {504, 333, 353, 73, 825, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt[6]{x^2+1}}dx$$

$$\Big\downarrow \text{504}$$

$$\int \frac{1}{(1-x^2)\sqrt[6]{x^2+1}}dx - \int \frac{x}{(1-x^2)\sqrt[6]{x^2+1}}dx$$

$$\Big\downarrow \text{333}$$

$$x\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right) - \int \frac{x}{(1-x^2)\sqrt[6]{x^2+1}}dx$$

$$\Big\downarrow \text{353}$$

$$x\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right) - \frac{1}{2}\int \frac{1}{(1-x^2)\sqrt[6]{x^2+1}}dx^2$$

$\Big\downarrow$ 73

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-3\int\frac{x^8}{2-x^{12}}d\sqrt[6]{x^2+1}$$

$\Big\downarrow$ 825

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-3\left(\frac{1}{3}\int\frac{1}{\sqrt[3]{2}-x^4}d\sqrt[6]{x^2+1}+\frac{\int-\frac{\sqrt[6]{x^2+1}+\sqrt[6]{2}}{2\left(x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}\right)}d\sqrt[6]{x^2+1}}{3\sqrt[6]{2}}+\frac{\int-\frac{\sqrt[6]{2}-\sqrt[6]{x^2+1}}{2\left(x^4+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}\right)}d\sqrt[6]{x^2+1}}{3\sqrt[6]{2}}\right)$$

$\Big\downarrow$ 27

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-3\left(\frac{1}{3}\int\frac{1}{\sqrt[3]{2}-x^4}d\sqrt[6]{x^2+1}-\frac{\int\frac{\sqrt[6]{x^2+1}+\sqrt[6]{2}}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{6\sqrt[6]{2}}-\frac{\int\frac{\sqrt[6]{2}-\sqrt[6]{x^2+1}}{x^4+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{6\sqrt[6]{2}}\right)$$

$\Big\downarrow$ 219

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-3\left(-\frac{\int\frac{\sqrt[6]{x^2+1}+\sqrt[6]{2}}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{6\sqrt[6]{2}}-\frac{\int\frac{\sqrt[6]{2}-\sqrt[6]{x^2+1}}{x^4+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{6\sqrt[6]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[6]{x^2+1}}{\sqrt[6]{2}}\right)}{3\sqrt[6]{2}}\right)$$

$\Big\downarrow$ 1142

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-3\left(-\frac{\frac{3\int\frac{1}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{2^{5/6}}+\frac{1}{2}\int-\frac{\sqrt[6]{2}\left(1-2^{5/6}\sqrt[6]{x^2+1}\right)}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{6\sqrt[6]{2}}-\frac{\frac{3\int\frac{1}{x^4+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2}}{2^{5/6}}}{}\right.$$

$\Big\downarrow$ 25

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-$$
$$3\left(-\frac{\frac{3\int\frac{1}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{2^{5/6}}-\frac{1}{2}\int\frac{\sqrt[6]{2}\left(1-2^{5/6}\sqrt[6]{x^2+1}\right)}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{6\sqrt[6]{2}}-\frac{\frac{3\int\frac{1}{x^4+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+\cdots}}{2^{5/6}}\cdots}{\cdots}\right.$$

$\downarrow$ 27

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-$$
$$3\left(-\frac{\frac{3\int\frac{1}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{2^{5/6}}-\frac{\int\frac{1-2^{5/6}\sqrt[6]{x^2+1}}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{2^{5/6}}}{6\sqrt[6]{2}}-\frac{\frac{3\int\frac{1}{x^4+\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{2^{5/6}}\cdots}{6\cdots}\right.$$

$\downarrow$ 1082

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-$$
$$3\left(-\frac{3\int\frac{1}{-x^4-3}d\left(1-2^{5/6}\sqrt[6]{x^2+1}\right)-\frac{\int\frac{1-2^{5/6}\sqrt[6]{x^2+1}}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{2^{5/6}}}{6\sqrt[6]{2}}-\frac{-3\int\frac{1}{-x^4-3}d\left(2^{5/6}\sqrt[6]{x^2+1}+1\right)-\cdots}{6\sqrt[6]{2}}\right.$$

$\downarrow$ 217

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-$$
$$3\left(-\frac{-\frac{\int\frac{1-2^{5/6}\sqrt[6]{x^2+1}}{x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}}d\sqrt[6]{x^2+1}}{2^{5/6}}-\sqrt{3}\arctan\left(\frac{1-2^{5/6}\sqrt[6]{x^2+1}}{\sqrt{3}}\right)}{6\sqrt[6]{2}}-\frac{\sqrt{3}\arctan\left(\frac{2^{5/6}\sqrt[6]{x^2+1}+1}{\sqrt{3}}\right)-\cdots}{6\sqrt[6]{2}}\right.$$

$\downarrow$ 1103

$$x\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},x^2,-x^2\right)-$$
$$3\left(-\frac{\frac{1}{2}\log\left(x^4-\sqrt[6]{2}\sqrt[6]{x^2+1}+\sqrt[3]{2}\right)-\sqrt{3}\arctan\left(\frac{1-2^{5/6}\sqrt[6]{x^2+1}}{\sqrt{3}}\right)}{6\sqrt[6]{2}}-\frac{\sqrt{3}\arctan\left(\frac{2^{5/6}\sqrt[6]{x^2+1}+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x\right.}{6\sqrt[6]{2}}\right.$$

input
```
Int[1/((1 + x)*(1 + x^2)^(1/6)),x]
```

output
```
x*AppellF1[1/2, 1, 1/6, 3/2, x^2, -x^2] - 3*(ArcTanh[(1 + x^2)^(1/6)/2^(1/
6)]/(3*2^(1/6)) - (-(Sqrt[3]*ArcTan[(1 - 2^(5/6)*(1 + x^2)^(1/6))/Sqrt[3]]
) + Log[2^(1/3) + x^4 - 2^(1/6)*(1 + x^2)^(1/6)]/2)/(6*2^(1/6)) - (Sqrt[3]
*ArcTan[(1 + 2^(5/6)*(1 + x^2)^(1/6))/Sqrt[3]] - Log[2^(1/3) + x^4 + 2^(1/
6)*(1 + x^2)^(1/6)]/2)/(6*2^(1/6)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{1}{(x+1)\,(x^2+1)^{\frac{1}{6}}}dx$$

input
```
int(1/(x+1)/(x^2+1)^(1/6),x)
```

output
```
int(1/(x+1)/(x^2+1)^(1/6),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+x)/(x^2+1)^(1/6),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = \int \frac{1}{(x+1)\,\sqrt[6]{x^2+1}}\,dx$$

input
```
integrate(1/(1+x)/(x**2+1)**(1/6),x)
```

output
```
Integral(1/((x + 1)*(x**2 + 1)**(1/6)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{1}{6}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(x^2+1)^(1/6),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)^(1/6)*(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{1}{6}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(x^2+1)^(1/6),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 1)^(1/6)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = \int \frac{1}{(x^2+1)^{1/6}\,(x+1)}\,dx$$

input
```
int(1/((x^2 + 1)^(1/6)*(x + 1)),x)
```

output
```
int(1/((x^2 + 1)^(1/6)*(x + 1)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)\sqrt[6]{1+x^2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{1}{6}}\,x+(x^2+1)^{\frac{1}{6}}}dx$$

input
```
int(1/(1+x)/(x^2+1)^(1/6),x)
```

output
```
int(1/((x**2 + 1)**(1/6)*x + (x**2 + 1)**(1/6)),x)
```

3.392 $\int (c+dx)^m \left(a+bx^2\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 223

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx = \frac{(bc^2+ad^2)^3 (c+dx)^{1+m}}{d^7(1+m)} - \frac{6bc(bc^2+ad^2)^2 (c+dx)^{2+m}}{d^7(2+m)} + \frac{3b(bc^2+ad^2)(5bc^2+ad^2)(c+dx)^{3+m}}{d^7(3+m)} - \frac{4b^2c(5bc^2+3ad^2)(c+dx)^{4+m}}{d^7(4+m)} + \frac{3b^2(5bc^2+ad^2)(c+dx)^{5+m}}{d^7(5+m)} - \frac{6b^3c(c+dx)^{6+m}}{d^7(6+m)} + \frac{b^3(c+dx)^{7+m}}{d^7(7+m)}$$

output
```
(a*d^2+b*c^2)^3*(d*x+c)^(1+m)/d^7/(1+m)-6*b*c*(a*d^2+b*c^2)^2*(d*x+c)^(2+m
)/d^7/(2+m)+3*b*(a*d^2+b*c^2)*(a*d^2+5*b*c^2)*(d*x+c)^(3+m)/d^7/(3+m)-4*b^
2*c*(3*a*d^2+5*b*c^2)*(d*x+c)^(4+m)/d^7/(4+m)+3*b^2*(a*d^2+5*b*c^2)*(d*x+c
)^(5+m)/d^7/(5+m)-6*b^3*c*(d*x+c)^(6+m)/d^7/(6+m)+b^3*(d*x+c)^(7+m)/d^7/(7
+m)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.70

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx$$

$$= \frac{(c+dx)^{1+m}\left(\left(a+bx^2\right)^3 + \frac{6\left(\left(bc^2+ad^2\right)(6+m)\left(d^4(1+m)(2+m)(3+m)(4+m)\left(a+bx^2\right)^2+4\left(bc^2+ad^2\right)(4+m)\left(ad^2\left(6+5m+m^2\right)+b(\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*x^2)^3,x]
```

output
```
((c + d*x)^(1 + m)*((a + b*x^2)^3 + (6*((b*c^2 + a*d^2)*(6 + m)*(d^4*(1 +
m)*(2 + m)*(3 + m)*(4 + m)*(a + b*x^2)^2 + 4*(b*c^2 + a*d^2)*(4 + m)*(a*d^
2*(6 + 5*m + m^2) + b*(2*c^2 - 2*c*d*(1 + m)*x + d^2*(2 + 3*m + m^2)*x^2))
 - 4*b*c*(1 + m)*(c + d*x)*(a*d^2*(12 + 7*m + m^2) + b*(2*c^2 - 2*c*d*(2 +
 m)*x + d^2*(6 + 5*m + m^2)*x^2))) - b*c*(1 + m)*(c + d*x)*(d^4*(2 + m)*(3
 + m)*(4 + m)*(5 + m)*(a + b*x^2)^2 + 4*(b*c^2 + a*d^2)*(5 + m)*(a*d^2*(12
 + 7*m + m^2) + b*(2*c^2 - 2*c*d*(2 + m)*x + d^2*(6 + 5*m + m^2)*x^2)) - 4
*b*c*(2 + m)*(c + d*x)*(a*d^2*(20 + 9*m + m^2) + b*(2*c^2 - 2*c*d*(3 + m)*
x + d^2*(12 + 7*m + m^2)*x^2)))))/(d^6*(1 + m)*(2 + m)*(3 + m)*(4 + m)*(5
+ m)*(6 + m))))/(d*(7 + m))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 (c+dx)^m \, dx$$

$$\Big\downarrow \; 476$$

$$\int \left(-\frac{4b^2c(3ad^2+5bc^2)(c+dx)^{m+3}}{d^6}+\frac{3b^2(ad^2+5bc^2)(c+dx)^{m+4}}{d^6}+\frac{(ad^2+bc^2)^3(c+dx)^m}{d^6}-\frac{6bc(ad^2+bc\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{4b^2c(3ad^2+5bc^2)(c+dx)^{m+4}}{d^7(m+4)}+\frac{3b^2(ad^2+5bc^2)(c+dx)^{m+5}}{d^7(m+5)}+\frac{(ad^2+bc^2)^3(c+dx)^{m+1}}{d^7(m+1)}-\frac{6bc(ad^2+bc^2)^2(c+dx)^{m+2}}{d^7(m+2)}+\frac{3b(ad^2+bc^2)(ad^2+5bc^2)(c+dx)^{m+3}}{d^7(m+3)}-\frac{6b^3c(c+dx)^{m+6}}{d^7(m+6)}+\frac{b^3(c+dx)^{m+7}}{d^7(m+7)}$$

input
```
Int[(c + d*x)^m*(a + b*x^2)^3,x]
```

output
```
((b*c^2 + a*d^2)^3*(c + d*x)^(1 + m))/(d^7*(1 + m)) - (6*b*c*(b*c^2 + a*d^
2)^2*(c + d*x)^(2 + m))/(d^7*(2 + m)) + (3*b*(b*c^2 + a*d^2)*(5*b*c^2 + a*
d^2)*(c + d*x)^(3 + m))/(d^7*(3 + m)) - (4*b^2*c*(5*b*c^2 + 3*a*d^2)*(c +
d*x)^(4 + m))/(d^7*(4 + m)) + (3*b^2*(5*b*c^2 + a*d^2)*(c + d*x)^(5 + m))/
(d^7*(5 + m)) - (6*b^3*c*(c + d*x)^(6 + m))/(d^7*(6 + m)) + (b^3*(c + d*x)
^(7 + m))/(d^7*(7 + m))
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(223) = 446.

Time = 0.30 (sec) , antiderivative size = 954, normalized size of antiderivative = 4.28

method	result
norman	$\frac{b^3x^7e^{m\ln(dx+c)}}{7+m}+\frac{c(a^3d^6m^6+27a^3d^6m^5+295a^3d^6m^4+6a^2bc^2d^4m^4+1665a^3d^6m^3+132a^2bc^2d^4m^3+5104a^3d^6m^2+1074}{d^7(m^7+28m^6+32}$
gosper	Expression too large to display
orering	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((d*x+c)^m*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3/(7+m)*x^7*exp(m*ln(d*x+c))+c*(a^3*d^6*m^6+27*a^3*d^6*m^5+295*a^3*d^6*m
^4+6*a^2*b*c^2*d^4*m^4+1665*a^3*d^6*m^3+132*a^2*b*c^2*d^4*m^3+5104*a^3*d^6
*m^2+1074*a^2*b*c^2*d^4*m^2+72*a*b^2*c^4*d^2*m^2+8028*a^3*d^6*m+3828*a^2*b
*c^2*d^4*m+936*a*b^2*c^4*d^2*m+5040*a^3*d^6+5040*a^2*b*c^2*d^4+3024*a*b^2*
c^4*d^2+720*b^3*c^6)/d^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+1
3068*m+5040)*exp(m*ln(d*x+c))+(a^3*d^6*m^6+27*a^3*d^6*m^5-6*a^2*b*c^2*d^4*
m^5+295*a^3*d^6*m^4-132*a^2*b*c^2*d^4*m^4+1665*a^3*d^6*m^3-1074*a^2*b*c^2*
d^4*m^3-72*a*b^2*c^4*d^2*m^3+5104*a^3*d^6*m^2-3828*a^2*b*c^2*d^4*m^2-936*a
*b^2*c^4*d^2*m^2+8028*a^3*d^6*m-5040*a^2*b*c^2*d^4*m-3024*a*b^2*c^4*d^2*m-
720*b^3*c^6*m+5040*a^3*d^6)/d^6/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+1313
2*m^2+13068*m+5040)*x*exp(m*ln(d*x+c))+b^3*c*m/d/(m^2+13*m+42)*x^6*exp(m*l
n(d*x+c))+3*(a*d^2*m^2+13*a*d^2*m-2*b*c^2*m+42*a*d^2)/d^2*b^2/(m^3+18*m^2+
107*m+210)*x^5*exp(m*ln(d*x+c))+3*(a^2*d^4*m^4+22*a^2*d^4*m^3-4*a*b*c^2*d^
2*m^3+179*a^2*d^4*m^2-52*a*b*c^2*d^2*m^2+638*a^2*d^4*m-168*a*b*c^2*d^2*m-4
0*b^2*c^4*m+840*a^2*d^4)/d^4*b/(m^5+25*m^4+245*m^3+1175*m^2+2754*m+2520)*x
^3*exp(m*ln(d*x+c))+3*(a*d^2*m^2+13*a*d^2*m+42*a*d^2+10*b*c^2)*b^2*c/d^3*m
/(m^4+22*m^3+179*m^2+638*m+840)*x^4*exp(m*ln(d*x+c))+3*(a^2*d^4*m^4+22*a^2
*d^4*m^3+179*a^2*d^4*m^2+12*a*b*c^2*d^2*m^2+638*a^2*d^4*m+156*a*b*c^2*d^2*
m+840*a^2*d^4+504*a*b*c^2*d^2+120*b^2*c^4)*b*c/d^5*m/(m^6+27*m^5+295*m^4+1
665*m^3+5104*m^2+8028*m+5040)*x^2*exp(m*ln(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(223) = 446$.

Time = 0.10 (sec) , antiderivative size = 1250, normalized size of antiderivative = 5.61

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^m*(b*x^2+a)^3,x, algorithm="fricas")
```

output

```
(a^3*c*d^6*m^6 + 27*a^3*c*d^6*m^5 + 720*b^3*c^7 + 3024*a*b^2*c^5*d^2 + 504
0*a^2*b*c^3*d^4 + 5040*a^3*c*d^6 + (b^3*d^7*m^6 + 21*b^3*d^7*m^5 + 175*b^3
*d^7*m^4 + 735*b^3*d^7*m^3 + 1624*b^3*d^7*m^2 + 1764*b^3*d^7*m + 720*b^3*d
^7)*x^7 + (b^3*c*d^6*m^6 + 15*b^3*c*d^6*m^5 + 85*b^3*c*d^6*m^4 + 225*b^3*c
*d^6*m^3 + 274*b^3*c*d^6*m^2 + 120*b^3*c*d^6*m)*x^6 + 3*(a*b^2*d^7*m^6 + 1
008*a*b^2*d^7 - (2*b^3*c^2*d^5 - 23*a*b^2*d^7)*m^5 - (20*b^3*c^2*d^5 - 207
*a*b^2*d^7)*m^4 - 5*(14*b^3*c^2*d^5 - 185*a*b^2*d^7)*m^3 - 4*(25*b^3*c^2*d
^5 - 536*a*b^2*d^7)*m^2 - 12*(4*b^3*c^2*d^5 - 201*a*b^2*d^7)*m)*x^5 + (6*a
^2*b*c^3*d^4 + 295*a^3*c*d^6)*m^4 + 3*(a*b^2*c*d^6*m^6 + 19*a*b^2*c*d^6*m^
5 + (10*b^3*c^3*d^4 + 131*a*b^2*c*d^6)*m^4 + (60*b^3*c^3*d^4 + 401*a*b^2*c
*d^6)*m^3 + 10*(11*b^3*c^3*d^4 + 54*a*b^2*c*d^6)*m^2 + 12*(5*b^3*c^3*d^4 +
 21*a*b^2*c*d^6)*m)*x^4 + 3*(44*a^2*b*c^3*d^4 + 555*a^3*c*d^6)*m^3 + 3*(a^
2*b*d^7*m^6 + 1680*a^2*b*d^7 - (4*a*b^2*c^2*d^5 - 25*a^2*b*d^7)*m^5 - (64*
a*b^2*c^2*d^5 - 247*a^2*b*d^7)*m^4 - (40*b^3*c^4*d^3 + 332*a*b^2*c^2*d^5 -
 1219*a^2*b*d^7)*m^3 - 8*(15*b^3*c^4*d^3 + 76*a*b^2*c^2*d^5 - 389*a^2*b*d^
7)*m^2 - 4*(20*b^3*c^4*d^3 + 84*a*b^2*c^2*d^5 - 949*a^2*b*d^7)*m)*x^3 + 2*
(36*a*b^2*c^5*d^2 + 537*a^2*b*c^3*d^4 + 2552*a^3*c*d^6)*m^2 + 3*(a^2*b*c*d
^6*m^6 + 23*a^2*b*c*d^6*m^5 + 3*(4*a*b^2*c^3*d^4 + 67*a^2*b*c*d^6)*m^4 + (
168*a*b^2*c^3*d^4 + 817*a^2*b*c*d^6)*m^3 + 2*(60*b^3*c^5*d^2 + 330*a*b^2*c
^3*d^4 + 739*a^2*b*c*d^6)*m^2 + 24*(5*b^3*c^5*d^2 + 21*a*b^2*c^3*d^4 + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15990 vs. 2(207) = 414.

Time = 3.86 (sec) , antiderivative size = 15990, normalized size of antiderivative = 71.70

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**m*(b*x**2+a)**3,x)
```

output
```
Piecewise((c**m*(a**3*x + a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7), Eq
(d, 0)), (-10*a**3*d**6/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*d**9*x*
*2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x**5 + 60*d*
*13*x**6) - 3*a**2*b*c**2*d**4/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*
d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x**5
+ 60*d**13*x**6) - 18*a**2*b*c*d**5*x/(60*c**6*d**7 + 360*c**5*d**8*x + 90
0*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**1
2*x**5 + 60*d**13*x**6) - 45*a**2*b*d**6*x**2/(60*c**6*d**7 + 360*c**5*d**
8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 36
0*c*d**12*x**5 + 60*d**13*x**6) - 6*a*b**2*c**4*d**2/(60*c**6*d**7 + 360*c
**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x*
*4 + 360*c*d**12*x**5 + 60*d**13*x**6) - 36*a*b**2*c**3*d**3*x/(60*c**6*d*
*7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**
2*d**11*x**4 + 360*c*d**12*x**5 + 60*d**13*x**6) - 90*a*b**2*c**2*d**4*x**
2/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x
**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x**5 + 60*d**13*x**6) - 120*a*b**2
*c*d**5*x**3/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c
**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x**5 + 60*d**13*x**6) -
 90*a*b**2*d**6*x**4/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2
+ 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x**5 + 60*d*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(223) = 446.

Time = 0.04 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.12

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx = \frac{(dx+c)^{m+1}a^3}{d(m+1)}$$
$$+ \frac{3\left((m^2+3m+2)d^3x^3 + (m^2+m)cd^2x^2 - 2c^2dmx + 2c^3\right)(dx+c)^m a^2 b}{(m^3+6m^2+11m+6)d^3}$$
$$+ \frac{3\left((m^4+10m^3+35m^2+50m+24)d^5x^5 + (m^4+6m^3+11m^2+6m)cd^4x^4 - 4(m^3+3m^2+2m)\right.}{(m^5+15m^4+85m^3+225m^2+274m+1}$$
$$+ \frac{\left((m^6+21m^5+175m^4+735m^3+1624m^2+1764m+720)d^7x^7 + (m^6+15m^5+85m^4+225m^3\right.}{}$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^3/(d*(m + 1)) + 3*((m^2 + 3*m + 2)*d^3*x^3 + (m^2 + m)
*c*d^2*x^2 - 2*c^2*d*m*x + 2*c^3)*(d*x + c)^m*a^2*b/((m^3 + 6*m^2 + 11*m +
 6)*d^3) + 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*d^5*x^5 + (m^4 + 6*m^3 +
 11*m^2 + 6*m)*c*d^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*c^2*d^3*x^3 + 12*(m^2 + m
)*c^3*d^2*x^2 - 24*c^4*d*m*x + 24*c^5)*(d*x + c)^m*a*b^2/((m^5 + 15*m^4 +
85*m^3 + 225*m^2 + 274*m + 120)*d^5) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3
+ 1624*m^2 + 1764*m + 720)*d^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 27
4*m^2 + 120*m)*c*d^6*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*c^2*d
^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*c^3*d^4*x^4 - 120*(m^3 + 3*m^2 +
2*m)*c^4*d^3*x^3 + 360*(m^2 + m)*c^5*d^2*x^2 - 720*c^6*d*m*x + 720*c^7)*(d
*x + c)^m*b^3/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 +
 13068*m + 5040)*d^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2085 vs. 2(223) = 446.

Time = 0.14 (sec) , antiderivative size = 2085, normalized size of antiderivative = 9.35

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
((d*x + c)^m*b^3*d^7*m^6*x^7 + (d*x + c)^m*b^3*c*d^6*m^6*x^6 + 21*(d*x + c
)^m*b^3*d^7*m^5*x^7 + 3*(d*x + c)^m*a*b^2*d^7*m^6*x^5 + 15*(d*x + c)^m*b^3
*c*d^6*m^5*x^6 + 175*(d*x + c)^m*b^3*d^7*m^4*x^7 + 3*(d*x + c)^m*a*b^2*c*d
^6*m^6*x^4 - 6*(d*x + c)^m*b^3*c^2*d^5*m^5*x^5 + 69*(d*x + c)^m*a*b^2*d^7*
m^5*x^5 + 85*(d*x + c)^m*b^3*c*d^6*m^4*x^6 + 735*(d*x + c)^m*b^3*d^7*m^3*x
^7 + 3*(d*x + c)^m*a^2*b*d^7*m^6*x^3 + 57*(d*x + c)^m*a*b^2*c*d^6*m^5*x^4
- 60*(d*x + c)^m*b^3*c^2*d^5*m^4*x^5 + 621*(d*x + c)^m*a*b^2*d^7*m^4*x^5 +
 225*(d*x + c)^m*b^3*c*d^6*m^3*x^6 + 1624*(d*x + c)^m*b^3*d^7*m^2*x^7 + 3*
(d*x + c)^m*a^2*b*c*d^6*m^6*x^2 - 12*(d*x + c)^m*a*b^2*c^2*d^5*m^5*x^3 + 7
5*(d*x + c)^m*a^2*b*d^7*m^5*x^3 + 30*(d*x + c)^m*b^3*c^3*d^4*m^4*x^4 + 393
*(d*x + c)^m*a*b^2*c*d^6*m^4*x^4 - 210*(d*x + c)^m*b^3*c^2*d^5*m^3*x^5 + 2
775*(d*x + c)^m*a*b^2*d^7*m^3*x^5 + 274*(d*x + c)^m*b^3*c*d^6*m^2*x^6 + 17
64*(d*x + c)^m*b^3*d^7*m*x^7 + (d*x + c)^m*a^3*d^7*m^6*x + 69*(d*x + c)^m*
a^2*b*c*d^6*m^5*x^2 - 192*(d*x + c)^m*a*b^2*c^2*d^5*m^4*x^3 + 741*(d*x + c
)^m*a^2*b*d^7*m^4*x^3 + 180*(d*x + c)^m*b^3*c^3*d^4*m^3*x^4 + 1203*(d*x +
c)^m*a*b^2*c*d^6*m^3*x^4 - 300*(d*x + c)^m*b^3*c^2*d^5*m^2*x^5 + 6432*(d*x
 + c)^m*a*b^2*d^7*m^2*x^5 + 120*(d*x + c)^m*b^3*c*d^6*m*x^6 + 720*(d*x + c
)^m*b^3*d^7*x^7 + (d*x + c)^m*a^3*c*d^6*m^6 - 6*(d*x + c)^m*a^2*b*c^2*d^5*
m^5*x + 27*(d*x + c)^m*a^3*d^7*m^5*x + 36*(d*x + c)^m*a*b^2*c^3*d^4*m^4*x^
2 + 603*(d*x + c)^m*a^2*b*c*d^6*m^4*x^2 - 120*(d*x + c)^m*b^3*c^4*d^3*m...
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 1144, normalized size of antiderivative = 5.13

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3*(c + d*x)^m,x)
```

output
```
((c + d*x)^m*(720*b^3*c^7 + 5040*a^3*c*d^6 + 3024*a*b^2*c^5*d^2 + 5040*a^2
*b*c^3*d^4 + 5104*a^3*c*d^6*m^2 + 1665*a^3*c*d^6*m^3 + 295*a^3*c*d^6*m^4 +
 27*a^3*c*d^6*m^5 + a^3*c*d^6*m^6 + 8028*a^3*c*d^6*m + 936*a*b^2*c^5*d^2*m
 + 3828*a^2*b*c^3*d^4*m + 72*a*b^2*c^5*d^2*m^2 + 1074*a^2*b*c^3*d^4*m^2 +
132*a^2*b*c^3*d^4*m^3 + 6*a^2*b*c^3*d^4*m^4))/(d^7*(13068*m + 13132*m^2 +
6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x*(c + d*x)^m*(72
0*b^3*c^6*d*m - 8028*a^3*d^7*m - 5104*a^3*d^7*m^2 - 1665*a^3*d^7*m^3 - 295
*a^3*d^7*m^4 - 27*a^3*d^7*m^5 - a^3*d^7*m^6 - 5040*a^3*d^7 + 3024*a*b^2*c^
4*d^3*m + 5040*a^2*b*c^2*d^5*m + 936*a*b^2*c^4*d^3*m^2 + 3828*a^2*b*c^2*d^
5*m^2 + 72*a*b^2*c^4*d^3*m^3 + 1074*a^2*b*c^2*d^5*m^3 + 132*a^2*b*c^2*d^5*
m^4 + 6*a^2*b*c^2*d^5*m^5))/(d^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^
4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (b^3*x^7*(c + d*x)^m*(1764*m + 1624*
m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769
*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (3*b^2*x^5*(c + d*x)^m*
(42*a*d^2 + a*d^2*m^2 + 13*a*d^2*m - 2*b*c^2*m)*(50*m + 35*m^2 + 10*m^3 +
m^4 + 24))/(d^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*
m^6 + m^7 + 5040)) + (3*b*x^3*(c + d*x)^m*(3*m + m^2 + 2)*(840*a^2*d^4 + 6
38*a^2*d^4*m - 40*b^2*c^4*m + 179*a^2*d^4*m^2 + 22*a^2*d^4*m^3 + a^2*d^4*m
^4 - 168*a*b*c^2*d^2*m - 52*a*b*c^2*d^2*m^2 - 4*a*b*c^2*d^2*m^3))/(d^4*(13
068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1411, normalized size of antiderivative = 6.33

$$\int (c+dx)^m \left(a+bx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*(b*x^2+a)^3,x)
```

output
```
((c + d*x)**m*(a**3*c*d**6*m**6 + 27*a**3*c*d**6*m**5 + 295*a**3*c*d**6*m*
*4 + 1665*a**3*c*d**6*m**3 + 5104*a**3*c*d**6*m**2 + 8028*a**3*c*d**6*m +
5040*a**3*c*d**6 + a**3*d**7*m**6*x + 27*a**3*d**7*m**5*x + 295*a**3*d**7*
m**4*x + 1665*a**3*d**7*m**3*x + 5104*a**3*d**7*m**2*x + 8028*a**3*d**7*m*
x + 5040*a**3*d**7*x + 6*a**2*b*c**3*d**4*m**4 + 132*a**2*b*c**3*d**4*m**3
 + 1074*a**2*b*c**3*d**4*m**2 + 3828*a**2*b*c**3*d**4*m + 5040*a**2*b*c**3
*d**4 - 6*a**2*b*c**2*d**5*m**5*x - 132*a**2*b*c**2*d**5*m**4*x - 1074*a**
2*b*c**2*d**5*m**3*x - 3828*a**2*b*c**2*d**5*m**2*x - 5040*a**2*b*c**2*d**
5*m*x + 3*a**2*b*c*d**6*m**6*x**2 + 69*a**2*b*c*d**6*m**5*x**2 + 603*a**2*
b*c*d**6*m**4*x**2 + 2451*a**2*b*c*d**6*m**3*x**2 + 4434*a**2*b*c*d**6*m**
2*x**2 + 2520*a**2*b*c*d**6*m*x**2 + 3*a**2*b*d**7*m**6*x**3 + 75*a**2*b*d
**7*m**5*x**3 + 741*a**2*b*d**7*m**4*x**3 + 3657*a**2*b*d**7*m**3*x**3 + 9
336*a**2*b*d**7*m**2*x**3 + 11388*a**2*b*d**7*m*x**3 + 5040*a**2*b*d**7*x*
*3 + 72*a*b**2*c**5*d**2*m**2 + 936*a*b**2*c**5*d**2*m + 3024*a*b**2*c**5*
d**2 - 72*a*b**2*c**4*d**3*m**3*x - 936*a*b**2*c**4*d**3*m**2*x - 3024*a*b
**2*c**4*d**3*m*x + 36*a*b**2*c**3*d**4*m**4*x**2 + 504*a*b**2*c**3*d**4*m
**3*x**2 + 1980*a*b**2*c**3*d**4*m**2*x**2 + 1512*a*b**2*c**3*d**4*m*x**2
- 12*a*b**2*c**2*d**5*m**5*x**3 - 192*a*b**2*c**2*d**5*m**4*x**3 - 996*a*b
**2*c**2*d**5*m**3*x**3 - 1824*a*b**2*c**2*d**5*m**2*x**3 - 1008*a*b**2*c*
*2*d**5*m*x**3 + 3*a*b**2*c*d**6*m**6*x**4 + 57*a*b**2*c*d**6*m**5*x**4...
```

3.393 $\int (c+dx)^m \left(a+bx^2\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx = \frac{(bc^2+ad^2)^2 (c+dx)^{1+m}}{d^5(1+m)} - \frac{4bc(bc^2+ad^2)(c+dx)^{2+m}}{d^5(2+m)} + \frac{2b(3bc^2+ad^2)(c+dx)^{3+m}}{d^5(3+m)} - \frac{4b^2c(c+dx)^{4+m}}{d^5(4+m)} + \frac{b^2(c+dx)^{5+m}}{d^5(5+m)}$$

output
```
(a*d^2+b*c^2)^2*(d*x+c)^(1+m)/d^5/(1+m)-4*b*c*(a*d^2+b*c^2)*(d*x+c)^(2+m)/
d^5/(2+m)+2*b*(a*d^2+3*b*c^2)*(d*x+c)^(3+m)/d^5/(3+m)-4*b^2*c*(d*x+c)^(4+m
)/d^5/(4+m)+b^2*(d*x+c)^(5+m)/d^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx = \frac{(c+dx)^{1+m} \left(\left(a+bx^2\right)^2 + \frac{4(bc^2+ad^2)\left(ad^2(6+5m+m^2)+b\left(2c^2-2cd(1+m)x+d^2(2+3m+m^2)x^2\right)\right)}{d^4(1+m)(2+m)(3+m)} - \frac{4bc(c+dx)\left(ad^2(12+7m+m\right.}{d^4} \right.}{d(5+m)}$$

input
```
Integrate[(c + d*x)^m*(a + b*x^2)^2,x]
```

output
```
((c + d*x)^(1 + m)*((a + b*x^2)^2 + (4*(b*c^2 + a*d^2)*(a*d^2*(6 + 5*m + m
^2) + b*(2*c^2 - 2*c*d*(1 + m)*x + d^2*(2 + 3*m + m^2)*x^2)))/(d^4*(1 + m)
*(2 + m)*(3 + m)) - (4*b*c*(c + d*x)*(a*d^2*(12 + 7*m + m^2) + b*(2*c^2 -
2*c*d*(2 + m)*x + d^2*(6 + 5*m + m^2)*x^2)))/(d^4*(2 + m)*(3 + m)*(4 + m))
))/(d*(5 + m))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 (c+dx)^m \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{\left(ad^2+bc^2\right)^2 (c+dx)^m}{d^4}-\frac{4bc\left(ad^2+bc^2\right) (c+dx)^{m+1}}{d^4}+\frac{2b\left(ad^2+3bc^2\right) (c+dx)^{m+2}}{d^4}-\frac{4b^2c(c+dx)^{m+3}}{d^4}+$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ad^2+bc^2\right)^2 (c+dx)^{m+1}}{d^5(m+1)}-\frac{4bc\left(ad^2+bc^2\right) (c+dx)^{m+2}}{d^5(m+2)}+\frac{2b\left(ad^2+3bc^2\right) (c+dx)^{m+3}}{d^5(m+3)}-\frac{4b^2c(c+dx)^{m+4}}{d^5(m+4)}+\frac{b^2(c+dx)^{m+5}}{d^5(m+5)}$$

input
```
Int[(c + d*x)^m*(a + b*x^2)^2,x]
```

output
```
((b*c^2 + a*d^2)^2*(c + d*x)^(1 + m))/(d^5*(1 + m)) - (4*b*c*(b*c^2 + a*d^
2)*(c + d*x)^(2 + m))/(d^5*(2 + m)) + (2*b*(3*b*c^2 + a*d^2)*(c + d*x)^(3
+ m))/(d^5*(3 + m)) - (4*b^2*c*(c + d*x)^(4 + m))/(d^5*(4 + m)) + (b^2*(c
+ d*x)^(5 + m))/(d^5*(5 + m))
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(140) = 280$.

Time = 0.23 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.00

method	result
gosper	$\frac{(dx+c)^{1+m}\left(b^2d^4m^4x^4+10b^2d^4m^3x^4+2ab\,d^4m^4x^2-4b^2c\,d^3m^3x^3+35b^2d^4m^2x^4+24ab\,d^4m^3x^2-24b^2c\,d^3m^2x^3+50b^2d^4m\,x\right.}{}$
orering	$\frac{(dx+c)\left(b^2d^4m^4x^4+10b^2d^4m^3x^4+2ab\,d^4m^4x^2-4b^2c\,d^3m^3x^3+35b^2d^4m^2x^4+24ab\,d^4m^3x^2-24b^2c\,d^3m^2x^3+50b^2d^4m\,x^4+a\right.}{}$
norman	$\frac{b^2x^5\mathrm{e}^{m\ln(dx+c)}}{5+m}+\frac{c\left(a^2d^4m^4+14a^2d^4m^3+71a^2d^4m^2+4ab\,c^2d^2m^2+154a^2d^4m+36ab\,c^2d^2m+120a^2d^4+80b\,c^2d^2a+24b^2c^4\right.}{d^5(m^5+15m^4+85m^3+225m^2+274m+120)}$
risch	$\frac{\left(b^2d^5m^4x^5+b^2c\,d^4m^4x^4+10b^2d^5m^3x^5+2ab\,d^5m^4x^3+6b^2c\,d^4m^3x^4+35b^2d^5m^2x^5+2abc\,d^4m^4x^2+24ab\,d^5m^3x^3-4b^2c^2d^3m\right.}{}$
parallelrisch	$\frac{x^5(dx+c)^mb^2c\,d^5m^4+10x^5(dx+c)^mb^2c\,d^5m^3+x^4(dx+c)^mb^2c^2d^4m^4+35x^5(dx+c)^mb^2c\,d^5m^2+6x^4(dx+c)^mb^2c^2d^4m^3+50}{}$

input
```
int((d*x+c)^m*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^5*(d*x+c)^(1+m)/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)*(b^2*d^4*m^4*x^4
+10*b^2*d^4*m^3*x^4+2*a*b*d^4*m^4*x^2-4*b^2*c*d^3*m^3*x^3+35*b^2*d^4*m^2*x
^4+24*a*b*d^4*m^3*x^2-24*b^2*c*d^3*m^2*x^3+50*b^2*d^4*m*x^4+a^2*d^4*m^4-4*
a*b*c*d^3*m^3*x+98*a*b*d^4*m^2*x^2+12*b^2*c^2*d^2*m^2*x^2-44*b^2*c*d^3*m*x
^3+24*b^2*d^4*x^4+14*a^2*d^4*m^3-40*a*b*c*d^3*m^2*x+156*a*b*d^4*m*x^2+36*b
^2*c^2*d^2*m*x^2-24*b^2*c*d^3*x^3+71*a^2*d^4*m^2+4*a*b*c^2*d^2*m^2-116*a*b
*c*d^3*m*x+80*a*b*d^4*x^2-24*b^2*c^3*d*m*x+24*b^2*c^2*d^2*x^2+154*a^2*d^4*
m+36*a*b*c^2*d^2*m-80*a*b*c*d^3*x-24*b^2*c^3*d*x+120*a^2*d^4+80*a*b*c^2*d^
2+24*b^2*c^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(140) = 280$.

Time = 0.09 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.71

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx = \frac{(a^2cd^4m^4 + 14\,a^2cd^4m^3 + 24\,b^2c^5 + 80\,abc^3d^2 + 120\,a^2cd^4 + (b^2d^5m^4 + 10\,b^2d^5m^3 + 35\,b^2d^5m^2 + 50\,b^2d^5}{}$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
(a^2*c*d^4*m^4 + 14*a^2*c*d^4*m^3 + 24*b^2*c^5 + 80*a*b*c^3*d^2 + 120*a^2*
c*d^4 + (b^2*d^5*m^4 + 10*b^2*d^5*m^3 + 35*b^2*d^5*m^2 + 50*b^2*d^5*m + 24
*b^2*d^5)*x^5 + (b^2*c*d^4*m^4 + 6*b^2*c*d^4*m^3 + 11*b^2*c*d^4*m^2 + 6*b^
2*c*d^4*m)*x^4 + 2*(a*b*d^5*m^4 + 40*a*b*d^5 - 2*(b^2*c^2*d^3 - 6*a*b*d^5)
*m^3 - (6*b^2*c^2*d^3 - 49*a*b*d^5)*m^2 - 2*(2*b^2*c^2*d^3 - 39*a*b*d^5)*m
)*x^3 + (4*a*b*c^3*d^2 + 71*a^2*c*d^4)*m^2 + 2*(a*b*c*d^4*m^4 + 10*a*b*c*d
^4*m^3 + (6*b^2*c^3*d^2 + 29*a*b*c*d^4)*m^2 + 2*(3*b^2*c^3*d^2 + 10*a*b*c*
d^4)*m)*x^2 + 2*(18*a*b*c^3*d^2 + 77*a^2*c*d^4)*m + (a^2*d^5*m^4 + 120*a^2
*d^5 - 2*(2*a*b*c^2*d^3 - 7*a^2*d^5)*m^3 - (36*a*b*c^2*d^3 - 71*a^2*d^5)*m
^2 - 2*(12*b^2*c^4*d + 40*a*b*c^2*d^3 - 77*a^2*d^5)*m)*x)*(d*x + c)^m/(d^5
*m^5 + 15*d^5*m^4 + 85*d^5*m^3 + 225*d^5*m^2 + 274*d^5*m + 120*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5097 vs. $2(128) = 256$.

Time = 1.49 (sec) , antiderivative size = 5097, normalized size of antiderivative = 36.41

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**m*(b*x**2+a)**2,x)
```

output
```
Piecewise((c**m*(a**2*x + 2*a*b*x**3/3 + b**2*x**5/5), Eq(d, 0)), (-3*a**2
*d**4/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3
+ 12*d**9*x**4) - 2*a*b*c**2*d**2/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2
*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) - 8*a*b*c*d**3*x/(12*c**4*d**5
 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) - 1
2*a*b*d**4*x**2/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*
d**8*x**3 + 12*d**9*x**4) + 12*b**2*c**4*log(c/d + x)/(12*c**4*d**5 + 48*c
**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) + 25*b**2*
c**4/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 +
 12*d**9*x**4) + 48*b**2*c**3*d*x*log(c/d + x)/(12*c**4*d**5 + 48*c**3*d**
6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) + 88*b**2*c**3*d*
x/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12
*d**9*x**4) + 72*b**2*c**2*d**2*x**2*log(c/d + x)/(12*c**4*d**5 + 48*c**3*
d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) + 108*b**2*c**
2*d**2*x**2/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8
*x**3 + 12*d**9*x**4) + 48*b**2*c*d**3*x**3*log(c/d + x)/(12*c**4*d**5 + 4
8*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) + 48*b*
*2*c*d**3*x**3/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d
**8*x**3 + 12*d**9*x**4) + 12*b**2*d**4*x**4*log(c/d + x)/(12*c**4*d**5 +
48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4), Eq...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.68

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx = \frac{(dx+c)^{m+1}a^2}{d(m+1)} + \frac{2\,((m^2+3\,m+2)d^3x^3+(m^2+m)cd^2x^2-2\,c^2dmx+2\,c^3)(dx+c)^m ab}{(m^3+6\,m^2+11\,m+6)d^3} + \frac{((m^4+10\,m^3+35\,m^2+50\,m+24)d^5x^5+(m^4+6\,m^3+11\,m^2+6\,m)cd^4x^4-4\,(m^3+3\,m^2+2\,m)c}{(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+12}$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^2/(d*(m + 1)) + 2*((m^2 + 3*m + 2)*d^3*x^3 + (m^2 + m)
*c*d^2*x^2 - 2*c^2*d*m*x + 2*c^3)*(d*x + c)^m*a*b/((m^3 + 6*m^2 + 11*m + 6
)*d^3) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*d^5*x^5 + (m^4 + 6*m^3 + 11*
m^2 + 6*m)*c*d^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*c^2*d^3*x^3 + 12*(m^2 + m)*c^
3*d^2*x^2 - 24*c^4*d*m*x + 24*c^5)*(d*x + c)^m*b^2/((m^5 + 15*m^4 + 85*m^3
 + 225*m^2 + 274*m + 120)*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(140) = 280.

Time = 0.13 (sec) , antiderivative size = 851, normalized size of antiderivative = 6.08

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
((d*x + c)^m*b^2*d^5*m^4*x^5 + (d*x + c)^m*b^2*c*d^4*m^4*x^4 + 10*(d*x + c
)^m*b^2*d^5*m^3*x^5 + 2*(d*x + c)^m*a*b*d^5*m^4*x^3 + 6*(d*x + c)^m*b^2*c*
d^4*m^3*x^4 + 35*(d*x + c)^m*b^2*d^5*m^2*x^5 + 2*(d*x + c)^m*a*b*c*d^4*m^4
*x^2 - 4*(d*x + c)^m*b^2*c^2*d^3*m^3*x^3 + 24*(d*x + c)^m*a*b*d^5*m^3*x^3
+ 11*(d*x + c)^m*b^2*c*d^4*m^2*x^4 + 50*(d*x + c)^m*b^2*d^5*m*x^5 + (d*x +
 c)^m*a^2*d^5*m^4*x + 20*(d*x + c)^m*a*b*c*d^4*m^3*x^2 - 12*(d*x + c)^m*b^
2*c^2*d^3*m^2*x^3 + 98*(d*x + c)^m*a*b*d^5*m^2*x^3 + 6*(d*x + c)^m*b^2*c*d
^4*m*x^4 + 24*(d*x + c)^m*b^2*d^5*x^5 + (d*x + c)^m*a^2*c*d^4*m^4 - 4*(d*x
 + c)^m*a*b*c^2*d^3*m^3*x + 14*(d*x + c)^m*a^2*d^5*m^3*x + 12*(d*x + c)^m*
b^2*c^3*d^2*m^2*x^2 + 58*(d*x + c)^m*a*b*c*d^4*m^2*x^2 - 8*(d*x + c)^m*b^2
*c^2*d^3*m*x^3 + 156*(d*x + c)^m*a*b*d^5*m*x^3 + 14*(d*x + c)^m*a^2*c*d^4*
m^3 - 36*(d*x + c)^m*a*b*c^2*d^3*m^2*x + 71*(d*x + c)^m*a^2*d^5*m^2*x + 12
*(d*x + c)^m*b^2*c^3*d^2*m*x^2 + 40*(d*x + c)^m*a*b*c*d^4*m*x^2 + 80*(d*x
+ c)^m*a*b*d^5*x^3 + 4*(d*x + c)^m*a*b*c^3*d^2*m^2 + 71*(d*x + c)^m*a^2*c*
d^4*m^2 - 24*(d*x + c)^m*b^2*c^4*d*m*x - 80*(d*x + c)^m*a*b*c^2*d^3*m*x +
154*(d*x + c)^m*a^2*d^5*m*x + 36*(d*x + c)^m*a*b*c^3*d^2*m + 154*(d*x + c)
^m*a^2*c*d^4*m + 120*(d*x + c)^m*a^2*d^5*x + 24*(d*x + c)^m*b^2*c^5 + 80*(
d*x + c)^m*a*b*c^3*d^2 + 120*(d*x + c)^m*a^2*c*d^4)/(d^5*m^5 + 15*d^5*m^4
+ 85*d^5*m^3 + 225*d^5*m^2 + 274*d^5*m + 120*d^5)
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.54

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx$$

$$= (c+d\,x)^m \left(\frac{c\,(a^2\,d^4\,m^4+14\,a^2\,d^4\,m^3+71\,a^2\,d^4\,m^2+154\,a^2\,d^4\,m+120\,a^2\,d^4+4\,a\,b\,c^2\,d^2\,m^2+36\,a\,b\,c^2\,d}{d^5\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{b^2\,x^5\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{x\,(a^2\,d^5\,m^4+14\,a^2\,d^5\,m^3+71\,a^2\,d^5\,m^2+154\,a^2\,d^5\,m+120\,a^2\,d^5-4\,a\,b\,c^2\,d^3\,m^3-36\,a\,b\,c^2\,d^3\,m^2-80}{d^5\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{2\,b\,x^3\,\left(m^2+3\,m+2\right)\,\left(-2\,b\,c^2\,m+a\,d^2\,m^2+9\,a\,d^2\,m+20\,a\,d^2\right)}{d^2\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{b^2\,c\,m\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{d\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{2\,b\,c\,m\,x^2\,\left(m+1\right)\,\left(6\,b\,c^2+a\,d^2\,m^2+9\,a\,d^2\,m+20\,a\,d^2\right)}{d^3\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} \right)$$

input
```
int((a + b*x^2)^2*(c + d*x)^m,x)
```

output
```
(c + d*x)^m*((c*(120*a^2*d^4 + 24*b^2*c^4 + 154*a^2*d^4*m + 71*a^2*d^4*m^2
 + 14*a^2*d^4*m^3 + a^2*d^4*m^4 + 80*a*b*c^2*d^2 + 36*a*b*c^2*d^2*m + 4*a*
b*c^2*d^2*m^2))/(d^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (b
^2*x^5*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15
*m^4 + m^5 + 120) + (x*(120*a^2*d^5 + 154*a^2*d^5*m + 71*a^2*d^5*m^2 + 14*
a^2*d^5*m^3 + a^2*d^5*m^4 - 24*b^2*c^4*d*m - 80*a*b*c^2*d^3*m - 36*a*b*c^2
*d^3*m^2 - 4*a*b*c^2*d^3*m^3))/(d^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m
^5 + 120)) + (2*b*x^3*(3*m + m^2 + 2)*(20*a*d^2 + a*d^2*m^2 + 9*a*d^2*m -
2*b*c^2*m))/(d^2*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (b^2*c
*m*x^4*(11*m + 6*m^2 + m^3 + 6))/(d*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m
^5 + 120)) + (2*b*c*m*x^2*(m + 1)*(20*a*d^2 + 6*b*c^2 + a*d^2*m^2 + 9*a*d^
2*m))/(d^3*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.96

$$\int (c+dx)^m \left(a+bx^2\right)^2 \, dx$$
$$= \frac{(dx+c)^m \left(b^2d^5m^4x^5+b^2c\,d^4m^4x^4+10b^2d^5m^3x^5+2ab\,d^5m^4x^3+6b^2c\,d^4m^3x^4+35b^2d^5m^2x^5+2abc\,d^4 \cdots\right.}{\cdots}$$

input
```
int((d*x+c)^m*(b*x^2+a)^2,x)
```

output
```
((c + d*x)**m*(a**2*c*d**4*m**4 + 14*a**2*c*d**4*m**3 + 71*a**2*c*d**4*m**
2 + 154*a**2*c*d**4*m + 120*a**2*c*d**4 + a**2*d**5*m**4*x + 14*a**2*d**5*
m**3*x + 71*a**2*d**5*m**2*x + 154*a**2*d**5*m*x + 120*a**2*d**5*x + 4*a*b
*c**3*d**2*m**2 + 36*a*b*c**3*d**2*m + 80*a*b*c**3*d**2 - 4*a*b*c**2*d**3*
m**3*x - 36*a*b*c**2*d**3*m**2*x - 80*a*b*c**2*d**3*m*x + 2*a*b*c*d**4*m**
4*x**2 + 20*a*b*c*d**4*m**3*x**2 + 58*a*b*c*d**4*m**2*x**2 + 40*a*b*c*d**4
*m*x**2 + 2*a*b*d**5*m**4*x**3 + 24*a*b*d**5*m**3*x**3 + 98*a*b*d**5*m**2*
x**3 + 156*a*b*d**5*m*x**3 + 80*a*b*d**5*x**3 + 24*b**2*c**5 - 24*b**2*c**
4*d*m*x + 12*b**2*c**3*d**2*m**2*x**2 + 12*b**2*c**3*d**2*m*x**2 - 4*b**2*
c**2*d**3*m**3*x**3 - 12*b**2*c**2*d**3*m**2*x**3 - 8*b**2*c**2*d**3*m*x**
3 + b**2*c*d**4*m**4*x**4 + 6*b**2*c*d**4*m**3*x**4 + 11*b**2*c*d**4*m**2*
x**4 + 6*b**2*c*d**4*m*x**4 + b**2*d**5*m**4*x**5 + 10*b**2*d**5*m**3*x**5
 + 35*b**2*d**5*m**2*x**5 + 50*b**2*d**5*m*x**5 + 24*b**2*d**5*x**5))/(d**
5*(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120))
```

3.394 $\int (c+dx)^m \left(a+bx^2\right) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (c+dx)^m \left(a+bx^2\right) \, dx = \frac{(bc^2+ad^2)\,(c+dx)^{1+m}}{d^3(1+m)} - \frac{2bc(c+dx)^{2+m}}{d^3(2+m)} + \frac{b(c+dx)^{3+m}}{d^3(3+m)}$$

output
```
(a*d^2+b*c^2)*(d*x+c)^(1+m)/d^3/(1+m)-2*b*c*(d*x+c)^(2+m)/d^3/(2+m)+b*(d*x
+c)^(3+m)/d^3/(3+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int (c+dx)^m \left(a+bx^2\right) \, dx = \frac{(c+dx)^{1+m}\left(\frac{bc^2+ad^2}{1+m} - \frac{2bc(c+dx)}{2+m} + \frac{b(c+dx)^2}{3+m}\right)}{d^3}$$

input
```
Integrate[(c + d*x)^m*(a + b*x^2),x]
```

output
```
((c + d*x)^(1 + m)*((b*c^2 + a*d^2)/(1 + m) - (2*b*c*(c + d*x))/(2 + m) +
(b*(c + d*x)^2)/(3 + m)))/d^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)(c+dx)^m\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{\left(ad^2+bc^2\right)(c+dx)^m}{d^2}-\frac{2bc(c+dx)^{m+1}}{d^2}+\frac{b(c+dx)^{m+2}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ad^2+bc^2\right)(c+dx)^{m+1}}{d^3(m+1)}-\frac{2bc(c+dx)^{m+2}}{d^3(m+2)}+\frac{b(c+dx)^{m+3}}{d^3(m+3)}$$

input
```
Int[(c + d*x)^m*(a + b*x^2),x]
```

output
```
((b*c^2 + a*d^2)*(c + d*x)^(1 + m))/(d^3*(1 + m)) - (2*b*c*(c + d*x)^(2 +
m))/(d^3*(2 + m)) + (b*(c + d*x)^(3 + m))/(d^3*(3 + m))
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

method	result
gosper	$\frac{(dx+c)^{1+m}\left(b\,d^{2}m^{2}x^{2}+3b\,d^{2}m\,x^{2}+a\,d^{2}m^{2}-2bcdmx+2b\,x^{2}d^{2}+5a\,d^{2}m-2bcdx+6a\,d^{2}+2b\,c^{2}\right)}{d^{3}\left(m^{3}+6m^{2}+11m+6\right)}$
orering	$\frac{(dx+c)\left(b\,d^{2}m^{2}x^{2}+3b\,d^{2}m\,x^{2}+a\,d^{2}m^{2}-2bcdmx+2b\,x^{2}d^{2}+5a\,d^{2}m-2bcdx+6a\,d^{2}+2b\,c^{2}\right)(dx+c)^{m}}{d^{3}\left(m^{3}+6m^{2}+11m+6\right)}$
risch	$\frac{(b\,d^{3}m^{2}x^{3}+bc\,d^{2}m^{2}x^{2}+3b\,d^{3}m\,x^{3}+a\,d^{3}m^{2}x+bcm\,x^{2}d^{2}+2b\,d^{3}x^{3}+ac\,d^{2}m^{2}+5a\,d^{3}mx-2b\,c^{2}dmx+5ac\,d^{2}m+6ax\,d^{3}+6a\,d^{2}c}{(2+m)(3+m)(1+m)d^{3}}$
norman	$\frac{b\,x^{3}\mathrm{e}^{m\ln(dx+c)}}{3+m}+\frac{c\left(a\,d^{2}m^{2}+5a\,d^{2}m+6a\,d^{2}+2b\,c^{2}\right)\mathrm{e}^{m\ln(dx+c)}}{d^{3}\left(m^{3}+6m^{2}+11m+6\right)}+\frac{\left(a\,d^{2}m^{2}+5a\,d^{2}m-2b\,c^{2}m+6a\,d^{2}\right)x\,\mathrm{e}^{m\ln(dx+c)}}{d^{2}\left(m^{3}+6m^{2}+11m+6\right)}+$
parallelrisch	$\frac{x^{3}(dx+c)^{m}b\,d^{3}m^{2}+3x^{3}(dx+c)^{m}b\,d^{3}m+x^{2}(dx+c)^{m}bc\,d^{2}m^{2}+2x^{3}(dx+c)^{m}b\,d^{3}+x^{2}(dx+c)^{m}bc\,d^{2}m+x(dx+c)^{m}a\,d^{3}m^{2}+5x}{d^{3}(m^{3}+6m}$

input
```
int((d*x+c)^m*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(d*x+c)^(1+m)/(m^3+6*m^2+11*m+6)*(b*d^2*m^2*x^2+3*b*d^2*m*x^2+a*d^2*
m^2-2*b*c*d*m*x+2*b*d^2*x^2+5*a*d^2*m-2*b*c*d*x+6*a*d^2+2*b*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(70) = 140.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.13

$$\int (c+dx)^m\left(a+bx^2\right)\,dx = \frac{(acd^2m^2+5\,acd^2m+2\,bc^3+6\,acd^2+(bd^3m^2+3\,bd^3m+2\,bd^3)x^3+(bcd^2m^2+bcd^2m)x^2+(ad^3m^2+6}{d^3m^3+6\,d^3m^2+11\,d^3m+6\,d^3}$$

input
```
integrate((d*x+c)^m*(b*x^2+a),x, algorithm="fricas")
```

output
```
(a*c*d^2*m^2 + 5*a*c*d^2*m + 2*b*c^3 + 6*a*c*d^2 + (b*d^3*m^2 + 3*b*d^3*m
+ 2*b*d^3)*x^3 + (b*c*d^2*m^2 + b*c*d^2*m)*x^2 + (a*d^3*m^2 + 6*a*d^3 - (2
*b*c^2*d - 5*a*d^3)*m)*x)*(d*x + c)^m/(d^3*m^3 + 6*d^3*m^2 + 11*d^3*m + 6*
d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(61) = 122$.

Time = 0.54 (sec) , antiderivative size = 952, normalized size of antiderivative = 13.60

$$\int (c+dx)^m \left(a+bx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**m*(b*x**2+a),x)
```

output
```
Piecewise((c**m*(a*x + b*x**3/3), Eq(d, 0)), (-a*d**2/(2*c**2*d**3 + 4*c*d
**4*x + 2*d**5*x**2) + 2*b*c**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d**4*x + 2
*d**5*x**2) + 3*b*c**2/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b*c*d*
x*log(c/d + x)/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b*c*d*x/(2*c**
2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 2*b*d**2*x**2*log(c/d + x)/(2*c**2*d*
*3 + 4*c*d**4*x + 2*d**5*x**2), Eq(m, -3)), (-a*d**2/(c*d**3 + d**4*x) - 2
*b*c**2*log(c/d + x)/(c*d**3 + d**4*x) - 2*b*c**2/(c*d**3 + d**4*x) - 2*b*
c*d*x*log(c/d + x)/(c*d**3 + d**4*x) + b*d**2*x**2/(c*d**3 + d**4*x), Eq(m
, -2)), (a*log(c/d + x)/d + b*c**2*log(c/d + x)/d**3 - b*c*x/d**2 + b*x**2
/(2*d), Eq(m, -1)), (a*c*d**2*m**2*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**2 +
 11*d**3*m + 6*d**3) + 5*a*c*d**2*m*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**2
+ 11*d**3*m + 6*d**3) + 6*a*c*d**2*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**2 +
 11*d**3*m + 6*d**3) + a*d**3*m**2*x*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**2
 + 11*d**3*m + 6*d**3) + 5*a*d**3*m*x*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**
2 + 11*d**3*m + 6*d**3) + 6*a*d**3*x*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**2
 + 11*d**3*m + 6*d**3) + 2*b*c**3*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**2 +
11*d**3*m + 6*d**3) - 2*b*c**2*d*m*x*(c + d*x)**m/(d**3*m**3 + 6*d**3*m**2
 + 11*d**3*m + 6*d**3) + b*c*d**2*m**2*x**2*(c + d*x)**m/(d**3*m**3 + 6*d*
*3*m**2 + 11*d**3*m + 6*d**3) + b*c*d**2*m*x**2*(c + d*x)**m/(d**3*m**3 +
6*d**3*m**2 + 11*d**3*m + 6*d**3) + b*d**3*m**2*x**3*(c + d*x)**m/(d**3...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int (c+dx)^m \left(a+bx^2\right) dx$$
$$= \frac{(dx+c)^{m+1}a}{d(m+1)} + \frac{((m^2+3m+2)d^3x^3 + (m^2+m)cd^2x^2 - 2c^2dmx + 2c^3)(dx+c)^m b}{(m^3+6m^2+11m+6)d^3}$$

input
```
integrate((d*x+c)^m*(b*x^2+a),x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a/(d*(m + 1)) + ((m^2 + 3*m + 2)*d^3*x^3 + (m^2 + m)*c*d
^2*x^2 - 2*c^2*d*m*x + 2*c^3)*(d*x + c)^m*b/((m^3 + 6*m^2 + 11*m + 6)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(70) = 140.

Time = 0.14 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.39

$$\int (c+dx)^m \left(a+bx^2\right) dx$$
$$= \frac{(dx+c)^m bd^3m^2x^3 + (dx+c)^m bcd^2m^2x^2 + 3(dx+c)^m bd^3mx^3 + (dx+c)^m ad^3m^2x + (dx+c)^m bcd^2mx}{}$$

input
```
integrate((d*x+c)^m*(b*x^2+a),x, algorithm="giac")
```

output
```
((d*x + c)^m*b*d^3*m^2*x^3 + (d*x + c)^m*b*c*d^2*m^2*x^2 + 3*(d*x + c)^m*b
*d^3*m*x^3 + (d*x + c)^m*a*d^3*m^2*x + (d*x + c)^m*b*c*d^2*m*x^2 + 2*(d*x
+ c)^m*b*d^3*x^3 + (d*x + c)^m*a*c*d^2*m^2 - 2*(d*x + c)^m*b*c^2*d*m*x + 5
*(d*x + c)^m*a*d^3*m*x + 5*(d*x + c)^m*a*c*d^2*m + 6*(d*x + c)^m*a*d^3*x +
 2*(d*x + c)^m*b*c^3 + 6*(d*x + c)^m*a*c*d^2)/(d^3*m^3 + 6*d^3*m^2 + 11*d^
3*m + 6*d^3)
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.33

$$\int (c+dx)^m \left(a+bx^2\right) \, dx = (c+d\,x)^m \left(\frac{b\,x^3\,(m^2+3\,m+2)}{m^3+6\,m^2+11\,m+6} + \frac{x\,(-2\,b\,c^2\,d\,m+a\,d^3\,m^2+5\,a\,d^3\,m+6\,a\,d^3)}{d^3\,(m^3+6\,m^2+11\,m+6)} + \frac{c\,(2\,b\,c^2+a\,d^2\,m^2+5\,a\,d^2\,m+6\,a\,d^2)}{d^3\,(m^3+6\,m^2+11\,m+6)} + \frac{b\,c\,m\,x^2\,(m+1)}{d\,(m^3+6\,m^2+11\,m+6)}\right)$$

input
```
int((a + b*x^2)*(c + d*x)^m,x)
```

output
```
(c + d*x)^m*((b*x^3*(3*m + m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) + (x*(6*a*d^
3 + a*d^3*m^2 + 5*a*d^3*m - 2*b*c^2*d*m))/(d^3*(11*m + 6*m^2 + m^3 + 6)) +
 (c*(6*a*d^2 + 2*b*c^2 + a*d^2*m^2 + 5*a*d^2*m))/(d^3*(11*m + 6*m^2 + m^3
+ 6)) + (b*c*m*x^2*(m + 1))/(d*(11*m + 6*m^2 + m^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.03

$$\int (c+dx)^m \left(a+bx^2\right) \, dx = \frac{(dx+c)^m \left(b\,d^3m^2x^3+bc\,d^2m^2x^2+3b\,d^3m\,x^3+a\,d^3m^2x+bc\,d^2m\,x^2+2b\,d^3x^3+ac\,d^2m^2+5a\,d^3mx-\right.}{d^3\,(m^3+6m^2+11m+6)}$$

input
```
int((d*x+c)^m*(b*x^2+a),x)
```

output
```
((c + d*x)**m*(a*c*d**2*m**2 + 5*a*c*d**2*m + 6*a*c*d**2 + a*d**3*m**2*x +
 5*a*d**3*m*x + 6*a*d**3*x + 2*b*c**3 - 2*b*c**2*d*m*x + b*c*d**2*m**2*x**
2 + b*c*d**2*m*x**2 + b*d**3*m**2*x**3 + 3*b*d**3*m*x**3 + 2*b*d**3*x**3))
/(d**3*(m**3 + 6*m**2 + 11*m + 6))
```

3.395 $\int \frac{(c+dx)^m}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int \frac{(c+dx)^m}{a+bx^2}\,dx = \frac{(c+dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{2\left(\sqrt{-a}\sqrt{b}c+ad\right)(1+m)} + \frac{(c+dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{2a\left(\frac{\sqrt{b}c}{\sqrt{-a}}+d\right)(1+m)}$$

output
```
1/2*(d*x+c)^(1+m)*hypergeom([1, 1+m],[2+m],b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)
^(1/2)*d))/((-a)^(1/2)*b^(1/2)*c+a*d)/(1+m)+1/2*(d*x+c)^(1+m)*hypergeom([1
, 1+m],[2+m],b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d))/a/(b^(1/2)*c/(-a)^(
1/2)+d)/(1+m)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^m}{a+bx^2}\,dx$$
$$=\frac{(c+dx)^{1+m}\left(\frac{\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{\sqrt{b}c-\sqrt{-a}d}-\frac{\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{\sqrt{b}c+\sqrt{-a}d}\right)}{2\sqrt{-a}(1+m)}$$

input
```
Integrate[(c + d*x)^m/(a + b*x^2),x]
```

output
```
((c + d*x)^(1 + m)*(Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*x))
/(Sqrt[b]*c - Sqrt[-a]*d)]/(Sqrt[b]*c - Sqrt[-a]*d) - Hypergeometric2F1[1,
 1 + m, 2 + m, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]/(Sqrt[b]*c +
Sqrt[-a]*d)))/(2*Sqrt[-a]*(1 + m))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+bx^2}\,dx$$
$$\Big\downarrow \text{485}$$
$$\int \left(\frac{\sqrt{-a}(c+dx)^m}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}(c+dx)^m}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{b}c-\sqrt{-a}d\right)}-\frac{(c+dx)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{-a}d+\sqrt{b}c\right)}$$

input
```
Int[(c + d*x)^m/(a + b*x^2),x]
```

output
```
((c + d*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*x))/
(Sqrt[b]*c - Sqrt[-a]*d)])/(2*Sqrt[-a]*(Sqrt[b]*c - Sqrt[-a]*d)*(1 + m)) -
 ((c + d*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*x))
/(Sqrt[b]*c + Sqrt[-a]*d)])/(2*Sqrt[-a]*(Sqrt[b]*c + Sqrt[-a]*d)*(1 + m))
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(dx+c)^m}{b\,x^2+a}dx$$

input
```
int((d*x+c)^m/(b*x^2+a),x)
```

output
```
int((d*x+c)^m/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{a+bx^2} \, dx = \int \frac{(dx+c)^m}{bx^2+a} \, dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{a+bx^2} \, dx = \int \frac{(c+dx)^m}{a+bx^2} \, dx$$

input
```
integrate((d*x+c)**m/(b*x**2+a),x)
```

output
```
Integral((c + d*x)**m/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^m}{a+bx^2} \, dx = \int \frac{(dx+c)^m}{bx^2+a} \, dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{a+bx^2}\,dx = \int \frac{(dx+c)^m}{bx^2+a}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{a+bx^2}\,dx = \int \frac{(c+d\,x)^m}{b\,x^2+a}\,dx$$

input
```
int((c + d*x)^m/(a + b*x^2),x)
```

output
```
int((c + d*x)^m/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{a+bx^2}\,dx = \int \frac{(dx+c)^m}{b\,x^2+a}dx$$

input
```
int((d*x+c)^m/(b*x^2+a),x)
```

output
```
int((c + d*x)**m/(a + b*x**2),x)
```

3.396 $\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 304

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx = \frac{(ad+bcx)(c+dx)^{1+m}}{2a\left(bc^2+ad^2\right)\left(a+bx^2\right)} - \frac{\left(bc^2+ad^2(1-m)+\sqrt{-a}\sqrt{b}cdm\right)(c+dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{4(-a)^{3/2}\left(\sqrt{b}c-\sqrt{-a}d\right)\left(bc^2+ad^2\right)(1+m)} + \frac{\left(bc^2+ad^2(1-m)-\sqrt{-a}\sqrt{b}cdm\right)(c+dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{4(-a)^{3/2}\left(\sqrt{b}c+\sqrt{-a}d\right)\left(bc^2+ad^2\right)(1+m)}$$

output

```
1/2*(b*c*x+a*d)*(d*x+c)^(1+m)/a/(a*d^2+b*c^2)/(b*x^2+a)-1/4*(b*c^2+a*d^2*(
1-m)+(-a)^(1/2)*b^(1/2)*c*d*m)*(d*x+c)^(1+m)*hypergeom([1, 1+m],[2+m],b^(1
/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d))/(-a)^(3/2)/(b^(1/2)*c-(-a)^(1/2)*d)/
(a*d^2+b*c^2)/(1+m)+1/4*(b*c^2+a*d^2*(1-m)-(-a)^(1/2)*b^(1/2)*c*d*m)*(d*x+
c)^(1+m)*hypergeom([1, 1+m],[2+m],b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d)
)/(-a)^(3/2)/(b^(1/2)*c+(-a)^(1/2)*d)/(a*d^2+b*c^2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx = \frac{(c+dx)^{1+m}\left(\frac{2(ad+bcx)}{a+bx^2}+\frac{\left(bc^2-ad^2(-1+m)+\sqrt{-a}\sqrt{b}cdm\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{\sqrt{-a}\left(\sqrt{b}c-\sqrt{-a}d\right)(1+m)}+\frac{\left(-bc^2+ad^2(-1+m)\cdots\right.}{\cdots}\right.}{4a\left(bc^2+ad^2\right)}$$

input
```
Integrate[(c + d*x)^m/(a + b*x^2)^2,x]
```

output
```
((c + d*x)^(1 + m)*((2*(a*d + b*c*x))/(a + b*x^2) + ((b*c^2 - a*d^2*(-1 +
m) + Sqrt[-a]*Sqrt[b]*c*d*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(
c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)])/(Sqrt[-a]*(Sqrt[b]*c - Sqrt[-a]*d)*(1
 + m)) + ((-(b*c^2) + a*d^2*(-1 + m) + Sqrt[-a]*Sqrt[b]*c*d*m)*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)])/(Sq
rt[-a]*(Sqrt[b]*c + Sqrt[-a]*d)*(1 + m))))/(4*a*(b*c^2 + a*d^2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 496$$

$$\frac{(c+dx)^{m+1}(ad+bcx)}{2a(a+bx^2)(ad^2+bc^2)}-\frac{\int -\frac{(c+dx)^m\left(bc^2-bdmxc+ad^2(1-m)\right)}{bx^2+a}dx}{2a(ad^2+bc^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(c+dx)^m (bc^2-bdmxc+ad^2(1-m))}{bx^2+a}dx}{2a(ad^2+bc^2)}+\frac{(c+dx)^{m+1}(ad+bcx)}{2a(a+bx^2)(ad^2+bc^2)}$$

$$\Big\downarrow \text{ 657}$$

$$\frac{\int \left(\frac{\left(\sqrt{-a}(bc^2+ad^2(1-m))+a\sqrt{b}cdm\right)(c+dx)^m}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\left(\sqrt{-a}(bc^2+ad^2(1-m))-a\sqrt{b}cdm\right)(c+dx)^m}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2a(ad^2+bc^2)}+\frac{(c+dx)^{m+1}(ad+bcx)}{2a(a+bx^2)(ad^2+bc^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(c+dx)^{m+1}\left(\sqrt{-a}\sqrt{b}cdm+ad^2(1-m)+bc^2\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{b}c-\sqrt{-a}d\right)}-\frac{(c+dx)^{m+1}\left(-\sqrt{-a}\sqrt{b}cdm+ad^2(1-m)+bc^2\right)\text{ H}}{2\sqrt{-a}(m+1)(}}{2a(ad^2+bc^2)}$$
$$\frac{(c+dx)^{m+1}(ad+bcx)}{2a(a+bx^2)(ad^2+bc^2)}$$

input
```
Int[(c + d*x)^m/(a + b*x^2)^2,x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(1 + m))/(2*a*(b*c^2 + a*d^2)*(a + b*x^2)) + (((b
*c^2 + a*d^2*(1 - m) + Sqrt[-a]*Sqrt[b]*c*d*m)*(c + d*x)^(1 + m)*Hypergeom
etric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)])/(
2*Sqrt[-a]*(Sqrt[b]*c - Sqrt[-a]*d)*(1 + m)) - ((b*c^2 + a*d^2*(1 - m) - S
qrt[-a]*Sqrt[b]*c*d*m)*(c + d*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)])/(2*Sqrt[-a]*(Sqrt[b]*c +
Sqrt[-a]*d)*(1 + m)))/(2*a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(dx+c)^m}{(b\,x^2+a)^2}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^2,x)
```

output
```
int((d*x+c)^m/(b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx=\int \frac{(dx+c)^m}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx=\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x+c)**m/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x)**m/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx=\int \frac{(dx+c)^m}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx = \int \frac{(dx+c)^m}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx = \int \frac{(c+d\,x)^m}{(b\,x^2+a)^2}\,dx$$

input
```
int((c + d*x)^m/(a + b*x^2)^2,x)
```

output
```
int((c + d*x)^m/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^2}\,dx = \int \frac{(dx+c)^m}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^2,x)
```

output
```
int((c + d*x)**m/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.397 $\int \frac{(c+dx)^m}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 474

$$\int \frac{(c+dx)^m}{(a+bx^2)^3} \, dx = \frac{(ad+bcx)(c+dx)^{1+m}}{4a(bc^2+ad^2)(a+bx^2)^2}$$
$$+ \frac{(c+dx)^{1+m}(ad(ad^2(3-m)+bc^2(1+m))+bc(3bc^2+ad^2(5-2m))x)}{8a^2(bc^2+ad^2)^2(a+bx^2)}$$
$$+ \frac{\left(3b^2c^4+\sqrt{-a}\sqrt{b}cd(3bc^2+ad^2(5-2m))m+abc^2d^2(6-2m-m^2)+a^2d^4(3-4m+m^2)\right)(c+dx)^{1}}{16(-a)^{5/2}\left(\sqrt{b}c-\sqrt{-a}d\right)(bc^2+ad^2)^2(1+m)}$$
$$- \frac{\left(3b^2c^4-\sqrt{-a}\sqrt{b}cd(3bc^2+ad^2(5-2m))m+abc^2d^2(6-2m-m^2)+a^2d^4(3-4m+m^2)\right)(c+dx)^{1}}{16(-a)^{5/2}\left(\sqrt{b}c+\sqrt{-a}d\right)(bc^2+ad^2)^2(1+m)}$$

output
```
1/4*(b*c*x+a*d)*(d*x+c)^(1+m)/a/(a*d^2+b*c^2)/(b*x^2+a)^2+1/8*(d*x+c)^(1+m
)*(a*d*(a*d^2*(3-m)+b*c^2*(1+m))+b*c*(3*b*c^2+a*d^2*(5-2*m))*x)/a^2/(a*d^2
+b*c^2)^2/(b*x^2+a)+1/16*(3*b^2*c^4+(-a)^(1/2)*b^(1/2)*c*d*(3*b*c^2+a*d^2*
(5-2*m))*m+a*b*c^2*d^2*(-m^2-2*m+6)+a^2*d^4*(m^2-4*m+3))*(d*x+c)^(1+m)*hyp
ergeom([1, 1+m],[2+m],b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d))/(-a)^(5/2)
/(b^(1/2)*c-(-a)^(1/2)*d)/(a*d^2+b*c^2)^2/(1+m)-1/16*(3*b^2*c^4-(-a)^(1/2)
*b^(1/2)*c*d*(3*b*c^2+a*d^2*(5-2*m))*m+a*b*c^2*d^2*(-m^2-2*m+6)+a^2*d^4*(m
^2-4*m+3))*(d*x+c)^(1+m)*hypergeom([1, 1+m],[2+m],b^(1/2)*(d*x+c)/(b^(1/2)
*c+(-a)^(1/2)*d))/(-a)^(5/2)/(b^(1/2)*c+(-a)^(1/2)*d)/(a*d^2+b*c^2)^2/(1+m
)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)^m}{(a+bx^2)^3}\,dx$$

$$= \frac{(c+dx)^{1+m}\left(\frac{4a(bc^2+ad^2)(ad+bcx)}{(a+bx^2)^2}+\frac{2(-a^2d^3(-3+m)+3b^2c^3x+abcd(c(1+m)+d(5-2m)x))}{a+bx^2}+\frac{\left(a\sqrt{b}cd\left(3bc^2+ad^2(5-2m)\right)m+\sqrt{-a}\right(}{}\right.}{}$$

input
```
Integrate[(c + d*x)^m/(a + b*x^2)^3,x]
```

output
```
((c + d*x)^(1 + m)*((4*a*(b*c^2 + a*d^2)*(a*d + b*c*x))/(a + b*x^2)^2 + (2
*(-(a^2*d^3*(-3 + m)) + 3*b^2*c^3*x + a*b*c*d*(c*(1 + m) + d*(5 - 2*m)*x))
)/(a + b*x^2) + (((a*Sqrt[b]*c*d*(3*b*c^2 + a*d^2*(5 - 2*m))*m + Sqrt[-a]*
(-3*b^2*c^4 - a^2*d^4*(3 - 4*m + m^2) + a*b*c^2*d^2*(-6 + 2*m + m^2)))*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*
d)])/(Sqrt[b]*c - Sqrt[-a]*d) + ((a*Sqrt[b]*c*d*(3*b*c^2 + a*d^2*(5 - 2*m)
)*m + Sqrt[-a]*(3*b^2*c^4 + a^2*d^4*(3 - 4*m + m^2) - a*b*c^2*d^2*(-6 + 2*
m + m^2)))*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*x))/(Sqrt[b]
*c + Sqrt[-a]*d)])/(Sqrt[b]*c + Sqrt[-a]*d))/(a*(1 + m))))/(16*a^2*(b*c^2
+ a*d^2)^2)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {496, 25, 686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{496}$$

$$\frac{(c+dx)^{m+1}(ad+bcx)}{4a(a+bx^2)^2(ad^2+bc^2)}-\frac{\int -\frac{(c+dx)^m(3bc^2+bd(2-m)xc+ad^2(3-m))}{(bx^2+a)^2}dx}{4a(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(c+dx)^m(3bc^2+bd(2-m)xc+ad^2(3-m))}{(bx^2+a)^2}dx}{4a(ad^2+bc^2)}+\frac{(c+dx)^{m+1}(ad+bcx)}{4a(a+bx^2)^2(ad^2+bc^2)}$$

$$\Big\downarrow \text{686}$$

$$\frac{\frac{(c+dx)^{m+1}(bcx(ad^2(5-2m)+3bc^2)+ad(ad^2(3-m)+bc^2(m+1)))}{2a(a+bx^2)(ad^2+bc^2)}-\frac{\int -\frac{b(c+dx)^m\left(3b^2c^4+abd^2\left(-m^2-2m+6\right)c^2-bd\left(3bc^2+ad^2(5-2m)\right)mxc+a^2d^4}{bx^2+a}}{2ab(ad^2+bc^2)}}{4a(ad^2+bc^2)}+\frac{(c+dx)^{m+1}(ad+bcx)}{4a(a+bx^2)^2(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b(c+dx)^m\left(3b^2c^4+abd^2\left(-m^2-2m+6\right)c^2-bd\left(3bc^2+ad^2(5-2m)\right)mxc+a^2d^4\left(m^2-4m+3\right)\right)}{bx^2+a}dx}{2ab(ad^2+bc^2)}+\frac{(c+dx)^{m+1}(bcx(ad^2(5-2m)+3bc^2)+ad(ad^2(3-m)}{2a(a+bx^2)(ad^2+bc^2)}}{4a(ad^2+bc^2)}+\frac{(c+dx)^{m+1}(ad+bcx)}{4a(a+bx^2)^2(ad^2+bc^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(c+dx)^m\left(3b^2c^4+abd^2\left(-m^2-2m+6\right)c^2-bd\left(3bc^2+ad^2(5-2m)\right)mxc+a^2d^4\left(m^2-4m+3\right)\right)}{bx^2+a}dx}{2a(ad^2+bc^2)}+\frac{(c+dx)^{m+1}\left(bcx\left(ad^2(5-2m)+3bc^2\right)+ad\left(ad^2(3-m)-\right.\right.}{2a(a+bx^2)(ad^2+bc^2)}}{4a\left(ad^2+bc^2\right)}$$

$$\frac{(c+dx)^{m+1}(ad+bcx)}{4a\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{657}$$

$$\frac{\frac{\int \left(\frac{\left(a\sqrt{b}cd\left(3bc^2+ad^2(5-2m)\right)m+\sqrt{-a}\left(3b^2c^4+abd^2\left(-m^2-2m+6\right)c^2+a^2d^4\left(m^2-4m+3\right)\right)\right)(c+dx)^m}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\left(\sqrt{-a}\left(3b^2c^4+abd^2\left(-m^2-2m+6\right)c^2+a^2d^4\left(m^2-4m\right.\right.\right.}{2a\left(\sqrt{b}x+\sqrt{\right.}}\right.}{2a(ad^2+bc^2)}}{4a\left(ad^2+bc^2\right)}$$

$$\frac{(c+dx)^{m+1}(ad+bcx)}{4a\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(c+dx)^{m+1}\left(a\sqrt{b}cdm\left(ad^2(5-2m)+3bc^2\right)-\sqrt{-a}\left(a^2d^4\left(m^2-4m+3\right)+abc^2d^2\left(-m^2-2m+6\right)+3b^2c^4\right)\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{2a(m+1)\left(\sqrt{b}c-\sqrt{-a}d\right)}+\frac{(c+da}{}}{2a(ad^2+bc^2)}$$

$$\frac{(c+dx)^{m+1}(ad+bcx)}{4a\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

input
```
Int[(c + d*x)^m/(a + b*x^2)^3,x]
```

output
```
((a*d + b*c*x)*(c + d*x)^(1 + m))/(4*a*(b*c^2 + a*d^2)*(a + b*x^2)^2) + ((
(c + d*x)^(1 + m)*(a*d*(a*d^2*(3 - m) + b*c^2*(1 + m)) + b*c*(3*b*c^2 + a*
d^2*(5 - 2*m))*x))/(2*a*(b*c^2 + a*d^2)*(a + b*x^2)) + (((a*Sqrt[b]*c*d*(3
*b*c^2 + a*d^2*(5 - 2*m))*m - Sqrt[-a]*(3*b^2*c^4 + a*b*c^2*d^2*(6 - 2*m -
 m^2) + a^2*d^4*(3 - 4*m + m^2)))*(c + d*x)^(1 + m)*Hypergeometric2F1[1, 1
 + m, 2 + m, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)])/(2*a*(Sqrt[b]*
c - Sqrt[-a]*d)*(1 + m)) + ((a*Sqrt[b]*c*d*(3*b*c^2 + a*d^2*(5 - 2*m))*m +
 Sqrt[-a]*(3*b^2*c^4 + a*b*c^2*d^2*(6 - 2*m - m^2) + a^2*d^4*(3 - 4*m + m^
2)))*(c + d*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[b]*(c + d*
x))/(Sqrt[b]*c + Sqrt[-a]*d)])/(2*a*(Sqrt[b]*c + Sqrt[-a]*d)*(1 + m)))/(2*
a*(b*c^2 + a*d^2)))/(4*a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(dx+c)^m}{(b\,x^2+a)^3}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^3,x)
```

output
```
int((d*x+c)^m/(b*x^2+a)^3,x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^3}\,dx=\int \frac{(dx+c)^m}{(bx^2+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+bx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**m/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^3}\,dx = \int \frac{(dx+c)^m}{(bx^2+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^3}\,dx = \int \frac{(dx+c)^m}{(bx^2+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+bx^2)^3}\,dx = \int \frac{(c+d\,x)^m}{(b\,x^2+a)^3}\,dx$$

input
```
int((c + d*x)^m/(a + b*x^2)^3,x)
```

output
```
int((c + d*x)^m/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^3} \, dx = \int \frac{(dx+c)^m}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^3,x)
```

output
```
int((c + d*x)**m/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)
```

3.398 $\int (c+dx)^m \left(a+bx^2\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} dx = \frac{(c+dx)^{1+m}\left(a+bx^2\right)^{3/2}\operatorname{AppellF1}\left(1+m,-\frac{3}{2},-\frac{3}{2},2+m,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(1+m)\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}}$$

output
```
(d*x+c)^(1+m)*(b*x^2+a)^(3/2)*AppellF1(1+m,-3/2,-3/2,2+m,(d*x+c)/(c-(-a)^(
1/2)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/(1+m)/(1-(d*x+c)/(c-(-
a)^(1/2)*d/b^(1/2)))^(3/2)/(1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^(3/2)
```

Mathematica [F]

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} dx = \int (c+dx)^m \left(a+bx^2\right)^{3/2} dx$$

input
```
Integrate[(c + d*x)^m*(a + b*x^2)^(3/2),x]
```

output
```
Integrate[(c + d*x)^m*(a + b*x^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (c+dx)^m \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a+bx^2\right)^{3/2} \int (c+dx)^m \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2} \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2} d(c+dx)}{d \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{3/2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(a+bx^2\right)^{3/2} (c+dx)^{m+1} \text{AppellF1}\left(m+1,-\frac{3}{2},-\frac{3}{2},m+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(m+1) \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{3/2}}$$

input
```
Int[(c + d*x)^m*(a + b*x^2)^(3/2),x]
```

output
```
((c + d*x)^(1 + m)*(a + b*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 + m, (c
 + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])
/(d*(1 + m)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^(3/2)*(1 - (c + d*x
)/(c + (Sqrt[-a]*d)/Sqrt[b]))^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{m} \left(b\,x^{2}+a\right)^{\frac{3}{2}}dx$$

input
```
int((d*x+c)^m*(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^m*(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}}(dx+c)^m\, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*(d*x + c)^m, x)
```

Sympy [F]

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} \, dx = \int \left(a+bx^2\right)^{\frac{3}{2}} (c+dx)^m \, dx$$

input
```
integrate((d*x+c)**m*(b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x)**m, x)
```

Maxima [F]

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} \, dx = \int (bx^2+a)^{\frac{3}{2}}(dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x + c)^m, x)
```

Giac [F]

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} \, dx = \int (bx^2+a)^{\frac{3}{2}}(dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} \, dx = \int {\left(b\,x^2+a\right)}^{3/2}\,{\left(c+d\,x\right)}^m \,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)^m,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \left(a+bx^2\right)^{3/2} \, dx = \int (dx+c)^m \left(b\,x^2+a\right)^{\frac{3}{2}} dx$$

input
```
int((d*x+c)^m*(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^m*(b*x^2+a)^(3/2),x)
```

3.399 $\int (c+dx)^m \sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int (c+dx)^m \sqrt{a+bx^2}\,dx = \frac{(c+dx)^{1+m}\sqrt{a+bx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(1+m)\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}}}$$

output
```
(d*x+c)^(1+m)*(b*x^2+a)^(1/2)*AppellF1(1+m,-1/2,-1/2,2+m,(d*x+c)/(c-(-a)^(
1/2)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/(1+m)/(1-(d*x+c)/(c-(-
a)^(1/2)*d/b^(1/2)))^(1/2)/(1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int (c+dx)^m \sqrt{a+bx^2}\,dx = \frac{(c+dx)^{1+m}\sqrt{a+bx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{-\frac{a}{b}}d}\right)}{d(1+m)\sqrt{\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}}\sqrt{\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}}}$$

input
```
Integrate[(c + d*x)^m*Sqrt[a + b*x^2],x]
```

output
```
((c + d*x)^(1 + m)*Sqrt[a + b*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (c +
 d*x)/(c - Sqrt[-(a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*(1 + m)*Sq
rt[(d*(Sqrt[-(a/b)] - x))/(c + Sqrt[-(a/b)]*d)]*Sqrt[(d*(Sqrt[-(a/b)] + x)
)/(-c + Sqrt[-(a/b)]*d)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^m \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\sqrt{a+bx^2}\int (c+dx)^m\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}}d(c+dx)}{d\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{a+bx^2}(c+dx)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(m+1)\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}}}$$

input
```
Int[(c + d*x)^m*Sqrt[a + b*x^2],x]
```

output
```
((c + d*x)^(1 + m)*Sqrt[a + b*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (c +
 d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(
d*(1 + m)*Sqrt[1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b])]*Sqrt[1 - (c + d*x
)/(c + (Sqrt[-a]*d)/Sqrt[b])])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^m \sqrt{b\,x^2+a}dx$$

input
```
int((d*x+c)^m*(b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^m*(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (c+dx)^m\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(d*x + c)^m, x)
```

Sympy [F]

$$\int (c+dx)^m\sqrt{a+bx^2}\,dx=\int \sqrt{a+bx^2}(c+dx)^m\,dx$$

input
```
integrate((d*x+c)**m*(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x)**m, x)
```

Maxima [F]

$$\int (c+dx)^m\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x + c)^m, x)
```

Giac [F]

$$\int (c+dx)^m\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m\sqrt{a+bx^2}\,dx=\int \sqrt{b\,x^2+a}\,(c+d\,x)^m\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^m,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m\sqrt{a+bx^2}\,dx=\int (dx+c)^m\sqrt{b\,x^2+a}dx$$

input
```
int((d*x+c)^m*(b*x^2+a)^(1/2),x)
```

output
```
int((c + d*x)**m*sqrt(a + b*x**2),x)
```

3.400 $\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx = \frac{(c+dx)^{1+m}\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(1+m)\sqrt{a+bx^2}}$$

output
```
(d*x+c)^(1+m)*(1-(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)))^(1/2)*(1-(d*x+c)/(c+(-a
)^(1/2)*d/b^(1/2)))^(1/2)*AppellF1(1+m,1/2,1/2,2+m,(d*x+c)/(c-(-a)^(1/2)*d
/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/(1+m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}}\sqrt{\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}}(c+dx)^{1+m}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{-\frac{a}{b}}d}\right)}{d(1+m)\sqrt{a+bx^2}}$$

input
```
Integrate[(c + d*x)^m/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[(d*(Sqrt[-(a/b)] - x))/(c + Sqrt[-(a/b)]*d)]*Sqrt[(d*(Sqrt[-(a/b)] +
 x))/(-c + Sqrt[-(a/b)]*d)]*(c + d*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2
+ m, (c + d*x)/(c - Sqrt[-(a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*(
1 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{514, 150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \; 514$$

$$\frac{\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}}\int \frac{(c+dx)^m}{\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}}}d(c+dx)}{d\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 150$$

$$\frac{(c+dx)^{m+1}\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}}\operatorname{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(m+1)\sqrt{a+bx^2}}$$

input
```
Int[(c + d*x)^m/Sqrt[a + b*x^2],x]
```

output
```
((c + d*x)^(1 + m)*Sqrt[1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b])]*Sqrt[1 -
 (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (c
 + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])
/(d*(1 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^m/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/sqrt(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x+c)**m/(b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x)**m/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx=\int \frac{{\left(c+d\,x\right)}^m}{\sqrt{b\,x^2+a}} \,d x$$

input
```
int((c + d*x)^m/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^m/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{\sqrt{a+bx^2}}\,dx=\int \frac{(dx+c)^m}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^(1/2),x)
```

output
```
int((c + d*x)**m/sqrt(a + b*x**2),x)
```

3.401 $\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx=\frac{(c+dx)^{1+m}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}\operatorname{AppellF1}\left(1+m,\frac{3}{2},\frac{3}{2},2+m,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\ldots\right)}{d(1+m)\left(a+bx^2\right)^{3/2}}$$

output
```
(d*x+c)^(1+m)*(1-(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)))^(3/2)*(1-(d*x+c)/(c+(-a
)^(1/2)*d/b^(1/2)))^(3/2)*AppellF1(1+m,3/2,3/2,2+m,(d*x+c)/(c-(-a)^(1/2)*d
/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/(1+m)/(b*x^2+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx=\frac{\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{3/2}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{3/2}(c+dx)^{1+m}\operatorname{AppellF1}\left(1+m,\frac{3}{2},\frac{3}{2},2+m,\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\ldots\right)}{d(1+m)\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(c + d*x)^m/(a + b*x^2)^(3/2),x]
```

output
```
(((d*(Sqrt[-(a/b)] - x))/(c + Sqrt[-(a/b)]*d))^(3/2)*((d*(Sqrt[-(a/b)] + x
))/(-c + Sqrt[-(a/b)]*d))^(3/2)*(c + d*x)^(1 + m)*AppellF1[1 + m, 3/2, 3/2
, 2 + m, (c + d*x)/(c - Sqrt[-(a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/
(d*(1 + m)*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{3/2}\int \frac{(c+dx)^m}{\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}}d(c+dx)}{d\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{(c+dx)^{m+1}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{3/2}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{3/2}\text{AppellF1}\left(m+1,\frac{3}{2},\frac{3}{2},m+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(m+1)\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(c + d*x)^m/(a + b*x^2)^(3/2),x]
```

output
```
((c + d*x)^(1 + m)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^(3/2)*(1 - (
c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b]))^(3/2)*AppellF1[1 + m, 3/2, 3/2, 2 + m
, (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b
])])/(d*(1 + m)*(a + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^m/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(d*x + c)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^m}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**m/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**m/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)^m}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c + d*x)^m/(a + b*x^2)^(3/2),x)
```

output
```
int((c + d*x)^m/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}dx$$

input
```
int((d*x+c)^m/(b*x^2+a)^(3/2),x)
```

output
```
int((c + d*x)**m/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)
```

3.402 $\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx = -2^{-\frac{1}{2}+m}11^m\sqrt{2-x}\,\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\frac{4(2-x)}{11},\frac{2-x}{4}\right)$$

output
```
-2^(-1/2+m)*11^m*(2-x)^(1/2)*AppellF1(1/2,1/2,-m,3/2,1/2-1/4*x,8/11-4/11*x
)
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx = \frac{2^{-\frac{1}{2}+m}(3+4x)^{1+m}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{1}{5}(-3-4x),\frac{1}{11}(3+4x)\right)}{\sqrt{55}(1+m)}$$

input
```
Integrate[(6 + 8*x)^m/Sqrt[8 - 2*x^2],x]
```

output
```
(2^(-1/2 + m)*(3 + 4*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (-3 - 4*x
)/5, (3 + 4*x)/11])/(Sqrt[55]*(1 + m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {513, 27, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(8x+6)^m}{\sqrt{8-2x^2}}dx$$

$$\Big\downarrow \text{ 513}$$

$$\frac{\int \frac{2(8x+6)^m}{\sqrt{2-x}\sqrt{x+2}}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(8x+6)^m}{\sqrt{2-x}\sqrt{x+2}}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{ 155}$$

$$-2^{m-\frac{1}{2}}11^m\sqrt{2-x}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{2-x}{4},\frac{4(2-x)}{11}\right)$$

input
```
Int[(6 + 8*x)^m/Sqrt[8 - 2*x^2],x]
```

output
```
-(2^(-1/2 + m)*11^m*Sqrt[2 - x]*AppellF1[1/2, 1/2, -m, 3/2, (2 - x)/4, (4*
(2 - x))/11])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(8x+6)^m}{\sqrt{-2x^2+8}}dx$$

input
```
int((8*x+6)^m/(-2*x^2+8)^(1/2),x)
```

output
```
int((8*x+6)^m/(-2*x^2+8)^(1/2),x)
```

Fricas [F]

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx = \int \frac{(8\,x+6)^m}{\sqrt{-2\,x^2+8}}\,dx$$

input
```
integrate((6+8*x)^m/(-2*x^2+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(-1/2*sqrt(-2*x^2 + 8)*(8*x + 6)^m/(x^2 - 4), x)
```

Sympy [F]

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx = \frac{\sqrt{2}\cdot 2^m \int \frac{(4x+3)^m}{\sqrt{4-x^2}}\,dx}{2}$$

input
```
integrate((6+8*x)**m/(-2*x**2+8)**(1/2),x)
```

output
```
sqrt(2)*2**m*Integral((4*x + 3)**m/sqrt(4 - x**2), x)/2
```

Maxima [F]

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx = \int \frac{(8\,x+6)^m}{\sqrt{-2\,x^2+8}}\,dx$$

input
```
integrate((6+8*x)^m/(-2*x^2+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate((8*x + 6)^m/sqrt(-2*x^2 + 8), x)
```

Giac [F]

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx = \int \frac{(8\,x+6)^m}{\sqrt{-2\,x^2+8}}\,dx$$

input
```
integrate((6+8*x)^m/(-2*x^2+8)^(1/2),x, algorithm="giac")
```

output
```
integrate((8*x + 6)^m/sqrt(-2*x^2 + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx=\int \frac{{\left(8\,x+6\right)}^m}{\sqrt{8-2\,x^2}}\,dx$$

input
```
int((8*x + 6)^m/(8 - 2*x^2)^(1/2),x)
```

output
```
int((8*x + 6)^m/(8 - 2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(6+8x)^m}{\sqrt{8-2x^2}}\,dx=\frac{\int \frac{(8x+6)^m}{\sqrt{-x^2+4}}dx}{\sqrt{2}}$$

input
```
int((6+8*x)^m/(-2*x^2+8)^(1/2),x)
```

output
```
int((8*x + 6)**m/sqrt( - x**2 + 4),x)/sqrt(2)
```

3.403 $\int \frac{(6+8x)^m}{\sqrt{8+2x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}} \, dx = \frac{i73^m\sqrt{2-ix}((3+8i)(3+4x))^{-m}(6+8x)^m \operatorname{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\left(\frac{32}{73}-\frac{12i}{73}\right)(2-ix),\frac{1}{4}(2-ix)\right)}{\sqrt{2}}$$

output
```
1/2*I*73^m*(2-I*x)^(1/2)*(6+8*x)^m*AppellF1(1/2,-m,1/2,3/2,(32/73-12/73*I)
*(2-I*x),1/2-1/4*I*x)*2^(1/2)/(((3+8*I)*(3+4*x))^m)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}} \, dx = \frac{2^{-\frac{1}{2}+m}\sqrt{(8+3i)(2+ix)}\sqrt{(-3-8i)(2i+x)}(3+4x)^{1+m} \operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\left(\frac{3}{73}+\frac{8i}{73}\right)(3+4\cdots}{73(1+m)\sqrt{4+x^2}}$$

input
```
Integrate[(6 + 8*x)^m/Sqrt[8 + 2*x^2],x]
```

output
```
(2^(-1/2 + m)*Sqrt[(8 + 3*I)*(2 + I*x)]*Sqrt[(-3 - 8*I)*(2*I + x)]*(3 + 4*
x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (3/73 + (8*I)/73)*(3 + 4*x), (
3/73 - (8*I)/73)*(3 + 4*x)])/(73*(1 + m)*Sqrt[4 + x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(8x+6)^m}{\sqrt{2x^2+8}}dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\sqrt{1-\left(\frac{3}{73}+\frac{8i}{73}\right)(4x+3)}\sqrt{1-\left(\frac{3}{73}-\frac{8i}{73}\right)(4x+3)}\int \frac{(8x+6)^m}{\sqrt{1-\left(\frac{3}{146}+\frac{4i}{73}\right)(8x+6)}\sqrt{1-\left(\frac{3}{146}-\frac{4i}{73}\right)(8x+6)}}d(8x+6)}{8\sqrt{2}\sqrt{x^2+4}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{1-\left(\frac{3}{73}+\frac{8i}{73}\right)(4x+3)}\sqrt{1-\left(\frac{3}{73}-\frac{8i}{73}\right)(4x+3)}(8x+6)^{m+1}\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\left(\frac{3}{146}+\frac{4i}{73}\right)(8x+6)\right.}{8\sqrt{2}(m+1)\sqrt{x^2+4}}$$

input
```
Int[(6 + 8*x)^m/Sqrt[8 + 2*x^2],x]
```

output
```
((6 + 8*x)^(1 + m)*Sqrt[1 - (3/73 + (8*I)/73)*(3 + 4*x)]*Sqrt[1 - (3/73 -
(8*I)/73)*(3 + 4*x)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (3/146 + (4*I)/73)*(
6 + 8*x), (3/146 - (4*I)/73)*(6 + 8*x)])/(8*Sqrt[2]*(1 + m)*Sqrt[4 + x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(8x+6)^m}{\sqrt{2x^2+8}}dx$$

input
```
int((8*x+6)^m/(2*x^2+8)^(1/2),x)
```

output
```
int((8*x+6)^m/(2*x^2+8)^(1/2),x)
```

Fricas [F]

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}}\,dx = \int \frac{(8\,x+6)^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
integrate((6+8*x)^m/(2*x^2+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2*x^2 + 8)*(8*x + 6)^m/(x^2 + 4), x)
```

Sympy [F]

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}}\,dx = \frac{\sqrt{2}\cdot 2^m \int \frac{(4x+3)^m}{\sqrt{x^2+4}}\,dx}{2}$$

input
```
integrate((6+8*x)**m/(2*x**2+8)**(1/2),x)
```

output
```
sqrt(2)*2**m*Integral((4*x + 3)**m/sqrt(x**2 + 4), x)/2
```

Maxima [F]

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}}\,dx = \int \frac{(8\,x+6)^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
integrate((6+8*x)^m/(2*x^2+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate((8*x + 6)^m/sqrt(2*x^2 + 8), x)
```

Giac [F]

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}}\,dx = \int \frac{(8\,x+6)^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
integrate((6+8*x)^m/(2*x^2+8)^(1/2),x, algorithm="giac")
```

output
```
integrate((8*x + 6)^m/sqrt(2*x^2 + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}}\,dx=\int \frac{{\left(8\,x+6\right)}^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
int((8*x + 6)^m/(2*x^2 + 8)^(1/2),x)
```

output
```
int((8*x + 6)^m/(2*x^2 + 8)^(1/2), x)
```

Reduce [F]

$$\int \frac{(6+8x)^m}{\sqrt{8+2x^2}}\,dx=\frac{\int \frac{(8x+6)^m}{\sqrt{x^2+4}}dx}{\sqrt{2}}$$

input
```
int((6+8*x)^m/(2*x^2+8)^(1/2),x)
```

output
```
int((8*x + 6)**m/sqrt(x**2 + 4),x)/sqrt(2)
```

3.404 $\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = \sqrt{2}7^m\sqrt{1+x}\,\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\frac{4(1+x)}{7},\frac{1+x}{2}\right)$$

output
```
2^(1/2)*7^m*(1+x)^(1/2)*AppellF1(1/2,1/2,-m,3/2,1/2+1/2*x,4/7+4/7*x)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = -\frac{(3-4x)^{1+m}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{1}{7}(3-4x),-3+4x\right)}{\sqrt{7}(1+m)}$$

input
```
Integrate[(3 - 4*x)^m/Sqrt[1 - x^2],x]
```

output
```
-(((3 - 4*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (3 - 4*x)/7, -3 + 4*
x])/(Sqrt[7]*(1 + m)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {513, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{513}$$
$$\int \frac{(3-4x)^m}{\sqrt{1-x}\sqrt{x+1}}dx$$
$$\Big\downarrow \text{155}$$
$$\sqrt{2}7^m\sqrt{x+1}\,\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\frac{4(x+1)}{7},\frac{x+1}{2}\right)$$

input
```
Int[(3 - 4*x)^m/Sqrt[1 - x^2],x]
```

output
```
Sqrt[2]*7^m*Sqrt[1 + x]*AppellF1[1/2, -m, 1/2, 3/2, (4*(1 + x))/7, (1 + x)
/2]
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(3-4x)^m}{\sqrt{-x^2+1}}dx$$

input
```
int((3-4*x)^m/(-x^2+1)^(1/2),x)
```

output
```
int((3-4*x)^m/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-4\,x+3)^m}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((3-4*x)^m/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*(-4*x + 3)^m/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(3-4x)^m}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate((3-4*x)**m/(-x**2+1)**(1/2),x)
```

output
```
Integral((3 - 4*x)**m/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-4\,x+3)^m}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((3-4*x)^m/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-4*x + 3)^m/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-4\,x+3)^m}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((3-4*x)^m/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((-4*x + 3)^m/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(3-4\,x)^m}{\sqrt{1-x^2}}\,dx$$

input
```
int((3 - 4*x)^m/(1 - x^2)^(1/2),x)
```

output
```
int((3 - 4*x)^m/(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(3-4x)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-4x+3)^m}{\sqrt{-x^2+1}}dx$$

input
```
int((3-4*x)^m/(-x^2+1)^(1/2),x)
```

output
```
int(( - 4*x + 3)**m/sqrt( - x**2 + 1),x)
```

3.405 $\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = i\sqrt{2}25^m(3-4x)^m((3-4i)(3-4x))^{-m}\sqrt{1-ix}\,\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\left(\frac{16}{25}+\frac{12i}{25}\right)(1-ix),\frac{1}{2}(1-ix)\right)$$

output
```
I*2^(1/2)*25^m*(3-4*x)^m*(1-I*x)^(1/2)*AppellF1(1/2,-m,1/2,3/2,(16/25+12/2
5*I)*(1-I*x),1/2-1/2*I*x)/(((3-4*I)*(3-4*x))^m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = -\frac{(3-4x)^{1+m}\sqrt{(4+3i)(1-ix)}\sqrt{(3+4i)(-i+x)}\,\text{AppellF1}\big(1+m,\frac{1}{2},\frac{1}{2},2+m,\left(\frac{3}{25}-\frac{4i}{25}\right)(3-4x),\big(}{25(1+m)\sqrt{1+x^2}}$$

input
```
Integrate[(3 - 4*x)^m/Sqrt[1 + x^2],x]
```

output
```
-1/25*((3 - 4*x)^(1 + m)*Sqrt[(4 + 3*I)*(1 - I*x)]*Sqrt[(3 + 4*I)*(-I + x)
]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (3/25 - (4*I)/25)*(3 - 4*x), (3/25 + (4
*I)/25)*(3 - 4*x)])/((1 + m)*Sqrt[1 + x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3-4x)^m}{\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \; 514$$

$$-\frac{\sqrt{1-\left(\frac{3}{25}+\frac{4i}{25}\right)(3-4x)}\sqrt{1-\left(\frac{3}{25}-\frac{4i}{25}\right)(3-4x)}\int \frac{(3-4x)^m}{\sqrt{1-\left(\frac{3}{25}+\frac{4i}{25}\right)(3-4x)}\sqrt{1-\left(\frac{3}{25}-\frac{4i}{25}\right)(3-4x)}}d(3-4x)}{4\sqrt{x^2+1}}$$

$$\Big\downarrow \; 150$$

$$-\frac{\sqrt{1-\left(\frac{3}{25}+\frac{4i}{25}\right)(3-4x)}\sqrt{1-\left(\frac{3}{25}-\frac{4i}{25}\right)(3-4x)}(3-4x)^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\left(\frac{3}{25}+\frac{4i}{25}\right)(3-4x\right.}{4(m+1)\sqrt{x^2+1}}$$

input
```
Int[(3 - 4*x)^m/Sqrt[1 + x^2],x]
```

output
```
-1/4*(Sqrt[1 - (3/25 + (4*I)/25)*(3 - 4*x)]*Sqrt[1 - (3/25 - (4*I)/25)*(3
- 4*x)]*(3 - 4*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (3/25 + (4*I)/2
5)*(3 - 4*x), (3/25 - (4*I)/25)*(3 - 4*x)])/((1 + m)*Sqrt[1 + x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(3-4x)^m}{\sqrt{x^2+1}}dx$$

input
```
int((3-4*x)^m/(x^2+1)^(1/2),x)
```

output
```
int((3-4*x)^m/(x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = \int \frac{(-4\,x+3)^m}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((3-4*x)^m/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral((-4*x + 3)^m/sqrt(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = \int \frac{(3-4x)^m}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((3-4*x)**m/(x**2+1)**(1/2),x)
```

output
```
Integral((3 - 4*x)**m/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = \int \frac{(-4\,x+3)^m}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((3-4*x)^m/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-4*x + 3)^m/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = \int \frac{(-4\,x+3)^m}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((3-4*x)^m/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((-4*x + 3)^m/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = \int \frac{(3-4\,x)^m}{\sqrt{x^2+1}}\,dx$$

input
```
int((3 - 4*x)^m/(x^2 + 1)^(1/2),x)
```

output
```
int((3 - 4*x)^m/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(3-4x)^m}{\sqrt{1+x^2}}\,dx = \int \frac{(-4x+3)^m}{\sqrt{x^2+1}}dx$$

input
```
int((3-4*x)^m/(x^2+1)^(1/2),x)
```

output
```
int(( - 4*x + 3)**m/sqrt(x**2 + 1),x)
```

3.406 $\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx=-\frac{\sqrt{2-x}(c+dx)^m\left(\frac{c+dx}{c+2d}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\frac{d(2-x)}{c+2d},\frac{2-x}{4}\right)}{\sqrt{2}}$$

output
```
-1/2*(2-x)^(1/2)*(d*x+c)^m*AppellF1(1/2,-m,1/2,3/2,d*(2-x)/(c+2*d),1/2-1/4
*x)*2^(1/2)/(((d*x+c)/(c+2*d))^m)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx=\frac{\sqrt{-\frac{d(-2+x)}{c+2d}}\sqrt{-\frac{d(2+x)}{c-2d}}(c+dx)^{1+m}\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{c+dx}{c-2d},\frac{c+dx}{c+2d}\right)}{d(1+m)\sqrt{8-2x^2}}$$

input
```
Integrate[(c + d*x)^m/Sqrt[8 - 2*x^2],x]
```

output
```
(Sqrt[-((d*(-2 + x))/(c + 2*d))]*Sqrt[-((d*(2 + x))/(c - 2*d))]*(c + d*x)^
(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (c + d*x)/(c - 2*d), (c + d*x)/(c
 + 2*d)])/(d*(1 + m)*Sqrt[8 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {513, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}dx$$

$$\Big\downarrow \text{513}$$

$$\frac{\int \frac{2(c+dx)^m}{\sqrt{2-x}\sqrt{x+2}}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^m}{\sqrt{2-x}\sqrt{x+2}}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{156}$$

$$\frac{(c+dx)^m\left(\frac{c+dx}{c+2d}\right)^{-m}\int \frac{\left(\frac{c}{c+2d}+\frac{dx}{c+2d}\right)^m}{\sqrt{2-x}\sqrt{x+2}}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2-x}(c+dx)^m\left(\frac{c+dx}{c+2d}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{2-x}{4},\frac{d(2-x)}{c+2d}\right)}{\sqrt{2}}$$

input
```
Int[(c + d*x)^m/Sqrt[8 - 2*x^2],x]
```

output
```
-((Sqrt[2 - x]*(c + d*x)^m*AppellF1[1/2, 1/2, -m, 3/2, (2 - x)/4, (d*(2 -
x))/(c + 2*d)])/(Sqrt[2]*((c + d*x)/(c + 2*d))^m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(dx+c)^m}{\sqrt{-2x^2+8}}dx$$

input
```
int((d*x+c)^m/(-2*x^2+8)^(1/2),x)
```

output
```
int((d*x+c)^m/(-2*x^2+8)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{-2\,x^2+8}}\,dx$$

input
```
integrate((d*x+c)^m/(-2*x^2+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(-1/2*sqrt(-2*x^2 + 8)*(d*x + c)^m/(x^2 - 4), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx = \frac{\sqrt{2}\int \frac{(c+dx)^m}{\sqrt{4-x^2}}\,dx}{2}$$

input
```
integrate((d*x+c)**m/(-2*x**2+8)**(1/2),x)
```

output
```
sqrt(2)*Integral((c + d*x)**m/sqrt(4 - x**2), x)/2
```

Maxima [F]

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx=\int \frac{(dx+c)^m}{\sqrt{-2\,x^2+8}}\,dx$$

input
```
integrate((d*x+c)^m/(-2*x^2+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/sqrt(-2*x^2 + 8), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx=\int \frac{(dx+c)^m}{\sqrt{-2\,x^2+8}}\,dx$$

input
```
integrate((d*x+c)^m/(-2*x^2+8)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/sqrt(-2*x^2 + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx=\int \frac{(c+d\,x)^m}{\sqrt{8-2\,x^2}}\,dx$$

input
```
int((c + d*x)^m/(8 - 2*x^2)^(1/2),x)
```

output
```
int((c + d*x)^m/(8 - 2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{\sqrt{8-2x^2}}\,dx = \frac{\int \frac{(dx+c)^m}{\sqrt{-x^2+4}}dx}{\sqrt{2}}$$

input
```
int((d*x+c)^m/(-2*x^2+8)^(1/2),x)
```

output
```
int((c + d*x)**m/sqrt( - x**2 + 4),x)/sqrt(2)
```

3.407 $\int \frac{(c+dx)^m}{\sqrt{8+2x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}} \, dx = \frac{i\sqrt{2-ix}(c+dx)^m \left(\frac{c+dx}{c-2id}\right)^{-m} \operatorname{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\frac{d(2-ix)}{ic+2d},\frac{1}{4}(2-ix)\right)}{\sqrt{2}}$$

output
```
1/2*I*(2-I*x)^(1/2)*(d*x+c)^m*AppellF1(1/2,-m,1/2,3/2,d*(2-I*x)/(I*c+2*d),
1/2-1/4*I*x)*2^(1/2)/(((d*x+c)/(c-2*I*d))^m)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}} \, dx = \frac{\sqrt{-\frac{d(-2i+x)}{c+2id}}\sqrt{-\frac{d(2i+x)}{c-2id}}(c+dx)^{1+m} \operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{c+dx}{c-2id},\frac{c+dx}{c+2id}\right)}{\sqrt{2}d(1+m)\sqrt{4+x^2}}$$

input
```
Integrate[(c + d*x)^m/Sqrt[8 + 2*x^2],x]
```

output
```
(Sqrt[-((d*(-2*I + x))/(c + (2*I)*d))]*Sqrt[-((d*(2*I + x))/(c - (2*I)*d))
]*(c + d*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (c + d*x)/(c - (2*I)*
d), (c + d*x)/(c + (2*I)*d)])/(Sqrt[2]*d*(1 + m)*Sqrt[4 + x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{\sqrt{2x^2+8}}dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\sqrt{1-\frac{c+dx}{c-2id}}\sqrt{1-\frac{c+dx}{c+2id}}\int \frac{(c+dx)^m}{\sqrt{1-\frac{c+dx}{c-2id}}\sqrt{1-\frac{c+dx}{c+2id}}}d(c+dx)}{\sqrt{2}d\sqrt{x^2+4}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{1-\frac{c+dx}{c-2id}}\sqrt{1-\frac{c+dx}{c+2id}}(c+dx)^{m+1}\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{c+dx}{c-2id},\frac{c+dx}{c+2id}\right)}{\sqrt{2}d(m+1)\sqrt{x^2+4}}$$

input
```
Int[(c + d*x)^m/Sqrt[8 + 2*x^2],x]
```

output
```
((c + d*x)^(1 + m)*Sqrt[1 - (c + d*x)/(c - (2*I)*d)]*Sqrt[1 - (c + d*x)/(c
 + (2*I)*d)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (c + d*x)/(c - (2*I)*d), (c
+ d*x)/(c + (2*I)*d)])/(Sqrt[2]*d*(1 + m)*Sqrt[4 + x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{\sqrt{2x^2+8}}dx$$

input
```
int((d*x+c)^m/(2*x^2+8)^(1/2),x)
```

output
```
int((d*x+c)^m/(2*x^2+8)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
integrate((d*x+c)^m/(2*x^2+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2*x^2 + 8)*(d*x + c)^m/(x^2 + 4), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}}\,dx = \frac{\sqrt{2}\int \frac{(c+dx)^m}{\sqrt{x^2+4}}\,dx}{2}$$

input
```
integrate((d*x+c)**m/(2*x**2+8)**(1/2),x)
```

output
```
sqrt(2)*Integral((c + d*x)**m/sqrt(x**2 + 4), x)/2
```

Maxima [F]

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
integrate((d*x+c)^m/(2*x^2+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/sqrt(2*x^2 + 8), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
integrate((d*x+c)^m/(2*x^2+8)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/sqrt(2*x^2 + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}}\,dx=\int \frac{(c+d\,x)^m}{\sqrt{2\,x^2+8}}\,dx$$

input
```
int((c + d*x)^m/(2*x^2 + 8)^(1/2),x)
```

output
```
int((c + d*x)^m/(2*x^2 + 8)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{\sqrt{8+2x^2}}\,dx=\frac{\int \frac{(dx+c)^m}{\sqrt{x^2+4}}dx}{\sqrt{2}}$$

input
```
int((d*x+c)^m/(2*x^2+8)^(1/2),x)
```

output
```
int((c + d*x)**m/sqrt(x**2 + 4),x)/sqrt(2)
```

3.408 $\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = -\frac{\sqrt{2-\sqrt{b}x}(c+dx)^m\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+2d}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},\frac{d\left(2-\sqrt{b}x\right)}{\sqrt{b}c+2d},\frac{1}{4}\left(2-\sqrt{b}x\right)\right)}{\sqrt{b}}$$

output
```
-(2-b^(1/2)*x)^(1/2)*(d*x+c)^m*AppellF1(1/2,-m,1/2,3/2,d*(2-b^(1/2)*x)/(b^
(1/2)*c+2*d),1/2-1/4*b^(1/2)*x)/b^(1/2)/((b^(1/2)*(d*x+c)/(b^(1/2)*c+2*d))
^m)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = \frac{\sqrt{\frac{d\left(2\sqrt{\frac{1}{b}}-x\right)}{c+2\sqrt{\frac{1}{b}}d}}\sqrt{\frac{d\left(2\sqrt{\frac{1}{b}}+x\right)}{-c+2\sqrt{\frac{1}{b}}d}}(c+dx)^{1+m}\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{c+dx}{c-2\sqrt{\frac{1}{b}}d},\frac{c+dx}{c+2\sqrt{\frac{1}{b}}d}\right)}{d(1+m)\sqrt{4-bx^2}}$$

input
```
Integrate[(c + d*x)^m/Sqrt[4 - b*x^2],x]
```

output
```
(Sqrt[(d*(2*Sqrt[b^(-1)] - x))/(c + 2*Sqrt[b^(-1)]*d)]*Sqrt[(d*(2*Sqrt[b^(
-1)] + x))/(-c + 2*Sqrt[b^(-1)]*d)]*(c + d*x)^(1 + m)*AppellF1[1 + m, 1/2,
 1/2, 2 + m, (c + d*x)/(c - 2*Sqrt[b^(-1)]*d), (c + d*x)/(c + 2*Sqrt[b^(-1
)]*d)])/(d*(1 + m)*Sqrt[4 - b*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {513, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}dx$$

$$\Big\downarrow \text{513}$$

$$\frac{1}{2}\int \frac{2(c+dx)^m}{\sqrt{2-\sqrt{b}x}\sqrt{\sqrt{b}x+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(c+dx)^m}{\sqrt{2-\sqrt{b}x}\sqrt{\sqrt{b}x+2}}dx$$

$$\Big\downarrow \text{156}$$

$$(c+dx)^m\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+2d}\right)^{-m}\int \frac{\left(\frac{\sqrt{b}c}{\sqrt{b}c+2d}+\frac{\sqrt{b}dx}{\sqrt{b}c+2d}\right)^m}{\sqrt{2-\sqrt{b}x}\sqrt{\sqrt{b}x+2}}dx$$

$$\Big\downarrow \text{155}$$

$$-\frac{\sqrt{2-\sqrt{b}x}(c+dx)^m\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+2d}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{4}\left(2-\sqrt{b}x\right),\frac{d\left(2-\sqrt{b}x\right)}{\sqrt{b}c+2d}\right)}{\sqrt{b}}$$

input
```
Int[(c + d*x)^m/Sqrt[4 - b*x^2],x]
```

output
```
-((Sqrt[2 - Sqrt[b]*x]*(c + d*x)^m*AppellF1[1/2, 1/2, -m, 3/2, (2 - Sqrt[b
]*x)/4, (d*(2 - Sqrt[b]*x))/(Sqrt[b]*c + 2*d)])/(Sqrt[b]*((Sqrt[b]*(c + d*
x))/(Sqrt[b]*c + 2*d))^m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(dx+c)^m}{\sqrt{-b\,x^2+4}}dx$$

input
```
int((d*x+c)^m/(-b*x^2+4)^(1/2),x)
```

output
```
int((d*x+c)^m/(-b*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{-bx^2+4}}\,dx$$

input
```
integrate((d*x+c)^m/(-b*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-b*x^2 + 4)*(d*x + c)^m/(b*x^2 - 4), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = \int \frac{(c+dx)^m}{\sqrt{-bx^2+4}}\,dx$$

input
```
integrate((d*x+c)**m/(-b*x**2+4)**(1/2),x)
```

output
```
Integral((c + d*x)**m/sqrt(-b*x**2 + 4), x)
```

Maxima [F]

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{-bx^2+4}}\,dx$$

input
```
integrate((d*x+c)^m/(-b*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/sqrt(-b*x^2 + 4), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{-bx^2+4}}\,dx$$

input
```
integrate((d*x+c)^m/(-b*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/sqrt(-b*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = \int \frac{(c+d\,x)^m}{\sqrt{4-b\,x^2}}\,dx$$

input
```
int((c + d*x)^m/(4 - b*x^2)^(1/2),x)
```

output
```
int((c + d*x)^m/(4 - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{\sqrt{4-bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{-b\,x^2+4}}dx$$

input
```
int((d*x+c)^m/(-b*x^2+4)^(1/2),x)
```

output
```
int((c + d*x)**m/sqrt( - b*x**2 + 4),x)
```

3.409 $\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = -\frac{\sqrt{2-\sqrt{-b}x}(c+dx)^m\left(\frac{\sqrt{-b}(c+dx)}{\sqrt{-b}c+2d}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},-\frac{\sqrt{-b}d(2-\sqrt{-b}x)}{bc-2\sqrt{-b}d},\frac{1}{4}(2-\sqrt{-b}x)\right)}{\sqrt{-b}}$$

output
```
-(2-(-b)^(1/2)*x)^(1/2)*(d*x+c)^m*AppellF1(1/2,-m,1/2,3/2,-(-b)^(1/2)*d*(2
-(-b)^(1/2)*x)/(b*c-2*(-b)^(1/2)*d),1/2-1/4*(-b)^(1/2)*x)/(-b)^(1/2)/(((-b
)^(1/2)*(d*x+c)/((-b)^(1/2)*c+2*d))^m)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = \frac{\sqrt{\frac{d\left(2\sqrt{-\frac{1}{b}}-x\right)}{c+2\sqrt{-\frac{1}{b}}d}}\sqrt{\frac{d\left(2\sqrt{-\frac{1}{b}}+x\right)}{-c+2\sqrt{-\frac{1}{b}}d}}(c+dx)^{1+m}\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{c+dx}{c-2\sqrt{-\frac{1}{b}}d},\frac{c+dx}{c+2\sqrt{-\frac{1}{b}}d}\right)}{d(1+m)\sqrt{4+bx^2}}$$

input
```
Integrate[(c + d*x)^m/Sqrt[4 + b*x^2],x]
```

output
```
(Sqrt[(d*(2*Sqrt[-b^(-1)] - x))/(c + 2*Sqrt[-b^(-1)]*d)]*Sqrt[(d*(2*Sqrt[-
b^(-1)] + x))/(-c + 2*Sqrt[-b^(-1)]*d)]*(c + d*x)^(1 + m)*AppellF1[1 + m,
1/2, 1/2, 2 + m, (c + d*x)/(c - 2*Sqrt[-b^(-1)]*d), (c + d*x)/(c + 2*Sqrt[
-b^(-1)]*d)])/(d*(1 + m)*Sqrt[4 + b*x^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{\sqrt{bx^2+4}}dx$$

$$\Big\downarrow \; 514$$

$$\frac{\sqrt{1-\frac{c+dx}{c-\frac{2d}{\sqrt{-b}}}}\sqrt{1-\frac{c+dx}{\frac{2d}{\sqrt{-b}}+c}}\int \frac{(c+dx)^m}{\sqrt{1-\frac{c+dx}{c-\frac{2d}{\sqrt{-b}}}}\sqrt{1-\frac{c+dx}{c+\frac{2d}{\sqrt{-b}}}}}d(c+dx)}{d\sqrt{bx^2+4}}$$

$$\Big\downarrow \; 150$$

$$\frac{\sqrt{1-\frac{c+dx}{c-\frac{2d}{\sqrt{-b}}}}\sqrt{1-\frac{c+dx}{\frac{2d}{\sqrt{-b}}+c}}(c+dx)^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{c+dx}{c-\frac{2d}{\sqrt{-b}}},\frac{c+dx}{c+\frac{2d}{\sqrt{-b}}}\right)}{d(m+1)\sqrt{bx^2+4}}$$

input
```
Int[(c + d*x)^m/Sqrt[4 + b*x^2],x]
```

output
```
((c + d*x)^(1 + m)*Sqrt[1 - (c + d*x)/(c - (2*d)/Sqrt[-b])]*Sqrt[1 - (c +
d*x)/(c + (2*d)/Sqrt[-b])]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (c + d*x)/(c -
 (2*d)/Sqrt[-b]), (c + d*x)/(c + (2*d)/Sqrt[-b])])/(d*(1 + m)*Sqrt[4 + b*x
^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{\sqrt{b\,x^2+4}}dx$$

input
```
int((d*x+c)^m/(b*x^2+4)^(1/2),x)
```

output
```
int((d*x+c)^m/(b*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{bx^2+4}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/sqrt(b*x^2 + 4), x)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = \int \frac{(c+dx)^m}{\sqrt{bx^2+4}}\,dx$$

input
```
integrate((d*x+c)**m/(b*x**2+4)**(1/2),x)
```

output
```
Integral((c + d*x)**m/sqrt(b*x**2 + 4), x)
```

Maxima [F]

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{bx^2+4}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/sqrt(b*x^2 + 4), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{bx^2+4}}\,dx$$

input
```
integrate((d*x+c)^m/(b*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/sqrt(b*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = \int \frac{(c+d\,x)^m}{\sqrt{b\,x^2+4}}\,dx$$

input
```
int((c + d*x)^m/(b*x^2 + 4)^(1/2),x)
```

output
```
int((c + d*x)^m/(b*x^2 + 4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{\sqrt{4+bx^2}}\,dx = \int \frac{(dx+c)^m}{\sqrt{b\,x^2+4}}dx$$

input
```
int((d*x+c)^m/(b*x^2+4)^(1/2),x)
```

output
```
int((c + d*x)**m/sqrt(b*x**2 + 4),x)
```

3.410 $\int (c+dx)^3 \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^p \, dx \\
&= \frac{d(c+dx)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} \\
&\quad - \frac{d((3+2p)\left(ad^2-bc^2(5+2p)\right)-2bcd(1+p)(3+p)x)\left(a+bx^2\right)^{1+p}}{2b^2(2+p)\left(3+5p+2p^2\right)} \\
&\quad + c\left(c^2-\frac{3ad^2}{3b+2bp}\right) x\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)
\end{aligned}$$

output
```
1/2*d*(d*x+c)^2*(b*x^2+a)^(p+1)/b/(2+p)-1/2*d*((3+2*p)*(a*d^2-b*c^2*(5+2*p
))-2*b*c*d*(p+1)*(3+p)*x)*(b*x^2+a)^(p+1)/b^2/(2+p)/(2*p^2+5*p+3)+c*(c^2-3
*a*d^2/(2*b*p+3*b))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+
b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.32

$$\int (c+dx)^3 \left(a+bx^2\right)^p \, dx$$
$$= \frac{\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(2b^2c^3(2+3p+p^2)\, x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+d\Big(b^2x^2\Big(1+\frac{bx^2}{a}\Big)^p\right.}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(2*b^2*c^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2
, -((b*x^2)/a)] + d*(b^2*x^2*(1 + (b*x^2)/a)^p*(3*c^2*(2 + p) + d^2*(1 + p
)*x^2) - a^2*d^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(d^2*p*x^2*(1 + (b*x^2)/a)
^p + 3*c^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 2*b^2*c*d*(2 + 3*p + p^2)*x
^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(2*b^2*(1 + p)*(2 + p)
*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {497, 27, 676, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+bx^2\right)^p \, dx$$
$$\Big\downarrow \; 497$$
$$\frac{\int -2(c+dx)\left(-b(p+2)c^2-bd(p+3)xc+ad^2\right)\left(bx^2+a\right)^p dx}{2b(p+2)}+\frac{d(c+dx)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$
$$\Big\downarrow \; 27$$
$$\frac{d(c+dx)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}-\frac{\int (c+dx)\left(-b(p+2)c^2-bd(p+3)xc+ad^2\right)\left(bx^2+a\right)^p dx}{b(p+2)}$$

$$\Big\downarrow \; 676$$

$$\frac{d(c+dx)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}-\frac{\frac{c(p+2)\left(3ad^2-bc^2(2p+3)\right)\int \left(bx^2+a\right)^p dx}{2p+3}+\frac{d\left(a+bx^2\right)^{p+1}\left(ad^2-bc^2(2p+5)\right)}{2b(p+1)}-\frac{cd^2(p+3)x\left(a+bx^2\right)^{p+1}}{2p+3}}{b(p+2)}$$

$$\Big\downarrow \; 238$$

$$\frac{d(c+dx)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}-\frac{\frac{c(p+2)\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ad^2-bc^2(2p+3)\right)\int \left(\frac{bx^2}{a}+1\right)^p dx}{2p+3}+\frac{d\left(a+bx^2\right)^{p+1}\left(ad^2-bc^2(2p+5)\right)}{2b(p+1)}-\frac{cd^2(p+3)x\left(a+bx^2\right)^{p+1}}{2p+3}}{b(p+2)}$$

$$\Big\downarrow \; 237$$

$$\frac{d(c+dx)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}-\frac{\frac{c(p+2)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ad^2-bc^2(2p+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{2p+3}+\frac{d\left(a+bx^2\right)^{p+1}\left(ad^2-bc^2(2p+5)\right)}{2b(p+1)}-\frac{cd^2(p+3)x(a+b}{2p+3}}{b(p+2)}$$

input
```
Int[(c + d*x)^3*(a + b*x^2)^p,x]
```

output
```
(d*(c + d*x)^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - ((d*(a*d^2 - b*c^2*(5
+ 2*p))*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) - (c*d^2*(3 + p)*x*(a + b*x^2)^
(1 + p))/(3 + 2*p) + (c*(2 + p)*(3*a*d^2 - b*c^2*(3 + 2*p))*x*(a + b*x^2)^
p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a
)^p))/(b*(2 + p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (dx + c)^3 \left(b\,x^2 + a\right)^p dx$$

input
```
int((d*x+c)^3*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^3 \left(a+bx^2\right)^p \,dx = \int (dx+c)^3 \left(bx^2+a\right)^p \,dx$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(146) = 292.

Time = 5.89 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.59

$$\int (c+dx)^3 \left(a+bx^2\right)^p \,dx = a^p c^3 x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -p \\ \frac{3}{2}\end{matrix}\middle|\, \frac{bx^2 e^{i\pi}}{a}\right) + a^p c d^2 x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\middle|\, \frac{bx^2 e^{i\pi}}{a}\right) + 3c^2 d \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + d^3 \left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\ b = 0 \\ \frac{a \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for}\ p = -2 \\ -\frac{a \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for}\ p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**p,x)
```

output
```
a**p*c**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*c*d*
*2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*c**2*d*Piec
ewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1),
Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + d**3*Piecewise((a**p*
x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log
(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b
*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-
a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**
2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p*
*2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*
p + 4*b**2), True))
```

Maxima [F]

$$\int (c+dx)^3 \left(a+bx^2\right)^p \, dx = \int (dx+c)^3 \left(bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^3 \left(a+bx^2\right)^p \, dx = \int (dx+c)^3 \left(bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+bx^2\right)^p \, dx = \int \left(b\,x^2+a\right)^p\,(c+d\,x)^3 \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^p*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \left(a+bx^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*(b*x^2+a)^p,x)
```

output
```
( - 4*(a + b*x**2)**p*a**2*d**3*p**2 - 8*(a + b*x**2)**p*a**2*d**3*p - 3*(
a + b*x**2)**p*a**2*d**3 + 12*(a + b*x**2)**p*a*b*c**2*d*p**3 + 48*(a + b*
x**2)**p*a*b*c**2*d*p**2 + 57*(a + b*x**2)**p*a*b*c**2*d*p + 18*(a + b*x**
2)**p*a*b*c**2*d + 12*(a + b*x**2)**p*a*b*c*d**2*p**3*x + 36*(a + b*x**2)*
*p*a*b*c*d**2*p**2*x + 24*(a + b*x**2)**p*a*b*c*d**2*p*x + 4*(a + b*x**2)*
*p*a*b*d**3*p**3*x**2 + 8*(a + b*x**2)**p*a*b*d**3*p**2*x**2 + 3*(a + b*x*
*2)**p*a*b*d**3*p*x**2 + 4*(a + b*x**2)**p*b**2*c**3*p**3*x + 18*(a + b*x*
*2)**p*b**2*c**3*p**2*x + 26*(a + b*x**2)**p*b**2*c**3*p*x + 12*(a + b*x**
2)**p*b**2*c**3*x + 12*(a + b*x**2)**p*b**2*c**2*d*p**3*x**2 + 48*(a + b*x
**2)**p*b**2*c**2*d*p**2*x**2 + 57*(a + b*x**2)**p*b**2*c**2*d*p*x**2 + 18
*(a + b*x**2)**p*b**2*c**2*d*x**2 + 12*(a + b*x**2)**p*b**2*c*d**2*p**3*x*
*3 + 42*(a + b*x**2)**p*b**2*c*d**2*p**2*x**3 + 42*(a + b*x**2)**p*b**2*c*
d**2*p*x**3 + 12*(a + b*x**2)**p*b**2*c*d**2*x**3 + 4*(a + b*x**2)**p*b**2
*d**3*p**3*x**4 + 12*(a + b*x**2)**p*b**2*d**3*p**2*x**4 + 11*(a + b*x**2)
**p*b**2*d**3*p*x**4 + 3*(a + b*x**2)**p*b**2*d**3*x**4 - 48*int((a + b*x*
*2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)
*a**2*b*c*d**2*p**5 - 240*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*
b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*b*c*d**2*p**4 - 420*int((a +
b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2
),x)*a**2*b*c*d**2*p**3 - 300*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3...
```

3.411 $\int (c+dx)^2 \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx = \frac{d(c(3+2p)+d(1+p)x)\left(a+bx^2\right)^{1+p}}{b\left(3+5p+2p^2\right)} + \left(c^2-\frac{ad^2}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
d*(c*(3+2*p)+d*(p+1)*x)*(b*x^2+a)^(p+1)/b/(2*p^2+5*p+3)+(c^2-a*d^2/(2*b*p+
3*b))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.24

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx = \frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(3bc^2(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+d\left(3c\left(bx^2\left(1+\frac{bx^2}{a}\right)^p+a\left(\right.\right.\right.\right.}{3b(1+p)}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(3*b*c^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2
)/a)] + d*(3*c*(b*x^2*(1 + (b*x^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + b*
d*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(3*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {497, 25, 455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{497}$$

$$\frac{\int -\left(\left(-b(2p+3)c^2-2bd(p+2)xc+ad^2\right)\left(bx^2+a\right)^p\right)dx}{b(2p+3)}+\frac{d(c+dx)\left(a+bx^2\right)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{d(c+dx)\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\int \left(-b(2p+3)c^2-2bd(p+2)xc+ad^2\right)\left(bx^2+a\right)^p dx}{b(2p+3)}$$

$$\Big\downarrow \text{455}$$

$$\frac{d(c+dx)\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\left(ad^2-bc^2(2p+3)\right)\int \left(bx^2+a\right)^p dx-\frac{cd(p+2)\left(a+bx^2\right)^{p+1}}{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{238}$$

$$\frac{d(c+dx)\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ad^2-bc^2(2p+3)\right)\int \left(\frac{bx^2}{a}+1\right)^p dx-\frac{cd(p+2)\left(a+bx^2\right)^{p+1}}{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{237}$$

$$\frac{d(c+dx)\left(a+bx^2\right)^{p+1}}{b(2p+3)}-$$

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ad^2-bc^2(2p+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)-\frac{cd(p+2)\left(a+bx^2\right)^{p+1}}{p+1}}{b(2p+3)}$$

input
```
Int[(c + d*x)^2*(a + b*x^2)^p,x]
```

output
```
(d*(c + d*x)*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) - (-((c*d*(2 + p)*(a + b*x
^2)^(1 + p))/(1 + p)) + ((a*d^2 - b*c^2*(3 + 2*p))*x*(a + b*x^2)^p*Hyperge
ometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p)/(b*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [F]

$$\int (dx+c)^2 \left(b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^2 \left(a+bx^2\right)^p \,dx = \int (dx+c)^2 \left(bx^2+a\right)^p \,dx$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx = a^p c^2 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p d^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + 2cd \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\: b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**p,x)
```

output
```
a**p*c**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d**2
*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 2*c*d*Piecewi
se((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx = \int (dx+c)^2 \left(bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx = \int (dx+c)^2 (bx^2+a)^p \, dx$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx = \int \left(b\,x^2+a\right)^p \, (c+d\,x)^2 \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a+bx^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^p,x)
```

output

```
(4*(a + b*x**2)**p*a*c*d*p**2 + 8*(a + b*x**2)**p*a*c*d*p + 3*(a + b*x**2)
**p*a*c*d + 2*(a + b*x**2)**p*a*d**2*p**2*x + 2*(a + b*x**2)**p*a*d**2*p*x
 + 2*(a + b*x**2)**p*b*c**2*p**2*x + 5*(a + b*x**2)**p*b*c**2*p*x + 3*(a +
 b*x**2)**p*b*c**2*x + 4*(a + b*x**2)**p*b*c*d*p**2*x**2 + 8*(a + b*x**2)*
*p*b*c*d*p*x**2 + 3*(a + b*x**2)**p*b*c*d*x**2 + 2*(a + b*x**2)**p*b*d**2*
p**2*x**3 + 3*(a + b*x**2)**p*b*d**2*p*x**3 + (a + b*x**2)**p*b*d**2*x**3
- 8*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x*
*2 + 3*b*x**2),x)*a**2*d**2*p**4 - 24*int((a + b*x**2)**p/(4*a*p**2 + 8*a*
p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*d**2*p**3 - 22*in
t((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3
*b*x**2),x)*a**2*d**2*p**2 - 6*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a
 + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*d**2*p + 16*int((a + b*x
**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x
)*a*b*c**2*p**5 + 72*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**
2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a*b*c**2*p**4 + 116*int((a + b*x**2)**p
/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a*b*c
**2*p**3 + 78*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2
+ 8*b*p*x**2 + 3*b*x**2),x)*a*b*c**2*p**2 + 18*int((a + b*x**2)**p/(4*a*p*
*2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a*b*c**2*p)/(
b*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.412 $\int (c+dx) \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (c+dx) \left(a+bx^2\right)^p \, dx = \frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)} + cx\left(a+bx^2\right)^p \left(1 + \frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*d*(b*x^2+a)^(p+1)/b/(p+1)+c*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b
*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int (c+dx) \left(a+bx^2\right)^p \, dx = \frac{\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(bdx^2\left(1+\frac{bx^2}{a}\right)^p + ad\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right) + 2bc(1+p)x \,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{2b(1+p)}$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(b*d*x^2*(1 + (b*x^2)/a)^p + a*d*(-1 + (1 + (b*x^2)/a)^p) +
 2*b*c*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(2*b*(1 +
 p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(bx^2+a\right)^p dx+\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{238}$$

$$c\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \left(\frac{bx^2}{a}+1\right)^p dx+\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{237}$$

$$cx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

input
```
Int[(c + d*x)*(a + b*x^2)^p,x]
```

output
```
(d*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (c*x*(a + b*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (dx + c) \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c + dx) \left(a + bx^2\right)^p \, dx = \int (dx + c)(bx^2 + a)^p \, dx$$

input
```
integrate((d*x+c)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (c+dx)\left(a+bx^2\right)^p\,dx = a^p c x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + d\left(\begin{cases}\frac{a^px^2}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(b*x**2+a)**p,x)
```

output
```
a**p*c*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + d*Piecewise(
(a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int (c+dx)\left(a+bx^2\right)^p\,dx = \int (dx+c)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx) \left(a+bx^2\right)^p \, dx = \int (dx+c)\left(bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (c+dx) \left(a+bx^2\right)^p \, dx = \frac{d\left(b\,x^2+a\right)^{p+1}}{2\,b\,\left(p+1\right)}+\frac{c\,x\,\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^p}$$

input
```
int((a + b*x^2)^p*(c + d*x),x)
```

output
```
(d*(a + b*x^2)^(p + 1))/(2*b*(p + 1)) + (c*x*(a + b*x^2)^p*hypergeom([1/2,
 -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

Reduce [F]

$$\int (c+dx) \left(a+bx^2\right)^p \, dx$$
$$= \frac{2(b\,x^2+a)^p\,adp+(b\,x^2+a)^p\,ad+2(b\,x^2+a)^p\,bcpx+2(b\,x^2+a)^p\,bcx+2(b\,x^2+a)^p\,bdp\,x^2+(b\,x^2+a}{2b\,(2p^2}$$

input
```
int((d*x+c)*(b*x^2+a)^p,x)
```

output
```
(2*(a + b*x**2)**p*a*d*p + (a + b*x**2)**p*a*d + 2*(a + b*x**2)**p*b*c*p*x
 + 2*(a + b*x**2)**p*b*c*x + 2*(a + b*x**2)**p*b*d*p*x**2 + (a + b*x**2)**
p*b*d*x**2 + 8*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*a*
b*c*p**3 + 12*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*a*b
*c*p**2 + 4*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*a*b*c
*p)/(2*b*(2*p**2 + 3*p + 1))
```

3.413 $\int (a+bx^2)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int (a+bx^2)^p \, dx = \frac{x(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1, \frac{3}{2}+p, \frac{3}{2}, -\frac{bx^2}{a}\right)}{a}$$

output
```
x*(b*x^2+a)^(p+1)*hypergeom([1, 3/2+p],[3/2],-b*x^2/a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int (a+bx^2)^p \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{bx^2}{a}\right)$$

input
```
Integrate[(a + b*x^2)^p,x]
```

output
```
(x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{238}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{237}$$

$$x\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[(a + b*x^2)^p,x]
```

output
```
(x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p}dx$$

input
```
int((b*x^2+a)^p,x)
```

output
```
int((b*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^{2}\right)^{p}\,dx=\int \left(bx^{2}+a\right)^{p}dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \left(a+bx^{2}\right)^{p}\,dx=a^{p}x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**p,x)
```

output
```
a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \left(a+bx^2\right)^p\,dx=\frac{x\,\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^p}$$

input
```
int((a + b*x^2)^p,x)
```

output
```
(x*(a + b*x^2)^p*hypergeom([1/2, -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

Reduce [F]

$$\int \left(a+bx^2\right)^p\,dx = \frac{(b\,x^2+a)^p\,x+4\left(\int \frac{(b\,x^2+a)^p}{2bp\,x^2+b\,x^2+2ap+a}dx\right)a\,p^2+2\left(\int \frac{(b\,x^2+a)^p}{2bp\,x^2+b\,x^2+2ap+a}dx\right)ap}{2p+1}$$

input
```
int((b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*x + 4*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**
2),x)*a*p**2 + 2*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*
a*p)/(2*p + 1)
```

3.414 $\int \frac{(a+bx^2)^p}{c+dx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 125

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c} - \frac{d(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^2)}{bc^2+ad^2}\right)}{2(bc^2+ad^2)(1+p)}$$

output
```
x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,d^2*x^2/c^2,-b*x^2/a)/c/((1+b*x^2/a)^p
)-1/2*d*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],d^2*(b*x^2+a)/(a*d^2+b*c^
2))/(a*d^2+b*c^2)/(p+1)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \frac{\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}(a+bx^2)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx},\frac{c+\sqrt{-\frac{a}{b}}d}{c+dx}\right)}{2dp}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x),x]
```

output
```
((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c - Sqrt[-(a/b)]*d)/(c + d
*x), (c + Sqrt[-(a/b)]*d)/(c + d*x)])/(2*d*p*((d*(-Sqrt[-(a/b)] + x))/(c +
 d*x))^p*((d*(Sqrt[-(a/b)] + x))/(c + d*x))^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{c+dx} \, dx$$

$$\Big\downarrow \text{504}$$

$$c\int \frac{(bx^2+a)^p}{c^2-d^2x^2}dx - d\int \frac{x(bx^2+a)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{334}$$

$$c(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{c^2-d^2x^2}dx - d\int \frac{x(bx^2+a)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{333}$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c} - d\int \frac{x(bx^2+a)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c} - \frac{1}{2}d\int \frac{(bx^2+a)^p}{c^2-d^2x^2}dx^2$$

$$\Big\downarrow \text{78}$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c}-\frac{d(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2(bx^2+a)}{bc^2+ad^2}\right)}{2(p+1)(ad^2+bc^2)}$$

input
```
Int[(a + b*x^2)^p/(c + d*x),x]
```

output
```
(x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (d^2*x^2)/c^2])/(
c*(1 + (b*x^2)/a)^p) - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p,
2 + p, (d^2*(a + b*x^2))/(b*c^2 + a*d^2)])/(2*(b*c^2 + a*d^2)*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{dx+c}dx$$

input
```
int((b*x^2+a)^p/(d*x+c),x)
```

output
```
int((b*x^2+a)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \int \frac{(bx^2+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(d*x + c), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \int \frac{(a+bx^2)^p}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**p/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**p/(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \int \frac{(bx^2+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \int \frac{(bx^2+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \int \frac{(b\,x^2+a)^p}{c+d\,x}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x),x)
```

output
```
int((a + b*x^2)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{c+dx}\,dx = \int \frac{(b\,x^2+a)^p}{dx+c}dx$$

input
```
int((b*x^2+a)^p/(d*x+c),x)
```

output
```
int((a + b*x**2)**p/(c + d*x),x)
```

3.415 $\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = -\frac{(a+bx^2)^p}{d(c+dx)} - \frac{2bpx^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},1-p,1,\frac{5}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{3ac^2} + \frac{bc(a+bx^2)^p\operatorname{Hypergeometric2F1}\left(1,p,1+p,\frac{d^2(a+bx^2)}{bc^2+ad^2}\right)}{d\left(bc^2+ad^2\right)}$$

output
```
-(b*x^2+a)^p/d/(d*x+c)-2/3*b*p*x^3*(b*x^2+a)^p*AppellF1(3/2,1,1-p,5/2,d^2*
x^2/c^2,-b*x^2/a)/a/c^2/((1+b*x^2/a)^p)+b*c*(b*x^2+a)^p*hypergeom([1, p],[
p+1],d^2*(b*x^2+a)/(a*d^2+b*c^2))/d/(a*d^2+b*c^2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = \frac{\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}(a+bx^2)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx},\frac{c+\sqrt{-\frac{a}{b}}d}{c+dx}\right)}{d(-1+2p)(c+dx)}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x)^2,x]
```

output
```
((a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c - Sqrt[-(a/b)]*d)/(c
+ d*x), (c + Sqrt[-(a/b)]*d)/(c + d*x)])/(d*(-1 + 2*p)*((d*(-Sqrt[-(a/b)]
+ x))/(c + d*x))^p*((d*(Sqrt[-(a/b)] + x))/(c + d*x))^p*(c + d*x))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx$$

$$\Big\downarrow \ 505$$

$$\int \left(\frac{c^2(a+bx^2)^p}{(c^2-d^2x^2)^2}-\frac{2cdx(a+bx^2)^p}{(c^2-d^2x^2)^2}+\frac{d^2x^2(a+bx^2)^p}{(d^2x^2-c^2)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c^2}+\frac{d^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{3c^4}-\frac{bcd(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{d^2(bx^2+a)}{bc^2+ad^2}\right)}{(p+1)(ad^2+bc^2)^2}$$

input
```
Int[(a + b*x^2)^p/(c + d*x)^2,x]
```

output
```
(x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (d^2*x^2)/c^2])/(
c^2*(1 + (b*x^2)/a)^p) + (d^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2,
-((b*x^2)/a), (d^2*x^2)/c^2])/(3*c^4*(1 + (b*x^2)/a)^p) - (b*c*d*(a + b*x^
2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (d^2*(a + b*x^2))/(b*c^2 + a
*d^2)])/((b*c^2 + a*d^2)^2*(1 + p))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{(dx+c)^2}dx$$

input
```
int((b*x^2+a)^p/(d*x+c)^2,x)
```

output
```
int((b*x^2+a)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = \int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**p/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**p/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^p}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^p/(d*x+c)^2,x)
```

output
```
((a + b*x**2)**p*x + 4*int((a + b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*
p*x + 2*a*c*d*x + 2*a*d**2*p*x**2 + a*d**2*x**2 + 2*b*c**2*p*x**2 + b*c**2
*x**2 + 4*b*c*d*p*x**3 + 2*b*c*d*x**3 + 2*b*d**2*p*x**4 + b*d**2*x**4),x)*
a*c**2*p**2 + 2*int((a + b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2
*a*c*d*x + 2*a*d**2*p*x**2 + a*d**2*x**2 + 2*b*c**2*p*x**2 + b*c**2*x**2 +
 4*b*c*d*p*x**3 + 2*b*c*d*x**3 + 2*b*d**2*p*x**4 + b*d**2*x**4),x)*a*c**2*
p + 4*int((a + b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x +
 2*a*d**2*p*x**2 + a*d**2*x**2 + 2*b*c**2*p*x**2 + b*c**2*x**2 + 4*b*c*d*p
*x**3 + 2*b*c*d*x**3 + 2*b*d**2*p*x**4 + b*d**2*x**4),x)*a*c*d*p**2*x + 2*
int((a + b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d
**2*p*x**2 + a*d**2*x**2 + 2*b*c**2*p*x**2 + b*c**2*x**2 + 4*b*c*d*p*x**3
+ 2*b*c*d*x**3 + 2*b*d**2*p*x**4 + b*d**2*x**4),x)*a*c*d*p*x - 4*int(((a +
 b*x**2)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d**
2*p*x**2 + a*d**2*x**2 + 2*b*c**2*p*x**2 + b*c**2*x**2 + 4*b*c*d*p*x**3 +
2*b*c*d*x**3 + 2*b*d**2*p*x**4 + b*d**2*x**4),x)*b*c*d*p**2 - 2*int(((a +
b*x**2)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d**2
*p*x**2 + a*d**2*x**2 + 2*b*c**2*p*x**2 + b*c**2*x**2 + 4*b*c*d*p*x**3 + 2
*b*c*d*x**3 + 2*b*d**2*p*x**4 + b*d**2*x**4),x)*b*c*d*p - 4*int(((a + b*x*
*2)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d**2*p*x
**2 + a*d**2*x**2 + 2*b*c**2*p*x**2 + b*c**2*x**2 + 4*b*c*d*p*x**3 + 2*...
```

3.416 $\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 216

$$\begin{aligned}\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx &= -\frac{(a+bx^2)^p}{2d(c+dx)^2}+\frac{p(a+bx^2)^p}{2d(1-p)\left(c^2-d^2x^2\right)} \\ &\quad -\frac{2bpx^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},1-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{3ac^3} \\ &\quad -\frac{b\left(ad^2-bc^2(1-2p)\right)(a+bx^2)^p\operatorname{Hypergeometric2F1}\left(2,p,1+p,\frac{d^2(a+bx^2)}{bc^2+ad^2}\right)}{2d\left(bc^2+ad^2\right)^2(1-p)}\end{aligned}$$

output
```
-1/2*(b*x^2+a)^p/d/(d*x+c)^2+1/2*p*(b*x^2+a)^p/d/(1-p)/(-d^2*x^2+c^2)-2/3*
b*p*x^3*(b*x^2+a)^p*AppellF1(3/2,2,1-p,5/2,d^2*x^2/c^2,-b*x^2/a)/a/c^3/((1
+b*x^2/a)^p)-1/2*b*(a*d^2-b*c^2*(1-2*p))*(b*x^2+a)^p*hypergeom([2, p],[p+1
],d^2*(b*x^2+a)/(a*d^2+b*c^2))/d/(a*d^2+b*c^2)^2/(1-p)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx = \frac{\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}(a+bx^2)^p \operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx},\frac{c+\sqrt{-\frac{a}{b}}d}{c+dx}\right)}{2d(-1+p)(c+dx)^2}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x)^3,x]
```

output
```
((a + b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (c - Sqrt[-(a/b)]*d)/(c
+ d*x), (c + Sqrt[-(a/b)]*d)/(c + d*x)])/(2*d*(-1 + p)*((d*(-Sqrt[-(a/b)]
+ x))/(c + d*x))^p*((d*(Sqrt[-(a/b)] + x))/(c + d*x))^p*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{505}$$

$$\int \left(\frac{3cd^2x^2(a+bx^2)^p}{(c^2-d^2x^2)^3}-\frac{3c^2dx(a+bx^2)^p}{(c^2-d^2x^2)^3}+\frac{d^3x^3(a+bx^2)^p}{(d^2x^2-c^2)^3}+\frac{c^3(a+bx^2)^p}{(c^2-d^2x^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c^5}+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c^3}-\frac{3b^2c^2d(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{d^2(bx^2+a)}{bc^2+ad^2}\right)}{2(p+1)\left(ad^2+bc^2\right)^3}+\frac{bd(a+bx^2)^{p+1}\left(2ad^2+bc^2(p+1)\right)\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{d^2(bx^2+a)}{bc^2+ad^2}\right)}{4(p+1)\left(ad^2+bc^2\right)^3}-\frac{c^2d(a+bx^2)^{p+1}}{4\left(c^2-d^2x^2\right)^2\left(ad^2+bc^2\right)}$$

input
```
Int[(a + b*x^2)^p/(c + d*x)^3,x]
```

output
```
-1/4*(c^2*d*(a + b*x^2)^(1 + p))/((b*c^2 + a*d^2)*(c^2 - d^2*x^2)^2) + (x*
(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (d^2*x^2)/c^2])/(c^3
*(1 + (b*x^2)/a)^p) + (d^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((
b*x^2)/a), (d^2*x^2)/c^2])/(c^5*(1 + (b*x^2)/a)^p) + (b*d*(2*a*d^2 + b*c^2
*(1 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (d^2*(a +
 b*x^2))/(b*c^2 + a*d^2)])/(4*(b*c^2 + a*d^2)^3*(1 + p)) - (3*b^2*c^2*d*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (d^2*(a + b*x^2))/(b*
c^2 + a*d^2)])/(2*(b*c^2 + a*d^2)^3*(1 + p))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{(dx+c)^3}dx$$

input
```
int((b*x^2+a)^p/(d*x+c)^3,x)
```

output
```
int((b*x^2+a)^p/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a)**p/(d*x+c)**3,x)
```

output
```
Integral((a + b*x**2)**p/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^3}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(b\,x^2+a)^p}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^3}\,dx=\int \frac{(b\,x^2+a)^p}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int((b*x^2+a)^p/(d*x+c)^3,x)
```

output
```
int((a + b*x**2)**p/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.417 $\int (c+dx)^3 \left(2+3x^2\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 143

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx = \frac{d(c+dx)^2 \left(2+3x^2\right)^{1+p}}{6(2+p)} - \frac{d\left((3+2p)\left(2d^2-3c^2(5+2p)\right)-6cd(1+p)(3+p)x\right)\left(2+3x^2\right)^{1+p}}{18\left(6+13p+9p^2+2p^3\right)} - \frac{2^p c\left(2d^2-c^2(3+2p)\right) x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)}{3+2p}$$

output
```
d*(d*x+c)^2*(3*x^2+2)^(p+1)/(12+6*p)-d*((3+2*p)*(2*d^2-3*c^2*(5+2*p))-6*c*
d*(p+1)*(3+p)*x)*(3*x^2+2)^(p+1)/(36*p^3+162*p^2+234*p+108)-2^p*c*(2*d^2-c
^2*(3+2*p))*x*hypergeom([1/2, -p],[3/2],-3/2*x^2)/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.27

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx = \frac{9\ 2^{1+p} c^3 (2+3p+p^2)\, x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)+d\Big(-9c^2(2+p)\left(2^{1+p}-2(2+3x^2)^p-3\right.}{}$$

input
```
Integrate[(c + d*x)^3*(2 + 3*x^2)^p,x]
```

output
```
(9*2^(1 + p)*c^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2, (-3*x^2
)/2] + d*(-9*c^2*(2 + p)*(2^(1 + p) - 2*(2 + 3*x^2)^p - 3*x^2*(2 + 3*x^2)^
p) + d^2*(2^(2 + p) - 4*(2 + 3*x^2)^p + 9*x^4*(2 + 3*x^2)^p + 3*p*x^2*(2 +
 3*x^2)^(1 + p)) + 9*2^(1 + p)*c*d*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3
/2, -p, 5/2, (-3*x^2)/2]))/(18*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {497, 27, 676, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+2\right)^p (c+dx)^3 \, dx$$

$$\Big\downarrow \ 497$$

$$\frac{\int -2(c+dx)\left(-3(p+2)c^2-3d(p+3)xc+2d^2\right)\left(3x^2+2\right)^p dx}{6(p+2)}+\frac{d\left(3x^2+2\right)^{p+1}(c+dx)^2}{6(p+2)}$$

$$\Big\downarrow \ 27$$

$$\frac{d\left(3x^2+2\right)^{p+1}(c+dx)^2}{6(p+2)}-\frac{\int (c+dx)\left(-3(p+2)c^2-3d(p+3)xc+2d^2\right)\left(3x^2+2\right)^p dx}{3(p+2)}$$

$$\Big\downarrow \ 676$$

$$\frac{d(3x^2+2)^{p+1}(c+dx)^2}{6(p+2)}-\frac{\frac{3c(p+2)(2d^2-c^2(2p+3))\int (3x^2+2)^p dx}{2p+3}+\frac{d(3x^2+2)^{p+1}(2d^2-3c^2(2p+5))}{6(p+1)}-\frac{cd^2(p+3)x(3x^2+2)^{p+1}}{2p+3}}{3(p+2)}$$

$$\Big\downarrow \text{237}$$

$$\frac{d(3x^2+2)^{p+1}(c+dx)^2}{6(p+2)}-\frac{\frac{3c2^p(p+2)x(2d^2-c^2(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)}{2p+3}+\frac{d(3x^2+2)^{p+1}(2d^2-3c^2(2p+5))}{6(p+1)}-\frac{cd^2(p+3)x(3x^2+2)^{p+1}}{2p+3}}{3(p+2)}$$

input
```
Int[(c + d*x)^3*(2 + 3*x^2)^p,x]
```

output
```
(d*(c + d*x)^2*(2 + 3*x^2)^(1 + p))/(6*(2 + p)) - ((d*(2*d^2 - 3*c^2*(5 +
2*p))*(2 + 3*x^2)^(1 + p))/(6*(1 + p)) - (c*d^2*(3 + p)*x*(2 + 3*x^2)^(1 +
 p))/(3 + 2*p) + (3*2^p*c*(2 + p)*(2*d^2 - c^2*(3 + 2*p))*x*Hypergeometric
2F1[1/2, -p, 3/2, (-3*x^2)/2])/(3 + 2*p))/(3*(2 + p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))    Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

method	result
meijerg	$2^{-2+p} d^3 x^4 \text{hypergeom}\left(\left[2,-p\right],\left[3\right],-\frac{3x^2}{2}\right)+c\,d^2 2^p x^3 \text{hypergeom}\left(\left[\frac{3}{2},-p\right],\left[\frac{5}{2}\right],-\frac{3x^2}{2}\right)+3\,2^{p-}$

input
```
int((d*x+c)^3*(3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
2^(-2+p)*d^3*x^4*hypergeom([2,-p],[3],-3/2*x^2)+c*d^2*2^p*x^3*hypergeom([3
/2,-p],[5/2],-3/2*x^2)+3*2^(p-1)*c^2*d*x^2*hypergeom([1,-p],[2],-3/2*x^2)+
c^3*2^p*x*hypergeom([1/2,-p],[3/2],-3/2*x^2)
```

Fricas [F]

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx = \int (dx+c)^3 \left(3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(3*x^2 + 2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.73

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx$$
$$= 2^p c^3 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right) + 2^p c d^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)$$
$$+ \frac{c^2 d \left(\begin{cases} \frac{(3x^2+2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(3x^2+2) & \text{otherwise} \end{cases}\right)}{2}$$
$$+ d^3 \left(\begin{cases} \frac{3x^2 \log(3x^2+2)}{54x^2+36} + \frac{2\log(3x^2+2)}{54x^2+36} + \frac{2}{54x^2+36} & \text{for } p = -2 \\ \frac{x^2}{6} - \frac{\log(3x^2+2)}{9} & \text{for } p = -1 \\ \frac{9px^4(3x^2+2)^p}{18p^2+54p+36} + \frac{6px^2(3x^2+2)^p}{18p^2+54p+36} + \frac{9x^4(3x^2+2)^p}{18p^2+54p+36} - \frac{4(3x^2+2)^p}{18p^2+54p+36} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**3*(3*x**2+2)**p,x)
```

output
```
2**p*c**3*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(I*pi)/2) + 2**p*c*d*
*2*x**3*hyper((3/2, -p), (5/2,), 3*x**2*exp_polar(I*pi)/2) + c**2*d*Piecew
ise(((3*x**2 + 2)**(p + 1)/(p + 1), Ne(p, -1)), (log(3*x**2 + 2), True))/2
 + d**3*Piecewise((3*x**2*log(3*x**2 + 2)/(54*x**2 + 36) + 2*log(3*x**2 +
2)/(54*x**2 + 36) + 2/(54*x**2 + 36), Eq(p, -2)), (x**2/6 - log(3*x**2 + 2
)/9, Eq(p, -1)), (9*p*x**4*(3*x**2 + 2)**p/(18*p**2 + 54*p + 36) + 6*p*x**
2*(3*x**2 + 2)**p/(18*p**2 + 54*p + 36) + 9*x**4*(3*x**2 + 2)**p/(18*p**2
+ 54*p + 36) - 4*(3*x**2 + 2)**p/(18*p**2 + 54*p + 36), True))
```

Maxima [F]

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx = \int (dx+c)^3 \left(3x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx = \int (dx+c)^3 \left(3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx = \int \left(3\,x^2+2\right)^p \, (c+d\,x)^3 \, dx$$

input
```
int((3*x^2 + 2)^p*(c + d*x)^3,x)
```

output
```
int((3*x^2 + 2)^p*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \left(2+3x^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*(3*x^2+2)^p,x)
```

output
```
(36*(3*x**2 + 2)**p*c**3*p**3*x + 162*(3*x**2 + 2)**p*c**3*p**2*x + 234*(3
*x**2 + 2)**p*c**3*p*x + 108*(3*x**2 + 2)**p*c**3*x + 108*(3*x**2 + 2)**p*
c**2*d*p**3*x**2 + 72*(3*x**2 + 2)**p*c**2*d*p**3 + 432*(3*x**2 + 2)**p*c*
*2*d*p**2*x**2 + 288*(3*x**2 + 2)**p*c**2*d*p**2 + 513*(3*x**2 + 2)**p*c**
2*d*p*x**2 + 342*(3*x**2 + 2)**p*c**2*d*p + 162*(3*x**2 + 2)**p*c**2*d*x**
2 + 108*(3*x**2 + 2)**p*c**2*d + 108*(3*x**2 + 2)**p*c*d**2*p**3*x**3 + 72
*(3*x**2 + 2)**p*c*d**2*p**3*x + 378*(3*x**2 + 2)**p*c*d**2*p**2*x**3 + 21
6*(3*x**2 + 2)**p*c*d**2*p**2*x + 378*(3*x**2 + 2)**p*c*d**2*p*x**3 + 144*
(3*x**2 + 2)**p*c*d**2*p*x + 108*(3*x**2 + 2)**p*c*d**2*x**3 + 36*(3*x**2
+ 2)**p*d**3*p**3*x**4 + 24*(3*x**2 + 2)**p*d**3*p**3*x**2 + 108*(3*x**2 +
 2)**p*d**3*p**2*x**4 + 48*(3*x**2 + 2)**p*d**3*p**2*x**2 - 16*(3*x**2 + 2
)**p*d**3*p**2 + 99*(3*x**2 + 2)**p*d**3*p*x**4 + 18*(3*x**2 + 2)**p*d**3*
p*x**2 - 32*(3*x**2 + 2)**p*d**3*p + 27*(3*x**2 + 2)**p*d**3*x**4 - 12*(3*
x**2 + 2)**p*d**3 + 576*int((3*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p*
x**2 + 16*p + 9*x**2 + 6),x)*c**3*p**6 + 3744*int((3*x**2 + 2)**p/(12*p**2
*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)*c**3*p**5 + 9360*int((3
*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)*c
**3*p**4 + 11160*int((3*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p*x**2 +
16*p + 9*x**2 + 6),x)*c**3*p**3 + 6264*int((3*x**2 + 2)**p/(12*p**2*x**2 +
 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)*c**3*p**2 + 1296*int((3*x**...
```

3.418 $\int (c+dx)^2 (2+3x^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int (c+dx)^2 (2+3x^2)^p \, dx = \frac{d(c(3+2p)+d(1+p)x)(2+3x^2)^{1+p}}{3(3+5p+2p^2)} - \frac{2^p(2d^2-3c^2(3+2p))\, x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)}{3(3+2p)}$$

output
```
d*(c*(3+2*p)+d*(p+1)*x)*(3*x^2+2)^(p+1)/(6*p^2+15*p+9)-2^p*(2*d^2-3*c^2*(3
+2*p))*x*hypergeom([1/2, -p],[3/2],-3/2*x^2)/(9+6*p)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int (c+dx)^2 (2+3x^2)^p \, dx = 2^p c^2 x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right) + \frac{d\left(3cx^2(2+3x^2)^p - 2c(2^p-(2+3x^2)^p) + 2^p d(1+p)x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{3x^2}{2}\right)\right)}{3(1+p)}$$

input
```
Integrate[(c + d*x)^2*(2 + 3*x^2)^p,x]
```

output
```
2^p*c^2*x*Hypergeometric2F1[1/2, -p, 3/2, (-3*x^2)/2] + (d*(3*c*x^2*(2 + 3
*x^2)^p - 2*c*(2^p - (2 + 3*x^2)^p) + 2^p*d*(1 + p)*x^3*Hypergeometric2F1[
3/2, -p, 5/2, (-3*x^2)/2]))/(3*(1 + p))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {497, 25, 455, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2)^p (c+dx)^2 \, dx$$

$$\Big\downarrow \; 497$$

$$\frac{\int -\left(\left(-\left((6p+9)c^2\right)-6d(p+2)xc+2d^2\right)\left(3x^2+2\right)^p\right)dx}{3(2p+3)}+\frac{d\left(3x^2+2\right)^{p+1}(c+dx)}{3(2p+3)}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(3x^2+2\right)^{p+1}(c+dx)}{3(2p+3)}-\frac{\int \left(-3(2p+3)c^2-6d(p+2)xc+2d^2\right)\left(3x^2+2\right)^p dx}{3(2p+3)}$$

$$\Big\downarrow \; 455$$

$$\frac{d\left(3x^2+2\right)^{p+1}(c+dx)}{3(2p+3)}-\frac{\left(2d^2-3c^2(2p+3)\right)\int \left(3x^2+2\right)^p dx-\frac{cd(p+2)\left(3x^2+2\right)^{p+1}}{p+1}}{3(2p+3)}$$

$$\Big\downarrow \; 237$$

$$\frac{d\left(3x^2+2\right)^{p+1}(c+dx)}{3(2p+3)}-\frac{2^p x\left(2d^2-3c^2(2p+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)-\frac{cd(p+2)\left(3x^2+2\right)^{p+1}}{p+1}}{3(2p+3)}$$

input
```
Int[(c + d*x)^2*(2 + 3*x^2)^p,x]
```

output
```
(d*(c + d*x)*(2 + 3*x^2)^(1 + p))/(3*(3 + 2*p)) - (-((c*d*(2 + p)*(2 + 3*x
^2)^(1 + p))/(1 + p)) + 2^p*(2*d^2 - 3*c^2*(3 + 2*p))*x*Hypergeometric2F1[
1/2, -p, 3/2, (-3*x^2)/2])/(3*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

method	result
meijerg	$\frac{d^{2} 2^{p} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{2},-p\right],\left[\frac{5}{2}\right],-\frac{3 x^{2}}{2}\right)}{3}+c d 2^{p} x^{2} \operatorname{hypergeom}\left(\left[1,-p\right],\left[2\right],-\frac{3 x^{2}}{2}\right)+c^{2} 2^{p} x \operatorname{hypergeom}\left(\left[\right.\right.$

input
```
int((d*x+c)^2*(3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*2^p*x^3*hypergeom([3/2,-p],[5/2],-3/2*x^2)+c*d*2^p*x^2*hypergeom([
1,-p],[2],-3/2*x^2)+c^2*2^p*x*hypergeom([1/2,-p],[3/2],-3/2*x^2)
```

Fricas [F]

$$\int (c+dx)^2 \left(2+3x^2\right)^p \, dx = \int (dx+c)^2 \left(3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(3*x^2 + 2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int (c+dx)^2 \left(2+3x^2\right)^p \, dx = 2^p c^2 x\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right) + \frac{2^p d^2 x^3\, {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{3} + \frac{cd \left(\begin{cases} \frac{\left(3x^2+2\right)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(3x^2+2\right) & \text{otherwise} \end{cases}\right)}{3}$$

input
```
integrate((d*x+c)**2*(3*x**2+2)**p,x)
```

output
```
2**p*c**2*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(I*pi)/2) + 2**p*d**2
*x**3*hyper((3/2, -p), (5/2,), 3*x**2*exp_polar(I*pi)/2)/3 + c*d*Piecewise
(((3*x**2 + 2)**(p + 1)/(p + 1), Ne(p, -1)), (log(3*x**2 + 2), True))/3
```

Maxima [F]

$$\int (c+dx)^2 \left(2+3x^2\right)^p \, dx = \int (dx+c)^2 \left(3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(2+3x^2\right)^p \, dx = \int (dx+c)^2 \left(3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(2+3x^2\right)^p \, dx = \int \left(3\,x^2+2\right)^p \left(c+d\,x\right)^2 \, dx$$

input
```
int((3*x^2 + 2)^p*(c + d*x)^2,x)
```

output
```
int((3*x^2 + 2)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(2+3x^2\right)^p \, dx$$
$$= \frac{6(3x^2+2)^p c^2p^2x + 15(3x^2+2)^p c^2px + 9(3x^2+2)^p c^2x + 12(3x^2+2)^p cd\,p^2x^2 + 8(3x^2+2)^p cd\,p^2 + 24}{}$$

input
```
int((d*x+c)^2*(3*x^2+2)^p,x)
```

output
```
(6*(3*x**2 + 2)**p*c**2*p**2*x + 15*(3*x**2 + 2)**p*c**2*p*x + 9*(3*x**2 +
 2)**p*c**2*x + 12*(3*x**2 + 2)**p*c*d*p**2*x**2 + 8*(3*x**2 + 2)**p*c*d*p
**2 + 24*(3*x**2 + 2)**p*c*d*p*x**2 + 16*(3*x**2 + 2)**p*c*d*p + 9*(3*x**2
 + 2)**p*c*d*x**2 + 6*(3*x**2 + 2)**p*c*d + 6*(3*x**2 + 2)**p*d**2*p**2*x*
*3 + 4*(3*x**2 + 2)**p*d**2*p**2*x + 9*(3*x**2 + 2)**p*d**2*p*x**3 + 4*(3*
x**2 + 2)**p*d**2*p*x + 3*(3*x**2 + 2)**p*d**2*x**3 + 96*int((3*x**2 + 2)*
*p/(12*p**2*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)*c**2*p**5 +
432*int((3*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2
 + 6),x)*c**2*p**4 + 696*int((3*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p
*x**2 + 16*p + 9*x**2 + 6),x)*c**2*p**3 + 468*int((3*x**2 + 2)**p/(12*p**2
*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)*c**2*p**2 + 108*int((3*
x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)*c*
*2*p - 32*int((3*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p*x**2 + 16*p +
9*x**2 + 6),x)*d**2*p**4 - 96*int((3*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 +
 24*p*x**2 + 16*p + 9*x**2 + 6),x)*d**2*p**3 - 88*int((3*x**2 + 2)**p/(12*
p**2*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)*d**2*p**2 - 24*int(
(3*x**2 + 2)**p/(12*p**2*x**2 + 8*p**2 + 24*p*x**2 + 16*p + 9*x**2 + 6),x)
*d**2*p)/(3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.419 $\int (c+dx)\left(2+3x^2\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int (c+dx)\left(2+3x^2\right)^p\,dx=\frac{d\left(2+3x^2\right)^{1+p}}{6(1+p)}+2^pcx\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)$$

output
```
d*(3*x^2+2)^(p+1)/(6*p+6)+2^p*c*x*hypergeom([1/2, -p],[3/2],-3/2*x^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int (c+dx)\left(2+3x^2\right)^p\,dx =\frac{3dx^2(2+3x^2)^p-2d\left(2^p-(2+3x^2)^p\right)+3\;2^{1+p}c(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)}{6(1+p)}$$

input
```
Integrate[(c + d*x)*(2 + 3*x^2)^p,x]
```

output
```
(3*d*x^2*(2 + 3*x^2)^p - 2*d*(2^p - (2 + 3*x^2)^p) + 3*2^(1 + p)*c*(1 + p)
*x*Hypergeometric2F1[1/2, -p, 3/2, (-3*x^2)/2])/(6*(1 + p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {455, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2)^p (c+dx)\,dx$$

$$\Big\downarrow \ 455$$

$$c\int (3x^2+2)^p\,dx+\frac{d(3x^2+2)^{p+1}}{6(p+1)}$$

$$\Big\downarrow \ 237$$

$$c2^p x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{3x^2}{2}\right)+\frac{d(3x^2+2)^{p+1}}{6(p+1)}$$

input
```
Int[(c + d*x)*(2 + 3*x^2)^p,x]
```

output
```
(d*(2 + 3*x^2)^(1 + p))/(6*(1 + p)) + 2^p*c*x*Hypergeometric2F1[1/2, -p, 3
/2, (-3*x^2)/2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	size
meijerg	$2^{p-1} d\,x^{2} \operatorname{hypergeom}\left(\left[1, -p\right], \left[2\right], -\frac{3 x^{2}}{2}\right)+2^{p} c x \operatorname{hypergeom}\left(\left[\frac{1}{2}, -p\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)$	44

input
```
int((d*x+c)*(3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
2^(p-1)*d*x^2*hypergeom([1,-p],[2],-3/2*x^2)+2^p*c*x*hypergeom([1/2,-p],[3
/2],-3/2*x^2)
```

Fricas [F]

$$\int (c+dx) \left(2+3x^2\right)^p \, dx = \int (dx+c){\left(3\,x^{2}+2\right)}^{p}\,dx$$

input
```
integrate((d*x+c)*(3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(3*x^2 + 2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c+dx) \left(2+3x^2\right)^p \, dx = 2^{p} c x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| {\frac{3 x^{2} e^{i \pi}}{2}} \right) + \frac{d \left(\begin{cases} \frac{\left(3 x^{2} + 2\right)^{p + 1}}{p + 1} & \text{for}\: p \neq -1 \\ \log{\left(3 x^{2} + 2 \right)} & \text{otherwise} \end{cases}\right)}{6}$$

input
```
integrate((d*x+c)*(3*x**2+2)**p,x)
```

output
```
2**p*c*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(I*pi)/2) + d*Piecewise(
((3*x**2 + 2)**(p + 1)/(p + 1), Ne(p, -1)), (log(3*x**2 + 2), True))/6
```

Maxima [F]

$$\int (c+dx)\left(2+3x^2\right)^p\,dx=\int (dx+c)\left(3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)*(3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)\left(2+3x^2\right)^p\,dx=\int (dx+c)\left(3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)*(3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(3*x^2 + 2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int (c+dx)\left(2+3x^2\right)^p\,dx=\frac{d\,\left(3\,x^2+2\right)^{p+1}}{6\,\left(p+1\right)}+2^p\,c\,x\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)$$

input
```
int((3*x^2 + 2)^p*(c + d*x),x)
```

output
```
(d*(3*x^2 + 2)^(p + 1))/(6*(p + 1)) + 2^p*c*x*hypergeom([1/2, -p], 3/2, -(
3*x^2)/2)
```

Reduce [F]

$$\int (c+dx)\left(2+3x^2\right)^p\,dx$$
$$=\frac{6(3x^2+2)^p\,cpx+6(3x^2+2)^p\,cx+6(3x^2+2)^p\,dp\,x^2+4(3x^2+2)^p\,dp+3(3x^2+2)^p\,d\,x^2+2(3x^2+2)^p}{12p^2+18p+}$$

input
```
int((d*x+c)*(3*x^2+2)^p,x)
```

output
```
(6*(3*x**2 + 2)**p*c*p*x + 6*(3*x**2 + 2)**p*c*x + 6*(3*x**2 + 2)**p*d*p*x
**2 + 4*(3*x**2 + 2)**p*d*p + 3*(3*x**2 + 2)**p*d*x**2 + 2*(3*x**2 + 2)**p
*d + 48*int((3*x**2 + 2)**p/(6*p*x**2 + 4*p + 3*x**2 + 2),x)*c*p**3 + 72*i
nt((3*x**2 + 2)**p/(6*p*x**2 + 4*p + 3*x**2 + 2),x)*c*p**2 + 24*int((3*x**
2 + 2)**p/(6*p*x**2 + 4*p + 3*x**2 + 2),x)*c*p)/(6*(2*p**2 + 3*p + 1))
```

3.420 $\int (2+3x^2)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int (2+3x^2)^p \, dx = 2^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{3x^2}{2}\right)$$

output
```
2^p*x*hypergeom([1/2, -p],[3/2],-3/2*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (2+3x^2)^p \, dx = 2^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{3x^2}{2}\right)$$

input
```
Integrate[(2 + 3*x^2)^p,x]
```

output
```
2^p*x*Hypergeometric2F1[1/2, -p, 3/2, (-3*x^2)/2]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2)^p \, dx$$

$$\Big\downarrow \; 237$$

$$2^p x \, \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{3x^2}{2}\right)$$

input
```
Int[(2 + 3*x^2)^p,x]
```

output
```
2^p*x*Hypergeometric2F1[1/2, -p, 3/2, (-3*x^2)/2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
meijerg	$2^p x \,\text{hypergeom}\left(\left[\frac{1}{2}, -p\right], \left[\frac{3}{2}\right], -\frac{3x^2}{2}\right)$	19

input
```
int((3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
2^p*x*hypergeom([1/2,-p],[3/2],-3/2*x^2)
```

Fricas [F]

$$\int (2+3x^2)^p \, dx = \int (3x^2+2)^p \, dx$$

input
```
integrate((3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (2+3x^2)^p \, dx = 2^p x {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\right| \frac{3x^2 e^{i\pi}}{2}\right)$$

input
```
integrate((3*x**2+2)**p,x)
```

output
```
2**p*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(I*pi)/2)
```

Maxima [F]

$$\int (2+3x^2)^p \, dx = \int (3x^2+2)^p \, dx$$

input
```
integrate((3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^p, x)
```

Giac [F]

$$\int \left(2+3x^2\right)^p\,dx=\int \left(3\,x^2+2\right)^p\,dx$$

input
```
integrate((3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \left(2+3x^2\right)^p\,dx=2^p\,x\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)$$

input
```
int((3*x^2 + 2)^p,x)
```

output
```
2^p*x*hypergeom([1/2, -p], 3/2, -(3*x^2)/2)
```

Reduce [F]

$$\int \left(2+3x^2\right)^p\,dx=\frac{\left(3x^2+2\right)^p x+8\left(\int \frac{\left(3x^2+2\right)^p}{6p\,x^2+3x^2+4p+2}dx\right)p^2+4\left(\int \frac{\left(3x^2+2\right)^p}{6p\,x^2+3x^2+4p+2}dx\right)p}{2p+1}$$

input
```
int((3*x^2+2)^p,x)
```

output
```
((3*x**2 + 2)**p*x + 8*int((3*x**2 + 2)**p/(6*p*x**2 + 4*p + 3*x**2 + 2),x
)*p**2 + 4*int((3*x**2 + 2)**p/(6*p*x**2 + 4*p + 3*x**2 + 2),x)*p)/(2*p +
1)
```

3.421 $\int \frac{(2+3x^2)^p}{c+dx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \frac{2^p x \operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c} - \frac{d(2+3x^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(2+3x^2)}{3c^2+2d^2}\right)}{2(3c^2+2d^2)(1+p)}$$

output
```
2^p*x*AppellF1(1/2,1,-p,3/2,d^2*x^2/c^2,-3/2*x^2)/c-1/2*d*(3*x^2+2)^(p+1)*
hypergeom([1, p+1],[2+p],d^2*(3*x^2+2)/(3*c^2+2*d^2))/(3*c^2+2*d^2)/(p+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \frac{9^p\left(\frac{d(-i\sqrt{6}+3x)}{c+dx}\right)^{-p}\left(\frac{d(i\sqrt{6}+3x)}{c+dx}\right)^{-p}(2+3x^2)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{3c-i\sqrt{6}d}{3c+3dx},\frac{3c+i\sqrt{6}d}{3c+3dx}\right)}{2dp}$$

input
```
Integrate[(2 + 3*x^2)^p/(c + d*x),x]
```

output
```
(9^p*(2 + 3*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (3*c - I*Sqrt[6]*d)/(3*
c + 3*d*x), (3*c + I*Sqrt[6]*d)/(3*c + 3*d*x)])/(2*d*p*((d*((-I)*Sqrt[6] +
 3*x))/(c + d*x))^p*((d*(I*Sqrt[6] + 3*x))/(c + d*x))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {504, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)^p}{c+dx}dx$$

$$\Big\downarrow \text{504}$$

$$c\int \frac{(3x^2+2)^p}{c^2-d^2x^2}dx-d\int \frac{x(3x^2+2)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{333}$$

$$\frac{2^px\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c}-d\int \frac{x(3x^2+2)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{2^px\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c}-\frac{1}{2}d\int \frac{(3x^2+2)^p}{c^2-d^2x^2}dx^2$$

$$\Big\downarrow \text{78}$$

$$\frac{2^px\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c}-\frac{d(3x^2+2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2(3x^2+2)}{3c^2+2d^2}\right)}{2(p+1)(3c^2+2d^2)}$$

input
```
Int[(2 + 3*x^2)^p/(c + d*x),x]
```

output
```
(2^p*x*AppellF1[1/2, -p, 1, 3/2, (-3*x^2)/2, (d^2*x^2)/c^2])/c - (d*(2 + 3
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (d^2*(2 + 3*x^2))/(3*c^2
+ 2*d^2)])/(2*(3*c^2 + 2*d^2)*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

Maple [F]

$$\int \frac{(3x^2+2)^p}{dx+c}dx$$

input
```
int((3*x^2+2)^p/(d*x+c),x)
```

output
```
int((3*x^2+2)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \int \frac{(3\,x^2+2)^p}{dx+c}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^p/(d*x + c), x)
```

Sympy [F]

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \int \frac{(3x^2+2)^p}{c+dx}\,dx$$

input
```
integrate((3*x**2+2)**p/(d*x+c),x)
```

output
```
Integral((3*x**2 + 2)**p/(c + d*x), x)
```

Maxima [F]

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \int \frac{(3\,x^2+2)^p}{dx+c}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \int \frac{(3\,x^2+2)^p}{dx+c}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \int \frac{(3\,x^2+2)^p}{c+d\,x}\,dx$$

input
```
int((3*x^2 + 2)^p/(c + d*x),x)
```

output
```
int((3*x^2 + 2)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(2+3x^2)^p}{c+dx}\,dx = \int \frac{(3x^2+2)^p}{dx+c}dx$$

input
```
int((3*x^2+2)^p/(d*x+c),x)
```

output
```
int((3*x**2 + 2)**p/(c + d*x),x)
```

3.422 $\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 122

$$\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx = -\frac{(2+3x^2)^p}{d(c+dx)} - \frac{2^p p x^3 \operatorname{AppellF1}\left(\frac{3}{2},1-p,1,\frac{5}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c^2} + \frac{3c(2+3x^2)^p \operatorname{Hypergeometric2F1}\left(1,p,1+p,\frac{d^2(2+3x^2)}{3c^2+2d^2}\right)}{d\left(3c^2+2d^2\right)}$$

output

```
-(3*x^2+2)^p/d/(d*x+c)-2^p*p*x^3*AppellF1(3/2,1,1-p,5/2,d^2*x^2/c^2,-3/2*x
^2)/c^2+3*c*(3*x^2+2)^p*hypergeom([1, p],[p+1],d^2*(3*x^2+2)/(3*c^2+2*d^2)
)/d/(3*c^2+2*d^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.22

$$\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx = \frac{9^p\left(\frac{d\left(-i\sqrt{6}+3x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(i\sqrt{6}+3x\right)}{c+dx}\right)^{-p}(2+3x^2)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{3c-i\sqrt{6}d}{3c+3dx},\frac{3c+i\sqrt{6}d}{3c+3dx}\right)}{d(-1+2p)(c+dx)}$$

input
```
Integrate[(2 + 3*x^2)^p/(c + d*x)^2,x]
```

output
```
(9^p*(2 + 3*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (3*c - I*Sqrt[6]*d)/
(3*c + 3*d*x), (3*c + I*Sqrt[6]*d)/(3*c + 3*d*x)])/(d*(-1 + 2*p)*((d*((-I)
*Sqrt[6] + 3*x))/(c + d*x))^p*((d*(I*Sqrt[6] + 3*x))/(c + d*x))^p*(c + d*x
))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)^p}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{505}$$

$$\int \left(\frac{c^2(3x^2+2)^p}{(c^2-d^2x^2)^2}-\frac{2cdx(3x^2+2)^p}{(c^2-d^2x^2)^2}+\frac{d^2x^2(3x^2+2)^p}{(d^2x^2-c^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2^p x \operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c^2}+\frac{d^2 2^p x^3 \operatorname{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{3c^4}-\frac{3cd(3x^2+2)^{p+1}\operatorname{Hypergeometric2F1}\left(2,p+1,p+2,\frac{d^2(3x^2+2)}{3c^2+2d^2}\right)}{(p+1)(3c^2+2d^2)^2}$$

input
```
Int[(2 + 3*x^2)^p/(c + d*x)^2,x]
```

output
```
(2^p*x*AppellF1[1/2, -p, 2, 3/2, (-3*x^2)/2, (d^2*x^2)/c^2])/c^2 + (2^p*d^
2*x^3*AppellF1[3/2, -p, 2, 5/2, (-3*x^2)/2, (d^2*x^2)/c^2])/(3*c^4) - (3*c
*d*(2 + 3*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (d^2*(2 + 3*x^2)
)/(3*c^2 + 2*d^2)])/((3*c^2 + 2*d^2)^2*(1 + p))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(3x^2+2)^p}{(dx+c)^2}dx$$

input
```
int((3*x^2+2)^p/(d*x+c)^2,x)
```

output
```
int((3*x^2+2)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^2} dx = \int \frac{(3x^2+2)^p}{(dx+c)^2} dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(3x^2+2)^p}{(c+dx)^2}\,dx$$

input
```
integrate((3*x**2+2)**p/(d*x+c)**2,x)
```

output
```
Integral((3*x**2 + 2)**p/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(3\,x^2+2)^p}{(dx+c)^2}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(3\,x^2+2)^p}{(dx+c)^2}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(3\,x^2+2)^p}{(c+d\,x)^2}\,dx$$

input
```
int((3*x^2 + 2)^p/(c + d*x)^2,x)
```

output
```
int((3*x^2 + 2)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^2}\,dx$$
$$= \frac{(3x^2+2)^p\,x + 8\left(\int \frac{(3x^2+2)^p}{6d^2p\,x^4+12cdp\,x^3+3d^2x^4+6c^2p\,x^2+6cd\,x^3+4d^2p\,x^2+3c^2x^2+8cdpx+2d^2x^2+4c^2p+4cdx+2c^2}dx\right)c^2p^2+4\left(\int\right.}{}$$

input
```
int((3*x^2+2)^p/(d*x+c)^2,x)
```

output
```
((3*x**2 + 2)**p*x + 8*int((3*x**2 + 2)**p/(6*c**2*p*x**2 + 4*c**2*p + 3*c
**2*x**2 + 2*c**2 + 12*c*d*p*x**3 + 8*c*d*p*x + 6*c*d*x**3 + 4*c*d*x + 6*d
**2*p*x**4 + 4*d**2*p*x**2 + 3*d**2*x**4 + 2*d**2*x**2),x)*c**2*p**2 + 4*i
nt((3*x**2 + 2)**p/(6*c**2*p*x**2 + 4*c**2*p + 3*c**2*x**2 + 2*c**2 + 12*c
*d*p*x**3 + 8*c*d*p*x + 6*c*d*x**3 + 4*c*d*x + 6*d**2*p*x**4 + 4*d**2*p*x*
*2 + 3*d**2*x**4 + 2*d**2*x**2),x)*c**2*p + 8*int((3*x**2 + 2)**p/(6*c**2*
p*x**2 + 4*c**2*p + 3*c**2*x**2 + 2*c**2 + 12*c*d*p*x**3 + 8*c*d*p*x + 6*c
*d*x**3 + 4*c*d*x + 6*d**2*p*x**4 + 4*d**2*p*x**2 + 3*d**2*x**4 + 2*d**2*x
**2),x)*c*d*p**2*x + 4*int((3*x**2 + 2)**p/(6*c**2*p*x**2 + 4*c**2*p + 3*c
**2*x**2 + 2*c**2 + 12*c*d*p*x**3 + 8*c*d*p*x + 6*c*d*x**3 + 4*c*d*x + 6*d
**2*p*x**4 + 4*d**2*p*x**2 + 3*d**2*x**4 + 2*d**2*x**2),x)*c*d*p*x - 12*in
t(((3*x**2 + 2)**p*x**3)/(6*c**2*p*x**2 + 4*c**2*p + 3*c**2*x**2 + 2*c**2
+ 12*c*d*p*x**3 + 8*c*d*p*x + 6*c*d*x**3 + 4*c*d*x + 6*d**2*p*x**4 + 4*d**
2*p*x**2 + 3*d**2*x**4 + 2*d**2*x**2),x)*c*d*p**2 - 6*int(((3*x**2 + 2)**p
*x**3)/(6*c**2*p*x**2 + 4*c**2*p + 3*c**2*x**2 + 2*c**2 + 12*c*d*p*x**3 +
8*c*d*p*x + 6*c*d*x**3 + 4*c*d*x + 6*d**2*p*x**4 + 4*d**2*p*x**2 + 3*d**2*
x**4 + 2*d**2*x**2),x)*c*d*p - 12*int(((3*x**2 + 2)**p*x**3)/(6*c**2*p*x**
2 + 4*c**2*p + 3*c**2*x**2 + 2*c**2 + 12*c*d*p*x**3 + 8*c*d*p*x + 6*c*d*x*
*3 + 4*c*d*x + 6*d**2*p*x**4 + 4*d**2*p*x**2 + 3*d**2*x**4 + 2*d**2*x**2),
x)*d**2*p**2*x - 6*int(((3*x**2 + 2)**p*x**3)/(6*c**2*p*x**2 + 4*c**2*p...
```

3.423 $\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx$$
$$= -\frac{(2+3x^2)^p}{2d(c+dx)^2}+\frac{p(2+3x^2)^p}{2d(1-p)\,(c^2-d^2x^2)}-\frac{2^p p x^3 \operatorname{AppellF1}\left(\frac{3}{2},1-p,2,\frac{5}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c^3}$$
$$-\frac{3(2d^2-c^2(3-6p))\,(2+3x^2)^p \operatorname{Hypergeometric2F1}\left(2,p,1+p,\frac{d^2(2+3x^2)}{3c^2+2d^2}\right)}{2d\,(3c^2+2d^2)^2\,(1-p)}$$

output
```
-1/2*(3*x^2+2)^p/d/(d*x+c)^2+1/2*p*(3*x^2+2)^p/d/(1-p)/(-d^2*x^2+c^2)-2^p*
p*x^3*AppellF1(3/2,2,1-p,5/2,d^2*x^2/c^2,-3/2*x^2)/c^3-3/2*(2*d^2-c^2*(3-6
*p))*(3*x^2+2)^p*hypergeom([2, p],[p+1],d^2*(3*x^2+2)/(3*c^2+2*d^2))/d/(3*
c^2+2*d^2)^2/(1-p)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx = \frac{9^p \left(\frac{d\left(-i\sqrt{6}+3x\right)}{c+dx}\right)^{-p} \left(\frac{d\left(i\sqrt{6}+3x\right)}{c+dx}\right)^{-p} (2+3x^2)^p \operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{3c-i\sqrt{6}d}{3c+3dx},\frac{3c+i\sqrt{6}d}{3c+3dx}\right)}{2d(-1+p)(c+dx)^2}$$

input
```
Integrate[(2 + 3*x^2)^p/(c + d*x)^3,x]
```

output
```
(9^p*(2 + 3*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (3*c - I*Sqrt[6]*d)/
(3*c + 3*d*x), (3*c + I*Sqrt[6]*d)/(3*c + 3*d*x)])/(2*d*(-1 + p)*((d*((-I)
*Sqrt[6] + 3*x))/(c + d*x))^p*((d*(I*Sqrt[6] + 3*x))/(c + d*x))^p*(c + d*x
)^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)^p}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{505}$$

$$\int \left(\frac{3cd^2x^2(3x^2+2)^p}{(c^2-d^2x^2)^3} - \frac{3c^2dx(3x^2+2)^p}{(c^2-d^2x^2)^3} + \frac{d^3x^3(3x^2+2)^p}{(d^2x^2-c^2)^3} + \frac{c^3(3x^2+2)^p}{(c^2-d^2x^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2 2^p x^3 \operatorname{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c^5}+\frac{2^p x \operatorname{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c^3}+\frac{3d(3x^2+2)^{p+1}(3c^2(p+1)+4d^2)\operatorname{Hypergeometric2F1}\left(2,p+1,p+2,\frac{d^2(3x^2+2)}{3c^2+2d^2}\right)}{4(p+1)(3c^2+2d^2)^3}-\frac{27c^2d(3x^2+2)^{p+1}\operatorname{Hypergeometric2F1}\left(3,p+1,p+2,\frac{d^2(3x^2+2)}{3c^2+2d^2}\right)}{2(p+1)(3c^2+2d^2)^3}-\frac{c^2d(3x^2+2)^{p+1}}{4(3c^2+2d^2)(c^2-d^2x^2)^2}$$

input
```
Int[(2 + 3*x^2)^p/(c + d*x)^3,x]
```

output
```
-1/4*(c^2*d*(2 + 3*x^2)^(1 + p))/((3*c^2 + 2*d^2)*(c^2 - d^2*x^2)^2) + (2^
p*x*AppellF1[1/2, -p, 3, 3/2, (-3*x^2)/2, (d^2*x^2)/c^2])/c^3 + (2^p*d^2*x
^3*AppellF1[3/2, -p, 3, 5/2, (-3*x^2)/2, (d^2*x^2)/c^2])/c^5 + (3*d*(4*d^2
 + 3*c^2*(1 + p))*(2 + 3*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (
d^2*(2 + 3*x^2))/(3*c^2 + 2*d^2)])/(4*(3*c^2 + 2*d^2)^3*(1 + p)) - (27*c^2
*d*(2 + 3*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (d^2*(2 + 3*x^2)
)/(3*c^2 + 2*d^2)])/(2*(3*c^2 + 2*d^2)^3*(1 + p))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(3x^2+2)^p}{(dx+c)^3}dx$$

input
```
int((3*x^2+2)^p/(d*x+c)^3,x)
```

output
```
int((3*x^2+2)^p/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(3\,x^2+2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^p/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(3x^2+2)^p}{(c+dx)^3}\,dx$$

input
```
integrate((3*x**2+2)**p/(d*x+c)**3,x)
```

output
```
Integral((3*x**2 + 2)**p/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(3\,x^2+2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(3\,x^2+2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(3\,x^2+2)^p}{(c+d\,x)^3}\,dx$$

input
```
int((3*x^2 + 2)^p/(c + d*x)^3,x)
```

output
```
int((3*x^2 + 2)^p/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(3x^2+2)^p}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int((3*x^2+2)^p/(d*x+c)^3,x)
```

output
```
int((3*x**2 + 2)**p/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.424 $\int (c+dx)^3 (a-bx^2)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 160

$$\int (c+dx)^3 (a-bx^2)^p \, dx = -\frac{d(3bc^2+ad^2)(a-bx^2)^{1+p}}{2b^2(1+p)} - \frac{3cd^2x(a-bx^2)^{1+p}}{b(3+2p)} + \frac{d^3(a-bx^2)^{2+p}}{2b^2(2+p)} + c\left(c^2+\frac{3ad^2}{3b+2bp}\right)x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)$$

output
```
-1/2*d*(a*d^2+3*b*c^2)*(-b*x^2+a)^(p+1)/b^2/(p+1)-3*c*d^2*x*(-b*x^2+a)^(p+
1)/b/(3+2*p)+1/2*d^3*(-b*x^2+a)^(2+p)/b^2/(2+p)+c*(c^2+3*a*d^2/(2*b*p+3*b)
)*x*(-b*x^2+a)^p*hypergeom([1/2, -p],[3/2],b*x^2/a)/((1-b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.42

$$\int (c+dx)^3 (a-bx^2)^p \, dx = \frac{(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\left(2b^2c^3(2+3p+p^2)\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)+d\left(b^2x^2\left(1-\frac{bx^2}{a}\right)^p(3\right.\right.}{}$$

input
```
Integrate[(c + d*x)^3*(a - b*x^2)^p,x]
```

output
```
((a - b*x^2)^p*(2*b^2*c^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2
, (b*x^2)/a] + d*(b^2*x^2*(1 - (b*x^2)/a)^p*(3*c^2*(2 + p) + d^2*(1 + p)*x
^2) - a^2*d^2*(-1 + (1 - (b*x^2)/a)^p) - a*b*(d^2*p*x^2*(1 - (b*x^2)/a)^p
+ 3*c^2*(2 + p)*(-1 + (1 - (b*x^2)/a)^p)) + 2*b^2*c*d*(2 + 3*p + p^2)*x^3*
Hypergeometric2F1[3/2, -p, 5/2, (b*x^2)/a])))/(2*b^2*(1 + p)*(2 + p)*(1 -
(b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {497, 27, 676, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a-bx^2\right)^p \, dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{\int -2(c+dx)\left(b(p+2)c^2+bd(p+3)xc+ad^2\right)\left(a-bx^2\right)^p dx}{2b(p+2)}-\frac{d(c+dx)^2\left(a-bx^2\right)^{p+1}}{2b(p+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c+dx)\left(b(p+2)c^2+bd(p+3)xc+ad^2\right)\left(a-bx^2\right)^p dx}{b(p+2)}-\frac{d(c+dx)^2\left(a-bx^2\right)^{p+1}}{2b(p+2)}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{c(p+2)\left(3ad^2+bc^2(2p+3)\right)\int \left(a-bx^2\right)^p dx}{2p+3}-\frac{d\left(a-bx^2\right)^{p+1}\left(ad^2+bc^2(2p+5)\right)}{2b(p+1)}-\frac{cd^2(p+3)x\left(a-bx^2\right)^{p+1}}{2p+3}}{b(p+2)}-\frac{d(c+dx)^2\left(a-bx^2\right)^{p+1}}{2b(p+2)}$$

$$\Big\downarrow \text{238}$$

$$\frac{\frac{c(p+2)(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}(3ad^2+bc^2(2p+3))\int\left(1-\frac{bx^2}{a}\right)^p dx}{2p+3}-\frac{d(a-bx^2)^{p+1}(ad^2+bc^2(2p+5))}{2b(p+1)}-\frac{cd^2(p+3)x(a-bx^2)^{p+1}}{2p+3}}{b(p+2)}-\frac{d(c+dx)^2(a-bx^2)^{p+1}}{2b(p+2)}$$

$$\Big\downarrow \text{237}$$

$$\frac{\frac{c(p+2)x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}(3ad^2+bc^2(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)}{2p+3}-\frac{d(a-bx^2)^{p+1}(ad^2+bc^2(2p+5))}{2b(p+1)}-\frac{cd^2(p+3)x(a-bx^2)^{p+1}}{2p+3}}{b(p+2)}-\frac{d(c+dx)^2(a-bx^2)^{p+1}}{2b(p+2)}$$

input
```
Int[(c + d*x)^3*(a - b*x^2)^p,x]
```

output
```
-1/2*(d*(c + d*x)^2*(a - b*x^2)^(1 + p))/(b*(2 + p)) + (-1/2*(d*(a*d^2 + b
*c^2*(5 + 2*p))*(a - b*x^2)^(1 + p))/(b*(1 + p)) - (c*d^2*(3 + p)*x*(a - b
*x^2)^(1 + p))/(3 + 2*p) + (c*(2 + p)*(3*a*d^2 + b*c^2*(3 + 2*p))*x*(a - b
*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (b*x^2)/a])/((3 + 2*p)*(1 - (b*x^2
)/a)^p))/(b*(2 + p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (dx+c)^3 \left(-b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^3*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^3*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^3 \left(a-bx^2\right)^p \, dx = \int (dx+c)^3 (-bx^2+a)^p \, dx$$

input
```
integrate((d*x+c)^3*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(133) = 266$.

Time = 6.02 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.71

$$\int (c+dx)^3 (a-bx^2)^p \, dx = a^p c^3 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{2i\pi}}{a}\right) + a^p c d^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{2i\pi}}{a}\right) + 3c^2 d \left(\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ -\frac{\begin{cases} \frac{(a-bx^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a-bx^2) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + d^3 \left(\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ -\frac{a \log\left(x - \sqrt{\frac{a}{b}}\right)}{-2ab^2 + 2b^3x^2} - \frac{a \log\left(x + \sqrt{\frac{a}{b}}\right)}{-2ab^2 + 2b^3x^2} - \frac{a}{-2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x - \sqrt{\frac{a}{b}}\right)}{-2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x + \sqrt{\frac{a}{b}}\right)}{-2ab^2 + 2b^3x^2} & \text{for } p = -2 \\ -\frac{a \log\left(x - \sqrt{\frac{a}{b}}\right)}{2b^2} - \frac{a \log\left(x + \sqrt{\frac{a}{b}}\right)}{2b^2} - \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2 (a-bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} - \frac{abpx^2 (a-bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2px^4 (a-bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2x^4 (a-bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**3*(-b*x**2+a)**p,x)
```

output
```
a**p*c**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + a**p*c*
d**2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 3*c**2*d*
Piecewise((a**p*x**2/2, Eq(b, 0)), (-Piecewise(((a - b*x**2)**(p + 1)/(p +
 1), Ne(p, -1)), (log(a - b*x**2), True))/(2*b), True)) + d**3*Piecewise((
a**p*x**4/4, Eq(b, 0)), (-a*log(x - sqrt(a/b))/(-2*a*b**2 + 2*b**3*x**2) -
 a*log(x + sqrt(a/b))/(-2*a*b**2 + 2*b**3*x**2) - a/(-2*a*b**2 + 2*b**3*x*
*2) + b*x**2*log(x - sqrt(a/b))/(-2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x +
 sqrt(a/b))/(-2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(a/b))/
(2*b**2) - a*log(x + sqrt(a/b))/(2*b**2) - x**2/(2*b), Eq(p, -1)), (-a**2*
(a - b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) - a*b*p*x**2*(a - b*x**2
)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a - b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a - b*x**2)**p/(2*b**2*p**2 + 6*
b**2*p + 4*b**2), True))
```

Maxima [F]

$$\int (c+dx)^3 \left(a-bx^2\right)^p \, dx = \int (dx+c)^3 \left(-bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(-b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^3 \left(a-bx^2\right)^p \, dx = \int (dx+c)^3 \left(-bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(-b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a-bx^2\right)^p \, dx = \int \left(a-b\,x^2\right)^p \,(c+d\,x)^3 \, dx$$

input
```
int((a - b*x^2)^p*(c + d*x)^3,x)
```

output
```
int((a - b*x^2)^p*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \left(a-bx^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*(-b*x^2+a)^p,x)
```

output
```
( - 4*(a - b*x**2)**p*a**2*d**3*p**2 - 8*(a - b*x**2)**p*a**2*d**3*p - 3*(
a - b*x**2)**p*a**2*d**3 - 12*(a - b*x**2)**p*a*b*c**2*d*p**3 - 48*(a - b*
x**2)**p*a*b*c**2*d*p**2 - 57*(a - b*x**2)**p*a*b*c**2*d*p - 18*(a - b*x**
2)**p*a*b*c**2*d - 12*(a - b*x**2)**p*a*b*c*d**2*p**3*x - 36*(a - b*x**2)*
*p*a*b*c*d**2*p**2*x - 24*(a - b*x**2)**p*a*b*c*d**2*p*x - 4*(a - b*x**2)*
*p*a*b*d**3*p**3*x**2 - 8*(a - b*x**2)**p*a*b*d**3*p**2*x**2 - 3*(a - b*x*
*2)**p*a*b*d**3*p*x**2 + 4*(a - b*x**2)**p*b**2*c**3*p**3*x + 18*(a - b*x*
*2)**p*b**2*c**3*p**2*x + 26*(a - b*x**2)**p*b**2*c**3*p*x + 12*(a - b*x**
2)**p*b**2*c**3*x + 12*(a - b*x**2)**p*b**2*c**2*d*p**3*x**2 + 48*(a - b*x
**2)**p*b**2*c**2*d*p**2*x**2 + 57*(a - b*x**2)**p*b**2*c**2*d*p*x**2 + 18
*(a - b*x**2)**p*b**2*c**2*d*x**2 + 12*(a - b*x**2)**p*b**2*c*d**2*p**3*x*
*3 + 42*(a - b*x**2)**p*b**2*c*d**2*p**2*x**3 + 42*(a - b*x**2)**p*b**2*c*
d**2*p*x**3 + 12*(a - b*x**2)**p*b**2*c*d**2*x**3 + 4*(a - b*x**2)**p*b**2
*d**3*p**3*x**4 + 12*(a - b*x**2)**p*b**2*d**3*p**2*x**4 + 11*(a - b*x**2)
**p*b**2*d**3*p*x**4 + 3*(a - b*x**2)**p*b**2*d**3*x**4 + 48*int((a - b*x*
*2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2),x)
*a**2*b*c*d**2*p**5 + 240*int((a - b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*
b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2),x)*a**2*b*c*d**2*p**4 + 420*int((a -
b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2
),x)*a**2*b*c*d**2*p**3 + 300*int((a - b*x**2)**p/(4*a*p**2 + 8*a*p + 3...
```

3.425 $\int (c+dx)^2 \left(a-bx^2\right)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int (c+dx)^2 \left(a-bx^2\right)^p \, dx = -\frac{d(c(3+2p)+d(1+p)x)\left(a-bx^2\right)^{1+p}}{b\left(3+5p+2p^2\right)} + \left(c^2+\frac{ad^2}{3b+2bp}\right)x\left(a-bx^2\right)^p\left(1-\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)$$

output
```
-d*(c*(3+2*p)+d*(p+1)*x)*(-b*x^2+a)^(p+1)/b/(2*p^2+5*p+3)+(c^2+a*d^2/(2*b*
p+3*b))*x*(-b*x^2+a)^p*hypergeom([1/2, -p],[3/2],b*x^2/a)/((1-b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.24

$$\int (c+dx)^2 \left(a-bx^2\right)^p \, dx = \frac{\left(a-bx^2\right)^p\left(1-\frac{bx^2}{a}\right)^{-p}\left(3bc^2(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)+d\left(3c\left(a-a\left(1-\frac{bx^2}{a}\right)^p+bx\right.\right.\right.}{3b(1+p)}$$

input
```
Integrate[(c + d*x)^2*(a - b*x^2)^p,x]
```

output
```
((a - b*x^2)^p*(3*b*c^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (b*x^2)/
a] + d*(3*c*(a - a*(1 - (b*x^2)/a)^p + b*x^2*(1 - (b*x^2)/a)^p) + b*d*(1 +
 p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, (b*x^2)/a])))/(3*b*(1 + p)*(1 - (b
*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {497, 25, 455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a-bx^2\right)^p \, dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{\int -\left(\left(b(2p+3)c^2+2bd(p+2)xc+ad^2\right)\left(a-bx^2\right)^p\right)dx}{b(2p+3)}-\frac{d(c+dx)\left(a-bx^2\right)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \left(b(2p+3)c^2+2bd(p+2)xc+ad^2\right)\left(a-bx^2\right)^p dx}{b(2p+3)}-\frac{d(c+dx)\left(a-bx^2\right)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(ad^2+bc^2(2p+3)\right)\int \left(a-bx^2\right)^p dx-\frac{cd(p+2)\left(a-bx^2\right)^{p+1}}{p+1}}{b(2p+3)}-\frac{d(c+dx)\left(a-bx^2\right)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(a-bx^2\right)^p\left(1-\frac{bx^2}{a}\right)^{-p}\left(ad^2+bc^2(2p+3)\right)\int \left(1-\frac{bx^2}{a}\right)^p dx-\frac{cd(p+2)\left(a-bx^2\right)^{p+1}}{p+1}}{b(2p+3)}-\frac{d(c+dx)\left(a-bx^2\right)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{237}$$

$$\frac{x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}(ad^2+bc^2(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)-\frac{cd(p+2)(a-bx^2)^{p+1}}{p+1}}{b(2p+3)}-\frac{d(c+dx)(a-bx^2)^{p+1}}{b(2p+3)}$$

input
```
Int[(c + d*x)^2*(a - b*x^2)^p,x]
```

output
```
-((d*(c + d*x)*(a - b*x^2)^(1 + p))/(b*(3 + 2*p))) + (-((c*d*(2 + p)*(a -
b*x^2)^(1 + p))/(1 + p)) + ((a*d^2 + b*c^2*(3 + 2*p))*x*(a - b*x^2)^p*Hype
rgeometric2F1[1/2, -p, 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^p)/(b*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [F]

$$\int (dx+c)^2 \left(-b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^2*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^2*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^2 \left(a-bx^2\right)^p \, dx = \int (dx+c)^2 \left(-bx^2+a\right)^p dx$$

input
```
integrate((d*x+c)^2*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(-b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx)^2\left(a-bx^2\right)^p\,dx = a^pc^2x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{a^pd^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3} + 2cd\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ -\frac{\begin{cases}\frac{(a-bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a-bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)**2*(-b*x**2+a)**p,x)
```

output
```
a**p*c**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + a**p*d*
*2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3 + 2*c*d*Pie
cewise((a**p*x**2/2, Eq(b, 0)), (-Piecewise(((a - b*x**2)**(p + 1)/(p + 1)
, Ne(p, -1)), (log(a - b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int (c+dx)^2\left(a-bx^2\right)^p\,dx = \int (dx+c)^2\left(-bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)^2*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(-b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(a-bx^2\right)^p \, dx = \int (dx+c)^2 (-bx^2+a)^p \, dx$$

input
```
integrate((d*x+c)^2*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(-b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a-bx^2\right)^p \, dx = \int \left(a-b\,x^2\right)^p\,(c+d\,x)^2 \, dx$$

input
```
int((a - b*x^2)^p*(c + d*x)^2,x)
```

output
```
int((a - b*x^2)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a-bx^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(-b*x^2+a)^p,x)
```

output
```
( - 4*(a - b*x**2)**p*a*c*d*p**2 - 8*(a - b*x**2)**p*a*c*d*p - 3*(a - b*x*
*2)**p*a*c*d - 2*(a - b*x**2)**p*a*d**2*p**2*x - 2*(a - b*x**2)**p*a*d**2*
p*x + 2*(a - b*x**2)**p*b*c**2*p**2*x + 5*(a - b*x**2)**p*b*c**2*p*x + 3*(
a - b*x**2)**p*b*c**2*x + 4*(a - b*x**2)**p*b*c*d*p**2*x**2 + 8*(a - b*x**
2)**p*b*c*d*p*x**2 + 3*(a - b*x**2)**p*b*c*d*x**2 + 2*(a - b*x**2)**p*b*d*
*2*p**2*x**3 + 3*(a - b*x**2)**p*b*d**2*p*x**3 + (a - b*x**2)**p*b*d**2*x*
*3 + 8*int((a - b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p
*x**2 - 3*b*x**2),x)*a**2*d**2*p**4 + 24*int((a - b*x**2)**p/(4*a*p**2 + 8
*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2),x)*a**2*d**2*p**3 + 22
*int((a - b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p*x**2
- 3*b*x**2),x)*a**2*d**2*p**2 + 6*int((a - b*x**2)**p/(4*a*p**2 + 8*a*p +
3*a - 4*b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2),x)*a**2*d**2*p + 16*int((a -
b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2
),x)*a*b*c**2*p**5 + 72*int((a - b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*
p**2*x**2 - 8*b*p*x**2 - 3*b*x**2),x)*a*b*c**2*p**4 + 116*int((a - b*x**2)
**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2),x)*a*
b*c**2*p**3 + 78*int((a - b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a - 4*b*p**2*x*
*2 - 8*b*p*x**2 - 3*b*x**2),x)*a*b*c**2*p**2 + 18*int((a - b*x**2)**p/(4*a
*p**2 + 8*a*p + 3*a - 4*b*p**2*x**2 - 8*b*p*x**2 - 3*b*x**2),x)*a*b*c**2*p
)/(b*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.426 $\int (c+dx) \left(a-bx^2\right)^p \, dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int (c+dx) \left(a-bx^2\right)^p \, dx = -\frac{d\left(a-bx^2\right)^{1+p}}{2b(1+p)} + cx\left(a-bx^2\right)^p \left(1 - \frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)$$

output
```
-1/2*d*(-b*x^2+a)^(p+1)/b/(p+1)+c*x*(-b*x^2+a)^p*hypergeom([1/2, -p],[3/2]
,b*x^2/a)/((1-b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.42

$$\int (c+dx) \left(a-bx^2\right)^p \, dx = \frac{\left(a-bx^2\right)^p \left(1-\frac{bx^2}{a}\right)^{-p} \left(d\left(a-a\left(1-\frac{bx^2}{a}\right)^p + bx^2\left(1-\frac{bx^2}{a}\right)^p\right) + 2bc(1+p)x \, \text{Hypergeometric2F1}\left(\frac{1}{2},-\right.\right.}{2b(1+p)}$$

input
```
Integrate[(c + d*x)*(a - b*x^2)^p,x]
```

output
```
((a - b*x^2)^p*(d*(a - a*(1 - (b*x^2)/a)^p + b*x^2*(1 - (b*x^2)/a)^p) + 2*
b*c*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (b*x^2)/a]))/(2*b*(1 + p)*(1
 - (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx) \left(a-bx^2\right)^p \, dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(a-bx^2\right)^p dx-\frac{d\left(a-bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{238}$$

$$c\left(a-bx^2\right)^p \left(1-\frac{bx^2}{a}\right)^{-p} \int \left(1-\frac{bx^2}{a}\right)^p dx-\frac{d\left(a-bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{237}$$

$$cx\left(a-bx^2\right)^p \left(1-\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)-\frac{d\left(a-bx^2\right)^{p+1}}{2b(p+1)}$$

input
```
Int[(c + d*x)*(a - b*x^2)^p,x]
```

output
```
-1/2*(d*(a - b*x^2)^(1 + p))/(b*(1 + p)) + (c*x*(a - b*x^2)^p*Hypergeometr
ic2F1[1/2, -p, 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (dx+c)\left(-b\,x^{2}+a\right)^{p}dx$$

input
```
int((d*x+c)*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)\left(a-bx^2\right)^p\,dx=\int (dx+c)\left(-bx^2+a\right)^p dx$$

input
```
integrate((d*x+c)*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(-b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (c+dx)\left(a-bx^2\right)^p\,dx = a^p c x {}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + d\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ -\frac{\begin{cases}\frac{(a-bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a-bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(-b*x**2+a)**p,x)
```

output
```
a**p*c*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + d*Piecewis
e((a**p*x**2/2, Eq(b, 0)), (-Piecewise(((a - b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a - b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int (c+dx)\left(a-bx^2\right)^p\,dx = \int (dx+c)\left(-bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(-b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx) \left(a-bx^2\right)^p \, dx = \int (dx+c)\left(-bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(-b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int (c+dx) \left(a-bx^2\right)^p \, dx = \frac{c\,x\,\left(a-b\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^p} - \frac{d\,\left(a-b\,x^2\right)^{p+1}}{2\,b\,\left(p+1\right)}$$

input
```
int((a - b*x^2)^p*(c + d*x),x)
```

output
```
(c*x*(a - b*x^2)^p*hypergeom([1/2, -p], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a)^p
 - (d*(a - b*x^2)^(p + 1))/(2*b*(p + 1))
```

Reduce [F]

$$\int (c+dx) \left(a-bx^2\right)^p \, dx$$
$$= \frac{-2(-b\,x^2+a)^p\,adp - (-b\,x^2+a)^p\,ad + 2(-b\,x^2+a)^p\,bcpx + 2(-b\,x^2+a)^p\,bcx + 2(-b\,x^2+a)^p\,bdp\,x^2}{}$$

input
```
int((d*x+c)*(-b*x^2+a)^p,x)
```

output
```
( - 2*(a - b*x**2)**p*a*d*p - (a - b*x**2)**p*a*d + 2*(a - b*x**2)**p*b*c*
p*x + 2*(a - b*x**2)**p*b*c*x + 2*(a - b*x**2)**p*b*d*p*x**2 + (a - b*x**2
)**p*b*d*x**2 + 8*int((a - b*x**2)**p/(2*a*p + a - 2*b*p*x**2 - b*x**2),x)
*a*b*c*p**3 + 12*int((a - b*x**2)**p/(2*a*p + a - 2*b*p*x**2 - b*x**2),x)*
a*b*c*p**2 + 4*int((a - b*x**2)**p/(2*a*p + a - 2*b*p*x**2 - b*x**2),x)*a*
b*c*p)/(2*b*(2*p**2 + 3*p + 1))
```

3.427 $\int (a-bx^2)^p \, dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int (a-bx^2)^p \, dx = x(a-bx^2)^p \left(1-\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)$$

output
```
x*(-b*x^2+a)^p*hypergeom([1/2, -p],[3/2],b*x^2/a)/((1-b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int (a-bx^2)^p \, dx = x(a-bx^2)^p \left(1-\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)$$

input
```
Integrate[(a - b*x^2)^p,x]
```

output
```
(x*(a - b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (b*x^2)/a])/(1 - (b*x^2)/
a)^p
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^2)^p\,dx$$

$$\Big\downarrow \text{238}$$

$$(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\int\left(1-\frac{bx^2}{a}\right)^p dx$$

$$\Big\downarrow \text{237}$$

$$x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{bx^2}{a}\right)$$

input
```
Int[(a - b*x^2)^p,x]
```

output
```
(x*(a - b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (b*x^2)/a])/(1 - (b*x^2)/
a)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{p}dx$$

input
```
int((-b*x^2+a)^p,x)
```

output
```
int((-b*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^p\,dx=\int \left(-bx^2+a\right)^p\,dx$$

input
```
integrate((-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \left(a-bx^2\right)^p\,dx=a^px\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)$$

input
```
integrate((-b*x**2+a)**p,x)
```

output
```
a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Maxima [F]

$$\int \left(a-bx^2\right)^p\,dx=\int \left(-bx^2+a\right)^p\,dx$$

input
```
integrate((-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a-bx^2\right)^p\,dx=\int \left(-bx^2+a\right)^p\,dx$$

input
```
integrate((-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \left(a-bx^2\right)^p\,dx=\frac{x\,\left(a-b\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^p}$$

input
```
int((a - b*x^2)^p,x)
```

output
```
(x*(a - b*x^2)^p*hypergeom([1/2, -p], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a)^p
```

Reduce [F]

$$\int (a - bx^2)^p \, dx$$
$$= \frac{(-b\,x^2 + a)^p\,x + 4\left(\int \frac{(-b\,x^2+a)^p}{-2bp\,x^2-b\,x^2+2ap+a}dx\right)a\,p^2 + 2\left(\int \frac{(-b\,x^2+a)^p}{-2bp\,x^2-b\,x^2+2ap+a}dx\right)ap}{2p+1}$$

input
```
int((-b*x^2+a)^p,x)
```

output
```
((a - b*x**2)**p*x + 4*int((a - b*x**2)**p/(2*a*p + a - 2*b*p*x**2 - b*x**
2),x)*a*p**2 + 2*int((a - b*x**2)**p/(2*a*p + a - 2*b*p*x**2 - b*x**2),x)*
a*p)/(2*p + 1)
```

3.428 $\int \frac{(a-bx^2)^p}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \frac{x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c} + \frac{d(a-bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,-\frac{d^2(a-bx^2)}{bc^2-ad^2}\right)}{2\,(bc^2-ad^2)\,(1+p)}$$

output
```
x*(-b*x^2+a)^p*AppellF1(1/2,-p,1,3/2,b*x^2/a,d^2*x^2/c^2)/c/((1-b*x^2/a)^p
)+1/2*d*(-b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],-d^2*(-b*x^2+a)/(-a*d^2+
b*c^2))/(-a*d^2+b*c^2)/(p+1)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \frac{\left(\frac{d\left(-\sqrt{\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}(a-bx^2)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c-\sqrt{\frac{a}{b}}d}{c+dx},\frac{c+\sqrt{\frac{a}{b}}d}{c+dx}\right)}{2dp}$$

input
```
Integrate[(a - b*x^2)^p/(c + d*x),x]
```

output
```
((a - b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c - Sqrt[a/b]*d)/(c + d*x)
, (c + Sqrt[a/b]*d)/(c + d*x)])/(2*d*p*((d*(-Sqrt[a/b] + x))/(c + d*x))^p*
((d*(Sqrt[a/b] + x))/(c + d*x))^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^p}{c+dx}dx$$

$$\Big\downarrow \text{504}$$

$$c\int \frac{(a-bx^2)^p}{c^2-d^2x^2}dx-d\int \frac{x(a-bx^2)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{334}$$

$$c(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\int \frac{\left(1-\frac{bx^2}{a}\right)^p}{c^2-d^2x^2}dx-d\int \frac{x(a-bx^2)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{333}$$

$$\frac{x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c}-d\int \frac{x(a-bx^2)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c}-\frac{1}{2}d\int \frac{(a-bx^2)^p}{c^2-d^2x^2}dx^2$$

$$\Big\downarrow \text{78}$$

$$\frac{x(a-bx^2)^p\left(1-\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{bx^2}{a},\frac{d^2x^2}{c^2}\right)}{c}+\frac{d(a-bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,-\frac{d^2(a-bx^2)}{bc^2-ad^2}\right)}{2(p+1)\left(bc^2-ad^2\right)}$$

input
```
Int[(a - b*x^2)^p/(c + d*x),x]
```

output
```
(x*(a - b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, (b*x^2)/a, (d^2*x^2)/c^2])/(c*(
1 - (b*x^2)/a)^p) + (d*(a - b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, -((d^2*(a - b*x^2))/(b*c^2 - a*d^2))])/(2*(b*c^2 - a*d^2)*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^p}{dx+c}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c),x)
```

output
```
int((-b*x^2+a)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \int \frac{(-bx^2+a)^p}{dx+c}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p/(d*x + c), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \int \frac{(a-bx^2)^p}{c+dx}\,dx$$

input
```
integrate((-b*x**2+a)**p/(d*x+c),x)
```

output
```
Integral((a - b*x**2)**p/(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \int \frac{(-bx^2+a)^p}{dx+c}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \int \frac{(-bx^2+a)^p}{dx+c}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \int \frac{(a-b\,x^2)^p}{c+d\,x}\,dx$$

input
```
int((a - b*x^2)^p/(c + d*x),x)
```

output
```
int((a - b*x^2)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^p}{c+dx}\,dx = \int \frac{(-b\,x^2+a)^p}{dx+c}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c),x)
```

output
```
int((a - b*x**2)**p/(c + d*x),x)
```

3.429 $\int \frac{(a-bx^2)^p}{(c+dx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{(a-bx^2)^p}{(c+dx)^2} dx = -\frac{2^p\sqrt{b}\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{-p}(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)\text{AppellF1}\left(1+p,2,-p,2+p,1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{\sqrt{a}+\sqrt{b}x}{2\sqrt{a}}\right)}{d\left(\sqrt{b}c-\sqrt{a}d\right)(1+p)}$$

output
```
-2^p*b^(1/2)*(-b*x^2+a)^p*(1-(d*x+c)/(c-a^(1/2)*d/b^(1/2)))*AppellF1(p+1,-
p,2,2+p,1/2*(a^(1/2)+b^(1/2)*x)/a^(1/2),1-(d*x+c)/(c-a^(1/2)*d/b^(1/2)))/d
/(b^(1/2)*c-a^(1/2)*d)/(p+1)/((1-b^(1/2)*x/a^(1/2))^p)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{(a-bx^2)^p}{(c+dx)^2} dx = \frac{\left(\frac{d\left(-\sqrt{\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}(a-bx^2)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c-\sqrt{\frac{a}{b}}d}{c+dx},\frac{c+\sqrt{\frac{a}{b}}d}{c+dx}\right)}{d(-1+2p)(c+dx)}$$

input
```
Integrate[(a - b*x^2)^p/(c + d*x)^2,x]
```

output
```
((a - b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c - Sqrt[a/b]*d)/(c + d
*x), (c + Sqrt[a/b]*d)/(c + d*x)])/(d*(-1 + 2*p)*((d*(-Sqrt[a/b] + x))/(c
+ d*x))^p*((d*(Sqrt[a/b] + x))/(c + d*x))^p*(c + d*x))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {506, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{506}$$

$$-\frac{(a-bx^2)^p\left(\frac{1}{c+dx}\right)^{2p}\left(1-\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{-p}\left(1-\frac{\frac{\sqrt{a}d}{\sqrt{b}}+c}{c+dx}\right)^{-p}\int\left(\frac{1}{c+dx}\right)^{-2p}\left(1-\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{p}\left(1-\frac{c+\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{p}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{(a-bx^2)^p\left(1-\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{-p}\left(1-\frac{\frac{\sqrt{a}d}{\sqrt{b}}+c}{c+dx}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx},\frac{c+\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)}{d(1-2p)(c+dx)}$$

input
```
Int[(a - b*x^2)^p/(c + d*x)^2,x]
```

output
```
-(((a - b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c - (Sqrt[a]*d)/Sqrt[
b])/(c + d*x), (c + (Sqrt[a]*d)/Sqrt[b])/(c + d*x)])/(d*(1 - 2*p)*(c + d*x
)*(1 - (c - (Sqrt[a]*d)/Sqrt[b])/(c + d*x))^p*(1 - (c + (Sqrt[a]*d)/Sqrt[b
])/(c + d*x))^p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^p}{(dx+c)^2}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c)^2,x)
```

output
```
int((-b*x^2+a)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx = \int \frac{(-bx^2+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx=\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx$$

input
```
integrate((-b*x**2+a)**p/(d*x+c)**2,x)
```

output
```
Integral((a - b*x**2)**p/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx=\int \frac{(-bx^2+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx=\int \frac{(-bx^2+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx=\int \frac{(a-b\,x^2)^p}{(c+d\,x)^2}\,dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^2,x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((-b*x^2+a)^p/(d*x+c)^2,x)
```

output

```
((a - b*x**2)**p*x + 4*int((a - b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*
p*x + 2*a*c*d*x + 2*a*d**2*p*x**2 + a*d**2*x**2 - 2*b*c**2*p*x**2 - b*c**2
*x**2 - 4*b*c*d*p*x**3 - 2*b*c*d*x**3 - 2*b*d**2*p*x**4 - b*d**2*x**4),x)*
a*c**2*p**2 + 2*int((a - b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2
*a*c*d*x + 2*a*d**2*p*x**2 + a*d**2*x**2 - 2*b*c**2*p*x**2 - b*c**2*x**2 -
 4*b*c*d*p*x**3 - 2*b*c*d*x**3 - 2*b*d**2*p*x**4 - b*d**2*x**4),x)*a*c**2*
p + 4*int((a - b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x +
 2*a*d**2*p*x**2 + a*d**2*x**2 - 2*b*c**2*p*x**2 - b*c**2*x**2 - 4*b*c*d*p
*x**3 - 2*b*c*d*x**3 - 2*b*d**2*p*x**4 - b*d**2*x**4),x)*a*c*d*p**2*x + 2*
int((a - b*x**2)**p/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d
**2*p*x**2 + a*d**2*x**2 - 2*b*c**2*p*x**2 - b*c**2*x**2 - 4*b*c*d*p*x**3
- 2*b*c*d*x**3 - 2*b*d**2*p*x**4 - b*d**2*x**4),x)*a*c*d*p*x + 4*int(((a -
 b*x**2)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d**
2*p*x**2 + a*d**2*x**2 - 2*b*c**2*p*x**2 - b*c**2*x**2 - 4*b*c*d*p*x**3 -
2*b*c*d*x**3 - 2*b*d**2*p*x**4 - b*d**2*x**4),x)*b*c*d*p**2 + 2*int(((a -
b*x**2)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d**2
*p*x**2 + a*d**2*x**2 - 2*b*c**2*p*x**2 - b*c**2*x**2 - 4*b*c*d*p*x**3 - 2
*b*c*d*x**3 - 2*b*d**2*p*x**4 - b*d**2*x**4),x)*b*c*d*p + 4*int(((a - b*x*
*2)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x + 2*a*c*d*x + 2*a*d**2*p*x
**2 + a*d**2*x**2 - 2*b*c**2*p*x**2 - b*c**2*x**2 - 4*b*c*d*p*x**3 - 2*...
```

3.430 $\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 146

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx = -\frac{2^p b\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{-p}(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)\operatorname{AppellF1}\left(1+p,3,-p,2+p,1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{\sqrt{a}+\sqrt{b}x}{2\sqrt{a}}\right)}{d\left(\sqrt{b}c-\sqrt{a}d\right)^2(1+p)}$$

output
```
-2^p*b*(-b*x^2+a)^p*(1-(d*x+c)/(c-a^(1/2)*d/b^(1/2)))*AppellF1(p+1,-p,3,2+
p,1/2*(a^(1/2)+b^(1/2)*x)/a^(1/2),1-(d*x+c)/(c-a^(1/2)*d/b^(1/2)))/d/(b^(1
/2)*c-a^(1/2)*d)^2/(p+1)/((1-b^(1/2)*x/a^(1/2))^p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx = \frac{\left(\frac{d\left(-\sqrt{\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{c+dx}\right)^{-p}(a-bx^2)^p\operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{c-\sqrt{\frac{a}{b}}d}{c+dx},\frac{c+\sqrt{\frac{a}{b}}d}{c+dx}\right)}{2d(-1+p)(c+dx)^2}$$

input
```
Integrate[(a - b*x^2)^p/(c + d*x)^3,x]
```

output
```
((a - b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (c - Sqrt[a/b]*d)/(c + d
*x), (c + Sqrt[a/b]*d)/(c + d*x)])/(2*d*(-1 + p)*((d*(-Sqrt[a/b] + x))/(c
+ d*x))^p*((d*(Sqrt[a/b] + x))/(c + d*x))^p*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {506, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{506}$$

$$-\frac{(a-bx^2)^p\left(\frac{1}{c+dx}\right)^{2p}\left(1-\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{-p}\left(1-\frac{\frac{\sqrt{a}d}{\sqrt{b}}+c}{c+dx}\right)^{-p}\int\left(\frac{1}{c+dx}\right)^{1-2p}\left(1-\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{p}\left(1-\frac{c+\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{p}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{(a-bx^2)^p\left(1-\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)^{-p}\left(1-\frac{\frac{\sqrt{a}d}{\sqrt{b}}+c}{c+dx}\right)^{-p}\text{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{c-\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx},\frac{c+\frac{\sqrt{a}d}{\sqrt{b}}}{c+dx}\right)}{2d(1-p)(c+dx)^2}$$

input
```
Int[(a - b*x^2)^p/(c + d*x)^3,x]
```

output
```
-1/2*((a - b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (c - (Sqrt[a]*d)/Sq
rt[b])/(c + d*x), (c + (Sqrt[a]*d)/Sqrt[b])/(c + d*x)])/(d*(1 - p)*(c + d*
x)^2*(1 - (c - (Sqrt[a]*d)/Sqrt[b])/(c + d*x))^p*(1 - (c + (Sqrt[a]*d)/Sqr
t[b])/(c + d*x))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^p}{(dx+c)^3}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c)^3,x)
```

output
```
int((-b*x^2+a)^p/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(-bx^2+a)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx$$

input
```
integrate((-b*x**2+a)**p/(d*x+c)**3,x)
```

output
```
Integral((a - b*x**2)**p/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(-bx^2+a)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx = \int \frac{(-bx^2+a)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx=\int \frac{{\left(a-b\,x^2\right)}^p}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^3,x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^3}\,dx=\int \frac{(-b\,x^2+a)^p}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c)^3,x)
```

output
```
int((a - b*x**2)**p/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.431 $\int (c+dx)^3 \left(2-3x^2\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx = -\frac{d(9c^2+2d^2)\left(2-3x^2\right)^{1+p}}{18(1+p)} - \frac{cd^2x\left(2-3x^2\right)^{1+p}}{3+2p} + \frac{d^3\left(2-3x^2\right)^{2+p}}{18(2+p)} + \frac{2^p c(2d^2+c^2(3+2p))\, x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)}{3+2p}$$

output

```
-1/18*d*(9*c^2+2*d^2)*(-3*x^2+2)^(p+1)/(p+1)-c*d^2*x*(-3*x^2+2)^(p+1)/(3+2
*p)+d^3*(-3*x^2+2)^(2+p)/(36+18*p)+2^p*c*(2*d^2+c^2*(3+2*p))*x*hypergeom([
1/2, -p],[3/2],3/2*x^2)/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx = \frac{9\; 2^{1+p}c^3(2+3p+p^2)\, x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right) + d\Big(9c^2(2+p)\left(2^{1+p}-2(2-3x^2)^p+3x^2(2\right.}{}$$

input
```
Integrate[(c + d*x)^3*(2 - 3*x^2)^p,x]
```

output
```
(9*2^(1 + p)*c^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2, (3*x^2)
/2] + d*(9*c^2*(2 + p)*(2^(1 + p) - 2*(2 - 3*x^2)^p + 3*x^2*(2 - 3*x^2)^p)
 + d^2*(2^(2 + p) - 4*(2 - 3*x^2)^p + 9*x^4*(2 - 3*x^2)^p - 3*p*x^2*(2 - 3
*x^2)^(1 + p)) + 9*2^(1 + p)*c*d*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2
, -p, 5/2, (3*x^2)/2]))/(18*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {497, 27, 676, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-3x^2\right)^p (c+dx)^3 \, dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{\int -2(c+dx)\left(3(p+2)c^2+3d(p+3)xc+2d^2\right)\left(2-3x^2\right)^p dx}{6(p+2)}-\frac{d\left(2-3x^2\right)^{p+1}(c+dx)^2}{6(p+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c+dx)\left(3(p+2)c^2+3d(p+3)xc+2d^2\right)\left(2-3x^2\right)^p dx}{3(p+2)}-\frac{d\left(2-3x^2\right)^{p+1}(c+dx)^2}{6(p+2)}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{3c(p+2)\left(c^2(2p+3)+2d^2\right)\int \left(2-3x^2\right)^p dx}{2p+3}-\frac{d\left(2-3x^2\right)^{p+1}\left(3c^2(2p+5)+2d^2\right)}{6(p+1)}-\frac{cd^2(p+3)x\left(2-3x^2\right)^{p+1}}{2p+3}}{3(p+2)}-\frac{d\left(2-3x^2\right)^{p+1}(c+dx)^2}{6(p+2)}$$

$$\Big\downarrow \text{237}$$

$$\frac{\frac{3c2^p(p+2)x(c^2(2p+3)+2d^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)}{2p+3}-\frac{d(2-3x^2)^{p+1}(3c^2(2p+5)+2d^2)}{6(p+1)}-\frac{cd^2(p+3)x(2-3x^2)^{p+1}}{2p+3}}{3(p+2)}-\frac{d(2-3x^2)^{p+1}(c+dx)^2}{6(p+2)}$$

input
```
Int[(c + d*x)^3*(2 - 3*x^2)^p,x]
```

output
```
-1/6*(d*(c + d*x)^2*(2 - 3*x^2)^(1 + p))/(2 + p) + (-1/6*(d*(2*d^2 + 3*c^2
*(5 + 2*p))*(2 - 3*x^2)^(1 + p))/(1 + p) - (c*d^2*(3 + p)*x*(2 - 3*x^2)^(1
 + p))/(3 + 2*p) + (3*2^p*c*(2 + p)*(2*d^2 + c^2*(3 + 2*p))*x*Hypergeometr
ic2F1[1/2, -p, 3/2, (3*x^2)/2])/(3 + 2*p))/(3*(2 + p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

method	result
meijerg	$2^{-2+p} d^{3} x^{4} \operatorname{hypergeom}\left(\left[2, -p\right], \left[3\right], \frac{3 x^{2}}{2}\right)+c\,d^{2} 2^{p} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{2}, -p\right], \left[\frac{5}{2}\right], \frac{3 x^{2}}{2}\right)+3\,2^{p -1} c^{2} d$

input
```
int((d*x+c)^3*(-3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
2^(-2+p)*d^3*x^4*hypergeom([2,-p],[3],3/2*x^2)+c*d^2*2^p*x^3*hypergeom([3/
2,-p],[5/2],3/2*x^2)+3*2^(p-1)*c^2*d*x^2*hypergeom([1,-p],[2],3/2*x^2)+c^3
*2^p*x*hypergeom([1/2,-p],[3/2],3/2*x^2)
```

Fricas [F]

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx = \int (dx+c)^3 \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(-3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-3*x^2 + 2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.01

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx$$
$$= 2^p c^3 x {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\right| \frac{3x^2 e^{2i\pi}}{2}\right) + 2^p c d^2 x^3 {}_2F_1\left(\left.\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\right| \frac{3x^2 e^{2i\pi}}{2}\right)$$
$$- \frac{c^2 d \left(\begin{cases} \frac{(2-3x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(2-3x^2) & \text{otherwise} \end{cases}\right)}{2}$$
$$+ d^3 \left(\begin{cases} \frac{3x^2 \log(3x^2-2)}{54x^2-36} - \frac{2\log(3x^2-2)}{54x^2-36} - \frac{2}{54x^2-36} & \text{for } p = -2 \\ -\frac{x^2}{6} - \frac{\log(3x^2-2)}{9} & \text{for } p = -1 \\ \frac{9px^4(2-3x^2)^p}{18p^2+54p+36} - \frac{6px^2(2-3x^2)^p}{18p^2+54p+36} + \frac{9x^4(2-3x^2)^p}{18p^2+54p+36} - \frac{4(2-3x^2)^p}{18p^2+54p+36} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**3*(-3*x**2+2)**p,x)
```

output
```
2**p*c**3*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(2*I*pi)/2) + 2**p*c*
d**2*x**3*hyper((3/2, -p), (5/2,), 3*x**2*exp_polar(2*I*pi)/2) - c**2*d*Pi
ecewise(((2 - 3*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(2 - 3*x**2), True
))/2 + d**3*Piecewise((3*x**2*log(3*x**2 - 2)/(54*x**2 - 36) - 2*log(3*x**
2 - 2)/(54*x**2 - 36) - 2/(54*x**2 - 36), Eq(p, -2)), (-x**2/6 - log(3*x**
2 - 2)/9, Eq(p, -1)), (9*p*x**4*(2 - 3*x**2)**p/(18*p**2 + 54*p + 36) - 6*
p*x**2*(2 - 3*x**2)**p/(18*p**2 + 54*p + 36) + 9*x**4*(2 - 3*x**2)**p/(18*
p**2 + 54*p + 36) - 4*(2 - 3*x**2)**p/(18*p**2 + 54*p + 36), True))
```

Maxima [F]

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx = \int (dx+c)^3 \left(-3x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^3*(-3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(-3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx = \int (dx+c)^3 (-3\,x^2+2)^p \, dx$$

input
```
integrate((d*x+c)^3*(-3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(-3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx = \int (2-3\,x^2)^p\,(c+d\,x)^3 \, dx$$

input
```
int((2 - 3*x^2)^p*(c + d*x)^3,x)
```

output
```
int((2 - 3*x^2)^p*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \left(2-3x^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*(-3*x^2+2)^p,x)
```

output

```
(36*( - 3*x**2 + 2)**p*c**3*p**3*x + 162*( - 3*x**2 + 2)**p*c**3*p**2*x +
234*( - 3*x**2 + 2)**p*c**3*p*x + 108*( - 3*x**2 + 2)**p*c**3*x + 108*( -
3*x**2 + 2)**p*c**2*d*p**3*x**2 - 72*( - 3*x**2 + 2)**p*c**2*d*p**3 + 432*
( - 3*x**2 + 2)**p*c**2*d*p**2*x**2 - 288*( - 3*x**2 + 2)**p*c**2*d*p**2 +
 513*( - 3*x**2 + 2)**p*c**2*d*p*x**2 - 342*( - 3*x**2 + 2)**p*c**2*d*p +
162*( - 3*x**2 + 2)**p*c**2*d*x**2 - 108*( - 3*x**2 + 2)**p*c**2*d + 108*(
 - 3*x**2 + 2)**p*c*d**2*p**3*x**3 - 72*( - 3*x**2 + 2)**p*c*d**2*p**3*x +
 378*( - 3*x**2 + 2)**p*c*d**2*p**2*x**3 - 216*( - 3*x**2 + 2)**p*c*d**2*p
**2*x + 378*( - 3*x**2 + 2)**p*c*d**2*p*x**3 - 144*( - 3*x**2 + 2)**p*c*d*
*2*p*x + 108*( - 3*x**2 + 2)**p*c*d**2*x**3 + 36*( - 3*x**2 + 2)**p*d**3*p
**3*x**4 - 24*( - 3*x**2 + 2)**p*d**3*p**3*x**2 + 108*( - 3*x**2 + 2)**p*d
**3*p**2*x**4 - 48*( - 3*x**2 + 2)**p*d**3*p**2*x**2 - 16*( - 3*x**2 + 2)*
*p*d**3*p**2 + 99*( - 3*x**2 + 2)**p*d**3*p*x**4 - 18*( - 3*x**2 + 2)**p*d
**3*p*x**2 - 32*( - 3*x**2 + 2)**p*d**3*p + 27*( - 3*x**2 + 2)**p*d**3*x**
4 - 12*( - 3*x**2 + 2)**p*d**3 - 576*int(( - 3*x**2 + 2)**p/(12*p**2*x**2
- 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)*c**3*p**6 - 3744*int(( - 3*x*
*2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)*c**3
*p**5 - 9360*int(( - 3*x**2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 24*p*x**2 - 1
6*p + 9*x**2 - 6),x)*c**3*p**4 - 11160*int(( - 3*x**2 + 2)**p/(12*p**2*x**
2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)*c**3*p**3 - 6264*int(( -...
```

3.432 $\int (c+dx)^2 \left(2-3x^2\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx = -\frac{d(c(3+2p)+d(1+p)x)\left(2-3x^2\right)^{1+p}}{3\left(3+5p+2p^2\right)} + \frac{2^p\left(2d^2+c^2(9+6p)\right)x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)}{3(3+2p)}$$

output
```
-1/3*d*(c*(3+2*p)+d*(p+1)*x)*(-3*x^2+2)^(p+1)/(2*p^2+5*p+3)+2^p*(2*d^2+c^2
*(9+6*p))*x*hypergeom([1/2, -p],[3/2],3/2*x^2)/(9+6*p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx = 2^p c^2 x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right) + \frac{d\left(c\left(2^{1+p}-2(2-3x^2)^p+3x^2(2-3x^2)^p\right)+2^p d(1+p)x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{3x^2}{2}\right)\right)}{3(1+p)}$$

input
```
Integrate[(c + d*x)^2*(2 - 3*x^2)^p,x]
```

output
```
2^p*c^2*x*Hypergeometric2F1[1/2, -p, 3/2, (3*x^2)/2] + (d*(c*(2^(1 + p) -
2*(2 - 3*x^2)^p + 3*x^2*(2 - 3*x^2)^p) + 2^p*d*(1 + p)*x^3*Hypergeometric2
F1[3/2, -p, 5/2, (3*x^2)/2]))/(3*(1 + p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {497, 25, 455, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-3x^2\right)^p (c+dx)^2 \, dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{\int -\left(\left(3(2p+3)c^2+6d(p+2)xc+2d^2\right)\left(2-3x^2\right)^p\right)dx}{3(2p+3)}-\frac{d\left(2-3x^2\right)^{p+1}(c+dx)}{3(2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \left((6p+9)c^2+6d(p+2)xc+2d^2\right)\left(2-3x^2\right)^p dx}{3(2p+3)}-\frac{d\left(2-3x^2\right)^{p+1}(c+dx)}{3(2p+3)}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(c^2(6p+9)+2d^2\right)\int \left(2-3x^2\right)^p dx-\frac{cd(p+2)\left(2-3x^2\right)^{p+1}}{p+1}}{3(2p+3)}-\frac{d\left(2-3x^2\right)^{p+1}(c+dx)}{3(2p+3)}$$

$$\Big\downarrow \text{237}$$

$$\frac{2^p x\left(c^2(6p+9)+2d^2\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)-\frac{cd(p+2)\left(2-3x^2\right)^{p+1}}{p+1}}{3(2p+3)}-\frac{d\left(2-3x^2\right)^{p+1}(c+dx)}{3(2p+3)}$$

input
```
Int[(c + d*x)^2*(2 - 3*x^2)^p,x]
```

output
```
-1/3*(d*(c + d*x)*(2 - 3*x^2)^(1 + p))/(3 + 2*p) + (-((c*d*(2 + p)*(2 - 3*
x^2)^(1 + p))/(1 + p)) + 2^p*(2*d^2 + c^2*(9 + 6*p))*x*Hypergeometric2F1[1
/2, -p, 3/2, (3*x^2)/2])/(3*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result
meijerg	$\frac{d^{2} 2^{p} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{2},-p\right],\left[\frac{5}{2}\right],\frac{3 x^{2}}{2}\right)}{3}+c d 2^{p} x^{2} \operatorname{hypergeom}\left(\left[1,-p\right],\left[2\right],\frac{3 x^{2}}{2}\right)+c^{2} 2^{p} x \operatorname{hypergeom}\left(\left[\frac{1}{2},-\right.\right.$

input
```
int((d*x+c)^2*(-3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*2^p*x^3*hypergeom([3/2,-p],[5/2],3/2*x^2)+c*d*2^p*x^2*hypergeom([1
,-p],[2],3/2*x^2)+c^2*2^p*x*hypergeom([1/2,-p],[3/2],3/2*x^2)
```

Fricas [F]

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx = \int (dx+c)^2 \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(-3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(-3*x^2 + 2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx = 2^p c^2 x\, {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, -p \\ \frac{3}{2}\end{matrix}\right| \frac{3x^2 e^{2i\pi}}{2}\right) + \frac{2^p d^2 x^3\, {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\right| \frac{3x^2 e^{2i\pi}}{2}\right)}{3} - \frac{cd\left(\begin{cases}\frac{(2-3x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(2-3x^2\right) & \text{otherwise}\end{cases}\right)}{3}$$

input
```
integrate((d*x+c)**2*(-3*x**2+2)**p,x)
```

output
```
2**p*c**2*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(2*I*pi)/2) + 2**p*d*
*2*x**3*hyper((3/2, -p), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/3 - c*d*Piece
wise(((2 - 3*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(2 - 3*x**2), True))/
3
```

Maxima [F]

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx = \int (dx+c)^2 \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(-3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(-3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx = \int (dx+c)^2 \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(-3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(-3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx = \int \left(2-3\,x^2\right)^p \,(c+d\,x)^2 \, dx$$

input
```
int((2 - 3*x^2)^p*(c + d*x)^2,x)
```

output
```
int((2 - 3*x^2)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(2-3x^2\right)^p \, dx$$
$$= \frac{6(-3x^2+2)^p c^2p^2x + 15(-3x^2+2)^p c^2px + 9(-3x^2+2)^p c^2x + 12(-3x^2+2)^p cd\,p^2x^2 - 8(-3x^2+2)^p}{}$$

input
```
int((d*x+c)^2*(-3*x^2+2)^p,x)
```

output
```
(6*( - 3*x**2 + 2)**p*c**2*p**2*x + 15*( - 3*x**2 + 2)**p*c**2*p*x + 9*( -
 3*x**2 + 2)**p*c**2*x + 12*( - 3*x**2 + 2)**p*c*d*p**2*x**2 - 8*( - 3*x**
2 + 2)**p*c*d*p**2 + 24*( - 3*x**2 + 2)**p*c*d*p*x**2 - 16*( - 3*x**2 + 2)
**p*c*d*p + 9*( - 3*x**2 + 2)**p*c*d*x**2 - 6*( - 3*x**2 + 2)**p*c*d + 6*(
 - 3*x**2 + 2)**p*d**2*p**2*x**3 - 4*( - 3*x**2 + 2)**p*d**2*p**2*x + 9*(
- 3*x**2 + 2)**p*d**2*p*x**3 - 4*( - 3*x**2 + 2)**p*d**2*p*x + 3*( - 3*x**
2 + 2)**p*d**2*x**3 - 96*int(( - 3*x**2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 2
4*p*x**2 - 16*p + 9*x**2 - 6),x)*c**2*p**5 - 432*int(( - 3*x**2 + 2)**p/(1
2*p**2*x**2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)*c**2*p**4 - 696*i
nt(( - 3*x**2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 -
 6),x)*c**2*p**3 - 468*int(( - 3*x**2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 24*
p*x**2 - 16*p + 9*x**2 - 6),x)*c**2*p**2 - 108*int(( - 3*x**2 + 2)**p/(12*
p**2*x**2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)*c**2*p - 32*int(( -
 3*x**2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)
*d**2*p**4 - 96*int(( - 3*x**2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 24*p*x**2
- 16*p + 9*x**2 - 6),x)*d**2*p**3 - 88*int(( - 3*x**2 + 2)**p/(12*p**2*x**
2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)*d**2*p**2 - 24*int(( - 3*x*
*2 + 2)**p/(12*p**2*x**2 - 8*p**2 + 24*p*x**2 - 16*p + 9*x**2 - 6),x)*d**2
*p)/(3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.433 $\int (c+dx) (2-3x^2)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int (c+dx) (2-3x^2)^p \, dx = -\frac{d(2-3x^2)^{1+p}}{6(1+p)} + 2^p cx \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)$$

output
```
-1/6*d*(-3*x^2+2)^(p+1)/(p+1)+2^p*c*x*hypergeom([1/2, -p],[3/2],3/2*x^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int (c+dx) (2-3x^2)^p \, dx = \frac{d(2^{1+p} - 2(2-3x^2)^p + 3x^2(2-3x^2)^p)}{6(1+p)} + 2^p cx \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)$$

input
```
Integrate[(c + d*x)*(2 - 3*x^2)^p,x]
```

output
```
(d*(2^(1 + p) - 2*(2 - 3*x^2)^p + 3*x^2*(2 - 3*x^2)^p))/(6*(1 + p)) + 2^p*
c*x*Hypergeometric2F1[1/2, -p, 3/2, (3*x^2)/2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {455, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-3x^2)^p (c+dx)\,dx$$

$$\Big\downarrow \text{ 455}$$

$$c\int (2-3x^2)^p\,dx-\frac{d(2-3x^2)^{p+1}}{6(p+1)}$$

$$\Big\downarrow \text{ 237}$$

$$c2^px\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)-\frac{d(2-3x^2)^{p+1}}{6(p+1)}$$

input
```
Int[(c + d*x)*(2 - 3*x^2)^p,x]
```

output
```
-1/6*(d*(2 - 3*x^2)^(1 + p))/(1 + p) + 2^p*c*x*Hypergeometric2F1[1/2, -p,
3/2, (3*x^2)/2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	size
meijerg	$2^{p-1} d\,x^{2} \operatorname{hypergeom}\left(\left[1,-p\right],\left[2\right],\frac{3 x^{2}}{2}\right)+2^{p} c x \operatorname{hypergeom}\left(\left[\frac{1}{2},-p\right],\left[\frac{3}{2}\right],\frac{3 x^{2}}{2}\right)$	44

input
```
int((d*x+c)*(-3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
2^(p-1)*d*x^2*hypergeom([1,-p],[2],3/2*x^2)+2^p*c*x*hypergeom([1/2,-p],[3/
2],3/2*x^2)
```

Fricas [F]

$$\int (c+dx)\left(2-3x^2\right)^p\,dx=\int (dx+c)\left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)*(-3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(-3*x^2 + 2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int (c+dx)\left(2-3x^2\right)^p\,dx=2^p c x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)-\frac{d\left(\begin{cases}\frac{\left(2-3x^2\right)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(2-3x^2\right) & \text{otherwise}\end{cases}\right)}{6}$$

input
```
integrate((d*x+c)*(-3*x**2+2)**p,x)
```

output
```
2**p*c*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(2*I*pi)/2) - d*Piecewis
e(((2 - 3*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(2 - 3*x**2), True))/6
```

Maxima [F]

$$\int (c+dx)\left(2-3x^2\right)^p\,dx = \int (dx+c)\left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)*(-3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(-3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)\left(2-3x^2\right)^p\,dx = \int (dx+c)\left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)*(-3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(-3*x^2 + 2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int (c+dx)\left(2-3x^2\right)^p\,dx = \frac{c\,x\,\left(2-3\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{\left(1-\frac{3\,x^2}{2}\right)^p} - \frac{d\,\left(2-3\,x^2\right)^{p+1}}{6\,\left(p+1\right)}$$

input
```
int((2 - 3*x^2)^p*(c + d*x),x)
```

output
```
(c*x*(2 - 3*x^2)^p*hypergeom([1/2, -p], 3/2, (3*x^2)/2))/(1 - (3*x^2)/2)^p
 - (d*(2 - 3*x^2)^(p + 1))/(6*(p + 1))
```

Reduce [F]

$$\int (c+dx)\left(2-3x^2\right)^p\,dx$$
$$= \frac{6(-3x^2+2)^p\,cpx+6(-3x^2+2)^p\,cx+6(-3x^2+2)^p\,dp\,x^2-4(-3x^2+2)^p\,dp+3(-3x^2+2)^p\,d\,x^2-2(\cdots}{12p^2+1\cdots}$$

input
```
int((d*x+c)*(-3*x^2+2)^p,x)
```

output
```
(6*( - 3*x**2 + 2)**p*c*p*x + 6*( - 3*x**2 + 2)**p*c*x + 6*( - 3*x**2 + 2)
**p*d*p*x**2 - 4*( - 3*x**2 + 2)**p*d*p + 3*( - 3*x**2 + 2)**p*d*x**2 - 2*
( - 3*x**2 + 2)**p*d - 48*int(( - 3*x**2 + 2)**p/(6*p*x**2 - 4*p + 3*x**2
- 2),x)*c*p**3 - 72*int(( - 3*x**2 + 2)**p/(6*p*x**2 - 4*p + 3*x**2 - 2),x
)*c*p**2 - 24*int(( - 3*x**2 + 2)**p/(6*p*x**2 - 4*p + 3*x**2 - 2),x)*c*p)
/(6*(2*p**2 + 3*p + 1))
```

3.434 $\int (2-3x^2)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int (2-3x^2)^p \, dx = 2^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)$$

output
```
2^p*x*hypergeom([1/2, -p],[3/2],3/2*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (2-3x^2)^p \, dx = 2^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{3x^2}{2}\right)$$

input
```
Integrate[(2 - 3*x^2)^p,x]
```

output
```
2^p*x*Hypergeometric2F1[1/2, -p, 3/2, (3*x^2)/2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-3x^2)^p \, dx$$
$$\Big\downarrow \text{237}$$
$$2^p x \,\text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, \frac{3x^2}{2}\right)$$

input
```
Int[(2 - 3*x^2)^p,x]
```

output
```
2^p*x*Hypergeometric2F1[1/2, -p, 3/2, (3*x^2)/2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
meijerg	$2^p x \,\text{hypergeom}\left(\left[\frac{1}{2}, -p\right], \left[\frac{3}{2}\right], \frac{3x^2}{2}\right)$	19

input
```
int((-3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
2^p*x*hypergeom([1/2,-p],[3/2],3/2*x^2)
```

Fricas [F]

$$\int \left(2-3x^2\right)^p\,dx=\int \left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((-3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(2-3x^2\right)^p\,dx=2^p x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{3x^2e^{2i\pi}}{2}\right)$$

input
```
integrate((-3*x**2+2)**p,x)
```

output
```
2**p*x*hyper((1/2, -p), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)
```

Maxima [F]

$$\int \left(2-3x^2\right)^p\,dx=\int \left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((-3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^p, x)
```

Giac [F]

$$\int \left(2-3x^2\right)^p \, dx = \int \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((-3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \left(2-3x^2\right)^p \, dx = \frac{x\,\left(2-3\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{\left(1-\frac{3\,x^2}{2}\right)^p}$$

input
```
int((2 - 3*x^2)^p,x)
```

output
```
(x*(2 - 3*x^2)^p*hypergeom([1/2, -p], 3/2, (3*x^2)/2))/(1 - (3*x^2)/2)^p
```

Reduce [F]

$$\int \left(2-3x^2\right)^p \, dx = \frac{\left(-3x^2+2\right)^p x-8\left(\int \frac{\left(-3x^2+2\right)^p}{6p\,x^2+3x^2-4p-2}dx\right)p^2-4\left(\int \frac{\left(-3x^2+2\right)^p}{6p\,x^2+3x^2-4p-2}dx\right)p}{2p+1}$$

input
```
int((-3*x^2+2)^p,x)
```

output
```
(( - 3*x**2 + 2)**p*x - 8*int(( - 3*x**2 + 2)**p/(6*p*x**2 - 4*p + 3*x**2
- 2),x)*p**2 - 4*int(( - 3*x**2 + 2)**p/(6*p*x**2 - 4*p + 3*x**2 - 2),x)*p
)/(2*p + 1)
```

3.435 $\int \frac{(2-3x^2)^p}{c+dx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \frac{2^p x\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c} + \frac{d(2-3x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,-\frac{d^2(2-3x^2)}{3c^2-2d^2}\right)}{2(3c^2-2d^2)(1+p)}$$

output
```
2^p*x*AppellF1(1/2,1,-p,3/2,d^2*x^2/c^2,3/2*x^2)/c+1/2*d*(-3*x^2+2)^(p+1)*
hypergeom([1, p+1],[2+p],-d^2*(-3*x^2+2)/(3*c^2-2*d^2))/(3*c^2-2*d^2)/(p+1
)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.22

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \frac{9^p\left(-\frac{d(\sqrt{6}-3x)}{c+dx}\right)^{-p}\left(\frac{d(\sqrt{6}+3x)}{c+dx}\right)^{-p}(2-3x^2)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{3c-\sqrt{6}d}{3c+3dx},\frac{3c+\sqrt{6}d}{3c+3dx}\right)}{2dp}$$

input
```
Integrate[(2 - 3*x^2)^p/(c + d*x),x]
```

output
```
(9^p*(2 - 3*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (3*c - Sqrt[6]*d)/(3*c
+ 3*d*x), (3*c + Sqrt[6]*d)/(3*c + 3*d*x)])/(2*d*p*(-((d*(Sqrt[6] - 3*x))/
(c + d*x)))^p*((d*(Sqrt[6] + 3*x))/(c + d*x))^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {504, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-3x^2)^p}{c+dx}dx$$

$$\Big\downarrow \text{504}$$

$$c\int \frac{(2-3x^2)^p}{c^2-d^2x^2}dx-d\int \frac{x(2-3x^2)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{333}$$

$$\frac{2^p x \operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c}-d\int \frac{x(2-3x^2)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{2^p x \operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c}-\frac{1}{2}d\int \frac{(2-3x^2)^p}{c^2-d^2x^2}dx^2$$

$$\Big\downarrow \text{78}$$

$$\frac{2^p x \operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},\frac{3x^2}{2},\frac{d^2x^2}{c^2}\right)}{c}+\frac{d(2-3x^2)^{p+1}\operatorname{Hypergeometric2F1}\left(1,p+1,p+2,-\frac{d^2(2-3x^2)}{3c^2-2d^2}\right)}{2(p+1)(3c^2-2d^2)}$$

input
```
Int[(2 - 3*x^2)^p/(c + d*x),x]
```

output
```
(2^p*x*AppellF1[1/2, -p, 1, 3/2, (3*x^2)/2, (d^2*x^2)/c^2])/c + (d*(2 - 3*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, -((d^2*(2 - 3*x^2))/(3*c^2
 - 2*d^2))])/(2*(3*c^2 - 2*d^2)*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

Maple [F]

$$\int \frac{(-3x^2+2)^p}{dx+c}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c),x)
```

output
```
int((-3*x^2+2)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \int \frac{(-3\,x^2+2)^p}{dx+c}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^p/(d*x + c), x)
```

Sympy [F]

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \int \frac{(2-3x^2)^p}{c+dx}\,dx$$

input
```
integrate((-3*x**2+2)**p/(d*x+c),x)
```

output
```
Integral((2 - 3*x**2)**p/(c + d*x), x)
```

Maxima [F]

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \int \frac{(-3\,x^2+2)^p}{dx+c}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \int \frac{(-3\,x^2+2)^p}{dx+c}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \int \frac{(2-3\,x^2)^p}{c+d\,x}\,dx$$

input
```
int((2 - 3*x^2)^p/(c + d*x),x)
```

output
```
int((2 - 3*x^2)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(2-3x^2)^p}{c+dx}\,dx = \int \frac{(-3x^2+2)^p}{dx+c}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c),x)
```

output
```
int(( - 3*x**2 + 2)**p/(c + d*x),x)
```

3.436 $\int \frac{(2-3x^2)^p}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 175

$$\int \frac{(2-3x^2)^p}{(c+dx)^2} \, dx = -\frac{3\, 4^p (2-3x^2)^p \left(1-\frac{3(c+dx)}{3c-\sqrt{6}d}\right) \left(\frac{\left(\sqrt{6}c+2d\right)\left(1-\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d}\right)^{-p} \operatorname{AppellF1}\left(1+p,2,-p,2+p,1-\frac{3(c+dx)}{3c-\sqrt{6}d},-\frac{(\sqrt{6}\cdots}{\cdots}\right)}{d\left(3c-\sqrt{6}d\right)(1+p)}$$

output

```
-3*4^p*(-3*x^2+2)^p*(1-3*(d*x+c)/(3*c-6^(1/2)*d))*AppellF1(p+1,-p,2,2+p,-1
/4*(6^(1/2)*c-2*d)*(1-3*(d*x+c)/(3*c-6^(1/2)*d))/d,1-3*(d*x+c)/(3*c-6^(1/2
)*d))/d/(3*c-6^(1/2)*d)/(p+1)/(((6^(1/2)*c+2*d)*(1-3*(d*x+c)/(6^(1/2)*d+3*
c))/d)^p)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.78

$$\int \frac{(2-3x^2)^p}{(c+dx)^2} \, dx = \frac{9^p \left(-\frac{d\left(\sqrt{6}-3x\right)}{c+dx}\right)^{-p} \left(\frac{d\left(\sqrt{6}+3x\right)}{c+dx}\right)^{-p} (2-3x^2)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{3c-\sqrt{6}d}{3c+3dx},\frac{3c+\sqrt{6}d}{3c+3dx}\right)}{d(-1+2p)(c+dx)}$$

input
```
Integrate[(2 - 3*x^2)^p/(c + d*x)^2,x]
```

output
```
(9^p*(2 - 3*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (3*c - Sqrt[6]*d)/(3
*c + 3*d*x), (3*c + Sqrt[6]*d)/(3*c + 3*d*x)])/(d*(-1 + 2*p)*(-((d*(Sqrt[6
] - 3*x))/(c + d*x)))^p*((d*(Sqrt[6] + 3*x))/(c + d*x))^p*(c + d*x))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {506, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-3x^2)^p}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{506}$$

$$-\frac{(2-3x^2)^p \left(\frac{1}{c+dx}\right)^{2p} \left(1-\frac{3c-\sqrt{6}d}{3(c+dx)}\right)^{-p} \left(1-\frac{3c+\sqrt{6}d}{3(c+dx)}\right)^{-p} \int \left(\frac{1}{c+dx}\right)^{-2p} \left(1-\frac{3c-\sqrt{6}d}{3(c+dx)}\right)^{p} \left(1-\frac{3c+\sqrt{6}d}{3(c+dx)}\right)^{p} d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{(2-3x^2)^p \left(1-\frac{3c-\sqrt{6}d}{3(c+dx)}\right)^{-p} \left(1-\frac{3c+\sqrt{6}d}{3(c+dx)}\right)^{-p} \text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{3c-\sqrt{6}d}{3(c+dx)},\frac{3c+\sqrt{6}d}{3(c+dx)}\right)}{d(1-2p)(c+dx)}$$

input
```
Int[(2 - 3*x^2)^p/(c + d*x)^2,x]
```

output
```
-(((2 - 3*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (3*c - Sqrt[6]*d)/(3*(
c + d*x)), (3*c + Sqrt[6]*d)/(3*(c + d*x))])/(d*(1 - 2*p)*(c + d*x)*(1 - (
3*c - Sqrt[6]*d)/(3*(c + d*x)))^p*(1 - (3*c + Sqrt[6]*d)/(3*(c + d*x)))^p)
)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{(-3x^2+2)^p}{(dx+c)^2}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c)^2,x)
```

output
```
int((-3*x^2+2)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(2-3x^2)^p}{(c+dx)^2}\,dx$$

input
```
integrate((-3*x**2+2)**p/(d*x+c)**2,x)
```

output
```
Integral((2 - 3*x**2)**p/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-3x^2)^p}{(c+dx)^2}\,dx = \int \frac{(2-3\,x^2)^p}{(c+d\,x)^2}\,dx$$

input
```
int((2 - 3*x^2)^p/(c + d*x)^2,x)
```

output
```
int((2 - 3*x^2)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^2}\,dx$$
$$= \frac{(-3x^2+2)^p x - 8\left(\int \frac{(-3x^2+2)^p}{6d^2p\,x^4+12cdp\,x^3+3d^2x^4+6c^2p\,x^2+6cd\,x^3-4d^2p\,x^2+3c^2x^2-8cdpx-2d^2x^2-4c^2p-4cdx-2c^2}dx\right)c^2p^2 - 4(}{}$$

input
```
int((-3*x^2+2)^p/(d*x+c)^2,x)
```

output
```
(( - 3*x**2 + 2)**p*x - 8*int(( - 3*x**2 + 2)**p/(6*c**2*p*x**2 - 4*c**2*p
 + 3*c**2*x**2 - 2*c**2 + 12*c*d*p*x**3 - 8*c*d*p*x + 6*c*d*x**3 - 4*c*d*x
 + 6*d**2*p*x**4 - 4*d**2*p*x**2 + 3*d**2*x**4 - 2*d**2*x**2),x)*c**2*p**2
 - 4*int(( - 3*x**2 + 2)**p/(6*c**2*p*x**2 - 4*c**2*p + 3*c**2*x**2 - 2*c*
*2 + 12*c*d*p*x**3 - 8*c*d*p*x + 6*c*d*x**3 - 4*c*d*x + 6*d**2*p*x**4 - 4*
d**2*p*x**2 + 3*d**2*x**4 - 2*d**2*x**2),x)*c**2*p - 8*int(( - 3*x**2 + 2)
**p/(6*c**2*p*x**2 - 4*c**2*p + 3*c**2*x**2 - 2*c**2 + 12*c*d*p*x**3 - 8*c
*d*p*x + 6*c*d*x**3 - 4*c*d*x + 6*d**2*p*x**4 - 4*d**2*p*x**2 + 3*d**2*x**
4 - 2*d**2*x**2),x)*c*d*p**2*x - 4*int(( - 3*x**2 + 2)**p/(6*c**2*p*x**2 -
 4*c**2*p + 3*c**2*x**2 - 2*c**2 + 12*c*d*p*x**3 - 8*c*d*p*x + 6*c*d*x**3
- 4*c*d*x + 6*d**2*p*x**4 - 4*d**2*p*x**2 + 3*d**2*x**4 - 2*d**2*x**2),x)*
c*d*p*x - 12*int((( - 3*x**2 + 2)**p*x**3)/(6*c**2*p*x**2 - 4*c**2*p + 3*c
**2*x**2 - 2*c**2 + 12*c*d*p*x**3 - 8*c*d*p*x + 6*c*d*x**3 - 4*c*d*x + 6*d
**2*p*x**4 - 4*d**2*p*x**2 + 3*d**2*x**4 - 2*d**2*x**2),x)*c*d*p**2 - 6*in
t((( - 3*x**2 + 2)**p*x**3)/(6*c**2*p*x**2 - 4*c**2*p + 3*c**2*x**2 - 2*c*
*2 + 12*c*d*p*x**3 - 8*c*d*p*x + 6*c*d*x**3 - 4*c*d*x + 6*d**2*p*x**4 - 4*
d**2*p*x**2 + 3*d**2*x**4 - 2*d**2*x**2),x)*c*d*p - 12*int((( - 3*x**2 + 2
)**p*x**3)/(6*c**2*p*x**2 - 4*c**2*p + 3*c**2*x**2 - 2*c**2 + 12*c*d*p*x**
3 - 8*c*d*p*x + 6*c*d*x**3 - 4*c*d*x + 6*d**2*p*x**4 - 4*d**2*p*x**2 + 3*d
**2*x**4 - 2*d**2*x**2),x)*d**2*p**2*x - 6*int((( - 3*x**2 + 2)**p*x**3...
```

3.437 $\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 175

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = -\frac{9\,4^p(2-3x^2)^p\left(1-\frac{3(c+dx)}{3c-\sqrt{6}d}\right)\left(\frac{\left(\sqrt{6}c+2d\right)\left(1-\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d}\right)^{-p}\operatorname{AppellF1}\left(1+p,3,-p,2+p,1-\frac{3(c+dx)}{3c-\sqrt{6}d},-\frac{(\sqrt{\cdots}}{\cdots}\right.}{d\left(3c-\sqrt{6}d\right)^2(1+p)}$$

output
```
-9*4^p*(-3*x^2+2)^p*(1-3*(d*x+c)/(3*c-6^(1/2)*d))*AppellF1(p+1,-p,3,2+p,-1
/4*(6^(1/2)*c-2*d)*(1-3*(d*x+c)/(3*c-6^(1/2)*d))/d,1-3*(d*x+c)/(3*c-6^(1/2
)*d))/d/(3*c-6^(1/2)*d)^2/(p+1)/(((6^(1/2)*c+2*d)*(1-3*(d*x+c)/(6^(1/2)*d+
3*c))/d)^p)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = \frac{9^p\left(-\frac{d\left(\sqrt{6}-3x\right)}{c+dx}\right)^{-p}\left(\frac{d\left(\sqrt{6}+3x\right)}{c+dx}\right)^{-p}(2-3x^2)^p\operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{3c-\sqrt{6}d}{3c+3dx},\frac{3c+\sqrt{6}d}{3c+3dx}\right)}{2d(-1+p)(c+dx)^2}$$

input
```
Integrate[(2 - 3*x^2)^p/(c + d*x)^3,x]
```

output
```
(9^p*(2 - 3*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (3*c - Sqrt[6]*d)/(3
*c + 3*d*x), (3*c + Sqrt[6]*d)/(3*c + 3*d*x)])/(2*d*(-1 + p)*(-((d*(Sqrt[6
] - 3*x))/(c + d*x)))^p*((d*(Sqrt[6] + 3*x))/(c + d*x))^p*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {506, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-3x^2)^p}{(c+dx)^3} \, dx$$

$$\Big\downarrow \textbf{506}$$

$$-\frac{(2-3x^2)^p \left(\frac{1}{c+dx}\right)^{2p} \left(1-\frac{3c-\sqrt{6}d}{3(c+dx)}\right)^{-p} \left(1-\frac{3c+\sqrt{6}d}{3(c+dx)}\right)^{-p} \int \left(\frac{1}{c+dx}\right)^{1-2p} \left(1-\frac{3c-\sqrt{6}d}{3(c+dx)}\right)^{p} \left(1-\frac{3c+\sqrt{6}d}{3(c+dx)}\right)^{p} d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \textbf{150}$$

$$-\frac{(2-3x^2)^p \left(1-\frac{3c-\sqrt{6}d}{3(c+dx)}\right)^{-p} \left(1-\frac{3c+\sqrt{6}d}{3(c+dx)}\right)^{-p} \text{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{3c-\sqrt{6}d}{3(c+dx)},\frac{3c+\sqrt{6}d}{3(c+dx)}\right)}{2d(1-p)(c+dx)^2}$$

input
```
Int[(2 - 3*x^2)^p/(c + d*x)^3,x]
```

output
```
-1/2*((2 - 3*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (3*c - Sqrt[6]*d)/(
3*(c + d*x)), (3*c + Sqrt[6]*d)/(3*(c + d*x))])/(d*(1 - p)*(c + d*x)^2*(1
- (3*c - Sqrt[6]*d)/(3*(c + d*x)))^p*(1 - (3*c + Sqrt[6]*d)/(3*(c + d*x)))
^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{(-3x^2+2)^p}{(dx+c)^3}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c)^3,x)
```

output
```
int((-3*x^2+2)^p/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^p/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx$$

input
```
integrate((-3*x**2+2)**p/(d*x+c)**3,x)
```

output
```
Integral((2 - 3*x**2)**p/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(2-3\,x^2)^p}{(c+d\,x)^3}\,dx$$

input
```
int((2 - 3*x^2)^p/(c + d*x)^3,x)
```

output
```
int((2 - 3*x^2)^p/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^3}\,dx = \int \frac{(-3x^2+2)^p}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c)^3,x)
```

output
```
int(( - 3*x**2 + 2)**p/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.438 $\int (c+dx)^{3/2} (a+bx^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int (c+dx)^{3/2} (a+bx^2)^p \, dx = \frac{2(c+dx)^{5/2}(a+bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c\cdots}{c+\cdots}\right.}{5d}$$

output
```
2/5*(d*x+c)^(5/2)*(b*x^2+a)^p*AppellF1(5/2,-p,-p,7/2,(d*x+c)/(c-(-a)^(1/2)
*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-(-a)^(1/2)*
d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.15

$$\int (c+dx)^{3/2} (a+bx^2)^p \, dx = \frac{2\left(\frac{d\left(\sqrt{-\frac{ab}{d^2}}d-bx\right)}{bc+\sqrt{-\frac{ab}{d^2}}d^2}\right)^{-p}(c+dx)^{5/2}\left(\frac{a-\sqrt{-\frac{ab}{d^2}}dx}{a+c\sqrt{-\frac{ab}{d^2}}}\right)^{-p}(a+bx^2)^p \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c+\frac{a}{\sqrt{-\frac{ab}{d^2}}}},\cdots\right.}{5d}$$

input
```
Integrate[(c + d*x)^(3/2)*(a + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(a + b*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (c + d*x)/(c +
 a/Sqrt[-((a*b)/d^2)]), (b*(c + d*x))/(b*c + Sqrt[-((a*b)/d^2)]*d^2)])/(5*
d*((d*(Sqrt[-((a*b)/d^2)]*d - b*x))/(b*c + Sqrt[-((a*b)/d^2)]*d^2))^p*((a
- Sqrt[-((a*b)/d^2)]*d*x)/(a + c*Sqrt[-((a*b)/d^2)]))^p)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{514, 150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 514}$$

$$\frac{\left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^{3/2} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{2(c+dx)^{5/2} \left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{5d}$$

input
```
Int[(c + d*x)^(3/2)*(a + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(a + b*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (c + d*x)/(c -
 (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(5*d*(1 - (
c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sq
rt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^(3/2)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(3/2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2} \left(a+bx^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(bx^2+a\right)^p dx$$

input
```
integrate((d*x+c)^(3/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(b*x^2 + a)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2}\left(a+bx^2\right)^p\,dx=\int \left(a+bx^2\right)^p(c+dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(b*x**2+a)**p,x)
```

output
```
Integral((a + b*x**2)**p*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2}\left(a+bx^2\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2}\left(a+bx^2\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\left(a+bx^2\right)^p\,dx=\int \left(b\,x^2+a\right)^p\,{\left(c+d\,x\right)}^{3/2}\,dx$$

input
```
int((a + b*x^2)^p*(c + d*x)^(3/2),x)
```

output
```
int((a + b*x^2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\left(a+bx^2\right)^p\,dx=\text{too large to display}$$

input
```
int((d*x+c)^(3/2)*(b*x^2+a)^p,x)
```

output
```
(16*sqrt(c + d*x)*(a + b*x**2)**p*a*d*p + 18*sqrt(c + d*x)*(a + b*x**2)**p
*a*d + 8*sqrt(c + d*x)*(a + b*x**2)**p*b*c*p*x + 12*sqrt(c + d*x)*(a + b*x
**2)**p*b*c*x + 8*sqrt(c + d*x)*(a + b*x**2)**p*b*d*p*x**2 + 6*sqrt(c + d*
x)*(a + b*x**2)**p*b*d*x**2 - 256*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)
/(16*a*c*p**2 + 32*a*c*p + 15*a*c + 16*a*d*p**2*x + 32*a*d*p*x + 15*a*d*x
+ 16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b
*d*p*x**3 + 15*b*d*x**3),x)*a*b*d**2*p**4 - 1024*int((sqrt(c + d*x)*(a + b
*x**2)**p*x**2)/(16*a*c*p**2 + 32*a*c*p + 15*a*c + 16*a*d*p**2*x + 32*a*d*
p*x + 15*a*d*x + 16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p
**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3),x)*a*b*d**2*p**3 - 1408*int((sqrt(
c + d*x)*(a + b*x**2)**p*x**2)/(16*a*c*p**2 + 32*a*c*p + 15*a*c + 16*a*d*p
**2*x + 32*a*d*p*x + 15*a*d*x + 16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*
x**2 + 16*b*d*p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3),x)*a*b*d**2*p**2 -
768*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)/(16*a*c*p**2 + 32*a*c*p + 15*
a*c + 16*a*d*p**2*x + 32*a*d*p*x + 15*a*d*x + 16*b*c*p**2*x**2 + 32*b*c*p*
x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3),x)*a*
b*d**2*p - 135*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)/(16*a*c*p**2 + 32*
a*c*p + 15*a*c + 16*a*d*p**2*x + 32*a*d*p*x + 15*a*d*x + 16*b*c*p**2*x**2
+ 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*
x**3),x)*a*b*d**2 + 48*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)/(16*a*c...
```

3.439 $\int \sqrt{c+dx}\,(a+bx^2)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \sqrt{c+dx}\,(a+bx^2)^p\,dx = \frac{2(c+dx)^{3/2}(a+bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{3d}$$

output
```
2/3*(d*x+c)^(3/2)*(b*x^2+a)^p*AppellF1(3/2,-p,-p,5/2,(d*x+c)/(c-(-a)^(1/2)
*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-(-a)^(1/2)*
d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03

$$\int \sqrt{c+dx}\,(a+bx^2)^p\,dx = \frac{2\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{-p}(c+dx)^{3/2}(a+bx^2)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{-\frac{a}{b}}d}\right)}{3d}$$

input
```
Integrate[Sqrt[c + d*x]*(a + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(a + b*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (c + d*x)/(c -
 Sqrt[-(a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(3*d*((d*(Sqrt[-(a/b)]
- x))/(c + Sqrt[-(a/b)]*d))^p*((d*(Sqrt[-(a/b)] + x))/(-c + Sqrt[-(a/b)]*d
))^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{514, 150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\left(a+bx^2\right)^p dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\int \sqrt{c+dx}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{2(c+dx)^{3/2}\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{3d}$$

input
```
Int[Sqrt[c + d*x]*(a + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(a + b*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (c + d*x)/(c -
 (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(3*d*(1 - (
c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sq
rt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \sqrt{dx+c}\,\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int((d*x+c)^(1/2)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(1/2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int \sqrt{c+dx}\left(a+bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(bx^2+a\right)^p dx$$

input
```
integrate((d*x+c)^(1/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(b*x^2 + a)^p, x)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(a+bx^2\right)^p\,dx=\int \left(a+bx^2\right)^p\sqrt{c+dx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(b*x**2+a)**p,x)
```

output
```
Integral((a + b*x**2)**p*sqrt(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(a+bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(a+bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(a+bx^2\right)^p\,dx=\int \left(b\,x^2+a\right)^p\,\sqrt{c+d\,x}\,dx$$

input
```
int((a + b*x^2)^p*(c + d*x)^(1/2),x)
```

output
```
int((a + b*x^2)^p*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(a+bx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(b*x^2+a)^p,x)
```

output
```
(2*sqrt(c + d*x)*(a + b*x**2)**p*a*d + 2*sqrt(c + d*x)*(a + b*x**2)**p*b*c
*x - 16*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)/(4*a*c*p + 3*a*c + 4*a*d*
p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3),x)*
a*b*d**2*p**2 - 16*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)/(4*a*c*p + 3*a
*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*
d*x**3),x)*a*b*d**2*p - 3*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)/(4*a*c*
p + 3*a*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**3
 + 3*b*d*x**3),x)*a*b*d**2 + 4*int((sqrt(c + d*x)*(a + b*x**2)**p*x**2)/(4
*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p
*x**3 + 3*b*d*x**3),x)*b**2*c**2*p + 3*int((sqrt(c + d*x)*(a + b*x**2)**p*
x**2)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*x**2 +
 4*b*d*p*x**3 + 3*b*d*x**3),x)*b**2*c**2 - 4*int((sqrt(c + d*x)*(a + b*x**
2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*x**2
+ 4*b*d*p*x**3 + 3*b*d*x**3),x)*a**2*d**2*p - 3*int((sqrt(c + d*x)*(a + b*
x**2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*x*
*2 + 4*b*d*p*x**3 + 3*b*d*x**3),x)*a**2*d**2 + 16*int((sqrt(c + d*x)*(a +
b*x**2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 + 3*b*c*
x**2 + 4*b*d*p*x**3 + 3*b*d*x**3),x)*a*b*c**2*p**2 + 16*int((sqrt(c + d*x)
*(a + b*x**2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x + 4*b*c*p*x**2 +
3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3),x)*a*b*c**2*p + 3*int((sqrt(c +...
```

3.440 $\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}(a+bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d}$$

output
```
2*(d*x+c)^(1/2)*(b*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,(d*x+c)/(c-(-a)^(1/2)*d
/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-(-a)^(1/2)*d/
b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{-p}\sqrt{c+dx}(a+bx^2)^p\,\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{-\frac{a}{b}}d}\right)}{d}$$

input
```
Integrate[(a + b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(a + b*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (c + d*x)/(c - S
qrt[-(a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*((d*(Sqrt[-(a/b)] - x)
)/(c + Sqrt[-(a/b)]*d))^p*((d*(Sqrt[-(a/b)] + x))/(-c + Sqrt[-(a/b)]*d))^p
)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{514, 150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \textbf{514}$$

$$\frac{(a+bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-ad}}{\sqrt{b}}+c}\right)^{-p}\int \frac{\left(1-\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^p}{\sqrt{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \textbf{150}$$

$$\frac{2\sqrt{c+dx}(a+bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-ad}}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-ad}}{\sqrt{b}}}\right)}{d}$$

input
```
Int[(a + b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(a + b*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (c + d*x)/(c - (
Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d*(1 - (c +
d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b
]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{\sqrt{dx+c}}dx$$

input
```
int((b*x^2+a)^p/(d*x+c)^(1/2),x)
```

output
```
int((b*x^2+a)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(bx^2+a)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((b*x**2+a)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(bx^2+a)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(bx^2+a)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(b\,x^2+a)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(1/2),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{dx+c}\,(b\,x^2+a)^p}{dx+c}\,dx$$

input
```
int((b*x^2+a)^p/(d*x+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*(a + b*x**2)**p)/(c + d*x),x)
```

3.441 $\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d\sqrt{c+dx}}$$

output
```
-2*(b*x^2+a)^p*AppellF1(-1/2,-p,-p,1/2,(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)),(d
*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/(d*x+c)^(1/2)/((1-(d*x+c)/(c-(-a)^(1/2)*
d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{-p}(a+bx^2)^p\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{-\frac{a}{b}}d}\right)}{d\sqrt{c+dx}}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(a + b*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (c + d*x)/(c - Sqrt[-(a/b)]*
d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*((d*(Sqrt[-(a/b)] - x))/(c + Sqrt[
-(a/b)]*d))^p*((d*(Sqrt[-(a/b)] + x))/(-c + Sqrt[-(a/b)]*d))^p*Sqrt[c + d*
x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(a+bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\int \frac{\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p}{(c+dx)^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2(a+bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d\sqrt{c+dx}}$$

input
```
Int[(a + b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(a + b*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (c + d*x)/(c - (Sqrt[-a]*d)/
Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d*Sqrt[c + d*x]*(1 - (c
+ d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt
[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{p}}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^p/(d*x+c)^(3/2),x)
```

output
```
int((b*x^2+a)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(b*x^2 + a)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(a+bx^2)^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(bx^2+a)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(3/2),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,(b\,x^2+a)^p}{d^2x^2+2cdx+c^2}dx$$

input
```
int((b*x^2+a)^p/(d*x+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*(a + b*x**2)**p)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.442 $\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \frac{2(c+dx)^{5/2}\left(2+bx^2\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^{-p} \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}},\right.}{5d}$$

output
```
2/5*(d*x+c)^(5/2)*(b*x^2+2)^p*AppellF1(5/2,-p,-p,7/2,(d*x+c)/(c+2^(1/2)*d/
(-b)^(1/2)),(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))/d/((1-(d*x+c)/(c+2^(1/2)*d
/(-b)^(1/2)))^p)/((1-(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.54 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \frac{2\left(\frac{d\left(\sqrt{2}\sqrt{-\frac{b}{d^2}}d-bx\right)}{bc+\sqrt{2}\sqrt{-\frac{b}{d^2}}d^2}\right)^{-p} (c+dx)^{5/2} \left(\frac{\sqrt{2}-\sqrt{-\frac{b}{d^2}}dx}{\sqrt{2}+c\sqrt{-\frac{b}{d^2}}}\right)^{-p} \left(2+bx^2\right)^p \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c+\frac{\sqrt{2}}{\sqrt{-\frac{b}{d^2}}}}\right.}{5d}$$

input
```
Integrate[(c + d*x)^(3/2)*(2 + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(2 + b*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (c + d*x)/(c +
 Sqrt[2]/Sqrt[-(b/d^2)]), (b*(c + d*x))/(b*c + Sqrt[2]*Sqrt[-(b/d^2)]*d^2)
])/(5*d*((d*(Sqrt[2]*Sqrt[-(b/d^2)]*d - b*x))/(b*c + Sqrt[2]*Sqrt[-(b/d^2)
]*d^2))^p*((Sqrt[2] - Sqrt[-(b/d^2)]*d*x)/(Sqrt[2] + c*Sqrt[-(b/d^2)]))^p)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+2)^p (c+dx)^{3/2}\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(bx^2+2)^p \left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p} \int (c+dx)^{3/2} \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{2(bx^2+2)^p (c+dx)^{5/2} \left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p} \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)}{5d}$$

input
```
Int[(c + d*x)^(3/2)*(2 + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(2 + b*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (c + d*x)/(c +
 (Sqrt[2]*d)/Sqrt[-b]), (c + d*x)/(c + (Sqrt[2]*b*d)/(-b)^(3/2))])/(5*d*(1
 - (c + d*x)/(c + (Sqrt[2]*d)/Sqrt[-b]))^p*(1 - (c + d*x)/(c + (Sqrt[2]*b*
d)/(-b)^(3/2)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(b\,x^{2}+2\right)^{p} dx$$

input
```
int((d*x+c)^(3/2)*(b*x^2+2)^p,x)
```

output
```
int((d*x+c)^(3/2)*(b*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(bx^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(b*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(b*x^2 + 2)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \int (c+dx)^{\frac{3}{2}} \left(bx^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)**(3/2)*(b*x**2+2)**p,x)
```

output
```
Integral((c + d*x)**(3/2)*(b*x**2 + 2)**p, x)
```

Maxima [F]

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(bx^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(b*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(b*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(bx^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(b*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(b*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \int \left(b\,x^2+2\right)^p \left(c+d\,x\right)^{3/2} \, dx$$

input
```
int((b*x^2 + 2)^p*(c + d*x)^(3/2),x)
```

output
```
int((b*x^2 + 2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2} \left(2+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^(3/2)*(b*x^2+2)^p,x)
```

output

```
(8*sqrt(c + d*x)*(b*x**2 + 2)**p*b*c*p*x + 12*sqrt(c + d*x)*(b*x**2 + 2)**
p*b*c*x + 8*sqrt(c + d*x)*(b*x**2 + 2)**p*b*d*p*x**2 + 6*sqrt(c + d*x)*(b*
x**2 + 2)**p*b*d*x**2 + 32*sqrt(c + d*x)*(b*x**2 + 2)**p*d*p + 36*sqrt(c +
 d*x)*(b*x**2 + 2)**p*d + 48*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(16*
b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*d*p*
x**3 + 15*b*d*x**3 + 32*c*p**2 + 64*c*p + 30*c + 32*d*p**2*x + 64*d*p*x +
30*d*x),x)*b**2*c**2*p**2 + 96*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(1
6*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*d*
p*x**3 + 15*b*d*x**3 + 32*c*p**2 + 64*c*p + 30*c + 32*d*p**2*x + 64*d*p*x
+ 30*d*x),x)*b**2*c**2*p + 45*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(16
*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*d*p
*x**3 + 15*b*d*x**3 + 32*c*p**2 + 64*c*p + 30*c + 32*d*p**2*x + 64*d*p*x +
 30*d*x),x)*b**2*c**2 - 512*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(16*b
*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*d*p*x
**3 + 15*b*d*x**3 + 32*c*p**2 + 64*c*p + 30*c + 32*d*p**2*x + 64*d*p*x + 3
0*d*x),x)*b*d**2*p**4 - 2048*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(16*
b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*d*p*
x**3 + 15*b*d*x**3 + 32*c*p**2 + 64*c*p + 30*c + 32*d*p**2*x + 64*d*p*x +
30*d*x),x)*b*d**2*p**3 - 2816*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(16
*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*p**2*x**3 + 32*b*...
```

3.443 $\int \sqrt{c+dx}(2+bx^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p \, dx = \frac{2(c+dx)^{3/2}\left(2+bx^2\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)}{3d}$$

output
```
2/3*(d*x+c)^(3/2)*(b*x^2+2)^p*AppellF1(3/2,-p,-p,5/2,(d*x+c)/(c+2^(1/2)*d/
(-b)^(1/2)),(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))/d/((1-(d*x+c)/(c+2^(1/2)*d
/(-b)^(1/2)))^p)/((1-(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.21

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p \, dx = \frac{2\left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p}(c+dx)^{3/2}\left(2+bx^2\right)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c+\sqrt{2}\left(-\frac{1}{b}\right)^{3/2}bd},\frac{c+}{c+\sqrt{2}}\right.}{3d}$$

input
```
Integrate[Sqrt[c + d*x]*(2 + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(2 + b*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (c + d*x)/(c +
 Sqrt[2]*(-b^(-1))^(3/2)*b*d), (c + d*x)/(c + Sqrt[2]*Sqrt[-b^(-1)]*d)])/(
3*d*((d*(Sqrt[2]*Sqrt[-b^(-1)] - x))/(c + Sqrt[2]*Sqrt[-b^(-1)]*d))^p*((d*
(Sqrt[2]*Sqrt[-b^(-1)] + x))/(-c + Sqrt[2]*Sqrt[-b^(-1)]*d))^p)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^2+2)^p \sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(bx^2+2)^p \left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p} \int \sqrt{c+dx}\left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{2(bx^2+2)^p (c+dx)^{3/2} \left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p} \text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)}{3d}$$

input
```
Int[Sqrt[c + d*x]*(2 + b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(2 + b*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (c + d*x)/(c +
 (Sqrt[2]*d)/Sqrt[-b]), (c + d*x)/(c + (Sqrt[2]*b*d)/(-b)^(3/2))])/(3*d*(1
 - (c + d*x)/(c + (Sqrt[2]*d)/Sqrt[-b]))^p*(1 - (c + d*x)/(c + (Sqrt[2]*b*
d)/(-b)^(3/2)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \sqrt{dx+c}\,\left(b\,x^{2}+2\right)^{p}dx$$

input
```
int((d*x+c)^(1/2)*(b*x^2+2)^p,x)
```

output
```
int((d*x+c)^(1/2)*(b*x^2+2)^p,x)
```

Fricas [F]

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(b*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(b*x^2 + 2)^p, x)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p\,dx=\int \sqrt{c+dx}\left(bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)**(1/2)*(b*x**2+2)**p,x)
```

output
```
Integral(sqrt(c + d*x)*(b*x**2 + 2)**p, x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(b*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*(b*x^2 + 2)^p, x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(b*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*(b*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p\,dx=\int \left(b\,x^2+2\right)^p\,\sqrt{c+d\,x}\,dx$$

input
```
int((b*x^2 + 2)^p*(c + d*x)^(1/2),x)
```

output
```
int((b*x^2 + 2)^p*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(2+bx^2\right)^p\,dx$$
$$=\frac{2\sqrt{dx+c}\,\left(b\,x^2+2\right)^p bcx+4\sqrt{dx+c}\,\left(b\,x^2+2\right)^p d+4\left(\int \frac{\sqrt{dx+c}\,\left(b\,x^2+2\right)^p x^2}{4bdp\,x^3+4bcp\,x^2+3bd\,x^3+3bc\,x^2+8dpx+8cp+6dx+6c}dx\right)b^2}{}$$

input
```
int((d*x+c)^(1/2)*(b*x^2+2)^p,x)
```

output

```
(2*sqrt(c + d*x)*(b*x**2 + 2)**p*b*c*x + 4*sqrt(c + d*x)*(b*x**2 + 2)**p*d
 + 4*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2 + 3*b*c*x**2 +
 4*b*d*p*x**3 + 3*b*d*x**3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b**2*c**2*p
 + 3*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2 + 3*b*c*x**2 +
 4*b*d*p*x**3 + 3*b*d*x**3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b**2*c**2 -
 32*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2 + 3*b*c*x**2 +
4*b*d*p*x**3 + 3*b*d*x**3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b*d**2*p**2
- 32*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2 + 3*b*c*x**2 +
 4*b*d*p*x**3 + 3*b*d*x**3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b*d**2*p -
6*int((sqrt(c + d*x)*(b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2 + 3*b*c*x**2 + 4*
b*d*p*x**3 + 3*b*d*x**3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b*d**2 + 32*in
t((sqrt(c + d*x)*(b*x**2 + 2)**p)/(4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**
3 + 3*b*d*x**3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b*c**2*p**2 + 32*int((s
qrt(c + d*x)*(b*x**2 + 2)**p)/(4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**3 +
3*b*d*x**3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b*c**2*p + 6*int((sqrt(c +
d*x)*(b*x**2 + 2)**p)/(4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x*
*3 + 8*c*p + 6*c + 8*d*p*x + 6*d*x),x)*b*c**2 - 16*int((sqrt(c + d*x)*(b*x
**2 + 2)**p)/(4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 + 8*c*
p + 6*c + 8*d*p*x + 6*d*x),x)*d**2*p - 12*int((sqrt(c + d*x)*(b*x**2 + 2)*
*p)/(4*b*c*p*x**2 + 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 + 8*c*p + 6*...
```

3.444 $\int \frac{(2+bx^2)^p}{\sqrt{c+dx}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}} \, dx = \frac{2\sqrt{c+dx}(2+bx^2)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)}{d}$$

output
```
2*(d*x+c)^(1/2)*(b*x^2+2)^p*AppellF1(1/2,-p,-p,3/2,(d*x+c)/(c+2^(1/2)*d/(-
b)^(1/2)),(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))/d/((1-(d*x+c)/(c+2^(1/2)*d/(
-b)^(1/2)))^p)/((1-(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.21

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}} \, dx = \frac{2\left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p} \sqrt{c+dx}(2+bx^2)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c+\sqrt{2}\left(-\frac{1}{b}\right)^{3/2}bd},\frac{c+dx}{c+\sqrt{2}\sqrt{-\cdots}}\right.}{d}$$

input
```
Integrate[(2 + b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(2 + b*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (c + d*x)/(c + S
qrt[2]*(-b^(-1))^(3/2)*b*d), (c + d*x)/(c + Sqrt[2]*Sqrt[-b^(-1)]*d)])/(d*
((d*(Sqrt[2]*Sqrt[-b^(-1)] - x))/(c + Sqrt[2]*Sqrt[-b^(-1)]*d))^p*((d*(Sqr
t[2]*Sqrt[-b^(-1)] + x))/(-c + Sqrt[2]*Sqrt[-b^(-1)]*d))^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+2)^p}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 514$$

$$\frac{(bx^2+2)^p\left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p}\int \frac{\left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^p}{\sqrt{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \; 150$$

$$\frac{2(bx^2+2)^p\sqrt{c+dx}\left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)}{d}$$

input
```
Int[(2 + b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(2 + b*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (c + d*x)/(c + (
Sqrt[2]*d)/Sqrt[-b]), (c + d*x)/(c + (Sqrt[2]*b*d)/(-b)^(3/2))])/(d*(1 - (
c + d*x)/(c + (Sqrt[2]*d)/Sqrt[-b]))^p*(1 - (c + d*x)/(c + (Sqrt[2]*b*d)/(
-b)^(3/2)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+2)^p}{\sqrt{dx+c}}dx$$

input
```
int((b*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((b*x^2+2)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(bx^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((b*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 2)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(bx^2+2)^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((b*x**2+2)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((b*x**2 + 2)**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(bx^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((b*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 2)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(bx^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((b*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 2)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(b\,x^2+2)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((b*x^2 + 2)^p/(c + d*x)^(1/2),x)
```

output
```
int((b*x^2 + 2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2+bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{dx+c}\,(b\,x^2+2)^p}{dx+c}\,dx$$

input
```
int((b*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*(b*x**2 + 2)**p)/(c + d*x),x)
```

3.445 $\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2(2+bx^2)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)}{d\sqrt{c+dx}}$$

output
```
-2*(b*x^2+2)^p*AppellF1(-1/2,-p,-p,1/2,(d*x+c)/(c+2^(1/2)*d/(-b)^(1/2)),(d
*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))/d/(d*x+c)^(1/2)/((1-(d*x+c)/(c+2^(1/2)*d
/(-b)^(1/2)))^p)/((1-(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.21

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2\left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p}(2+bx^2)^p\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c+\sqrt{2}\left(-\frac{1}{b}\right)^{3/2}bd},\frac{c+dx}{c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)}{d\sqrt{c+dx}}$$

input
```
Integrate[(2 + b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(2 + b*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (c + d*x)/(c + Sqrt[2]*(-b^(
-1))^(3/2)*b*d), (c + d*x)/(c + Sqrt[2]*Sqrt[-b^(-1)]*d)])/(d*((d*(Sqrt[2]
*Sqrt[-b^(-1)] - x))/(c + Sqrt[2]*Sqrt[-b^(-1)]*d))^p*((d*(Sqrt[2]*Sqrt[-b
^(-1)] + x))/(-c + Sqrt[2]*Sqrt[-b^(-1)]*d))^p*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+2)^p}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(bx^2+2)^p\left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p}\int \frac{\left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^p}{(c+dx)^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2(bx^2+2)^p\left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)}{d\sqrt{c+dx}}$$

input
```
Int[(2 + b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(2 + b*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (c + d*x)/(c + (Sqrt[2]*d)/S
qrt[-b]), (c + d*x)/(c + (Sqrt[2]*b*d)/(-b)^(3/2))])/(d*Sqrt[c + d*x]*(1 -
 (c + d*x)/(c + (Sqrt[2]*d)/Sqrt[-b]))^p*(1 - (c + d*x)/(c + (Sqrt[2]*b*d)
/(-b)^(3/2)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((b*x^2+2)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(bx^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(b*x^2 + 2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(bx^2+2)^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+2)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((b*x**2 + 2)**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(bx^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(bx^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(b\,x^2+2)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((b*x^2 + 2)^p/(c + d*x)^(3/2),x)
```

output
```
int((b*x^2 + 2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2+bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,(b\,x^2+2)^p}{d^2x^2+2cdx+c^2}dx$$

input
```
int((b*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*(b*x**2 + 2)**p)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.446 $\int (c+dx)^{3/2} (2+3x^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int (c+dx)^{3/2} (2+3x^2)^p \, dx = \frac{2(c+dx)^{5/2} (2+3x^2)^p \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c\cdots}{3c+\cdots}\right.}{5d}$$

output
```
2/5*(d*x+c)^(5/2)*(3*x^2+2)^p*AppellF1(5/2,-p,-p,7/2,3*(d*x+c)/(3*c-I*6^(1
/2)*d),3*(d*x+c)/(3*c+I*6^(1/2)*d))/d/((1-3*(d*x+c)/(3*c-I*6^(1/2)*d))^p)/
((1-3*(d*x+c)/(3*c+I*6^(1/2)*d))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.35

$$\int (c+dx)^{3/2} (2+3x^2)^p \, dx = \frac{2\left(\frac{d\left(\sqrt{6}\sqrt{-\frac{1}{d^2}}d-3x\right)}{3c+\sqrt{6}\sqrt{-\frac{1}{d^2}}d^2}\right)^{-p} (c+dx)^{5/2} \left(\frac{\sqrt{6}-3\sqrt{-\frac{1}{d^2}}dx}{\sqrt{6}+3c\sqrt{-\frac{1}{d^2}}}\right)^{-p} (2+3x^2)^p \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{3(c\cdots}{3c+\cdots}\right.}{5d}$$

input
```
Integrate[(c + d*x)^(3/2)*(2 + 3*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(2 + 3*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (3*(c + d*x))/
(3*c + Sqrt[6]/Sqrt[-d^(-2)]), (3*(c + d*x))/(3*c + Sqrt[6]*Sqrt[-d^(-2)]*
d^2)])/(5*d*((d*(Sqrt[6]*Sqrt[-d^(-2)]*d - 3*x))/(3*c + Sqrt[6]*Sqrt[-d^(-
2)]*d^2))^p*((Sqrt[6] - 3*Sqrt[-d^(-2)]*d*x)/(Sqrt[6] + 3*c*Sqrt[-d^(-2)])
)^p)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+2\right)^p (c+dx)^{3/2}dx$$

$$\Big\downarrow \text{ 514}$$

$$\frac{\left(3x^2+2\right)^p \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \int (c+dx)^{3/2} \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^p \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{2\left(3x^2+2\right)^p (c+dx)^{5/2} \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{5d}$$

input
```
Int[(c + d*x)^(3/2)*(2 + 3*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(2 + 3*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (3*(c + d*x))/
(3*c - I*Sqrt[6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(5*d*(1 - (3*(c +
 d*x))/(3*c - I*Sqrt[6]*d))^p*(1 - (3*(c + d*x))/(3*c + I*Sqrt[6]*d))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(3x^2+2\right)^p dx$$

input
```
int((d*x+c)^(3/2)*(3*x^2+2)^p,x)
```

output
```
int((d*x+c)^(3/2)*(3*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2} \left(2+3x^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(3\,x^2+2\right)^p dx$$

input
```
integrate((d*x+c)^(3/2)*(3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(3*x^2 + 2)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2} \left(2+3x^2\right)^p \, dx = \int (c+dx)^{\frac{3}{2}} \left(3x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)**(3/2)*(3*x**2+2)**p,x)
```

output
```
Integral((c + d*x)**(3/2)*(3*x**2 + 2)**p, x)
```

Maxima [F]

$$\int (c+dx)^{3/2} \left(2+3x^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2} \left(2+3x^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \left(2+3x^2\right)^p \, dx = \int \left(3\,x^2+2\right)^p\,(c+d\,x)^{3/2}\,dx$$

input
```
int((3*x^2 + 2)^p*(c + d*x)^(3/2),x)
```

output
```
int((3*x^2 + 2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2} \left(2+3x^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^(3/2)*(3*x^2+2)^p,x)
```

output

```
(24*sqrt(c + d*x)*(3*x**2 + 2)**p*c*p*x + 36*sqrt(c + d*x)*(3*x**2 + 2)**p
*c*x + 24*sqrt(c + d*x)*(3*x**2 + 2)**p*d*p*x**2 + 32*sqrt(c + d*x)*(3*x**
2 + 2)**p*d*p + 18*sqrt(c + d*x)*(3*x**2 + 2)**p*d*x**2 + 36*sqrt(c + d*x)
*(3*x**2 + 2)**p*d + 432*int((sqrt(c + d*x)*(3*x**2 + 2)**p*x**2)/(48*c*p*
*2*x**2 + 32*c*p**2 + 96*c*p*x**2 + 64*c*p + 45*c*x**2 + 30*c + 48*d*p**2*
x**3 + 32*d*p**2*x + 96*d*p*x**3 + 64*d*p*x + 45*d*x**3 + 30*d*x),x)*c**2*
p**2 + 864*int((sqrt(c + d*x)*(3*x**2 + 2)**p*x**2)/(48*c*p**2*x**2 + 32*c
*p**2 + 96*c*p*x**2 + 64*c*p + 45*c*x**2 + 30*c + 48*d*p**2*x**3 + 32*d*p*
*2*x + 96*d*p*x**3 + 64*d*p*x + 45*d*x**3 + 30*d*x),x)*c**2*p + 405*int((s
qrt(c + d*x)*(3*x**2 + 2)**p*x**2)/(48*c*p**2*x**2 + 32*c*p**2 + 96*c*p*x*
*2 + 64*c*p + 45*c*x**2 + 30*c + 48*d*p**2*x**3 + 32*d*p**2*x + 96*d*p*x**
3 + 64*d*p*x + 45*d*x**3 + 30*d*x),x)*c**2 - 1536*int((sqrt(c + d*x)*(3*x*
*2 + 2)**p*x**2)/(48*c*p**2*x**2 + 32*c*p**2 + 96*c*p*x**2 + 64*c*p + 45*c
*x**2 + 30*c + 48*d*p**2*x**3 + 32*d*p**2*x + 96*d*p*x**3 + 64*d*p*x + 45*
d*x**3 + 30*d*x),x)*d**2*p**4 - 6144*int((sqrt(c + d*x)*(3*x**2 + 2)**p*x*
*2)/(48*c*p**2*x**2 + 32*c*p**2 + 96*c*p*x**2 + 64*c*p + 45*c*x**2 + 30*c
+ 48*d*p**2*x**3 + 32*d*p**2*x + 96*d*p*x**3 + 64*d*p*x + 45*d*x**3 + 30*d
*x),x)*d**2*p**3 - 8448*int((sqrt(c + d*x)*(3*x**2 + 2)**p*x**2)/(48*c*p**
2*x**2 + 32*c*p**2 + 96*c*p*x**2 + 64*c*p + 45*c*x**2 + 30*c + 48*d*p**2*x
**3 + 32*d*p**2*x + 96*d*p*x**3 + 64*d*p*x + 45*d*x**3 + 30*d*x),x)*d**...
```

3.447 $\int \sqrt{c+dx}(2+3x^2)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \sqrt{c+dx}\left(2+3x^2\right)^p\,dx = \frac{2(c+dx)^{3/2}\left(2+3x^2\right)^p\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{3d}$$

output
```
2/3*(d*x+c)^(3/2)*(3*x^2+2)^p*AppellF1(3/2,-p,-p,5/2,3*(d*x+c)/(3*c-I*6^(1
/2)*d),3*(d*x+c)/(3*c+I*6^(1/2)*d))/d/((1-3*(d*x+c)/(3*c-I*6^(1/2)*d))^p)/
((1-3*(d*x+c)/(3*c+I*6^(1/2)*d))^p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.04

$$\int \sqrt{c+dx}\left(2+3x^2\right)^p\,dx = \frac{2\left(\frac{d\left(\sqrt{6}-3ix\right)}{3ic+\sqrt{6}d}\right)^{-p}\left(\frac{d\left(\sqrt{6}+3ix\right)}{-3ic+\sqrt{6}d}\right)^{-p}(c+dx)^{3/2}\left(2+3x^2\right)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{3d}$$

input
```
Integrate[Sqrt[c + d*x]*(2 + 3*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(2 + 3*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (3*(c + d*x))/
(3*c - I*Sqrt[6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(3*d*((d*(Sqrt[6]
 - (3*I)*x))/((3*I)*c + Sqrt[6]*d))^p*((d*(Sqrt[6] + (3*I)*x))/((-3*I)*c +
 Sqrt[6]*d))^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2)^p \sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(3x^2+2)^p \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \int \sqrt{c+dx}\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^p \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{2(3x^2+2)^p (c+dx)^{3/2} \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{3d}$$

input
```
Int[Sqrt[c + d*x]*(2 + 3*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(2 + 3*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (3*(c + d*x))/
(3*c - I*Sqrt[6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(3*d*(1 - (3*(c +
 d*x))/(3*c - I*Sqrt[6]*d))^p*(1 - (3*(c + d*x))/(3*c + I*Sqrt[6]*d))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \sqrt{dx+c}\,(3x^2+2)^p\,dx$$

input
```
int((d*x+c)^(1/2)*(3*x^2+2)^p,x)
```

output
```
int((d*x+c)^(1/2)*(3*x^2+2)^p,x)
```

Fricas [F]

$$\int \sqrt{c+dx}(2+3x^2)^p\,dx = \int \sqrt{dx+c}(3\,x^2+2)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(3*x^2 + 2)^p, x)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(2+3x^2\right)^p\,dx=\int \sqrt{c+dx}\left(3x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)**(1/2)*(3*x**2+2)**p,x)
```

output
```
Integral(sqrt(c + d*x)*(3*x**2 + 2)**p, x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(2+3x^2\right)^p\,dx=\int \sqrt{dx+c}\left(3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*(3*x^2 + 2)^p, x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(2+3x^2\right)^p\,dx=\int \sqrt{dx+c}\left(3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*(3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(2+3x^2\right)^p\,dx=\int {\left(3\,x^2+2\right)}^p\,\sqrt{c+d\,x}\,dx$$

input
```
int((3*x^2 + 2)^p*(c + d*x)^(1/2),x)
```

output
```
int((3*x^2 + 2)^p*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(2+3x^2\right)^p\,dx$$
$$=\frac{6\sqrt{dx+c}\,(3x^2+2)^p\,cx+4\sqrt{dx+c}\,(3x^2+2)^p\,d+36\left(\int \frac{\sqrt{dx+c}\,(3x^2+2)^p\,x^2}{12dp\,x^3+12cp\,x^2+9d\,x^3+9c\,x^2+8dpx+8cp+6dx+6c}dx\right)c^2p$$

input
```
int((d*x+c)^(1/2)*(3*x^2+2)^p,x)
```

output
```
(6*sqrt(c + d*x)*(3*x**2 + 2)**p*c*x + 4*sqrt(c + d*x)*(3*x**2 + 2)**p*d +
 36*int((sqrt(c + d*x)*(3*x**2 + 2)**p*x**2)/(12*c*p*x**2 + 8*c*p + 9*c*x*
*2 + 6*c + 12*d*p*x**3 + 8*d*p*x + 9*d*x**3 + 6*d*x),x)*c**2*p + 27*int((s
qrt(c + d*x)*(3*x**2 + 2)**p*x**2)/(12*c*p*x**2 + 8*c*p + 9*c*x**2 + 6*c +
 12*d*p*x**3 + 8*d*p*x + 9*d*x**3 + 6*d*x),x)*c**2 - 96*int((sqrt(c + d*x)
*(3*x**2 + 2)**p*x**2)/(12*c*p*x**2 + 8*c*p + 9*c*x**2 + 6*c + 12*d*p*x**3
 + 8*d*p*x + 9*d*x**3 + 6*d*x),x)*d**2*p**2 - 96*int((sqrt(c + d*x)*(3*x**
2 + 2)**p*x**2)/(12*c*p*x**2 + 8*c*p + 9*c*x**2 + 6*c + 12*d*p*x**3 + 8*d*
p*x + 9*d*x**3 + 6*d*x),x)*d**2*p - 18*int((sqrt(c + d*x)*(3*x**2 + 2)**p*
x**2)/(12*c*p*x**2 + 8*c*p + 9*c*x**2 + 6*c + 12*d*p*x**3 + 8*d*p*x + 9*d*
x**3 + 6*d*x),x)*d**2 + 96*int((sqrt(c + d*x)*(3*x**2 + 2)**p)/(12*c*p*x**
2 + 8*c*p + 9*c*x**2 + 6*c + 12*d*p*x**3 + 8*d*p*x + 9*d*x**3 + 6*d*x),x)*
c**2*p**2 + 96*int((sqrt(c + d*x)*(3*x**2 + 2)**p)/(12*c*p*x**2 + 8*c*p +
9*c*x**2 + 6*c + 12*d*p*x**3 + 8*d*p*x + 9*d*x**3 + 6*d*x),x)*c**2*p + 18*
int((sqrt(c + d*x)*(3*x**2 + 2)**p)/(12*c*p*x**2 + 8*c*p + 9*c*x**2 + 6*c
+ 12*d*p*x**3 + 8*d*p*x + 9*d*x**3 + 6*d*x),x)*c**2 - 16*int((sqrt(c + d*x
)*(3*x**2 + 2)**p)/(12*c*p*x**2 + 8*c*p + 9*c*x**2 + 6*c + 12*d*p*x**3 + 8
*d*p*x + 9*d*x**3 + 6*d*x),x)*d**2*p - 12*int((sqrt(c + d*x)*(3*x**2 + 2)*
*p)/(12*c*p*x**2 + 8*c*p + 9*c*x**2 + 6*c + 12*d*p*x**3 + 8*d*p*x + 9*d*x*
*3 + 6*d*x),x)*d**2)/(3*c*(4*p + 3))
```

3.448 $\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}(2+3x^2)^p\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{d}$$

output
```
2*(d*x+c)^(1/2)*(3*x^2+2)^p*AppellF1(1/2,-p,-p,3/2,3*(d*x+c)/(3*c-I*6^(1/2
)*d),3*(d*x+c)/(3*c+I*6^(1/2)*d))/d/((1-3*(d*x+c)/(3*c-I*6^(1/2)*d))^p)/((
1-3*(d*x+c)/(3*c+I*6^(1/2)*d))^p)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\left(\frac{d\left(\sqrt{6}-3ix\right)}{3ic+\sqrt{6}d}\right)^{-p}\left(\frac{d\left(\sqrt{6}+3ix\right)}{-3ic+\sqrt{6}d}\right)^{-p}\sqrt{c+dx}(2+3x^2)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{d}$$

input
```
Integrate[(2 + 3*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(2 + 3*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (3*(c + d*x))/(3
*c - I*Sqrt[6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(d*((d*(Sqrt[6] - (
3*I)*x))/((3*I)*c + Sqrt[6]*d))^p*((d*(Sqrt[6] + (3*I)*x))/((-3*I)*c + Sqr
t[6]*d))^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{514, 150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)^p}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 514}$$

$$\frac{(3x^2+2)^p\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p}\int \frac{\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^p\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^p}{\sqrt{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{2(3x^2+2)^p\sqrt{c+dx}\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{d}$$

input
```
Int[(2 + 3*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(2 + 3*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (3*(c + d*x))/(3
*c - I*Sqrt[6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(d*(1 - (3*(c + d*x
))/(3*c - I*Sqrt[6]*d))^p*(1 - (3*(c + d*x))/(3*c + I*Sqrt[6]*d))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(3x^2+2)^p}{\sqrt{dx+c}}dx$$

input
```
int((3*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((3*x^2+2)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(3\,x^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(3x^2+2)^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((3*x**2+2)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(3\,x^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(3\,x^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(3\,x^2+2)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((3*x^2 + 2)^p/(c + d*x)^(1/2),x)
```

output
```
int((3*x^2 + 2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2+3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{dx+c}\,(3x^2+2)^p}{dx+c}dx$$

input
```
int((3*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*(3*x**2 + 2)**p)/(c + d*x),x)
```

3.449 $\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2(2+3x^2)^p\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{d\sqrt{c+dx}}$$

output
```
-2*(3*x^2+2)^p*AppellF1(-1/2,-p,-p,1/2,3*(d*x+c)/(3*c-I*6^(1/2)*d),3*(d*x+
c)/(3*c+I*6^(1/2)*d))/d/(d*x+c)^(1/2)/((1-3*(d*x+c)/(3*c-I*6^(1/2)*d))^p)/
((1-3*(d*x+c)/(3*c+I*6^(1/2)*d))^p)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2\left(\frac{d(\sqrt{6}-3ix)}{3ic+\sqrt{6}d}\right)^{-p}\left(\frac{d(\sqrt{6}+3ix)}{-3ic+\sqrt{6}d}\right)^{-p}(2+3x^2)^p\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{d\sqrt{c+dx}}$$

input
```
Integrate[(2 + 3*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(2 + 3*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (3*(c + d*x))/(3*c - I*Sqrt[
6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(d*((d*(Sqrt[6] - (3*I)*x))/((3
*I)*c + Sqrt[6]*d))^p*((d*(Sqrt[6] + (3*I)*x))/((-3*I)*c + Sqrt[6]*d))^p*S
qrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)^p}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(3x^2+2)^p\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p}\int \frac{\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^p\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^p}{(c+dx)^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2(3x^2+2)^p\left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{d\sqrt{c+dx}}$$

input
```
Int[(2 + 3*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(2 + 3*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (3*(c + d*x))/(3*c - I*Sqrt[
6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(d*Sqrt[c + d*x]*(1 - (3*(c + d
*x))/(3*c - I*Sqrt[6]*d))^p*(1 - (3*(c + d*x))/(3*c + I*Sqrt[6]*d))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(3x^2+2)^p}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((3*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((3*x^2+2)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx=\int \frac{(3\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(3*x^2 + 2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(3x^2+2)^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x**2+2)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((3*x**2 + 2)**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(3\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(3\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(3\,x^2+2)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)^p/(c + d*x)^(3/2),x)
```

output
```
int((3*x^2 + 2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2+3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,(3x^2+2)^p}{d^2x^2+2cdx+c^2}dx$$

input
```
int((3*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*(3*x**2 + 2)**p)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.450 $\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx = \frac{2(c+dx)^{5/2}\left(a-bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{5d}$$

output
```
2/5*(d*x+c)^(5/2)*(-b*x^2+a)^p*AppellF1(5/2,-p,-p,7/2,(d*x+c)/(c-a^(1/2)*d
/b^(1/2)),(d*x+c)/(c+a^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-a^(1/2)*d/b^(1/2
)))^p)/((1-(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx = \frac{2\left(\frac{d\left(\sqrt{\frac{ab}{d^2}}d-bx\right)}{bc+\sqrt{\frac{ab}{d^2}}d^2}\right)^{-p} \left(\frac{d\left(\sqrt{\frac{ab}{d^2}}d+bx\right)}{-bc+\sqrt{\frac{ab}{d^2}}d^2}\right)^{-p} (c+dx)^{5/2}\left(a-bx^2\right)^p \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{b(c+dx)}{bc+\sqrt{\frac{ab}{d^2}}d^2},\ldots\right)}{5d}$$

input
```
Integrate[(c + d*x)^(3/2)*(a - b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(a - b*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (b*(c + d*x))/
(b*c + Sqrt[(a*b)/d^2]*d^2), (b*(c + d*x))/(b*c - Sqrt[(a*b)/d^2]*d^2)])/(
5*d*((d*(Sqrt[(a*b)/d^2]*d - b*x))/(b*c + Sqrt[(a*b)/d^2]*d^2))^p*((d*(Sqr
t[(a*b)/d^2]*d + b*x))/(-(b*c) + Sqrt[(a*b)/d^2]*d^2))^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a-bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^{3/2} \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{2(c+dx)^{5/2} \left(a-bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p} \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{5d}$$

input
```
Int[(c + d*x)^(3/2)*(a - b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(a - b*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (c + d*x)/(c -
 (Sqrt[a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])])/(5*d*(1 - (c
+ d*x)/(c - (Sqrt[a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b
]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(-b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^(3/2)*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(3/2)*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \,dx = \int (dx+c)^{\frac{3}{2}} \left(-bx^2+a\right)^p dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(-b*x^2 + a)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx = \int \left(a-bx^2\right)^p (c+dx)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x+c)**(3/2)*(-b*x**2+a)**p,x)
```

output
```
Integral((a - b*x**2)**p*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(-bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(-b*x^2 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(-bx^2+a\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(-b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx = \int \left(a-b\,x^2\right)^p \, (c+d\,x)^{3/2} \, dx$$

input
```
int((a - b*x^2)^p*(c + d*x)^(3/2),x)
```

output
```
int((a - b*x^2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2} \left(a-bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^(3/2)*(-b*x^2+a)^p,x)
```

output
```
( - 16*sqrt(c + d*x)*(a - b*x**2)**p*a*d*p - 18*sqrt(c + d*x)*(a - b*x**2)
**p*a*d + 8*sqrt(c + d*x)*(a - b*x**2)**p*b*c*p*x + 12*sqrt(c + d*x)*(a -
b*x**2)**p*b*c*x + 8*sqrt(c + d*x)*(a - b*x**2)**p*b*d*p*x**2 + 6*sqrt(c +
 d*x)*(a - b*x**2)**p*b*d*x**2 - 256*int((sqrt(c + d*x)*(a - b*x**2)**p*x*
*2)/(16*a*c*p**2 + 32*a*c*p + 15*a*c + 16*a*d*p**2*x + 32*a*d*p*x + 15*a*d
*x - 16*b*c*p**2*x**2 - 32*b*c*p*x**2 - 15*b*c*x**2 - 16*b*d*p**2*x**3 - 3
2*b*d*p*x**3 - 15*b*d*x**3),x)*a*b*d**2*p**4 - 1024*int((sqrt(c + d*x)*(a
- b*x**2)**p*x**2)/(16*a*c*p**2 + 32*a*c*p + 15*a*c + 16*a*d*p**2*x + 32*a
*d*p*x + 15*a*d*x - 16*b*c*p**2*x**2 - 32*b*c*p*x**2 - 15*b*c*x**2 - 16*b*
d*p**2*x**3 - 32*b*d*p*x**3 - 15*b*d*x**3),x)*a*b*d**2*p**3 - 1408*int((sq
rt(c + d*x)*(a - b*x**2)**p*x**2)/(16*a*c*p**2 + 32*a*c*p + 15*a*c + 16*a*
d*p**2*x + 32*a*d*p*x + 15*a*d*x - 16*b*c*p**2*x**2 - 32*b*c*p*x**2 - 15*b
*c*x**2 - 16*b*d*p**2*x**3 - 32*b*d*p*x**3 - 15*b*d*x**3),x)*a*b*d**2*p**2
 - 768*int((sqrt(c + d*x)*(a - b*x**2)**p*x**2)/(16*a*c*p**2 + 32*a*c*p +
15*a*c + 16*a*d*p**2*x + 32*a*d*p*x + 15*a*d*x - 16*b*c*p**2*x**2 - 32*b*c
*p*x**2 - 15*b*c*x**2 - 16*b*d*p**2*x**3 - 32*b*d*p*x**3 - 15*b*d*x**3),x)
*a*b*d**2*p - 135*int((sqrt(c + d*x)*(a - b*x**2)**p*x**2)/(16*a*c*p**2 +
32*a*c*p + 15*a*c + 16*a*d*p**2*x + 32*a*d*p*x + 15*a*d*x - 16*b*c*p**2*x*
*2 - 32*b*c*p*x**2 - 15*b*c*x**2 - 16*b*d*p**2*x**3 - 32*b*d*p*x**3 - 15*b
*d*x**3),x)*a*b*d**2 - 48*int((sqrt(c + d*x)*(a - b*x**2)**p*x**2)/(16*...
```

3.451 $\int \sqrt{c+dx}\left(a-bx^2\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \sqrt{c+dx}\left(a-bx^2\right)^p\,dx = \frac{2(c+dx)^{3/2}\left(a-bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{3d}$$

output
```
2/3*(d*x+c)^(3/2)*(-b*x^2+a)^p*AppellF1(3/2,-p,-p,5/2,(d*x+c)/(c-a^(1/2)*d
/b^(1/2)),(d*x+c)/(c+a^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-a^(1/2)*d/b^(1/2
)))^p)/((1-(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05

$$\int \sqrt{c+dx}\left(a-bx^2\right)^p\,dx = \frac{2\left(\frac{d\left(\sqrt{\frac{a}{b}}-x\right)}{c+\sqrt{\frac{a}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{-c+\sqrt{\frac{a}{b}}d}\right)^{-p}(c+dx)^{3/2}\left(a-bx^2\right)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\sqrt{\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{\frac{a}{b}}d}\right)}{3d}$$

input
```
Integrate[Sqrt[c + d*x]*(a - b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(a - b*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (c + d*x)/(c -
 Sqrt[a/b]*d), (c + d*x)/(c + Sqrt[a/b]*d)])/(3*d*((d*(Sqrt[a/b] - x))/(c
+ Sqrt[a/b]*d))^p*((d*(Sqrt[a/b] + x))/(-c + Sqrt[a/b]*d))^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\left(a-bx^2\right)^p\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a-bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{ad}}{\sqrt{b}}+c}\right)^{-p}\int \sqrt{c+dx}\left(1-\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{ad}}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{2(c+dx)^{3/2}\left(a-bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{ad}}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{ad}}{\sqrt{b}}}\right)}{3d}$$

input
```
Int[Sqrt[c + d*x]*(a - b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(a - b*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (c + d*x)/(c -
 (Sqrt[a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])])/(3*d*(1 - (c
+ d*x)/(c - (Sqrt[a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b
]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \sqrt{dx+c}\,(-b\,x^{2}+a)^{p}\,dx$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(1/2)*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int \sqrt{c+dx}(a-bx^2)^p\ dx=\int \sqrt{dx+c}(-bx^2+a)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-b*x^2 + a)^p, x)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^p\,dx=\int \left(a-bx^2\right)^p\sqrt{c+dx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-b*x**2+a)**p,x)
```

output
```
Integral((a - b*x**2)**p*sqrt(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(-bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*(-b*x^2 + a)^p, x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(-bx^2+a\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*(-b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\,(a-bx^2)^p\,dx = \int (a-b\,x^2)^p\,\sqrt{c+d\,x}\,dx$$

input
```
int((a - b*x^2)^p*(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^p*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\,(a-bx^2)^p\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^p,x)
```

output
```
( - 2*sqrt(c + d*x)*(a - b*x**2)**p*a*d + 2*sqrt(c + d*x)*(a - b*x**2)**p*
b*c*x - 16*int((sqrt(c + d*x)*(a - b*x**2)**p*x**2)/(4*a*c*p + 3*a*c + 4*a
*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b*c*x**2 - 4*b*d*p*x**3 - 3*b*d*x**3),
x)*a*b*d**2*p**2 - 16*int((sqrt(c + d*x)*(a - b*x**2)**p*x**2)/(4*a*c*p +
3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b*c*x**2 - 4*b*d*p*x**3 - 3
*b*d*x**3),x)*a*b*d**2*p - 3*int((sqrt(c + d*x)*(a - b*x**2)**p*x**2)/(4*a
*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b*c*x**2 - 4*b*d*p*x
**3 - 3*b*d*x**3),x)*a*b*d**2 - 4*int((sqrt(c + d*x)*(a - b*x**2)**p*x**2)
/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b*c*x**2 - 4*b*
d*p*x**3 - 3*b*d*x**3),x)*b**2*c**2*p - 3*int((sqrt(c + d*x)*(a - b*x**2)*
*p*x**2)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b*c*x**
2 - 4*b*d*p*x**3 - 3*b*d*x**3),x)*b**2*c**2 + 4*int((sqrt(c + d*x)*(a - b*
x**2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b*c*x*
*2 - 4*b*d*p*x**3 - 3*b*d*x**3),x)*a**2*d**2*p + 3*int((sqrt(c + d*x)*(a -
 b*x**2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b*c
*x**2 - 4*b*d*p*x**3 - 3*b*d*x**3),x)*a**2*d**2 + 16*int((sqrt(c + d*x)*(a
 - b*x**2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2 - 3*b
*c*x**2 - 4*b*d*p*x**3 - 3*b*d*x**3),x)*a*b*c**2*p**2 + 16*int((sqrt(c + d
*x)*(a - b*x**2)**p)/(4*a*c*p + 3*a*c + 4*a*d*p*x + 3*a*d*x - 4*b*c*p*x**2
 - 3*b*c*x**2 - 4*b*d*p*x**3 - 3*b*d*x**3),x)*a*b*c**2*p + 3*int((sqrt(...
```

3.452 $\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d}$$

output
```
2*(d*x+c)^(1/2)*(-b*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,(d*x+c)/(c-a^(1/2)*d/b
^(1/2)),(d*x+c)/(c+a^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-a^(1/2)*d/b^(1/2))
)^p)/((1-(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\left(\frac{d\left(\sqrt{\frac{a}{b}}-x\right)}{c+\sqrt{\frac{a}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{-c+\sqrt{\frac{a}{b}}d}\right)^{-p}\sqrt{c+dx}(a-bx^2)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\sqrt{\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{\frac{a}{b}}d}\right)}{d}$$

input
```
Integrate[(a - b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(a - b*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (c + d*x)/(c - S
qrt[a/b]*d), (c + d*x)/(c + Sqrt[a/b]*d)])/(d*((d*(Sqrt[a/b] - x))/(c + Sq
rt[a/b]*d))^p*((d*(Sqrt[a/b] + x))/(-c + Sqrt[a/b]*d))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p}\int \frac{\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p}{\sqrt{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\sqrt{c+dx}(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d}$$

input
```
Int[(a - b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(a - b*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (c + d*x)/(c - (
Sqrt[a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])])/(d*(1 - (c + d*
x)/(c - (Sqrt[a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b]))^
p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^p}{\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+a)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+a)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+a)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(a-b\,x^2)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^(1/2),x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{dx+c}\,(-b\,x^2+a)^p}{dx+c}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*(a - b*x**2)**p)/(c + d*x),x)
```

3.453 $\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d\sqrt{c+dx}}$$

output
```
-2*(-b*x^2+a)^p*AppellF1(-1/2,-p,-p,1/2,(d*x+c)/(c-a^(1/2)*d/b^(1/2)),(d*x
+c)/(c+a^(1/2)*d/b^(1/2)))/d/(d*x+c)^(1/2)/((1-(d*x+c)/(c-a^(1/2)*d/b^(1/2
)))^p)/((1-(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2\left(\frac{d\left(\sqrt{\frac{a}{b}}-x\right)}{c+\sqrt{\frac{a}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{-c+\sqrt{\frac{a}{b}}d}\right)^{-p}(a-bx^2)^p\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\sqrt{\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{\frac{a}{b}}d}\right)}{d\sqrt{c+dx}}$$

input
```
Integrate[(a - b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(a - b*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (c + d*x)/(c - Sqrt[a/b]*d),
 (c + d*x)/(c + Sqrt[a/b]*d)])/(d*((d*(Sqrt[a/b] - x))/(c + Sqrt[a/b]*d))^
p*((d*(Sqrt[a/b] + x))/(-c + Sqrt[a/b]*d))^p*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p}\int \frac{\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p}{(c+dx)^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2(a-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d\sqrt{c+dx}}$$

input
```
Int[(a - b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(a - b*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (c + d*x)/(c - (Sqrt[a]*d)/S
qrt[b]), (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])])/(d*Sqrt[c + d*x]*(1 - (c +
d*x)/(c - (Sqrt[a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])
)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^p}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c)^(3/2),x)
```

output
```
int((-b*x^2+a)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-b*x^2 + a)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(a-bx^2)^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((a - b*x**2)**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx=\int \frac{(a-b\,x^2)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^(3/2),x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^p}{(c+dx)^{3/2}}\,dx=\int \frac{\sqrt{dx+c}\,(-b\,x^2+a)^p}{d^2x^2+2cdx+c^2}dx$$

input
```
int((-b*x^2+a)^p/(d*x+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*(a - b*x**2)**p)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.454 $\int (c+dx)^{3/2} \left(2-bx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int (c+dx)^{3/2} \left(2-bx^2\right)^p \, dx = \frac{2(c+dx)^{5/2} \left(2-bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)}{5d}$$

output
```
2/5*(d*x+c)^(5/2)*(-b*x^2+2)^p*AppellF1(5/2,-p,-p,7/2,(d*x+c)/(c-2^(1/2)*d
/b^(1/2)),(d*x+c)/(c+2^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-2^(1/2)*d/b^(1/2
)))^p)/((1-(d*x+c)/(c+2^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.23

$$\int (c+dx)^{3/2} \left(2-bx^2\right)^p \, dx = \frac{2(c+dx)^{5/2} \left(\frac{\sqrt{2}-\sqrt{\frac{b}{d^2}}dx}{\sqrt{2}+c\sqrt{\frac{b}{d^2}}}\right)^{-p} \left(\frac{\sqrt{2}+\sqrt{\frac{b}{d^2}}dx}{\sqrt{2}-c\sqrt{\frac{b}{d^2}}}\right)^{-p} \left(2-bx^2\right)^p \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{c+dx}{c+\frac{\sqrt{2}}{\sqrt{\frac{b}{d^2}}}},\frac{c+dx}{c-\frac{\sqrt{2}}{\sqrt{\frac{b}{d^2}}}}\right)}{5d}$$

input
```
Integrate[(c + d*x)^(3/2)*(2 - b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(2 - b*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (c + d*x)/(c +
 Sqrt[2]/Sqrt[b/d^2]), (c + d*x)/(c - Sqrt[2]/Sqrt[b/d^2])])/(5*d*((Sqrt[2
] - Sqrt[b/d^2]*d*x)/(Sqrt[2] + c*Sqrt[b/d^2]))^p*((Sqrt[2] + Sqrt[b/d^2]*
d*x)/(Sqrt[2] - c*Sqrt[b/d^2]))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-bx^2\right)^p (c+dx)^{3/2} \, dx$$

$$\Big\downarrow \; 513$$

$$2^p \int \left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p \left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p (c+dx)^{3/2}dx$$

$$\Big\downarrow \; 156$$

$$\frac{2^{p-\frac{1}{4}}\left(\frac{\sqrt{2}d}{\sqrt{b}}+c\right)\sqrt{c+dx}\int \left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p \left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p \left(\frac{\sqrt{2}\sqrt{b}c}{\sqrt{2}\sqrt{b}c+2d}+\frac{\sqrt{2}\sqrt{b}dx}{\sqrt{2}\sqrt{b}c+2d}\right)^{3/2}dx}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}}$$

$$\Big\downarrow \; 155$$

$$-\frac{2^{2p+\frac{1}{4}}\left(\frac{\sqrt{2}d}{\sqrt{b}}+c\right)\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^{p+1}\sqrt{c+dx}\,\text{AppellF1}\left(p+1,-p,-\frac{3}{2},p+2,\frac{\sqrt{2}-\sqrt{b}x}{2\sqrt{2}},\frac{d\left(2\sqrt{2}-2\sqrt{b}x\right)}{2\left(\sqrt{b}c+\sqrt{2}d\right)}\right)}{\sqrt{b}(p+1)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}}$$

input
```
Int[(c + d*x)^(3/2)*(2 - b*x^2)^p,x]
```

output
```
-((2^(1/4 + 2*p)*(c + (Sqrt[2]*d)/Sqrt[b])*(1 - (Sqrt[b]*x)/Sqrt[2])^(1 +
p)*Sqrt[c + d*x]*AppellF1[1 + p, -p, -3/2, 2 + p, (Sqrt[2] - Sqrt[b]*x)/(2
*Sqrt[2]), (d*(2*Sqrt[2] - 2*Sqrt[b]*x))/(2*(Sqrt[b]*c + Sqrt[2]*d))])/(Sq
rt[b]*(1 + p)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[2]*Sqrt[b]*c + 2*d)]))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(-b\,x^{2}+2\right)^{p} dx$$

input
```
int((d*x+c)^(3/2)*(-b*x^2+2)^p,x)
```

output
```
int((d*x+c)^(3/2)*(-b*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2}\left(2-bx^2\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(-b*x^2 + 2)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2}\left(2-bx^2\right)^p\,dx=\int (c+dx)^{\frac{3}{2}}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)**(3/2)*(-b*x**2+2)**p,x)
```

output
```
Integral((c + d*x)**(3/2)*(-b*x**2 + 2)**p, x)
```

Maxima [F]

$$\int (c+dx)^{3/2}\left(2-bx^2\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(-b*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2}\left(2-bx^2\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(-b*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(-b*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\left(2-bx^2\right)^p\,dx=\int \left(2-b\,x^2\right)^p\,(c+d\,x)^{3/2}\,dx$$

input
```
int((2 - b*x^2)^p*(c + d*x)^(3/2),x)
```

output
```
int((2 - b*x^2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\left(2-bx^2\right)^p\,dx=\text{too large to display}$$

input
```
int((d*x+c)^(3/2)*(-b*x^2+2)^p,x)
```

output

```
(8*sqrt(c + d*x)*( - b*x**2 + 2)**p*b*c*p*x + 12*sqrt(c + d*x)*( - b*x**2
+ 2)**p*b*c*x + 8*sqrt(c + d*x)*( - b*x**2 + 2)**p*b*d*p*x**2 + 6*sqrt(c +
 d*x)*( - b*x**2 + 2)**p*b*d*x**2 - 32*sqrt(c + d*x)*( - b*x**2 + 2)**p*d*
p - 36*sqrt(c + d*x)*( - b*x**2 + 2)**p*d + 48*int((sqrt(c + d*x)*( - b*x*
*2 + 2)**p*x**2)/(16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16*b*d*
p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3 - 32*c*p**2 - 64*c*p - 30*c - 32*d
*p**2*x - 64*d*p*x - 30*d*x),x)*b**2*c**2*p**2 + 96*int((sqrt(c + d*x)*( -
 b*x**2 + 2)**p*x**2)/(16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 + 16
*b*d*p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3 - 32*c*p**2 - 64*c*p - 30*c -
 32*d*p**2*x - 64*d*p*x - 30*d*x),x)*b**2*c**2*p + 45*int((sqrt(c + d*x)*(
 - b*x**2 + 2)**p*x**2)/(16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 +
16*b*d*p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3 - 32*c*p**2 - 64*c*p - 30*c
 - 32*d*p**2*x - 64*d*p*x - 30*d*x),x)*b**2*c**2 + 512*int((sqrt(c + d*x)*
( - b*x**2 + 2)**p*x**2)/(16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x**2 +
 16*b*d*p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3 - 32*c*p**2 - 64*c*p - 30*
c - 32*d*p**2*x - 64*d*p*x - 30*d*x),x)*b*d**2*p**4 + 2048*int((sqrt(c + d
*x)*( - b*x**2 + 2)**p*x**2)/(16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15*b*c*x*
*2 + 16*b*d*p**2*x**3 + 32*b*d*p*x**3 + 15*b*d*x**3 - 32*c*p**2 - 64*c*p -
 30*c - 32*d*p**2*x - 64*d*p*x - 30*d*x),x)*b*d**2*p**3 + 2816*int((sqrt(c
 + d*x)*( - b*x**2 + 2)**p*x**2)/(16*b*c*p**2*x**2 + 32*b*c*p*x**2 + 15...
```

3.455 $\int \sqrt{c+dx}(2-bx^2)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \sqrt{c+dx}(2-bx^2)^p \, dx = \frac{2(c+dx)^{3/2}(2-bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p} \operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)}{3d}$$

output
```
2/3*(d*x+c)^(3/2)*(-b*x^2+2)^p*AppellF1(3/2,-p,-p,5/2,(d*x+c)/(c-2^(1/2)*d
/b^(1/2)),(d*x+c)/(c+2^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-2^(1/2)*d/b^(1/2
)))^p)/((1-(d*x+c)/(c+2^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int \sqrt{c+dx}(2-bx^2)^p \, dx = \frac{2\left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p} (c+dx)^{3/2}(2-bx^2)^p \operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{c+dx}{c-\sqrt{2}\sqrt{\frac{1}{b}}d},\frac{c+dx}{c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)}{3d}$$

input
```
Integrate[Sqrt[c + d*x]*(2 - b*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(2 - b*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (c + d*x)/(c -
 Sqrt[2]*Sqrt[b^(-1)]*d), (c + d*x)/(c + Sqrt[2]*Sqrt[b^(-1)]*d)])/(3*d*((
d*(Sqrt[2]*Sqrt[b^(-1)] - x))/(c + Sqrt[2]*Sqrt[b^(-1)]*d))^p*((d*(Sqrt[2]
*Sqrt[b^(-1)] + x))/(-c + Sqrt[2]*Sqrt[b^(-1)]*d))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-bx^2)^p \sqrt{c+dx}\,dx$$

$$\Big\downarrow \text{513}$$

$$2^p\int \left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p\left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p\sqrt{c+dx}dx$$

$$\Big\downarrow \text{156}$$

$$\frac{2^{p-\frac{1}{4}}\sqrt{c+dx}\int \left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p\left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p\sqrt{\frac{\sqrt{2}\sqrt{b}c}{\sqrt{2}\sqrt{b}c+2d}+\frac{\sqrt{2}\sqrt{b}dx}{\sqrt{2}\sqrt{b}c+2d}}dx}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{2^{2p+\frac{1}{4}}\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^{p+1}\sqrt{c+dx}\,\text{AppellF1}\left(p+1,-p,-\frac{1}{2},p+2,\frac{\sqrt{2}-\sqrt{b}x}{2\sqrt{2}},\frac{d\left(2\sqrt{2}-2\sqrt{b}x\right)}{2\left(\sqrt{b}c+\sqrt{2}d\right)}\right)}{\sqrt{b}(p+1)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}}$$

input
```
Int[Sqrt[c + d*x]*(2 - b*x^2)^p,x]
```

output
```
-((2^(1/4 + 2*p)*(1 - (Sqrt[b]*x)/Sqrt[2])^(1 + p)*Sqrt[c + d*x]*AppellF1[
1 + p, -p, -1/2, 2 + p, (Sqrt[2] - Sqrt[b]*x)/(2*Sqrt[2]), (d*(2*Sqrt[2] -
 2*Sqrt[b]*x))/(2*(Sqrt[b]*c + Sqrt[2]*d))])/(Sqrt[b]*(1 + p)*Sqrt[(Sqrt[b
]*(c + d*x))/(Sqrt[2]*Sqrt[b]*c + 2*d)]))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \sqrt{dx+c}\,(-b\,x^{2}+2)^{p}\,dx$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+2)^p,x)
```

output
```
int((d*x+c)^(1/2)*(-b*x^2+2)^p,x)
```

Fricas [F]

$$\int \sqrt{c+dx}\left(2-bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-b*x^2 + 2)^p, x)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(2-bx^2\right)^p\,dx=\int \sqrt{c+dx}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-b*x**2+2)**p,x)
```

output
```
Integral(sqrt(c + d*x)*(-b*x**2 + 2)**p, x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(2-bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*(-b*x^2 + 2)^p, x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(2-bx^2\right)^p\,dx=\int \sqrt{dx+c}\left(-bx^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*(-b*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(2-bx^2\right)^p\,dx=\int \left(2-b\,x^2\right)^p\,\sqrt{c+d\,x}\,dx$$

input
```
int((2 - b*x^2)^p*(c + d*x)^(1/2),x)
```

output
```
int((2 - b*x^2)^p*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(2-bx^2\right)^p\,dx$$
$$=\frac{2\sqrt{dx+c}\,\left(-b\,x^2+2\right)^p bcx-4\sqrt{dx+c}\,\left(-b\,x^2+2\right)^p d+4\left(\int \frac{\sqrt{dx+c}\,\left(-b\,x^2+2\right)^p x^2}{4bdp\,x^3+4bcp\,x^2+3bd\,x^3+3bc\,x^2-8dpx-8cp-6dx-6c}dx\right.}{}$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+2)^p,x)
```

output
```
(2*sqrt(c + d*x)*( - b*x**2 + 2)**p*b*c*x - 4*sqrt(c + d*x)*( - b*x**2 + 2
)**p*d + 4*int((sqrt(c + d*x)*( - b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2 + 3*b
*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d*x),x)*b*
*2*c**2*p + 3*int((sqrt(c + d*x)*( - b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2 +
3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d*x),x)
*b**2*c**2 + 32*int((sqrt(c + d*x)*( - b*x**2 + 2)**p*x**2)/(4*b*c*p*x**2
+ 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d*x),
x)*b*d**2*p**2 + 32*int((sqrt(c + d*x)*( - b*x**2 + 2)**p*x**2)/(4*b*c*p*x
**2 + 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d
*x),x)*b*d**2*p + 6*int((sqrt(c + d*x)*( - b*x**2 + 2)**p*x**2)/(4*b*c*p*x
**2 + 3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d
*x),x)*b*d**2 - 32*int((sqrt(c + d*x)*( - b*x**2 + 2)**p)/(4*b*c*p*x**2 +
3*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d*x),x)
*b*c**2*p**2 - 32*int((sqrt(c + d*x)*( - b*x**2 + 2)**p)/(4*b*c*p*x**2 + 3
*b*c*x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d*x),x)*
b*c**2*p - 6*int((sqrt(c + d*x)*( - b*x**2 + 2)**p)/(4*b*c*p*x**2 + 3*b*c*
x**2 + 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d*x),x)*b*c**
2 - 16*int((sqrt(c + d*x)*( - b*x**2 + 2)**p)/(4*b*c*p*x**2 + 3*b*c*x**2 +
 4*b*d*p*x**3 + 3*b*d*x**3 - 8*c*p - 6*c - 8*d*p*x - 6*d*x),x)*d**2*p - 12
*int((sqrt(c + d*x)*( - b*x**2 + 2)**p)/(4*b*c*p*x**2 + 3*b*c*x**2 + 4*...
```

3.456 $\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}(2-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)}{d}$$

output
```
2*(d*x+c)^(1/2)*(-b*x^2+2)^p*AppellF1(1/2,-p,-p,3/2,(d*x+c)/(c-2^(1/2)*d/b
^(1/2)),(d*x+c)/(c+2^(1/2)*d/b^(1/2)))/d/((1-(d*x+c)/(c-2^(1/2)*d/b^(1/2))
)^p)/((1-(d*x+c)/(c+2^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.19

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p}\sqrt{c+dx}(2-bx^2)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{c+dx}{c-\sqrt{2}\sqrt{\frac{1}{b}}d},\frac{c+dx}{c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)}{d}$$

input
```
Integrate[(2 - b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(2 - b*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (c + d*x)/(c - S
qrt[2]*Sqrt[b^(-1)]*d), (c + d*x)/(c + Sqrt[2]*Sqrt[b^(-1)]*d)])/(d*((d*(S
qrt[2]*Sqrt[b^(-1)] - x))/(c + Sqrt[2]*Sqrt[b^(-1)]*d))^p*((d*(Sqrt[2]*Sqr
t[b^(-1)] + x))/(-c + Sqrt[2]*Sqrt[b^(-1)]*d))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2-bx^2\right)^p}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{513}$$

$$2^p\int \frac{\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p\left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{156}$$

$$\frac{2^{p+\frac{1}{4}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}\int \frac{\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p\left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p}{\sqrt{\frac{\sqrt{2}\sqrt{b}c}{\sqrt{2}\sqrt{b}c+2d}+\frac{\sqrt{2}\sqrt{b}dx}{\sqrt{2}\sqrt{b}c+2d}}}dx}{\sqrt{c+dx}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{2^{2p+\frac{3}{4}}\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^{p+1}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}\text{AppellF1}\left(p+1,-p,\frac{1}{2},p+2,\frac{\sqrt{2}-\sqrt{b}x}{2\sqrt{2}},\frac{d\left(2\sqrt{2}-2\sqrt{b}x\right)}{2\left(\sqrt{b}c+\sqrt{2}d\right)}\right)}{\sqrt{b}(p+1)\sqrt{c+dx}}$$

input
```
Int[(2 - b*x^2)^p/Sqrt[c + d*x],x]
```

output
```
-((2^(3/4 + 2*p)*(1 - (Sqrt[b]*x)/Sqrt[2])^(1 + p)*Sqrt[(Sqrt[b]*(c + d*x)
)/(Sqrt[2]*Sqrt[b]*c + 2*d)]*AppellF1[1 + p, -p, 1/2, 2 + p, (Sqrt[2] - Sq
rt[b]*x)/(2*Sqrt[2]), (d*(2*Sqrt[2] - 2*Sqrt[b]*x))/(2*(Sqrt[b]*c + Sqrt[2
]*d))]])/(Sqrt[b]*(1 + p)*Sqrt[c + d*x]))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(-b\,x^2+2)^p}{\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+2)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + 2)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+2)^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+2)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((-b*x**2 + 2)**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + 2)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-bx^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + 2)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(2-b\,x^2)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((2 - b*x^2)^p/(c + d*x)^(1/2),x)
```

output
```
int((2 - b*x^2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2-bx^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{dx+c}\,(-b\,x^2+2)^p}{dx+c}dx$$

input
```
int((-b*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*( - b*x**2 + 2)**p)/(c + d*x),x)
```

3.457 $\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2(2-bx^2)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)}{d\sqrt{c+dx}}$$

output

```
-2*(-b*x^2+2)^p*AppellF1(-1/2,-p,-p,1/2,(d*x+c)/(c-2^(1/2)*d/b^(1/2)),(d*x
+c)/(c+2^(1/2)*d/b^(1/2)))/d/(d*x+c)^(1/2)/((1-(d*x+c)/(c-2^(1/2)*d/b^(1/2
)))^p)/((1-(d*x+c)/(c+2^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.19

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2\left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p}\left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p}(2-bx^2)^p\,\text{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{c+dx}{c-\sqrt{2}\sqrt{\frac{1}{b}}d},\frac{c+dx}{c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)}{d\sqrt{c+dx}}$$

input
```
Integrate[(2 - b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(2 - b*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (c + d*x)/(c - Sqrt[2]*Sqrt[
b^(-1)]*d), (c + d*x)/(c + Sqrt[2]*Sqrt[b^(-1)]*d)])/(d*((d*(Sqrt[2]*Sqrt[
b^(-1)] - x))/(c + Sqrt[2]*Sqrt[b^(-1)]*d))^p*((d*(Sqrt[2]*Sqrt[b^(-1)] +
x))/(-c + Sqrt[2]*Sqrt[b^(-1)]*d))^p*Sqrt[c + d*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}dx$$
$$\Big\downarrow \ 513$$
$$2^p\int \frac{\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p\left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p}{(c+dx)^{3/2}}dx$$
$$\Big\downarrow \ 156$$
$$\frac{\sqrt{b}2^{p+\frac{3}{4}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}\int \frac{\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p\left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p}{\left(\frac{\sqrt{2}\sqrt{b}c}{\sqrt{2}\sqrt{b}c+2d}+\frac{\sqrt{2}\sqrt{b}dx}{\sqrt{2}\sqrt{b}c+2d}\right)^{3/2}}dx}{\left(\sqrt{2}\sqrt{b}c+2d\right)\sqrt{c+dx}}$$
$$\Big\downarrow \ 155$$
$$-\frac{2^{2p+\frac{5}{4}}\left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^{p+1}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}}\text{AppellF1}\left(p+1,-p,\frac{3}{2},p+2,\frac{\sqrt{2}-\sqrt{b}x}{2\sqrt{2}},\frac{d\left(2\sqrt{2}-2\sqrt{b}x\right)}{2\left(\sqrt{b}c+\sqrt{2}d\right)}\right)}{(p+1)\left(\sqrt{2}\sqrt{b}c+2d\right)\sqrt{c+dx}}$$

input
```
Int[(2 - b*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
-((2^(5/4 + 2*p)*(1 - (Sqrt[b]*x)/Sqrt[2])^(1 + p)*Sqrt[(Sqrt[b]*(c + d*x)
)/(Sqrt[2]*Sqrt[b]*c + 2*d)]*AppellF1[1 + p, -p, 3/2, 2 + p, (Sqrt[2] - Sq
rt[b]*x)/(2*Sqrt[2]), (d*(2*Sqrt[2] - 2*Sqrt[b]*x))/(2*(Sqrt[b]*c + Sqrt[2
]*d))])/((Sqrt[2]*Sqrt[b]*c + 2*d)*(1 + p)*Sqrt[c + d*x]))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(-b\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((-b*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((-b*x^2+2)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-b*x^2 + 2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+2)^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x**2+2)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((-b*x**2 + 2)**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-bx^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(2-b\,x^2)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((2 - b*x^2)^p/(c + d*x)^(3/2),x)
```

output
```
int((2 - b*x^2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-bx^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,(-b\,x^2+2)^p}{d^2x^2+2cdx+c^2}dx$$

input
```
int((-b*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*( - b*x**2 + 2)**p)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.458 $\int (c+dx)^{3/2} (2-3x^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int (c+dx)^{3/2} (2-3x^2)^p \, dx = \frac{2(c+dx)^{5/2} (2-3x^2)^p \left(1-\frac{3(c+dx)}{3c-\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+\sqrt{6}d}\right)^{-p} \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{5d}$$

output
```
2/5*(d*x+c)^(5/2)*(-3*x^2+2)^p*AppellF1(5/2,-p,-p,7/2,3*(d*x+c)/(3*c-6^(1/
2)*d),3*(d*x+c)/(6^(1/2)*d+3*c))/d/((1-3*(d*x+c)/(3*c-6^(1/2)*d))^p)/((1-3
*(d*x+c)/(6^(1/2)*d+3*c))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.59 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.38

$$\int (c+dx)^{3/2} (2-3x^2)^p \, dx = \frac{2(c+dx)^{5/2} \left(\frac{\sqrt{6}-3\sqrt{\frac{1}{d^2}}dx}{\sqrt{6}+3c\sqrt{\frac{1}{d^2}}}\right)^{-p} \left(\frac{\sqrt{6}+3\sqrt{\frac{1}{d^2}}dx}{\sqrt{6}-3c\sqrt{\frac{1}{d^2}}}\right)^{-p} (2-3x^2)^p \operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{3\sqrt{\frac{1}{d^2}}(c+dx)}{\sqrt{6}+3c\sqrt{\frac{1}{d^2}}}, \ldots\right)}{5d}$$

input
```
Integrate[(c + d*x)^(3/2)*(2 - 3*x^2)^p,x]
```

output
```
(2*(c + d*x)^(5/2)*(2 - 3*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (3*Sqrt[d^(-2)
]*(c + d*x))/(Sqrt[6] + 3*c*Sqrt[d^(-2)]), (-3*Sqrt[d^(-2)]*(c + d*x))/(Sq
rt[6] - 3*c*Sqrt[d^(-2)])])/(5*d*((Sqrt[6] - 3*Sqrt[d^(-2)]*d*x)/(Sqrt[6]
+ 3*c*Sqrt[d^(-2)]))^p*((Sqrt[6] + 3*Sqrt[d^(-2)]*d*x)/(Sqrt[6] - 3*c*Sqrt
[d^(-2)]))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-3x^2\right)^p (c+dx)^{3/2} \, dx$$

$$\Big\downarrow \text{513}$$

$$2^p \int \left(1-\sqrt{\frac{3}{2}}x\right)^p \left(\sqrt{\frac{3}{2}}x+1\right)^p (c+dx)^{3/2}dx$$

$$\Big\downarrow \text{156}$$

$$\frac{2^{p-\frac{1}{4}}\left(\sqrt{3}c+\sqrt{2}d\right)\sqrt{c+dx}\int \left(1-\sqrt{\frac{3}{2}}x\right)^p\left(\sqrt{\frac{3}{2}}x+1\right)^p\left(\frac{\sqrt{6}c}{\sqrt{6}c+2d}+\frac{\sqrt{6}dx}{\sqrt{6}c+2d}\right)^{3/2}dx}{3^{3/4}\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{2^{2p+\frac{1}{4}}\left(\sqrt{3}c+\sqrt{2}d\right)\left(1-\sqrt{\frac{3}{2}}x\right)^{p+1}\sqrt{c+dx}\,\text{AppellF1}\left(p+1,-p,-\frac{3}{2},p+2,\frac{1}{4}\left(2-\sqrt{6}x\right),\frac{\sqrt{\frac{3}{2}}d\left(2-\sqrt{6}x\right)}{3c+\sqrt{6}d}\right)}{3\sqrt[4]{3}(p+1)\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}}$$

input
```
Int[(c + d*x)^(3/2)*(2 - 3*x^2)^p,x]
```

output
```
-1/3*(2^(1/4 + 2*p)*(Sqrt[3]*c + Sqrt[2]*d)*(1 - Sqrt[3/2]*x)^(1 + p)*Sqrt
[c + d*x]*AppellF1[1 + p, -p, -3/2, 2 + p, (2 - Sqrt[6]*x)/4, (Sqrt[3/2]*d
*(2 - Sqrt[6]*x))/(3*c + Sqrt[6]*d)])/(3^(1/4)*(1 + p)*Sqrt[(c + d*x)/(Sqr
t[6]*c + 2*d)])
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int (dx + c)^{\frac{3}{2}} \left(-3x^2 + 2\right)^p dx$$

input
```
int((d*x+c)^(3/2)*(-3*x^2+2)^p,x)
```

output
```
int((d*x+c)^(3/2)*(-3*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2} \left(2-3x^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(-3*x^2 + 2)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2} \left(2-3x^2\right)^p \, dx = \int \left(2-3x^2\right)^p (c+dx)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x+c)**(3/2)*(-3*x**2+2)**p,x)
```

output
```
Integral((2 - 3*x**2)**p*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2} \left(2-3x^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(-3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2} \left(2-3x^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(-3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \left(2-3x^2\right)^p \, dx = \int \left(2-3\,x^2\right)^p \left(c+d\,x\right)^{3/2} \, dx$$

input
```
int((2 - 3*x^2)^p*(c + d*x)^(3/2),x)
```

output
```
int((2 - 3*x^2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2} \left(2-3x^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^(3/2)*(-3*x^2+2)^p,x)
```

output
```
(24*sqrt(c + d*x)*( - 3*x**2 + 2)**p*c*p*x + 36*sqrt(c + d*x)*( - 3*x**2 +
 2)**p*c*x + 24*sqrt(c + d*x)*( - 3*x**2 + 2)**p*d*p*x**2 - 32*sqrt(c + d*
x)*( - 3*x**2 + 2)**p*d*p + 18*sqrt(c + d*x)*( - 3*x**2 + 2)**p*d*x**2 - 3
6*sqrt(c + d*x)*( - 3*x**2 + 2)**p*d + 432*int((sqrt(c + d*x)*( - 3*x**2 +
 2)**p*x**2)/(48*c*p**2*x**2 - 32*c*p**2 + 96*c*p*x**2 - 64*c*p + 45*c*x**
2 - 30*c + 48*d*p**2*x**3 - 32*d*p**2*x + 96*d*p*x**3 - 64*d*p*x + 45*d*x*
*3 - 30*d*x),x)*c**2*p**2 + 864*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p*x**2
)/(48*c*p**2*x**2 - 32*c*p**2 + 96*c*p*x**2 - 64*c*p + 45*c*x**2 - 30*c +
48*d*p**2*x**3 - 32*d*p**2*x + 96*d*p*x**3 - 64*d*p*x + 45*d*x**3 - 30*d*x
),x)*c**2*p + 405*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p*x**2)/(48*c*p**2*x
**2 - 32*c*p**2 + 96*c*p*x**2 - 64*c*p + 45*c*x**2 - 30*c + 48*d*p**2*x**3
 - 32*d*p**2*x + 96*d*p*x**3 - 64*d*p*x + 45*d*x**3 - 30*d*x),x)*c**2 + 15
36*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p*x**2)/(48*c*p**2*x**2 - 32*c*p**2
 + 96*c*p*x**2 - 64*c*p + 45*c*x**2 - 30*c + 48*d*p**2*x**3 - 32*d*p**2*x
+ 96*d*p*x**3 - 64*d*p*x + 45*d*x**3 - 30*d*x),x)*d**2*p**4 + 6144*int((sq
rt(c + d*x)*( - 3*x**2 + 2)**p*x**2)/(48*c*p**2*x**2 - 32*c*p**2 + 96*c*p*
x**2 - 64*c*p + 45*c*x**2 - 30*c + 48*d*p**2*x**3 - 32*d*p**2*x + 96*d*p*x
**3 - 64*d*p*x + 45*d*x**3 - 30*d*x),x)*d**2*p**3 + 8448*int((sqrt(c + d*x
)*( - 3*x**2 + 2)**p*x**2)/(48*c*p**2*x**2 - 32*c*p**2 + 96*c*p*x**2 - 64*
c*p + 45*c*x**2 - 30*c + 48*d*p**2*x**3 - 32*d*p**2*x + 96*d*p*x**3 - 6...
```

3.459 $\int \sqrt{c+dx}(2-3x^2)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx = \frac{2(c+dx)^{3/2}\left(2-3x^2\right)^p\left(1-\frac{3(c+dx)}{3c-\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+\sqrt{6}d}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{3d}$$

output
```
2/3*(d*x+c)^(3/2)*(-3*x^2+2)^p*AppellF1(3/2,-p,-p,5/2,3*(d*x+c)/(3*c-6^(1/
2)*d),3*(d*x+c)/(6^(1/2)*d+3*c))/d/((1-3*(d*x+c)/(3*c-6^(1/2)*d))^p)/((1-3
*(d*x+c)/(6^(1/2)*d+3*c))^p)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx = \frac{2\left(\frac{d\left(\sqrt{6}-3x\right)}{3c+\sqrt{6}d}\right)^{-p}\left(\frac{d\left(\sqrt{6}+3x\right)}{-3c+\sqrt{6}d}\right)^{-p}(c+dx)^{3/2}\left(2-3x^2\right)^p\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{3d}$$

input
```
Integrate[Sqrt[c + d*x]*(2 - 3*x^2)^p,x]
```

output
```
(2*(c + d*x)^(3/2)*(2 - 3*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (3*(c + d*x))/
(3*c - Sqrt[6]*d), (3*(c + d*x))/(3*c + Sqrt[6]*d)])/(3*d*((d*(Sqrt[6] - 3
*x))/(3*c + Sqrt[6]*d))^p*((d*(Sqrt[6] + 3*x))/(-3*c + Sqrt[6]*d))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-3x^2)^p \sqrt{c+dx}\,dx$$

$$\Big\downarrow \; 513$$

$$2^p \int \left(1-\sqrt{\frac{3}{2}}x\right)^p \left(\sqrt{\frac{3}{2}}x+1\right)^p \sqrt{c+dx}dx$$

$$\Big\downarrow \; 156$$

$$\frac{2^{p-\frac{1}{4}}\sqrt{c+dx}\int \left(1-\sqrt{\frac{3}{2}}x\right)^p \left(\sqrt{\frac{3}{2}}x+1\right)^p \sqrt{\frac{\sqrt{6}c}{\sqrt{6}c+2d}+\frac{\sqrt{6}dx}{\sqrt{6}c+2d}}dx}{\sqrt[4]{3}\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}}$$

$$\Big\downarrow \; 155$$

$$-\frac{2^{2p+\frac{1}{4}}\left(1-\sqrt{\frac{3}{2}}x\right)^{p+1}\sqrt{c+dx}\,\text{AppellF1}\left(p+1,-p,-\frac{1}{2},p+2,\frac{1}{4}(2-\sqrt{6}x),\frac{\sqrt{\frac{3}{2}}d\left(2-\sqrt{6}x\right)}{3c+\sqrt{6}d}\right)}{3^{3/4}(p+1)\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}}$$

input
```
Int[Sqrt[c + d*x]*(2 - 3*x^2)^p,x]
```

output
```
-((2^(1/4 + 2*p)*(1 - Sqrt[3/2]*x)^(1 + p)*Sqrt[c + d*x]*AppellF1[1 + p, -
p, -1/2, 2 + p, (2 - Sqrt[6]*x)/4, (Sqrt[3/2]*d*(2 - Sqrt[6]*x))/(3*c + Sq
rt[6]*d)])/(3^(3/4)*(1 + p)*Sqrt[(c + d*x)/(Sqrt[6]*c + 2*d)]))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \sqrt{dx+c}\,\left(-3x^{2}+2\right)^{p}dx$$

input
```
int((d*x+c)^(1/2)*(-3*x^2+2)^p,x)
```

output
```
int((d*x+c)^(1/2)*(-3*x^2+2)^p,x)
```

Fricas [F]

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx=\int \sqrt{dx+c}\left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-3*x^2 + 2)^p, x)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx=\int \left(2-3x^2\right)^p\sqrt{c+dx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-3*x**2+2)**p,x)
```

output
```
Integral((2 - 3*x**2)**p*sqrt(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx=\int \sqrt{dx+c}\left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*(-3*x^2 + 2)^p, x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx=\int \sqrt{dx+c}\left(-3\,x^2+2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*(-3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx=\int \left(2-3\,x^2\right)^p\,\sqrt{c+d\,x}\,dx$$

input
```
int((2 - 3*x^2)^p*(c + d*x)^(1/2),x)
```

output
```
int((2 - 3*x^2)^p*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(2-3x^2\right)^p\,dx$$
$$=\frac{6\sqrt{dx+c}\,(-3x^2+2)^p\,cx-4\sqrt{dx+c}\,(-3x^2+2)^p\,d+36\left(\int \frac{\sqrt{dx+c}\,(-3x^2+2)^p\,x^2}{12dp\,x^3+12cp\,x^2+9d\,x^3+9c\,x^2-8dpx-8cp-6dx-6c}dx\right)}{}$$

input
```
int((d*x+c)^(1/2)*(-3*x^2+2)^p,x)
```

output
```
(6*sqrt(c + d*x)*( - 3*x**2 + 2)**p*c*x - 4*sqrt(c + d*x)*( - 3*x**2 + 2)*
*p*d + 36*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p*x**2)/(12*c*p*x**2 - 8*c*p
 + 9*c*x**2 - 6*c + 12*d*p*x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),x)*c**2*p +
27*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p*x**2)/(12*c*p*x**2 - 8*c*p + 9*c*
x**2 - 6*c + 12*d*p*x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),x)*c**2 + 96*int((s
qrt(c + d*x)*( - 3*x**2 + 2)**p*x**2)/(12*c*p*x**2 - 8*c*p + 9*c*x**2 - 6*
c + 12*d*p*x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),x)*d**2*p**2 + 96*int((sqrt(
c + d*x)*( - 3*x**2 + 2)**p*x**2)/(12*c*p*x**2 - 8*c*p + 9*c*x**2 - 6*c +
12*d*p*x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),x)*d**2*p + 18*int((sqrt(c + d*x
)*( - 3*x**2 + 2)**p*x**2)/(12*c*p*x**2 - 8*c*p + 9*c*x**2 - 6*c + 12*d*p*
x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),x)*d**2 - 96*int((sqrt(c + d*x)*( - 3*x
**2 + 2)**p)/(12*c*p*x**2 - 8*c*p + 9*c*x**2 - 6*c + 12*d*p*x**3 - 8*d*p*x
 + 9*d*x**3 - 6*d*x),x)*c**2*p**2 - 96*int((sqrt(c + d*x)*( - 3*x**2 + 2)*
*p)/(12*c*p*x**2 - 8*c*p + 9*c*x**2 - 6*c + 12*d*p*x**3 - 8*d*p*x + 9*d*x*
*3 - 6*d*x),x)*c**2*p - 18*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p)/(12*c*p*
x**2 - 8*c*p + 9*c*x**2 - 6*c + 12*d*p*x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),
x)*c**2 - 16*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p)/(12*c*p*x**2 - 8*c*p +
 9*c*x**2 - 6*c + 12*d*p*x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),x)*d**2*p - 12
*int((sqrt(c + d*x)*( - 3*x**2 + 2)**p)/(12*c*p*x**2 - 8*c*p + 9*c*x**2 -
6*c + 12*d*p*x**3 - 8*d*p*x + 9*d*x**3 - 6*d*x),x)*d**2)/(3*c*(4*p + 3)...
```

3.460 $\int \frac{(2-3x^2)^p}{\sqrt{c+dx}} dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}} dx = \frac{2\sqrt{c+dx}(2-3x^2)^p \left(1-\frac{3(c+dx)}{3c-\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+\sqrt{6}d}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d}$$

output
```
2*(d*x+c)^(1/2)*(-3*x^2+2)^p*AppellF1(1/2,-p,-p,3/2,3*(d*x+c)/(3*c-6^(1/2)
*d),3*(d*x+c)/(6^(1/2)*d+3*c))/d/((1-3*(d*x+c)/(3*c-6^(1/2)*d))^p)/((1-3*(
d*x+c)/(6^(1/2)*d+3*c))^p)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}} dx = \frac{2\left(\frac{d\left(\sqrt{6}-3x\right)}{3c+\sqrt{6}d}\right)^{-p} \left(\frac{d\left(\sqrt{6}+3x\right)}{-3c+\sqrt{6}d}\right)^{-p} \sqrt{c+dx}(2-3x^2)^p \text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d}$$

input
```
Integrate[(2 - 3*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(2 - 3*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (3*(c + d*x))/(3
*c - Sqrt[6]*d), (3*(c + d*x))/(3*c + Sqrt[6]*d)])/(d*((d*(Sqrt[6] - 3*x))
/(3*c + Sqrt[6]*d))^p*((d*(Sqrt[6] + 3*x))/(-3*c + Sqrt[6]*d))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{513, 156, 155\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{513}$$

$$2^p\int \frac{\left(1-\sqrt{\frac{3}{2}}x\right)^p\left(\sqrt{\frac{3}{2}}x+1\right)^p}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{156}$$

$$\frac{\sqrt[4]{3}2^{p+\frac{1}{4}}\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}\int \frac{\left(1-\sqrt{\frac{3}{2}}x\right)^p\left(\sqrt{\frac{3}{2}}x+1\right)^p}{\sqrt{\frac{\sqrt{6}c}{\sqrt{6}c+2d}+\frac{\sqrt{6}dx}{\sqrt{6}c+2d}}}dx}{\sqrt{c+dx}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{2^{2p+\frac{3}{4}}\left(1-\sqrt{\frac{3}{2}}x\right)^{p+1}\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}\,\text{AppellF1}\left(p+1,-p,\frac{1}{2},p+2,\frac{1}{4}\left(2-\sqrt{6}x\right),\frac{\sqrt{\frac{3}{2}}d\left(2-\sqrt{6}x\right)}{3c+\sqrt{6}d}\right)}{\sqrt[4]{3}(p+1)\sqrt{c+dx}}$$

input
```
Int[(2 - 3*x^2)^p/Sqrt[c + d*x],x]
```

output
```
-((2^(3/4 + 2*p)*(1 - Sqrt[3/2]*x)^(1 + p)*Sqrt[(c + d*x)/(Sqrt[6]*c + 2*d
)]*AppellF1[1 + p, -p, 1/2, 2 + p, (2 - Sqrt[6]*x)/4, (Sqrt[3/2]*d*(2 - Sq
rt[6]*x))/(3*c + Sqrt[6]*d)])/(3^(1/4)*(1 + p)*Sqrt[c + d*x]))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(-3x^2+2)^p}{\sqrt{dx+c}}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((-3*x^2+2)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-3\,x^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-3*x**2+2)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((2 - 3*x**2)**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-3\,x^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-3\,x^2+2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(2-3\,x^2)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((2 - 3*x^2)^p/(c + d*x)^(1/2),x)
```

output
```
int((2 - 3*x^2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2-3x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{dx+c}\,(-3x^2+2)^p}{dx+c}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*( - 3*x**2 + 2)**p)/(c + d*x),x)
```

3.461 $\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2(2-3x^2)^p\left(1-\frac{3(c+dx)}{3c-\sqrt{6}d}\right)^{-p}\left(1-\frac{3(c+dx)}{3c+\sqrt{6}d}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d\sqrt{c+dx}}$$

output
```
-2*(-3*x^2+2)^p*AppellF1(-1/2,-p,-p,1/2,3*(d*x+c)/(3*c-6^(1/2)*d),3*(d*x+c
)/(6^(1/2)*d+3*c))/d/(d*x+c)^(1/2)/((1-3*(d*x+c)/(3*c-6^(1/2)*d))^p)/((1-3
*(d*x+c)/(6^(1/2)*d+3*c))^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2\left(\frac{d(\sqrt{6}-3x)}{3c+\sqrt{6}d}\right)^{-p}\left(\frac{d(\sqrt{6}+3x)}{-3c+\sqrt{6}d}\right)^{-p}(2-3x^2)^p\operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d\sqrt{c+dx}}$$

input
```
Integrate[(2 - 3*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
(-2*(2 - 3*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (3*(c + d*x))/(3*c - Sqrt[6]
*d), (3*(c + d*x))/(3*c + Sqrt[6]*d)])/(d*((d*(Sqrt[6] - 3*x))/(3*c + Sqrt
[6]*d))^p*((d*(Sqrt[6] + 3*x))/(-3*c + Sqrt[6]*d))^p*Sqrt[c + d*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \; 513$$

$$2^p\int \frac{\left(1-\sqrt{\frac{3}{2}}x\right)^p\left(\sqrt{\frac{3}{2}}x+1\right)^p}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \; 156$$

$$\frac{3^{3/4}2^{p+\frac{3}{4}}\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}\int \frac{\left(1-\sqrt{\frac{3}{2}}x\right)^p\left(\sqrt{\frac{3}{2}}x+1\right)^p}{\left(\frac{\sqrt{6}c}{\sqrt{6}c+2d}+\frac{\sqrt{6}dx}{\sqrt{6}c+2d}\right)^{3/2}}dx}{\left(\sqrt{6}c+2d\right)\sqrt{c+dx}}$$

$$\Big\downarrow \; 155$$

$$-\frac{\sqrt[4]{3}2^{2p+\frac{5}{4}}\left(1-\sqrt{\frac{3}{2}}x\right)^{p+1}\sqrt{\frac{c+dx}{\sqrt{6}c+2d}}\operatorname{AppellF1}\left(p+1,-p,\frac{3}{2},p+2,\frac{1}{4}\left(2-\sqrt{6}x\right),\frac{\sqrt{\frac{3}{2}}d\left(2-\sqrt{6}x\right)}{3c+\sqrt{6}d}\right)}{(p+1)\left(\sqrt{6}c+2d\right)\sqrt{c+dx}}$$

input
```
Int[(2 - 3*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
-((2^(5/4 + 2*p)*3^(1/4)*(1 - Sqrt[3/2]*x)^(1 + p)*Sqrt[(c + d*x)/(Sqrt[6]
*c + 2*d)]*AppellF1[1 + p, -p, 3/2, 2 + p, (2 - Sqrt[6]*x)/4, (Sqrt[3/2]*d
*(2 - Sqrt[6]*x))/(3*c + Sqrt[6]*d)])/((Sqrt[6]*c + 2*d)*(1 + p)*Sqrt[c +
d*x]))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int \frac{(-3x^2+2)^p}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((-3*x^2+2)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-3*x^2 + 2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(2-3x^2)^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-3*x**2+2)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((2 - 3*x**2)**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-3\,x^2+2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-3*x^2+2)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(2-3\,x^2)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((2 - 3*x^2)^p/(c + d*x)^(3/2),x)
```

output
```
int((2 - 3*x^2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-3x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,(-3x^2+2)^p}{d^2x^2+2cdx+c^2}dx$$

input
```
int((-3*x^2+2)^p/(d*x+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*( - 3*x**2 + 2)**p)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.462 $\int (c+dx)^m (a+bx^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 152

$$\int (c+dx)^m (a+bx^2)^p \, dx$$
$$= \frac{(c+dx)^{1+m}(a+bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+}{c+}\right.}{d(1+m)}$$

output
```
(d*x+c)^(1+m)*(b*x^2+a)^p*AppellF1(1+m,-p,-p,2+m,(d*x+c)/(c-(-a)^(1/2)*d/b
^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/(1+m)/((1-(d*x+c)/(c-(-a)^(1/2
)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

$$\int (c+dx)^m (a+bx^2)^p \, dx$$
$$= \frac{\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{-p} (c+dx)^{1+m}(a+bx^2)^p \text{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+d}{c+\sqrt{-}}\right.}{d(1+m)}$$

input
```
Integrate[(c + d*x)^m*(a + b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(a + b*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (c + d*x)/
(c - Sqrt[-(a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*(1 + m)*((d*(Sqr
t[-(a/b)] - x))/(c + Sqrt[-(a/b)]*d))^p*((d*(Sqrt[-(a/b)] + x))/(-c + Sqrt
[-(a/b)]*d))^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^m \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^m \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(a+bx^2\right)^p (c+dx)^{m+1} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(a + b*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (c + d*x)/
(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d*(1 +
 m)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt
[-a]*d)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{m} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^m*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^m*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^m \left(a+bx^2\right)^p \, dx = \int \left(bx^{2}+a\right)^{p}(dx+c)^{m} \, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \left(a+bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^m \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^m, x)
```

Giac [F]

$$\int (c+dx)^m \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \left(a+bx^2\right)^p \, dx = \int \left(b\,x^2+a\right)^p\,\left(c+d\,x\right)^m \,dx$$

input
```
int((a + b*x^2)^p*(c + d*x)^m,x)
```

output
```
int((a + b*x^2)^p*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \left(a+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^m*(b*x^2+a)^p,x)
```

output
```
((c + d*x)**m*(a + b*x**2)**p*a*d + (c + d*x)**m*(a + b*x**2)**p*b*c*x - i
nt(((c + d*x)**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x +
2*a*d*p*x + a*d*x + b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*
b*d*p*x**3 + b*d*x**3),x)*a*b*d**2*m**2 - 4*int(((c + d*x)**m*(a + b*x**2)
**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*x + b*c*m*x**
2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3),x)*a*b
*d**2*m*p - int(((c + d*x)**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c
 + a*d*m*x + 2*a*d*p*x + a*d*x + b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*
d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3),x)*a*b*d**2*m - 4*int(((c + d*x)**m*(a
 + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*x +
 b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x*
*3),x)*a*b*d**2*p**2 - 2*int(((c + d*x)**m*(a + b*x**2)**p*x**2)/(a*c*m +
2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*x + b*c*m*x**2 + 2*b*c*p*x**2 +
b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3),x)*a*b*d**2*p + int(((c +
 d*x)**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*
x + a*d*x + b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x*
*3 + b*d*x**3),x)*b**2*c**2*m**2 + 2*int(((c + d*x)**m*(a + b*x**2)**p*x**
2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*x + b*c*m*x**2 + 2*b
*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3),x)*b**2*c**2*
m*p + int(((c + d*x)**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + ...
```

3.463 $\int (c+dx)^m (2+bx^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 152

$$\int (c+dx)^m (2+bx^2)^p \, dx = \frac{(c+dx)^{1+m} (2+bx^2)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^{-p} \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}},\ldots\right)}{d(1+m)}$$

output
```
(d*x+c)^(1+m)*(b*x^2+2)^p*AppellF1(1+m,-p,-p,2+m,(d*x+c)/(c+2^(1/2)*d/(-b)
^(1/2)),(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))/d/(1+m)/((1-(d*x+c)/(c+2^(1/2)
*d/(-b)^(1/2)))^p)/((1-(d*x+c)/(c+2^(1/2)*b*d/(-b)^(3/2)))^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.20

$$\int (c+dx)^m (2+bx^2)^p \, dx = \frac{\left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{2}\sqrt{-\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{-\frac{1}{b}}d}\right)^{-p} (c+dx)^{1+m} (2+bx^2)^p \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c+\sqrt{2}\left(-\frac{1}{b}\right)^{3\ldots}}\ldots\right)}{d(1+m)}$$

input
```
Integrate[(c + d*x)^m*(2 + b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(2 + b*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (c + d*x)/
(c + Sqrt[2]*(-b^(-1))^(3/2)*b*d), (c + d*x)/(c + Sqrt[2]*Sqrt[-b^(-1)]*d)
])/(d*(1 + m)*((d*(Sqrt[2]*Sqrt[-b^(-1)] - x))/(c + Sqrt[2]*Sqrt[-b^(-1)]*
d))^p*((d*(Sqrt[2]*Sqrt[-b^(-1)] + x))/(-c + Sqrt[2]*Sqrt[-b^(-1)]*d))^p)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+2\right)^p (c+dx)^m \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(bx^2+2\right)^p \left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p} \int (c+dx)^m \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(bx^2+2\right)^p (c+dx)^{m+1} \left(1-\frac{c+dx}{\frac{\sqrt{2}d}{\sqrt{-b}}+c}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{2}bd}{(-b)^{3/2}}+c}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{-b}}},\frac{c+dx}{c+\frac{\sqrt{2}bd}{(-b)^{3/2}}}\right)}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(2 + b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(2 + b*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (c + d*x)/
(c + (Sqrt[2]*d)/Sqrt[-b]), (c + d*x)/(c + (Sqrt[2]*b*d)/(-b)^(3/2))])/(d*
(1 + m)*(1 - (c + d*x)/(c + (Sqrt[2]*d)/Sqrt[-b]))^p*(1 - (c + d*x)/(c + (
Sqrt[2]*b*d)/(-b)^(3/2)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^m \left(b\,x^2+2\right)^p dx$$

input
```
int((d*x+c)^m*(b*x^2+2)^p,x)
```

output
```
int((d*x+c)^m*(b*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^m \left(2+bx^2\right)^p \, dx = \int (bx^2+2)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + 2)^p*(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2+bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(b*x**2+2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^m \left(2+bx^2\right)^p \, dx = \int (bx^2+2)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(b*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 2)^p*(d*x + c)^m, x)
```

Giac [F]

$$\int (c+dx)^m \left(2+bx^2\right)^p \,dx=\int (bx^2+2)^p(dx+c)^m \,dx$$

input
```
integrate((d*x+c)^m*(b*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + 2)^p*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2+bx^2\right)^p \,dx=\int \left(b\,x^2+2\right)^p\,\left(c+d\,x\right)^m \,dx$$

input
```
int((b*x^2 + 2)^p*(c + d*x)^m,x)
```

output
```
int((b*x^2 + 2)^p*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \left(2+bx^2\right)^p \,dx=\text{too large to display}$$

input
```
int((d*x+c)^m*(b*x^2+2)^p,x)
```

output
```
((c + d*x)**m*(b*x**2 + 2)**p*b*c*x + 2*(c + d*x)**m*(b*x**2 + 2)**p*d + i
nt(((c + d*x)**m*(b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x*
*2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3 + 2*c*m + 4*c*p + 2*c + 2*d*m*x
+ 4*d*p*x + 2*d*x),x)*b**2*c**2*m**2 + 2*int(((c + d*x)**m*(b*x**2 + 2)**p
*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 +
 b*d*x**3 + 2*c*m + 4*c*p + 2*c + 2*d*m*x + 4*d*p*x + 2*d*x),x)*b**2*c**2*
m*p + int(((c + d*x)**m*(b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 +
 b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3 + 2*c*m + 4*c*p + 2*c + 2
*d*m*x + 4*d*p*x + 2*d*x),x)*b**2*c**2*m - 2*int(((c + d*x)**m*(b*x**2 + 2
)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x*
*3 + b*d*x**3 + 2*c*m + 4*c*p + 2*c + 2*d*m*x + 4*d*p*x + 2*d*x),x)*b*d**2
*m**2 - 8*int(((c + d*x)**m*(b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x*
*2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3 + 2*c*m + 4*c*p + 2*c
 + 2*d*m*x + 4*d*p*x + 2*d*x),x)*b*d**2*m*p - 2*int(((c + d*x)**m*(b*x**2
+ 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p
*x**3 + b*d*x**3 + 2*c*m + 4*c*p + 2*c + 2*d*m*x + 4*d*p*x + 2*d*x),x)*b*d
**2*m - 8*int(((c + d*x)**m*(b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x*
*2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3 + 2*c*m + 4*c*p + 2*c
 + 2*d*m*x + 4*d*p*x + 2*d*x),x)*b*d**2*p**2 - 4*int(((c + d*x)**m*(b*x**2
 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b...
```

3.464 $\int (c+dx)^m (2+3x^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int (c+dx)^m (2+3x^2)^p \, dx = \frac{(c+dx)^{1+m} (2+3x^2)^p \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \text{AppellF1}\left(1+m,-p,-p,2+m,\frac{3(c+dx)}{3c-i\sqrt{6}d},\right.}{d(1+m)}$$

output
```
(d*x+c)^(1+m)*(3*x^2+2)^p*AppellF1(1+m,-p,-p,2+m,3*(d*x+c)/(3*c-I*6^(1/2)*
d),3*(d*x+c)/(3*c+I*6^(1/2)*d))/d/(1+m)/((1-3*(d*x+c)/(3*c-I*6^(1/2)*d))^p
)/((1-3*(d*x+c)/(3*c+I*6^(1/2)*d))^p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04

$$\int (c+dx)^m (2+3x^2)^p \, dx = \frac{\left(\frac{d(\sqrt{6}-3ix)}{3ic+\sqrt{6}d}\right)^{-p} \left(\frac{d(\sqrt{6}+3ix)}{-3ic+\sqrt{6}d}\right)^{-p} (c+dx)^{1+m} (2+3x^2)^p \text{AppellF1}\left(1+m,-p,-p,2+m,\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right.}{d(1+m)}$$

input
```
Integrate[(c + d*x)^m*(2 + 3*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(2 + 3*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (3*(c + d*
x))/(3*c - I*Sqrt[6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(d*(1 + m)*((
d*(Sqrt[6] - (3*I)*x))/((3*I)*c + Sqrt[6]*d))^p*((d*(Sqrt[6] + (3*I)*x))/(
(-3*I)*c + Sqrt[6]*d))^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+2\right)^p (c+dx)^m \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(3x^2+2\right)^p \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \int (c+dx)^m \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^p \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(3x^2+2\right)^p (c+dx)^{m+1} \left(1-\frac{3(c+dx)}{3c-i\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{3(c+dx)}{3c-i\sqrt{6}d},\frac{3(c+dx)}{3c+i\sqrt{6}d}\right)}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(2 + 3*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(2 + 3*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (3*(c + d*
x))/(3*c - I*Sqrt[6]*d), (3*(c + d*x))/(3*c + I*Sqrt[6]*d)])/(d*(1 + m)*(1
 - (3*(c + d*x))/(3*c - I*Sqrt[6]*d))^p*(1 - (3*(c + d*x))/(3*c + I*Sqrt[6
]*d))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^m \left(3x^2+2\right)^p dx$$

input
```
int((d*x+c)^m*(3*x^2+2)^p,x)
```

output
```
int((d*x+c)^m*(3*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^m \left(2+3x^2\right)^p \, dx = \int (dx+c)^m \left(3\,x^2+2\right)^p dx$$

input
```
integrate((d*x+c)^m*(3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(3*x^2 + 2)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2+3x^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(3*x**2+2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^m \left(2+3x^2\right)^p \, dx = \int (dx+c)^m \left(3\,x^2+2\right)^p \,dx$$

input
```
integrate((d*x+c)^m*(3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^m \left(2+3x^2\right)^p \, dx = \int (dx+c)^m \left(3\,x^2+2\right)^p \,dx$$

input
```
integrate((d*x+c)^m*(3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2+3x^2\right)^p \, dx = \int \left(3\,x^2+2\right)^p\,\left(c+d\,x\right)^m \,dx$$

input
```
int((3*x^2 + 2)^p*(c + d*x)^m,x)
```

output
```
int((3*x^2 + 2)^p*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \left(2+3x^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*(3*x^2+2)^p,x)
```

output

```
(3*(c + d*x)**m*(3*x**2 + 2)**p*c*x + 2*(c + d*x)**m*(3*x**2 + 2)**p*d + 9
*int(((c + d*x)**m*(3*x**2 + 2)**p*x**2)/(3*c*m*x**2 + 2*c*m + 6*c*p*x**2
+ 4*c*p + 3*c*x**2 + 2*c + 3*d*m*x**3 + 2*d*m*x + 6*d*p*x**3 + 4*d*p*x + 3
*d*x**3 + 2*d*x),x)*c**2*m**2 + 18*int(((c + d*x)**m*(3*x**2 + 2)**p*x**2)
/(3*c*m*x**2 + 2*c*m + 6*c*p*x**2 + 4*c*p + 3*c*x**2 + 2*c + 3*d*m*x**3 +
2*d*m*x + 6*d*p*x**3 + 4*d*p*x + 3*d*x**3 + 2*d*x),x)*c**2*m*p + 9*int(((c
 + d*x)**m*(3*x**2 + 2)**p*x**2)/(3*c*m*x**2 + 2*c*m + 6*c*p*x**2 + 4*c*p
+ 3*c*x**2 + 2*c + 3*d*m*x**3 + 2*d*m*x + 6*d*p*x**3 + 4*d*p*x + 3*d*x**3
+ 2*d*x),x)*c**2*m - 6*int(((c + d*x)**m*(3*x**2 + 2)**p*x**2)/(3*c*m*x**2
 + 2*c*m + 6*c*p*x**2 + 4*c*p + 3*c*x**2 + 2*c + 3*d*m*x**3 + 2*d*m*x + 6*
d*p*x**3 + 4*d*p*x + 3*d*x**3 + 2*d*x),x)*d**2*m**2 - 24*int(((c + d*x)**m
*(3*x**2 + 2)**p*x**2)/(3*c*m*x**2 + 2*c*m + 6*c*p*x**2 + 4*c*p + 3*c*x**2
 + 2*c + 3*d*m*x**3 + 2*d*m*x + 6*d*p*x**3 + 4*d*p*x + 3*d*x**3 + 2*d*x),x
)*d**2*m*p - 6*int(((c + d*x)**m*(3*x**2 + 2)**p*x**2)/(3*c*m*x**2 + 2*c*m
 + 6*c*p*x**2 + 4*c*p + 3*c*x**2 + 2*c + 3*d*m*x**3 + 2*d*m*x + 6*d*p*x**3
 + 4*d*p*x + 3*d*x**3 + 2*d*x),x)*d**2*m - 24*int(((c + d*x)**m*(3*x**2 +
2)**p*x**2)/(3*c*m*x**2 + 2*c*m + 6*c*p*x**2 + 4*c*p + 3*c*x**2 + 2*c + 3*
d*m*x**3 + 2*d*m*x + 6*d*p*x**3 + 4*d*p*x + 3*d*x**3 + 2*d*x),x)*d**2*p**2
 - 12*int(((c + d*x)**m*(3*x**2 + 2)**p*x**2)/(3*c*m*x**2 + 2*c*m + 6*c*p*
x**2 + 4*c*p + 3*c*x**2 + 2*c + 3*d*m*x**3 + 2*d*m*x + 6*d*p*x**3 + 4*d...
```

3.465 $\int (c+dx)^{-2p} (a+bx^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int (c+dx)^{-2p} (a+bx^2)^p \, dx = \frac{(c+dx)^{1-2p} (a+bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(1-2p)}$$

output
```
(d*x+c)^(1-2*p)*(b*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,(d*x+c)/(c-(-a)^(1/
2)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/(1-2*p)/((1-(d*x+c)/(c-(
-a)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int (c+dx)^{-2p} (a+bx^2)^p \, dx = -\frac{\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{-p} (c+dx)^{1-2p} (a+bx^2)^p \text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{-\frac{a}{b}}d}\right)}{d(-1+2p)}$$

input
```
Integrate[(a + b*x^2)^p/(c + d*x)^(2*p),x]
```

output
```
-(((c + d*x)^(1 - 2*p)*(a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c
 + d*x)/(c - Sqrt[-(a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*(-1 + 2*
p)*((d*(Sqrt[-(a/b)] - x))/(c + Sqrt[-(a/b)]*d))^p*((d*(Sqrt[-(a/b)] + x))
/(-c + Sqrt[-(a/b)]*d))^p))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^{-2p} \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^{-2p} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(a+bx^2\right)^p (c+dx)^{1-2p} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d(1-2p)}$$

input
```
Int[(a + b*x^2)^p/(c + d*x)^(2*p),x]
```

output
```
((c + d*x)^(1 - 2*p)*(a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c +
 d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(
d*(1 - 2*p)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c
 + (Sqrt[-a]*d)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(dx+c\right)^{-2p}dx$$

input
```
int((b*x^2+a)^p/((d*x+c)^(2*p)),x)
```

output
```
int((b*x^2+a)^p/((d*x+c)^(2*p)),x)
```

Fricas [F]

$$\int (c+dx)^{-2p} \left(a+bx^2\right)^p \, dx = \int \frac{\left(bx^2+a\right)^p}{(dx+c)^{2\,p}}\,dx$$

input
```
integrate((b*x^2+a)^p/((d*x+c)^(2*p)),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(d*x + c)^(2*p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-2p} \left(a+bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/((d*x+c)**(2*p)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-2p} \left(a+bx^2\right)^p \, dx = \int \frac{(bx^2+a)^p}{(dx+c)^{2p}} \, dx$$

input
```
integrate((b*x^2+a)^p/((d*x+c)^(2*p)),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^(2*p), x)
```

Giac [F]

$$\int (c+dx)^{-2p} \left(a+bx^2\right)^p \, dx = \int \frac{(bx^2+a)^p}{(dx+c)^{2p}} \, dx$$

input
```
integrate((b*x^2+a)^p/((d*x+c)^(2*p)),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(d*x + c)^(2*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{\left(c+d\,x\right)^{2\,p}}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(2*p),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(2*p), x)
```

Reduce [F]

$$\int (c+dx)^{-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}}dx$$

input
```
int((b*x^2+a)^p/((d*x+c)^(2*p)),x)
```

output
```
int((a + b*x**2)**p/(c + d*x)**(2*p),x)
```

3.466 $\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx =$$

$$-\frac{(c+dx)^{-2p} \left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+}{c+}\right.}{2dp}$$

output
```
-1/2*(b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)
),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/p/((d*x+c)^(2*p))/((1-(d*x+c)/(c-(-a
)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx =$$

$$-\frac{\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{-p} (c+dx)^{-2p} \left(a+bx^2\right)^p \text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\frac{c+}{c+\sqrt{}}\right.}{2dp}$$

input
```
Integrate[(c + d*x)^(-1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - Sqrt[-(
a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*p*((d*(Sqrt[-(a/b)] - x))/(c
 + Sqrt[-(a/b)]*d))^p*((d*(Sqrt[-(a/b)] + x))/(-c + Sqrt[-(a/b)]*d))^p*(c
+ d*x)^(2*p))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^{-2p-1}\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^{-2p-1} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\left(a+bx^2\right)^p (c+dx)^{-2p} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{2dp}$$

input
```
Int[(c + d*x)^(-1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - (Sqrt[-
a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d*p*(c + d*x)^(2*p
)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-
a]*d)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{-1-2p} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-1} \, dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Sympy [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \left(a+bx^2\right)^p(c+dx)^{-2p-1}\,dx$$

input
```
integrate((d*x+c)**(-1-2*p)*(b*x**2+a)**p,x)
```

output
```
Integral((a + b*x**2)**p*(c + d*x)**(-2*p - 1), x)
```

Maxima [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p(dx+c)^{-2\,p-1}\,dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Giac [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p(dx+c)^{-2\,p-1}\,dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{\left(c+d\,x\right)^{2\,p+1}}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 1),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 1), x)
```

Reduce [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}c+\left(dx+c\right)^{2p}dx}dx$$

input
```
int((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c + (c + d*x)**(2*p)*d*x),x)
```

3.467 $\int (c+dx)^{-2-2p} \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 209

$$\int (c+dx)^{-2-2p} \left(a+bx^2\right)^p \, dx = -\frac{\left(\sqrt{-a}-\sqrt{b}x\right)\left(-\frac{\left(\sqrt{b}c+\sqrt{-a}d\right)\left(\sqrt{-a}+\sqrt{b}x\right)}{\left(\sqrt{b}c-\sqrt{-a}d\right)\left(\sqrt{-a}-\sqrt{b}x\right)}\right)^{-p} (c+dx)^{-1-2p} \left(a+bx^2\right)^p \text{Hypergeometric2F1}\left(-1-2p,\right.}{\left(\sqrt{b}c+\sqrt{-a}d\right)(1+2p)}$$

output
```
-((-a)^(1/2)-b^(1/2)*x)*(d*x+c)^(-1-2*p)*(b*x^2+a)^p*hypergeom([-p, -1-2*p
],[-2*p],2*(-a)^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d)/((-a)^(1/2)
-b^(1/2)*x))/(b^(1/2)*c+(-a)^(1/2)*d)/(1+2*p)/((-(b^(1/2)*c+(-a)^(1/2)*d)*
((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*c-(-a)^(1/2)*d)/((-a)^(1/2)-b^(1/2)*x))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.96

$$\int (c+dx)^{-2-2p} \left(a+bx^2\right)^p \, dx = \frac{\left(-\sqrt{-a}+\sqrt{b}x\right)(c+dx)^{-1-2p}\left(a+bx^2\right)^p\left(1-\frac{b(c+dx)}{bc-\sqrt{-a}\sqrt{b}d}\right)^{-p}\left(1-\frac{b(c+dx)}{bc+\sqrt{-a}\sqrt{b}d}\right)^{p}\text{Hypergeometric2F1}\left(-\right.}{\left(\sqrt{b}c+\sqrt{-a}d\right)(1+2p)}$$

input
```
Integrate[(c + d*x)^(-2 - 2*p)*(a + b*x^2)^p,x]
```

output
```
((-Sqrt[-a] + Sqrt[b]*x)*(c + d*x)^(-1 - 2*p)*(a + b*x^2)^p*(1 - (b*(c + d
*x))/(b*c + Sqrt[-a]*Sqrt[b]*d))^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (
2*Sqrt[-a]*Sqrt[b]*(c + d*x))/((Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b
]*x))])/((Sqrt[b]*c + Sqrt[-a]*d)*(1 + 2*p)*(1 - (b*(c + d*x))/(b*c - Sqrt
[-a]*Sqrt[b]*d))^p)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^{-2p-2} \, dx$$

$$\Big\downarrow \text{ 489}$$

$$-\frac{\left(\sqrt{-a}-\sqrt{b}x\right)\left(a+bx^2\right)^p (c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{b}x\right)\left(\sqrt{-a}d+\sqrt{b}c\right)}{\left(\sqrt{-a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{-a}d\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,-\right.}{(2p+1)\left(\sqrt{-a}d+\sqrt{b}c\right)}$$

input
```
Int[(c + d*x)^(-2 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-(((Sqrt[-a] - Sqrt[b]*x)*(c + d*x)^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometri
c2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[b]*(c + d*x))/((Sqrt[b]*c - Sqrt
[-a]*d)*(Sqrt[-a] - Sqrt[b]*x))])/((Sqrt[b]*c + Sqrt[-a]*d)*(1 + 2*p)*(-((
(Sqrt[b]*c + Sqrt[-a]*d)*(Sqrt[-a] + Sqrt[b]*x))/((Sqrt[b]*c - Sqrt[-a]*d)
*(Sqrt[-a] - Sqrt[b]*x))))^p))
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

Maple [F]

$$\int (dx+c)^{-2p-2} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^(-2*p-2)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-2*p-2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-2-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-2} \, dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-2-2p}\left(a+bx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(-2-2*p)*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-2-2p}\left(a+bx^2\right)^p\,dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-2}\,dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Giac [F]

$$\int (c+dx)^{-2-2p}\left(a+bx^2\right)^p\,dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-2}\,dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-2-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{\left(c+d\,x\right)^{2\,p+2}}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 2),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 2), x)
```

Reduce [F]

$$\int (c+dx)^{-2-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}c^2+2\left(dx+c\right)^{2p}cdx+\left(dx+c\right)^{2p}d^2x^2}dx$$

input
```
int((d*x+c)^(-2-2*p)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**2 + 2*(c + d*x)**(2*p)*c*d*x + (c
 + d*x)**(2*p)*d**2*x**2),x)
```

3.468 $\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 270

$$\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{d(c+dx)^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2\left(bc^2+ad^2\right)(1+p)}$$
$$-\frac{bc\left(\sqrt{-a}-\sqrt{b}x\right)\left(-\frac{\left(\sqrt{b}c+\sqrt{-a}d\right)\left(\sqrt{-a}+\sqrt{b}x\right)}{\left(\sqrt{b}c-\sqrt{-a}d\right)\left(\sqrt{-a}-\sqrt{b}x\right)}\right)^{-p}(c+dx)^{-1-2p}\left(a+bx^2\right)^p \text{Hypergeometric2F1}\left(-1-2\right.}{\left(\sqrt{b}c+\sqrt{-a}d\right)\left(bc^2+ad^2\right)(1+2p)}$$

output
```
-1/2*d*(b*x^2+a)^(p+1)/(a*d^2+b*c^2)/(p+1)/((d*x+c)^(2*p+2))-b*c*((-a)^(1/
2)-b^(1/2)*x)*(d*x+c)^(-1-2*p)*(b*x^2+a)^p*hypergeom([-p, -1-2*p],[-2*p],2
*(-a)^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d)/((-a)^(1/2)-b^(1/2)*x
))/(b^(1/2)*c+(-a)^(1/2)*d)/(a*d^2+b*c^2)/(1+2*p)/((-(b^(1/2)*c+(-a)^(1/2)
*d)*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*c-(-a)^(1/2)*d)/((-a)^(1/2)-b^(1/2)*x)
)^p)
```

Mathematica [F]

$$\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx=\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx$$

input
```
Integrate[(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
Integrate[(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {491, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^{-2p-3}\,dx$$

$$\Big\downarrow \text{491}$$

$$\frac{bc\int (c+dx)^{-2(p+1)}\left(bx^2+a\right)^p dx}{ad^2+bc^2}-\frac{d\left(a+bx^2\right)^{p+1}(c+dx)^{-2(p+1)}}{2(p+1)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{489}$$

$$-\frac{bc\left(\sqrt{-a}-\sqrt{b}x\right)\left(a+bx^2\right)^p (c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{b}x\right)\left(\sqrt{-a}d+\sqrt{b}c\right)}{\left(\sqrt{-a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{-a}d\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,\right.}{(2p+1)\left(\sqrt{-a}d+\sqrt{b}c\right)\left(ad^2+bc^2\right)}$$

$$\frac{d\left(a+bx^2\right)^{p+1}(c+dx)^{-2(p+1)}}{2(p+1)\left(ad^2+bc^2\right)}$$

input
```
Int[(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*(d*(a + b*x^2)^(1 + p))/((b*c^2 + a*d^2)*(1 + p)*(c + d*x)^(2*(1 + p)
)) - (b*c*(Sqrt[-a] - Sqrt[b]*x)*(c + d*x)^(-1 - 2*p)*(a + b*x^2)^p*Hyperg
eometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[b]*(c + d*x))/((Sqrt[b]*c
 - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b]*x))])/((Sqrt[b]*c + Sqrt[-a]*d)*(b*c^2
+ a*d^2)*(1 + 2*p)*(-(((Sqrt[b]*c + Sqrt[-a]*d)*(Sqrt[-a] + Sqrt[b]*x))/((
Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b]*x))))^p)
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

Maple [F]

$$\int (dx+c)^{-3-2p} \left(b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^(-3-2*p)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-3-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \, dx = \int (bx^2+a)^p (dx+c)^{-2\,p-3} \, dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(-3-2*p)*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \, dx = \int (bx^2+a)^p (dx+c)^{-2\,p-3} \, dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Giac [F]

$$\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx=\int (bx^2+a)^p(dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx=\int \frac{(b\,x^2+a)^p}{(c+d\,x)^{2\,p+3}}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 3),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 3), x)
```

Reduce [F]

$$\begin{aligned}&\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx\\&=\int \frac{(b\,x^2+a)^p}{(dx+c)^{2p}\,c^3+3\,(dx+c)^{2p}\,c^2dx+3\,(dx+c)^{2p}\,c\,d^2x^2+(dx+c)^{2p}\,d^3x^3}dx\end{aligned}$$

input
```
int((d*x+c)^(-3-2*p)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*x +
 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)
```

3.469 $\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 347

$$\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx = -\frac{d(c+dx)^{-3-2p}\left(a+bx^2\right)^{1+p}}{\left(bc^2+ad^2\right)(3+2p)} - \frac{bcd(2+p)(c+dx)^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{\left(bc^2+ad^2\right)^2(1+p)(3+2p)} + \frac{b\left(ad^2-bc^2(3+2p)\right)\left(\sqrt{-a}-\sqrt{b}x\right)\left(-\frac{\left(\sqrt{b}c+\sqrt{-a}d\right)\left(\sqrt{-a}+\sqrt{b}x\right)}{\left(\sqrt{b}c-\sqrt{-a}d\right)\left(\sqrt{-a}-\sqrt{b}x\right)}\right)^{-p}(c+dx)^{-1-2p}\left(a+bx^2\right)^p \text{Hypergeor}}{\left(\sqrt{b}c+\sqrt{-a}d\right)\left(bc^2+ad^2\right)^2(1+2p)(3+2p)}$$

output
```
-d*(d*x+c)^(-3-2*p)*(b*x^2+a)^(p+1)/(a*d^2+b*c^2)/(3+2*p)-b*c*d*(2+p)*(b*x
^2+a)^(p+1)/(a*d^2+b*c^2)^2/(p+1)/(3+2*p)/((d*x+c)^(2*p+2))+b*(a*d^2-b*c^2
*(3+2*p))*((-a)^(1/2)-b^(1/2)*x)*(d*x+c)^(-1-2*p)*(b*x^2+a)^p*hypergeom([-
p, -1-2*p],[-2*p],2*(-a)^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d)/((
-a)^(1/2)-b^(1/2)*x))/(b^(1/2)*c+(-a)^(1/2)*d)/(a*d^2+b*c^2)^2/(1+2*p)/(3+
2*p)/((-(b^(1/2)*c+(-a)^(1/2)*d)*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*c-(-a)^(1
/2)*d)/((-a)^(1/2)-b^(1/2)*x))^p)
```

Mathematica [F]

$$\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx = \int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx$$

input
```
Integrate[(c + d*x)^(-4 - 2*p)*(a + b*x^2)^p,x]
```

output
```
Integrate[(c + d*x)^(-4 - 2*p)*(a + b*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {498, 25, 679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^{-2p-4} \, dx$$

$$\Big\downarrow \; 498$$

$$-\frac{b \int -\left((c(2p+3)-dx)(c+dx)^{-2p-3} \left(bx^2+a\right)^p\right) dx}{(2p+3) \left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{p+1} (c+dx)^{-2p-3}}{(2p+3) \left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{b \int (c(2p+3)-dx)(c+dx)^{-2p-3} \left(bx^2+a\right)^p dx}{(2p+3) \left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{p+1} (c+dx)^{-2p-3}}{(2p+3) \left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 679$$

$$\frac{b \left(-\frac{\left(ad^2-bc^2(2p+3)\right) \int (c+dx)^{-2(p+1)} \left(bx^2+a\right)^p dx}{ad^2+bc^2}-\frac{cd(p+2)\left(a+bx^2\right)^{p+1}(c+dx)^{-2(p+1)}}{(p+1)\left(ad^2+bc^2\right)}\right)}{(2p+3) \left(ad^2+bc^2\right)}-\frac{d\left(a+bx^2\right)^{p+1} (c+dx)^{-2p-3}}{(2p+3) \left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 489$$

$$b\left(\frac{\left(\sqrt{-a}-\sqrt{b}x\right)(a+bx^2)^p(c+dx)^{-2p-1}(ad^2-bc^2(2p+3))\left(-\frac{\left(\sqrt{-a}+\sqrt{b}x\right)\left(\sqrt{-a}d+\sqrt{b}c\right)}{\left(\sqrt{-a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{-a}d\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,\frac{2\sqrt{-a}\cdots}{\left(\sqrt{b}c-\sqrt{-a}\cdots\right.}\right.}{(2p+1)\left(\sqrt{-a}d+\sqrt{b}c\right)(ad^2+bc^2)}\right.$$
$$\overline{\qquad (2p+3)(ad^2+bc^2)}$$

$$\frac{d(a+bx^2)^{p+1}(c+dx)^{-2p-3}}{(2p+3)(ad^2+bc^2)}$$

input
```
Int[(c + d*x)^(-4 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-((d*(c + d*x)^(-3 - 2*p)*(a + b*x^2)^(1 + p))/((b*c^2 + a*d^2)*(3 + 2*p))
) + (b*(-((c*d*(2 + p)*(a + b*x^2)^(1 + p))/((b*c^2 + a*d^2)*(1 + p)*(c +
d*x)^(2*(1 + p)))) + ((a*d^2 - b*c^2*(3 + 2*p))*(Sqrt[-a] - Sqrt[b]*x)*(c
+ d*x)^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*S
qrt[-a]*Sqrt[b]*(c + d*x))/((Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b]*x
))])/((Sqrt[b]*c + Sqrt[-a]*d)*(b*c^2 + a*d^2)*(1 + 2*p)*(-(((Sqrt[b]*c +
Sqrt[-a]*d)*(Sqrt[-a] + Sqrt[b]*x))/((Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] -
Sqrt[b]*x))))^p)))/((b*c^2 + a*d^2)*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int (dx+c)^{-4-2p} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^(-4-2*p)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-4-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-4} \, dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(-4-2*p)*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-4} \, dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Giac [F]

$$\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-4} \, dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-4-2p} \left(a+bx^2\right)^p \, dx = \int \frac{\left(b\,x^2+a\right)^p}{\left(c+d\,x\right)^{2\,p+4}} \, dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 4),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 4), x)
```

Reduce [F]

$$\int (c+dx)^{-4-2p}\left(a+bx^2\right)^p\,dx$$
$$=\int \frac{\left(b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}c^4+4\left(dx+c\right)^{2p}c^3dx+6\left(dx+c\right)^{2p}c^2d^2x^2+4\left(dx+c\right)^{2p}c\,d^3x^3+\left(dx+c\right)^{2p}d^4x^4}dx$$

input
```
int((d*x+c)^(-4-2*p)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**4 + 4*(c + d*x)**(2*p)*c**3*d*x +
 6*(c + d*x)**(2*p)*c**2*d**2*x**2 + 4*(c + d*x)**(2*p)*c*d**3*x**3 + (c +
 d*x)**(2*p)*d**4*x**4),x)
```

3.470 $\int (c+dx)^m (a-bx^2)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int (c+dx)^m (a-bx^2)^p \, dx = \frac{(c+dx)^{1+m}(a-bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d(1+m)}$$

output
```
(d*x+c)^(1+m)*(-b*x^2+a)^p*AppellF1(1+m,-p,-p,2+m,(d*x+c)/(c-a^(1/2)*d/b^(
1/2)),(d*x+c)/(c+a^(1/2)*d/b^(1/2)))/d/(1+m)/((1-(d*x+c)/(c-a^(1/2)*d/b^(1
/2)))^p)/((1-(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

$$\int (c+dx)^m (a-bx^2)^p \, dx = \frac{\left(\frac{d\left(\sqrt{\frac{a}{b}}-x\right)}{c+\sqrt{\frac{a}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{-c+\sqrt{\frac{a}{b}}d}\right)^{-p} (c+dx)^{1+m}(a-bx^2)^p \text{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c-\sqrt{\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{\frac{a}{b}}d}\right)}{d(1+m)}$$

input
```
Integrate[(c + d*x)^m*(a - b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(a - b*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (c + d*x)/
(c - Sqrt[a/b]*d), (c + d*x)/(c + Sqrt[a/b]*d)])/(d*(1 + m)*((d*(Sqrt[a/b]
 - x))/(c + Sqrt[a/b]*d))^p*((d*(Sqrt[a/b] + x))/(-c + Sqrt[a/b]*d))^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^p (c+dx)^m \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(a-bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^m \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(a-bx^2\right)^p (c+dx)^{m+1} \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(a - b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(a - b*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (c + d*x)/
(c - (Sqrt[a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])])/(d*(1 + m
)*(1 - (c + d*x)/(c - (Sqrt[a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[a]
*d)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{m} \left(-b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^m*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^m*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^m \left(a-bx^2\right)^p \, dx = \int \left(-bx^2+a\right)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p*(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c + dx)^m \left(a - bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(-b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c + dx)^m \left(a - bx^2\right)^p \, dx = \int \left(-bx^2 + a\right)^p (dx + c)^m \, dx$$

input
```
integrate((d*x+c)^m*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^m, x)
```

Giac [F]

$$\int (c + dx)^m \left(a - bx^2\right)^p \, dx = \int \left(-bx^2 + a\right)^p (dx + c)^m \, dx$$

input
```
integrate((d*x+c)^m*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \left(a-bx^2\right)^p \, dx=\int \left(a-b\,x^2\right)^p\,{\left(c+d\,x\right)}^m \,dx$$

input
```
int((a - b*x^2)^p*(c + d*x)^m,x)
```

output
```
int((a - b*x^2)^p*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \left(a-bx^2\right)^p \, dx=\text{too large to display}$$

input
```
int((d*x+c)^m*(-b*x^2+a)^p,x)
```

output

```
( - (c + d*x)**m*(a - b*x**2)**p*a*d + (c + d*x)**m*(a - b*x**2)**p*b*c*x
- int(((c + d*x)**m*(a - b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x
 + 2*a*d*p*x + a*d*x - b*c*m*x**2 - 2*b*c*p*x**2 - b*c*x**2 - b*d*m*x**3 -
 2*b*d*p*x**3 - b*d*x**3),x)*a*b*d**2*m**2 - 4*int(((c + d*x)**m*(a - b*x*
*2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*x - b*c*m*
x**2 - 2*b*c*p*x**2 - b*c*x**2 - b*d*m*x**3 - 2*b*d*p*x**3 - b*d*x**3),x)*
a*b*d**2*m*p - int(((c + d*x)**m*(a - b*x**2)**p*x**2)/(a*c*m + 2*a*c*p +
a*c + a*d*m*x + 2*a*d*p*x + a*d*x - b*c*m*x**2 - 2*b*c*p*x**2 - b*c*x**2 -
 b*d*m*x**3 - 2*b*d*p*x**3 - b*d*x**3),x)*a*b*d**2*m - 4*int(((c + d*x)**m
*(a - b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*
x - b*c*m*x**2 - 2*b*c*p*x**2 - b*c*x**2 - b*d*m*x**3 - 2*b*d*p*x**3 - b*d
*x**3),x)*a*b*d**2*p**2 - 2*int(((c + d*x)**m*(a - b*x**2)**p*x**2)/(a*c*m
 + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*x - b*c*m*x**2 - 2*b*c*p*x**2
 - b*c*x**2 - b*d*m*x**3 - 2*b*d*p*x**3 - b*d*x**3),x)*a*b*d**2*p - int(((
c + d*x)**m*(a - b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d
*p*x + a*d*x - b*c*m*x**2 - 2*b*c*p*x**2 - b*c*x**2 - b*d*m*x**3 - 2*b*d*p
*x**3 - b*d*x**3),x)*b**2*c**2*m**2 - 2*int(((c + d*x)**m*(a - b*x**2)**p*
x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x + 2*a*d*p*x + a*d*x - b*c*m*x**2 -
2*b*c*p*x**2 - b*c*x**2 - b*d*m*x**3 - 2*b*d*p*x**3 - b*d*x**3),x)*b**2*c*
*2*m*p - int(((c + d*x)**m*(a - b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c...
```

3.471 $\int (c+dx)^m (2-bx^2)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int (c+dx)^m (2-bx^2)^p \, dx$$
$$= \frac{(c+dx)^{1+m}(2-bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)^{-p} \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c-\frac{\sqrt{2}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{2}d}{\sqrt{b}}}\right)}{d(1+m)}$$

output
```
(d*x+c)^(1+m)*(-b*x^2+2)^p*AppellF1(1+m,-p,-p,2+m,(d*x+c)/(c-2^(1/2)*d/b^(
1/2)),(d*x+c)/(c+2^(1/2)*d/b^(1/2)))/d/(1+m)/((1-(d*x+c)/(c-2^(1/2)*d/b^(1
/2)))^p)/((1-(d*x+c)/(c+2^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19

$$\int (c+dx)^m (2-bx^2)^p \, dx$$
$$= \frac{\left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}-x\right)}{c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{2}\sqrt{\frac{1}{b}}+x\right)}{-c+\sqrt{2}\sqrt{\frac{1}{b}}d}\right)^{-p} (c+dx)^{1+m}(2-bx^2)^p \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{c+dx}{c-\sqrt{2}\sqrt{\frac{1}{b}}d}, \ldots\right)}{d(1+m)}$$

input
```
Integrate[(c + d*x)^m*(2 - b*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(2 - b*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (c + d*x)/
(c - Sqrt[2]*Sqrt[b^(-1)]*d), (c + d*x)/(c + Sqrt[2]*Sqrt[b^(-1)]*d)])/(d*
(1 + m)*((d*(Sqrt[2]*Sqrt[b^(-1)] - x))/(c + Sqrt[2]*Sqrt[b^(-1)]*d))^p*((
d*(Sqrt[2]*Sqrt[b^(-1)] + x))/(-c + Sqrt[2]*Sqrt[b^(-1)]*d))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-bx^2\right)^p (c+dx)^m \, dx$$

$$\Big\downarrow \text{513}$$

$$2^p \int \left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p \left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p (c+dx)^m dx$$

$$\Big\downarrow \text{156}$$

$$2^{p-\frac{m}{2}}(c+dx)^m \left(\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}\right)^{-m} \int \left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^p \left(\frac{\sqrt{b}x}{\sqrt{2}}+1\right)^p \left(\frac{\sqrt{2}\sqrt{b}c}{\sqrt{2}\sqrt{b}c+2d}+\frac{\sqrt{2}\sqrt{b}dx}{\sqrt{2}\sqrt{b}c+2d}\right)^m dx$$

$$\Big\downarrow \text{155}$$

$$-\frac{2^{-\frac{m}{2}+2p+\frac{1}{2}} \left(1-\frac{\sqrt{b}x}{\sqrt{2}}\right)^{p+1} (c+dx)^m \left(\frac{\sqrt{b}(c+dx)}{\sqrt{2}\sqrt{b}c+2d}\right)^{-m} \text{AppellF1}\left(p+1,-p,-m,p+2,\frac{\sqrt{2}-\sqrt{b}x}{2\sqrt{2}},\frac{d\left(2\sqrt{2}-2\sqrt{b}x\right)}{2\left(\sqrt{b}c+\sqrt{2}d\right)}\right)}{\sqrt{b}(p+1)}$$

input
```
Int[(c + d*x)^m*(2 - b*x^2)^p,x]
```

output
```
-((2^(1/2 - m/2 + 2*p)*(1 - (Sqrt[b]*x)/Sqrt[2])^(1 + p)*(c + d*x)^m*Appel
lF1[1 + p, -p, -m, 2 + p, (Sqrt[2] - Sqrt[b]*x)/(2*Sqrt[2]), (d*(2*Sqrt[2]
 - 2*Sqrt[b]*x))/(2*(Sqrt[b]*c + Sqrt[2]*d))])/(Sqrt[b]*(1 + p)*((Sqrt[b]*
(c + d*x))/(Sqrt[2]*Sqrt[b]*c + 2*d))^m))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int (dx+c)^m \left(-b\,x^2+2\right)^p dx$$

input
```
int((d*x+c)^m*(-b*x^2+2)^p,x)
```

output
```
int((d*x+c)^m*(-b*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^m \left(2-bx^2\right)^p \, dx = \int \left(-bx^2+2\right)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(-b*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + 2)^p*(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2-bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(-b*x**2+2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^m \left(2-bx^2\right)^p \, dx = \int \left(-bx^2+2\right)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(-b*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + 2)^p*(d*x + c)^m, x)
```

Giac [F]

$$\int (c+dx)^m \left(2-bx^2\right)^p \, dx = \int \left(-bx^2+2\right)^p (dx+c)^m \, dx$$

input
```
integrate((d*x+c)^m*(-b*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + 2)^p*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2-bx^2\right)^p \, dx = \int \left(2-b\,x^2\right)^p \, (c+d\,x)^m \, dx$$

input
```
int((2 - b*x^2)^p*(c + d*x)^m,x)
```

output
```
int((2 - b*x^2)^p*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \left(2-bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^m*(-b*x^2+2)^p,x)
```

output

```
((c + d*x)**m*( - b*x**2 + 2)**p*b*c*x - 2*(c + d*x)**m*( - b*x**2 + 2)**p
*d + int(((c + d*x)**m*( - b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2
 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3 - 2*c*m - 4*c*p - 2*c -
 2*d*m*x - 4*d*p*x - 2*d*x),x)*b**2*c**2*m**2 + 2*int(((c + d*x)**m*( - b*
x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*
b*d*p*x**3 + b*d*x**3 - 2*c*m - 4*c*p - 2*c - 2*d*m*x - 4*d*p*x - 2*d*x),x
)*b**2*c**2*m*p + int(((c + d*x)**m*( - b*x**2 + 2)**p*x**2)/(b*c*m*x**2 +
 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3 - 2*c*m -
4*c*p - 2*c - 2*d*m*x - 4*d*p*x - 2*d*x),x)*b**2*c**2*m + 2*int(((c + d*x)
**m*( - b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m
*x**3 + 2*b*d*p*x**3 + b*d*x**3 - 2*c*m - 4*c*p - 2*c - 2*d*m*x - 4*d*p*x
- 2*d*x),x)*b*d**2*m**2 + 8*int(((c + d*x)**m*( - b*x**2 + 2)**p*x**2)/(b*
c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3
- 2*c*m - 4*c*p - 2*c - 2*d*m*x - 4*d*p*x - 2*d*x),x)*b*d**2*m*p + 2*int((
(c + d*x)**m*( - b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**
2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d*x**3 - 2*c*m - 4*c*p - 2*c - 2*d*m*x -
 4*d*p*x - 2*d*x),x)*b*d**2*m + 8*int(((c + d*x)**m*( - b*x**2 + 2)**p*x**
2)/(b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**3 + 2*b*d*p*x**3 + b*d
*x**3 - 2*c*m - 4*c*p - 2*c - 2*d*m*x - 4*d*p*x - 2*d*x),x)*b*d**2*p**2 +
4*int(((c + d*x)**m*( - b*x**2 + 2)**p*x**2)/(b*c*m*x**2 + 2*b*c*p*x**2...
```

3.472 $\int (c+dx)^m (2-3x^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int (c+dx)^m (2-3x^2)^p \, dx = \frac{(c+dx)^{1+m} (2-3x^2)^p \left(1-\frac{3(c+dx)}{3c-\sqrt{6}d}\right)^{-p} \left(1-\frac{3(c+dx)}{3c+\sqrt{6}d}\right)^{-p} \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d(1+m)}$$

output
```
(d*x+c)^(1+m)*(-3*x^2+2)^p*AppellF1(1+m,-p,-p,2+m,3*(d*x+c)/(3*c-6^(1/2)*d
),3*(d*x+c)/(6^(1/2)*d+3*c))/d/(1+m)/((1-3*(d*x+c)/(3*c-6^(1/2)*d))^p)/((1
-3*(d*x+c)/(6^(1/2)*d+3*c))^p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02

$$\int (c+dx)^m (2-3x^2)^p \, dx = \frac{\left(\frac{d(\sqrt{6}-3x)}{3c+\sqrt{6}d}\right)^{-p} \left(\frac{d(\sqrt{6}+3x)}{-3c+\sqrt{6}d}\right)^{-p} (c+dx)^{1+m} (2-3x^2)^p \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{3(c+dx)}{3c-\sqrt{6}d},\frac{3(c+dx)}{3c+\sqrt{6}d}\right)}{d(1+m)}$$

input
```
Integrate[(c + d*x)^m*(2 - 3*x^2)^p,x]
```

output
```
((c + d*x)^(1 + m)*(2 - 3*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (3*(c + d*
x))/(3*c - Sqrt[6]*d), (3*(c + d*x))/(3*c + Sqrt[6]*d)])/(d*(1 + m)*((d*(S
qrt[6] - 3*x))/(3*c + Sqrt[6]*d))^p*((d*(Sqrt[6] + 3*x))/(-3*c + Sqrt[6]*d
))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {513, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-3x^2\right)^p (c+dx)^m \, dx$$

$$\Big\downarrow \text{513}$$

$$2^p \int \left(1-\sqrt{\frac{3}{2}}x\right)^p \left(\sqrt{\frac{3}{2}}x+1\right)^p (c+dx)^m dx$$

$$\Big\downarrow \text{156}$$

$$3^{-m/2} 2^{p-\frac{m}{2}} (c+dx)^m \left(\frac{c+dx}{\sqrt{6}c+2d}\right)^{-m} \int \left(1-\sqrt{\frac{3}{2}}x\right)^p \left(\sqrt{\frac{3}{2}}x+1\right)^p \left(\frac{\sqrt{6}c}{\sqrt{6}c+2d}+\frac{\sqrt{6}dx}{\sqrt{6}c+2d}\right)^m dx$$

$$\Big\downarrow \text{155}$$

$$-\frac{3^{-\frac{m}{2}-\frac{1}{2}} 2^{-\frac{m}{2}+2p+\frac{1}{2}} \left(1-\sqrt{\frac{3}{2}}x\right)^{p+1} (c+dx)^m \left(\frac{c+dx}{\sqrt{6}c+2d}\right)^{-m} \text{AppellF1}\left(p+1,-p,-m,p+2,\frac{1}{4}\left(2-\sqrt{6}x\right),\frac{\sqrt{\frac{3}{2}}d(\ldots}{3c+\ldots}\right.}{p+1}$$

input
```
Int[(c + d*x)^m*(2 - 3*x^2)^p,x]
```

output
```
-((2^(1/2 - m/2 + 2*p)*3^(-1/2 - m/2)*(1 - Sqrt[3/2]*x)^(1 + p)*(c + d*x)^
m*AppellF1[1 + p, -p, -m, 2 + p, (2 - Sqrt[6]*x)/4, (Sqrt[3/2]*d*(2 - Sqrt
[6]*x))/(3*c + Sqrt[6]*d)])/((1 + p)*((c + d*x)/(Sqrt[6]*c + 2*d))^m))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 513
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_^2))^(p_), x_Symbol] :> Simp[
a^p   Int[(c + d*x)^n*(1 + Rt[-b/a, 2]*x)^p*(1 - Rt[-b/a, 2]*x)^p, x], x] /
; FreeQ[{a, b, c, d, n, p}, x] && GtQ[a, 0] && NegQ[b/a]
```

Maple [F]

$$\int (dx+c)^m \left(-3x^2+2\right)^p dx$$

input
```
int((d*x+c)^m*(-3*x^2+2)^p,x)
```

output
```
int((d*x+c)^m*(-3*x^2+2)^p,x)
```

Fricas [F]

$$\int (c+dx)^m \left(2-3x^2\right)^p \, dx = \int (dx+c)^m \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^m*(-3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(-3*x^2 + 2)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2-3x^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(-3*x**2+2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^m \left(2-3x^2\right)^p \, dx = \int (dx+c)^m \left(-3\,x^2+2\right)^p \, dx$$

input
```
integrate((d*x+c)^m*(-3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(-3*x^2 + 2)^p, x)
```

Giac [F]

$$\int (c+dx)^m \left(2-3x^2\right)^p \, dx = \int (dx+c)^m (-3x^2+2)^p \, dx$$

input
```
integrate((d*x+c)^m*(-3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(-3*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \left(2-3x^2\right)^p \, dx = \int (2-3x^2)^p (c+dx)^m \, dx$$

input
```
int((2 - 3*x^2)^p*(c + d*x)^m,x)
```

output
```
int((2 - 3*x^2)^p*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \left(2-3x^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*(-3*x^2+2)^p,x)
```

output
```
(3*(c + d*x)**m*( - 3*x**2 + 2)**p*c*x - 2*(c + d*x)**m*( - 3*x**2 + 2)**p
*d + 9*int(((c + d*x)**m*( - 3*x**2 + 2)**p*x**2)/(3*c*m*x**2 - 2*c*m + 6*
c*p*x**2 - 4*c*p + 3*c*x**2 - 2*c + 3*d*m*x**3 - 2*d*m*x + 6*d*p*x**3 - 4*
d*p*x + 3*d*x**3 - 2*d*x),x)*c**2*m**2 + 18*int(((c + d*x)**m*( - 3*x**2 +
 2)**p*x**2)/(3*c*m*x**2 - 2*c*m + 6*c*p*x**2 - 4*c*p + 3*c*x**2 - 2*c + 3
*d*m*x**3 - 2*d*m*x + 6*d*p*x**3 - 4*d*p*x + 3*d*x**3 - 2*d*x),x)*c**2*m*p
 + 9*int(((c + d*x)**m*( - 3*x**2 + 2)**p*x**2)/(3*c*m*x**2 - 2*c*m + 6*c*
p*x**2 - 4*c*p + 3*c*x**2 - 2*c + 3*d*m*x**3 - 2*d*m*x + 6*d*p*x**3 - 4*d*
p*x + 3*d*x**3 - 2*d*x),x)*c**2*m + 6*int(((c + d*x)**m*( - 3*x**2 + 2)**p
*x**2)/(3*c*m*x**2 - 2*c*m + 6*c*p*x**2 - 4*c*p + 3*c*x**2 - 2*c + 3*d*m*x
**3 - 2*d*m*x + 6*d*p*x**3 - 4*d*p*x + 3*d*x**3 - 2*d*x),x)*d**2*m**2 + 24
*int(((c + d*x)**m*( - 3*x**2 + 2)**p*x**2)/(3*c*m*x**2 - 2*c*m + 6*c*p*x*
*2 - 4*c*p + 3*c*x**2 - 2*c + 3*d*m*x**3 - 2*d*m*x + 6*d*p*x**3 - 4*d*p*x
+ 3*d*x**3 - 2*d*x),x)*d**2*m*p + 6*int(((c + d*x)**m*( - 3*x**2 + 2)**p*x
**2)/(3*c*m*x**2 - 2*c*m + 6*c*p*x**2 - 4*c*p + 3*c*x**2 - 2*c + 3*d*m*x**
3 - 2*d*m*x + 6*d*p*x**3 - 4*d*p*x + 3*d*x**3 - 2*d*x),x)*d**2*m + 24*int(
((c + d*x)**m*( - 3*x**2 + 2)**p*x**2)/(3*c*m*x**2 - 2*c*m + 6*c*p*x**2 -
4*c*p + 3*c*x**2 - 2*c + 3*d*m*x**3 - 2*d*m*x + 6*d*p*x**3 - 4*d*p*x + 3*d
*x**3 - 2*d*x),x)*d**2*p**2 + 12*int(((c + d*x)**m*( - 3*x**2 + 2)**p*x**2
)/(3*c*m*x**2 - 2*c*m + 6*c*p*x**2 - 4*c*p + 3*c*x**2 - 2*c + 3*d*m*x**...
```

3.473 $\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 153

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = \frac{(c+dx)^{1-2p} \left(a-bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d(1-2p)}$$

output
```
(d*x+c)^(1-2*p)*(-b*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,(d*x+c)/(c-a^(1/2)
*d/b^(1/2)),(d*x+c)/(c+a^(1/2)*d/b^(1/2)))/d/(1-2*p)/((1-(d*x+c)/(c-a^(1/2
)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = -\frac{\left(\frac{d\left(\sqrt{\frac{a}{b}}-x\right)}{c+\sqrt{\frac{a}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{-c+\sqrt{\frac{a}{b}}d}\right)^{-p} (c+dx)^{1-2p} \left(a-bx^2\right)^p \text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c+dx}{c-\sqrt{\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{\frac{a}{b}}}\right)}{d(-1+2p)}$$

input
```
Integrate[(a - b*x^2)^p/(c + d*x)^(2*p),x]
```

output
```
-(((c + d*x)^(1 - 2*p)*(a - b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c
 + d*x)/(c - Sqrt[a/b]*d), (c + d*x)/(c + Sqrt[a/b]*d)])/(d*(-1 + 2*p)*((d
*(Sqrt[a/b] - x))/(c + Sqrt[a/b]*d))^p*((d*(Sqrt[a/b] + x))/(-c + Sqrt[a/b
]*d))^p))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^2)^p (c+dx)^{-2p} \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(a-bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^{-2p} \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{(a-bx^2)^p (c+dx)^{1-2p} \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{a}d}{\sqrt{b}}+c}\right)^{-p} \text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{d(1-2p)}$$

input
```
Int[(a - b*x^2)^p/(c + d*x)^(2*p),x]
```

output
```
((c + d*x)^(1 - 2*p)*(a - b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c +
 d*x)/(c - (Sqrt[a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])])/(d*
(1 - 2*p)*(1 - (c + d*x)/(c - (Sqrt[a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c +
(Sqrt[a]*d)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \left(-b\,x^{2}+a \right)^{p} \left(d x +c \right)^{-2 p}d x$$

input
```
int((-b*x^2+a)^p/((d*x+c)^(2*p)),x)
```

output
```
int((-b*x^2+a)^p/((d*x+c)^(2*p)),x)
```

Fricas [F]

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = \int \frac{(-bx^2+a)^p}{(dx+c)^{2\,p}} \, dx$$

input
```
integrate((-b*x^2+a)^p/((d*x+c)^(2*p)),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p/(d*x + c)^(2*p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((-b*x**2+a)**p/((d*x+c)**(2*p)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = \int \frac{(-bx^2+a)^p}{(dx+c)^{2p}} \, dx$$

input
```
integrate((-b*x^2+a)^p/((d*x+c)^(2*p)),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^(2*p), x)
```

Giac [F]

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = \int \frac{(-bx^2+a)^p}{(dx+c)^{2p}} \, dx$$

input
```
integrate((-b*x^2+a)^p/((d*x+c)^(2*p)),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p/(d*x + c)^(2*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = \int \frac{\left(a-b\,x^2\right)^p}{\left(c+d\,x\right)^{2\,p}} \,dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^(2*p),x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^(2*p), x)
```

Reduce [F]

$$\int (c+dx)^{-2p} \left(a-bx^2\right)^p \, dx = \int \frac{\left(-b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}}dx$$

input
```
int((-b*x^2+a)^p/((d*x+c)^(2*p)),x)
```

output
```
int((a - b*x**2)**p/(c + d*x)**(2*p),x)
```

3.474 $\int (c+dx)^{-1-2p} \left(a-bx^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int (c+dx)^{-1-2p} \left(a-bx^2\right)^p \, dx = -\frac{(c+dx)^{-2p} \left(a-bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)^{-p} \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}\right)}{2dp}$$

output
```
-1/2*(-b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(d*x+c)/(c-a^(1/2)*d/b^(1/2)),
(d*x+c)/(c+a^(1/2)*d/b^(1/2)))/d/p/((d*x+c)^(2*p))/((1-(d*x+c)/(c-a^(1/2)*
d/b^(1/2)))^p)/((1-(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int (c+dx)^{-1-2p} \left(a-bx^2\right)^p \, dx = -\frac{\left(\frac{d\left(\sqrt{\frac{a}{b}}-x\right)}{c+\sqrt{\frac{a}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{\frac{a}{b}}+x\right)}{-c+\sqrt{\frac{a}{b}}d}\right)^{-p} (c+dx)^{-2p} \left(a-bx^2\right)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\sqrt{\frac{a}{b}}d},\frac{c+dx}{c+\sqrt{\frac{a}{b}}d}\right)}{2dp}$$

input
```
Integrate[(c + d*x)^(-1 - 2*p)*(a - b*x^2)^p,x]
```

output
```
-1/2*((a - b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - Sqrt[a/
b]*d), (c + d*x)/(c + Sqrt[a/b]*d)])/(d*p*((d*(Sqrt[a/b] - x))/(c + Sqrt[a
/b]*d))^p*((d*(Sqrt[a/b] + x))/(-c + Sqrt[a/b]*d))^p*(c + d*x)^(2*p))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^2)^p (c+dx)^{-2p-1} \, dx$$

$$\Big\downarrow \ 514$$

$$\frac{(a-bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{ad}}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^{-2p-1} \left(1-\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}}\right)^p \left(1-\frac{c+dx}{c+\frac{\sqrt{ad}}{\sqrt{b}}}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \ 150$$

$$-\frac{(a-bx^2)^p (c+dx)^{-2p} \left(1-\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{ad}}{\sqrt{b}}+c}\right)^{-p} \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{ad}}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{ad}}{\sqrt{b}}}\right)}{2dp}$$

input
```
Int[(c + d*x)^(-1 - 2*p)*(a - b*x^2)^p,x]
```

output
```
-1/2*((a - b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - (Sqrt[a
]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[a]*d)/Sqrt[b])])/(d*p*(c + d*x)^(2*p)*
(1 - (c + d*x)/(c - (Sqrt[a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[a]*d
)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{-1-2p} \left(-b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^(-1-2*p)*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-1-2*p)*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-1-2p} \left(a-bx^2\right)^p \, dx = \int \left(-bx^2+a\right)^p (dx+c)^{-2\,p-1} \, dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Sympy [F]

$$\int (c+dx)^{-1-2p}\left(a-bx^2\right)^p\,dx=\int \left(a-bx^2\right)^p(c+dx)^{-2p-1}\,dx$$

input
```
integrate((d*x+c)**(-1-2*p)*(-b*x**2+a)**p,x)
```

output
```
Integral((a - b*x**2)**p*(c + d*x)**(-2*p - 1), x)
```

Maxima [F]

$$\int (c+dx)^{-1-2p}\left(a-bx^2\right)^p\,dx=\int (-bx^2+a)^p(dx+c)^{-2\,p-1}\,dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Giac [F]

$$\int (c+dx)^{-1-2p}\left(a-bx^2\right)^p\,dx=\int (-bx^2+a)^p(dx+c)^{-2\,p-1}\,dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-1-2p} \left(a-bx^2\right)^p \, dx = \int \frac{\left(a-b\,x^2\right)^p}{\left(c+d\,x\right)^{2\,p+1}} \, dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 1),x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 1), x)
```

Reduce [F]

$$\int (c+dx)^{-1-2p} \left(a-bx^2\right)^p \, dx = \int \frac{\left(-b\,x^2+a\right)^p}{\left(dx+c\right)^{2p} c+\left(dx+c\right)^{2p} dx} dx$$

input
```
int((d*x+c)^(-1-2*p)*(-b*x^2+a)^p,x)
```

output
```
int((a - b*x**2)**p/((c + d*x)**(2*p)*c + (c + d*x)**(2*p)*d*x),x)
```

3.475 $\int (c+dx)^{-2-2p} \left(a-bx^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\int (c+dx)^{-2-2p} \left(a-bx^2\right)^p \, dx$$
$$= \frac{\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\left(\sqrt{a}-\sqrt{b}x\right)}{\left(\sqrt{b}c+\sqrt{a}d\right)\left(\sqrt{a}+\sqrt{b}x\right)}\right)^{-p} \left(\sqrt{a}+\sqrt{b}x\right)(c+dx)^{-1-2p}\left(a-bx^2\right)^p \text{Hypergeometric2F1}\left(-1-2p,-p,-2\right.}{\left(\sqrt{b}c-\sqrt{a}d\right)(1+2p)}$$

output
```
(a^(1/2)+b^(1/2)*x)*(d*x+c)^(-1-2*p)*(-b*x^2+a)^p*hypergeom([-p, -1-2*p],[
-2*p],2*a^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d)/(a^(1/2)+b^(1/2)*x))
/(b^(1/2)*c-a^(1/2)*d)/(1+2*p)/((-(b^(1/2)*c-a^(1/2)*d)*(-b^(1/2)*x+a^(1/2
))/(b^(1/2)*c+a^(1/2)*d)/(a^(1/2)+b^(1/2)*x))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97

$$\int (c+dx)^{-2-2p} \left(a-bx^2\right)^p \, dx$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}x\right)(c+dx)^{-1-2p}\left(a-bx^2\right)^p \left(1+\frac{b(c+dx)}{-bc+\sqrt{a}\sqrt{b}d}\right)^p \left(1-\frac{b(c+dx)}{bc+\sqrt{a}\sqrt{b}d}\right)^{-p} \text{Hypergeometric2F1}\left(-1-\right.}{\left(\sqrt{b}c-\sqrt{a}d\right)(1+2p)}$$

input
```
Integrate[(c + d*x)^(-2 - 2*p)*(a - b*x^2)^p,x]
```

output
```
((Sqrt[a] + Sqrt[b]*x)*(c + d*x)^(-1 - 2*p)*(a - b*x^2)^p*(1 + (b*(c + d*x
))/(-(b*c) + Sqrt[a]*Sqrt[b]*d))^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (
2*Sqrt[a]*Sqrt[b]*(c + d*x))/((Sqrt[b]*c + Sqrt[a]*d)*(Sqrt[a] + Sqrt[b]*x
))])/((Sqrt[b]*c - Sqrt[a]*d)*(1 + 2*p)*(1 - (b*(c + d*x))/(b*c + Sqrt[a]*
Sqrt[b]*d))^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{489\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^2)^p (c+dx)^{-2p-2}\,dx$$

$$\Big\downarrow \ 489$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x\right)(a-bx^2)^p (c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{a}d\right)}{\left(\sqrt{a}+\sqrt{b}x\right)\left(\sqrt{a}d+\sqrt{b}c\right)}\right)^{-p} \text{Hypergeometric2F1}\left(-2p-1,-p,-2p,\frac{}{(\sqrt{b}}\right.}{(2p+1)\left(\sqrt{b}c-\sqrt{a}d\right)}$$

input
```
Int[(c + d*x)^(-2 - 2*p)*(a - b*x^2)^p,x]
```

output
```
((Sqrt[a] + Sqrt[b]*x)*(c + d*x)^(-1 - 2*p)*(a - b*x^2)^p*Hypergeometric2F
1[-1 - 2*p, -p, -2*p, (2*Sqrt[a]*Sqrt[b]*(c + d*x))/((Sqrt[b]*c + Sqrt[a]*
d)*(Sqrt[a] + Sqrt[b]*x))])/((Sqrt[b]*c - Sqrt[a]*d)*(1 + 2*p)*(-(((Sqrt[b
]*c - Sqrt[a]*d)*(Sqrt[a] - Sqrt[b]*x))/((Sqrt[b]*c + Sqrt[a]*d)*(Sqrt[a]
+ Sqrt[b]*x))))^p)
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

Maple [F]

$$\int (dx+c)^{-2p-2} \left(-b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^(-2*p-2)*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-2*p-2)*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-2-2p} \left(a-bx^2\right)^p \, dx = \int \left(-bx^2+a\right)^p (dx+c)^{-2\,p-2} \, dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-2-2p}\left(a-bx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(-2-2*p)*(-b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-2-2p}\left(a-bx^2\right)^p\,dx=\int (-bx^2+a)^p(dx+c)^{-2\,p-2}\,dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Giac [F]

$$\int (c+dx)^{-2-2p}\left(a-bx^2\right)^p\,dx=\int (-bx^2+a)^p(dx+c)^{-2\,p-2}\,dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-2-2p}\left(a-bx^2\right)^p\,dx=\int \frac{\left(a-b\,x^2\right)^p}{\left(c+d\,x\right)^{2\,p+2}}\,dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 2),x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 2), x)
```

Reduce [F]

$$\int (c+dx)^{-2-2p}\left(a-bx^2\right)^p\,dx=\int \frac{\left(-b\,x^2+a\right)^p}{(dx+c)^{2p}\,c^2+2\,(dx+c)^{2p}\,cdx+(dx+c)^{2p}\,d^2x^2}dx$$

input
```
int((d*x+c)^(-2-2*p)*(-b*x^2+a)^p,x)
```

output
```
int((a - b*x**2)**p/((c + d*x)**(2*p)*c**2 + 2*(c + d*x)**(2*p)*c*d*x + (c
 + d*x)**(2*p)*d**2*x**2),x)
```

3.476 $\int (c+dx)^{-3-2p}\left(a-bx^2\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 256

$$\int (c+dx)^{-3-2p}\left(a-bx^2\right)^p\,dx=\frac{d(c+dx)^{-2(1+p)}\left(a-bx^2\right)^{1+p}}{2\left(bc^2-ad^2\right)(1+p)}$$

$$-\frac{bc\left(\sqrt{a}-\sqrt{b}x\right)\left(-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\left(\sqrt{a}+\sqrt{b}x\right)}{\left(\sqrt{b}c-\sqrt{a}d\right)\left(\sqrt{a}-\sqrt{b}x\right)}\right)^{-p}(c+dx)^{-1-2p}\left(a-bx^2\right)^p\,\text{Hypergeometric2F1}\left(-1-2p,-\right.}{\left(\sqrt{b}c+\sqrt{a}d\right)\left(bc^2-ad^2\right)(1+2p)}$$

output
```
1/2*d*(-b*x^2+a)^(p+1)/(-a*d^2+b*c^2)/(p+1)/((d*x+c)^(2*p+2))-b*c*(-b^(1/2
)*x+a^(1/2))*(d*x+c)^(-1-2*p)*(-b*x^2+a)^p*hypergeom([-p, -1-2*p],[-2*p],2
*a^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-a^(1/2)*d)/(-b^(1/2)*x+a^(1/2)))/(b^(1
/2)*c+a^(1/2)*d)/(-a*d^2+b*c^2)/(1+2*p)/((-(b^(1/2)*c+a^(1/2)*d)*(a^(1/2)+
b^(1/2)*x)/(b^(1/2)*c-a^(1/2)*d)/(-b^(1/2)*x+a^(1/2)))^p)
```

Mathematica [F]

$$\int (c+dx)^{-3-2p}\left(a-bx^2\right)^p\,dx=\int (c+dx)^{-3-2p}\left(a-bx^2\right)^p\,dx$$

input
```
Integrate[(c + d*x)^(-3 - 2*p)*(a - b*x^2)^p,x]
```

output
```
Integrate[(c + d*x)^(-3 - 2*p)*(a - b*x^2)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {491, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^p (c+dx)^{-2p-3}\,dx$$

$$\Big\downarrow \text{491}$$

$$\frac{bc\int (c+dx)^{-2(p+1)}\left(a-bx^2\right)^p dx}{bc^2-ad^2}+\frac{d\left(a-bx^2\right)^{p+1}(c+dx)^{-2(p+1)}}{2(p+1)\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{489}$$

$$\frac{bc\left(\sqrt{a}+\sqrt{b}x\right)\left(a-bx^2\right)^p(c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{a}d\right)}{\left(\sqrt{a}+\sqrt{b}x\right)\left(\sqrt{a}d+\sqrt{b}c\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,\right.}{(2p+1)\left(\sqrt{b}c-\sqrt{a}d\right)\left(bc^2-ad^2\right)}$$
$$\frac{d\left(a-bx^2\right)^{p+1}(c+dx)^{-2(p+1)}}{2(p+1)\left(bc^2-ad^2\right)}$$

input
```
Int[(c + d*x)^(-3 - 2*p)*(a - b*x^2)^p,x]
```

output
```
(d*(a - b*x^2)^(1 + p))/(2*(b*c^2 - a*d^2)*(1 + p)*(c + d*x)^(2*(1 + p)))
+ (b*c*(Sqrt[a] + Sqrt[b]*x)*(c + d*x)^(-1 - 2*p)*(a - b*x^2)^p*Hypergeome
tric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[a]*Sqrt[b]*(c + d*x))/((Sqrt[b]*c + Sq
rt[a]*d)*(Sqrt[a] + Sqrt[b]*x))])/((Sqrt[b]*c - Sqrt[a]*d)*(b*c^2 - a*d^2)
*(1 + 2*p)*(-(((Sqrt[b]*c - Sqrt[a]*d)*(Sqrt[a] - Sqrt[b]*x))/((Sqrt[b]*c
+ Sqrt[a]*d)*(Sqrt[a] + Sqrt[b]*x))))^p)
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

Maple [F]

$$\int (dx + c)^{-3-2p} \left(-b\,x^2 + a\right)^p dx$$

input
```
int((d*x+c)^(-3-2*p)*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-3-2*p)*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-3-2p}\left(a-bx^2\right)^p\,dx=\int \left(-bx^2+a\right)^p(dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p}\left(a-bx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(-3-2*p)*(-b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-3-2p}\left(a-bx^2\right)^p\,dx=\int \left(-bx^2+a\right)^p(dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Giac [F]

$$\int (c+dx)^{-3-2p} \left(a-bx^2\right)^p \, dx = \int (-bx^2+a)^p (dx+c)^{-2\,p-3} \, dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p} \left(a-bx^2\right)^p \, dx = \int \frac{\left(a-b\,x^2\right)^p}{\left(c+d\,x\right)^{2\,p+3}} \, dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 3),x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 3), x)
```

Reduce [F]

$$\int (c+dx)^{-3-2p} \left(a-bx^2\right)^p \, dx$$
$$= \int \frac{\left(-b\,x^2+a\right)^p}{\left(dx+c\right)^{2p} c^3+3\left(dx+c\right)^{2p} c^2dx+3\left(dx+c\right)^{2p} c\,d^2x^2+\left(dx+c\right)^{2p} d^3x^3}dx$$

input
```
int((d*x+c)^(-3-2*p)*(-b*x^2+a)^p,x)
```

output
```
int((a - b*x**2)**p/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*x +
 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)
```

3.477 $\int (c+dx)^{-4-2p} \left(a-bx^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 335

$$\int (c+dx)^{-4-2p} \left(a-bx^2\right)^p \, dx = \frac{d(c+dx)^{-3-2p}\left(a-bx^2\right)^{1+p}}{\left(bc^2-ad^2\right)(3+2p)}+\frac{bcd(2+p)(c+dx)^{-2(1+p)}\left(a-bx^2\right)^{1+p}}{\left(bc^2-ad^2\right)^2(1+p)(3+2p)} - \frac{b\left(ad^2+bc^2(3+2p)\right)\left(\sqrt{a}-\sqrt{b}x\right)\left(-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\left(\sqrt{a}+\sqrt{b}x\right)}{\left(\sqrt{b}c-\sqrt{a}d\right)\left(\sqrt{a}-\sqrt{b}x\right)}\right)^{-p}(c+dx)^{-1-2p}\left(a-bx^2\right)^p \text{Hypergeometri}}{\left(\sqrt{b}c-\sqrt{a}d\right)^2\left(\sqrt{b}c+\sqrt{a}d\right)^3\left(3+8p+4p^2\right)}$$

output
```
d*(d*x+c)^(-3-2*p)*(-b*x^2+a)^(p+1)/(-a*d^2+b*c^2)/(3+2*p)+b*c*d*(2+p)*(-b
*x^2+a)^(p+1)/(-a*d^2+b*c^2)^2/(p+1)/(3+2*p)/((d*x+c)^(2*p+2))-b*(a*d^2+b*
c^2*(3+2*p))*(-b^(1/2)*x+a^(1/2))*(d*x+c)^(-1-2*p)*(-b*x^2+a)^p*hypergeom(
[-p, -1-2*p],[-2*p],2*a^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-a^(1/2)*d)/(-b^(1
/2)*x+a^(1/2)))/(b^(1/2)*c-a^(1/2)*d)^2/(b^(1/2)*c+a^(1/2)*d)^3/(4*p^2+8*p
+3)/((-(b^(1/2)*c+a^(1/2)*d)*(a^(1/2)+b^(1/2)*x)/(b^(1/2)*c-a^(1/2)*d)/(-b
^(1/2)*x+a^(1/2)))^p)
```

Mathematica [F]

$$\int (c+dx)^{-4-2p} \left(a-bx^2\right)^p \, dx = \int (c+dx)^{-4-2p} \left(a-bx^2\right)^p \, dx$$

input
```
Integrate[(c + d*x)^(-4 - 2*p)*(a - b*x^2)^p,x]
```

output
```
Integrate[(c + d*x)^(-4 - 2*p)*(a - b*x^2)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {498, 25, 679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^p (c+dx)^{-2p-4} dx$$

$$\Big\downarrow \text{ 498}$$

$$\frac{d\left(a-bx^2\right)^{p+1} (c+dx)^{-2p-3}}{(2p+3) \left(bc^2-ad^2\right)}-\frac{b\int -\left((c(2p+3)-dx)(c+dx)^{-2p-3} \left(a-bx^2\right)^p\right)dx}{(2p+3) \left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\int (c(2p+3)-dx)(c+dx)^{-2p-3} \left(a-bx^2\right)^p dx}{(2p+3) \left(bc^2-ad^2\right)}+\frac{d\left(a-bx^2\right)^{p+1} (c+dx)^{-2p-3}}{(2p+3) \left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 679}$$

$$\frac{b\left(\frac{\left(ad^2+bc^2(2p+3)\right)\int (c+dx)^{-2(p+1)}\left(a-bx^2\right)^p dx}{bc^2-ad^2}+\frac{cd(p+2)\left(a-bx^2\right)^{p+1}(c+dx)^{-2(p+1)}}{(p+1)\left(bc^2-ad^2\right)}\right)}{(2p+3) \left(bc^2-ad^2\right)}+\frac{d\left(a-bx^2\right)^{p+1} (c+dx)^{-2p-3}}{(2p+3) \left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 489}$$

$$b\left(\frac{\left(\sqrt{a}+\sqrt{b}x\right)(a-bx^2)^p(c+dx)^{-2p-1}(ad^2+bc^2(2p+3))\left(-\frac{\left(\sqrt{a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{a}d\right)}{\left(\sqrt{a}+\sqrt{b}x\right)\left(\sqrt{a}d+\sqrt{b}c\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,\frac{2\sqrt{a}\sqrt{b}(c+dx}{\left(\sqrt{b}c+\sqrt{a}d\right)\left(\sqrt{b}x\right.}\right.}{(2p+1)\left(\sqrt{b}c-\sqrt{a}d\right)(bc^2-ad^2)}\right.$$
$$\overline{\qquad\qquad (2p+3)\,(bc^2-ad^2)}$$

$$\frac{d(a-bx^2)^{p+1}(c+dx)^{-2p-3}}{(2p+3)\,(bc^2-ad^2)}$$

input
```
Int[(c + d*x)^(-4 - 2*p)*(a - b*x^2)^p,x]
```

output
```
(d*(c + d*x)^(-3 - 2*p)*(a - b*x^2)^(1 + p))/((b*c^2 - a*d^2)*(3 + 2*p)) +
 (b*((c*d*(2 + p)*(a - b*x^2)^(1 + p))/((b*c^2 - a*d^2)*(1 + p)*(c + d*x)^
(2*(1 + p))) + ((a*d^2 + b*c^2*(3 + 2*p))*(Sqrt[a] + Sqrt[b]*x)*(c + d*x)^
(-1 - 2*p)*(a - b*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[a]*
Sqrt[b]*(c + d*x))/((Sqrt[b]*c + Sqrt[a]*d)*(Sqrt[a] + Sqrt[b]*x))])/((Sqr
t[b]*c - Sqrt[a]*d)*(b*c^2 - a*d^2)*(1 + 2*p)*(-(((Sqrt[b]*c - Sqrt[a]*d)*
(Sqrt[a] - Sqrt[b]*x))/((Sqrt[b]*c + Sqrt[a]*d)*(Sqrt[a] + Sqrt[b]*x))))^p
)))/((b*c^2 - a*d^2)*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int (dx+c)^{-4-2p} \left(-b\,x^{2}+a\right)^{p} dx$$

input
```
int((d*x+c)^(-4-2*p)*(-b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-4-2*p)*(-b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-4-2p} \left(a-bx^2\right)^p \,dx = \int \left(-bx^{2}+a\right)^{p} (dx+c)^{-2\,p-4} \,dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(-b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-4-2p} \left(a-bx^2\right)^p \,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(-4-2*p)*(-b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-4-2p}\left(a-bx^2\right)^p\,dx=\int \left(-bx^2+a\right)^p(dx+c)^{-2p-4}\,dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(-b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Giac [F]

$$\int (c+dx)^{-4-2p}\left(a-bx^2\right)^p\,dx=\int \left(-bx^2+a\right)^p(dx+c)^{-2p-4}\,dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(-b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-4-2p}\left(a-bx^2\right)^p\,dx=\int \frac{\left(a-b\,x^2\right)^p}{\left(c+d\,x\right)^{2\,p+4}}\,dx$$

input
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 4),x)
```

output
```
int((a - b*x^2)^p/(c + d*x)^(2*p + 4), x)
```

Reduce [F]

$$\int (c+dx)^{-4-2p} \left(a-bx^2\right)^p \, dx$$
$$= \int \frac{(-b\,x^2+a)^p}{(dx+c)^{2p}\,c^4+4\,(dx+c)^{2p}\,c^3dx+6\,(dx+c)^{2p}\,c^2d^2x^2+4\,(dx+c)^{2p}\,c\,d^3x^3+(dx+c)^{2p}\,d^4x^4}dx$$

input
```
int((d*x+c)^(-4-2*p)*(-b*x^2+a)^p,x)
```

output
```
int((a - b*x**2)**p/((c + d*x)**(2*p)*c**4 + 4*(c + d*x)**(2*p)*c**3*d*x +
 6*(c + d*x)**(2*p)*c**2*d**2*x**2 + 4*(c + d*x)**(2*p)*c*d**3*x**3 + (c +
 d*x)**(2*p)*d**4*x**4),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file