Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.5-Nested-general-binomial/77-1.1.5.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**386**]. This is test number [77].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	84.20 (325)	15.80 (61)
Mathematica	83.42 (322)	16.58 (64)
Fricas	63.47 (245)	36.53 (141)
Maple	59.59 (230)	40.41 (156)
Reduce	54.15 (209)	45.85 (177)
Giac	46.63 (180)	53.37 (206)
Maxima	29.27 (113)	70.73 (273)
Mupad	27.72 (107)	72.28 (279)
Sympy	20.47 (79)	79.53 (307)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

<table>
<tr><th>grade</th><th>description</th></tr>
<tr><td>A</td><td>Integral was solved and antiderivative is optimal in quality and leaf size.</td></tr>
<tr><td>B</td><td>Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.</td></tr>
<tr><td>C</td><td>Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons
1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.
2. antiderivative contains a special function and the optimal antiderivative does not.
3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</td></tr>
<tr><td>F</td><td>Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.</td></tr>
</table>

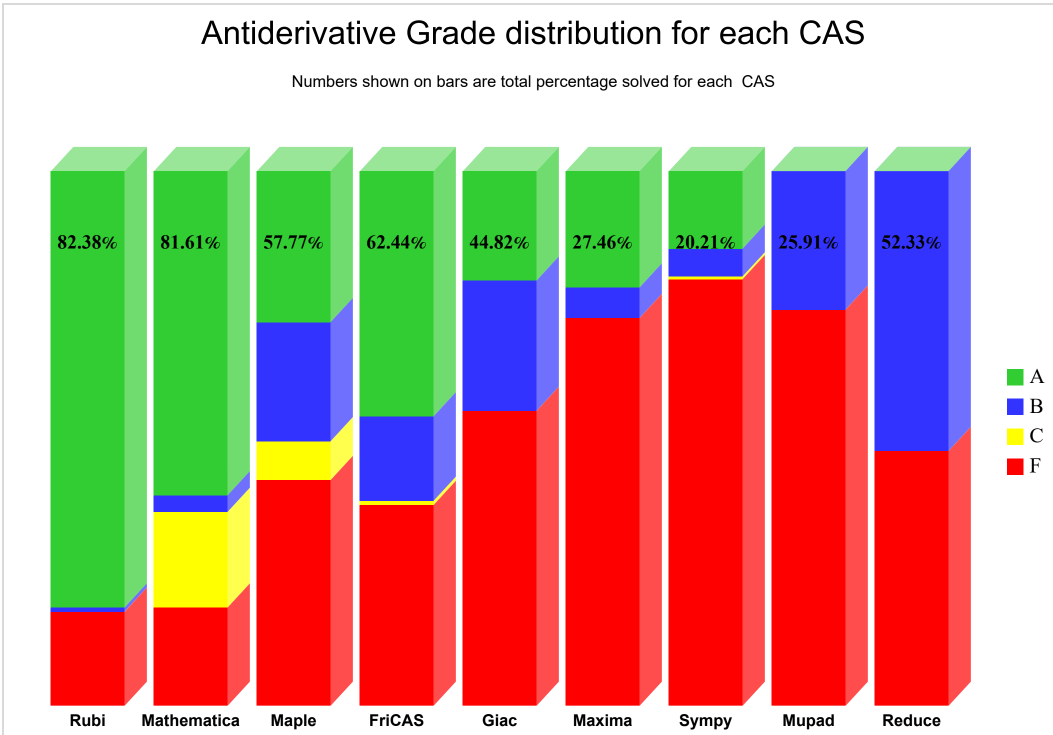
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

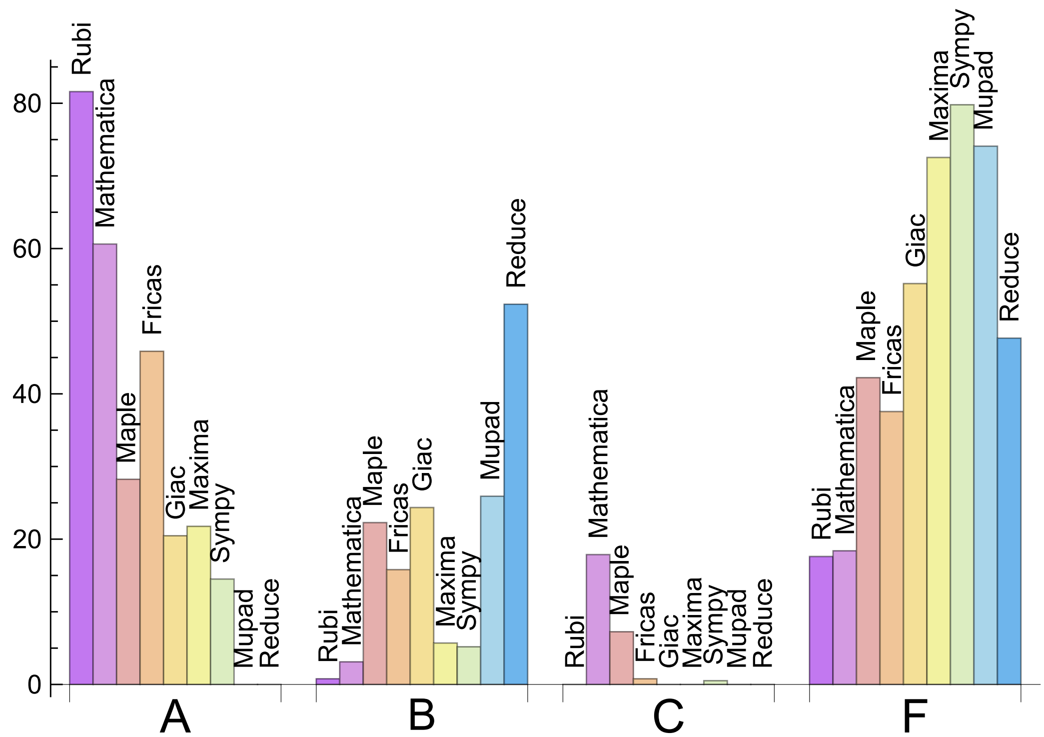
System	% A grade	% B grade	% C grade	% F grade
Rubi	81.606	0.777	0.000	17.617
Mathematica	60.622	3.109	17.876	18.394
Fricas	45.855	15.803	0.777	37.565
Maple	28.238	22.280	7.254	42.228
Maxima	21.762	5.699	0.000	72.539
Giac	20.466	24.352	0.000	55.181
Sympy	14.508	5.181	0.518	79.793
Mupad	0.000	25.907	0.000	74.093
Reduce	0.000	52.332	0.000	47.668

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	61	100.00	0.00	0.00
Mathematica	64	100.00	0.00	0.00
Fricas	141	65.96	9.93	24.11
Maple	156	100.00	0.00	0.00
Reduce	177	100.00	0.00	0.00
Giac	206	80.58	3.40	16.02
Maxima	273	100.00	0.00	0.00
Mupad	279	0.00	100.00	0.00
Sympy	307	77.52	22.15	0.33

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.10
Giac	0.38
Reduce	0.48
Rubi	0.80
Fricas	1.26
Maple	1.29
Sympy	3.55
Mathematica	5.41
Mupad	7.48

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	156.82	1.28	113.00	1.11
Rubi	205.36	1.10	137.00	1.04
Sympy	239.52	2.44	124.00	1.25
Mathematica	259.54	1.29	140.50	1.00
Mupad	422.06	2.42	89.00	1.16
Reduce	438.25	2.90	260.00	1.78
Giac	495.61	2.90	232.00	1.76
Maple	512.81	3.26	195.50	1.43
Fricas	3293.39	7.35	284.00	2.24

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

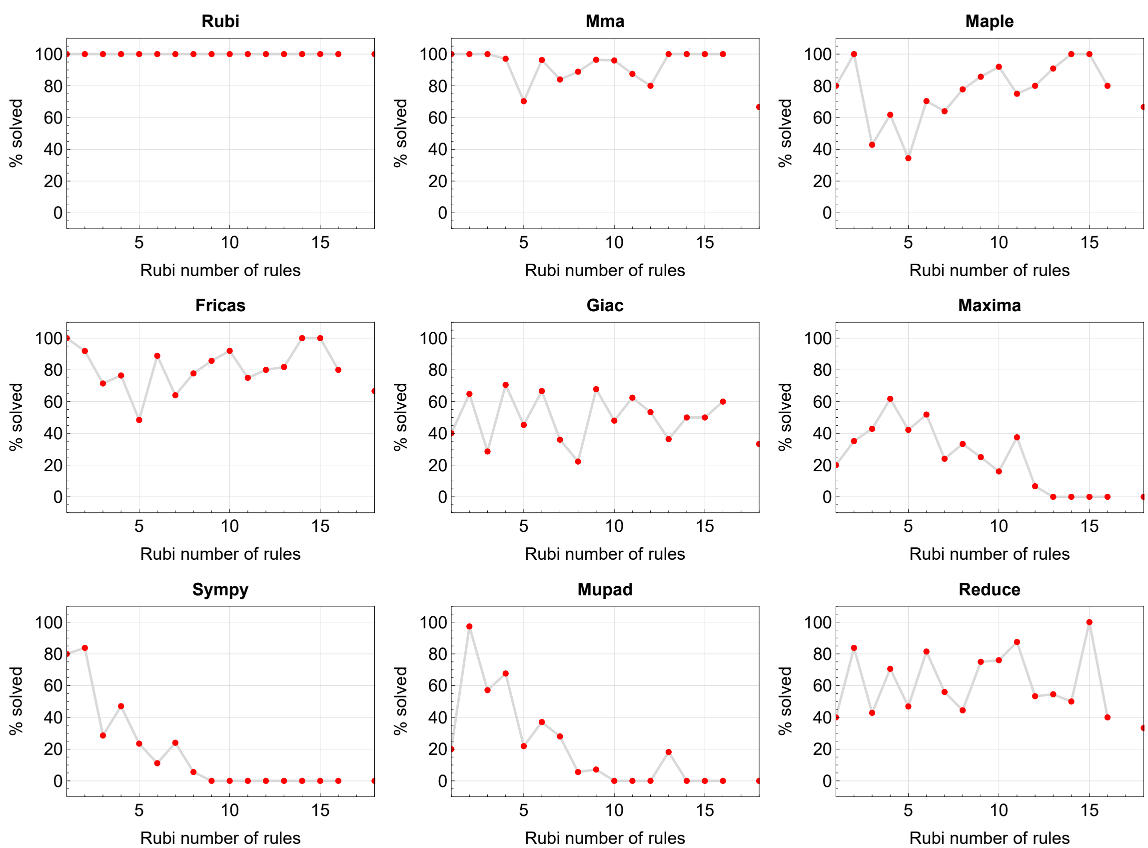


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

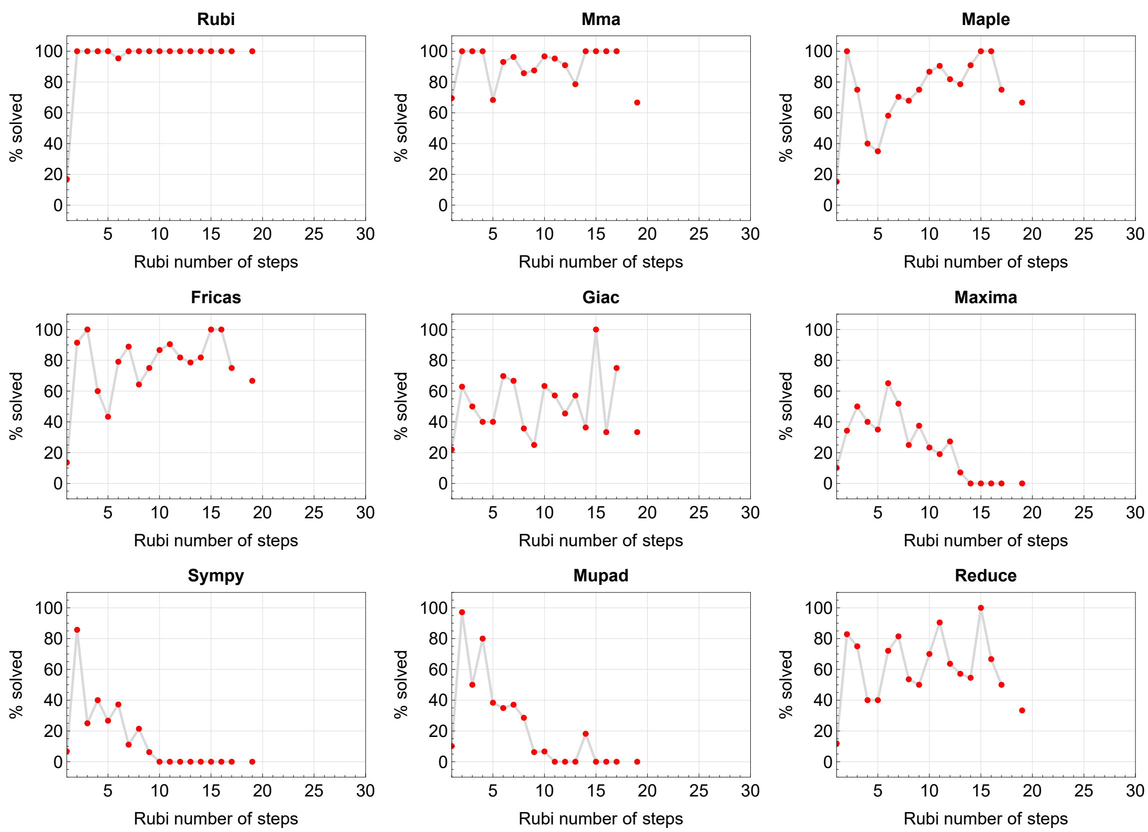


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

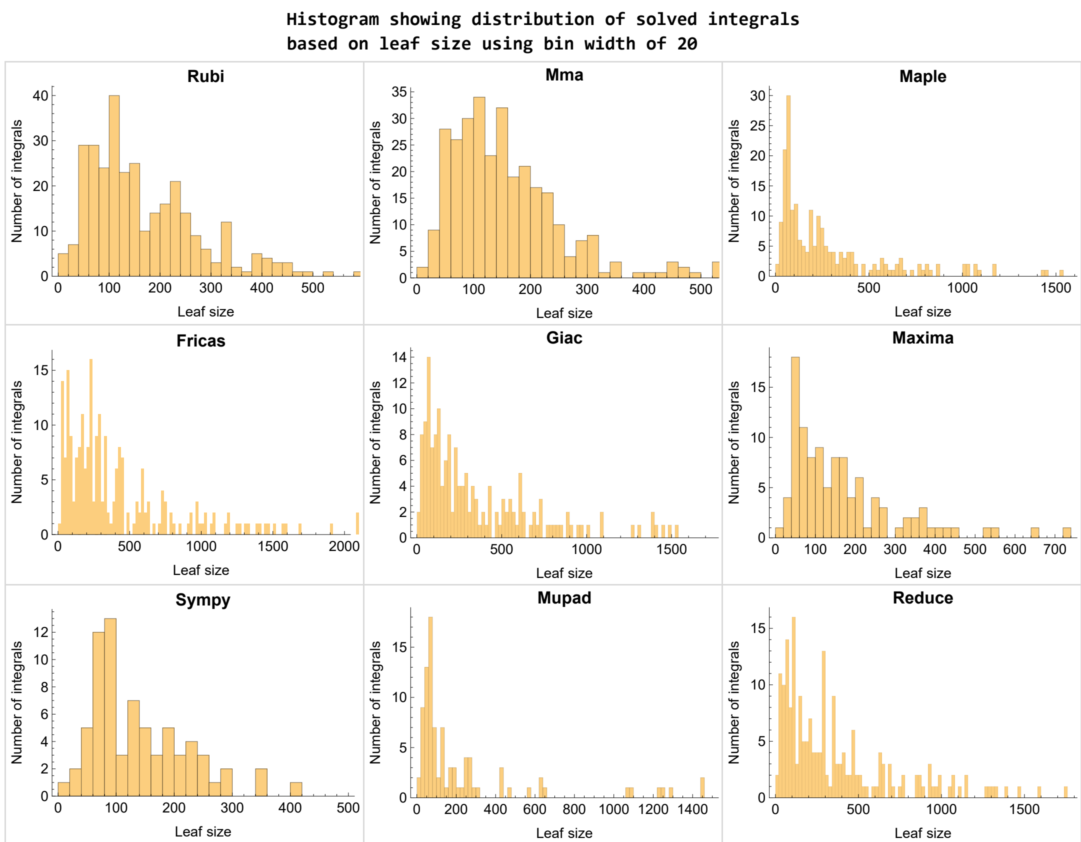


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

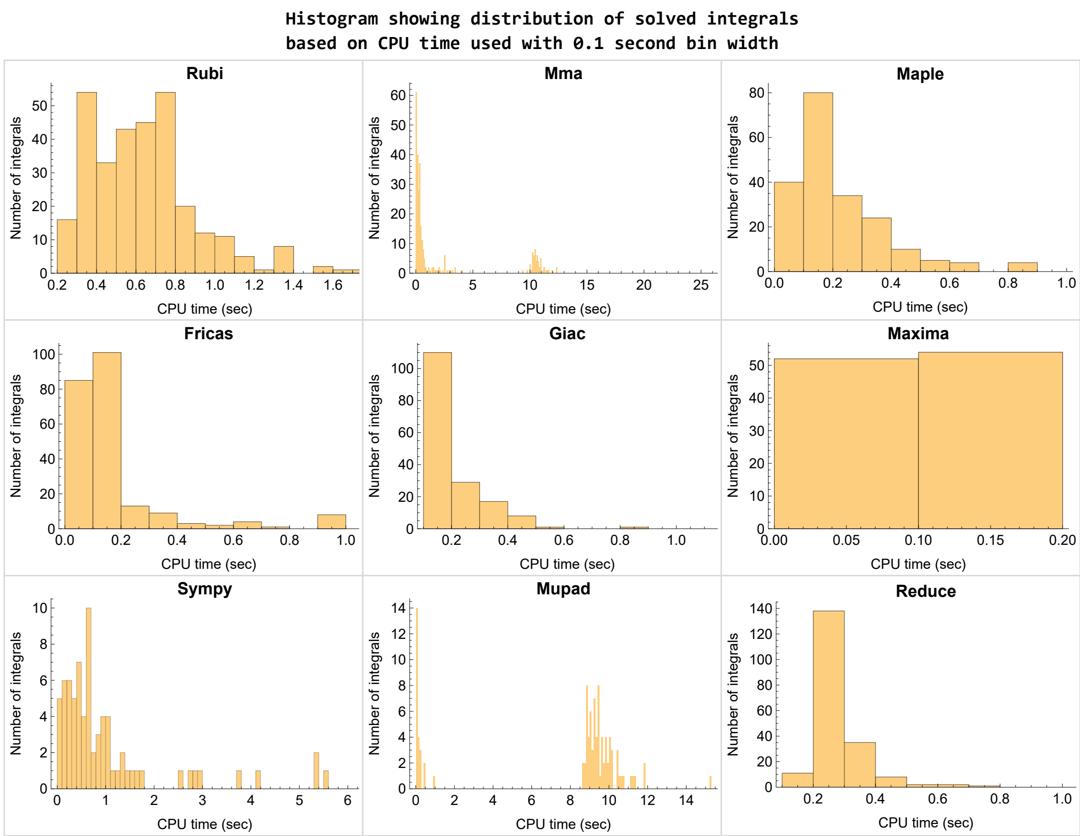


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

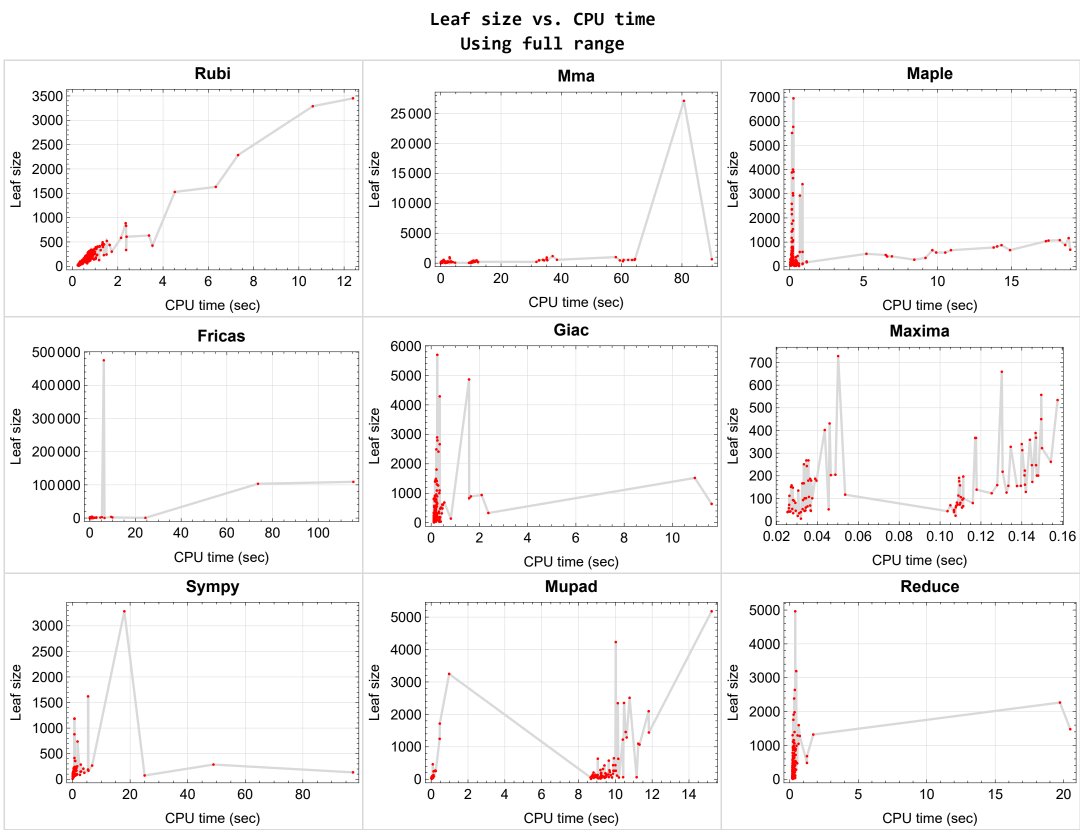


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{103, 366, 367, 368, 384, 385, 386}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {28, 29, 30, 31, 33, 34, 35, 96, 130, 131, 137, 138, 163, 164, 167, 168, 169, 176, 177, 180, 181, 182, 191, 192, 195, 196, 202, 203, 205, 206, 207, 234, 235, 258, 283, 294, 306, 307, 310, 311, 312, 313, 331, 354, 358, 359, 360}

Mathematica {275, 319, 320, 322, 323, 329, 330, 332, 333, 344, 345}

Maple $\{8, 225\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

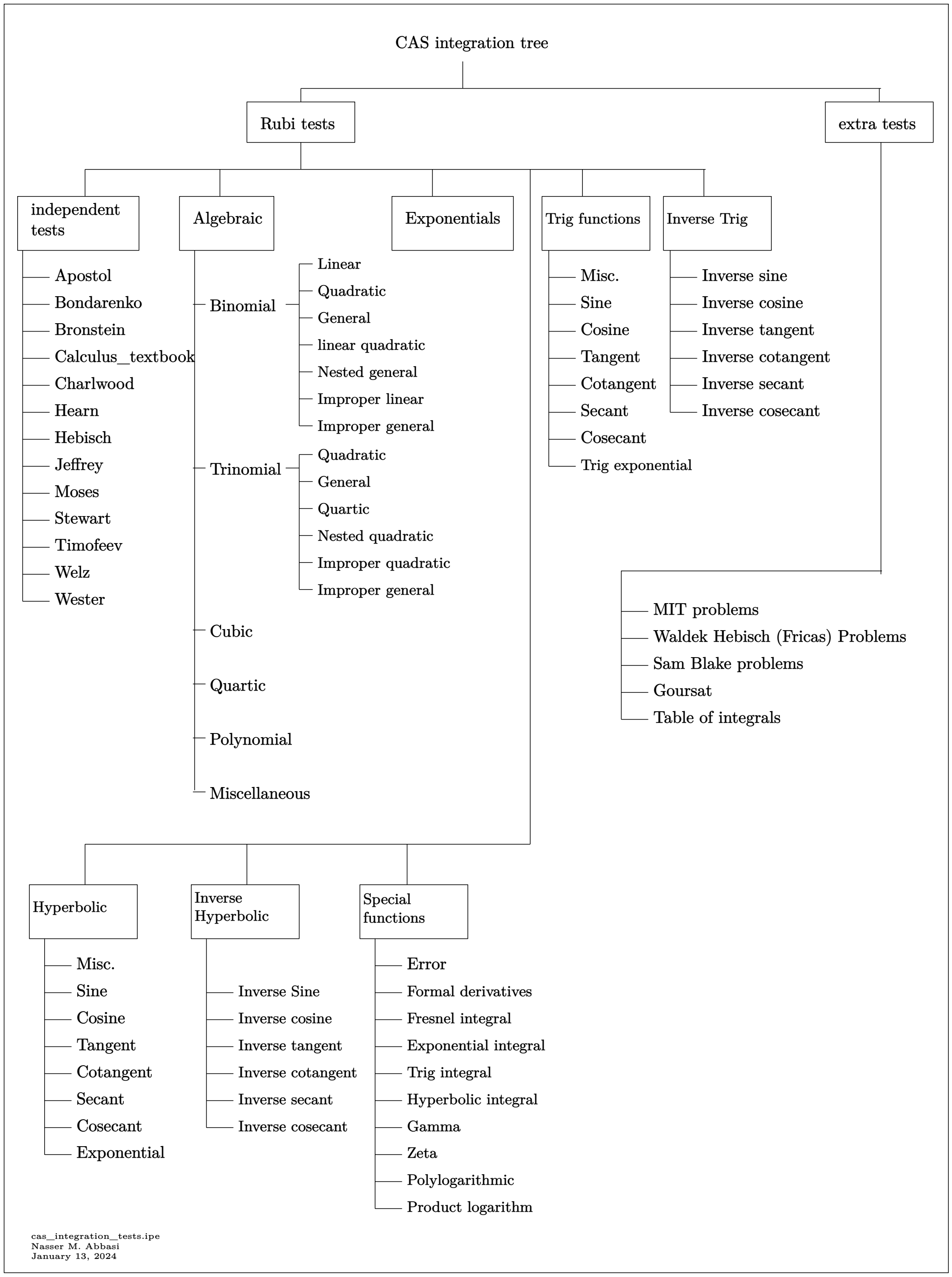
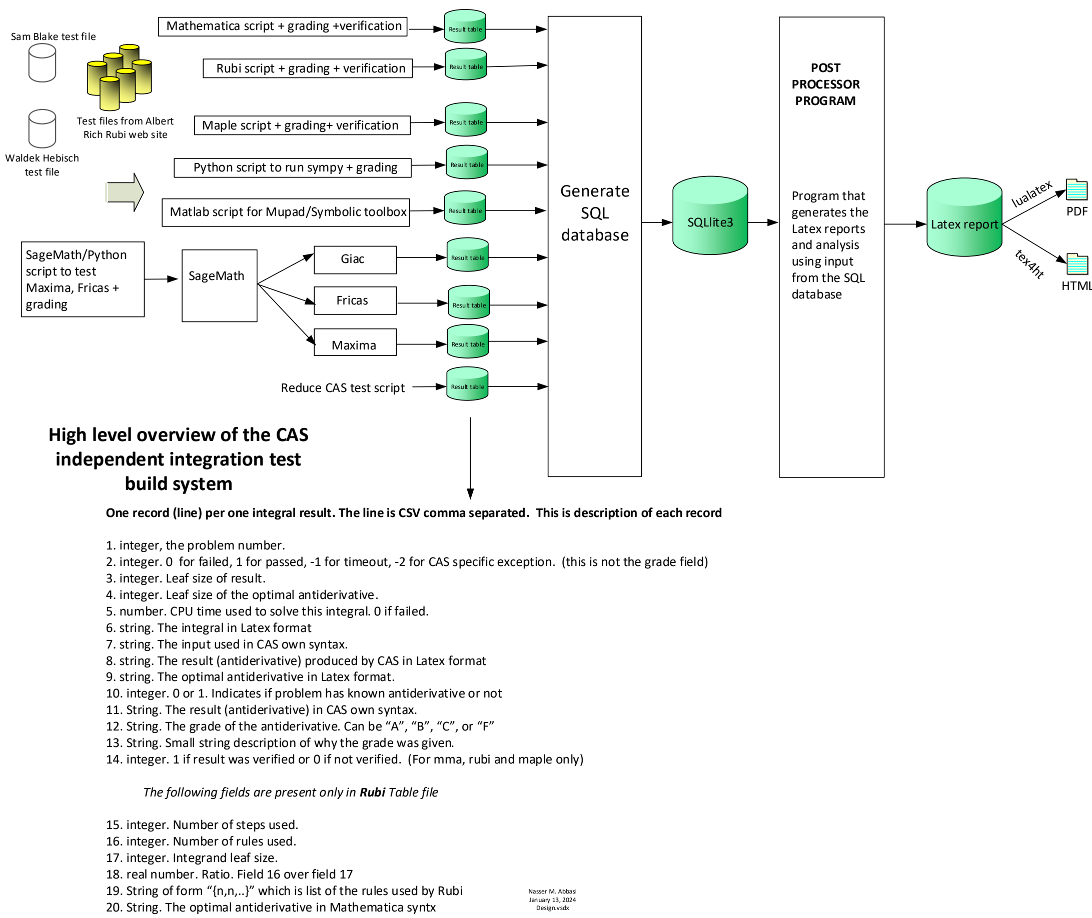


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 280, 281, 282, 283, 290, 291, 292, 293, 294, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 354, 358, 359, 360, 372, 378 }

B grade { 156, 157, 159 }

C grade { }

F normal fail { 61, 62, 213, 215, 216, 217, 248, 249, 250, 251, 252, 253, 254, 259, 260, 261, 262, 263, 264, 265, 274, 275, 276, 277, 278, 279, 284, 285, 286, 287, 288, 289, 295, 296, 297, 298, 299, 336, 352, 353, 355, 356, 357, 361, 362, 363, 364, 365, 369, 370, 371, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 163, 164, 165, 166, 167, 168, 169, 172, 173, 176, 177, 178, 179, 180, 181, 182, 191, 192, 193, 194, 195, 196, 199, 200, 202, 203, 204, 205, 206, 207, 214, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 260, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 290, 291, 292, 293, 294, 300, 301, 302, 303, 304, 305, 308, 309, 310, 311, 312, 313, 316, 321, 322, 323, 326, 331, 342, 343, 344, 345, 348, 349, 350, 351, 354, 360, 365, 371, 372, 373, 377, 378, 383 }

B grade { 10, 31, 189, 190, 319, 320, 329, 330, 332, 333, 346, 347 }

C grade { 74, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 170, 171, 174, 175, 183, 184, 185, 186, 187, 188, 197, 198, 201, 208, 209, 210, 211, 212, 234, 235, 237, 238, 239, 240, 241, 242, 243, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 275, 276, 277, 278, 279, 285, 286, 287, 288, 289, 340, 341 }

F normal fail { 53, 54, 55, 57, 58, 59, 60, 61, 62, 96, 97, 100, 101, 102, 213, 215, 216, 217, 218, 219, 220, 221, 222, 295, 296, 297, 298, 299, 306, 307, 314, 315, 317, 318, 324, 325, 327, 328, 334, 335, 336, 337, 338, 339, 352, 353, 355, 356, 357, 358, 359, 361, 362, 363, 364, 369, 370, 374, 375, 376, 379, 380, 381, 382 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 28, 29, 32, 33, 34, 35, 63, 64, 68, 71, 72, 76, 77, 79, 80, 82, 85, 86, 87, 88, 89, 90, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 163, 164, 168, 169, 171, 172, 173, 176, 177, 181, 182, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 207, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 246, 257, 272, 282, 312 }

B grade { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 65, 66, 67, 69, 70, 73, 74, 75, 78, 81, 83, 84, 91, 92, 93, 94, 95, 131, 137, 165, 166, 167, 170, 174, 175, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 204, 205, 206, 208, 209, 210, 211, 212, 316, 321, 326, 331 }

C grade { 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 239, 240, 241, 242, 243, 266, 267, 268, 313 }

F normal fail { 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 96, 97, 98, 99, 100, 101, 102, 139, 140, 141, 142, 143, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 317, 318, 319, 320, 322, 323, 324, 325, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 63, 64, 70, 71, 72, 74, 77, 79, 80, 81, 86, 87, 88, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 132, 133, 134, 135, 142, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 163, 164, 165, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 197, 198, 199, 200, 201, 202, 203, 208, 209, 210, 211, 212, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 241, 244, 245, 246, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 280, 281, 282, 292, 303, 312, 313, 316, 321, 326, 346, 347, 348 }

B grade { 17, 30, 31, 41, 48, 50, 51, 52, 65, 66, 67, 68, 69, 73, 75, 76, 78, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 123, 124, 129, 130, 131, 136, 137, 138, 139, 140, 141, 146, 166, 167, 179, 189, 190, 194, 195, 196, 204, 205, 206, 207, 239, 240, 242, 243, 290, 291, 304, 305, 331 }

C grade { 157, 158, 160 }

F normal fail { 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 96, 97, 98, 99, 100, 101, 102, 143, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 275, 276, 277, 278, 279, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 381, 382, 383 }

F(-1) timedout fail { 156, 159, 161, 162, 247, 248, 249, 258, 259, 260, 273, 274, 283, 284 }

F(-2) exception fail { 314, 315, 317, 318, 319, 320, 322, 323, 324, 325, 327, 328, 329, 330, 332, 333, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 132, 133, 134, 135, 140, 141, 142, 163, 176, 177, 178, 191, 202, 203, 204, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 244, 245, 246, 255, 256, 257, 270, 271, 272, 280, 281, 282, 290, 291, 292, 312, 313, 346, 347, 348 }

B grade { 10, 11, 124, 139, 164, 165, 166, 167, 168, 169, 179, 180, 181, 182, 192, 193, 194, 195, 196, 205, 206, 207 }

C grade { }

F normal fail { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 129, 130, 131, 136, 137, 138, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 170, 171, 172, 173, 174, 175, 183, 184, 185, 186, 187, 188, 189, 190, 197, 198, 199, 200, 201, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 283, 284, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 28, 29, 36, 37, 38, 66, 74, 76, 79, 80, 81, 82, 87, 88, 89, 90, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 132, 133, 134, 135, 136, 151, 152, 153, 154, 155, 163, 164, 191, 192, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 266, 272, 312 }

B grade { 15, 17, 18, 19, 20, 21, 22, 23, 30, 31, 32, 33, 34, 35, 39, 41, 42, 43, 44, 45, 46, 47, 68, 69, 70, 71, 72, 73, 77, 84, 85, 86, 92, 93, 94, 95, 107, 125, 126, 127, 128, 130, 131, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 165, 167, 168, 169, 176, 177, 178, 193, 195, 196, 202, 203, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 260, 270, 271, 273, 274, 280, 281, 282, 283, 284, 290, 291, 292, 303, 313 }

C grade { }

F normal fail { 25, 26, 27, 49, 50, 56, 58, 59, 60, 61, 62, 96, 97, 98, 99, 100, 101, 102, 143, 157, 158, 159, 160, 161, 162, 170, 171, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 197, 198, 199, 200, 201, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 267, 268, 269, 275, 276, 277, 278, 279, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383 }

F(-1) timedout fail { 51, 52, 63, 64, 65, 78, 156 }

F(-2) exception fail { 16, 24, 40, 48, 53, 54, 55, 57, 67, 75, 83, 91, 166, 179, 194, 204, 205, 213, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 285, 286, 287, 288, 289 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 23, 30, 32, 33, 34, 35, 39, 47, 56, 66, 74, 82, 90, 99, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 135, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 165, 178, 193, 204, 214, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 257, 272, 282, 292, 303, 309 }

C grade { }

F normal fail { }

F(-1) timedout fail { 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 31, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 125, 126, 127, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 143, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383 }

F(-2) exception fail { }

Sympy

A grade { 8, 9, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 122, 125, 126, 127, 128, 132, 133, 134, 135, 144, 145, 146, 147, 148, 149, 150, 154, 155, 157, 160, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 236, 239, 240, 241, 242, 243, 282, 312 }

B grade { 1, 2, 3, 4, 5, 6, 7, 10, 11, 121, 151, 152, 153, 231, 235, 237, 257, 267, 268, 313 }

C grade { 30, 266 }

F normal fail { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 116, 117, 123, 124, 129, 130, 136, 137, 139, 140, 141, 142, 143, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 314, 315, 316, 317, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 336, 346, 347, 348, 349, 350, 351, 355, 361, 372 }

F(-1) timedout fail { 96, 97, 98, 101, 102, 103, 131, 138, 156, 158, 159, 161, 162, 216, 217, 220, 221, 222, 238, 310, 311, 318, 319, 320, 322, 323, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 352, 353, 354, 356, 357, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 385, 386 }

F(-2) exception fail { 295 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 169, 176, 177, 178, 179, 180, 181, 182, 191, 192, 193, 194, 195, 196, 202, 203, 204, 205, 206, 207, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 255, 256, 257, 258, 266, 270, 271, 272, 273, 280, 281, 282, 283, 291, 292, 312, 313, 346, 347, 348 }

C grade { }

F normal fail { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 71, 79, 87, 96, 97, 98, 99, 100, 101, 102, 143, 156, 157, 158, 159, 160, 161, 162, 170, 171, 172, 173, 174, 175, 183, 184, 185, 186, 187, 188, 189, 190, 197, 198, 199, 200, 201, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 248, 249, 250, 251, 252, 253, 254, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 274, 275, 276, 277, 278, 279, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column N.S. means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name Mathematica was abbreviated to MMA.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	111	81	198	209	77	105	87
N.S.	1	1.00	0.99	1.50	1.09	2.68	2.82	1.04	1.42	1.18
time (sec)	N/A	0.342	0.051	0.529	0.110	0.105	0.375	0.121	0.224	0.153

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	54	70	61	157	153	54	67	206
N.S.	1	0.94	1.08	1.40	1.22	3.14	3.06	1.08	1.34	4.12
time (sec)	N/A	0.310	0.033	0.513	0.108	0.080	0.237	0.128	0.223	8.818

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	39	38	54	50	136	124	43	45	46
N.S.	1	0.95	0.93	1.32	1.22	3.32	3.02	1.05	1.10	1.12
time (sec)	N/A	0.276	0.018	0.468	0.107	0.080	0.107	0.121	0.214	8.809

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	24	83	54	17	20	17
N.S.	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.95	0.81
time (sec)	N/A	0.229	0.006	0.454	0.108	0.076	0.079	0.118	0.219	0.037

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	48	74	68	154	738	62	55	173
N.S.	1	0.97	0.81	1.25	1.15	2.61	12.51	1.05	0.93	2.93
time (sec)	N/A	0.313	0.041	0.473	0.108	0.090	1.727	0.119	0.220	9.473

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	93	81	96	123	229	1620	117	102	425
N.S.	1	1.18	1.03	1.22	1.56	2.90	20.51	1.48	1.29	5.38
time (sec)	N/A	0.423	0.052	0.537	0.125	0.092	5.394	0.119	0.233	9.410

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	134	106	151	197	371	3284	195	199	573
N.S.	1	1.11	0.88	1.25	1.63	3.07	27.14	1.61	1.64	4.74
time (sec)	N/A	0.524	0.144	0.499	0.111	0.107	17.993	0.134	0.241	9.646

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	65	64	61	54	54	58	62	63	55
N.S.	1	1.10	1.08	1.03	0.92	0.92	0.98	1.05	1.07	0.93
time (sec)	N/A	0.338	0.030	0.531	0.028	0.106	0.071	0.119	0.233	0.056

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	11	11	12	11	11	10	11	11	11
N.S.	1	1.10	1.10	1.20	1.10	1.10	1.00	1.10	1.10	1.10
time (sec)	N/A	0.264	0.010	0.628	0.032	0.097	0.034	0.117	0.209	8.682

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	163	107	139	92	231	64	75	0
N.S.	1	0.94	2.43	1.60	2.07	1.37	3.45	0.96	1.12	0.00
time (sec)	N/A	0.323	0.443	0.628	0.118	0.282	0.730	0.130	0.223	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	60	86	87	135	70	228	87	103	0
N.S.	1	0.95	1.37	1.38	2.14	1.11	3.62	1.38	1.63	0.00
time (sec)	N/A	0.324	0.317	0.858	0.031	0.101	0.607	0.132	0.261	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	260	182	803	0	443	0	350	409	0
N.S.	1	1.07	0.75	3.29	0.00	1.82	0.00	1.43	1.68	0.00
time (sec)	N/A	0.701	0.444	0.204	0.000	0.105	0.000	0.166	0.350	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	155	131	545	0	328	0	249	258	0
N.S.	1	0.93	0.78	3.26	0.00	1.96	0.00	1.49	1.54	0.00
time (sec)	N/A	0.501	0.310	0.124	0.000	0.100	0.000	0.153	0.226	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	100	95	333	0	229	0	168	139	0
N.S.	1	0.93	0.89	3.11	0.00	2.14	0.00	1.57	1.30	0.00
time (sec)	N/A	0.397	0.178	0.120	0.000	0.088	0.000	0.149	0.244	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	69	136	0	172	0	115	56	136
N.S.	1	1.02	1.21	2.39	0.00	3.02	0.00	2.02	0.98	2.39
time (sec)	N/A	0.294	0.127	0.106	0.000	0.091	0.000	0.169	0.233	8.768

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	95	521	0	459	0	0	185	0
N.S.	1	1.00	1.23	6.77	0.00	5.96	0.00	0.00	2.40	0.00
time (sec)	N/A	0.460	0.199	0.125	0.000	0.109	0.000	0.000	0.260	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	85	96	766	0	276	0	279	217	0
N.S.	1	1.10	1.25	9.95	0.00	3.58	0.00	3.62	2.82	0.00
time (sec)	N/A	0.411	0.242	0.125	0.000	0.090	0.000	0.155	0.283	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	152	139	1434	0	425	0	729	857	0
N.S.	1	1.09	0.99	10.24	0.00	3.04	0.00	5.21	6.12	0.00
time (sec)	N/A	0.491	0.440	0.128	0.000	0.089	0.000	0.160	0.419	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	224	200	2156	0	602	0	1447	1294	0
N.S.	1	1.06	0.94	10.17	0.00	2.84	0.00	6.83	6.10	0.00
time (sec)	N/A	0.611	0.573	0.140	0.000	0.096	0.000	0.178	0.566	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	296	187	2361	0	443	0	560	680	0
N.S.	1	1.12	0.71	8.94	0.00	1.68	0.00	2.12	2.58	0.00
time (sec)	N/A	0.783	0.429	0.148	0.000	0.092	0.000	0.198	1.257	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	177	138	1762	0	332	0	461	474	0
N.S.	1	0.93	0.72	9.23	0.00	1.74	0.00	2.41	2.48	0.00
time (sec)	N/A	0.535	0.334	0.141	0.000	0.094	0.000	0.196	0.222	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	122	103	1173	0	239	0	425	284	0
N.S.	1	0.95	0.80	9.09	0.00	1.85	0.00	3.29	2.20	0.00
time (sec)	N/A	0.427	0.231	0.128	0.000	0.105	0.000	0.191	0.207	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	76	359	0	174	0	340	151	70
N.S.	1	1.00	0.95	4.49	0.00	2.18	0.00	4.25	1.89	0.88
time (sec)	N/A	0.350	0.135	0.118	0.000	0.092	0.000	0.186	0.208	9.236

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	105	129	2584	0	591	0	0	753	0
N.S.	1	1.07	1.32	26.37	0.00	6.03	0.00	0.00	7.68	0.00
time (sec)	N/A	0.505	0.313	0.136	0.000	0.107	0.000	0.000	0.282	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	108	2349	0	238	0	0	928	0
N.S.	1	1.08	1.03	22.37	0.00	2.27	0.00	0.00	8.84	0.00
time (sec)	N/A	0.463	0.300	0.133	0.000	0.086	0.000	0.000	0.325	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	180	143	3882	0	405	0	0	1598	0
N.S.	1	1.11	0.88	23.96	0.00	2.50	0.00	0.00	9.86	0.00
time (sec)	N/A	0.535	0.306	0.142	0.000	0.088	0.000	0.000	0.652	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	252	212	5518	0	580	0	0	2266	0
N.S.	1	1.08	0.91	23.58	0.00	2.48	0.00	0.00	9.68	0.00
time (sec)	N/A	0.694	0.525	0.155	0.000	0.095	0.000	0.000	19.729	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	92	83	74	0	72	0	84	56	0
N.S.	1	1.07	0.97	0.86	0.00	0.84	0.00	0.98	0.65	0.00
time (sec)	N/A	0.355	0.120	0.145	0.000	0.074	0.000	0.132	0.242	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	78	69	0	67	0	72	41	0
N.S.	1	1.00	1.24	1.10	0.00	1.06	0.00	1.14	0.65	0.00
time (sec)	N/A	0.331	0.093	0.102	0.000	0.089	0.000	0.130	0.218	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	39	72	64	0	62	90	59	28	57
N.S.	1	1.05	1.95	1.73	0.00	1.68	2.43	1.59	0.76	1.54
time (sec)	N/A	0.278	0.073	0.089	0.000	0.071	0.829	0.137	0.256	9.202

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	16	52	40	0	39	0	34	15	0
N.S.	1	1.14	3.71	2.86	0.00	2.79	0.00	2.43	1.07	0.00
time (sec)	N/A	0.284	0.049	0.086	0.000	0.092	0.000	0.136	0.244	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	17	15	24	0	24	0	42	21	21
N.S.	1	0.81	0.71	1.14	0.00	1.14	0.00	2.00	1.00	1.00
time (sec)	N/A	0.302	0.026	0.058	0.000	0.090	0.000	0.121	0.236	9.195

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	33	25	29	0	29	0	73	36	28
N.S.	1	0.70	0.53	0.62	0.00	0.62	0.00	1.55	0.77	0.60
time (sec)	N/A	0.359	0.071	0.060	0.000	0.089	0.000	0.130	0.214	9.088

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	48	30	34	0	34	0	101	51	33
N.S.	1	0.69	0.43	0.49	0.00	0.49	0.00	1.44	0.73	0.47
time (sec)	N/A	0.369	0.092	0.065	0.000	0.088	0.000	0.136	0.277	9.053

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	62	35	39	0	39	0	135	66	85
N.S.	1	0.67	0.38	0.42	0.00	0.42	0.00	1.45	0.71	0.91
time (sec)	N/A	0.392	0.091	0.063	0.000	0.088	0.000	0.136	0.246	8.708

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	217	184	1078	0	446	0	339	409	0
N.S.	1	0.87	0.74	4.33	0.00	1.79	0.00	1.36	1.64	0.00
time (sec)	N/A	0.680	0.451	0.220	0.000	0.114	0.000	0.177	0.393	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	161	134	721	0	329	0	245	258	0
N.S.	1	0.91	0.76	4.10	0.00	1.87	0.00	1.39	1.47	0.00
time (sec)	N/A	0.526	0.318	0.211	0.000	0.101	0.000	0.168	0.228	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	104	96	418	0	238	0	170	140	0
N.S.	1	0.91	0.84	3.67	0.00	2.09	0.00	1.49	1.23	0.00
time (sec)	N/A	0.416	0.184	0.212	0.000	0.115	0.000	0.158	0.237	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	73	140	0	170	0	122	57	53
N.S.	1	0.98	1.20	2.30	0.00	2.79	0.00	2.00	0.93	0.87
time (sec)	N/A	0.320	0.128	0.187	0.000	0.102	0.000	0.181	0.219	8.656

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	95	526	0	481	0	0	227	0
N.S.	1	1.00	1.23	6.83	0.00	6.25	0.00	0.00	2.95	0.00
time (sec)	N/A	0.461	0.177	0.201	0.000	0.118	0.000	0.000	0.222	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	88	100	629	0	294	0	282	227	0
N.S.	1	1.07	1.22	7.67	0.00	3.59	0.00	3.44	2.77	0.00
time (sec)	N/A	0.431	0.265	0.205	0.000	0.100	0.000	0.136	0.262	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	153	135	1030	0	440	0	737	866	0
N.S.	1	1.07	0.94	7.20	0.00	3.08	0.00	5.15	6.06	0.00
time (sec)	N/A	0.509	0.419	0.211	0.000	0.110	0.000	0.149	0.395	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	225	198	1534	0	621	0	1397	1278	0
N.S.	1	1.05	0.92	7.13	0.00	2.89	0.00	6.50	5.94	0.00
time (sec)	N/A	0.635	0.512	0.211	0.000	0.114	0.000	0.163	0.731	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	269	217	3910	0	734	0	685	482	0
N.S.	1	0.96	0.78	13.96	0.00	2.62	0.00	2.45	1.72	0.00
time (sec)	N/A	0.733	0.645	0.251	0.000	0.116	0.000	0.229	1.253	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	190	167	2932	0	571	0	599	335	0
N.S.	1	0.92	0.81	14.16	0.00	2.76	0.00	2.89	1.62	0.00
time (sec)	N/A	0.572	0.482	0.225	0.000	0.094	0.000	0.217	0.225	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	133	130	1888	0	438	0	500	211	0
N.S.	1	0.93	0.91	13.20	0.00	3.06	0.00	3.50	1.48	0.00
time (sec)	N/A	0.446	0.323	0.229	0.000	0.100	0.000	0.199	0.219	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	95	652	0	290	0	420	112	89
N.S.	1	1.05	1.12	7.67	0.00	3.41	0.00	4.94	1.32	1.05
time (sec)	N/A	0.335	0.205	0.196	0.000	0.090	0.000	0.187	0.246	9.324

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	124	129	3024	0	938	0	0	417	0
N.S.	1	1.18	1.23	28.80	0.00	8.93	0.00	0.00	3.97	0.00
time (sec)	N/A	0.525	0.617	0.214	0.000	0.131	0.000	0.000	0.301	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	125	128	3646	0	417	0	0	649	0
N.S.	1	1.15	1.17	33.45	0.00	3.83	0.00	0.00	5.95	0.00
time (sec)	N/A	0.500	0.332	0.219	0.000	0.091	0.000	0.000	0.335	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	190	176	4002	0	810	0	0	1046	0
N.S.	1	1.13	1.05	23.82	0.00	4.82	0.00	0.00	6.23	0.00
time (sec)	N/A	0.562	0.477	0.215	0.000	0.104	0.000	0.000	0.640	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	288	252	5770	0	1085	0	0	1479	0
N.S.	1	1.18	1.03	23.65	0.00	4.45	0.00	0.00	6.06	0.00
time (sec)	N/A	0.703	0.713	0.233	0.000	0.117	0.000	0.000	20.481	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	332	351	6950	0	1398	0	0	19	0
N.S.	1	1.01	1.07	21.12	0.00	4.25	0.00	0.00	0.06	0.00
time (sec)	N/A	0.805	1.279	0.249	0.000	0.146	0.000	0.000	200.031	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	259	0	0	0	0	0	0	26	0
N.S.	1	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.722	0.000	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	162	0	0	0	0	0	0	26	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.502	0.000	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.380	0.000	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	69	0	0	0	0	0	23	70
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.45	1.37
time (sec)	N/A	0.287	0.101	0.000	0.000	0.000	0.000	0.000	0.188	9.399

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	0	0	0	0	0	0	26	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.421	0.000	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	0	0	0	0	0	0	26	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.344	0.000	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	120	0	0	0	0	0	0	26	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.415	0.000	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	203	0	0	0	0	0	0	26	0
N.S.	1	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.594	0.000	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	26	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	197	198	320	0	1166	0	0	19	0
N.S.	1	0.98	0.99	1.59	0.00	5.80	0.00	0.00	0.09	0.00
time (sec)	N/A	0.868	10.695	0.226	0.000	0.556	0.000	0.000	200.039	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	144	235	235	0	1028	0	0	260	0
N.S.	1	0.97	1.58	1.58	0.00	6.90	0.00	0.00	1.74	0.00
time (sec)	N/A	0.597	10.217	0.149	0.000	0.224	0.000	0.000	0.242	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	155	213	0	989	0	0	197	0
N.S.	1	0.99	1.34	1.84	0.00	8.53	0.00	0.00	1.70	0.00
time (sec)	N/A	0.510	10.424	0.140	0.000	0.177	0.000	0.000	0.250	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	63	106	133	0	272	0	107	116	64
N.S.	1	0.98	1.66	2.08	0.00	4.25	0.00	1.67	1.81	1.00
time (sec)	N/A	0.310	10.159	0.132	0.000	0.117	0.000	0.139	0.300	8.828

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	125	177	249	0	2695	0	0	179	0
N.S.	1	1.01	1.43	2.01	0.00	21.73	0.00	0.00	1.44	0.00
time (sec)	N/A	0.674	10.478	0.132	0.000	0.918	0.000	0.000	0.275	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	142	207	224	0	1334	0	299	289	0
N.S.	1	1.06	1.54	1.67	0.00	9.96	0.00	2.23	2.16	0.00
time (sec)	N/A	0.549	10.465	0.151	0.000	0.195	0.000	0.196	0.264	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	236	402	0	1687	0	685	638	0
N.S.	1	1.00	1.11	1.90	0.00	7.96	0.00	3.23	3.01	0.00
time (sec)	N/A	0.716	10.331	0.164	0.000	0.316	0.000	0.544	0.248	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	298	312	603	0	2097	0	1521	1061	0
N.S.	1	1.02	1.06	2.06	0.00	7.16	0.00	5.19	3.62	0.00
time (sec)	N/A	0.874	10.563	0.175	0.000	0.485	0.000	10.931	0.341	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	255	234	436	0	1509	0	940	19	0
N.S.	1	0.94	0.87	1.61	0.00	5.59	0.00	3.48	0.07	0.00
time (sec)	N/A	0.996	10.326	0.181	0.000	1.582	0.000	2.094	200.039	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	220	211	352	0	1307	0	893	986	0
N.S.	1	1.06	1.01	1.69	0.00	6.28	0.00	4.29	4.74	0.00
time (sec)	N/A	0.730	10.235	0.183	0.000	0.977	0.000	1.646	0.287	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	146	181	284	0	1189	0	832	643	0
N.S.	1	0.94	1.17	1.83	0.00	7.67	0.00	5.37	4.15	0.00
time (sec)	N/A	0.566	10.788	0.167	0.000	0.607	0.000	1.575	0.242	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	184	0	347	0	34	436	56
N.S.	1	1.00	0.68	1.90	0.00	3.58	0.00	0.35	4.49	0.58
time (sec)	N/A	0.331	10.050	0.152	0.000	0.381	0.000	0.334	0.225	9.440

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	211	239	1455	0	3665	0	0	1155	0
N.S.	1	1.13	1.28	7.78	0.00	19.60	0.00	0.00	6.18	0.00
time (sec)	N/A	0.870	11.212	0.162	0.000	9.350	0.000	0.000	0.272	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	192	235	349	0	1562	0	330	930	0
N.S.	1	0.96	1.18	1.75	0.00	7.85	0.00	1.66	4.67	0.00
time (sec)	N/A	0.656	10.412	0.174	0.000	0.725	0.000	2.371	0.226	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	238	277	390	0	2095	0	639	1757	0
N.S.	1	0.74	0.87	1.22	0.00	6.55	0.00	2.00	5.49	0.00
time (sec)	N/A	0.747	10.471	0.187	0.000	2.889	0.000	11.626	0.262	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	355	345	792	0	2690	0	0	2639	0
N.S.	1	0.99	0.96	2.20	0.00	7.47	0.00	0.00	7.33	0.00
time (sec)	N/A	1.053	10.999	0.204	0.000	5.319	0.000	0.000	0.370	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	183	128	243	0	409	0	232	19	0
N.S.	1	0.99	0.69	1.31	0.00	2.21	0.00	1.25	0.10	0.00
time (sec)	N/A	0.841	10.335	0.103	0.000	0.163	0.000	0.128	200.044	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	134	104	213	0	346	0	172	163	0
N.S.	1	0.98	0.76	1.55	0.00	2.53	0.00	1.26	1.19	0.00
time (sec)	N/A	0.599	10.275	0.092	0.000	0.119	0.000	0.308	0.221	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	105	196	0	325	0	131	104	0
N.S.	1	0.98	1.08	2.02	0.00	3.35	0.00	1.35	1.07	0.00
time (sec)	N/A	0.476	10.319	0.087	0.000	0.103	0.000	0.138	0.199	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	40	49	0	54	0	38	29	58
N.S.	1	1.00	1.48	1.81	0.00	2.00	0.00	1.41	1.07	2.15
time (sec)	N/A	0.250	9.347	0.065	0.000	0.081	0.000	0.121	0.209	8.867

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	152	203	0	1089	0	0	1148	0
N.S.	1	1.00	1.81	2.42	0.00	12.96	0.00	0.00	13.67	0.00
time (sec)	N/A	0.549	10.412	0.078	0.000	0.148	0.000	0.000	0.307	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	101	117	205	0	498	0	220	154	0
N.S.	1	1.02	1.18	2.07	0.00	5.03	0.00	2.22	1.56	0.00
time (sec)	N/A	0.482	10.346	0.092	0.000	0.164	0.000	0.134	0.322	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	168	152	230	0	620	0	575	304	0
N.S.	1	1.04	0.94	1.43	0.00	3.85	0.00	3.57	1.89	0.00
time (sec)	N/A	0.607	10.491	0.092	0.000	0.301	0.000	0.127	0.233	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	260	205	289	0	789	0	1005	588	0
N.S.	1	1.08	0.85	1.20	0.00	3.27	0.00	4.17	2.44	0.00
time (sec)	N/A	0.830	10.267	0.102	0.000	0.611	0.000	0.339	0.206	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	178	399	0	791	0	320	19	0
N.S.	1	1.00	0.74	1.66	0.00	3.28	0.00	1.33	0.08	0.00
time (sec)	N/A	1.502	10.288	0.152	0.000	1.702	0.000	0.258	200.030	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	180	145	278	0	638	0	263	508	0
N.S.	1	0.98	0.79	1.51	0.00	3.47	0.00	1.43	2.76	0.00
time (sec)	N/A	1.025	10.257	0.118	0.000	0.990	0.000	0.194	0.366	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	114	230	0	545	0	223	442	0
N.S.	1	1.01	0.86	1.74	0.00	4.13	0.00	1.69	3.35	0.00
time (sec)	N/A	0.723	10.355	0.112	0.000	0.581	0.000	0.173	0.375	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	42	72	0	122	0	99	75	55
N.S.	1	1.04	0.74	1.26	0.00	2.14	0.00	1.74	1.32	0.96
time (sec)	N/A	0.298	10.043	0.093	0.000	0.330	0.000	0.144	0.303	9.823

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	149	193	474	0	2298	0	0	4963	0
N.S.	1	1.17	1.52	3.73	0.00	18.09	0.00	0.00	39.08	0.00
time (sec)	N/A	0.699	10.755	0.099	0.000	0.697	0.000	0.000	0.399	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	183	176	373	0	1022	0	523	842	0
N.S.	1	1.08	1.04	2.21	0.00	6.05	0.00	3.09	4.98	0.00
time (sec)	N/A	0.645	10.406	0.124	0.000	0.922	0.000	0.208	0.395	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	242	244	574	0	1260	0	961	1394	0
N.S.	1	1.03	1.04	2.45	0.00	5.38	0.00	4.11	5.96	0.00
time (sec)	N/A	0.768	10.532	0.129	0.000	2.169	0.000	0.214	0.346	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	334	336	805	0	1599	0	1804	1979	0
N.S.	1	1.04	1.05	2.51	0.00	4.98	0.00	5.62	6.17	0.00
time (sec)	N/A	2.368	10.671	0.151	0.000	4.623	0.000	0.222	0.351	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	425	428	1001	0	1916	0	2896	2386	0
N.S.	1	1.05	1.05	2.47	0.00	4.72	0.00	7.13	5.88	0.00
time (sec)	N/A	3.532	10.945	0.159	0.000	9.910	0.000	0.246	0.323	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	211	0	0	0	0	0	0	49	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.650	0.000	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	165	0	0	0	0	0	0	49	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.580	0.000	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	117	132	0	0	0	0	0	47	0
N.S.	1	0.98	1.11	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.472	0.578	0.000	0.000	0.000	0.000	0.000	0.289	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	46	76
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.73	1.21
time (sec)	N/A	0.312	0.149	0.000	0.000	0.000	0.000	0.000	0.314	9.182

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	49	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.665	0.000	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	222	0	0	0	0	0	0	49	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.720	0.000	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	375	356	0	0	0	0	0	0	49	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.971	0.000	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	50	0	21	53	21
N.S.	1	1.00	1.11	1.00	1.11	2.63	0.00	1.11	2.79	1.11
time (sec)	N/A	0.436	1.033	0.057	0.093	0.099	0.000	0.164	0.238	9.228

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	171	105	78	151	94	177	151	120	167
N.S.	1	1.51	0.93	0.69	1.34	0.83	1.57	1.34	1.06	1.48
time (sec)	N/A	0.656	0.108	0.381	0.027	0.073	0.565	0.130	0.214	9.233

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	129	93	66	112	81	133	127	101	124
N.S.	1	1.40	1.01	0.72	1.22	0.88	1.45	1.38	1.10	1.35
time (sec)	N/A	0.502	0.084	0.365	0.026	0.073	0.537	0.115	0.224	9.232

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	85	68	54	72	67	88	131	82	79
N.S.	1	0.96	0.76	0.61	0.81	0.75	0.99	1.47	0.92	0.89
time (sec)	N/A	0.413	0.058	0.369	0.026	0.100	0.530	0.161	0.228	0.030

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	47	41	35	35	49	68	82	55	36
N.S.	1	1.15	1.00	0.85	0.85	1.20	1.66	2.00	1.34	0.88
time (sec)	N/A	0.315	0.027	0.336	0.028	0.080	0.096	0.145	0.233	0.046

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	71	63	51	70	115	87	59	64	130
N.S.	1	1.25	1.11	0.89	1.23	2.02	1.53	1.04	1.12	2.28
time (sec)	N/A	0.421	0.082	0.358	0.105	0.085	1.480	0.123	0.242	9.042

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	66	64	63	73	144	73	62	79	131
N.S.	1	1.22	1.19	1.17	1.35	2.67	1.35	1.15	1.46	2.43
time (sec)	N/A	0.383	0.139	0.363	0.110	0.107	25.038	0.125	0.263	0.121

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	91	77	81	113	178	134	105	106	80
N.S.	1	1.14	0.96	1.01	1.41	2.22	1.68	1.31	1.32	1.00
time (sec)	N/A	0.384	0.265	0.372	0.109	0.140	97.423	0.110	0.263	9.028

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	216	235	288	243	228	252	341	349	317
N.S.	1	0.94	1.02	1.25	1.06	0.99	1.10	1.48	1.52	1.38
time (sec)	N/A	0.683	0.298	0.043	0.035	0.307	1.612	0.108	0.281	9.331

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	143	151	166	148	138	167	203	202	184
N.S.	1	0.95	1.00	1.10	0.98	0.91	1.11	1.34	1.34	1.22
time (sec)	N/A	0.547	0.169	0.040	0.028	0.099	1.366	0.112	0.250	9.462

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	85	85	81	71	104	105	95	89
N.S.	1	0.98	0.94	0.94	0.90	0.79	1.16	1.17	1.06	0.99
time (sec)	N/A	0.438	0.093	0.036	0.029	0.083	1.250	0.224	0.222	0.054

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	39	40	36	35	33	49	38	31	33
N.S.	1	0.95	0.98	0.88	0.85	0.80	1.20	0.93	0.76	0.80
time (sec)	N/A	0.304	0.011	0.033	0.031	0.078	0.285	0.204	0.225	0.003

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	79	62	69	95	122	97	88	85	181
N.S.	1	0.96	0.76	0.84	1.16	1.49	1.18	1.07	1.04	2.21
time (sec)	N/A	0.411	0.076	0.070	0.111	0.116	1.553	0.257	0.217	9.229

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	151	118	128	191	280	0	191	231	220
N.S.	1	1.16	0.91	0.98	1.47	2.15	0.00	1.47	1.78	1.69
time (sec)	N/A	0.577	0.247	0.080	0.109	0.227	0.000	0.218	0.259	9.704

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	244	198	227	367	531	0	375	458	1094
N.S.	1	1.20	0.97	1.11	1.80	2.60	0.00	1.84	2.25	5.36
time (sec)	N/A	0.761	0.552	0.088	0.118	0.460	0.000	0.148	0.298	11.248

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	228	283	277	251	392	267	324	463	461
N.S.	1	0.95	1.18	1.15	1.05	1.63	1.11	1.35	1.93	1.92
time (sec)	N/A	0.739	0.285	0.054	0.033	0.114	6.796	0.324	0.278	0.089

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	158	194	161	158	269	189	195	288	197
N.S.	1	0.95	1.17	0.97	0.95	1.62	1.14	1.17	1.73	1.19
time (sec)	N/A	0.568	0.194	0.046	0.027	0.106	5.373	0.122	0.241	9.233

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	108	87	90	163	126	102	151	98
N.S.	1	1.00	1.14	0.92	0.95	1.72	1.33	1.07	1.59	1.03
time (sec)	N/A	0.451	0.088	0.044	0.029	0.108	4.105	0.117	0.230	0.062

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	44	43	41	43	75	124	44	62	43
N.S.	1	0.94	0.91	0.87	0.91	1.60	2.64	0.94	1.32	0.91
time (sec)	N/A	0.324	0.054	0.043	0.027	0.097	0.542	0.113	0.228	0.055

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	152	107	118	176	441	167	174	408	125
N.S.	1	1.18	0.83	0.91	1.36	3.42	1.29	1.35	3.16	0.97
time (sec)	N/A	0.576	0.243	0.083	0.109	0.132	5.527	0.113	0.240	9.752

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	219	177	182	367	851	0	311	696	275
N.S.	1	1.12	0.91	0.93	1.88	4.36	0.00	1.59	3.57	1.41
time (sec)	N/A	0.678	0.398	0.095	0.117	0.311	0.000	0.116	0.224	9.683

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	336	301	303	659	1249	0	521	1103	1441
N.S.	1	1.12	1.01	1.01	2.20	4.18	0.00	1.74	3.69	4.82
time (sec)	N/A	0.945	0.839	0.114	0.130	0.950	0.000	0.129	0.238	11.823

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	307	232	385	268	286	359	915	354	0
N.S.	1	0.94	0.71	1.18	0.82	0.88	1.10	2.81	1.09	0.00
time (sec)	N/A	0.727	0.260	0.204	0.036	0.120	0.883	0.155	0.235	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	211	147	184	167	184	241	549	212	0
N.S.	1	0.94	0.66	0.82	0.75	0.82	1.08	2.45	0.95	0.00
time (sec)	N/A	0.579	0.182	0.204	0.033	0.122	0.739	0.128	0.242	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	126	84	93	93	103	156	279	110	0
N.S.	1	0.95	0.63	0.70	0.70	0.77	1.17	2.10	0.83	0.00
time (sec)	N/A	0.462	0.092	0.201	0.033	0.117	0.661	0.165	0.219	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	55	41	43	50	70	99	47	44
N.S.	1	0.98	0.98	0.73	0.77	0.89	1.25	1.77	0.84	0.79
time (sec)	N/A	0.314	0.048	0.141	0.026	0.113	0.422	0.151	0.229	9.251

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	134	124	154	0	194	0	157	108	0
N.S.	1	1.16	1.07	1.33	0.00	1.67	0.00	1.35	0.93	0.00
time (sec)	N/A	0.636	0.351	0.398	0.000	0.131	0.000	0.135	0.236	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	186	144	166	0	1003	0	250	281	0
N.S.	1	1.36	1.05	1.21	0.00	7.32	0.00	1.82	2.05	0.00
time (sec)	N/A	0.586	0.514	0.398	0.000	0.143	0.000	0.224	0.285	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	336	217	372	0	2856	0	895	719	0
N.S.	1	1.50	0.97	1.66	0.00	12.75	0.00	4.00	3.21	0.00
time (sec)	N/A	0.913	2.501	0.409	0.000	0.319	0.000	0.250	0.430	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	305	232	384	268	231	357	409	287	0
N.S.	1	0.94	0.72	1.19	0.83	0.71	1.10	1.26	0.89	0.00
time (sec)	N/A	0.733	0.283	0.224	0.035	0.127	0.905	0.154	0.236	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	209	147	184	167	140	240	238	162	0
N.S.	1	0.94	0.66	0.83	0.75	0.63	1.08	1.07	0.73	0.00
time (sec)	N/A	0.568	0.144	0.216	0.033	0.120	0.949	0.111	0.233	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	124	84	92	93	71	155	115	77	0
N.S.	1	0.95	0.64	0.70	0.71	0.54	1.18	0.88	0.59	0.00
time (sec)	N/A	0.443	0.089	0.146	0.028	0.112	0.700	0.347	0.233	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	42	41	42	34	68	38	31	44
N.S.	1	0.98	0.78	0.76	0.78	0.63	1.26	0.70	0.57	0.81
time (sec)	N/A	0.305	0.038	0.096	0.027	0.110	0.383	0.398	0.243	9.430

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	112	105	92	0	743	0	140	214	0
N.S.	1	1.15	1.08	0.95	0.00	7.66	0.00	1.44	2.21	0.00
time (sec)	N/A	0.482	0.209	0.395	0.000	0.128	0.000	0.816	0.225	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	235	170	259	0	2493	0	637	469	0
N.S.	1	1.44	1.04	1.59	0.00	15.29	0.00	3.91	2.88	0.00
time (sec)	N/A	0.682	0.740	0.401	0.000	0.202	0.000	0.241	0.270	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	391	249	427	0	4390	0	1303	987	0
N.S.	1	1.45	0.93	1.59	0.00	16.32	0.00	4.84	3.67	0.00
time (sec)	N/A	1.027	2.028	0.529	0.000	0.970	0.000	0.237	0.491	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	330	555	0	728	1416	0	5699	1909	0
N.S.	1	0.94	1.59	0.00	2.08	4.05	0.00	16.28	5.45	0.00
time (sec)	N/A	0.846	0.947	0.000	0.050	0.210	0.000	0.255	0.279	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	227	284	0	402	712	0	2490	865	0
N.S.	1	0.94	1.17	0.00	1.66	2.94	0.00	10.29	3.57	0.00
time (sec)	N/A	0.650	0.546	0.000	0.044	0.222	0.000	0.217	0.239	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	137	128	0	187	294	0	806	300	0
N.S.	1	0.94	0.88	0.00	1.29	2.03	0.00	5.56	2.07	0.00
time (sec)	N/A	0.491	0.261	0.000	0.039	0.116	0.000	0.106	0.234	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	53	0	60	81	0	129	70	146
N.S.	1	0.97	0.85	0.00	0.97	1.31	0.00	2.08	1.13	2.35
time (sec)	N/A	0.324	0.123	0.000	0.034	0.108	0.000	0.172	0.273	9.612

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	145	136	0	0	0	0	0	18	0
N.S.	1	1.04	0.98	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.520	0.219	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	264	187	76	0	284	92	619	351	261
N.S.	1	1.21	0.85	0.35	0.00	1.30	0.42	2.83	1.60	1.19
time (sec)	N/A	0.946	0.368	0.244	0.000	0.091	1.069	0.460	0.320	10.027

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	257	180	71	0	302	87	612	346	256
N.S.	1	1.21	0.85	0.33	0.00	1.42	0.41	2.89	1.63	1.21
time (sec)	N/A	0.796	0.263	0.109	0.000	0.091	1.036	0.488	0.281	0.229

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	254	167	68	0	299	85	609	343	253
N.S.	1	1.22	0.80	0.33	0.00	1.43	0.41	2.91	1.64	1.21
time (sec)	N/A	0.781	0.304	0.098	0.000	0.117	1.019	0.475	0.257	0.249

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	254	172	68	0	262	85	609	343	256
N.S.	1	1.22	0.82	0.33	0.00	1.25	0.41	2.91	1.64	1.22
time (sec)	N/A	0.795	0.277	0.099	0.000	0.089	2.569	0.472	0.259	9.950

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	246	149	62	0	229	71	485	282	170
N.S.	1	1.22	0.74	0.31	0.00	1.14	0.35	2.41	1.40	0.85
time (sec)	N/A	0.689	0.269	0.101	0.000	0.091	0.916	0.468	0.238	9.878

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	250	140	62	0	268	71	317	343	262
N.S.	1	1.20	0.67	0.30	0.00	1.28	0.34	1.52	1.64	1.25
time (sec)	N/A	0.671	0.082	0.092	0.000	0.084	1.018	0.440	0.239	9.897

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	275	176	71	0	262	90	610	362	267
N.S.	1	1.21	0.77	0.31	0.00	1.15	0.39	2.68	1.59	1.17
time (sec)	N/A	0.786	0.337	0.101	0.000	0.085	0.811	0.476	0.300	0.220

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	223	77	68	0	232	1187	190	263	1287
N.S.	1	1.23	0.42	0.37	0.00	1.27	6.52	1.04	1.45	7.07
time (sec)	N/A	0.790	0.070	0.280	0.000	0.083	0.672	0.359	0.253	10.615

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	229	84	68	0	234	1182	185	260	1222
N.S.	1	1.37	0.50	0.41	0.00	1.40	7.08	1.11	1.56	7.32
time (sec)	N/A	0.717	0.059	0.161	0.000	0.088	0.683	0.349	0.248	10.409

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	215	77	68	0	206	879	190	262	243
N.S.	1	1.22	0.44	0.39	0.00	1.17	4.99	1.08	1.49	1.38
time (sec)	N/A	0.699	0.063	0.155	0.000	0.078	0.655	0.335	0.287	0.129

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	237	85	68	0	218	65	185	259	1069
N.S.	1	1.37	0.49	0.39	0.00	1.26	0.38	1.07	1.50	6.18
time (sec)	N/A	0.692	0.063	0.150	0.000	0.105	0.369	0.264	0.274	11.315

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	246	90	73	0	203	71	187	286	1457
N.S.	1	1.22	0.45	0.36	0.00	1.01	0.35	0.93	1.42	7.25
time (sec)	N/A	0.743	0.078	0.194	0.000	0.083	0.416	0.372	0.232	10.563

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	2285	112	107	0	0	0	0	22	2511
N.S.	1	3.59	0.18	0.17	0.00	0.00	0.00	0.00	0.03	3.95
time (sec)	N/A	7.314	0.059	0.296	0.000	0.000	0.000	0.000	200.050	10.775

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	1527	110	107	0	103351	150	0	22	646
N.S.	1	2.78	0.20	0.19	0.00	187.91	0.27	0.00	0.04	1.17
time (sec)	N/A	4.518	0.051	0.096	0.000	73.642	2.983	0.000	200.043	9.924

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	831	111	104	0	475271	0	0	18	433
N.S.	1	1.37	0.18	0.17	0.00	782.98	0.00	0.00	0.03	0.71
time (sec)	N/A	2.366	0.048	0.105	0.000	6.136	0.000	0.000	200.021	10.004

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	752	3288	176	156	0	0	0	0	22	2353
N.S.	1	4.37	0.23	0.21	0.00	0.00	0.00	0.00	0.03	3.13
time (sec)	N/A	10.616	0.115	0.125	0.000	0.000	0.000	0.000	200.022	10.473

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1845	1631	108	103	0	109156	148	0	55	629
N.S.	1	0.88	0.06	0.06	0.00	59.16	0.08	0.00	0.03	0.34
time (sec)	N/A	6.329	0.048	0.311	0.000	115.395	2.777	0.000	0.183	10.161

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1693	885	109	100	0	0	0	0	51	433
N.S.	1	0.52	0.06	0.06	0.00	0.00	0.00	0.00	0.03	0.26
time (sec)	N/A	2.347	0.039	0.090	0.000	0.000	0.000	0.000	0.193	9.881

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2262	3451	174	149	0	0	0	0	1074	2345
N.S.	1	1.53	0.08	0.07	0.00	0.00	0.00	0.00	0.47	1.04
time (sec)	N/A	12.394	0.097	0.128	0.000	0.000	0.000	0.000	0.210	10.135

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	203	145	237	328	423	0	218	295	0
N.S.	1	1.12	0.80	1.31	1.81	2.34	0.00	1.20	1.63	0.00
time (sec)	N/A	0.748	0.308	0.241	0.135	0.113	0.000	0.161	0.388	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	144	107	193	218	325	0	158	156	0
N.S.	1	1.23	0.91	1.65	1.86	2.78	0.00	1.35	1.33	0.00
time (sec)	N/A	0.608	0.195	0.125	0.131	0.095	0.000	0.162	0.307	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	85	137	126	267	0	123	64	120
N.S.	1	0.96	1.23	1.99	1.83	3.87	0.00	1.78	0.93	1.74
time (sec)	N/A	0.357	0.129	0.198	0.133	0.095	0.000	0.172	0.226	9.516

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	104	102	235	159	927	0	0	108	0
N.S.	1	1.30	1.28	2.94	1.99	11.59	0.00	0.00	1.35	0.00
time (sec)	N/A	0.619	0.197	0.096	0.128	0.138	0.000	0.000	0.222	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	109	110	189	156	433	0	281	144	0
N.S.	1	1.24	1.25	2.15	1.77	4.92	0.00	3.19	1.64	0.00
time (sec)	N/A	0.551	0.310	0.144	0.133	0.120	0.000	0.178	0.236	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	184	152	239	322	577	0	713	381	0
N.S.	1	1.23	1.01	1.59	2.15	3.85	0.00	4.75	2.54	0.00
time (sec)	N/A	0.699	0.384	0.155	0.150	0.185	0.000	0.203	0.292	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	263	216	317	557	755	0	1414	691	0
N.S.	1	1.16	0.96	1.40	2.46	3.34	0.00	6.26	3.06	0.00
time (sec)	N/A	0.827	0.555	0.170	0.150	0.366	0.000	0.233	0.294	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	403	294	665	0	238	0	0	471	0
N.S.	1	1.23	0.90	2.03	0.00	0.73	0.00	0.00	1.44	0.00
time (sec)	N/A	1.198	10.884	9.644	0.000	0.087	0.000	0.000	0.564	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	329	243	406	0	166	0	0	276	0
N.S.	1	1.32	0.97	1.62	0.00	0.66	0.00	0.00	1.10	0.00
time (sec)	N/A	0.891	10.565	6.589	0.000	0.087	0.000	0.000	0.440	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	281	98	199	0	148	0	0	32	0
N.S.	1	1.49	0.52	1.05	0.00	0.78	0.00	0.00	0.17	0.00
time (sec)	N/A	0.674	10.077	1.139	0.000	0.088	0.000	0.000	0.221	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	328	136	272	0	196	0	0	36	0
N.S.	1	1.64	0.68	1.36	0.00	0.98	0.00	0.00	0.18	0.00
time (sec)	N/A	0.842	10.641	8.425	0.000	0.095	0.000	0.000	0.233	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	394	307	568	0	292	0	0	697	0
N.S.	1	1.44	1.12	2.08	0.00	1.07	0.00	0.00	2.55	0.00
time (sec)	N/A	1.076	11.128	9.907	0.000	0.098	0.000	0.000	0.783	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	493	400	776	0	411	0	0	36	0
N.S.	1	1.38	1.12	2.17	0.00	1.15	0.00	0.00	0.10	0.00
time (sec)	N/A	1.331	11.673	13.792	0.000	0.097	0.000	0.000	1.902	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	236	150	295	368	427	0	517	469	0
N.S.	1	1.14	0.72	1.43	1.78	2.06	0.00	2.50	2.27	0.00
time (sec)	N/A	0.795	0.322	0.182	0.147	0.212	0.000	0.402	0.547	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	175	113	242	247	335	0	475	281	0
N.S.	1	1.25	0.81	1.73	1.76	2.39	0.00	3.39	2.01	0.00
time (sec)	N/A	0.640	0.250	0.161	0.147	0.142	0.000	0.376	0.463	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	87	187	156	269	0	381	160	61
N.S.	1	0.97	0.94	2.01	1.68	2.89	0.00	4.10	1.72	0.66
time (sec)	N/A	0.365	0.146	0.242	0.140	0.137	0.000	0.320	0.269	10.424

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	140	139	652	201	1073	0	0	427	0
N.S.	1	1.37	1.36	6.39	1.97	10.52	0.00	0.00	4.19	0.00
time (sec)	N/A	0.678	0.314	0.127	0.147	0.202	0.000	0.000	0.245	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	122	124	251	202	404	0	0	292	0
N.S.	1	1.04	1.06	2.15	1.73	3.45	0.00	0.00	2.50	0.00
time (sec)	N/A	0.577	0.303	0.171	0.141	0.190	0.000	0.000	0.251	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	212	153	274	313	557	0	0	640	0
N.S.	1	1.23	0.88	1.58	1.81	3.22	0.00	0.00	3.70	0.00
time (sec)	N/A	0.774	0.329	0.178	0.140	0.405	0.000	0.000	0.307	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	297	226	371	534	733	0	0	1063	0
N.S.	1	1.19	0.90	1.48	2.14	2.93	0.00	0.00	4.25	0.00
time (sec)	N/A	0.991	0.650	0.210	0.157	0.675	0.000	0.000	0.328	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	434	312	1062	0	233	0	0	1049	0
N.S.	1	1.22	0.87	2.97	0.00	0.65	0.00	0.00	2.94	0.00
time (sec)	N/A	1.341	10.929	17.503	0.000	0.093	0.000	0.000	1.093	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	371	256	820	0	204	0	0	1024	0
N.S.	1	1.27	0.88	2.82	0.00	0.70	0.00	0.00	3.52	0.00
time (sec)	N/A	1.023	10.615	14.029	0.000	0.091	0.000	0.000	0.938	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	325	229	515	0	160	0	0	245	0
N.S.	1	1.66	1.17	2.63	0.00	0.82	0.00	0.00	1.25	0.00
time (sec)	N/A	0.793	10.654	5.186	0.000	0.089	0.000	0.000	0.651	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	393	270	873	0	251	0	0	642	0
N.S.	1	1.67	1.15	3.71	0.00	1.07	0.00	0.00	2.73	0.00
time (sec)	N/A	1.090	10.705	14.325	0.000	0.087	0.000	0.000	0.899	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	440	352	1037	0	284	0	0	0	0
N.S.	1	1.48	1.18	3.48	0.00	0.95	0.00	0.00	0.00	0.00
time (sec)	N/A	1.307	11.484	17.328	0.000	0.096	0.000	0.000	1.846	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	521	472	1166	0	393	0	0	0	0
N.S.	1	1.38	1.25	3.08	0.00	1.04	0.00	0.00	0.00	0.00
time (sec)	N/A	1.516	11.928	18.869	0.000	0.091	0.000	0.000	2.774	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	57	110	0	74	0	0	28	0
N.S.	1	1.00	3.00	5.79	0.00	3.89	0.00	0.00	1.47	0.00
time (sec)	N/A	0.258	1.466	0.872	0.000	0.092	0.000	0.000	0.200	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	57	110	0	74	0	0	28	0
N.S.	1	1.00	3.00	5.79	0.00	3.89	0.00	0.00	1.47	0.00
time (sec)	N/A	0.246	0.008	0.411	0.000	0.089	0.000	0.000	0.199	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	227	148	239	340	425	0	225	292	0
N.S.	1	1.19	0.78	1.26	1.79	2.24	0.00	1.18	1.54	0.00
time (sec)	N/A	0.722	0.345	0.142	0.140	0.105	0.000	0.174	0.371	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	146	108	195	223	333	0	166	156	0
N.S.	1	1.18	0.87	1.57	1.80	2.69	0.00	1.34	1.26	0.00
time (sec)	N/A	0.558	0.208	0.127	0.141	0.102	0.000	0.162	0.272	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	88	134	129	267	0	128	63	111
N.S.	1	0.94	1.22	1.86	1.79	3.71	0.00	1.78	0.88	1.54
time (sec)	N/A	0.345	0.138	0.428	0.142	0.104	0.000	0.163	0.245	10.094

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	104	101	312	155	972	0	0	147	0
N.S.	1	1.30	1.26	3.90	1.94	12.15	0.00	0.00	1.84	0.00
time (sec)	N/A	0.575	0.195	0.101	0.138	0.144	0.000	0.000	0.212	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	113	114	197	173	451	0	292	157	0
N.S.	1	1.23	1.24	2.14	1.88	4.90	0.00	3.17	1.71	0.00
time (sec)	N/A	0.537	0.355	0.145	0.145	0.138	0.000	0.166	0.253	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	188	149	236	359	593	0	778	392	0
N.S.	1	1.23	0.97	1.54	2.35	3.88	0.00	5.08	2.56	0.00
time (sec)	N/A	0.629	0.391	0.157	0.144	0.212	0.000	0.192	0.350	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	413	298	664	0	240	0	0	472	0
N.S.	1	1.02	0.74	1.65	0.00	0.60	0.00	0.00	1.17	0.00
time (sec)	N/A	1.198	10.950	10.903	0.000	0.115	0.000	0.000	0.704	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	335	248	409	0	168	0	0	280	0
N.S.	1	1.04	0.77	1.27	0.00	0.52	0.00	0.00	0.87	0.00
time (sec)	N/A	0.910	10.234	6.909	0.000	0.087	0.000	0.000	0.411	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	287	107	164	0	127	0	0	36	0
N.S.	1	1.28	0.48	0.73	0.00	0.56	0.00	0.00	0.16	0.00
time (sec)	N/A	0.695	9.704	1.151	0.000	0.087	0.000	0.000	0.214	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	329	151	345	0	239	0	0	43	0
N.S.	1	1.42	0.65	1.49	0.00	1.03	0.00	0.00	0.19	0.00
time (sec)	N/A	0.854	12.356	9.184	0.000	0.111	0.000	0.000	0.265	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	402	299	567	0	297	0	0	581	0
N.S.	1	1.20	0.89	1.69	0.00	0.88	0.00	0.00	1.73	0.00
time (sec)	N/A	1.116	10.941	10.520	0.000	0.089	0.000	0.000	0.813	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	254	183	303	389	675	0	663	538	0
N.S.	1	1.14	0.82	1.36	1.75	3.04	0.00	2.99	2.42	0.00
time (sec)	N/A	0.734	0.394	0.209	0.147	0.199	0.000	0.251	0.393	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	170	144	248	262	541	0	552	351	0
N.S.	1	1.10	0.93	1.60	1.69	3.49	0.00	3.56	2.26	0.00
time (sec)	N/A	0.712	0.262	0.171	0.154	0.174	0.000	0.233	0.292	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	99	114	217	161	395	0	0	207	61
N.S.	1	1.02	1.18	2.24	1.66	4.07	0.00	0.00	2.13	0.63
time (sec)	N/A	0.392	0.145	0.461	0.142	0.157	0.000	0.000	0.264	11.158

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	137	140	1015	201	1477	0	0	627	0
N.S.	1	1.25	1.27	9.23	1.83	13.43	0.00	0.00	5.70	0.00
time (sec)	N/A	0.707	0.494	0.128	0.148	0.215	0.000	0.000	0.317	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	112	148	244	247	599	0	0	519	0
N.S.	1	0.93	1.23	2.03	2.06	4.99	0.00	0.00	4.32	0.00
time (sec)	N/A	0.628	0.282	0.182	0.145	0.192	0.000	0.000	0.451	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	199	192	279	450	961	0	0	896	0
N.S.	1	1.11	1.07	1.55	2.50	5.34	0.00	0.00	4.98	0.00
time (sec)	N/A	0.765	0.384	0.196	0.149	0.920	0.000	0.000	0.432	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	450	296	879	0	436	0	0	1284	0
N.S.	1	1.04	0.68	2.03	0.00	1.00	0.00	0.00	2.96	0.00
time (sec)	N/A	1.392	10.724	18.634	0.000	0.109	0.000	0.000	1.061	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	386	253	667	0	326	0	0	0	0
N.S.	1	1.36	0.89	2.35	0.00	1.15	0.00	0.00	0.00	0.00
time (sec)	N/A	1.034	10.594	14.902	0.000	0.116	0.000	0.000	0.820	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	339	240	467	0	327	0	0	930	0
N.S.	1	1.44	1.02	1.99	0.00	1.39	0.00	0.00	3.96	0.00
time (sec)	N/A	0.822	10.699	6.493	0.000	0.091	0.000	0.000	0.581	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	405	261	686	0	432	0	0	0	0
N.S.	1	1.61	1.04	2.73	0.00	1.72	0.00	0.00	0.00	0.00
time (sec)	N/A	1.106	10.805	18.975	0.000	0.113	0.000	0.000	0.780	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	470	306	1080	0	601	0	0	0	0
N.S.	1	1.20	0.78	2.76	0.00	1.53	0.00	0.00	0.00	0.00
time (sec)	N/A	1.328	11.254	18.266	0.000	0.133	0.000	0.000	1.321	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	79	0	0	0	0	0	28	78
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.49	1.37
time (sec)	N/A	0.315	0.174	0.000	0.000	0.000	0.000	0.000	0.203	9.496

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	95	0	0	0	0	0	0	30	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.497	0.000	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	90	0	0	0	0	0	0	27	0
N.S.	1	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.412	0.000	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	93	0	0	0	0	0	0	30	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.454	0.000	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	95	0	0	0	0	0	0	30	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.480	0.000	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	0	0	0	0	0	0	34	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.495	0.000	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	50	57	81	51	68	79	72
N.S.	1	1.00	0.93	0.91	1.04	1.47	0.93	1.24	1.44	1.31
time (sec)	N/A	0.350	0.056	0.309	0.026	0.091	0.143	0.128	0.195	0.079

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	38	40	59	39	38	66	37
N.S.	1	1.00	0.93	0.88	0.93	1.37	0.91	0.88	1.53	0.86
time (sec)	N/A	0.320	0.025	0.305	0.025	0.078	0.309	0.123	0.197	0.063

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	33	23	23	34	20	22	31	22
N.S.	1	1.00	1.27	0.88	0.88	1.31	0.77	0.85	1.19	0.85
time (sec)	N/A	0.280	0.015	0.305	0.030	0.089	0.100	0.117	0.208	0.045

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	42	46	65	42	57	84	43
N.S.	1	1.00	0.94	0.89	0.98	1.38	0.89	1.21	1.79	0.91
time (sec)	N/A	0.328	0.040	0.295	0.029	0.091	0.385	0.123	0.206	9.058

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	64	69	92	70	70	104	70
N.S.	1	1.00	0.86	1.08	1.17	1.56	1.19	1.19	1.76	1.19
time (sec)	N/A	0.362	0.066	0.307	0.036	0.075	0.272	0.118	0.201	8.807

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	65	73	214	121	75	107	103
N.S.	1	1.00	1.00	0.84	0.95	2.78	1.57	0.97	1.39	1.34
time (sec)	N/A	0.383	0.088	0.316	0.109	0.085	0.162	0.119	0.205	8.840

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	55	60	184	95	60	92	69
N.S.	1	1.00	0.95	0.89	0.97	2.97	1.53	0.97	1.48	1.11
time (sec)	N/A	0.339	0.067	0.309	0.111	0.080	0.163	0.124	0.193	8.804

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	45	44	174	88	43	89	41
N.S.	1	1.00	1.00	0.82	0.80	3.16	1.60	0.78	1.62	0.75
time (sec)	N/A	0.323	0.046	0.328	0.104	0.077	0.127	0.346	0.202	0.058

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	40	168	85	40	84	38
N.S.	1	1.00	1.00	0.82	0.80	3.36	1.70	0.80	1.68	0.76
time (sec)	N/A	0.273	0.045	0.325	0.107	0.103	0.127	0.110	0.197	0.055

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	66	173	112	66	92	64
N.S.	1	1.00	1.03	0.84	1.05	2.75	1.78	1.05	1.46	1.02
time (sec)	N/A	0.345	0.093	0.309	0.111	0.097	0.206	0.113	0.235	8.873

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	64	82	210	136	67	109	77
N.S.	1	1.00	0.97	0.83	1.06	2.73	1.77	0.87	1.42	1.00
time (sec)	N/A	0.366	0.088	0.382	0.108	0.099	0.282	0.115	0.208	8.952

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	F(-2)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	102	138	108	106	237	233	0	496	633
N.S.	1	0.78	1.05	0.82	0.81	1.81	1.78	0.00	3.79	4.83
time (sec)	N/A	0.704	0.119	0.112	0.110	0.103	0.928	0.000	0.220	9.050

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F(-2)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	78	112	86	80	205	182	0	290	135
N.S.	1	0.81	1.17	0.90	0.83	2.14	1.90	0.00	3.02	1.41
time (sec)	N/A	0.628	0.091	0.077	0.116	0.092	0.494	0.000	0.205	8.917

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	50	54	44	46	142	71	0	197	39
N.S.	1	1.06	1.15	0.94	0.98	3.02	1.51	0.00	4.19	0.83
time (sec)	N/A	0.311	0.025	0.092	0.107	0.101	0.236	0.000	0.211	0.101

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	100	113	99	100	217	287	0	294	2098
N.S.	1	1.11	1.26	1.10	1.11	2.41	3.19	0.00	3.27	23.31
time (sec)	N/A	0.718	0.109	0.096	0.111	0.089	48.940	0.000	0.205	11.812

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	186	124	158	296	0	0	546	5179
N.S.	1	1.09	1.59	1.06	1.35	2.53	0.00	0.00	4.67	44.26
time (sec)	N/A	0.789	0.198	0.109	0.110	0.119	0.000	0.000	0.211	15.230

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	632	224	111	0	1444	286	0	1301	3247
N.S.	1	1.41	0.50	0.25	0.00	3.22	0.64	0.00	2.90	7.25
time (sec)	N/A	3.379	0.229	0.253	0.000	0.135	2.853	0.000	0.209	0.978

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	607	199	89	0	962	184	0	957	1715
N.S.	1	1.47	0.48	0.21	0.00	2.32	0.44	0.00	2.31	4.14
time (sec)	N/A	2.389	0.213	0.165	0.000	0.114	1.123	0.000	0.237	0.468

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	449	185	67	0	405	76	0	632	285
N.S.	1	1.31	0.54	0.20	0.00	1.18	0.22	0.00	1.85	0.83
time (sec)	N/A	1.382	0.178	0.155	0.000	0.105	0.407	0.000	0.261	9.190

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	158	158	61	0	903	100	0	962	1245
N.S.	1	0.43	0.43	0.16	0.00	2.44	0.27	0.00	2.60	3.36
time (sec)	N/A	0.908	0.199	0.170	0.000	0.107	0.615	0.000	0.252	0.462

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	584	200	243	0	2314	206	0	1320	4231
N.S.	1	1.32	0.45	0.55	0.00	5.21	0.46	0.00	2.97	9.53
time (sec)	N/A	2.144	0.481	0.169	0.000	0.100	3.758	0.000	1.713	10.025

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	222	202	0	179	198	0	855	607	0
N.S.	1	0.94	0.86	0.00	0.76	0.84	0.00	3.62	2.57	0.00
time (sec)	N/A	0.872	0.460	0.000	0.037	0.167	0.000	0.145	0.266	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	134	121	0	101	112	0	433	342	0
N.S.	1	0.95	0.86	0.00	0.72	0.79	0.00	3.07	2.43	0.00
time (sec)	N/A	0.728	0.248	0.000	0.037	0.150	0.000	0.142	0.191	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	61	45	47	56	0	133	163	65
N.S.	1	0.97	1.02	0.75	0.78	0.93	0.00	2.22	2.72	1.08
time (sec)	N/A	0.349	0.129	0.316	0.034	0.137	0.000	0.121	0.194	9.646

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	130	0	0	0	0	288	392	0
N.S.	1	1.14	1.07	0.00	0.00	0.00	0.00	2.36	3.21	0.00
time (sec)	N/A	0.948	0.456	0.000	0.000	0.000	0.000	0.153	0.191	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-1)	F	B	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	150	0	0	0	0	1093	0	0
N.S.	1	0.00	1.03	0.00	0.00	0.00	0.00	7.54	0.00	0.00
time (sec)	N/A	0.000	0.728	0.000	0.000	0.000	0.000	0.236	5.458	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-1)	F	B	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	0	262	0	0	0	0	2413	0	0
N.S.	1	0.00	1.12	0.00	0.00	0.00	0.00	10.31	0.00	0.00
time (sec)	N/A	0.000	3.257	0.000	0.000	0.000	0.000	0.302	24.588	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	406	0	551	0	0	0	0	0	19	0
N.S.	1	0.00	1.36	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	33.878	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	0	454	0	0	0	0	0	15	0
N.S.	1	0.00	1.51	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	32.593	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	207	0	0	0	0	0	19	0
N.S.	1	0.00	1.38	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	31.736	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	357	0	474	0	0	0	0	0	19	0
N.S.	1	0.00	1.33	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	32.656	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	478	0	665	0	0	0	0	0	19	0
N.S.	1	0.00	1.39	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	35.092	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	222	254	0	179	247	0	2794	770	0
N.S.	1	0.94	1.08	0.00	0.76	1.05	0.00	11.84	3.26	0.00
time (sec)	N/A	0.843	0.716	0.000	0.036	0.197	0.000	0.258	0.309	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	134	162	0	101	148	0	1488	461	0
N.S.	1	0.95	1.15	0.00	0.72	1.05	0.00	10.55	3.27	0.00
time (sec)	N/A	0.730	0.476	0.000	0.034	0.171	0.000	0.199	0.224	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	80	45	47	77	223	552	238	65
N.S.	1	0.97	1.33	0.75	0.78	1.28	3.72	9.20	3.97	1.08
time (sec)	N/A	0.350	0.382	0.323	0.037	0.177	0.449	0.142	0.224	9.372

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	B	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	150	147	0	0	0	0	420	924	0
N.S.	1	1.03	1.01	0.00	0.00	0.00	0.00	2.88	6.33	0.00
time (sec)	N/A	1.019	0.692	0.000	0.000	0.000	0.000	0.161	0.351	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-1)	F	B	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	157	0	0	0	0	1265	0	0
N.S.	1	0.00	1.08	0.00	0.00	0.00	0.00	8.72	0.00	0.00
time (sec)	N/A	0.000	0.850	0.000	0.000	0.000	0.000	0.292	4.535	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-1)	F	B	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	0	215	0	0	0	0	2664	0	0
N.S.	1	0.00	1.03	0.00	0.00	0.00	0.00	12.75	0.00	0.00
time (sec)	N/A	0.000	2.600	0.000	0.000	0.000	0.000	0.354	12.161	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	455	0	1169	0	0	0	0	0	0	0
N.S.	1	0.00	2.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	37.108	0.000	0.000	0.000	0.000	0.000	2.562	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	0	950	0	0	0	0	0	0	0
N.S.	1	0.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	35.160	0.000	0.000	0.000	0.000	0.000	0.559	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	445	0	0	0	0	0	784	0
N.S.	1	0.00	2.97	0.00	0.00	0.00	0.00	0.00	5.23	0.00
time (sec)	N/A	0.000	34.297	0.000	0.000	0.000	0.000	0.000	0.667	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	0	485	0	0	0	0	0	0	0
N.S.	1	0.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	35.346	0.000	0.000	0.000	0.000	0.000	1.433	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	429	0	559	0	0	0	0	0	0	0
N.S.	1	0.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	38.544	0.000	0.000	0.000	0.000	0.000	1.824	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	60	0	34	415	25	33	0
N.S.	1	1.00	0.78	1.33	0.00	0.76	9.22	0.56	0.73	0.00
time (sec)	N/A	0.275	0.068	0.042	0.000	0.098	0.657	0.121	0.178	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	55	0	28	197	0	11	0
N.S.	1	1.00	0.76	1.34	0.00	0.68	4.80	0.00	0.27	0.00
time (sec)	N/A	0.263	0.079	0.041	0.000	0.091	0.611	0.000	0.202	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	64	0	30	197	0	11	0
N.S.	1	1.00	0.76	1.56	0.00	0.73	4.80	0.00	0.27	0.00
time (sec)	N/A	0.267	0.062	0.045	0.000	0.106	0.620	0.000	0.176	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	72	0	0	70	0	0	18	0
N.S.	1	1.06	1.16	0.00	0.00	1.13	0.00	0.00	0.29	0.00
time (sec)	N/A	0.335	4.772	0.000	0.000	0.141	0.000	0.000	0.186	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	220	163	0	179	150	0	514	461	0
N.S.	1	0.94	0.70	0.00	0.76	0.64	0.00	2.20	1.97	0.00
time (sec)	N/A	0.877	0.277	0.000	0.040	0.176	0.000	0.146	0.205	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	132	94	0	101	79	0	253	240	0
N.S.	1	0.95	0.68	0.00	0.73	0.57	0.00	1.82	1.73	0.00
time (sec)	N/A	0.768	0.177	0.000	0.034	0.156	0.000	0.235	0.182	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	56	46	45	46	38	0	66	105	65
N.S.	1	0.97	0.79	0.78	0.79	0.66	0.00	1.14	1.81	1.12
time (sec)	N/A	0.355	0.072	0.278	0.037	0.144	0.000	0.112	0.196	9.473

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	107	0	0	0	0	227	778	0
N.S.	1	1.15	1.06	0.00	0.00	0.00	0.00	2.25	7.70	0.00
time (sec)	N/A	0.779	0.246	0.000	0.000	0.000	0.000	0.133	0.207	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-1)	F	B	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	178	0	0	0	0	798	0	0
N.S.	1	0.00	1.03	0.00	0.00	0.00	0.00	4.61	0.00	0.00
time (sec)	N/A	0.000	0.744	0.000	0.000	0.000	0.000	0.208	8.341	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	458	0	27114	0	0	0	0	0	0	0
N.S.	1	0.00	59.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	80.648	0.000	0.000	0.000	0.000	0.000	3.670	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	0	540	0	0	0	0	0	1489	0
N.S.	1	0.00	1.61	0.00	0.00	0.00	0.00	0.00	4.43	0.00
time (sec)	N/A	0.000	64.286	0.000	0.000	0.000	0.000	0.000	0.529	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	0	458	0	0	0	0	0	88	0
N.S.	1	0.00	1.75	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.000	59.411	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	0	1005	0	0	0	0	0	100	0
N.S.	1	0.00	5.32	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.000	58.065	0.000	0.000	0.000	0.000	0.000	0.453	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	418	0	535	0	0	0	0	0	100	0
N.S.	1	0.00	1.28	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	60.650	0.000	0.000	0.000	0.000	0.000	0.892	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	218	161	0	187	225	0	736	628	0
N.S.	1	0.94	0.69	0.00	0.81	0.97	0.00	3.17	2.71	0.00
time (sec)	N/A	0.898	0.612	0.000	0.036	0.387	0.000	0.214	0.265	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	128	85	0	107	139	0	388	362	0
N.S.	1	0.95	0.63	0.00	0.79	1.03	0.00	2.87	2.68	0.00
time (sec)	N/A	0.768	0.442	0.000	0.036	0.292	0.000	0.185	0.234	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	54	44	42	45	76	92	127	182	53
N.S.	1	0.96	0.79	0.75	0.80	1.36	1.64	2.27	3.25	0.95
time (sec)	N/A	0.355	0.333	0.282	0.034	0.231	0.466	0.140	0.206	10.196

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	B	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	171	143	0	0	0	0	1095	3195	0
N.S.	1	1.27	1.06	0.00	0.00	0.00	0.00	8.11	23.67	0.00
time (sec)	N/A	1.031	0.621	0.000	0.000	0.000	0.000	0.351	0.471	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-1)	F	B	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	0	208	0	0	0	0	4863	0	0
N.S.	1	0.00	0.99	0.00	0.00	0.00	0.00	23.05	0.00	0.00
time (sec)	N/A	0.000	1.529	0.000	0.000	0.000	0.000	1.571	59.995	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	448	0	646	0	0	0	0	0	0	0
N.S.	1	0.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	89.909	0.000	0.000	0.000	0.000	0.000	8.461	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	0	538	0	0	0	0	0	0	0
N.S.	1	0.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	62.154	0.000	0.000	0.000	0.000	0.000	1.273	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	0	315	0	0	0	0	0	310	0
N.S.	1	0.00	1.02	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.000	60.435	0.000	0.000	0.000	0.000	0.000	0.911	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	367	0	529	0	0	0	0	0	337	0
N.S.	1	0.00	1.44	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.000	63.443	0.000	0.000	0.000	0.000	0.000	1.134	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	0	678	0	0	0	0	0	347	0
N.S.	1	0.00	1.38	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.000	64.442	0.000	0.000	0.000	0.000	0.000	1.569	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	238	299	0	431	740	0	4289	21	0
N.S.	1	0.94	1.19	0.00	1.71	2.94	0.00	17.02	0.08	0.00
time (sec)	N/A	0.923	0.640	0.000	0.046	24.358	0.000	0.355	200.030	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	145	137	0	203	304	0	1396	372	0
N.S.	1	0.96	0.91	0.00	1.34	2.01	0.00	9.25	2.46	0.00
time (sec)	N/A	0.737	0.361	0.000	0.047	6.432	0.000	0.186	0.323	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	63	56	0	65	86	0	232	120	69
N.S.	1	0.97	0.86	0.00	1.00	1.32	0.00	3.57	1.85	1.06
time (sec)	N/A	0.369	0.196	0.000	0.035	1.532	0.000	0.122	0.367	8.954

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	144	0	0	0	0	0	20	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.767	0.303	0.000	0.000	0.000	0.000	0.000	0.328	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	261	203	0	0	0	0	0	20	0
N.S.	1	1.53	1.19	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.876	0.392	0.000	0.000	0.000	0.000	0.000	0.484	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	0	0	0	0	0	0	0	21	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.051	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	0	0	0	0	0	0	0	20	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.531	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	0	0	0	0	0	0	0	16	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.399	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	0	0	0	0	0	0	0	20	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.359	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	0	0	0	0	0	0	0	20	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.686	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	397	300	0	0	0	0	0	26	0
N.S.	1	1.52	1.15	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.343	2.381	0.000	0.000	0.000	0.000	0.000	0.333	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	257	195	0	0	0	0	0	25	0
N.S.	1	1.55	1.17	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.701	0.138	0.000	0.000	0.000	0.000	0.000	0.365	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	153	141	0	0	0	0	0	28	0
N.S.	1	1.06	0.97	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.746	1.817	0.000	0.000	0.000	0.000	0.000	0.387	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	56	0	0	93	0	161	28	69
N.S.	1	0.97	0.85	0.00	0.00	1.41	0.00	2.44	0.42	1.05
time (sec)	N/A	0.639	1.617	0.000	0.000	0.289	0.000	0.125	0.544	8.994

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	144	139	0	0	309	0	0	28	0
N.S.	1	0.94	0.91	0.00	0.00	2.02	0.00	0.00	0.18	0.00
time (sec)	N/A	0.740	2.708	0.000	0.000	0.909	0.000	0.000	0.652	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	254	239	303	0	0	726	0	0	28	0
N.S.	1	0.94	1.19	0.00	0.00	2.86	0.00	0.00	0.11	0.00
time (sec)	N/A	0.912	3.450	0.000	0.000	2.815	0.000	0.000	0.581	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	437	0	0	0	0	0	0	30	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.640	0.000	0.000	0.000	0.000	0.000	0.000	0.390	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	268	0	0	0	0	0	0	29	0
N.S.	1	1.49	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.882	0.000	0.000	0.000	0.000	0.000	0.000	0.443	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	228	176	0	0	0	0	0	32	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.080	2.104	0.000	0.000	0.000	0.000	0.000	0.404	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	32	83
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.52	1.36
time (sec)	N/A	0.650	2.088	0.000	0.000	0.000	0.000	0.000	0.565	9.451

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	174	122	0	0	0	0	0	32	0
N.S.	1	1.04	0.73	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.846	3.430	0.000	0.000	0.000	0.000	0.000	0.762	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	330	158	0	0	0	0	0	32	0
N.S.	1	1.04	0.50	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.245	4.046	0.000	0.000	0.000	0.000	0.000	0.957	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	61	58	54	53	35	71	82	32	0
N.S.	1	0.73	0.70	0.65	0.64	0.42	0.86	0.99	0.39	0.00
time (sec)	N/A	0.456	0.063	0.338	0.033	0.104	0.410	0.143	0.318	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	48	62	17	40	39	216	268	35	0
N.S.	1	0.75	0.97	0.27	0.62	0.61	3.38	4.19	0.55	0.00
time (sec)	N/A	0.440	0.062	0.409	0.032	0.133	1.330	0.200	0.387	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	107	0	0	0	0	0	0	33	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.653	0.000	0.000	0.000	0.000	0.000	0.000	0.488	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	98	0	0	0	0	0	0	32	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.517	0.000	0.000	0.000	0.000	0.000	0.000	0.456	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	108	107	590	0	581	0	0	35	0
N.S.	1	1.24	1.23	6.78	0.00	6.68	0.00	0.00	0.40	0.00
time (sec)	N/A	0.644	0.357	0.672	0.000	0.149	0.000	0.000	0.366	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	0	0	0	0	0	0	39	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.710	0.000	0.000	0.000	0.000	0.000	0.000	0.437	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	0	0	0	0	0	0	39	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.725	0.000	0.000	0.000	0.000	0.000	0.000	0.393	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	219	0	0	0	0	0	0	0
N.S.	1	1.07	2.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	12.389	0.000	0.000	0.000	0.000	0.000	0.771	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	983	0	0	0	0	0	0	0
N.S.	1	1.08	10.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.540	2.927	0.000	0.000	0.000	0.000	0.000	0.865	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	144	143	2925	0	723	0	0	902	0
N.S.	1	1.27	1.27	25.88	0.00	6.40	0.00	0.00	7.98	0.00
time (sec)	N/A	0.719	1.123	0.685	0.000	0.125	0.000	0.000	0.487	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	116	216	0	0	0	0	0	0	0
N.S.	1	1.07	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.723	2.551	0.000	0.000	0.000	0.000	0.000	0.813	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	118	220	0	0	0	0	0	0	0
N.S.	1	1.07	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.707	2.600	0.000	0.000	0.000	0.000	0.000	0.957	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	107	0	0	0	0	0	0	37	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.612	0.000	0.000	0.000	0.000	0.000	0.000	0.489	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	98	0	0	0	0	0	0	36	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.497	0.000	0.000	0.000	0.000	0.000	0.000	0.452	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	108	106	595	0	599	0	0	40	0
N.S.	1	1.24	1.22	6.84	0.00	6.89	0.00	0.00	0.46	0.00
time (sec)	N/A	0.615	0.335	0.865	0.000	0.102	0.000	0.000	0.372	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	0	0	0	0	0	0	46	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.686	0.000	0.000	0.000	0.000	0.000	0.000	0.483	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	0	0	0	0	0	0	46	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.677	0.000	0.000	0.000	0.000	0.000	0.000	0.462	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	228	0	0	0	0	0	0	0
N.S.	1	1.07	2.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.643	11.512	0.000	0.000	0.000	0.000	0.000	0.649	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	784	0	0	0	0	0	0	0
N.S.	1	1.08	8.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	3.019	0.000	0.000	0.000	0.000	0.000	0.627	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	143	145	3399	0	1187	0	0	837	0
N.S.	1	1.19	1.21	28.32	0.00	9.89	0.00	0.00	6.98	0.00
time (sec)	N/A	0.709	1.574	0.861	0.000	0.143	0.000	0.000	0.332	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	116	237	0	0	0	0	0	0	0
N.S.	1	1.07	2.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.721	2.501	0.000	0.000	0.000	0.000	0.000	0.683	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	118	229	0	0	0	0	0	0	0
N.S.	1	1.07	2.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.713	2.537	0.000	0.000	0.000	0.000	0.000	0.750	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	101	0	0	0	0	0	0	28	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.551	0.000	0.000	0.000	0.000	0.000	0.000	0.352	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	92	0	0	0	0	0	0	27	0
N.S.	1	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.443	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.410	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	102	0	0	0	0	0	0	30	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.584	0.000	0.000	0.000	0.000	0.000	0.000	0.370	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	0	0	0	0	0	0	30	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.579	0.000	0.000	0.000	0.000	0.000	0.000	0.347	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	0	0	0	0	0	0	34	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.624	0.000	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	278	187	0	0	0	0	0	0	0
N.S.	1	0.89	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.922	1.455	0.000	0.000	0.000	0.000	0.000	0.453	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	205	187	0	0	0	0	0	547	0
N.S.	1	0.92	0.84	0.00	0.00	0.00	0.00	0.00	2.46	0.00
time (sec)	N/A	0.645	1.175	0.000	0.000	0.000	0.000	0.000	0.418	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	88	0	0	0	0	0	348	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	2.22	0.00
time (sec)	N/A	0.550	0.253	0.000	0.000	0.000	0.000	0.000	0.454	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	179	0	0	0	0	0	248	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.693	0.393	0.000	0.000	0.000	0.000	0.000	0.385	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	206	183	0	0	0	0	0	0	0
N.S.	1	0.99	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	1.740	0.000	0.000	0.000	0.000	0.000	0.418	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	208	184	0	0	0	0	0	0	0
N.S.	1	0.99	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.708	1.926	0.000	0.000	0.000	0.000	0.000	0.459	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	114	395	0	205	222	0	0	214	0
N.S.	1	0.77	2.65	0.00	1.38	1.49	0.00	0.00	1.44	0.00
time (sec)	N/A	0.518	0.637	0.000	0.049	0.093	0.000	0.000	0.378	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	80	249	0	117	124	0	0	103	0
N.S.	1	0.82	2.57	0.00	1.21	1.28	0.00	0.00	1.06	0.00
time (sec)	N/A	0.454	0.542	0.000	0.054	0.117	0.000	0.000	0.401	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	0	52	49	0	0	45	0
N.S.	1	1.00	1.02	0.00	1.21	1.14	0.00	0.00	1.05	0.00
time (sec)	N/A	0.346	0.072	0.000	0.046	0.095	0.000	0.000	0.383	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	99	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.355	0.103	0.000	0.000	0.000	0.000	0.000	0.371	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	89	0	0	0	0	0	74	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.379	0.532	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	89	0	0	0	0	0	374	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	4.92	0.00
time (sec)	N/A	0.395	0.528	0.000	0.000	0.000	0.000	0.000	0.324	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	0	0	0	0	0	0	0	44	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.455	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	0	0	0	0	0	0	0	44	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.420	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	101	94	0	0	0	0	0	40	0
N.S.	1	1.42	1.32	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.643	0.491	0.000	0.000	0.000	0.000	0.000	0.426	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	0	0	0	0	0	0	0	34	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.405	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	44	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.355	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	0	0	0	0	0	0	0	48	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.315	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	301	0	0	0	0	0	0	71	0
N.S.	1	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.734	0.000	0.000	0.000	0.000	0.000	0.000	0.364	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	228	0	0	0	0	0	0	71	0
N.S.	1	1.37	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.385	0.000	0.000	0.000	0.000	0.000	0.000	0.405	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	126	88	0	0	0	0	0	67	0
N.S.	1	1.45	1.01	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	1.175	0.325	0.000	0.000	0.000	0.000	0.000	0.417	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	0	0	0	0	0	0	0	61	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.362	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	0	0	0	0	0	0	0	71	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.301	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	0	0	0	0	0	0	0	49	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.399	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0	49	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.434	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	94	0	0	0	0	0	45	0
N.S.	1	0.00	1.01	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.000	0.329	0.000	0.000	0.000	0.000	0.000	0.428	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	27	22	25	0	0	24	39	24
N.S.	1	1.00	1.12	0.92	1.04	0.00	0.00	1.00	1.62	1.00
time (sec)	N/A	1.376	0.289	0.012	0.386	0.000	0.000	0.483	0.389	8.813

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	0	0	27	49	27
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.00	1.00	1.81	1.00
time (sec)	N/A	0.582	0.694	0.015	0.403	0.000	0.000	0.526	0.375	8.859

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	27	0	0	27	53	29
N.S.	1	1.00	1.07	0.93	1.00	0.00	0.00	1.00	1.96	1.07
time (sec)	N/A	0.574	0.587	0.014	0.405	0.000	0.000	0.506	0.324	8.862

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	0	0	0	0	0	0	0	34	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.383	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	225	0	0	0	0	0	0	0	34	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.438	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	73	0	0	0	0	0	30	0
N.S.	1	0.00	0.74	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.189	0.000	0.000	0.000	0.000	0.000	0.434	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	148	0	0	0	0	0	24	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.856	0.180	0.000	0.000	0.000	0.000	0.000	0.389	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	0	213	0	0	0	0	0	34	0
N.S.	1	0.00	1.01	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.322	0.000	0.000	0.000	0.000	0.000	0.318	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	323	0	0	0	0	0	0	0	38	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.459	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	406	0	0	0	0	0	0	0	47	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.427	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	0	0	0	0	0	0	0	47	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.437	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	103	0	0	0	0	0	43	0
N.S.	1	0.00	1.34	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.000	0.211	0.000	0.000	0.000	0.000	0.000	0.452	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	234	220	0	0	0	0	0	37	0
N.S.	1	0.98	0.92	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.089	0.403	0.000	0.000	0.000	0.000	0.000	0.406	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	0	0	0	0	0	0	0	47	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.370	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	483	0	0	0	0	0	0	0	51	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.419	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.881	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0	834	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.30	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.445	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	90	0	0	0	0	0	190	0
N.S.	1	0.00	1.01	0.00	0.00	0.00	0.00	0.00	2.13	0.00
time (sec)	N/A	0.000	0.104	0.000	0.000	0.000	0.000	0.000	0.391	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	22	25	24	17	24	242	24
N.S.	1	1.00	1.14	1.00	1.14	1.09	0.77	1.09	11.00	1.09
time (sec)	N/A	0.670	0.285	0.027	0.357	0.088	9.223	0.182	0.420	8.780

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	114	27
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	4.56	1.08
time (sec)	N/A	0.461	0.584	1.099	0.354	0.094	0.000	0.234	0.426	8.532

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	397	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	15.88	1.16
time (sec)	N/A	0.466	0.523	0.121	0.359	0.096	0.000	0.239	0.457	9.056

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [102] had the largest ratio of [1.05882000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	15	0.267
2	A	4	3	0.94	15	0.200
3	A	6	5	0.95	13	0.385
4	A	3	2	1.00	11	0.182
5	A	7	6	0.97	15	0.400
6	A	5	4	1.18	15	0.267
7	A	6	5	1.11	15	0.333
8	A	5	4	1.10	15	0.267
9	A	4	3	1.10	13	0.231
10	A	6	5	0.94	19	0.263
11	A	6	5	0.95	17	0.294
12	A	13	12	1.07	19	0.632
13	A	10	9	0.93	19	0.474
14	A	10	9	0.93	17	0.529
15	A	6	5	1.02	15	0.333
16	A	9	8	1.00	19	0.421
17	A	7	6	1.10	19	0.316
18	A	10	9	1.09	19	0.474
19	A	10	9	1.06	19	0.474
20	A	15	14	1.12	19	0.737
21	A	11	10	0.93	19	0.526

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	11	10	0.95	17	0.588
23	A	7	6	1.00	15	0.400
24	A	11	10	1.07	19	0.526
25	A	8	7	1.08	19	0.368
26	A	11	10	1.11	19	0.526
27	A	11	10	1.08	19	0.526
28	A	8	7	1.07	19	0.368
29	A	7	6	1.00	17	0.353
30	A	6	5	1.05	15	0.333
31	A	5	4	1.14	19	0.211
32	A	5	4	0.81	19	0.211
33	A	8	7	0.70	19	0.368
34	A	7	6	0.69	19	0.316
35	A	8	7	0.67	19	0.368
36	A	12	11	0.87	19	0.579
37	A	10	9	0.91	19	0.474
38	A	10	9	0.91	17	0.529
39	A	6	5	0.98	15	0.333
40	A	9	8	1.00	19	0.421
41	A	7	6	1.07	19	0.316
42	A	10	9	1.07	19	0.474
43	A	10	9	1.05	19	0.474
44	A	13	12	0.96	19	0.632
45	A	11	10	0.92	19	0.526
46	A	11	10	0.93	17	0.588
47	A	7	6	1.05	15	0.400
48	A	11	10	1.18	19	0.526
49	A	8	7	1.15	19	0.368
50	A	11	10	1.13	19	0.526
51	A	11	10	1.18	19	0.526
52	A	13	12	1.01	19	0.632
53	A	10	9	0.92	17	0.529

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	7	1.03	17	0.412
55	A	8	7	1.00	15	0.467
56	A	4	3	1.00	13	0.231
57	A	9	8	1.00	17	0.471
58	A	5	4	1.00	17	0.235
59	A	8	7	1.04	17	0.412
60	A	7	6	1.10	17	0.353
61	F	0	0	N/A	0.000	N/A
62	F	0	0	N/A	0.000	N/A
63	A	17	16	0.98	19	0.842
64	A	14	13	0.97	19	0.684
65	A	14	13	0.99	17	0.765
66	A	6	5	0.98	15	0.333
67	A	16	15	1.01	19	0.789
68	A	10	9	1.06	19	0.474
69	A	13	12	1.00	19	0.632
70	A	13	12	1.02	19	0.632
71	A	19	18	0.94	19	0.947
72	A	14	13	1.06	19	0.684
73	A	16	15	0.94	17	0.882
74	A	7	6	1.00	15	0.400
75	A	19	18	1.13	19	0.947
76	A	11	10	0.96	19	0.526
77	A	15	14	0.74	19	0.737
78	A	14	13	0.99	19	0.684
79	A	14	13	0.99	19	0.684
80	A	11	10	0.98	19	0.526
81	A	11	10	0.98	17	0.588
82	A	3	2	1.00	15	0.133
83	A	8	7	1.00	19	0.368
84	A	7	6	1.02	19	0.316
85	A	10	9	1.04	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	10	1.08	19	0.526
87	A	17	16	1.00	19	0.842
88	A	13	12	0.98	19	0.632
89	A	13	12	1.01	17	0.706
90	A	5	4	1.04	15	0.267
91	A	8	7	1.17	19	0.368
92	A	10	9	1.08	19	0.474
93	A	13	12	1.03	19	0.632
94	A	14	13	1.04	19	0.684
95	A	17	16	1.05	19	0.842
96	A	13	12	0.99	17	0.706
97	A	11	10	0.98	17	0.588
98	A	11	10	0.98	15	0.667
99	A	5	4	1.00	13	0.308
100	A	13	12	1.00	17	0.706
101	A	6	5	0.99	17	0.294
102	A	19	18	0.95	17	1.059
103	N/A	1	0	1.00	19	0.000
104	A	6	5	1.51	19	0.263
105	A	5	4	1.40	19	0.211
106	A	6	5	0.96	17	0.294
107	A	5	4	1.15	15	0.267
108	A	8	7	1.25	19	0.368
109	A	8	7	1.22	19	0.368
110	A	8	7	1.14	19	0.368
111	A	6	5	0.94	19	0.263
112	A	5	4	0.95	19	0.211
113	A	6	5	0.98	17	0.294
114	A	5	4	0.95	15	0.267
115	A	9	8	0.96	19	0.421
116	A	7	6	1.16	19	0.316
117	A	10	9	1.20	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	0.95	19	0.263
119	A	5	4	0.95	19	0.211
120	A	6	5	1.00	17	0.294
121	A	5	4	0.94	15	0.267
122	A	8	7	1.18	19	0.368
123	A	7	6	1.12	19	0.316
124	A	10	9	1.12	19	0.474
125	A	6	5	0.94	21	0.238
126	A	5	4	0.94	21	0.190
127	A	6	5	0.95	19	0.263
128	A	5	4	0.98	17	0.235
129	A	11	10	1.16	21	0.476
130	A	10	9	1.36	21	0.429
131	A	12	11	1.50	21	0.524
132	A	6	5	0.94	21	0.238
133	A	5	4	0.94	21	0.190
134	A	6	5	0.95	19	0.263
135	A	5	4	0.98	17	0.235
136	A	8	7	1.15	21	0.333
137	A	10	9	1.44	21	0.429
138	A	12	11	1.45	21	0.524
139	A	6	5	0.94	19	0.263
140	A	5	4	0.94	19	0.211
141	A	6	5	0.94	17	0.294
142	A	5	4	0.97	15	0.267
143	A	6	5	1.04	19	0.263
144	A	2	2	1.21	17	0.118
145	A	2	2	1.21	17	0.118
146	A	2	2	1.22	17	0.118
147	A	2	2	1.22	17	0.118
148	A	2	2	1.22	17	0.118
149	A	2	2	1.20	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.21	17	0.118
151	A	2	2	1.23	15	0.133
152	A	2	2	1.37	15	0.133
153	A	2	2	1.22	15	0.133
154	A	2	2	1.37	11	0.182
155	A	2	2	1.22	15	0.133
156	B	2	2	3.59	20	0.100
157	B	2	2	2.78	20	0.100
158	A	14	13	1.37	16	0.812
159	B	2	2	4.37	20	0.100
160	A	2	2	0.88	19	0.105
161	A	14	13	0.52	15	0.867
162	A	2	2	1.53	19	0.105
163	A	11	10	1.12	21	0.476
164	A	9	8	1.23	21	0.381
165	A	6	5	0.96	19	0.263
166	A	9	8	1.30	21	0.381
167	A	6	5	1.24	21	0.238
168	A	10	9	1.23	21	0.429
169	A	11	10	1.16	21	0.476
170	A	10	10	1.23	21	0.476
171	A	7	7	1.32	21	0.333
172	A	6	6	1.49	17	0.353
173	A	8	8	1.64	21	0.381
174	A	12	12	1.44	21	0.571
175	A	14	14	1.38	21	0.667
176	A	11	10	1.14	21	0.476
177	A	12	11	1.25	21	0.524
178	A	7	6	0.97	19	0.316
179	A	10	9	1.37	21	0.429
180	A	7	6	1.04	21	0.286
181	A	11	10	1.23	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	12	11	1.19	21	0.524
183	A	12	12	1.22	21	0.571
184	A	9	9	1.27	21	0.429
185	A	8	8	1.66	17	0.471
186	A	12	12	1.67	21	0.571
187	A	14	14	1.48	21	0.667
188	A	16	16	1.38	21	0.762
189	A	3	3	1.00	21	0.143
190	A	2	2	1.00	21	0.095
191	A	10	9	1.19	21	0.429
192	A	9	8	1.18	21	0.381
193	A	6	5	0.94	19	0.263
194	A	9	8	1.30	21	0.381
195	A	6	5	1.23	21	0.238
196	A	9	8	1.23	21	0.381
197	A	9	9	1.02	21	0.429
198	A	7	7	1.04	21	0.333
199	A	6	6	1.28	17	0.353
200	A	8	8	1.42	21	0.381
201	A	10	10	1.20	21	0.476
202	A	10	9	1.14	21	0.429
203	A	13	12	1.10	21	0.571
204	A	7	6	1.02	19	0.316
205	A	10	9	1.25	21	0.429
206	A	7	6	0.93	21	0.286
207	A	12	11	1.11	21	0.524
208	A	13	13	1.04	21	0.619
209	A	10	10	1.36	21	0.476
210	A	8	8	1.44	17	0.471
211	A	10	10	1.61	21	0.476
212	A	13	13	1.20	21	0.619
213	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	4	3	1.00	17	0.176
215	F	0	0	N/A	0.000	N/A
216	F	0	0	N/A	0.000	N/A
217	F	0	0	N/A	0.000	N/A
218	A	5	5	1.09	19	0.263
219	A	5	5	1.10	15	0.333
220	A	5	5	1.09	19	0.263
221	A	5	5	1.09	19	0.263
222	A	5	5	1.08	21	0.238
223	A	2	2	1.00	17	0.118
224	A	2	2	1.00	17	0.118
225	A	2	2	1.00	15	0.133
226	A	2	2	1.00	17	0.118
227	A	2	2	1.00	17	0.118
228	A	2	2	1.00	17	0.118
229	A	2	2	1.00	17	0.118
230	A	2	2	1.00	17	0.118
231	A	1	1	1.00	13	0.077
232	A	2	2	1.00	17	0.118
233	A	2	2	1.00	17	0.118
234	A	8	7	0.78	19	0.368
235	A	7	6	0.81	19	0.316
236	A	5	4	1.06	17	0.235
237	A	2	2	1.11	19	0.105
238	A	2	2	1.09	19	0.105
239	A	2	2	1.41	19	0.105
240	A	2	2	1.47	19	0.105
241	A	2	2	1.31	19	0.105
242	A	2	2	0.43	15	0.133
243	A	2	2	1.32	19	0.105
244	A	6	5	0.94	23	0.217
245	A	7	6	0.95	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	5	4	0.97	21	0.190
247	A	14	13	1.14	23	0.565
248	F	0	0	N/A	0.000	N/A
249	F	0	0	N/A	0.000	N/A
250	F	0	0	N/A	0.000	N/A
251	F	0	0	N/A	0.000	N/A
252	F	0	0	N/A	0.000	N/A
253	F	0	0	N/A	0.000	N/A
254	F	0	0	N/A	0.000	N/A
255	A	6	5	0.94	23	0.217
256	A	7	6	0.95	23	0.261
257	A	5	4	0.97	21	0.190
258	A	17	16	1.03	23	0.696
259	F	0	0	N/A	0.000	N/A
260	F	0	0	N/A	0.000	N/A
261	F	0	0	N/A	0.000	N/A
262	F	0	0	N/A	0.000	N/A
263	F	0	0	N/A	0.000	N/A
264	F	0	0	N/A	0.000	N/A
265	F	0	0	N/A	0.000	N/A
266	A	1	1	1.00	17	0.059
267	A	1	1	1.00	15	0.067
268	A	1	1	1.00	15	0.067
269	A	1	1	1.06	25	0.040
270	A	6	5	0.94	23	0.217
271	A	7	6	0.95	23	0.261
272	A	5	4	0.97	21	0.190
273	A	10	9	1.15	23	0.391
274	F	0	0	N/A	0.000	N/A
275	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
276	F	0	0	N/A	0.000	N/A
277	F	0	0	N/A	0.000	N/A
278	F	0	0	N/A	0.000	N/A
279	F	0	0	N/A	0.000	N/A
280	A	6	5	0.94	23	0.217
281	A	7	6	0.95	23	0.261
282	A	5	4	0.96	21	0.190
283	A	12	11	1.27	23	0.478
284	F	0	0	N/A	0.000	N/A
285	F	0	0	N/A	0.000	N/A
286	F	0	0	N/A	0.000	N/A
287	F	0	0	N/A	0.000	N/A
288	F	0	0	N/A	0.000	N/A
289	F	0	0	N/A	0.000	N/A
290	A	6	5	0.94	21	0.238
291	A	7	6	0.96	21	0.286
292	A	5	4	0.97	19	0.211
293	A	7	6	1.00	21	0.286
294	A	8	7	1.53	21	0.333
295	F	0	0	N/A	0.000	N/A
296	F	0	0	N/A	0.000	N/A
297	F	0	0	N/A	0.000	N/A
298	F	0	0	N/A	0.000	N/A
299	F	0	0	N/A	0.000	N/A
300	A	9	8	1.52	19	0.421
301	A	6	5	1.55	17	0.294
302	A	5	4	1.06	21	0.190
303	A	5	4	0.97	21	0.190
304	A	5	4	0.94	21	0.190
305	A	4	3	0.94	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	13	12	1.13	19	0.632
307	A	8	7	1.49	17	0.412
308	A	8	7	1.03	21	0.333
309	A	5	4	1.00	21	0.190
310	A	9	8	1.04	21	0.381
311	A	10	9	1.04	21	0.429
312	A	7	6	0.73	17	0.353
313	A	8	7	0.75	17	0.412
314	A	5	5	1.08	19	0.263
315	A	5	5	1.09	17	0.294
316	A	9	8	1.24	21	0.381
317	A	5	5	1.08	21	0.238
318	A	5	5	1.08	21	0.238
319	A	5	5	1.07	19	0.263
320	A	5	5	1.08	17	0.294
321	A	10	9	1.27	21	0.429
322	A	5	5	1.07	21	0.238
323	A	5	5	1.07	21	0.238
324	A	5	5	1.08	19	0.263
325	A	5	5	1.09	17	0.294
326	A	9	8	1.24	21	0.381
327	A	5	5	1.08	21	0.238
328	A	5	5	1.08	21	0.238
329	A	5	5	1.07	19	0.263
330	A	5	5	1.08	17	0.294
331	A	10	9	1.19	21	0.429
332	A	5	5	1.07	21	0.238
333	A	5	5	1.07	21	0.238
334	A	5	5	1.09	17	0.294
335	A	5	5	1.10	15	0.333
336	F	0	0	N/A	0.000	N/A
337	A	5	5	1.09	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	5	5	1.08	19	0.263
339	A	5	5	1.08	21	0.238
340	A	8	7	0.89	25	0.280
341	A	6	5	0.92	25	0.200
342	A	5	4	1.00	23	0.174
343	A	6	5	1.00	22	0.227
344	A	6	5	0.99	25	0.200
345	A	6	5	0.99	25	0.200
346	A	6	5	0.77	23	0.217
347	A	6	5	0.82	23	0.217
348	A	3	3	1.00	21	0.143
349	A	5	4	1.00	20	0.200
350	A	5	4	1.00	23	0.174
351	A	5	4	1.00	23	0.174
352	F	0	0	N/A	0.000	N/A
353	F	0	0	N/A	0.000	N/A
354	A	7	6	1.42	23	0.261
355	F	0	0	N/A	0.000	N/A
356	F	0	0	N/A	0.000	N/A
357	F	0	0	N/A	0.000	N/A
358	A	12	11	1.26	25	0.440
359	A	9	8	1.37	25	0.320
360	A	8	7	1.45	23	0.304
361	F	0	0	N/A	0.000	N/A
362	F	0	0	N/A	0.000	N/A
363	F	0	0	N/A	0.000	N/A
364	F	0	0	N/A	0.000	N/A
365	F	0	0	N/A	0.000	N/A
366	N/A	8	0	1.00	24	0.000
367	N/A	1	0	1.00	27	0.000
368	N/A	1	0	1.00	27	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
369	F	0	0	N/A	0.000	N/A
370	F	0	0	N/A	0.000	N/A
371	F	0	0	N/A	0.000	N/A
372	A	8	7	1.00	24	0.292
373	F	0	0	N/A	0.000	N/A
374	F	0	0	N/A	0.000	N/A
375	F	0	0	N/A	0.000	N/A
376	F	0	0	N/A	0.000	N/A
377	F	0	0	N/A	0.000	N/A
378	A	10	9	0.98	24	0.375
379	F	0	0	N/A	0.000	N/A
380	F	0	0	N/A	0.000	N/A
381	F	0	0	N/A	0.000	N/A
382	F	0	0	N/A	0.000	N/A
383	F	0	0	N/A	0.000	N/A
384	N/A	6	0	1.00	22	0.000
385	N/A	1	0	1.00	25	0.000
386	N/A	1	0	1.00	25	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{x^3}{c+(a+bx)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^3}{c+(a+bx)^2} \, dx = -\frac{2ax}{b^3}+\frac{x^2}{2b^2}-\frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(3a^2-c)\log\left(c+(a+bx)^2\right)}{2b^4}$$

output
```
-2*a*x/b^3+1/2*x^2/b^2-a*(a^2-3*c)*arctan((b*x+a)/c^(1/2))/b^4/c^(1/2)+1/2
*(3*a^2-c)*ln(c+(b*x+a)^2)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{c+(a+bx)^2} \, dx = \frac{bx(-4a+bx)-\frac{2(a^3-3ac)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+(3a^2-c)\log\left(a^2+c+2abx+b^2x^2\right)}{2b^4}$$

input
```
Integrate[x^3/(c + (a + b*x)^2),x]
```

output
```
(b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] +
(3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {896, 25, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^2+c}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{b^3x^3}{(a+bx)^2+c}d(a+bx)}{b^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{b^3x^3}{(a+bx)^2+c}d(a+bx)}{b^4}$$

$$\Big\downarrow \text{478}$$

$$-\frac{\int \left(2a-bx+\frac{a^3-3ca-(3a^2-c)(a+bx)}{(a+bx)^2+c}\right)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{1}{2}(3a^2-c)\log\left((a+bx)^2+c\right)+\frac{1}{2}(a+bx)^2-3a(a+bx)}{b^4}$$

input
```
Int[x^3/(c + (a + b*x)^2),x]
```

output
```
(-3*a*(a + b*x) + (a + b*x)^2/2 - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c]]
)/Sqrt[c] + ((3*a^2 - c)*Log[c + (a + b*x)^2])/2)/b^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

method	result
default	$-\frac{-\frac{1}{2}b\,x^{2}+2ax}{b^{3}}+\frac{\frac{\left(3a^{2}b-bc\right)\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{2b^{2}}+\frac{\left(2a^{3}+2ac-\frac{\left(3a^{2}b-bc\right)a}{b}\right)\arctan\left(\frac{2b^{2}x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}}{b^{3}}$
risch	$\frac{x^{2}}{2b^{2}}-\frac{2ax}{b^{3}}+\frac{3\ln\left(-c\,a^{3}-\sqrt{-a^{2}c\left(a^{2}-3c\right)^{2}}\,bx+3a\,c^{2}-\sqrt{-a^{2}c\left(a^{2}-3c\right)^{2}}\,a\right)a^{2}}{2b^{4}}-\frac{c\ln\left(-c\,a^{3}-\sqrt{-a^{2}c\left(a^{2}-3c\right)^{2}}\,bx+3a\,c^{2}-\sqrt{-}\right.}{2b^{4}}$

input
```
int(x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/2*b*x^2+2*a*x)+1/b^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+
a^2+c)+(2*a^3+2*a*c-(3*a^2*b-b*c)*a/b)/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b
)/b/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.68

$$\int \frac{x^3}{c+(a+bx)^2}\,dx$$
$$= \left[\frac{b^2cx^2 - 4\,abcx + (a^3 - 3\,ac)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right) + (3\,a^2c - c^2)\log\left(b^2x^2 + 2\,abx + a^2\right.}{2\,b^4c}\right.$$

input
```
integrate(x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*
x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^
2*c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a
*b*c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^
2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(68) = 136.

Time = 0.38 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.82

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = -\frac{2ax}{b^3} + \left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \frac{x^2}{2b^2}$$

input
```
integrate(x**3/(c+(b*x+a)**2),x)
```

output
```
-2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)
)*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2
- c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b
**4*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2
 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) +
x**2/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{bx^2-4\,ax}{2\,b^3} + \frac{(3\,a^2-c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4} - \frac{(a^3-3\,ac)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^4\sqrt{c}}$$

input
```
integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)
/b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{(3\,a^2-c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4} - \frac{(a^3-3\,ac)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{b^2x^2-4\,abx}{2\,b^4}$$

input
```
integrate(x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arcta
n((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{x^2}{2\,b^2} - \frac{2\,a\,x}{b^3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)\,\left(4\,b^4\,c^2-12\,a^2\,b^4\,c\right)}{8\,b^8\,c} + \frac{a\,\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)\,\left(3\,c-a^2\right)}{b^4\,\sqrt{c}}$$

input
```
int(x^3/(c + (a + b*x)^2),x)
```

output
```
x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 -
 12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{-2\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)a^3+6\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)ac+3\log(b^2x^2+2abx+a^2+c)\,a^2c-\log(b^2x^2+2abx+a^2+}{2b^4c}$$

input
```
int(x^3/(c+(b*x+a)^2),x)
```

output
```
( - 2*sqrt(c)*atan((a + b*x)/sqrt(c))*a**3 + 6*sqrt(c)*atan((a + b*x)/sqrt
(c))*a*c + 3*log(a**2 + 2*a*b*x + b**2*x**2 + c)*a**2*c - log(a**2 + 2*a*b
*x + b**2*x**2 + c)*c**2 - 4*a*b*c*x + b**2*c*x**2)/(2*b**4*c)
```

3.2 $\int \frac{x^2}{c+(a+bx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{x^2}{c+(a+bx)^2} \, dx = \frac{x}{b^2} + \frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}} - \frac{a\log\left(c+(a+bx)^2\right)}{b^3}$$

output
```
x/b^2+(a^2-c)*arctan((b*x+a)/c^(1/2))/b^3/c^(1/2)-a*ln(c+(b*x+a)^2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{c+(a+bx)^2} \, dx = \frac{bx + \frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} - a\log\left(a^2+c+2abx+b^2x^2\right)}{b^3}$$

input
```
Integrate[x^2/(c + (a + b*x)^2),x]
```

output
```
(b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a
*b*x + b^2*x^2])/b^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {896, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^2+c}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{b^2x^2}{(a+bx)^2+c}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 478}$$

$$\frac{\int \left(\frac{a^2-2(a+bx)a-c}{(a+bx)^2+c}+1\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+a\left(-\log\left((a+bx)^2+c\right)\right)+a+bx}{b^3}$$

input
```
Int[x^2/(c + (a + b*x)^2),x]
```

output
```
(a + b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[c + (a +
b*x)^2])/b^3
```

Defintions of rubi rules used

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

method	result
default	$\frac{x}{b^{2}}+\frac{-\frac{a\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{b}+\frac{\left(a^{2}-c\right)\arctan\left(\frac{2b^{2}x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}}{b^{2}}$
risch	$\frac{x}{b^{2}}-\frac{\ln\left(-\sqrt{-c\left(a^{2}-c\right)^{2}}\,bx+a^{2}c-\sqrt{-c\left(a^{2}-c\right)^{2}}\,a-c^{2}\right)a}{b^{3}}+\frac{\ln\left(-\sqrt{-c\left(a^{2}-c\right)^{2}}\,bx+a^{2}c-\sqrt{-c\left(a^{2}-c\right)^{2}}\,a-c^{2}\right)\sqrt{-c\left(a^{2}-c\right)^{2}}}{2b^{3}c}-$

input
```
int(x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b^2+1/b^2*(-a/b*ln(b^2*x^2+2*a*b*x+a^2+c)+(a^2-c)/b/c^(1/2)*arctan(1/2*(
2*b^2*x+2*a*b)/b/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.14

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \left[\frac{2\,bcx - 2\,ac\log\left(b^2x^2+2\,abx+a^2+c\right)+\left(a^2-c\right)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,b^3c}, \frac{bcx - ac\log\left(b\right.}{}\right.$$

input
```
integrate(x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c
)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*
b*x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) +
(a^2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(44) = 88$.

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \left(-\frac{a}{b^3} - \frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)}{a^2b-bc}\right) + \left(-\frac{a}{b^3} + \frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}+\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)}{a^2b-bc}\right) + \frac{x}{b^2}$$

input
```
integrate(x**2/(c+(b*x+a)**2),x)
```

output
```
(-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*
(-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + s
qrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + s
qrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3} + \frac{(a^2-c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^3\sqrt{c}}$$

input
```
integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x +
 a*b)/(b*sqrt(c)))/(b^3*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3} + \frac{(a^2-c)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

input
```
integrate(x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a
)/sqrt(c))/(b^3*sqrt(c))
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.12

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{b^3} + \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,(c-a^2)}-\frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}}-\frac{a\,\sqrt{c}}{c-a^2}+\frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3} - \frac{a^2\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,(c-a^2)}-\frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}}-\frac{a\,\sqrt{c}}{c-a^2}+\frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3\,\sqrt{c}}$$

input
```
int(x^2/(c + (a + b*x)^2),x)
```

output
```
x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1
/2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*
x)/(c^(1/2)*(c/b - a^2/b))))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2)) - (c^
(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a
^2/b))))/(b^3*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)a^2-\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)c-\log(b^2x^2+2abx+a^2+c)\,ac+bcx}{b^3c}$$

input
```
int(x^2/(c+(b*x+a)^2),x)
```

output
```
(sqrt(c)*atan((a + b*x)/sqrt(c))*a**2 - sqrt(c)*atan((a + b*x)/sqrt(c))*c
- log(a**2 + 2*a*b*x + b**2*x**2 + c)*a*c + b*c*x)/(b**3*c)
```

3.3 $\int \frac{x}{c+(a+bx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{x}{c+(a+bx)^2}\,dx=-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(c+(a+bx)^2\right)}{2b^2}$$

output
```
-a*arctan((b*x+a)/c^(1/2))/b^2/c^(1/2)+1/2*ln(c+(b*x+a)^2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x}{c+(a+bx)^2}\,dx=\frac{-\frac{2a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\log\left(c+(a+bx)^2\right)}{2b^2}$$

input
```
Integrate[x/(c + (a + b*x)^2),x]
```

output
```
((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {896, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^2+c}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{bx}{(a+bx)^2+c}d(a+bx)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{bx}{(a+bx)^2+c}d(a+bx)}{b^2}$$

$$\Big\downarrow \text{452}$$

$$\frac{\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)-a\int \frac{1}{(a+bx)^2+c}d(a+bx)}{b^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{b^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{2}\log\left((a+bx)^2+c\right)-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{b^2}$$

input
```
Int[x/(c + (a + b*x)^2),x]
```

output
```
(-((a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c]) + Log[c + (a + b*x)^2]/2)/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\ln(b^2x^2+2abx+a^2+c)}{2b^2}-\frac{a\arctan\left(\frac{2b^2x+2ab}{2b\sqrt{c}}\right)}{b^2\sqrt{c}}$	54
risch	$\frac{\ln(-\sqrt{-c}\,bx-a\sqrt{-c}-c)a\sqrt{-c}}{2c\,b^2}+\frac{\ln(-\sqrt{-c}\,bx-a\sqrt{-c}-c)}{2b^2}-\frac{\ln(\sqrt{-c}\,bx+a\sqrt{-c}-c)a\sqrt{-c}}{2c\,b^2}+\frac{\ln(\sqrt{-c}\,bx+a\sqrt{-c}-c)}{2b^2}$	124

input
```
int(x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)
/b/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.32

$$\int \frac{x}{c+(a+bx)^2}\,dx = \left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c},\ -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}\right]$$

input
```
integrate(x/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)
/(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2
*c), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x
 + a^2 + c))/(b^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(36) = 72.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.02

$$\int \frac{x}{c+(a+bx)^2}\,dx = \left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)+\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

input
```
integrate(x/(c+(b*x+a)**2),x)
```

output
```
(-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-a*sqrt(-c
)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*b**2*c) + 1/(2*b**
2))*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*
b))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{x}{c+(a+bx)^2}\,dx = -\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

input
```
integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-a*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a
*b*x + a^2 + c)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{x}{c+(a+bx)^2}\,dx = -\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

input
```
integrate(x/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
-a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a
^2 + c)/b^2
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{c+(a+bx)^2}\,dx = \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{2\,b^2} - \frac{a\,\mathrm{atan}\left(\frac{a}{\sqrt{c}}+\frac{b\,x}{\sqrt{c}}\right)}{b^2\,\sqrt{c}}$$

input
```
int(x/(c + (a + b*x)^2),x)
```

output
```
log(c + a^2 + b^2*x^2 + 2*a*b*x)/(2*b^2) - (a*atan(a/c^(1/2) + (b*x)/c^(1/
2)))/(b^2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{x}{c+(a+bx)^2}\,dx = \frac{-2\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)a+\log(b^2x^2+2abx+a^2+c)\,c}{2b^2c}$$

input
```
int(x/(c+(b*x+a)^2),x)
```

output
```
( - 2*sqrt(c)*atan((a + b*x)/sqrt(c))*a + log(a**2 + 2*a*b*x + b**2*x**2 +
 c)*c)/(2*b**2*c)
```

3.4 $\int \frac{1}{c+(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{1}{c+(a+bx)^2}\,dx=\frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

output
```
arctan((b*x+a)/c^(1/2))/b/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{c+(a+bx)^2}\,dx=\frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
Integrate[(c + (a + b*x)^2)^(-1),x]
```

output
```
ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {239, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2+c}\,dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{(a+bx)^2+c}d(a+bx)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
Int[(c + (a + b*x)^2)^(-1),x]
```

output
```
ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{\arctan\left(\frac{2b^2x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}$	28
risch	$-\frac{\ln\left(bx+\sqrt{-c}+a\right)}{2\sqrt{-c}\,b}+\frac{\ln\left(-bx+\sqrt{-c}-a\right)}{2\sqrt{-c}\,b}$	47

input
```
int(1/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.95

$$\int \frac{1}{c+(a+bx)^2}\,dx = \left[-\frac{\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,bc}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}\right]$$

input
```
integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b
^2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

input
```
integrate(1/(c+(b*x+a)**2),x)
```

output
```
(-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*s
qrt(-1/c))/b)/2)/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
integrate(1/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{c+(a+bx)^2}\,dx=\frac{\operatorname{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)}{b\,\sqrt{c}}$$

input
```
int(1/(c + (a + b*x)^2),x)
```

output
```
atan((a + b*x)/c^(1/2))/(b*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{c+(a+bx)^2}\,dx=\frac{\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)}{bc}$$

input
```
int(1/(c+(b*x+a)^2),x)
```

output
```
(sqrt(c)*atan((a + b*x)/sqrt(c)))/(b*c)
```

3.5 $\int \frac{1}{x(c+(a+bx)^2)} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x\left(c+(a+bx)^2\right)} dx = -\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}$$

output
```
-a*arctan((b*x+a)/c^(1/2))/c^(1/2)/(a^2+c)+ln(x)/(a^2+c)-ln(c+(b*x+a)^2)/(
2*a^2+2*c)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(c+(a+bx)^2\right)} dx = -\frac{\frac{2a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)+\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}$$

input
```
Integrate[1/(x*(c + (a + b*x)^2)),x]
```

output
```
-1/2*((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a +
b*x)^2])/(a^2 + c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {896, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left((a+bx)^2+c\right)} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{1}{bx \left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{1}{bx \left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{ 479}$$

$$\frac{\log(-bx)}{a^2+c}-\frac{\int \frac{2a+bx}{(a+bx)^2+c}d(a+bx)}{a^2+c}$$

$$\Big\downarrow \text{ 452}$$

$$\frac{\log(-bx)}{a^2+c}-\frac{a\int \frac{1}{(a+bx)^2+c}d(a+bx)+\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)}{a^2+c}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\log(-bx)}{a^2+c}-\frac{\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)+\frac{a \arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{a^2+c}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\log(-bx)}{a^2+c}-\frac{\frac{a \arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{1}{2}\log\left((a+bx)^2+c\right)}{a^2+c}$$

input
```
Int[1/(x*(c + (a + b*x)^2)),x]
```

output
```
Log[-(b*x)]/(a^2 + c) - ((a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (
a + b*x)^2]/2)/(a^2 + c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{b\left(\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}\right)}{a^2+c}+\frac{\ln(x)}{a^2+c}$	74
risch	$\frac{\ln(x)}{a^2+c}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^2c+c^2\right)\_Z^2+2\_Zc\right)}\_R\ln\left(\left(\left(-a^2b+3bc\right)\_R+3b\right)x+\left(-a^3-ac\right)\_R+2a\right)\right)}{2}$	75

input
```
int(1/x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-b/(a^2+c)*(1/2/b*ln(b^2*x^2+2*a*b*x+a^2+c)+a/b/c^(1/2)*arctan(1/2*(2*b^2*
x+2*a*b)/b/c^(1/2)))+ln(x)/(a^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.61

$$\int\frac{1}{x\left(c+(a+bx)^2\right)}\,dx=\left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log\left(x\right)}{2\,(a^2c+c^2)},\ -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log\left(x\right)}{2\,(a^2c+c^2)}\right]$$

input
```
integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)
/(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c
*log(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*lo
g(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(49) = 98.

Time = 1.73 (sec) , antiderivative size = 738, normalized size of antiderivative = 12.51

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c+(b*x+a)**2),x)
```

output
```
(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**6*c*(-a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)
/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**
2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) -
 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*
(a**2 + c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a
**2 + c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) -
 6*c**2)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 +
 c)))*log(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*
*2 + 4*a**4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a
**4*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)
/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2
*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**6*c/(
a**2 + c)**2 + 4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c
**3/(a**2 + c)**2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 +
c)**2 - 6*c**3/(a**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = -\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} - \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)} + \frac{\log\left(x\right)}{a^2+c}$$

input
```
integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-a*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2
 + 2*a*b*x + a^2 + c)/(a^2 + c) + log(x)/(a^2 + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\,(c+(a+bx)^2)}\,dx = -\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} - \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)} + \frac{\log\left(|x|\right)}{a^2+c}$$

input
```
integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
-a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b
*x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \frac{1}{x\,(c+(a+bx)^2)}\,dx &= \frac{\ln(x)}{a^2+c} - \frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c+a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,(a^2+c)}\right)\,\left(c+a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)} \\ &\quad - \frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c-a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,(a^2+c)}\right)\,\left(c-a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)}\end{aligned}$$

input
```
int(1/(x*(c + (a + b*x)^2)),x)
```

output
```
log(x)/(c + a^2) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c + a*(-c)^(1/2))*(a*c +
 a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c + a*(-c)^(1/2)))/(2*(a^2*c +
c^2)) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c - a*(-c)^(1/2))*(a*c + a^3 - 3*b*
c*x + a^2*b*x))/(c*(c + a^2)))*(c - a*(-c)^(1/2)))/(2*(a^2*c + c^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \frac{-2\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)a-\log\left(b^2x^2+2abx+a^2+c\right)c+2\log(x)\,c}{2c\left(a^2+c\right)}$$

input
```
int(1/x/(c+(b*x+a)^2),x)
```

output
```
( - 2*sqrt(c)*atan((a + b*x)/sqrt(c))*a - log(a**2 + 2*a*b*x + b**2*x**2 +
 c)*c + 2*log(x)*c)/(2*c*(a**2 + c))
```

3.6 $\int \frac{1}{x^2(c+(a+bx)^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx=-\frac{1}{\left(a^2+c\right)x}+\frac{b\left(a^2-c\right)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left(c+(a+bx)^2\right)}{\left(a^2+c\right)^2}$$

output
```
-1/(a^2+c)/x+b*(a^2-c)*arctan((b*x+a)/c^(1/2))/c^(1/2)/(a^2+c)^2-2*a*b*ln(
x)/(a^2+c)^2+a*b*ln(c+(b*x+a)^2)/(a^2+c)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx=\frac{b\left(a^2-c\right)x\arctan\left(\frac{a+bx}{\sqrt{c}}\right)-\sqrt{c}\left(a^2+c+2abx\log(x)-abx\log\left(a^2+c+2abx+b^2x^2\right)\right)}{\sqrt{c}\left(a^2+c\right)^2x}$$

input
```
Integrate[1/(x^2*(c + (a + b*x)^2)),x]
```

output
```
(b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[
x] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {896, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left((a+bx)^2+c\right)}dx$$

$$\Big\downarrow \text{896}$$

$$b\int \frac{1}{b^2x^2\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{480}$$

$$b\left(\frac{\int -\frac{2a+bx}{bx((a+bx)^2+c)}d(a+bx)}{a^2+c}-\frac{1}{bx\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$b\left(\frac{\int \left(\frac{a^2+2(a+bx)a-c}{(a^2+c)((a+bx)^2+c)}-\frac{2a}{b(a^2+c)x}\right)d(a+bx)}{a^2+c}-\frac{1}{bx\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{\frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)}-\frac{2a\log(-bx)}{a^2+c}+\frac{a\log((a+bx)^2+c)}{a^2+c}}{a^2+c}-\frac{1}{bx\left(a^2+c\right)}\right)$$

input
```
Int[1/(x^2*(c + (a + b*x)^2)),x]
```

output
```
b*(-(1/(b*(a^2 + c)*x)) + (((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*
(a^2 + c)) - (2*a*Log[-(b*x)])/(a^2 + c) + (a*Log[c + (a + b*x)^2])/(a^2 +
 c))/(a^2 + c))
```

Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

method	result
default	$\frac{b^{2}\left(\frac{a\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{b}+\frac{\left(a^{2}-c\right)\arctan\left(\frac{2b^{2}x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}\right)}{\left(a^{2}+c\right)^{2}}-\frac{1}{\left(a^{2}+c\right)x}-\frac{2ab\ln\left(x\right)}{\left(a^{2}+c\right)^{2}}$
risch	$-\frac{1}{\left(a^{2}+c\right)x}+\frac{b\ln\left(\left(a^{6}b-15a^{4}bc-8\sqrt{-c\left(a^{2}-c\right)^{2}}\,a^{3}b+15a^{2}b\,c^{2}+8\sqrt{-c\left(a^{2}-c\right)^{2}}\,abc-b\,c^{3}\right)x+a^{7}-7c\,a^{5}-7\sqrt{-c\left(a^{2}-c\right)^{2}}\,a^{4}-c^{2}a^{3}\right.}{a^{4}+2a^{2}c+c^{2}}$

input
```
int(1/x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
b^2/(a^2+c)^2*(a/b*ln(b^2*x^2+2*a*b*x+a^2+c)+(a^2-c)/b/c^(1/2)*arctan(1/2*
(2*b^2*x+2*a*b)/b/c^(1/2)))-1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.90

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)} dx = \left[\frac{2\,abcx\log\left(b^2x^2+2\,abx+a^2+c\right)-4\,abcx\log\left(x\right)+\left(a^2b-bc\right)\sqrt{-c}x\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-}{2\,(a^4c+2\,a^2c^2+c^3)x}\right.$$

input
```
integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2
*b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) -
 c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2
+ c^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) +
(a^2*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c +
 2*a^2*c^2 + c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(73) = 146$.

Time = 5.39 (sec) , antiderivative size = 1620, normalized size of antiderivative = 20.51

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)} dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(c+(b*x+a)**2),x)
```

output
```
-2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c**2/(a**2
+ c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**2*c/(a**2 + c)**2
+ 608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b**2*c**2/(a**2 + c)**2 + 432
*a**5*b**2*c**5/(a**2 + c)**4 - 72*a**5*b**2*c**3/(a**2 + c)**2 + 36*a**5*
b**2*c + 112*a**3*b**2*c**6/(a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**
2 - 88*a**3*b**2*c**2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6
*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (
a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
log(x + (-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a*
*2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**
2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(
a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 32*a**6*b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 -
 b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3
*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))
) - 88*a**3*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**
2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^2 (c+(a+bx)^2)}\,dx = \frac{ab\log\left(b^2x^2+2\,abx+a^2+c\right)}{a^4+2\,a^2c+c^2} - \frac{2\,ab\log\left(x\right)}{a^4+2\,a^2c+c^2} + \frac{(a^2b^2-b^2c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^4+2\,a^2c+c^2)b\sqrt{c}} - \frac{1}{(a^2+c)x}$$

input
```
integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(x)/
(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b^2*x + a*b)/(b*sqrt(c))
)/((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2 (c+(a+bx)^2)} dx = \frac{ab \log (b^2x^2 + 2\,abx + a^2 + c)}{a^4 + 2\,a^2c + c^2} - \frac{2\,ab \log (|x|)}{a^4 + 2\,a^2c + c^2} + \frac{(a^2b^2 - b^2c) \arctan \left(\frac{bx+a}{\sqrt{c}}\right)}{(a^4 + 2\,a^2c + c^2)b\sqrt{c}} - \frac{1}{(a^2 + c)x}$$

input
```
integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs
(x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/(
(a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.38

$$\int \frac{1}{x^2 (c+(a+bx)^2)} dx = \frac{\ln \left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} + a\,c^6 - a^{11}\,c\right.}{} - \frac{1}{x\,(a^2 + c)} - \frac{\ln \left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} - a\,c^6 + a^{11}\right.}{} - \frac{2\,a\,b\,\ln (x)}{(a^2 + c)^2}$$

input
```
int(1/(x^2*(c + (a + b*x)^2)),x)
```

output
```
(log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7
/2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) + a*c^6 - a^11*c + 35*a^3*c^5 +
34*a^5*c^4 - 34*a^7*c^3 - 35*a^9*c^2 + b*c^6*x - a^10*b*c*x + 35*a^2*b*c^5
*x + 34*a^4*b*c^4*x - 34*a^6*b*c^3*x - 35*a^8*b*c^2*x)*(b*(-c)^(3/2) + 2*a
*b*c + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2))
- (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^
(7/2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) - a*c^6 + a^11*c - 35*a^3*c^5
- 34*a^5*c^4 + 34*a^7*c^3 + 35*a^9*c^2 - b*c^6*x + a^10*b*c*x - 35*a^2*b*c
^5*x - 34*a^4*b*c^4*x + 34*a^6*b*c^3*x + 35*a^8*b*c^2*x)*(b*(-c)^(3/2) - 2
*a*b*c + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - (2*a*b*log(x))
/(c + a^2)^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2 (c + (a + bx)^2)} \, dx$$
$$= \frac{\sqrt{c}\, atan\left(\frac{bx+a}{\sqrt{c}}\right) a^2bx - \sqrt{c}\, atan\left(\frac{bx+a}{\sqrt{c}}\right) bcx + \log(b^2x^2 + 2abx + a^2 + c)\, abcx - 2\log(x)\, abcx - a^2c - c^2}{cx\,(a^4 + 2a^2c + c^2)}$$

input
```
int(1/x^2/(c+(b*x+a)^2),x)
```

output
```
(sqrt(c)*atan((a + b*x)/sqrt(c))*a**2*b*x - sqrt(c)*atan((a + b*x)/sqrt(c)
)*b*c*x + log(a**2 + 2*a*b*x + b**2*x**2 + c)*a*b*c*x - 2*log(x)*a*b*c*x -
 a**2*c - c**2)/(c*x*(a**4 + 2*a**2*c + c**2))
```

3.7 $\int \frac{1}{x^3(c+(a+bx)^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx=-\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}-\frac{ab^2(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{b^2(3a^2-c)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2(3a^2-c)\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)^3}$$

output
```
-1/2/(a^2+c)/x^2+2*a*b/(a^2+c)^2/x-a*b^2*(a^2-3*c)*arctan((b*x+a)/c^(1/2))
/c^(1/2)/(a^2+c)^3+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3-1/2*b^2*(3*a^2-c)*ln(c+(b
*x+a)^2)/(a^2+c)^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx=-\frac{\frac{(a^2+c)(a^2+c-4abx)}{x^2}+\frac{2ab^2(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+2b^2(-3a^2+c)\log(x)+b^2(3a^2-c)\log\left(a^2+c+2abx+b^2\right.}{2\left(a^2+c\right)^3}$$

input
```
Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

output
```
-1/2*(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a
 + b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Lo
g[a^2 + c + 2*a*b*x + b^2*x^2])/(a^2 + c)^3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {896, 25, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left((a+bx)^2+c\right)}dx$$

$$\Big\downarrow \text{896}$$

$$b^2\int \frac{1}{b^3x^3\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{25}$$

$$-b^2\int -\frac{1}{b^3x^3\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{480}$$

$$b^2\left(-\frac{\int \frac{2a+bx}{b^2x^2((a+bx)^2+c)}d(a+bx)}{a^2+c}-\frac{1}{2b^2x^2\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$b^2\left(-\frac{\int \left(\frac{2a}{b^2(a^2+c)x^2}-\frac{3a^2-c}{b(a^2+c)^2x}+\frac{a(a^2-3c)+(3a^2-c)(a+bx)}{(a^2+c)^2((a+bx)^2+c)}\right)d(a+bx)}{a^2+c}-\frac{1}{2b^2x^2\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$b^2\left(-\frac{\frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^2}-\frac{2a}{bx(a^2+c)}-\frac{(3a^2-c)\log(-bx)}{(a^2+c)^2}+\frac{(3a^2-c)\log((a+bx)^2+c)}{2(a^2+c)^2}}{a^2+c}-\frac{1}{2b^2x^2(a^2+c)}\right)$$

input
```
Int[1/(x^3*(c + (a + b*x)^2)),x]
```

output
```
b^2*(-1/2*1/(b^2*(a^2 + c)*x^2) - ((-2*a)/(b*(a^2 + c)*x) + (a*(a^2 - 3*c)
*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^2) - ((3*a^2 - c)*Log[-(b*x
)])/(a^2 + c)^2 + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^2))/(a^2
 + c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

method	result
default	$-\frac{b^3\left(\frac{(3a^2b-bc)\ln(b^2x^2+2abx+a^2+c)}{2b^2}+\frac{\left(4a^3-4ac-\frac{(3a^2b-bc)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}\right)}{(a^2+c)^3}-\frac{1}{2(a^2+c)x^2}+\frac{b^2(3a^2-c)\ln(x)}{(a^2+c)^3}+\frac{}{(a^2}$
risch	$\frac{\frac{2abx}{a^4+2a^2c+c^2}-\frac{1}{2(a^2+c)}}{x^2}+\frac{3b^2\ln(x)a^2}{a^6+3ca^4+3a^2c^2+c^3}-\frac{b^2\ln(x)c}{a^6+3ca^4+3a^2c^2+c^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^6c+3a^4c^2+3a^2c^3+c^4\right)\_Z^2+\left(6a^2b^2c\right.\right.}\right.}{}$

input
```
int(1/x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-b^3/(a^2+c)^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)+(4*a^3-4*a
*c-(3*a^2*b-b*c)*a/b)/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2)))-1/2
/(a^2+c)/x^2+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3+2*a*b/(a^2+c)^2/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^3(c+(a+bx)^2)}\,dx$$
$$=\left[-\frac{a^4c-(a^3b^2-3\,ab^2c)\sqrt{-c}x^2\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log(b^2x^2+}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2}\right.$$
$$-\frac{a^4c+2\,(a^3b^2-3\,ab^2c)\sqrt{c}x^2\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log(b^2x^2+2\,abx+a^2+c)-}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2}$$

input
```
integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x +
 a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^
2 + (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^
2*b^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3
*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sq
rt(c)*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*
x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x
) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*
x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3284 vs. $2(109) = 218$.

Time = 17.99 (sec) , antiderivative size = 3284, normalized size of antiderivative = 27.14

$$\int \frac{1}{x^3 \left(c + (a + bx)^2\right)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(c+(b*x+a)**2),x)
```

output
```
b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
 24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3
*a**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 +
 568*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3
*a**2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**
6 - 42*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a*
*6*b**4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)
/(a**2 + c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(
a**2 + c)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4
*c**3 - 24*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**
6*(3*a**2 - c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 -
c)**2/(a**2 + c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5
)/(a**9*b**5 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 -
27*a*b**5*c**4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6
 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*
log(x + (-4*a**16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14
*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 +
 c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b*
*2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^3 (c+(a+bx)^2)} dx = -\frac{(3a^2b^2-b^2c)\log(b^2x^2+2abx+a^2+c)}{2(a^6+3a^4c+3a^2c^2+c^3)} + \frac{(3a^2b^2-b^2c)\log(x)}{a^6+3a^4c+3a^2c^2+c^3} - \frac{(a^3b^3-3ab^3c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^6+3a^4c+3a^2c^2+c^3)b\sqrt{c}} + \frac{4abx-a^2-c}{2(a^4+2a^2c+c^2)x^2}$$

input
```
integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2
+ c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^6 + 3
*a^4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a
^2*c + c^2)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = -\frac{(3\,a^2b^2-b^2c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)} + \frac{(3\,a^2b^2-b^2c)\log\left(|x|\right)}{a^6+3\,a^4c+3\,a^2c^2+c^3} - \frac{(a^3b^3-3\,ab^3c)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^6+3\,a^4c+3\,a^2c^2+c^3)b\sqrt{c}} - \frac{a^4+2\,a^2c+c^2-4\,(a^3b+abc)x}{2\left(a^2+c\right)^3x^2}$$

input
```
integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
-1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2
*c^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^
4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b +
a*b*c)*x)/((a^2 + c)^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = \ln(x)\left(\frac{3\,b^2}{\left(a^2+c\right)^2} - \frac{4\,b^2\,c}{\left(a^2+c\right)^3}\right) - \frac{\frac{1}{2\,(a^2+c)} - \frac{2\,a\,b\,x}{(a^2+c)^2}}{x^2}$$

$$- \frac{\ln\left(27\,(-c)^{15/2} + 90\,a^2\,(-c)^{13/2} + 9\,a^4\,(-c)^{11/2} - 324\,a^6\,(-c)^{9/2} + 125\,a^8\,(-c)^{7/2} + 74\,a^{10}\,(-c)^{5/2} - a\right.}{}$$

$$+ \frac{\ln\left(27\,(-c)^{15/2} + 90\,a^2\,(-c)^{13/2} + 9\,a^4\,(-c)^{11/2} - 324\,a^6\,(-c)^{9/2} + 125\,a^8\,(-c)^{7/2} + 74\,a^{10}\,(-c)^{5/2} - a\right.}{}$$

input
```
int(1/(x^3*(c + (a + b*x)^2)),x)
```

output
```
log(x)*((3*b^2)/(c + a^2)^2 - (4*b^2*c)/(c + a^2)^3) - (1/(2*(c + a^2)) -
(2*a*b*x)/(c + a^2)^2)/x^2 - (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) + 9*
a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(
5/2) - a^12*(-c)^(3/2) - 27*a*c^7 + a^13*c + 90*a^3*c^6 - 9*a^5*c^5 - 324*
a^7*c^4 - 125*a^9*c^3 + 74*a^11*c^2 - 27*b*c^7*x + a^12*b*c*x + 90*a^2*b*c
^6*x - 9*a^4*b*c^5*x - 324*a^6*b*c^4*x - 125*a^8*b*c^3*x + 74*a^10*b*c^2*x
)*(a^3*b^2*(-c)^(1/2) - b^2*c^2 + 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a
^6*c + c^4 + 3*a^2*c^3 + 3*a^4*c^2)) + (log(27*(-c)^(15/2) + 90*a^2*(-c)^(
13/2) + 9*a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a
^10*(-c)^(5/2) - a^12*(-c)^(3/2) + 27*a*c^7 - a^13*c - 90*a^3*c^6 + 9*a^5*
c^5 + 324*a^7*c^4 + 125*a^9*c^3 - 74*a^11*c^2 + 27*b*c^7*x - a^12*b*c*x -
90*a^2*b*c^6*x + 9*a^4*b*c^5*x + 324*a^6*b*c^4*x + 125*a^8*b*c^3*x - 74*a^
10*b*c^2*x)*(b^2*c^2 + a^3*b^2*(-c)^(1/2) - 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/
2)))/(2*(a^6*c + c^4 + 3*a^2*c^3 + 3*a^4*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.64

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx$$
$$= \frac{-2\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)a^3b^2x^2+6\sqrt{c}\,atan\left(\frac{bx+a}{\sqrt{c}}\right)a\,b^2c\,x^2-3\log(b^2x^2+2abx+a^2+c)\,a^2b^2c\,x^2+\log(b^2x^2 -}{2c\,x^2\left(a^6+3a^4c+\right.}$$

input
```
int(1/x^3/(c+(b*x+a)^2),x)
```

output
```
( - 2*sqrt(c)*atan((a + b*x)/sqrt(c))*a**3*b**2*x**2 + 6*sqrt(c)*atan((a +
 b*x)/sqrt(c))*a*b**2*c*x**2 - 3*log(a**2 + 2*a*b*x + b**2*x**2 + c)*a**2*
b**2*c*x**2 + log(a**2 + 2*a*b*x + b**2*x**2 + c)*b**2*c**2*x**2 + 6*log(x
)*a**2*b**2*c*x**2 - 2*log(x)*b**2*c**2*x**2 - a**4*c + 4*a**3*b*c*x - 2*a
**2*c**2 + 4*a*b*c**2*x - c**3)/(2*c*x**2*(a**6 + 3*a**4*c + 3*a**2*c**2 +
 c**3))
```

3.8 $\int \frac{(1+(a+bx)^2)^2}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{(1+(a+bx)^2)^2}{x} dx = a(2+a^2) bx + \frac{1}{2}(2+a^2)(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2 \log(x)$$

output
```
a*(a^2+2)*b*x+1/2*(a^2+2)*(b*x+a)^2+1/3*a*(b*x+a)^3+1/4*(b*x+a)^4+(a^2+1)^
2*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{(1+(a+bx)^2)^2}{x} dx = a(2+a^2)(a+bx) + \frac{1}{2}(2+a^2)(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2 \log(bx)$$

input
```
Integrate[(1 + (a + b*x)^2)^2/x,x]
```

output
```
a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
 + b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {896, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left((a+bx)^2+1\right)^2}{x} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{\left((a+bx)^2+1\right)^2}{bx}d(a+bx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{\left((a+bx)^2+1\right)^2}{bx}d(a+bx)$$

$$\Big\downarrow \text{ 476}$$

$$-\int \left(-(a+bx)^3-a(a+bx)^2-\left(a^2+2\right)(a+bx)-a\left(a^2+2\right)-\frac{\left(a^2+1\right)^2}{bx}\right)d(a+bx)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)(a+bx)+\left(a^2+1\right)^2\log(-bx)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

input
```
Int[(1 + (a + b*x)^2)^2/x,x]
```

output
```
a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
 + b*x)^4/4 + (1 + a^2)^2*Log[-(b*x)]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

method	result	size
norman	$(3a^2b^2+b^2)\,x^2+(4a^3b+4ab)\,x+\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+(a^4+2a^2+1)\ln(x)$	61
default	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+4a^3bx+b^2x^2+4abx+(a^4+2a^2+1)\ln(x)$	62
risch	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+b^2x^2+4a^3bx+4abx+\ln(x)\,a^4+2\ln(x)\,a^2+\ln(x)$	64
parallelrisch	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+b^2x^2+4a^3bx+4abx+\ln(x)\,a^4+2\ln(x)\,a^2+\ln(x)$	64

input
```
int((1+(b*x+a)^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(3*a^2*b^2+b^2)*x^2+(4*a^3*b+4*a*b)*x+1/4*b^4*x^4+4/3*a*b^3*x^3+(a^4+2*a^2
+1)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2+1\right)b^2x^2 + 4\left(a^3+a\right)bx + \left(a^4+2\,a^2+1\right)\log\left(x\right)$$

input
```
integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
 + 2*a^2 + 1)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2\cdot\left(3a^2b^2+b^2\right) + x\left(4a^3b+4ab\right) + \left(a^2+1\right)^2\log\left(x\right)$$

input
```
integrate((1+(b*x+a)**2)**2/x,x)
```

output
```
4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b +
4*a*b) + (a**2 + 1)**2*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2+1\right)b^2x^2 + 4\left(a^3+a\right)bx + \left(a^4+2\,a^2+1\right)\log\left(x\right)$$

input
```
integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
 + 2*a^2 + 1)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + b^2x^2 + 4\,abx + \left(a^4 + 2\,a^2 + 1\right)\log\left(|x|\right)$$

input
```
integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*
x + (a^4 + 2*a^2 + 1)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \ln(x)\,\left(a^4 + 2\,a^2 + 1\right) + \frac{b^4\,x^4}{4} + \frac{4\,a\,b^3\,x^3}{3} + b^2\,x^2\,\left(3\,a^2 + 1\right) + 4\,a\,b\,x\,\left(a^2 + 1\right)$$

input
```
int(((a + b*x)^2 + 1)^2/x,x)
```

output
```
log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2
+ 1) + 4*a*b*x*(a^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx = \log(x)\,a^4 + 2\log(x)\,a^2 + \log(x) + 4a^3bx + 3a^2b^2x^2 + \frac{4a\,b^3x^3}{3} + 4abx + \frac{b^4x^4}{4} + b^2x^2$$

input
```
int((1+(b*x+a)^2)^2/x,x)
```

output
```
(12*log(x)*a**4 + 24*log(x)*a**2 + 12*log(x) + 48*a**3*b*x + 36*a**2*b**2*
x**2 + 16*a*b**3*x**3 + 48*a*b*x + 3*b**4*x**4 + 12*b**2*x**2)/12
```

3.9 $\int \frac{x^2}{1+(-1+x)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{x^2}{1+(-1+x)^2} dx = x + \log\left(1+(-1+x)^2\right)$$

output
```
x+ln(1+(-1+x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2} dx = x + \log\left(2-2x+x^2\right)$$

input
```
Integrate[x^2/(1 + (-1 + x)^2),x]
```

output
```
x + Log[2 - 2*x + x^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {896, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x-1)^2+1} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{x^2}{(x-1)^2+1} d(x-1)$$

$$\Big\downarrow \text{ 478}$$

$$\int \left(\frac{2(x-1)}{(x-1)^2+1}+1\right) d(x-1)$$

$$\Big\downarrow \text{ 2009}$$

$$x+\log\left((x-1)^2+1\right)-1$$

input
```
Int[x^2/(1 + (-1 + x)^2),x]
```

output
```
-1 + x + Log[1 + (-1 + x)^2]
```

Defintions of rubi rules used

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
default	$x+\ln\left(x^{2}-2x+2\right)$	12
norman	$x+\ln\left(x^{2}-2x+2\right)$	12
risch	$x+\ln\left(x^{2}-2x+2\right)$	12
parallelrisch	$x+\ln\left(x^{2}-2x+2\right)$	12

input
```
int(x^2/(1+(x-1)^2),x,method=_RETURNVERBOSE)
```

output
```
x+ln(x^2-2*x+2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^{2} - 2\,x + 2\right)$$

input
```
integrate(x^2/(1+(x-1)^2),x, algorithm="fricas")
```

output
```
x + log(x^2 - 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2 - 2x + 2\right)$$

input
```
integrate(x**2/(1+(x-1)**2),x)
```

output
```
x + log(x**2 - 2*x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2 - 2\,x + 2\right)$$

input
```
integrate(x^2/(1+(x-1)^2),x, algorithm="maxima")
```

output
```
x + log(x^2 - 2*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2 - 2\,x + 2\right)$$

input
```
integrate(x^2/(1+(x-1)^2),x, algorithm="giac")
```

output
```
x + log(x^2 - 2*x + 2)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x+\ln\left(x^2-2\,x+2\right)$$

input
```
int(x^2/((x - 1)^2 + 1),x)
```

output
```
x + log(x^2 - 2*x + 2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = \log\left(x^2-2x+2\right)+x$$

input
```
int(x^2/(1+(x-1)^2),x)
```

output
```
log(x**2 - 2*x + 2) + x
```

3.10 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\arcsin(a+bx)}{2b^3}$$

output
```
3/2*a*(1-(b*x+a)^2)^(1/2)/b^3-1/2*x*(1-(b*x+a)^2)^(1/2)/b^2+1/2*(2*a^2+1)*
arcsin(b*x+a)/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(67) = 134.

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.43

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \frac{-2b(-3a+bx)\sqrt{1-a^2-2abx-b^2x^2} - 2(1+2a^2)\,b\arctan\left(\frac{-\sqrt{-b^2}x+\sqrt{1-a^2-2abx-b^2x^2}}{a}\right) + (1+2a^2)\,\sqrt{-}}{4b^4}$$

input
```
Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
```

output
```
(-2*b*(-3*a + b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] - 2*(1 + 2*a^2)*b*Arc
Tan[(-(Sqrt[-b^2]*x) + Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a] + (1 + 2*a^2)
*Sqrt[-b^2]*Log[-1 + 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 - a^2 - 2
*a*b*x - b^2*x^2]])/(4*b^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 497, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{b^2x^2}{\sqrt{1-(a+bx)^2}}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{497}$$

$$\frac{-\frac{1}{2}\int -\frac{2a^2-3(a+bx)a+1}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{2a^2-3(a+bx)a+1}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(\left(2a^2+1\right)\int \frac{1}{\sqrt{1-(a+bx)^2}}d(a+bx)+3a\sqrt{1-(a+bx)^2}\right)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{2}\left((2a^2+1)\arcsin(a+bx)+3a\sqrt{1-(a+bx)^2}\right)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

input
```
Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

output
```
(-1/2*(b*x*Sqrt[1 - (a + b*x)^2]) + (3*a*Sqrt[1 - (a + b*x)^2] + (1 + 2*a^
2)*ArcSin[a + b*x])/2)/b^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{(-bx+3a)(b^2x^2+2abx+a^2-1)}{2b^3\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{(2a^2+1)\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b^2\sqrt{b^2}}$
default	$-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}-\frac{3a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b\sqrt{b^2}}\right)}{2b}+\frac{(-a^2+1)\arctan\left(\frac{\sqrt{b^2}\,(x+}{\sqrt{-b^2x^2-2ab}}\right.}{2b^2\sqrt{b^2}}$

input
```
int(x^2/(1-(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x+3*a)*(b^2*x^2+2*a*b*x+a^2-1)/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
+1/2/b^2*(2*a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{(2\,a^2+1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}(bx-3\,a)}{2\,b^3}$$

input
```
integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2
*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a
))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(54) = 108$.

Time = 0.73 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \begin{cases} \left(\frac{3a}{2b^3} - \frac{x}{2b^2}\right)\sqrt{-a^2-2abx-b^2x^2+1} + \frac{\left(\frac{3a^2}{2b^2}+\frac{1-a^2}{2b^2}\right)\log\left(-2ab-2b^2x+2\sqrt{-b^2}\sqrt{-a^2-2abx-b^2x^2+1}\right)}{\sqrt{-b^2}} & \text{for } b^2 \neq 0 \\ -\frac{a^4\sqrt{-a^2-2abx+1}-2a^2\sqrt{-a^2-2abx+1}+\frac{(2a^2-2)(-a^2-2abx+1)^{\frac{3}{2}}}{3}+\frac{(-a^2-2abx+1)^{\frac{5}{2}}}{5}+\sqrt{-a^2-2abx+1}}{4a^3b^3} & \text{for } ab \neq 0 \\ \frac{x^3}{3\sqrt{1-a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise(((3*a/(2*b**3) - x/(2*b**2))*sqrt(-a**2 - 2*a*b*x - b**2*x**2 +
1) + (3*a**2/(2*b**2) + (1 - a**2)/(2*b**2))*log(-2*a*b - 2*b**2*x + 2*sqr
t(-b**2)*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1))/sqrt(-b**2), Ne(b**2, 0)),
 (-(a**4*sqrt(-a**2 - 2*a*b*x + 1) - 2*a**2*sqrt(-a**2 - 2*a*b*x + 1) + (2
*a**2 - 2)*(-a**2 - 2*a*b*x + 1)**(3/2)/3 + (-a**2 - 2*a*b*x + 1)**(5/2)/5
 + sqrt(-a**2 - 2*a*b*x + 1))/(4*a**3*b**3), Ne(a*b, 0)), (x**3/(3*sqrt(1
- a**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(57) = 114$.

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{3\,a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3} - \frac{\sqrt{-b^2x^2-2\,abx-a^2+1}x}{2\,b^2} + \frac{(a^2-1)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3} + \frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}a}{2\,b^3}$$

input
```
integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 1/2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + 1/2*(a^2 - 1)*arcsin(-(b^2*x + a*
b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{1}{2}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right) - \frac{(2\,a^2+1)\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{2\,b^2|b|}$$

input
```
integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1
)*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \int \frac{x^2}{\sqrt{1-{\left(a+b\,x\right)}^2}}\,dx$$

input
```
int(x^2/(1 - (a + b*x)^2)^(1/2),x)
```

output
```
int(x^2/(1 - (a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx$$
$$= \frac{2asin(bx+a)\,a^2+asin(bx+a)+3\sqrt{-b^2x^2-2abx-a^2+1}\,a-\sqrt{-b^2x^2-2abx-a^2+1}\,bx-4a}{2b^3}$$

input
```
int(x^2/(1-(b*x+a)^2)^(1/2),x)
```

output
```
(2*asin(a + b*x)*a**2 + asin(a + b*x) + 3*sqrt( - a**2 - 2*a*b*x - b**2*x*
*2 + 1)*a - sqrt( - a**2 - 2*a*b*x - b**2*x**2 + 1)*b*x - 4*a)/(2*b**3)
```

3.11 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx=-\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{(1-2a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

output
```
-3/2*a*(1+(b*x+a)^2)^(1/2)/b^3+1/2*x*(1+(b*x+a)^2)^(1/2)/b^2-1/2*(-2*a^2+1
)*arcsinh(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx=\frac{(-3a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{2b^3}+\frac{(1-2a^2)\operatorname{arctanh}\left(\frac{bx}{\sqrt{1+a^2}-\sqrt{1+a^2+2abx+b^2x^2}}\right)}{b^3}$$

input
```
Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
```

output
```
((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^3) + ((1 - 2*a^2)*Ar
cTanh[(b*x)/(Sqrt[1 + a^2] - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])/b^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {896, 497, 25, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{(a+bx)^2+1}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{b^2x^2}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{497}$$

$$\frac{\frac{1}{2}\int -\frac{-2a^2+3(a+bx)a+1}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}}{b^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}bx\sqrt{(a+bx)^2+1}-\frac{1}{2}\int \frac{-2a^2+3(a+bx)a+1}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(-\left(1-2a^2\right)\int \frac{1}{\sqrt{(a+bx)^2+1}}d(a+bx)-3a\sqrt{(a+bx)^2+1}\right)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}}{b^3}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{1}{2}\left(-\left(1-2a^2\right)\text{arcsinh}(a+bx)-3a\sqrt{(a+bx)^2+1}\right)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}}{b^3}$$

input
```
Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

output
```
((b*x*Sqrt[1 + (a + b*x)^2])/2 + (-3*a*Sqrt[1 + (a + b*x)^2] - (1 - 2*a^2)
*ArcSinh[a + b*x])/2)/b^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{(-bx+3a)\sqrt{b^2x^2+2abx+a^2+1}}{2b^3}+\frac{(2a^2-1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$
default	$\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}-\frac{(a^2+1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a\right.}{2b^2\sqrt{b^2}}$

input
```
int(x^2/(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x+3*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^3+1/2/b^2*(2*a^2-1)*ln((b^
2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = -\frac{(2\,a^2-1)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)}{2\,b^3}$$

input
```
integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(54) = 108$.

Time = 0.61 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.62

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \begin{cases} \left(-\frac{3a}{2b^3}+\frac{x}{2b^2}\right)\sqrt{a^2+2abx+b^2x^2+1}+\frac{\left(\frac{3a^2}{2b^2}-\frac{a^2+1}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} & \text{for } b^2\neq 0 \\ \frac{a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{(-2a^2-2)(a^2+2abx+1)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx+1)^{\frac{5}{2}}}{5}+\sqrt{a^2+2abx+1}}{4a^3b^3} & \text{for } ab\neq 0 \\ \frac{x^3}{3\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise(((-3*a/(2*b**3) + x/(2*b**2))*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + (3*a**2/(2*b**2) - (a**2 + 1)/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2), Ne(b**2, 0)), ((a
**4*sqrt(a**2 + 2*a*b*x + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-2*a**2
- 2)*(a**2 + 2*a*b*x + 1)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5 + sqrt(
a**2 + 2*a*b*x + 1))/(4*a**3*b**3), Ne(a*b, 0)), (x**3/(3*sqrt(a**2 + 1)),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.14

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{3\,a^2\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b^2} - \frac{(a^2+1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} - \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^3}$$

input
```
integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
3/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 +
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 1/2*(a^2 + 1)*arcsinh(2*(b^2
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3/2*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{1}{2}\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right) - \frac{(2\,a^2-1)\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right|\right)}{2\,b^2|b|}$$

input
```
integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*
log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b)))/(b^
2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \int \frac{x^2}{\sqrt{{\left(a+b\,x\right)}^2+1}}\,dx$$

input
```
int(x^2/((a + b*x)^2 + 1)^(1/2),x)
```

output
```
int(x^2/((a + b*x)^2 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.63

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx$$
$$= \frac{-3\sqrt{b^2x^2+2abx+a^2+1}\,a+\sqrt{b^2x^2+2abx+a^2+1}\,bx+2\log\left(\sqrt{b^2x^2+2abx+a^2+1}+a+bx\right)a^2-}{2b^3}$$

input
```
int(x^2/(1+(b*x+a)^2)^(1/2),x)
```

output
```
( - 3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a + sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*b*x + 2*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a**2
 - log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x))/(2*b**3)
```

3.12 $\int x^3 \sqrt{a+\frac{b}{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 244

$$\int x^3 \sqrt{a+\frac{b}{c+dx}}\,dx = \frac{(5b^3+24ab^2c+48a^2bc^2-64a^3c^3)\,(c+dx)\sqrt{a+\frac{b}{c+dx}}}{64a^3d^4} - \frac{(5b^2+24abc-144a^2c^2)\,(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{96a^2d^4} + \frac{(b-24ac)(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{24ad^4} + \frac{(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4d^4} - \frac{b(5b^3+24ab^2c+48a^2bc^2+64a^3c^3)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{64a^{7/2}d^4}$$

output
```
1/64*(-64*a^3*c^3+48*a^2*b*c^2+24*a*b^2*c+5*b^3)*(d*x+c)*(a+b/(d*x+c))^(1/
2)/a^3/d^4-1/96*(-144*a^2*c^2+24*a*b*c+5*b^2)*(d*x+c)^2*(a+b/(d*x+c))^(1/2
)/a^2/d^4+1/24*(-24*a*c+b)*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/a/d^4+1/4*(d*x+c)
^4*(a+b/(d*x+c))^(1/2)/d^4-1/64*b*(64*a^3*c^3+48*a^2*b*c^2+24*a*b^2*c+5*b^
3)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(7/2)/d^4
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.75

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(15b^3+2ab^2(31c-5dx)+8a^2b(13c^2-4cdx+d^2x^2)-48a^3(c^3-c^2dx+cd^2x^2-\right.}{192a^{7/2}d^4}$$

input
```
Integrate[x^3*Sqrt[a + b/(c + d*x)],x]
```

output
```
(Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(15*b^3 + 2*a*b^2*(31
*c - 5*d*x) + 8*a^2*b*(13*c^2 - 4*c*d*x + d^2*x^2) - 48*a^3*(c^3 - c^2*d*x
 + c*d^2*x^2 - d^3*x^3)) - 3*b*(5*b^3 + 24*a*b^2*c + 48*a^2*b*c^2 + 64*a^3
*c^3)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(192*a^(7/2)*d^4
)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {896, 25, 941, 948, 25, 108, 27, 166, 27, 162, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int d^3x^3\sqrt{a+\frac{b}{c+dx}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -d^3x^3\sqrt{a+\frac{b}{c+dx}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{941}$$

$$-\frac{\int (c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(\frac{c}{c+dx}-1\right)^3d(c+dx)}{d^4}$$

$$\Big\downarrow \text{948}$$

$$\frac{\int -(c+dx)^5\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (c+dx)^5\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{108}$$

$$\frac{\frac{1}{4}(c+dx)^4\left(1-\frac{c}{c+dx}\right)^3\sqrt{a+\frac{b}{c+dx}}-\frac{1}{4}\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2\left(-\frac{7cb}{c+dx}+b-6ac\right)}{2\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}(c+dx)^4\left(1-\frac{c}{c+dx}\right)^3\sqrt{a+\frac{b}{c+dx}}-\frac{1}{8}\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2\left(-\frac{7cb}{c+dx}+b-6ac\right)}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{166}$$

$$\frac{\frac{1}{8}\left(\frac{(b-6ac)(c+dx)^3\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{c+dx}}}{3a}-\frac{\int -\frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)\left(5b^2+16acb-\frac{c(b+36ac)b}{c+dx}-24a^2c^2\right)}{2\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{3a}\right)+\frac{1}{4}\left(1-\frac{c}{c+dx}\right)^3(c+dx}{d^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{8}\left(\frac{\int \frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)\left(5b^2+16acb-\frac{c(b+36ac)b}{c+dx}-24a^2c^2\right)}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{6a}+\frac{(b-6ac)\left(1-\frac{c}{c+dx}\right)^2(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a}\right)+\frac{1}{4}\left(1-\frac{c}{c+dx}\right)^3(c+dx)}{d^4}$$

$$\Big\downarrow \text{162}$$

$$\frac{\frac{1}{8}\left(\frac{\frac{3b\left(64a^3c^3+48a^2bc^2+24ab^2c+5b^3\right)\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{8a^2}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}\left(2a\left(-24a^2c^2+16abc+5b^2\right)-\frac{-96a^3c^3+136a^2bc^2+72ab^2c+15b^3}{c+dx}\right)}{4a^2}}{6a}+\frac{(b-}{}\right.}{d^4}$$

$$\downarrow 73$$

$$\frac{\frac{1}{8}\left(\frac{\frac{3\left(64a^3c^3+48a^2bc^2+24ab^2c+5b^3\right)\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{4a^2}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}\left(2a\left(-24a^2c^2+16abc+5b^2\right)-\frac{-96a^3c^3+136a^2bc^2+72ab^2c+15b^3}{c+dx}\right)}{4a^2}}{6a}\right.}{d^4}$$

$$\downarrow 221$$

$$\frac{\frac{1}{8}\left(\frac{-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}\left(2a\left(-24a^2c^2+16abc+5b^2\right)-\frac{-96a^3c^3+136a^2bc^2+72ab^2c+15b^3}{c+dx}\right)}{4a^2}-\frac{3b\left(64a^3c^3+48a^2bc^2+24ab^2c+5b^3\right)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{4a^{5/2}}}{6a}\right.}{d^4}$$

input
```
Int[x^3*Sqrt[a + b/(c + d*x)],x]
```

output
```
(((c + d*x)^4*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))^3)/4 + (((b - 6*a*c)
*(c + d*x)^3*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))^2)/(3*a) + (-1/4*((c
+ d*x)^2*Sqrt[a + b/(c + d*x)]*(2*a*(5*b^2 + 16*a*b*c - 24*a^2*c^2) - (15*
b^3 + 72*a*b^2*c + 136*a^2*b*c^2 - 96*a^3*c^3)/(c + d*x)))/a^2 - (3*b*(5*b
^3 + 24*a*b^2*c + 48*a^2*b*c^2 + 64*a^3*c^3)*ArcTanh[Sqrt[a + b/(c + d*x)]
/Sqrt[a]])/(4*a^(5/2)))/(6*a))/8)/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(220) = 440$.

Time = 0.20 (sec) , antiderivative size = 803, normalized size of antiderivative = 3.29

method	result
default	$-\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(192\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^3b\,c^3d-576\sqrt{a\,d^2}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,a^3$

input
```
int(x^3*(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/d^4*(192*ln(1/2*(2*a*d^2*x+2*
a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^
3*b*c^3*d-576*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^
3*c^2*d*x+384*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^3*c^3+144*ln(1
/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2
)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d-96*x*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(3/2)*a^2*d*(a*d^2)^(1/2)-576*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*
x+a*c^2+b*d*x+b*c)^(1/2)*a^3*c^3-288*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(1/2)*a^2*b*c*d*x+72*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2
+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^3*
c*d+288*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a^2*c-57
6*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*b*c^2-60*(
a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a*b^2*d*x+15*ln(1
/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2
)^(1/2)+b*d)/(a*d^2)^(1/2))*b^4*d+80*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(3/2)*a*b-204*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*
x+b*c)^(1/2)*a*b^2*c-30*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c
)^(1/2)*b^3)/((a*d*x+a*c+b)*(d*x+c))^(1/2)/a^3/(a*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.82

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx$$

$$= \left[\frac{3\left(64\,a^3bc^3+48\,a^2b^2c^2+24\,ab^3c+5\,b^4\right)\sqrt{a}\log\left(2\,adx+2\,ac-2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(48\,a^{\cdots}}{}\right.$$

input
```
integrate(x^3*(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(64*a^3*b*c^3 + 48*a^2*b^2*c^2 + 24*a*b^3*c + 5*b^4)*sqrt(a)*log
(2*a*d*x + 2*a*c - 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) +
 b) + 2*(48*a^4*d^4*x^4 + 8*a^3*b*d^3*x^3 - 48*a^4*c^4 + 104*a^3*b*c^3 + 6
2*a^2*b^2*c^2 + 15*a*b^3*c - 2*(12*a^3*b*c + 5*a^2*b^2)*d^2*x^2 + (72*a^3*
b*c^2 + 52*a^2*b^2*c + 15*a*b^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(
a^4*d^4), 1/192*(3*(64*a^3*b*c^3 + 48*a^2*b^2*c^2 + 24*a*b^3*c + 5*b^4)*sq
rt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x
+ a*c + b)) + (48*a^4*d^4*x^4 + 8*a^3*b*d^3*x^3 - 48*a^4*c^4 + 104*a^3*b*c
^3 + 62*a^2*b^2*c^2 + 15*a*b^3*c - 2*(12*a^3*b*c + 5*a^2*b^2)*d^2*x^2 + (7
2*a^3*b*c^2 + 52*a^2*b^2*c + 15*a*b^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x +
c)))/(a^4*d^4)]
```

Sympy [F]

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx=\int x^3\sqrt{\frac{ac+adx+b}{c+dx}}\,dx$$

input
```
integrate(x**3*(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(x**3*sqrt((a*c + a*d*x + b)/(c + d*x)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx=\int\sqrt{a+\frac{b}{dx+c}}x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))*x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.43

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{1}{192}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(2\left(4\,x\left(\frac{6\,x\text{sgn}(dx+c)}{d}-\frac{6\,a^3cd^5\text{sgn}(dx+c)-a^2bd^5\text{sgn}(dx+}{a^3d^7}\right.\right.\right.$$
$$+\frac{(64\,a^3bc^3\text{sgn}(dx+c)+48\,a^2b^2c^2\text{sgn}(dx+c)+24\,ab^3c\text{sgn}(dx+c)+5\,b^4\text{sgn}(dx+c))\log\left(\left|2\,acd+2\,(\right.\right.}{128\,a^{\frac{7}{2}}d^3|d|}$$

input
```
integrate(x^3*(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*(4*x*(6*x*sgn(d
*x + c)/d - (6*a^3*c*d^5*sgn(d*x + c) - a^2*b*d^5*sgn(d*x + c))/(a^3*d^7))
 + (24*a^3*c^2*d^4*sgn(d*x + c) - 16*a^2*b*c*d^4*sgn(d*x + c) - 5*a*b^2*d^
4*sgn(d*x + c))/(a^3*d^7))*x - (48*a^3*c^3*d^3*sgn(d*x + c) - 104*a^2*b*c^
2*d^3*sgn(d*x + c) - 62*a*b^2*c*d^3*sgn(d*x + c) - 15*b^3*d^3*sgn(d*x + c)
)/(a^3*d^7)) + 1/128*(64*a^3*b*c^3*sgn(d*x + c) + 48*a^2*b^2*c^2*sgn(d*x +
 c) + 24*a*b^3*c*sgn(d*x + c) + 5*b^4*sgn(d*x + c))*log(abs(2*a*c*d + 2*(s
qrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*
abs(d) + b*d))/(a^(7/2)*d^3*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx=\int x^3\,\sqrt{a+\frac{b}{c+d\,x}}\,dx$$

input
```
int(x^3*(a + b/(c + d*x))^(1/2),x)
```

output
```
int(x^3*(a + b/(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.68

$$\int x^3\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{-48\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4c^3+48\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4c^2dx-48\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4c\,d^2}{}$$

input
```
int(x^3*(a+b/(d*x+c))^(1/2),x)
```

output
```
( - 48*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*c**3 + 48*sqrt(c + d*x)*sq
rt(a*c + a*d*x + b)*a**4*c**2*d*x - 48*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)
*a**4*c*d**2*x**2 + 48*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*d**3*x**3
+ 104*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**2 - 32*sqrt(c + d*x)*s
qrt(a*c + a*d*x + b)*a**3*b*c*d*x + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*
a**3*b*d**2*x**2 + 62*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*c - 10
*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*d*x + 15*sqrt(c + d*x)*sqrt
(a*c + a*d*x + b)*a*b**3 - 192*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a
)*sqrt(c + d*x))/sqrt(b))*a**3*b*c**3 - 144*sqrt(a)*log((sqrt(a*c + a*d*x
+ b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*b**2*c**2 - 72*sqrt(a)*log((sq
rt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**3*c - 15*sqrt(a
)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**4)/(192*
a**4*d**4)
```

3.13 $\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx = -\frac{(b^2+4abc-8a^2c^2)\,(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8a^2d^3} + \frac{(b-12ac)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12ad^3} + \frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3d^3} + \frac{b(b^2+4abc+8a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{8a^{5/2}d^3}$$

output
```
-1/8*(-8*a^2*c^2+4*a*b*c+b^2)*(d*x+c)*(a+b/(d*x+c))^(1/2)/a^2/d^3+1/12*(-1
2*a*c+b)*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/a/d^3+1/3*(d*x+c)^3*(a+b/(d*x+c))^(
1/2)/d^3+1/8*b*(8*a^2*c^2+4*a*b*c+b^2)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2)
)/a^(5/2)/d^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(-3b^2+2ab(-5c+dx)+8a^2(c^2-cdx+d^2x^2)\right)+3b(b^2+4abc+8a^2c^2)\,\text{arctanh}}{24a^{5/2}d^3}$$

input
```
Integrate[x^2*Sqrt[a + b/(c + d*x)],x]
```

output
```
(Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(-3*b^2 + 2*a*b*(-5*c
 + d*x) + 8*a^2*(c^2 - c*d*x + d^2*x^2)) + 3*b*(b^2 + 4*a*b*c + 8*a^2*c^2)
*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(24*a^(5/2)*d^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 941, 948, 100, 27, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$\Big\downarrow \text{896}$$
$$\frac{\int d^2x^2\sqrt{a+\frac{b}{c+dx}}d(c+dx)}{d^3}$$
$$\Big\downarrow \text{941}$$
$$\frac{\int (c+dx)^2\sqrt{a+\frac{b}{c+dx}}\left(\frac{c}{c+dx}-1\right)^2d(c+dx)}{d^3}$$
$$\Big\downarrow \text{948}$$

$$-\frac{\int (c+dx)^4\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^2d\frac{1}{c+dx}}{d^3}$$

$\Big\downarrow$ 100

$$-\frac{\frac{\int -\frac{3}{2}(c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(-\frac{2ac^2}{c+dx}+4ac+b\right)d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}}{3a}}{d^3}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int (c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(-\frac{2ac^2}{c+dx}+4ac+b\right)d\frac{1}{c+dx}}{2a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}}{3a}}{d^3}$$

$\Big\downarrow$ 87

$$-\frac{-\frac{-\frac{\left(8a^2c^2+4abc+b^2\right)\int (c+dx)^2\sqrt{a+\frac{b}{c+dx}}d\frac{1}{c+dx}}{4a}-\frac{(4ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{2a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}}{3a}}{d^3}$$

$\Big\downarrow$ 51

$$-\frac{-\frac{-\frac{\left(8a^2c^2+4abc+b^2\right)\left(\frac{1}{2}b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}-(c+dx)\sqrt{a+\frac{b}{c+dx}}\right)}{4a}-\frac{(4ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{2a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}}{3a}}{d^3}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{-\frac{\left(8a^2c^2+4abc+b^2\right)\left(\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}-(c+dx)\sqrt{a+\frac{b}{c+dx}}\right)}{4a}-\frac{(4ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{2a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}}{3a}}{d^3}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{-\frac{\left(8a^2c^2+4abc+b^2\right)\left((c+dx)\left(-\sqrt{a+\frac{b}{c+dx}}\right)-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{4a}-\frac{(4ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{2a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}}{3a}}{d^3}$$

input

```
Int[x^2*Sqrt[a + b/(c + d*x)],x]
```

output
```
-((-1/3*((c + d*x)^3*(a + b/(c + d*x))^(3/2))/a - (-1/2*((b + 4*a*c)*(c +
d*x)^2*(a + b/(c + d*x))^(3/2))/a - ((b^2 + 4*a*b*c + 8*a^2*c^2)*(-((c + d
*x)*Sqrt[a + b/(c + d*x)]) - (b*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/Sq
rt[a]))/(4*a))/(2*a))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(147) = 294$.

Time = 0.12 (sec) , antiderivative size = 545, normalized size of antiderivative = 3.26

method	result
default	$\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(-48\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,d^2}\,a^2cdx+24\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^2b\,c^2 \cdots\right.}{}$

input
```
int(x^2*(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/d^3*(-48*(a*d^2*x^2+2*a*c*d*x+a
*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c*d*x+24*ln(1/2*(2*a*d^2*x+2*a*c*d
+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c
^2*d+48*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*c^2-48*(a*d^2*x^2+
2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c^2-12*(a*d^2*x^2+2*a*c
*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*d*x+12*ln(1/2*(2*a*d^2*x+2*a
*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*a*b^2*c*d+16*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a*(a*d
^2)^(1/2)-36*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*b
*c+3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/
2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d-6*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*
d*x+b*c)^(1/2)*(a*d^2)^(1/2)*b^2)/((a*d*x+a*c+b)*(d*x+c))^(1/2)/a^2/(a*d^2
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.96

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx$$

$$= \left[\frac{3\left(8a^2bc^2+4ab^2c+b^3\right)\sqrt{a}\log\left(2adx+2ac+2(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\left(8a^3d^3x^3+2a^2bd^2x^2\right.}{48a^3d^3}\right.$$

$$-\frac{3\left(8a^2bc^2+4ab^2c+b^3\right)\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-\left(8a^3d^3x^3+2a^2bd^2x^2+8a^3c^3-10a^2bc\right.}{24a^3d^3}$$

input
```
integrate(x^2*(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(2*a*d*x + 2*a*c + 2*(
d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(8*a^3*d^3*x^3
 + 2*a^2*b*d^2*x^2 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c - (8*a^2*b*c + 3
*a*b^2)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^3*d^3), -1/24*(3*(8*a^2
*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x +
 a*c + b)/(d*x + c))/(a*d*x + a*c + b)) - (8*a^3*d^3*x^3 + 2*a^2*b*d^2*x^2
 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x)*sqrt(
(a*d*x + a*c + b)/(d*x + c)))/(a^3*d^3)]
```

Sympy [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx=\int x^2\sqrt{\frac{ac+adx+b}{c+dx}}\,dx$$

input
```
integrate(x**2*(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*c + a*d*x + b)/(c + d*x)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx=\int\sqrt{a+\frac{b}{dx+c}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.49

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$= \frac{1}{24}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(2\,x\left(\frac{4\,x\mathrm{sgn}(dx+c)}{d}-\frac{4\,a^2cd^3\mathrm{sgn}(dx+c)-abd^3\mathrm{sgn}(dx+c)}{a^2d^5}\right)\right.$$
$$-\frac{(8\,a^2bc^2\mathrm{sgn}(dx+c)+4\,ab^2c\mathrm{sgn}(dx+c)+b^3\mathrm{sgn}(dx+c))\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx\right.\right.\right.}{16\,a^{\frac{5}{2}}d^2|d|}$$

input
```
integrate(x^2*(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*x*(4*x*sgn(d*x +
 c)/d - (4*a^2*c*d^3*sgn(d*x + c) - a*b*d^3*sgn(d*x + c))/(a^2*d^5)) + (8*
a^2*c^2*d^2*sgn(d*x + c) - 10*a*b*c*d^2*sgn(d*x + c) - 3*b^2*d^2*sgn(d*x +
 c))/(a^2*d^5)) - 1/16*(8*a^2*b*c^2*sgn(d*x + c) + 4*a*b^2*c*sgn(d*x + c)
+ b^3*sgn(d*x + c))*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))/(a^(5/2)*d^2*abs(
d))
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx=\int x^2\,\sqrt{a+\frac{b}{c+d\,x}}\,dx$$

input
```
int(x^2*(a + b/(c + d*x))^(1/2),x)
```

output
```
int(x^2*(a + b/(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.54

$$\int x^2\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{8\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3c^2-8\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3cdx+8\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3d^2x^2-10}{}$$

input
```
int(x^2*(a+b/(d*x+c))^(1/2),x)
```

output
```
(8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**2 - 8*sqrt(c + d*x)*sqrt(a*
c + a*d*x + b)*a**3*c*d*x + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*d**
2*x**2 - 10*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c + 2*sqrt(c + d*x)
*sqrt(a*c + a*d*x + b)*a**2*b*d*x - 3*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*
a*b**2 + 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sq
rt(b))*a**2*b*c**2 + 12*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(
c + d*x))/sqrt(b))*a*b**2*c + 3*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(
a)*sqrt(c + d*x))/sqrt(b))*b**3)/(24*a**3*d**3)
```

3.14 $\int x\sqrt{a+\frac{b}{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx=\frac{(b-4ac)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4ad^2}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2d^2}-\frac{b(b+4ac)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{4a^{3/2}d^2}$$

output
```
1/4*(-4*a*c+b)*(d*x+c)*(a+b/(d*x+c))^(1/2)/a/d^2+1/2*(d*x+c)^2*(a+b/(d*x+c
))^(1/2)/d^2-1/4*b*(4*a*c+b)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/
d^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx=\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(b-2ac+2adx)}{4ad^2}-\frac{b(b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{4a^{3/2}d^2}$$

input
```
Integrate[x*Sqrt[a + b/(c + d*x)],x]
```

output
```
((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(b - 2*a*c + 2*a*d*x))/(4*a*d
^2) - (b*(b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(
4*a^(3/2)*d^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {896, 25, 941, 948, 25, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{c+dx}}dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int dx\sqrt{a+\frac{b}{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -dx\sqrt{a+\frac{b}{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \ 941$$

$$-\frac{\int (c+dx)\sqrt{a+\frac{b}{c+dx}}\left(\frac{c}{c+dx}-1\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \ 948$$

$$\frac{\int -(c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int (c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{(4ac+b)\int (c+dx)^2\sqrt{a+\frac{b}{c+dx}}d\frac{1}{c+dx}}{4a}+\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{d^2}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{(4ac+b)\left(\frac{1}{2}b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}-(c+dx)\sqrt{a+\frac{b}{c+dx}}\right)}{4a}+\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{d^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{(4ac+b)\left(\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}-(c+dx)\sqrt{a+\frac{b}{c+dx}}\right)}{4a}+\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{d^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(4ac+b)\left((c+dx)\left(-\sqrt{a+\frac{b}{c+dx}}\right)-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{4a}+\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}}{2a}}{d^2}$$

input

```
Int[x*Sqrt[a + b/(c + d*x)],x]
```

output

```
(((c + d*x)^2*(a + b/(c + d*x))^(3/2))/(2*a) + ((b + 4*a*c)*(-((c + d*x)*S
qrt[a + b/(c + d*x)]) - (b*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/Sqrt[a]
))/(4*a))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.11

method	result
default	$-\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(-4\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,d^2}\,adx+4\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)abcd+8\sqrt{}\right.}{}$

input
```
int(x*(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/d^2*(-4*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x+4*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a
*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d+8*((a
*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a*c-4*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*c+b^2*d*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^
2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))-2
*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((a*d*x+a*c+
b)*(d*x+c))^(1/2)/a/(a*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.14

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx$$

$$=\left[\frac{(4\,abc+b^2)\sqrt{a}\log\left(2\,adx+2\,ac-2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(2\,a^2d^2x^2-2\,a^2c^2+abdx+abc)}{8\,a^2d^2}\right.$$

input
```
integrate(x*(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((4*a*b*c + b^2)*sqrt(a)*log(2*a*d*x + 2*a*c - 2*(d*x + c)*sqrt(a)*sq
rt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(2*a^2*d^2*x^2 - 2*a^2*c^2 + a*b*
d*x + a*b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^2*d^2), 1/4*((4*a*b*c +
 b^2)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))
/(a*d*x + a*c + b)) + (2*a^2*d^2*x^2 - 2*a^2*c^2 + a*b*d*x + a*b*c)*sqrt((
a*d*x + a*c + b)/(d*x + c)))/(a^2*d^2)]
```

Sympy [F]

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx=\int x\sqrt{\frac{ac+adx+b}{c+dx}}\,dx$$

input
```
integrate(x*(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(x*sqrt((a*c + a*d*x + b)/(c + d*x)), x)
```

Maxima [F]

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx=\int \sqrt{a+\frac{b}{dx+c}}x\,dx$$

input
```
integrate(x*(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))*x, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.57

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{1}{4}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(\frac{2\,x\mathrm{sgn}(dx+c)}{d}-\frac{2\,acd\mathrm{sgn}(dx+c)-bd\mathrm{sgn}(dx+c)}{ad^3}\right)$$
$$+\frac{(4\,abc\mathrm{sgn}(dx+c)+b^2\mathrm{sgn}(dx+c))\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)\sqrt{a}\right.\right.}{8\,a^{\frac{3}{2}}d|d|}$$

input
```
integrate(x*(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*x*sgn(d*x + c)/d
- (2*a*c*d*sgn(d*x + c) - b*d*sgn(d*x + c))/(a*d^3)) + 1/8*(4*a*b*c*sgn(d*
x + c) + b^2*sgn(d*x + c))*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))/(a^(3/2)*d
*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx=\int x\,\sqrt{a+\frac{b}{c+d\,x}}\,dx$$

input
```
int(x*(a + b/(c + d*x))^(1/2),x)
```

output
```
int(x*(a + b/(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int x\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{-2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^2c+2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^2dx+\sqrt{dx+c}\,\sqrt{adx+ac+b}\,ab-4\sqrt{a}\log}{4a^2d^2}$$

input
```
int(x*(a+b/(d*x+c))^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*c + 2*sqrt(c + d*x)*sqrt(a*
c + a*d*x + b)*a**2*d*x + sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b - 4*sqrt
(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b*c - s
qrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**2)/
(4*a**2*d**2)
```

3.15 $\int \sqrt{a+\frac{b}{c+dx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \sqrt{a+\frac{b}{c+dx}}\,dx=\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
(d*x+c)*(a+b/(d*x+c))^(1/2)/d+b*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(1/
2)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \sqrt{a+\frac{b}{c+dx}}\,dx=\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}}{d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)],x]
```

output
```
((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/d + (b*ArcTanh[Sqrt[(b + a*c
 + a*d*x)/(c + d*x)]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {239, 773, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{c+dx}}dx$$

$$\Big\downarrow \text{ 239}$$

$$\frac{\int \sqrt{a+\frac{b}{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 773}$$

$$-\frac{\int (c+dx)^2\sqrt{a+\frac{b}{c+dx}}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{ 51}$$

$$-\frac{\frac{1}{2}b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}-(c+dx)\sqrt{a+\frac{b}{c+dx}}}{d}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}-(c+dx)\sqrt{a+\frac{b}{c+dx}}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{(c+dx)\left(-\sqrt{a+\frac{b}{c+dx}}\right)-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

input
```
Int[Sqrt[a + b/(c + d*x)],x]
```

output
```
-((-((c + d*x)*Sqrt[a + b/(c + d*x)]) - (b*ArcTanh[Sqrt[a + b/(c + d*x)]/S
qrt[a]])/Sqrt[a])/d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(49) = 98.

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

method	result	size
default	$\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(bd\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}\right)}{2\sqrt{(adx+ac+b)(dx+c)}\,d\sqrt{a\,d^2}}$	136

input
```
int((a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(b*d*ln(1/2*(2*a*d^2*x+2*a*c*d+2
*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))+2*((a*d*x
+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2))/((a*d*x+a*c+b)*(d*x+c))^(1/2)/d/(a*d
^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.02

$$\int\sqrt{a+\frac{b}{c+dx}}\,dx = \left[\frac{\sqrt{a}b\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(adx+ac)\sqrt{\frac{adx+ac+b}{dx+c}}}{2\,ad},\ -\frac{\sqrt{-a}b\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-(adx+ac)\sqrt{\frac{adx+ac+b}{dx+c}}}{ad}\right]$$

input
```
integrate((a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*
c + b)/(d*x + c)) + b) + 2*(a*d*x + a*c)*sqrt((a*d*x + a*c + b)/(d*x + c))
)/(a*d), -(sqrt(-a)*b*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*
x + c))/(a*d*x + a*c + b)) - (a*d*x + a*c)*sqrt((a*d*x + a*c + b)/(d*x + c
)))/(a*d)]
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{c+dx}}\,dx=\int \sqrt{a+\frac{b}{c+dx}}\,dx$$

input
```
integrate((a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b/(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{c+dx}}\,dx=\int \sqrt{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate((a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(49) = 98$.

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \sqrt{a+\frac{b}{c+dx}}\,dx = -\frac{b\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)\sqrt{a}|d|+bd\right|\right)\mathrm{sgn}(dx+c)}{2\sqrt{a}|d|} + \frac{\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\,\mathrm{sgn}(dx+c)}{d}$$

input

```
integrate((a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
-1/2*b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))*sgn(d*x + c)/(sqrt(a)*abs(d))
+ sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*sgn(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \sqrt{a+\frac{b}{c+dx}}\,dx = \frac{\sqrt{\frac{a\,(c+d\,x)^2+b\,(c+d\,x)}{(c+d\,x)^2}}\,(c+d\,x)}{d} + \frac{b\,\ln\left(\frac{\frac{b}{2}+\sqrt{a}\,\sqrt{a\,(c+d\,x)^2+b\,(c+d\,x)}+a\,(c+d\,x)}{\sqrt{a}}\right)\sqrt{\frac{a\,(c+d\,x)^2+b\,(c+d\,x)}{(c+d\,x)^2}}\,(c+d\,x)}{2\sqrt{a}\,d\,\sqrt{a\,(c+d\,x)^2+b\,(c+d\,x)}}$$

input

```
int((a + b/(c + d*x))^(1/2),x)
```

output
```
(((a*(c + d*x)^2 + b*(c + d*x))/(c + d*x)^2)^(1/2)*(c + d*x))/d + (b*log((
b/2 + a^(1/2)*(a*(c + d*x)^2 + b*(c + d*x))^(1/2) + a*(c + d*x))/a^(1/2))*
((a*(c + d*x)^2 + b*(c + d*x))/(c + d*x)^2)^(1/2)*(c + d*x))/(2*a^(1/2)*d*
(a*(c + d*x)^2 + b*(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \sqrt{a+\frac{b}{c+dx}}\,dx=\frac{\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a+\sqrt{a}\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)b}{ad}$$

input
```
int((a+b/(d*x+c))^(1/2),x)
```

output
```
(sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a + sqrt(a)*log((sqrt(a*c + a*d*x + b
) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b)/(a*d)
```

3.16 $\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x} dx = 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right) - \frac{2\sqrt{b+ac}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{\sqrt{c}}$$

output
```
2*a^(1/2)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))-2*(a*c+b)^(1/2)*arctanh(c^(
1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x} dx = -\frac{2\sqrt{-b-ac}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}} + 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b/(c + d*x)]/x,x]
```

output
```
(-2*Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt
[-b - a*c]])/Sqrt[c] + 2*Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]
/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {896, 25, 941, 948, 25, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x} \, dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{dx} d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\sqrt{a+\frac{b}{c+dx}}}{dx} d(c+dx)$$

$$\Big\downarrow \text{941}$$

$$-\int \frac{\sqrt{a+\frac{b}{c+dx}}}{(c+dx)\left(\frac{c}{c+dx}-1\right)} d(c+dx)$$

$$\Big\downarrow \text{948}$$

$$\int -\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{1-\frac{c}{c+dx}} d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{1-\frac{c}{c+dx}} d\frac{1}{c+dx}$$

$$\Big\downarrow \; 94$$

$$-a\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}-(ac+b)\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$$\Big\downarrow \; 73$$

$$-\frac{2a\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{b}-\frac{2(ac+b)\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{b}$$

$$\Big\downarrow \; 221$$

$$2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)-\frac{2\sqrt{ac+b}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

input
```
Int[Sqrt[a + b/(c + d*x)]/x,x]
```

output
```
2*Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]] - (2*Sqrt[b + a*c]*ArcTan
h[(Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/Sqrt[c]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(61) = 122$.

Time = 0.12 (sec) , antiderivative size = 521, normalized size of antiderivative = 6.77

method	result
default	$-\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(-2\sqrt{(ac+b)c}\,\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)acd+2\sqrt{a\,d^2}\,\ln\left(\frac{2adxc+2a\,c^2+bdx\cdots}{}\right.\right.}{}$

input
```
int((a+b/(d*x+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(-2*((a*c+b)*c)^(1/2)*ln(1/2*(2
*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/
2)+b*d)/(a*d^2)^(1/2))*a*c*d+2*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2
*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a
*c^2-((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b*d+((a*c+b)*c)^(1/
2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)
+b*d)/(a*d^2)^(1/2))*b*d+2*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a
*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*b*c+2
*((a*d*x+a*c+b)*(d*x+c))^(1/2)*((a*c+b)*c)^(1/2)*(a*d^2)^(1/2)-2*(a*d^2*x^
2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2))/((a*d*
x+a*c+b)*(d*x+c))^(1/2)/c/(a*d^2)^(1/2)/((a*c+b)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.96

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x}\,dx = \left[\sqrt{a}\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right) + \sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,(cdx+c^2)\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{\frac{ac+b}{c}}}{x}\right), -2\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right) + \sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,(cdx+c^2)\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{\frac{ac+b}{c}}}{x}\right), 2\sqrt{-\frac{ac+b}{c}}\arctan\left(\right. + \sqrt{a}\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right), -2\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right) + 2\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{c\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{-\frac{ac+b}{c}}}{ac+b}\right)\right]$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/
(d*x + c)) + b) + sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*
c - 2*(c*d*x + c^2)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt((a*c + b)/c))/x
), -2*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))
/(a*d*x + a*c + b)) + sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*c + b)*d*x +
2*b*c - 2*(c*d*x + c^2)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt((a*c + b)/c
))/x), 2*sqrt(-(a*c + b)/c)*arctan(c*sqrt((a*d*x + a*c + b)/(d*x + c))*sqr
t(-(a*c + b)/c)/(a*c + b)) + sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqr
t(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b), -2*sqrt(-a)*arctan((d*x + c)*
sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)) + 2*sqrt(-(a
*c + b)/c)*arctan(c*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt(-(a*c + b)/c)/(
a*c + b))]
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx}}}{x}\, dx = \int \frac{\sqrt{\frac{ac+adx+b}{c+dx}}}{x}\, dx$$

input
```
integrate((a+b/(d*x+c))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*c + a*d*x + b)/(c + d*x))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx}}}{x}\, dx = \int \frac{\sqrt{a + \frac{b}{dx+c}}}{x}\, dx$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x}\,dx = \int \frac{\sqrt{a+\frac{b}{c+d\,x}}}{x}\,dx$$

input
```
int((a + b/(c + d*x))^(1/2)/x,x)
```

output
```
int((a + b/(c + d*x))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x}\,dx = \frac{\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{adx+ac+b}-\sqrt{2\sqrt{c}\,\sqrt{a}\,\sqrt{ac+b}+2ac+b}+\sqrt{a}\,\sqrt{dx+c}\right)+\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{a}\right.}{}$$

input
```
int((a+b/(d*x+c))^(1/2)/x,x)
```

output
```
(sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*
sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x)) + sqrt(c)*sqrt(a*c + b
)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c
 + b) + sqrt(a)*sqrt(c + d*x)) - sqrt(c)*sqrt(a*c + b)*log(2*sqrt(a)*sqrt(
c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*d*x
) + 2*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))
*c)/c
```

3.17 $\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2} \, dx = -\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{cx} + \frac{bd\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{c^{3/2}\sqrt{b+ac}}$$

output
```
-(d*x+c)*(a+b/(d*x+c))^(1/2)/c/x+b*d*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(
a*c+b)^(1/2))/c^(3/2)/(a*c+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2} \, dx = -\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}}{cx} - \frac{bd \arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{c^{3/2}\sqrt{-b-ac}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)]/x^2,x]
```

output
```
-(((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/(c*x)) - (b*d*ArcTan[(Sqrt
[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(c^(3/2)*Sqrt[-b -
 a*c])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {896, 941, 946, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2} \, dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{\sqrt{a+\frac{b}{c+dx}}}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \text{941}$$

$$d\int \frac{\sqrt{a+\frac{b}{c+dx}}}{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{946}$$

$$-d\int \frac{\sqrt{a+\frac{b}{c+dx}}}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{51}$$

$$-d\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2c}\right)$$

$$\Big\downarrow \text{73}$$

$$-d\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{c}\right)$$

$$\Big\downarrow \text{221}$$

$$-d\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{c^{3/2}\sqrt{ac+b}}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x)]/x^2,x]
```

output
```
-(d*(Sqrt[a + b/(c + d*x)]/(c*(1 - c/(c + d*x))) - (b*ArcTanh[(Sqrt[c]*Sqr
t[a + b/(c + d*x)])/Sqrt[b + a*c]])/(c^(3/2)*Sqrt[b + a*c])))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(65) = 130$.

Time = 0.12 (sec) , antiderivative size = 766, normalized size of antiderivative = 9.95

method	result
default	$\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(2\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,d^2}\,\sqrt{(ac+b)c}\,a\,d^2x^2-\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,\ldots}}{2\sqrt{a\,d^2}}\right.\right.}{\ldots}$

input
```
int((a+b/(d*x+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*
d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*d^2*x^2-ln(1/2*(2*a*d^2*x
+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a*b*c*d^2*x+(a*d^2)^(1/2)*ln((2*a*d*x*c+2
*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/
2)+2*b*c)/x)*a*b*c^2*d*x+((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a
*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d^2*x+2
*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*c*d*x+2*(
a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)
*a*c*d*x-ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)
^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^2*d^2*x+(a*d^
2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*
d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*b^2*c*d*x+((a*c+b)*c)^(1/2)*ln(1/2*(2
*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2
)^(1/2))*b^2*d^2*x+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*((a*c+b)*
c)^(1/2)*b*d*x-2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)
*((a*c+b)*c)^(1/2))/((a*d*x+a*c+b)*(d*x+c))^(1/2)/c^2/(a*c+b)/x/(a*d^2)^(1
/2)/((a*c+b)*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.58

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2}\,dx = \left[\frac{\sqrt{ac^2+bc}bdx\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc+2\,\sqrt{ac^2+bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{x}\right)-2\,(ac^3+bc^2+(ac^2+bc)dx)\sqrt{\frac{adx+ac+b}{dx+c}}}{2\,(ac^3+bc^2)x}\right.$$
$$\left.-\frac{\sqrt{-ac^2-bc}bdx\arctan\left(\frac{\sqrt{-ac^2-bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{acdx+ac^2+bc}\right)+(ac^3+bc^2+(ac^2+bc)dx)\sqrt{\frac{adx+ac+b}{dx+c}}}{(ac^3+bc^2)x}\right]$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a*c^2 + b*c)*b*d*x*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c + 2*
sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/x) - 2*(a*c
^3 + b*c^2 + (a*c^2 + b*c)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a*c^3
 + b*c^2)*x), -(sqrt(-a*c^2 - b*c)*b*d*x*arctan(sqrt(-a*c^2 - b*c)*(d*x +
c)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) + (a*c^3 + b
*c^2 + (a*c^2 + b*c)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a*c^3 + b*c
^2)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx}}}{x^2} \, dx = \int \frac{\sqrt{\frac{ac+adx+b}{c+dx}}}{x^2} \, dx$$

input
```
integrate((a+b/(d*x+c))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*c + a*d*x + b)/(c + d*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx}}}{x^2} \, dx = \int \frac{\sqrt{a + \frac{b}{dx+c}}}{x^2} \, dx$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(65) = 130$.

Time = 0.15 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.62

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2}\,dx = -\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}}{\sqrt{-ac^2-bc}}\right)\mathrm{sgn}(dx+c)}{\sqrt{-ac^2-bc}c} - \frac{2\,a^{\frac{3}{2}}c^2|d|\mathrm{sgn}(dx+c)+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)acd\mathrm{sgn}(dx+c)+2\,\sqrt{a}bc|d|\mathrm{sg}}{\left(ac^2-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+}\right.\right.}$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
-b*d*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x +
 b*c))/sqrt(-a*c^2 - b*c))*sgn(d*x + c)/(sqrt(-a*c^2 - b*c)*c) - (2*a^(3/2
)*c^2*abs(d)*sgn(d*x + c) + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b*d*x + b*c))*a*c*d*sgn(d*x + c) + 2*sqrt(a)*b*c*abs(d)*sgn(d*x
+ c) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))
*b*d*sgn(d*x + c))/((a*c^2 - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b*d*x + b*c))^2 + b*c)*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{c+d\,x}}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x))^(1/2)/x^2,x)
```

output
```
int((a + b/(c + d*x))^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^2}\,dx$$

$$= \frac{-2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a\,c^2-2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,bc-\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{adx+ac+b}-\sqrt{2\sqrt{}}\right.}{}$$

input
```
int((a+b/(d*x+c))^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*c**2 - 2*sqrt(c + d*x)*sqrt(a*
c + a*d*x + b)*b*c - sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqr
t(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b*
d*x - sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqr
t(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b*d*x + sqrt(c)*s
qrt(a*c + b)*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)
*sqrt(a)*sqrt(a*c + b) + 2*a*d*x)*b*d*x)/(2*c**2*x*(a*c + b))
```

3.18 $\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3}\,dx = \frac{(5b+4ac)d(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4c^2(b+ac)x} - \frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2c^2x^2} - \frac{b(3b+4ac)d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{4c^{5/2}(b+ac)^{3/2}}$$

output
```
1/4*(4*a*c+5*b)*d*(d*x+c)*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)/x-1/2*(d*x+c)^2*
(a+b/(d*x+c))^(1/2)/c^2/x^2-1/4*b*(4*a*c+3*b)*d^2*arctanh(c^(1/2)*(a+b/(d*
x+c))^(1/2)/(a*c+b)^(1/2))/c^(5/2)/(a*c+b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3}\,dx = \frac{\frac{\sqrt{c}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(-2ac(c-dx)+b(-2c+3dx))}{(b+ac)x^2} - \frac{b(3b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}}}{4c^{5/2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)]/x^3,x]
```

output
```
((Sqrt[c]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(-2*a*c*(c - d*x) +
b*(-2*c + 3*d*x)))/((b + a*c)*x^2) - (b*(3*b + 4*a*c)*d^2*ArcTan[(Sqrt[c]*
Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(-b - a*c)^(3/2))/(4*c
^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 25, 941, 948, 25, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2 \int \frac{\sqrt{a+\frac{b}{c+dx}}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^2 \int -\frac{\sqrt{a+\frac{b}{c+dx}}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$-d^2 \int \frac{\sqrt{a+\frac{b}{c+dx}}}{(c+dx)^3 \left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \text{ 948}$$

$$d^2 \int -\frac{\sqrt{a+\frac{b}{c+dx}}}{(c+dx) \left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 25$$

$$-d^2 \int \frac{\sqrt{a+\frac{b}{c+dx}}}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3} d\frac{1}{c+dx}$$

$$\Big\downarrow \ 87$$

$$d^2 \left(\frac{(4ac+3b)\int \frac{\sqrt{a+\frac{b}{c+dx}}}{\left(1-\frac{c}{c+dx}\right)^2} d\frac{1}{c+dx}}{4c(ac+b)} - \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \ 51$$

$$d^2 \left(\frac{(4ac+3b)\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)} - \frac{b\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)} d\frac{1}{c+dx}}{2c}\right)}{4c(ac+b)} - \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \ 73$$

$$d^2 \left(\frac{(4ac+3b)\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)} - \frac{\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1} d\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{4c(ac+b)} - \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \ 221$$

$$d^2 \left(\frac{(4ac+3b)\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{c^{3/2}\sqrt{ac+b}}\right)}{4c(ac+b)} - \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x)]/x^3,x]
```

output
```
d^2*(-1/2*(a + b/(c + d*x))^(3/2)/(c*(b + a*c)*(1 - c/(c + d*x))^2) + ((3*
b + 4*a*c)*(Sqrt[a + b/(c + d*x)]/(c*(1 - c/(c + d*x))) - (b*ArcTanh[(Sqrt
[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/(c^(3/2)*Sqrt[b + a*c])))/(4*c*
(b + a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1433 vs. $2(120) = 240$.

Time = 0.13 (sec) , antiderivative size = 1434, normalized size of antiderivative = 10.24

method	result	size
default	Expression too large to display	1434

input
```
int((a+b/(d*x+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(12*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*c*d^3*x^3-4*ln(1/2*(2
*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/
2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a^2*b*c^2*d^3*x^2+4*(a*d^2)^(1/2)
*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^
2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a^2*b*c^3*d^2*x^2+4*((a*c+b)*c)^(1/2)*ln(1/2*
(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*a^2*b*c^2*d^3*x^2+12*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2
)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*c^2*d^2*x^2+10*(a*d^2*x^2+2*a*c*d*x+
a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b*d^3*x^3-8*ln(1/
2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)
^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a*b^2*c*d^3*x^2+8*(a*d^2)^(1/
2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a^2*c^2*d^2*x^2+7*(a*d^
2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*
d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a*b^2*c^2*d^2*x^2+8*((a*c+b)*c)^(1/2)
*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b
*d)/(a*d^2)^(1/2))*a*b^2*c*d^3*x^2+12*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c
)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b*c*d^2*x^2-4*ln(1/2*(2*a*d^2*x+
2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(
a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^3*d^3*x^2+16*(a*d^2)^(1/2)*((a*c+b)*c...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.04

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3}\,dx$$
$$= \left[\frac{(4\,abc+3\,b^2)\sqrt{ac^2+bc}d^2x^2\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,\sqrt{ac^2+bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{x}\right)-2\,(2\,a^2c^5+4\,abc^4+2\right.}{8\,(a^2c^5+2\,abc^4+b^2c^3)x^2}$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^2*log(-(2*a*c^2 + (2*a*c +
 b)*d*x + 2*b*c - 2*sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*
x + c)))/x) - 2*(2*a^2*c^5 + 4*a*b*c^4 + 2*b^2*c^3 - (2*a^2*c^3 + 5*a*b*c^
2 + 3*b^2*c)*d^2*x^2 - (a*b*c^3 + b^2*c^2)*d*x)*sqrt((a*d*x + a*c + b)/(d*
x + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^2), 1/4*((4*a*b*c + 3*b^2)*sqr
t(-a*c^2 - b*c)*d^2*x^2*arctan(sqrt(-a*c^2 - b*c)*(d*x + c)*sqrt((a*d*x +
a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) - (2*a^2*c^5 + 4*a*b*c^4 + 2*
b^2*c^3 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^2 - (a*b*c^3 + b^2*c^2)*
d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x
^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{\frac{ac+adx+b}{c+dx}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a*c + a*d*x + b)/(c + d*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{a + \frac{b}{dx+c}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(120) = 240$.

Time = 0.16 (sec) , antiderivative size = 729, normalized size of antiderivative = 5.21

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(4*a*b*c*d^2*sgn(d*x + c) + 3*b^2*d^2*sgn(d*x + c))*arctan(-(sqrt(a*d^
2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))/sqrt(-a*c^2 - b*
c))/((a*c^3 + b*c^2)*sqrt(-a*c^2 - b*c)) + 1/4*(8*a^(7/2)*c^5*d*abs(d)*sgn
(d*x + c) + 16*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x
 + b*c))*a^3*c^4*d^2*sgn(d*x + c) + 8*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b*d*x + b*c))^2*a^(5/2)*c^3*d*abs(d)*sgn(d*x + c) + 24*a
^(5/2)*b*c^4*d*abs(d)*sgn(d*x + c) + 36*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^2*b*c^3*d^2*sgn(d*x + c) + 8*(sqrt(a*d
^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*a^(3/2)*b*c^2
*d*abs(d)*sgn(d*x + c) + 24*a^(3/2)*b^2*c^3*d*abs(d)*sgn(d*x + c) - 4*(sqr
t(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*a*b*c*d^
2*sgn(d*x + c) + 25*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b*d*x + b*c))*a*b^2*c^2*d^2*sgn(d*x + c) + 8*sqrt(a)*b^3*c^2*d*abs(d)*sgn(
d*x + c) - 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x +
 b*c))^3*b^2*d^2*sgn(d*x + c) + 5*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b*d*x + b*c))*b^3*c*d^2*sgn(d*x + c))/((a*c^3 + b*c^2)*(a*c^
2 - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2
+ b*c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{a+\frac{b}{c+d\,x}}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x))^(1/2)/x^3,x)
```

output
```
int((a + b/(c + d*x))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 857, normalized size of antiderivative = 6.12

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b/(d*x+c))^(1/2)/x^3,x)
```

output
```
( - 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**5 + 8*sqrt(c + d*x)*sqrt
(a*c + a*d*x + b)*a**3*c**4*d*x - 20*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a
**2*b*c**4 + 24*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c**3*d*x - 16*s
qrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**2*c**3 + 22*sqrt(c + d*x)*sqrt(a*c
 + a*d*x + b)*a*b**2*c**2*d*x - 4*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**3
*c**2 + 6*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**3*c*d*x + 8*sqrt(c)*sqrt(
a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b)
+ 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b*c**2*d**2*x**2 + 10*sqrt(c)*s
qrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c +
 b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**2*c*d**2*x**2 + 3*sqrt(c)*s
qrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c +
 b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**3*d**2*x**2 + 8*sqrt(c)*sqrt(
a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b)
+ 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b*c**2*d**2*x**2 + 10*sqrt(c)*s
qrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c +
 b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**2*c*d**2*x**2 + 3*sqrt(c)*s
qrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c +
 b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**3*d**2*x**2 - 8*sqrt(c)*sqrt(
a*c + b)*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqr
t(a)*sqrt(a*c + b) + 2*a*d*x)*a**2*b*c**2*d**2*x**2 - 10*sqrt(c)*sqrt(a...
```

3.19 $\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 212

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4} \, dx = -\frac{(11b^2+20abc+8a^2c^2)\, d^2(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8c^3(b+ac)^2x} + \frac{(13b+12ac)d(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12c^3(b+ac)x^2} - \frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3c^3x^3} + \frac{b(5b^2+12abc+8a^2c^2)\, d^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{8c^{7/2}(b+ac)^{5/2}}$$

output
```
-1/8*(8*a^2*c^2+20*a*b*c+11*b^2)*d^2*(d*x+c)*(a+b/(d*x+c))^(1/2)/c^3/(a*c+
b)^2/x+1/12*(12*a*c+13*b)*d*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/c^3/(a*c+b)/x^2-
1/3*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/c^3/x^3+1/8*b*(8*a^2*c^2+12*a*b*c+5*b^2)
*d^3*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/c^(7/2)/(a*c+b)^(5
/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx = -\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(8a^2c^2(c^2-cdx+d^2x^2)+2abc(8c^2-9cdx+13d^2x^2)+b^2(8c^2-10cdx+15d^2x^2)\right)}{24c^3(b+ac)^2x^3} - \frac{b(5b^2+12abc+8a^2c^2)\,d^3\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{8c^{7/2}(-b-ac)^{5/2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)]/x^4,x]
```

output
```
-1/24*((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(8*a^2*c^2*(c^2 - c*d*x
 + d^2*x^2) + 2*a*b*c*(8*c^2 - 9*c*d*x + 13*d^2*x^2) + b^2*(8*c^2 - 10*c*d
*x + 15*d^2*x^2)))/(c^3*(b + a*c)^2*x^3) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^
2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])
/(8*c^(7/2)*(-b - a*c)^(5/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 941, 948, 100, 27, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx$$

$$\Big\downarrow \; 896$$

$$d^3\int \frac{\sqrt{a+\frac{b}{c+dx}}}{d^4x^4}d(c+dx)$$

$$\Big\downarrow \ 941$$

$$d^3 \int \frac{\sqrt{a+\frac{b}{c+dx}}}{(c+dx)^4 \left(\frac{c}{c+dx}-1\right)^4} d(c+dx)$$

$$\Big\downarrow \ 948$$

$$-d^3 \int \frac{\sqrt{a+\frac{b}{c+dx}}}{(c+dx)^2 \left(1-\frac{c}{c+dx}\right)^4} d\frac{1}{c+dx}$$

$$\Big\downarrow \ 100$$

$$-d^3 \left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3} - \frac{\int \frac{3\sqrt{a+\frac{b}{c+dx}}\left(b+2ac+\frac{2c(b+ac)}{c+dx}\right)}{2\left(1-\frac{c}{c+dx}\right)^3} d\frac{1}{c+dx}}{3c^2(ac+b)}\right)$$

$$\Big\downarrow \ 27$$

$$-d^3 \left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3} - \frac{\int \frac{\sqrt{a+\frac{b}{c+dx}}\left(b+2ac+\frac{2c(b+ac)}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^3} d\frac{1}{c+dx}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \ 87$$

$$-d^3 \left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3} - \frac{\frac{(4ac+3b)\left(a+\frac{b}{c+dx}\right)^{3/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2} - \frac{(8a^2c^2+12abc+5b^2)\int \frac{\sqrt{a+\frac{b}{c+dx}}}{\left(1-\frac{c}{c+dx}\right)^2} d\frac{1}{c+dx}}{4(ac+b)}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \ 51$$

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(4ac+3b)\left(a+\frac{b}{c+dx}\right)^{3/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(8a^2c^2+12abc+5b^2)\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\int\frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2c}\right)}{4(ac+b)}}{2c^2(ac+b)}\right)$$

$\downarrow$ 73

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(4ac+3b)\left(a+\frac{b}{c+dx}\right)^{3/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(8a^2c^2+12abc+5b^2)\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{\int\frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{4(ac+b)}}{2c^2(ac+b)}\right)$$

$\downarrow$ 221

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(4ac+3b)\left(a+\frac{b}{c+dx}\right)^{3/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(8a^2c^2+12abc+5b^2)\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{c^{3/2}\sqrt{ac+b}}\right)}{4(ac+b)}}{2c^2(ac+b)}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x)]/x^4,x]
```

output
```
-(d^3*((a + b/(c + d*x))^(3/2)/(3*c^2*(b + a*c)*(1 - c/(c + d*x))^3) - (((
3*b + 4*a*c)*(a + b/(c + d*x))^(3/2))/(2*(b + a*c)*(1 - c/(c + d*x))^2) -
((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(Sqrt[a + b/(c + d*x)]/(c*(1 - c/(c + d*x)
)) - (b*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/(c^(3/2)*S
qrt[b + a*c])))/(4*(b + a*c)))/(2*c^2*(b + a*c))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2155 vs. $2(188) = 376$.

Time = 0.14 (sec) , antiderivative size = 2156, normalized size of antiderivative = 10.17

method	result	size
default	Expression too large to display	2156

input
```
int((a+b/(d*x+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(-72*ln(1/2*(2*a*d^2*x+2*a*c*d+
2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(
1/2))*((a*c+b)*c)^(1/2)*a*b^3*c*d^4*x^3+51*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a
*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)
+2*b*c)/x)*a*b^3*c^2*d^3*x^3+72*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*
d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^3*
c*d^4*x^3-96*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a
*c+b)*c)^(1/2)*a^2*c^2*d^2*x^2+48*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3
/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*c^3*d*x+36*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*b^2*c*d*x-66*(a*d^2*x
^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*b^2*d^
2*x^2-32*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b
)*c)^(1/2)*a*b*c^3+96*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^
(1/2)*((a*c+b)*c)^(1/2)*a^3*c^2*d^4*x^4+15*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a
*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)
+2*b*c)/x)*b^4*c*d^3*x^3-72*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d
*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1
/2)*a^2*b^2*c^2*d^4*x^3+60*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a
*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a^2*b
^2*c^3*d^3*x^3+72*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx = \left[\frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\sqrt{ac^2+bc}\,d^3x^3\log\left(-\frac{2ac^2+(2ac+b)dx+2bc+2\sqrt{ac^2+bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{x}\right)-2\left(8a^3c^7\right.\cdots}{\cdots}\right.$$

$$-\frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\sqrt{-ac^2-bc}\,d^3x^3\arctan\left(\frac{\sqrt{-ac^2-bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{acdx+ac^2+bc}\right)+\left(8a^3c^7+24a^2bc^6+2\cdots\right.}{\cdots}$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^3*log(
-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c + 2*sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((
a*d*x + a*c + b)/(d*x + c)))/x) - 2*(8*a^3*c^7 + 24*a^2*b*c^6 + 24*a*b^2*c
^5 + 8*b^3*c^4 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*
x^3 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^2 - 2*(a^2*b*c^5 + 2*
a*b^2*c^4 + b^3*c^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^3*c^7 + 3
*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^3), -1/24*(3*(8*a^2*b*c^2 + 12*a*b^2
*c + 5*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^3*arctan(sqrt(-a*c^2 - b*c)*(d*x + c)
*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) + (8*a^3*c^7 +
 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*
a*b^2*c^2 + 15*b^3*c)*d^3*x^3 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d
^2*x^2 - 2*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x)*sqrt((a*d*x + a*c + b)
/(d*x + c)))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx=\int \frac{\sqrt{\frac{ac+adx+b}{c+dx}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a*c + a*d*x + b)/(c + d*x))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{dx+c}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))/x^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(188) = 376$.

Time = 0.18 (sec) , antiderivative size = 1447, normalized size of antiderivative = 6.83

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c))^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/8*(8*a^2*b*c^2*d^3*sgn(d*x + c) + 12*a*b^2*c*d^3*sgn(d*x + c) + 5*b^3*d
^3*sgn(d*x + c))*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c
^2 + b*d*x + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sq
rt(-a*c^2 - b*c)) - 1/24*(64*a^(11/2)*c^8*d^2*abs(d)*sgn(d*x + c) + 192*(s
qrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^5*c^7*
d^3*sgn(d*x + c) + 192*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b*d*x + b*c))^2*a^(9/2)*c^6*d^2*abs(d)*sgn(d*x + c) + 304*a^(9/2)*b*c^7
*d^2*abs(d)*sgn(d*x + c) + 64*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b*d*x + b*c))^3*a^4*c^5*d^3*sgn(d*x + c) + 744*(sqrt(a*d^2)*x -
sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^4*b*c^6*d^3*sgn(d*x +
 c) + 528*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*
c))^2*a^(7/2)*b*c^5*d^2*abs(d)*sgn(d*x + c) + 576*a^(7/2)*b^2*c^6*d^2*abs(
d)*sgn(d*x + c) + 64*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b*d*x + b*c))^3*a^3*b*c^4*d^3*sgn(d*x + c) + 1116*(sqrt(a*d^2)*x - sqrt(a
*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^3*b^2*c^5*d^3*sgn(d*x + c)
+ 480*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^
2*a^(5/2)*b^2*c^4*d^2*abs(d)*sgn(d*x + c) + 544*a^(5/2)*b^3*c^5*d^2*abs(d)
*sgn(d*x + c) + 24*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b
*d*x + b*c))^5*a^2*b*c^2*d^3*sgn(d*x + c) - 96*(sqrt(a*d^2)*x - sqrt(a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*a^2*b^2*c^3*d^3*sgn(d*x + c)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{c+d\,x}}}{x^4}\,dx$$

input
```
int((a + b/(c + d*x))^(1/2)/x^4,x)
```

output
```
int((a + b/(c + d*x))^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 1294, normalized size of antiderivative = 6.10

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{x^4}\,dx=\text{Too large to display}$$

input
```
int((a+b/(d*x+c))^(1/2)/x^4,x)
```

output
```
( - 32*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*c**7 + 32*sqrt(c + d*x)*sq
rt(a*c + a*d*x + b)*a**4*c**6*d*x - 32*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)
*a**4*c**5*d**2*x**2 - 112*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**6
 + 120*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**5*d*x - 152*sqrt(c +
d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**4*d**2*x**2 - 144*sqrt(c + d*x)*sqrt(
a*c + a*d*x + b)*a**2*b**2*c**5 + 164*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*
a**2*b**2*c**4*d*x - 232*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*c**
3*d**2*x**2 - 80*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**3*c**4 + 96*sqrt
(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**3*c**3*d*x - 142*sqrt(c + d*x)*sqrt(a
*c + a*d*x + b)*a*b**3*c**2*d**2*x**2 - 16*sqrt(c + d*x)*sqrt(a*c + a*d*x
+ b)*b**4*c**3 + 20*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**4*c**2*d*x - 30
*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**4*c*d**2*x**2 - 48*sqrt(c)*sqrt(a*
c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) +
2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**3*b*c**3*d**3*x**3 - 96*sqrt(c)*sqr
t(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b
) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b**2*c**2*d**3*x**3 - 66*sqrt
(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(
a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**3*c*d**3*x**3 - 15*sqr
t(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt
(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**4*d**3*x**3 - 48*sqr...
```

3.20 $\int x^3 \left(a+\frac{b}{c+dx}\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 264

$$\int x^3 \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \frac{2bc^3\sqrt{a+\frac{b}{c+dx}}}{d^4} - \frac{(3b^3+24ab^2c-240a^2bc^2+64a^3c^3)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{64a^2d^4} + \frac{(b^2-56abc+48a^2c^2)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{32ad^4} + \frac{(3b-8ac)(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{8d^4} + \frac{a(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4d^4} + \frac{3b(b^3+8ab^2c+48a^2bc^2-64a^3c^3)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{64a^{5/2}d^4}$$

output

```
2*b*c^3*(a+b/(d*x+c))^(1/2)/d^4-1/64*(64*a^3*c^3-240*a^2*b*c^2+24*a*b^2*c+
3*b^3)*(d*x+c)*(a+b/(d*x+c))^(1/2)/a^2/d^4+1/32*(48*a^2*c^2-56*a*b*c+b^2)*
(d*x+c)^2*(a+b/(d*x+c))^(1/2)/a/d^4+1/8*(-8*a*c+3*b)*(d*x+c)^3*(a+b/(d*x+c
))^(1/2)/d^4+1/4*a*(d*x+c)^4*(a+b/(d*x+c))^(1/2)/d^4+3/64*b*(-64*a^3*c^3+4
8*a^2*b*c^2+8*a*b^2*c+b^3)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d^
4
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

$$\int x^3\left(a+\frac{b}{c+dx}\right)^{3/2}dx=\frac{\sqrt{a}\sqrt{\frac{b+ac+adx}{c+dx}}(-3b^3(c+dx)+2ab^2(-11c^2-10cdx+d^2x^2)+8a^2b(35c^3+11c^2dx-5c\cdots}{64\cdots}$$

input
```
Integrate[x^3*(a + b/(c + d*x))^(3/2),x]
```

output
```
(Sqrt[a]*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(-3*b^3*(c + d*x) + 2*a*b^2*(-1
1*c^2 - 10*c*d*x + d^2*x^2) + 8*a^2*b*(35*c^3 + 11*c^2*d*x - 5*c*d^2*x^2 +
 3*d^3*x^3) - 16*a^3*(c^4 - d^4*x^4)) + 3*b*(b^3 + 8*a*b^2*c + 48*a^2*b*c^
2 - 64*a^3*c^3)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(64*a^
(5/2)*d^4)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {896, 25, 941, 948, 25, 108, 27, 166, 27, 166, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{c+dx}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int d^3x^3\left(a+\frac{b}{c+dx}\right)^{3/2}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -d^3x^3\left(a+\frac{b}{c+dx}\right)^{3/2}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{941}$$

$$-\frac{\int (c+dx)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(\frac{c}{c+dx}-1\right)^3 d(c+dx)}{d^4}$$

$$\Big\downarrow \text{948}$$

$$\frac{\int -(c+dx)^5 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^3 d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (c+dx)^5 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^3 d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{108}$$

$$\frac{\frac{1}{4}(c+dx)^4 \left(1-\frac{c}{c+dx}\right)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} - \frac{1}{4}\int \frac{3}{2}(c+dx)^4 \sqrt{a+\frac{b}{c+dx}} \left(1-\frac{c}{c+dx}\right)^2 \left(-\frac{3cb}{c+dx}+b-2ac\right) d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}(c+dx)^4 \left(1-\frac{c}{c+dx}\right)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} - \frac{3}{8}\int (c+dx)^4 \sqrt{a+\frac{b}{c+dx}} \left(1-\frac{c}{c+dx}\right)^2 \left(-\frac{3cb}{c+dx}+b-2ac\right) d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{166}$$

$$\frac{\frac{1}{4}(c+dx)^4 \left(1-\frac{c}{c+dx}\right)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} - \frac{3}{8}\left(\frac{1}{3}\int \frac{b(c+dx)^3 \left(1-\frac{c}{c+dx}\right)^2 \left(b-20ac-\frac{c(19b-2ac)}{c+dx}\right)}{2\sqrt{a+\frac{b}{c+dx}}} d\frac{1}{c+dx} - \frac{1}{3}(b-2ac)(c+dx)^3}{d^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}(c+dx)^4 \left(1-\frac{c}{c+dx}\right)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} - \frac{3}{8}\left(\frac{1}{6}b\int \frac{(c+dx)^3 \left(1-\frac{c}{c+dx}\right)^2 \left(b-20ac-\frac{c(19b-2ac)}{c+dx}\right)}{\sqrt{a+\frac{b}{c+dx}}} d\frac{1}{c+dx} - \frac{1}{3}(b-2ac)(c+dx)^3}{d^4}$$

$$\Big\downarrow \text{166}$$

$$\frac{\frac{1}{4}(c+dx)^4\left(1-\frac{c}{c+dx}\right)^3\left(a+\frac{b}{c+dx}\right)^{3/2}-\frac{3}{8}\left(\frac{1}{6}b\left(\frac{\int -\frac{(c+dx)^2\left(1-\frac{c}{c+dx}\right)\left(3b^2+20acb-88a^2c^2+\frac{c\left(b^2-96acb+8a^2c^2\right)}{c+dx}\right)}{2\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a}-\frac{(b-2}{}\right.\right.}{d^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}(c+dx)^4\left(1-\frac{c}{c+dx}\right)^3\left(a+\frac{b}{c+dx}\right)^{3/2}-\frac{3}{8}\left(\frac{1}{6}b\left(-\frac{\int \frac{(c+dx)^2\left(1-\frac{c}{c+dx}\right)\left(3b^2+20acb-88a^2c^2+\frac{c\left(b^2-96acb+8a^2c^2\right)}{c+dx}\right)}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{4a}-\frac{(b-2}{}\right.\right.}{d^4}$$

$$\Big\downarrow \; 163$$

$$\frac{\frac{1}{4}(c+dx)^4\left(1-\frac{c}{c+dx}\right)^3\left(a+\frac{b}{c+dx}\right)^{3/2}-\frac{3}{8}\left(\frac{1}{6}b\left(-\frac{-\frac{3\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(\frac{2ac^2(}{}\right.}{}}{4a}\right.\right.}{}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{4}(c+dx)^4\left(1-\frac{c}{c+dx}\right)^3\left(a+\frac{b}{c+dx}\right)^{3/2}-\frac{3}{8}\left(\frac{1}{6}b\left(-\frac{-\frac{3\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{ab}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{}}{4a}\right.\right.}{}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{4}(c+dx)^4\left(1-\frac{c}{c+dx}\right)^3\left(a+\frac{b}{c+dx}\right)^{3/2}-\frac{3}{8}\left(\frac{1}{6}b\left(-\frac{\frac{3\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(\frac{2a}{}\right.}{}}{4a}\right.\right.}{}$$

input
```
Int[x^3*(a + b/(c + d*x))^(3/2),x]
```

output
```
(((c + d*x)^4*(a + b/(c + d*x))^(3/2)*(1 - c/(c + d*x))^3)/4 - (3*(-1/3*((
b - 2*a*c)*(c + d*x)^3*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))^3) + (b*(-1
/2*((b - 20*a*c)*(c + d*x)^2*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))^2)/a
- (-(((c + d*x)*Sqrt[a + b/(c + d*x)]*(b*(3*b^2 + 20*a*b*c - 88*a^2*c^2) +
 (2*a*c^2*(b^2 - 96*a*b*c + 8*a^2*c^2))/(c + d*x)))/(a*b)) + (3*(b^3 + 8*a
*b^2*c + 48*a^2*b*c^2 - 64*a^3*c^3)*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]]
)/a^(3/2))/(4*a)))/6))/8)/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2360 vs. $2(238) = 476$.

Time = 0.15 (sec) , antiderivative size = 2361, normalized size of antiderivative = 8.94

method	result	size
default	Expression too large to display	2361

input
```
int(x^3*(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(-192*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*
b*c^2*d^2*x^2+768*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*b*c^3*d*
x-96*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*b*c^3*d
*x-84*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^2*c*d^
2*x^2+32*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*a*b*c*d
*x+16*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*a*b*c^2-13
2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^2*c^2*d*x-
48*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)
*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d^3*x^2-384*ln(1/2*(2*a*d^2
*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2
))*a^3*b*c^4*d^2*x+192*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))
^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d^3*x^2-96*(a*d^2*x^2
+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*b*c*d^3*x^3+384*((a*d*
x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2*d^2*x^2+6*ln(1/2*(2*a*d^2*
x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)
/(a*d^2)^(1/2))*b^4*c*d^2*x+3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c
*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^4*d^3*x^2+
256*((a*d*x+a*c+b)*(d*x+c))^(3/2)*(a*d^2)^(1/2)*a^2*c^3-96*(a*d^2*x^2+2*a*
c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*a^2*c^3+3*ln(1/2*(2*a*d^2*x+2*a
*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.68

$$\int x^3\left(a+\frac{b}{c+dx}\right)^{3/2}dx = \left[\frac{3\left(64a^3bc^3-48a^2b^2c^2-8ab^3c-b^4\right)\sqrt{a}\log\left(2adx+2ac-2(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}\right.}{}\right.$$

input
```
integrate(x^3*(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(3*(64*a^3*b*c^3 - 48*a^2*b^2*c^2 - 8*a*b^3*c - b^4)*sqrt(a)*log(2*
a*d*x + 2*a*c - 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b)
 + 2*(16*a^4*d^4*x^4 + 24*a^3*b*d^3*x^3 - 16*a^4*c^4 + 280*a^3*b*c^3 - 22*
a^2*b^2*c^2 - 3*a*b^3*c - 2*(20*a^3*b*c - a^2*b^2)*d^2*x^2 + (88*a^3*b*c^2
 - 20*a^2*b^2*c - 3*a*b^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^3*d^
4), 1/64*(3*(64*a^3*b*c^3 - 48*a^2*b^2*c^2 - 8*a*b^3*c - b^4)*sqrt(-a)*arc
tan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)
) + (16*a^4*d^4*x^4 + 24*a^3*b*d^3*x^3 - 16*a^4*c^4 + 280*a^3*b*c^3 - 22*a
^2*b^2*c^2 - 3*a*b^3*c - 2*(20*a^3*b*c - a^2*b^2)*d^2*x^2 + (88*a^3*b*c^2
- 20*a^2*b^2*c - 3*a*b^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^3*d^4
)]
```

Sympy [F]

$$\int x^3\left(a+\frac{b}{c+dx}\right)^{3/2}dx = \int x^3\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x**3*(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(x**3*((a*c + a*d*x + b)/(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int x^3\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2)*x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(240) = 480.

Time = 0.20 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.12

$$\int x^3\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\frac{1}{64}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(2\left(4\left(\frac{2\,ax\mathrm{sgn}(dx+c)}{d}-\frac{2\,a^4cd^{11}\mathrm{sgn}(dx+c)-}{a^3c}\right.\right.\right.$$
$$+\frac{(64\,a^3bc^3\mathrm{sgn}(dx+c)-48\,a^2b^2c^2\mathrm{sgn}(dx+c)-8\,ab^3c\mathrm{sgn}(dx+c)-b^4\mathrm{sgn}(dx+c))\log\left(\left|2\,a^2c^3d+6\left(\sqrt{c}\right.\right.\right.}{}$$

input
```
integrate(x^3*(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/64*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*(4*(2*a*x*sgn(d*
x + c)/d - (2*a^4*c*d^11*sgn(d*x + c) - 3*a^3*b*d^11*sgn(d*x + c))/(a^3*d^
13))*x + (8*a^4*c^2*d^10*sgn(d*x + c) - 32*a^3*b*c*d^10*sgn(d*x + c) + a^2
*b^2*d^10*sgn(d*x + c))/(a^3*d^13))*x - (16*a^4*c^3*d^9*sgn(d*x + c) - 152
*a^3*b*c^2*d^9*sgn(d*x + c) + 22*a^2*b^2*c*d^9*sgn(d*x + c) + 3*a*b^3*d^9*
sgn(d*x + c))/(a^3*d^13)) + 1/128*(64*a^3*b*c^3*sgn(d*x + c) - 48*a^2*b^2*
c^2*sgn(d*x + c) - 8*a*b^3*c*sgn(d*x + c) - b^4*sgn(d*x + c))*log(abs(2*a^
2*c^3*d + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x +
b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b*d*x + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*sqrt(a)*abs(d) + 2*(sqrt(a*d^2)
*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*b*c*abs(d)
 + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*b
*d))/(a^(5/2)*d^3*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\int x^3\,\left(a+\frac{b}{c+d\,x}\right)^{3/2}\,dx$$

input
```
int(x^3*(a + b/(c + d*x))^(3/2),x)
```

output
```
int(x^3*(a + b/(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 680, normalized size of antiderivative = 2.58

$$\int x^3\Big(a+\frac{b}{c+dx}\Big)^{3/2}\,dx=\frac{-20\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^2b^2cdx-192\sqrt{a}\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)a^3b\,c^3dx+144\sqrt{}}{}$$

input
```
int(x^3*(a+b/(d*x+c))^(3/2),x)
```

output

```
( - 16*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*c**4 + 16*sqrt(c + d*x)*sq
rt(a*c + a*d*x + b)*a**4*d**4*x**4 + 280*sqrt(c + d*x)*sqrt(a*c + a*d*x +
b)*a**3*b*c**3 + 88*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**2*d*x -
40*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c*d**2*x**2 + 24*sqrt(c + d*
x)*sqrt(a*c + a*d*x + b)*a**3*b*d**3*x**3 - 22*sqrt(c + d*x)*sqrt(a*c + a*
d*x + b)*a**2*b**2*c**2 - 20*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2
*c*d*x + 2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*d**2*x**2 - 3*sqr
t(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**3*c - 3*sqrt(c + d*x)*sqrt(a*c + a*d
*x + b)*a*b**3*d*x - 192*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt
(c + d*x))/sqrt(b))*a**3*b*c**4 - 192*sqrt(a)*log((sqrt(a*c + a*d*x + b) +
 sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**3*b*c**3*d*x + 144*sqrt(a)*log((sqrt(a
*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*b**2*c**3 + 144*sqr
t(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*b**
2*c**2*d*x + 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x)
)/sqrt(b))*a*b**3*c**2 + 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*s
qrt(c + d*x))/sqrt(b))*a*b**3*c*d*x + 3*sqrt(a)*log((sqrt(a*c + a*d*x + b)
 + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**4*c + 3*sqrt(a)*log((sqrt(a*c + a*d*
x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**4*d*x + 144*sqrt(a)*a**3*b*c**
4 + 144*sqrt(a)*a**3*b*c**3*d*x - 48*sqrt(a)*a**2*b**2*c**3 - 48*sqrt(a)*a
**2*b**2*c**2*d*x - 3*sqrt(a)*a*b**3*c**2 - 3*sqrt(a)*a*b**3*c*d*x)/(64...
```

3.21 $\int x^2 \left(a+\frac{b}{c+dx}\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 191

$$\int x^2 \left(a+\frac{b}{c+dx}\right)^{3/2} dx = -\frac{2bc^2\sqrt{a+\frac{b}{c+dx}}}{d^3} + \frac{(b^2-20abc+8a^2c^2)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8ad^3} + \frac{(7b-12ac)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12d^3} + \frac{a(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3d^3} - \frac{b(b^2+12abc-24a^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{8a^{3/2}d^3}$$

output
```
-2*b*c^2*(a+b/(d*x+c))^(1/2)/d^3+1/8*(8*a^2*c^2-20*a*b*c+b^2)*(d*x+c)*(a+b
/(d*x+c))^(1/2)/a/d^3+1/12*(-12*a*c+7*b)*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/d^3
+1/3*a*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/d^3-1/8*b*(-24*a^2*c^2+12*a*b*c+b^2)*
arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int x^2\left(a+\frac{b}{c+dx}\right)^{3/2}dx=\frac{\sqrt{a}\sqrt{\frac{b+ac+adx}{c+dx}}(3b^2(c+dx)-2ab(47c^2+16cdx-7d^2x^2)+8a^2(c^3+d^3x^3))-3b(b^2+1}{24a^{3/2}d^3}$$

input
```
Integrate[x^2*(a + b/(c + d*x))^(3/2),x]
```

output
```
(Sqrt[a]*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(3*b^2*(c + d*x) - 2*a*b*(47*c^
2 + 16*c*d*x - 7*d^2*x^2) + 8*a^2*(c^3 + d^3*x^3)) - 3*b*(b^2 + 12*a*b*c -
 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(24*a^(3/
2)*d^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {896, 941, 948, 100, 27, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{c+dx}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int d^2x^2\left(a+\frac{b}{c+dx}\right)^{3/2}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 941}$$

$$\frac{\int (c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)^2d(c+dx)}{d^3}$$

$$\Big\downarrow \; 948$$

$$-\frac{\int (c+dx)^4 \left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^2 d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \; 100$$

$$-\frac{\frac{\int -\frac{1}{2}(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}\left(-\frac{6ac^2}{c+dx}+12ac+b\right)d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{5/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int (c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}\left(-\frac{6ac^2}{c+dx}+12ac+b\right)d\frac{1}{c+dx}}{6a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{5/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 87$$

$$-\frac{-\frac{\frac{\left(-24a^2c^2+12abc+b^2\right)\int (c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}d\frac{1}{c+dx}}{4a}-\frac{(12ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}}{6a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{5/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 51$$

$$-\frac{-\frac{\frac{\left(-24a^2c^2+12abc+b^2\right)\left(\frac{3}{2}b\int (c+dx)\sqrt{a+\frac{b}{c+dx}}\,d\frac{1}{c+dx}-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}-\frac{(12ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}}{6a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{5/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 60$$

$$-\frac{-\frac{\frac{\left(-24a^2c^2+12abc+b^2\right)\left(\frac{3}{2}b\left(a\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}+2\sqrt{a+\frac{b}{c+dx}}\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}-\frac{(12ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}}{6a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\cdots\right.}{3a}}{d^3}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{\frac{\left(-24a^2c^2+12abc+b^2\right)\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{b}+2\sqrt{a+\frac{b}{c+dx}}\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}-\frac{(12ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}}{6a}-(c+dx)^3\left(\cdots\right.}{d^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{\left(-24a^2c^2+12abc+b^2\right)\left(\frac{3}{2}b\left(2\sqrt{a+\frac{b}{c+dx}}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}-\frac{(12ac+b)(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}}{6a}-\frac{(c+dx)^3\left(a\right.}{3}}{d^3}$$

input
```
Int[x^2*(a + b/(c + d*x))^(3/2),x]
```

output
```
-((-1/3*((c + d*x)^3*(a + b/(c + d*x))^(5/2))/a - (-1/2*((b + 12*a*c)*(c +
 d*x)^2*(a + b/(c + d*x))^(5/2))/a + ((b^2 + 12*a*b*c - 24*a^2*c^2)*(-((c
+ d*x)*(a + b/(c + d*x))^(3/2)) + (3*b*(2*Sqrt[a + b/(c + d*x)] - 2*Sqrt[a
]*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]]))/2))/(4*a))/(6*a))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1761 vs. $2(169) = 338$.

Time = 0.14 (sec) , antiderivative size = 1762, normalized size of antiderivative = 9.23

method	result	size
default	Expression too large to display	1762

input
```
int(x^2*(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-16*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a*d^2
*x^2+48*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^
(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c^3*d-6*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x
+a*c^2+b*d*x+b*c)^(1/2)*b^2*d^2*x^2-12*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*
x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b
^2*c^3*d+96*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a*b*c*d^2*x^2-12*(
a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a*b*c*d^2*x^2+192
*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a*b*c^2*d*x+12*(a*d^2)^(1/2)*
(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a*b*c^2*d*x+96*ln(1/2*(2*a*d^2
*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2
))*a*b^2*c^2*d^2*x+144*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)
^(1/2)*a^2*c^3*d*x+48*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^
(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d^3*x^2-32*(a*d^2)^(1/2)*(
a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a*c*d*x+6*ln(1/2*(2*a*d^2*x+2*a
*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*b^3*c*d^2*x+96*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a*b*
c^3+12*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a*b*c^3-2
4*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*
(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c^2*d^2*x+48*(a*d^2)^(1/2)*(a*d^2*
x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*c*d^3*x^3-144*((a*d*x+a*c+b)*(...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.74

$$\int x^2\left(a+\frac{b}{c+dx}\right)^{3/2}dx = \left[\frac{3\,(24\,a^2bc^2-12\,ab^2c-b^3)\sqrt{a}\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(8\ldots}{48\,a^2\ldots}\right.$$

$$-\frac{3\,(24\,a^2bc^2-12\,ab^2c-b^3)\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-(8\,a^3d^3x^3+14\,a^2bd^2x^2+8\,a^3c^3-94\,a^2b\ldots}{24\,a^2d^3}$$

input
```
integrate(x^2*(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(2*a*d*x + 2*a*c + 2
*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(8*a^3*d^3*x
^3 + 14*a^2*b*d^2*x^2 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c
 - 3*a*b^2)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^2*d^3), -1/24*(3*(2
4*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a
*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)) - (8*a^3*d^3*x^3 + 14*a^2*b*
d^2*x^2 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*
x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^2*d^3)]
```

Sympy [F]

$$\int x^2\left(a+\frac{b}{c+dx}\right)^{3/2}dx=\int x^2\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x**2*(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(x**2*((a*c + a*d*x + b)/(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \int \left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}} x^2\, dx$$

input
```
integrate(x^2*(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2)*x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(171) = 342$.

Time = 0.20 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.41

$$\int x^2 \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \frac{1}{24}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(2\left(\frac{4\,ax\mathrm{sgn}(dx+c)}{d} - \frac{4\,a^3cd^6\mathrm{sgn}(dx+c)-7\,a}{a^2d^8}\right.\right.$$
$$-\frac{(24\,a^2bc^2\mathrm{sgn}(dx+c)-12\,ab^2c\mathrm{sgn}(dx+c)-b^3\mathrm{sgn}(dx+c))\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,ac}\right.\right.\right.}{}$$

input
```
integrate(x^2*(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*(4*a*x*sgn(d*x +
 c)/d - (4*a^3*c*d^6*sgn(d*x + c) - 7*a^2*b*d^6*sgn(d*x + c))/(a^2*d^8))*x
 + (8*a^3*c^2*d^5*sgn(d*x + c) - 46*a^2*b*c*d^5*sgn(d*x + c) + 3*a*b^2*d^5
*sgn(d*x + c))/(a^2*d^8)) - 1/48*(24*a^2*b*c^2*sgn(d*x + c) - 12*a*b^2*c*s
gn(d*x + c) - b^3*sgn(d*x + c))*log(abs(2*a^2*c^3*d + 6*(sqrt(a*d^2)*x - s
qrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^(3/2)*c^2*abs(d) + 6*(
sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*a*c*d
 + a*b*c^2*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d
*x + b*c))^3*sqrt(a)*abs(d) + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b*d*x + b*c))*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*b*d))/(a^(3/2)*d^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \int x^2 \left(a+\frac{b}{c+d\,x}\right)^{3/2} dx$$

input
```
int(x^2*(a + b/(c + d*x))^(3/2),x)
```

output
```
int(x^2*(a + b/(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.48

$$\int x^2 \left(a + \frac{b}{c+dx}\right)^{3/2} dx = \frac{64\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3c^3 + 64\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3d^3x^3 - 752\sqrt{dx+c}\,\sqrt{ad}$$

input
```
int(x^2*(a+b/(d*x+c))^(3/2),x)
```

output
```
(64*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**3 + 64*sqrt(c + d*x)*sqrt(
a*c + a*d*x + b)*a**3*d**3*x**3 - 752*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*
a**2*b*c**2 - 256*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c*d*x + 112*s
qrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*d**2*x**2 + 24*sqrt(c + d*x)*sqr
t(a*c + a*d*x + b)*a*b**2*c + 24*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**
2*d*x + 576*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sq
rt(b))*a**2*b*c**3 + 576*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt
(c + d*x))/sqrt(b))*a**2*b*c**2*d*x - 288*sqrt(a)*log((sqrt(a*c + a*d*x +
b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**2*c**2 - 288*sqrt(a)*log((sqrt(a
*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**2*c*d*x - 24*sqrt(a
)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**3*c - 24
*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**3
*d*x - 432*sqrt(a)*a**2*b*c**3 - 432*sqrt(a)*a**2*b*c**2*d*x + 96*sqrt(a)*
a*b**2*c**2 + 96*sqrt(a)*a*b**2*c*d*x + 3*sqrt(a)*b**3*c + 3*sqrt(a)*b**3*
d*x)/(192*a**2*d**3*(c + d*x))
```

3.22 $\int x\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 129

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\frac{2bc\sqrt{a+\frac{b}{c+dx}}}{d^2}+\frac{(5b-4ac)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4d^2}+\frac{a(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2d^2}+\frac{3b(b-4ac)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{4\sqrt{a}d^2}$$

output
```
2*b*c*(a+b/(d*x+c))^(1/2)/d^2+1/4*(-4*a*c+5*b)*(d*x+c)*(a+b/(d*x+c))^(1/2)
/d^2+1/2*a*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/d^2+3/4*b*(-4*a*c+b)*arctanh((a+b
/(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\frac{\sqrt{\frac{b+ac+adx}{c+dx}}(13bc-2ac^2+5bdx+2ad^2x^2)}{4d^2}-\frac{3b(-b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{4\sqrt{a}d^2}$$

input
```
Integrate[x*(a + b/(c + d*x))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(13*b*c - 2*a*c^2 + 5*b*d*x + 2*a*d^2*x
^2))/(4*d^2) - (3*b*(-b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]
/Sqrt[a]])/(4*Sqrt[a]*d^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {896, 25, 941, 948, 25, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int dx\left(a+\frac{b}{c+dx}\right)^{3/2}d(c+dx)}{d^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -dx\left(a+\frac{b}{c+dx}\right)^{3/2}d(c+dx)}{d^2}$$

$$\Big\downarrow \; 941$$

$$-\frac{\int (c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \; 948$$

$$\frac{\int -(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int (c+dx)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(1-\frac{c}{c+dx}\right) d\frac{1}{c+dx}}{d^2}$$

$\Big\downarrow$ 87

$$\frac{\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}-\frac{(b-4ac)\int (c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}d\frac{1}{c+dx}}{4a}}{d^2}$$

$\Big\downarrow$ 51

$$\frac{\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}-\frac{(b-4ac)\left(\frac{3}{2}b\int (c+dx)\sqrt{a+\frac{b}{c+dx}}d\frac{1}{c+dx}-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}}{d^2}$$

$\Big\downarrow$ 60

$$\frac{\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}-\frac{(b-4ac)\left(\frac{3}{2}b\left(a\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}+2\sqrt{a+\frac{b}{c+dx}}\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}}{d^2}$$

$\Big\downarrow$ 73

$$\frac{\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}-\frac{(b-4ac)\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{b}+2\sqrt{a+\frac{b}{c+dx}}\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}}{d^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{5/2}}{2a}-\frac{(b-4ac)\left(\frac{3}{2}b\left(2\sqrt{a+\frac{b}{c+dx}}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\right)}{4a}}{d^2}$$

input
```
Int[x*(a + b/(c + d*x))^(3/2),x]
```

output
```
(((c + d*x)^2*(a + b/(c + d*x))^(5/2))/(2*a) - ((b - 4*a*c)*(-((c + d*x)*(
a + b/(c + d*x))^(3/2)) + (3*b*(2*Sqrt[a + b/(c + d*x)] - 2*Sqrt[a]*ArcTan
h[Sqrt[a + b/(c + d*x)]/Sqrt[a]]))/2))/(4*a))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. $2(111) = 222$.

Time = 0.13 (sec) , antiderivative size = 1173, normalized size of antiderivative = 9.09

method	result	size
default	Expression too large to display	1173

input
```
int(x*(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(4*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*d^3*x^3
-12*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2
)+b*d)/(a*d^2)^(1/2))*a*b*c*d^3*x^2-24*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^
2)^(1/2)*a*c*d^2*x^2+12*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2
)^(1/2)*a*c*d^2*x^2-ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2
+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d^3*x^2-24*ln(1/2*
(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*a*b*c^2*d^2*x+4*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x
+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d^3*x^2-48*((a*d*x+a*c+b)
*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a*c^2*d*x+8*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a
*d^2)^(1/2)*b*d^2*x^2+12*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^
2)^(1/2)*a*c^2*d*x+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(
1/2)*b*d^2*x^2-2*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*
d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*d^2*x-12*ln(1/2*(2*
a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)
^(1/2))*a*b*c^3*d+8*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1
/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*d^2*x-24*((a*d*x+a*c+b)*(d*x+c
))^(1/2)*(a*d^2)^(1/2)*a*c^3+16*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2
)*b*c*d*x+4*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*c^
3+4*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*b*c*d*x-l...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.85

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}dx=\left[\frac{3\left(4\,abc-b^2\right)\sqrt{a}\log\left(2\,adx+2\,ac-2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\left(2\,a^2d^2x^2-2\,a\right.}{8\,ad^2}\right.$$

input
```
integrate(x*(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(4*a*b*c - b^2)*sqrt(a)*log(2*a*d*x + 2*a*c - 2*(d*x + c)*sqrt(a)*
sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(2*a^2*d^2*x^2 - 2*a^2*c^2 + 5*
a*b*d*x + 13*a*b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a*d^2), 1/4*(3*(4*
a*b*c - b^2)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*
x + c))/(a*d*x + a*c + b)) + (2*a^2*d^2*x^2 - 2*a^2*c^2 + 5*a*b*d*x + 13*a
*b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a*d^2)]
```

Sympy [F]

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\int x\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(x*((a*c + a*d*x + b)/(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2)*x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(113) = 226$.

Time = 0.19 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.29

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}dx = \frac{1}{4}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(\frac{2\,ax\mathrm{sgn}(dx+c)}{d}-\frac{2\,a^2cd^2\mathrm{sgn}(dx+c)-5\,abd^2\mathrm{sg}}{ad^4}\right.$$
$$+\frac{(4\,abc\mathrm{sgn}(dx+c)-b^2\mathrm{sgn}(dx+c))\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)a^{\frac{3}{2}}c^2\right.\right.}{}$$
$$+\frac{(4\,abcd^2|d|\mathrm{sgn}(dx+c)-b^2d^2|d|\mathrm{sgn}(dx+c))\log(48)}{4\,\sqrt{a}d^5}$$

input
```
integrate(x*(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*a*x*sgn(d*x + c)/
d - (2*a^2*c*d^2*sgn(d*x + c) - 5*a*b*d^2*sgn(d*x + c))/(a*d^4)) + 1/8*(4*
a*b*c*sgn(d*x + c) - b^2*sgn(d*x + c))*log(abs(2*a^2*c^3*d + 6*(sqrt(a*d^2
)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^(3/2)*c^2*abs(d
) + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^
2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^
2 + b*d*x + b*c))^3*sqrt(a)*abs(d) + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2
*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x - sqr
t(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*b*d))/(sqrt(a)*d*abs(d))
 + 1/4*(4*a*b*c*d^2*abs(d)*sgn(d*x + c) - b^2*d^2*abs(d)*sgn(d*x + c))*log
(48)/(sqrt(a)*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}dx = \int x\left(a+\frac{b}{c+d\,x}\right)^{3/2}dx$$

input
```
int(x*(a + b/(c + d*x))^(3/2),x)
```

output
```
int(x*(a + b/(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.20

$$\int x\left(a+\frac{b}{c+dx}\right)^{3/2}dx = \frac{-2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^2c^2+2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^2d^2x^2+13\sqrt{dx+c}\,\sqrt{adx}$$

input
```
int(x*(a+b/(d*x+c))^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*c**2 + 2*sqrt(c + d*x)*sqrt
(a*c + a*d*x + b)*a**2*d**2*x**2 + 13*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*
a*b*c + 5*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b*d*x - 12*sqrt(a)*log((sq
rt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b*c**2 - 12*sqrt(a
)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b*c*d*x +
 3*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b*
*2*c + 3*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(
b))*b**2*d*x + 9*sqrt(a)*a*b*c**2 + 9*sqrt(a)*a*b*c*d*x - sqrt(a)*b**2*c -
 sqrt(a)*b**2*d*x)/(4*a*d**2*(c + d*x))
```

3.23 $\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx = -\frac{2b\sqrt{a+\frac{b}{c+dx}}}{d} + \frac{a(c+dx)\sqrt{a+\frac{b}{c+dx}}}{d} + \frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{d}$$

output
```
-2*b*(a+b/(d*x+c))^(1/2)/d+a*(d*x+c)*(a+b/(d*x+c))^(1/2)/d+3*a^(1/2)*b*arc
tanh((a+b/(d*x+c))^(1/2)/a^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \frac{(-2b+ac+adx)\sqrt{\frac{b+ac+adx}{c+dx}}}{d} + \frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{d}$$

input
```
Integrate[(a + b/(c + d*x))^(3/2),x]
```

output
```
((-2*b + a*c + a*d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/d + (3*Sqrt[a]*b*
ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {239, 773, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \left(a+\frac{b}{c+dx}\right)^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int (c+dx)^2\left(a+\frac{b}{c+dx}\right)^{3/2}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{3}{2}b\int (c+dx)\sqrt{a+\frac{b}{c+dx}}d\frac{1}{c+dx}-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}}{d}$$

$$\Big\downarrow \text{60}$$

$$-\frac{\frac{3}{2}b\left(a\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}+2\sqrt{a+\frac{b}{c+dx}}\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{b}+2\sqrt{a+\frac{b}{c+dx}}\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}}{d}$$

$$\downarrow \text{221}$$

$$-\frac{\frac{3}{2}b\left(2\sqrt{a+\frac{b}{c+dx}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)\right)-(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}}{d}$$

input
```
Int[(a + b/(c + d*x))^(3/2),x]
```

output
```
-((-((c + d*x)*(a + b/(c + d*x))^(3/2)) + (3*b*(2*Sqrt[a + b/(c + d*x)] -
2*Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]]))/2)/d)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 4.49

method	result
default	$-\frac{\left(-3\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)ab\,d^3x^2-6\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}\,a\,d^2x^2-6\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx}}{2\sqrt{}}\right.\right.}{}$

input
```
int((a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)
^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*d^3*x^2-6*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*
d^2)^(1/2)*a*d^2*x^2-6*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))
^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d^2*x-12*((a*d*x+a*c+b)*(d*
x+c))^(1/2)*(a*d^2)^(1/2)*a*c*d*x-3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*
c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c^2*d-6*((a*d*x+
a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a*c^2+4*((a*d*x+a*c+b)*(d*x+c))^(3/2)*
(a*d^2)^(1/2))/d*((a*d*x+a*c+b)/(d*x+c))^(1/2)/(a*d^2)^(1/2)/(d*x+c)/((a*d
*x+a*c+b)*(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.18

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \left[\frac{3\sqrt{a}b\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(adx+ac-2\,b)\sqrt{\frac{adx+ac+b}{dx+c}}}{2\,d}, -\frac{3\sqrt{-a}b\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-(adx+ac-2\,b)\sqrt{\frac{adx+ac+b}{dx+c}}}{d}\right]$$

input
```
integrate((a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*sqrt(a)*b*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x +
a*c + b)/(d*x + c)) + b) + 2*(a*d*x + a*c - 2*b)*sqrt((a*d*x + a*c + b)/(d
*x + c)))/d, -(3*sqrt(-a)*b*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c +
b)/(d*x + c))/(a*d*x + a*c + b)) - (a*d*x + a*c - 2*b)*sqrt((a*d*x + a*c +
 b)/(d*x + c)))/d]
```

Sympy [F]

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \int \left(a+\frac{b}{c+dx}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x+c))**(3/2),x)
```

output
```
Integral((a + b/(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx = \int \left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(70) = 140$.

Time = 0.19 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.25

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2} dx = -\frac{\sqrt{a}b|d|\log(12)\,\mathrm{sgn}(dx+c)}{d^2}$$
$$-\frac{\sqrt{a}b\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)a^{\frac{3}{2}}c^2|d|+6\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\right.\right.}{}$$
$$+\frac{\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}a\,\mathrm{sgn}(dx+c)}{d}$$

input
```
integrate((a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-sqrt(a)*b*abs(d)*log(12)*sgn(d*x + c)/d^2 - 1/2*sqrt(a)*b*log(abs(2*a^2*c
^3*d + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c
))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b*d*x + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*sqrt(a)*abs(d) + 2*(sqrt(a*d^2)*x
- sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*b*c*abs(d) +
(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*b*d)
)*sgn(d*x + c)/abs(d) + sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*
a*sgn(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\frac{3\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{c+d\,x}}}{\sqrt{a}}\right)}{d}-\frac{2\,b\,\sqrt{a+\frac{b}{c+d\,x}}}{d}+\frac{a\,\sqrt{a+\frac{b}{c+d\,x}}\,(c+d\,x)}{d}$$

input
```
int((a + b/(c + d*x))^(3/2),x)
```

output
```
(3*a^(1/2)*b*atanh((a + b/(c + d*x))^(1/2)/a^(1/2)))/d - (2*b*(a + b/(c +
d*x))^(1/2))/d + (a*(a + b/(c + d*x))^(1/2)*(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.89

$$\int \left(a+\frac{b}{c+dx}\right)^{3/2}\,dx=\frac{4\sqrt{dx+c}\,\sqrt{adx+ac+b}\,ac+4\sqrt{dx+c}\,\sqrt{adx+ac+b}\,adx-8\sqrt{dx+c}\,\sqrt{adx+ac+}}{}$$

input
```
int((a+b/(d*x+c))^(3/2),x)
```

output
```
(4*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*c + 4*sqrt(c + d*x)*sqrt(a*c + a*
d*x + b)*a*d*x - 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b + 12*sqrt(a)*log(
(sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b*c + 12*sqrt(a)*
log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b*d*x - 9*sqr
t(a)*b*c - 9*sqrt(a)*b*d*x)/(4*d*(c + d*x))
```

3.24 $\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x} dx = \frac{2b\sqrt{a+\frac{b}{c+dx}}}{c} + 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right) - \frac{2(b+ac)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{c^{3/2}}$$

output
```
2*b*(a+b/(d*x+c))^(1/2)/c+2*a^(3/2)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))-2
*(a*c+b)^(3/2)*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x} dx = \frac{2b\sqrt{\frac{b+ac+adx}{c+dx}}}{c} - \frac{2(-b-ac)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{-b-ac}\sqrt{\frac{b+ac+adx}{c+dx}}}{b+ac}\right)}{c^{3/2}} + 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/(c + d*x))^(3/2)/x,x]
```

output
```
(2*b*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/c - (2*(-b - a*c)^(3/2)*ArcTan[(Sq
rt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/(b + a*c)])/c^(3/2
) + 2*a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 25, 941, 948, 25, 95, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x} \, dx$$

$$\Big\downarrow \; 896$$

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{dx} d(c+dx)$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{dx} d(c+dx)$$

$$\Big\downarrow \; 941$$

$$-\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{(c+dx)\left(\frac{c}{c+dx}-1\right)} d(c+dx)$$

$$\Big\downarrow \; 948$$

$$\int -\frac{(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}}{1-\frac{c}{c+dx}} d\frac{1}{c+dx}$$

$$\Big\downarrow \; 25$$

$$-\int \frac{(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{95}$$

$$\frac{\int -\frac{(c+dx)\left(ca^2+\frac{b(b+2ac)}{c+dx}\right)}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}+\frac{2b\sqrt{a+\frac{b}{c+dx}}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\sqrt{a+\frac{b}{c+dx}}}{c}-\frac{\int \frac{(c+dx)\left(ca^2+\frac{b(b+2ac)}{c+dx}\right)}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}$$

$$\Big\downarrow \text{174}$$

$$\frac{2b\sqrt{a+\frac{b}{c+dx}}}{c}-\frac{a^2c\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}+(ac+b)^2\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}$$

$$\Big\downarrow \text{73}$$

$$\frac{2b\sqrt{a+\frac{b}{c+dx}}}{c}-\frac{\frac{2a^2c\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{b}+\frac{2(ac+b)^2\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{b}}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2b\sqrt{a+\frac{b}{c+dx}}}{c}-\frac{\frac{2(ac+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}-2a^{3/2}c\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{c}$$

input
```
Int[(a + b/(c + d*x))^(3/2)/x,x]
```

output
```
(2*b*Sqrt[a + b/(c + d*x)])/c - (-2*a^(3/2)*c*ArcTanh[Sqrt[a + b/(c + d*x)
]/Sqrt[a]] + (2*(b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sq
rt[b + a*c]])/Sqrt[c])/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2583 vs. $2(80) = 160$.

Time = 0.14 (sec) , antiderivative size = 2584, normalized size of antiderivative = 26.37

method	result	size
default	Expression too large to display	2584

input
```
int((a+b/(d*x+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-3*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+
c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c^3*d+3*ln(1/2*(2*a*d^2*x+
2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(
a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a*b*c*d^3*x^2-4*(a*d^2)^(1/2)*ln((2*a*d*x*
c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^
(1/2)+2*b*c)/x)*a*b*c^2*d^2*x^2-3*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*
c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*
c*d^3*x^2+2*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b
*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^2*c*d^2*x+
2*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*
(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a^2*c^2*d^3*x^2+4*(a*d
^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*
c^2*d*x-12*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a
*c^2*d*x+4*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c
+b)*c)^(1/2)*b*c*d*x+ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^
2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^2
*c^2*d-2*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*
d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*b^2*c*d^2*x^2+2*(a*d^2*
x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*b*d^2
*x^2+3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 591, normalized size of antiderivative = 6.03

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x}\,dx = \left[\frac{a^{\frac{3}{2}}c\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+(ac+b)\sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^{2}}{}\right.}{c}\right.$$

$$-\frac{2\sqrt{-a}ac\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-(ac+b)\sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^{2}+(2\,ac+b)dx+2\,bc-2\,(cdx+c^{2})\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{\frac{ac+b}{c}}}{x}\right.}{c}$$

$$-\frac{2\left(\sqrt{-a}ac\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-(ac+b)\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{c\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{-\frac{ac+b}{c}}}{ac+b}\right)-b\sqrt{\frac{adx+ac+b}{dx+c}}\right)}{c}$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x,x, algorithm="fricas")
```

output
```
[(a^(3/2)*c*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c +
b)/(d*x + c)) + b) + (a*c + b)*sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*c +
b)*d*x + 2*b*c - 2*(c*d*x + c^2)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt((a
*c + b)/c))/x) + 2*b*sqrt((a*d*x + a*c + b)/(d*x + c)))/c, -(2*sqrt(-a)*a*
c*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c
 + b)) - (a*c + b)*sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b
*c - 2*(c*d*x + c^2)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt((a*c + b)/c))/
x) - 2*b*sqrt((a*d*x + a*c + b)/(d*x + c)))/c, (a^(3/2)*c*log(2*a*d*x + 2*
a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(a*c
+ b)*sqrt(-(a*c + b)/c)*arctan(c*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt(-(
a*c + b)/c)/(a*c + b)) + 2*b*sqrt((a*d*x + a*c + b)/(d*x + c)))/c, -2*(sqr
t(-a)*a*c*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d
*x + a*c + b)) - (a*c + b)*sqrt(-(a*c + b)/c)*arctan(c*sqrt((a*d*x + a*c +
 b)/(d*x + c))*sqrt(-(a*c + b)/c)/(a*c + b)) - b*sqrt((a*d*x + a*c + b)/(d
*x + c)))/c]
```

Sympy [F]

$$\int \frac{\left(a + \frac{b}{c+dx}\right)^{3/2}}{x} \, dx = \int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}{x} \, dx$$

input
```
integrate((a+b/(d*x+c))**(3/2)/x,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(a + \frac{b}{c+dx}\right)^{3/2}}{x} \, dx = \int \frac{\left(a + \frac{b}{dx+c}\right)^{\frac{3}{2}}}{x} \, dx$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a + \frac{b}{c+dx}\right)^{3/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x}\right)^{3/2}}{x}\,dx$$

input
```
int((a + b/(c + d*x))^(3/2)/x,x)
```

output
```
int((a + b/(c + d*x))^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 753, normalized size of antiderivative = 7.68

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x}\,dx=\text{Too large to display}$$

input
```
int((a+b/(d*x+c))^(3/2)/x,x)
```

output

```
(2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b*c + sqrt(c)*sqrt(a*c + b)*log(sqr
t(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + s
qrt(a)*sqrt(c + d*x))*a*c**2 + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x
+ b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c
+ d*x))*a*c*d*x + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2
*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b*c +
 sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*
sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b*d*x + sqrt(c)*sqrt(a
*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) +
 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*c**2 + sqrt(c)*sqrt(a*c + b)*log(sq
rt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) +
sqrt(a)*sqrt(c + d*x))*a*c*d*x + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*
x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(
c + d*x))*b*c + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*s
qrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b*d*x -
 sqrt(c)*sqrt(a*c + b)*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) +
 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*d*x)*a*c**2 - sqrt(c)*sqrt(a*c + b)
*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqr
t(a*c + b) + 2*a*d*x)*a*c*d*x - sqrt(c)*sqrt(a*c + b)*log(2*sqrt(a)*sqrt(c
 + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*d...
```

3.25 $\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx = -\frac{2bd\sqrt{a+\frac{b}{c+dx}}}{c^2} - \frac{(b+ac)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{c^2x} + \frac{3b\sqrt{b+ac}\,d\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{c^{5/2}}$$

output

```
-2*b*d*(a+b/(d*x+c))^(1/2)/c^2-(a*c+b)*(d*x+c)*(a+b/(d*x+c))^(1/2)/c^2/x+3
*b*(a*c+b)^(1/2)*d*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/c^(5
/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx = -\frac{\sqrt{\frac{b+ac+adx}{c+dx}}\,(ac(c+dx)+b(c+3dx))}{c^2x} + \frac{3b\sqrt{-b-ac}\,d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{c^{5/2}}$$

input
```
Integrate[(a + b/(c + d*x))^(3/2)/x^2,x]
```

output
```
-((Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(a*c*(c + d*x) + b*(c + 3*d*x)))/(c^2
*x)) + (3*b*Sqrt[-b - a*c]*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d
*x)])/Sqrt[-b - a*c]])/c^(5/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {896, 941, 946, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 896}$$

$$d\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$d\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{ 946}$$

$$-d\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 51}$$

$$-d\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\int \frac{\sqrt{a+\frac{b}{c+dx}}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{2c}\right)$$

$$\Big\downarrow \text{60}$$

$$-d\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{(ac+b)\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{2c}\right)$$

$$\Big\downarrow \text{73}$$

$$-d\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{2(ac+b)\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{bc}-\frac{2\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$-d\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{2\sqrt{ac+b}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{c^{3/2}}-\frac{2\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{2c}\right)$$

input
```
Int[(a + b/(c + d*x))^(3/2)/x^2,x]
```

output
```
-(d*((a + b/(c + d*x))^(3/2)/(c*(1 - c/(c + d*x))) - (3*b*((-2*Sqrt[a + b/
(c + d*x)])/c + (2*Sqrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x)])/S
qrt[b + a*c]])/c^(3/2)))/(2*c))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2348 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 2349, normalized size of antiderivative = 22.37

method	result	size
default	Expression too large to display	2349

input
```
int((a+b/(d*x+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c
)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*d^2*x^2-3*(a*d^2)^(1/2)*ln((2*a*d*
x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c
)^(1/2)+2*b*c)/x)*a*b*c^2*d^3*x^3-3*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*
a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*
b*c*d^4*x^3-6*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((
a*c+b)*c)^(1/2)*a*c*d^3*x^3+6*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c
*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^
(1/2)*a*b*c^2*d^3*x^2-6*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c+b)*(d*
x+c))^(1/2)*a*c*d^3*x^3-6*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*
c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a*b*c^
3*d^2*x^2-6*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(
d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c^2*d^3*x^2-6*(a*d^2*x
^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*c^2*
d^2*x^2+3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c
)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a*b*c*d^4*x^3-
2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1
/2)*a*c^3*d*x+4*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*
((a*c+b)*c)^(1/2)*b*c*d^2*x^2-6*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*
c+b)*(d*x+c))^(1/2)*a*c^3*d*x-8*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.27

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx = \left[\frac{3\,bdx\sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc+2\,(cdx+c^2)\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{\frac{ac+b}{c}}}{x}\right)-2\,(ac^2+(ac+3}{2\,c^2x}\right.$$

$$\left.-\frac{3\,bdx\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{c\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{-\frac{ac+b}{c}}}{ac+b}\right)+(ac^2+(ac+3\,b)dx+bc)\sqrt{\frac{adx+ac+b}{dx+c}}}{c^2x}\right]$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(3*b*d*x*sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c +
2*(c*d*x + c^2)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt((a*c + b)/c))/x) -
2*(a*c^2 + (a*c + 3*b)*d*x + b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(c^2*
x), -(3*b*d*x*sqrt(-(a*c + b)/c)*arctan(c*sqrt((a*d*x + a*c + b)/(d*x + c)
)*sqrt(-(a*c + b)/c)/(a*c + b)) + (a*c^2 + (a*c + 3*b)*d*x + b*c)*sqrt((a*
d*x + a*c + b)/(d*x + c)))/(c^2*x)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c))**(3/2)/x**2,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x))^(3/2)/x^2,x)
```

output
```
int((a + b/(c + d*x))^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 928, normalized size of antiderivative = 8.84

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
int((a+b/(d*x+c))^(3/2)/x^2,x)
```

output
```
( - 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*c**4 - 8*sqrt(c + d*x)*sqrt
(a*c + a*d*x + b)*a**2*c**3*d*x - 14*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a
*b*c**3 - 30*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b*c**2*d*x - 6*sqrt(c +
 d*x)*sqrt(a*c + a*d*x + b)*b**2*c**2 - 18*sqrt(c + d*x)*sqrt(a*c + a*d*x
+ b)*b**2*c*d*x - 12*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqr
t(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*
b*c**2*d*x - 12*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*s
qrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b*c*d
**2*x**2 - 9*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt
(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**2*c*d*x
 - 9*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt
(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**2*d**2*x**2 - 1
2*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)
*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b*c**2*d*x - 12*sqr
t(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt
(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b*c*d**2*x**2 - 9*sqrt(c
)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*
c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**2*c*d*x - 9*sqrt(c)*sqrt(a
*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) +
 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**2*d**2*x**2 + 12*sqrt(c)*sqrt(a...
```

3.26 $\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 162

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx=\frac{2bd^2\sqrt{a+\frac{b}{c+dx}}}{c^3}+\frac{(9b+4ac)d(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4c^3x}-\frac{(b+ac)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2c^3x^2}-\frac{3b(5b+4ac)d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{4c^{7/2}\sqrt{b+ac}}$$

output
```
2*b*d^2*(a+b/(d*x+c))^(1/2)/c^3+1/4*(4*a*c+9*b)*d*(d*x+c)*(a+b/(d*x+c))^(1
/2)/c^3/x-1/2*(a*c+b)*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/c^3/x^2-3/4*b*(4*a*c+5
*b)*d^2*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/c^(7/2)/(a*c+b)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx=\frac{\sqrt{\frac{b+ac+adx}{c+dx}}\left(-2bc^2-2ac^3+5bcdx+15bd^2x^2+2acd^2x^2\right)}{4c^3x^2}+\frac{3b(5b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{4c^{7/2}\sqrt{-b-ac}}$$

input
```
Integrate[(a + b/(c + d*x))^(3/2)/x^3,x]
```

output
```
(Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(-2*b*c^2 - 2*a*c^3 + 5*b*c*d*x + 15*b*
d^2*x^2 + 2*a*c*d^2*x^2))/(4*c^3*x^2) + (3*b*(5*b + 4*a*c)*d^2*ArcTan[(Sqr
t[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(4*c^(7/2)*Sqrt[-
b - a*c])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.526$, Rules used = {896, 25, 941, 948, 25, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\int -\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$-d^2\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \text{ 948}$$

$$d^2\int -\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 87}$$

$$d^2\left(\frac{(4ac+5b)\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4c(ac+b)}-\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{ 51}$$

$$d^2\left(\frac{(4ac+5b)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\int \frac{\sqrt{a+\frac{b}{c+dx}}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{2c}\right)}{4c(ac+b)}-\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{ 60}$$

$$d^2\left(\frac{(4ac+5b)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{(ac+b)\int\frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{2c}\right)}{4c(ac+b)}-\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$\downarrow$ 73

$$d^2\left(\frac{(4ac+5b)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{2(ac+b)\int\frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{bc}-\frac{2\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{2c}\right)}{4c(ac+b)}-\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$\downarrow$ 221

$$d^2\left(\frac{(4ac+5b)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{2\sqrt{ac+b}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{c^{3/2}}-\frac{2\sqrt{a+\frac{b}{c+dx}}}{c}\right)}{2c}\right)}{4c(ac+b)}-\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

input
```
Int[(a + b/(c + d*x))^(3/2)/x^3,x]
```

output
```
d^2*(-1/2*(a + b/(c + d*x))^(5/2)/(c*(b + a*c)*(1 - c/(c + d*x))^2) + ((5*
b + 4*a*c)*((a + b/(c + d*x))^(3/2)/(c*(1 - c/(c + d*x))) - (3*b*((-2*Sqrt
[a + b/(c + d*x)])/c + (2*Sqrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d
*x)])/Sqrt[b + a*c]])/c^(3/2)))/(2*c)))/(4*c*(b + a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3881 vs. $2(140) = 280$.

Time = 0.14 (sec) , antiderivative size = 3882, normalized size of antiderivative = 23.96

method	result	size
default	Expression too large to display	3882

input
```
int((a+b/(d*x+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(24*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x
+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))
*b^3*c*d^4*x^3-18*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2
)*((a*c+b)*c)^(1/2)*b*d^3*x^3-12*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*
a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*
c)^(1/2)*b^3*c^2*d^3*x^2-32*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a
*d^2)^(1/2)*((a*c+b)*c)^(1/2)*b*c*d^2*x^2-6*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d
*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*b^2*c^2*d^2*x^2-16*(a*d^2)^(
1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(3/2)*b*c*d^2*x^2+24*(a*d^2
)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*b^2*c^2*d^2*x^2-10
*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/
2)*b*c^2*d*x-12*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*
((a*c+b)*c)^(1/2)*a*c*d^3*x^3+12*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/
2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*c^4*d^2*x^2-24*ln(1/2*(2*a*d^2*x+2*
a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*
d^2)^(1/2))*((a*c+b)*c)^(1/2)*a*b^2*c^3*d^3*x^2+24*(a*d^2)^(1/2)*((a*c+b)*
c)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a^2*c^4*d^2*x^2+27*(a*d^2)^(1/2)*ln
((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b
*d*x+b*c)^(1/2)+2*b*c)/x)*a*b^2*c^4*d^2*x^2+15*(a*d^2)^(1/2)*ln((2*a*d*x*c
+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.50

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,(4\,abc+5\,b^2)\sqrt{ac^2+bc}d^2x^2\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,\sqrt{ac^2+bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{x}\right)-}{8\,(}\right.$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^2*log(-(2*a*c^2 + (2*a*c
 + b)*d*x + 2*b*c - 2*sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(
d*x + c)))/x) - 2*(2*a^2*c^5 + 4*a*b*c^4 + 2*b^2*c^3 - (2*a^2*c^3 + 17*a*b
*c^2 + 15*b^2*c)*d^2*x^2 - 5*(a*b*c^3 + b^2*c^2)*d*x)*sqrt((a*d*x + a*c +
b)/(d*x + c)))/((a*c^5 + b*c^4)*x^2), 1/4*(3*(4*a*b*c + 5*b^2)*sqrt(-a*c^2
 - b*c)*d^2*x^2*arctan(sqrt(-a*c^2 - b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)
/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) - (2*a^2*c^5 + 4*a*b*c^4 + 2*b^2*c^3
- (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^2 - 5*(a*b*c^3 + b^2*c^2)*d*x)
*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a*c^5 + b*c^4)*x^2)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))**(3/2)/x**3,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x))^(3/2)/x^3,x)
```

output
```
int((a + b/(c + d*x))^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 1598, normalized size of antiderivative = 9.86

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b/(d*x+c))^(3/2)/x^3,x)
```

output
```
( - 32*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**6 + 32*sqrt(c + d*x)*sq
rt(a*c + a*d*x + b)*a**3*c**4*d**2*x**2 - 84*sqrt(c + d*x)*sqrt(a*c + a*d*
x + b)*a**2*b*c**5 + 80*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c**4*d*
x + 292*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c**3*d**2*x**2 - 72*sqr
t(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**2*c**4 + 130*sqrt(c + d*x)*sqrt(a*c
+ a*d*x + b)*a*b**2*c**3*d*x + 410*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b
**2*c**2*d**2*x**2 - 20*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**3*c**3 + 50
*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**3*c**2*d*x + 150*sqrt(c + d*x)*sqr
t(a*c + a*d*x + b)*b**3*c*d**2*x**2 + 96*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*
c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(
a)*sqrt(c + d*x))*a**2*b*c**3*d**2*x**2 + 96*sqrt(c)*sqrt(a*c + b)*log(sqr
t(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + s
qrt(a)*sqrt(c + d*x))*a**2*b*c**2*d**3*x**3 + 180*sqrt(c)*sqrt(a*c + b)*lo
g(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b
) + sqrt(a)*sqrt(c + d*x))*a*b**2*c**2*d**2*x**2 + 180*sqrt(c)*sqrt(a*c +
b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*
c + b) + sqrt(a)*sqrt(c + d*x))*a*b**2*c*d**3*x**3 + 75*sqrt(c)*sqrt(a*c +
 b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a
*c + b) + sqrt(a)*sqrt(c + d*x))*b**3*c*d**2*x**2 + 75*sqrt(c)*sqrt(a*c +
b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2...
```

3.27 $\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 234

$$\begin{aligned}\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx = &-\frac{2bd^3\sqrt{a+\frac{b}{c+dx}}}{c^4} \\ &-\frac{\left(29b^2+36abc+8a^2c^2\right)d^2(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8c^4(b+ac)x} \\ &+\frac{(19b+12ac)d(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12c^4x^2}-\frac{(b+ac)(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3c^4x^3} \\ &+\frac{b\left(35b^2+60abc+24a^2c^2\right)d^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{8c^{9/2}(b+ac)^{3/2}}\end{aligned}$$

output

```
-2*b*d^3*(a+b/(d*x+c))^(1/2)/c^4-1/8*(8*a^2*c^2+36*a*b*c+29*b^2)*d^2*(d*x+
c)*(a+b/(d*x+c))^(1/2)/c^4/(a*c+b)/x+1/12*(12*a*c+19*b)*d*(d*x+c)^2*(a+b/(
d*x+c))^(1/2)/c^4/x^2-1/3*(a*c+b)*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/c^4/x^3+1/
8*b*(24*a^2*c^2+60*a*b*c+35*b^2)*d^3*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(
a*c+b)^(1/2))/c^(9/2)/(a*c+b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx = -\frac{\sqrt{\frac{b+ac+adx}{c+dx}}\left(8a^2c^2(c^3+d^3x^3)+2abc(8c^3-7c^2dx+16cd^2x^2+55d^3x^3)+b^2(8c^3-14c^2dx+35cd^2x^2+10\right.}{24c^4(b+ac)x^3} + \frac{b(35b^2+60abc+24a^2c^2)\,d^3\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{8c^{9/2}(-b-ac)^{3/2}}$$

input
```
Integrate[(a + b/(c + d*x))^(3/2)/x^4,x]
```

output
```
-1/24*(Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(8*a^2*c^2*(c^3 + d^3*x^3) + 2*a*
b*c*(8*c^3 - 7*c^2*d*x + 16*c*d^2*x^2 + 55*d^3*x^3) + b^2*(8*c^3 - 14*c^2*
d*x + 35*c*d^2*x^2 + 105*d^3*x^3)))/(c^4*(b + a*c)*x^3) + (b*(35*b^2 + 60*
a*b*c + 24*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])
/Sqrt[-b - a*c]])/(8*c^(9/2)*(-b - a*c)^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 941, 948, 100, 27, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{896}$$

$$d^3\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{d^4x^4}d(c+dx)$$

$$\Big\downarrow \text{941}$$

$$d^3\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{(c+dx)^4\left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \text{948}$$

$$-d^3\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{(c+dx)^2\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{100}$$

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}\left(5b+6ac+\frac{6c(b+ac)}{c+dx}\right)}{2\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3c^2(ac+b)}\right)$$

$$\Big\downarrow \text{27}$$

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}\left(5b+6ac+\frac{6c(b+ac)}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \text{87}$$

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+11b)\left(a+\frac{b}{c+dx}\right)^{5/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(24a^2c^2+60abc+35b^2)\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4(ac+b)}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \text{51}$$

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+11b)\left(a+\frac{b}{c+dx}\right)^{5/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(24a^2c^2+60abc+35b^2)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\int\frac{\sqrt{a+\frac{b}{c+dx}}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{2c}\right)}{4(ac+b)}}{6c^2(ac+b)}\right)$$

$\downarrow$ 60

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+11b)\left(a+\frac{b}{c+dx}\right)^{5/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(24a^2c^2+60abc+35b^2)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{(ac+b)\int\frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+\cdots}\right)}\cdots}{c}\cdots\right)}{2c}\cdots\right)}{4(ac+b)}}{6c^2(ac+b)}\cdots\right.$$

$\downarrow$ 73

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+11b)\left(a+\frac{b}{c+dx}\right)^{5/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(24a^2c^2+60abc+35b^2)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{2(ac+b)\int\frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+\cdots}\cdots}{bc}\cdots\right)}{2c}\cdots\right)}{4(ac+b)}}{6c^2(ac+b)}\cdots\right.$$

$\downarrow$ 221

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{5/2}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+11b)\left(a+\frac{b}{c+dx}\right)^{5/2}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(\frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{2\sqrt{ac+b}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a\cdots}}{\sqrt{\cdots}}\right)}{c^{3/2}}\cdots\right)}{2c\cdots}\right)}{4(ac+b)\cdots}}{6c^2(ac+b)\cdots}\right.$$

input
```
Int[(a + b/(c + d*x))^(3/2)/x^4,x]
```

output
```
-(d^3*((a + b/(c + d*x))^(5/2)/(3*c^2*(b + a*c)*(1 - c/(c + d*x))^3) - (((
11*b + 12*a*c)*(a + b/(c + d*x))^(5/2))/(2*(b + a*c)*(1 - c/(c + d*x))^2)
- ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*((a + b/(c + d*x))^(3/2)/(c*(1 - c/(c
+ d*x))) - (3*b*((-2*Sqrt[a + b/(c + d*x)])/c + (2*Sqrt[b + a*c]*ArcTanh[(
Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/c^(3/2)))/(2*c)))/(4*(b + a
*c)))/(6*c^2*(b + a*c))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5517 vs. $2(208) = 416$.

Time = 0.16 (sec) , antiderivative size = 5518, normalized size of antiderivative = 23.58

method	result	size
default	Expression too large to display	5518

input
```
int((a+b/(d*x+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.48

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx = \left[\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)\sqrt{ac^2+bc}d^3x^3\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc+2\,\sqrt{ac^2+bc}(dx+c}{x}\right.}{}\right.$$

$$-\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)\sqrt{-ac^2-bc}d^3x^3\arctan\left(\frac{\sqrt{-ac^2-bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{acdx+ac^2+bc}\right)+(8\,a^3c^7+24\,a^2bc^6+2}{}$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^3*lo
g(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c + 2*sqrt(a*c^2 + b*c)*(d*x + c)*sqrt
((a*d*x + a*c + b)/(d*x + c)))/x) - 2*(8*a^3*c^7 + 24*a^2*b*c^6 + 24*a*b^2
*c^5 + 8*b^3*c^4 + (8*a^3*c^4 + 118*a^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)
*d^3*x^3 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^2 - 14*(a^2*b*
c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^2
*c^7 + 2*a*b*c^6 + b^2*c^5)*x^3), -1/24*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35
*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^3*arctan(sqrt(-a*c^2 - b*c)*(d*x + c)*sqrt(
(a*d*x + a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) + (8*a^3*c^7 + 24*a^
2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*a^3*c^4 + 118*a^2*b*c^3 + 215*a*b^
2*c^2 + 105*b^3*c)*d^3*x^3 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^
2*x^2 - 14*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x)*sqrt((a*d*x + a*c + b)
/(d*x + c)))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^3)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))**(3/2)/x**4,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a + b/(c + d*x))^(3/2)/x^4,x)
```

output
```
int((a + b/(c + d*x))^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 19.73 (sec) , antiderivative size = 2266, normalized size of antiderivative = 9.68

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^{3/2}}{x^4}\,dx=\text{Too large to display}$$

input
```
int((a+b/(d*x+c))^(3/2)/x^4,x)
```

output

```
( - 192*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*c**8 - 192*sqrt(c + d*x)*
sqrt(a*c + a*d*x + b)*a**4*c**5*d**3*x**3 - 688*sqrt(c + d*x)*sqrt(a*c + a
*d*x + b)*a**3*b*c**7 + 336*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**
6*d*x - 768*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**5*d**2*x**2 - 29
44*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**4*d**3*x**3 - 912*sqrt(c
+ d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*c**6 + 868*sqrt(c + d*x)*sqrt(a*c +
 a*d*x + b)*a**2*b**2*c**5*d*x - 2056*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*
a**2*b**2*c**4*d**2*x**2 - 6812*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b
**2*c**3*d**3*x**3 - 528*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**3*c**5 +
 728*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**3*c**4*d*x - 1778*sqrt(c + d
*x)*sqrt(a*c + a*d*x + b)*a*b**3*c**3*d**2*x**2 - 5530*sqrt(c + d*x)*sqrt(
a*c + a*d*x + b)*a*b**3*c**2*d**3*x**3 - 112*sqrt(c + d*x)*sqrt(a*c + a*d*
x + b)*b**4*c**4 + 196*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**4*c**3*d*x -
 490*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**4*c**2*d**2*x**2 - 1470*sqrt(c
 + d*x)*sqrt(a*c + a*d*x + b)*b**4*c*d**3*x**3 - 864*sqrt(c)*sqrt(a*c + b)
*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c
+ b) + sqrt(a)*sqrt(c + d*x))*a**3*b*c**4*d**3*x**3 - 864*sqrt(c)*sqrt(a*c
 + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2
*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**3*b*c**3*d**4*x**4 - 2664*sqrt(c)*sq
rt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c...
```

3.28 $\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx=\frac{5\sqrt{x}\sqrt{1+2x}}{32\sqrt{2}}-\frac{5x^{3/2}\sqrt{1+2x}}{24\sqrt{2}}+\frac{x^{5/2}\sqrt{1+2x}}{3\sqrt{2}}-\frac{5}{64}\text{arcsinh}\left(\sqrt{2}\sqrt{x}\right)$$

output
```
5/64*x^(1/2)*(1+2*x)^(1/2)*2^(1/2)-5/48*x^(3/2)*(1+2*x)^(1/2)*2^(1/2)+1/6*
x^(5/2)*(1+2*x)^(1/2)*2^(1/2)-5/64*arcsinh(2^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx=\frac{\sqrt{\frac{x}{1+2x}}\left(\sqrt{2}\sqrt{x}(15+10x-8x^2+64x^3)+15\sqrt{1+2x}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{1+2x}\right)\right)}{192\sqrt{x}}$$

input
```
Integrate[x^2*Sqrt[1 - (1 + 2*x)^(-1)],x]
```

output
```
(Sqrt[x/(1 + 2*x)]*(Sqrt[2]*Sqrt[x]*(15 + 10*x - 8*x^2 + 64*x^3) + 15*Sqrt
[1 + 2*x]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[1 + 2*x]]))/(192*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 798, 51, 51, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{1-\frac{1}{2x+1}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{8}\int (2x+1)^2\left(1-\frac{1}{2x+1}\right)^{5/2}d(2x+1)$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{8}\int 4\sqrt{2}(-x)^{5/2}(2x+1)^4d\frac{1}{2x+1}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{8}\left(\frac{5}{6}\int 2\sqrt{2}(-x)^{3/2}(2x+1)^3d\frac{1}{2x+1}+\frac{4}{3}\sqrt{2}(-x)^{5/2}(2x+1)^3\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{8}\left(\frac{5}{6}\left(-\frac{3}{4}\int \sqrt{2}\sqrt{-x}(2x+1)^2d\frac{1}{2x+1}-\sqrt{2}(-x)^{3/2}(2x+1)^2\right)+\frac{4}{3}\sqrt{2}(-x)^{5/2}(2x+1)^3\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{8}\left(\frac{5}{6}\left(-\frac{3}{4}\left(-\frac{1}{2}\int \frac{2x+1}{\sqrt{2}\sqrt{-x}}d\frac{1}{2x+1}-\sqrt{2}\sqrt{-x}(2x+1)\right)-\sqrt{2}(-x)^{3/2}(2x+1)^2\right)+\frac{4}{3}\sqrt{2}(-x)^{5/2}(2x+1)^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(\frac{5}{6}\left(-\frac{3}{4}\left(\int \frac{1}{1-\frac{1}{(2x+1)^2}}d\left(\sqrt{2}\sqrt{-x}\right)-\sqrt{2}\sqrt{-x}(2x+1)\right)-\sqrt{2}(-x)^{3/2}(2x+1)^2\right)+\frac{4}{3}\sqrt{2}(-x)^{5/2}(2x+1)^3\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{8}\left(\frac{5}{6}\left(-\frac{3}{4}\left(\operatorname{arctanh}\left(\sqrt{2}\sqrt{-x}\right)-\sqrt{2}\sqrt{-x}(2x+1)\right)-\sqrt{2}(-x)^{3/2}(2x+1)^2\right)+\frac{4}{3}\sqrt{2}(-x)^{5/2}(2x+1)^3\right)$$

input
```
Int[x^2*Sqrt[1 - (1 + 2*x)^(-1)],x]
```

output
```
((4*Sqrt[2]*(-x)^(5/2)*(1 + 2*x)^3)/3 + (5*(-(Sqrt[2]*(-x)^(3/2)*(1 + 2*x)
^2) - (3*(-(Sqrt[2]*Sqrt[-x]*(1 + 2*x)) + ArcTanh[Sqrt[2]*Sqrt[-x]]))/4))/
6)/8
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
trager	$\frac{(1+2x)(32x^2-20x+15)\sqrt{2}\sqrt{\frac{x}{1+2x}}}{192}-\frac{5\ln\left(4x\sqrt{2}\sqrt{\frac{x}{1+2x}}+2\sqrt{2}\sqrt{\frac{x}{1+2x}}+4x+1\right)}{128}$	74
risch	$\frac{(1+2x)(32x^2-20x+15)\sqrt{2}\sqrt{\frac{x}{1+2x}}}{192}-\frac{5\ln\left(\frac{\left(\frac{1}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^2+x}\right)\sqrt{\frac{x}{1+2x}}\sqrt{x(1+2x)}}{128x}$	79
default	$-\frac{\sqrt{2}\sqrt{\frac{x}{1+2x}}(1+2x)\left(-64(2x^2+x)^{\frac{3}{2}}+15\ln\left(\frac{\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2x^2+x}\right)\sqrt{2}+144\sqrt{2x^2+x}\,x-60\sqrt{2x^2+x}\right)}{768\sqrt{x(1+2x)}}$	92

input
```
int(x^2*(1-1/(1+2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(1+2*x)*(32*x^2-20*x+15)*2^(1/2)*(x/(1+2*x))^(1/2)-5/128*ln(4*x*2^(1
/2)*(x/(1+2*x))^(1/2)+2*2^(1/2)*(x/(1+2*x))^(1/2)+4*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx=\frac{1}{192}\sqrt{2}(64\,x^3-8\,x^2+10\,x+15)\sqrt{\frac{x}{2\,x+1}}-\frac{5}{128}\log\left(\sqrt{2}\sqrt{\frac{x}{2\,x+1}}+1\right)+\frac{5}{128}\log\left(\sqrt{2}\sqrt{\frac{x}{2\,x+1}}-1\right)$$

input
```
integrate(x^2*(1-1/(1+2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/192*sqrt(2)*(64*x^3 - 8*x^2 + 10*x + 15)*sqrt(x/(2*x + 1)) - 5/128*log(s
qrt(2)*sqrt(x/(2*x + 1)) + 1) + 5/128*log(sqrt(2)*sqrt(x/(2*x + 1)) - 1)
```

Sympy [F]

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx=\sqrt{2}\int x^2\sqrt{\frac{x}{2x+1}}\,dx$$

input
```
integrate(x**2*(1-1/(1+2*x))**(1/2),x)
```

output
```
sqrt(2)*Integral(x**2*sqrt(x/(2*x + 1)), x)
```

Maxima [F]

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx=\int x^2\sqrt{-\frac{1}{2\,x+1}+1}\,dx$$

input
```
integrate(x^2*(1-1/(1+2*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(-1/(2*x + 1) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx$$
$$=\frac{1}{768}\sqrt{2}\Big(15\sqrt{2}\log\left(\left|-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2+x}\right)-1\right|\right)\mathrm{sgn}(2\,x+1)+4\,(4\,(8\,x\mathrm{sgn}(2\,x+1)-5\,\mathrm{sgn}(2\,x+$$

input
```
integrate(x^2*(1-1/(1+2*x))^(1/2),x, algorithm="giac")
```

output
```
1/768*sqrt(2)*(15*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + x))
 - 1))*sgn(2*x + 1) + 4*(4*(8*x*sgn(2*x + 1) - 5*sgn(2*x + 1))*x + 15*sgn(
2*x + 1))*sqrt(2*x^2 + x))
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx=\int x^2\sqrt{1-\frac{1}{2\,x+1}}\,dx$$

input
```
int(x^2*(1 - 1/(2*x + 1))^(1/2),x)
```

output
```
int(x^2*(1 - 1/(2*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{1-\frac{1}{1+2x}}\,dx=\frac{\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x^2}{6}-\frac{5\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x}{48}+\frac{5\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}}{64}-\frac{5\log\left(\sqrt{2x+1}+\sqrt{x}\,\sqrt{2}\right)}{64}$$

input
```
int(x^2*(1-1/(1+2*x))^(1/2),x)
```

output
```
(32*sqrt(x)*sqrt(2*x + 1)*sqrt(2)*x**2 - 20*sqrt(x)*sqrt(2*x + 1)*sqrt(2)*
x + 15*sqrt(x)*sqrt(2*x + 1)*sqrt(2) - 15*log(sqrt(2*x + 1) + sqrt(x)*sqrt
(2)))/192
```

3.29 $\int x\sqrt{1-\frac{1}{1+2x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx=-\frac{3\sqrt{x}\sqrt{1+2x}}{8\sqrt{2}}+\frac{x^{3/2}\sqrt{1+2x}}{2\sqrt{2}}+\frac{3}{16}\text{arcsinh}\left(\sqrt{2}\sqrt{x}\right)$$

output
```
-3/16*x^(1/2)*(1+2*x)^(1/2)*2^(1/2)+1/4*x^(3/2)*(1+2*x)^(1/2)*2^(1/2)+3/16
*arcsinh(2^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx = \frac{\sqrt{\frac{x}{1+2x}}\left(\sqrt{2}\sqrt{x}(-3-2x+8x^2)-3\sqrt{1+2x}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{1+2x}\right)\right)}{16\sqrt{x}}$$

input
```
Integrate[x*Sqrt[1 - (1 + 2*x)^(-1)],x]
```

output
```
(Sqrt[x/(1 + 2*x)]*(Sqrt[2]*Sqrt[x]*(-3 - 2*x + 8*x^2) - 3*Sqrt[1 + 2*x]*L
og[-(Sqrt[2]*Sqrt[x]) + Sqrt[1 + 2*x]]))/(16*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {896, 798, 51, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{1-\frac{1}{2x+1}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{4}\int (2x+1)\left(1-\frac{1}{2x+1}\right)^{3/2}d(2x+1)$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int 2\sqrt{2}(-x)^{3/2}(2x+1)^3d\frac{1}{2x+1}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{3}{4}\int \sqrt{2}\sqrt{-x}(2x+1)^2d\frac{1}{2x+1}+\sqrt{2}(-x)^{3/2}(2x+1)^2\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(-\frac{1}{2}\int \frac{2x+1}{\sqrt{2}\sqrt{-x}}d\frac{1}{2x+1}-\sqrt{2}\sqrt{-x}(2x+1)\right)+\sqrt{2}(-x)^{3/2}(2x+1)^2\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\int \frac{1}{1-\frac{1}{(2x+1)^2}}d\left(\sqrt{2}\sqrt{-x}\right)-\sqrt{2}\sqrt{-x}(2x+1)\right)+\sqrt{2}(-x)^{3/2}(2x+1)^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\text{arctanh}\left(\sqrt{2}\sqrt{-x}\right)-\sqrt{2}\sqrt{-x}(2x+1)\right)+\sqrt{2}(-x)^{3/2}(2x+1)^2\right)$$

input
```
Int[x*Sqrt[1 - (1 + 2*x)^(-1)],x]
```

output
```
(Sqrt[2]*(-x)^(3/2)*(1 + 2*x)^2 + (3*(-(Sqrt[2]*Sqrt[-x]*(1 + 2*x)) + ArcT
anh[Sqrt[2]*Sqrt[-x]]))/4)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

method	result	size
trager	$\frac{(1+2x)(4x-3)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{16}+\frac{3\ln\left(4x\sqrt{2}\,\sqrt{\frac{x}{1+2x}}+2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}+4x+1\right)}{32}$	69
risch	$\frac{(1+2x)(4x-3)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{16}+\frac{3\ln\left(\frac{\left(\frac{1}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^2+x}\right)\sqrt{\frac{x}{1+2x}}\,\sqrt{x(1+2x)}}{32x}$	74
default	$\frac{\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,(1+2x)\left(3\ln\left(\frac{\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2x^2+x}\right)\sqrt{2}+16\sqrt{2x^2+x}\,x-12\sqrt{2x^2+x}\right)}{64\sqrt{x(1+2x)}}$	81

input
```
int(x*(1-1/(1+2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(1+2*x)*(4*x-3)*2^(1/2)*(x/(1+2*x))^(1/2)+3/32*ln(4*x*2^(1/2)*(x/(1+2
*x))^(1/2)+2*2^(1/2)*(x/(1+2*x))^(1/2)+4*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx=\frac{1}{16}\sqrt{2}\left(8x^2-2x-3\right)\sqrt{\frac{x}{2x+1}}+\frac{3}{32}\log\left(\sqrt{2}\sqrt{\frac{x}{2x+1}}+1\right)-\frac{3}{32}\log\left(\sqrt{2}\sqrt{\frac{x}{2x+1}}-1\right)$$

input
```
integrate(x*(1-1/(1+2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*(8*x^2 - 2*x - 3)*sqrt(x/(2*x + 1)) + 3/32*log(sqrt(2)*sqrt(x
/(2*x + 1)) + 1) - 3/32*log(sqrt(2)*sqrt(x/(2*x + 1)) - 1)
```

Sympy [F]

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx=\sqrt{2}\int x\sqrt{\frac{x}{2x+1}}\,dx$$

input
```
integrate(x*(1-1/(1+2*x))**(1/2),x)
```

output
```
sqrt(2)*Integral(x*sqrt(x/(2*x + 1)), x)
```

Maxima [F]

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx=\int x\sqrt{-\frac{1}{2\,x+1}+1}\,dx$$

input
```
integrate(x*(1-1/(1+2*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(-1/(2*x + 1) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx = -\frac{1}{64}\,\sqrt{2}\Big(3\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2+x}\right)-1\right|\right)\mathrm{sgn}(2\,x+1)-4\,\sqrt{2\,x^2+x}(4\,x\mathrm{sgn}(2\,x+1)-3\,\mathrm{s}$$

input
```
integrate(x*(1-1/(1+2*x))^(1/2),x, algorithm="giac")
```

output
```
-1/64*sqrt(2)*(3*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + x))
- 1))*sgn(2*x + 1) - 4*sqrt(2*x^2 + x)*(4*x*sgn(2*x + 1) - 3*sgn(2*x + 1))
)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx=\int x\,\sqrt{1-\frac{1}{2\,x+1}}\,dx$$

input
```
int(x*(1 - 1/(2*x + 1))^(1/2),x)
```

output
```
int(x*(1 - 1/(2*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int x\sqrt{1-\frac{1}{1+2x}}\,dx=\frac{\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x}{4}-\frac{3\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}}{16}+\frac{3\log\left(\sqrt{2x+1}+\sqrt{x}\,\sqrt{2}\right)}{16}$$

input
```
int(x*(1-1/(1+2*x))^(1/2),x)
```

output
```
(4*sqrt(x)*sqrt(2*x + 1)*sqrt(2)*x - 3*sqrt(x)*sqrt(2*x + 1)*sqrt(2) + 3*l
og(sqrt(2*x + 1) + sqrt(x)*sqrt(2)))/16
```

3.30 $\int \sqrt{1-\frac{1}{1+2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \sqrt{1-\frac{1}{1+2x}}\,dx=\frac{\sqrt{x}\sqrt{1+2x}}{\sqrt{2}}-\frac{1}{2}\text{arcsinh}\left(\sqrt{2}\sqrt{x}\right)$$

output
```
1/2*x^(1/2)*(1+2*x)^(1/2)*2^(1/2)-1/2*arcsinh(2^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

$$\int \sqrt{1-\frac{1}{1+2x}}\,dx=\frac{\sqrt{\frac{x}{1+2x}}\left(\sqrt{2}\sqrt{x}(1+2x)+\sqrt{1+2x}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{1+2x}\right)\right)}{2\sqrt{x}}$$

input
```
Integrate[Sqrt[1 - (1 + 2*x)^(-1)],x]
```

output
```
(Sqrt[x/(1 + 2*x)]*(Sqrt[2]*Sqrt[x]*(1 + 2*x) + Sqrt[1 + 2*x]*Log[-(Sqrt[2
]*Sqrt[x]) + Sqrt[1 + 2*x]]))/(2*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {239, 773, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\frac{1}{2x+1}}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{1}{2}\int \sqrt{1-\frac{1}{2x+1}}d(2x+1)$$

$$\Big\downarrow \text{773}$$

$$-\frac{1}{2}\int \sqrt{2}\sqrt{-x}(2x+1)^2d\frac{1}{2x+1}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{\sqrt{2}\sqrt{-x}}d\frac{1}{2x+1}+\sqrt{2}\sqrt{-x}(2x+1)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\sqrt{2}\sqrt{-x}(2x+1)-\int \frac{1}{1-\frac{1}{(2x+1)^2}}d\left(\sqrt{2}\sqrt{-x}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\sqrt{2}\sqrt{-x}(2x+1)-\text{arctanh}\left(\sqrt{2}\sqrt{-x}\right)\right)$$

input
```
Int[Sqrt[1 - (1 + 2*x)^(-1)],x]
```

output
```
(Sqrt[2]*Sqrt[-x]*(1 + 2*x) - ArcTanh[Sqrt[2]*Sqrt[-x]])/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

method	result	size
trager	$3\left(\frac{1}{6}+\frac{x}{3}\right)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}-\frac{\ln\left(4x\sqrt{2}\,\sqrt{\frac{x}{1+2x}}+2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}+4x+1\right)}{4}$	64
default	$\frac{\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,(1+2x)\left(-\ln\left(\frac{\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2x^2+x}\right)\sqrt{2}+4\sqrt{2x^2+x}\right)}{8\sqrt{x(1+2x)}}$	69
risch	$\frac{(1+2x)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{2}-\frac{\ln\left(\frac{\left(\frac{1}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^2+x}\right)\sqrt{\frac{x}{1+2x}}\,\sqrt{x(1+2x)}}{4x}$	69

input
```
int((1-1/(1+2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3*(1/6+1/3*x)*2^(1/2)*(x/(1+2*x))^(1/2)-1/4*ln(4*x*2^(1/2)*(x/(1+2*x))^(1/
2)+2*2^(1/2)*(x/(1+2*x))^(1/2)+4*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int\sqrt{1-\frac{1}{1+2x}}\,dx=\frac{1}{2}\sqrt{2}(2x+1)\sqrt{\frac{x}{2x+1}}-\frac{1}{4}\log\left(\sqrt{2}\sqrt{\frac{x}{2x+1}}+1\right)+\frac{1}{4}\log\left(\sqrt{2}\sqrt{\frac{x}{2x+1}}-1\right)$$

input
```
integrate((1-1/(1+2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*(2*x + 1)*sqrt(x/(2*x + 1)) - 1/4*log(sqrt(2)*sqrt(x/(2*x + 1)
) + 1) + 1/4*log(sqrt(2)*sqrt(x/(2*x + 1)) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \sqrt{1-\frac{1}{1+2x}}\,dx = \begin{cases} \sqrt{x}\sqrt{x+\frac{1}{2}} - \frac{\operatorname{acosh}\left(\sqrt{2}\sqrt{x+\frac{1}{2}}\right)}{2} & \text{for } \left|x+\frac{1}{2}\right| > \frac{1}{2} \\ \frac{i\operatorname{asin}\left(\sqrt{2}\sqrt{x+\frac{1}{2}}\right)}{2} - \frac{i\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{\sqrt{-x}} + \frac{i\sqrt{x+\frac{1}{2}}}{2\sqrt{-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-1/(1+2*x))**(1/2),x)
```

output
```
Piecewise((sqrt(x)*sqrt(x + 1/2) - acosh(sqrt(2)*sqrt(x + 1/2))/2, Abs(x +
 1/2) > 1/2), (I*asin(sqrt(2)*sqrt(x + 1/2))/2 - I*(x + 1/2)**(3/2)/sqrt(-
x) + I*sqrt(x + 1/2)/(2*sqrt(-x)), True))
```

Maxima [F]

$$\int \sqrt{1-\frac{1}{1+2x}}\,dx = \int \sqrt{-\frac{1}{2\,x+1}+1}\,dx$$

input
```
integrate((1-1/(1+2*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(2*x + 1) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \sqrt{1-\frac{1}{1+2x}}\,dx$$
$$= \frac{1}{8}\sqrt{2}\left(\sqrt{2}\log\left(\left|-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2+x}\right)-1\right|\right)\operatorname{sgn}(2\,x+1)+4\sqrt{2\,x^2+x}\operatorname{sgn}(2\,x+1)\right)$$

input
```
integrate((1-1/(1+2*x))^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + x)) - 1)
)*sgn(2*x + 1) + 4*sqrt(2*x^2 + x)*sgn(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \sqrt{1-\frac{1}{1+2x}}\,dx=-\frac{(2\,x+1)\,\left(\frac{\ln\left(2\,x+\sqrt{{(2\,x+1)}^2-2\,x-1}+\frac{1}{2}\right)}{\sqrt{{(2\,x+1)}^2-2\,x-1}}-2\right)\,\sqrt{1-\frac{1}{2\,x+1}}}{4}$$

input
```
int((1 - 1/(2*x + 1))^(1/2),x)
```

output
```
-((2*x + 1)*(log(2*x + ((2*x + 1)^2 - 2*x - 1)^(1/2) + 1/2)/((2*x + 1)^2 -
 2*x - 1)^(1/2) - 2)*(1 - 1/(2*x + 1))^(1/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \sqrt{1-\frac{1}{1+2x}}\,dx=\frac{\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}}{2}-\frac{\log\left(\sqrt{2x+1}+\sqrt{x}\,\sqrt{2}\right)}{2}$$

input
```
int((1-1/(1+2*x))^(1/2),x)
```

output
```
(sqrt(x)*sqrt(2*x + 1)*sqrt(2) - log(sqrt(2*x + 1) + sqrt(x)*sqrt(2)))/2
```

3.31 $\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 14

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x} dx = 2\text{arcsinh}\left(\sqrt{2}\sqrt{x}\right)$$

output
```
2*arcsinh(2^(1/2)*x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(14) = 28.

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.71

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x} dx = -\frac{2\sqrt{\frac{x}{1+2x}}\sqrt{1+2x}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{1+2x}\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[1 - (1 + 2*x)^(-1)]/x,x]
```

output
```
(-2*Sqrt[x/(1 + 2*x)]*Sqrt[1 + 2*x]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[1 + 2*x]
])/Sqrt[x]
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{2x+1}}}{x} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{1}{(2x+1)\sqrt{1-\frac{1}{2x+1}}}d(2x+1)$$

$$\Big\downarrow \text{ 798}$$

$$-\int \frac{2x+1}{\sqrt{2}\sqrt{-x}}d\frac{1}{2x+1}$$

$$\Big\downarrow \text{ 73}$$

$$2\int \frac{1}{1-\frac{1}{(2x+1)^2}}d\left(\sqrt{2}\sqrt{-x}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\text{arctanh}\left(\sqrt{2}\sqrt{-x}\right)$$

input
```
Int[Sqrt[1 - (1 + 2*x)^(-1)]/x,x]
```

output
```
2*ArcTanh[Sqrt[2]*Sqrt[-x]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

method	result	size
trager	$\ln\left(4x\sqrt{2}\,\sqrt{\frac{x}{1+2x}}+2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}+4x+1\right)$	40
default	$\frac{\sqrt{\frac{x}{1+2x}}\,(1+2x)\ln\left(\frac{\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2x^2+x}\right)}{\sqrt{x(1+2x)}}$	48

input
```
int((1-1/(1+2*x))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(4*x*2^(1/2)*(x/(1+2*x))^(1/2)+2*2^(1/2)*(x/(1+2*x))^(1/2)+4*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x}\,dx = \log\left(\sqrt{2}\sqrt{\frac{x}{2\,x+1}}+1\right) - \log\left(\sqrt{2}\sqrt{\frac{x}{2\,x+1}}-1\right)$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x,x, algorithm="fricas")
```

output
```
log(sqrt(2)*sqrt(x/(2*x + 1)) + 1) - log(sqrt(2)*sqrt(x/(2*x + 1)) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x}\,dx = \sqrt{2}\int \frac{\sqrt{\frac{x}{2x+1}}}{x}\,dx$$

input
```
integrate((1-1/(1+2*x))**(1/2)/x,x)
```

output
```
sqrt(2)*Integral(sqrt(x/(2*x + 1))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x}\,dx = \int \frac{\sqrt{-\frac{1}{2\,x+1}+1}}{x}\,dx$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(2*x + 1) + 1)/x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x}\,dx = -\log\left(\left|-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2+x}\right)-1\right|\right)\operatorname{sgn}(2\,x+1)$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x,x, algorithm="giac")
```

output
```
-log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + x)) - 1))*sgn(2*x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x}\,dx = \int \frac{\sqrt{1-\frac{1}{2\,x+1}}}{x}\,dx$$

input
```
int((1 - 1/(2*x + 1))^(1/2)/x,x)
```

output
```
int((1 - 1/(2*x + 1))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x}\,dx = 2\log\left(\sqrt{2x+1}+\sqrt{x}\,\sqrt{2}\right)$$

input
```
int((1-1/(1+2*x))^(1/2)/x,x)
```

output
```
2*log(sqrt(2*x + 1) + sqrt(x)*sqrt(2))
```

3.32 $\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2} \, dx = -\frac{2\sqrt{2}\sqrt{1+2x}}{\sqrt{x}}$$

output
```
-2*2^(1/2)*(1+2*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2} \, dx = -\frac{2}{\sqrt{\frac{x}{2+4x}}}$$

input
```
Integrate[Sqrt[1 - (1 + 2*x)^(-1)]/x^2,x]
```

output
```
-2/Sqrt[x/(2 + 4*x)]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 941, 281, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{2x+1}}}{x^2} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$2\int \frac{\sqrt{1-\frac{1}{2x+1}}}{4x^2}d(2x+1)$$

$$\Big\downarrow \text{ 941}$$

$$2\int \frac{\sqrt{1-\frac{1}{2x+1}}}{(2x+1)^2\left(\frac{1}{2x+1}-1\right)^2}d(2x+1)$$

$$\Big\downarrow \text{ 281}$$

$$2\int \frac{1}{(2x+1)^2\left(1-\frac{1}{2x+1}\right)^{3/2}}d(2x+1)$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{4}{\sqrt{1-\frac{1}{2x+1}}}$$

input
```
Int[Sqrt[1 - (1 + 2*x)^(-1)]/x^2,x]
```

output
```
-4/Sqrt[1 - (1 + 2*x)^(-1)]
```

Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
orering	$-\frac{2(1+2x)\sqrt{1-\frac{1}{1+2x}}}{x}$	24
gosper	$-\frac{2(1+2x)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{x}$	25
trager	$-\frac{2(1+2x)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{x}$	25
risch	$-\frac{2(1+2x)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{x}$	25
default	$-\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,(1+2x)\sqrt{2x^{2}+x}}{x\sqrt{x(1+2x)}}$	43

input
```
int((1-1/(1+2*x))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/x*(1+2*x)*(1-1/(1+2*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2}\,dx = -\frac{2\sqrt{2}(2x+1)\sqrt{\frac{x}{2x+1}}}{x}$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2*sqrt(2)*(2*x + 1)*sqrt(x/(2*x + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2}\,dx = \sqrt{2}\int \frac{\sqrt{\frac{x}{2x+1}}}{x^2}\,dx$$

input
```
integrate((1-1/(1+2*x))**(1/2)/x**2,x)
```

output
```
sqrt(2)*Integral(sqrt(x/(2*x + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2}\,dx=\int \frac{\sqrt{-\frac{1}{2\,x+1}+1}}{x^2}\,dx$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(2*x + 1) + 1)/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15)=30$.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2}\,dx=2\sqrt{2}\left(\sqrt{2}\mathrm{sgn}(2\,x+1)+\frac{\mathrm{sgn}(2\,x+1)}{\sqrt{2}x-\sqrt{2\,x^2+x}}\right)$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*sqrt(2)*(sqrt(2)*sgn(2*x + 1) + sgn(2*x + 1)/(sqrt(2)*x - sqrt(2*x^2 + x
)))
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2}\,dx=-\frac{4\left(x+\frac{1}{2}\right)\sqrt{1-\frac{1}{2\,x+1}}}{x}$$

input
```
int((1 - 1/(2*x + 1))^(1/2)/x^2,x)
```

output
```
-(4*(x + 1/2)*(1 - 1/(2*x + 1))^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^2}\,dx = \frac{-2\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}-4x}{x}$$

input
```
int((1-1/(1+2*x))^(1/2)/x^2,x)
```

output
```
(2*( - sqrt(x)*sqrt(2*x + 1)*sqrt(2) - 2*x))/x
```

3.33 $\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3} \, dx = -\frac{2\sqrt{2}\sqrt{1+2x}}{3x^{3/2}} + \frac{8\sqrt{2}\sqrt{1+2x}}{3\sqrt{x}}$$

output
```
-2/3*2^(1/2)*(1+2*x)^(1/2)/x^(3/2)+8/3*2^(1/2)*(1+2*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3} \, dx = \frac{2(-1+4x)}{3x\sqrt{\frac{x}{2+4x}}}$$

input
```
Integrate[Sqrt[1 - (1 + 2*x)^(-1)]/x^3,x]
```

output
```
(2*(-1 + 4*x))/(3*x*Sqrt[x/(2 + 4*x)])
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 25, 941, 281, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{2x+1}}}{x^3}dx$$

$$\Big\downarrow \text{ 896}$$

$$4\int \frac{\sqrt{1-\frac{1}{2x+1}}}{8x^3}d(2x+1)$$

$$\Big\downarrow \text{ 25}$$

$$-4\int -\frac{\sqrt{1-\frac{1}{2x+1}}}{8x^3}d(2x+1)$$

$$\Big\downarrow \text{ 941}$$

$$-4\int \frac{\sqrt{1-\frac{1}{2x+1}}}{(2x+1)^3\left(\frac{1}{2x+1}-1\right)^3}d(2x+1)$$

$$\Big\downarrow \text{ 281}$$

$$4\int \frac{1}{(2x+1)^3\left(1-\frac{1}{2x+1}\right)^{5/2}}d(2x+1)$$

$$\Big\downarrow \text{ 798}$$

$$-4\int \frac{1}{4\sqrt{2}(-x)^{5/2}(2x+1)}d\frac{1}{2x+1}$$

$$\Big\downarrow \text{ 53}$$

$$-4\int \left(\frac{1}{4\sqrt{2}(-x)^{5/2}}-\frac{1}{2\sqrt{2}(-x)^{3/2}}\right)d\frac{1}{2x+1}$$

$$\Big\downarrow \text{ 2009}$$

$$-4\left(\frac{1}{3\sqrt{2}(-x)^{3/2}}-\frac{\sqrt{2}}{\sqrt{-x}}\right)$$

input
```
Int[Sqrt[1 - (1 + 2*x)^(-1)]/x^3,x]
```

output
```
-4*(1/(3*Sqrt[2]*(-x)^(3/2)) - Sqrt[2]/Sqrt[-x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

method	result	size
orering	$\frac{2(4x-1)(1+2x)\sqrt{1-\frac{1}{1+2x}}}{3x^2}$	29
gosper	$\frac{2(1+2x)(4x-1)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{3x^2}$	30
trager	$\frac{2(1+2x)(4x-1)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{3x^2}$	30
risch	$\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,(8x^2+2x-1)}{3x^2}$	30
default	$\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,(1+2x)\sqrt{2x^2+x}\,(4x-1)}{3x^2\sqrt{x(1+2x)}}$	48

input
```
int((1-1/(1+2*x))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(4*x-1)*(1+2*x)/x^2*(1-1/(1+2*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3}\,dx = \frac{2\sqrt{2}(8x^2+2x-1)\sqrt{\frac{x}{2x+1}}}{3x^2}$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/3*sqrt(2)*(8*x^2 + 2*x - 1)*sqrt(x/(2*x + 1))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3}\,dx=\sqrt{2}\int \frac{\sqrt{\frac{x}{2x+1}}}{x^3}\,dx$$

input
```
integrate((1-1/(1+2*x))**(1/2)/x**3,x)
```

output
```
sqrt(2)*Integral(sqrt(x/(2*x + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3}\,dx=\int \frac{\sqrt{-\frac{1}{2\,x+1}+1}}{x^3}\,dx$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(2*x + 1) + 1)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(31) = 62.

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3}\,dx=-\frac{2}{3}\sqrt{2}\left(4\sqrt{2}\mathrm{sgn}(2\,x+1)-\frac{3\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2+x}\right)\mathrm{sgn}(2\,x+1)+\mathrm{sgn}(2\,x+1)}{\left(\sqrt{2}x-\sqrt{2\,x^2+x}\right)^3}\right)$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-2/3*sqrt(2)*(4*sqrt(2)*sgn(2*x + 1) - (3*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
+ x))*sgn(2*x + 1) + sgn(2*x + 1))/(sqrt(2)*x - sqrt(2*x^2 + x))^3)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3}\,dx = \frac{2\sqrt{1-\frac{1}{2\,x+1}}\,\left(8\,x^2+2\,x-1\right)}{3\,x^2}$$

input
```
int((1 - 1/(2*x + 1))^(1/2)/x^3,x)
```

output
```
(2*(1 - 1/(2*x + 1))^(1/2)*(2*x + 8*x^2 - 1))/(3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^3}\,dx = \frac{\frac{8\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x}{3}-\frac{2\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}}{3}-\frac{16x^2}{3}}{x^2}$$

input
```
int((1-1/(1+2*x))^(1/2)/x^3,x)
```

output
```
(2*(4*sqrt(x)*sqrt(2*x + 1)*sqrt(2)*x - sqrt(x)*sqrt(2*x + 1)*sqrt(2) - 8*
x**2))/(3*x**2)
```

3.34 $\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4} dx = -\frac{2\sqrt{2}\sqrt{1+2x}}{5x^{5/2}} + \frac{16\sqrt{2}\sqrt{1+2x}}{15x^{3/2}} - \frac{64\sqrt{2}\sqrt{1+2x}}{15\sqrt{x}}$$

output
```
-2/5*2^(1/2)*(1+2*x)^(1/2)/x^(5/2)+16/15*2^(1/2)*(1+2*x)^(1/2)/x^(3/2)-64/
15*2^(1/2)*(1+2*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4} dx = -\frac{2(3-8x+32x^2)}{15x^2\sqrt{\frac{x}{2+4x}}}$$

input
```
Integrate[Sqrt[1 - (1 + 2*x)^(-1)]/x^4,x]
```

output
```
(-2*(3 - 8*x + 32*x^2))/(15*x^2*Sqrt[x/(2 + 4*x)])
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {896, 941, 281, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{2x+1}}}{x^4}dx$$

$$\Big\downarrow \text{ 896}$$

$$8\int \frac{\sqrt{1-\frac{1}{2x+1}}}{16x^4}d(2x+1)$$

$$\Big\downarrow \text{ 941}$$

$$8\int \frac{\sqrt{1-\frac{1}{2x+1}}}{(2x+1)^4\left(\frac{1}{2x+1}-1\right)^4}d(2x+1)$$

$$\Big\downarrow \text{ 281}$$

$$8\int \frac{1}{(2x+1)^4\left(1-\frac{1}{2x+1}\right)^{7/2}}d(2x+1)$$

$$\Big\downarrow \text{ 798}$$

$$-8\int \frac{1}{8\sqrt{2}(-x)^{7/2}(2x+1)^2}d\frac{1}{2x+1}$$

$$\Big\downarrow \text{ 53}$$

$$-8\int \left(\frac{1}{2\sqrt{2}(-x)^{3/2}}-\frac{1}{2\sqrt{2}(-x)^{5/2}}+\frac{1}{8\sqrt{2}(-x)^{7/2}}\right)d\frac{1}{2x+1}$$

$$\Big\downarrow \text{ 2009}$$

$$-8\left(\frac{\sqrt{2}}{\sqrt{-x}}-\frac{\sqrt{2}}{3(-x)^{3/2}}+\frac{1}{10\sqrt{2}(-x)^{5/2}}\right)$$

input
```
Int[Sqrt[1 - (1 + 2*x)^(-1)]/x^4,x]
```

output
```
-8*(1/(10*Sqrt[2]*(-x)^(5/2)) - Sqrt[2]/(3*(-x)^(3/2)) + Sqrt[2]/Sqrt[-x])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

method	result	size
orering	$-\frac{2(32x^2-8x+3)(1+2x)\sqrt{1-\frac{1}{1+2x}}}{15x^3}$	34
gosper	$-\frac{2(1+2x)(32x^2-8x+3)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{15x^3}$	35
trager	$-\frac{2(1+2x)(32x^2-8x+3)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{15x^3}$	35
risch	$-\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,(64x^3+16x^2-2x+3)}{15x^3}$	35
default	$-\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,(1+2x)\sqrt{2x^2+x}\,(32x^2-8x+3)}{15x^3\sqrt{x(1+2x)}}$	53

input
```
int((1-1/(1+2*x))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(32*x^2-8*x+3)*(1+2*x)/x^3*(1-1/(1+2*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4}\,dx = -\frac{2\,\sqrt{2}(64\,x^3+16\,x^2-2\,x+3)\sqrt{\frac{x}{2\,x+1}}}{15\,x^3}$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/15*sqrt(2)*(64*x^3 + 16*x^2 - 2*x + 3)*sqrt(x/(2*x + 1))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4}\,dx = \sqrt{2}\int \frac{\sqrt{\frac{x}{2x+1}}}{x^4}\,dx$$

input
```
integrate((1-1/(1+2*x))**(1/2)/x**4,x)
```

output
```
sqrt(2)*Integral(sqrt(x/(2*x + 1))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4}\,dx = \int \frac{\sqrt{-\frac{1}{2x+1}+1}}{x^4}\,dx$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(2*x + 1) + 1)/x^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(46) = 92.

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4}\,dx$$
$$= \frac{2}{15}\sqrt{2}\left(32\sqrt{2}\text{sgn}(2x+1) + \frac{40\left(\sqrt{2}x-\sqrt{2x^2+x}\right)^2\text{sgn}(2x+1) + 15\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2+x}\right)\text{sgn}(2x+}{\left(\sqrt{2}x-\sqrt{2x^2+x}\right)^5}\right.$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^4,x, algorithm="giac")
```

output
```
2/15*sqrt(2)*(32*sqrt(2)*sgn(2*x + 1) + (40*(sqrt(2)*x - sqrt(2*x^2 + x))^
2*sgn(2*x + 1) + 15*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + x))*sgn(2*x + 1) + 3
*sgn(2*x + 1))/(sqrt(2)*x - sqrt(2*x^2 + x))^5)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4}\,dx = -\frac{2\sqrt{1-\frac{1}{2\,x+1}}\,\left(64\,x^3+16\,x^2-2\,x+3\right)}{15\,x^3}$$

input
```
int((1 - 1/(2*x + 1))^(1/2)/x^4,x)
```

output
```
-(2*(1 - 1/(2*x + 1))^(1/2)*(16*x^2 - 2*x + 64*x^3 + 3))/(15*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^4}\,dx = \frac{-\frac{64\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x^2}{15}+\frac{16\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x}{15}-\frac{2\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}}{5}+\frac{128x^3}{15}}{x^3}$$

input
```
int((1-1/(1+2*x))^(1/2)/x^4,x)
```

output
```
(2*( - 32*sqrt(x)*sqrt(2*x + 1)*sqrt(2)*x**2 + 8*sqrt(x)*sqrt(2*x + 1)*sqr
t(2)*x - 3*sqrt(x)*sqrt(2*x + 1)*sqrt(2) + 64*x**3))/(15*x**3)
```

3.35 $\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx=-\frac{2\sqrt{2}\sqrt{1+2x}}{7x^{7/2}}+\frac{24\sqrt{2}\sqrt{1+2x}}{35x^{5/2}}-\frac{64\sqrt{2}\sqrt{1+2x}}{35x^{3/2}}+\frac{256\sqrt{2}\sqrt{1+2x}}{35\sqrt{x}}$$

output
```
-2/7*2^(1/2)*(1+2*x)^(1/2)/x^(7/2)+24/35*2^(1/2)*(1+2*x)^(1/2)/x^(5/2)-64/
35*2^(1/2)*(1+2*x)^(1/2)/x^(3/2)+256/35*2^(1/2)*(1+2*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx=\frac{2(-5+12x-32x^2+128x^3)}{35x^3\sqrt{\frac{x}{2+4x}}}$$

input
```
Integrate[Sqrt[1 - (1 + 2*x)^(-1)]/x^5,x]
```

output
```
(2*(-5 + 12*x - 32*x^2 + 128*x^3))/(35*x^3*Sqrt[x/(2 + 4*x)])
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {896, 25, 941, 281, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{2x+1}}}{x^5}dx$$

$$\Big\downarrow \text{896}$$

$$16\int \frac{\sqrt{1-\frac{1}{2x+1}}}{32x^5}d(2x+1)$$

$$\Big\downarrow \text{25}$$

$$-16\int -\frac{\sqrt{1-\frac{1}{2x+1}}}{32x^5}d(2x+1)$$

$$\Big\downarrow \text{941}$$

$$-16\int \frac{\sqrt{1-\frac{1}{2x+1}}}{(2x+1)^5\left(\frac{1}{2x+1}-1\right)^5}d(2x+1)$$

$$\Big\downarrow \text{281}$$

$$16\int \frac{1}{(2x+1)^5\left(1-\frac{1}{2x+1}\right)^{9/2}}d(2x+1)$$

$$\Big\downarrow \text{798}$$

$$-16\int \frac{1}{16\sqrt{2}(-x)^{9/2}(2x+1)^3}d\frac{1}{2x+1}$$

$$\Big\downarrow \text{53}$$

$$-16\int \left(-\frac{1}{2\sqrt{2}(-x)^{3/2}}+\frac{3}{4\sqrt{2}(-x)^{5/2}}-\frac{3}{8\sqrt{2}(-x)^{7/2}}+\frac{1}{16\sqrt{2}(-x)^{9/2}}\right)d\frac{1}{2x+1}$$

$$\Big\downarrow \text{2009}$$

$$-16\left(-\frac{\sqrt{2}}{\sqrt{-x}}+\frac{1}{\sqrt{2}(-x)^{3/2}}-\frac{3}{10\sqrt{2}(-x)^{5/2}}+\frac{1}{28\sqrt{2}(-x)^{7/2}}\right)$$

input
```
Int[Sqrt[1 - (1 + 2*x)^(-1)]/x^5,x]
```

output
```
-16*(1/(28*Sqrt[2]*(-x)^(7/2)) - 3/(10*Sqrt[2]*(-x)^(5/2)) + 1/(Sqrt[2]*(-
x)^(3/2)) - Sqrt[2]/Sqrt[-x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

method	result	size
orering	$\frac{2\left(128x^{3}-32x^{2}+12x-5\right)\left(1+2x\right)\sqrt{1-\frac{1}{1+2x}}}{35x^{4}}$	39
gosper	$\frac{2\left(1+2x\right)\left(128x^{3}-32x^{2}+12x-5\right)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{35x^{4}}$	40
trager	$\frac{2\left(1+2x\right)\left(128x^{3}-32x^{2}+12x-5\right)\sqrt{2}\,\sqrt{\frac{x}{1+2x}}}{35x^{4}}$	40
risch	$\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,\left(256x^{4}+64x^{3}-8x^{2}+2x-5\right)}{35x^{4}}$	40
default	$\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+2x}}\,\left(1+2x\right)\sqrt{2x^{2}+x}\,\left(128x^{3}-32x^{2}+12x-5\right)}{35x^{4}\sqrt{x\left(1+2x\right)}}$	58

input
```
int((1-1/(1+2*x))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/35*(128*x^3-32*x^2+12*x-5)*(1+2*x)/x^4*(1-1/(1+2*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx = \frac{2\,\sqrt{2}(256\,x^4+64\,x^3-8\,x^2+2\,x-5)\sqrt{\frac{x}{2\,x+1}}}{35\,x^4}$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
2/35*sqrt(2)*(256*x^4 + 64*x^3 - 8*x^2 + 2*x - 5)*sqrt(x/(2*x + 1))/x^4
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx=\sqrt{2}\int \frac{\sqrt{\frac{x}{2x+1}}}{x^5}\,dx$$

input
```
integrate((1-1/(1+2*x))**(1/2)/x**5,x)
```

output
```
sqrt(2)*Integral(sqrt(x/(2*x + 1))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx=\int \frac{\sqrt{-\frac{1}{2x+1}+1}}{x^5}\,dx$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(2*x + 1) + 1)/x^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(61) = 122.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx=$$

$$-\frac{1}{35}\sqrt{2}\left(256\sqrt{2}\mathrm{sgn}(2x+1)-\frac{\sqrt{2}\left(280\left(\sqrt{2}x-\sqrt{2x^2+x}\right)^3\mathrm{sgn}(2x+1)+168\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2+x}\right.\right.}{\left(\sqrt{2}x-\right.}\right.$$

input
```
integrate((1-1/(1+2*x))^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/35*sqrt(2)*(256*sqrt(2)*sgn(2*x + 1) - sqrt(2)*(280*(sqrt(2)*x - sqrt(2
*x^2 + x))^3*sgn(2*x + 1) + 168*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + x))^2*sg
n(2*x + 1) + 70*(sqrt(2)*x - sqrt(2*x^2 + x))*sgn(2*x + 1) + 5*sqrt(2)*sgn
(2*x + 1))/(sqrt(2)*x - sqrt(2*x^2 + x))^7)
```

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx = \frac{512\left(x+\frac{1}{2}\right)\sqrt{1-\frac{1}{2\,x+1}}}{35\,x} - \frac{128\left(x+\frac{1}{2}\right)\sqrt{1-\frac{1}{2\,x+1}}}{35\,x^2} + \frac{48\left(x+\frac{1}{2}\right)\sqrt{1-\frac{1}{2\,x+1}}}{35\,x^3} - \frac{4\left(x+\frac{1}{2}\right)\sqrt{1-\frac{1}{2\,x+1}}}{7\,x^4}$$

input
```
int((1 - 1/(2*x + 1))^(1/2)/x^5,x)
```

output
```
(512*(x + 1/2)*(1 - 1/(2*x + 1))^(1/2))/(35*x) - (128*(x + 1/2)*(1 - 1/(2*
x + 1))^(1/2))/(35*x^2) + (48*(x + 1/2)*(1 - 1/(2*x + 1))^(1/2))/(35*x^3)
- (4*(x + 1/2)*(1 - 1/(2*x + 1))^(1/2))/(7*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{1-\frac{1}{1+2x}}}{x^5}\,dx = \frac{\frac{256\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x^3}{35} - \frac{64\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x^2}{35} + \frac{24\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}\,x}{35} - \frac{2\sqrt{x}\,\sqrt{2x+1}\,\sqrt{2}}{7} - \frac{512x^4}{35}}{x^4}$$

input
```
int((1-1/(1+2*x))^(1/2)/x^5,x)
```

output
```
(2*(128*sqrt(x)*sqrt(2*x + 1)*sqrt(2)*x**3 - 32*sqrt(x)*sqrt(2*x + 1)*sqrt
(2)*x**2 + 12*sqrt(x)*sqrt(2*x + 1)*sqrt(2)*x - 5*sqrt(x)*sqrt(2*x + 1)*sq
rt(2) - 256*x**4))/(35*x**4)
```

3.36 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 249

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx = & -\frac{(35b^3+120ab^2c+144a^2bc^2+64a^3c^3)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{64a^4d^4} \\ & +\frac{(35b^2+120abc+144a^2c^2)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{96a^3d^4} \\ & -\frac{(7b+24ac)(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{24a^2d^4}+\frac{(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4ad^4} \\ & +\frac{b(35b^3+120ab^2c+144a^2bc^2+64a^3c^3)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{64a^{9/2}d^4}\end{aligned}$$

output
```
-1/64*(64*a^3*c^3+144*a^2*b*c^2+120*a*b^2*c+35*b^3)*(d*x+c)*(a+b/(d*x+c))^
(1/2)/a^4/d^4+1/96*(144*a^2*c^2+120*a*b*c+35*b^2)*(d*x+c)^2*(a+b/(d*x+c))^
(1/2)/a^3/d^4-1/24*(24*a*c+7*b)*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/a^2/d^4+1/4*
(d*x+c)^4*(a+b/(d*x+c))^(1/2)/a/d^4+1/64*b*(64*a^3*c^3+144*a^2*b*c^2+120*a
*b^2*c+35*b^3)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(9/2)/d^4
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= \frac{-\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(105b^3+10ab^2(29c-7dx)+8a^2b(31c^2-16cdx+7d^2x^2)+48a^3(c^3-c^2dx+c}{192a^{9/2}d^4}$$

input
```
Integrate[x^3/Sqrt[a + b/(c + d*x)],x]
```

output
```
(-(Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(105*b^3 + 10*a*b^2
*(29*c - 7*d*x) + 8*a^2*b*(31*c^2 - 16*c*d*x + 7*d^2*x^2) + 48*a^3*(c^3 -
c^2*d*x + c*d^2*x^2 - d^3*x^3))) + 3*b*(35*b^3 + 120*a*b^2*c + 144*a^2*b*c
^2 + 64*a^3*c^3)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(192*
a^(9/2)*d^4)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {896, 25, 941, 948, 25, 109, 27, 162, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{d^3x^3}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{d^3x^3}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^4}$$

$$\big\downarrow \; 941$$

$$-\frac{\int \frac{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^4}$$

$$\big\downarrow \; 948$$

$$\frac{\int -\frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d^4}$$

$$\big\downarrow \; 25$$

$$-\frac{\int \frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d^4}$$

$$\big\downarrow \; 109$$

$$\frac{\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)\left(7b+12ac-\frac{c(3b+8ac)}{c+dx}\right)}{2\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{4a}+\frac{\left(1-\frac{c}{c+dx}\right)^2(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4a}}{d^4}$$

$$\big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)\left(7b+12ac-\frac{c(3b+8ac)}{c+dx}\right)}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{8a}+\frac{\left(1-\frac{c}{c+dx}\right)^2(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4a}}{d^4}$$

$$\big\downarrow \; 162$$

$$\frac{\frac{\frac{\left(64a^3c^3+144a^2bc^2+120ab^2c+35b^3\right)\int \frac{(c+dx)^2}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{8a^2}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(4a(12ac+7b)-\frac{5\left(24a^2c^2+24abc+7b^2\right)}{c+dx}\right)}{12a^2}}{8a}+\frac{\left(1-\frac{c}{c+dx}\right)^2(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4a}}{d^4}$$

$$\big\downarrow \; 52$$

$$\frac{\frac{\left(64a^3c^3+144a^2bc^2+120ab^2c+35b^3\right)\left(-\frac{b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{8a^2}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(4a(12ac+7b)-\frac{5\left(24a^2c^2+24abc+7b^2\right)}{c+dx}\right)}{12a^2}}{8a}+\frac{\left(1-\ldots\right.}{}$$
$$\overline{\qquad\qquad d^4\qquad\qquad}$$

$\Big\downarrow$ 73

$$\frac{\frac{\left(64a^3c^3+144a^2bc^2+120ab^2c+35b^3\right)\left(-\frac{\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{8a^2}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(4a(12ac+7b)-\frac{5\left(24a^2c^2+24abc+7b^2\right)}{c+dx}\right)}{12a^2}}{8a}+\ldots$$
$$\overline{\qquad\qquad d^4\qquad\qquad}$$

$\Big\downarrow$ 221

$$\frac{\frac{\left(64a^3c^3+144a^2bc^2+120ab^2c+35b^3\right)\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{8a^2}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(4a(12ac+7b)-\frac{5\left(24a^2c^2+24abc+7b^2\right)}{c+dx}\right)}{12a^2}}{8a}+\ldots$$
$$\overline{\qquad\qquad d^4\qquad\qquad}$$

input
```
Int[x^3/Sqrt[a + b/(c + d*x)],x]
```

output
```
(((c + d*x)^4*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))^2)/(4*a) + (-1/12*((
c + d*x)^3*Sqrt[a + b/(c + d*x)]*(4*a*(7*b + 12*a*c) - (5*(7*b^2 + 24*a*b*
c + 24*a^2*c^2))/(c + d*x)))/a^2 + ((35*b^3 + 120*a*b^2*c + 144*a^2*b*c^2
+ 64*a^3*c^3)*(-(((c + d*x)*Sqrt[a + b/(c + d*x)])/a) + (b*ArcTanh[Sqrt[a
+ b/(c + d*x)]/Sqrt[a]])/a^(3/2)))/(8*a^2))/(8*a))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(225) = 450$.

Time = 0.22 (sec) , antiderivative size = 1078, normalized size of antiderivative = 4.33

method	result	size
default	Expression too large to display	1078

input
```
int(x^3/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a^4/d^4*(-576*(a*d^2)^(1/2)*(
a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^3*c^2*d*x-192*ln(1/2*(2*a*d^2
*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2
))*a^3*b*c^3*d+384*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^3*c^3-96*
x*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a^2*d*(a*d^2)^(1/2)-576*(a*d
^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^3*c^3-864*(a*d^2)^
(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*b*c*d*x+144*ln(1/2*(
2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1
/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d-576*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*
d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d+
1152*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a^2*b*c^2+288*(a*d^2)^(1/
2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a^2*c-1152*(a*d^2)^(1/2)*(a
*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*b*c^2-348*(a*d^2)^(1/2)*(a*d
^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a*b^2*d*x+216*ln(1/2*(2*a*d^2*x+2*
a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*
d^2)^(1/2))*a*b^3*c*d-576*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+
c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^3*c*d+1152*(a*d^2)^(1/2)*(
(a*d*x+a*c+b)*(d*x+c))^(1/2)*a*b^2*c+208*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*
x+a*c^2+b*d*x+b*c)^(3/2)*a*b-780*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(1/2)*a*b^2*c+87*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.79

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \left[\frac{3\left(64\,a^3bc^3+144\,a^2b^2c^2+120\,ab^3c+35\,b^4\right)\sqrt{a}\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(4\right.$$

$$-\frac{3\left(64\,a^3bc^3+144\,a^2b^2c^2+120\,ab^3c+35\,b^4\right)\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-\left(48\,a^4d^4x^4-56\,a^3bd\right.}{}$$

input
```
integrate(x^3/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(64*a^3*b*c^3 + 144*a^2*b^2*c^2 + 120*a*b^3*c + 35*b^4)*sqrt(a)*
log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)
) + b) + 2*(48*a^4*d^4*x^4 - 56*a^3*b*d^3*x^3 - 48*a^4*c^4 - 248*a^3*b*c^3
 - 290*a^2*b^2*c^2 - 105*a*b^3*c + 2*(36*a^3*b*c + 35*a^2*b^2)*d^2*x^2 - 5
*(24*a^3*b*c^2 + 44*a^2*b^2*c + 21*a*b^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x
 + c)))/(a^5*d^4), -1/192*(3*(64*a^3*b*c^3 + 144*a^2*b^2*c^2 + 120*a*b^3*c
 + 35*b^4)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x
+ c))/(a*d*x + a*c + b)) - (48*a^4*d^4*x^4 - 56*a^3*b*d^3*x^3 - 48*a^4*c^4
 - 248*a^3*b*c^3 - 290*a^2*b^2*c^2 - 105*a*b^3*c + 2*(36*a^3*b*c + 35*a^2*
b^2)*d^2*x^2 - 5*(24*a^3*b*c^2 + 44*a^2*b^2*c + 21*a*b^3)*d*x)*sqrt((a*d*x
 + a*c + b)/(d*x + c)))/(a^5*d^4)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{x^3}{\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(x**3/sqrt((a*c + a*d*x + b)/(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{dx+c}}}\,dx$$

input
```
integrate(x^3/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a + b/(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= \frac{1}{192}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\Bigg(2\left(4\,x\left(\frac{6\,x}{ad\mathrm{sgn}\,(dx+c)}-\frac{6\,a^3cd^5\mathrm{sgn}(dx+c)+7\,a^2bd^5\mathrm{sgn}(dx-}{a^4d^7}\right.\right.$$
$$-\frac{(64\,a^3bc^3+144\,a^2b^2c^2+120\,ab^3c+35\,b^4)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+b}\right.\right.\right.}{128\,a^{\frac{9}{2}}d^3|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(x^3/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*(4*x*(6*x/(a*d*
sgn(d*x + c)) - (6*a^3*c*d^5*sgn(d*x + c) + 7*a^2*b*d^5*sgn(d*x + c))/(a^4
*d^7)) + (24*a^3*c^2*d^4*sgn(d*x + c) + 64*a^2*b*c*d^4*sgn(d*x + c) + 35*a
*b^2*d^4*sgn(d*x + c))/(a^4*d^7))*x - (48*a^3*c^3*d^3*sgn(d*x + c) + 248*a
^2*b*c^2*d^3*sgn(d*x + c) + 290*a*b^2*c*d^3*sgn(d*x + c) + 105*b^3*d^3*sgn
(d*x + c))/(a^4*d^7)) - 1/128*(64*a^3*b*c^3 + 144*a^2*b^2*c^2 + 120*a*b^3*
c + 35*b^4)*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))/(a^(9/2)*d^3*abs(d)*sgn(d
*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x))^(1/2),x)
```

output
```
int(x^3/(a + b/(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \frac{-48\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4c^3+48\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4c^2dx-48\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4c\,d^2}{}$$

input
```
int(x^3/(a+b/(d*x+c))^(1/2),x)
```

output
```
( - 48*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*c**3 + 48*sqrt(c + d*x)*sq
rt(a*c + a*d*x + b)*a**4*c**2*d*x - 48*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)
*a**4*c*d**2*x**2 + 48*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*d**3*x**3
- 248*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**2 + 128*sqrt(c + d*x)*
sqrt(a*c + a*d*x + b)*a**3*b*c*d*x - 56*sqrt(c + d*x)*sqrt(a*c + a*d*x + b
)*a**3*b*d**2*x**2 - 290*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*c +
 70*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*d*x - 105*sqrt(c + d*x)*
sqrt(a*c + a*d*x + b)*a*b**3 + 192*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sq
rt(a)*sqrt(c + d*x))/sqrt(b))*a**3*b*c**3 + 432*sqrt(a)*log((sqrt(a*c + a*
d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*b**2*c**2 + 360*sqrt(a)*lo
g((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**3*c + 105*
sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**4)
/(192*a**5*d**4)
```

3.37 $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 176

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \frac{(5b^2+12abc+8a^2c^2)\,(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8a^3d^3} - \frac{(5b+12ac)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12a^2d^3} + \frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3ad^3} - \frac{b(5b^2+12abc+8a^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{8a^{7/2}d^3}$$

output
```
1/8*(8*a^2*c^2+12*a*b*c+5*b^2)*(d*x+c)*(a+b/(d*x+c))^(1/2)/a^3/d^3-1/12*(1
2*a*c+5*b)*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/a^2/d^3+1/3*(d*x+c)^3*(a+b/(d*x+c
))^(1/2)/a/d^3-1/8*b*(8*a^2*c^2+12*a*b*c+5*b^2)*arctanh((a+b/(d*x+c))^(1/2
)/a^(1/2))/a^(7/2)/d^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$=\frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(15b^2+2ab(13c-5dx)+8a^2(c^2-cdx+d^2x^2))-3b(5b^2+12abc+8a^2c^2)\operatorname{arctan}}{24a^{7/2}d^3}$$

input
```
Integrate[x^2/Sqrt[a + b/(c + d*x)],x]
```

output
```
(Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(15*b^2 + 2*a*b*(13*c
 - 5*d*x) + 8*a^2*(c^2 - c*d*x + d^2*x^2)) - 3*b*(5*b^2 + 12*a*b*c + 8*a^2
*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(24*a^(7/2)*d^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {896, 941, 948, 100, 27, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$\Big\downarrow \text{896}$$
$$\frac{\int \frac{d^2x^2}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^3}$$
$$\Big\downarrow \text{941}$$
$$\frac{\int \frac{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{948}$$

$$-\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\frac{\int -\frac{(c+dx)^3\left(-\frac{6ac^2}{c+dx}+12ac+5b\right)}{2\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a}}{d^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{(c+dx)^3\left(-\frac{6ac^2}{c+dx}+12ac+5b\right)}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{6a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a}}{d^3}$$

$$\Big\downarrow \text{87}$$

$$-\frac{-\frac{-\frac{3\left(8a^2c^2+12abc+5b^2\right)\int \frac{(c+dx)^2}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{4a}-\frac{(12ac+5b)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{6a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a}}{d^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{-\frac{3\left(8a^2c^2+12abc+5b^2\right)\left(-\frac{b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}-\frac{(12ac+5b)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{6a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a}}{d^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{-\frac{3\left(8a^2c^2+12abc+5b^2\right)\left(-\frac{\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}-\frac{(12ac+5b)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{6a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a}}{d^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{-\frac{3\left(8a^2c^2+12abc+5b^2\right)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}-\frac{(12ac+5b)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{6a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a}}{d^3}$$

input
```
Int[x^2/Sqrt[a + b/(c + d*x)],x]
```

output
```
-((-1/3*((c + d*x)^3*Sqrt[a + b/(c + d*x)])/a - (-1/2*((5*b + 12*a*c)*(c +
 d*x)^2*Sqrt[a + b/(c + d*x)])/a - (3*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*(-(((
c + d*x)*Sqrt[a + b/(c + d*x)])/a) + (b*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt
[a]])/a^(3/2)))/(4*a))/(6*a))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(156) = 312.

Time = 0.21 (sec) , antiderivative size = 721, normalized size of antiderivative = 4.10

method	result
default	$-\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(24\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^2b\,c^2d+48\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,d^2}\,a^2cd}{}$

input
```
int(x^2/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a^3/d^3*(24*ln(1/2*(2*a*d^2*x+
2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*
a^2*b*c^2*d+48*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a
^2*c*d*x+48*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d
^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d+48*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*
x+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c^2+36*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c
)^(1/2)*(a*d^2)^(1/2)*a*b*d*x-48*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/
2)*a^2*c^2-12*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x
+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d+24*ln(1/2*(2*a*d^2
*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2
))*b^3*d-16*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a*(a*d^2)^(1/2)+60
*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c-96*(a*d^2
)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a*b*c-9*ln(1/2*(2*a*d^2*x+2*a*c*d+2*
(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/
2))*b^3*d+18*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*b^2
-48*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*b^2)/((a*d*x+a*c+b)*(d*x+c
))^(1/2)/(a*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.87

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= \left[\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\sqrt{a}\log\left(2\,adx+2\,ac-2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(8\,a^3d^3x^3-10\,a^2b\right.}{48\,a^4d^3}$$

input
```
integrate(x^2/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(2*a*d*x + 2*a*c -
2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(8*a^3*d^3*
x^3 - 10*a^2*b*d^2*x^2 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b
*c + 15*a*b^2)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^4*d^3), 1/24*(3*
(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt
((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)) + (8*a^3*d^3*x^3 - 10*a^2
*b*d^2*x^2 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^
2)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^4*d^3)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{x^2}{\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt((a*c + a*d*x + b)/(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{x^2}{\sqrt{a+\frac{b}{dx+c}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a + b/(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= \frac{1}{24}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(2\,x\left(\frac{4\,x}{ad\mathrm{sgn}\,(dx+c)}-\frac{4\,a^2cd^3\mathrm{sgn}(dx+c)+5\,abd^3\mathrm{sgn}(dx+c)}{a^3d^5}\right)\right.$$
$$+\frac{(8\,a^2bc^2+12\,ab^2c+5\,b^3)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)\sqrt{a}|d|+bd\right|\right)}{16\,a^{\frac{7}{2}}d^2|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(x^2/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*x*(4*x/(a*d*sgn(
d*x + c)) - (4*a^2*c*d^3*sgn(d*x + c) + 5*a*b*d^3*sgn(d*x + c))/(a^3*d^5))
 + (8*a^2*c^2*d^2*sgn(d*x + c) + 26*a*b*c*d^2*sgn(d*x + c) + 15*b^2*d^2*sg
n(d*x + c))/(a^3*d^5)) + 1/16*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*log(abs(2
*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b
*c))*sqrt(a)*abs(d) + b*d))/(a^(7/2)*d^2*abs(d)*sgn(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int(x^2/(a + b/(c + d*x))^(1/2),x)
```

output
```
int(x^2/(a + b/(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \frac{8\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3c^2-8\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3cdx+8\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3d^2x^2+26}{}$$

input
```
int(x^2/(a+b/(d*x+c))^(1/2),x)
```

output
```
(8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**2 - 8*sqrt(c + d*x)*sqrt(a*
c + a*d*x + b)*a**3*c*d*x + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*d**
2*x**2 + 26*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c - 10*sqrt(c + d*x
)*sqrt(a*c + a*d*x + b)*a**2*b*d*x + 15*sqrt(c + d*x)*sqrt(a*c + a*d*x + b
)*a*b**2 - 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/
sqrt(b))*a**2*b*c**2 - 36*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqr
t(c + d*x))/sqrt(b))*a*b**2*c - 15*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sq
rt(a)*sqrt(c + d*x))/sqrt(b))*b**3)/(24*a**4*d**3)
```

3.38 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx = -\frac{(3b+4ac)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4a^2d^2}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2ad^2} + \frac{b(3b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{4a^{5/2}d^2}$$

output
```
-1/4*(4*a*c+3*b)*(d*x+c)*(a+b/(d*x+c))^(1/2)/a^2/d^2+1/2*(d*x+c)^2*(a+b/(d
*x+c))^(1/2)/a/d^2+1/4*b*(4*a*c+3*b)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/
a^(5/2)/d^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.84

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(-3b-2ac+2adx)+b(3b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{4a^{5/2}d^2}$$

input
```
Integrate[x/Sqrt[a + b/(c + d*x)],x]
```

output
```
(Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(-3*b - 2*a*c + 2*a*d
*x) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/
(4*a^(5/2)*d^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {896, 25, 941, 948, 25, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{dx}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{dx}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 941}$$

$$-\frac{\int \frac{(c+dx)\left(\frac{c}{c+dx}-1\right)}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int -\frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d^2}$$

$\Big\downarrow$ 87

$$\frac{\frac{(4ac+3b)\int \frac{(c+dx)^2}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{4a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{d^2}$$

$\Big\downarrow$ 52

$$\frac{\frac{(4ac+3b)\left(-\frac{b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{d^2}$$

$\Big\downarrow$ 73

$$\frac{\frac{(4ac+3b)\left(-\frac{\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{d^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{(4ac+3b)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}}{d^2}$$

input
```
Int[x/Sqrt[a + b/(c + d*x)],x]
```

output
```
(((c + d*x)^2*Sqrt[a + b/(c + d*x)])/(2*a) + ((3*b + 4*a*c)*(-(((c + d*x)*
Sqrt[a + b/(c + d*x)])/a) + (b*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/a^(
3/2)))/(4*a))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(98) = 196$.

Time = 0.21 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.67

method	result
default	$-\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(-4\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,d^2}\,adx-4\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)abcd+8\sqrt{}$

input
```
int(x/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/d^2*(-4*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x-4*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a
*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d+8*((a
*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a*c-4*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*c+b^2*d*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^
2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))-4
*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b
*d)/(a*d^2)^(1/2))*b^2*d+8*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*b-2
*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((a*d*x+a*c+
b)*(d*x+c))^(1/2)/a^2/(a*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.09

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \left[\frac{(4\,abc+3\,b^2)\sqrt{a}\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(2\,a^2d^2x^2-2\,a^2c^2-3\,abdx-3}{8\,a^3d^2}\right.$$

$$\left. -\frac{(4\,abc+3\,b^2)\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-(2\,a^2d^2x^2-2\,a^2c^2-3\,abdx-3\,abc)\sqrt{\frac{adx+ac+b}{dx+c}}}{4\,a^3d^2}\right]$$

input
```
integrate(x/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((4*a*b*c + 3*b^2)*sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*
sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(2*a^2*d^2*x^2 - 2*a^2*c^2 - 3*
a*b*d*x - 3*a*b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^3*d^2), -1/4*((4*
a*b*c + 3*b^2)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(
d*x + c))/(a*d*x + a*c + b)) - (2*a^2*d^2*x^2 - 2*a^2*c^2 - 3*a*b*d*x - 3*
a*b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^3*d^2)]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{x}{\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(x/sqrt((a*c + a*d*x + b)/(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{x}{\sqrt{a+\frac{b}{dx+c}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a + b/(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \frac{1}{4}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(\frac{2\,x}{ad\mathrm{sgn}\,(dx+c)}-\frac{2\,acd\mathrm{sgn}(dx+c)+3\,bd\mathrm{sgn}(dx+c)}{a^2d^3}\right) - \frac{(4\,abc+3\,b^2)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)\sqrt{a}|d|+bd\right|\right)}{8\,a^{\frac{5}{2}}d|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(x/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*x/(a*d*sgn(d*x +
c)) - (2*a*c*d*sgn(d*x + c) + 3*b*d*sgn(d*x + c))/(a^2*d^3)) - 1/8*(4*a*b*
c + 3*b^2)*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x
 + a*c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))/(a^(5/2)*d*abs(d)*sgn(d*x
+ c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int(x/(a + b/(c + d*x))^(1/2),x)
```

output
```
int(x/(a + b/(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$=\frac{-2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^2c+2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^2dx-3\sqrt{dx+c}\,\sqrt{adx+ac+b}\,ab+4\sqrt{a}\,\mathrm{lo}}{4a^3d^2}$$

input
```
int(x/(a+b/(d*x+c))^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*c + 2*sqrt(c + d*x)*sqrt(a*
c + a*d*x + b)*a**2*d*x - 3*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b + 4*sq
rt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b*c +
 3*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b*
*2)/(4*a**3*d**2)
```

3.39 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}} dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}} dx = \frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{ad} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}d}$$

output
```
(d*x+c)*(a+b/(d*x+c))^(1/2)/a/d-b*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(
3/2)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}} dx = \frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}}{ad} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}d}$$

input
```
Integrate[1/Sqrt[a + b/(c + d*x)],x]
```

output
```
((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/(a*d) - (b*ArcTanh[Sqrt[(b +
 a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {239, 773, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}dx$$

$$\Big\downarrow \ 239$$

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}d(c+dx)}{d}$$

$$\Big\downarrow \ 773$$

$$-\frac{\int \frac{(c+dx)^2}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \ 52$$

$$-\frac{-\frac{b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}}{d}$$

$$\Big\downarrow \ 73$$

$$-\frac{-\frac{\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}}{d}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}}{d}$$

input
```
Int[1/Sqrt[a + b/(c + d*x)],x]
```

output
```
-((-(((c + d*x)*Sqrt[a + b/(c + d*x)])/a) + (b*ArcTanh[Sqrt[a + b/(c + d*x
)]/Sqrt[a]])/a^(3/2))/d)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(53) = 106$.

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.30

method	result	size
default	$\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(-bd\ln\left(\frac{2a\,d^{2}x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^{2}}+bd}{2\sqrt{a\,d^{2}}}\right)+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^{2}}\right)}{2\sqrt{(adx+ac+b)(dx+c)}\,ad\sqrt{a\,d^{2}}}$	140

input
```
int(1/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(-b*d*ln(1/2*(2*a*d^2*x+2*a*c*d+
2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))+2*((a*d*
x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2))/((a*d*x+a*c+b)*(d*x+c))^(1/2)/a/d/(
a*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \left[\frac{\sqrt{a}b\log\left(2\,adx+2\,ac-2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(adx+ac)\sqrt{\frac{adx+ac+b}{dx+c}}}{2\,a^{2}d}, \frac{\sqrt{-a}b\arctan\left(\frac{(dx+c)}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(1/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*log(2*a*d*x + 2*a*c - 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*
c + b)/(d*x + c)) + b) + 2*(a*d*x + a*c)*sqrt((a*d*x + a*c + b)/(d*x + c))
)/(a^2*d), (sqrt(-a)*b*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d
*x + c))/(a*d*x + a*c + b)) + (a*d*x + a*c)*sqrt((a*d*x + a*c + b)/(d*x +
c)))/(a^2*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b/(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx+c}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(53) = 106$.

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \frac{b\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)\sqrt{a}|d|+bd\right|\right)}{2\,a^{\frac{3}{2}}|d|\mathrm{sgn}\left(dx+c\right)} + \frac{\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}}{ad\mathrm{sgn}\left(dx+c\right)}$$

input
```
integrate(1/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))/(a^(3/2)*abs(d)*sgn(d*x + c)) +
 sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)/(a*d*sgn(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}\,dx=\frac{\sqrt{a+\frac{b}{c+d\,x}}\,(c+d\,x)}{a\,d}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{c+d\,x}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

input
```
int(1/(a + b/(c + d*x))^(1/2),x)
```

output
```
((a + b/(c + d*x))^(1/2)*(c + d*x))/(a*d) - (b*atanh((a + b/(c + d*x))^(1/
2)/a^(1/2)))/(a^(3/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}}\,dx=\frac{\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a-\sqrt{a}\,\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)b}{a^2d}$$

input
```
int(1/(a+b/(d*x+c))^(1/2),x)
```

output
```
(sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a - sqrt(a)*log((sqrt(a*c + a*d*x + b
) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b)/(a**2*d)
```

3.40 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{\sqrt{b+ac}}$$

output
```
2*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(1/2)-2*c^(1/2)*arctanh(c^(1/2)*(
a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx=\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{\sqrt{-b-ac}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b/(c + d*x)]),x]
```

output
```
(2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*
c]])/Sqrt[-b - a*c] + (2*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]
])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {896, 25, 941, 948, 25, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{1}{dx\sqrt{a+\frac{b}{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{1}{dx\sqrt{a+\frac{b}{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$-\int \frac{1}{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$

$$\Big\downarrow \text{ 948}$$

$$\int -\frac{c+dx}{\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 97}$$

$$-\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}-c\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{b}-\frac{2c\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

input
```
Int[1/(x*Sqrt[a + b/(c + d*x)]),x]
```

output
```
(2*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[c]*ArcTanh[(S
qrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/Sqrt[b + a*c]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(61) = 122$.

Time = 0.20 (sec) , antiderivative size = 526, normalized size of antiderivative = 6.83

method	result
default	$-\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(-2\sqrt{(ac+b)c}\,\ln\left(\frac{2a\,d^{2}x+2acd+2\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+bdx+bc}\,\sqrt{a\,d^{2}}+bd}{2\sqrt{a\,d^{2}}}\right)acd+2\sqrt{a\,d^{2}}\,\ln\left(\frac{2adxc+2a\,c^{2}+bdx}{}\right.\right.}{}$

input
```
int(1/x/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(-2*((a*c+b)*c)^(1/2)*ln(1/2*(2
*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/
2)+b*d)/(a*d^2)^(1/2))*a*c*d+2*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2
*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a
*c^2-((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b*d+2*(a*d^2)^(1/2)
*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^
2+b*d*x+b*c)^(1/2)+2*b*c)/x)*b*c-((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c
*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b*d+2
*((a*d*x+a*c+b)*(d*x+c))^(1/2)*((a*c+b)*c)^(1/2)*(a*d^2)^(1/2)-2*(a*d^2*x^
2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2))/((a*d*
x+a*c+b)*(d*x+c))^(1/2)/(a*c+b)/(a*d^2)^(1/2)/((a*c+b)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.25

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \left[\frac{a\sqrt{\frac{c}{ac+b}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,\left(ac^2+(ac+b)dx+bc\right)\sqrt{\frac{adx+ac+b}{dx+c}}\sqrt{\frac{c}{ac+b}}}{x}\right)+\sqrt{a}\log\left(2\,adx+2\,ac+2\,(dx+c\right.}{a} \right.$$

input
```
integrate(1/x/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[(a*sqrt(c/(a*c + b))*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c - 2*(a*c^2 +
 (a*c + b)*d*x + b*c)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt(c/(a*c + b)))
/x) + sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c
+ b)/(d*x + c)) + b))/a, (2*a*sqrt(-c/(a*c + b))*arctan(sqrt((a*d*x + a*c
+ b)/(d*x + c))*sqrt(-c/(a*c + b))) + sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x
 + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b))/a, (a*sqrt(c/(a*c +
b))*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c - 2*(a*c^2 + (a*c + b)*d*x + b
*c)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt(c/(a*c + b)))/x) - 2*sqrt(-a)*a
rctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c +
b)))/a, 2*(a*sqrt(-c/(a*c + b))*arctan(sqrt((a*d*x + a*c + b)/(d*x + c))*s
qrt(-c/(a*c + b))) - sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c
+ b)/(d*x + c))/(a*d*x + a*c + b)))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{x\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a*c + a*d*x + b)/(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx+c}}\,x}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c))*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{x\,\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x))^(1/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.95

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \frac{\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{adx+ac+b}-\sqrt{2\sqrt{c}\,\sqrt{a}\,\sqrt{ac+b}+2ac+b}+\sqrt{a}\,\sqrt{dx+c}\right)a+\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{}\right.}{}$$

input
```
int(1/x/(a+b/(d*x+c))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*
sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a + sqrt(c)*sqrt(a*c +
 b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a
*c + b) + sqrt(a)*sqrt(c + d*x))*a - sqrt(c)*sqrt(a*c + b)*log(2*sqrt(a)*s
qrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a
*d*x)*a + 2*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sq
rt(b))*a*c + 2*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))
/sqrt(b))*b)/(a*(a*c + b))
```

3.41 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx = -\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{(b+ac)x} - \frac{bd\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{\sqrt{c}(b+ac)^{3/2}}$$

output
```
-(d*x+c)*(a+b/(d*x+c))^(1/2)/(a*c+b)/x-b*d*arctanh(c^(1/2)*(a+b/(d*x+c))^(
1/2)/(a*c+b)^(1/2))/c^(1/2)/(a*c+b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx = -\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}}{(b+ac)x} - \frac{bd\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}(-b-ac)^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b/(c + d*x)]),x]
```

output
```
-(((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/((b + a*c)*x)) - (b*d*ArcT
an[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(Sqrt[c]*(
-b - a*c)^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {896, 941, 946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{1}{d^2x^2\sqrt{a+\frac{b}{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{941}$$

$$d\int \frac{1}{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{946}$$

$$-d\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{52}$$

$$-d\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$-d\left(\frac{\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{ac+b}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)$$

$$\downarrow 221$$

$$-d\left(\frac{b\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)$$

input
```
Int[1/(x^2*Sqrt[a + b/(c + d*x)]),x]
```

output
```
-(d*(Sqrt[a + b/(c + d*x)]/((b + a*c)*(1 - c/(c + d*x))) + (b*ArcTanh[(Sqr
t[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/(Sqrt[c]*(b + a*c)^(3/2))))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(70) = 140$.

Time = 0.20 (sec) , antiderivative size = 629, normalized size of antiderivative = 7.67

method	result
default	$\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(2\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,d^2}\,\sqrt{(ac+b)c}\,a\,d^2x^2+\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,\sqrt{a\,\cdots}}{2\sqrt{a\,d^2}}\cdots\right.\right.}{\cdots}$

input
```
int(1/x^2/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*
d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*d^2*x^2+ln(1/2*(2*a*d^2*x
+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a*b*c*d^2*x-(a*d^2)^(1/2)*ln((2*a*d*x*c+2
*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/
2)+2*b*c)/x)*a*b*c^2*d*x-((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a
*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d^2*x+2
*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*c*d*x+2*(
a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)
*a*c*d*x-(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*
d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*b^2*c*d*x+2*(a*d^2*x^2+
2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*b*d*x-2*(
a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)
)/((a*d*x+a*c+b)*(d*x+c))^(1/2)/(a*c+b)^2/c/x/(a*d^2)^(1/2)/((a*c+b)*c)^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(70) = 140.

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.59

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= \left[\frac{\sqrt{ac^2+bc}bdx\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,\sqrt{ac^2+bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{x}\right)-2\,(ac^3+bc^2+(ac^2+bc)dx)\sqrt{\frac{adx+c}{dx+}}}{2\,(a^2c^3+2\,abc^2+b^2c)x}\right.$$

input
```
integrate(1/x^2/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a*c^2 + b*c)*b*d*x*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c - 2*
sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/x) - 2*(a*c
^3 + b*c^2 + (a*c^2 + b*c)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^2*c
^3 + 2*a*b*c^2 + b^2*c)*x), (sqrt(-a*c^2 - b*c)*b*d*x*arctan(sqrt(-a*c^2 -
 b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c))
 - (a*c^3 + b*c^2 + (a*c^2 + b*c)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/
((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{x^2\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a*c + a*d*x + b)/(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx+c}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c))*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(70) = 140$.

Time = 0.14 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.44

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx = \frac{bd\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}(ac\text{sgn}\,(dx+c)+b\text{sgn}\,(dx+c))} - \frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)acd+2\,\sqrt{a}bc|d|+\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\cdots}\right.}{\left(ac^2-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)^2+bc\right)(ac\text{sgn}\,(dx+c)+b\text{sg}\cdots}$$

input
```
integrate(1/x^2/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
b*d*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x +
b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c*sgn(d*x + c) + b*sgn(d*
x + c))) - (2*a^(3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b*d*x + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)
*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*b*d)/((a*c^2 - (sq
rt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2 + b*c)*
(a*c*sgn(d*x + c) + b*sgn(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{1}{x^2\,\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.77

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$=\frac{-2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a\,c^2-2\sqrt{dx+c}\,\sqrt{adx+ac+b}\,bc+\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{adx+ac+b}-\sqrt{2\sqrt{}}\right.}{}$$

input
```
int(1/x^2/(a+b/(d*x+c))^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*c**2 - 2*sqrt(c + d*x)*sqrt(a*
c + a*d*x + b)*b*c + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqr
t(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b*
d*x + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqr
t(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b*d*x - sqrt(c)*s
qrt(a*c + b)*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)
*sqrt(a)*sqrt(a*c + b) + 2*a*d*x)*b*d*x)/(2*c*x*(a**2*c**2 + 2*a*b*c + b**
2))
```

3.42 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx = \frac{(b+4ac)d(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4c(b+ac)^2x} - \frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2c(b+ac)x^2} + \frac{b(b+4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{4c^{3/2}(b+ac)^{5/2}}$$

output
```
1/4*(4*a*c+b)*d*(d*x+c)*(a+b/(d*x+c))^(1/2)/c/(a*c+b)^2/x-1/2*(d*x+c)^2*(a
+b/(d*x+c))^(1/2)/c/(a*c+b)/x^2+1/4*b*(4*a*c+b)*d^2*arctanh(c^(1/2)*(a+b/(
d*x+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)/(a*c+b)^(5/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx = -\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(2ac(c-dx)+b(2c+dx))}{4c(b+ac)^2x^2} - \frac{b(b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{4c^{3/2}(-b-ac)^{5/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b/(c + d*x)]),x]
```

output
```
-1/4*((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(2*a*c*(c - d*x) + b*(2*
c + d*x)))/(c*(b + a*c)^2*x^2) - (b*(b + 4*a*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[(
b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(4*c^(3/2)*(-b - a*c)^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 25, 941, 948, 25, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}dx$$

$$\Big\downarrow \; 896$$

$$d^2\int \frac{1}{d^3x^3\sqrt{a+\frac{b}{c+dx}}}d(c+dx)$$

$$\Big\downarrow \; 25$$

$$-d^2\int -\frac{1}{d^3x^3\sqrt{a+\frac{b}{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{941}$$

$$-d^2\int \frac{1}{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \text{948}$$

$$d^2\int -\frac{1}{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-d^2\int \frac{1}{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{87}$$

$$d^2\left(\frac{(4ac+b)\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4c(ac+b)}-\frac{\sqrt{a+\frac{b}{c+dx}}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{52}$$

$$d^2\left(\frac{(4ac+b)\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4c(ac+b)}-\frac{\sqrt{a+\frac{b}{c+dx}}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{73}$$

$$d^2\left(\frac{(4ac+b)\left(\frac{\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{ac+b}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4c(ac+b)}-\frac{\sqrt{a+\frac{b}{c+dx}}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{221}$$

$$d^2\left(\frac{(4ac+b)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4c(ac+b)}-\frac{\sqrt{a+\frac{b}{c+dx}}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

input
```
Int[1/(x^3*Sqrt[a + b/(c + d*x)]),x]
```

output
```
d^2*(-1/2*Sqrt[a + b/(c + d*x)]/(c*(b + a*c)*(1 - c/(c + d*x))^2) + ((b +
4*a*c)*(Sqrt[a + b/(c + d*x)]/((b + a*c)*(1 - c/(c + d*x))) + (b*ArcTanh[(
Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/(Sqrt[c]*(b + a*c)^(3/2))))
/(4*c*(b + a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(123) = 246$.

Time = 0.21 (sec) , antiderivative size = 1030, normalized size of antiderivative = 7.20

method	result	size
default	Expression too large to display	1030

input
```
int(1/x^3/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(12*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*c*d^3*x^3+4*ln(1/2*(2
*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/
2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a^2*b*c^2*d^3*x^2-4*(a*d^2)^(1/2)
*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^
2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a^2*b*c^3*d^2*x^2-4*((a*c+b)*c)^(1/2)*ln(1/2*
(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*a^2*b*c^2*d^3*x^2+8*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c
+b)*(d*x+c))^(1/2)*a^2*c^2*d^2*x^2+12*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c
)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*c^2*d^2*x^2+2*(a*d^2*x^2+2*a*c
*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b*d^3*x^3-5*
(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2
*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a*b^2*c^2*d^2*x^2+12*(a*d^2*x^2+
2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b*c*d^2
*x^2-(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*
x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*b^3*c*d^2*x^2-12*(a*d^2*x^2
+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*c*d*x+
2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1
/2)*b^2*d^2*x^2+4*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2
)*((a*c+b)*c)^(1/2)*a*c^2-2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.08

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \left[\frac{(4\,abc+b^2)\sqrt{ac^2+bc}d^2x^2\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc+2\,\sqrt{ac^2+bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{x}\right)-2\,(2\,a^2c^5+4\,abc^4+2\,b^2 \cdots}{8\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)x^2 \cdots}\right.$$

$$-\frac{(4\,abc+b^2)\sqrt{-ac^2-bc}d^2x^2\arctan\left(\frac{\sqrt{-ac^2-bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{acdx+ac^2+bc}\right)+(2\,a^2c^5+4\,abc^4+2\,b^2c^3-(2\,a^2c^3+ \cdots}{4\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)x^2 \cdots}$$

input
```
integrate(1/x^3/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^2*log(-(2*a*c^2 + (2*a*c + b
)*d*x + 2*b*c + 2*sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x
+ c)))/x) - 2*(2*a^2*c^5 + 4*a*b*c^4 + 2*b^2*c^3 - (2*a^2*c^3 + a*b*c^2 -
b^2*c)*d^2*x^2 + 3*(a*b*c^3 + b^2*c^2)*d*x)*sqrt((a*d*x + a*c + b)/(d*x +
c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^2), -1/4*((4*a*b*c
 + b^2)*sqrt(-a*c^2 - b*c)*d^2*x^2*arctan(sqrt(-a*c^2 - b*c)*(d*x + c)*sqr
t((a*d*x + a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) + (2*a^2*c^5 + 4*a
*b*c^4 + 2*b^2*c^3 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^2 + 3*(a*b*c^3 +
b^2*c^2)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^3*c^5 + 3*a^2*b*c^4 +
 3*a*b^2*c^3 + b^3*c^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{x^3\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a*c + a*d*x + b)/(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx+c}}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c))*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(123) = 246$.

Time = 0.15 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.15

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= -\frac{(4\,abcd^2+b^2d^2)\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}}{\sqrt{-ac^2-bc}}\right)}{4\,(a^2c^3\mathrm{sgn}\,(dx+c)+2\,abc^2\mathrm{sgn}\,(dx+c)+b^2c\mathrm{sgn}\,(dx+c))\sqrt{-ac^2-bc}}$$
$$+\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)a^3c^4d^2+8\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx\right.}{}$$

input
```
integrate(1/x^3/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b*d*x + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3*sgn(d*x + c)
+ 2*a*b*c^2*sgn(d*x + c) + b^2*c*sgn(d*x + c))*sqrt(-a*c^2 - b*c)) + 1/4*(
8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b*d*x + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b*d*x + b*c))^2*a^(5/2)*c^3*d*abs(d) + 16*a^(5/2)*b*c^4*
d*abs(d) + 28*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x
+ b*c))*a^2*b*c^3*d^2 + 16*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b*d*x + b*c))^2*a^(3/2)*b*c^2*d*abs(d) + 8*a^(3/2)*b^2*c^3*d*abs(d)
 + 4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3
*a*b*c*d^2 + 13*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*
x + b*c))*a*b^2*c^2*d^2 + 8*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b*d*x + b*c))^2*sqrt(a)*b^2*c*d*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d
^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*b^2*d^2 + (sqrt(a*d^2)*x - sq
rt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*b^3*c*d^2)/((a^2*c^3*sgn(
d*x + c) + 2*a*b*c^2*sgn(d*x + c) + b^2*c*sgn(d*x + c))*(a*c^2 - (sqrt(a*d
^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2 + b*c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{1}{x^3\,\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 866, normalized size of antiderivative = 6.06

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx}}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(a+b/(d*x+c))^(1/2),x)
```

output
```
( - 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**5 + 8*sqrt(c + d*x)*sqrt
(a*c + a*d*x + b)*a**3*c**4*d*x - 20*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a
**2*b*c**4 + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c**3*d*x - 16*sq
rt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**2*c**3 - 2*sqrt(c + d*x)*sqrt(a*c +
 a*d*x + b)*a*b**2*c**2*d*x - 4*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**3*c
**2 - 2*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**3*c*d*x - 8*sqrt(c)*sqrt(a*
c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) +
2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b*c**2*d**2*x**2 - 6*sqrt(c)*sqrt
(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b)
 + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**2*c*d**2*x**2 - sqrt(c)*sqrt(a
*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) +
 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**3*d**2*x**2 - 8*sqrt(c)*sqrt(a*c +
 b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a
*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b*c**2*d**2*x**2 - 6*sqrt(c)*sqrt(a*
c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) +
2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**2*c*d**2*x**2 - sqrt(c)*sqrt(a*c
+ b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*
a*c + b) + sqrt(a)*sqrt(c + d*x))*b**3*d**2*x**2 + 8*sqrt(c)*sqrt(a*c + b)
*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqr
t(a*c + b) + 2*a*d*x)*a**2*b*c**2*d**2*x**2 + 6*sqrt(c)*sqrt(a*c + b)*l...
```

3.43 $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 215

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx = -\frac{(b^2+4abc+8a^2c^2)\,d^2(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8c^2(b+ac)^3x} + \frac{(7b+12ac)d(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12c^2(b+ac)^2x^2} - \frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3c^2(b+ac)x^3} - \frac{b(b^2+4abc+8a^2c^2)\,d^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{8c^{5/2}(b+ac)^{7/2}}$$

output
```
-1/8*(8*a^2*c^2+4*a*b*c+b^2)*d^2*(d*x+c)*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)^3
/x+1/12*(12*a*c+7*b)*d*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)^2/x^2-1/3
*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)/x^3-1/8*b*(8*a^2*c^2+4*a*b*c+b^
2)*d^3*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/c^(5/2)/(a*c+b)^
(7/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx = -\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(2abc(8c^2-3cdx-5d^2x^2)+b^2(8c^2+2cdx-3d^2x^2)+8a^2c^2(c^2-cdx+d^2x^2)\right)}{24c^2(b+ac)^3x^3} - \frac{b(b^2+4abc+8a^2c^2)\,d^3\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{8c^{5/2}(-b-ac)^{7/2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b/(c + d*x)]),x]
```

output
```
-1/24*((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(2*a*b*c*(8*c^2 - 3*c*d
*x - 5*d^2*x^2) + b^2*(8*c^2 + 2*c*d*x - 3*d^2*x^2) + 8*a^2*c^2*(c^2 - c*d
*x + d^2*x^2)))/(c^2*(b + a*c)^3*x^3) - (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*d^3
*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(8*c^
(5/2)*(-b - a*c)^(7/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 941, 948, 100, 27, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}dx$$

$$\Big\downarrow \text{896}$$

$$d^3\int \frac{1}{d^4x^4\sqrt{a+\frac{b}{c+dx}}}d(c+dx)$$

$$\Big\downarrow \; 941$$

$$d^3\int \frac{1}{(c+dx)^4\sqrt{a+\frac{b}{c+dx}}\left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \; 948$$

$$-d^3\int \frac{1}{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \; 100$$

$$-d^3\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{b+6ac+\frac{6c(b+ac)}{c+dx}}{2\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3c^2(ac+b)}\right)$$

$$\Big\downarrow \; 27$$

$$-d^3\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{b+6ac+\frac{6c(b+ac)}{c+dx}}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \; 87$$

$$-d^3\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+7b)\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(8a^2c^2+4abc+b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4(ac+b)}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \; 52$$

$$-d^3\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+7b)\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(8a^2c^2+4abc+b^2\right)\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4(ac+b)}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \; 73$$

$$-d^3\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+7b)\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{3(8a^2c^2+4abc+b^2)\left(\frac{\int\frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{ac+b}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4(ac+b)}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow 221$$

$$-d^3\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(12ac+7b)\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{3(8a^2c^2+4abc+b^2)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4(ac+b)}}{6c^2(ac+b)}\right)$$

input
```
Int[1/(x^4*Sqrt[a + b/(c + d*x)]),x]
```

output
```
-(d^3*(Sqrt[a + b/(c + d*x)]/(3*c^2*(b + a*c)*(1 - c/(c + d*x))^3) - (((7*
b + 12*a*c)*Sqrt[a + b/(c + d*x)])/(2*(b + a*c)*(1 - c/(c + d*x))^2) - (3*
(b^2 + 4*a*b*c + 8*a^2*c^2)*(Sqrt[a + b/(c + d*x)]/((b + a*c)*(1 - c/(c +
d*x))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])/(Sqrt[
c]*(b + a*c)^(3/2))))/(4*(b + a*c)))/(6*c^2*(b + a*c))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1533 vs. $2(191) = 382$.

Time = 0.21 (sec) , antiderivative size = 1534, normalized size of antiderivative = 7.13

method	result	size
default	Expression too large to display	1534

input
```
int(1/x^4/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(-96*(a*d^2*x^2+2*a*c*d*x+a*c^
2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^3*c^2*d^4*x^4-24*ln(1
/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2
)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a^3*b*c^3*d^4*x^3+24*(a*d^2)
^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*
x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a^3*b*c^4*d^3*x^3+24*((a*c+b)*c)^(1/2)*
ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*
d)/(a*d^2)^(1/2))*a^3*b*c^3*d^4*x^3-96*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*
c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^3*c^3*d^3*x^3-36*(a*d^2*x^2+2*a
*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*b*c*d^4*
x^4-48*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^3*c
^3*d^3*x^3+36*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2
)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a^2*b^2*c^3*d^3*x^
3-108*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c
)^(1/2)*a^2*b*c^2*d^3*x^3-6*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a
*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b^2*d^4*x^4+15*(a*d^2)^(1/2)*ln((2*a*d*x*c
+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(
1/2)+2*b*c)/x)*a*b^3*c^2*d^3*x^3+96*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^
(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*c^2*d^2*x^2-36*(a*d^2*x^2+2*a*c*
d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b^2*c*d^3*...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \left[\frac{3\left(8\,a^2bc^2+4\,ab^2c+b^3\right)\sqrt{ac^2+bc}d^3x^3\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,\sqrt{ac^2+bc}(dx+c)\sqrt{\frac{adx+ac+b}{dx+c}}}{x}\right)-2\left(8\,a^3c^7+\right.}{}\right.$$

input
```
integrate(1/x^4/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a*c^2 + b*c)*d^3*x^3*log(-(2
*a*c^2 + (2*a*c + b)*d*x + 2*b*c - 2*sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((a*d
*x + a*c + b)/(d*x + c)))/x) - 2*(8*a^3*c^7 + 24*a^2*b*c^6 + 24*a*b^2*c^5
+ 8*b^3*c^4 + (8*a^3*c^4 - 2*a^2*b*c^3 - 13*a*b^2*c^2 - 3*b^3*c)*d^3*x^3 -
 (16*a^2*b*c^4 + 17*a*b^2*c^3 + b^3*c^2)*d^2*x^2 + 10*(a^2*b*c^5 + 2*a*b^2
*c^4 + b^3*c^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^4*c^7 + 4*a^3*
b*c^6 + 6*a^2*b^2*c^5 + 4*a*b^3*c^4 + b^4*c^3)*x^3), 1/24*(3*(8*a^2*b*c^2
+ 4*a*b^2*c + b^3)*sqrt(-a*c^2 - b*c)*d^3*x^3*arctan(sqrt(-a*c^2 - b*c)*(d
*x + c)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) - (8*a^
3*c^7 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*a^3*c^4 - 2*a^2*b*c^3
 - 13*a*b^2*c^2 - 3*b^3*c)*d^3*x^3 - (16*a^2*b*c^4 + 17*a*b^2*c^3 + b^3*c^
2)*d^2*x^2 + 10*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x)*sqrt((a*d*x + a*c
 + b)/(d*x + c)))/((a^4*c^7 + 4*a^3*b*c^6 + 6*a^2*b^2*c^5 + 4*a*b^3*c^4 +
b^4*c^3)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{1}{x^4\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate(1/x**4/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a*c + a*d*x + b)/(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx+c}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c))*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(191) = 382$.

Time = 0.16 (sec) , antiderivative size = 1397, normalized size of antiderivative = 6.50

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/8*(8*a^2*b*c^2*d^3 + 4*a*b^2*c*d^3 + b^3*d^3)*arctan(-(sqrt(a*d^2)*x - s
qrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))/sqrt(-a*c^2 - b*c))/((a^
3*c^5*sgn(d*x + c) + 3*a^2*b*c^4*sgn(d*x + c) + 3*a*b^2*c^3*sgn(d*x + c) +
 b^3*c^2*sgn(d*x + c))*sqrt(-a*c^2 - b*c)) - 1/24*(64*a^(11/2)*c^8*d^2*abs
(d) + 192*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*
c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b*d*x + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 208*a^(9/2)*b*c^7*d^2*abs(d) +
64*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*a
^4*c^5*d^3 + 600*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d
*x + b*c))*a^4*b*c^6*d^3 + 624*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x
 + a*c^2 + b*d*x + b*c))^2*a^(7/2)*b*c^5*d^2*abs(d) + 240*a^(7/2)*b^2*c^6*
d^2*abs(d) + 256*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d
*x + b*c))^3*a^3*b*c^4*d^3 + 684*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b*d*x + b*c))*a^3*b^2*c^5*d^3 + 720*(sqrt(a*d^2)*x - sqrt(a*d
^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*a^(5/2)*b^2*c^4*d^2*abs(d) +
112*a^(5/2)*b^3*c^5*d^2*abs(d) - 24*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b*d*x + b*c))^5*a^2*b*c^2*d^3 + 288*(sqrt(a*d^2)*x - sqrt(
a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*a^2*b^2*c^3*d^3 + 339*(sqr
t(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^2*b^3*c^
4*d^3 + 336*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{1}{x^4\,\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int(1/(x^4*(a + b/(c + d*x))^(1/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 1278, normalized size of antiderivative = 5.94

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx}}}\,dx = \text{Too large to display}$$

input
```
int(1/x^4/(a+b/(d*x+c))^(1/2),x)
```

output
```
( - 32*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*c**7 + 32*sqrt(c + d*x)*sq
rt(a*c + a*d*x + b)*a**4*c**6*d*x - 32*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)
*a**4*c**5*d**2*x**2 - 112*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**6
 + 72*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*b*c**5*d*x - 8*sqrt(c + d*x
)*sqrt(a*c + a*d*x + b)*a**3*b*c**4*d**2*x**2 - 144*sqrt(c + d*x)*sqrt(a*c
 + a*d*x + b)*a**2*b**2*c**5 + 44*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2
*b**2*c**4*d*x + 56*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*c**3*d**
2*x**2 - 80*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*b**3*c**4 + 38*sqrt(c +
d*x)*sqrt(a*c + a*d*x + b)*a*b**3*c**2*d**2*x**2 - 16*sqrt(c + d*x)*sqrt(a
*c + a*d*x + b)*b**4*c**3 - 4*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**4*c**
2*d*x + 6*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*b**4*c*d**2*x**2 + 48*sqrt(c
)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*
c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**3*b*c**3*d**3*x**3 + 48*sq
rt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqr
t(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b**2*c**2*d**3*x**3
+ 18*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt
(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**3*c*d**3*x**3
 + 3*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt
(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**4*d**3*x**3 + 4
8*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt...
```

3.44 $\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 280

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}} \, dx = & -\frac{2b(b+ac)^3}{a^5d^4\sqrt{a+\frac{b}{c+dx}}} \\ & -\frac{(187b^3+456ab^2c+336a^2bc^2+64a^3c^3)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{64a^5d^4} \\ & +\frac{(41b^2+88abc+48a^2c^2)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{32a^4d^4} \\ & -\frac{(5b+8ac)(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{8a^3d^4}+\frac{(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4a^2d^4} \\ & +\frac{3b(105b^3+280ab^2c+240a^2bc^2+64a^3c^3)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{64a^{11/2}d^4}\end{aligned}$$

output
```
-2*b*(a*c+b)^3/a^5/d^4/(a+b/(d*x+c))^(1/2)-1/64*(64*a^3*c^3+336*a^2*b*c^2+
456*a*b^2*c+187*b^3)*(d*x+c)*(a+b/(d*x+c))^(1/2)/a^5/d^4+1/32*(48*a^2*c^2+
88*a*b*c+41*b^2)*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/a^4/d^4-1/8*(8*a*c+5*b)*(d*
x+c)^3*(a+b/(d*x+c))^(1/2)/a^3/d^4+1/4*(d*x+c)^4*(a+b/(d*x+c))^(1/2)/a^2/d
^4+3/64*b*(64*a^3*c^3+240*a^2*b*c^2+280*a*b^2*c+105*b^3)*arctanh((a+b/(d*x
+c))^(1/2)/a^(1/2))/a^(11/2)/d^4
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(315b^4+105ab^3(9c+dx)+2a^2b^2\left(479c^2+98cdx-21d^2x^2\right)+8a^3b\left(43c^3+11c^2dx-5cd^2x^2+3c\right.\right.}{b+a(c+dx)}}{64a^{11}}$$

input

```
Integrate[x^3/(a + b/(c + d*x))^(3/2),x]
```

output

```
(-((Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(315*b^4 + 105*a*b
^3*(9*c + d*x) + 2*a^2*b^2*(479*c^2 + 98*c*d*x - 21*d^2*x^2) + 8*a^3*b*(43
*c^3 + 11*c^2*d*x - 5*c*d^2*x^2 + 3*d^3*x^3) + 16*a^4*(c^4 - d^4*x^4)))/(b
 + a*(c + d*x))) + 3*b*(105*b^3 + 280*a*b^2*c + 240*a^2*b*c^2 + 64*a^3*c^3
)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(64*a^(11/2)*d^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {896, 25, 941, 948, 25, 109, 27, 161, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int \frac{d^3x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{d^3x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^4}$$

$$\downarrow \text{941}$$

$$-\frac{\int \frac{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^4}$$

$$\downarrow \text{948}$$

$$\frac{\int -\frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{d^4}$$

$$\downarrow \text{25}$$

$$-\frac{\int \frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{d^4}$$

$$\downarrow \text{109}$$

$$\frac{\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)\left(3(3b+4ac)-\frac{c(5b+8ac)}{c+dx}\right)}{2\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{4a}+\frac{\left(1-\frac{c}{c+dx}\right)^2(c+dx)^4}{4a\sqrt{a+\frac{b}{c+dx}}}}{d^4}$$

$$\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)\left(3(3b+4ac)-\frac{c(5b+8ac)}{c+dx}\right)}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{8a}+\frac{\left(1-\frac{c}{c+dx}\right)^2(c+dx)^4}{4a\sqrt{a+\frac{b}{c+dx}}}}{d^4}$$

$$\downarrow \text{161}$$

$$\frac{\frac{-\frac{\left(64a^3c^3+240a^2bc^2+280ab^2c+105b^3\right)\int \frac{(c+dx)^3}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a^2b}-\frac{(c+dx)^3\left(\frac{16a^3c^3+50a^2bc^2+56ab^2c+21b^3}{c+dx}+ab(4ac+3b)\right)}{a^2b\sqrt{a+\frac{b}{c+dx}}}}{8a}+\frac{\left(1-\frac{c}{c+dx}\right)^2(c+dx)^4}{4a\sqrt{a+\frac{b}{c+dx}}}}{d^4}$$

$$\downarrow \text{52}$$

$$\frac{\frac{-\frac{\left(64a^3c^3+240a^2bc^2+280ab^2c+105b^3\right)\left(-\frac{3b\int \frac{(c+dx)^2}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{4a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}\right)}{2a^2b}-\frac{(c+dx)^3\left(\frac{16a^3c^3+50a^2bc^2+56ab^2c+21b^3}{c+dx}+ab(4ac+3b)\right)}{a^2b\sqrt{a+\frac{b}{c+dx}}}}{8a}+\frac{(1\cdots}{\cdots}}{d^4}$$

$\downarrow$ 52

$$\frac{\frac{-\frac{\left(64a^3c^3+240a^2bc^2+280ab^2c+105b^3\right)\left(-\frac{3b\left(-\frac{b\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}\right)}{2a^2b}-\frac{(c+dx)^3\left(\frac{16a^3c^3+50a^2bc^2+56ab^2c+2\cdots}{c+dx}\cdots\right)}{a^2b\sqrt{a+\frac{b}{c+dx}}}\cdots}{8a}\cdots}{d^4}$$

$\downarrow$ 73

$$\frac{\frac{-\frac{\left(64a^3c^3+240a^2bc^2+280ab^2c+105b^3\right)\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a}\right)}{2a^2b}-\frac{(c+dx)^3\left(\frac{16a^3c^3+50a^2bc^2+56\cdots}{c+dx}\cdots\right)}{a^2b\sqrt{a+\frac{\cdots}{c\cdots}}}\cdots}{8a}\cdots}{d^4}$$

$\downarrow$ 221

$$\frac{-\frac{(c+dx)^3\left(\frac{16a^3c^3+50a^2bc^2+56ab^2c+21b^3}{c+dx}+ab(4ac+3b)\right)}{a^2b\sqrt{a+\frac{b}{c+dx}}}-\frac{\left(64a^3c^3+240a^2bc^2+280ab^2c+105b^3\right)\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a}\right)}{4a}-\right.}{2a^2b}}{8a}}{d^4}$$

input

```
Int[x^3/(a + b/(c + d*x))^(3/2),x]
```

output

```
(((c + d*x)^4*(1 - c/(c + d*x))^2)/(4*a*Sqrt[a + b/(c + d*x)]) + (-(((c +
d*x)^3*(a*b*(3*b + 4*a*c) + (21*b^3 + 56*a*b^2*c + 50*a^2*b*c^2 + 16*a^3*c
^3)/(c + d*x)))/(a^2*b*Sqrt[a + b/(c + d*x)])) - ((105*b^3 + 280*a*b^2*c +
 240*a^2*b*c^2 + 64*a^3*c^3)*(-1/2*((c + d*x)^2*Sqrt[a + b/(c + d*x)])/a -
 (3*b*(-(((c + d*x)*Sqrt[a + b/(c + d*x)])/a) + (b*ArcTanh[Sqrt[a + b/(c +
 d*x)]/Sqrt[a]])/a^(3/2)))/(4*a)))/(2*a^2*b))/(8*a))/d^4
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3909 vs. $2(254) = 508$.

Time = 0.25 (sec) , antiderivative size = 3910, normalized size of antiderivative = 13.96

method	result	size
default	Expression too large to display	3910

input
```
int(x^3/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a^5/d^4*(378*ln(1/2*(2*a*d^2*
x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)
/(a*d^2)^(1/2))*a^2*b^4*c*d^2*x-276*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^
(1/2)*(a*d^2)^(1/2)*a^3*b^2*d^3*x^3-576*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b
*c)^(1/2)*(a*d^2)^(1/2)*a^5*c^4*d*x+6912*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d*x
+c))^(1/2)*a^3*b^2*c^2*d*x-2340*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2
)*(a*d^2)^(1/2)*a^2*b^3*c*d*x+5376*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(
1/2)*a^2*b^3*c*d*x+96*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c
^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^4*b^2*c^3*d^2*x-26
88*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)
+b*d)/(a*d^2)^(1/2))*a^2*b^4*c*d^2*x+768*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d*x
+c))^(1/2)*a^2*b^3*d^2*x^2+336*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*
c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*b^3*c^2
*d^2*x-768*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^
2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^4*b^2*c^2*d^3*x^2-552*(a*d^2*x^2+2*a*c*d*x+
a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^4*d*x+1536*(a*d^2)^(1/2)*((a*d*x+
a*c+b)*(d*x+c))^(1/2)*a*b^4*d*x+357*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2
+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^
4*c^2*d+138*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b
*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^5*d^2*x-768*(a*d^2)^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.62

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(3*(64*a^4*b*c^4 + 304*a^3*b^2*c^3 + 520*a^2*b^3*c^2 + 385*a*b^4*c
+ 105*b^5 + (64*a^4*b*c^3 + 240*a^3*b^2*c^2 + 280*a^2*b^3*c + 105*a*b^4)*d
*x)*sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c +
b)/(d*x + c)) + b) + 2*(16*a^5*d^5*x^5 - 16*a^5*c^5 - 344*a^4*b*c^4 + 8*(2
*a^5*c - 3*a^4*b)*d^4*x^4 - 958*a^3*b^2*c^3 - 945*a^2*b^3*c^2 + 2*(8*a^4*b
*c + 21*a^3*b^2)*d^3*x^3 - 315*a*b^4*c - (48*a^4*b*c^2 + 154*a^3*b^2*c + 1
05*a^2*b^3)*d^2*x^2 - (16*a^5*c^4 + 432*a^4*b*c^3 + 1154*a^3*b^2*c^2 + 105
0*a^2*b^3*c + 315*a*b^4)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^7*d^5*
x + (a^7*c + a^6*b)*d^4), -1/64*(3*(64*a^4*b*c^4 + 304*a^3*b^2*c^3 + 520*a
^2*b^3*c^2 + 385*a*b^4*c + 105*b^5 + (64*a^4*b*c^3 + 240*a^3*b^2*c^2 + 280
*a^2*b^3*c + 105*a*b^4)*d*x)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*
x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)) - (16*a^5*d^5*x^5 - 16*a^5*c^5
- 344*a^4*b*c^4 + 8*(2*a^5*c - 3*a^4*b)*d^4*x^4 - 958*a^3*b^2*c^3 - 945*a^
2*b^3*c^2 + 2*(8*a^4*b*c + 21*a^3*b^2)*d^3*x^3 - 315*a*b^4*c - (48*a^4*b*c
^2 + 154*a^3*b^2*c + 105*a^2*b^3)*d^2*x^2 - (16*a^5*c^4 + 432*a^4*b*c^3 +
1154*a^3*b^2*c^2 + 1050*a^2*b^3*c + 315*a*b^4)*d*x)*sqrt((a*d*x + a*c + b)
/(d*x + c)))/(a^7*d^5*x + (a^7*c + a^6*b)*d^4)]
```

Sympy [F]

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(x**3/((a*c + a*d*x + b)/(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a + b/(d*x + c))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(254) = 508.

Time = 0.23 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.45

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/64*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*(4*x*(2*x/(a^2*d
*sgn(d*x + c)) - (2*a^17*c*d^11 + 5*a^16*b*d^11)/(a^19*d^13*sgn(d*x + c)))
 + (8*a^17*c^2*d^10 + 48*a^16*b*c*d^10 + 41*a^15*b^2*d^10)/(a^19*d^13*sgn(
d*x + c)))*x - (16*a^17*c^3*d^9 + 200*a^16*b*c^2*d^9 + 374*a^15*b^2*c*d^9
+ 187*a^14*b^3*d^9)/(a^19*d^13*sgn(d*x + c))) - 1/128*(64*a^3*b*c^3 + 240*
a^2*b^2*c^2 + 280*a*b^3*c + 105*b^4)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*
x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^(5/2)*c^2*abs(d)
+ 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*
a^2*c*d + 5*a^2*b*c^2*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b*d*x + b*c))^3*a^(3/2)*abs(d) + 10*(sqrt(a*d^2)*x - sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt(a*d^2)*
x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*a*b*d + 4*a*b^2*c
*d + 4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))
*sqrt(a)*b^2*abs(d) + b^3*d))/(a^(11/2)*d^3*abs(d)*sgn(d*x + c)) - 1/128*(
64*a^(17/2)*b*c^3*d^4*abs(d)*sgn(d*x + c) + 240*a^(15/2)*b^2*c^2*d^4*abs(d
)*sgn(d*x + c) + 280*a^(13/2)*b^3*c*d^4*abs(d)*sgn(d*x + c) + 105*a^(11/2)
*b^4*d^4*abs(d)*sgn(d*x + c))*log(abs(a))/(a^11*d^9)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a+\frac{b}{c+d\,x}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x))^(3/2),x)
```

output
```
int(x^3/(a + b/(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.72

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{192\sqrt{a}\,\sqrt{adx+ac+b}\,\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)a^3b\,c^3+720\sqrt{a}\,\sqrt{adx+ac+b}\,\log\left(\right.}{}$$

input
```
int(x^3/(a+b/(d*x+c))^(3/2),x)
```

output
```
(192*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sq
rt(c + d*x))/sqrt(b))*a**3*b*c**3 + 720*sqrt(a)*sqrt(a*c + a*d*x + b)*log(
(sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*b**2*c**2 +
840*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqr
t(c + d*x))/sqrt(b))*a*b**3*c + 315*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqr
t(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**4 - 144*sqrt(a)*sq
rt(a*c + a*d*x + b)*a**3*b*c**3 - 480*sqrt(a)*sqrt(a*c + a*d*x + b)*a**2*b
**2*c**2 - 525*sqrt(a)*sqrt(a*c + a*d*x + b)*a*b**3*c - 189*sqrt(a)*sqrt(a
*c + a*d*x + b)*b**4 - 16*sqrt(c + d*x)*a**5*c**4 + 16*sqrt(c + d*x)*a**5*
d**4*x**4 - 344*sqrt(c + d*x)*a**4*b*c**3 - 88*sqrt(c + d*x)*a**4*b*c**2*d
*x + 40*sqrt(c + d*x)*a**4*b*c*d**2*x**2 - 24*sqrt(c + d*x)*a**4*b*d**3*x*
*3 - 958*sqrt(c + d*x)*a**3*b**2*c**2 - 196*sqrt(c + d*x)*a**3*b**2*c*d*x
+ 42*sqrt(c + d*x)*a**3*b**2*d**2*x**2 - 945*sqrt(c + d*x)*a**2*b**3*c - 1
05*sqrt(c + d*x)*a**2*b**3*d*x - 315*sqrt(c + d*x)*a*b**4)/(64*sqrt(a*c +
a*d*x + b)*a**6*d**4)
```

3.45 $\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 207

$$\begin{aligned}\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx &= \frac{2b(b+ac)^2}{a^4d^3\sqrt{a+\frac{b}{c+dx}}} \\ &+ \frac{(19b^2+28abc+8a^2c^2)\,(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8a^4d^3} - \frac{(11b+12ac)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12a^3d^3} \\ &+ \frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3a^2d^3} - \frac{b(35b^2+60abc+24a^2c^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{8a^{9/2}d^3}\end{aligned}$$

output
```
2*b*(a*c+b)^2/a^4/d^3/(a+b/(d*x+c))^(1/2)+1/8*(8*a^2*c^2+28*a*b*c+19*b^2)*
(d*x+c)*(a+b/(d*x+c))^(1/2)/a^4/d^3-1/12*(12*a*c+11*b)*(d*x+c)^2*(a+b/(d*x
+c))^(1/2)/a^3/d^3+1/3*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/a^2/d^3-1/8*b*(24*a^2
*c^2+60*a*b*c+35*b^2)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(9/2)/d^3
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{\frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(105b^3+5ab^2(43c+7dx)+2a^2b(59c^2+16cdx-7d^2x^2)+8a^3(c^3+d^3x^3)\right)}{b+a(c+dx)} - 3b(35b^2+60}{24a^{9/2}d^3}$$

input
```
Integrate[x^2/(a + b/(c + d*x))^(3/2),x]
```

output
```
((Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(105*b^3 + 5*a*b^2*(
43*c + 7*d*x) + 2*a^2*b*(59*c^2 + 16*c*d*x - 7*d^2*x^2) + 8*a^3*(c^3 + d^3
*x^3)))/(b + a*(c + d*x)) - 3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*ArcTanh[S
qrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(24*a^(9/2)*d^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 941, 948, 100, 27, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^2x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{941}$$

$$\frac{\int \frac{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{948}$$

$$-\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\frac{\int -\frac{(c+dx)^3\left(-\frac{6ac^2}{c+dx}+12ac+7b\right)}{2\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3}{3a\sqrt{a+\frac{b}{c+dx}}}}{d^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{(c+dx)^3\left(-\frac{6ac^2}{c+dx}+12ac+7b\right)}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{6a}-\frac{(c+dx)^3}{3a\sqrt{a+\frac{b}{c+dx}}}}{d^3}$$

$$\Big\downarrow \text{87}$$

$$-\frac{-\frac{-\frac{\left(24a^2c^2+60abc+35b^2\right)\int \frac{(c+dx)^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{4a}-\frac{(12ac+7b)(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{6a}-\frac{(c+dx)^3}{3a\sqrt{a+\frac{b}{c+dx}}}}{d^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(-\frac{3b\int \frac{c+dx}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}-\frac{(12ac+7b)(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{6a}-\frac{(c+dx)^3}{3a\sqrt{a+\frac{b}{c+dx}}}}{d^3}$$

$$\Big\downarrow \text{61}$$

$$-\frac{-\frac{-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(-\frac{3b\left(\frac{\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{a}+\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}-\frac{(12ac+7b)(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{6a}-\frac{(c+dx)^3}{3a\sqrt{a+\frac{b}{c+dx}}}}{d^3}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}-\frac{(12ac+7b)(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{6a}-\frac{(c+dx)^3}{3a\sqrt{a+\frac{b}{c+dx}}}}{d^3}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}-\frac{(12ac+7b)(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{6a}-\frac{(c+dx)^3}{3a\sqrt{a+\frac{b}{c+dx}}}}{d^3}$$

input
```
Int[x^2/(a + b/(c + d*x))^(3/2),x]
```

output
```
-((-1/3*(c + d*x)^3/(a*Sqrt[a + b/(c + d*x)]) - (-1/2*((7*b + 12*a*c)*(c +
 d*x)^2)/(a*Sqrt[a + b/(c + d*x)]) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(-(
(c + d*x)/(a*Sqrt[a + b/(c + d*x)])) - (3*b*(2/(a*Sqrt[a + b/(c + d*x)]) -
 (2*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a))/(6*a)
)/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2931 vs. $2(185) = 370$.

Time = 0.22 (sec) , antiderivative size = 2932, normalized size of antiderivative = 14.16

method	result	size
default	Expression too large to display	2932

input
```
int(x^2/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a^4/d^3*(72*ln(1/2*(2*a*d^2*x+
2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*
a^4*b*c^2*d^3*x^2+624*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^
(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^3*c*d^2*x-32*(a*d^2)^(1/2)*(
a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a^3*c*d*x-32*(a*d^2)^(1/2)*(a*d
^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*a^2*b*d*x-480*((a*d*x+a*c+b)*(d*x+
c))^(1/2)*(a*d^2)^(1/2)*a*b^3*d*x+150*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a
*c^2+b*d*x+b*c)^(1/2)*a^2*b^2*d^2*x^2-54*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^
2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a
^2*b^3*c*d^2*x-288*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^4*c^3*d*x
-384*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^3*b*c*d^2*x^2+300*(a*d^
2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^3*b*c*d^2*x^2-1056*
((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^3*b*c^2*d*x+420*(a*d^2)^(1/2
)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^3*b*c^2*d*x-1248*((a*d*x+a
*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*b^2*c*d*x+396*(a*d^2)^(1/2)*(a*d^2*
x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*b^2*c*d*x+192*ln(1/2*(2*a*d^2*x+2
*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a
^3*b^2*c*d^3*x^2-16*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3
/2)*a*b^2+144*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a
*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^4*b*c^3*d^2*x+120*(a*d^2)^(1/2)*(a*d^...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.76

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \left[\frac{3\,(24\,a^3bc^3+84\,a^2b^2c^2+95\,ab^3c+35\,b^4+(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3)dx)\sqrt{a}\log}{}\right.$$

input
```
integrate(x^2/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*
c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x)*sqrt(a)*log(2*a*d*x + 2*a*c - 2*(d*x +
 c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(8*a^4*d^4*x^4 + 8*
a^4*c^4 + 118*a^3*b*c^3 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^3 + 215*a^2*b^2*c^2
+ 105*a*b^3*c + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^2 + (8*a^4*c^3 + 150*a^3*b
*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/
(a^6*d^4*x + (a^6*c + a^5*b)*d^3), 1/24*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2
+ 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x)*sqrt
(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x +
a*c + b)) + (8*a^4*d^4*x^4 + 8*a^4*c^4 + 118*a^3*b*c^3 + 2*(4*a^4*c - 7*a^
3*b)*d^3*x^3 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (18*a^3*b*c + 35*a^2*b^2)*d
^2*x^2 + (8*a^4*c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x)*sqrt
((a*d*x + a*c + b)/(d*x + c)))/(a^6*d^4*x + (a^6*c + a^5*b)*d^3)]
```

Sympy [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(x**2/((a*c + a*d*x + b)/(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a + b/(d*x + c))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(185) = 370.

Time = 0.22 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.89

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{1}{24}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(2\,x\left(\frac{4\,x}{a^2d\mathrm{sgn}\left(dx+c\right)}-\frac{4\,a^{11}cd^6\mathrm{sgn}(dx+c)}{a}\right.\right.$$
$$+\frac{\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)a^{\frac{5}{2}}c^2|d|+6\right.\right.}{}$$
$$+\frac{\left(24\,a^{\frac{13}{2}}bc^2d^3|d|\mathrm{sgn}(dx+c)+60\,a^{\frac{11}{2}}b^2cd^3|d|\mathrm{sgn}(dx+c)+35\,a^{\frac{9}{2}}b^3d^3|d|\mathrm{sgn}(dx+c)\right)\log\left(|a|\right)}{48\,a^9d^7}$$

input
```
integrate(x^2/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*x*(4*x/(a^2*d*sg
n(d*x + c)) - (4*a^11*c*d^6*sgn(d*x + c) + 11*a^10*b*d^6*sgn(d*x + c))/(a^
13*d^8)) + (8*a^11*c^2*d^5*sgn(d*x + c) + 62*a^10*b*c*d^5*sgn(d*x + c) + 5
7*a^9*b^2*d^5*sgn(d*x + c))/(a^13*d^8)) + 1/48*(24*a^2*b*c^2 + 60*a*b^2*c
+ 35*b^3)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b*d*x + b*c))*a^(5/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x - sqrt(a
*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*a^2*c*d + 5*a^2*b*c^2*d + 2
*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*a^(
3/2)*abs(d) + 10*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d
*x + b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b*d*x + b*c))^2*a*b*d + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x - sqr
t(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*b^2*abs(d) + b^3*d
))/(a^(9/2)*d^2*abs(d)*sgn(d*x + c)) + 1/48*(24*a^(13/2)*b*c^2*d^3*abs(d)*
sgn(d*x + c) + 60*a^(11/2)*b^2*c*d^3*abs(d)*sgn(d*x + c) + 35*a^(9/2)*b^3*
d^3*abs(d)*sgn(d*x + c))*log(abs(a))/(a^9*d^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{x^2}{{\left(a+\frac{b}{c+d\,x}\right)}^{3/2}} \,d x$$

input
```
int(x^2/(a + b/(c + d*x))^(3/2),x)
```

output
```
int(x^2/(a + b/(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{-576\sqrt{a}\,\sqrt{adx+ac+b}\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)a^2b\,c^2-1440\sqrt{a}\,\sqrt{adx+ac+b}\,\mathrm{lo}$$

input
```
int(x^2/(a+b/(d*x+c))^(3/2),x)
```

output
```
( - 576*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)
*sqrt(c + d*x))/sqrt(b))*a**2*b*c**2 - 1440*sqrt(a)*sqrt(a*c + a*d*x + b)*
log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**2*c - 84
0*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(
c + d*x))/sqrt(b))*b**3 + 432*sqrt(a)*sqrt(a*c + a*d*x + b)*a**2*b*c**2 +
960*sqrt(a)*sqrt(a*c + a*d*x + b)*a*b**2*c + 525*sqrt(a)*sqrt(a*c + a*d*x
+ b)*b**3 + 64*sqrt(c + d*x)*a**4*c**3 + 64*sqrt(c + d*x)*a**4*d**3*x**3 +
 944*sqrt(c + d*x)*a**3*b*c**2 + 256*sqrt(c + d*x)*a**3*b*c*d*x - 112*sqrt
(c + d*x)*a**3*b*d**2*x**2 + 1720*sqrt(c + d*x)*a**2*b**2*c + 280*sqrt(c +
 d*x)*a**2*b**2*d*x + 840*sqrt(c + d*x)*a*b**3)/(192*sqrt(a*c + a*d*x + b)
*a**5*d**3)
```

3.46 $\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 143

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = -\frac{2b(b+ac)}{a^3d^2\sqrt{a+\frac{b}{c+dx}}} - \frac{(7b+4ac)(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4a^3d^2} + \frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2a^2d^2} + \frac{3b(5b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{4a^{7/2}d^2}$$

output

```
-2*b*(a*c+b)/a^3/d^2/(a+b/(d*x+c))^(1/2)-1/4*(4*a*c+7*b)*(d*x+c)*(a+b/(d*x
+c))^(1/2)/a^3/d^2+1/2*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/a^2/d^2+3/4*b*(4*a*c+
5*b)*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(7/2)/d^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{-\frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(15b^2+ab(17c+5dx)+2a^2\left(c^2-d^2x^2\right)\right)}{b+a(c+dx)}+3b(5b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{4a^{7/2}d^2}$$

input
```
Integrate[x/(a + b/(c + d*x))^(3/2),x]
```

output
```
(-((Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(15*b^2 + a*b*(17*
c + 5*d*x) + 2*a^2*(c^2 - d^2*x^2)))/(b + a*(c + d*x))) + 3*b*(5*b + 4*a*c
)*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(4*a^(7/2)*d^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {896, 25, 941, 948, 25, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{dx}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{dx}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{941}$$

$$-\frac{\int \frac{(c+dx)\left(\frac{c}{c+dx}-1\right)}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d^2}$$

$\Big\downarrow$ 948

$$\frac{\int -\frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{d^2}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{d^2}$$

$\Big\downarrow$ 87

$$\frac{\frac{(4ac+5b)\int \frac{(c+dx)^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{4a}+\frac{(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{d^2}$$

$\Big\downarrow$ 52

$$\frac{\frac{(4ac+5b)\left(-\frac{3b\int \frac{c+dx}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}+\frac{(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{d^2}$$

$\Big\downarrow$ 61

$$\frac{\frac{(4ac+5b)\left(-\frac{3b\left(\frac{\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{a}+\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}+\frac{(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{d^2}$$

$\Big\downarrow$ 73

$$\frac{\frac{(4ac+5b)\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}+\frac{(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{d^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(4ac+5b)\left(-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{4a}+\frac{(c+dx)^2}{2a\sqrt{a+\frac{b}{c+dx}}}}{d^2}$$

input
```
Int[x/(a + b/(c + d*x))^(3/2),x]
```

output
```
((c + d*x)^2/(2*a*Sqrt[a + b/(c + d*x)]) + ((5*b + 4*a*c)*(-((c + d*x)/(a*
Sqrt[a + b/(c + d*x)])) - (3*b*(2/(a*Sqrt[a + b/(c + d*x)]) - (2*ArcTanh[S
qrt[a + b/(c + d*x)]/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1887 vs. $2(125) = 250$.

Time = 0.23 (sec) , antiderivative size = 1888, normalized size of antiderivative = 13.20

method	result	size
default	Expression too large to display	1888

input
```
int(x/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a^3/d^2*(20*(a*d^2)^(1/2)*(a*d^2
*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*b*c*d*x-112*(a*d^2)^(1/2)*((a*d*
x+a*c+b)*(d*x+c))^(1/2)*a^2*b*c*d*x+12*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+
a*c^2+b*d*x+b*c)^(1/2)*a^3*c^2*d*x+8*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(1/2)*a*b^2*d*x+56*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+
b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c*d^2*x-64*(a*
d^2)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a*b^2*d*x+40*ln(1/2*(2*a*d^2*x+2*
a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^
2*b^2*c^2*d-80*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a^2*b*c^2+44*ln
(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)
/(a*d^2)^(1/2))*a*b^3*c*d-ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x
+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d-2*
ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a
*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^3*c*d+10*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*
c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*b*c^2+8*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*
x+a*c^2+b*d*x+b*c)^(1/2)*a*b^2*c-24*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^
(1/2)*a^3*c^3+4*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*
a^3*c^3+12*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^
2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*b*c*d^3*x^2+16*(a*d^2)^(1/2)*((a*d*x+a*c+
b)*(d*x+c))^(3/2)*a^2*c+16*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(3/2)*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.06

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \left[\frac{3\left(4a^2bc^2+9ab^2c+5b^3+(4a^2bc+5ab^2)dx\right)\sqrt{a}\log\left(2adx+2ac+2(dx+c)\sqrt{\right.}\right.}{}\right.$$

$$-\frac{3\left(4a^2bc^2+9ab^2c+5b^3+(4a^2bc+5ab^2)dx\right)\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-(2a^3d^3x^3-2a^3c^3-}{4\left(a^5d^3x+(a^5c+a^4b)\right.}$$

input
```
integrate(x/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(4*a^2*b*c^2 + 9*a*b^2*c + 5*b^3 + (4*a^2*b*c + 5*a*b^2)*d*x)*sqrt
(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x
+ c)) + b) + 2*(2*a^3*d^3*x^3 - 2*a^3*c^3 - 17*a^2*b*c^2 + (2*a^3*c - 5*a^
2*b)*d^2*x^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x)*sqrt(
(a*d*x + a*c + b)/(d*x + c)))/(a^5*d^3*x + (a^5*c + a^4*b)*d^2), -1/4*(3*(
4*a^2*b*c^2 + 9*a*b^2*c + 5*b^3 + (4*a^2*b*c + 5*a*b^2)*d*x)*sqrt(-a)*arct
an((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b))
 - (2*a^3*d^3*x^3 - 2*a^3*c^3 - 17*a^2*b*c^2 + (2*a^3*c - 5*a^2*b)*d^2*x^2
 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x)*sqrt((a*d*x + a*c
 + b)/(d*x + c)))/(a^5*d^3*x + (a^5*c + a^4*b)*d^2)]
```

Sympy [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{x}{\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(x/((a*c + a*d*x + b)/(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{x}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a + b/(d*x + c))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(125) = 250$.

Time = 0.20 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.50

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{1}{4}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(\frac{2\,x}{a^2d\mathrm{sgn}\left(dx+c\right)}-\frac{2\,a^6cd^2+7\,a^5bd^2}{a^8d^4\mathrm{sgn}\left(dx+c\right)}\right)$$
$$-\frac{\left(4\,abc+5\,b^2\right)\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)a^{\frac{5}{2}}c^2|d|+6\left(\sqrt{ad^2}x-\sqrt{a\cdots}\right.\right.\right.}{\cdots}$$
$$-\frac{\left(4\,a^{\frac{9}{2}}bcd^2|d|\mathrm{sgn}(dx+c)+5\,a^{\frac{7}{2}}b^2d^2|d|\mathrm{sgn}(dx+c)\right)\log\left(|a|\right)}{8\,a^7d^5}$$

input
```
integrate(x/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*x/(a^2*d*sgn(d*x
+ c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x + c))) - 1/8*(4*a*b*c
 + 5*b^2)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b*d*x + b*c))*a^(5/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x - sqrt(a
*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*a^2*c*d + 5*a^2*b*c^2*d + 2
*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*a^(
3/2)*abs(d) + 10*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d
*x + b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b*d*x + b*c))^2*a*b*d + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x - sqr
t(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*b^2*abs(d) + b^3*d
))/(a^(7/2)*d*abs(d)*sgn(d*x + c)) - 1/8*(4*a^(9/2)*b*c*d^2*abs(d)*sgn(d*x
 + c) + 5*a^(7/2)*b^2*d^2*abs(d)*sgn(d*x + c))*log(abs(a))/(a^7*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{x}{\left(a+\frac{b}{c+d\,x}\right)^{3/2}}\,dx$$

input
```
int(x/(a + b/(c + d*x))^(3/2),x)
```

output
```
int(x/(a + b/(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.48

$$\int \frac{x}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{12\sqrt{a}\,\sqrt{adx+ac+b}\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)abc+15\sqrt{a}\,\sqrt{adx+ac+b}\log\left(\frac{\sqrt{adx}\cdots}{}\right.}{}$$

input
```
int(x/(a+b/(d*x+c))^(3/2),x)
```

output
```
(12*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqr
t(c + d*x))/sqrt(b))*a*b*c + 15*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*
c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**2 - 9*sqrt(a)*sqrt(a*c
 + a*d*x + b)*a*b*c - 10*sqrt(a)*sqrt(a*c + a*d*x + b)*b**2 - 2*sqrt(c + d
*x)*a**3*c**2 + 2*sqrt(c + d*x)*a**3*d**2*x**2 - 17*sqrt(c + d*x)*a**2*b*c
 - 5*sqrt(c + d*x)*a**2*b*d*x - 15*sqrt(c + d*x)*a*b**2)/(4*sqrt(a*c + a*d
*x + b)*a**4*d**2)
```

3.47 $\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{2b}{a^2d\sqrt{a+\frac{b}{c+dx}}}+\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{a^2d}-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{5/2}d}$$

output
```
2*b/a^2/d/(a+b/(d*x+c))^(1/2)+(d*x+c)*(a+b/(d*x+c))^(1/2)/a^2/d-3*b*arctan
h((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(3b+ac+adx)}{a^2d(b+ac+adx)}-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{a^{5/2}d}$$

input
```
Integrate[(a + b/(c + d*x))^(-3/2),x]
```

output
```
((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(3*b + a*c + a*d*x))/(a^2*d*(
b + a*c + a*d*x)) - (3*b*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]
])/(a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {239, 773, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int \frac{(c+dx)^2}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{3b\int \frac{c+dx}{\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{-\frac{3b\left(\frac{\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{a}+\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}}{d}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{c+dx}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{c+dx}{a\sqrt{a+\frac{b}{c+dx}}}}{d}$$

input
```
Int[(a + b/(c + d*x))^(-3/2),x]
```

output
```
-((-((c + d*x)/(a*Sqrt[a + b/(c + d*x)])) - (3*b*(2/(a*Sqrt[a + b/(c + d*x
)]) - (2*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/a^(3/2)))/(2*a))/d)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(75) = 150$.

Time = 0.20 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

method	result
default	$-\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(3\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^2b\,d^3x^2-6\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}\,a^2d^2x^2+6\ln\left(\right.\right.}{}$

input
```
int(1/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a^2/d*(3*ln(1/2*(2*a*d^2*x+2*a*
c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*
b*d^3*x^2-6*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*d^2*x^2+6*ln(1
/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(
a*d^2)^(1/2))*a^2*b*c*d^2*x-12*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)
*a^2*c*d*x+3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*
d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d+6*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(
(a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*d^2*x
-6*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*c^2-12*((a*d*x+a*c+b)*(
d*x+c))^(1/2)*(a*d^2)^(1/2)*a*b*d*x+6*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+
a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d+4*a*((a*
d*x+a*c+b)*(d*x+c))^(3/2)*(a*d^2)^(1/2)-12*(a*d^2)^(1/2)*((a*d*x+a*c+b)*(d
*x+c))^(1/2)*a*b*c+3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(
1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d-6*(a*d^2)^(1/2)*((a*d*x+a*c+b
)*(d*x+c))^(1/2)*b^2)/((a*d*x+a*c+b)*(d*x+c))^(1/2)/(a*d*x+a*c+b)^2/(a*d^2
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.41

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \left[\frac{3\left(abdx+abc+b^2\right)\sqrt{a}\log\left(2\,adx+2\,ac-2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)+2\,(a^2d^2}{2\,(a^4d^2x+(a^4c+a^3b)d)}\right.$$

input
```
integrate(1/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*d*x + a*b*c + b^2)*sqrt(a)*log(2*a*d*x + 2*a*c - 2*(d*x + c)*
sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(a^2*d^2*x^2 + a^2*c^2
+ 3*a*b*c + (2*a^2*c + 3*a*b)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^4
*d^2*x + (a^4*c + a^3*b)*d), (3*(a*b*d*x + a*b*c + b^2)*sqrt(-a)*arctan((d
*x + c)*sqrt(-a)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)) + (a
^2*d^2*x^2 + a^2*c^2 + 3*a*b*c + (2*a^2*c + 3*a*b)*d*x)*sqrt((a*d*x + a*c
+ b)/(d*x + c)))/(a^4*d^2*x + (a^4*c + a^3*b)*d)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral((a + b/(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^(-3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(75) = 150.

Time = 0.19 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.94

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{b\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)a^{\frac{5}{2}}c^2|d|+6\left(\sqrt{ad\right.\right.}{}$$

$$+\frac{b|d|\log\left(|a|\right)}{2\,a^{\frac{5}{2}}d^2\mathrm{sgn}\left(dx+c\right)}+\frac{\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}}{a^2d\mathrm{sgn}\left(dx+c\right)}$$

input
```
integrate(1/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/2*b*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b*d*x + b*c))*a^(5/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x - sqrt(a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^2*a^2*c*d + 5*a^2*b*c^2*d + 2*(sq
rt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))^3*a^(3/2)
*abs(d) + 10*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x +
 b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b*d*x + b*c))^2*a*b*d + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x - sqrt(a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*b^2*abs(d) + b^3*d))/(
a^(5/2)*abs(d)*sgn(d*x + c)) + 1/2*b*abs(d)*log(abs(a))/(a^(5/2)*d^2*sgn(d
*x + c)) + sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)/(a^2*d*sgn(d*
x + c))
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = -\frac{\frac{2\,b}{a}-\frac{3\,b\,\left(a+\frac{b}{c+d\,x}\right)}{a^2}}{d\,\left(a+\frac{b}{c+d\,x}\right)^{3/2}-a\,d\,\sqrt{a+\frac{b}{c+d\,x}}}-\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{c+d\,x}}}{\sqrt{a}}\right)}{a^{5/2}\,d}$$

input
```
int(1/(a + b/(c + d*x))^(3/2),x)
```

output
```
- ((2*b)/a - (3*b*(a + b/(c + d*x)))/a^2)/(d*(a + b/(c + d*x))^(3/2) - a*d
*(a + b/(c + d*x))^(1/2)) - (3*b*atanh((a + b/(c + d*x))^(1/2)/a^(1/2)))/(
a^(5/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{-12\sqrt{a}\,\sqrt{adx+ac+b}\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)b+9\sqrt{a}\,\sqrt{adx+ac+b}\,b+4\sqrt{dx-}}{4\sqrt{adx+ac+b}\,a^3d}$$

input
```
int(1/(a+b/(d*x+c))^(3/2),x)
```

output
```
( - 12*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*
sqrt(c + d*x))/sqrt(b))*b + 9*sqrt(a)*sqrt(a*c + a*d*x + b)*b + 4*sqrt(c +
 d*x)*a**2*c + 4*sqrt(c + d*x)*a**2*d*x + 12*sqrt(c + d*x)*a*b)/(4*sqrt(a*
c + a*d*x + b)*a**3*d)
```

3.48 $\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=-\frac{2b}{a(b+ac)\sqrt{a+\frac{b}{c+dx}}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{(b+ac)^{3/2}}$$

output
```
-2*b/a/(a*c+b)/(a+b/(d*x+c))^(1/2)+2*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/
a^(3/2)-2*c^(3/2)*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/(a*c+
b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=-\frac{2b}{a(b+ac)\sqrt{\frac{b+ac+adx}{c+dx}}}-\frac{2c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a + b/(c + d*x))^(3/2)),x]
```

output
```
(-2*b)/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]) - (2*c^(3/2)*ArcTan
[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(-b - a*c)^(
3/2) + (2*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 25, 941, 948, 25, 96, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \ 896$$

$$\int \frac{1}{dx\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \ 25$$

$$-\int -\frac{1}{dx\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \ 941$$

$$-\int \frac{1}{(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$

$$\Big\downarrow \ 948$$

$$\int -\frac{c+dx}{\left(1-\frac{c}{c+dx}\right)\left(a+\frac{b}{c+dx}\right)^{3/2}}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 25$$

$$-\int \frac{c+dx}{\left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$\Big\downarrow$ 96

$$\frac{\int -\frac{(c+dx)\left(-\frac{cb}{c+dx}+b+ac\right)}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{a(ac+b)}-\frac{2b}{a(ac+b)\sqrt{a+\frac{b}{c+dx}}}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{(c+dx)\left(-\frac{cb}{c+dx}+b+ac\right)}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{a(ac+b)}-\frac{2b}{a(ac+b)\sqrt{a+\frac{b}{c+dx}}}$$

$\Big\downarrow$ 174

$$-\frac{ac^2\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}+(ac+b)\int \frac{c+dx}{\sqrt{a+\frac{b}{c+dx}}}d\frac{1}{c+dx}}{a(ac+b)}-\frac{2b}{a(ac+b)\sqrt{a+\frac{b}{c+dx}}}$$

$\Big\downarrow$ 73

$$-\frac{\frac{2ac^2\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{b}+\frac{2(ac+b)\int \frac{1}{\frac{1}{b(c+dx)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{c+dx}}}{b}}{a(ac+b)}-\frac{2b}{a(ac+b)\sqrt{a+\frac{b}{c+dx}}}$$

$\Big\downarrow$ 221

$$-\frac{\frac{2ac^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}-\frac{2(ac+b)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(ac+b)}-\frac{2b}{a(ac+b)\sqrt{a+\frac{b}{c+dx}}}$$

input
```
Int[1/(x*(a + b/(c + d*x))^(3/2)),x]
```

output
```
(-2*b)/(a*(b + a*c)*Sqrt[a + b/(c + d*x)]) - ((-2*(b + a*c)*ArcTanh[Sqrt[a
 + b/(c + d*x)]/Sqrt[a]])/Sqrt[a] + (2*a*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a +
 b/(c + d*x)])/Sqrt[b + a*c]])/Sqrt[b + a*c])/(a*(b + a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3023 vs. $2(87) = 174$.

Time = 0.21 (sec) , antiderivative size = 3024, normalized size of antiderivative = 28.80

method	result	size
default	Expression too large to display	3024

input
```
int(1/x/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a*(4*ln(1/2*(2*a*d^2*x+2*a*c*d+2
*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1
/2))*((a*c+b)*c)^(1/2)*a^4*c^3*d^2*x-4*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2
+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b
*c)/x)*a^4*c^4*d*x+4*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(
1/2)*((a*c+b)*c)^(1/2)*a^3*c^2*d*x+ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+
2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b
)*c)^(1/2)*a^3*b*c*d^3*x^2+7*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2
*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^3*c*d
-8*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b^2*d*x
+2*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)
*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a^4*c^2*d^3*x^2-8*(a*
d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*
c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a^3*b*c^3*d*x+6*((a*c+b)*c)^(1/2)*l
n(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d
)/(a*d^2)^(1/2))*a^3*b*c^2*d^2*x-12*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^
(1/2)*((a*c+b)*c)^(1/2)*a^3*c^2*d*x-4*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2
)^(1/2)*((a*c+b)*c)^(1/2)*a^2*b*d^2*x^2+10*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d
^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1
/2))*a^2*b^2*c*d^2*x+3*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(87) = 174$.

Time = 0.13 (sec) , antiderivative size = 938, normalized size of antiderivative = 8.93

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[((a^2*c^2 + 2*a*b*c + (a^2*c + a*b)*d*x + b^2)*sqrt(a)*log(2*a*d*x + 2*a*
c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + (a^3*c*d*
x + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(-(2*a*c^2 + (2*a*c + b)*d*x +
 2*b*c - 2*(a*c^2 + (a*c + b)*d*x + b*c)*sqrt((a*d*x + a*c + b)/(d*x + c))
*sqrt(c/(a*c + b)))/x) - 2*(a*b*d*x + a*b*c)*sqrt((a*d*x + a*c + b)/(d*x +
 c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x), (2*(a^3*c*d*x
 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(sqrt((a*d*x + a*c + b)/(d*
x + c))*sqrt(-c/(a*c + b))) + (a^2*c^2 + 2*a*b*c + (a^2*c + a*b)*d*x + b^2
)*sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)
/(d*x + c)) + b) - 2*(a*b*d*x + a*b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/
(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x), -(2*(a^2*c^2 + 2*a*
b*c + (a^2*c + a*b)*d*x + b^2)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*
d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)) - (a^3*c*d*x + a^3*c^2 + a^2*
b*c)*sqrt(c/(a*c + b))*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c - 2*(a*c^2
+ (a*c + b)*d*x + b*c)*sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt(c/(a*c + b))
)/x) + 2*(a*b*d*x + a*b*c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^4*c^2 + 2
*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x), -2*((a^2*c^2 + 2*a*b*c + (a^2*c
 + a*b)*d*x + b^2)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c +
b)/(d*x + c))/(a*d*x + a*c + b)) - (a^3*c*d*x + a^3*c^2 + a^2*b*c)*sqrt(-c
/(a*c + b))*arctan(sqrt((a*d*x + a*c + b)/(d*x + c))*sqrt(-c/(a*c + b))...
```

Sympy [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{x\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(1/(x*((a*c + a*d*x + b)/(c + d*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c))^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x))^(3/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.97

$$\int \frac{1}{x\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{ac+b}\,\sqrt{adx+ac+b}\log\left(\sqrt{adx+ac+b}-\sqrt{2\sqrt{c}\,\sqrt{a}\,\sqrt{ac+b}+2ac+b+}\right.}{}$$

input
```
int(1/x/(a+b/(d*x+c))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b) - s
qrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*
a**2*c + sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x
+ b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c
+ d*x))*a**2*c - sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(2*sqrt(a)
*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2
*a*d*x)*a**2*c + 2*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b
) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*c**2 + 4*sqrt(a)*sqrt(a*c + a*d*x
 + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b*c +
 2*sqrt(a)*sqrt(a*c + a*d*x + b)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt
(c + d*x))/sqrt(b))*b**2 - 2*sqrt(a)*sqrt(a*c + a*d*x + b)*a*b*c - 2*sqrt(
a)*sqrt(a*c + a*d*x + b)*b**2 - 2*sqrt(c + d*x)*a**2*b*c - 2*sqrt(c + d*x)
*a*b**2)/(sqrt(a*c + a*d*x + b)*a**2*(a**2*c**2 + 2*a*b*c + b**2))
```

3.49 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\frac{2bd}{(b+ac)^2\sqrt{a+\frac{b}{c+dx}}}-\frac{c(c+dx)\sqrt{a+\frac{b}{c+dx}}}{(b+ac)^2x}-\frac{3b\sqrt{c}d\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{(b+ac)^{5/2}}$$

output
```
2*b*d/(a*c+b)^2/(a+b/(d*x+c))^(1/2)-c*(d*x+c)*(a+b/(d*x+c))^(1/2)/(a*c+b)^
2/x-3*b*c^(1/2)*d*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/(a*c+
b)^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=-\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(b(c-2dx)+ac(c+dx))}{(b+ac)^2x(b+a(c+dx))}+\frac{3b\sqrt{c}d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{5/2}}$$

input
```
Integrate[1/(x^2*(a + b/(c + d*x))^(3/2)),x]
```

output
```
-(((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(b*(c - 2*d*x) + a*c*(c + d
*x)))/((b + a*c)^2*x*(b + a*(c + d*x)))) + (3*b*Sqrt[c]*d*ArcTan[(Sqrt[c]*
Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(-b - a*c)^(5/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 941, 946, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 896}$$

$$d\int \frac{1}{d^2x^2 \left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$d\int \frac{1}{(c+dx)^2 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{ 946}$$

$$-d\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 52}$$

$$-d\left(\frac{3b\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \text{ 61}$$

$$-d\left(\frac{3b\left(\frac{c\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{ac+b}-\frac{2}{(ac+b)\sqrt{a+\frac{b}{c+dx}}}\right)}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \text{73}$$

$$-d\left(\frac{3b\left(\frac{2c\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{b(ac+b)}-\frac{2}{(ac+b)\sqrt{a+\frac{b}{c+dx}}}\right)}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \text{221}$$

$$-d\left(\frac{3b\left(\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{2}{(ac+b)\sqrt{a+\frac{b}{c+dx}}}\right)}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)$$

input
```
Int[1/(x^2*(a + b/(c + d*x))^(3/2)),x]
```

output
```
-(d*(1/((b + a*c)*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))) + (3*b*(-2/((b
+ a*c)*Sqrt[a + b/(c + d*x)]) + (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c
+ d*x)])/Sqrt[b + a*c]])/(b + a*c)^(3/2)))/(2*(b + a*c))))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3645 vs. $2(95) = 190$.

Time = 0.22 (sec) , antiderivative size = 3646, normalized size of antiderivative = 33.45

method	result	size
default	Expression too large to display	3646

input
```
int(1/x^2/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*(-10*(a*d^2*x^2+2*a*c*d*x+a*c^2
+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b^2*d^2*x^2-5*ln(1/2*(
2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1
/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a*b^3*c*d^2*x+9*(a*d^2)^(1/2)*ln
((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b
*d*x+b*c)^(1/2)+2*b*c)/x)*a*b^3*c^2*d*x+5*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^
2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/
2))*a*b^3*c*d^2*x-3*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2
+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a^3*
b*c*d^4*x^3+3*(a*d^2)^(1/2)*ln((2*a*d*x*c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2
)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)+2*b*c)/x)*a^3*b*c^2*d^3*x^3+
3*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1
/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*b*c*d^4*x^3-6*((a*d*x+a*c+b)*(d*
x+c))^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^3*c*d^3*x^3-2*(a*d^2*x^2+2*a
*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^3*d^4*x^4-
ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a
*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a^2*b^2*d^4*x^3+((a*c+b)
*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2
)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*d^4*x^3+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b
*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*d^2*x^2-2*((a*d*x+a...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.83

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \left[\frac{3\left(abd^2x^2+(abc+b^2)dx\right)\sqrt{\frac{c}{ac+b}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx+2\,bc-2\,(ac^2+(ac+b)dx+bc)\sqrt{\frac{adx\cdots}{d\cdots}}\cdots}{x}\cdots\right.}{2\left((a^3c^2+2\,a^2bc+ab^2)dx^2+(a^3c\cdots\right.}\cdots\right.$$

input
```
integrate(1/x^2/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*d^2*x^2 + (a*b*c + b^2)*d*x)*sqrt(c/(a*c + b))*log(-(2*a*c^2
+ (2*a*c + b)*d*x + 2*b*c - 2*(a*c^2 + (a*c + b)*d*x + b*c)*sqrt((a*d*x +
a*c + b)/(d*x + c))*sqrt(c/(a*c + b)))/x) - 2*((a*c - 2*b)*d^2*x^2 + a*c^3
 + b*c^2 + (2*a*c^2 - b*c)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^3*c
^2 + 2*a^2*b*c + a*b^2)*d*x^2 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*
x), (3*(a*b*d^2*x^2 + (a*b*c + b^2)*d*x)*sqrt(-c/(a*c + b))*arctan(sqrt((a
*d*x + a*c + b)/(d*x + c))*sqrt(-c/(a*c + b))) - ((a*c - 2*b)*d^2*x^2 + a*
c^3 + b*c^2 + (2*a*c^2 - b*c)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^
3*c^2 + 2*a^2*b*c + a*b^2)*d*x^2 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^
3)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(1/(x**2*((a*c + a*d*x + b)/(c + d*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+\frac{b}{c+d\,x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 649, normalized size of antiderivative = 5.95

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{12\sqrt{c}\,\sqrt{ac+b}\,\sqrt{adx+ac+b}\,\log\left(\sqrt{adx+ac+b}-\sqrt{2\sqrt{c}\,\sqrt{a}\,\sqrt{ac+b}+2ac+b}\right.}{}$$

input
```
int(1/x^2/(a+b/(d*x+c))^(3/2),x)
```

output
```
(12*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b)
- sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x
))*a**2*b*c*d*x + 3*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a
*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt
(a)*sqrt(c + d*x))*a*b**2*d*x + 12*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x
+ b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*
a*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b*c*d*x + 3*sqrt(c)*sqrt(a*c + b)*s
qrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sq
rt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**2*d*x - 12*sqrt(c)*
sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c +
 a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*d*x)*a**2*b*c*d*x - 3*
sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(2*sqrt(a)*sqrt(c + d*x)*sq
rt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*d*x)*a*b**2*d*
x - 18*sqrt(a)*sqrt(a*c + a*d*x + b)*a**2*b*c**2*d*x - 30*sqrt(a)*sqrt(a*c
 + a*d*x + b)*a*b**2*c*d*x - 12*sqrt(a)*sqrt(a*c + a*d*x + b)*b**3*d*x - 8
*sqrt(c + d*x)*a**4*c**4 - 8*sqrt(c + d*x)*a**4*c**3*d*x - 18*sqrt(c + d*x
)*a**3*b*c**3 + 6*sqrt(c + d*x)*a**3*b*c**2*d*x - 12*sqrt(c + d*x)*a**2*b*
*2*c**2 + 18*sqrt(c + d*x)*a**2*b**2*c*d*x - 2*sqrt(c + d*x)*a*b**3*c + 4*
sqrt(c + d*x)*a*b**3*d*x)/(2*sqrt(a*c + a*d*x + b)*a*x*(4*a**4*c**4 + 13*a
**3*b*c**3 + 15*a**2*b**2*c**2 + 7*a*b**3*c + b**4))
```

3.50 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 168

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=-\frac{2abd^2}{(b+ac)^3\sqrt{a+\frac{b}{c+dx}}}-\frac{(3b-4ac)d(c+dx)\sqrt{a+\frac{b}{c+dx}}}{4(b+ac)^3x}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{2(b+ac)^2x^2}-\frac{3b(b-4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{4\sqrt{c}(b+ac)^{7/2}}$$

output
```
-2*a*b*d^2/(a*c+b)^3/(a+b/(d*x+c))^(1/2)-1/4*(-4*a*c+3*b)*d*(d*x+c)*(a+b/(
d*x+c))^(1/2)/(a*c+b)^3/x-1/2*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/(a*c+b)^2/x^2-
3/4*b*(-4*a*c+b)*d^2*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/c^
(1/2)/(a*c+b)^(7/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = -\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(b^2(2c+5dx)+2a^2c(c^2-d^2x^2)+ab(4c^2+5cdx+13d^2x^2)\right)}{4(b+ac)^3x^2(b+a(c+dx))} - \frac{3b(b-4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{4\sqrt{c}(-b-ac)^{7/2}}$$

input
```
Integrate[1/(x^3*(a + b/(c + d*x))^(3/2)),x]
```

output
```
-1/4*((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(b^2*(2*c + 5*d*x) + 2*a
^2*c*(c^2 - d^2*x^2) + a*b*(4*c^2 + 5*c*d*x + 13*d^2*x^2)))/((b + a*c)^3*x
^2*(b + a*(c + d*x))) - (3*b*(b - 4*a*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[(b + a*c
 + a*d*x)/(c + d*x)])/Sqrt[-b - a*c]])/(4*Sqrt[c]*(-b - a*c)^(7/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 25, 941, 948, 25, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \text{896}$$

$$d^2\int \frac{1}{d^3x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-d^2\int -\frac{1}{d^3x^3\left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \ 941$$

$$-d^2\int \frac{1}{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \ 948$$

$$d^2\int -\frac{1}{(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 25$$

$$-d^2\int \frac{1}{(c+dx)\left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 87$$

$$d^2\left(-\frac{(b-4ac)\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4c(ac+b)}-\frac{1}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \ 52$$

$$d^2\left(-\frac{(b-4ac)\left(\frac{3b\int \frac{1}{\left(a+\frac{b}{c+dx}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)}{4c(ac+b)}-\frac{1}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{c+dx}}}\right.$$

$$\Big\downarrow \ 61$$

$$d^2\left(-\frac{(b-4ac)\left(\frac{3b\left(\frac{c\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{ac+b}-\frac{2}{(ac+b)\sqrt{a+\frac{b}{c+dx}}}\right)}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)}{4c(ac+b)}-\frac{1}{2c(ac+b)\left(1-\frac{c}{c+d}\right.}\right.$$

$$\Big\downarrow \; 73$$

$$d^2\left(-\frac{(b-4ac)\left(\frac{3b\left(\frac{2c\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{b(ac+b)}-\frac{2}{(ac+b)\sqrt{a+\frac{b}{c+dx}}}\right)}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)}{4c(ac+b)}-\frac{1}{2c(ac+b)\left(1-\frac{}{c+}\right.}\right.$$

$$\Big\downarrow \; 221$$

$$d^2\left(-\frac{(b-4ac)\left(\frac{3b\left(\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{2}{(ac+b)\sqrt{a+\frac{b}{c+dx}}}\right)}{2(ac+b)}+\frac{1}{(ac+b)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{c+dx}}}\right)}{4c(ac+b)}-\frac{1}{2c(ac+b)\left(1-\frac{c}{c+d}\right.}\right.$$

input
```
Int[1/(x^3*(a + b/(c + d*x))^(3/2)),x]
```

output
```
d^2*(-1/2*1/(c*(b + a*c)*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))^2) - ((b
- 4*a*c)*(1/((b + a*c)*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))) + (3*b*(-2
/((b + a*c)*Sqrt[a + b/(c + d*x)]) + (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a +
b/(c + d*x)])/Sqrt[b + a*c]])/(b + a*c)^(3/2)))/(2*(b + a*c))))/(4*c*(b +
a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. $2(146) = 292$.

Time = 0.22 (sec) , antiderivative size = 4002, normalized size of antiderivative = 23.82

method	result	size
default	Expression too large to display	4002

input
```
int(1/x^3/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/c*(6*(a*d^2)^(1/2)*ln((2*a*d*x*
c+2*a*c^2+b*d*x+2*((a*c+b)*c)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^
(1/2)+2*b*c)/x)*a*b^4*c*d^3*x^3-12*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x+2*a
*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2
*b^3*c^2*d^3*x^2+30*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1
/2)*((a*c+b)*c)^(1/2)*a^2*b^2*c^2*d^2*x^2+48*(a*d^2)^(1/2)*((a*c+b)*c)^(1/
2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a^3*b*c^2*d^3*x^3-8*(a*d^2*x^2+2*a*c*d*x+
a*c^2+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*b*c*d^2*x^2+24*
(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c+b)*(d*x+c))^(1/2)*a^2*b^2*c^2*
d^2*x^2+66*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c
+b)*c)^(1/2)*a^3*b*c^2*d^3*x^3+24*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1
/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^3*b*c*d^4*x^4-18*(a*d^2*x^2+2*a*c*d*
x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b^3*d^3*x^3+36*
(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2
)*a^3*b*c^3*d^2*x^2+24*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*a
^3*b^2*c^3*d^3*x^2+48*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((a*d*x+a*c+b)*(d*x+
c))^(1/2)*a^3*b*c^3*d^2*x^2-2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(3/2)*
(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a^2*b*c^2*d*x+8*(a*d^2*x^2+2*a*c*d*x+a*c^2
+b*d*x+b*c)^(3/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*a*b^2*c*d*x+12*(a*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(148) = 296$.

Time = 0.10 (sec) , antiderivative size = 810, normalized size of antiderivative = 4.82

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*((4*a^2*b*c - a*b^2)*d^3*x^3 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2
*x^2)*sqrt(a*c^2 + b*c)*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c + 2*sqrt(a
*c^2 + b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/x) - 2*(2*a^3*c^6
 + 6*a^2*b*c^5 + 6*a*b^2*c^4 - (2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3
*x^3 + 2*b^3*c^3 - (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2
*x^2 + (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x)*sqrt((a*
d*x + a*c + b)/(d*x + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2
*b^3*c^2 + a*b^4*c)*d*x^3 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a
^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^2), -1/4*(3*((4*a^2*b*c - a*b^2)*d^3*x
^3 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^2)*sqrt(-a*c^2 - b*c)*arctan(sq
rt(-a*c^2 - b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x + c))/(a*c*d*x + a*
c^2 + b*c)) + (2*a^3*c^6 + 6*a^2*b*c^5 + 6*a*b^2*c^4 - (2*a^3*c^3 - 11*a^2
*b*c^2 - 13*a*b^2*c)*d^3*x^3 + 2*b^3*c^3 - (2*a^3*c^4 - 16*a^2*b*c^3 - 23*
a*b^2*c^2 - 5*b^3*c)*d^2*x^2 + (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 +
7*b^3*c^2)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^5*c^5 + 4*a^4*b*c^4
 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^3 + (a^5*c^6 + 5*a^4*b*c^5
 + 10*a^3*b^2*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}} \, dx = \int \frac{1}{x^3 \left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(1/(x**3*((a*c + a*d*x + b)/(c + d*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c))^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{x^3\,{\left(a+\frac{b}{c+d\,x}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x))^(3/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 1046, normalized size of antiderivative = 6.23

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx}\right)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/x^3/(a+b/(d*x+c))^(3/2),x)
```

output
```
( - 96*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x +
b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c +
d*x))*a**2*b*c**2*d**2*x**2 - 12*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x +
b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*
c + b) + sqrt(a)*sqrt(c + d*x))*a*b**2*c*d**2*x**2 + 9*sqrt(c)*sqrt(a*c +
b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a
)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**3*d**2*x**2 - 96*
sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b) + sq
rt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a
**2*b*c**2*d**2*x**2 - 12*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(
sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b)
 + sqrt(a)*sqrt(c + d*x))*a*b**2*c*d**2*x**2 + 9*sqrt(c)*sqrt(a*c + b)*sqrt
(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(
a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*b**3*d**2*x**2 + 96*sqrt(c)
*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c
 + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*d*x)*a**2*b*c**2*d**2
*x**2 + 12*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(2*sqrt(a)*sqrt(
c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*d*x
)*a*b**2*c*d**2*x**2 - 9*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(2
*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x + b) + 2*sqrt(c)*sqrt(a)*sqrt(a...
```

3.51 $\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 244

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{2a^2bd^3}{(b+ac)^4\sqrt{a+\frac{b}{c+dx}}} + \frac{(b^2+12abc-8a^2c^2)\,d^2(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8c(b+ac)^4x} + \frac{(b+12ac)d(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12c(b+ac)^3x^2} - \frac{(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3c(b+ac)^2x^3} + \frac{b(b^2+12abc-24a^2c^2)\,d^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{8c^{3/2}(b+ac)^{9/2}}$$

output
```
2*a^2*b*d^3/(a*c+b)^4/(a+b/(d*x+c))^(1/2)+1/8*(-8*a^2*c^2+12*a*b*c+b^2)*d^
2*(d*x+c)*(a+b/(d*x+c))^(1/2)/c/(a*c+b)^4/x+1/12*(12*a*c+b)*d*(d*x+c)^2*(a
+b/(d*x+c))^(1/2)/c/(a*c+b)^3/x^2-1/3*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/c/(a*c
+b)^2/x^3+1/8*b*(-24*a^2*c^2+12*a*b*c+b^2)*d^3*arctanh(c^(1/2)*(a+b/(d*x+c
))^(1/2)/(a*c+b)^(1/2))/c^(3/2)/(a*c+b)^(9/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx =$$

$$-\frac{(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(b^3(8c^2+14cdx+3d^2x^2)+2a^2bc(12c^3+7c^2dx-16cd^2x^2-47d^3x^3)+8a^3c^2(c^3+d^3}{24c(b+ac)^4x^3(b+a(c+dx))}$$

$$-\frac{b(b^2+12abc-24a^2c^2)\,d^3\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{-b-ac}}\right)}{8c^{3/2}(-b-ac)^{9/2}}$$

input
```
Integrate[1/(x^4*(a + b/(c + d*x))^(3/2)),x]
```

output
```
-1/24*((c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(b^3*(8*c^2 + 14*c*d*x
+ 3*d^2*x^2) + 2*a^2*b*c*(12*c^3 + 7*c^2*d*x - 16*c*d^2*x^2 - 47*d^3*x^3)
+ 8*a^3*c^2*(c^3 + d^3*x^3) + a*b^2*(24*c^3 + 28*c^2*d*x - 29*c*d^2*x^2 +
3*d^3*x^3)))/(c*(b + a*c)^4*x^3*(b + a*(c + d*x))) - (b*(b^2 + 12*a*b*c -
24*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/Sqrt[-b
 - a*c]])/(8*c^(3/2)*(-b - a*c)^(9/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 941, 948, 100, 27, 87, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 896}$$

$$d^3 \int \frac{1}{d^4 x^4 \left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \; 941$$

$$d^3 \int \frac{1}{(c+dx)^4 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \; 948$$

$$-d^3 \int \frac{1}{(c+dx)^2 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \; 100$$

$$-d^3 \left(\frac{2\int -\frac{a(b-6ac)-\frac{b(b+ac)}{c+dx}}{2\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}}{b^2(ac+b)}-\frac{2a^2}{b^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3\sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \; 27$$

$$-d^3 \left(-\frac{\int \frac{a(b-6ac)-\frac{b(b+ac)}{c+dx}}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}}{b^2(ac+b)}-\frac{2a^2}{b^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3\sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \; 87$$

$$-d^3 \left(-\frac{\frac{b\left(-24a^2c^2+12abc+b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{6c(ac+b)}-\frac{\left(6a^2c^2+b^2\right)\sqrt{a+\frac{b}{c+dx}}}{3c(ac+b)\left(1-\frac{c}{c+dx}\right)^3}}{b^2(ac+b)}-\frac{2a^2}{b^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3\sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \; 52$$

$$-d^3\left(-\frac{\frac{b\left(-24a^2c^2+12abc+b^2\right)\left(\frac{3b\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)}{6c(ac+b)}-\frac{\left(6a^2c^2+b^2\right)\sqrt{a+\frac{b}{c+dx}}}{3c(ac+b)\left(1-\frac{c}{c+dx}\right)^3}}{b^2(ac+b)}-\frac{\cdots}{b^2(ac+b)\left(1-\cdots\right.}\right.$$

$$\Big\downarrow \; 52$$

$$-d^3\left(-\frac{\frac{b\left(-24a^2c^2+12abc+b^2\right)\left(\frac{3b\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)}{6c(ac+b)}-\frac{\left(6a^2c^2+b^2\right)\sqrt{a+\frac{b}{c+dx}}}{3c(ac+b)\left(1-\frac{c}{c+dx}\right)^3}}{b^2(ac+b)}-\cdots\right.$$

$$\Big\downarrow \; 73$$

$$-d^3\left(-\frac{\frac{b\left(-24a^2c^2+12abc+b^2\right)\left(\frac{3b\left(\frac{\int \frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{ac+b}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)}{6c(ac+b)}-\frac{\left(6a^2c^2+b^2\right)\sqrt{a+\frac{b}{c+dx}}}{3c(ac+b)\left(1-\frac{c}{c+dx}\right)^3}}{b^2(ac+b)}-\cdots\right.$$

$$\Big\downarrow \; 221$$

$$-d^3\left(-\frac{\frac{b(-24a^2c^2+12abc+b^2)\left(\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{a+\frac{b}{c+dx}}}{(ac+b)\left(1-\frac{c}{c+dx}\right)}\right)}{4(ac+b)}+\frac{\sqrt{a+\frac{b}{c+dx}}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)}{6c(ac+b)}-\frac{(6a^2c^2+b^2)\sqrt{a+\frac{b}{c+dx}}}{3c(ac+b)\left(1-\frac{c}{c+dx}\right)^3}}{b^2(ac+b)} - \ldots\right.$$

input
```
Int[1/(x^4*(a + b/(c + d*x))^(3/2)),x]
```

output
```
-(d^3*((-2*a^2)/(b^2*(b + a*c)*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x))^3)
- (-1/3*((b^2 + 6*a^2*c^2)*Sqrt[a + b/(c + d*x)])/(c*(b + a*c)*(1 - c/(c +
 d*x))^3) + (b*(b^2 + 12*a*b*c - 24*a^2*c^2)*(Sqrt[a + b/(c + d*x)]/(2*(b
+ a*c)*(1 - c/(c + d*x))^2) + (3*b*(Sqrt[a + b/(c + d*x)]/((b + a*c)*(1 -
c/(c + d*x))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]])
/(Sqrt[c]*(b + a*c)^(3/2))))/(4*(b + a*c))))/(6*c*(b + a*c)))/(b^2*(b + a*
c))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5769 vs. $2(218) = 436$.

Time = 0.23 (sec) , antiderivative size = 5770, normalized size of antiderivative = 23.65

method	result	size
default	Expression too large to display	5770

input
```
int(1/x^4/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(222) = 444$.

Time = 0.12 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.45

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx}\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((24*a^3*b*c^2 - 12*a^2*b^2*c - a*b^3)*d^4*x^4 + (24*a^3*b*c^3 +
12*a^2*b^2*c^2 - 13*a*b^3*c - b^4)*d^3*x^3)*sqrt(a*c^2 + b*c)*log(-(2*a*c^
2 + (2*a*c + b)*d*x + 2*b*c - 2*sqrt(a*c^2 + b*c)*(d*x + c)*sqrt((a*d*x +
a*c + b)/(d*x + c)))/x) - 2*(8*a^4*c^8 + 32*a^3*b*c^7 + 48*a^2*b^2*c^6 + 3
2*a*b^3*c^5 + (8*a^4*c^4 - 86*a^3*b*c^3 - 91*a^2*b^2*c^2 + 3*a*b^3*c)*d^4*
x^4 + 8*b^4*c^4 + (8*a^4*c^5 - 118*a^3*b*c^4 - 152*a^2*b^2*c^3 - 23*a*b^3*
c^2 + 3*b^4*c)*d^3*x^3 - (18*a^3*b*c^5 + 19*a^2*b^2*c^4 - 16*a*b^3*c^3 - 1
7*b^4*c^2)*d^2*x^2 + 2*(4*a^4*c^7 + 23*a^3*b*c^6 + 45*a^2*b^2*c^5 + 37*a*b
^3*c^4 + 11*b^4*c^3)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^6*c^7 + 5
*a^5*b*c^6 + 10*a^4*b^2*c^5 + 10*a^3*b^3*c^4 + 5*a^2*b^4*c^3 + a*b^5*c^2)*
d*x^4 + (a^6*c^8 + 6*a^5*b*c^7 + 15*a^4*b^2*c^6 + 20*a^3*b^3*c^5 + 15*a^2*
b^4*c^4 + 6*a*b^5*c^3 + b^6*c^2)*x^3), 1/24*(3*((24*a^3*b*c^2 - 12*a^2*b^2
*c - a*b^3)*d^4*x^4 + (24*a^3*b*c^3 + 12*a^2*b^2*c^2 - 13*a*b^3*c - b^4)*d
^3*x^3)*sqrt(-a*c^2 - b*c)*arctan(sqrt(-a*c^2 - b*c)*(d*x + c)*sqrt((a*d*x
 + a*c + b)/(d*x + c))/(a*c*d*x + a*c^2 + b*c)) - (8*a^4*c^8 + 32*a^3*b*c^
7 + 48*a^2*b^2*c^6 + 32*a*b^3*c^5 + (8*a^4*c^4 - 86*a^3*b*c^3 - 91*a^2*b^2
*c^2 + 3*a*b^3*c)*d^4*x^4 + 8*b^4*c^4 + (8*a^4*c^5 - 118*a^3*b*c^4 - 152*a
^2*b^2*c^3 - 23*a*b^3*c^2 + 3*b^4*c)*d^3*x^3 - (18*a^3*b*c^5 + 19*a^2*b^2*
c^4 - 16*a*b^3*c^3 - 17*b^4*c^2)*d^2*x^2 + 2*(4*a^4*c^7 + 23*a^3*b*c^6 + 4
5*a^2*b^2*c^5 + 37*a*b^3*c^4 + 11*b^4*c^3)*d*x)*sqrt((a*d*x + a*c + b)/...
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a + \frac{b}{c+dx}\right)^{3/2}}\, dx = \int \frac{1}{x^4 \left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/x**4/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(1/(x**4*((a*c + a*d*x + b)/(c + d*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c))^(3/2)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(a+\frac{b}{c+d\,x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a + b/(c + d*x))^(3/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 20.48 (sec) , antiderivative size = 1479, normalized size of antiderivative = 6.06

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^4/(a+b/(d*x+c))^(3/2),x)
```

output
```
(864*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b)
 - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*
x))*a**3*b*c**3*d**3*x**3 - 72*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)
*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c
+ b) + sqrt(a)*sqrt(c + d*x))*a**2*b**2*c**2*d**3*x**3 - 216*sqrt(c)*sqrt(
a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b) - sqrt(2*sqrt(c)*
sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a*b**3*c*d**3*
x**3 - 15*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x
 + b) - sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c
 + d*x))*b**4*d**3*x**3 + 864*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*
log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c +
 b) + sqrt(a)*sqrt(c + d*x))*a**3*b*c**3*d**3*x**3 - 72*sqrt(c)*sqrt(a*c +
 b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(
a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt(c + d*x))*a**2*b**2*c**2*d**3
*x**3 - 216*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x + b)*log(sqrt(a*c + a*d
*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*a*c + b) + sqrt(a)*sqrt
(c + d*x))*a*b**3*c*d**3*x**3 - 15*sqrt(c)*sqrt(a*c + b)*sqrt(a*c + a*d*x
+ b)*log(sqrt(a*c + a*d*x + b) + sqrt(2*sqrt(c)*sqrt(a)*sqrt(a*c + b) + 2*
a*c + b) + sqrt(a)*sqrt(c + d*x))*b**4*d**3*x**3 - 864*sqrt(c)*sqrt(a*c +
b)*sqrt(a*c + a*d*x + b)*log(2*sqrt(a)*sqrt(c + d*x)*sqrt(a*c + a*d*x +...
```

3.52 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 329

$$\begin{aligned}\int \frac{1}{x^5\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = &-\frac{2a^3bd^4}{(b+ac)^5\sqrt{a+\frac{b}{c+dx}}} \\ &-\frac{(3b^3+24ab^2c+144a^2bc^2-64a^3c^3)\,d^3(c+dx)\sqrt{a+\frac{b}{c+dx}}}{64c^2(b+ac)^5x} \\ &-\frac{(b^2+8abc+48a^2c^2)\,d^2(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{32c^2(b+ac)^4x^2} \\ &+\frac{(3b+8ac)d(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{8c^2(b+ac)^3x^3}-\frac{(c+dx)^4\sqrt{a+\frac{b}{c+dx}}}{4c^2(b+ac)^2x^4} \\ &-\frac{3b(b^3+8ab^2c+48a^2bc^2-64a^3c^3)\,d^4\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{b+ac}}\right)}{64c^{5/2}(b+ac)^{11/2}}\end{aligned}$$

output
```
-2*a^3*b*d^4/(a*c+b)^5/(a+b/(d*x+c))^(1/2)-1/64*(-64*a^3*c^3+144*a^2*b*c^2
+24*a*b^2*c+3*b^3)*d^3*(d*x+c)*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)^5/x-1/32*(4
8*a^2*c^2+8*a*b*c+b^2)*d^2*(d*x+c)^2*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)^4/x^2
+1/8*(8*a*c+3*b)*d*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)^3/x^3-1/4*(d*
x+c)^4*(a+b/(d*x+c))^(1/2)/c^2/(a*c+b)^2/x^4-3/64*b*(-64*a^3*c^3+48*a^2*b*
c^2+8*a*b^2*c+b^3)*d^4*arctanh(c^(1/2)*(a+b/(d*x+c))^(1/2)/(a*c+b)^(1/2))/
c^(5/2)/(a*c+b)^(11/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{c}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(b^4\left(16c^3+24c^2dx+2cd^2x^2-3d^3x^3\right)+2a^2b^2c\left(48c^4+36c^3dx-39c^2d^2x^2+34cd^3x^3-11d^4\right.\right.}{(b+}}{}$$

input
```
Integrate[1/(x^5*(a + b/(c + d*x))^(3/2)),x]
```

output
```
(-((Sqrt[c]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(b^4*(16*c^3 + 24*
c^2*d*x + 2*c*d^2*x^2 - 3*d^3*x^3) + 2*a^2*b^2*c*(48*c^4 + 36*c^3*d*x - 39
*c^2*d^2*x^2 + 34*c*d^3*x^3 - 11*d^4*x^4) + a*b^3*(64*c^4 + 72*c^3*d*x - 3
6*c^2*d^2*x^2 - 23*c*d^3*x^3 - 3*d^4*x^4) + 16*a^4*c^3*(c^4 - d^4*x^4) + 8
*a^3*b*c^2*(8*c^4 + 3*c^3*d*x - 5*c^2*d^2*x^2 + 11*c*d^3*x^3 + 35*d^4*x^4)
))/((b + a*c)^5*x^4*(b + a*(c + d*x)))) - (3*b*(b^3 + 8*a*b^2*c + 48*a^2*b
*c^2 - 64*a^3*c^3)*d^4*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x)/(c + d*x)])/
Sqrt[-b - a*c]])/(-b - a*c)^(11/2))/(64*c^(5/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {896, 25, 941, 948, 25, 109, 27, 162, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}dx$$

$$\Big\downarrow \text{896}$$

$$d^4 \int \frac{1}{d^5 x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-d^4 \int -\frac{1}{d^5 x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{941}$$

$$-d^4 \int \frac{1}{(c+dx)^5 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(\frac{c}{c+dx}-1\right)^5}d(c+dx)$$

$$\Big\downarrow \text{948}$$

$$d^4 \int -\frac{1}{(c+dx)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^5}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-d^4 \int \frac{1}{(c+dx)^3 \left(a+\frac{b}{c+dx}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^5}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{109}$$

$$d^4 \left(\frac{2\int \frac{4a-\frac{b-4ac}{c+dx}}{2(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^5}d\frac{1}{c+dx}}{b(ac+b)}-\frac{2a}{b(ac+b)(c+dx)^2 \left(1-\frac{c}{c+dx}\right)^4 \sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \text{27}$$

$$d^4 \left(\frac{\int \frac{4a-\frac{b-4ac}{c+dx}}{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^5}d\frac{1}{c+dx}}{b(ac+b)}-\frac{2a}{b(ac+b)(c+dx)^2 \left(1-\frac{c}{c+dx}\right)^4 \sqrt{a+\frac{b}{c+dx}}}\right)$$

$$\Big\downarrow \text{162}$$

$$d^4\left(\frac{\frac{\sqrt{a+\frac{b}{c+dx}}\left((b-2ac)(8ac+b)-\frac{c\left(-32a^2c^2-8abc+3b^2\right)}{c+dx}\right)}{8c^2(ac+b)^2\left(1-\frac{c}{c+dx}\right)^4}-\frac{\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\int\frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{16c^2(ac+b)^2}}{b(ac+b)}-\frac{}{b(ac+b)}\right.$$

$\downarrow$ 52

$$d^4\left(\frac{\frac{\sqrt{a+\frac{b}{c+dx}}\left((b-2ac)(8ac+b)-\frac{c\left(-32a^2c^2-8abc+3b^2\right)}{c+dx}\right)}{8c^2(ac+b)^2\left(1-\frac{c}{c+dx}\right)^4}-\frac{\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\left(\frac{3b\int\frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4(ac+b)}+\frac{\sqrt{a+\cdots}}{2(ac+b)\left(1-\cdots\right.}\right.}{16c^2(ac+b)^2}}{b(ac+b)}\right.$$

$\downarrow$ 52

$$d^4\left(\frac{\frac{\sqrt{a+\frac{b}{c+dx}}\left((b-2ac)(8ac+b)-\frac{c\left(-32a^2c^2-8abc+3b^2\right)}{c+dx}\right)}{8c^2(ac+b)^2\left(1-\frac{c}{c+dx}\right)^4}-\frac{\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\left(\frac{3b\left(\frac{b\int\frac{1}{\sqrt{a+\frac{b}{c+dx}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2(ac+b)}+\frac{\sqrt{a+\cdots}}{(ac+b)\left(1\cdots\right.}\right.}{4(ac+b)}\right.}{16c^2(ac+b)^2}}{b(ac+b)}\right.$$

$\downarrow$ 73

$$d^4\left(\frac{\frac{\sqrt{a+\frac{b}{c+dx}}\left((b-2ac)(8ac+b)-\frac{c\left(-32a^2c^2-8abc+3b^2\right)}{c+dx}\right)}{8c^2(ac+b)^2\left(1-\frac{c}{c+dx}\right)^4}-\frac{\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\left(\frac{3b\left(\frac{\int\frac{1}{\frac{ac}{b}-\frac{c}{b(c+dx)^2}+1}d\sqrt{a+\frac{b}{c+dx}}}{ac+b}+\frac{\sqrt{a+\frac{b}{c\cdots}}}{(ac+b)\left(1-\cdots\right.}\right.}{4(ac+b)}\cdots\right.}{16c^2(ac+b)^2}\cdots}{b(ac+b)}\cdots\right.$$

$\downarrow$ 221

$$d^4\left(\frac{\frac{\sqrt{a+\frac{b}{c+dx}}\left((b-2ac)(8ac+b)-\frac{c\left(-32a^2c^2-8abc+3b^2\right)}{c+dx}\right)}{8c^2(ac+b)^2\left(1-\frac{c}{c+dx}\right)^4}-\frac{\left(-64a^3c^3+48a^2bc^2+8ab^2c+b^3\right)\left(\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx}}}{\sqrt{ac+b}}\right)}{\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{a+\frac{b}{c+\cdots}}}{(ac+b)\left(1-\cdots\right.}\right.}{4(ac+b)}\cdots\right.}{16c^2(ac+b)^2}\cdots}{b(ac+b)}\cdots\right.$$

input

```
Int[1/(x^5*(a + b/(c + d*x))^(3/2)),x]
```

output
```
d^4*((-2*a)/(b*(b + a*c)*(c + d*x)^2*Sqrt[a + b/(c + d*x)]*(1 - c/(c + d*x
))^4) + ((Sqrt[a + b/(c + d*x)]*((b - 2*a*c)*(b + 8*a*c) - (c*(3*b^2 - 8*a
*b*c - 32*a^2*c^2))/(c + d*x)))/(8*c^2*(b + a*c)^2*(1 - c/(c + d*x))^4) -
((b^3 + 8*a*b^2*c + 48*a^2*b*c^2 - 64*a^3*c^3)*(Sqrt[a + b/(c + d*x)]/(2*(
b + a*c)*(1 - c/(c + d*x))^2) + (3*b*(Sqrt[a + b/(c + d*x)]/((b + a*c)*(1
- c/(c + d*x))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x)])/Sqrt[b + a*c]
])/(Sqrt[c]*(b + a*c)^(3/2))))/(4*(b + a*c))))/(16*c^2*(b + a*c)^2))/(b*(b
 + a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6949 vs. $2(299) = 598$.

Time = 0.25 (sec) , antiderivative size = 6950, normalized size of antiderivative = 21.12

method	result	size
default	Expression too large to display	6950

input
```
int(1/x^5/(a+b/(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(301) = 602$.

Time = 0.15 (sec) , antiderivative size = 1398, normalized size of antiderivative = 4.25

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(a+b/(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(3*((64*a^4*b*c^3 - 48*a^3*b^2*c^2 - 8*a^2*b^3*c - a*b^4)*d^5*x^5 +
 (64*a^4*b*c^4 + 16*a^3*b^2*c^3 - 56*a^2*b^3*c^2 - 9*a*b^4*c - b^5)*d^4*x^
4)*sqrt(a*c^2 + b*c)*log(-(2*a*c^2 + (2*a*c + b)*d*x + 2*b*c + 2*sqrt(a*c^
2 + b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x + c)))/x) - 2*(16*a^5*c^10
+ 80*a^4*b*c^9 + 160*a^3*b^2*c^8 + 160*a^2*b^3*c^7 + 80*a*b^4*c^6 - (16*a^
5*c^5 - 264*a^4*b*c^4 - 258*a^3*b^2*c^3 + 25*a^2*b^3*c^2 + 3*a*b^4*c)*d^5*
x^5 + 16*b^5*c^5 - (16*a^5*c^6 - 352*a^4*b*c^5 - 414*a^3*b^2*c^4 - 20*a^2*
b^3*c^3 + 29*a*b^4*c^2 + 3*b^5*c)*d^4*x^4 + (48*a^4*b*c^6 + 38*a^3*b^2*c^5
 - 69*a^2*b^3*c^4 - 60*a*b^4*c^3 - b^5*c^2)*d^3*x^3 - 2*(8*a^4*b*c^7 + 11*
a^3*b^2*c^6 - 15*a^2*b^3*c^5 - 31*a*b^4*c^4 - 13*b^5*c^3)*d^2*x^2 + 8*(2*a
^5*c^9 + 13*a^4*b*c^8 + 32*a^3*b^2*c^7 + 38*a^2*b^3*c^6 + 22*a*b^4*c^5 + 5
*b^5*c^4)*d*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/((a^7*c^9 + 6*a^6*b*c^8
+ 15*a^5*b^2*c^7 + 20*a^4*b^3*c^6 + 15*a^3*b^4*c^5 + 6*a^2*b^5*c^4 + a*b^6
*c^3)*d*x^5 + (a^7*c^10 + 7*a^6*b*c^9 + 21*a^5*b^2*c^8 + 35*a^4*b^3*c^7 +
35*a^3*b^4*c^6 + 21*a^2*b^5*c^5 + 7*a*b^6*c^4 + b^7*c^3)*x^4), -1/64*(3*((
64*a^4*b*c^3 - 48*a^3*b^2*c^2 - 8*a^2*b^3*c - a*b^4)*d^5*x^5 + (64*a^4*b*c
^4 + 16*a^3*b^2*c^3 - 56*a^2*b^3*c^2 - 9*a*b^4*c - b^5)*d^4*x^4)*sqrt(-a*c
^2 - b*c)*arctan(sqrt(-a*c^2 - b*c)*(d*x + c)*sqrt((a*d*x + a*c + b)/(d*x
+ c))/(a*c*d*x + a*c^2 + b*c)) + (16*a^5*c^10 + 80*a^4*b*c^9 + 160*a^3*b^2
*c^8 + 160*a^2*b^3*c^7 + 80*a*b^4*c^6 - (16*a^5*c^5 - 264*a^4*b*c^4 - 2...
```

Sympy [F]

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{1}{x^5 \left(\frac{ac+adx+b}{c+dx}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**5/(a+b/(d*x+c))**(3/2),x)
```

output
```
Integral(1/(x**5*((a*c + a*d*x + b)/(c + d*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}} x^5}\,dx$$

input
```
integrate(1/x^5/(a+b/(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c))^(3/2)*x^5), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^5/(a+b/(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx=\int \frac{1}{x^5\,{\left(a+\frac{b}{c+d\,x}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^5*(a + b/(c + d*x))^(3/2)),x)
```

output
```
int(1/(x^5*(a + b/(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx}\right)^{3/2}}\,dx = \int \frac{1}{x^5 \left(a+\frac{b}{dx+c}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^5/(a+b/(d*x+c))^(3/2),x)
```

output
```
int(1/x^5/(a+b/(d*x+c))^(3/2),x)
```

3.53 $\int x^3 \left(a+\frac{b}{c+dx}\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 283

$$\int x^3\left(a+\frac{b}{c+dx}\right)^p\,dx=-\frac{cx^2(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{1+p}}{bd^2(1-p)}$$
$$-\frac{\left(24a^2c^2+12abc(1-p)+b^2(3-4p+p^2)\right)(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{1+p}}{12a^2bd^4(1-p)}$$
$$+\frac{(4ac+b(1-p))(c+dx)^4\left(a+\frac{b}{c+dx}\right)^{1+p}}{4abd^4(1-p)}$$
$$+\frac{b\left(24a^3c^3+36a^2bc^2(1-p)+12ab^2c(2-3p+p^2)+b^3(6-11p+6p^2-p^3)\right)\left(a+\frac{b}{c+dx}\right)^{1+p}\text{Hypergeom}}{12a^5d^4\left(1-p^2\right)}$$

output
```
-c*x^2*(d*x+c)^2*(a+b/(d*x+c))^(p+1)/b/d^2/(1-p)-1/12*(24*a^2*c^2+12*a*b*c
*(1-p)+b^2*(p^2-4*p+3))*(d*x+c)^3*(a+b/(d*x+c))^(p+1)/a^2/b/d^4/(1-p)+1/4*
(4*a*c+b*(1-p))*(d*x+c)^4*(a+b/(d*x+c))^(p+1)/a/b/d^4/(1-p)+1/12*b*(24*a^3
*c^3+36*a^2*b*c^2*(1-p)+12*a*b^2*c*(p^2-3*p+2)+b^3*(-p^3+6*p^2-11*p+6))*(a
+b/(d*x+c))^(p+1)*hypergeom([3, p+1],[2+p],1+b/a/(d*x+c))/a^5/d^4/(-p^2+1)
```

Mathematica [F]

$$\int x^3\left(a+\frac{b}{c+dx}\right)^p\,dx=\int x^3\left(a+\frac{b}{c+dx}\right)^p\,dx$$

input
```
Integrate[x^3*(a + b/(c + d*x))^p,x]
```

output
```
Integrate[x^3*(a + b/(c + d*x))^p, x]
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {896, 25, 941, 948, 25, 111, 25, 162, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{c+dx}\right)^p\,dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int d^3x^3\left(a+\frac{b}{c+dx}\right)^p\,d(c+dx)}{d^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -d^3x^3\left(a+\frac{b}{c+dx}\right)^p\,d(c+dx)}{d^4}$$

$$\Big\downarrow \ 941$$

$$-\frac{\int (c+dx)^3\left(a+\frac{b}{c+dx}\right)^p\left(\frac{c}{c+dx}-1\right)^3\,d(c+dx)}{d^4}$$

$$\Big\downarrow \ 948$$

$$\frac{\int -(c+dx)^5\left(a+\frac{b}{c+dx}\right)^p\left(1-\frac{c}{c+dx}\right)^3\,d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int (c+dx)^5 \left(a+\frac{b}{c+dx}\right)^p \left(1-\frac{c}{c+dx}\right)^3 d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \ 111$$

$$\frac{\frac{\int -(c+dx)^5 \left(a+\frac{b}{c+dx}\right)^p \left(1-\frac{c}{c+dx}\right)\left(4ac+\frac{(pb+b-2ac)c}{c+dx}+b(1-p)\right)d\frac{1}{c+dx}}{b(1-p)}-\frac{c(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2\left(a+\frac{b}{c+dx}\right)^{p+1}}{b(1-p)}}{d^4}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{\int (c+dx)^5 \left(a+\frac{b}{c+dx}\right)^p \left(1-\frac{c}{c+dx}\right)\left(-pb+b+4ac+\frac{c(pb+b-2ac)}{c+dx}\right)d\frac{1}{c+dx}}{b(1-p)}-\frac{c(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2\left(a+\frac{b}{c+dx}\right)^{p+1}}{b(1-p)}}{d^4}$$

$$\Big\downarrow \ 162$$

$$-\frac{\frac{\frac{\left(24a^3c^3+36a^2bc^2(1-p)+12ab^2c\left(p^2-3p+2\right)+b^3\left(-p^3+6p^2-11p+6\right)\right)\int (c+dx)^3\left(a+\frac{b}{c+dx}\right)^p d\frac{1}{c+dx}}{12a^2}-\frac{(c+dx)^4\left(a+\frac{b}{c+dx}\right)^{p+1}\left(3a(4ac+b(-p)+b)-\frac{24a^2c^2+\cdots}{\cdots}\right.}{12a^2}}{b(1-p)}}{d^4}$$

$$\Big\downarrow \ 75$$

$$-\frac{\frac{-\frac{(c+dx)^4\left(3a(4ac+b(-p)+b)-\frac{24a^2c^2+12abc(1-p)+b^2\left(p^2-4p+3\right)}{c+dx}\right)\left(a+\frac{b}{c+dx}\right)^{p+1}}{12a^2}-\frac{b^2\left(24a^3c^3+36a^2bc^2(1-p)+12ab^2c\left(p^2-3p+2\right)+b^3\left(-p^3+6p^2-11p\cdots\right.\right.}{12a^5(\cdots}}{b(1-p)}}{d^4}$$

input
```
Int[x^3*(a + b/(c + d*x))^p,x]
```

output
```
(-((c*(c + d*x)^4*(a + b/(c + d*x))^(1 + p)*(1 - c/(c + d*x))^2)/(b*(1 - p
))) - (-1/12*((c + d*x)^4*(a + b/(c + d*x))^(1 + p)*(3*a*(b + 4*a*c - b*p)
 - (24*a^2*c^2 + 12*a*b*c*(1 - p) + b^2*(3 - 4*p + p^2))/(c + d*x)))/a^2 -
 (b^2*(24*a^3*c^3 + 36*a^2*b*c^2*(1 - p) + 12*a*b^2*c*(2 - 3*p + p^2) + b^
3*(6 - 11*p + 6*p^2 - p^3))*(a + b/(c + d*x))^(1 + p)*Hypergeometric2F1[3,
 1 + p, 2 + p, 1 + b/(a*(c + d*x))])/(12*a^5*(1 + p)))/(b*(1 - p)))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^3\left(a+\frac{b}{dx+c}\right)^p dx$$

input
```
int(x^3*(a+b/(d*x+c))^p,x)
```

output
```
int(x^3*(a+b/(d*x+c))^p,x)
```

Fricas [F]

$$\int x^3\left(a+\frac{b}{c+dx}\right)^p dx=\int\left(a+\frac{b}{dx+c}\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral(x^3*((a*d*x + a*c + b)/(d*x + c))^p, x)
```

Sympy [F]

$$\int x^3 \left(a + \frac{b}{c + dx}\right)^p dx = \int x^3 \left(\frac{ac + adx + b}{c + dx}\right)^p dx$$

input
```
integrate(x**3*(a+b/(d*x+c))**p,x)
```

output
```
Integral(x**3*((a*c + a*d*x + b)/(c + d*x))**p, x)
```

Maxima [F]

$$\int x^3 \left(a + \frac{b}{c + dx}\right)^p dx = \int \left(a + \frac{b}{dx + c}\right)^p x^3\, dx$$

input
```
integrate(x^3*(a+b/(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p*x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(a + \frac{b}{c + dx}\right)^p dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b/(d*x+c))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{-1,[
0,3,1,0]%%%} / %%%{1,[0,0,0,3]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + \frac{b}{c + dx}\right)^p dx = \int x^3 \left(a + \frac{b}{c + d\,x}\right)^p dx$$

input
```
int(x^3*(a + b/(c + d*x))^p,x)
```

output
```
int(x^3*(a + b/(c + d*x))^p, x)
```

Reduce [F]

$$\int x^3 \left(a + \frac{b}{c + dx}\right)^p dx = \int \frac{(adx + ac + b)^p x^3}{(dx + c)^p} dx$$

input
```
int(x^3*(a+b/(d*x+c))^p,x)
```

output
```
int(((a*c + a*d*x + b)**p*x**3)/(c + d*x)**p,x)
```

3.54 $\int x^2 \left(a+\frac{b}{c+dx}\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int x^2 \left(a+\frac{b}{c+dx}\right)^p \, dx = -\frac{(6ac+b(2-p))(c+dx)^2 \left(a+\frac{b}{c+dx}\right)^{1+p}}{6a^2d^3}+\frac{(c+dx)^3 \left(a+\frac{b}{c+dx}\right)^{1+p}}{3ad^3} - \frac{b(6a^2c^2+b(6ac+b(2-p))(1-p)) \left(a+\frac{b}{c+dx}\right)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx)}\right)}{6a^4d^3(1+p)}$$

output
```
-1/6*(6*a*c+b*(2-p))*(d*x+c)^2*(a+b/(d*x+c))^(p+1)/a^2/d^3+1/3*(d*x+c)^3*(
a+b/(d*x+c))^(p+1)/a/d^3-1/6*b*(6*a^2*c^2+b*(6*a*c+b*(2-p))*(1-p))*(a+b/(d
*x+c))^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(d*x+c))/a^4/d^3/(p+1)
```

Mathematica [F]

$$\int x^2 \left(a+\frac{b}{c+dx}\right)^p \, dx = \int x^2 \left(a+\frac{b}{c+dx}\right)^p \, dx$$

input
```
Integrate[x^2*(a + b/(c + d*x))^p,x]
```

output
```
Integrate[x^2*(a + b/(c + d*x))^p, x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {896, 941, 948, 100, 25, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{c+dx}\right)^p dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int d^2x^2\left(a+\frac{b}{c+dx}\right)^p d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 941}$$

$$\frac{\int (c+dx)^2\left(a+\frac{b}{c+dx}\right)^p\left(\frac{c}{c+dx}-1\right)^2 d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{\int (c+dx)^4\left(a+\frac{b}{c+dx}\right)^p\left(1-\frac{c}{c+dx}\right)^2 d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{\frac{\int -(c+dx)^3\left(a+\frac{b}{c+dx}\right)^p\left(-\frac{3ac^2}{c+dx}+6ac+b(2-p)\right)d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{p+1}}{3a}}{d^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int (c+dx)^3\left(a+\frac{b}{c+dx}\right)^p\left(-\frac{3ac^2}{c+dx}+6ac+b(2-p)\right)d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{p+1}}{3a}}{d^3}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{-\frac{-\frac{\left(6a^2c^2+b(1-p)(6ac+b(2-p))\right)\int (c+dx)^2\left(a+\frac{b}{c+dx}\right)^p d\frac{1}{c+dx}}{2a}-\frac{(c+dx)^2(6ac+b(2-p))\left(a+\frac{b}{c+dx}\right)^{p+1}}{2a}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{p+1}}{3a}}{d^3}$$

$$\Big\downarrow \textcolor{blue}{75}$$

$$-\frac{-\frac{-\frac{b\left(6a^2c^2+b(1-p)(6ac+b(2-p))\right)\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{2a^3(p+1)}-\frac{(c+dx)^2(6ac+b(2-p))\left(a+\frac{b}{c+dx}\right)^{p+1}}{2a}}{3a}-\frac{(c+dx)^3\left(a\ldots\right.}{3\ldots}}{d^3}$$

input
```
Int[x^2*(a + b/(c + d*x))^p,x]
```

output
```
-((-1/3*((c + d*x)^3*(a + b/(c + d*x))^(1 + p))/a - (-1/2*((6*a*c + b*(2 -
 p))*(c + d*x)^2*(a + b/(c + d*x))^(1 + p))/a - (b*(6*a^2*c^2 + b*(6*a*c +
 b*(2 - p))*(1 - p))*(a + b/(c + d*x))^(1 + p)*Hypergeometric2F1[2, 1 + p,
 2 + p, 1 + b/(a*(c + d*x))])/(2*a^3*(1 + p)))/(3*a))/d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^2 \left(a + \frac{b}{dx + c}\right)^p dx$$

input
```
int(x^2*(a+b/(d*x+c))^p,x)
```

output
```
int(x^2*(a+b/(d*x+c))^p,x)
```

Fricas [F]

$$\int x^2\left(a+\frac{b}{c+dx}\right)^p\,dx=\int \left(a+\frac{b}{dx+c}\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral(x^2*((a*d*x + a*c + b)/(d*x + c))^p, x)
```

Sympy [F]

$$\int x^2\left(a+\frac{b}{c+dx}\right)^p\,dx=\int x^2\left(\frac{ac+adx+b}{c+dx}\right)^p\,dx$$

input
```
integrate(x**2*(a+b/(d*x+c))**p,x)
```

output
```
Integral(x**2*((a*c + a*d*x + b)/(c + d*x))**p, x)
```

Maxima [F]

$$\int x^2\left(a+\frac{b}{c+dx}\right)^p\,dx=\int \left(a+\frac{b}{dx+c}\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p*x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\left(a+\frac{b}{c+dx}\right)^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b/(d*x+c))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,2,1,0]%%%} / %%%{1,[0,0,0,2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+\frac{b}{c+dx}\right)^p\,dx=\int x^2\,\left(a+\frac{b}{c+d\,x}\right)^p\,dx$$

input
```
int(x^2*(a + b/(c + d*x))^p,x)
```

output
```
int(x^2*(a + b/(c + d*x))^p, x)
```

Reduce [F]

$$\int x^2\left(a+\frac{b}{c+dx}\right)^p\,dx=\int\frac{(adx+ac+b)^p\,x^2}{(dx+c)^p}dx$$

input
```
int(x^2*(a+b/(d*x+c))^p,x)
```

output
```
int(((a*c + a*d*x + b)**p*x**2)/(c + d*x)**p,x)
```

3.55 $\int x\left(a+\frac{b}{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int x\left(a+\frac{b}{c+dx}\right)^p\,dx=\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{1+p}}{2ad^2}+\frac{b(b+2ac-bp)\left(a+\frac{b}{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx)}\right)}{2a^3d^2(1+p)}$$

output
```
1/2*(d*x+c)^2*(a+b/(d*x+c))^(p+1)/a/d^2+1/2*b*(2*a*c-b*p+b)*(a+b/(d*x+c))^
(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(d*x+c))/a^3/d^2/(p+1)
```

Mathematica [F]

$$\int x\left(a+\frac{b}{c+dx}\right)^p\,dx=\int x\left(a+\frac{b}{c+dx}\right)^p\,dx$$

input
```
Integrate[x*(a + b/(c + d*x))^p,x]
```

output
```
Integrate[x*(a + b/(c + d*x))^p, x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {896, 25, 941, 948, 25, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{c+dx}\right)^p dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int dx\left(a+\frac{b}{c+dx}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -dx\left(a+\frac{b}{c+dx}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \ 941$$

$$-\frac{\int (c+dx)\left(a+\frac{b}{c+dx}\right)^p\left(\frac{c}{c+dx}-1\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \ 948$$

$$\frac{\int -(c+dx)^3\left(a+\frac{b}{c+dx}\right)^p\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int (c+dx)^3\left(a+\frac{b}{c+dx}\right)^p\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \ 87$$

$$\frac{\frac{(2ac+b(1-p))\int (c+dx)^2\left(a+\frac{b}{c+dx}\right)^p d\frac{1}{c+dx}}{2a}+\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{p+1}}{2a}}{d^2}$$

$$\Big\downarrow \, 75$$

$$\frac{\frac{b(2ac+b(1-p))\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{2a^3(p+1)}+\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{p+1}}{2a}}{d^2}$$

input
```
Int[x*(a + b/(c + d*x))^p,x]
```

output
```
(((c + d*x)^2*(a + b/(c + d*x))^(1 + p))/(2*a) + (b*(2*a*c + b*(1 - p))*(a
 + b/(c + d*x))^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + b/(a*(c + d
*x))])/(2*a^3*(1 + p)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x\left(a+\frac{b}{dx+c}\right)^{p}dx$$

input
```
int(x*(a+b/(d*x+c))^p,x)
```

output
```
int(x*(a+b/(d*x+c))^p,x)
```

Fricas [F]

$$\int x\left(a+\frac{b}{c+dx}\right)^{p}dx=\int\left(a+\frac{b}{dx+c}\right)^{p}x\,dx$$

input
```
integrate(x*(a+b/(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral(x*((a*d*x + a*c + b)/(d*x + c))^p, x)
```

Sympy [F]

$$\int x\left(a+\frac{b}{c+dx}\right)^p\,dx=\int x\left(\frac{ac+adx+b}{c+dx}\right)^p\,dx$$

input
```
integrate(x*(a+b/(d*x+c))**p,x)
```

output
```
Integral(x*((a*c + a*d*x + b)/(c + d*x))**p, x)
```

Maxima [F]

$$\int x\left(a+\frac{b}{c+dx}\right)^p\,dx=\int\left(a+\frac{b}{dx+c}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x\left(a+\frac{b}{c+dx}\right)^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b/(d*x+c))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{-1,[
0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{c+dx}\right)^p\,dx=\int x\,\left(a+\frac{b}{c+d\,x}\right)^p\,dx$$

input
```
int(x*(a + b/(c + d*x))^p,x)
```

output
```
int(x*(a + b/(c + d*x))^p, x)
```

Reduce [F]

$$\int x\left(a+\frac{b}{c+dx}\right)^p\,dx=\int\frac{(adx+ac+b)^p\,x}{(dx+c)^p}dx$$

input
```
int(x*(a+b/(d*x+c))^p,x)
```

output
```
int(((a*c + a*d*x + b)**p*x)/(c + d*x)**p,x)
```

3.56 $\int \left(a+\frac{b}{c+dx}\right)^p dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \left(a+\frac{b}{c+dx}\right)^p dx = -\frac{b\left(a+\frac{b}{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx)}\right)}{a^2d(1+p)}$$

output
```
-b*(a+b/(d*x+c))^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(d*x+c))/a^2/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \left(a+\frac{b}{c+dx}\right)^p dx = -\frac{(c+dx)\left(a+\frac{b}{c+dx}\right)^p\left(1+\frac{a(c+dx)}{b}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{a(c+dx)}{b}\right)}{d(-1+p)}$$

input
```
Integrate[(a + b/(c + d*x))^p,x]
```

output
```
-(((c + d*x)*(a + b/(c + d*x))^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((a*
(c + d*x))/b)])/(d*(-1 + p)*(1 + (a*(c + d*x))/b)^p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 773, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{c+dx}\right)^p dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \left(a+\frac{b}{c+dx}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int (c+dx)^2 \left(a+\frac{b}{c+dx}\right)^p d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{75}$$

$$-\frac{b\left(a+\frac{b}{c+dx}\right)^{p+1} \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{a^2d(p+1)}$$

input
```
Int[(a + b/(c + d*x))^p,x]
```

output
```
-((b*(a + b/(c + d*x))^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + b/(a
*(c + d*x))])/(a^2*d*(1 + p)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+\frac{b}{dx+c}\right)^{p} dx$$

input
```
int((a+b/(d*x+c))^p,x)
```

output
```
int((a+b/(d*x+c))^p,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{c+dx}\right)^{p} dx = \int \left(a+\frac{b}{dx+c}\right)^{p} dx$$

input
```
integrate((a+b/(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral(((a*d*x + a*c + b)/(d*x + c))^p, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{c+dx}\right)^p \,dx=\int \left(a+\frac{b}{c+dx}\right)^p\,dx$$

input
```
integrate((a+b/(d*x+c))**p,x)
```

output
```
Integral((a + b/(c + d*x))**p, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{c+dx}\right)^p \,dx=\int \left(a+\frac{b}{dx+c}\right)^p\,dx$$

input
```
integrate((a+b/(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p, x)
```

Giac [F]

$$\int \left(a+\frac{b}{c+dx}\right)^p \,dx=\int \left(a+\frac{b}{dx+c}\right)^p\,dx$$

input
```
integrate((a+b/(d*x+c))^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c))^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \left(a+\frac{b}{c+dx}\right)^p dx = -\frac{\left(a+\frac{b}{c+d\,x}\right)^p (c+d\,x)\ {}_2\mathrm{F}_1\left(1-p,-p;\ 2-p;\ -\frac{a\,(c+d\,x)}{b}\right)}{d\,(p-1)\,\left(\frac{a\,(c+d\,x)}{b}+1\right)^p}$$

input
```
int((a + b/(c + d*x))^p,x)
```

output
```
-((a + b/(c + d*x))^p*(c + d*x)*hypergeom([1 - p, -p], 2 - p, -(a*(c + d*x
))/b))/(d*(p - 1)*((a*(c + d*x))/b + 1)^p)
```

Reduce [F]

$$\int \left(a+\frac{b}{c+dx}\right)^p dx = \int \frac{(adx+ac+b)^p}{(dx+c)^p}dx$$

input
```
int((a+b/(d*x+c))^p,x)
```

output
```
int((a*c + a*d*x + b)**p/(c + d*x)**p,x)
```

3.57 $\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 105

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x} dx = -\frac{c\left(a+\frac{b}{c+dx}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx}\right)}{b+ac}\right)}{(b+ac)(1+p)} + \frac{\left(a+\frac{b}{c+dx}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a(c+dx)}\right)}{a(1+p)}$$

output
```
-c*(a+b/(d*x+c))^(p+1)*hypergeom([1, p+1],[2+p],c*(a+b/(d*x+c))/(a*c+b))/(
a*c+b)/(p+1)+(a+b/(d*x+c))^(p+1)*hypergeom([1, p+1],[2+p],1+b/a/(d*x+c))/a
/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x} dx = \int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x} dx$$

input
```
Integrate[(a + b/(c + d*x))^p/x,x]
```

output
```
Integrate[(a + b/(c + d*x))^p/x, x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {896, 25, 941, 948, 25, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{\left(a+\frac{b}{c+dx}\right)^p}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$-\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{(c+dx)\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$

$$\Big\downarrow \text{ 948}$$

$$\int -\frac{(c+dx)\left(a+\frac{b}{c+dx}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{(c+dx)\left(a+\frac{b}{c+dx}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 97}$$

$$-\int (c+dx)\left(a+\frac{b}{c+dx}\right)^p d\frac{1}{c+dx}-c\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow 75$$

$$\frac{\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{a(p+1)}-c\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow 78$$

$$\frac{\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{a(p+1)}-\frac{c\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{c\left(a+\frac{b}{c+dx}\right)}{b+ac}\right)}{(p+1)(ac+b)}$$

input
```
Int[(a + b/(c + d*x))^p/x,x]
```

output
```
-((c*(a + b/(c + d*x))^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (c*(a +
b/(c + d*x)))/(b + a*c)])/((b + a*c)*(1 + p))) + ((a + b/(c + d*x))^(1 + p
)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + b/(a*(c + d*x))])/(a*(1 + p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{dx+c}\right)^{p}}{x}dx$$

input
```
int((a+b/(d*x+c))^p/x,x)
```

output
```
int((a+b/(d*x+c))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x,x, algorithm="fricas")
```

output
```
integral(((a*d*x + a*c + b)/(d*x + c))^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x+c))**p/x,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b/(d*x+c))^p/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{-1,[
0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x}\,dx = \int \frac{\left(a+\frac{b}{c+d\,x}\right)^p}{x}\,dx$$

input
```
int((a + b/(c + d*x))^p/x,x)
```

output
```
int((a + b/(c + d*x))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x}\,dx = \int \frac{(adx+ac+b)^p}{(dx+c)^p\,x}dx$$

input
```
int((a+b/(d*x+c))^p/x,x)
```

output
```
int((a*c + a*d*x + b)**p/((c + d*x)**p*x),x)
```

3.58 $\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx = -\frac{bd\left(a+\frac{b}{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx}\right)}{b+ac}\right)}{(b+ac)^2(1+p)}$$

output
```
-b*d*(a+b/(d*x+c))^(p+1)*hypergeom([2, p+1],[2+p],c*(a+b/(d*x+c))/(a*c+b))
/(a*c+b)^2/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx = \int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b/(c + d*x))^p/x^2,x]
```

output
```
Integrate[(a + b/(c + d*x))^p/x^2, x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {896, 941, 946, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2} \, dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \text{941}$$

$$d\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{946}$$

$$-d\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{78}$$

$$-\frac{bd\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{c\left(a+\frac{b}{c+dx}\right)}{b+ac}\right)}{(p+1)(ac+b)^2}$$

input
```
Int[(a + b/(c + d*x))^p/x^2,x]
```

output
```
-((b*d*(a + b/(c + d*x))^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (c*(a
+ b/(c + d*x)))/(b + a*c)])/((b + a*c)^2*(1 + p)))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{dx+c}\right)^{p}}{x^{2}}dx$$

input
```
int((a+b/(d*x+c))^p/x^2,x)
```

output
```
int((a+b/(d*x+c))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^2,x, algorithm="fricas")
```

output
```
integral(((a*d*x + a*c + b)/(d*x + c))^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx=\int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c))**p/x**2,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c))^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x}\right)^p}{x^2}\,dx$$

input
```
int((a + b/(c + d*x))^p/x^2,x)
```

output
```
int((a + b/(c + d*x))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^2}\,dx=\int \frac{(adx+ac+b)^p}{(dx+c)^p\,x^2}dx$$

input
```
int((a+b/(d*x+c))^p/x^2,x)
```

output
```
int((a*c + a*d*x + b)**p/((c + d*x)**p*x**2),x)
```

3.59 $\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx = -\frac{(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{1+p}}{2c(b+ac)x^2} + \frac{bd^2(b+2ac+bp)\left(a+\frac{b}{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx}\right)}{b+ac}\right)}{2c(b+ac)^3(1+p)}$$

output
```
-1/2*(d*x+c)^2*(a+b/(d*x+c))^(p+1)/c/(a*c+b)/x^2+1/2*b*d^2*(2*a*c+b*p+b)*(
a+b/(d*x+c))^(p+1)*hypergeom([2, p+1],[2+p],c*(a+b/(d*x+c))/(a*c+b))/c/(a*
c+b)^3/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx$$

input
```
Integrate[(a + b/(c + d*x))^p/x^3,x]
```

output
```
Integrate[(a + b/(c + d*x))^p/x^3, x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {896, 25, 941, 948, 25, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\int -\frac{\left(a+\frac{b}{c+dx}\right)^p}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$-d^2\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \text{ 948}$$

$$d^2\int -\frac{\left(a+\frac{b}{c+dx}\right)^p}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$\Big\downarrow$ 25

$$-d^2\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$\Big\downarrow$ 87

$$d^2\left(\frac{(2ac+bp+b)\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2c(ac+b)}-\frac{\left(a+\frac{b}{c+dx}\right)^{p+1}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$\Big\downarrow$ 78

$$d^2\left(\frac{b(2ac+bp+b)\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{c\left(a+\frac{b}{c+dx}\right)}{b+ac}\right)}{2c(p+1)(ac+b)^3}-\frac{\left(a+\frac{b}{c+dx}\right)^{p+1}}{2c(ac+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

input

```
Int[(a + b/(c + d*x))^p/x^3,x]
```

output

```
d^2*(-1/2*(a + b/(c + d*x))^(1 + p)/(c*(b + a*c)*(1 - c/(c + d*x))^2) + (b
*(b + 2*a*c + b*p)*(a + b/(c + d*x))^(1 + p)*Hypergeometric2F1[2, 1 + p, 2
 + p, (c*(a + b/(c + d*x)))/(b + a*c)])/(2*c*(b + a*c)^3*(1 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^3}dx$$

input
```
int((a+b/(d*x+c))^p/x^3,x)
```

output
```
int((a+b/(d*x+c))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^3,x, algorithm="fricas")
```

output
```
integral(((a*d*x + a*c + b)/(d*x + c))^p/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx=\int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))**p/x**3,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**p/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x}\right)^p}{x^3}\,dx$$

input
```
int((a + b/(c + d*x))^p/x^3,x)
```

output
```
int((a + b/(c + d*x))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^3}\,dx = \int \frac{(adx+ac+b)^p}{(dx+c)^p\,x^3}dx$$

input
```
int((a+b/(d*x+c))^p/x^3,x)
```

output
```
int((a*c + a*d*x + b)**p/((c + d*x)**p*x**3),x)
```

3.60 $\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx = \frac{d(6ac+b(4+p))(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{1+p}}{6c^2(b+ac)^2x^2} - \frac{(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{1+p}}{3c^2(b+ac)x^3} - \frac{bd^3(6a^2c^2+6abc(1+p)+b^2(2+3p+p^2))\left(a+\frac{b}{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx}\right)}{b+ac}\right)}{6c^2(b+ac)^4(1+p)}$$

output
```
1/6*d*(6*a*c+b*(4+p))*(d*x+c)^2*(a+b/(d*x+c))^(p+1)/c^2/(a*c+b)^2/x^2-1/3*
(d*x+c)^3*(a+b/(d*x+c))^(p+1)/c^2/(a*c+b)/x^3-1/6*b*d^3*(6*a^2*c^2+6*a*b*c
*(p+1)+b^2*(p^2+3*p+2))*(a+b/(d*x+c))^(p+1)*hypergeom([2, p+1],[2+p],c*(a+
b/(d*x+c))/(a*c+b))/c^2/(a*c+b)^4/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx$$

input
```
Integrate[(a + b/(c + d*x))^p/x^4,x]
```

output
```
Integrate[(a + b/(c + d*x))^p/x^4, x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {896, 941, 948, 100, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$d^3\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{d^4x^4}d(c+dx)$$

$$\Big\downarrow \text{ 941}$$

$$d^3\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{(c+dx)^4\left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \text{ 948}$$

$$-d^3\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{(c+dx)^2\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 100}$$

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{p+1}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{\left(a+\frac{b}{c+dx}\right)^p\left(pb+b+3ac+\frac{3c(b+ac)}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3c^2(ac+b)}\right)$$

$\big\downarrow$ 87

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{p+1}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(6ac+b(p+4))\left(a+\frac{b}{c+dx}\right)^{p+1}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{(6a^2c^2+6abc(p+1)+b^2(p^2+3p+2))\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2(ac+b)}}{3c^2(ac+b)}\right)$$

$\big\downarrow$ 78

$$-d^3\left(\frac{\left(a+\frac{b}{c+dx}\right)^{p+1}}{3c^2(ac+b)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(6ac+b(p+4))\left(a+\frac{b}{c+dx}\right)^{p+1}}{2(ac+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b(6a^2c^2+6abc(p+1)+b^2(p^2+3p+2))\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometr}}{2(p+1)(ac+b)^3}}{3c^2(ac+b)}\right.$$

input
```
Int[(a + b/(c + d*x))^p/x^4,x]
```

output
```
-(d^3*((a + b/(c + d*x))^(1 + p)/(3*c^2*(b + a*c)*(1 - c/(c + d*x))^3) - (
((6*a*c + b*(4 + p))*(a + b/(c + d*x))^(1 + p))/(2*(b + a*c)*(1 - c/(c + d
*x))^2) - (b*(6*a^2*c^2 + 6*a*b*c*(1 + p) + b^2*(2 + 3*p + p^2))*(a + b/(c
 + d*x))^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (c*(a + b/(c + d*x)))/
(b + a*c)])/(2*(b + a*c)^3*(1 + p)))/(3*c^2*(b + a*c)))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^4}dx$$

input
```
int((a+b/(d*x+c))^p/x^4,x)
```

output
```
int((a+b/(d*x+c))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^4,x, algorithm="fricas")
```

output
```
integral(((a*d*x + a*c + b)/(d*x + c))^p/x^4, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx=\int \frac{\left(\frac{ac+adx+b}{c+dx}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))**p/x**4,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**p/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx = \int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx = \int \frac{\left(a+\frac{b}{dx+c}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c))^p/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c))^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx = \int \frac{\left(a+\frac{b}{c+d\,x}\right)^p}{x^4}\,dx$$

input
```
int((a + b/(c + d*x))^p/x^4,x)
```

output
```
int((a + b/(c + d*x))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx}\right)^p}{x^4}\,dx=\int \frac{(adx+ac+b)^p}{(dx+c)^p\,x^4}dx$$

input
```
int((a+b/(d*x+c))^p/x^4,x)
```

output
```
int((a*c + a*d*x + b)**p/((c + d*x)**p*x**4),x)
```

3.61 $\int x^m \left(a+\frac{b}{c+dx}\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int x^m \left(a+\frac{b}{c+dx}\right)^p \, dx = \frac{x^{1+m}\left(1+\frac{dx}{c}\right)^p \left(1+\frac{adx}{b+ac}\right)^{-p} \left(a+\frac{b}{c+dx}\right)^p \operatorname{AppellF1}\left(1+m,p,-p,2+m,-\frac{dx}{c},-\frac{adx}{b+ac}\right)}{1+m}$$

output
```
x^(1+m)*(1+d*x/c)^p*(a+b/(d*x+c))^p*AppellF1(1+m,p,-p,2+m,-d*x/c,-a*d*x/(a
*c+b))/(1+m)/((1+a*d*x/(a*c+b))^p)
```

Mathematica [F]

$$\int x^m \left(a+\frac{b}{c+dx}\right)^p \, dx = \int x^m \left(a+\frac{b}{c+dx}\right)^p \, dx$$

input
```
Integrate[x^m*(a + b/(c + d*x))^p,x]
```

output
```
Integrate[x^m*(a + b/(c + d*x))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+\frac{b}{c+dx}\right)^p \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^m \left(a+\frac{b}{c+dx}\right)^p \, dx$$

input
```
Int[x^m*(a + b/(c + d*x))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^m \left(a+\frac{b}{dx+c}\right)^p \, dx$$

input
```
int(x^m*(a+b/(d*x+c))^p,x)
```

output
```
int(x^m*(a+b/(d*x+c))^p,x)
```

Fricas [F]

$$\int x^m\left(a+\frac{b}{c+dx}\right)^p\,dx=\int\left(a+\frac{b}{dx+c}\right)^p x^m\,dx$$

input
```
integrate(x^m*(a+b/(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral(x^m*((a*d*x + a*c + b)/(d*x + c))^p, x)
```

Sympy [F]

$$\int x^m\left(a+\frac{b}{c+dx}\right)^p\,dx=\int x^m\left(\frac{ac+adx+b}{c+dx}\right)^p\,dx$$

input
```
integrate(x**m*(a+b/(d*x+c))**p,x)
```

output
```
Integral(x**m*((a*c + a*d*x + b)/(c + d*x))**p, x)
```

Maxima [F]

$$\int x^m\left(a+\frac{b}{c+dx}\right)^p\,dx=\int\left(a+\frac{b}{dx+c}\right)^p x^m\,dx$$

input
```
integrate(x^m*(a+b/(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c))^p*x^m, x)
```

Giac [F]

$$\int x^m \left(a+\frac{b}{c+dx}\right)^p \,dx=\int \left(a+\frac{b}{dx+c}\right)^p x^m \,dx$$

input
```
integrate(x^m*(a+b/(d*x+c))^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c))^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(a+\frac{b}{c+dx}\right)^p \,dx=\int x^m \left(a+\frac{b}{c+d\,x}\right)^p \,dx$$

input
```
int(x^m*(a + b/(c + d*x))^p,x)
```

output
```
int(x^m*(a + b/(c + d*x))^p, x)
```

Reduce [F]

$$\int x^m \left(a+\frac{b}{c+dx}\right)^p \,dx=\int \frac{x^m(adx+ac+b)^p}{(dx+c)^p}dx$$

input
```
int(x^m*(a+b/(d*x+c))^p,x)
```

output
```
int((x**m*(a*c + a*d*x + b)**p)/(c + d*x)**p,x)
```

3.62 $\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p \, dx = \frac{(ex)^{1+m} \left(1+\frac{dx}{c}\right)^p \left(1+\frac{adx}{b+ac}\right)^{-p} \left(a+\frac{b}{c+dx}\right)^p \text{AppellF1}\left(1+m,p,-p,2+m,-\frac{dx}{c},-\frac{adx}{b+ac}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(1+d*x/c)^p*(a+b/(d*x+c))^p*AppellF1(1+m,p,-p,2+m,-d*x/c,-a*d*
x/(a*c+b))/e/(1+m)/((1+a*d*x/(a*c+b))^p)
```

Mathematica [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p \, dx = \int (ex)^m \left(a+\frac{b}{c+dx}\right)^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b/(c + d*x))^p,x]
```

output
```
Integrate[(e*x)^m*(a + b/(c + d*x))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p \, dx$$

input
```
Int[(e*x)^m*(a + b/(c + d*x))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^m \left(a+\frac{b}{dx+c}\right)^p \, dx$$

input
```
int((e*x)^m*(a+b/(d*x+c))^p,x)
```

output
```
int((e*x)^m*(a+b/(d*x+c))^p,x)
```

Fricas [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*((a*d*x + a*c + b)/(d*x + c))^p, x)
```

Sympy [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p dx = \int (ex)^m \left(\frac{ac+adx+b}{c+dx}\right)^p dx$$

input
```
integrate((e*x)**m*(a+b/(d*x+c))**p,x)
```

output
```
Integral((e*x)**m*((a*c + a*d*x + b)/(c + d*x))**p, x)
```

Maxima [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(a + b/(d*x + c))^p, x)
```

Giac [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x+c))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(a + b/(d*x + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p dx = \int (e\,x)^m \left(a+\frac{b}{c+d\,x}\right)^p dx$$

input
```
int((e*x)^m*(a + b/(c + d*x))^p,x)
```

output
```
int((e*x)^m*(a + b/(c + d*x))^p, x)
```

Reduce [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx}\right)^p dx = e^m \left(\int \frac{x^m (adx+ac+b)^p}{(dx+c)^p} dx\right)$$

input
```
int((e*x)^m*(a+b/(d*x+c))^p,x)
```

output
```
e**m*int((x**m*(a*c + a*d*x + b)**p)/(c + d*x)**p,x)
```

3.63 $\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 201

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=-\frac{c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{ad^4}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4ad^4}-\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(b-12ac^2+\frac{8ac^3}{c+dx}\right)}{8ad^4}+\frac{\sqrt{b}c^3\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{d^4}-\frac{b(b-12ac^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^4}$$

output

```
-c*(d*x+c)^3*(a+b/(d*x+c)^2)^(3/2)/a/d^4+1/4*(d*x+c)^4*(a+b/(d*x+c)^2)^(3/
2)/a/d^4-1/8*(d*x+c)^2*(a+b/(d*x+c)^2)^(1/2)*(b-12*a*c^2+8*a*c^3/(d*x+c))/
a/d^4+b^(1/2)*c^3*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/d^4-1/8*b
*(-12*a*c^2+b)*arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(3/2)/d^4
```

Mathematica [A] (verified)

Time = 10.70 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \frac{-8a^{3/2}\sqrt{b}c^3\log(c+dx)-b(b-12ac^2)\log\left((c+dx)\left(a+\sqrt{a}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)+\sqrt{a}\Big((-7bc^2-2ac^4-6b}{8a^{3/2}d^4}$$

input
```
Integrate[x^3*Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(-8*a^(3/2)*Sqrt[b]*c^3*Log[c + d*x] - b*(b - 12*a*c^2)*Log[(c + d*x)*(a +
 Sqrt[a]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])] + Sqrt[a]*((-7*b*c^2 - 2*
a*c^4 - 6*b*c*d*x + b*d^2*x^2 + 2*a*d^4*x^4)*Sqrt[(b + a*(c + d*x)^2)/(c +
 d*x)^2] + 8*a*Sqrt[b]*c^3*Log[b + Sqrt[b]*(c + d*x)*Sqrt[(b + a*(c + d*x)
^2)/(c + d*x)^2]]))/(8*a^(3/2)*d^4)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {896, 25, 1774, 1803, 25, 540, 2338, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int d^3x^3\sqrt{a+\frac{b}{(c+dx)^2}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -d^3x^3\sqrt{a+\frac{b}{(c+dx)^2}}d(c+dx)}{d^4}$$

$$\Big\downarrow \; 1774$$

$$-\frac{\int (c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{c}{c+dx}-1\right)^3d(c+dx)}{d^4}$$

$$\Big\downarrow \; 1803$$

$$\frac{\int -(c+dx)^5\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int (c+dx)^5\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \; 540$$

$$\frac{\frac{\int (c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{4ac^3}{(c+dx)^2}+12ac+\frac{b-12ac^2}{c+dx}\right)d\frac{1}{c+dx}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}}{d^4}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{-\frac{\int -3a(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{4ac^3}{c+dx}-12ac^2+b\right)d\frac{1}{c+dx}}{3a}-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}}{d^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int (c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{4ac^3}{c+dx}-12ac^2+b\right)d\frac{1}{c+dx}-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}}{d^4}$$

$$\Big\downarrow \; 537$$

$$\frac{\frac{-\frac{1}{2}b\int -\frac{(c+dx)\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}}{d^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}b\int \frac{(c+dx)\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}}{d^4}$$

$$\Big\downarrow \; 538$$

$$\frac{\frac{\frac{1}{2}b\left(8ac^3\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+(b-12ac^2)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{4a}}{d^4}$$

$\downarrow$ 224

$$\frac{\frac{\frac{1}{2}b\left(8ac^3\int\frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}+(b-12ac^2)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+dx)^3\left(a+\frac{b}{(c\right.}}{4a}}{d^4}$$

$\downarrow$ 219

$$\frac{\frac{\frac{1}{2}b\left((b-12ac^2)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\frac{8ac^3\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+dx)^3\left(a+\frac{}{(c+\right.}}{4a}}{d^4}$$

$\downarrow$ 243

$$\frac{\frac{\frac{1}{2}b\left(\frac{1}{2}(b-12ac^2)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}+\frac{8ac^3\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+dx)^3\left(a\right.}{4a}}{d^4}$$

$\downarrow$ 73

$$\frac{\frac{\frac{1}{2}b\left(\frac{(b-12ac^2)\int\frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}+\frac{8ac^3\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+d}{4a}}{d^4}$$

$\downarrow$ 221

$$\frac{\frac{\frac{1}{2}b\left(\frac{8ac^3\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}-\frac{(b-12ac^2)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-\frac{1}{2}(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{8ac^3}{c+dx}-12ac^2+b\right)-4c(c+dx)^3\left(a+\right.}{4a}}{d^4}$$

input
```
Int[x^3*Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(((c + d*x)^4*(a + b/(c + d*x)^2)^(3/2))/(4*a) + (-4*c*(c + d*x)^3*(a + b/
(c + d*x)^2)^(3/2) - ((c + d*x)^2*Sqrt[a + b/(c + d*x)^2]*(b - 12*a*c^2 +
(8*a*c^3)/(c + d*x)))/2 + (b*((8*a*c^3*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a +
 b/(c + d*x)^2])])/Sqrt[b] - ((b - 12*a*c^2)*ArcTanh[Sqrt[a + b/(c + d*x)^
2]/Sqrt[a]])/Sqrt[a]))/2)/(4*a))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{\left(-2 a \,x^{3} d^{3}+2 a \,d^{2} x^{2} c -2 a d x \,c^{2}+2 c^{3} a -b d x +7 b c \right) \sqrt{\frac{a \,d^{2} x^{2}+2 a d x c +a \,c^{2}+b}{\left(d x +c \right)^{2}}}\, \left(d x +c \right)}{8 a \,d^{4}}+\frac{\left(-\frac{b^{2} \ln \left(\frac{a \,d^{2} x +a c d}{\sqrt{a \,d^{2}}}+\sqrt{a \,d^{2} x^{2}+2 a d x c +a \,c^{2}+\cdots}\right)}{8 d^{3} a \sqrt{a \,d^{2}}} \cdots\right.}{\cdots}$
default	$-\frac{\sqrt{\frac{a \,d^{2} x^{2}+2 a d x c +a \,c^{2}+b}{\left(d x +c \right)^{2}}}\, \left(d x +c \right) \left(-12 \sqrt{a \,d^{2} x^{2}+2 a d x c +a \,c^{2}+b}\, \sqrt{a \,d^{2}}\, a \,c^{2} d x -8 \sqrt{a \,d^{2}}\, \sqrt{b}\, \ln \left(\frac{2 \left(\sqrt{b}\, \sqrt{a \,d^{2} x^{2}+2 a d x c +a \,c^{2}+b}+b \right) d}{d x +c}\right) \cdots\right.}{\cdots}$

input
```
int(x^3*(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*a*d^3*x^3+2*a*c*d^2*x^2-2*a*c^2*d*x+2*a*c^3-b*d*x+7*b*c)/a/d^4*((
a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(-1/8*b^2/d^3/a*ln((
a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/(a*d^2)^
(1/2)+3/2*b/d^3*c^2*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+
a*c^2+b)^(1/2))/(a*d^2)^(1/2)+b^(1/2)/d^4*c^3*ln((2*b+2*b^(1/2)*(a*d^2*(x+
c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)
*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1166, normalized size of antiderivative = 5.80

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*a^2*sqrt(b)*c^3*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x + c)
*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2
)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + (12*a*b*c^2 - b^2)*sqrt(a)*log(-2*a
*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt(
(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 2*(2
*a^2*d^4*x^4 - 2*a^2*c^4 + a*b*d^2*x^2 - 6*a*b*c*d*x - 7*a*b*c^2)*sqrt((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^2*d^4), 1/
8*(4*a^2*sqrt(b)*c^3*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x + c)*sqr
t(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) +
 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) - (12*a*b*c^2 - b^2)*sqrt(-a)*arctan((d^2
*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d
^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)) + (2*a^2*d^4
*x^4 - 2*a^2*c^4 + a*b*d^2*x^2 - 6*a*b*c*d*x - 7*a*b*c^2)*sqrt((a*d^2*x^2
+ 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^2*d^4), -1/16*(16*
a^2*sqrt(-b)*c^3*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) - (12*a*b*c^2 - b^2)*sqrt(a)*log(-
2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sq
rt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) - 2
*(2*a^2*d^4*x^4 - 2*a^2*c^4 + a*b*d^2*x^2 - 6*a*b*c*d*x - 7*a*b*c^2)*sqrt(
(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^2*d^...
```

Sympy [F]

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \int x^3\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}\,dx$$

input
```
integrate(x**3*(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(x**3*sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \int \sqrt{a+\frac{b}{(dx+c)^2}}x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2)*x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int x^3\,\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}\,dx$$

input
```
int(x^3*(a + b/(c + d*x)^2)^(1/2),x)
```

output
```
int(x^3*(a + b/(c + d*x)^2)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int x^3\sqrt{a+\frac{b}{\left(dx+c\right)^2}}dx$$

input
```
int(x^3*(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
int(x^3*(a+b/(d*x+c)^2)^(1/2),x)
```

3.64 $\int x^2 \sqrt{a+\frac{b}{(c+dx)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3ad^3}-\frac{c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}{d^3}-\frac{\sqrt{b}c^2\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{d^3}-\frac{bc\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

output
```
1/3*(d*x+c)^3*(a+b/(d*x+c)^2)^(3/2)/a/d^3-c*(d*x+c)^2*(a+b/(d*x+c)^2)^(1/2
)*(1-c/(d*x+c))/d^3-b^(1/2)*c^2*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1
/2))/d^3-b*c*arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 10.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.58

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx$$
$$=\frac{bc\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}+ac^3\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}+bdx\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}+ad^3x^3\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}+3a\sqrt{b}c^2\log(c+dx)-3\sqrt{a}bc\,\mathrm{l}}{3ad^3}$$

input
```
Integrate[x^2*Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(b*c*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2] + a*c^3*Sqrt[(b + a*(c + d*x)^2
)/(c + d*x)^2] + b*d*x*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2] + a*d^3*x^3*S
qrt[(b + a*(c + d*x)^2)/(c + d*x)^2] + 3*a*Sqrt[b]*c^2*Log[c + d*x] - 3*Sq
rt[a]*b*c*Log[(c + d*x)*(a + Sqrt[a]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]
)] - 3*a*Sqrt[b]*c^2*Log[b + Sqrt[b]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c
 + d*x)^2]])/(3*a*d^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {896, 1774, 1803, 540, 27, 537, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx$$
$$\Big\downarrow \text{ 896}$$
$$\frac{\int d^2x^2\sqrt{a+\frac{b}{(c+dx)^2}}d(c+dx)}{d^3}$$
$$\Big\downarrow \text{ 1774}$$

$$\frac{\int (c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{c}{c+dx}-1\right)^2d(c+dx)}{d^3}$$

$$\Big\downarrow \; 1803$$

$$-\frac{\int (c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \; 540$$

$$-\frac{-\frac{\int 3ac(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(2-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-c\int (c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(2-\frac{c}{c+dx}\right)d\frac{1}{c+dx}-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 537$$

$$-\frac{-c\left(\left(1-\frac{c}{c+dx}\right)(c+dx)^2\left(-\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{1}{2}b\int -\frac{2(c+dx)\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-c\left(b\int \frac{(c+dx)\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-(c+dx)^2\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 538$$

$$-\frac{-c\left(b\left(\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-c\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)-(c+dx)^2\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 224$$

$$-\frac{-c\left(b\left(\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-c\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)-(c+dx)^2\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3a}}{d^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{-c\left(b\left(\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)-(c+dx)^2\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{(c+dx)^3\left(\right.}{}}{d^3}$$

$\Big\downarrow$ 243

$$-\frac{-c\left(b\left(\frac{1}{2}\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\frac{c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)-(c+dx)^2\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{(c+}{}}{d^3}$$

$\Big\downarrow$ 73

$$-\frac{-c\left(b\left(\frac{\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)-(c+dx)^2\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-}{d^3}$$

$\Big\downarrow$ 221

$$-\frac{-c\left(b\left(-\frac{c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-(c+dx)^2\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{(c+}{}}{d^3}$$

input
```
Int[x^2*Sqrt[a + b/(c + d*x)^2],x]
```

output
```
-((-1/3*((c + d*x)^3*(a + b/(c + d*x)^2)^(3/2))/a - c*(-((c + d*x)^2*Sqrt[
a + b/(c + d*x)^2]*(1 - c/(c + d*x))) + b*(-((c*ArcTanh[Sqrt[b]/((c + d*x)
*Sqrt[a + b/(c + d*x)^2])])/Sqrt[b]) - ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqr
t[a]]/Sqrt[a])))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.58

method	result
risch	$\frac{\left(a\,d^2x^2-adxc+a\,c^2+b\right)\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}{3a\,d^3}+\frac{\left(-\frac{bc\ln\left(\frac{a\,d^2x+acd}{\sqrt{a\,d^2}}+\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\right)}{d^2\sqrt{a\,d^2}}-\frac{\sqrt{b}\,c^2\ln\left(\frac{2b+2\sqrt{b}\sqrt{a\,d^2(}}{x+\frac{c}{d}}\right.}{d^3}\right.}{\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}$
default	$\frac{\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)\left(-3\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,\sqrt{a\,d^2}\,acdx-3\sqrt{a\,d^2}\,\sqrt{b}\,\ln\left(\frac{2\left(\sqrt{b}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}+b\right)d}{dx+c}\right)a\,c^2-3\right.}{3d^3\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,a\sqrt{a\,d^2}}$

input
```
int(x^2*(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d^2*x^2-a*c*d*x+a*c^2+b)/a/d^3*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+
c)^2)^(1/2)*(d*x+c)+(-b*c/d^2*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+
2*a*c*d*x+a*c^2+b)^(1/2))/(a*d^2)^(1/2)-b^(1/2)*c^2/d^3*ln((2*b+2*b^(1/2)*
(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c
)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1028, normalized size of antiderivative = 6.90

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*a*sqrt(b)*c^2*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x + c)*sq
rt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + 3*sqrt(a)*b*c*log(-2*a*d^2*x^2 - 4*a*c
*d*x - 2*a*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 2*(a*d^3*x^3 + a*c^3
 + b*d*x + b*c)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*
x + c^2)))/(a*d^3), 1/6*(3*a*sqrt(b)*c^2*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c
^2 - 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2
 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + 6*sqrt(-a)*b*c*arct
an((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)) + 2*(
a*d^3*x^3 + a*c^3 + b*d*x + b*c)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/
(d^2*x^2 + 2*c*d*x + c^2)))/(a*d^3), 1/6*(6*a*sqrt(-b)*c^2*arctan((d*x + c
)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c
^2))/b) + 3*sqrt(a)*b*c*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 + 2*(d^2*x^
2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x
^2 + 2*c*d*x + c^2)) - b) + 2*(a*d^3*x^3 + a*c^3 + b*d*x + b*c)*sqrt((a*d^
2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a*d^3), 1/3*(3
*a*sqrt(-b)*c^2*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) + 3*sqrt(-a)*b*c*arctan((d^2*x^2...
```

Sympy [F]

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int x^2\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}\,dx$$

input
```
integrate(x**2*(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int \sqrt{a+\frac{b}{(dx+c)^2}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2)*x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int x^2\,\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}\,dx$$

input
```
int(x^2*(a + b/(c + d*x)^2)^(1/2),x)
```

output
```
int(x^2*(a + b/(c + d*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.74

$$\int x^2\sqrt{a+\frac{b}{(c+dx)^2}}\,dx$$
$$=\frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,c^2-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,acdx+\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,d^2x^2}{}$$

input
```
int(x^2*(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**2 - sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b)*a*c*d*x + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
*a*d**2*x**2 + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b - 3*sqrt(a)*lo
g((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/s
qrt(b))*b*c + 3*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) +
sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/sqrt(b))*a*c**2 - 3*sqrt(b)*log((sqrt(a
*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x + sqrt(b))/
sqrt(b))*a*c**2)/(3*a*d**3)
```

3.65 $\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{2c}{c+dx}\right)}{2d^2}+\frac{\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{d^2}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}$$

output
```
1/2*(d*x+c)^2*(a+b/(d*x+c)^2)^(1/2)*(1-2*c/(d*x+c))/d^2+b^(1/2)*c*arctanh(
b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/d^2+1/2*b*arctanh((a+b/(d*x+c)^2)^(
1/2)/a^(1/2))/a^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 10.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.34

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=-\frac{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(\sqrt{a}(c-dx)\sqrt{b+a(c+dx)^2}+4\sqrt{a}\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{a}(c+dx)-\sqrt{b+a(c+dx)^2}}{\sqrt{b}}\right)+b\log\right.}{2\sqrt{a}d^2\sqrt{b+a(c+dx)^2}}$$

input
```
Integrate[x*Sqrt[a + b/(c + d*x)^2],x]
```

output
```
-1/2*((c + d*x)*Sqrt[a + b/(c + d*x)^2]*(Sqrt[a]*(c - d*x)*Sqrt[b + a*(c +
 d*x)^2] + 4*Sqrt[a]*Sqrt[b]*c*ArcTanh[(Sqrt[a]*(c + d*x) - Sqrt[b + a*(c
+ d*x)^2])/Sqrt[b]] + b*Log[-(Sqrt[a]*(c + d*x)) + Sqrt[b + a*(c + d*x)^2]
]))/(Sqrt[a]*d^2*Sqrt[b + a*(c + d*x)^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {896, 25, 1774, 1803, 25, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int dx\sqrt{a+\frac{b}{(c+dx)^2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -dx\sqrt{a+\frac{b}{(c+dx)^2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \ 1774$$

$$-\frac{\int (c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{c}{c+dx}-1\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \ 1803$$

$$\frac{\int -(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int (c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$\Big\downarrow$ 537

$$\frac{\frac{1}{2}b\int -\frac{(c+dx)\left(1-\frac{2c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\frac{1}{2}\left(1-\frac{2c}{c+dx}\right)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{d^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{1}{2}b\int \frac{(c+dx)\left(1-\frac{2c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{d^2}$$

$\Big\downarrow$ 538

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{1}{2}b\left(\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-2c\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)}{d^2}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{1}{2}b\left(\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-2c\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{d^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{1}{2}b\left(\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{2c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)}{d^2}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{1}{2}b\left(\frac{1}{2}\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\frac{2c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)}{d^2}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{1}{2}b\left(\frac{\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{2c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)}{d^2}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{1}{2}b\left(-\frac{2c\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d^2}$$

input
```
Int[x*Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(((c + d*x)^2*Sqrt[a + b/(c + d*x)^2]*(1 - (2*c)/(c + d*x)))/2 - (b*((-2*c
*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/Sqrt[b] - ArcTanh[S
qrt[a + b/(c + d*x)^2]/Sqrt[a]]/Sqrt[a]))/2)/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(98) = 196.

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{(-dx+c)\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}{2d^2}+\frac{\left(\frac{b\ln\left(\frac{a\,d^2x+acd}{\sqrt{a\,d^2}}+\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\right)}{2d\sqrt{a\,d^2}}+\frac{\sqrt{b}\,c\ln\left(\frac{2b+2\sqrt{b}\sqrt{a\,d^2\left(x+\frac{c}{d}\right)^2+b}}{x+\frac{c}{d}}\right)}{d^2}\right)\sqrt{\cdots}}{\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}$
default	$-\frac{\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)\left(-\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,\sqrt{a\,d^2}\,dx-2\sqrt{a\,d^2}\,\sqrt{b}\ln\left(\frac{2\left(\sqrt{b}\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}+b\right)d}{dx+c}\right)c+\sqrt{a\cdots}\right.}{2d^2\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,\sqrt{a\,d^2}}$

input
```
int(x*(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+c)/d^2*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+
(1/2*b/d*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1
/2))/(a*d^2)^(1/2)+b^(1/2)/d^2*c*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/
2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d
^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(98) = 196.

Time = 0.18 (sec) , antiderivative size = 989, normalized size of antiderivative = 8.53

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*sqrt(b)*c*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x + c)*sqrt
(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) +
2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + sqrt(a)*b*log(-2*a*d^2*x^2 - 4*a*c*d*x -
 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x
 + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 2*(a*d^2*x^2 - a*c^2)*sqrt
((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a*d^2),
1/2*(a*sqrt(b)*c*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x + c)*sqrt(b)
*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b
)/(d^2*x^2 + 2*c*d*x + c^2)) - sqrt(-a)*b*arctan((d^2*x^2 + 2*c*d*x + c^2)
*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^
2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)) + (a*d^2*x^2 - a*c^2)*sqrt((a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a*d^2), -1/4*(4
*a*sqrt(-b)*c*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^
2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) - sqrt(a)*b*log(-2*a*d^2*x^2 - 4*a*c*
d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) - 2*(a*d^2*x^2 - a*c^2)
*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a*d
^2), -1/2*(2*a*sqrt(-b)*c*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) + sqrt(-a)*b*arctan((d^2*
x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/...
```

Sympy [F]

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int x\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}\,dx$$

input
```
integrate(x*(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(x*sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d
**2*x**2)), x)
```

Maxima [F]

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int \sqrt{a+\frac{b}{(dx+c)^2}}x\,dx$$

input
```
integrate(x*(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2)*x, x)
```

Giac [F(-1)]

Timed out.

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int x\,\sqrt{a+\frac{b}{(c+d\,x)^2}}\,dx$$

input
```
int(x*(a + b/(c + d*x)^2)^(1/2),x)
```

output
```
int(x*(a + b/(c + d*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int x\sqrt{a+\frac{b}{(c+dx)^2}}\,dx$$

$$= \frac{-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,ac+\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,adx+\sqrt{a}\log\left(\frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}+\sqrt{a}\,c+\sqrt{\cdots}}{\sqrt{b}}\right.\cdots}{2a\cdots}$$

input
```
int(x*(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c + sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b)*a*d*x + sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**
2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b - 2*sqrt(b)*log((sqrt(a*
c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/s
qrt(b))*a*c + 2*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) +
sqrt(a)*c + sqrt(a)*d*x + sqrt(b))/sqrt(b))*a*c)/(2*a*d**2)
```

3.66 $\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\frac{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{d}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{d}$$

output
```
(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/d-b^(1/2)*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+
c)^2)^(1/2))/d
```

Mathematica [A] (verified)

Time = 10.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\sqrt{a}(c+dx)\sqrt{\frac{b+a(c+dx)^2}{a(c+dx)^2}}-\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}}{\sqrt{a}(c+dx)}\right)\right)}{\sqrt{a}d\sqrt{1+\frac{b}{a(c+dx)^2}}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(Sqrt[a + b/(c + d*x)^2]*(Sqrt[a]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(a*(c
 + d*x)^2)] - Sqrt[b]*ArcSinh[Sqrt[b]/(Sqrt[a]*(c + d*x))]))/(Sqrt[a]*d*Sq
rt[1 + b/(a*(c + d*x)^2)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {239, 773, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \sqrt{a+\frac{b}{(c+dx)^2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int (c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{247}$$

$$-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{d}$$

$$\Big\downarrow \text{224}$$

$$-\frac{b\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}-(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)-(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{d}$$

input
```
Int[Sqrt[a + b/(c + d*x)^2],x]
```

output
```
-((-((c + d*x)*Sqrt[a + b/(c + d*x)^2]) + Sqrt[b]*ArcTanh[Sqrt[b]/((c + d*
x)*Sqrt[a + b/(c + d*x)^2])])/d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.08

method	result	size
default	$\frac{\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)\left(\sqrt{a d^2x^2+2adxc+a c^2+b}-\sqrt{b}\,\ln\left(\frac{2\left(\sqrt{b}\,\sqrt{a d^2x^2+2adxc+a c^2+b}+b\right)d}{dx+c}\right)\right)}{\sqrt{a d^2x^2+2adxc+a c^2+b}\,d}$	133
risch	$\frac{\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)}{d}-\frac{\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,d^2\left(x+\frac{c}{d}\right)^2+b}}{x+\frac{c}{d}}\right)\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)}{d\sqrt{a d^2x^2+2adxc+a c^2+b}}$	147

input
```
int((a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d
*x+a*c^2+b)^(1/2)/d*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)-b^(1/2)*ln(2*(b^(
1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)*d/(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.25

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx$$

$$= \left[\frac{2\,(dx+c)\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}+\sqrt{b}\log\left(-\frac{ad^2x^2+2\,acdx+ac^2-2\,(dx+c)\sqrt{b}\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}+2\,b}{d^2x^2+2\,cdx+c^2}\right)}{2\,d}, \frac{(dx+c)\sqrt{\cdots}}{\cdots}\right.$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*(d*x + c)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*
d*x + c^2)) + sqrt(b)*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x + c)*sq
rt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d, ((d*x + c)*sqrt((a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + sqrt(-b)*arctan((d*x + c)*sqr
t(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/
b))/d]
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int \sqrt{a+\frac{b}{\left(c+dx\right)^2}}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b/(c + d*x)**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx=\int \sqrt{a+\frac{b}{\left(dx+c\right)^2}}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \frac{2\,b\arctan\left(-\frac{\sqrt{a}c|d|+\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)d}{\sqrt{-b}d}\right)\mathrm{sgn}(dx+c)}{\sqrt{-b}d} + \frac{\sqrt{ad^2x^2+2\,acdx+ac^2+b}\,\mathrm{sgn}(dx+c)}{d}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
2*b*arctan(-(sqrt(a)*c*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b))*d)/(sqrt(-b)*d))*sgn(d*x + c)/(sqrt(-b)*d) + sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b)*sgn(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx = \frac{\sqrt{a+\frac{b}{(c+d\,x)^2}}\left(c+d\,x+\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{a}\,(c+d\,x)}\right)\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b}{a\,(c+d\,x)^2}+1}}\right)}{d}$$

input
```
int((a + b/(c + d*x)^2)^(1/2),x)
```

output
```
((a + b/(c + d*x)^2)^(1/2)*(c + d*x + (b^(1/2)*asin((b^(1/2)*1i)/(a^(1/2)*
(c + d*x)))*1i)/(a^(1/2)*(b/(a*(c + d*x)^2) + 1)^(1/2))))/d
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.81

$$\int \sqrt{a+\frac{b}{(c+dx)^2}}\,dx$$

$$= \frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}+\sqrt{b}\log\left(\frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}+\sqrt{a}\,c+\sqrt{a}\,dx-\sqrt{b}}{\sqrt{b}}\right)-\sqrt{b}\log\left(\frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}+\sqrt{\ldots}}{\sqrt{b}}\right.}{d}$$

input
```
int((a+b/(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(b)*log((sqrt(a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/sqrt(b))
- sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sq
rt(a)*d*x + sqrt(b))/sqrt(b)))/d
```

3.67 $\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x} \, dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c} + \sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right) - \frac{\sqrt{b+ac^2}\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}$$

output
```
b^(1/2)*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/c+a^(1/2)*arctanh((
a+b/(d*x+c)^2)^(1/2)/a^(1/2))-(a*c^2+b)^(1/2)*arctanh((a*c+b/(d*x+c))/(a*c
^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/c
```

Mathematica [A] (verified)

Time = 10.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x} \, dx =$$
$$-\frac{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(2\sqrt{-b-ac^2}\arctan\left(\frac{-\sqrt{a}dx+\sqrt{b+a(c+dx)^2}}{\sqrt{-b-ac^2}}\right)+2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{a}(c+dx)-\sqrt{b+a(c+dx)^2}}{\sqrt{b}}\right)\right.}{c\sqrt{b+a(c+dx)^2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)^2]/x,x]
```

output
```
-(((c + d*x)*Sqrt[a + b/(c + d*x)^2]*(2*Sqrt[-b - a*c^2]*ArcTan[(-(Sqrt[a]
*d*x) + Sqrt[b + a*(c + d*x)^2])/Sqrt[-b - a*c^2]] + 2*Sqrt[b]*ArcTanh[(Sq
rt[a]*(c + d*x) - Sqrt[b + a*(c + d*x)^2])/Sqrt[b]] + Sqrt[a]*c*Log[-(Sqrt
[a]*(c + d*x)) + Sqrt[b + a*(c + d*x)^2]]))/(c*Sqrt[b + a*(c + d*x)^2]))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {896, 25, 1774, 1803, 25, 606, 25, 243, 73, 221, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x} \, dx$$
$$\Big\downarrow \text{896}$$
$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{dx} d(c+dx)$$
$$\Big\downarrow \text{25}$$
$$-\int -\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{dx} d(c+dx)$$

$$\Big\downarrow \text{1774}$$

$$-\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$

$$\Big\downarrow \text{1803}$$

$$\int -\frac{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{606}$$

$$\int -\frac{\frac{b}{c+dx}+ac}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}-a\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-a\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\int \frac{\frac{b}{c+dx}+ac}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}a\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\int \frac{\frac{b}{c+dx}+ac}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{\frac{b}{c+dx}+ac}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}-\frac{a\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}$$

$$\Big\downarrow \text{221}$$

$$\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)-\int \frac{\frac{b}{c+dx}+ac}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{719}$$

$$-\frac{(ac^2+b)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}+\frac{b\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{c}+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)$$

$$\Big\downarrow \text{224}$$

$$-\frac{(ac^2+b)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}+\frac{b\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{(ac^2+b)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{(ac^2+b)\int \frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ac^2+b}\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x)^2]/x,x]
```

output
```
(Sqrt[b]*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/c + Sqrt[a]
*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]] + (Sqrt[b + a*c^2]*ArcTanh[(-(a*
c) - b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(106) = 212$.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.01

method	result
default	$-\frac{\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)\left(-acd\ln\left(\frac{a\,d^{2}x+acd+\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\,\sqrt{a\,d^{2}}}{\sqrt{a\,d^{2}}}\right)+\sqrt{a\,c^{2}+b}\,\ln\left(\frac{2a\,c^{2}+2b+2adxc+2\sqrt{a\,c^{2}+b}\,\sqrt{}}{x}\right.\right.}{\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\,c\sqrt{a\,d^{2}}}$

input
```
int((a+b/(d*x+c)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)*(-a*c*d*ln((a*d^2
*x+a*c*d+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))
+(a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+
a*c^2+b)^(1/2)+b)/x)*(a*d^2)^(1/2)-(a*d^2)^(1/2)*b^(1/2)*ln(2*(b^(1/2)*(a*
d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)*d/(d*x+c)))/(a*d^2*x^2+2*a*c*d*x+a*c^2
+b)^(1/2)/c/(a*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(106) = 212$.

Time = 0.92 (sec) , antiderivative size = 2695, normalized size of antiderivative = 21.73

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*c*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*
d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2)) - b) + sqrt(b)*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x +
 c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + sqrt(a*c^2 + b)*log(-(2*a^2*c^4
+ (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2
- 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((
a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2))/c, 1/
2*(sqrt(a)*c*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x
 + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*
x + c^2)) - b) - 2*sqrt(-b)*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) + sqrt(a*c^2 + b)*log(-
(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d
*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2
+ b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/
x^2))/c, -1/2*(2*sqrt(-a)*c*arctan((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt
((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2
 + 2*a*c*d*x + a*c^2 + b)) - sqrt(b)*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2
*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) - sqrt(a*c^2 + b)*log(...
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x}\,dx=\int \frac{\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**
2*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x}\,dx = \int \frac{\sqrt{a+\frac{b}{(dx+c)^2}}}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x}\,dx = \int \frac{\sqrt{a+\frac{b}{(c+d\,x)^2}}}{x}\,dx$$

input
```
int((a + b/(c + d*x)^2)^(1/2)/x,x)
```

output
```
int((a + b/(c + d*x)^2)^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x}\,dx$$
$$= \frac{2\sqrt{a\,c^2+b}\log\left(\sqrt{a\,c^2+b}\,\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}-a\,c^2-acdx-b\right)-2\sqrt{a\,c^2+b}\log(x)+2\sqrt{a}\log(}{}$$

input
```
int((a+b/(d*x+c)^2)^(1/2)/x,x)
```

output
```
(2*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*
x**2 + b) - a*c**2 - a*c*d*x - b) - 2*sqrt(a*c**2 + b)*log(x) + 2*sqrt(a)*
log( - sqrt(a)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c - a*d*x)*c
 - sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(b)) + sqr
t(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(b)))/(2*c)
```

3.68 $\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx = -\frac{d\sqrt{a+\frac{b}{(c+dx)^2}}}{c\left(1-\frac{c}{c+dx}\right)} - \frac{\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c^2} + \frac{bd\text{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c^2\sqrt{b+ac^2}}$$

output
```
-d*(a+b/(d*x+c)^2)^(1/2)/c/(1-c/(d*x+c))-b^(1/2)*d*arctanh(b^(1/2)/(d*x+c)
/(a+b/(d*x+c)^2)^(1/2))/c^2+b*d*arctanh((a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a
+b/(d*x+c)^2)^(1/2))/c^2/(a*c^2+b)^(1/2)
```

Mathematica [A] (verified)

Time = 10.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx = -\frac{d\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}}{c} - \frac{\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}}{x} - \frac{bd\log(x)}{c^2\sqrt{b+ac^2}} + \frac{\sqrt{b}d\log(c+dx)}{c^2} - \frac{\sqrt{b}d\log\left(b+\sqrt{b}(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)}{c^2} + \frac{bd\log\left(b+(c+dx)\left(ac+\sqrt{b+ac^2}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{c^2\sqrt{b+ac^2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)^2]/x^2,x]
```

output
```
-((d*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])/c) - Sqrt[(b + a*(c + d*x)^2)/
(c + d*x)^2]/x - (b*d*Log[x])/(c^2*Sqrt[b + a*c^2]) + (Sqrt[b]*d*Log[c + d
*x])/c^2 - (Sqrt[b]*d*Log[b + Sqrt[b]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(
c + d*x)^2]])/c^2 + (b*d*Log[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b
+ a*(c + d*x)^2)/(c + d*x)^2])])/(c^2*Sqrt[b + a*c^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 1774, 1799, 492, 605, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx$$

$$\Big\downarrow \ 896$$

$$d\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \text{1774}$$

$$d\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{1799}$$

$$-d\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{492}$$

$$-d\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\int \frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}\right)$$

$$\Big\downarrow \text{605}$$

$$-d\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(\frac{\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{c}\right)}{c}\right)$$

$$\Big\downarrow \text{224}$$

$$-d\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(\frac{\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}\right)}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$-d\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(\frac{\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}\right)}{c}\right)$$

$$\Big\downarrow \ 488$$

$$-d\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(-\frac{\int \frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}-\frac{\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}\right)}{c}\right)$$

$$\Big\downarrow \ 219$$

$$-d\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(-\frac{\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c\sqrt{ac^2+b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}\right)}{c}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x)^2]/x^2,x]
```

output
```
-(d*(Sqrt[a + b/(c + d*x)^2]/(c*(1 - c/(c + d*x))) - (b*(-(ArcTanh[Sqrt[b]
/((c + d*x)*Sqrt[a + b/(c + d*x)^2])]/(Sqrt[b]*c)) - ArcTanh[(-(a*c) - b/(
c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])]/(c*Sqrt[b + a*c^2])))
/c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}{cx}+\frac{\left(\frac{db\ln\left(\frac{2a\,c^2+2b+2adxc+2\sqrt{a\,c^2+b}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}{x}\right)}{c^2\sqrt{a\,c^2+b}}-\frac{d\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,d^2\left(x+\frac{c}{d}\right)^2+b}}{x+\frac{c}{d}}\right.}{c^2}\right.}{\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}$
default	$-\frac{\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)\left(-\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,acd^2x^2+\sqrt{b}\,\ln\left(\frac{2\left(\sqrt{b}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}+b\right)d}{dx+c}\right)a\,c^2dx-2\sqrt{a\,d^2}\right.}{}$

input
```
int((a+b/(d*x+c)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c/x*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(1/c^2*d*b/
(a*c^2+b)^(1/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a
*c*d*x+a*c^2+b)^(1/2))/x)-1/c^2*d*b^(1/2)*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)
^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*
x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(118) = 236.

Time = 0.19 (sec) , antiderivative size = 1334, normalized size of antiderivative = 9.96

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*((a*c^2 + b)*sqrt(b)*d*x*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x
 + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
+ c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + sqrt(a*c^2 + b)*b*d*x*log(-(2*
a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x
+ 2*b^2 + 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b
)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2
) - 2*(a*c^4 + b*c^2 + (a*c^3 + b*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a*c^4 + b*c^2)*x), 1/2*(2*(a*c^2 +
b)*sqrt(-b)*d*x*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) + sqrt(a*c^2 + b)*b*d*x*log(-(2*a^2
*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2
*b^2 + 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*s
qrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) -
 2*(a*c^4 + b*c^2 + (a*c^3 + b*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a*c^4 + b*c^2)*x), -1/2*(2*sqrt(-a*c^2
 - b)*b*d*x*arctan((a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a
*c^2 - b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^
2))/(a^2*c^4 + (a^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 2*(a^2*c^3 + a*b*c)*d
*x + b^2)) - (a*c^2 + b)*sqrt(b)*d*x*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 -
 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 ...
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx=\int \frac{\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**
2*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{(dx+c)^2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2)/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(118) = 236.

Time = 0.20 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx = -\frac{2\,bd\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}}{\sqrt{-ac^2-b}}\right)\operatorname{sgn}(dx+c)}{\sqrt{-ac^2-b}c^2} + \frac{2\,bd\arctan\left(-\frac{\sqrt{a}c|d|+\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)d}{\sqrt{-b}d}\right)\operatorname{sgn}(dx+c)}{\sqrt{-b}c^2} - \frac{2\left(a^{\frac{3}{2}}b^2c^2d^4|d|\operatorname{sgn}(dx+c)+\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)ab^2cd^5\operatorname{sgn}(dx+c)+\sqrt{a}b^3d^4|d|\operatorname{sg}\right.}{\left(ac^2-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)^2+b\right)b^2cd^4}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-2*b*d*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))/s
qrt(-a*c^2 - b))*sgn(d*x + c)/(sqrt(-a*c^2 - b)*c^2) + 2*b*d*arctan(-(sqrt
(a)*c*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*d
)/(sqrt(-b)*d))*sgn(d*x + c)/(sqrt(-b)*c^2) - 2*(a^(3/2)*b^2*c^2*d^4*abs(d
)*sgn(d*x + c) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))
*a*b^2*c*d^5*sgn(d*x + c) + sqrt(a)*b^3*d^4*abs(d)*sgn(d*x + c))/((a*c^2 -
 (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2 + b)*b^2*c*d^
4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{(c+d\,x)^2}}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x)^2)^(1/2)/x^2,x)
```

output
```
int((a + b/(c + d*x)^2)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^2}\,dx$$
$$= \frac{-2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,c^3-2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,bc+2\sqrt{a\,c^2+b}\log\left(-\sqrt{a\,c^2+b}\,\sqrt{a\,d\right.}{}$$

input
```
int((a+b/(d*x+c)^2)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**3 - 2*sqrt(a*c**2 +
2*a*c*d*x + a*d**2*x**2 + b)*b*c + 2*sqrt(a*c**2 + b)*log( - sqrt(a*c**2 +
 b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b*d
*x - 2*sqrt(a*c**2 + b)*log(x)*b*d*x + sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x
 + a*d**2*x**2 + b) - sqrt(b))*a*c**2*d*x + sqrt(b)*log(sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b) - sqrt(b))*b*d*x - sqrt(b)*log(sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b) + sqrt(b))*a*c**2*d*x - sqrt(b)*log(sqrt(a*c**2 +
 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(b))*b*d*x)/(2*c**2*x*(a*c**2 + b))
```

3.69 $\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 212

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx = -\frac{d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^2\left(1-\frac{c}{c+dx}\right)^2}+\frac{(3b+2ac^2)\,d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^2(b+ac^2)\left(1-\frac{c}{c+dx}\right)} + \frac{\sqrt{b}d^2\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c^3} - \frac{b(2b+3ac^2)\,d^2\text{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^3(b+ac^2)^{3/2}}$$

output
```
-1/2*d^2*(a+b/(d*x+c)^2)^(1/2)/c^2/(1-c/(d*x+c))^2+1/2*(2*a*c^2+3*b)*d^2*(
a+b/(d*x+c)^2)^(1/2)/c^2/(a*c^2+b)/(1-c/(d*x+c))+b^(1/2)*d^2*arctanh(b^(1/
2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/c^3-1/2*b*(3*a*c^2+2*b)*d^2*arctanh((a*c
+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/c^3/(a*c^2+b)^(3/2)
```

Mathematica [A] (verified)

Time = 10.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx = -\frac{\frac{c(c+dx)\left(b(c-2dx)+ac^2(c-dx)\right)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}}{(b+ac^2)x^2}-\frac{b(2b+3ac^2)d^2\log(x)}{(b+ac^2)^{3/2}}+2\sqrt{b}d^2\log(c+dx)-2\sqrt{b}d^2\log\left(b+\sqrt{b}(c+d\right.}{2c^3}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)^2]/x^3,x]
```

output
```
-1/2*((c*(c + d*x)*(b*(c - 2*d*x) + a*c^2*(c - d*x))*Sqrt[(b + a*(c + d*x)
^2)/(c + d*x)^2])/((b + a*c^2)*x^2) - (b*(2*b + 3*a*c^2)*d^2*Log[x])/(b +
a*c^2)^(3/2) + 2*Sqrt[b]*d^2*Log[c + d*x] - 2*Sqrt[b]*d^2*Log[b + Sqrt[b]*
(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]] + (b*(2*b + 3*a*c^2)*d^2*
Log[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x
)^2])])/(b + a*c^2)^(3/2))/c^3
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {896, 25, 1774, 1803, 25, 589, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx$$

$$\Big\downarrow \text{896}$$

$$d^2\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{d^3x^3}\,d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-d^2\int -\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \ 1774$$

$$-d^2\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \ 1803$$

$$d^2\int -\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 25$$

$$-d^2\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 589$$

$$d^2\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{c\left(2ac^2+3b\right)}{c+dx}+ac^2+2b\right)}{2c^2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b\int \frac{2\left(ac+\frac{2\left(ac^2+b\right)}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{4c^2\left(ac^2+b\right)}\right)$$

$$\Big\downarrow \ 27$$

$$d^2\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{c\left(2ac^2+3b\right)}{c+dx}+ac^2+2b\right)}{2c^2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b\int \frac{ac+\frac{2\left(ac^2+b\right)}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2c^2\left(ac^2+b\right)}\right)$$

$$\Big\downarrow \ 719$$

$$d^2\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{c\left(2ac^2+3b\right)}{c+dx}+ac^2+2b\right)}{2c^2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b\left(\frac{\left(3ac^2+2b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2\left(ac^2+b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{c}\right.}{2c^2\left(ac^2+b\right)}\right.$$

$$\Big\downarrow \text{224}$$

$$d^2\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{c(2ac^2+3b)}{c+dx}+ac^2+2b\right)}{2c^2(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b\left(\frac{(3ac^2+2b)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2(ac^2+b)\int\frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{\cdots}{(c+dx)\sqrt{\cdots}}}{c}\cdots\right.}{2c^2(ac^2+b)}\cdots\right.$$

$$\Big\downarrow \text{219}$$

$$d^2\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{c(2ac^2+3b)}{c+dx}+ac^2+2b\right)}{2c^2(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b\left(\frac{(3ac^2+2b)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2(ac^2+b)\text{arctanh}\left(\frac{\sqrt{\cdots}}{(c+dx)\sqrt{a+\cdots}}\cdots\right.}{\sqrt{b}c}\cdots\right.}{2c^2(ac^2+b)}\cdots\right.$$

$$\Big\downarrow \text{488}$$

$$d^2\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{c(2ac^2+3b)}{c+dx}+ac^2+2b\right)}{2c^2(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b\left(-\frac{(3ac^2+2b)\int\frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}-\frac{2(ac^2+b)\text{arctanh}\left(\frac{\cdots}{(c+dx)\sqrt{\cdots}}\cdots\right.}{\sqrt{b}c}\cdots\right.}{2c^2(ac^2+b)}\cdots\right.$$

$$\Big\downarrow \text{219}$$

$$d^2\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{c(2ac^2+3b)}{c+dx}+ac^2+2b\right)}{2c^2(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{b\left(-\frac{2(ac^2+b)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}-\frac{(3ac^2+2b)\text{arctanh}\left(\frac{-\cdots}{\sqrt{ac^2+\cdots}}\cdots\right.}{c\sqrt{ac^2+b}}\cdots\right.}{2c^2(ac^2+b)}\cdots\right.$$

input
```
Int[Sqrt[a + b/(c + d*x)^2]/x^3,x]
```

output
```
d^2*((Sqrt[a + b/(c + d*x)^2]*(2*b + a*c^2 - (c*(3*b + 2*a*c^2))/(c + d*x)
))/(2*c^2*(b + a*c^2)*(1 - c/(c + d*x))^2) - (b*((-2*(b + a*c^2)*ArcTanh[S
qrt[b]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/(Sqrt[b]*c) - ((2*b + 3*a*c^2
)*ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])
])/(c*Sqrt[b + a*c^2])))/(2*c^2*(b + a*c^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 589
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(a*d^2 + b*c^2*(2*p + 1)) - d*(
a*d^2*(n + 1) + b*c^2*(n - 2*p + 1))*x)/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2
))), x] + Simp[b*(p/(d^2*(n + 1)*(n + 2)*(b*c^2 + a*d^2)))   Int[(c + d*x)^
(n + 2)*(a + b*x^2)^(p - 1)*Simp[2*a*c*d*(n + 2) - (2*a*d^2*(n + 1) - 2*b*c
^2*(2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n
, -2] && LtQ[n + 2*p, 0] &&  !ILtQ[n + 2*p + 3, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(190) = 380$.

Time = 0.16 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{\left(-adx\,c^{2}+c^{3}a-2bdx+bc\right)\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}{2(a\,c^{2}+b)c^{2}x^{2}}+\frac{\left(-\frac{3d^{2}b\ln\left(\frac{2a\,c^{2}+2b+2adxc+2\sqrt{a\,c^{2}+b}\,\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}}{x}\right)a}{2c\left(a\,c^{2}+b\right)^{\frac{3}{2}}}-\right.}{}$
default	$-\frac{\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)\left(3\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\,a^{2}c^{3}d^{3}x^{3}-2\sqrt{b}\,\ln\left(\frac{2\left(\sqrt{b}\,\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}+b\right)d}{dx+c}\right)a^{2}c^{4}d^{2}x^{2}+5\sqrt{\right.}}{}$

input
```
int((a+b/(d*x+c)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a*c^2*d*x+a*c^3-2*b*d*x+b*c)/(a*c^2+b)/c^2/x^2*((a*d^2*x^2+2*a*c*d*
x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(-3/2/c*d^2*b/(a*c^2+b)^(3/2)*ln((2*a*
c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x
)*a-1/c^3*d^2*b^2/(a*c^2+b)^(3/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1
/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x)+1/c*d^2*b^(1/2)/(a*c^2+b)*ln((
2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d))*a+1/c^3*d^2*b^(3/2)/(a*c
^2+b)*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2
*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(
1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(188) = 376$.

Time = 0.32 (sec) , antiderivative size = 1687, normalized size of antiderivative = 7.96

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*c^4 + 2*a*b*c^2 + b^2)*sqrt(b)*d^2*x^2*log(-(a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + (3*a*b*c
^2 + 2*b^2)*sqrt(a*c^2 + b)*d^2*x^2*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^
2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c
^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(a^2*c^7 + 2*a*b*c^5 + b
^2*c^3 - (a^2*c^5 + 3*a*b*c^3 + 2*b^2*c)*d^2*x^2 - (a*b*c^4 + b^2*c^2)*d*x
)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a
^2*c^7 + 2*a*b*c^5 + b^2*c^3)*x^2), -1/4*(4*(a^2*c^4 + 2*a*b*c^2 + b^2)*sq
rt(-b)*d^2*x^2*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c
^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) - (3*a*b*c^2 + 2*b^2)*sqrt(a*c^2 + b
)*d^2*x^2*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2
*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b
*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*
c*d*x + c^2)))/x^2) + 2*(a^2*c^7 + 2*a*b*c^5 + b^2*c^3 - (a^2*c^5 + 3*a*b*
c^3 + 2*b^2*c)*d^2*x^2 - (a*b*c^4 + b^2*c^2)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^2*c^7 + 2*a*b*c^5 + b^2*c
^3)*x^2), 1/2*((3*a*b*c^2 + 2*b^2)*sqrt(-a*c^2 - b)*d^2*x^2*arctan((a*c*d^
2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a*c^2 - b)*sqrt((a*d^2*x...
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{(c+dx)^2}}}{x^3}\,dx = \int \frac{\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**
2*x**2))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{(dx+c)^2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(188) = 376$.

Time = 0.54 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-2*b*d^2*arctan(-(sqrt(a)*c*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b))*d)/(sqrt(-b)*d))*sgn(d*x + c)/(sqrt(-b)*c^3) + (3*a*b
^4*c^2*d^8*sgn(d*x + c) + 2*b^5*d^8*sgn(d*x + c))*arctan(-(sqrt(a*d^2)*x -
 sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))/sqrt(-a*c^2 - b))/((a*b^3*c^5*d^
6 + b^4*c^3*d^6)*sqrt(-a*c^2 - b)) + (2*a^(7/2)*b^3*c^6*d^7*abs(d)*sgn(d*x
 + c) + 4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^3*b^
3*c^5*d^8*sgn(d*x + c) + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b))^2*a^(5/2)*b^3*c^4*d^7*abs(d)*sgn(d*x + c) + 6*a^(5/2)*b^4*c^4*d
^7*abs(d)*sgn(d*x + c) + 7*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b))*a^2*b^4*c^3*d^8*sgn(d*x + c) - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2
+ 2*a*c*d*x + a*c^2 + b))^3*a*b^4*c*d^8*sgn(d*x + c) + 6*a^(3/2)*b^5*c^2*d
^7*abs(d)*sgn(d*x + c) + 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b))*a*b^5*c*d^8*sgn(d*x + c) - 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b))^2*sqrt(a)*b^5*d^7*abs(d)*sgn(d*x + c) + 2*sqrt(a)*
b^6*d^7*abs(d)*sgn(d*x + c))/((a*b^3*c^4*d^6 + b^4*c^2*d^6)*(a*c^2 - (sqrt
(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2 + b)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}}{x^3} \,d x$$

input
```
int((a + b/(c + d*x)^2)^(1/2)/x^3,x)
```

output
```
int((a + b/(c + d*x)^2)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^3}\,dx = \frac{-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^6+\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^5dx-2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,\cdots}{}$$

input
```
int((a+b/(d*x+c)^2)^(1/2)/x^3,x)
```

output

```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*c**6 + sqrt(a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b)*a**2*c**5*d*x - 2*sqrt(a*c**2 + 2*a*c*d*x + a*
d**2*x**2 + b)*a*b*c**4 + 3*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b
*c**3*d*x - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**2*c**2 + 2*sqrt(
a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**2*c*d*x + 3*sqrt(a*c**2 + b)*log(
sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c
*d*x - b)*a*b*c**2*d**2*x**2 + 2*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqr
t(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b**2*d**2*
x**2 - 3*sqrt(a*c**2 + b)*log(x)*a*b*c**2*d**2*x**2 - 2*sqrt(a*c**2 + b)*l
og(x)*b**2*d**2*x**2 - sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b) - sqrt(b))*a**2*c**4*d**2*x**2 - 2*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x
 + a*d**2*x**2 + b) - sqrt(b))*a*b*c**2*d**2*x**2 - sqrt(b)*log(sqrt(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(b))*b**2*d**2*x**2 + sqrt(b)*log(s
qrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(b))*a**2*c**4*d**2*x**2 +
 2*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(b))*a*b*c
**2*d**2*x**2 + sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + s
qrt(b))*b**2*d**2*x**2)/(2*c**3*x**2*(a**2*c**4 + 2*a*b*c**2 + b**2))
```

3.70 $\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 293

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx = -\frac{d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3c^3\left(1-\frac{c}{c+dx}\right)^3}+\frac{(7b+6ac^2)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6c^3\,(b+ac^2)\left(1-\frac{c}{c+dx}\right)^2} - \frac{(11b^2+20abc^2+6a^2c^4)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6c^3\,(b+ac^2)^2\left(1-\frac{c}{c+dx}\right)} - \frac{\sqrt{b}d^3\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c^4} + \frac{b(2b^2+5abc^2+4a^2c^4)\,d^3\text{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^4\,(b+ac^2)^{5/2}}$$

output
```
-1/3*d^3*(a+b/(d*x+c)^2)^(1/2)/c^3/(1-c/(d*x+c))^3+1/6*(6*a*c^2+7*b)*d^3*(
a+b/(d*x+c)^2)^(1/2)/c^3/(a*c^2+b)/(1-c/(d*x+c))^2-1/6*(6*a^2*c^4+20*a*b*c
^2+11*b^2)*d^3*(a+b/(d*x+c)^2)^(1/2)/c^3/(a*c^2+b)^2/(1-c/(d*x+c))-b^(1/2)
*d^3*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/c^4+1/2*b*(4*a^2*c^4+5
*a*b*c^2+2*b^2)*d^3*arctanh((a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2
)^(1/2))/c^4/(a*c^2+b)^(5/2)
```

Mathematica [A] (verified)

Time = 10.56 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx = -\frac{\frac{c(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\left(2a^2c^4\left(c^2-cdx+d^2x^2\right)+b^2\left(2c^2-3cdx+6d^2x^2\right)+abc^2\left(4c^2-5cdx+11d^2x^2\right)\right)}{(b+ac^2)^2x^3}+\frac{3b(2b^2+5abc^2+4a^2c^4)d^3\log(x)}{(b+ac^2)^{5/2}}-\cdots}{}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)^2]/x^4,x]
```

output
```
-1/6*((c*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]*(2*a^2*c^4*(c^2 -
 c*d*x + d^2*x^2) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2) + a*b*c^2*(4*c^2 - 5
*c*d*x + 11*d^2*x^2)))/((b + a*c^2)^2*x^3) + (3*b*(2*b^2 + 5*a*b*c^2 + 4*a
^2*c^4)*d^3*Log[x])/(b + a*c^2)^(5/2) - 6*Sqrt[b]*d^3*Log[c + d*x] + 6*Sqr
t[b]*d^3*Log[b + Sqrt[b]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]]
- (3*b*(2*b^2 + 5*a*b*c^2 + 4*a^2*c^4)*d^3*Log[b + (c + d*x)*(a*c + Sqrt[b
 + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])/(b + a*c^2)^(5/2))/c^4
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {896, 1774, 1803, 603, 27, 680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$d^3\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{d^4x^4}d(c+dx)$$

$$\Big\downarrow \text{ 1774}$$

$$d^3 \int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4 \left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \text{1803}$$

$$-d^3 \int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2 \left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{603}$$

$$-d^3 \left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{3\sqrt{a+\frac{b}{(c+dx)^2}}\left(a+\frac{\frac{b}{c}+ac}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3\left(ac^2+b\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$-d^3 \left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(a+\frac{\frac{b}{c}+ac}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{ac^2+b}\right)$$

$$\Big\downarrow \text{680}$$

$$-d^3 \left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{-\frac{\int -\frac{2b\left(\frac{2\left(ac^2+b\right)^2}{c+dx}+ac\left(2ac^2+b\right)\right)}{c\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{4c^2\left(ac^2+b\right)}-\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(b\left(3ac+\frac{2b}{c}\right)-\frac{2a^2c^4+6abc^2+3b^2}{c+dx}\right)}{2c^2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}}{ac^2+b}\right)$$

$$\Big\downarrow \text{27}$$

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{b\int\frac{\frac{2\left(ac^2+b\right)^2}{c+dx}+ac\left(2ac^2+b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2c^3(ac^2+b)}-\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(b\left(3ac+\frac{2b}{c}\right)-\frac{2a^2c^4+6abc^2+3b^2}{c+dx}\right)}{2c^2(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}}{ac^2+b}\right)$$

$\downarrow$ 719

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{b\left(\frac{\left(4a^2c^4+5abc^2+2b^2\right)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2\left(ac^2+b\right)^2\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{c}\right)}{2c^3(ac^2+b)}-\frac{\sqrt{a-}}{}}{ac^2+b}$$

$\downarrow$ 224

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{b\left(\frac{\left(4a^2c^4+5abc^2+2b^2\right)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2\left(ac^2+b\right)^2\int\frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}\right.}{2c^3(ac^2+b)}}{ac^2+b}$$

$\downarrow$ 219

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{b\left(\frac{\left(4a^2c^4+5abc^2+2b^2\right)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{2\left(ac^2+b\right)^2\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}\right.}{2c^3(ac^2+b)}}{ac^2+b}\right.$$

$\downarrow$ 488

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{b\left(-\frac{\left(4a^2c^4+5abc^2+2b^2\right)\int\frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}-\frac{2\left(ac^2+b\right)^2\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right.}{\sqrt{b}c}\right.}{2c^3(ac^2+b)}}{ac^2+b}\right.$$

$\downarrow$ 219

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{b\left(-\frac{\left(4a^2c^4+5abc^2+2b^2\right)\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c\sqrt{ac^2+b}}-\frac{2\left(ac^2+b\right)^2\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right.}{\sqrt{b}c}\right.}{2c^3(ac^2+b)}}{ac^2+b}\right.$$

input
```
Int[Sqrt[a + b/(c + d*x)^2]/x^4,x]
```

output
```
-(d^3*((a + b/(c + d*x)^2)^(3/2)/(3*c*(b + a*c^2)*(1 - c/(c + d*x))^3) - (
-1/2*(Sqrt[a + b/(c + d*x)^2]*(b*((2*b)/c + 3*a*c) - (3*b^2 + 6*a*b*c^2 +
2*a^2*c^4)/(c + d*x)))/(c^2*(b + a*c^2)*(1 - c/(c + d*x))^2) + (b*((-2*(b
 + a*c^2)^2*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/(Sqrt[b]*
c) - ((2*b^2 + 5*a*b*c^2 + 4*a^2*c^4)*ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt
[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/(c*Sqrt[b + a*c^2])))/(2*c^3*(b + a
*c^2)))/(b + a*c^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(267) = 534.

Time = 0.18 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{\left(2a^{2}c^{4}d^{2}x^{2}-2a^{2}c^{5}dx+2a^{2}c^{6}+11ab\,c^{2}d^{2}x^{2}-5ab\,c^{3}dx+4ab\,c^{4}+6b^{2}d^{2}x^{2}-3b^{2}cxd+2b^{2}c^{2}\right)\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}{6\left(a\,c^{2}+b\right)^{2}c^{3}x^{3}}+$
default	Expression too large to display

input
```
int((a+b/(d*x+c)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*a^2*c^4*d^2*x^2-2*a^2*c^5*d*x+2*a^2*c^6+11*a*b*c^2*d^2*x^2-5*a*b*c
^3*d*x+4*a*b*c^4+6*b^2*d^2*x^2-3*b^2*c*d*x+2*b^2*c^2)/(a*c^2+b)^2/c^3/x^3*
((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(2*d^3*b/(a*c^2+b)
^(5/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*
c^2+b)^(1/2))/x)*a^2+5/2*d^3*b^2/c^2/(a*c^2+b)^(5/2)*ln((2*a*c^2+2*b+2*a*d
*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x)*a+d^3*b^3/c
^4/(a*c^2+b)^(5/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+
2*a*c*d*x+a*c^2+b)^(1/2))/x)-d^3*b^(1/2)/(a*c^2+b)^2*ln((2*b+2*b^(1/2)*(a*
d^2*(x+c/d)^2+b)^(1/2))/(x+c/d))*a^2-2*d^3*b^(3/2)/c^2/(a*c^2+b)^2*ln((2*b
+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d))*a-d^3*b^(5/2)/c^4/(a*c^2+b)
^2*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*
c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 2097, normalized size of antiderivative = 7.16

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(6*(a^3*c^6 + 3*a^2*b*c^4 + 3*a*b^2*c^2 + b^3)*sqrt(b)*d^3*x^3*log(-
(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x +
 c^2)) + 3*(4*a^2*b*c^4 + 5*a*b^2*c^2 + 2*b^3)*sqrt(a*c^2 + b)*d^3*x^3*log
(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)
*d*x + 2*b^2 + 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^
2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
)/x^2) - 2*(2*a^3*c^10 + 6*a^2*b*c^8 + 6*a*b^2*c^6 + 2*b^3*c^4 + (2*a^3*c^
7 + 13*a^2*b*c^5 + 17*a*b^2*c^3 + 6*b^3*c)*d^3*x^3 + 3*(2*a^2*b*c^6 + 3*a*
b^2*c^4 + b^3*c^2)*d^2*x^2 - (a^2*b*c^7 + 2*a*b^2*c^5 + b^3*c^3)*d*x)*sqrt
((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^3*c^1
0 + 3*a^2*b*c^8 + 3*a*b^2*c^6 + b^3*c^4)*x^3), 1/12*(12*(a^3*c^6 + 3*a^2*b
*c^4 + 3*a*b^2*c^2 + b^3)*sqrt(-b)*d^3*x^3*arctan((d*x + c)*sqrt(-b)*sqrt(
(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) + 3*(4*a
^2*b*c^4 + 5*a*b^2*c^2 + 2*b^3)*sqrt(a*c^2 + b)*d^3*x^3*log(-(2*a^2*c^4 +
(2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 +
2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(2*a
^3*c^10 + 6*a^2*b*c^8 + 6*a*b^2*c^6 + 2*b^3*c^4 + (2*a^3*c^7 + 13*a^2*b*c^
5 + 17*a*b^2*c^3 + 6*b^3*c)*d^3*x^3 + 3*(2*a^2*b*c^6 + 3*a*b^2*c^4 + b^...
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx=\int \frac{\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**
2*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx = \int \frac{\sqrt{a+\frac{b}{(dx+c)^2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c)^2)/x^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1521 vs. $2(264) = 528$.

Time = 10.93 (sec) , antiderivative size = 1521, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(1/2)/x^4,x, algorithm="giac")
```

output

```
2*b*d^3*arctan(-(sqrt(a)*c*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b))*d)/(sqrt(-b)*d))*sgn(d*x + c)/(sqrt(-b)*c^4) - (4*a^2*
b^5*c^4*d^11*sgn(d*x + c) + 5*a*b^6*c^2*d^11*sgn(d*x + c) + 2*b^7*d^11*sgn
(d*x + c))*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b
))/sqrt(-a*c^2 - b))/((a^2*b^4*c^8*d^8 + 2*a*b^5*c^6*d^8 + b^6*c^4*d^8)*sq
rt(-a*c^2 - b)) - 1/3*(8*a^(11/2)*b^4*c^10*d^10*abs(d)*sgn(d*x + c) + 24*(
sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^5*b^4*c^9*d^11*
sgn(d*x + c) + 24*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)
)^2*a^(9/2)*b^4*c^8*d^10*abs(d)*sgn(d*x + c) + 8*(sqrt(a*d^2)*x - sqrt(a*d
^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*a^4*b^4*c^7*d^11*sgn(d*x + c) + 38*a^(9
/2)*b^5*c^8*d^10*abs(d)*sgn(d*x + c) + 78*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2
+ 2*a*c*d*x + a*c^2 + b))*a^4*b^5*c^7*d^11*sgn(d*x + c) + 36*(sqrt(a*d^2)*
x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(7/2)*b^5*c^6*d^10*abs(d)
*sgn(d*x + c) - 4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)
)^3*a^3*b^5*c^5*d^11*sgn(d*x + c) + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b))^4*a^(5/2)*b^5*c^4*d^10*abs(d)*sgn(d*x + c) + 72*a^(7
/2)*b^6*c^6*d^10*abs(d)*sgn(d*x + c) + 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
 2*a*c*d*x + a*c^2 + b))^5*a^2*b^5*c^3*d^11*sgn(d*x + c) + 93*(sqrt(a*d^2)
*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^3*b^6*c^5*d^11*sgn(d*x + c
) - 12*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{(c+d\,x)^2}}}{x^4}\,dx$$

input

```
int((a + b/(c + d*x)^2)^(1/2)/x^4,x)
```

output

```
int((a + b/(c + d*x)^2)^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1061, normalized size of antiderivative = 3.62

$$\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a+b/(d*x+c)^2)^(1/2)/x^4,x)
```

output
```
( - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**9 + 2*sqrt(a*c**2
 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**8*d*x - 2*sqrt(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)*a**3*c**7*d**2*x**2 - 6*sqrt(a*c**2 + 2*a*c*d*x + a*d**
2*x**2 + b)*a**2*b*c**7 + 7*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**
2*b*c**6*d*x - 13*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**5*d
**2*x**2 - 6*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**5 + 8*sq
rt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**4*d*x - 17*sqrt(a*c**2
+ 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**3*d**2*x**2 - 2*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*b**3*c**3 + 3*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)*b**3*c**2*d*x - 6*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3
*c*d**2*x**2 + 12*sqrt(a*c**2 + b)*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a**2*b*c**4*d**3*x**3
+ 15*sqrt(a*c**2 + b)*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*
d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a*b**2*c**2*d**3*x**3 + 6*sqrt(a*c*
*2 + b)*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
 - a*c**2 - a*c*d*x - b)*b**3*d**3*x**3 - 12*sqrt(a*c**2 + b)*log(x)*a**2*
b*c**4*d**3*x**3 - 15*sqrt(a*c**2 + b)*log(x)*a*b**2*c**2*d**3*x**3 - 6*sq
rt(a*c**2 + b)*log(x)*b**3*d**3*x**3 + 3*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b) - sqrt(b))*a**3*c**6*d**3*x**3 + 9*sqrt(b)*log(sqrt(
a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(b))*a**2*b*c**4*d**3*x**3 ...
```

3.71 $\int x^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = -\frac{c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{ad^4} + \frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{4ad^4} + \frac{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(b+12ac^2-\frac{8c(2b+ac^2)}{c+dx}\right)}{8ad^4} - \frac{3b\sqrt{a+\frac{b}{(c+dx)^2}}\left(b+12ac^2-\frac{4c(2b+ac^2)}{c+dx}\right)}{8ad^4} + \frac{3\sqrt{b}c(2b+ac^2)\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2d^4} + \frac{3b(b+12ac^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^4}$$

output
```
-c*(d*x+c)^3*(a+b/(d*x+c)^2)^(5/2)/a/d^4+1/4*(d*x+c)^4*(a+b/(d*x+c)^2)^(5/
2)/a/d^4+1/8*(d*x+c)^2*(a+b/(d*x+c)^2)^(3/2)*(b+12*a*c^2-8*c*(a*c^2+2*b)/(
d*x+c))/a/d^4-3/8*b*(a+b/(d*x+c)^2)^(1/2)*(b+12*a*c^2-4*c*(a*c^2+2*b)/(d*x
+c))/a/d^4+3/2*b^(1/2)*c*(a*c^2+2*b)*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^
2)^(1/2))/d^4+3/8*b*(12*a*c^2+b)*arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/a^
(1/2)/d^4
```

Mathematica [A] (verified)

Time = 10.33 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.87

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx = \frac{-\frac{\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\left(b\left(47c^3+73c^2dx+17cd^2x^2-5d^3x^3\right)+2a\left(c^5+c^4dx-cd^4x^4-d^5x^5\right)\right)}{c+dx}-12\sqrt{b}c\left(2b+ac^2\right)\log(}{}$$

input
```
Integrate[x^3*(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
(-((Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]*(b*(47*c^3 + 73*c^2*d*x + 17*c*d
^2*x^2 - 5*d^3*x^3) + 2*a*(c^5 + c^4*d*x - c*d^4*x^4 - d^5*x^5)))/(c + d*x
)) - 12*Sqrt[b]*c*(2*b + a*c^2)*Log[c + d*x] + (3*b*(b + 12*a*c^2)*Log[(c
+ d*x)*(a + Sqrt[a]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])/Sqrt[a] + 12*
Sqrt[b]*c*(2*b + a*c^2)*Log[b + Sqrt[b]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)
/(c + d*x)^2]])/(8*d^4)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.947$, Rules used = {896, 25, 1774, 1803, 25, 540, 2338, 27, 537, 25, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int d^3x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -d^3x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}d(c+dx)}{d^4}$$

$$\Big\downarrow \ 1774$$

$$-\frac{\int (c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)^3d(c+dx)}{d^4}$$

$$\Big\downarrow \ 1803$$

$$\frac{\int -(c+dx)^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^3d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int (c+dx)^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^3d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \ 540$$

$$\frac{\frac{\int (c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{4ac^3}{(c+dx)^2}+12ac-\frac{12ac^2+b}{c+dx}\right)d\frac{1}{c+dx}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{4a}}{d^4}$$

$$\Big\downarrow \ 2338$$

$$\frac{\frac{-\frac{\int 3a(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(12ac^2-\frac{4\left(ac^2+2b\right)c}{c+dx}+b\right)d\frac{1}{c+dx}}{3a}-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{4a}}{d^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\int (c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(12ac^2-\frac{4\left(ac^2+2b\right)c}{c+dx}+b\right)d\frac{1}{c+dx}-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{4a}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{4a}}{d^4}$$

$$\Big\downarrow \ 537$$

$$\frac{\frac{\frac{3}{2}b\int -\left((c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(12ac^2-\frac{8\left(ac^2+2b\right)c}{c+dx}+b\right)\right)d\frac{1}{c+dx}+\frac{1}{2}(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{8c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)-4c(c+dx)^3\left(a+\frac{b}{(c+dx)}\right.}{4a}}{d^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{-\frac{3}{2}b\int(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(12ac^2-\frac{8\left(ac^2+2b\right)c}{c+dx}+b\right)d\frac{1}{c+dx}+\frac{1}{2}(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{8c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)-4c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5}}{4a}}{d^4}$$

$\downarrow$ 535

$$\frac{\frac{-\frac{3}{2}b\left(\frac{1}{2}a\int\frac{2(c+dx)\left(12ac^2-\frac{4\left(ac^2+2b\right)c}{c+dx}+b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)\right)+\frac{1}{2}(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{8c\left(ac^2+2b\right)}{c+dx}+12\right.}{4a}}{d^4}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{3}{2}b\left(a\int\frac{(c+dx)\left(12ac^2-\frac{4\left(ac^2+2b\right)c}{c+dx}+b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)\right)+\frac{1}{2}(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{8c\left(ac^2+2b\right)}{c+dx}+12ac\right.}{4a}}{d^4}$$

$\downarrow$ 538

$$\frac{\frac{-\frac{3}{2}b\left(a\left((12ac^2+b)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-4c(ac^2+2b)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)\right)+\frac{1}{2}(c+dx)^2\left(a+\right.}{4a}}{d^4}$$

$\downarrow$ 224

$$\frac{\frac{-\frac{3}{2}b\left(a\left((12ac^2+b)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-4c(ac^2+2b)\int\frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)\right)+\frac{1}{2}(c\ldots}{4a}}{d^4}$$

$\downarrow$ 219

$$\frac{\frac{-\frac{3}{2}b\left(a\left((12ac^2+b)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{4c\left(ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)\right)+\frac{1}{2}(c+\ldots}{4a}}{d^4}$$

$\downarrow$ 243

$$-\frac{\frac{3}{2}b\left(\frac{a\left(\frac{1}{2}\left(12ac^2+b\right)\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\frac{4c\left(ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)}{4a}\right)+\frac{1}{2}}{d^4}$$

$\downarrow$ 73

$$-\frac{\frac{3}{2}b\left(\frac{a\left(\frac{\left(12ac^2+b\right)\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{4c\left(ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)}{4a}\right)}{d^4}$$

$\downarrow$ 221

$$-\frac{\frac{3}{2}b\left(\frac{a\left(-\frac{4c\left(ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}-\frac{\left(12ac^2+b\right)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\sqrt{a+\frac{b}{(c+dx)^2}}\left(-\frac{4c\left(ac^2+2b\right)}{c+dx}+12ac^2+b\right)}{4a}\right)+}{d^4}$$

input
```
Int[x^3*(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
(((c + d*x)^4*(a + b/(c + d*x)^2)^(5/2))/(4*a) + (-4*c*(c + d*x)^3*(a + b/
(c + d*x)^2)^(5/2) + ((c + d*x)^2*(a + b/(c + d*x)^2)^(3/2)*(b + 12*a*c^2
- (8*c*(2*b + a*c^2))/(c + d*x)))/2 - (3*b*(Sqrt[a + b/(c + d*x)^2]*(b + 1
2*a*c^2 - (4*c*(2*b + a*c^2))/(c + d*x)) + a*((-4*c*(2*b + a*c^2)*ArcTanh[
Sqrt[b]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/Sqrt[b] - ((b + 12*a*c^2)*Ar
cTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]])/Sqrt[a])))/2)/(4*a))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.61

method	result
risch	$-\frac{\left(-2a\,x^3d^3+2a\,d^2x^2c-2adx\,c^2+2c^3a-5bdx+27bc\right)\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}{8d^4}+\frac{\left(\frac{3b^2\ln\left(\frac{a\,d^2x+acd}{\sqrt{a\,d^2}}+\sqrt{a\,d^2x^2+2adxc+a\,c^2\cdots}\right)}{8d^3\sqrt{a\,d^2}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int(x^3*(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*a*d^3*x^3+2*a*c*d^2*x^2-2*a*c^2*d*x+2*a*c^3-5*b*d*x+27*b*c)/d^4*(
(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(3/8*b^2/d^3*ln((a*
d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/(a*d^2)^(1
/2)+9/2*b/d^3*a*c^2*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+
a*c^2+b)^(1/2))/(a*d^2)^(1/2)-3*b/d^5*c^2/(x+c/d)*(a*d^2*(x+c/d)^2+b)^(1/2
)+1/2*b/d^6*c^3/(x+c/d)^2*(a*d^2*(x+c/d)^2+b)^(1/2)+3/2*b^(1/2)/d^4*c^3*ln
((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d))*a+3*b^(3/2)/d^4*c*ln((
2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a
*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 1509, normalized size of antiderivative = 5.59

$$\int x^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(12*a*b*c^3 + b^2*c + (12*a*b*c^2 + b^2)*d*x)*sqrt(a)*log(-2*a*d^
2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 12*(a^2
*c^4 + 2*a*b*c^2 + (a^2*c^3 + 2*a*b*c)*d*x)*sqrt(b)*log(-(a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(2*a^2
*d^5*x^5 + 2*a^2*c*d^4*x^4 + 5*a*b*d^3*x^3 - 2*a^2*c^5 - 17*a*b*c*d^2*x^2
- 47*a*b*c^3 - (2*a^2*c^4 + 73*a*b*c^2)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a*d^5*x + a*c*d^4), -1/8*(3*(12*a
*b*c^3 + b^2*c + (12*a*b*c^2 + b^2)*d*x)*sqrt(-a)*arctan((d^2*x^2 + 2*c*d*
x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*
d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)) - 6*(a^2*c^4 + 2*a*b*c^2
+ (a^2*c^3 + 2*a*b*c)*d*x)*sqrt(b)*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + 2
*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) - (2*a^2*d^5*x^5 + 2*a^2*c*
d^4*x^4 + 5*a*b*d^3*x^3 - 2*a^2*c^5 - 17*a*b*c*d^2*x^2 - 47*a*b*c^3 - (2*a
^2*c^4 + 73*a*b*c^2)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^
2 + 2*c*d*x + c^2)))/(a*d^5*x + a*c*d^4), -1/16*(24*(a^2*c^4 + 2*a*b*c^2 +
 (a^2*c^3 + 2*a*b*c)*d*x)*sqrt(-b)*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x
^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) - 3*(12*a*b*c...
```

Sympy [F]

$$\int x^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \int x^3 \left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**3*(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(x**3*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \int \left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2)*x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(240) = 480$.

Time = 2.09 (sec) , antiderivative size = 940, normalized size of antiderivative = 3.48

$$\int x^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)*((2*(a*x*sgn(d*x + c)/d - a*c*
sgn(d*x + c)/d^2)*x + (2*a^3*c^2*d^10*sgn(d*x + c) + 5*a^2*b*d^10*sgn(d*x
+ c))/(a^2*d^13))*x - (2*a^3*c^3*d^9*sgn(d*x + c) + 27*a^2*b*c*d^9*sgn(d*x
 + c))/(a^2*d^13)) - 3/56*(12*a*b*c^2*sgn(d*x + c) + b^2*sgn(d*x + c))*log
(abs(16*a^(7/2)*c^7*d + 112*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b))*a^3*c^6*abs(d) + 336*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b))^2*a^(5/2)*c^5*d + 560*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2
*a*c*d*x + a*c^2 + b))^3*a^2*c^4*abs(d) + 560*(sqrt(a*d^2)*x - sqrt(a*d^2*
x^2 + 2*a*c*d*x + a*c^2 + b))^4*a^(3/2)*c^3*d - 48*a^(5/2)*b*c^5*d + 336*(
sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^5*a*c^2*abs(d) -
240*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^2*b*c^4*ab
s(d) + 112*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^6*sqr
t(a)*c*d - 480*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2
*a^(3/2)*b*c^3*d + 16*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b))^7*abs(d) - 480*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b))^3*a*b*c^2*abs(d) - 240*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b))^4*sqrt(a)*b*c*d + 48*a^(3/2)*b^2*c^3*d - 48*(sqrt(a*d^2)*x - s
qrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^5*b*abs(d) + 144*(sqrt(a*d^2)*x -
sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a*b^2*c^2*abs(d) + 144*(sqrt(a*d^
2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*sqrt(a)*b^2*c*d + 48*...
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\int x^3\,\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)^{3/2}\,dx$$

input
```
int(x^3*(a + b/(c + d*x)^2)^(3/2),x)
```

output
```
int(x^3*(a + b/(c + d*x)^2)^(3/2), x)
```

Reduce [F]

$$\int x^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \int x^3 \left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}} dx$$

input
```
int(x^3*(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
int(x^3*(a+b/(d*x+c)^2)^(3/2),x)
```

3.72 $\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 208

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx=\frac{(c+dx)(2b-3acdx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{3ad^3}+\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3ad^3}-\frac{b\sqrt{a+\frac{b}{(c+dx)^2}}(2b+3ac(c-2(c+dx)))}{2ad^3(c+dx)}-\frac{\sqrt{b}(2b+3ac^2)\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2d^3}-\frac{3\sqrt{a}bc\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{d^3}$$

output
```
1/3*(d*x+c)*(-3*a*c*d*x+2*b)*(a+b/(d*x+c)^2)^(3/2)/a/d^3+1/3*(d*x+c)^3*(a+
b/(d*x+c)^2)^(5/2)/a/d^3-1/2*b*(a+b/(d*x+c)^2)^(1/2)*(2*b+3*a*c*(-2*d*x-c)
)/a/d^3/(d*x+c)-1/2*b^(1/2)*(3*a*c^2+2*b)*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*
x+c)^2)^(1/2))/d^3-3*a^(1/2)*b*c*arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/d^
3
```

Mathematica [A] (verified)

Time = 10.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.01

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx=\frac{\frac{\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}(2a(c+dx)^2(c^2-cdx+d^2x^2)+b(17c^2+28cdx+8d^2x^2))}{c+dx}+3\sqrt{b}(2b+3ac^2)\log(c+dx)-1$$

input
```
Integrate[x^2*(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
((Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]*(2*a*(c + d*x)^2*(c^2 - c*d*x + d^
2*x^2) + b*(17*c^2 + 28*c*d*x + 8*d^2*x^2)))/(c + d*x) + 3*Sqrt[b]*(2*b +
3*a*c^2)*Log[c + d*x] - 18*Sqrt[a]*b*c*Log[(c + d*x)*(a + Sqrt[a]*Sqrt[(b
+ a*(c + d*x)^2)/(c + d*x)^2])] - 3*Sqrt[b]*(2*b + 3*a*c^2)*Log[b + Sqrt[b
]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]])/(6*d^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {896, 1774, 1803, 540, 537, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int d^2x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1774}$$

$$\frac{\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)^2d(c+dx)}{d^3}$$

$\Big\downarrow$ 1803

$$-\frac{\int (c+dx)^4 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^2 d\frac{1}{c+dx}}{d^3}$$

$\Big\downarrow$ 540

$$-\frac{-\frac{\int (c+dx)^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3a}}{d^3}$$

$\Big\downarrow$ 537

$$-\frac{-\frac{(c+dx)^2\left(-\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\right)\left(3ac-\frac{3ac^2+2b}{c+dx}\right)-\frac{3}{2}b\int -2(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(3ac-\frac{3ac^2+2b}{c+dx}\right)d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3a}}{d^3}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{3b\int (c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(3ac-\frac{3ac^2+2b}{c+dx}\right)d\frac{1}{c+dx}-(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(3ac-\frac{3ac^2+2b}{c+dx}\right)}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3a}}{d^3}$$

$\Big\downarrow$ 535

$$-\frac{-\frac{3b\left(\frac{1}{2}a\int \frac{(c+dx)\left(6ac-\frac{3ac^2+2b}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\frac{1}{2}\sqrt{a+\frac{b}{(c+dx)^2}}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)\right)-(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(3ac-\frac{3ac^2+2b}{c+dx}\right)}{3a}-\frac{(c+dx)^3\left(a+\frac{b}{(c+}\right.}{3a}}{d^3}$$

$\Big\downarrow$ 538

$$-\frac{-\frac{3b\left(\frac{1}{2}a\left(6ac\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-(3ac^2+2b)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)+\frac{1}{2}\sqrt{a+\frac{b}{(c+dx)^2}}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)\right)-(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(3ac\right.}{3a}}{d^3}$$

$\Big\downarrow$ 224

$$-\frac{-\frac{3b\left(\frac{1}{2}a\left(6ac\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-(3ac^2+2b)\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)+\frac{1}{2}\sqrt{a+\frac{b}{(c+dx)^2}}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)\right)-(c+dx)^2\left(a+\frac{b}{(c+d}\right.}{3a}}{d^3}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{3b\left(\frac{1}{2}a\left(6ac\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{\left(3ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+\frac{b}{(c+dx)^2}}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)\right)-(c+dx)^2\left(a+\frac{b}{(c+dx)}\right.}{3a}}{d^3}$$

$\downarrow$ 243

$$-\frac{-\frac{3b\left(\frac{1}{2}a\left(3ac\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\frac{\left(3ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+\frac{b}{(c+dx)^2}}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)\right)-(c+dx)^2\left(a+\frac{}{(c+}\right.}{3a}}{d^3}$$

$\downarrow$ 73

$$-\frac{-\frac{3b\left(\frac{1}{2}a\left(\frac{6ac\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{\left(3ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+\frac{b}{(c+dx)^2}}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)\right)-(c+dx)^2}{3a}}{d^3}$$

$\downarrow$ 221

$$-\frac{-\frac{3b\left(\frac{1}{2}a\left(-\frac{\left(3ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}-6\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{1}{2}\sqrt{a+\frac{b}{(c+dx)^2}}\left(6ac-\frac{3ac^2+2b}{c+dx}\right)\right)-(c+dx)^2\left(\right.}{3a}}{d^3}$$

input

```
Int[x^2*(a + b/(c + d*x)^2)^(3/2),x]
```

output

```
-((-1/3*((c + d*x)^3*(a + b/(c + d*x)^2)^(5/2))/a - (-((c + d*x)^2*(a + b/
(c + d*x)^2)^(3/2)*(3*a*c - (2*b + 3*a*c^2)/(c + d*x))) + 3*b*((Sqrt[a + b
/(c + d*x)^2]*(6*a*c - (2*b + 3*a*c^2)/(c + d*x)))/2 + (a*(-(((2*b + 3*a*c
^2)*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/Sqrt[b]) - 6*Sqr
t[a]*c*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]]))/2))/(3*a))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.69

method	result
risch	$\frac{\left(a\,d^{2}x^{2}-adxc+a\,c^{2}+4b\right)\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}{3d^{3}}+\frac{\left(-\frac{3\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,d^{2}\left(x+\frac{c}{d}\right)^{2}+b}}{x+\frac{c}{d}}\right)a\,c^{2}}{2d^{3}}-\frac{b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,d^{2}\left(x+\frac{c}{d}\right.}}{x+\frac{c}{d}}\right.}{d^{3}}\right.}{}$
default	Expression too large to display

input
```
int(x^2*(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d^2*x^2-a*c*d*x+a*c^2+4*b)/d^3*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+
c)^2)^(1/2)*(d*x+c)+(-3/2*b^(1/2)/d^3*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b
)^(1/2))/(x+c/d))*a*c^2-b^(3/2)/d^3*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^
(1/2))/(x+c/d))-3*b/d^2*a*c*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*
a*c*d*x+a*c^2+b)^(1/2))/(a*d^2)^(1/2)+2*b/d^4*c/(x+c/d)*(a*d^2*(x+c/d)^2+b
)^(1/2)-1/2*b/d^5*c^2/(x+c/d)^2*(a*d^2*(x+c/d)^2+b)^(1/2))*((a*d^2*x^2+2*a
*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 1307, normalized size of antiderivative = 6.28

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(18*(b*c*d*x + b*c^2)*sqrt(a)*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2
 + 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 3*(3*a*c^3 + (3*a*c^2 + 2*b)*d*x +
 2*b*c)*sqrt(b)*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x + c)*sqrt(b)*
sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)
/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(2*a*d^4*x^4 + 2*a*c*d^3*x^3 + 2*a*c^4 + 8
*b*d^2*x^2 + 17*b*c^2 + 2*(a*c^3 + 14*b*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^4*x + c*d^3), 1/12*(36*(b*c*
d*x + b*c^2)*sqrt(-a)*arctan((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^
2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)) + 3*(3*a*c^3 + (3*a*c^2 + 2*b)*d*x + 2*b*c)*sqrt(b)*l
og(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 +
 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d
*x + c^2)) + 2*(2*a*d^4*x^4 + 2*a*c*d^3*x^3 + 2*a*c^4 + 8*b*d^2*x^2 + 17*b
*c^2 + 2*(a*c^3 + 14*b*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d
^2*x^2 + 2*c*d*x + c^2)))/(d^4*x + c*d^3), 1/6*(3*(3*a*c^3 + (3*a*c^2 + 2*
b)*d*x + 2*b*c)*sqrt(-b)*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) + 9*(b*c*d*x + b*c^2)*sqrt
(a)*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*s
qrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^...
```

Sympy [F]

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\int x^2\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(x**2*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2)*x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(179) = 358$.

Time = 1.65 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.29

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)*((a*x*sgn(d*x + c)/d - a*c*sgn
(d*x + c)/d^2)*x + (a^2*c^2*d^5*sgn(d*x + c) + 4*a*b*d^5*sgn(d*x + c))/(a*
d^8)) + 18/7*sqrt(a)*b*c*abs(d)*log(28)*sgn(d*x + c)/d^4 + 3/7*sqrt(a)*b*c
*log(abs(-a^(7/2)*c^7*d - 7*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b))*a^3*c^6*abs(d) - 21*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b))^2*a^(5/2)*c^5*d - 35*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b))^3*a^2*c^4*abs(d) - 35*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2
 + 2*a*c*d*x + a*c^2 + b))^4*a^(3/2)*c^3*d + 3*a^(5/2)*b*c^5*d - 21*(sqrt(
a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^5*a*c^2*abs(d) + 15*(s
qrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^2*b*c^4*abs(d) -
 7*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^6*sqrt(a)*c*d
 + 30*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(3/2)*
b*c^3*d - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^7*abs(
d) + 30*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*a*b*c^
2*abs(d) + 15*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^4*
sqrt(a)*b*c*d - 3*a^(3/2)*b^2*c^3*d + 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b))^5*b*abs(d) - 9*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2
*a*c*d*x + a*c^2 + b))*a*b^2*c^2*abs(d) - 9*(sqrt(a*d^2)*x - sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b))^2*sqrt(a)*b^2*c*d - 3*(sqrt(a*d^2)*x - sqrt(a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*b^2*abs(d) + sqrt(a)*b^3*c*d + (sqr...
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\int x^2\,\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)^{3/2}\,dx$$

input
```
int(x^2*(a + b/(c + d*x)^2)^(3/2),x)
```

output
```
int(x^2*(a + b/(c + d*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 986, normalized size of antiderivative = 4.74

$$\int x^2 \left(a + \frac{b}{(c + dx)^2}\right)^{3/2} dx = \text{Too large to display}$$

input

```
int(x^2*(a+b/(d*x+c)^2)^(3/2),x)
```

output

```
(2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**4 + 2*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*a*c**3*d*x + 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*
x**2 + b)*a*c*d**3*x**3 + 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*d
**4*x**4 + 17*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b*c**2 + 28*sqrt(
a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b*c*d*x + 8*sqrt(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)*b*d**2*x**2 - 18*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b*c**3 - 36*sqrt(a)*
log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)
/sqrt(b))*b*c**2*d*x - 18*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b*c*d**2*x**2 + 9*sqrt(b)*log(
(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sq
rt(b))/sqrt(b))*a*c**4 + 18*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*
x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/sqrt(b))*a*c**3*d*x + 9*sqr
t(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)
*d*x - sqrt(b))/sqrt(b))*a*c**2*d**2*x**2 + 6*sqrt(b)*log((sqrt(a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/sqrt(b))*
b*c**2 + 12*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt
(a)*c + sqrt(a)*d*x - sqrt(b))/sqrt(b))*b*c*d*x + 6*sqrt(b)*log((sqrt(a*c*
*2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/sqr
t(b))*b*d**2*x**2 - 9*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**...
```

3.73 $\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\frac{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{2c}{c+dx}\right)}{2d^2}-\frac{3b\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}{2d^2}+\frac{3a\sqrt{b}c\,\mathrm{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2d^2}+\frac{3\sqrt{a}b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{2d^2}$$

output
```
1/2*(d*x+c)^2*(a+b/(d*x+c)^2)^(3/2)*(1-2*c/(d*x+c))/d^2-3/2*b*(a+b/(d*x+c)
^2)^(1/2)*(1-c/(d*x+c))/d^2+3/2*a*b^(1/2)*c*arctanh(b^(1/2)/(d*x+c)/(a+b/(
d*x+c)^2)^(1/2))/d^2+3/2*a^(1/2)*b*arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/
d^2
```

Mathematica [A] (verified)

Time = 10.79 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.17

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx = -\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\sqrt{b+a(c+dx)^2}(a(c-dx)(c+dx)^2+b(c+2dx))+6a\sqrt{b}c(c+dx)^2\operatorname{arctanh}\left(\frac{\sqrt{a}(c+dx)-\sqrt{\ }}{\sqrt{\ }}\right.\right.}{2d^2(c+dx)\sqrt{b+a(c+dx)^2}}$$

input
```
Integrate[x*(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[a + b/(c + d*x)^2]*(Sqrt[b + a*(c + d*x)^2]*(a*(c - d*x)*(c + d
*x)^2 + b*(c + 2*d*x)) + 6*a*Sqrt[b]*c*(c + d*x)^2*ArcTanh[(Sqrt[a]*(c + d
*x) - Sqrt[b + a*(c + d*x)^2])/Sqrt[b]] + 3*Sqrt[a]*b*(c + d*x)^2*Log[-(Sq
rt[a]*(c + d*x)) + Sqrt[b + a*(c + d*x)^2]]))/(d^2*(c + d*x)*Sqrt[b + a*(c
 + d*x)^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {896, 25, 1774, 1803, 25, 537, 25, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int dx\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1774}$$

$$-\frac{\int (c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1803}$$

$$\frac{\int -(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \text{537}$$

$$\frac{\frac{3}{2}b\int -\left((c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{2c}{c+dx}\right)\right)d\frac{1}{c+dx}+\frac{1}{2}\left(1-\frac{2c}{c+dx}\right)(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\int (c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{2c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(\frac{1}{2}a\int \frac{2(c+dx)\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(a\int \frac{(c+dx)\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}\right)}{d^2}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(a\left(\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-c\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)+\left(1-\frac{c}{c+dx}\right)\sqrt{a\ldots}\right.}{d^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(a\left(\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-c\int\frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)+\left(1-\right.\right.}{d^2}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(a\left(\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\left(1-\frac{}{c}\right.\right.}{d^2}$$

$\downarrow$ 243

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(a\left(\frac{1}{2}\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\left(1\right.\right.}{d^2}$$

$\downarrow$ 73

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(a\left(\frac{\int\frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}\right)+\right.}{d^2}$$

$\downarrow$ 221

$$\frac{\frac{1}{2}(c+dx)^2\left(1-\frac{2c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}-\frac{3}{2}b\left(a\left(-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\left(1\right.\right.}{d^2}$$

input

```
Int[x*(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
(((c + d*x)^2*(a + b/(c + d*x)^2)^(3/2)*(1 - (2*c)/(c + d*x)))/2 - (3*b*(S
qrt[a + b/(c + d*x)^2]*(1 - c/(c + d*x)) + a*(-((c*ArcTanh[Sqrt[b]/((c + d
*x)*Sqrt[a + b/(c + d*x)^2])])/Sqrt[b]) - ArcTanh[Sqrt[a + b/(c + d*x)^2]/
Sqrt[a]]/Sqrt[a])))/2)/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(131) = 262$.

Time = 0.17 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{(-dx+c)a\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}{2d^2}+\frac{\left(\frac{3ba\ln\left(\frac{a\,d^2x+acd}{\sqrt{a\,d^2}}+\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\right)}{2d\sqrt{a\,d^2}}-\frac{b\sqrt{a\,d^2\left(x+\frac{c}{d}\right)^2+b}}{d^3\left(x+\frac{c}{d}\right)}+\frac{3\sqrt{b}\,ac\ln\left(\frac{2b+2\cdots}{\cdots}\right.}{\cdots}\right.}{\sqrt{a\,d^2x^2+2adxc\cdots}}$
default	$-\frac{\left(-2\left(a\,d^2x^2+2adxc+a\,c^2+b\right)^{\frac{3}{2}}\sqrt{a\,d^2}\,a\,d^3x^3-5\left(a\,d^2x^2+2adxc+a\,c^2+b\right)^{\frac{3}{2}}\sqrt{a\,d^2}\,ac\,d^2x^2-3\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,\sqrt{a\,d^2}\,ab\,d^3\cdots\right.}{\cdots}$

input
```
int(x*(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+c)*a/d^2*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c
)+(3/2*b/d*a*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+a*c^2+b
)^(1/2))/(a*d^2)^(1/2)-b/d^3/(x+c/d)*(a*d^2*(x+c/d)^2+b)^(1/2)+3/2*b^(1/2)
/d^2*a*c*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d))+1/2*b/d^4*c
/(x+c/d)^2*(a*d^2*(x+c/d)^2+b)^(1/2))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+
c)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(130) = 260$.

Time = 0.61 (sec) , antiderivative size = 1189, normalized size of antiderivative = 7.67

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(b*d*x + b*c)*sqrt(a)*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(
d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/
(d^2*x^2 + 2*c*d*x + c^2)) - b) + 3*(a*c*d*x + a*c^2)*sqrt(b)*log(-(a*d^2*
x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*(a*d^3*x^3 + a*c*d^2*x^2 - a*c^3 - (a*c^2 + 2*b)*d*x - b*c)*sqrt((a*d^
2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^3*x + c*d^2)
, -1/4*(6*(b*d*x + b*c)*sqrt(-a)*arctan((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)
*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^
2*x^2 + 2*a*c*d*x + a*c^2 + b)) - 3*(a*c*d*x + a*c^2)*sqrt(b)*log(-(a*d^2*
x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2))
- 2*(a*d^3*x^3 + a*c*d^2*x^2 - a*c^3 - (a*c^2 + 2*b)*d*x - b*c)*sqrt((a*d^
2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^3*x + c*d^2)
, -1/4*(6*(a*c*d*x + a*c^2)*sqrt(-b)*arctan((d*x + c)*sqrt(-b)*sqrt((a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b) - 3*(b*d*x + b
*c)*sqrt(a)*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x
+ c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
 + c^2)) - b) - 2*(a*d^3*x^3 + a*c*d^2*x^2 - a*c^3 - (a*c^2 + 2*b)*d*x - b
*c)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))...
```

Sympy [F]

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\int x\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(x*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*
x**2))**(3/2), x)
```

Maxima [F]

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2)*x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(130) = 260$.

Time = 1.58 (sec) , antiderivative size = 832, normalized size of antiderivative = 5.37

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)*(a*x*sgn(d*x + c)/d - a*c*sgn(
d*x + c)/d^2) - 3/14*sqrt(a)*b*log(abs(-a^(7/2)*c^7*d - 7*(sqrt(a*d^2)*x -
 sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^3*c^6*abs(d) - 21*(sqrt(a*d^2)
*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(5/2)*c^5*d - 35*(sqrt(a
*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*a^2*c^4*abs(d) - 35*(
sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^4*a^(3/2)*c^3*d +
 3*a^(5/2)*b*c^5*d - 21*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^
2 + b))^5*a*c^2*abs(d) + 15*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b))*a^2*b*c^4*abs(d) - 7*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b))^6*sqrt(a)*c*d + 30*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b))^2*a^(3/2)*b*c^3*d - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b))^7*abs(d) + 30*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b))^3*a*b*c^2*abs(d) + 15*(sqrt(a*d^2)*x - sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b))^4*sqrt(a)*b*c*d - 3*a^(3/2)*b^2*c^3*d + 3*(sqr
t(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^5*b*abs(d) - 9*(sqrt
(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a*b^2*c^2*abs(d) - 9*
(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*sqrt(a)*b^2*c*
d - 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*b^2*abs(
d) + sqrt(a)*b^3*c*d + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b))*b^3*abs(d)))*sgn(d*x + c)/(d*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\,dx=\int x\,\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)^{3/2}\,dx$$

input
```
int(x*(a + b/(c + d*x)^2)^(3/2),x)
```

output
```
int(x*(a + b/(c + d*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 643, normalized size of antiderivative = 4.15

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx = \frac{-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,c^3-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,c^2dx+\sqrt{a\,d^2x^2+}$$

input
```
int(x*(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**3 - sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b)*a*c**2*d*x + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**
2 + b)*a*c*d**2*x**2 + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*d**3*x
**3 - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b*c - 2*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*b*d*x + 3*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b*c**2 + 6*sqrt(a)*l
og((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/
sqrt(b))*b*c*d*x + 3*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b*d**2*x**2 - 3*sqrt(b)*log((sqrt(a
*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/
sqrt(b))*a*c**3 - 6*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/sqrt(b))*a*c**2*d*x - 3*sqrt(b)*log
((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - s
qrt(b))/sqrt(b))*a*c*d**2*x**2 + 3*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x +
a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x + sqrt(b))/sqrt(b))*a*c**3 + 6*
sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt
(a)*d*x + sqrt(b))/sqrt(b))*a*c**2*d*x + 3*sqrt(b)*log((sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x + sqrt(b))/sqrt(b))*a*c
*d**2*x**2)/(2*d**2*(c**2 + 2*c*d*x + d**2*x**2))
```

3.74 $\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = -\frac{3b\sqrt{a+\frac{b}{(c+dx)^2}}}{2d(c+dx)} + \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d} - \frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2d}$$

output
```
-3/2*b*(a+b/(d*x+c)^2)^(1/2)/d/(d*x+c)+(d*x+c)*(a+b/(d*x+c)^2)^(3/2)/d-3/2
*a*b^(1/2)*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \frac{a(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-\frac{b}{a(c+dx)^2}\right)}{d\sqrt{1+\frac{b}{a(c+dx)^2}}}$$

input
```
Integrate[(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
(a*(c + d*x)*Sqrt[a + b/(c + d*x)^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -(
b/(a*(c + d*x)^2))])/(d*Sqrt[1 + b/(a*(c + d*x)^2)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {239, 773, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}dx$$

$$\Big\downarrow \; 239$$

$$\frac{\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 773$$

$$-\frac{\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \; 247$$

$$-\frac{3b\int \sqrt{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}-(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d}$$

$$\Big\downarrow \; 211$$

$$-\frac{3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{2(c+dx)}\right)-(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d}$$

$$\Big\downarrow \; 224$$

$$-\frac{3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{2(c+dx)}\right)-(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{2(c+dx)}\right)-(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d}$$

input
```
Int[(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
-((-((c + d*x)*(a + b/(c + d*x)^2)^(3/2)) + 3*b*(Sqrt[a + b/(c + d*x)^2]/(
2*(c + d*x)) + (a*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/(2
*Sqrt[b])))/d)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(83) = 166$.

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.90

method	result
risch	$\frac{a\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)}{d}+\frac{\left(-\frac{b\sqrt{a d^2\left(x+\frac{c}{d}\right)^2+b}}{2d^3\left(x+\frac{c}{d}\right)^2}-\frac{3\sqrt{b}\,a\ln\left(\frac{2b+2\sqrt{b}\sqrt{a d^2\left(x+\frac{c}{d}\right)^2+b}}{x+\frac{c}{d}}\right)}{2d}\right)\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)}{\sqrt{a d^2x^2+2adxc+a c^2+b}}$
default	$-\Big(-\left(a d^2x^2+2adxc+a c^2+b\right)^{\frac{3}{2}} a d^2x^2+3b^{\frac{3}{2}}\ln\left(\frac{2\left(\sqrt{b}\sqrt{a d^2x^2+2adxc+a c^2+b}+b\right)d}{dx+c}\right) a d^2x^2-2\left(a d^2x^2+2adxc+a c^2+b\right)^{\frac{3}{2}} acdx-3\sqrt{}$

input
```
int((a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(-1/2*b/d^3/
(x+c/d)^2*(a*d^2*(x+c/d)^2+b)^(1/2)-3/2*b^(1/2)*a/d*ln((2*b+2*b^(1/2)*(a*d
^2*(x+c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)
^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.58

$$\int \left(a + \frac{b}{(c+dx)^2}\right)^{3/2} dx = \left[\frac{3\,(adx + ac)\sqrt{b}\log\left(-\frac{ad^2x^2 + 2\,acdx + ac^2 - 2\,(dx+c)\sqrt{b}\sqrt{\frac{ad^2x^2 + 2\,acdx + ac^2 + b}{d^2x^2 + 2\,cdx + c^2}} + 2\,b}{d^2x^2 + 2\,cdx + c^2}\right) + 2\,(2\,ad^2x^2}{4\,(d^2x + cd)}\right.$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*d*x + a*c)*sqrt(b)*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x
 + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
+ c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(2*a*d^2*x^2 + 4*a*c*d*x + 2
*a*c^2 - b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
c^2)))/(d^2*x + c*d), 1/2*(3*(a*d*x + a*c)*sqrt(-b)*arctan((d*x + c)*sqrt(
-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/b)
 + (2*a*d^2*x^2 + 4*a*c*d*x + 2*a*c^2 - b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^2*x + c*d)]
```

Sympy [F]

$$\int \left(a + \frac{b}{(c+dx)^2}\right)^{3/2} dx = \int \left(a + \frac{b}{(c+dx)^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral((a + b/(c + d*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \int \left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.35

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \frac{\sqrt{ad^2x^2+2\,acdx+ac^2+b}a\mathrm{sgn}(dx+c)}{d}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)*a*sgn(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \frac{\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\;\frac{1}{2};\;-\frac{b}{a\,(c+d\,x)^2}\right)}{d\left(\frac{b}{a\,(c+d\,x)^2}+1\right)^{3/2}}$$

input
```
int((a + b/(c + d*x)^2)^(3/2),x)
```

output
```
((a + b/(c + d*x)^2)^(3/2)*(c + d*x)*hypergeom([-3/2, -1/2], 1/2, -b/(a*(c
 + d*x)^2)))/(d*(b/(a*(c + d*x)^2) + 1)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 436, normalized size of antiderivative = 4.49

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} dx = \frac{2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,c^2+4\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,acdx+2\sqrt{a\,d^2x^2+}}{}$$

input
```
int((a+b/(d*x+c)^2)^(3/2),x)
```

output
```
(2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**2 + 4*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*a*c*d*x + 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**
2 + b)*a*d**2*x**2 - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b + 3*sqrt
(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*
d*x - sqrt(b))/sqrt(b))*a*c**2 + 6*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x +
a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x - sqrt(b))/sqrt(b))*a*c*d*x + 3
*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqr
t(a)*d*x - sqrt(b))/sqrt(b))*a*d**2*x**2 - 3*sqrt(b)*log((sqrt(a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x + sqrt(b))/sqrt(b))*a
*c**2 - 6*sqrt(b)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a
)*c + sqrt(a)*d*x + sqrt(b))/sqrt(b))*a*c*d*x - 3*sqrt(b)*log((sqrt(a*c**2
 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x + sqrt(b))/sqrt(
b))*a*d**2*x**2)/(2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.75 $\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 187

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \frac{b\sqrt{a+\frac{b}{(c+dx)^2}}}{c^2}+\frac{b\sqrt{a+\frac{b}{(c+dx)^2}}}{2c(c+dx)} + \frac{\sqrt{b}(2b+3ac^2)\,\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^3} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right) - \frac{(b+ac^2)^{3/2}\,\text{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c^3}$$

output
```
b*(a+b/(d*x+c)^2)^(1/2)/c^2+1/2*b*(a+b/(d*x+c)^2)^(1/2)/c/(d*x+c)+1/2*b^(1
/2)*(3*a*c^2+2*b)*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/c^3+a^(3/
2)*arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))-(a*c^2+b)^(3/2)*arctanh((a*c+b/(
d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 11.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.28

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(bc(3c+2dx)\sqrt{b+a(c+dx)^2}+4\left(-b-ac^2\right)^{3/2}(c+d\ldots\right.}{}$$

input
```
Integrate[(a + b/(c + d*x)^2)^(3/2)/x,x]
```

output
```
((c + d*x)*(a + b/(c + d*x)^2)^(3/2)*(b*c*(3*c + 2*d*x)*Sqrt[b + a*(c + d*
x)^2] + 4*(-b - a*c^2)^(3/2)*(c + d*x)^2*ArcTan[(-(Sqrt[a]*d*x) + Sqrt[b +
 a*(c + d*x)^2])/Sqrt[-b - a*c^2]] - 2*Sqrt[b]*(2*b + 3*a*c^2)*(c + d*x)^2
*ArcTanh[(Sqrt[a]*(c + d*x) - Sqrt[b + a*(c + d*x)^2])/Sqrt[b]] - 2*a^(3/2
)*c^3*(c + d*x)^2*Log[-(Sqrt[a]*(c + d*x)) + Sqrt[b + a*(c + d*x)^2]]))/(2
*c^3*(b + a*(c + d*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.947$, Rules used = {896, 25, 1774, 1803, 25, 606, 25, 243, 60, 73, 221, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{dx}d(c+dx)$$

$\downarrow$ 1774

$$-\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{(c+dx)\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$

$\downarrow$ 1803

$$\int -\frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$\downarrow$ 25

$$-\int \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$\downarrow$ 606

$$\int -\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{b}{c+dx}+ac\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}-a\int (c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}$$

$\downarrow$ 25

$$-a\int (c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}-\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{b}{c+dx}+ac\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$\downarrow$ 243

$$-\frac{1}{2}a\int (c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)^2}-\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{b}{c+dx}+ac\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$\downarrow$ 60

$$-\frac{1}{2}a\left(a\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}+2\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{b}{c+dx}+ac\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$\downarrow$ 73

$$-\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{b}{c+dx}+ac\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}-\frac{1}{2}a\left(\frac{2a\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}}{b}+2\sqrt{a+\frac{b}{(c+dx)^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$-\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{b}{c+dx}+ac\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}-\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)$$

$$\Big\downarrow \text{682}$$

$$-\frac{\int \frac{b\left(ac(2ac^2+b)+\frac{b(3ac^2+2b)}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2bc^2}-\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(2(ac^2+b)+\frac{bc}{c+dx}\right)}{2c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{ac(2ac^2+b)+\frac{b(3ac^2+2b)}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2c^2}-\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(2(ac^2+b)+\frac{bc}{c+dx}\right)}{2c^2}$$

$$\Big\downarrow \text{719}$$

$$-\frac{\frac{2(ac^2+b)^2\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{b(3ac^2+2b)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{c}}{2c^2}-\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(2(ac^2+b)+\frac{bc}{c+dx}\right)}{2c^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(2\left(ac^2+b\right)+\frac{bc}{c+dx}\right)}{2c^2}-\frac{-\frac{2\left(ac^2+b\right)^2\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{b\left(3ac^2+2b\right)\int\frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}}{2c^2}$$

$\downarrow$ 219

$$\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(2\left(ac^2+b\right)+\frac{bc}{c+dx}\right)}{2c^2}-\frac{-\frac{2\left(ac^2+b\right)^2\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\sqrt{b}\left(3ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}}{2c^2}$$

$\downarrow$ 488

$$\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(2\left(ac^2+b\right)+\frac{bc}{c+dx}\right)}{2c^2}-\frac{-\frac{2\left(ac^2+b\right)^2\int\frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}-\frac{\sqrt{b}\left(3ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}}{2c^2}$$

$\downarrow$ 219

$$\frac{1}{2}a\left(2\sqrt{a+\frac{b}{(c+dx)^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(2\left(ac^2+b\right)+\frac{bc}{c+dx}\right)}{2c^2}-\frac{-\frac{2\left(ac^2+b\right)^{3/2}\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}-\frac{\sqrt{b}\left(3ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}}{2c^2}$$

input

```
Int[(a + b/(c + d*x)^2)^(3/2)/x,x]
```

output

```
(Sqrt[a + b/(c + d*x)^2]*(2*(b + a*c^2) + (b*c)/(c + d*x)))/(2*c^2) - (a*(
2*Sqrt[a + b/(c + d*x)^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt
[a]]))/2 - (-((Sqrt[b]*(2*b + 3*a*c^2)*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a +
 b/(c + d*x)^2])])/c) - (2*(b + a*c^2)^(3/2)*ArcTanh[(-(a*c) - b/(c + d*x)
)/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/c)/(2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1454 vs. $2(161) = 322$.

Time = 0.16 (sec) , antiderivative size = 1455, normalized size of antiderivative = 7.78

method	result	size
default	Expression too large to display	1455

input
```
int((a+b/(d*x+c)^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*ln((a*d^2*x+a*c*d+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2)
)/(a*d^2)^(1/2))*a^2*b*c^5*d-2*(a*d^2)^(1/2)*(a*c^2+b)^(1/2)*ln(2*(a*d*x*c
+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*b^2*c^2+6
*ln(2*(b^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)*d/(d*x+c))*(a*d^2)^(
1/2)*b^(3/2)*a*c^3*d*x+3*ln(2*(b^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
+b)*d/(d*x+c))*(a*d^2)^(1/2)*b^(3/2)*a*c^2*d^2*x^2-4*(a*d^2)^(1/2)*(a*c^2+
b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)
^(1/2)+b)/x)*b^2*c*d*x-7*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2)
*a*b*c^2*d^2*x^2-8*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2)*a*b*c
^3*d*x+4*ln(2*(b^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)*d/(d*x+c))*(
a*d^2)^(1/2)*b^(5/2)*c*d*x+3*ln(2*(b^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(
1/2)+b)*d/(d*x+c))*(a*d^2)^(1/2)*b^(3/2)*a*c^4+2*ln(2*(b^(1/2)*(a*d^2*x^2+
2*a*c*d*x+a*c^2+b)^(1/2)+b)*d/(d*x+c))*(a*d^2)^(1/2)*b^(5/2)*d^2*x^2-7*(a*
d^2*x^2+2*a*c*d*x+a*c^2+b)^(3/2)*(a*d^2)^(1/2)*a*c^2*d^2*x^2-4*(a*d^2)^(1/
2)*(a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*
x+a*c^2+b)^(1/2)+b)/x)*a*b*c^3*d*x-2*(a*d^2)^(1/2)*(a*c^2+b)^(1/2)*ln(2*(a
*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*a*b
*c^2*d^2*x^2-2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2)*a*b*c*d^3
*x^3-2*(a*d^2)^(1/2)*(a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(
a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*a*b*c^4-2*(a*d^2)^(1/2)*(a*c^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(161) = 322$.

Time = 9.35 (sec) , antiderivative size = 3665, normalized size of antiderivative = 19.60

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(3/2)/x,x)
```

output
```
Integral(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**
*2))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}{x}\,dx$$

input
```
int((a + b/(c + d*x)^2)^(3/2)/x,x)
```

output
```
int((a + b/(c + d*x)^2)^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1155, normalized size of antiderivative = 6.18

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
int((a+b/(d*x+c)^2)^(3/2)/x,x)
```

output

```
(6*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b*c**2 + 4*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*b*c*d*x + 4*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b
)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a*c**
4 + 8*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d*
*2*x**2 + b) - a*c**2 - a*c*d*x - b)*a*c**3*d*x + 4*sqrt(a*c**2 + b)*log(s
qrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*
d*x - b)*a*c**2*d**2*x**2 + 4*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a
*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b*c**2 + 8*sq
rt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b) - a*c**2 - a*c*d*x - b)*b*c*d*x + 4*sqrt(a*c**2 + b)*log(sqrt(a*c**2
+ b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b*
d**2*x**2 - 4*sqrt(a*c**2 + b)*log(x)*a*c**4 - 8*sqrt(a*c**2 + b)*log(x)*a
*c**3*d*x - 4*sqrt(a*c**2 + b)*log(x)*a*c**2*d**2*x**2 - 4*sqrt(a*c**2 + b
)*log(x)*b*c**2 - 8*sqrt(a*c**2 + b)*log(x)*b*c*d*x - 4*sqrt(a*c**2 + b)*l
og(x)*b*d**2*x**2 + 4*sqrt(a)*log( - sqrt(a)*sqrt(a*c**2 + 2*a*c*d*x + a*d
**2*x**2 + b) - a*c - a*d*x)*a*c**5 + 8*sqrt(a)*log( - sqrt(a)*sqrt(a*c**2
 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c - a*d*x)*a*c**4*d*x + 4*sqrt(a)*log(
 - sqrt(a)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c - a*d*x)*a*c**
3*d**2*x**2 - 3*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - s
qrt(b))*a*c**4 - 6*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + ...
```

3.76 $\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 199

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx = -\frac{3bd\sqrt{a+\frac{b}{(c+dx)^2}}}{c^3}-\frac{3bd\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^2(c+dx)} -\frac{d\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3\sqrt{b}(2b+ac^2)\,d\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^4} +\frac{3b\sqrt{b+ac^2}d\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c^4}$$

output
```
-3*b*d*(a+b/(d*x+c)^2)^(1/2)/c^3-3/2*b*d*(a+b/(d*x+c)^2)^(1/2)/c^2/(d*x+c)
-d*(a+b/(d*x+c)^2)^(3/2)/c/(1-c/(d*x+c))-3/2*b^(1/2)*(a*c^2+2*b)*d*arctanh
(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/c^4+3*b*(a*c^2+b)^(1/2)*d*arctanh(
(a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/c^4
```

Mathematica [A] (verified)

Time = 10.41 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx =$$

$$-\frac{\frac{c\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\left(2ac^2(c+dx)^2+b(2c^2+9cdx+6d^2x^2)\right)}{x(c+dx)}+6b\sqrt{b+ac^2}d\log(x)-3\sqrt{b}(2b+ac^2)\,d\log(c+dx)+3\sqrt{b}(2b+}{2}$$

input
```
Integrate[(a + b/(c + d*x)^2)^(3/2)/x^2,x]
```

output
```
-1/2*((c*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]*(2*a*c^2*(c + d*x)^2 + b*(2
*c^2 + 9*c*d*x + 6*d^2*x^2)))/(x*(c + d*x)) + 6*b*Sqrt[b + a*c^2]*d*Log[x]
 - 3*Sqrt[b]*(2*b + a*c^2)*d*Log[c + d*x] + 3*Sqrt[b]*(2*b + a*c^2)*d*Log[
b + Sqrt[b]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]] - 6*b*Sqrt[b
+ a*c^2]*d*Log[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^
2)/(c + d*x)^2])])/c^4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {896, 1774, 1799, 492, 591, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \ 896$$

$$d\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \ 1774$$

$$d\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{ 1799}$$

$$-d\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 492}$$

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{(c+dx)\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}\right)$$

$$\Big\downarrow \text{ 591}$$

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{\int \frac{ac+\frac{ac^2+2b}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2c^2}-\frac{\left(\frac{c}{c+dx}+2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^2}\right)}{c}\right)$$

$$\Big\downarrow \text{ 719}$$

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{\frac{2\left(ac^2+b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\left(ac^2+2b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{c}}{2c^2}-\frac{\left(\frac{c}{c+dx}+2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^2}\right)}{c}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{\frac{2\left(ac^2+b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\left(ac^2+2b\right)\int \frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}}{2c^2}-\frac{\left(\frac{c}{c+dx}+2\right)\sqrt{\cdots}}{2c^2}\cdots\right.}{c}\cdots\right.$$

$\Big\downarrow$ 219

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{\frac{2\left(ac^2+b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\left(ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}}{2c^2}-\frac{\left(\frac{c}{c+dx}+2\right)\sqrt{a\cdots}}{2c^2}\cdots\right.}{c}\cdots\right.$$

$\Big\downarrow$ 488

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{-\frac{2\left(ac^2+b\right)\int \frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}-\frac{\left(ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}}{2c^2}-\frac{\left(\frac{c}{c+dx}+2\right)\sqrt{\cdots}}{2c\cdots}\cdots\right.}{c}\cdots\right.$$

$\Big\downarrow$ 219

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{c\left(1-\frac{c}{c+dx}\right)}-\frac{3b\left(\frac{-\frac{\left(ac^2+2b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}-\frac{2\sqrt{ac^2+b}\,\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c}}{2c^2}-\frac{\left(\frac{c}{c+dx}+2\right)\cdots}{2\cdots}\right.}{c}\right.$$

input
```
Int[(a + b/(c + d*x)^2)^(3/2)/x^2,x]
```

output
```
-(d*((a + b/(c + d*x)^2)^(3/2)/(c*(1 - c/(c + d*x))) - (3*b*(-1/2*(Sqrt[a
+ b/(c + d*x)^2]*(2 + c/(c + d*x)))/c^2 + (-(((2*b + a*c^2)*ArcTanh[Sqrt[b
]/((c + d*x)*Sqrt[a + b/(c + d*x)^2])])/(Sqrt[b]*c)) - (2*Sqrt[b + a*c^2]*
ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])
/c)/(2*c^2)))/c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.75

method	result
risch	$-\frac{(a\,c^2+b)\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}{c^3x}+\frac{\left(\frac{3db\sqrt{a\,c^2+b}\,\ln\left(\frac{2a\,c^2+2b+2adxc+2\sqrt{a\,c^2+b}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}{x}\right)}{c^4}-\frac{2b\sqrt{a\,d^2\left(x+\frac{c}{d}\right.}}{c^3\left(x+\frac{c}{d}\right)}\right.}{}$
default	Expression too large to display

input
```
int((a+b/(d*x+c)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*c^2+b)/c^3/x*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(
3*d/c^4*b*(a*c^2+b)^(1/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d
^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x)-2/c^3*b/(x+c/d)*(a*d^2*(x+c/d)^2+b)^(1
/2)-1/2/d/c^2*b/(x+c/d)^2*(a*d^2*(x+c/d)^2+b)^(1/2)-3/2*d/c^2*b^(1/2)*ln((
2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d))*a-3*d/c^4*b^(3/2)*ln((2*
b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a*c
^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(175) = 350$.

Time = 0.72 (sec) , antiderivative size = 1562, normalized size of antiderivative = 7.85

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(3*((a*c^2 + 2*b)*d^2*x^2 + (a*c^3 + 2*b*c)*d*x)*sqrt(b)*log(-(a*d^2*
x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2))
+ 6*(b*d^2*x^2 + b*c*d*x)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2*a^2*c^2 + a
*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 + 2*(a*c*d^2*x^2
 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(2*a*c^5 + 2*(a*c^
3 + 3*b*c)*d^2*x^2 + 2*b*c^3 + (4*a*c^4 + 9*b*c^2)*d*x)*sqrt((a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(c^4*d*x^2 + c^5*x), 1/
2*(3*((a*c^2 + 2*b)*d^2*x^2 + (a*c^3 + 2*b*c)*d*x)*sqrt(-b)*arctan((d*x +
c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
c^2))/b) + 3*(b*d^2*x^2 + b*c*d*x)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2*a^
2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 + 2*(a*
c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x
^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - (2*a*c^5 +
2*(a*c^3 + 3*b*c)*d^2*x^2 + 2*b*c^3 + (4*a*c^4 + 9*b*c^2)*d*x)*sqrt((a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(c^4*d*x^2 + c^5
*x), -1/4*(12*(b*d^2*x^2 + b*c*d*x)*sqrt(-a*c^2 - b)*arctan((a*c*d^2*x^2 +
 a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a*c^2 - b)*sqrt((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a^2*c^4 + (a^2*c^2 + a*b)...
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(3/2)/x**2,x)
```

output
```
Integral(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.66

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx=$$

$$-\frac{6\left(ab^7c^2d^5\operatorname{sgn}(dx+c)+b^8d^5\operatorname{sgn}(dx+c)\right)\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}}{\sqrt{-ac^2-b}}\right)}{\sqrt{-ac^2-b}\,b^6c^4d^4}$$

$$-\frac{2\left(a^{\frac{5}{2}}b^6c^4d^4|d|\operatorname{sgn}(dx+c)+\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)a^2b^6c^3d^5\operatorname{sgn}(dx+c)+2\,a^{\frac{3}{2}}b^7c^2d^4|d\right.}{\left(ac^2-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,a\right.\right.}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
-6*(a*b^7*c^2*d^5*sgn(d*x + c) + b^8*d^5*sgn(d*x + c))*arctan(-(sqrt(a*d^2
)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))/sqrt(-a*c^2 - b))/(sqrt(-a*
c^2 - b)*b^6*c^4*d^4) - 2*(a^(5/2)*b^6*c^4*d^4*abs(d)*sgn(d*x + c) + (sqrt
(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^2*b^6*c^3*d^5*sgn(d
*x + c) + 2*a^(3/2)*b^7*c^2*d^4*abs(d)*sgn(d*x + c) + (sqrt(a*d^2)*x - sqr
t(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a*b^7*c*d^5*sgn(d*x + c) + sqrt(a)*b
^8*d^4*abs(d)*sgn(d*x + c))/((a*c^2 - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b))^2 + b)*b^6*c^3*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x)^2)^(3/2)/x^2,x)
```

output
```
int((a + b/(c + d*x)^2)^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 930, normalized size of antiderivative = 4.67

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^2}\,dx=\text{Too large to display}$$

input
```
int((a+b/(d*x+c)^2)^(3/2)/x^2,x)
```

output
```
( - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**5 - 8*sqrt(a*c**2 +
2*a*c*d*x + a*d**2*x**2 + b)*a*c**4*d*x - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d*
*2*x**2 + b)*a*c**3*d**2*x**2 - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
b)*b*c**3 - 18*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b*c**2*d*x - 12*
sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b*c*d**2*x**2 + 12*sqrt(a*c**2
+ b)*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) -
a*c**2 - a*c*d*x - b)*b*c**2*d*x + 24*sqrt(a*c**2 + b)*log( - sqrt(a*c**2
+ b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b*
c*d**2*x**2 + 12*sqrt(a*c**2 + b)*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b*d**3*x**3 - 12*sqrt(a
*c**2 + b)*log(x)*b*c**2*d*x - 24*sqrt(a*c**2 + b)*log(x)*b*c*d**2*x**2 -
12*sqrt(a*c**2 + b)*log(x)*b*d**3*x**3 + 3*sqrt(b)*log(sqrt(a*c**2 + 2*a*c
*d*x + a*d**2*x**2 + b) - sqrt(b))*a*c**4*d*x + 6*sqrt(b)*log(sqrt(a*c**2
+ 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(b))*a*c**3*d**2*x**2 + 3*sqrt(b)*log
(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(b))*a*c**2*d**3*x**3 +
6*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(b))*b*c**2
*d*x + 12*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(b)
)*b*c*d**2*x**2 + 6*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
 - sqrt(b))*b*d**3*x**3 - 3*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b) + sqrt(b))*a*c**4*d*x - 6*sqrt(b)*log(sqrt(a*c**2 + 2*a*c*d*x ...
```

3.77 $\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 320

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx=\frac{3b(4b+3ac^2)\,d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^4(b+ac^2)}+\frac{3b(2b+ac^2)\,d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^3(b+ac^2)(c+dx)}-\frac{d^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{2c^2\left(1-\frac{c}{c+dx}\right)^2}+\frac{(5b+2ac^2)\,d^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{2c^2(b+ac^2)\left(1-\frac{c}{c+dx}\right)}+\frac{3\sqrt{b}(4b+ac^2)\,d^2\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^5}-\frac{3b(4b+3ac^2)\,d^2\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^5\sqrt{b+ac^2}}$$

output
```
3/2*b*(3*a*c^2+4*b)*d^2*(a+b/(d*x+c)^2)^(1/2)/c^4/(a*c^2+b)+3/2*b*(a*c^2+2
*b)*d^2*(a+b/(d*x+c)^2)^(1/2)/c^3/(a*c^2+b)/(d*x+c)-1/2*d^2*(a+b/(d*x+c)^2
)^(3/2)/c^2/(1-c/(d*x+c))^2+1/2*(2*a*c^2+5*b)*d^2*(a+b/(d*x+c)^2)^(3/2)/c^
2/(a*c^2+b)/(1-c/(d*x+c))+3/2*b^(1/2)*(a*c^2+4*b)*d^2*arctanh(b^(1/2)/(d*x
+c)/(a+b/(d*x+c)^2)^(1/2))/c^5-3/2*b*(3*a*c^2+4*b)*d^2*arctanh((a*c+b/(d*x
+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/c^5/(a*c^2+b)^(1/2)
```

Mathematica [A] (verified)

Time = 10.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3} dx = -\frac{\frac{c\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}(ac^2(c-dx)(c+dx)^2+b(c^3-4c^2dx-18cd^2x^2-12d^3x^3))}{x^2(c+dx)} - \frac{3b(4b+3ac^2)d^2\log(x)}{\sqrt{b+ac^2}} + 3\sqrt{b}(4b+ac^2)d^2\log(c+dx) -}{2c^5}$$

input
```
Integrate[(a + b/(c + d*x)^2)^(3/2)/x^3,x]
```

output
```
-1/2*((c*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]*(a*c^2*(c - d*x)*(c + d*x)^
2 + b*(c^3 - 4*c^2*d*x - 18*c*d^2*x^2 - 12*d^3*x^3)))/(x^2*(c + d*x)) - (3
*b*(4*b + 3*a*c^2)*d^2*Log[x])/Sqrt[b + a*c^2] + 3*Sqrt[b]*(4*b + a*c^2)*d
^2*Log[c + d*x] - 3*Sqrt[b]*(4*b + a*c^2)*d^2*Log[b + Sqrt[b]*(c + d*x)*Sq
rt[(b + a*(c + d*x)^2)/(c + d*x)^2]] + (3*b*(4*b + 3*a*c^2)*d^2*Log[b + (c
 + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])/Sq
rt[b + a*c^2])/c^5
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.74, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {896, 25, 1774, 1803, 25, 590, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3} dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2 \int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d^3x^3} d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-d^2\int -\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{1774}$$

$$-d^2\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \text{1803}$$

$$d^2\int -\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-d^2\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{590}$$

$$d^2\left(\frac{3\int \frac{2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{2b}{c+dx}+ac\right)}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{4c^2}-\frac{\left(2-\frac{c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{2c^2\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$d^2\left(\frac{3\int \frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{2b}{c+dx}+ac\right)}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2c^2}-\frac{\left(2-\frac{c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{2c^2\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{681}$$

$$d^2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac^2-\frac{2bc}{c+dx}+4b\right)}{c^2\left(1-\frac{c}{c+dx}\right)}-\frac{\int \frac{2b\left(2ac+\frac{ac^2+4b}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{2c^2}\right)}{2c^2}-\frac{\left(2-\frac{c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{2c^2\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$d^2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac^2-\frac{2bc}{c+dx}+4b\right)}{c^2\left(1-\frac{c}{c+dx}\right)}-\frac{b\int \frac{2ac+\frac{ac^2+4b}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c^2}\right)}{2c^2}-\frac{\left(2-\frac{c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{2c^2\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{719}$$

$$d^2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac^2-\frac{2bc}{c+dx}+4b\right)}{c^2\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(\frac{\left(3ac^2+4b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\left(ac^2+4b\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{c}\right)}{c^2}\right)}{2c^2}-\frac{\left(2-\frac{c}{c+}\right.}{2c}\right.$$

$$\Big\downarrow \text{224}$$

$$d^2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac^2-\frac{2bc}{c+dx}+4b\right)}{c^2\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(\frac{\left(3ac^2+4b\right)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\left(ac^2+4b\right)\int\frac{1}{1-\frac{b}{(c+dx)^2}}d\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}\right)}{c^2}\right)}{2c^2}-\right.$$

$\downarrow$ 219

$$d^2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac^2-\frac{2bc}{c+dx}+4b\right)}{c^2\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(\frac{\left(3ac^2+4b\right)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{c}-\frac{\left(ac^2+4b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}\right)}{c^2}\right)}{2c^2}-\right.$$

$\downarrow$ 488

$$d^2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac^2-\frac{2bc}{c+dx}+4b\right)}{c^2\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(-\frac{\left(3ac^2+4b\right)\int \frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{c}-\frac{\left(ac^2+4b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}\right)}{c^2}\right)}{2c^2}\right.$$

$$\Big\downarrow 219$$

$$d^2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac^2-\frac{2bc}{c+dx}+4b\right)}{c^2\left(1-\frac{c}{c+dx}\right)}-\frac{b\left(-\frac{\left(ac^2+4b\right)\text{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b}c}-\frac{\left(3ac^2+4b\right)\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{c\sqrt{ac^2+b}}\right)}{c^2}\right)}{2c^2}\right.$$

input
```
Int[(a + b/(c + d*x)^2)^(3/2)/x^3,x]
```

output
```
d^2*(-1/2*((a + b/(c + d*x)^2)^(3/2)*(2 - c/(c + d*x)))/(c^2*(1 - c/(c + d
*x))^2) + (3*((Sqrt[a + b/(c + d*x)^2]*(4*b + a*c^2 - (2*b*c)/(c + d*x)))/
(c^2*(1 - c/(c + d*x))) - (b*(-(((4*b + a*c^2)*ArcTanh[Sqrt[b]/((c + d*x)*
Sqrt[a + b/(c + d*x)^2])])/(Sqrt[b]*c)) - ((4*b + 3*a*c^2)*ArcTanh[(-(a*c)
 - b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/(c*Sqrt[b + a*
c^2])))/c^2))/(2*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\left(-adx\,c^{2}+c^{3}a-6bdx+bc\right)\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}}\,\left(dx+c\right)}{2c^{4}x^{2}}+\frac{b\,d^{2}\left(-\frac{3\left(3a\,c^{2}+4b\right)\ln\left(\frac{2a\,c^{2}+2b+2adxc+2\sqrt{a\,c^{2}+b}\,\sqrt{a\,d^{2}x^{2}+2adxc+\cdots}}{x}\right)}{c\sqrt{a\,c^{2}+b}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((a+b/(d*x+c)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a*c^2*d*x+a*c^3-6*b*d*x+b*c)/c^4/x^2*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)
/(d*x+c)^2)^(1/2)*(d*x+c)+1/2*b/c^4*d^2*(-3/c*(3*a*c^2+4*b)/(a*c^2+b)^(1/2
)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b
)^(1/2))/x)+6/d/(x+c/d)*(a*d^2*(x+c/d)^2+b)^(1/2)-2/d^2*b*c*(-1/2/b/(x+c/d
)^2*(a*d^2*(x+c/d)^2+b)^(1/2)+1/2*a*d^2/b^(3/2)*ln((2*b+2*b^(1/2)*(a*d^2*(
x+c/d)^2+b)^(1/2))/(x+c/d)))+4*(a*c^2+3*b)/c/b^(1/2)*ln((2*b+2*b^(1/2)*(a*
d^2*(x+c/d)^2+b)^(1/2))/(x+c/d)))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2
)^(1/2)*(d*x+c)/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 2095, normalized size of antiderivative = 6.55

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(3*((a^2*c^4 + 5*a*b*c^2 + 4*b^2)*d^3*x^3 + (a^2*c^5 + 5*a*b*c^3 + 4*
b^2*c)*d^2*x^2)*sqrt(b)*log(-(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + 2*(d*x + c)*
sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*b)/(d^2*x^2 + 2*c*d*x + c^2)) + 3*((3*a*b*c^2 + 4*b^2)*d^3*x^3 + (3*
a*b*c^3 + 4*b^2*c)*d^2*x^2)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2*a^2*c^2 +
 a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x
^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(a^2*c^8 + 2*a*b
*c^6 - (a^2*c^5 + 13*a*b*c^3 + 12*b^2*c)*d^3*x^3 + b^2*c^4 - (a^2*c^6 + 19
*a*b*c^4 + 18*b^2*c^2)*d^2*x^2 + (a^2*c^7 - 3*a*b*c^5 - 4*b^2*c^3)*d*x)*sq
rt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a*c^7
 + b*c^5)*d*x^3 + (a*c^8 + b*c^6)*x^2), -1/4*(6*((a^2*c^4 + 5*a*b*c^2 + 4*
b^2)*d^3*x^3 + (a^2*c^5 + 5*a*b*c^3 + 4*b^2*c)*d^2*x^2)*sqrt(-b)*arctan((d
*x + c)*sqrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d
*x + c^2))/b) - 3*((3*a*b*c^2 + 4*b^2)*d^3*x^3 + (3*a*b*c^3 + 4*b^2*c)*d^2
*x^2)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*
c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2
+ b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(
d^2*x^2 + 2*c*d*x + c^2)))/x^2) + 2*(a^2*c^8 + 2*a*b*c^6 - (a^2*c^5 + 13*a
*b*c^3 + 12*b^2*c)*d^3*x^3 + b^2*c^4 - (a^2*c^6 + 19*a*b*c^4 + 18*b^2*c...
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(3/2)/x**3,x)
```

output
```
Integral(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(286) = 572.

Time = 11.63 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx=\frac{3\,(3\,ab^{10}c^2d^8\mathrm{sgn}(dx+c)+4\,b^{11}d^8\mathrm{sgn}(dx+c))\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}}{\sqrt{-ac^2-b}}\right)}{\sqrt{-ac^2-b}b^9c^5d^6}+\frac{2\,a^{\frac{7}{2}}b^9c^6d^7|d|\mathrm{sgn}(dx+c)+4\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)a^3b^9c^5d^8\mathrm{sgn}(dx+c)+2\left(\sqrt{ad^2}x-\right.}{}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
3*(3*a*b^10*c^2*d^8*sgn(d*x + c) + 4*b^11*d^8*sgn(d*x + c))*arctan(-(sqrt(
a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))/sqrt(-a*c^2 - b))/(sqr
t(-a*c^2 - b)*b^9*c^5*d^6) + (2*a^(7/2)*b^9*c^6*d^7*abs(d)*sgn(d*x + c) +
4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^3*b^9*c^5*d^
8*sgn(d*x + c) + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b
))^2*a^(5/2)*b^9*c^4*d^7*abs(d)*sgn(d*x + c) + 10*a^(5/2)*b^10*c^4*d^7*abs
(d)*sgn(d*x + c) + 11*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b))*a^2*b^10*c^3*d^8*sgn(d*x + c) - 4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b))^2*a^(3/2)*b^10*c^2*d^7*abs(d)*sgn(d*x + c) - 5*(sq
rt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*a*b^10*c*d^8*sgn(
d*x + c) + 14*a^(3/2)*b^11*c^2*d^7*abs(d)*sgn(d*x + c) + 7*(sqrt(a*d^2)*x
- sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a*b^11*c*d^8*sgn(d*x + c) - 6*(
sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*sqrt(a)*b^11*d^
7*abs(d)*sgn(d*x + c) + 6*sqrt(a)*b^12*d^7*abs(d)*sgn(d*x + c))/((a*c^2 -
(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2 + b)^2*b^9*c^4
*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x)^2)^(3/2)/x^3,x)
```

output
```
int((a + b/(c + d*x)^2)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1757, normalized size of antiderivative = 5.49

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b/(d*x+c)^2)^(3/2)/x^3,x)
```

output
```
( - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*c**8 - 2*sqrt(a*c**2
 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*c**7*d*x + 2*sqrt(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)*a**2*c**6*d**2*x**2 + 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**
2*x**2 + b)*a**2*c**5*d**3*x**3 - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b)*a*b*c**6 + 6*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b*c**5*d*x
+ 38*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b*c**4*d**2*x**2 + 26*sq
rt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b*c**3*d**3*x**3 - 2*sqrt(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**2*c**4 + 8*sqrt(a*c**2 + 2*a*c*d*x + a
*d**2*x**2 + b)*b**2*c**3*d*x + 36*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)*b**2*c**2*d**2*x**2 + 24*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b*
*2*c*d**3*x**3 + 18*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a*b*c**4*d**2*x**2 + 36
*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b) - a*c**2 - a*c*d*x - b)*a*b*c**3*d**3*x**3 + 18*sqrt(a*c**2 + b)*l
og(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 -
a*c*d*x - b)*a*b*c**2*d**4*x**4 + 24*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)
*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b**2*c
**2*d**2*x**2 + 48*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b**2*c*d**3*x**3 + 24*sq
rt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x*...
```

3.78 $\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 360

$$\begin{aligned}\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx &= -\frac{4bd^3\sqrt{a+\frac{b}{(c+dx)^2}}}{c^5}-\frac{bd^3\sqrt{a+\frac{b}{(c+dx)^2}}}{2c^4(c+dx)}\\
&-\frac{(b+ac^2)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3c^5\left(1-\frac{c}{c+dx}\right)^3}+\frac{(13b+6ac^2)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6c^5\left(1-\frac{c}{c+dx}\right)^2}\\
&-\frac{(47b^2+50abc^2+6a^2c^4)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6c^5(b+ac^2)\left(1-\frac{c}{c+dx}\right)}\\
&-\frac{\sqrt{b}(20b+3ac^2)\,d^3\operatorname{arctanh}\left(\frac{\sqrt{b}}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^6}\\
&+\frac{b(20b^2+33abc^2+12a^2c^4)\,d^3\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2c^6(b+ac^2)^{3/2}}\end{aligned}$$

output
```
-4*b*d^3*(a+b/(d*x+c)^2)^(1/2)/c^5-1/2*b*d^3*(a+b/(d*x+c)^2)^(1/2)/c^4/(d*
x+c)-1/3*(a*c^2+b)*d^3*(a+b/(d*x+c)^2)^(1/2)/c^5/(1-c/(d*x+c))^3+1/6*(6*a*
c^2+13*b)*d^3*(a+b/(d*x+c)^2)^(1/2)/c^5/(1-c/(d*x+c))^2-1/6*(6*a^2*c^4+50*
a*b*c^2+47*b^2)*d^3*(a+b/(d*x+c)^2)^(1/2)/c^5/(a*c^2+b)/(1-c/(d*x+c))-1/2*
b^(1/2)*(3*a*c^2+20*b)*d^3*arctanh(b^(1/2)/(d*x+c)/(a+b/(d*x+c)^2)^(1/2))/
c^6+1/2*b*(12*a^2*c^4+33*a*b*c^2+20*b^2)*d^3*arctanh((a*c+b/(d*x+c))/(a*c^
2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/c^6/(a*c^2+b)^(3/2)
```

Mathematica [A] (verified)

Time = 11.00 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx =$$

$$-\frac{c\left(\frac{(60b^2+59abc^2+2a^2c^4)d^3}{b+ac^2}+\frac{2c^3(b+ac^2)}{x^3}-\frac{7bc^2d}{x^2}+\frac{3bc(9b+8ac^2)d^2}{(b+ac^2)x}+\frac{3bcd^3}{c+dx}\right)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}+\frac{3b(20b^2+33abc^2+12a^2c^4)d^3\log(}{(b+ac^2)^{3/2}}}{}$$

input
```
Integrate[(a + b/(c + d*x)^2)^(3/2)/x^4,x]
```

output
```
-1/6*(c*(((60*b^2 + 59*a*b*c^2 + 2*a^2*c^4)*d^3)/(b + a*c^2) + (2*c^3*(b +
 a*c^2))/x^3 - (7*b*c^2*d)/x^2 + (3*b*c*(9*b + 8*a*c^2)*d^2)/((b + a*c^2)*
x) + (3*b*c*d^3)/(c + d*x))*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2] + (3*b*(
20*b^2 + 33*a*b*c^2 + 12*a^2*c^4)*d^3*Log[x])/(b + a*c^2)^(3/2) - 3*Sqrt[b
]*(20*b + 3*a*c^2)*d^3*Log[c + d*x] + 3*Sqrt[b]*(20*b + 3*a*c^2)*d^3*Log[b
 + Sqrt[b]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]] - (3*b*(20*b^2
 + 33*a*b*c^2 + 12*a^2*c^4)*d^3*Log[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*S
qrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])/(b + a*c^2)^(3/2))/c^6
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {896, 1774, 1803, 603, 681, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{ 896}$$

$$d^3\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{d^4x^4}d(c+dx)$$

$$\Big\downarrow \text{ 1774}$$

$$d^3\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{(c+dx)^4\left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \text{ 1803}$$

$$-d^3\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{(c+dx)^2\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 603}$$

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(3a+\frac{\frac{5b}{c}+3ac}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3\left(ac^2+b\right)}\right)$$

$$\Big\downarrow \text{ 681}$$

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\int\frac{4\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac\left(3ac^2+5b\right)+\frac{b\left(9ac^2+10b\right)}{c+dx}\right)}{c\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{8c^2}}{3\left(ac^2+b\right)}\right.$$

$\downarrow$ 27

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\int\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(ac\left(3ac^2+5b\right)+\frac{b\left(9ac^2+10b\right)}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2c^3}}{3\left(ac^2+b\right)}\right)$$

$\downarrow$ 681

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\left(ac^2+b\right)\left(3ac^2+20b\right)-\frac{bc\left(9ac^2+10b\right)}{c+dx}\right)}{c^2\left(1-\frac{c}{c+dx}\right)}\cdots\right.}{2c^3\cdots}}{3\left(ac^2+b\right)}\right.$$

$\downarrow$ 27

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\left(ac^2+b\right)\left(3ac^2+20b\right)-\frac{bc\left(9ac^2+10b\right)}{c+dx}\right)}{c^2\left(1-\frac{c}{c+dx}\right)}\cdots\right.}{2c^3\cdots}}{3\left(ac^2+b\right)\cdots}\right.$$

$\downarrow$ 719

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\left(ac^2+b\right)\left(3ac^2+20b\right)-\frac{bc\left(9ac^2+10b\right)}{c+dx}\right)}{c^2\left(1-\frac{c}{c+dx}\right)}\cdots\right.}{\cdots}}{3(\cdots}\right.$$

$\downarrow$ 224

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\left(ac^2+b\right)\left(3ac^2+20b\right)-\frac{bc\left(9ac^2+10b\right)}{c+dx}\right)}{c^2\left(1-\frac{c}{c+dx}\right)}\cdot\right.}{}}{}\right.$$

$\downarrow$ 219

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\left(ac^2+b\right)\left(3ac^2+20b\right)-\frac{bc\left(9ac^2+10b\right)}{c+dx}\right)}{c^2\left(1-\frac{c}{c+dx}\right)}\cdot\right.}{}}{}\right.$$

$\downarrow$ 488

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\left(ac^2+b\right)\left(3ac^2+20b\right)-\frac{bc\left(9ac^2+10b\right)}{c+dx}\right)}{c^2\left(1-\frac{c}{c+dx}\right)}\right.}{}}{}\right.$$

$\downarrow$ 219

$$-d^3\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{5/2}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(-\frac{3ac^2+5b}{c+dx}+9ac+\frac{10b}{c}\right)}{2c^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\left(ac^2+b\right)\left(3ac^2+20b\right)-\frac{bc\left(9ac^2+10b\right)}{c+dx}\right)}{c^2\left(1-\frac{c}{c+dx}\right)}\right.}{}}{}\right.$$

input
```
Int[(a + b/(c + d*x)^2)^(3/2)/x^4,x]
```

output
```
-(d^3*((a + b/(c + d*x)^2)^(5/2)/(3*c*(b + a*c^2)*(1 - c/(c + d*x))^3) - (
((a + b/(c + d*x)^2)^(3/2)*((10*b)/c + 9*a*c - (5*b + 3*a*c^2)/(c + d*x)))
/(2*c^2*(1 - c/(c + d*x))^2) - (3*((Sqrt[a + b/(c + d*x)^2]*((b + a*c^2)*(
20*b + 3*a*c^2) - (b*c*(10*b + 9*a*c^2))/(c + d*x)))/(c^2*(1 - c/(c + d*x)
)) - (b*(-(((b + a*c^2)*(20*b + 3*a*c^2)*ArcTanh[Sqrt[b]/((c + d*x)*Sqrt[a
 + b/(c + d*x)^2])])/(Sqrt[b]*c)) - ((20*b^2 + 33*a*b*c^2 + 12*a^2*c^4)*Ar
cTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/(
c*Sqrt[b + a*c^2])))/c^2))/(2*c^3))/(3*(b + a*c^2))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(326) = 652$.

Time = 0.20 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.20

method	result
risch	$-\frac{\left(2a^2c^4d^2x^2-2a^2c^5dx+2a^2c^6+35ab\,c^2d^2x^2-11ab\,c^3dx+4ab\,c^4+36b^2d^2x^2-9b^2cxd+2b^2c^2\right)\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}{6(a\,c^2+b)c^5x^3}+$
default	Expression too large to display

input
```
int((a+b/(d*x+c)^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*a^2*c^4*d^2*x^2-2*a^2*c^5*d*x+2*a^2*c^6+35*a*b*c^2*d^2*x^2-11*a*b*
c^3*d*x+4*a*b*c^4+36*b^2*d^2*x^2-9*b^2*c*d*x+2*b^2*c^2)/(a*c^2+b)/c^5/x^3*
((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)+(6*b*d^3/c^2/(a*c^
2+b)^(3/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*
x+a*c^2+b)^(1/2))/x)*a^2+33/2*b^2*d^3/c^4/(a*c^2+b)^(3/2)*ln((2*a*c^2+2*b+
2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x)*a+10*b
^3*d^3/c^6/(a*c^2+b)^(3/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*
d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x)-4*b*d^2/c^3/(a*c^2+b)/(x+c/d)*(a*d^2*
(x+c/d)^2+b)^(1/2)*a-4*b^2*d^2/c^5/(a*c^2+b)/(x+c/d)*(a*d^2*(x+c/d)^2+b)^(
1/2)-3/2*b^(1/2)*d^3/c^2/(a*c^2+b)*ln((2*b+2*b^(1/2)*(a*d^2*(x+c/d)^2+b)^(
1/2))/(x+c/d))*a^2-23/2*b^(3/2)*d^3/c^4/(a*c^2+b)*ln((2*b+2*b^(1/2)*(a*d^2
*(x+c/d)^2+b)^(1/2))/(x+c/d))*a-10*b^(5/2)*d^3/c^6/(a*c^2+b)*ln((2*b+2*b^(
1/2)*(a*d^2*(x+c/d)^2+b)^(1/2))/(x+c/d))-1/2*b*d/c^2/(a*c^2+b)/(x+c/d)^2*(
a*d^2*(x+c/d)^2+b)^(1/2)*a-1/2*b^2*d/c^4/(a*c^2+b)/(x+c/d)^2*(a*d^2*(x+c/d
)^2+b)^(1/2))*((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)*(d*x+c)/(a*d
^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(323) = 646$.

Time = 5.32 (sec) , antiderivative size = 2690, normalized size of antiderivative = 7.47

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4} \, dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(3*((3*a^3*c^6 + 26*a^2*b*c^4 + 43*a*b^2*c^2 + 20*b^3)*d^4*x^4 + (3*
a^3*c^7 + 26*a^2*b*c^5 + 43*a*b^2*c^3 + 20*b^3*c)*d^3*x^3)*sqrt(b)*log(-(a
*d^2*x^2 + 2*a*c*d*x + a*c^2 - 2*(d*x + c)*sqrt(b)*sqrt((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*b)/(d^2*x^2 + 2*c*d*x + c
^2)) + 3*((12*a^2*b*c^4 + 33*a*b^2*c^2 + 20*b^3)*d^4*x^4 + (12*a^2*b*c^5 +
 33*a*b^2*c^3 + 20*b^3*c)*d^3*x^3)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2*a^
2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 + 2*(a*
c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x
^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(2*a^3*c^
11 + 6*a^2*b*c^9 + 6*a*b^2*c^7 + (2*a^3*c^7 + 61*a^2*b*c^5 + 119*a*b^2*c^3
 + 60*b^3*c)*d^4*x^4 + 2*b^3*c^5 + 2*(a^3*c^8 + 44*a^2*b*c^6 + 88*a*b^2*c^
4 + 45*b^3*c^2)*d^3*x^3 + (17*a^2*b*c^7 + 37*a*b^2*c^5 + 20*b^3*c^3)*d^2*x
^2 + (2*a^3*c^10 - a^2*b*c^8 - 8*a*b^2*c^6 - 5*b^3*c^4)*d*x)*sqrt((a*d^2*x
^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^2*c^10 + 2*a*b
*c^8 + b^2*c^6)*d*x^4 + (a^2*c^11 + 2*a*b*c^9 + b^2*c^7)*x^3), 1/12*(6*((3
*a^3*c^6 + 26*a^2*b*c^4 + 43*a*b^2*c^2 + 20*b^3)*d^4*x^4 + (3*a^3*c^7 + 26
*a^2*b*c^5 + 43*a*b^2*c^3 + 20*b^3*c)*d^3*x^3)*sqrt(-b)*arctan((d*x + c)*s
qrt(-b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)
)/b) + 3*((12*a^2*b*c^4 + 33*a*b^2*c^2 + 20*b^3)*d^4*x^4 + (12*a^2*b*c^5 +
 33*a*b^2*c^3 + 20*b^3*c)*d^3*x^3)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2...
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**(3/2)/x**4,x)
```

output
```
Integral(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(3/2)/x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(d*x+c)^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a + b/(c + d*x)^2)^(3/2)/x^4,x)
```

output
```
int((a + b/(c + d*x)^2)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 2639, normalized size of antiderivative = 7.33

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a+b/(d*x+c)^2)^(3/2)/x^4,x)
```

output
```
( - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**11 - 4*sqrt(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**10*d*x - 4*sqrt(a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)*a**3*c**8*d**3*x**3 - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d
**2*x**2 + b)*a**3*c**7*d**4*x**4 - 12*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b)*a**2*b*c**9 + 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*
c**8*d*x - 34*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**7*d**2*
x**2 - 176*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**6*d**3*x**
3 - 122*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**5*d**4*x**4 -
 12*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**7 + 16*sqrt(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**6*d*x - 74*sqrt(a*c**2 + 2*a*c*
d*x + a*d**2*x**2 + b)*a*b**2*c**5*d**2*x**2 - 352*sqrt(a*c**2 + 2*a*c*d*x
 + a*d**2*x**2 + b)*a*b**2*c**4*d**3*x**3 - 238*sqrt(a*c**2 + 2*a*c*d*x +
a*d**2*x**2 + b)*a*b**2*c**3*d**4*x**4 - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**
2*x**2 + b)*b**3*c**5 + 10*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3
*c**4*d*x - 40*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3*c**3*d**2*x
**2 - 180*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3*c**2*d**3*x**3 -
 120*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3*c*d**4*x**4 + 72*sqrt
(a*c**2 + b)*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2
 + b) - a*c**2 - a*c*d*x - b)*a**2*b*c**6*d**3*x**3 + 144*sqrt(a*c**2 + b)
*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a...
```

3.79 $\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 185

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx &= \frac{c(2b-ac^2)\,(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a^2d^4} \\ &\quad - \frac{3(b-4ac^2)\,(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{8a^2d^4} - \frac{c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{ad^4} \\ &\quad + \frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4ad^4} + \frac{3b(b-4ac^2)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^4}\end{aligned}$$

output
```
c*(-a*c^2+2*b)*(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a^2/d^4-3/8*(-4*a*c^2+b)*(d*x
+c)^2*(a+b/(d*x+c)^2)^(1/2)/a^2/d^4-c*(d*x+c)^3*(a+b/(d*x+c)^2)^(1/2)/a/d^
4+1/4*(d*x+c)^4*(a+b/(d*x+c)^2)^(1/2)/a/d^4+3/8*b*(-4*a*c^2+b)*arctanh((a+
b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(5/2)/d^4
```

Mathematica [A] (verified)

Time = 10.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$

$$= \frac{\sqrt{a}(13bc^2-2ac^4+10bcdx-3bd^2x^2+2ad^4x^4)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}+3b(b-4ac^2)\log\left((c+dx)\left(a+\sqrt{a}\sqrt{\frac{b+a(}{(c-}}\right.\right.}{8a^{5/2}d^4}$$

input
```
Integrate[x^3/Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(Sqrt[a]*(13*b*c^2 - 2*a*c^4 + 10*b*c*d*x - 3*b*d^2*x^2 + 2*a*d^4*x^4)*Sqr
t[(b + a*(c + d*x)^2)/(c + d*x)^2] + 3*b*(b - 4*a*c^2)*Log[(c + d*x)*(a +
Sqrt[a]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])/(8*a^(5/2)*d^4)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {896, 25, 1774, 1803, 25, 540, 2338, 27, 539, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{d^3x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{d^3x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^4}$$

$\Big\downarrow$ 1774

$$-\frac{\int \frac{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^4}$$

$\Big\downarrow$ 1803

$$\frac{\int -\frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{d^4}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{d^4}$$

$\Big\downarrow$ 540

$$\frac{\frac{\int \frac{(c+dx)^4\left(\frac{4ac^3}{(c+dx)^2}+12ac+\frac{3\left(b-4ac^2\right)}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{4a}+\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{d^4}$$

$\Big\downarrow$ 2338

$$\frac{\frac{-\frac{\int -\frac{3a(c+dx)^3\left(3\left(b-4ac^2\right)-\frac{4c\left(2b-ac^2\right)}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{3a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}+\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{d^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(c+dx)^3\left(3\left(b-4ac^2\right)-\frac{4c\left(2b-ac^2\right)}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}+\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{d^4}$$

$\Big\downarrow$ 539

$$\frac{\frac{-\frac{\int \frac{(c+dx)^2\left(\frac{3b\left(b-4ac^2\right)}{c+dx}+8ac\left(2b-ac^2\right)\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{2a}-\frac{3\left(b-4ac^2\right)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}+\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{d^4}$$

$\Big\downarrow$ 534

$$\frac{-\frac{-\frac{3b(b-4ac^2)\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-8c(2b-ac^2)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{3(b-4ac^2)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}+\frac{(c+dx)^4\sqrt{a+}}{4a}}{d^4}$$

$\Big\downarrow$ 243

$$\frac{-\frac{-\frac{\frac{3}{2}b(b-4ac^2)\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-8c(2b-ac^2)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{3(b-4ac^2)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}+\frac{(c+dx)^4\sqrt{}}{4}}{d^4}$$

$\Big\downarrow$ 73

$$\frac{-\frac{-\frac{3(b-4ac^2)\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}-8c(2b-ac^2)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{3(b-4ac^2)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}+\frac{(c+}{}}{d^4}$$

$\Big\downarrow$ 221

$$\frac{-\frac{-\frac{-\frac{3b(b-4ac^2)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-8c(2b-ac^2)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{3(b-4ac^2)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}+\frac{(c+dx)}{}}{d^4}$$

input

```
Int[x^3/Sqrt[a + b/(c + d*x)^2],x]
```

output

```
(((c + d*x)^4*Sqrt[a + b/(c + d*x)^2])/(4*a) + ((-3*(b - 4*a*c^2)*(c + d*x
)^2*Sqrt[a + b/(c + d*x)^2])/(2*a) - 4*c*(c + d*x)^3*Sqrt[a + b/(c + d*x)^
2] - (-8*c*(2*b - a*c^2)*(c + d*x)*Sqrt[a + b/(c + d*x)^2] - (3*b*(b - 4*a
*c^2)*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]])/Sqrt[a])/(2*a))/(4*a))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{\left(-2a\,x^{3}d^{3}+2a\,d^{2}x^{2}c-2adx\,c^{2}+2c^{3}a+3bdx-13bc\right)\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)}{8a^{2}d^{4}\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}-\frac{3b\left(4a\,c^{2}-b\right)\ln\left(\frac{a\,d^{2}x+acd}{\sqrt{a\,d^{2}}}+\sqrt{a\,d^{2}x^{2}+2adxc+c\right.}{8d^{3}a^{2}\sqrt{a\,d^{2}}\,\sqrt{\frac{a\,d^{2}x^{2}+2adxc+}{(dx+c)^{2}}}}$
default	$-\frac{\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\left(-2x^{3}\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\,d^{3}a\sqrt{a\,d^{2}}+2\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\,\sqrt{a\,d^{2}}\,ac\,d^{2}x^{2}-2\sqrt{a\,d^{2}x^{2}+2adx\right.}{}$

input
```
int(x^3/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*a*d^3*x^3+2*a*c*d^2*x^2-2*a*c^2*d*x+2*a*c^3+3*b*d*x-13*b*c)*(a*d^
2*x^2+2*a*c*d*x+a*c^2+b)/a^2/d^4/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)
^(1/2)/(d*x+c)-3/8*b/d^3*(4*a*c^2-b)/a^2*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+
(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/(a*d^2)^(1/2)/((a*d^2*x^2+2*a*c*d*x+a
*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.21

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$

$$=\left[\frac{3\left(4\,abc^2-b^2\right)\sqrt{a}\log\left(-2\,ad^2x^2-4\,acdx-2\,ac^2+2\left(d^2x^2+2\,cdx+c^2\right)\sqrt{a}\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}-b\right)+}{16\,a^3d^4}\right.$$

input
```
integrate(x^3/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(4*a*b*c^2 - b^2)*sqrt(a)*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2
+ 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 2*(2*a^2*d^4*x^4 - 2*a^2*c^4 - 3*a*
b*d^2*x^2 + 10*a*b*c*d*x + 13*a*b*c^2)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^3*d^4), 1/8*(3*(4*a*b*c^2 - b^2)*sqrt
(-a)*arctan((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x
 + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b)) + (2*a^2*d^4*x^4 - 2*a^2*c^4 - 3*a*b*d^2*x^2 + 10*a*b*c*d*x + 13*a*b*c
^2)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(
a^3*d^4)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{x^3}{\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{x^3}{\sqrt{a+\frac{b}{(dx+c)^2}}}\,dx$$

input
```
integrate(x^3/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a + b/(d*x + c)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.25

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= \frac{1}{8}\sqrt{ad^2x^2+2\,acdx+ac^2+b}\left(\left(2\,x\left(\frac{x}{ad\mathrm{sgn}\,(dx+c)}-\frac{c}{ad^2\mathrm{sgn}\,(dx+c)}\right)+\frac{2\,a^3c^2d^4\mathrm{sgn}(dx+c)-3\,a^2b}{a^4d^7}\right.\right.$$
$$+\frac{3\,(4\,abc^2-b^2)\log\left(\left|-acd-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)\sqrt{a}|d|\right|\right)}{8\,a^{\frac{5}{2}}d^3|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(x^3/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)*((2*x*(x/(a*d*sgn(d*x + c)) -
c/(a*d^2*sgn(d*x + c))) + (2*a^3*c^2*d^4*sgn(d*x + c) - 3*a^2*b*d^4*sgn(d*
x + c))/(a^4*d^7))*x - (2*a^3*c^3*d^3*sgn(d*x + c) - 13*a^2*b*c*d^3*sgn(d*
x + c))/(a^4*d^7)) + 3/8*(4*a*b*c^2 - b^2)*log(abs(-a*c*d - (sqrt(a*d^2)*x
 - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*sqrt(a)*abs(d)))/(a^(5/2)*d^3*
abs(d)*sgn(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x)^2)^(1/2),x)
```

output
```
int(x^3/(a + b/(c + d*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{(dx+c)^2}}}dx$$

input
```
int(x^3/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
int(x^3/(a+b/(d*x+c)^2)^(1/2),x)
```

3.80 $\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=-\frac{(2b-3ac^2)\,(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{3a^2d^3}-\frac{c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{ad^3}+\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3ad^3}+\frac{bc\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{a^{3/2}d^3}$$

output
```
-1/3*(-3*a*c^2+2*b)*(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a^2/d^3-c*(d*x+c)^2*(a+b
/(d*x+c)^2)^(1/2)/a/d^3+1/3*(d*x+c)^3*(a+b/(d*x+c)^2)^(1/2)/a/d^3+b*c*arct
anh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(3/2)/d^3
```

Mathematica [A] (verified)

Time = 10.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \frac{\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\left(-2b(c+dx)+a(c^3+d^3x^3)\right)+3\sqrt{a}bc\log\left((c+dx)\left(a+\sqrt{a}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{3a^2d^3}$$

input
```
Integrate[x^2/Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]*(-2*b*(c + d*x) + a*(c^3 + d^3*x^3)
) + 3*Sqrt[a]*b*c*Log[(c + d*x)*(a + Sqrt[a]*Sqrt[(b + a*(c + d*x)^2)/(c +
 d*x)^2])])/(3*a^2*d^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 1774, 1803, 540, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^2x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1774}$$

$$\frac{\int \frac{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^3}$$

$$\Big\downarrow \ 1803$$

$$-\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \ 540$$

$$-\frac{-\frac{\int \frac{(c+dx)^3\left(6ac+\frac{2b-3ac^2}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{d^3}$$

$$\Big\downarrow \ 539$$

$$-\frac{-\frac{-\frac{\int -\frac{2a(c+dx)^2\left(-3ac^2-\frac{3bc}{c+dx}+2b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{2a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{d^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\int \frac{(c+dx)^2\left(-3ac^2-\frac{3bc}{c+dx}+2b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{d^3}$$

$$\Big\downarrow \ 534$$

$$-\frac{-\frac{-3bc\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{\left(2b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{d^3}$$

$$\Big\downarrow \ 243$$

$$-\frac{-\frac{-\frac{3}{2}bc\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\frac{\left(2b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{d^3}$$

$$\Big\downarrow \ 73$$

$$-\frac{-\frac{-3c\int \frac{1}{\frac{a+\frac{b}{(c+dx)^2}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{\left(2b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{d^3}$$

$$\Big\downarrow \ 221$$

$$-\frac{-\frac{\frac{3bc\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\left(2b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{d^3}$$

input
```
Int[x^2/Sqrt[a + b/(c + d*x)^2],x]
```

output
```
-((-1/3*((c + d*x)^3*Sqrt[a + b/(c + d*x)^2])/a - (-(((2*b - 3*a*c^2)*(c +
 d*x)*Sqrt[a + b/(c + d*x)^2])/a) - 3*c*(c + d*x)^2*Sqrt[a + b/(c + d*x)^2
] + (3*b*c*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]])/Sqrt[a])/(3*a))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.55

method	result
risch	$\frac{\left(a\,d^{2} x^{2}-a d x c+a\,c^{2}-2 b\right) \left(a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b\right)}{3 a^{2} d^{3} \sqrt{\frac{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}{\left(d x+c\right)^{2}}}\, \left(d x+c\right)}+\frac{b c \ln\left(\frac{a\,d^{2} x+a c d}{\sqrt{a\,d^{2}}}+\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}\right) \sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}}{a\,d^{2} \sqrt{a\,d^{2}}\, \sqrt{\frac{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}{\left(d x+c\right)^{2}}}\, \left(d x+c\right)}$
default	$\frac{\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}\, \left(x^{2} \sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}\, a\,d^{2} \sqrt{a\,d^{2}}-\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}\, \sqrt{a\,d^{2}}\, a c d x+\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}\right.}{3 d^{3} \sqrt{\frac{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}{\left(d x+c\right)^{2}}}\, \left(d x+c\right) a}$

input
```
int(x^2/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d^2*x^2-a*c*d*x+a*c^2-2*b)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/a^2/d^3/((
a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)+1/a*b*c/d^2*ln((a*d^
2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/(a*d^2)^(1/2
)/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)*(a*d^2*x^2+2*a*c
*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.53

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= \left[\frac{3\sqrt{a}bc\log\left(-2\,ad^2x^2-4\,acdx-2\,ac^2-2\left(d^2x^2+2\,cdx+c^2\right)\sqrt{a}\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}-b\right)+2\left(ad^3x^3\right.}{6\,a^2d^3}\right.$$
$$-\frac{3\sqrt{-a}bc\arctan\left(\frac{\left(d^2x^2+2\,cdx+c^2\right)\sqrt{-a}\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}}{ad^2x^2+2\,acdx+ac^2+b}\right)-\left(ad^3x^3+ac^3-2\,bdx-2\,bc\right)\sqrt{\frac{ad^2x^2+2\,acdx+a}{d^2x^2+2\,cdx+c}}}{3\,a^2d^3}$$

input
```
integrate(x^2/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a)*b*c*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 +
2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 +
 2*c*d*x + c^2)) - b) + 2*(a*d^3*x^3 + a*c^3 - 2*b*d*x - 2*b*c)*sqrt((a*d^
2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^2*d^3), -1/3
*(3*sqrt(-a)*b*c*arctan((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2
 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b)) - (a*d^3*x^3 + a*c^3 - 2*b*d*x - 2*b*c)*sqrt((a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^2*d^3)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{x^2}{\sqrt{a+\frac{b}{(dx+c)^2}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a + b/(d*x + c)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$=\frac{1}{3}\sqrt{ad^2x^2+2\,acdx+ac^2+b}\left(x\left(\frac{x}{ad\mathrm{sgn}\,(dx+c)}-\frac{c}{ad^2\mathrm{sgn}\,(dx+c)}\right)+\frac{a^2c^2d^2\mathrm{sgn}(dx+c)-2\,abd^2\mathrm{sgn}}{a^3d^5}\right.$$
$$-\frac{bc\log\left(\left|-acd-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)\sqrt{a}|d|\right|\right)}{a^{\frac{3}{2}}d^2|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(x^2/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)*(x*(x/(a*d*sgn(d*x + c)) - c/(
a*d^2*sgn(d*x + c))) + (a^2*c^2*d^2*sgn(d*x + c) - 2*a*b*d^2*sgn(d*x + c))
/(a^3*d^5)) - b*c*log(abs(-a*c*d - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b))*sqrt(a)*abs(d)))/(a^(3/2)*d^2*abs(d)*sgn(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{{(c+d\,x)}^2}}}\,dx$$

input
```
int(x^2/(a + b/(c + d*x)^2)^(1/2),x)
```

output
```
int(x^2/(a + b/(c + d*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= \frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,c^2-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,acdx+\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,d^2x^2}{3a^2d^3}$$

input
```
int(x^2/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**2 - sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b)*a*c*d*x + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
*a*d**2*x**2 - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b + 3*sqrt(a)*
log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)
/sqrt(b))*b*c)/(3*a**2*d**3)
```

3.81 $\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=-\frac{c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{ad^2}+\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2ad^2}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{2a^{3/2}d^2}$$

output
```
-c*(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a/d^2+1/2*(d*x+c)^2*(a+b/(d*x+c)^2)^(1/2)
/a/d^2-1/2*b*arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 10.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$=\frac{-\sqrt{a}(c-dx)\left(b+a(c+dx)^2\right)+b\sqrt{b+a(c+dx)^2}\log\left(-\sqrt{a}(c+dx)+\sqrt{b+a(c+dx)^2}\right)}{2a^{3/2}d^2(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}$$

input
```
Integrate[x/Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(-(Sqrt[a]*(c - d*x)*(b + a*(c + d*x)^2)) + b*Sqrt[b + a*(c + d*x)^2]*Log[
-(Sqrt[a]*(c + d*x)) + Sqrt[b + a*(c + d*x)^2]])/(2*a^(3/2)*d^2*(c + d*x)*
Sqrt[a + b/(c + d*x)^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {896, 25, 1774, 1803, 25, 539, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1774}$$

$$-\frac{\int \frac{(c+dx)\left(\frac{c}{c+dx}-1\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1803}$$

$$\frac{\int -\frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{\frac{\int \frac{(c+dx)^2\left(\frac{b}{c+dx}+2ac\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{2a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{d^2}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{\frac{b\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{d^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{\frac{1}{2}b\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{d^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{\int \frac{1}{\frac{a+\frac{b}{(c+dx)^2}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{d^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{d^2}$$

input
```
Int[x/Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(((c + d*x)^2*Sqrt[a + b/(c + d*x)^2])/(2*a) + (-2*c*(c + d*x)*Sqrt[a + b/
(c + d*x)^2] - (b*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]])/Sqrt[a])/(2*a)
)/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(83) = 166$.

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.02

method	result
risch	$-\frac{(-dx+c)(a\,d^2x^2+2adxc+a\,c^2+b)}{2a\,d^2\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}-\frac{b\ln\left(\frac{a\,d^2x+acd}{\sqrt{a\,d^2}}+\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\right)\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}{2da\sqrt{a\,d^2}\,\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}$
default	$-\frac{\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\left(-\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,\sqrt{a\,d^2}\,dx+\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,\sqrt{a\,d^2}\,c+bd\ln\left(\frac{a\,d^2x+acd+\sqrt{a\,d^2x^2+2\ldots}}{\sqrt{a\,d^2}}\ldots\right.\right.}{2d^2\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)a\sqrt{a\,d^2}}$

input
```
int(x/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/a/d^2/((a*d^2*x^2+2*a*c*d*x+a*
c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)-1/2*b/d/a*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)
+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/(a*d^2)^(1/2)/((a*d^2*x^2+2*a*c*d*x+
a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.35

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$

$$= \left[\frac{\sqrt{a}b\log\left(-2\,ad^2x^2-4\,acdx-2\,ac^2+2\,(d^2x^2+2\,cdx+c^2)\sqrt{a}\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}-b\right)+2\,(ad^2x^2-a}{4\,a^2d^2}\right.$$

input
```
integrate(x/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b*log(-2*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 + 2*(d^2*x^2 + 2*c*
d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2)) - b) + 2*(a*d^2*x^2 - a*c^2)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^2*d^2), 1/2*(sqrt(-a)*b*arctan((d^
2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(
d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)) + (a*d^2*x^
2 - a*c^2)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c
^2)))/(a^2*d^2)]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{x}{\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(x/sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d
**2*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{x}{\sqrt{a+\frac{b}{(dx+c)^2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a + b/(d*x + c)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.35

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \frac{1}{2}\sqrt{ad^2x^2+2\,acdx+ac^2+b}\left(\frac{x}{ad\mathrm{sgn}\,(dx+c)}-\frac{c}{ad^2\mathrm{sgn}\,(dx+c)}\right) + \frac{b\log\left(\left|-acd-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)\sqrt{a}|d|\right|\right)}{2\,a^{\frac{3}{2}}d|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(x/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)*(x/(a*d*sgn(d*x + c)) - c/(a*d
^2*sgn(d*x + c))) + 1/2*b*log(abs(-a*c*d - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2
 + 2*a*c*d*x + a*c^2 + b))*sqrt(a)*abs(d)))/(a^(3/2)*d*abs(d)*sgn(d*x + c)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}}\,dx$$

input
```
int(x/(a + b/(c + d*x)^2)^(1/2),x)
```

output
```
int(x/(a + b/(c + d*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \frac{x}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$=\frac{-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,ac+\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,adx-\sqrt{a}\log\left(\frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}+\sqrt{a}\,c+\sqrt{}}{\sqrt{b}}\right.}{2a^2d^2}$$

input
```
int(x/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c + sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b)*a*d*x - sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**
2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b)/(2*a**2*d**2)
```

3.82 $\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \frac{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{ad}$$

output
```
(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 9.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \frac{b+a(c+dx)^2}{ad(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}$$

input
```
Integrate[1/Sqrt[a + b/(c + d*x)^2],x]
```

output
```
(b + a*(c + d*x)^2)/(a*d*(c + d*x)*Sqrt[a + b/(c + d*x)^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {239, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}} \, dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)}{d}$$

$$\Big\downarrow \text{746}$$

$$\frac{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{ad}$$

input
```
Int[1/Sqrt[a + b/(c + d*x)^2],x]
```

output
```
((c + d*x)*Sqrt[a + b/(c + d*x)^2])/(a*d)
```

Defintions of rubi rules used

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

method	result	size
orering	$\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{ad(dx+c)\sqrt{a+\frac{b}{(dx+c)^{2}}}}$	49
trager	$\frac{(dx+c)\sqrt{-\frac{-a\,d^{2}x^{2}-2adxc-a\,c^{2}-b}{d^{2}x^{2}+2cdx+c^{2}}}}{ad}$	60
gosper	$\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{da\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}$	67
default	$\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{da\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}$	67
risch	$\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{da\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}$	67

input
```
int(1/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/a/d/(d*x+c)/(a+b/(d*x+c)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \frac{(dx+c)\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}}{ad}$$

input
```
integrate(1/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
(d*x + c)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^
2))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b/(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{(dx+c)^2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/(d*x + c)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\frac{\sqrt{ad^2x^2+2\,acdx+ac^2+b}}{ad\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(1/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(a*d*sgn(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\frac{\sqrt{\frac{a\,(c+d\,x)^2}{b}+1}\,(c+d\,x)}{d\,\sqrt{a+\frac{b}{(c+d\,x)^2}}\,\left(\sqrt{\frac{a\,(c+d\,x)^2}{b}+1}+1\right)}$$

input
```
int(1/(a + b/(c + d*x)^2)^(1/2),x)
```

output
```
(((a*(c + d*x)^2)/b + 1)^(1/2)*(c + d*x))/(d*(a + b/(c + d*x)^2)^(1/2)*(((
a*(c + d*x)^2)/b + 1)^(1/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}}{ad}$$

input
```
int(1/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(a*d)
```

3.83 $\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{c\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{b+ac^2}}$$

output
```
arctanh((a+b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(1/2)-c*arctanh((a*c+b/(d*x+c))/(
a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/(a*c^2+b)^(1/2)
```

Mathematica [A] (verified)

Time = 10.41 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$=\frac{\sqrt{b+a(c+dx)^2}\left(2\sqrt{a}c\arctan\left(\frac{-\sqrt{a}dx+\sqrt{b+a(c+dx)^2}}{\sqrt{-b-ac^2}}\right)-\sqrt{-b-ac^2}\log\left(-\sqrt{a}(c+dx)+\sqrt{b+a(c+dx)}\right.\right.}{\sqrt{a}\sqrt{-b-ac^2}(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}$$

input
```
Integrate[1/(x*Sqrt[a + b/(c + d*x)^2]),x]
```

output
```
(Sqrt[b + a*(c + d*x)^2]*(2*Sqrt[a]*c*ArcTan[(-(Sqrt[a]*d*x) + Sqrt[b + a*
(c + d*x)^2])/Sqrt[-b - a*c^2]] - Sqrt[-b - a*c^2]*Log[-(Sqrt[a]*(c + d*x)
) + Sqrt[b + a*(c + d*x)^2]]))/(Sqrt[a]*Sqrt[-b - a*c^2]*(c + d*x)*Sqrt[a
+ b/(c + d*x)^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 25, 1774, 1803, 25, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{1}{dx\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{1}{dx\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)$$

$$\Big\downarrow \text{ 1774}$$

$$-\int \frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$

$$\Big\downarrow \text{ 1803}$$

$$\int -\frac{c+dx}{\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{617}$$

$$-\int \left(\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{c}{c+dx}-1\right)}\right)d\frac{1}{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{c\operatorname{arctan}h\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{ac^2+b}}$$

input
```
Int[1/(x*Sqrt[a + b/(c + d*x)^2]),x]
```

output
```
ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]]/Sqrt[a] - (c*ArcTanh[(a*c + b/(c
+ d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/Sqrt[b + a*c^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(72) = 144$.

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.42

method	result
default	$\frac{\sqrt{a d^2x^2+2adxc+a c^2+b}\left(d\ln\left(\frac{a d^2x+acd+\sqrt{a d^2x^2+2adxc+a c^2+b}\sqrt{a d^2}}{\sqrt{a d^2}}\right)\sqrt{a c^2+b}-c\ln\left(\frac{2a c^2+2b+2adxc+2\sqrt{a c^2+b}\sqrt{a d^2x^2+2adxc}}{x}\right.\right.}{\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}(dx+c)\sqrt{a d^2}\sqrt{a c^2+b}}$

input
```
int(1/x/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(d*ln((a*d^2*x+a*c*d+(a*d^2*x^2+2*a*c*
d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*(a*c^2+b)^(1/2)-c*ln(2*(a
*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*
d^2)^(1/2))/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)/(a*d^2
)^(1/2)/(a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(72) = 144$.

Time = 0.15 (sec) , antiderivative size = 1089, normalized size of antiderivative = 12.96

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a*c^2 + b)*a*c*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*
a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*a*
c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) + (a*c^2 + b)*sqrt(a)*log(-2*a*d^2*x^2
 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x
^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b))/(a^2*c^2 + a*
b), 1/2*(sqrt(a*c^2 + b)*a*c*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 +
 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2
*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(a*c^2 + b)*sqrt(-a)*arctan((d^
2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(
d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)))/(a^2*c^2 +
 a*b), 1/2*(2*sqrt(-a*c^2 - b)*a*c*arctan((a*c*d^2*x^2 + a*c^3 + (2*a*c^2
+ b)*d*x + b*c)*sqrt(-a*c^2 - b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/
(d^2*x^2 + 2*c*d*x + c^2))/(a^2*c^4 + (a^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2
+ 2*(a^2*c^3 + a*b*c)*d*x + b^2)) + (a*c^2 + b)*sqrt(a)*log(-2*a*d^2*x^2 -
 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2
 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b))/(a^2*c^2 + a*b)
, (sqrt(-a*c^2 - b)*a*c*arctan((a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x +
b*c)*sqrt(-a*c^2 - b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2...
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{x\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{(dx+c)^2}}\,x}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c)^2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{1}{x\,\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x)^2)^(1/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1148, normalized size of antiderivative = 13.67

$$\int \frac{1}{x\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*sqrt(a*c**
2 + b)*atan((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt
(a)*d*x)/sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a*c**2 - 2*sqr
t(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*atan((sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(2*sqrt(a)*sqrt(a*c**
2 + b)*c - 2*a*c**2 - b))*a**2*c**3 - 2*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c
- 2*a*c**2 - b)*atan((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)
*c + sqrt(a)*d*x)/sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a*b*c
 - sqrt(a)*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c + 2*a*c**2 + b)*sqrt(a*c**2 +
 b)*log(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(2*sqrt(a)*sqrt(a
*c**2 + b)*c + 2*a*c**2 + b) + sqrt(a)*c + sqrt(a)*d*x)*a*c**2 + sqrt(a)*s
qrt(2*sqrt(a)*sqrt(a*c**2 + b)*c + 2*a*c**2 + b)*sqrt(a*c**2 + b)*log(sqrt
(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c
 + 2*a*c**2 + b) + sqrt(a)*c + sqrt(a)*d*x)*a*c**2 + sqrt(a*c**2 + b)*log(
sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - sqrt(2*sqrt(a)*sqrt(a*c**2 +
b)*c + 2*a*c**2 + b) + sqrt(a)*c + sqrt(a)*d*x)*a*b*c + sqrt(a*c**2 + b)*l
og(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(2*sqrt(a)*sqrt(a*c**2
 + b)*c + 2*a*c**2 + b) + sqrt(a)*c + sqrt(a)*d*x)*a*b*c - sqrt(a*c**2 + b
)*log(2*sqrt(a)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*c + 2*sqrt(a)*s
qrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*d*x + 2*sqrt(a)*sqrt(a*c**2 +...
```

3.84 $\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=-\frac{cd\sqrt{a+\frac{b}{(c+dx)^2}}}{(b+ac^2)\left(1-\frac{c}{c+dx}\right)}-\frac{bd\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{(b+ac^2)^{3/2}}$$

output
```
-c*d*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)/(1-c/(d*x+c))-b*d*arctanh((a*c+b/(d*x
+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/(a*c^2+b)^(3/2)
```

Mathematica [A] (verified)

Time = 10.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$=\frac{-c\sqrt{b+ac^2}(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}+bdx\log(x)-bdx\log\left(b+(c+dx)\left(ac+\sqrt{b+ac^2}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{(b+ac^2)^{3/2}\,x}$$

input
```
Integrate[1/(x^2*Sqrt[a + b/(c + d*x)^2]),x]
```

output
```
(-(c*Sqrt[b + a*c^2]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]) + b*
d*x*Log[x] - b*d*x*Log[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c
 + d*x)^2)/(c + d*x)^2])])/((b + a*c^2)^(3/2)*x)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {896, 1774, 1799, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{1}{d^2x^2\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)$$

$$\Big\downarrow \text{1774}$$

$$d\int \frac{1}{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{1799}$$

$$-d\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{491}$$

$$-d\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{ac^2+b}+\frac{c\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}\right)$$

$$\Big\downarrow \text{488}$$

$$-d\left(\frac{c\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}-\frac{b\int\frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{ac^2+b}\right)$$

$$\Big\downarrow \text{219}$$

$$-d\left(\frac{c\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}-\frac{b\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{(ac^2+b)^{3/2}}\right)$$

input

```
Int[1/(x^2*Sqrt[a + b/(c + d*x)^2]),x]
```

output

```
-(d*((c*Sqrt[a + b/(c + d*x)^2])/((b + a*c^2)*(1 - c/(c + d*x))) - (b*ArcT
anh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/(b
+ a*c^2)^(3/2)))
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(91) = 182$.

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.07

method	result
risch	$-\frac{c(a d^2x^2+2adxc+a c^2+b)}{(a c^2+b)x\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)}-\frac{bd\ln\left(\frac{2a c^2+2b+2adxc+2\sqrt{a c^2+b}\,\sqrt{a d^2x^2+2adxc+a c^2+b}}{x}\right)\sqrt{a d^2x^2+2adxc+a c^2+b}}{(a c^2+b)^{\frac{3}{2}}\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)}$
default	$-\frac{\sqrt{a d^2x^2+2adxc+a c^2+b}\left(\ln\left(\frac{2a c^2+2b+2adxc+2\sqrt{a c^2+b}\,\sqrt{a d^2x^2+2adxc+a c^2+b}}{x}\right)ab c^2dx+\sqrt{a d^2x^2+2adxc+a c^2+b}\,(a c^2+b)^{\frac{3}{2}}c\right)}{\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)(a c^2+b)^{\frac{5}{2}}x}$

input
```
int(1/x^2/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*c^2+b)*c*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/x/((a*d^2*x^2+2*a*c*d*x+a*c^2
+b)/(d*x+c)^2)^(1/2)/(d*x+c)-b*d/(a*c^2+b)^(3/2)*ln((2*a*c^2+2*b+2*a*d*x*c
+2*(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x)/((a*d^2*x^2+2*a
*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(89) = 178$.

Time = 0.16 (sec) , antiderivative size = 498, normalized size of antiderivative = 5.03

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= \left[\frac{\sqrt{ac^2+b}bdx\log\left(-\frac{2a^2c^4+(2a^2c^2+ab)d^2x^2+4abc^2+4(a^2c^3+abc)dx+2b^2-2(acd^2x^2+ac^3+(2ac^2+b)dx+bc)\sqrt{ac^2+b}\sqrt{\frac{ad^2x^2+2}{d^2x^2+}}}{x^2}\right.}{2(a^2c^4+2abc^2+b^2)x}\right.$$

input
```
integrate(1/x^2/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a*c^2 + b)*b*d*x*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 +
4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*
a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(a*c^4 + b*c^2 + (a*c^3 + b*c)*d
*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(
(a^2*c^4 + 2*a*b*c^2 + b^2)*x), (sqrt(-a*c^2 - b)*b*d*x*arctan((a*c*d^2*x^
2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a*c^2 - b)*sqrt((a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a^2*c^4 + (a^2*c^2 + a*b)
*d^2*x^2 + 2*a*b*c^2 + 2*(a^2*c^3 + a*b*c)*d*x + b^2)) - (a*c^4 + b*c^2 +
(a*c^3 + b*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2)))/((a^2*c^4 + 2*a*b*c^2 + b^2)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{1}{x^2\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d
*x + d**2*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{(dx+c)^2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c)^2)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(89) = 178.

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.22

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \frac{2\,bd\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}}{\sqrt{-ac^2-b}}\right)}{(ac^2\mathrm{sgn}\,(dx+c)+b\mathrm{sgn}\,(dx+c))\sqrt{-ac^2-b}} - \frac{2\left(a^{\frac{3}{2}}c^3|d|+\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)ac^2d+\sqrt{a}bc|d|\right)}{(ac^2\mathrm{sgn}\,(dx+c)+b\mathrm{sgn}\,(dx+c))\left(ac^2-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)^2+b\right)}$$

input
```
integrate(1/x^2/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
2*b*d*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))/sq
rt(-a*c^2 - b))/((a*c^2*sgn(d*x + c) + b*sgn(d*x + c))*sqrt(-a*c^2 - b)) -
 2*(a^(3/2)*c^3*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c
^2 + b))*a*c^2*d + sqrt(a)*b*c*abs(d))/((a*c^2*sgn(d*x + c) + b*sgn(d*x +
c))*(a*c^2 - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2 +
 b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{x^2\,\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x)^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$=\frac{-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a\,c^3-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,bc+\sqrt{a\,c^2+b}\log\left(\sqrt{a\,c^2+b}\,\sqrt{a\,d^2x^2+\cdots}\right.}{x\left(a^2c^4+2ab\,c^2+b^2\right)}$$

input
```
int(1/x^2/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*c**3 - sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b)*b*c + sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(
a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*b*d*x - sqrt
(a*c**2 + b)*log(x)*b*d*x)/(x*(a**2*c**4 + 2*a*b*c**2 + b**2))
```

3.85 $\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = -\frac{d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2(b+ac^2)\left(1-\frac{c}{c+dx}\right)^2} - \frac{(b-2ac^2)\,d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2(b+ac^2)^2\left(1-\frac{c}{c+dx}\right)} + \frac{3abcd^2\,\mathrm{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2(b+ac^2)^{5/2}}$$

output
```
-1/2*d^2*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)/(1-c/(d*x+c))^2-1/2*(-2*a*c^2+b)*
d^2*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^2/(1-c/(d*x+c))+3/2*a*b*c*d^2*arctanh(
(a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/(a*c^2+b)^(5/2)
```

Mathematica [A] (verified)

Time = 10.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= \frac{-\sqrt{b+ac^2}(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}(ac^2(c-dx)+b(c+2dx)) - 3abcd^2x^2\log(x) + 3abcd^2x^2\log\Big(b+(c+d}{2\left(b+ac^2\right)^{5/2}x^2}$$

input
```
Integrate[1/(x^3*Sqrt[a + b/(c + d*x)^2]),x]
```

output
```
(-(Sqrt[b + a*c^2]*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]*(a*c^2*
(c - d*x) + b*(c + 2*d*x))) - 3*a*b*c*d^2*x^2*Log[x] + 3*a*b*c*d^2*x^2*Log
[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2
])])/(2*(b + a*c^2)^(5/2)*x^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 25, 1774, 1803, 25, 594, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$\Big\downarrow \; 896$$
$$d^2\int \frac{1}{d^3x^3\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)$$
$$\Big\downarrow \; 25$$
$$-d^2\int -\frac{1}{d^3x^3\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)$$

$$\Big\downarrow \text{1774}$$

$$-d^2\int \frac{1}{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \text{1803}$$

$$d^2\int -\frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-d^2\int \frac{1}{(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{594}$$

$$d^2\left(\frac{\int \frac{2ac-\frac{b}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)}-\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{679}$$

$$d^2\left(\frac{\frac{3abc\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{ac^2+b}-\frac{\left(b-2ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}}{2\left(ac^2+b\right)}-\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{488}$$

$$d^2\left(\frac{-\frac{3abc\int \frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{ac^2+b}-\frac{\left(b-2ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}}{2\left(ac^2+b\right)}-\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$d^2\left(\frac{-\frac{3abc\,\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{(ac^2+b)^{3/2}}-\frac{(b-2ac^2)\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}}{2\,(ac^2+b)}-\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{2\,(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

input
```
Int[1/(x^3*Sqrt[a + b/(c + d*x)^2]),x]
```

output
```
d^2*(-1/2*Sqrt[a + b/(c + d*x)^2]/((b + a*c^2)*(1 - c/(c + d*x))^2) + (-((
(b - 2*a*c^2)*Sqrt[a + b/(c + d*x)^2])/((b + a*c^2)*(1 - c/(c + d*x)))) -
(3*a*b*c*ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d
*x)^2])])/(b + a*c^2)^(3/2))/(2*(b + a*c^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_^2)]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{\left(a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b\right) \left(-a d x\,c^{2}+c^{3} a+2 b d x+b c\right)}{2 \left(a\,c^{2}+b\right)^{2} x^{2} \sqrt{\frac{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}{\left(d x+c\right)^{2}}}\, \left(d x+c\right)}+\frac{3 a b c\,d^{2} \ln\left(\frac{2 a\,c^{2}+2 b+2 a d x c+2 \sqrt{a\,c^{2}+b}\,\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}}{x}\right) \sqrt{a\,d^{2} x^{2}+}}{2 \left(a\,c^{2}+b\right)^{\frac{5}{2}} \sqrt{\frac{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}{\left(d x+c\right)^{2}}}\, \left(d x+c\right)}$
default	$-\frac{\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}\, \left(-3 \ln\left(\frac{2 a\,c^{2}+2 b+2 a d x c+2 \sqrt{a\,c^{2}+b}\,\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}}{x}\right) a^{2} b\,c^{3} d^{2} x^{2}-\sqrt{a\,d^{2} x^{2}+2 a d x c+a\,c^{2}+b}\, \left(a\,c^{2}\right.\right.}{}$

input
```
int(1/x^3/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)*(-a*c^2*d*x+a*c^3+2*b*d*x+b*c)/(a*c^2+b
)^2/x^2/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)+3/2/(a*c^2
+b)^(5/2)*a*b*c*d^2*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d^2*x^2
+2*a*c*d*x+a*c^2+b)^(1/2))/x)/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1
/2)/(d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(145) = 290$.

Time = 0.30 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.85

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$

$$= \left[\frac{3\sqrt{ac^2+b}abcd^2x^2\log\left(-\frac{2a^2c^4+(2a^2c^2+ab)d^2x^2+4abc^2+4(a^2c^3+abc)dx+2b^2+2(acd^2x^2+ac^3+(2ac^2+b)dx+bc)\sqrt{ac^2+b}\sqrt{\cdots}}{x^2}\cdots\right.}{4(a^3c^6+3a^2b\cdots}\right.$$

$$-\frac{3\sqrt{-ac^2-b}abcd^2x^2\arctan\left(\frac{(acd^2x^2+ac^3+(2ac^2+b)dx+bc)\sqrt{-ac^2-b}\sqrt{\frac{ad^2x^2+2acdx+ac^2+b}{d^2x^2+2cdx+c^2}}}{a^2c^4+(a^2c^2+ab)d^2x^2+2abc^2+2(a^2c^3+abc)dx+b^2}\right)+(a^2c^6+2abc^4-(\cdots}{2(a^3c^6+3a^2bc^4+3ab^2c^2+b^3)x\cdots}$$

input
```
integrate(1/x^3/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*sqrt(a*c^2 + b)*a*b*c*d^2*x^2*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*
d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 + 2*(a*c*d^2*x^2 + a
*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(a^2*c^6 + 2*a*b*c^4 -
 (a^2*c^4 - a*b*c^2 - 2*b^2)*d^2*x^2 + b^2*c^2 + 3*(a*b*c^3 + b^2*c)*d*x)*
sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^3
*c^6 + 3*a^2*b*c^4 + 3*a*b^2*c^2 + b^3)*x^2), -1/2*(3*sqrt(-a*c^2 - b)*a*b
*c*d^2*x^2*arctan((a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a*
c^2 - b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2
))/(a^2*c^4 + (a^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 2*(a^2*c^3 + a*b*c)*d*
x + b^2)) + (a^2*c^6 + 2*a*b*c^4 - (a^2*c^4 - a*b*c^2 - 2*b^2)*d^2*x^2 + b
^2*c^2 + 3*(a*b*c^3 + b^2*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)
/(d^2*x^2 + 2*c*d*x + c^2)))/((a^3*c^6 + 3*a^2*b*c^4 + 3*a*b^2*c^2 + b^3)*
x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{x^3\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d
*x + d**2*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{(dx+c)^2}}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c)^2)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(145) = 290.

Time = 0.13 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.57

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= -\frac{3\,abcd^2\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}}{\sqrt{-ac^2-b}}\right)}{(a^2c^4\mathrm{sgn}\,(dx+c)+2\,abc^2\mathrm{sgn}\,(dx+c)+b^2\mathrm{sgn}\,(dx+c))\sqrt{-ac^2-b}} + \frac{2\,a^{\frac{7}{2}}c^6d|d|+4\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)a^3c^5d^2+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+\right.}$$

input
```
integrate(1/x^3/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-3*a*b*c*d^2*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b))/sqrt(-a*c^2 - b))/((a^2*c^4*sgn(d*x + c) + 2*a*b*c^2*sgn(d*x + c) + b
^2*sgn(d*x + c))*sqrt(-a*c^2 - b)) + (2*a^(7/2)*c^6*d*abs(d) + 4*(sqrt(a*d
^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^3*c^5*d^2 + 2*(sqrt(a*d
^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(5/2)*c^4*d*abs(d) +
2*a^(5/2)*b*c^4*d*abs(d) + 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b))*a^2*b*c^3*d^2 + 4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x
 + a*c^2 + b))^2*a^(3/2)*b*c^2*d*abs(d) + 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b))^3*a*b*c*d^2 - 2*a^(3/2)*b^2*c^2*d*abs(d) - (sq
rt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a*b^2*c*d^2 + 2*(sq
rt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*sqrt(a)*b^2*d*abs
(d) - 2*sqrt(a)*b^3*d*abs(d))/((a^2*c^4*sgn(d*x + c) + 2*a*b*c^2*sgn(d*x +
 c) + b^2*sgn(d*x + c))*(a*c^2 - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b))^2 + b)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{x^3\,\sqrt{a+\frac{b}{{\left(c+d\,x\right)}^2}}} \,d x$$

input
```
int(1/(x^3*(a + b/(c + d*x)^2)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$=\frac{-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^5+\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^4dx-2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}}{}$$

input
```
int(1/x^3/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*c**5 + sqrt(a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b)*a**2*c**4*d*x - 2*sqrt(a*c**2 + 2*a*c*d*x + a*
d**2*x**2 + b)*a*b*c**3 - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b*c
**2*d*x - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**2*c - 2*sqrt(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**2*d*x + 3*sqrt(a*c**2 + b)*log( - sqrt
(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x
 - b)*a*b*c*d**2*x**2 - 3*sqrt(a*c**2 + b)*log(x)*a*b*c*d**2*x**2)/(2*x**2
*(a**3*c**6 + 3*a**2*b*c**4 + 3*a*b**2*c**2 + b**3))
```

3.86 $\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 241

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = -\frac{d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3c(b+ac^2)\left(1-\frac{c}{c+dx}\right)^3}+\frac{(b+6ac^2)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6c(b+ac^2)^2\left(1-\frac{c}{c+dx}\right)^2} + \frac{(b^2+10abc^2-6a^2c^4)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6c(b+ac^2)^3\left(1-\frac{c}{c+dx}\right)} + \frac{ab(b-4ac^2)\,d^3\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2(b+ac^2)^{7/2}}$$

output
```
-1/3*d^3*(a+b/(d*x+c)^2)^(1/2)/c/(a*c^2+b)/(1-c/(d*x+c))^3+1/6*(6*a*c^2+b)
*d^3*(a+b/(d*x+c)^2)^(1/2)/c/(a*c^2+b)^2/(1-c/(d*x+c))^2+1/6*(-6*a^2*c^4+1
0*a*b*c^2+b^2)*d^3*(a+b/(d*x+c)^2)^(1/2)/c/(a*c^2+b)^3/(1-c/(d*x+c))+1/2*a
*b*(-4*a*c^2+b)*d^3*arctanh((a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2
)^(1/2))/(a*c^2+b)^(7/2)
```

Mathematica [A] (verified)

Time = 10.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= \frac{-\sqrt{b+ac^2}\left(2c^2(b+ac^2)^2+5bc(b+ac^2)\,dx+3b(b-4ac^2)\,d^2x^2+ac(-13b+2ac^2)\,d^3x^3\right)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}-}{6\,(b+ac^2)^{7/}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b/(c + d*x)^2]),x]
```

output
```
(-(Sqrt[b + a*c^2]*(2*c^2*(b + a*c^2)^2 + 5*b*c*(b + a*c^2)*d*x + 3*b*(b -
 4*a*c^2)*d^2*x^2 + a*c*(-13*b + 2*a*c^2)*d^3*x^3)*Sqrt[(b + a*(c + d*x)^2
)/(c + d*x)^2]) - 3*a*b*(b - 4*a*c^2)*d^3*x^3*Log[x] + 3*a*b*(b - 4*a*c^2)
*d^3*x^3*Log[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^2)
/(c + d*x)^2])])/(6*(b + a*c^2)^(7/2)*x^3)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 1774, 1803, 603, 688, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$\Big\downarrow \text{ 896}$$
$$d^3\int \frac{1}{d^4x^4\sqrt{a+\frac{b}{(c+dx)^2}}}d(c+dx)$$
$$\Big\downarrow \text{ 1774}$$

$$d^3 \int \frac{1}{(c+dx)^4 \sqrt{a+\frac{b}{(c+dx)^2}} \left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \text{ 1803}$$

$$-d^3 \int \frac{1}{(c+dx)^2 \sqrt{a+\frac{b}{(c+dx)^2}} \left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 603}$$

$$-d^3 \left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int \frac{3a+\frac{\frac{b}{c}+3ac}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3\left(ac^2+b\right)}\right)$$

$$\Big\downarrow \text{ 688}$$

$$-d^3 \left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{(6ac^2+b)\sqrt{a+\frac{b}{(c+dx)^2}}}{2c(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{\int -\frac{2ac\left(2b-3ac^2\right)+\frac{b\left(6ac^2+b\right)}{c+dx}}{c\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2(ac^2+b)}}{3\left(ac^2+b\right)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-d^3 \left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\int \frac{2ac\left(2b-3ac^2\right)+\frac{b\left(6ac^2+b\right)}{c+dx}}{c\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2(ac^2+b)}+\frac{(6ac^2+b)\sqrt{a+\frac{b}{(c+dx)^2}}}{2c(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}}{3\left(ac^2+b\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-d^3 \left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\int \frac{2ac\left(2b-3ac^2\right)+\frac{b\left(6ac^2+b\right)}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2c(ac^2+b)}+\frac{(6ac^2+b)\sqrt{a+\frac{b}{(c+dx)^2}}}{2c(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}}{3\left(ac^2+b\right)}\right)$$

$\Big\downarrow$ 679

$$-d^3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\frac{3abc\left(b-4ac^2\right)\int\frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{ac^2+b}+\frac{\left(-6a^2c^4+10abc^2+b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}}{2c(ac^2+b)}+\frac{(6ac^2+b)\sqrt{a-}}{2c(ac^2+b)\left(1-\right.}}{3\left(ac^2+b\right)}\right.$$

$\Big\downarrow$ 488

$$-d^3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\frac{\left(-6a^2c^4+10abc^2+b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}-\frac{3abc\left(b-4ac^2\right)\int\frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{ac^2+b}}{2c(ac^2+b)}+\frac{(6ac^2+b)\sqrt{a-}}{2c(ac^2+b)\left(1-\right.}}{3\left(ac^2+b\right)}\right.$$

$\Big\downarrow$ 219

$$-d^3\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{3c\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^3}-\frac{\frac{\frac{\left(-6a^2c^4+10abc^2+b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}-\frac{3abc\left(b-4ac^2\right)\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\left(ac^2+b\right)^{3/2}}}{2c(ac^2+b)}+\frac{(6ac^2+b)\sqrt{}}{2c(ac^2+b)\left(\right.}}{3\left(ac^2+b\right)}\right.$$

input
```
Int[1/(x^4*Sqrt[a + b/(c + d*x)^2]),x]
```

output
```
-(d^3*(Sqrt[a + b/(c + d*x)^2]/(3*c*(b + a*c^2)*(1 - c/(c + d*x))^3) - (((
b + 6*a*c^2)*Sqrt[a + b/(c + d*x)^2])/(2*c*(b + a*c^2)*(1 - c/(c + d*x))^2
) + (((b^2 + 10*a*b*c^2 - 6*a^2*c^4)*Sqrt[a + b/(c + d*x)^2])/((b + a*c^2)
*(1 - c/(c + d*x))) - (3*a*b*c*(b - 4*a*c^2)*ArcTanh[(-(a*c) - b/(c + d*x)
)/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)^2])])/(b + a*c^2)^(3/2))/(2*c*(b +
 a*c^2)))/(3*(b + a*c^2))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{(a\,d^2x^2+2adxc+a\,c^2+b)(2a^2c^3d^2x^2-2a^2c^4dx+2a^2c^5-13abc\,d^2x^2+ab\,c^2dx+4ab\,c^3+3b^2dx+2b^2c)}{6(a\,c^2+b)^3x^3\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)} - \frac{d^3ba(4a\,c^2-b)\ln\left(\frac{2a\,c^2}{}\right.}{}$
default	$-\frac{\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\left(12\ln\left(\frac{2a\,c^2+2b+2adxc+2\sqrt{a\,c^2+b}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}{x}\right)a^3b\,c^4d^3x^3+2\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,(a\,c^2\right.}{}$

input
```
int(1/x^4/(a+b/(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)*(2*a^2*c^3*d^2*x^2-2*a^2*c^4*d*x+2*a^2*
c^5-13*a*b*c*d^2*x^2+a*b*c^2*d*x+4*a*b*c^3+3*b^2*d*x+2*b^2*c)/(a*c^2+b)^3/
x^3/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)-1/2*d^3*b*a*(4
*a*c^2-b)/(a*c^2+b)^(7/2)*ln((2*a*c^2+2*b+2*a*d*x*c+2*(a*c^2+b)^(1/2)*(a*d
^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2))/x)/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)
^2)^(1/2)/(d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.27

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b/(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(4*a^2*b*c^2 - a*b^2)*sqrt(a*c^2 + b)*d^3*x^3*log(-(2*a^2*c^4 + (
2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2
*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d
^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(2*a^
3*c^8 + 6*a^2*b*c^6 + 6*a*b^2*c^4 + (2*a^3*c^5 - 11*a^2*b*c^3 - 13*a*b^2*c
)*d^3*x^3 + 2*b^3*c^2 - 3*(4*a^2*b*c^4 + 3*a*b^2*c^2 - b^3)*d^2*x^2 + 5*(a
^2*b*c^5 + 2*a*b^2*c^3 + b^3*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^4*c^8 + 4*a^3*b*c^6 + 6*a^2*b^2*c^4 +
4*a*b^3*c^2 + b^4)*x^3), 1/6*(3*(4*a^2*b*c^2 - a*b^2)*sqrt(-a*c^2 - b)*d^3
*x^3*arctan((a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a*c^2 -
b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a^
2*c^4 + (a^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 2*(a^2*c^3 + a*b*c)*d*x + b^
2)) - (2*a^3*c^8 + 6*a^2*b*c^6 + 6*a*b^2*c^4 + (2*a^3*c^5 - 11*a^2*b*c^3 -
 13*a*b^2*c)*d^3*x^3 + 2*b^3*c^2 - 3*(4*a^2*b*c^4 + 3*a*b^2*c^2 - b^3)*d^2
*x^2 + 5*(a^2*b*c^5 + 2*a*b^2*c^3 + b^3*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^4*c^8 + 4*a^3*b*c^6 + 6*a^2
*b^2*c^4 + 4*a*b^3*c^2 + b^4)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx=\int \frac{1}{x^4\sqrt{\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}}}\,dx$$

input
```
integrate(1/x**4/(a+b/(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d
*x + d**2*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{(dx+c)^2}}x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x + c)^2)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. 2(222) = 444.

Time = 0.34 (sec) , antiderivative size = 1005, normalized size of antiderivative = 4.17

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b/(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
(4*a^2*b*c^2*d^3 - a*b^2*d^3)*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b))/sqrt(-a*c^2 - b))/((a^3*c^6*sgn(d*x + c) + 3*a^2*b*c
^4*sgn(d*x + c) + 3*a*b^2*c^2*sgn(d*x + c) + b^3*sgn(d*x + c))*sqrt(-a*c^2
 - b)) - 1/3*(8*a^(11/2)*c^9*d^2*abs(d) + 24*(sqrt(a*d^2)*x - sqrt(a*d^2*x
^2 + 2*a*c*d*x + a*c^2 + b))*a^5*c^8*d^3 + 24*(sqrt(a*d^2)*x - sqrt(a*d^2*
x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(9/2)*c^7*d^2*abs(d) + 8*(sqrt(a*d^2)*x
- sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*a^4*c^6*d^3 + 8*a^(9/2)*b*c^7
*d^2*abs(d) + 36*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))
*a^4*b*c^6*d^3 + 72*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b))^2*a^(7/2)*b*c^5*d^2*abs(d) + 56*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b))^3*a^3*b*c^4*d^3 - 24*a^(7/2)*b^2*c^5*d^2*abs(d) - 12*(
sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^5*a^2*b*c^2*d^3 -
 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^3*b^2*c^4*d
^3 + 72*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(5/2
)*b^2*c^3*d^2*abs(d) + 48*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b))^3*a^2*b^2*c^2*d^3 - 40*a^(5/2)*b^3*c^3*d^2*abs(d) + 3*(sqrt(a*d^
2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^5*a*b^2*d^3 - 18*(sqrt(a*d
^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^2*b^3*c^2*d^3 + 24*(sqr
t(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(3/2)*b^3*c*d^2*
abs(d) - 16*a^(3/2)*b^4*c*d^2*abs(d) - 3*(sqrt(a*d^2)*x - sqrt(a*d^2*x^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx = \int \frac{1}{x^4\,\sqrt{a+\frac{b}{{(c+d\,x)}^2}}}\,dx$$

input
```
int(1/(x^4*(a + b/(c + d*x)^2)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{(c+dx)^2}}}\,dx$$
$$= \frac{-2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^3c^7+2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^3c^6dx-2\sqrt{a\,d^2x^2+2acdx+a\,c^2+}}{}$$

input
```
int(1/x^4/(a+b/(d*x+c)^2)^(1/2),x)
```

output
```
( - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**7 + 2*sqrt(a*c**2
 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**6*d*x - 2*sqrt(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)*a**3*c**5*d**2*x**2 - 6*sqrt(a*c**2 + 2*a*c*d*x + a*d**
2*x**2 + b)*a**2*b*c**5 + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*
b*c**4*d*x + 11*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**3*d**
2*x**2 - 6*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**3 - 4*sqrt
(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**2*d*x + 13*sqrt(a*c**2 +
2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c*d**2*x**2 - 2*sqrt(a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)*b**3*c - 3*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
*b**3*d*x + 12*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a**2*b*c**2*d**3*x**3 - 3*sq
rt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b) - a*c**2 - a*c*d*x - b)*a*b**2*d**3*x**3 - 12*sqrt(a*c**2 + b)*log(x)
*a**2*b*c**2*d**3*x**3 + 3*sqrt(a*c**2 + b)*log(x)*a*b**2*d**3*x**3)/(6*x*
*3*(a**4*c**8 + 4*a**3*b*c**6 + 6*a**2*b**2*c**4 + 4*a*b**3*c**2 + b**4))
```

3.87 $\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 241

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx =\ & \frac{c(5b-ac^2)\,(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a^3d^4} \\ & -\frac{(7b-12ac^2)\,(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{8a^3d^4}-\frac{c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{a^2d^4} \\ & +\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a^2d^4}-\frac{b\left(b-3ac^2-\frac{c(3b-ac^2)}{c+dx}\right)}{a^3d^4\sqrt{a+\frac{b}{(c+dx)^2}}} \\ & +\frac{3b(5b-12ac^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^4}\end{aligned}$$

output
```
c*(-a*c^2+5*b)*(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a^3/d^4-1/8*(-12*a*c^2+7*b)*(
d*x+c)^2*(a+b/(d*x+c)^2)^(1/2)/a^3/d^4-c*(d*x+c)^3*(a+b/(d*x+c)^2)^(1/2)/a
^2/d^4+1/4*(d*x+c)^4*(a+b/(d*x+c)^2)^(1/2)/a^2/d^4-b*(b-3*a*c^2-c*(-a*c^2+
3*b)/(d*x+c))/a^3/d^4/(a+b/(d*x+c)^2)^(1/2)+3/8*b*(-12*a*c^2+5*b)*arctanh(
(a+b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(7/2)/d^4
```

Mathematica [A] (verified)

Time = 10.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{\frac{\sqrt{a}\left(b^2(49c-15dx)+ab\left(47c^3+85c^2dx+17cd^2x^2-5d^3x^3\right)-2a^2\left(c^5+c^4dx-cd^4x^4-d^5x^5\right)\right)}{(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}}+3b(5b-12ac^2}{8a^{7/2}d^4}$$

input
```
Integrate[x^3/(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
((Sqrt[a]*(b^2*(49*c - 15*d*x) + a*b*(47*c^3 + 85*c^2*d*x + 17*c*d^2*x^2 -
 5*d^3*x^3) - 2*a^2*(c^5 + c^4*d*x - c*d^4*x^4 - d^5*x^5)))/((c + d*x)*Sqr
t[(b + a*(c + d*x)^2)/(c + d*x)^2]) + 3*b*(5*b - 12*a*c^2)*Log[(c + d*x)*(
a + Sqrt[a]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])/(8*a^(7/2)*d^4)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.842, Rules used = {896, 25, 1774, 1803, 25, 532, 25, 2338, 2338, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^3x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{d^3x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{1774}$$

$$-\frac{\int \frac{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{1803}$$

$$\frac{\int -\frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{(c+dx)^5\left(1-\frac{c}{c+dx}\right)^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{\int -\frac{(c+dx)^5\left(-\frac{3c}{c+dx}+\frac{\left(\frac{3b}{a}-c^2\right)c}{(c+dx)^3}-\frac{\frac{b}{a}-3c^2}{(c+dx)^2}+\frac{b\left(b-3ac^2\right)}{a^2(c+dx)^4}+1\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{a}-\frac{b\left(-\frac{c\left(3b-ac^2\right)}{c+dx}-3ac^2+b\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{(c+dx)^5\left(-\frac{3c}{c+dx}+\frac{\left(\frac{3b}{a}-c^2\right)c}{(c+dx)^3}-\frac{\frac{b}{a}-3c^2}{(c+dx)^2}+\frac{b\left(b-3ac^2\right)}{a^2(c+dx)^4}+1\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{a}-\frac{b\left(-\frac{c\left(3b-ac^2\right)}{c+dx}-3ac^2+b\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^4}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{-\frac{\int \frac{(c+dx)^4\left(12ac-\frac{4\left(3b-ac^2\right)c}{(c+dx)^2}+\frac{7b-12ac^2}{c+dx}-\frac{4b\left(b-3ac^2\right)}{a(c+dx)^3}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{4a}-\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{a}-\frac{b\left(-\frac{c\left(3b-ac^2\right)}{c+dx}-3ac^2+b\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^4}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{-\frac{-\frac{-\frac{\int -\frac{3(c+dx)^3\left(a\left(7b-12ac^2\right)-\frac{4ac\left(5b-ac^2\right)}{c+dx}-\frac{4b\left(b-3ac^2\right)}{(c+dx)^2}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{3a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}-\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{a}-\frac{b\left(-\frac{c\left(3b-ac^2\right)}{c+dx}-3ac^2+b\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^4}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{\frac{\int \frac{(c+dx)^3\left(a\left(7b-12ac^2\right)-\frac{4ac\left(5b-ac^2\right)}{c+dx}-\frac{4b\left(b-3ac^2\right)}{(c+dx)^2}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}-\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{a}-\frac{b\left(-\frac{c\left(3b-ac^2\right)}{c+dx}-3ac^2+b\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^4}$$

$\Big\downarrow$ 2338

$$-\frac{-\frac{-\frac{\frac{-\frac{\int \frac{a(c+dx)^2\left(\frac{3b\left(5b-12ac^2\right)}{c+dx}+8ac\left(5b-ac^2\right)\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{2a}-\frac{1}{2}\left(7b-12ac^2\right)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}-\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{a}-b\left(\ldots\right.}{d^4}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{\frac{-\frac{1}{2}\int \frac{(c+dx)^2\left(\frac{3b\left(5b-12ac^2\right)}{c+dx}+8ac\left(5b-ac^2\right)\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{1}{2}\left(7b-12ac^2\right)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}-\frac{(c+dx)^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}}{a}-b\left(\ldots\right.}{d^4}$$

$\Big\downarrow$ 534

$$-\frac{-\frac{\frac{\frac{1}{2}\left(8c\left(5b-ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}-3b\left(5b-12ac^2\right)\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}\right)-\frac{1}{2}\left(7b-12ac^2\right)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}-\frac{(c+dx)^{\ldots}}{\ldots}}{a}}{d^4}$$

$\Big\downarrow$ 243

$$-\frac{-\frac{-\frac{\frac{1}{2}\left(8c\left(5b-ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{3}{2}b\left(5b-12ac^2\right)\int\frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}\right)-\frac{1}{2}\left(7b-12ac^2\right)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{4a}-\frac{(c+\cdots}{\cdots}}{a}}{d^4}$$

$\downarrow$ 73

$$-\frac{-\frac{-\frac{\frac{1}{2}\left(8c\left(5b-ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}-3\left(5b-12ac^2\right)\int\frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{1}{2}\left(7b-12ac^2\right)(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-4c(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}\cdots}{4a}}{a}}{d^4}$$

$\downarrow$ 221

$$\frac{-\frac{b\left(-\frac{c\left(3b-ac^2\right)}{c+dx}-3ac^2+b\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{-\frac{\frac{1}{2}\left(\frac{3b\left(5b-12ac^2\right)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}+8c\left(5b-ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}\right)-\frac{1}{2}\left(7b-12ac^2\right)(c+dx)^2\sqrt{a+\frac{b}{(c+d\cdots}}}{a}\cdots}{4a}}{a}}{d^4}$$

input

```
Int[x^3/(a + b/(c + d*x)^2)^(3/2),x]
```

output

```
(-((b*(b - 3*a*c^2 - (c*(3*b - a*c^2))/(c + d*x)))/(a^3*Sqrt[a + b/(c + d*
x)^2])) - (-1/4*((c + d*x)^4*Sqrt[a + b/(c + d*x)^2])/a - (-4*c*(c + d*x)^
3*Sqrt[a + b/(c + d*x)^2] + (-1/2*((7*b - 12*a*c^2)*(c + d*x)^2*Sqrt[a + b
/(c + d*x)^2]) + (8*c*(5*b - a*c^2)*(c + d*x)*Sqrt[a + b/(c + d*x)^2] + (3
*b*(5*b - 12*a*c^2)*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]])/Sqrt[a])/2)/
a)/(4*a))/a)/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)\left(-2x^{5}d^{5}a^{2}\sqrt{a\,d^{2}}-2\sqrt{a\,d^{2}}\,a^{2}c\,d^{4}x^{4}+2\sqrt{a\,d^{2}}\,a^{2}c^{4}dx+5x^{3}d^{3}ab\sqrt{a\,d^{2}}+2\sqrt{a\,d^{2}}\,a^{2}c^{5}-17\sqrt{a\,d^{2}}\,abc\,d^{2}x^{2}-\right.}{}$
risch	$-\frac{\left(-2a\,x^{3}d^{3}+2a\,d^{2}x^{2}c-2adx\,c^{2}+2c^{3}a+7bdx-33bc\right)\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)}{8d^{4}a^{3}\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}+\frac{\left(\frac{3bx\,c^{2}}{a^{2}d^{3}\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}}-\frac{b^{2}x}{a^{3}d^{3}\sqrt{a\,d^{2}x^{2}+2}}\right.}{}$

input
```
int(x^3/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/d^4*(-2*x^5*d^5*a^2*(a*d^2)^(1/2)-2*(a*
d^2)^(1/2)*a^2*c*d^4*x^4+2*(a*d^2)^(1/2)*a^2*c^4*d*x+5*x^3*d^3*a*b*(a*d^2)
^(1/2)+2*(a*d^2)^(1/2)*a^2*c^5-17*(a*d^2)^(1/2)*a*b*c*d^2*x^2+36*(a*d^2*x^
2+2*a*c*d*x+a*c^2+b)^(1/2)*ln((a*d^2*x+a*c*d+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)
^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a*b*c^2*d-85*(a*d^2)^(1/2)*a*b*c^2*d*
x-47*(a*d^2)^(1/2)*a*b*c^3-15*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*ln((a*d^
2*x+a*c*d+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2)
)*b^2*d+15*(a*d^2)^(1/2)*b^2*d*x-49*(a*d^2)^(1/2)*b^2*c)/((a*d^2*x^2+2*a*c
*d*x+a*c^2+b)/(d*x+c)^2)^(3/2)/(d*x+c)^3/a^3/(a*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 791, normalized size of antiderivative = 3.28

$$\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(12*a^2*b*c^4 + 7*a*b^2*c^2 + (12*a^2*b*c^2 - 5*a*b^2)*d^2*x^2 -
5*b^3 + 2*(12*a^2*b*c^3 - 5*a*b^2*c)*d*x)*sqrt(a)*log(-2*a*d^2*x^2 - 4*a*c
*d*x - 2*a*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 2*(2*a^3*d^6*x^6 + 4
*a^3*c*d^5*x^5 + 12*a^2*b*c*d^3*x^3 - 2*a^3*c^6 + (2*a^3*c^2 - 5*a^2*b)*d^
4*x^4 + 47*a^2*b*c^4 + 49*a*b^2*c^2 - (2*a^3*c^4 - 102*a^2*b*c^2 + 15*a*b^
2)*d^2*x^2 - 2*(2*a^3*c^5 - 66*a^2*b*c^3 - 17*a*b^2*c)*d*x)*sqrt((a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^5*d^6*x^2 + 2*a^
5*c*d^5*x + (a^5*c^2 + a^4*b)*d^4), 1/8*(3*(12*a^2*b*c^4 + 7*a*b^2*c^2 + (
12*a^2*b*c^2 - 5*a*b^2)*d^2*x^2 - 5*b^3 + 2*(12*a^2*b*c^3 - 5*a*b^2*c)*d*x
)*sqrt(-a)*arctan((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)) + (2*a^3*d^6*x^6 + 4*a^3*c*d^5*x^5 + 12*a^2*b*c*d^3*x^3 - 2*a^3*
c^6 + (2*a^3*c^2 - 5*a^2*b)*d^4*x^4 + 47*a^2*b*c^4 + 49*a*b^2*c^2 - (2*a^3
*c^4 - 102*a^2*b*c^2 + 15*a*b^2)*d^2*x^2 - 2*(2*a^3*c^5 - 66*a^2*b*c^3 - 1
7*a*b^2*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*
x + c^2)))/(a^5*d^6*x^2 + 2*a^5*c*d^5*x + (a^5*c^2 + a^4*b)*d^4)]
```

Sympy [F]

$$\int \frac{x^3}{\left(a + \frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(x**3/((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a + b/(d*x + c)^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{\left(\left(\left(2\,x\left(\frac{dx}{a\mathrm{sgn}(dx+c)}+\frac{c}{a\mathrm{sgn}(dx+c)}\right)-\frac{5\,b}{a^2d\mathrm{sgn}(dx+c)}\right)x+\frac{17\,bc}{a^2d^2\mathrm{sgn}(dx+c)}\right)x-\frac{2\,a^5bc^4d^6\mathrm{sgn}(dx-}{}\right.}{8\,\sqrt{ad^2x^2}}$$
$$+\frac{3\,(12\,abc^2-5\,b^2)\log\left(\left|-acd-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)\sqrt{a}|d|\right|\right)}{8\,a^{\frac{7}{2}}d^3|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate(x^3/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*((((2*x*(d*x/(a*sgn(d*x + c)) + c/(a*sgn(d*x + c))) - 5*b/(a^2*d*sgn(d
*x + c)))*x + 17*b*c/(a^2*d^2*sgn(d*x + c)))*x - (2*a^5*b*c^4*d^6*sgn(d*x
+ c) - 85*a^4*b^2*c^2*d^6*sgn(d*x + c) + 15*a^3*b^3*d^6*sgn(d*x + c))/(a^6
*b*d^9))*x - (2*a^5*b*c^5*d^5*sgn(d*x + c) - 47*a^4*b^2*c^3*d^5*sgn(d*x +
c) - 49*a^3*b^3*c*d^5*sgn(d*x + c))/(a^6*b*d^9))/sqrt(a*d^2*x^2 + 2*a*c*d*
x + a*c^2 + b) + 3/8*(12*a*b*c^2 - 5*b^2)*log(abs(-a*c*d - (sqrt(a*d^2)*x
- sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*sqrt(a)*abs(d)))/(a^(7/2)*d^3*a
bs(d)*sgn(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x)^2)^(3/2),x)
```

output
```
int(x^3/(a + b/(c + d*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
int(x^3/(a+b/(d*x+c)^2)^(3/2),x)
```

3.88 $\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = -\frac{(5b-3ac^2)\,(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{3a^3d^3} - \frac{c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{a^2d^3} + \frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a^2d^3} - \frac{b(b-ac(c-2(c+dx)))}{a^3d^3(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}} + \frac{3bc\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{a^{5/2}d^3}$$

output

```
-1/3*(-3*a*c^2+5*b)*(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a^3/d^3-c*(d*x+c)^2*(a+b
/(d*x+c)^2)^(1/2)/a^2/d^3+1/3*(d*x+c)^3*(a+b/(d*x+c)^2)^(1/2)/a^2/d^3-b*(b
-a*c*(-2*d*x-c))/a^3/d^3/(d*x+c)/(a+b/(d*x+c)^2)^(1/2)+3*b*c*arctanh((a+b/
(d*x+c)^2)^(1/2)/a^(1/2))/a^(5/2)/d^3
```

Mathematica [A] (verified)

Time = 10.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{\frac{-8b^2+a^2(c+dx)^2(c^2-cdx+d^2x^2)-ab(7c^2+17cdx+4d^2x^2)}{(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}}+9\sqrt{a}bc\log\left((c+dx)\left(a+\sqrt{a}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{3a^3d^3}$$

input
```
Integrate[x^2/(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
((-8*b^2 + a^2*(c + d*x)^2*(c^2 - c*d*x + d^2*x^2) - a*b*(7*c^2 + 17*c*d*x
 + 4*d^2*x^2))/((c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2]) + 9*Sqrt[
a]*b*c*Log[(c + d*x)*(a + Sqrt[a]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])
/(3*a^3*d^3)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {896, 1774, 1803, 532, 25, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^2x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1774}$$

$$\frac{\int \frac{(c+dx)^2\left(\frac{c}{c+dx}-1\right)^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1803}$$

$$-\frac{\int \frac{(c+dx)^4\left(1-\frac{c}{c+dx}\right)^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{\int -\frac{(c+dx)^4\left(-\frac{2c}{c+dx}+\frac{2bc}{a(c+dx)^3}-\frac{\frac{b}{a}-c^2}{(c+dx)^2}+1\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{a}}{d^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{(c+dx)^4\left(-\frac{2c}{c+dx}+\frac{2bc}{a(c+dx)^3}-\frac{\frac{b}{a}-c^2}{(c+dx)^2}+1\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{a}+\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^3}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\frac{-\frac{\int \frac{(c+dx)^3\left(6ac-\frac{6bc}{(c+dx)^2}+\frac{5b-3ac^2}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{a}+\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^3}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\frac{-\frac{-\frac{\int -\frac{2a(c+dx)^2\left(-3ac^2-\frac{9bc}{c+dx}+5b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{2a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{a}+\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{-\frac{\int \frac{(c+dx)^2\left(-3ac^2-\frac{9bc}{c+dx}+5b\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{a}+\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^3}$$

$$\Big\downarrow \text{534}$$

$$-\frac{-\frac{-\frac{-9bc\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-\frac{\left(5b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{a}+\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^3}$$

$$\Big\downarrow \; 243$$

$$-\frac{-\frac{-\frac{-\frac{9}{2}bc\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-\frac{\left(5b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{a}+\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^3}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{-\frac{-9c\int \frac{1}{\frac{a+\frac{b}{(c+dx)^2}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}-\frac{\left(5b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{a}+\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{b\left(\frac{b-ac^2}{c+dx}+2ac\right)}{a^3\sqrt{a+\frac{b}{(c+dx)^2}}}+\frac{-\frac{\frac{9bc\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\left(5b-3ac^2\right)(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a}-3c(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}-\frac{(c+dx)^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3a}}{a}}{d^3}$$

input

```
Int[x^2/(a + b/(c + d*x)^2)^(3/2),x]
```

output

```
-(((b*(2*a*c + (b - a*c^2)/(c + d*x)))/(a^3*Sqrt[a + b/(c + d*x)^2]) + (-1
/3*((c + d*x)^3*Sqrt[a + b/(c + d*x)^2])/a - (-(((5*b - 3*a*c^2)*(c + d*x)
*Sqrt[a + b/(c + d*x)^2])/a) - 3*c*(c + d*x)^2*Sqrt[a + b/(c + d*x)^2] + (
9*b*c*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]])/Sqrt[a])/(3*a))/a)/d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.51

method	result
default	$\frac{(a\,d^2x^2+2adxc+a\,c^2+b)\left(x^4a^2d^4\sqrt{a\,d^2}+\sqrt{a\,d^2}\,a^2c\,d^3x^3+\sqrt{a\,d^2}\,a^2c^3dx+\sqrt{a\,d^2}\,a^2c^4-4x^2a\,d^2b\sqrt{a\,d^2}+9\ln\left(\frac{a\,d^2x+acd+\sqrt{a\,d^2x^2+2}}{\sqrt{a\,d}}\right.\right.}{3d^3\left(\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}\right)^{\frac{3}{2}}(dx+c)^3a^3\sqrt{}}$
risch	$\frac{(a\,d^2x^2-adxc+a\,c^2-5b)(a\,d^2x^2+2adxc+a\,c^2+b)}{3a^3d^3\sqrt{\frac{a\,d^2x^2+2adxc+a\,c^2+b}{(dx+c)^2}}\,(dx+c)}+\frac{\left(-\frac{b\,c^2}{a^2d^3\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}-\frac{b^2}{a^3d^3\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}-\frac{2}{a^2d^2\sqrt{a\,d^2x^2}}\right.}{\sqrt{\frac{a\,d^2x^2+}{(}}}$

input
```
int(x^2/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/d^3*(x^4*a^2*d^4*(a*d^2)^(1/2)+(a*d^2)^(
1/2)*a^2*c*d^3*x^3+(a*d^2)^(1/2)*a^2*c^3*d*x+(a*d^2)^(1/2)*a^2*c^4-4*x^2*a
*d^2*b*(a*d^2)^(1/2)+9*ln((a*d^2*x+a*c*d+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/
2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*a*b*c
*d-17*(a*d^2)^(1/2)*a*b*c*d*x-7*(a*d^2)^(1/2)*a*b*c^2-8*(a*d^2)^(1/2)*b^2)
/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(3/2)/(d*x+c)^3/a^3/(a*d^2)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.47

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \left[\frac{9\left(abcd^2x^2+2\,abc^2dx+abc^3+b^2c\right)\sqrt{a}\log\left(-2\,ad^2x^2-4\,acdx-2\,ac^2-2\,(d^2x\right.}{}\right.$$

$$-\frac{9\left(abcd^2x^2+2\,abc^2dx+abc^3+b^2c\right)\sqrt{-a}\arctan\left(\frac{(d^2x^2+2\,cdx+c^2)\sqrt{-a}\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}}{ad^2x^2+2\,acdx+ac^2+b}\right)-\left(a^2d^5x^5+2\,a^2c\right.}{3\left(a^4d^5x^2+\right.}$$

input
```
integrate(x^2/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(9*(a*b*c*d^2*x^2 + 2*a*b*c^2*d*x + a*b*c^3 + b^2*c)*sqrt(a)*log(-2*a
*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt(
(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 2*(a
^2*d^5*x^5 + 2*a^2*c*d^4*x^4 + a^2*c^5 + (a^2*c^2 - 4*a*b)*d^3*x^3 - 7*a*b
*c^3 + (a^2*c^3 - 21*a*b*c)*d^2*x^2 - 8*b^2*c + 2*(a^2*c^4 - 12*a*b*c^2 -
4*b^2)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
c^2)))/(a^4*d^5*x^2 + 2*a^4*c*d^4*x + (a^4*c^2 + a^3*b)*d^3), -1/3*(9*(a*b
*c*d^2*x^2 + 2*a*b*c^2*d*x + a*b*c^3 + b^2*c)*sqrt(-a)*arctan((d^2*x^2 + 2
*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 +
 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)) - (a^2*d^5*x^5 + 2*a
^2*c*d^4*x^4 + a^2*c^5 + (a^2*c^2 - 4*a*b)*d^3*x^3 - 7*a*b*c^3 + (a^2*c^3
- 21*a*b*c)*d^2*x^2 - 8*b^2*c + 2*(a^2*c^4 - 12*a*b*c^2 - 4*b^2)*d*x)*sqrt
((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^4*d^5*
x^2 + 2*a^4*c*d^4*x + (a^4*c^2 + a^3*b)*d^3)]
```

Sympy [F]

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(x**2/((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a + b/(d*x + c)^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{\left(\left(x\left(\frac{dx}{a\text{sgn}(dx+c)}+\frac{c}{a\text{sgn}(dx+c)}\right)-\frac{4\,b}{a^2d\text{sgn}(dx+c)}\right)x+\frac{a^4bc^3d^5\text{sgn}(dx+c)-17\,a^3b^2cd^5\text{sgn}(dx+c)}{a^5bd^7}\right)x}{3\sqrt{ad^2x^2+2\,acdx+ac^2+b}}$$
$$-\frac{3\,bc\log\left(\left|-acd-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)\sqrt{a}|d|\right|\right)}{a^{\frac{5}{2}}d^2|d|\text{sgn}\,(dx+c)}$$

input
```
integrate(x^2/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*(((x*(d*x/(a*sgn(d*x + c)) + c/(a*sgn(d*x + c))) - 4*b/(a^2*d*sgn(d*x
+ c)))*x + (a^4*b*c^3*d^5*sgn(d*x + c) - 17*a^3*b^2*c*d^5*sgn(d*x + c))/(a
^5*b*d^7))*x + (a^4*b*c^4*d^4*sgn(d*x + c) - 7*a^3*b^2*c^2*d^4*sgn(d*x + c
) - 8*a^2*b^3*d^4*sgn(d*x + c))/(a^5*b*d^7))/sqrt(a*d^2*x^2 + 2*a*c*d*x +
a*c^2 + b) - 3*b*c*log(abs(-a*c*d - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*
c*d*x + a*c^2 + b))*sqrt(a)*abs(d)))/(a^(5/2)*d^2*abs(d)*sgn(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{x^2}{{\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(x^2/(a + b/(c + d*x)^2)^(3/2),x)
```

output
```
int(x^2/(a + b/(c + d*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.76

$$\int \frac{x^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^4+\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^3dx+\sqrt{a\,d^2x^2}}{}$$

input
```
int(x^2/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*c**4 + sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b)*a**2*c**3*d*x + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*
x**2 + b)*a**2*c*d**3*x**3 + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*
*2*d**4*x**4 - 7*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b*c**2 - 17*
sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b*c*d*x - 4*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*a*b*d**2*x**2 - 8*sqrt(a*c**2 + 2*a*c*d*x + a*d*
*2*x**2 + b)*b**2 + 9*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*a*b*c**3 + 18*sqrt(a)*log((sqrt(a*
c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*a*
b*c**2*d*x + 9*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + s
qrt(a)*c + sqrt(a)*d*x)/sqrt(b))*a*b*c*d**2*x**2 + 9*sqrt(a)*log((sqrt(a*c
**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b**
2*c - 6*sqrt(a)*a*b*c**3 - 12*sqrt(a)*a*b*c**2*d*x - 6*sqrt(a)*a*b*c*d**2*
x**2 - 6*sqrt(a)*b**2*c)/(3*a**3*d**3*(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
b))
```

3.89 $\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 132

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}} \, dx = -\frac{c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a^2d^2} + \frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a^2d^2} + \frac{b\left(1-\frac{c}{c+dx}\right)}{a^2d^2\sqrt{a+\frac{b}{(c+dx)^2}}} - \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{2a^{5/2}d^2}$$

output
```
-c*(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a^2/d^2+1/2*(d*x+c)^2*(a+b/(d*x+c)^2)^(1/
2)/a^2/d^2+b*(1-c/(d*x+c))/a^2/d^2/(a+b/(d*x+c)^2)^(1/2)-3/2*b*arctanh((a+
b/(d*x+c)^2)^(1/2)/a^(1/2))/a^(5/2)/d^2
```

Mathematica [A] (verified)

Time = 10.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{\sqrt{a}(-b(c-3dx)-a(c-dx)(c+dx)^2)+3b\sqrt{b+a(c+dx)^2}\log\left(-\sqrt{a}(c+dx)\right.}{2a^{5/2}d^2(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}$$

input
```
Integrate[x/(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
(Sqrt[a]*(-(b*(c - 3*d*x)) - a*(c - d*x)*(c + d*x)^2) + 3*b*Sqrt[b + a*(c
+ d*x)^2]*Log[-(Sqrt[a]*(c + d*x)) + Sqrt[b + a*(c + d*x)^2]])/(2*a^(5/2)*
d^2*(c + d*x)*Sqrt[a + b/(c + d*x)^2])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {896, 25, 1774, 1803, 25, 532, 25, 2338, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{dx}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{dx}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 1774}$$

$$-\frac{\int \frac{(c+dx)\left(\frac{c}{c+dx}-1\right)}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \; 1803$$

$$\frac{\int -\frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{(c+dx)^3\left(1-\frac{c}{c+dx}\right)}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}}{d^2}$$

$$\Big\downarrow \; 532$$

$$\frac{\frac{\int -\frac{(c+dx)^3\left(-\frac{b}{a(c+dx)^2}-\frac{c}{c+dx}+1\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{a}+\frac{b\left(1-\frac{c}{c+dx}\right)}{a^2\sqrt{a+\frac{b}{(c+dx)^2}}}}{d^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{b\left(1-\frac{c}{c+dx}\right)}{a^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{\int \frac{(c+dx)^3\left(-\frac{b}{a(c+dx)^2}-\frac{c}{c+dx}+1\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{a}}{d^2}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{b\left(1-\frac{c}{c+dx}\right)}{a^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{-\frac{\int \frac{(c+dx)^2\left(\frac{3b}{c+dx}+2ac\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}}{2a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{a}}{d^2}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{b\left(1-\frac{c}{c+dx}\right)}{a^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{-\frac{3b\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{c+dx}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{a}}{d^2}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{b\left(1-\frac{c}{c+dx}\right)}{a^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{-\frac{\frac{3}{2}b\int \frac{c+dx}{\sqrt{a+\frac{b}{(c+dx)^2}}}d\frac{1}{(c+dx)^2}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{a}}{d^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{b\left(1-\frac{c}{c+dx}\right)}{a^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{-\frac{3\int \frac{1}{\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{b}-\frac{a}{b}}d\sqrt{a+\frac{b}{(c+dx)^2}}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{a}}{d^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{b\left(1-\frac{c}{c+dx}\right)}{a^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{-\frac{-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-2c(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2a}}{a}}{d^2}$$

input
```
Int[x/(a + b/(c + d*x)^2)^(3/2),x]
```

output
```
((b*(1 - c/(c + d*x)))/(a^2*Sqrt[a + b/(c + d*x)^2]) - (-1/2*((c + d*x)^2*
Sqrt[a + b/(c + d*x)^2])/a - (-2*c*(c + d*x)*Sqrt[a + b/(c + d*x)^2] - (3*
b*ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]])/Sqrt[a])/(2*a))/a)/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{\left(-dx+c\right)\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)}{2a^{2}d^{2}\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}}\,\left(dx+c\right)}+\frac{\left(\frac{bx}{a^{2}d\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}}-\frac{3b\ln\left(\frac{a\,d^{2}x+acd}{\sqrt{a\,d^{2}}}+\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\right)}{2a^{2}d\sqrt{a\,d^{2}}}\right)\sqrt{a\,d^{2}x^{2}+2a}}{\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}}\,\left(dx+c\right)}$
default	$-\frac{\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)\left(-x^{3}d^{3}a\sqrt{a\,d^{2}}-\sqrt{a\,d^{2}}\,ac\,d^{2}x^{2}+\sqrt{a\,d^{2}}\,a\,c^{2}dx+c^{3}a\sqrt{a\,d^{2}}+3\ln\left(\frac{a\,d^{2}x+acd+\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\,\sqrt{a\,d}}{\sqrt{a\,d^{2}}}\right.\right.}{2d^{2}\left(\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}\right)^{\frac{3}{2}}\left(dx+c\right)^{3}a^{2}\sqrt{a\,d^{2}}}$

input
```
int(x/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/a^2/d^2/((a*d^2*x^2+2*a*c*d*x+
a*c^2+b)/(d*x+c)^2)^(1/2)/(d*x+c)+(b/a^2/d*x/(a*d^2*x^2+2*a*c*d*x+a*c^2+b)
^(1/2)-3/2*b/a^2/d*ln((a*d^2*x+a*c*d)/(a*d^2)^(1/2)+(a*d^2*x^2+2*a*c*d*x+a
*c^2+b)^(1/2))/(a*d^2)^(1/2))/((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(1
/2)/(d*x+c)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(116) = 232.

Time = 0.58 (sec) , antiderivative size = 545, normalized size of antiderivative = 4.13

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \left[\frac{3\left(abd^2x^2+2\,abcdx+abc^2+b^2\right)\sqrt{a}\log\left(-2\,ad^2x^2-4\,acdx-2\,ac^2+2\left(d^2x^2+\right.\right.}{}\right.$$

input
```
integrate(x/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 + b^2)*sqrt(a)*log(-2*a*d^2*x
^2 - 4*a*c*d*x - 2*a*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqrt((a*d^2
*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + 2*(a^2*d^4
*x^4 + 2*a^2*c*d^3*x^3 - a^2*c^4 + 3*a*b*d^2*x^2 - a*b*c^2 - 2*(a^2*c^3 -
a*b*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
c^2)))/(a^4*d^4*x^2 + 2*a^4*c*d^3*x + (a^4*c^2 + a^3*b)*d^2), 1/2*(3*(a*b*
d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 + b^2)*sqrt(-a)*arctan((d^2*x^2 + 2*c*d*x
+ c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*
x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)) + (a^2*d^4*x^4 + 2*a^2*c*d^
3*x^3 - a^2*c^4 + 3*a*b*d^2*x^2 - a*b*c^2 - 2*(a^2*c^3 - a*b*c)*d*x)*sqrt(
(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(a^4*d^4*x
^2 + 2*a^4*c*d^3*x + (a^4*c^2 + a^3*b)*d^2)]
```

Sympy [F]

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{x}{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(x/((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*
x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{x}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a + b/(d*x + c)^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.69

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{\left(x\left(\frac{dx}{a\mathrm{sgn}(dx+c)}+\frac{c}{a\mathrm{sgn}(dx+c)}\right)-\frac{a^3bc^2d^4\mathrm{sgn}(dx+c)-3\,a^2b^2d^4\mathrm{sgn}(dx+c)}{a^4bd^5}\right)x-\frac{a^3bc^3d^3\mathrm{sgn}(dx+c)+a}{a^4bd^5}}{2\sqrt{ad^2x^2+2\,acdx+ac^2+b}} + \frac{3\,b\log\left(\left|-acd-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)\sqrt{a}|d|\right|\right)}{2\,a^{\frac{5}{2}}d|d|\mathrm{sgn}\left(dx+c\right)}$$

input
```
integrate(x/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*((x*(d*x/(a*sgn(d*x + c)) + c/(a*sgn(d*x + c))) - (a^3*b*c^2*d^4*sgn(d
*x + c) - 3*a^2*b^2*d^4*sgn(d*x + c))/(a^4*b*d^5))*x - (a^3*b*c^3*d^3*sgn(
d*x + c) + a^2*b^2*c*d^3*sgn(d*x + c))/(a^4*b*d^5))/sqrt(a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b) + 3/2*b*log(abs(-a*c*d - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2
 + 2*a*c*d*x + a*c^2 + b))*sqrt(a)*abs(d)))/(a^(5/2)*d*abs(d)*sgn(d*x + c)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{x}{\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(x/(a + b/(c + d*x)^2)^(3/2),x)
```

output
```
int(x/(a + b/(c + d*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.35

$$\int \frac{x}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{-4\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^3-4\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^2c^2dx+4\sqrt{a}}{}$$

input
```
int(x/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
( - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*c**3 - 4*sqrt(a*c**2
 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*c**2*d*x + 4*sqrt(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)*a**2*c*d**2*x**2 + 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)*a**2*d**3*x**3 - 4*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b
*c + 12*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b*d*x - 12*sqrt(a)*lo
g((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/s
qrt(b))*a*b*c**2 - 24*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*a*b*c*d*x - 12*sqrt(a)*log((sqrt(a
*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*a
*b*d**2*x**2 - 12*sqrt(a)*log((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
+ sqrt(a)*c + sqrt(a)*d*x)/sqrt(b))*b**2 + 9*sqrt(a)*a*b*c**2 + 18*sqrt(a)
*a*b*c*d*x + 9*sqrt(a)*a*b*d**2*x**2 + 9*sqrt(a)*b**2)/(8*a**3*d**2*(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b))
```

3.90 $\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}} dx = -\frac{c+dx}{ad\sqrt{a+\frac{b}{(c+dx)^2}}} + \frac{2(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}{a^2d}$$

output
```
-(d*x+c)/a/d/(a+b/(d*x+c)^2)^(1/2)+2*(d*x+c)*(a+b/(d*x+c)^2)^(1/2)/a^2/d
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}} dx = \frac{2b+a(c+dx)^2}{a^2d(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}$$

input
```
Integrate[(a + b/(c + d*x)^2)^(-3/2),x]
```

output
```
(2*b + a*(c + d*x)^2)/(a^2*d*(c + d*x)*Sqrt[a + b/(c + d*x)^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int \frac{(c+dx)^2}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{245}$$

$$-\frac{-\frac{2b\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}}{a}-\frac{c+dx}{a\sqrt{a+\frac{b}{(c+dx)^2}}}}{d}$$

$$\Big\downarrow \text{208}$$

$$-\frac{-\frac{2b}{a^2(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c+dx}{a\sqrt{a+\frac{b}{(c+dx)^2}}}}{d}$$

input
```
Int[(a + b/(c + d*x)^2)^(-3/2),x]
```

output
```
-(((-2*b)/(a^2*(c + d*x)*Sqrt[a + b/(c + d*x)^2]) - (c + d*x)/(a*Sqrt[a +
b/(c + d*x)^2]))/d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

method	result	size
orering	$\frac{\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+2b\right)\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)}{a^{2}d\left(dx+c\right)^{3}\left(a+\frac{b}{\left(dx+c\right)^{2}}\right)^{\frac{3}{2}}}$	72
gosper	$\frac{\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+2b\right)}{a^{2}d\left(dx+c\right)^{3}\left(\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}\right)^{\frac{3}{2}}}$	90
default	$\frac{\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+2b\right)}{a^{2}d\left(dx+c\right)^{3}\left(\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}\right)^{\frac{3}{2}}}$	90
trager	$\frac{\left(dx+c\right)\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+2b\right)\sqrt{-\frac{-a\,d^{2}x^{2}-2adxc-a\,c^{2}-b}{d^{2}x^{2}+2cdx+c^{2}}}}{a^{2}d\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)}$	106
risch	$\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{d\,a^{2}\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}}\,\left(dx+c\right)}+\frac{b}{a^{2}d\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{\left(dx+c\right)^{2}}}\,\left(dx+c\right)}$	114

input
```
int(1/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*d^2*x^2+2*a*c*d*x+a*c^2+2*b)/a^2/d*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c
)^3/(a+b/(d*x+c)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(53) = 106$.

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{(ad^3x^3+3\,acd^2x^2+ac^3+(3\,ac^2+2\,b)dx+2\,bc)\sqrt{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}}{a^3d^3x^2+2\,a^3cd^2x+(a^3c^2+a^2b)d}$$

input
```
integrate(1/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
(a*d^3*x^3 + 3*a*c*d^2*x^2 + a*c^3 + (3*a*c^2 + 2*b)*d*x + 2*b*c)*sqrt((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a^3*d^3*x^2 +
 2*a^3*c*d^2*x + (a^3*c^2 + a^2*b)*d)
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral((a + b/(c + d*x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{x\left(\frac{dx}{a\mathrm{sgn}(dx+c)}+\frac{2c}{a\mathrm{sgn}(dx+c)}\right)+\frac{a^2bc^2d^2\mathrm{sgn}(dx+c)+2ab^2d^2\mathrm{sgn}(dx+c)}{a^3bd^3}}{\sqrt{ad^2x^2+2acdx+ac^2+b}}$$

input
```
integrate(1/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
(x*(d*x/(a*sgn(d*x + c)) + 2*c/(a*sgn(d*x + c))) + (a^2*b*c^2*d^2*sgn(d*x
+ c) + 2*a*b^2*d^2*sgn(d*x + c))/(a^3*b*d^3))/sqrt(a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b)
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{a^2\,(c+d\,x)^4+2\,b^2+3\,a\,b\,(c+d\,x)^2}{a^2\,d\,\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}\,(c+d\,x)^3}$$

input
```
int(1/(a + b/(c + d*x)^2)^(3/2),x)
```

output
```
(a^2*(c + d*x)^4 + 2*b^2 + 3*a*b*(c + d*x)^2)/(a^2*d*(a + b/(c + d*x)^2)^(
3/2)*(c + d*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,(a\,d^2x^2+2acdx+a\,c^2+2b)}{a^2d\,(a\,d^2x^2+2acdx+a\,c^2+b)}$$

input
```
int(1/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*(a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + 2*b))/(a**2*d*(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b))
```

3.91 $\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=-\frac{b\left(1+\frac{c}{c+dx}\right)}{a\left(b+ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^3\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\left(b+ac^2\right)^{3/2}}$$

output
```
-b*(1+c/(d*x+c))/a/(a*c^2+b)/(a+b/(d*x+c)^2)^(1/2)+arctanh((a+b/(d*x+c)^2)
^(1/2)/a^(1/2))/a^(3/2)-c^3*arctanh((a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(
d*x+c)^2)^(1/2))/(a*c^2+b)^(3/2)
```

Mathematica [A] (verified)

Time = 10.76 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.52

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx =$$

$$-\frac{2a^{3/2}c^3\sqrt{b+a(c+dx)^2}\arctan\left(\frac{-\sqrt{a}dx+\sqrt{b+a(c+dx)^2}}{\sqrt{-b-ac^2}}\right)-\sqrt{-b-ac^2}\left(\sqrt{a}b(2c+dx)+(b+ac^2)\sqrt{b+a(c\cdots\right.}{a^{3/2}(-b-ac^2)^{3/2}(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}}$$

input
```
Integrate[1/(x*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
-((2*a^(3/2)*c^3*Sqrt[b + a*(c + d*x)^2]*ArcTan[(-(Sqrt[a]*d*x) + Sqrt[b +
 a*(c + d*x)^2])/Sqrt[-b - a*c^2]] - Sqrt[-b - a*c^2]*(Sqrt[a]*b*(2*c + d*
x) + (b + a*c^2)*Sqrt[b + a*(c + d*x)^2]*Log[-(Sqrt[a]*(c + d*x)) + Sqrt[b
 + a*(c + d*x)^2]]))/(a^(3/2)*(-b - a*c^2)^(3/2)*(c + d*x)*Sqrt[a + b/(c +
 d*x)^2]))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {896, 25, 1774, 1803, 25, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 896$$

$$\int \frac{1}{dx\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{1}{dx\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$\Big\downarrow$ 1774

$$-\int \frac{1}{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$

$\Big\downarrow$ 1803

$$\int -\frac{c+dx}{\left(1-\frac{c}{c+dx}\right)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d\frac{1}{c+dx}$$

$\Big\downarrow$ 25

$$-\int \frac{c+dx}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}$$

$\Big\downarrow$ 617

$$-\int \left(\frac{c+dx}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}-\frac{c}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)}\right)d\frac{1}{c+dx}$$

$\Big\downarrow$ 2009

$$\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{(c+dx)^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^3\text{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{(ac^2+b)^{3/2}}+\frac{c\left(ac-\frac{b}{c+dx}\right)}{a\,(ac^2+b)\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{1}{a\sqrt{a+\frac{b}{(c+dx)^2}}}$$

input
```
Int[1/(x*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
-(1/(a*Sqrt[a + b/(c + d*x)^2])) + (c*(a*c - b/(c + d*x)))/(a*(b + a*c^2)*
Sqrt[a + b/(c + d*x)^2]) + ArcTanh[Sqrt[a + b/(c + d*x)^2]/Sqrt[a]]/a^(3/2
) - (c^3*ArcTanh[(a*c + b/(c + d*x))/(Sqrt[b + a*c^2]*Sqrt[a + b/(c + d*x)
^2])])/(b + a*c^2)^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.73

method	result
default	$\frac{(a\,d^2x^2+2adxc+a\,c^2+b)\left(-\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}\,\ln\left(\frac{2a\,c^2+2b+2adxc+2\sqrt{a\,c^2+b}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}{x}\right)\sqrt{a\,d^2}\,a^2c^5+\sqrt{a\,d^2x^2\cdots}\right.}{}$

input
```
int(1/x/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*d^2*x^2+2*a*c*d*x+a*c^2+b)*(-(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*ln(2*(
a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a
*d^2)^(1/2)*a^2*c^5+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*c^2+b)^(3/2)*ln
((a*d^2*x+a*c*d+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2))/(a*d^2)
^(1/2))*a*c^2*d-(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a
*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*d^2)^(1/2)*a*b*
c^3+(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*c^2+b)^(3/2)*ln((a*d^2*x+a*c*d+
(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*b*d-(a*c
^2+b)^(3/2)*(a*d^2)^(1/2)*b*d*x-2*(a*c^2+b)^(3/2)*(a*d^2)^(1/2)*b*c)/((a*d
^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)^(3/2)/(d*x+c)^3/(a*c^2+b)^(5/2)/a/(a*
d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(113) = 226$.

Time = 0.70 (sec) , antiderivative size = 2298, normalized size of antiderivative = 18.09

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a^3*c^6 + 3*a^2*b*c^4 + 3*a*b^2*c^2 + (a^3*c^4 + 2*a^2*b*c^2 + a*b^
2)*d^2*x^2 + b^3 + 2*(a^3*c^5 + 2*a^2*b*c^3 + a*b^2*c)*d*x)*sqrt(a)*log(-2
*a*d^2*x^2 - 4*a*c*d*x - 2*a*c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(a)*sqr
t((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) - b) + (a
^3*c^3*d^2*x^2 + 2*a^3*c^4*d*x + a^3*c^5 + a^2*b*c^3)*sqrt(a*c^2 + b)*log(
-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*
d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2
 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))
/x^2) - 2*(2*a^2*b*c^4 + 2*a*b^2*c^2 + (a^2*b*c^2 + a*b^2)*d^2*x^2 + 3*(a^
2*b*c^3 + a*b^2*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2
+ 2*c*d*x + c^2)))/(a^5*c^6 + 3*a^4*b*c^4 + 3*a^3*b^2*c^2 + a^2*b^3 + (a^5
*c^4 + 2*a^4*b*c^2 + a^3*b^2)*d^2*x^2 + 2*(a^5*c^5 + 2*a^4*b*c^3 + a^3*b^2
*c)*d*x), -1/2*(2*(a^3*c^6 + 3*a^2*b*c^4 + 3*a*b^2*c^2 + (a^3*c^4 + 2*a^2*
b*c^2 + a*b^2)*d^2*x^2 + b^3 + 2*(a^3*c^5 + 2*a^2*b*c^3 + a*b^2*c)*d*x)*sq
rt(-a)*arctan((d^2*x^2 + 2*c*d*x + c^2)*sqrt(-a)*sqrt((a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b)) - (a^3*c^3*d^2*x^2 + 2*a^3*c^4*d*x + a^3*c^5 + a^2*b*c^3)*sqrt(a*c^2
 + b)*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3
 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*
sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c...
```

Sympy [F]

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{x\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(1/(x*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c)^2)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x)^2)^(3/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x)^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 4963, normalized size of antiderivative = 39.08

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*sqrt(a*c**
2 + b)*atan((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt
(a)*d*x)/sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a**3*c**6 - 4*
sqrt(a)*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*sqrt(a*c**2 + b)
*atan((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*
x)/sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a**3*c**5*d*x - 2*sq
rt(a)*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*sqrt(a*c**2 + b)*a
tan((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)
/sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a**3*c**4*d**2*x**2 -
2*sqrt(a)*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*sqrt(a*c**2 +
b)*atan((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*
d*x)/sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a**2*b*c**4 - 2*sq
rt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*atan((sqrt(a*c**2 + 2*a*c*
d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(2*sqrt(a)*sqrt(a*c*
*2 + b)*c - 2*a*c**2 - b))*a**4*c**7 - 4*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c
 - 2*a*c**2 - b)*atan((sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a
)*c + sqrt(a)*d*x)/sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a**4
*c**6*d*x - 2*sqrt(2*sqrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b)*atan((sqrt
(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) + sqrt(a)*c + sqrt(a)*d*x)/sqrt(2*s
qrt(a)*sqrt(a*c**2 + b)*c - 2*a*c**2 - b))*a**4*c**5*d**2*x**2 - 4*sqrt...
```

3.92 $\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 169

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{c(b-2ac^2)\,d\sqrt{a+\frac{b}{(c+dx)^2}}}{a\,(b+ac^2)^2\left(1-\frac{c}{c+dx}\right)}+\frac{d\left(ac-\frac{b}{c+dx}\right)}{a\,(b+ac^2)\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}-\frac{3bc^2d\,\mathrm{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{(b+ac^2)^{5/2}}$$

output
```
c*(-2*a*c^2+b)*d*(a+b/(d*x+c)^2)^(1/2)/a/(a*c^2+b)^2/(1-c/(d*x+c))+d*(a*c-
b/(d*x+c))/a/(a*c^2+b)/(a+b/(d*x+c)^2)^(1/2)/(1-c/(d*x+c))-3*b*c^2*d*arcta
nh((a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/(a*c^2+b)^(5/2)
```

Mathematica [A] (verified)

Time = 10.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = -\frac{b^2dx+a^2c^3(c+dx)^2+abc(c^2-3cdx-2d^2x^2)}{a\left(b+ac^2\right)^2x(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}} + \frac{3bc^2d\log(x)}{(b+ac^2)^{5/2}} - \frac{3bc^2d\log\left(b+(c+dx)\left(ac+\sqrt{b+ac^2}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{(b+ac^2)^{5/2}}$$

input
```
Integrate[1/(x^2*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
-((b^2*d*x + a^2*c^3*(c + d*x)^2 + a*b*c*(c^2 - 3*c*d*x - 2*d^2*x^2))/(a*(
b + a*c^2)^2*x*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])) + (3*b*c^
2*d*Log[x])/(b + a*c^2)^(5/2) - (3*b*c^2*d*Log[b + (c + d*x)*(a*c + Sqrt[b
 + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])/(b + a*c^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 1774, 1799, 496, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 896}$$

$$d\int \frac{1}{d^2x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{ 1774}$$

$$d\int \frac{1}{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)^2}d(c+dx)$$

$$\Big\downarrow \text{1799}$$

$$-d\int \frac{1}{\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{496}$$

$$-d\left(-\frac{\int -\frac{c\left(2ac-\frac{b}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{a\left(ac^2+b\right)}-\frac{ac-\frac{b}{c+dx}}{a\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)$$

$$\Big\downarrow \text{25}$$

$$-d\left(\frac{\int \frac{c\left(2ac-\frac{b}{c+dx}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{a\left(ac^2+b\right)}-\frac{ac-\frac{b}{c+dx}}{a\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)$$

$$\Big\downarrow \text{27}$$

$$-d\left(\frac{c\int \frac{2ac-\frac{b}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{a\left(ac^2+b\right)}-\frac{ac-\frac{b}{c+dx}}{a\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)$$

$$\Big\downarrow \text{679}$$

$$-d\left(\frac{c\left(\frac{3abc\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{ac^2+b}-\frac{\left(b-2ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}\right)}{a\left(ac^2+b\right)}-\frac{ac-\frac{b}{c+dx}}{a\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)$$

$$\Big\downarrow \text{488}$$

$$-d\left(\frac{c\left(-\frac{3abc\int \frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{ac^2+b}-\frac{(b-2ac^2)\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}\right)}{a\left(ac^2+b\right)}-\frac{ac-\frac{b}{c+dx}}{a\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-d\left(\frac{c\left(-\frac{3abc\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{(ac^2+b)^{3/2}}-\frac{(b-2ac^2)\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}\right)}{a\left(ac^2+b\right)}-\frac{ac-\frac{b}{c+dx}}{a\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)\sqrt{a+\frac{b}{(c+dx)^2}}}\right)$$

input
```
Int[1/(x^2*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
-(d*(-((a*c - b/(c + d*x))/(a*(b + a*c^2)*Sqrt[a + b/(c + d*x)^2]*(1 - c/(
c + d*x)))) + (c*(-(((b - 2*a*c^2)*Sqrt[a + b/(c + d*x)^2])/((b + a*c^2)*(
1 - c/(c + d*x)))) - (3*a*b*c*ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c
^2]*Sqrt[a + b/(c + d*x)^2])])/(b + a*c^2)^(3/2)))/(a*(b + a*c^2))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(159) = 318$.

Time = 0.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.21

method	result
default	$-\frac{(a d^2x^2+2adxc+a c^2+b)\left((a c^2+b)^{\frac{3}{2}} a^2c^3d^2x^2+3\sqrt{a d^2x^2+2adxc+a c^2+b}\,\ln\left(\frac{2a c^2+2b+2adxc+2\sqrt{a c^2+b}\,\sqrt{a d^2x^2+2adxc+a c^2+b}}{x}\right.\right.}{}$
risch	$-\frac{c^3(a d^2x^2+2adxc+a c^2+b)}{(a c^2+b)^2x\sqrt{\frac{a d^2x^2+2adxc+a c^2+b}{(dx+c)^2}}\,(dx+c)}+\frac{\left(-\frac{b^2d}{(a c^2+b)^2a\sqrt{a d^2x^2+2adxc+a c^2+b}}+\frac{2b d^2cx}{(a c^2+b)^2\sqrt{a d^2x^2+2adxc+a c^2+b}}+\frac{}{(a c^2+b}\right.}{}$

input
```
int(1/x^2/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*d^2*x^2+2*a*c*d*x+a*c^2+b)*((a*c^2+b)^(3/2)*a^2*c^3*d^2*x^2+3*(a*d^2*x
^2+2*a*c*d*x+a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2
+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*a^2*b*c^4*d*x+2*(a*c^2+b)^(3/2)*a^2*c^4*d*
x+(a*c^2+b)^(3/2)*a^2*c^5-2*(a*c^2+b)^(3/2)*a*b*c*d^2*x^2+3*(a*d^2*x^2+2*a
*c*d*x+a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c
*d*x+a*c^2+b)^(1/2)+b)/x)*a*b^2*c^2*d*x-3*(a*c^2+b)^(3/2)*a*b*c^2*d*x+(a*c
^2+b)^(3/2)*a*b*c^3+(a*c^2+b)^(3/2)*b^2*d*x)/((a*d^2*x^2+2*a*c*d*x+a*c^2+b
)/(d*x+c)^2)^(3/2)/(d*x+c)^3/(a*c^2+b)^(7/2)/x/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(160) = 320$.

Time = 0.92 (sec) , antiderivative size = 1022, normalized size of antiderivative = 6.05

$$\int \frac{1}{x^2 \left(a + \frac{b}{(c+dx)^2}\right)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a^2*b*c^2*d^3*x^3 + 2*a^2*b*c^3*d^2*x^2 + (a^2*b*c^4 + a*b^2*c^2)
*d*x)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*
c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2
+ b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(
d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(a^3*c^8 + 2*a^2*b*c^6 + a*b^2*c^4 + (
a^3*c^5 - a^2*b*c^3 - 2*a*b^2*c)*d^3*x^3 + (3*a^3*c^6 - 2*a^2*b*c^4 - 4*a*
b^2*c^2 + b^3)*d^2*x^2 + (3*a^3*c^7 + a^2*b*c^5 - a*b^2*c^3 + b^3*c)*d*x)*
sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^5
*c^6 + 3*a^4*b*c^4 + 3*a^3*b^2*c^2 + a^2*b^3)*d^2*x^3 + 2*(a^5*c^7 + 3*a^4
*b*c^5 + 3*a^3*b^2*c^3 + a^2*b^3*c)*d*x^2 + (a^5*c^8 + 4*a^4*b*c^6 + 6*a^3
*b^2*c^4 + 4*a^2*b^3*c^2 + a*b^4)*x), (3*(a^2*b*c^2*d^3*x^3 + 2*a^2*b*c^3*
d^2*x^2 + (a^2*b*c^4 + a*b^2*c^2)*d*x)*sqrt(-a*c^2 - b)*arctan((a*c*d^2*x^
2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a*c^2 - b)*sqrt((a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a^2*c^4 + (a^2*c^2 + a*b)
*d^2*x^2 + 2*a*b*c^2 + 2*(a^2*c^3 + a*b*c)*d*x + b^2)) - (a^3*c^8 + 2*a^2*
b*c^6 + a*b^2*c^4 + (a^3*c^5 - a^2*b*c^3 - 2*a*b^2*c)*d^3*x^3 + (3*a^3*c^6
 - 2*a^2*b*c^4 - 4*a*b^2*c^2 + b^3)*d^2*x^2 + (3*a^3*c^7 + a^2*b*c^5 - a*b
^2*c^3 + b^3*c)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2
*c*d*x + c^2)))/((a^5*c^6 + 3*a^4*b*c^4 + 3*a^3*b^2*c^2 + a^2*b^3)*d^2*x^3
 + 2*(a^5*c^7 + 3*a^4*b*c^5 + 3*a^3*b^2*c^3 + a^2*b^3*c)*d*x^2 + (a^5*c...
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
 d**2*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c)^2)^(3/2)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(160) = 320$.

Time = 0.21 (sec) , antiderivative size = 523, normalized size of antiderivative = 3.09

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{6\,bc^2d\arctan\left(-\frac{\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}}{\sqrt{-ac^2-b}}\right)}{\left(a^2c^4\mathrm{sgn}\left(dx+c\right)+2\,abc^2\mathrm{sgn}\left(dx+c\right)+b^2\mathrm{sgn}\left(dx+c\right)\right)\sqrt{-ac^2-b}}$$
$$+\frac{\frac{2\left(a^3b^2c^5d^3\mathrm{sgn}(dx+c)+2\,a^2b^3c^3d^3\mathrm{sgn}(dx+c)+ab^4cd^3\mathrm{sgn}(dx+c)\right)x}{a^5bc^8d+4\,a^4b^2c^6d+6\,a^3b^3c^4d+4\,a^2b^4c^2d+ab^5d}+\frac{3\,a^3b^2c^6d^2\mathrm{sgn}(dx+c)+5\,a^2b^3c^4d^2\mathrm{sgn}(dx+c)+ab^4c^2d^2\mathrm{sgn}(dx+c)-b^5d^2}{a^5bc^8d+4\,a^4b^2c^6d+6\,a^3b^3c^4d+4\,a^2b^4c^2d+ab^5d}}{\sqrt{ad^2x^2+2\,acdx+ac^2+b}}$$
$$-\frac{2\left(a^{\frac{3}{2}}c^5|d|+\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)ac^4d+\sqrt{a}bc^3|d|\right)}{\left(a^2c^4\mathrm{sgn}\left(dx+c\right)+2\,abc^2\mathrm{sgn}\left(dx+c\right)+b^2\mathrm{sgn}\left(dx+c\right)\right)\left(ac^2-\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+b}\right)\right.}$$

input
```
integrate(1/x^2/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
6*b*c^2*d*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)
)/sqrt(-a*c^2 - b))/((a^2*c^4*sgn(d*x + c) + 2*a*b*c^2*sgn(d*x + c) + b^2*
sgn(d*x + c))*sqrt(-a*c^2 - b)) + (2*(a^3*b^2*c^5*d^3*sgn(d*x + c) + 2*a^2
*b^3*c^3*d^3*sgn(d*x + c) + a*b^4*c*d^3*sgn(d*x + c))*x/(a^5*b*c^8*d + 4*a
^4*b^2*c^6*d + 6*a^3*b^3*c^4*d + 4*a^2*b^4*c^2*d + a*b^5*d) + (3*a^3*b^2*c
^6*d^2*sgn(d*x + c) + 5*a^2*b^3*c^4*d^2*sgn(d*x + c) + a*b^4*c^2*d^2*sgn(d
*x + c) - b^5*d^2*sgn(d*x + c))/(a^5*b*c^8*d + 4*a^4*b^2*c^6*d + 6*a^3*b^3
*c^4*d + 4*a^2*b^4*c^2*d + a*b^5*d))/sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b) - 2*(a^(3/2)*c^5*abs(d) + (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b))*a*c^4*d + sqrt(a)*b*c^3*abs(d))/((a^2*c^4*sgn(d*x + c) + 2*a*
b*c^2*sgn(d*x + c) + b^2*sgn(d*x + c))*(a*c^2 - (sqrt(a*d^2)*x - sqrt(a*d^
2*x^2 + 2*a*c*d*x + a*c^2 + b))^2 + b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x)^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x)^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 842, normalized size of antiderivative = 4.98

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{-\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^3c^7-2\sqrt{a\,d^2x^2+2acdx+a\,c^2+b}\,a^3c^6dx-\sqrt{}}{}$$

input
```
int(1/x^2/(a+b/(d*x+c)^2)^(3/2),x)
```

output

```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**7 - 2*sqrt(a*c**2 +
 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**6*d*x - sqrt(a*c**2 + 2*a*c*d*x + a*
d**2*x**2 + b)*a**3*c**5*d**2*x**2 - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b)*a**2*b*c**5 + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c*
*4*d*x + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**3*d**2*x**2
- sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**3 + 2*sqrt(a*c**2 +
 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**2*d*x + 2*sqrt(a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)*a*b**2*c*d**2*x**2 - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)*b**3*d*x + 3*sqrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 +
2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a**2*b*c**4*d*x + 6*s
qrt(a*c**2 + b)*log(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2
 + b) - a*c**2 - a*c*d*x - b)*a**2*b*c**3*d**2*x**2 + 3*sqrt(a*c**2 + b)*l
og(sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 -
a*c*d*x - b)*a**2*b*c**2*d**3*x**3 + 3*sqrt(a*c**2 + b)*log(sqrt(a*c**2 +
b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a*b*
*2*c**2*d*x - 3*sqrt(a*c**2 + b)*log(x)*a**2*b*c**4*d*x - 6*sqrt(a*c**2 +
b)*log(x)*a**2*b*c**3*d**2*x**2 - 3*sqrt(a*c**2 + b)*log(x)*a**2*b*c**2*d*
*3*x**3 - 3*sqrt(a*c**2 + b)*log(x)*a*b**2*c**2*d*x)/(a*x*(a**4*c**8 + 2*a
**4*c**7*d*x + a**4*c**6*d**2*x**2 + 4*a**3*b*c**6 + 6*a**3*b*c**5*d*x + 3
*a**3*b*c**4*d**2*x**2 + 6*a**2*b**2*c**4 + 6*a**2*b**2*c**3*d*x + 3*a*...
```

3.93 $\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 234

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = -\frac{c^2d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(b+ac^2\right)^2\left(1-\frac{c}{c+dx}\right)^2} - \frac{c^2\left(5b-2ac^2\right)d^2\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(b+ac^2\right)^3\left(1-\frac{c}{c+dx}\right)} + \frac{bd^2\left(b-3ac^2+\frac{c(3b-ac^2)}{c+dx}\right)}{\left(b+ac^2\right)^3\sqrt{a+\frac{b}{(c+dx)^2}}} - \frac{3bc\left(2b-3ac^2\right)d^2\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2\left(b+ac^2\right)^{7/2}}$$

output
```
-1/2*c^2*d^2*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^2/(1-c/(d*x+c))^2-1/2*c^2*(-2
*a*c^2+5*b)*d^2*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^3/(1-c/(d*x+c))+b*d^2*(b-3
*a*c^2+c*(-a*c^2+3*b)/(d*x+c))/(a*c^2+b)^3/(a+b/(d*x+c)^2)^(1/2)-3/2*b*c*(
-3*a*c^2+2*b)*d^2*arctanh((a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^
(1/2))/(a*c^2+b)^(7/2)
```

Mathematica [A] (verified)

Time = 10.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{1}{2}\left(-\frac{a^2c^4(c-dx)(c+dx)^2+b^2(c^3+6c^2dx-8cd^2x^2-2d^3x^3)+abc^2(2c^3+7}{(b+ac^2)^3\,x^2(c+dx)\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}}\right.$$
$$+\frac{3bc(2b-3ac^2)\,d^2\log(x)}{(b+ac^2)^{7/2}}$$
$$\left.-\frac{3bc(2b-3ac^2)\,d^2\log\left(b+(c+dx)\left(ac+\sqrt{b+ac^2}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{(b+ac^2)^{7/2}}\right)$$

input
```
Integrate[1/(x^3*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
(-((a^2*c^4*(c - d*x)*(c + d*x)^2 + b^2*(c^3 + 6*c^2*d*x - 8*c*d^2*x^2 - 2
*d^3*x^3) + a*b*c^2*(2*c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 12*d^3*x^3))/((b +
 a*c^2)^3*x^2*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])) + (3*b*c*(
2*b - 3*a*c^2)*d^2*Log[x])/(b + a*c^2)^(7/2) - (3*b*c*(2*b - 3*a*c^2)*d^2*
Log[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x
)^2])])/(b + a*c^2)^(7/2))/2
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {896, 25, 1774, 1803, 25, 593, 25, 688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 896$$

$$d^2\int \frac{1}{d^3x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$\Big\downarrow$ 25

$$-d^2\int -\frac{1}{d^3x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$\Big\downarrow$ 1774

$$-d^2\int \frac{1}{(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$\Big\downarrow$ 1803

$$d^2\int -\frac{1}{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$\Big\downarrow$ 25

$$-d^2\int \frac{1}{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$\Big\downarrow$ 593

$$d^2\left(\frac{c\int -\frac{\frac{2c}{c+dx}+3}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{ac^2+b}+\frac{\frac{c}{c+dx}+1}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}}\right)$$

$\Big\downarrow$ 25

$$d^2\left(\frac{\frac{c}{c+dx}+1}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c\int \frac{\frac{2c}{c+dx}+3}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{ac^2+b}\right)$$

$\Big\downarrow$ 688

$$d^2\left(\frac{\frac{c}{c+dx}+1}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c\left(\frac{5c\sqrt{a+\frac{b}{(c+dx)^2}}}{2(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}-\frac{\int -\frac{\frac{5bc}{c+dx}+2\left(3b-2ac^2\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2(ac^2+b)}\right)}{ac^2+b}\right)$$

$\downarrow$ 25

$$d^2\left(\frac{\frac{c}{c+dx}+1}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c\left(\frac{\int \frac{\frac{5bc}{c+dx}+2\left(3b-2ac^2\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2(ac^2+b)}+\frac{5c\sqrt{a+\frac{b}{(c+dx)^2}}}{2(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2}\right)}{ac^2+b}\right)$$

$\downarrow$ 679

$$d^2\left(\frac{\frac{c}{c+dx}+1}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c\left(\frac{\frac{3b\left(2b-3ac^2\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}}{ac^2+b}+\frac{c\left(11b-4ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}}{2(ac^2+b)}+\frac{5c\sqrt{\ldots}}{2(ac^2+\ldots}\right.}{ac^2+b\ldots}\right.$$

$\downarrow$ 488

$$d^2\left(\frac{\frac{c}{c+dx}+1}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c\left(\frac{\frac{c(11b-4ac^2)\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}-\frac{3b(2b-3ac^2)\int\frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}}{ac^2+b}}{2(ac^2+b)}+\frac{5c\sqrt{}}{2(ac^2+}\right.}{ac^2+b}\right.$$

$\downarrow$ 219

$$d^2\left(\frac{\frac{c}{c+dx}+1}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{c\left(\frac{\frac{c(11b-4ac^2)\sqrt{a+\frac{b}{(c+dx)^2}}}{(ac^2+b)\left(1-\frac{c}{c+dx}\right)}-\frac{3b(2b-3ac^2)\operatorname{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{(ac^2+b)^{3/2}}}{2(ac^2+b)}+\frac{5c}{2(ac^2}\right.}{ac^2+b}\right.$$

input
```
Int[1/(x^3*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
d^2*((1 + c/(c + d*x))/((b + a*c^2)*Sqrt[a + b/(c + d*x)^2]*(1 - c/(c + d*
x))^2) - (c*((5*c*Sqrt[a + b/(c + d*x)^2])/(2*(b + a*c^2)*(1 - c/(c + d*x)
)^2) + ((c*(11*b - 4*a*c^2)*Sqrt[a + b/(c + d*x)^2])/((b + a*c^2)*(1 - c/(
c + d*x))) - (3*b*(2*b - 3*a*c^2)*ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b +
 a*c^2]*Sqrt[a + b/(c + d*x)^2])])/(b + a*c^2)^(3/2))/(2*(b + a*c^2))))/(b
 + a*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(216) = 432$.

Time = 0.13 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.45

method	result
default	$\frac{\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)\left(\left(a\,c^{2}+b\right)^{\frac{3}{2}}a^{2}c^{4}d^{3}x^{3}+9\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}\,\ln\left(\frac{2a\,c^{2}+2b+2adxc+2\sqrt{a\,c^{2}+b}\,\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}}{x}\right)\right.}{}$
risch	$-\frac{c^{2}\left(a\,d^{2}x^{2}+2adxc+a\,c^{2}+b\right)\left(-adx\,c^{2}+c^{3}a+6bdx+bc\right)}{2\left(a\,c^{2}+b\right)^{3}x^{2}\sqrt{\frac{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}{(dx+c)^{2}}}\,(dx+c)}+\frac{\left(\frac{d^{3}b^{2}x}{\left(a\,c^{2}+b\right)^{3}\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}}+\frac{d^{2}b^{2}c}{\left(a\,c^{2}+b\right)^{3}\sqrt{a\,d^{2}x^{2}+2adxc+a\,c^{2}+b}}\right.}{}$

input
```
int(1/x^3/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)*((a*c^2+b)^(3/2)*a^2*c^4*d^3*x^3+9*(a*d^
2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*
x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*a^2*b*c^5*d^2*x^2+(a*c^2+b)^(3/2)*a^2*c
^5*d^2*x^2-(a*c^2+b)^(3/2)*a^2*c^6*d*x-12*(a*c^2+b)^(3/2)*a*b*c^2*d^3*x^3+
3*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*
(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*a*b^2*c^3*d^2*x^2-(a*c^2+b)^(3/2
)*a^2*c^7-21*(a*c^2+b)^(3/2)*a*b*c^3*d^2*x^2-7*(a*c^2+b)^(3/2)*a*b*c^4*d*x
+2*(a*c^2+b)^(3/2)*b^2*d^3*x^3-6*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*ln(2*
(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*b
^3*c*d^2*x^2-2*(a*c^2+b)^(3/2)*a*b*c^5+8*(a*c^2+b)^(3/2)*b^2*c*d^2*x^2-6*(
a*c^2+b)^(3/2)*b^2*c^2*d*x-(a*c^2+b)^(3/2)*b^2*c^3)/((a*d^2*x^2+2*a*c*d*x+
a*c^2+b)/(d*x+c)^2)^(3/2)/(d*x+c)^3/(a*c^2+b)^(9/2)/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(216) = 432$.

Time = 2.17 (sec) , antiderivative size = 1260, normalized size of antiderivative = 5.38

$$\int \frac{1}{x^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*((3*a^2*b*c^3 - 2*a*b^2*c)*d^4*x^4 + 2*(3*a^2*b*c^4 - 2*a*b^2*c^2)
*d^3*x^3 + (3*a^2*b*c^5 + a*b^2*c^3 - 2*b^3*c)*d^2*x^2)*sqrt(a*c^2 + b)*lo
g(-(2*a^2*c^4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c
)*d*x + 2*b^2 + 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c
^2 + b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)
))/x^2) - 2*(a^3*c^10 + 3*a^2*b*c^8 + 3*a*b^2*c^6 - (a^3*c^6 - 11*a^2*b*c^
4 - 10*a*b^2*c^2 + 2*b^3)*d^4*x^4 + b^3*c^4 - (2*a^3*c^7 - 31*a^2*b*c^5 -
23*a*b^2*c^3 + 10*b^3*c)*d^3*x^3 + 2*(14*a^2*b*c^6 + 13*a*b^2*c^4 - b^3*c^
2)*d^2*x^2 + (2*a^3*c^9 + 11*a^2*b*c^7 + 16*a*b^2*c^5 + 7*b^3*c^3)*d*x)*sq
rt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^5*c
^8 + 4*a^4*b*c^6 + 6*a^3*b^2*c^4 + 4*a^2*b^3*c^2 + a*b^4)*d^2*x^4 + 2*(a^5
*c^9 + 4*a^4*b*c^7 + 6*a^3*b^2*c^5 + 4*a^2*b^3*c^3 + a*b^4*c)*d*x^3 + (a^5
*c^10 + 5*a^4*b*c^8 + 10*a^3*b^2*c^6 + 10*a^2*b^3*c^4 + 5*a*b^4*c^2 + b^5)
*x^2), -1/2*(3*((3*a^2*b*c^3 - 2*a*b^2*c)*d^4*x^4 + 2*(3*a^2*b*c^4 - 2*a*b
^2*c^2)*d^3*x^3 + (3*a^2*b*c^5 + a*b^2*c^3 - 2*b^3*c)*d^2*x^2)*sqrt(-a*c^2
 - b)*arctan((a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(-a*c^2 -
 b)*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(a
^2*c^4 + (a^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 2*(a^2*c^3 + a*b*c)*d*x + b
^2)) + (a^3*c^10 + 3*a^2*b*c^8 + 3*a*b^2*c^6 - (a^3*c^6 - 11*a^2*b*c^4 - 1
0*a*b^2*c^2 + 2*b^3)*d^4*x^4 + b^3*c^4 - (2*a^3*c^7 - 31*a^2*b*c^5 - 23...
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a + \frac{b}{(c+dx)^2}\right)^{3/2}}\, dx = \int \frac{1}{x^3 \left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/x**3/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(1/(x**3*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
 d**2*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c)^2)^(3/2)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(216) = 432.

Time = 0.21 (sec) , antiderivative size = 961, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^3/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-((3*a^4*b^2*c^8*d^4*sgn(d*x + c) + 8*a^3*b^3*c^6*d^4*sgn(d*x + c) + 6*a^2
*b^4*c^4*d^4*sgn(d*x + c) - b^6*d^4*sgn(d*x + c))*x/(a^6*b*c^12*d + 6*a^5*
b^2*c^10*d + 15*a^4*b^3*c^8*d + 20*a^3*b^4*c^6*d + 15*a^2*b^5*c^4*d + 6*a*
b^6*c^2*d + b^7*d) + 4*(a^4*b^2*c^9*d^3*sgn(d*x + c) + 2*a^3*b^3*c^7*d^3*s
gn(d*x + c) - 2*a*b^5*c^3*d^3*sgn(d*x + c) - b^6*c*d^3*sgn(d*x + c))/(a^6*
b*c^12*d + 6*a^5*b^2*c^10*d + 15*a^4*b^3*c^8*d + 20*a^3*b^4*c^6*d + 15*a^2
*b^5*c^4*d + 6*a*b^6*c^2*d + b^7*d))/sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
b) - 3*(3*a*b*c^3*d^2 - 2*b^2*c*d^2)*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x
^2 + 2*a*c*d*x + a*c^2 + b))/sqrt(-a*c^2 - b))/((a^3*c^6*sgn(d*x + c) + 3*
a^2*b*c^4*sgn(d*x + c) + 3*a*b^2*c^2*sgn(d*x + c) + b^3*sgn(d*x + c))*sqrt
(-a*c^2 - b)) + (2*a^(7/2)*c^8*d*abs(d) + 4*(sqrt(a*d^2)*x - sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b))*a^3*c^7*d^2 + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^
2 + 2*a*c*d*x + a*c^2 + b))^2*a^(5/2)*c^6*d*abs(d) - 2*a^(5/2)*b*c^6*d*abs
(d) - (sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^2*b*c^5*
d^2 + 8*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(3/2
)*b*c^4*d*abs(d) + 7*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b))^3*a*b*c^3*d^2 - 10*a^(3/2)*b^2*c^4*d*abs(d) - 5*(sqrt(a*d^2)*x - sqrt
(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a*b^2*c^3*d^2 + 6*(sqrt(a*d^2)*x - sq
rt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*sqrt(a)*b^2*c^2*d*abs(d) - 6*sqrt
(a)*b^3*c^2*d*abs(d))/((a^3*c^6*sgn(d*x + c) + 3*a^2*b*c^4*sgn(d*x + c)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x)^2)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x)^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1394, normalized size of antiderivative = 5.96

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
( - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*c**9 - sqrt(a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b)*a**3*c**8*d*x + sqrt(a*c**2 + 2*a*c*d*x + a*d*
*2*x**2 + b)*a**3*c**7*d**2*x**2 + sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)*a**3*c**6*d**3*x**3 - 3*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**
2*b*c**7 - 8*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**6*d*x -
20*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**5*d**2*x**2 - 11*s
qrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b*c**4*d**3*x**3 - 3*sqrt(a
*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**2*c**5 - 13*sqrt(a*c**2 + 2*a*c*
d*x + a*d**2*x**2 + b)*a*b**2*c**4*d*x - 13*sqrt(a*c**2 + 2*a*c*d*x + a*d*
*2*x**2 + b)*a*b**2*c**3*d**2*x**2 - 10*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)*a*b**2*c**2*d**3*x**3 - sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)*b**3*c**3 - 6*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3*c**2*d*x +
 8*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3*c*d**2*x**2 + 2*sqrt(a*
c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**3*d**3*x**3 + 9*sqrt(a*c**2 + b)*lo
g( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2
- a*c*d*x - b)*a**2*b*c**5*d**2*x**2 + 18*sqrt(a*c**2 + b)*log( - sqrt(a*c
**2 + b)*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b
)*a**2*b*c**4*d**3*x**3 + 9*sqrt(a*c**2 + b)*log( - sqrt(a*c**2 + b)*sqrt(
a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b) - a*c**2 - a*c*d*x - b)*a**2*b*c**3*
d**4*x**4 + 3*sqrt(a*c**2 + b)*log( - sqrt(a*c**2 + b)*sqrt(a*c**2 + 2*...
```

3.94 $\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 321

$$\begin{aligned}
\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = &-\frac{cd^3\sqrt{a+\frac{b}{(c+dx)^2}}}{3\left(b+ac^2\right)^2\left(1-\frac{c}{c+dx}\right)^3} \\
&-\frac{c(5b-6ac^2)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6\left(b+ac^2\right)^3\left(1-\frac{c}{c+dx}\right)^2}-\frac{c(11b^2-40abc^2+6a^2c^4)\,d^3\sqrt{a+\frac{b}{(c+dx)^2}}}{6\left(b+ac^2\right)^4\left(1-\frac{c}{c+dx}\right)} \\
&-\frac{bd^3\left(4ac(b-ac^2)-\frac{b^2-6abc^2+a^2c^4}{c+dx}\right)}{\left(b+ac^2\right)^4\sqrt{a+\frac{b}{(c+dx)^2}}} \\
&-\frac{b(2b^2-21abc^2+12a^2c^4)\,d^3\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2\left(b+ac^2\right)^{9/2}}
\end{aligned}$$

output
```
-1/3*c*d^3*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^2/(1-c/(d*x+c))^3-1/6*c*(-6*a*c
^2+5*b)*d^3*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^3/(1-c/(d*x+c))^2-1/6*c*(6*a^2
*c^4-40*a*b*c^2+11*b^2)*d^3*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^4/(1-c/(d*x+c)
)-b*d^3*(4*a*c*(-a*c^2+b)-(a^2*c^4-6*a*b*c^2+b^2)/(d*x+c))/(a*c^2+b)^4/(a+
b/(d*x+c)^2)^(1/2)-1/2*b*(12*a^2*c^4-21*a*b*c^2+2*b^2)*d^3*arctanh((a*c+b/
(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d*x+c)^2)^(1/2))/(a*c^2+b)^(9/2)
```

Mathematica [A] (verified)

Time = 10.67 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\frac{1}{6}\left(-\frac{2a^3c^5(c+dx)^2\left(c^2-cdx+d^2x^2\right)+b^3(2c^3+9c^2dx+18cd^2x^2-6d^3x^3)+}{}\right.$$
$$+\frac{3b(2b^2-21abc^2+12a^2c^4)\,d^3\log(x)}{(b+ac^2)^{9/2}}$$
$$\left.-\frac{3b(2b^2-21abc^2+12a^2c^4)\,d^3\log\left(b+(c+dx)\left(ac+\sqrt{b+ac^2}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{(b+ac^2)^{9/2}}\right)$$

input
```
Integrate[1/(x^4*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
(-((2*a^3*c^5*(c + d*x)^2*(c^2 - c*d*x + d^2*x^2) + b^3*(2*c^3 + 9*c^2*d*x
 + 18*c*d^2*x^2 - 6*d^3*x^3) + a^2*b*c^3*(6*c^4 + 13*c^3*d*x - 17*c^2*d^2*
x^2 - 97*c*d^3*x^3 - 61*d^4*x^4) + a*b^2*c*(6*c^4 + 20*c^3*d*x + c^2*d^2*x
^2 + 105*c*d^3*x^3 + 42*d^4*x^4))/((b + a*c^2)^4*x^3*(c + d*x)*Sqrt[(b + a
*(c + d*x)^2)/(c + d*x)^2])) + (3*b*(2*b^2 - 21*a*b*c^2 + 12*a^2*c^4)*d^3*
Log[x])/(b + a*c^2)^(9/2) - (3*b*(2*b^2 - 21*a*b*c^2 + 12*a^2*c^4)*d^3*Log
[b + (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2
])])/(b + a*c^2)^(9/2))/6
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {896, 1774, 1803, 601, 25, 2182, 25, 2182, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 896}$$

$$d^3\int \frac{1}{d^4x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{ 1774}$$

$$d^3\int \frac{1}{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(\frac{c}{c+dx}-1\right)^4}d(c+dx)$$

$$\Big\downarrow \text{ 1803}$$

$$-d^3\int \frac{1}{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 601}$$

$$-d^3\left(\frac{b\left(4ac(b-ac^2)-\frac{a^2c^4-6abc^2+b^2}{c+dx}\right)}{(ac^2+b)^4\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{\int -\frac{\frac{4a^2b\left(b-ac^2\right)c^5}{\left(ac^2+b\right)^4(c+dx)^3}-\frac{a^2\left(-a^2c^4-10abc^2+15b^2\right)c^4}{\left(ac^2+b\right)^4(c+dx)^2}+\frac{4a^2b\left(5b-ac^2\right)c^3}{\left(ac^2+b\right)^4(c+dx)}+\frac{ab\left(a^2c^4-6abc^2+b^2\right)}{\left(ac^2+b\right)^4}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^4}}{a}\right.$$

$$\Big\downarrow \text{ 25}$$

$$-d^3\left(\frac{\int \frac{\frac{4a^2b\left(b-ac^2\right)c^5}{\left(ac^2+b\right)^4(c+dx)^3}-\frac{a^2\left(-a^2c^4-10abc^2+15b^2\right)c^4}{\left(ac^2+b\right)^4(c+dx)^2}+\frac{4a^2b\left(5b-ac^2\right)c^3}{\left(ac^2+b\right)^4(c+dx)}+\frac{ab\left(a^2c^4-6abc^2+b^2\right)}{\left(ac^2+b\right)^4}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^4}d\frac{1}{c+dx}}{a}+\frac{b\left(4ac\left(b-ac^2\right)-\frac{a^2c^4-6abc^2}{c+dx}\right.}{\left(ac^2+b\right)^4\sqrt{a+\frac{b}{(c+dx)^2}}}\right.$$

$\downarrow$ 2182

$$-d^3\left(\frac{\frac{ac\sqrt{a+\frac{b}{(c+dx)^2}}}{3(ac^2+b)^2\left(1-\frac{c}{c+dx}\right)^3}-\frac{\int -\frac{-\frac{12a^2b\left(b-ac^2\right)c^4}{\left(ac^2+b\right)^3(c+dx)^2}+\frac{a\left(-3a^3c^6-16a^2bc^4+37ab^2c^2+2b^3\right)c}{\left(ac^2+b\right)^3(c+dx)}+\frac{3a\left(-a^3c^6-a^2bc^4-7ab^2c^2+b^3\right)}{\left(ac^2+b\right)^3}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3(ac^2+b)}}{a}+\frac{b\left(4ac\right.}{(a}\right.$$

$\downarrow$ 25

$$-d^3\left(\frac{\frac{\int \frac{-\frac{12a^2b\left(b-ac^2\right)c^4}{\left(ac^2+b\right)^3(c+dx)^2}+\frac{a\left(-3a^3c^6-16a^2bc^4+37ab^2c^2+2b^3\right)c}{\left(ac^2+b\right)^3(c+dx)}+\frac{3a\left(-a^3c^6-a^2bc^4-7ab^2c^2+b^3\right)}{\left(ac^2+b\right)^3}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3(ac^2+b)}+\frac{ac\sqrt{a+\frac{b}{(c+dx)^2}}}{3(ac^2+b)^2\left(1-\frac{c}{c+dx}\right)^3}}{a}+\frac{b\left(4ac(\right.}{(ac}\right.$$

$\downarrow$ 2182

$$-d^3\left(\frac{\frac{\frac{ac\left(5b-6ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}-\frac{\int -\frac{a\left(\frac{bc\left(-30a^2c^4+23abc^2+5b^2\right)}{c+dx}+2\left(3a^3c^6-2a^2bc^4-26ab^2c^2+3b^3\right)\right)}{\left(ac^2+b\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)}}{3(ac^2+b)}+\frac{ac\sqrt{a+\frac{b}{(c+dx)^2}}}{3(ac^2+b)^2\left(1-\frac{c}{c+dx}\right)^3}}{a}+\frac{b\left(4\right.}{}\right.$$

$\downarrow$ 25

$$-d^3\left(\frac{\frac{\frac{\int \frac{a\left(\frac{bc\left(-30a^2c^4+23abc^2+5b^2\right)}{c+dx}+2\left(3a^3c^6-2a^2bc^4-26ab^2c^2+3b^3\right)\right)}{\left(ac^2+b\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)}+\frac{ac\left(5b-6ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{3(ac^2+b)}+\frac{ac\sqrt{a+\frac{b}{(c+dx)^2}}}{3(ac^2+b)^2\left(1-\frac{c}{c+dx}\right)^3}}{a}+\frac{b\left(4a\cdots\right.}{(\cdots}\right.$$

$\downarrow$ 27

$$-d^3\left(\frac{\frac{\frac{a\int \frac{\frac{bc\left(-30a^2c^4+23abc^2+5b^2\right)}{c+dx}+2\left(3a^3c^6-2a^2bc^4-26ab^2c^2+3b^3\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)^3}+\frac{ac\left(5b-6ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{3(ac^2+b)}+\frac{ac\sqrt{a+\frac{b}{(c+dx)^2}}}{3(ac^2+b)^2\left(1-\frac{c}{c+dx}\right)^3}}{a}+\frac{b\left(4ac\left(\cdots\right.\right.}{(ac\cdots}\right.$$

$\downarrow$ 679

$$-d^3\left(\frac{\frac{\frac{a\left(3b\left(12a^2c^4-21abc^2+2b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}+\frac{c\left(6a^2c^4-40abc^2+11b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}\right)}{2\left(ac^2+b\right)^3}+\frac{ac\left(5b-6ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{3(ac^2+b)}+\frac{ac\sqrt{\cdots}}{3(ac^2+\cdots}}{a}\right.$$

$\downarrow$ 488

$$-d^3\left(\frac{\frac{\frac{a\left(\frac{c\left(6a^2c^4-40abc^2+11b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}-3b\left(12a^2c^4-21abc^2+2b^2\right)\int\frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{2\left(ac^2+b\right)^3}+\frac{ac\left(5b-6ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{3(ac^2+b)}+\frac{ac\sqrt{\ }}{3(ac^2+}}{a}\right.$$

$\downarrow$ 219

$$-d^3\left(\frac{\frac{\frac{a\left(\frac{c\left(6a^2c^4-40abc^2+11b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}-\frac{3b\left(12a^2c^4-21abc^2+2b^2\right)\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{ac^2+b}}\right)}{2\left(ac^2+b\right)^3}+\frac{ac\left(5b-6ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{3(ac^2+b)}+\frac{a}{3(ac}}{a}\right.$$

input
```
Int[1/(x^4*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
-(d^3*((b*(4*a*c*(b - a*c^2) - (b^2 - 6*a*b*c^2 + a^2*c^4)/(c + d*x)))/((b
 + a*c^2)^4*Sqrt[a + b/(c + d*x)^2]) + ((a*c*Sqrt[a + b/(c + d*x)^2])/(3*(
b + a*c^2)^2*(1 - c/(c + d*x))^3) + ((a*c*(5*b - 6*a*c^2)*Sqrt[a + b/(c +
d*x)^2])/(2*(b + a*c^2)^2*(1 - c/(c + d*x))^2) + (a*((c*(11*b^2 - 40*a*b*c
^2 + 6*a^2*c^4)*Sqrt[a + b/(c + d*x)^2])/(1 - c/(c + d*x)) - (3*b*(2*b^2 -
 21*a*b*c^2 + 12*a^2*c^4)*ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c^2]*
Sqrt[a + b/(c + d*x)^2])])/Sqrt[b + a*c^2]))/(2*(b + a*c^2)^3))/(3*(b + a*
c^2)))/a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(299) = 598$.

Time = 0.15 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.51

method	result
default	$-\frac{(a\,d^2x^2+2adxc+a\,c^2+b)\left(2(a\,c^2+b)^{\frac{3}{2}}a^3c^5d^4x^4+2(a\,c^2+b)^{\frac{3}{2}}a^3c^6d^3x^3+36\ln\left(\frac{2a\,c^2+2b+2adxc+2\sqrt{a\,c^2+b}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+b}}{x}\right.\right.}{}$
risch	Expression too large to display

input
```
int(1/x^4/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)*(2*(a*c^2+b)^(3/2)*a^3*c^5*d^4*x^4+2*(a
*c^2+b)^(3/2)*a^3*c^6*d^3*x^3+36*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^
2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*a
^3*b*c^6*d^3*x^3-61*(a*c^2+b)^(3/2)*a^2*b*c^3*d^4*x^4+2*(a*c^2+b)^(3/2)*a^
3*c^8*d*x-97*(a*c^2+b)^(3/2)*a^2*b*c^4*d^3*x^3-27*ln(2*(a*d*x*c+a*c^2+(a*c
^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*d^2*x^2+2*a*c*d*x
+a*c^2+b)^(1/2)*a^2*b^2*c^4*d^3*x^3+2*(a*c^2+b)^(3/2)*a^3*c^9-17*(a*c^2+b)
^(3/2)*a^2*b*c^5*d^2*x^2+42*(a*c^2+b)^(3/2)*a*b^2*c*d^4*x^4+13*(a*c^2+b)^(
3/2)*a^2*b*c^6*d*x+105*(a*c^2+b)^(3/2)*a*b^2*c^2*d^3*x^3-57*ln(2*(a*d*x*c+
a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*d^2*x^2
+2*a*c*d*x+a*c^2+b)^(1/2)*a*b^3*c^2*d^3*x^3+6*(a*c^2+b)^(3/2)*a^2*b*c^7+(a
*c^2+b)^(3/2)*a*b^2*c^3*d^2*x^2+20*(a*c^2+b)^(3/2)*a*b^2*c^4*d*x-6*(a*c^2+
b)^(3/2)*b^3*d^3*x^3+6*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*
c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*b^4*d^3*x^3
+6*(a*c^2+b)^(3/2)*a*b^2*c^5+18*(a*c^2+b)^(3/2)*b^3*c*d^2*x^2+9*(a*c^2+b)^
(3/2)*b^3*c^2*d*x+2*(a*c^2+b)^(3/2)*b^3*c^3)/((a*d^2*x^2+2*a*c*d*x+a*c^2+b
)/(d*x+c)^2)^(3/2)/(d*x+c)^3/(a*c^2+b)^(11/2)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(295) = 590$.

Time = 4.62 (sec) , antiderivative size = 1599, normalized size of antiderivative = 4.98

$$\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((12*a^3*b*c^4 - 21*a^2*b^2*c^2 + 2*a*b^3)*d^5*x^5 + 2*(12*a^3*b*
c^5 - 21*a^2*b^2*c^3 + 2*a*b^3*c)*d^4*x^4 + (12*a^3*b*c^6 - 9*a^2*b^2*c^4
- 19*a*b^3*c^2 + 2*b^4)*d^3*x^3)*sqrt(a*c^2 + b)*log(-(2*a^2*c^4 + (2*a^2*
c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^2 - 2*(a*c*
d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt((a*d^2*x^2
 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*(2*a^4*c^12
 + 8*a^3*b*c^10 + 12*a^2*b^2*c^8 + (2*a^4*c^7 - 59*a^3*b*c^5 - 19*a^2*b^2*
c^3 + 42*a*b^3*c)*d^5*x^5 + 8*a*b^3*c^6 + (4*a^4*c^8 - 154*a^3*b*c^6 - 11*
a^2*b^2*c^4 + 141*a*b^3*c^2 - 6*b^4)*d^4*x^4 + 2*b^4*c^4 + 2*(a^4*c^9 - 56
*a^3*b*c^7 - 4*a^2*b^2*c^5 + 59*a*b^3*c^3 + 6*b^4*c)*d^3*x^3 + (2*a^4*c^10
 - 2*a^3*b*c^8 + 17*a^2*b^2*c^6 + 48*a*b^3*c^4 + 27*b^4*c^2)*d^2*x^2 + (4*
a^4*c^11 + 23*a^3*b*c^9 + 45*a^2*b^2*c^7 + 37*a*b^3*c^5 + 11*b^4*c^3)*d*x)
*sqrt((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^
6*c^10 + 5*a^5*b*c^8 + 10*a^4*b^2*c^6 + 10*a^3*b^3*c^4 + 5*a^2*b^4*c^2 + a
*b^5)*d^2*x^5 + 2*(a^6*c^11 + 5*a^5*b*c^9 + 10*a^4*b^2*c^7 + 10*a^3*b^3*c^
5 + 5*a^2*b^4*c^3 + a*b^5*c)*d*x^4 + (a^6*c^12 + 6*a^5*b*c^10 + 15*a^4*b^2
*c^8 + 20*a^3*b^3*c^6 + 15*a^2*b^4*c^4 + 6*a*b^5*c^2 + b^6)*x^3), 1/6*(3*(
(12*a^3*b*c^4 - 21*a^2*b^2*c^2 + 2*a*b^3)*d^5*x^5 + 2*(12*a^3*b*c^5 - 21*a
^2*b^2*c^3 + 2*a*b^3*c)*d^4*x^4 + (12*a^3*b*c^6 - 9*a^2*b^2*c^4 - 19*a*b^3
*c^2 + 2*b^4)*d^3*x^3)*sqrt(-a*c^2 - b)*arctan((a*c*d^2*x^2 + a*c^3 + (...
```

Sympy [F]

$$\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{x^4\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(1/(x**4*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
 d**2*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c)^2)^(3/2)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(295) = 590$.

Time = 0.22 (sec) , antiderivative size = 1804, normalized size of antiderivative = 5.62

$$\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^4/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
(4*(a^6*b^2*c^11*d^5*sgn(d*x + c) + 3*a^5*b^3*c^9*d^5*sgn(d*x + c) + 2*a^4
*b^4*c^7*d^5*sgn(d*x + c) - 2*a^3*b^5*c^5*d^5*sgn(d*x + c) - 3*a^2*b^6*c^3
*d^5*sgn(d*x + c) - a*b^7*c*d^5*sgn(d*x + c))*x/(a^8*b*c^16*d + 8*a^7*b^2*
c^14*d + 28*a^6*b^3*c^12*d + 56*a^5*b^4*c^10*d + 70*a^4*b^5*c^8*d + 56*a^3
*b^6*c^6*d + 28*a^2*b^7*c^4*d + 8*a*b^8*c^2*d + b^9*d) + (5*a^6*b^2*c^12*d
^4*sgn(d*x + c) + 10*a^5*b^3*c^10*d^4*sgn(d*x + c) - 9*a^4*b^4*c^8*d^4*sgn
(d*x + c) - 36*a^3*b^5*c^6*d^4*sgn(d*x + c) - 29*a^2*b^6*c^4*d^4*sgn(d*x +
 c) - 6*a*b^7*c^2*d^4*sgn(d*x + c) + b^8*d^4*sgn(d*x + c))/(a^8*b*c^16*d +
 8*a^7*b^2*c^14*d + 28*a^6*b^3*c^12*d + 56*a^5*b^4*c^10*d + 70*a^4*b^5*c^8
*d + 56*a^3*b^6*c^6*d + 28*a^2*b^7*c^4*d + 8*a*b^8*c^2*d + b^9*d))/sqrt(a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b) + (12*a^2*b*c^4*d^3 - 21*a*b^2*c^2*d^3 +
2*b^3*d^3)*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b
))/sqrt(-a*c^2 - b))/((a^4*c^8*sgn(d*x + c) + 4*a^3*b*c^6*sgn(d*x + c) + 6
*a^2*b^2*c^4*sgn(d*x + c) + 4*a*b^3*c^2*sgn(d*x + c) + b^4*sgn(d*x + c))*s
qrt(-a*c^2 - b)) - 1/3*(8*a^(11/2)*c^11*d^2*abs(d) + 24*(sqrt(a*d^2)*x - s
qrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))*a^5*c^10*d^3 + 24*(sqrt(a*d^2)*x -
 sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^2*a^(9/2)*c^9*d^2*abs(d) + 8*(sq
rt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))^3*a^4*c^8*d^3 - 22*
a^(9/2)*b*c^9*d^2*abs(d) - 6*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b))*a^4*b*c^8*d^3 + 108*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{x^4\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a + b/(c + d*x)^2)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x)^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1979, normalized size of antiderivative = 6.17

$$\int \frac{1}{x^4 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^4/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
( - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**4*c**11 - 2*sqrt(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**4*c**10*d*x - 2*sqrt(a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)*a**4*c**8*d**3*x**3 - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d
**2*x**2 + b)*a**4*c**7*d**4*x**4 - 8*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**
2 + b)*a**3*b*c**9 - 15*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*b*
c**8*d*x + 17*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*b*c**7*d**2*
x**2 + 95*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*b*c**6*d**3*x**3
 + 59*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*b*c**5*d**4*x**4 - 1
2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b**2*c**7 - 33*sqrt(a*c*
*2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**2*b**2*c**6*d*x + 16*sqrt(a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b)*a**2*b**2*c**5*d**2*x**2 - 8*sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b)*a**2*b**2*c**4*d**3*x**3 + 19*sqrt(a*c**2 + 2*a*c
*d*x + a*d**2*x**2 + b)*a**2*b**2*c**3*d**4*x**4 - 8*sqrt(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b)*a*b**3*c**5 - 29*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b)*a*b**3*c**4*d*x - 19*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*
b**3*c**3*d**2*x**2 - 99*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**3
*c**2*d**3*x**3 - 42*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a*b**3*c*d
**4*x**4 - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**4*c**3 - 9*sqrt
(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*b**4*c**2*d*x - 18*sqrt(a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b)*b**4*c*d**2*x**2 + 6*sqrt(a*c**2 + 2*a*c*d*x...
```

3.95 $\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 406

$$\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = -\frac{d^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4\left(b+ac^2\right)^2\left(1-\frac{c}{c+dx}\right)^4} - \frac{\left(b-4ac^2\right)d^4\sqrt{a+\frac{b}{(c+dx)^2}}}{4\left(b+ac^2\right)^3\left(1-\frac{c}{c+dx}\right)^3} - \frac{\left(2b^2-27abc^2+12a^2c^4\right)d^4\sqrt{a+\frac{b}{(c+dx)^2}}}{8\left(b+ac^2\right)^4\left(1-\frac{c}{c+dx}\right)^2} - \frac{\left(2b^3-77ab^2c^2+100a^2bc^4-8a^3c^6\right)d^4\sqrt{a+\frac{b}{(c+dx)^2}}}{8\left(b+ac^2\right)^5\left(1-\frac{c}{c+dx}\right)} + \frac{abd^4(10abc^2(2c+dx)-b^2(6c+dx)-a^2c^4(c+5(c+dx)))}{\left(b+ac^2\right)^5(c+dx)\sqrt{a+\frac{b}{(c+dx)^2}}} + \frac{15abc\left(4b^2-13abc^2+4a^2c^4\right)d^4\operatorname{arctanh}\left(\frac{ac+\frac{b}{c+dx}}{\sqrt{b+ac^2}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{8\left(b+ac^2\right)^{11/2}}$$

output
```
-1/4*d^4*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^2/(1-c/(d*x+c))^4-1/4*(-4*a*c^2+b
)*d^4*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^3/(1-c/(d*x+c))^3-1/8*(12*a^2*c^4-27
*a*b*c^2+2*b^2)*d^4*(a+b/(d*x+c)^2)^(1/2)/(a*c^2+b)^4/(1-c/(d*x+c))^2-1/8*
(-8*a^3*c^6+100*a^2*b*c^4-77*a*b^2*c^2+2*b^3)*d^4*(a+b/(d*x+c)^2)^(1/2)/(a
*c^2+b)^5/(1-c/(d*x+c))+a*b*d^4*(10*a*b*c^2*(d*x+2*c)-b^2*(d*x+6*c)-a^2*c^
4*(5*d*x+6*c))/(a*c^2+b)^5/(d*x+c)/(a+b/(d*x+c)^2)^(1/2)+15/8*a*b*c*(4*a^2
*c^4-13*a*b*c^2+4*b^2)*d^4*arctanh((a*c+b/(d*x+c))/(a*c^2+b)^(1/2)/(a+b/(d
*x+c)^2)^(1/2))/(a*c^2+b)^(11/2)
```

Mathematica [A] (verified)

Time = 10.94 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \frac{1}{8}\left(-\frac{2b^4(c^3+4c^2dx+6cd^2x^2+4d^3x^3)+3a^2b^2c^2(4c^5+10c^4dx-2c^3d^2x^2-1\cdots}{\cdots} - \frac{15abc(4b^2-13abc^2+4a^2c^4)\,d^4\log(x)}{(b+ac^2)^{11/2}} + \frac{15abc(4b^2-13abc^2+4a^2c^4)\,d^4\log\left(b+(c+dx)\left(ac+\sqrt{b+ac^2}\sqrt{\frac{b+a(c+dx)^2}{(c+dx)^2}}\right)\right)}{(b+ac^2)^{11/2}}\right)$$

input
```
Integrate[1/(x^5*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
(-((2*b^4*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3) + 3*a^2*b^2*c^2*(4*c
^5 + 10*c^4*d*x - 2*c^3*d^2*x^2 - 13*c^2*d^3*x^3 - 125*c*d^4*x^4 - 60*d^5*
x^5) + 2*a^4*c^6*(c^5 + c^4*d*x - c*d^4*x^4 - d^5*x^5) + a*b^3*(8*c^5 + 26
*c^4*d*x + 15*c^3*d^2*x^2 - 60*c^2*d^3*x^3 + 76*c*d^4*x^4 + 16*d^5*x^5) +
a^3*b*c^4*(8*c^5 + 14*c^4*d*x - 9*c^3*d^2*x^2 + 29*c^2*d^3*x^3 + 177*c*d^4
*x^4 + 117*d^5*x^5))/((b + a*c^2)^5*x^4*(c + d*x)*Sqrt[(b + a*(c + d*x)^2)
/(c + d*x)^2])) - (15*a*b*c*(4*b^2 - 13*a*b*c^2 + 4*a^2*c^4)*d^4*Log[x])/(
b + a*c^2)^(11/2) + (15*a*b*c*(4*b^2 - 13*a*b*c^2 + 4*a^2*c^4)*d^4*Log[b +
 (c + d*x)*(a*c + Sqrt[b + a*c^2]*Sqrt[(b + a*(c + d*x)^2)/(c + d*x)^2])])
/(b + a*c^2)^(11/2))/8
```

Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.842, Rules used = {896, 25, 1774, 1803, 25, 601, 2182, 25, 2182, 27, 2182, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 896}$$

$$d^4 \int \frac{1}{d^5 x^5 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^4 \int -\frac{1}{d^5 x^5 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}d(c+dx)$$

$$\Big\downarrow \text{ 1774}$$

$$-d^4 \int \frac{1}{(c+dx)^5 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} \left(\frac{c}{c+dx}-1\right)^5}d(c+dx)$$

$$\Big\downarrow \text{ 1803}$$

$$d^4 \int -\frac{1}{(c+dx)^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^5}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 25}$$

$$-d^4 \int \frac{1}{(c+dx)^3 \left(a+\frac{b}{(c+dx)^2}\right)^{3/2} \left(1-\frac{c}{c+dx}\right)^5}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{ 601}$$

$$d^4\left(\frac{\int \frac{\frac{a^2b\left(5a^2c^4-10abc^2+b^2\right)c^5}{\left(ac^2+b\right)^5(c+dx)^4}-\frac{a^2\left(a^3c^6+15a^2bc^4-45ab^2c^2+5b^3\right)c^4}{\left(ac^2+b\right)^5(c+dx)^3}+\frac{2a^2b\left(5a^2c^4-38abc^2+5b^2\right)c^3}{\left(ac^2+b\right)^5(c+dx)^2}+\frac{a^2b\left(a^2c^4-10abc^2+5b^2\right)c}{\left(ac^2+b\right)^5}-\frac{ab\left(5a^3c^6-45a^2bc^4\cdots\right.}{\left(ac^2+b\right)^5(\cdots}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^5}}{a}\cdots\right.$$

$\Big\downarrow$ 2182

$$d^4\left(\frac{-\frac{\int -\frac{-\frac{4a^2b\left(5a^2c^4-10abc^2+b^2\right)c^4}{\left(ac^2+b\right)^4(c+dx)^3}+\frac{4a^2\left(a^3c^6+10a^2bc^4-35ab^2c^2+4b^3\right)c^3}{\left(ac^2+b\right)^4(c+dx)^2}+\frac{4a^2\left(a^3c^6+4a^2bc^4-7ab^2c^2+6b^3\right)c}{\left(ac^2+b\right)^4}-\frac{a\left(-4a^4c^8+3a^3bc^6-155a^2b^2c^4+33a\cdots\right.}{\left(ac^2+b\right)^4(c+dx)}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^4}}{4(ac^2+b)}}{a}\cdots\right.$$

$\Big\downarrow$ 25

$$d^4\left(\frac{\frac{\int \frac{-\frac{4a^2b\left(5a^2c^4-10abc^2+b^2\right)c^4}{\left(ac^2+b\right)^4(c+dx)^3}+\frac{4a^2\left(a^3c^6+10a^2bc^4-35ab^2c^2+4b^3\right)c^3}{\left(ac^2+b\right)^4(c+dx)^2}+\frac{4a^2\left(a^3c^6+4a^2bc^4-7ab^2c^2+6b^3\right)c}{\left(ac^2+b\right)^4}-\frac{a\left(-4a^4c^8+3a^3bc^6-155a^2b^2c^4+33ab^3c^2\cdots\right.}{\left(ac^2+b\right)^4(c+dx)}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^4}}{4(ac^2+b)}}{a}\cdots\right.$$

$\Big\downarrow$ 2182

$$d^4\left(\frac{\frac{-\frac{\int -\frac{3\left(\frac{4a^2b\left(5a^2c^4-10abc^2+b^2\right)c^3}{\left(ac^2+b\right)^3(c+dx)^2}+\frac{a^2\left(-8a^3c^6-5a^2bc^4-34ab^2c^2+27b^3\right)c}{\left(ac^2+b\right)^3}-\frac{2a\left(2a^4c^8+6a^3bc^6-57a^2b^2c^4+4ab^3c^2+b^4\right)}{\left(ac^2+b\right)^3(c+dx)}\right)}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{3\left(ac^2+b\right)}-\frac{a\left(b-4ac^2\right)\sqrt{a+\cdots}}{\left(ac^2+b\right)^2\left(1-\cdots\right.}}{4(ac^2+b)}}{a}\cdots\right.$$

$\downarrow$ 27

$$d^4\left(\frac{\frac{\frac{\int \frac{\frac{4a^2b\left(5a^2c^4-10abc^2+b^2\right)c^3}{\left(ac^2+b\right)^3(c+dx)^2}+\frac{a^2\left(-8a^3c^6-5a^2bc^4-34ab^2c^2+27b^3\right)c}{\left(ac^2+b\right)^3}-\frac{2a\left(2a^4c^8+6a^3bc^6-57a^2b^2c^4+4ab^3c^2+b^4\right)}{\left(ac^2+b\right)^3(c+dx)}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}}{ac^2+b}-\frac{a\left(b-4ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^3}}{4(ac^2+b)}}{a}\right.$$

$\downarrow$ 2182

$$d^4\left(\frac{\frac{\frac{-\frac{\int -\frac{a\left(2ac\left(4a^3c^6-20a^2bc^4-59ab^2c^2+29b^3\right)-\frac{b\left(52a^3c^6-95a^2bc^4-17ab^2c^2+2b^3\right)}{c+dx}\right)}{\left(ac^2+b\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)}-\frac{a\left(12a^2c^4-27abc^2+2b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{ac^2+b}-\frac{a\left(b-4ac^2\right)\sqrt{a}}{\left(ac^2+b\right)^2\left(1\right.}}{4(ac^2+b)}}{a}\right.$$

$\downarrow$ 25

$$d^4\left(\frac{\frac{\frac{\frac{\int \frac{a\left(2ac\left(4a^3c^6-20a^2bc^4-59ab^2c^2+29b^3\right)-\frac{b\left(52a^3c^6-95a^2bc^4-17ab^2c^2+2b^3\right)}{c+dx}\right)}{\left(ac^2+b\right)^2\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)}-\frac{a\left(12a^2c^4-27abc^2+2b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{ac^2+b}-\frac{a\left(b-4ac^2\right)\sqrt{a+\frac{}{(}}}{\left(ac^2+b\right)^2\left(1-\frac{c}{c-}\right.}}{4(ac^2+b)}}{a}\right.$$

$\Big\downarrow$ 27

$$d^4\left(\frac{\frac{\frac{\frac{a\int \frac{2ac\left(4a^3c^6-20a^2bc^4-59ab^2c^2+29b^3\right)-\frac{b\left(52a^3c^6-95a^2bc^4-17ab^2c^2+2b^3\right)}{c+dx}}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)^3}-\frac{a\left(12a^2c^4-27abc^2+2b^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)^2}}{ac^2+b}-\frac{a\left(b-4ac^2\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)\cdots}}{4(ac^2+b)}\cdots}{a}\cdots\right.$$

$\Big\downarrow$ 679

$$d^4\left(\frac{\frac{\frac{a\left(15abc\left(4a^2c^4-13abc^2+4b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{(c+dx)^2}}\left(1-\frac{c}{c+dx}\right)}d\frac{1}{c+dx}-\frac{\left(-8a^3c^6+100a^2bc^4-77ab^2c^2+2b^3\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}\right)}{2\left(ac^2+b\right)^3}-\frac{a\left(12a^2c^4-27abc^2+2b^2\right)\sqrt{a+\cdots}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+dx}\right)\cdots}}{ac^2+b}\cdots}{4(ac^2+b)}\cdots}{a}\cdots\right.$$

$\Big\downarrow$ 488

$$d^4\left(\frac{\frac{\frac{a\left(-15abc\left(4a^2c^4-13abc^2+4b^2\right)\int \frac{1}{ac^2+b-\frac{1}{(c+dx)^2}}d\frac{-\frac{b}{c+dx}-ac}{\sqrt{a+\frac{b}{(c+dx)^2}}}-\frac{\left(-8a^3c^6+100a^2bc^4-77ab^2c^2+2b^3\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}\right)}{2\left(ac^2+b\right)^3}-\frac{a\left(12a^2c^4-27abc^2+2b^2\right)\sqrt{\cdots}}{2\left(ac^2+b\right)^2\left(1-\frac{c}{c+\cdots}\right)\cdots}}{ac^2+b}\cdots}{4(ac^2+b)}\cdots}{a}\cdots\right.$$

$\Big\downarrow$ 219

$$d^4\left(\frac{\frac{\frac{a\left(-\frac{15abc\left(4a^2c^4-13abc^2+4b^2\right)\text{arctanh}\left(\frac{-ac-\frac{b}{c+dx}}{\sqrt{ac^2+b}\sqrt{a+\frac{b}{(c+dx)^2}}}\right)}{\sqrt{ac^2+b}}-\frac{\left(-8a^3c^6+100a^2bc^4-77ab^2c^2+2b^3\right)\sqrt{a+\frac{b}{(c+dx)^2}}}{1-\frac{c}{c+dx}}\right)}{2\left(ac^2+b\right)^3}-\frac{a\left(12a^2c^4-27abc^2+2b^2\right.}{2\left(ac^2+b\right)^2\left(1-\frac{c}{}\right.}}{ac^2+b}}{4(ac^2+b)}}{a}\right.$$

input
```
Int[1/(x^5*(a + b/(c + d*x)^2)^(3/2)),x]
```

output
```
d^4*(-((a*b*(b^2 - 10*a*b*c^2 + 5*a^2*c^4 + (c*(5*b^2 - 10*a*b*c^2 + a^2*c
^4))/(c + d*x)))/((b + a*c^2)^5*Sqrt[a + b/(c + d*x)^2])) + (-1/4*(a*Sqrt[
a + b/(c + d*x)^2])/((b + a*c^2)^2*(1 - c/(c + d*x))^4) + (-((a*(b - 4*a*c
^2)*Sqrt[a + b/(c + d*x)^2])/((b + a*c^2)^2*(1 - c/(c + d*x))^3)) + (-1/2*
(a*(2*b^2 - 27*a*b*c^2 + 12*a^2*c^4)*Sqrt[a + b/(c + d*x)^2])/((b + a*c^2)
^2*(1 - c/(c + d*x))^2) + (a*(-(((2*b^3 - 77*a*b^2*c^2 + 100*a^2*b*c^4 - 8
*a^3*c^6)*Sqrt[a + b/(c + d*x)^2])/(1 - c/(c + d*x))) - (15*a*b*c*(4*b^2 -
 13*a*b*c^2 + 4*a^2*c^4)*ArcTanh[(-(a*c) - b/(c + d*x))/(Sqrt[b + a*c^2]*S
qrt[a + b/(c + d*x)^2])])/Sqrt[b + a*c^2]))/(2*(b + a*c^2)^3))/(b + a*c^2)
)/(4*(b + a*c^2)))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(379) = 758$.

Time = 0.16 (sec) , antiderivative size = 1001, normalized size of antiderivative = 2.47

method	result
default	$-\frac{(a\,d^2x^2+2adxc+a\,c^2+b)\left(-2(a\,c^2+b)^{\frac{3}{2}}a^4c^6d^5x^5-2(a\,c^2+b)^{\frac{3}{2}}a^4c^7d^4x^4+2(a\,c^2+b)^{\frac{3}{2}}a^4c^{10}dx+16(a\,c^2+b)^{\frac{3}{2}}a\,b^3d^5x^5+12(a\,c^2\right.}{}$
risch	Expression too large to display

input
```
int(1/x^5/(a+b/(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)*(-2*(a*c^2+b)^(3/2)*a^4*c^6*d^5*x^5-2*(
a*c^2+b)^(3/2)*a^4*c^7*d^4*x^4+2*(a*c^2+b)^(3/2)*a^4*c^10*d*x+16*(a*c^2+b)
^(3/2)*a*b^3*d^5*x^5+12*(a*c^2+b)^(3/2)*b^4*c*d^2*x^2+8*(a*c^2+b)^(3/2)*b^
4*c^2*d*x+135*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c
^2+b)^(1/2)+b)/x)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*a^2*b^3*c^3*d^4*x^4-
60*ln(2*(a*d*x*c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)
+b)/x)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)*a*b^4*c*d^4*x^4-60*ln(2*(a*d*x*
c+a*c^2+(a*c^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*d^2*x
^2+2*a*c*d*x+a*c^2+b)^(1/2)*a^4*b*c^7*d^4*x^4+135*ln(2*(a*d*x*c+a*c^2+(a*c
^2+b)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b)^(1/2)+b)/x)*(a*d^2*x^2+2*a*c*d*x
+a*c^2+b)^(1/2)*a^3*b^2*c^5*d^4*x^4+8*(a*c^2+b)^(3/2)*a^3*b*c^9+12*(a*c^2+
b)^(3/2)*a^2*b^2*c^7+8*(a*c^2+b)^(3/2)*b^4*d^3*x^3+8*(a*c^2+b)^(3/2)*a*b^3
*c^5+76*(a*c^2+b)^(3/2)*a*b^3*c*d^4*x^4+30*(a*c^2+b)^(3/2)*a^2*b^2*c^6*d*x
-60*(a*c^2+b)^(3/2)*a*b^3*c^2*d^3*x^3+15*(a*c^2+b)^(3/2)*a*b^3*c^3*d^2*x^2
+26*(a*c^2+b)^(3/2)*a*b^3*c^4*d*x+117*(a*c^2+b)^(3/2)*a^3*b*c^4*d^5*x^5+17
7*(a*c^2+b)^(3/2)*a^3*b*c^5*d^4*x^4+29*(a*c^2+b)^(3/2)*a^3*b*c^6*d^3*x^3-1
80*(a*c^2+b)^(3/2)*a^2*b^2*c^2*d^5*x^5-9*(a*c^2+b)^(3/2)*a^3*b*c^7*d^2*x^2
-375*(a*c^2+b)^(3/2)*a^2*b^2*c^3*d^4*x^4+14*(a*c^2+b)^(3/2)*a^3*b*c^8*d*x-
39*(a*c^2+b)^(3/2)*a^2*b^2*c^4*d^3*x^3-6*(a*c^2+b)^(3/2)*a^2*b^2*c^5*d^2*x
^2+2*(a*c^2+b)^(3/2)*b^4*c^3+2*(a*c^2+b)^(3/2)*a^4*c^11)/((a*d^2*x^2+2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(376) = 752$.

Time = 9.91 (sec) , antiderivative size = 1916, normalized size of antiderivative = 4.72

$$\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(a+b/(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(15*((4*a^4*b*c^5 - 13*a^3*b^2*c^3 + 4*a^2*b^3*c)*d^6*x^6 + 2*(4*a^4
*b*c^6 - 13*a^3*b^2*c^4 + 4*a^2*b^3*c^2)*d^5*x^5 + (4*a^4*b*c^7 - 9*a^3*b^
2*c^5 - 9*a^2*b^3*c^3 + 4*a*b^4*c)*d^4*x^4)*sqrt(a*c^2 + b)*log(-(2*a^2*c^
4 + (2*a^2*c^2 + a*b)*d^2*x^2 + 4*a*b*c^2 + 4*(a^2*c^3 + a*b*c)*d*x + 2*b^
2 + 2*(a*c*d^2*x^2 + a*c^3 + (2*a*c^2 + b)*d*x + b*c)*sqrt(a*c^2 + b)*sqrt
((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/x^2) - 2*
(2*a^5*c^14 + 10*a^4*b*c^12 + 20*a^3*b^2*c^10 + 20*a^2*b^3*c^8 - (2*a^5*c^
8 - 115*a^4*b*c^6 + 63*a^3*b^2*c^4 + 164*a^2*b^3*c^2 - 16*a*b^4)*d^6*x^6 +
 10*a*b^4*c^6 - (4*a^5*c^9 - 290*a^4*b*c^7 + 261*a^3*b^2*c^5 + 463*a^2*b^3
*c^3 - 92*a*b^4*c)*d^5*x^5 + 2*b^5*c^4 - 2*(a^5*c^10 - 102*a^4*b*c^8 + 104
*a^3*b^2*c^6 + 199*a^2*b^3*c^4 - 12*a*b^4*c^2 - 4*b^5)*d^4*x^4 + 5*(4*a^4*
b*c^9 - 5*a^3*b^2*c^7 - 18*a^2*b^3*c^5 - 5*a*b^4*c^3 + 4*b^5*c)*d^3*x^3 +
(2*a^5*c^12 + 7*a^4*b*c^10 + 29*a^3*b^2*c^8 + 65*a^2*b^3*c^6 + 61*a*b^4*c^
4 + 20*b^5*c^2)*d^2*x^2 + 2*(2*a^5*c^13 + 13*a^4*b*c^11 + 32*a^3*b^2*c^9 +
 38*a^2*b^3*c^7 + 22*a*b^4*c^5 + 5*b^5*c^3)*d*x)*sqrt((a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((a^7*c^12 + 6*a^6*b*c^10 + 15
*a^5*b^2*c^8 + 20*a^4*b^3*c^6 + 15*a^3*b^4*c^4 + 6*a^2*b^5*c^2 + a*b^6)*d^
2*x^6 + 2*(a^7*c^13 + 6*a^6*b*c^11 + 15*a^5*b^2*c^9 + 20*a^4*b^3*c^7 + 15*
a^3*b^4*c^5 + 6*a^2*b^5*c^3 + a*b^6*c)*d*x^5 + (a^7*c^14 + 7*a^6*b*c^12 +
21*a^5*b^2*c^10 + 35*a^4*b^3*c^8 + 35*a^3*b^4*c^6 + 21*a^2*b^5*c^4 + 7*...
```

Sympy [F]

$$\int \frac{1}{x^5 \left(a + \frac{b}{(c+dx)^2}\right)^{3/2}}\, dx = \int \frac{1}{x^5 \left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/x**5/(a+b/(d*x+c)**2)**(3/2),x)
```

output
```
Integral(1/(x**5*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
 d**2*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{(dx+c)^2}\right)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate(1/x^5/(a+b/(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x + c)^2)^(3/2)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2896 vs. $2(376) = 752$.

Time = 0.25 (sec) , antiderivative size = 2896, normalized size of antiderivative = 7.13

$$\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^5/(a+b/(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-((5*a^8*b^2*c^14*d^6*sgn(d*x + c) + 15*a^7*b^3*c^12*d^6*sgn(d*x + c) + a^
6*b^4*c^10*d^6*sgn(d*x + c) - 45*a^5*b^5*c^8*d^6*sgn(d*x + c) - 65*a^4*b^6
*c^6*d^6*sgn(d*x + c) - 35*a^3*b^7*c^4*d^6*sgn(d*x + c) - 5*a^2*b^8*c^2*d^
6*sgn(d*x + c) + a*b^9*d^6*sgn(d*x + c))*x/(a^10*b*c^20*d + 10*a^9*b^2*c^1
8*d + 45*a^8*b^3*c^16*d + 120*a^7*b^4*c^14*d + 210*a^6*b^5*c^12*d + 252*a^
5*b^6*c^10*d + 210*a^4*b^7*c^8*d + 120*a^3*b^8*c^6*d + 45*a^2*b^9*c^4*d +
10*a*b^10*c^2*d + b^11*d) + 2*(3*a^8*b^2*c^15*d^5*sgn(d*x + c) + 5*a^7*b^3
*c^13*d^5*sgn(d*x + c) - 17*a^6*b^4*c^11*d^5*sgn(d*x + c) - 55*a^5*b^5*c^9
*d^5*sgn(d*x + c) - 55*a^4*b^6*c^7*d^5*sgn(d*x + c) - 17*a^3*b^7*c^5*d^5*s
gn(d*x + c) + 5*a^2*b^8*c^3*d^5*sgn(d*x + c) + 3*a*b^9*c*d^5*sgn(d*x + c))
/(a^10*b*c^20*d + 10*a^9*b^2*c^18*d + 45*a^8*b^3*c^16*d + 120*a^7*b^4*c^14
*d + 210*a^6*b^5*c^12*d + 252*a^5*b^6*c^10*d + 210*a^4*b^7*c^8*d + 120*a^3
*b^8*c^6*d + 45*a^2*b^9*c^4*d + 10*a*b^10*c^2*d + b^11*d))/sqrt(a*d^2*x^2
+ 2*a*c*d*x + a*c^2 + b) - 15/4*(4*a^3*b*c^5*d^4 - 13*a^2*b^2*c^3*d^4 + 4*
a*b^3*c*d^4)*arctan(-(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b))/sqrt(-a*c^2 - b))/((a^5*c^10*sgn(d*x + c) + 5*a^4*b*c^8*sgn(d*x + c)
+ 10*a^3*b^2*c^6*sgn(d*x + c) + 10*a^2*b^3*c^4*sgn(d*x + c) + 5*a*b^4*c^2*
sgn(d*x + c) + b^5*sgn(d*x + c))*sqrt(-a*c^2 - b)) + 1/4*(16*a^(15/2)*c^14
*d^3*abs(d) + 64*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b))
*a^7*c^13*d^4 + 96*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx=\int \frac{1}{x^5\left(a+\frac{b}{(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(x^5*(a + b/(c + d*x)^2)^(3/2)),x)
```

output
```
int(1/(x^5*(a + b/(c + d*x)^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2386, normalized size of antiderivative = 5.88

$$\int \frac{1}{x^5\left(a+\frac{b}{(c+dx)^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^5/(a+b/(d*x+c)^2)^(3/2),x)
```

output
```
( - 2*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**5*c**13 - 2*sqrt(a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**5*c**12*d*x + 2*sqrt(a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)*a**5*c**9*d**4*x**4 + 2*sqrt(a*c**2 + 2*a*c*d*x + a*d
**2*x**2 + b)*a**5*c**8*d**5*x**5 - 10*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b)*a**4*b*c**11 - 16*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**4*
b*c**10*d*x + 9*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**4*b*c**9*d**
2*x**2 - 29*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**4*b*c**8*d**3*x*
*3 - 175*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**4*b*c**7*d**4*x**4
- 115*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**4*b*c**6*d**5*x**5 - 2
0*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*b**2*c**9 - 44*sqrt(a*c*
*2 + 2*a*c*d*x + a*d**2*x**2 + b)*a**3*b**2*c**8*d*x + 15*sqrt(a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b)*a**3*b**2*c**7*d**2*x**2 + 10*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*a**3*b**2*c**6*d**3*x**3 + 198*sqrt(a*c**2 + 2*a
*c*d*x + a*d**2*x**2 + b)*a**3*b**2*c**5*d**4*x**4 + 63*sqrt(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b)*a**3*b**2*c**4*d**5*x**5 - 20*sqrt(a*c**2 + 2*a*c
*d*x + a*d**2*x**2 + b)*a**2*b**3*c**7 - 56*sqrt(a*c**2 + 2*a*c*d*x + a*d*
*2*x**2 + b)*a**2*b**3*c**6*d*x - 9*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b)*a**2*b**3*c**5*d**2*x**2 + 99*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)*a**2*b**3*c**4*d**3*x**3 + 299*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)*a**2*b**3*c**3*d**4*x**4 + 164*sqrt(a*c**2 + 2*a*c*d*x + a*d**2*x**...
```

3.96 $\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 214

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p dx=-\frac{c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}}{ad^4}+\frac{(c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}}{4ad^4}$$
$$-\frac{c(ac^2-b(1-2p))\,(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx)^2}\right)}{ad^4}$$
$$-\frac{b(6ac^2-b(1-p))\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx)^2}\right)}{4a^3d^4(1+p)}$$

output
```
-c*(d*x+c)^3*(a+b/(d*x+c)^2)^(p+1)/a/d^4+1/4*(d*x+c)^4*(a+b/(d*x+c)^2)^(p+
1)/a/d^4-c*(a*c^2-b*(1-2*p))*(d*x+c)*(a+b/(d*x+c)^2)^p*hypergeom([-1/2, -p
],[1/2],-b/a/(d*x+c)^2)/a/d^4/((1+b/a/(d*x+c)^2)^p)-1/4*b*(6*a*c^2-b*(1-p)
)*(a+b/(d*x+c)^2)^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(d*x+c)^2)/a^3/d^4/
(p+1)
```

Mathematica [F]

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$$

input
```
Integrate[x^3*(a + b/(c + d*x)^2)^p,x]
```

output
```
Integrate[x^3*(a + b/(c + d*x)^2)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {896, 25, 1774, 1803, 25, 543, 354, 87, 75, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int d^3x^3\left(a+\frac{b}{(c+dx)^2}\right)^p d(c+dx)}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -d^3x^3\left(a+\frac{b}{(c+dx)^2}\right)^p d(c+dx)}{d^4}$$

$$\Big\downarrow \text{1774}$$

$$-\frac{\int (c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{c}{c+dx}-1\right)^3 d(c+dx)}{d^4}$$

$$\Big\downarrow \text{1803}$$

$$\frac{\int -(c+dx)^5\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1-\frac{c}{c+dx}\right)^3 d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow 25$$

$$-\frac{\int (c+dx)^5 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(1-\frac{c}{c+dx}\right)^3 d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow 543$$

$$\frac{-\int (c+dx)^4 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(-\frac{c^3}{(c+dx)^2}-3c\right) d\frac{1}{c+dx}-\int (c+dx)^5 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(\frac{3c^2}{(c+dx)^2}+1\right) d\frac{1}{c+dx}}{d^4}$$

$$\Big\downarrow 354$$

$$\frac{-\int (c+dx)^4 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(-\frac{c^3}{(c+dx)^2}-3c\right) d\frac{1}{c+dx}-\frac{1}{2}\int (c+dx)^3 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(\frac{3c^2}{(c+dx)^2}+1\right) d\frac{1}{(c+dx)^2}}{d^4}$$

$$\Big\downarrow 87$$

$$\frac{\frac{1}{2}\left(\frac{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2a}-\frac{(6ac^2-b(1-p))\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{(c+dx)^2}}{2a}\right)-\int (c+dx)^4 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(-\frac{c^3}{(c+dx)^2}-3\right.}{d^4}$$

$$\Big\downarrow 75$$

$$\frac{\frac{1}{2}\left(\frac{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2a}-\frac{b(6ac^2-b(1-p))\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{2a^3(p+1)}\right)-\int (c+dx)^4 \left(a+\right.}{d^4}$$

$$\Big\downarrow 359$$

$$\frac{\frac{c(ac^2-b(1-2p))\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}}{a}+\frac{1}{2}\left(\frac{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2a}-\frac{b(6ac^2-b(1-p))\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(\right.}{2a^3(p+1)}\right.}{d^4}$$

$$\Big\downarrow 279$$

$$\frac{\frac{c(ac^2-b(1-2p))\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\int (c+dx)^2\left(\frac{b}{a(c+dx)^2}+1\right)^p d\frac{1}{c+dx}}{a}+\frac{1}{2}\left(\frac{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2a}-\frac{b(6ac^2-b(1-p))\left(a+\right.}{}\right.}{d^4}$$

$$\Big\downarrow 278$$

$$\frac{\frac{1}{2}\left(\frac{(c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2a}-\frac{b(6ac^2-b(1-p))\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{2a^3(p+1)}\right)-\frac{c(c+dx)(ac^2-b(1-2}{d^4}$$

input
```
Int[x^3*(a + b/(c + d*x)^2)^p,x]
```

output
```
(-((c*(c + d*x)^3*(a + b/(c + d*x)^2)^(1 + p))/a) - (c*(a*c^2 - b*(1 - 2*p
))*(c + d*x)*(a + b/(c + d*x)^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -(b/(a
*(c + d*x)^2))])/(a*(1 + b/(a*(c + d*x)^2))^p) + (((c + d*x)^2*(a + b/(c +
 d*x)^2)^(1 + p))/(2*a) - (b*(6*a*c^2 - b*(1 - p))*(a + b/(c + d*x)^2)^(1
+ p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + b/(a*(c + d*x)^2)])/(2*a^3*(1
+ p)))/2)/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^3\left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
int(x^3*(a+b/(d*x+c)^2)^p,x)
```

output
```
int(x^3*(a+b/(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral(x^3*((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2
))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b/(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c)^2)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int x^3\left(a+\frac{b}{(c+d\,x)^2}\right)^p\,dx$$

input
```
int(x^3*(a + b/(c + d*x)^2)^p,x)
```

output
```
int(x^3*(a + b/(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int x^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\frac{(a\,d^2x^2+2acdx+a\,c^2+b)^p\,x^3}{(d^2x^2+2cdx+c^2)^p}dx$$

input
```
int(x^3*(a+b/(d*x+c)^2)^p,x)
```

output
```
int(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p*x**3)/(c**2 + 2*c*d*x + d**
2*x**2)**p,x)
```

3.97 $\int x^2 \left(a+\frac{b}{(c+dx)^2}\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 168

$$\int x^2 \left(a+\frac{b}{(c+dx)^2}\right)^p \, dx = \frac{(c+dx)^3 \left(a+\frac{b}{(c+dx)^2}\right)^{1+p}}{3ad^3} + \frac{(3ac^2-b(1-2p))\,(c+dx) \left(a+\frac{b}{(c+dx)^2}\right)^p \left(1+\frac{b}{a(c+dx)^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx)^2}\right.}{3ad^3} + \frac{bc\left(a+\frac{b}{(c+dx)^2}\right)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx)^2}\right)}{a^2d^3(1+p)}$$

output
```
1/3*(d*x+c)^3*(a+b/(d*x+c)^2)^(p+1)/a/d^3+1/3*(3*a*c^2-b*(1-2*p))*(d*x+c)*
(a+b/(d*x+c)^2)^p*hypergeom([-1/2, -p],[1/2],-b/a/(d*x+c)^2)/a/d^3/((1+b/a
/(d*x+c)^2)^p)+b*c*(a+b/(d*x+c)^2)^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(d
*x+c)^2)/a^2/d^3/(p+1)
```

Mathematica [F]

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$$

input
```
Integrate[x^2*(a + b/(c + d*x)^2)^p,x]
```

output
```
Integrate[x^2*(a + b/(c + d*x)^2)^p, x]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {896, 1774, 1803, 543, 27, 243, 75, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int d^2x^2\left(a+\frac{b}{(c+dx)^2}\right)^p\,d(c+dx)}{d^3}$$

$$\Big\downarrow \ 1774$$

$$\frac{\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{c}{c+dx}-1\right)^2\,d(c+dx)}{d^3}$$

$$\Big\downarrow \ 1803$$

$$-\frac{\int (c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1-\frac{c}{c+dx}\right)^2\,d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \ 543$$

$$-\frac{\int (c+dx)^4\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{c^2}{(c+dx)^2}+1\right)\,d\frac{1}{c+dx}+\int -2c(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^p\,d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int (c+dx)^4 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(\frac{c^2}{(c+dx)^2}+1\right)d\frac{1}{c+dx}-2c\int (c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}}{d^3}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{\int (c+dx)^4 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(\frac{c^2}{(c+dx)^2}+1\right)d\frac{1}{c+dx}-c\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{(c+dx)^2}}{d^3}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{\int (c+dx)^4 \left(a+\frac{b}{(c+dx)^2}\right)^p \left(\frac{c^2}{(c+dx)^2}+1\right)d\frac{1}{c+dx}-\frac{bc\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{a^2(p+1)}}{d^3}$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{\frac{(3ac^2-b(1-2p))\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}}{3a}-\frac{bc\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{a^2(p+1)}-\frac{(c+dx)^3\left(a+\cdots\right.}{3a}}{d^3}$$

$$\Big\downarrow \text{ 279}$$

$$-\frac{\frac{(3ac^2-b(1-2p))\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\int (c+dx)^2\left(\frac{b}{a(c+dx)^2}+1\right)^p d\frac{1}{c+dx}}{3a}-\frac{bc\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\cdots\right.}{a^2(p+1)}}{d^3}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{-\frac{bc\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{a^2(p+1)}-\frac{(c+dx)(3ac^2-b(1-2p))\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{Hypergeor}\cdots}{3a}}{d^3}$$

input

```
Int[x^2*(a + b/(c + d*x)^2)^p,x]
```

output

```
-((-1/3*((c + d*x)^3*(a + b/(c + d*x)^2)^(1 + p))/a - ((3*a*c^2 - b*(1 - 2
*p))*(c + d*x)*(a + b/(c + d*x)^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -(b/
(a*(c + d*x)^2))])/(3*a*(1 + b/(a*(c + d*x)^2))^p) - (b*c*(a + b/(c + d*x)
^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + b/(a*(c + d*x)^2)])/(a^
2*(1 + p)))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^2 \left(a + \frac{b}{(dx+c)^2}\right)^p dx$$

input
```
int(x^2*(a+b/(d*x+c)^2)^p,x)
```

output
```
int(x^2*(a+b/(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral(x^2*((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2
))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b/(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^p*x^2, x)
```

Giac [F]

$$\int x^2 \left(a + \frac{b}{(c+dx)^2}\right)^p dx = \int \left(a + \frac{b}{(dx+c)^2}\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b/(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c)^2)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + \frac{b}{(c+dx)^2}\right)^p dx = \int x^2 \left(a + \frac{b}{(c+d\,x)^2}\right)^p dx$$

input
```
int(x^2*(a + b/(c + d*x)^2)^p,x)
```

output
```
int(x^2*(a + b/(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int x^2 \left(a + \frac{b}{(c+dx)^2}\right)^p dx = \int \frac{(a\,d^2x^2 + 2acdx + a\,c^2 + b)^p\,x^2}{(d^2x^2 + 2cdx + c^2)^p} dx$$

input
```
int(x^2*(a+b/(d*x+c)^2)^p,x)
```

output
```
int(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p*x**2)/(c**2 + 2*c*d*x + d**
2*x**2)**p,x)
```

3.98 $\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 119

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx = -\frac{c(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx)^2}\right)}{d^2} - \frac{b\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx)^2}\right)}{2a^2d^2(1+p)}$$

output

```
-c*(d*x+c)*(a+b/(d*x+c)^2)^p*hypergeom([-1/2, -p],[1/2],-b/a/(d*x+c)^2)/d^
2/((1+b/a/(d*x+c)^2)^p)-1/2*b*(a+b/(d*x+c)^2)^(p+1)*hypergeom([2, p+1],[2+
p],1+b/a/(d*x+c)^2)/a^2/d^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$$
$$=\frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{a(c+dx)^2}{b}\right)^{-p}\left(2c(-1+p)\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,\frac{3}{2}-p,-\frac{a(c+dx)^2}{b}\right)\right.}{2d^2(-1+p)(-1+2p)}$$

input
```
Integrate[x*(a + b/(c + d*x)^2)^p,x]
```

output
```
((c + d*x)*(a + b/(c + d*x)^2)^p*(2*c*(-1 + p)*Hypergeometric2F1[1/2 - p,
-p, 3/2 - p, -((a*(c + d*x)^2)/b)] - (-1 + 2*p)*(c + d*x)*Hypergeometric2F
1[1 - p, -p, 2 - p, -((a*(c + d*x)^2)/b)]))/(2*d^2*(-1 + p)*(-1 + 2*p)*(1
+ (a*(c + d*x)^2)/b)^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {896, 25, 1774, 1803, 25, 542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx$$
$$\Big\downarrow\ 896$$
$$\frac{\int dx\left(a+\frac{b}{(c+dx)^2}\right)^p d(c+dx)}{d^2}$$
$$\Big\downarrow\ 25$$
$$-\frac{\int -dx\left(a+\frac{b}{(c+dx)^2}\right)^p d(c+dx)}{d^2}$$
$$\Big\downarrow\ 1774$$

$$-\frac{\int (c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{c}{c+dx}-1\right)d(c+dx)}{d^2}$$

$\big\downarrow$ 1803

$$\frac{\int -(c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$\big\downarrow$ 25

$$-\frac{\int (c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1-\frac{c}{c+dx}\right)d\frac{1}{c+dx}}{d^2}$$

$\big\downarrow$ 542

$$\frac{c\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}-\int (c+dx)^3\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}}{d^2}$$

$\big\downarrow$ 243

$$\frac{c\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}-\frac{1}{2}\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{(c+dx)^2}}{d^2}$$

$\big\downarrow$ 75

$$\frac{c\int (c+dx)^2\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}-\frac{b\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{2a^2(p+1)}}{d^2}$$

$\big\downarrow$ 279

$$\frac{c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\int (c+dx)^2\left(\frac{b}{a(c+dx)^2}+1\right)^p d\frac{1}{c+dx}-\frac{b\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\right.}{2a^2(p+1)}}{d^2}$$

$\big\downarrow$ 278

$$\frac{-\frac{b\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{2a^2(p+1)}-c(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{Hypergeome}}{d^2}$$

input
```
Int[x*(a + b/(c + d*x)^2)^p,x]
```

output
```
(-((c*(c + d*x)*(a + b/(c + d*x)^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -(b
/(a*(c + d*x)^2))])/(1 + b/(a*(c + d*x)^2))^p) - (b*(a + b/(c + d*x)^2)^(1
 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + b/(a*(c + d*x)^2)])/(2*a^2*(1
 + p)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x\left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
int(x*(a+b/(d*x+c)^2)^p,x)
```

output
```
int(x*(a+b/(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p dx=\int \left(a+\frac{b}{(dx+c)^2}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral(x*((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b/(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^p*x, x)
```

Giac [F]

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\left(a+\frac{b}{(dx+c)^2}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c)^2)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int x\,\left(a+\frac{b}{{\left(c+d\,x\right)}^2}\right)^p\,dx$$

input
```
int(x*(a + b/(c + d*x)^2)^p,x)
```

output
```
int(x*(a + b/(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int x\left(a+\frac{b}{(c+dx)^2}\right)^p\,dx=\int\frac{\left(a\,d^2x^2+2acdx+a\,c^2+b\right)^p x}{\left(d^2x^2+2cdx+c^2\right)^p}dx$$

input
```
int(x*(a+b/(d*x+c)^2)^p,x)
```

output
```
int(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p*x)/(c**2 + 2*c*d*x + d**2*x
**2)**p,x)
```

3.99 $\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx)^2}\right)}{d}$$

output
```
(d*x+c)*(a+b/(d*x+c)^2)^p*hypergeom([-1/2, -p],[1/2],-b/a/(d*x+c)^2)/d/((1
+b/a/(d*x+c)^2)^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx)^2}\right)}{d}$$

input
```
Integrate[(a + b/(c + d*x)^2)^p,x]
```

output
```
((c + d*x)*(a + b/(c + d*x)^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -(b/(a*(
c + d*x)^2))])/(d*(1 + b/(a*(c + d*x)^2))^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {239, 773, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \left(a+\frac{b}{(c+dx)^2}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int (c+dx)^2 \left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{279}$$

$$-\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p \left(\frac{b}{a(c+dx)^2}+1\right)^{-p} \int (c+dx)^2 \left(\frac{b}{a(c+dx)^2}+1\right)^p d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p \left(\frac{b}{a(c+dx)^2}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx)^2}\right)}{d}$$

input
```
Int[(a + b/(c + d*x)^2)^p,x]
```

output
```
((c + d*x)*(a + b/(c + d*x)^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -(b/(a*(
c + d*x)^2))])/(d*(1 + b/(a*(c + d*x)^2))^p)
```

Defintions of rubi rules used

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
int((a+b/(d*x+c)^2)^p,x)
```

output
```
int((a+b/(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
integrate((a+b/(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral(((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^p
, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int \left(a+\frac{b}{(c+dx)^2}\right)^p dx$$

input
```
integrate((a+b/(d*x+c)**2)**p,x)
```

output
```
Integral((a + b/(c + d*x)**2)**p, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
integrate((a+b/(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^p, x)
```

Giac [F]

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
integrate((a+b/(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c)^2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = -\frac{\left(a+\frac{b}{(c+d\,x)^2}\right)^p (c+d\,x)\ {}_2\mathrm{F}_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a\,(c+d\,x)^2}{b}\right)}{d\,(2\,p-1)\,\left(\frac{a\,(c+d\,x)^2}{b}+1\right)^p}$$

input
```
int((a + b/(c + d*x)^2)^p,x)
```

output
```
-((a + b/(c + d*x)^2)^p*(c + d*x)*hypergeom([1/2 - p, -p], 3/2 - p, -(a*(c
 + d*x)^2)/b))/(d*(2*p - 1)*((a*(c + d*x)^2)/b + 1)^p)
```

Reduce [F]

$$\int \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int \frac{(a\,d^2x^2+2acdx+a\,c^2+b)^p}{(d^2x^2+2cdx+c^2)^p}dx$$

input
```
int((a+b/(d*x+c)^2)^p,x)
```

output
```
int((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p/(c**2 + 2*c*d*x + d**2*x**2)
**p,x)
```

3.100 $\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 194

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx = -\frac{c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx} - \frac{c^2\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c^2\left(a+\frac{b}{(c+dx)^2}\right)}{b+ac^2}\right)}{2\,(b+ac^2)\,(1+p)} + \frac{\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a(c+dx)^2}\right)}{2a(1+p)}$$

output
```
-c*(a+b/(d*x+c)^2)^p*AppellF1(1/2,1,-p,3/2,c^2/(d*x+c)^2,-b/a/(d*x+c)^2)/(
d*x+c)/((1+b/a/(d*x+c)^2)^p)-1/2*c^2*(a+b/(d*x+c)^2)^(p+1)*hypergeom([1, p
+1],[2+p],c^2*(a+b/(d*x+c)^2)/(a*c^2+b))/(a*c^2+b)/(p+1)+1/2*(a+b/(d*x+c)^
2)^(p+1)*hypergeom([1, p+1],[2+p],1+b/a/(d*x+c)^2)/a/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx$$

input
```
Integrate[(a + b/(c + d*x)^2)^p/x,x]
```

output
```
Integrate[(a + b/(c + d*x)^2)^p/x, x]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {896, 25, 1774, 1803, 25, 621, 334, 333, 354, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}dx$$
$$\Big\downarrow \text{ 896}$$
$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{dx}d(c+dx)$$
$$\Big\downarrow \text{ 25}$$
$$-\int -\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{dx}d(c+dx)$$
$$\Big\downarrow \text{ 1774}$$
$$-\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)\left(\frac{c}{c+dx}-1\right)}d(c+dx)$$
$$\Big\downarrow \text{ 1803}$$

$$\int -\frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 25$$

$$-\int \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 621$$

$$-c\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{c+dx}-\int \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 334$$

$$-c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\int \frac{\left(\frac{b}{a(c+dx)^2}+1\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{c+dx}-\int \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{c+dx}$$

$$\Big\downarrow \ 333$$

$$-\int \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{c+dx}-\frac{c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx}$$

$$\Big\downarrow \ 354$$

$$-\frac{1}{2}\int \frac{(c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{(c+dx)^2}-\frac{c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx}$$

$$\Big\downarrow \ 97$$

$$\frac{1}{2}\left(c^2\left(-\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{(c+dx)^2}\right)-\int (c+dx)\left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{(c+dx)^2}\right)-\frac{c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx}$$

$$\downarrow 75$$

$$\frac{1}{2}\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{a(p+1)}-c^2\int\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{(c+dx)^2}\right)-$$
$$\frac{c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx}$$

$$\downarrow 78$$

$$\frac{1}{2}\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b}{a(c+dx)^2}+1\right)}{a(p+1)}-\frac{c^2\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}\text{Hypergeometric2}}{(p+1)\,(c}\right.$$
$$\frac{c\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx}$$

input
```
Int[(a + b/(c + d*x)^2)^p/x,x]
```

output
```
-((c*(a + b/(c + d*x)^2)^p*AppellF1[1/2, -p, 1, 3/2, -(b/(a*(c + d*x)^2)),
 c^2/(c + d*x)^2])/((c + d*x)*(1 + b/(a*(c + d*x)^2))^p)) + (-((c^2*(a + b
/(c + d*x)^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (c^2*(a + b/(c +
d*x)^2))/(b + a*c^2)])/((b + a*c^2)*(1 + p))) + ((a + b/(c + d*x)^2)^(1 +
p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + b/(a*(c + d*x)^2)])/(a*(1 + p)))
/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x}dx$$

input
```
int((a+b/(d*x+c)^2)^p/x,x)
```

output
```
int((a+b/(d*x+c)^2)^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x,x, algorithm="fricas")
```

output
```
integral(((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^p
/x, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx=\int \frac{\left(\frac{ac^2+2acdx+ad^2x^2+b}{c^2+2cdx+d^2x^2}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)**2)**p/x,x)
```

output
```
Integral(((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**
*2))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c)^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx = \int \frac{\left(a+\frac{b}{(c+d\,x)^2}\right)^p}{x}\,dx$$

input
```
int((a + b/(c + d*x)^2)^p/x,x)
```

output
```
int((a + b/(c + d*x)^2)^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x}\,dx = \int \frac{(a\,d^2x^2+2acdx+a\,c^2+b)^p}{(d^2x^2+2cdx+c^2)^p\,x}dx$$

input
```
int((a+b/(d*x+c)^2)^p/x,x)
```

output
```
int((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p/((c**2 + 2*c*d*x + d**2*x**2
)**p*x),x)
```

3.101 $\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 224

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx
&= -\frac{d\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx}\\
&\quad-\frac{c^2d\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{3(c+dx)^3}\\
&\quad-\frac{bcd\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c^2\left(a+\frac{b}{(c+dx)^2}\right)}{b+ac^2}\right)}{\left(b+ac^2\right)^2(1+p)}
\end{aligned}$$

output
```
-d*(a+b/(d*x+c)^2)^p*AppellF1(1/2,2,-p,3/2,c^2/(d*x+c)^2,-b/a/(d*x+c)^2)/(
d*x+c)/((1+b/a/(d*x+c)^2)^p)-1/3*c^2*d*(a+b/(d*x+c)^2)^p*AppellF1(3/2,2,-p
,5/2,c^2/(d*x+c)^2,-b/a/(d*x+c)^2)/(d*x+c)^3/((1+b/a/(d*x+c)^2)^p)-b*c*d*(
a+b/(d*x+c)^2)^(p+1)*hypergeom([2, p+1],[2+p],c^2*(a+b/(d*x+c)^2)/(a*c^2+b
))/(a*c^2+b)^2/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2} \, dx = \int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2} \, dx$$

input
```
Integrate[(a + b/(c + d*x)^2)^p/x^2,x]
```

output
```
Integrate[(a + b/(c + d*x)^2)^p/x^2, x]
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {896, 1774, 1799, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2} \, dx$$

$$\Big\downarrow \; 896$$

$$d \int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{d^2 x^2} d(c+dx)$$

$$\Big\downarrow \; 1774$$

$$d \int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)^2 \left(\frac{c}{c+dx}-1\right)^2} d(c+dx)$$

$$\Big\downarrow \; 1799$$

$$-d \int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{\left(1-\frac{c}{c+dx}\right)^2} d\frac{1}{c+dx}$$

$$\Big\downarrow 505$$

$$-d\int \left(\frac{2c\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)\left(\frac{c^2}{(c+dx)^2}-1\right)^2}+\frac{c^2\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)^2\left(\frac{c^2}{(c+dx)^2}-1\right)^2}+\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{\left(\frac{c^2}{(c+dx)^2}-1\right)^2}\right)d\frac{1}{c+dx}$$

$$\Big\downarrow 2009$$

$$-d\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c+dx}+\frac{c^2\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+\right.}{}\right.$$

input
```
Int[(a + b/(c + d*x)^2)^p/x^2,x]
```

output
```
-(d*(((a + b/(c + d*x)^2)^p*AppellF1[1/2, -p, 2, 3/2, -(b/(a*(c + d*x)^2))
, c^2/(c + d*x)^2])/((c + d*x)*(1 + b/(a*(c + d*x)^2))^p) + (c^2*(a + b/(c
 + d*x)^2)^p*AppellF1[3/2, -p, 2, 5/2, -(b/(a*(c + d*x)^2)), c^2/(c + d*x)
^2])/(3*(c + d*x)^3*(1 + b/(a*(c + d*x)^2))^p) + (b*c*(a + b/(c + d*x)^2)^
(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (c^2*(a + b/(c + d*x)^2))/(b +
a*c^2)])/((b + a*c^2)^2*(1 + p))))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^2}dx$$

input
```
int((a+b/(d*x+c)^2)^p/x^2,x)
```

output
```
int((a+b/(d*x+c)^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral(((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^p
/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b/(d*x+c)**2)**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c)^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{(c+d\,x)^2}\right)^p}{x^2}\,dx$$

input
```
int((a + b/(c + d*x)^2)^p/x^2,x)
```

output
```
int((a + b/(c + d*x)^2)^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^2}\,dx=\int \frac{(a\,d^2x^2+2acdx+a\,c^2+b)^p}{(d^2x^2+2cdx+c^2)^p\,x^2}dx$$

input
```
int((a+b/(d*x+c)^2)^p/x^2,x)
```

output
```
int((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p/((c**2 + 2*c*d*x + d**2*x**2
)**p*x**2),x)
```

3.102 $\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 375

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3} dx = -\frac{d^2\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}}{2(b+ac^2)\left(1-\frac{c}{c+dx}\right)^2}+\frac{d^2(ac^2+bp)\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}}{(b+ac^2)^2\left(1-\frac{c}{c+dx}\right)} - \frac{bd^2p(b+3ac^2+2bp)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx)^2},\frac{c^2}{(c+dx)^2}\right)}{c(b+ac^2)^2(c+dx)} + \frac{bd^2(1+2p)(ac^2+bp)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(1+\frac{b}{a(c+dx)^2}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{a(c+dx)^2}\right)}{c(b+ac^2)^2(c+dx)} - \frac{bd^2p(b+3ac^2+2bp)\left(a+\frac{b}{(c+dx)^2}\right)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c^2\left(a+\frac{b}{(c+dx)^2}\right)}{b+ac^2}\right)}{2(b+ac^2)^3(1+p)}$$

output

```
-1/2*d^2*(a+b/(d*x+c)^2)^(p+1)/(a*c^2+b)/(1-c/(d*x+c))^2+d^2*(a*c^2+b*p)*(
a+b/(d*x+c)^2)^(p+1)/(a*c^2+b)^2/(1-c/(d*x+c))-b*d^2*p*(3*a*c^2+2*b*p+b)*(
a+b/(d*x+c)^2)^p*AppellF1(1/2,1,-p,3/2,c^2/(d*x+c)^2,-b/a/(d*x+c)^2)/c/(a*
c^2+b)^2/(d*x+c)/((1+b/a/(d*x+c)^2)^p)+b*d^2*(1+2*p)*(a*c^2+b*p)*(a+b/(d*x
+c)^2)^p*hypergeom([1/2, -p],[3/2],-b/a/(d*x+c)^2)/c/(a*c^2+b)^2/(d*x+c)/(
(1+b/a/(d*x+c)^2)^p)-1/2*b*d^2*p*(3*a*c^2+2*b*p+b)*(a+b/(d*x+c)^2)^(p+1)*h
ypergeom([1, p+1],[2+p],c^2*(a+b/(d*x+c)^2)/(a*c^2+b))/(a*c^2+b)^3/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx = \int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx$$

input
```
Integrate[(a + b/(c + d*x)^2)^p/x^3,x]
```

output
```
Integrate[(a + b/(c + d*x)^2)^p/x^3, x]
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.059$, Rules used = {896, 25, 1774, 1803, 25, 594, 27, 688, 25, 27, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx$$

$$\Big\downarrow \ 896$$

$$d^2\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \ 25$$

$$-d^2\int -\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \ 1774$$

$$-d^2\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)^3\left(\frac{c}{c+dx}-1\right)^3}d(c+dx)$$

$$\Big\downarrow \text{1803}$$

$$d^2\int -\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{25}$$

$$-d^2\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)\left(1-\frac{c}{c+dx}\right)^3}d\frac{1}{c+dx}$$

$$\Big\downarrow \text{594}$$

$$d^2\left(\frac{\int \frac{2\left(a+\frac{b}{(c+dx)^2}\right)^p\left(ac+\frac{bp}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{2\left(ac^2+b\right)}-\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$d^2\left(\frac{\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p\left(ac+\frac{bp}{c+dx}\right)}{\left(1-\frac{c}{c+dx}\right)^2}d\frac{1}{c+dx}}{ac^2+b}-\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{688}$$

$$d^2\left(\frac{\frac{\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}-\frac{\int -\frac{b\left(a+\frac{b}{(c+dx)^2}\right)^p\left(ac(1-p)-\frac{(2p+1)\left(ac^2+bp\right)}{c+dx}\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{ac^2+b}}{ac^2+b}-\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$d^2\left(\frac{\frac{\int \frac{b\left(a+\frac{b}{(c+dx)^2}\right)^p\left(ac(1-p)-\frac{(2p+1)\left(ac^2+bp\right)}{c+dx}\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{ac^2+b}+\frac{\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}}{ac^2+b}-\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \; 27$$

$$d^2\left(\frac{\frac{b\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p\left(ac(1-p)-\frac{(2p+1)\left(ac^2+bp\right)}{c+dx}\right)}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{ac^2+b}+\frac{\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}}{ac^2+b}-\frac{\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{2\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)^2}\right)$$

$$\Big\downarrow \; 719$$

$$d^2\left(\frac{\frac{b\left(\frac{(2p+1)\left(ac^2+bp\right)\int \left(a+\frac{b}{(c+dx)^2}\right)^p d\frac{1}{c+dx}}{c}-\frac{p\left(3ac^2+2bp+b\right)\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{c}\right)}{ac^2+b}+\frac{\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^{p+1}}{\left(ac^2+b\right)\left(1-\frac{c}{c+dx}\right)}}{ac^2+b}-\frac{\left(a+\frac{b}{(c+d}{2\left(ac^2+b\right)\left(1\right.}\right.$$

$$\Big\downarrow \; 238$$

$$d^2\left(\frac{\frac{b\left(\frac{(2p+1)\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\int \left(\frac{b}{a(c+dx)^2}+1\right)^p d\frac{1}{c+dx}}{c}-\frac{p\left(3ac^2+2bp+b\right)\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{c}\right)}{ac^2+b}+\frac{\left(ac^2+bp\right)\left(a+\frac{}{(}\right.}{\left(ac^2+b\right)\left(1-\right.}}{ac^2+b}\right.$$

$\downarrow$ 237

$$d^2\left(\frac{\frac{b\left(\frac{(2p+1)\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{a(c+dx)^2}\right)}{c(c+dx)}-\frac{p\left(3ac^2+2bp+b\right)\int\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c}{c+dx}}d\frac{1}{c+dx}}{c}\right)}{ac^2+b}+}{ac^2+b}\right.$$

$\downarrow$ 504

$$d^2\left(\frac{\frac{b\left(\frac{(2p+1)\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{a(c+dx)^2}\right)}{c(c+dx)}-\frac{p\left(3ac^2+2bp+b\right)\left(\int\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{c+dx}+c\int\right.}{c}}{ac^2+b}}{ac^2+b}\right.$$

$\downarrow$ 334

$$d^2\left(\frac{\frac{b\left(\frac{(2p+1)\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{a(c+dx)^2}\right)}{c(c+dx)}-\frac{p\left(3ac^2+2bp+b\right)\left(\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)\right.}{}}{ac^2+b}}{ac^2+b}\right.$$

$\downarrow$ 333

$$d^2\left(\frac{\frac{b\left(\frac{(2p+1)\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{a(c+dx)^2}\right)}{c(c+dx)}-\frac{p\left(3ac^2+2bp+b\right)\left(c\int\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{(c+dx)\left(1-\frac{c^2}{(c+dx)^2}\right)}d\frac{1}{c+dx}\right.}{}\right.}{ac^2+b}}{ac^2+b}\right.$$

$\downarrow$ 353

$$d^2\left(\frac{\frac{b\left(\frac{(2p+1)\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{a(c+dx)^2}\right)}{c(c+dx)}-\frac{p\left(3ac^2+2bp+b\right)\left(\frac{1}{2}c\int\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{1-\frac{c^2}{(c+dx)^2}}d\frac{1}{(c+dx)^2}\right.}{}\right.}{ac^2+b}}{ac^2+b}\right.$$

$\downarrow$ 78

$$d^2 \Bigg(\frac{b \left(\frac{(2p+1)\left(ac^2+bp\right)\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{a(c+dx)^2}\right)}{c(c+dx)} - \frac{p\left(3ac^2+2bp+b\right)\left(\frac{\left(a+\frac{b}{(c+dx)^2}\right)^p\left(\frac{b}{a(c+dx)^2}+1\right)}{\;}\right.}{\;} \right.}{ac^2+b}$$

input
```
Int[(a + b/(c + d*x)^2)^p/x^3,x]
```

output
```
d^2*(-1/2*(a + b/(c + d*x)^2)^(1 + p)/((b + a*c^2)*(1 - c/(c + d*x))^2) +
(((a*c^2 + b*p)*(a + b/(c + d*x)^2)^(1 + p))/((b + a*c^2)*(1 - c/(c + d*x)
)) + (b*(((1 + 2*p)*(a*c^2 + b*p)*(a + b/(c + d*x)^2)^p*Hypergeometric2F1[
1/2, -p, 3/2, -(b/(a*(c + d*x)^2))])/(c*(c + d*x)*(1 + b/(a*(c + d*x)^2))^
p) - (p*(b + 3*a*c^2 + 2*b*p)*(((a + b/(c + d*x)^2)^p*AppellF1[1/2, -p, 1,
 3/2, -(b/(a*(c + d*x)^2)), c^2/(c + d*x)^2])/((c + d*x)*(1 + b/(a*(c + d*
x)^2))^p) + (c*(a + b/(c + d*x)^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, (c^2*(a + b/(c + d*x)^2))/(b + a*c^2)])/(2*(b + a*c^2)*(1 + p))))/c))/
(b + a*c^2))/(b + a*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^3}dx$$

input
```
int((a+b/(d*x+c)^2)^p/x^3,x)
```

output
```
int((a+b/(d*x+c)^2)^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x^3,x, algorithm="fricas")
```

output
```
integral((((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^p
/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(d*x+c)**2)**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx = \int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x + c)^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx = \int \frac{\left(a+\frac{b}{(dx+c)^2}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x+c)^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x + c)^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{(c+d\,x)^2}\right)^p}{x^3}\,dx$$

input
```
int((a + b/(c + d*x)^2)^p/x^3,x)
```

output
```
int((a + b/(c + d*x)^2)^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{(c+dx)^2}\right)^p}{x^3}\,dx=\int \frac{(a\,d^2x^2+2acdx+a\,c^2+b)^p}{(d^2x^2+2cdx+c^2)^p\,x^3}dx$$

input
```
int((a+b/(d*x+c)^2)^p/x^3,x)
```

output
```
int((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p/((c**2 + 2*c*d*x + d**2*x**2
)**p*x**3),x)
```

3.103 $\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p \, dx = \frac{x^{-1-m}(ex)^{1+m}\mathrm{Int}\left(x^m\left(a+\frac{b}{(c+dx)^2}\right)^p,x\right)}{e}$$

output
```
x^(-1-m)*(e*x)^(1+m)*Defer(Int)(x^m*(a+b/(d*x+c)^2)^p,x)/e
```

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p \, dx = \int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b/(c + d*x)^2)^p,x]
```

output
```
Integrate[(e*x)^m*(a + b/(c + d*x)^2)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx$$

input
```
Int[(e*x)^m*(a + b/(c + d*x)^2)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (ex)^m \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
int((e*x)^m*(a+b/(d*x+c)^2)^p,x)
```

output
```
int((e*x)^m*(a+b/(d*x+c)^2)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.63

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx=\int (ex)^m \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx=\text{Timed out}$$

input
```
integrate((e*x)**m*(a+b/(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int (ex)^m \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(a + b/(d*x + c)^2)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int (ex)^m \left(a+\frac{b}{(dx+c)^2}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(a + b/(d*x + c)^2)^p, x)
```

Mupad [N/A]

Not integrable

Time = 9.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx = \int (e\,x)^m \left(a+\frac{b}{(c+d\,x)^2}\right)^p dx$$

input
```
int((e*x)^m*(a + b/(c + d*x)^2)^p,x)
```

output
```
int((e*x)^m*(a + b/(c + d*x)^2)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int (ex)^m \left(a+\frac{b}{(c+dx)^2}\right)^p dx = e^m \left(\int \frac{x^m (a\,d^2x^2+2acdx+a\,c^2+b)^p}{(d^2x^2+2cdx+c^2)^p} dx\right)$$

input
```
int((e*x)^m*(a+b/(d*x+c)^2)^p,x)
```

output
```
e**m*int((x**m*(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)**p)/(c**2 + 2*c*d*x
+ d**2*x**2)**p,x)
```

3.104 $\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 dx = \frac{1}{20}\left(5a^2+b^2c\right)x^4+\frac{1}{5}b^2x^4(c+dx)-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{12abc(c+dx)^{7/2}}{7d^4}+\frac{4ab(c+dx)^{9/2}}{9d^4}$$

output
```
1/20*(b^2*c+5*a^2)*x^4+1/5*b^2*x^4*(d*x+c)-4/3*a*b*c^3*(d*x+c)^(3/2)/d^4+1
2/5*a*b*c^2*(d*x+c)^(5/2)/d^4-12/7*a*b*c*(d*x+c)^(7/2)/d^4+4/9*a*b*(d*x+c)
^(9/2)/d^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 dx = -\frac{16ab\sqrt{c+dx}\left(16c^4-8c^3dx+6c^2d^2x^2-5cd^3x^3-35d^4x^4\right)+315a^2\left(c^4-d^4x^4\right)+63b^2\left(c^5-5cd^4x^4-4\right.}{1260d^4}$$

input
```
Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
-1/1260*(16*a*b*Sqrt[c + d*x]*(16*c^4 - 8*c^3*d*x + 6*c^2*d^2*x^2 - 5*c*d^
3*x^3 - 35*d^4*x^4) + 315*a^2*(c^4 - d^4*x^4) + 63*b^2*(c^5 - 5*c*d^4*x^4
- 4*d^5*x^5))/d^4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.51, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2 dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int d^3x^3\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -d^3x^3\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^4}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -d^3x^3\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2 d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(-b^2(c+dx)^{9/2}-2ab(c+dx)^4-\left(a^2-3b^2c\right)(c+dx)^{7/2}+6abc(c+dx)^3-3c\left(b^2c-a^2\right)(c+dx)^{5/2}-6a}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{1}{4}c^2\left(3a^2-b^2c\right)(c+dx)^2-\frac{1}{8}\left(a^2-3b^2c\right)(c+dx)^4+\frac{1}{2}c\left(a^2-b^2c\right)(c+dx)^3+\frac{1}{2}a^2c^3(c+dx)+\frac{2}{3}abc^3(c+\right.}{d^4}$$

input
```
Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((a^2*c^3*(c + d*x))/2 + (2*a*b*c^3*(c + d*x)^(3/2))/3 - (c^2*(3*a^2 -
 b^2*c)*(c + d*x)^2)/4 - (6*a*b*c^2*(c + d*x)^(5/2))/5 + (c*(a^2 - b^2*c)*
(c + d*x)^3)/2 + (6*a*b*c*(c + d*x)^(7/2))/7 - ((a^2 - 3*b^2*c)*(c + d*x)^
4)/8 - (2*a*b*(c + d*x)^(9/2))/9 - (b^2*(c + d*x)^5)/10))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

method	result
default	$b^{2}\left(\frac{1}{5}d\,x^{5}+\frac{1}{4}c\,x^{4}\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{9}{2}}}{9}-\frac{3c(dx+c)^{\frac{7}{2}}}{7}+\frac{3c^{2}(dx+c)^{\frac{5}{2}}}{5}-\frac{c^{3}(dx+c)^{\frac{3}{2}}}{3}\right)}{d^{4}}+\frac{x^{4}a^{2}}{4}$
trager	$\frac{(4b^{2}dx+5b^{2}c+5a^{2})x^{4}}{20}-\frac{4ab(-35d^{4}x^{4}-5c\,d^{3}x^{3}+6d^{2}c^{2}x^{2}-8c^{3}dx+16c^{4})\sqrt{dx+c}}{315d^{4}}$
derivativedivides	$\frac{\frac{b^{2}(dx+c)^{5}}{5}+\frac{4ab(dx+c)^{\frac{9}{2}}}{9}+\frac{(-3b^{2}c+a^{2})(dx+c)^{4}}{4}-\frac{12cab(dx+c)^{\frac{7}{2}}}{7}+\frac{(3b^{2}c^{2}-3a^{2}c)(dx+c)^{3}}{3}+\frac{12c^{2}ab(dx+c)^{\frac{5}{2}}}{5}+\frac{(-c^{3}b^{2}+3a^{2}c^{2}}{2}}{d^{4}}$
orering	$-\frac{(476b^{2}x^{5}d^{5}+523b^{2}c\,x^{4}d^{4}-525a^{2}d^{4}x^{4}-34c^{2}d^{3}x^{3}b^{2}+30a^{2}c\,d^{3}x^{3}+80b^{2}c^{3}d^{2}x^{2}-48a^{2}c^{2}d^{2}x^{2}-416b^{2}c^{4}dx+96a^{2}d\,c^{3}}{1260d^{4}(-b^{2}dx-b^{2}c+a^{2})}$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/5*d*x^5+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)
+3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*x^4*a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int x^{3}\left(a+b\sqrt{c+dx}\right)^{2}\,dx$$
$$=\frac{252\,b^{2}d^{5}x^{5}+315\,(b^{2}c+a^{2})d^{4}x^{4}+16\,(35\,abd^{4}x^{4}+5\,abcd^{3}x^{3}-6\,abc^{2}d^{2}x^{2}+8\,abc^{3}dx-16\,abc^{4})\sqrt{dx+c}}{1260\,d^{4}}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 +
 5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*
x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.57

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{a^2c^3(c+dx)}{2}-\frac{2abc^3(c+dx)^{\frac{3}{2}}}{3}+\frac{6abc^2(c+dx)^{\frac{5}{2}}}{5}-\frac{6abc(c+dx)^{\frac{7}{2}}}{7}+\frac{2ab(c+dx)^{\frac{9}{2}}}{9}+\frac{b^2(c+dx)^5}{10}+\frac{\left(a^2-3b^2c\right)(c+dx)^4}{8}+\frac{(c+dx)^3\left(-3a^2c+3b^2c^2\right)}{6}+\cdots\right.}{d^4} \\ \frac{x^4\left(a+b\sqrt{c}\right)^2}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-a**2*c**3*(c + d*x)/2 - 2*a*b*c**3*(c + d*x)**(3/2)/3 + 6*a
*b*c**2*(c + d*x)**(5/2)/5 - 6*a*b*c*(c + d*x)**(7/2)/7 + 2*a*b*(c + d*x)*
*(9/2)/9 + b**2*(c + d*x)**5/10 + (a**2 - 3*b**2*c)*(c + d*x)**4/8 + (c +
d*x)**3*(-3*a**2*c + 3*b**2*c**2)/6 + (c + d*x)**2*(3*a**2*c**2 - b**2*c**
3)/4)/d**4, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$= \frac{252\,(dx+c)^5b^2+560\,(dx+c)^{\frac{9}{2}}ab-2160\,(dx+c)^{\frac{7}{2}}abc+3024\,(dx+c)^{\frac{5}{2}}abc^2-1680\,(dx+c)^{\frac{3}{2}}abc^3-12\cdots}{1260\cdots}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/
2)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 1
260*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 -
a^2*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$$

$$=\frac{252\,b^2d^2x^5+315\,b^2cdx^4+315\,a^2dx^4+\frac{144\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}c^3\right)abc}{d^3}+\frac{16\left(35\,(dx+c)^{\frac{9}{2}}-1\right.}{}}{1260\,d}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/1260*(252*b^2*d^2*x^5 + 315*b^2*c*d*x^4 + 315*a^2*d*x^4 + 144*(5*(d*x +
c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a*b*c/d^3 + 16*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d
*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d
^3)/d
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,{\left(c+d\,x\right)}^5}{5\,d^4}-\frac{\left(6\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^4}{8\,d^4}+\frac{\left(6\,a^2\,c^2-2\,b^2\,c^3\right)\,{\left(c+d\,x\right)}^2}{4\,d^4}-\frac{a^2\,c^3\,x}{d^3}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}+\frac{c\,\left(b^2\,c-a^2\right)\,{\left(c+d\,x\right)}^3}{d^4}-\frac{4\,a\,b\,c^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}+\frac{12\,a\,b\,c^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}-\frac{12\,a\,b\,c\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}$$

input
```
int(x^3*(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(b^2*(c + d*x)^5)/(5*d^4) - ((6*b^2*c - 2*a^2)*(c + d*x)^4)/(8*d^4) + ((6*
a^2*c^2 - 2*b^2*c^3)*(c + d*x)^2)/(4*d^4) - (a^2*c^3*x)/d^3 + (4*a*b*(c +
d*x)^(9/2))/(9*d^4) + (c*(b^2*c - a^2)*(c + d*x)^3)/d^4 - (4*a*b*c^3*(c +
d*x)^(3/2))/(3*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (12*a*b*c*(c
+ d*x)^(7/2))/(7*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{-256\sqrt{dx+c}\,ab\,c^4+128\sqrt{dx+c}\,ab\,c^3dx-96\sqrt{dx+c}\,ab\,c^2d^2x^2+80\sqrt{dx+c}\,abc\,d^3x^3+560\sqrt{dx+c}\,a}{1260d^4}$$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
( - 256*sqrt(c + d*x)*a*b*c**4 + 128*sqrt(c + d*x)*a*b*c**3*d*x - 96*sqrt(
c + d*x)*a*b*c**2*d**2*x**2 + 80*sqrt(c + d*x)*a*b*c*d**3*x**3 + 560*sqrt(
c + d*x)*a*b*d**4*x**4 + 315*a**2*d**4*x**4 + 315*b**2*c*d**4*x**4 + 252*b
**2*d**5*x**5)/(1260*d**4)
```

3.105 $\int x^2 \left(a+b\sqrt{c+dx}\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int x^2 \left(a+b\sqrt{c+dx}\right)^2 \, dx = \frac{1}{12}\left(4a^2+b^2c\right)x^3+\frac{1}{4}b^2x^3(c+dx)+\frac{4abc^2(c+dx)^{3/2}}{3d^3} - \frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}$$

output
```
1/12*(b^2*c+4*a^2)*x^3+1/4*b^2*x^3*(d*x+c)+4/3*a*b*c^2*(d*x+c)^(3/2)/d^3-8
/5*a*b*c*(d*x+c)^(5/2)/d^3+4/7*a*b*(d*x+c)^(7/2)/d^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int x^2 \left(a+b\sqrt{c+dx}\right)^2 \, dx = \frac{140a^2(c^3+d^3x^3)+16ab\sqrt{c+dx}(8c^3-4c^2dx+3cd^2x^2+15d^3x^3)+35b^2(c^4+4cd^3x^3+3d^4x^4)}{420d^3}$$

input
```
Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(140*a^2*(c^3 + d^3*x^3) + 16*a*b*Sqrt[c + d*x]*(8*c^3 - 4*c^2*d*x + 3*c*d
^2*x^2 + 15*d^3*x^3) + 35*b^2*(c^4 + 4*c*d^3*x^3 + 3*d^4*x^4))/(420*d^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int d^2x^2\left(a+b\sqrt{c+dx}\right)^2d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1732}$$

$$\frac{2\int d^2x^2\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{522}$$

$$\frac{2\int \left(b^2(c+dx)^{7/2}+2ab(c+dx)^3+\left(a^2-2b^2c\right)(c+dx)^{5/2}-4abc(c+dx)^2+c\left(b^2c-2a^2\right)(c+dx)^{3/2}+2abc^2(\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{6}\left(a^2-2b^2c\right)(c+dx)^3-\frac{1}{4}c\left(2a^2-b^2c\right)(c+dx)^2+\frac{1}{2}a^2c^2(c+dx)+\frac{2}{3}abc^2(c+dx)^{3/2}+\frac{2}{7}ab(c+dx)^{7/2}-\frac{4}{5}ab\right.}{d^3}$$

input
```
Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*((a^2*c^2*(c + d*x))/2 + (2*a*b*c^2*(c + d*x)^(3/2))/3 - (c*(2*a^2 - b^
2*c)*(c + d*x)^2)/4 - (4*a*b*c*(c + d*x)^(5/2))/5 + ((a^2 - 2*b^2*c)*(c +
d*x)^3)/6 + (2*a*b*(c + d*x)^(7/2))/7 + (b^2*(c + d*x)^4)/8))/d^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

method	result
default	$b^{2} \left(\frac{1}{4} d \,x^{4}+\frac{1}{3} c \,x^{3}\right)+\frac{4 a b \left(\frac{\left(d x +c \right)^{\frac{7}{2}}}{7}-\frac{2 c \left(d x +c \right)^{\frac{5}{2}}}{5}+\frac{c^{2} \left(d x +c \right)^{\frac{3}{2}}}{3}\right)}{d^{3}}+\frac{a^{2} x^{3}}{3}$
trager	$\frac{\left(3 b^{2} d x +4 b^{2} c +4 a^{2}\right) x^{3}}{12}+\frac{4 a b \left(15 d^{3} x^{3}+3 c \,d^{2} x^{2}-4 c^{2} d x +8 c^{3}\right) \sqrt{d x +c}}{105 d^{3}}$
derivativedivides	$\frac{\frac{b^{2} \left(d x +c \right)^{4}}{4}+\frac{4 a b \left(d x +c \right)^{\frac{7}{2}}}{7}+\frac{\left(-2 b^{2} c +a^{2}\right) \left(d x +c \right)^{3}}{3}-\frac{8 c a b \left(d x +c \right)^{\frac{5}{2}}}{5}+\frac{\left(b^{2} c^{2}-2 a^{2} c \right) \left(d x +c \right)^{2}}{2}+\frac{4 c^{2} a b \left(d x +c \right)^{\frac{3}{2}}}{3}+c^{2} a^{2} \left(d x +c \right)}{d^{3}}$
orering	$\frac{\left(-195 b^{2} d^{4} x^{4}-224 b^{2} c \,d^{3} x^{3}+220 a^{2} d^{3} x^{3}+32 d^{2} c^{2} x^{2} b^{2}-16 a^{2} c \,d^{2} x^{2}-160 b^{2} c^{3} d x +32 a^{2} c^{2} d x -192 b^{2} c^{4}+192 a^{2} c^{3}\right) \left(a +b \sqrt{\right.}{420 d^{3} \left(-b^{2} d x -b^{2} c +a^{2}\right)}$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/4*d*x^4+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*c*(d*x+c)^(5/2)
+1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{105\,b^2d^4x^4+140\,(b^2c+a^2)d^3x^3+16\,(15\,abd^3x^3+3\,abcd^2x^2-4\,abc^2dx+8\,abc^3)\sqrt{dx+c}}{420\,d^3}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 +
3*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.45

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx = \begin{cases} \frac{2\left(\frac{a^2c^2(c+dx)}{2}+\frac{2abc^2(c+dx)^{\frac{3}{2}}}{3}-\frac{4abc(c+dx)^{\frac{5}{2}}}{5}+\frac{2ab(c+dx)^{\frac{7}{2}}}{7}+\frac{b^2(c+dx)^4}{8}+\frac{(a^2-2b^2c)(c+dx)^3}{6}+\frac{(c+dx)^2(-2a^2c+b^2c^2)}{4}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^3(a+b\sqrt{c})^2}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(a**2*c**2*(c + d*x)/2 + 2*a*b*c**2*(c + d*x)**(3/2)/3 - 4*a*
b*c*(c + d*x)**(5/2)/5 + 2*a*b*(c + d*x)**(7/2)/7 + b**2*(c + d*x)**4/8 +
(a**2 - 2*b**2*c)*(c + d*x)**3/6 + (c + d*x)**2*(-2*a**2*c + b**2*c**2)/4)
/d**3, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{105\,(dx+c)^4b^2+240\,(dx+c)^{\frac{7}{2}}ab-672\,(dx+c)^{\frac{5}{2}}abc+560\,(dx+c)^{\frac{3}{2}}abc^2+420\,(dx+c)a^2c^2-140\,(2\,b}{420\,d^3}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x + c)^(5/2)
*a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*
c - a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.38

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{105\,b^2d^2x^4+140\,b^2cdx^3+140\,a^2dx^3+\frac{112\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}c^2\right)abc}{d^2}+\frac{48\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(}{d^2}}{420\,d}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/420*(105*b^2*d^2*x^4 + 140*b^2*c*d*x^3 + 140*a^2*d*x^3 + 112*(3*(d*x + c
)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d^2 + 48*(5*(
d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d
*x + c)*c^3)*a*b/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.35

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,(c+d\,x)^4}{4\,d^3}-\frac{\left(4\,a^2\,c-2\,b^2\,c^2\right)\,(c+d\,x)^2}{4\,d^3}-\frac{\left(4\,b^2\,c-2\,a^2\right)\,(c+d\,x)^3}{6\,d^3}+\frac{a^2\,c^2\,x}{d^2}+\frac{4\,a\,b\,(c+d\,x)^{7/2}}{7\,d^3}+\frac{4\,a\,b\,c^2\,(c+d\,x)^{3/2}}{3\,d^3}-\frac{8\,a\,b\,c\,(c+d\,x)^{5/2}}{5\,d^3}$$

input
```
int(x^2*(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(b^2*(c + d*x)^4)/(4*d^3) - ((4*a^2*c - 2*b^2*c^2)*(c + d*x)^2)/(4*d^3) -
((4*b^2*c - 2*a^2)*(c + d*x)^3)/(6*d^3) + (a^2*c^2*x)/d^2 + (4*a*b*(c + d*
x)^(7/2))/(7*d^3) + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (8*a*b*c*(c + d*
x)^(5/2))/(5*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{128\sqrt{dx+c}\,ab\,c^3-64\sqrt{dx+c}\,ab\,c^2dx+48\sqrt{dx+c}\,abc\,d^2x^2+240\sqrt{dx+c}\,ab\,d^3x^3+140a^2d^3x^3+140b}{420d^3}$$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
(128*sqrt(c + d*x)*a*b*c**3 - 64*sqrt(c + d*x)*a*b*c**2*d*x + 48*sqrt(c +
d*x)*a*b*c*d**2*x**2 + 240*sqrt(c + d*x)*a*b*d**3*x**3 + 140*a**2*d**3*x**
3 + 140*b**2*c*d**3*x**3 + 105*b**2*d**4*x**4)/(420*d**3)
```

3.106 $\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=-\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{(a^2-b^2c)\,(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

output
```
-a^2*c*x/d-4/3*a*b*c*(d*x+c)^(3/2)/d^2+1/2*(-b^2*c+a^2)*(d*x+c)^2/d^2+4/5*
a*b*(d*x+c)^(5/2)/d^2+1/3*b^2*(d*x+c)^3/d^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{(c+dx)\left(-15a^2(c-dx)-8ab(2c-3dx)\sqrt{c+dx}+5b^2(-c^2+cdx+2d^2x^2)\right)}{30d^2}$$

input
```
Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
((c + d*x)*(-15*a^2*(c - d*x) - 8*a*b*(2*c - 3*d*x)*Sqrt[c + d*x] + 5*b^2*
(-c^2 + c*d*x + 2*d^2*x^2)))/(30*d^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{c+dx}\right)^2 dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int dx\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -dx\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2 d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(-b^2(c+dx)^{5/2}-2ab(c+dx)^2-\left(a^2-b^2c\right)(c+dx)^{3/2}+2abc(c+dx)+a^2c\sqrt{c+dx}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{1}{4}\left(a^2-b^2c\right)(c+dx)^2+\frac{1}{2}a^2c(c+dx)-\frac{2}{5}ab(c+dx)^{5/2}+\frac{2}{3}abc(c+dx)^{3/2}-\frac{1}{6}b^2(c+dx)^3\right)}{d^2}$$

input
```
Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((a^2*c*(c + d*x))/2 + (2*a*b*c*(c + d*x)^(3/2))/3 - ((a^2 - b^2*c)*(c
 + d*x)^2)/4 - (2*a*b*(c + d*x)^(5/2))/5 - (b^2*(c + d*x)^3)/6))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

method	result
default	$b^{2}\left(\frac{1}{3}d\,x^{3}+\frac{1}{2}c\,x^{2}\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{c(dx+c)^{\frac{3}{2}}}{3}\right)}{d^{2}}+\frac{a^{2}x^{2}}{2}$
trager	$\frac{\left(2b^{2}dx+3b^{2}c+3a^{2}\right)x^{2}}{6}-\frac{4ab\left(-3d^{2}x^{2}-cdx+2c^{2}\right)\sqrt{dx+c}}{15d^{2}}$
derivativedivides	$\frac{\frac{b^{2}(dx+c)^{3}}{3}+\frac{4ab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)(dx+c)^{2}}{2}-\frac{4abc(dx+c)^{\frac{3}{2}}}{3}-c\,a^{2}(dx+c)}{d^{2}}$
orering	$-\frac{\left(18b^{2}d^{3}x^{3}+23b^{2}c\,d^{2}x^{2}-21a^{2}d^{2}x^{2}-14b^{2}c^{2}dx+2a^{2}cdx-16c^{3}b^{2}+16a^{2}c^{2}\right)\left(a+b\sqrt{dx+c}\right)^{2}}{30d^{2}\left(-b^{2}dx-b^{2}c+a^{2}\right)}+\frac{(2b^{2}d^{2}x^{2}+b^{2}cxd-3a^{2}}{}$

input
```
int(x*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/3*d*x^3+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*c*(d*x+c)^(3/2)
)+1/2*a^2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{10\,b^2d^3x^3+15\,(b^2c+a^2)d^2x^2+8\,(3\,abd^2x^2+abcdx-2\,abc^2)\sqrt{dx+c}}{30\,d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c
*d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \begin{cases} \frac{2\left(-\frac{a^2c(c+dx)}{2}-\frac{2abc(c+dx)^{\frac{3}{2}}}{3}+\frac{2ab(c+dx)^{\frac{5}{2}}}{5}+\frac{b^2(c+dx)^3}{6}+\frac{\left(a^2-b^2c\right)(c+dx)^2}{4}\right)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\left(a+b\sqrt{c}\right)^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-a**2*c*(c + d*x)/2 - 2*a*b*c*(c + d*x)**(3/2)/3 + 2*a*b*(c
+ d*x)**(5/2)/5 + b**2*(c + d*x)**3/6 + (a**2 - b**2*c)*(c + d*x)**2/4)/d*
*2, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{10\,(dx+c)^3b^2+24\,(dx+c)^{\frac{5}{2}}ab-40\,(dx+c)^{\frac{3}{2}}abc-30\,(dx+c)a^2c-15\,(b^2c-a^2)(dx+c)^2}{30\,d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b
*c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{10\,b^2d^2x^3+\frac{40\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)abc}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)b^2c}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)a^2}{d}+\frac{8\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)\right.}{d}}{30\,d}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/30*(10*b^2*d^2*x^3 + 40*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d +
15*((d*x + c)^2 - 2*(d*x + c)*c)*b^2*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c
)*a^2/d + 8*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a*b/d)/d
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,{\left(c+d\,x\right)}^3}{3\,d^2}-\frac{\left(2\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^2}{4\,d^2}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{5/2}}{5\,d^2}-\frac{a^2\,c\,x}{d}-\frac{4\,a\,b\,c\,{\left(c+d\,x\right)}^{3/2}}{3\,d^2}$$

input
```
int(x*(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(b^2*(c + d*x)^3)/(3*d^2) - ((2*b^2*c - 2*a^2)*(c + d*x)^2)/(4*d^2) + (4*a
*b*(c + d*x)^(5/2))/(5*d^2) - (a^2*c*x)/d - (4*a*b*c*(c + d*x)^(3/2))/(3*d
^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{-16\sqrt{dx+c}\,ab\,c^2+8\sqrt{dx+c}\,abcdx+24\sqrt{dx+c}\,ab\,d^2x^2+15a^2d^2x^2+15b^2c\,d^2x^2+10b^2d^3x^3}{30d^2}$$

input
```
int(x*(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
( - 16*sqrt(c + d*x)*a*b*c**2 + 8*sqrt(c + d*x)*a*b*c*d*x + 24*sqrt(c + d*
x)*a*b*d**2*x**2 + 15*a**2*d**2*x**2 + 15*b**2*c*d**2*x**2 + 10*b**2*d**3*
x**3)/(30*d**2)
```

3.107 $\int \left(a+b\sqrt{c+dx}\right)^2 dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx = a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

output
```
a^2*x+4/3*a*b*(d*x+c)^(3/2)/d+1/2*b^2*(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx = \frac{(c+dx)\left(6a^2+8ab\sqrt{c+dx}+3b^2(c+dx)\right)}{6d}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2,x]
```

output
```
((c + d*x)*(6*a^2 + 8*a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)))/(6*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx$$

$$\Big\downarrow \ 239$$

$$\frac{\int \left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d}$$

$$\Big\downarrow \ 774$$

$$\frac{2\int \sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2 d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \ 49$$

$$\frac{2\int \left(\sqrt{c+dx}a^2+2b(c+dx)a+b^2(c+dx)^{3/2}\right) d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(\frac{1}{2}a^2(c+dx)+\frac{2}{3}ab(c+dx)^{3/2}+\frac{1}{4}b^2(c+dx)^2\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*((a^2*(c + d*x))/2 + (2*a*b*(c + d*x)^(3/2))/3 + (b^2*(c + d*x)^2)/4))/
d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$b^2\left(\frac{1}{2}d\,x^2+cx\right)+\frac{4ab(dx+c)^{\frac{3}{2}}}{3d}+a^2x$	35
trager	$\frac{(b^2dx+2b^2c+2a^2)x}{2}+\frac{4ab(dx+c)^{\frac{3}{2}}}{3d}$	37
derivativedivides	$\frac{\frac{b^2(dx+c)^2}{2}+\frac{4ab(dx+c)^{\frac{3}{2}}}{3}+a^2(dx+c)}{d}$	40
orering	$\frac{(-5b^2d^2x^2-10b^2cxd+6a^2dx-4b^2c^2+4a^2c)\left(a+b\sqrt{dx+c}\right)^2}{6d(-b^2dx-b^2c+a^2)}-\frac{x(-b^2dx-2b^2c+2a^2)\sqrt{dx+c}\,\left(a+b\sqrt{dx+c}\right)b}{3(-b^2dx-b^2c+a^2)}$	140

input
```
int((a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/2*d*x^2+c*x)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{3\,b^2d^2x^2+6\,(b^2c+a^2)dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x +
c))/d
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\begin{cases} a^2x+\frac{4abc\sqrt{c+dx}}{3d}+\frac{4abx\sqrt{c+dx}}{3}+b^2cx+\frac{b^2dx^2}{2} & \text{for } d\neq 0\\ x\left(a+b\sqrt{c}\right)^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3
+ b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{1}{2}\,(dx^2+2\,cx)b^2+a^2x+\frac{4\,(dx+c)^{\frac{3}{2}}ab}{3\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(35) = 70$.

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$= \frac{6\,(dx+c)b^2c+24\,\sqrt{dx+c}abc+6\,(dx+c)a^2+8\,\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)ab+3\,\left((dx+c)^2-2\,(dx+\right.}{6\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/6*(6*(d*x + c)*b^2*c + 24*sqrt(d*x + c)*a*b*c + 6*(d*x + c)*a^2 + 8*((d*
x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b + 3*((d*x + c)^2 - 2*(d*x + c)*c)*b^
2)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{3\,b^2\,{\left(c+d\,x\right)}^2+8\,a\,b\,{\left(c+d\,x\right)}^{3/2}+6\,a^2\,d\,x}{6\,d}$$

input
```
int((a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(3*b^2*(c + d*x)^2 + 8*a*b*(c + d*x)^(3/2) + 6*a^2*d*x)/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx = \frac{8\sqrt{dx+c}\,abc+8\sqrt{dx+c}\,abdx+6a^2dx+6b^2cdx+3b^2d^2x^2}{6d}$$

input
```
int((a+b*(d*x+c)^(1/2))^2,x)
```

output
```
(8*sqrt(c + d*x)*a*b*c + 8*sqrt(c + d*x)*a*b*d*x + 6*a**2*d*x + 6*b**2*c*d
*x + 3*b**2*d**2*x**2)/(6*d)
```

3.108 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \left(a^2+b^2c\right)\log(x)$$

output
```
b^2*d*x+4*a*b*(d*x+c)^(1/2)-4*a*b*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))+(
b^2*c+a^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = b\left(bc+bdx+4a\sqrt{c+dx}\right) - 4ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \left(a^2+b^2c\right)\log(-dx)$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
```

output
```
b*(b*c + b*d*x + 4*a*Sqrt[c + d*x]) - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/
Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {896, 25, 1732, 525, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{dx}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\left(a+b\sqrt{c+dx}\right)^2}{dx}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2}{dx}d\sqrt{c+dx}$$

$$\Big\downarrow \text{525}$$

$$-2\left(-\int \frac{\sqrt{c+dx}\left(a^2+2b\sqrt{c+dx}a+b^2c\right)}{dx}d\sqrt{c+dx}-\frac{1}{2}b^2(c+dx)\right)$$

$$\Big\downarrow \text{25}$$

$$-2\left(\int -\frac{\sqrt{c+dx}\left(a^2+2b\sqrt{c+dx}a+b^2c\right)}{dx}d\sqrt{c+dx}-\frac{1}{2}b^2(c+dx)\right)$$

$$\Big\downarrow \text{523}$$

$$-2\left(\int \left(-2ab-\frac{2abc+\left(a^2+b^2c\right)\sqrt{c+dx}}{dx}\right)d\sqrt{c+dx}-\frac{1}{2}b^2(c+dx)\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{1}{2}(a^2+b^2c)\log(-dx)+2ab\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)-2ab\sqrt{c+dx}-\frac{1}{2}b^2(c+dx)\right)$$

input
```
Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

output
```
-2*(-2*a*b*Sqrt[c + d*x] - (b^2*(c + d*x))/2 + 2*a*b*Sqrt[c]*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]] - ((a^2 + b^2*c)*Log[-(d*x)])/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

method	result	size
default	$b^2(dx+c\ln(x))+2ab\left(2\sqrt{dx+c}-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\right)+\ln(x)\,a^2$	51
derivativedivides	$b^2(dx+c)+4ab\sqrt{dx+c}-(-b^2c-a^2)\ln(-dx)-4ab\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)$	60

input
```
int((a+b*(d*x+c)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
b^2*(d*x+c*ln(x))+2*a*b*(2*(d*x+c)^(1/2)-2*c^(1/2)*arctanh((d*x+c)^(1/2)/c
^(1/2)))+ln(x)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx=\left[b^2dx+2\,ab\sqrt{c}\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+4\,\sqrt{dx+c}ab+\left(b^2c+a^2\right)\log(x)\,,b^2dx+4\,ab\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx+c}}\right)+4\,\sqrt{dx+c}ab+\left(b^2c+a^2\right)\log(x)\right]$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")
```

output
```
[b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*
sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x + c)) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \begin{cases} \frac{4abc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 4ab\sqrt{c+dx} + b^2(c+dx) - 2\left(-\frac{a^2}{2}-\frac{b^2c}{2}\right)\log(-dx) & \text{for } d \neq 0 \\ \left(a+b\sqrt{c}\right)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

output
```
Piecewise((4*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 4*a*b*sqrt(c +
d*x) + b**2*(c + d*x) - 2*(-a**2/2 - b**2*c/2)*log(-d*x), Ne(d, 0)), ((a +
 b*sqrt(c))**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = 2\,ab\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + (dx+c)b^2 + 4\,\sqrt{dx+c}ab + \left(b^2c+a^2\right)\log(dx)$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")
```

output
```
2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + (
d*x + c)*b^2 + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + (dx+c)b^2 + 4\sqrt{dx+c}ab + \left(b^2c+a^2\right)\log\left(dx\right)$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")
```

output
```
4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + (d*x + c)*b^2 + 4*sqrt(d
*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.28

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,\left(a+b\,\sqrt{c}\right)^2\,\sqrt{c+d\,x} + 4\,a\,b\,c\right)\,\left(a+b\,\sqrt{c}\right)^2 + \ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,\left(a-b\,\sqrt{c}\right)^2\,\sqrt{c+d\,x} + 4\,a\,b\,c\right)\,\left(a-b\,\sqrt{c}\right)^2 + 4\,a\,b\,\sqrt{c+d\,x} + b^2\,d\,x$$

input
```
int((a + b*(c + d*x)^(1/2))^2/x,x)
```

output
```
log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a + b*c^(1/2))^2*(c + d*x)^(1/2
) + 4*a*b*c)*(a + b*c^(1/2))^2 + log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2
*(a - b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a - b*c^(1/2))^2 + 4*a*b*(c
 + d*x)^(1/2) + b^2*d*x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = 4\sqrt{dx+c}\,ab + 2\sqrt{c}\log\left(\sqrt{dx+c}-\sqrt{c}\right)ab - 2\sqrt{c}\log\left(\sqrt{dx+c}+\sqrt{c}\right)ab + \log(x)\,a^2 + \log(x)\,b^2c + b^2dx$$

input

```
int((a+b*(d*x+c)^(1/2))^2/x,x)
```

output

```
4*sqrt(c + d*x)*a*b + 2*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a*b - 2*sqrt(
c)*log(sqrt(c + d*x) + sqrt(c))*a*b + log(x)*a**2 + log(x)*b**2*c + b**2*d
*x
```

3.109 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx=-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

output
```
-(a+b*(d*x+c)^(1/2))^2/x-2*a*b*d*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(1/2)+b^
2*d*ln(x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$$
$$=-\frac{a^2+b^2c+2ab\sqrt{c+dx}}{x}-\frac{2abd\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(-dx)$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

output
```
-((a^2 + b^2*c + 2*a*b*Sqrt[c + d*x])/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/
Sqrt[c]])/Sqrt[c] + b^2*d*Log[-(d*x)]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 1732, 531, 27, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{ 896}$$

$$d\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$2d\int \frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2}{d^2x^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 531}$$

$$2d\left(\frac{\int \frac{2bc\left(a+b\sqrt{c+dx}\right)}{dx}d\sqrt{c+dx}}{2c}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2d\left(-b\int -\frac{a+b\sqrt{c+dx}}{dx}d\sqrt{c+dx}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow \text{ 452}$$

$$2d\left(-b\left(a\int -\frac{1}{dx}d\sqrt{c+dx}+b\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}\right)-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2d\left(-b\left(b\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}+\frac{a\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow 240$$

$$2d\left(-b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{1}{2}b\log(-dx)\right)-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

input
```
Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

output
```
2*d*(-1/2*(a + b*Sqrt[c + d*x])^2/(d*x) - b*((a*ArcTanh[Sqrt[c + d*x]/Sqrt
[c]])/Sqrt[c] - (b*Log[-(d*x)])/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

method	result	size
default	$b^{2}\left(d \ln \left(x \right)-\frac{c}{x}\right)+4 a b d \left(-\frac{\sqrt{d x +c}}{2 d x}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x +c}}{\sqrt{c}}\right)}{2 \sqrt{c}}\right)-\frac{a^{2}}{x}$	63
derivativedivides	$2 d \left(-\frac{a b \sqrt{d x +c}+\frac{b^{2} c}{2}+\frac{a^{2}}{2}}{d x}+b \left(\frac{b \ln \left(-d x \right)}{2}-\frac{a \,\operatorname{arctanh}\left(\frac{\sqrt{d x +c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)\right)$	64

input
```
int((a+b*(d*x+c)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(d*ln(x)-1/x*c)+4*a*b*d*(-1/2*(d*x+c)^(1/2)/d/x-1/2/c^(1/2)*arctanh((d
*x+c)^(1/2)/c^(1/2)))-a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.67

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = \left[\frac{b^2cdx\log(x)+ab\sqrt{c}dx\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-b^2c^2-2\sqrt{dx+c}abc-a^2c}{cx}, \frac{b^2cdx\log(x)+2ab\sqrt{-c}d\ldots}{\ldots}\right.$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
[(b^2*c*d*x*log(x) + a*b*sqrt(c)*d*x*log((d*x - 2*sqrt(d*x + c)*sqrt(c) +
2*c)/x) - b^2*c^2 - 2*sqrt(d*x + c)*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*log(x
) + 2*a*b*sqrt(-c)*d*x*arctan(sqrt(-c)/sqrt(d*x + c)) - b^2*c^2 - 2*sqrt(d
*x + c)*a*b*c - a^2*c)/(c*x)]
```

Sympy [A] (verification not implemented)

Time = 25.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = -\frac{a^2}{x}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx}+1}}{\sqrt{x}}-\frac{2abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\sqrt{x}}\right)}{\sqrt{c}}-\frac{b^2c}{x}+b^2d\log(x)$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

output
```
-a**2/x - 2*a*b*sqrt(d)*sqrt(c/(d*x) + 1)/sqrt(x) - 2*a*b*d*asinh(sqrt(c)/
(sqrt(d)*sqrt(x)))/sqrt(c) - b**2*c/x + b**2*d*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$$
$$=\left(b^2\log\left(dx\right)+\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{b^2c+2\sqrt{dx+c}ab+a^2}{dx}\right)d$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
(b^2*log(d*x) + a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)
))/sqrt(c) - (b^2*c + 2*sqrt(d*x + c)*a*b + a^2)/(d*x))*d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$$
$$=\left(b^2\log\left(dx\right)+\frac{2\,ab\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{b^2c+2\sqrt{dx+c}ab+a^2}{dx}\right)d$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
(b^2*log(d*x) + 2*a*b*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) - (b^2*c + 2
*sqrt(d*x + c)*a*b + a^2)/(d*x))*d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = b\,d\,\ln\left(2\,b\,d\left(b+\frac{a}{\sqrt{c}}\right)\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\left(b+\frac{a}{\sqrt{c}}\right)-\frac{a^2\,d+b^2\,c\,d+2\,a\,b\,d\,\sqrt{c+d\,x}}{d\,x}+b\,d\,\ln\left(2\,b\,d\left(b-\frac{a}{\sqrt{c}}\right)\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\left(b-\frac{a}{\sqrt{c}}\right)$$

input
```
int((a + b*(c + d*x)^(1/2))^2/x^2,x)
```

output
```
b*d*log(2*b*d*(b + a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) -
2*a*b*d)*(b + a/c^(1/2)) - (a^2*d + b^2*c*d + 2*a*b*d*(c + d*x)^(1/2))/(d*
x) + b*d*log(2*b*d*(b - a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/
2) - 2*a*b*d)*(b - a/c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = \frac{-2\sqrt{dx+c}\,abc+\sqrt{c}\log\left(\sqrt{dx+c}-\sqrt{c}\right)abdx-\sqrt{c}\log\left(\sqrt{dx+c}+\sqrt{c}\right)abdx+\log(x)\,b^2cdx-a^2c-b^2}{cx}$$

input
```
int((a+b*(d*x+c)^(1/2))^2/x^2,x)
```

output
```
( - 2*sqrt(c + d*x)*a*b*c + sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a*b*d*x -
 sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*a*b*d*x + log(x)*b**2*c*d*x - a**2*c
 - b**2*c**2)/(c*x)
```

3.110 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}+\frac{abd^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

output
```
-1/2*b*d*(b*c+a*(d*x+c)^(1/2))/c/x-1/2*(a+b*(d*x+c)^(1/2))^2/x^2+1/2*a*b*d
^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{a^2c+ab\sqrt{c+dx}(2c+dx)+b^2c(c+2dx)}{2cx^2}+\frac{abd^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

output
```
-1/2*(a^2*c + a*b*Sqrt[c + d*x]*(2*c + d*x) + b^2*c*(c + 2*d*x))/(c*x^2) +
 (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 25, 1732, 531, 27, 454, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\int -\frac{\left(a+b\sqrt{c+dx}\right)^2}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2d^2\int -\frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 531}$$

$$-2d^2\left(\frac{\int -\frac{2bc\left(a+b\sqrt{c+dx}\right)}{d^2x^2}d\sqrt{c+dx}}{4c}+\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-2d^2\left(\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}-\frac{1}{2}b\int \frac{a+b\sqrt{c+dx}}{d^2x^2}d\sqrt{c+dx}\right)$$

$$\Big\downarrow \text{ 454}$$

$$-2d^2\left(\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}-\frac{1}{2}b\left(\frac{a\int -\frac{1}{dx}d\sqrt{c+dx}}{2c}-\frac{a\sqrt{c+dx}+bc}{2cdx}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$-2d^2\left(\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}-\frac{1}{2}b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{a\sqrt{c+dx}+bc}{2cdx}\right)\right)$$

input
```
Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

output
```
-2*d^2*((a + b*Sqrt[c + d*x])^2/(4*d^2*x^2) - (b*(-1/2*(b*c + a*Sqrt[c + d
*x])/(c*d*x) + (a*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$2d^2\left(-\frac{\frac{ab(dx+c)^{\frac{3}{2}}}{4c}+\frac{b^2(dx+c)}{2}+\frac{ab\sqrt{dx+c}}{4}-\frac{b^2c}{4}+\frac{a^2}{4}}{d^2x^2}+\frac{ab\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4c^{\frac{3}{2}}}\right)$	81
default	$b^2\left(-\frac{c}{2x^2}-\frac{d}{x}\right)+4ab\,d^2\left(-\frac{\frac{(dx+c)^{\frac{3}{2}}}{8c}+\frac{\sqrt{dx+c}}{8}}{d^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)-\frac{a^2}{2x^2}$	82

input
```
int((a+b*(d*x+c)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^2*(-(1/4*a*b/c*(d*x+c)^(3/2)+1/2*b^2*(d*x+c)+1/4*a*b*(d*x+c)^(1/2)-1/4
*b^2*c+1/4*a^2)/d^2/x^2+1/4*a*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$$
$$=\left[\frac{ab\sqrt{c}d^2x^2\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-4b^2c^2dx-2b^2c^3-2a^2c^2-2\left(abcdx+2abc^2\right)\sqrt{dx+c}}{4c^2x^2},\right.$$
$$\left.-\frac{ab\sqrt{-c}d^2x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx+c}}\right)+2b^2c^2dx+b^2c^3+a^2c^2+\left(abcdx+2abc^2\right)\sqrt{dx+c}}{2c^2x^2}\right]$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")
```

output
```
[1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4
*b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x
+ c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(-c)/sqrt(d*x + c))
 + 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x +
c))/(c^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 97.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx=-\frac{a^2}{2x^2}-\frac{abc}{\sqrt{d}x^{\frac{5}{2}}\sqrt{\frac{c}{dx}+1}}-\frac{3ab\sqrt{d}}{2x^{\frac{3}{2}}\sqrt{\frac{c}{dx}+1}}-\frac{abd^{\frac{3}{2}}}{2c\sqrt{x}\sqrt{\frac{c}{dx}+1}}+\frac{abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\sqrt{x}}\right)}{2c^{\frac{3}{2}}}-\frac{b^2c}{2x^2}-\frac{b^2d}{x}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

output
```
-a**2/(2*x**2) - a*b*c/(sqrt(d)*x**(5/2)*sqrt(c/(d*x) + 1)) - 3*a*b*sqrt(d
)/(2*x**(3/2)*sqrt(c/(d*x) + 1)) - a*b*d**(3/2)/(2*c*sqrt(x)*sqrt(c/(d*x)
+ 1)) + a*b*d**2*asinh(sqrt(c)/(sqrt(d)*sqrt(x)))/(2*c**(3/2)) - b**2*c/(2
*x**2) - b**2*d/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{1}{4}\left(\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(2\,(dx+c)b^2c-b^2c^2+(dx+c)^{\frac{3}{2}}ab+\sqrt{dx+c}abc+a^2c\right)}{(dx+c)^2c-2\,(dx+c)c^2+c^3}\right)d^2$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")
```

output
```
-1/4*(a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/c^(3/2)
 + 2*(2*(d*x + c)*b^2*c - b^2*c^2 + (d*x + c)^(3/2)*a*b + sqrt(d*x + c)*a*
b*c + a^2*c)/((d*x + c)^2*c - 2*(d*x + c)*c^2 + c^3))*d^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{2\,(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}abcd^3+a^2cd^3}{cd^2x^2}}{2\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")
```

output
```
-1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (2*(d*x + c)*b
^2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3
 + a^2*c*d^3)/(c*d^2*x^2))/d
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = \frac{a\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c}}\right)}{2\,c^{3/2}} - \frac{b^2\,c}{2\,x^2} - \frac{b^2\,d}{x} - \frac{a\,b\,\sqrt{c+d\,x}}{2\,x^2} - \frac{a\,b\,(c+d\,x)^{3/2}}{2\,c\,x^2} - \frac{a^2}{2\,x^2}$$

input
```
int((a + b*(c + d*x)^(1/2))^2/x^3,x)
```

output
```
(a*b*d^2*atanh((c + d*x)^(1/2)/c^(1/2)))/(2*c^(3/2)) - (b^2*c)/(2*x^2) - (
b^2*d)/x - (a*b*(c + d*x)^(1/2))/(2*x^2) - (a*b*(c + d*x)^(3/2))/(2*c*x^2)
 - a^2/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = \frac{-4\sqrt{dx+c}\,ab\,c^2 - 2\sqrt{dx+c}\,abcdx - \sqrt{c}\log\left(\sqrt{dx+c}-\sqrt{c}\right)ab\,d^2x^2 + \sqrt{c}\log\left(\sqrt{dx+c}+\sqrt{c}\right)ab\,d^2x^2 +}{4c^2x^2}$$

input
```
int((a+b*(d*x+c)^(1/2))^2/x^3,x)
```

output
```
( - 4*sqrt(c + d*x)*a*b*c**2 - 2*sqrt(c + d*x)*a*b*c*d*x - sqrt(c)*log(sqr
t(c + d*x) - sqrt(c))*a*b*d**2*x**2 + sqrt(c)*log(sqrt(c + d*x) + sqrt(c))
*a*b*d**2*x**2 - 2*a**2*c**2 - 2*b**2*c**3 - 4*b**2*c**2*d*x)/(4*c**2*x**2
)
```

3.111 $\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 230

$$\begin{aligned}\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = &-\frac{a(a^4-3a^2b^2c+3b^4c^2)\,x}{b^6d^3}+\frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4}\\&+\frac{2(a^4-3a^2b^2c+3b^4c^2)\,(c+dx)^{3/2}}{3b^5d^4}-\frac{a(a^2-3b^2c)\,(c+dx)^2}{2b^4d^4}\\&+\frac{2(a^2-3b^2c)\,(c+dx)^{5/2}}{5b^3d^4}-\frac{a(c+dx)^3}{3b^2d^4}\\&+\frac{2(c+dx)^{7/2}}{7bd^4}-\frac{2a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}\end{aligned}$$

output
```
-a*(3*b^4*c^2-3*a^2*b^2*c+a^4)*x/b^6/d^3+2*(-b^2*c+a^2)^3*(d*x+c)^(1/2)/b^
7/d^4+2/3*(3*b^4*c^2-3*a^2*b^2*c+a^4)*(d*x+c)^(3/2)/b^5/d^4-1/2*a*(-3*b^2*
c+a^2)*(d*x+c)^2/b^4/d^4+2/5*(-3*b^2*c+a^2)*(d*x+c)^(5/2)/b^3/d^4-1/3*a*(d
*x+c)^3/b^2/d^4+2/7*(d*x+c)^(7/2)/b/d^4-2*a*(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^
(1/2))/b^8/d^4
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{b\left(420a^6\sqrt{c+dx}-140a^4b^2(8c-dx)\sqrt{c+dx}-210a^5b(c+dx)+105a^3b^3(5c^2+4cdx-d^2x^2)+84a^2b^4\cdots\right.}{}$$

input
```
Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

output
```
(b*(420*a^6*Sqrt[c + d*x] - 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] - 210*a^
5*b*(c + d*x) + 105*a^3*b^3*(5*c^2 + 4*c*d*x - d^2*x^2) + 84*a^2*b^4*Sqrt[
c + d*x]*(11*c^2 - 3*c*d*x + d^2*x^2) - 35*a*b^5*(11*c^3 + 6*c^2*d*x - 3*c
*d^2*x^2 + 2*d^3*x^3) + 12*b^6*Sqrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^
2*x^2 + 5*d^3*x^3)) - 420*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210
*b^8*d^4)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{896}$$
$$\frac{\int \frac{d^3x^3}{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -\frac{d^3x^3}{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$

$$\Big\downarrow \ 1732$$

$$-\frac{2\int -\frac{d^3x^3\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow \ 522$$

$$-\frac{2\int \left(\frac{a(a^2-b^2c)^3}{b^7(a+b\sqrt{c+dx})}+\frac{(b^2c-a^2)^3}{b^7}-\frac{(c+dx)^3}{b}+\frac{a(c+dx)^{5/2}}{b^2}+\frac{(3b^2c-a^2)(c+dx)^2}{b^3}+\frac{a(a^2-3b^2c)(c+dx)^{3/2}}{b^4}-\frac{(a^4-3b^2ca^2+3b^4c^2)}{b^5}\right.}{d^4}$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\left(\frac{a(a^2-b^2c)^3\log(a+b\sqrt{c+dx})}{b^8}-\frac{(a^2-b^2c)^3\sqrt{c+dx}}{b^7}+\frac{a(a^2-3b^2c)(c+dx)^2}{4b^4}-\frac{(a^2-3b^2c)(c+dx)^{5/2}}{5b^3}+\frac{a(a^4-3a^2b^2c+3b^4c^2)(c+dx)}{2b^6}-\right.}{d^4}$$

input
```
Int[x^3/(a + b*Sqrt[c + d*x]),x]
```

output
```
(-2*(-(((a^2 - b^2*c)^3*Sqrt[c + d*x])/b^7) + (a*(a^4 - 3*a^2*b^2*c + 3*b^
4*c^2)*(c + d*x))/(2*b^6) - ((a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/
2))/(3*b^5) + (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(4*b^4) - ((a^2 - 3*b^2*c)*(
c + d*x)^(5/2))/(5*b^3) + (a*(c + d*x)^3)/(6*b^2) - (c + d*x)^(7/2)/(7*b)
+ (a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/b^8))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{2\left(\frac{(dx+c)^{\frac{7}{2}}b^6}{7}-\frac{a(dx+c)^3b^5}{6}-\frac{3b^6c(dx+c)^{\frac{5}{2}}}{5}+\frac{a^2b^4(dx+c)^{\frac{5}{2}}}{5}+\frac{3a\,b^5c(dx+c)^2}{4}+b^6c^2(dx+c)^{\frac{3}{2}}-\frac{a^3b^3(dx+c)^2}{4}-a^2b^4c(dx+c)^{\frac{3}{2}}-3}{b^7}$
default	$\frac{2\left(\frac{(dx+c)^{\frac{7}{2}}b^6}{7}-\frac{a(dx+c)^3b^5}{6}-\frac{3b^6c(dx+c)^{\frac{5}{2}}}{5}+\frac{a^2b^4(dx+c)^{\frac{5}{2}}}{5}+\frac{3a\,b^5c(dx+c)^2}{4}+b^6c^2(dx+c)^{\frac{3}{2}}-\frac{a^3b^3(dx+c)^2}{4}-a^2b^4c(dx+c)^{\frac{3}{2}}-3}{b^7}$

input
```
int(x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^4*(1/b^7*(1/7*(d*x+c)^(7/2)*b^6-1/6*a*(d*x+c)^3*b^5-3/5*b^6*c*(d*x+c)^
(5/2)+1/5*a^2*b^4*(d*x+c)^(5/2)+3/4*a*b^5*c*(d*x+c)^2+b^6*c^2*(d*x+c)^(3/2
)-1/4*a^3*b^3*(d*x+c)^2-a^2*b^4*c*(d*x+c)^(3/2)-3/2*a*b^5*c^2*(d*x+c)-b^6*
c^3*(d*x+c)^(1/2)+1/3*a^4*b^2*(d*x+c)^(3/2)+3/2*a^3*b^3*c*(d*x+c)+3*a^2*b^
4*c^2*(d*x+c)^(1/2)-1/2*a^5*b*(d*x+c)-3*a^4*b^2*c*(d*x+c)^(1/2)+a^6*(d*x+c
)^(1/2))-a*(-b^6*c^3+3*a^2*b^4*c^2-3*a^4*b^2*c+a^6)/b^8*ln(a+b*(d*x+c)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx =$$
$$-\frac{70\,ab^6d^3x^3 - 105\,(ab^6c - a^3b^4)d^2x^2 + 210\,(ab^6c^2 - 2\,a^3b^4c + a^5b^2)dx - 420\,(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^
2 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^
2*c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231
*a^2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2
 + (24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{a(c+dx)^3}{6b^2} - \frac{a\left(a^2-b^2c\right)^3\left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for } b=0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise} \end{cases}\right)}{b^7} + \frac{(c+dx)^{\frac{7}{2}}}{7b} + \frac{\left(a^2-3b^2c\right)(c+dx)^{\frac{5}{2}}}{5b^3} + \frac{\left(-a^3+3ab^2c\right)(c+dx)^2}{4b^4} + \frac{(c+dx)^{\frac{3}{2}}\left(a^4-3a\right.}{3b^5}\right.}{d^4} \\ \frac{x^4}{4(a+b\sqrt{c})} \end{cases}$$

input
```
integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((2*(-a*(c + d*x)**3/(6*b**2) - a*(a**2 - b**2*c)**3*Piecewise((s
qrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**7 + (c +
 d*x)**(7/2)/(7*b) + (a**2 - 3*b**2*c)*(c + d*x)**(5/2)/(5*b**3) + (-a**3
+ 3*a*b**2*c)*(c + d*x)**2/(4*b**4) + (c + d*x)**(3/2)*(a**4 - 3*a**2*b**2
*c + 3*b**4*c**2)/(3*b**5) + (c + d*x)*(-a**5 + 3*a**3*b**2*c - 3*a*b**4*c
**2)/(2*b**6) + sqrt(c + d*x)*(a**6 - 3*a**4*b**2*c + 3*a**2*b**4*c**2 - b
**6*c**3)/b**7)/d**4, Ne(d, 0)), (x**4/(4*(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\,(3\,b^6c-a^2b^4)(dx+c)^{\frac{5}{2}}+105\,(3\,ab^5c-a^3b^3)(dx+c)^2+140\,(3\,b^6c^2-3\,a^2b^4c+a^4b^2)(dx+c)^{\frac{3}{2}}-210\,(3\,ab^5c^2-3\,a}{b^7}}{210\,d^4}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*
b^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*
c^2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^
3*c + a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)
*sqrt(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*
log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.48

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{2\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^8d^4} + \frac{60\,(dx+c)^{\frac{7}{2}}b^6d^{24}-252\,(dx+c)^{\frac{5}{2}}b^6cd^{24}+420\,(dx+c)^{\frac{3}{2}}b^6c^2d^{24}-420\,\sqrt{dx+c}b^6c^3d^{24}-70\,(dx+c)^3a}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b
+ a))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)
*b^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c^3
*d^24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^24 - 630*(d*
x + c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^24 - 420*(d*x + c)^(3
/2)*a^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b^4*c^2*d^24 - 105*(d*x + c)^2
*a^3*b^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2
*d^24 - 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420
*sqrt(d*x + c)*a^6*d^24)/(b^7*d^28)
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,b\,d^4}-\left(\frac{a^2\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{b^2}+\frac{2\,c^3}{b\,d^4}\right)\,\sqrt{c+d\,x}-\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{3\,b^2}-\frac{2\,c^2}{b\,d^4}\right)\,{\left(c+d\,x\right)}^{3/2}-\left(\frac{6\,c}{5\,b\,d^4}-\frac{2\,a^2}{5\,b^3\,d^4}\right)\,{\left(c+d\,x\right)}^{5/2}+\frac{a\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)\,{\left(c+d\,x\right)}^2}{4\,b}-\frac{a\,{\left(c+d\,x\right)}^3}{3\,b^2\,d^4}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^7-6\,a^5\,b^2\,c+6\,a^3\,b^4\,c^2-2\,a\,b^6\,c^3\right)}{b^8\,d^4}+\frac{a\,d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{2\,b}$$

input
```
int(x^3/(a + b*(c + d*x)^(1/2)),x)
```

output
```
(2*(c + d*x)^(7/2))/(7*b*d^4) - ((a^2*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*
d^4)))/b^2 - (6*c^2)/(b*d^4)))/b^2 + (2*c^3)/(b*d^4))*(c + d*x)^(1/2) - ((
a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/(3*b^2) - (2*c^2)/(b*d^4))*(c + d
*x)^(3/2) - ((6*c)/(5*b*d^4) - (2*a^2)/(5*b^3*d^4))*(c + d*x)^(5/2) + (a*(
(6*c)/(b*d^4) - (2*a^2)/(b^3*d^4))*(c + d*x)^2)/(4*b) - (a*(c + d*x)^3)/(3
*b^2*d^4) - (log(a + b*(c + d*x)^(1/2))*(2*a^7 - 6*a^5*b^2*c - 2*a*b^6*c^3
 + 6*a^3*b^4*c^2))/(b^8*d^4) + (a*d*x*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*
d^4)))/b^2 - (6*c^2)/(b*d^4)))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{420\sqrt{dx+c}\,a^6b - 1120\sqrt{dx+c}\,a^4b^3c + 140\sqrt{dx+c}\,a^4b^3dx + 924\sqrt{dx+c}\,a^2b^5c^2 - 252\sqrt{dx+c}\,a^2b^5cdx$$

input
```
int(x^3/(a+b*(d*x+c)^(1/2)),x)
```

output
```
(420*sqrt(c + d*x)*a**6*b - 1120*sqrt(c + d*x)*a**4*b**3*c + 140*sqrt(c +
d*x)*a**4*b**3*d*x + 924*sqrt(c + d*x)*a**2*b**5*c**2 - 252*sqrt(c + d*x)*
a**2*b**5*c*d*x + 84*sqrt(c + d*x)*a**2*b**5*d**2*x**2 - 192*sqrt(c + d*x)
*b**7*c**3 + 96*sqrt(c + d*x)*b**7*c**2*d*x - 72*sqrt(c + d*x)*b**7*c*d**2
*x**2 + 60*sqrt(c + d*x)*b**7*d**3*x**3 - 420*log(sqrt(c + d*x)*b + a)*a**
7 + 1260*log(sqrt(c + d*x)*b + a)*a**5*b**2*c - 1260*log(sqrt(c + d*x)*b +
 a)*a**3*b**4*c**2 + 420*log(sqrt(c + d*x)*b + a)*a*b**6*c**3 - 210*a**5*b
**2*c - 210*a**5*b**2*d*x + 525*a**3*b**4*c**2 + 420*a**3*b**4*c*d*x - 105
*a**3*b**4*d**2*x**2 - 385*a*b**6*c**3 - 210*a*b**6*c**2*d*x + 105*a*b**6*
c*d**2*x**2 - 70*a*b**6*d**3*x**3)/(210*b**8*d**4)
```

3.112 $\int \frac{x^2}{a+b\sqrt{c+dx}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{x^2}{a+b\sqrt{c+dx}} \, dx = -\frac{a(a^2-2b^2c)\,x}{b^4d^2} + \frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3} + \frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3} - \frac{a(c+dx)^2}{2b^2d^3} + \frac{2(c+dx)^{5/2}}{5bd^3} - \frac{2a(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

output
```
-a*(-2*b^2*c+a^2)*x/b^4/d^2+2*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^5/d^3+2/3*(-2
*b^2*c+a^2)*(d*x+c)^(3/2)/b^3/d^3-1/2*a*(d*x+c)^2/b^2/d^3+2/5*(d*x+c)^(5/2
)/b/d^3-2*a*(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))/b^6/d^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sqrt{c+dx}} \, dx = \frac{b\left(60a^4\sqrt{c+dx} - 20a^2b^2(5c-dx)\sqrt{c+dx} - 30a^3b(c+dx) + 15ab^3(3c^2+2cdx-d^2x^2) + 4b^4\sqrt{c+dx}(\right.}{30b^6d^3}$$

input
```
Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
```

output
```
(b*(60*a^4*Sqrt[c + d*x] - 20*a^2*b^2*(5*c - d*x)*Sqrt[c + d*x] - 30*a^3*b
*(c + d*x) + 15*a*b^3*(3*c^2 + 2*c*d*x - d^2*x^2) + 4*b^4*Sqrt[c + d*x]*(8
*c^2 - 4*c*d*x + 3*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x
]])/(30*b^6*d^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^2x^2}{a+b\sqrt{c+dx}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1732}$$

$$\frac{2\int \frac{d^2x^2\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{522}$$

$$\frac{2\int \left(-\frac{a(a^2-b^2c)^2}{b^5(a+b\sqrt{c+dx})}+\frac{(b^2c-a^2)^2}{b^5}+\frac{(c+dx)^2}{b}-\frac{a(c+dx)^{3/2}}{b^2}-\frac{(2b^2c-a^2)(c+dx)}{b^3}-\frac{a(a^2-2b^2c)\sqrt{c+dx}}{b^4}\right)d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{a(a^2-b^2c)^2\log(a+b\sqrt{c+dx})}{b^6}+\frac{(a^2-b^2c)^2\sqrt{c+dx}}{b^5}-\frac{a(a^2-2b^2c)(c+dx)}{2b^4}+\frac{(a^2-2b^2c)(c+dx)^{3/2}}{3b^3}-\frac{a(c+dx)^2}{4b^2}+\frac{(c+dx)^{5/2}}{5b}\right)}{d^3}$$

input
```
Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

output
```
(2*(((a^2 - b^2*c)^2*Sqrt[c + d*x])/b^5 - (a*(a^2 - 2*b^2*c)*(c + d*x))/(2
*b^4) + ((a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3) - (a*(c + d*x)^2)/(4*b^2
) + (c + d*x)^(5/2)/(5*b) - (a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/b
^6))/d^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}} b^4}{5}-\frac{a(dx+c)^2 b^3}{4}-\frac{2b^4 c(dx+c)^{\frac{3}{2}}}{3}+\frac{a^2 b^2 (dx+c)^{\frac{3}{2}}}{3}+a\,b^3 c(dx+c)+b^4 c^2\sqrt{dx+c}-\frac{a^3 b(dx+c)}{2}-2a^2 b^2 c\sqrt{dx+c}+a^4\sqrt{dx+c}\right.}{b^5}}{d^3}$
default	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}} b^4}{5}-\frac{a(dx+c)^2 b^3}{4}-\frac{2b^4 c(dx+c)^{\frac{3}{2}}}{3}+\frac{a^2 b^2 (dx+c)^{\frac{3}{2}}}{3}+a\,b^3 c(dx+c)+b^4 c^2\sqrt{dx+c}-\frac{a^3 b(dx+c)}{2}-2a^2 b^2 c\sqrt{dx+c}+a^4\sqrt{dx+c}\right.}{b^5}}{d^3}$

input
```
int(x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^3*(1/b^5*(1/5*(d*x+c)^(5/2)*b^4-1/4*a*(d*x+c)^2*b^3-2/3*b^4*c*(d*x+c)^
(3/2)+1/3*a^2*b^2*(d*x+c)^(3/2)+a*b^3*c*(d*x+c)+b^4*c^2*(d*x+c)^(1/2)-1/2*
a^3*b*(d*x+c)-2*a^2*b^2*c*(d*x+c)^(1/2)+a^4*(d*x+c)^(1/2))-a*(b^4*c^2-2*a^
2*b^2*c+a^4)/b^6*ln(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{15\,ab^4d^2x^2 - 30\,(ab^4c - a^3b^2)dx + 60\,(ab^4c^2 - 2\,a^3b^2c + a^5)\log\left(\sqrt{dx+c}b + a\right) - 4\,(3\,b^5d^2x^2 + 8\,b^5c^2}{30\,b^6d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a
^3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 -
25*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d
^3)
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{a(c+dx)^2}{4b^2}-\frac{a\left(a^2-b^2c\right)^2\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^5}+\frac{(c+dx)^{\frac{5}{2}}}{5b}+\frac{\left(a^2-2b^2c\right)(c+dx)^{\frac{3}{2}}}{3b^3}+\frac{\left(-a^3+2ab^2c\right)(c+dx)}{2b^4}+\frac{\sqrt{c+dx}\left(a^4-2a^2b^2\right.}{b^5}\right.}{d^3} \\ \frac{x^3}{3\left(a+b\sqrt{c}\right)} \end{cases}$$

input
```
integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((2*(-a*(c + d*x)**2/(4*b**2) - a*(a**2 - b**2*c)**2*Piecewise((s
qrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**5 + (c +
 d*x)**(5/2)/(5*b) + (a**2 - 2*b**2*c)*(c + d*x)**(3/2)/(3*b**3) + (-a**3
+ 2*a*b**2*c)*(c + d*x)/(2*b**4) + sqrt(c + d*x)*(a**4 - 2*a**2*b**2*c + b
**4*c**2)/b**5)/d**3, Ne(d, 0)), (x**3/(3*(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{\frac{12\,(dx+c)^{\frac{5}{2}}b^4-15\,(dx+c)^2ab^3-20\,\left(2\,b^4c-a^2b^2\right)(dx+c)^{\frac{3}{2}}+30\,\left(2\,ab^3c-a^3b\right)(dx+c)+60\,\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{dx+c}}{b^5}-\frac{60\,\left(ab^4c^2-2\,a^3b^2c+a^5\right)}{b^6}}{30\,d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b
^2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a
^2*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*lo
g(sqrt(d*x + c)*b + a)/b^6)/d^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{\frac{60\,(ab^4c^2-2\,a^3b^2c+a^5)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^6d} - \frac{12\,(dx+c)^{\frac{5}{2}}b^4d^4-40\,(dx+c)^{\frac{3}{2}}b^4cd^4+60\,\sqrt{dx+c}b^4c^2d^4-15\,(dx+c)^2ab^3d^4+60\,(dx+c)ab^3cd^4-}{b^5d^5}}{30\,d^2}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-1/30*(60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b
^6*d) - (12*(d*x + c)^(5/2)*b^4*d^4 - 40*(d*x + c)^(3/2)*b^4*c*d^4 + 60*sq
rt(d*x + c)*b^4*c^2*d^4 - 15*(d*x + c)^2*a*b^3*d^4 + 60*(d*x + c)*a*b^3*c*
d^4 + 20*(d*x + c)^(3/2)*a^2*b^2*d^4 - 120*sqrt(d*x + c)*a^2*b^2*c*d^4 - 3
0*(d*x + c)*a^3*b*d^4 + 60*sqrt(d*x + c)*a^4*d^4)/(b^5*d^5))/d^2
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx &= \frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b\,d^3} - \left(\frac{a^2\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{b^2} - \frac{2\,c^2}{b\,d^3}\right)\,\sqrt{c+d\,x} \\ &\quad - \left(\frac{4\,c}{3\,b\,d^3}-\frac{2\,a^2}{3\,b^3\,d^3}\right)\,{\left(c+d\,x\right)}^{3/2} \\ &\quad - \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^5-4\,a^3\,b^2\,c+2\,a\,b^4\,c^2\right)}{b^6\,d^3} \\ &\quad - \frac{a\,{\left(c+d\,x\right)}^2}{2\,b^2\,d^3} + \frac{a\,d\,x\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{2\,b}\end{aligned}$$

input
```
int(x^2/(a + b*(c + d*x)^(1/2)),x)
```

output
```
(2*(c + d*x)^(5/2))/(5*b*d^3) - ((a^2*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))
/b^2 - (2*c^2)/(b*d^3))*(c + d*x)^(1/2) - ((4*c)/(3*b*d^3) - (2*a^2)/(3*b^
3*d^3))*(c + d*x)^(3/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^5 - 4*a^3*b^2*c
 + 2*a*b^4*c^2))/(b^6*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (a*d*x*((4*c)/(
b*d^3) - (2*a^2)/(b^3*d^3)))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{60\sqrt{dx+c}\,a^4b - 100\sqrt{dx+c}\,a^2b^3c + 20\sqrt{dx+c}\,a^2b^3dx + 32\sqrt{dx+c}\,b^5c^2 - 16\sqrt{dx+c}\,b^5cdx + 12\sqrt{dx}}{}$$

input
```
int(x^2/(a+b*(d*x+c)^(1/2)),x)
```

output
```
(60*sqrt(c + d*x)*a**4*b - 100*sqrt(c + d*x)*a**2*b**3*c + 20*sqrt(c + d*x
)*a**2*b**3*d*x + 32*sqrt(c + d*x)*b**5*c**2 - 16*sqrt(c + d*x)*b**5*c*d*x
 + 12*sqrt(c + d*x)*b**5*d**2*x**2 - 60*log(sqrt(c + d*x)*b + a)*a**5 + 12
0*log(sqrt(c + d*x)*b + a)*a**3*b**2*c - 60*log(sqrt(c + d*x)*b + a)*a*b**
4*c**2 - 30*a**3*b**2*c - 30*a**3*b**2*d*x + 45*a*b**4*c**2 + 30*a*b**4*c*
d*x - 15*a*b**4*d**2*x**2)/(30*b**6*d**3)
```

3.113 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = -\frac{ax}{b^2d}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}+\frac{2(c+dx)^{3/2}}{3bd^2} - \frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

output
```
-a*x/b^2/d+2*(-b^2*c+a^2)*(d*x+c)^(1/2)/b^3/d^2+2/3*(d*x+c)^(3/2)/b/d^2-2*
a*(-b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/b^4/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{b\left(6a^2\sqrt{c+dx}+2b^2(-2c+dx)\sqrt{c+dx}-3ab(c+dx)\right)-6(a^3-ab^2c)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}$$

input
```
Integrate[x/(a + b*Sqrt[c + d*x]),x]
```

output
```
(b*(6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x] - 3*a*b*(c + d*
x)) - 6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int \frac{dx}{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{dx}{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \ 1732$$

$$-\frac{2\int -\frac{dx\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \ 522$$

$$-\frac{2\int \left(\frac{\sqrt{c+dx}a}{b^2}+\frac{b^2c-a^2}{b^3}-\frac{c+dx}{b}+\frac{a^3-ab^2c}{b^3\left(a+b\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\left(\frac{a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4}-\frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3}+\frac{a(c+dx)}{2b^2}-\frac{(c+dx)^{3/2}}{3b}\right)}{d^2}$$

input
```
Int[x/(a + b*Sqrt[c + d*x]),x]
```

output
```
(-2*(-(((a^2 - b^2*c)*Sqrt[c + d*x])/b^3) + (a*(c + d*x))/(2*b^2) - (c + d
*x)^(3/2)/(3*b) + (a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/b^4))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{2\left(\frac{b^{2}(dx+c)^{\frac{3}{2}}}{3}-\frac{a(dx+c)b}{2}-b^{2}c\sqrt{dx+c}+a^{2}\sqrt{dx+c}\right)}{b^{3}}-\frac{2a\left(-b^{2}c+a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^{4}}}{d^{2}}$	85
default	$\frac{\frac{2\left(\frac{b^{2}(dx+c)^{\frac{3}{2}}}{3}-\frac{a(dx+c)b}{2}-b^{2}c\sqrt{dx+c}+a^{2}\sqrt{dx+c}\right)}{b^{3}}-\frac{2a\left(-b^{2}c+a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^{4}}}{d^{2}}$	85

input
```
int(x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(1/b^3*(1/3*b^2*(d*x+c)^(3/2)-1/2*a*(d*x+c)*b-b^2*c*(d*x+c)^(1/2)+a^
2*(d*x+c)^(1/2))-a*(-b^2*c+a^2)/b^4*ln(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = -\frac{3\,ab^2dx-6\,(ab^2c-a^3)\log\left(\sqrt{dx+c}b+a\right)-2\,(b^3dx-2\,b^3c+3\,a^2b)\sqrt{dx+c}}{3\,b^4d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*
x - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(-\frac{a(c+dx)}{2b^2}-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\: b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3}+\frac{(c+dx)^{\frac{3}{2}}}{3b}+\frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3}\right)}{d^2} & \text{for}\: d\neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)} & \text{otherwise}\end{cases}$$

input
```
integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((2*(-a*(c + d*x)/(2*b**2) - a*(a**2 - b**2*c)*Piecewise((sqrt(c
+ d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**3 + (c + d*x)*
*(3/2)/(3*b) + (a**2 - b**2*c)*sqrt(c + d*x)/b**3)/d**2, Ne(d, 0)), (x**2/
(2*(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{2\,(dx+c)^{\frac{3}{2}}b^2-3\,(dx+c)ab-6\,(b^2c-a^2)\sqrt{dx+c}}{b^3}+\frac{6\,(ab^2c-a^3)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{3\,d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x +
 c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{6\,(ab^2c-a^3)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}a^2d^2}{b^3d^3}}{3\,d}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) + (2*(d*x + c
)^(3/2)*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt
(d*x + c)*a^2*d^2)/(b^3*d^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}-\frac{2\,a^2}{b^3\,d^2}\right)\,\sqrt{c+d\,x}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^3-2\,a\,b^2\,c\right)}{b^4\,d^2}-\frac{a\,x}{b^2\,d}$$

input
```
int(x/(a + b*(c + d*x)^(1/2)),x)
```

output
```
(2*(c + d*x)^(3/2))/(3*b*d^2) - ((2*c)/(b*d^2) - (2*a^2)/(b^3*d^2))*(c + d
*x)^(1/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^3 - 2*a*b^2*c))/(b^4*d^2) - (
a*x)/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{6\sqrt{dx+c}\,a^2b-4\sqrt{dx+c}\,b^3c+2\sqrt{dx+c}\,b^3dx-6\log\left(\sqrt{dx+c}\,b+a\right)a^3+6\log\left(\sqrt{dx+c}\,b+a\right)a\,b^2c-}{3b^4d^2}$$

input
```
int(x/(a+b*(d*x+c)^(1/2)),x)
```

output
```
(6*sqrt(c + d*x)*a**2*b - 4*sqrt(c + d*x)*b**3*c + 2*sqrt(c + d*x)*b**3*d*
x - 6*log(sqrt(c + d*x)*b + a)*a**3 + 6*log(sqrt(c + d*x)*b + a)*a*b**2*c
- 3*a*b**2*c - 3*a*b**2*d*x)/(3*b**4*d**2)
```

3.114 $\int \frac{1}{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx=\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

output
```
2*(d*x+c)^(1/2)/b/d-2*a*ln(a+b*(d*x+c)^(1/2))/b^2/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx=\frac{2b\sqrt{c+dx}-2a\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
```

output
```
(2*b*Sqrt[c + d*x] - 2*a*Log[b*d*(a + b*Sqrt[c + d*x])])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{239}$$
$$\frac{\int \frac{1}{a+b\sqrt{c+dx}}d(c+dx)}{d}$$
$$\Big\downarrow \text{774}$$
$$\frac{2\int \frac{\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d}$$
$$\Big\downarrow \text{49}$$
$$\frac{2\int \left(\frac{1}{b}-\frac{a}{b\left(a+b\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{d}$$
$$\Big\downarrow \text{2009}$$
$$\frac{2\left(\frac{\sqrt{c+dx}}{b}-\frac{a\log\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^(-1),x]
```

output
```
(2*(Sqrt[c + d*x]/b - (a*Log[a + b*Sqrt[c + d*x]])/b^2))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{2\sqrt{dx+c}}{b}-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{b^{2}}}{d}$	36
default	$\frac{2\sqrt{dx+c}}{bd}-\frac{a\ln\left(a+b\sqrt{dx+c}\right)}{b^{2}d}+\frac{a\ln\left(b\sqrt{dx+c}-a\right)}{b^{2}d}-\frac{a\ln\left(b^{2}dx+b^{2}c-a^{2}\right)}{b^{2}d}$	87

input
```
int(1/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d*((d*x+c)^(1/2)/b-a/b^2*ln(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\left(a\log\left(\sqrt{dx+c}b+a\right)-\sqrt{dx+c}b\right)}{b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{x}{a} & \text{for}\ b=0\wedge(b=0\vee d=0) \\ \frac{x}{a+b\sqrt{c}} & \text{for}\ d=0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(
d, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\left(\frac{a\log\left(\sqrt{dx+c}b+a\right)}{b^2}-\frac{\sqrt{dx+c}}{b}\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
-2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\,a\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d} + \frac{2\,\sqrt{dx+c}}{bd}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*sqrt(d*x + c)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\,\left(a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-b\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

input
```
int(1/(a + b*(c + d*x)^(1/2)),x)
```

output
```
-(2*(a*log(a + b*(c + d*x)^(1/2)) - b*(c + d*x)^(1/2)))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \frac{2\sqrt{dx+c}\,b-2\log\left(\sqrt{dx+c}\,b+a\right)a}{b^2d}$$

input
```
int(1/(a+b*(d*x+c)^(1/2)),x)
```

output
```
(2*(sqrt(c + d*x)*b - log(sqrt(c + d*x)*b + a)*a))/(b**2*d)
```

3.115 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx=\frac{2b\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

output
```
2*b*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))/(-b^2*c+a^2)+a*ln(x)/(-b^2*c+a^
2)-2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx=\frac{2b\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+a\log(-dx)-2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

input
```
Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
```

output
```
(2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[-(d*x)] - 2*a*Log[a +
b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {896, 25, 1732, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{1}{dx\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{1}{dx\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{587}$$

$$-2\left(\frac{ab\int \frac{1}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{a^2-b^2c}-\frac{\int -\frac{bc-a\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\Big\downarrow \text{16}$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{\int -\frac{bc-a\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\Big\downarrow \text{452}$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{bc\int -\frac{1}{dx}d\sqrt{c+dx}-a\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)-a\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\downarrow 240$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{\frac{1}{2}a\log(-dx)+b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}\right)$$

input
```
Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

output
```
-2*(-((b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + (a*Log[-(d*x)])/2)/(a^2
- b^2*c)) + (a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^{2}c+a^{2}}+\frac{a\ln\left(-dx\right)+2b\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^{2}c+a^{2}}$	69
default	$-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^{2}c+a^{2}}+\frac{a\ln\left(-dx\right)+2b\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^{2}c+a^{2}}$	69

input
```
int(1/x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)+2/(-b^2*c+a^2)*(1/2*a*ln(-d*x)+b*c
^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \left[\frac{b\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2a\log\left(\sqrt{dx+c}b+a\right)-a\log\left(x\right)}{b^2c-a^2}, \frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx+c}}\right)+2a\log\left(\sqrt{d\right.}}{b^2c-a^2}\right.$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*log(sqrt(d*
x + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqrt(-c)*arctan(sqrt(-c)/sqr
t(d*x + c)) + 2*a*log(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]
```

Sympy [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \begin{cases} -\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{a^2-b^2c}-\frac{2\left(-\frac{a\log\left(-dx\right)}{2}+\frac{bc\,\text{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{a^2-b^2c} & \text{for}\ d\neq 0\\ \frac{\log\left(x\right)}{a+b\sqrt{c}} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((-2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c
 + d*x))/b, True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c +
 d*x)/sqrt(-c))/sqrt(-c))/(a**2 - b**2*c), Ne(d, 0)), (log(x)/(a + b*sqrt(
c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx=\frac{b\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^2c-a^2}-\frac{a\log\left(dx\right)}{b^2c-a^2}+\frac{2\,a\log\left(\sqrt{dx+c}b+a\right)}{b^2c-a^2}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^2*c
- a^2) - a*log(d*x)/(b^2*c - a^2) + 2*a*log(sqrt(d*x + c)*b + a)/(b^2*c -
a^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx=\frac{2\,ab\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^3c-a^2b}+\frac{2\,bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^2c-a^2\right)\sqrt{-c}}-\frac{a\log\left(dx\right)}{b^2c-a^2}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*
x + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{\ln\left(2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c+6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a+b\,\sqrt{c}} + \frac{\ln\left(-2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c-6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a-b\,\sqrt{c}} + \frac{2\,a\,\ln\left(4\,b^5\,c^2\,\sqrt{c+d\,x}-36\,a^3\,b^2\,c+4\,a\,b^4\,c^2-36\,a^2\,b^3\,c\,\sqrt{c+d\,x}\right)}{b^2\,c-a^2}$$

input
```
int(1/(x*(a + b*(c + d*x)^(1/2))),x)
```

output
```
log(2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c + 6*a*b^2*c^(1/2)*
(c + d*x)^(1/2))/(a + b*c^(1/2)) + log(- 2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)
^(1/2) - 6*a*b^2*c - 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a - b*c^(1/2)) + (2
*a*log(4*b^5*c^2*(c + d*x)^(1/2) - 36*a^3*b^2*c + 4*a*b^4*c^2 - 36*a^2*b^3
*c*(c + d*x)^(1/2)))/(b^2*c - a^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{-\sqrt{c}\log\left(\sqrt{dx+c}-\sqrt{c}\right)b+\sqrt{c}\log\left(\sqrt{dx+c}+\sqrt{c}\right)b+\log\left(\sqrt{dx+c}-\sqrt{c}\right)a+\log\left(\sqrt{dx+c}+\sqrt{c}\right)a-}{-b^2c+a^2}$$

input
```
int(1/x/(a+b*(d*x+c)^(1/2)),x)
```

output
```
( - sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*b + sqrt(c)*log(sqrt(c + d*x) + s
qrt(c))*b + log(sqrt(c + d*x) - sqrt(c))*a + log(sqrt(c + d*x) + sqrt(c))*
a - 2*log(sqrt(c + d*x)*b + a)*a)/(a**2 - b**2*c)
```

3.116 $\int \frac{1}{x^2(a+b\sqrt{c+dx})}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x}+\frac{b(a^2+b^2c)\,d\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^2}+\frac{ab^2d\log(x)}{(a^2-b^2c)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

output
```
-(a-b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x+b*(b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/
c^(1/2))/c^(1/2)/(-b^2*c+a^2)^2+a*b^2*d*ln(x)/(-b^2*c+a^2)^2-2*a*b^2*d*ln(
a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

$$= \frac{b(a^2+b^2c)\,dx\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(-\left((a^2-b^2c)\left(a-b\sqrt{c+dx}\right)\right)+ab^2dx\log(-dx)-2ab^2dx\log\left(a-\right.\right.}{\sqrt{c}\,(a^2-b^2c)^2\,x}$$

input
```
Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

output
```
(b*(a^2 + b^2*c)*d*x*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^
2*c)*(a - b*Sqrt[c + d*x])) + a*b^2*d*x*Log[-(d*x)] - 2*a*b^2*d*x*Log[a +
b*Sqrt[c + d*x]]))/(Sqrt[c]*(a^2 - b^2*c)^2*x)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {896, 1732, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)}dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{1}{d^2x^2 \left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$2d\int \frac{\sqrt{c+dx}}{d^2x^2 \left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{593}$$

$$2d\left(-\frac{b\int \frac{a-b\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2d\left(\frac{b\int -\frac{a-b\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$2d\left(\frac{b\int\left(-\frac{2ab^2}{(a^2-b^2c)(a+b\sqrt{c+dx})}-\frac{a^2-2b\sqrt{c+dx}a+b^2c}{(a^2-b^2c)dx}\right)d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2d\left(\frac{b\left(\frac{(a^2+b^2c)\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(a^2-b^2c)}+\frac{ab\log(-dx)}{a^2-b^2c}-\frac{2ab\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}\right)}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

input
```
Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

output
```
2*d*(-1/2*(a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*d*x) + (b*(((a^2 + b^2*c)*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)) + (a*b*Log[-(d*x)])
/(a^2 - b^2*c) - (2*a*b*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)))/(2*(a^2
- b^2*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

method	result
derivativedivides	$2d\left(-\frac{a\,b^{2}\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{2}}+\frac{-\frac{\left(\frac{1}{2}b^{3}c-\frac{1}{2}a^{2}b\right)\sqrt{dx+c}-\frac{ca\,b^{2}}{2}+\frac{a^{3}}{2}}{dx}+\frac{b\left(ab\ln\left(-dx\right)+\frac{\left(b^{2}c+a^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2}}{\left(-b^{2}c+a^{2}\right)^{2}}\right)$
default	$2d\left(-\frac{a\,b^{2}\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{2}}+\frac{-\frac{\left(\frac{1}{2}b^{3}c-\frac{1}{2}a^{2}b\right)\sqrt{dx+c}-\frac{ca\,b^{2}}{2}+\frac{a^{3}}{2}}{dx}+\frac{b\left(ab\ln\left(-dx\right)+\frac{\left(b^{2}c+a^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2}}{\left(-b^{2}c+a^{2}\right)^{2}}\right)$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*d*(-a*b^2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^2*(-(((1/2*
b^3*c-1/2*a^2*b)*(d*x+c)^(1/2)-1/2*c*a*b^2+1/2*a^3)/d/x+1/2*b*(a*b*ln(-d*x
)+(b^2*c+a^2)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$= \left[-\frac{4\,ab^2cdx\log\left(\sqrt{dx+c}b+a\right)-2\,ab^2cdx\log\left(x\right)-2\,ab^2c^2-(b^3c+a^2b)\sqrt{c}dx\log\left(\frac{dx+2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+}{2\,(b^4c^3-2\,a^2b^2c^2+a^4c)x}\right.$$
$$-\frac{2\,ab^2cdx\log\left(\sqrt{dx+c}b+a\right)-ab^2cdx\log\left(x\right)-ab^2c^2+(b^3c+a^2b)\sqrt{-c}dx\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx+c}}\right)+a^3c+(b}{(b^4c^3-2\,a^2b^2c^2+a^4c)x}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a
*b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c)
+ 2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a
^2*b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c
*d*x*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(-c)/sqr
t(d*x + c)) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2
*b^2*c^2 + a^4*c)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx=\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$=\frac{1}{2}\left(\frac{2\,ab^2\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{4\,ab^2\log\left(\sqrt{dx+c}b+a\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{(b^3c+a^2b)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{(b^4c^2-2\,a^2b^2c+a^4)\sqrt{c}}+\frac{2\left(\sqrt{dx+c}b\right.}{b^2c^2-a^2c-(b^2c-}\right.$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*(2*a*b^2*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 4*a*b^2*log(sqrt(d*x
 + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c + a^2*b)*log((sqrt(d*x
 + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^4*c^2 - 2*a^2*b^2*c + a^4)
*sqrt(c)) + 2*(sqrt(d*x + c)*b - a)/(b^2*c^2 - a^2*c - (b^2*c - a^2)*(d*x
+ c)))*d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx=-\frac{2\,ab^3d\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}+\frac{ab^2d\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{(b^3cd+a^2bd)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^4c^2-2\,a^2b^2c+a^4)\sqrt{-c}}+\frac{ab^2cd-a^3d-(b^3cd-a^2bd)\sqrt{dx+c}}{(b^2c-a^2)^2dx}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) +
 a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arc
tan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (a*
b^2*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d*x)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d-b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3} + \frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d+b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3} + \frac{\frac{a\,d}{b^2\,c-a^2}-\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}}{d\,x} - \frac{2\,a\,b^2\,d\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{\left(b^2\,c-a^2\right)^2}$$

input
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))),x)
```

output
```
(log((c + d*x)^(1/2) - c^(1/2))*(4*a*b^2*c*d - b*c^(1/2)*d*(2*b^2*c + 2*a^
2)))/(4*a^4*c + 4*b^4*c^3 - 8*a^2*b^2*c^2) + (log((c + d*x)^(1/2) + c^(1/2
))*(4*a*b^2*c*d + b*c^(1/2)*d*(2*b^2*c + 2*a^2)))/(4*a^4*c + 4*b^4*c^3 - 8
*a^2*b^2*c^2) + ((a*d)/(b^2*c - a^2) - (b*d*(c + d*x)^(1/2))/(b^2*c - a^2)
)/(d*x) - (2*a*b^2*d*log(a + b*(c + d*x)^(1/2)))/(b^2*c - a^2)^2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{2\sqrt{dx+c}\,a^2bc-2\sqrt{dx+c}\,b^3c^2-\sqrt{c}\log\left(\sqrt{dx+c}-\sqrt{c}\right)a^2bdx-\sqrt{c}\log\left(\sqrt{dx+c}-\sqrt{c}\right)b^3cdx+\sqrt{c}\,\mathrm{lo}}{}$$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2)),x)
```

output
```
(2*sqrt(c + d*x)*a**2*b*c - 2*sqrt(c + d*x)*b**3*c**2 - sqrt(c)*log(sqrt(c
 + d*x) - sqrt(c))*a**2*b*d*x - sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*b**3*
c*d*x + sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*a**2*b*d*x + sqrt(c)*log(sqrt
(c + d*x) + sqrt(c))*b**3*c*d*x + 2*log(sqrt(c + d*x) - sqrt(c))*a*b**2*c*
d*x + 2*log(sqrt(c + d*x) + sqrt(c))*a*b**2*c*d*x - 4*log(sqrt(c + d*x)*b
+ a)*a*b**2*c*d*x - 2*a**3*c - 2*a**3*d*x + 2*a*b**2*c**2 + 2*a*b**2*c*d*x
)/(2*c*x*(a**4 - 2*a**2*b**2*c + b**4*c**2))
```

3.117 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 204

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{b(a^4-6a^2b^2c-3b^4c^2)\,d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3} + \frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}$$

output

```
-1/2*(a-b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x^2-1/4*b*d*(4*a*b*c-(3*b^2*c+a^2)*(
d*x+c)^(1/2))/c/(-b^2*c+a^2)^2/x-1/4*b*(-3*b^4*c^2-6*a^2*b^2*c+a^4)*d^2*ar
ctanh((d*x+c)^(1/2)/c^(1/2))/c^(3/2)/(-b^2*c+a^2)^3+a*b^4*d^2*ln(x)/(-b^2*
c+a^2)^3-2*a*b^4*d^2*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^3
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)}\,dx$$
$$=\frac{b(a^4-6a^2b^2c-3b^4c^2)\,d^2x^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left((a^2-b^2c)\left(2a^3c-2ab^2c(c-2dx)+b^3c(2c-3dx)\sqrt{c}\right.\right.}{4c^{3/2}\left(-a^2+b^2c\right)^3}$$

input
```
Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

output
```
(b*(a^4 - 6*a^2*b^2*c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
+ Sqrt[c]*((a^2 - b^2*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3
*d*x)*Sqrt[c + d*x] - a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2
*Log[-(d*x)] + 8*a*b^4*c*d^2*x^2*Log[a + b*Sqrt[c + d*x]]))/(4*c^(3/2)*(-a
^2 + b^2*c)^3*x^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 25, 1732, 593, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)}\,dx$$
$$\Big\downarrow \text{896}$$
$$d^2\int \frac{1}{d^3x^3\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$
$$\Big\downarrow \text{25}$$
$$-d^2\int -\frac{1}{d^3x^3\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2d^2 \int -\frac{\sqrt{c+dx}}{d^3x^3\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{593}$$

$$-2d^2\left(\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}-\frac{b\int -\frac{a-3b\sqrt{c+dx}}{d^2x^2\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{4\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$-2d^2\left(\frac{b\int \frac{a-3b\sqrt{c+dx}}{d^2x^2\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{4\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{686}$$

$$-2d^2\left(\frac{b\left(\frac{4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}}{2cdx\left(a^2-b^2c\right)}-\frac{\int \frac{a\left(a^2-5b^2c\right)+b\left(a^2+3b^2c\right)\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{2c\left(a^2-b^2c\right)}\right)}{4\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$-2d^2\left(\frac{b\left(\frac{\int -\frac{a\left(a^2-5b^2c\right)+b\left(a^2+3b^2c\right)\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{2c\left(a^2-b^2c\right)}+\frac{4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}}{2cdx\left(a^2-b^2c\right)}\right)}{4\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$-2d^2\left(\frac{b\left(\frac{\int \left(\frac{8ab^4c}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{a^4-6b^2ca^2+8b^3c\sqrt{c+dx}a-3b^4c^2}{\left(a^2-b^2c\right)dx}\right)d\sqrt{c+dx}}{2c\left(a^2-b^2c\right)}+\frac{4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}}{2cdx\left(a^2-b^2c\right)}\right)}{4\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2d^2\left(\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}+\frac{b\left(\frac{4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}}{2cdx\left(a^2-b^2c\right)}+\frac{-\frac{4ab^3c\log(-dx)}{a^2-b^2c}+\frac{8ab^3c\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{\left(a^4-6a^2b^2c-3b^4c^2\right)\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)}}{2c\left(a^2-b^2c\right)}\right)}{4\left(a^2-b^2c\right)}\right)$$

input
```
Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

output
```
-2*d^2*((a - b*Sqrt[c + d*x])/(4*(a^2 - b^2*c)*d^2*x^2) + (b*((4*a*b*c - (
a^2 + 3*b^2*c)*Sqrt[c + d*x])/(2*c*(a^2 - b^2*c)*d*x) + (((a^4 - 6*a^2*b^2
*c - 3*b^4*c^2)*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)) -
(4*a*b^3*c*Log[-(d*x)])/(a^2 - b^2*c) + (8*a*b^3*c*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c))/(2*c*(a^2 - b^2*c))))/(4*(a^2 - b^2*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11

method	result
derivativedivides	$2d^2\left(-\frac{a\,b^4\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^3}-\frac{\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}a^3b^2\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}a^4b-\frac{5}{8}b^5c^2\right)}{d^2x^2}}{(-}\right.$
default	$2d^2\left(-\frac{a\,b^4\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^3}-\frac{\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}a^3b^2\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}a^4b-\frac{5}{8}b^5c^2\right)}{d^2x^2}}{(-}\right.$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*d^2*(-a*b^4/(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))-1/(-b^2*c+a^2)^3*((-1/8
*b*(-3*b^4*c^2+2*a^2*b^2*c+a^4)/c*(d*x+c)^(3/2)+(-1/2*a*b^4*c+1/2*a^3*b^2)
*(d*x+c)+(3/4*a^2*b^3*c-1/8*a^4*b-5/8*b^5*c^2)*(d*x+c)^(1/2)+3/4*a*b^4*c^2
-a^3*b^2*c+1/4*a^5)/d^2/x^2+1/8*b/c*(-4*a*b^3*c*ln(-d*x)+(-3*b^4*c^2-6*a^2
*b^2*c+a^4)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^3 \left(a + b\sqrt{c+dx}\right)}\,dx$$

$$= \left[\frac{16\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}b+a\right) - 8\,ab^4c^2d^2x^2\log\left(x\right) + 4\,ab^4c^4 - 8\,a^3b^2c^3 + 4\,a^5c^2 + \left(3\,b^5c^2 + 6\,a^2b^3c\right.}{8\,(b^6}\right.$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*
log(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*
c - a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) -
8*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c
^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 -
 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*l
og(sqrt(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3
*b^2*c^3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*
arctan(sqrt(-c)/sqrt(d*x + c)) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^5*
c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*
d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x
^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = \int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Integral(1/(x**3*(a + b*sqrt(c + d*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx =$$
$$-\frac{1}{8}\left(\frac{8\,ab^4\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{16\,ab^4\log\left(\sqrt{dx+c}b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\log\left(\frac{\sqrt{dx}}{\sqrt{dx}}\right.}{\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a\right.}\right.$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
-1/8*(8*a*b^4*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - 16*
a*b^4*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^
6) - (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt
(d*x + c) + sqrt(c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*s
qrt(c)) + 2*(4*(d*x + c)*a*b^2*c - 6*a*b^2*c^2 + 2*a^3*c - (3*b^3*c + a^2*
b)*(d*x + c)^(3/2) + (5*b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/(b^4*c^5 - 2*a^2
*b^2*c^4 + a^4*c^3 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(d*x + c)^2 - 2*(b^
4*c^4 - 2*a^2*b^2*c^3 + a^4*c^2)*(d*x + c)))*d^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{2\,ab^5d^2\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b} - \frac{ab^4d^2\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6} + \frac{\left(3\,b^5c^2d^2+6\,a^2b^3cd^2-a^4bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{-c}} + \frac{6\,ab^4c^3d^2-8\,a^3b^2c^2d^2+2\,a^5cd^2+\left(3\,b^5c^2d^2-2\,a^2b^3cd^2-a^4bd^2\right)(dx+c)^{\frac{3}{2}}-4\left(ab^4c^2d^2-a^3b^2cd^2\right)(dx}{4\left(b^2c-a^2\right)^3cd^2x^2}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4
*b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*
c - a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d
*x + c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(
-c)) + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^5*c^2
*d^2 - 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2 - a
^3*b^2*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*d^2
)*sqrt(d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 11.25 (sec) , antiderivative size = 1094, normalized size of antiderivative = 5.36

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))),x)
```

output
```
(log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2*(b^2*c - a^2)^4)
- (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) -
(b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d
^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(
3*b^4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)
^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c - a^2)^2))*(
3*b^4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(b^2*c - a^2)
^3))*(8*a*b^4*c^3*d^2 - a^4*b*d^2*(c^3)^(1/2) + 3*b^5*c^2*d^2*(c^3)^(1/2)
+ 6*a^2*b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*
a^2*b^4*c^5)) - ((a^3*d^2 - 3*a*b^2*c*d^2)/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c
)) - ((a^2*b*d^2 + 3*b^3*c*d^2)*(c + d*x)^(3/2))/(4*c*(a^4 + b^4*c^2 - 2*a
^2*b^2*c)) + (b*d^2*(5*b^2*c - a^2)*(c + d*x)^(1/2))/(4*(a^4 + b^4*c^2 - 2
*a^2*b^2*c)) + (a*b^2*d^2*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/((c +
d*x)^2 - 2*c*(c + d*x) + c^2) + (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^
(1/2))/(16*c^2*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2
*c))/(16*c^2*(b^2*c - a^2)^4) - (b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^
2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*
b*c + 3*b^2*c*(c + d*x)^(1/2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c
^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)
^(1/2))/(2*c*(b^2*c - a^2)^2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.25

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$= \frac{4\sqrt{dx+c}\,a^4b\,c^2+2\sqrt{dx+c}\,a^4bcdx-8\sqrt{dx+c}\,a^2b^3c^3+4\sqrt{dx+c}\,a^2b^3c^2dx+4\sqrt{dx+c}\,b^5c^4-6\sqrt{dx+\cdots}}{}$$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2)),x)
```

output

```
(4*sqrt(c + d*x)*a**4*b*c**2 + 2*sqrt(c + d*x)*a**4*b*c*d*x - 8*sqrt(c + d
*x)*a**2*b**3*c**3 + 4*sqrt(c + d*x)*a**2*b**3*c**2*d*x + 4*sqrt(c + d*x)*
b**5*c**4 - 6*sqrt(c + d*x)*b**5*c**3*d*x + sqrt(c)*log(sqrt(c + d*x) - sq
rt(c))*a**4*b*d**2*x**2 - 6*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a**2*b**3
*c*d**2*x**2 - 3*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*b**5*c**2*d**2*x**2
- sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*a**4*b*d**2*x**2 + 6*sqrt(c)*log(sq
rt(c + d*x) + sqrt(c))*a**2*b**3*c*d**2*x**2 + 3*sqrt(c)*log(sqrt(c + d*x)
 + sqrt(c))*b**5*c**2*d**2*x**2 + 8*log(sqrt(c + d*x) - sqrt(c))*a*b**4*c*
*2*d**2*x**2 + 8*log(sqrt(c + d*x) + sqrt(c))*a*b**4*c**2*d**2*x**2 - 16*l
og(sqrt(c + d*x)*b + a)*a*b**4*c**2*d**2*x**2 - 4*a**5*c**2 + 8*a**3*b**2*
c**3 - 8*a**3*b**2*c**2*d*x - 4*a**3*b**2*c*d**2*x**2 - 4*a*b**4*c**4 + 8*
a*b**4*c**3*d*x + 4*a*b**4*c**2*d**2*x**2)/(8*c**2*x**2*(a**6 - 3*a**4*b**
2*c + 3*a**2*b**4*c**2 - b**6*c**3))
```

3.118 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 240

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{(5a^4-9a^2b^2c+3b^4c^2)\,x}{b^6d^3} - \frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4} - \frac{4a(2a^2-3b^2c)\,(c+dx)^{3/2}}{3b^5d^4} + \frac{3(a^2-b^2c)\,(c+dx)^2}{2b^4d^4} - \frac{4a(c+dx)^{5/2}}{5b^3d^4} + \frac{(c+dx)^3}{3b^2d^4} + \frac{2a(a^2-b^2c)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)} + \frac{2(a^2-b^2c)^2\,(7a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}$$

output
```
(3*b^4*c^2-9*a^2*b^2*c+5*a^4)*x/b^6/d^3-12*a*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/
b^7/d^4-4/3*a*(-3*b^2*c+2*a^2)*(d*x+c)^(3/2)/b^5/d^4+3/2*(-b^2*c+a^2)*(d*x
+c)^2/b^4/d^4-4/5*a*(d*x+c)^(5/2)/b^3/d^4+1/3*(d*x+c)^3/b^2/d^4+2*a*(-b^2*
c+a^2)^3/b^8/d^4/(a+b*(d*x+c)^(1/2))+2*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*ln(a+
b*(d*x+c)^(1/2))/b^8/d^4
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{60a^7-360a^6b\sqrt{c+dx}-30a^5b^2(13c+7dx)+10a^4b^3\sqrt{c+dx}(79c+7dx)-3a^2b^5\sqrt{c+dx}(163c^2+36c}{}$$

input
```
Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(60*a^7 - 360*a^6*b*Sqrt[c + d*x] - 30*a^5*b^2*(13*c + 7*d*x) + 10*a^4*b^3
*Sqrt[c + d*x]*(79*c + 7*d*x) - 3*a^2*b^5*Sqrt[c + d*x]*(163*c^2 + 36*c*d*
x - 7*d^2*x^2) + 5*a^3*b^4*(119*c^2 + 76*c*d*x - 7*d^2*x^2) + 5*b^7*Sqrt[c
 + d*x]*(11*c^3 + 6*c^2*d*x - 3*c*d^2*x^2 + 2*d^3*x^3) - a*b^6*(269*c^3 +
162*c^2*d*x - 33*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2
*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqr
t[c + d*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}dx$$
$$\Big\downarrow \ 896$$
$$\frac{\int \frac{d^3x^3}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^4}$$
$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{d^3x^3}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^4}$$

$\downarrow$ 1732

$$-\frac{2\int -\frac{d^3x^3\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d^4}$$

$\downarrow$ 522

$$-\frac{2\int \left(\frac{a\left(a^2-b^2c\right)^3}{b^7\left(a+b\sqrt{c+dx}\right)^2}+\frac{6a\left(a^2-b^2c\right)^2}{b^7}-\frac{(c+dx)^{5/2}}{b^2}+\frac{2a(c+dx)^2}{b^3}+\frac{3\left(b^2c-a^2\right)(c+dx)^{3/2}}{b^4}+\frac{2a\left(2a^2-3b^2c\right)(c+dx)}{b^5}-\frac{(5a^4-9b^2ca^2+}{b}\right.}{d^4}$$

$\downarrow$ 2009

$$-\frac{2\left(-\frac{a\left(a^2-b^2c\right)^3}{b^8\left(a+b\sqrt{c+dx}\right)}-\frac{\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8}+\frac{6a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7}+\frac{2a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5}-\frac{3\left(a^2-b^2c\right)(c+dx)^{2}}{4b^4}\right.}{d^4}$$

input
```
Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((6*a*(a^2 - b^2*c)^2*Sqrt[c + d*x])/b^7 - ((5*a^4 - 9*a^2*b^2*c + 3*b
^4*c^2)*(c + d*x))/(2*b^6) + (2*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^
5) - (3*(a^2 - b^2*c)*(c + d*x)^2)/(4*b^4) + (2*a*(c + d*x)^(5/2))/(5*b^3)
 - (c + d*x)^3/(6*b^2) - (a*(a^2 - b^2*c)^3)/(b^8*(a + b*Sqrt[c + d*x])) -
 ((a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/b^8))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2b^3(dx+c)^2}{4}-2a\,b^4c(dx+c)^{\frac{3}{2}}-\frac{3b^5c^2(dx+c)}{2}+\frac{4a^3b^2(dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2b^3c(dx+\right.}{b^7}$
default	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2b^3(dx+c)^2}{4}-2a\,b^4c(dx+c)^{\frac{3}{2}}-\frac{3b^5c^2(dx+c)}{2}+\frac{4a^3b^2(dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2b^3c(dx+\right.}{b^7}$

input
```
int(x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d^4*(-1/b^7*(-1/6*(d*x+c)^3*b^5+2/5*a*(d*x+c)^(5/2)*b^4+3/4*b^5*c*(d*x+c
)^2-3/4*a^2*b^3*(d*x+c)^2-2*a*b^4*c*(d*x+c)^(3/2)-3/2*b^5*c^2*(d*x+c)+4/3*
a^3*b^2*(d*x+c)^(3/2)+9/2*a^2*b^3*c*(d*x+c)+6*a*c^2*b^4*(d*x+c)^(1/2)-5/2*
a^4*b*(d*x+c)-12*a^3*c*b^2*(d*x+c)^(1/2)+6*a^5*(d*x+c)^(1/2))+1/b^8*(-b^6*
c^3+9*a^2*b^4*c^2-15*a^4*b^2*c+7*a^6)*ln(a+b*(d*x+c)^(1/2))+a*(-b^6*c^3+3*
a^2*b^4*c^2-3*a^4*b^2*c+a^6)/b^8/(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.63

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{10\,b^8d^4x^4 + 55\,b^8c^4 - 220\,a^2b^6c^3 + 195\,a^4b^4c^2 + 30\,a^6b^2c - 60\,a^8 - 5\,(b^8c - 7\,a^2b^6)d^3x^3 + 15\,(b^8c^2 - 8\,a^2\cdots}{\cdots}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30
*a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*
b^6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c
 - 30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^
6*b^2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x
)*log(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b
^5*c^2 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2
*(24*a*b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*
x + (b^10*c - a^2*b^8)*d^4)
```

Sympy [A] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{2a(c+dx)^{\frac{5}{2}}}{5b^3} - \frac{a\left(a^2-b^2c\right)^3\left(\begin{cases} \frac{\sqrt{c+dx}}{a^2} & \text{for } b=0 \\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise} \end{cases}\right)}{b^7} + \frac{(c+dx)^3}{6b^2} + \frac{\left(3a^2-3b^2c\right)(c+dx)^2}{4b^4} + \frac{\left(-4a^3+6ab^2c\right)(c+dx)^{\frac{3}{2}}}{3b^5} + \frac{(c+dx)\left(5a^4 - \cdots\right)}{2\cdots} \cdots\right)}{d^4} & \cdots \\ \frac{x^4}{4\left(a+b\sqrt{c}\right)^2} & \cdots \end{cases}$$

input
```
integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-2*a*(c + d*x)**(5/2)/(5*b**3) - a*(a**2 - b**2*c)**3*Piecew
ise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x))), True))/
b**7 + (c + d*x)**3/(6*b**2) + (3*a**2 - 3*b**2*c)*(c + d*x)**2/(4*b**4) +
 (-4*a**3 + 6*a*b**2*c)*(c + d*x)**(3/2)/(3*b**5) + (c + d*x)*(5*a**4 - 9*
a**2*b**2*c + 3*b**4*c**2)/(2*b**6) + (a**2 - b**2*c)**2*(7*a**2 - b**2*c)
*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True)
)/b**7 + sqrt(c + d*x)*(-6*a**5 + 12*a**3*b**2*c - 6*a*b**4*c**2)/b**7)/d*
*4, Ne(d, 0)), (x**4/(4*(a + b*sqrt(c))**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx =$$

$$-\frac{\frac{60\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)}{\sqrt{dx+c}b^9+ab^8}-\frac{10\,(dx+c)^3b^5-24\,(dx+c)^{\frac{5}{2}}ab^4-45\,(b^5c-a^2b^3)(dx+c)^2+40\,(3\,ab^4c-2\,a^3b^2)(dx+c)^{\frac{3}{2}}+30\,(3\,b^5c^2-9\,a}{b^7}}{30\,d^4}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b
^9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c -
 a^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3
*b^5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c
 + a^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c -
7*a^6)*log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{2\,(b^6c^3-9\,a^2b^4c^2+15\,a^4b^2c-7\,a^6)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^8d^4}$$

$$-\frac{2\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)}{\left(\sqrt{dx+c}b+a\right)b^8d^4}$$

$$+\frac{10\,(dx+c)^3b^{10}d^{20}-45\,(dx+c)^2b^{10}cd^{20}+90\,(dx+c)b^{10}c^2d^{20}-24\,(dx+c)^{\frac{5}{2}}ab^9d^{20}+120\,(dx+c)^{\frac{3}{2}}ab}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-2*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(abs(sqrt(d*x + c)*
b + a))/(b^8*d^4) - 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/((sq
rt(d*x + c)*b + a)*b^8*d^4) + 1/30*(10*(d*x + c)^3*b^10*d^20 - 45*(d*x + c
)^2*b^10*c*d^20 + 90*(d*x + c)*b^10*c^2*d^20 - 24*(d*x + c)^(5/2)*a*b^9*d^
20 + 120*(d*x + c)^(3/2)*a*b^9*c*d^20 - 360*sqrt(d*x + c)*a*b^9*c^2*d^20 +
 45*(d*x + c)^2*a^2*b^8*d^20 - 270*(d*x + c)*a^2*b^8*c*d^20 - 80*(d*x + c)
^(3/2)*a^3*b^7*d^20 + 720*sqrt(d*x + c)*a^3*b^7*c*d^20 + 150*(d*x + c)*a^4
*b^6*d^20 - 360*sqrt(d*x + c)*a^5*b^5*d^20)/(b^12*d^24)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.92

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \left(\frac{4\,a^3}{3\,b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{3\,b}\right)\,{\left(c+d\,x\right)}^{3/2} - \left(\frac{3\,c}{2\,b^2\,d^4}-\frac{3\,a^2}{2\,b^4\,d^4}\right)\,{\left(c+d\,x\right)}^2 - \left(\frac{2\,a\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b}+\frac{6\,c^2}{b^2\,d^4}\right)}{b} + \frac{a^2\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b^2}\right)\,\sqrt{c+d\,x} + \frac{{\left(c+d\,x\right)}^3}{3\,b^2\,d^4} + \frac{2\,\left(a^7-3\,a^5\,b^2\,c+3\,a^3\,b^4\,c^2-a\,b^6\,c^3\right)}{b\,\left(b^8\,d^4\,\sqrt{c+d\,x}+a\,b^7\,d^4\right)} + d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{2\,b^2}-\frac{a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b} + \frac{3\,c^2}{b^2\,d^4}\right) + \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(14\,a^6-30\,a^4\,b^2\,c+18\,a^2\,b^4\,c^2-2\,b^6\,c^3\right)}{b^8\,d^4} - \frac{4\,a\,{\left(c+d\,x\right)}^{5/2}}{5\,b^3\,d^4}$$

input
```
int(x^3/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
((4*a^3)/(3*b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(3*b))*
(c + d*x)^(3/2) - ((3*c)/(2*b^2*d^4) - (3*a^2)/(2*b^4*d^4))*(c + d*x)^2 -
((2*a*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b^2 - (2*a*((4*a^3)/(b^
5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (6*c^2)/(b^2*
d^4)))/b + (a^2*((4*a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*
d^4)))/b))/b^2)*(c + d*x)^(1/2) + (c + d*x)^3/(3*b^2*d^4) + (2*(a^7 - 3*a^
5*b^2*c - a*b^6*c^3 + 3*a^3*b^4*c^2))/(b*(b^8*d^4*(c + d*x)^(1/2) + a*b^7*
d^4)) + d*x*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(2*b^2) - (a*((4*
a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (3*c^
2)/(b^2*d^4)) + (log(a + b*(c + d*x)^(1/2))*(14*a^6 - 2*b^6*c^3 - 30*a^4*b
^2*c + 18*a^2*b^4*c^2))/(b^8*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.93

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{420\sqrt{dx+c}\log\left(\sqrt{dx+c}\,b+a\right)a^6b-60\sqrt{dx+c}\log\left(\sqrt{dx+c}\,b+a\right)b^7c^3+540\sqrt{dx+c}\log\left(\sqrt{dx+c}\,b\right.}{}$$

input
```
int(x^3/(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
(420*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*a**6*b - 900*sqrt(c + d*x)*log
(sqrt(c + d*x)*b + a)*a**4*b**3*c + 540*sqrt(c + d*x)*log(sqrt(c + d*x)*b
+ a)*a**2*b**5*c**2 - 60*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*b**7*c**3
- 420*sqrt(c + d*x)*a**6*b + 970*sqrt(c + d*x)*a**4*b**3*c + 70*sqrt(c + d
*x)*a**4*b**3*d*x - 669*sqrt(c + d*x)*a**2*b**5*c**2 - 108*sqrt(c + d*x)*a
**2*b**5*c*d*x + 21*sqrt(c + d*x)*a**2*b**5*d**2*x**2 + 115*sqrt(c + d*x)*
b**7*c**3 + 30*sqrt(c + d*x)*b**7*c**2*d*x - 15*sqrt(c + d*x)*b**7*c*d**2*
x**2 + 10*sqrt(c + d*x)*b**7*d**3*x**3 + 420*log(sqrt(c + d*x)*b + a)*a**7
 - 900*log(sqrt(c + d*x)*b + a)*a**5*b**2*c + 540*log(sqrt(c + d*x)*b + a)
*a**3*b**4*c**2 - 60*log(sqrt(c + d*x)*b + a)*a*b**6*c**3 - 210*a**5*b**2*
c - 210*a**5*b**2*d*x + 415*a**3*b**4*c**2 + 380*a**3*b**4*c*d*x - 35*a**3
*b**4*d**2*x**2 - 209*a*b**6*c**3 - 162*a*b**6*c**2*d*x + 33*a*b**6*c*d**2
*x**2 - 14*a*b**6*d**3*x**3)/(30*b**8*d**4*(sqrt(c + d*x)*b + a))
```

3.119 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 166

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{(3a^2-2b^2c)\,x}{b^4d^2} - \frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3} + \frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)} + \frac{2(5a^4-6a^2b^2c+b^4c^2)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

output
```
(-2*b^2*c+3*a^2)*x/b^4/d^2-8*a*(-b^2*c+a^2)*(d*x+c)^(1/2)/b^5/d^3-4/3*a*(d
*x+c)^(3/2)/b^3/d^3+1/2*(d*x+c)^2/b^2/d^3+2*a*(-b^2*c+a^2)^2/b^6/d^3/(a+b*
(d*x+c)^(1/2))+2*(b^4*c^2-6*a^2*b^2*c+5*a^4)*ln(a+b*(d*x+c)^(1/2))/b^6/d^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{12a^5 - 48a^4b\sqrt{c+dx} - 6a^3b^2(9c+5dx) + 2a^2b^3\sqrt{c+dx}(29c+5dx) + ab^4(43c^2+26cdx-5d^2x^2) + 3b}{6b^6d^3\left(a+b\sqrt{c}\right.}$$

input
```
Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(12*a^5 - 48*a^4*b*Sqrt[c + d*x] - 6*a^3*b^2*(9*c + 5*d*x) + 2*a^2*b^3*Sqr
t[c + d*x]*(29*c + 5*d*x) + a*b^4*(43*c^2 + 26*c*d*x - 5*d^2*x^2) + 3*b^5*
Sqrt[c + d*x]*(-3*c^2 - 2*c*d*x + d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4
*c^2)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sq
rt[c + d*x]))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}dx$$
$$\Big\downarrow \; 896$$
$$\frac{\int \frac{d^2x^2}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^3}$$
$$\Big\downarrow \; 1732$$
$$\frac{2\int \frac{d^2x^2\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{522}$$

$$\frac{2\int \left(-\frac{a(a^2-b^2c)^2}{b^5\left(a+b\sqrt{c+dx}\right)^2}-\frac{4a(a^2-b^2c)}{b^5}+\frac{(c+dx)^{3/2}}{b^2}-\frac{2a(c+dx)}{b^3}-\frac{(2b^2c-3a^2)\sqrt{c+dx}}{b^4}+\frac{5a^4-6b^2ca^2+b^4c^2}{b^5\left(a+b\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{a(a^2-b^2c)^2}{b^6\left(a+b\sqrt{c+dx}\right)}-\frac{4a(a^2-b^2c)\sqrt{c+dx}}{b^5}+\frac{(3a^2-2b^2c)(c+dx)}{2b^4}+\frac{(5a^4-6a^2b^2c+b^4c^2)\log\left(a+b\sqrt{c+dx}\right)}{b^6}-\frac{2a(c+dx)^{3/2}}{3b^3}+\frac{(c+dx)^2}{4b^2}\right)}{d^3}$$

input
```
Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*((-4*a*(a^2 - b^2*c)*Sqrt[c + d*x])/b^5 + ((3*a^2 - 2*b^2*c)*(c + d*x))
/(2*b^4) - (2*a*(c + d*x)^(3/2))/(3*b^3) + (c + d*x)^2/(4*b^2) + (a*(a^2 -
 b^2*c)^2)/(b^6*(a + b*Sqrt[c + d*x])) + ((5*a^4 - 6*a^2*b^2*c + b^4*c^2)*
Log[a + b*Sqrt[c + d*x]])/b^6))/d^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(dx+c)^2 b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}} b^2}{3}+b^3 c(dx+c)-\frac{3a^2 b(dx+c)}{2}-4acb^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}+\frac{2\left(b^4c^2-6a^2\cdots\right.}{\cdots}}{d^3}$
default	$\frac{-\frac{2\left(-\frac{(dx+c)^2 b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}} b^2}{3}+b^3 c(dx+c)-\frac{3a^2 b(dx+c)}{2}-4acb^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}+\frac{2\left(b^4c^2-6a^2\cdots\right.}{\cdots}}{d^3}$

input
```
int(x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d^3*(-1/b^5*(-1/4*(d*x+c)^2*b^3+2/3*a*(d*x+c)^(3/2)*b^2+b^3*c*(d*x+c)-3/
2*a^2*b*(d*x+c)-4*a*c*b^2*(d*x+c)^(1/2)+4*a^3*(d*x+c)^(1/2))+a*(b^4*c^2-2*
a^2*b^2*c+a^4)/b^6/(a+b*(d*x+c)^(1/2))+1/b^6*(b^4*c^2-6*a^2*b^2*c+5*a^4)*l
n(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$=\frac{3\,b^6d^3x^3-9\,b^6c^3+15\,a^2b^4c^2+6\,a^4b^2c-12\,a^6-3\,(b^6c-5\,a^2b^4)d^2x^2-3\,(5\,b^6c^2-14\,a^2b^4c+6\,a^4b^2)dx\cdots}{\cdots}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3
*(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*
x + 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*
b^4*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13
*a*b^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt
(d*x + c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)
```

Sympy [A] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{2a(c+dx)^{\frac{3}{2}}}{3b^3} - \frac{a\left(a^2-b^2c\right)^2\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0 \\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^5} + \frac{(c+dx)^2}{4b^2} + \frac{\left(3a^2-2b^2c\right)(c+dx)}{2b^4} + \frac{\left(a^2-b^2c\right)\left(5a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} \\ \frac{\log\left(a+b\sqrt{c+d\ldots}\right)}{b}\end{cases}\right.}{b^5} \ldots\right.}{d^3} \\ \frac{x^3}{3\left(a+b\sqrt{c}\right)^2} \end{cases}$$

input
```
integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-2*a*(c + d*x)**(3/2)/(3*b**3) - a*(a**2 - b**2*c)**2*Piecew
ise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x))), True))/
b**5 + (c + d*x)**2/(4*b**2) + (3*a**2 - 2*b**2*c)*(c + d*x)/(2*b**4) + (a
**2 - b**2*c)*(5*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (lo
g(a + b*sqrt(c + d*x))/b, True))/b**5 + (-4*a**3 + 4*a*b**2*c)*sqrt(c + d*
x)/b**5)/d**3, Ne(d, 0)), (x**3/(3*(a + b*sqrt(c))**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\frac{12\,(ab^4c^2-2\,a^3b^2c+a^5)}{\sqrt{dx+c}b^7+ab^6}+\frac{3\,(dx+c)^2b^3-8\,(dx+c)^{\frac{3}{2}}ab^2-6\,(2\,b^3c-3\,a^2b)(dx+c)+48\,(ab^2c-a^3)\sqrt{dx+c}}{b^5}+\frac{12\,(b^4c^2-6\,a^2b^2c+5\,a^4)\log\left(\sqrt{dx+c}b+a\right)}{b^6}}{6\,d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^6) + (3*(
d*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c)
 + 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a
^4)*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\frac{12\,(b^4c^2-6\,a^2b^2c+5\,a^4)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^6d}+\frac{12\,(ab^4c^2-2\,a^3b^2c+a^5)}{(\sqrt{dx+c}b+a)b^6d}+\frac{3\,(dx+c)^2b^6d^3-12\,(dx+c)b^6cd^3-8\,(dx+c)^{\frac{3}{2}}ab^5d^3+48\,\sqrt{dx+c}ab^5cd^3+18\,(dx+c)a^2b^4d^3-48\,\sqrt{dx+c}a^3b^3d^3}{b^8d^4}}{6\,d^2}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/6*(12*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*log(abs(sqrt(d*x + c)*b + a))/(b^6
*d) + 12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/((sqrt(d*x + c)*b + a)*b^6*d) + (
3*(d*x + c)^2*b^6*d^3 - 12*(d*x + c)*b^6*c*d^3 - 8*(d*x + c)^(3/2)*a*b^5*d
^3 + 48*sqrt(d*x + c)*a*b^5*c*d^3 + 18*(d*x + c)*a^2*b^4*d^3 - 48*sqrt(d*x
 + c)*a^3*b^3*d^3)/(b^8*d^4))/d^2
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \left(\frac{4\,a^3}{b^5\,d^3}+\frac{2\,a\,\left(\frac{4\,c}{b^2\,d^3}-\frac{6\,a^2}{b^4\,d^3}\right)}{b}\right)\,\sqrt{c+d\,x} + \frac{2\,\left(a^5-2\,a^3\,b^2\,c+a\,b^4\,c^2\right)}{b\,\left(b^6\,d^3\,\sqrt{c+d\,x}+a\,b^5\,d^3\right)}+\frac{{\left(c+d\,x\right)}^2}{2\,b^2\,d^3} - d\,x\,\left(\frac{2\,c}{b^2\,d^3}-\frac{3\,a^2}{b^4\,d^3}\right)-\frac{4\,a\,{\left(c+d\,x\right)}^{3/2}}{3\,b^3\,d^3} + \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(10\,a^4-12\,a^2\,b^2\,c+2\,b^4\,c^2\right)}{b^6\,d^3}$$

input
```
int(x^2/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
((4*a^3)/(b^5*d^3) + (2*a*((4*c)/(b^2*d^3) - (6*a^2)/(b^4*d^3)))/b)*(c + d
*x)^(1/2) + (2*(a^5 - 2*a^3*b^2*c + a*b^4*c^2))/(b*(b^6*d^3*(c + d*x)^(1/2
) + a*b^5*d^3)) + (c + d*x)^2/(2*b^2*d^3) - d*x*((2*c)/(b^2*d^3) - (3*a^2)
/(b^4*d^3)) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (log(a + b*(c + d*x)^(1/
2))*(10*a^4 + 2*b^4*c^2 - 12*a^2*b^2*c))/(b^6*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{60\sqrt{dx+c}\,\log\left(\sqrt{dx+c}\,b+a\right)a^4b-72\sqrt{dx+c}\,\log\left(\sqrt{dx+c}\,b+a\right)a^2b^3c+12\sqrt{dx+c}\,\log\left(\sqrt{dx+c}\,b+\right.}{}$$

input
```
int(x^2/(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
(60*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*a**4*b - 72*sqrt(c + d*x)*log(s
qrt(c + d*x)*b + a)*a**2*b**3*c + 12*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a
)*b**5*c**2 - 60*sqrt(c + d*x)*a**4*b + 82*sqrt(c + d*x)*a**2*b**3*c + 10*
sqrt(c + d*x)*a**2*b**3*d*x - 21*sqrt(c + d*x)*b**5*c**2 - 6*sqrt(c + d*x)
*b**5*c*d*x + 3*sqrt(c + d*x)*b**5*d**2*x**2 + 60*log(sqrt(c + d*x)*b + a)
*a**5 - 72*log(sqrt(c + d*x)*b + a)*a**3*b**2*c + 12*log(sqrt(c + d*x)*b +
 a)*a*b**4*c**2 - 30*a**3*b**2*c - 30*a**3*b**2*d*x + 31*a*b**4*c**2 + 26*
a*b**4*c*d*x - 5*a*b**4*d**2*x**2)/(6*b**6*d**3*(sqrt(c + d*x)*b + a))
```

3.120 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{x}{b^2d}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

output
```
x/b^2/d-4*a*(d*x+c)^(1/2)/b^3/d^2+2*a*(-b^2*c+a^2)/b^4/d^2/(a+b*(d*x+c)^(1
/2))+2*(-b^2*c+3*a^2)*ln(a+b*(d*x+c)^(1/2))/b^4/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2a^3-2ab^2c-4a^2b\sqrt{c+dx}-3ab^2(c+dx)+b^3(c+dx)^{3/2}}{b^4d^2\left(a+b\sqrt{c+dx}\right)}-\frac{2(-3a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

input
```
Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*a^3 - 2*a*b^2*c - 4*a^2*b*Sqrt[c + d*x] - 3*a*b^2*(c + d*x) + b^3*(c +
d*x)^(3/2))/(b^4*d^2*(a + b*Sqrt[c + d*x])) - (2*(-3*a^2 + b^2*c)*Log[a +
b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}dx$$
$$\Big\downarrow \text{896}$$
$$\frac{\int \frac{dx}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^2}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -\frac{dx}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^2}$$
$$\Big\downarrow \text{1732}$$
$$-\frac{2\int -\frac{dx\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d^2}$$
$$\Big\downarrow \text{522}$$
$$-\frac{2\int \left(\frac{2a}{b^3}-\frac{\sqrt{c+dx}}{b^2}+\frac{b^2c-3a^2}{b^3\left(a+b\sqrt{c+dx}\right)}+\frac{a^3-ab^2c}{b^3\left(a+b\sqrt{c+dx}\right)^2}\right)d\sqrt{c+dx}}{d^2}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2\left(-\frac{a\left(a^2-b^2c\right)}{b^4\left(a+b\sqrt{c+dx}\right)}-\frac{\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4}+\frac{2a\sqrt{c+dx}}{b^3}-\frac{c+dx}{2b^2}\right)}{d^2}$$

input
```
Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((2*a*Sqrt[c + d*x])/b^3 - (c + d*x)/(2*b^2) - (a*(a^2 - b^2*c))/(b^4*
(a + b*Sqrt[c + d*x])) - ((3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/b^4))/
d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{2\left(-\frac{b(dx+c)}{2}+2a\sqrt{dx+c}\right)}{b^3}+\frac{2\left(-b^2c+3a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}+\frac{2a\left(-b^2c+a^2\right)}{b^4\left(a+b\sqrt{dx+c}\right)}}{d^2}$	87
default	$\frac{-\frac{2\left(-\frac{b(dx+c)}{2}+2a\sqrt{dx+c}\right)}{b^3}+\frac{2\left(-b^2c+3a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}+\frac{2a\left(-b^2c+a^2\right)}{b^4\left(a+b\sqrt{dx+c}\right)}}{d^2}$	87

input
```
int(x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(-1/b^3*(-1/2*b*(d*x+c)+2*a*(d*x+c)^(1/2))+1/b^4*(-b^2*c+3*a^2)*ln(a
+b*(d*x+c)^(1/2))+a*(-b^2*c+a^2)/b^4/(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.72

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{b^4d^2x^2+b^4c^2+a^2b^2c-2\,a^4+(2\,b^4c-a^2b^2)dx-2\,(b^4c^2-4\,a^2b^2c+3\,a^4+(b^4c-3\,a^2b^2)dx)\log\left(\sqrt{dx-\right.}{b^6d^3x+(b^6c-a^2b^4)d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
(b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(
b^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)
*b + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x
+ (b^6*c - a^2*b^4)*d^2)
```

Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{a(a^2-b^2c)\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b(a+b\sqrt{c+dx})} & \text{otherwise}\end{cases}\right)}{b^3}-\frac{2a\sqrt{c+dx}}{b^3}+\frac{c+dx}{2b^2}+\frac{(3a^2-b^2c)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log(a+b\sqrt{c+dx})}{b} & \text{otherwise}\end{cases}\right)}{b^3}\right)}{d^2} & \text{for } d \ne 0\\ \frac{x^2}{2(a+b\sqrt{c})^2} & \text{otherwise}\end{cases}$$

input
```
integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)),
 (-1/(b*(a + b*sqrt(c + d*x))), True))/b**3 - 2*a*sqrt(c + d*x)/b**3 + (c
+ d*x)/(2*b**2) + (3*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)),
 (log(a + b*sqrt(c + d*x))/b, True))/b**3)/d**2, Ne(d, 0)), (x**2/(2*(a +
b*sqrt(c))**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{\frac{2\,(ab^2c-a^3)}{\sqrt{dx+c}b^5+ab^4}-\frac{(dx+c)b-4\sqrt{dx+c}a}{b^3}+\frac{2\,(b^2c-3\,a^2)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b - 4*sqrt(d*
x + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{\frac{2\,(b^2c-3\,a^2)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d}-\frac{(dx+c)b^2d-4\,\sqrt{dx+c}abd}{b^4d^2}+\frac{2\,(ab^2c-a^3)}{(\sqrt{dx+c}b+a)b^4d}}{d}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-(2*(b^2*c - 3*a^2)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) - ((d*x + c)*b^2
*d - 4*sqrt(d*x + c)*a*b*d)/(b^4*d^2) + 2*(a*b^2*c - a^3)/((sqrt(d*x + c)*
b + a)*b^4*d))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{x}{b^2\,d}+\frac{2\,(a^3-a\,b^2\,c)}{b\,\left(b^4\,d^2\,\sqrt{c+d\,x}+a\,b^3\,d^2\right)} - \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,(2\,b^2\,c-6\,a^2)}{b^4\,d^2}-\frac{4\,a\,\sqrt{c+d\,x}}{b^3\,d^2}$$

input
```
int(x/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
x/(b^2*d) + (2*(a^3 - a*b^2*c))/(b*(b^4*d^2*(c + d*x)^(1/2) + a*b^3*d^2))
- (log(a + b*(c + d*x)^(1/2))*(2*b^2*c - 6*a^2))/(b^4*d^2) - (4*a*(c + d*x
)^(1/2))/(b^3*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.59

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{6\sqrt{dx+c}\log\left(\sqrt{dx+c}\,b+a\right)a^2b-2\sqrt{dx+c}\log\left(\sqrt{dx+c}\,b+a\right)b^3c-6\sqrt{dx+c}\,a^2b+3\sqrt{dx+c}\,b^3c+}{b^4d^2\left(\sqrt{dx+c}\,b+\right.}$$

input
```
int(x/(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
(6*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*a**2*b - 2*sqrt(c + d*x)*log(sqr
t(c + d*x)*b + a)*b**3*c - 6*sqrt(c + d*x)*a**2*b + 3*sqrt(c + d*x)*b**3*c
 + sqrt(c + d*x)*b**3*d*x + 6*log(sqrt(c + d*x)*b + a)*a**3 - 2*log(sqrt(c
 + d*x)*b + a)*a*b**2*c - 3*a*b**2*c - 3*a*b**2*d*x)/(b**4*d**2*(sqrt(c +
d*x)*b + a))
```

3.121 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

output
```
2*a/b^2/d/(a+b*(d*x+c)^(1/2))+2*ln(a+b*(d*x+c)^(1/2))/b^2/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)\right)}{b^2d}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
```

output
```
(2*(a/(a + b*Sqrt[c + d*x]) + Log[b*d*(a + b*Sqrt[c + d*x])]))/(b^2*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \frac{\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{2\int \left(\frac{1}{b\left(a+b\sqrt{c+dx}\right)}-\frac{a}{b\left(a+b\sqrt{c+dx}\right)^2}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{a}{b^2\left(a+b\sqrt{c+dx}\right)}+\frac{\log\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^(-2),x]
```

output
```
(2*(a/(b^2*(a + b*Sqrt[c + d*x])) + Log[a + b*Sqrt[c + d*x]]/b^2))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{2\ln\left(a+b\sqrt{dx+c}\right)}{b^2}+\frac{2a}{b^2\left(a+b\sqrt{dx+c}\right)}}{d}$
default	$\frac{a^2}{\left(-b^2dx-b^2c+a^2\right)b^2d}+\frac{c}{\left(-b^2dx-b^2c+a^2\right)d}+b^2d\left(\frac{\ln\left(b^2dx+b^2c-a^2\right)}{b^4d^2}-\frac{-b^2c+a^2}{b^4d^2\left(b^2dx+b^2c-a^2\right)}\right)+\frac{a}{b^2d\left(a+b\sqrt{d\right.}}$

input
```
int(1/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/b^2*ln(a+b*(d*x+c)^(1/2))+a/b^2/(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\left(\sqrt{dx+c}ab-a^2+(b^2dx+b^2c-a^2)\log\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2x+(b^4c-a^2b^2)d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b +
 a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(39) = 78.

Time = 0.54 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.64

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for } b=0 \wedge (b=0 \vee d=0) \\ \frac{x}{(a+b\sqrt{c})^2} & \text{for } d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} + \frac{2a}{ab^2d+b^3d\sqrt{c+dx}} + \frac{2b\sqrt{c+dx}\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**
2, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*
x)) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b +
sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2\left(\frac{a}{\sqrt{dx+c}b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}b+a\right)}{b^2}\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d}+\frac{2a}{\left(\sqrt{dx+c}b+a\right)b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
2*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a/((sqrt(d*x + c)*b + a)*b^2*d
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2\ln\left(a+b\sqrt{c+d\,x}\right)}{b^2\,d}+\frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{c+d\,x}\right)}$$

input
```
int(1/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(2*log(a + b*(c + d*x)^(1/2)))/(b^2*d) + (2*a)/(b^2*(a*d + b*d*(c + d*x)^(
1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\sqrt{dx+c}\log\left(\sqrt{dx+c}\,b+a\right)b-2\sqrt{dx+c}\,b+2\log\left(\sqrt{dx+c}\,b+a\right)a}{b^2d\left(\sqrt{dx+c}\,b+a\right)}$$

input

```
int(1/(a+b*(d*x+c)^(1/2))^2,x)
```

output

```
(2*(sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*b - sqrt(c + d*x)*b + log(sqrt(
c + d*x)*b + a)*a))/(b**2*d*(sqrt(c + d*x)*b + a))
```

3.122 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}+\frac{4ab\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{(a^2+b^2c)\log(x)}{(a^2-b^2c)^2}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

output
```
2*a/(-b^2*c+a^2)/(a+b*(d*x+c)^(1/2))+4*a*b*c^(1/2)*arctanh((d*x+c)^(1/2)/c
^(1/2))/(-b^2*c+a^2)^2+(b^2*c+a^2)*ln(x)/(-b^2*c+a^2)^2-2*(b^2*c+a^2)*ln(a
+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\frac{2a(a^2-b^2c)}{a+b\sqrt{c+dx}}+4ab\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+(a^2+b^2c)\log(-dx)-2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

input
```
Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
((2*a*(a^2 - b^2*c))/(a + b*Sqrt[c + d*x]) + 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
+ d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)] - 2*(a^2 + b^2*c)*Log[a + b*Sq
rt[c + d*x]])/(a^2 - b^2*c)^2
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 25, 1732, 594, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b\sqrt{c+dx}\right)^2}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{1}{dx \left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{1}{dx \left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2\int -\frac{\sqrt{c+dx}}{dx \left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 594}$$

$$-2\left(\frac{\int \frac{bc-a\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-2\left(-\frac{\int -\frac{bc-a\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$-2\left(-\frac{\int \left(-\frac{b(a^2+b^2c)}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{2abc-(a^2+b^2c)\sqrt{c+dx}}{(a^2-b^2c)dx}\right)d\sqrt{c+dx}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{\frac{2ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{(a^2+b^2c)\log(-dx)}{2(a^2-b^2c)}-\frac{(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

input
```
Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
-2*(-(a/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x]))) - ((2*a*b*Sqrt[c]*ArcTanh[S
qrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + ((a^2 + b^2*c)*Log[-(d*x)])/(2*(a^2
 - b^2*c)) - ((a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c))/(a^2
- b^2*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2a}{(-b^2c+a^2)(a+b\sqrt{dx+c})}-\frac{2(b^2c+a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(-b^2c+a^2)^2}$	118
default	$\frac{2a}{(-b^2c+a^2)(a+b\sqrt{dx+c})}-\frac{2(b^2c+a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(-b^2c+a^2)^2}$	118

input
```
int(1/x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*a/(-b^2*c+a^2)/(a+b*(d*x+c)^(1/2))-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(
-b^2*c+a^2)^2+2/(-b^2*c+a^2)^2*(-1/2*(-b^2*c-a^2)*ln(-d*x)+2*a*b*c^(1/2)*a
rctanh((d*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.42

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \left[\frac{2a^2b^2c - 2a^4 + 2\left(ab^3dx + ab^3c - a^3b\right)\sqrt{c}\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right) - 2\left(b^4c^2 - a^4 + \left(b^4c + a^2b^2\right)dx\right)\log\left(\right.}{b^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^6 + \left(b^6c^2 - 2\right.}\right.$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x +
 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*
d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*lo
g(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4
*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^
4 - 4*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x + c)
) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (
b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d
*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b
^4*c + a^4*b^2)*d*x)]
```

Sympy [A] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \begin{cases} -\frac{2ab\left(\begin{cases} \frac{\sqrt{c+dx}}{a^2} & \text{for } b=0 \\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise} \end{cases}\right)}{a^2-b^2c} - \frac{2b\left(a^2+b^2c\right)\left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for } b=0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise} \end{cases}\right)}{\left(a^2-b^2c\right)^2} - \frac{2\cdot\left(\frac{2abc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \left(-\frac{a^2}{2}\right.\right.}{\left(a^2-b^2c\right)^2} \\ \frac{\log(x)}{\left(a+b\sqrt{c}\right)^2} \end{cases}$$

input
```
integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((-2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*
sqrt(c + d*x))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((s
qrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**
2*c)**2 - 2*(2*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b*
*2*c/2)*log(-d*x))/(a**2 - b**2*c)**2, Ne(d, 0)), (log(x)/(a + b*sqrt(c))*
*2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{2\,ab\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^4c^2-2\,a^2b^2c+a^4}+\frac{\left(b^2c+a^2\right)\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\left(b^2c+a^2\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\,a}{ab^2c-a^3+\left(b^3c-a^2b\right)\sqrt{dx+c}}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b
^4*c^2 - 2*a^2*b^2*c + a^4) + (b^2*c + a^2)*log(d*x)/(b^4*c^2 - 2*a^2*b^2*
c + a^4) - 2*(b^2*c + a^2)*log(sqrt(d*x + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c
 + a^4) - 2*a/(a*b^2*c - a^3 + (b^3*c - a^2*b)*sqrt(d*x + c))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}}+\frac{\left(b^2c+a^2\right)\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\left(b^3c+a^2b\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b} - \frac{2\left(ab^2c-a^3\right)}{\left(b^2c-a^2\right)^2\left(\sqrt{dx+c}b+a\right)}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqr
t(-c)) + (b^2*c + a^2)*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^3*c
+ a^2*b)*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) - 2
*(a*b^2*c - a^3)/((b^2*c - a^2)^2*(sqrt(d*x + c)*b + a))
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\left(a+b\,\sqrt{c}\right)^2} + \ln\left(a+b\,\sqrt{c+d\,x}\right)\left(\frac{2}{b^2\,c-a^2}-\frac{4\,b^2\,c}{\left(b^2\,c-a^2\right)^2}\right) + \frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{\left(a-b\,\sqrt{c}\right)^2} - \frac{2\,a}{\left(b^2\,c-a^2\right)\left(a+b\,\sqrt{c+d\,x}\right)}$$

input
```
int(1/(x*(a + b*(c + d*x)^(1/2))^2),x)
```

output
```
log((c + d*x)^(1/2) - c^(1/2))/(a + b*c^(1/2))^2 + log(a + b*(c + d*x)^(1/
2))*(2/(b^2*c - a^2) - (4*b^2*c)/(b^2*c - a^2)^2) + log((c + d*x)^(1/2) +
c^(1/2))/(a - b*c^(1/2))^2 - (2*a)/((b^2*c - a^2)*(a + b*(c + d*x)^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.16

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{-2\sqrt{c}\,\sqrt{dx+c}\log\left(\sqrt{dx+c}-\sqrt{c}\right)a\,b^2+2\sqrt{c}\,\sqrt{dx+c}\log\left(\sqrt{dx+c}+\sqrt{c}\right)a\,b^2+\sqrt{dx+c}\log\left(\sqrt{dx+c}\right.}{}$$

input
```
int(1/x/(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
( - 2*sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*a*b**2 + 2*sqrt(c
)*sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a*b**2 + sqrt(c + d*x)*log(sq
rt(c + d*x) - sqrt(c))*a**2*b + sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))
*b**3*c + sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a**2*b + sqrt(c + d*x
)*log(sqrt(c + d*x) + sqrt(c))*b**3*c - 2*sqrt(c + d*x)*log(sqrt(c + d*x)*
b + a)*a**2*b - 2*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*b**3*c - 2*sqrt(c
 + d*x)*a**2*b + 2*sqrt(c + d*x)*b**3*c - 2*sqrt(c)*log(sqrt(c + d*x) - sq
rt(c))*a**2*b + 2*sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*a**2*b + log(sqrt(c
 + d*x) - sqrt(c))*a**3 + log(sqrt(c + d*x) - sqrt(c))*a*b**2*c + log(sqrt
(c + d*x) + sqrt(c))*a**3 + log(sqrt(c + d*x) + sqrt(c))*a*b**2*c - 2*log(
sqrt(c + d*x)*b + a)*a**3 - 2*log(sqrt(c + d*x)*b + a)*a*b**2*c)/(sqrt(c +
 d*x)*a**4*b - 2*sqrt(c + d*x)*a**2*b**3*c + sqrt(c + d*x)*b**5*c**2 + a**
5 - 2*a**3*b**2*c + a*b**4*c**2)
```

3.123 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 195

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a^2+b^2c-2ab\sqrt{c+dx}}{(a^2-b^2c)^2\,x} + \frac{2ab(a^2+3b^2c)\,d\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3} + \frac{b^2(3a^2+b^2c)\,d\log(x)}{(a^2-b^2c)^3} - \frac{2b^2(3a^2+b^2c)\,d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}$$

output
```
2*a*b^2*d/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))-(a^2+b^2*c-2*a*b*(d*x+c)^(1/2
))/(-b^2*c+a^2)^2/x+2*a*b*(3*b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2))/c
^(1/2)/(-b^2*c+a^2)^3+b^2*(b^2*c+3*a^2)*d*ln(x)/(-b^2*c+a^2)^3-2*b^2*(b^2*
c+3*a^2)*d*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^3
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)^2} \, dx$$
$$= \frac{\frac{(a^2-b^2c)\left(-a^3+a^2b\sqrt{c+dx}-b^3c\sqrt{c+dx}+ab^2(c+4dx)\right)}{x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2c)d\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2(3a^2+b^2c)\,d\log(-dx) - 2b^2(}{(a^2-b^2c)^3}$$

input
```
Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
(((a^2 - b^2*c)*(-a^3 + a^2*b*Sqrt[c + d*x] - b^3*c*Sqrt[c + d*x] + a*b^2*
(c + 4*d*x)))/(x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/Sqrt[c] + b^2*(3*a^2 + b^2*c)*d*Log[-(d*x)] - 2*b
^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {896, 1732, 593, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)^2} \, dx$$
$$\Big\downarrow \; 896$$
$$d\int \frac{1}{d^2x^2 \left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$
$$\Big\downarrow \; 1732$$
$$2d\int \frac{\sqrt{c+dx}}{d^2x^2 \left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{593}$$

$$2d\left(-\frac{b\int \frac{2\left(a-b\sqrt{c+dx}\right)}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2d\left(\frac{b\int -\frac{a-b\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{a^2-b^2c}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$2d\left(\frac{b\int \left(-\frac{\left(3a^2+b^2c\right)b^2}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{2ab^2}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^2}-\frac{a\left(a^2+3b^2c\right)-b\left(3a^2+b^2c\right)\sqrt{c+dx}}{\left(a^2-b^2c\right)^2dx}\right)d\sqrt{c+dx}}{a^2-b^2c}-\frac{a-b\sqrt{}}{2dx\left(a^2-b^2c\right)(}\right.$$

$$\Big\downarrow \text{2009}$$

$$2d\left(\frac{b\left(\frac{a\left(a^2+3b^2c\right)\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}+\frac{2ab}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b\left(3a^2+b^2c\right)\log(-dx)}{2\left(a^2-b^2c\right)^2}-\frac{b\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}\right)}{a^2-b^2c}-\frac{}{2dx\left(a^2\right.}\right.$$

input
```
Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
2*d*(-1/2*(a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*d*x*(a + b*Sqrt[c + d*x]))
+ (b*((2*a*b)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (a*(a^2 + 3*b^2*c)*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (b*(3*a^2 + b^2
*c)*Log[-(d*x)])/(2*(a^2 - b^2*c)^2) - (b*(3*a^2 + b^2*c)*Log[a + b*Sqrt[c
 + d*x]])/(a^2 - b^2*c)^2))/(a^2 - b^2*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93

method	result
derivativedivides	$2d\left(\frac{a\,b^{2}}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+b\sqrt{dx+c}\right)}-\frac{b^{2}\left(b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{-\frac{\left(a\,b^{3}c-a^{3}b\right)\sqrt{dx+c}-\frac{b^{4}c^{2}}{2}+\frac{a^{4}}{2}}{dx}+b\left(-\frac{\left(-b^{3}c-3 \ldots\right.}{}\right.}{\left(-b^{2}c- \ldots\right.}\right.$
default	$2d\left(\frac{a\,b^{2}}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+b\sqrt{dx+c}\right)}-\frac{b^{2}\left(b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{-\frac{\left(a\,b^{3}c-a^{3}b\right)\sqrt{dx+c}-\frac{b^{4}c^{2}}{2}+\frac{a^{4}}{2}}{dx}+b\left(-\frac{\left(-b^{3}c-3 \ldots\right.}{}\right.}{\left(-b^{2}c- \ldots\right.}\right.$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*d*(a*b^2/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))-b^2*(b^2*c+3*a^2)/(-b^2*c+a^
2)^3*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^3*(-((a*b^3*c-a^3*b)*(d*x+c)^(1/
2)-1/2*b^4*c^2+1/2*a^4)/d/x+b*(-1/2*(-b^3*c-3*a^2*b)*ln(-d*x)+(3*a*b^2*c+a
^3)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(189) = 378.

Time = 0.31 (sec) , antiderivative size = 851, normalized size of antiderivative = 4.36

$$\int \frac{1}{x^{2}\left(a+b\sqrt{c+dx}\right)^{2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2
- 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3
*b^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
- 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^
2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b
^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b
^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4
 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6
*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3
- a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((
3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sq
rt(-c)*arctan(sqrt(-c)/sqrt(d*x + c)) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2
 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) +
 ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c
)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^
3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a
^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2
 + a^8*c)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(a + b*sqrt(c + d*x))**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.88

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx =$$

$$-d\left(\frac{\left(b^4c+3\,a^2b^2\right)\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{2\left(b^4c+3\,a^2b^2\right)\log\left(\sqrt{dx+c}b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,ab^3c+a^3b\right)\log\left(\frac{\sqrt{dx+c}}{\sqrt{dx+c}}\right.}{\left(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-\right.}\right.$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-d*((b^4*c + 3*a^2*b^2)*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c -
a^6) - 2*(b^4*c + 3*a^2*b^2)*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4
*c^2 + 3*a^4*b^2*c - a^6) - (3*a*b^3*c + a^3*b)*log((sqrt(d*x + c) - sqrt(
c))/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a
^6)*sqrt(c)) + (4*(d*x + c)*a*b^2 - 3*a*b^2*c - a^3 - (b^3*c - a^2*b)*sqrt
(d*x + c))/(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - (b^5*c^2 - 2*a^2*b^3*c + a
^4*b)*(d*x + c)^(3/2) - (a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(d*x + c) + (b^5*c
^3 - 2*a^2*b^3*c^2 + a^4*b*c)*sqrt(d*x + c)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

$$= -\frac{\left(b^4cd+3\,a^2b^2d\right)\log\left(-dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{2\left(b^5cd+3\,a^2b^3d\right)\log\left(\left|-\sqrt{dx+c}b-a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}$$
$$+\frac{2\left(3\,ab^3cd+a^3bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6\right)\sqrt{-c}}$$
$$-\frac{\sqrt{dx+c}b^3cd-4\left(dx+c\right)ab^2d+3\,ab^2cd-\sqrt{dx+c}a^2bd+a^3d}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\left(\left(dx+c\right)^{\frac{3}{2}}b-\sqrt{dx+c}bc+\left(dx+c\right)a-ac\right)}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-(b^4*c*d + 3*a^2*b^2*d)*log(-d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c
- a^6) + 2*(b^5*c*d + 3*a^2*b^3*d)*log(abs(-sqrt(d*x + c)*b - a))/(b^7*c^3
 - 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) + 2*(3*a*b^3*c*d + a^3*b*d)*arctan
(sqrt(d*x + c)/sqrt(-c))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sq
rt(-c)) - (sqrt(d*x + c)*b^3*c*d - 4*(d*x + c)*a*b^2*d + 3*a*b^2*c*d - sqr
t(d*x + c)*a^2*b*d + a^3*d)/((b^4*c^2 - 2*a^2*b^2*c + a^4)*((d*x + c)^(3/2
)*b - sqrt(d*x + c)*b*c + (d*x + c)*a - a*c))
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{b\,d\,\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{a^3\,\sqrt{c}-b^3\,c^2+3\,a\,b^2\,c^{3/2}-3\,a^2\,b\,c} - \frac{\frac{a\,d\,\left(a^2+3\,c\,b^2\right)}{\left(b^2\,c-a^2\right)^2}+\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}-\frac{4\,a\,b^2\,d\,\left(c+d\,x\right)}{a^4-2\,a^2\,b^2\,c+b^4\,c^2}}{b\,\left(c+d\,x\right)^{3/2}-a\,c+a\,\left(c+d\,x\right)-b\,c\,\sqrt{c+d\,x}} - \frac{b\,d\,\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{a^3\,\sqrt{c}+b^3\,c^2+3\,a\,b^2\,c^{3/2}+3\,a^2\,b\,c} - \ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(\frac{6\,b^2\,d}{\left(b^2\,c-a^2\right)^2}-\frac{8\,b^4\,c\,d}{\left(b^2\,c-a^2\right)^3}\right)$$

input
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))^2),x)
```

output
```
(b*d*log((c + d*x)^(1/2) + c^(1/2)))/(a^3*c^(1/2) - b^3*c^2 + 3*a*b^2*c^(3
/2) - 3*a^2*b*c) - ((a*d*(3*b^2*c + a^2))/(b^2*c - a^2)^2 + (b*d*(c + d*x)
^(1/2))/(b^2*c - a^2) - (4*a*b^2*d*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c
))/(b*(c + d*x)^(3/2) - a*c + a*(c + d*x) - b*c*(c + d*x)^(1/2)) - (b*d*lo
g((c + d*x)^(1/2) - c^(1/2)))/(a^3*c^(1/2) + b^3*c^2 + 3*a*b^2*c^(3/2) + 3
*a^2*b*c) - log(a + b*(c + d*x)^(1/2))*((6*b^2*d)/(b^2*c - a^2)^2 - (8*b^4
*c*d)/(b^2*c - a^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 696, normalized size of antiderivative = 3.57

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2))^2,x)
```

output
```
( - sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*a**3*b**2*d*x - 3*s
qrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*a*b**4*c*d*x + sqrt(c)*s
qrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a**3*b**2*d*x + 3*sqrt(c)*sqrt(c
 + d*x)*log(sqrt(c + d*x) + sqrt(c))*a*b**4*c*d*x + 3*sqrt(c + d*x)*log(sq
rt(c + d*x) - sqrt(c))*a**2*b**3*c*d*x + sqrt(c + d*x)*log(sqrt(c + d*x) -
 sqrt(c))*b**5*c**2*d*x + 3*sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a**
2*b**3*c*d*x + sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*b**5*c**2*d*x -
6*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*a**2*b**3*c*d*x - 2*sqrt(c + d*x)
*log(sqrt(c + d*x)*b + a)*b**5*c**2*d*x + sqrt(c + d*x)*a**4*b*c - 2*sqrt(
c + d*x)*a**2*b**3*c**2 - 4*sqrt(c + d*x)*a**2*b**3*c*d*x + sqrt(c + d*x)*
b**5*c**3 + 4*sqrt(c + d*x)*b**5*c**2*d*x - sqrt(c)*log(sqrt(c + d*x) - sq
rt(c))*a**4*b*d*x - 3*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a**2*b**3*c*d*x
 + sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*a**4*b*d*x + 3*sqrt(c)*log(sqrt(c
+ d*x) + sqrt(c))*a**2*b**3*c*d*x + 3*log(sqrt(c + d*x) - sqrt(c))*a**3*b*
*2*c*d*x + log(sqrt(c + d*x) - sqrt(c))*a*b**4*c**2*d*x + 3*log(sqrt(c + d
*x) + sqrt(c))*a**3*b**2*c*d*x + log(sqrt(c + d*x) + sqrt(c))*a*b**4*c**2*
d*x - 6*log(sqrt(c + d*x)*b + a)*a**3*b**2*c*d*x - 2*log(sqrt(c + d*x)*b +
 a)*a*b**4*c**2*d*x - a**5*c + 2*a**3*b**2*c**2 - a*b**4*c**3)/(c*x*(sqrt(
c + d*x)*a**6*b - 3*sqrt(c + d*x)*a**4*b**3*c + 3*sqrt(c + d*x)*a**2*b**5*
c**2 - sqrt(c + d*x)*b**7*c**3 + a**7 - 3*a**5*b**2*c + 3*a**3*b**4*c**...
```

3.124 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\begin{aligned}\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = {} & \frac{ab^2(a^2+11b^2c)\,d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a^2+b^2c-2ab\sqrt{c+dx}}{2\left(a^2-b^2c\right)^2x^2} \\ & - \frac{bd\left(5abc-(a^2+2b^2c)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} \\ & - \frac{ab(a^4-10a^2b^2c-15b^4c^2)\,d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4} \\ & + \frac{b^4(5a^2+b^2c)\,d^2\log(x)}{\left(a^2-b^2c\right)^4} \\ & - \frac{2b^4(5a^2+b^2c)\,d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}\end{aligned}$$

output
```
1/2*a*b^2*(11*b^2*c+a^2)*d^2/c/(-b^2*c+a^2)^3/(a+b*(d*x+c)^(1/2))-1/2*(a^2
+b^2*c-2*a*b*(d*x+c)^(1/2))/(-b^2*c+a^2)^2/x^2-1/2*b*d*(5*a*b*c-(2*b^2*c+a
^2)*(d*x+c)^(1/2))/c/(-b^2*c+a^2)^2/x/(a+b*(d*x+c)^(1/2))-1/2*a*b*(-15*b^4
*c^2-10*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3/2)/(-b^2*c+
a^2)^4+b^4*(b^2*c+5*a^2)*d^2*ln(x)/(-b^2*c+a^2)^4-2*b^4*(b^2*c+5*a^2)*d^2*
ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^4
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$=\frac{1}{2}\left(\frac{a^5c-b^5c^2(c-2dx)\sqrt{c+dx}+a^2b^3c(2c-dx)\sqrt{c+dx}-a^4b(c+dx)^{3/2}+ab^4c(c^2-3cdx-11d^2x^2)}{c\left(-a^2+b^2c\right)^3x^2\left(a+b\sqrt{c+dx}\right)}\right.$$
$$+\frac{\left(-a^5b+10a^3b^3c+15ab^5c^2\right)d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{c^{3/2}\left(a^2-b^2c\right)^4}+\frac{2b^4\left(5a^2+b^2c\right)d^2\log(-dx)}{\left(a^2-b^2c\right)^4}$$
$$\left.-\frac{4b^4\left(5a^2+b^2c\right)d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}\right)$$

input
```
Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
((a^5*c - b^5*c^2*(c - 2*d*x)*Sqrt[c + d*x] + a^2*b^3*c*(2*c - d*x)*Sqrt[c
 + d*x] - a^4*b*(c + d*x)^(3/2) + a*b^4*c*(c^2 - 3*c*d*x - 11*d^2*x^2) - a
^3*b^2*(2*c^2 - 3*c*d*x + d^2*x^2))/(c*(-a^2 + b^2*c)^3*x^2*(a + b*Sqrt[c
+ d*x])) + ((-(a^5*b) + 10*a^3*b^3*c + 15*a*b^5*c^2)*d^2*ArcTanh[Sqrt[c +
d*x]/Sqrt[c]])/(c^(3/2)*(a^2 - b^2*c)^4) + (2*b^4*(5*a^2 + b^2*c)*d^2*Log[
-(d*x)])/(a^2 - b^2*c)^4 - (4*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d
*x]])/(a^2 - b^2*c)^4)/2
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {896, 25, 1732, 593, 27, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

$$\Big\downarrow \text{896}$$

$$d^2\int \frac{1}{d^3x^3\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-d^2\int -\frac{1}{d^3x^3\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2d^2\int -\frac{\sqrt{c+dx}}{d^3x^3\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{593}$$

$$-2d^2\left(\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{b\int -\frac{2\left(a-2b\sqrt{c+dx}\right)}{d^2x^2\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{4\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$-2d^2\left(\frac{b\int \frac{a-2b\sqrt{c+dx}}{d^2x^2\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{686}$$

$$-2d^2\left(\frac{b\left(\frac{3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}}{2cdx\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{\int \frac{a\left(a^2-7b^2c\right)+2b\left(a^2+2b^2c\right)\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2c\left(a^2-b^2c\right)}\right)}{2\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$-2d^2\left(\frac{b\left(\frac{\int -\frac{a\left(a^2-7b^2c\right)+2b\left(a^2+2b^2c\right)\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2c\left(a^2-b^2c\right)}+\frac{3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}}{2cdx\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)}{2\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$-2d^2\left(\frac{b\left(\frac{\int \left(\frac{a\left(a^2+11b^2c\right)b^2}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^2}-\frac{4c\left(5a^2+b^2c\right)\sqrt{c+dx}b^3+a\left(a^4-10b^2ca^2-15b^4c^2\right)}{\left(a^2-b^2c\right)^2dx}+\frac{4c\left(cb^6+5a^2b^4\right)}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{2c(a^2-b^2c)}+\frac{3abc-(a^2+2b^2c}{2cdx(a^2-b^2c)(a+}\right.}{2\left(a^2-b^2c\right)}\right.$$

$$\Big\downarrow \; 2009$$

$$-2d^2\left(\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b\left(\frac{3abc-(a^2+2b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)(a+b\sqrt{c+dx})}+\frac{-\frac{ab\left(a^2+11b^2c\right)}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2b^3c\left(5a^2+b^2c\right)\log(-dx)}{\left(a^2-b^2c\right)^2}+\frac{4b}{}}{}\right.}{2\left(a^2-b^2c\right.}\right.$$

input
```
Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
-2*d^2*((a - b*Sqrt[c + d*x])/(4*(a^2 - b^2*c)*d^2*x^2*(a + b*Sqrt[c + d*x
])) + (b*((3*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x])/(2*c*(a^2 - b^2*c)*d*x
*(a + b*Sqrt[c + d*x])) + (-((a*b*(a^2 + 11*b^2*c))/((a^2 - b^2*c)*(a + b*
Sqrt[c + d*x]))) + (a*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) - (2*b^3*c*(5*a^2 + b^2*c)*Log[-(d
*x)])/(a^2 - b^2*c)^2 + (4*b^3*c*(5*a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(a^2 - b^2*c)^2)/(2*c*(a^2 - b^2*c))))/(2*(a^2 - b^2*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.01

method	result
derivativedivides	$2d^2\left(\frac{b^4 a}{(-b^2c+a^2)^3(a+b\sqrt{dx+c})} - \frac{b^4(b^2c+5a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^4} - \frac{-\frac{ab(-7b^4c^2+6a^2b^2c+a^4)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}b^6c^2-\right.}{}\right.$
default	$2d^2\left(\frac{b^4 a}{(-b^2c+a^2)^3(a+b\sqrt{dx+c})} - \frac{b^4(b^2c+5a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^4} - \frac{-\frac{ab(-7b^4c^2+6a^2b^2c+a^4)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}b^6c^2-\right.}{}\right.$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^2*(b^4/(-b^2*c+a^2)^3*a/(a+b*(d*x+c)^(1/2))-b^4*(b^2*c+5*a^2)/(-b^2*c+
a^2)^4*ln(a+b*(d*x+c)^(1/2))-1/(-b^2*c+a^2)^4*((-1/4*a*b*(-7*b^4*c^2+6*a^2
*b^2*c+a^4)/c*(d*x+c)^(3/2)+(-1/2*b^6*c^2-a^2*c*b^4+3/2*a^4*b^2)*(d*x+c)+(
-9/4*a*b^5*c^2+5/2*a^3*b^3*c-1/4*a^5*b)*(d*x+c)^(1/2)+3/4*b^6*c^3+3/4*a^2*
b^4*c^2-7/4*a^4*b^2*c+1/4*a^6)/d^2/x^2+1/4*b/c*(-1/2*(4*b^5*c^2+20*a^2*b^3
*c)*ln(-d*x)+(-15*a*b^4*c^2-10*a^3*b^2*c+a^5)/c^(1/2)*arctanh((d*x+c)^(1/2
)/c^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(283) = 566$.

Time = 0.95 (sec) , antiderivative size = 1249, normalized size of antiderivative = 4.18

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4
+ 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*
a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3
)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)
*sqrt(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 8*((b^8*c^3 + 5*a^
2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*lo
g(sqrt(d*x + c)*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 +
 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b
^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^
5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)
*d*x)*sqrt(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^
4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 -
10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6
*c^5 + 2*a^6*b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^
2)*d^2*x^2 - (b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x
 + ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^
3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d
*x + c)) + 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3
 - 5*a^4*b^4*c^2)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - 2*((b^8*c^3 + 5*a^2*
b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*l...
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)^2}\,dx=\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)^2}\,dx$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x**3*(a + b*sqrt(c + d*x))**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(283) = 566$.

Time = 0.13 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$=\frac{1}{4}\,d^2\left(\frac{4\,(b^6c+5\,a^2b^4)\log(dx)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{8\,(b^6c+5\,a^2b^4)\log\left(\sqrt{dx+c}b+a\right)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{(15\,ab^5c^2}{(b^8c^5-4\,a}\right.$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/4*d^2*(4*(b^6*c + 5*a^2*b^4)*log(d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b
^4*c^2 - 4*a^6*b^2*c + a^8) - 8*(b^6*c + 5*a^2*b^4)*log(sqrt(d*x + c)*b +
a)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - (15*a*b
^5*c^2 + 10*a^3*b^3*c - a^5*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c
) + sqrt(c)))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 +
a^8*c)*sqrt(c)) - 2*(7*a*b^4*c^3 + 6*a^3*b^2*c^2 - a^5*c + (11*a*b^4*c + a
^3*b^2)*(d*x + c)^2 - (2*b^5*c^2 - a^2*b^3*c - a^4*b)*(d*x + c)^(3/2) - (1
9*a*b^4*c^2 + 5*a^3*b^2*c)*(d*x + c) + 3*(b^5*c^3 - a^2*b^3*c^2)*sqrt(d*x
+ c))/(a*b^6*c^6 - 3*a^3*b^4*c^5 + 3*a^5*b^2*c^4 - a^7*c^3 + (b^7*c^4 - 3*
a^2*b^5*c^3 + 3*a^4*b^3*c^2 - a^6*b*c)*(d*x + c)^(5/2) + (a*b^6*c^4 - 3*a^
3*b^4*c^3 + 3*a^5*b^2*c^2 - a^7*c)*(d*x + c)^2 - 2*(b^7*c^5 - 3*a^2*b^5*c^
4 + 3*a^4*b^3*c^3 - a^6*b*c^2)*(d*x + c)^(3/2) - 2*(a*b^6*c^5 - 3*a^3*b^4*
c^4 + 3*a^5*b^2*c^3 - a^7*c^2)*(d*x + c) + (b^7*c^6 - 3*a^2*b^5*c^5 + 3*a^
4*b^3*c^4 - a^6*b*c^3)*sqrt(d*x + c)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.74

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{(b^6cd^2+5a^2b^4d^2)\log(-dx)}{b^8c^4-4a^2b^6c^3+6a^4b^4c^2-4a^6b^2c+a^8} - \frac{2(b^7cd^2+5a^2b^5d^2)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^9c^4-4a^2b^7c^3+6a^4b^5c^2-4a^6b^3c+a^8b}$$
$$- \frac{(15ab^5c^2d^2+10a^3b^3cd^2-a^5bd^2)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{2(b^8c^5-4a^2b^6c^4+6a^4b^4c^3-4a^6b^2c^2+a^8c)\sqrt{-c}}$$
$$- \frac{7ab^6c^4d^2-a^3b^4c^3d^2-7a^5b^2c^2d^2+a^7cd^2+(11ab^6c^2d^2-10a^3b^4cd^2-a^5b^2d^2)(dx+c)^2-(2b^7c^3d^2-}{2(b^2c}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
(b^6*c*d^2 + 5*a^2*b^4*d^2)*log(-d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4
*c^2 - 4*a^6*b^2*c + a^8) - 2*(b^7*c*d^2 + 5*a^2*b^5*d^2)*log(abs(sqrt(d*x
 + c)*b + a))/(b^9*c^4 - 4*a^2*b^7*c^3 + 6*a^4*b^5*c^2 - 4*a^6*b^3*c + a^8
*b) - 1/2*(15*a*b^5*c^2*d^2 + 10*a^3*b^3*c*d^2 - a^5*b*d^2)*arctan(sqrt(d*
x + c)/sqrt(-c))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2
 + a^8*c)*sqrt(-c)) - 1/2*(7*a*b^6*c^4*d^2 - a^3*b^4*c^3*d^2 - 7*a^5*b^2*c
^2*d^2 + a^7*c*d^2 + (11*a*b^6*c^2*d^2 - 10*a^3*b^4*c*d^2 - a^5*b^2*d^2)*(
d*x + c)^2 - (2*b^7*c^3*d^2 - 3*a^2*b^5*c^2*d^2 + a^6*b*d^2)*(d*x + c)^(3/
2) - (19*a*b^6*c^3*d^2 - 14*a^3*b^4*c^2*d^2 - 5*a^5*b^2*c*d^2)*(d*x + c) +
 3*(b^7*c^4*d^2 - 2*a^2*b^5*c^3*d^2 + a^4*b^3*c^2*d^2)*sqrt(d*x + c))/((b^
2*c - a^2)^4*(sqrt(d*x + c)*b + a)*c*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 11.82 (sec) , antiderivative size = 1441, normalized size of antiderivative = 4.82

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))^2),x)
```

output
```
(((5*a^3*b^2*d^2 + 19*a*b^4*c*d^2)*(c + d*x))/(2*(b^2*c - a^2)*(a^4 + b^4*
c^2 - 2*a^2*b^2*c)) + ((a^3*b^2*d^2 + 11*a*b^4*c*d^2)*(c + d*x)^2)/(2*c*(a
^6 - b^6*c^3 - 3*a^4*b^2*c + 3*a^2*b^4*c^2)) - (a*(7*b^4*c^2*d^2 - a^4*d^2
 + 6*a^2*b^2*c*d^2))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*
(a^2*d^2 + 2*b^2*c*d^2)*(c + d*x)^(3/2))/(2*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c
)) - (3*b^3*c*d^2*(c + d*x)^(1/2))/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)))/(a*(
c + d*x)^2 + b*(c + d*x)^(5/2) + a*c^2 - 2*a*c*(c + d*x) - 2*b*c*(c + d*x)
^(3/2) + b*c^2*(c + d*x)^(1/2)) + log(a + b*(c + d*x)^(1/2))*((10*b^4*d^2)
/(b^2*c - a^2)^3 - (12*b^6*c*d^2)/(b^2*c - a^2)^4) + (log((a*b^4*d^4*(a^6
- 44*b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (
b*d^2*((b^2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*
(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) -
 15*a*b^4*c^2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(
1/2)*(6*b^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7
*b^2*c - a^2))/(2*c*(b^2*c - a^2)^2))*(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^
2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(
b^2*c - a^2)^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*
(b^2*c - a^2)^6))*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 + a^5*b*d^2*(c^3)^(1
/2) - 10*a^3*b^3*c*d^2*(c^3)^(1/2) - 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^
8*c^3 + b^8*c^7 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) + (lo...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1103, normalized size of antiderivative = 3.69

$$\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)^2}\, dx = \text{Too large to display}$$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2))^2,x)
```

output

```
(sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*a**5*b**2*d**2*x**2 -
10*sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*a**3*b**4*c*d**2*x**
2 - 15*sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*a*b**6*c**2*d**2
*x**2 - sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a**5*b**2*d**2*
x**2 + 10*sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a**3*b**4*c*d
**2*x**2 + 15*sqrt(c)*sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a*b**6*c*
*2*d**2*x**2 + 20*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*a**2*b**5*c**
2*d**2*x**2 + 4*sqrt(c + d*x)*log(sqrt(c + d*x) - sqrt(c))*b**7*c**3*d**2*
x**2 + 20*sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*a**2*b**5*c**2*d**2*x
**2 + 4*sqrt(c + d*x)*log(sqrt(c + d*x) + sqrt(c))*b**7*c**3*d**2*x**2 - 4
0*sqrt(c + d*x)*log(sqrt(c + d*x)*b + a)*a**2*b**5*c**2*d**2*x**2 - 8*sqrt
(c + d*x)*log(sqrt(c + d*x)*b + a)*b**7*c**3*d**2*x**2 + 2*sqrt(c + d*x)*a
**6*b*c**2 + 2*sqrt(c + d*x)*a**6*b*c*d*x - 6*sqrt(c + d*x)*a**4*b**3*c**3
 - 2*sqrt(c + d*x)*a**4*b**3*c*d**2*x**2 + 6*sqrt(c + d*x)*a**2*b**5*c**4
- 6*sqrt(c + d*x)*a**2*b**5*c**3*d*x - 20*sqrt(c + d*x)*a**2*b**5*c**2*d**
2*x**2 - 2*sqrt(c + d*x)*b**7*c**5 + 4*sqrt(c + d*x)*b**7*c**4*d*x + 22*sq
rt(c + d*x)*b**7*c**3*d**2*x**2 + sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a**
6*b*d**2*x**2 - 10*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a**4*b**3*c*d**2*x
**2 - 15*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a**2*b**5*c**2*d**2*x**2 - s
qrt(c)*log(sqrt(c + d*x) + sqrt(c))*a**6*b*d**2*x**2 + 10*sqrt(c)*log(s...
```

3.125 $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 326

$$\begin{aligned}\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = &-\frac{4a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}\\&+\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}\\&-\frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}\\&+\frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}\\&-\frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}\\&+\frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}\\&-\frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}\end{aligned}$$

output
```
-4/3*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4+4/5*(-b^2*c+a^2)^2
*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4-12/7*a*(-3*b^2*c+7*a^2)*
(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4+4/9*(3*b^4*c^2-30*a^2*b^2*c
+35*a^4)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4-20/11*a*(-3*b^2*c+7*a^2)*(a+b*(
d*x+c)^(1/2))^(11/2)/b^8/d^4+12/13*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(13/
2)/b^8/d^4-28/15*a*(a+b*(d*x+c)^(1/2))^(15/2)/b^8/d^4+4/17*(a+b*(d*x+c)^(1
/2))^(17/2)/b^8/d^4
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-14336a^7+3840a^5b^2(10c-7dx)+21504a^6b\sqrt{c+dx}-640a^4b^3(104c-49dx)\sqrt{c\right.}{}$$

input
```
Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) +
 21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] -
48*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]
*(2960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x
+ 5754*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^
2*d*x + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int d^3x^3\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -d^3x^3\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 1732}$$

$$-\frac{2\int -d^3x^3\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{2\int \left(-\frac{\left(a+b\sqrt{c+dx}\right)^{15/2}}{b^7}+\frac{7a\left(a+b\sqrt{c+dx}\right)^{13/2}}{b^7}+\frac{3\left(b^2c-7a^2\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{b^7}-\frac{5\left(3ab^2c-7a^3\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^7}+\frac{\left(-35a^4+30b^2ca\right.}{}\right.}{}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\left(-\frac{6\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8}+\frac{10a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8}+\frac{6a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8}-\frac{2\left(a^2-b^2c\right)^2\left(7a^2-\right.}{}\right.}{}$$

input
```
Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8) - (2*(a^2 -
 b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8) + (6*a*(7*a
^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8) - (2*(35*
a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8) + (10
*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8) - (6*(7*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8) + (14*a*(a + b*Sqrt[c + d*x]
)^(15/2))/(15*b^8) - (2*(a + b*Sqrt[c + d*x])^(17/2))/(17*b^8)))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.18

method	result
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{17}{2}}}{17}+\frac{7a\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{\left(3b^{2}c-21a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{\left(8\left(-b^{2}c+a^{2}\right)a+2a\left(-2b^{2}c+6a^{2}\right)+\left(-3b^{2}c+15a\right.\right.}{11}}{}$
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{17}{2}}}{17}+\frac{7a\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{\left(3b^{2}c-21a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{\left(8\left(-b^{2}c+a^{2}\right)a+2a\left(-2b^{2}c+6a^{2}\right)+\left(-3b^{2}c+15a\right.\right.}{11}}{}$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^4/b^8*(-1/17*(a+b*(d*x+c)^(1/2))^(17/2)+7/15*a*(a+b*(d*x+c)^(1/2))^(1
5/2)+1/13*(3*b^2*c-21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(8*(-b^2*c+a^2)
*a+2*a*(-2*b^2*c+6*a^2)+(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/
9*(-(-b^2*c+a^2)*(-2*b^2*c+6*a^2)-8*a^2*(-b^2*c+a^2)-(-b^2*c+a^2)^2+(-8*(-
b^2*c+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(6*(-b
^2*c+a^2)^2*a+((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^
2)^2)*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-(-b^2*c+a^2)^3-6*(-b^2*c+a^2)^2*a
^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/
2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(45045\,b^8d^4x^4-29568\,b^8c^4+72960\,a^2b^6c^3-96128\,a^4b^4c^2+59904\,a^6b^2c-14336\,a^8+231\,(15\,b^8c-14\right.}{}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^
4*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*
x^3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8
*c^3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^
3*x^3 - 27648*a*b^7*c^3 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b
 - 3528*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c
+ 140*a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.10

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{7a(a+b\sqrt{c+dx})^{\frac{15}{2}}}{15b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{17}{2}}}{17b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{13}{2}}\cdot(21a^2-3b^2c)}{13b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{11}{2}}(-35a^3+15ab^2c)}{11b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}\cdot(35a^4-30a^2b^2c+\cdots}{9b^6}\cdots\right)}{\cdots} \\ \frac{\sqrt{a}d^4x^4}{8} \end{cases}\right)}{\cdots} \\ \frac{x^4\sqrt{a+b\sqrt{c}}}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-7*a*(a + b*sqrt(c + d*x))**(15/2)/(15*b**6) +
(a + b*sqrt(c + d*x))**(17/2)/(17*b**6) + (a + b*sqrt(c + d*x))**(13/2)*(2
1*a**2 - 3*b**2*c)/(13*b**6) + (a + b*sqrt(c + d*x))**(11/2)*(-35*a**3 + 1
5*a*b**2*c)/(11*b**6) + (a + b*sqrt(c + d*x))**(9/2)*(35*a**4 - 30*a**2*b*
*2*c + 3*b**4*c**2)/(9*b**6) + (a + b*sqrt(c + d*x))**(7/2)*(-21*a**5 + 30
*a**3*b**2*c - 9*a*b**4*c**2)/(7*b**6) + (a + b*sqrt(c + d*x))**(5/2)*(7*a
**6 - 15*a**4*b**2*c + 9*a**2*b**4*c**2 - b**6*c**3)/(5*b**6) + (a + b*sqr
t(c + d*x))**(3/2)*(-a**7 + 3*a**5*b**2*c - 3*a**3*b**4*c**2 + a*b**6*c**3
)/(3*b**6))/b**2, Ne(b, 0)), (sqrt(a)*d**4*x**4/8, True))/d**4, Ne(d, 0)),
 (x**4*sqrt(a + b*sqrt(c))/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.82

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \frac{4\left(45045\left(\sqrt{dx+c}b+a\right)^{\frac{17}{2}}-357357\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}a-176715\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}+3480\cdots\right.}{\cdots}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a
)^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*
(3*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a
^2*b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*
a^3*b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b
^4*c^2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6
*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^
8*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(279) = 558$.

Time = 0.16 (sec) , antiderivative size = 915, normalized size of antiderivative = 2.81

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/765765*(17*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt
(d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 8
1081*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^
(3/2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(
sqrt(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^
2*c + 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)
*b + a)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 1
35135*sqrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15
/2) + 24255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(
11/2)*a^2 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)
*b + a)^(7/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(
d*x + c)*b + a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)*a/(b^7*d^
3) + (153153*(sqrt(d*x + c)*b + a)^(5/2)*b^6*c^3 - 510510*(sqrt(d*x + c)*b
 + a)^(3/2)*a*b^6*c^3 + 765765*sqrt(sqrt(d*x + c)*b + a)*a^2*b^6*c^3 - 255
255*(sqrt(d*x + c)*b + a)^(9/2)*b^4*c^2 + 1312740*(sqrt(d*x + c)*b + a)^(7
/2)*a*b^4*c^2 - 2756754*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^4*c^2 + 3063060*
(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^4*c^2 - 2297295*sqrt(sqrt(d*x + c)*b + a
)*a^4*b^4*c^2 + 176715*(sqrt(d*x + c)*b + a)^(13/2)*b^2*c - 1253070*(sqrt(
d*x + c)*b + a)^(11/2)*a*b^2*c + 3828825*(sqrt(d*x + c)*b + a)^(9/2)*a^2*b
^2*c - 6563700*(sqrt(d*x + c)*b + a)^(7/2)*a^3*b^2*c + 6891885*(sqrt(d*...
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx=\int x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^3*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.09

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\sqrt{\sqrt{dx+c}\,b+a}\,\left(-28160\sqrt{dx+c}\,a^5b^3c+41472\sqrt{dx+c}\,a^3b^5c^2-27648\sqrt{dx+c}\,a\,b^7c^3-5376a^6b^2dx\right.}{}$$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(7168*sqrt(c + d*x)*a**7*b - 28160*sqrt(c + d
*x)*a**5*b**3*c + 4480*sqrt(c + d*x)*a**5*b**3*d*x + 41472*sqrt(c + d*x)*a
**3*b**5*c**2 - 13344*sqrt(c + d*x)*a**3*b**5*c*d*x + 3528*sqrt(c + d*x)*a
**3*b**5*d**2*x**2 - 27648*sqrt(c + d*x)*a*b**7*c**3 + 13344*sqrt(c + d*x)
*a*b**7*c**2*d*x - 7056*sqrt(c + d*x)*a*b**7*c*d**2*x**2 + 3003*sqrt(c + d
*x)*a*b**7*d**3*x**3 - 14336*a**8 + 59904*a**6*b**2*c - 5376*a**6*b**2*d*x
 - 96128*a**4*b**4*c**2 + 16640*a**4*b**4*c*d*x - 3920*a**4*b**4*d**2*x**2
 + 72960*a**2*b**6*c**3 - 17760*a**2*b**6*c**2*d*x + 8148*a**2*b**6*c*d**2
*x**2 - 3234*a**2*b**6*d**3*x**3 - 29568*b**8*c**4 + 7392*b**8*c**3*d*x -
4620*b**8*c**2*d**2*x**2 + 3465*b**8*c*d**3*x**3 + 45045*b**8*d**4*x**4))/
(765765*b**8*d**4)
```

3.126 $\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = -\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} - \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} + \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3} - \frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}$$

output
```
-4/3*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3+4/5*(b^4*c^2-6*a^2
*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3-8/7*a*(-3*b^2*c+5*a^2)*(a+
b*(d*x+c)^(1/2))^(7/2)/b^6/d^3+8/9*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(9/2
)/b^6/d^3-20/11*a*(a+b*(d*x+c)^(1/2))^(11/2)/b^6/d^3+4/13*(a+b*(d*x+c)^(1/
2))^(13/2)/b^6/d^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-1280a^5+32a^3b^2(68c-75dx)+1920a^4b\sqrt{c+dx}+16a^2b^3\sqrt{c+dx}(-254c+175\right.}{45045b^6d^3}$$

input
```
Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 1
920*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77
*b^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 38
0*c*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{896}$$
$$\frac{\int d^2x^2\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^3}$$
$$\Big\downarrow \text{1732}$$
$$\frac{2\int d^2x^2\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^3}$$
$$\Big\downarrow \text{522}$$

$$\frac{2\int\left(\frac{\left(a+b\sqrt{c+dx}\right)^{11/2}}{b^5}-\frac{5a\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^5}-\frac{2\left(b^2c-5a^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^5}+\frac{\left(5a^4-6b^2ca^2+b^4c^2\right)\left(a\right.}{b^5}\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{4\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6}-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/}}{5b^6}\right.}{d^3}$$

input
```
Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6) + (2*(5*a^4
 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6) - (4*a*(5*a
^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6) + (4*(5*a^2 - b^2*c)*(a
 + b*Sqrt[c + d*x])^(9/2))/(9*b^6) - (10*a*(a + b*Sqrt[c + d*x])^(11/2))/(
11*b^6) + (2*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6)))/d^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-4}{d^3b^6}$
default	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-4}{d^3b^6}$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d^3/b^6*(1/13*(a+b*(d*x+c)^(1/2))^(13/2)-5/11*a*(a+b*(d*x+c)^(1/2))^(11/
2)-1/9*(2*b^2*c-10*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)-1/7*(4*(-b^2*c+a^2)*a+a*
(-2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(7/2)-1/5*(-(-b^2*c+a^2)^2-4*a^2*(-b
^2*c+a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/
2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\,(3465\,b^6d^3x^3+2464\,b^6c^3-4640\,a^2b^4c^2+4096\,a^4b^2c-1280\,a^6+35\,(11\,b^6c-10\,a^2b^4)d^2x^2-8\,(77\,b^6c^2}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2
*c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a
^2*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^
3*b^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(s
qrt(d*x + c)*b + a)/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{5a(a+b\sqrt{c+dx})^{\frac{11}{2}}}{11b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{13}{2}}}{13b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}\cdot(10a^2-2b^2c)}{9b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}(-10a^3+6ab^2c)}{7b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}\cdot(5a^4-6a^2b^2c+b^4c^2)}{5b^4} \cdots}{b^2} \\ \frac{\sqrt{a}d^3x^3}{6} \end{cases}\right)}{d^3} \\ \frac{x^3\sqrt{a+b\sqrt{c}}}{3} \end{cases}$$

input
```
integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-5*a*(a + b*sqrt(c + d*x))**(11/2)/(11*b**4) +
(a + b*sqrt(c + d*x))**(13/2)/(13*b**4) + (a + b*sqrt(c + d*x))**(9/2)*(10
*a**2 - 2*b**2*c)/(9*b**4) + (a + b*sqrt(c + d*x))**(7/2)*(-10*a**3 + 6*a*
b**2*c)/(7*b**4) + (a + b*sqrt(c + d*x))**(5/2)*(5*a**4 - 6*a**2*b**2*c +
b**4*c**2)/(5*b**4) + (a + b*sqrt(c + d*x))**(3/2)*(-a**5 + 2*a**3*b**2*c
- a*b**4*c**2)/(3*b**4))/b**2, Ne(b, 0)), (sqrt(a)*d**3*x**3/6, True))/d**
3, Ne(d, 0)), (x**3*sqrt(a + b*sqrt(c))/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(3465\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}-20475\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}a-10010\left(b^2c-5a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}+12870\left(3\right.\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)*b + a)^(
11/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9/2) + 12870*(3*a*b
^2*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c +
5*a^4)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)
*(sqrt(d*x + c)*b + a)^(3/2))/(b^6*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(188) = 376$.

Time = 0.13 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.45

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(\frac{13\left(1155\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}b+a}\,ab^4c^2-990\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}ab^2c-6930\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a^2b^2c+6\right.}{}\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/45045*(13*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x
 + c)*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqr
t(d*x + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2
*c + 6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^
(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(
7/2)*a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b +
a)^(3/2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)*a/(b^5*d^2) + (9009*(sq
rt(d*x + c)*b + a)^(5/2)*b^4*c^2 - 30030*(sqrt(d*x + c)*b + a)^(3/2)*a*b^4
*c^2 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^2*b^4*c^2 - 10010*(sqrt(d*x + c)*
b + a)^(9/2)*b^2*c + 51480*(sqrt(d*x + c)*b + a)^(7/2)*a*b^2*c - 108108*(s
qrt(d*x + c)*b + a)^(5/2)*a^2*b^2*c + 120120*(sqrt(d*x + c)*b + a)^(3/2)*a
^3*b^2*c - 90090*sqrt(sqrt(d*x + c)*b + a)*a^4*b^2*c + 3465*(sqrt(d*x + c)
*b + a)^(13/2) - 24570*(sqrt(d*x + c)*b + a)^(11/2)*a + 75075*(sqrt(d*x +
c)*b + a)^(9/2)*a^2 - 128700*(sqrt(d*x + c)*b + a)^(7/2)*a^3 + 135135*(sqr
t(d*x + c)*b + a)^(5/2)*a^4 - 90090*(sqrt(d*x + c)*b + a)^(3/2)*a^5 + 4504
5*sqrt(sqrt(d*x + c)*b + a)*a^6)/(b^5*d^2))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx=\int x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^2*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.95

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\sqrt{\sqrt{dx+c}\,b+a}\,\left(640\sqrt{dx+c}\,a^5b-1888\sqrt{dx+c}\,a^3b^3c+400\sqrt{dx+c}\,a^3b^3dx+1888\sqrt{dx+c}\,a\,b^5c^2-\right.}{}$$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(640*sqrt(c + d*x)*a**5*b - 1888*sqrt(c + d*x
)*a**3*b**3*c + 400*sqrt(c + d*x)*a**3*b**3*d*x + 1888*sqrt(c + d*x)*a*b**
5*c**2 - 800*sqrt(c + d*x)*a*b**5*c*d*x + 315*sqrt(c + d*x)*a*b**5*d**2*x*
*2 - 1280*a**6 + 4096*a**4*b**2*c - 480*a**4*b**2*d*x - 4640*a**2*b**4*c**
2 + 1016*a**2*b**4*c*d*x - 350*a**2*b**4*d**2*x**2 + 2464*b**6*c**3 - 616*
b**6*c**2*d*x + 385*b**6*c*d**2*x**2 + 3465*b**6*d**3*x**3))/(45045*b**6*d
**3)
```

3.127 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = -\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} + \frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2} - \frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2} + \frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}$$

output
```
-4/3*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2+4/5*(-b^2*c+3*a^2)*(
a+b*(d*x+c)^(1/2))^(5/2)/b^4/d^2-12/7*a*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2+
4/9*(a+b*(d*x+c)^(1/2))^(9/2)/b^4/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-16a^3+6ab^2(2c-5dx)+24a^2b\sqrt{c+dx}+7b^3\sqrt{c+dx}(-4c+5dx)\right)}{315b^4d^2}$$

input
```
Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b
*Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int dx\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -dx\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(-\frac{\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^3}+\frac{3a\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^3}+\frac{(b^2c-3a^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{b^3}+\frac{(a^3-ab^2c)\sqrt{a+b\sqrt{c+dx}}}{b^3}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4}+\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4}-\frac{2\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4}+\frac{6a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4}\right)}{d^2}$$

input
```
Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4) - (2*(3*a^2 -
 b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4) + (6*a*(a + b*Sqrt[c + d*x])^
(7/2))/(7*b^4) - (2*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}\right)}{d^{2}b^{4}}$	93
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}\right)}{d^{2}b^{4}}$	93

input
```
int(x*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^2/b^4*(-1/9*(a+b*(d*x+c)^(1/2))^(9/2)+3/7*a*(a+b*(d*x+c)^(1/2))^(7/2)
+1/5*(b^2*c-3*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-b^2*c+a^2)*a*(a+b*(d*x+
c)^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(35\,b^{4}d^{2}x^{2}-28\,b^{4}c^{2}+36\,a^{2}b^{2}c-16\,a^{4}+(7\,b^{4}c-6\,a^{2}b^{2})dx+(5\,ab^{3}dx-16\,ab^{3}c+8\,a^{3}b)\sqrt{dx+c}\right)\sqrt{\sqrt{}}}{315\,b^{4}d^{2}}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*
a^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sq
rt(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{3a(a+b\sqrt{c+dx})^{\frac{7}{2}}}{7b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}}{9b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}\cdot(3a^2-b^2c)}{5b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}(-a^3+ab^2c)}{3b^2}\right)}{b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}\right) & \text{otherwise} \end{cases}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^2\sqrt{a+b\sqrt{c}}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-3*a*(a + b*sqrt(c + d*x))**(7/2)/(7*b**2) + (a
 + b*sqrt(c + d*x))**(9/2)/(9*b**2) + (a + b*sqrt(c + d*x))**(5/2)*(3*a**2
 - b**2*c)/(5*b**2) + (a + b*sqrt(c + d*x))**(3/2)*(-a**3 + a*b**2*c)/(3*b
**2))/b**2, Ne(b, 0)), (sqrt(a)*(-c*(c + d*x)/2 + (c + d*x)**2/4), True))/
d**2, Ne(d, 0)), (x**2*sqrt(a + b*sqrt(c))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}a-63\left(b^2c-3a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}+105\left(ab^2c-a^3\right)\left(\sqrt{\right.}\right.}{315\,b^4d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)^(7/2)*a
- 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(sq
rt(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(109) = 218$.

Time = 0.17 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.10

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx =$$

$$-\frac{4\left(\frac{3\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}b+a}\,ab^2c-15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+c}b+a}\,aa}{b^3d}\right.}{}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/315*(3*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b
 + a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^
(5/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b +
 a)*a^3)*a/(b^3*d) + (63*(sqrt(d*x + c)*b + a)^(5/2)*b^2*c - 210*(sqrt(d*x
 + c)*b + a)^(3/2)*a*b^2*c + 315*sqrt(sqrt(d*x + c)*b + a)*a^2*b^2*c - 35*
(sqrt(d*x + c)*b + a)^(9/2) + 180*(sqrt(d*x + c)*b + a)^(7/2)*a - 378*(sqr
t(d*x + c)*b + a)^(5/2)*a^2 + 420*(sqrt(d*x + c)*b + a)^(3/2)*a^3 - 315*sq
rt(sqrt(d*x + c)*b + a)*a^4)/(b^3*d))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \int x\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\sqrt{\sqrt{dx+c}\,b+a}\,\left(8\sqrt{dx+c}\,a^3b-16\sqrt{dx+c}\,a\,b^3c+5\sqrt{dx+c}\,a\,b^3dx-16a^4+36a^2b^2c-6a^2b^2dx-2\right.}{315b^4d^2}$$

input
```
int(x*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(8*sqrt(c + d*x)*a**3*b - 16*sqrt(c + d*x)*a*
b**3*c + 5*sqrt(c + d*x)*a*b**3*d*x - 16*a**4 + 36*a**2*b**2*c - 6*a**2*b*
*2*d*x - 28*b**4*c**2 + 7*b**4*c*d*x + 35*b**4*d**2*x**2))/(315*b**4*d**2)
```

3.128 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx=-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}$$

output
```
-4/3*a*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d+4/5*(a+b*(d*x+c)^(1/2))^(5/2)/b^2/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx=\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-2a^2+ab\sqrt{c+dx}+3b^2(c+dx)\right)}{15b^2d}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-2*a^2 + a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)
))/(15*b^2*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \left(\frac{\left(a+b\sqrt{c+dx}\right)^{3/2}}{b}-\frac{a\sqrt{a+b\sqrt{c+dx}}}{b}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{2\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2}\right)}{d}$$

input
```
Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-2*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2) + (2*(a + b*Sqrt[c + d*x])^
(5/2))/(5*b^2)))/d
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^{2}d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^{2}d}$	41

input
```
int((a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d/b^2*(1/5*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(3\,b^2dx+3\,b^2c+\sqrt{dx+c}ab-2\,a^2\right)\sqrt{\sqrt{dx+c}b+a}}{15\,b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*
b + a)/(b^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{a(a+b\sqrt{c+dx})^{\frac{3}{2}}}{3}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}}{5}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{\sqrt{a}(c+dx)}{2} & \text{otherwise} \end{cases}\right)}{d} & \text{for } d\neq 0 \\ x\sqrt{a+b\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-a*(a + b*sqrt(c + d*x))**(3/2)/3 + (a + b*sqrt
(c + d*x))**(5/2)/5)/b**2, Ne(b, 0)), (sqrt(a)*(c + d*x)/2, True))/d, Ne(d
, 0)), (x*sqrt(a + b*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(\frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/
b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(44) = 88$.

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.77

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(\frac{5\left(\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}b+a}a\right)a}{b}+\frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}-10\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a+15\sqrt{\sqrt{dx+c}b+a}a^2}{b}\right)}{15\,bd}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/15*(5*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)*a/b
+ (3*(sqrt(d*x + c)*b + a)^(5/2) - 10*(sqrt(d*x + c)*b + a)^(3/2)*a + 15*s
qrt(sqrt(d*x + c)*b + a)*a^2)/b)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5\,b^2\,d} - \frac{4\,a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3\,b^2\,d}$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
(4*(a + b*(c + d*x)^(1/2))^(5/2))/(5*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))
^(3/2))/(3*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\sqrt{\sqrt{dx+c}\,b+a}\,\left(\sqrt{dx+c}\,ab-2a^2+3b^2c+3b^2dx\right)}{15b^2d}$$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(sqrt(c + d*x)*a*b - 2*a**2 + 3*b**2*c + 3*b*
*2*d*x))/(15*b**2*d)
```

3.129 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = 4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

output
```
4*(a+b*(d*x+c)^(1/2))^(1/2)-2*(a-b*c^(1/2))^(1/2)*arctanh((a+b*(d*x+c)^(1/
2))^(1/2)/(a-b*c^(1/2))^(1/2))-2*(a+b*c^(1/2))^(1/2)*arctanh((a+b*(d*x+c)^
(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = 4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{-a-b\sqrt{c}}\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right) - 2\sqrt{-a+b\sqrt{c}}\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

output
```
4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[-a - b*Sqrt[c]]*ArcTan[Sqrt[a + b*Sqr
t[c + d*x]]/Sqrt[-a - b*Sqrt[c]]] - 2*Sqrt[-a + b*Sqrt[c]]*ArcTan[Sqrt[a +
 b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {896, 25, 1732, 561, 27, 1602, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{\sqrt{a+b\sqrt{c+dx}}}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2\int -\frac{\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}}{dx}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 561}$$

$$-\frac{4\int \frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4\int \frac{(a-c-dx)(c+dx)}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \text{1602}$$

$$-\frac{4\left(b^2\left(-\int -\frac{b^2\left(\frac{a^2}{b^2}-c\right)-a(c+dx)}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}\right)-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4\left(b^2\int \frac{a^2-(c+dx)a-b^2c}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\left(\int \frac{a^2-(c+dx)a-b^2c}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{4\left(-\frac{1}{2}\left(a-b\sqrt{c}\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}-\frac{1}{2}\left(a+b\sqrt{c}\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}+b^2\left(-\sqrt{a+b\sqrt{}}\right.\right.}{b^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{4\left(\frac{1}{2}b^2\sqrt{a-b\sqrt{c}}\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)+\frac{1}{2}b^2\sqrt{a+b\sqrt{c}}\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

input
```
Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

output
```
(-4*(-(b^2*Sqrt[a + b*Sqrt[c + d*x]]) + (b^2*Sqrt[a - b*Sqrt[c]]*ArcTanh[S
qrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/2 + (b^2*Sqrt[a + b*Sqrt[c]
]*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/2))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

method	result	si
derivativedivides	$4\sqrt{a+b\sqrt{dx+c}}-\frac{2\left(-b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{-\sqrt{b^2c}-a}}-\frac{2\left(b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{\sqrt{b^2c}-a}}$	1
default	$4\sqrt{a+b\sqrt{dx+c}}-\frac{2\left(-b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{-\sqrt{b^2c}-a}}-\frac{2\left(b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{\sqrt{b^2c}-a}}$	1

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
4*(a+b*(d*x+c)^(1/2))^(1/2)-2*(-b^2*c-a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/(-(b^
2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1
/2))-2*(b^2*c-a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arcta
n((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(88) = 176$.

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = -\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a+\sqrt{b^2c}}\right) + \sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a+\sqrt{b^2c}}\right) - \sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a-\sqrt{b^2c}}\right) + \sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a-\sqrt{b^2c}}\right) + 4\sqrt{\sqrt{dx+c}b+a}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
-sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b
^2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a
 + sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) +
 2*sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)
*b + a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \frac{2\left(2\sqrt{\sqrt{dx+c}b+a}b - \frac{(b^3c-a^2b)\sqrt{b\sqrt{c}-a}\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{b^2c-a^2} - \frac{(b^3c-a^2b)\sqrt{-b\sqrt{c}-a}\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{b^2c-a^2}\right)}{b}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
2*(2*sqrt(sqrt(d*x + c)*b + a)*b - (b^3*c - a^2*b)*sqrt(b*sqrt(c) - a)*arc
tan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/(b^2*c - a^2) - (b^3
*c - a^2*b)*sqrt(-b*sqrt(c) - a)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a
- sqrt(b^2*c)))/(b^2*c - a^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{c+d\,x}}}{x}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = -2\sqrt{\sqrt{c}\,b-a}\,atan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{\sqrt{c}\,b-a}}\right)+4\sqrt{\sqrt{dx+c}\,b+a} + \sqrt{\sqrt{c}\,b+a}\log\left(\sqrt{\sqrt{dx+c}\,b+a}-\sqrt{\sqrt{c}\,b+a}\right) - \sqrt{\sqrt{c}\,b+a}\log\left(\sqrt{\sqrt{dx+c}\,b+a}+\sqrt{\sqrt{c}\,b+a}\right)$$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x,x)
```

output
```
- 2*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt(c)*b - a
)) + 4*sqrt(sqrt(c + d*x)*b + a) + sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d
*x)*b + a) - sqrt(sqrt(c)*b + a)) - sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c +
d*x)*b + a) + sqrt(sqrt(c)*b + a))
```

3.130 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2} \, dx = -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\sqrt{c}} - \frac{bd\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\sqrt{c}}$$

output

```
-(a+b*(d*x+c)^(1/2))^(1/2)/x+1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-
b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(1/2)/c^(1/2)-1/2*b*d*arctanh((a+b*(d*x+c)
^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+b*c^(1/2))^(1/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2} \, dx = \frac{1}{2}\left(-\frac{2\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}\sqrt{c}} - \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

output
```
((-2*Sqrt[a + b*Sqrt[c + d*x]])/x + (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/
Sqrt[-a - b*Sqrt[c]]])/(Sqrt[-a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTan[Sqrt[a
 + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(Sqrt[-a + b*Sqrt[c]]*Sqrt[c]))
/2
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {896, 1732, 561, 25, 27, 1598, 27, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{\sqrt{a+b\sqrt{c+dx}}}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$2d\int \frac{\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}}{d^2x^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{561}$$

$$\frac{4d\int -\frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4d\int \frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d\int \frac{(a-c-dx)(c+dx)}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \text{1598}$$

$$-\frac{4d\left(\frac{b^2\int -\frac{2c}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{8c}+\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{1}{4}b^2\int \frac{1}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \text{1406}$$

$$-\frac{4d\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{1}{4}b^2\left(\frac{\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b\sqrt{c}}-\frac{\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b\sqrt{c}}\right)\right)}{b^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{4d\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{1}{4}b^2\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}\right)\right)}{b^2}$$

input
```
Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

output
```
(-4*d*((b^2*Sqrt[a + b*Sqrt[c + d*x]])/(4*(a^2/b^2 - c - (2*a*(c + d*x))/b
^2 + (c + d*x)^2/b^2)) - (b^2*((b*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a
 - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*ArcTanh[Sqrt[a + b*Sq
rt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])))/4))/b
^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

method	result
derivativedivides	$4 d\,b^{2} \left(-\frac{\sqrt{a +b \sqrt{d x +c}}}{4 \left(\left(a +b \sqrt{d x +c}\right)^{2}-2 a \left(a +b \sqrt{d x +c}\right)-b^{2} c +a^{2}\right)}+\frac{\arctan\left(\frac{\sqrt{a +b \sqrt{d x +c}}}{\sqrt{-\sqrt{b^{2} c}-a}}\right)}{8 \sqrt{b^{2} c}\, \sqrt{-\sqrt{b^{2} c}-a}}-\frac{\arctan\left(\frac{\sqrt{a +b \sqrt{d x +c}}}{\sqrt{\sqrt{b^{2} c}-a}}\right)}{8 \sqrt{b^{2} c}\, \sqrt{\sqrt{b^{2} c}-a}}\right)$
default	$4 d\,b^{2} \left(-\frac{\sqrt{a +b \sqrt{d x +c}}}{4 \left(\left(a +b \sqrt{d x +c}\right)^{2}-2 a \left(a +b \sqrt{d x +c}\right)-b^{2} c +a^{2}\right)}+\frac{\arctan\left(\frac{\sqrt{a +b \sqrt{d x +c}}}{\sqrt{-\sqrt{b^{2} c}-a}}\right)}{8 \sqrt{b^{2} c}\, \sqrt{-\sqrt{b^{2} c}-a}}-\frac{\arctan\left(\frac{\sqrt{a +b \sqrt{d x +c}}}{\sqrt{\sqrt{b^{2} c}-a}}\right)}{8 \sqrt{b^{2} c}\, \sqrt{\sqrt{b^{2} c}-a}}\right)$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
4*d*b^2*(-1/4*(a+b*(d*x+c)^(1/2))^(1/2)/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d
*x+c)^(1/2))-b^2*c+a^2)+1/8/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan(
(a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))-1/8/(b^2*c)^(1/2)/((b^
2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(101) = 202.

Time = 0.14 (sec) , antiderivative size = 1003, normalized size of antiderivative = 7.32

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d
^3 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c
^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^
4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b
^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^
2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b
^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + s
qrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2
 - a^2*c))) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 +
 a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b +
a)*b^4*d^3 + (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*
(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*
c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2
- sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*
c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^
6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2
*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/
(b^2*c^2 - a^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a))/x
```

Sympy [F]

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2}\, dx = \int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2}\, dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(101) = 202.

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$$
$$= \frac{\left(\frac{2\sqrt{\sqrt{dx+c}b+a}\,b^3}{b^2c-\left(\sqrt{dx+c}b+a\right)^2+2\left(\sqrt{dx+c}b+a\right)a-a^2} - \frac{\left(\sqrt{b\sqrt{c}-a}\,b^3\sqrt{c}|b|+\sqrt{b\sqrt{c}-a}\,ab^3\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b^2c^{\frac{3}{2}}-a^2\sqrt{c}\right)|b|} - \frac{\left(\sqrt{-b\sqrt{c}-a}\,b^3\sqrt{c}|b|-\sqrt{-b\sqrt{c}}\right.}{\left(b^2c^{\frac{3}{2}}-\right.}\right.}{2b}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*(2*sqrt(sqrt(d*x + c)*b + a)*b^3/(b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*
(sqrt(d*x + c)*b + a)*a - a^2) - (sqrt(b*sqrt(c) - a)*b^3*sqrt(c)*abs(b) +
 sqrt(b*sqrt(c) - a)*a*b^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqr
t(b^2*c)))/((b^2*c^(3/2) - a^2*sqrt(c))*abs(b)) - (sqrt(-b*sqrt(c) - a)*b^
3*sqrt(c)*abs(b) - sqrt(-b*sqrt(c) - a)*a*b^3)*arctan(sqrt(sqrt(d*x + c)*b
 + a)/sqrt(-a - sqrt(b^2*c)))/((b^2*c^(3/2) - a^2*sqrt(c))*abs(b)))*d/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{c+d\,x}}}{x^2}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^2,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \frac{2\sqrt{c}\,\sqrt{\sqrt{c}\,b-a}\,atan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{\sqrt{c}\,b-a}}\right)abdx+2\sqrt{\sqrt{c}\,b-a}\,atan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{\sqrt{c}\,b-a}}\right)b^2cdx-4\sqrt{\sqrt{dx+c}\,b+a}\,a^2c}{}$$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x)
```

output
```
(2*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt(c)
*b - a))*a*b*d*x + 2*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sq
rt(sqrt(c)*b - a))*b**2*c*d*x - 4*sqrt(sqrt(c + d*x)*b + a)*a**2*c + 4*sqr
t(sqrt(c + d*x)*b + a)*b**2*c**2 + sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sq
rt(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*a*b*d*x - sqrt(c)*sqrt(sqrt(c)*b
 + a)*log(sqrt(sqrt(c + d*x)*b + a) + sqrt(sqrt(c)*b + a))*a*b*d*x - sqrt(
sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*b**2*c
*d*x + sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) + sqrt(sqrt(c)*b
+ a))*b**2*c*d*x)/(4*c*x*(a**2 - b**2*c))
```

3.131 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}+\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} - \frac{b\left(2a-3b\sqrt{c}\right)d^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{3/2}c^{3/2}} + \frac{b\left(2a+3b\sqrt{c}\right)d^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{3/2}c^{3/2}}$$

output
```
-1/2*(a+b*(d*x+c)^(1/2))^(1/2)/x^2+1/8*b*d*(b*c-a*(d*x+c)^(1/2))*(a+b*(d*x
+c)^(1/2))^(1/2)/c/(-b^2*c+a^2)/x-1/16*b*(2*a-3*b*c^(1/2))*d^2*arctanh((a+
b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(3/2)/c^(3/2)+1/
16*b*(2*a+3*b*c^(1/2))*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))
^(1/2))/(a+b*c^(1/2))^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}(4a^2c+abdx\sqrt{c+dx}-b^2c(4c+dx))}{(a^2-b^2c)x^2}+\frac{b(2a+3b\sqrt{c})d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{(-a-b\sqrt{c})^{3/2}}+\frac{b(-2a+3b\sqrt{c})d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{(-a+b\sqrt{c})^{3/2}}}{16c^{3/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

output
```
((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^2*c + a*b*d*x*Sqrt[c + d*x] -
b^2*c*(4*c + d*x)))/((a^2 - b^2*c)*x^2) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTa
n[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(3/2)
+ (b*(-2*a + 3*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b
*Sqrt[c]]])/(-a + b*Sqrt[c])^(3/2))/(16*c^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {896, 25, 1732, 561, 27, 1598, 27, 1405, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$$

$$\Big\downarrow \text{896}$$

$$d^2\int \frac{\sqrt{a+b\sqrt{c+dx}}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-d^2\int -\frac{\sqrt{a+b\sqrt{c+dx}}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2d^2\int -\frac{\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{561}$$

$$-\frac{4d^2\int \frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d^2\int \frac{(a-c-dx)(c+dx)}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \text{1598}$$

$$-\frac{4d^2\left(\frac{b^2\int -\frac{2c}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{16c}+\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}\right)}{b^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\int \frac{1}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \text{1405}$$

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a^2-a(c+dx)+b^2c\right)}{4c(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{b^4\int \frac{2\left(a^2+(c+dx)a-3b^2c\right)}{b^4\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{8c(a^2-b^2c)}\right)\right)}{b^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a^2-a(c+dx)+b^2c\right)}{4c(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\int \frac{a^2+(c+dx)a-3b^2c}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{4c(a^2-b^2c)}\right)\right)}{b^2}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a^2-a(c+dx)+b^2c\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\frac{1}{2}\left(-\frac{2a^2}{b\sqrt{c}}+a+3b\sqrt{c}\right)\int\frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\cdots}}{\cdots}\right.\right.}{b^2}$$

$\Big\downarrow$ 221

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a^2-a(c+dx)+b^2c\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{-\frac{b^2\left(-\frac{2a^2}{b\sqrt{c}}+a+3b\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\cdots\right.}{2\sqrt{a-b\sqrt{c}}}\cdots}{4c(a^2\cdots}\right.\right.}{b^2}$$

input
```
Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

output
```
(-4*d^2*((b^2*Sqrt[a + b*Sqrt[c + d*x]])/(8*(a^2/b^2 - c - (2*a*(c + d*x))
/b^2 + (c + d*x)^2/b^2)^2) - (b^2*((Sqrt[a + b*Sqrt[c + d*x]]*(a^2 + b^2*c
 - a*(c + d*x)))/(4*c*(a^2 - b^2*c)*(a^2/b^2 - c - (2*a*(c + d*x))/b^2 + (
c + d*x)^2/b^2)) - (-1/2*(b^2*(a - (2*a^2)/(b*Sqrt[c]) + 3*b*Sqrt[c])*ArcT
anh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt[c]] -
(b^2*(a + (2*a^2)/(b*Sqrt[c]) - 3*b*Sqrt[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d
*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]))/(4*c*(a^2 - b^2*c))))/
8))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(174) = 348$.

Time = 0.41 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.66

method	result
derivativedivides	$-4d^2b^4\left(\frac{\frac{a(a+b\sqrt{dx+c})^{\frac{7}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{32b^2c(-b^2c+a^2)}+\frac{a(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{3}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(-3b^2c+a^2)\sqrt{a+b\sqrt{dx+c}}}{32b^2c}}{\left((a+b\sqrt{dx+c})^2-2a(a+b\sqrt{dx+c})-b^2c+a^2\right)^2}+\frac{(3b}{}\right.$
default	$-4d^2b^4\left(\frac{\frac{a(a+b\sqrt{dx+c})^{\frac{7}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{32b^2c(-b^2c+a^2)}+\frac{a(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{3}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(-3b^2c+a^2)\sqrt{a+b\sqrt{dx+c}}}{32b^2c}}{\left((a+b\sqrt{dx+c})^2-2a(a+b\sqrt{dx+c})-b^2c+a^2\right)^2}+\frac{(3b}{}\right.$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-4*d^2*b^4*((1/32*a/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)-1/32*(b^2
*c+3*a^2)/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/32*a*(b^2*c+3*a^2
)/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-1/32*(-3*b^2*c+a^2)/b^2/c*(
a+b*(d*x+c)^(1/2))^(1/2))/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d*x+c)^(1/2))-b
^2*c+a^2)^2+1/32/b^2/c/(-b^2*c+a^2)*(1/2*(3*b^2*c+a*(b^2*c)^(1/2)-2*a^2)/(
b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^
2*c)^(1/2)-a)^(1/2))+1/2*(-3*b^2*c+a*(b^2*c)^(1/2)+2*a^2)/(b^2*c)^(1/2)/(-
(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. $2(175) = 350$.

Time = 0.32 (sec) , antiderivative size = 2856, normalized size of antiderivative = 12.75

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^
2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14
*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^
4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))
/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 8
1*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 -
24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a
^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c +
25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*
c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 -
 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4
- a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9
- 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^
10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^
3))) - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b
^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^1
4*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a
^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))
)/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 -
81*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^3}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(175) = 350$.

Time = 0.25 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/16*(((b^3*c^2 - a^2*b*c)^2*a*b^3*sqrt(c)*d^3 - (3*b^7*c^3 - 4*a^2*b^5*c^
2 + a^4*b^3*c)*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^(9/2) - 8*a^3*b^7*c
^(7/2) + 7*a^5*b^5*c^(5/2) - 2*a^7*b^3*c^(3/2))*d^3)*arctan(sqrt(sqrt(d*x
+ c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c + sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c
^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*
c^(9/2) - a*b^4*c^4 - 2*a^2*b^3*c^(7/2) + 2*a^3*b^2*c^3 + a^4*b*c^(5/2) -
a^5*c^2)*sqrt(-b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) + ((b^3*c^2 - a^2*b*
c)^2*a*b^3*d^3 + (3*b^7*c^(5/2) - 4*a^2*b^5*c^(3/2) + a^4*b^3*sqrt(c))*d^3
*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^4 - 8*a^3*b^7*c^3 + 7*a^5*b^5*c^2 - 2
*a^7*b^3*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c
 - sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2
 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*c^4 + a*b^4*c^(7/2) - 2*a^2*b^3*c^3
- 2*a^3*b^2*c^(5/2) + a^4*b*c^2 + a^5*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^3
*c^2 - a^2*b*c)) - 2*(3*sqrt(sqrt(d*x + c)*b + a)*b^7*c^2*d^3 + (sqrt(d*x
+ c)*b + a)^(5/2)*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(3/2)*a*b^5*c*d^3 - 4*
sqrt(sqrt(d*x + c)*b + a)*a^2*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a*b^
3*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^3*d^3 - 3*(sqrt(d*x + c)*b + a
)^(3/2)*a^3*b^3*d^3 + sqrt(sqrt(d*x + c)*b + a)*a^4*b^3*d^3)/((b^2*c^2 - a
^2*c)*(b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^
2))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x^3}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^3,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 719, normalized size of antiderivative = 3.21

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x)
```

output
```
( - 4*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt
(c)*b - a))*a**3*b*d**2*x**2 + 8*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqr
t(c + d*x)*b + a)/sqrt(sqrt(c)*b - a))*a*b**3*c*d**2*x**2 - 2*sqrt(sqrt(c)
*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt(c)*b - a))*a**2*b**2*c*d*
*2*x**2 + 6*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt(c
)*b - a))*b**4*c**2*d**2*x**2 - 4*sqrt(c + d*x)*sqrt(sqrt(c + d*x)*b + a)*
a**3*b*c*d*x + 4*sqrt(c + d*x)*sqrt(sqrt(c + d*x)*b + a)*a*b**3*c**2*d*x -
 16*sqrt(sqrt(c + d*x)*b + a)*a**4*c**2 + 32*sqrt(sqrt(c + d*x)*b + a)*a**
2*b**2*c**3 + 4*sqrt(sqrt(c + d*x)*b + a)*a**2*b**2*c**2*d*x - 16*sqrt(sqr
t(c + d*x)*b + a)*b**4*c**4 - 4*sqrt(sqrt(c + d*x)*b + a)*b**4*c**3*d*x -
2*sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) - sqrt(sqrt(c)
*b + a))*a**3*b*d**2*x**2 + 4*sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c
+ d*x)*b + a) - sqrt(sqrt(c)*b + a))*a*b**3*c*d**2*x**2 + 2*sqrt(c)*sqrt(s
qrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) + sqrt(sqrt(c)*b + a))*a**3*b*
d**2*x**2 - 4*sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) +
sqrt(sqrt(c)*b + a))*a*b**3*c*d**2*x**2 + sqrt(sqrt(c)*b + a)*log(sqrt(sqr
t(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*a**2*b**2*c*d**2*x**2 - 3*sqrt(sq
rt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*b**4*c**
2*d**2*x**2 - sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) + sqrt(sqr
t(c)*b + a))*a**2*b**2*c*d**2*x**2 + 3*sqrt(sqrt(c)*b + a)*log(sqrt(sqr...
```

3.132 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 324

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = & -\frac{4a(a^2-b^2c)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} \\
& + \frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} \\
& - \frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} \\
& + \frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} \\
& - \frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} \\
& + \frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} \\
& - \frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4} + \frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}
\end{aligned}
$$

output
```
-4*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(1/2)/b^8/d^4+4/3*(-b^2*c+a^2)^2*(
-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4-12/5*a*(-3*b^2*c+7*a^2)*(-
b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4+4/7*(3*b^4*c^2-30*a^2*b^2*c+3
5*a^4)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4-20/9*a*(-3*b^2*c+7*a^2)*(a+b*(d*x
+c)^(1/2))^(9/2)/b^8/d^4+12/11*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(11/2)/b
^8/d^4-28/13*a*(a+b*(d*x+c)^(1/2))^(13/2)/b^8/d^4+4/15*(a+b*(d*x+c)^(1/2))
^(15/2)/b^8/d^4
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-14336a^7+768a^5b^2(58c-7dx)+7168a^6b\sqrt{c+dx}-640a^4b^3(32c-7dx)\sqrt{c+dx}+\right.}{}$$

input
```
Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 71
68*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2
*b^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*
c^2 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d
^2*x^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*
d^2*x^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}dx$$

$\downarrow$ 896

$$\frac{\int \frac{d^3x^3}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^4}$$

$\downarrow$ 25

$$-\frac{\int -\frac{d^3x^3}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^4}$$

$\downarrow$ 1732

$$-\frac{2\int -\frac{d^3x^3\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d^4}$$

$\downarrow$ 522

$$-\frac{2\int \left(-\frac{\left(a+b\sqrt{c+dx}\right)^{13/2}}{b^7}+\frac{7a\left(a+b\sqrt{c+dx}\right)^{11/2}}{b^7}+\frac{3\left(b^2c-7a^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^7}-\frac{5\left(3ab^2c-7a^3\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^7}+\frac{\left(-35a^4+30b^2ca^2\right.}{}\right.}{}$$

$\downarrow$ 2009

$$-\frac{2\left(-\frac{6\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8}+\frac{10a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8}+\frac{6a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8}-\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b\right.}{3}\right.}{}$$

input
```
Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/b^8 - (2*(a^2 - b^2*c
)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8) + (6*a*(7*a^2 - 3
*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8) - (2*(35*a^4 -
30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8) + (10*a*(7*
a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8) - (6*(7*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(11/2))/(11*b^8) + (14*a*(a + b*Sqrt[c + d*x])^(13/2)
)/(13*b^8) - (2*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8)))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{7a\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{\left(3b^2c-21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11}+\frac{\left(8\left(-b^2c+a^2\right)a+2a\left(-2b^2c+6a^2\right)+\left(-3b^2c+15a\right.\right.}{9}\right.}{}$
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{7a\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{\left(3b^2c-21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11}+\frac{\left(8\left(-b^2c+a^2\right)a+2a\left(-2b^2c+6a^2\right)+\left(-3b^2c+15a\right.\right.}{9}\right.}{}$

input
```
int(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^4/b^8*(-1/15*(a+b*(d*x+c)^(1/2))^(15/2)+7/13*a*(a+b*(d*x+c)^(1/2))^(1
3/2)+1/11*(3*b^2*c-21*a^2)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(8*(-b^2*c+a^2)*
a+2*a*(-2*b^2*c+6*a^2)+(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*
(-(-b^2*c+a^2)*(-2*b^2*c+6*a^2)-8*a^2*(-b^2*c+a^2)-(-b^2*c+a^2)^2+(-8*(-b^
2*c+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(6*(-b^2
*c+a^2)^2*a+((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)
^2)*a)*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-(-b^2*c+a^2)^3-6*(-b^2*c+a^2)^2*a^2
)*(a+b*(d*x+c)^(1/2))^(3/2)+(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\,\left(3234\,ab^6d^3x^3 - 17280\,ab^6c^3 + 46976\,a^3b^4c^2 - 44544\,a^5b^2c + 14336\,a^7 - 28\,(141\,ab^6c - 140\,a^3b^4)d^2x\right.}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544
*a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a
*b^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*
c^3 + 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 1
4*a^2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*s
qrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{7a(a+b\sqrt{c+dx})^{\frac{13}{2}}}{13b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{15}{2}}}{15b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{11}{2}}\cdot(21a^2-3b^2c)}{11b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}(-35a^3+15ab^2c)}{9b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}\cdot(35a^4-30a^2b^2c+3}{7b^6}\right.}{} \\ \frac{d^4x^4}{8\sqrt{a}} \end{cases}\right)}{} \\ \frac{x^4}{4\sqrt{a+b\sqrt{c}}} \end{cases}$$

input
```
integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-7*a*(a + b*sqrt(c + d*x))**(13/2)/(13*b**6) +
(a + b*sqrt(c + d*x))**(15/2)/(15*b**6) + (a + b*sqrt(c + d*x))**(11/2)*(2
1*a**2 - 3*b**2*c)/(11*b**6) + (a + b*sqrt(c + d*x))**(9/2)*(-35*a**3 + 15
*a*b**2*c)/(9*b**6) + (a + b*sqrt(c + d*x))**(7/2)*(35*a**4 - 30*a**2*b**2
*c + 3*b**4*c**2)/(7*b**6) + (a + b*sqrt(c + d*x))**(5/2)*(-21*a**5 + 30*a
**3*b**2*c - 9*a*b**4*c**2)/(5*b**6) + (a + b*sqrt(c + d*x))**(3/2)*(7*a**
6 - 15*a**4*b**2*c + 9*a**2*b**4*c**2 - b**6*c**3)/(3*b**6) + sqrt(a + b*s
qrt(c + d*x))*(-a**7 + 3*a**5*b**2*c - 3*a**3*b**4*c**2 + a*b**6*c**3)/b**
6)/b**2, Ne(b, 0)), (d**4*x**4/(8*sqrt(a)), True))/d**4, Ne(d, 0)), (x**4/
(4*sqrt(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \frac{4\left(3003\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}-24255\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}a-12285\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}+25025\,(\right.}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(
13/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*
b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*
c + 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*
c + 7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 +
15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a
^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx =$$
$$-\frac{4\left(15015\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^6c^3-45045\sqrt{\sqrt{dx+c}b+a}ab^6c^3-19305\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^4c^2+81081\left(\right.$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/45045*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x
+ c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 81081*
(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)
*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(
d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c +
 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b +
a)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135
*sqrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) +
 24255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)
*a^2 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b +
a)^(7/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x +
 c)*b + a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/(b^8*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx=\int \frac{x^3}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(x^3/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{\sqrt{dx+c}\,b+a}\,\left(7168\sqrt{dx+c}\,a^6b-20480\sqrt{dx+c}\,a^4b^3c+4480\sqrt{dx+c}\,a^4b^3dx+18816\sqrt{dx+c}\,a^2b^5\right.}{}$$

input
```
int(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(7168*sqrt(c + d*x)*a**6*b - 20480*sqrt(c + d
*x)*a**4*b**3*c + 4480*sqrt(c + d*x)*a**4*b**3*d*x + 18816*sqrt(c + d*x)*a
**2*b**5*c**2 - 8544*sqrt(c + d*x)*a**2*b**5*c*d*x + 3528*sqrt(c + d*x)*a*
*2*b**5*d**2*x**2 - 4992*sqrt(c + d*x)*b**7*c**3 + 3744*sqrt(c + d*x)*b**7
*c**2*d*x - 3276*sqrt(c + d*x)*b**7*c*d**2*x**2 + 3003*sqrt(c + d*x)*b**7*
d**3*x**3 - 14336*a**7 + 44544*a**5*b**2*c - 5376*a**5*b**2*d*x - 46976*a*
*3*b**4*c**2 + 10880*a**3*b**4*c*d*x - 3920*a**3*b**4*d**2*x**2 + 17280*a*
b**6*c**3 - 5568*a*b**6*c**2*d*x + 3948*a*b**6*c*d**2*x**2 - 3234*a*b**6*d
**3*x**3))/(45045*b**8*d**4)
```

3.133 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = &-\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} \\ &+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} \\ &-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} \\ &+\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} \\ &-\frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}\end{aligned}$$

output
```
-4*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(1/2)/b^6/d^3+4/3*(b^4*c^2-6*a^2*b
^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3-8/5*a*(-3*b^2*c+5*a^2)*(a+b*
(d*x+c)^(1/2))^(5/2)/b^6/d^3+8/7*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(7/2)/
b^6/d^3-20/9*a*(a+b*(d*x+c)^(1/2))^(9/2)/b^6/d^3+4/11*(a+b*(d*x+c)^(1/2))^
(11/2)/b^6/d^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-1280a^5+96a^3b^2(28c-5dx)+640a^4b\sqrt{c+dx}-16a^2b^3(74c-25dx)\sqrt{c+dx}+15b\right.}{3465b^6d^3}$$

input
```
Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*
a^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sq
rt[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*
x + 175*d^2*x^2)))/(3465*b^6*d^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{d^2x^2}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^3}$$

$$\Big\downarrow \; 1732$$

$$\frac{2\int \frac{d^2x^2\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \; 522$$

$$\frac{2\int\left(\frac{\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^5}-\frac{5a\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^5}-\frac{2\left(b^2c-5a^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{b^5}+\frac{\left(5a^4-6b^2ca^2+b^4c^2\right)\sqrt{c}}{b^5}\right.}{d^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(\frac{4\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6}-\frac{2a\left(a^2-b^2c\right)^2\sqrt{a+b\sqrt{c+dx}}}{b^6}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6}\right.}{d^3}$$

input
```
Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-2*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/b^6 + (2*(5*a^4 - 6*a
^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6) - (4*a*(5*a^2 - 3
*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6) + (4*(5*a^2 - b^2*c)*(a + b*S
qrt[c + d*x])^(7/2))/(7*b^6) - (10*a*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6)
+ (2*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6)))/d^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4\left(2b^2c-10a^2\right)(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4\left(4\left(-b^2c+a^2\right)a+a\left(-2b^2c+6a^2\right)\right)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{5}-4(}{d^3b^6}$
default	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4\left(2b^2c-10a^2\right)(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4\left(4\left(-b^2c+a^2\right)a+a\left(-2b^2c+6a^2\right)\right)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{5}-4(}{d^3b^6}$

input
```
int(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d^3/b^6*(1/11*(a+b*(d*x+c)^(1/2))^(11/2)-5/9*a*(a+b*(d*x+c)^(1/2))^(9/2)
-1/7*(2*b^2*c-10*a^2)*(a+b*(d*x+c)^(1/2))^(7/2)-1/5*(4*(-b^2*c+a^2)*a+a*(-
2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-(-b^2*c+a^2)^2-4*a^2*(-b^2
*c+a^2))*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(350\,ab^4d^2x^2+1472\,ab^4c^2-2688\,a^3b^2c+1280\,a^5-8\,(61\,ab^4c-60\,a^3b^2)dx-(315\,b^5d^2x^2+480\,b^5c^2\right.}{3465\,b^6d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 -
8*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^
2*b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(s
qrt(d*x + c)*b + a)/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{5a(a+b\sqrt{c+dx})^{\frac{9}{2}}}{9b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{11}{2}}}{11b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}\cdot(10a^2-2b^2c)}{7b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}(-10a^3+6ab^2c)}{5b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}\cdot(5a^4-6a^2b^2c+b^4c^2)}{3b^4}\right.}{b^2} \\ \frac{d^3x^3}{6\sqrt{a}} \end{cases}\right.}{d^3} \\ \frac{x^3}{3\sqrt{a+b\sqrt{c}}} \end{cases}$$

input
```
integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-5*a*(a + b*sqrt(c + d*x))**(9/2)/(9*b**4) + (a
 + b*sqrt(c + d*x))**(11/2)/(11*b**4) + (a + b*sqrt(c + d*x))**(7/2)*(10*a
**2 - 2*b**2*c)/(7*b**4) + (a + b*sqrt(c + d*x))**(5/2)*(-10*a**3 + 6*a*b*
*2*c)/(5*b**4) + (a + b*sqrt(c + d*x))**(3/2)*(5*a**4 - 6*a**2*b**2*c + b*
*4*c**2)/(3*b**4) + sqrt(a + b*sqrt(c + d*x))*(-a**5 + 2*a**3*b**2*c - a*b
**4*c**2)/b**4)/b**2, Ne(b, 0)), (d**3*x**3/(6*sqrt(a)), True))/d**3, Ne(d
, 0)), (x**3/(3*sqrt(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{4\left(315\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}} - 1925\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}a - 990\left(b^2c-5a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}} + 1386\left(3ab^2c-\right.\right.}{}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2
)*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c -
5*a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*
(sqrt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sq
rt(d*x + c)*b + a))/(b^6*d^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{4\left(1155\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^4c^2 - 3465\sqrt{\sqrt{dx+c}b+a}\,ab^4c^2 - 990\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^2c + 4158\left(\sqrt{dx+c}\right.\right.}{}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/3465*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)
*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x
 + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c +
6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2
) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*
a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3
/2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)/(b^6*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(x^2/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^2/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{\sqrt{dx+c}\,b+a}\,\left(640\sqrt{dx+c}\,a^4b-1184\sqrt{dx+c}\,a^2b^3c+400\sqrt{dx+c}\,a^2b^3dx+480\sqrt{dx+c}\,b^5c^2-360\right.}{}$$

input
```
int(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(640*sqrt(c + d*x)*a**4*b - 1184*sqrt(c + d*x
)*a**2*b**3*c + 400*sqrt(c + d*x)*a**2*b**3*d*x + 480*sqrt(c + d*x)*b**5*c
**2 - 360*sqrt(c + d*x)*b**5*c*d*x + 315*sqrt(c + d*x)*b**5*d**2*x**2 - 12
80*a**5 + 2688*a**3*b**2*c - 480*a**3*b**2*d*x - 1472*a*b**4*c**2 + 488*a*
b**4*c*d*x - 350*a*b**4*d**2*x**2))/(3465*b**6*d**3)
```

3.134 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2} + \frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} - \frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2} + \frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

output
```
-4*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(1/2)/b^4/d^2+4/3*(-b^2*c+3*a^2)*(a+
b*(d*x+c)^(1/2))^(3/2)/b^4/d^2-12/5*a*(a+b*(d*x+c)^(1/2))^(5/2)/b^4/d^2+4/
7*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-48a^3+2ab^2(26c-9dx)+24a^2b\sqrt{c+dx}+5b^3\sqrt{c+dx}(-4c+3dx)\right)}{105b^4d^2}$$

input
```
Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*
Sqrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{dx}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{dx}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 1732}$$

$$-\frac{2\int -\frac{dx\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{2\int \left(-\frac{\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^3}+\frac{3a\left(a+b\sqrt{c+dx}\right)^{3/2}}{b^3}+\frac{(b^2c-3a^2)\sqrt{a+b\sqrt{c+dx}}}{b^3}+\frac{a^3-ab^2c}{b^3\sqrt{a+b\sqrt{c+dx}}}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\left(-\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4}+\frac{2a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4}-\frac{2\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4}+\frac{6a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4}\right)}{d^2}$$

input
```
Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/b^4 - (2*(3*a^2 - b^2*c
)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4) + (6*a*(a + b*Sqrt[c + d*x])^(5/2))
/(5*b^4) - (2*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}+\left(-b^{2}c+a^{2}\right)a\sqrt{a+b\sqrt{dx+c}}\right)}{d^{2}b^{4}}$	92
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}+\left(-b^{2}c+a^{2}\right)a\sqrt{a+b\sqrt{dx+c}}\right)}{d^{2}b^{4}}$	92

input
```
int(x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^2/b^4*(-1/7*(a+b*(d*x+c)^(1/2))^(7/2)+3/5*a*(a+b*(d*x+c)^(1/2))^(5/2)
+1/3*(b^2*c-3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+(-b^2*c+a^2)*a*(a+b*(d*x+c)^(
1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(18\,ab^{2}dx-52\,ab^{2}c+48\,a^{3}-\left(15\,b^{3}dx-20\,b^{3}c+24\,a^{2}b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}b+a}}{105\,b^{4}d^{2}}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a
^2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{3a(a+b\sqrt{c+dx})^{\frac{5}{2}}}{5b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}}{7b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}\cdot(3a^2-b^2c)}{3b^2}+\frac{\sqrt{a+b\sqrt{c+dx}}(-a^3+ab^2c)}{b^2}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^2}{2\sqrt{a+b\sqrt{c}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-3*a*(a + b*sqrt(c + d*x))**(5/2)/(5*b**2) + (a
 + b*sqrt(c + d*x))**(7/2)/(7*b**2) + (a + b*sqrt(c + d*x))**(3/2)*(3*a**2
 - b**2*c)/(3*b**2) + sqrt(a + b*sqrt(c + d*x))*(-a**3 + a*b**2*c)/b**2)/b
**2, Ne(b, 0)), ((-c*(c + d*x)/2 + (c + d*x)**2/4)/sqrt(a), True))/d**2, N
e(d, 0)), (x**2/(2*sqrt(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{4\left(15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}-63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-35\left(b^2c-3a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}+105\left(ab^2c-a^3\right)\sqrt{\sqrt{}}\right.}{105\,b^4d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^(5/2)*a -
 35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a*b^2*c - a^3)*sqrt
(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}b+a}\,ab^2c-15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a\right.}{105\,b^4d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/105*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b +
a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/
2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)
*a^3)/(b^4*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(x/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{\sqrt{dx+c}\,b+a}\left(24\sqrt{dx+c}\,a^2b-20\sqrt{dx+c}\,b^3c+15\sqrt{dx+c}\,b^3dx-48a^3+52a\,b^2c-18a\,b^2dx\right)}{105b^4d^2}$$

input
```
int(x/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(24*sqrt(c + d*x)*a**2*b - 20*sqrt(c + d*x)*b
**3*c + 15*sqrt(c + d*x)*b**3*d*x - 48*a**3 + 52*a*b**2*c - 18*a*b**2*d*x)
)/(105*b**4*d**2)
```

3.135 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} \, dx = -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d} + \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

output
```
-4*a*(a+b*(d*x+c)^(1/2))^(1/2)/b^2/d+4/3*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} \, dx = \frac{4\left(-2a+b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

input
```
Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \frac{\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{b}-\frac{a}{b\sqrt{a+b\sqrt{c+dx}}}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a+b\sqrt{c+dx}}}{b^2}\right)}{d}$$

input
```
Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-2*a*Sqrt[a + b*Sqrt[c + d*x]])/b^2 + (2*(a + b*Sqrt[c + d*x])^(3/2))
/(3*b^2)))/d
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^{2}d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^{2}d}$	41

input
```
int(1/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d/b^2*(1/3*(a+b*(d*x+c)^(1/2))^(3/2)-a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\,\sqrt{\sqrt{dx+c}b+a}\left(\sqrt{dx+c}b-2\,a\right)}{3\,b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-a\sqrt{a+b\sqrt{c+dx}}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}}{3}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{c+dx}{2\sqrt{a}} & \text{otherwise} \end{cases}\right)}{d} & \text{for } d\neq 0 \\ \frac{x}{\sqrt{a+b\sqrt{c}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-a*sqrt(a + b*sqrt(c + d*x)) + (a + b*sqrt(c +
d*x))**(3/2)/3)/b**2, Ne(b, 0)), ((c + d*x)/(2*sqrt(a)), True))/d, Ne(d, 0
)), (x/sqrt(a + b*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(\frac{(\sqrt{dx+c}b+a)^{\frac{3}{2}}}{b^2} - \frac{3\sqrt{\sqrt{dx+c}b+a}a}{b^2}\right)}{3\,d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/
d
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left((\sqrt{dx+c}b+a)^{\frac{3}{2}} - 3\sqrt{\sqrt{dx+c}b+a}a\right)}{3\,b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/3*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d} - \frac{4\,a\,\sqrt{a+b\sqrt{c+d\,x}}}{b^2\,d}$$

input
```
int(1/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
(4*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))
^(1/2))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{\sqrt{dx+c}\,b+a}\,\left(\sqrt{dx+c}\,b-2a\right)}{3b^2d}$$

input
```
int(1/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(c + d*x)*b + a)*(sqrt(c + d*x)*b - 2*a))/(3*b**2*d)
```

3.136 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

output
```
-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(1
/2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+b*c^(1/2))
^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}} + \frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}}$$

input
```
Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/Sqrt[-a - b*Sqr
t[c]] + (2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/Sqrt[-a
 + b*Sqrt[c]]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {896, 25, 1732, 561, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{1}{dx\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{1}{dx\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}}{dx\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{561}$$

$$-\frac{4\int \frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{a-c-dx}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{4\left(-\frac{1}{2}\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}-\frac{1}{2}\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{4\left(\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}}\right)}{b^2}$$

input
```
Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(-4*((b^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*Sqrt[
a - b*Sqrt[c]]) + (b^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c
]]])/(2*Sqrt[a + b*Sqrt[c]])))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{\sqrt{-\sqrt{b^{2}c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^{2}c}-a}}\right)}{\sqrt{\sqrt{b^{2}c}-a}}$	92
default	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{\sqrt{-\sqrt{b^{2}c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^{2}c}-a}}\right)}{\sqrt{\sqrt{b^{2}c}-a}}$	92

input
```
int(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2
)-a)^(1/2))+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b
^2*c)^(1/2)-a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 743, normalized size of antiderivative = 7.66

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-\cdots}}}{b^2c-\cdots}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

$$-\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\cdots}}{b^2\cdots}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

$$-\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b}{b^4c^2-2\cdots}}}{b^2c-a^2\cdots}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

$$+\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{\cdots}{b^4c^2-\cdots}}}{b^2c-\cdots}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) -
a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^
2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2
*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2
)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*
x + c)*b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) - a)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*
b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c +
 a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c -
 a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4
*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*
c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4
*sqrt(sqrt(d*x + c)*b + a))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx=\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}b+ax}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{2\left(\frac{(b^2\sqrt{c}|b|+ab^2)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{(b\sqrt{c}+a)\sqrt{b\sqrt{c}-a}}+\frac{(b^2\sqrt{c}|b|-ab^2)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{(b\sqrt{c}-a)\sqrt{-b\sqrt{c}-a}}\right)}{b^2}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2*((b^2*sqrt(c)*abs(b) + a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a +
 sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^2*sqrt(c)*abs(b)
 - a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqr
t(c) - a)*sqrt(-b*sqrt(c) - a)))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.21

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{-2\sqrt{c}\,\sqrt{\sqrt{c}\,b-a}\,atan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{\sqrt{c}\,b-a}}\right)b-2\sqrt{\sqrt{c}\,b-a}\,atan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{\sqrt{c}\,b-a}}\right)a-\sqrt{c}\,\sqrt{\sqrt{c}\,b+a}\log\left(\sqrt{\sqrt{dx-}}\right.}{}$$

input
```
int(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt
(c)*b - a))*b - 2*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(
sqrt(c)*b - a))*a - sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b +
 a) - sqrt(sqrt(c)*b + a))*b + sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c
 + d*x)*b + a) + sqrt(sqrt(c)*b + a))*b + sqrt(sqrt(c)*b + a)*log(sqrt(sqr
t(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*a - sqrt(sqrt(c)*b + a)*log(sqrt(
sqrt(c + d*x)*b + a) + sqrt(sqrt(c)*b + a))*a)/(a**2 - b**2*c)
```

3.137 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} - \frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}} + \frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\left(a+b\sqrt{c}\right)^{3/2}\sqrt{c}}$$

output
```
-(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/(-b^2*c+a^2)/x-1/2*b*d*arct
anh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(3/2)/c^(
1/2)+1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+b*c
^(1/2))^(3/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{1}{2}\left(-\frac{2\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} + \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{3/2}\sqrt{c}} - \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{3/2}\sqrt{c}}\right)$$

input
```
Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
((-2*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x) +
(b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/((-a - b*Sqrt
[c])^(3/2)*Sqrt[c]) - (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sq
rt[c]]])/((-a + b*Sqrt[c])^(3/2)*Sqrt[c]))/2
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {896, 1732, 561, 25, 27, 1492, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{ 896}$$

$$d\int \frac{1}{d^2x^2\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$2d\int \frac{\sqrt{c+dx}}{d^2x^2\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 561}$$

$$\frac{4d\int -\frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4d\int \frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4d\int \frac{a-c-dx}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \ 1492$$

$$-\frac{4d\left(\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{b^4\int -\frac{2c(2a-c-dx)}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{8c(a^2-b^2c)}\right)}{b^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{4d\left(\frac{b^2\int \frac{2a-c-dx}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

$$\Big\downarrow \ 1480$$

$$-\frac{4d\left(\frac{b^2\left(-\frac{\left(\frac{a}{\sqrt{c}}+b\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b}-\frac{1}{2}\left(1-\frac{a}{b\sqrt{c}}\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}\right)}{4(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

$$\Big\downarrow \ 221$$

$$-\frac{4d\left(\frac{b^2\left(\frac{b^2\left(1-\frac{a}{b\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}}+\frac{b\left(\frac{a}{\sqrt{c}}+b\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}}\right)}{4(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

input

```
Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(-4*d*((b^2*(2*a - c - d*x)*Sqrt[a + b*Sqrt[c + d*x]])/(4*(a^2 - b^2*c)*(a
^2/b^2 - c - (2*a*(c + d*x))/b^2 + (c + d*x)^2/b^2)) + (b^2*((b*(b + a/Sqr
t[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*Sqrt[a -
b*Sqrt[c]]) + (b^2*(1 - a/(b*Sqrt[c]))*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/S
qrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]])))/(4*(a^2 - b^2*c))))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(125) = 250$.

Time = 0.40 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.59

method	result
derivativedivides	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}\left(-\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(-4\sqrt{b^2c}-4a\right)\left(-b\sqrt{dx+c}+\sqrt{\right.}}\right.}{4b^2}\right.$
default	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}\left(-\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(-4\sqrt{b^2c}-4a\right)\left(-b\sqrt{dx+c}+\sqrt{\right.}}\right.}{4b^2}\right.$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*d*b^2*(-1/4*(b^2*c)^(1/2)/b^2/c*(2*(a+b*(d*x+c)^(1/2))^(1/2)/(4*(b^2*c)^
(1/2)-4*a)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(4*(b^2*c)^(1/2)-4*a)/((b^2*c
)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
)+1/4*(b^2*c)^(1/2)/b^2/c*(-2*(a+b*(d*x+c)^(1/2))^(1/2)/(-4*(b^2*c)^(1/2)-
4*a)/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(-4*(b^2*c)^(1/2)-4*a)/(-(b^2*c)^(
1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2493 vs. $2(127) = 254$.

Time = 0.20 (sec) , antiderivative size = 2493, normalized size of antiderivative = 15.29

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b
^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*
d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*
b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*
c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(
a*b^6*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 -
a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^1
0*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2
+ a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 +
3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12
*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 -
 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6
*c))) - (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^
2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^
6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a
^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b
^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (
2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2
 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*
b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^...
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}\,b+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(127) = 254.

Time = 0.24 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.91

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{d\left(\frac{\left(\left(b^3c-a^2b\right)^2\sqrt{b\sqrt{c}-a}\,b^4c-2\left(ab^6c^{\frac{3}{2}}-a^3b^4\sqrt{c}\right)\sqrt{b\sqrt{c}-a}\,|-b^3c+a^2b|+\left(a^2b^8c^2-2\,a^4b^6c+a^6b^4\right)\sqrt{b\sqrt{c}-a}\right)\arctan\left(\frac{\cdots}{\sqrt{-\frac{ab^2c-a^3+\sqrt{\left(ab^2c-\cdots\right.}}{\cdots}}}\right.}{\left(b^6c^{\frac{7}{2}}-3\,a^2b^4c^{\frac{5}{2}}+3\,a^4b^2c^{\frac{3}{2}}-a^6\sqrt{c}\right)|-b^3c+a^2b|}\cdots\right.}{\cdots}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/2*d*(((b^3*c - a^2*b)^2*sqrt(b*sqrt(c) - a)*b^4*c - 2*(a*b^6*c^(3/2) - a
^3*b^4*sqrt(c))*sqrt(b*sqrt(c) - a)*abs(-b^3*c + a^2*b) + (a^2*b^8*c^2 - 2
*a^4*b^6*c + a^6*b^4)*sqrt(b*sqrt(c) - a))*arctan(sqrt(sqrt(d*x + c)*b + a
)/sqrt(-(a*b^2*c - a^3 + sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c +
 a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((b^6*c^(7/2) - 3*a^2*b^4*c^(5/2) +
3*a^4*b^2*c^(3/2) - a^6*sqrt(c))*abs(-b^3*c + a^2*b)) - ((b^3*c - a^2*b)^2
*sqrt(-b*sqrt(c) - a)*b^4*c + 2*(a*b^6*c^(3/2) - a^3*b^4*sqrt(c))*sqrt(-b*
sqrt(c) - a)*abs(-b^3*c + a^2*b) + (a^2*b^8*c^2 - 2*a^4*b^6*c + a^6*b^4)*s
qrt(-b*sqrt(c) - a))*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3
 - sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2)))/
(b^2*c - a^2)))/((b^6*c^(7/2) - 3*a^2*b^4*c^(5/2) + 3*a^4*b^2*c^(3/2) - a^
6*sqrt(c))*abs(-b^3*c + a^2*b)) + 2*((sqrt(d*x + c)*b + a)^(3/2)*b^4 - 2*s
qrt(sqrt(d*x + c)*b + a)*a*b^4)/((b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqr
t(d*x + c)*b + a)*a - a^2)*(b^2*c - a^2)))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.88

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{\sqrt{c}\,b-a}\,atan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{\sqrt{c}\,b-a}}\right)a^2bdx-2\sqrt{c}\,\sqrt{\sqrt{c}\,b-a}\,atan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{\sqrt{c}\,b-a}}\right)b^3cdx-4\sqrt{\sqrt{c}\,b-a}\,ata}{}$$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt
(c)*b - a))*a**2*b*d*x - 2*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c +
d*x)*b + a)/sqrt(sqrt(c)*b - a))*b**3*c*d*x - 4*sqrt(sqrt(c)*b - a)*atan(s
qrt(sqrt(c + d*x)*b + a)/sqrt(sqrt(c)*b - a))*a*b**2*c*d*x + 4*sqrt(c + d*
x)*sqrt(sqrt(c + d*x)*b + a)*a**2*b*c - 4*sqrt(c + d*x)*sqrt(sqrt(c + d*x)
*b + a)*b**3*c**2 - 4*sqrt(sqrt(c + d*x)*b + a)*a**3*c + 4*sqrt(sqrt(c + d
*x)*b + a)*a*b**2*c**2 - sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x
)*b + a) - sqrt(sqrt(c)*b + a))*a**2*b*d*x - sqrt(c)*sqrt(sqrt(c)*b + a)*l
og(sqrt(sqrt(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*b**3*c*d*x + sqrt(c)*s
qrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) + sqrt(sqrt(c)*b + a))*a*
*2*b*d*x + sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) + sqr
t(sqrt(c)*b + a))*b**3*c*d*x + 2*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x
)*b + a) - sqrt(sqrt(c)*b + a))*a*b**2*c*d*x - 2*sqrt(sqrt(c)*b + a)*log(s
qrt(sqrt(c + d*x)*b + a) + sqrt(sqrt(c)*b + a))*a*b**2*c*d*x)/(4*c*x*(a**4
 - 2*a**2*b**2*c + b**4*c**2))
```

3.138 $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 269

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-a^2\sqrt{c+dx}-5b^2c\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x} + \frac{b\left(2a-5b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{5/2}c^{3/2}} - \frac{b\left(2a+5b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{5/2}c^{3/2}}$$

output

```
-1/2*(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/(-b^2*c+a^2)/x^2-1/8*b*
d*(a+b*(d*x+c)^(1/2))^(1/2)*(6*a*b*c-a^2*(d*x+c)^(1/2)-5*b^2*c*(d*x+c)^(1/
2))/c/(-b^2*c+a^2)^2/x+1/16*b*(2*a-5*b*c^(1/2))*d^2*arctanh((a+b*(d*x+c)^(
1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(5/2)/c^(3/2)-1/16*b*(2*a+5
*b*c^(1/2))*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+
b*c^(1/2))^(5/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}\left(4a^3c+b^3c(4c-5dx)\sqrt{c+dx}-a^2b\sqrt{c+dx}(4c+dx)+2ab^2c(-2c+3dx)\right)}{(a^2-b^2c)^2x^2}+\frac{b(2a+5b\sqrt{c})d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{5/2}}+\frac{b(-2a}{}}{16c^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^3*c + b^3*c*(4*c - 5*d*x)*Sqrt
[c + d*x] - a^2*b*Sqrt[c + d*x]*(4*c + d*x) + 2*a*b^2*c*(-2*c + 3*d*x)))/(
(a^2 - b^2*c)^2*x^2) + (b*(2*a + 5*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c
 + d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(5/2) + (b*(-2*a + 5*b*Sq
rt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(-a + b
*Sqrt[c])^(5/2))/(16*c^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.45, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {896, 25, 1732, 561, 27, 1492, 27, 1492, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$\Big\downarrow \text{896}$$
$$d^2\int \frac{1}{d^3x^3\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$
$$\Big\downarrow \text{25}$$
$$-d^2\int -\frac{1}{d^3x^3\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2d^2\int -\frac{\sqrt{c+dx}}{d^3x^3\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{561}$$

$$-\frac{4d^2\int \frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d^2\int \frac{a-c-dx}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \text{1492}$$

$$-\frac{4d^2\left(\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{b^4\int -\frac{2c(6a-5(c+dx))}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{16c(a^2-b^2c)}\right)}{b^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d^2\left(\frac{b^2\int \frac{6a-5(c+dx)}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{8(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}\right)}{b^2}$$

$$\Big\downarrow \text{1492}$$

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{b^4\int \frac{2\left(a\left(a^2-13b^2c\right)+\left(a^2+5b^2c\right)(c+dx)\right)}{b^4\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)}\right)}{8(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a-}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}\right.}\right.}{b^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\int\frac{a\left(a^2-13b^2c\right)+\left(a^2+5b^2c\right)(c+dx)}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{4c\left(a^2-b^2c\right)}\right)}{8(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c-}}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+}{}\right.}\right.}{b^2}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\frac{(a-b\sqrt{c})^2(2a+5b\sqrt{c})\int\frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b\sqrt{c}}-\frac{(2a-5b\sqrt{c})(a+b\sqrt{c})^2\int\frac{1}{\frac{c+dx}{b^2}-\frac{a-}{b}}}{2b\sqrt{c}}}{4c\left(a^2-b^2c\right)}\right.}{8(a^2-b^2c)}\right.}{b^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\frac{b(2a-5b\sqrt{c})(a+b\sqrt{c})^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{b(a-b\sqrt{c})^2(2a+5b\sqrt{c})\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{}}\right.}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}}{4c\left(a^2-b^2c\right)}\right.}{8(a^2-b^2c)}\right.}{b^2}$$

input
```
Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(-4*d^2*((b^2*(2*a - c - d*x)*Sqrt[a + b*Sqrt[c + d*x]])/(8*(a^2 - b^2*c)*
(a^2/b^2 - c - (2*a*(c + d*x))/b^2 + (c + d*x)^2/b^2)^2) + (b^2*((Sqrt[a +
 b*Sqrt[c + d*x]]*(a*(a^2 + 11*b^2*c) - (a^2 + 5*b^2*c)*(c + d*x)))/(4*c*(
a^2 - b^2*c)*(a^2/b^2 - c - (2*a*(c + d*x))/b^2 + (c + d*x)^2/b^2)) - ((b*
(2*a - 5*b*Sqrt[c])*(a + b*Sqrt[c])^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*(a - b*Sqrt[c])^2
*(2*a + 5*b*Sqrt[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]
])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c]))/(4*c*(a^2 - b^2*c))))/(8*(a^2 - b^2*c)
)))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.59

method	result
derivativedivides	$-4d^2b^4\left(-\frac{\frac{-\frac{\left(-5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c-2a\sqrt{b^2c}+a^2\right)}+\frac{\left(-7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{-4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}-\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}-2a\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{4\left(-b^2c+2a\sqrt{b^2c}-a^2\right)\sqrt{\sqrt{b^2c}-a}}}{16b^2c\sqrt{b^2c}}+\right.$
default	$-4d^2b^4\left(-\frac{\frac{-\frac{\left(-5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c-2a\sqrt{b^2c}+a^2\right)}+\frac{\left(-7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{-4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}-\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}-2a\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{4\left(-b^2c+2a\sqrt{b^2c}-a^2\right)\sqrt{\sqrt{b^2c}-a}}}{16b^2c\sqrt{b^2c}}+\right.$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*d^2*b^4*(-1/16/b^2/c/(b^2*c)^(1/2)*((-1/4*(-5*(b^2*c)^(1/2)+2*a)/(b^2*c
-2*a*(b^2*c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(-7*(b^2*c)^(1/2)+2*
a)/(-(b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)-(b^2*c)
^(1/2))^2-1/4*(5*(b^2*c)^(1/2)-2*a)/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/((b^2*c
)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
)+1/16/b^2/c/(b^2*c)^(1/2)*((-1/4*(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2*a*(b^2*c)
^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(7*(b^2*c)^(1/2)+2*a)/((b^2*c)^(
1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2))^2-1/4*
(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)/(-(b^2*c)^(1/2)-a)^(1/
2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4390 vs. $2(217) = 434$.

Time = 0.97 (sec) , antiderivative size = 4390, normalized size of antiderivative = 16.32

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}b+ax^3}}\,dx$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(217) = 434$.

Time = 0.24 (sec) , antiderivative size = 1303, normalized size of antiderivative = 4.84

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/16*(((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 - (13
*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c
))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4
*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c
)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 +
 a^5*c + sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4
*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3
 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 - a*b^8*c^(11/2) - 4*a^2*b^7*c^5 + 4
*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 - 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 + 4*a
^7*b^2*c^(5/2) + a^8*b*c^2 - a^9*c^(3/2))*sqrt(-b*sqrt(c) - a)*abs(b^5*c^3
 - 2*a^2*b^3*c^2 + a^4*b*c)) + ((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b
^6*c + a^2*b^4)*d^3 + (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6
*c^(3/2) - a^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2
*(4*a^2*b^14*c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*
a^10*b^6*c^2 - a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*
b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)
^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^
2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 + a*b^8*c^(
11/2) - 4*a^2*b^7*c^5 - 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 + 6*a^5*b^4*c^(7
/2) - 4*a^6*b^3*c^3 - 4*a^7*b^2*c^(5/2) + a^8*b*c^2 + a^9*c^(3/2))*sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 987, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt(c)
*b - a))*a**4*b*d**2*x**2 - 18*sqrt(c)*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(
c + d*x)*b + a)/sqrt(sqrt(c)*b - a))*a**2*b**3*c*d**2*x**2 - 10*sqrt(c)*sq
rt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqrt(sqrt(c)*b - a))*b**5
*c**2*d**2*x**2 + 2*sqrt(sqrt(c)*b - a)*atan(sqrt(sqrt(c + d*x)*b + a)/sqr
t(sqrt(c)*b - a))*a**3*b**2*c*d**2*x**2 - 26*sqrt(sqrt(c)*b - a)*atan(sqrt
(sqrt(c + d*x)*b + a)/sqrt(sqrt(c)*b - a))*a*b**4*c**2*d**2*x**2 + 16*sqrt
(c + d*x)*sqrt(sqrt(c + d*x)*b + a)*a**4*b*c**2 + 4*sqrt(c + d*x)*sqrt(sqr
t(c + d*x)*b + a)*a**4*b*c*d*x - 32*sqrt(c + d*x)*sqrt(sqrt(c + d*x)*b + a
)*a**2*b**3*c**3 + 16*sqrt(c + d*x)*sqrt(sqrt(c + d*x)*b + a)*a**2*b**3*c*
*2*d*x + 16*sqrt(c + d*x)*sqrt(sqrt(c + d*x)*b + a)*b**5*c**4 - 20*sqrt(c
+ d*x)*sqrt(sqrt(c + d*x)*b + a)*b**5*c**3*d*x - 16*sqrt(sqrt(c + d*x)*b +
 a)*a**5*c**2 + 32*sqrt(sqrt(c + d*x)*b + a)*a**3*b**2*c**3 - 24*sqrt(sqrt
(c + d*x)*b + a)*a**3*b**2*c**2*d*x - 16*sqrt(sqrt(c + d*x)*b + a)*a*b**4*
c**4 + 24*sqrt(sqrt(c + d*x)*b + a)*a*b**4*c**3*d*x + 2*sqrt(c)*sqrt(sqrt(
c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*a**4*b*d**2
*x**2 - 9*sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) - sqrt
(sqrt(c)*b + a))*a**2*b**3*c*d**2*x**2 - 5*sqrt(c)*sqrt(sqrt(c)*b + a)*log
(sqrt(sqrt(c + d*x)*b + a) - sqrt(sqrt(c)*b + a))*b**5*c**2*d**2*x**2 - 2*
sqrt(c)*sqrt(sqrt(c)*b + a)*log(sqrt(sqrt(c + d*x)*b + a) + sqrt(sqrt(c...
```

3.139 $\int x^3 \left(a+b\sqrt{c+dx}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 350

$$\begin{aligned}\int x^3 \left(a+b\sqrt{c+dx}\right)^p \, dx = & -\frac{2a(a^2-b^2c)^3 \left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)} \\ & +\frac{2(a^2-b^2c)^2 (7a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)} \\ & -\frac{6a(7a^2-3b^2c) (a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{3+p}}{b^8d^4(3+p)} \\ & +\frac{2(35a^4-30a^2b^2c+3b^4c^2) \left(a+b\sqrt{c+dx}\right)^{4+p}}{b^8d^4(4+p)} \\ & -\frac{10a(7a^2-3b^2c) \left(a+b\sqrt{c+dx}\right)^{5+p}}{b^8d^4(5+p)} \\ & +\frac{6(7a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{6+p}}{b^8d^4(6+p)} \\ & -\frac{14a\left(a+b\sqrt{c+dx}\right)^{7+p}}{b^8d^4(7+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{8+p}}{b^8d^4(8+p)}\end{aligned}$$

output
```
-2*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(p+1)/b^8/d^4/(p+1)+2*(-b^2*c+a^2)
^2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(2+p)/b^8/d^4/(2+p)-6*a*(-3*b^2*c+7*
a^2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3+p)/b^8/d^4/(3+p)+2*(3*b^4*c^2-30*
a^2*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(4+p)/b^8/d^4/(4+p)-10*a*(-3*b^2*c+7
*a^2)*(a+b*(d*x+c)^(1/2))^(5+p)/b^8/d^4/(5+p)+6*(-b^2*c+7*a^2)*(a+b*(d*x+c
)^(1/2))^(6+p)/b^8/d^4/(6+p)-14*a*(a+b*(d*x+c)^(1/2))^(7+p)/b^8/d^4/(7+p)+
2*(a+b*(d*x+c)^(1/2))^(8+p)/b^8/d^4/(8+p)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.59

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx = -\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(5040a^7-5040a^6b(1+p)\sqrt{c+dx}+360a^5b^2(6c(-7+p+p^2)+7d(2+3p+p^2)\right.}{}$$

input
```
Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c +
d*x] + 360*a^5*b^2*(6*c*(-7 + p + p^2) + 7*d*(2 + 3*p + p^2)*x) - 120*a^4*
b^3*(1 + p)*Sqrt[c + d*x]*(2*c*(-63 - 5*p + 2*p^2) + 7*d*(6 + 5*p + p^2)*x
) + 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42
- 61*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p
^4)*x^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 +
 p^3) + 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p +
 71*p^2 + 14*p^3 + p^4)*x^2) - b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*S
qrt[c + d*x]*(-48*c^3 + 24*c^2*d*(2 + p)*x - 6*c*d^2*(8 + 6*p + p^2)*x^2 +
 d^3*(48 + 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p
^2 + 38*p^3 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 +
 2*p^5)*x + 6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5
+ p^6)*x^2 + 7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 +
 p^6)*x^3)))/(b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 +
 p)*(8 + p))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int d^3x^3\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -d^3x^3\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 1732}$$

$$-\frac{2\int -d^3x^3\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{2\int \left(\frac{a(a^2-b^2c)^3(a+b\sqrt{c+dx})^p}{b^7}+\frac{(b^2c-7a^2)(b^2c-a^2)^2(a+b\sqrt{c+dx})^{p+1}}{b^7}+\frac{3(7a^5-10b^2ca^3+3b^4c^2a)(a+b\sqrt{c+dx})^{p+2}}{b^7}+\frac{(-35a^4+30b^2\cdots}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\left(\frac{a(a^2-b^2c)^3(a+b\sqrt{c+dx})^{p+1}}{b^8(p+1)}-\frac{(a^2-b^2c)^2(7a^2-b^2c)(a+b\sqrt{c+dx})^{p+2}}{b^8(p+2)}+\frac{3a(7a^2-3b^2c)(a^2-b^2c)(a+b\sqrt{c+dx})^{p+3}}{b^8(p+3)}+\frac{5a(7a^2-3b^2c)(\cdots}{b^8(\cdots}\right.}{\cdots}$$

input
```
Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*((a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*(1 + p)) - ((a
^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*(2 + p))
 + (3*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^
8*(3 + p)) - ((35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(4
 + p))/(b^8*(4 + p)) + (5*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5 + p
))/(b^8*(5 + p)) - (3*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(6 + p))/(b^8*
(6 + p)) + (7*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*(7 + p)) - (a + b*Sqrt
[c + d*x])^(8 + p)/(b^8*(8 + p))))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^3\left(a+b\sqrt{dx+c}\right)^p\,dx$$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(335) = 670$.

Time = 0.21 (sec) , antiderivative size = 1416, normalized size of antiderivative = 4.05

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
-2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c
 + 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^
7 + 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3
 + 13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^
4 + 7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^
2*b^6)*d^3*p^6 + 5*(38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^
2*b^6)*d^3*p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959
*a^2*b^6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c^4 + 81
*a^2*b^6*c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18*b^8*c^2*d^2*p^5
 + (b^8*c^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 - 95*a^2*b^6*c + 35*a^4*b^
4)*d^2*p^4 + 6*(58*b^8*c^2 - 80*a^2*b^6*c + 35*a^4*b^4)*d^2*p^3 + (457*b^8
*c^2 - 806*a^2*b^6*c + 385*a^4*b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c +
 a^4*b^4)*d^2*p)*x^2 + 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 -
 15*a^6*b^2*c)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*
a^2*b^6*c^2 - 5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*a^4*b
^4*c)*d*p^3 + (176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c - 105*a^6*b^2
)*d*p^2 + 105*(b^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*c - a^6*b^2)*d*p)*x + (
192*(a*b^7*c^3 + a^3*b^5*c^2)*p^4 + 96*(27*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a
^5*b^3*c)*p^3 - (a*b^7*d^3*p^7 + 21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 73
5*a*b^7*d^3*p^4 + 1624*a*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d...
```

Sympy [F]

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx$$

input
```
integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral(x**3*(a + b*sqrt(c + d*x))**p, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(335) = 670$.

Time = 0.05 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.08

$$\int x^3 \left(a + b\sqrt{c + dx}\right)^p \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
-2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
 a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*
b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^
2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4
 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2
 + 274*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*
(d*x + c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4
 + 20*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)
*a^4*b^2 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p*c/
((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7
 + 28*p^6 + 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d
*x + c)^4*b^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 +
720*p)*(d*x + c)^(7/2)*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^
2 + 120*p)*(d*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p
)*(d*x + c)^(5/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a
^4*b^4 + 840*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2520*(p^2 + p)*
(d*x + c)*a^6*b^2 + 5040*sqrt(d*x + c)*a^7*b*p - 5040*a^8)*(sqrt(d*x + c)*
b + a)^p/((p^8 + 36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118
124*p^2 + 109584*p + 40320)*b^8))/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5699 vs. $2(335) = 670$.

Time = 0.25 (sec) , antiderivative size = 5699, normalized size of antiderivative = 16.28

$$\int x^3 \left(a + b\sqrt{c + dx}\right)^p \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
-2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^7 - (sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^7 + 34*(sqrt(d*x + c)*b
+ a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^6 - 35*(sqrt(d*x + c)*b + a)*(sqr
t(d*x + c)*b + a)^p*a*b^6*c^3*p^6 - 3*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
c)*b + a)^p*b^4*c^2*p^7 + 9*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^
p*a*b^4*c^2*p^7 - 9*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^
4*c^2*p^7 + 3*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^
7 + 478*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^5 - 511*
(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^5 - 96*(sqrt(d*x
 + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^6 + 297*(sqrt(d*x + c)*b
+ a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^6 - 306*(sqrt(d*x + c)*b + a)^2
*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^6 + 105*(sqrt(d*x + c)*b + a)*(sqrt
(d*x + c)*b + a)^p*a^3*b^4*c^2*p^6 + 3*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x +
 c)*b + a)^p*b^2*c*p^7 - 15*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^
p*a*b^2*c*p^7 + 30*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2
*c*p^7 - 30*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^7
+ 15*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^7 - 3*(sq
rt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^7 + 3580*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^4 - 4025*(sqrt(d*x + c)*b
+ a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^4 - 1254*(sqrt(d*x + c)*b + a)...
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^3\left(a+b\sqrt{c+d\,x}\right)^p\,dx$$

input
```
int(x^3*(a + b*(c + d*x)^(1/2))^p,x)
```

output
```
int(x^3*(a + b*(c + d*x)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1909, normalized size of antiderivative = 5.45

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=\text{Too large to display}$$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
(2*(sqrt(c + d*x)*b + a)**p*(5040*sqrt(c + d*x)*a**7*b*p + 480*sqrt(c + d*
x)*a**5*b**3*c*p**3 - 2880*sqrt(c + d*x)*a**5*b**3*c*p**2 - 18480*sqrt(c +
 d*x)*a**5*b**3*c*p + 840*sqrt(c + d*x)*a**5*b**3*d*p**3*x + 2520*sqrt(c +
 d*x)*a**5*b**3*d*p**2*x + 1680*sqrt(c + d*x)*a**5*b**3*d*p*x - 192*sqrt(c
 + d*x)*a**3*b**5*c**2*p**4 - 192*sqrt(c + d*x)*a**3*b**5*c**2*p**3 + 9408
*sqrt(c + d*x)*a**3*b**5*c**2*p**2 + 24528*sqrt(c + d*x)*a**3*b**5*c**2*p
+ 24*sqrt(c + d*x)*a**3*b**5*c*d*p**5*x - 240*sqrt(c + d*x)*a**3*b**5*c*d*
p**4*x - 3240*sqrt(c + d*x)*a**3*b**5*c*d*p**3*x - 7680*sqrt(c + d*x)*a**3
*b**5*c*d*p**2*x - 4704*sqrt(c + d*x)*a**3*b**5*c*d*p*x + 42*sqrt(c + d*x)
*a**3*b**5*d**2*p**5*x**2 + 420*sqrt(c + d*x)*a**3*b**5*d**2*p**4*x**2 + 1
470*sqrt(c + d*x)*a**3*b**5*d**2*p**3*x**2 + 2100*sqrt(c + d*x)*a**3*b**5*
d**2*p**2*x**2 + 1008*sqrt(c + d*x)*a**3*b**5*d**2*p*x**2 - 192*sqrt(c + d
*x)*a*b**7*c**3*p**4 - 2592*sqrt(c + d*x)*a*b**7*c**3*p**3 - 10752*sqrt(c
+ d*x)*a*b**7*c**3*p**2 - 13392*sqrt(c + d*x)*a*b**7*c**3*p + 72*sqrt(c +
d*x)*a*b**7*c**2*d*p**5*x + 1008*sqrt(c + d*x)*a*b**7*c**2*d*p**4*x + 4608
*sqrt(c + d*x)*a*b**7*c**2*d*p**3*x + 7848*sqrt(c + d*x)*a*b**7*c**2*d*p**
2*x + 4176*sqrt(c + d*x)*a*b**7*c**2*d*p*x - 12*sqrt(c + d*x)*a*b**7*c*d**
2*p**6*x**2 - 198*sqrt(c + d*x)*a*b**7*c*d**2*p**5*x**2 - 1200*sqrt(c + d*
x)*a*b**7*c*d**2*p**4*x**2 - 3330*sqrt(c + d*x)*a*b**7*c*d**2*p**3*x**2 -
4188*sqrt(c + d*x)*a*b**7*c*d**2*p**2*x**2 - 1872*sqrt(c + d*x)*a*b**7*...
```

3.140 $\int x^2 \left(a+b\sqrt{c+dx}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 242

$$\int x^2 \left(a+b\sqrt{c+dx}\right)^p \, dx = -\frac{2a(a^2-b^2c)^2 \left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)} + \frac{2(5a^4-6a^2b^2c+b^4c^2) \left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)} - \frac{4a(5a^2-3b^2c) \left(a+b\sqrt{c+dx}\right)^{3+p}}{b^6d^3(3+p)} + \frac{4(5a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{4+p}}{b^6d^3(4+p)} - \frac{10a\left(a+b\sqrt{c+dx}\right)^{5+p}}{b^6d^3(5+p)} + \frac{2\left(a+b\sqrt{c+dx}\right)^{6+p}}{b^6d^3(6+p)}$$

output
```
-2*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(p+1)/b^6/d^3/(p+1)+2*(b^4*c^2-6*a
^2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(2+p)/b^6/d^3/(2+p)-4*a*(-3*b^2*c+5*a^
2)*(a+b*(d*x+c)^(1/2))^(3+p)/b^6/d^3/(3+p)+4*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(
1/2))^(4+p)/b^6/d^3/(4+p)-10*a*(a+b*(d*x+c)^(1/2))^(5+p)/b^6/d^3/(5+p)+2*(
a+b*(d*x+c)^(1/2))^(6+p)/b^6/d^3/(6+p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.17

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx =$$
$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(120a^5-120a^4b(1+p)\sqrt{c+dx}+12a^3b^2\left(4c\left(-5+p+p^2\right)+5d\left(2+3p+p^2\right)x\right)\right.}{}$$

input
```
Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*(a + b*Sqrt[c + d*x])^(1 + p)*(120*a^5 - 120*a^4*b*(1 + p)*Sqrt[c + d*
x] + 12*a^3*b^2*(4*c*(-5 + p + p^2) + 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(
1 + p)*Sqrt[c + d*x]*(2*c*(-30 - 4*p + p^2) + 5*d*(6 + 5*p + p^2)*x) - b^5
*(15 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8
 + 6*p + p^2)*x^2) + a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(
-30 - 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3
+ p^4)*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$\Big\downarrow \; 896$$
$$\frac{\int d^2x^2\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^3}$$
$$\Big\downarrow \; 1732$$
$$\frac{2\int d^2x^2\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d^3}$$

$$\downarrow \text{522}$$

$$\frac{2\int \left(-\frac{a(a^2-b^2c)^2(a+b\sqrt{c+dx})^p}{b^5}+\frac{(5a^4-6b^2ca^2+b^4c^2)(a+b\sqrt{c+dx})^{p+1}}{b^5}-\frac{2(5a^3-3ab^2c)(a+b\sqrt{c+dx})^{p+2}}{b^5}-\frac{2(b^2c-5a^2)(a+b\sqrt{c+dx})}{b^5}\right.}{d^3}$$

$$\downarrow \text{2009}$$

$$\frac{2\left(-\frac{a(a^2-b^2c)^2(a+b\sqrt{c+dx})^{p+1}}{b^6(p+1)}-\frac{2a(5a^2-3b^2c)(a+b\sqrt{c+dx})^{p+3}}{b^6(p+3)}+\frac{2(5a^2-b^2c)(a+b\sqrt{c+dx})^{p+4}}{b^6(p+4)}+\frac{(5a^4-6a^2b^2c+b^4c^2)(a+b\sqrt{c+dx})}{b^6(p+2)}\right.}{d^3}$$

input
```
Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(2*(-((a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*(1 + p))) + (
(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*(2 + p
)) - (2*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*(3 + p)) +
 (2*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*(4 + p)) - (5*a*(a
 + b*Sqrt[c + d*x])^(5 + p))/(b^6*(5 + p)) + (a + b*Sqrt[c + d*x])^(6 + p)
/(b^6*(6 + p))))/d^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^2 \left(a+b\sqrt{dx+c}\right)^p dx$$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(230) = 460.

Time = 0.22 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.94

$$\int x^2 \left(a+b\sqrt{c+dx}\right)^p dx$$
$$= \frac{2\left(120\,b^6c^3-360\,a^2b^4c^2+360\,a^4b^2c-120\,a^6+8\,(b^6c^3+3\,a^2b^4c^2)p^3+(b^6d^3p^5+15\,b^6d^3p^4+85\,b^6d^3p^3+\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 +
3*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*
b^6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4
*c^2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4
+ (41*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(
b^6*c - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c
)*p - 4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (
23*b^6*c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c
+ a^4*b^2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3
*a^3*b^3*c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 5
0*a*b^5*d^2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 1
5*a^5*b)*p - 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^
5*c - 15*a^3*b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))
*(sqrt(d*x + c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 +
 735*b^6*d^3*p^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)
```

Sympy [F]

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$

input
```
integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral(x**2*(a + b*sqrt(c + d*x))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.66

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$=\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^pc^2}{(p^2+3p+2)b^2}-\frac{2\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b
^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2
*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 +
10*p^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 27
4*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x
+ c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20
*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*
b^2 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 +
 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. $2(230) = 460$.

Time = 0.22 (sec) , antiderivative size = 2490, normalized size of antiderivative = 10.29

$$\int x^2 \left(a + b\sqrt{c + dx}\right)^p dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 - (sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 + 19*(sqrt(d*x + c)*b +
 a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt
(d*x + c)*b + a)^p*a*b^4*c^2*p^4 - 2*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c
)*b + a)^p*b^2*c*p^5 + 6*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a
*b^2*c*p^5 - 6*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p
^5 + 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 137*(
sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 - 155*(sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 34*(sqrt(d*x + c)*b +
 a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p^4 + 108*(sqrt(d*x + c)*b + a)^3*(sqr
t(d*x + c)*b + a)^p*a*b^2*c*p^4 - 114*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x +
c)*b + a)^p*a^2*b^2*c*p^4 + 40*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)
^p*a^3*b^2*c*p^4 + (sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^5 - 5
*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^5 + 10*(sqrt(d*x + c)
*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^5 - 10*(sqrt(d*x + c)*b + a)^3*(sq
rt(d*x + c)*b + a)^p*a^3*p^5 + 5*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b
+ a)^p*a^4*p^5 - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^5 + 4
61*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^2 - 580*(sqrt
(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^2 - 214*(sqrt(d*x + c
)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 + 726*(sqrt(d*x + c)*b + a...
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^2\,\left(a+b\,\sqrt{c+d\,x}\right)^p\,dx$$

input
```
int(x^2*(a + b*(c + d*x)^(1/2))^p,x)
```

output
```
int(x^2*(a + b*(c + d*x)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 865, normalized size of antiderivative = 3.57

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$=\frac{2\left(\sqrt{dx+c}\,b+a\right)^p\left(-360a^2b^4c^2+8b^6c^3p^3+72b^6c^3p^2+184b^6c^3p-120a^6+8\sqrt{dx+c}\,a^3b^3cp^3-72\sqrt{dx}\right.}{}$$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
(2*(sqrt(c + d*x)*b + a)**p*(120*sqrt(c + d*x)*a**5*b*p + 8*sqrt(c + d*x)*
a**3*b**3*c*p**3 - 72*sqrt(c + d*x)*a**3*b**3*c*p**2 - 320*sqrt(c + d*x)*a
**3*b**3*c*p + 20*sqrt(c + d*x)*a**3*b**3*d*p**3*x + 60*sqrt(c + d*x)*a**3
*b**3*d*p**2*x + 40*sqrt(c + d*x)*a**3*b**3*d*p*x + 24*sqrt(c + d*x)*a*b**
5*c**2*p**3 + 168*sqrt(c + d*x)*a*b**5*c**2*p**2 + 264*sqrt(c + d*x)*a*b**
5*c**2*p - 8*sqrt(c + d*x)*a*b**5*c*d*p**4*x - 60*sqrt(c + d*x)*a*b**5*c*d
*p**3*x - 124*sqrt(c + d*x)*a*b**5*c*d*p**2*x - 72*sqrt(c + d*x)*a*b**5*c*
d*p*x + sqrt(c + d*x)*a*b**5*d**2*p**5*x**2 + 10*sqrt(c + d*x)*a*b**5*d**2
*p**4*x**2 + 35*sqrt(c + d*x)*a*b**5*d**2*p**3*x**2 + 50*sqrt(c + d*x)*a*b
**5*d**2*p**2*x**2 + 24*sqrt(c + d*x)*a*b**5*d**2*p*x**2 - 120*a**6 - 48*a
**4*b**2*c*p**2 + 72*a**4*b**2*c*p + 360*a**4*b**2*c - 60*a**4*b**2*d*p**2
*x - 60*a**4*b**2*d*p*x + 24*a**2*b**4*c**2*p**3 + 72*a**2*b**4*c**2*p**2
- 192*a**2*b**4*c**2*p - 360*a**2*b**4*c**2 - 4*a**2*b**4*c*d*p**4*x + 12*
a**2*b**4*c*d*p**3*x + 136*a**2*b**4*c*d*p**2*x + 120*a**2*b**4*c*d*p*x -
5*a**2*b**4*d**2*p**4*x**2 - 30*a**2*b**4*d**2*p**3*x**2 - 55*a**2*b**4*d*
*2*p**2*x**2 - 30*a**2*b**4*d**2*p*x**2 + 8*b**6*c**3*p**3 + 72*b**6*c**3*
p**2 + 184*b**6*c**3*p + 120*b**6*c**3 - 4*b**6*c**2*d*p**4*x - 36*b**6*c*
*2*d*p**3*x - 92*b**6*c**2*d*p**2*x - 60*b**6*c**2*d*p*x + b**6*c*d**2*p**
5*x**2 + 11*b**6*c*d**2*p**4*x**2 + 41*b**6*c*d**2*p**3*x**2 + 61*b**6*c*d
**2*p**2*x**2 + 30*b**6*c*d**2*p*x**2 + b**6*d**3*p**5*x**3 + 15*b**6*d...
```

3.141 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^4d^2(3+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{4+p}}{b^4d^2(4+p)}$$

output
```
-2*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(p+1)/b^4/d^2/(p+1)+2*(-b^2*c+3*a^2)
*(a+b*(d*x+c)^(1/2))^(2+p)/b^4/d^2/(2+p)-6*a*(a+b*(d*x+c)^(1/2))^(3+p)/b^4
/d^2/(3+p)+2*(a+b*(d*x+c)^(1/2))^(4+p)/b^4/d^2/(4+p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=$$
$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(6a^3-6a^2b(1+p)\sqrt{c+dx}-b^3(3+4p+p^2)\sqrt{c+dx}(-2c+d(2+p)x)+ab^2(2\right.}{b^4d^2(1+p)(2+p)(3+p)(4+p)}$$

input
```
Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] -
 b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 +
 p + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 +
 p))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int dx\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -dx\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(\frac{(a^3-ab^2c)\left(a+b\sqrt{c+dx}\right)^p}{b^3}+\frac{(b^2c-3a^2)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^3}+\frac{3a\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^3}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^3}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(\frac{a(a^2-b^2c)(a+b\sqrt{c+dx})^{p+1}}{b^4(p+1)}-\frac{(3a^2-b^2c)(a+b\sqrt{c+dx})^{p+2}}{b^4(p+2)}+\frac{3a(a+b\sqrt{c+dx})^{p+3}}{b^4(p+3)}-\frac{(a+b\sqrt{c+dx})^{p+4}}{b^4(p+4)}\right)}{d^2}$$

input
```
Int[x*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*((a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*(1 + p)) - ((3*a
^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*(2 + p)) + (3*a*(a + b*Sqr
t[c + d*x])^(3 + p))/(b^4*(3 + p)) - (a + b*Sqrt[c + d*x])^(4 + p)/(b^4*(4
 + p))))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x\left(a+b\sqrt{dx+c}\right)^p dx$$

input
```
int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(137) = 274.

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.03

$$\int x\left(a+b\sqrt{c+dx}\right)^p dx = -\frac{2\,(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\,(b^4c^2+a^2b^2c)p^2-(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2)x^2+4\,(2\,b^4c^2}{}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
-2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*
d^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a
^2*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*
b^2)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 +
3*a*b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^
4*d^2*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

Sympy [F]

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

input
```
integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral(x*(a + b*sqrt(c + d*x))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=$$
$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^p c}{(p^2+3p+2)b^2}-\frac{\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
-2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
 a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4
+ (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^
2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^
3 + 35*p^2 + 50*p + 24)*b^4))/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(137) = 274$.

Time = 0.11 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.56

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
-2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 - (sqrt(d*x
+ c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 + 8*(sqrt(d*x + c)*b + a)^
2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x +
c)*b + a)^p*a*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
p^3 + 3*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^3 - 3*(sqrt(d*
x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + (sqrt(d*x + c)*b + a)*(s
qrt(d*x + c)*b + a)^p*a^3*p^3 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*
b + a)^p*b^2*c*p - 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*
c*p - 6*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^2 + 21*(sqrt(d*x
 + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^2 - 24*(sqrt(d*x + c)*b + a)^2*
(sqrt(d*x + c)*b + a)^p*a^2*p^2 + 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b
 + a)^p*a^3*p^2 + 12*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c
 - 24*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 11*(sqrt(d*x
 + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p + 42*(sqrt(d*x + c)*b + a)^3*(sqr
t(d*x + c)*b + a)^p*a*p - 57*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)
^p*a^2*p + 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p - 6*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p + 24*(sqrt(d*x + c)*b + a)^3*(
sqrt(d*x + c)*b + a)^p*a - 36*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^2 + 24*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3)/((b^2*p^4
+ 10*b^2*p^3 + 35*b^2*p^2 + 50*b^2*p + 24*b^2)*b^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x\,\left(a+b\,\sqrt{c+d\,x}\right)^p\,dx$$

input
```
int(x*(a + b*(c + d*x)^(1/2))^p,x)
```

output
```
int(x*(a + b*(c + d*x)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.07

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$=\frac{2\left(\sqrt{dx+c}\,b+a\right)^p\left(6\sqrt{dx+c}\,a^3bp-4\sqrt{dx+c}\,a\,b^3c\,p^2-10\sqrt{dx+c}\,a\,b^3cp+\sqrt{dx+c}\,a\,b^3d\,p^3x+3\sqrt{dx}\right.}{}$$

input
```
int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
(2*(sqrt(c + d*x)*b + a)**p*(6*sqrt(c + d*x)*a**3*b*p - 4*sqrt(c + d*x)*a*
b**3*c*p**2 - 10*sqrt(c + d*x)*a*b**3*c*p + sqrt(c + d*x)*a*b**3*d*p**3*x
+ 3*sqrt(c + d*x)*a*b**3*d*p**2*x + 2*sqrt(c + d*x)*a*b**3*d*p*x - 6*a**4
- 2*a**2*b**2*c*p**2 + 4*a**2*b**2*c*p + 12*a**2*b**2*c - 3*a**2*b**2*d*p*
*2*x - 3*a**2*b**2*d*p*x - 2*b**4*c**2*p**2 - 8*b**4*c**2*p - 6*b**4*c**2
+ b**4*c*d*p**3*x + 4*b**4*c*d*p**2*x + 3*b**4*c*d*p*x + b**4*d**2*p**3*x*
*2 + 6*b**4*d**2*p**2*x**2 + 11*b**4*d**2*p*x**2 + 6*b**4*d**2*x**2))/(b**
4*d**2*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.142 $\int \left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}$$

output
```
-2*a*(a+b*(d*x+c)^(1/2))^(p+1)/b^2/d/(p+1)+2*(a+b*(d*x+c)^(1/2))^(2+p)/b^2
/d/(2+p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(-a+b(1+p)\sqrt{c+dx}\right)}{b^2d(1+p)(2+p)}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1
 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{c+dx}\right)^p dx$$

$$\Big\downarrow \text{ 239}$$

$$\frac{\int \left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{2\int \sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{2\int \left(\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}}{b}-\frac{a\left(a+b\sqrt{c+dx}\right)^p}{b}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(\frac{\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2(p+1)}\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(2*(-((a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*(1 + p))) + (a + b*Sqrt[c + d
*x])^(2 + p)/(b^2*(2 + p))))/d
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a+b\sqrt{dx+c}\right)^{p} dx$$

input
```
int((a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int((a+b*(d*x+c)^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \left(a+b\sqrt{c+dx}\right)^p dx = \frac{2\left(b^2cp+\sqrt{dx+c}abp+b^2c-a^2+(b^2dp+b^2d)x\right)\left(\sqrt{dx+c}b+a\right)^p}{b^2dp^2+3b^2dp+2b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqr
t(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

Sympy [F]

$$\int \left(a+b\sqrt{c+dx}\right)^p\, dx=\int \left(a+b\sqrt{c+dx}\right)^p\, dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral((a + b*sqrt(c + d*x))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{c+dx}\right)^p\, dx=\frac{2\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^p}{(p^2+3\,p+2)b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a
)^p/((p^2 + 3*p + 2)*b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(58) = 116$.

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.08

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$= \frac{2\left(\left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+c}b+a\right)^p p - \left(\sqrt{dx+c}b+a\right)\left(\sqrt{dx+c}b+a\right)^p ap + \left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+\right.\right.}{(p^2+3p+2)b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*p - (sqrt(d*x + c)*b +
a)*(sqrt(d*x + c)*b + a)^p*a*p + (sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b
+ a)^p - 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a)/((p^2 + 3*p +
2)*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.35

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \begin{cases} -\frac{2\,a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-2\,b\,\sqrt{c+d\,x}}{b^2\,d} & \text{if } p=-1 \\ \frac{2\,\left(\ln\left(a+b\,\sqrt{c+d\,x}\right)+\frac{a}{a+b\,\sqrt{c+d\,x}}\right)}{b^2\,d} & \text{if } p=-2 \\ \frac{4\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+2}}{b^2\,d\,(2\,p+4)}-\frac{4\,a\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+1}}{b^2\,d\,(2\,p+2)} & \text{if } p\neq -1 \wedge p\neq -2 \end{cases}$$

input
```
int((a + b*(c + d*x)^(1/2))^p,x)
```

output
```
piecewise(p == -1, -(2*a*log(a + b*(c + d*x)^(1/2)) - 2*b*(c + d*x)^(1/2))
/(b^2*d), p == -2, (2*(log(a + b*(c + d*x)^(1/2)) + a/(a + b*(c + d*x)^(1/
2))))/(b^2*d), p ~= -1 & p ~= -2, (4*(a + b*(c + d*x)^(1/2))^(p + 2))/(b^2
*d*(2*p + 4)) - (4*a*(a + b*(c + d*x)^(1/2))^(p + 1))/(b^2*d*(2*p + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{2\left(\sqrt{dx+c}\,b+a\right)^p\left(\sqrt{dx+c}\,abp-a^2+b^2cp+b^2c+b^2dpx+b^2dx\right)}{b^2d\,(p^2+3p+2)}$$

input
```
int((a+b*(d*x+c)^(1/2))^p,x)
```

output
```
(2*(sqrt(c + d*x)*b + a)**p*(sqrt(c + d*x)*a*b*p - a**2 + b**2*c*p + b**2*
c + b**2*d*p*x + b**2*d*x))/(b**2*d*(p**2 + 3*p + 2))
```

3.143 $\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)} - \frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{\left(a+b\sqrt{c}\right)(1+p)}$$

output
```
-(a+b*(d*x+c)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],(a+b*(d*x+c)^(1/2))/(a
-b*c^(1/2)))/(a-b*c^(1/2))/(p+1)-(a+b*(d*x+c)^(1/2))^(p+1)*hypergeom([1, p
+1],[2+p],(a+b*(d*x+c)^(1/2))/(a+b*c^(1/2)))/(a+b*c^(1/2))/(p+1)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\left(\left(a+b\sqrt{c}\right)\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right)\text{Hyperge}\right.}{\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c}\right)(1+p)}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^p/x,x]
```

output
```
-(((a + b*Sqrt[c + d*x])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1 +
 p, 2 + p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hyperg
eometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])]))/((a
- b*Sqrt[c])*(a + b*Sqrt[c])*(1 + p)))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{\left(a+b\sqrt{c+dx}\right)^p}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2\int -\frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p}{dx}d\sqrt{c+dx}$$

$$\Big\downarrow \text{615}$$

$$-2\int \left(\frac{\left(a+b\sqrt{c+dx}\right)^p}{2\left(-c+\sqrt{c}-dx\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^p}{2\left(\sqrt{c}+\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{2(p+1)\left(a-b\sqrt{c}\right)}+\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\text{Hypergeometric2}}{2(p+1)\left(a-\right.}\right.$$

input
```
Int[(a + b*Sqrt[c + d*x])^p/x,x]
```

output
```
-2*(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a +
 b*Sqrt[c + d*x])/(a - b*Sqrt[c])])/(2*(a - b*Sqrt[c])*(1 + p)) + ((a + b*
Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c +
d*x])/(a + b*Sqrt[c])])/(2*(a + b*Sqrt[c])*(1 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{dx+c}\right)^{p}}{x}dx$$

input
```
int((a+b*(d*x+c)^(1/2))^p/x,x)
```

output
```
int((a+b*(d*x+c)^(1/2))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="fricas")
```

output
```
integral((sqrt(d*x + c)*b + a)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**p/x,x)
```

output
```
Integral((a + b*sqrt(c + d*x))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="maxima")
```

output
```
integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="giac")
```

output
```
integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = \int \frac{\left(a+b\,\sqrt{c+d\,x}\right)^p}{x}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^p/x,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = \int \frac{\left(\sqrt{dx+c}\,b+a\right)^p}{x}dx$$

input
```
int((a+b*(d*x+c)^(1/2))^p/x,x)
```

output
```
int((sqrt(c + d*x)*b + a)**p/x,x)
```

3.144 $\int \frac{x^{10}}{4-\left(1+x^2\right)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 219

$$\begin{aligned}
\int \frac{x^{10}}{4-(1+x^2)^4}\,dx = {} & 4x-\frac{x^3}{3}+\frac{1}{16}\sqrt{95+81\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
& -\frac{\arctan\left(\sqrt{-1+\sqrt{2}}x\right)}{8\sqrt{2\left(-1393+985\sqrt{2}\right)}} \\
& -\frac{1}{16}\sqrt{95+81\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
& +\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt{2}}x\right)}{8\sqrt{2\left(1393+985\sqrt{2}\right)}} \\
& +\frac{1}{16}\sqrt{-95+81\sqrt{3}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
4*x-1/3*x^3+1/16*(95+81*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(
2+2*3^(1/2))^(1/2))-1/8*arctan((2^(1/2)-1)^(1/2)*x)/(-2786+1970*2^(1/2))^(
1/2)-1/16*(95+81*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(
1/2))^(1/2))+1/8*arctanh((1+2^(1/2))^(1/2)*x)/(2786+1970*2^(1/2))^(1/2)+1/
16*(-95+81*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{x^{10}}{4-(1+x^2)^4}\,dx = 4x-\frac{x^3}{3}-\frac{(-i+11\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{8\sqrt{2-2i\sqrt{2}}}-\frac{(i+11\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{8\sqrt{2+2i\sqrt{2}}}-\frac{(41+29\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{(-41+29\sqrt{2})\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{8\sqrt{2\left(-1+\sqrt{2}\right)}}$$

input
```
Integrate[x^10/(4 - (1 + x^2)^4),x]
```

output
```
4*x - x^3/3 - ((-I + 11*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(8*Sqrt[2
- (2*I)*Sqrt[2]]) - ((I + 11*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(8*Sq
rt[2 + (2*I)*Sqrt[2]]) - ((41 + 29*Sqrt[2])*ArcTan[x/Sqrt[1 + Sqrt[2]]])/(
8*Sqrt[2*(1 + Sqrt[2])]) + ((-41 + 29*Sqrt[2])*ArcTanh[x/Sqrt[-1 + Sqrt[2]
]])/(8*Sqrt[2*(-1 + Sqrt[2])])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{4-\left(x^2+1\right)^4}dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(-x^2+\frac{12-29x^2}{4\left(x^4+2x^2-1\right)}+\frac{-11x^2-12}{4\left(x^4+2x^2+3\right)}+4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\sqrt{95+81\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(1393+985\sqrt{2}\right)}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)-\frac{1}{16}\sqrt{95+81\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(985\sqrt{2}-1393\right)}\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\frac{x^3}{3}-\frac{1}{32}\sqrt{81\sqrt{3}-95}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{81\sqrt{3}-95}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+4x$$

input
```
Int[x^10/(4 - (1 + x^2)^4),x]
```

output
```
4*x - x^3/3 + (Sqrt[95 + 81*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)
/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(1393 + 985*Sqrt[2])/2]*ArcTan[x/Sqrt[
1 + Sqrt[2]]])/8 - (Sqrt[95 + 81*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] +
 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 + (Sqrt[(-1393 + 985*Sqrt[2])/2]*ArcTanh[
x/Sqrt[-1 + Sqrt[2]]])/8 - (Sqrt[-95 + 81*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[-95 + 81*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*
(-1 + Sqrt[3])]*x + x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{x^3}{3}+4x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4+380\_Z^2+19683\right)}\_R\ln\left(-14\_R^3-1249\_R+5913x\right)\right)}{16}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4+5572\_Z\right.}\right.}{}$
default	$-\frac{x^3}{3}+4x-\frac{\left(7\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2-x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{64}-\frac{\left(16\sqrt{3}+\frac{\left(7\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{2}\right)}{16\sqrt{2+2\sqrt{3}}}$

input
```
int(x^10/(4-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+4*x+1/16*sum(_R*ln(-14*_R^3-1249*_R+5913*x),_R=RootOf(4*_Z^4+380*
_Z^2+19683))+1/16*sum(_R*ln(34*_R^3+47321*_R+985*x),_R=RootOf(4*_Z^4+5572*
_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{x^{10}}{4-(1+x^2)^4}\,dx &= -\frac{1}{3}\,x^3 \\ &+\frac{1}{16}\,\sqrt{81\,\sqrt{3}+95}\arctan\left(\frac{1}{146}\left(8\,\sqrt{3}x+\sqrt{81\,\sqrt{3}-95}\left(\sqrt{3}-1\right)-22\,x\right)\sqrt{81\,\sqrt{3}+95}\right) \\ &-\frac{1}{16}\,\sqrt{81\,\sqrt{3}+95}\arctan\left(-\frac{1}{146}\left(8\,\sqrt{3}x-\sqrt{81\,\sqrt{3}-95}\left(\sqrt{3}-1\right)-22\,x\right)\sqrt{81\,\sqrt{3}+95}\right) \\ &+\frac{1}{8}\,\sqrt{\frac{985}{2}\,\sqrt{2}+\frac{1393}{2}}\arctan\left(\left(41\,\sqrt{2}x-58\,x\right)\sqrt{\frac{985}{2}\,\sqrt{2}+\frac{1393}{2}}\right) \\ &+\frac{1}{32}\,\sqrt{81\,\sqrt{3}-95}\log\left(73\,x^2+\left(7\,\sqrt{3}x+x\right)\sqrt{81\,\sqrt{3}-95}+73\,\sqrt{3}\right) \\ &-\frac{1}{32}\,\sqrt{81\,\sqrt{3}-95}\log\left(73\,x^2-\left(7\,\sqrt{3}x+x\right)\sqrt{81\,\sqrt{3}-95}+73\,\sqrt{3}\right) \\ &+\frac{1}{16}\,\sqrt{\frac{985}{2}\,\sqrt{2}-\frac{1393}{2}}\log\left(\sqrt{\frac{985}{2}\,\sqrt{2}-\frac{1393}{2}}\left(17\,\sqrt{2}+24\right)+x\right) \\ &-\frac{1}{16}\,\sqrt{\frac{985}{2}\,\sqrt{2}-\frac{1393}{2}}\log\left(-\sqrt{\frac{985}{2}\,\sqrt{2}-\frac{1393}{2}}\left(17\,\sqrt{2}+24\right)+x\right)+4\,x\end{aligned}$$

input
```
integrate(x^10/(4-(x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/3*x^3 + 1/16*sqrt(81*sqrt(3) + 95)*arctan(1/146*(8*sqrt(3)*x + sqrt(81*
sqrt(3) - 95)*(sqrt(3) - 1) - 22*x)*sqrt(81*sqrt(3) + 95)) - 1/16*sqrt(81*
sqrt(3) + 95)*arctan(-1/146*(8*sqrt(3)*x - sqrt(81*sqrt(3) - 95)*(sqrt(3)
- 1) - 22*x)*sqrt(81*sqrt(3) + 95)) + 1/8*sqrt(985/2*sqrt(2) + 1393/2)*arc
tan((41*sqrt(2)*x - 58*x)*sqrt(985/2*sqrt(2) + 1393/2)) + 1/32*sqrt(81*sqr
t(3) - 95)*log(73*x^2 + (7*sqrt(3)*x + x)*sqrt(81*sqrt(3) - 95) + 73*sqrt(
3)) - 1/32*sqrt(81*sqrt(3) - 95)*log(73*x^2 - (7*sqrt(3)*x + x)*sqrt(81*sq
rt(3) - 95) + 73*sqrt(3)) + 1/16*sqrt(985/2*sqrt(2) - 1393/2)*log(sqrt(985
/2*sqrt(2) - 1393/2)*(17*sqrt(2) + 24) + x) - 1/16*sqrt(985/2*sqrt(2) - 13
93/2)*log(-sqrt(985/2*sqrt(2) - 1393/2)*(17*sqrt(2) + 24) + x) + 4*x
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.42

$$\int \frac{x^{10}}{4-(1+x^2)^4}\,dx = -\frac{x^3}{3} + 4x$$
$$-\operatorname{RootSum}\left(262144t^4 + 97280t^2 + 19683, \left(t \mapsto t\log\left(-\frac{6726445525693890 56t^7}{355901897526345} - \frac{1303034319856467}{1186339658421}\right.\right.\right.$$
$$-\operatorname{RootSum}\left(262144t^4 + 1426432t^2 - 1, \left(t \mapsto t\log\left(-\frac{672644552569389056t^7}{355901897526345} - \frac{130303431985646796}{118633965842115}\right.\right.\right.$$

input
```
integrate(x**10/(4-(x**2+1)**4),x)
```

output
```
-x**3/3 + 4*x - RootSum(262144*_t**4 + 97280*_t**2 + 19683, Lambda(_t, _t*
log(-672644552569389056*_t**7/355901897526345 - 1303034319856467968*_t**5/
118633965842115 - 1405067195890829312*_t**3/355901897526345 - 273568734602
968628*_t/355901897526345 + x))) - RootSum(262144*_t**4 + 1426432*_t**2 -
1, Lambda(_t, _t*log(-672644552569389056*_t**7/355901897526345 - 130303431
9856467968*_t**5/118633965842115 - 1405067195890829312*_t**3/3559018975263
45 - 273568734602968628*_t/355901897526345 + x)))
```

Maxima [F]

$$\int \frac{x^{10}}{4-(1+x^2)^4}\,dx = \int -\frac{x^{10}}{(x^2+1)^4-4}\,dx$$

input
```
integrate(x^10/(4-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-1/3*x^3 + 4*x - 1/4*integrate((29*x^2 - 12)/(x^4 + 2*x^2 - 1), x) - 1/4*i
ntegrate((11*x^2 + 12)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(156) = 312$.

Time = 0.46 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.83

$$\int \frac{x^{10}}{4-(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^10/(4-(x^2+1)^4),x, algorithm="giac")
```

output
```
-1/3*x^3 + 1/10368*sqrt(2)*(11*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 19
8*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 198*3^(3/4)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) - 11*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 432*3^
(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 432*3^(1/4)*sqrt(-6*sqrt(3) + 18))*ar
ctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
 1/2)) + 1/10368*sqrt(2)*(11*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 198*
3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 198*3^(3/4)*(sqrt(3)
+ 3)*sqrt(-6*sqrt(3) + 18) - 11*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 432*3^(1
/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 432*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1
/2)) + 1/20736*sqrt(2)*(198*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) - 11*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) - 11*3^(3/4)*(6*sqrt(3)
 + 18)^(3/2) - 198*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 432*3^(1/4
)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 432*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^
2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/20736*sqrt(2)*(198
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 11*3^(3/4)*sqrt(2)*
(-6*sqrt(3) + 18)^(3/2) - 11*3^(3/4)*(6*sqrt(3) + 18)^(3/2) - 198*3^(3/4)*
sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 432*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) +
 18) + 432*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*s
qrt(3) + 1/2) + sqrt(3)) - 1/16*sqrt(1970*sqrt(2) + 2786)*arctan(x/sqrt...
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.19

$$\int \frac{x^{10}}{4-(1+x^2)^4}\,dx$$
$$= 4\,x - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-190-\sqrt{2}\,146\mathrm{i}}\,16637138\mathrm{i}}{370131024+\sqrt{2}\,28616730\mathrm{i}} - \frac{2056994\,\sqrt{2}\,x\,\sqrt{-190-\sqrt{2}\,146\mathrm{i}}}{370131024+\sqrt{2}\,28616730\mathrm{i}}\right)\,\sqrt{-190-\sqrt{2}\,146\mathrm{i}}\,1\mathrm{i}}{16}$$
$$+ \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-190+\sqrt{2}\,146\mathrm{i}}\,16637138\mathrm{i}}{-370131024+\sqrt{2}\,28616730\mathrm{i}} + \frac{2056994\,\sqrt{2}\,x\,\sqrt{-190+\sqrt{2}\,146\mathrm{i}}}{-370131024+\sqrt{2}\,28616730\mathrm{i}}\right)\,\sqrt{-190+\sqrt{2}\,146\mathrm{i}}\,1\mathrm{i}}{16} - \frac{x^3}{3}$$
$$- \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1970\,\sqrt{2}-2786}\,1594003830\mathrm{i}}{128691674310\,\sqrt{2}+181998367260} + \frac{\sqrt{2}\,x\,\sqrt{-1970\,\sqrt{2}-2786}\,1092026160\mathrm{i}}{128691674310\,\sqrt{2}+181998367260}\right)\,\sqrt{-1970\,\sqrt{2}-2786}\,1\mathrm{i}}{16}$$
$$+ \frac{\mathrm{atan}\left(\frac{x\,\sqrt{1970\,\sqrt{2}-2786}\,1594003830\mathrm{i}}{128691674310\,\sqrt{2}-181998367260} - \frac{\sqrt{2}\,x\,\sqrt{1970\,\sqrt{2}-2786}\,1092026160\mathrm{i}}{128691674310\,\sqrt{2}-181998367260}\right)\,\sqrt{1970\,\sqrt{2}-2786}\,1\mathrm{i}}{16}$$

input
```
int(-x^10/((x^2 + 1)^4 - 4),x)
```

output
```
4*x - (atan((x*(- 2^(1/2)*146i - 190)^(1/2)*16637138i)/(2^(1/2)*28616730i
+ 370131024) - (2056994*2^(1/2)*x*(- 2^(1/2)*146i - 190)^(1/2))/(2^(1/2)*2
8616730i + 370131024))*(- 2^(1/2)*146i - 190)^(1/2)*1i)/16 + (atan((x*(2^(
1/2)*146i - 190)^(1/2)*16637138i)/(2^(1/2)*28616730i - 370131024) + (20569
94*2^(1/2)*x*(2^(1/2)*146i - 190)^(1/2))/(2^(1/2)*28616730i - 370131024))*
(2^(1/2)*146i - 190)^(1/2)*1i)/16 - x^3/3 - (atan((x*(- 1970*2^(1/2) - 278
6)^(1/2)*1594003830i)/(128691674310*2^(1/2) + 181998367260) + (2^(1/2)*x*(
- 1970*2^(1/2) - 2786)^(1/2)*1092026160i)/(128691674310*2^(1/2) + 18199836
7260))*(- 1970*2^(1/2) - 2786)^(1/2)*1i)/16 + (atan((x*(1970*2^(1/2) - 278
6)^(1/2)*1594003830i)/(128691674310*2^(1/2) - 181998367260) - (2^(1/2)*x*(
1970*2^(1/2) - 2786)^(1/2)*1092026160i)/(128691674310*2^(1/2) - 1819983672
60))*(1970*2^(1/2) - 2786)^(1/2)*1i)/16
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.60

$$\int \frac{x^{10}}{4-(1+x^2)^4}\,dx = \frac{7\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} + \frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{7\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{17\sqrt{\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{16} - \frac{3\sqrt{\sqrt{2}+1}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{2} - \frac{7\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{7\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{17\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{32} + \frac{17\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{32} + \frac{3\sqrt{\sqrt{2}-1}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{4} - \frac{3\sqrt{\sqrt{2}-1}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{4} - \frac{x^3}{3} + 4x$$

input
```
int(x^10/(4-(x^2+1)^4),x)
```

output
```
(42*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt
(sqrt(3) + 1)*sqrt(2))) + 6*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) -
 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 42*sqrt(sqrt(3) + 1)*sqr
t(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) -
 6*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(
sqrt(3) + 1)*sqrt(2))) - 204*sqrt(sqrt(2) + 1)*sqrt(2)*atan(x/sqrt(sqrt(2)
 + 1)) - 288*sqrt(sqrt(2) + 1)*atan(x/sqrt(sqrt(2) + 1)) - 21*sqrt(sqrt(3)
 - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + 21*sq
rt(sqrt(3) - 1)*sqrt(6)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)
+ 3*sqrt(sqrt(3) - 1)*sqrt(2)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3)
 + x**2) - 3*sqrt(sqrt(3) - 1)*sqrt(2)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + s
qrt(3) + x**2) - 102*sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(sqrt(2) - 1) +
x) + 102*sqrt(sqrt(2) - 1)*sqrt(2)*log(sqrt(sqrt(2) - 1) + x) + 144*sqrt(s
qrt(2) - 1)*log( - sqrt(sqrt(2) - 1) + x) - 144*sqrt(sqrt(2) - 1)*log(sqrt
(sqrt(2) - 1) + x) - 64*x**3 + 768*x)/192
```

3.145 $\int \frac{x^8}{4-(1+x^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 212

$$
\begin{aligned}
\int \frac{x^8}{4-(1+x^2)^4}\,dx = {} & -x-\frac{1}{16}\sqrt{-43+27\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
& +\frac{\arctan\left(\sqrt{-1+\sqrt{2}}x\right)}{8\sqrt{2\left(-239+169\sqrt{2}\right)}} \\
& +\frac{1}{16}\sqrt{-43+27\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
& +\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt{2}}x\right)}{8\sqrt{2\left(239+169\sqrt{2}\right)}} \\
& -\frac{1}{16}\sqrt{43+27\sqrt{3}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}
$$

output
```
-x-1/16*(-43+27*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1
/2))^(1/2))+1/8*arctan((2^(1/2)-1)^(1/2)*x)/(-478+338*2^(1/2))^(1/2)+1/16*
(-43+27*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/
2))+1/8*arctanh((1+2^(1/2))^(1/2)*x)/(478+338*2^(1/2))^(1/2)-1/16*(43+27*3
^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{4-(1+x^2)^4}\,dx = -x+\frac{(7i+4\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{8\sqrt{2-2i\sqrt{2}}} + \frac{(-7i+4\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{8\sqrt{2+2i\sqrt{2}}} + \frac{(17+12\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{2(1+\sqrt{2})}} - \frac{(-17+12\sqrt{2})\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{8\sqrt{2(-1+\sqrt{2})}}$$

input
```
Integrate[x^8/(4 - (1 + x^2)^4),x]
```

output
```
-x + ((7*I + 4*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(8*Sqrt[2 - (2*I)*S
qrt[2]]) + ((-7*I + 4*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(8*Sqrt[2 +
(2*I)*Sqrt[2]]) + ((17 + 12*Sqrt[2])*ArcTan[x/Sqrt[1 + Sqrt[2]]])/(8*Sqrt[
2*(1 + Sqrt[2])]) - ((-17 + 12*Sqrt[2])*ArcTanh[x/Sqrt[-1 + Sqrt[2]]])/(8*
Sqrt[2*(-1 + Sqrt[2])])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{4-(x^2+1)^4}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{4x^2-3}{4(x^4+2x^2+3)}+\frac{12x^2-5}{4(x^4+2x^2-1)}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{16}\sqrt{27\sqrt{3}-43}\arctan\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(239+169\sqrt{2}\right)}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\frac{1}{16}\sqrt{27\sqrt{3}-43}\arctan\left(\frac{2x+\sqrt{2(\sqrt{3}-1)}}{\sqrt{2(1+\sqrt{3})}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(169\sqrt{2}-239\right)}\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\frac{1}{32}\sqrt{43+27\sqrt{3}}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{32}\sqrt{43+27\sqrt{3}}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-x$$

input
```
Int[x^8/(4 - (1 + x^2)^4),x]
```

output
```
-x - (Sqrt[-43 + 27*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 + (Sqrt[(239 + 169*Sqrt[2])/2]*ArcTan[x/Sqrt[1 + Sqrt[
2]]])/8 + (Sqrt[-43 + 27*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sq
rt[2*(1 + Sqrt[3])]])/16 + (Sqrt[(-239 + 169*Sqrt[2])/2]*ArcTanh[x/Sqrt[-1
 + Sqrt[2]]])/8 + (Sqrt[43 + 27*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/32 - (Sqrt[43 + 27*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[
3])]*x + x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.33

method	result
risch	$-x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4-172\_Z^2+2187\right)}\_R\ln\left(10\_R^3-241\_R+351x\right)\right)}{16}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+956\_Z^2-1\right)}\_R\ln\left(1\right.\right.}{16}$
default	$-x-\frac{\left(-5\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-7\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2-x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{64}-\frac{\left(4\sqrt{3}+\frac{\left(-5\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-7\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{2}\right)\mathrm{arc}}{16\sqrt{2+2\sqrt{3}}}$

input
```
int(x^8/(4-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
-x+1/16*sum(_R*ln(10*_R^3-241*_R+351*x),_R=RootOf(4*_Z^4-172*_Z^2+2187))+1
/16*sum(_R*ln(14*_R^3+3363*_R+169*x),_R=RootOf(4*_Z^4+956*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.42

$$\int \frac{x^8}{4-(1+x^2)^4}\,dx = \frac{1}{16}\sqrt{27\sqrt{3}-43}\arctan\left(\frac{1}{26}\sqrt{27\sqrt{3}+43}\sqrt{27\sqrt{3}-43}\left(\sqrt{3}-1\right)+\frac{1}{13}\left(\sqrt{3}x+4x\right)\sqrt{27\sqrt{3}-43}\right) - \frac{1}{16}\sqrt{27\sqrt{3}-43}\arctan\left(\frac{1}{26}\sqrt{27\sqrt{3}+43}\sqrt{27\sqrt{3}-43}\left(\sqrt{3}-1\right)-\frac{1}{13}\left(\sqrt{3}x+4x\right)\sqrt{27\sqrt{3}-43}\right) + \frac{1}{8}\sqrt{\frac{169}{2}\sqrt{2}+\frac{239}{2}}\arctan\left(\left(17\sqrt{2}x-24x\right)\sqrt{\frac{169}{2}\sqrt{2}+\frac{239}{2}}\right) - \frac{1}{32}\sqrt{27\sqrt{3}+43}\log\left(13x^2+\left(5\sqrt{3}x-7x\right)\sqrt{27\sqrt{3}+43}+13\sqrt{3}\right) + \frac{1}{32}\sqrt{27\sqrt{3}+43}\log\left(13x^2-\left(5\sqrt{3}x-7x\right)\sqrt{27\sqrt{3}+43}+13\sqrt{3}\right) + \frac{1}{16}\sqrt{\frac{169}{2}\sqrt{2}-\frac{239}{2}}\log\left(\sqrt{\frac{169}{2}\sqrt{2}-\frac{239}{2}}\left(7\sqrt{2}+10\right)+x\right) - \frac{1}{16}\sqrt{\frac{169}{2}\sqrt{2}-\frac{239}{2}}\log\left(-\sqrt{\frac{169}{2}\sqrt{2}-\frac{239}{2}}\left(7\sqrt{2}+10\right)+x\right) - x$$

input
```
integrate(x^8/(4-(x^2+1)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(27*sqrt(3) - 43)*arctan(1/26*sqrt(27*sqrt(3) + 43)*sqrt(27*sqrt(
3) - 43)*(sqrt(3) - 1) + 1/13*(sqrt(3)*x + 4*x)*sqrt(27*sqrt(3) - 43)) - 1
/16*sqrt(27*sqrt(3) - 43)*arctan(1/26*sqrt(27*sqrt(3) + 43)*sqrt(27*sqrt(3
) - 43)*(sqrt(3) - 1) - 1/13*(sqrt(3)*x + 4*x)*sqrt(27*sqrt(3) - 43)) + 1/
8*sqrt(169/2*sqrt(2) + 239/2)*arctan((17*sqrt(2)*x - 24*x)*sqrt(169/2*sqrt
(2) + 239/2)) - 1/32*sqrt(27*sqrt(3) + 43)*log(13*x^2 + (5*sqrt(3)*x - 7*x
)*sqrt(27*sqrt(3) + 43) + 13*sqrt(3)) + 1/32*sqrt(27*sqrt(3) + 43)*log(13*
x^2 - (5*sqrt(3)*x - 7*x)*sqrt(27*sqrt(3) + 43) + 13*sqrt(3)) + 1/16*sqrt(
169/2*sqrt(2) - 239/2)*log(sqrt(169/2*sqrt(2) - 239/2)*(7*sqrt(2) + 10) +
x) - 1/16*sqrt(169/2*sqrt(2) - 239/2)*log(-sqrt(169/2*sqrt(2) - 239/2)*(7*
sqrt(2) + 10) + x) - x
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{x^8}{4-(1+x^2)^4}\,dx = -x$$
$$-\mathrm{RootSum}\left(262144t^4-44032t^2+2187,\left(t\mapsto t\log\left(-\frac{3573552913580032t^7}{84748676571}-\frac{2745951176359936t^5}{84748676571}\right.\right.\right.$$
$$-\mathrm{RootSum}\left(262144t^4+244736t^2-1,\left(t\mapsto t\log\left(-\frac{3573552913580032t^7}{84748676571}-\frac{2745951176359936t^5}{84748676571}+\right.\right.\right.$$

input
```
integrate(x**8/(4-(x**2+1)**4),x)
```

output
```
-x - RootSum(262144*_t**4 - 44032*_t**2 + 2187, Lambda(_t, _t*log(-3573552
913580032*_t**7/84748676571 - 2745951176359936*_t**5/84748676571 + 1741178
10636800*_t**3/28249558857 - 26985428064460*_t/84748676571 + x))) - RootSu
m(262144*_t**4 + 244736*_t**2 - 1, Lambda(_t, _t*log(-3573552913580032*_t*
*7/84748676571 - 2745951176359936*_t**5/84748676571 + 174117810636800*_t**
3/28249558857 - 26985428064460*_t/84748676571 + x)))
```

Maxima [F]

$$\int \frac{x^8}{4-(1+x^2)^4}\,dx = \int -\frac{x^8}{(x^2+1)^4-4}\,dx$$

input
```
integrate(x^8/(4-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-x + 1/4*integrate((12*x^2 - 5)/(x^4 + 2*x^2 - 1), x) + 1/4*integrate((4*x
^2 - 3)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(151) = 302$.

Time $= 0.49$ (sec) , antiderivative size $= 612$, normalized size of antiderivative $= 2.89$

$$\int \frac{x^8}{4-(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(4-(x^2+1)^4),x, algorithm="giac")
```

output
```
-1/2592*sqrt(2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) - 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 27*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) + 27*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x +
 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/2592*sqrt(
2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(2)*sqrt(6*sqr
t(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18)
- 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 27*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18
) + 27*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt
(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/5184*sqrt(2)*(18*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt(3
) + 18)^(3/2) - 3^(3/4)*(6*sqrt(3) + 18)^(3/2) - 18*3^(3/4)*sqrt(6*sqrt(3)
 + 18)*(sqrt(3) - 3) + 27*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 27*3^(1/
4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) +
sqrt(3)) + 1/5184*sqrt(2)*(18*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3
) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) - 3^(3/4)*(6*sqrt(3) + 1
8)^(3/2) - 18*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 27*3^(1/4)*sqrt
(2)*sqrt(-6*sqrt(3) + 18) - 27*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3
^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/16*sqrt(338*sqrt(2) + 478
)*arctan(x/sqrt(sqrt(2) + 1)) + 1/32*sqrt(338*sqrt(2) - 478)*log(abs(x ...
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.21

$$\int \frac{x^8}{4-(1+x^2)^4}\,dx$$
$$= -x + \frac{\operatorname{atan}\left(\frac{x\sqrt{-338\sqrt{2}-478}\,10562838i}{342765462\sqrt{2}+484704168} + \frac{\sqrt{2}\,x\sqrt{-338\sqrt{2}-478}\,6799884i}{342765462\sqrt{2}+484704168}\right)\sqrt{-338\sqrt{2}-478}\,1i}{16}$$
$$- \frac{\operatorname{atan}\left(\frac{x\sqrt{338\sqrt{2}-478}\,10562838i}{342765462\sqrt{2}-484704168} - \frac{\sqrt{2}\,x\sqrt{338\sqrt{2}-478}\,6799884i}{342765462\sqrt{2}-484704168}\right)\sqrt{338\sqrt{2}-478}\,1i}{16}$$
$$- \frac{\operatorname{atan}\left(\frac{x\sqrt{86-\sqrt{2}\,26i}\,95758i}{-2437188+\sqrt{2}\,284622i} + \frac{119366\sqrt{2}\,x\sqrt{86-\sqrt{2}\,26i}}{-2437188+\sqrt{2}\,284622i}\right)\sqrt{86-\sqrt{2}\,26i}\,1i}{16}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\sqrt{86+\sqrt{2}\,26i}\,95758i}{2437188+\sqrt{2}\,284622i} - \frac{119366\sqrt{2}\,x\sqrt{86+\sqrt{2}\,26i}}{2437188+\sqrt{2}\,284622i}\right)\sqrt{86+\sqrt{2}\,26i}\,1i}{16}$$

input
```
int(-x^8/((x^2 + 1)^4 - 4),x)
```

output
```
(atan((x*(- 338*2^(1/2) - 478)^(1/2)*10562838i)/(342765462*2^(1/2) + 48470
4168) + (2^(1/2)*x*(- 338*2^(1/2) - 478)^(1/2)*6799884i)/(342765462*2^(1/2
) + 484704168))*(- 338*2^(1/2) - 478)^(1/2)*1i)/16 - x - (atan((x*(338*2^(
1/2) - 478)^(1/2)*10562838i)/(342765462*2^(1/2) - 484704168) - (2^(1/2)*x*
(338*2^(1/2) - 478)^(1/2)*6799884i)/(342765462*2^(1/2) - 484704168))*(338*
2^(1/2) - 478)^(1/2)*1i)/16 - (atan((x*(86 - 2^(1/2)*26i)^(1/2)*95758i)/(2
^(1/2)*284622i - 2437188) + (119366*2^(1/2)*x*(86 - 2^(1/2)*26i)^(1/2))/(2
^(1/2)*284622i - 2437188))*(86 - 2^(1/2)*26i)^(1/2)*1i)/16 + (atan((x*(2^(
1/2)*26i + 86)^(1/2)*95758i)/(2^(1/2)*284622i + 2437188) - (119366*2^(1/2)
*x*(2^(1/2)*26i + 86)^(1/2))/(2^(1/2)*284622i + 2437188))*(2^(1/2)*26i + 8
6)^(1/2)*1i)/16
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.63

$$\int \frac{x^8}{4-(1+x^2)^4}\,dx = -\frac{5\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} + \frac{7\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} + \frac{5\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{7\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} + \frac{7\sqrt{\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{16} + \frac{5\sqrt{\sqrt{2}+1}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{8} + \frac{5\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{5\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{7\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{7\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{7\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{32} - \frac{7\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{32} - \frac{5\sqrt{\sqrt{2}-1}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{16} + \frac{5\sqrt{\sqrt{2}-1}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{16} - x$$

input
```
int(x^8/(4-(x^2+1)^4),x)
```

output
```
( - 10*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(s
qrt(sqrt(3) + 1)*sqrt(2))) + 14*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(
3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 10*sqrt(sqrt(3) + 1)
*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)
)) - 14*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(
sqrt(sqrt(3) + 1)*sqrt(2))) + 28*sqrt(sqrt(2) + 1)*sqrt(2)*atan(x/sqrt(sqr
t(2) + 1)) + 40*sqrt(sqrt(2) + 1)*atan(x/sqrt(sqrt(2) + 1)) + 5*sqrt(sqrt(
3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 5*s
qrt(sqrt(3) - 1)*sqrt(6)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)
 + 7*sqrt(sqrt(3) - 1)*sqrt(2)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3
) + x**2) - 7*sqrt(sqrt(3) - 1)*sqrt(2)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x +
sqrt(3) + x**2) + 14*sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(sqrt(2) - 1) +
x) - 14*sqrt(sqrt(2) - 1)*sqrt(2)*log(sqrt(sqrt(2) - 1) + x) - 20*sqrt(sqr
t(2) - 1)*log( - sqrt(sqrt(2) - 1) + x) + 20*sqrt(sqrt(2) - 1)*log(sqrt(sq
rt(2) - 1) + x) - 64*x)/64
```

3.146 $\int \frac{x^6}{4-(1+x^2)^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 209

$$\begin{aligned}\int \frac{x^6}{4-(1+x^2)^4} \, dx = & -\frac{1}{16}\sqrt{-1+9\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ & -\frac{\arctan\left(\sqrt{-1+\sqrt{2}}x\right)}{8\sqrt{2\left(-41+29\sqrt{2}\right)}} \\ & +\frac{1}{16}\sqrt{-1+9\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ & +\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt{2}}x\right)}{8\sqrt{2\left(41+29\sqrt{2}\right)}} \\ & +\frac{1}{16}\sqrt{1+9\sqrt{3}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)\end{aligned}$$

output
```
-1/16*(-1+9*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))
^(1/2))-1/8*arctan((2^(1/2)-1)^(1/2)*x)/(-82+58*2^(1/2))^(1/2)+1/16*(-1+9*
3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/8*
arctanh((1+2^(1/2))^(1/2)*x)/(82+58*2^(1/2))^(1/2)+1/16*(1+9*3^(1/2))^(1/2
)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{4-(1+x^2)^4}\,dx = \frac{\frac{\left(-5i+\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(5i+\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}-\frac{\left(7+5\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}+\frac{\left(-7+5\sqrt{2}\right)\text{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}}}{8\sqrt{2}}$$

input
```
Integrate[x^6/(4 - (1 + x^2)^4),x]
```

output
```
(((-5*I + Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((
5*I + Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]] - ((7 +
5*Sqrt[2])*ArcTan[x/Sqrt[1 + Sqrt[2]]])/Sqrt[1 + Sqrt[2]] + ((-7 + 5*Sqrt[
2])*ArcTanh[x/Sqrt[-1 + Sqrt[2]]])/Sqrt[-1 + Sqrt[2]])/(8*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{4-(x^2+1)^4}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{2-5x^2}{4\left(x^4+2x^2-1\right)}+\frac{x^2+6}{4\left(x^4+2x^2+3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{16}\sqrt{9\sqrt{3}-1}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(41+29\sqrt{2}\right)}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\frac{1}{16}\sqrt{9\sqrt{3}-1}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(29\sqrt{2}-41\right)}\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\frac{1}{32}\sqrt{1+9\sqrt{3}}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{1+9\sqrt{3}}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$

input
```
Int[x^6/(4 - (1 + x^2)^4),x]
```

output
```
-1/16*(Sqrt[-1 + 9*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(
1 + Sqrt[3])]]) - (Sqrt[(41 + 29*Sqrt[2])/2]*ArcTan[x/Sqrt[1 + Sqrt[2]]])/
8 + (Sqrt[-1 + 9*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/16 + (Sqrt[(-41 + 29*Sqrt[2])/2]*ArcTanh[x/Sqrt[-1 + Sqrt[2]
]])/8 - (Sqrt[1 + 9*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2]
)/32 + (Sqrt[1 + 9*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4-4\_Z^2+243\right)}\_R\ln\left(2\_R^3+43\_R+99x\right)\right)}{16}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+164\_Z^2-1\right)}\_R\ln\left(6\_R^3+239\_R\right.\right.}{16}$
default	$-\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+5\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2-x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{64}-\frac{\left(-8\sqrt{3}+\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+5\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{2}\right)\arctan\left(\frac{2x-}{\sqrt{}}\right.}{16\sqrt{2+2\sqrt{3}}}$

input
```
int(x^6/(4-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(2*_R^3+43*_R+99*x),_R=RootOf(4*_Z^4-4*_Z^2+243))+1/16*sum(_
R*ln(6*_R^3+239*_R+29*x),_R=RootOf(4*_Z^4+164*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(148) = 296.

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.43

$$\int \frac{x^6}{4-(1+x^2)^4}\,dx = \frac{1}{16}\sqrt{9\sqrt{3}-1}\arctan\left(\frac{1}{22}\sqrt{9\sqrt{3}+1}\sqrt{9\sqrt{3}-1}\left(\sqrt{3}-1\right)+\frac{1}{11}\left(2\sqrt{3}x-x\right)\sqrt{9\sqrt{3}-1}\right) - \frac{1}{16}\sqrt{9\sqrt{3}-1}\arctan\left(\frac{1}{22}\sqrt{9\sqrt{3}+1}\sqrt{9\sqrt{3}-1}\left(\sqrt{3}-1\right)-\frac{1}{11}\left(2\sqrt{3}x-x\right)\sqrt{9\sqrt{3}-1}\right) + \frac{1}{8}\sqrt{\frac{29}{2}\sqrt{2}+\frac{41}{2}}\arctan\left(\left(7\sqrt{2}x-10\,x\right)\sqrt{\frac{29}{2}\sqrt{2}+\frac{41}{2}}\right) - \frac{1}{32}\sqrt{9\sqrt{3}+1}\log\left(11\,x^2+\left(\sqrt{3}x-5\,x\right)\sqrt{9\sqrt{3}+1}+11\sqrt{3}\right) + \frac{1}{32}\sqrt{9\sqrt{3}+1}\log\left(11\,x^2-\left(\sqrt{3}x-5\,x\right)\sqrt{9\sqrt{3}+1}+11\sqrt{3}\right) + \frac{1}{16}\sqrt{\frac{29}{2}\sqrt{2}-\frac{41}{2}}\log\left(\sqrt{\frac{29}{2}\sqrt{2}-\frac{41}{2}}\left(3\sqrt{2}+4\right)+x\right) - \frac{1}{16}\sqrt{\frac{29}{2}\sqrt{2}-\frac{41}{2}}\log\left(-\sqrt{\frac{29}{2}\sqrt{2}-\frac{41}{2}}\left(3\sqrt{2}+4\right)+x\right)$$

input
```
integrate(x^6/(4-(x^2+1)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(9*sqrt(3) - 1)*arctan(1/22*sqrt(9*sqrt(3) + 1)*sqrt(9*sqrt(3) -
1)*(sqrt(3) - 1) + 1/11*(2*sqrt(3)*x - x)*sqrt(9*sqrt(3) - 1)) - 1/16*sqrt
(9*sqrt(3) - 1)*arctan(1/22*sqrt(9*sqrt(3) + 1)*sqrt(9*sqrt(3) - 1)*(sqrt(
3) - 1) - 1/11*(2*sqrt(3)*x - x)*sqrt(9*sqrt(3) - 1)) + 1/8*sqrt(29/2*sqrt
(2) + 41/2)*arctan((7*sqrt(2)*x - 10*x)*sqrt(29/2*sqrt(2) + 41/2)) - 1/32*
sqrt(9*sqrt(3) + 1)*log(11*x^2 + (sqrt(3)*x - 5*x)*sqrt(9*sqrt(3) + 1) + 1
1*sqrt(3)) + 1/32*sqrt(9*sqrt(3) + 1)*log(11*x^2 - (sqrt(3)*x - 5*x)*sqrt(
9*sqrt(3) + 1) + 11*sqrt(3)) + 1/16*sqrt(29/2*sqrt(2) - 41/2)*log(sqrt(29/
2*sqrt(2) - 41/2)*(3*sqrt(2) + 4) + x) - 1/16*sqrt(29/2*sqrt(2) - 41/2)*lo
g(-sqrt(29/2*sqrt(2) - 41/2)*(3*sqrt(2) + 4) + x)
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.41

$$\int \frac{x^6}{4-(1+x^2)^4}\,dx = -\operatorname{RootSum}\left(262144t^4 - 1024t^2 + 243, \left(t \mapsto t\log\left(-\frac{130739341361152t^7}{155496231} - \frac{20444980707328t^5}{155496231} - \frac{174}{5}\right.\right.\right. - \operatorname{RootSum}\left(262144t^4 + 41984t^2 - 1, \left(t \mapsto t\log\left(-\frac{130739341361152t^7}{155496231} - \frac{20444980707328t^5}{155496231} - \frac{1740}{51}\right.\right.\right.$$

input
```
integrate(x**6/(4-(x**2+1)**4),x)
```

output
```
-RootSum(262144*_t**4 - 1024*_t**2 + 243, Lambda(_t, _t*log(-1307393413611
52*_t**7/155496231 - 20444980707328*_t**5/155496231 - 17400141824*_t**3/51
832077 - 20505938212*_t/155496231 + x))) - RootSum(262144*_t**4 + 41984*_t
**2 - 1, Lambda(_t, _t*log(-130739341361152*_t**7/155496231 - 204449807073
28*_t**5/155496231 - 17400141824*_t**3/51832077 - 20505938212*_t/155496231
 + x)))
```

Maxima [F]

$$\int \frac{x^6}{4-(1+x^2)^4}\,dx = \int -\frac{x^6}{(x^2+1)^4-4}\,dx$$

input
```
integrate(x^6/(4-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate(x^6/((x^2 + 1)^4 - 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(148) = 296$.

Time = 0.47 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.91

$$\int \frac{x^6}{4-(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(4-(x^2+1)^4),x, algorithm="giac")
```

output
```
-1/10368*sqrt(2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*
sqrt(3) + 18) - 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 216*3^(1/4)*sqrt(2)*sqrt
(6*sqrt(3) + 18) - 216*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(
x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/10368*s
qrt(2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) +
18) - 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 216*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3)
 + 18) - 216*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4
)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/20736*sqrt(2)*(18
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 3^(3/4)*sqrt(2)*(-6
*sqrt(3) + 18)^(3/2) - 3^(3/4)*(6*sqrt(3) + 18)^(3/2) - 18*3^(3/4)*sqrt(6*
sqrt(3) + 18)*(sqrt(3) - 3) - 216*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) +
216*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3)
+ 1/2) + sqrt(3)) + 1/20736*sqrt(2)*(18*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) - 3^(3/4)*(6*s
qrt(3) + 18)^(3/2) - 18*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 216*3
^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 216*3^(1/4)*sqrt(6*sqrt(3) + 18))*l
og(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/16*sqrt(58*sq
rt(2) + 82)*arctan(x/sqrt(sqrt(2) + 1)) + 1/32*sqrt(58*sqrt(2) - 82)*lo...
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{4-(1+x^2)^4}\,dx = \frac{\operatorname{atan}\left(\frac{x\sqrt{2-\sqrt{2}\,22\mathrm{i}}\,4202\mathrm{i}}{63624+\sqrt{2}\,9966\mathrm{i}}+\frac{5522\sqrt{2}\,x\sqrt{2-\sqrt{2}\,22\mathrm{i}}}{63624+\sqrt{2}\,9966\mathrm{i}}\right)\sqrt{2-\sqrt{2}\,22\mathrm{i}}\,1\mathrm{i}}{16} - \frac{\operatorname{atan}\left(\frac{x\sqrt{2+\sqrt{2}\,22\mathrm{i}}\,4202\mathrm{i}}{-63624+\sqrt{2}\,9966\mathrm{i}}-\frac{5522\sqrt{2}\,x\sqrt{2+\sqrt{2}\,22\mathrm{i}}}{-63624+\sqrt{2}\,9966\mathrm{i}}\right)\sqrt{2+\sqrt{2}\,22\mathrm{i}}\,1\mathrm{i}}{16} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-58\sqrt{2}-82}\,54462\mathrm{i}}{638754\sqrt{2}+905148}+\frac{\sqrt{2}\,x\sqrt{-58\sqrt{2}-82}\,25752\mathrm{i}}{638754\sqrt{2}+905148}\right)\sqrt{-58\sqrt{2}-82}\,1\mathrm{i}}{16} + \frac{\operatorname{atan}\left(\frac{x\sqrt{58\sqrt{2}-82}\,54462\mathrm{i}}{638754\sqrt{2}-905148}-\frac{\sqrt{2}\,x\sqrt{58\sqrt{2}-82}\,25752\mathrm{i}}{638754\sqrt{2}-905148}\right)\sqrt{58\sqrt{2}-82}\,1\mathrm{i}}{16}$$

input
```
int(-x^6/((x^2 + 1)^4 - 4),x)
```

output
```
(atan((x*(2 - 2^(1/2)*22i)^(1/2)*4202i)/(2^(1/2)*9966i + 63624) + (5522*2^
(1/2)*x*(2 - 2^(1/2)*22i)^(1/2))/(2^(1/2)*9966i + 63624))*(2 - 2^(1/2)*22i
)^(1/2)*1i)/16 - (atan((x*(2^(1/2)*22i + 2)^(1/2)*4202i)/(2^(1/2)*9966i -
63624) - (5522*2^(1/2)*x*(2^(1/2)*22i + 2)^(1/2))/(2^(1/2)*9966i - 63624))
*(2^(1/2)*22i + 2)^(1/2)*1i)/16 - (atan((x*(- 58*2^(1/2) - 82)^(1/2)*54462
i)/(638754*2^(1/2) + 905148) + (2^(1/2)*x*(- 58*2^(1/2) - 82)^(1/2)*25752i
)/(638754*2^(1/2) + 905148))*(- 58*2^(1/2) - 82)^(1/2)*1i)/16 + (atan((x*(
58*2^(1/2) - 82)^(1/2)*54462i)/(638754*2^(1/2) - 905148) - (2^(1/2)*x*(58*
2^(1/2) - 82)^(1/2)*25752i)/(638754*2^(1/2) - 905148))*(58*2^(1/2) - 82)^(
1/2)*1i)/16
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.64

$$\int \frac{x^6}{4-(1+x^2)^4}\,dx = \frac{\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{5\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} + \frac{5\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{3\sqrt{\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{16} - \frac{\sqrt{\sqrt{2}+1}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{4} - \frac{\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{5\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{5\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{3\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{32} + \frac{3\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{32} + \frac{\sqrt{\sqrt{2}-1}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{8} - \frac{\sqrt{\sqrt{2}-1}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{8}$$

input

```
int(x^6/(4-(x^2+1)^4),x)
```

output

```
(2*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(
sqrt(3) + 1)*sqrt(2))) - 10*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) -
 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 2*sqrt(sqrt(3) + 1)*sqrt
(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) +
10*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(
sqrt(3) + 1)*sqrt(2))) - 12*sqrt(sqrt(2) + 1)*sqrt(2)*atan(x/sqrt(sqrt(2)
+ 1)) - 16*sqrt(sqrt(2) + 1)*atan(x/sqrt(sqrt(2) + 1)) - sqrt(sqrt(3) - 1)
*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + sqrt(sqrt(
3) - 1)*sqrt(6)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 5*sqrt
(sqrt(3) - 1)*sqrt(2)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)
 + 5*sqrt(sqrt(3) - 1)*sqrt(2)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) +
 x**2) - 6*sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(sqrt(2) - 1) + x) + 6*sqr
t(sqrt(2) - 1)*sqrt(2)*log(sqrt(sqrt(2) - 1) + x) + 8*sqrt(sqrt(2) - 1)*lo
g( - sqrt(sqrt(2) - 1) + x) - 8*sqrt(sqrt(2) - 1)*log(sqrt(sqrt(2) - 1) +
x))/64
```

3.147 $\int \frac{x^4}{4-(1+x^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 209

$$\begin{aligned}\int \frac{x^4}{4-(1+x^2)^4}\,dx &= \frac{1}{16}\sqrt{5+3\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &+\frac{\arctan\left(\sqrt{-1+\sqrt{2}}x\right)}{8\sqrt{2\left(-7+5\sqrt{2}\right)}} \\ &-\frac{1}{16}\sqrt{5+3\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &+\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt{2}}x\right)}{8\sqrt{2\left(7+5\sqrt{2}\right)}} \\ &+\frac{1}{16}\sqrt{-5+3\sqrt{3}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)\end{aligned}$$

output
```
1/16*(5+3*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(
1/2))+1/8*arctan((2^(1/2)-1)^(1/2)*x)/(-14+10*2^(1/2))^(1/2)-1/16*(5+3*3^(
1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/8*arc
tanh((1+2^(1/2))^(1/2)*x)/(14+10*2^(1/2))^(1/2)+1/16*(-5+3*3^(1/2))^(1/2)*
arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{4-(1+x^2)^4}\,dx = \frac{\frac{(i-2\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}-\frac{(i+2\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}+\frac{(3+2\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}-\frac{(-3+2\sqrt{2})\text{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}}}{8\sqrt{2}}$$

input
```
Integrate[x^4/(4 - (1 + x^2)^4),x]
```

output
```
(((I - 2*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] - ((I
 + 2*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]] + ((3 + 2
*Sqrt[2])*ArcTan[x/Sqrt[1 + Sqrt[2]]])/Sqrt[1 + Sqrt[2]] - ((-3 + 2*Sqrt[2
])*ArcTanh[x/Sqrt[-1 + Sqrt[2]]])/Sqrt[-1 + Sqrt[2]])/(8*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{4-(x^2+1)^4}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{-2x^2-3}{4\left(x^4+2x^2+3\right)}+\frac{2x^2-1}{4\left(x^4+2x^2-1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\sqrt{5+3\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(7+5\sqrt{2}\right)}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)-\frac{1}{16}\sqrt{5+3\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(5\sqrt{2}-7\right)}\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\frac{1}{32}\sqrt{3\sqrt{3}-5}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{3\sqrt{3}-5}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$

input
```
Int[x^4/(4 - (1 + x^2)^4),x]
```

output
```
(Sqrt[5 + 3*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqr
t[3])]])/16 + (Sqrt[(7 + 5*Sqrt[2])/2]*ArcTan[x/Sqrt[1 + Sqrt[2]]])/8 - (S
qrt[5 + 3*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[
3])]])/16 + (Sqrt[(-7 + 5*Sqrt[2])/2]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]])/8 - (
Sqrt[-5 + 3*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (
Sqrt[-5 + 3*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^{4}+20\_Z^{2}+27\right)}\_R\ln\left(-2\_R^{3}-7\_R+3x\right)\right)}{16}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^{4}+28\_Z^{2}-1\right)}\_R\ln\left(2\_R^{3}+17\_R+\right.\right.}{16}$
default	$-\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^{2}-x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{64}-\frac{\left(4\sqrt{3}+\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{2}\right)\arctan\left(\frac{2x-\sqrt{-2}}{\sqrt{2+2}}\right.}{16\sqrt{2+2\sqrt{3}}}$

input
```
int(x^4/(4-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(-2*_R^3-7*_R+3*x),_R=RootOf(4*_Z^4+20*_Z^2+27))+1/16*sum(_R
*ln(2*_R^3+17*_R+5*x),_R=RootOf(4*_Z^4+28*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

$$\int \frac{x^4}{4-(1+x^2)^4}\,dx$$
$$= \frac{1}{16}\sqrt{3\sqrt{3}+5}\arctan\left(\frac{1}{2}\left(2\sqrt{3}x+\sqrt{3\sqrt{3}-5}\left(\sqrt{3}-1\right)-4x\right)\sqrt{3\sqrt{3}+5}\right)$$
$$-\frac{1}{16}\sqrt{3\sqrt{3}+5}\arctan\left(-\frac{1}{2}\left(2\sqrt{3}x-\sqrt{3\sqrt{3}-5}\left(\sqrt{3}-1\right)-4x\right)\sqrt{3\sqrt{3}+5}\right)$$
$$+\frac{1}{8}\sqrt{\frac{5}{2}\sqrt{2}+\frac{7}{2}}\arctan\left(\left(3\sqrt{2}x-4x\right)\sqrt{\frac{5}{2}\sqrt{2}+\frac{7}{2}}\right)$$
$$+\frac{1}{32}\sqrt{3\sqrt{3}-5}\log\left(x^2+\left(\sqrt{3}x+x\right)\sqrt{3\sqrt{3}-5}+\sqrt{3}\right)$$
$$-\frac{1}{32}\sqrt{3\sqrt{3}-5}\log\left(x^2-\left(\sqrt{3}x+x\right)\sqrt{3\sqrt{3}-5}+\sqrt{3}\right)$$
$$+\frac{1}{16}\sqrt{\frac{5}{2}\sqrt{2}-\frac{7}{2}}\log\left(\sqrt{\frac{5}{2}\sqrt{2}-\frac{7}{2}}\left(\sqrt{2}+2\right)+x\right)$$
$$-\frac{1}{16}\sqrt{\frac{5}{2}\sqrt{2}-\frac{7}{2}}\log\left(-\sqrt{\frac{5}{2}\sqrt{2}-\frac{7}{2}}\left(\sqrt{2}+2\right)+x\right)$$

input
```
integrate(x^4/(4-(x^2+1)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(3*sqrt(3) + 5)*arctan(1/2*(2*sqrt(3)*x + sqrt(3*sqrt(3) - 5)*(sq
rt(3) - 1) - 4*x)*sqrt(3*sqrt(3) + 5)) - 1/16*sqrt(3*sqrt(3) + 5)*arctan(-
1/2*(2*sqrt(3)*x - sqrt(3*sqrt(3) - 5)*(sqrt(3) - 1) - 4*x)*sqrt(3*sqrt(3)
 + 5)) + 1/8*sqrt(5/2*sqrt(2) + 7/2)*arctan((3*sqrt(2)*x - 4*x)*sqrt(5/2*s
qrt(2) + 7/2)) + 1/32*sqrt(3*sqrt(3) - 5)*log(x^2 + (sqrt(3)*x + x)*sqrt(3
*sqrt(3) - 5) + sqrt(3)) - 1/32*sqrt(3*sqrt(3) - 5)*log(x^2 - (sqrt(3)*x +
 x)*sqrt(3*sqrt(3) - 5) + sqrt(3)) + 1/16*sqrt(5/2*sqrt(2) - 7/2)*log(sqrt
(5/2*sqrt(2) - 7/2)*(sqrt(2) + 2) + x) - 1/16*sqrt(5/2*sqrt(2) - 7/2)*log(
-sqrt(5/2*sqrt(2) - 7/2)*(sqrt(2) + 2) + x)
```

Sympy [A] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.41

$$\int \frac{x^4}{4-(1+x^2)^4}\,dx =$$
$$-\operatorname{RootSum}\left(262144t^4+5120t^2+27,\left(t\mapsto t\log\left(-\frac{38117834752t^7}{1095}-\frac{565182464t^5}{365}-\frac{19511296t^3}{1095}-\right.\right.\right.$$
$$-\operatorname{RootSum}\left(262144t^4+7168t^2-1,\left(t\mapsto t\log\left(-\frac{38117834752t^7}{1095}-\frac{565182464t^5}{365}-\frac{19511296t^3}{1095}-\right.\right.\right.$$

input
```
integrate(x**4/(4-(x**2+1)**4),x)
```

output
```
-RootSum(262144*_t**4 + 5120*_t**2 + 27, Lambda(_t, _t*log(-38117834752*_t
**7/1095 - 565182464*_t**5/365 - 19511296*_t**3/1095 - 57076*_t/1095 + x))
) - RootSum(262144*_t**4 + 7168*_t**2 - 1, Lambda(_t, _t*log(-38117834752*
_t**7/1095 - 565182464*_t**5/365 - 19511296*_t**3/1095 - 57076*_t/1095 + x
)))
```

Maxima [F]

$$\int \frac{x^4}{4-(1+x^2)^4}\,dx = \int -\frac{x^4}{(x^2+1)^4-4}\,dx$$

input
```
integrate(x^4/(4-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate(x^4/((x^2 + 1)^4 - 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(148) = 296$.

Time = 0.47 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.91

$$\int \frac{x^4}{4-(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(4-(x^2+1)^4),x, algorithm="giac")
```

output
```
1/5184*sqrt(2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sq
rt(3) + 18) - 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 54*3^(1/4)*sqrt(2)*sqrt(6*
sqrt(3) + 18) - 54*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x +
3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/5184*sqrt(2
)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(2)*sqrt(6*sqrt
(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) -
 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 54*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)
 - 54*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(
-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/10368*sqrt(2)*(18*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt(3
) + 18)^(3/2) - 3^(3/4)*(6*sqrt(3) + 18)^(3/2) - 18*3^(3/4)*sqrt(6*sqrt(3)
 + 18)*(sqrt(3) - 3) - 54*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 54*3^(1/
4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) +
sqrt(3)) - 1/10368*sqrt(2)*(18*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(
3) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) - 3^(3/4)*(6*sqrt(3) +
18)^(3/2) - 18*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 54*3^(1/4)*sqr
t(2)*sqrt(-6*sqrt(3) + 18) + 54*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*
3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/16*sqrt(10*sqrt(2) + 14)
*arctan(x/sqrt(sqrt(2) + 1)) + 1/32*sqrt(10*sqrt(2) - 14)*log(abs(x + s...
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{4-(1+x^2)^4}\,dx = \frac{\operatorname{atan}\left(\frac{x\sqrt{-10-\sqrt{2}\,2\mathrm{i}}\,26\mathrm{i}}{-84+\sqrt{2}\,66\mathrm{i}}+\frac{10\sqrt{2}\,x\sqrt{-10-\sqrt{2}\,2\mathrm{i}}}{-84+\sqrt{2}\,66\mathrm{i}}\right)\sqrt{-10-\sqrt{2}\,2\mathrm{i}}\,1\mathrm{i}}{16} + \frac{\operatorname{atan}\left(\frac{x\sqrt{-10\sqrt{2}-14}\,270\mathrm{i}}{570\sqrt{2}+720}-\frac{\sqrt{2}\,x\sqrt{-10\sqrt{2}-14}\,60\mathrm{i}}{570\sqrt{2}+720}\right)\sqrt{-10\sqrt{2}-14}\,1\mathrm{i}}{16} - \frac{\operatorname{atan}\left(\frac{x\sqrt{10\sqrt{2}-14}\,270\mathrm{i}}{570\sqrt{2}-720}+\frac{\sqrt{2}\,x\sqrt{10\sqrt{2}-14}\,60\mathrm{i}}{570\sqrt{2}-720}\right)\sqrt{10\sqrt{2}-14}\,1\mathrm{i}}{16} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{20\,x\sqrt{-5+\sqrt{2}\,1\mathrm{i}}}{84+\sqrt{2}\,66\mathrm{i}}-\frac{\sqrt{2}\,x\sqrt{-5+\sqrt{2}\,1\mathrm{i}}\,26\mathrm{i}}{84+\sqrt{2}\,66\mathrm{i}}\right)\sqrt{-5+\sqrt{2}\,1\mathrm{i}}\,1\mathrm{i}}{16}$$

input
```
int(-x^4/((x^2 + 1)^4 - 4),x)
```

output
```
(atan((x*(- 2^(1/2)*2i - 10)^(1/2)*26i)/(2^(1/2)*66i - 84) + (10*2^(1/2)*x
*(- 2^(1/2)*2i - 10)^(1/2))/(2^(1/2)*66i - 84))*(- 2^(1/2)*2i - 10)^(1/2)*
1i)/16 + (atan((x*(- 10*2^(1/2) - 14)^(1/2)*270i)/(570*2^(1/2) + 720) - (2
^(1/2)*x*(- 10*2^(1/2) - 14)^(1/2)*60i)/(570*2^(1/2) + 720))*(- 10*2^(1/2)
 - 14)^(1/2)*1i)/16 - (atan((x*(10*2^(1/2) - 14)^(1/2)*270i)/(570*2^(1/2)
- 720) + (2^(1/2)*x*(10*2^(1/2) - 14)^(1/2)*60i)/(570*2^(1/2) - 720))*(10*
2^(1/2) - 14)^(1/2)*1i)/16 + (2^(1/2)*atan((20*x*(2^(1/2)*1i - 5)^(1/2))/(
2^(1/2)*66i + 84) - (2^(1/2)*x*(2^(1/2)*1i - 5)^(1/2)*26i)/(2^(1/2)*66i +
84))*(2^(1/2)*1i - 5)^(1/2)*1i)/16
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.64

$$\int \frac{x^4}{4-(1+x^2)^4}\,dx = \frac{\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} + \frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} + \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{16} + \frac{\sqrt{\sqrt{2}+1}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{8} - \frac{\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{32} - \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{32} - \frac{\sqrt{\sqrt{2}-1}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{16} + \frac{\sqrt{\sqrt{2}-1}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{16}$$

input
```
int(x^4/(4-(x^2+1)^4),x)
```

output
```
(2*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(
sqrt(3) + 1)*sqrt(2))) + 2*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) -
1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 2*sqrt(sqrt(3) + 1)*sqrt(
6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 2
*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sq
rt(3) + 1)*sqrt(2))) + 4*sqrt(sqrt(2) + 1)*sqrt(2)*atan(x/sqrt(sqrt(2) + 1
)) + 8*sqrt(sqrt(2) + 1)*atan(x/sqrt(sqrt(2) + 1)) - sqrt(sqrt(3) - 1)*sqr
t(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + sqrt(sqrt(3) -
 1)*sqrt(6)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + sqrt(sqrt(
3) - 1)*sqrt(2)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - sqr
t(sqrt(3) - 1)*sqrt(2)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) +
 2*sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(sqrt(2) - 1) + x) - 2*sqrt(sqrt(2
) - 1)*sqrt(2)*log(sqrt(sqrt(2) - 1) + x) - 4*sqrt(sqrt(2) - 1)*log( - sqr
t(sqrt(2) - 1) + x) + 4*sqrt(sqrt(2) - 1)*log(sqrt(sqrt(2) - 1) + x))/64
```

3.148 $\int \frac{x^2}{4-(1+x^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 201

$$\begin{aligned}\int \frac{x^2}{4-(1+x^2)^4}\,dx = &-\frac{1}{16}\sqrt{-1+\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &-\frac{\arctan\left(\sqrt{-1+\sqrt{2}}x\right)}{8\sqrt{2\left(-1+\sqrt{2}\right)}}\\ &+\frac{1}{16}\sqrt{-1+\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\ &+\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt{2}}x\right)}{8\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)}{8\sqrt{2\left(-1+\sqrt{3}\right)}}\end{aligned}$$

output
```
-1/16*(3^(1/2)-1)^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1
/2))-1/8*arctan((2^(1/2)-1)^(1/2)*x)/(-2+2*2^(1/2))^(1/2)+1/16*(3^(1/2)-1)
^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/8*arctanh(
(1+2^(1/2))^(1/2)*x)/(2+2*2^(1/2))^(1/2)-1/8*arctanh((-2+2*3^(1/2))^(1/2)*
x/(3^(1/2)+x^2))/(-2+2*3^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{4-(1+x^2)^4}\,dx = \frac{\frac{(i+\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{(-i+\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}-\sqrt{1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+}}\right.}{8\sqrt{2}}$$

input
```
Integrate[x^2/(4 - (1 + x^2)^4),x]
```

output
```
(((I + Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((-I
+ Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]] - Sqrt[1 + S
qrt[2]]*ArcTan[x/Sqrt[1 + Sqrt[2]]] + Sqrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1
 + Sqrt[2]]])/(8*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{4-(x^2+1)^4}\,dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{x^2}{4(x^4+2x^2+3)}-\frac{x^2}{4(x^4+2x^2-1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{16}\sqrt{\sqrt{3}-1}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\frac{1}{16}\sqrt{\sqrt{3}-1}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\frac{\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{16\sqrt{2\left(\sqrt{3}-1\right)}}-\frac{\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{16\sqrt{2\left(\sqrt{3}-1\right)}}$$

input
```
Int[x^2/(4 - (1 + x^2)^4),x]
```

output
```
-1/16*(Sqrt[-1 + Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
+ Sqrt[3])]]) - (Sqrt[(1 + Sqrt[2])/2]*ArcTan[x/Sqrt[1 + Sqrt[2]]])/8 + (S
qrt[-1 + Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/16 + (Sqrt[(-1 + Sqrt[2])/2]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]])/8 + Log[
Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2]/(16*Sqrt[2*(-1 + Sqrt[3])]) - Lo
g[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2]/(16*Sqrt[2*(-1 + Sqrt[3])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4+4\_Z^2-1\right)}\_R\ln\left(2\_R^3+\_R+x\right)\right)}{16}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-4\_Z^2+3\right)}\_R\ln\left(2\_R^3-\_R+x\right)\right)}{16}$
default	$-\frac{\sqrt{-2+2\sqrt{3}}\left(1+\sqrt{3}\right)\left(-\frac{\ln\left(x^2-x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{2}-\frac{\sqrt{-2+2\sqrt{3}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{\sqrt{2+2\sqrt{3}}}\right)}{32}-\frac{\sqrt{-2+2\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{\ln\left(x^2+x\sqrt{-2+\dots}\right.}{2}\right.}{}$

input
```
int(x^2/(4-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(2*_R^3+_R+x),_R=RootOf(4*_Z^4+4*_Z^2-1))+1/16*sum(_R*ln(2*_
R^3-_R+x),_R=RootOf(4*_Z^4-4*_Z^2+3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{x^2}{4-(1+x^2)^4}\,dx =\ & \frac{1}{16}\sqrt{\sqrt{3}-1}\arctan\left(\frac{1}{2}\sqrt{\sqrt{3}+1}\left(\sqrt{3}-1\right)^{\frac{3}{2}}+x\sqrt{\sqrt{3}-1}\right)\\ & -\frac{1}{16}\sqrt{\sqrt{3}-1}\arctan\left(\frac{1}{2}\sqrt{\sqrt{3}+1}\left(\sqrt{3}-1\right)^{\frac{3}{2}}-x\sqrt{\sqrt{3}-1}\right)\\ & +\frac{1}{8}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(\left(\sqrt{2}x-2\,x\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right)\\ & -\frac{1}{32}\sqrt{\sqrt{3}+1}\log\left(x^2+\left(\sqrt{3}x-x\right)\sqrt{\sqrt{3}+1}+\sqrt{3}\right)\\ & +\frac{1}{32}\sqrt{\sqrt{3}+1}\log\left(x^2-\left(\sqrt{3}x-x\right)\sqrt{\sqrt{3}+1}+\sqrt{3}\right)\\ & +\frac{1}{16}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(x+\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\right)\\ & -\frac{1}{16}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(x-\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\right)\end{aligned}$$

input
```
integrate(x^2/(4-(x^2+1)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(sqrt(3) - 1)*arctan(1/2*sqrt(sqrt(3) + 1)*(sqrt(3) - 1)^(3/2) +
x*sqrt(sqrt(3) - 1)) - 1/16*sqrt(sqrt(3) - 1)*arctan(1/2*sqrt(sqrt(3) + 1)
*(sqrt(3) - 1)^(3/2) - x*sqrt(sqrt(3) - 1)) + 1/8*sqrt(1/2*sqrt(2) + 1/2)*
arctan((sqrt(2)*x - 2*x)*sqrt(1/2*sqrt(2) + 1/2)) - 1/32*sqrt(sqrt(3) + 1)
*log(x^2 + (sqrt(3)*x - x)*sqrt(sqrt(3) + 1) + sqrt(3)) + 1/32*sqrt(sqrt(3
) + 1)*log(x^2 - (sqrt(3)*x - x)*sqrt(sqrt(3) + 1) + sqrt(3)) + 1/16*sqrt(
1/2*sqrt(2) - 1/2)*log(x + sqrt(2)*sqrt(1/2*sqrt(2) - 1/2)) - 1/16*sqrt(1/
2*sqrt(2) - 1/2)*log(x - sqrt(2)*sqrt(1/2*sqrt(2) - 1/2))
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.35

$$\int \frac{x^2}{4-(1+x^2)^4}\,dx = -\operatorname{RootSum}\left(262144t^4-1024t^2+3,\left(t\mapsto t\log\left(-536870912t^7-1048576t^5-2048t^3-20t+x\right)\right)\right) - \operatorname{RootSum}\left(262144t^4+1024t^2-1,\left(t\mapsto t\log\left(-536870912t^7-1048576t^5-2048t^3-20t+x\right)\right)\right)$$

input
```
integrate(x**2/(4-(x**2+1)**4),x)
```

output
```
-RootSum(262144*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(-536870912*_t**7
 - 1048576*_t**5 - 2048*_t**3 - 20*_t + x))) - RootSum(262144*_t**4 + 1024
*_t**2 - 1, Lambda(_t, _t*log(-536870912*_t**7 - 1048576*_t**5 - 2048*_t**
3 - 20*_t + x)))
```

Maxima [F]

$$\int \frac{x^2}{4-(1+x^2)^4}\,dx = \int -\frac{x^2}{(x^2+1)^4-4}\,dx$$

input
```
integrate(x^2/(4-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((x^2 + 1)^4 - 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(144) = 288$.

Time = 0.47 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.41

$$\int \frac{x^2}{4-(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(4-(x^2+1)^4),x, algorithm="giac")
```

output
```
-1/10368*sqrt(2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*
sqrt(3) + 18) - 3^(3/4)*(-6*sqrt(3) + 18)^(3/2))*arctan(1/3*3^(3/4)*(x + 3
^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/10368*sqrt(2
)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(2)*sqrt(6*sqrt
(3) + 18)*(sqrt(3) - 3) + 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) -
 3^(3/4)*(-6*sqrt(3) + 18)^(3/2))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/
6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/20736*sqrt(2)*(18*3^(3/4)*s
qrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt(3) +
 18)^(3/2) - 3^(3/4)*(6*sqrt(3) + 18)^(3/2) - 18*3^(3/4)*sqrt(6*sqrt(3) +
18)*(sqrt(3) - 3))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3
)) + 1/20736*sqrt(2)*(18*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 1
8) - 3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) - 3^(3/4)*(6*sqrt(3) + 18)^(3
/2) - 18*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3))*log(x^2 - 2*3^(1/4)*x
*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/16*sqrt(2*sqrt(2) + 2)*arctan(x/s
qrt(sqrt(2) + 1)) + 1/32*sqrt(2*sqrt(2) - 2)*log(abs(x + sqrt(sqrt(2) - 1)
)) - 1/32*sqrt(2*sqrt(2) - 2)*log(abs(x - sqrt(sqrt(2) - 1)))
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{4-(1+x^2)^4}\,dx = \frac{\operatorname{atan}\left(\frac{x\sqrt{2-\sqrt{2}\,2i}\,1i}{3}+\frac{\sqrt{2}\,x\sqrt{2-\sqrt{2}\,2i}}{6}\right)\sqrt{2-\sqrt{2}\,2i}\,1i}{16} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{x\sqrt{1+\sqrt{2}\,1i}}{3}-\frac{\sqrt{2}\,x\sqrt{1+\sqrt{2}\,1i}\,1i}{3}\right)\sqrt{1+\sqrt{2}\,1i}\,1i}{16} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{6x\sqrt{2-2\sqrt{2}}}{6\sqrt{2}-12}\right)\sqrt{\sqrt{2}-1}\,1i}{16} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x\sqrt{-\sqrt{2}-1}\,6i}{6\sqrt{2}+12}\right)\sqrt{-\sqrt{2}-1}\,1i}{16}$$

input
```
int(-x^2/((x^2 + 1)^4 - 4),x)
```

output
```
(atan((x*(2 - 2^(1/2)*2i)^(1/2)*1i)/3 + (2^(1/2)*x*(2 - 2^(1/2)*2i)^(1/2))
/6)*(2 - 2^(1/2)*2i)^(1/2)*1i)/16 - (2^(1/2)*atan((x*(2^(1/2)*1i + 1)^(1/2
))/3 - (2^(1/2)*x*(2^(1/2)*1i + 1)^(1/2)*1i)/3)*(2^(1/2)*1i + 1)^(1/2)*1i)
/16 + (2^(1/2)*atan((6*x*(2 - 2*2^(1/2))^(1/2))/(6*2^(1/2) - 12))*(2^(1/2)
 - 1)^(1/2)*1i)/16 - (2^(1/2)*atan((2^(1/2)*x*(- 2^(1/2) - 1)^(1/2)*6i)/(6
*2^(1/2) + 12))*(- 2^(1/2) - 1)^(1/2)*1i)/16
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{4-(1+x^2)^4}\,dx = \frac{\sqrt{2}\left(-2\sqrt{\sqrt{3}+1}\,\sqrt{3}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)+2\sqrt{\sqrt{3}+1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)+2\sqrt{\sqrt{3}+1}\,\sqrt{3}\,atan\left(\cdots\right.\right.}{}$$

input
```
int(x^2/(4-(x^2+1)^4),x)
```

output

```
(sqrt(2)*( - 2*sqrt(sqrt(3) + 1)*sqrt(3)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 2*sqrt(sqrt(3) + 1)*atan((sqrt(sqrt(3
) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 2*sqrt(sqrt(3) + 1)*s
qrt(3)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))
 - 2*sqrt(sqrt(3) + 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3
) + 1)*sqrt(2))) - 4*sqrt(sqrt(2) + 1)*atan(x/sqrt(sqrt(2) + 1)) + sqrt(sq
rt(3) - 1)*sqrt(3)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) -
sqrt(sqrt(3) - 1)*sqrt(3)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2
) + sqrt(sqrt(3) - 1)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)
 - sqrt(sqrt(3) - 1)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 2
*sqrt(sqrt(2) - 1)*log( - sqrt(sqrt(2) - 1) + x) + 2*sqrt(sqrt(2) - 1)*log
(sqrt(sqrt(2) - 1) + x)))/64
```

3.149 $\int \frac{1}{4-(1+x^2)^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 209

$$\int \frac{1}{4-(1+x^2)^4}\,dx = -\frac{1}{16}\sqrt{\frac{1}{3}\left(-1+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{1}{16}\sqrt{\frac{1}{3}\left(-1+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{8\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)}{8\sqrt{6\left(-1+\sqrt{3}\right)}}$$

output

```
-1/48*(-3+3*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))
^(1/2))+1/8*arctan(x/(1+2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)+1/48*(-3+3*3^(
1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/8*arc
tanh(x/(2^(1/2)-1)^(1/2))/(-2+2*2^(1/2))^(1/2)+1/8*arctanh((-2+2*3^(1/2))^
(1/2)*x/(3^(1/2)+x^2))/(-6+6*3^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.67

$$\int \frac{1}{4-(1+x^2)^4}\,dx = \frac{-i\sqrt{1+i\sqrt{2}}\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)+i\sqrt{1-i\sqrt{2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)+\sqrt{3\left(-1+\sqrt{2}\right)}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{3}}{8\sqrt{6}}$$

input
```
Integrate[(4 - (1 + x^2)^4)^(-1),x]
```

output
```
((-I)*Sqrt[1 + I*Sqrt[2]]*ArcTan[x/Sqrt[1 - I*Sqrt[2]]] + I*Sqrt[1 - I*Sqr
t[2]]*ArcTan[x/Sqrt[1 + I*Sqrt[2]]] + Sqrt[3*(-1 + Sqrt[2])]*ArcTan[x/Sqrt
[1 + Sqrt[2]]] + Sqrt[3*(1 + Sqrt[2])]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]])/(8*S
qrt[6])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-(x^2+1)^4}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{1}{4(x^4+2x^2+3)}-\frac{1}{4(x^4+2x^2-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{16}\sqrt{\frac{1}{3}\left(\sqrt{3}-1\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{1}{16}\sqrt{\frac{1}{3}\left(\sqrt{3}-1\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{16\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{16\sqrt{6\left(\sqrt{3}-1\right)}}$$

input
```
Int[(4 - (1 + x^2)^4)^(-1),x]
```

output
```
-1/16*(Sqrt[(-1 + Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2
*(1 + Sqrt[3])]]) + ArcTan[x/Sqrt[1 + Sqrt[2]]]/(8*Sqrt[2*(1 + Sqrt[2])])
+ (Sqrt[(-1 + Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/16 + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(8*Sqrt[2*(-1 + Sqrt[2])]
) - Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2]/(16*Sqrt[6*(-1 + Sqrt[3]
)]) + Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2]/(16*Sqrt[6*(-1 + Sqrt[
3])])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4-4\_Z^2-1\right)}\_R\ln\left(-2\_R^3+3\_R+x\right)\right)}{16}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(12\_Z^4-4\_Z^2+1\right)}\_R\ln\left(6\_R^3+\_R+x\right)\right)}{16}$
default	$-\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+3\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2-x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{192}-\frac{\left(-4\sqrt{3}+\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+3\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{2}\right)\arctan\left(\frac{2x-}{\sqrt{}}\right.}{48\sqrt{2+2\sqrt{3}}}$

input
```
int(1/(4-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(-2*_R^3+3*_R+x),_R=RootOf(4*_Z^4-4*_Z^2-1))+1/16*sum(_R*ln(
6*_R^3+_R+x),_R=RootOf(12*_Z^4-4*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.28

$$\int \frac{1}{4-(1+x^2)^4}\,dx = \frac{1}{16}\sqrt{\frac{1}{3}\sqrt{3}-\frac{1}{3}}\arctan\left(\sqrt{3}x\sqrt{\frac{1}{3}\sqrt{3}-\frac{1}{3}} + \frac{3}{2}\left(\sqrt{3}-1\right)\sqrt{\frac{1}{3}\sqrt{3}+\frac{1}{3}}\sqrt{\frac{1}{3}\sqrt{3}-\frac{1}{3}}\right) - \frac{1}{16}\sqrt{\frac{1}{3}\sqrt{3}-\frac{1}{3}}\arctan\left(-\sqrt{3}x\sqrt{\frac{1}{3}\sqrt{3}-\frac{1}{3}} + \frac{3}{2}\left(\sqrt{3}-1\right)\sqrt{\frac{1}{3}\sqrt{3}+\frac{1}{3}}\sqrt{\frac{1}{3}\sqrt{3}-\frac{1}{3}}\right) + \frac{1}{8}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(\sqrt{2}x\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\right) - \frac{1}{32}\sqrt{\frac{1}{3}\sqrt{3}+\frac{1}{3}}\log\left(x^2+\left(\sqrt{3}x-3\,x\right)\sqrt{\frac{1}{3}\sqrt{3}+\frac{1}{3}}+\sqrt{3}\right) + \frac{1}{32}\sqrt{\frac{1}{3}\sqrt{3}+\frac{1}{3}}\log\left(x^2-\left(\sqrt{3}x-3\,x\right)\sqrt{\frac{1}{3}\sqrt{3}+\frac{1}{3}}+\sqrt{3}\right) - \frac{1}{16}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(\left(\sqrt{2}-2\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+x\right) + \frac{1}{16}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(-\left(\sqrt{2}-2\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+x\right)$$

input
```
integrate(1/(4-(x^2+1)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(1/3*sqrt(3) - 1/3)*arctan(sqrt(3)*x*sqrt(1/3*sqrt(3) - 1/3) + 3/
2*(sqrt(3) - 1)*sqrt(1/3*sqrt(3) + 1/3)*sqrt(1/3*sqrt(3) - 1/3)) - 1/16*sq
rt(1/3*sqrt(3) - 1/3)*arctan(-sqrt(3)*x*sqrt(1/3*sqrt(3) - 1/3) + 3/2*(sqr
t(3) - 1)*sqrt(1/3*sqrt(3) + 1/3)*sqrt(1/3*sqrt(3) - 1/3)) + 1/8*sqrt(1/2*
sqrt(2) - 1/2)*arctan(sqrt(2)*x*sqrt(1/2*sqrt(2) - 1/2)) - 1/32*sqrt(1/3*s
qrt(3) + 1/3)*log(x^2 + (sqrt(3)*x - 3*x)*sqrt(1/3*sqrt(3) + 1/3) + sqrt(3
)) + 1/32*sqrt(1/3*sqrt(3) + 1/3)*log(x^2 - (sqrt(3)*x - 3*x)*sqrt(1/3*sqr
t(3) + 1/3) + sqrt(3)) - 1/16*sqrt(1/2*sqrt(2) + 1/2)*log((sqrt(2) - 2)*sq
rt(1/2*sqrt(2) + 1/2) + x) + 1/16*sqrt(1/2*sqrt(2) + 1/2)*log(-(sqrt(2) -
2)*sqrt(1/2*sqrt(2) + 1/2) + x)
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.34

$$\int \frac{1}{4-(1+x^2)^4}\,dx = -\operatorname{RootSum}\left(262144t^4-1024t^2-1,\left(t\mapsto t\log\left(4831838208t^7-22020096t^5+2048t^3-36t+x\right)\right)\right) - \operatorname{RootSum}\left(786432t^4-1024t^2+1,\left(t\mapsto t\log\left(4831838208t^7-22020096t^5+2048t^3-36t+x\right)\right)\right)$$

input
```
integrate(1/(4-(x**2+1)**4),x)
```

output
```
-RootSum(262144*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log(4831838208*_t**7
 - 22020096*_t**5 + 2048*_t**3 - 36*_t + x))) - RootSum(786432*_t**4 - 102
4*_t**2 + 1, Lambda(_t, _t*log(4831838208*_t**7 - 22020096*_t**5 + 2048*_t
**3 - 36*_t + x)))
```

Maxima [F]

$$\int \frac{1}{4-(1+x^2)^4}\,dx = \int -\frac{1}{(x^2+1)^4-4}\,dx$$

input
```
integrate(1/(4-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate(1/((x^2 + 1)^4 - 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(148) = 296.

Time = 0.44 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.52

$$\int \frac{1}{4-(1+x^2)^4}\,dx$$
$$= \frac{1}{288}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}+3^{\frac{1}{4}}\sqrt{-6\sqrt{3}+18}\right)\arctan\left(\frac{3^{\frac{3}{4}}\left(x+3^{\frac{1}{4}}\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}\right)}{3\sqrt{\frac{1}{6}\sqrt{3}+\frac{1}{2}}}\right)$$
$$+\frac{1}{288}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}+3^{\frac{1}{4}}\sqrt{-6\sqrt{3}+18}\right)\arctan\left(\frac{3^{\frac{3}{4}}\left(x-3^{\frac{1}{4}}\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}\right)}{3\sqrt{\frac{1}{6}\sqrt{3}+\frac{1}{2}}}\right)$$
$$+\frac{1}{576}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}\sqrt{-6\sqrt{3}+18}-3^{\frac{1}{4}}\sqrt{6\sqrt{3}+18}\right)\log\left(x^2+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{1}{576}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}\sqrt{-6\sqrt{3}+18}-3^{\frac{1}{4}}\sqrt{6\sqrt{3}+18}\right)\log\left(x^2-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)+\frac{1}{16}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)$$
$$+\frac{1}{32}\sqrt{2\sqrt{2}+2}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right)-\frac{1}{32}\sqrt{2\sqrt{2}+2}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

input
```
integrate(1/(4-(x^2+1)^4),x, algorithm="giac")
```

output
```
1/288*sqrt(2)*(3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 3^(1/4)*sqrt(-6*sqrt
(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(
1/6*sqrt(3) + 1/2)) + 1/288*sqrt(2)*(3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)
+ 3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6
*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/576*sqrt(2)*(3^(1/4)*sqrt(2)
*sqrt(-6*sqrt(3) + 18) - 3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)
*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/576*sqrt(2)*(3^(1/4)*sqrt(2)*sq
rt(-6*sqrt(3) + 18) - 3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*
sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/16*sqrt(2*sqrt(2) - 2)*arctan(x/sq
rt(sqrt(2) + 1)) + 1/32*sqrt(2*sqrt(2) + 2)*log(abs(x + sqrt(sqrt(2) - 1))
) - 1/32*sqrt(2*sqrt(2) + 2)*log(abs(x - sqrt(sqrt(2) - 1)))
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

$$\int \frac{1}{4-(1+x^2)^4}\,dx = \frac{\mathrm{atan}\left(\frac{x\sqrt{6-\sqrt{2}\,6\mathrm{i}}\,10\mathrm{i}}{81\left(-\frac{4}{27}+\frac{\sqrt{2}\,10\mathrm{i}}{27}\right)}-\frac{2\sqrt{2}\,x\sqrt{6-\sqrt{2}\,6\mathrm{i}}}{81\left(-\frac{4}{27}+\frac{\sqrt{2}\,10\mathrm{i}}{27}\right)}\right)\sqrt{6-\sqrt{2}\,6\mathrm{i}}\,1\mathrm{i}}{48} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\sqrt{1-\sqrt{2}}\,8\mathrm{i}}{6\sqrt{2}-8}-\frac{\sqrt{2}\,x\sqrt{1-\sqrt{2}}\,6\mathrm{i}}{6\sqrt{2}-8}\right)\sqrt{1-\sqrt{2}}\,1\mathrm{i}}{16} - \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\sqrt{1+\sqrt{2}\,1\mathrm{i}}\,10\mathrm{i}}{81\left(\frac{4}{27}+\frac{\sqrt{2}\,10\mathrm{i}}{27}\right)}+\frac{2\sqrt{2}\sqrt{6}\,x\sqrt{1+\sqrt{2}\,1\mathrm{i}}}{81\left(\frac{4}{27}+\frac{\sqrt{2}\,10\mathrm{i}}{27}\right)}\right)\sqrt{1+\sqrt{2}\,1\mathrm{i}}\,1\mathrm{i}}{48} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\sqrt{\sqrt{2}+1}\,8\mathrm{i}}{6\sqrt{2}+8}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+1}\,6\mathrm{i}}{6\sqrt{2}+8}\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{16}$$

input
```
int(-1/((x^2 + 1)^4 - 4),x)
```

output
```
(atan((x*(6 - 2^(1/2)*6i)^(1/2)*10i)/(81*((2^(1/2)*10i)/27 - 4/27)) - (2*2
^(1/2)*x*(6 - 2^(1/2)*6i)^(1/2))/(81*((2^(1/2)*10i)/27 - 4/27)))*(6 - 2^(1
/2)*6i)^(1/2)*1i)/48 - (2^(1/2)*atan((x*(1 - 2^(1/2))^(1/2)*8i)/(6*2^(1/2)
 - 8) - (2^(1/2)*x*(1 - 2^(1/2))^(1/2)*6i)/(6*2^(1/2) - 8))*(1 - 2^(1/2))^
(1/2)*1i)/16 - (6^(1/2)*atan((6^(1/2)*x*(2^(1/2)*1i + 1)^(1/2)*10i)/(81*((
2^(1/2)*10i)/27 + 4/27)) + (2*2^(1/2)*6^(1/2)*x*(2^(1/2)*1i + 1)^(1/2))/(8
1*((2^(1/2)*10i)/27 + 4/27)))*(2^(1/2)*1i + 1)^(1/2)*1i)/48 - (2^(1/2)*ata
n((x*(2^(1/2) + 1)^(1/2)*8i)/(6*2^(1/2) + 8) + (2^(1/2)*x*(2^(1/2) + 1)^(1
/2)*6i)/(6*2^(1/2) + 8))*(2^(1/2) + 1)^(1/2)*1i)/16
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.64

$$\int \frac{1}{4-(1+x^2)^4}\,dx = \frac{\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{96} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{96} + \frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)}{32} - \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{16} + \frac{\sqrt{\sqrt{2}+1}\,atan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)}{8} - \frac{\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{192} + \frac{\sqrt{\sqrt{3}-1}\,\sqrt{6}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{192} - \frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} + \frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{3}-1}\,\sqrt{2}\,x+\sqrt{3}+x^2\right)}{64} - \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{32} + \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{32} - \frac{\sqrt{\sqrt{2}-1}\log\left(-\sqrt{\sqrt{2}-1}+x\right)}{16} + \frac{\sqrt{\sqrt{2}-1}\log\left(\sqrt{\sqrt{2}-1}+x\right)}{16}$$

input
```
int(1/(4-(x^2+1)^4),x)
```

output
```
(2*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(
sqrt(3) + 1)*sqrt(2))) - 6*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) -
1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 2*sqrt(sqrt(3) + 1)*sqrt(
6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 6
*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sq
rt(3) + 1)*sqrt(2))) - 12*sqrt(sqrt(2) + 1)*sqrt(2)*atan(x/sqrt(sqrt(2) +
1)) + 24*sqrt(sqrt(2) + 1)*atan(x/sqrt(sqrt(2) + 1)) - sqrt(sqrt(3) - 1)*s
qrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + sqrt(sqrt(3)
 - 1)*sqrt(6)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 3*sqrt(s
qrt(3) - 1)*sqrt(2)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) +
 3*sqrt(sqrt(3) - 1)*sqrt(2)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x
**2) - 6*sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(sqrt(2) - 1) + x) + 6*sqrt(
sqrt(2) - 1)*sqrt(2)*log(sqrt(sqrt(2) - 1) + x) - 12*sqrt(sqrt(2) - 1)*log
( - sqrt(sqrt(2) - 1) + x) + 12*sqrt(sqrt(2) - 1)*log(sqrt(sqrt(2) - 1) +
x))/192
```

3.150 $\int \frac{1}{x^2\left(4-\left(1+x^2\right)^4\right)} dx$

Optimal result

Integrand size = 17, antiderivative size = 228

$$\begin{aligned}\int \frac{1}{x^2\left(4-\left(1+x^2\right)^4\right)} dx = & -\frac{1}{3x}+\frac{1}{48}\sqrt{\frac{1}{3}\left(5+3\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ & -\frac{\arctan\left(\sqrt{-1+\sqrt{2}}x\right)}{8\sqrt{2\left(7+5\sqrt{2}\right)}} \\ & -\frac{1}{48}\sqrt{\frac{1}{3}\left(5+3\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ & +\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt{2}}x\right)}{8\sqrt{2\left(-7+5\sqrt{2}\right)}} \\ & -\frac{1}{48}\sqrt{\frac{1}{3}\left(-5+3\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)\end{aligned}$$

output
```
-1/3/x+1/144*(15+9*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3
^(1/2))^(1/2))-1/8*arctan((2^(1/2)-1)^(1/2)*x)/(14+10*2^(1/2))^(1/2)-1/144
*(15+9*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2
))+1/8*arctanh((1+2^(1/2))^(1/2)*x)/(-14+10*2^(1/2))^(1/2)-1/144*(-15+9*3^
(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2\left(4-(1+x^2)^4\right)}\,dx = -\frac{1}{3x}-\frac{\left(-i+\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{24\sqrt{2-2i\sqrt{2}}}-\frac{\left(i+\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{24\sqrt{2+2i\sqrt{2}}}-\frac{\left(-1+\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\left(1+\sqrt{2}\right)\text{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{8\sqrt{2\left(-1+\sqrt{2}\right)}}$$

input
```
Integrate[1/(x^2*(4 - (1 + x^2)^4)),x]
```

output
```
-1/3*1/x - ((-I + Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(24*Sqrt[2 - (2*
I)*Sqrt[2]]) - ((I + Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(24*Sqrt[2 +
(2*I)*Sqrt[2]]) - ((-1 + Sqrt[2])*ArcTan[x/Sqrt[1 + Sqrt[2]]])/(8*Sqrt[2*(
1 + Sqrt[2])]) + ((1 + Sqrt[2])*ArcTanh[x/Sqrt[-1 + Sqrt[2]]])/(8*Sqrt[2*(
-1 + Sqrt[2])])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(4-\left(x^2+1\right)^4\right)}dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{1}{3x^2}+\frac{-x^2-2}{4\left(x^4+2x^2-1\right)}+\frac{-x^2-2}{12\left(x^4+2x^2+3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{48}\sqrt{\frac{1}{3}\left(5+3\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{\left(\sqrt{2}-1\right)^{3/2}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{2}}-\frac{1}{48}\sqrt{\frac{1}{3}\left(5+3\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{\left(1+\sqrt{2}\right)^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{2}}+\frac{1}{96}\sqrt{\frac{1}{3}\left(3\sqrt{3}-5\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{3}\left(3\sqrt{3}-5\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{3x}$$

input
```
Int[1/(x^2*(4 - (1 + x^2)^4)),x]
```

output
```
-1/3*1/x + (Sqrt[(5 + 3*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/
Sqrt[2*(1 + Sqrt[3])]])/48 - ((-1 + Sqrt[2])^(3/2)*ArcTan[x/Sqrt[1 + Sqrt[
2]]])/(8*Sqrt[2]) - (Sqrt[(5 + 3*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])
] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + ((1 + Sqrt[2])^(3/2)*ArcTanh[x/Sqrt[
-1 + Sqrt[2]]])/(8*Sqrt[2]) + (Sqrt[(-5 + 3*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/96 - (Sqrt[(-5 + 3*Sqrt[3])/3]*Log[Sqrt[3] +
Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.31

method	result
risch	$-\frac{1}{3x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(12\_Z^4+20\_Z^2+9\right)}\_R\ln\left(6\_R^3+\_R+3x\right)\right)}{48}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-28\_Z^2-1\right)}\_R\ln\left(6\_R^3-41\right.\right.}{16}$
default	$-\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-3\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2-x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{576}-\frac{\left(8\sqrt{3}+\frac{\left(\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-3\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{2}\right)\arctan\left(\frac{2x-\sqrt{-}}{\sqrt{2-}}\right.}{144\sqrt{2+2\sqrt{3}}}$

input
```
int(1/x^2/(4-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x+1/48*sum(_R*ln(6*_R^3+_R+3*x),_R=RootOf(12*_Z^4+20*_Z^2+9))+1/16*su
m(_R*ln(6*_R^3-41*_R+5*x),_R=RootOf(4*_Z^4-28*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2\left(4-(1+x^2)^4\right)}\,dx =$$

$$-\frac{2\,x\sqrt{\sqrt{3}+\frac{5}{3}}\arctan\left(\frac{1}{2}\left(4\sqrt{3}x+3\left(\sqrt{3}-1\right)\sqrt{\sqrt{3}-\frac{5}{3}}-6\,x\right)\sqrt{\sqrt{3}+\frac{5}{3}}\right)-2\,x\sqrt{\sqrt{3}+\frac{5}{3}}\arctan\left(-\right.}{}$$

input
```
integrate(1/x^2/(4-(x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/96*(2*x*sqrt(sqrt(3) + 5/3)*arctan(1/2*(4*sqrt(3)*x + 3*(sqrt(3) - 1)*s
qrt(sqrt(3) - 5/3) - 6*x)*sqrt(sqrt(3) + 5/3)) - 2*x*sqrt(sqrt(3) + 5/3)*a
rctan(-1/2*(4*sqrt(3)*x - 3*(sqrt(3) - 1)*sqrt(sqrt(3) - 5/3) - 6*x)*sqrt(
sqrt(3) + 5/3)) + 12*x*sqrt(5/2*sqrt(2) - 7/2)*arctan((sqrt(2)*x + 2*x)*sq
rt(5/2*sqrt(2) - 7/2)) + x*sqrt(sqrt(3) - 5/3)*log(x^2 + (sqrt(3)*x + 3*x)
*sqrt(sqrt(3) - 5/3) + sqrt(3)) - x*sqrt(sqrt(3) - 5/3)*log(x^2 - (sqrt(3)
*x + 3*x)*sqrt(sqrt(3) - 5/3) + sqrt(3)) - 6*x*sqrt(5/2*sqrt(2) + 7/2)*log
((3*sqrt(2) - 4)*sqrt(5/2*sqrt(2) + 7/2) + x) + 6*x*sqrt(5/2*sqrt(2) + 7/2
)*log(-(3*sqrt(2) - 4)*sqrt(5/2*sqrt(2) + 7/2) + x) + 32)/x
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^2\left(4-(1+x^2)^4\right)}\,dx =$$
$$-\operatorname{RootSum}\left(262144t^4-7168t^2-1,\left(t\mapsto t\log\left(-\frac{25642565369856t^7}{365}+\frac{689041309696t^5}{365}+\frac{42750156\ldots}{365}\right.\right.\right.$$
$$-\operatorname{RootSum}\left(7077888t^4+5120t^2+1,\left(t\mapsto t\log\left(-\frac{25642565369856t^7}{365}+\frac{689041309696t^5}{365}+\frac{4275015\ldots}{365}\right.\right.\right.$$
$$-\frac{1}{3x}$$

input
```
integrate(1/x**2/(4-(x**2+1)**4),x)
```

output
```
-RootSum(262144*_t**4 - 7168*_t**2 - 1, Lambda(_t, _t*log(-25642565369856*
_t**7/365 + 689041309696*_t**5/365 + 427501568*_t**3/365 + 94132*_t/365 +
x))) - RootSum(7077888*_t**4 + 5120*_t**2 + 1, Lambda(_t, _t*log(-25642565
369856*_t**7/365 + 689041309696*_t**5/365 + 427501568*_t**3/365 + 94132*_t
/365 + x))) - 1/(3*x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(4-(1+x^2)^4\right)}\,dx = \int -\frac{1}{\left((x^2+1)^4-4\right)x^2}\,dx$$

input
```
integrate(1/x^2/(4-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-1/3/x - 1/12*integrate((x^2 + 2)/(x^4 + 2*x^2 + 3), x) - 1/4*integrate((x
^2 + 2)/(x^4 + 2*x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(153) = 306$.

Time = 0.48 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.68

$$\int \frac{1}{x^2\left(4-(1+x^2)^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(4-(x^2+1)^4),x, algorithm="giac")
```

output
```
1/31104*sqrt(2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) + 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 72*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) + 72*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x +
 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/31104*sqrt
(2)*(3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(2)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) - 18*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18)
 + 3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 72*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 1
8) + 72*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqr
t(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/62208*sqrt(2)*(18*3^(3
/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 18*3^(3/4)*sqrt(6*sqrt(
3) + 18)*(sqrt(3) - 3) - 72*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 72*3^(
1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2)
+ sqrt(3)) - 1/62208*sqrt(2)*(18*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqr
t(3) + 18) - 3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 3^(3/4)*(6*sqrt(3)
+ 18)^(3/2) + 18*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 72*3^(1/4)*s
qrt(2)*sqrt(-6*sqrt(3) + 18) - 72*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 -
2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/16*sqrt(10*sqrt(2) - 1
4)*arctan(x/sqrt(sqrt(2) + 1)) + 1/32*sqrt(10*sqrt(2) + 14)*log(abs(x +...
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\left(4-(1+x^2)^4\right)}\,dx = \frac{\operatorname{atan}\left(\frac{x\sqrt{14-10\sqrt{2}}\,11710\mathrm{i}}{9\left(\frac{28270\sqrt{2}}{9}-\frac{39980}{9}\right)}-\frac{\sqrt{2}\,x\sqrt{14-10\sqrt{2}}\,920\mathrm{i}}{\frac{28270\sqrt{2}}{9}-\frac{39980}{9}}\right)\sqrt{14-10\sqrt{2}}\,1\mathrm{i}}{16} - \frac{\operatorname{atan}\left(\frac{x\sqrt{10\sqrt{2}+14}\,11710\mathrm{i}}{9\left(\frac{28270\sqrt{2}}{9}+\frac{39980}{9}\right)}+\frac{\sqrt{2}\,x\sqrt{10\sqrt{2}+14}\,920\mathrm{i}}{\frac{28270\sqrt{2}}{9}+\frac{39980}{9}}\right)\sqrt{10\sqrt{2}+14}\,1\mathrm{i}}{16} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-30-\sqrt{2}\,6\mathrm{i}}\,398\mathrm{i}}{177147\left(-\frac{184}{59049}+\frac{\sqrt{2}\,1378\mathrm{i}}{59049}\right)}-\frac{490\sqrt{2}\,x\sqrt{-30-\sqrt{2}\,6\mathrm{i}}}{177147\left(-\frac{184}{59049}+\frac{\sqrt{2}\,1378\mathrm{i}}{59049}\right)}\right)\sqrt{-30-\sqrt{2}\,6\mathrm{i}}\,1\mathrm{i}}{144} - \frac{1}{3x} + \frac{\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}\,x\sqrt{-5+\sqrt{2}\,1\mathrm{i}}\,398\mathrm{i}}{177147\left(\frac{184}{59049}+\frac{\sqrt{2}\,1378\mathrm{i}}{59049}\right)}+\frac{490\sqrt{2}\sqrt{6}\,x\sqrt{-5+\sqrt{2}\,1\mathrm{i}}}{177147\left(\frac{184}{59049}+\frac{\sqrt{2}\,1378\mathrm{i}}{59049}\right)}\right)\sqrt{-5+\sqrt{2}\,1\mathrm{i}}\,1\mathrm{i}}{144}$$

input
```
int(-1/(x^2*((x^2 + 1)^4 - 4)),x)
```

output
```
(atan((x*(14 - 10*2^(1/2))^(1/2)*11710i)/(9*((28270*2^(1/2))/9 - 39980/9))
 - (2^(1/2)*x*(14 - 10*2^(1/2))^(1/2)*920i)/((28270*2^(1/2))/9 - 39980/9))
*(14 - 10*2^(1/2))^(1/2)*1i)/16 - (atan((x*(10*2^(1/2) + 14)^(1/2)*11710i)
/(9*((28270*2^(1/2))/9 + 39980/9)) + (2^(1/2)*x*(10*2^(1/2) + 14)^(1/2)*92
0i)/((28270*2^(1/2))/9 + 39980/9))*(10*2^(1/2) + 14)^(1/2)*1i)/16 - (atan(
(x*(- 2^(1/2)*6i - 30)^(1/2)*398i)/(177147*((2^(1/2)*1378i)/59049 - 184/59
049)) - (490*2^(1/2)*x*(- 2^(1/2)*6i - 30)^(1/2))/(177147*((2^(1/2)*1378i)
/59049 - 184/59049)))*(- 2^(1/2)*6i - 30)^(1/2)*1i)/144 - 1/(3*x) + (6^(1/
2)*atan((6^(1/2)*x*(2^(1/2)*1i - 5)^(1/2)*398i)/(177147*((2^(1/2)*1378i)/5
9049 + 184/59049)) + (490*2^(1/2)*6^(1/2)*x*(2^(1/2)*1i - 5)^(1/2))/(17714
7*((2^(1/2)*1378i)/59049 + 184/59049)))*(2^(1/2)*1i - 5)^(1/2)*1i)/144
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^2\left(4-(1+x^2)^4\right)}\,dx = \frac{2\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x+6\sqrt{\sqrt{3}+1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x-2\sqrt{\sqrt{3}+1}\,\sqrt{6}\,atan\left(\cdots\right.}{\cdots}$$

input
```
int(1/x^2/(4-(x^2+1)^4),x)
```

output
```
(2*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(
sqrt(3) + 1)*sqrt(2)))*x + 6*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3)
- 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x - 2*sqrt(sqrt(3) + 1)*s
qrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))
*x - 6*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(s
qrt(sqrt(3) + 1)*sqrt(2)))*x - 108*sqrt(sqrt(2) + 1)*sqrt(2)*atan(x/sqrt(s
qrt(2) + 1))*x + 144*sqrt(sqrt(2) + 1)*atan(x/sqrt(sqrt(2) + 1))*x - sqrt(
sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*
x + sqrt(sqrt(3) - 1)*sqrt(6)*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) +
x**2)*x + 3*sqrt(sqrt(3) - 1)*sqrt(2)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x +
 sqrt(3) + x**2)*x - 3*sqrt(sqrt(3) - 1)*sqrt(2)*log(sqrt(sqrt(3) - 1)*sqr
t(2)*x + sqrt(3) + x**2)*x - 54*sqrt(sqrt(2) - 1)*sqrt(2)*log( - sqrt(sqrt
(2) - 1) + x)*x + 54*sqrt(sqrt(2) - 1)*sqrt(2)*log(sqrt(sqrt(2) - 1) + x)*
x - 72*sqrt(sqrt(2) - 1)*log( - sqrt(sqrt(2) - 1) + x)*x + 72*sqrt(sqrt(2)
 - 1)*log(sqrt(sqrt(2) - 1) + x)*x - 192)/(576*x)
```

3.151 $\int \frac{x^6}{4+(1+x^2)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 182

$$\int \frac{x^6}{4+(1+x^2)^4} dx = -\frac{1}{16}\sqrt{-41+25\sqrt{5}}\arctan\left(\frac{\sqrt{2(-2+\sqrt{5})}-2x}{\sqrt{2(2+\sqrt{5})}}\right) + \frac{1}{16}\sqrt{-41+25\sqrt{5}}\arctan\left(\frac{\sqrt{2(-2+\sqrt{5})}+2x}{\sqrt{2(2+\sqrt{5})}}\right) + \frac{\arctan(1-\sqrt{2}x)}{8\sqrt{2}} - \frac{\arctan(1+\sqrt{2}x)}{8\sqrt{2}} - \frac{1}{16}\sqrt{41+25\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2(-2+\sqrt{5})}x}{\sqrt{5}+x^2}\right)$$

output
```
-1/16*(-41+25*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)-2*x)/(4+2*5^(1/2
))^(1/2))+1/16*(-41+25*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)+2*x)/(4
+2*5^(1/2))^(1/2))-1/16*arctan(-1+x*2^(1/2))*2^(1/2)-1/16*arctan(1+x*2^(1/
2))*2^(1/2)-1/16*(41+25*5^(1/2))^(1/2)*arctanh((-4+2*5^(1/2))^(1/2)*x/(5^(
1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.42

$$\int \frac{x^6}{4+(1+x^2)^4}\,dx = \frac{1}{16}\left((1+7i)\sqrt{2-i}\arctan\left(\frac{x}{\sqrt{2-i}}\right)\right.$$
$$+(1-7i)\sqrt{2+i}\arctan\left(\frac{x}{\sqrt{2+i}}\right)$$
$$\left.+\sqrt{2}\left(\arctan\left(1-\sqrt{2}x\right)-\arctan\left(1+\sqrt{2}x\right)\right)\right)$$

input
```
Integrate[x^6/(4 + (1 + x^2)^4),x]
```

output
```
((1 + 7*I)*Sqrt[2 - I]*ArcTan[x/Sqrt[2 - I]] + (1 - 7*I)*Sqrt[2 + I]*ArcTa
n[x/Sqrt[2 + I]] + Sqrt[2]*(ArcTan[1 - Sqrt[2]*x] - ArcTan[1 + Sqrt[2]*x])
)/16
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^2+1)^4+4}\,dx$$
$$\Big\downarrow \text{2460}$$
$$\int \left(\frac{-x^2-1}{8(x^4+1)}+\frac{9x^2+5}{8(x^4+4x^2+5)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{16}\sqrt{25\sqrt{5}-41}\arctan\left(\frac{\sqrt{2\left(\sqrt{5}-2\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)+\frac{1}{16}\sqrt{25\sqrt{5}-41}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{5}-2\right)}}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)+\frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}+\frac{1}{32}\sqrt{41+25\sqrt{5}}\log\left(x^2-\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)-\frac{1}{32}\sqrt{41+25\sqrt{5}}\log\left(x^2+\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)$$

input
```
Int[x^6/(4 + (1 + x^2)^4),x]
```

output
```
-1/16*(Sqrt[-41 + 25*Sqrt[5]]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] - 2*x)/Sqrt[2
*(2 + Sqrt[5])]]) + (Sqrt[-41 + 25*Sqrt[5]]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])]
 + 2*x)/Sqrt[2*(2 + Sqrt[5])]])/16 + ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) - A
rcTan[1 + Sqrt[2]*x]/(8*Sqrt[2]) + (Sqrt[41 + 25*Sqrt[5]]*Log[Sqrt[5] - Sq
rt[2*(-2 + Sqrt[5])]*x + x^2])/32 - (Sqrt[41 + 25*Sqrt[5]]*Log[Sqrt[5] + S
qrt[2*(-2 + Sqrt[5])]*x + x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-164\_Z^2+3125\right)}\_R\ln\left(14\_R^3-249\_R+950x\right)\right)}{16}-\frac{\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{16}-\frac{\sqrt{2}\,\arctan\left(\frac{x^3\sqrt{2}}{2}+\frac{x\sqrt{2}}{2}\right)}{16}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right.}{64}$

input
```
int(x^6/(4+(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(14*_R^3-249*_R+950*x),_R=RootOf(4*_Z^4-164*_Z^2+3125))-1/16
*2^(1/2)*arctan(1/2*x*2^(1/2))-1/16*2^(1/2)*arctan(1/2*x^3*2^(1/2)+1/2*x*2
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.27

$$\int \frac{x^6}{4+(1+x^2)^4}\,dx$$
$$= -\frac{1}{16}\sqrt{25\sqrt{5}-41}\arctan\left(\frac{1}{38}\sqrt{25\sqrt{5}+41}\sqrt{25\sqrt{5}-41}\left(\sqrt{5}-2\right)+\frac{1}{38}\left(\sqrt{5}x-9\,x\right)\sqrt{25\sqrt{5}-41}\right)+\frac{1}{16}\sqrt{25\sqrt{5}-41}\arctan\left(\frac{1}{38}\sqrt{25\sqrt{5}+41}\sqrt{25\sqrt{5}-41}\left(\sqrt{5}-2\right)-\frac{1}{38}\left(\sqrt{5}x-9\,x\right)\sqrt{25\sqrt{5}-41}\right)-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^3+x)\right)-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{32}\sqrt{25\sqrt{5}+41}\log\left(38\,x^2+\left(7\sqrt{5}x-13\,x\right)\sqrt{25\sqrt{5}+41}+38\sqrt{5}\right)+\frac{1}{32}\sqrt{25\sqrt{5}+41}\log\left(38\,x^2-\left(7\sqrt{5}x-13\,x\right)\sqrt{25\sqrt{5}+41}+38\sqrt{5}\right)$$

input
```
integrate(x^6/(4+(x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/16*sqrt(25*sqrt(5) - 41)*arctan(1/38*sqrt(25*sqrt(5) + 41)*sqrt(25*sqrt
(5) - 41)*(sqrt(5) - 2) + 1/38*(sqrt(5)*x - 9*x)*sqrt(25*sqrt(5) - 41)) +
1/16*sqrt(25*sqrt(5) - 41)*arctan(1/38*sqrt(25*sqrt(5) + 41)*sqrt(25*sqrt(
5) - 41)*(sqrt(5) - 2) - 1/38*(sqrt(5)*x - 9*x)*sqrt(25*sqrt(5) - 41)) - 1
/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)) - 1/16*sqrt(2)*arctan(1/2*sqrt(2
)*x) - 1/32*sqrt(25*sqrt(5) + 41)*log(38*x^2 + (7*sqrt(5)*x - 13*x)*sqrt(2
5*sqrt(5) + 41) + 38*sqrt(5)) + 1/32*sqrt(25*sqrt(5) + 41)*log(38*x^2 - (7
*sqrt(5)*x - 13*x)*sqrt(25*sqrt(5) + 41) + 38*sqrt(5))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1187 vs. $2(155) = 310$.

Time = 0.67 (sec) , antiderivative size = 1187, normalized size of antiderivative = 6.52

$$\int \frac{x^6}{4+(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**6/(4+(x**2+1)**4),x)
```

output
```
sqrt(2)*(-2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/32 +
 sqrt(41/1024 + 25*sqrt(5)/1024)*log(x**2 + x*(-14*sqrt(5)*sqrt(41 + 25*sq
rt(5))/19 - 268*sqrt(41 + 25*sqrt(5))/475 + 21*sqrt(2)*sqrt(41 + 25*sqrt(5
))*sqrt(1025*sqrt(5) + 2403)/950) - 631257*sqrt(2)*sqrt(1025*sqrt(5) + 240
3)/451250 - 4823*sqrt(10)*sqrt(1025*sqrt(5) + 2403)/9025 + 56025287/451250
 + 1044793*sqrt(5)/18050) - sqrt(41/1024 + 25*sqrt(5)/1024)*log(x**2 + x*(
-21*sqrt(2)*sqrt(41 + 25*sqrt(5))*sqrt(1025*sqrt(5) + 2403)/950 + 268*sqrt
(41 + 25*sqrt(5))/475 + 14*sqrt(5)*sqrt(41 + 25*sqrt(5))/19) - 631257*sqrt
(2)*sqrt(1025*sqrt(5) + 2403)/451250 - 4823*sqrt(10)*sqrt(1025*sqrt(5) + 2
403)/9025 + 56025287/451250 + 1044793*sqrt(5)/18050) + 2*sqrt(-sqrt(2)*sqr
t(1025*sqrt(5) + 2403)/512 + 41/1024 + 75*sqrt(5)/1024)*atan(1900*x/(38*sq
rt(-2*sqrt(2)*sqrt(1025*sqrt(5) + 2403) + 41 + 75*sqrt(5)) + 7*sqrt(2)*sqr
t(1025*sqrt(5) + 2403)*sqrt(-2*sqrt(2)*sqrt(1025*sqrt(5) + 2403) + 41 + 75
*sqrt(5))) - 700*sqrt(5)*sqrt(41 + 25*sqrt(5))/(38*sqrt(-2*sqrt(2)*sqrt(10
25*sqrt(5) + 2403) + 41 + 75*sqrt(5)) + 7*sqrt(2)*sqrt(1025*sqrt(5) + 2403
)*sqrt(-2*sqrt(2)*sqrt(1025*sqrt(5) + 2403) + 41 + 75*sqrt(5))) - 536*sqrt
(41 + 25*sqrt(5))/(38*sqrt(-2*sqrt(2)*sqrt(1025*sqrt(5) + 2403) + 41 + 75*
sqrt(5)) + 7*sqrt(2)*sqrt(1025*sqrt(5) + 2403)*sqrt(-2*sqrt(2)*sqrt(1025*s
qrt(5) + 2403) + 41 + 75*sqrt(5))) + 21*sqrt(2)*sqrt(41 + 25*sqrt(5))*sqrt
(1025*sqrt(5) + 2403)/(38*sqrt(-2*sqrt(2)*sqrt(1025*sqrt(5) + 2403) + 4...
```

Maxima [F]

$$\int \frac{x^6}{4+(1+x^2)^4}\,dx = \int \frac{x^6}{(x^2+1)^4+4}\,dx$$

input
```
integrate(x^6/(4+(x^2+1)^4),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 1/8*integrate((9*x^2 + 5)/(x^4 + 4*x^2 + 5),
x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{x^6}{4+(1+x^2)^4}\,dx =& -\frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)\\ &-\frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)\\ &-\frac{1}{80}\,\sqrt{625\,\sqrt{5}+1199}\arctan\left(\frac{5^{\frac{3}{4}}\left(x+5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\,\sqrt{5}+\frac{1}{2}}\right)}{5\,\sqrt{\frac{1}{5}\,\sqrt{5}+\frac{1}{2}}}\right)\\ &-\frac{1}{80}\,\sqrt{625\,\sqrt{5}+1199}\arctan\left(\frac{5^{\frac{3}{4}}\left(x-5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\,\sqrt{5}+\frac{1}{2}}\right)}{5\,\sqrt{\frac{1}{5}\,\sqrt{5}+\frac{1}{2}}}\right)\\ &+\frac{1}{160}\,\sqrt{625\,\sqrt{5}-1199}\log\left(x^2+2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\,\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right)\\ &-\frac{1}{160}\,\sqrt{625\,\sqrt{5}-1199}\log\left(x^2-2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\,\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right)\end{aligned}$$

input
```
integrate(x^6/(4+(x^2+1)^4),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 1/80*sqrt(625*sqrt(5) + 1199)*arctan(1/5*5^(3
/4)*(x + 5^(1/4)*sqrt(-1/5*sqrt(5) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)) - 1/80
*sqrt(625*sqrt(5) + 1199)*arctan(1/5*5^(3/4)*(x - 5^(1/4)*sqrt(-1/5*sqrt(5
) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)) + 1/160*sqrt(625*sqrt(5) - 1199)*log(x^
2 + 2*5^(1/4)*x*sqrt(-1/5*sqrt(5) + 1/2) + sqrt(5)) - 1/160*sqrt(625*sqrt(
5) - 1199)*log(x^2 - 2*5^(1/4)*x*sqrt(-1/5*sqrt(5) + 1/2) + sqrt(5))
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 1287, normalized size of antiderivative = 7.07

$$\int \frac{x^6}{4+(1+x^2)^4} \, dx = \text{Too large to display}$$

input
```
int(x^6/((x^2 + 1)^4 + 4),x)
```

output
```
- (2^(1/2)*(2*atan((2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) + 2*atan((2^(1/2)*x)/2
)))/32 - atan(((62968*x + (((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(
1/2))/1024 + 41/1024)^(1/2))*(31637504*x - (((41/1024 - (25*5^(1/2))/1024)
^(1/2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2))*(304087040*x - (2147483648*x
*((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2
)) - 671088640)*((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(1/2))/1024
+ 41/1024)^(1/2))) - 64225280)*((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25
*5^(1/2))/1024 + 41/1024)^(1/2))) - 768000)*((41/1024 - (25*5^(1/2))/1024)
^(1/2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2)))*((41/1024 - (25*5^(1/2))/10
24)^(1/2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2))*1i + (62968*x + (((41/102
4 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2))*(31637
504*x - (((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(1/2))/1024 + 41/10
24)^(1/2))*(304087040*x - (2147483648*x*((41/1024 - (25*5^(1/2))/1024)^(1/
2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2)) + 671088640)*((41/1024 - (25*5^(
1/2))/1024)^(1/2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2))) + 64225280)*((41
/1024 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(1/2))/1024 + 41/1024)^(1/2))) +
 768000)*((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(1/2))/1024 + 41/10
24)^(1/2)))*((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25*5^(1/2))/1024 + 41
/1024)^(1/2))*1i)/((62968*x + (((41/1024 - (25*5^(1/2))/1024)^(1/2) - ((25
*5^(1/2))/1024 + 41/1024)^(1/2))*(31637504*x - (((41/1024 - (25*5^(1/2)...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.45

$$\int \frac{x^6}{4+(1+x^2)^4}\,dx$$
$$= \frac{\sqrt{2}\left(4atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)-4atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)-14\sqrt{\sqrt{5}+2}\,\sqrt{5}\,atan\left(\frac{\sqrt{\sqrt{5}-2}\,\sqrt{2}-2x}{\sqrt{\sqrt{5}+2}\,\sqrt{2}}\right)+26\sqrt{\sqrt{5}+2}\,atan\left(\frac{\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}}}\right.\right.}{}$$

input
```
int(x^6/(4+(x^2+1)^4),x)
```

output
```
(sqrt(2)*(4*atan((sqrt(2) - 2*x)/sqrt(2)) - 4*atan((sqrt(2) + 2*x)/sqrt(2)
) - 14*sqrt(sqrt(5) + 2)*sqrt(5)*atan((sqrt(sqrt(5) - 2)*sqrt(2) - 2*x)/(s
qrt(sqrt(5) + 2)*sqrt(2))) + 26*sqrt(sqrt(5) + 2)*atan((sqrt(sqrt(5) - 2)*
sqrt(2) - 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) + 14*sqrt(sqrt(5) + 2)*sqrt(5)
*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) - 26*
sqrt(sqrt(5) + 2)*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5) + 2
)*sqrt(2))) + 7*sqrt(sqrt(5) - 2)*sqrt(5)*log( - sqrt(sqrt(5) - 2)*sqrt(2)
*x + sqrt(5) + x**2) - 7*sqrt(sqrt(5) - 2)*sqrt(5)*log(sqrt(sqrt(5) - 2)*s
qrt(2)*x + sqrt(5) + x**2) + 13*sqrt(sqrt(5) - 2)*log( - sqrt(sqrt(5) - 2)
*sqrt(2)*x + sqrt(5) + x**2) - 13*sqrt(sqrt(5) - 2)*log(sqrt(sqrt(5) - 2)*
sqrt(2)*x + sqrt(5) + x**2)))/64
```

3.152 $\int \frac{x^4}{4+\left(1+x^2\right)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 167

$$\begin{aligned}\int \frac{x^4}{4+\left(1+x^2\right)^4} dx = &-\frac{1}{16}\sqrt{-11+5\sqrt{5}}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)\\ &+\frac{1}{16}\sqrt{-11+5\sqrt{5}}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}+2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)\\ &-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}+\frac{1}{16}\sqrt{11+5\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}x}{\sqrt{5}+x^2}\right)\end{aligned}$$

output
```
-1/16*(-11+5*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)-2*x)/(4+2*5^(1/2)
)^(1/2))+1/16*(-11+5*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)+2*x)/(4+2
*5^(1/2))^(1/2))-1/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)+1/16*(11+5*5^(1/2
))^(1/2)*arctanh((-4+2*5^(1/2))^(1/2)*x/(5^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{4+(1+x^2)^4}\,dx = \frac{1}{32}\left((2-6i)\sqrt{2-i}\arctan\left(\frac{x}{\sqrt{2-i}}\right) + (2+6i)\sqrt{2+i}\arctan\left(\frac{x}{\sqrt{2+i}}\right) + \sqrt{2}\left(\log\left(-1+\sqrt{2}x-x^2\right)-\log\left(1+\sqrt{2}x+x^2\right)\right)\right)$$

input
```
Integrate[x^4/(4 + (1 + x^2)^4),x]
```

output
```
((2 - 6*I)*Sqrt[2 - I]*ArcTan[x/Sqrt[2 - I]] + (2 + 6*I)*Sqrt[2 + I]*ArcTa
n[x/Sqrt[2 + I]] + Sqrt[2]*(Log[-1 + Sqrt[2]*x - x^2] - Log[1 + Sqrt[2]*x
+ x^2]))/32
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(x^2+1)^4+4}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{5-x^2}{8(x^4+4x^2+5)}+\frac{x^2-1}{8(x^4+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{16}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{\sqrt{2\left(\sqrt{5}-2\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)+\frac{1}{16}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{5}-2\right)}}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{1}{32}\sqrt{11+5\sqrt{5}}\log\left(x^2-\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)+\frac{1}{32}\sqrt{11+5\sqrt{5}}\log\left(x^2+\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)$$

input
```
Int[x^4/(4 + (1 + x^2)^4),x]
```

output
```
-1/16*(Sqrt[-11 + 5*Sqrt[5]]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] - 2*x)/Sqrt[2*
(2 + Sqrt[5])]]) + (Sqrt[-11 + 5*Sqrt[5]]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] +
 2*x)/Sqrt[2*(2 + Sqrt[5])]])/16 + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) -
 Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2]) - (Sqrt[11 + 5*Sqrt[5]]*Log[Sqrt[5]
 - Sqrt[2*(-2 + Sqrt[5])]*x + x^2])/32 + (Sqrt[11 + 5*Sqrt[5]]*Log[Sqrt[5]
 + Sqrt[2*(-2 + Sqrt[5])]*x + x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4-44\_Z^2+125\right)} \_R\ln\left(6\_R^3-31\_R+10x\right)\right)}{16}+\frac{\sqrt{2}\,\ln\left(x^2-x\sqrt{2}+1\right)}{32}-\frac{\sqrt{2}\,\ln\left(x^2+x\sqrt{2}+1\right)}{32}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right.\right.}{64}$

input
```
int(x^4/(4+(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(6*_R^3-31*_R+10*x),_R=RootOf(4*_Z^4-44*_Z^2+125))+1/32*2^(1
/2)*ln(x^2-x*2^(1/2)+1)-1/32*2^(1/2)*ln(x^2+x*2^(1/2)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.40

$$\int \frac{x^4}{4+\left(1+x^2\right)^4}\,dx = \frac{1}{16}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{1}{2}\sqrt{5\sqrt{5}+11}\sqrt{5\sqrt{5}-11}\left(\sqrt{5}-2\right)+\frac{1}{2}\left(\sqrt{5}x+x\right)\sqrt{5\sqrt{5}-11}\right) - \frac{1}{16}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{1}{2}\sqrt{5\sqrt{5}+11}\sqrt{5\sqrt{5}-11}\left(\sqrt{5}-2\right)-\frac{1}{2}\left(\sqrt{5}x+x\right)\sqrt{5\sqrt{5}-11}\right) - \frac{1}{32}\sqrt{5\sqrt{5}+11}\log\left(2x^2+\left(3\sqrt{5}x-7x\right)\sqrt{5\sqrt{5}+11}+2\sqrt{5}\right) + \frac{1}{32}\sqrt{5\sqrt{5}+11}\log\left(2x^2-\left(3\sqrt{5}x-7x\right)\sqrt{5\sqrt{5}+11}+2\sqrt{5}\right) + \frac{1}{32}\sqrt{2}\log\left(\frac{x^4+4x^2-2\sqrt{2}(x^3+x)+1}{x^4+1}\right)$$

input
```
integrate(x^4/(4+(x^2+1)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(5*sqrt(5) - 11)*arctan(1/2*sqrt(5*sqrt(5) + 11)*sqrt(5*sqrt(5) -
 11)*(sqrt(5) - 2) + 1/2*(sqrt(5)*x + x)*sqrt(5*sqrt(5) - 11)) - 1/16*sqrt
(5*sqrt(5) - 11)*arctan(1/2*sqrt(5*sqrt(5) + 11)*sqrt(5*sqrt(5) - 11)*(sqr
t(5) - 2) - 1/2*(sqrt(5)*x + x)*sqrt(5*sqrt(5) - 11)) - 1/32*sqrt(5*sqrt(5
) + 11)*log(2*x^2 + (3*sqrt(5)*x - 7*x)*sqrt(5*sqrt(5) + 11) + 2*sqrt(5))
+ 1/32*sqrt(5*sqrt(5) + 11)*log(2*x^2 - (3*sqrt(5)*x - 7*x)*sqrt(5*sqrt(5)
 + 11) + 2*sqrt(5)) + 1/32*sqrt(2)*log((x^4 + 4*x^2 - 2*sqrt(2)*(x^3 + x)
+ 1)/(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. 2(141) = 282.

Time = 0.68 (sec) , antiderivative size = 1182, normalized size of antiderivative = 7.08

$$\int \frac{x^4}{4+(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(4+(x**2+1)**4),x)
```

output
```
sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**2 + sqrt(2)*x + 1)/3
2 + sqrt(11/1024 + 5*sqrt(5)/1024)*log(x**2 + x*(-6*sqrt(5)*sqrt(11 + 5*sq
rt(5)) - 32*sqrt(11 + 5*sqrt(5))/5 + 9*sqrt(2)*sqrt(11 + 5*sqrt(5))*sqrt(5
5*sqrt(5) + 123)/10) - 5497*sqrt(2)*sqrt(55*sqrt(5) + 123)/50 - 243*sqrt(1
0)*sqrt(55*sqrt(5) + 123)/5 + 121217/50 + 10853*sqrt(5)/10) - sqrt(11/1024
 + 5*sqrt(5)/1024)*log(x**2 + x*(-9*sqrt(2)*sqrt(11 + 5*sqrt(5))*sqrt(55*s
qrt(5) + 123)/10 + 32*sqrt(11 + 5*sqrt(5))/5 + 6*sqrt(5)*sqrt(11 + 5*sqrt(
5))) - 5497*sqrt(2)*sqrt(55*sqrt(5) + 123)/50 - 243*sqrt(10)*sqrt(55*sqrt(
5) + 123)/5 + 121217/50 + 10853*sqrt(5)/10) + 2*sqrt(-sqrt(2)*sqrt(55*sqrt
(5) + 123)/512 + 11/1024 + 15*sqrt(5)/1024)*atan(20*x/(2*sqrt(-2*sqrt(2)*s
qrt(55*sqrt(5) + 123) + 11 + 15*sqrt(5)) + 3*sqrt(2)*sqrt(55*sqrt(5) + 123
)*sqrt(-2*sqrt(2)*sqrt(55*sqrt(5) + 123) + 11 + 15*sqrt(5))) - 60*sqrt(5)*
sqrt(11 + 5*sqrt(5))/(2*sqrt(-2*sqrt(2)*sqrt(55*sqrt(5) + 123) + 11 + 15*s
qrt(5)) + 3*sqrt(2)*sqrt(55*sqrt(5) + 123)*sqrt(-2*sqrt(2)*sqrt(55*sqrt(5)
 + 123) + 11 + 15*sqrt(5))) - 64*sqrt(11 + 5*sqrt(5))/(2*sqrt(-2*sqrt(2)*s
qrt(55*sqrt(5) + 123) + 11 + 15*sqrt(5)) + 3*sqrt(2)*sqrt(55*sqrt(5) + 123
)*sqrt(-2*sqrt(2)*sqrt(55*sqrt(5) + 123) + 11 + 15*sqrt(5))) + 9*sqrt(2)*s
qrt(11 + 5*sqrt(5))*sqrt(55*sqrt(5) + 123)/(2*sqrt(-2*sqrt(2)*sqrt(55*sqrt
(5) + 123) + 11 + 15*sqrt(5)) + 3*sqrt(2)*sqrt(55*sqrt(5) + 123)*sqrt(-2*s
qrt(2)*sqrt(55*sqrt(5) + 123) + 11 + 15*sqrt(5))) + 2*sqrt(-sqrt(2)*sq...
```

Maxima [F]

$$\int \frac{x^4}{4+(1+x^2)^4}\,dx = \int \frac{x^4}{(x^2+1)^4+4}\,dx$$

input
```
integrate(x^4/(4+(x^2+1)^4),x, algorithm="maxima")
```

output
```
-1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x
+ 1) - 1/8*integrate((x^2 - 5)/(x^4 + 4*x^2 + 5), x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{4+(1+x^2)^4}\,dx = \frac{1}{80}\sqrt{125\sqrt{5}+29}\arctan\left(\frac{5^{\frac{3}{4}}\left(x+5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{80}\sqrt{125\sqrt{5}+29}\arctan\left(\frac{5^{\frac{3}{4}}\left(x-5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{160}\sqrt{125\sqrt{5}-29}\log\left(x^2+2\cdot5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right) + \frac{1}{160}\sqrt{125\sqrt{5}-29}\log\left(x^2-2\cdot5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right) - \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^4/(4+(x^2+1)^4),x, algorithm="giac")
```

output
```
1/80*sqrt(125*sqrt(5) + 29)*arctan(1/5*5^(3/4)*(x + 5^(1/4)*sqrt(-1/5*sqrt
(5) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)) + 1/80*sqrt(125*sqrt(5) + 29)*arctan(
1/5*5^(3/4)*(x - 5^(1/4)*sqrt(-1/5*sqrt(5) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)
) - 1/160*sqrt(125*sqrt(5) - 29)*log(x^2 + 2*5^(1/4)*x*sqrt(-1/5*sqrt(5) +
 1/2) + sqrt(5)) + 1/160*sqrt(125*sqrt(5) - 29)*log(x^2 - 2*5^(1/4)*x*sqrt
(-1/5*sqrt(5) + 1/2) + sqrt(5)) - 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) +
1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 1222, normalized size of antiderivative = 7.32

$$\int \frac{x^4}{4+(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
int(x^4/((x^2 + 1)^4 + 4),x)
```

output
```
atan(((2472*x - ((1474560*x - ((111149056*x - (2147483648*x*((11/1024 - (5
*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1024 + 11/1024)^(1/2)) - 335544320)*(
(11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1024 + 11/1024)^(1/2)))*
((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1024 + 11/1024)^(1/2))
- 10485760)*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1024 + 11/1
024)^(1/2)))*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1024 + 11/
1024)^(1/2)) - 70400)*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1
024 + 11/1024)^(1/2)))*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/
1024 + 11/1024)^(1/2))*1i + (2472*x - ((1474560*x - ((111149056*x - (21474
83648*x*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1024 + 11/1024)
^(1/2)) + 335544320)*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/10
24 + 11/1024)^(1/2)))*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^(1/2))/1
024 + 11/1024)^(1/2)) + 10485760)*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((
5*5^(1/2))/1024 + 11/1024)^(1/2)))*((11/1024 - (5*5^(1/2))/1024)^(1/2) + (
(5*5^(1/2))/1024 + 11/1024)^(1/2)) + 70400)*((11/1024 - (5*5^(1/2))/1024)^
(1/2) + ((5*5^(1/2))/1024 + 11/1024)^(1/2)))*((11/1024 - (5*5^(1/2))/1024)
^(1/2) + ((5*5^(1/2))/1024 + 11/1024)^(1/2))*1i)/((2472*x - ((1474560*x -
((111149056*x - (2147483648*x*((11/1024 - (5*5^(1/2))/1024)^(1/2) + ((5*5^
(1/2))/1024 + 11/1024)^(1/2)) + 335544320)*((11/1024 - (5*5^(1/2))/1024)^(
1/2) + ((5*5^(1/2))/1024 + 11/1024)^(1/2)))*((11/1024 - (5*5^(1/2))/102...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.56

$$\int \frac{x^4}{4+(1+x^2)^4}\,dx$$
$$= \frac{\sqrt{2}\left(6\sqrt{\sqrt{5}+2}\,\sqrt{5}\,atan\left(\frac{\sqrt{\sqrt{5}-2}\,\sqrt{2}-2x}{\sqrt{\sqrt{5}+2}\,\sqrt{2}}\right)-14\sqrt{\sqrt{5}+2}\,atan\left(\frac{\sqrt{\sqrt{5}-2}\,\sqrt{2}-2x}{\sqrt{\sqrt{5}+2}\,\sqrt{2}}\right)-6\sqrt{\sqrt{5}+2}\,\sqrt{5}\,atan\left(\frac{\sqrt{\sqrt{5}-}}{\sqrt{\sqrt{}}}\right.\right.}{}$$

input
```
int(x^4/(4+(x^2+1)^4),x)
```

output
```
(sqrt(2)*(6*sqrt(sqrt(5) + 2)*sqrt(5)*atan((sqrt(sqrt(5) - 2)*sqrt(2) - 2*
x)/(sqrt(sqrt(5) + 2)*sqrt(2))) - 14*sqrt(sqrt(5) + 2)*atan((sqrt(sqrt(5)
- 2)*sqrt(2) - 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) - 6*sqrt(sqrt(5) + 2)*sqr
t(5)*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) +
 14*sqrt(sqrt(5) + 2)*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5)
 + 2)*sqrt(2))) - 3*sqrt(sqrt(5) - 2)*sqrt(5)*log( - sqrt(sqrt(5) - 2)*sqr
t(2)*x + sqrt(5) + x**2) + 3*sqrt(sqrt(5) - 2)*sqrt(5)*log(sqrt(sqrt(5) -
2)*sqrt(2)*x + sqrt(5) + x**2) - 7*sqrt(sqrt(5) - 2)*log( - sqrt(sqrt(5) -
 2)*sqrt(2)*x + sqrt(5) + x**2) + 7*sqrt(sqrt(5) - 2)*log(sqrt(sqrt(5) - 2
)*sqrt(2)*x + sqrt(5) + x**2) + 2*log( - sqrt(2)*x + x**2 + 1) - 2*log(sqr
t(2)*x + x**2 + 1)))/64
```

3.153 $\int \frac{x^2}{4+(1+x^2)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 176

$$\begin{aligned}\int \frac{x^2}{4+(1+x^2)^4}\,dx &= \frac{1}{16}\sqrt{-1+\sqrt{5}}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)\\ &\quad-\frac{1}{16}\sqrt{-1+\sqrt{5}}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}+2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)\\ &\quad-\frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}\\ &\quad-\frac{1}{16}\sqrt{1+\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}x}{\sqrt{5}+x^2}\right)\end{aligned}$$

output
```
1/16*(5^(1/2)-1)^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)-2*x)/(4+2*5^(1/2))^(1/
2))-1/16*(5^(1/2)-1)^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)+2*x)/(4+2*5^(1/2))
^(1/2))+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*2^(1/2)
-1/16*(5^(1/2)+1)^(1/2)*arctanh((-4+2*5^(1/2))^(1/2)*x/(5^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.44

$$\int \frac{x^2}{4+(1+x^2)^4}\,dx = \frac{1}{16}\left((-1+i)\sqrt{2-i}\arctan\left(\frac{x}{\sqrt{2-i}}\right) - (1+i)\sqrt{2+i}\arctan\left(\frac{x}{\sqrt{2+i}}\right) + \sqrt{2}\left(-\arctan\left(1-\sqrt{2}x\right)+\arctan\left(1+\sqrt{2}x\right)\right)\right)$$

input
```
Integrate[x^2/(4 + (1 + x^2)^4),x]
```

output
```
((-1 + I)*Sqrt[2 - I]*ArcTan[x/Sqrt[2 - I]] - (1 + I)*Sqrt[2 + I]*ArcTan[x
/Sqrt[2 + I]] + Sqrt[2]*(-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x]))/
16
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)^4+4}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{-x^2-5}{8(x^4+4x^2+5)}+\frac{x^2+1}{8(x^4+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\sqrt{\sqrt{5}-1}\arctan\left(\frac{\sqrt{2\left(\sqrt{5}-2\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)-\frac{1}{16}\sqrt{\sqrt{5}-1}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{5}-2\right)}}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)-\frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}+\frac{1}{32}\sqrt{1+\sqrt{5}}\log\left(x^2-\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)-\frac{1}{32}\sqrt{1+\sqrt{5}}\log\left(x^2+\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)$$

input
```
Int[x^2/(4 + (1 + x^2)^4),x]
```

output
```
(Sqrt[-1 + Sqrt[5]]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] - 2*x)/Sqrt[2*(2 + Sqrt
[5])]])/16 - (Sqrt[-1 + Sqrt[5]]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] + 2*x)/Sqr
t[2*(2 + Sqrt[5])]])/16 - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + S
qrt[2]*x]/(8*Sqrt[2]) + (Sqrt[1 + Sqrt[5]]*Log[Sqrt[5] - Sqrt[2*(-2 + Sqrt
[5])]*x + x^2])/32 - (Sqrt[1 + Sqrt[5]]*Log[Sqrt[5] + Sqrt[2*(-2 + Sqrt[5]
)]*x + x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^{4}-4\_Z^{2}+5\right)}\_R\ln\left(-2\_R^{3}-\_R+2x\right)\right)}{16}+\frac{\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{16}+\frac{\sqrt{2}\,\arctan\left(\frac{x^{3}\sqrt{2}}{2}+\frac{x\sqrt{2}}{2}\right)}{16}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}+x\sqrt{2}+1}{x^{2}-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}+\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-x\sqrt{2}+1}{x^{2}+x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}$

input
```
int(x^2/(4+(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(-2*_R^3-_R+2*x),_R=RootOf(4*_Z^4-4*_Z^2+5))+1/16*2^(1/2)*ar
ctan(1/2*x*2^(1/2))+1/16*2^(1/2)*arctan(1/2*x^3*2^(1/2)+1/2*x*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{4+(1+x^2)^4}\,dx = -\frac{1}{16}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{2}\sqrt{\sqrt{5}+1}\sqrt{\sqrt{5}-1}\left(\sqrt{5}-2\right)+\frac{1}{2}\left(\sqrt{5}x-x\right)\sqrt{\sqrt{5}-1}\right)+\frac{1}{16}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{2}\sqrt{\sqrt{5}+1}\sqrt{\sqrt{5}-1}\left(\sqrt{5}-2\right)-\frac{1}{2}\left(\sqrt{5}x-x\right)\sqrt{\sqrt{5}-1}\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^3+x)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{32}\sqrt{\sqrt{5}+1}\log\left(2\,x^2+\left(\sqrt{5}x-3\,x\right)\sqrt{\sqrt{5}+1}+2\sqrt{5}\right)-\frac{1}{32}\sqrt{\sqrt{5}+1}\log\left(2\,x^2-\left(\sqrt{5}x-3\,x\right)\sqrt{\sqrt{5}+1}+2\sqrt{5}\right)$$

input
```
integrate(x^2/(4+(x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/16*sqrt(sqrt(5) - 1)*arctan(1/2*sqrt(sqrt(5) + 1)*sqrt(sqrt(5) - 1)*(sq
rt(5) - 2) + 1/2*(sqrt(5)*x - x)*sqrt(sqrt(5) - 1)) + 1/16*sqrt(sqrt(5) -
1)*arctan(1/2*sqrt(sqrt(5) + 1)*sqrt(sqrt(5) - 1)*(sqrt(5) - 2) - 1/2*(sqr
t(5)*x - x)*sqrt(sqrt(5) - 1)) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)
) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*sqrt(sqrt(5) + 1)*log(2*x^2
+ (sqrt(5)*x - 3*x)*sqrt(sqrt(5) + 1) + 2*sqrt(5)) - 1/32*sqrt(sqrt(5) + 1
)*log(2*x^2 - (sqrt(5)*x - 3*x)*sqrt(sqrt(5) + 1) + 2*sqrt(5))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(150) = 300$.

Time = 0.65 (sec) , antiderivative size = 879, normalized size of antiderivative = 4.99

$$\int \frac{x^2}{4+(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**2/(4+(x**2+1)**4),x)
```

output
```
sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/32 -
sqrt(1/1024 + sqrt(5)/1024)*log(x**2 + x*(-2*sqrt(5)*sqrt(1 + sqrt(5)) + 3
*sqrt(2)*sqrt(1 + sqrt(5))*sqrt(sqrt(5) + 3)/2) - 17*sqrt(2)*sqrt(sqrt(5)
+ 3)/2 - sqrt(10)*sqrt(sqrt(5) + 3) + 27/2 + 21*sqrt(5)/2) + sqrt(1/1024 +
 sqrt(5)/1024)*log(x**2 + x*(-3*sqrt(2)*sqrt(1 + sqrt(5))*sqrt(sqrt(5) + 3
)/2 + 2*sqrt(5)*sqrt(1 + sqrt(5))) - 17*sqrt(2)*sqrt(sqrt(5) + 3)/2 - sqrt
(10)*sqrt(sqrt(5) + 3) + 27/2 + 21*sqrt(5)/2) - 2*sqrt(-sqrt(2)*sqrt(sqrt(
5) + 3)/512 + 1/1024 + 3*sqrt(5)/1024)*atan(4*x/(2*sqrt(-2*sqrt(2)*sqrt(sq
rt(5) + 3) + 1 + 3*sqrt(5)) + sqrt(2)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(2)*sq
rt(sqrt(5) + 3) + 1 + 3*sqrt(5))) - 4*sqrt(5)*sqrt(1 + sqrt(5))/(2*sqrt(-2
*sqrt(2)*sqrt(sqrt(5) + 3) + 1 + 3*sqrt(5)) + sqrt(2)*sqrt(sqrt(5) + 3)*sq
rt(-2*sqrt(2)*sqrt(sqrt(5) + 3) + 1 + 3*sqrt(5))) + 3*sqrt(2)*sqrt(1 + sqr
t(5))*sqrt(sqrt(5) + 3)/(2*sqrt(-2*sqrt(2)*sqrt(sqrt(5) + 3) + 1 + 3*sqrt(
5)) + sqrt(2)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(2)*sqrt(sqrt(5) + 3) + 1 + 3*
sqrt(5)))) - 2*sqrt(-sqrt(2)*sqrt(sqrt(5) + 3)/512 + 1/1024 + 3*sqrt(5)/10
24)*atan(4*x/(2*sqrt(-2*sqrt(2)*sqrt(sqrt(5) + 3) + 1 + 3*sqrt(5)) + sqrt(
2)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(2)*sqrt(sqrt(5) + 3) + 1 + 3*sqrt(5))) -
 3*sqrt(2)*sqrt(1 + sqrt(5))*sqrt(sqrt(5) + 3)/(2*sqrt(-2*sqrt(2)*sqrt(sqr
t(5) + 3) + 1 + 3*sqrt(5)) + sqrt(2)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(2)*sqr
t(sqrt(5) + 3) + 1 + 3*sqrt(5))) + 4*sqrt(5)*sqrt(1 + sqrt(5))/(2*sqrt(...
```

Maxima [F]

$$\int \frac{x^2}{4+(1+x^2)^4}\,dx=\int \frac{x^2}{(x^2+1)^4+4}\,dx$$

input
```
integrate(x^2/(4+(x^2+1)^4),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 1/8*integrate((x^2 + 5)/(x^4 + 4*x^2 + 5), x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{4+(1+x^2)^4}\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{80}\sqrt{25\sqrt{5}-41}\arctan\left(\frac{5^{\frac{3}{4}}\left(x+5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{80}\sqrt{25\sqrt{5}-41}\arctan\left(\frac{5^{\frac{3}{4}}\left(x-5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{160}\sqrt{25\sqrt{5}+41}\log\left(x^2+2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right) - \frac{1}{160}\sqrt{25\sqrt{5}+41}\log\left(x^2-2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right)$$

input

```
integrate(x^2/(4+(x^2+1)^4),x, algorithm="giac")
```

output

```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 1/80*sqrt(25*sqrt(5) - 41)*arctan(1/5*5^(3/4)*
(x + 5^(1/4)*sqrt(-1/5*sqrt(5) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)) - 1/80*sqr
t(25*sqrt(5) - 41)*arctan(1/5*5^(3/4)*(x - 5^(1/4)*sqrt(-1/5*sqrt(5) + 1/2
))/sqrt(1/5*sqrt(5) + 1/2)) + 1/160*sqrt(25*sqrt(5) + 41)*log(x^2 + 2*5^(1
/4)*x*sqrt(-1/5*sqrt(5) + 1/2) + sqrt(5)) - 1/160*sqrt(25*sqrt(5) + 41)*lo
g(x^2 - 2*5^(1/4)*x*sqrt(-1/5*sqrt(5) + 1/2) + sqrt(5))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{4+(1+x^2)^4}\,dx = \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^3}{2}+\frac{\sqrt{2}\,x}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{32} - \mathrm{atanh}\left(\frac{128\sqrt{5}\,x\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}}}{10240\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}}\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}-20} + \frac{128\sqrt{5}\,x\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}}{10240\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}}\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}-20}\right)\left(2\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}} - 2\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}\right) + \mathrm{atanh}\left(\frac{128\sqrt{5}\,x\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}}}{10240\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}}\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}+20} - \frac{128\sqrt{5}\,x\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}}{10240\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}}\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}+20}\right)\left(2\sqrt{\frac{1}{1024}-\frac{\sqrt{5}}{1024}} + 2\sqrt{\frac{\sqrt{5}}{1024}+\frac{1}{1024}}\right)$$

input
```
int(x^2/((x^2 + 1)^4 + 4),x)
```

output
```
(2^(1/2)*(2*atan((2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) + 2*atan((2^(1/2)*x)/2))
)/32 - atanh((128*5^(1/2)*x*(1/1024 - 5^(1/2)/1024)^(1/2))/(10240*(1/1024
- 5^(1/2)/1024)^(1/2)*(5^(1/2)/1024 + 1/1024)^(1/2) - 20) + (128*5^(1/2)*x
*(5^(1/2)/1024 + 1/1024)^(1/2))/(10240*(1/1024 - 5^(1/2)/1024)^(1/2)*(5^(1
/2)/1024 + 1/1024)^(1/2) - 20))*(2*(1/1024 - 5^(1/2)/1024)^(1/2) - 2*(5^(1
/2)/1024 + 1/1024)^(1/2)) + atanh((128*5^(1/2)*x*(1/1024 - 5^(1/2)/1024)^(
1/2))/(10240*(1/1024 - 5^(1/2)/1024)^(1/2)*(5^(1/2)/1024 + 1/1024)^(1/2) +
 20) - (128*5^(1/2)*x*(5^(1/2)/1024 + 1/1024)^(1/2))/(10240*(1/1024 - 5^(1
/2)/1024)^(1/2)*(5^(1/2)/1024 + 1/1024)^(1/2) + 20))*(2*(1/1024 - 5^(1/2)/
1024)^(1/2) + 2*(5^(1/2)/1024 + 1/1024)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.49

$$\int \frac{x^2}{4+(1+x^2)^4}\,dx$$

$$= \frac{\sqrt{2}\left(-4atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)+4atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)-2\sqrt{\sqrt{5}+2}\,\sqrt{5}\,atan\left(\frac{\sqrt{\sqrt{5}-2}\,\sqrt{2}-2x}{\sqrt{\sqrt{5}+2}\,\sqrt{2}}\right)+6\sqrt{\sqrt{5}+2}\,atan\left(\frac{\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+}}\right.\right.}{}$$

input
```
int(x^2/(4+(x^2+1)^4),x)
```

output
```
(sqrt(2)*( - 4*atan((sqrt(2) - 2*x)/sqrt(2)) + 4*atan((sqrt(2) + 2*x)/sqrt
(2)) - 2*sqrt(sqrt(5) + 2)*sqrt(5)*atan((sqrt(sqrt(5) - 2)*sqrt(2) - 2*x)/
(sqrt(sqrt(5) + 2)*sqrt(2))) + 6*sqrt(sqrt(5) + 2)*atan((sqrt(sqrt(5) - 2)
*sqrt(2) - 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) + 2*sqrt(sqrt(5) + 2)*sqrt(5)
*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) - 6*s
qrt(sqrt(5) + 2)*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5) + 2)
*sqrt(2))) + sqrt(sqrt(5) - 2)*sqrt(5)*log( - sqrt(sqrt(5) - 2)*sqrt(2)*x
+ sqrt(5) + x**2) - sqrt(sqrt(5) - 2)*sqrt(5)*log(sqrt(sqrt(5) - 2)*sqrt(2
)*x + sqrt(5) + x**2) + 3*sqrt(sqrt(5) - 2)*log( - sqrt(sqrt(5) - 2)*sqrt(
2)*x + sqrt(5) + x**2) - 3*sqrt(sqrt(5) - 2)*log(sqrt(sqrt(5) - 2)*sqrt(2)
*x + sqrt(5) + x**2)))/64
```

3.154 $\int \frac{1}{4+(1+x^2)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 173

$$\int \frac{1}{4+(1+x^2)^4} dx = -\frac{1}{16}\sqrt{\frac{1}{5}\left(1+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right) + \frac{1}{16}\sqrt{\frac{1}{5}\left(1+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}+2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}} + \frac{1}{16}\sqrt{\frac{1}{5}\left(-1+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}x}{\sqrt{5}+x^2}\right)$$

output
```
-1/80*(5+5*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)-2*x)/(4+2*5^(1/2))^
(1/2))+1/80*(5+5*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)+2*x)/(4+2*5^(
1/2))^(1/2))+1/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)+1/80*(-5+5*5^(1/2))^(
1/2)*arctanh((-4+2*5^(1/2))^(1/2)*x/(5^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.49

$$\int \frac{1}{4+(1+x^2)^4}\,dx = \frac{1}{160}\left((6-2i)\sqrt{2-i}\arctan\left(\frac{x}{\sqrt{2-i}}\right)+(6+2i)\sqrt{2+i}\arctan\left(\frac{x}{\sqrt{2+i}}\right)+5\sqrt{2}\left(-\log\left(-1+\sqrt{2}x-x^2\right)+\log\left(1+\sqrt{2}x+x^2\right)\right)\right)$$

input
```
Integrate[(4 + (1 + x^2)^4)^(-1),x]
```

output
```
((6 - 2*I)*Sqrt[2 - I]*ArcTan[x/Sqrt[2 - I]] + (6 + 2*I)*Sqrt[2 + I]*ArcTa
n[x/Sqrt[2 + I]] + 5*Sqrt[2]*(-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]
*x + x^2]))/160
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^4+4}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{1-x^2}{8(x^4+1)}+\frac{x^2+3}{8(x^4+4x^2+5)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{16}\sqrt{\frac{1}{5}\left(1+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{5}-2\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)+\frac{1}{16}\sqrt{\frac{1}{5}\left(1+\sqrt{5}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{5}-2\right)}}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{1}{32}\sqrt{\frac{1}{5}\left(\sqrt{5}-1\right)}\log\left(x^2-\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)+\frac{1}{32}\sqrt{\frac{1}{5}\left(\sqrt{5}-1\right)}\log\left(x^2+\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)$$

input
```
Int[(4 + (1 + x^2)^4)^(-1),x]
```

output
```
-1/16*(Sqrt[(1 + Sqrt[5])/5]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] - 2*x)/Sqrt[2*
(2 + Sqrt[5])]]) + (Sqrt[(1 + Sqrt[5])/5]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] +
 2*x)/Sqrt[2*(2 + Sqrt[5])]])/16 - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) +
 Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2]) - (Sqrt[(-1 + Sqrt[5])/5]*Log[Sqrt[
5] - Sqrt[2*(-2 + Sqrt[5])]*x + x^2])/32 + (Sqrt[(-1 + Sqrt[5])/5]*Log[Sqr
t[5] + Sqrt[2*(-2 + Sqrt[5])]*x + x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(20\_Z^4+4\_Z^2+1\right)}\_R\ln\left(10\_R^3+7\_R+2x\right)\right)}{16}+\frac{\sqrt{2}\,\ln\left(x^2+x\sqrt{2}+1\right)}{32}-\frac{\sqrt{2}\,\ln\left(x^2-x\sqrt{2}+1\right)}{32}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}$

input
```
int(1/(4+(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(10*_R^3+7*_R+2*x),_R=RootOf(20*_Z^4+4*_Z^2+1))+1/32*2^(1/2)
*ln(x^2+x*2^(1/2)+1)-1/32*2^(1/2)*ln(x^2-x*2^(1/2)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.26

$$\begin{aligned}
&\int \frac{1}{4+(1+x^2)^4}\,dx \\
&= \frac{1}{16}\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{5}}\arctan\left(\frac{1}{2}\left(3\sqrt{5}x+5\left(\sqrt{5}-2\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{1}{5}}-5x\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{5}}\right) \\
&\quad-\frac{1}{16}\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{5}}\arctan\left(-\frac{1}{2}\left(3\sqrt{5}x-5\left(\sqrt{5}-2\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{1}{5}}-5x\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{5}}\right) \\
&\quad-\frac{1}{32}\sqrt{\frac{1}{5}\sqrt{5}-\frac{1}{5}}\log\left(2x^2+\left(\sqrt{5}x-5x\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{1}{5}}+2\sqrt{5}\right) \\
&\quad+\frac{1}{32}\sqrt{\frac{1}{5}\sqrt{5}-\frac{1}{5}}\log\left(2x^2-\left(\sqrt{5}x-5x\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{1}{5}}+2\sqrt{5}\right) \\
&\quad+\frac{1}{32}\sqrt{2}\log\left(\frac{x^4+4x^2+2\sqrt{2}(x^3+x)+1}{x^4+1}\right)
\end{aligned}$$

input
```
integrate(1/(4+(x^2+1)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(1/5*sqrt(5) + 1/5)*arctan(1/2*(3*sqrt(5)*x + 5*(sqrt(5) - 2)*sqr
t(1/5*sqrt(5) - 1/5) - 5*x)*sqrt(1/5*sqrt(5) + 1/5)) - 1/16*sqrt(1/5*sqrt(
5) + 1/5)*arctan(-1/2*(3*sqrt(5)*x - 5*(sqrt(5) - 2)*sqrt(1/5*sqrt(5) - 1/
5) - 5*x)*sqrt(1/5*sqrt(5) + 1/5)) - 1/32*sqrt(1/5*sqrt(5) - 1/5)*log(2*x^
2 + (sqrt(5)*x - 5*x)*sqrt(1/5*sqrt(5) - 1/5) + 2*sqrt(5)) + 1/32*sqrt(1/5
*sqrt(5) - 1/5)*log(2*x^2 - (sqrt(5)*x - 5*x)*sqrt(1/5*sqrt(5) - 1/5) + 2*
sqrt(5)) + 1/32*sqrt(2)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 1)/(x^4 +
 1))
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

$$\int \frac{1}{4+(1+x^2)^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \operatorname{RootSum}\left(1310720t^4+1024t^2+1,\left(t\mapsto t\log\left(20480t^3+56t+x\right)\right)\right)$$

input
```
integrate(1/(4+(x**2+1)**4),x)
```

output
```
-sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/
32 + RootSum(1310720*_t**4 + 1024*_t**2 + 1, Lambda(_t, _t*log(20480*_t**3
 + 56*_t + x)))
```

Maxima [F]

$$\int \frac{1}{4+(1+x^2)^4}\,dx = \int \frac{1}{(x^2+1)^4+4}\,dx$$

input
```
integrate(1/(4+(x^2+1)^4),x, algorithm="maxima")
```

output
```
1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqrt(2)*log(x^2 - sqrt(2)*x +
 1) + 1/8*integrate((x^2 + 3)/(x^4 + 4*x^2 + 5), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{1}{4+(1+x^2)^4}\,dx = &-\frac{1}{80}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{5^{\frac{3}{4}}\left(x+5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right)\\
&-\frac{1}{80}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{5^{\frac{3}{4}}\left(x-5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right)\\
&-\frac{1}{160}\sqrt{5\sqrt{5}+11}\log\left(x^2+2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right)\\
&+\frac{1}{160}\sqrt{5\sqrt{5}+11}\log\left(x^2-2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right)\\
&+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)\end{aligned}$$

input
```
integrate(1/(4+(x^2+1)^4),x, algorithm="giac")
```

output
```
-1/80*sqrt(5*sqrt(5) - 11)*arctan(1/5*5^(3/4)*(x + 5^(1/4)*sqrt(-1/5*sqrt(
5) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)) - 1/80*sqrt(5*sqrt(5) - 11)*arctan(1/5
*5^(3/4)*(x - 5^(1/4)*sqrt(-1/5*sqrt(5) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)) -
 1/160*sqrt(5*sqrt(5) + 11)*log(x^2 + 2*5^(1/4)*x*sqrt(-1/5*sqrt(5) + 1/2)
 + sqrt(5)) + 1/160*sqrt(5*sqrt(5) + 11)*log(x^2 - 2*5^(1/4)*x*sqrt(-1/5*s
qrt(5) + 1/2) + sqrt(5)) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 1069, normalized size of antiderivative = 6.18

$$\int \frac{1}{4+\left(1+x^2\right)^4}\,dx = \text{Too large to display}$$

input
```
int(1/((x^2 + 1)^4 + 4),x)
```

output
```
atan((((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2))*(8
*x + ((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2))*((2
097152*x - (2147483648*x*((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120
- 1/5120)^(1/2)) - 67108864)*((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5
120 - 1/5120)^(1/2)))*((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1
/5120)^(1/2))^3 - 256))*1i + ((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5
120 - 1/5120)^(1/2))*(8*x + ((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/51
20 - 1/5120)^(1/2))*((2097152*x - (2147483648*x*((- 5^(1/2)/5120 - 1/5120)
^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2)) + 67108864)*((- 5^(1/2)/5120 - 1/5
120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2)))*((- 5^(1/2)/5120 - 1/5120)^(1
/2) + (5^(1/2)/5120 - 1/5120)^(1/2))^3 + 256))*1i)/((((- 5^(1/2)/5120 - 1/5
120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2))*(8*x + ((- 5^(1/2)/5120 - 1/51
20)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2))*((2097152*x - (2147483648*x*((-
 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2)) - 67108864)
*((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2)))*((- 5^
(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2))^3 - 256)) - ((
- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2))*(8*x + ((-
 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120)^(1/2))*((2097152*x
 - (2147483648*x*((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 - 1/5120
)^(1/2)) + 67108864)*((- 5^(1/2)/5120 - 1/5120)^(1/2) + (5^(1/2)/5120 -...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.50

$$\int \frac{1}{4+\left(1+x^2\right)^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{\sqrt{5}+2}\,\sqrt{5}\,atan\left(\frac{\sqrt{\sqrt{5}-2}\,\sqrt{2}-2x}{\sqrt{\sqrt{5}+2}\,\sqrt{2}}\right)-10\sqrt{\sqrt{5}+2}\,atan\left(\frac{\sqrt{\sqrt{5}-2}\,\sqrt{2}-2x}{\sqrt{\sqrt{5}+2}\,\sqrt{2}}\right)-2\sqrt{\sqrt{5}+2}\,\sqrt{5}\,atan\left(\frac{\sqrt{\sqrt{5}-}}{\sqrt{\sqrt{}}}\right.\right.}{}$$

input
```
int(1/(4+(x^2+1)^4),x)
```

output
```
(sqrt(2)*(2*sqrt(sqrt(5) + 2)*sqrt(5)*atan((sqrt(sqrt(5) - 2)*sqrt(2) - 2*
x)/(sqrt(sqrt(5) + 2)*sqrt(2))) - 10*sqrt(sqrt(5) + 2)*atan((sqrt(sqrt(5)
- 2)*sqrt(2) - 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) - 2*sqrt(sqrt(5) + 2)*sqr
t(5)*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2))) +
 10*sqrt(sqrt(5) + 2)*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5)
 + 2)*sqrt(2))) - sqrt(sqrt(5) - 2)*sqrt(5)*log( - sqrt(sqrt(5) - 2)*sqrt(
2)*x + sqrt(5) + x**2) + sqrt(sqrt(5) - 2)*sqrt(5)*log(sqrt(sqrt(5) - 2)*s
qrt(2)*x + sqrt(5) + x**2) - 5*sqrt(sqrt(5) - 2)*log( - sqrt(sqrt(5) - 2)*
sqrt(2)*x + sqrt(5) + x**2) + 5*sqrt(sqrt(5) - 2)*log(sqrt(sqrt(5) - 2)*sq
rt(2)*x + sqrt(5) + x**2) - 10*log( - sqrt(2)*x + x**2 + 1) + 10*log(sqrt(
2)*x + x**2 + 1)))/320
```

3.155 $\int \frac{1}{x^2\left(4+\left(1+x^2\right)^4\right)} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 201

$$\int \frac{1}{x^2\left(4+\left(1+x^2\right)^4\right)} \, dx = -\frac{1}{5x} + \frac{1}{80}\sqrt{\frac{1}{5}\left(11+5\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right) - \frac{1}{80}\sqrt{\frac{1}{5}\left(11+5\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}+2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right) + \frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{1}{80}\sqrt{\frac{1}{5}\left(-11+5\sqrt{5}\right)}\text{arctanh}\left(\frac{\sqrt{2\left(-2+\sqrt{5}\right)}x}{\sqrt{5}+x^2}\right)$$

output
```
-1/5/x+1/400*(55+25*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)-2*x)/(4+2*
5^(1/2))^(1/2))-1/400*(55+25*5^(1/2))^(1/2)*arctan(((-4+2*5^(1/2))^(1/2)+2
*x)/(4+2*5^(1/2))^(1/2))-1/16*arctan(-1+x*2^(1/2))*2^(1/2)-1/16*arctan(1+x
*2^(1/2))*2^(1/2)-1/400*(-55+25*5^(1/2))^(1/2)*arctanh((-4+2*5^(1/2))^(1/2
)*x/(5^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2 \left(4+(1+x^2)^4\right)} \, dx = -\frac{80+(7+i)\sqrt{2-i}x \arctan\left(\frac{x}{\sqrt{2-i}}\right)+(7-i)\sqrt{2+i}x \arctan\left(\frac{x}{\sqrt{2+i}}\right)-25\sqrt{2}x \arctan\left(1-\sqrt{2}x\right)+25}{400x}$$

input
```
Integrate[1/(x^2*(4 + (1 + x^2)^4)),x]
```

output
```
-1/400*(80 + (7 + I)*Sqrt[2 - I]*x*ArcTan[x/Sqrt[2 - I]] + (7 - I)*Sqrt[2
+ I]*x*ArcTan[x/Sqrt[2 + I]] - 25*Sqrt[2]*x*ArcTan[1 - Sqrt[2]*x] + 25*Sqr
t[2]*x*ArcTan[1 + Sqrt[2]*x])/x
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left((x^2+1)^4+4\right)} \, dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{1}{5x^2}+\frac{-3x^2-7}{40\left(x^4+4x^2+5\right)}+\frac{-x^2-1}{8\left(x^4+1\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{80}\sqrt{\frac{1}{5}\left(11+5\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{5}-2\right)}-2x}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)-\frac{1}{80}\sqrt{\frac{1}{5}\left(11+5\sqrt{5}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{5}-2\right)}}{\sqrt{2\left(2+\sqrt{5}\right)}}\right)+\frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}+\frac{1}{160}\sqrt{\frac{1}{5}\left(5\sqrt{5}-11\right)}\log\left(x^2-\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)-\frac{1}{160}\sqrt{\frac{1}{5}\left(5\sqrt{5}-11\right)}\log\left(x^2+\sqrt{2\left(\sqrt{5}-2\right)}x+\sqrt{5}\right)-\frac{1}{5x}$$

input
```
Int[1/(x^2*(4 + (1 + x^2)^4)),x]
```

output
```
-1/5*1/x + (Sqrt[(11 + 5*Sqrt[5])/5]*ArcTan[(Sqrt[2*(-2 + Sqrt[5])] - 2*x)
/Sqrt[2*(2 + Sqrt[5])]])/80 - (Sqrt[(11 + 5*Sqrt[5])/5]*ArcTan[(Sqrt[2*(-2
 + Sqrt[5])] + 2*x)/Sqrt[2*(2 + Sqrt[5])]])/80 + ArcTan[1 - Sqrt[2]*x]/(8*
Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/(8*Sqrt[2]) + (Sqrt[(-11 + 5*Sqrt[5])/5]*
Log[Sqrt[5] - Sqrt[2*(-2 + Sqrt[5])]*x + x^2])/160 - (Sqrt[(-11 + 5*Sqrt[5
])/5]*Log[Sqrt[5] + Sqrt[2*(-2 + Sqrt[5])]*x + x^2])/160
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result
risch	$-\frac{1}{5x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(20\_Z^4+44\_Z^2+25\right)}\_R\ln\left(10\_R^3-3\_R+10x\right)\right)}{80}-\frac{\sqrt{2}\arctan\left(\frac{x\sqrt{2}}{2}\right)}{16}-\frac{\sqrt{2}\arctan\left(\frac{x^3\sqrt{2}}{2}+\frac{x\sqrt{2}}{2}\right)}{16}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}$

input
```
int(1/x^2/(4+(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x+1/80*sum(_R*ln(10*_R^3-3*_R+10*x),_R=RootOf(20*_Z^4+44*_Z^2+25))-1/
16*2^(1/2)*arctan(1/2*x*2^(1/2))-1/16*2^(1/2)*arctan(1/2*x^3*2^(1/2)+1/2*x
*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2\left(4+(1+x^2)^4\right)}\,dx =$$

$$-\frac{10\sqrt{2}x\arctan\left(\frac{1}{2}\sqrt{2}(x^3+x)\right)+10\sqrt{2}x\arctan\left(\frac{1}{2}\sqrt{2}x\right)+2x\sqrt{\sqrt{5}+\frac{11}{5}}\arctan\left(\frac{1}{2}\left(7\sqrt{5}x+5\left(\sqrt{5}\right.\right.\right.}{}$$

input
```
integrate(1/x^2/(4+(x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/160*(10*sqrt(2)*x*arctan(1/2*sqrt(2)*(x^3 + x)) + 10*sqrt(2)*x*arctan(1
/2*sqrt(2)*x) + 2*x*sqrt(sqrt(5) + 11/5)*arctan(1/2*(7*sqrt(5)*x + 5*(sqrt
(5) - 2)*sqrt(sqrt(5) - 11/5) - 15*x)*sqrt(sqrt(5) + 11/5)) - 2*x*sqrt(sqr
t(5) + 11/5)*arctan(-1/2*(7*sqrt(5)*x - 5*(sqrt(5) - 2)*sqrt(sqrt(5) - 11/
5) - 15*x)*sqrt(sqrt(5) + 11/5)) + x*sqrt(sqrt(5) - 11/5)*log(2*x^2 + (sqr
t(5)*x + 5*x)*sqrt(sqrt(5) - 11/5) + 2*sqrt(5)) - x*sqrt(sqrt(5) - 11/5)*l
og(2*x^2 - (sqrt(5)*x + 5*x)*sqrt(sqrt(5) - 11/5) + 2*sqrt(5)) + 32)/x
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^2\left(4+(1+x^2)^4\right)}\,dx = \frac{\sqrt{2}\left(-2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)-2\operatorname{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{\sqrt{2}x}{2}\right)\right)}{32} + \operatorname{RootSum}\left(32768000t^4+11264t^2+1,\left(t\mapsto t\log\left(512000t^3-24t+x\right)\right)\right)-\frac{1}{5x}$$

input
```
integrate(1/x**2/(4+(x**2+1)**4),x)
```

output
```
sqrt(2)*(-2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/32 +
 RootSum(32768000*_t**4 + 11264*_t**2 + 1, Lambda(_t, _t*log(512000*_t**3
- 24*_t + x))) - 1/(5*x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(4+(1+x^2)^4\right)}\,dx = \int \frac{1}{\left((x^2+1)^4+4\right)x^2}\,dx$$

input
```
integrate(1/x^2/(4+(x^2+1)^4),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 1/5/x - 1/40*integrate((3*x^2 + 7)/(x^4 + 4*x
^2 + 5), x)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(4+\left(1+x^2\right)^4\right)}\,dx = -\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) - \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{80}\sqrt{\sqrt{5}-1}\arctan\left(\frac{5^{\frac{3}{4}}\left(x+5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{80}\sqrt{\sqrt{5}-1}\arctan\left(\frac{5^{\frac{3}{4}}\left(x-5^{\frac{1}{4}}\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}\right)}{5\sqrt{\frac{1}{5}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{160}\sqrt{\sqrt{5}+1}\log\left(x^2+2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right) - \frac{1}{160}\sqrt{\sqrt{5}+1}\log\left(x^2-2\cdot 5^{\frac{1}{4}}x\sqrt{-\frac{1}{5}\sqrt{5}+\frac{1}{2}}+\sqrt{5}\right) - \frac{1}{5\,x}$$

input
```
integrate(1/x^2/(4+(x^2+1)^4),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 1/80*sqrt(sqrt(5) - 1)*arctan(1/5*5^(3/4)*(x
+ 5^(1/4)*sqrt(-1/5*sqrt(5) + 1/2))/sqrt(1/5*sqrt(5) + 1/2)) + 1/80*sqrt(s
qrt(5) - 1)*arctan(1/5*5^(3/4)*(x - 5^(1/4)*sqrt(-1/5*sqrt(5) + 1/2))/sqrt
(1/5*sqrt(5) + 1/2)) + 1/160*sqrt(sqrt(5) + 1)*log(x^2 + 2*5^(1/4)*x*sqrt(
-1/5*sqrt(5) + 1/2) + sqrt(5)) - 1/160*sqrt(sqrt(5) + 1)*log(x^2 - 2*5^(1/
4)*x*sqrt(-1/5*sqrt(5) + 1/2) + sqrt(5)) - 1/5/x
```

Mupad [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 1457, normalized size of antiderivative = 7.25

$$\int \frac{1}{x^2 \left(4+\left(1+x^2\right)^4\right)}\, dx = \text{Too large to display}$$

input
```
int(1/(x^2*((x^2 + 1)^4 + 4)),x)
```

output
```
atan((((8*x)/25 + (((16384*x)/25 - (((98566144*x)/25 + (2147483648*x*((- 5
^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2)) - 402
653184/5)*((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/12800
0)^(1/2)))*((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/1280
00)^(1/2)) - 4456448/25)*((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/2
5600 - 11/128000)^(1/2)))*((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/
25600 - 11/128000)^(1/2)) + 1024/25)*((- 5^(1/2)/25600 - 11/128000)^(1/2)
+ (5^(1/2)/25600 - 11/128000)^(1/2)))*((- 5^(1/2)/25600 - 11/128000)^(1/2)
 + (5^(1/2)/25600 - 11/128000)^(1/2))*1i + ((8*x)/25 + (((16384*x)/25 - ((
(98566144*x)/25 + (2147483648*x*((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^
(1/2)/25600 - 11/128000)^(1/2)) + 402653184/5)*((- 5^(1/2)/25600 - 11/1280
00)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2)))*((- 5^(1/2)/25600 - 11/128
000)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2)) + 4456448/25)*((- 5^(1/2)/
25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2)))*((- 5^(1/2)
/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2)) - 1024/25)*
((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2)))
*((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2))
*1i)/(((8*x)/25 + (((16384*x)/25 - (((98566144*x)/25 + (2147483648*x*((- 5
^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/128000)^(1/2)) - 402
653184/5)*((- 5^(1/2)/25600 - 11/128000)^(1/2) + (5^(1/2)/25600 - 11/12...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2 \left(4+\left(1+x^2\right)^4\right)}\, dx = \frac{100\sqrt{2}\, atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) x - 100\sqrt{2}\, atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) x + 2\sqrt{\sqrt{5}+2}\,\sqrt{10}\, atan\left(\frac{\sqrt{\sqrt{5}-2}\,\sqrt{2}-2x}{\sqrt{\sqrt{5}+2}\,\sqrt{2}}\right) x + 10\sqrt{\sqrt{5}+2}}{}$$

input
```
int(1/x^2/(4+(x^2+1)^4),x)
```

output
```
(100*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x - 100*sqrt(2)*atan((sqrt(2) +
 2*x)/sqrt(2))*x + 2*sqrt(sqrt(5) + 2)*sqrt(10)*atan((sqrt(sqrt(5) - 2)*sq
rt(2) - 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2)))*x + 10*sqrt(sqrt(5) + 2)*sqrt(2)
*atan((sqrt(sqrt(5) - 2)*sqrt(2) - 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2)))*x - 2
*sqrt(sqrt(5) + 2)*sqrt(10)*atan((sqrt(sqrt(5) - 2)*sqrt(2) + 2*x)/(sqrt(s
qrt(5) + 2)*sqrt(2)))*x - 10*sqrt(sqrt(5) + 2)*sqrt(2)*atan((sqrt(sqrt(5)
- 2)*sqrt(2) + 2*x)/(sqrt(sqrt(5) + 2)*sqrt(2)))*x - sqrt(sqrt(5) - 2)*sqr
t(10)*log( - sqrt(sqrt(5) - 2)*sqrt(2)*x + sqrt(5) + x**2)*x + sqrt(sqrt(5
) - 2)*sqrt(10)*log(sqrt(sqrt(5) - 2)*sqrt(2)*x + sqrt(5) + x**2)*x + 5*sq
rt(sqrt(5) - 2)*sqrt(2)*log( - sqrt(sqrt(5) - 2)*sqrt(2)*x + sqrt(5) + x**
2)*x - 5*sqrt(sqrt(5) - 2)*sqrt(2)*log(sqrt(sqrt(5) - 2)*sqrt(2)*x + sqrt(
5) + x**2)*x - 320)/(1600*x)
```

3.156 $\int \frac{x^4}{a-b(c+dx^2)^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 636

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx$$
$$= \frac{\left(\sqrt[4]{a}+\sqrt[4]{b}c\right)^{3/2}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{4a^{3/4}b^{5/8}d^{5/2}}$$
$$+\frac{\left(2\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}\right)\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}b^{5/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}d^{5/2}}$$
$$-\frac{\left(2\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}\right)\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}+\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}b^{5/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}d^{5/2}}$$
$$+\frac{\left(\sqrt[4]{a}-\sqrt[4]{b}c\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{4a^{3/4}b^{5/8}d^{5/2}}$$
$$+\frac{\left(2\sqrt[4]{b}c-\sqrt{\sqrt{a}+\sqrt{b}c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}\sqrt{d}x}{\sqrt{\sqrt{a}+\sqrt{b}c^2}+\sqrt[4]{b}dx^2}\right)}{4\sqrt{2}\sqrt{a}b^{5/8}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}d^{5/2}}$$

output
```
1/4*(a^(1/4)+b^(1/4)*c)^(3/2)*arctan(b^(1/8)*d^(1/2)*x/(a^(1/4)+b^(1/4)*c)
^(1/2))/a^(3/4)/b^(5/8)/d^(5/2)+1/8*(2*b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/
2))*arctan(((-b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)-2^(1/2)*b^(1/8)
*d^(1/2)*x)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2))*2^(1/2)/a^(1/2)
/b^(5/8)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)/d^(5/2)-1/8*(2*b^(1
/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))*arctan(((-b^(1/4)*c+(a^(1/2)+b^(1/2)*c^
2)^(1/2))^(1/2)+2^(1/2)*b^(1/8)*d^(1/2)*x)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2
)^(1/2))^(1/2))*2^(1/2)/a^(1/2)/b^(5/8)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(
1/2))^(1/2)/d^(5/2)+1/4*(a^(1/4)-b^(1/4)*c)^(3/2)*arctanh(b^(1/8)*d^(1/2)*
x/(a^(1/4)-b^(1/4)*c)^(1/2))/a^(3/4)/b^(5/8)/d^(5/2)+1/8*(2*b^(1/4)*c-(a^(
1/2)+b^(1/2)*c^2)^(1/2))*arctanh(2^(1/2)*b^(1/8)*(-b^(1/4)*c+(a^(1/2)+b^(1
/2)*c^2)^(1/2))^(1/2)*d^(1/2)*x/((a^(1/2)+b^(1/2)*c^2)^(1/2)+b^(1/4)*d*x^2
))*2^(1/2)/a^(1/2)/b^(5/8)/(-b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)/
d^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.18

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx = -\frac{\text{RootSum}\left[a-bc^4-4bc^3d\#1^2-6bc^2d^2\#1^4-4bcd^3\#1^6-bd^4\#1^8\&,\frac{\log(x-\#1)\#1^3}{c^3+3c^2d\#1^2+3cd^2\#1^4+d^3\#1^6}\&\right]}{8bd}$$

input
```
Integrate[x^4/(a - b*(c + d*x^2)^4),x]
```

output
```
-1/8*RootSum[a - b*c^4 - 4*b*c^3*d*#1^2 - 6*b*c^2*d^2*#1^4 - 4*b*c*d^3*#1^
6 - b*d^4*#1^8 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1^2 + 3*c*d^2*#1^4 +
 d^3*#1^6) & ]/(b*d)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2285 vs. 2(636) = 1272.

Time = 7.31 (sec) , antiderivative size = 2285, normalized size of antiderivative = 3.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx$$

$$\Big\downarrow \text{ 7291}$$

$$\int \left(\frac{c^2}{d^2\left(a-b\left(c+dx^2\right)^4\right)}-\frac{2c\left(c+dx^2\right)}{d^2\left(a-b\left(c+dx^2\right)^4\right)}+\frac{\left(c+dx^2\right)^2}{d^2\left(a-b\left(c+dx^2\right)^4\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)c^2}{4a^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}d^{5/2}}-\frac{\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c^2}{4\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}d^{5/2}}+$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c^2}{4\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}d^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)c^2}{4a^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}d^{5/2}}-$$

$$\frac{\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c^2}{8\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}d^{5/2}}+$$

$$\frac{\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c^2}{8\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}d^{5/2}}+$$

$$\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)c}{2\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}d^{5/2}}+\frac{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c}{2\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{5/2}}-$$

$$\frac{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c}{2\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)c}{2\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}d^{5/2}}-$$

$$\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{5/2}}+$$

$$\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{5/2}}+$$

$$\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{4\sqrt[4]{a}b^{5/8}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}d^{5/2}}+\frac{\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{4\sqrt{2}b^{5/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}d^{5/2}}-$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{4\sqrt{2}b^{5/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}d^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{4\sqrt[4]{a}b^{5/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}d^{5/2}}+$$

$$\frac{\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)}{\cdots}-$$

input
```
Int[x^4/(a - b*(c + d*x^2)^4),x]
```

output
```
ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4) + b^(1/4)*c]]/(4*a^(1/4)*b^(5/8)*S
qrt[a^(1/4) + b^(1/4)*c]*d^(5/2)) + (c*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(
1/4) + b^(1/4)*c]])/(2*Sqrt[a]*b^(3/8)*Sqrt[a^(1/4) + b^(1/4)*c]*d^(5/2))
+ (c^2*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4) + b^(1/4)*c]])/(4*a^(3/4)*b
^(1/8)*Sqrt[a^(1/4) + b^(1/4)*c]*d^(5/2)) + ArcTan[(Sqrt[-(b^(1/4)*c) + Sq
rt[Sqrt[a] + Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + S
qrt[Sqrt[a] + Sqrt[b]*c^2]]]/(4*Sqrt[2]*b^(5/8)*Sqrt[Sqrt[a] + Sqrt[b]*c^2
]*Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*d^(5/2)) - (c^2*ArcTan[(Sq
rt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x
)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[a]*b^(1/
8)*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2
]]*d^(5/2)) + (c*Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*ArcTan[(Sqr
t[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)
/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(2*Sqrt[2]*Sqrt[a]*b^(3/8
)*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*d^(5/2)) - ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[
Sqrt[a] + Sqrt[b]*c^2]] + Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt
[Sqrt[a] + Sqrt[b]*c^2]]]/(4*Sqrt[2]*b^(5/8)*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*S
qrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*d^(5/2)) + (c^2*ArcTan[(Sqrt[
-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] + Sqrt[2]*b^(1/8)*Sqrt[d]*x)/S
qrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[a]*b^(1/...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^4\_Z^8+4bc\,d^3\_Z^6+6b\,c^2d^2\_Z^4+4b\,c^3d\_Z^2+b\,c^4-a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{-d^3\_R^7-3c\,d^2\_R^5-3c^2d\_R^3-c^3\_R}}{8db}$	107
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^4\_Z^8+4bc\,d^3\_Z^6+6b\,c^2d^2\_Z^4+4b\,c^3d\_Z^2+b\,c^4-a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{-d^3\_R^7-3c\,d^2\_R^5-3c^2d\_R^3-c^3\_R}}{8db}$	107

input
```
int(x^4/(a-b*(d*x^2+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/b*sum(_R^4/(-_R^7*d^3-3*_R^5*c*d^2-3*_R^3*c^2*d-_R*c^3)*ln(x-_R),_R=
RootOf(_Z^8*b*d^4+4*_Z^6*b*c*d^3+6*_Z^4*b*c^2*d^2+4*_Z^2*b*c^3*d+b*c^4-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(a-b*(d*x^2+c)^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(a-b*(d*x**2+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx = \int -\frac{x^4}{\left(dx^2+c\right)^4b-a}\,dx$$

input
```
integrate(x^4/(a-b*(d*x^2+c)^4),x, algorithm="maxima")
```

output
```
-integrate(x^4/((d*x^2 + c)^4*b - a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(a-b*(d*x^2+c)^4),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 2511, normalized size of antiderivative = 3.95

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a - b*(c + d*x^2)^4),x)
```

output
```
symsum(log(8*a*b^5*c^7*d^13 - 4*b^6*c^11*d^13 - 262144*root(16777216*a^6*b
^5*d^20*z^8 + 3145728*a^5*b^4*c*d^15*z^6 + 172032*a^4*b^3*c^2*d^10*z^4 - 8
192*a^3*b^4*c^6*d^10*z^4 + 2560*a^3*b^2*c^3*d^5*z^2 + 1536*a^2*b^3*c^7*d^5
*z^2 - 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 - a^3, z, k)^6*a^5*b^7*d^28 -
4*a^2*b^4*c^3*d^13 - 960*root(16777216*a^6*b^5*d^20*z^8 + 3145728*a^5*b^4*
c*d^15*z^6 + 172032*a^4*b^3*c^2*d^10*z^4 - 8192*a^3*b^4*c^6*d^10*z^4 + 256
0*a^3*b^2*c^3*d^5*z^2 + 1536*a^2*b^3*c^7*d^5*z^2 - 3*a*b^2*c^8 + 3*a^2*b*c
^4 + b^3*c^12 - a^3, z, k)^2*a^3*b^5*c^2*d^18 + 640*root(16777216*a^6*b^5*
d^20*z^8 + 3145728*a^5*b^4*c*d^15*z^6 + 172032*a^4*b^3*c^2*d^10*z^4 - 8192
*a^3*b^4*c^6*d^10*z^4 + 2560*a^3*b^2*c^3*d^5*z^2 + 1536*a^2*b^3*c^7*d^5*z^
2 - 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 - a^3, z, k)^2*a^2*b^6*c^6*d^18 +
 32768*root(16777216*a^6*b^5*d^20*z^8 + 3145728*a^5*b^4*c*d^15*z^6 + 17203
2*a^4*b^3*c^2*d^10*z^4 - 8192*a^3*b^4*c^6*d^10*z^4 + 2560*a^3*b^2*c^3*d^5*
z^2 + 1536*a^2*b^3*c^7*d^5*z^2 - 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 - a^
3, z, k)^4*a^3*b^7*c^5*d^23 + 262144*root(16777216*a^6*b^5*d^20*z^8 + 3145
728*a^5*b^4*c*d^15*z^6 + 172032*a^4*b^3*c^2*d^10*z^4 - 8192*a^3*b^4*c^6*d^
10*z^4 + 2560*a^3*b^2*c^3*d^5*z^2 + 1536*a^2*b^3*c^7*d^5*z^2 - 3*a*b^2*c^8
 + 3*a^2*b*c^4 + b^3*c^12 - a^3, z, k)^6*a^4*b^8*c^4*d^28 - 8*root(1677721
6*a^6*b^5*d^20*z^8 + 3145728*a^5*b^4*c*d^15*z^6 + 172032*a^4*b^3*c^2*d^10*
z^4 - 8192*a^3*b^4*c^6*d^10*z^4 + 2560*a^3*b^2*c^3*d^5*z^2 + 1536*a^2*b...
```

Reduce [F]

$$\int \frac{x^4}{a-b\left(c+dx^2\right)^4}\,dx = \int \frac{x^4}{a-b\left(d\,x^2+c\right)^4}dx$$

input
```
int(x^4/(a-b*(d*x^2+c)^4),x)
```

output
```
int(x^4/(a-b*(d*x^2+c)^4),x)
```

3.157 $\int \frac{x^2}{a-b(c+dx^2)^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 550

$$\int \frac{x^2}{a-b\left(c+dx^2\right)^4}\,dx=-\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{4a^{3/4}b^{3/8}d^{3/2}}-\frac{\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}d^{3/2}}+\frac{\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}+\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}d^{3/2}}+\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{4a^{3/4}b^{3/8}d^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}\sqrt{d}x}{\sqrt{\sqrt{a}+\sqrt{b}c^2}+\sqrt[4]{b}dx^2}\right)}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}d^{3/2}}$$

output
```
-1/4*(a^(1/4)+b^(1/4)*c)^(1/2)*arctan(b^(1/8)*d^(1/2)*x/(a^(1/4)+b^(1/4)*c
)^(1/2))/a^(3/4)/b^(3/8)/d^(3/2)-1/8*arctan(((-b^(1/4)*c+(a^(1/2)+b^(1/2)*
c^2)^(1/2))^(1/2)-2^(1/2)*b^(1/8)*d^(1/2)*x)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c
^2)^(1/2))^(1/2))*2^(1/2)/a^(1/2)/b^(3/8)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)
^(1/2))^(1/2)/d^(3/2)+1/8*arctan(((-b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))
^(1/2)+2^(1/2)*b^(1/8)*d^(1/2)*x)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^
(1/2))*2^(1/2)/a^(1/2)/b^(3/8)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/
2)/d^(3/2)+1/4*(a^(1/4)-b^(1/4)*c)^(1/2)*arctanh(b^(1/8)*d^(1/2)*x/(a^(1/4
)-b^(1/4)*c)^(1/2))/a^(3/4)/b^(3/8)/d^(3/2)-1/8*arctanh(2^(1/2)*b^(1/8)*(-
b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)*d^(1/2)*x/((a^(1/2)+b^(1/2)*c
^2)^(1/2)+b^(1/4)*d*x^2))*2^(1/2)/a^(1/2)/b^(3/8)/(-b^(1/4)*c+(a^(1/2)+b^(
1/2)*c^2)^(1/2))^(1/2)/d^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.20

$$\int \frac{x^2}{a-b\left(c+dx^2\right)^4}\,dx = -\frac{\text{RootSum}\left[a-bc^4-4bc^3d\#1^2-6bc^2d^2\#1^4-4bcd^3\#1^6-bd^4\#1^8\&,\frac{\log(x-\#1)\#1}{c^3+3c^2d\#1^2+3cd^2\#1^4+d^3\#1^6}\&\right]}{8bd}$$

input
```
Integrate[x^2/(a - b*(c + d*x^2)^4),x]
```

output
```
-1/8*RootSum[a - b*c^4 - 4*b*c^3*d*#1^2 - 6*b*c^2*d^2*#1^4 - 4*b*c*d^3*#1^
6 - b*d^4*#1^8 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1^2 + 3*c*d^2*#1^4 + d
^3*#1^6) & ]/(b*d)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1527 vs. 2(550) = 1100.

Time = 4.52 (sec) , antiderivative size = 1527, normalized size of antiderivative = 2.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a-b\left(c+dx^2\right)^4}dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(\frac{c+dx^2}{d\left(a-b\left(c+dx^2\right)^4\right)}-\frac{c}{d\left(a-b\left(c+dx^2\right)^4\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{4a^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}d^{3/2}}-\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{4\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}d^{3/2}}-$$

$$\frac{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{3/2}}+$$

$$\frac{c\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{4\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}d^{3/2}}+$$

$$\frac{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{3/2}}-$$

$$\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{4\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}d^{3/2}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{4a^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}d^{3/2}}+$$

$$\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{4\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}d^{3/2}}+$$

$$\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{3/2}}+$$

$$\frac{c\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}d^{3/2}}-$$

$$\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{a}}d^{3/2}}-$$

$$\frac{c\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}d^{3/2}}$$

input
```
Int[x^2/(a - b*(c + d*x^2)^4),x]
```

output
```
-1/4*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4) + b^(1/4)*c]]/(Sqrt[a]*b^(3/8
)*Sqrt[a^(1/4) + b^(1/4)*c]*d^(3/2)) - (c*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[
a^(1/4) + b^(1/4)*c]])/(4*a^(3/4)*b^(1/8)*Sqrt[a^(1/4) + b^(1/4)*c]*d^(3/2
)) + (c*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] - Sqrt[2]
*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqr
t[2]*Sqrt[a]*b^(1/8)*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*Sqrt[b^(1/4)*c + Sqrt[Sqr
t[a] + Sqrt[b]*c^2]]*d^(3/2)) - (Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c
^2]]*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] - Sqrt[2]*b^
(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqrt[2
]*Sqrt[a]*b^(3/8)*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*d^(3/2)) - (c*ArcTan[(Sqrt[-
(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] + Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sq
rt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[a]*b^(1/8)*S
qrt[Sqrt[a] + Sqrt[b]*c^2]*Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*d
^(3/2)) + (Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*ArcTan[(Sqrt[-(b^
(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] + Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[
b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[a]*b^(3/8)*Sqrt
[Sqrt[a] + Sqrt[b]*c^2]*d^(3/2)) + ArcTanh[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4
) - b^(1/4)*c]]/(4*Sqrt[a]*b^(3/8)*Sqrt[a^(1/4) - b^(1/4)*c]*d^(3/2)) - (c
*ArcTanh[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4) - b^(1/4)*c]])/(4*a^(3/4)*b^(1/8
)*Sqrt[a^(1/4) - b^(1/4)*c]*d^(3/2)) + (c*Log[Sqrt[Sqrt[a] + Sqrt[b]*c^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^8+4bc\,d^3\_Z^6+6b\,c^2d^2\_Z^4+4b\,c^3d\_Z^2+b\,c^4-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-d^3\_R^7-3c\,d^2\_R^5-3c^2d\_R^3-c^3\_R}}{8db}$	107
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^8+4bc\,d^3\_Z^6+6b\,c^2d^2\_Z^4+4b\,c^3d\_Z^2+b\,c^4-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-d^3\_R^7-3c\,d^2\_R^5-3c^2d\_R^3-c^3\_R}}{8db}$	107

input
```
int(x^2/(a-b*(d*x^2+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/b*sum(_R^2/(-_R^7*d^3-3*_R^5*c*d^2-3*_R^3*c^2*d-_R*c^3)*ln(x-_R),_R=
RootOf(_Z^8*b*d^4+4*_Z^6*b*c*d^3+6*_Z^4*b*c^2*d^2+4*_Z^2*b*c^3*d+b*c^4-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 73.64 (sec) , antiderivative size = 103351, normalized size of antiderivative = 187.91

$$\int \frac{x^2}{a-b\left(c+dx^2\right)^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^2/(a-b*(d*x^2+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.27

$$\int \frac{x^2}{a-b(c+dx^2)^4}\,dx =$$
$$-\operatorname{RootSum}\left(16777216t^8a^6b^3d^{12}-8192t^4a^3b^2c^2d^6+256t^2a^2bcd^3-a+bc^4,\left(t\mapsto t\log\left(x+\frac{-4194304}{}\right.\right.\right.$$

input
```
integrate(x**2/(a-b*(d*x**2+c)**4),x)
```

output
```
-RootSum(16777216*_t**8*a**6*b**3*d**12 - 8192*_t**4*a**3*b**2*c**2*d**6 +
 256*_t**2*a**2*b*c*d**3 - a + b*c**4, Lambda(_t, _t*log(x + (-4194304*_t*
*7*a**5*b**3*c**2*d**10 - 32768*_t**5*a**4*b**2*c*d**7 - 512*_t**3*a**3*b*
d**4 + 1024*_t**3*a**2*b**2*c**4*d**4 - 40*_t*a*b*c**3*d)/(a + 4*b*c**4)))
)
```

Maxima [F]

$$\int \frac{x^2}{a-b(c+dx^2)^4}\,dx = \int -\frac{x^2}{(dx^2+c)^4b-a}\,dx$$

input
```
integrate(x^2/(a-b*(d*x^2+c)^4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((d*x^2 + c)^4*b - a), x)
```

Giac [F]

$$\int \frac{x^2}{a-b(c+dx^2)^4}\,dx = \int -\frac{x^2}{(dx^2+c)^4b-a}\,dx$$

input
```
integrate(x^2/(a-b*(d*x^2+c)^4),x, algorithm="giac")
```

output
```
integrate(-x^2/((d*x^2 + c)^4*b - a), x)
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{a-b\,(c+dx^2)^4}\,dx=\sum_{k=1}^{8}\ln\Big(-a\,b^5\,d^{20}+b^6\,c^4\,d^{20}-\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)\,b^7\,c^6\,d^{22}\,x\,8$$
$$+\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)^2\,a^2\,b^6\,c\,d^{23}\,128$$
$$-\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)^2\,a\,b^7\,c^5\,d^{23}\,128$$
$$-\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)^5\,a^4\,b^7\,d^{28}\,x\,32768$$
$$-\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)^7\,a^5\,b^8\,c\,d^{31}\,x\,2097152$$
$$+\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)^3\,a^2\,b^7\,c^3\,d^{25}\,x\,1024$$
$$+\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)^5\,a^3\,b^8\,c^4\,d^{28}\,x\,32768$$
$$-\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)\,a\,b^6\,c^2\,d^{22}\,x\,24\Big)\,\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8-8192\,a^3\,b^2\,c^2\,d^6\,z^4+256\,a^2\,b\,c\,d^3\,z^2+b\,c^4-a,z,k\big)$$

input
```
int(x^2/(a - b*(c + d*x^2)^4),x)
```

output
```
symsum(log(b^6*c^4*d^20 - a*b^5*d^20 - 8*root(16777216*a^6*b^3*d^12*z^8 -
8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 - a, z, k)*b^7*c^6*
d^22*x + 128*root(16777216*a^6*b^3*d^12*z^8 - 8192*a^3*b^2*c^2*d^6*z^4 + 2
56*a^2*b*c*d^3*z^2 + b*c^4 - a, z, k)^2*a^2*b^6*c*d^23 - 128*root(16777216
*a^6*b^3*d^12*z^8 - 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4
 - a, z, k)^2*a*b^7*c^5*d^23 - 32768*root(16777216*a^6*b^3*d^12*z^8 - 8192
*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 - a, z, k)^5*a^4*b^7*d^
28*x - 2097152*root(16777216*a^6*b^3*d^12*z^8 - 8192*a^3*b^2*c^2*d^6*z^4 +
 256*a^2*b*c*d^3*z^2 + b*c^4 - a, z, k)^7*a^5*b^8*c*d^31*x + 1024*root(167
77216*a^6*b^3*d^12*z^8 - 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 +
b*c^4 - a, z, k)^3*a^2*b^7*c^3*d^25*x + 32768*root(16777216*a^6*b^3*d^12*z
^8 - 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 - a, z, k)^5*a
^3*b^8*c^4*d^28*x - 24*root(16777216*a^6*b^3*d^12*z^8 - 8192*a^3*b^2*c^2*d
^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 - a, z, k)*a*b^6*c^2*d^22*x)*root(167
77216*a^6*b^3*d^12*z^8 - 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 +
b*c^4 - a, z, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{x^2}{a-b\left(c+dx^2\right)^4}\,dx=\int \frac{x^2}{a-b\left(d\,x^2+c\right)^4}dx$$

input
```
int(x^2/(a-b*(d*x^2+c)^4),x)
```

output
```
int(x^2/(a-b*(d*x^2+c)^4),x)
```

3.158 $\int \frac{1}{a-b(c+dx^2)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 607

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}\,dx=\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{4a^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{a}+\sqrt{b}c^2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}\sqrt{d}}+\frac{\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}+\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{a}+\sqrt{b}c^2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{4a^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}\sqrt{d}x}{\sqrt{\sqrt{a}+\sqrt{b}c^2}+\sqrt[4]{b}dx^2}\right)}{4\sqrt{2}\sqrt{a}\sqrt[8]{b}\sqrt{\sqrt{a}+\sqrt{b}c^2}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}\sqrt{d}}$$

output
```
1/4*arctan(b^(1/8)*d^(1/2)*x/(a^(1/4)+b^(1/4)*c)^(1/2))/a^(3/4)/b^(1/8)/(a
^(1/4)+b^(1/4)*c)^(1/2)/d^(1/2)-1/8*arctan(((-b^(1/4)*c+(a^(1/2)+b^(1/2)*c
^2)^(1/2))^(1/2)-2^(1/2)*b^(1/8)*d^(1/2)*x)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^
2)^(1/2))^(1/2))*2^(1/2)/a^(1/2)/b^(1/8)/(a^(1/2)+b^(1/2)*c^2)^(1/2)/(b^(1
/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)/d^(1/2)+1/8*arctan(((-b^(1/4)*c+(
a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)+2^(1/2)*b^(1/8)*d^(1/2)*x)/(b^(1/4)*c+(a
^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2))*2^(1/2)/a^(1/2)/b^(1/8)/(a^(1/2)+b^(1/2)
*c^2)^(1/2)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)/d^(1/2)+1/4*arct
anh(b^(1/8)*d^(1/2)*x/(a^(1/4)-b^(1/4)*c)^(1/2))/a^(3/4)/b^(1/8)/(a^(1/4)-
b^(1/4)*c)^(1/2)/d^(1/2)+1/8*arctanh(2^(1/2)*b^(1/8)*(-b^(1/4)*c+(a^(1/2)+
b^(1/2)*c^2)^(1/2))^(1/2)*d^(1/2)*x/((a^(1/2)+b^(1/2)*c^2)^(1/2)+b^(1/4)*d
*x^2))*2^(1/2)/a^(1/2)/b^(1/8)/(a^(1/2)+b^(1/2)*c^2)^(1/2)/(-b^(1/4)*c+(a^
(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)/d^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.18

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}\,dx = -\frac{\text{RootSum}\left[a-bc^4-4bc^3d\#1^2-6bc^2d^2\#1^4-4bcd^3\#1^6-bd^4\#1^8\&,\frac{\log(x-\#1)}{c^3\#1+3c^2d\#1^3+3cd^2\#1^5+d^3\#1^7}\&\right]}{8bd}$$

input
```
Integrate[(a - b*(c + d*x^2)^4)^(-1),x]
```

output
```
-1/8*RootSum[a - b*c^4 - 4*b*c^3*d*#1^2 - 6*b*c^2*d^2*#1^4 - 4*b*c*d^3*#1^
6 - b*d^4*#1^8 & , Log[x - #1]/(c^3*#1 + 3*c^2*d*#1^3 + 3*c*d^2*#1^5 + d^3
*#1^7) & ]/(b*d)
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.37, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {7289, 27, 1406, 218, 221, 1407, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}dx$$

$$\Big\downarrow \text{7289}$$

$$\frac{\int \frac{\sqrt{a}}{-\sqrt{b}d^2x^4-2\sqrt{b}cdx^2-\sqrt{b}c^2+\sqrt{a}}dx}{2a}+\frac{\int \frac{\sqrt{a}}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{a}}dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{-\sqrt{b}d^2x^4-2\sqrt{b}cdx^2-\sqrt{b}c^2+\sqrt{a}}dx}{2\sqrt{a}}+\frac{\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{a}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1406}$$

$$\frac{\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{a}}dx}{2\sqrt{a}}+\frac{\frac{\sqrt[4]{b}d\int \frac{1}{\sqrt[4]{b}\left(\sqrt[4]{a}-\sqrt[4]{b}c\right)d-\sqrt{b}d^2x^2}dx}{2\sqrt[4]{a}}-\frac{\sqrt[4]{b}d\int \frac{1}{-\sqrt{b}d^2x^2-\sqrt[4]{b}\left(\sqrt[4]{b}c+\sqrt[4]{a}\right)d}dx}{2\sqrt[4]{a}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt[4]{b}d\int \frac{1}{\sqrt[4]{b}\left(\sqrt[4]{a}-\sqrt[4]{b}c\right)d-\sqrt{b}d^2x^2}dx}{2\sqrt[4]{a}}+\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}}{2\sqrt{a}}+\frac{\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{a}}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{a}}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{1407}$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt[8]{b}\sqrt{d}x}{\sqrt[8]{b}\sqrt{d}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}\right)}dx}{2\sqrt{2}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt{a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}+\frac{\int \frac{\sqrt[8]{b}\sqrt{d}x+\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}\right)}dx}{2\sqrt{2}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt{a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}}{2\sqrt{a}}+$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{2\sqrt{2}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}+\frac{\int \frac{\sqrt[8]{b}\sqrt{d}x+\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{2\sqrt{2}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}}{2\sqrt{a}}+$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\sqrt{d}}}{2\sqrt{a}}+$$

$$\frac{\frac{\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}-\frac{1}{2}\sqrt[8]{b}\sqrt{d}\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x\right)}{\sqrt[8]{b}\sqrt{d}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}\right)}dx}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}cd}}+\cdots}{2\sqrt{a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\sqrt{d}}}{2\sqrt{a}}+\frac{\frac{\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}+\frac{1}{2}\sqrt[8]{b}\sqrt{d}\int\frac{\sqrt{2}\left(\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x\right)}{\sqrt[8]{b}\sqrt{d}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}\right)}dx}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}cd}}+\cdots}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\sqrt{d}}}{2\sqrt{a}}+\frac{\frac{\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}+\frac{\int\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}cd}}+\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\int\frac{\cdots}{x^2+\cdots}}{\cdots}}{2\sqrt{a}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\sqrt{d}}}{2\sqrt{a}}+\frac{\frac{\int\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\int\frac{1}{-\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}}\right)^2-\frac{2\left(c+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}}\right)}{d}}d\left(2x-\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right)\cdots}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}cd}}\cdots$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\sqrt{d}}}{2\sqrt{a}}+$$

$$\frac{\frac{\frac{\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}}\right)}{\sqrt{2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}+\frac{\int\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}cd}}+\frac{\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}\cdots}}{\cdots}}{2\sqrt{a}}$$

$\downarrow$ 1103

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{a}\sqrt[8]{b}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\sqrt{d}}}{2\sqrt{a}}+$$

$$\frac{\frac{\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}}\right)}{\sqrt{2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}-\frac{1}{2}\sqrt[8]{b}\sqrt{d}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2\cdots}\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}cd}}$$

input
```
Int[(a - b*(c + d*x^2)^4)^(-1),x]
```

output
```
(ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4) + b^(1/4)*c]]/(2*a^(1/4)*b^(1/8)*
Sqrt[a^(1/4) + b^(1/4)*c]*Sqrt[d]) + ArcTanh[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1
/4) - b^(1/4)*c]]/(2*a^(1/4)*b^(1/8)*Sqrt[a^(1/4) - b^(1/4)*c]*Sqrt[d]))/(
2*Sqrt[a]) + (((b^(1/8)*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*S
qrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*(-((Sqrt[2]*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a
] + Sqrt[b]*c^2]])/(b^(1/8)*Sqrt[d])) + 2*x))/(Sqrt[2]*Sqrt[b^(1/4)*c + Sq
rt[Sqrt[a] + Sqrt[b]*c^2]])])/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]
] - (b^(1/8)*Sqrt[d]*Log[Sqrt[Sqrt[a] + Sqrt[b]*c^2] - Sqrt[2]*b^(1/8)*Sqr
t[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*Sqrt[d]*x + b^(1/4)*d*x^2])/
2)/(2*Sqrt[2]*b^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*Sqrt[-(b^(1/4)*c) + Sqrt
[Sqrt[a] + Sqrt[b]*c^2]]*d) + ((b^(1/8)*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a] +
 Sqrt[b]*c^2]]*Sqrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*((Sqrt[2]*Sqrt[-(b^(1/4)*c)
 + Sqrt[Sqrt[a] + Sqrt[b]*c^2]])/(b^(1/8)*Sqrt[d]) + 2*x))/(Sqrt[2]*Sqrt[b
^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]])])/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] +
 Sqrt[b]*c^2]] + (b^(1/8)*Sqrt[d]*Log[Sqrt[Sqrt[a] + Sqrt[b]*c^2] + Sqrt[2
]*b^(1/8)*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*Sqrt[d]*x + b^(
1/4)*d*x^2])/2)/(2*Sqrt[2]*b^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*Sqrt[-(b^(1
/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*d))/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 7289
```
Int[((a_) + (b_.)*(v_)^(n_))^(-1), x_Symbol] :> Simp[2/(a*n)   Sum[Int[Toge
ther[1/(1 - v^2/((-1)^(4*(k/n))*Rt[-a/b, n/2]))], x], {k, 1, n/2}], x] /; F
reeQ[{a, b}, x] && IGtQ[n/2, 1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{8}+4bc\,d^{3}\_Z^{6}+6b\,c^{2}d^{2}\_Z^{4}+4b\,c^{3}d\_Z^{2}+b\,c^{4}-a\right)}\frac{\ln\left(x-\_R\right)}{-d^{3}\_R^{7}-3c\,d^{2}\_R^{5}-3c^{2}d\_R^{3}-c^{3}\_R}}{8db}$	104
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{8}+4bc\,d^{3}\_Z^{6}+6b\,c^{2}d^{2}\_Z^{4}+4b\,c^{3}d\_Z^{2}+b\,c^{4}-a\right)}\frac{\ln\left(x-\_R\right)}{-d^{3}\_R^{7}-3c\,d^{2}\_R^{5}-3c^{2}d\_R^{3}-c^{3}\_R}}{8db}$	104

input
```
int(1/(a-b*(d*x^2+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/b*sum(1/(-_R^7*d^3-3*_R^5*c*d^2-3*_R^3*c^2*d-_R*c^3)*ln(x-_R),_R=Roo
tOf(_Z^8*b*d^4+4*_Z^6*b*c*d^3+6*_Z^4*b*c^2*d^2+4*_Z^2*b*c^3*d+b*c^4-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.14 (sec) , antiderivative size = 475271, normalized size of antiderivative = 782.98

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*(d*x^2+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*(d*x**2+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}\,dx = \int -\frac{1}{\left(dx^2+c\right)^4 b-a}\,dx$$

input
```
integrate(1/(a-b*(d*x^2+c)^4),x, algorithm="maxima")
```

output
```
-integrate(1/((d*x^2 + c)^4*b - a), x)
```

Giac [F]

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}\,dx = \int -\frac{1}{\left(dx^2+c\right)^4 b-a}\,dx$$

input
```
integrate(1/(a-b*(d*x^2+c)^4),x, algorithm="giac")
```

output
```
integrate(-1/((d*x^2 + c)^4*b - a), x)
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.71

$$\int \frac{1}{a-b\left(c+dx^2\right)^4}\,dx = \sum_{k=1}^{8}\ln\Big(-\mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8-16777216\,a^7\,b\,d^4\,z^8+1048576\,a^5\,b\,c\,d^3\,z^6-8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)\,b^7\,d^{28}\Big(x+\mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8-16777216\,a^7\,b\,d^4\,z^8+1048576\,a^5\,b\,c\,d^3\,z^6-8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)\,a\,8+\mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8-16777216\,a^7\,b\,d^4\,z^8+1048576\,a^5\,b\,c\,d^3\,z^6-8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)^5-\mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8-16777216\,a^7\,b\,d^4\,z^8+1048576\,a^5\,b\,c\,d^3\,z^6-8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)^4+\mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8-16777216\,a^7\,b\,d^4\,z^8+1048576\,a^5\,b\,c\,d^3\,z^6-8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)^6-16777216\,a^7\,b\,d^4\,z^8+1048576\,a^5\,b\,c\,d^3\,z^6-8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)$$

input
```
int(1/(a - b*(c + d*x^2)^4),x)
```

output
```
symsum(log(-8*root(16777216*a^6*b^2*c^4*d^4*z^8 - 16777216*a^7*b*d^4*z^8 +
 1048576*a^5*b*c*d^3*z^6 - 8192*a^3*b*c^2*d^2*z^4 + 1, z, k)*b^7*d^28*(x +
 8*root(16777216*a^6*b^2*c^4*d^4*z^8 - 16777216*a^7*b*d^4*z^8 + 1048576*a^
5*b*c*d^3*z^6 - 8192*a^3*b*c^2*d^2*z^4 + 1, z, k)*a + 32768*root(16777216*
a^6*b^2*c^4*d^4*z^8 - 16777216*a^7*b*d^4*z^8 + 1048576*a^5*b*c*d^3*z^6 - 8
192*a^3*b*c^2*d^2*z^4 + 1, z, k)^5*a^4*b*c^2*d^2 - 4096*root(16777216*a^6*
b^2*c^4*d^4*z^8 - 16777216*a^7*b*d^4*z^8 + 1048576*a^5*b*c*d^3*z^6 - 8192*
a^3*b*c^2*d^2*z^4 + 1, z, k)^4*a^3*b*c^2*d^2*x + 262144*root(16777216*a^6*
b^2*c^4*d^4*z^8 - 16777216*a^7*b*d^4*z^8 + 1048576*a^5*b*c*d^3*z^6 - 8192*
a^3*b*c^2*d^2*z^4 + 1, z, k)^6*a^5*b*c*d^3*x))*root(16777216*a^6*b^2*c^4*d
^4*z^8 - 16777216*a^7*b*d^4*z^8 + 1048576*a^5*b*c*d^3*z^6 - 8192*a^3*b*c^2
*d^2*z^4 + 1, z, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{1}{a - b\left(c + dx^2\right)^4}\,dx = \int \frac{1}{a - b\left(d\,x^2 + c\right)^4}dx$$

input
```
int(1/(a-b*(d*x^2+c)^4),x)
```

output
```
int(1/(a-b*(d*x^2+c)^4),x)
```

3.159 $\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 752

$$\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}\,dx$$

$$= -\frac{1}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}c^2\right)x} - \frac{1}{2\sqrt{a}\left(\sqrt{a}+\sqrt{b}c^2\right)x} - \frac{\sqrt[8]{b}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}c}}\right)}{4a^{3/4}\left(\sqrt[4]{a}+\sqrt[4]{b}c\right)^{3/2}}$$

$$+ \frac{\sqrt[8]{b}\left(2\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}\right)\sqrt{d}\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}\left(\sqrt{a}+\sqrt{b}c^2\right)^{3/2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}$$

$$- \frac{\sqrt[8]{b}\left(2\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}\right)\sqrt{d}\arctan\left(\frac{\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}+\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}\right)}{4\sqrt{2}\sqrt{a}\left(\sqrt{a}+\sqrt{b}c^2\right)^{3/2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}$$

$$+ \frac{\sqrt[8]{b}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)}{4a^{3/4}\left(\sqrt[4]{a}-\sqrt[4]{b}c\right)^{3/2}}$$

$$- \frac{\sqrt[8]{b}\left(2\sqrt[4]{b}c-\sqrt{\sqrt{a}+\sqrt{b}c^2}\right)\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}\sqrt{d}x}{\sqrt{\sqrt{a}+\sqrt{b}c^2}+\sqrt[4]{b}dx^2}\right)}{4\sqrt{2}\sqrt{a}\left(\sqrt{a}+\sqrt{b}c^2\right)^{3/2}\sqrt{-\sqrt[4]{b}c+\sqrt{\sqrt{a}+\sqrt{b}c^2}}}$$

output
```
-1/2/a^(1/2)/(a^(1/2)-b^(1/2)*c^2)/x-1/2/a^(1/2)/(a^(1/2)+b^(1/2)*c^2)/x-1
/4*b^(1/8)*d^(1/2)*arctan(b^(1/8)*d^(1/2)*x/(a^(1/4)+b^(1/4)*c)^(1/2))/a^(
3/4)/(a^(1/4)+b^(1/4)*c)^(3/2)+1/8*b^(1/8)*(2*b^(1/4)*c+(a^(1/2)+b^(1/2)*c
^2)^(1/2))*d^(1/2)*arctan(((-b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)-
2^(1/2)*b^(1/8)*d^(1/2)*x)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2))*
2^(1/2)/a^(1/2)/(a^(1/2)+b^(1/2)*c^2)^(3/2)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^
2)^(1/2))^(1/2)-1/8*b^(1/8)*(2*b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))*d^(1
/2)*arctan(((-b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)+2^(1/2)*b^(1/8)
*d^(1/2)*x)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2))*2^(1/2)/a^(1/2)
/(a^(1/2)+b^(1/2)*c^2)^(3/2)/(b^(1/4)*c+(a^(1/2)+b^(1/2)*c^2)^(1/2))^(1/2)
+1/4*b^(1/8)*d^(1/2)*arctanh(b^(1/8)*d^(1/2)*x/(a^(1/4)-b^(1/4)*c)^(1/2))/
a^(3/4)/(a^(1/4)-b^(1/4)*c)^(3/2)-1/8*b^(1/8)*(2*b^(1/4)*c-(a^(1/2)+b^(1/2
)*c^2)^(1/2))*d^(1/2)*arctanh(2^(1/2)*b^(1/8)*(-b^(1/4)*c+(a^(1/2)+b^(1/2)
*c^2)^(1/2))^(1/2)*d^(1/2)*x/((a^(1/2)+b^(1/2)*c^2)^(1/2)+b^(1/4)*d*x^2))*
2^(1/2)/a^(1/2)/(a^(1/2)+b^(1/2)*c^2)^(3/2)/(-b^(1/4)*c+(a^(1/2)+b^(1/2)*c
^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}\,dx =$$
$$-\frac{8+x\text{RootSum}\left[a-bc^4-4bc^3d\#1^2-6bc^2d^2\#1^4-4bcd^3\#1^6-bd^4\#1^8\&,\frac{4c^3\log(x-\#1)+6c^2d\log(x-\#1)\cdots}{c^3\#1+3c^2\cdots}\right.}{8ax-8bc^4x}$$

input
```
Integrate[1/(x^2*(a - b*(c + d*x^2)^4)),x]
```

output
```
-((8 + x*RootSum[a - b*c^4 - 4*b*c^3*d*#1^2 - 6*b*c^2*d^2*#1^4 - 4*b*c*d^3
*#1^6 - b*d^4*#1^8 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1^2 + 4*c
*d^2*Log[x - #1]*#1^4 + d^3*Log[x - #1]*#1^6)/(c^3*#1 + 3*c^2*d*#1^3 + 3*c
*d^2*#1^5 + d^3*#1^7) & ])/(8*a*x - 8*b*c^4*x))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3288 vs. 2(752) = 1504.

Time = 10.62 (sec) , antiderivative size = 3288, normalized size of antiderivative = 4.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{1}{x^2\left(a-bc^4\right)}+\frac{bd\left(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6\right)}{\left(a-bc^4\right)\left(a\left(1-\frac{bc^4}{a}\right)-4bc^3dx^2-6bc^2d^2x^4-4bcd^3x^6-bd^4x^8\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^{7/8}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)c^3}{4a^{3/4}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\,(a-bc^4)} - \frac{b^{7/8}\sqrt{d}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c^3}{4\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}} +$$

$$\frac{b^{7/8}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c^3}{4\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}} + \frac{b^{7/8}\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)c^3}{4a^{3/4}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\,(a-bc^4)} -$$

$$\frac{b^{7/8}\sqrt{d}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c^3}{8\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}} +$$

$$\frac{b^{7/8}\sqrt{d}\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c^3}{8\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}} -$$

$$\frac{b^{5/8}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)c^2}{4\sqrt{a}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\,(a-bc^4)} -$$

$$\frac{b^{5/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}\sqrt{d}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c^2}{4\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)} +$$

$$\frac{b^{5/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c^2}{4\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)} +$$

$$\frac{b^{5/8}\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}}\right)c^2}{4\sqrt{a}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}c}\,(a-bc^4)} +$$

$$\frac{b^{5/8}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c^2}{8\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)} -$$

$$\frac{b^{5/8}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{a}}\right)c^2}{8\sqrt{2}\sqrt{a}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)} +$$

$$\frac{b^{3/8}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}}\right)c}{4\sqrt[4]{a}\sqrt{\sqrt[4]{b}c+\sqrt[4]{a}}\,(a-bc^4)} + \frac{b^{3/8}\sqrt{d}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}}\right)c}{4\sqrt{2}\sqrt{\sqrt{b}c^2+\sqrt{a}}\,(a-bc^4)\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{a}}}} -$$

input
```
Int[1/(x^2*(a - b*(c + d*x^2)^4)),x]
```

output
```
-(1/((a - b*c^4)*x)) - (b^(1/8)*Sqrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^
(1/4) + b^(1/4)*c]])/(4*Sqrt[a^(1/4) + b^(1/4)*c]*(a - b*c^4)) + (b^(3/8)*
c*Sqrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4) + b^(1/4)*c]])/(4*a^(1/4
)*Sqrt[a^(1/4) + b^(1/4)*c]*(a - b*c^4)) - (b^(5/8)*c^2*Sqrt[d]*ArcTan[(b^
(1/8)*Sqrt[d]*x)/Sqrt[a^(1/4) + b^(1/4)*c]])/(4*Sqrt[a]*Sqrt[a^(1/4) + b^(
1/4)*c]*(a - b*c^4)) + (b^(7/8)*c^3*Sqrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqr
t[a^(1/4) + b^(1/4)*c]])/(4*a^(3/4)*Sqrt[a^(1/4) + b^(1/4)*c]*(a - b*c^4))
 + (b^(3/8)*c*Sqrt[d]*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c
^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*
c^2]]])/(4*Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[b]*c^2]*(a - b*c^4)*Sqrt[b^(1/4)*c
+ Sqrt[Sqrt[a] + Sqrt[b]*c^2]]) - (b^(7/8)*c^3*Sqrt[d]*ArcTan[(Sqrt[-(b^(1
/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^
(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[a]*Sqrt[Sqrt[a] +
 Sqrt[b]*c^2]*(a - b*c^4)*Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]) +
 (b^(1/8)*Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]*Sqrt[d]*ArcTan[(Sq
rt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x
)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[Sqrt[a]
+ Sqrt[b]*c^2]*(a - b*c^4)) - (b^(5/8)*c^2*Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] +
 Sqrt[b]*c^2]]*Sqrt[d]*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[a] + Sqrt[b]*
c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[a] + Sqrt...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.21

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^8+4bc\,d^3\_Z^6+6b\,c^2d^2\_Z^4+4b\,c^3d\_Z^2+b\,c^4-a\right)}\frac{\left(-\_R^6d^3-4\_R^4c\,d^2-6\_R^2c^2d-4c^3\right)\ln\left(x-\_R\right)}{-d^3\_R^7-3c\,d^2\_R^5-3c^2d\_R^3-c^3\_R}}{8\left(-b\,c^4+a\right)}-\frac{}{(-}$
risch	$-\frac{1}{\left(-b\,c^4+a\right)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(-a^6b^3c^{12}+3a^7b^2c^8-3a^8b\,c^4+a^9\right)\_Z^8+\left(-24a^5b^2c^7d-40a^6b\,c^3d\right)\_Z^6+\left(2a^3b^2c^6d^2-42a^4b\,c^2d^2\right)\_Z^4\right.}\right.}{}$

input
```
int(1/x^2/(a-b*(d*x^2+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(-b*c^4+a)*sum((-_R^6*d^3-4*_R^4*c*d^2-6*_R^2*c^2*d-4*c^3)/(-_R^7*d^3
-3*_R^5*c*d^2-3*_R^3*c^2*d-_R*c^3)*ln(x-_R),_R=RootOf(_Z^8*b*d^4+4*_Z^6*b*
c*d^3+6*_Z^4*b*c^2*d^2+4*_Z^2*b*c^3*d+b*c^4-a))-1/(-b*c^4+a)/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(a-b*(d*x^2+c)^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**2/(a-b*(d*x**2+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}\,dx=\int -\frac{1}{\left((dx^2+c)^4b-a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a-b*(d*x^2+c)^4),x, algorithm="maxima")
```

output
```
b*d*integrate((d^3*x^6 + 4*c*d^2*x^4 + 6*c^2*d*x^2 + 4*c^3)/(b*d^4*x^8 + 4
*b*c*d^3*x^6 + 6*b*c^2*d^2*x^4 + 4*b*c^3*d*x^2 + b*c^4 - a), x)/(b*c^4 - a
) + 1/((b*c^4 - a)*x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a-b\left(c+dx^2\right)^4\right)}\,dx=\int -\frac{1}{\left((dx^2+c)^4b-a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a-b*(d*x^2+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 2353, normalized size of antiderivative = 3.13

$$\int \frac{1}{x^2 \left(a-b \left(c+dx^2\right)^4\right)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a - b*(c + d*x^2)^4)),x)
```

output
```
symsum(log(-root(50331648*a^7*b^2*c^8*z^8 - 16777216*a^6*b^3*c^12*z^8 - 50
331648*a^8*b*c^4*z^8 + 16777216*a^9*z^8 - 6291456*a^5*b^2*c^7*d*z^6 - 1048
5760*a^6*b*c^3*d*z^6 - 172032*a^4*b*c^2*d^2*z^4 + 8192*a^3*b^2*c^6*d^2*z^4
 - 768*a^2*b*c*d^3*z^2 - b*d^4, z, k)*(root(50331648*a^7*b^2*c^8*z^8 - 167
77216*a^6*b^3*c^12*z^8 - 50331648*a^8*b*c^4*z^8 + 16777216*a^9*z^8 - 62914
56*a^5*b^2*c^7*d*z^6 - 10485760*a^6*b*c^3*d*z^6 - 172032*a^4*b*c^2*d^2*z^4
 + 8192*a^3*b^2*c^6*d^2*z^4 - 768*a^2*b*c*d^3*z^2 - b*d^4, z, k)*(root(503
31648*a^7*b^2*c^8*z^8 - 16777216*a^6*b^3*c^12*z^8 - 50331648*a^8*b*c^4*z^8
 + 16777216*a^9*z^8 - 6291456*a^5*b^2*c^7*d*z^6 - 10485760*a^6*b*c^3*d*z^6
 - 172032*a^4*b*c^2*d^2*z^4 + 8192*a^3*b^2*c^6*d^2*z^4 - 768*a^2*b*c*d^3*z
^2 - b*d^4, z, k)*(x*(1024*a^13*b^9*c^3*d^33 - 11264*a^12*b^10*c^7*d^33 +
56320*a^11*b^11*c^11*d^33 - 168960*a^10*b^12*c^15*d^33 + 337920*a^9*b^13*c
^19*d^33 - 473088*a^8*b^14*c^23*d^33 + 473088*a^7*b^15*c^27*d^33 - 337920*
a^6*b^16*c^31*d^33 + 168960*a^5*b^17*c^35*d^33 - 56320*a^4*b^18*c^39*d^33
+ 11264*a^3*b^19*c^43*d^33 - 1024*a^2*b^20*c^47*d^33) + root(50331648*a^7*
b^2*c^8*z^8 - 16777216*a^6*b^3*c^12*z^8 - 50331648*a^8*b*c^4*z^8 + 1677721
6*a^9*z^8 - 6291456*a^5*b^2*c^7*d*z^6 - 10485760*a^6*b*c^3*d*z^6 - 172032*
a^4*b*c^2*d^2*z^4 + 8192*a^3*b^2*c^6*d^2*z^4 - 768*a^2*b*c*d^3*z^2 - b*d^4
, z, k)*(root(50331648*a^7*b^2*c^8*z^8 - 16777216*a^6*b^3*c^12*z^8 - 50331
648*a^8*b*c^4*z^8 + 16777216*a^9*z^8 - 6291456*a^5*b^2*c^7*d*z^6 - 1048...
```

Reduce [F]

$$\int \frac{1}{x^2 \left(a-b \left(c+dx^2\right)^4\right)} \, dx = \int \frac{1}{x^2 \left(a-b \left(d\,x^2+c\right)^4\right)}dx$$

input
```
int(1/x^2/(a-b*(d*x^2+c)^4),x)
```

output
```
int(1/x^2/(a-b*(d*x^2+c)^4),x)
```

3.160 $\int \frac{x^2}{a+b(c+dx^2)^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1845

$$\int \frac{x^2}{a+b\left(c+dx^2\right)^4} \, dx = \text{Too large to display}$$

output
```
1/8*(2^(1/2)*b^(1/4)*c+(a^(1/2)*2^(1/2)+2*a^(1/4)*b^(1/4)*c+2^(1/2)*b^(1/2
)*c^2)/(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))*arctan(((-2^
(1/2)*a^(1/4)-2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2
)^(1/2))^(1/2)-2*b^(1/8)*d^(1/2)*x)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2
)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2))/a^(3/4)/b^(3/8)/(2^
(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2
)^(1/2))^(1/2)/d^(3/2)-1/8*(2^(1/2)*b^(1/4)*c+(a^(1/2)*2^(1/2)+2*a^(1/4)*b
^(1/4)*c+2^(1/2)*b^(1/2)*c^2)/(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c
^2)^(1/2))*arctan(((-2^(1/2)*a^(1/4)-2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4
)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)+2*b^(1/8)*d^(1/2)*x)/(2^(1/2)*a^(1/4
)+2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/
2))/a^(3/4)/b^(3/8)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4
)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)/d^(3/2)+1/8*(2^(1/2)*b^(1/4)*c-(a^(1
/2)*2^(1/2)+2*a^(1/4)*b^(1/4)*c+2^(1/2)*b^(1/2)*c^2)/(a^(1/2)+2^(1/2)*a^(1
/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))*arctan(b^(1/8)*(2^(1/2)*a^(1/4)+2*b^(1/4
)*c-2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)*d^(1/2)
*x/((a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2)+b^(1/4)*d*x^2))/
a^(3/4)/b^(3/8)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c-2*(a^(1/2)+2^(1/2)*a^(1/4)*b^
(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)/d^(3/2)+1/8*(2^(1/2)*b^(1/4)*c+(a^(1/2)*
2^(1/2)-2*a^(1/4)*b^(1/4)*c+2^(1/2)*b^(1/2)*c^2)/(a^(1/2)-2^(1/2)*a^(1/...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{a+b\left(c+dx^2\right)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1^2+6bc^2d^2\#1^4+4bcd^3\#1^6+bd^4\#1^8\&,\frac{\log(x-\#1)\#1}{c^3+3c^2d\#1^2+3cd^2\#1^4+d^3\#1^6}\&\right]}{8bd}$$

input
```
Integrate[x^2/(a + b*(c + d*x^2)^4),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1^2 + 6*b*c^2*d^2*#1^4 + 4*b*c*d^3*#1^6 + b
*d^4*#1^8 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1^2 + 3*c*d^2*#1^4 + d^3*#1
^6) & ]/(8*b*d)
```

Rubi [A] (verified)

Time = 6.33 (sec) , antiderivative size = 1631, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\left(c+dx^2\right)^4}dx$$

$$\Big\downarrow \text{ 7291}$$

$$\int \left(\frac{c+dx^2}{d\left(a+b\left(c+dx^2\right)^4\right)}-\frac{c}{d\left(a+b\left(c+dx^2\right)^4\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{4(-a)^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}d^{3/2}}+\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}d^{3/2}}+$$

$$\frac{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{-a}}d^{3/2}}-$$

$$\frac{c\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}d^{3/2}}-$$

$$\frac{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{-a}}d^{3/2}}+$$

$$\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}d^{3/2}}+\frac{c\,\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{4(-a)^{3/4}\sqrt[8]{b}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}d^{3/2}}-$$

$$\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}d^{3/2}}-$$

$$\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{-a}}d^{3/2}}-$$

$$\frac{c\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)}{8\sqrt{2}\sqrt{-a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}d^{3/2}}+$$

$$\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{b}c^2+\sqrt{-a}}d^{3/2}}+$$

$$\frac{c\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)}{8\sqrt{2}\sqrt{-a}\sqrt[8]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}d^{3/2}}$$

input
```
Int[x^2/(a + b*(c + d*x^2)^4),x]
```

output
```
ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[(-a)^(1/4) + b^(1/4)*c]]/(4*Sqrt[-a]*b^(3/
8)*Sqrt[(-a)^(1/4) + b^(1/4)*c]*d^(3/2)) + (c*ArcTan[(b^(1/8)*Sqrt[d]*x)/S
qrt[(-a)^(1/4) + b^(1/4)*c]])/(4*(-a)^(3/4)*b^(1/8)*Sqrt[(-a)^(1/4) + b^(1
/4)*c]*d^(3/2)) - (c*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c
^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]
*c^2]]])/(4*Sqrt[2]*Sqrt[-a]*b^(1/8)*Sqrt[Sqrt[-a] + Sqrt[b]*c^2]*Sqrt[b^(
1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]*d^(3/2)) + (Sqrt[b^(1/4)*c + Sqrt[S
qrt[-a] + Sqrt[b]*c^2]]*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b
]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt
[b]*c^2]]])/(4*Sqrt[2]*Sqrt[-a]*b^(3/8)*Sqrt[Sqrt[-a] + Sqrt[b]*c^2]*d^(3/
2)) + (c*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]] + Sqrt[
2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]])/(4*
Sqrt[2]*Sqrt[-a]*b^(1/8)*Sqrt[Sqrt[-a] + Sqrt[b]*c^2]*Sqrt[b^(1/4)*c + Sqr
t[Sqrt[-a] + Sqrt[b]*c^2]]*d^(3/2)) - (Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sq
rt[b]*c^2]]*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]] + Sq
rt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]])/
(4*Sqrt[2]*Sqrt[-a]*b^(3/8)*Sqrt[Sqrt[-a] + Sqrt[b]*c^2]*d^(3/2)) - ArcTan
h[(b^(1/8)*Sqrt[d]*x)/Sqrt[(-a)^(1/4) - b^(1/4)*c]]/(4*Sqrt[-a]*b^(3/8)*Sq
rt[(-a)^(1/4) - b^(1/4)*c]*d^(3/2)) + (c*ArcTanh[(b^(1/8)*Sqrt[d]*x)/Sqrt[
(-a)^(1/4) - b^(1/4)*c]])/(4*(-a)^(3/4)*b^(1/8)*Sqrt[(-a)^(1/4) - b^(1/...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.06

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{8}+4bc\,d^{3}\_Z^{6}+6b\,c^{2}d^{2}\_Z^{4}+4b\,c^{3}d\,\_Z^{2}+b\,c^{4}+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{d^{3}\_R^{7}+3c\,d^{2}\_R^{5}+3c^{2}d\,\_R^{3}+c^{3}\_R}}{8db}$	103
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{8}+4bc\,d^{3}\_Z^{6}+6b\,c^{2}d^{2}\_Z^{4}+4b\,c^{3}d\,\_Z^{2}+b\,c^{4}+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{d^{3}\_R^{7}+3c\,d^{2}\_R^{5}+3c^{2}d\,\_R^{3}+c^{3}\_R}}{8db}$	103

input
```
int(x^2/(a+b*(d*x^2+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/b*sum(_R^2/(_R^7*d^3+3*_R^5*c*d^2+3*_R^3*c^2*d+_R*c^3)*ln(x-_R),_R=R
ootOf(_Z^8*b*d^4+4*_Z^6*b*c*d^3+6*_Z^4*b*c^2*d^2+4*_Z^2*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 115.39 (sec) , antiderivative size = 109156, normalized size of antiderivative = 59.16

$$\int \frac{x^2}{a+b\left(c+dx^2\right)^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^2/(a+b*(d*x^2+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.08

$$\int \frac{x^2}{a+b(c+dx^2)^4}\,dx$$
$$= \mathrm{RootSum}\left(16777216t^8a^6b^3d^{12}+8192t^4a^3b^2c^2d^6+256t^2a^2bcd^3+a+bc^4, \left(t\mapsto t\log\left(x+\frac{-4194304t^7c\right.\right.\right.$$

input
```
integrate(x**2/(a+b*(d*x**2+c)**4),x)
```

output
```
RootSum(16777216*_t**8*a**6*b**3*d**12 + 8192*_t**4*a**3*b**2*c**2*d**6 +
256*_t**2*a**2*b*c*d**3 + a + b*c**4, Lambda(_t, _t*log(x + (-4194304*_t**
7*a**5*b**3*c**2*d**10 + 32768*_t**5*a**4*b**2*c*d**7 - 512*_t**3*a**3*b*d
**4 - 1024*_t**3*a**2*b**2*c**4*d**4 - 40*_t*a*b*c**3*d)/(a - 4*b*c**4))))
```

Maxima [F]

$$\int \frac{x^2}{a+b(c+dx^2)^4}\,dx = \int \frac{x^2}{(dx^2+c)^4b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x^2+c)^4),x, algorithm="maxima")
```

output
```
integrate(x^2/((d*x^2 + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+b(c+dx^2)^4}\,dx = \int \frac{x^2}{(dx^2+c)^4b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x^2+c)^4),x, algorithm="giac")
```

output
```
integrate(x^2/((d*x^2 + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 629, normalized size of antiderivative = 0.34

$$\int \frac{x^2}{a+b\left(c+dx^2\right)^4}\,dx = \sum_{k=1}^{8} \ln\Big(-a\,b^5\,d^{20} - b^6\,c^4\,d^{20} - \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)\,b^7\,c^6\,d^{22}\,x\,8 - \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)^2\,a^2\,b^6\,c\,d^{23}\,128 - \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)^2\,a\,b^7\,c^5\,d^{23}\,128 - \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)^5\,a^4\,b^7\,d^{28}\,x\,32768 + \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)^7\,a^5\,b^8\,c\,d^{31}\,x\,2097152 + \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)^3\,a^2\,b^7\,c^3\,d^{25}\,x\,1024 - \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)^5\,a^3\,b^8\,c^4\,d^{28}\,x\,32768 + \mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)\,a\,b^6\,c^2\,d^{22}\,x\,24\Big)\,\mathrm{root}\big(16777216\,a^6\,b^3\,d^{12}\,z^8 + 8192\,a^3\,b^2\,c^2\,d^6\,z^4 + 256\,a^2\,b\,c\,d^3\,z^2 + b\,c^4 + a, z, k\big)$$

input
```
int(x^2/(a + b*(c + d*x^2)^4),x)
```

output
```
symsum(log(2097152*root(16777216*a^6*b^3*d^12*z^8 + 8192*a^3*b^2*c^2*d^6*z
^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 + a, z, k)^7*a^5*b^8*c*d^31*x - b^6*c^4*d
^20 - 8*root(16777216*a^6*b^3*d^12*z^8 + 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^
2*b*c*d^3*z^2 + b*c^4 + a, z, k)*b^7*c^6*d^22*x - 128*root(16777216*a^6*b^
3*d^12*z^8 + 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 + a, z
, k)^2*a^2*b^6*c*d^23 - 128*root(16777216*a^6*b^3*d^12*z^8 + 8192*a^3*b^2*
c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 + a, z, k)^2*a*b^7*c^5*d^23 - 32
768*root(16777216*a^6*b^3*d^12*z^8 + 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*
c*d^3*z^2 + b*c^4 + a, z, k)^5*a^4*b^7*d^28*x - a*b^5*d^20 + 1024*root(167
77216*a^6*b^3*d^12*z^8 + 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 +
b*c^4 + a, z, k)^3*a^2*b^7*c^3*d^25*x - 32768*root(16777216*a^6*b^3*d^12*z
^8 + 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 + a, z, k)^5*a
^3*b^8*c^4*d^28*x + 24*root(16777216*a^6*b^3*d^12*z^8 + 8192*a^3*b^2*c^2*d
^6*z^4 + 256*a^2*b*c*d^3*z^2 + b*c^4 + a, z, k)*a*b^6*c^2*d^22*x)*root(167
77216*a^6*b^3*d^12*z^8 + 8192*a^3*b^2*c^2*d^6*z^4 + 256*a^2*b*c*d^3*z^2 +
b*c^4 + a, z, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{x^2}{a+b\left(c+dx^2\right)^4}\,dx = \int \frac{x^2}{b\,d^4x^8+4bc\,d^3x^6+6b\,c^2d^2x^4+4b\,c^3d\,x^2+b\,c^4+a}dx$$

input
```
int(x^2/(a+b*(d*x^2+c)^4),x)
```

output
```
int(x**2/(a + b*c**4 + 4*b*c**3*d*x**2 + 6*b*c**2*d**2*x**4 + 4*b*c*d**3*x
**6 + b*d**4*x**8),x)
```

3.161 $\int \frac{1}{a+b(c+dx^2)^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 1693

$$\int \frac{1}{a+b(c+dx^2)^4} \, dx = \text{Too large to display}$$

output
```
-1/8*(2^(1/2)+(2*a^(1/4)+2^(1/2)*b^(1/4)*c)/(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/
4)*c+b^(1/2)*c^2)^(1/2))*arctan(((-2^(1/2)*a^(1/4)-2*b^(1/4)*c+2*(a^(1/2)+
2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)-2*b^(1/8)*d^(1/2)*x)/(
2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c
^2)^(1/2))^(1/2))/a^(3/4)/b^(1/8)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2)+
2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)/d^(1/2)+1/8*(2^(1/2)+(
2*a^(1/4)+2^(1/2)*b^(1/4)*c)/(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^
2)^(1/2))*arctan(((-2^(1/2)*a^(1/4)-2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)
*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)+2*b^(1/8)*d^(1/2)*x)/(2^(1/2)*a^(1/4)
+2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2
))/a^(3/4)/b^(1/8)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)
*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)/d^(1/2)-1/8*(2^(1/2)-(2*a^(1/4)+2^(1/
2)*b^(1/4)*c)/(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))*arcta
n(b^(1/8)*(2^(1/2)*a^(1/4)+2*b^(1/4)*c-2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*
c+b^(1/2)*c^2)^(1/2))^(1/2)*d^(1/2)*x/((a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+
b^(1/2)*c^2)^(1/2)+b^(1/4)*d*x^2))/a^(3/4)/b^(1/8)/(2^(1/2)*a^(1/4)+2*b^(1
/4)*c-2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)/d^(1/
2)-1/8*(2^(1/2)-(2*a^(1/4)-2^(1/2)*b^(1/4)*c)/(a^(1/2)-2^(1/2)*a^(1/4)*b^(
1/4)*c+b^(1/2)*c^2)^(1/2))*arctanh(((2^(1/2)*a^(1/4)-2*b^(1/4)*c+2*(a^(1/2
)-2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)-2*b^(1/8)*d^(1/2)...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.06

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1^2+6bc^2d^2\#1^4+4bcd^3\#1^6+bd^4\#1^8\&,\frac{\log(x-\#1)}{c^3\#1+3c^2d\#1^3+3cd^2\#1^5+d^3\#1^7}\&\right]}{8bd}$$

input
```
Integrate[(a + b*(c + d*x^2)^4)^(-1),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1^2 + 6*b*c^2*d^2*#1^4 + 4*b*c*d^3*#1^6 + b
*d^4*#1^8 & , Log[x - #1]/(c^3*#1 + 3*c^2*d*#1^3 + 3*c*d^2*#1^5 + d^3*#1^7
) & ]/(8*b*d)
```

Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 885, normalized size of antiderivative = 0.52, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {7289, 27, 1406, 218, 221, 1407, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}dx$$

$$\Big\downarrow \; 7289$$

$$\frac{\int \frac{\sqrt{-a}}{-\sqrt{b}d^2x^4-2\sqrt{b}cdx^2-\sqrt{b}c^2+\sqrt{-a}}dx}{2a}+\frac{\int \frac{\sqrt{-a}}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{-a}}dx}{2a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{-a}\int \frac{1}{-\sqrt{b}d^2x^4-2\sqrt{b}cdx^2-\sqrt{b}c^2+\sqrt{-a}}dx}{2a}+\frac{\sqrt{-a}\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{-a}}dx}{2a}$$

$$\Big\downarrow \; 1406$$

$$\frac{\sqrt{-a}\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{-a}}dx}{2a}+\frac{\sqrt{-a}\left(\frac{\sqrt[4]{b}d\int \frac{1}{\sqrt[4]{b}\left(\sqrt[4]{-a}-\sqrt[4]{b}c\right)d-\sqrt{b}d^2x^2}dx}{2\sqrt[4]{-a}}-\frac{\sqrt[4]{b}d\int \frac{1}{-\sqrt{b}d^2x^2-\sqrt[4]{b}\left(\sqrt[4]{b}c+\sqrt[4]{-a}\right)d}dx}{2\sqrt[4]{-a}}\right)}{2a}$$

$$\Big\downarrow \; 218$$

$$\frac{\sqrt{-a}\left(\frac{\sqrt[4]{b}d\int \frac{1}{\sqrt[4]{b}\left(\sqrt[4]{-a}-\sqrt[4]{b}c\right)d-\sqrt{b}d^2x^2}dx}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}\right)}{2a}+\frac{\sqrt{-a}\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{-a}}dx}{2a}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{-a}\int \frac{1}{\sqrt{b}d^2x^4+2\sqrt{b}cdx^2+\sqrt{b}c^2+\sqrt{-a}}dx}{2a}+\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2a}$$

$\Big\downarrow$ 1407

$$\frac{\sqrt{-a}\left(\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt[8]{b}\sqrt{d}x}{\sqrt[8]{b}\sqrt{d}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}\right)}dx}{2\sqrt{2}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt{-a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}+\frac{\int \frac{\sqrt[8]{b}\sqrt{d}x+\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}\right)}dx}{2\sqrt{2}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt{-a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}\right)}{2a}+\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2a}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{-a}\left(\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{2\sqrt{2}\sqrt[4]{b}d\sqrt{\sqrt{-a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}+\frac{\int \frac{\sqrt[8]{b}\sqrt{d}x+\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{2\sqrt{2}\sqrt[4]{b}d\sqrt{\sqrt{-a}+\sqrt{b}c^2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}c^2}-\sqrt[4]{b}c}}\right)}{2a}+\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{d}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2a}$$

$\Big\downarrow$ 1142

$$\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\sqrt{d}}\right)}{2a}+$$

$$\sqrt{-a}\left(\frac{\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}-\frac{1}{2}\sqrt[8]{b}\sqrt{d}\int-\frac{\sqrt{2}\left(\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x\right)}{\sqrt[8]{b}\sqrt{d}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\cdots}}{\sqrt[4]{b}d}\right.}}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}cd}}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\sqrt{d}}\right)}{2a}+$$

$$\sqrt{-a}\left(\frac{\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}+\frac{1}{2}\sqrt[8]{b}\sqrt{d}\int\frac{\sqrt{2}\left(\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x\right)}{\sqrt[8]{b}\sqrt{d}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\cdots}}{\sqrt[4]{b}d}\right.}}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}cd}}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\sqrt{d}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\sqrt{d}}\right)}{2a}+$$

$$\frac{\sqrt{-a}\left(\frac{\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}+\frac{\int\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,d}+\frac{\sqrt{\sqrt{\sqrt{b}c^2\cdots}}}{\cdots}\cdots\right)}{2a}$$

$\downarrow$ 1083

$$\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\sqrt{d}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\sqrt{d}}\right)}{2a}+$$

$$\frac{\sqrt{-a}\left(\frac{\frac{\int\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\int\frac{1}{-\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}}\right)^2-\frac{2\left(c+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}\cdots}}{\sqrt[4]{b}}\cdots\right)}{d}\cdots}\cdots}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,d\cdots}\cdots\right)}{\cdots}$$

$\downarrow$ 217

$$\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\sqrt{d}}\right)}{2a}+$$

$$\frac{\sqrt{-a}\left(\frac{\frac{\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}}\right)}{\sqrt{2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}+\frac{\int\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\,x}{\sqrt[8]{b}\sqrt{d}}+\frac{\sqrt{\sqrt{b}c^2+\sqrt{-a}}}{\sqrt[4]{b}d}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}cd}}+-\ldots\right)}{2a}$$

$\downarrow$ 1103

$$\frac{\sqrt{-a}\left(\frac{\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\sqrt{d}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)}{2\sqrt[4]{-a}\sqrt[8]{b}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\sqrt{d}}\right)}{2a}+$$

$$\frac{\sqrt{-a}\left(\frac{\frac{\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt[8]{b}\sqrt{d}}\right)}{\sqrt{2}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}-\frac{1}{2}\sqrt[8]{b}\sqrt{d}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c\ldots}\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}cd}}\right)}{\ldots}$$

input

```
Int[(a + b*(c + d*x^2)^4)^(-1),x]
```

output
```
(Sqrt[-a]*(ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[(-a)^(1/4) + b^(1/4)*c]]/(2*(-a
)^(1/4)*b^(1/8)*Sqrt[(-a)^(1/4) + b^(1/4)*c]*Sqrt[d]) + ArcTanh[(b^(1/8)*S
qrt[d]*x)/Sqrt[(-a)^(1/4) - b^(1/4)*c]]/(2*(-a)^(1/4)*b^(1/8)*Sqrt[(-a)^(1
/4) - b^(1/4)*c]*Sqrt[d])))/(2*a) + (Sqrt[-a]*(((b^(1/8)*Sqrt[-(b^(1/4)*c)
 + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]*Sqrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*(-((Sqrt[
2]*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]])/(b^(1/8)*Sqrt[d])) +
 2*x))/(Sqrt[2]*Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]])])/Sqrt[b^(
1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]] - (b^(1/8)*Sqrt[d]*Log[Sqrt[Sqrt[-a
] + Sqrt[b]*c^2] - Sqrt[2]*b^(1/8)*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqr
t[b]*c^2]]*Sqrt[d]*x + b^(1/4)*d*x^2])/2)/(2*Sqrt[2]*b^(1/4)*Sqrt[Sqrt[-a]
 + Sqrt[b]*c^2]*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]*d) + ((b
^(1/8)*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]*Sqrt[d]*ArcTan[(b
^(1/8)*Sqrt[d]*((Sqrt[2]*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]
)/(b^(1/8)*Sqrt[d]) + 2*x))/(Sqrt[2]*Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt
[b]*c^2]])])/Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]] + (b^(1/8)*Sqr
t[d]*Log[Sqrt[Sqrt[-a] + Sqrt[b]*c^2] + Sqrt[2]*b^(1/8)*Sqrt[-(b^(1/4)*c)
+ Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]*Sqrt[d]*x + b^(1/4)*d*x^2])/2)/(2*Sqrt[2]*
b^(1/4)*Sqrt[Sqrt[-a] + Sqrt[b]*c^2]*Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + S
qrt[b]*c^2]]*d)))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 7289
```
Int[((a_) + (b_.)*(v_)^(n_))^(-1), x_Symbol] :> Simp[2/(a*n)   Sum[Int[Toge
ther[1/(1 - v^2/((-1)^(4*(k/n))*Rt[-a/b, n/2]))], x], {k, 1, n/2}], x] /; F
reeQ[{a, b}, x] && IGtQ[n/2, 1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.06

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{8}+4bc\,d^{3}\_Z^{6}+6b\,c^{2}d^{2}\_Z^{4}+4b\,c^{3}d\,\_Z^{2}+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{7}+3c\,d^{2}\_R^{5}+3c^{2}d\,\_R^{3}+c^{3}\_R}}{8db}$	100
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{8}+4bc\,d^{3}\_Z^{6}+6b\,c^{2}d^{2}\_Z^{4}+4b\,c^{3}d\,\_Z^{2}+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{7}+3c\,d^{2}\_R^{5}+3c^{2}d\,\_R^{3}+c^{3}\_R}}{8db}$	100

input
```
int(1/(a+b*(d*x^2+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/b*sum(1/(_R^7*d^3+3*_R^5*c*d^2+3*_R^3*c^2*d+_R*c^3)*ln(x-_R),_R=Root
Of(_Z^8*b*d^4+4*_Z^6*b*c*d^3+6*_Z^4*b*c^2*d^2+4*_Z^2*b*c^3*d+b*c^4+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*(d*x^2+c)^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*(d*x**2+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}\,dx=\int \frac{1}{\left(dx^2+c\right)^4b+a}\,dx$$

input
```
integrate(1/(a+b*(d*x^2+c)^4),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^2 + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}\,dx=\int \frac{1}{\left(dx^2+c\right)^4b+a}\,dx$$

input
```
integrate(1/(a+b*(d*x^2+c)^4),x, algorithm="giac")
```

output
```
integrate(1/((d*x^2 + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.26

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}\,dx = \sum_{k=1}^{8} \ln\Big(-\mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8+16777216\,a^7\,b\,d^4\,z^8 - 1048576\,a^5\,b\,c\,d^3\,z^6+8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)\,b^7\,d^{28}\Big(x + \mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8+16777216\,a^7\,b\,d^4\,z^8-1048576\,a^5\,b\,c\,d^3\,z^6+8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)\,a\,8 - \mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8+16777216\,a^7\,b\,d^4\,z^8-1048576\,a^5\,b\,c\,d^3\,z^6+8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)^5 + \mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8+16777216\,a^7\,b\,d^4\,z^8-1048576\,a^5\,b\,c\,d^3\,z^6+8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)^4 - \mathrm{root}\big(16777216\,a^6\,b^2\,c^4\,d^4\,z^8+16777216\,a^7\,b\,d^4\,z^8-1048576\,a^5\,b\,c\,d^3\,z^6+8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)^6 + 16777216\,a^7\,b\,d^4\,z^8-1048576\,a^5\,b\,c\,d^3\,z^6+8192\,a^3\,b\,c^2\,d^2\,z^4+1,z,k\big)$$

input
```
int(1/(a + b*(c + d*x^2)^4),x)
```

output
```
symsum(log(-8*root(16777216*a^6*b^2*c^4*d^4*z^8 + 16777216*a^7*b*d^4*z^8 -
 1048576*a^5*b*c*d^3*z^6 + 8192*a^3*b*c^2*d^2*z^4 + 1, z, k)*b^7*d^28*(x +
 8*root(16777216*a^6*b^2*c^4*d^4*z^8 + 16777216*a^7*b*d^4*z^8 - 1048576*a^
5*b*c*d^3*z^6 + 8192*a^3*b*c^2*d^2*z^4 + 1, z, k)*a - 32768*root(16777216*
a^6*b^2*c^4*d^4*z^8 + 16777216*a^7*b*d^4*z^8 - 1048576*a^5*b*c*d^3*z^6 + 8
192*a^3*b*c^2*d^2*z^4 + 1, z, k)^5*a^4*b*c^2*d^2 + 4096*root(16777216*a^6*
b^2*c^4*d^4*z^8 + 16777216*a^7*b*d^4*z^8 - 1048576*a^5*b*c*d^3*z^6 + 8192*
a^3*b*c^2*d^2*z^4 + 1, z, k)^4*a^3*b*c^2*d^2*x - 262144*root(16777216*a^6*
b^2*c^4*d^4*z^8 + 16777216*a^7*b*d^4*z^8 - 1048576*a^5*b*c*d^3*z^6 + 8192*
a^3*b*c^2*d^2*z^4 + 1, z, k)^6*a^5*b*c*d^3*x))*root(16777216*a^6*b^2*c^4*d
^4*z^8 + 16777216*a^7*b*d^4*z^8 - 1048576*a^5*b*c*d^3*z^6 + 8192*a^3*b*c^2
*d^2*z^4 + 1, z, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{1}{a+b\left(c+dx^2\right)^4}\,dx=\int \frac{1}{b\,d^4x^8+4bc\,d^3x^6+6b\,c^2d^2x^4+4b\,c^3d\,x^2+b\,c^4+a}dx$$

input
```
int(1/(a+b*(d*x^2+c)^4),x)
```

output
```
int(1/(a + b*c**4 + 4*b*c**3*d*x**2 + 6*b*c**2*d**2*x**4 + 4*b*c*d**3*x**6
 + b*d**4*x**8),x)
```

3.162 $\int \frac{1}{x^2\left(a+b\left(c+dx^2\right)^4\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 2262

$$\int \frac{1}{x^2\left(a+b\left(c+dx^2\right)^4\right)}\,dx = \text{Too large to display}$$

output
```
-1/4*(2*a^(1/4)-2^(1/2)*b^(1/4)*c)/a^(3/4)/(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4
)*c+b^(1/2)*c^2)/x-1/4*(2*a^(1/4)+2^(1/2)*b^(1/4)*c)/a^(3/4)/(a^(1/2)+2^(1
/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)/x+1/8*(2*a^(1/4)/b^(1/4)+2^(1/2)*c+(a^(
1/2)*2^(1/2)+4*a^(1/4)*b^(1/4)*c+2^(1/2)*b^(1/2)*c^2)/b^(1/4)/(a^(1/2)+2^(
1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))*d^(1/2)*arctan(((-2^(1/2)*a^(1/
4)-2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1
/2)-2*b^(1/8)*d^(1/2)*x)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a
^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2))/a^(3/4)/b^(1/8)/(a^(1/2)/b^(1/
2)+2^(1/2)*a^(1/4)*c/b^(1/4)+c^2)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^(1/2)+
2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)-1/8*(2*a^(1/4)/b^(1/4)
+2^(1/2)*c+(a^(1/2)*2^(1/2)+4*a^(1/4)*b^(1/4)*c+2^(1/2)*b^(1/2)*c^2)/b^(1/
4)/(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))*d^(1/2)*arctan((
(-2^(1/2)*a^(1/4)-2*b^(1/4)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)
*c^2)^(1/2))^(1/2)+2*b^(1/8)*d^(1/2)*x)/(2^(1/2)*a^(1/4)+2*b^(1/4)*c+2*(a^
(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2))/a^(3/4)/b^(1/8)
/(a^(1/2)/b^(1/2)+2^(1/2)*a^(1/4)*c/b^(1/4)+c^2)/(2^(1/2)*a^(1/4)+2*b^(1/4
)*c+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))^(1/2)+1/8*(2*
a^(1/4)/b^(1/4)+2^(1/2)*c-(a^(1/2)*2^(1/2)+4*a^(1/4)*b^(1/4)*c+2^(1/2)*b^(
1/2)*c^2)/b^(1/4)/(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*c+b^(1/2)*c^2)^(1/2))*d
^(1/2)*arctan(b^(1/8)*(2^(1/2)*a^(1/4)+2*b^(1/4)*c-2*(a^(1/2)+2^(1/2)*a...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2 \left(a+b \left(c+dx^2\right)^4\right)} \, dx =$$

$$-\frac{8+x\text{RootSum}\left[a+bc^4+4bc^3d\#1^2+6bc^2d^2\#1^4+4bcd^3\#1^6+bd^4\#1^8\&, \frac{4c^3\log(x-\#1)+6c^2d\log(x-\#1)\ldots}{c^3\#1+3c^2\ldots}\right.}{8ax+8bc^4x}$$

input
```
Integrate[1/(x^2*(a + b*(c + d*x^2)^4)),x]
```

output
```
-((8 + x*RootSum[a + b*c^4 + 4*b*c^3*d*#1^2 + 6*b*c^2*d^2*#1^4 + 4*b*c*d^3
*#1^6 + b*d^4*#1^8 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1^2 + 4*c
*d^2*Log[x - #1]*#1^4 + d^3*Log[x - #1]*#1^6)/(c^3*#1 + 3*c^2*d*#1^3 + 3*c
*d^2*#1^5 + d^3*#1^7) & ])/(8*a*x + 8*b*c^4*x))
```

Rubi [A] (verified)

Time = 12.39 (sec) , antiderivative size = 3451, normalized size of antiderivative = 1.53, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b \left(c+d x^2\right)^4\right)} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{1}{x^2 (a+b c^4)}+\frac{b d \left(-4 c^3-6 c^2 d x^2-4 c d^2 x^4-d^3 x^6\right)}{(a+b c^4) \left(a \left(\frac{b c^4}{a}+1\right)+4 b c^3 d x^2+6 b c^2 d^2 x^4+4 b c d^3 x^6+b d^4 x^8\right)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^{7/8}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)c^3}{4(-a)^{3/4}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\,(bc^4+a)}-$$

$$\frac{b^{7/8}\sqrt{d}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)c^3}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\,(bc^4+a)\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}+$$

$$\frac{b^{7/8}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)c^3}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\,(bc^4+a)\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}+$$

$$\frac{b^{7/8}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)c^3}{4(-a)^{3/4}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\,(bc^4+a)}-$$

$$\frac{b^{7/8}\sqrt{d}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)c^3}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\,(bc^4+a)\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}+$$

$$\frac{b^{7/8}\sqrt{d}\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)c^3}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\,(bc^4+a)\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}-$$

$$\frac{b^{5/8}\sqrt{d}\arctan\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}}\right)c^2}{4\sqrt{-a}\sqrt{\sqrt[4]{b}c+\sqrt[4]{-a}}\,(bc^4+a)}-$$

$$\frac{b^{5/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}\sqrt{d}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}-\sqrt{2}\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)c^2}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\,(bc^4+a)}+$$

$$\frac{b^{5/8}\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt{d}x+\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}}{\sqrt{\sqrt[4]{b}c+\sqrt{\sqrt{b}c^2+\sqrt{-a}}}}\right)c^2}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\,(bc^4+a)}+$$

$$\frac{b^{5/8}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt[8]{b}\sqrt{d}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}}\right)c^2}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}c}\,(bc^4+a)}+$$

$$\frac{b^{5/8}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}\log\left(\sqrt[4]{b}dx^2-\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)c^2}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{b}c^2+\sqrt{-a}}\,(bc^4+a)}-$$

$$b^{5/8}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}\log\left(\sqrt[4]{b}dx^2+\sqrt{2}\sqrt[8]{b}\sqrt{\sqrt{\sqrt{b}c^2+\sqrt{-a}}-\sqrt[4]{b}c}\sqrt{d}x+\sqrt{\sqrt{b}c^2+\sqrt{-a}}\right)c^2$$

input
```
Int[1/(x^2*(a + b*(c + d*x^2)^4)),x]
```

output
```
-(1/((a + b*c^4)*x)) - (b^(1/8)*Sqrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[(-
a)^(1/4) + b^(1/4)*c]])/(4*Sqrt[(-a)^(1/4) + b^(1/4)*c]*(a + b*c^4)) + (b^
(3/8)*c*Sqrt[d]*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[(-a)^(1/4) + b^(1/4)*c]])/
(4*(-a)^(1/4)*Sqrt[(-a)^(1/4) + b^(1/4)*c]*(a + b*c^4)) - (b^(5/8)*c^2*Sqr
t[d]*ArcTan[(b^(1/8)*Sqrt[d]*x)/Sqrt[(-a)^(1/4) + b^(1/4)*c]])/(4*Sqrt[-a]
*Sqrt[(-a)^(1/4) + b^(1/4)*c]*(a + b*c^4)) + (b^(7/8)*c^3*Sqrt[d]*ArcTan[(
b^(1/8)*Sqrt[d]*x)/Sqrt[(-a)^(1/4) + b^(1/4)*c]])/(4*(-a)^(3/4)*Sqrt[(-a)^
(1/4) + b^(1/4)*c]*(a + b*c^4)) + (b^(3/8)*c*Sqrt[d]*ArcTan[(Sqrt[-(b^(1/4
)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(
1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[Sqrt[-a] + Sqrt[b
]*c^2]*(a + b*c^4)*Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]) - (b^(7
/8)*c^3*Sqrt[d]*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]
- Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]
]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]*c^2]*(a + b*c^4)*Sqrt[b^(1
/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]) + (b^(1/8)*Sqrt[b^(1/4)*c + Sqrt[Sq
rt[-a] + Sqrt[b]*c^2]]*Sqrt[d]*ArcTan[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] +
 Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*Sqrt[d]*x)/Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a]
 + Sqrt[b]*c^2]]])/(4*Sqrt[2]*Sqrt[Sqrt[-a] + Sqrt[b]*c^2]*(a + b*c^4)) -
(b^(5/8)*c^2*Sqrt[b^(1/4)*c + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]]*Sqrt[d]*ArcTan
[(Sqrt[-(b^(1/4)*c) + Sqrt[Sqrt[-a] + Sqrt[b]*c^2]] - Sqrt[2]*b^(1/8)*S...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.07

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{8}+4bc\,d^{3}\_Z^{6}+6b\,c^{2}d^{2}\_Z^{4}+4b\,c^{3}d\,\_Z^{2}+b\,c^{4}+a\right)}\frac{\left(\_R^{6}d^{3}+4\_R^{4}c\,d^{2}+6\_R^{2}c^{2}d+4c^{3}\right)\ln\left(x-\_R\right)}{d^{3}\_R^{7}+3c\,d^{2}\_R^{5}+3c^{2}d\,\_R^{3}+c^{3}\_R}}{8(b\,c^{4}+a)}-\frac{}{(b\,c^{4}}$
risch	$-\frac{1}{(b\,c^{4}+a)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{6}b^{3}c^{12}+3a^{7}b^{2}c^{8}+3a^{8}b\,c^{4}+a^{9}\right)\_Z^{8}+\left(-24a^{5}b^{2}c^{7}d+40a^{6}b\,c^{3}d\right)\_Z^{6}+\left(2a^{3}b^{2}c^{6}d^{2}+42a^{4}b\,c^{2}d^{2}\right)\_Z^{4}+12\right.}\right.}{}$

input
```
int(1/x^2/(a+b*(d*x^2+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(b*c^4+a)*sum((_R^6*d^3+4*_R^4*c*d^2+6*_R^2*c^2*d+4*c^3)/(_R^7*d^3+3*
_R^5*c*d^2+3*_R^3*c^2*d+_R*c^3)*ln(x-_R),_R=RootOf(_Z^8*b*d^4+4*_Z^6*b*c*d
^3+6*_Z^4*b*c^2*d^2+4*_Z^2*b*c^3*d+b*c^4+a))-1/(b*c^4+a)/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+b\left(c+dx^2\right)^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(a+b*(d*x^2+c)^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a + b \left(c + dx^2\right)^4\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a+b*(d*x**2+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a + b \left(c + dx^2\right)^4\right)} \, dx = \int \frac{1}{\left(\left(dx^2 + c\right)^4 b + a\right) x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*(d*x^2+c)^4),x, algorithm="maxima")
```

output
```
-b*d*integrate((d^3*x^6 + 4*c*d^2*x^4 + 6*c^2*d*x^2 + 4*c^3)/(b*d^4*x^8 +
4*b*c*d^3*x^6 + 6*b*c^2*d^2*x^4 + 4*b*c^3*d*x^2 + b*c^4 + a), x)/(b*c^4 +
a) - 1/((b*c^4 + a)*x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a + b \left(c + dx^2\right)^4\right)} \, dx = \int \frac{1}{\left(\left(dx^2 + c\right)^4 b + a\right) x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*(d*x^2+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 2345, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 \left(a+b \left(c+dx^2\right)^4\right)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*(c + d*x^2)^4)),x)
```

output
```
symsum(log(root(50331648*a^7*b^2*c^8*z^8 + 16777216*a^6*b^3*c^12*z^8 + 503
31648*a^8*b*c^4*z^8 + 16777216*a^9*z^8 - 6291456*a^5*b^2*c^7*d*z^6 + 10485
760*a^6*b*c^3*d*z^6 + 172032*a^4*b*c^2*d^2*z^4 + 8192*a^3*b^2*c^6*d^2*z^4
+ 768*a^2*b*c*d^3*z^2 + b*d^4, z, k)*(root(50331648*a^7*b^2*c^8*z^8 + 1677
7216*a^6*b^3*c^12*z^8 + 50331648*a^8*b*c^4*z^8 + 16777216*a^9*z^8 - 629145
6*a^5*b^2*c^7*d*z^6 + 10485760*a^6*b*c^3*d*z^6 + 172032*a^4*b*c^2*d^2*z^4
+ 8192*a^3*b^2*c^6*d^2*z^4 + 768*a^2*b*c*d^3*z^2 + b*d^4, z, k)*(256*a*b^2
0*c^47*d^33 - root(50331648*a^7*b^2*c^8*z^8 + 16777216*a^6*b^3*c^12*z^8 +
50331648*a^8*b*c^4*z^8 + 16777216*a^9*z^8 - 6291456*a^5*b^2*c^7*d*z^6 + 10
485760*a^6*b*c^3*d*z^6 + 172032*a^4*b*c^2*d^2*z^4 + 8192*a^3*b^2*c^6*d^2*z
^4 + 768*a^2*b*c*d^3*z^2 + b*d^4, z, k)*(x*(1024*a^13*b^9*c^3*d^33 + 11264
*a^12*b^10*c^7*d^33 + 56320*a^11*b^11*c^11*d^33 + 168960*a^10*b^12*c^15*d^
33 + 337920*a^9*b^13*c^19*d^33 + 473088*a^8*b^14*c^23*d^33 + 473088*a^7*b^
15*c^27*d^33 + 337920*a^6*b^16*c^31*d^33 + 168960*a^5*b^17*c^35*d^33 + 563
20*a^4*b^18*c^39*d^33 + 11264*a^3*b^19*c^43*d^33 + 1024*a^2*b^20*c^47*d^33
) - root(50331648*a^7*b^2*c^8*z^8 + 16777216*a^6*b^3*c^12*z^8 + 50331648*a
^8*b*c^4*z^8 + 16777216*a^9*z^8 - 6291456*a^5*b^2*c^7*d*z^6 + 10485760*a^6
*b*c^3*d*z^6 + 172032*a^4*b*c^2*d^2*z^4 + 8192*a^3*b^2*c^6*d^2*z^4 + 768*a
^2*b*c*d^3*z^2 + b*d^4, z, k)*(root(50331648*a^7*b^2*c^8*z^8 + 16777216*a^
6*b^3*c^12*z^8 + 50331648*a^8*b*c^4*z^8 + 16777216*a^9*z^8 - 6291456*a^...
```

Reduce [F]

$$\int \frac{1}{x^2 \left(a+b \left(c+dx^2\right)^4\right)} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(a+b*(d*x^2+c)^4),x)
```

output
```
( - int(x**6/(a**2 + 2*a*b*c**4 + 4*a*b*c**3*d*x**2 + 6*a*b*c**2*d**2*x**4
 + 4*a*b*c*d**3*x**6 + a*b*d**4*x**8 + b**2*c**8 + 4*b**2*c**7*d*x**2 + 6*
b**2*c**6*d**2*x**4 + 4*b**2*c**5*d**3*x**6 + b**2*c**4*d**4*x**8),x)*a*b*
d**4*x - int(x**6/(a**2 + 2*a*b*c**4 + 4*a*b*c**3*d*x**2 + 6*a*b*c**2*d**2
*x**4 + 4*a*b*c*d**3*x**6 + a*b*d**4*x**8 + b**2*c**8 + 4*b**2*c**7*d*x**2
 + 6*b**2*c**6*d**2*x**4 + 4*b**2*c**5*d**3*x**6 + b**2*c**4*d**4*x**8),x)
*b**2*c**4*d**4*x - 4*int(x**4/(a**2 + 2*a*b*c**4 + 4*a*b*c**3*d*x**2 + 6*
a*b*c**2*d**2*x**4 + 4*a*b*c*d**3*x**6 + a*b*d**4*x**8 + b**2*c**8 + 4*b**
2*c**7*d*x**2 + 6*b**2*c**6*d**2*x**4 + 4*b**2*c**5*d**3*x**6 + b**2*c**4*
d**4*x**8),x)*a*b*c*d**3*x - 4*int(x**4/(a**2 + 2*a*b*c**4 + 4*a*b*c**3*d*
x**2 + 6*a*b*c**2*d**2*x**4 + 4*a*b*c*d**3*x**6 + a*b*d**4*x**8 + b**2*c**
8 + 4*b**2*c**7*d*x**2 + 6*b**2*c**6*d**2*x**4 + 4*b**2*c**5*d**3*x**6 + b
**2*c**4*d**4*x**8),x)*b**2*c**5*d**3*x - 6*int(x**2/(a**2 + 2*a*b*c**4 +
4*a*b*c**3*d*x**2 + 6*a*b*c**2*d**2*x**4 + 4*a*b*c*d**3*x**6 + a*b*d**4*x*
*8 + b**2*c**8 + 4*b**2*c**7*d*x**2 + 6*b**2*c**6*d**2*x**4 + 4*b**2*c**5*
d**3*x**6 + b**2*c**4*d**4*x**8),x)*a*b*c**2*d**2*x - 6*int(x**2/(a**2 + 2
*a*b*c**4 + 4*a*b*c**3*d*x**2 + 6*a*b*c**2*d**2*x**4 + 4*a*b*c*d**3*x**6 +
 a*b*d**4*x**8 + b**2*c**8 + 4*b**2*c**7*d*x**2 + 6*b**2*c**6*d**2*x**4 +
4*b**2*c**5*d**3*x**6 + b**2*c**4*d**4*x**8),x)*b**2*c**6*d**2*x - 4*int(1
/(a**2 + 2*a*b*c**4 + 4*a*b*c**3*d*x**2 + 6*a*b*c**2*d**2*x**4 + 4*a*b*...
```

3.163 $\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 181

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{(b^2+4abc-8a^2c^2)\,(c+dx^2)\,\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}+\frac{(b-12ac)\,(c+dx^2)^2\,\sqrt{a+\frac{b}{c+dx^2}}}{24ad^3}+\frac{(c+dx^2)^3\,\sqrt{a+\frac{b}{c+dx^2}}}{6d^3}+\frac{b(b^2+4abc+8a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}$$

output
```
-1/16*(-8*a^2*c^2+4*a*b*c+b^2)*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a^2/d^3+1/2
4*(-12*a*c+b)*(d*x^2+c)^2*(a+b/(d*x^2+c))^(1/2)/a/d^3+1/6*(d*x^2+c)^3*(a+b
/(d*x^2+c))^(1/2)/d^3+1/16*b*(8*a^2*c^2+4*a*b*c+b^2)*arctanh((a+b/(d*x^2+c
))^(1/2)/a^(1/2))/a^(5/2)/d^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.80

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-3b^2+2ab(-5c+dx^2)+8a^2(c^2-cdx^2+d^2x^4)\right)+3b(b^2+4abc+8a^2c^2)\,\text{arct}}{48a^{5/2}d^3}$$

input
```
Integrate[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b
*(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c +
8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(
5/2)*d^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2057, 2053, 2052, 27, 366, 27, 360, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$\Big\downarrow \text{2057}$$
$$\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$
$$\Big\downarrow \text{2053}$$
$$\frac{1}{2}\int x^4\sqrt{\frac{adx^2+b+ac}{dx^2+c}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{x^4\left(-cx^4+b+ac\right)^2}{d^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{x^4\left(-cx^4+b+ac\right)^2}{\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$$\Big\downarrow \text{366}$$

$$-\frac{b\left(\frac{b^2x^6}{6a\left(a-x^4\right)^3}-\frac{\int \frac{3x^4\left(2ac^2x^4+b^2-2(b+ac)^2\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}\right)}{d^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\left(\frac{b^2x^6}{6a\left(a-x^4\right)^3}-\frac{\int \frac{x^4\left(2ac^2x^4+b^2-2(b+ac)^2\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}\right)}{d^3}$$

$$\Big\downarrow \text{360}$$

$$-\frac{b\left(\frac{b^2x^6}{6a\left(a-x^4\right)^3}-\frac{-\frac{1}{4}\int -\frac{b(b+4ac)-8ac^2x^4}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{2a}\right)}{d^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(\frac{b^2x^6}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\int \frac{b(b+4ac)-8ac^2x^4}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{2a}\right)}{d^3}$$

$$\Big\downarrow \text{298}$$

$$-\frac{b\left(\frac{b^2x^6}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\left(\frac{\left(8a^2c^2+4abc+b^2\right)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\left(-8a^2c^2+4abc+b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{2a}\right)}{d^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\left(\frac{b^2x^6}{6a(a-x^4)^3}-\frac{\frac{1}{4}\left(\frac{\left(-8a^2c^2+4abc+b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}+\frac{\left(8a^2c^2+4abc+b^2\right)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}\right)-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{2a}\right)}{d^3}$$

input
```
Int[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-((b*((b^2*x^6)/(6*a*(a - x^4)^3) - (-1/4*(b*(b + 4*a*c)*Sqrt[(b + a*c + a
*d*x^2)/(c + d*x^2)])/(a - x^4)^2 + (((b^2 + 4*a*b*c - 8*a^2*c^2)*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)])/(2*a*(a - x^4)) + ((b^2 + 4*a*b*c + 8*a^2*c
^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)))/4
)/(2*a)))/d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.31

method	result
risch	$\frac{(8a^2d^2x^4-8a^2cd\,x^2+2abd\,x^2+8a^2c^2-10abc-3b^2)(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48d^3a^2}+\frac{b(8a^2c^2+4abc+b^2)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc}\right.}{32d^2a^2}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-48\sqrt{a\,d^2}\,\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,a^2cd\,x^2+24\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2}}{2\sqrt{a\,d^2}}\right.\right.}{}$

input
```
int(x^5*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2+2*a*b*d*x^2+8*a^2*c^2-10*a*b*c-3*b^2
)*(d*x^2+c)/a^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/32*b/d^2*(8*a^2*c^2+4*
a*b*c+b^2)/a^2*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*a*
c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.34

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$=\left[\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac\right.}{}\right.$$

$$-\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(8\,a^3d^3x^6+2\,a^2bd^2x^4+8\,a^3c^3}{96\,a^3d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^
2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c
+ b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
 + 4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2
*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(
a^3*d^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d
*x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10
*a^2*b*c^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^3*d^3)]
```

Sympy [F]

$$\int x^5 \sqrt{a + \frac{b}{c + dx^2}}\, dx = \int x^5 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\, dx$$

input
```
integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.81

$$\int x^5 \sqrt{a + \frac{b}{c + dx^2}}\, dx = -\frac{3\,(8\,a^2bc^2 - 4\,ab^2c - b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}} - 8\,(6\,a^3bc^2 - ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} + 3\,(8\,a^4bc^2 + 4\,a^3b^2c + a^2b^3)\sqrt{}}{48\left(a^5d^3 - \frac{3\,(adx^2+ac+b)a^4d^3}{dx^2+c} + \frac{3\,(adx^2+ac+b)^2a^3d^3}{(dx^2+c)^2} - \frac{(adx^2+ac+b)^3a^2d^3}{(dx^2+c)^3}\right)} - \frac{(8\,a^2c^2 + 4\,abc + b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{5}{2}}d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(8*a^2*b*c^2 - 4*a*b^2*c - b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))
^(5/2) - 8*(6*a^3*b*c^2 - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) +
 3*(8*a^4*b*c^2 + 4*a^3*b^2*c + a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(a^5*d^3 - 3*(a*d*x^2 + a*c + b)*a^4*d^3/(d*x^2 + c) + 3*(a*d*x^2 +
a*c + b)^2*a^3*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^2*d^3/(d*x^2 +
c)^3) - 1/32*(8*a^2*c^2 + 4*a*b*c + b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
/(a^(5/2)*d^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.20

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{1}{96}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{4\,a^2cd^3-abd^3}{a^2d^5}\right)+\frac{8\,a^2c^2d^2-10\,abcd^2-3\,b^2d^2}{a^2d^5}\right.\right.$$
$$+c)$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^
2/d - (4*a^2*c*d^3 - a*b*d^3)/(a^2*d^5)) + (8*a^2*c^2*d^2 - 10*a*b*c*d^2 -
 3*b^2*d^2)/(a^2*d^5)) - 3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*log(abs(2*a*c*d
 + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b
*c))*sqrt(a)*abs(d) + b*d))/(a^(5/2)*d^2*abs(d)))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^5\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^5*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5*(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.63

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3c^2-8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3cd\,x^2+8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3}{}$$

input
```
int(x^5*(a+b/(d*x^2+c))^(1/2),x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**2 - 8*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*a**3*c*d*x**2 + 8*sqrt(c + d*x**2)*sqrt(a*c + a
*d*x**2 + b)*a**3*d**2*x**4 - 10*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)
*a**2*b*c + 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b*d*x**2 - 3*
sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2 + 24*sqrt(a)*log( - sqrt(
a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*a**2*b*c**2 + 12*sqrt(a)
*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*a*b**2*c +
3*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*b*
*3)/(48*a**3*d**3)
```

3.164 $\int x^3 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)^2\sqrt{a+\frac{b}{c+dx^2}}}{4d^2}-\frac{b(b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

output
```
1/8*(-4*a*c+b)*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a/d^2+1/4*(d*x^2+c)^2*(a+b/
(d*x^2+c))^(1/2)/d^2-1/8*b*(4*a*c+b)*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2)
)/a^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b-2ac+2adx^2\right)}{8ad^2}-\frac{b(b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

input
```
Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a*d*x^2)
)/(8*a*d^2) - (b*(b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
/Sqrt[a]])/(8*a^(3/2)*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2053, 2052, 25, 27, 360, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\sqrt{\frac{adx^2+b+ac}{dx^2+c}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{x^4\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{x^4\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{x^4\left(-cx^4+b+ac\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{360}$$

$$\frac{b\left(\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}-\frac{1}{4}\int \frac{b-4cx^4}{(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)}{d^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{b\left(\frac{1}{4}\left(-\frac{(4ac+b)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}-\frac{(b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a(a-x^4)}\right)+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{1}{4}\left(-\frac{(4ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{(b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a(a-x^4)}\right)+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

input
```
Int[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*(a - x^4)^2) + (-1/2*((b
- 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(a*(a - x^4)) - ((b + 4*a*
c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)))/4)
)/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{\left(-2ad\,x^{2}+2ac-b\right)\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{8d^{2}a}-\frac{b(4ac+b)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^{2}x^{2}}{\sqrt{a\,d^{2}}}+\sqrt{a\,c^{2}+bc+(2acd+bd)x^{2}+a\,d^{2}x^{4}}\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{16da\sqrt{a\,d^{2}}\,(ad\,x^{2}+ac+b)}$
default	$-\frac{\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(-4\sqrt{a\,d^{2}x^{4}+2ad\,x^{2}c+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,ad\,x^{2}+4\ln\left(\frac{2a\,d^{2}x^{2}+2acd+2\sqrt{a\,d^{2}x^{4}+2ad\,x^{2}c+bd\,x^{2}+a\,c^{2}+b}}{2\sqrt{a\,d^{2}}}\right.\right.}{}$

input
```
int(x^3*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c-b)*(d*x^2+c)/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)-1/16*b/d*(4*a*c+b)/a*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b
*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.78

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$=\left[\frac{(4\,abc+b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\right.}{32\,a^2d^2}\right.$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
+ a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(
a^2*d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b
)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)
) + 2*(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^2*d^2)]
```

Sympy [F]

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(103) = 206$.

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.86

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{(4\,abc-b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc+ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3d^2-\frac{2\,(adx^2+ac+b)a^2d^2}{dx^2+c}+\frac{(adx^2+ac+b)^2ad^2}{(dx^2+c)^2}\right)}+\frac{(4\,ac+b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{3}{2}}d^2}$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/8*((4*a*b*c - b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c
 + a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2 - 2*(a*d*x^2 + a
*c + b)*a^2*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a*d^2/(d*x^2 + c)^2) +
 1/16*(4*a*c + b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.35

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{1}{16}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{d}-\frac{2\,acd-bd}{ad^3}\right)+\frac{(4\,abc+b^2)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2\right.\right.\right.}{}\right.$$
$$+c)$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (
2*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(2*a*c*d + 2*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(
d) + b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^3*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3*(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.33

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$= \frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c+2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2d\,x^2+\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,ab\,-}{8a^2}$$

input
```
int(x^3*(a+b/(d*x^2+c))^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c + 2*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*a**2*d*x**2 + sqrt(c + d*x**2)*sqrt(a*c + a*d*x
**2 + b)*a*b + 4*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c
+ d*x**2)*a)*a*b*c + sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqr
t(c + d*x**2)*a)*b**2)/(8*a**2*d**2)
```

3.165 $\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

output
```
1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/d+1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/
a^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{2d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

input
```
Integrate[x*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqr
t[(b + a*(c + d*x^2))/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2024, 773, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \sqrt{a+\frac{b}{dx^2+c}}d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int \left(dx^2+c\right)^2\sqrt{a+\frac{b}{dx^2+c}}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{1}{2}b\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}-\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{b\left(dx^2+c\right)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}-\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(c+dx^2\right)\left(-\sqrt{a+\frac{b}{c+dx^2}}\right)-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d}$$

input
```
Int[x*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-1/2*(-((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (b*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/Sqrt[a])/d
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(57) = 114$.

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{\sqrt{\frac{(d x^2+c)a+b}{d x^2+c}}\,(d x^2+c)\left(b\ln\left(\frac{2\sqrt{a(d x^2+c)^2+b(d x^2+c)}\,\sqrt{a}+2(d x^2+c)a+b}{2\sqrt{a}}\right)+2\sqrt{a(d x^2+c)^2+b(d x^2+c)}\,\sqrt{a}\right)}{4d\sqrt{(d x^2+c)((d x^2+c)a+b)}\,\sqrt{a}}$
risch	$\frac{(d x^2+c)\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}}{2d}+\frac{b\ln\left(\frac{acd+\frac{1}{2}bd+a d^2x^2}{\sqrt{a d^2}}+\sqrt{a c^2+bc+(2acd+bd)x^2+a d^2x^4}\right)\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}\,\sqrt{(d x^2+c)(ad x^2}}{4\sqrt{a d^2}\,(ad x^2+ac+b)}$
default	$\frac{\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(b\ln\left(\frac{2a d^2x^2+2acd+2\sqrt{a d^2x^4+2ad x^2c+bd x^2+a c^2+bc}\,\sqrt{a d^2}+bd}{2\sqrt{a d^2}}\right)d+2\sqrt{a d^2x^4+2ad x^2c+bd x^2}\right.}{4\sqrt{(d x^2+c)(ad x^2+ac+b)}\,\sqrt{a d^2}\,d}$

input
```
int(x*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(((d*x^2+c)*a+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(b*ln(1/2*(2*(a*(d*x^2+c
)^2+b*(d*x^2+c))^(1/2)*a^(1/2)+2*(d*x^2+c)*a+b)/a^(1/2))+2*(a*(d*x^2+c)^2+
b*(d*x^2+c))^(1/2)*a^(1/2))/((d*x^2+c)*((d*x^2+c)*a+b))^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.87

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$= \left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{\right.}{8\,ad}\right.$$

$$\left. -\frac{\sqrt{-a}b\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(adx^2+ac)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,ad}\right]$$

input
```
integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 +
2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2
*c + a*b)) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d
)]
```

Sympy [F]

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(57) = 114$.

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.83

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)}-\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\sqrt{a}d}$$

input
```
integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d/
(d*x^2 + c)) - 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(57) = 114.

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx = -\frac{1}{4}\left(\frac{b\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}|d|+bd\right|\right)}{\sqrt{a}|d|}-\frac{2\sqrt{ad^2x^4+2}\cdots}{}\right. \left. + c\right)$$

input
```
integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
-1/4*(b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^
2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(sqrt(a)*abs(d)) - 2*sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 + c)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\sqrt{\frac{b\,(d\,x^2+c)+a\,(d\,x^2+c)^2}{(d\,x^2+c)^2}}\,(d\,x^2+c)\left(\frac{b\,\ln\left(\frac{\frac{b}{2}+a\,(d\,x^2+c)+\sqrt{a}\,\sqrt{b\,(d\,x^2+c)+a\,(d\,x^2+c)^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b\,(d\,x^2+c)+a\,(d\,x^2+c)^2}}+2\right)}{4\,d}$$

input
```
int(x*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
(((b*(c + d*x^2) + a*(c + d*x^2)^2)/(c + d*x^2)^2)^(1/2)*(c + d*x^2)*((b*l
og((b/2 + a*(c + d*x^2) + a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2))
/a^(1/2)))/(a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2)) + 2))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a+\sqrt{a}\,\log\left(-\sqrt{a}\,\sqrt{ad\,x^2+ac+b}-\sqrt{d\,x^2+c}\,a\right)b}{2ad}$$

input
```
int(x*(a+b/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a + sqrt(a)*log( - sqrt(a)*sqrt
(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*b)/(2*a*d)
```

3.166 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx=\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{\sqrt{b+ac}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{c}}$$

output
```
a^(1/2)*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))-(a*c+b)^(1/2)*arctanh(c^(1/
2)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx=-\frac{\sqrt{-b-ac}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}}+\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]
```

output
```
-((Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/S
qrt[-b - a*c]])/Sqrt[c]) + Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2053, 2052, 25, 27, 383, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{x^4}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{x^4}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{x^4}{\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{383}$$

$$b\left(\frac{a\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{(ac+b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{ 219}$$

$$b\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}-\frac{(ac+b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$b\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}-\frac{\sqrt{ac+b}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{c}}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x,x]
```

output
```
b*((Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/b - (S
qrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[
b + a*c]])/(b*Sqrt[c]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.94

method	result
default	$\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(\ln\left(\frac{2a d^2 x^2+2acd+2\sqrt{a d^2 x^4+2ad x^2 c+bd x^2+a c^2+bc}\,\sqrt{a d^2}+bd}{2\sqrt{a d^2}}\right)acd-\sqrt{a c^2+bc}\,\ln\left(\frac{2ad x^2 c+bd x^2+2a c^2+2\sqrt{}\right.\right.}{2\sqrt{(d x^2+c)(ad x^2+ac+b)}\,c\sqrt{a d^2}}$

input
```
int((a+b/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(ln(1/2*(2*a*d^2*x^2+2*a*c
*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*a*c*d-(a*c^2+b*c)^(1/2)*ln((2*a*d*x^2*c+b*d*x^2+2*a*c^2+2*(a*
c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)
*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c/(a*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(64) = 128$.

Time = 0.14 (sec) , antiderivative size = 927, normalized size of antiderivative = 11.59

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} \, dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a
*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*
c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 +
 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
)*sqrt((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b
)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)
) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2
*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 -
4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(
-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x
^2 + 2*a*b*c + b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^
2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*
a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)
*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((
2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c...
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(64) = 128$.

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.99

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{(ac+b)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}} - \frac{1}{2}\sqrt{a}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*(a*c + b)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b
)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt(
(a*c + b)*c) - 1/2*sqrt(a)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{ac+b}\,\sqrt{a d\,x^2+ac+b}\,c-\sqrt{c}\,\sqrt{d\,x^2+c}\,ac-\sqrt{c}\,\sqrt{d\,x^2+c}\,b\right)-\sqrt{c}\,\sqrt{ac+b}\log(x)+}{c}$$

input
```
int((a+b/(d*x^2+c))^(1/2)/x,x)
```

output

```
(sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt
(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b) - sqrt(c)*sqrt(a*c
+ b)*log(x) + sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d
*x**2)*a)*c)/c
```

3.167 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx=-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}+\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{3/2}\sqrt{b+ac}}$$

output
```
-1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c/x^2+1/2*b*d*arctanh(c^(1/2)*(a+b/(d
*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)/(a*c+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx=-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2}-\frac{bd\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{3/2}\sqrt{-b-ac}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

output
```
-1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^2) - (b*d*Ar
cTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c
^(3/2)*Sqrt[-b - a*c])
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2053, 2052, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^4}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd\int \frac{x^4}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 252}$$

$$-bd\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c\left(ac+b-cx^4\right)}-\frac{\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2c}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-bd\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c\left(ac+b-cx^4\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{3/2}\sqrt{ac+b}}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

output
```
-(b*d*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*c*(b + a*c - c*x^4)) - Arc
Tanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]]/(2*c^(
3/2)*Sqrt[b + a*c])))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(72) = 144$.

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

method	result
risch	$-\frac{(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{2c\,x^2}+\frac{db\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}}{x^2}\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(a}}{4c\sqrt{a\,c^2+bc}\,(ad\,x^2+ac+b)}$
default	$-\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-2a\,d^2\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,x^4\sqrt{a\,c^2+bc}-\ln\left(\frac{2ad\,x^2c+bd\,x^2+2a\,c^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2a}}{x^2}\right.\right.$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c*(d*x^2+c)/x^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/4/c*d*b/(a*c^2+b*
c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*
c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(72) = 144$.

Time = 0.12 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.92

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$$

$$= \left[\frac{\sqrt{ac^2+bc}bdx^2\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2+4((2ac+b)d^2x^4+2ac^3+(4ac^2+3}{x^4}\right.}{8(ac^3+bc^2)x^2}\right.$$

$$-\frac{\sqrt{-ac^2-bc}bdx^2\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2c^3+2abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{}}{4(ac^3+bc^2)x^2}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2
)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3
 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a
*c^3 + b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c +
b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a
*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/((a*c^3 + b*c^2)*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx=\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(72) = 144$.

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx=-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2+bc-\frac{(adx^2+ac+b)c^2}{dx^2+c}\right)}-\frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^2 + b*c - (a*d*x^2 + a
*c + b)*c^2/(d*x^2 + c)) - 1/4*b*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt
((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(72) = 144$.

Time = 0.18 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = -\frac{1}{2}\left(\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bcc}} + \frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+b\cdots}\right.}{\left(ac^2-\left(\sqrt{ad\cdots}\right.\right.} \cdots + c\right)$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*c) + (2*a^(3/2)*
c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*c))*
sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^3,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$$
$$= \frac{-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a\,c^2-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,bc+\sqrt{c}\,\sqrt{ac+b}\,\log\left(\sqrt{ac+b}\,\sqrt{ad\,x^2+ac}\right.}{2c^2x^2\,(ac+b)}$$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^3,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c**2 - sqrt(c + d*x**2)*sq
rt(a*c + a*d*x**2 + b)*b*c + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(
a*c + a*d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt(c + d*
x**2)*b)*b*d*x**2 - sqrt(c)*sqrt(a*c + b)*log(x)*b*d*x**2)/(2*c**2*x**2*(a
*c + b))
```

3.168 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{(5b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2)^2\sqrt{a+\frac{b}{c+dx^2}}}{4c^2x^4} - \frac{b(3b+4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{5/2}(b+ac)^{3/2}}$$

output

```
1/8*(4*a*c+5*b)*d*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^2-1/4*(d*x
^2+c)^2*(a+b/(d*x^2+c))^(1/2)/c^2/x^4-1/8*b*(4*a*c+3*b)*d^2*arctanh(c^(1/2
)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(5/2)/(a*c+b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b(2c-3dx^2)+2ac(c-dx^2)\right)}{8c^2(b+ac)x^4} - \frac{b(3b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{5/2}(-b-ac)^{3/2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

output
```
-1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(2*c - 3*d*x^2)
 + 2*a*c*(c - d*x^2)))/(c^2*(b + a*c)*x^4) - (b*(3*b + 4*a*c)*d^2*ArcTan[(
Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(5/2)
*(-b - a*c)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2057, 2053, 2052, 25, 27, 360, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^6}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd\int -\frac{dx^4(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$bd\int \frac{dx^4(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 27}$$

$$bd^2\int \frac{x^4(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 360}$$

$$bd^2\left(-\frac{\int -\frac{4cx^4+b}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$bd^2\left(\frac{\int \frac{4cx^4+b}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{ 298}$$

$$bd^2\left(\frac{\frac{(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}-\frac{(4ac+3b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$bd^2\left(\frac{\frac{(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}-\frac{(4ac+3b)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

output
```
b*d^2*(-1/4*(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c^2*(b + a*c - c*x^
4)^2) + (((5*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(b + a*c
)*(b + a*c - c*x^4)) - ((3*b + 4*a*c)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*Sqrt[c]*(b + a*c)^(3/2)))/(4*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(-2ad\,x^{2}c-3bd\,x^{2}+2a\,c^{2}+2bc\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{8c^{2}x^{4}\left(ac+b\right)}-\frac{d^{2}b\left(4ac+3b\right)\ln\left(\frac{2a\,c^{2}+2bc+\left(2acd+bd\right)x^{2}+2\sqrt{a\,c^{2}+bc}\,\sqrt{a\,c^{2}+bc+\left(2acd+\right.}}{x^{2}}\right.}{16c^{2}\left(ac+b\right)\sqrt{a\,c^{2}+bc}\,\left(ad\right.}$
default	$-\frac{\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(12a^{2}d^{3}\sqrt{a\,d^{2}x^{4}+2ad\,x^{2}c+bd\,x^{2}+a\,c^{2}+bc}\,x^{6}c\left(a\,c^{2}+bc\right)^{\frac{3}{2}}+4\ln\left(\frac{2ad\,x^{2}c+bd\,x^{2}+2a\,c^{2}+2\sqrt{a\,c^{2}+bc}\,\sqrt{a\,d^{2}x}}{x^{2}}\right.\right.}{}$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)*(-2*a*c*d*x^2-3*b*d*x^2+2*a*c^2+2*b*c)/c^2/x^4/(a*c+b)*((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*d^2*b*(4*a*c+3*b)/c^2/(a*c+b)/(a*c^2+b*
c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*
c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.85

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$

$$= \left[\frac{(4\,abc+3\,b^2)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)\,d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)\,dx^2-4\,((2\,ac+b)d^2x^4}{x^4}\right.}{}\right.$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b
*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b
*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*
b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)
*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*
c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a
*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5
*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*
d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*
c^3)*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**5,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(130) = 260.

Time = 0.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{(4\,abc+3\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(ac^3+bc^2)\sqrt{(ac+b)c}} - \frac{(4\,abc^2+5\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4\,a^2bc^2+7\,ab^2c+3\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2+\frac{(ac^5+bc^4)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)}{dx^2+c}\right)}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/16*(4*a*b*c + 3*b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) -
sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c +
b)*c)))/((a*c^3 + b*c^2)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 + 5*b^2*c)*d
^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 7*a*b^2*c + 3*
b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*c^5 + 3*a^2*b*c^4 + 3
*a*b^2*c^3 + b^3*c^2 + (a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2
 - 2*(a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(130) = 260.

Time = 0.20 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.75

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$
$$=\frac{1}{8}\left(\frac{(4\,abcd^2+3\,b^2d^2)\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{(ac^3+bc^2)\sqrt{-ac^2-bc}}+\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4}\right.}{}\right.$$
$$+c)$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/8*((4*a*b*c*d^2 + 3*b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a*c^3 + b*c^2
)*sqrt(-a*c^2 - b*c)) + (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(
5/2)*c^3*d*abs(d) + 24*a^(5/2)*b*c^4*d*abs(d) + 36*(sqrt(a*d^2)*x^2 - sqrt
(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 8*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(
3/2)*b*c^2*d*abs(d) + 24*a^(3/2)*b^2*c^3*d*abs(d) - 4*(sqrt(a*d^2)*x^2 - s
qrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 25*(sq
rt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b
^2*c^2*d^2 + 8*sqrt(a)*b^3*c^2*d*abs(d) - 3*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*
x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + 5*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a*c
^3 + b*c^2)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^2 + b*c)^2))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx=\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^5}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^5,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.54

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$
$$= \frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^4+2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^3d\,x^2-4\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{}$$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^5,x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**4 + 2*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**3*d*x**2 - 4*sqrt(c + d*x**2)*sqrt(a
*c + a*d*x**2 + b)*a*b*c**3 + 5*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*
a*b*c**2*d*x**2 - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*c**2 +
3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*c*d*x**2 + 4*sqrt(c)*sqrt
(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c +
d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*a*b*c*d**2*x**4 + 3*sqrt(c)*sqrt
(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c +
d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*b**2*d**2*x**4 - 4*sqrt(c)*sqrt(
a*c + b)*log(x)*a*b*c*d**2*x**4 - 3*sqrt(c)*sqrt(a*c + b)*log(x)*b**2*d**2
*x**4)/(8*c**3*x**4*(a**2*c**2 + 2*a*b*c + b**2))
```

3.169 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 226

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7} \, dx = -\frac{(11b^2+20abc+8a^2c^2)\, d^2(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16c^3(b+ac)^2x^2} + \frac{(13b+12ac)d(c+dx^2)^2\sqrt{a+\frac{b}{c+dx^2}}}{24c^3(b+ac)x^4} - \frac{(c+dx^2)^3\sqrt{a+\frac{b}{c+dx^2}}}{6c^3x^6} + \frac{b(5b^2+12abc+8a^2c^2)\, d^3 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{7/2}(b+ac)^{5/2}}$$

output
```
-1/16*(8*a^2*c^2+20*a*b*c+11*b^2)*d^2*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c^3/
(a*c+b)^2/x^2+1/24*(12*a*c+13*b)*d*(d*x^2+c)^2*(a+b/(d*x^2+c))^(1/2)/c^3/(
a*c+b)/x^4-1/6*(d*x^2+c)^3*(a+b/(d*x^2+c))^(1/2)/c^3/x^6+1/16*b*(8*a^2*c^2
+12*a*b*c+5*b^2)*d^3*arctanh(c^(1/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/
c^(7/2)/(a*c+b)^(5/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(8a^2c^2(c^2-cdx^2+d^2x^4)+2abc(8c^2-9cdx^2+13d^2x^4)+b^2(8c^2-10cdx^2+15\right.}{48c^3(b+ac)^2x^6} - \frac{b(5b^2+12abc+8a^2c^2)\,d^3\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{16c^{7/2}(-b-ac)^{5/2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

output
```
-1/48*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^2 -
 c*d*x^2 + d^2*x^4) + 2*a*b*c*(8*c^2 - 9*c*d*x^2 + 13*d^2*x^4) + b^2*(8*c^
2 - 10*c*d*x^2 + 15*d^2*x^4)))/(c^3*(b + a*c)^2*x^6) - (b*(5*b^2 + 12*a*b*
c + 8*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/
Sqrt[-b - a*c]])/(16*c^(7/2)*(-b - a*c)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2053, 2052, 27, 366, 27, 360, 27, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^8}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{d^2x^4\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$-bd^3\int \frac{x^4\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{366}$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\int \frac{3x^4\left(2c(b+ac)x^4+b^2-2a^2c^2\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \text{27}$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\int \frac{x^4\left(2c(b+ac)x^4+b^2-2a^2c^2\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \text{360}$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c\left(ac+b-cx^4\right)^2}-\frac{\int \frac{c\left(8c(b+ac)x^4+b(3b+4ac)\right)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^2}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \text{27}$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c\left(ac+b-cx^4\right)^2}-\frac{\int \frac{8c(b+ac)x^4+b(3b+4ac)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \text{298}$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b-cx^4)^2}-\frac{\frac{\left(8a^2c^2+20abc+11b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)\left(ac+b-cx^4\right)}-\frac{\left(8a^2c^2+12abc+5b^2\right)\int\frac{1}{-cx^4+b+ac}}{2(ac+b)}}{4c}}{2c^2(ac+b)}\right.$$

$\downarrow$ 221

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b-cx^4)^2}-\frac{\frac{\left(8a^2c^2+20abc+11b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)\left(ac+b-cx^4\right)}-\frac{\left(8a^2c^2+12abc+5b^2\right)\text{arctanh}\left(\right.}{2\sqrt{c}(ac+b)^{3/2}}}{4c}}{2c^2(ac+b)}\right.$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

output
```
-(b*d^3*((b^2*x^6)/(6*c^2*(b + a*c)*(b + a*c - c*x^4)^3) - ((b*(3*b + 4*a*
c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c*(b + a*c - c*x^4)^2) - (((1
1*b^2 + 20*a*b*c + 8*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(b
 + a*c)*(b + a*c - c*x^4)) - ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*ArcTanh[(Sqrt
[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*Sqrt[c]*(b +
 a*c)^(3/2)))/(4*c))/(2*c^2*(b + a*c))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.40

method	result
risch	$-\frac{(d\,x^2+c)\left(8a^2c^2d^2x^4+26abc\,d^2x^4-8a^2c^3d\,x^2+15b^2d^2x^4-18ab\,c^2x^2d+8a^2c^4-10b^2c\,x^2d+16ab\,c^3+8b^2c^2\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48c^3x^6(ac+b)^2}+\frac{d^3b}{}$
default	Expression too large to display

input
```
int((a+b/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(8*a^2*c^2*d^2*x^4+26*a*b*c*d^2*x^4-8*a^2*c^3*d*x^2+15*b^2
*d^2*x^4-18*a*b*c^2*d*x^2+8*a^2*c^4-10*b^2*c*d*x^2+16*a*b*c^3+8*b^2*c^2)/c
^3/x^6/(a*c+b)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/32*d^3*b*(8*a^2*c^2+1
2*a*b*c+5*b^2)/(a*c+b)^2/c^3/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+
b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2)
)/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.34

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$$

$$= \left[\frac{3\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)\sqrt{ac^2+bc}d^3x^6\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx}{x}\right.}{}\right.$$

$$-\frac{3\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)\sqrt{-ac^2-bc}d^3x^6\arctan\left(\frac{\left((2\,ac+b)dx^2+2\,ac^2+2\,bc\right)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2\,abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)+2\left(8\,a\right.}{}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log
(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*
c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 +
41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4
 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b
^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7
+ 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*
b^2*c + 5*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 +
2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 +
(8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^
6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^
2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^7}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(202) = 404$.

Time = 0.15 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = -\frac{(8\,a^2bc^2+12\,ab^2c+5\,b^3)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\,(a^2c^5+2\,abc^4+b^2c^3)\sqrt{(ac+b)c}} - \frac{3\,(8\,a^2bc^4+20\,ab^2c^3+11\,b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^4+18\,a^2b^2c^3+17\,ab^3c^2+5\,b^4c)d^3\left(\frac{adx^2+ac+}{dx^2+c}\right.}{48\left(a^5c^8+5\,a^4bc^7+10\,a^3b^2c^6+10\,a^2b^3c^5+5\,ab^4c^4+b^5c^3-\frac{(a^2c^8+2\,abc^7+b^2c^6)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{3\,(a^3c^8+3\,a^2b}{}\right.}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) + sqrt((a*c + b)*c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt((a*c + b)*
c)) - 1/48*(3*(8*a^2*b*c^4 + 20*a*b^2*c^3 + 11*b^3*c^2)*d^3*((a*d*x^2 + a*
c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 18*a^2*b^2*c^3 + 17*a*b^3*c^2
 + 5*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 +
 28*a^3*b^2*c^3 + 37*a^2*b^3*c^2 + 22*a*b^4*c + 5*b^5)*d^3*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a^5*c^8 + 5*a^4*b*c^7 + 10*a^3*b^2*c^6 + 10*a^2*b
^3*c^5 + 5*a*b^4*c^4 + b^5*c^3 - (a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2
+ a*c + b)^3/(d*x^2 + c)^3 + 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*
c^5)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^4*c^8 + 4*a^3*b*c^7 + 6*a^
2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(202) = 404$.

Time = 0.23 (sec) , antiderivative size = 1414, normalized size of antiderivative = 6.26

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/48*(3*(8*a^2*b*c^2*d^3 + 12*a*b^2*c*d^3 + 5*b^3*d^3)*arctan(-(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^
2 - b*c))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt(-a*c^2 - b*c)) + (64*a^(11
/2)*c^8*d^2*abs(d) + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 304*
a^(9/2)*b*c^7*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*
x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^4*c^5*d^3 + 744*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^4*b*c^6*d^3 + 528*(sqr
t(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^
(7/2)*b*c^5*d^2*abs(d) + 576*a^(7/2)*b^2*c^6*d^2*abs(d) + 64*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^3*b*c^4*d
^3 + 1116*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^
2 + b*c))*a^3*b^2*c^5*d^3 + 480*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*
d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*b^2*c^4*d^2*abs(d) + 544*a^(5/2)
*b^3*c^5*d^2*abs(d) + 24*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))^5*a^2*b*c^2*d^3 - 96*(sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*b^2*c^3*d^3 + 801*(sqr
t(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*
b^3*c^4*d^3 + 144*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^7}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^7,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 691, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \frac{-8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3c^6+8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3c^5d\,x^2-8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{}$$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^7,x)
```

output
```
( - 8*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**6 + 8*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**5*d*x**2 - 8*sqrt(c + d*x**2)*sqrt(a
*c + a*d*x**2 + b)*a**3*c**4*d**2*x**4 - 24*sqrt(c + d*x**2)*sqrt(a*c + a*
d*x**2 + b)*a**2*b*c**5 + 26*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**
2*b*c**4*d*x**2 - 34*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b*c**3
*d**2*x**4 - 24*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c**4 + 28
*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c**3*d*x**2 - 41*sqrt(c
+ d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c**2*d**2*x**4 - 8*sqrt(c + d*x*
*2)*sqrt(a*c + a*d*x**2 + b)*b**3*c**3 + 10*sqrt(c + d*x**2)*sqrt(a*c + a*
d*x**2 + b)*b**3*c**2*d*x**2 - 15*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b
)*b**3*c*d**2*x**4 + 24*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c +
 a*d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt(c + d*x**2)
*b)*a**2*b*c**2*d**3*x**6 + 36*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqr
t(a*c + a*d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt(c +
d*x**2)*b)*a*b**2*c*d**3*x**6 + 15*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)
*sqrt(a*c + a*d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt(
c + d*x**2)*b)*b**3*d**3*x**6 - 24*sqrt(c)*sqrt(a*c + b)*log(x)*a**2*b*c**
2*d**3*x**6 - 36*sqrt(c)*sqrt(a*c + b)*log(x)*a*b**2*c*d**3*x**6 - 15*sqrt
(c)*sqrt(a*c + b)*log(x)*b**3*d**3*x**6)/(48*c**4*x**6*(a**3*c**3 + 3*a**2
*b*c**2 + 3*a*b**2*c + b**3))
```

3.170 $\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 328

$$\begin{aligned}\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx = & -\frac{(2b^2+7abc-3a^2c^2)\,x\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2} \\ & +\frac{(b-3ac)x(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2}+\frac{x^3(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{5d} \\ & +\frac{\sqrt{c}(2b^2+7abc-3a^2c^2)\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ & -\frac{c^{3/2}(b-3ac)\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
-1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*x*(a+b/(d*x^2+c))^(1/2)/a^2/d^2+1/15*(-3*
a*c+b)*x*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a/d^2+1/5*x^3*(d*x^2+c)*(a+b/(d*x
^2+c))^(1/2)/d+1/15*c^(1/2)*(-3*a^2*c^2+7*a*b*c+2*b^2)*(a+b/(d*x^2+c))^(1/
2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^2/d^
(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/15*c^(3/2)*(-3*a*c+b)*
(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b)
)^(1/2))/a/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.88 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.90

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b^2-2ab(c-2dx^2)-3a^2(c^2-d^2x^4)\right)+i(2b^3+9ab^2c+4a^2bc^2-3a^3c^3)\right.}{15}$$

input
```
Integrate[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b^2 - 2
*a*b*(c - 2*d*x^2) - 3*a^2*(c^2 - d^2*x^4)) + I*(2*b^3 + 9*a*b^2*c + 4*a^2
*b*c^2 - 3*a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (2*I)*b*(b^2 + 4*a*
b*c + 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(15*a^2*c^2*(d/c)^(5/2
)*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2057, 2058, 380, 444, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$\Big\downarrow \; 2057$$
$$\int x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{x^4\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 380}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\int \frac{x^2\left(3c(b+ac)-(b-3ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{-\frac{\int -\frac{d\left(\left(2b^2+7acb-3a^2c^2\right)dx^2+c(b-3ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}-\frac{x(b-3ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{\int \frac{d\left(\left(2b^2+7acb-3a^2c^2\right)dx^2+c(b-3ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}-\frac{x(b-3ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{\int \frac{\left(2b^2+7acb-3a^2c^2\right)dx^2+c(b-3ac)(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}-\frac{x(b-3ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(b-3ac)(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}}{3ad}}{5d}\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(b-3ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\text{arct}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2\right.}{(ac+b)\left(c+d\right.}}}}{3ad}}{5d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(b-3ac)\sqrt{ac+adx^2+b}}{\sqrt{d}\sqrt{c+dx^2}}}{3ad}}{5d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}}{}}{3ad}}{5d}\right.}{\sqrt{ac+adx^2+b}}$$

input

```
Int[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((x^3*Sqrt[c + d*x^
2]*Sqrt[b + a*c + a*d*x^2])/(5*d) - (-1/3*((b - 3*a*c)*x*Sqrt[c + d*x^2]*S
qrt[b + a*c + a*d*x^2])/(a*d) + ((2*b^2 + 7*a*b*c - 3*a^2*c^2)*d*((x*Sqrt[
b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c
+ d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2
)*(b - 3*a*c)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b
 + a*c)*(c + d*x^2))]))/(3*a*d))/(5*d)))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(303) = 606$.

Time = 9.64 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\left(-3\sqrt{-\frac{ad}{ac+b}}\,a^2d^3x^7-3\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^2x^5-4\sqrt{-\frac{ad}{ac+b}}\,ab\,d^2x^5+3\sqrt{-\frac{ad}{ac+b}}\,a^2c^2d\,x^3-2\sqrt{-\frac{ad}{ac+b}}\,abcd\,x^3-3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x}{}}\right.}{}$
risch	$-\frac{x\left(-3ad\,x^2+3ac-b\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{15d^2a}+\frac{\left(-\frac{2d\left(3a^2c^2-7abc-2b^2\right)\left(a\,c^2+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{x^2d}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+}\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+b}}\right.}{}$

input

```
int(x^4*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/15*(-3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7-3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^
2*x^5-4*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5+3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*
x^3-2*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a
^2*c^3+3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x-(-a*d/(a*c+b))^(1/2)*b^2*d*x^3-9*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a
*b*c^2+2*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^
2*c+2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-(-a*d/(a*c+b))^(1/2)*b^2*c*x)*
(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.73

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx =$$

$$-\frac{(3\,a^2c^3-7\,abc^2-2\,b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(3\,a^2c^3-7\,abc^2-2\,b^2c+(3\,a^2c^2+2\,ab}{}$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((3*a^2*c^3 - 7*a*b*c^2 - 2*b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(a
rcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*c^3 - 7*a*b*c^2 - 2*b^2*c +
 (3*a^2*c^2 + 2*a*b*c - b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqr
t(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*d^3*x^6 + a*b*d^2*x^4 + 3*a^2*c^3 -
7*a*b*c^2 - 2*(3*a*b*c + b^2)*d*x^2 - 2*b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^2*d^3*x)
```

Sympy [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x**4*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^4\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^4*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4*(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$= \frac{-3\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,acx+3\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,ad\,x^3+\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,bx+$$

input
```
int(x^4*(a+b/(d*x^2+c))^(1/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c*x + 3*sqrt(c + d*x**2)
*sqrt(a*c + a*d*x**2 + b)*a*d*x**3 + sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
+ b)*b*x + 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/(a*c**2
+ 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*a**2*c**2*d - 7*int((sqr
t(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d*
*2*x**4 + b*c + b*d*x**2),x)*a*b*c*d - 2*int((sqrt(c + d*x**2)*sqrt(a*c +
a*d*x**2 + b)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2)
,x)*b**2*d + 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**2 + 2
*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*a**2*c**3 + 2*int((sqrt(c +
 d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 +
b*c + b*d*x**2),x)*a*b*c**2 - int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 +
b))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*b**2*c)/(15*
a*d**2)
```

3.171 $\int x^2 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(b-ac)x\sqrt{a+\frac{b}{c+dx^2}}}{3ad}+\frac{x(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{3d}-\frac{\sqrt{c}(b-ac)\sqrt{a+\frac{b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
1/3*(-a*c+b)*x*(a+b/(d*x^2+c))^(1/2)/a/d+1/3*x*(d*x^2+c)*(a+b/(d*x^2+c))^(
1/2)/d-1/3*c^(1/2)*(-a*c+b)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1
/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c
+b)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arc
tan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+
b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.97

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b+a(c+dx^2)\right)+i(-b^2+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)\right.\right.}{3ad\sqrt{\frac{d}{c}}\left(b+a\left(c+dx^2\right)\right)}$$

input
```
Integrate[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b + a*(
c + d*x^2)) + I*(-b^2 + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[
1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] + I*b*(b
 + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a*d*Sqrt[d/c]*(b + a*(c + d*
x^2)))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2057, 2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{x^2\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 380

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2 \cdots}}{\left(dx^2+ \cdots\right.}}{ac \cdots}\right.}{3d}\right.}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+ac\ldots}}{ad^{3/2}\ldots}\right)}{3d}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

input

```
Int[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

output

```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((x*Sqrt[c + d*x^2]
*Sqrt[b + a*c + a*d*x^2])/(3*d) - (-((b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^
2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c
*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a*c
+ a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*S
qrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*
d)))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.62

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^{2} x^{5}+2 \sqrt{-\frac{ad}{ac+b}}\, acd\, x^{3}+\sqrt{-\frac{ad}{ac+b}}\, bd\, x^{3}-\sqrt{\frac{ad\, x^{2}+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, \mathrm{EllipticE}\left(x \sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right) a\, c^{2}+\sqrt{-\frac{ad}{ac+b}}\, a\, c^{}\right.}{3d\sqrt{a\, d^{2} x^{4}+2ad\, x^{2} c}}$
risch	$\frac{x \left(d\, x^{2}+c \right) \sqrt{\frac{ad\, x^{2}+ac+b}{d\, x^{2}+c}}}{3d}-\frac{\left(\frac{a\, c^{2} \sqrt{1+\frac{ad\, x^{2}}{ac+b}}\, \sqrt{1+\frac{x^{2} d}{c}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2ad\, x^{2} c+bd\, x^{2}+a\, c^{2}+bc}}+\frac{bc \sqrt{1+\frac{ad\, x^{2}}{ac+b}}\, \sqrt{1+\frac{x^{2} d}{c}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{}{ac}}\right.}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2ad\, x^{2} c+bd}}\right.}{}$

input
```
int(x^2*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d
/(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b
))^(1/2)*a*c^2*x-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*
c+b))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{(ac^2-bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(ac^2-bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}}{3\,ad^2x}$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*((a*c^2 - b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (
a*c + b)/(a*c)) - (a*c^2 - b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*ellipti
c_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a*d^2*x^4 + b*d*x^2 - a*c^2
+ b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^2*x)
```

Sympy [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x**2*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^2\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^2*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2*(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x^2}{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}dx\right)acd+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x^2}{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}dx\right)bd}{3d}$$

input
```
int(x^2*(a+b/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x - int((sqrt(c + d*x**2)*sqrt(
a*c + a*d*x**2 + b)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d
*x**2),x)*a*c*d + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/(a*
c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*b*d - int((sqrt(c +
 d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 +
b*c + b*d*x**2),x)*a*c**2 - int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)
)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*b*c)/(3*d)
```

3.172 $\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 189

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = x\sqrt{a+\frac{b}{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
x*(a+b/(d*x^2+c))^(1/2)-c^(1/2)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/
c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a
*c+b)/(d*x^2+c))^(1/2)+c^(1/2)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arcta
n(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)
/(d*x^2+c))^(1/2)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSi
n[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/(Sqrt[-(d/c)]*Sqrt[(b + a*c + a*d*x^2
)/(b + a*c)])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.49, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2057, 2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left((ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ad\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{\sqrt{ac+adx^2+b}}$$

input
```
Int[Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*d*((x*Sqrt[b + a
*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x
^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (Sqrt[c]*Sqr
t[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^
2))])))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{\left(ac\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)b\right)\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{\sqrt{a\,d^{2}x^{4}+2ad\,x^{2}c+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^{2}+c\right)\left(ad\,x^{2}+ac+b\right)}}$	199

input
```
int((a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*c*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+EllipticF(x*(-a
*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b)*((d*x^2+c)/c)^(1/2)*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*x^2+c)*(a*d
*x^2+a*c+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.78

$$\int\sqrt{a+\frac{b}{c+dx^2}}\,dx = -\frac{a^{\frac{3}{2}}c^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}) - (ac^2+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}) - (acdx^2}{acdx}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(a^(3/2)*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c
)) - (a*c^2 + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
d)/x), (a*c + b)/(a*c)) - (a*c*d*x^2 + a*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*
x^2 + c)))/(a*c*d*x)
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b/(c + d*x**2)), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = \int \sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2),x)
```

output
```
int((a + b/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{d\,x^2+c}dx$$

input
```
int((a+b/(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(c + d*x**2),x)
```

3.173 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = -\frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} - \frac{\sqrt{d}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
-(a+b/(d*x^2+c))^(1/2)/x-d^(1/2)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x
/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/c^(1/2)/(c*(a*d*x^2+a*c+b)/(
a*c+b)/(d*x^2+c))^(1/2)+a*c^(1/2)*d^(1/2)*(a+b/(d*x^2+c))^(1/2)*InverseJac
obiAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/(a*c+b)/(c*(a*d*x^2+a*c
+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx=\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-\frac{1}{x}-\frac{dx}{c}+\frac{ad\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)}{\sqrt{-\frac{ad}{b+ac}}\,(b+a(c+dx^2))}\right)$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

output
```
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-x^(-1) - (d*x)/c + (a*d*Sqrt[(b +
a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-((a*d
)/(b + a*c))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*(b + a*(c + d*x^
2))))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.64, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{x^2\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 377$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{ad\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\int \frac{\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 324$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(c\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)\right)}{c}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-((Sqrt[c + d*x^2]
*Sqrt[b + a*c + a*d*x^2])/(c*x)) + (a*d*(d*((x*Sqrt[b + a*c + a*d*x^2])/(a
*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b +
a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x
^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d
]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))
/c))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 8.42 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2 x^4 - adc\sqrt{\frac{a d\, x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\, x^2+c}{c}}\, x\, \text{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right) + 2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^2 + \sqrt{-\frac{ad}{ac+b}}\, bd\, x^2 + \sqrt{-\frac{ad}{ac+b}}}{\sqrt{a\, d^2 x^4 + 2ad\, x^2 c + bd\, x^2 + a\, c^2 + bc}\,\sqrt{-\frac{ad}{ac+b}}\, xc\sqrt{(d\, x^2+c)(ad\, x^2+ac+b)}}$
risch	$-\frac{(d\, x^2+c)\sqrt{\frac{ad\, x^2+ac+b}{d\, x^2+c}}}{cx} + \frac{ad\left(\frac{c\sqrt{1+\frac{ad\, x^2}{ac+b}}\,\sqrt{1+\frac{x^2 d}{c}}\, \text{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\, d^2 x^4+2ad\, x^2 c+bd\, x^2+a\, c^2+bc}} - \frac{2d\left(a\, c^2+bc\right)\sqrt{1+\frac{ad\, x^2}{ac+b}}\,\sqrt{1+\frac{x^2 d}{c}}\,\left(\text{Ellip}\right.}{\sqrt{-\frac{ad}{ac+b}}}\right.}{c(}$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2
*(-a*d/(a*c+b))^(1/2)*a*c*d*x^2+(-a*d/(a*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b)
)^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(
1/2)/x/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\, dx$$
$$= \frac{a\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - (ad^2+(ac+b)d)\sqrt{-\frac{ad}{ac+b}}x\sqrt{\frac{ac^2+bc}{d^2}}F(\arcsin\left(\sqrt{-\right.\right.}{(ac^2+bc)x}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(a*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sq
rt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a*d^2 + (a*c + b)*d)*sqrt(-a*d/
(a*c + b))*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b)
)*x), (a*c + b)/(a*c)) - ((a*c + b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a*c^2 + b*c)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^2,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{d\,x^4+c\,x^2}dx$$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(c*x**2 + d*x**4),x)
```

3.174 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 273

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4} dx = \frac{(2b+ac)d\sqrt{a+\frac{b}{c+dx^2}}}{3c(b+ac)x} - \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d^{3/2}\sqrt{a+\frac{b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{ad^{3/2}\sqrt{a+\frac{b}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
1/3*(a*c+2*b)*d*(a+b/(d*x^2+c))^(1/2)/c/(a*c+b)/x-1/3*(d*x^2+c)*(a+b/(d*x^
2+c))^(1/2)/c/x^3+1/3*(a*c+2*b)*d^(3/2)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^
(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/c^(3/2)/(a*c+b)/(c*(a
*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(a+b/(d*x^2+c))^(1/2)
*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/c^(1/2)/(a*c
+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.13 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c-2dx^2)+a^2c(c^2-d^2x^4)+2ab(c^2-cdx^2-d^2x^4)\right)-i(2b^2+3abc+\right.}{3}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

output
```
-1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^2*(c
 - 2*d*x^2) + a^2*c*(c^2 - d^2*x^4) + 2*a*b*(c^2 - c*d*x^2 - d^2*x^4)) - I
*(2*b^2 + 3*a*b*c + a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] + (2
*I)*b*(b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(c^2*(b + a*c)*
Sqrt[d/c]*x^3*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2057, 2058, 377, 25, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4}dx$$

$\Big\downarrow$ 2058

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{x^4\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 377

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int -\frac{d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{\int \frac{d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\int \frac{adx^2+2b+ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{\int -\frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{\int \frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\int \frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{a}}{cx(ac+b)}\right.}{3c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c(ac+b)}-\frac{(ac+\cdots}{\cdots}\right)}{3c}\cdots\right)$$

$$\overline{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\cdots\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2\cdots\right)}{(ac+b)\left(c+\cdots\right)}}}\cdots\right)\right)}{c(ac+b)}\cdots\right)}{3c}\cdots\right)$$

$$\overline{\sqrt{ac+adx^2+b}}$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-1/3*(Sqrt[c + d*x
^2]*Sqrt[b + a*c + a*d*x^2])/(c*x^3) - (d*(-(((2*b + a*c)*Sqrt[c + d*x^2]*
Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) + (a*d*((2*b + a*c)*d*((x*Sqrt[b
 + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c +
 d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)
*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c
)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
d*x^2))])))/(c*(b + a*c))))/(3*c)))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(252) = 504$.

Time = 9.91 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.08

method	result
risch	$-\frac{(d\,x^2+c)(-a d\,x^2 c-2 b d\,x^2+a\,c^2+b c)\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}}{3 c^2 x^3 (a c+b)} - \frac{a\,d^2\left(\frac{a\,c^2\sqrt{1+\frac{a d\,x^2}{a c+b}}\,\sqrt{1+\frac{x^2 d}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{a d}{a c+b}},\sqrt{-1+\frac{2 a c d+b d}{d c a}}\right)}{\sqrt{-\frac{a d}{a c+b}}\,\sqrt{a\,d^2 x^4+2 a d\,x^2 c+b d\,x^2+a\,c^2+b c}}+\frac{b c\sqrt{\ldots}}{\ldots}\right.}{\ldots}$
default	$-\frac{\left(-\sqrt{-\frac{a d}{a c+b}}\,a^2 c\,d^3 x^6-2\sqrt{-\frac{a d}{a c+b}}\,a b\,d^3 x^6+\sqrt{\frac{a d\,x^2+a c+b}{a c+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\;\mathrm{EllipticE}\left(x\sqrt{-\frac{a d}{a c+b}},\sqrt{\frac{a c+b}{a c}}\right)a^2 c^2 d^2 x^3-\sqrt{-\frac{a d}{a c+b}}\,a^2 c^2 d^2 \ldots\right.}{\ldots}$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2+c)*(-a*c*d*x^2-2*b*d*x^2+a*c^2+b*c)/c^2/x^3/(a*c+b)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^2/c^2/(a*c+b)*(a*c^2/(-a*d/(a*c+b))^(1/2)*
(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c
/a)^(1/2))+b*c/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d
)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/
(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*d*(a*c+2*b)*(a*c^2+b*c)/(
-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d
/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))
^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx =$$

$$-\frac{(a^2c+2\,ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-((a^2c+2\,ab)d^3+(a^2c^2+2\,abc+b^2)\ldots}{\ldots}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*((a^2*c + 2*a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)
*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 2
*a*b)*d^3 + (a^2*c^2 + 2*a*b*c + b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt((
a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*
c)) - ((a^2*c^2 + 3*a*b*c + 2*b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c^2
 + (a*b*c^2 + b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c
^4 + 2*a*b*c^3 + b^2*c^2)*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^4}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^4,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$$
$$=\frac{-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,ad+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x^2}{a^2c\,d^2x^4+ab\,d^2x^4+2a^2c^2d\,x^2+3abcd\,x^2+a^2c^3+b^2d\,x^2+2ab\,c^2+b^2c}dx\right)a^3c\,d^3x+\Big(\int}{}$$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^4,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*d + int((sqrt(c + d*x**2)*
sqrt(a*c + a*d*x**2 + b)*x**2)/(a**2*c**3 + 2*a**2*c**2*d*x**2 + a**2*c*d*
*2*x**4 + 2*a*b*c**2 + 3*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c + b**2*d*x*
*2),x)*a**3*c*d**3*x + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2
)/(a**2*c**3 + 2*a**2*c**2*d*x**2 + a**2*c*d**2*x**4 + 2*a*b*c**2 + 3*a*b*
c*d*x**2 + a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*a**2*b*d**3*x + int((s
qrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a**2*c**3*x**4 + 2*a**2*c**2*d*
x**6 + a**2*c*d**2*x**8 + 2*a*b*c**2*x**4 + 3*a*b*c*d*x**6 + a*b*d**2*x**8
 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*c**4*x + 3*int((sqrt(c + d*x**2)*sqr
t(a*c + a*d*x**2 + b))/(a**2*c**3*x**4 + 2*a**2*c**2*d*x**6 + a**2*c*d**2*
x**8 + 2*a*b*c**2*x**4 + 3*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c*x**4 + b*
*2*d*x**6),x)*a**2*b*c**3*x + 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
+ b))/(a**2*c**3*x**4 + 2*a**2*c**2*d*x**6 + a**2*c*d**2*x**8 + 2*a*b*c**2
*x**4 + 3*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c*x**4 + b**2*d*x**6),x)*a*b
**2*c**2*x + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a**2*c**3*x*
*4 + 2*a**2*c**2*d*x**6 + a**2*c*d**2*x**8 + 2*a*b*c**2*x**4 + 3*a*b*c*d*x
**6 + a*b*d**2*x**8 + b**2*c*x**4 + b**2*d*x**6),x)*b**3*c*x)/(c*x*(a*c +
b))
```

3.175 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 357

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6} \, dx = &-\frac{(8b^2+13abc+3a^2c^2)\, d^2\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)^2x} \\ &-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5}+\frac{(4b+3ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)x^3} \\ &-\frac{(8b^2+13abc+3a^2c^2)\, d^{5/2}\sqrt{a+\frac{b}{c+dx^2}}\, E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{5/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &+\frac{a(4b+3ac)d^{5/2}\sqrt{a+\frac{b}{c+dx^2}}\, \text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15c^{3/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^2*(a+b/(d*x^2+c))^(1/2)/c^2/(a*c+b)^2/x
-1/5*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c/x^5+1/15*(3*a*c+4*b)*d*(d*x^2+c)*(a
+b/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^3-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^(5/2
)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(
a*c+b))^(1/2))/c^(5/2)/(a*c+b)^2/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/
2)+1/15*a*(3*a*c+4*b)*d^(5/2)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan
(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/c^(3/2)/(a*c+b)^2/(c*(a*d*x^2+a*c+b
)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.67 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^3(3c^2-4cdx^2+8d^2x^4)+3a^3c^2(c^3+d^3x^6)+ab^2(9c^3-8c^2dx^2+17cd^2x\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

output
```
-1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^3*(
3*c^2 - 4*c*d*x^2 + 8*d^2*x^4) + 3*a^3*c^2*(c^3 + d^3*x^6) + a*b^2*(9*c^3
- 8*c^2*d*x^2 + 17*c*d^2*x^4 + 8*d^3*x^6) + a^2*b*c*(9*c^3 - 4*c^2*d*x^2 +
 9*c*d^2*x^4 + 13*d^3*x^6)) + I*(8*b^3 + 21*a*b^2*c + 16*a^2*b*c^2 + 3*a^3
*c^3)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Elli
pticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(8*b^2 + 17*a*b*c + 9
*a^2*c^2)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(c^3*(b + a*c)^2*Sqrt
[d/c]*x^5*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2057, 2058, 377, 25, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6} \, dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{x^6\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int -\frac{d(3adx^2+4b+3ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{\int \frac{d(3adx^2+4b+3ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\int \frac{3adx^2+4b+3ac}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{\int \frac{d\left(8b^2+13acb+3a^2c^2+a(4b+3ac)dx^2\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3(ac+b)}\right)}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\int \frac{8b^2+13acb+3a^2c^2+a(4b+3ac)dx^2}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3(ac+b)}\right)}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 445

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{ad\left(\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(3a^2c^2+13abc+8b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c(ac+b)}-\frac{(3ac+ \cdots}{\cdots}\right)}{5c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{ad\left(\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(3a^2c^2+13abc+8b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c(ac+b)}-\frac{(3ac+4b)\cdots}{\cdots}\right)}{5c}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{ad\int \frac{\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(3a^2c^2+13abc+8b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c(ac+b)}-\frac{(3ac+4b)\sqrt{c\cdots}}{3c\cdots}\right)}{5c}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{ad\left(d\left(3a^2c^2+13abc+8b^2\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(3ac+4b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{\left(3a^2c^2+\cdots\right.}{\cdots}\right)}{3c(ac+b)}\cdots\right)}{5c}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{ad\left(d\left(3a^2c^2+13abc+8b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(3ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+a}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right.}{c(ac+b)}\right.}{3c(ac+b)}\right.}{5c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{ad\left(d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(3ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\text{arcta}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right.}{c(ac+b)}\right.}{3c(ac+b)}\right.}{5c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{ad\left(d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}(3ac+4b)\sqrt{ac+a\cdots}}{\sqrt{d}\sqrt{c\cdots}}\cdots}{c(ac+b)}\cdots\right)}{3c(ac+b)}\cdots\right)}{\cdots}\cdots\right)}{\sqrt{ac+a\cdots}}$$

input

```
Int[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-1/5*(Sqrt[c + d*x
^2]*Sqrt[b + a*c + a*d*x^2])/(c*x^5) - (d*(-1/3*((4*b + 3*a*c)*Sqrt[c + d*
x^2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x^3) - (d*(-(((8*b^2 + 13*a*b*c
 + 3*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) +
(a*d*((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*S
qrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c
+ a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*(4*b + 3*a*c)*Sqrt[b + a
*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d
]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))
/(c*(b + a*c))))/(3*c*(b + a*c))))/(5*c)))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(332) = 664$.

Time = 13.79 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{(d\,x^2+c)\left(3a^2c^2d^2x^4+13abc\,d^2x^4-3a^2c^3d\,x^2+8b^2d^2x^4-7ab\,c^2x^2d+3a^2c^4-4b^2c\,x^2d+6ab\,c^3+3b^2c^2\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{15c^3x^5(ac+b)^2}+\frac{d^3a\left(-\right.}{}$
default	$-\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\,a^3c^2d^4x^8+13\sqrt{-\frac{ad}{ac+b}}\,a^2bc\,d^4x^8-3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\text{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^3c^3d^3x^5+3\sqrt{-\frac{ad}{ac+b}}\,c\right.}{}$

input

```
int((a+b/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output

```
-1/15*(d*x^2+c)*(3*a^2*c^2*d^2*x^4+13*a*b*c*d^2*x^4-3*a^2*c^3*d*x^2+8*b^2*
d^2*x^4-7*a*b*c^2*d*x^2+3*a^2*c^4-4*b^2*c*d*x^2+6*a*b*c^3+3*b^2*c^2)/c^3/x
^5/(a*c+b)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/15*d^3*a/(a*c+b)^2/c^3*(-
2*d*(3*a^2*c^2+13*a*b*c+8*b^2)*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2
/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b
*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a
)^(1/2)))+3*a^2*c^3/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*
x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(
-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+4*b^2*c/(-a*d/(a*c+b))
^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b
*d)/d/c/a)^(1/2))+7*a*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)
*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*Ellip
ticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2)))*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \frac{(3\,a^3c^2+13\,a^2bc+8\,ab^2)\sqrt{-\frac{ad}{ac+b}}d^4x^5\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-((3\,a^3c^2+13\,a^2bc+8\,ab^2}{}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/15*((3*a^3*c^2 + 13*a^2*b*c + 8*a*b^2)*sqrt(-a*d/(a*c + b))*d^4*x^5*sqrt
((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(
a*c)) - ((3*a^3*c^2 + 13*a^2*b*c + 8*a*b^2)*d^4 + (3*a^3*c^3 + 10*a^2*b*c^
2 + 11*a*b^2*c + 4*b^3)*d^3)*sqrt(-a*d/(a*c + b))*x^5*sqrt((a*c^2 + b*c)/d
^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((3*a^3*
c^3 + 16*a^2*b*c^2 + 21*a*b^2*c + 8*b^3)*d^3*x^6 + 3*a^3*c^6 + 9*a^2*b*c^5
 + 9*a*b^2*c^4 + 2*(3*a^2*b*c^3 + 5*a*b^2*c^2 + 2*b^3*c)*d^2*x^4 + 3*b^3*c
^3 - (a^2*b*c^4 + 2*a*b^2*c^3 + b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(
d*x^2 + c)))/((a^3*c^6 + 3*a^2*b*c^5 + 3*a*b^2*c^4 + b^3*c^3)*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**6,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx=\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^6}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^6,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{d\,x^8+c\,x^6}dx$$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^6,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(c*x**6 + d*x**8),x)
```

3.176 $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 207

$$\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = -\frac{bc^2\sqrt{a+\frac{b}{c+dx^2}}}{d^3} + \frac{(b^2-20abc+8a^2c^2)\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3} + \frac{(7b-12ac)\,(c+dx^2)^2\sqrt{a+\frac{b}{c+dx^2}}}{24d^3} + \frac{a(c+dx^2)^3\sqrt{a+\frac{b}{c+dx^2}}}{6d^3} - \frac{b(b^2+12abc-24a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}$$

output

```
-b*c^2*(a+b/(d*x^2+c))^(1/2)/d^3+1/16*(8*a^2*c^2-20*a*b*c+b^2)*(d*x^2+c)*(
a+b/(d*x^2+c))^(1/2)/a/d^3+1/24*(-12*a*c+7*b)*(d*x^2+c)^2*(a+b/(d*x^2+c))^
(1/2)/d^3+1/6*a*(d*x^2+c)^3*(a+b/(d*x^2+c))^(1/2)/d^3-1/16*b*(-24*a^2*c^2+
12*a*b*c+b^2)*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.72

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{a}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(3b^2(c+dx^2)-2ab(47c^2+16cdx^2-7d^2x^4)+8a^2(c^3+d^3x^6))-3b(b^2}{48a^{3/2}d^3}$$

input
```
Integrate[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*
(47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12
*a*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]
])/(48*a^(3/2)*d^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2057, 2053, 2052, 27, 366, 360, 1471, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^4\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{x^8\left(-cx^4+b+ac\right)^2}{d^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{x^8\left(-cx^4+b+ac\right)^2}{\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$$\Big\downarrow \text{366}$$

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\int \frac{x^8\left(6ac^2x^4+5b^2-6(b+ac)^2\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}\right)}{d^3}$$

$$\Big\downarrow \text{360}$$

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\int \frac{-24ac^2x^8+4b(b+12ac)x^4+ab(b+12ac)}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{ab(12ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{6a}\right)}{d^3}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\left(\frac{\left(-24a^2c^2+60abc+5b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(a-x^4\right)}-\frac{\int \frac{3a\left(-16ac^2x^4+b^2-8a^2c^2+12abc\right)}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}\right)-\frac{ab(12ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{6a}\right)}{d^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\left(\frac{\left(-24a^2c^2+60abc+5b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(a-x^4\right)}-\frac{3}{2}\int \frac{-16ac^2x^4+b^2-8a^2c^2+12abc}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)-\frac{ab(12ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{6a}\right)}{d^3}$$

$$\Big\downarrow \text{299}$$

$$-\frac{b\left(\frac{b^2x^{10}}{6a(a-x^4)^3}-\frac{\frac{1}{4}\left(\frac{(-24a^2c^2+60abc+5b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}-\frac{3}{2}\left((-24a^2c^2+12abc+b^2)\int\frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+16ac^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)\right)-\frac{ab(12a}{}}{6a}\right.}{d^3}$$

$\downarrow$ 219

$$-\frac{b\left(\frac{b^2x^{10}}{6a(a-x^4)^3}-\frac{\frac{1}{4}\left(\frac{(-24a^2c^2+60abc+5b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}-\frac{3}{2}\left(\frac{(-24a^2c^2+12abc+b^2)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}+16ac^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)\right)-\frac{ab}{}}{6a}\right.}{d^3}$$

input
```
Int[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-((b*((b^2*x^10)/(6*a*(a - x^4)^3) - (-1/4*(a*b*(b + 12*a*c)*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)])/(a - x^4)^2 + (((5*b^2 + 60*a*b*c - 24*a^2*c^2)*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(a - x^4)) - (3*(16*a*c^2*Sqrt[(
b + a*c + a*d*x^2)/(c + d*x^2)] + ((b^2 + 12*a*b*c - 24*a^2*c^2)*ArcTanh[S
qrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/Sqrt[a]))/2)/4)/(6*a)))/d^3
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.43

method	result
risch	$\frac{\left(8a^2d^2x^4-8a^2cd\,x^2+14abd\,x^2+8a^2c^2-46abc+3b^2\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48a\,d^3}+\frac{b\left(\frac{\left(24a^2c^2-12abc-b^2\right)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+\right.}{2\sqrt{a\,d^2}}\right.}{}$
default	Expression too large to display

input
```
int(x^5*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/a/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2+14*a*b*d*x^2+8*a^2*c^2-46*a*b*c+3*
b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/16*b/a/d^2*(1/2*(24*a^2
*c^2-12*a*b*c-b^2)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(
2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-16*a*c^2*(a*d*x^2+a*c+b)/
d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.06

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\left[\frac{3\left(24a^2bc^2-12ab^2c-b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2\right.}{}\right.$$

$$-\frac{3\left(24a^2bc^2-12ab^2c-b^3\right)\sqrt{-a}\arctan\left(\frac{\left(2adx^2+2ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(8a^3d^3x^6+14a^2bd^2x^4+8a^3c\right.}{96a^2d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*
a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*
c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
)) + 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*
b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1
/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d*x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c
^3 - 94*a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)))/(a^2*d^3)]
```

Sympy [F]

$$\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int x^5 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.78

$$\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = -\frac{bc^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^3} - \frac{3\,(8\,a^2bc^2-20\,ab^2c+b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}} - 8\,(6\,a^3bc^2-12\,a^2b^2c-ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} + 3\,(8\,a^4bc^2-12\,a^3b^2}{48\left(a^4d^3-\frac{3\,(adx^2+ac+b)a^3d^3}{dx^2+c}+\frac{3\,(adx^2+ac+b)^2a^2d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3ad^3}{(dx^2+c)^3}\right)} - \frac{(24\,a^2c^2-12\,abc-b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{3}{2}}d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-b*c^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^3 - 1/48*(3*(8*a^2*b*c^2 -
20*a*b^2*c + b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2
 - 12*a^2*b^2*c - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^
4*b*c^2 - 12*a^3*b^2*c - a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(
a^4*d^3 - 3*(a*d*x^2 + a*c + b)*a^3*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b
)^2*a^2*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a*d^3/(d*x^2 + c)^3) - 1
/32*(24*a^2*c^2 - 12*a*b*c - b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/
2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(187) = 374$.

Time = 0.40 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.50

$$\int x^5\left(a + \frac{b}{c+dx^2}\right)^{3/2} dx = \frac{1}{48}\sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\left(2\left(\frac{4\,ax^2\mathrm{sgn}(dx^2+c)}{d} - \frac{4\,a^3cd^6\mathrm{sgn}(dx^2+c)}{a\cdots}\right.\right.$$
$$- \frac{(24\,a^2bc^2\mathrm{sgn}(dx^2+c) - 12\,ab^2c\mathrm{sgn}(dx^2+c) - b^3\mathrm{sgn}(dx^2+c))\log\left(\left|2\,a^2c^3d + 6\left(\sqrt{ad^2}x^2 - \sqrt{ad^2x^4+\cdots}\right.\right.\right.}{\cdots}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*a*x^2*sgn
(d*x^2 + c)/d - (4*a^3*c*d^6*sgn(d*x^2 + c) - 7*a^2*b*d^6*sgn(d*x^2 + c))/
(a^2*d^8))*x^2 + (8*a^3*c^2*d^5*sgn(d*x^2 + c) - 46*a^2*b*c*d^5*sgn(d*x^2
+ c) + 3*a*b^2*d^5*sgn(d*x^2 + c))/(a^2*d^8)) - 1/96*(24*a^2*b*c^2*sgn(d*x
^2 + c) - 12*a*b^2*c*sgn(d*x^2 + c) - b^3*sgn(d*x^2 + c))*log(abs(2*a^2*c^
3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x^2 -
sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*abs(d) +
2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)
)*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b
*d*x^2 + a*c^2 + b*c))^2*b*d))/(a^(3/2)*d^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int x^5\left(a+\frac{b}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(x^5*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5*(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.27

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \frac{8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3c^3+8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3d^3x^6-94\sqrt{d\,x^2+c}\cdots}{}$$

input
```
int(x^5*(a+b/(d*x^2+c))^(3/2),x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**3 + 8*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*a**3*d**3*x**6 - 94*sqrt(c + d*x**2)*sqrt(a*c +
 a*d*x**2 + b)*a**2*b*c**2 - 32*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*
a**2*b*c*d*x**2 + 14*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b*d**2
*x**4 + 3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c + 3*sqrt(c +
d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*d*x**2 + 72*sqrt(a)*log( - sqrt(a)
*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*a**2*b*c**3 + 72*sqrt(a)*l
og( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*a**2*b*c**2*d
*x**2 - 36*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x*
*2)*a)*a*b**2*c**2 - 36*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) -
sqrt(c + d*x**2)*a)*a*b**2*c*d*x**2 - 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c +
a*d*x**2 + b) - sqrt(c + d*x**2)*a)*b**3*c - 3*sqrt(a)*log( - sqrt(a)*sqrt
(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*b**3*d*x**2)/(48*a**2*d**3*(c +
 d*x**2))
```

3.177 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{bc\sqrt{a+\frac{b}{c+dx^2}}}{d^2} + \frac{(5b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2} + \frac{a\left(c+dx^2\right)^2\sqrt{a+\frac{b}{c+dx^2}}}{4d^2} + \frac{3b(b-4ac)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

output
```
b*c*(a+b/(d*x^2+c))^(1/2)/d^2+1/8*(-4*a*c+5*b)*(d*x^2+c)*(a+b/(d*x^2+c))^(
1/2)/d^2+1/4*a*(d*x^2+c)^2*(a+b/(d*x^2+c))^(1/2)/d^2+3/8*b*(-4*a*c+b)*arct
anh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(13bc-2ac^2+5bdx^2+2ad^2x^4\right)}{8d^2}-\frac{3b(-b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

input
```
Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^2 + 2*a
*d^2*x^4))/(8*d^2) - (3*b*(-b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2057, 2053, 2052, 25, 27, 360, 25, 1471, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{x^8\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{x^8\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{x^8\left(-cx^4+b+ac\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{360}$$

$$\frac{b\left(\frac{1}{4}\int -\frac{-4cx^8+4bx^4+ab}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}\right)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}-\frac{1}{4}\int \frac{-4cx^8+4bx^4+ab}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)}{d^2}$$

$$\Big\downarrow \text{1471}$$

$$\frac{b\left(\frac{1}{4}\left(\frac{\int \frac{a\left(-8cx^4+3b-4ac\right)}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(a-x^4\right)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{-8cx^4+3b-4ac}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(a-x^4\right)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}\right)}{d^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{1}{4}\left(\frac{1}{2}\left(3(b-4ac)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+8c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(a-x^4\right)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}\right)}{d^2}$$

$\downarrow$ 219

$$\frac{b\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{3(b-4ac)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}+8c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

input
```
Int[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(b*((a*b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*(a - x^4)^2) + (-1/2*((
5*b - 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(a - x^4) + (8*c*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)] + (3*(b - 4*a*c)*ArcTanh[Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/Sqrt[a])/2)/4))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{(-2ad\,x^2+2ac-5b)(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{8d^2}-\frac{b\left(\frac{(12ac-3b)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)}{2\sqrt{a\,d^2}}-\frac{8c(\cdots}{d\sqrt{a\,d^2x^4+2\cdots}}\right.}{8d(ad\,x^2+ac+b)}$
default	$\frac{\left(4\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,a\,d^2x^4-12\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)abc\,d^2x^2+3\ln\cdots}{\cdots}$

input
```
int(x^3*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c-5*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)-1/8*b/d*(1/2*(12*a*c-3*b)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*
c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-8*c*(a*d*x^2+a*c
+b)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.39

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\left[\frac{3\left(4\,abc-b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4\cdots\right.\right.}{\cdots}\right.$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*
c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x
^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*
a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c
 + a*b)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^2)]
```

Sympy [F]

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.76

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{bc\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^2}+\frac{3\,(4\,ac-b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,\sqrt{a}d^2}-\frac{(4\,abc-5\,b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc-3\,ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2d^2-\frac{2\,(adx^2+ac+b)ad^2}{dx^2+c}+\frac{(adx^2+ac+b)^2d^2}{(dx^2+c)^2}\right)}$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
b*c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^2 + 3/16*(4*a*c - b)*b*log(-(s
qrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))))/(sqrt(a)*d^2) - 1/8*((4*a*b*c - 5*b^2)*((a*d*x^2
+ a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c - 3*a*b^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^2*d^2 - 2*(a*d*x^2 + a*c + b)*a*d^2/(d*x^2 + c) + (a
*d*x^2 + a*c + b)^2*d^2/(d*x^2 + c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(124) = 248$.

Time = 0.38 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.39

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \frac{1}{8}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\left(\frac{2ax^2\operatorname{sgn}(dx^2+c)}{d}-\frac{2a^2cd^2\operatorname{sgn}(dx^2+c)-5\ldots}{ad^4}\right.$$
$$+\frac{(4abc\operatorname{sgn}(dx^2+c)-b^2\operatorname{sgn}(dx^2+c))\log\left(\left|2a^2c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)\ldots\right.\right.}{\ldots}$$
$$+\frac{(4abcd^2|d|\operatorname{sgn}(dx^2+c)-b^2d^2|d|\operatorname{sgn}(dx^2+c))\log(96)}{8\sqrt{a}d^5}$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2*sgn(d*x
^2 + c)/d - (2*a^2*c*d^2*sgn(d*x^2 + c) - 5*a*b*d^2*sgn(d*x^2 + c))/(a*d^4
)) + 1/16*(4*a*b*c*sgn(d*x^2 + c) - b^2*sgn(d*x^2 + c))*log(abs(2*a^2*c^3*
d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*abs(d) + 2*
(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*
sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^2*b*d))/(sqrt(a)*d*abs(d)) + 1/8*(4*a*b*c*d^2*abs(d)*
sgn(d*x^2 + c) - b^2*d^2*abs(d)*sgn(d*x^2 + c))*log(96)/(sqrt(a)*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int x^3 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(x^3*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3*(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.01

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^2+2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2d^2x^4+13\sqrt{d\,x^2+}}{}$$

input
```
int(x^3*(a+b/(d*x^2+c))^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**2 + 2*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**2*d**2*x**4 + 13*sqrt(c + d*x**2)*sqrt(a*
c + a*d*x**2 + b)*a*b*c + 5*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b*
d*x**2 + 12*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x
**2)*a)*a*b*c**2 + 12*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sq
rt(c + d*x**2)*a)*a*b*c*d*x**2 - 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x
**2 + b) + sqrt(c + d*x**2)*a)*b**2*c - 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c
+ a*d*x**2 + b) + sqrt(c + d*x**2)*a)*b**2*d*x**2)/(8*a*d**2*(c + d*x**2))
```

3.178 $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{b\sqrt{a+\frac{b}{c+dx^2}}}{d}+\frac{a(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

output
```
-b*(a+b/(d*x^2+c))^(1/2)/d+1/2*a*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/d+3/2*a^(
1/2)*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-2b+a(c+dx^2)\right)+3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

input
```
Integrate[x*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*
b*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2024, 773, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \left(a+\frac{b}{dx^2+c}\right)^{3/2}d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int (dx^2+c)^2\left(a+\frac{b}{dx^2+c}\right)^{3/2}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{3}{2}b\int (dx^2+c)\sqrt{a+\frac{b}{dx^2+c}}d\frac{1}{dx^2+c}-(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

$$\Big\downarrow \text{60}$$

$$-\frac{\frac{3}{2}b\left(a\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}+2\sqrt{a+\frac{b}{c+dx^2}}\right)-(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{b(dx^2+c)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}}{b}+2\sqrt{a+\frac{b}{c+dx^2}}\right)-(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3}{2}b\left(2\sqrt{a+\frac{b}{c+dx^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)\right)-\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

input
```
Int[x*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-1/2*(-((c + d*x^2)*(a + b/(c + d*x^2))^(3/2)) + (3*b*(2*Sqrt[a + b/(c + d
*x^2)] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]))/2)/d
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(79) = 158$.

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{\sqrt{\frac{\left(dx^2+c\right)a+b}{dx^2+c}}\left(6a^{\frac{3}{2}}\sqrt{a(dx^2+c)^2+b(dx^2+c)}\,(dx^2+c)^2+3\ln\left(\frac{2\sqrt{a\left(dx^2+c\right)^2+b\left(dx^2+c\right)}\,\sqrt{a}+2\left(dx^2+c\right)a+b}{2\sqrt{a}}\right)ab(dx^2}{4d(dx^2+c)\sqrt{(dx^2+c)((dx^2+c)a+b)}\,\sqrt{a}}$
risch	$\frac{(dx^2+c)a\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2d}+\frac{b\left(\frac{3a\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)}{2\sqrt{a\,d^2}}-\frac{2\left(adx^2+ac+b\right)}{d\sqrt{a\,d^2x^4+2adx^2c+bdx^2+a\,c}}\right.}{2adx^2+2ac+2b}$
default	$\frac{\left(3\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2adx^2c+bdx^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)ab\,d^2x^2+2\sqrt{a\,d^2x^4+2adx^2c+bdx^2+a\,c^2+bc}\,\sqrt{a\,d^2}\right.}{}$

input
```
int(x*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(((d*x^2+c)*a+b)/(d*x^2+c))^(1/2)*(6*a^(3/2)*(a*(d*x^2+c)^2+b*(d*x^2
+c))^(1/2)*(d*x^2+c)^2+3*ln(1/2*(2*(a*(d*x^2+c)^2+b*(d*x^2+c))^(1/2)*a^(1/
2)+2*(d*x^2+c)*a+b)/a^(1/2))*a*b*(d*x^2+c)^2-4*(a*(d*x^2+c)^2+b*(d*x^2+c))
^(3/2)*a^(1/2))/(d*x^2+c)/((d*x^2+c)*((d*x^2+c)*a+b))^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.89

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\left[\frac{3\,\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b\right.}{8\,d}\right.$$

$$\left.-\frac{3\,\sqrt{-a}b\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(adx^2+ac-2\,b)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,d}\right]$$

input
```
integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2
+ 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt
(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c - 2*b)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c)))/d, -1/4*(3*sqrt(-a)*b*arctan(1/2*(2*a*d*
x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2
 + a^2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/d]
```

Sympy [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{ab\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)}-\frac{3\sqrt{a}b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,d}-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d}$$

input
```
integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/2*a*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*
d/(d*x^2 + c)) - 3/4*sqrt(a)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/d - b*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(79) = 158.

Time = 0.32 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.10

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{\sqrt{a}b|d|\log(24)\,\mathrm{sgn}(dx^2+c)}{2\,d^2}-\frac{\sqrt{a}b\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{3}{2}}c^2|d|+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4}\right.\right.\right.}{}+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\,a\,\mathrm{sgn}(dx^2+c)}{2\,d}$$

input
```
integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*b*abs(d)*log(24)*sgn(d*x^2 + c)/d^2 - 1/4*sqrt(a)*b*log(abs(2
*a^2*c^3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*ab
s(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*
x^2 + b*d*x^2 + a*c^2 + b*c))^2*b*d))*sgn(d*x^2 + c)/abs(d) + 1/2*sqrt(a*d
^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*a*sgn(d*x^2 + c)/d
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,\left(d\,x^2+c\right)}{b}\right)}{d\,{\left(\frac{a\,\left(d\,x^2+c\right)}{b}+1\right)}^{3/2}}$$

input
```
int(x*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
-((a + b/(c + d*x^2))^(3/2)*(c + d*x^2)*hypergeom([-3/2, -1/2], 1/2, -(a*(
c + d*x^2))/b))/(d*((a*(c + d*x^2))/b + 1)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.72

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,ac+\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,ad\,x^2-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2}$$

input
```
int(x*(a+b/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c + sqrt(c + d*x**2)*sqrt(a*c
 + a*d*x**2 + b)*a*d*x**2 - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b
+ 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)*a)*
b*c + 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c + d*x**2)
*a)*b*d*x**2)/(2*d*(c + d*x**2))
```

3.179 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx = \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{(b+ac)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{c^{3/2}}$$

output
```
b*(a+b/(d*x^2+c))^(1/2)/c+a^(3/2)*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))-(
a*c+b)^(3/2)*arctanh(c^(1/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(-b-ac)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{-b-ac}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{b+ac}\right)}{c^{3/2}} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x,x]
```

output
```
(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((-b - a*c)^(3/2)*ArcTan[(Sq
rt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/c^
(3/2) + a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2057, 2053, 2052, 25, 27, 381, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^2}dx^2$$

$\big\downarrow$ 2052

$$-bd\int -\frac{x^8}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\big\downarrow$ 25

$$bd\int \frac{x^8}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\big\downarrow$ 27

$$b\int \frac{x^8}{\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\big\downarrow$ 381

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\int \frac{a(b+ac)-(b+2ac)x^4}{(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{c}\right)$$

$\big\downarrow$ 397

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\frac{(ac+b)^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{a^2c\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}}{c}\right)$$

$\big\downarrow$ 219

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\frac{(ac+b)^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{a^{3/2}c\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}}{c}\right)$$

$\big\downarrow$ 221

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\frac{(ac+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{c}}-\frac{a^{3/2}c\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}}{c}\right)$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x,x]
```

output
```
b*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/c - (-((a^(3/2)*c*ArcTanh[Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/b) + ((b + a*c)^(3/2)*ArcTanh[(Sq
rt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(b*Sqrt[c]))/
c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 652, normalized size of antiderivative = 6.39

method	result
default	$\frac{\left(\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^2c^2d^2x^2-\sqrt{a\,c^2+bc}\,\ln\left(\frac{2ad\,x^2c+bd\,x^2+2a\,c^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+}}{x^2}\right.}{}$

input
```
int((a+b/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^2*d^2*x^2-(a*c^2+b*c)^(1/
2)*ln((2*a*d*x^2*c+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2)*a*c*d*x^2+ln(1/2*(2
*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^
2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^3*d-(a*c^2+b*c)^(1/2)*ln((2*a*d*x^2*c+b
*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2)*b*d*x^2-(a*c^2+b*c)^(1/2)*ln((2*a*d*x^2
*c+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2)*a*c^2-(a*c^2+b*c)^(1/2)*ln((2*a*d*x
^2*c+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2)*b*c+2*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*(a*d^2)^(1/2)*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/
c^2/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(84) = 168$.

Time = 0.20 (sec) , antiderivative size = 1073, normalized size of antiderivative = 10.52

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log
(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 +
2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c))*sqrt((a*c + b)/c))/x^4) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/c, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c +
b)*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4
+ 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2
*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*b*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c)))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*
c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c +
b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) +
 2*(a*c + b)*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 +
2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 +
 (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/c, -1/2*(sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt...
```

Sympy [F]

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x}\, dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x}\, dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(84) = 168$.

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.97

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = -\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right) + \frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c} + \frac{(a^2c^2+2\,abc+b^2)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\,\sqrt{(ac+b)c}c}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/2*a^(3/2)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(
a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))) + b*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c))/c + 1/2*(a^2*c^2 + 2*a*b*c + b^2)*log((c*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.19

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx=\frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,bc+\sqrt{c}\,\sqrt{ac+b}\,\log\left(\sqrt{ac+b}\,\sqrt{ad\,x^2+ac+b}\,c-\sqrt{c}\,\sqrt{}\right.}{}$$

input
```
int((a+b/(d*x^2+c))^(3/2)/x,x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b*c + sqrt(c)*sqrt(a*c + b)*log
(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c -
 sqrt(c)*sqrt(c + d*x**2)*b)*a*c**2 + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c +
 b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sq
rt(c + d*x**2)*b)*a*c*d*x**2 + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqr
t(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c +
d*x**2)*b)*b*c + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x*
*2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*b*d
*x**2 - sqrt(c)*sqrt(a*c + b)*log(x)*a*c**2 - sqrt(c)*sqrt(a*c + b)*log(x)
*a*c*d*x**2 - sqrt(c)*sqrt(a*c + b)*log(x)*b*c - sqrt(c)*sqrt(a*c + b)*log
(x)*b*d*x**2 + sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt(c +
d*x**2)*a)*a*c**3 + sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) - sqrt
(c + d*x**2)*a)*a*c**2*d*x**2)/(c**2*(c + d*x**2))
```

3.180 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx = -\frac{bd\sqrt{a+\frac{b}{c+dx^2}}}{c^2} - \frac{(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2c^2x^2} + \frac{3b\sqrt{b+ac}d\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{5/2}}$$

output
```
-b*d*(a+b/(d*x^2+c))^(1/2)/c^2-1/2*(a*c+b)*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)
/c^2/x^2+3/2*b*(a*c+b)^(1/2)*d*arctanh(c^(1/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+
b)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(ac\left(c+dx^2\right)+b\left(c+3dx^2\right)\right)}{2c^2x^2} + \frac{3b\sqrt{-b-ac}d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{5/2}}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
-1/2*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*c*(c + d*x^2) + b*(c + 3*d*
x^2)))/(c^2*x^2) + (3*b*Sqrt[-b - a*c]*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a
*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2057, 2053, 2052, 252, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd\int \frac{x^8}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 252}$$

$$-bd\left(\frac{x^6}{2c\left(ac+b-cx^4\right)}-\frac{3\int \frac{x^4}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2c}\right)$$

$$\Big\downarrow \text{ 262}$$

$$-bd\left(\frac{x^6}{2c\left(ac+b-cx^4\right)}-\frac{3\left(\frac{(ac+b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{c}-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}\right)}{2c}\right)$$

$\downarrow$ 221

$$-bd\left(\frac{x^6}{2c\left(ac+b-cx^4\right)}-\frac{3\left(\frac{\sqrt{ac+b}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}\right)}{2c}\right)$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
-(b*d*(x^6/(2*c*(b + a*c - c*x^4)) - (3*(-(Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)]/c) + (Sqrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)])/Sqrt[b + a*c]])/c^(3/2)))/(2*c)))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(99) = 198$.

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.15

method	result
risch	$-\frac{(ac+b)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{2c^2x^2}-\frac{db\left(-\frac{(3ac+3b)\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}}{x^2}\right)}{2\sqrt{a\,c^2+bc}}+\frac{}{\sqrt{a\,d^2 x}}\right.}{2c^2(ad\,x^2+ac+b)}$
default	$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\left(-2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,a\,d^3x^6-3\ln\left(\frac{2ad\,x^2c+bd\,x^2+2a\,c^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2ad\,x^2c+b}}{x^2}\right.\right.}{}$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*c+b)/c^2*(d*x^2+c)/x^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/2/c^2*d
*b*(-1/2*(3*a*c+3*b)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2
+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+2
*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b
)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 404, normalized size of antiderivative = 3.45

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,bdx^2\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac\right.}{x^4}\right.$$

$$-\frac{3\,bdx^2\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{ac+b}{c}}}{2\,(a^2c^2+(a^2c+ab)dx^2+2\,abc+b^2)}\right)+2\,((ac+3\,b)dx^2+ac^2+bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,c^2x^2}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4
 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*
d*x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^
2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) -
4*((a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(c^2*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x
^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b
)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x
^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(99) = 198.

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = -\frac{(abc+b^2)d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^3+bc^2-\frac{(adx^2+ac+b)c^3}{dx^2+c}\right)} - \frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2} - \frac{3\,(abc+b^2)d\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c^2}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(a*b*c + b^2)*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^3 + b*c^2
- (a*d*x^2 + a*c + b)*c^3/(d*x^2 + c)) - b*d*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))/c^2 - 3/4*(a*b*c + b^2)*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt
((a*c + b)*c)))/(sqrt((a*c + b)*c)*c^2)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^3,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.50

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\frac{-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a\,c^3-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a\,c^2d\,x^2-\sqrt{d\,x^2+c}}{}$$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^3,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c**3 - sqrt(c + d*x**2)*sq
rt(a*c + a*d*x**2 + b)*a*c**2*d*x**2 - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**
2 + b)*b*c**2 - 3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b*c*d*x**2 + 3
*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c + sqrt
(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt(c + d*x**2)*b)*b*c*d*x**2 + 3*sqrt
(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c + sqrt(c)*s
qrt(c + d*x**2)*a*c + sqrt(c)*sqrt(c + d*x**2)*b)*b*d**2*x**4 - 3*sqrt(c)*
sqrt(a*c + b)*log(x)*b*c*d*x**2 - 3*sqrt(c)*sqrt(a*c + b)*log(x)*b*d**2*x*
*4)/(2*c**3*x**2*(c + d*x**2))
```

3.181 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5} \, dx = \frac{bd^2\sqrt{a+\frac{b}{c+dx^2}}}{c^3} + \frac{(9b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8c^3x^2} - \frac{(b+ac)\left(c+dx^2\right)^2\sqrt{a+\frac{b}{c+dx^2}}}{4c^3x^4} - \frac{3b(5b+4ac)d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{7/2}\sqrt{b+ac}}$$

output
```
b*d^2*(a+b/(d*x^2+c))^(1/2)/c^3+1/8*(4*a*c+9*b)*d*(d*x^2+c)*(a+b/(d*x^2+c)
)^(1/2)/c^3/x^2-1/4*(a*c+b)*(d*x^2+c)^2*(a+b/(d*x^2+c))^(1/2)/c^3/x^4-3/8*
b*(4*a*c+5*b)*d^2*arctanh(c^(1/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(
7/2)/(a*c+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-2bc^2-2ac^3+5bcdx^2+15bd^2x^4+2acd^2x^4\right)}{8c^3x^4} + \frac{3b(5b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{7/2}\sqrt{-b-ac}}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + 5*b*c*d*x^2 +
 15*b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^3*x^4) + (3*b*(5*b + 4*a*c)*d^2*ArcTa
n[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(7
/2)*Sqrt[-b - a*c])
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2053, 2052, 25, 27, 360, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^6}dx^2$$

$\big\downarrow$ 2052

$$-bd\int -\frac{dx^8\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\big\downarrow$ 25

$$bd\int \frac{dx^8\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\big\downarrow$ 27

$$bd^2\int \frac{x^8\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\big\downarrow$ 360

$$bd^2\left(\frac{\int \frac{4c^2x^8+4bcx^4+b(b+ac)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$\big\downarrow$ 1471

$$bd^2\left(\frac{\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}-\frac{\int \frac{(b+ac)\left(8cx^4+7b+4ac\right)}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$\big\downarrow$ 27

$$bd^2\left(\frac{\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}-\frac{1}{2}\int \frac{8cx^4+7b+4ac}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$\big\downarrow$ 299

$$bd^2\left(\frac{\frac{1}{2}\left(8\sqrt{\frac{ac+adx^2+b}{c+dx^2}}-3(4ac+5b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)+\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$\big\downarrow$ 221

$$bd^2\left(\frac{\frac{1}{2}\left(8\sqrt{\frac{ac+adx^2+b}{c+dx^2}}-\frac{3(4ac+5b)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}\sqrt{ac+b}}\right)+\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\,(ac+b-cx^4)^2}\right)$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

output
```
b*d^2*(-1/4*(b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c^3*(b +
a*c - c*x^4)^2) + (((9*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(
2*(b + a*c - c*x^4)) + (8*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] - (3*(5*b
+ 4*a*c)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b +
a*c]])/(Sqrt[c]*Sqrt[b + a*c]))/2)/(4*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(-2 a d\,x^{2} c-7 b d\,x^{2}+2 a\,c^{2}+2 b c\right)\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}{8 c^{3} x^{4}}+\frac{d^{2} b\left(-\frac{\left(12 a c+15 b\right)\ln\left(\frac{2 a\,c^{2}+2 b c+\left(2 a c d+b d\right)x^{2}+2\sqrt{a\,c^{2}+b c}\,\sqrt{a\,c^{2}+b c+(2 \cdots}{x^{2}}\right)}{2\sqrt{a\,c^{2}+b c}} \cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((a+b/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)*(-2*a*c*d*x^2-7*b*d*x^2+2*a*c^2+2*b*c)/c^3/x^4*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)+1/8/c^3*d^2*b*(-1/2*(12*a*c+15*b)/(a*c^2+b*c)^(1/2)*
ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*
d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+8*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.22

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \left[\frac{3\left(4\,abc+5\,b^2\right)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,\left(2\,a^2c^3+3\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a
*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 +
3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3
*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 +
4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a
*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 +
 (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^
2 + 15*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^
2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**5,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(151) = 302$.

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.81

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{bd^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^3} + \frac{3\,(4\,abc+5\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,\sqrt{(ac+b)c}c^3} - \frac{(4\,abc^2+9\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4\,a^2bc^2+11\,ab^2c+7\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2c^5+2\,abc^4+b^2c^3+\frac{(adx^2+ac+b)^2c^5}{(dx^2+c)^2}-\frac{2\,(ac^5+bc^4)(adx^2+ac+b)}{dx^2+c}\right)}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
b*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^3 + 3/16*(4*a*b*c + 5*b^2)*d
^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*
c)*c^3) - 1/8*((4*a*b*c^2 + 9*b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))
^(3/2) - (4*a^2*b*c^2 + 11*a*b^2*c + 7*b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(
d*x^2 + c)))/(a^2*c^5 + 2*a*b*c^4 + b^2*c^3 + (a*d*x^2 + a*c + b)^2*c^5/(d
*x^2 + c)^2 - 2*(a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^5,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 640, normalized size of antiderivative = 3.70

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx=\frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^5+2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^3d^2x^4-4\sqrt{d\,x}}{}$$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^5,x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**5 + 2*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**3*d**2*x**4 - 4*sqrt(c + d*x**2)*sqr
t(a*c + a*d*x**2 + b)*a*b*c**4 + 5*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 +
b)*a*b*c**3*d*x**2 + 17*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b*c**2
*d**2*x**4 - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*c**3 + 5*sqr
t(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*c**2*d*x**2 + 15*sqrt(c + d*x*
*2)*sqrt(a*c + a*d*x**2 + b)*b**2*c*d**2*x**4 + 12*sqrt(c)*sqrt(a*c + b)*l
og(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c
 - sqrt(c)*sqrt(c + d*x**2)*b)*a*b*c**2*d**2*x**4 + 12*sqrt(c)*sqrt(a*c +
b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)
*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*a*b*c*d**3*x**6 + 15*sqrt(c)*sqrt(a*c +
 b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2
)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*b**2*c*d**2*x**4 + 15*sqrt(c)*sqrt(a*c
 + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x*
*2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*b**2*d**3*x**6 - 12*sqrt(c)*sqrt(a*c
 + b)*log(x)*a*b*c**2*d**2*x**4 - 12*sqrt(c)*sqrt(a*c + b)*log(x)*a*b*c*d*
*3*x**6 - 15*sqrt(c)*sqrt(a*c + b)*log(x)*b**2*c*d**2*x**4 - 15*sqrt(c)*sq
rt(a*c + b)*log(x)*b**2*d**3*x**6)/(8*c**4*x**4*(a*c**2 + a*c*d*x**2 + b*c
 + b*d*x**2))
```

3.182 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7} \, dx = -\frac{bd^3\sqrt{a+\frac{b}{c+dx^2}}}{c^4} - \frac{(29b^2+36abc+8a^2c^2)\,d^2(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16c^4(b+ac)x^2} + \frac{(19b+12ac)d(c+dx^2)^2\sqrt{a+\frac{b}{c+dx^2}}}{24c^4x^4} - \frac{(b+ac)\,(c+dx^2)^3\sqrt{a+\frac{b}{c+dx^2}}}{6c^4x^6} + \frac{b(35b^2+60abc+24a^2c^2)\,d^3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{9/2}(b+ac)^{3/2}}$$

output

```
-b*d^3*(a+b/(d*x^2+c))^(1/2)/c^4-1/16*(8*a^2*c^2+36*a*b*c+29*b^2)*d^2*(d*x
^2+c)*(a+b/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x^2+1/24*(12*a*c+19*b)*d*(d*x^2+c)
^2*(a+b/(d*x^2+c))^(1/2)/c^4/x^4-1/6*(a*c+b)*(d*x^2+c)^3*(a+b/(d*x^2+c))^(
1/2)/c^4/x^6+1/16*b*(24*a^2*c^2+60*a*b*c+35*b^2)*d^3*arctanh(c^(1/2)*(a+b/
(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(9/2)/(a*c+b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{-\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(8a^2c^2\left(c^3+d^3x^6\right)+2abc\left(8c^3-7c^2dx^2+16cd^2x^4+55d^3x^6\right)+b^2\left(8c^3-14c^2dx^2+35cd^2x^4+105d^3x\right.\right.}{(b+ac)x^6}}{48c^{9/2}}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

output
```
(-((Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^3 + d^3*x^
6) + 2*a*b*c*(8*c^3 - 7*c^2*d*x^2 + 16*c*d^2*x^4 + 55*d^3*x^6) + b^2*(8*c^
3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6)))/((b + a*c)*x^6)) + (3*b*(
35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*c)^(3/2))/(48*c^(9/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2057, 2053, 2052, 27, 366, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \; 2052$$

$$-bd\int \frac{d^2x^8\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \; 27$$

$$-bd^3\int \frac{x^8\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \; 366$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\int \frac{x^8\left(6c(b+ac)x^4+5b^2-6a^2c^2\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \; 360$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{\int -\frac{24c^3(b+ac)x^8+4bc^2(11b+12ac)x^4+bc(b+ac)(11b+12ac)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^3}+\frac{b(ac+b)(12ac+11b)\sqrt{\cdots}}{4c^2(ac+b-cx\cdots}}{6c^2(ac+b)}\right.$$

$$\Big\downarrow \; 25$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}-\frac{\int \frac{24c^3(b+ac)x^8+4bc^2(11b+12ac)x^4+bc(b+ac)(11b+12ac)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\cdots}}{4c^3}}{6c^2(ac+b)}\right.$$

$$\Big\downarrow \; 1471$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\int \frac{3c(b+ac)\left(16c(b+ac\cdots\right.}{-c\cdots}}{4c^3}}{6c^2(ac+b)}\right.$$

$$\Big\downarrow \; 27$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\frac{3}{2}c\int\frac{16c(b+ac)x^4+19}{-cx^4}}{4c^3}}{6c^2(ac+b)}\right.$$

$\downarrow$ 299

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\frac{3}{2}c\left(\left(24a^2c^2+60abc\right.\right.}{}}{6c^2(ac+b)}\right.$$

$\downarrow$ 221

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\frac{3}{2}c\left(\frac{\left(24a^2c^2+60abc-\right.}{}\right.}{}}{6c^2(ac+b)}\right.$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

output
```
-(b*d^3*((b^2*x^10)/(6*c^2*(b + a*c)*(b + a*c - c*x^4)^3) - ((b*(b + a*c)*
(11*b + 12*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c^2*(b + a*c - c
*x^4)^2) - ((c*(79*b^2 + 108*a*b*c + 24*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)])/(2*(b + a*c - c*x^4)) - (3*c*(-16*(b + a*c)*Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)] + ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*ArcTanh[(Sqrt[c]
*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(Sqrt[c]*Sqrt[b +
a*c])))/2)/(4*c^3))/(6*c^2*(b + a*c))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(8a^{2}c^{2}d^{2}x^{4}+62abc\,d^{2}x^{4}-8a^{2}c^{3}d\,x^{2}+57b^{2}d^{2}x^{4}-30ab\,c^{2}x^{2}d+8a^{2}c^{4}-22b^{2}c\,x^{2}d+16ab\,c^{3}+8b^{2}c^{2}\right)\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}{48c^{4}x^{6}\left(a c+b\right)}-\frac{d^{3}b\cdots}{\cdots}$
default	Expression too large to display

input
```
int((a+b/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(8*a^2*c^2*d^2*x^4+62*a*b*c*d^2*x^4-8*a^2*c^3*d*x^2+57*b^2
*d^2*x^4-30*a*b*c^2*d*x^2+8*a^2*c^4-22*b^2*c*d*x^2+16*a*b*c^3+8*b^2*c^2)/c
^4/x^6/(a*c+b)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*d^3*b/c^4/(a*c+b)*(-
1/2*(24*a^2*c^2+60*a*b*c+35*b^2)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*
c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(
1/2))/x^2)+16*(a*c+b)*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.93

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \left[\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{ac^2+bc}d^3x^6\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8}{}\right.}{}\right.$$

$$-\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{-ac^2-bc}d^3x^6\arctan\left(\frac{\left((2\,ac+b)dx^2+2\,ac^2+2\,bc\right)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2\,abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)+2\left(8\,a^3\right.}{}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*l
og(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c
^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*
a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3
 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^
3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5
 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((
a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c +
 35*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^
2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c
^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3
*c^4 + 118*a^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 +
 24*a*b^2*c^5 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2
*x^4 - 14*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]
```

Sympy [F]

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**7,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(224) = 448$.

Time = 0.16 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.14

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx =$$

$$-\frac{(24\,a^2bc^2+60\,ab^2c+35\,b^3)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\,(ac^5+bc^4)\sqrt{(ac+b)c}}-\frac{bd^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^4}$$

$$-\frac{3\,(8\,a^2bc^4+36\,ab^2c^3+29\,b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^4+30\,a^2b^2c^3+41\,ab^3c^2+17\,b^4c)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right.}{48\left(a^4c^8+4\,a^3bc^7+6\,a^2b^2c^6+4\,ab^3c^5+b^4c^4-\frac{(ac^8+bc^7)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{3\,(a^2c^8+2\,abc^7+b^2c}{(dx^2+}\right.}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/32*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) + sqrt((a*c + b)*c)))/((a*c^5 + b*c^4)*sqrt((a*c + b)*c)) - b*d^3*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^4 - 1/48*(3*(8*a^2*b*c^4 + 36*a*b^2*
c^3 + 29*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b
*c^4 + 30*a^2*b^2*c^3 + 41*a*b^3*c^2 + 17*b^4*c)*d^3*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 44*a^3*b^2*c^3 + 83*a^2*b^3*c^2 + 66
*a*b^4*c + 19*b^5)*d^3*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^8 + 4
*a^3*b*c^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4 - (a*c^8 + b*c^7)*(a*d*
x^2 + a*c + b)^3/(d*x^2 + c)^3 + 3*(a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^
2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^
3*c^5)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^7,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1063, normalized size of antiderivative = 4.25

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=\text{Too large to display}$$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^7,x)
```

output
```
( - 8*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**7 - 8*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**4*d**3*x**6 - 24*sqrt(c + d*x**2)*sq
rt(a*c + a*d*x**2 + b)*a**2*b*c**6 + 14*sqrt(c + d*x**2)*sqrt(a*c + a*d*x*
*2 + b)*a**2*b*c**5*d*x**2 - 32*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*
a**2*b*c**4*d**2*x**4 - 118*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2
*b*c**3*d**3*x**6 - 24*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c*
*5 + 28*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c**4*d*x**2 - 67*
sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c**3*d**2*x**4 - 215*sqrt
(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c**2*d**3*x**6 - 8*sqrt(c + d
*x**2)*sqrt(a*c + a*d*x**2 + b)*b**3*c**4 + 14*sqrt(c + d*x**2)*sqrt(a*c +
 a*d*x**2 + b)*b**3*c**3*d*x**2 - 35*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
+ b)*b**3*c**2*d**2*x**4 - 105*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b
**3*c*d**3*x**6 + 72*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*
d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt(c + d*x**2)*b)
*a**2*b*c**3*d**3*x**6 + 72*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a
*c + a*d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt(c + d*x
**2)*b)*a**2*b*c**2*d**4*x**8 + 180*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b
)*sqrt(a*c + a*d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + sqrt(c)*sqrt
(c + d*x**2)*b)*a*b**2*c**2*d**3*x**6 + 180*sqrt(c)*sqrt(a*c + b)*log(sqrt
(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c + sqrt(c)*sqrt(c + d*x**2)*a*c + s...
```

3.183 $\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 357

$$\begin{aligned}\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx &= \frac{(b^2-14abc+a^2c^2)\,x\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2} \\ &+ \frac{(7b-ac)x(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{5d^2} \\ &+ \frac{6ax^3(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{5d} - \frac{x^3(b+ac+adx^2)\sqrt{a+\frac{b}{c+dx^2}}}{d} \\ &- \frac{\sqrt{c}(b^2-14abc+a^2c^2)\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &- \frac{c^{3/2}(7b-ac)\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
1/5*(a^2*c^2-14*a*b*c+b^2)*x*(a+b/(d*x^2+c))^(1/2)/a/d^2+1/5*(-a*c+7*b)*x*
(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/d^2+6/5*a*x^3*(d*x^2+c)*(a+b/(d*x^2+c))^(1
/2)/d-x^3*(a*d*x^2+a*c+b)*(a+b/(d*x^2+c))^(1/2)/d-1/5*c^(1/2)*(a^2*c^2-14*
a*b*c+b^2)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(
1/2),(b/(a*c+b))^(1/2))/a/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1
/2)-1/5*c^(3/2)*(-a*c+7*b)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^
(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*
x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.93 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.87

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x\left(-a^2(c-dx^2)\left(c+dx^2\right)^2+b^2(7c+2dx^2)+3ab(2c^2+3cdx^2+d^2\right.\right.}{}$$

input
```
Integrate[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(-(a^2*(c - d*x^2)*(
c + d*x^2)^2) + b^2*(7*c + 2*d*x^2) + 3*a*b*(2*c^2 + 3*c*d*x^2 + d^2*x^4))
 - I*(b^3 - 13*a*b^2*c - 13*a^2*b*c^2 + a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/
(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b
+ a*c)] + I*b*(b^2 - 6*a*b*c - 7*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*
c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]
))/(5*a*c^2*(d/c)^(5/2)*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2057, 2058, 369, 27, 443, 27, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int\frac{x^4(adx^2+b+ac)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int\frac{3x^2\sqrt{adx^2+b+ac}(2adx^2+b+ac)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(ac+adx^2+b)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\int\frac{x^2\sqrt{adx^2+b+ac}(2adx^2+b+ac)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(ac+adx^2+b)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{443}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{\int\frac{dx^2(a(7b-ac)dx^2+(5b-ac)(b+ac))}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5d}+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}\right)}{d}-\frac{x^3(ac+adx^2+b)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\int\frac{x^2\left(a(7b-ac)dx^2+(5b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}\right)}{d}-\frac{x^3\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\downarrow \text{444}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\int\frac{ad\left(c(7b-ac)(b+ac)-\left(b^2-14acb+a^2c^2\right)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}\right)+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2-}\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$$\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\int\frac{c(7b-ac)(b+ac)-\left(b^2-14acb+a^2c^2\right)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}\right)}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$$\downarrow \text{406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{c(7b-ac)(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d\left(a^2c^2-14abc+b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2}}\right.}{3d}\right.\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$$\downarrow \text{320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}(7b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d\left(a^2c^2-14abc+b^2\right)\int}{3d}\right.\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}(7b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d\left(a^2c^2-14abc+b^2\right)\left(\right.}{3d}\right.\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}(7b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d\left(a^2c^2-14abc+b^2\right)\left(\cdots\right.}{3d}\cdots\right.\right.}{d}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-((x^3*(b + a*c +
a*d*x^2)^(3/2))/(d*Sqrt[c + d*x^2])) + (3*((2*a*x^3*Sqrt[c + d*x^2]*Sqrt[b
 + a*c + a*d*x^2])/5 + (((7*b - a*c)*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*
x^2])/(3*d) - (-((b^2 - 14*a*b*c + a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])
/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b
 + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*(7*b - a*c)*Sqrt
[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(
Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2
))]))/(3*d))/5))/d))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(330) = 660$.

Time = 17.50 (sec) , antiderivative size = 1062, normalized size of antiderivative = 2.97

method	result	size
risch	Expression too large to display	1062
default	Expression too large to display	1101

input
```
int(x^4*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d^2*x*(-a*d*x^2+a*c-2*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+
1/5/d^2*(a^2*c^3/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2
*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*d*(a^2*c^2-9*a*b*c+b^2)
*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^
(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(Ell
ipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*
(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2)))-7*b^2*c/(-a*d/(a*c+b
))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d
+b*d)/d/c/a)^(1/2))-a*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)
*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*Ellip
ticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+5*b^2*c^2*((a*
d^2*x^2+a*c*d+b*d)/c/b*x/d/((x^2+c/d)*(a*d^2*x^2+a*c*d+b*d))^(1/2)+(1/c-(a
*c*d+b*d)/c/b/d)/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2
*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+2*a*d/b/c*(a*c^2+b*c)/(-a
*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(
a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b)...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.65

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx =$$

$$-\frac{(a^2c^3-14\,abc^2+b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac})-(a^2c^3-14\,abc^2+b^2c+(a^2c^2-6\,abc-7\,b^2)\cdots}{\cdots}$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*((a^2*c^3 - 14*a*b*c^2 + b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsi
n(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a^2*c^3 - 14*a*b*c^2 + b^2*c + (a^2*c
^2 - 6*a*b*c - 7*b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), (a*c + b)/(a*c)) - (a^2*d^3*x^6 + 2*a*b*d^2*x^4 + a^2*c^3 - 14*a*b*c^
2 - (7*a*b*c - b^2)*d*x^2 + b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
(a*d^3*x)
```

Sympy [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x**4*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^4 \, dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int x^4 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(x^4*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4*(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \text{Too large to display}$$

input
```
int(x^4*(a+b/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*d**2*x**5 - 3*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a*b*c*x + 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*
x**2 + b)*a*b*d*x**3 - 3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*x
- 4*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a*c**3 + 3*a*c**
2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*
x**4),x)*a**2*b*c**2*d**2 - 4*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 +
b)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**
2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a**2*b*c*d**3*x**2 + 4*int((sqrt(c + d*
x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**
2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a*b**2*c*d*
*2 + 4*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a*c**3 + 3*a*
c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d*
*2*x**4),x)*a*b**2*d**3*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
 + b))/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2
+ 2*b*c*d*x**2 + b*d**2*x**4),x)*a**2*b*c**4 + 3*int((sqrt(c + d*x**2)*sqr
t(a*c + a*d*x**2 + b))/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**
3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a**2*b*c**3*d*x**2 + 6*in
t((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**3 + 3*a*c**2*d*x**2 +
3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a*
b**2*c**3 + 6*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**3 +...
```

3.184 $\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$

Optimal result

Integrand size = 21, antiderivative size = 291

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{(7b-ac)x\sqrt{a+\frac{b}{c+dx^2}}}{3d}+\frac{4ax(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{3d}-\frac{x(b+ac+adx^2)\sqrt{a+\frac{b}{c+dx^2}}}{d}-\frac{\sqrt{c}(7b-ac)\sqrt{a+\frac{b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{\sqrt{c}(3b-ac)\sqrt{a+\frac{b}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output

```
1/3*(-a*c+7*b)*x*(a+b/(d*x^2+c))^(1/2)/d+4/3*a*x*(d*x^2+c)*(a+b/(d*x^2+c))
^(1/2)/d-x*(a*d*x^2+a*c+b)*(a+b/(d*x^2+c))^(1/2)/d-1/3*c^(1/2)*(-a*c+7*b)*
(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*
c+b))^(1/2))/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*c^(1/
2)*(-a*c+3*b)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/
2)),(b/(a*c+b))^(1/2))/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.62 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.88

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}x\left(-3b^2-2ab(c+dx^2)+a^2(c+dx^2)^2\right)+i(-7b^2-6abc+a^2c^2)\sqrt{\frac{b}{}}\right.}{}$$

input
```
Integrate[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*x*(-3*b^2 - 2*a*b*(c + d
*x^2) + a^2*(c + d*x^2)^2) + I*(-7*b^2 - 6*a*b*c + a^2*c^2)*Sqrt[(b + a*c
+ a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x]
, (a*c)/(b + a*c)] + (4*I)*b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/
(3*d*Sqrt[d/c]*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2058, 369, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$\Big\downarrow$ 2058

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{x^2\left(adx^2+b+ac\right)^{3/2}}{\left(dx^2+c\right)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 369

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{\sqrt{adx^2+b+ac}\left(4adx^2+b+ac\right)}{\sqrt{dx^2+c}}dx}{d}-\frac{x\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 403

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{\int \frac{d\left(a(7b-ac)dx^2+(3b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{d}-\frac{x\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\int \frac{a(7b-ac)dx^2+(3b-ac)(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{d}-\frac{x\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left((3b-ac)(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad(7b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left(ad(7b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{\cdots}}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{4}{3}}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right.}{(ac+b)\left(c+dx^2\right.}}}\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

input

```
Int[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

output

```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-((x*(b + a*c + a*
d*x^2)^(3/2))/(d*Sqrt[c + d*x^2])) + ((4*a*x*Sqrt[c + d*x^2]*Sqrt[b + a*c
+ a*d*x^2])/3 + (a*(7*b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c
+ d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*
x^2))/((b + a*c)*(c + d*x^2))])) + (Sqrt[c]*(3*b - a*c)*Sqrt[b + a*c + a*d
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c
 + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/3)/d))/S
qrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(268) = 536$.

Time = 14.03 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.82

method	result
default	$\left(\sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{-\frac{a d}{a c+b}}\, a^2 d^2 x^5+2\sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{-\frac{a d}{a c+b}}\, a^2 c d x^3+\sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{-\frac{a d}{a c+b}}\, ab\right.$
risch	Expression too large to display

input
```
int(x^2*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*d^2*x^5+2*
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d*x^3+((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3-((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2-3*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3+((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*x-5*((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+7*((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+((d*x^2+c)*(a*d*x^
2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x+3*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a
*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x-3*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b
))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.70

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{(a^2c^3-7\,abc^2)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(a^2c^3-7\,abc^2+(a^2c^2-2\,abc-3}{3}$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((a^2*c^3 - 7*a*b*c^2)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/
d)/x), (a*c + b)/(a*c)) - (a^2*c^3 - 7*a*b*c^2 + (a^2*c^2 - 2*a*b*c - 3*b^
2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c
)) + (a^2*c*d^2*x^4 + 4*a*b*c*d*x^2 - a^2*c^3 + 7*a*b*c^2)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a*c*d^2*x)
```

Sympy [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x**2*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int x^2 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(x^2*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2*(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \text{Too large to display}$$

input
```
int(x^2*(a+b/(d*x^2+c))^(3/2),x)
```

output

```
(2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c*d*x**3 + 3*sqrt(c + d*
x**2)*sqrt(a*c + a*d*x**2 + b)*a*b*c*x + 3*sqrt(c + d*x**2)*sqrt(a*c + a*d
*x**2 + b)*b**2*x + 5*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)
/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*
c*d*x**2 + b*d**2*x**4),x)*a**2*b*c**2*d**2 + 5*int((sqrt(c + d*x**2)*sqrt
(a*c + a*d*x**2 + b)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a
*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a**2*b*c*d**3*x**2 -
3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a*c**3 + 3*a*c**2*
d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x*
*4),x)*a*b**2*c*d**2 - 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x*
*4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2
*b*c*d*x**2 + b*d**2*x**4),x)*a*b**2*d**3*x**2 - 3*int((sqrt(c + d*x**2)*s
qrt(a*c + a*d*x**2 + b))/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d
**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a**2*b*c**4 - 3*int((sq
rt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c
*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a**2*b*
c**3*d*x**2 - 6*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**3 +
3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 +
b*d**2*x**4),x)*a*b**2*c**3 - 6*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
+ b))/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**...
```

3.185 $\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 196

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = ax\sqrt{a+\frac{b}{c+dx^2}} + \frac{(b-ac)\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
a*x*(a+b/(d*x^2+c))^(1/2)+(-a*c+b)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)
*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/c^(1/2)/d^(1/2)/(c*(a*d*x^
2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*c^(1/2)*(a+b/(d*x^2+c))^(1/2)*InverseJ
acobiAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/d^(1/2)/(c*(a*d*x^2+a
*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.17

$$\int \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b\sqrt{\frac{d}{c}}x(b+a(c+dx^2)) + i(b^2-a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsin}\right.}{d\,(b+a(}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[d/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[d/c]*x*(b + a*(c
+ d*x^2)) + I*(b^2 - a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(b +
a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(d*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.66, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2057, 2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx$$

$$\Big\downarrow \; 2057$$

$$\int \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2} dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\left(adx^2+b+ac\right)^{3/2}}{\left(dx^2+c\right)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{ad\left(c(b+ac)-(b-ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)\right)}{c}+\frac{bx\sqrt{ac-}}{c\sqrt{c}}\right.}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+}\right.}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right.\right)}{c}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*x*Sqrt[b + a*c
+ a*d*x^2])/(c*Sqrt[c + d*x^2]) + (a*(-((b - a*c)*d*((x*Sqrt[b + a*c + a*d
*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a
*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d
]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))
/c))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(185) = 370$.

Time = 5.19 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.63

method	result
default	$\frac{\left(\sqrt{(d\,x^2+c)(ad\,x^2+ac+b)}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\operatorname{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^2c^2+\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{a}{}}$

input
```
int((a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2
+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*
d*x^3+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*
a*b*c-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*
b*c+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a
*b*c*x+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2
)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/c/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx =$$

$$-\frac{(ac^2-bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(ac^2-bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}}{cdx}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-((a*c^2 - b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c
 + b)/(a*c)) - (a*c^2 - b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f
(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a*c*d*x^2 + a*c^2 - b*c)*sqrt((
a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c*d*x)
```

Sympy [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral((a + b/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int((a + b/(c + d*x^2))^(3/2),x)
```

output
```
int((a + b/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,acx+\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,bx-\left(\int \frac{\sqrt{d\,x^2+\cdots}}{a\,d^3x^6+3ac\,d^2x^4+b\,d^2\cdots}\right.}{c\cdots}$$

input
```
int((a+b/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c*x + sqrt(c + d*x**2)*sqrt(a
*c + a*d*x**2 + b)*b*x - int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x*
*4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2
*b*c*d*x**2 + b*d**2*x**4),x)*a*b*c*d**2 - int((sqrt(c + d*x**2)*sqrt(a*c
+ a*d*x**2 + b)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3
*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a*b*d**3*x**2)/(c*(c + d*x
**2))
```

3.186 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx &= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{cx} - \frac{(2b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{cx} \\ &\quad - \frac{(2b+ac)\sqrt{d}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &\quad + \frac{a\sqrt{d}\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
b*(a+b/(d*x^2+c))^(1/2)/c/x-(a*c+2*b)*(a+b/(d*x^2+c))^(1/2)/c/x-(a*c+2*b)*
d^(1/2)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2
),(b/(a*c+b))^(1/2))/c^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a
*d^(1/2)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(
b/(a*c+b))^(1/2))/c^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}\left(2ab(c+dx^2)^2+a^2c(c+dx^2)^2+b^2(c+2dx^2)\right)+i(2b^2+3abc+a^2c^2)\,dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{\ldots}\right.}{c^2\sqrt{\frac{d}{c}}x\,(b\ldots}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

output
```
-((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(2*a*b*(c + d*x^2)^2 +
 a^2*c*(c + d*x^2)^2 + b^2*(c + 2*d*x^2)) + I*(2*b^2 + 3*a*b*c + a^2*c^2)*
d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (2*I)*b*(b + a*c)*d*x*Sqrt[(b + a*c
 + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x
], (a*c)/(b + a*c)]))/(c^2*Sqrt[d/c]*x*(b + a*(c + d*x^2))))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.67, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2057, 2058, 370, 25, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \; 2057$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\left(adx^2+b+ac\right)^{3/2}}{x^2\left(dx^2+c\right)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}-\frac{\int -\frac{(b+ac)d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{(b+ac)d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\int \frac{adx^2+2b+ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(-\frac{\int -\frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{\int \frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\int\frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(c(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(d(ac+2b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}}{cx(ac}\right.}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c(ac+b)}-\ldots\right)}{c}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

$$\downarrow \; 313$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,E\left(\arctan\left(\ldots\right)\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+\ldots\right)}{(ac+b)\ldots}}}\right)\right)}{c(ac+b)}\ldots\right)}{c}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*Sqrt[b + a*c +
a*d*x^2])/(c*x*Sqrt[c + d*x^2]) + ((b + a*c)*(-(((2*b + a*c)*Sqrt[c + d*x^
2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) + (a*d*((2*b + a*c)*d*((x*Sqr
t[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[
c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3
/2)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c
 + d*x^2))])))/(c*(b + a*c))))/c))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(222) = 444$.

Time = 14.32 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.71

method	result
default	$-\frac{\left(\sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{-\frac{a d}{a c+b}}\,a^2 c\,d^2 x^4+\sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{-\frac{a d}{a c+b}}\,a b\,d^2 x^4-\sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{\frac{a d x^2+\cdots}{a c+\cdots}}\right.}{}$
risch	$-\frac{(a c+b)(d x^2+c)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{c^2 x}+\frac{d\left(\frac{a^2 c^2\sqrt{1+\frac{a d x^2}{a c+b}}\,\sqrt{1+\frac{x^2 d}{c}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{a d}{a c+b}},\sqrt{-1+\frac{2 a c d+b d}{d c a}}\right)}{\sqrt{-\frac{a d}{a c+b}}\,\sqrt{a\,d^2 x^4+2 a d\,x^2 c+b d\,x^2+a\,c^2+b c}}-\frac{2 d a b\left(a\,c^2+b c\right)\sqrt{1+\frac{a d x^2}{a c+b}}\,\sqrt{1\cdots}}{}\right.}{}$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^2*x^4+((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^4-((d*x^2+c)
*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d*x+(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^4
+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^2+((
d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*d*x-
2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*
d*x+3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^2+(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*
d*x^2+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^3+((d*x
^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*d*x^2+(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*d*x^2+2*((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c^2+((d*x^2+c)*(a*d*x^
2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x
/c^2/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{(a^2c+2\,ab)\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - ((a^2c+2\,ab)d^2+($$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
((a^2*c + 2*a*b)*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*ellipt
ic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 2*a*b)*d
^2 + (a^2*c^2 + 2*a*b*c + b^2)*d)*sqrt(-a*d/(a*c + b))*x*sqrt((a*c^2 + b*c
)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*
c^3 + 2*a*b*c^2 + (a^2*c^2 + 3*a*b*c + 2*b^2)*d*x^2 + b^2*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/((a*c^3 + b*c^2)*x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**2,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^2,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a\,c^2-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,bc-\sqrt{d\,x^2+c}\,\sqrt{a}$$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^2,x)
```

output

```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c**2 - 2*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*b*c - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)
*b*d*x**2 + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a*c**3 +
 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 +
 b*d**2*x**4),x)*a*b*c*d**3*x + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
+ b)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c
**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a*b*d**4*x**3 - 3*int((sqrt(c + d*x**
2)*sqrt(a*c + a*d*x**2 + b))/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 +
 a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a*b*c**3*d*x - 3*in
t((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**3 + 3*a*c**2*d*x**2 +
3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*a*
b*c**2*d**2*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c*
*3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**2 + 2*b*c*d*x*
*2 + b*d**2*x**4),x)*b**2*c**2*d*x - 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*
d*x**2 + b))/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b
*c**2 + 2*b*c*d*x**2 + b*d**2*x**4),x)*b**2*c*d**2*x**3)/(2*c**2*x*(c + d*
x**2))
```

3.187 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 298

$$\begin{aligned}\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4} dx &= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{cx^3} \\ &+ \frac{(8b+ac)d\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x^3} \\ &+ \frac{(8b+ac)d^{3/2}\sqrt{a+\frac{b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &- \frac{a(4b+ac)d^{3/2}\sqrt{a+\frac{b}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
b*(a+b/(d*x^2+c))^(1/2)/c/x^3+1/3*(a*c+8*b)*d*(a+b/(d*x^2+c))^(1/2)/c^2/x-
1/3*(a*c+4*b)*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c^2/x^3+1/3*(a*c+8*b)*d^(3/2
)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(
a*c+b))^(1/2))/c^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*(
a*c+4*b)*d^(3/2)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^
(1/2)),(b/(a*c+b))^(1/2))/c^(3/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^
2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.48 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-b^2c^2-2abc^3-a^2c^4+4b^2cdx^2+3abc^2dx^2-a^2c^3dx^2+8b^2d^2x^4+13\right.}{}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b^2*c^2) - 2*a*b*c^3 - a^2*c^4 +
 4*b^2*c*d*x^2 + 3*a*b*c^2*d*x^2 - a^2*c^3*d*x^2 + 8*b^2*d^2*x^4 + 13*a*b*
c*d^2*x^4 + a^2*c^2*d^2*x^4 + 8*a*b*d^3*x^6 + a^2*c*d^3*x^6 + I*c*(8*b^2 +
 9*a*b*c + a^2*c^2)*d*Sqrt[d/c]*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sq
rt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b
*c*(8*b + 5*a*c)*d*Sqrt[d/c]*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[
1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*c^3
*x^3*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2057, 2058, 370, 25, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \; 2057$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{(adx^2+b+ac)^{3/2}}{x^4(dx^2+c)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}-\frac{\int -\frac{d\left(a(3b+ac)dx^2+(b+ac)(4b+ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{d\left(a(3b+ac)dx^2+(b+ac)(4b+ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{a(3b+ac)dx^2+(b+ac)(4b+ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{\int \frac{(b+ac)d\left(a(4b+ac)dx^2+(b+ac)(8b+ac)\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\int \frac{a(4b+ac)dx^2+(b+ac)(8b+ac)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(-\frac{\int -\frac{a(b+ac)d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{a}}{cx}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(\frac{\int \frac{a(b+ac)d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+}}{cx^3\sqrt{}}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(\frac{ad\int \frac{(8b+ac)dx^2+c(4b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(\frac{ad\left(c(ac+4b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+8b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\cdots}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(d(ac+8b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}-\frac{(ac+8b)\sqrt{c+}}{}\right)}{3c}}{c}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}\right)}{3c}}{c}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{\frac{d\left(ad\left(\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\text{arcta}\right.}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(\ldots\right.}{(ac\ldots}}}\right.\right.\right.}{c}}{3c}}{c}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*Sqrt[b + a*c +
a*d*x^2])/(c*x^3*Sqrt[c + d*x^2]) + (-1/3*((4*b + a*c)*Sqrt[c + d*x^2]*Sqr
t[b + a*c + a*d*x^2])/(c*x^3) - (d*(-(((8*b + a*c)*Sqrt[c + d*x^2]*Sqrt[b
+ a*c + a*d*x^2])/(c*x)) + (a*d*((8*b + a*c)*d*((x*Sqrt[b + a*c + a*d*x^2]
)/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(
b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*(4*b + a*c)*Sqrt
[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(
(b + a*c)*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*
(c + d*x^2))])))/c))/(3*c))/c))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(275) = 550$.

Time = 17.33 (sec) , antiderivative size = 1037, normalized size of antiderivative = 3.48

method	result	size
default	Expression too large to display	1037
risch	Expression too large to display	1122

input
```
int((a+b/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^
6-5*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^6+((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2
*x^3-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2
)*a*b*d^3*x^6-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c
^2*d^2*x^4-4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1
/2))*a*b*c*d^2*x^3+8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*a*b*c*d^2*x^3-10*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c
+b))^(1/2)*a*b*c*d^2*x^4-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*(-a*d/(a*c+b))^(1/2)*a*b*c*d^2*x^4+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*
d/(a*c+b))^(1/2)*a^2*c^3*d*x^2-5*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(
a*c+b))^(1/2)*b^2*d^2*x^4-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(-a*d/(a*c+b))^(1/2)*b^2*d^2*x^4-3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a
*d/(a*c+b))^(1/2)*a*b*c^2*d*x^2+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a
*c+b))^(1/2)*a^2*c^4-4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1
/2)*b^2*c*d*x^2+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a
*b*c^3+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c^2)*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = -\frac{(a^2c+8\,ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac}) - ((a^2c+8\,ab)d^3+(a^2c^2+5\,abc+4\,b^2)d}{}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*((a^2*c + 8*a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)
*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 8
*a*b)*d^3 + (a^2*c^2 + 5*a*b*c + 4*b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt
((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(
a*c)) - ((a^2*c^2 + 9*a*b*c + 8*b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c
^2 + 4*(a*b*c^2 + b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a
*c^4 + b*c^3)*x^3)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**4,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^4,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\text{too large to display}$$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^4,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c - sqrt(c + d*x**2)*sqrt(
a*c + a*d*x**2 + b)*b - 12*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))
/(3*a**3*c**5*x**2 + 9*a**3*c**4*d*x**4 + 9*a**3*c**3*d**2*x**6 + 3*a**3*c
**2*d**3*x**8 + 8*a**2*b*c**4*x**2 + 21*a**2*b*c**3*d*x**4 + 18*a**2*b*c**
2*d**2*x**6 + 5*a**2*b*c*d**3*x**8 + 7*a*b**2*c**3*x**2 + 16*a*b**2*c**2*d
*x**4 + 11*a*b**2*c*d**2*x**6 + 2*a*b**2*d**3*x**8 + 2*b**3*c**2*x**2 + 4*
b**3*c*d*x**4 + 2*b**3*d**2*x**6),x)*a**3*b*c**4*d*x**3 - 12*int((sqrt(c +
 d*x**2)*sqrt(a*c + a*d*x**2 + b))/(3*a**3*c**5*x**2 + 9*a**3*c**4*d*x**4
+ 9*a**3*c**3*d**2*x**6 + 3*a**3*c**2*d**3*x**8 + 8*a**2*b*c**4*x**2 + 21*
a**2*b*c**3*d*x**4 + 18*a**2*b*c**2*d**2*x**6 + 5*a**2*b*c*d**3*x**8 + 7*a
*b**2*c**3*x**2 + 16*a*b**2*c**2*d*x**4 + 11*a*b**2*c*d**2*x**6 + 2*a*b**2
*d**3*x**8 + 2*b**3*c**2*x**2 + 4*b**3*c*d*x**4 + 2*b**3*d**2*x**6),x)*a**
3*b*c**3*d**2*x**5 - 32*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(3
*a**3*c**5*x**2 + 9*a**3*c**4*d*x**4 + 9*a**3*c**3*d**2*x**6 + 3*a**3*c**2
*d**3*x**8 + 8*a**2*b*c**4*x**2 + 21*a**2*b*c**3*d*x**4 + 18*a**2*b*c**2*d
**2*x**6 + 5*a**2*b*c*d**3*x**8 + 7*a*b**2*c**3*x**2 + 16*a*b**2*c**2*d*x*
*4 + 11*a*b**2*c*d**2*x**6 + 2*a*b**2*d**3*x**8 + 2*b**3*c**2*x**2 + 4*b**
3*c*d*x**4 + 2*b**3*d**2*x**6),x)*a**2*b**2*c**3*d*x**3 - 32*int((sqrt(c +
 d*x**2)*sqrt(a*c + a*d*x**2 + b))/(3*a**3*c**5*x**2 + 9*a**3*c**4*d*x**4
+ 9*a**3*c**3*d**2*x**6 + 3*a**3*c**2*d**3*x**8 + 8*a**2*b*c**4*x**2 + ...
```

3.188 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 21, antiderivative size = 378

$$\begin{aligned}\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6} dx = {} & \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{cx^5} - \frac{(16b^2+16abc+a^2c^2)\,d^2\sqrt{a+\frac{b}{c+dx^2}}}{5c^3(b+ac)x} \\ & - \frac{(6b+ac)\,(c+dx^2)\,\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5} + \frac{(8b+ac)d(c+dx^2)\,\sqrt{a+\frac{b}{c+dx^2}}}{5c^3x^3} \\ & - \frac{(16b^2+16abc+a^2c^2)\,d^{5/2}\sqrt{a+\frac{b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{7/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ & + \frac{a(8b+ac)d^{5/2}\sqrt{a+\frac{b}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5c^{5/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
b*(a+b/(d*x^2+c))^(1/2)/c/x^5-1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^2*(a+b/(d*x^
2+c))^(1/2)/c^3/(a*c+b)/x-1/5*(a*c+6*b)*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c^
2/x^5+1/5*(a*c+8*b)*d*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(a^2*c^2
+16*a*b*c+16*b^2)*d^(5/2)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2
)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/c^(7/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/
(a*c+b)/(d*x^2+c))^(1/2)+1/5*a*(a*c+8*b)*d^(5/2)*(a+b/(d*x^2+c))^(1/2)*Inv
erseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/c^(5/2)/(a*c+b)/
(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.93 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b^3c^3+3ab^2c^4+3a^2bc^5+a^3c^6-2b^3c^2dx^2-3ab^2c^3dx^2+a^3c^5dx^2+8b^3cd^2x^4+13ab^2c^2d^2x^4\right.}{}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

output
```
-1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b^3*c^3 + 3*a*b^2*c^4 + 3*a^2
*b*c^5 + a^3*c^6 - 2*b^3*c^2*d*x^2 - 3*a*b^2*c^3*d*x^2 + a^3*c^5*d*x^2 + 8
*b^3*c*d^2*x^4 + 13*a*b^2*c^2*d^2*x^4 + 5*a^2*b*c^3*d^2*x^4 + 16*b^3*d^3*x
^6 + 40*a*b^2*c*d^3*x^6 + 24*a^2*b*c^2*d^3*x^6 + a^3*c^3*d^3*x^6 + 16*a*b^
2*d^4*x^8 + 16*a^2*b*c*d^4*x^8 + a^3*c^2*d^4*x^8 + I*c*(16*b^3 + 32*a*b^2*
c + 17*a^2*b*c^2 + a^3*c^3)*d^2*Sqrt[d/c]*x^5*Sqrt[(b + a*c + a*d*x^2)/(b
+ a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a
*c)] - (8*I)*b*c*(2*b^2 + 3*a*b*c + a^2*c^2)*d^2*Sqrt[d/c]*x^5*Sqrt[(b + a
*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (a*c)/(b + a*c)]))/(c^4*(b + a*c)*x^5*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.38, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {2057, 2058, 370, 25, 27, 445, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6} \, dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{(adx^2+b+ac)^{3/2}}{x^6(dx^2+c)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 370}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}-\frac{\int -\frac{d\left(a(5b+ac)dx^2+(b+ac)(6b+ac)\right)}{x^6\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{d\left(a(5b+ac)dx^2+(b+ac)(6b+ac)\right)}{x^6\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{a(5b+ac)dx^2+(b+ac)(6b+ac)}{x^6\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{\int \frac{3(b+ac)d\left(a(6b+ac)dx^2+(b+ac)(8b+ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c(ac+b)}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\int \frac{a(6b+ac)dx^2+(b+ac)(8b+ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 445

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\left(-\frac{\int \frac{(b+ac)d\left(16b^2+16acb+a^2c^2+a(8b+ac)dx^2\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{5c}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\left(-\frac{d\int \frac{16b^2+16acb+a^2c^2+a(8b+ac)dx^2}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{5c}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}+\frac{b\sqrt{a}}{cx}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 445

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{d\left(-\frac{\int -\frac{ad\left(\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(a^2c^2+16abc+16b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{(ac+8b)\cdots}{\cdots}\cdots\right)}{5c}\cdots}{c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{d\left(\frac{\int \frac{ad\left(\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(a^2c^2+16abc+16b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{(ac+8b)\sqrt{c}\cdots}{\cdots}\cdots\right)}{5c}\cdots}{c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{3d\left(-\frac{d\left(\frac{ad\int\frac{\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(a^2c^2+16abc+16b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{(ac+8b)\sqrt{c+\cdots}}{3\cdots}\cdots\right)}{5c}\cdots}{c}\cdots\right)}{\sqrt{ac+adx^2+b}\cdots}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{3d\left(-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(ac+8b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{\left(a^2c^2+16\cdots\right)\cdots}{\cdots}\cdots\right)}{3c}\cdots\right)}{5c}\cdots}{c}\cdots\right)}{\sqrt{ac+adx^2+b}\cdots}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\left(-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+8b)\sqrt{ac+adx^2+b}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right.}{c(ac+b)}}{3c}\right.}{5c}}{c}\right.}{\sqrt{ac+adx^2}}$$

↓ 388

$$\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{3d\left(-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(ac+8b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\ldots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\ldots\right)}{(ac+b)\left(c+dx^2\ldots\right)}}}\right.}{c(ac+b)}\ldots}{3c}\ldots\right.}{5c}\ldots\right)}{\sqrt{ac+a\ldots}}$$

$\downarrow$ 313

$$\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{3d\left(-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}(ac+8b)\sqrt{ac+ad}}{\sqrt{d}\sqrt{c}}\right.}{c(ac+b)}}{3c}\right.}{5}\right.$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*Sqrt[b + a*c +
a*d*x^2])/(c*x^5*Sqrt[c + d*x^2]) + (-1/5*((6*b + a*c)*Sqrt[c + d*x^2]*Sqr
t[b + a*c + a*d*x^2])/(c*x^5) - (3*d*(-1/3*((8*b + a*c)*Sqrt[c + d*x^2]*Sq
rt[b + a*c + a*d*x^2])/(c*x^3) - (d*(-(((16*b^2 + 16*a*b*c + a^2*c^2)*Sqrt
[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) + (a*d*((16*b^2 + 16
*a*b*c + a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (
Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(
b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a
*c)*(c + d*x^2))])) + (c^(3/2)*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(c*(b + a*c))))/(3*c
)))/(5*c))/c))/Sqrt[b + a*c + a*d*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(351) = 702$.

Time = 18.87 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.08

method	result	size
risch	Expression too large to display	1166
default	Expression too large to display	1666

input
```
int((a+b/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(d*x^2+c)*(a^2*c^2*d^2*x^4+11*a*b*c*d^2*x^4-a^2*c^3*d*x^2+11*b^2*d^2*
x^4-4*a*b*c^2*d*x^2+a^2*c^4-3*b^2*c*d*x^2+2*a*b*c^3+b^2*c^2)/c^4/x^5/(a*c+
b)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5/c^4*d^3/(a*c+b)*(a^3*c^3/(-a*d/(a
*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a
*c*d+b*d)/d/c/a)^(1/2))-2*a*d*(a^2*c^2+11*a*b*c+11*b^2)*(a*c^2+b*c)/(-a*d/
(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c
+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2
),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2)))+4*a^2*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d
*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1
/2))+3*c*a*b^2/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d
)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/
(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-5*b^2*c*(a*c+b)*((a*d^2*x^2
+a*c*d+b*d)/c/b*x/d/((x^2+c/d)*(a*d^2*x^2+a*c*d+b*d))^(1/2)+(1/c-(a*c*d+b*
d)/c/b/d)/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/
2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+
b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+2*a*d/b/c*(a*c^2+b*c)/(-a*d/(a*c
+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{(a^3c^2+16\,a^2bc+16\,ab^2)\sqrt{-\frac{ad}{ac+b}}d^4x^5\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-((a^3c$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/5*((a^3*c^2 + 16*a^2*b*c + 16*a*b^2)*sqrt(-a*d/(a*c + b))*d^4*x^5*sqrt((
a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*
c)) - ((a^3*c^2 + 16*a^2*b*c + 16*a*b^2)*d^4 + (a^3*c^3 + 10*a^2*b*c^2 + 1
7*a*b^2*c + 8*b^3)*d^3)*sqrt(-a*d/(a*c + b))*x^5*sqrt((a*c^2 + b*c)/d^2)*e
lliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^3*c^3 + 1
7*a^2*b*c^2 + 32*a*b^2*c + 16*b^3)*d^3*x^6 + a^3*c^6 + 3*a^2*b*c^5 + 3*a*b
^2*c^4 + (7*a^2*b*c^3 + 15*a*b^2*c^2 + 8*b^3*c)*d^2*x^4 + b^3*c^3 - 2*(a^2
*b*c^4 + 2*a*b^2*c^3 + b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/((a^2*c^6 + 2*a*b*c^5 + b^2*c^4)*x^5)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**6,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^6}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^6,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \text{too large to display}$$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^6,x)
```

output
```
( - 5*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**3 - 13*sqrt(c + d*
x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b*c**2 + 10*sqrt(c + d*x**2)*sqrt(a*c
+ a*d*x**2 + b)*a**2*b*d**2*x**4 - 11*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
 + b)*a*b**2*c - 3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**3 + 50*int
((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/(5*a**3*c**5 + 15*a**3*c
**4*d*x**2 + 15*a**3*c**3*d**2*x**4 + 5*a**3*c**2*d**3*x**6 + 13*a**2*b*c*
*4 + 34*a**2*b*c**3*d*x**2 + 29*a**2*b*c**2*d**2*x**4 + 8*a**2*b*c*d**3*x*
*6 + 11*a*b**2*c**3 + 25*a*b**2*c**2*d*x**2 + 17*a*b**2*c*d**2*x**4 + 3*a*
b**2*d**3*x**6 + 3*b**3*c**2 + 6*b**3*c*d*x**2 + 3*b**3*d**2*x**4),x)*a**5
*b*c**3*d**4*x**5 + 50*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2
)/(5*a**3*c**5 + 15*a**3*c**4*d*x**2 + 15*a**3*c**3*d**2*x**4 + 5*a**3*c**
2*d**3*x**6 + 13*a**2*b*c**4 + 34*a**2*b*c**3*d*x**2 + 29*a**2*b*c**2*d**2
*x**4 + 8*a**2*b*c*d**3*x**6 + 11*a*b**2*c**3 + 25*a*b**2*c**2*d*x**2 + 17
*a*b**2*c*d**2*x**4 + 3*a*b**2*d**3*x**6 + 3*b**3*c**2 + 6*b**3*c*d*x**2 +
 3*b**3*d**2*x**4),x)*a**5*b*c**2*d**5*x**7 + 80*int((sqrt(c + d*x**2)*sqr
t(a*c + a*d*x**2 + b)*x**2)/(5*a**3*c**5 + 15*a**3*c**4*d*x**2 + 15*a**3*c
**3*d**2*x**4 + 5*a**3*c**2*d**3*x**6 + 13*a**2*b*c**4 + 34*a**2*b*c**3*d*
x**2 + 29*a**2*b*c**2*d**2*x**4 + 8*a**2*b*c*d**3*x**6 + 11*a*b**2*c**3 +
25*a*b**2*c**2*d*x**2 + 17*a*b**2*c*d**2*x**4 + 3*a*b**2*d**3*x**6 + 3*b**
3*c**2 + 6*b**3*c*d*x**2 + 3*b**3*d**2*x**4),x)*a**4*b**2*c**2*d**4*x**...
```

3.189 $\int \sqrt{\frac{5}{4}-\frac{7}{4(3-4x^2)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \sqrt{\frac{5}{4}-\frac{7}{4(3-4x^2)}}\,dx=\frac{E\left(\arcsin\left(\frac{2x}{\sqrt{3}}\right)|\frac{15}{8}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticE(2/3*x*3^(1/2),1/4*30^(1/2))*2^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(19) = 38.

Time = 1.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \sqrt{\frac{5}{4}-\frac{7}{4(3-4x^2)}}\,dx=\frac{\sqrt{\frac{2-5x^2}{3-4x^2}}\sqrt{3-4x^2}E\left(\arcsin\left(\frac{2x}{\sqrt{3}}\right)|\frac{15}{8}\right)}{\sqrt{4-10x^2}}$$

input
```
Integrate[Sqrt[5/4 - 7/(4*(3 - 4*x^2))],x]
```

output
```
(Sqrt[(2 - 5*x^2)/(3 - 4*x^2)]*Sqrt[3 - 4*x^2]*EllipticE[ArcSin[(2*x)/Sqrt
[3]], 15/8])/Sqrt[4 - 10*x^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2057, 2050, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{5}{4}-\frac{7}{4(3-4x^2)}}dx$$
$$\Big\downarrow \text{2057}$$
$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}dx$$
$$\Big\downarrow \text{2050}$$
$$\int \frac{\sqrt{2-5x^2}}{\sqrt{3-4x^2}}dx$$
$$\Big\downarrow \text{327}$$
$$\frac{E\left(\arcsin\left(\frac{2x}{\sqrt{3}}\right)|\frac{15}{8}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[5/4 - 7/(4*(3 - 4*x^2))],x]
```

output
```
EllipticE[ArcSin[(2*x)/Sqrt[3]], 15/8]/Sqrt[2]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(17) = 34$.

Time = 0.87 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.79

method	result
default	$\frac{\sqrt{\frac{5x^2-2}{4x^2-3}}\,(4x^2-3)\sqrt{10}\,\sqrt{-10x^2+4}\,\sqrt{-12x^2+9}\left(7\,\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{30}}{15}\right)-15\,\mathrm{EllipticE}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{30}}{15}\right)\right)}{120\sqrt{(4x^2-3)(5x^2-2)}\,\sqrt{20x^4-23x^2+6}}$
elliptic	$\frac{\sqrt{\frac{5x^2-2}{4x^2-3}}\,\sqrt{(4x^2-3)(5x^2-2)}\left(-\frac{\sqrt{10}\,\sqrt{-10x^2+4}\,\sqrt{-12x^2+9}\,\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{30}}{15}\right)}{15\sqrt{20x^4-23x^2+6}}+\frac{\sqrt{10}\,\sqrt{-10x^2+4}\,\sqrt{-12x^2+9}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{3}}{15}\right.\right.}{8\sqrt{20x^4-23x^2+6}}\right.}{5x^2-2}$

input
```
int(1/2*(5-28/(-16*x^2+12))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*((5*x^2-2)/(4*x^2-3))^(1/2)*(4*x^2-3)*10^(1/2)*(-10*x^2+4)^(1/2)*(-1
2*x^2+9)^(1/2)*(7*EllipticF(1/2*x*10^(1/2),2/15*30^(1/2))-15*EllipticE(1/2
*x*10^(1/2),2/15*30^(1/2)))/((4*x^2-3)*(5*x^2-2))^(1/2)/(20*x^4-23*x^2+6)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.89

$$\int \sqrt{\frac{5}{4}-\frac{7}{4(3-4x^2)}}\,dx$$
$$=\frac{45\sqrt{5}\sqrt{3}xE(\arcsin\left(\frac{\sqrt{3}}{2x}\right)\mid\frac{8}{15})-13\sqrt{5}\sqrt{3}xF(\arcsin\left(\frac{\sqrt{3}}{2x}\right)\mid\frac{8}{15})+60\,(4\,x^2-3)\sqrt{\frac{5\,x^2-2}{4\,x^2-3}}}{240\,x}$$

input
```
integrate(1/2*(5-28/(-16*x^2+12))^(1/2),x, algorithm="fricas")
```

output
```
1/240*(45*sqrt(5)*sqrt(3)*x*elliptic_e(arcsin(1/2*sqrt(3)/x), 8/15) - 13*s
qrt(5)*sqrt(3)*x*elliptic_f(arcsin(1/2*sqrt(3)/x), 8/15) + 60*(4*x^2 - 3)*
sqrt((5*x^2 - 2)/(4*x^2 - 3)))/x
```

Sympy [F]

$$\int \sqrt{\frac{5}{4}-\frac{7}{4(3-4x^2)}}\,dx=\int\frac{\sqrt{5-\frac{28}{12-16x^2}}}{2}\,dx$$

input
```
integrate(1/2*(5-28/(-16*x**2+12))**(1/2),x)
```

output
```
Integral(sqrt(5 - 28/(12 - 16*x**2))/2, x)
```

Maxima [F]

$$\int \sqrt{\frac{5}{4}-\frac{7}{4\,(3-4x^2)}}\,dx=\int \frac{1}{2}\,\sqrt{\frac{7}{4\,x^2-3}+5}\,dx$$

input
```
integrate(1/2*(5-28/(-16*x^2+12))^(1/2),x, algorithm="maxima")
```

output
```
1/2*integrate(sqrt(7/(4*x^2 - 3) + 5), x)
```

Giac [F]

$$\int \sqrt{\frac{5}{4}-\frac{7}{4\,(3-4x^2)}}\,dx=\int \frac{1}{2}\,\sqrt{\frac{7}{4\,x^2-3}+5}\,dx$$

input
```
integrate(1/2*(5-28/(-16*x^2+12))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/2*sqrt(7/(4*x^2 - 3) + 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{5}{4}-\frac{7}{4\,(3-4x^2)}}\,dx=\int \frac{\sqrt{\frac{28}{16\,x^2-12}+5}}{2}\,dx$$

input
```
int((28/(16*x^2 - 12) + 5)^(1/2)/2,x)
```

output
```
int((28/(16*x^2 - 12) + 5)^(1/2)/2, x)
```

Reduce [F]

$$\int \sqrt{\frac{5}{4}-\frac{7}{4\left(3-4x^2\right)}}\,dx=\int \frac{\sqrt{4x^2-3}\,\sqrt{5x^2-2}}{4x^2-3}dx$$

input
```
int(1/2*(5-28/(-16*x^2+12))^(1/2),x)
```

output
```
int((sqrt(4*x**2 - 3)*sqrt(5*x**2 - 2))/(4*x**2 - 3),x)
```

3.190 $\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \frac{E\left(\arcsin\left(\frac{2x}{\sqrt{3}}\right)|\frac{15}{8}\right)}{\sqrt{2}}$$

output
```
1/2*EllipticE(2/3*x*3^(1/2),1/4*30^(1/2))*2^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(19) = 38.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \frac{\sqrt{\frac{2-5x^2}{3-4x^2}}\sqrt{3-4x^2}E\left(\arcsin\left(\frac{2x}{\sqrt{3}}\right)|\frac{15}{8}\right)}{\sqrt{4-10x^2}}$$

input
```
Integrate[Sqrt[(2 - 5*x^2)/(3 - 4*x^2)],x]
```

output
```
(Sqrt[(2 - 5*x^2)/(3 - 4*x^2)]*Sqrt[3 - 4*x^2]*EllipticE[ArcSin[(2*x)/Sqrt
[3]], 15/8])/Sqrt[4 - 10*x^2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2050, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{\sqrt{2-5x^2}}{\sqrt{3-4x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{E\left(\arcsin\left(\frac{2x}{\sqrt{3}}\right)|\frac{15}{8}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[(2 - 5*x^2)/(3 - 4*x^2)],x]
```

output
```
EllipticE[ArcSin[(2*x)/Sqrt[3]], 15/8]/Sqrt[2]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(17) = 34$.

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.79

method	result
default	$\frac{\sqrt{\frac{5x^2-2}{4x^2-3}}\,(4x^2-3)\sqrt{10}\,\sqrt{-10x^2+4}\,\sqrt{-12x^2+9}\left(7\,\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{30}}{15}\right)-15\,\mathrm{EllipticE}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{30}}{15}\right)\right)}{120\sqrt{(4x^2-3)(5x^2-2)}\,\sqrt{20x^4-23x^2+6}}$
elliptic	$\frac{\sqrt{\frac{5x^2-2}{4x^2-3}}\,\sqrt{(4x^2-3)(5x^2-2)}\left(-\frac{\sqrt{10}\,\sqrt{-10x^2+4}\,\sqrt{-12x^2+9}\,\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{30}}{15}\right)}{15\sqrt{20x^4-23x^2+6}}+\frac{\sqrt{10}\,\sqrt{-10x^2+4}\,\sqrt{-12x^2+9}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{10}}{2},\frac{2\sqrt{3}}{15}\right.\right.}{8\sqrt{20x^4-23x^2+6}}\right.}{5x^2-2}$

input
```
int(((-5*x^2+2)/(-4*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*((5*x^2-2)/(4*x^2-3))^(1/2)*(4*x^2-3)*10^(1/2)*(-10*x^2+4)^(1/2)*(-1
2*x^2+9)^(1/2)*(7*EllipticF(1/2*x*10^(1/2),2/15*30^(1/2))-15*EllipticE(1/2
*x*10^(1/2),2/15*30^(1/2)))/((4*x^2-3)*(5*x^2-2))^(1/2)/(20*x^4-23*x^2+6)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.89

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \frac{45\sqrt{5}\sqrt{3}xE\left(\arcsin\left(\frac{\sqrt{3}}{2x}\right) \mid \frac{8}{15}\right) - 13\sqrt{5}\sqrt{3}xF\left(\arcsin\left(\frac{\sqrt{3}}{2x}\right) \mid \frac{8}{15}\right) + 60\,(4\,x^2-3)\sqrt{\frac{5\,x^2-2}{4\,x^2-3}}}{240\,x}$$

input
```
integrate(((-5*x^2+2)/(-4*x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
1/240*(45*sqrt(5)*sqrt(3)*x*elliptic_e(arcsin(1/2*sqrt(3)/x), 8/15) - 13*s
qrt(5)*sqrt(3)*x*elliptic_f(arcsin(1/2*sqrt(3)/x), 8/15) + 60*(4*x^2 - 3)*
sqrt((5*x^2 - 2)/(4*x^2 - 3)))/x
```

Sympy [F]

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx$$

input
```
integrate(((-5*x**2+2)/(-4*x**2+3))**(1/2),x)
```

output
```
Integral(sqrt((2 - 5*x**2)/(3 - 4*x**2)), x)
```

Maxima [F]

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \int \sqrt{\frac{5\,x^2-2}{4\,x^2-3}}\,dx$$

input
```
integrate(((-5*x^2+2)/(-4*x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((5*x^2 - 2)/(4*x^2 - 3)), x)
```

Giac [F]

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \int \sqrt{\frac{5\,x^2-2}{4\,x^2-3}}\,dx$$

input
```
integrate(((-5*x^2+2)/(-4*x^2+3))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((5*x^2 - 2)/(4*x^2 - 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \int \sqrt{\frac{5\,x^2-2}{4\,x^2-3}}\,dx$$

input
```
int(((5*x^2 - 2)/(4*x^2 - 3))^(1/2),x)
```

output
```
int(((5*x^2 - 2)/(4*x^2 - 3))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{2-5x^2}{3-4x^2}}\,dx = \int \frac{\sqrt{4x^2-3}\,\sqrt{5x^2-2}}{4x^2-3}dx$$

input
```
int(((-5*x^2+2)/(-4*x^2+3))^(1/2),x)
```

output
```
int((sqrt(4*x**2 - 3)*sqrt(5*x**2 - 2))/(4*x**2 - 3),x)
```

3.191 $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 190

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{(5b^2+12abc+8a^2c^2)\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16a^3d^3} - \frac{(5b+12ac)\,(c+dx^2)^2\sqrt{a+\frac{b}{c+dx^2}}}{24a^2d^3} + \frac{(c+dx^2)^3\sqrt{a+\frac{b}{c+dx^2}}}{6ad^3} - \frac{b(5b^2+12abc+8a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}$$

output
```
1/16*(8*a^2*c^2+12*a*b*c+5*b^2)*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a^3/d^3-1/
24*(12*a*c+5*b)*(d*x^2+c)^2*(a+b/(d*x^2+c))^(1/2)/a^2/d^3+1/6*(d*x^2+c)^3*
(a+b/(d*x^2+c))^(1/2)/a/d^3-1/16*b*(8*a^2*c^2+12*a*b*c+5*b^2)*arctanh((a+b
/(d*x^2+c))^(1/2)/a^(1/2))/a^(7/2)/d^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$=\frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(15b^2+2ab(13c-5dx^2)+8a^2(c^2-cdx^2+d^2x^4))-3b(5b^2+12abc+8a^2c^2)\,\mathrm{ar}}{48a^{7/2}d^3}$$

input
```
Integrate[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b
*(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b
*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(4
8*a^(7/2)*d^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2053, 2052, 27, 315, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$\downarrow$ 2052

$$-bd\int \frac{\left(-cx^4+b+ac\right)^2}{d^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\downarrow$ 27

$$-\frac{b\int \frac{\left(-cx^4+b+ac\right)^2}{(a-x^4)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$\downarrow$ 315

$$-\frac{b\left(\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a(a-x^4)^3}-\frac{\int -\frac{(b+ac)(5b+6ac)-3c(b+2ac)x^4}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}\right)}{d^3}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{\int \frac{(b+ac)(5b+6ac)-3c(b+2ac)x^4}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a(a-x^4)^3}\right)}{d^3}$$

$\downarrow$ 298

$$-\frac{b\left(\frac{\frac{3\left(8a^2c^2+12abc+5b^2\right)\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a}+\frac{b(8ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a(a-x^4)^3}\right)}{d^3}$$

$\downarrow$ 215

$$-\frac{b\left(\frac{\frac{3\left(8a^2c^2+12abc+5b^2\right)\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b(8ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a(a-x^4)^3}\right)}{d^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\left(\frac{\frac{3\left(8a^2c^2+12abc+5b^2\right)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b(8ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}\right)}{d^3}$$

input
```
Int[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-((b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b + a*c - c*x^4))/(6*a*(a
- x^4)^3) + ((b*(5*b + 8*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a*
(a - x^4)^2) + (3*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*(Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]/(2*a*(a - x^4)) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^
2)]/Sqrt[a]]/(2*a^(3/2))))/(4*a))/(6*a)))/d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.26

method	result
risch	$\frac{(8a^2d^2x^4-8a^2cd\,x^2-10abd\,x^2+8a^2c^2+26abc+15b^2)(ad\,x^2+ac+b)}{48d^3a^3\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{b(8a^2c^2+12abc+5b^2)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2}\right.}{32d^2a^3\sqrt{a\,d^2}\,\sqrt{\frac{ad\,x^2+a}{d\,x^2+}}}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-48\sqrt{a\,d^2}\,\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,a^2cd\,x^2-36\sqrt{a\,d^2}\,\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,abd\,x^2\right.}{}$

input
```
int(x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2-10*a*b*d*x^2+8*a^2*c^2+26*a*b*c+15*b
^2)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/32*b/d^2*(8*a^
2*c^2+12*a*b*c+5*b^2)/a^3*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^
2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)/((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.24

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\right.}{}\right.$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8
*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a
*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
))) + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*
a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arcta
n(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^
3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3)]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.79

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6a^3bc^2+12a^2b^2c+5ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8a^4bc^2+2\right.}{48\left(a^6d^3-\frac{3(adx^2+ac+b)a^5d^3}{dx^2+c}+\frac{3(adx^2+ac+b)^2a^4d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3a^3d^3}{(dx^2+c)^3}\right)} + \frac{(8a^2c^2+12abc+5b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32a^{\frac{7}{2}}d^3}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 +
c))^(5/2) - 8*(6*a^3*b*c^2 + 12*a^2*b^2*c + 5*a*b^3)*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^2 + 20*a^3*b^2*c + 11*a^2*b^3)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c)))/(a^6*d^3 - 3*(a*d*x^2 + a*c + b)*a^5*d^3/(d*
x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a^4*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c
+ b)^3*a^3*d^3/(d*x^2 + c)^3) + 1/32*(8*a^2*c^2 + 12*a*b*c + 5*b^2)*b*log(
-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{ad}-\frac{4\,a^2cd^3+5\,abd^3}{a^3d^5}\right)+\frac{8\,a^2c^2d^2+26\,abcd^2+15\,b^2d^2}{a^3d^5}\right)+\frac{3\,(8\,a^2bc^2+12\,a}{}}{96\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^
2/(a*d) - (4*a^2*c*d^3 + 5*a*b*d^3)/(a^3*d^5)) + (8*a^2*c^2*d^2 + 26*a*b*c
*d^2 + 15*b^2*d^2)/(a^3*d^5)) + 3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*log(a
bs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(a^(7/2)*d^2*abs(d)))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^5}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^5/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5/(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.54

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3c^2-8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3cd\,x^2+8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3}{}$$

input
```
int(x^5/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**2 - 8*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*a**3*c*d*x**2 + 8*sqrt(c + d*x**2)*sqrt(a*c + a
*d*x**2 + b)*a**3*d**2*x**4 + 26*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)
*a**2*b*c - 10*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b*d*x**2 + 1
5*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2 + 24*sqrt(a)*log( - sqr
t(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a**2*b*c**2 + 36*sqrt(
a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a*b**2*c
+ 15*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)
*b**3)/(48*a**4*d**3)
```

3.192 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(3b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{a+\frac{b}{c+dx^2}}}{4ad^2}+\frac{b(3b+4ac)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}$$

output
```
-1/8*(4*a*c+3*b)*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a^2/d^2+1/4*(d*x^2+c)^2*(
a+b/(d*x^2+c))^(1/2)/a/d^2+1/8*b*(4*a*c+3*b)*arctanh((a+b/(d*x^2+c))^(1/2)
/a^(1/2))/a^(5/2)/d^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(-3b-2ac+2adx^2)+b(3b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}$$

input
```
Integrate[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b - 2*a*c +
 2*a*d*x^2) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]/Sqrt[a]])/(8*a^(5/2)*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2053, 2052, 25, 27, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{-cx^4+b+ac}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{-cx^4+b+ac}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{-cx^4+b+ac}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{b\left(\frac{(4ac+3b)\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{b\left(\frac{(4ac+3b)\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{(4ac+3b)\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a(a-x^4)^2}\right)}{d^2}$$

input
```
Int[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a*(a - x^4)^2) + ((3*b +
4*a*c)*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*a*(a - x^4)) + ArcTanh[Sq
rt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/(2*a^(3/2))))/(4*a)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{(-2adx^2+2ac+3b)(adx^2+ac+b)}{8d^2a^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}+\frac{b(4ac+3b)\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{ac^2+bc+(2acd+bd)x^2+ad^2x^4}\right)\sqrt{(dx^2+c)(adx^2+a\cdots}}{16da^2\sqrt{ad^2}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}(dx^2+c)}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}(dx^2+c)\left(-4\sqrt{ad^2x^4+2adx^2c+bdx^2+ac^2+bc}\sqrt{ad^2}\,adx^2-4\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2adx^2c+bdx^2+ac^2+b\cdots}}{2\sqrt{ad^2}}\cdots\right.\right.}{\cdots}$

input
```
int(x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c+3*b)*(a*d*x^2+a*c+b)/a^2/((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)+1/16*b/d*(4*a*c+3*b)/a^2*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^
(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)/((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.69

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{(4\,abc+3\,b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+\right.}{32\,a^3d^2}\right.$$

$$-\frac{(4\,abc+3\,b^2)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(2\,a^2d^2x^4-3\,abdx^2-2\,a^2c^2-3\,abc)\sqrt{}}{16\,a^3d^2}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*
c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x
^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 +
2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2
*c + a*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(108) = 216.

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.80

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(4\,abc+3\,b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc+5\,ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2-\frac{2\,(adx^2+ac+b)a^3d^2}{dx^2+c}+\frac{(adx^2+ac+b)^2a^2d^2}{(dx^2+c)^2}\right)}-\frac{(4\,ac+3\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{5}{2}}d^2}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/8*((4*a*b*c + 3*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b
*c + 5*a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2 - 2*(a*d*x^2
 + a*c + b)*a^3*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a^2*d^2/(d*x^2 + c
)^2) - 1/16*(4*a*c + 3*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.34

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{ad}-\frac{2\,acd+3\,bd}{a^2d^3}\right)-\frac{(4\,abc+3\,b^2)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2}\right)\right|\right)}{a^{\frac{5}{2}}d|d|}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d)
 - (2*a*c*d + 3*b*d)/(a^2*d^3)) - (4*a*b*c + 3*b^2)*log(abs(2*a*c*d + 2*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sq
rt(a)*abs(d) + b*d))/(a^(5/2)*d*abs(d)))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3/(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c+2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2d\,x^2-3\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,ab}{8a^3d}$$

input
```
int(x^3/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c + 2*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*a**2*d*x**2 - 3*sqrt(c + d*x**2)*sqrt(a*c + a*d
*x**2 + b)*a*b + 4*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c +
 d*x**2)*a)*a*b*c + 3*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(
c + d*x**2)*a)*b**2)/(8*a**3*d**2)
```

3.193 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

output
```
1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a/d-1/2*b*arctanh((a+b/(d*x^2+c))^(1/2
)/a^(1/2))/a^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{2ad}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

input
```
Integrate[x/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[S
qrt[(b + a*(c + d*x^2))/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2024, 773, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2024}$$

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{ 773}$$

$$-\frac{\int \frac{(dx^2+c)^2}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{-\frac{b\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}}{2a}-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{a}}{2d}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{-\frac{\int \frac{1}{\frac{1}{b(dx^2+c)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}}{a}-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{a}}{2d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{a}}{2d}$$

input
```
Int[x/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-1/2*(-(((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/a) + (b*ArcTanh[Sqrt[a + b/(
c + d*x^2)]/Sqrt[a]])/a^(3/2))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(60) = 120$.

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\sqrt{\frac{(d x^2+c)a+b}{d x^2+c}}\,(d x^2+c)\left(2\sqrt{(d x^2+c)((d x^2+c)a+b)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{(d x^2+c)((d x^2+c)a+b)}\,\sqrt{a}+2(d x^2+c)a+b}{2\sqrt{a}}\right)\right)}{4d\sqrt{(d x^2+c)((d x^2+c)a+b)}\,a^{\frac{3}{2}}}$
risch	$\frac{a d x^2+ac+b}{2da\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}}-\frac{\ln\left(\frac{acd+\frac{1}{2}bd+a d^2x^2}{\sqrt{a d^2}}+\sqrt{a c^2+bc+(2acd+bd)x^2+a d^2x^4}\right)b\sqrt{(d x^2+c)(a d x^2+ac+b)}}{4a\sqrt{a d^2}\,\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)}$
default	$\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(-\ln\left(\frac{2a d^2x^2+2acd+2\sqrt{a d^2x^4+2ad x^2c+bd x^2+a c^2+bc}\,\sqrt{a d^2}+bd}{2\sqrt{a d^2}}\right)bd+2\sqrt{a d^2x^4+2ad x^2c+bd x^2 \cdots}\right.}{4\sqrt{(d x^2+c)(a d x^2+ac+b)}\,ad\sqrt{a d^2}}$

input
```
int(x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(((d*x^2+c)*a+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*((d*x^2+c)*((d*x^2+c)
*a+b))^(1/2)*a^(1/2)-b*ln(1/2*(2*((d*x^2+c)*((d*x^2+c)*a+b))^(1/2)*a^(1/2)
+2*(d*x^2+c)*a+b)/a^(1/2)))/((d*x^2+c)*((d*x^2+c)*a+b))^(1/2)/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.71

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{\cdots}\right.}{8\,a^2d}\right.$$

input
```
integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 +
 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^
2*c + a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^
2*d)]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(60) = 120$.

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2d-\frac{(adx^2+ac+b)ad}{dx^2+c}\right)}+\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{3}{2}}d}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d - (a*d*x^2 + a*c + b)*
a*d/(d*x^2 + c)) + 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\frac{b\log\left(\left|2\,acd+2\,\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}|d|+bd\right|\right)}{a^{\frac{3}{2}}|d|}+\frac{2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{ad}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(a^(3/2)*abs(d)) + 2*sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a*d))/sgn(d*x^2 + c)
```

Mupad [B] (verification not implemented)

Time = 10.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{a\,(d\,x^2+c)}{b}+1}\,(d\,x^2+c)\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,(d\,x^2+c)}}{2\,a\,(d\,x^2+c)}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,(d\,x^2+c)^{3/2}}\right)}{3\,d\,\sqrt{a+\frac{b}{d\,x^2+c}}}$$

input
```
int(x/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
(((a*(c + d*x^2))/b + 1)^(1/2)*(c + d*x^2)*((b^(3/2)*asin((a^(1/2)*(c + d*
x^2)^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2)*(c + d*x^2)^(3/2)) + (3*b^(1/2)*(b
+ a*(c + d*x^2))^(1/2))/(2*a*(c + d*x^2))))/(3*d*(a + b/(c + d*x^2))^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a+\sqrt{a}\,\log\left(-\sqrt{a}\,\sqrt{ad\,x^2+ac+b}+\sqrt{d\,x^2+c}\,a\right)b}{2a^2d}$$

input
```
int(x/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a + sqrt(a)*log( - sqrt(a)*sqrt
(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*b)/(2*a**2*d)
```

3.194 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{b+ac}}$$

output
```
arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(1/2)-c^(1/2)*arctanh(c^(1/2)*(a+
b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{-b-ac}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b -
a*c]])/Sqrt[-b - a*c] + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt
[a]]/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2053, 2052, 25, 27, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{1}{d(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{1}{d(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{1}{(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{303}$$

$$b\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{c\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{ 219}$$

$$b\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{c\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$b\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{ac+b}}\right)$$

input
```
Int[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
b*(ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/(Sqrt[a]*b) - (S
qrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*
c]])/(b*Sqrt[b + a*c]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.90

method	result
default	$\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(\ln\left(\frac{2a d^2 x^2+2acd+2\sqrt{a d^2 x^4+2ad x^2 c+bd x^2+a c^2+bc}\,\sqrt{a d^2}+bd}{2\sqrt{a d^2}}\right)acd+\ln\left(\frac{2a d^2 x^2+2acd+2\sqrt{a d^2 x^4+2ad x^2 c+b}}{2\sqrt{a d^2}}\right.}{2\sqrt{(d x^2+c)(ad x^2+ac+b}}$

input
```
int(1/x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(ln(1/2*(2*a*d^2*x^2+2*a*c
*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*a*c*d+ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b*d-(a*c^2+b*c)^(
1/2)*ln((2*a*d*x^2*c+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)/(a*c+b)/(a*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(64) = 128$.

Time = 0.14 (sec) , antiderivative size = 972, normalized size of antiderivative = 12.15

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2
*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 -
4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2
 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c))*sqrt(c/(a*c + b)))/x^4) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 +
8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x
^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/4
*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4
 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((
2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (
4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(c/(a*c + b)))/x^4) - 2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b
)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)
))/a, 1/4*(2*a*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2
+ 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x
^2 + a*c^2 + b*c)) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c +
a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2
+ b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/2*(a*sqrt(-c/(
a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) - ...
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(64) = 128$.

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.94

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{c\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}} - \frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\sqrt{a}}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*c*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a*c + b
)*c) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a)
 + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/sqrt(a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x^2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.84

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{ac+b}\,\sqrt{ad\,x^2+ac+b}\,c-\sqrt{c}\,\sqrt{d\,x^2+c}\,ac-\sqrt{c}\,\sqrt{d\,x^2+c}\,b\right)a-\sqrt{c}\,\sqrt{ac+b}\log(x)\,a}{a\,(ac+}$$

input
```
int(1/x/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt
(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*a - sqrt(c)*sqrt(a*
c + b)*log(x)*a + sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c +
d*x**2)*a)*a*c + sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d
*x**2)*a)*b)/(a*(a*c + b))
```

3.195 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2(b+ac)x^2}-\frac{bd\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2\sqrt{c}(b+ac)^{3/2}}$$

output
```
-1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)/x^2-1/2*b*d*arctanh(c^(1/2)*(
a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(1/2)/(a*c+b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{1}{2}\left(-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x^2}-\frac{bd\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}(-b-ac)^{3/2}}\right)$$

input
```
Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(-(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x^2)) -
(b*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]
])/(Sqrt[c]*(-b - a*c)^(3/2)))/2
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2053, 2052, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{1}{(cx^4-b-ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{215}$$

$$-bd\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}-\frac{\int \frac{1}{cx^4-b-ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}\right)$$

$$\Big\downarrow \text{221}$$

$$-bd\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)\left(ac+b-cx^4\right)}\right)$$

input
```
Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
-(b*d*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*(b + a*c)*(b + a*c - c*x^4
)) + ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]
]/(2*Sqrt[c]*(b + a*c)^(3/2))))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(76) = 152$.

Time = 0.14 (sec), antiderivative size = 197, normalized size of antiderivative = 2.14

method	result
risch	$-\frac{a d x^2+a c+b}{2(ac+b)x^2\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}}-\frac{bd\ln\left(\frac{2a c^2+2bc+(2acd+bd)x^2+2\sqrt{a c^2+bc}\sqrt{a c^2+bc+(2acd+bd)x^2+a d^2x^4}}{x^2}\right)\sqrt{(d x^2+c)(ad x^2+ac+b)}}{4(ac+b)\sqrt{a c^2+bc}\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}(d x^2+c)}$
default	$-\frac{\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}(d x^2+c)\left(-2a d^2\sqrt{a d^2x^4+2ad x^2c+bd x^2+a c^2+bc}\,x^4\sqrt{a c^2+bc}+\ln\left(\frac{2ad x^2c+bd x^2+2a c^2+2\sqrt{a c^2+bc}\sqrt{a d^2x^4+2a}}{x^2}\right.\right.}{}$

input
```
int(1/x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*c+b)*(a*d*x^2+a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/4*b*d
/(a*c+b)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*
c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)/((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(76) = 152$.

Time = 0.14 (sec), antiderivative size = 451, normalized size of antiderivative = 4.90

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$=\left[\frac{\sqrt{ac^2+bc}bdx^2\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2-4((2ac+b)d^2x^4+2ac^3+(4ac^2+3}{x^4}\right.}{8(a^2c^3+2abc^2+b^2c)x^2}\right.$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2
)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3
 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a
^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1
/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b
^2*c)) - 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(76) = 152$.

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.88

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^2+2\,abc+b^2-\frac{(adx^2+ac+b)(ac^2+bc)}{dx^2+c}\right)}+\frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}(ac+b)}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^2 + 2*a*b*c + b^2 -
(a*d*x^2 + a*c + b)*(a*c^2 + b*c)/(d*x^2 + c)) + 1/4*b*d*log((c*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*(a*c + b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(76) = 152$.

Time = 0.17 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.17

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \frac{\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}(ac+b)} - \frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)acd+2\sqrt{a}bc|d|+(\sqrt{ad^2}x^2}{(ac^2-\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2+}}{2\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c + b)) - (2*a
^(3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*
d*x^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(
a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*
c)*(a*c + b)))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x^2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$=\frac{-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a\,c^2-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,bc+\sqrt{c}\,\sqrt{ac+b}\log\left(\sqrt{ac+b}\,\sqrt{ad\,x^2+a\right.}{2c\,x^2\,(a^2c^2+2abc+b^2)}$$

input
```
int(1/x^3/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c**2 - sqrt(c + d*x**2)*sq
rt(a*c + a*d*x**2 + b)*b*c + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(
a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c + d*
x**2)*b)*b*d*x**2 - sqrt(c)*sqrt(a*c + b)*log(x)*b*d*x**2)/(2*c*x**2*(a**2
*c**2 + 2*a*b*c + b**2))
```

3.196 $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8c(b+ac)^2x^2}-\frac{(c+dx^2)^2\sqrt{a+\frac{b}{c+dx^2}}}{4c(b+ac)x^4}+\frac{b(b+4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{3/2}(b+ac)^{5/2}}$$

output
```
1/8*(4*a*c+b)*d*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/c/(a*c+b)^2/x^2-1/4*(d*x^2
+c)^2*(a+b/(d*x^2+c))^(1/2)/c/(a*c+b)/x^4+1/8*b*(4*a*c+b)*d^2*arctanh(c^(1
/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)/(a*c+b)^(5/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(2ac(c-dx^2)+b(2c+dx^2)\right)}{8c(b+ac)^2x^4}-\frac{b(b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{3/2}(-b-ac)^{5/2}}$$

input

```
Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

output

```
-1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*a*c*(c - d*x^2)
 + b*(2*c + d*x^2)))/(c*(b + a*c)^2*x^4) - (b*(b + 4*a*c)*d^2*ArcTan[(Sqrt
[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(3/2)*(-b
 - a*c)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2053, 2052, 25, 27, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$\Big\downarrow$ 2052

$$-bd\int -\frac{d(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 25

$$bd\int \frac{d(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 27

$$bd^2\int \frac{a-x^4}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 298

$$bd^2\left(\frac{(4ac+b)\int \frac{1}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c(ac+b)}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b)\left(ac+b-cx^4\right)^2}\right)$$

$\Big\downarrow$ 215

$$bd^2\left(\frac{(4ac+b)\left(\frac{\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}\right)}{4c(ac+b)}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b)\left(ac+b-cx^4\right)^2}\right)$$

$\Big\downarrow$ 221

$$bd^2\left(\frac{(4ac+b)\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}\right)}{4c(ac+b)}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b)\left(ac+b-cx^4\right)^2}\right)$$

input
```
Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
b*d^2*(-1/4*(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*(b + a*c)*(b + a*
c - c*x^4)^2) + ((b + 4*a*c)*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*(b
+ a*c)*(b + a*c - c*x^4)) + ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)])/Sqrt[b + a*c]]/(2*Sqrt[c]*(b + a*c)^(3/2))))/(4*c*(b + a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{(a d x^2+ac+b)(-2 a d x^2 c+b d x^2+2 a c^2+2bc)}{8(ac+b)^2 x^4 c\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}}+\frac{(4ac+b) b d^2 \ln\left(\frac{2 a c^2+2bc+(2acd+bd)x^2+2\sqrt{a c^2+bc}\sqrt{a c^2+bc+(2acd+bd)x^2+a d^2}}{x^2}\right.}{16(ac+b)^2 c\sqrt{a c^2+bc}\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}(d x^2+c)}$
default	$-\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}(d x^2+c)\left(12 a^2 d^3\sqrt{a d^2 x^4+2 a d x^2 c+b d x^2+a c^2+bc}\, x^6 c (a c^2+bc)^{\frac{3}{2}}-4\ln\left(\frac{2 a d x^2 c+b d x^2+2 a c^2+2\sqrt{a c^2+bc}\sqrt{a d^2 x}}{x^2}\right.\right.}{}$

input
```
int(1/x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*d*x^2+a*c+b)*(-2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c)/(a*c+b)^2/x^4/c/
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/16*(4*a*c+b)*b*d^2/(a*c+b)^2/c/(a*c^2+
b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+
b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(133) = 266$.

Time = 0.21 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.88

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{(4\,abc+b^2)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+}{x^4}\right.}{32\,(} \right.$$

$$-\frac{(4\,abc+b^2)\sqrt{-ac^2-bc}d^2x^4\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(2\,a^2c^5-(2\,a^2c^3+}{16\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)x}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c
 + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*
b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c
)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4
+ 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a
*b*c + b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a
*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^
2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*
a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 +
b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b
*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(133) = 266.

Time = 0.14 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.35

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(4\,abc+b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^2c^3+2\,abc^2+b^2c)\sqrt{(ac+b)c}}-\frac{(4\,abc^2+b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+3\,ab^2c-b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c+\frac{(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)(ad}{dx^2+c}\right.}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(4*a*b*c + b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b
)*c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2
 + b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 3*a
*b^2*c - b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^5 + 4*a^3*
b*c^4 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c + (a^2*c^5 + 2*a*b*c^4 + b^2*c
^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b
^2*c^3 + b^3*c^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(133) = 266$.

Time = 0.19 (sec) , antiderivative size = 778, normalized size of antiderivative = 5.08

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\frac{(4\,abcd^2+b^2d^2)\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{(a^2c^3+2\,abc^2+b^2c)\sqrt{-ac^2-bc}} - \frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^3c^4d}{}}{}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
-1/8*((4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3 + 2*a*
b*c^2 + b^2*c)*sqrt(-a*c^2 - b*c)) - (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*
d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*
d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))^2*a^(5/2)*c^3*d*abs(d) + 16*a^(5/2)*b*c^4*d*abs(d) + 28*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*
d^2 + 16*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))^2*a^(3/2)*b*c^2*d*abs(d) + 8*a^(3/2)*b^2*c^3*d*abs(d) + 4*(sqrt(a
*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c
*d^2 + 13*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^
2 + b*c))*a*b^2*c^2*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^
2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b^2*c*d*abs(d) + (sqrt(a*d^2)*x^2 -
sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + (sqrt(a
*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d
^2)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*
x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)^2))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^5*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^5*(a + b/(c + d*x^2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.56

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^4+2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^3d\,x^2-4\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{}$$

input
```
int(1/x^5/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**4 + 2*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**3*d*x**2 - 4*sqrt(c + d*x**2)*sqrt(a
*c + a*d*x**2 + b)*a*b*c**3 + sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*
b*c**2*d*x**2 - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*c**2 - sq
rt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*c*d*x**2 + 4*sqrt(c)*sqrt(a*c
 + b)*log( - sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d
*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*a*b*c*d**2*x**4 + sqrt(c)*sqrt(a*
c + b)*log( - sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c +
d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b)*b**2*d**2*x**4 - 4*sqrt(c)*sqrt(
a*c + b)*log(x)*a*b*c*d**2*x**4 - sqrt(c)*sqrt(a*c + b)*log(x)*b**2*d**2*x
**4)/(8*c**2*x**4*(a**3*c**3 + 3*a**2*b*c**2 + 3*a*b**2*c + b**3))
```

3.197 $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 403

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = &-\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{x^3(b+ac+adx^2)}{5ad\sqrt{a+\frac{b}{c+dx^2}}}\\ &+\frac{(8b^2+13abc+3a^2c^2)\,x(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\,\sqrt{a+\frac{b}{c+dx^2}}}\\ &-\frac{\sqrt{c}(8b^2+13abc+3a^2c^2)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^3d^{5/2}\,(c+dx^2)\,\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}\\ &+\frac{c^{3/2}(4b+3ac)\,(b+ac+adx^2)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15a^2d^{5/2}\,(c+dx^2)\,\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}\end{aligned}$$

output
```
-1/15*(3*a*c+4*b)*x*(a*d*x^2+a*c+b)/a^2/d^2/(a+b/(d*x^2+c))^(1/2)+1/5*x^3*
(a*d*x^2+a*c+b)/a/d/(a+b/(d*x^2+c))^(1/2)+1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*
x*(a*d*x^2+a*c+b)/a^3/d^2/(d*x^2+c)/(a+b/(d*x^2+c))^(1/2)-1/15*c^(1/2)*(3*
a^2*c^2+13*a*b*c+8*b^2)*(a*d*x^2+a*c+b)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x
^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^3/d^(5/2)/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a
*c+b)/(d*x^2+c))^(1/2)/(a+b/(d*x^2+c))^(1/2)+1/15*c^(3/2)*(3*a*c+4*b)*(a*d
*x^2+a*c+b)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/a
^2/d^(5/2)/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)/(a+b/(d*x
^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(4b^2+ab(7c+dx^2)+3a^2(c^2-d^2x^4)\right)+i(8b^3+21ab^2c+16a^2bc^2+3a^3\right.}{}$$

input
```
Integrate[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(4
*b^2 + a*b*(7*c + d*x^2) + 3*a^2*(c^2 - d^2*x^4)) + I*(8*b^3 + 21*a*b^2*c
+ 16*a^2*b*c^2 + 3*a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(8*b^2
+ 17*a*b*c + 9*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^3*c^2*(d/c)^
(5/2)*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2057, 2058, 380, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}} \, dx$$

$$\Big\downarrow \ 2057$$

$$\int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \ 2058$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^4\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 380$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\int \frac{x^2\left((4b+3ac)dx^2+3c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 444$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{d\left(\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}}{5ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int\frac{\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}}{5ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 406

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(3ac+\cdots}{3ad}}{5ad}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(3ac+4b)\cdots}{\cdots}}{3ad}}{5ad}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\cdots}{3ad}}{5ad}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\mathrm{arct}\cdots\right.}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(\cdots}{(a\cdots}}}\right.\cdots}{\cdots}}{5ad}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*((x^3*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(5
*a*d) - (((4*b + 3*a*c)*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a*d)
 - ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqr
t[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c +
a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*(4*b + 3*a*c)*Sqrt[b + a*c
 + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*
Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3
*a*d))/(5*a*d)))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 10.90 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.65

method	result
default	$-\frac{\left(-3\sqrt{-\frac{ad}{ac+b}}\,a^2d^3x^7-3\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^2x^5+\sqrt{-\frac{ad}{ac+b}}\,ab\,d^2x^5+3\sqrt{-\frac{ad}{ac+b}}\,a^2c^2d\,x^3+8\sqrt{-\frac{ad}{ac+b}}\,abcd\,x^3-3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2}{c}}\right.}{}$
risch	$-\frac{x\left(-3ad\,x^2+3ac+4b\right)\left(ad\,x^2+ac+b\right)}{15d^2a^2\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(-\frac{2d\left(3a^2c^2+13abc+8b^2\right)\left(a\,c^2+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{x^2d}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+}{dca}}\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}\,(2acd}\right.}{}$

input
```
int(x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7-3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^
2*x^5+(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5+3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^
3+8*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2
*c^3+3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x+4*(-a*d/(a*c+b))^(1/2)*b^2*d*x^3+6*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-13*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*
a*b*c^2+7*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))
*b^2*c-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c+4*(-a*d/(a*c+b))^(1/2)*b^2*
c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2/(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(3\,a^2c^3+13\,abc^2+8\,b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(3\,a^2c^3+13\,abc^2+8\,b^2c+(3\,a^2c^2+7\,\cdots}{\cdots}$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((3*a^2*c^3 + 13*a*b*c^2 + 8*b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(
arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*c^3 + 13*a*b*c^2 + 8*b^2*c
 + (3*a^2*c^2 + 7*a*b*c + 4*b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin
(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*d^3*x^6 - 4*a*b*d^2*x^4 + 3*a^2*
c^3 + 13*a*b*c^2 + (9*a*b*c + 8*b^2)*d*x^2 + 8*b^2*c)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^3*d^3*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**4/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**4/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^4/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4/(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{-3\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,acx+3\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,ad\,x^3-4\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,bx}{}$$

input
```
int(x^4/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c*x + 3*sqrt(c + d*x**2)
*sqrt(a*c + a*d*x**2 + b)*a*d*x**3 - 4*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**
2 + b)*b*x + 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/(a*c**
2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*a**2*c**2*d + 13*int((
sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a
*d**2*x**4 + b*c + b*d*x**2),x)*a*b*c*d + 8*int((sqrt(c + d*x**2)*sqrt(a*c
 + a*d*x**2 + b)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x*
*2),x)*b**2*d + 3*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**2
+ 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*a**2*c**3 + 7*int((sqrt(
c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
 + b*c + b*d*x**2),x)*a*b*c**2 + 4*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x*
*2 + b))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*b**2*c)
/(15*a**2*d**2)
```

3.198 $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 322

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{x(b+ac+adx^2)}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x(b+ac+adx^2)}{3a^2d\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{c}(2b+ac)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\,(c+dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3ad^{3/2}\,(c+dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

output

```
1/3*x*(a*d*x^2+a*c+b)/a/d/(a+b/(d*x^2+c))^(1/2)-1/3*(a*c+2*b)*x*(a*d*x^2+a
*c+b)/a^2/d/(d*x^2+c)/(a+b/(d*x^2+c))^(1/2)+1/3*c^(1/2)*(a*c+2*b)*(a*d*x^2
+a*c+b)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a
^2/d^(3/2)/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)/(a+b/(d*x
^2+c))^(1/2)-1/3*c^(3/2)*(a*d*x^2+a*c+b)*InverseJacobiAM(arctan(d^(1/2)*x/
c^(1/2)),(b/(a*c+b))^(1/2))/a/d^(3/2)/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a*c+b)
/(d*x^2+c))^(1/2)/(a+b/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b+a(c+dx^2)\right)+i(2b^2+3abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\right.\right.\right.}{3a^2d\sqrt{\frac{d}{c}}\left(b+a\left(c+dx\right.\right.}$$

input
```
Integrate[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b + a*(
c + d*x^2)) + I*(2*b^2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c
)] - (2*I)*b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a^2*d*Sqrt[d/
c]*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2057, 2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^2\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{380}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\overline{b+}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{3ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E}{ad^{3/2}\sqrt{c+dx}}\right.}{3ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*((x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a
*d) - ((2*b + a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) -
(Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a
*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*a*d)))/(Sqrt[c + d*x^2]*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2 x^5+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^3+\sqrt{-\frac{ad}{ac+b}}\, bd\, x^3-\sqrt{\frac{ad\, x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^2+c}{c}}\, \operatorname{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right) a\, c^2+\sqrt{-\frac{ad}{ac+b}}\, a\, c^2 \right.}{3d\sqrt{a\, d^2 x^4+2ad\, x^2 c}}$
risch	$\frac{x\left(ad\, x^2+ac+b\right)}{3da\sqrt{\frac{ad\, x^2+ac+b}{d\, x^2+c}}}-\frac{\left(\frac{a\, c^2\sqrt{1+\frac{ad\, x^2}{ac+b}}\, \sqrt{1+\frac{x^2 d}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^2 x^4+2ad\, x^2 c+bd\, x^2+a\, c^2+bc}}+\frac{bc\sqrt{1+\frac{ad\, x^2}{ac+b}}\, \sqrt{1+\frac{x^2 d}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-\right.}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^2 x^4+2ad\, x^2 c+bd\, x^2+a\, c}}\right.}{}$

input
```
int(x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d
/(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b
))^(1/2)*a*c^2*x+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-2*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*
c+b))^(1/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\, dx = \frac{(ac^2+2\,bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (ac^2+2\,bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{a}{}}{3\,a^2d^2x}$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*((a*c^2 + 2*b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x),
 (a*c + b)/(a*c)) - (a*c^2 + 2*b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*ell
iptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a*d^2*x^4 - 2*b*d*x^2 -
a*c^2 - 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^2/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2/(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x^2}{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}dx\right)acd-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x^2}{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}dx\right)bc}{3ad}$$

input
```
int(x^2/(a+b/(d*x^2+c))^(1/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x - int((sqrt(c + d*x**2)*sqrt(
a*c + a*d*x**2 + b)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d
*x**2),x)*a*c*d - 2*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/(
a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*b*d - int((sqrt(c
 + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
+ b*c + b*d*x**2),x)*a*c**2 - int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 +
b))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c + b*d*x**2),x)*b*c)/(3*a*d)
```

3.199 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 225

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+ac}\,E\left(\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{b+ac}}\right)|-\frac{b}{ac}\right)}{\sqrt{a}\sqrt{d}\sqrt{\frac{(b+ac)(c+dx^2)}{c(b+ac+adx^2)}}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+ac}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{b+ac}}\right),-\frac{b}{ac}\right)}{\sqrt{a}\sqrt{d}\sqrt{\frac{(b+ac)(c+dx^2)}{c(b+ac+adx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

output
```
x/(a+b/(d*x^2+c))^(1/2)-(a*c+b)^(1/2)*EllipticE(a^(1/2)*d^(1/2)*x/(a*c+b)^
(1/2)/(1+a*d*x^2/(a*c+b))^(1/2),(-b/a/c)^(1/2))/a^(1/2)/d^(1/2)/((a*c+b)*(
d*x^2+c)/c/(a*d*x^2+a*c+b))^(1/2)/(a+b/(d*x^2+c))^(1/2)+(a*c+b)^(1/2)*Inve
rseJacobiAM(arctan(a^(1/2)*d^(1/2)*x/(a*c+b)^(1/2)),(-b/a/c)^(1/2))/a^(1/2
)/d^(1/2)/((a*c+b)*(d*x^2+c)/c/(a*d*x^2+a*c+b))^(1/2)/(a+b/(d*x^2+c))^(1/2
)
```

Mathematica [A] (verified)

Time = 9.70 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\sqrt{\frac{b+ac+adx^2}{b+ac}}E\left(\arcsin\left(\sqrt{-\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)}{\sqrt{-\frac{ad}{b+ac}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{1+\frac{dx^2}{c}}}$$

input
```
Integrate[1/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*EllipticE[ArcSin[Sqrt[-((a*d)/(b + a*
c))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)]*Sqrt[1 + (d*x^2)/c])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2057, 2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{\sqrt{ac+adx^2+b}\left(c\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(d\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x
^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d]*Sqrt[c + d*x^
2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(Sqrt[c + d*x^
2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,c\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{\sqrt{a\,d^{2}x^{4}+2ad\,x^{2}c+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^{2}+c\right)\left(ad\,x^{2}+ac+b\right)}}$	164

input
```
int(1/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)/c)^(1/2)*
((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*c*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx =$$
$$-\frac{\sqrt{a}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - \sqrt{a}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (dx^2+c)\sqrt{\frac{adx^2+a}{dx^2+}}}{adx}$$

input
```
integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c))
 - sqrt(a)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)
/(a*c)) - (d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b/(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(1/(a + b/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{ad\,x^2+ac+b}dx$$

input
```
int(1/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c + a*d*x**2 + b),x)
```

3.200 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{a}\sqrt{d}E\left(\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{b+ac}}\right)|-\frac{b}{ac}\right)}{\sqrt{b+ac}\sqrt{\frac{(b+ac)(c+dx^2)}{c(b+ac+adx^2)}}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+ac}\sqrt{d}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{b+ac}}\right),-\frac{b}{ac}\right)}{\sqrt{ac}\sqrt{\frac{(b+ac)(c+dx^2)}{c(b+ac+adx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

output
```
-1/x/(a+b/(d*x^2+c))^(1/2)-a^(1/2)*d^(1/2)*EllipticE(a^(1/2)*d^(1/2)*x/(a*
c+b)^(1/2)/(1+a*d*x^2/(a*c+b))^(1/2),(-b/a/c)^(1/2))/(a*c+b)^(1/2)/((a*c+b
)*(d*x^2+c)/c/(a*d*x^2+a*c+b))^(1/2)/(a+b/(d*x^2+c))^(1/2)+(a*c+b)^(1/2)*d
^(1/2)*InverseJacobiAM(arctan(a^(1/2)*d^(1/2)*x/(a*c+b)^(1/2)),(-b/a/c)^(1
/2))/a^(1/2)/c/((a*c+b)*(d*x^2+c)/c/(a*d*x^2+a*c+b))^(1/2)/(a+b/(d*x^2+c))
^(1/2)
```

Mathematica [A] (verified)

Time = 12.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x} + \frac{d\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)\mid\frac{ac}{b+ac}\right)}{(b+ac)\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
-(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x)) + (d*
Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin
[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/((b + a*c)*Sqrt[-(d/c)]*Sqrt[(b + a*c
+ a*d*x^2)/(b + a*c)])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\sqrt{dx^2+c}}{x^2\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{d\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\int \frac{\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left((ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(ad\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{(dx^2+c)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{ac+b}-\sqrt{\cdots}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/((b
+ a*c)*x)) + (d*(a*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) -
(Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))])) + (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a
*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(b + a*c)))/(Sqrt[c + d*x^2]*Sq
rt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 9.18 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^{2} x^{4}-adc\sqrt{\frac{a d\, x^{2}+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, x\, \operatorname{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^{2}-\sqrt{\frac{a d\, x^{2}+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, \operatorname{E}\right.}{\sqrt{a\, d^{2} x^{4}+2ad\, x^{2} c+bd\, x^{2}+a\, c^{2}+bc}\, \sqrt{-\frac{ad}{ac+b}}\, x(ac+}$
risch	$-\frac{a d\, x^{2}+ac+b}{(ac+b) x \sqrt{\frac{a d\, x^{2}+ac+b}{d\, x^{2}+c}}}+\frac{d\left(\frac{b\sqrt{1+\frac{a d\, x^{2}}{ac+b}}\, \sqrt{1+\frac{x^{2} d}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2ad\, x^{2} c+bd\, x^{2}+a\, c^{2}+bc}}+\frac{ac\sqrt{1+\frac{a d\, x^{2}}{ac+b}}\, \sqrt{1+\frac{x^{2} d}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{a}{ac-}}\right.}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2ad\, x^{2} c+bd\, x}}\right.}{}$

input
```
int(1/x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2
*(-a*d/(a*c+b))^(1/2)*a*c*d*x^2-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*d*x+(-a*
d/(a*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b))^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b
*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/(a*c+b)/((d*x^2+c)*(a*d*x^2+a
*c+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{a^2c\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E\left(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}\right) - (a^2cd^2+(a^2c^2+2\,abc+b^2)d)\sqrt{-\frac{ad}{ac+b}}x\sqrt{\frac{ac^2+bc}{d^2}}}{(a^3c^3+2\,a^2bc^2+ab^2c)x}$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
(a^2*c*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsi
n(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*c*d^2 + (a^2*c^2 + 2*a*
b*c + b^2)*d)*sqrt(-a*d/(a*c + b))*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(ar
csin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*c^3 + a*b*c^2 + (a^2
*c^2 + a*b*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^3 + 2*
a^2*b*c^2 + a*b^2*c)*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^2\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}}{ad\,x^4+ac\,x^2+b\,x^2}dx$$

input
```
int(1/x^2/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a*c*x**2 + a*d*x**4 + b*x
**2),x)
```

3.201 $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 336

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx =& -\frac{b+ac+adx^2}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(b-ac)d(b+ac+adx^2)}{3(b+ac)^2x\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}\\ &-\frac{(b-ac)d^{3/2}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}\\ &-\frac{a\sqrt{c}d^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}\end{aligned}$$

output

```
-1/3*(a*d*x^2+a*c+b)/(a*c+b)/x^3/(a+b/(d*x^2+c))^(1/2)-1/3*(-a*c+b)*d*(a*d
*x^2+a*c+b)/(a*c+b)^2/x/(d*x^2+c)/(a+b/(d*x^2+c))^(1/2)-1/3*(-a*c+b)*d^(3/
2)*(a*d*x^2+a*c+b)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))/c^(1/2)/(a*c+b)^2/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c)
)^(1/2)/(a+b/(d*x^2+c))^(1/2)-1/3*a*c^(1/2)*d^(3/2)*(a*d*x^2+a*c+b)*Invers
eJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/(a*c+b)^2/(d*x^2+c)
/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)/(a+b/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx =$$

$$-\frac{\sqrt{\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c+dx^2)+a^2c(c^2-d^2x^4)+ab(2c^2+cdx^2+d^2x^4)\right)+i(b^2-a^2c^2)\right.}{3(b}$$

input
```
Integrate[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
-1/3*(Sqrt[d/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^
2)*(b^2*(c + d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + c*d*x^2 + d^2*x
^4)) + I*(b^2 - a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[
1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(b
 + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/((b + a*c)^2*d*x^3*(b +
 a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2058, 377, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\sqrt{dx^2+c}}{x^4\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{d\left(-adx^2+b-ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\int \frac{-adx^2+b-ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{\int \frac{ad\left(c(b+ac)-(b-ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(c(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3(ac+b)}-\frac{\sqrt{c+dx^2}\ldots}{3x^{\ldots}}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\ldots\right.}{3(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}}{cx(a\ldots}\right.}{3(ac+b)}\ldots}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\ldots\right.}{c(ac+b)}\ldots\right.}{3(ac+b)}\ldots}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-1/3*(Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(
(b + a*c)*x^3) + (d*(-(((b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])
/(c*(b + a*c)*x)) - (a*d*(-((b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*
Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c
 + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d
*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(c*(b + a*c
))))/(3*(b + a*c))))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 10.52 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{(a d x^2+ac+b)(-a d x^2 c+b d x^2+a c^2+bc)}{3(ac+b)^2 x^3 c\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}}-\frac{d^2 a\left(\frac{a c^2\sqrt{1+\frac{a d x^2}{ac+b}}\sqrt{1+\frac{x^2 d}{c}}\operatorname{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\sqrt{a d^2 x^4+2ad x^2 c+bd x^2+a c^2+bc}}+\frac{bc\sqrt{1+\frac{a d x^2}{ac+b}}\sqrt{}}{\sqrt{-\frac{a}{ac}}}\right.}{}$
default	$-\frac{\left(-\sqrt{-\frac{ad}{ac+b}}\,a^2 c d^3 x^6+\sqrt{-\frac{ad}{ac+b}}\,ab d^3 x^6+\sqrt{\frac{a d x^2+ac+b}{ac+b}}\sqrt{\frac{d x^2+c}{c}}\operatorname{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^2 c^2 d^2 x^3-\sqrt{-\frac{ad}{ac+b}}\,a^2 c^2 d^2 x\right.}{}$

input
```
int(1/x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*d*x^2+a*c+b)*(-a*c*d*x^2+b*d*x^2+a*c^2+b*c)/(a*c+b)^2/x^3/c/((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*d^2*a/(a*c+b)^2/c*(a*c^2/(-a*d/(a*c+b))^(1
/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)
/d/c/a)^(1/2))+b*c/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x
^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-
a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*d*(a*c-b)*(a*c^2+b*c)
/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a
*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b
))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx =$$

$$-\frac{(a^2c-ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-((a^2c-ab)d^3+(a^2c^2+2\,abc+b^2)d^2)}{3\,(a^3c^4}$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((a^2*c - a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)*e
lliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c - a*b
)*d^3 + (a^2*c^2 + 2*a*b*c + b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt((a*c^
2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c))
- ((a^2*c^2 - b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c^2 - 2*(a*b*c^2 +
b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^4 + 3*a^2*b*c
^3 + 3*a*b^2*c^2 + b^3*c)*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**4/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^4\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^4*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,d+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,x^2}{a^2c\,d^2x^4+ab\,d^2x^4+2a^2c^2d\,x^2+3abcd\,x^2+a^2c^3+b^2d\,x^2+2ab\,c^2+b^2c}dx\right)a^2c\,d^3x+\left(\int\right.}{}$$

input
```
int(1/x^4/(a+b/(d*x^2+c))^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*d + int((sqrt(c + d*x**2)*sq
rt(a*c + a*d*x**2 + b)*x**2)/(a**2*c**3 + 2*a**2*c**2*d*x**2 + a**2*c*d**2
*x**4 + 2*a*b*c**2 + 3*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c + b**2*d*x**2
),x)*a**2*c*d**3*x + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**2)/
(a**2*c**3 + 2*a**2*c**2*d*x**2 + a**2*c*d**2*x**4 + 2*a*b*c**2 + 3*a*b*c*
d*x**2 + a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*a*b*d**3*x + int((sqrt(c
 + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a**2*c**3*x**4 + 2*a**2*c**2*d*x**6
+ a**2*c*d**2*x**8 + 2*a*b*c**2*x**4 + 3*a*b*c*d*x**6 + a*b*d**2*x**8 + b*
*2*c*x**4 + b**2*d*x**6),x)*a**2*c**4*x + 2*int((sqrt(c + d*x**2)*sqrt(a*c
 + a*d*x**2 + b))/(a**2*c**3*x**4 + 2*a**2*c**2*d*x**6 + a**2*c*d**2*x**8
+ 2*a*b*c**2*x**4 + 3*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c*x**4 + b**2*d*
x**6),x)*a*b*c**3*x + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a**
2*c**3*x**4 + 2*a**2*c**2*d*x**6 + a**2*c*d**2*x**8 + 2*a*b*c**2*x**4 + 3*
a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c**2*x)/
(c*x*(a*c + b))
```

3.202 $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= \frac{b(b+ac)^2}{a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} \\ &+ \frac{(19b^2+28abc+8a^2c^2)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^4d^3} \\ &- \frac{(11b+12ac)\left(c+dx^2\right)^2\sqrt{a+\frac{b}{c+dx^2}}}{24a^3d^3} + \frac{\left(c+dx^2\right)^3\sqrt{a+\frac{b}{c+dx^2}}}{6a^2d^3} \\ &- \frac{b(35b^2+60abc+24a^2c^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}\end{aligned}$$

output

```
b*(a*c+b)^2/a^4/d^3/(a+b/(d*x^2+c))^(1/2)+1/16*(8*a^2*c^2+28*a*b*c+19*b^2)
*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a^4/d^3-1/24*(12*a*c+11*b)*(d*x^2+c)^2*(a
+b/(d*x^2+c))^(1/2)/a^3/d^3+1/6*(d*x^2+c)^3*(a+b/(d*x^2+c))^(1/2)/a^2/d^3-
1/16*b*(24*a^2*c^2+60*a*b*c+35*b^2)*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))
/a^(9/2)/d^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(105b^3+5ab^2(43c+7dx^2)+2a^2b(59c^2+16cdx^2-7d^2x^4)+8a^3(c^3+d^3x^6)\right)}{b+a(c+dx^2)} - 3b(35b^2}{48a^{9/2}d^3}$$

input
```
Integrate[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(105*b^3 + 5*a
*b^2*(43*c + 7*d*x^2) + 2*a^2*b*(59*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^3*
(c^3 + d^3*x^6)))/(b + a*(c + d*x^2)) - 3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^
2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(9/2)*d^3
)
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2053, 2052, 27, 365, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 2057$$

$$\int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{x^4}{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$\Big\downarrow$ 2052

$$-bd\int \frac{\left(-cx^4+b+ac\right)^2}{d^4x^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 27

$$-\frac{b\int \frac{\left(-cx^4+b+ac\right)^2}{x^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$\Big\downarrow$ 365

$$-\frac{b\left(\frac{\int \frac{ac^2x^4+(b+ac)(7b+5ac)}{\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{a}-\frac{(ac+b)^2}{ax^2\left(a-x^4\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 298

$$-\frac{b\left(\frac{\frac{\left(24a^2c^2+60abc+35b^2\right)\int \frac{1}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}+\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}}{a}-\frac{(ac+b)^2}{ax^2\left(a-x^4\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 215

$$-\frac{b\left(\frac{\frac{\left(24a^2c^2+60abc+35b^2\right)\left(\frac{3\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}\right)}{6a}+\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}}{a}-\frac{(ac+b)^2}{ax^2\left(a-x^4\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 215

$$-\frac{b\left(\frac{\frac{\left(24a^2c^2+60abc+35b^2\right)\left(\frac{3\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}\right)}{6a}+\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}}{a}-\frac{(ac+b)^2}{ax^2(a-x^4)^3}\right)}{d^3}$$

$\downarrow$ 219

$$-\frac{b\left(\frac{\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}+\frac{\left(24a^2c^2+60abc+35b^2\right)\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}\right)}{6a}}{a}-\frac{(ac+b)^2}{ax^2(a-x^4}\right.}{d^3}$$

input

```
Int[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-((b*(-((b + a*c)^2/(a*x^2*(a - x^4)^3)) + (((7*b^2 + 12*a*b*c + 6*a^2*c^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(6*a*(a - x^4)^3) + ((35*b^2 + 60
*a*b*c + 24*a^2*c^2)*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(4*a*(a - x^4)
^2) + (3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*a*(a - x^4)) + ArcTanh[
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/(2*a^(3/2))))/(4*a)))/(6*a)
)/a))/d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.36

method	result
risch	$\frac{(8a^2d^2x^4-8a^2cd\,x^2-22abd\,x^2+8a^2c^2+62abc+57b^2)(ad\,x^2+ac+b)}{48d^3a^4\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{b\left(\frac{(24a^2c^2+60abc+35b^2)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2}\right.}{2\sqrt{a\,d^2}}\right.}{16a}$
default	Expression too large to display

input
```
int(x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2-22*a*b*d*x^2+8*a^2*c^2+62*a*b*c+57*b
^2)*(a*d*x^2+a*c+b)/a^4/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*b/a^4/d^2*(
1/2*(24*a^2*c^2+60*a*b*c+35*b^2)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2
)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-16*(a^2*c^2
+2*a*b*c+b^2)*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))
/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^
2+c)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 675, normalized size of antiderivative = 3.04

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(24\,a^3bc^3+84\,a^2b^2c^2+95\,ab^3c+35\,b^4+(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3)dx^2)\sqrt{a}\,\mathrm{l}}{}\right.$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b
*c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c
^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b
)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) +
4*(8*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c
^3 + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (
8*a^4*c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(2
4*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^
2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2
*(8*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^
3 + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8
*a^4*c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]
```

Sympy [F]

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.75

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{48\,a^5bc^2+96\,a^4b^2c+48\,a^3b^3-\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)\,(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{8\,(24\,a^3bc^2+60\,a^2b^2c+}{(dx^2}}{48\left(a^7d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-3\,a^6d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,a^5d^3\left(\frac{adx^2+}{dx^2}\right.\right.} + \frac{(24\,a^2c^2+60\,abc+35\,b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{9}{2}}d^3}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
1/48*(48*a^5*b*c^2 + 96*a^4*b^2*c + 48*a^3*b^3 - 3*(24*a^2*b*c^2 + 60*a*b^
2*c + 35*b^3)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 8*(24*a^3*b*c^2 + 60*a
^2*b^2*c + 35*a*b^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(56*a^4*b*c^2
 + 132*a^3*b^2*c + 77*a^2*b^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^7*d^3*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 3*a^6*d^3*((a*d*x^2 + a*c + b)/(d*x
^2 + c))^(3/2) + 3*a^5*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - a^4*d
^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(7/2)) + 1/32*(24*a^2*c^2 + 60*a*b*c
+ 35*b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a
) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(9/2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(200) = 400.

Time = 0.25 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.99

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(
a^2*d*sgn(d*x^2 + c)) - (4*a^11*c*d^6*sgn(d*x^2 + c) + 11*a^10*b*d^6*sgn(d
*x^2 + c))/(a^13*d^8)) + (8*a^11*c^2*d^5*sgn(d*x^2 + c) + 62*a^10*b*c*d^5*
sgn(d*x^2 + c) + 57*a^9*b^2*d^5*sgn(d*x^2 + c))/(a^13*d^8)) + 1/96*(24*a^2
*b*c^2 + 60*a*b^2*c + 35*b^3)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x^2 - s
qrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(5/2)*c^2*abs(d) +
 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))^2*a^2*c*d + 5*a^2*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c
*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^(3/2)*abs(d) + 10*(sqrt(a*d^2)*x^2 -
sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(3/2)*b*c*abs(d)
+ 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^2*a*b*d + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*
x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*b^2*abs(d) + b^3*d))/(a^(9/2)*d^2*ab
s(d)*sgn(d*x^2 + c)) + 1/96*(24*a^(13/2)*b*c^2*d^3*abs(d)*sgn(d*x^2 + c) +
 60*a^(11/2)*b^2*c*d^3*abs(d)*sgn(d*x^2 + c) + 35*a^(9/2)*b^3*d^3*abs(d)*s
gn(d*x^2 + c))*log(abs(a))/(a^9*d^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^5}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^5/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5/(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 538, normalized size of antiderivative = 2.42

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^4c^3+8\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^4d^3x^6+118\sqrt{d\,x^2}}{}$$

input
```
int(x^5/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**4*c**3 + 8*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*a**4*d**3*x**6 + 118*sqrt(c + d*x**2)*sqrt(a*c
+ a*d*x**2 + b)*a**3*b*c**2 + 32*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)
*a**3*b*c*d*x**2 - 14*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*b*d**
2*x**4 + 215*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b**2*c + 35*sq
rt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b**2*d*x**2 + 105*sqrt(c + d*
x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**3 + 72*sqrt(a)*log( - sqrt(a)*sqrt(a*c
 + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a**3*b*c**3 + 72*sqrt(a)*log( - sqr
t(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a**3*b*c**2*d*x**2 + 2
52*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a
**2*b**2*c**2 + 180*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt
(c + d*x**2)*a)*a**2*b**2*c*d*x**2 + 285*sqrt(a)*log( - sqrt(a)*sqrt(a*c +
 a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a*b**3*c + 105*sqrt(a)*log( - sqrt(a)
*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a*b**3*d*x**2 + 105*sqrt(a
)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*b**4)/(48*
a**5*d**3*(a*c + a*d*x**2 + b))
```

3.203 $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx = -\frac{b(b+ac)}{a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(7b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8a^3d^2} + \frac{\left(c+dx^2\right)^2\sqrt{a+\frac{b}{c+dx^2}}}{4a^2d^2} + \frac{3b(5b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}$$

output
```
-b*(a*c+b)/a^3/d^2/(a+b/(d*x^2+c))^(1/2)-1/8*(4*a*c+7*b)*(d*x^2+c)*(a+b/(d
*x^2+c))^(1/2)/a^3/d^2+1/4*(d*x^2+c)^2*(a+b/(d*x^2+c))^(1/2)/a^2/d^2+3/8*b
*(4*a*c+5*b)*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(7/2)/d^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(15b^2+ab(17c+5dx^2)+2a^2(c^2-d^2x^4)\right)}{b+a(c+dx^2)}+3b(5b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}$$

input
```
Integrate[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(-((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + a*
b*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4)))/(b + a*(c + d*x^2))) + 3*b*(5
*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(
7/2)*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2057, 2053, 2052, 25, 27, 361, 25, 27, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{x^2}{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{-cx^4+b+ac}{d^3x^4\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{-cx^4+b+ac}{d^3x^4\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{-cx^4+b+ac}{x^4(a-x^4)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{b\left(\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}-\frac{1}{4}\int -\frac{3bx^4+4a(b+ac)}{a^2x^4(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{1}{4}\int \frac{3bx^4+4a(b+ac)}{a^2x^4(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\int \frac{3bx^4+4a(b+ac)}{x^4\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{b\left(\frac{\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}-\frac{1}{2}\int -\frac{\left(\frac{7b}{a}+4c\right)x^4+8(b+ac)}{x^4\left(a-x^4\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\left(\frac{7b}{a}+4c\right)x^4+8(b+ac)}{x^4\left(a-x^4\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 359$$

$$\frac{b\left(\frac{\frac{1}{2}\left(\frac{3(4ac+5b)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{a}-\frac{8(ac+b)}{ax^2}\right)+\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}+\frac{\frac{1}{2}\left(\frac{3(4ac+5b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{8(ac+b)}{ax^2}\right)+\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}}{4a^2}\right)}{d^2}$$

input
```
Int[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a^2*(a - x^4)^2) + (((7*b
 + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*a*(a - x^4)) + ((-8*(b
 + a*c))/(a*x^2) + (3*(5*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)]/Sqrt[a]])/a^(3/2))/2)/(4*a^2)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{(-2adx^2+2ac+7b)(adx^2+ac+b)}{8d^2a^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}+\frac{b\left(\frac{(12ac+15b)\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{ac^2+bc+(2acd+bd)x^2+ad^2x^4}\right)}{2\sqrt{ad^2}}-\frac{8(ac+b)(dx^2}{d\sqrt{ad^2x^4+2adx^2c+b}}\right.}{8a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}(dx^2+c)}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}(dx^2+c)\left(-4\sqrt{ad^2x^4+2adx^2c+bdx^2+ac^2+bc}\sqrt{ad^2}a^2d^2x^4-12\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2adx^2c+bdx^2+ac^2}}{2\sqrt{ad^2}}\right.\right.}{}$

input
```
int(x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c+7*b)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)+1/8*b/a^3/d*(1/2*(12*a*c+15*b)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a
*d^2)^(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-8
*(a*c+b)*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.49

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(4\,a^2bc^2+9\,ab^2c+(4\,a^2bc+5\,ab^2)dx^2+5\,b^3)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a\right.}{}\right.$$

$$-\frac{3\,(4\,a^2bc^2+9\,ab^2c+(4\,a^2bc+5\,ab^2)dx^2+5\,b^3)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(2\,a^3d^3x^6}{16\,(a^5d^3x^2+(a^5c}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*s
qrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c +
 b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d
^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c +
 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a
^5*c + a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b
^2)*d*x^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^3*
d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^
2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]
```

Sympy [F]

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.69

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{8\,a^3bc+8\,a^2b^2+\frac{3\,(adx^2+ac+b)^2(4\,abc+5\,b^2)}{(dx^2+c)^2}-\frac{5\,(4\,a^2bc+5\,ab^2)(adx^2+ac+b)}{dx^2+c}}{8\left(a^5d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-2\,a^4d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+a^3d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}\right)} - \frac{3\,(4\,ac+5\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{7}{2}}d^2}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(8*a^3*b*c + 8*a^2*b^2 + 3*(a*d*x^2 + a*c + b)^2*(4*a*b*c + 5*b^2)/(d
*x^2 + c)^2 - 5*(4*a^2*b*c + 5*a*b^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^
5*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 2*a^4*d^2*((a*d*x^2 + a*c +
b)/(d*x^2 + c))^(3/2) + a^3*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2)) -
 3/16*(4*a*c + 5*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(137) = 274$.

Time = 0.23 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.56

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{a^2d\mathrm{sgn}\,(dx^2+c)}-\frac{2\,a^6cd^2+7\,a^5bd^2}{a^8d^4\mathrm{sgn}\,(dx^2+c)}\right)$$
$$-\frac{(4\,abc+5\,b^2)\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{5}{2}}c^2|d|+6\left(\sqrt{ad^2}x^2-\right.\right.\right.}{}$$
$$-\frac{\left(4\,a^{\frac{9}{2}}bcd^2|d|\mathrm{sgn}(dx^2+c)+5\,a^{\frac{7}{2}}b^2d^2|d|\mathrm{sgn}(dx^2+c)\right)\log\left(|a|\right)}{16\,a^7d^5}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sg
n(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/
16*(4*a*b*c + 5*b^2)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(5/2)*c^2*abs(d) + 6*(sqrt(
a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^2*
c*d + 5*a^2*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
b*d*x^2 + a*c^2 + b*c))^3*a^(3/2)*abs(d) + 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*b
*d + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*sqrt(a)*b^2*abs(d) + b^3*d))/(a^(7/2)*d*abs(d)*sgn(d*
x^2 + c)) - 1/16*(4*a^(9/2)*b*c*d^2*abs(d)*sgn(d*x^2 + c) + 5*a^(7/2)*b^2*
d^2*abs(d)*sgn(d*x^2 + c))*log(abs(a))/(a^7*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3/(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.26

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{-2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3c^2+2\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^3d^2x^4-17\sqrt{d\,x^2}}{}$$

input
```
int(x^3/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**2 + 2*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**3*d**2*x**4 - 17*sqrt(c + d*x**2)*sqrt(a*
c + a*d*x**2 + b)*a**2*b*c - 5*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a
**2*b*d*x**2 - 15*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2 + 12*sq
rt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a**2*b*c*
*2 + 12*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)
*a**2*b*c*d*x**2 + 27*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(
c + d*x**2)*a)*a*b**2*c + 15*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b)
+ sqrt(c + d*x**2)*a)*a*b**2*d*x**2 + 15*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*
d*x**2 + b) + sqrt(c + d*x**2)*a)*b**3)/(8*a**4*d**2*(a*c + a*d*x**2 + b))
```

3.204 $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx = \frac{b}{a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d}-\frac{3b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

output
```
b/a^2/d/(a+b/(d*x^2+c))^(1/2)+1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a^2/d-3/
2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx = \frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}(3b+a(c+dx^2))}{2a^2d\,(b+a\,(c+dx^2))}-\frac{3b\text{arctanh}\left(\frac{\sqrt{\frac{b+a\left(c+dx^2\right)}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

input
```
Integrate[x/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)]*(3*b + a*(c + d*x^2)))/
(2*a^2*d*(b + a*(c + d*x^2))) - (3*b*ArcTanh[Sqrt[(b + a*(c + d*x^2))/(c +
 d*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2024, 773, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{3/2}}d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int \frac{(dx^2+c)^2}{\left(a+\frac{b}{dx^2+c}\right)^{3/2}}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{3b\int \frac{dx^2+c}{\left(a+\frac{b}{dx^2+c}\right)^{3/2}}d\frac{1}{dx^2+c}}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{-\frac{3b\left(\frac{\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}}{a}+\frac{2}{a\sqrt{a+\frac{b}{c+dx^2}}}\right)}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{b\left(dx^2+c\right)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{c+dx^2}}}\right)}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{c+dx^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

input
```
Int[x/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-1/2*(-((c + d*x^2)/(a*Sqrt[a + b/(c + d*x^2)])) - (3*b*(2/(a*Sqrt[a + b/(
c + d*x^2)]) - (2*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/a^(3/2)))/(2*a
))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(83) = 166$.

Time = 0.46 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.24

method	result
risch	$\frac{a d x^2+a c+b}{2 d a^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}-\frac{b \left(\frac{3 \ln \left(\frac{a c d+\frac{1}{2} b d+a d^2 x^2}{\sqrt{a d^2}}+\sqrt{a c^2+b c+(2 a c d+b d) x^2+a d^2 x^4}\right)}{2 \sqrt{a d^2}}-\frac{2 \left(d x^2+c\right)}{d \sqrt{a d^2 x^4+2 a d x^2 c+b d x^2+a c^2+b c}}\right.}{2 a^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}} \left(d x^2+c\right)}$
derivativedivides	$-\frac{\sqrt{\frac{\left(d x^2+c\right) a+b}{d x^2+c}} \left(d x^2+c\right) \left(-6 \sqrt{(d x^2+c)((d x^2+c) a+b)} a^{\frac{5}{2}} \left(d x^2+c\right)^2+3 \ln \left(\frac{2 \sqrt{\left(d x^2+c\right) \left(\left(d x^2+c\right) a+b\right)} \sqrt{a}+2 \left(d x^2+\right.}{2 \sqrt{a}}\right.\right.}{}$
default	$\frac{\sqrt{\frac{a d x^2+a c+b}{d x^2+c}} \left(d x^2+c\right) \left(-3 \ln \left(\frac{2 a d^2 x^2+2 a c d+2 \sqrt{a d^2 x^4+2 a d x^2 c+b d x^2+a c^2+b c} \sqrt{a d^2}+b d}{2 \sqrt{a d^2}}\right) a b d^2 x^2+2 \sqrt{a d^2 x^4+2 a d x^2}\right.}{}$

input
```
int(x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(a*d*x^2+a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/2*b/a^2*(3/2
*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a
*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-2*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+
a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(83) = 166$.

Time = 0.16 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.07

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left(abdx^2+abc+b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-\right.}{}\right.$$

input
```
integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 +
8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x
^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^
2*d^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2
 + a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x
^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]
```

Sympy [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.66

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{2\,ab-\frac{3\,(adx^2+ac+b)b}{dx^2+c}}{2\left(a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-a^2d\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}\right)}+\frac{3\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{5}{2}}d}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a*b - 3*(a*d*x^2 + a*c + b)*b/(d*x^2 + c))/(a^3*d*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)) - a^2*d*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2)) + 3/
4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\left(\frac{a\,(d\,x^2+c)}{b}+1\right)^{3/2}\,(d\,x^2+c)\;{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,(d\,x^2+c)}{b}\right)}{5\,d\,\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}$$

input
```
int(x/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
(((a*(c + d*x^2))/b + 1)^(3/2)*(c + d*x^2)*hypergeom([3/2, 5/2], 7/2, -(a*
(c + d*x^2))/b))/(5*d*(a + b/(c + d*x^2))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.13

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c+\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2d\,x^2+3\sqrt{d\,x^2+c}\,\sqrt{}}{}$$

input
```
int(x/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c + sqrt(c + d*x**2)*sqrt(
a*c + a*d*x**2 + b)*a**2*d*x**2 + 3*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 +
 b)*a*b + 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x
**2)*a)*a*b*c + 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c
 + d*x**2)*a)*a*b*d*x**2 + 3*sqrt(a)*log( - sqrt(a)*sqrt(a*c + a*d*x**2 +
b) + sqrt(c + d*x**2)*a)*b**2)/(2*a**3*d*(a*c + a*d*x**2 + b))
```

3.205 $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx = -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{(b+ac)^{3/2}}$$

output
```
-b/a/(a*c+b)/(a+b/(d*x^2+c))^(1/2)+arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/
a^(3/2)-c^(3/2)*arctanh(c^(1/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(a*c+
b)^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=-\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - (c^(3/2)*ArcTan
[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*
c)^(3/2) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2)
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {2057, 2053, 2052, 25, 27, 382, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$\Big\downarrow$ 2052

$$-bd\int -\frac{1}{dx^4\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 25

$$bd\int \frac{1}{dx^4\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 27

$$b\int \frac{1}{x^4\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 382

$$b\left(\frac{\int \frac{-cx^4+b+2ac}{(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

$\Big\downarrow$ 397

$$b\left(\frac{\frac{(ac+b)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{ac^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

$\Big\downarrow$ 219

$$b\left(\frac{\frac{(ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{ac^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

$\Big\downarrow$ 221

$$b\left(\frac{\frac{(ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{ac^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{ac+b}}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

input
```
Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
b*(-(1/(a*(b + a*c)*x^2)) + (((b + a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(
c + d*x^2)]/Sqrt[a]])/(Sqrt[a]*b) - (a*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(b +
a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(b*Sqrt[b + a*c]))/(a*(b + a*
c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(92) = 184$.

Time = 0.13 (sec) , antiderivative size = 1015, normalized size of antiderivative = 9.23

method	result	size
default	Expression too large to display	1015

input
```
int(1/x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a*(-ln(1/2*(2*a*d^2*x^2+2
*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d
)/(a*d^2)^(1/2))*a^3*c^2*d^2*x^2+(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*d
*x^2*c+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*c*d*x^2-2*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^
(1/2))*a^2*b*c*d^2*x^2-ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*c^3*d-ln(
1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*d^2*x^2+(a*d^2)^(1/2)*(a*c^2+b*c)
^(1/2)*ln((2*a*d*x^2*c+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*c^2-3*ln(1/2*(2*a*d^2*x^2
+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b
*d)/(a*d^2)^(1/2))*a^2*b*c^2*d+(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*d*x
^2*c+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b*c-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a
*b^2*c*d+2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c-ln(1/2*(2
*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^
2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d+2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(92) = 184$.

Time = 0.21 (sec) , antiderivative size = 1477, normalized size of antiderivative = 13.43

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^
2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x
^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(
d*x^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8
*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8
*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^
2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2
*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) -
 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2
*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a
*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (
a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b
*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^
4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 +
a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b
^2 + (a^4*c + a^3*b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt
(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a...
```

Sympy [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(92) = 184$.

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.83

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{c^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}(ac+b)} - \frac{b}{(a^2c+ab)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}} - \frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\,a^{\frac{3}{2}}}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*c^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/
(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c
+ b)*c)*(a*c + b)) - b/((a^2*c + a*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
)) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) +
 sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a^(3/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x^2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 627, normalized size of antiderivative = 5.70

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2bc-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a\,b^2+\sqrt{c}\,\sqrt{ac+b}}{}$$

input
```
int(1/x/(a+b/(d*x^2+c))^(3/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*b*c - sqrt(c + d*x**2)*
sqrt(a*c + a*d*x**2 + b)*a*b**2 + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*
sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c
 + d*x**2)*b)*a**3*c**2 + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c
 + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**
2)*b)*a**3*c*d*x**2 + sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)*sqrt(a*c + a
*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(c + d*x**2)*b
)*a**2*b*c - sqrt(c)*sqrt(a*c + b)*log(x)*a**3*c**2 - sqrt(c)*sqrt(a*c + b
)*log(x)*a**3*c*d*x**2 - sqrt(c)*sqrt(a*c + b)*log(x)*a**2*b*c + sqrt(a)*l
og(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a**3*c**3 + sqrt
(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*a)*a**3*c**2*d
*x**2 + 3*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c + d*x**2)*
a)*a**2*b*c**2 + 2*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 + b) + sqrt(c +
 d*x**2)*a)*a**2*b*c*d*x**2 + 3*sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**2 +
b) + sqrt(c + d*x**2)*a)*a*b**2*c + sqrt(a)*log(sqrt(a)*sqrt(a*c + a*d*x**
2 + b) + sqrt(c + d*x**2)*a)*a*b**2*d*x**2 + sqrt(a)*log(sqrt(a)*sqrt(a*c
+ a*d*x**2 + b) + sqrt(c + d*x**2)*a)*b**3)/(a**2*(a**3*c**3 + a**3*c**2*d
*x**2 + 3*a**2*b*c**2 + 2*a**2*b*c*d*x**2 + 3*a*b**2*c + a*b**2*d*x**2 + b
**3))
```

3.206 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{bd}{(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2(b+ac)^2x^2} - \frac{3b\sqrt{c}d\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2(b+ac)^{5/2}}$$

output
```
b*d/(a*c+b)^2/(a+b/(d*x^2+c))^(1/2)-1/2*c*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/
(a*c+b)^2/x^2-3/2*b*c^(1/2)*d*arctanh(c^(1/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+b
)^(1/2))/(a*c+b)^(5/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(b(c-2dx^2)+ac(c+dx^2))}{2(b+ac)^2x^2\,(b+a\,(c+dx^2))} + \frac{3b\sqrt{c}d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2(-b-ac)^{5/2}}$$

input
```
Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(c - 2*d*x^2) +
 a*c*(c + d*x^2)))/((b + a*c)^2*x^2*(b + a*(c + d*x^2))) + (3*b*Sqrt[c]*d*
ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2
*(-b - a*c)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2057, 2053, 2052, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$\downarrow$ 2052

$$-bd\int \frac{1}{x^4\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\downarrow$ 253

$$-bd\left(\frac{3\int \frac{1}{x^4(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}+\frac{1}{2x^2(ac+b)\left(ac+b-cx^4\right)}\right)$$

$\downarrow$ 264

$$-bd\left(\frac{3\left(\frac{c\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{ac+b}-\frac{1}{x^2(ac+b)}\right)}{2(ac+b)}+\frac{1}{2x^2(ac+b)\left(ac+b-cx^4\right)}\right)$$

$\downarrow$ 221

$$-bd\left(\frac{3\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{1}{x^2(ac+b)}\right)}{2(ac+b)}+\frac{1}{2x^2(ac+b)\left(ac+b-cx^4\right)}\right)$$

input
```
Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-(b*d*(1/(2*(b + a*c)*x^2*(b + a*c - c*x^4)) + (3*(-(1/((b + a*c)*x^2)) +
(Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b +
a*c]])/(b + a*c)^(3/2)))/(2*(b + a*c))))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(102) = 204.

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{c(ad\,x^2+ac+b)}{2(ac+b)^2 x^2 \sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{db\left(-\frac{3c\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}}{x^2}\right)}{2\sqrt{a\,c^2+bc}}+\frac{2d\,x^2\cdots}{\sqrt{a\,d^2x^4+2ad\,x^2c\cdots}}\right.}{2(ac+b)^2\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)}$
default	Expression too large to display

input
```
int(1/x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*c+b)^2*c*(a*d*x^2+a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/2
*d*b/(a*c+b)^2*(-3/2*c/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x
^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)
+2*(d*x^2+c)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(102) = 204.

Time = 0.19 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.99

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left(abd^2x^4+(abc+b^2)dx^2\right)\sqrt{\frac{c}{ac+b}}\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\,(2\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*
c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4
 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d
*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*((
a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 +
 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*
d*x^2)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c
^2 + b*c)) - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^
2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d
*x^4 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(102) = 204.

Time = 0.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{3\,bcd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\,(a^2c^2+2\,abc+b^2)\sqrt{(ac+b)c}}+\frac{\frac{3\,(adx^2+ac+b)bcd}{dx^2+c}-2\,(abc+b^2)d}{2\left((a^2c^3+2\,abc^2+b^2c)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
3/4*b*c*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c)
)/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/((a^2*c^2
 + 2*a*b*c + b^2)*sqrt((a*c + b)*c)) + 1/2*(3*(a*d*x^2 + a*c + b)*b*c*d/(d
*x^2 + c) - 2*(a*b*c + b^2)*d)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*((a*d*x^2 +
a*c + b)/(d*x^2 + c))^(3/2) - (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

Giac [F]

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x^2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.32

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^3-\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,a^2c^2d\,x^2-2\sqrt{d\,x}}{}$$

input
```
int(1/x^3/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**2*c**3 - sqrt(c + d*x**2)
*sqrt(a*c + a*d*x**2 + b)*a**2*c**2*d*x**2 - 2*sqrt(c + d*x**2)*sqrt(a*c +
 a*d*x**2 + b)*a*b*c**2 + sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b*c*
d*x**2 - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**2*c + 2*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*b**2*d*x**2 + 3*sqrt(c)*sqrt(a*c + b)*log(sq
rt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sq
rt(c)*sqrt(c + d*x**2)*b)*a*b*c*d*x**2 + 3*sqrt(c)*sqrt(a*c + b)*log(sqrt(
a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(
c)*sqrt(c + d*x**2)*b)*a*b*d**2*x**4 + 3*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*
c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)
*sqrt(c + d*x**2)*b)*b**2*d*x**2 - 3*sqrt(c)*sqrt(a*c + b)*log(x)*a*b*c*d*
x**2 - 3*sqrt(c)*sqrt(a*c + b)*log(x)*a*b*d**2*x**4 - 3*sqrt(c)*sqrt(a*c +
 b)*log(x)*b**2*d*x**2)/(2*x**2*(a**4*c**4 + a**4*c**3*d*x**2 + 4*a**3*b*c
**3 + 3*a**3*b*c**2*d*x**2 + 6*a**2*b**2*c**2 + 3*a**2*b**2*c*d*x**2 + 4*a
*b**3*c + a*b**3*d*x**2 + b**4))
```

3.207 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 180

$$\begin{aligned}\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{abd^2}{(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}}\\ &\quad -\frac{(3b-4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8(b+ac)^3x^2}-\frac{(c+dx^2)^2\sqrt{a+\frac{b}{c+dx^2}}}{4(b+ac)^2x^4}\\ &\quad -\frac{3b(b-4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8\sqrt{c}(b+ac)^{7/2}}\end{aligned}$$

output
```
-a*b*d^2/(a*c+b)^3/(a+b/(d*x^2+c))^(1/2)-1/8*(-4*a*c+3*b)*d*(d*x^2+c)*(a+b
/(d*x^2+c))^(1/2)/(a*c+b)^3/x^2-1/4*(d*x^2+c)^2*(a+b/(d*x^2+c))^(1/2)/(a*c
+b)^2/x^4-3/8*b*(-4*a*c+b)*d^2*arctanh(c^(1/2)*(a+b/(d*x^2+c))^(1/2)/(a*c+
b)^(1/2))/c^(1/2)/(a*c+b)^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b^2(2c+5dx^2)+2a^2c(c^2-d^2x^4)+ab(4c^2+5cdx^2+13d^2x^4)\right)}{8(b+ac)^3x^4\left(b+a\left(c+dx^2\right)\right)} - \frac{3b(b-4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8\sqrt{c}(-b-ac)^{7/2}}$$

input
```
Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b^2*(2*c + 5*d*x^
2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + 5*c*d*x^2 + 13*d^2*x^4)))/((b
+ a*c)^3*x^4*(b + a*(c + d*x^2))) - (3*b*(b - 4*a*c)*d^2*ArcTan[(Sqrt[c]*S
qrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*Sqrt[c]*(-b - a*
c)^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2057, 2053, 2052, 25, 27, 361, 25, 361, 25, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 2057$$

$$\int \frac{1}{x^5 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^6 \left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{d(a-x^4)}{x^4\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{d(a-x^4)}{x^4\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$bd^2\int \frac{a-x^4}{x^4\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{361}$$

$$bd^2\left(-\frac{1}{4}\int -\frac{\frac{4a}{b+ac}-\frac{3bx^4}{(b+ac)^2}}{x^4\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$bd^2\left(\frac{1}{4}\int \frac{\frac{4a}{b+ac}-\frac{3bx^4}{(b+ac)^2}}{x^4\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{361}$$

$$bd^2\left(\frac{1}{4}\left(-\frac{1}{2}\int -\frac{\frac{8a}{(b+ac)^2}-\frac{(3b-4ac)x^4}{(b+ac)^3}}{x^4\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$bd^2\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{\frac{8a}{(b+ac)^2}-\frac{(3b-4ac)x^4}{(b+ac)^3}}{x^4\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)^2}\right)$$

$\Big\downarrow$ 359

$$bd^2\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{3(b-4ac)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{(ac+b)^3}-\frac{8a}{x^2(ac+b)^3}\right)-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{a}}{4(ac+b)^2}\right.$$

$\Big\downarrow$ 221

$$bd^2\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{3(b-4ac)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}(ac+b)^{7/2}}-\frac{8a}{x^2(ac+b)^3}\right)-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{a}}{4(ac+b)^2\,(}\right.$$

input
```
Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
b*d^2*(-1/4*(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)^2*(b + a*
c - c*x^4)^2) + (-1/2*((3*b - 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/((b + a*c)^3*(b + a*c - c*x^4)) + ((-8*a)/((b + a*c)^3*x^2) - (3*(b - 4*
a*c)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]
])/(Sqrt[c]*(b + a*c)^(7/2)))/2)/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{(a d x^2+ac+b)(-2ad x^2 c+5bd x^2+2a c^2+2bc)}{8(ac+b)^3 x^4\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}}-\frac{d^2 b\left(-\frac{(12ac-3b)\ln\left(\frac{2a c^2+2bc+(2acd+bd)x^2+2\sqrt{a c^2+bc}\sqrt{a c^2+bc+(2acd+bd)x^2}}{x^2}\right)}{2\sqrt{a c^2+bc}}\right.}{8(ac+b)^3\sqrt{\frac{ad x^2+a}{d x^2+}}}$
default	Expression too large to display

input
```
int(1/x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*d*x^2+a*c+b)*(-2*a*c*d*x^2+5*b*d*x^2+2*a*c^2+2*b*c)/(a*c+b)^3/x^4/
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*d^2*b/(a*c+b)^3*(-1/2*(12*a*c-3*b)/(
a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(
a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+8*a*(d*x^2+c)/(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(160) = 320.

Time = 0.92 (sec) , antiderivative size = 961, normalized size of antiderivative = 5.34

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^
2*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*
c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4
*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(
a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 -
 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^
4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^
3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*
a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 1
0*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d
^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arcta
n(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2
+ b^2*c)) - 2*((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6
 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 -
5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 +
7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4
*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4
*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^5 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(160) = 320$.

Time = 0.15 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.50

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{3\,(4\,abc-b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{(ac+b)c}} - \frac{8\,(a^3bc^2+2\,a^2b^2c+ab^3)d^2+\frac{3\,(4\,abc^2-b^2c)\left(adx^2+ac+b\right)^2d^2}{(dx^2+c)^2}-\frac{5\,(\cdots}{\cdots}}{8\left((a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-2\,(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c)\left(\frac{adx^2+ac+\cdots}{dx^2+c}\right.\cdots\right.}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-3/16*(4*a*b*c - b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b
)*c)))/((a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a*c + b)*c)) - 1/8
*(8*(a^3*b*c^2 + 2*a^2*b^2*c + a*b^3)*d^2 + 3*(4*a*b*c^2 - b^2*c)*(a*d*x^2
 + a*c + b)^2*d^2/(d*x^2 + c)^2 - 5*(4*a^2*b*c^2 + 3*a*b^2*c - b^3)*(a*d*x
^2 + a*c + b)*d^2/(d*x^2 + c))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3
*c^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 2*(a^4*c^5 + 4*a^3*b*c^4 +
 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3
/2) + (a^5*c^5 + 5*a^4*b*c^4 + 10*a^3*b^2*c^3 + 10*a^2*b^3*c^2 + 5*a*b^4*c
 + b^5)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

Giac [F]

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} \, dx = \int \frac{1}{x^5 \left(a + \frac{b}{d x^2+c}\right)^{3/2}} \, dx$$

input
```
int(1/(x^5*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^5*(a + b/(c + d*x^2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.98

$$\int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/x^5/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**5 + 2*sqrt(c + d*x
**2)*sqrt(a*c + a*d*x**2 + b)*a**3*c**3*d**2*x**4 - 6*sqrt(c + d*x**2)*sqr
t(a*c + a*d*x**2 + b)*a**2*b*c**4 - 5*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
 + b)*a**2*b*c**3*d*x**2 - 11*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*
*2*b*c**2*d**2*x**4 - 6*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c
**3 - 10*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c**2*d*x**2 - 13
*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*b**2*c*d**2*x**4 - 2*sqrt(c +
 d*x**2)*sqrt(a*c + a*d*x**2 + b)*b**3*c**2 - 5*sqrt(c + d*x**2)*sqrt(a*c
+ a*d*x**2 + b)*b**3*c*d*x**2 - 12*sqrt(c)*sqrt(a*c + b)*log(sqrt(a*c + b)
*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c)*sqrt(
c + d*x**2)*b)*a**2*b*c**2*d**2*x**4 - 12*sqrt(c)*sqrt(a*c + b)*log(sqrt(a
*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqrt(c
)*sqrt(c + d*x**2)*b)*a**2*b*c*d**3*x**6 - 9*sqrt(c)*sqrt(a*c + b)*log(sqr
t(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c - sqr
t(c)*sqrt(c + d*x**2)*b)*a*b**2*c*d**2*x**4 + 3*sqrt(c)*sqrt(a*c + b)*log(
sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c -
sqrt(c)*sqrt(c + d*x**2)*b)*a*b**2*d**3*x**6 + 3*sqrt(c)*sqrt(a*c + b)*log
(sqrt(a*c + b)*sqrt(a*c + a*d*x**2 + b)*c - sqrt(c)*sqrt(c + d*x**2)*a*c -
 sqrt(c)*sqrt(c + d*x**2)*b)*b**3*d**2*x**4 + 12*sqrt(c)*sqrt(a*c + b)*log
(x)*a**2*b*c**2*d**2*x**4 + 12*sqrt(c)*sqrt(a*c + b)*log(x)*a**2*b*c*d*...
```

3.208 $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 434

$$\begin{aligned}\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{x^3(c+dx^2)}{ad\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}\\
&\quad+\frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{(16b^2+16abc+a^2c^2)\,x(b+ac+adx^2)}{5a^4d^2\,(c+dx^2)\,\sqrt{a+\frac{b}{c+dx^2}}}\\
&\quad-\frac{\sqrt{c}(16b^2+16abc+a^2c^2)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^4d^{5/2}\,(c+dx^2)\,\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}\\
&\quad+\frac{c^{3/2}(8b+ac)\,(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5a^3d^{5/2}\,(c+dx^2)\,\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}\end{aligned}$$

output
```
-x^3*(d*x^2+c)/a/d/(a+b/(d*x^2+c))^(1/2)-1/5*(a*c+8*b)*x*(a*d*x^2+a*c+b)/a
^3/d^2/(a+b/(d*x^2+c))^(1/2)+6/5*x^3*(a*d*x^2+a*c+b)/a^2/d/(a+b/(d*x^2+c))
^(1/2)+1/5*(a^2*c^2+16*a*b*c+16*b^2)*x*(a*d*x^2+a*c+b)/a^4/d^2/(d*x^2+c)/(
a+b/(d*x^2+c))^(1/2)-1/5*c^(1/2)*(a^2*c^2+16*a*b*c+16*b^2)*(a*d*x^2+a*c+b)
*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^4/d^(5
/2)/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)/(a+b/(d*x^2+c))^
(1/2)+1/5*c^(3/2)*(a*c+8*b)*(a*d*x^2+a*c+b)*InverseJacobiAM(arctan(d^(1/2)
*x/c^(1/2)),(b/(a*c+b))^(1/2))/a^3/d^(5/2)/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a
*c+b)/(d*x^2+c))^(1/2)/(a+b/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.72 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(8b^2+ab(9c+2dx^2)+a^2(c^2-d^2x^4)\right)+i(16b^3+32ab^2c+17a^2bc^2+a^3c^3)\right.}{5}$$

input
```
Integrate[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(8*
b^2 + a*b*(9*c + 2*d*x^2) + a^2*(c^2 - d^2*x^4)) + I*(16*b^3 + 32*a*b^2*c
+ 17*a^2*b*c^2 + a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (8*I)*b*(2*b^
2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^4*c^2*(d/c)^(
5/2)*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {2057, 2058, 369, 27, 443, 25, 27, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^4\left(dx^2+c\right)^{3/2}}{\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{3x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{\sqrt{adx^2+b+ac}}dx}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\int \frac{x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{\sqrt{adx^2+b+ac}}dx}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 443}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{\int -\frac{dx^2\left((8b+ac)dx^2+c(6b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5ad}+\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 25

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\int \frac{dx^2\left((8b+ac)dx^2+c(6b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5ad}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\int \frac{x^2\left((8b+ac)dx^2+c(6b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5a}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 444

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{d\left(\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}}{5a}\right)}{ad}-\frac{x^3\left(c+\right.}{ad\sqrt{ac-}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int\frac{\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}}{5a}\right)}{ad}-\frac{x^3\left(c+dx^2\cdots\right.}{ad\sqrt{ac+ad\cdots}}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$\downarrow$ 406

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(a^2c^2+16abc+16b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(ac+8b)\cdots}{3ad}}{5a}\cdots\right.}{ad}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$\downarrow$ 320

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(a^2c^2+16abc+16b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+8b)\sqrt{\cdots}}{\cdots}}{3ad}}{5a}\cdots\right.}{ad}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\frac{d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\cdots}{3ad}}{5a}\right)}{ad}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,E\left(\arctan\cdots\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(\cdots)}{(ac\cdots)}}}\cdots\right)}{5a}\cdots\right)}{ad}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

input

```
Int[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((x^3*(c + d*x^2)^(3/2))/(a*d*Sqrt[b + a*c + a*
d*x^2])) + (3*((2*x^3*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(5*a) - (((
8*b + a*c)*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a*d) - ((16*b^2 +
 16*a*b*c + a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2])
- (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b
+ a*c)*(c + d*x^2))])) + (c^(3/2)*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Elli
pticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*
Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*a*d))/(5*a)))/(
a*d)))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(407) = 814$.

Time = 18.63 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{x(-adx^2+ac+3b)(adx^2+ac+b)}{5a^3d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}+\frac{\left(\frac{(a^3c^3+4a^2bc^2-2cab^2-5b^3)\sqrt{1+\frac{adx^2}{ac+b}}\sqrt{1+\frac{x^2d}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{a\sqrt{-\frac{ad}{ac+b}}\sqrt{ad^2x^4+2adx^2c+bdx^2+ac^2+bc}}-\frac{2d(\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input

```
int(x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/5/a^3/d^2*x*(-a*d*x^2+a*c+3*b)*(a*d*x^2+a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)+1/5/a^3/d^2*((a^3*c^3+4*a^2*b*c^2-2*a*b^2*c-5*b^3)/a/(-a*d/(a*c+
b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*
d+b*d)/d/c/a)^(1/2))-2*d*(a^2*c^2+11*a*b*c+11*b^2)*(a*c^2+b*c)/(-a*d/(a*c+
b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^
(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1
+(2*a*c*d+b*d)/d/c/a)^(1/2)))+5*b^2*(a^2*c^2+2*a*b*c+b^2)/a*(-(a*d^2*x^2+a
*c*d)/(a*c+b)/b*x/d/((x^2+(a*c+b)/a/d)*(a*d^2*x^2+a*c*d))^(1/2)+(1/(a*c+b)
+a*c/(a*c+b)/b)/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*
d)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d
/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*a/b*d/(a*c+b)*(a*c^2+b*c
)/(-a*d/(a*c+b))^(1/2)*(1+a*d*x^2/(a*c+b))^(1/2)*(1+1/c*x^2*d)^(1/2)/(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-
a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+
b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2)))))/((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{\left((a^3c^3+16\,a^2bc^2+16\,ab^2c)dx^3+(a^3c^4+17\,a^2bc^3+32\,ab^2c^2+16\,b^3c)x\right)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+}{ac}}{}$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(((a^3*c^3 + 16*a^2*b*c^2 + 16*a*b^2*c)*d*x^3 + (a^3*c^4 + 17*a^2*b*c
^3 + 32*a*b^2*c^2 + 16*b^3*c)*x)*sqrt(a)*sqrt(-c/d)*elliptic_e(arcsin(sqrt
(-c/d)/x), (a*c + b)/(a*c)) - (((a^3*c^2 + 9*a^2*b*c + 8*a*b^2)*d^2 + (a^3
*c^3 + 16*a^2*b*c^2 + 16*a*b^2*c)*d)*x^3 + (a^3*c^4 + 17*a^2*b*c^3 + 32*a*
b^2*c^2 + 16*b^3*c + (a^3*c^3 + 10*a^2*b*c^2 + 17*a*b^2*c + 8*b^3)*d)*x)*s
qrt(a)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a^3
*d^4*x^8 + (a^3*c - 2*a^2*b)*d^3*x^6 + a^3*c^4 + (5*a^2*b*c + 8*a*b^2)*d^2
*x^4 + 17*a^2*b*c^3 + 32*a*b^2*c^2 + 16*b^3*c + (a^3*c^3 + 24*a^2*b*c^2 +
40*a*b^2*c + 16*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^
4*x^3 + (a^5*c + a^4*b)*d^3*x)
```

Sympy [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**4/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^4/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4/(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*d**2*x**5 + 3*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*b*c*x - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
+ b)*b*d*x**3 + 4*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a*
*2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x**6 + 2*a*b
*c**2 + 4*a*b*c*d*x**2 + 2*a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*a**2*b
*c**2*d**2 + 4*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**2*
c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x**6 + 2*a*b*c*
*2 + 4*a*b*c*d*x**2 + 2*a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*a**2*b*c*
d**3*x**2 + 12*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**2*
c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x**6 + 2*a*b*c*
*2 + 4*a*b*c*d*x**2 + 2*a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*a*b**2*c*
d**2 + 8*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**2*c**3 +
 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x**6 + 2*a*b*c**2 + 4
*a*b*c*d*x**2 + 2*a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*a*b**2*d**3*x**
2 + 8*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**2*c**3 + 3*
a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x**6 + 2*a*b*c**2 + 4*a*
b*c*d*x**2 + 2*a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*b**3*d**2 - 3*int(
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(a**2*c**3 + 3*a**2*c**2*d*x**
2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x**6 + 2*a*b*c**2 + 4*a*b*c*d*x**2 + 2*
a*b*d**2*x**4 + b**2*c + b**2*d*x**2),x)*a**2*b*c**4 - 3*int((sqrt(c + ...
```

3.209 $\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 284

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{x(c+dx^2)}{ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{a+\frac{b}{c+dx^2}}}{3a^3d} + \frac{4x(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{3a^2d} + \frac{\sqrt{c}(8b+ac)\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\sqrt{\frac{c\left(a+\frac{b}{c+dx^2}\right)}{b+ac}}} - \frac{\sqrt{c}(4b+ac)\sqrt{\frac{c\left(a+\frac{b}{c+dx^2}\right)}{b+ac}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3a^2d^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}$$

output

```
-x*(d*x^2+c)/a/d/(a+b/(d*x^2+c))^(1/2)-1/3*(a*c+8*b)*x*(a+b/(d*x^2+c))^(1/
2)/a^3/d+4/3*x*(d*x^2+c)*(a+b/(d*x^2+c))^(1/2)/a^2/d+1/3*c^(1/2)*(a*c+8*b)
*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a
*c+b))^(1/2))/a^3/d^(3/2)/(c*(a+b/(d*x^2+c))/(a*c+b))^(1/2)-1/3*c^(1/2)*(a
*c+4*b)*(c*(a+b/(d*x^2+c))/(a*c+b))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x
/c^(1/2)),(b/(a*c+b))^(1/2))/a^2/d^(3/2)/(a+b/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(4b+a(c+dx^2)\right)+i(8b^2+9abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\right.}{3}$$

input
```
Integrate[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(4*b + a
*(c + d*x^2)) + I*(8*b^2 + 9*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b
+ a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a
*c)] - I*b*(8*b + 5*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a^3*d*Sqrt[
d/c]*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2058, 369, 403, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 2057$$

$$\int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^2\left(dx^2+c\right)^{3/2}}{\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 369

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{\sqrt{dx^2+c}\left(4dx^2+c\right)}{\sqrt{adx^2+b+ac}}dx}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 403

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{\int -\frac{d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}+\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{\int \frac{d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{\int \frac{(8b+ac)dx^2+c(4b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3a}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{c(ac+4b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+8b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3a}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{d(ac+8b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}}{3a}}{ad}-\frac{3}{ac}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{(dx^2+c)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}}{3a}}{ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+}}{ad^{3/2}\sqrt{c+}}\right.}{3a}}{ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((x*(c + d*x^2)^(3/2))/(a*d*Sqrt[b + a*c + a*d*
x^2])) + ((4*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a) - ((8*b + a*
c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b
+ a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d
^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)])) + (c^(3/2)*(4*b + a*c)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*
(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*a))/(a*d)))/(Sqrt[c + d
*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(261) = 522$.

Time = 14.90 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.35

method	result
default	$\frac{\left(\sqrt{(d\,x^2+c)(ad\,x^2+ac+b)}\,\sqrt{-\frac{ad}{ac+b}}\,a\,d^2x^5+2\sqrt{(d\,x^2+c)(ad\,x^2+ac+b)}\,\sqrt{-\frac{ad}{ac+b}}\,acd\,x^3+\sqrt{(d\,x^2+c)(ad\,x^2+ac+b)}\,\sqrt{-\frac{ad}{ac+b}}\,bd\,x^3\right.}{}$
risch	$\frac{x(ad\,x^2+ac+b)}{3d\,a^2\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{\left(\frac{\left(a^2c^2+abc-3b^2\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{x^2d}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{a\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2ad\,x^2c+bd\,x^2+a\,c^2+bc}}-\frac{2d(ac+5b)\left(a\,c^2+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\right.}{}$

input

```
int(x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*((
d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+((d*x^2+c)*
(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d*x^3-((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+3*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d*x^3+((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^2*x+4*((d*x^2+c)*(a*d*x^2+a*c+b))^
(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1
/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/
(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*
(-a*d/(a*c+b))^(1/2)*b*c*x+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2)*(-a*d/(a*c+b))^(1/2)*b*c*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d/(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2
+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{((a^2c^2+8\,abc)dx^3+(a^2c^3+9\,abc^2+8\,b^2c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(}{}$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((a^2*c^2 + 8*a*b*c)*d*x^3 + (a^2*c^3 + 9*a*b*c^2 + 8*b^2*c)*x)*sqrt(
a)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (((a^2*c
 + 4*a*b)*d^2 + (a^2*c^2 + 8*a*b*c)*d)*x^3 + (a^2*c^3 + 9*a*b*c^2 + 8*b^2*
c + (a^2*c^2 + 5*a*b*c + 4*b^2)*d)*x)*sqrt(a)*sqrt(-c/d)*elliptic_f(arcsin
(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a^2*d^3*x^6 + (a^2*c - 4*a*b)*d^2*x^4
- a^2*c^3 - 9*a*b*c^2 - (a^2*c^2 + 13*a*b*c + 8*b^2)*d*x^2 - 8*b^2*c)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3*x^3 + (a^4*c + a^3*b)*d^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**2/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2/(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(x^2/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
(2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c*d*x**3 - 3*sqrt(c + d*x**
2)*sqrt(a*c + a*d*x**2 + b)*b*c*x + 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2
 + b)*b*d*x**3 - 5*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a
**3*c**4 + 3*a**3*c**3*d*x**2 + 3*a**3*c**2*d**2*x**4 + a**3*c*d**3*x**6 +
 3*a**2*b*c**3 + 7*a**2*b*c**2*d*x**2 + 5*a**2*b*c*d**2*x**4 + a**2*b*d**3
*x**6 + 3*a*b**2*c**2 + 5*a*b**2*c*d*x**2 + 2*a*b**2*d**2*x**4 + b**3*c +
b**3*d*x**2),x)*a**3*b*c**3*d**2 - 5*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*
x**2 + b)*x**4)/(a**3*c**4 + 3*a**3*c**3*d*x**2 + 3*a**3*c**2*d**2*x**4 +
a**3*c*d**3*x**6 + 3*a**2*b*c**3 + 7*a**2*b*c**2*d*x**2 + 5*a**2*b*c*d**2*
x**4 + a**2*b*d**3*x**6 + 3*a*b**2*c**2 + 5*a*b**2*c*d*x**2 + 2*a*b**2*d**
2*x**4 + b**3*c + b**3*d*x**2),x)*a**3*b*c**2*d**3*x**2 - 18*int((sqrt(c +
 d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**3*c**4 + 3*a**3*c**3*d*x**2 +
3*a**3*c**2*d**2*x**4 + a**3*c*d**3*x**6 + 3*a**2*b*c**3 + 7*a**2*b*c**2*d
*x**2 + 5*a**2*b*c*d**2*x**4 + a**2*b*d**3*x**6 + 3*a*b**2*c**2 + 5*a*b**2
*c*d*x**2 + 2*a*b**2*d**2*x**4 + b**3*c + b**3*d*x**2),x)*a**2*b**2*c**2*d
**2 - 13*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**3*c**4 +
 3*a**3*c**3*d*x**2 + 3*a**3*c**2*d**2*x**4 + a**3*c*d**3*x**6 + 3*a**2*b*
c**3 + 7*a**2*b*c**2*d*x**2 + 5*a**2*b*c*d**2*x**4 + a**2*b*d**3*x**6 + 3*
a*b**2*c**2 + 5*a*b**2*c*d*x**2 + 2*a*b**2*d**2*x**4 + b**3*c + b**3*d*x**
2),x)*a**2*b**2*c*d**3*x**2 - 21*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x...
```

3.210 $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 235

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{x}{a\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(2b+ac)E\left(\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{b+ac}}\right)|-\frac{b}{ac}\right)}{a^{3/2}\sqrt{b+ac}\sqrt{d}\sqrt{\frac{(b+ac)(c+dx^2)}{c(b+ac+adx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{b+ac}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{b+ac}}\right),-\frac{b}{ac}\right)}{a^{3/2}\sqrt{d}\sqrt{\frac{(b+ac)(c+dx^2)}{c(b+ac+adx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

output
```
x/a/(a+b/(d*x^2+c))^(1/2)-(a*c+2*b)*EllipticE(a^(1/2)*d^(1/2)*x/(a*c+b)^(1
/2)/(1+a*d*x^2/(a*c+b))^(1/2),(-b/a/c)^(1/2))/a^(3/2)/(a*c+b)^(1/2)/d^(1/2
)/((a*c+b)*(d*x^2+c)/c/(a*d*x^2+a*c+b))^(1/2)/(a+b/(d*x^2+c))^(1/2)+(a*c+b
)^(1/2)*InverseJacobiAM(arctan(a^(1/2)*d^(1/2)*x/(a*c+b)^(1/2)),(-b/a/c)^(
1/2))/a^(3/2)/d^(1/2)/((a*c+b)*(d*x^2+c)/c/(a*d*x^2+a*c+b))^(1/2)/(a+b/(d*
x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{i\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-iab\sqrt{\frac{d}{c}}x(c+dx^2)+(2b^2+3abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)\right.}{a^2(b+ac)\sqrt{\frac{d}{c}}\left(b+a\left(c+dx^2\right)\right)}$$

input
```
Integrate[(a + b/(c + d*x^2))^(-3/2),x]
```

output
```
((-I)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((-I)*a*b*Sqrt[d/c]*x*(c + d*x
^2) + (2*b^2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - 2*b*(
b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^2*(b + a*c)*Sqrt[d/c]*(b +
a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2057, 2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\left(dx^2+c\right)^{3/2}}{\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{d\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ad(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2}}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{a(ac+b)}-\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[(a + b/(c + d*x^2))^(-3/2),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((b*x*Sqrt[c + d*x^2])/(a*(b + a*c)*Sqrt[b + a*
c + a*d*x^2])) + ((2*b + a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c +
 d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x
^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(a*(b + a*c))))/(Sqrt
[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(221) = 442$.

Time = 6.49 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.99

method	result
default	$\frac{\left(\sqrt{(d x^2+c)(ad x^2+ac+b)}\sqrt{\frac{ad x^2+ac+b}{ac+b}}\sqrt{\frac{d x^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a c^2-\sqrt{a d^2x^4+2ad x^2c+bd x^2+a c^2+bc}\,\sqrt{-\frac{a}{ac}}\right.}{}$

input
```
int(1/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2-(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d*x^
3+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*x)/a*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*c+b)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((a^2c^2+2\,abc)dx^3+(a^2c^3+3\,abc^2+2\,b^2c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(((a^2c+ab)d^2+(a^2c^2}{}$$

input
```
integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-(((a^2*c^2 + 2*a*b*c)*d*x^3 + (a^2*c^3 + 3*a*b*c^2 + 2*b^2*c)*x)*sqrt(a)*
sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (((a^2*c +
a*b)*d^2 + (a^2*c^2 + 2*a*b*c)*d)*x^3 + (a^2*c^3 + 3*a*b*c^2 + 2*b^2*c + (
a^2*c^2 + 2*a*b*c + b^2)*d)*x)*sqrt(a)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-
c/d)/x), (a*c + b)/(a*c)) - ((a^2*c + a*b)*d^2*x^4 + a^2*c^3 + 3*a*b*c^2 +
 2*(a^2*c^2 + 2*a*b*c + b^2)*d*x^2 + 2*b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d*
x^2 + c)))/((a^4*c + a^3*b)*d^2*x^3 + (a^4*c^2 + 2*a^3*b*c + a^2*b^2)*d*x)
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral((a + b/(c + d*x**2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(1/(a + b/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{\sqrt{d\,x^2+c}\,\sqrt{ad\,x^2+ac+b}\,cx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{a}}{a^3c\,d^3x^6+a^2b\,d^3x^6+3a^3c^2d^2x^4+5a^2bc\,d^2x^4+3a^3c^3d\,x^2+2a\,b^2d^2}\right.}{}$$

input
```
int(1/(a+b/(d*x^2+c))^(3/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*c*x + int((sqrt(c + d*x**2)*sqr
t(a*c + a*d*x**2 + b)*x**4)/(a**3*c**4 + 3*a**3*c**3*d*x**2 + 3*a**3*c**2*
d**2*x**4 + a**3*c*d**3*x**6 + 3*a**2*b*c**3 + 7*a**2*b*c**2*d*x**2 + 5*a*
*2*b*c*d**2*x**4 + a**2*b*d**3*x**6 + 3*a*b**2*c**2 + 5*a*b**2*c*d*x**2 +
2*a*b**2*d**2*x**4 + b**3*c + b**3*d*x**2),x)*a**2*b*c**2*d**2 + int((sqrt
(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**3*c**4 + 3*a**3*c**3*d*x**
2 + 3*a**3*c**2*d**2*x**4 + a**3*c*d**3*x**6 + 3*a**2*b*c**3 + 7*a**2*b*c*
*2*d*x**2 + 5*a**2*b*c*d**2*x**4 + a**2*b*d**3*x**6 + 3*a*b**2*c**2 + 5*a*
b**2*c*d*x**2 + 2*a*b**2*d**2*x**4 + b**3*c + b**3*d*x**2),x)*a**2*b*c*d**
3*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**3*c**4
 + 3*a**3*c**3*d*x**2 + 3*a**3*c**2*d**2*x**4 + a**3*c*d**3*x**6 + 3*a**2*
b*c**3 + 7*a**2*b*c**2*d*x**2 + 5*a**2*b*c*d**2*x**4 + a**2*b*d**3*x**6 +
3*a*b**2*c**2 + 5*a*b**2*c*d*x**2 + 2*a*b**2*d**2*x**4 + b**3*c + b**3*d*x
**2),x)*a*b**2*c*d**2 + int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**
4)/(a**3*c**4 + 3*a**3*c**3*d*x**2 + 3*a**3*c**2*d**2*x**4 + a**3*c*d**3*x
**6 + 3*a**2*b*c**3 + 7*a**2*b*c**2*d*x**2 + 5*a**2*b*c*d**2*x**4 + a**2*b
*d**3*x**6 + 3*a*b**2*c**2 + 5*a*b**2*c*d*x**2 + 2*a*b**2*d**2*x**4 + b**3
*c + b**3*d*x**2),x)*a*b**2*d**3*x**2 + int((sqrt(c + d*x**2)*sqrt(a*c + a
*d*x**2 + b)*x**4)/(a**3*c**4 + 3*a**3*c**3*d*x**2 + 3*a**3*c**2*d**2*x**4
 + a**3*c*d**3*x**6 + 3*a**2*b*c**3 + 7*a**2*b*c**2*d*x**2 + 5*a**2*b*c...
```

3.211 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 251

$$\begin{aligned}\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = &-\frac{b}{a(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c(b-ac)\sqrt{a+\frac{b}{c+dx^2}}}{a(b+ac)^2x}\\ &-\frac{\sqrt{c}(-b+ac)\sqrt{d}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(b+ac)^2\sqrt{\frac{c\left(a+\frac{b}{c+dx^2}\right)}{b+ac}}}\\ &+\frac{c^{3/2}\sqrt{d}\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)^2\sqrt{\frac{c\left(a+\frac{b}{c+dx^2}\right)}{b+ac}}}\end{aligned}$$

output

```
-b/a/(a*c+b)/x/(a+b/(d*x^2+c))^(1/2)+c*(-a*c+b)*(a+b/(d*x^2+c))^(1/2)/a/(a
*c+b)^2/x-c^(1/2)*(a*c-b)*d^(1/2)*(a+b/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*
x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/(a*c+b)^2/(c*(a+b/(d*x^2+
c))/(a*c+b))^(1/2)+c^(3/2)*d^(1/2)*(a+b/(d*x^2+c))^(1/2)*InverseJacobiAM(a
rctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/(a*c+b)^2/(c*(a+b/(d*x^2+c))/(
a*c+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.80 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}(c+dx^2)\left(b(c-dx^2)+ac(c+dx^2)\right)+i(-b^2+a^2c^2)\,dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsi}\right.}{a(b+ac)^2\sqrt{\frac{d}{c}}x\,(b+a\,(}$$

input
```
Integrate[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*(c + d*x^2)*(b*(c -
d*x^2) + a*c*(c + d*x^2)) + I*(-b^2 + a^2*c^2)*d*x*Sqrt[(b + a*c + a*d*x^2
)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(
b + a*c)] + I*b*(b + a*c)*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a*(b + a
*c)^2*Sqrt[d/c]*x*(b + a*(c + d*x^2))))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.61, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2058, 370, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x^2 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}} dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\left(dx^2+c\right)^{3/2}}{x^2\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{\int \frac{cd\left(-adx^2+b-ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ad(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\int \frac{-adx^2+b-ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{\int \frac{ad\left(c(b+ac)-(b-ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{a(ac+b)}-\frac{\cdots}{ax(ac\cdots}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2\cdots}}{cx(ac+b)}\right.}{a(ac+b)}\cdots\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+\cdots}}{cx\cdots}\cdots\right)}{a(ac+b)}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+a\cdots}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\cdots\right)\right)}{c(ac+b)}\cdots\right)}{a(ac+b)}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

input

```
Int[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((b*Sqrt[c + d*x^2])/(a*(b + a*c)*x*Sqrt[b + a*
c + a*d*x^2])) - (c*(-(((b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])
/(c*(b + a*c)*x)) - (a*d*(-((b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*
Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c
 + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d
*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(c*(b + a*c
))))/(a*(b + a*c))))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(238) = 476$.

Time = 18.98 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.73

method	result
default	$\frac{\left(-\sqrt{(d\,x^2+c)(ad\,x^2+ac+b)}\,\sqrt{-\frac{ad}{ac+b}}\,ac\,d^2x^4+a\,c^2d\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\;\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x\sqrt{(d\,x^2+c)(ad\,x^2+}\right.}{}$
risch	Expression too large to display

input
```
int(1/x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c*d^2*x^4+a*c^2
*d*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(
a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)+(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d^2*x^
4-EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^
2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*b*c*d*
x-2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^2*d*x^2+2*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c*d*x-(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*d*x^2+(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*d*x^2-((d*x
^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^3-((d*x^2+c)*(a*d*x^
2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x
/(a*c+b)^2/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\, dx = \frac{((a^3c-a^2b)d^3x^3+(a^3c^2-ab^2)d^2x)\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+}{ac}}{}$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
(((a^3*c - a^2*b)*d^3*x^3 + (a^3*c^2 - a*b^2)*d^2*x)*sqrt(-a*d/(a*c + b))*
sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c +
b)/(a*c)) - (((a^3*c - a^2*b)*d^3 + (a^3*c^2 + 2*a^2*b*c + a*b^2)*d^2)*x^3
 + ((a^3*c^2 - a*b^2)*d^2 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*d)*x
)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d
/(a*c + b))*x), (a*c + b)/(a*c)) - (a^3*c^4 + (a^3*c^2 - a*b^2)*d^2*x^4 +
2*a^2*b*c^3 + a*b^2*c^2 + 2*(a^3*c^3 + a^2*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a^5*c^3 + 3*a^4*b*c^2 + 3*a^3*b^2*c + a^2*b^3)*d*x
^3 + (a^5*c^4 + 4*a^4*b*c^3 + 6*a^3*b^2*c^2 + 4*a^2*b^3*c + a*b^4)*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x^2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\text{too large to display}$$

input
```
int(1/x^2/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*a*c**2 - 2*sqrt(c + d*x**2
)*sqrt(a*c + a*d*x**2 + b)*b*c + sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)
*b*d*x**2 - int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**4*c**
5 + 3*a**4*c**4*d*x**2 + 3*a**4*c**3*d**2*x**4 + a**4*c**2*d**3*x**6 + 4*a
**3*b*c**4 + 10*a**3*b*c**3*d*x**2 + 8*a**3*b*c**2*d**2*x**4 + 2*a**3*b*c*
d**3*x**6 + 6*a**2*b**2*c**3 + 12*a**2*b**2*c**2*d*x**2 + 7*a**2*b**2*c*d*
*2*x**4 + a**2*b**2*d**3*x**6 + 4*a*b**3*c**2 + 6*a*b**3*c*d*x**2 + 2*a*b*
*3*d**2*x**4 + b**4*c + b**4*d*x**2),x)*a**4*b*c**3*d**3*x - int((sqrt(c +
 d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**4)/(a**4*c**5 + 3*a**4*c**4*d*x**2 +
3*a**4*c**3*d**2*x**4 + a**4*c**2*d**3*x**6 + 4*a**3*b*c**4 + 10*a**3*b*c*
*3*d*x**2 + 8*a**3*b*c**2*d**2*x**4 + 2*a**3*b*c*d**3*x**6 + 6*a**2*b**2*c
**3 + 12*a**2*b**2*c**2*d*x**2 + 7*a**2*b**2*c*d**2*x**4 + a**2*b**2*d**3*
x**6 + 4*a*b**3*c**2 + 6*a*b**3*c*d*x**2 + 2*a*b**3*d**2*x**4 + b**4*c + b
**4*d*x**2),x)*a**4*b*c**2*d**4*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a*c +
a*d*x**2 + b)*x**4)/(a**4*c**5 + 3*a**4*c**4*d*x**2 + 3*a**4*c**3*d**2*x**
4 + a**4*c**2*d**3*x**6 + 4*a**3*b*c**4 + 10*a**3*b*c**3*d*x**2 + 8*a**3*b
*c**2*d**2*x**4 + 2*a**3*b*c*d**3*x**6 + 6*a**2*b**2*c**3 + 12*a**2*b**2*c
**2*d*x**2 + 7*a**2*b**2*c*d**2*x**4 + a**2*b**2*d**3*x**6 + 4*a*b**3*c**2
 + 6*a*b**3*c*d*x**2 + 2*a*b**3*d**2*x**4 + b**4*c + b**4*d*x**2),x)*a**3*
b**2*c**2*d**3*x - 2*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*x**...
```

3.212 $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 392

$$\begin{aligned}\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{b}{a(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} \\ &+\frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} \\ &-\frac{\sqrt{c}(7b-ac)d^{3/2}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}} \\ &+\frac{\sqrt{c}(3b-ac)d^{3/2}(b+ac+adx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}\end{aligned}$$

output
```
-b/a/(a*c+b)/x^3/(a+b/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*(a*d*x^2+a*c+b)/a/(a
*c+b)^2/x^3/(a+b/(d*x^2+c))^(1/2)-1/3*c*(-a*c+7*b)*d*(a*d*x^2+a*c+b)/(a*c+
b)^3/x/(d*x^2+c)/(a+b/(d*x^2+c))^(1/2)-1/3*c^(1/2)*(-a*c+7*b)*d^(3/2)*(a*d
*x^2+a*c+b)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2
))/(a*c+b)^3/(d*x^2+c)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)/(a+b/(d
*x^2+c))^(1/2)+1/3*c^(1/2)*(-a*c+3*b)*d^(3/2)*(a*d*x^2+a*c+b)*InverseJacob
iAM(arctan(d^(1/2)*x/c^(1/2)),(b/(a*c+b))^(1/2))/(a*c+b)^3/(d*x^2+c)/(c*(a
*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)/(a+b/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.25 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c+4dx^2)+a^2c(c^2-d^2x^4)+ab(2c^2+4cdx^2+7d^2x^4)\right)+i(7b^2+6abc-\cdots\right.}{3(\cdots}$$

input
```
Integrate[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^2*(c
 + 4*d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + 4*c*d*x^2 + 7*d^2*x^4))
 + I*(7*b^2 + 6*a*b*c - a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c
)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]
- (4*I)*b*(b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/((b + a*c)^
3*Sqrt[d/c]*x^3*(b + a*(c + d*x^2)))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {2057, 2058, 370, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^4 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{(dx^2+c)^{3/2}}{x^4(adx^2+b+ac)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{\int \frac{d\left((2b-ac)dx^2+c(3b-ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ad(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{\int \frac{(2b-ac)dx^2+c(3b-ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{\int \frac{acd\left((3b-ac)dx^2+c(7b-ac)\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\int \frac{(3b-ac)dx^2+c(7b-ac)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(-\frac{\int -\frac{cd\left(a(7b-ac)dx^2+(3b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{}{ax^3(ac\cdots}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{\int \frac{cd\left(a(7b-ac)dx^2+(3b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{\cdots}}{ax^3(ac+b)\cdots}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\int \frac{a(7b-ac)dx^2+(3b-ac)(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ac+b}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+\cdots}}{ax^3(ac+b)\sqrt{a\cdots}}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left((3b-ac)(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad(7b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{ac+b}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-}{a(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left(ad(7b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{}}{x(ac+b)}\right.}{3(ac+b)}}{a(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\frac{(7b-}{}}{3(ac+b)}}{a(ac+b)}}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{}}{}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}}{3(ac+b)}}{a(ac+b)}}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((b*Sqrt[c + d*x^2])/(a*(b + a*c)*x^3*Sqrt[b +
a*c + a*d*x^2])) - (-1/3*((3*b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x
^2])/((b + a*c)*x^3) - (a*d*(-(((7*b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c +
 a*d*x^2])/((b + a*c)*x)) + (d*(a*(7*b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2
])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*
(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (Sqrt[c]*(3*b - a*c)*Sqr
t[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^
2))])))/(b + a*c)))/(3*(b + a*c)))/(a*(b + a*c))))/(Sqrt[c + d*x^2]*Sqrt[(
b + a*c + a*d*x^2)/(c + d*x^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(369) = 738.

Time = 18.27 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	1080
risch	Expression too large to display	1142

input
```
int(1/x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^
6+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^6+((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2
*x^3+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2
)*a*b*d^3*x^6-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c
^2*d^2*x^4+5*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1
/2))*a*b*c*d^2*x^3-7*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*a*b*c*d^2*x^3+8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+
b))^(1/2)*a*b*c*d^2*x^4-3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+
b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a
*c+b)/a/c)^(1/2))*b^2*d^2*x^3+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d^2*x^4+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
*(-a*d/(a*c+b))^(1/2)*a^2*c^3*d*x^2+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-
a*d/(a*c+b))^(1/2)*b^2*d^2*x^4+6*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(
a*c+b))^(1/2)*a*b*c^2*d*x^2+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b
))^(1/2)*a^2*c^4+5*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*
b^2*c*d*x^2+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.53

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$
$$-\frac{((a^4c^2-7\,a^3bc)d^4x^5+(a^4c^3-6\,a^3bc^2-7\,a^2b^2c)d^3x^3)\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac^2+bc}{d^2}}E\left(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac}\right)-((}{}$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((a^4*c^2 - 7*a^3*b*c)*d^4*x^5 + (a^4*c^3 - 6*a^3*b*c^2 - 7*a^2*b^2*
c)*d^3*x^3)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin
(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (((a^4*c^2 - 7*a^3*b*c)*d^4 +
 (a^4*c^3 - a^3*b*c^2 - 5*a^2*b^2*c - 3*a*b^3)*d^3)*x^5 + ((a^4*c^3 - 6*a^
3*b*c^2 - 7*a^2*b^2*c)*d^3 + (a^4*c^4 - 6*a^2*b^2*c^2 - 8*a*b^3*c - 3*b^4)
*d^2)*x^3)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(
sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) + (a^4*c^6 - (a^4*c^3 - 6*a^3*b*
c^2 - 7*a^2*b^2*c)*d^3*x^6 + 3*a^3*b*c^5 + 3*a^2*b^2*c^4 + a*b^3*c^3 - (a^
4*c^4 - 10*a^3*b*c^3 - 15*a^2*b^2*c^2 - 4*a*b^3*c)*d^2*x^4 + (a^4*c^5 + 7*
a^3*b*c^4 + 11*a^2*b^2*c^3 + 5*a*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)))/((a^6*c^5 + 4*a^5*b*c^4 + 6*a^4*b^2*c^3 + 4*a^3*b^3*c^2 + a^
2*b^4*c)*d*x^5 + (a^6*c^6 + 5*a^5*b*c^5 + 10*a^4*b^2*c^4 + 10*a^3*b^3*c^3
+ 5*a^2*b^4*c^2 + a*b^5*c)*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x^2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(1/x^4/(a+b/(d*x^2+c))^(3/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b)*c + 12*int((sqrt(c + d*x**2)
*sqrt(a*c + a*d*x**2 + b))/(3*a**4*c**5*x**2 + 9*a**4*c**4*d*x**4 + 9*a**4
*c**3*d**2*x**6 + 3*a**4*c**2*d**3*x**8 + 10*a**3*b*c**4*x**2 + 24*a**3*b*
c**3*d*x**4 + 18*a**3*b*c**2*d**2*x**6 + 4*a**3*b*c*d**3*x**8 + 12*a**2*b*
*2*c**3*x**2 + 22*a**2*b**2*c**2*d*x**4 + 11*a**2*b**2*c*d**2*x**6 + a**2*
b**2*d**3*x**8 + 6*a*b**3*c**2*x**2 + 8*a*b**3*c*d*x**4 + 2*a*b**3*d**2*x*
*6 + b**4*c*x**2 + b**4*d*x**4),x)*a**3*b*c**4*d*x**3 + 12*int((sqrt(c + d
*x**2)*sqrt(a*c + a*d*x**2 + b))/(3*a**4*c**5*x**2 + 9*a**4*c**4*d*x**4 +
9*a**4*c**3*d**2*x**6 + 3*a**4*c**2*d**3*x**8 + 10*a**3*b*c**4*x**2 + 24*a
**3*b*c**3*d*x**4 + 18*a**3*b*c**2*d**2*x**6 + 4*a**3*b*c*d**3*x**8 + 12*a
**2*b**2*c**3*x**2 + 22*a**2*b**2*c**2*d*x**4 + 11*a**2*b**2*c*d**2*x**6 +
 a**2*b**2*d**3*x**8 + 6*a*b**3*c**2*x**2 + 8*a*b**3*c*d*x**4 + 2*a*b**3*d
**2*x**6 + b**4*c*x**2 + b**4*d*x**4),x)*a**3*b*c**3*d**2*x**5 + 28*int((s
qrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(3*a**4*c**5*x**2 + 9*a**4*c**4*
d*x**4 + 9*a**4*c**3*d**2*x**6 + 3*a**4*c**2*d**3*x**8 + 10*a**3*b*c**4*x*
*2 + 24*a**3*b*c**3*d*x**4 + 18*a**3*b*c**2*d**2*x**6 + 4*a**3*b*c*d**3*x*
*8 + 12*a**2*b**2*c**3*x**2 + 22*a**2*b**2*c**2*d*x**4 + 11*a**2*b**2*c*d*
*2*x**6 + a**2*b**2*d**3*x**8 + 6*a*b**3*c**2*x**2 + 8*a*b**3*c*d*x**4 + 2
*a*b**3*d**2*x**6 + b**4*c*x**2 + b**4*d*x**4),x)*a**2*b**2*c**3*d*x**3 +
16*int((sqrt(c + d*x**2)*sqrt(a*c + a*d*x**2 + b))/(3*a**4*c**5*x**2 + ...
```

3.213 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^p \, dx = \frac{(c+dx^2)^2 \left(a+\frac{b}{c+dx^2}\right)^{1+p}}{4ad^2} + \frac{b(b+2ac-bp)\left(a+\frac{b}{c+dx^2}\right)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx^2)}\right)}{4a^3d^2(1+p)}$$

output
```
1/4*(d*x^2+c)^2*(a+b/(d*x^2+c))^(p+1)/a/d^2+1/4*b*(2*a*c-b*p+b)*(a+b/(d*x^
2+c))^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(d*x^2+c))/a^3/d^2/(p+1)
```

Mathematica [F]

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^p \, dx = \int x^3 \left(a+\frac{b}{c+dx^2}\right)^p \, dx$$

input
```
Integrate[x^3*(a + b/(c + d*x^2))^p,x]
```

output
```
Integrate[x^3*(a + b/(c + d*x^2))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^p dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\left(\frac{adx^2+b+ac}{dx^2+c}\right)^p dx^2$$

input
```
Int[x^3*(a + b/(c + d*x^2))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int x^3 \left(a + \frac{b}{d\,x^2 + c}\right)^p dx$$

input
```
int(x^3*(a+b/(d*x^2+c))^p,x)
```

output
```
int(x^3*(a+b/(d*x^2+c))^p,x)
```

Fricas [F]

$$\int x^3 \left(a + \frac{b}{c + dx^2}\right)^p dx = \int \left(a + \frac{b}{dx^2 + c}\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^p,x, algorithm="fricas")
```

output
```
integral(x^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^p, x)
```

Sympy [F]

$$\int x^3 \left(a + \frac{b}{c + dx^2}\right)^p dx = \int x^3 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^p dx$$

input
```
integrate(x**3*(a+b/(d*x**2+c))**p,x)
```

output
```
Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**p, x)
```

Maxima [F]

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\int\left(a+\frac{b}{dx^2+c}\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p*x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{-1,[
0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\int x^3\left(a+\frac{b}{d\,x^2+c}\right)^p\,dx$$

input
```
int(x^3*(a + b/(c + d*x^2))^p,x)
```

output
```
int(x^3*(a + b/(c + d*x^2))^p, x)
```

Reduce [F]

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^p dx=\int \frac{(ad\,x^2+ac+b)^p\,x^3}{(d\,x^2+c)^p}dx$$

input
```
int(x^3*(a+b/(d*x^2+c))^p,x)
```

output
```
int(((a*c + a*d*x**2 + b)**p*x**3)/(c + d*x**2)**p,x)
```

3.214 $\int x\left(a+\frac{b}{c+dx^2}\right)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p\,dx = -\frac{b\left(a+\frac{b}{c+dx^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx^2)}\right)}{2a^2d(1+p)}$$

output
```
-1/2*b*(a+b/(d*x^2+c))^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(d*x^2+c))/a^2
/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p\,dx = -\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^p\left(1+\frac{a(c+dx^2)}{b}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{a(c+dx^2)}{b}\right)}{2d(-1+p)}$$

input
```
Integrate[x*(a + b/(c + d*x^2))^p,x]
```

output
```
-1/2*((c + d*x^2)*(a + b/(c + d*x^2))^p*Hypergeometric2F1[1 - p, -p, 2 - p
, -((a*(c + d*x^2))/b)])/(d*(-1 + p)*(1 + (a*(c + d*x^2))/b)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2024, 773, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \left(a+\frac{b}{dx^2+c}\right)^p d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int (dx^2+c)^2\left(a+\frac{b}{dx^2+c}\right)^p d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{75}$$

$$-\frac{b\left(a+\frac{b}{c+dx^2}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(dx^2+c)}+1\right)}{2a^2d(p+1)}$$

input
```
Int[x*(a + b/(c + d*x^2))^p,x]
```

output
```
-1/2*(b*(a + b/(c + d*x^2))^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
 b/(a*(c + d*x^2))])/(a^2*d*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [F]

$$\int x\left(a+\frac{b}{d\,x^{2}+c}\right)^{p}dx$$

input
```
int(x*(a+b/(d*x^2+c))^p,x)
```

output
```
int(x*(a+b/(d*x^2+c))^p,x)
```

Fricas [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\int\left(a+\frac{b}{dx^2+c}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(d*x^2+c))^p,x, algorithm="fricas")
```

output
```
integral(x*((a*d*x^2 + a*c + b)/(d*x^2 + c))^p, x)
```

Sympy [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p dx=\int x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p dx$$

input
```
integrate(x*(a+b/(d*x**2+c))**p,x)
```

output
```
Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**p, x)
```

Maxima [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p dx=\int\left(a+\frac{b}{dx^2+c}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(d*x^2+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p*x, x)
```

Giac [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p dx=\int\left(a+\frac{b}{dx^2+c}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(d*x^2+c))^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p*x, x)
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p\,dx=-\frac{\left(a+\frac{b}{d\,x^2+c}\right)^p\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(1-p,-p;\ 2-p;\ -\frac{a\,\left(d\,x^2+c\right)}{b}\right)}{2\,d\,{\left(\frac{a\,\left(d\,x^2+c\right)}{b}+1\right)}^p\,\left(p-1\right)}$$

input
```
int(x*(a + b/(c + d*x^2))^p,x)
```

output
```
-((a + b/(c + d*x^2))^p*(c + d*x^2)*hypergeom([1 - p, -p], 2 - p, -(a*(c +
 d*x^2))/b))/(2*d*((a*(c + d*x^2))/b + 1)^p*(p - 1))
```

Reduce [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\int\frac{\left(ad\,x^2+ac+b\right)^p x}{\left(d\,x^2+c\right)^p}dx$$

input
```
int(x*(a+b/(d*x^2+c))^p,x)
```

output
```
int(((a*c + a*d*x**2 + b)**p*x)/(c + d*x**2)**p,x)
```

3.215 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x} dx = -\frac{c\left(a+\frac{b}{c+dx^2}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx^2}\right)}{b+ac}\right)}{2(b+ac)(1+p)} + \frac{\left(a+\frac{b}{c+dx^2}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a(c+dx^2)}\right)}{2a(1+p)}$$

output
```
-1/2*c*(a+b/(d*x^2+c))^(p+1)*hypergeom([1, p+1],[2+p],c*(a+b/(d*x^2+c))/(a
*c+b))/(a*c+b)/(p+1)+1/2*(a+b/(d*x^2+c))^(p+1)*hypergeom([1, p+1],[2+p],1+
b/a/(d*x^2+c))/a/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x} dx = \int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x} dx$$

input
```
Integrate[(a + b/(c + d*x^2))^p/x,x]
```

output
```
Integrate[(a + b/(c + d*x^2))^p/x, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x} \, dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p}{x} dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2} \int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^p}{x^2} dx^2$$

input
```
Int[(a + b/(c + d*x^2))^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int \frac{\left(a + \frac{b}{d x^2 + c}\right)^p}{x} dx$$

input
```
int((a+b/(d*x^2+c))^p/x,x)
```

output
```
int((a+b/(d*x^2+c))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a + \frac{b}{c + d x^2}\right)^p}{x} dx = \int \frac{\left(a + \frac{b}{d x^2 + c}\right)^p}{x} dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x,x, algorithm="fricas")
```

output
```
integral(((a*d*x^2 + a*c + b)/(d*x^2 + c))^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a + \frac{b}{c + d x^2}\right)^p}{x} dx = \int \frac{\left(\frac{ac + a d x^2 + b}{c + d x^2}\right)^p}{x} dx$$

input
```
integrate((a+b/(d*x**2+c))**p/x,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^p}{x}\,dx$$

input
```
int((a + b/(c + d*x^2))^p/x,x)
```

output
```
int((a + b/(c + d*x^2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x}\,dx = \int \frac{(ad\,x^2+ac+b)^p}{(d\,x^2+c)^p\,x}dx$$

input
```
int((a+b/(d*x^2+c))^p/x,x)
```

output
```
int((a*c + a*d*x**2 + b)**p/((c + d*x**2)**p*x),x)
```

3.216 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx = -\frac{bd\left(a+\frac{b}{c+dx^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx^2}\right)}{b+ac}\right)}{2(b+ac)^2(1+p)}$$

output
```
-1/2*b*d*(a+b/(d*x^2+c))^(p+1)*hypergeom([2, p+1],[2+p],c*(a+b/(d*x^2+c))/
(a*c+b))/(a*c+b)^2/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx = \int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx$$

input
```
Integrate[(a + b/(c + d*x^2))^p/x^3,x]
```

output
```
Integrate[(a + b/(c + d*x^2))^p/x^3, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3} \, dx$$

$$\Big\downarrow \ 2057$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p}{x^3}dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^p}{x^4}dx^2$$

input
```
Int[(a + b/(c + d*x^2))^p/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int \frac{\left(a + \frac{b}{d x^2+c}\right)^p}{x^3} dx$$

input
```
int((a+b/(d*x^2+c))^p/x^3,x)
```

output
```
int((a+b/(d*x^2+c))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^p}{x^3} dx = \int \frac{\left(a + \frac{b}{dx^2+c}\right)^p}{x^3} dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^3,x, algorithm="fricas")
```

output
```
integral(((a*d*x^2 + a*c + b)/(d*x^2 + c))^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^p}{x^3} dx = \text{Timed out}$$

input
```
integrate((a+b/(d*x**2+c))**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^p}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^2))^p/x^3,x)
```

output
```
int((a + b/(c + d*x^2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^3}\,dx = \int \frac{(ad\,x^2+ac+b)^p}{(d\,x^2+c)^p\,x^3}dx$$

input
```
int((a+b/(d*x^2+c))^p/x^3,x)
```

output
```
int((a*c + a*d*x**2 + b)**p/((c + d*x**2)**p*x**3),x)
```

3.217 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx = -\frac{\left(c+dx^2\right)^2\left(a+\frac{b}{c+dx^2}\right)^{1+p}}{4c(b+ac)x^4} + \frac{bd^2(b+2ac+bp)\left(a+\frac{b}{c+dx^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx^2}\right)}{b+ac}\right)}{4c(b+ac)^3(1+p)}$$

output
```
-1/4*(d*x^2+c)^2*(a+b/(d*x^2+c))^(p+1)/c/(a*c+b)/x^4+1/4*b*d^2*(2*a*c+b*p+
b)*(a+b/(d*x^2+c))^(p+1)*hypergeom([2, p+1],[2+p],c*(a+b/(d*x^2+c))/(a*c+b
))/c/(a*c+b)^3/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx=\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx$$

input
```
Integrate[(a + b/(c + d*x^2))^p/x^5,x]
```

output
```
Integrate[(a + b/(c + d*x^2))^p/x^5, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p}{x^5}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^p}{x^6}dx^2$$

input
```
Int[(a + b/(c + d*x^2))^p/x^5,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{d x^{2}+c}\right)^{p}}{x^{5}}d x$$

input
```
int((a+b/(d*x^2+c))^p/x^5,x)
```

output
```
int((a+b/(d*x^2+c))^p/x^5,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^5}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^5,x, algorithm="fricas")
```

output
```
integral(((a*d*x^2 + a*c + b)/(d*x^2 + c))^p/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(d*x**2+c))**p/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx = \int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^5}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^5,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^5, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx = \int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^5}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^5,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx = \int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^p}{x^5}\,dx$$

input
```
int((a + b/(c + d*x^2))^p/x^5,x)
```

output
```
int((a + b/(c + d*x^2))^p/x^5, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^5}\,dx = \int \frac{\left(ad\,x^2+ac+b\right)^p}{\left(d\,x^2+c\right)^p x^5}dx$$

input
```
int((a+b/(d*x^2+c))^p/x^5,x)
```

output
```
int((a*c + a*d*x**2 + b)**p/((c + d*x**2)**p*x**5),x)
```

3.218 $\int x^2 \left(a+\frac{b}{c+dx^2}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^p \, dx = \frac{1}{3}x^3\left(1+\frac{dx^2}{c}\right)^p\left(1+\frac{adx^2}{b+ac}\right)^{-p}\left(a+\frac{b}{c+dx^2}\right)^p \operatorname{AppellF1}\left(\frac{3}{2},p,-p,\frac{5}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)$$

output
```
1/3*x^3*(1+d*x^2/c)^p*(a+b/(d*x^2+c))^p*AppellF1(3/2,p,-p,5/2,-d*x^2/c,-a*
d*x^2/(a*c+b))/((1+a*d*x^2/(a*c+b))^p)
```

Mathematica [F]

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^p \, dx = \int x^2 \left(a+\frac{b}{c+dx^2}\right)^p \, dx$$

input
```
Integrate[x^2*(a + b/(c + d*x^2))^p,x]
```

output
```
Integrate[x^2*(a + b/(c + d*x^2))^p, x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2057, 2058, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^p dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^2 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$(c+dx^2)^p (ac+adx^2+b)^{-p} \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \int x^2 (dx^2+c)^{-p} (adx^2+b+ac)^p dx$$

$$\Big\downarrow \text{395}$$

$$\left(\frac{dx^2}{c}+1\right)^p (ac+adx^2+b)^{-p} \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \int x^2 (adx^2+b+ac)^p \left(\frac{dx^2}{c}+1\right)^{-p} dx$$

$$\Big\downarrow \text{395}$$

$$\left(\frac{dx^2}{c}+1\right)^p \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \left(\frac{adx^2}{ac+b}+1\right)^{-p} \int x^2 \left(\frac{dx^2}{c}+1\right)^{-p} \left(\frac{adx^2}{b+ac}+1\right)^p dx$$

$$\Big\downarrow \text{394}$$

$$\frac{1}{3}x^3 \left(\frac{dx^2}{c}+1\right)^p \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \left(\frac{adx^2}{ac+b}+1\right)^{-p} \text{AppellF1}\left(\frac{3}{2},p,-p,\frac{5}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)$$

input
```
Int[x^2*(a + b/(c + d*x^2))^p,x]
```

output
```
(x^3*((b + a*c + a*d*x^2)/(c + d*x^2))^p*(1 + (d*x^2)/c)^p*AppellF1[3/2, p
, -p, 5/2, -((d*x^2)/c), -((a*d*x^2)/(b + a*c))])/(3*(1 + (a*d*x^2)/(b + a
*c))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int x^2\left(a+\frac{b}{d\,x^2+c}\right)^p dx$$

input
```
int(x^2*(a+b/(d*x^2+c))^p,x)
```

output
```
int(x^2*(a+b/(d*x^2+c))^p,x)
```

Fricas [F]

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^p dx = \int \left(a+\frac{b}{dx^2+c}\right)^p x^2 dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^p,x, algorithm="fricas")
```

output
```
integral(x^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^p, x)
```

Sympy [F]

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^p dx = \int x^2 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p dx$$

input
```
integrate(x**2*(a+b/(d*x**2+c))**p,x)
```

output
```
Integral(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**p, x)
```

Maxima [F]

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^p dx = \int \left(a+\frac{b}{dx^2+c}\right)^p x^2 dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p*x^2, x)
```

Giac [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\int\left(a+\frac{b}{dx^2+c}\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\int x^2\left(a+\frac{b}{d\,x^2+c}\right)^p\,dx$$

input
```
int(x^2*(a + b/(c + d*x^2))^p,x)
```

output
```
int(x^2*(a + b/(c + d*x^2))^p, x)
```

Reduce [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^p\,dx=\int\frac{(ad\,x^2+ac+b)^p\,x^2}{(d\,x^2+c)^p}dx$$

input
```
int(x^2*(a+b/(d*x^2+c))^p,x)
```

output
```
int(((a*c + a*d*x**2 + b)**p*x**2)/(c + d*x**2)**p,x)
```

3.219 $\int \left(a+\frac{b}{c+dx^2}\right)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \left(a+\frac{b}{c+dx^2}\right)^p \, dx = x\left(1+\frac{dx^2}{c}\right)^p \left(1+\frac{adx^2}{b+ac}\right)^{-p} \left(a+\frac{b}{c+dx^2}\right)^p \operatorname{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)$$

output
```
x*(1+d*x^2/c)^p*(a+b/(d*x^2+c))^p*AppellF1(1/2,p,-p,3/2,-d*x^2/c,-a*d*x^2/
(a*c+b))/((1+a*d*x^2/(a*c+b))^p)
```

Mathematica [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^p \, dx = \int \left(a+\frac{b}{c+dx^2}\right)^p \, dx$$

input
```
Integrate[(a + b/(c + d*x^2))^p,x]
```

output
```
Integrate[(a + b/(c + d*x^2))^p, x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2057, 2058, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{c+dx^2}\right)^p dx$$

$$\Big\downarrow \text{2057}$$

$$\int \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$\left(c+dx^2\right)^p \left(ac+adx^2+b\right)^{-p} \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \int \left(dx^2+c\right)^{-p} \left(adx^2+b+ac\right)^p dx$$

$$\Big\downarrow \text{334}$$

$$\left(\frac{dx^2}{c}+1\right)^p \left(ac+adx^2+b\right)^{-p} \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \int \left(adx^2+b+ac\right)^p \left(\frac{dx^2}{c}+1\right)^{-p} dx$$

$$\Big\downarrow \text{334}$$

$$\left(\frac{dx^2}{c}+1\right)^p \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \left(\frac{adx^2}{ac+b}+1\right)^{-p} \int \left(\frac{dx^2}{c}+1\right)^{-p} \left(\frac{adx^2}{b+ac}+1\right)^p dx$$

$$\Big\downarrow \text{333}$$

$$x\left(\frac{dx^2}{c}+1\right)^p \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \left(\frac{adx^2}{ac+b}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)$$

input
```
Int[(a + b/(c + d*x^2))^p,x]
```

output
```
(x*((b + a*c + a*d*x^2)/(c + d*x^2))^p*(1 + (d*x^2)/c)^p*AppellF1[1/2, p,
-p, 3/2, -((d*x^2)/c), -((a*d*x^2)/(b + a*c))])/(1 + (a*d*x^2)/(b + a*c))^
p
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \left(a+\frac{b}{d\,x^{2}+c}\right)^{p}dx$$

input
```
int((a+b/(d*x^2+c))^p,x)
```

output
```
int((a+b/(d*x^2+c))^p,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^p dx = \int \left(a+\frac{b}{dx^2+c}\right)^p dx$$

input
```
integrate((a+b/(d*x^2+c))^p,x, algorithm="fricas")
```

output
```
integral(((a*d*x^2 + a*c + b)/(d*x^2 + c))^p, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^p dx = \int \left(a+\frac{b}{c+dx^2}\right)^p dx$$

input
```
integrate((a+b/(d*x**2+c))**p,x)
```

output
```
Integral((a + b/(c + d*x**2))**p, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^p dx = \int \left(a+\frac{b}{dx^2+c}\right)^p dx$$

input
```
integrate((a+b/(d*x^2+c))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p, x)
```

Giac [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^p dx=\int \left(a+\frac{b}{dx^2+c}\right)^p dx$$

input
```
integrate((a+b/(d*x^2+c))^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^2}\right)^p dx=\int \left(a+\frac{b}{d\,x^2+c}\right)^p dx$$

input
```
int((a + b/(c + d*x^2))^p,x)
```

output
```
int((a + b/(c + d*x^2))^p, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^p dx=\int \frac{(ad\,x^2+ac+b)^p}{(d\,x^2+c)^p}dx$$

input
```
int((a+b/(d*x^2+c))^p,x)
```

output
```
int((a*c + a*d*x**2 + b)**p/(c + d*x**2)**p,x)
```

3.220 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2} dx = -\frac{\left(1+\frac{dx^2}{c}\right)^p \left(1+\frac{adx^2}{b+ac}\right)^{-p} \left(a+\frac{b}{c+dx^2}\right)^p \text{AppellF1}\left(-\frac{1}{2},p,-p,\frac{1}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)}{x}$$

output
```
-(1+d*x^2/c)^p*(a+b/(d*x^2+c))^p*AppellF1(-1/2,p,-p,1/2,-d*x^2/c,-a*d*x^2/
(a*c+b))/x/((1+a*d*x^2/(a*c+b))^p)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2} dx = \int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2} dx$$

input
```
Integrate[(a + b/(c + d*x^2))^p/x^2,x]
```

output
```
Integrate[(a + b/(c + d*x^2))^p/x^2, x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2057, 2058, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{2058}$$

$$(c+dx^2)^p\left(ac+adx^2+b\right)^{-p}\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\int \frac{(dx^2+c)^{-p}\left(adx^2+b+ac\right)^p}{x^2}dx$$

$$\Big\downarrow \text{395}$$

$$\left(\frac{dx^2}{c}+1\right)^p\left(ac+adx^2+b\right)^{-p}\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\int \frac{\left(adx^2+b+ac\right)^p\left(\frac{dx^2}{c}+1\right)^{-p}}{x^2}dx$$

$$\Big\downarrow \text{395}$$

$$\left(\frac{dx^2}{c}+1\right)^p\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\left(\frac{adx^2}{ac+b}+1\right)^{-p}\int \frac{\left(\frac{dx^2}{c}+1\right)^{-p}\left(\frac{adx^2}{b+ac}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{394}$$

$$-\frac{\left(\frac{dx^2}{c}+1\right)^p\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\left(\frac{adx^2}{ac+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},p,-p,\frac{1}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)}{x}$$

input
```
Int[(a + b/(c + d*x^2))^p/x^2,x]
```

output
```
-((((b + a*c + a*d*x^2)/(c + d*x^2))^p*(1 + (d*x^2)/c)^p*AppellF1[-1/2, p,
 -p, 1/2, -((d*x^2)/c), -((a*d*x^2)/(b + a*c))])/(x*(1 + (a*d*x^2)/(b + a*
c))^p))
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{d x^2+c}\right)^p}{x^2}dx$$

input
```
int((a+b/(d*x^2+c))^p/x^2,x)
```

output
```
int((a+b/(d*x^2+c))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^2,x, algorithm="fricas")
```

output
```
integral(((a*d*x^2 + a*c + b)/(d*x^2 + c))^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b/(d*x**2+c))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^p}{x^2}\,dx$$

input
```
int((a + b/(c + d*x^2))^p/x^2,x)
```

output
```
int((a + b/(c + d*x^2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^2}\,dx=\int \frac{(ad\,x^2+ac+b)^p}{(d\,x^2+c)^p\,x^2}dx$$

input
```
int((a+b/(d*x^2+c))^p/x^2,x)
```

output
```
int((a*c + a*d*x**2 + b)**p/((c + d*x**2)**p*x**2),x)
```

3.221 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx = -\frac{\left(1+\frac{dx^2}{c}\right)^p\left(1+\frac{adx^2}{b+ac}\right)^{-p}\left(a+\frac{b}{c+dx^2}\right)^p \operatorname{AppellF1}\left(-\frac{3}{2},p,-p,-\frac{1}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)}{3x^3}$$

output
```
-1/3*(1+d*x^2/c)^p*(a+b/(d*x^2+c))^p*AppellF1(-3/2,p,-p,-1/2,-d*x^2/c,-a*d
*x^2/(a*c+b))/x^3/((1+a*d*x^2/(a*c+b))^p)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx = \int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx$$

input
```
Integrate[(a + b/(c + d*x^2))^p/x^4,x]
```

output
```
Integrate[(a + b/(c + d*x^2))^p/x^4, x]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2057, 2058, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p}{x^4}dx$$

$$\Big\downarrow \text{2058}$$

$$(c+dx^2)^p\,(ac+adx^2+b)^{-p}\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\int \frac{(dx^2+c)^{-p}\,(adx^2+b+ac)^p}{x^4}dx$$

$$\Big\downarrow \text{395}$$

$$\left(\frac{dx^2}{c}+1\right)^p(ac+adx^2+b)^{-p}\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\int \frac{(adx^2+b+ac)^p\left(\frac{dx^2}{c}+1\right)^{-p}}{x^4}dx$$

$$\Big\downarrow \text{395}$$

$$\left(\frac{dx^2}{c}+1\right)^p\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\left(\frac{adx^2}{ac+b}+1\right)^{-p}\int \frac{\left(\frac{dx^2}{c}+1\right)^{-p}\left(\frac{adx^2}{b+ac}+1\right)^p}{x^4}dx$$

$$\Big\downarrow \text{394}$$

$$-\frac{\left(\frac{dx^2}{c}+1\right)^p\left(\frac{ac+adx^2+b}{c+dx^2}\right)^p\left(\frac{adx^2}{ac+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{3}{2},p,-p,-\frac{1}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)}{3x^3}$$

input
```
Int[(a + b/(c + d*x^2))^p/x^4,x]
```

output
```
-1/3*(((b + a*c + a*d*x^2)/(c + d*x^2))^p*(1 + (d*x^2)/c)^p*AppellF1[-3/2,
 p, -p, -1/2, -((d*x^2)/c), -((a*d*x^2)/(b + a*c))])/(x^3*(1 + (a*d*x^2)/(
b + a*c))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{d x^2+c}\right)^p}{x^4}dx$$

input
```
int((a+b/(d*x^2+c))^p/x^4,x)
```

output
```
int((a+b/(d*x^2+c))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^4,x, algorithm="fricas")
```

output
```
integral(((a*d*x^2 + a*c + b)/(d*x^2 + c))^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b/(d*x**2+c))**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^p/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^p}{x^4}\,dx$$

input
```
int((a + b/(c + d*x^2))^p/x^4,x)
```

output
```
int((a + b/(c + d*x^2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^p}{x^4}\,dx=\int \frac{(ad\,x^2+ac+b)^p}{(d\,x^2+c)^p\,x^4}dx$$

input
```
int((a+b/(d*x^2+c))^p/x^4,x)
```

output
```
int((a*c + a*d*x**2 + b)**p/((c + d*x**2)**p*x**4),x)
```

3.222 $\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p \, dx = \frac{(ex)^{1+m}\left(1+\frac{dx^2}{c}\right)^p\left(1+\frac{adx^2}{b+ac}\right)^{-p}\left(a+\frac{b}{c+dx^2}\right)^p \text{AppellF1}\left(\frac{1+m}{2},p,-p,\frac{3+m}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(1+d*x^2/c)^p*(a+b/(d*x^2+c))^p*AppellF1(1/2+1/2*m,p,-p,3/2+1/
2*m,-d*x^2/c,-a*d*x^2/(a*c+b))/e/(1+m)/((1+a*d*x^2/(a*c+b))^p)
```

Mathematica [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p \, dx = \int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b/(c + d*x^2))^p,x]
```

output
```
Integrate[(e*x)^m*(a + b/(c + d*x^2))^p, x]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2057, 2058, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p dx$$
$$\Big\downarrow \text{2057}$$
$$\int (ex)^m \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p dx$$
$$\Big\downarrow \text{2058}$$
$$(c+dx^2)^p (ac+adx^2+b)^{-p} \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \int (ex)^m (dx^2+c)^{-p} (adx^2+b+ac)^p dx$$
$$\Big\downarrow \text{395}$$
$$\left(\frac{dx^2}{c}+1\right)^p (ac+adx^2+b)^{-p} \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \int (ex)^m (adx^2+b+ac)^p \left(\frac{dx^2}{c}+1\right)^{-p} dx$$
$$\Big\downarrow \text{395}$$
$$\left(\frac{dx^2}{c}+1\right)^p \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \left(\frac{adx^2}{ac+b}+1\right)^{-p} \int (ex)^m \left(\frac{dx^2}{c}+1\right)^{-p} \left(\frac{adx^2}{b+ac}+1\right)^p dx$$
$$\Big\downarrow \text{394}$$
$$\frac{(ex)^{m+1} \left(\frac{dx^2}{c}+1\right)^p \left(\frac{ac+adx^2+b}{c+dx^2}\right)^p \left(\frac{adx^2}{ac+b}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2},p,-p,\frac{m+3}{2},-\frac{dx^2}{c},-\frac{adx^2}{b+ac}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b/(c + d*x^2))^p,x]
```

output
```
((e*x)^(1 + m)*((b + a*c + a*d*x^2)/(c + d*x^2))^p*(1 + (d*x^2)/c)^p*Appel
lF1[(1 + m)/2, p, -p, (3 + m)/2, -((d*x^2)/c), -((a*d*x^2)/(b + a*c))])/(e
*(1 + m)*(1 + (a*d*x^2)/(b + a*c))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int (ex)^m \left(a+\frac{b}{d\,x^2+c}\right)^p dx$$

input
```
int((e*x)^m*(a+b/(d*x^2+c))^p,x)
```

output
```
int((e*x)^m*(a+b/(d*x^2+c))^p,x)
```

Fricas [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx^2+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x^2+c))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*((a*d*x^2 + a*c + b)/(d*x^2 + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b/(d*x**2+c))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx^2+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x^2+c))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(a + b/(d*x^2 + c))^p, x)
```

Giac [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx^2+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(d*x^2+c))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(a + b/(d*x^2 + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p dx = \int (e\,x)^m \left(a+\frac{b}{d\,x^2+c}\right)^p dx$$

input
```
int((e*x)^m*(a + b/(c + d*x^2))^p,x)
```

output
```
int((e*x)^m*(a + b/(c + d*x^2))^p, x)
```

Reduce [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^2}\right)^p dx = e^m \left(\int \frac{x^m (ad\,x^2+ac+b)^p}{(d\,x^2+c)^p}dx\right)$$

input
```
int((e*x)^m*(a+b/(d*x^2+c))^p,x)
```

output
```
e**m*int((x**m*(a*c + a*d*x**2 + b)**p)/(c + d*x**2)**p,x)
```

3.223 $\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{bx^2}{2d^2}+\frac{ax^6}{6}-\frac{bc^2}{2d^3(c+dx^2)}-\frac{bc\log(c+dx^2)}{d^3}$$

output
```
1/2*b*x^2/d^2+1/6*a*x^6-1/2*b*c^2/d^3/(d*x^2+c)-b*c*ln(d*x^2+c)/d^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{1}{6}\left(ax^6+\frac{3b\left(dx^2-\frac{c^2}{c+dx^2}\right)}{d^3}-\frac{6bc\log(c+dx^2)}{d^3}\right)$$

input
```
Integrate[x^5*(a + b/(c + d*x^2)^2),x]
```

output
```
(a*x^6 + (3*b*(d*x^2 - c^2/(c + d*x^2)))/d^3 - (6*b*c*Log[c + d*x^2])/d^3)
/6
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{(c+dx^2)^2}\right) \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^5+\frac{bc^2x}{d^2(c+dx^2)^2}-\frac{2bcx}{d^2(c+dx^2)}+\frac{bx}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^6}{6}-\frac{bc^2}{2d^3(c+dx^2)}-\frac{bc\log(c+dx^2)}{d^3}+\frac{bx^2}{2d^2}$$

input
```
Int[x^5*(a + b/(c + d*x^2)^2),x]
```

output
```
(b*x^2)/(2*d^2) + (a*x^6)/6 - (b*c^2)/(2*d^3*(c + d*x^2)) - (b*c*Log[c + d
*x^2])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{b\,x^{2}}{2 d^{2}}+\frac{a\,x^{6}}{6}-\frac{b\,c^{2}}{2 d^{3} \left(d\,x^{2}+c \right)}-\frac{b c \ln \left(d\,x^{2}+c \right)}{d^{3}}$	50
default	$\frac{\frac{1}{6} a\,x^{6} d^{2}+\frac{1}{2} b\,x^{2}}{d^{2}}-\frac{b c \left(\frac{c}{d \left(d\,x^{2}+c \right)}+\frac{2 \ln \left(d\,x^{2}+c \right)}{d}\right)}{2 d^{2}}$	57
norman	$\frac{\frac{a c\,x^{6}}{6}+\frac{a d\,x^{8}}{6}+\frac{b\,x^{4}}{2 d}-\frac{c^{2} b}{d^{3}}}{d\,x^{2}+c}-\frac{b c \ln \left(d\,x^{2}+c \right)}{d^{3}}$	60
parallelrisch	$-\frac{-a\,d^{4} x^{8}-a c\,x^{6} d^{3}-3 b\,d^{2} x^{4}+6 \ln \left(d\,x^{2}+c \right) x^{2} b c d+6 \ln \left(d\,x^{2}+c \right) b\,c^{2}+6 b\,c^{2}}{6 d^{3} \left(d\,x^{2}+c \right)}$	80

input
```
int(x^5*(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2/d^2+1/6*a*x^6-1/2*b*c^2/d^3/(d*x^2+c)-b*c*ln(d*x^2+c)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx = \frac{ad^4x^8+acd^3x^6+3\,bd^2x^4+3\,bcdx^2-3\,bc^2-6\,(bcdx^2+bc^2)\log\left(dx^2+c\right)}{6\,(d^4x^2+cd^3)}$$

input
```
integrate(x^5*(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
1/6*(a*d^4*x^8 + a*c*d^3*x^6 + 3*b*d^2*x^4 + 3*b*c*d*x^2 - 3*b*c^2 - 6*(b*
c*d*x^2 + b*c^2)*log(d*x^2 + c))/(d^4*x^2 + c*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^6}{6}-\frac{bc^2}{2cd^3+2d^4x^2}-\frac{bc\log{(c+dx^2)}}{d^3}+\frac{bx^2}{2d^2}$$

input
```
integrate(x**5*(a+b/(d*x**2+c)**2),x)
```

output
```
a*x**6/6 - b*c**2/(2*c*d**3 + 2*d**4*x**2) - b*c*log(c + d*x**2)/d**3 + b*
x**2/(2*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=-\frac{bc^2}{2\,(d^4x^2+cd^3)}-\frac{bc\log{(dx^2+c)}}{d^3}+\frac{ad^2x^6+3\,bx^2}{6\,d^2}$$

input
```
integrate(x^5*(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
-1/2*b*c^2/(d^4*x^2 + c*d^3) - b*c*log(d*x^2 + c)/d^3 + 1/6*(a*d^2*x^6 + 3
*b*x^2)/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=-\frac{bc\log{(|dx^2+c|)}}{d^3}+\frac{2\,bcdx^2+bc^2}{2\,(dx^2+c)d^3}+\frac{ad^6x^6+3\,bd^4x^2}{6\,d^6}$$

input
```
integrate(x^5*(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
-b*c*log(abs(d*x^2 + c))/d^3 + 1/2*(2*b*c*d*x^2 + b*c^2)/((d*x^2 + c)*d^3)
 + 1/6*(a*d^6*x^6 + 3*b*d^4*x^2)/d^6
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx = x^2\left(\frac{a\,c^2+b}{2\,d^2}-\frac{a\,c^2}{2\,d^2}\right)+\frac{a\,x^6}{6}-\frac{b\,c^2}{2\,d\,\left(d^3\,x^2+c\,d^2\right)}-\frac{b\,c\,\ln\left(d\,x^2+c\right)}{d^3}$$

input
```
int(x^5*(a + b/(c + d*x^2)^2),x)
```

output
```
x^2*((b + a*c^2)/(2*d^2) - (a*c^2)/(2*d^2)) + (a*x^6)/6 - (b*c^2)/(2*d*(c*
d^2 + d^3*x^2)) - (b*c*log(c + d*x^2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int x^5\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx = \frac{-6\log(d\,x^2+c)\,b\,c^2-6\log(d\,x^2+c)\,bcd\,x^2+ac\,d^3x^6+a\,d^4x^8+6bcd\,x^2+3b\,d^2x^4}{6d^3\,(d\,x^2+c)}$$

input
```
int(x^5*(a+b/(d*x^2+c)^2),x)
```

output
```
( - 6*log(c + d*x**2)*b*c**2 - 6*log(c + d*x**2)*b*c*d*x**2 + a*c*d**3*x**
6 + a*d**4*x**8 + 6*b*c*d*x**2 + 3*b*d**2*x**4)/(6*d**3*(c + d*x**2))
```

3.224 $\int x^3 \left(a+\frac{b}{(c+dx^2)^2}\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int x^3 \left(a+\frac{b}{(c+dx^2)^2}\right) dx = \frac{ax^4}{4}+\frac{bc}{2d^2(c+dx^2)}+\frac{b\log(c+dx^2)}{2d^2}$$

output
```
1/4*a*x^4+1/2*b*c/d^2/(d*x^2+c)+1/2*b*ln(d*x^2+c)/d^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^3 \left(a+\frac{b}{(c+dx^2)^2}\right) dx = \frac{1}{4}\left(ax^4+\frac{2bc}{d^2(c+dx^2)}+\frac{2b\log(c+dx^2)}{d^2}\right)$$

input
```
Integrate[x^3*(a + b/(c + d*x^2)^2),x]
```

output
```
(a*x^4 + (2*b*c)/(d^2*(c + d*x^2)) + (2*b*Log[c + d*x^2])/d^2)/4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a + \frac{b}{(c + dx^2)^2}\right) dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^3 + \frac{bx}{d (c + dx^2)} - \frac{bcx}{d (c + dx^2)^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^4}{4} + \frac{bc}{2d^2 (c + dx^2)} + \frac{b \log (c + dx^2)}{2d^2}$$

input
```
Int[x^3*(a + b/(c + d*x^2)^2),x]
```

output
```
(a*x^4)/4 + (b*c)/(2*d^2*(c + d*x^2)) + (b*Log[c + d*x^2])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{a\,x^{4}}{4}+\frac{b\left(\frac{c}{d^{2}\left(d\,x^{2}+c\right)}+\frac{\ln\left(d\,x^{2}+c\right)}{d^{2}}\right)}{2}$	38
risch	$\frac{a\,x^{4}}{4}+\frac{bc}{2d^{2}\left(d\,x^{2}+c\right)}+\frac{b\ln\left(d\,x^{2}+c\right)}{2d^{2}}$	38
norman	$\frac{\frac{ac\,x^{4}}{4}+\frac{ad\,x^{6}}{4}+\frac{bc}{2d^{2}}}{d\,x^{2}+c}+\frac{b\ln\left(d\,x^{2}+c\right)}{2d^{2}}$	48
parallelrisch	$\frac{a\,d^{3}x^{6}+ac\,d^{2}x^{4}+2\ln\left(d\,x^{2}+c\right)x^{2}bd+2\ln\left(d\,x^{2}+c\right)bc+2bc}{4d^{2}\left(d\,x^{2}+c\right)}$	64

input
```
int(x^3*(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/2*b*(c/d^2/(d*x^2+c)+1/d^2*ln(d*x^2+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int x^3\left(a+\frac{b}{\left(c+dx^2\right)^2}\right)\,dx=\frac{ad^3x^6+acd^2x^4+2\,bc+2\left(bdx^2+bc\right)\log\left(dx^2+c\right)}{4\left(d^3x^2+cd^2\right)}$$

input
```
integrate(x^3*(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*x^6 + a*c*d^2*x^4 + 2*b*c + 2*(b*d*x^2 + b*c)*log(d*x^2 + c))/(
d^3*x^2 + c*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x^3\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^4}{4}+\frac{bc}{2cd^2+2d^3x^2}+\frac{b\log{(c+dx^2)}}{2d^2}$$

input
```
integrate(x**3*(a+b/(d*x**2+c)**2),x)
```

output
```
a*x**4/4 + b*c/(2*c*d**2 + 2*d**3*x**2) + b*log(c + d*x**2)/(2*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^3\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{1}{4}\,ax^4+\frac{bc}{2\,(d^3x^2+cd^2)}+\frac{b\log{(dx^2+c)}}{2\,d^2}$$

input
```
integrate(x^3*(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/2*b*c/(d^3*x^2 + c*d^2) + 1/2*b*log(d*x^2 + c)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int x^3\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{1}{4}\,ax^4+\frac{b\log{(|dx^2+c|)}}{2\,d^2}+\frac{bc}{2\,(dx^2+c)d^2}$$

input
```
integrate(x^3*(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
1/4*a*x^4 + 1/2*b*log(abs(d*x^2 + c))/d^2 + 1/2*b*c/((d*x^2 + c)*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^3\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{a\,x^4}{4}+\frac{b\,\ln\left(d\,x^2+c\right)}{2\,d^2}+\frac{b\,c}{2\,d^2\,\left(d\,x^2+c\right)}$$

input
```
int(x^3*(a + b/(c + d*x^2)^2),x)
```

output
```
(a*x^4)/4 + (b*log(c + d*x^2))/(2*d^2) + (b*c)/(2*d^2*(c + d*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int x^3\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$
$$=\frac{2\log(d\,x^2+c)\,bc+2\log(d\,x^2+c)\,bd\,x^2+ac\,d^2x^4+a\,d^3x^6-2bd\,x^2}{4d^2\,(d\,x^2+c)}$$

input
```
int(x^3*(a+b/(d*x^2+c)^2),x)
```

output
```
(2*log(c + d*x**2)*b*c + 2*log(c + d*x**2)*b*d*x**2 + a*c*d**2*x**4 + a*d*
*3*x**6 - 2*b*d*x**2)/(4*d**2*(c + d*x**2))
```

3.225 $\int x\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int x\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^2}{2}-\frac{b}{2d\,(c+dx^2)}$$

output
```
1/2*a*x^2-1/2*b/d/(d*x^2+c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int x\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=-\frac{b}{2d\,(c+dx^2)}+\frac{a(c+dx^2)}{2d}$$

input
```
Integrate[x*(a + b/(c + d*x^2)^2),x]
```

output
```
-1/2*b/(d*(c + d*x^2)) + (a*(c + d*x^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$

$$\Big\downarrow \; 2010$$

$$\int \left(ax+\frac{bx}{(c+dx^2)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^2}{2}-\frac{b}{2d\,(c+dx^2)}$$

input
```
Int[x*(a + b/(c + d*x^2)^2),x]
```

output
```
(a*x^2)/2 - b/(2*d*(c + d*x^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{a\,x^{2}}{2}-\frac{b}{2d\left(d\,x^{2}+c\right)}$	23
risch	$\frac{a\,x^{2}}{2}-\frac{b}{2d\left(d\,x^{2}+c\right)}$	23
derivativedivides	$\frac{\left(d\,x^{2}+c\right)a-\frac{b}{d\,x^{2}+c}}{2d}$	28
gosper	$-\frac{-a\,d^{2}x^{4}+a\,c^{2}+b}{2\left(d\,x^{2}+c\right)d}$	31
norman	$\frac{\frac{a d\,x^{4}}{2}-\frac{a\,c^{2}+b}{2d}}{d\,x^{2}+c}$	31
parallelrisch	$\frac{a\,d^{2}x^{4}-a\,c^{2}-b}{2d\left(d\,x^{2}+c\right)}$	33
orering	$-\frac{\left(-a\,d^{2}x^{4}+a\,c^{2}+b\right)\left(d\,x^{2}+c\right)\left(a+\frac{b}{\left(d\,x^{2}+c\right)^{2}}\right)}{2d\left(a\,d^{2}x^{4}+2ad\,x^{2}c+a\,c^{2}+b\right)}$	67

input
```
int(x*(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2-1/2*b/d/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int x\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ad^2x^4+acdx^2-b}{2\,(d^2x^2+cd)}$$

input
```
integrate(x*(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
1/2*(a*d^2*x^4 + a*c*d*x^2 - b)/(d^2*x^2 + c*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int x\left(a+\frac{b}{\left(c+dx^2\right)^2}\right)\,dx=\frac{ax^2}{2}-\frac{b}{2cd+2d^2x^2}$$

input
```
integrate(x*(a+b/(d*x**2+c)**2),x)
```

output
```
a*x**2/2 - b/(2*c*d + 2*d**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int x\left(a+\frac{b}{\left(c+dx^2\right)^2}\right)\,dx=\frac{1}{2}\,ax^2-\frac{b}{2\,(d^2x^2+cd)}$$

input
```
integrate(x*(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/2*a*x^2 - 1/2*b/(d^2*x^2 + c*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x\left(a+\frac{b}{\left(c+dx^2\right)^2}\right)\,dx=\frac{1}{2}\,ax^2-\frac{b}{2\,(dx^2+c)d}$$

input
```
integrate(x*(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
1/2*a*x^2 - 1/2*b/((d*x^2 + c)*d)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{a\,x^2}{2}-\frac{b}{2\,d\,\left(d\,x^2+c\right)}$$

input
```
int(x*(a + b/(c + d*x^2)^2),x)
```

output
```
(a*x^2)/2 - b/(2*d*(c + d*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int x\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{x^2(acd\,x^2+a\,c^2+b)}{2c\,(d\,x^2+c)}$$

input
```
int(x*(a+b/(d*x^2+c)^2),x)
```

output
```
(x**2*(a*c**2 + a*c*d*x**2 + b))/(2*c*(c + d*x**2))
```

3.226 $\int \frac{a+\frac{b}{\left(c+dx^2\right)^2}}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x} \, dx = \frac{b}{2c\left(c+dx^2\right)}+\frac{(b+ac^2)\log(x)}{c^2}-\frac{b\log\left(c+dx^2\right)}{2c^2}$$

output
```
1/2*b/c/(d*x^2+c)+(a*c^2+b)*ln(x)/c^2-1/2*b*ln(d*x^2+c)/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x} \, dx = \frac{b}{2c^2+2cdx^2}+\left(a+\frac{b}{c^2}\right)\log(x)-\frac{b\log\left(c+dx^2\right)}{2c^2}$$

input
```
Integrate[(a + b/(c + d*x^2)^2)/x,x]
```

output
```
b/(2*c^2 + 2*c*d*x^2) + (a + b/c^2)*Log[x] - (b*Log[c + d*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x} \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{ac^2+b}{c^2x}-\frac{bdx}{c^2(c+dx^2)}-\frac{bdx}{c(c+dx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(x)\left(ac^2+b\right)}{c^2}-\frac{b\log\left(c+dx^2\right)}{2c^2}+\frac{b}{2c\left(c+dx^2\right)}$$

input
```
Int[(a + b/(c + d*x^2)^2)/x,x]
```

output
```
b/(2*c*(c + d*x^2)) + ((b + a*c^2)*Log[x])/c^2 - (b*Log[c + d*x^2])/(2*c^2
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{b}{2c(d\,x^{2}+c)}+\ln\left(x\right)a+\frac{\ln(x)b}{c^{2}}-\frac{b\ln\left(d\,x^{2}+c\right)}{2c^{2}}$	42
norman	$-\frac{db\,x^{2}}{2c^{2}\left(d\,x^{2}+c\right)}+\frac{\left(a\,c^{2}+b\right)\ln\left(x\right)}{c^{2}}-\frac{b\ln\left(d\,x^{2}+c\right)}{2c^{2}}$	48
default	$-\frac{db\left(-\frac{c}{d\left(d\,x^{2}+c\right)}+\frac{\ln\left(d\,x^{2}+c\right)}{d}\right)}{2c^{2}}+\frac{\left(a\,c^{2}+b\right)\ln\left(x\right)}{c^{2}}$	50
parallelrisch	$\frac{2\ln\left(x\right)x^{2}a\,c^{2}d+2\ln\left(x\right)x^{2}bd+2a\,c^{3}\ln\left(x\right)-\ln\left(d\,x^{2}+c\right)x^{2}bd-bd\,x^{2}+2bc\ln\left(x\right)-\ln\left(d\,x^{2}+c\right)bc}{2c^{2}\left(d\,x^{2}+c\right)}$	85

input
```
int((a+b/(d*x^2+c)^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*b/c/(d*x^2+c)+ln(x)*a+ln(x)/c^2*b-1/2*b*ln(d*x^2+c)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x}\,dx=\frac{bc-(bdx^2+bc)\log\left(dx^2+c\right)+2\left(ac^3+(ac^2+b)dx^2+bc\right)\log\left(x\right)}{2\left(c^2dx^2+c^3\right)}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x,x, algorithm="fricas")
```

output
```
1/2*(b*c - (b*d*x^2 + b*c)*log(d*x^2 + c) + 2*(a*c^3 + (a*c^2 + b)*d*x^2 +
 b*c)*log(x))/(c^2*d*x^2 + c^3)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x}\,dx = \frac{b}{2c^2+2cdx^2} - \frac{b\log\left(\frac{c}{d}+x^2\right)}{2c^2} + \frac{(ac^2+b)\log(x)}{c^2}$$

input
```
integrate((a+b/(d*x**2+c)**2)/x,x)
```

output
```
b/(2*c**2 + 2*c*d*x**2) - b*log(c/d + x**2)/(2*c**2) + (a*c**2 + b)*log(x)
/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x}\,dx = \frac{b}{2\,(cdx^2+c^2)} - \frac{b\log(dx^2+c)}{2\,c^2} + \frac{(ac^2+b)\log(x^2)}{2\,c^2}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x,x, algorithm="maxima")
```

output
```
1/2*b/(c*d*x^2 + c^2) - 1/2*b*log(d*x^2 + c)/c^2 + 1/2*(a*c^2 + b)*log(x^2
)/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x}\,dx = \frac{(ac^2+b)\log(x^2)}{2\,c^2} - \frac{b\log(|dx^2+c|)}{2\,c^2} + \frac{bdx^2+2\,bc}{2\,(dx^2+c)c^2}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x,x, algorithm="giac")
```

output
```
1/2*(a*c^2 + b)*log(x^2)/c^2 - 1/2*b*log(abs(d*x^2 + c))/c^2 + 1/2*(b*d*x^
2 + 2*b*c)/((d*x^2 + c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x}\,dx = \frac{b}{2\,c\,\left(d\,x^2+c\right)}+\frac{\ln\left(x\right)\,\left(a\,c^2+b\right)}{c^2}-\frac{b\,\ln\left(d\,x^2+c\right)}{2\,c^2}$$

input
```
int((a + b/(c + d*x^2)^2)/x,x)
```

output
```
b/(2*c*(c + d*x^2)) + (log(x)*(b + a*c^2))/c^2 - (b*log(c + d*x^2))/(2*c^2
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x}\,dx$$
$$= \frac{-\log(d\,x^2+c)\,bc-\log(d\,x^2+c)\,bd\,x^2+2\log(x)\,a\,c^3+2\log(x)\,a\,c^2d\,x^2+2\log(x)\,bc+2\log(x)\,bd\,x^2-}{2c^2\left(d\,x^2+c\right)}$$

input
```
int((a+b/(d*x^2+c)^2)/x,x)
```

output
```
( - log(c + d*x**2)*b*c - log(c + d*x**2)*b*d*x**2 + 2*log(x)*a*c**3 + 2*l
og(x)*a*c**2*d*x**2 + 2*log(x)*b*c + 2*log(x)*b*d*x**2 - b*d*x**2)/(2*c**2
*(c + d*x**2))
```

3.227 $\int \frac{a+\frac{b}{\left(c+dx^2\right)^2}}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = -\frac{b+ac^2}{2c^2x^2}-\frac{bd}{2c^2\left(c+dx^2\right)}-\frac{2bd\log(x)}{c^3}+\frac{bd\log\left(c+dx^2\right)}{c^3}$$

output
```
-1/2*(a*c^2+b)/c^2/x^2-1/2*b*d/c^2/(d*x^2+c)-2*b*d*ln(x)/c^3+b*d*ln(d*x^2+
c)/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = -\frac{\frac{c(b+ac^2)}{x^2}+\frac{bcd}{c+dx^2}+4bd\log(x)-2bd\log\left(c+dx^2\right)}{2c^3}$$

input
```
Integrate[(a + b/(c + d*x^2)^2)/x^3,x]
```

output
```
-1/2*((c*(b + a*c^2))/x^2 + (b*c*d)/(c + d*x^2) + 4*b*d*Log[x] - 2*b*d*Log
[c + d*x^2])/c^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3} \, dx$$

$$\Big\downarrow \ 2010$$

$$\int \left(\frac{ac^2+b}{c^2x^3}+\frac{2bd^2x}{c^3\left(c+dx^2\right)}-\frac{2bd}{c^3x}+\frac{bd^2x}{c^2\left(c+dx^2\right)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{ac^2+b}{2c^2x^2}+\frac{bd\log\left(c+dx^2\right)}{c^3}-\frac{2bd\log(x)}{c^3}-\frac{bd}{2c^2\left(c+dx^2\right)}$$

input
```
Int[(a + b/(c + d*x^2)^2)/x^3,x]
```

output
```
-1/2*(b + a*c^2)/(c^2*x^2) - (b*d)/(2*c^2*(c + d*x^2)) - (2*b*d*Log[x])/c^
3 + (b*d*Log[c + d*x^2])/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{d^{2} b \left(-\frac{c}{d \left(d \,x^{2}+c \right)}+\frac{2 \ln \left(d \,x^{2}+c \right)}{d}\right)}{2 c^{3}}-\frac{a \,c^{2}+b}{2 c^{2} x^{2}}-\frac{2 b d \ln \left(x \right)}{c^{3}}$	64
norman	$\frac{-\frac{a \,c^{2}+b}{2 c}+\frac{d \left(a \,c^{2} d +2 b d \right) x^{4}}{2 c^{3}}}{\left(d \,x^{2}+c \right) x^{2}}+\frac{b d \ln \left(d \,x^{2}+c \right)}{c^{3}}-\frac{2 b d \ln \left(x \right)}{c^{3}}$	71
risch	$\frac{-\frac{\left(a \,c^{2}+2 b \right) d \,x^{2}}{2 c^{2}}-\frac{a \,c^{2}+b}{2 c}}{\left(d \,x^{2}+c \right) x^{2}}-\frac{2 b d \ln \left(x \right)}{c^{3}}+\frac{d b \ln \left(-d \,x^{2}-c \right)}{c^{3}}$	72
parallelrisch	$-\frac{-a \,c^{2} d^{2} x^{4}+4 \ln \left(x \right) x^{4} b \,d^{2}-2 \ln \left(d \,x^{2}+c \right) x^{4} b \,d^{2}-2 b \,d^{2} x^{4}+4 \ln \left(x \right) x^{2} b c d -2 \ln \left(d \,x^{2}+c \right) x^{2} b c d +a \,c^{4}+b \,c^{2}}{2 c^{3} x^{2} \left(d \,x^{2}+c \right)}$	104

input
```
int((a+b/(d*x^2+c)^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/c^3*d^2*b*(-c/d/(d*x^2+c)+2/d*ln(d*x^2+c))-1/2*(a*c^2+b)/c^2/x^2-2*b*d
*ln(x)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.56

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = -\frac{ac^4+(ac^3+2\,bc)dx^2+bc^2-2\,(bd^2x^4+bcdx^2)\log\left(dx^2+c\right)+4\,(bd^2x^4+bcdx^2)\log\left(x\right)}{2\,(c^3dx^4+c^4x^2)}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*c^4 + (a*c^3 + 2*b*c)*d*x^2 + b*c^2 - 2*(b*d^2*x^4 + b*c*d*x^2)*lo
g(d*x^2 + c) + 4*(b*d^2*x^4 + b*c*d*x^2)*log(x))/(c^3*d*x^4 + c^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = -\frac{2bd\log(x)}{c^3} + \frac{bd\log\left(\frac{c}{d}+x^2\right)}{c^3} + \frac{-ac^3-bc+x^2(-ac^2d-2bd)}{2c^3x^2+2c^2dx^4}$$

input
```
integrate((a+b/(d*x**2+c)**2)/x**3,x)
```

output
```
-2*b*d*log(x)/c**3 + b*d*log(c/d + x**2)/c**3 + (-a*c**3 - b*c + x**2*(-a*
c**2*d - 2*b*d))/(2*c**3*x**2 + 2*c**2*d*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = -\frac{ac^3+(ac^2+2b)dx^2+bc}{2(c^2dx^4+c^3x^2)} + \frac{bd\log(dx^2+c)}{c^3} - \frac{bd\log(x^2)}{c^3}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(a*c^3 + (a*c^2 + 2*b)*d*x^2 + b*c)/(c^2*d*x^4 + c^3*x^2) + b*d*log(d
*x^2 + c)/c^3 - b*d*log(x^2)/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = -\frac{bd\log(x^2)}{c^3} + \frac{bd\log(|dx^2+c|)}{c^3} - \frac{ac^2dx^2+ac^3+2bdx^2+bc}{2(dx^4+cx^2)c^2}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^3,x, algorithm="giac")
```

output
```
-b*d*log(x^2)/c^3 + b*d*log(abs(d*x^2 + c))/c^3 - 1/2*(a*c^2*d*x^2 + a*c^3
 + 2*b*d*x^2 + b*c)/((d*x^4 + c*x^2)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = \frac{b\,d\,\ln\left(d\,x^2+c\right)}{c^3} - \frac{\frac{a\,c^2+b}{2\,c}+\frac{d\,x^2\,\left(a\,c^2+2\,b\right)}{2\,c^2}}{d\,x^4+c\,x^2} - \frac{2\,b\,d\,\ln\left(x\right)}{c^3}$$

input
```
int((a + b/(c + d*x^2)^2)/x^3,x)
```

output
```
(b*d*log(c + d*x^2))/c^3 - ((b + a*c^2)/(2*c) + (d*x^2*(2*b + a*c^2))/(2*c
^2))/(c*x^2 + d*x^4) - (2*b*d*log(x))/c^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^3}\,dx = \frac{2\log(d\,x^2+c)\,bcd\,x^2+2\log(d\,x^2+c)\,b\,d^2x^4-4\log(x)\,bcd\,x^2-4\log(x)\,b\,d^2x^4-a\,c^4+a\,c^2d^2x^4-b\,c^2 -}{2c^3x^2\,(d\,x^2+c)}$$

input
```
int((a+b/(d*x^2+c)^2)/x^3,x)
```

output
```
(2*log(c + d*x**2)*b*c*d*x**2 + 2*log(c + d*x**2)*b*d**2*x**4 - 4*log(x)*b
*c*d*x**2 - 4*log(x)*b*d**2*x**4 - a*c**4 + a*c**2*d**2*x**4 - b*c**2 + 2*
b*d**2*x**4)/(2*c**3*x**2*(c + d*x**2))
```

3.228 $\int x^6 \left(a+\frac{b}{(c+dx^2)^2}\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int x^6 \left(a+\frac{b}{(c+dx^2)^2}\right) dx = -\frac{2bcx}{d^3}+\frac{bx^3}{3d^2}+\frac{ax^7}{7}-\frac{bc^2x}{2d^3(c+dx^2)}+\frac{5bc^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{7/2}}$$

output
```
-2*b*c*x/d^3+1/3*b*x^3/d^2+1/7*a*x^7-1/2*b*c^2*x/d^3/(d*x^2+c)+5/2*b*c^(3/
2)*arctan(d^(1/2)*x/c^(1/2))/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int x^6 \left(a+\frac{b}{(c+dx^2)^2}\right) dx = -\frac{2bcx}{d^3}+\frac{bx^3}{3d^2}+\frac{ax^7}{7}-\frac{bc^2x}{2d^3(c+dx^2)}+\frac{5bc^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{7/2}}$$

input
```
Integrate[x^6*(a + b/(c + d*x^2)^2),x]
```

output
```
(-2*b*c*x)/d^3 + (b*x^3)/(3*d^2) + (a*x^7)/7 - (b*c^2*x)/(2*d^3*(c + d*x^2
)) + (5*b*c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+\frac{b}{(c+dx^2)^2}\right) \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(ax^6-\frac{bc^3}{d^3 (c+dx^2)^2}+\frac{3bc^2}{d^3 (c+dx^2)}-\frac{2bc}{d^3}+\frac{bx^2}{d^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^7}{7}+\frac{5bc^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{7/2}}-\frac{bc^2x}{2d^3 (c+dx^2)}-\frac{2bcx}{d^3}+\frac{bx^3}{3d^2}$$

input
```
Int[x^6*(a + b/(c + d*x^2)^2),x]
```

output
```
(-2*b*c*x)/d^3 + (b*x^3)/(3*d^2) + (a*x^7)/7 - (b*c^2*x)/(2*d^3*(c + d*x^2
)) + (5*b*c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(7/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{1}{7} a\, d^{3} x^{7}+\frac{1}{3} b d\, x^{3}-2 c b x}{d^{3}}+\frac{c^{2} b \left(-\frac{x}{2 \left(d\, x^{2}+c \right)}+\frac{5 \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{2 \sqrt{c d}}\right)}{d^{3}}$	65
risch	$\frac{a\, x^{7}}{7}+\frac{b\, x^{3}}{3 d^{2}}-\frac{2 b c x}{d^{3}}-\frac{b\, c^{2} x}{2 d^{3} \left(d\, x^{2}+c \right)}+\frac{5 \sqrt{-c d}\, b c \ln \left(-\sqrt{-c d}\, x +c \right)}{4 d^{4}}-\frac{5 \sqrt{-c d}\, b c \ln \left(\sqrt{-c d}\, x +c \right)}{4 d^{4}}$	93

input
```
int(x^6*(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(1/7*a*d^3*x^7+1/3*b*d*x^3-2*c*b*x)+c^2/d^3*b*(-1/2*x/(d*x^2+c)+5/2/
(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.78

$$\int x^6\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$
$$=\left[\frac{12\,ad^4x^9+12\,acd^3x^7+28\,bd^2x^5-140\,bcdx^3-210\,bc^2x+105\,(bcdx^2+bc^2)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{84\,(d^4x^2+cd^3)}\right.$$

input
```
integrate(x^6*(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[1/84*(12*a*d^4*x^9 + 12*a*c*d^3*x^7 + 28*b*d^2*x^5 - 140*b*c*d*x^3 - 210*
b*c^2*x + 105*(b*c*d*x^2 + b*c^2)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d)
 - c)/(d*x^2 + c)))/(d^4*x^2 + c*d^3), 1/42*(6*a*d^4*x^9 + 6*a*c*d^3*x^7 +
 14*b*d^2*x^5 - 70*b*c*d*x^3 - 105*b*c^2*x + 105*(b*c*d*x^2 + b*c^2)*sqrt(
c/d)*arctan(d*x*sqrt(c/d)/c))/(d^4*x^2 + c*d^3)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int x^6\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^7}{7}-\frac{bc^2x}{2cd^3+2d^4x^2}-\frac{2bcx}{d^3}+b\left(-\frac{5\sqrt{-\frac{c^3}{d^7}}\log\left(x-\frac{d^3\sqrt{-\frac{c^3}{d^7}}}{c}\right)}{4}+\frac{5\sqrt{-\frac{c^3}{d^7}}\log\left(x+\frac{d^3\sqrt{-\frac{c^3}{d^7}}}{c}\right)}{4}\right)+\frac{bx^3}{3d^2}$$

input
```
integrate(x**6*(a+b/(d*x**2+c)**2),x)
```

output
```
a*x**7/7 - b*c**2*x/(2*c*d**3 + 2*d**4*x**2) - 2*b*c*x/d**3 + b*(-5*sqrt(-
c**3/d**7)*log(x - d**3*sqrt(-c**3/d**7)/c)/4 + 5*sqrt(-c**3/d**7)*log(x +
 d**3*sqrt(-c**3/d**7)/c)/4) + b*x**3/(3*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int x^6\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=-\frac{bc^2x}{2\,(d^4x^2+cd^3)}+\frac{5\,bc^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^3}+\frac{3\,ad^3x^7+7\,bdx^3-42\,bcx}{21\,d^3}$$

input
```
integrate(x^6*(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
-1/2*b*c^2*x/(d^4*x^2 + c*d^3) + 5/2*b*c^2*arctan(d*x/sqrt(c*d))/(sqrt(c*d
)*d^3) + 1/21*(3*a*d^3*x^7 + 7*b*d*x^3 - 42*b*c*x)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int x^6 \left(a+\frac{b}{(c+dx^2)^2}\right)\,dx = \frac{5\,bc^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^3} - \frac{bc^2x}{2\,(dx^2+c)d^3} + \frac{3\,ad^{14}x^7+7\,bd^{12}x^3-42\,bcd^{11}x}{21\,d^{14}}$$

input
```
integrate(x^6*(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
5/2*b*c^2*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3) - 1/2*b*c^2*x/((d*x^2 + c)
*d^3) + 1/21*(3*a*d^14*x^7 + 7*b*d^12*x^3 - 42*b*c*d^11*x)/d^14
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.34

$$\int x^6 \left(a+\frac{b}{(c+dx^2)^2}\right)\,dx = x^3\left(\frac{a\,c^2+b}{3\,d^2}-\frac{a\,c^2}{3\,d^2}\right)+\frac{a\,x^7}{7}+\frac{5\,b\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,d^{7/2}} - \frac{2\,c\,x\left(\frac{a\,c^2+b}{d^2}-\frac{a\,c^2}{d^2}\right)}{d} - \frac{b\,c^2\,x}{2\,\left(d^4\,x^2+c\,d^3\right)}$$

input
```
int(x^6*(a + b/(c + d*x^2)^2),x)
```

output
```
x^3*((b + a*c^2)/(3*d^2) - (a*c^2)/(3*d^2)) + (a*x^7)/7 + (5*b*c^(3/2)*ata
n((d^(1/2)*x)/c^(1/2)))/(2*d^(7/2)) - (2*c*x*((b + a*c^2)/d^2 - (a*c^2)/d^
2))/d - (b*c^2*x)/(2*(c*d^3 + d^4*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int x^6\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$
$$=\frac{105\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)b\,c^2+105\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bcd\,x^2+6ac\,d^4x^7+6a\,d^5x^9-105b\,c^2dx-70bc\,d^2}{42d^4\,(d\,x^2+c)}$$

input
```
int(x^6*(a+b/(d*x^2+c)^2),x)
```

output
```
(105*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c**2 + 105*sqrt(d)*sq
rt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*d*x**2 + 6*a*c*d**4*x**7 + 6*a*d**
5*x**9 - 105*b*c**2*d*x - 70*b*c*d**2*x**3 + 14*b*d**3*x**5)/(42*d**4*(c +
 d*x**2))
```

3.229 $\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{bx}{d^2}+\frac{ax^5}{5}+\frac{bcx}{2d^2\,(c+dx^2)}-\frac{3b\sqrt{c}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{5/2}}$$

output
```
b*x/d^2+1/5*a*x^5+1/2*b*c*x/d^2/(d*x^2+c)-3/2*b*c^(1/2)*arctan(d^(1/2)*x/c
^(1/2))/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^5}{5}+\frac{bx\left(2+\frac{c}{c+dx^2}\right)}{2d^2}-\frac{3b\sqrt{c}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{5/2}}$$

input
```
Integrate[x^4*(a + b/(c + d*x^2)^2),x]
```

output
```
(a*x^5)/5 + (b*x*(2 + c/(c + d*x^2)))/(2*d^2) - (3*b*Sqrt[c]*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(2*d^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^4+\frac{bc^2}{d^2(c+dx^2)^2}-\frac{2bc}{d^2(c+dx^2)}+\frac{b}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^5}{5}-\frac{3b\sqrt{c}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{5/2}}+\frac{bcx}{2d^2(c+dx^2)}+\frac{bx}{d^2}$$

input
```
Int[x^4*(a + b/(c + d*x^2)^2),x]
```

output
```
(b*x)/d^2 + (a*x^5)/5 + (b*c*x)/(2*d^2*(c + d*x^2)) - (3*b*Sqrt[c]*ArcTan[
(Sqrt[d]*x)/Sqrt[c]])/(2*d^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{5} a\,x^{5} d^{2}+b x}{d^{2}}-\frac{b c \left(-\frac{x}{2 \left(d\,x^{2}+c \right)}+\frac{3 \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{2 \sqrt{c d}}\right)}{d^{2}}$	55
risch	$\frac{a\,x^{5}}{5}+\frac{b x}{d^{2}}+\frac{b c x}{2 d^{2} \left(d\,x^{2}+c \right)}+\frac{3 \sqrt{-c d}\, b \ln \left(-\sqrt{-c d}\, x -c \right)}{4 d^{3}}-\frac{3 \sqrt{-c d}\, b \ln \left(\sqrt{-c d}\, x -c \right)}{4 d^{3}}$	82

input
```
int(x^4*(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(1/5*a*x^5*d^2+b*x)-b*c/d^2*(-1/2*x/(d*x^2+c)+3/2/(c*d)^(1/2)*arctan
(d*x/(c*d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.97

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$
$$=\left[\frac{4\,ad^3x^7+4\,acd^2x^5+20\,bdx^3+30\,bcx+15\,(bdx^2+bc)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{20\,(d^3x^2+cd^2)}, \frac{2\,ad^3x^7+2\,acd^2x^5}{}\right.$$

input
```
integrate(x^4*(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[1/20*(4*a*d^3*x^7 + 4*a*c*d^2*x^5 + 20*b*d*x^3 + 30*b*c*x + 15*(b*d*x^2 +
 b*c)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)))/(d^3*x^2
 + c*d^2), 1/10*(2*a*d^3*x^7 + 2*a*c*d^2*x^5 + 10*b*d*x^3 + 15*b*c*x - 15*
(b*d*x^2 + b*c)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c))/(d^3*x^2 + c*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^5}{5}+\frac{bcx}{2cd^2+2d^3x^2}+b\left(\frac{3\sqrt{-\frac{c}{d^5}}\log\left(-d^2\sqrt{-\frac{c}{d^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{c}{d^5}}\log\left(d^2\sqrt{-\frac{c}{d^5}}+x\right)}{4}\right)+\frac{bx}{d^2}$$

input
```
integrate(x**4*(a+b/(d*x**2+c)**2),x)
```

output
```
a*x**5/5 + b*c*x/(2*c*d**2 + 2*d**3*x**2) + b*(3*sqrt(-c/d**5)*log(-d**2*s
qrt(-c/d**5) + x)/4 - 3*sqrt(-c/d**5)*log(d**2*sqrt(-c/d**5) + x)/4) + b*x
/d**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{bcx}{2\left(d^3x^2+cd^2\right)}-\frac{3\,bc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}d^2}+\frac{ad^2x^5+5\,bx}{5\,d^2}$$

input
```
integrate(x^4*(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*c*x/(d^3*x^2 + c*d^2) - 3/2*b*c*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2
) + 1/5*(a*d^2*x^5 + 5*b*x)/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=-\frac{3\,bc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^2}+\frac{bcx}{2\,(dx^2+c)d^2}+\frac{ad^{10}x^5+5\,bd^8x}{5\,d^{10}}$$

input
```
integrate(x^4*(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
-3/2*b*c*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/2*b*c*x/((d*x^2 + c)*d^
2) + 1/5*(a*d^10*x^5 + 5*b*d^8*x)/d^10
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{a\,x^5}{5}+x\left(\frac{a\,c^2+b}{d^2}-\frac{a\,c^2}{d^2}\right)-\frac{3\,b\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,d^{5/2}}+\frac{b\,c\,x}{2\,\left(d^3\,x^2+c\,d^2\right)}$$

input
```
int(x^4*(a + b/(c + d*x^2)^2),x)
```

output
```
(a*x^5)/5 + x*((b + a*c^2)/d^2 - (a*c^2)/d^2) - (3*b*c^(1/2)*atan((d^(1/2)
*x)/c^(1/2)))/(2*d^(5/2)) + (b*c*x)/(2*(c*d^2 + d^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int x^4\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{-15\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bc-15\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bd\,x^2+2ac\,d^3x^5+2a\,d^4x^7+15bcdx+10b\,d^2x^3}{10d^3\,(d\,x^2+c)}$$

input
```
int(x^4*(a+b/(d*x^2+c)^2),x)
```

output
```
( - 15*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c - 15*sqrt(d)*sqrt
(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*d*x**2 + 2*a*c*d**3*x**5 + 2*a*d**4*x*
*7 + 15*b*c*d*x + 10*b*d**2*x**3)/(10*d**3*(c + d*x**2))
```

3.230 $\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^3}{3}-\frac{bx}{2d\,(c+dx^2)}+\frac{b\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{3/2}}$$

output
```
1/3*a*x^3-1/2*b*x/d/(d*x^2+c)+1/2*b*arctan(d^(1/2)*x/c^(1/2))/c^(1/2)/d^(3
/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^3}{3}-\frac{bx}{2d\,(c+dx^2)}+\frac{b\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{3/2}}$$

input
```
Integrate[x^2*(a + b/(c + d*x^2)^2),x]
```

output
```
(a*x^3)/3 - (b*x)/(2*d*(c + d*x^2)) + (b*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*S
qrt[c]*d^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^2+\frac{b}{d(c+dx^2)}-\frac{bc}{d(c+dx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{b\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{3/2}}-\frac{bx}{2d(c+dx^2)}$$

input
```
Int[x^2*(a + b/(c + d*x^2)^2),x]
```

output
```
(a*x^3)/3 - (b*x)/(2*d*(c + d*x^2)) + (b*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*S
qrt[c]*d^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{a\,x^3}{3}+b\left(-\frac{x}{2d(d\,x^2+c)}+\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2d\sqrt{cd}}\right)$	45
risch	$\frac{a\,x^3}{3}-\frac{bx}{2d(d\,x^2+c)}-\frac{b\ln\left(dx+\sqrt{-cd}\right)}{4\sqrt{-cd}\,d}+\frac{b\ln\left(-dx+\sqrt{-cd}\right)}{4\sqrt{-cd}\,d}$	71

input
```
int(x^2*(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+b*(-1/2/d*x/(d*x^2+c)+1/2/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.16

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$

$$=\left[\frac{4\,acd^3x^5+4\,ac^2d^2x^3-6\,bcdx-3\,(bdx^2+bc)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)}{12\,(cd^3x^2+c^2d^2)},\ \frac{2\,acd^3x^5+2\,ac^2d^2x^3-3\,bcdx}{6\,(cd}\right.$$

input
```
integrate(x^2*(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[1/12*(4*a*c*d^3*x^5 + 4*a*c^2*d^2*x^3 - 6*b*c*d*x - 3*(b*d*x^2 + b*c)*sqr
t(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c*d^3*x^2 + c^2*d^
2), 1/6*(2*a*c*d^3*x^5 + 2*a*c^2*d^2*x^3 - 3*b*c*d*x + 3*(b*d*x^2 + b*c)*s
qrt(c*d)*arctan(sqrt(c*d)*x/c))/(c*d^3*x^2 + c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{ax^3}{3}-\frac{bx}{2cd+2d^2x^2}+b\left(-\frac{\sqrt{-\frac{1}{cd^3}}\log\left(-cd\sqrt{-\frac{1}{cd^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{cd^3}}\log\left(cd\sqrt{-\frac{1}{cd^3}}+x\right)}{4}\right)$$

input
```
integrate(x**2*(a+b/(d*x**2+c)**2),x)
```

output
```
a*x**3/3 - b*x/(2*c*d + 2*d**2*x**2) + b*(-sqrt(-1/(c*d**3))*log(-c*d*sqrt
(-1/(c*d**3)) + x)/4 + sqrt(-1/(c*d**3))*log(c*d*sqrt(-1/(c*d**3)) + x)/4)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{1}{3}\,ax^3-\frac{bx}{2\,(d^2x^2+cd)}+\frac{b\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d}$$

input
```
integrate(x^2*(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/3*a*x^3 - 1/2*b*x/(d^2*x^2 + c*d) + 1/2*b*arctan(d*x/sqrt(c*d))/(sqrt(c*
d)*d)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{1}{3}\,ax^3+\frac{b\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}d}-\frac{bx}{2\,(dx^2+c)d}$$

input
```
integrate(x^2*(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
1/3*a*x^3 + 1/2*b*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d) - 1/2*b*x/((d*x^2 +
c)*d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{a\,x^3}{3}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,\sqrt{c}\,d^{3/2}}-\frac{b\,x}{2\,d\,\left(d\,x^2+c\right)}$$

input
```
int(x^2*(a + b/(c + d*x^2)^2),x)
```

output
```
(a*x^3)/3 + (b*atan((d^(1/2)*x)/c^(1/2)))/(2*c^(1/2)*d^(3/2)) - (b*x)/(2*d
*(c + d*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int x^2\left(a+\frac{b}{(c+dx^2)^2}\right)\,dx=\frac{3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bc+3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bd\,x^2+2a\,c^2d^2x^3+2ac\,d^3x^5-3bcdx}{6c\,d^2\left(d\,x^2+c\right)}$$

input
```
int(x^2*(a+b/(d*x^2+c)^2),x)
```

output
```
(3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c + 3*sqrt(d)*sqrt(c)*a
tan((d*x)/(sqrt(d)*sqrt(c)))*b*d*x**2 + 2*a*c**2*d**2*x**3 + 2*a*c*d**3*x*
*5 - 3*b*c*d*x)/(6*c*d**2*(c + d*x**2))
```

3.231 $\int \left(a+\frac{b}{(c+dx^2)^2}\right) dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right) dx = ax+\frac{bx}{2c(c+dx^2)}+\frac{b\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}$$

output
```
a*x+1/2*b*x/c/(d*x^2+c)+1/2*b*arctan(d^(1/2)*x/c^(1/2))/c^(3/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right) dx = ax+\frac{bx}{2c(c+dx^2)}+\frac{b\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}$$

input
```
Integrate[a + b/(c + d*x^2)^2,x]
```

output
```
a*x + (b*x)/(2*c*(c + d*x^2)) + (b*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)
*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{bx}{2c\,(c+dx^2)}$$

input
```
Int[a + b/(c + d*x^2)^2,x]
```

output
```
a*x + (b*x)/(2*c*(c + d*x^2)) + (b*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)
*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
default	$ax+\frac{bx}{2c(d\,x^2+c)}+\frac{b\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}$	41
risch	$ax+\frac{bx}{2c(d\,x^2+c)}-\frac{b\ln\left(dx+\sqrt{-cd}\right)}{4\sqrt{-cd}\,c}+\frac{b\ln\left(-dx+\sqrt{-cd}\right)}{4\sqrt{-cd}\,c}$	68

input
```
int(a+b/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x/c/(d*x^2+c)+1/2*b/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.36

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right)\,dx$$
$$=\left[\frac{4\,ac^2d^2x^3+2\,(2\,ac^3+bc)dx-(bdx^2+bc)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)}{4\,(c^2d^2x^2+c^3d)},\ \frac{2\,ac^2d^2x^3+(2\,ac^3+bc)dx+(bd}{2\,(c^2d^2x^2+}\right.$$

input
```
integrate(a+b/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*c^2*d^2*x^3 + 2*(2*a*c^3 + b*c)*d*x - (b*d*x^2 + b*c)*sqrt(-c*d)
*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^2*d^2*x^2 + c^3*d), 1/2
*(2*a*c^2*d^2*x^3 + (2*a*c^3 + b*c)*d*x + (b*d*x^2 + b*c)*sqrt(c*d)*arctan
(sqrt(c*d)*x/c))/(c^2*d^2*x^2 + c^3*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.70

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right)\,dx = ax+\frac{bx}{2c^2+2cdx^2}+b\left(-\frac{\sqrt{-\frac{1}{c^3d}}\log\left(-c^2\sqrt{-\frac{1}{c^3d}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^3d}}\log\left(c^2\sqrt{-\frac{1}{c^3d}}+x\right)}{4}\right)$$

input
```
integrate(a+b/(d*x**2+c)**2,x)
```

output
```
a*x + b*x/(2*c**2 + 2*c*d*x**2) + b*(-sqrt(-1/(c**3*d))*log(-c**2*sqrt(-1/
(c**3*d)) + x)/4 + sqrt(-1/(c**3*d))*log(c**2*sqrt(-1/(c**3*d)) + x)/4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right) dx = ax+\frac{1}{2}b\left(\frac{x}{cdx^2+c^2}+\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c}\right)$$

input
```
integrate(a+b/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
a*x + 1/2*b*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right) dx = ax+\frac{1}{2}b\left(\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c}+\frac{x}{(dx^2+c)c}\right)$$

input
```
integrate(a+b/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
a*x + 1/2*b*(arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c) + x/((d*x^2 + c)*c))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right) dx = a\,x+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,c^{3/2}\,\sqrt{d}}+\frac{b\,x}{2\,c\,\left(d\,x^2+c\right)}$$

input
```
int(a + b/(c + d*x^2)^2,x)
```

output
```
a*x + (b*atan((d^(1/2)*x)/c^(1/2)))/(2*c^(3/2)*d^(1/2)) + (b*x)/(2*c*(c +
d*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

$$\int \left(a+\frac{b}{(c+dx^2)^2}\right) dx = \frac{\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right) bc+\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right) bd\,x^2+2a\,c^3dx+2a\,c^2d^2x^3+bcdx}{2c^2d\left(d\,x^2+c\right)}$$

input
```
int(a+b/(d*x^2+c)^2,x)
```

output
```
(sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c + sqrt(d)*sqrt(c)*atan(
(d*x)/(sqrt(d)*sqrt(c)))*b*d*x**2 + 2*a*c**3*d*x + 2*a*c**2*d**2*x**3 + b*
c*d*x)/(2*c**2*d*(c + d*x**2))
```

3.232 $\int \frac{a+\frac{b}{\left(c+dx^2\right)^2}}{x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{a+\frac{b}{\left(c+dx^2\right)^2}}{x^2} \, dx = -\frac{b+ac^2}{c^2x} - \frac{bdx}{2c^2\left(c+dx^2\right)} - \frac{3b\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}}$$

output
```
-(a*c^2+b)/c^2/x-1/2*b*d*x/c^2/(d*x^2+c)-3/2*b*d^(1/2)*arctan(d^(1/2)*x/c^
(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{a+\frac{b}{\left(c+dx^2\right)^2}}{x^2} \, dx = \frac{-b-ac^2}{c^2x} - \frac{bdx}{2c^2\left(c+dx^2\right)} - \frac{3b\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}}$$

input
```
Integrate[(a + b/(c + d*x^2)^2)/x^2,x]
```

output
```
(-b - a*c^2)/(c^2*x) - (b*d*x)/(2*c^2*(c + d*x^2)) - (3*b*Sqrt[d]*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^2} \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\frac{ac^2+b}{c^2x^2}-\frac{bd}{c^2\,(c+dx^2)}-\frac{bd}{c\,(c+dx^2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ac^2+b}{c^2x}-\frac{3b\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}}-\frac{bdx}{2c^2\,(c+dx^2)}$$

input
```
Int[(a + b/(c + d*x^2)^2)/x^2,x]
```

output
```
-((b + a*c^2)/(c^2*x)) - (b*d*x)/(2*c^2*(c + d*x^2)) - (3*b*Sqrt[d]*ArcTan
[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{db\left(\frac{x}{2d\,x^2+2c}+\frac{3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{c^2}-\frac{a\,c^2+b}{c^2x}$	53
risch	$\frac{-\frac{d\left(2a\,c^2+3b\right)x^2}{2c^2}-\frac{a\,c^2+b}{c}}{x(d\,x^2+c)}+\frac{3\sqrt{-cd}\,b\ln\left(-dx+\sqrt{-cd}\right)}{4c^3}-\frac{3\sqrt{-cd}\,b\ln\left(-dx-\sqrt{-cd}\right)}{4c^3}$	97

input
```
int((a+b/(d*x^2+c)^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*d*b*(1/2*x/(d*x^2+c)+3/2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-(a*c^
2+b)/c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.75

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^2}\,dx$$
$$=\left[-\frac{4\,ac^3+2\,(2\,ac^2+3\,b)dx^2-3\,(bdx^3+bcx)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+4\,bc}{4\,(c^2dx^3+c^3x)},\right.$$
$$\left.-\frac{2\,ac^3+(2\,ac^2+3\,b)dx^2+3\,(bdx^3+bcx)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+2\,bc}{2\,(c^2dx^3+c^3x)}\right]$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a*c^3 + 2*(2*a*c^2 + 3*b)*d*x^2 - 3*(b*d*x^3 + b*c*x)*sqrt(-d/c)*
log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 4*b*c)/(c^2*d*x^3 + c^3*
x), -1/2*(2*a*c^3 + (2*a*c^2 + 3*b)*d*x^2 + 3*(b*d*x^3 + b*c*x)*sqrt(d/c)*
arctan(x*sqrt(d/c)) + 2*b*c)/(c^2*d*x^3 + c^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^2}\,dx = b\left(\frac{3\sqrt{-\frac{d}{c^5}}\log\left(-\frac{c^3\sqrt{-\frac{d}{c^5}}}{d}+x\right)}{4}-\frac{3\sqrt{-\frac{d}{c^5}}\log\left(\frac{c^3\sqrt{-\frac{d}{c^5}}}{d}+x\right)}{4}\right)+\frac{-2ac^3-2bc+x^2(-2ac^2d-3bd)}{2c^3x+2c^2dx^3}$$

input
```
integrate((a+b/(d*x**2+c)**2)/x**2,x)
```

output
```
b*(3*sqrt(-d/c**5)*log(-c**3*sqrt(-d/c**5)/d + x)/4 - 3*sqrt(-d/c**5)*log(
c**3*sqrt(-d/c**5)/d + x)/4) + (-2*a*c**3 - 2*b*c + x**2*(-2*a*c**2*d - 3*
b*d))/(2*c**3*x + 2*c**2*d*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^2}\,dx = -\frac{3\,bd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}c^2}-\frac{2\,ac^3+(2\,ac^2+3\,b)dx^2+2\,bc}{2\,(c^2dx^3+c^3x)}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^2,x, algorithm="maxima")
```

output
```
-3/2*b*d*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/2*(2*a*c^3 + (2*a*c^2 +
 3*b)*d*x^2 + 2*b*c)/(c^2*d*x^3 + c^3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^2}\,dx = -\frac{3\,bd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^2} - \frac{2\,ac^2dx^2+2\,ac^3+3\,bdx^2+2\,bc}{2\,(dx^3+cx)c^2}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^2,x, algorithm="giac")
```

output
```
-3/2*b*d*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/2*(2*a*c^2*d*x^2 + 2*a*
c^3 + 3*b*d*x^2 + 2*b*c)/((d*x^3 + c*x)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^2}\,dx = -\frac{\frac{a\,c^2+b}{c}+\frac{x^2\,(2\,a\,d\,c^2+3\,b\,d)}{2\,c^2}}{d\,x^3+c\,x} - \frac{3\,b\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,c^{5/2}}$$

input
```
int((a + b/(c + d*x^2)^2)/x^2,x)
```

output
```
- ((b + a*c^2)/c + (x^2*(3*b*d + 2*a*c^2*d))/(2*c^2))/(c*x + d*x^3) - (3*b
*d^(1/2)*atan((d^(1/2)*x)/c^(1/2)))/(2*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^2}\,dx = \frac{-3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bcx-3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bd\,x^3-2a\,c^4-2a\,c^3d\,x^2-2b\,c^2-3bcd\,x^2}{2c^3x\,(d\,x^2+c)}$$

input
```
int((a+b/(d*x^2+c)^2)/x^2,x)
```

output
```
( - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*x - 3*sqrt(d)*sqrt
(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*d*x**3 - 2*a*c**4 - 2*a*c**3*d*x**2 -
2*b*c**2 - 3*b*c*d*x**2)/(2*c**3*x*(c + d*x**2))
```

3.233 $\int \frac{a+\frac{b}{\left(c+dx^2\right)^2}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx = -\frac{b+ac^2}{3c^2x^3}+\frac{2bd}{c^3x}+\frac{bd^2x}{2c^3\left(c+dx^2\right)}+\frac{5bd^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}}$$

output
```
-1/3*(a*c^2+b)/c^2/x^3+2*b*d/c^3/x+1/2*b*d^2*x/c^3/(d*x^2+c)+5/2*b*d^(3/2)
*arctan(d^(1/2)*x/c^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx = \frac{-2a+\frac{b(-2c^2+10cdx^2+15d^2x^4)}{c^3(c+dx^2)}}{6x^3}+\frac{5bd^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}}$$

input
```
Integrate[(a + b/(c + d*x^2)^2)/x^4,x]
```

output
```
(-2*a + (b*(-2*c^2 + 10*c*d*x^2 + 15*d^2*x^4))/(c^3*(c + d*x^2)))/(6*x^3)
+ (5*b*d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\frac{ac^2+b}{c^2x^4}+\frac{2bd^2}{c^3(c+dx^2)}-\frac{2bd}{c^3x^2}+\frac{bd^2}{c^2(c+dx^2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ac^2+b}{3c^2x^3}+\frac{5bd^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}}+\frac{bd^2x}{2c^3(c+dx^2)}+\frac{2bd}{c^3x}$$

input
```
Int[(a + b/(c + d*x^2)^2)/x^4,x]
```

output
```
-1/3*(b + a*c^2)/(c^2*x^3) + (2*b*d)/(c^3*x) + (b*d^2*x)/(2*c^3*(c + d*x^2
)) + (5*b*d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{d^{2} b \left(\frac{x}{2 d \,x^{2}+2 c}+\frac{5 \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{2 \sqrt{c d}}\right)}{c^{3}}-\frac{a \,c^{2}+b}{3 c^{2} x^{3}}+\frac{2 b d}{c^{3} x}$	64
risch	$\frac{\frac{5 d^{2} b \,x^{4}}{2 c^{3}}-\frac{d \left(a \,c^{2}-5 b \right) x^{2}}{3 c^{2}}-\frac{a \,c^{2}+b}{3 c}}{\left(d \,x^{2}+c \right) x^{3}}+\frac{5 \left(\sum_{\_R =\operatorname{RootOf}\left(c^{7} \_Z^{2}+b^{2} d^{3}\right)}\_R \ln \left(\left(3 \_R^{2} c^{7}+2 b^{2} d^{3}\right) x -b \,c^{4} d \_R \right)\right)}{4}$	110

input
```
int((a+b/(d*x^2+c)^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*d^2*b*(1/2*x/(d*x^2+c)+5/2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-1/3*
(a*c^2+b)/c^2/x^3+2*b*d/c^3/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.73

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx$$
$$= \left[\frac{30\,bd^2x^4-4\,ac^4-4\,(ac^3-5\,bc)dx^2-4\,bc^2+15\,(bd^2x^5+bcdx^3)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{12\,(c^3dx^5+c^4x^3)}, \frac{15\,bd^2x^4 \cdots}{\cdots}\right]$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(30*b*d^2*x^4 - 4*a*c^4 - 4*(a*c^3 - 5*b*c)*d*x^2 - 4*b*c^2 + 15*(b*
d^2*x^5 + b*c*d*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2
+ c)))/(c^3*d*x^5 + c^4*x^3), 1/6*(15*b*d^2*x^4 - 2*a*c^4 - 2*(a*c^3 - 5*b
*c)*d*x^2 - 2*b*c^2 + 15*(b*d^2*x^5 + b*c*d*x^3)*sqrt(d/c)*arctan(x*sqrt(d
/c)))/(c^3*d*x^5 + c^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx = b\left(-\frac{5\sqrt{-\frac{d^3}{c^7}}\log\left(-\frac{c^4\sqrt{-\frac{d^3}{c^7}}}{d^2}+x\right)}{4}+\frac{5\sqrt{-\frac{d^3}{c^7}}\log\left(\frac{c^4\sqrt{-\frac{d^3}{c^7}}}{d^2}+x\right)}{4}\right) + \frac{-2ac^4-2bc^2+15bd^2x^4+x^2(-2ac^3d+10bcd)}{6c^4x^3+6c^3dx^5}$$

input
```
integrate((a+b/(d*x**2+c)**2)/x**4,x)
```

output
```
b*(-5*sqrt(-d**3/c**7)*log(-c**4*sqrt(-d**3/c**7)/d**2 + x)/4 + 5*sqrt(-d*
*3/c**7)*log(c**4*sqrt(-d**3/c**7)/d**2 + x)/4) + (-2*a*c**4 - 2*b*c**2 +
15*b*d**2*x**4 + x**2*(-2*a*c**3*d + 10*b*c*d))/(6*c**4*x**3 + 6*c**3*d*x*
*5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx = \frac{5\,bd^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}c^3} + \frac{15\,bd^2x^4-2\,ac^4-2\,(ac^3-5\,bc)dx^2-2\,bc^2}{6\,(c^3dx^5+c^4x^3)}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^4,x, algorithm="maxima")
```

output
```
5/2*b*d^2*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3) + 1/6*(15*b*d^2*x^4 - 2*a*
c^4 - 2*(a*c^3 - 5*b*c)*d*x^2 - 2*b*c^2)/(c^3*d*x^5 + c^4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx = \frac{5\,bd^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^3} + \frac{bd^2x}{2\,(dx^2+c)c^3} - \frac{ac^3-6\,bdx^2+bc}{3\,c^3x^3}$$

input
```
integrate((a+b/(d*x^2+c)^2)/x^4,x, algorithm="giac")
```

output
```
5/2*b*d^2*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3) + 1/2*b*d^2*x/((d*x^2 + c)
*c^3) - 1/3*(a*c^3 - 6*b*d*x^2 + b*c)/(c^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx = \frac{\frac{d\,x^2\,(5\,b-a\,c^2)}{3\,c^2}-\frac{a\,c^2+b}{3\,c}+\frac{5\,b\,d^2\,x^4}{2\,c^3}}{d\,x^5+c\,x^3} + \frac{5\,b\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\,c^{7/2}}$$

input
```
int((a + b/(c + d*x^2)^2)/x^4,x)
```

output
```
((d*x^2*(5*b - a*c^2))/(3*c^2) - (b + a*c^2)/(3*c) + (5*b*d^2*x^4)/(2*c^3)
)/(c*x^3 + d*x^5) + (5*b*d^(3/2)*atan((d^(1/2)*x)/c^(1/2)))/(2*c^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

$$\int \frac{a+\frac{b}{(c+dx^2)^2}}{x^4}\,dx$$
$$= \frac{15\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bcd\,x^3+15\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)b\,d^2x^5-2a\,c^5-2a\,c^4d\,x^2-2b\,c^3+10b\,c^2d\,x^2+1}{6c^4x^3\,(d\,x^2+c)}$$

input
```
int((a+b/(d*x^2+c)^2)/x^4,x)
```

output
```
(15*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*d*x**3 + 15*sqrt(d)*
sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*d**2*x**5 - 2*a*c**5 - 2*a*c**4*d*
x**2 - 2*b*c**3 + 10*b*c**2*d*x**2 + 15*b*c*d**2*x**4)/(6*c**4*x**3*(c + d
*x**2))
```

3.234 $\int \frac{x^5}{a+\frac{b}{\left(c+dx^2\right)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx = -\frac{(b-ac^2)\,x^2}{2a^2d^2} - \frac{c(c+dx^2)^2}{2ad^3} + \frac{(c+dx^2)^3}{6ad^3} + \frac{\sqrt{b}(b-ac^2)\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2a^{5/2}d^3} + \frac{bc\log\left(b+a(c+dx^2)^2\right)}{2a^2d^3}$$

output
```
-1/2*(-a*c^2+b)*x^2/a^2/d^2-1/2*c*(d*x^2+c)^2/a/d^3+1/6*(d*x^2+c)^3/a/d^3+
1/2*b^(1/2)*(-a*c^2+b)*arctan(a^(1/2)*(d*x^2+c)/b^(1/2))/a^(5/2)/d^3+1/2*b
*c*ln(b+a*(d*x^2+c)^2)/a^2/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{2\sqrt{a}dx^2(-3b+ad^2x^4) - 3i\sqrt{b}\left(\sqrt{b}+i\sqrt{a}c\right)^2\log\left(-i\sqrt{b}+\sqrt{a}(c+dx^2)\right) + 3i\sqrt{b}\left(\sqrt{b}-i\sqrt{a}c\right)^2\log\left(i\sqrt{\right.}}{12a^{5/2}d^3}$$

input
```
Integrate[x^5/(a + b/(c + d*x^2)^2),x]
```

output
```
(2*Sqrt[a]*d*x^2*(-3*b + a*d^2*x^4) - (3*I)*Sqrt[b]*(Sqrt[b] + I*Sqrt[a]*c
)^2*Log[(-I)*Sqrt[b] + Sqrt[a]*(c + d*x^2)] + (3*I)*Sqrt[b]*(Sqrt[b] - I*S
qrt[a]*c)^2*Log[I*Sqrt[b] + Sqrt[a]*(c + d*x^2)])/(12*a^(5/2)*d^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {7283, 896, 1776, 525, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{x^4}{a+\frac{b}{(dx^2+c)^2}}dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^2x^4}{a+\frac{b}{x^4}}d(dx^2+c)}{2d^3}$$

$$\Big\downarrow \text{1776}$$

$$\frac{\int \frac{d^2x^8}{ax^4+b}d(dx^2+c)}{2d^3}$$

$$\Big\downarrow \text{525}$$

$$\frac{\frac{\int -\frac{x^4\left(-ac^2+2a\left(dx^2+c\right)c+b\right)}{ax^4+b}d(dx^2+c)}{a}+\frac{x^6}{3a}}{2d^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x^6}{3a}-\frac{\int \frac{x^4\left(-ac^2+2a\left(dx^2+c\right)c+b\right)}{ax^4+b}d(dx^2+c)}{a}}{2d^3}$$

$$\Big\downarrow \; 523$$

$$\frac{\frac{x^6}{3a}-\frac{\int \left(-c^2+2(dx^2+c)c+\frac{b}{a}-\frac{b\left(b-ac^2\right)+2abc\left(dx^2+c\right)}{a\left(ax^4+b\right)}\right)d(dx^2+c)}{a}}{2d^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{x^6}{3a}-\frac{-\frac{\sqrt{b}\left(b-ac^2\right)\arctan\left(\frac{\sqrt{a}\left(c+dx^2\right)}{\sqrt{b}}\right)}{a^{3/2}}+\left(\frac{b}{a}-c^2\right)\left(c+dx^2\right)-\frac{bc\log\left(ax^4+b\right)}{a}+cx^4}{a}}{2d^3}$$

input
```
Int[x^5/(a + b/(c + d*x^2)^2),x]
```

output
```
(x^6/(3*a) - (c*x^4 + (b/a - c^2)*(c + d*x^2) - (Sqrt[b]*(b - a*c^2)*ArcTa
n[(Sqrt[a]*(c + d*x^2))/Sqrt[b]])/a^(3/2) - (b*c*Log[b + a*x^4])/a)/a)/(2*
d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1776
```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{1}{3} a\,x^{6} d^{2}-b\,x^{2}}{2 a^{2} d^{2}}+\frac{b \left(\frac{c \ln \left(a\,d^{2} x^{4}+2 a d\,x^{2} c+a\,c^{2}+b \right)}{d}+\frac{\left(-a\,c^{2}+b \right) \arctan \left(\frac{2 a\,d^{2} x^{2}+2 a c d}{2 d \sqrt{a b}}\right)}{d \sqrt{a b}}\right)}{2 a^{2} d^{2}}$
risch	$\frac{x^{6}}{6 a}-\frac{b\,x^{2}}{2 a^{2} d^{2}}+\frac{\ln \left(\left(\left(a^{2} b\,c^{2} d -\sqrt{-a b \left(a\,c^{2}-b \right)^{2}}\,a c d -d a\,b^{2}\right) x^{2}-\sqrt{-a b \left(a\,c^{2}-b \right)^{2}}\,a\,c^{2}-\sqrt{-a b \left(a\,c^{2}-b \right)^{2}}\,b \right) b c}{2 a^{2} d^{3}}+\frac{\ln \left(\left(\left(a^{2} b\,c^{2} d -\sqrt{\right.\right.\right.}{}$

input
```
int(x^5/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/d^2*(1/3*a*x^6*d^2-b*x^2)+1/2*b/a^2/d^2*(c/d*ln(a*d^2*x^4+2*a*c*d*
x^2+a*c^2+b)+(-a*c^2+b)/d/(a*b)^(1/2)*arctan(1/2*(2*a*d^2*x^2+2*a*c*d)/d/(
a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.81

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx$$

$$= \left[\frac{2\,ad^3x^6 - 6\,bdx^2 + 6\,bc\log\left(ad^2x^4 + 2\,acdx^2 + ac^2 + b\right) + 3\left(ac^2 - b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{ad^2x^4+2\,acdx^2+ac^2-2\,(adx^2\cdots}{ad^2x^4+2\,acdx^2+ac^2\cdots}\right.}{12\,a^2d^3}\right.$$

input
```
integrate(x^5/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[1/12*(2*a*d^3*x^6 - 6*b*d*x^2 + 6*b*c*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2
 + b) + 3*(a*c^2 - b)*sqrt(-b/a)*log((a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 - 2*
(a*d*x^2 + a*c)*sqrt(-b/a) - b)/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)))/(a
^2*d^3), 1/6*(a*d^3*x^6 - 3*b*d*x^2 + 3*b*c*log(a*d^2*x^4 + 2*a*c*d*x^2 +
a*c^2 + b) - 3*(a*c^2 - b)*sqrt(b/a)*arctan((a*d*x^2 + a*c)*sqrt(b/a)/b))/
(a^2*d^3)]
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.78

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx$$

$$= \left(\frac{bc}{2a^2d^3} - \frac{\sqrt{-a^5b}(ac^2-b)}{4a^5d^3}\right)\log\left(x^2 + \frac{-4a^2d^3\left(\frac{bc}{2a^2d^3} - \frac{\sqrt{-a^5b}(ac^2-b)}{4a^5d^3}\right) + ac^3 + bc}{ac^2d - bd}\right)$$

$$+ \left(\frac{bc}{2a^2d^3} + \frac{\sqrt{-a^5b}(ac^2-b)}{4a^5d^3}\right)\log\left(x^2 + \frac{-4a^2d^3\left(\frac{bc}{2a^2d^3} + \frac{\sqrt{-a^5b}(ac^2-b)}{4a^5d^3}\right) + ac^3 + bc}{ac^2d - bd}\right)$$

$$+ \frac{x^6}{6a} - \frac{bx^2}{2a^2d^2}$$

input
```
integrate(x**5/(a+b/(d*x**2+c)**2),x)
```

output
```
(b*c/(2*a**2*d**3) - sqrt(-a**5*b)*(a*c**2 - b)/(4*a**5*d**3))*log(x**2 +
(-4*a**2*d**3*(b*c/(2*a**2*d**3) - sqrt(-a**5*b)*(a*c**2 - b)/(4*a**5*d**3
)) + a*c**3 + b*c)/(a*c**2*d - b*d)) + (b*c/(2*a**2*d**3) + sqrt(-a**5*b)*
(a*c**2 - b)/(4*a**5*d**3))*log(x**2 + (-4*a**2*d**3*(b*c/(2*a**2*d**3) +
sqrt(-a**5*b)*(a*c**2 - b)/(4*a**5*d**3)) + a*c**3 + b*c)/(a*c**2*d - b*d)
) + x**6/(6*a) - b*x**2/(2*a**2*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{bc\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)}{2\,a^2d^3} + \frac{ad^2x^6-3\,bx^2}{6\,a^2d^2} - \frac{\left(abc^2-b^2\right)\arctan\left(\frac{ad^2x^2+acd}{\sqrt{abd}}\right)}{2\,\sqrt{ab}a^2d^3}$$

input
```
integrate(x^5/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*c*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)/(a^2*d^3) + 1/6*(a*d^2*x^
6 - 3*b*x^2)/(a^2*d^2) - 1/2*(a*b*c^2 - b^2)*arctan((a*d^2*x^2 + a*c*d)/(s
qrt(a*b)*d))/(sqrt(a*b)*a^2*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.83

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{x^6}{6\,a} - x^2\left(\frac{a\,c^2+b}{2\,a^2\,d^2} - \frac{c^2}{2\,a\,d^2}\right)$$

$$+ \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{a^3\,d^2\left(x^2\left(\frac{\frac{2\,b^{3/2}\,c^2\,\left(b-a\,c^2\right)}{a^{3/2}\,d}+\frac{\sqrt{b}\,\left(b-a\,c^2\right)\,\left(\frac{2\,\left(2\,a^2\,b^2\,d^3-10\,a^3\,b\,c^2\,d^3\right)}{a^2\,d}+8\,a\,b\,c^2\,d^2\right)}{4\,a^{5/2}\,d^3}}{a\,c^2+b} + \frac{\sqrt{a}\,c\,\left(\frac{2\,\left(b^3\,c-3\,a\,b^2\,c^3\right)}{a^2\,d}+\frac{b\,c\,\left(b-a\,c^2\right)^2}{a^2\,d} - \cdots\right)}{\sqrt{b}\,(\cdots}\right.\right.}{\cdots}\right)}{\cdots}$$

$$+ \frac{b\,c\,\ln\left(a\,c^2+2\,a\,c\,d\,x^2+a\,d^2\,x^4+b\right)}{2\,a^2\,d^3}$$

input
```
int(x^5/(a + b/(c + d*x^2)^2),x)
```

output
```
x^6/(6*a) - x^2*((b + a*c^2)/(2*a^2*d^2) - c^2/(2*a*d^2)) + (b^(1/2)*atan(
(a^3*d^2*(x^2*(((2*b^(3/2)*c^2*(b - a*c^2))/(a^(3/2)*d) + (b^(1/2)*(b - a*
c^2)*((2*(2*a^2*b^2*d^3 - 10*a^3*b*c^2*d^3))/(a^2*d) + 8*a*b*c^2*d^2))/(4*
a^(5/2)*d^3))/(b + a*c^2) + (a^(1/2)*c*((2*(b^3*c - 3*a*b^2*c^3))/(a^2*d)
+ (b*c*(b - a*c^2)^2)/(a^2*d) - (b*c*((2*(2*a^2*b^2*d^3 - 10*a^3*b*c^2*d^3
))/(a^2*d) + 8*a*b*c^2*d^2))/(2*a^2*d^3)))/(b^(1/2)*(b + a*c^2))) - ((b^(1
/2)*(b - a*c^2)*((4*(4*a^2*b^2*c*d^3 + 4*a^3*b*c^3*d^3))/(a^2*d^2) - (2*b*
c*(4*a^4*b*d^6 + 4*a^5*c^2*d^6))/(a^4*d^5)))/(4*a^(5/2)*d^3) - (b^(3/2)*c*
(b - a*c^2)*(4*a^4*b*d^6 + 4*a^5*c^2*d^6))/(2*a^(13/2)*d^8))/(b + a*c^2) +
 (a^(1/2)*c*((b*(b - a*c^2)^2*(4*a^4*b*d^6 + 4*a^5*c^2*d^6))/(4*a^7*d^8) -
 (4*(b^3*c^2 + a*b^2*c^4))/(a^2*d^2) + (b*c*((4*(4*a^2*b^2*c*d^3 + 4*a^3*b
*c^3*d^3))/(a^2*d^2) - (2*b*c*(4*a^4*b*d^6 + 4*a^5*c^2*d^6))/(a^4*d^5)))/(
2*a^2*d^3)))/(b^(1/2)*(b + a*c^2))))/(b^3 - 2*a*b^2*c^2 + a^2*b*c^4))*(b -
 a*c^2))/(2*a^(5/2)*d^3) + (b*c*log(b + a*c^2 + a*d^2*x^4 + 2*a*c*d*x^2))/
(2*a^2*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.79

$$\int \frac{x^5}{a+\frac{b}{(c+dx^2)^2}}\,dx$$

$$= \frac{3\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,atan\left(\frac{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,\sqrt{2}-2\sqrt{a}\,dx}{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{2}}\right)a\,c^2-3\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+}$$

input
```
int(x^5/(a+b/(d*x^2+c)^2),x)
```

output
```
(3*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*
c)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*
d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2 - 3*sq
rt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*at
an((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/
(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b + 3*sqrt(sqrt(a)
*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*atan((sqrt(d
)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*s
qrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2 - 3*sqrt(sqrt(a)*sqrt
(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*atan((sqrt(d)*sqr
t(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(s
qrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b + 3*log( - sqrt(d)*sqrt(sqrt(a)
*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)*d*x**2)*a*
b*c + 3*log(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(
a*c**2 + b) + sqrt(a)*d*x**2)*a*b*c + a**2*d**3*x**6 - 3*a*b*d*x**2)/(6*a*
*3*d**3)
```

3.235 $\int \frac{x^3}{a+\frac{b}{\left(c+dx^2\right)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx = -\frac{cx^2}{2ad}+\frac{(c+dx^2)^2}{4ad^2}+\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2a^{3/2}d^2}-\frac{b\log\left(b+a(c+dx^2)^2\right)}{4a^2d^2}$$

output
```
-1/2*c*x^2/a/d+1/4*(d*x^2+c)^2/a/d^2+1/2*b^(1/2)*c*arctan(a^(1/2)*(d*x^2+c
)/b^(1/2))/a^(3/2)/d^2-1/4*b*ln(b+a*(d*x^2+c)^2)/a^2/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx = -\frac{-ad^2x^4+\left(b+i\sqrt{a}\sqrt{b}c\right)\log\left(\frac{-i\sqrt{b}+\sqrt{a}(c+dx^2)}{d}\right)+\left(b-i\sqrt{a}\sqrt{b}c\right)\log\left(\frac{i\sqrt{b}+\sqrt{a}(c+dx^2)}{d}\right)}{4a^2d^2}$$

input
```
Integrate[x^3/(a + b/(c + d*x^2)^2),x]
```

output
```
-1/4*(-(a*d^2*x^4) + (b + I*Sqrt[a]*Sqrt[b]*c)*Log[((-I)*Sqrt[b] + Sqrt[a]
*(c + d*x^2))/d] + (b - I*Sqrt[a]*Sqrt[b]*c)*Log[(I*Sqrt[b] + Sqrt[a]*(c +
 d*x^2))/d])/(a^2*d^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {7283, 896, 25, 1776, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{x^2}{a+\frac{b}{(dx^2+c)^2}}dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{dx^2}{a+\frac{b}{x^4}}d(dx^2+c)}{2d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{dx^2}{a+\frac{b}{x^4}}d(dx^2+c)}{2d^2}$$

$$\Big\downarrow \text{1776}$$

$$-\frac{\int -\frac{dx^6}{ax^4+b}d(dx^2+c)}{2d^2}$$

$$\Big\downarrow \text{523}$$

$$-\frac{\int \left(\frac{c}{a}-\frac{dx^2+c}{a}-\frac{bc-b(dx^2+c)}{a(ax^4+b)}\right)d(dx^2+c)}{2d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{a^{3/2}}-\frac{b\log(ax^4+b)}{2a^2}-\frac{c(c+dx^2)}{a}+\frac{x^4}{2a}}{2d^2}$$

input
```
Int[x^3/(a + b/(c + d*x^2)^2),x]
```

output
```
(x^4/(2*a) - (c*(c + d*x^2))/a + (Sqrt[b]*c*ArcTan[(Sqrt[a]*(c + d*x^2))/S
qrt[b]])/a^(3/2) - (b*Log[b + a*x^4])/(2*a^2))/(2*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1776
```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

method	result
default	$\frac{x^4}{4a}-\frac{b\left(\frac{\ln\left(a\,d^2x^4+2ad\,x^2c+a\,c^2+b\right)}{2a\,d^2}-\frac{c\arctan\left(\frac{2a\,d^2x^2+2acd}{2d\sqrt{ab}}\right)}{d^2\sqrt{ab}}\right)}{2a}$
risch	$\frac{x^4}{4a}+\frac{\ln((\sqrt{-ab}\,acd+dab)x^2+\sqrt{-ab}\,a\,c^2+\sqrt{-ab}\,b)c\sqrt{-ab}}{4a^2d^2}-\frac{\ln((\sqrt{-ab}\,acd+dab)x^2+\sqrt{-ab}\,a\,c^2+\sqrt{-ab}\,b)b}{4a^2d^2}-\frac{\ln((-\sqrt{-ab}\,ac$

input
```
int(x^3/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*x^4-1/2*b/a*(1/2/a/d^2*ln(a*d^2*x^4+2*a*c*d*x^2+a*c^2+b)-c/d^2/(a*b)
^(1/2)*arctan(1/2*(2*a*d^2*x^2+2*a*c*d)/d/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.14

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx = \left[\frac{ad^2x^4+ac\sqrt{-\frac{b}{a}}\log\left(\frac{ad^2x^4+2\,acdx^2+ac^2+2\,(adx^2+ac)\sqrt{-\frac{b}{a}}-b}{ad^2x^4+2\,acdx^2+ac^2+b}\right)-b\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)}{4\,a^2d^2}, \frac{ad^2x^4+}{}\right.$$

input
```
integrate(x^3/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(a*d^2*x^4 + a*c*sqrt(-b/a)*log((a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + 2*
(a*d*x^2 + a*c)*sqrt(-b/a) - b)/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)) - b
*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b))/(a^2*d^2), 1/4*(a*d^2*x^4 + 2*a
*c*sqrt(b/a)*arctan((a*d*x^2 + a*c)*sqrt(b/a)/b) - b*log(a*d^2*x^4 + 2*a*c
*d*x^2 + a*c^2 + b))/(a^2*d^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(83) = 166$.

Time = 0.49 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx = \left(-\frac{b}{4a^2d^2} - \frac{c\sqrt{-a^5b}}{4a^4d^2}\right)\log\left(x^2 + \frac{4a^2d^2\left(-\frac{b}{4a^2d^2} - \frac{c\sqrt{-a^5b}}{4a^4d^2}\right) + ac^2 + b}{acd}\right) + \left(-\frac{b}{4a^2d^2} + \frac{c\sqrt{-a^5b}}{4a^4d^2}\right)\log\left(x^2 + \frac{4a^2d^2\left(-\frac{b}{4a^2d^2} + \frac{c\sqrt{-a^5b}}{4a^4d^2}\right) + ac^2 + b}{acd}\right) + \frac{x^4}{4a}$$

input
```
integrate(x**3/(a+b/(d*x**2+c)**2),x)
```

output
```
(-b/(4*a**2*d**2) - c*sqrt(-a**5*b)/(4*a**4*d**2))*log(x**2 + (4*a**2*d**2
*(-b/(4*a**2*d**2) - c*sqrt(-a**5*b)/(4*a**4*d**2)) + a*c**2 + b)/(a*c*d))
 + (-b/(4*a**2*d**2) + c*sqrt(-a**5*b)/(4*a**4*d**2))*log(x**2 + (4*a**2*d
**2*(-b/(4*a**2*d**2) + c*sqrt(-a**5*b)/(4*a**4*d**2)) + a*c**2 + b)/(a*c*
d)) + x**4/(4*a)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{x^4}{4\,a} + \frac{bc\arctan\left(\frac{ad^2x^2+acd}{\sqrt{ab}d}\right)}{2\,\sqrt{ab}ad^2} - \frac{b\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)}{4\,a^2d^2}$$

input
```
integrate(x^3/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/4*x^4/a + 1/2*b*c*arctan((a*d^2*x^2 + a*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a
*d^2) - 1/4*b*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)/(a^2*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{x^4}{4\,a} - \frac{b\,\ln\left(a\,c^2+2\,a\,c\,d\,x^2+a\,d^2\,x^4+b\right)}{4\,a^2\,d^2} + \frac{\sqrt{b}\,c\,\mathrm{atan}\left(\frac{a^{3/2}\,c^3}{\sqrt{b}\,(a\,c^2+b)}+\frac{\sqrt{a}\,\sqrt{b}\,c}{a\,c^2+b}+\frac{\sqrt{a}\,\sqrt{b}\,d\,x^2}{a\,c^2+b}+\frac{a^{3/2}\,c^2\,d\,x^2}{\sqrt{b}\,(a\,c^2+b)}\right)}{2\,a^{3/2}\,d^2}$$

input
```
int(x^3/(a + b/(c + d*x^2)^2),x)
```

output
```
x^4/(4*a) - (b*log(b + a*c^2 + a*d^2*x^4 + 2*a*c*d*x^2))/(4*a^2*d^2) + (b^
(1/2)*c*atan((a^(3/2)*c^3)/(b^(1/2)*(b + a*c^2)) + (a^(1/2)*b^(1/2)*c)/(b
+ a*c^2) + (a^(1/2)*b^(1/2)*d*x^2)/(b + a*c^2) + (a^(3/2)*c^2*d*x^2)/(b^(1
/2)*(b + a*c^2))))/(2*a^(3/2)*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.02

$$\int \frac{x^3}{a+\frac{b}{(c+dx^2)^2}}\,dx$$
$$= \frac{-2\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,atan\left(\frac{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,\sqrt{2}-2\sqrt{a}\,dx}{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{2}}\right)c-2\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+}$$

input
```
int(x^3/(a+b/(d*x^2+c)^2),x)
```

output
```
( - 2*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) -
 a*c)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(
a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*c - 2*sqrt
(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*atan
((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(s
qrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*c - log( - sqrt(d)*s
qrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)
*d*x**2)*b - log(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x +
sqrt(a*c**2 + b) + sqrt(a)*d*x**2)*b + a*d**2*x**4)/(4*a**2*d**2)
```

3.236 $\int \frac{x}{a+\frac{b}{\left(c+dx^2\right)^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}} dx = \frac{x^2}{2a} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2a^{3/2}d}$$

output
```
1/2*x^2/a-1/2*b^(1/2)*arctan(a^(1/2)*(d*x^2+c)/b^(1/2))/a^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}} dx = \frac{c+dx^2}{2ad} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2a^{3/2}d}$$

input
```
Integrate[x/(a + b/(c + d*x^2)^2),x]
```

output
```
(c + d*x^2)/(2*a*d) - (Sqrt[b]*ArcTan[(Sqrt[a]*(c + d*x^2))/Sqrt[b]])/(2*a
^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2024, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}}dx$$
$$\Big\downarrow \ 2024$$
$$\frac{\int \frac{1}{a+\frac{b}{\left(dx^2+c\right)^2}}d\left(dx^2+c\right)}{2d}$$
$$\Big\downarrow \ 772$$
$$\frac{\int \frac{\left(dx^2+c\right)^2}{a\left(dx^2+c\right)^2+b}d\left(dx^2+c\right)}{2d}$$
$$\Big\downarrow \ 262$$
$$\frac{\frac{c+dx^2}{a}-\frac{b\int \frac{1}{a\left(dx^2+c\right)^2+b}d(dx^2+c)}{a}}{2d}$$
$$\Big\downarrow \ 218$$
$$\frac{\frac{c+dx^2}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\left(c+dx^2\right)}{\sqrt{b}}\right)}{a^{3/2}}}{2d}$$

input
```
Int[x/(a + b/(c + d*x^2)^2),x]
```

output
```
((c + d*x^2)/a - (Sqrt[b]*ArcTan[(Sqrt[a]*(c + d*x^2))/Sqrt[b]])/a^(3/2))/
(2*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{d\,x^{2}+c}{a}-\frac{b \arctan\left(\frac{a\left(d\,x^{2}+c\right)}{\sqrt{a b}}\right)}{a \sqrt{a b}}}{2 d}$
default	$\frac{x^{2}}{2 a}-\frac{b \arctan\left(\frac{2 a\,d^{2} x^{2}+2 a c d}{2 d \sqrt{a b}}\right)}{2 a d \sqrt{a b}}$
risch	$\frac{x^{2}}{2 a}+\frac{\sqrt{-a b}\,\ln\left(\left(\sqrt{-a b}\,a c d-d a b\right) x^{2}+\sqrt{-a b}\,a\,c^{2}+\sqrt{-a b}\,b\right)}{4 a^{2} d}-\frac{\sqrt{-a b}\,\ln\left(\left(-\sqrt{-a b}\,a c d-d a b\right) x^{2}-\sqrt{-a b}\,a\,c^{2}-\sqrt{-a b}\,b\right.}{4 a^{2} d}$

input
```
int(x/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(1/a*(d*x^2+c)-b/a/(a*b)^(1/2)*arctan(a*(d*x^2+c)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.02

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}}\,dx = \left[\frac{2\,dx^2+\sqrt{-\frac{b}{a}}\log\left(\frac{ad^2x^4+2\,acdx^2+ac^2-2\,(adx^2+ac)\sqrt{-\frac{b}{a}}-b}{ad^2x^4+2\,acdx^2+ac^2+b}\right)}{4\,ad}, \frac{dx^2-\sqrt{\frac{b}{a}}\arctan\left(\frac{(adx^2+ac)\sqrt{\frac{b}{a}}}{b}\right)}{2\,ad}\right]$$

input
```
integrate(x/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*d*x^2 + sqrt(-b/a)*log((a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 - 2*(a*d*x
^2 + a*c)*sqrt(-b/a) - b)/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)))/(a*d), 1
/2*(d*x^2 - sqrt(b/a)*arctan((a*d*x^2 + a*c)*sqrt(b/a)/b))/(a*d)]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{\frac{\sqrt{-\frac{b}{a^3}}\log\left(x^2+\frac{-a\sqrt{-\frac{b}{a^3}}+c}{d}\right)}{4}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(x^2+\frac{a\sqrt{-\frac{b}{a^3}}+c}{d}\right)}{4}}{d}+\frac{x^2}{2a}$$

input
```
integrate(x/(a+b/(d*x**2+c)**2),x)
```

output
```
(sqrt(-b/a**3)*log(x**2 + (-a*sqrt(-b/a**3) + c)/d)/4 - sqrt(-b/a**3)*log(
x**2 + (a*sqrt(-b/a**3) + c)/d)/4)/d + x**2/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{x^2}{2\,a} - \frac{b\arctan\left(\frac{ad^2x^2+acd}{\sqrt{ab}d}\right)}{2\,\sqrt{ab}ad}$$

input
```
integrate(x/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/2*x^2/a - 1/2*b*arctan((a*d^2*x^2 + a*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{x^2}{2\,a} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,c+\sqrt{a}\,d\,x^2}{\sqrt{b}}\right)}{2\,a^{3/2}\,d}$$

input
```
int(x/(a + b/(c + d*x^2)^2),x)
```

output
```
x^2/(2*a) - (b^(1/2)*atan((a^(1/2)*c + a^(1/2)*d*x^2)/b^(1/2)))/(2*a^(3/2)
*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.19

$$\int \frac{x}{a+\frac{b}{(c+dx^2)^2}}\,dx$$

$$= \frac{\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,atan\left(\frac{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,\sqrt{2}-2\sqrt{a}\,dx}{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{2}}\right)+\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{}}{2a^2d}$$

input
```
int(x/(a+b/(d*x^2+c)^2),x)
```

output
```
(sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)
*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*
x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2))) + sqrt(sqrt(a)*
sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*atan((sqrt(d)
*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sq
rt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2))) + a*d*x**2)/(2*a**2*d)
```

3.237 $\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx=\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,(b+ac^2)}+\frac{c^2\log(x)}{b+ac^2}+\frac{b\log\left(b+a(c+dx^2)^2\right)}{4a\,(b+ac^2)}$$

output
```
1/2*b^(1/2)*c*arctan(a^(1/2)*(d*x^2+c)/b^(1/2))/a^(1/2)/(a*c^2+b)+c^2*ln(x
)/(a*c^2+b)+1/4*b*ln(b+a*(d*x^2+c)^2)/a/(a*c^2+b)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx=\frac{4ac^2\log(x)+\left(b-i\sqrt{a}\sqrt{b}c\right)\log\left(-i\sqrt{b}+\sqrt{a}(c+dx^2)\right)+\sqrt{b}\left(\sqrt{b}+i\sqrt{a}c\right)\log\left(i\sqrt{b}+\sqrt{a}(c+dx^2)\right)}{4a\,(b+ac^2)}$$

input
```
Integrate[1/(x*(a + b/(c + d*x^2)^2)),x]
```

output
```
(4*a*c^2*Log[x] + (b - I*Sqrt[a]*Sqrt[b]*c)*Log[(-I)*Sqrt[b] + Sqrt[a]*(c
+ d*x^2)] + Sqrt[b]*(Sqrt[b] + I*Sqrt[a]*c)*Log[I*Sqrt[b] + Sqrt[a]*(c + d
*x^2)])/(4*a*(b + a*c^2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{bdx(2c+dx^2)}{(ac^2+b)(ac^2+2acdx^2+ad^2x^4+b)}+\frac{c^2}{x(ac^2+b)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2\sqrt{a}(ac^2+b)}+\frac{b\log(ac^2+2acdx^2+ad^2x^4+b)}{4a(ac^2+b)}+\frac{c^2\log(x)}{ac^2+b}$$

input
```
Int[1/(x*(a + b/(c + d*x^2)^2)),x]
```

output
```
(Sqrt[b]*c*ArcTan[(Sqrt[a]*(c + d*x^2))/Sqrt[b]])/(2*Sqrt[a]*(b + a*c^2))
+ (c^2*Log[x])/(b + a*c^2) + (b*Log[b + a*c^2 + 2*a*c*d*x^2 + a*d^2*x^4])/
(4*a*(b + a*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

method	result
default	$\frac{bd\left(\frac{\ln\left(a\,d^2x^4+2ad\,x^2c+a\,c^2+b\right)}{2da}+\frac{c\arctan\left(\frac{2a\,d^2x^2+2acd}{2d\sqrt{ab}}\right)}{d\sqrt{ab}}\right)}{2a\,c^2+2b}+\frac{c^2\ln(x)}{a\,c^2+b}$
risch	$\frac{c^2\ln(x)}{a\,c^2+b}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(c^2a^3+a^2b\right)\_Z^2-2a\_Zb+b\right)}\_R\ln\left(\left(\left(-a^3c^2d+5a^2bd\right)\_R^2+\left(4a^2c^2d-9dab\right)\_R+4bd\right)x^2+\left(-a^3c^3-\right.\right.\right.}{4}$

input
```
int(1/x/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*d/(a*c^2+b)*(1/2/d/a*ln(a*d^2*x^4+2*a*c*d*x^2+a*c^2+b)+c/d/(a*b)^(1/
2)*arctan(1/2*(2*a*d^2*x^2+2*a*c*d)/d/(a*b)^(1/2)))+c^2*ln(x)/(a*c^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.41

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx$$
$$=\left[\frac{4\,ac^2\log(x)+ac\sqrt{-\frac{b}{a}}\log\left(\frac{ad^2x^4+2\,acdx^2+ac^2+2\,(adx^2+ac)\sqrt{-\frac{b}{a}}-b}{ad^2x^4+2\,acdx^2+ac^2+b}\right)+b\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)\quad 4\,a}{4\,(a^2c^2+ab)},\,-\right.$$

input
```
integrate(1/x/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*c^2*log(x) + a*c*sqrt(-b/a)*log((a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2
 + 2*(a*d*x^2 + a*c)*sqrt(-b/a) - b)/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)
) + b*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b))/(a^2*c^2 + a*b), 1/4*(4*a*
c^2*log(x) + 2*a*c*sqrt(b/a)*arctan((a*d*x^2 + a*c)*sqrt(b/a)/b) + b*log(a
*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b))/(a^2*c^2 + a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(76) = 152$.

Time = 48.94 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.19

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \frac{c^2\log(x)}{ac^2+b} + \left(\frac{b}{4a(ac^2+b)} - \frac{c\sqrt{-a^3b}}{4a^2(ac^2+b)}\right)\log\left(x^2 + \frac{4a^2c^2\left(\frac{b}{4a(ac^2+b)} - \frac{c\sqrt{-a^3b}}{4a^2(ac^2+b)}\right) + 4ab\left(\frac{b}{4a(ac^2+b)} - \frac{c\sqrt{-a^3b}}{4a^2(ac^2+b)}\right) + ac^2 - b}{acd}\right) + \left(\frac{b}{4a(ac^2+b)} + \frac{c\sqrt{-a^3b}}{4a^2(ac^2+b)}\right)\log\left(x^2 + \frac{4a^2c^2\left(\frac{b}{4a(ac^2+b)} + \frac{c\sqrt{-a^3b}}{4a^2(ac^2+b)}\right) + 4ab\left(\frac{b}{4a(ac^2+b)} + \frac{c\sqrt{-a^3b}}{4a^2(ac^2+b)}\right) + ac^2 - b}{acd}\right)$$

input
```
integrate(1/x/(a+b/(d*x**2+c)**2),x)
```

output
```
c**2*log(x)/(a*c**2 + b) + (b/(4*a*(a*c**2 + b)) - c*sqrt(-a**3*b)/(4*a**2
*(a*c**2 + b)))*log(x**2 + (4*a**2*c**2*(b/(4*a*(a*c**2 + b)) - c*sqrt(-a*
*3*b)/(4*a**2*(a*c**2 + b))) + 4*a*b*(b/(4*a*(a*c**2 + b)) - c*sqrt(-a**3*
b)/(4*a**2*(a*c**2 + b))) + a*c**2 - b)/(a*c*d)) + (b/(4*a*(a*c**2 + b)) +
 c*sqrt(-a**3*b)/(4*a**2*(a*c**2 + b)))*log(x**2 + (4*a**2*c**2*(b/(4*a*(a
*c**2 + b)) + c*sqrt(-a**3*b)/(4*a**2*(a*c**2 + b))) + 4*a*b*(b/(4*a*(a*c*
*2 + b)) + c*sqrt(-a**3*b)/(4*a**2*(a*c**2 + b))) + a*c**2 - b)/(a*c*d))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \frac{c^2\log\left(x^2\right)}{2\left(ac^2+b\right)} + \frac{bc\arctan\left(\frac{ad^2x^2+acd}{\sqrt{abd}}\right)}{2\left(ac^2+b\right)\sqrt{ab}} + \frac{b\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)}{4\left(a^2c^2+ab\right)}$$

input
```
integrate(1/x/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/2*c^2*log(x^2)/(a*c^2 + b) + 1/2*b*c*arctan((a*d^2*x^2 + a*c*d)/(sqrt(a*
b)*d))/((a*c^2 + b)*sqrt(a*b)) + 1/4*b*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2
 + b)/(a^2*c^2 + a*b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 2098, normalized size of antiderivative = 23.31

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)} dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b/(c + d*x^2)^2)),x)
```

output
```
(c^2*log(x))/(b + a*c^2) + (4*a*b*log(b + a*c^2 + a*d^2*x^4 + 2*a*c*d*x^2)
)/(16*a^2*b + 16*a^3*c^2) + (b^(1/2)*c*atan(((x^2*((a*(b + a*c^2)^(3/2)*((
b^(1/2)*c*(4*a^2*b*c*d^7 - (4*a*b*(36*a^3*b*c*d^7 - 16*a^4*c^3*d^7 + (4*a*
b*(16*a^5*c^3*d^7 - 80*a^4*b*c*d^7))/(16*a^2*b + 16*a^3*c^2)))/(16*a^2*b +
 16*a^3*c^2)))/(4*(b + a*c^2)^(1/2)*(a*b + a^2*c^2)^(1/2)) - (4*a*b*((b^(1
/2)*c*(36*a^3*b*c*d^7 - 16*a^4*c^3*d^7 + (4*a*b*(16*a^5*c^3*d^7 - 80*a^4*b
*c*d^7))/(16*a^2*b + 16*a^3*c^2)))/(4*(b + a*c^2)^(1/2)*(a*b + a^2*c^2)^(1
/2)) + (a*b^(3/2)*c*(16*a^5*c^3*d^7 - 80*a^4*b*c*d^7))/((b + a*c^2)^(1/2)*
(a*b + a^2*c^2)^(1/2)*(16*a^2*b + 16*a^3*c^2))))/(16*a^2*b + 16*a^3*c^2) +
 (b^(3/2)*c^3*(16*a^5*c^3*d^7 - 80*a^4*b*c*d^7))/(64*(b + a*c^2)^(3/2)*(a*
b + a^2*c^2)^(3/2)))*(b^2 + 13*a^2*c^4 - 10*a*b*c^2))/((a*(b + a*c^2))^(5/
2)*(b + a*c^2)*(b^6 + 16*a^6*c^12 - 4*a*b^5*c^2 + 48*a^5*b*c^10 - a^2*b^4*
c^4 + 22*a^3*b^3*c^6 + 51*a^4*b^2*c^8) + b*c^2*(a*(b + a*c^2))^(7/2)*(b^4
+ 9*a^4*c^8 - 5*a*b^3*c^2 + 18*a^3*b*c^6 + 4*a^2*b^2*c^4)) - (4*c*((4*a*b*
(4*a^2*b*c*d^7 - (4*a*b*(36*a^3*b*c*d^7 - 16*a^4*c^3*d^7 + (4*a*b*(16*a^5*
c^3*d^7 - 80*a^4*b*c*d^7))/(16*a^2*b + 16*a^3*c^2)))/(16*a^2*b + 16*a^3*c^
2)))/(16*a^2*b + 16*a^3*c^2) + (b^(1/2)*c*((b^(1/2)*c*(36*a^3*b*c*d^7 - 16
*a^4*c^3*d^7 + (4*a*b*(16*a^5*c^3*d^7 - 80*a^4*b*c*d^7))/(16*a^2*b + 16*a^
3*c^2)))/(4*(b + a*c^2)^(1/2)*(a*b + a^2*c^2)^(1/2)) + (a*b^(3/2)*c*(16*a^
5*c^3*d^7 - 80*a^4*b*c*d^7))/((b + a*c^2)^(1/2)*(a*b + a^2*c^2)^(1/2)*(...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.27

$$\int \frac{1}{x\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx$$
$$= \frac{-2\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,atan\left(\frac{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,\sqrt{2}-2\sqrt{a}\,dx}{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{2}}\right)c-2\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+}{}$$

input
```
int(1/x/(a+b/(d*x^2+c)^2),x)
```

output
```
( - 2*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) -
 a*c)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(
a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*c - 2*sqrt
(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*atan
((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(s
qrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*c + log( - sqrt(d)*s
qrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)
*d*x**2)*b + log(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x +
sqrt(a*c**2 + b) + sqrt(a)*d*x**2)*b + 4*log(x)*a*c**2)/(4*a*(a*c**2 + b))
```

3.238 $\int \frac{1}{x^3\left(a+\frac{b}{\left(c+dx^2\right)^2}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 117

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx=-\frac{c^2}{2\left(b+ac^2\right)x^2}+\frac{\sqrt{b}(b-ac^2)\,d\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2\sqrt{a}\left(b+ac^2\right)^2}+\frac{2bcd\log(x)}{\left(b+ac^2\right)^2}-\frac{bcd\log\left(b+a(c+dx^2)^2\right)}{2\left(b+ac^2\right)^2}$$

output

```
-1/2*c^2/(a*c^2+b)/x^2+1/2*b^(1/2)*(-a*c^2+b)*d*arctan(a^(1/2)*(d*x^2+c)/b
^(1/2))/a^(1/2)/(a*c^2+b)^2+2*b*c*d*ln(x)/(a*c^2+b)^2-1/2*b*c*d*ln(b+a*(d*
x^2+c)^2)/(a*c^2+b)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx$$
$$= \frac{8\sqrt{a}bcdx^2\log(x)+i\left(-\sqrt{b}\left(\sqrt{b}-i\sqrt{a}c\right)^2dx^2\log\left(-i\sqrt{b}+\sqrt{a}(c+dx^2)\right)+\left(\sqrt{b}+i\sqrt{a}c\right)\left(2i\sqrt{a}\sqrt{b}c^2\right.\right.}{4\sqrt{a}\left(b+ac^2\right)^2x^2}$$

input
```
Integrate[1/(x^3*(a + b/(c + d*x^2)^2)),x]
```

output
```
(8*Sqrt[a]*b*c*d*x^2*Log[x] + I*(-(Sqrt[b]*(Sqrt[b] - I*Sqrt[a]*c)^2*d*x^2
*Log[(-I)*Sqrt[b] + Sqrt[a]*(c + d*x^2)]) + (Sqrt[b] + I*Sqrt[a]*c)*((2*I)
*Sqrt[a]*Sqrt[b]*c^2 + 2*a*c^3 + Sqrt[b]*(Sqrt[b] + I*Sqrt[a]*c)*d*x^2*Log
[I*Sqrt[b] + Sqrt[a]*(c + d*x^2)])))/(4*Sqrt[a]*(b + a*c^2)^2*x^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(\frac{bd^2x\left(-3ac^2-2acdx^2+b\right)}{\left(ac^2+b\right)^2\left(ac^2+2acdx^2+ad^2x^4+b\right)}+\frac{2bcd}{x\left(ac^2+b\right)^2}+\frac{c^2}{x^3\left(ac^2+b\right)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{b}d(b-ac^2)\arctan\left(\frac{\sqrt{a}(c+dx^2)}{\sqrt{b}}\right)}{2\sqrt{a}\left(ac^2+b\right)^2}-\frac{bcd\log\left(ac^2+2acdx^2+ad^2x^4+b\right)}{2\left(ac^2+b\right)^2}+\frac{2bcd\log(x)}{\left(ac^2+b\right)^2}-\frac{c^2}{2x^2\left(ac^2+b\right)}$$

input
```
Int[1/(x^3*(a + b/(c + d*x^2)^2)),x]
```

output
```
-1/2*c^2/((b + a*c^2)*x^2) + (Sqrt[b]*(b - a*c^2)*d*ArcTan[(Sqrt[a]*(c + d
*x^2))/Sqrt[b]])/(2*Sqrt[a]*(b + a*c^2)^2) + (2*b*c*d*Log[x])/(b + a*c^2)^
2 - (b*c*d*Log[b + a*c^2 + 2*a*c*d*x^2 + a*d^2*x^4])/(2*(b + a*c^2)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

method	result
default	$-\frac{b\,d^2\left(\frac{c\ln\left(a\,d^2x^4+2ad\,x^2c+a\,c^2+b\right)}{d}+\frac{\left(a\,c^2-b\right)\arctan\left(\frac{2a\,d^2x^2+2acd}{2d\sqrt{ab}}\right)}{d\sqrt{ab}}\right)}{2(a\,c^2+b)^2}-\frac{c^2}{2(a\,c^2+b)x^2}+\frac{2bcd\ln(x)}{(a\,c^2+b)^2}$
risch	$-\frac{c^2}{2(a\,c^2+b)x^2}+\frac{2dbc\ln(x)}{a^2c^4+2ab\,c^2+b^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^3c^4+2a^2b\,c^2+a\,b^2\right)\_Z^2+4abcd\_Z+b\,d^2\right)}\_R\ln\left(\left(\left(-a^4c^6d+3a^3b\,c^4d+9a^2\right.\right.\right.\right.}{}$

input
```
int(1/x^3/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*d^2/(a*c^2+b)^2*(c/d*ln(a*d^2*x^4+2*a*c*d*x^2+a*c^2+b)+(a*c^2-b)/d/
(a*b)^(1/2)*arctan(1/2*(2*a*d^2*x^2+2*a*c*d)/d/(a*b)^(1/2)))-1/2*c^2/(a*c^
2+b)/x^2+2*b*c*d*ln(x)/(a*c^2+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.53

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \left[-\frac{2\,bcdx^2\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)-8\,bcdx^2\log\left(x\right)-(ac^2-b)dx^2\sqrt{-\frac{b}{a}}\log\left(\frac{ad^2x^4+2\,acdx^2+ac^2-2}{ad^2x^4+2\,acd}\right.}{4\,(a^2c^4+2\,abc^2+b^2)x^2}\right.$$

$$-\frac{bcdx^2\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)-4\,bcdx^2\log\left(x\right)+(ac^2-b)dx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{(adx^2+ac)\sqrt{\frac{b}{a}}}{b}\right)+ac}{2\,(a^2c^4+2\,abc^2+b^2)x^2}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
[-1/4*(2*b*c*d*x^2*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b) - 8*b*c*d*x^2*
log(x) - (a*c^2 - b)*d*x^2*sqrt(-b/a)*log((a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2
 - 2*(a*d*x^2 + a*c)*sqrt(-b/a) - b)/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)
) + 2*a*c^4 + 2*b*c^2)/((a^2*c^4 + 2*a*b*c^2 + b^2)*x^2), -1/2*(b*c*d*x^2*
log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b) - 4*b*c*d*x^2*log(x) + (a*c^2 - b
)*d*x^2*sqrt(b/a)*arctan((a*d*x^2 + a*c)*sqrt(b/a)/b) + a*c^4 + b*c^2)/((a
^2*c^4 + 2*a*b*c^2 + b^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a+b/(d*x**2+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = -\frac{bcd\log\left(ad^2x^4+2\,acdx^2+ac^2+b\right)}{2\left(a^2c^4+2\,abc^2+b^2\right)}+\frac{bcd\log\left(x^2\right)}{a^2c^4+2\,abc^2+b^2} - \frac{(abc^2-b^2)d\arctan\left(\frac{ad^2x^2+acd}{\sqrt{ab}d}\right)}{2\left(a^2c^4+2\,abc^2+b^2\right)\sqrt{ab}} - \frac{c^2}{2\left(ac^2+b\right)x^2}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
-1/2*b*c*d*log(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b)/(a^2*c^4 + 2*a*b*c^2 +
 b^2) + b*c*d*log(x^2)/(a^2*c^4 + 2*a*b*c^2 + b^2) - 1/2*(a*b*c^2 - b^2)*d
*arctan((a*d^2*x^2 + a*c*d)/(sqrt(a*b)*d))/((a^2*c^4 + 2*a*b*c^2 + b^2)*sq
rt(a*b)) - 1/2*c^2/((a*c^2 + b)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 5179, normalized size of antiderivative = 44.26

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b/(c + d*x^2)^2)),x)
```

output
```
(2*b*c*d*log(x))/(b^2 + a^2*c^4 + 2*a*b*c^2) - c^2/(2*x^2*(b + a*c^2)) - (
b^(1/2)*d*atan((x^2*(((b^2 + 13*a^2*c^4 - 34*a*b*c^2)*((a^2*b^3*d^10)/(b^3
 + a^3*c^6 + 3*a*b^2*c^2 + 3*a^2*b*c^4) - (b^(1/2)*d*(b - a*c^2)*((b^(1/2)
*d*(b - a*c^2)*((20*a^3*b^4*d^8 + 28*a^6*b*c^6*d^8 + 68*a^4*b^3*c^2*d^8 +
76*a^5*b^2*c^4*d^8)/(b^3 + a^3*c^6 + 3*a*b^2*c^2 + 3*a^2*b*c^4) - (8*a*b*c
*d*(80*a^4*b^4*c*d^7 - 16*a^8*c^9*d^7 + 32*a^7*b*c^7*d^7 + 224*a^5*b^3*c^3
*d^7 + 192*a^6*b^2*c^5*d^7))/((16*a*b^2 + 16*a^3*c^4 + 32*a^2*b*c^2)*(b^3
+ a^3*c^6 + 3*a*b^2*c^2 + 3*a^2*b*c^4))))/(4*a^(1/2)*(b^2 + a^2*c^4 + 2*a*
b*c^2)) - (2*a^(1/2)*b^(3/2)*c*d^2*(b - a*c^2)*(80*a^4*b^4*c*d^7 - 16*a^8*
c^9*d^7 + 32*a^7*b*c^7*d^7 + 224*a^5*b^3*c^3*d^7 + 192*a^6*b^2*c^5*d^7))/(
(16*a*b^2 + 16*a^3*c^4 + 32*a^2*b*c^2)*(b^2 + a^2*c^4 + 2*a*b*c^2)*(b^3 +
a^3*c^6 + 3*a*b^2*c^2 + 3*a^2*b*c^4))))/(4*a^(1/2)*(b^2 + a^2*c^4 + 2*a*b*
c^2)) - (8*a*b*c*d*((12*a^3*b^3*c*d^9 + 12*a^4*b^2*c^3*d^9)/(b^3 + a^3*c^6
 + 3*a*b^2*c^2 + 3*a^2*b*c^4) - (8*a*b*c*d*((20*a^3*b^4*d^8 + 28*a^6*b*c^6
*d^8 + 68*a^4*b^3*c^2*d^8 + 76*a^5*b^2*c^4*d^8)/(b^3 + a^3*c^6 + 3*a*b^2*c
^2 + 3*a^2*b*c^4) - (8*a*b*c*d*(80*a^4*b^4*c*d^7 - 16*a^8*c^9*d^7 + 32*a^7
*b*c^7*d^7 + 224*a^5*b^3*c^3*d^7 + 192*a^6*b^2*c^5*d^7))/((16*a*b^2 + 16*a
^3*c^4 + 32*a^2*b*c^2)*(b^3 + a^3*c^6 + 3*a*b^2*c^2 + 3*a^2*b*c^4))))/(16*
a*b^2 + 16*a^3*c^4 + 32*a^2*b*c^2)))/(16*a*b^2 + 16*a^3*c^4 + 32*a^2*b*c^2
) + (b^2*c*d^3*(b - a*c^2)^2*(80*a^4*b^4*c*d^7 - 16*a^8*c^9*d^7 + 32*a^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 546, normalized size of antiderivative = 4.67

$$\int \frac{1}{x^3\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx$$

$$= \frac{\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,atan\left(\frac{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,\sqrt{2}-2\sqrt{a}\,dx}{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{2}}\right)a\,c^2 d\,x^2-\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}}}{}$$

input
```
int(1/x^3/(a+b/(d*x^2+c)^2),x)
```

output

```
(sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)
*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*
x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2*d*x**2 -
 sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)
*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*
x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b*d*x**2 + sqrt
(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*atan
((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(s
qrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2*d*x**2 - sqrt
(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*atan
((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(s
qrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b*d*x**2 - log( - sq
rt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) +
sqrt(a)*d*x**2)*a*b*c*d*x**2 - log(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) -
 a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)*d*x**2)*a*b*c*d*x**2 + 4*log(
x)*a*b*c*d*x**2 - a**2*c**4 - a*b*c**2)/(2*a*x**2*(a**2*c**4 + 2*a*b*c**2
+ b**2))
```

3.239 $\int \frac{x^6}{a+\frac{b}{\left(c+dx^2\right)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 448

$$\int \frac{x^6}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{2bcx}{a^2d^3}-\frac{bx^3}{3a^2d^2}+\frac{x^7}{7a}$$
$$-\frac{b\left(b-3ac^2-2\sqrt{a}c\sqrt{b+ac^2}\right)\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{11/4}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}\,d^{7/2}}$$
$$+\frac{b\left(b-3ac^2-2\sqrt{a}c\sqrt{b+ac^2}\right)\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{11/4}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}\,d^{7/2}}$$
$$-\frac{b\left(b-3ac^2+2\sqrt{a}c\sqrt{b+ac^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}x}{\sqrt{b+ac^2}+\sqrt{a}dx^2}\right)}{2\sqrt{2}a^{11/4}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\,d^{7/2}}$$

output
```
2*b*c*x/a^2/d^3-1/3*b*x^3/a^2/d^2+1/7*x^7/a-1/4*b*(b-3*a*c^2-2*a^(1/2)*c*(
a*c^2+b)^(1/2))*arctan(((-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)-2^(1/2)*a^(1/4)
*d^(1/2)*x)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2))*2^(1/2)/a^(11/4)/(a^(1/2)*c
+(a*c^2+b)^(1/2))^(1/2)/d^(7/2)+1/4*b*(b-3*a*c^2-2*a^(1/2)*c*(a*c^2+b)^(1/
2))*arctan(((-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)+2^(1/2)*a^(1/4)*d^(1/2)*x)/
(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2))*2^(1/2)/a^(11/4)/(a^(1/2)*c+(a*c^2+b)^(
1/2))^(1/2)/d^(7/2)-1/4*b*(b-3*a*c^2+2*a^(1/2)*c*(a*c^2+b)^(1/2))*arctanh(
2^(1/2)*a^(1/4)*(-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)*d^(1/2)*x/((a*c^2+b)^(1
/2)+a^(1/2)*d*x^2))*2^(1/2)/a^(11/4)/(-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^
(7/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.50

$$\int \frac{x^6}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{2bcx}{a^2d^3}-\frac{bx^3}{3a^2d^2}+\frac{x^7}{7a}+\frac{\sqrt{b}\left(\sqrt{b}+i\sqrt{a}c\right)^3\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{-i\sqrt{a}\sqrt{b}+ac}}\right)}{2a^{5/2}\sqrt{-i\sqrt{a}\sqrt{b}+ac}d^{7/2}} + \frac{\sqrt{b}\left(\sqrt{b}-i\sqrt{a}c\right)^3\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{i\sqrt{a}\sqrt{b}+ac}}\right)}{2a^{5/2}\sqrt{i\sqrt{a}\sqrt{b}+ac}d^{7/2}}$$

input
```
Integrate[x^6/(a + b/(c + d*x^2)^2),x]
```

output
```
(2*b*c*x)/(a^2*d^3) - (b*x^3)/(3*a^2*d^2) + x^7/(7*a) + (Sqrt[b]*(Sqrt[b]
+ I*Sqrt[a]*c)^3*ArcTan[(Sqrt[a]*Sqrt[d]*x)/Sqrt[(-I)*Sqrt[a]*Sqrt[b] + a*
c]])/(2*a^(5/2)*Sqrt[(-I)*Sqrt[a]*Sqrt[b] + a*c]*d^(7/2)) + (Sqrt[b]*(Sqrt
[b] - I*Sqrt[a]*c)^3*ArcTan[(Sqrt[a]*Sqrt[d]*x)/Sqrt[I*Sqrt[a]*Sqrt[b] + a
*c]])/(2*a^(5/2)*Sqrt[I*Sqrt[a]*Sqrt[b] + a*c]*d^(7/2))
```

Rubi [A] (verified)

Time = 3.38 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+\frac{b}{(c+dx^2)^2}}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{b\left(dx^2\left(b-3ac^2\right)-2c\left(ac^2+b\right)\right)}{a^2d^3\left(ac^2+2acdx^2+ad^2x^4+b\right)}+\frac{2bc}{a^2d^3}-\frac{bx^2}{a^2d^2}+\frac{x^6}{a}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(2a^{3/2}c^3-\left(b-3ac^2\right)\sqrt{ac^2+b}+2\sqrt{a}bc\right)\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{11/4}d^{7/2}\sqrt{ac^2+b}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}-\frac{b\left(2a^{3/2}c^3-\left(b-3ac^2\right)\sqrt{ac^2+b}+2\sqrt{a}bc\right)\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{11/4}d^{7/2}\sqrt{ac^2+b}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}+\frac{b\left(2\sqrt{a}c\sqrt{ac^2+b}-3ac^2+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{11/4}d^{7/2}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}-\frac{b\left(2\sqrt{a}c\sqrt{ac^2+b}-3ac^2+b\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{11/4}d^{7/2}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}+\frac{2bcx}{a^2d^3}-\frac{bx^3}{3a^2d^2}+\frac{x^7}{7a}$$

input
```
Int[x^6/(a + b/(c + d*x^2)^2),x]
```

output
```
(2*b*c*x)/(a^2*d^3) - (b*x^3)/(3*a^2*d^2) + x^7/(7*a) + (b*(2*Sqrt[a]*b*c
+ 2*a^(3/2)*c^3 - (b - 3*a*c^2)*Sqrt[b + a*c^2])*ArcTan[(Sqrt[-(Sqrt[a]*c)
 + Sqrt[b + a*c^2]] - Sqrt[2]*a^(1/4)*Sqrt[d]*x)/Sqrt[Sqrt[a]*c + Sqrt[b +
 a*c^2]]])/(2*Sqrt[2]*a^(11/4)*Sqrt[b + a*c^2]*Sqrt[Sqrt[a]*c + Sqrt[b + a
*c^2]]*d^(7/2)) - (b*(2*Sqrt[a]*b*c + 2*a^(3/2)*c^3 - (b - 3*a*c^2)*Sqrt[b
 + a*c^2])*ArcTan[(Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]] + Sqrt[2]*a^(1/4)*
Sqrt[d]*x)/Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]])/(2*Sqrt[2]*a^(11/4)*Sqrt[b
+ a*c^2]*Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]*d^(7/2)) + (b*(b - 3*a*c^2 + 2*
Sqrt[a]*c*Sqrt[b + a*c^2])*Log[Sqrt[b + a*c^2] - Sqrt[2]*a^(1/4)*Sqrt[-(Sq
rt[a]*c) + Sqrt[b + a*c^2]]*Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqrt[2]*a^(11/4
)*Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]*d^(7/2)) - (b*(b - 3*a*c^2 + 2*Sqrt
[a]*c*Sqrt[b + a*c^2])*Log[Sqrt[b + a*c^2] + Sqrt[2]*a^(1/4)*Sqrt[-(Sqrt[a
]*c) + Sqrt[b + a*c^2]]*Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqrt[2]*a^(11/4)*Sq
rt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]*d^(7/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{x^7}{7a}-\frac{b\,x^3}{3a^2d^2}+\frac{2bcx}{a^2d^3}+\frac{b\left(\sum_{\_R=\operatorname{RootOf}\left(a\,d^2\_Z^4+2acd\_Z^2+a\,c^2+b\right)}\frac{\left(d\left(-3a\,c^2+b\right)\_R^2-2c^3a-2bc\right)\ln\left(x-\_R\right)}{d\_R^3+c\_R}\right)}{4a^3d^4}$	111
default	Expression too large to display	1742

input
```
int(x^6/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7/a-1/3*b*x^3/a^2/d^2+2*b*c*x/a^2/d^3+1/4/a^3/d^4*b*sum((d*(-3*a*c^2
+b)*_R^2-2*c^3*a-2*b*c)/(_R^3*d+_R*c)*ln(x-_R),_R=RootOf(_Z^4*a*d^2+2*_Z^2
*a*c*d+a*c^2+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1444 vs. $2(346) = 692$.

Time = 0.14 (sec) , antiderivative size = 1444, normalized size of antiderivative = 3.22

$$\int \frac{x^6}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
1/84*(12*a*d^3*x^7 - 21*a^2*d^3*sqrt((a^5*d^7*sqrt(-(25*a^4*b^3*c^8 - 100*
a^3*b^4*c^6 + 110*a^2*b^5*c^4 - 20*a*b^6*c^2 + b^7)/(a^11*d^14)) + a^2*b*c
^5 - 10*a*b^2*c^3 + 5*b^3*c)/(a^5*d^7))*log((5*a^4*b^2*c^8 - 14*a^2*b^4*c^
4 - 8*a*b^5*c^2 + b^6)*x + ((a^9*c^2 - a^8*b)*d^10*sqrt(-(25*a^4*b^3*c^8 -
 100*a^3*b^4*c^6 + 110*a^2*b^5*c^4 - 20*a*b^6*c^2 + b^7)/(a^11*d^14)) + 2*
(5*a^5*b^2*c^5 - 10*a^4*b^3*c^3 + a^3*b^4*c)*d^3)*sqrt((a^5*d^7*sqrt(-(25*
a^4*b^3*c^8 - 100*a^3*b^4*c^6 + 110*a^2*b^5*c^4 - 20*a*b^6*c^2 + b^7)/(a^1
1*d^14)) + a^2*b*c^5 - 10*a*b^2*c^3 + 5*b^3*c)/(a^5*d^7))) + 21*a^2*d^3*sq
rt((a^5*d^7*sqrt(-(25*a^4*b^3*c^8 - 100*a^3*b^4*c^6 + 110*a^2*b^5*c^4 - 20
*a*b^6*c^2 + b^7)/(a^11*d^14)) + a^2*b*c^5 - 10*a*b^2*c^3 + 5*b^3*c)/(a^5*
d^7))*log((5*a^4*b^2*c^8 - 14*a^2*b^4*c^4 - 8*a*b^5*c^2 + b^6)*x - ((a^9*c
^2 - a^8*b)*d^10*sqrt(-(25*a^4*b^3*c^8 - 100*a^3*b^4*c^6 + 110*a^2*b^5*c^4
 - 20*a*b^6*c^2 + b^7)/(a^11*d^14)) + 2*(5*a^5*b^2*c^5 - 10*a^4*b^3*c^3 +
a^3*b^4*c)*d^3)*sqrt((a^5*d^7*sqrt(-(25*a^4*b^3*c^8 - 100*a^3*b^4*c^6 + 11
0*a^2*b^5*c^4 - 20*a*b^6*c^2 + b^7)/(a^11*d^14)) + a^2*b*c^5 - 10*a*b^2*c^
3 + 5*b^3*c)/(a^5*d^7))) + 21*a^2*d^3*sqrt(-(a^5*d^7*sqrt(-(25*a^4*b^3*c^8
 - 100*a^3*b^4*c^6 + 110*a^2*b^5*c^4 - 20*a*b^6*c^2 + b^7)/(a^11*d^14)) -
a^2*b*c^5 + 10*a*b^2*c^3 - 5*b^3*c)/(a^5*d^7))*log((5*a^4*b^2*c^8 - 14*a^2
*b^4*c^4 - 8*a*b^5*c^2 + b^6)*x + ((a^9*c^2 - a^8*b)*d^10*sqrt(-(25*a^4*b^
3*c^8 - 100*a^3*b^4*c^6 + 110*a^2*b^5*c^4 - 20*a*b^6*c^2 + b^7)/(a^11*d...
```

Sympy [A] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.64

$$\int \frac{x^6}{a+\frac{b}{(c+dx^2)^2}}\,dx$$
$$= \text{RootSum}\left(256t^4a^{11}d^{14} + t^2\left(-32a^8bc^5d^7 + 320a^7b^2c^3d^7 - 160a^6b^3cd^7\right) + a^5b^2c^{10} + 5a^4b^3c^8 + 10a^3b^4c^6 + \right.$$
$$+ \frac{x^7}{7a} + \frac{2bcx}{a^2d^3} - \frac{bx^3}{3a^2d^2}$$

input
```
integrate(x**6/(a+b/(d*x**2+c)**2),x)
```

output
```
RootSum(256*_t**4*a**11*d**14 + _t**2*(-32*a**8*b*c**5*d**7 + 320*a**7*b**
2*c**3*d**7 - 160*a**6*b**3*c*d**7) + a**5*b**2*c**10 + 5*a**4*b**3*c**8 +
 10*a**3*b**4*c**6 + 10*a**2*b**5*c**4 + 5*a*b**6*c**2 + b**7, Lambda(_t,
_t*log(x + (-64*_t**3*a**9*c**2*d**10 + 64*_t**3*a**8*b*d**10 + 4*_t*a**6*
b*c**7*d**3 - 84*_t*a**5*b**2*c**5*d**3 + 140*_t*a**4*b**3*c**3*d**3 - 28*
_t*a**3*b**4*c*d**3)/(5*a**4*b**2*c**8 - 14*a**2*b**4*c**4 - 8*a*b**5*c**2
 + b**6))))) + x**7/(7*a) + 2*b*c*x/(a**2*d**3) - b*x**3/(3*a**2*d**2)
```

Maxima [F]

$$\int \frac{x^6}{a+\frac{b}{(c+dx^2)^2}}\,dx = \int \frac{x^6}{a+\frac{b}{(dx^2+c)^2}}\,dx$$

input
```
integrate(x^6/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
-b*integrate((2*a*c^3 + (3*a*c^2 - b)*d*x^2 + 2*b*c)/(a*d^2*x^4 + 2*a*c*d*
x^2 + a*c^2 + b), x)/(a^2*d^3) + 1/21*(3*a*d^3*x^7 - 7*b*d*x^3 + 42*b*c*x)
/(a^2*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6}{a + \frac{b}{(c+dx^2)^2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 3247, normalized size of antiderivative = 7.25

$$\int \frac{x^6}{a + \frac{b}{(c+dx^2)^2}} \, dx = \text{Too large to display}$$

input
```
int(x^6/(a + b/(c + d*x^2)^2),x)
```

output
```
atan(((((32*a^6*b^3*c*d^7 + 32*a^7*b^2*c^3*d^7)/(a^5*d^4) - 64*a^4*b*c*d^7
*x*((b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3*c + a
^8*b*c^5 - 10*a^7*b^2*c^3 - 10*a*b*c^2*(-a^11*b^3)^(1/2))/(16*a^11*d^7))^(
1/2))*((b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3*c
+ a^8*b*c^5 - 10*a^7*b^2*c^3 - 10*a*b*c^2*(-a^11*b^3)^(1/2))/(16*a^11*d^7)
)^(1/2) - (4*x*(b^5 - 15*a*b^4*c^2 + 15*a^2*b^3*c^4 - a^3*b^2*c^6))/a^2)*(
(b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3*c + a^8*b
*c^5 - 10*a^7*b^2*c^3 - 10*a*b*c^2*(-a^11*b^3)^(1/2))/(16*a^11*d^7))^(1/2)
*1i - (((32*a^6*b^3*c*d^7 + 32*a^7*b^2*c^3*d^7)/(a^5*d^4) + 64*a^4*b*c*d^7
*x*((b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3*c + a
^8*b*c^5 - 10*a^7*b^2*c^3 - 10*a*b*c^2*(-a^11*b^3)^(1/2))/(16*a^11*d^7))^(
1/2))*((b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3*c
+ a^8*b*c^5 - 10*a^7*b^2*c^3 - 10*a*b*c^2*(-a^11*b^3)^(1/2))/(16*a^11*d^7)
)^(1/2) + (4*x*(b^5 - 15*a*b^4*c^2 + 15*a^2*b^3*c^4 - a^3*b^2*c^6))/a^2)*(
(b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3*c + a^8*b
*c^5 - 10*a^7*b^2*c^3 - 10*a*b*c^2*(-a^11*b^3)^(1/2))/(16*a^11*d^7))^(1/2)
*1i)/((((32*a^6*b^3*c*d^7 + 32*a^7*b^2*c^3*d^7)/(a^5*d^4) - 64*a^4*b*c*d^7
*x*((b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3*c + a
^8*b*c^5 - 10*a^7*b^2*c^3 - 10*a*b*c^2*(-a^11*b^3)^(1/2))/(16*a^11*d^7))^(
1/2))*((b^2*(-a^11*b^3)^(1/2) + 5*a^2*c^4*(-a^11*b^3)^(1/2) + 5*a^6*b^3...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1301, normalized size of antiderivative = 2.90

$$\int \frac{x^6}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Too large to display}$$

input
```
int(x^6/(a+b/(d*x^2+c)^2),x)
```

output
```
(42*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*
atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x
)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2 - 42*sqrt
(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sq
rt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(
d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b - 42*sqrt(d)*sqrt(a)*s
qrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqr
t(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a
*c**2 + b) + a*c)*sqrt(2)))*a*c**3 + 126*sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqrt
(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) -
a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c
)*sqrt(2)))*b*c - 42*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b
) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2
) + 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))
*a*c**2 + 42*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)
*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sq
rt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b + 42*
sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)
*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sq
rt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**3 - 126*sqrt(d)*sqrt(...
```

3.240 $\int \frac{x^4}{a+\frac{b}{\left(c+dx^2\right)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 414

$$\int \frac{x^4}{a+\frac{b}{(c+dx^2)^2}}\,dx = -\frac{bx}{a^2d^2}+\frac{x^5}{5a} - \frac{b\left(2\sqrt{a}c+\sqrt{b+ac^2}\right)\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{9/4}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}\,d^{5/2}} + \frac{b\left(2\sqrt{a}c+\sqrt{b+ac^2}\right)\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{9/4}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}\,d^{5/2}} - \frac{b\left(2\sqrt{a}c-\sqrt{b+ac^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}x}{\sqrt{b+ac^2}+\sqrt{a}dx^2}\right)}{2\sqrt{2}a^{9/4}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\,d^{5/2}}$$

output
```
-b*x/a^2/d^2+1/5*x^5/a-1/4*b*(2*a^(1/2)*c+(a*c^2+b)^(1/2))*arctan(((-a^(1/
2)*c+(a*c^2+b)^(1/2))^(1/2)-2^(1/2)*a^(1/4)*d^(1/2)*x)/(a^(1/2)*c+(a*c^2+b
)^(1/2))^(1/2))*2^(1/2)/a^(9/4)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^(5/2)+
1/4*b*(2*a^(1/2)*c+(a*c^2+b)^(1/2))*arctan(((-a^(1/2)*c+(a*c^2+b)^(1/2))^(
1/2)+2^(1/2)*a^(1/4)*d^(1/2)*x)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2))*2^(1/2)
/a^(9/4)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^(5/2)-1/4*b*(2*a^(1/2)*c-(a*c
^2+b)^(1/2))*arctanh(2^(1/2)*a^(1/4)*(-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)*d^
(1/2)*x/((a*c^2+b)^(1/2)+a^(1/2)*d*x^2))*2^(1/2)/a^(9/4)/(-a^(1/2)*c+(a*c^
2+b)^(1/2))^(1/2)/d^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.48

$$\int \frac{x^4}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{-\frac{10bx}{d^2}+2ax^5-\frac{5i\sqrt{b}\left(\sqrt{b}+i\sqrt{a}c\right)^2\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{-i\sqrt{a}\sqrt{b}+ac}}\right)}{\sqrt{-i\sqrt{a}\sqrt{b}+ac}\,d^{5/2}}+\frac{5i\sqrt{b}\left(\sqrt{b}-i\sqrt{a}c\right)^2\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{i\sqrt{a}\sqrt{b}+ac}}\right)}{\sqrt{i\sqrt{a}\sqrt{b}+ac}\,d^{5/2}}}{10a^2}$$

input
```
Integrate[x^4/(a + b/(c + d*x^2)^2),x]
```

output
```
((-10*b*x)/d^2 + 2*a*x^5 - ((5*I)*Sqrt[b]*(Sqrt[b] + I*Sqrt[a]*c)^2*ArcTan
[(Sqrt[a]*Sqrt[d]*x)/Sqrt[(-I)*Sqrt[a]*Sqrt[b] + a*c]])/(Sqrt[(-I)*Sqrt[a]
*Sqrt[b] + a*c]*d^(5/2)) + ((5*I)*Sqrt[b]*(Sqrt[b] - I*Sqrt[a]*c)^2*ArcTan
[(Sqrt[a]*Sqrt[d]*x)/Sqrt[I*Sqrt[a]*Sqrt[b] + a*c]])/(Sqrt[I*Sqrt[a]*Sqrt[
b] + a*c]*d^(5/2)))/(10*a^2)
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+\frac{b}{(c+dx^2)^2}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{b(ac^2+2acdx^2+b)}{a^2d^2(ac^2+2acdx^2+ad^2x^4+b)}-\frac{b}{a^2d^2}+\frac{x^4}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(2\sqrt{a}c\sqrt{ac^2+b}+ac^2+b\right)\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{9/4}d^{5/2}\sqrt{ac^2+b}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}+\frac{b\left(2\sqrt{a}c\sqrt{ac^2+b}+ac^2+b\right)\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{9/4}d^{5/2}\sqrt{ac^2+b}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}-\frac{b\left(-2\sqrt{a}c\sqrt{ac^2+b}+ac^2+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{9/4}d^{5/2}\sqrt{ac^2+b}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}+\frac{b\left(-2\sqrt{a}c\sqrt{ac^2+b}+ac^2+b\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{9/4}d^{5/2}\sqrt{ac^2+b}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}-\frac{bx}{a^2d^2}+\frac{x^5}{5a}$$

input
```
Int[x^4/(a + b/(c + d*x^2)^2),x]
```

output
```
-((b*x)/(a^2*d^2)) + x^5/(5*a) - (b*(b + a*c^2 + 2*Sqrt[a]*c*Sqrt[b + a*c^
2])*ArcTan[(Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]] - Sqrt[2]*a^(1/4)*Sqrt[d]
*x)/Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]])/(2*Sqrt[2]*a^(9/4)*Sqrt[b + a*c^2]
*Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]*d^(5/2)) + (b*(b + a*c^2 + 2*Sqrt[a]*c*
Sqrt[b + a*c^2])*ArcTan[(Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]] + Sqrt[2]*a^
(1/4)*Sqrt[d]*x)/Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]])/(2*Sqrt[2]*a^(9/4)*Sq
rt[b + a*c^2]*Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]*d^(5/2)) - (b*(b + a*c^2 -
 2*Sqrt[a]*c*Sqrt[b + a*c^2])*Log[Sqrt[b + a*c^2] - Sqrt[2]*a^(1/4)*Sqrt[-
(Sqrt[a]*c) + Sqrt[b + a*c^2]]*Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqrt[2]*a^(9
/4)*Sqrt[b + a*c^2]*Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]*d^(5/2)) + (b*(b
+ a*c^2 - 2*Sqrt[a]*c*Sqrt[b + a*c^2])*Log[Sqrt[b + a*c^2] + Sqrt[2]*a^(1/
4)*Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]*Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqr
t[2]*a^(9/4)*Sqrt[b + a*c^2]*Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]*d^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.21

method	result	size
risch	$\frac{x^{5}}{5 a}-\frac{b x}{a^{2} d^{2}}+\frac{b \left(\sum_{\_R =\operatorname{RootOf}\left(a \,d^{2} \_Z^{4}+2 a c d \_Z^{2}+a \,c^{2}+b \right)}\frac{\left(2 a c d \_R^{2}+a \,c^{2}+b \right) \ln \left(x -\_R \right)}{d \_R^{3}+c \_R}\right)}{4 a^{3} d^{3}}$	89
default	Expression too large to display	1355

input
```
int(x^4/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5/a-b*x/a^2/d^2+1/4/a^3/d^3*b*sum((2*_R^2*a*c*d+a*c^2+b)/(_R^3*d+_R*
c)*ln(x-_R),_R=RootOf(_Z^4*a*d^2+2*_Z^2*a*c*d+a*c^2+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(308) = 616$.

Time = 0.11 (sec) , antiderivative size = 962, normalized size of antiderivative = 2.32

$$\int \frac{x^4}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
1/20*(4*a*d^2*x^5 + 5*a^2*d^2*sqrt((a^4*d^5*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4
*c^2 + b^5)/(a^9*d^10)) + a*b*c^3 - 3*b^2*c)/(a^4*d^5))*log((3*a^2*b^2*c^4
 + 2*a*b^3*c^2 - b^4)*x + (a^7*c*d^7*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 +
b^5)/(a^9*d^10)) + (3*a^3*b^2*c^2 - a^2*b^3)*d^2)*sqrt((a^4*d^5*sqrt(-(9*a
^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^9*d^10)) + a*b*c^3 - 3*b^2*c)/(a^4*d^5)
)) - 5*a^2*d^2*sqrt((a^4*d^5*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^
9*d^10)) + a*b*c^3 - 3*b^2*c)/(a^4*d^5))*log((3*a^2*b^2*c^4 + 2*a*b^3*c^2
- b^4)*x - (a^7*c*d^7*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^9*d^10)
) + (3*a^3*b^2*c^2 - a^2*b^3)*d^2)*sqrt((a^4*d^5*sqrt(-(9*a^2*b^3*c^4 - 6*
a*b^4*c^2 + b^5)/(a^9*d^10)) + a*b*c^3 - 3*b^2*c)/(a^4*d^5))) - 5*a^2*d^2*
sqrt(-(a^4*d^5*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^9*d^10)) - a*b
*c^3 + 3*b^2*c)/(a^4*d^5))*log((3*a^2*b^2*c^4 + 2*a*b^3*c^2 - b^4)*x + (a^
7*c*d^7*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^9*d^10)) - (3*a^3*b^2
*c^2 - a^2*b^3)*d^2)*sqrt(-(a^4*d^5*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b
^5)/(a^9*d^10)) - a*b*c^3 + 3*b^2*c)/(a^4*d^5))) + 5*a^2*d^2*sqrt(-(a^4*d^
5*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^9*d^10)) - a*b*c^3 + 3*b^2*
c)/(a^4*d^5))*log((3*a^2*b^2*c^4 + 2*a*b^3*c^2 - b^4)*x - (a^7*c*d^7*sqrt(
-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^9*d^10)) - (3*a^3*b^2*c^2 - a^2*b^
3)*d^2)*sqrt(-(a^4*d^5*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)/(a^9*d^10
)) - a*b*c^3 + 3*b^2*c)/(a^4*d^5))) - 20*b*x)/(a^2*d^2)
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.44

$$\int \frac{x^4}{a+\frac{b}{(c+dx^2)^2}}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^9d^{10} + t^2\left(-32a^6bc^3d^5 + 96a^5b^2cd^5\right) + a^3b^2c^6 + 3a^2b^3c^4 + 3ab^4c^2 + b^5, \left(t \mapsto t\log\left(x +\right.\right.\right.$$
$$+ \frac{x^5}{5a} - \frac{bx}{a^2d^2}$$

input
```
integrate(x**4/(a+b/(d*x**2+c)**2),x)
```

output
```
RootSum(256*_t**4*a**9*d**10 + _t**2*(-32*a**6*b*c**3*d**5 + 96*a**5*b**2*
c*d**5) + a**3*b**2*c**6 + 3*a**2*b**3*c**4 + 3*a*b**4*c**2 + b**5, Lambda
(_t, _t*log(x + (64*_t**3*a**7*c*d**7 - 4*_t*a**4*b*c**4*d**2 + 24*_t*a**3
*b**2*c**2*d**2 - 4*_t*a**2*b**3*d**2)/(3*a**2*b**2*c**4 + 2*a*b**3*c**2 -
 b**4)))) + x**5/(5*a) - b*x/(a**2*d**2)
```

Maxima [F]

$$\int \frac{x^4}{a+\frac{b}{(c+dx^2)^2}}\,dx = \int \frac{x^4}{a+\frac{b}{(dx^2+c)^2}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
b*integrate((2*a*c*d*x^2 + a*c^2 + b)/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b
), x)/(a^2*d^2) + 1/5*(a*d^2*x^5 - 5*b*x)/(a^2*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{a + \frac{b}{(c+dx^2)^2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1715, normalized size of antiderivative = 4.14

$$\int \frac{x^4}{a + \frac{b}{(c+dx^2)^2}} \, dx = \text{Too large to display}$$

input
```
int(x^4/(a + b/(c + d*x^2)^2),x)
```

output
```
x^5/(5*a) - atan((b^4*d^2*x*((b*c^3)/(16*a^3*d^5) - (3*b^2*c)/(16*a^4*d^5)
 + (b*(-a^9*b^3)^(1/2))/(16*a^9*d^5) - (3*c^2*(-a^9*b^3)^(1/2))/(16*a^8*d^
5))^(1/2)*8i)/((6*b^3*c^5)/d - (2*b^5*c)/(a^2*d) + (4*b^4*c^3)/(a*d) + (2*
b^4*(-a^9*b^3)^(1/2))/(a^7*d) - (6*b^2*c^4*(-a^9*b^3)^(1/2))/(a^5*d) - (4*
b^3*c^2*(-a^9*b^3)^(1/2))/(a^6*d)) + (b^3*c^2*d^2*x*((b*c^3)/(16*a^3*d^5)
- (3*b^2*c)/(16*a^4*d^5) + (b*(-a^9*b^3)^(1/2))/(16*a^9*d^5) - (3*c^2*(-a^
9*b^3)^(1/2))/(16*a^8*d^5))^(1/2)*24i)/((2*b^5*c)/(a^3*d) - (6*b^3*c^5)/(a
*d) - (4*b^4*c^3)/(a^2*d) - (2*b^4*(-a^9*b^3)^(1/2))/(a^8*d) + (6*b^2*c^4*
(-a^9*b^3)^(1/2))/(a^6*d) + (4*b^3*c^2*(-a^9*b^3)^(1/2))/(a^7*d)) + (b^2*c
*d^2*x*(-a^9*b^3)^(1/2)*((b*c^3)/(16*a^3*d^5) - (3*b^2*c)/(16*a^4*d^5) + (
b*(-a^9*b^3)^(1/2))/(16*a^9*d^5) - (3*c^2*(-a^9*b^3)^(1/2))/(16*a^8*d^5))^
(1/2)*8i)/((2*a^2*b^5*c)/d - (4*a^3*b^4*c^3)/d - (6*a^4*b^3*c^5)/d - (2*b^
4*(-a^9*b^3)^(1/2))/(a^3*d) + (6*b^2*c^4*(-a^9*b^3)^(1/2))/(a*d) + (4*b^3*
c^2*(-a^9*b^3)^(1/2))/(a^2*d)) + (b*c^3*d^2*x*(-a^9*b^3)^(1/2)*((b*c^3)/(1
6*a^3*d^5) - (3*b^2*c)/(16*a^4*d^5) + (b*(-a^9*b^3)^(1/2))/(16*a^9*d^5) -
(3*c^2*(-a^9*b^3)^(1/2))/(16*a^8*d^5))^(1/2)*24i)/((4*a^2*b^4*c^3)/d + (6*
a^3*b^3*c^5)/d - (2*a*b^5*c)/d + (2*b^4*(-a^9*b^3)^(1/2))/(a^4*d) - (6*b^2
*c^4*(-a^9*b^3)^(1/2))/(a^2*d) - (4*b^3*c^2*(-a^9*b^3)^(1/2))/(a^3*d)))*((
b*(-a^9*b^3)^(1/2) - 3*a^5*b^2*c + a^6*b*c^3 - 3*a*c^2*(-a^9*b^3)^(1/2))/(
16*a^9*d^5))^(1/2)*2i - atan((b^4*d^2*x*((b*c^3)/(16*a^3*d^5) - (3*b^2*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 957, normalized size of antiderivative = 2.31

$$\int \frac{x^4}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a+b/(d*x^2+c)^2),x)
```

output

```
( - 10*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(
2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*
d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c + 10*sqrt
(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqr
t(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(s
qrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2 - 10*sqrt(d)*sqrt(a)*sqrt(
sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*
c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**
2 + b) + a*c)*sqrt(2)))*b + 10*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(
a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a
*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)
*sqrt(2)))*a*c - 10*sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*s
qrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt
(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2 +
10*sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt
(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)
*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b + 5*sqrt(d)*sqrt(a*c**2
+ b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*log( - sqrt(d)*sqrt(sqrt
(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)*d*x**2)
*a*c - 5*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*...
```

3.241 $\int \frac{x^2}{a+\frac{b}{\left(c+dx^2\right)^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 342

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{x^3}{3a}+\frac{b\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{7/4}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}d^{3/2}} - \frac{b\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{7/4}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}d^{3/2}} + \frac{b\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}x}{\sqrt{b+ac^2}+\sqrt{a}dx^2}\right)}{2\sqrt{2}a^{7/4}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}d^{3/2}}$$

output
```
1/3*x^3/a+1/4*b*arctan(((-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)-2^(1/2)*a^(1/4)
*d^(1/2)*x)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2))*2^(1/2)/a^(7/4)/(a^(1/2)*c+
(a*c^2+b)^(1/2))^(1/2)/d^(3/2)-1/4*b*arctan(((-a^(1/2)*c+(a*c^2+b)^(1/2))^
(1/2)+2^(1/2)*a^(1/4)*d^(1/2)*x)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2))*2^(1/2
)/a^(7/4)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^(3/2)+1/4*b*arctanh(2^(1/2)*
a^(1/4)*(-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)*d^(1/2)*x/((a*c^2+b)^(1/2)+a^(1
/2)*d*x^2))*2^(1/2)/a^(7/4)/(-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{2ad^{3/2}x^3+3i\sqrt{b}\sqrt{-i\sqrt{a}\sqrt{b}+ac}\arctan\left(\frac{\sqrt{-i\sqrt{a}\sqrt{b}+ac}\sqrt{d}x}{i\sqrt{b}-\sqrt{ac}}\right)+3i\sqrt{b}\sqrt{i\sqrt{a}\sqrt{b}+ac}\arctan\left(\frac{\sqrt{i\sqrt{a}\sqrt{b}+ac}\sqrt{d}x}{i\sqrt{b}+\sqrt{ac}}\right.}{6a^2d^{3/2}}$$

input
```
Integrate[x^2/(a + b/(c + d*x^2)^2),x]
```

output
```
(2*a*d^(3/2)*x^3 + (3*I)*Sqrt[b]*Sqrt[(-I)*Sqrt[a]*Sqrt[b] + a*c]*ArcTan[(
Sqrt[(-I)*Sqrt[a]*Sqrt[b] + a*c]*Sqrt[d]*x)/(I*Sqrt[b] - Sqrt[a]*c)] + (3*
I)*Sqrt[b]*Sqrt[I*Sqrt[a]*Sqrt[b] + a*c]*ArcTan[(Sqrt[I*Sqrt[a]*Sqrt[b] +
a*c]*Sqrt[d]*x)/(I*Sqrt[b] + Sqrt[a]*c)])/(6*a^2*d^(3/2))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{x^2}{a}-\frac{bx^2}{a\,(ac^2+2acdx^2+ad^2x^4+b)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{7/4}d^{3/2}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}-\frac{b\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{7/4}d^{3/2}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}-\frac{b\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{7/4}d^{3/2}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}+\frac{b\log\left(\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{7/4}d^{3/2}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}+\frac{x^3}{3a}$$

input
```
Int[x^2/(a + b/(c + d*x^2)^2),x]
```

output
```
x^3/(3*a) + (b*ArcTan[(Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]] - Sqrt[2]*a^(1
/4)*Sqrt[d]*x)/Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]])/(2*Sqrt[2]*a^(7/4)*Sqrt
[Sqrt[a]*c + Sqrt[b + a*c^2]]*d^(3/2)) - (b*ArcTan[(Sqrt[-(Sqrt[a]*c) + Sq
rt[b + a*c^2]] + Sqrt[2]*a^(1/4)*Sqrt[d]*x)/Sqrt[Sqrt[a]*c + Sqrt[b + a*c^
2]]])/(2*Sqrt[2]*a^(7/4)*Sqrt[Sqrt[a]*c + Sqrt[b + a*c^2]]*d^(3/2)) - (b*L
og[Sqrt[b + a*c^2] - Sqrt[2]*a^(1/4)*Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]*
Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqrt[2]*a^(7/4)*Sqrt[-(Sqrt[a]*c) + Sqrt[b
+ a*c^2]]*d^(3/2)) + (b*Log[Sqrt[b + a*c^2] + Sqrt[2]*a^(1/4)*Sqrt[-(Sqrt[
a]*c) + Sqrt[b + a*c^2]]*Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqrt[2]*a^(7/4)*Sq
rt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]*d^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.20

method	result
risch	$\frac{x^3}{3a} - \frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\,d^2\_Z^4+2acd\_Z^2+a\,c^2+b\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d\_R^3+c\_R}\right)}{4a^2d}$
default	$\frac{x^3}{3a} - \frac{b\left(\frac{\sqrt{2\sqrt{a^2c^2d^2+ab\,d^2}-2acd}\left(acd+\sqrt{a^2c^2d^2+ab\,d^2}\right)\left(\frac{\ln\left(\sqrt{a\,d^2}\,x^2-x\sqrt{2\sqrt{a\,d^2\left(a\,c^2+b\right)}-2acd}+\sqrt{a\,c^2+b}\right)}{2\sqrt{a\,d^2}}+\frac{\sqrt{2\sqrt{a\,d^2\left(a\,c^2+b\right)}-2acd}}{\sqrt{a\,d^2}\sqrt{4}}\right.}{4ab\,d^2}\right.}{}$

input
```
int(x^2/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*x^3-1/4/a^2*b/d*sum(_R^2/(_R^3*d+_R*c)*ln(x-_R),_R=RootOf(_Z^4*a*d^2
+2*_Z^2*a*c*d+a*c^2+b))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{4\,x^3 - 3\,a\sqrt{\frac{a^3d^3\sqrt{-\frac{b^3}{a^7d^6}}+bc}{a^3d^3}}\log\left(a^5d^4\sqrt{\frac{a^3d^3\sqrt{-\frac{b^3}{a^7d^6}}+bc}{a^3d^3}}\sqrt{-\frac{b^3}{a^7d^6}}+b^2x\right)+3\,a\sqrt{\frac{a^3d^3\sqrt{-\frac{b^3}{a^7d^6}}+bc}{a^3d^3}}\log\left(-a^5d^4\sqrt{\right.}{}$$

input
```
integrate(x^2/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
1/12*(4*x^3 - 3*a*sqrt((a^3*d^3*sqrt(-b^3/(a^7*d^6)) + b*c)/(a^3*d^3))*log
(a^5*d^4*sqrt((a^3*d^3*sqrt(-b^3/(a^7*d^6)) + b*c)/(a^3*d^3))*sqrt(-b^3/(a
^7*d^6)) + b^2*x) + 3*a*sqrt((a^3*d^3*sqrt(-b^3/(a^7*d^6)) + b*c)/(a^3*d^3
))*log(-a^5*d^4*sqrt((a^3*d^3*sqrt(-b^3/(a^7*d^6)) + b*c)/(a^3*d^3))*sqrt(
-b^3/(a^7*d^6)) + b^2*x) + 3*a*sqrt(-(a^3*d^3*sqrt(-b^3/(a^7*d^6)) - b*c)/
(a^3*d^3))*log(a^5*d^4*sqrt(-(a^3*d^3*sqrt(-b^3/(a^7*d^6)) - b*c)/(a^3*d^3
))*sqrt(-b^3/(a^7*d^6)) + b^2*x) - 3*a*sqrt(-(a^3*d^3*sqrt(-b^3/(a^7*d^6))
 - b*c)/(a^3*d^3))*log(-a^5*d^4*sqrt(-(a^3*d^3*sqrt(-b^3/(a^7*d^6)) - b*c)
/(a^3*d^3))*sqrt(-b^3/(a^7*d^6)) + b^2*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.22

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^7d^6 - 32t^2a^4bcd^3 + ab^2c^2 + b^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^5d^4 + 4ta^2bcd}{b^2}\right)\right)\right) + \frac{x^3}{3a}$$

input
```
integrate(x**2/(a+b/(d*x**2+c)**2),x)
```

output
```
RootSum(256*_t**4*a**7*d**6 - 32*_t**2*a**4*b*c*d**3 + a*b**2*c**2 + b**3,
 Lambda(_t, _t*log(x + (-64*_t**3*a**5*d**4 + 4*_t*a**2*b*c*d)/b**2))) + x
**3/(3*a)
```

Maxima [F]

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx = \int \frac{x^2}{a+\frac{b}{(dx^2+c)^2}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
1/3*x^3/a - b*integrate(x^2/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b), x)/a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx$$
$$= 2\,\mathrm{atanh}\left(\frac{a^2\left(x\left(4\,b^3\,d^4-4\,a\,b^2\,c^2\,d^4\right)+\frac{4\,b\,c\,d^4\,x\left(\sqrt{-a^7\,b^3}+a^4\,b\,c\right)}{a^3}\right)\sqrt{\frac{\sqrt{-a^7\,b^3}+a^4\,b\,c}{16\,a^7\,d^3}}}{b^4\,d^2+a\,b^3\,c^2\,d^2}\right)\sqrt{\frac{\sqrt{-a^7\,b^3}+a^4\,b\,c}{16\,a^7\,d^3}}$$
$$+2\,\mathrm{atanh}\left(\frac{a^2\left(x\left(4\,b^3\,d^4-4\,a\,b^2\,c^2\,d^4\right)-\frac{4\,b\,c\,d^4\,x\left(\sqrt{-a^7\,b^3}-a^4\,b\,c\right)}{a^3}\right)\sqrt{-\frac{\sqrt{-a^7\,b^3}-a^4\,b\,c}{16\,a^7\,d^3}}}{b^4\,d^2+a\,b^3\,c^2\,d^2}\right)\sqrt{-\frac{\sqrt{-a^7\,b^3}-a^4\,b}{16\,a^7\,d^3}}$$
$$+\frac{x^3}{3\,a}$$

input
```
int(x^2/(a + b/(c + d*x^2)^2),x)
```

output
```
2*atanh((a^2*(x*(4*b^3*d^4 - 4*a*b^2*c^2*d^4) + (4*b*c*d^4*x*((-a^7*b^3)^(
1/2) + a^4*b*c))/a^3)*(((-a^7*b^3)^(1/2) + a^4*b*c)/(16*a^7*d^3))^(1/2))/(
b^4*d^2 + a*b^3*c^2*d^2))*(((-a^7*b^3)^(1/2) + a^4*b*c)/(16*a^7*d^3))^(1/2
) + 2*atanh((a^2*(x*(4*b^3*d^4 - 4*a*b^2*c^2*d^4) - (4*b*c*d^4*x*((-a^7*b^
3)^(1/2) - a^4*b*c))/a^3)*(-((-a^7*b^3)^(1/2) - a^4*b*c)/(16*a^7*d^3))^(1/
2))/(b^4*d^2 + a*b^3*c^2*d^2))*(-((-a^7*b^3)^(1/2) - a^4*b*c)/(16*a^7*d^3)
)^(1/2) + x^3/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.85

$$\int \frac{x^2}{a+\frac{b}{(c+dx^2)^2}}\,dx$$

$$= \frac{6\sqrt{d}\,\sqrt{a\,c^2+b}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{2}\,atan\left(\frac{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}-ac}\,\sqrt{2}-2\sqrt{a}\,dx}{\sqrt{d}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+ac}\,\sqrt{2}}\right)-6\sqrt{d}\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{a\,c^2+b}+\cdots}}{\cdots}$$

input
```
int(x^2/(a+b/(d*x^2+c)^2),x)
```

output
```
(6*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*a
tan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)
/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2))) - 6*sqrt(d)*sqrt(
a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)
*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sq
rt(a*c**2 + b) + a*c)*sqrt(2)))*c - 6*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a
)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 +
 b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b)
 + a*c)*sqrt(2))) + 6*sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)
*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sq
rt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*c - 3*s
qrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*log(
- sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b
) + sqrt(a)*d*x**2) + 3*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2
+ b) - a*c)*sqrt(2)*log(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(
2)*x + sqrt(a*c**2 + b) + sqrt(a)*d*x**2) - 3*sqrt(d)*sqrt(a)*sqrt(sqrt(a)
*sqrt(a*c**2 + b) - a*c)*sqrt(2)*log( - sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 +
 b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)*d*x**2)*c + 3*sqrt(d)*sq
rt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*log(sqrt(d)*sqrt(sqrt(a
)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)*d*x**2...
```

3.242 $\int \frac{1}{a+\frac{b}{\left(c+dx^2\right)^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 370

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}}\,dx = \frac{x}{a}+\frac{b\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{b+ac^2}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}} - \frac{b\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{b+ac^2}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}x}{\sqrt{b+ac^2}+\sqrt{a}dx^2}\right)}{2\sqrt{2}a^{5/4}\sqrt{b+ac^2}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}}$$

output
```
x/a+1/4*b*arctan(((-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)-2^(1/2)*a^(1/4)*d^(1/
2)*x)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2))*2^(1/2)/a^(5/4)/(a*c^2+b)^(1/2)/(
a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^(1/2)-1/4*b*arctan(((-a^(1/2)*c+(a*c^2+
b)^(1/2))^(1/2)+2^(1/2)*a^(1/4)*d^(1/2)*x)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/
2))*2^(1/2)/a^(5/4)/(a*c^2+b)^(1/2)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^(1
/2)-1/4*b*arctanh(2^(1/2)*a^(1/4)*(-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)*d^(1/
2)*x/((a*c^2+b)^(1/2)+a^(1/2)*d*x^2))*2^(1/2)/a^(5/4)/(a*c^2+b)^(1/2)/(-a^
(1/2)*c+(a*c^2+b)^(1/2))^(1/2)/d^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.43

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}}\,dx=\frac{x}{a}+\frac{i\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{-i\sqrt{a}\sqrt{b}+ac}}\right)}{2a\sqrt{-i\sqrt{a}\sqrt{b}+ac}\sqrt{d}}-\frac{i\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{i\sqrt{a}\sqrt{b}+ac}}\right)}{2a\sqrt{i\sqrt{a}\sqrt{b}+ac}\sqrt{d}}$$

input
```
Integrate[(a + b/(c + d*x^2)^2)^(-1),x]
```

output
```
x/a + ((I/2)*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[d]*x)/Sqrt[(-I)*Sqrt[a]*Sqrt[b]
+ a*c]])/(a*Sqrt[(-I)*Sqrt[a]*Sqrt[b] + a*c]*Sqrt[d]) - ((I/2)*Sqrt[b]*Arc
Tan[(Sqrt[a]*Sqrt[d]*x)/Sqrt[I*Sqrt[a]*Sqrt[b] + a*c]])/(a*Sqrt[I*Sqrt[a]*
Sqrt[b] + a*c]*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.43, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{\sqrt{b}}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-\frac{b}{c+dx^2}\right)}-\frac{\sqrt{b}}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+\frac{b}{c+dx^2}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{\sqrt{-a}\sqrt{b}+ac}}\right)}{2\sqrt{-a^2}\sqrt{d}\sqrt{\sqrt{-a}\sqrt{b}+ac}}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt[4]{-a}\sqrt{d}x}{\sqrt{\sqrt{-a}c+\sqrt{b}}}\right)}{2(-a)^{5/4}\sqrt{d}\sqrt{\sqrt{-a}c+\sqrt{b}}}+\frac{x}{a}$$

input
```
Int[(a + b/(c + d*x^2)^2)^(-1),x]
```

output
```
x/a + (Sqrt[b]*ArcTan[((-a)^(1/4)*Sqrt[d]*x)/Sqrt[Sqrt[b] + Sqrt[-a]*c]])/
(2*(-a)^(5/4)*Sqrt[Sqrt[b] + Sqrt[-a]*c]*Sqrt[d]) + (Sqrt[b]*ArcTan[(Sqrt[
a]*Sqrt[d]*x)/Sqrt[Sqrt[-a]*Sqrt[b] + a*c]])/(2*Sqrt[-a^2]*Sqrt[Sqrt[-a]*S
qrt[b] + a*c]*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

method	result
risch	$\frac{x}{a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\,d^{2}\_Z^{4}+2acd\_Z^{2}+a\,c^{2}+b\right)}\frac{\ln\left(x-\_R\right)}{d\_R^{3}+c\_R}\right)}{4a^{2}d}$
default	$\frac{x}{a}-\frac{b\left(\frac{\left(-\sqrt{a\,d^{2}}\,\sqrt{2\sqrt{a^{2}c^{2}d^{2}+ab\,d^{2}}-2acd}\,acd-\sqrt{a\,d^{2}}\,\sqrt{2\sqrt{a^{2}c^{2}d^{2}+ab\,d^{2}}-2acd}\,\sqrt{a^{2}c^{2}d^{2}+ab\,d^{2}}\right)\ln\left(\sqrt{a\,d^{2}}\,x^{2}-x\sqrt{2\sqrt{a\,d^{2}\left(a\,c^{2}+b\right)}-2acd}\right.}{2\sqrt{a\,d^{2}}}\right.}{}$

input
```
int(1/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*x-1/4/a^2*b/d*sum(1/(_R^3*d+_R*c)*ln(x-_R),_R=RootOf(_Z^4*a*d^2+2*_Z^2
*a*c*d+a*c^2+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(272) = 544.

Time = 0.11 (sec) , antiderivative size = 903, normalized size of antiderivative = 2.44

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
-1/4*(a*sqrt(((a^3*c^2 + a^2*b)*d*sqrt(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*
b^2)*d^2)) + b*c)/((a^3*c^2 + a^2*b)*d))*log(b^2*x + (a*b^2 + (a^5*c^3 + a
^4*b*c)*d*sqrt(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^2)*d^2)))*sqrt(((a^3*c
^2 + a^2*b)*d*sqrt(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^2)*d^2)) + b*c)/((
a^3*c^2 + a^2*b)*d))) - a*sqrt(((a^3*c^2 + a^2*b)*d*sqrt(-b^3/((a^7*c^4 +
2*a^6*b*c^2 + a^5*b^2)*d^2)) + b*c)/((a^3*c^2 + a^2*b)*d))*log(b^2*x - (a*
b^2 + (a^5*c^3 + a^4*b*c)*d*sqrt(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^2)*d
^2)))*sqrt(((a^3*c^2 + a^2*b)*d*sqrt(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^
2)*d^2)) + b*c)/((a^3*c^2 + a^2*b)*d))) + a*sqrt(-((a^3*c^2 + a^2*b)*d*sqr
t(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^2)*d^2)) - b*c)/((a^3*c^2 + a^2*b)*
d))*log(b^2*x + (a*b^2 - (a^5*c^3 + a^4*b*c)*d*sqrt(-b^3/((a^7*c^4 + 2*a^6
*b*c^2 + a^5*b^2)*d^2)))*sqrt(-((a^3*c^2 + a^2*b)*d*sqrt(-b^3/((a^7*c^4 +
2*a^6*b*c^2 + a^5*b^2)*d^2)) - b*c)/((a^3*c^2 + a^2*b)*d))) - a*sqrt(-((a^
3*c^2 + a^2*b)*d*sqrt(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^2)*d^2)) - b*c)
/((a^3*c^2 + a^2*b)*d))*log(b^2*x - (a*b^2 - (a^5*c^3 + a^4*b*c)*d*sqrt(-b
^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^2)*d^2)))*sqrt(-((a^3*c^2 + a^2*b)*d*sq
rt(-b^3/((a^7*c^4 + 2*a^6*b*c^2 + a^5*b^2)*d^2)) - b*c)/((a^3*c^2 + a^2*b)
*d))) - 4*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.27

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^6c^2d^2+256a^5bd^2\right)-32t^2a^3bcd+b^2,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^3d-64t^3a^4bcd+4t\right.\right.\right.$$
$$+\frac{x}{a}$$

input
```
integrate(1/(a+b/(d*x**2+c)**2),x)
```

output
```
RootSum(_t**4*(256*a**6*c**2*d**2 + 256*a**5*b*d**2) - 32*_t**2*a**3*b*c*d
 + b**2, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**3*d - 64*_t**3*a**4*b*c*
d + 4*_t*a**2*b*c**2 - 4*_t*a*b**2)/b**2))) + x/a
```

Maxima [F]

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}}\,dx = \int \frac{1}{a+\frac{b}{(dx^2+c)^2}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
-b*integrate(1/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b), x)/a + x/a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1245, normalized size of antiderivative = 3.36

$$\int \frac{1}{a+\frac{b}{(c+dx^2)^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/(c + d*x^2)^2),x)
```

output
```
x/a + 2*atanh((8*a^7*b^2*c^2*d^7*x*((-a^5*b^3)^(1/2)/(16*(a^6*c^2*d + a^5*
b*d)) + (a^3*b*c)/(16*(a^6*c^2*d + a^5*b*d)))^(1/2))/((2*a^11*b^3*c^3*d^7)
/(a^6*c^2*d + a^5*b*d) + (2*a^7*b^3*d^7*(-a^5*b^3)^(1/2))/(a^6*c^2*d + a^5
*b*d) + (2*a^10*b^4*c*d^7)/(a^6*c^2*d + a^5*b*d) + (2*a^8*b^2*c^2*d^7*(-a^
5*b^3)^(1/2))/(a^6*c^2*d + a^5*b*d)) - (8*a*b^2*d^6*x*((-a^5*b^3)^(1/2)/(1
6*(a^6*c^2*d + a^5*b*d)) + (a^3*b*c)/(16*(a^6*c^2*d + a^5*b*d)))^(1/2))/((
2*a^2*b^2*d^6*(-a^5*b^3)^(1/2))/(a^6*c^2*d + a^5*b*d) + (2*a^5*b^3*c*d^6)/
(a^6*c^2*d + a^5*b*d)) + (8*a^4*b*c*d^7*x*((-a^5*b^3)^(1/2)/(16*(a^6*c^2*d
 + a^5*b*d)) + (a^3*b*c)/(16*(a^6*c^2*d + a^5*b*d)))^(1/2)*(-a^5*b^3)^(1/2
))/((2*a^11*b^3*c^3*d^7)/(a^6*c^2*d + a^5*b*d) + (2*a^7*b^3*d^7*(-a^5*b^3)
^(1/2))/(a^6*c^2*d + a^5*b*d) + (2*a^10*b^4*c*d^7)/(a^6*c^2*d + a^5*b*d) +
 (2*a^8*b^2*c^2*d^7*(-a^5*b^3)^(1/2))/(a^6*c^2*d + a^5*b*d)))*(((-a^5*b^3)
^(1/2) + a^3*b*c)/(16*(a^6*c^2*d + a^5*b*d)))^(1/2) + 2*atanh((8*a*b^2*d^6
*x*((a^3*b*c)/(16*(a^6*c^2*d + a^5*b*d)) - (-a^5*b^3)^(1/2)/(16*(a^6*c^2*d
 + a^5*b*d)))^(1/2))/((2*a^2*b^2*d^6*(-a^5*b^3)^(1/2))/(a^6*c^2*d + a^5*b*
d) - (2*a^5*b^3*c*d^6)/(a^6*c^2*d + a^5*b*d)) + (8*a^7*b^2*c^2*d^7*x*((a^3
*b*c)/(16*(a^6*c^2*d + a^5*b*d)) - (-a^5*b^3)^(1/2)/(16*(a^6*c^2*d + a^5*b
*d)))^(1/2))/((2*a^11*b^3*c^3*d^7)/(a^6*c^2*d + a^5*b*d) - (2*a^7*b^3*d^7*
(-a^5*b^3)^(1/2))/(a^6*c^2*d + a^5*b*d) + (2*a^10*b^4*c*d^7)/(a^6*c^2*d +
a^5*b*d) - (2*a^8*b^2*c^2*d^7*(-a^5*b^3)^(1/2))/(a^6*c^2*d + a^5*b*d)) ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 962, normalized size of antiderivative = 2.60

$$\int \frac{1}{a + \frac{b}{(c+dx^2)^2}} \, dx = \text{Too large to display}$$

input
```
int(1/(a+b/(d*x^2+c)^2),x)
```

output
```
( - 2*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2
)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d
*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c + 2*sqrt(d
)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(
sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqr
t(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2 + 2*sqrt(d)*sqrt(a)*sqrt(sqr
t(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**
2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 +
 b) + a*c)*sqrt(2)))*b + 2*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c*
*2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*
sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqr
t(2)))*a*c - 2*sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2
)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d
*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2 - 2*sqr
t(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sq
rt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(
sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b + sqrt(d)*sqrt(a*c**2 + b)*sqr
t(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*log( - sqrt(d)*sqrt(sqrt(a)*sqrt
(a*c**2 + b) - a*c)*sqrt(2)*x + sqrt(a*c**2 + b) + sqrt(a)*d*x**2)*a*c - s
qrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2)*lo...
```

3.243 $\int \frac{1}{x^2\left(a+\frac{b}{\left(c+dx^2\right)^2}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 444

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = -\frac{c^2}{(b+ac^2)\,x} - \frac{b\left(2\sqrt{a}c+\sqrt{b+ac^2}\right)\sqrt{d}\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{3/4}\left(b+ac^2\right)^{3/2}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}} + \frac{b\left(2\sqrt{a}c+\sqrt{b+ac^2}\right)\sqrt{d}\arctan\left(\frac{\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}}\right)}{2\sqrt{2}a^{3/4}\left(b+ac^2\right)^{3/2}\sqrt{\sqrt{a}c+\sqrt{b+ac^2}}} + \frac{b\left(2\sqrt{a}c-\sqrt{b+ac^2}\right)\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}\sqrt{d}x}{\sqrt{b+ac^2}+\sqrt{a}dx^2}\right)}{2\sqrt{2}a^{3/4}\left(b+ac^2\right)^{3/2}\sqrt{-\sqrt{a}c+\sqrt{b+ac^2}}}$$

output
```
-c^2/(a*c^2+b)/x-1/4*b*(2*a^(1/2)*c+(a*c^2+b)^(1/2))*d^(1/2)*arctan(((-a^(
1/2)*c+(a*c^2+b)^(1/2))^(1/2)-2^(1/2)*a^(1/4)*d^(1/2)*x)/(a^(1/2)*c+(a*c^2
+b)^(1/2))^(1/2))*2^(1/2)/a^(3/4)/(a*c^2+b)^(3/2)/(a^(1/2)*c+(a*c^2+b)^(1/
2))^(1/2)+1/4*b*(2*a^(1/2)*c+(a*c^2+b)^(1/2))*d^(1/2)*arctan(((-a^(1/2)*c+
(a*c^2+b)^(1/2))^(1/2)+2^(1/2)*a^(1/4)*d^(1/2)*x)/(a^(1/2)*c+(a*c^2+b)^(1/
2))^(1/2))*2^(1/2)/a^(3/4)/(a*c^2+b)^(3/2)/(a^(1/2)*c+(a*c^2+b)^(1/2))^(1/
2)+1/4*b*(2*a^(1/2)*c-(a*c^2+b)^(1/2))*d^(1/2)*arctanh(2^(1/2)*a^(1/4)*(-a
^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)*d^(1/2)*x/((a*c^2+b)^(1/2)+a^(1/2)*d*x^2))
*2^(1/2)/a^(3/4)/(a*c^2+b)^(3/2)/(-a^(1/2)*c+(a*c^2+b)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \frac{-\frac{2c^2}{x}+\frac{\sqrt{b}\left(\sqrt{b}-i\sqrt{a}c\right)\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{-i\sqrt{a}\sqrt{b}+ac}}\right)}{\sqrt{a}\sqrt{-i\sqrt{a}\sqrt{b}+ac}}+\frac{\sqrt{b}\left(\sqrt{b}+i\sqrt{a}c\right)\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}x}{\sqrt{i\sqrt{a}\sqrt{b}+ac}}\right)}{\sqrt{a}\sqrt{i\sqrt{a}\sqrt{b}+ac}}}{2\left(b+ac^2\right)}$$

input
```
Integrate[1/(x^2*(a + b/(c + d*x^2)^2)),x]
```

output
```
((-2*c^2)/x + (Sqrt[b]*(Sqrt[b] - I*Sqrt[a]*c)*Sqrt[d]*ArcTan[(Sqrt[a]*Sqr
t[d]*x)/Sqrt[(-I)*Sqrt[a]*Sqrt[b] + a*c]])/(Sqrt[a]*Sqrt[(-I)*Sqrt[a]*Sqrt
[b] + a*c]) + (Sqrt[b]*(Sqrt[b] + I*Sqrt[a]*c)*Sqrt[d]*ArcTan[(Sqrt[a]*Sqr
t[d]*x)/Sqrt[I*Sqrt[a]*Sqrt[b] + a*c]])/(Sqrt[a]*Sqrt[I*Sqrt[a]*Sqrt[b] +
a*c]))/(2*(b + a*c^2))
```

Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{bd(2c+dx^2)}{(ac^2+b)(ac^2+2acdx^2+ad^2x^4+b)}+\frac{c^2}{x^2(ac^2+b)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\sqrt{d}\left(\sqrt{ac^2+b}+2\sqrt{a}c\right)\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}-\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{3/4}(ac^2+b)^{3/2}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}+\frac{b\sqrt{d}\left(\sqrt{ac^2+b}+2\sqrt{a}c\right)\arctan\left(\frac{\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}\right)}{2\sqrt{2}a^{3/4}(ac^2+b)^{3/2}\sqrt{\sqrt{ac^2+b}+\sqrt{a}c}}-\frac{b\sqrt{d}\left(2\sqrt{a}c-\sqrt{ac^2+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{3/4}(ac^2+b)^{3/2}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}+\frac{b\sqrt{d}\left(2\sqrt{a}c-\sqrt{ac^2+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt{d}x\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}+\sqrt{ac^2+b}+\sqrt{a}dx^2\right)}{4\sqrt{2}a^{3/4}(ac^2+b)^{3/2}\sqrt{\sqrt{ac^2+b}-\sqrt{a}c}}-\frac{c^2}{x(ac^2+b)}$$

input
```
Int[1/(x^2*(a + b/(c + d*x^2)^2)),x]
```

output
```
-(c^2/((b + a*c^2)*x)) - (b*(2*Sqrt[a]*c + Sqrt[b + a*c^2])*Sqrt[d]*ArcTan
[(Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]] - Sqrt[2]*a^(1/4)*Sqrt[d]*x)/Sqrt[S
qrt[a]*c + Sqrt[b + a*c^2]]])/(2*Sqrt[2]*a^(3/4)*(b + a*c^2)^(3/2)*Sqrt[Sq
rt[a]*c + Sqrt[b + a*c^2]]) + (b*(2*Sqrt[a]*c + Sqrt[b + a*c^2])*Sqrt[d]*A
rcTan[(Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c^2]] + Sqrt[2]*a^(1/4)*Sqrt[d]*x)/S
qrt[Sqrt[a]*c + Sqrt[b + a*c^2]]])/(2*Sqrt[2]*a^(3/4)*(b + a*c^2)^(3/2)*Sq
rt[Sqrt[a]*c + Sqrt[b + a*c^2]]) - (b*(2*Sqrt[a]*c - Sqrt[b + a*c^2])*Sqrt
[d]*Log[Sqrt[b + a*c^2] - Sqrt[2]*a^(1/4)*Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*c
^2]]*Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqrt[2]*a^(3/4)*(b + a*c^2)^(3/2)*Sqrt
[-(Sqrt[a]*c) + Sqrt[b + a*c^2]]) + (b*(2*Sqrt[a]*c - Sqrt[b + a*c^2])*Sqr
t[d]*Log[Sqrt[b + a*c^2] + Sqrt[2]*a^(1/4)*Sqrt[-(Sqrt[a]*c) + Sqrt[b + a*
c^2]]*Sqrt[d]*x + Sqrt[a]*d*x^2])/(4*Sqrt[2]*a^(3/4)*(b + a*c^2)^(3/2)*Sqr
t[-(Sqrt[a]*c) + Sqrt[b + a*c^2]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{c^2}{(a\,c^2+b)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^6c^6+3a^5b\,c^4+3a^4b^2c^2+a^3b^3\right)\_Z^4+\left(-2a^3b\,c^3d+6a^2b^2cd\right)\_Z^2+b^2d^2\right)}\_R\ln\left(\left(\left(-a^7c^8+2a^6b\,c^6+1\right.\right.\right.\right.}{}$
default	Expression too large to display

input
```
int(1/x^2/(a+b/(d*x^2+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-c^2/(a*c^2+b)/x+1/4*sum(_R*ln(((-a^7*c^8+2*a^6*b*c^6+12*a^5*b^2*c^4+14*a^
4*b^3*c^2+5*a^3*b^4)*_R^4+(a^4*b*c^5*d-6*a^3*b^2*c^3*d+25*a^2*b^3*c*d)*_R^
2+4*b^3*d^2)*x+(3*a^5*b*c^6+5*a^4*b^2*c^4+a^3*b^3*c^2-a^2*b^4)*_R^3),_R=Ro
otOf((a^6*c^6+3*a^5*b*c^4+3*a^4*b^2*c^2+a^3*b^3)*_Z^4+(-2*a^3*b*c^3*d+6*a^
2*b^2*c*d)*_Z^2+b^2*d^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2314 vs. $2(334) = 668$.

Time $= 0.10$ (sec) , antiderivative size $= 2314$, normalized size of antiderivative $= 5.21$

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c)^2),x, algorithm="fricas")
```

output
```
1/4*((a*c^2 + b)*x*sqrt(((a*b*c^3 - 3*b^2*c)*d + (a^4*c^6 + 3*a^3*b*c^4 +
3*a^2*b^2*c^2 + a*b^3)*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)*d^2/(a^9*
c^12 + 6*a^8*b*c^10 + 15*a^7*b^2*c^8 + 20*a^6*b^3*c^6 + 15*a^5*b^4*c^4 + 6
*a^4*b^5*c^2 + a^3*b^6)))/(a^4*c^6 + 3*a^3*b*c^4 + 3*a^2*b^2*c^2 + a*b^3))
*log((3*a*b^2*c^2 - b^3)*d^2*x + (2*(3*a^2*b^2*c^3 - a*b^3*c)*d + (a^6*c^8
 + 2*a^5*b*c^6 - 2*a^3*b^3*c^2 - a^2*b^4)*sqrt(-(9*a^2*b^3*c^4 - 6*a*b^4*c
^2 + b^5)*d^2/(a^9*c^12 + 6*a^8*b*c^10 + 15*a^7*b^2*c^8 + 20*a^6*b^3*c^6 +
 15*a^5*b^4*c^4 + 6*a^4*b^5*c^2 + a^3*b^6)))*sqrt(((a*b*c^3 - 3*b^2*c)*d +
 (a^4*c^6 + 3*a^3*b*c^4 + 3*a^2*b^2*c^2 + a*b^3)*sqrt(-(9*a^2*b^3*c^4 - 6*
a*b^4*c^2 + b^5)*d^2/(a^9*c^12 + 6*a^8*b*c^10 + 15*a^7*b^2*c^8 + 20*a^6*b^
3*c^6 + 15*a^5*b^4*c^4 + 6*a^4*b^5*c^2 + a^3*b^6)))/(a^4*c^6 + 3*a^3*b*c^4
 + 3*a^2*b^2*c^2 + a*b^3))) - (a*c^2 + b)*x*sqrt(((a*b*c^3 - 3*b^2*c)*d +
(a^4*c^6 + 3*a^3*b*c^4 + 3*a^2*b^2*c^2 + a*b^3)*sqrt(-(9*a^2*b^3*c^4 - 6*a
*b^4*c^2 + b^5)*d^2/(a^9*c^12 + 6*a^8*b*c^10 + 15*a^7*b^2*c^8 + 20*a^6*b^3
*c^6 + 15*a^5*b^4*c^4 + 6*a^4*b^5*c^2 + a^3*b^6)))/(a^4*c^6 + 3*a^3*b*c^4
+ 3*a^2*b^2*c^2 + a*b^3))*log((3*a*b^2*c^2 - b^3)*d^2*x - (2*(3*a^2*b^2*c^
3 - a*b^3*c)*d + (a^6*c^8 + 2*a^5*b*c^6 - 2*a^3*b^3*c^2 - a^2*b^4)*sqrt(-(
9*a^2*b^3*c^4 - 6*a*b^4*c^2 + b^5)*d^2/(a^9*c^12 + 6*a^8*b*c^10 + 15*a^7*b
^2*c^8 + 20*a^6*b^3*c^6 + 15*a^5*b^4*c^4 + 6*a^4*b^5*c^2 + a^3*b^6)))*sqrt
(((a*b*c^3 - 3*b^2*c)*d + (a^4*c^6 + 3*a^3*b*c^4 + 3*a^2*b^2*c^2 + a*b^...
```

Sympy [A] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = -\frac{c^2}{x\,(ac^2+b)}$$
$$+\operatorname{RootSum}\left(t^4\cdot\left(256a^6c^6+768a^5bc^4+768a^4b^2c^2+256a^3b^3\right)+t^2\left(-32a^3bc^3d+96a^2b^2cd\right)+b^2d^2,\left(t\mapsto\right.\right.$$

input
```
integrate(1/x**2/(a+b/(d*x**2+c)**2),x)
```

output
```
-c**2/(x*(a*c**2 + b)) + RootSum(_t**4*(256*a**6*c**6 + 768*a**5*b*c**4 +
768*a**4*b**2*c**2 + 256*a**3*b**3) + _t**2*(-32*a**3*b*c**3*d + 96*a**2*b
**2*c*d) + b**2*d**2, Lambda(_t, _t*log(x + (64*_t**3*a**6*c**8 + 128*_t**
3*a**5*b*c**6 - 128*_t**3*a**3*b**3*c**2 - 64*_t**3*a**2*b**4 - 4*_t*a**3*
b*c**5*d + 40*_t*a**2*b**2*c**3*d - 20*_t*a*b**3*c*d)/(3*a*b**2*c**2*d**2
- b**3*d**2))))
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \int \frac{1}{\left(a+\frac{b}{(dx^2+c)^2}\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c)^2),x, algorithm="maxima")
```

output
```
b*d*integrate((d*x^2 + 2*c)/(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2 + b), x)/(a*c
^2 + b) - c^2/((a*c^2 + b)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,%%%{1,[1,1]%%%}]%%},[0,1]%%%}+%%%{%%%{1,[0,1
]%%%},[0,
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 4231, normalized size of antiderivative = 9.53

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b/(c + d*x^2)^2)),x)
```

output
```
atan(((x*(4*a^2*b^6*d^8 + 8*a^3*b^5*c^2*d^8 - 8*a^5*b^3*c^6*d^8 - 4*a^6*b^
2*c^8*d^8) + ((b*d*(-a^3*b^3)^(1/2) - 3*a*c^2*d*(-a^3*b^3)^(1/2) - 3*a^2*b
^2*c*d + a^3*b*c^3*d)/(16*(a^3*b^3 + a^6*c^6 + 3*a^5*b*c^4 + 3*a^4*b^2*c^2
)))^(1/2)*(x*((b*d*(-a^3*b^3)^(1/2) - 3*a*c^2*d*(-a^3*b^3)^(1/2) - 3*a^2*b
^2*c*d + a^3*b*c^3*d)/(16*(a^3*b^3 + a^6*c^6 + 3*a^5*b*c^4 + 3*a^4*b^2*c^2
)))^(1/2)*(64*a^4*b^6*c*d^7 + 64*a^9*b*c^11*d^7 + 320*a^5*b^5*c^3*d^7 + 64
0*a^6*b^4*c^5*d^7 + 640*a^7*b^3*c^7*d^7 + 320*a^8*b^2*c^9*d^7) + 32*a^3*b^
6*c*d^7 + 128*a^4*b^5*c^3*d^7 + 192*a^5*b^4*c^5*d^7 + 128*a^6*b^3*c^7*d^7
+ 32*a^7*b^2*c^9*d^7))*((b*d*(-a^3*b^3)^(1/2) - 3*a*c^2*d*(-a^3*b^3)^(1/2)
 - 3*a^2*b^2*c*d + a^3*b*c^3*d)/(16*(a^3*b^3 + a^6*c^6 + 3*a^5*b*c^4 + 3*a
^4*b^2*c^2)))^(1/2)*1i + (x*(4*a^2*b^6*d^8 + 8*a^3*b^5*c^2*d^8 - 8*a^5*b^3
*c^6*d^8 - 4*a^6*b^2*c^8*d^8) - ((b*d*(-a^3*b^3)^(1/2) - 3*a*c^2*d*(-a^3*b
^3)^(1/2) - 3*a^2*b^2*c*d + a^3*b*c^3*d)/(16*(a^3*b^3 + a^6*c^6 + 3*a^5*b*
c^4 + 3*a^4*b^2*c^2)))^(1/2)*(32*a^3*b^6*c*d^7 - x*((b*d*(-a^3*b^3)^(1/2)
- 3*a*c^2*d*(-a^3*b^3)^(1/2) - 3*a^2*b^2*c*d + a^3*b*c^3*d)/(16*(a^3*b^3 +
 a^6*c^6 + 3*a^5*b*c^4 + 3*a^4*b^2*c^2)))^(1/2)*(64*a^4*b^6*c*d^7 + 64*a^9
*b*c^11*d^7 + 320*a^5*b^5*c^3*d^7 + 640*a^6*b^4*c^5*d^7 + 640*a^7*b^3*c^7*
d^7 + 320*a^8*b^2*c^9*d^7) + 128*a^4*b^5*c^3*d^7 + 192*a^5*b^4*c^5*d^7 + 1
28*a^6*b^3*c^7*d^7 + 32*a^7*b^2*c^9*d^7))*((b*d*(-a^3*b^3)^(1/2) - 3*a*c^2
*d*(-a^3*b^3)^(1/2) - 3*a^2*b^2*c*d + a^3*b*c^3*d)/(16*(a^3*b^3 + a^6*c...
```

Reduce [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 1320, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^2\left(a+\frac{b}{(c+dx^2)^2}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(a+b/(d*x^2+c)^2),x)
```

output
```
(2*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*a
tan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)
/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**2*x - 2*sqrt
(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sq
rt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(
d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b*x - 2*sqrt(d)*sqrt(a)*
sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sq
rt(a*c**2 + b) - a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(
a*c**2 + b) + a*c)*sqrt(2)))*a*c**3*x - 2*sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqr
t(a*c**2 + b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) -
 a*c)*sqrt(2) - 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*
c)*sqrt(2)))*b*c*x - 2*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 +
 b) + a*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt
(2) + 2*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)
))*a*c**2*x + 2*sqrt(d)*sqrt(a*c**2 + b)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a
*c)*sqrt(2)*atan((sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2
*sqrt(a)*d*x)/(sqrt(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*b*x
+ 2*sqrt(d)*sqrt(a)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)*atan((sqr
t(d)*sqrt(sqrt(a)*sqrt(a*c**2 + b) - a*c)*sqrt(2) + 2*sqrt(a)*d*x)/(sqrt(d
)*sqrt(sqrt(a)*sqrt(a*c**2 + b) + a*c)*sqrt(2)))*a*c**3*x + 2*sqrt(d)*s...
```

3.244 $\int x^5 \sqrt{c+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 236

$$\int x^5 \sqrt{c+d\sqrt{a+bx^2}}\,dx = -\frac{2c(c^2-ad^2)^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3b^3d^6} + \frac{2(5c^4-6ac^2d^2+a^2d^4)\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^3d^6} - \frac{4c(5c^2-3ad^2)\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7b^3d^6} + \frac{4(5c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{9/2}}{9b^3d^6} - \frac{10c\left(c+d\sqrt{a+bx^2}\right)^{11/2}}{11b^3d^6} + \frac{2\left(c+d\sqrt{a+bx^2}\right)^{13/2}}{13b^3d^6}$$

output
```
-2/3*c*(-a*d^2+c^2)^2*(c+d*(b*x^2+a)^(1/2))^(3/2)/b^3/d^6+2/5*(a^2*d^4-6*a
*c^2*d^2+5*c^4)*(c+d*(b*x^2+a)^(1/2))^(5/2)/b^3/d^6-4/7*c*(-3*a*d^2+5*c^2)
*(c+d*(b*x^2+a)^(1/2))^(7/2)/b^3/d^6+4/9*(-a*d^2+5*c^2)*(c+d*(b*x^2+a)^(1/
2))^(9/2)/b^3/d^6-10/11*c*(c+d*(b*x^2+a)^(1/2))^(11/2)/b^3/d^6+2/13*(c+d*(
b*x^2+a)^(1/2))^(13/2)/b^3/d^6
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.86

$$\int x^5\sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-1280c^6+32c^4d^2(128a-15bx^2)+640c^5d\sqrt{a+bx^2}+16c^3d^3\sqrt{a+bx^2}(-118a+25\right.}{}$$

input
```
Integrate[x^5*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-1280*c^6 + 32*c^4*d^2*(128*a - 15*b*x^2)
+ 640*c^5*d*Sqrt[a + b*x^2] + 16*c^3*d^3*Sqrt[a + b*x^2]*(-118*a + 25*b*x^
2) - 2*c^2*d^4*(2320*a^2 - 508*a*b*x^2 + 175*b^2*x^4) + c*d^5*Sqrt[a + b*x
^2]*(1888*a^2 - 800*a*b*x^2 + 315*b^2*x^4) + 77*d^6*(32*a^3 - 8*a^2*b*x^2
+ 5*a*b^2*x^4 + 45*b^3*x^6)))/(45045*b^3*d^6)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {7283, 896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{d\sqrt{a+bx^2}+c}\,dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^4\sqrt{c+d\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{\int b^2x^4\sqrt{c+d\sqrt{bx^2+a}}d(bx^2+a)}{2b^3}$$

$$\Big\downarrow \text{1732}$$

$$\frac{\int \sqrt{bx^2+a}\left(a-x^4\right)^2\sqrt{c+d\sqrt{bx^2+a}}d\sqrt{bx^2+a}}{b^3}$$

$$\Big\downarrow \textbf{522}$$

$$\frac{\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^{11/2}}{d^5}-\frac{5c\left(c+d\sqrt{bx^2+a}\right)^{9/2}}{d^5}-\frac{2(ad^2-5c^2)\left(c+d\sqrt{bx^2+a}\right)^{7/2}}{d^5}-\frac{2(5c^3-3acd^2)\left(c+d\sqrt{bx^2+a}\right)^{5/2}}{d^5}+\frac{(5c^4-6ad^2c^2+a^2}{}\right.}{b^3}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{2(a^2d^4-6ac^2d^2+5c^4)\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^6}+\frac{4(5c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{9/2}}{9d^6}-\frac{4c(5c^2-3ad^2)\left(d\sqrt{a+bx^2}+c\right)^{7/2}}{7d^6}-\frac{2c(c^2-ad^2)^2\left(d\sqrt{a+bx^2}+\right.}{3d^6}}{b^3}$$

input
```
Int[x^5*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
((-2*c*(c^2 - a*d^2)^2*(c + d*Sqrt[a + b*x^2])^(3/2))/(3*d^6) + (2*(5*c^4
- 6*a*c^2*d^2 + a^2*d^4)*(c + d*Sqrt[a + b*x^2])^(5/2))/(5*d^6) - (4*c*(5*
c^2 - 3*a*d^2)*(c + d*Sqrt[a + b*x^2])^(7/2))/(7*d^6) + (4*(5*c^2 - a*d^2)
*(c + d*Sqrt[a + b*x^2])^(9/2))/(9*d^6) - (10*c*(c + d*Sqrt[a + b*x^2])^(1
1/2))/(11*d^6) + (2*(c + d*Sqrt[a + b*x^2])^(13/2))/(13*d^6))/b^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^5\sqrt{c+d\sqrt{b\,x^2+a}}dx$$

input
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.84

$$\int x^5\sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{2\,(3465\,b^3d^6x^6+2464\,a^3d^6-4640\,a^2c^2d^4+4096\,ac^4d^2-1280\,c^6+35\,(11\,ab^2d^6-10\,b^2c^2d^4)x^4-8\,(77\,a$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3465*b^3*d^6*x^6 + 2464*a^3*d^6 - 4640*a^2*c^2*d^4 + 4096*a*c^4*d
^2 - 1280*c^6 + 35*(11*a*b^2*d^6 - 10*b^2*c^2*d^4)*x^4 - 8*(77*a^2*b*d^6 -
 127*a*b*c^2*d^4 + 60*b*c^4*d^2)*x^2 + (315*b^2*c*d^5*x^4 + 1888*a^2*c*d^5
 - 1888*a*c^3*d^3 + 640*c^5*d - 400*(2*a*b*c*d^5 - b*c^3*d^3)*x^2)*sqrt(b*
x^2 + a))*sqrt(sqrt(b*x^2 + a)*d + c)/(b^3*d^6)
```

Sympy [F]

$$\int x^5\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int x^5\sqrt{c+d\sqrt{a+bx^2}}\,dx$$

input
```
integrate(x**5*(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x**5*sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.76

$$\int x^5\sqrt{c+d\sqrt{a+bx^2}}\,dx$$
$$=\frac{2\left(3465\left(\sqrt{bx^2+a}d+c\right)^{\frac{13}{2}}-20475\left(\sqrt{bx^2+a}d+c\right)^{\frac{11}{2}}c-10010\left(ad^2-5c^2\right)\left(\sqrt{bx^2+a}d+c\right)^{\frac{9}{2}}+12870\right.}{}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/45045*(3465*(sqrt(b*x^2 + a)*d + c)^(13/2) - 20475*(sqrt(b*x^2 + a)*d +
c)^(11/2)*c - 10010*(a*d^2 - 5*c^2)*(sqrt(b*x^2 + a)*d + c)^(9/2) + 12870*
(3*a*c*d^2 - 5*c^3)*(sqrt(b*x^2 + a)*d + c)^(7/2) + 9009*(a^2*d^4 - 6*a*c^
2*d^2 + 5*c^4)*(sqrt(b*x^2 + a)*d + c)^(5/2) - 15015*(a^2*c*d^4 - 2*a*c^3*
d^2 + c^5)*(sqrt(b*x^2 + a)*d + c)^(3/2))/(b^3*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(200) = 400$.

Time = 0.15 (sec) , antiderivative size = 855, normalized size of antiderivative = 3.62

$$\int x^5 \sqrt{c + d\sqrt{a + bx^2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/45045*(13*(1155*(sqrt(b*x^2 + a)*d + c)^(3/2)*a^2*d^4*sgn((sqrt(b*x^2 +
a)*d + c)*d - c*d) - 3465*sqrt(sqrt(b*x^2 + a)*d + c)*a^2*c*d^4*sgn((sqrt(
b*x^2 + a)*d + c)*d - c*d) - 990*(sqrt(b*x^2 + a)*d + c)^(7/2)*a*d^2*sgn((
sqrt(b*x^2 + a)*d + c)*d - c*d) + 4158*(sqrt(b*x^2 + a)*d + c)^(5/2)*a*c*d
^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 6930*(sqrt(b*x^2 + a)*d + c)^(3/
2)*a*c^2*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 6930*sqrt(sqrt(b*x^2 +
 a)*d + c)*a*c^3*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 315*(sqrt(b*x^
2 + a)*d + c)^(11/2)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 1925*(sqrt(b*x
^2 + a)*d + c)^(9/2)*c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 4950*(sqrt(b
*x^2 + a)*d + c)^(7/2)*c^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 6930*(sq
rt(b*x^2 + a)*d + c)^(5/2)*c^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 5775
*(sqrt(b*x^2 + a)*d + c)^(3/2)*c^4*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) -
3465*sqrt(sqrt(b*x^2 + a)*d + c)*c^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))
*c/(b^2*d^5) + (9009*(sqrt(b*x^2 + a)*d + c)^(5/2)*a^2*d^4 - 30030*(sqrt(b
*x^2 + a)*d + c)^(3/2)*a^2*c*d^4 + 45045*sqrt(sqrt(b*x^2 + a)*d + c)*a^2*c
^2*d^4 - 10010*(sqrt(b*x^2 + a)*d + c)^(9/2)*a*d^2 + 51480*(sqrt(b*x^2 + a
)*d + c)^(7/2)*a*c*d^2 - 108108*(sqrt(b*x^2 + a)*d + c)^(5/2)*a*c^2*d^2 +
120120*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*c^3*d^2 - 90090*sqrt(sqrt(b*x^2 + a
)*d + c)*a*c^4*d^2 + 3465*(sqrt(b*x^2 + a)*d + c)^(13/2) - 24570*(sqrt(b*x
^2 + a)*d + c)^(11/2)*c + 75075*(sqrt(b*x^2 + a)*d + c)^(9/2)*c^2 - 128...
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{c+d\sqrt{a+bx^2}}\,dx = \int x^5\,\sqrt{c+d\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^5*(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^5*(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.57

$$\int x^5\sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\left(-640\sqrt{b}\,\sqrt{b\,x^2+a}\,c^5dx-\right.}{}$$

input
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output

```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - 1888*sqrt(b)*sqrt(a
 + b*x**2)*a**2*c*d**5*x + 800*sqrt(b)*sqrt(a + b*x**2)*a*b*c*d**5*x**3 +
1888*sqrt(b)*sqrt(a + b*x**2)*a*c**3*d**3*x - 315*sqrt(b)*sqrt(a + b*x**2)
*b**2*c*d**5*x**5 - 400*sqrt(b)*sqrt(a + b*x**2)*b*c**3*d**3*x**3 - 640*sq
rt(b)*sqrt(a + b*x**2)*c**5*d*x + 2464*sqrt(a + b*x**2)*a**3*d**6 - 616*sq
rt(a + b*x**2)*a**2*b*d**6*x**2 - 4640*sqrt(a + b*x**2)*a**2*c**2*d**4 + 3
85*sqrt(a + b*x**2)*a*b**2*d**6*x**4 + 1016*sqrt(a + b*x**2)*a*b*c**2*d**4
*x**2 + 4096*sqrt(a + b*x**2)*a*c**4*d**2 + 3465*sqrt(a + b*x**2)*b**3*d**
6*x**6 - 350*sqrt(a + b*x**2)*b**2*c**2*d**4*x**4 - 480*sqrt(a + b*x**2)*b
*c**4*d**2*x**2 - 1280*sqrt(a + b*x**2)*c**6 - 2464*sqrt(b)*a**3*d**6*x +
616*sqrt(b)*a**2*b*d**6*x**3 + 4640*sqrt(b)*a**2*c**2*d**4*x - 385*sqrt(b)
*a*b**2*d**6*x**5 - 1016*sqrt(b)*a*b*c**2*d**4*x**3 - 4096*sqrt(b)*a*c**4*
d**2*x - 3465*sqrt(b)*b**3*d**6*x**7 + 350*sqrt(b)*b**2*c**2*d**4*x**5 + 4
80*sqrt(b)*b*c**4*d**2*x**3 + 1280*sqrt(b)*c**6*x + 1888*a**3*c*d**5 + 108
8*a**2*b*c*d**5*x**2 - 1888*a**2*c**3*d**3 - 485*a*b**2*c*d**5*x**4 - 1488
*a*b*c**3*d**3*x**2 + 640*a*c**5*d + 315*b**3*c*d**5*x**6 + 400*b**2*c**3*
d**3*x**4 + 640*b*c**5*d*x**2))/(45045*a*b**3*d**6)
```

3.245 $\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx=-\frac{2c(c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3b^2d^4}+\frac{2(3c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^2d^4}-\frac{6c\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7b^2d^4}+\frac{2\left(c+d\sqrt{a+bx^2}\right)^{9/2}}{9b^2d^4}$$

output
```
-2/3*c*(-a*d^2+c^2)*(c+d*(b*x^2+a)^(1/2))^(3/2)/b^2/d^4+2/5*(-a*d^2+3*c^2)
*(c+d*(b*x^2+a)^(1/2))^(5/2)/b^2/d^4-6/7*c*(c+d*(b*x^2+a)^(1/2))^(7/2)/b^2
/d^4+2/9*(c+d*(b*x^2+a)^(1/2))^(9/2)/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx$$
$$=\frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-16c^4+6c^2d^2(6a-bx^2)+8c^3d\sqrt{a+bx^2}+cd^3\sqrt{a+bx^2}(-16a+5bx^2)+7d^4(-4a^2\right.}{315b^2d^4}$$

input
```
Integrate[x^3*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-16*c^4 + 6*c^2*d^2*(6*a - b*x^2) + 8*c^3*
d*Sqrt[a + b*x^2] + c*d^3*Sqrt[a + b*x^2]*(-16*a + 5*b*x^2) + 7*d^4*(-4*a^
2 + a*b*x^2 + 5*b^2*x^4)))/(315*b^2*d^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {7283, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d\sqrt{a+bx^2}+c}\,dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^2\sqrt{c+d\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{\int bx^2\sqrt{c+d\sqrt{bx^2+a}}d(bx^2+a)}{2b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -bx^2\sqrt{c+d\sqrt{bx^2+a}}d(bx^2+a)}{2b^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{\int \sqrt{bx^2+a}\left(a-x^4\right)\sqrt{c+d\sqrt{bx^2+a}}d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(-\frac{\left(c+d\sqrt{bx^2+a}\right)^{7/2}}{d^3}+\frac{3c\left(c+d\sqrt{bx^2+a}\right)^{5/2}}{d^3}+\frac{\left(ad^2-3c^2\right)\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d^3}+\frac{\left(c^3-acd^2\right)\sqrt{c+d\sqrt{bx^2+a}}}{d^3}\right)d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2(3c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^4}+\frac{2c(c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{3d^4}-\frac{2\left(d\sqrt{a+bx^2}+c\right)^{9/2}}{9d^4}+\frac{6c\left(d\sqrt{a+bx^2}+c\right)^{7/2}}{7d^4}}{b^2}$$

input
```
Int[x^3*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
-(((2*c*(c^2 - a*d^2)*(c + d*Sqrt[a + b*x^2])^(3/2))/(3*d^4) - (2*(3*c^2 -
 a*d^2)*(c + d*Sqrt[a + b*x^2])^(5/2))/(5*d^4) + (6*c*(c + d*Sqrt[a + b*x^
2])^(7/2))/(7*d^4) - (2*(c + d*Sqrt[a + b*x^2])^(9/2))/(9*d^4))/b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^3\sqrt{c+d\sqrt{b\,x^2+a}}dx$$

input
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{2\left(35\,b^2d^4x^4-28\,a^2d^4+36\,ac^2d^2-16\,c^4+(7\,abd^4-6\,bc^2d^2)x^2+(5\,bcd^3x^2-16\,acd^3+8\,c^3d)\sqrt{bx^2+a}\right.}{315\,b^2d^4}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^2*d^4*x^4 - 28*a^2*d^4 + 36*a*c^2*d^2 - 16*c^4 + (7*a*b*d^4 -
6*b*c^2*d^2)*x^2 + (5*b*c*d^3*x^2 - 16*a*c*d^3 + 8*c^3*d)*sqrt(b*x^2 + a))
*sqrt(sqrt(b*x^2 + a)*d + c)/(b^2*d^4)
```

Sympy [F]

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx$$

input
```
integrate(x**3*(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x**3*sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx$$
$$=\frac{2\left(35\left(\sqrt{bx^2+a}d+c\right)^{\frac{9}{2}}-135\left(\sqrt{bx^2+a}d+c\right)^{\frac{7}{2}}c-63\left(ad^2-3c^2\right)\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}+105\left(acd^2-c^3\right)\right.}{315\,b^2d^4}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(sqrt(b*x^2 + a)*d + c)^(9/2) - 135*(sqrt(b*x^2 + a)*d + c)^(7/2
)*c - 63*(a*d^2 - 3*c^2)*(sqrt(b*x^2 + a)*d + c)^(5/2) + 105*(a*c*d^2 - c^
3)*(sqrt(b*x^2 + a)*d + c)^(3/2))/(b^2*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(117) = 234.

Time = 0.14 (sec) , antiderivative size = 433, normalized size of antiderivative = 3.07

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx=$$
$$-\frac{2\left(\frac{3\left(35\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}ad^2\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-105\sqrt{\sqrt{bx^2+a}d+c}\,acd^2\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-15\left(\sqrt{bx^2+a}d+c\right)^{\frac{7}{2}}\mathrm{sgn}\left(\left(\right.\right.\right.}{}\right.}{}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-2/315*(3*(35*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*d^2*sgn((sqrt(b*x^2 + a)*d +
 c)*d - c*d) - 105*sqrt(sqrt(b*x^2 + a)*d + c)*a*c*d^2*sgn((sqrt(b*x^2 + a
)*d + c)*d - c*d) - 15*(sqrt(b*x^2 + a)*d + c)^(7/2)*sgn((sqrt(b*x^2 + a)*
d + c)*d - c*d) + 63*(sqrt(b*x^2 + a)*d + c)^(5/2)*c*sgn((sqrt(b*x^2 + a)*
d + c)*d - c*d) - 105*(sqrt(b*x^2 + a)*d + c)^(3/2)*c^2*sgn((sqrt(b*x^2 +
a)*d + c)*d - c*d) + 105*sqrt(sqrt(b*x^2 + a)*d + c)*c^3*sgn((sqrt(b*x^2 +
 a)*d + c)*d - c*d))*c/(b*d^3) + (63*(sqrt(b*x^2 + a)*d + c)^(5/2)*a*d^2 -
 210*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*c*d^2 + 315*sqrt(sqrt(b*x^2 + a)*d +
c)*a*c^2*d^2 - 35*(sqrt(b*x^2 + a)*d + c)^(9/2) + 180*(sqrt(b*x^2 + a)*d +
 c)^(7/2)*c - 378*(sqrt(b*x^2 + a)*d + c)^(5/2)*c^2 + 420*(sqrt(b*x^2 + a)
*d + c)^(3/2)*c^3 - 315*sqrt(sqrt(b*x^2 + a)*d + c)*c^4)/(b*d^3))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int x^3\,\sqrt{c+d\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^3*(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^3*(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.43

$$\int x^3\sqrt{c+d\sqrt{a+bx^2}}\,dx$$

$$=\frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\left(16\sqrt{b}\,\sqrt{b\,x^2+a}\,ac\,d^3x-5\right.}{}$$

input
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*(16*sqrt(b)*sqrt(a + b*
x**2)*a*c*d**3*x - 5*sqrt(b)*sqrt(a + b*x**2)*b*c*d**3*x**3 - 8*sqrt(b)*sq
rt(a + b*x**2)*c**3*d*x - 28*sqrt(a + b*x**2)*a**2*d**4 + 7*sqrt(a + b*x**
2)*a*b*d**4*x**2 + 36*sqrt(a + b*x**2)*a*c**2*d**2 + 35*sqrt(a + b*x**2)*b
**2*d**4*x**4 - 6*sqrt(a + b*x**2)*b*c**2*d**2*x**2 - 16*sqrt(a + b*x**2)*
c**4 + 28*sqrt(b)*a**2*d**4*x - 7*sqrt(b)*a*b*d**4*x**3 - 36*sqrt(b)*a*c**
2*d**2*x - 35*sqrt(b)*b**2*d**4*x**5 + 6*sqrt(b)*b*c**2*d**2*x**3 + 16*sqr
t(b)*c**4*x - 16*a**2*c*d**3 - 11*a*b*c*d**3*x**2 + 8*a*c**3*d + 5*b**2*c*
d**3*x**4 + 8*b*c**3*d*x**2))/(315*a*b**2*d**4)
```

3.246 $\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx=-\frac{2c\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3bd^2}+\frac{2\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5bd^2}$$

output
```
-2/3*c*(c+d*(b*x^2+a)^(1/2))^(3/2)/b/d^2+2/5*(c+d*(b*x^2+a)^(1/2))^(5/2)/b
/d^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx=\frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-2c^2+cd\sqrt{a+bx^2}+3d^2(a+bx^2)\right)}{15bd^2}$$

input
```
Integrate[x*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-2*c^2 + c*d*Sqrt[a + b*x^2] + 3*d^2*(a +
b*x^2)))/(15*b*d^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d\sqrt{a+bx^2}+c}\,dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \sqrt{c+d\sqrt{bx^2+a}}d(bx^2+a)}{2b}$$

$$\Big\downarrow \text{774}$$

$$\frac{\int \sqrt{bx^2+a}\sqrt{c+d\sqrt{bx^2+a}}d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d}-\frac{c\sqrt{c+d\sqrt{bx^2+a}}}{d}\right)d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^2}-\frac{2c\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{3d^2}}{b}$$

input
```
Int[x*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
((-2*c*(c + d*Sqrt[a + b*x^2])^(3/2))/(3*d^2) + (2*(c + d*Sqrt[a + b*x^2])
^(5/2))/(5*d^2))/b
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{2\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{5}{2}}}{5}-\frac{2c\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{3}}{b\,d^{2}}$	45
default	$\frac{\frac{2\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{5}{2}}}{5}-\frac{2c\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{3}}{b\,d^{2}}$	45

input
```
int(x*(c+d*(b*x^2+a)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^2*(1/5*(c+d*(b*x^2+a)^(1/2))^(5/2)-1/3*c*(c+d*(b*x^2+a)^(1/2))^(3/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx=\frac{2\left(3\,bd^2x^2+3\,ad^2+\sqrt{bx^2+a}cd-2\,c^2\right)\sqrt{\sqrt{bx^2+a}d+c}}{15\,bd^2}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b*d^2*x^2 + 3*a*d^2 + sqrt(b*x^2 + a)*c*d - 2*c^2)*sqrt(sqrt(b*x^2
 + a)*d + c)/(b*d^2)
```

Sympy [F]

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx$$

input
```
integrate(x*(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x*sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx=\frac{2\left(\frac{3\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}c}{d^2}\right)}{15\,b}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*(sqrt(b*x^2 + a)*d + c)^(5/2)/d^2 - 5*(sqrt(b*x^2 + a)*d + c)^(3/2
)*c/d^2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(48) = 96$.

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.22

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{2\left(\frac{5\left(\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}d-3\sqrt{\sqrt{bx^2+a}d+c}cd\right)\operatorname{csgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)}{d^2}+\frac{3\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}-10\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}c+15\sqrt{\sqrt{bx^2+a}d+c}\cdots}{d}\right.}{15\,bd}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((sqrt(b*x^2 + a)*d + c)^(3/2)*d - 3*sqrt(sqrt(b*x^2 + a)*d + c)*c
*d)*c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d)/d^2 + (3*(sqrt(b*x^2 + a)*d + c
)^(5/2) - 10*(sqrt(b*x^2 + a)*d + c)^(3/2)*c + 15*sqrt(sqrt(b*x^2 + a)*d +
 c)*c^2)/d)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c+d\sqrt{b\,x^2+a}}\,(b\,x^2+a)\;{}_2\mathrm{F}_1\left(-\frac{1}{2},2;\;3;\;-\frac{d\sqrt{b\,x^2+a}}{c}\right)}{2\,b\sqrt{\frac{d\sqrt{b\,x^2+a}}{c}+1}}$$

input
```
int(x*(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
((c + d*(a + b*x^2)^(1/2))^(1/2)*(a + b*x^2)*hypergeom([-1/2, 2], 3, -(d*(
a + b*x^2)^(1/2))/c))/(2*b*((d*(a + b*x^2)^(1/2))/c + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.72

$$\int x\sqrt{c+d\sqrt{a+bx^2}}\,dx$$
$$=\frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\left(-\sqrt{b}\,\sqrt{b\,x^2+a}\,cdx+3\sqrt{\right.}{15ab}$$

input
```
int(x*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - sqrt(b)*sqrt(a + b*
x**2)*c*d*x + 3*sqrt(a + b*x**2)*a*d**2 + 3*sqrt(a + b*x**2)*b*d**2*x**2 -
 2*sqrt(a + b*x**2)*c**2 - 3*sqrt(b)*a*d**2*x - 3*sqrt(b)*b*d**2*x**3 + 2*
sqrt(b)*c**2*x + a*c*d + b*c*d*x**2))/(15*a*b*d**2)
```

3.247 $\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx = 2\sqrt{c+d\sqrt{a+bx^2}} - \sqrt{c-\sqrt{a}d}\,\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right) - \sqrt{c+\sqrt{a}d}\,\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)$$

output
```
2*(c+d*(b*x^2+a)^(1/2))^(1/2)-(c-a^(1/2)*d)^(1/2)*arctanh((c+d*(b*x^2+a)^(
1/2))^(1/2)/(c-a^(1/2)*d)^(1/2))-(c+a^(1/2)*d)^(1/2)*arctanh((c+d*(b*x^2+a
)^(1/2))^(1/2)/(c+a^(1/2)*d)^(1/2))
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx = 2\sqrt{c+d\sqrt{a+bx^2}} - \sqrt{-c-\sqrt{a}d}\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right) - \sqrt{-c+\sqrt{a}d}\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)$$

input
```
Integrate[Sqrt[c + d*Sqrt[a + b*x^2]]/x,x]
```

output
```
2*Sqrt[c + d*Sqrt[a + b*x^2]] - Sqrt[-c - Sqrt[a]*d]*ArcTan[Sqrt[c + d*Sqr
t[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]] - Sqrt[-c + Sqrt[a]*d]*ArcTan[Sqrt[c +
 d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]]
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {7282, 896, 25, 1732, 561, 25, 27, 1602, 25, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{2}\int \frac{\sqrt{c+d\sqrt{bx^2+a}}}{x^2}dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{2}\int \frac{\sqrt{c+d\sqrt{bx^2+a}}}{bx^2}d(bx^2+a)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int -\frac{\sqrt{c+d\sqrt{bx^2+a}}}{bx^2}d(bx^2+a)$$

$$\Big\downarrow \text{1732}$$

$$-\int \frac{\sqrt{bx^2+a}\sqrt{c+d\sqrt{bx^2+a}}}{a-x^4}d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{561}$$

$$-\frac{2\int -\frac{x^4(c-x^4)}{d\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int \frac{x^4(c-x^4)}{d\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{x^4(c-x^4)}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}}{d^2}$$

$$\Big\downarrow \; 1602$$

$$\frac{2\left(d^2\int -\frac{cx^4+\left(a-\frac{c^2}{d^2}\right)d^2}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}+d^2\sqrt{d\sqrt{a+bx^2}+c}\right)}{d^2}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(d^2\sqrt{d\sqrt{a+bx^2}+c}-d^2\int -\frac{-cx^4+c^2-ad^2}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}\right)}{d^2}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(d^2\int \frac{-cx^4+c^2-ad^2}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}+d^2\sqrt{d\sqrt{a+bx^2}+c}\right)}{d^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\int \frac{-cx^4+c^2-ad^2}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}+d^2\sqrt{d\sqrt{a+bx^2}+c}\right)}{d^2}$$

$$\Big\downarrow \; 1480$$

$$\frac{2\left(-\frac{1}{2}\left(c-\sqrt{a}d\right)\int \frac{1}{\frac{c-\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}-\frac{1}{2}\left(\sqrt{a}d+c\right)\int \frac{1}{\frac{c+\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}+d^2\sqrt{d\sqrt{a+bx^2}+\right.}{d^2}$$

$$\Big\downarrow \; 221$$

$$\frac{2\left(-\frac{1}{2}d^2\sqrt{c-\sqrt{a}d}\,\text{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{c-\sqrt{a}d}}\right)-\frac{1}{2}d^2\sqrt{\sqrt{a}d+c}\,\text{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{\sqrt{a}d+c}}\right)+d^2\sqrt{d\sqrt{a+bx^2}+c}\right)}{d^2}$$

input
```
Int[Sqrt[c + d*Sqrt[a + b*x^2]]/x,x]
```

output
```
(2*(d^2*Sqrt[c + d*Sqrt[a + b*x^2]] - (d^2*Sqrt[c - Sqrt[a]*d]*ArcTanh[Sqr
t[c + d*Sqrt[a + b*x^2]]/Sqrt[c - Sqrt[a]*d]])/2 - (d^2*Sqrt[c + Sqrt[a]*d
]*ArcTanh[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[c + Sqrt[a]*d]])/2))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{\sqrt{c + d\sqrt{b\,x^2 + a}}}{x}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx = \int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(c + d*sqrt(a + b*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(94) = 188$.

Time = 0.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx$$
$$= \frac{2\sqrt{\sqrt{bx^2+a}d+c}d + \frac{\left(\sqrt{a}cd^3\text{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-ad^3|d|+c^2d|d|\text{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-\sqrt{a}cd^3\right)\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}d+c}}{\sqrt{-c+\sqrt{ad^2}}}\right)}{\left(\sqrt{a}d+c\right)\sqrt{\sqrt{a}d-c}|d|}}{d}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
(2*sqrt(sqrt(b*x^2 + a)*d + c)*d + (sqrt(a)*c*d^3*sgn((sqrt(b*x^2 + a)*d +
 c)*d - c*d) - a*d^3*abs(d) + c^2*d*abs(d)*sgn((sqrt(b*x^2 + a)*d + c)*d -
 c*d) - sqrt(a)*c*d^3)*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-c + sqrt(a
*d^2)))/((sqrt(a)*d + c)*sqrt(sqrt(a)*d - c)*abs(d)) + (sqrt(a)*c*d^3*sgn(
(sqrt(b*x^2 + a)*d + c)*d - c*d) + a*d^3*abs(d) - c^2*d*abs(d)*sgn((sqrt(b
*x^2 + a)*d + c)*d - c*d) - sqrt(a)*c*d^3)*arctan(sqrt(sqrt(b*x^2 + a)*d +
 c)/sqrt(-c - sqrt(a*d^2)))/((sqrt(a)*d - c)*sqrt(-sqrt(a)*d - c)*abs(d)))
/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx = \int \frac{\sqrt{c+d\sqrt{b\,x^2+a}}}{x}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.21

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x}\,dx$$

$$= \frac{\sqrt{\sqrt{a}\,d-c}\,atan\left(\frac{\sqrt{b\,x^2+a}\,\sqrt{\sqrt{a}\,d-c}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}\,a\,d^3-\sqrt{b\,x^2+a}\,\sqrt{\sqrt{a}\,d-c}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}\,c^2d-\sqrt{a}\,\sqrt{\sqrt{a}\,d-c}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}\,a\,d^3-}{}\right.}{}$$

$$+2\sqrt{\sqrt{b\,x^2+a}\,d+c}+\frac{\sqrt{\sqrt{a}\,d+c}\log\left(\sqrt{\sqrt{b\,x^2+a}\,d+c}-\sqrt{\sqrt{a}\,d+c}\right)}{2}$$

$$-\frac{\sqrt{\sqrt{a}\,d+c}\log\left(\sqrt{\sqrt{b\,x^2+a}\,d+c}+\sqrt{\sqrt{a}\,d+c}\right)}{2}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x,x)
```

output
```
(sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(
a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt
(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x
**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d +
 c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)
*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*c*d**2 - sq
rt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 + 2*sqrt(sqrt
(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*a*b*d**4*x*
*2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4)) + 4*sqrt(sqrt(a + b*x**
2)*d + c) + sqrt(sqrt(a)*d + c)*log(sqrt(sqrt(a + b*x**2)*d + c) - sqrt(sq
rt(a)*d + c)) - sqrt(sqrt(a)*d + c)*log(sqrt(sqrt(a + b*x**2)*d + c) + sqr
t(sqrt(a)*d + c)))/2
```

3.248 $\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = -\frac{\sqrt{c+d\sqrt{a+bx^2}}}{2x^2} + \frac{bd\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{4\sqrt{a}\sqrt{c-\sqrt{a}d}} - \frac{bd\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{4\sqrt{a}\sqrt{c+\sqrt{a}d}}$$

output
```
-1/2*(c+d*(b*x^2+a)^(1/2))^(1/2)/x^2+1/4*b*d*arctanh((c+d*(b*x^2+a)^(1/2))
^(1/2)/(c-a^(1/2)*d)^(1/2))/a^(1/2)/(c-a^(1/2)*d)^(1/2)-1/4*b*d*arctanh((c
+d*(b*x^2+a)^(1/2))^(1/2)/(c+a^(1/2)*d)^(1/2))/a^(1/2)/(c+a^(1/2)*d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = \frac{1}{4}\left(-\frac{2\sqrt{c+d\sqrt{a+bx^2}}}{x^2} + \frac{bd\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right)}{\sqrt{a}\sqrt{-c-\sqrt{a}d}} - \frac{bd\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)}{\sqrt{a}\sqrt{-c+\sqrt{a}d}}\right)$$

input
```
Integrate[Sqrt[c + d*Sqrt[a + b*x^2]]/x^3,x]
```

output
```
((-2*Sqrt[c + d*Sqrt[a + b*x^2]])/x^2 + (b*d*ArcTan[Sqrt[c + d*Sqrt[a + b*
x^2]]/Sqrt[-c - Sqrt[a]*d]])/(Sqrt[a]*Sqrt[-c - Sqrt[a]*d]) - (b*d*ArcTan[
Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]])/(Sqrt[a]*Sqrt[-c + Sqrt
[a]*d]))/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^3}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^3}dx$$

input
```
Int[Sqrt[c + d*Sqrt[a + b*x^2]]/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\sqrt{c+d\sqrt{b\,x^2+a}}}{x^3}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^3,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = \int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c + d*sqrt(a + b*x**2))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^3}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. 2(107) = 214.

Time = 0.24 (sec) , antiderivative size = 1093, normalized size of antiderivative = 7.54

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*b*((a^(7/2)*c*d^9 - 4*a^(5/2)*c^3*d^7 + 5*a^(3/2)*c^5*d^5 - 2*sqrt(a)
*c^7*d^3 + (a*d^3 - c^2*d)^2*a^(3/2)*c*d^3*sgn((sqrt(b*x^2 + a)*d + c)*d -
 c*d) - (a*d^3 - c^2*d)^2*a^(3/2)*c*d^3 - 2*(a^2*c^2*d^5 - a*c^4*d^3)*abs(
a*d^3 - c^2*d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - (a^3*d^7 - 4*a^2*c^2
*d^5 + 3*a*c^4*d^3)*abs(a*d^3 - c^2*d) + (a^(5/2)*c^3*d^7 - 2*a^(3/2)*c^5*
d^5 + sqrt(a)*c^7*d^3)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))*arctan(sqrt(s
qrt(b*x^2 + a)*d + c)/sqrt(-(a*c*d^2 - c^3 + sqrt((a*c*d^2 - c^3)^2 + (a^2
*d^4 - 2*a*c^2*d^2 + c^4)*(a*d^2 - c^2)))/(a*d^2 - c^2)))/((a^(7/2)*d^5 -
a^3*c*d^4 - 2*a^(5/2)*c^2*d^3 + 2*a^2*c^3*d^2 + a^(3/2)*c^4*d - a*c^5)*sqr
t(-sqrt(a)*d - c)*abs(a*d^3 - c^2*d)) + (a^(7/2)*c*d^9 - 4*a^(5/2)*c^3*d^7
 + 5*a^(3/2)*c^5*d^5 - 2*sqrt(a)*c^7*d^3 + (a*d^3 - c^2*d)^2*a^(3/2)*c*d^3
*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - (a*d^3 - c^2*d)^2*a^(3/2)*c*d^3 +
2*(a^2*c^2*d^5 - a*c^4*d^3)*abs(a*d^3 - c^2*d)*sgn((sqrt(b*x^2 + a)*d + c)
*d - c*d) + (a^3*d^7 - 4*a^2*c^2*d^5 + 3*a*c^4*d^3)*abs(a*d^3 - c^2*d) + (
a^(5/2)*c^3*d^7 - 2*a^(3/2)*c^5*d^5 + sqrt(a)*c^7*d^3)*sgn((sqrt(b*x^2 + a
)*d + c)*d - c*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-(a*c*d^2 - c^3
 - sqrt((a*c*d^2 - c^3)^2 + (a^2*d^4 - 2*a*c^2*d^2 + c^4)*(a*d^2 - c^2)))/
(a*d^2 - c^2)))/((a^(7/2)*d^5 + a^3*c*d^4 - 2*a^(5/2)*c^2*d^3 - 2*a^2*c^3*
d^2 + a^(3/2)*c^4*d + a*c^5)*sqrt(sqrt(a)*d - c)*abs(a*d^3 - c^2*d)) - 2*(
sqrt(sqrt(b*x^2 + a)*d + c)*a*d^5 + (sqrt(b*x^2 + a)*d + c)^(3/2)*c*d^3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = \int \frac{\sqrt{c+d\,\sqrt{b\,x^2+a}}}{x^3}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^3,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^3}\,dx = \text{too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^3,x)
```

output
```
(15*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)
*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c
)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(s
qrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a +
b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x
**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*
c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 +
2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*
a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**2*b*c*d**
5*x**2 - 16*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a
)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(
a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c
)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(s
qrt(a + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt
(a + b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d
 + c)*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2
*x**2 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d
**4 + 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a*b*
c**3*d**3*x**2 + 4*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt
(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*...
```

3.249 $\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 234

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^5}\,dx = -\frac{\sqrt{c+d\sqrt{a+bx^2}}}{4x^4} + \frac{bd\left(ad-c\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{16a\left(c^2-ad^2\right)x^2} - \frac{b^2d\left(2c-3\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{32a^{3/2}\left(c-\sqrt{a}d\right)^{3/2}} + \frac{b^2d\left(2c+3\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{32a^{3/2}\left(c+\sqrt{a}d\right)^{3/2}}$$

output
```
-1/4*(c+d*(b*x^2+a)^(1/2))^(1/2)/x^4+1/16*b*d*(a*d-c*(b*x^2+a)^(1/2))*(c+d
*(b*x^2+a)^(1/2))^(1/2)/a/(-a*d^2+c^2)/x^2-1/32*b^2*d*(2*c-3*a^(1/2)*d)*ar
ctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c-a^(1/2)*d)^(1/2))/a^(3/2)/(c-a^(1/2)*
d)^(3/2)+1/32*b^2*d*(2*c+3*a^(1/2)*d)*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/
(c+a^(1/2)*d)^(1/2))/a^(3/2)/(c+a^(1/2)*d)^(3/2)
```

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^5}\,dx = \frac{1}{32}\left(\frac{2bcd\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{a(-c^2+ad^2)x^2} + \frac{2\sqrt{c+d\sqrt{a+bx^2}}(-4c^2+d^2(4a+bx^2))}{(c^2-ad^2)x^4} + \frac{b^2d(2c+3\sqrt{a}d)\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right)}{a^{3/2}\left(-c-\sqrt{a}d\right)^{3/2}} + \frac{b^2d(-2c+3\sqrt{a}d)\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)}{a^{3/2}\left(-c+\sqrt{a}d\right)^{3/2}}\right)$$

input
```
Integrate[Sqrt[c + d*Sqrt[a + b*x^2]]/x^5,x]
```

output
```
((2*b*c*d*Sqrt[a + b*x^2]*Sqrt[c + d*Sqrt[a + b*x^2]])/(a*(-c^2 + a*d^2)*x
^2) + (2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-4*c^2 + d^2*(4*a + b*x^2)))/((c^2 -
 a*d^2)*x^4) + (b^2*d*(2*c + 3*Sqrt[a]*d)*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2
]]/Sqrt[-c - Sqrt[a]*d]])/(a^(3/2)*(-c - Sqrt[a]*d)^(3/2)) + (b^2*d*(-2*c
+ 3*Sqrt[a]*d)*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]])/(
a^(3/2)*(-c + Sqrt[a]*d)^(3/2)))/32
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^5}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^5}dx$$

input
```
Int[Sqrt[c + d*Sqrt[a + b*x^2]]/x^5,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\sqrt{c+d\sqrt{b\,x^{2}+a}}}{x^{5}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^5,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^5,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c + d\sqrt{a + bx^2}}}{x^5} dx = \int \frac{\sqrt{c + d\sqrt{a + bx^2}}}{x^5} dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(c + d*sqrt(a + b*x**2))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{c + d\sqrt{a + bx^2}}}{x^5} dx = \int \frac{\sqrt{\sqrt{bx^2 + a}d + c}}{x^5} dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2413 vs. $2(185) = 370$.

Time = 0.30 (sec) , antiderivative size = 2413, normalized size of antiderivative = 10.31

$$\int \frac{\sqrt{c + d\sqrt{a + bx^2}}}{x^5} dx = \text{Too large to display}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/32*(((a^3*d^5 - 2*a^2*c^2*d^3 + a*c^4*d)^2*(5*a*c*d^5 + c^3*d^3)*b^3*sgn
((sqrt(b*x^2 + a)*d + c)*d - c*d) - 2*(a^3*d^5 - 2*a^2*c^2*d^3 + a*c^4*d)^
2*(2*a*c*d^5 + c^3*d^3)*b^3 - (13*a^(7/2)*c^2*d^9 - 27*a^(5/2)*c^4*d^7 + 1
5*a^(3/2)*c^6*d^5 - sqrt(a)*c^8*d^3)*b^3*abs(a^3*d^5 - 2*a^2*c^2*d^3 + a*c
^4*d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - (3*a^(9/2)*d^11 - 23*a^(7/2)*
c^2*d^9 + 39*a^(5/2)*c^4*d^7 - 21*a^(3/2)*c^6*d^5 + 2*sqrt(a)*c^8*d^3)*b^3
*abs(a^3*d^5 - 2*a^2*c^2*d^3 + a*c^4*d) + 2*(4*a^6*c^3*d^13 - 17*a^5*c^5*d
^11 + 28*a^4*c^7*d^9 - 22*a^3*c^9*d^7 + 8*a^2*c^11*d^5 - a*c^13*d^3)*b^3*s
gn((sqrt(b*x^2 + a)*d + c)*d - c*d) + (3*a^7*c*d^15 - 25*a^6*c^3*d^13 + 74
*a^5*c^5*d^11 - 106*a^4*c^7*d^9 + 79*a^3*c^9*d^7 - 29*a^2*c^11*d^5 + 4*a*c
^13*d^3)*b^3)*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-(a^3*c*d^4 - 2*a^2*
c^3*d^2 + a*c^5 + sqrt((a^3*c*d^4 - 2*a^2*c^3*d^2 + a*c^5)^2 + (a^4*d^6 -
3*a^3*c^2*d^4 + 3*a^2*c^4*d^2 - a*c^6)*(a^3*d^4 - 2*a^2*c^2*d^2 + a*c^4)))
/(a^3*d^4 - 2*a^2*c^2*d^2 + a*c^4)))/((a^6*d^9 - a^(11/2)*c*d^8 - 4*a^5*c^
2*d^7 + 4*a^(9/2)*c^3*d^6 + 6*a^4*c^4*d^5 - 6*a^(7/2)*c^5*d^4 - 4*a^3*c^6*
d^3 + 4*a^(5/2)*c^7*d^2 + a^2*c^8*d - a^(3/2)*c^9)*sqrt(-sqrt(a)*d - c)*ab
s(a^3*d^5 - 2*a^2*c^2*d^3 + a*c^4*d)) + ((a^3*d^5 - 2*a^2*c^2*d^3 + a*c^4*
d)^2*(5*a*c*d^5 + c^3*d^3)*b^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 2*(a
^3*d^5 - 2*a^2*c^2*d^3 + a*c^4*d)^2*(2*a*c*d^5 + c^3*d^3)*b^3 + (13*a^(7/2
)*c^2*d^9 - 27*a^(5/2)*c^4*d^7 + 15*a^(3/2)*c^6*d^5 - sqrt(a)*c^8*d^3)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^5}\,dx = \int \frac{\sqrt{c+d\,\sqrt{b\,x^2+a}}}{x^5}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^5,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^5}\,dx = \text{too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^5,x)
```

output
```
( - 10140*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*
d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)
*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*
sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqr
t(a + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a
 + b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d +
 c)*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x
**2 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**
4 + 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**5*b
**2*c*d**12*x**4 + 25246*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2
)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**
2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(
sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*
d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d -
 c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(
a + b*x**2)*d + c)*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d
+ c)*b*c*d**2*x**2 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c*
*3)/(2*a**2*d**4 + 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 +
2*c**4))*a**4*b**2*c**3*d**10*x**4 - 24936*sqrt(a)*sqrt(sqrt(a)*d - c)*ata
n((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*...
```

3.250 $\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 406

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{2\left(5a-\frac{4c^2}{d^2}\right)x\sqrt{c+d\sqrt{a+bx^2}}}{105b} + \frac{2}{7}x^3\sqrt{c+d\sqrt{a+bx^2}} + \frac{2cx\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{35bd}$$
$$- \frac{16\sqrt{a}c(c^2-2ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{105b^2d^3x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}}$$
$$+ \frac{4\sqrt{a}(4c^4-9ac^2d^2+5a^2d^4)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{105b^2d^3x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
2/105*(5*a-4*c^2/d^2)*x*(c+d*(b*x^2+a)^(1/2))^(1/2)/b+2/7*x^3*(c+d*(b*x^2+
a)^(1/2))^(1/2)+2/35*c*x*(b*x^2+a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)/b/d-1
6/105*a^(1/2)*c*(-2*a*d^2+c^2)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2
)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)
*d/(c+a^(1/2)*d))^(1/2))/b^2/d^3/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(
1/2)+4/105*a^(1/2)*(5*a^2*d^4-9*a*c^2*d^2+4*c^4)*(-b*x^2/a)^(1/2)*((c+d*(b
*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/
2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b^2/d^3/x/(c+d*
(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.88 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.36

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx$$

$$= \frac{2\left(8bcd^2(c^2-2ad^2)\,x^2+bd^2x^2\left(c+d\sqrt{a+bx^2}\right)\left(-4c^2+3cd\sqrt{a+bx^2}+5d^2(a+3bx^2)\right)-\frac{8ic\left(c^3+\sqrt{a}c^2d-\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[x^2*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*(8*b*c*d^2*(c^2 - 2*a*d^2)*x^2 + b*d^2*x^2*(c + d*Sqrt[a + b*x^2])*(-4*
c^2 + 3*c*d*Sqrt[a + b*x^2] + 5*d^2*(a + 3*b*x^2)) - ((8*I)*c*(c^3 + Sqrt[
a]*c^2*d - 2*a*c*d^2 - 2*a^(3/2)*d^3)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2])
)/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqr
t[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c -
 Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)]
)/Sqrt[-c - Sqrt[a]*d] + ((2*I)*Sqrt[a]*d*(4*c^3 + Sqrt[a]*c^2*d - 8*a*c*d
^2 - 5*a^(3/2)*d^3)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a +
b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c
+ d*Sqrt[a + b*x^2])^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c
 + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/Sqrt[-c - Sqrt[a
]*d]))/(105*b^2*d^4*x*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d\sqrt{a+bx^2}+c}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^2\sqrt{d\sqrt{a+bx^2}+c}dx$$

input
```
Int[x^2*Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^2\sqrt{c+d\sqrt{b\,x^2+a}}dx$$

input
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int\sqrt{\sqrt{bx^2+a}d+c}\,x^2\,dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c)*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx$$

input
```
integrate(x**2*(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x**2*sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int\sqrt{\sqrt{bx^2+a}d+c}\,x^2\,dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int \sqrt{\sqrt{bx^2+a}d+c}x^2\,dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int x^2\,\sqrt{c+d\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^2*(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^2*(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{c+d\sqrt{a+bx^2}}\,dx=\int \sqrt{\sqrt{b\,x^2+a}\,d+c}\,x^2dx$$

input
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c)*x**2,x)
```

3.251 $\int \sqrt{c+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 300

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx = \frac{2}{3}x\sqrt{c+d\sqrt{a+bx^2}} - \frac{2\sqrt{a}c\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{3bdx\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} + \frac{2\sqrt{a}(c^2-ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{3bdx\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
2/3*x*(c+d*(b*x^2+a)^(1/2))^(1/2)-2/3*a^(1/2)*c*(-b*x^2/a)^(1/2)*(c+d*(b*x
^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2)
,2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b/d/x/((c+d*(b*x^2+a)^(1/2))/(c+
a^(1/2)*d))^(1/2)+2/3*a^(1/2)*(-a*d^2+c^2)*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a
)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b/d/x/(c+d*(b*x^2+a)^
(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.59 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.51

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx$$
$$= \frac{2bd^2\sqrt{-c-\sqrt{a}d}x^2\left(2c+d\sqrt{a+bx^2}\right)-2ic\left(c+\sqrt{a}d\right)\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\left(c+d\sqrt{a+bx^2}\right)}{}$$

input
```
Integrate[Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*b*d^2*Sqrt[-c - Sqrt[a]*d]*x^2*(2*c + d*Sqrt[a + b*x^2]) - (2*I)*c*(c +
 Sqrt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]
*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt
[a + b*x^2])^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqr
t[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + (2*I)*Sqrt[a]*d*(c + Sq
rt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sq
rt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a
+ b*x^2])^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a
 + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/(3*b*d^2*Sqrt[-c - Sqrt[a]*
d]*x*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sqrt{a+bx^2}+c}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \sqrt{d\sqrt{a+bx^2}+c}dx$$

input
```
Int[Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sqrt{c+d\sqrt{b\,x^{2}+a}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx=\int \sqrt{\sqrt{bx^2+a}d+c}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Sympy [F]

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx = \int \sqrt{c+d\sqrt{a+bx^2}}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [F]

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx = \int \sqrt{\sqrt{bx^2+a}d+c}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Giac [F]

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx = \int \sqrt{\sqrt{bx^2+a}d+c}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx = \int \sqrt{c+d\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+d\sqrt{a+bx^2}}\,dx = \int \sqrt{\sqrt{b\,x^2+a}\,d+c}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c),x)
```

3.252 $\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = -\frac{\sqrt{c+d\sqrt{a+bx^2}}}{x} - \frac{\sqrt{a}d\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-(c+d*(b*x^2+a)^(1/2))^(1/2)/x-a^(1/2)*d*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^
(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2
)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/x/(c+d*(b*x^2+a)^(1/2))
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.74 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = \frac{-\sqrt{c+d\sqrt{a+bx^2}}+\frac{i\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\left(c+d\sqrt{a+bx^2}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c-\sqrt{a}d}}{\sqrt{c+d\sqrt{a+bx^2}}}\right),\frac{c-\sqrt{a}d}{c+\sqrt{a}d}\right)}{\sqrt{-c-\sqrt{a}d}}}{x}$$

input
```
Integrate[Sqrt[c + d*Sqrt[a + b*x^2]]/x^2,x]
```

output
```
(-Sqrt[c + d*Sqrt[a + b*x^2]] + (I*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(
c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a
 + b*x^2])]*(c + d*Sqrt[a + b*x^2])*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*
d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/Sqrt[-c
 - Sqrt[a]*d])/x
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^2}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^2}dx$$

input
```
Int[Sqrt[c + d*Sqrt[a + b*x^2]]/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\sqrt{c+d\sqrt{b\,x^2+a}}}{x^2}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^2,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = \int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c + d*sqrt(a + b*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = \int \frac{\sqrt{c+d\sqrt{b\,x^2+a}}}{x^2}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^2,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{x^2}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^2,x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c)/x**2,x)
```

3.253 $\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 357

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx = -\frac{\sqrt{c+d\sqrt{a+bx^2}}}{3x^3}+\frac{bd\left(ad-c\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{6a\left(c^2-ad^2\right)x} - \frac{bcd\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{6\sqrt{a}\left(c^2-ad^2\right)x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} + \frac{bd\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{6\sqrt{a}x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-1/3*(c+d*(b*x^2+a)^(1/2))^(1/2)/x^3+1/6*b*d*(a*d-c*(b*x^2+a)^(1/2))*(c+d*
(b*x^2+a)^(1/2))^(1/2)/a/(-a*d^2+c^2)/x-1/6*b*c*d*(-b*x^2/a)^(1/2)*(c+d*(b
*x^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/
2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a*d^2+c^2)/x/((c+d*(
b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)+1/6*b*d*(-b*x^2/a)^(1/2)*((c+d*(b*x^2
+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(1/2)/x/(c+d*(b*x
^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.66 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx =$$

$$-\frac{\sqrt{-c-\sqrt{a}d}\left(c-\sqrt{a}d\right)\left(bdx^2\sqrt{a+bx^2}+2a\left(c+d\sqrt{a+bx^2}\right)\right)+ibcx^2\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\cdots}{\cdots}$$

input
```
Integrate[Sqrt[c + d*Sqrt[a + b*x^2]]/x^4,x]
```

output
```
-1/6*(Sqrt[-c - Sqrt[a]*d]*(c - Sqrt[a]*d)*(b*d*x^2*Sqrt[a + b*x^2] + 2*a*
(c + d*Sqrt[a + b*x^2])) + I*b*c*x^2*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))
/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt
[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c -
Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)]
- I*Sqrt[a]*b*d*x^2*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a +
b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c
+ d*Sqrt[a + b*x^2])^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c
 + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/(a*Sqrt[-c - Sqr
t[a]*d]*(c - Sqrt[a]*d)*x^3*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^4} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^4} dx$$

input
```
Int[Sqrt[c + d*Sqrt[a + b*x^2]]/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\sqrt{c+d\sqrt{b\,x^{2}+a}}}{x^{4}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^4,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx = \int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(c + d*sqrt(a + b*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx=\int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx=\int \frac{\sqrt{c+d\,\sqrt{b\,x^2+a}}}{x^4}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^4,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^4}\,dx=\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{x^4}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^4,x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c)/x**4,x)
```

3.254 $\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 478

$$\begin{aligned}
\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx
&= -\frac{\sqrt{c+d\sqrt{a+bx^2}}}{5x^5} + \frac{bd\left(ad-c\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{30a\left(c^2-ad^2\right)x^3} \\
&\quad - \frac{b^2d\sqrt{c+d\sqrt{a+bx^2}}\left(ad(c^2-5ad^2)-4c(c^2-2ad^2)\sqrt{a+bx^2}\right)}{60a^2\left(c^2-ad^2\right)^2x} \\
&\quad + \frac{b^2cd(c^2-2ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{15a^{3/2}\left(c^2-ad^2\right)^2x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} \\
&\quad - \frac{b^2d(4c^2-5ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{60a^{3/2}\left(c^2-ad^2\right)x\sqrt{c+d\sqrt{a+bx^2}}}
\end{aligned}$$

output
```
-1/5*(c+d*(b*x^2+a)^(1/2))^(1/2)/x^5+1/30*b*d*(a*d-c*(b*x^2+a)^(1/2))*(c+d
*(b*x^2+a)^(1/2))^(1/2)/a/(-a*d^2+c^2)/x^3-1/60*b^2*d*(c+d*(b*x^2+a)^(1/2)
)^(1/2)*(a*d*(-5*a*d^2+c^2)-4*c*(-2*a*d^2+c^2)*(b*x^2+a)^(1/2))/a^2/(-a*d^
2+c^2)^2/x+1/15*b^2*c*d*(-2*a*d^2+c^2)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/
2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*
(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(3/2)/(-a*d^2+c^2)^2/x/((c+d*(b*x^2+a)^
(1/2))/(c+a^(1/2)*d))^(1/2)-1/60*b^2*d*(-5*a*d^2+4*c^2)*(-b*x^2/a)^(1/2)*(
(c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2
)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(3/2)/
(-a*d^2+c^2)/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.09 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx$$

$$= \frac{-12a^2(c^2-ad^2)^2\left(c+d\sqrt{a+bx^2}\right)-2abd(-c^2+ad^2)\,x^2\left(ad-c\sqrt{a+bx^2}\right)\left(c+d\sqrt{a+bx^2}\right)+b^2dx^4\big(c}{}$$

input
```
Integrate[Sqrt[c + d*Sqrt[a + b*x^2]]/x^6,x]
```

output
```
(-12*a^2*(c^2 - a*d^2)^2*(c + d*Sqrt[a + b*x^2]) - 2*a*b*d*(-c^2 + a*d^2)*
x^2*(a*d - c*Sqrt[a + b*x^2])*(c + d*Sqrt[a + b*x^2]) + b^2*d*x^4*(c + d*S
qrt[a + b*x^2])*(5*a^2*d^3 + 4*c^3*Sqrt[a + b*x^2] - a*c*d*(c + 8*d*Sqrt[a
 + b*x^2])) - (b^2*x^4*(4*b*c*d^2*Sqrt[-c - Sqrt[a]*d]*(c^2 - 2*a*d^2)*x^2
 - (4*I)*c*(c^3 + Sqrt[a]*c^2*d - 2*a*c*d^2 - 2*a^(3/2)*d^3)*Sqrt[(d*(-Sqr
t[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[
a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*Ellipt
icE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt
[a]*d)/(c + Sqrt[a]*d)] + I*Sqrt[a]*d*(4*c^3 + Sqrt[a]*c^2*d - 8*a*c*d^2 -
 5*a^(3/2)*d^3)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^
2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*
Sqrt[a + b*x^2])^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d
*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)]))/Sqrt[-c - Sqrt[a]*d
])/(60*a^2*(c^2 - a*d^2)^2*x^5*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^6} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\sqrt{d\sqrt{a+bx^2}+c}}{x^6}dx$$

input
```
Int[Sqrt[c + d*Sqrt[a + b*x^2]]/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\sqrt{c+d\sqrt{b\,x^{2}+a}}}{x^{6}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^6,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c)/x^6, x)
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx = \int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(1/2)/x**6,x)
```

output
```
Integral(sqrt(c + d*sqrt(a + b*x**2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx = \int \frac{\sqrt{\sqrt{bx^2+a}d+c}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(b*x^2 + a)*d + c)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx = \int \frac{\sqrt{c+d\,\sqrt{b\,x^2+a}}}{x^6}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^6,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sqrt{a+bx^2}}}{x^6}\,dx = \int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{x^6}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(1/2)/x^6,x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c)/x**6,x)
```

3.255 $\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 236

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=-\frac{2c(c^2-ad^2)^2\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^3d^6}+\frac{2(5c^4-6ac^2d^2+a^2d^4)\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7b^3d^6}-\frac{4c(5c^2-3ad^2)\left(c+d\sqrt{a+bx^2}\right)^{9/2}}{9b^3d^6}+\frac{4(5c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{11/2}}{11b^3d^6}-\frac{10c\left(c+d\sqrt{a+bx^2}\right)^{13/2}}{13b^3d^6}+\frac{2\left(c+d\sqrt{a+bx^2}\right)^{15/2}}{15b^3d^6}$$

output
```
-2/5*c*(-a*d^2+c^2)^2*(c+d*(b*x^2+a)^(1/2))^(5/2)/b^3/d^6+2/7*(a^2*d^4-6*a
*c^2*d^2+5*c^4)*(c+d*(b*x^2+a)^(1/2))^(7/2)/b^3/d^6-4/9*c*(-3*a*d^2+5*c^2)
*(c+d*(b*x^2+a)^(1/2))^(9/2)/b^3/d^6+4/11*(-a*d^2+5*c^2)*(c+d*(b*x^2+a)^(1
/2))^(11/2)/b^3/d^6-10/13*c*(c+d*(b*x^2+a)^(1/2))^(13/2)/b^3/d^6+2/15*(c+d
*(b*x^2+a)^(1/2))^(15/2)/b^3/d^6
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.08

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}dx=\frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-256c^7+96c^5d^2(12a-bx^2)+128c^6d\sqrt{a+bx^2}+16c^4d^3\sqrt{a+b\right.}{}$$

input
```
Integrate[x^5*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-256*c^7 + 96*c^5*d^2*(12*a - b*x^2) + 128
*c^6*d*Sqrt[a + b*x^2] + 16*c^4*d^3*Sqrt[a + b*x^2]*(-34*a + 5*b*x^2) + 3*
c^2*d^5*Sqrt[a + b*x^2]*(320*a^2 - 88*a*b*x^2 + 21*b^2*x^4) - 2*c^3*d^4*(1
088*a^2 - 164*a*b*x^2 + 35*b^2*x^4) + 39*d^7*Sqrt[a + b*x^2]*(32*a^3 - 24*
a^2*b*x^2 + 21*a*b^2*x^4 + 77*b^3*x^6) + 24*c*d^6*(128*a^3 - 19*a^2*b*x^2
+ 7*a*b^2*x^4 + 154*b^3*x^6)))/(45045*b^3*d^6)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {7283, 896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(d\sqrt{a+bx^2}+c\right)^{3/2}dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2}\int x^4\left(c+d\sqrt{bx^2+a}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int b^2x^4\left(c+d\sqrt{bx^2+a}\right)^{3/2}d\left(bx^2+a\right)}{2b^3}$$

$$\Big\downarrow \ 1732$$

$$\frac{\int \sqrt{bx^2+a}\left(a-x^4\right)^2\left(c+d\sqrt{bx^2+a}\right)^{3/2}d\sqrt{bx^2+a}}{b^3}$$

$$\Big\downarrow \ 522$$

$$\frac{\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^{13/2}}{d^5}-\frac{5c\left(c+d\sqrt{bx^2+a}\right)^{11/2}}{d^5}-\frac{2\left(ad^2-5c^2\right)\left(c+d\sqrt{bx^2+a}\right)^{9/2}}{d^5}-\frac{2\left(5c^3-3acd^2\right)\left(c+d\sqrt{bx^2+a}\right)^{7/2}}{d^5}+\frac{\left(5c^4-6ad^2c^2+a\right.}{}\right.}{b^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2\left(a^2d^4-6ac^2d^2+5c^4\right)\left(d\sqrt{a+bx^2}+c\right)^{7/2}}{7d^6}+\frac{4\left(5c^2-ad^2\right)\left(d\sqrt{a+bx^2}+c\right)^{11/2}}{11d^6}-\frac{4c\left(5c^2-3ad^2\right)\left(d\sqrt{a+bx^2}+c\right)^{9/2}}{9d^6}-\frac{2c\left(c^2-ad^2\right)^2\left(d\sqrt{a+bx^2}+\right.}{5d^6}}{b^3}$$

input
```
Int[x^5*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((-2*c*(c^2 - a*d^2)^2*(c + d*Sqrt[a + b*x^2])^(5/2))/(5*d^6) + (2*(5*c^4
- 6*a*c^2*d^2 + a^2*d^4)*(c + d*Sqrt[a + b*x^2])^(7/2))/(7*d^6) - (4*c*(5*
c^2 - 3*a*d^2)*(c + d*Sqrt[a + b*x^2])^(9/2))/(9*d^6) + (4*(5*c^2 - a*d^2)
*(c + d*Sqrt[a + b*x^2])^(11/2))/(11*d^6) - (10*c*(c + d*Sqrt[a + b*x^2])^
(13/2))/(13*d^6) + (2*(c + d*Sqrt[a + b*x^2])^(15/2))/(15*d^6))/b^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^5\left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}dx$$

input
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.05

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}dx = \frac{2\left(3696\,b^3cd^6x^6+3072\,a^3cd^6-2176\,a^2c^3d^4+1152\,ac^5d^2-256\,c^7+14\left(12\,ab^2cd^6-\right.\right.}{}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/45045*(3696*b^3*c*d^6*x^6 + 3072*a^3*c*d^6 - 2176*a^2*c^3*d^4 + 1152*a*c
^5*d^2 - 256*c^7 + 14*(12*a*b^2*c*d^6 - 5*b^2*c^3*d^4)*x^4 - 8*(57*a^2*b*c
*d^6 - 41*a*b*c^3*d^4 + 12*b*c^5*d^2)*x^2 + (3003*b^3*d^7*x^6 + 1248*a^3*d
^7 + 960*a^2*c^2*d^5 - 544*a*c^4*d^3 + 128*c^6*d + 63*(13*a*b^2*d^7 + b^2*
c^2*d^5)*x^4 - 8*(117*a^2*b*d^7 + 33*a*b*c^2*d^5 - 10*b*c^4*d^3)*x^2)*sqrt
(b*x^2 + a))*sqrt(sqrt(b*x^2 + a)*d + c)/(b^3*d^6)
```

Sympy [F]

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\int x^5\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**5*(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**5*(c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.76

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(3003\left(\sqrt{bx^2+a}d+c\right)^{\frac{15}{2}}-17325\left(\sqrt{bx^2+a}d+c\right)^{\frac{13}{2}}c-8190\left(ad^2-5c^2\right)\left(\sqrt{bx^2\right.\right.}{}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2/45045*(3003*(sqrt(b*x^2 + a)*d + c)^(15/2) - 17325*(sqrt(b*x^2 + a)*d +
c)^(13/2)*c - 8190*(a*d^2 - 5*c^2)*(sqrt(b*x^2 + a)*d + c)^(11/2) + 10010*
(3*a*c*d^2 - 5*c^3)*(sqrt(b*x^2 + a)*d + c)^(9/2) + 6435*(a^2*d^4 - 6*a*c^
2*d^2 + 5*c^4)*(sqrt(b*x^2 + a)*d + c)^(7/2) - 9009*(a^2*c*d^4 - 2*a*c^3*d
^2 + c^5)*(sqrt(b*x^2 + a)*d + c)^(5/2))/(b^3*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2794 vs. $2(200) = 400$.

Time = 0.26 (sec) , antiderivative size = 2794, normalized size of antiderivative = 11.84

$$\int x^5 \left(c + d\sqrt{a + bx^2}\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
2/45045*(143*(315*sqrt(sqrt(b*x^2 + a)*d + c)*a^2*d^4 - 126*(sqrt(b*x^2 +
a)*d + c)^(5/2)*a*d^2 + 420*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*c*d^2 - 630*sq
rt(sqrt(b*x^2 + a)*d + c)*a*c^2*d^2 + 35*(sqrt(b*x^2 + a)*d + c)^(9/2) - 1
80*(sqrt(b*x^2 + a)*d + c)^(7/2)*c + 378*(sqrt(b*x^2 + a)*d + c)^(5/2)*c^2
 - 420*(sqrt(b*x^2 + a)*d + c)^(3/2)*c^3 + 315*sqrt(sqrt(b*x^2 + a)*d + c)
*c^4)*a*c/(b^2*d^3) + 13*(1155*(sqrt(b*x^2 + a)*d + c)^(3/2)*a^2*d^4*sgn((
sqrt(b*x^2 + a)*d + c)*d - c*d) - 3465*sqrt(sqrt(b*x^2 + a)*d + c)*a^2*c*d
^4*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 990*(sqrt(b*x^2 + a)*d + c)^(7/2
)*a*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 4158*(sqrt(b*x^2 + a)*d + c
)^(5/2)*a*c*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 6930*(sqrt(b*x^2 +
a)*d + c)^(3/2)*a*c^2*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 6930*sqrt
(sqrt(b*x^2 + a)*d + c)*a*c^3*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 3
15*(sqrt(b*x^2 + a)*d + c)^(11/2)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 1
925*(sqrt(b*x^2 + a)*d + c)^(9/2)*c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) +
 4950*(sqrt(b*x^2 + a)*d + c)^(7/2)*c^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*
d) - 6930*(sqrt(b*x^2 + a)*d + c)^(5/2)*c^3*sgn((sqrt(b*x^2 + a)*d + c)*d
- c*d) + 5775*(sqrt(b*x^2 + a)*d + c)^(3/2)*c^4*sgn((sqrt(b*x^2 + a)*d + c
)*d - c*d) - 3465*sqrt(sqrt(b*x^2 + a)*d + c)*c^5*sgn((sqrt(b*x^2 + a)*d +
 c)*d - c*d))*a/(b^2*d^3) + 13*(1155*(sqrt(b*x^2 + a)*d + c)^(3/2)*a^2*d^4
*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 3465*sqrt(sqrt(b*x^2 + a)*d + c...
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\int x^5\,\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}\,dx$$

input
```
int(x^5*(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^5*(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 770, normalized size of antiderivative = 3.26

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\Big(-256}{}$$

input
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - 1248*sqrt(b)*sqrt(a
 + b*x**2)*a**3*d**7*x + 936*sqrt(b)*sqrt(a + b*x**2)*a**2*b*d**7*x**3 - 9
60*sqrt(b)*sqrt(a + b*x**2)*a**2*c**2*d**5*x - 819*sqrt(b)*sqrt(a + b*x**2
)*a*b**2*d**7*x**5 + 264*sqrt(b)*sqrt(a + b*x**2)*a*b*c**2*d**5*x**3 + 544
*sqrt(b)*sqrt(a + b*x**2)*a*c**4*d**3*x - 3003*sqrt(b)*sqrt(a + b*x**2)*b*
*3*d**7*x**7 - 63*sqrt(b)*sqrt(a + b*x**2)*b**2*c**2*d**5*x**5 - 80*sqrt(b
)*sqrt(a + b*x**2)*b*c**4*d**3*x**3 - 128*sqrt(b)*sqrt(a + b*x**2)*c**6*d*
x + 3072*sqrt(a + b*x**2)*a**3*c*d**6 - 456*sqrt(a + b*x**2)*a**2*b*c*d**6
*x**2 - 2176*sqrt(a + b*x**2)*a**2*c**3*d**4 + 168*sqrt(a + b*x**2)*a*b**2
*c*d**6*x**4 + 328*sqrt(a + b*x**2)*a*b*c**3*d**4*x**2 + 1152*sqrt(a + b*x
**2)*a*c**5*d**2 + 3696*sqrt(a + b*x**2)*b**3*c*d**6*x**6 - 70*sqrt(a + b*
x**2)*b**2*c**3*d**4*x**4 - 96*sqrt(a + b*x**2)*b*c**5*d**2*x**2 - 256*sqr
t(a + b*x**2)*c**7 - 3072*sqrt(b)*a**3*c*d**6*x + 456*sqrt(b)*a**2*b*c*d**
6*x**3 + 2176*sqrt(b)*a**2*c**3*d**4*x - 168*sqrt(b)*a*b**2*c*d**6*x**5 -
328*sqrt(b)*a*b*c**3*d**4*x**3 - 1152*sqrt(b)*a*c**5*d**2*x - 3696*sqrt(b)
*b**3*c*d**6*x**7 + 70*sqrt(b)*b**2*c**3*d**4*x**5 + 96*sqrt(b)*b*c**5*d**
2*x**3 + 256*sqrt(b)*c**7*x + 1248*a**4*d**7 + 312*a**3*b*d**7*x**2 + 960*
a**3*c**2*d**5 - 117*a**2*b**2*d**7*x**4 + 696*a**2*b*c**2*d**5*x**2 - 544
*a**2*c**4*d**3 + 3822*a*b**3*d**7*x**6 - 201*a*b**2*c**2*d**5*x**4 - 4...
```

3.256 $\int x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=-\frac{2c(c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^2d^4}+\frac{2(3c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7b^2d^4}-\frac{2c\left(c+d\sqrt{a+bx^2}\right)^{9/2}}{3b^2d^4}+\frac{2\left(c+d\sqrt{a+bx^2}\right)^{11/2}}{11b^2d^4}$$

output
```
-2/5*c*(-a*d^2+c^2)*(c+d*(b*x^2+a)^(1/2))^(5/2)/b^2/d^4+2/7*(-a*d^2+3*c^2)
*(c+d*(b*x^2+a)^(1/2))^(7/2)/b^2/d^4-2/3*c*(c+d*(b*x^2+a)^(1/2))^(9/2)/b^2
/d^4+2/11*(c+d*(b*x^2+a)^(1/2))^(11/2)/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.15

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-16c^5+6c^3d^2(10a-bx^2)+8c^4d\sqrt{a+bx^2}+c^2d^3\sqrt{a+bx^2}(-28\right.}{1155}$$

input
```
Integrate[x^3*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-16*c^5 + 6*c^3*d^2*(10*a - b*x^2) + 8*c^4
*d*Sqrt[a + b*x^2] + c^2*d^3*Sqrt[a + b*x^2]*(-28*a + 5*b*x^2) - 4*c*d^4*(
31*a^2 - 4*a*b*x^2 - 35*b^2*x^4) + 15*d^5*Sqrt[a + b*x^2]*(-4*a^2 + 3*a*b*
x^2 + 7*b^2*x^4)))/(1155*b^2*d^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {7283, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d\sqrt{a+bx^2}+c\right)^{3/2} dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^2 \left(c+d\sqrt{bx^2+a}\right)^{3/2} dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{\int bx^2 \left(c+d\sqrt{bx^2+a}\right)^{3/2} d\left(bx^2+a\right)}{2b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -bx^2 \left(c+d\sqrt{bx^2+a}\right)^{3/2} d\left(bx^2+a\right)}{2b^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{\int \sqrt{bx^2+a}\left(a-x^4\right) \left(c+d\sqrt{bx^2+a}\right)^{3/2} d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(-\frac{\left(c+d\sqrt{bx^2+a}\right)^{9/2}}{d^3}+\frac{3c\left(c+d\sqrt{bx^2+a}\right)^{7/2}}{d^3}+\frac{\left(ad^2-3c^2\right)\left(c+d\sqrt{bx^2+a}\right)^{5/2}}{d^3}+\frac{\left(c^3-acd^2\right)\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d^3}\right)d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2(3c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{7/2}}{7d^4}+\frac{2c(c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^4}-\frac{2\left(d\sqrt{a+bx^2}+c\right)^{11/2}}{11d^4}+\frac{2c\left(d\sqrt{a+bx^2}+c\right)^{9/2}}{3d^4}}{b^2}$$

input
```
Int[x^3*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
-(((2*c*(c^2 - a*d^2)*(c + d*Sqrt[a + b*x^2])^(5/2))/(5*d^4) - (2*(3*c^2 -
 a*d^2)*(c + d*Sqrt[a + b*x^2])^(7/2))/(7*d^4) + (2*c*(c + d*Sqrt[a + b*x^
2])^(9/2))/(3*d^4) - (2*(c + d*Sqrt[a + b*x^2])^(11/2))/(11*d^4))/b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^3 \left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}} dx$$

input
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int x^3 \Big(c + d\sqrt{a+bx^2}\Big)^{3/2} dx = \frac{2\left(140\,b^2cd^4x^4 - 124\,a^2cd^4 + 60\,ac^3d^2 - 16\,c^5 + 2\,(8\,abcd^4 - 3\,bc^3d^2)x^2 + (105\,b^2d^5x\right.}{1155\,b}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(140*b^2*c*d^4*x^4 - 124*a^2*c*d^4 + 60*a*c^3*d^2 - 16*c^5 + 2*(8*a
*b*c*d^4 - 3*b*c^3*d^2)*x^2 + (105*b^2*d^5*x^4 - 60*a^2*d^5 - 28*a*c^2*d^3
 + 8*c^4*d + 5*(9*a*b*d^5 + b*c^2*d^3)*x^2)*sqrt(b*x^2 + a))*sqrt(sqrt(b*x
^2 + a)*d + c)/(b^2*d^4)
```

Sympy [F]

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\int x^3\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**3*(c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(105\left(\sqrt{bx^2+a}d+c\right)^{\frac{11}{2}}-385\left(\sqrt{bx^2+a}d+c\right)^{\frac{9}{2}}c-165\left(ad^2-3c^2\right)\left(\sqrt{bx^2+a}d-\right.}{1155\,b^2d^4}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2/1155*(105*(sqrt(b*x^2 + a)*d + c)^(11/2) - 385*(sqrt(b*x^2 + a)*d + c)^(
9/2)*c - 165*(a*d^2 - 3*c^2)*(sqrt(b*x^2 + a)*d + c)^(7/2) + 231*(a*c*d^2
- c^3)*(sqrt(b*x^2 + a)*d + c)^(5/2))/(b^2*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. 2(117) = 234.

Time = 0.20 (sec) , antiderivative size = 1488, normalized size of antiderivative = 10.55

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
-2/3465*(231*(15*sqrt(sqrt(b*x^2 + a)*d + c)*a - (3*(sqrt(b*x^2 + a)*d + c
)^(5/2) - 10*(sqrt(b*x^2 + a)*d + c)^(3/2)*c + 15*sqrt(sqrt(b*x^2 + a)*d +
 c)*c^2)/d^2)*a*c*d/b + 33*(35*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*d^2*sgn((sq
rt(b*x^2 + a)*d + c)*d - c*d) - 105*sqrt(sqrt(b*x^2 + a)*d + c)*a*c*d^2*sg
n((sqrt(b*x^2 + a)*d + c)*d - c*d) - 15*(sqrt(b*x^2 + a)*d + c)^(7/2)*sgn(
(sqrt(b*x^2 + a)*d + c)*d - c*d) + 63*(sqrt(b*x^2 + a)*d + c)^(5/2)*c*sgn(
(sqrt(b*x^2 + a)*d + c)*d - c*d) - 105*(sqrt(b*x^2 + a)*d + c)^(3/2)*c^2*s
gn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 105*sqrt(sqrt(b*x^2 + a)*d + c)*c^3*
sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))*a/(b*d) + 33*(35*(sqrt(b*x^2 + a)*d
+ c)^(3/2)*a*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 105*sqrt(sqrt(b*x^
2 + a)*d + c)*a*c*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 15*(sqrt(b*x^
2 + a)*d + c)^(7/2)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 63*(sqrt(b*x^2
+ a)*d + c)^(5/2)*c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 105*(sqrt(b*x^2
 + a)*d + c)^(3/2)*c^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 105*sqrt(sqr
t(b*x^2 + a)*d + c)*c^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))*c^2/(b*d^3)
- 11*(315*sqrt(sqrt(b*x^2 + a)*d + c)*a^2*d^4 - 126*(sqrt(b*x^2 + a)*d + c
)^(5/2)*a*d^2 + 420*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*c*d^2 - 630*sqrt(sqrt(
b*x^2 + a)*d + c)*a*c^2*d^2 + 35*(sqrt(b*x^2 + a)*d + c)^(9/2) - 180*(sqrt
(b*x^2 + a)*d + c)^(7/2)*c + 378*(sqrt(b*x^2 + a)*d + c)^(5/2)*c^2 - 420*(
sqrt(b*x^2 + a)*d + c)^(3/2)*c^3 + 315*sqrt(sqrt(b*x^2 + a)*d + c)*c^4)...
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\int x^3\left(c+d\sqrt{b\,x^2+a}\right)^{3/2}\,dx$$

input
```
int(x^3*(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^3*(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.27

$$\int x^3\Big(c+d\sqrt{a+bx^2}\Big)^{3/2}\,dx = \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\Big(-124}{}$$

input

```
int(x^3*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output

```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*(60*sqrt(b)*sqrt(a + b*
x**2)*a**2*d**5*x - 45*sqrt(b)*sqrt(a + b*x**2)*a*b*d**5*x**3 + 28*sqrt(b)
*sqrt(a + b*x**2)*a*c**2*d**3*x - 105*sqrt(b)*sqrt(a + b*x**2)*b**2*d**5*x
**5 - 5*sqrt(b)*sqrt(a + b*x**2)*b*c**2*d**3*x**3 - 8*sqrt(b)*sqrt(a + b*x
**2)*c**4*d*x - 124*sqrt(a + b*x**2)*a**2*c*d**4 + 16*sqrt(a + b*x**2)*a*b
*c*d**4*x**2 + 60*sqrt(a + b*x**2)*a*c**3*d**2 + 140*sqrt(a + b*x**2)*b**2
*c*d**4*x**4 - 6*sqrt(a + b*x**2)*b*c**3*d**2*x**2 - 16*sqrt(a + b*x**2)*c
**5 + 124*sqrt(b)*a**2*c*d**4*x - 16*sqrt(b)*a*b*c*d**4*x**3 - 60*sqrt(b)*
a*c**3*d**2*x - 140*sqrt(b)*b**2*c*d**4*x**5 + 6*sqrt(b)*b*c**3*d**2*x**3
+ 16*sqrt(b)*c**5*x - 60*a**3*d**5 - 15*a**2*b*d**5*x**2 - 28*a**2*c**2*d*
*3 + 150*a*b**2*d**5*x**4 - 23*a*b*c**2*d**3*x**2 + 8*a*c**4*d + 105*b**3*
d**5*x**6 + 5*b**2*c**2*d**3*x**4 + 8*b*c**4*d*x**2))/(1155*a*b**2*d**4)
```

3.257 $\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=-\frac{2c\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5bd^2}+\frac{2\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7bd^2}$$

output
```
-2/5*c*(c+d*(b*x^2+a)^(1/2))^(5/2)/b/d^2+2/7*(c+d*(b*x^2+a)^(1/2))^(7/2)/b
/d^2
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-2c^3+c^2d\sqrt{a+bx^2}+8cd^2(a+bx^2)+5d^3(a+bx^2)^{3/2}\right)}{35bd^2}$$

input
```
Integrate[x*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-2*c^3 + c^2*d*Sqrt[a + b*x^2] + 8*c*d^2*(
a + b*x^2) + 5*d^3*(a + b*x^2)^(3/2)))/(35*b*d^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d\sqrt{a+bx^2}+c\right)^{3/2}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \left(c+d\sqrt{bx^2+a}\right)^{3/2}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{774}$$

$$\frac{\int \sqrt{bx^2+a}\left(c+d\sqrt{bx^2+a}\right)^{3/2}d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^{5/2}}{d}-\frac{c\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d}\right)d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\left(d\sqrt{a+bx^2}+c\right)^{7/2}}{7d^2}-\frac{2c\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^2}}{b}$$

input
```
Int[x*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((-2*c*(c + d*Sqrt[a + b*x^2])^(5/2))/(5*d^2) + (2*(c + d*Sqrt[a + b*x^2])
^(7/2))/(7*d^2))/b
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{2\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{7}{2}}}{7}-\frac{2c\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{5}{2}}}{5}}{b\,d^{2}}$	45
default	$\frac{\frac{2\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{7}{2}}}{7}-\frac{2c\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{5}{2}}}{5}}{b\,d^{2}}$	45

input
```
int(x*(c+d*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^2*(1/7*(c+d*(b*x^2+a)^(1/2))^(7/2)-1/5*c*(c+d*(b*x^2+a)^(1/2))^(5/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28

$$\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(8\,bcd^2x^2+8\,acd^2-2\,c^3+\left(5\,bd^3x^2+5\,ad^3+c^2d\right)\sqrt{bx^2+a}\right)\sqrt{\sqrt{bx^2+a}d+c}}{35\,bd^2}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/35*(8*b*c*d^2*x^2 + 8*a*c*d^2 - 2*c^3 + (5*b*d^3*x^2 + 5*a*d^3 + c^2*d)*
sqrt(b*x^2 + a))*sqrt(sqrt(b*x^2 + a)*d + c)/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(51) = 102.

Time = 0.45 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.72

$$\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}\frac{c^{\frac{3}{2}}x^2}{2}\\ \frac{x^2\left(\sqrt{a}d+c\right)^{\frac{3}{2}}}{2}\\ \frac{c^{\frac{3}{2}}x^2}{2}\\ \frac{16ac\sqrt{c+d\sqrt{a+bx^2}}}{35b}+\frac{2ad\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{7b}+\frac{16cx^2\sqrt{c+d\sqrt{a+bx^2}}}{35}+\frac{2dx^2\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{7}-\end{cases}$$

input
```
integrate(x*(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Piecewise((c**(3/2)*x**2/2, Eq(b, 0) & Eq(d, 0)), (x**2*(sqrt(a)*d + c)**(
3/2)/2, Eq(b, 0)), (c**(3/2)*x**2/2, Eq(d, 0)), (16*a*c*sqrt(c + d*sqrt(a
+ b*x**2))/(35*b) + 2*a*d*sqrt(a + b*x**2)*sqrt(c + d*sqrt(a + b*x**2))/(7
*b) + 16*c*x**2*sqrt(c + d*sqrt(a + b*x**2))/35 + 2*d*x**2*sqrt(a + b*x**2
)*sqrt(c + d*sqrt(a + b*x**2))/7 - 4*c**3*sqrt(c + d*sqrt(a + b*x**2))/(35
*b*d**2) + 2*c**2*sqrt(a + b*x**2)*sqrt(c + d*sqrt(a + b*x**2))/(35*b*d),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(\frac{5\left(\sqrt{bx^2+a}d+c\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}c}{d^2}\right)}{35\,b}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2/35*(5*(sqrt(b*x^2 + a)*d + c)^(7/2)/d^2 - 7*(sqrt(b*x^2 + a)*d + c)^(5/2
)*c/d^2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(48) = 96.

Time = 0.14 (sec) , antiderivative size = 552, normalized size of antiderivative = 9.20

$$\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(105\sqrt{\sqrt{bx^2+a}d+c}acd-7\left(15\sqrt{\sqrt{bx^2+a}d+c}a-\frac{3\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}-10\left(\sqrt{bx^2+ad}\right.}{d^2}\right.\right.}{}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(sqrt(b*x^2 + a)*d + c)*a*c*d - 7*(15*sqrt(sqrt(b*x^2 + a)*
d + c)*a - (3*(sqrt(b*x^2 + a)*d + c)^(5/2) - 10*(sqrt(b*x^2 + a)*d + c)^(
3/2)*c + 15*sqrt(sqrt(b*x^2 + a)*d + c)*c^2)/d^2)*c*d + 35*((sqrt(b*x^2 +
a)*d + c)^(3/2)*d - 3*sqrt(sqrt(b*x^2 + a)*d + c)*c*d)*a*sgn((sqrt(b*x^2 +
 a)*d + c)*d - c*d) + 35*((sqrt(b*x^2 + a)*d + c)^(3/2)*d - 3*sqrt(sqrt(b*
x^2 + a)*d + c)*c*d)*c^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d)/d^2 + 7*(3*(
sqrt(b*x^2 + a)*d + c)^(5/2) - 10*(sqrt(b*x^2 + a)*d + c)^(3/2)*c + 15*sqr
t(sqrt(b*x^2 + a)*d + c)*c^2)*c/d - (35*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*d^
2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 105*sqrt(sqrt(b*x^2 + a)*d + c)*a
*c*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 15*(sqrt(b*x^2 + a)*d + c)^(
7/2)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 63*(sqrt(b*x^2 + a)*d + c)^(5/
2)*c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 105*(sqrt(b*x^2 + a)*d + c)^(3
/2)*c^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 105*sqrt(sqrt(b*x^2 + a)*d
+ c)*c^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))/d)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int x\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}\,\left(b\,x^2+a\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},2;\ 3;\ -\frac{d\,\sqrt{b\,x^2+a}}{c}\right)}{2\,b\,\left(\frac{d\,\sqrt{b\,x^2+a}}{c}+1\right)^{3/2}}$$

input
```
int(x*(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
((c + d*(a + b*x^2)^(1/2))^(3/2)*(a + b*x^2)*hypergeom([-3/2, 2], 3, -(d*(
a + b*x^2)^(1/2))/c))/(2*b*((d*(a + b*x^2)^(1/2))/c + 1)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.97

$$\int x\Big(c+d\sqrt{a+bx^2}\Big)^{3/2}\,dx=\frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\Big(-5\sqrt{b}}{}$$

input
```
int(x*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - 5*sqrt(b)*sqrt(a +
b*x**2)*a*d**3*x - 5*sqrt(b)*sqrt(a + b*x**2)*b*d**3*x**3 - sqrt(b)*sqrt(a
 + b*x**2)*c**2*d*x + 8*sqrt(a + b*x**2)*a*c*d**2 + 8*sqrt(a + b*x**2)*b*c
*d**2*x**2 - 2*sqrt(a + b*x**2)*c**3 - 8*sqrt(b)*a*c*d**2*x - 8*sqrt(b)*b*
c*d**2*x**3 + 2*sqrt(b)*c**3*x + 5*a**2*d**3 + 10*a*b*d**3*x**2 + a*c**2*d
 + 5*b**2*d**3*x**4 + b*c**2*d*x**2))/(35*a*b*d**2)
```

3.258 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = 2c\sqrt{c+d\sqrt{a+bx^2}}+\frac{2}{3}\left(c+d\sqrt{a+bx^2}\right)^{3/2} - \left(c-\sqrt{a}d\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)-\left(c+\sqrt{a}d\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)$$

output
```
2*c*(c+d*(b*x^2+a)^(1/2))^(1/2)+2/3*(c+d*(b*x^2+a)^(1/2))^(3/2)-(c-a^(1/2)
*d)^(3/2)*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c-a^(1/2)*d)^(1/2))-(c+a^(1
/2)*d)^(3/2)*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c+a^(1/2)*d)^(1/2))
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{2}{3}\sqrt{c+d\sqrt{a+bx^2}}\left(4c+d\sqrt{a+bx^2}\right) + \left(-c-\sqrt{a}d\right)^{3/2}\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right) + \left(-c+\sqrt{a}d\right)^{3/2}\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^(3/2)/x,x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(4*c + d*Sqrt[a + b*x^2]))/3 + (-c - Sqrt[a
]*d)^(3/2)*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]] + (-c
+ Sqrt[a]*d)^(3/2)*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]
]
```

Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {7282, 896, 25, 1732, 561, 25, 27, 1602, 27, 25, 1602, 25, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{2}\int \frac{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{2}\int \frac{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{bx^2}d\left(bx^2+a\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int -\frac{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{bx^2}d\left(bx^2+a\right)$$

$$\Big\downarrow \text{1732}$$

$$-\int \frac{\sqrt{bx^2+a}\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{a-x^4}d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{561}$$

$$-\frac{2\int -\frac{x^8\left(c-x^4\right)}{d\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{x^8\left(c-x^4\right)}{d\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^8\left(c-x^4\right)}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}}{d^2}$$

$$\Big\downarrow \text{1602}$$

$$\frac{2\left(\frac{1}{3}d^2\int -\frac{3x^4\left(cx^4+\left(a-\frac{c^2}{d^2}\right)d^2\right)}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}+\frac{d^2x^6}{3}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{d^2x^6}{3}-\int -\frac{x^4\left(-cx^4+c^2-ad^2\right)}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}\right)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\int \frac{x^4\left(-cx^4+c^2-ad^2\right)}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}+\frac{d^2x^6}{3}\right)}{d^2}$$

$$\Big\downarrow \text{1602}$$

$$\frac{2\left(d^2\int -\frac{(c^2+ad^2)x^4+c\left(a-\frac{c^2}{d^2}\right)d^2}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}+cd^2\sqrt{d\sqrt{a+bx^2}+c}+\frac{d^2x^6}{3}\right)}{d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-d^2\int -\frac{c(c^2-ad^2)-(c^2+ad^2)x^4}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}+cd^2\sqrt{d\sqrt{a+bx^2}+c}+\frac{d^2x^6}{3}\right)}{d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(d^2\int \frac{c(c^2-ad^2)-(c^2+ad^2)x^4}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}+cd^2\sqrt{d\sqrt{a+bx^2}+c}+\frac{d^2x^6}{3}\right)}{d^2}$$

$$\Big\downarrow \ 27$$

$$\frac{2\left(\int \frac{c(c^2-ad^2)-(c^2+ad^2)x^4}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}+cd^2\sqrt{d\sqrt{a+bx^2}+c}+\frac{d^2x^6}{3}\right)}{d^2}$$

$$\Big\downarrow \ 1480$$

$$\frac{2\left(-\frac{1}{2}\left(c-\sqrt{a}d\right)^2\int \frac{1}{\frac{c-\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}-\frac{1}{2}\left(\sqrt{a}d+c\right)^2\int \frac{1}{\frac{c+\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}+cd^2\sqrt{d\sqrt{a+bx}}\right.}{d^2}$$

$$\Big\downarrow \ 221$$

$$\frac{2\left(-\frac{1}{2}d^2\left(c-\sqrt{a}d\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{c-\sqrt{a}d}}\right)-\frac{1}{2}d^2\left(\sqrt{a}d+c\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{\sqrt{a}d+c}}\right)+cd^2\sqrt{d\sqrt{a+bx^2}+c}\right.}{d^2}$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(3/2)/x,x]
```

output
```
(2*((d^2*x^6)/3 + c*d^2*Sqrt[c + d*Sqrt[a + b*x^2]] - (d^2*(c - Sqrt[a]*d)
^(3/2)*ArcTanh[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[c - Sqrt[a]*d]])/2 - (d^2*
(c + Sqrt[a]*d)^(3/2)*ArcTanh[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[c + Sqrt[a]
*d]])/2))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx=\text{Timed out}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(3/2)/x,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(112) = 224.

Time = 0.16 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.88

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{2\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}d\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-6\sqrt{\sqrt{bx^2+a}d+c}\,cd\mathrm{sgn}}{}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

output
```
1/3*(2*(sqrt(b*x^2 + a)*d + c)^(3/2)*d*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d
) - 6*sqrt(sqrt(b*x^2 + a)*d + c)*c*d*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d)
 + 12*sqrt(sqrt(b*x^2 + a)*d + c)*c*d - 3*(2*a*c*d^3*abs(d) + 2*sqrt(a)*c^
2*d^3 - (a^(3/2)*d^3 + sqrt(a)*c^2*d)*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d -
c*d) - (a*c*d^3 + c^3*d)*abs(d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))*arct
an(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-c + sqrt(a*d^2)))/((sqrt(a)*d + c)*sq
rt(sqrt(a)*d - c)*abs(d)) + 3*(2*a*c*d^3*abs(d) - 2*sqrt(a)*c^2*d^3 + (a^(
3/2)*d^3 + sqrt(a)*c^2*d)*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - (a*c*
d^3 + c^3*d)*abs(d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))*arctan(sqrt(sqrt
(b*x^2 + a)*d + c)/sqrt(-c - sqrt(a*d^2)))/((sqrt(a)*d - c)*sqrt(-sqrt(a)*
d - c)*abs(d)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx=\int \frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}{x}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 924, normalized size of antiderivative = 6.33

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx=\text{Too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

output
```
( - 3*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
 c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt
(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a
+ b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b
*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*
a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2
+ 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 +
2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a*d + 3*sq
rt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a +
 b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a
+ b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2
)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)
*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c*
*2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*c*d**2 - sqrt(
sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 + 2*sqrt(sqrt(a)
*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*a*b*d**4*x**2
- 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a*c + 16*sqrt(a + b*x**2)*
sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x + a*d
 + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*c - 16*sqrt(b)*sqrt(sqr...
```

3.259 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{2x^2} + \frac{3bd\sqrt{c-\sqrt{a}d}\,\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{4\sqrt{a}} - \frac{3bd\sqrt{c+\sqrt{a}d}\,\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{4\sqrt{a}}$$

output
```
-1/2*(c+d*(b*x^2+a)^(1/2))^(3/2)/x^2+3/4*b*d*(c-a^(1/2)*d)^(1/2)*arctanh((
c+d*(b*x^2+a)^(1/2))^(1/2)/(c-a^(1/2)*d)^(1/2))/a^(1/2)-3/4*b*d*(c+a^(1/2)
*d)^(1/2)*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c+a^(1/2)*d)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{2\sqrt{a}\left(c+d\sqrt{a+bx^2}\right)^{3/2} + 3bd\sqrt{-c-\sqrt{a}d}\,x^2\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right) - 3bd\sqrt{-c+\sqrt{a}d}\,x^2\arctan\left(\frac{\sqrt{c+d\sqrt{a}}}{\sqrt{-c+}}\right.}{4\sqrt{a}x^2}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

output
```
-1/4*(2*Sqrt[a]*(c + d*Sqrt[a + b*x^2])^(3/2) + 3*b*d*Sqrt[-c - Sqrt[a]*d]
*x^2*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]] - 3*b*d*Sqrt
[-c + Sqrt[a]*d]*x^2*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*
d]])/(Sqrt[a]*x^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^3}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^3}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(3/2)/x**3,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(107) = 214$.

Time = 0.29 (sec) , antiderivative size = 1265, normalized size of antiderivative = 8.72

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*b*((2*a^(7/2)*c^2*d^9 - 8*a^(5/2)*c^4*d^7 + 10*a^(3/2)*c^6*d^5 - 4*sq
rt(a)*c^8*d^3 - 2*(a*d^3 - c^2*d)^2*a^(3/2)*c^2*d^3 - (3*a^(5/2)*d^5 - 5*a
^(3/2)*c^2*d^3)*(a*d^3 - c^2*d)^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 2
*(a^3*c*d^7 - 4*a^2*c^3*d^5 + 3*a*c^5*d^3)*abs(a*d^3 - c^2*d)*sgn((sqrt(b*
x^2 + a)*d + c)*d - c*d) - 2*(a^3*c*d^7 - 4*a^2*c^3*d^5 + 3*a*c^5*d^3)*abs
(a*d^3 - c^2*d) + (a^(7/2)*c^2*d^9 - a^(5/2)*c^4*d^7 - a^(3/2)*c^6*d^5 + s
qrt(a)*c^8*d^3)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))*arctan(sqrt(sqrt(b*x
^2 + a)*d + c)/sqrt(-(a*c*d^2 - c^3 + sqrt((a*c*d^2 - c^3)^2 + (a^2*d^4 -
2*a*c^2*d^2 + c^4)*(a*d^2 - c^2)))/(a*d^2 - c^2)))/((a^(7/2)*d^5 - a^3*c*d
^4 - 2*a^(5/2)*c^2*d^3 + 2*a^2*c^3*d^2 + a^(3/2)*c^4*d - a*c^5)*sqrt(-sqrt
(a)*d - c)*abs(a*d^3 - c^2*d)) + (2*a^(7/2)*c^2*d^9 - 8*a^(5/2)*c^4*d^7 +
10*a^(3/2)*c^6*d^5 - 4*sqrt(a)*c^8*d^3 - 2*(a*d^3 - c^2*d)^2*a^(3/2)*c^2*d
^3 - (3*a^(5/2)*d^5 - 5*a^(3/2)*c^2*d^3)*(a*d^3 - c^2*d)^2*sgn((sqrt(b*x^2
 + a)*d + c)*d - c*d) - 2*(a^3*c*d^7 - 4*a^2*c^3*d^5 + 3*a*c^5*d^3)*abs(a*
d^3 - c^2*d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 2*(a^3*c*d^7 - 4*a^2*c
^3*d^5 + 3*a*c^5*d^3)*abs(a*d^3 - c^2*d) + (a^(7/2)*c^2*d^9 - a^(5/2)*c^4*
d^7 - a^(3/2)*c^6*d^5 + sqrt(a)*c^8*d^3)*sgn((sqrt(b*x^2 + a)*d + c)*d - c
*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-(a*c*d^2 - c^3 - sqrt((a*c*d
^2 - c^3)^2 + (a^2*d^4 - 2*a*c^2*d^2 + c^4)*(a*d^2 - c^2)))/(a*d^2 - c^2))
)/((a^(7/2)*d^5 + a^3*c*d^4 - 2*a^(5/2)*c^2*d^3 - 2*a^2*c^3*d^2 + a^(3/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^3,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\left(c + d\sqrt{a + bx^2}\right)^{3/2}}{x^3}\,dx = \text{too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

output
```
( - 45*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
 c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d
- c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqr
t(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a
 + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a +
b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)
*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2
 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 +
 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**3*b*d*
*7*x**2 + 93*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(
a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt
(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d -
c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(
sqrt(a + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqr
t(a + b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*
d + c)*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**
2*x**2 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*
d**4 + 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**
2*b*c**2*d**5*x**2 - 60*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)
*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x...
```

3.260 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 209

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx = -\frac{3bd\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{16ax^2} - \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{4x^4} - \frac{3b^2\left(\frac{2c}{\sqrt{a}}-d\right)d\,\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{32a\sqrt{c-\sqrt{a}d}} + \frac{3b^2d\left(\frac{2c}{\sqrt{a}}+d\right)\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{32a\sqrt{c+\sqrt{a}d}}$$

output
```
-3/16*b*d*(b*x^2+a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)/a/x^2-1/4*(c+d*(b*x^
2+a)^(1/2))^(3/2)/x^4-3/32*b^2*(2*c/a^(1/2)-d)*d*arctanh((c+d*(b*x^2+a)^(1
/2))^(1/2)/(c-a^(1/2)*d)^(1/2))/a/(c-a^(1/2)*d)^(1/2)+3/32*b^2*d*(2*c/a^(1
/2)+d)*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c+a^(1/2)*d)^(1/2))/a/(c+a^(1/
2)*d)^(1/2)
```

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx = \frac{-\frac{2\sqrt{a}\sqrt{c+d\sqrt{a+bx^2}}\left(3bdx^2\sqrt{a+bx^2}+4a\left(c+d\sqrt{a+bx^2}\right)\right)}{x^4}-\frac{3b^2d(2c+\sqrt{a}d)\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right)}{\sqrt{-c-\sqrt{a}d}}+}{32a^{3/2}}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^(3/2)/x^5,x]
```

output
```
((-2*Sqrt[a]*Sqrt[c + d*Sqrt[a + b*x^2]]*(3*b*d*x^2*Sqrt[a + b*x^2] + 4*a*
(c + d*Sqrt[a + b*x^2])))/x^4 - (3*b^2*d*(2*c + Sqrt[a]*d)*ArcTan[Sqrt[c +
 d*Sqrt[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]])/Sqrt[-c - Sqrt[a]*d] + (3*b^2*d
*(2*c - Sqrt[a]*d)*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]
])/Sqrt[-c + Sqrt[a]*d])/(32*a^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^5}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(3/2)/x^5,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{5}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^5,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^5,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx=\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(3/2)/x**5,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2664 vs. $2(161) = 322$.

Time = 0.35 (sec) , antiderivative size = 2664, normalized size of antiderivative = 12.75

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

output
```
-1/32*(((3*a^2*d^7 - 14*a*c^2*d^5 - c^4*d^3)*(a^3*d^5 - 2*a^2*c^2*d^3 + a*
c^4*d)^2*b^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 4*(a^3*d^5 - 2*a^2*c^2
*d^3 + a*c^4*d)^2*(2*a*c^2*d^5 + c^4*d^3)*b^3 - (3*a^(9/2)*c*d^11 - 34*a^(
7/2)*c^3*d^9 + 60*a^(5/2)*c^5*d^7 - 30*a^(3/2)*c^7*d^5 + sqrt(a)*c^9*d^3)*
b^3*abs(a^3*d^5 - 2*a^2*c^2*d^3 + a*c^4*d)*sgn((sqrt(b*x^2 + a)*d + c)*d -
 c*d) + 2*(3*a^(9/2)*c*d^11 - 23*a^(7/2)*c^3*d^9 + 39*a^(5/2)*c^5*d^7 - 21
*a^(3/2)*c^7*d^5 + 2*sqrt(a)*c^9*d^3)*b^3*abs(a^3*d^5 - 2*a^2*c^2*d^3 + a*
c^4*d) - 2*(7*a^6*c^4*d^13 - 29*a^5*c^6*d^11 + 46*a^4*c^8*d^9 - 34*a^3*c^1
0*d^7 + 11*a^2*c^12*d^5 - a*c^14*d^3)*b^3*sgn((sqrt(b*x^2 + a)*d + c)*d -
c*d) - 2*(3*a^7*c^2*d^15 - 25*a^6*c^4*d^13 + 74*a^5*c^6*d^11 - 106*a^4*c^8
*d^9 + 79*a^3*c^10*d^7 - 29*a^2*c^12*d^5 + 4*a*c^14*d^3)*b^3)*arctan(sqrt(
sqrt(b*x^2 + a)*d + c)/sqrt(-(a^3*c*d^4 - 2*a^2*c^3*d^2 + a*c^5 + sqrt((a^
3*c*d^4 - 2*a^2*c^3*d^2 + a*c^5)^2 + (a^4*d^6 - 3*a^3*c^2*d^4 + 3*a^2*c^4*
d^2 - a*c^6)*(a^3*d^4 - 2*a^2*c^2*d^2 + a*c^4)))/(a^3*d^4 - 2*a^2*c^2*d^2
+ a*c^4)))/((a^6*d^9 - a^(11/2)*c*d^8 - 4*a^5*c^2*d^7 + 4*a^(9/2)*c^3*d^6
+ 6*a^4*c^4*d^5 - 6*a^(7/2)*c^5*d^4 - 4*a^3*c^6*d^3 + 4*a^(5/2)*c^7*d^2 +
a^2*c^8*d - a^(3/2)*c^9)*sqrt(-sqrt(a)*d - c)*abs(a^3*d^5 - 2*a^2*c^2*d^3
+ a*c^4*d)) + ((3*a^2*d^7 - 14*a*c^2*d^5 - c^4*d^3)*(a^3*d^5 - 2*a^2*c^2*d
^3 + a*c^4*d)^2*b^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 4*(a^3*d^5 - 2*
a^2*c^2*d^3 + a*c^4*d)^2*(2*a*c^2*d^5 + c^4*d^3)*b^3 + (3*a^(9/2)*c*d^1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^5}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^5,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^5}\,dx = \text{too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^5,x)
```

output
```
(7605*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
 c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt
(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a
+ b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b
*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*
a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2
+ 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 +
2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**6*b**2*
d**14*x**4 - 37947*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt
(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqr
t(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a
)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)
*sqrt(sqrt(a + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sq
rt(sqrt(a + b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*
x**2)*d + c)*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b
*c*d**2*x**2 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2
*a**2*d**4 + 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4
))*a**5*b**2*c**2*d**12*x**4 + 71742*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqr
t(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 -...
```

3.261 $\int x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}dx$

Optimal result

Integrand size = 23, antiderivative size = 455

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}dx = -\frac{8c(c^2-3ad^2)\,x\sqrt{c+d\sqrt{a+bx^2}}}{315bd^2}+\frac{2}{21}cx^3\sqrt{c+d\sqrt{a+bx^2}}$$
$$+\frac{2(c^2+7ad^2)\,x\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{105bd}+\frac{2}{9}x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}$$
$$-\frac{4\sqrt{a}(4c^4-15ac^2d^2-21a^2d^4)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{315b^2d^3x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}}$$
$$+\frac{16\sqrt{a}c(c^4-4ac^2d^2+3a^2d^4)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{315b^2d^3x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-8/315*c*(-3*a*d^2+c^2)*x*(c+d*(b*x^2+a)^(1/2))^(1/2)/b/d^2+2/21*c*x^3*(c+
d*(b*x^2+a)^(1/2))^(1/2)+2/105*(7*a*d^2+c^2)*x*(b*x^2+a)^(1/2)*(c+d*(b*x^2
+a)^(1/2))^(1/2)/b/d+2/9*x^3*(c+d*(b*x^2+a)^(1/2))^(3/2)-4/315*a^(1/2)*(-2
1*a^2*d^4-15*a*c^2*d^2+4*c^4)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)
*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
d/(c+a^(1/2)*d))^(1/2))/b^2/d^3/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1
/2)+16/315*a^(1/2)*c*(3*a^2*d^4-4*a*c^2*d^2+c^4)*(-b*x^2/a)^(1/2)*((c+d*(b
*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/
2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b^2/d^3/x/(c+d*
(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 37.11 (sec) , antiderivative size = 1169, normalized size of antiderivative = 2.57

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \text{Too large to display}$$

input
```
Integrate[x^2*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((-4*b*d^2*(8*c^4 - 9*a*c^2*d^2 + 21*a^2*d^4)*x^2)/Sqrt[c + d*Sqrt[a + b*x
^2]] + 2*b*d^2*x^2*Sqrt[c + d*Sqrt[a + b*x^2]]*(8*c^3 - 6*c^2*d*Sqrt[a + b
*x^2] + c*d^2*(-3*a + 5*b*x^2) + 7*d^3*Sqrt[a + b*x^2]*(3*a + 5*b*x^2)) +
((4*I)*(8*c^5 + 8*Sqrt[a]*c^4*d - 9*a*c^3*d^2 - 9*a^(3/2)*c^2*d^3 + 21*a^2
*c*d^4 + 21*a^(5/2)*d^5)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt
[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])
]*(c + d*Sqrt[a + b*x^2])*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c
+ d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/Sqrt[-c - Sqrt[a]
*d] - ((4*I)*Sqrt[a]*d*(8*c^4 + 2*Sqrt[a]*c^3*d - 9*a*c^2*d^2 + 18*a^(3/2)
*c*d^3 + 21*a^2*d^4)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a +
 b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c
 + d*Sqrt[a + b*x^2])*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*
Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/Sqrt[-c - Sqrt[a]*d]
+ (6*c*(8*b*c*d^2*(c^2 - 2*a*d^2)*x^2 + b*d^2*x^2*(c + d*Sqrt[a + b*x^2])*
(-4*c^2 + 3*c*d*Sqrt[a + b*x^2] + 5*d^2*(a + 3*b*x^2)) - ((8*I)*c*(c^3 + S
qrt[a]*c^2*d - 2*a*c*d^2 - 2*a^(3/2)*d^3)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x
^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d
*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*ArcSinh[Sqrt[
-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]
*d)])/Sqrt[-c - Sqrt[a]*d] + ((2*I)*Sqrt[a]*d*(4*c^3 + Sqrt[a]*c^2*d - ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d\sqrt{a+bx^2}+c\right)^{3/2}dx$$
$$\Big\downarrow \text{7299}$$
$$\int x^2\left(d\sqrt{a+bx^2}+c\right)^{3/2}dx$$

input
```
Int[x^2*(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^{2} \left(c +d \sqrt{b \,x^{2}+a}\right)^{\frac{3}{2}}d x$$

input
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int x^2 \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d*x^2 + c*x^2)*sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Sympy [F]

$$\int x^2 \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \int x^{2} \left(c+d\sqrt{a+bx^{2}}\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**2*(c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\int\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\int\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}\,dx=\int x^2\left(c+d\sqrt{b\,x^2+a}\right)^{3/2}\,dx$$

input
```
int(x^2*(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^2*(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int x^2 \left(c + d\sqrt{a + bx^2}\right)^{3/2} dx = \text{too large to display}$$

input
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
(2*(3*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a**2*d**3*x + 7*sqrt(a
 + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*b*d**3*x**3 - 3*sqrt(a + b*x**2)
*sqrt(sqrt(a + b*x**2)*d + c)*a*c**2*d*x - 4*sqrt(a + b*x**2)*sqrt(sqrt(a
+ b*x**2)*d + c)*b*c**2*d*x**3 + 245*int((sqrt(sqrt(a + b*x**2)*d + c)*x**
6)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*
x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4
*x**2),x)*a**2*b**3*c*d**6 - 280*int((sqrt(sqrt(a + b*x**2)*d + c)*x**6)/(
7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4
 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**
2),x)*a*b**3*c**3*d**4 + 80*int((sqrt(sqrt(a + b*x**2)*d + c)*x**6)/(7*a**
3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15
*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)
*b**3*c**5*d**2 + 476*int((sqrt(sqrt(a + b*x**2)*d + c)*x**4)/(7*a**3*d**4
 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c
**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a**3*
b**2*c*d**6 - 769*int((sqrt(sqrt(a + b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 1
4*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*
d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a**2*b**2
*c**3*d**4 + 410*int((sqrt(sqrt(a + b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 14
*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**...
```

3.262 $\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 335

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2}{5}cx\sqrt{c+d\sqrt{a+bx^2}}+\frac{2}{5}x\left(c+d\sqrt{a+bx^2}\right)^{3/2} - \frac{2\sqrt{a}(c^2+3ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{5bdx\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} + \frac{2\sqrt{a}c(c^2-ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{5bdx\sqrt{c+d\sqrt{a+bx^2}}}$$

output

```
2/5*c*x*(c+d*(b*x^2+a)^(1/2))^(1/2)+2/5*x*(c+d*(b*x^2+a)^(1/2))^(3/2)-2/5*
a^(1/2)*(3*a*d^2+c^2)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*Ellipti
cE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(
1/2)*d))^(1/2))/b/d/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)+2/5*a^(1
/2)*c*(-a*d^2+c^2)*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^
(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(
1/2)*d/(c+a^(1/2)*d))^(1/2))/b/d/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.16 (sec) , antiderivative size = 950, normalized size of antiderivative = 2.84

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \frac{-2bd^2\sqrt{-c-\sqrt{a}d}(2c^2-9ad^2)\,x^2+2bd^2\sqrt{-c-\sqrt{a}d}x^2\left(c+d\sqrt{a+bx^2}\right)\left(c+3d\sqrt{a+bx^2}\right)\cdots}{\cdots}$$

input

```
Integrate[(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output

```
(-2*b*d^2*Sqrt[-c - Sqrt[a]*d]*(2*c^2 - 9*a*d^2)*x^2 + 2*b*d^2*Sqrt[-c - S
qrt[a]*d]*x^2*(c + d*Sqrt[a + b*x^2])*(c + 3*d*Sqrt[a + b*x^2]) + (2*I)*(2
*c^3 + 2*Sqrt[a]*c^2*d - 9*a*c*d^2 - 9*a^(3/2)*d^3)*Sqrt[(d*(-Sqrt[a] + Sq
rt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2
]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c
 + Sqrt[a]*d)] + (2*I)*Sqrt[a]*d*(-2*c^2 + 7*Sqrt[a]*c*d + 9*a*d^2)*Sqrt[(
d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a]
+ Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)
*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c
 - Sqrt[a]*d)/(c + Sqrt[a]*d)] + 5*c*(2*b*d^2*Sqrt[-c - Sqrt[a]*d]*x^2*(2*
c + d*Sqrt[a + b*x^2]) - (2*I)*c*(c + Sqrt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[
a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))
/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*ArcSin
h[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c +
Sqrt[a]*d)] + (2*I)*Sqrt[a]*d*(c + Sqrt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a +
 b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c
 + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticF[I*ArcSinh[S
qrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqr
t[a]*d)]))/(15*b*d^2*Sqrt[-c - Sqrt[a]*d]*x*Sqrt[c + d*Sqrt[a + b*x^2]]...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d\sqrt{a+bx^2}+c\right)^{3/2} dx$$

$$\Big\downarrow \; 7299$$

$$\int \left(d\sqrt{a+bx^2}+c\right)^{3/2} dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}} dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^(3/2), x)
```

Sympy [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}} dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2), x)
```

Giac [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}} dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(c+d\sqrt{b\,x^2+a}\right)^{3/2} dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^{3/2} dx = \text{too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output

```
(4*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3*x - 2*sqrt(a + b*x
**2)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d*x + 49*int(sqrt(sqrt(a + b*x**2)*
d + c)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d
**4*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*
c**4*x**2),x)*a**4*c*d**6 - 105*int(sqrt(sqrt(a + b*x**2)*d + c)/(7*a**3*d
**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*
b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a*
*3*c**3*d**4 + 72*int(sqrt(sqrt(a + b*x**2)*d + c)/(7*a**3*d**4 + 14*a**2*
b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2*x*
*2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a**2*c**5*d**2 -
 16*int(sqrt(sqrt(a + b*x**2)*d + c)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 -
11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4
- 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a*c**7 + 63*int((sqrt(sqrt(a +
 b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d*
*2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d
**2*x**4 + 4*b*c**4*x**2),x)*a**2*b**2*c*d**6 - 71*int((sqrt(sqrt(a + b*x*
*2)*d + c)*x**4)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 +
7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x
**4 + 4*b*c**4*x**2),x)*a*b**2*c**3*d**4 + 20*int((sqrt(sqrt(a + b*x**2)*d
 + c)*x**4)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*...
```

3.263 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2} dx = -\frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x} - \frac{3\sqrt{a}d\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}}$$

output
```
-(c+d*(b*x^2+a)^(1/2))^(3/2)/x-3*a^(1/2)*d*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)
^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1
/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d
))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.30 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.97

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \frac{-\sqrt{-c-\sqrt{a}d}\left(c^2+d^2(a-2bx^2)+2cd\sqrt{a+bx^2}\right)-3i\left(c+\sqrt{a}d\right)\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\cdots}{\cdots}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

output
```
(-(Sqrt[-c - Sqrt[a]*d]*(c^2 + d^2*(a - 2*b*x^2) + 2*c*d*Sqrt[a + b*x^2]))
 - (3*I)*(c + Sqrt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt
[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])
]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/S
qrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + (3*I)*(c +
 Sqrt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]
*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt
[a + b*x^2])^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqr
t[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/(Sqrt[-c - Sqrt[a]*d]*x*
Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^2}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^(3/2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(3/2)/x**2,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^2}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^2,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \frac{-4\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}\,a\,d^2+2\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}\,c^2+\left(\int \frac{}{b}\right.}{}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

output
```
( - 4*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**2 + 2*sqrt(a + b*
x**2)*sqrt(sqrt(a + b*x**2)*d + c)*c**2 + int(sqrt(sqrt(a + b*x**2)*d + c)
/(a**2*d**2*x**2 + 2*a*b*d**2*x**4 - a*c**2*x**2 + b**2*d**2*x**6 - b*c**2
*x**4),x)*a**3*c*d**3*x - int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**2*x**2
 + 2*a*b*d**2*x**4 - a*c**2*x**2 + b**2*d**2*x**6 - b*c**2*x**4),x)*a**2*c
**3*d*x - int((sqrt(sqrt(a + b*x**2)*d + c)*x**2)/(a**2*d**2 + 2*a*b*d**2*
x**2 - a*c**2 + b**2*d**2*x**4 - b*c**2*x**2),x)*a*b**2*c*d**3*x + int((sq
rt(sqrt(a + b*x**2)*d + c)*x**2)/(a**2*d**2 + 2*a*b*d**2*x**2 - a*c**2 + b
**2*d**2*x**4 - b*c**2*x**2),x)*b**2*c**3*d*x + 3*int((sqrt(a + b*x**2)*sq
rt(sqrt(a + b*x**2)*d + c)*x**2)/(a**2*d**2 + 2*a*b*d**2*x**2 - a*c**2 + b
**2*d**2*x**4 - b*c**2*x**2),x)*a*b**2*d**4*x - int((sqrt(a + b*x**2)*sqrt
(sqrt(a + b*x**2)*d + c)*x**2)/(a**2*d**2 + 2*a*b*d**2*x**2 - a*c**2 + b**
2*d**2*x**4 - b*c**2*x**2),x)*b**2*c**2*d**2*x - 3*int((sqrt(a + b*x**2)*s
qrt(sqrt(a + b*x**2)*d + c))/(a**2*d**2*x**2 + 2*a*b*d**2*x**4 - a*c**2*x*
*2 + b**2*d**2*x**6 - b*c**2*x**4),x)*a**3*d**4*x + 5*int((sqrt(a + b*x**2
)*sqrt(sqrt(a + b*x**2)*d + c))/(a**2*d**2*x**2 + 2*a*b*d**2*x**4 - a*c**2
*x**2 + b**2*d**2*x**6 - b*c**2*x**4),x)*a**2*c**2*d**2*x - 2*int((sqrt(a
+ b*x**2)*sqrt(sqrt(a + b*x**2)*d + c))/(a**2*d**2*x**2 + 2*a*b*d**2*x**4
- a*c**2*x**2 + b**2*d**2*x**6 - b*c**2*x**4),x)*a*c**4*x)/(a*d*x)
```

3.264 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 326

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = -\frac{bd\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{2ax} - \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3x^3} - \frac{bd\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{2\sqrt{a}x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} + \frac{bcd\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{2\sqrt{a}x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-1/2*b*d*(b*x^2+a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)/a/x-1/3*(c+d*(b*x^2+a
)^(1/2))^(3/2)/x^3-1/2*b*d*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*El
lipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(
c+a^(1/2)*d))^(1/2))/a^(1/2)/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)
+1/2*b*c*d*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*El
lipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(
c+a^(1/2)*d))^(1/2))/a^(1/2)/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.35 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.49

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \frac{-\sqrt{-c-\sqrt{a}d}\left(2a^2d^2+3bcdx^2\sqrt{a+bx^2}+a\left(2c^2+5bd^2x^2+4cd\sqrt{a+bx^2}\right)\right)}{}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

output
```
(-(Sqrt[-c - Sqrt[a]*d]*(2*a^2*d^2 + 3*b*c*d*x^2*Sqrt[a + b*x^2] + a*(2*c^
2 + 5*b*d^2*x^2 + 4*c*d*Sqrt[a + b*x^2]))) - (3*I)*b*(c + Sqrt[a]*d)*x^2*S
qrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqr
t[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^
(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]
], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + (3*I)*Sqrt[a]*b*d*x^2*Sqrt[(d*(-Sqrt
[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a
 + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*Ellipti
cF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[
a]*d)/(c + Sqrt[a]*d)])/(6*a*Sqrt[-c - Sqrt[a]*d]*x^3*Sqrt[c + d*Sqrt[a +
b*x^2]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^4}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{4}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

Fricas [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^(3/2)/x^4, x)
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(3/2)/x**4,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^4}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^4,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx=\text{too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

output

```
( - 2*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*d + 3*int(sqrt(sqrt(a
+ b*x**2)*d + c)/(9*a**4*d**6*x**2 + 18*a**3*b*d**6*x**4 - 22*a**3*c**2*d*
*4*x**2 + 9*a**2*b**2*d**6*x**6 - 35*a**2*b*c**2*d**4*x**4 + 17*a**2*c**4*
d**2*x**2 - 13*a*b**2*c**2*d**4*x**6 + 21*a*b*c**4*d**2*x**4 - 4*a*c**6*x*
*2 + 4*b**2*c**4*d**2*x**6 - 4*b*c**6*x**4),x)*a**3*b*c*d**6*x**3 - 5*int(
sqrt(sqrt(a + b*x**2)*d + c)/(9*a**4*d**6*x**2 + 18*a**3*b*d**6*x**4 - 22*
a**3*c**2*d**4*x**2 + 9*a**2*b**2*d**6*x**6 - 35*a**2*b*c**2*d**4*x**4 + 1
7*a**2*c**4*d**2*x**2 - 13*a*b**2*c**2*d**4*x**6 + 21*a*b*c**4*d**2*x**4 -
 4*a*c**6*x**2 + 4*b**2*c**4*d**2*x**6 - 4*b*c**6*x**4),x)*a**2*b*c**3*d**
4*x**3 + 2*int(sqrt(sqrt(a + b*x**2)*d + c)/(9*a**4*d**6*x**2 + 18*a**3*b*
d**6*x**4 - 22*a**3*c**2*d**4*x**2 + 9*a**2*b**2*d**6*x**6 - 35*a**2*b*c**
2*d**4*x**4 + 17*a**2*c**4*d**2*x**2 - 13*a*b**2*c**2*d**4*x**6 + 21*a*b*c
**4*d**2*x**4 - 4*a*c**6*x**2 + 4*b**2*c**4*d**2*x**6 - 4*b*c**6*x**4),x)*
a*b*c**5*d**2*x**3 + 3*int(sqrt(sqrt(a + b*x**2)*d + c)/(9*a**4*d**6 + 18*
a**3*b*d**6*x**2 - 22*a**3*c**2*d**4 + 9*a**2*b**2*d**6*x**4 - 35*a**2*b*c
**2*d**4*x**2 + 17*a**2*c**4*d**2 - 13*a*b**2*c**2*d**4*x**4 + 21*a*b*c**4
*d**2*x**2 - 4*a*c**6 + 4*b**2*c**4*d**2*x**4 - 4*b*c**6*x**2),x)*a**2*b**
2*c*d**6*x**3 - 5*int(sqrt(sqrt(a + b*x**2)*d + c)/(9*a**4*d**6 + 18*a**3*
b*d**6*x**2 - 22*a**3*c**2*d**4 + 9*a**2*b**2*d**6*x**4 - 35*a**2*b*c**2*d
**4*x**2 + 17*a**2*c**4*d**2 - 13*a*b**2*c**2*d**4*x**4 + 21*a*b*c**4*d...
```

3.265 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 429

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx = -\frac{bd\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{10ax^3} - \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{5x^5} - \frac{b^2d\sqrt{c+d\sqrt{a+bx^2}}\left(acd-(4c^2-3ad^2)\sqrt{a+bx^2}\right)}{20a^2(c^2-ad^2)x} + \frac{b^2d(4c^2-3ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{20a^{3/2}(c^2-ad^2)x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} - \frac{b^2cd\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{5a^{3/2}x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-1/10*b*d*(b*x^2+a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)/a/x^3-1/5*(c+d*(b*x^
2+a)^(1/2))^(3/2)/x^5-1/20*b^2*d*(c+d*(b*x^2+a)^(1/2))^(1/2)*(a*c*d-(-3*a*
d^2+4*c^2)*(b*x^2+a)^(1/2))/a^2/(-a*d^2+c^2)/x+1/20*b^2*d*(-3*a*d^2+4*c^2)
*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(
1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(3/
2)/(-a*d^2+c^2)/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)-1/5*b^2*c*d*
(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2
*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d
))^(1/2))/a^(3/2)/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 38.54 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.30

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{-c-\sqrt{a}d}\left(-c+\sqrt{a}d\right)\left(4a^3d^2-4b^2cdx^4\sqrt{a+bx^2}+abdx^2\left(-bdx^2+2c\sqrt{a\right.}\right.}{}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^(3/2)/x^6,x]
```

output
```
(Sqrt[-c - Sqrt[a]*d]*(-c + Sqrt[a]*d)*(4*a^3*d^2 - 4*b^2*c*d*x^4*Sqrt[a +
 b*x^2] + a*b*d*x^2*(-(b*d*x^2) + 2*c*Sqrt[a + b*x^2]) + a^2*(4*c^2 + 6*b*
d^2*x^2 + 8*c*d*Sqrt[a + b*x^2])) - I*b^2*(-4*c^2 + 3*a*d^2)*x^4*Sqrt[(d*(
-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + S
qrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*El
lipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c -
Sqrt[a]*d)/(c + Sqrt[a]*d)] + I*Sqrt[a]*b^2*d*(-4*c + 3*Sqrt[a]*d)*x^4*Sqr
t[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[
a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3
/2)*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]],
 (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/(20*a^2*Sqrt[-c - Sqrt[a]*d]*(c - Sqrt[
a]*d)*x^5*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^6}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{x^6}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(3/2)/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{6}}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^6,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^6,x)
```

Fricas [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^(3/2)/x^6, x)
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**(3/2)/x**6,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(c+d\sqrt{b\,x^2+a}\right)^{3/2}}{x^6}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^6,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{x^6}\,dx = \text{too large to display}$$

input
```
int((c+d*(b*x^2+a)^(1/2))^(3/2)/x^6,x)
```

output

```
( - 2*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*d + 3*int(sqrt(sqrt(a
+ b*x**2)*d + c)/(17*a**4*d**6*x**4 + 34*a**3*b*d**6*x**6 - 42*a**3*c**2*d
**4*x**4 + 17*a**2*b**2*d**6*x**8 - 67*a**2*b*c**2*d**4*x**6 + 33*a**2*c**
4*d**2*x**4 - 25*a*b**2*c**2*d**4*x**8 + 41*a*b*c**4*d**2*x**6 - 8*a*c**6*
x**4 + 8*b**2*c**4*d**2*x**8 - 8*b*c**6*x**6),x)*a**3*b*c*d**6*x**5 - 5*in
t(sqrt(sqrt(a + b*x**2)*d + c)/(17*a**4*d**6*x**4 + 34*a**3*b*d**6*x**6 -
42*a**3*c**2*d**4*x**4 + 17*a**2*b**2*d**6*x**8 - 67*a**2*b*c**2*d**4*x**6
 + 33*a**2*c**4*d**2*x**4 - 25*a*b**2*c**2*d**4*x**8 + 41*a*b*c**4*d**2*x*
*6 - 8*a*c**6*x**4 + 8*b**2*c**4*d**2*x**8 - 8*b*c**6*x**6),x)*a**2*b*c**3
*d**4*x**5 + 2*int(sqrt(sqrt(a + b*x**2)*d + c)/(17*a**4*d**6*x**4 + 34*a*
*3*b*d**6*x**6 - 42*a**3*c**2*d**4*x**4 + 17*a**2*b**2*d**6*x**8 - 67*a**2
*b*c**2*d**4*x**6 + 33*a**2*c**4*d**2*x**4 - 25*a*b**2*c**2*d**4*x**8 + 41
*a*b*c**4*d**2*x**6 - 8*a*c**6*x**4 + 8*b**2*c**4*d**2*x**8 - 8*b*c**6*x**
6),x)*a*b*c**5*d**2*x**5 + 3*int(sqrt(sqrt(a + b*x**2)*d + c)/(17*a**4*d**
6*x**2 + 34*a**3*b*d**6*x**4 - 42*a**3*c**2*d**4*x**2 + 17*a**2*b**2*d**6*
x**6 - 67*a**2*b*c**2*d**4*x**4 + 33*a**2*c**4*d**2*x**2 - 25*a*b**2*c**2*
d**4*x**6 + 41*a*b*c**4*d**2*x**4 - 8*a*c**6*x**2 + 8*b**2*c**4*d**2*x**6
- 8*b*c**6*x**4),x)*a**2*b**2*c*d**6*x**5 - 5*int(sqrt(sqrt(a + b*x**2)*d
+ c)/(17*a**4*d**6*x**2 + 34*a**3*b*d**6*x**4 - 42*a**3*c**2*d**4*x**2 + 1
7*a**2*b**2*d**6*x**6 - 67*a**2*b*c**2*d**4*x**4 + 33*a**2*c**4*d**2*x*...
```

3.266 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = -\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

output
```
-2/3*x^3/(1+(-x^2+1)^(1/2))^(3/2)+2*x/(1+(-x^2+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{2x\left(2+\sqrt{1-x^2}\right)}{3\sqrt{1+\sqrt{1-x^2}}}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]
```

output
```
(2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{1-x^2}+1}\,dx$$

$$\Big\downarrow \ 2554$$

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

input
```
Int[Sqrt[1 + Sqrt[1 - x^2]],x]
```

output
```
(-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

method	result	size
meijerg	$\frac{i\left(\frac{32 i \sqrt{\pi}\,\sqrt{2}\,x^{3} \cos\left(\frac{3 \arcsin\left(x\right)}{2}\right)}{3}-\frac{8 i \sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3} x^{4}+\frac{2}{3} x^{2}+\frac{2}{3}\right) \sin\left(\frac{3 \arcsin\left(x\right)}{2}\right)}{\sqrt{-x^{2}+1}}\right)}{8 \sqrt{\pi}}$	60

input
```
int(((-x^2+1)^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*I*Pi^(1/2
)*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{2\left(x^2-\sqrt{-x^2+1}+1\right)\sqrt{\sqrt{-x^2+1}+1}}{3\,x}$$

input
```
integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 415, normalized size of antiderivative = 9.22

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx$$
$$= \begin{cases} -\frac{\sqrt{2}ix^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-1}+1}} - \frac{3\sqrt{2}x\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-1}+1}} + \frac{3\sqrt{2}ix\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2}}} \\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\sqrt{1-x^2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} \end{cases}$$

input
```
integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

output
```
Piecewise((-sqrt(2)*I*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sq
rt(I*sqrt(x**2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) - 3*sqrt(2)
*x*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt
(x**2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*I*x*gamm
a(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*I
*pi*sqrt(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)
*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sq
rt(1 - x**2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12
*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) +
1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1
 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))
```

Maxima [F]

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

input
```
integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{2}{3}\,(x+1)\sqrt{\frac{1}{2}\,x+\frac{1}{2}}+\frac{2}{3}\,(x-1)\sqrt{-\frac{1}{2}\,x+\frac{1}{2}}$$

input
```
integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/3*(x + 1)*sqrt(1/2*x + 1/2) + 2/3*(x - 1)*sqrt(-1/2*x + 1/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{1-x^2}+1}\,dx$$

input
```
int(((1 - x^2)^(1/2) + 1)^(1/2),x)
```

output
```
int(((1 - x^2)^(1/2) + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{\frac{2\sqrt{1-x}\,x}{3}-\frac{2\sqrt{1-x}}{3}+\frac{2\sqrt{x+1}\,x}{3}+\frac{2\sqrt{x+1}}{3}}{\sqrt{2}}$$

input
```
int((1+(-x^2+1)^(1/2))^(1/2),x)
```

output
```
(2*(sqrt( - x + 1)*x - sqrt( - x + 1) + sqrt(x + 1)*x + sqrt(x + 1)))/(3*s
qrt(2))
```

3.267 $\int \sqrt{1+\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2x}{3\sqrt{1+\sqrt{1+x^2}}}+\frac{2}{3}x\sqrt{1+\sqrt{1+x^2}}$$

output
```
2/3*x/(1+(x^2+1)^(1/2))^(1/2)+2/3*x*(1+(x^2+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
```

output
```
(2*x*(2 + Sqrt[1 + x^2]))/(3*Sqrt[1 + Sqrt[1 + x^2]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

$$\Big\downarrow \; 2554$$

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

input
```
Int[Sqrt[1 + Sqrt[1 + x^2]],x]
```

output
```
(2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

method	result	size
meijerg	$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^{3}\cosh\left(\frac{3\,\operatorname{arcsinh}(x)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3}x^{4}-\frac{2}{3}x^{2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\operatorname{arcsinh}(x)}{2}\right)}{\sqrt{x^{2}+1}}}{8\sqrt{\pi}}$	55

input
```
int((1+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*
2^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

input
```
integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(36) = 72.

Time = 0.61 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.80

$$\begin{aligned}\int \sqrt{1+\sqrt{1+x^2}}\,dx = &-\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}\\&-\frac{3\sqrt{2}x\sqrt{x^2+1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}\\&-\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}\end{aligned}$$

input
```
integrate((1+(x**2+1)**(1/2))**(1/2),x)
```

output
```
-sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2
+ 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*g
amma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*
pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*s
qrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

Maxima [F]

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Giac [F]

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

input
```
int(((x^2 + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int(((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}dx$$

input
```
int((1+(x^2+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**2 + 1) + 1),x)
```

3.268 $\int \sqrt{5+\sqrt{25+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{10x}{3\sqrt{5+\sqrt{25+x^2}}}+\frac{2}{3}x\sqrt{5+\sqrt{25+x^2}}$$

output
```
10/3*x/(5+(x^2+25)^(1/2))^(1/2)+2/3*x*(5+(x^2+25)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x\left(10+\sqrt{25+x^2}\right)}{3\sqrt{5+\sqrt{25+x^2}}}$$

input
```
Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
```

output
```
(2*x*(10 + Sqrt[25 + x^2]))/(3*Sqrt[5 + Sqrt[25 + x^2]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

$$\Big\downarrow \text{ 2554}$$

$$\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

input
```
Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

output
```
(2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

method	result	size
meijerg	$-\frac{5\sqrt{5}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{375}-\frac{8\sqrt{\pi}\,\sqrt{2}\left(-\frac{4}{1875}x^4-\frac{2}{75}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{\sqrt{\frac{x^2}{25}+1}}\right)}{8\sqrt{\pi}}$	64

input
```
int((5+(x^2+25)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x
))-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/2*arcsinh(1/5*x))/
(1/25*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int\sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2\left(x^2+5\sqrt{x^2+25}-25\right)\sqrt{\sqrt{x^2+25}+5}}{3\,x}$$

input
```
integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(36) = 72.

Time = 0.62 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.80

$$\begin{aligned}\int\sqrt{5+\sqrt{25+x^2}}\,dx=&-\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}\\&-\frac{15\sqrt{2}x\sqrt{x^2+25}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}\\&-\frac{75\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}\end{aligned}$$

input
```
integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

output
```
-sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2
 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 +
25)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5
) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)
/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 +
 25) + 5))
```

Maxima [F]

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

input
```
integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Giac [F]

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

input
```
integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

input
```
int(((x^2 + 25)^(1/2) + 5)^(1/2),x)
```

output
```
int(((x^2 + 25)^(1/2) + 5)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\int \sqrt{\sqrt{x^2+25}+5}dx$$

input
```
int((5+(x^2+25)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**2 + 25) + 5),x)
```

3.269 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2ax}{3\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}+\frac{2}{3}x\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}$$

output
```
2/3*a*x/(a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2)+2/3*x*(a+b*(a^2/b^2+c*x^2)^(1/2)
)^(1/2)
```

Mathematica [A] (verified)

Time = 4.77 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\left(-a^2+b^2cx^2+ab\sqrt{\frac{a^2}{b^2}+cx^2}\right)}{3b^2cx}$$

input
```
Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

output
```
(2*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]]*(-a^2 + b^2*c*x^2 + a*b*Sqrt[a^2/b^2
+ c*x^2]))/(3*b^2*c*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}\,dx$$

$$\Big\downarrow \text{2554}$$

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

input
```
Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

output
```
(2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b
*Sqrt[a^2/b^2 + c*x^2]]
```

Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Maple [F]

$$\int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + c\,x^2}}dx$$

input
```
int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

output
```
int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}}\,dx = \frac{2\left(b^2cx^2 + ab\sqrt{\frac{b^2cx^2+a^2}{b^2}} - a^2\right)\sqrt{b\sqrt{\frac{b^2cx^2+a^2}{b^2}} + a}}{3\,b^2cx}$$

input
```
integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c
*x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

Sympy [F]

$$\int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}}\,dx = \int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}}\,dx$$

input
```
integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}b+a}\,dx$$

input
```
integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}b+a}\,dx$$

input
```
integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{a+b\,\sqrt{c\,x^2+\frac{a^2}{b^2}}}\,dx$$

input
```
int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2),x)
```

output
```
int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{\sqrt{b^2c\,x^2+a^2}+a}dx$$

input
```
int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(a**2 + b**2*c*x**2) + a),x)
```

3.270 $\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 234

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{2c(c^2-ad^2)^2\sqrt{c+d\sqrt{a+bx^2}}}{b^3d^6} + \frac{2(5c^4-6ac^2d^2+a^2d^4)\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3b^3d^6} - \frac{4c(5c^2-3ad^2)\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^3d^6} + \frac{4(5c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7b^3d^6} - \frac{10c\left(c+d\sqrt{a+bx^2}\right)^{9/2}}{9b^3d^6} + \frac{2\left(c+d\sqrt{a+bx^2}\right)^{11/2}}{11b^3d^6}$$

output

```
-2*c*(-a*d^2+c^2)^2*(c+d*(b*x^2+a)^(1/2))^(1/2)/b^3/d^6+2/3*(a^2*d^4-6*a*c
^2*d^2+5*c^4)*(c+d*(b*x^2+a)^(1/2))^(3/2)/b^3/d^6-4/5*c*(-3*a*d^2+5*c^2)*(
c+d*(b*x^2+a)^(1/2))^(5/2)/b^3/d^6+4/7*(-a*d^2+5*c^2)*(c+d*(b*x^2+a)^(1/2)
)^(7/2)/b^3/d^6-10/9*c*(c+d*(b*x^2+a)^(1/2))^(9/2)/b^3/d^6+2/11*(c+d*(b*x^
2+a)^(1/2))^(11/2)/b^3/d^6
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$
$$= \frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-1280c^5+96c^3d^2(28a-5bx^2)+640c^4d\sqrt{a+bx^2}-16c^2d^3(74a-25bx^2)\sqrt{a+bx^2}\right.}{3465b^3d^6}$$

input
```
Integrate[x^5/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-1280*c^5 + 96*c^3*d^2*(28*a - 5*b*x^2) +
640*c^4*d*Sqrt[a + b*x^2] - 16*c^2*d^3*(74*a - 25*b*x^2)*Sqrt[a + b*x^2] +
 15*d^5*Sqrt[a + b*x^2]*(32*a^2 - 24*a*b*x^2 + 21*b^2*x^4) - 2*c*d^4*(736*
a^2 - 244*a*b*x^2 + 175*b^2*x^4)))/(3465*b^3*d^6)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {7283, 896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{d\sqrt{a+bx^2}+c}}\,dx$$
$$\Big\downarrow \ 7283$$
$$\frac{1}{2}\int \frac{x^4}{\sqrt{c+d\sqrt{bx^2+a}}}\,dx^2$$
$$\Big\downarrow \ 896$$
$$\frac{\int \frac{b^2x^4}{\sqrt{c+d\sqrt{bx^2+a}}}\,d(bx^2+a)}{2b^3}$$
$$\Big\downarrow \ 1732$$

$$\frac{\int \frac{\sqrt{bx^2+a}(a-x^4)^2}{\sqrt{c+d\sqrt{bx^2+a}}}d\sqrt{bx^2+a}}{b^3}$$

$$\Big\downarrow \; 522$$

$$\frac{\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^{9/2}}{d^5}-\frac{5c\left(c+d\sqrt{bx^2+a}\right)^{7/2}}{d^5}-\frac{2(ad^2-5c^2)\left(c+d\sqrt{bx^2+a}\right)^{5/2}}{d^5}-\frac{2(5c^3-3acd^2)\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d^5}+\frac{(5c^4-6ad^2c^2+a^2c}{d}\right.}{b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2(a^2d^4-6ac^2d^2+5c^4)\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{3d^6}+\frac{4(5c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{7/2}}{7d^6}-\frac{4c(5c^2-3ad^2)\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^6}-\frac{2c(c^2-ad^2)^2\sqrt{d\sqrt{a+bx^2}+}}{d^6}}{b^3}$$

input
```
Int[x^5/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
((-2*c*(c^2 - a*d^2)^2*Sqrt[c + d*Sqrt[a + b*x^2]])/d^6 + (2*(5*c^4 - 6*a*
c^2*d^2 + a^2*d^4)*(c + d*Sqrt[a + b*x^2])^(3/2))/(3*d^6) - (4*c*(5*c^2 -
3*a*d^2)*(c + d*Sqrt[a + b*x^2])^(5/2))/(5*d^6) + (4*(5*c^2 - a*d^2)*(c +
d*Sqrt[a + b*x^2])^(7/2))/(7*d^6) - (10*c*(c + d*Sqrt[a + b*x^2])^(9/2))/(
9*d^6) + (2*(c + d*Sqrt[a + b*x^2])^(11/2))/(11*d^6))/b^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^5}{\sqrt{c+d\sqrt{b\,x^2+a}}}dx$$

input
```
int(x^5/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(x^5/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{2\left(350\,b^2cd^4x^4+1472\,a^2cd^4-2688\,ac^3d^2+1280\,c^5-8\left(61\,abcd^4-60\,bc^3d^2\right)x^2-\left(315\,b^2d^5x^4+480\,a\right.\right.}{3465\,b^3d^6}$$

input
```
integrate(x^5/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-2/3465*(350*b^2*c*d^4*x^4 + 1472*a^2*c*d^4 - 2688*a*c^3*d^2 + 1280*c^5 -
8*(61*a*b*c*d^4 - 60*b*c^3*d^2)*x^2 - (315*b^2*d^5*x^4 + 480*a^2*d^5 - 118
4*a*c^2*d^3 + 640*c^4*d - 40*(9*a*b*d^5 - 10*b*c^2*d^3)*x^2)*sqrt(b*x^2 +
a))*sqrt(sqrt(b*x^2 + a)*d + c)/(b^3*d^6)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(x**5/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x**5/sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.76

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\left(315\left(\sqrt{bx^2+a}d+c\right)^{\frac{11}{2}}-1925\left(\sqrt{bx^2+a}d+c\right)^{\frac{9}{2}}c-990\left(ad^2-5c^2\right)\left(\sqrt{bx^2+a}d+c\right)^{\frac{7}{2}}+1386\left(3\,acd\right.\right.}{}$$

input
```
integrate(x^5/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(sqrt(b*x^2 + a)*d + c)^(11/2) - 1925*(sqrt(b*x^2 + a)*d + c)^
(9/2)*c - 990*(a*d^2 - 5*c^2)*(sqrt(b*x^2 + a)*d + c)^(7/2) + 1386*(3*a*c*
d^2 - 5*c^3)*(sqrt(b*x^2 + a)*d + c)^(5/2) + 1155*(a^2*d^4 - 6*a*c^2*d^2 +
 5*c^4)*(sqrt(b*x^2 + a)*d + c)^(3/2) - 3465*(a^2*c*d^4 - 2*a*c^3*d^2 + c^
5)*sqrt(sqrt(b*x^2 + a)*d + c))/(b^3*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(200) = 400$.

Time = 0.15 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.20

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\left(1155\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}a^2d^4\text{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-3465\sqrt{\sqrt{bx^2+a}d+c}a^2cd^4\text{sgn}\left(\left(\sqrt{bx^2+\right.\right.}{}$$

input
```
integrate(x^5/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/3465*(1155*(sqrt(b*x^2 + a)*d + c)^(3/2)*a^2*d^4*sgn((sqrt(b*x^2 + a)*d
+ c)*d - c*d) - 3465*sqrt(sqrt(b*x^2 + a)*d + c)*a^2*c*d^4*sgn((sqrt(b*x^2
 + a)*d + c)*d - c*d) - 990*(sqrt(b*x^2 + a)*d + c)^(7/2)*a*d^2*sgn((sqrt(
b*x^2 + a)*d + c)*d - c*d) + 4158*(sqrt(b*x^2 + a)*d + c)^(5/2)*a*c*d^2*sg
n((sqrt(b*x^2 + a)*d + c)*d - c*d) - 6930*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*
c^2*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 6930*sqrt(sqrt(b*x^2 + a)*d
 + c)*a*c^3*d^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 315*(sqrt(b*x^2 + a
)*d + c)^(11/2)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 1925*(sqrt(b*x^2 +
a)*d + c)^(9/2)*c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 4950*(sqrt(b*x^2
+ a)*d + c)^(7/2)*c^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 6930*(sqrt(b*
x^2 + a)*d + c)^(5/2)*c^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 5775*(sqr
t(b*x^2 + a)*d + c)^(3/2)*c^4*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 3465*
sqrt(sqrt(b*x^2 + a)*d + c)*c^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))/(b^3
*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^5}{\sqrt{c+d\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(x^5/(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^5/(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.97

$$\int \frac{x^5}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\left(-1472\sqrt{b\,x^2+a}\,a^2c\,d^4+\right.}{}$$

input
```
int(x^5/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - 480*sqrt(b)*sqrt(a
+ b*x**2)*a**2*d**5*x + 360*sqrt(b)*sqrt(a + b*x**2)*a*b*d**5*x**3 + 1184*
sqrt(b)*sqrt(a + b*x**2)*a*c**2*d**3*x - 315*sqrt(b)*sqrt(a + b*x**2)*b**2
*d**5*x**5 - 400*sqrt(b)*sqrt(a + b*x**2)*b*c**2*d**3*x**3 - 640*sqrt(b)*s
qrt(a + b*x**2)*c**4*d*x - 1472*sqrt(a + b*x**2)*a**2*c*d**4 + 488*sqrt(a
+ b*x**2)*a*b*c*d**4*x**2 + 2688*sqrt(a + b*x**2)*a*c**3*d**2 - 350*sqrt(a
 + b*x**2)*b**2*c*d**4*x**4 - 480*sqrt(a + b*x**2)*b*c**3*d**2*x**2 - 1280
*sqrt(a + b*x**2)*c**5 + 1472*sqrt(b)*a**2*c*d**4*x - 488*sqrt(b)*a*b*c*d*
*4*x**3 - 2688*sqrt(b)*a*c**3*d**2*x + 350*sqrt(b)*b**2*c*d**4*x**5 + 480*
sqrt(b)*b*c**3*d**2*x**3 + 1280*sqrt(b)*c**5*x + 480*a**3*d**5 + 120*a**2*
b*d**5*x**2 - 1184*a**2*c**2*d**3 - 45*a*b**2*d**5*x**4 - 784*a*b*c**2*d**
3*x**2 + 640*a*c**4*d + 315*b**3*d**5*x**6 + 400*b**2*c**2*d**3*x**4 + 640
*b*c**4*d*x**2))/(3465*a*b**3*d**6)
```

3.271 $\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}} dx$

Optimal result

Integrand size = 23, antiderivative size = 139

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}} dx = -\frac{2c(c^2-ad^2)\sqrt{c+d\sqrt{a+bx^2}}}{b^2d^4} + \frac{2(3c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3b^2d^4} - \frac{6c\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^2d^4} + \frac{2\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7b^2d^4}$$

output
```
-2*c*(-a*d^2+c^2)*(c+d*(b*x^2+a)^(1/2))^(1/2)/b^2/d^4+2/3*(-a*d^2+3*c^2)*(
c+d*(b*x^2+a)^(1/2))^(3/2)/b^2/d^4-6/5*c*(c+d*(b*x^2+a)^(1/2))^(5/2)/b^2/d
^4+2/7*(c+d*(b*x^2+a)^(1/2))^(7/2)/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\sqrt{c+d\sqrt{a+bx^2}}\left(-48c^3+2cd^2(26a-9bx^2)+24c^2d\sqrt{a+bx^2}+5d^3\sqrt{a+bx^2}(-4a+3bx^2)\right)}{105b^2d^4}$$

input
```
Integrate[x^3/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*Sqrt[c + d*Sqrt[a + b*x^2]]*(-48*c^3 + 2*c*d^2*(26*a - 9*b*x^2) + 24*c^
2*d*Sqrt[a + b*x^2] + 5*d^3*Sqrt[a + b*x^2]*(-4*a + 3*b*x^2)))/(105*b^2*d^
4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {7283, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{d\sqrt{a+bx^2}+c}}dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{c+d\sqrt{bx^2+a}}}dx^2$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{bx^2}{\sqrt{c+d\sqrt{bx^2+a}}}d(bx^2+a)}{2b^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{bx^2}{\sqrt{c+d\sqrt{bx^2+a}}}d(bx^2+a)}{2b^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}(a-x^4)}{\sqrt{c+d\sqrt{bx^2+a}}}d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(-\frac{\left(c+d\sqrt{bx^2+a}\right)^{5/2}}{d^3}+\frac{3c\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d^3}+\frac{(ad^2-3c^2)\sqrt{c+d\sqrt{bx^2+a}}}{d^3}+\frac{c^3-acd^2}{d^3\sqrt{c+d\sqrt{bx^2+a}}}\right)d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2(3c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{3d^4}+\frac{2c(c^2-ad^2)\sqrt{d\sqrt{a+bx^2}+c}}{d^4}-\frac{2\left(d\sqrt{a+bx^2}+c\right)^{7/2}}{7d^4}+\frac{6c\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^4}}{b^2}$$

input
```
Int[x^3/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
-(((2*c*(c^2 - a*d^2)*Sqrt[c + d*Sqrt[a + b*x^2]])/d^4 - (2*(3*c^2 - a*d^2
)*(c + d*Sqrt[a + b*x^2])^(3/2))/(3*d^4) + (6*c*(c + d*Sqrt[a + b*x^2])^(5
/2))/(5*d^4) - (2*(c + d*Sqrt[a + b*x^2])^(7/2))/(7*d^4))/b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^3}{\sqrt{c+d\sqrt{b\,x^2+a}}}dx$$

input
```
int(x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{2\left(18\,bcd^2x^2-52\,acd^2+48\,c^3-\left(15\,bd^3x^2-20\,ad^3+24\,c^2d\right)\sqrt{bx^2+a}\right)\sqrt{\sqrt{bx^2+a}d+c}}{105\,b^2d^4}$$

input
```
integrate(x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(18*b*c*d^2*x^2 - 52*a*c*d^2 + 48*c^3 - (15*b*d^3*x^2 - 20*a*d^3 +
24*c^2*d)*sqrt(b*x^2 + a))*sqrt(sqrt(b*x^2 + a)*d + c)/(b^2*d^4)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(x**3/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x**3/sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\left(15\left(\sqrt{bx^2+a}d+c\right)^{\frac{7}{2}}-63\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}c-35\left(ad^2-3\,c^2\right)\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}+105\left(acd^2-c^3\right)\right.}{105\,b^2d^4}$$

input
```
integrate(x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(sqrt(b*x^2 + a)*d + c)^(7/2) - 63*(sqrt(b*x^2 + a)*d + c)^(5/2)
*c - 35*(a*d^2 - 3*c^2)*(sqrt(b*x^2 + a)*d + c)^(3/2) + 105*(a*c*d^2 - c^3
)*sqrt(sqrt(b*x^2 + a)*d + c))/(b^2*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(117) = 234$.

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{2\left(35\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}ad^2\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-105\sqrt{\sqrt{bx^2+a}d+c}\,acd^2\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d\right.\right.\right.}{}$$

input
```
integrate(x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-2/105*(35*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*d^2*sgn((sqrt(b*x^2 + a)*d + c)
*d - c*d) - 105*sqrt(sqrt(b*x^2 + a)*d + c)*a*c*d^2*sgn((sqrt(b*x^2 + a)*d
 + c)*d - c*d) - 15*(sqrt(b*x^2 + a)*d + c)^(7/2)*sgn((sqrt(b*x^2 + a)*d +
 c)*d - c*d) + 63*(sqrt(b*x^2 + a)*d + c)^(5/2)*c*sgn((sqrt(b*x^2 + a)*d +
 c)*d - c*d) - 105*(sqrt(b*x^2 + a)*d + c)^(3/2)*c^2*sgn((sqrt(b*x^2 + a)*
d + c)*d - c*d) + 105*sqrt(sqrt(b*x^2 + a)*d + c)*c^3*sgn((sqrt(b*x^2 + a)
*d + c)*d - c*d))/(b^2*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^3}{\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(x^3/(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^3/(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.73

$$\int \frac{x^3}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$
$$= \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\left(20\sqrt{b}\,\sqrt{b\,x^2+a}\,a\,d^3x-1\right.}{}$$

input
```
int(x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*(20*sqrt(b)*sqrt(a + b*
x**2)*a*d**3*x - 15*sqrt(b)*sqrt(a + b*x**2)*b*d**3*x**3 - 24*sqrt(b)*sqrt
(a + b*x**2)*c**2*d*x + 52*sqrt(a + b*x**2)*a*c*d**2 - 18*sqrt(a + b*x**2)
*b*c*d**2*x**2 - 48*sqrt(a + b*x**2)*c**3 - 52*sqrt(b)*a*c*d**2*x + 18*sqr
t(b)*b*c*d**2*x**3 + 48*sqrt(b)*c**3*x - 20*a**2*d**3 - 5*a*b*d**3*x**2 +
24*a*c**2*d + 15*b**2*d**3*x**4 + 24*b*c**2*d*x**2))/(105*a*b**2*d**4)
```

3.272 $\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=-\frac{2c\sqrt{c+d\sqrt{a+bx^2}}}{bd^2}+\frac{2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3bd^2}$$

output
```
-2*c*(c+d*(b*x^2+a)^(1/2))^(1/2)/b/d^2+2/3*(c+d*(b*x^2+a)^(1/2))^(3/2)/b/d
^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\frac{2\left(-2c+d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{3bd^2}$$

input
```
Integrate[x/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*(-2*c + d*Sqrt[a + b*x^2])*Sqrt[c + d*Sqrt[a + b*x^2]])/(3*b*d^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{d\sqrt{a+bx^2}+c}}dx$$

$$\Big\downarrow \text{ 2024}$$

$$\frac{\int \frac{1}{\sqrt{c+d\sqrt{bx^2+a}}}d(bx^2+a)}{2b}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{\int \frac{\sqrt{bx^2+a}}{\sqrt{c+d\sqrt{bx^2+a}}}d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{\sqrt{c+d\sqrt{bx^2+a}}}{d}-\frac{c}{d\sqrt{c+d\sqrt{bx^2+a}}}\right)d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{3d^2}-\frac{2c\sqrt{d\sqrt{a+bx^2}+c}}{d^2}}{b}$$

input
```
Int[x/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
((-2*c*Sqrt[c + d*Sqrt[a + b*x^2]])/d^2 + (2*(c + d*Sqrt[a + b*x^2])^(3/2)
)/(3*d^2))/b
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{3}-2c\sqrt{c+d\sqrt{b\,x^{2}+a}}}{b\,d^{2}}$	45
default	$\frac{\frac{2\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{3}-2c\sqrt{c+d\sqrt{b\,x^{2}+a}}}{b\,d^{2}}$	45

input
```
int(x/(c+d*(b*x^2+a)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^2*(1/3*(c+d*(b*x^2+a)^(1/2))^(3/2)-c*(c+d*(b*x^2+a)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\sqrt{\sqrt{bx^2+a}d+c}\left(\sqrt{bx^2+a}d-2\,c\right)}{3\,bd^2}$$

input
```
integrate(x/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d - 2*c)/(b*d^2)
```

Sympy [F]

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(x/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x/sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\left(\frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{\sqrt{bx^2+a}d+c}c}{d^2}\right)}{3\,b}$$

input
```
integrate(x/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/3*((sqrt(b*x^2 + a)*d + c)^(3/2)/d^2 - 3*sqrt(sqrt(b*x^2 + a)*d + c)*c/d
^2)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\left(\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}d-3\sqrt{\sqrt{bx^2+a}d+c}cd\right)\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)}{3\,bd^3}$$

input
```
integrate(x/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/3*((sqrt(b*x^2 + a)*d + c)^(3/2)*d - 3*sqrt(sqrt(b*x^2 + a)*d + c)*c*d)*
sgn((sqrt(b*x^2 + a)*d + c)*d - c*d)/(b*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{(b\,x^2+a)\,\sqrt{\frac{d\,\sqrt{b\,x^2+a}}{c}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{d\,\sqrt{b\,x^2+a}}{c}\right)}{2\,b\,\sqrt{c+d\,\sqrt{b\,x^2+a}}}$$

input
```
int(x/(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
((a + b*x^2)*((d*(a + b*x^2)^(1/2))/c + 1)^(1/2)*hypergeom([1/2, 2], 3, -(
d*(a + b*x^2)^(1/2))/c))/(2*b*(c + d*(a + b*x^2)^(1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.81

$$\int \frac{x}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$
$$= \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,\Big(-\sqrt{b}\,\sqrt{b\,x^2+a}\,dx-2\sqrt{b}}{3ab\,d^2}$$

input
```
int(x/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - sqrt(b)*sqrt(a + b*
x**2)*d*x - 2*sqrt(a + b*x**2)*c + 2*sqrt(b)*c*x + a*d + b*d*x**2))/(3*a*b
*d**2)
```

3.273 $\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{\sqrt{c-\sqrt{a}d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{\sqrt{c+\sqrt{a}d}}$$

output

```
-arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c-a^(1/2)*d)^(1/2))/(c-a^(1/2)*d)^(1
/2)-arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c+a^(1/2)*d)^(1/2))/(c+a^(1/2)*d)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right)}{\sqrt{-c-\sqrt{a}d}}+\frac{\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)}{\sqrt{-c+\sqrt{a}d}}$$

input

```
Integrate[1/(x*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]]/Sqrt[-c - Sqrt[a]
*d] + ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]]/Sqrt[-c + S
qrt[a]*d]
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {7282, 896, 25, 1732, 561, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{d\sqrt{a+bx^2}+c}}dx$$

$$\Big\downarrow \text{ 7282}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{c+d\sqrt{bx^2+a}}}dx^2$$

$$\Big\downarrow \text{ 896}$$

$$\frac{1}{2}\int \frac{1}{bx^2\sqrt{c+d\sqrt{bx^2+a}}}d(bx^2+a)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int -\frac{1}{bx^2\sqrt{c+d\sqrt{bx^2+a}}}d(bx^2+a)$$

$$\Big\downarrow \text{ 1732}$$

$$-\int \frac{\sqrt{bx^2+a}}{(a-x^4)\sqrt{c+d\sqrt{bx^2+a}}}d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{ 561}$$

$$-\frac{2\int -\frac{c-x^4}{d\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\int \frac{c-x^4}{d\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{c-x^4}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}}{d^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{2\left(-\frac{1}{2}\int \frac{1}{\frac{c-\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}-\frac{1}{2}\int \frac{1}{\frac{c+\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}\right)}{d^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(-\frac{d^2\text{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{c-\sqrt{a}d}}\right)}{2\sqrt{c-\sqrt{a}d}}-\frac{d^2\text{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{\sqrt{a}d+c}}\right)}{2\sqrt{\sqrt{a}d+c}}\right)}{d^2}$$

input
```
Int[1/(x*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
(2*(-1/2*(d^2*ArcTanh[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[c - Sqrt[a]*d]])/Sq
rt[c - Sqrt[a]*d] - (d^2*ArcTanh[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[c + Sqrt
[a]*d]])/(2*Sqrt[c + Sqrt[a]*d])))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{1}{x\sqrt{c+d\sqrt{b\,x^{2}+a}}}dx$$

input
```
int(1/x/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(1/x/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(1/x/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c + d*sqrt(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+c}x}\,dx$$

input
```
integrate(1/x/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(b*x^2 + a)*d + c)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{\frac{\left(\sqrt{a}d^3|d|\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)+cd^3\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)\right)\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}d+c}}{\sqrt{-c+\sqrt{ad^2}}}\right)}{\left(\sqrt{a}d+c\right)\sqrt{\sqrt{a}d-c}}+\frac{\left(\sqrt{a}d^3|d|\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-cd^3\mathrm{sg}\cdots}{\left(\sqrt{a}d-\cdots\right.}}{d^3}$$

input
```
integrate(1/x/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
((sqrt(a)*d^3*abs(d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + c*d^3*sgn((sqr
t(b*x^2 + a)*d + c)*d - c*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-c +
 sqrt(a*d^2)))/((sqrt(a)*d + c)*sqrt(sqrt(a)*d - c)) + (sqrt(a)*d^3*abs(d)
*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c*d^3*sgn((sqrt(b*x^2 + a)*d + c)*
d - c*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-c - sqrt(a*d^2)))/((sqr
t(a)*d - c)*sqrt(-sqrt(a)*d - c)))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{x\,\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(1/(x*(c + d*(a + b*x^2)^(1/2))^(1/2)),x)
```

output
```
int(1/(x*(c + d*(a + b*x^2)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 778, normalized size of antiderivative = 7.70

$$\int \frac{1}{x\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
( - sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)
*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c
)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(s
qrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a +
b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x
**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*
c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 +
2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*
a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*d - sqrt(sqr
t(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**
2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x*
*2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d +
c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*d**
3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d -
 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*c*d**2 - sqrt(sqrt(a
)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 + 2*sqrt(sqrt(a)*d - c
)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*a*b*d**4*x**2 - 4*a*
c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*c + sqrt(a)*sqrt(sqrt(a)*d + c)*
log(sqrt(sqrt(a + b*x**2)*d + c) - sqrt(sqrt(a)*d + c))*d - sqrt(a)*sqrt(s
qrt(a)*d + c)*log(sqrt(sqrt(a + b*x**2)*d + c) + sqrt(sqrt(a)*d + c))*d...
```

3.274 $\int \frac{1}{x^3\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 173

$$\int \frac{1}{x^3\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{\left(c-d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{2\left(c^2-ad^2\right)x^2} - \frac{bd\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{4\sqrt{a}\left(c-\sqrt{a}d\right)^{3/2}} + \frac{bd\text{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{4\sqrt{a}\left(c+\sqrt{a}d\right)^{3/2}}$$

output
```
-1/2*(c-d*(b*x^2+a)^(1/2))*(c+d*(b*x^2+a)^(1/2))^(1/2)/(-a*d^2+c^2)/x^2-1/
4*b*d*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c-a^(1/2)*d)^(1/2))/a^(1/2)/(c-
a^(1/2)*d)^(3/2)+1/4*b*d*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c+a^(1/2)*d)
^(1/2))/a^(1/2)/(c+a^(1/2)*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{1}{4}\left(-\frac{2\left(c-d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{\left(c^2-ad^2\right)x^2} + \frac{bd\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right)}{\sqrt{a}\left(-c-\sqrt{a}d\right)^{3/2}} - \frac{bd\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)}{\sqrt{a}\left(-c+\sqrt{a}d\right)^{3/2}}\right)$$

input
```
Integrate[1/(x^3*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
((-2*(c - d*Sqrt[a + b*x^2])*Sqrt[c + d*Sqrt[a + b*x^2]])/((c^2 - a*d^2)*x
^2) + (b*d*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]])/(Sqrt
[a]*(-c - Sqrt[a]*d)^(3/2)) - (b*d*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt
[-c + Sqrt[a]*d]])/(Sqrt[a]*(-c + Sqrt[a]*d)^(3/2)))/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{d\sqrt{a+bx^2}+c}}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{1}{x^3\sqrt{d\sqrt{a+bx^2}+c}}dx$$

input
```
Int[1/(x^3*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^3 \sqrt{c+d\sqrt{b\,x^2+a}}}dx$$

input
```
int(1/x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(1/x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \sqrt{c+d\sqrt{a+bx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 \sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{x^3 \sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(1/x**3/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(c + d*sqrt(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+c}x^3}\,dx$$

input
```
integrate(1/x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(b*x^2 + a)*d + c)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(135) = 270$.

Time = 0.21 (sec) , antiderivative size = 798, normalized size of antiderivative = 4.61

$$\int \frac{1}{x^3\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*(((a*d^3 - c^2*d)^2*a*d^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 2*
(a^(3/2)*c*d^7 - sqrt(a)*c^3*d^5)*abs(a*d^3 - c^2*d)*sgn((sqrt(b*x^2 + a)*
d + c)*d - c*d) + (a^2*c^2*d^9 - 2*a*c^4*d^7 + c^6*d^5)*sgn((sqrt(b*x^2 +
a)*d + c)*d - c*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-(a*c*d^2 - c^
3 + sqrt((a*c*d^2 - c^3)^2 + (a^2*d^4 - 2*a*c^2*d^2 + c^4)*(a*d^2 - c^2)))
/(a*d^2 - c^2)))/((a^3*d^5 - a^(5/2)*c*d^4 - 2*a^2*c^2*d^3 + 2*a^(3/2)*c^3
*d^2 + a*c^4*d - sqrt(a)*c^5)*sqrt(-sqrt(a)*d - c)*abs(a*d^3 - c^2*d)) + (
(a*d^3 - c^2*d)^2*a*d^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 2*(a^(3/2)*
c*d^7 - sqrt(a)*c^3*d^5)*abs(a*d^3 - c^2*d)*sgn((sqrt(b*x^2 + a)*d + c)*d
 - c*d) + (a^2*c^2*d^9 - 2*a*c^4*d^7 + c^6*d^5)*sgn((sqrt(b*x^2 + a)*d + c)
*d - c*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-(a*c*d^2 - c^3 - sqrt(
(a*c*d^2 - c^3)^2 + (a^2*d^4 - 2*a*c^2*d^2 + c^4)*(a*d^2 - c^2)))/(a*d^2 -
 c^2)))/((a^3*d^5 + a^(5/2)*c*d^4 - 2*a^2*c^2*d^3 - 2*a^(3/2)*c^3*d^2 + a*
c^4*d + sqrt(a)*c^5)*sqrt(sqrt(a)*d - c)*abs(a*d^3 - c^2*d)) - 2*((sqrt(b*
x^2 + a)*d + c)^(3/2)*d^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 2*sqrt(sq
rt(b*x^2 + a)*d + c)*c*d^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))/((a*d^2 -
 (sqrt(b*x^2 + a)*d + c)^2 + 2*(sqrt(b*x^2 + a)*d + c)*c - c^2)*(a*d^2 - c
^2)))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{x^3\,\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(1/(x^3*(c + d*(a + b*x^2)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*(c + d*(a + b*x^2)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \text{too large to display}$$

input
```
int(1/x^3/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(15*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)
*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c
)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(s
qrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a +
b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x
**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*
c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 +
2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*
a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**3*b*d**7*
x**2 - sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
 c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d
- c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqr
t(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a
 + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a +
b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)
*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2
 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 +
 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**2*b*c*
*2*d**5*x**2 - 12*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(
sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*s...
```

3.275 $\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 458

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$
$$= -\frac{4c(32c^2-33ad^2)\,x\sqrt{c+d\sqrt{a+bx^2}}}{315b^2d^4}+\frac{4(8c^2-7ad^2)\,x\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{105b^2d^3}$$
$$-\frac{2x^3\left(8c-7d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{63bd^2}$$
$$-\frac{8\sqrt{a}(32c^4-57ac^2d^2+21a^2d^4)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{315b^3d^5x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}}$$
$$+\frac{8\sqrt{a}c(32c^4-65ac^2d^2+33a^2d^4)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{315b^3d^5x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-4/315*c*(-33*a*d^2+32*c^2)*x*(c+d*(b*x^2+a)^(1/2))^(1/2)/b^2/d^4+4/105*(-
7*a*d^2+8*c^2)*x*(b*x^2+a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)/b^2/d^3-2/63*
x^3*(8*c-7*d*(b*x^2+a)^(1/2))*(c+d*(b*x^2+a)^(1/2))^(1/2)/b/d^2-8/315*a^(1
/2)*(21*a^2*d^4-57*a*c^2*d^2+32*c^4)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2)
)^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a
^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b^3/d^5/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)
*d))^(1/2)+8/315*a^(1/2)*c*(33*a^2*d^4-65*a*c^2*d^2+32*c^4)*(-b*x^2/a)^(1/
2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^
(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b^3/
d^5/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 80.65 (sec) , antiderivative size = 27114, normalized size of antiderivative = 59.20

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^4/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{d\sqrt{a+bx^2}+c}}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{x^4}{\sqrt{d\sqrt{a+bx^2}+c}}dx$$

input
```
Int[x^4/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^4}{\sqrt{c+d\sqrt{b\,x^2+a}}}dx$$

input
```
int(x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^4}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d*x^4 - c*x^4)*sqrt(sqrt(b*x^2 + a)*d + c)/(b*d^
2*x^2 + a*d^2 - c^2), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(x**4/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x**4/sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^4}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^4}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\int \frac{x^4}{\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(x^4/(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^4/(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\text{too large to display}$$

input
```
int(x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(2*( - 6*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a**2*d**2*x + 7*sqr
t(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*b*d**2*x**3 + 6*sqrt(a + b*x*
*2)*sqrt(sqrt(a + b*x**2)*d + c)*a*c**2*x - 4*sqrt(a + b*x**2)*sqrt(sqrt(a
 + b*x**2)*d + c)*b*c**2*x**3 - 196*int((sqrt(sqrt(a + b*x**2)*d + c)*x**6
)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x
**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*
x**2),x)*a**2*b**3*c*d**5 + 224*int((sqrt(sqrt(a + b*x**2)*d + c)*x**6)/(7
*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4
- 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2
),x)*a*b**3*c**3*d**3 - 64*int((sqrt(sqrt(a + b*x**2)*d + c)*x**6)/(7*a**3
*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*
a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*
b**3*c**5*d - 217*int((sqrt(sqrt(a + b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 1
4*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*
d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a**3*b**2
*c*d**5 + 257*int((sqrt(sqrt(a + b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 14*a*
*2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2
*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a**2*b**2*c**
3*d**3 - 76*int((sqrt(sqrt(a + b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 14*a**2
*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**...
```

3.276 $\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 336

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{2x\left(4c-3d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{15bd^2} - \frac{4\sqrt{a}(4c^2-3ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{15b^2d^3x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} + \frac{16\sqrt{a}c(c^2-ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{15b^2d^3x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-2/15*x*(4*c-3*d*(b*x^2+a)^(1/2))*(c+d*(b*x^2+a)^(1/2))^(1/2)/b/d^2-4/15*a
^(1/2)*(-3*a*d^2+4*c^2)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*Ellip
ticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a
^(1/2)*d))^(1/2))/b^2/d^3/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)+16
/15*a^(1/2)*c*(-a*d^2+c^2)*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1
/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1
/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b^2/d^3/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 64.29 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.61

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \frac{2\left(c-d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}\left(2i\left(-4c^3-4\sqrt{a}c^2d+3acd^2+3a^{3/2}d^3\right)\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a\right.}}{c+d\sqrt{a-}}}\right.}{}$$

input
```
Integrate[x^2/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*(c - d*Sqrt[a + b*x^2])*Sqrt[c + d*Sqrt[a + b*x^2]]*((2*I)*(-4*c^3 - 4*
Sqrt[a]*c^2*d + 3*a*c*d^2 + 3*a^(3/2)*d^3)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*
x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c +
d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*ArcSinh[Sqrt
[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a
]*d)] + d*(b*d*Sqrt[-c - Sqrt[a]*d]*x^2*(4*c^2 - 3*d^2*(a - b*x^2) - c*d*S
qrt[a + b*x^2]) - (2*I)*Sqrt[a]*(-4*c^2 - Sqrt[a]*c*d + 3*a*d^2)*Sqrt[(d*(
-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + S
qrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*El
lipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c -
Sqrt[a]*d)/(c + Sqrt[a]*d)])))/(15*b^2*d^4*Sqrt[-c - Sqrt[a]*d]*x*(c^2 - d
^2*(a + b*x^2)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{d\sqrt{a+bx^2}+c}}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^2}{\sqrt{d\sqrt{a+bx^2}+c}}dx$$

input
```
Int[x^2/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{c+d\sqrt{b\,x^2+a}}}dx$$

input
```
int(x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^2}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d*x^2 - c*x^2)*sqrt(sqrt(b*x^2 + a)*d + c)/(b*d^
2*x^2 + a*d^2 - c^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(x**2/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(x**2/sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^2}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^2}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{x^2}{\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(x^2/(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^2/(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \text{Too large to display}$$

input
```
int(x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
(2*(sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*d*x - 21*int((sqrt(sqr
t(a + b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c*
*2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c
**2*d**2*x**4 + 4*b*c**4*x**2),x)*a**2*b**2*c*d**4 + 26*int((sqrt(sqrt(a +
 b*x**2)*d + c)*x**4)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d*
*2 + 7*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d
**2*x**4 + 4*b*c**4*x**2),x)*a*b**2*c**3*d**2 - 8*int((sqrt(sqrt(a + b*x**
2)*d + c)*x**4)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7
*a*b**2*d**4*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x*
*4 + 4*b*c**4*x**2),x)*b**2*c**5 - 21*int((sqrt(sqrt(a + b*x**2)*d + c)*x*
*2)/(7*a**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4
*x**4 - 15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**
4*x**2),x)*a**3*b*c*d**4 + 26*int((sqrt(sqrt(a + b*x**2)*d + c)*x**2)/(7*a
**3*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 -
15*a*b*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),
x)*a**2*b*c**3*d**2 - 8*int((sqrt(sqrt(a + b*x**2)*d + c)*x**2)/(7*a**3*d*
*4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - 15*a*b
*c**2*d**2*x**2 + 4*a*c**4 - 4*b**2*c**2*d**2*x**4 + 4*b*c**4*x**2),x)*a*b
*c**5 + 7*int((sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*x**4)/(7*a**3
*d**4 + 14*a**2*b*d**4*x**2 - 11*a**2*c**2*d**2 + 7*a*b**2*d**4*x**4 - ...
```

3.277 $\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 262

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}} \, dx = -\frac{2\sqrt{a}\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{bdx\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} + \frac{2\sqrt{a}c\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{bdx\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
-2*a^(1/2)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(
b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1
/2))/b/d/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)+2*a^(1/2)*c*(-b*x^2
/a)^(1/2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*
x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2
))/b/d/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 59.41 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$
$$= \frac{2\left(c-d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}\left(-i\left(c+\sqrt{a}d\right)\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\left(c+d\sqrt{a+bx^2}\right)^{3/2}\right.}{}$$

input
```
Integrate[1/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
(2*(c - d*Sqrt[a + b*x^2])*Sqrt[c + d*Sqrt[a + b*x^2]]*((-I)*(c + Sqrt[a]*
d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*
(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^
2])^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x
^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + d*(b*d*Sqrt[-c - Sqrt[a]*d]*x^2
+ I*Sqrt[a]*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]
*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt
[a + b*x^2])^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqr
t[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])))/(b*d^2*Sqrt[-c - Sqrt[
a]*d]*x*(c^2 - d^2*(a + b*x^2)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\sqrt{a+bx^2}+c}}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{1}{\sqrt{d\sqrt{a+bx^2}+c}}dx$$

input
```
Int[1/Sqrt[c + d*Sqrt[a + b*x^2]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{c+d\sqrt{b\,x^{2}+a}}}dx$$

input
```
int(1/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(1/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\int \frac{1}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(1/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d - c)/(b*d^2*x^2 +
a*d^2 - c^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(1/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(c + d*sqrt(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(1/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+c}}\,dx$$

input
```
integrate(1/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sqrt(b*x^2 + a)*d + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\int \frac{1}{\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(1/(c + d*(a + b*x^2)^(1/2))^(1/2),x)
```

output
```
int(1/(c + d*(a + b*x^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c+d\sqrt{a+bx^2}}}\,dx=-\left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b\,d^2x^2+a\,d^2-c^2}dx\right)c+\left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b\,d^2x^2+a\,d^2-c^2}dx\right)d$$

input
```
int(1/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
- int(sqrt(sqrt(a + b*x**2)*d + c)/(a*d**2 + b*d**2*x**2 - c**2),x)*c + i
nt((sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c))/(a*d**2 + b*d**2*x**2 -
 c**2),x)*d
```

3.278 $\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 189

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\frac{\left(c-d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{(c^2-ad^2)\,x} + \frac{\sqrt{a}d\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{(c^2-ad^2)\,x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}}$$

output
```
-(c-d*(b*x^2+a)^(1/2))*(c+d*(b*x^2+a)^(1/2))^(1/2)/(-a*d^2+c^2)/x+a^(1/2)*
d*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^
(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/(-a*
d^2+c^2)/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 58.06 (sec) , antiderivative size = 1005, normalized size of antiderivative = 5.32

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx =$$

$$-\frac{-c^2+ad^2+c\left(c+d\sqrt{a+bx^2}\right)-i\sqrt{-c-\sqrt{a}d}\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\left(c+d\sqrt{a+bx^2}\right)^{3/2}E}{}$$

input
```
Integrate[1/(x^2*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
-((-c^2 + a*d^2 + c*(c + d*Sqrt[a + b*x^2]) - I*Sqrt[-c - Sqrt[a]*d]*Sqrt[
(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a]
 + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2
)*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (
c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + I*Sqrt[-c - Sqrt[a]*d]*Sqrt[(d*(-Sqrt[a]
 + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a +
b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticF[
I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*
d)/(c + Sqrt[a]*d)] + c*(c - Sqrt[a]*d)*Sqrt[-((c + d*Sqrt[a + b*x^2])/(c
- Sqrt[a]*d))]*(Sqrt[-((c + d*Sqrt[a + b*x^2])/(c - Sqrt[a]*d))] - I*Sqrt[
(d*(Sqrt[a] - Sqrt[a + b*x^2]))/(c + Sqrt[a]*d)]*Sqrt[(d*(Sqrt[a] + Sqrt[a
 + b*x^2]))/(-c + Sqrt[a]*d)]*EllipticF[I*ArcSinh[Sqrt[(c + d*Sqrt[a + b*x
^2])/(-c + Sqrt[a]*d)]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + I*Sqrt[(d*(Sqr
t[a] - Sqrt[a + b*x^2]))/(c + Sqrt[a]*d)]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^
2]))/(-c + Sqrt[a]*d)]*EllipticPi[(c - Sqrt[a]*d)/(2*c), I*ArcSinh[Sqrt[(c
 + d*Sqrt[a + b*x^2])/(-c + Sqrt[a]*d)]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)]
) + (I*c*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sq
rt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*(c + d*Sqrt[a
+ b*x^2])^(3/2)*EllipticPi[(2*c)/(c + Sqrt[a]*d), I*ArcSinh[Sqrt[-c - Sqrt
[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])/...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{d\sqrt{a+bx^2}+c}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{x^2\sqrt{d\sqrt{a+bx^2}+c}}dx$$

input
```
Int[1/(x^2*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{b\,x^2+a}}}dx$$

input
```
int(1/x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(1/x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+cx^2}}\,dx$$

input
```
integrate(1/x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d - c)/(b*d^2*x^4 +
(a*d^2 - c^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(1/x**2/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(c + d*sqrt(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+cx^2}}\,dx$$

input
```
integrate(1/x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(b*x^2 + a)*d + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+c}\,x^2}\,dx$$

input
```
integrate(1/x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(sqrt(b*x^2 + a)*d + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{x^2\,\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(1/(x^2*(c + d*(a + b*x^2)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*(c + d*(a + b*x^2)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{c+d\sqrt{a+bx^2}}}\,dx = -\left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b\,d^2x^4+a\,d^2x^2-c^2x^2}dx\right)c + \left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b\,d^2x^4+a\,d^2x^2-c^2x^2}dx\right)d$$

input
```
int(1/x^2/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
 - int(sqrt(sqrt(a + b*x**2)*d + c)/(a*d**2*x**2 + b*d**2*x**4 - c**2*x**2
),x)*c + int((sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c))/(a*d**2*x**2
+ b*d**2*x**4 - c**2*x**2),x)*d
```

3.279 $\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 418

$$\begin{aligned}
&\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx \\
&= -\frac{\left(c-d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}}{3\left(c^2-ad^2\right)x^3} \\
&\quad -\frac{bd\sqrt{c+d\sqrt{a+bx^2}}\left(4acd-\left(c^2+3ad^2\right)\sqrt{a+bx^2}\right)}{6a\left(c^2-ad^2\right)^2x} \\
&\quad +\frac{bd(c^2+3ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{6\sqrt{a}\left(c^2-ad^2\right)^2x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} \\
&\quad -\frac{bcd\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{6\sqrt{a}\left(c^2-ad^2\right)x\sqrt{c+d\sqrt{a+bx^2}}}
\end{aligned}$$

output
```
-1/3*(c-d*(b*x^2+a)^(1/2))*(c+d*(b*x^2+a)^(1/2))^(1/2)/(-a*d^2+c^2)/x^3-1/
6*b*d*(c+d*(b*x^2+a)^(1/2))^(1/2)*(4*a*c*d-(3*a*d^2+c^2)*(b*x^2+a)^(1/2))/
a/(-a*d^2+c^2)^2/x+1/6*b*d*(3*a*d^2+c^2)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(
1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a*d^2+c^2)^2/x/((c+d*(b*x^2+a
)^(1/2))/(c+a^(1/2)*d))^(1/2)-1/6*b*c*d*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^(
1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)
*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a*d^2+c^2)/x/(
c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 60.65 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx =$$

$$-\frac{\left(c-d\sqrt{a+bx^2}\right)\sqrt{c+d\sqrt{a+bx^2}}\left(-\sqrt{-c-\sqrt{a}d}(c-\sqrt{a}d)\left(2ac^2-2a^2d^2+abd^2x^2-bcdx^2\sqrt{a+b\right.\right.}{}$$

input
```
Integrate[1/(x^4*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
-1/6*((c - d*Sqrt[a + b*x^2])*Sqrt[c + d*Sqrt[a + b*x^2]]*(-(Sqrt[-c - Sqr
t[a]*d]*(c - Sqrt[a]*d)*(2*a*c^2 - 2*a^2*d^2 + a*b*d^2*x^2 - b*c*d*x^2*Sqr
t[a + b*x^2])) + I*b*(c^2 + 3*a*d^2)*x^2*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^
2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*
Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticE[I*ArcSinh[Sqrt[-
c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*
d)] - I*Sqrt[a]*b*d*(c + 3*Sqrt[a]*d)*x^2*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x
^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d
*Sqrt[a + b*x^2])]*(c + d*Sqrt[a + b*x^2])^(3/2)*EllipticF[I*ArcSinh[Sqrt[
-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]
*d)]))/(a*(-c - Sqrt[a]*d)^(3/2)*(c - Sqrt[a]*d)^2*x^3*(c^2 - d^2*(a + b*x
^2)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{d\sqrt{a+bx^2}+c}}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{1}{x^4\sqrt{d\sqrt{a+bx^2}+c}}dx$$

input
```
Int[1/(x^4*Sqrt[c + d*Sqrt[a + b*x^2]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{b\,x^2+a}}}dx$$

input
```
int(1/x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
int(1/x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+cx^4}}\,dx$$

input
```
integrate(1/x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d - c)/(b*d^2*x^6 +
(a*d^2 - c^2)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx$$

input
```
integrate(1/x**4/(c+d*(b*x**2+a)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(c + d*sqrt(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx = \int \frac{1}{\sqrt{\sqrt{bx^2+a}d+cx^4}}\,dx$$

input
```
integrate(1/x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(b*x^2 + a)*d + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\int \frac{1}{\sqrt{\sqrt{bx^2+a}d+c}\,x^4}\,dx$$

input
```
integrate(1/x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(sqrt(b*x^2 + a)*d + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx=\int \frac{1}{x^4\,\sqrt{c+d\,\sqrt{b\,x^2+a}}}\,dx$$

input
```
int(1/(x^4*(c + d*(a + b*x^2)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^4*(c + d*(a + b*x^2)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{c+d\sqrt{a+bx^2}}}\,dx=-\left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b\,d^2x^6+a\,d^2x^4-c^2x^4}dx\right)c+\left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b\,d^2x^6+a\,d^2x^4-c^2x^4}dx\right)d$$

input
```
int(1/x^4/(c+d*(b*x^2+a)^(1/2))^(1/2),x)
```

output
```
- int(sqrt(sqrt(a + b*x**2)*d + c)/(a*d**2*x**4 + b*d**2*x**6 - c**2*x**4
),x)*c + int((sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c))/(a*d**2*x**4
+ b*d**2*x**6 - c**2*x**4),x)*d
```

3.280 $\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2c(c^2-ad^2)^2}{b^3d^6\sqrt{c+d\sqrt{a+bx^2}}} + \frac{2(5c^4-6ac^2d^2+a^2d^4)\sqrt{c+d\sqrt{a+bx^2}}}{b^3d^6} - \frac{4c(5c^2-3ad^2)\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{3b^3d^6} + \frac{4(5c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^3d^6} - \frac{10c\left(c+d\sqrt{a+bx^2}\right)^{7/2}}{7b^3d^6} + \frac{2\left(c+d\sqrt{a+bx^2}\right)^{9/2}}{9b^3d^6}$$

output

```
2*c*(-a*d^2+c^2)^2/b^3/d^6/(c+d*(b*x^2+a)^(1/2))^(1/2)+2*(a^2*d^4-6*a*c^2*
d^2+5*c^4)*(c+d*(b*x^2+a)^(1/2))^(1/2)/b^3/d^6-4/3*c*(-3*a*d^2+5*c^2)*(c+d
*(b*x^2+a)^(1/2))^(3/2)/b^3/d^6+4/5*(-a*d^2+5*c^2)*(c+d*(b*x^2+a)^(1/2))^(
5/2)/b^3/d^6-10/7*c*(c+d*(b*x^2+a)^(1/2))^(7/2)/b^3/d^6+2/9*(c+d*(b*x^2+a)
^(1/2))^(9/2)/b^3/d^6
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2(1280c^5-2176ac^3d^2+832a^2cd^4-160bc^3d^2x^2+152abcd^4x^2-50b^2cd^4x^4)+}{315b^3d^6\sqrt{c}}$$

input
```
Integrate[x^5/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*(1280*c^5 - 2176*a*c^3*d^2 + 832*a^2*c*d^4 - 160*b*c^3*d^2*x^2 + 152*a*
b*c*d^4*x^2 - 50*b^2*c*d^4*x^4) + 2*Sqrt[a + b*x^2]*(640*c^4*d - 928*a*c^2
*d^3 + 224*a^2*d^5 + 80*b*c^2*d^3*x^2 - 56*a*b*d^5*x^2 + 35*b^2*d^5*x^4))/
(315*b^3*d^6*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {7283, 896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2}\int \frac{x^4}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{b^2x^4}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\left(bx^2+a\right)}{2b^3}$$

$$\Big\downarrow \text{ 1732}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(a-x^4)^2}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\sqrt{bx^2+a}}{b^3}$$

$\big\downarrow$ 522

$$\frac{\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^{7/2}}{d^5}-\frac{5c\left(c+d\sqrt{bx^2+a}\right)^{5/2}}{d^5}-\frac{2(ad^2-5c^2)\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d^5}-\frac{2(5c^3-3acd^2)\sqrt{c+d\sqrt{bx^2+a}}}{d^5}+\frac{5c^4-6ad^2c^2+a^2d^4}{d^5\sqrt{c+d\sqrt{bx^2+a}}}-\right.}{b^3}$$

$\big\downarrow$ 2009

$$\frac{\frac{2(a^2d^4-6ac^2d^2+5c^4)\sqrt{d\sqrt{a+bx^2}+c}}{d^6}+\frac{4(5c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^6}-\frac{4c(5c^2-3ad^2)\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{3d^6}+\frac{2c(c^2-ad^2)^2}{d^6\sqrt{d\sqrt{a+bx^2}+c}}+\frac{2\left(d\sqrt{a+\right.}{}}{b^3}$$

input
```
Int[x^5/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((2*c*(c^2 - a*d^2)^2)/(d^6*Sqrt[c + d*Sqrt[a + b*x^2]]) + (2*(5*c^4 - 6*a
*c^2*d^2 + a^2*d^4)*Sqrt[c + d*Sqrt[a + b*x^2]])/d^6 - (4*c*(5*c^2 - 3*a*d
^2)*(c + d*Sqrt[a + b*x^2])^(3/2))/(3*d^6) + (4*(5*c^2 - a*d^2)*(c + d*Sqr
t[a + b*x^2])^(5/2))/(5*d^6) - (10*c*(c + d*Sqrt[a + b*x^2])^(7/2))/(7*d^6
) + (2*(c + d*Sqrt[a + b*x^2])^(9/2))/(9*d^6))/b^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^5}{\left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^5/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^5/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(35\,b^3d^6x^6+224\,a^3d^6-1760\,a^2c^2d^4+2816\,ac^4d^2-1280\,c^6-\left(21\,ab^2d^6-1\right.\right.}{}$$

input
```
integrate(x^5/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^3*d^6*x^6 + 224*a^3*d^6 - 1760*a^2*c^2*d^4 + 2816*a*c^4*d^2 -
1280*c^6 - (21*a*b^2*d^6 - 130*b^2*c^2*d^4)*x^4 + 8*(21*a^2*b*d^6 - 125*a*
b*c^2*d^4 + 100*b*c^4*d^2)*x^2 - (85*b^2*c*d^5*x^4 - 608*a^2*c*d^5 + 1248*
a*c^3*d^3 - 640*c^5*d - 16*(13*a*b*c*d^5 - 15*b*c^3*d^3)*x^2)*sqrt(b*x^2 +
 a))*sqrt(sqrt(b*x^2 + a)*d + c)/(b^4*d^8*x^2 + a*b^3*d^8 - b^3*c^2*d^6)
```

Sympy [F]

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**5/(c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(\frac{35\left(\sqrt{bx^2+a}d+c\right)^{\frac{9}{2}}-225\left(\sqrt{bx^2+a}d+c\right)^{\frac{7}{2}}c-126\left(ad^2-5c^2\right)\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}+210\left(3acd^2-5c^3\right)\left(\sqrt{\right.}\right.}{d^4}}{315\,b^3d^2}$$

input
```
integrate(x^5/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(sqrt(b*x^2 + a)*d + c)^(9/2) - 225*(sqrt(b*x^2 + a)*d + c)^(7/
2)*c - 126*(a*d^2 - 5*c^2)*(sqrt(b*x^2 + a)*d + c)^(5/2) + 210*(3*a*c*d^2
- 5*c^3)*(sqrt(b*x^2 + a)*d + c)^(3/2) + 315*(a^2*d^4 - 6*a*c^2*d^2 + 5*c^
4)*sqrt(sqrt(b*x^2 + a)*d + c))/d^4 + 315*(a^2*c*d^4 - 2*a*c^3*d^2 + c^5)/
(sqrt(sqrt(b*x^2 + a)*d + c)*d^4))/(b^3*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(200) = 400$.

Time = 0.21 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.17

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(x^5/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output

```
-2/315*(315*(a^2*c*d^4 - 2*a*c^3*d^2 + c^5)*arctan(sqrt(sqrt(b*x^2 + a)*d
+ c)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d)/sqrt(c*sgn((sqrt(b*x^2 + a)*d +
c)*d - c*d) - c))/(sqrt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c)*b*d^4)
 - (315*sqrt(sqrt(b*x^2 + a)*d + c)*a^2*b^8*d^36*sgn((sqrt(b*x^2 + a)*d +
c)*d - c*d) - 126*(sqrt(b*x^2 + a)*d + c)^(5/2)*a*b^8*d^34*sgn((sqrt(b*x^2
 + a)*d + c)*d - c*d) + 420*(sqrt(b*x^2 + a)*d + c)^(3/2)*a*b^8*c*d^34*sgn
((sqrt(b*x^2 + a)*d + c)*d - c*d) - 1260*sqrt(sqrt(b*x^2 + a)*d + c)*a*b^8
*c^2*d^34*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 210*(sqrt(b*x^2 + a)*d +
c)^(3/2)*a*b^8*c*d^34 - 630*sqrt(sqrt(b*x^2 + a)*d + c)*a*b^8*c^2*d^34 + 3
5*(sqrt(b*x^2 + a)*d + c)^(9/2)*b^8*d^32*sgn((sqrt(b*x^2 + a)*d + c)*d - c
*d) - 180*(sqrt(b*x^2 + a)*d + c)^(7/2)*b^8*c*d^32*sgn((sqrt(b*x^2 + a)*d
+ c)*d - c*d) + 441*(sqrt(b*x^2 + a)*d + c)^(5/2)*b^8*c^2*d^32*sgn((sqrt(b
*x^2 + a)*d + c)*d - c*d) - 630*(sqrt(b*x^2 + a)*d + c)^(3/2)*b^8*c^3*d^32
*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 945*sqrt(sqrt(b*x^2 + a)*d + c)*b^
8*c^4*d^32*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 45*(sqrt(b*x^2 + a)*d +
c)^(7/2)*b^8*c*d^32 + 189*(sqrt(b*x^2 + a)*d + c)^(5/2)*b^8*c^2*d^32 - 420
*(sqrt(b*x^2 + a)*d + c)^(3/2)*b^8*c^3*d^32 + 630*sqrt(sqrt(b*x^2 + a)*d +
 c)*b^8*c^4*d^32)/(b^9*d^36*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d)))/(b^2*d^
2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^5}{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(x^5/(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^5/(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.71

$$\int \frac{x^5}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,(}{}$$

input
```
int(x^5/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output

```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - 608*sqrt(b)*sqrt(a
+ b*x**2)*a**2*c*d**5*x - 208*sqrt(b)*sqrt(a + b*x**2)*a*b*c*d**5*x**3 + 1
248*sqrt(b)*sqrt(a + b*x**2)*a*c**3*d**3*x + 85*sqrt(b)*sqrt(a + b*x**2)*b
**2*c*d**5*x**5 + 240*sqrt(b)*sqrt(a + b*x**2)*b*c**3*d**3*x**3 - 640*sqrt
(b)*sqrt(a + b*x**2)*c**5*d*x + 224*sqrt(a + b*x**2)*a**3*d**6 + 168*sqrt(
a + b*x**2)*a**2*b*d**6*x**2 - 1760*sqrt(a + b*x**2)*a**2*c**2*d**4 - 21*s
qrt(a + b*x**2)*a*b**2*d**6*x**4 - 1000*sqrt(a + b*x**2)*a*b*c**2*d**4*x**
2 + 2816*sqrt(a + b*x**2)*a*c**4*d**2 + 35*sqrt(a + b*x**2)*b**3*d**6*x**6
 + 130*sqrt(a + b*x**2)*b**2*c**2*d**4*x**4 + 800*sqrt(a + b*x**2)*b*c**4*
d**2*x**2 - 1280*sqrt(a + b*x**2)*c**6 - 224*sqrt(b)*a**3*d**6*x - 168*sqr
t(b)*a**2*b*d**6*x**3 + 1760*sqrt(b)*a**2*c**2*d**4*x + 21*sqrt(b)*a*b**2*
d**6*x**5 + 1000*sqrt(b)*a*b*c**2*d**4*x**3 - 2816*sqrt(b)*a*c**4*d**2*x -
 35*sqrt(b)*b**3*d**6*x**7 - 130*sqrt(b)*b**2*c**2*d**4*x**5 - 800*sqrt(b)
*b*c**4*d**2*x**3 + 1280*sqrt(b)*c**6*x + 608*a**3*c*d**5 + 816*a**2*b*c*d
**5*x**2 - 1248*a**2*c**3*d**3 + 123*a*b**2*c*d**5*x**4 - 1488*a*b*c**3*d*
*3*x**2 + 640*a*c**5*d - 85*b**3*c*d**5*x**6 - 240*b**2*c**3*d**3*x**4 + 6
40*b*c**5*d*x**2))/(315*a*b**3*d**6*(a*d**2 + b*d**2*x**2 - c**2))
```

3.281 $\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2c(c^2-ad^2)}{b^2d^4\sqrt{c+d\sqrt{a+bx^2}}} + \frac{2(3c^2-ad^2)\sqrt{c+d\sqrt{a+bx^2}}}{b^2d^4} - \frac{2c\left(c+d\sqrt{a+bx^2}\right)^{3/2}}{b^2d^4} + \frac{2\left(c+d\sqrt{a+bx^2}\right)^{5/2}}{5b^2d^4}$$

output
```
2*c*(-a*d^2+c^2)/b^2/d^4/(c+d*(b*x^2+a)^(1/2))^(1/2)+2*(-a*d^2+3*c^2)*(c+d
*(b*x^2+a)^(1/2))^(1/2)/b^2/d^4-2*c*(c+d*(b*x^2+a)^(1/2))^(3/2)/b^2/d^4+2/
5*(c+d*(b*x^2+a)^(1/2))^(5/2)/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{32c^3-24acd^2-4bcd^2x^2+2\sqrt{a+bx^2}(8c^2d+d^3(-4a+bx^2))}{5b^2d^4\sqrt{c+d\sqrt{a+bx^2}}}$$

input
```
Integrate[x^3/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(32*c^3 - 24*a*c*d^2 - 4*b*c*d^2*x^2 + 2*Sqrt[a + b*x^2]*(8*c^2*d + d^3*(-
4*a + b*x^2)))/(5*b^2*d^4*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {7283, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2}\int \frac{x^2}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{bx^2}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\left(bx^2+a\right)}{2b^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{bx^2}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\left(bx^2+a\right)}{2b^2}$$

$$\Big\downarrow \text{ 1732}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\left(a-x^4\right)}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{\int \left(-\frac{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}{d^3}+\frac{3c\sqrt{c+d\sqrt{bx^2+a}}}{d^3}+\frac{ad^2-3c^2}{d^3\sqrt{c+d\sqrt{bx^2+a}}}+\frac{c^3-acd^2}{d^3\left(c+d\sqrt{bx^2+a}\right)^{3/2}}\right)d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2(3c^2-ad^2)\sqrt{d\sqrt{a+bx^2}+c}}{d^4}-\frac{2c(c^2-ad^2)}{d^4\sqrt{d\sqrt{a+bx^2}+c}}-\frac{2\left(d\sqrt{a+bx^2}+c\right)^{5/2}}{5d^4}+\frac{2c\left(d\sqrt{a+bx^2}+c\right)^{3/2}}{d^4}}{b^2}$$

input
```
Int[x^3/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
-(((-2*c*(c^2 - a*d^2))/(d^4*Sqrt[c + d*Sqrt[a + b*x^2]]) - (2*(3*c^2 - a*
d^2)*Sqrt[c + d*Sqrt[a + b*x^2]])/d^4 + (2*c*(c + d*Sqrt[a + b*x^2])^(3/2)
)/d^4 - (2*(c + d*Sqrt[a + b*x^2])^(5/2))/(5*d^4))/b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^3}{\left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(b^2d^4x^4-4\,a^2d^4+20\,ac^2d^2-16\,c^4-(3\,abd^4-10\,bc^2d^2)x^2-(3\,bcd^3x^2+8\right.}{5\left(b^3d^6x^2+ab^2d^6-b^2c^2d^4\right)}$$

input
```
integrate(x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^2*d^4*x^4 - 4*a^2*d^4 + 20*a*c^2*d^2 - 16*c^4 - (3*a*b*d^4 - 10*b*c
^2*d^2)*x^2 - (3*b*c*d^3*x^2 + 8*a*c*d^3 - 8*c^3*d)*sqrt(b*x^2 + a))*sqrt(
sqrt(b*x^2 + a)*d + c)/(b^3*d^6*x^2 + a*b^2*d^6 - b^2*c^2*d^4)
```

Sympy [F]

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**3/(c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(\frac{\left(\sqrt{bx^2+a}d+c\right)^{\frac{5}{2}}-5\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}c-5\,(ad^2-3\,c^2)\sqrt{\sqrt{bx^2+a}d+c}}{d^2}-\frac{5\,(acd^2-c^3)}{\sqrt{\sqrt{bx^2+a}d+c}d^2}\right)}{5\,b^2d^2}$$

input
```
integrate(x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2/5*(((sqrt(b*x^2 + a)*d + c)^(5/2) - 5*(sqrt(b*x^2 + a)*d + c)^(3/2)*c -
5*(a*d^2 - 3*c^2)*sqrt(sqrt(b*x^2 + a)*d + c))/d^2 - 5*(a*c*d^2 - c^3)/(sq
rt(sqrt(b*x^2 + a)*d + c)*d^2))/(b^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(117) = 234.

Time = 0.18 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.87

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(\frac{15\,(acd^2-c^3)\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}d+c}\operatorname{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)}{\sqrt{c\operatorname{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-c}}\right)}{\sqrt{c\operatorname{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-c}bd^2}-\frac{15\sqrt{\sqrt{bx^2+a}d+c}ab^4d^{10}\operatorname{sgn}\left(\left(\sqrt{bx^2+a}\right.\right.}{}\right.}{}$$

input
```
integrate(x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
2/15*(15*(a*c*d^2 - c^3)*arctan(sqrt(sqrt(b*x^2 + a)*d + c)*sgn((sqrt(b*x^
2 + a)*d + c)*d - c*d)/sqrt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c))/(
sqrt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c)*b*d^2) - (15*sqrt(sqrt(b*
x^2 + a)*d + c)*a*b^4*d^10*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 3*(sqrt(
b*x^2 + a)*d + c)^(5/2)*b^4*d^8*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 10*
(sqrt(b*x^2 + a)*d + c)^(3/2)*b^4*c*d^8*sgn((sqrt(b*x^2 + a)*d + c)*d - c*
d) - 30*sqrt(sqrt(b*x^2 + a)*d + c)*b^4*c^2*d^8*sgn((sqrt(b*x^2 + a)*d + c
)*d - c*d) + 5*(sqrt(b*x^2 + a)*d + c)^(3/2)*b^4*c*d^8 - 15*sqrt(sqrt(b*x^
2 + a)*d + c)*b^4*c^2*d^8)/(b^5*d^10*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))
)/(b*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(x^3/(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^3/(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.68

$$\int \frac{x^3}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,(}{}$$

input
```
int(x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output

```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*(8*sqrt(b)*sqrt(a + b*x
**2)*a*c*d**3*x + 3*sqrt(b)*sqrt(a + b*x**2)*b*c*d**3*x**3 - 8*sqrt(b)*sqr
t(a + b*x**2)*c**3*d*x - 4*sqrt(a + b*x**2)*a**2*d**4 - 3*sqrt(a + b*x**2)
*a*b*d**4*x**2 + 20*sqrt(a + b*x**2)*a*c**2*d**2 + sqrt(a + b*x**2)*b**2*d
**4*x**4 + 10*sqrt(a + b*x**2)*b*c**2*d**2*x**2 - 16*sqrt(a + b*x**2)*c**4
 + 4*sqrt(b)*a**2*d**4*x + 3*sqrt(b)*a*b*d**4*x**3 - 20*sqrt(b)*a*c**2*d**
2*x - sqrt(b)*b**2*d**4*x**5 - 10*sqrt(b)*b*c**2*d**2*x**3 + 16*sqrt(b)*c*
*4*x - 8*a**2*c*d**3 - 11*a*b*c*d**3*x**2 + 8*a*c**3*d - 3*b**2*c*d**3*x**
4 + 8*b*c**3*d*x**2))/(5*a*b**2*d**4*(a*d**2 + b*d**2*x**2 - c**2))
```

3.282 $\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2c}{bd^2\sqrt{c+d\sqrt{a+bx^2}}}+\frac{2\sqrt{c+d\sqrt{a+bx^2}}}{bd^2}$$

output
```
2*c/b/d^2/(c+d*(b*x^2+a)^(1/2))^(1/2)+2*(c+d*(b*x^2+a)^(1/2))^(1/2)/b/d^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{4c+2d\sqrt{a+bx^2}}{bd^2\sqrt{c+d\sqrt{a+bx^2}}}$$

input
```
Integrate[x/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(4*c + 2*d*Sqrt[a + b*x^2])/(b*d^2*Sqrt[c + d*Sqrt[a + b*x^2]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2024, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \frac{1}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{774}$$

$$\frac{\int \frac{\sqrt{bx^2+a}}{\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{1}{d\sqrt{c+d\sqrt{bx^2+a}}}-\frac{c}{d\left(c+d\sqrt{bx^2+a}\right)^{3/2}}\right)d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2c}{d^2\sqrt{d\sqrt{a+bx^2}+c}}+\frac{2\sqrt{d\sqrt{a+bx^2}+c}}{d^2}}{b}$$

input
```
Int[x/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((2*c)/(d^2*Sqrt[c + d*Sqrt[a + b*x^2]]) + (2*Sqrt[c + d*Sqrt[a + b*x^2]])
/d^2)/b
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2\sqrt{c+d\sqrt{b\,x^{2}+a}}+\frac{2c}{\sqrt{c+d\sqrt{b\,x^{2}+a}}}}{b\,d^{2}}$	42
default	$\frac{2\sqrt{c+d\sqrt{b\,x^{2}+a}}+\frac{2c}{\sqrt{c+d\sqrt{b\,x^{2}+a}}}}{b\,d^{2}}$	42

input
```
int(x/(c+d*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^2*((c+d*(b*x^2+a)^(1/2))^(1/2)+c/(c+d*(b*x^2+a)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(bd^2x^2+ad^2+\sqrt{bx^2+a}cd-2c^2\right)\sqrt{\sqrt{bx^2+a}d+c}}{b^2d^4x^2+abd^4-bc^2d^2}$$

input
```
integrate(x/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2*(b*d^2*x^2 + a*d^2 + sqrt(b*x^2 + a)*c*d - 2*c^2)*sqrt(sqrt(b*x^2 + a)*d
 + c)/(b^2*d^4*x^2 + a*b*d^4 - b*c^2*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.64

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \begin{cases} \frac{x^2}{2c^{\frac{3}{2}}} & \text{for } b=0 \wedge d=0 \\ \frac{x^2}{2(\sqrt{a}d+c)^{\frac{3}{2}}} & \text{for } b=0 \\ \frac{x^2}{2c^{\frac{3}{2}}} & \text{for } d=0 \\ \frac{4c}{bd^2\sqrt{c+d\sqrt{a+bx^2}}}+\frac{2\sqrt{a+bx^2}}{bd\sqrt{c+d\sqrt{a+bx^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Piecewise((x**2/(2*c**(3/2)), Eq(b, 0) & Eq(d, 0)), (x**2/(2*(sqrt(a)*d +
c)**(3/2)), Eq(b, 0)), (x**2/(2*c**(3/2)), Eq(d, 0)), (4*c/(b*d**2*sqrt(c
+ d*sqrt(a + b*x**2))) + 2*sqrt(a + b*x**2)/(b*d*sqrt(c + d*sqrt(a + b*x**
2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{\sqrt{bx^2+a}d+c}}{d^2}+\frac{c}{\sqrt{\sqrt{bx^2+a}d+c}d^2}\right)}{b}$$

input
```
integrate(x/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2*(sqrt(sqrt(b*x^2 + a)*d + c)/d^2 + c/(sqrt(sqrt(b*x^2 + a)*d + c)*d^2))/
b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(48) = 96.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = -\frac{2\left(\frac{c\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}d+c}\,\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)}{\sqrt{c\,\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-c}}\right)}{\sqrt{c\,\mathrm{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-c}\,b}-\frac{\sqrt{\sqrt{bx^2+a}d+c}}{b}\right)}{d^2}$$

input
```
integrate(x/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
-2*(c*arctan(sqrt(sqrt(b*x^2 + a)*d + c)*sgn((sqrt(b*x^2 + a)*d + c)*d - c
*d)/sqrt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c))/(sqrt(c*sgn((sqrt(b*
x^2 + a)*d + c)*d - c*d) - c)*b) - sqrt(sqrt(b*x^2 + a)*d + c)/b)/d^2
```

Mupad [B] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\,d^2\,\left(b\,x^2+a\right)+4\,c^2+6\,c\,d\,\sqrt{b\,x^2+a}}{b\,d^2\,\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}$$

input
```
int(x/(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
(2*d^2*(a + b*x^2) + 4*c^2 + 6*c*d*(a + b*x^2)^(1/2))/(b*d^2*(c + d*(a + b
*x^2)^(1/2))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.25

$$\int \frac{x}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\sqrt{\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2}\,\sqrt{\sqrt{b\,x^2+a}+\sqrt{b}\,x}\,(}{}$$

input
```
int(x/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
(2*sqrt(sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x +
a*d + b*d*x**2)*sqrt(sqrt(a + b*x**2) + sqrt(b)*x)*( - sqrt(b)*sqrt(a + b*
x**2)*c*d*x + sqrt(a + b*x**2)*a*d**2 + sqrt(a + b*x**2)*b*d**2*x**2 - 2*s
qrt(a + b*x**2)*c**2 - sqrt(b)*a*d**2*x - sqrt(b)*b*d**2*x**3 + 2*sqrt(b)*
c**2*x + a*c*d + b*c*d*x**2))/(a*b*d**2*(a*d**2 + b*d**2*x**2 - c**2))
```

3.283 $\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\frac{2c}{(c^2-ad^2)\sqrt{c+d\sqrt{a+bx^2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{\left(c-\sqrt{a}d\right)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{\left(c+\sqrt{a}d\right)^{3/2}}$$

output
```
2*c/(-a*d^2+c^2)/(c+d*(b*x^2+a)^(1/2))^(1/2)-arctanh((c+d*(b*x^2+a)^(1/2))
^(1/2)/(c-a^(1/2)*d)^(1/2))/(c-a^(1/2)*d)^(3/2)-arctanh((c+d*(b*x^2+a)^(1/
2))^(1/2)/(c+a^(1/2)*d)^(1/2))/(c+a^(1/2)*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\frac{2c}{(c^2-ad^2)\sqrt{c+d\sqrt{a+bx^2}}}-\frac{\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right)}{\left(-c-\sqrt{a}d\right)^{3/2}}-\frac{\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)}{\left(-c+\sqrt{a}d\right)^{3/2}}$$

input
```
Integrate[1/(x*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
(2*c)/((c^2 - a*d^2)*Sqrt[c + d*Sqrt[a + b*x^2]]) - ArcTan[Sqrt[c + d*Sqrt
[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]]/(-c - Sqrt[a]*d)^(3/2) - ArcTan[Sqrt[c
+ d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]]/(-c + Sqrt[a]*d)^(3/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {7282, 896, 25, 1732, 561, 25, 27, 1604, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(c+d\sqrt{bx^2+a}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{2}\int \frac{1}{bx^2\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\left(bx^2+a\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int -\frac{1}{bx^2\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\left(bx^2+a\right)$$

$$\Big\downarrow \text{1732}$$

$$-\int \frac{\sqrt{bx^2+a}}{(a-x^4)\left(c+d\sqrt{bx^2+a}\right)^{3/2}}d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{561}$$

$$-\frac{2\int -\frac{c-x^4}{dx^4\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow 25$$

$$\frac{2\int \frac{c-x^4}{dx^4\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow 27$$

$$\frac{2\int \frac{c-x^4}{x^4\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{d^2}$$

$$\Big\downarrow 1604$$

$$\frac{2\left(-\frac{\int \frac{\left(\frac{c^2}{d^2}+a\right)d^2-cx^4}{d^2\left(-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}\right)}d\sqrt{c+d\sqrt{bx^2+a}}}{a-\frac{c^2}{d^2}}-\frac{c}{x^2\left(a-\frac{c^2}{d^2}\right)}\right)}{d^2}$$

$$\Big\downarrow 27$$

$$\frac{2\left(-\frac{\int \frac{-cx^4+c^2+ad^2}{-\frac{x^8}{d^2}+\frac{2cx^4}{d^2}+a-\frac{c^2}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}}{d^2\left(a-\frac{c^2}{d^2}\right)}-\frac{c}{x^2\left(a-\frac{c^2}{d^2}\right)}\right)}{d^2}$$

$$\Big\downarrow 1480$$

$$\frac{2\left(-\frac{-\frac{1}{2}\left(\sqrt{a}d+c\right)\int \frac{1}{\frac{c-\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}-\frac{1}{2}\left(c-\sqrt{a}d\right)\int \frac{1}{\frac{c+\sqrt{a}d}{d^2}-\frac{x^4}{d^2}}d\sqrt{c+d\sqrt{bx^2+a}}}{d^2\left(a-\frac{c^2}{d^2}\right)}-\frac{c}{x^2\left(a-\frac{c^2}{d^2}\right)}\right)}{d^2}$$

$$\Big\downarrow 221$$

$$\frac{2\left(-\frac{-\frac{d^2\left(\sqrt{a}d+c\right)\text{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{c-\sqrt{a}d}}\right)}{2\sqrt{c-\sqrt{a}d}}-\frac{d^2\left(c-\sqrt{a}d\right)\text{arctanh}\left(\frac{\sqrt{d\sqrt{a+bx^2}+c}}{\sqrt{\sqrt{a}d+c}}\right)}{2\sqrt{\sqrt{a}d+c}}}{d^2\left(a-\frac{c^2}{d^2}\right)}-\frac{c}{x^2\left(a-\frac{c^2}{d^2}\right)}\right)}{d^2}$$

input
```
Int[1/(x*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
(2*(-(c/((a - c^2/d^2)*x^2)) - (-1/2*(d^2*(c + Sqrt[a]*d)*ArcTanh[Sqrt[c +
 d*Sqrt[a + b*x^2]]/Sqrt[c - Sqrt[a]*d]])/Sqrt[c - Sqrt[a]*d] - (d^2*(c -
Sqrt[a]*d)*ArcTanh[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[c + Sqrt[a]*d]])/(2*Sq
rt[c + Sqrt[a]*d]))/((a - c^2/d^2)*d^2)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{1}{x\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(1/x/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(1/(x*(c + d*sqrt(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((sqrt(b*x^2 + a)*d + c)^(3/2)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(108) = 216$.

Time = 0.35 (sec) , antiderivative size = 1095, normalized size of antiderivative = 8.11

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
(2*c*d^2*arctan(sqrt(sqrt(b*x^2 + a)*d + c)*sgn((sqrt(b*x^2 + a)*d + c)*d
- c*d)/sqrt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c))/((a*d^2 - c^2)*sq
rt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c)) + (sqrt(-sqrt(a)*d - c)*a^
(5/2)*c*d^8 - 2*sqrt(-sqrt(a)*d - c)*a^(3/2)*c^3*d^6 + sqrt(-sqrt(a)*d - c
)*sqrt(a)*c^5*d^4 - sqrt(-sqrt(a)*d - c)*a^2*d^6*abs(a*d^3 - c^2*d) - sqrt
(-sqrt(a)*d - c)*a*c^2*d^4*abs(a*d^3 - c^2*d)*sgn((sqrt(b*x^2 + a)*d + c)*
d - c*d) + sqrt(-sqrt(a)*d - c)*a*c^2*d^4*abs(a*d^3 - c^2*d) + sqrt(-sqrt(
a)*d - c)*c^4*d^2*abs(a*d^3 - c^2*d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d)
+ (a*d^3 - c^2*d)^2*sqrt(-sqrt(a)*d - c)*sqrt(a)*c*d^2*sgn((sqrt(b*x^2 + a
)*d + c)*d - c*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c)/sqrt(-(a*c*d^2 - c^3
 + sqrt((a*c*d^2 - c^3)^2 + (a^2*d^4 - 2*a*c^2*d^2 + c^4)*(a*d^2 - c^2)))/
(a*d^2 - c^2)))/((a^3*d^6 - a^2*c^2*d^4)*abs(a*d^3 - c^2*d) - 2*(a^2*c^2*d
^4 - a*c^4*d^2)*abs(a*d^3 - c^2*d) + (a*c^4*d^2 - c^6)*abs(a*d^3 - c^2*d))
 - (sqrt(sqrt(a)*d - c)*a^(5/2)*c*d^8 - 2*sqrt(sqrt(a)*d - c)*a^(3/2)*c^3*
d^6 + sqrt(sqrt(a)*d - c)*sqrt(a)*c^5*d^4 + sqrt(sqrt(a)*d - c)*a^2*d^6*ab
s(a*d^3 - c^2*d) + sqrt(sqrt(a)*d - c)*a*c^2*d^4*abs(a*d^3 - c^2*d)*sgn((s
qrt(b*x^2 + a)*d + c)*d - c*d) - sqrt(sqrt(a)*d - c)*a*c^2*d^4*abs(a*d^3 -
 c^2*d) - sqrt(sqrt(a)*d - c)*c^4*d^2*abs(a*d^3 - c^2*d)*sgn((sqrt(b*x^2 +
 a)*d + c)*d - c*d) + (a*d^3 - c^2*d)^2*sqrt(sqrt(a)*d - c)*sqrt(a)*c*d^2*
sgn((sqrt(b*x^2 + a)*d + c)*d - c*d))*arctan(sqrt(sqrt(b*x^2 + a)*d + c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(c + d*(a + b*x^2)^(1/2))^(3/2)),x)
```

output
```
int(1/(x*(c + d*(a + b*x^2)^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 3195, normalized size of antiderivative = 23.67

$$\int \frac{1}{x\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
(2*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*
sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)
*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sq
rt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b
*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x*
*2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*c
*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 + 2
*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*a
*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**2*c*d**3 +
 2*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)*
sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d - c)
*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sq
rt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b
*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x*
*2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*c
*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2 + 2
*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 + 2*a
*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a*b*c*d**3*x*
*2 - 2*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
 c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)...
```

3.284 $\int \frac{1}{x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\int \frac{1}{x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{3bcd^2}{\left(c^2-ad^2\right)^2\sqrt{c+d\sqrt{a+bx^2}}} - \frac{c-d\sqrt{a+bx^2}}{2\left(c^2-ad^2\right)x^2\sqrt{c+d\sqrt{a+bx^2}}} - \frac{3bd\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c-\sqrt{a}d}}\right)}{4\sqrt{a}\left(c-\sqrt{a}d\right)^{5/2}} + \frac{3bd\operatorname{arctanh}\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{c+\sqrt{a}d}}\right)}{4\sqrt{a}\left(c+\sqrt{a}d\right)^{5/2}}$$

output

```
3*b*c*d^2/(-a*d^2+c^2)^2/(c+d*(b*x^2+a)^(1/2))^(1/2)-1/2*(c-d*(b*x^2+a)^(1
/2))/(-a*d^2+c^2)/x^2/(c+d*(b*x^2+a)^(1/2))^(1/2)-3/4*b*d*arctanh((c+d*(b*
x^2+a)^(1/2))^(1/2)/(c-a^(1/2)*d)^(1/2))/a^(1/2)/(c-a^(1/2)*d)^(5/2)+3/4*b
*d*arctanh((c+d*(b*x^2+a)^(1/2))^(1/2)/(c+a^(1/2)*d)^(1/2))/a^(1/2)/(c+a^(
1/2)*d)^(5/2)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\frac{1}{4}\left(\frac{2\left(-c^3+acd^2+6bcd^2x^2+d(c^2-ad^2)\sqrt{a+bx^2}\right)}{(c^2-ad^2)^2x^2\sqrt{c+d\sqrt{a+bx^2}}}-\frac{3bd\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c-\sqrt{a}d}}\right)}{\sqrt{a}\left(-c-\sqrt{a}d\right)^{5/2}}+\frac{3bd\arctan\left(\frac{\sqrt{c+d\sqrt{a+bx^2}}}{\sqrt{-c+\sqrt{a}d}}\right)}{\sqrt{a}\left(-c+\sqrt{a}d\right)^{5/2}}\right)$$

input
```
Integrate[1/(x^3*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
((2*(-c^3 + a*c*d^2 + 6*b*c*d^2*x^2 + d*(c^2 - a*d^2)*Sqrt[a + b*x^2]))/((
c^2 - a*d^2)^2*x^2*Sqrt[c + d*Sqrt[a + b*x^2]]) - (3*b*d*ArcTan[Sqrt[c + d
*Sqrt[a + b*x^2]]/Sqrt[-c - Sqrt[a]*d]])/(Sqrt[a]*(-c - Sqrt[a]*d)^(5/2))
+ (3*b*d*ArcTan[Sqrt[c + d*Sqrt[a + b*x^2]]/Sqrt[-c + Sqrt[a]*d]])/(Sqrt[a
]*(-c + Sqrt[a]*d)^(5/2)))/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{1}{x^3\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

input
```
Int[1/(x^3*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(c +d \sqrt{b \,x^{2}+a}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(1/x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 \left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^{3} \left(c + d\sqrt{a + bx^{2}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(1/(x**3*(c + d*sqrt(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((sqrt(b*x^2 + a)*d + c)^(3/2)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4863 vs. $2(170) = 340$.

Time = 1.57 (sec) , antiderivative size = 4863, normalized size of antiderivative = 23.05

$$\int \frac{1}{x^3\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
-1/4*(8*b*c*d^4*arctan(sqrt(sqrt(b*x^2 + a)*d + c)*sgn((sqrt(b*x^2 + a)*d
+ c)*d - c*d)/sqrt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c))/((a^2*d^4
- 2*a*c^2*d^2 + c^4)*sqrt(c*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - c)) - (
(a^3*d^7 - 3*a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d)^2*a^(5/2)*b*c*d^6 - (a^3*d
^7 - 3*a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d)^2*a^(3/2)*b*c^3*d^4*sgn((sqrt(b*
x^2 + a)*d + c)*d - c*d) - (a^3*d^7 - 3*a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d)
^2*a^(3/2)*b*c^3*d^4 + (a^3*d^7 - 3*a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d)^2*(
5*a^(5/2)*c*d^6 - 4*a^(3/2)*c^3*d^4)*b*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d
) - 2*(3*a^5*c^2*d^12 - 5*a^4*c^4*d^10 + 2*a^3*c^6*d^8)*b*abs(a^3*d^7 - 3*
a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) +
2*(7*a^4*c^4*d^10 - 11*a^3*c^6*d^8 + 4*a^2*c^8*d^6)*b*abs(a^3*d^7 - 3*a^2*
c^2*d^5 + 3*a*c^4*d^3 - c^6*d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 2*(5
*a^3*c^6*d^8 - 7*a^2*c^8*d^6 + 2*a*c^10*d^4)*b*abs(a^3*d^7 - 3*a^2*c^2*d^5
 + 3*a*c^4*d^3 - c^6*d)*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 2*(a^2*c^8*
d^6 - a*c^10*d^4)*b*abs(a^3*d^7 - 3*a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d)*sgn
((sqrt(b*x^2 + a)*d + c)*d - c*d) - (3*a^6*d^14 - 4*a^5*c^2*d^12 + a^4*c^4
*d^10)*b*abs(a^3*d^7 - 3*a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d) + (5*a^5*c^2*d
^12 - 4*a^4*c^4*d^10 - a^3*c^6*d^8)*b*abs(a^3*d^7 - 3*a^2*c^2*d^5 + 3*a*c^
4*d^3 - c^6*d) - (a^4*c^4*d^10 + 4*a^3*c^6*d^8 - 5*a^2*c^8*d^6)*b*abs(a^3*
d^7 - 3*a^2*c^2*d^5 + 3*a*c^4*d^3 - c^6*d) - (a^3*c^6*d^8 - 4*a^2*c^8*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(c + d*(a + b*x^2)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^3*(c + d*(a + b*x^2)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \left(c + d\sqrt{a + bx^2}\right)^{3/2}} \, dx = \text{too large to display}$$

input
```
int(1/x^3/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
( - 81*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqrt(a)*d -
 c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sqrt(a)*d
- c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d - c)*sqr
t(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a
 + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a +
b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)
*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d**2*x**2
 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**2*d**4 +
 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a**4*b*c*
d**9*x**2 - 81*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**2)*sqrt(sqr
t(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b*x**2)*sqrt(sq
rt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**2*d - sqrt(a)*sqrt(sqrt(a)*d
- c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a)*sqrt(sqrt(a)*d - c)*sqr
t(sqrt(a + b*x**2)*d + c)*b*d**3*x**2 + sqrt(a)*sqrt(sqrt(a)*d - c)*sqrt(s
qrt(a + b*x**2)*d + c)*c**2*d - 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2
)*d + c)*a*c*d**2 - sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*b*c*d
**2*x**2 + 2*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*c**3)/(2*a**
2*d**4 + 2*a*b*d**4*x**2 - 4*a*c**2*d**2 - 2*b*c**2*d**2*x**2 + 2*c**4))*a
**3*b**2*c*d**9*x**4 - 90*sqrt(a)*sqrt(sqrt(a)*d - c)*atan((sqrt(a + b*x**
2)*sqrt(sqrt(a)*d - c)*sqrt(sqrt(a + b*x**2)*d + c)*a*d**3 - sqrt(a + b...
```

3.285 $\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 448

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2cx^3}{bd^2\sqrt{c+d\sqrt{a+bx^2}}}+\frac{4(32c^2-5ad^2)\,x\sqrt{c+d\sqrt{a+bx^2}}}{35b^2d^4}$$
$$+\frac{2x^3\sqrt{c+d\sqrt{a+bx^2}}}{7bd^2}-\frac{96cx\sqrt{a+bx^2}\sqrt{c+d\sqrt{a+bx^2}}}{35b^2d^3}$$
$$+\frac{8\sqrt{a}c(32c^2-29ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{35b^3d^5x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}}$$
$$-\frac{8\sqrt{a}(32c^4-37ac^2d^2+5a^2d^4)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{35b^3d^5x\sqrt{c+d\sqrt{a+bx^2}}}$$

output
```
2*c*x^3/b/d^2/(c+d*(b*x^2+a)^(1/2))^(1/2)+4/35*(-5*a*d^2+32*c^2)*x*(c+d*(b
*x^2+a)^(1/2))^(1/2)/b^2/d^4+2/7*x^3*(c+d*(b*x^2+a)^(1/2))^(1/2)/b/d^2-96/
35*c*x*(b*x^2+a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)/b^2/d^3+8/35*a^(1/2)*c*
(-29*a*d^2+32*c^2)*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*EllipticE(
1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2
)*d))^(1/2))/b^3/d^5/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)-8/35*a^
(1/2)*(5*a^2*d^4-37*a*c^2*d^2+32*c^4)*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^(1/
2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2
^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b^3/d^5/x/(c+d*(b*x^2+a)^(
1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 89.91 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.44

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2\left(c-d\sqrt{a+bx^2}\right)\left(c+d\sqrt{a+bx^2}\right)^{3/2}\left(4ic\left(-32c^3-32\sqrt{a}c^2d+29acd^2+29\right.\right.}{}$$

input
```
Integrate[x^4/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*(c - d*Sqrt[a + b*x^2])*(c + d*Sqrt[a + b*x^2])^(3/2)*((4*I)*c*(-32*c^3
 - 32*Sqrt[a]*c^2*d + 29*a*c*d^2 + 29*a^(3/2)*d^3)*Sqrt[(d*(-Sqrt[a] + Sqr
t[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]
))/(c + d*Sqrt[a + b*x^2])]*Sqrt[c + d*Sqrt[a + b*x^2]]*(-c^2 + d^2*(a + b
*x^2))*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]
]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + d*(b*d*Sqrt[-c - Sqrt[a]*d]*x^2*(-6
4*c^4 + 6*c^2*d^2*(7*a - 4*b*x^2) + 80*c^3*d*Sqrt[a + b*x^2] + c*d^3*Sqrt[
a + b*x^2]*(-68*a + 13*b*x^2) + 5*d^4*(2*a^2 + a*b*x^2 - b^2*x^4)) - (4*I)
*Sqrt[a]*(-32*c^3 - 8*Sqrt[a]*c^2*d + 29*a*c*d^2 + 5*a^(3/2)*d^3)*Sqrt[(d*
(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] +
Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[c + d*Sqrt[a + b*x^2]]*(-c
^2 + d^2*(a + b*x^2))*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*
Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)])))/(35*b^3*d^6*Sqrt[-c
 - Sqrt[a]*d]*x*(c^2 - d^2*(a + b*x^2))^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}\,dx$$

$$\downarrow \text{7299}$$

$$\int \frac{x^4}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

input
```
Int[x^4/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^4}{\left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*d^2*x^6 - 2*sqrt(b*x^2 + a)*c*d*x^4 + (a*d^2 + c^2)*x^4)*sqrt(
sqrt(b*x^2 + a)*d + c)/(b^2*d^4*x^4 + a^2*d^4 - 2*a*c^2*d^2 + c^4 + 2*(a*b
*d^4 - b*c^2*d^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**4/(c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*d + c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Recursive assumption sageVARc>=(-`
u`*sageVARd) ignoredRecursive assumption sageVARc>=(-` u`*sageVARd) ignore
dRecursiv
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(x^4/(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^4/(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output

```
(24*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a**2*c*d**2*x - 12*sqrt(
a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*b*c*d**2*x**3 - 24*sqrt(a + b*x
**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*c**3*x + 16*sqrt(a + b*x**2)*sqrt(sqrt
(a + b*x**2)*d + c)*b*c**3*x**3 + 45*int((sqrt(sqrt(a + b*x**2)*d + c)*x**
8)/(3*a**4*d**6 + 9*a**3*b*d**6*x**2 - 10*a**3*c**2*d**4 + 9*a**2*b**2*d**
6*x**4 - 24*a**2*b*c**2*d**4*x**2 + 11*a**2*c**4*d**2 + 3*a*b**3*d**6*x**6
 - 18*a*b**2*c**2*d**4*x**4 + 19*a*b*c**4*d**2*x**2 - 4*a*c**6 - 4*b**3*c*
*2*d**4*x**6 + 8*b**2*c**4*d**2*x**4 - 4*b*c**6*x**2),x)*a**3*b**4*d**9 +
45*int((sqrt(sqrt(a + b*x**2)*d + c)*x**8)/(3*a**4*d**6 + 9*a**3*b*d**6*x*
*2 - 10*a**3*c**2*d**4 + 9*a**2*b**2*d**6*x**4 - 24*a**2*b*c**2*d**4*x**2
+ 11*a**2*c**4*d**2 + 3*a*b**3*d**6*x**6 - 18*a*b**2*c**2*d**4*x**4 + 19*a
*b*c**4*d**2*x**2 - 4*a*c**6 - 4*b**3*c**2*d**4*x**6 + 8*b**2*c**4*d**2*x*
*4 - 4*b*c**6*x**2),x)*a**2*b**5*d**9*x**2 - 165*int((sqrt(sqrt(a + b*x**2
)*d + c)*x**8)/(3*a**4*d**6 + 9*a**3*b*d**6*x**2 - 10*a**3*c**2*d**4 + 9*a
**2*b**2*d**6*x**4 - 24*a**2*b*c**2*d**4*x**2 + 11*a**2*c**4*d**2 + 3*a*b*
*3*d**6*x**6 - 18*a*b**2*c**2*d**4*x**4 + 19*a*b*c**4*d**2*x**2 - 4*a*c**6
 - 4*b**3*c**2*d**4*x**6 + 8*b**2*c**4*d**2*x**4 - 4*b*c**6*x**2),x)*a**2*
b**4*c**2*d**7 - 120*int((sqrt(sqrt(a + b*x**2)*d + c)*x**8)/(3*a**4*d**6
+ 9*a**3*b*d**6*x**2 - 10*a**3*c**2*d**4 + 9*a**2*b**2*d**6*x**4 - 24*a**2
*b*c**2*d**4*x**2 + 11*a**2*c**4*d**2 + 3*a*b**3*d**6*x**6 - 18*a*b**2*...
```

3.286 $\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 337

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2cx}{bd^2\sqrt{c+d\sqrt{a+bx^2}}}+\frac{2x\sqrt{c+d\sqrt{a+bx^2}}}{3bd^2} + \frac{16\sqrt{a}c\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{3b^2d^3x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} - \frac{4\sqrt{a}(4c^2-ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{3b^2d^3x\sqrt{c+d\sqrt{a+bx^2}}}$$

output

```
2*c*x/b/d^2/(c+d*(b*x^2+a)^(1/2))^(1/2)+2/3*x*(c+d*(b*x^2+a)^(1/2))^(1/2)/
b/d^2+16/3*a^(1/2)*c*(-b*x^2/a)^(1/2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*Elliptic
E(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1
/2)*d))^(1/2))/b^2/d^3/x/((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)-4/3*a
^(1/2)*(-a*d^2+4*c^2)*(-b*x^2/a)^(1/2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d
))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b^2/d^3/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 62.15 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.60

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx =$$

$$-\frac{2\left(c-d\sqrt{a+bx^2}\right)\left(c+d\sqrt{a+bx^2}\right)^{3/2}\left(8ic(c+\sqrt{a}d)\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{c+d\sqrt{a+bx^2}}\right.}{}$$

input
```
Integrate[x^2/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(-2*(c - d*Sqrt[a + b*x^2])*(c + d*Sqrt[a + b*x^2])^(3/2)*((8*I)*c*(c + Sq
rt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sq
rt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[c + d*Sqr
t[a + b*x^2]]*(-c^2 + d^2*(a + b*x^2))*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[
a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + d*(
b*d*Sqrt[-c - Sqrt[a]*d]*x^2*(4*c^2 - 5*c*d*Sqrt[a + b*x^2] + d^2*(a + b*x
^2)) - (2*I)*Sqrt[a]*(4*c + Sqrt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]
))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sq
rt[a + b*x^2])]*Sqrt[c + d*Sqrt[a + b*x^2]]*(-c^2 + d^2*(a + b*x^2))*Ellip
ticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqr
t[a]*d)/(c + Sqrt[a]*d)])))/(3*b^2*d^4*Sqrt[-c - Sqrt[a]*d]*x*(c^2 - d^2*(
a + b*x^2))^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{x^2}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

input
```
Int[x^2/(c + d*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^2}{\left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*d^2*x^4 - 2*sqrt(b*x^2 + a)*c*d*x^2 + (a*d^2 + c^2)*x^2)*sqrt(
sqrt(b*x^2 + a)*d + c)/(b^2*d^4*x^4 + a^2*d^4 - 2*a*c^2*d^2 + c^4 + 2*(a*b
*d^4 - b*c^2*d^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**2/(c + d*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*d + c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Recursive assumption sageVARc>=(-`
u`*sageVARd) ignoredRecursive assumption sageVARc>=(-` u`*sageVARd) ignore
dRecursiv
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{x^2}{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(x^2/(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^2/(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\text{too large to display}$$

input
```
int(x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output

```
( - 4*sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c)*a*c*d*x + 9*int((sqrt(
sqrt(a + b*x**2)*d + c)*x**6)/(3*a**4*d**6 + 9*a**3*b*d**6*x**2 - 10*a**3*
c**2*d**4 + 9*a**2*b**2*d**6*x**4 - 24*a**2*b*c**2*d**4*x**2 + 11*a**2*c**
4*d**2 + 3*a*b**3*d**6*x**6 - 18*a*b**2*c**2*d**4*x**4 + 19*a*b*c**4*d**2*
x**2 - 4*a*c**6 - 4*b**3*c**2*d**4*x**6 + 8*b**2*c**4*d**2*x**4 - 4*b*c**6
*x**2),x)*a**3*b**3*d**8 + 9*int((sqrt(sqrt(a + b*x**2)*d + c)*x**6)/(3*a*
*4*d**6 + 9*a**3*b*d**6*x**2 - 10*a**3*c**2*d**4 + 9*a**2*b**2*d**6*x**4 -
 24*a**2*b*c**2*d**4*x**2 + 11*a**2*c**4*d**2 + 3*a*b**3*d**6*x**6 - 18*a*
b**2*c**2*d**4*x**4 + 19*a*b*c**4*d**2*x**2 - 4*a*c**6 - 4*b**3*c**2*d**4*
x**6 + 8*b**2*c**4*d**2*x**4 - 4*b*c**6*x**2),x)*a**2*b**4*d**8*x**2 - 33*
int((sqrt(sqrt(a + b*x**2)*d + c)*x**6)/(3*a**4*d**6 + 9*a**3*b*d**6*x**2
- 10*a**3*c**2*d**4 + 9*a**2*b**2*d**6*x**4 - 24*a**2*b*c**2*d**4*x**2 + 1
1*a**2*c**4*d**2 + 3*a*b**3*d**6*x**6 - 18*a*b**2*c**2*d**4*x**4 + 19*a*b*
c**4*d**2*x**2 - 4*a*c**6 - 4*b**3*c**2*d**4*x**6 + 8*b**2*c**4*d**2*x**4
- 4*b*c**6*x**2),x)*a**2*b**3*c**2*d**6 - 24*int((sqrt(sqrt(a + b*x**2)*d
+ c)*x**6)/(3*a**4*d**6 + 9*a**3*b*d**6*x**2 - 10*a**3*c**2*d**4 + 9*a**2*
b**2*d**6*x**4 - 24*a**2*b*c**2*d**4*x**2 + 11*a**2*c**4*d**2 + 3*a*b**3*d
**6*x**6 - 18*a*b**2*c**2*d**4*x**4 + 19*a*b*c**4*d**2*x**2 - 4*a*c**6 - 4
*b**3*c**2*d**4*x**6 + 8*b**2*c**4*d**2*x**4 - 4*b*c**6*x**2),x)*a*b**4*c*
*2*d**6*x**2 + 40*int((sqrt(sqrt(a + b*x**2)*d + c)*x**6)/(3*a**4*d**6 ...
```

3.287 $\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 309

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2cx}{(c^2-ad^2)\sqrt{c+d\sqrt{a+bx^2}}} + \frac{2\sqrt{a}c\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{bd(c^2-ad^2)x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} - \frac{2\sqrt{a}\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{bdx\sqrt{c+d\sqrt{a+bx^2}}}$$

output

```
2*c*x/(-a*d^2+c^2)/(c+d*(b*x^2+a)^(1/2))^(1/2)+2*a^(1/2)*c*(-b*x^2/a)^(1/2
)*(c+d*(b*x^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b/d/(-a*d^2+c^2)/x/((
c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)-2*a^(1/2)*(-b*x^2/a)^(1/2)*((c+d
*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^
(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/b/d/x/(c+d*(
b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 60.44 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{2i\left(c-d\sqrt{a+bx^2}\right)^2\sqrt{\frac{d\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\sqrt{\frac{d\left(\sqrt{a}+\sqrt{a+bx^2}\right)}{c+d\sqrt{a+bx^2}}}\left(c+d\sqrt{a+bx^2}\right)^3\left(cE\left(i\right.\right.}{bd^2\sqrt{-c-\sqrt{a}d}\,(c-\sqrt{}}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^(-3/2),x]
```

output
```
((2*I)*(c - d*Sqrt[a + b*x^2])^2*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c
+ d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a +
 b*x^2])]*(c + d*Sqrt[a + b*x^2])^3*(c*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[
a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] - Sqr
t[a]*d*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]
]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)]))/(b*d^2*Sqrt[-c - Sqrt[a]*d]*(c - Sq
rt[a]*d)*x*(c^2 - d^2*(a + b*x^2))^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{1}{\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^(-3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{\left(c+d\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(1/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*d^2*x^2 + a*d^2 - 2*sqrt(b*x^2 + a)*c*d + c^2)*sqrt(sqrt(b*x^2
 + a)*d + c)/(b^2*d^4*x^4 + a^2*d^4 - 2*a*c^2*d^2 + c^4 + 2*(a*b*d^4 - b*c
^2*d^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(1/(c + d*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(1/(c + d*(a + b*x^2)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^4+2ab\,d^4x^2-2b\,c^2d^2x^2+a^2d^4-2a\,c^2d^2+c^4}dx\right)a\,d^2 + \left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^4+2ab\,d^4x^2-2b\,c^2d^2x^2+a^2d^4-2a\,c^2d^2+c^4}dx\right)c^2 + \left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}\,x^2}{b^2d^4x^4+2ab\,d^4x^2-2b\,c^2d^2x^2+a^2d^4-2a\,c^2d^2+c^4}dx\right)b\,d^2 - 2\left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^4+2ab\,d^4x^2-2b\,c^2d^2x^2+a^2d^4-2a\,c^2d^2+c^4}dx\right)cd$$

input
```
int(1/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**4 + 2*a*b*d**4*x**2 - 2*a*c**2*d
**2 + b**2*d**4*x**4 - 2*b*c**2*d**2*x**2 + c**4),x)*a*d**2 + int(sqrt(sqr
t(a + b*x**2)*d + c)/(a**2*d**4 + 2*a*b*d**4*x**2 - 2*a*c**2*d**2 + b**2*d
**4*x**4 - 2*b*c**2*d**2*x**2 + c**4),x)*c**2 + int((sqrt(sqrt(a + b*x**2)
*d + c)*x**2)/(a**2*d**4 + 2*a*b*d**4*x**2 - 2*a*c**2*d**2 + b**2*d**4*x**
4 - 2*b*c**2*d**2*x**2 + c**4),x)*b*d**2 - 2*int((sqrt(a + b*x**2)*sqrt(sq
rt(a + b*x**2)*d + c))/(a**2*d**4 + 2*a*b*d**4*x**2 - 2*a*c**2*d**2 + b**2
*d**4*x**4 - 2*b*c**2*d**2*x**2 + c**4),x)*c*d
```

3.288 $\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 367

$$\begin{aligned}\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx &= \frac{4bcd^2x}{(c^2-ad^2)^2\sqrt{c+d\sqrt{a+bx^2}}} \\ &- \frac{c-d\sqrt{a+bx^2}}{(c^2-ad^2)\,x\sqrt{c+d\sqrt{a+bx^2}}} \\ &+ \frac{4\sqrt{a}cd\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{(c^2-ad^2)^2\,x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} \\ &- \frac{\sqrt{a}d\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{(c^2-ad^2)\,x\sqrt{c+d\sqrt{a+bx^2}}}\end{aligned}$$

output
```
4*b*c*d^2*x/(-a*d^2+c^2)^2/(c+d*(b*x^2+a)^(1/2))^(1/2)-(c-d*(b*x^2+a)^(1/2
))/(-a*d^2+c^2)/x/(c+d*(b*x^2+a)^(1/2))^(1/2)+4*a^(1/2)*c*d*(-b*x^2/a)^(1/
2)*(c+d*(b*x^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(
1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/(-a*d^2+c^2)^2/x/((c
+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)-a^(1/2)*d*(-b*x^2/a)^(1/2)*((c+d*
(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2+a)^(1/2)/a^(
1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/(-a*d^2+c^2)/
x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 63.44 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx =$$

$$-\frac{\left(c-d\sqrt{a+bx^2}\right)\left(c+d\sqrt{a+bx^2}\right)^{3/2}\left(-\sqrt{-c-\sqrt{a}d}(c-\sqrt{a}d)\left(c^2-2cd\sqrt{a+bx^2}+d^2(a+bx^2)\right)-4i\right.}{}$$

input
```
Integrate[1/(x^2*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
-(((c - d*Sqrt[a + b*x^2])*(c + d*Sqrt[a + b*x^2])^(3/2)*(-(Sqrt[-c - Sqrt
[a]*d]*(c - Sqrt[a]*d)*(c^2 - 2*c*d*Sqrt[a + b*x^2] + d^2*(a + b*x^2))) -
(4*I)*c*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqr
t[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[c + d*Sqrt
[a + b*x^2]]*(-c^2 + d^2*(a + b*x^2))*EllipticE[I*ArcSinh[Sqrt[-c - Sqrt[a
]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)] + I*(3
*c + Sqrt[a]*d)*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^
2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[c
+ d*Sqrt[a + b*x^2]]*(-c^2 + d^2*(a + b*x^2))*EllipticF[I*ArcSinh[Sqrt[-c
- Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)
]))/((-c - Sqrt[a]*d)^(3/2)*(c - Sqrt[a]*d)^2*x*(c^2 - d^2*(a + b*x^2))^2)
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{1}{x^2 \left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

input
```
Int[1/(x^2*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^2 \left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(1/x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*d^2*x^2 + a*d^2 - 2*sqrt(b*x^2 + a)*c*d + c^2)*sqrt(sqrt(b*x^2
 + a)*d + c)/(b^2*d^4*x^6 + 2*(a*b*d^4 - b*c^2*d^2)*x^4 + (a^2*d^4 - 2*a*c
^2*d^2 + c^4)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(1/(x**2*(c + d*sqrt(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((sqrt(b*x^2 + a)*d + c)^(3/2)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^2\,\left(c+d\,\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(c + d*(a + b*x^2)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^2*(c + d*(a + b*x^2)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^6+2ab\,d^4x^4-2b\,c^2d^2x^4+a^2d^4x^2-2a\,c^2d^2x^2+c^4x^2}dx\right)a\,d^2 + \left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^6+2ab\,d^4x^4-2b\,c^2d^2x^4+a^2d^4x^2-2a\,c^2d^2x^2+c^4x^2}dx\right)c^2 + \left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^4+2ab\,d^4x^2-2b\,c^2d^2x^2+a^2d^4-2a\,c^2d^2+c^4}dx\right)b\,d^2 - 2\left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^6+2ab\,d^4x^4-2b\,c^2d^2x^4+a^2d^4x^2-2a\,c^2d^2x^2+c^4x^2}dx\right)cd$$

input
```
int(1/x^2/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**4*x**2 + 2*a*b*d**4*x**4 - 2*a*c
**2*d**2*x**2 + b**2*d**4*x**6 - 2*b*c**2*d**2*x**4 + c**4*x**2),x)*a*d**2
 + int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**4*x**2 + 2*a*b*d**4*x**4 - 2*
a*c**2*d**2*x**2 + b**2*d**4*x**6 - 2*b*c**2*d**2*x**4 + c**4*x**2),x)*c**
2 + int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**4 + 2*a*b*d**4*x**2 - 2*a*c*
*2*d**2 + b**2*d**4*x**4 - 2*b*c**2*d**2*x**2 + c**4),x)*b*d**2 - 2*int((s
qrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c))/(a**2*d**4*x**2 + 2*a*b*d**4
*x**4 - 2*a*c**2*d**2*x**2 + b**2*d**4*x**6 - 2*b*c**2*d**2*x**4 + c**4*x*
*2),x)*c*d
```

3.289 $\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 492

$$\begin{aligned}\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx &= \frac{b^2cd^2(3c^2+29ad^2)\,x}{6a\left(c^2-ad^2\right)^3\sqrt{c+d\sqrt{a+bx^2}}} \\ &- \frac{c-d\sqrt{a+bx^2}}{3\left(c^2-ad^2\right)x^3\sqrt{c+d\sqrt{a+bx^2}}} - \frac{bd\left(8acd-(3c^2+5ad^2)\sqrt{a+bx^2}\right)}{6a\left(c^2-ad^2\right)^2x\sqrt{c+d\sqrt{a+bx^2}}} \\ &+ \frac{bcd(3c^2+29ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{c+d\sqrt{a+bx^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{6\sqrt{a}\left(c^2-ad^2\right)^3x\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}} \\ &- \frac{bd(3c^2+5ad^2)\sqrt{-\frac{bx^2}{a}}\sqrt{\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{c+\sqrt{a}d}\right)}{6\sqrt{a}\left(c^2-ad^2\right)^2x\sqrt{c+d\sqrt{a+bx^2}}}\end{aligned}$$

output
```
1/6*b^2*c*d^2*(29*a*d^2+3*c^2)*x/a/(-a*d^2+c^2)^3/(c+d*(b*x^2+a)^(1/2))^(1
/2)-1/3*(c-d*(b*x^2+a)^(1/2))/(-a*d^2+c^2)/x^3/(c+d*(b*x^2+a)^(1/2))^(1/2)
-1/6*b*d*(8*a*c*d-(5*a*d^2+3*c^2)*(b*x^2+a)^(1/2))/a/(-a*d^2+c^2)^2/x/(c+d
*(b*x^2+a)^(1/2))^(1/2)+1/6*b*c*d*(29*a*d^2+3*c^2)*(-b*x^2/a)^(1/2)*(c+d*(
b*x^2+a)^(1/2))^(1/2)*EllipticE(1/2*(1-(b*x^2+a)^(1/2)/a^(1/2))^(1/2)*2^(1
/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a*d^2+c^2)^3/x/((c+
d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)-1/6*b*d*(5*a*d^2+3*c^2)*(-b*x^2/a)
^(1/2)*((c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*EllipticF(1/2*(1-(b*x^2
+a)^(1/2)/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(c+a^(1/2)*d))^(1/2))/
a^(1/2)/(-a*d^2+c^2)^2/x/(c+d*(b*x^2+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 64.44 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \frac{\left(c-d\sqrt{a+bx^2}\right)\left(c+d\sqrt{a+bx^2}\right)^{3/2}\left(-\sqrt{-c-\sqrt{a}d}(c-\sqrt{a}d)\left(-2a^3d^4+\right.\right.}{}$$

input
```
Integrate[1/(x^4*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
((c - d*Sqrt[a + b*x^2])*(c + d*Sqrt[a + b*x^2])^(3/2)*(-(Sqrt[-c - Sqrt[a
]*d]*(c - Sqrt[a]*d)*(-2*a^3*d^4 + 3*b*c^2*d*x^2*(b*d*x^2 - c*Sqrt[a + b*x
^2]) + a^2*d^3*(3*b*d*x^2 + 4*c*Sqrt[a + b*x^2]) + a*(2*c^4 + 13*b*c^2*d^2
*x^2 + 5*b^2*d^4*x^4 - 4*c^3*d*Sqrt[a + b*x^2] - 13*b*c*d^3*x^2*Sqrt[a + b
*x^2]))) - I*b*c*(3*c^2 + 29*a*d^2)*x^2*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2
]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*S
qrt[a + b*x^2])]*Sqrt[c + d*Sqrt[a + b*x^2]]*(-c^2 + d^2*(a + b*x^2))*Elli
pticE[I*ArcSinh[Sqrt[-c - Sqrt[a]*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sq
rt[a]*d)/(c + Sqrt[a]*d)] + I*Sqrt[a]*b*d*(3*c^2 + 24*Sqrt[a]*c*d + 5*a*d^
2)*x^2*Sqrt[(d*(-Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt
[(d*(Sqrt[a] + Sqrt[a + b*x^2]))/(c + d*Sqrt[a + b*x^2])]*Sqrt[c + d*Sqrt[
a + b*x^2]]*(-c^2 + d^2*(a + b*x^2))*EllipticF[I*ArcSinh[Sqrt[-c - Sqrt[a]
*d]/Sqrt[c + d*Sqrt[a + b*x^2]]], (c - Sqrt[a]*d)/(c + Sqrt[a]*d)]))/(6*a*
(-c - Sqrt[a]*d)^(5/2)*(c - Sqrt[a]*d)^3*x^3*(c^2 - d^2*(a + b*x^2))^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{1}{x^4\left(d\sqrt{a+bx^2}+c\right)^{3/2}}dx$$

input
```
Int[1/(x^4*(c + d*Sqrt[a + b*x^2])^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^4\left(c+d\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(1/x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*d^2*x^2 + a*d^2 - 2*sqrt(b*x^2 + a)*c*d + c^2)*sqrt(sqrt(b*x^2
 + a)*d + c)/(b^2*d^4*x^8 + 2*(a*b*d^4 - b*c^2*d^2)*x^6 + (a^2*d^4 - 2*a*c
^2*d^2 + c^4)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(c+d*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(1/(x**4*(c + d*sqrt(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{bx^2+a}d+c\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((sqrt(b*x^2 + a)*d + c)^(3/2)*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^4\left(c+d\sqrt{b\,x^2+a}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(c + d*(a + b*x^2)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^4*(c + d*(a + b*x^2)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{1}{x^4\left(c+d\sqrt{a+bx^2}\right)^{3/2}}\,dx&=\left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^8+2ab\,d^4x^6-2b\,c^2d^2x^6+a^2d^4x^4-2a\,c^2d^2x^4+c^4x^4}dx\right)a\,d^2\\&+\left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^8+2ab\,d^4x^6-2b\,c^2d^2x^6+a^2d^4x^4-2a\,c^2d^2x^4+c^4x^4}dx\right)c^2\\&+\left(\int \frac{\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^6+2ab\,d^4x^4-2b\,c^2d^2x^4+a^2d^4x^2-2a\,c^2d^2x^2+c^4x^2}dx\right)b\,d^2\\&-2\left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{\sqrt{b\,x^2+a}\,d+c}}{b^2d^4x^8+2ab\,d^4x^6-2b\,c^2d^2x^6+a^2d^4x^4-2a\,c^2d^2x^4+c^4x^4}dx\right)cd\end{aligned}$$

input
```
int(1/x^4/(c+d*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**4*x**4 + 2*a*b*d**4*x**6 - 2*a*c
**2*d**2*x**4 + b**2*d**4*x**8 - 2*b*c**2*d**2*x**6 + c**4*x**4),x)*a*d**2
 + int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**4*x**4 + 2*a*b*d**4*x**6 - 2*
a*c**2*d**2*x**4 + b**2*d**4*x**8 - 2*b*c**2*d**2*x**6 + c**4*x**4),x)*c**
2 + int(sqrt(sqrt(a + b*x**2)*d + c)/(a**2*d**4*x**2 + 2*a*b*d**4*x**4 - 2
*a*c**2*d**2*x**2 + b**2*d**4*x**6 - 2*b*c**2*d**2*x**4 + c**4*x**2),x)*b*
d**2 - 2*int((sqrt(a + b*x**2)*sqrt(sqrt(a + b*x**2)*d + c))/(a**2*d**4*x*
*4 + 2*a*b*d**4*x**6 - 2*a*c**2*d**2*x**4 + b**2*d**4*x**8 - 2*b*c**2*d**2
*x**6 + c**4*x**4),x)*c*d
```

3.290 $\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 252

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx=-\frac{c(c^2-ad^2)^2\left(c+d\sqrt{a+bx^2}\right)^{1+p}}{b^3d^6(1+p)}+\frac{(5c^4-6ac^2d^2+a^2d^4)\left(c+d\sqrt{a+bx^2}\right)^{2+p}}{b^3d^6(2+p)}-\frac{2c(5c^2-3ad^2)\left(c+d\sqrt{a+bx^2}\right)^{3+p}}{b^3d^6(3+p)}+\frac{2(5c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{4+p}}{b^3d^6(4+p)}-\frac{5c\left(c+d\sqrt{a+bx^2}\right)^{5+p}}{b^3d^6(5+p)}+\frac{\left(c+d\sqrt{a+bx^2}\right)^{6+p}}{b^3d^6(6+p)}$$

output
```
-c*(-a*d^2+c^2)^2*(c+d*(b*x^2+a)^(1/2))^(p+1)/b^3/d^6/(p+1)+(a^2*d^4-6*a*c
^2*d^2+5*c^4)*(c+d*(b*x^2+a)^(1/2))^(2+p)/b^3/d^6/(2+p)-2*c*(-3*a*d^2+5*c^
2)*(c+d*(b*x^2+a)^(1/2))^(3+p)/b^3/d^6/(3+p)+2*(-a*d^2+5*c^2)*(c+d*(b*x^2+
a)^(1/2))^(4+p)/b^3/d^6/(4+p)-5*c*(c+d*(b*x^2+a)^(1/2))^(5+p)/b^3/d^6/(5+p
)+(c+d*(b*x^2+a)^(1/2))^(6+p)/b^3/d^6/(6+p)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.19

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$
$$=\frac{\left(c+d\sqrt{a+bx^2}\right)^{1+p}\left(-120c^5+120c^4d(1+p)\sqrt{a+bx^2}-12c^3d^2(4a(-5+p+p^2)+5b(2+3p+p^2)\,x^2)\right.}{}$$

input
```
Integrate[x^5*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
((c + d*Sqrt[a + b*x^2])^(1 + p)*(-120*c^5 + 120*c^4*d*(1 + p)*Sqrt[a + b*
x^2] - 12*c^3*d^2*(4*a*(-5 + p + p^2) + 5*b*(2 + 3*p + p^2)*x^2) + 4*c^2*d
^3*(1 + p)*Sqrt[a + b*x^2]*(2*a*(-30 - 4*p + p^2) + 5*b*(6 + 5*p + p^2)*x^
2) + d^5*(15 + 23*p + 9*p^2 + p^3)*Sqrt[a + b*x^2]*(8*a^2 - 4*a*b*(2 + p)*
x^2 + b^2*(8 + 6*p + p^2)*x^4) - c*d^4*(-8*a^2*(-15 + 10*p + 12*p^2 + 2*p^
3) + 4*a*b*(-30 - 43*p - 10*p^2 + 4*p^3 + p^4)*x^2 + 5*b^2*(24 + 50*p + 35
*p^2 + 10*p^3 + p^4)*x^4)))/(b^3*d^6*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 +
p)*(6 + p))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {7283, 896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(d\sqrt{a+bx^2}+c\right)^p\,dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2}\int x^4\left(c+d\sqrt{bx^2+a}\right)^p\,dx^2$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int b^2x^4\left(c+d\sqrt{bx^2+a}\right)^p d(bx^2+a)}{2b^3}$$

$$\Big\downarrow \; 1732$$

$$\frac{\int \sqrt{bx^2+a}\left(a-x^4\right)^2\left(c+d\sqrt{bx^2+a}\right)^p d\sqrt{bx^2+a}}{b^3}$$

$$\Big\downarrow \; 522$$

$$\frac{\int \left(-\frac{c(c^2-ad^2)^2\left(c+d\sqrt{bx^2+a}\right)^p}{d^5}+\frac{(5c^4-6ad^2c^2+a^2d^4)\left(c+d\sqrt{bx^2+a}\right)^{p+1}}{d^5}-\frac{2(5c^3-3acd^2)\left(c+d\sqrt{bx^2+a}\right)^{p+2}}{d^5}-\frac{2(ad^2-5c^2)\left(c+d\sqrt{bx\right.}}{d^5}}{b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(a^2d^4-6ac^2d^2+5c^4)\left(d\sqrt{a+bx^2}+c\right)^{p+2}}{d^6(p+2)}-\frac{c(c^2-ad^2)^2\left(d\sqrt{a+bx^2}+c\right)^{p+1}}{d^6(p+1)}-\frac{2c(5c^2-3ad^2)\left(d\sqrt{a+bx^2}+c\right)^{p+3}}{d^6(p+3)}+\frac{2(5c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right.}{d^6(p+4)}}{b^3}$$

input
```
Int[x^5*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
(-((c*(c^2 - a*d^2)^2*(c + d*Sqrt[a + b*x^2])^(1 + p))/(d^6*(1 + p))) + ((
5*c^4 - 6*a*c^2*d^2 + a^2*d^4)*(c + d*Sqrt[a + b*x^2])^(2 + p))/(d^6*(2 +
p)) - (2*c*(5*c^2 - 3*a*d^2)*(c + d*Sqrt[a + b*x^2])^(3 + p))/(d^6*(3 + p)
) + (2*(5*c^2 - a*d^2)*(c + d*Sqrt[a + b*x^2])^(4 + p))/(d^6*(4 + p)) - (5
*c*(c + d*Sqrt[a + b*x^2])^(5 + p))/(d^6*(5 + p)) + (c + d*Sqrt[a + b*x^2]
)^(6 + p)/(d^6*(6 + p)))/b^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^5 \left(c+d\sqrt{b\,x^2+a}\right)^p dx$$

input
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(240) = 480.

Time = 24.36 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.94

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx = \frac{(120\,a^3d^6-360\,a^2c^2d^4+360\,ac^4d^2+(b^3d^6p^5+15\,b^3d^6p^4+85\,b^3d^6p^3+225\,b^3d^6p^2+274\,b^3d^6p+120\,b^3c}{}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
(120*a^3*d^6 - 360*a^2*c^2*d^4 + 360*a*c^4*d^2 + (b^3*d^6*p^5 + 15*b^3*d^6
*p^4 + 85*b^3*d^6*p^3 + 225*b^3*d^6*p^2 + 274*b^3*d^6*p + 120*b^3*d^6)*x^6
 - 120*c^6 + (a*b^2*d^6*p^5 + (11*a*b^2*d^6 - 5*b^2*c^2*d^4)*p^4 + (41*a*b
^2*d^6 - 30*b^2*c^2*d^4)*p^3 + (61*a*b^2*d^6 - 55*b^2*c^2*d^4)*p^2 + 30*(a
*b^2*d^6 - b^2*c^2*d^4)*p)*x^4 + 8*(a^3*d^6 + 3*a^2*c^2*d^4)*p^3 + 24*(3*a
^3*d^6 + 3*a^2*c^2*d^4 - 2*a*c^4*d^2)*p^2 - 4*((a^2*b*d^6 + a*b*c^2*d^4)*p
^4 + 3*(3*a^2*b*d^6 - a*b*c^2*d^4)*p^3 + (23*a^2*b*d^6 - 34*a*b*c^2*d^4 +
15*b*c^4*d^2)*p^2 + 15*(a^2*b*d^6 - 2*a*b*c^2*d^4 + b*c^4*d^2)*p)*x^2 + 8*
(23*a^3*d^6 - 24*a^2*c^2*d^4 + 9*a*c^4*d^2)*p + ((b^2*c*d^5*p^5 + 10*b^2*c
*d^5*p^4 + 35*b^2*c*d^5*p^3 + 50*b^2*c*d^5*p^2 + 24*b^2*c*d^5*p)*x^4 + 8*(
3*a^2*c*d^5 + a*c^3*d^3)*p^3 + 24*(7*a^2*c*d^5 - 3*a*c^3*d^3)*p^2 - 4*(2*a
*b*c*d^5*p^4 + 5*(3*a*b*c*d^5 - b*c^3*d^3)*p^3 + (31*a*b*c*d^5 - 15*b*c^3*
d^3)*p^2 + 2*(9*a*b*c*d^5 - 5*b*c^3*d^3)*p)*x^2 + 8*(33*a^2*c*d^5 - 40*a*c
^3*d^3 + 15*c^5*d)*p)*sqrt(b*x^2 + a))*(sqrt(b*x^2 + a)*d + c)^p/(b^3*d^6*
p^6 + 21*b^3*d^6*p^5 + 175*b^3*d^6*p^4 + 735*b^3*d^6*p^3 + 1624*b^3*d^6*p^
2 + 1764*b^3*d^6*p + 720*b^3*d^6)
```

Sympy [F]

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$

input
```
integrate(x**5*(c+d*(b*x**2+a)**(1/2))**p,x)
```

output
```
Integral(x**5*(c + d*sqrt(a + b*x**2))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.71

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$
$$=\frac{\frac{\left((bx^2+a)d^2(p+1)+\sqrt{bx^2+a}cdp-c^2\right)\left(\sqrt{bx^2+a}d+c\right)^pa^2}{(p^2+3p+2)d^2}-\frac{2\left((p^3+6p^2+11p+6)(bx^2+a)^2d^4+(p^3+3p^2+2p)(bx^2+a)^{\frac{3}{2}}cd^3-3(bx^2+a)(p^2\cdots\right.}{(p^4+10p^3+35p^2+50p+24)d^4}\cdots}{\cdots}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
(((b*x^2 + a)*d^2*(p + 1) + sqrt(b*x^2 + a)*c*d*p - c^2)*(sqrt(b*x^2 + a)*
d + c)^p*a^2/((p^2 + 3*p + 2)*d^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(b*x^2 +
a)^2*d^4 + (p^3 + 3*p^2 + 2*p)*(b*x^2 + a)^(3/2)*c*d^3 - 3*(b*x^2 + a)*(p^
2 + p)*c^2*d^2 + 6*sqrt(b*x^2 + a)*c^3*d*p - 6*c^4)*(sqrt(b*x^2 + a)*d + c
)^p*a/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*d^4) + ((p^5 + 15*p^4 + 85*p^3
+ 225*p^2 + 274*p + 120)*(b*x^2 + a)^3*d^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p
^2 + 24*p)*(b*x^2 + a)^(5/2)*c*d^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(b*x^2
 + a)^2*c^2*d^4 + 20*(p^3 + 3*p^2 + 2*p)*(b*x^2 + a)^(3/2)*c^3*d^3 - 60*(b
*x^2 + a)*(p^2 + p)*c^4*d^2 + 120*sqrt(b*x^2 + a)*c^5*d*p - 120*c^6)*(sqrt
(b*x^2 + a)*d + c)^p/((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*
p + 720)*d^6))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4289 vs. 2(240) = 480.

Time = 0.35 (sec) , antiderivative size = 4289, normalized size of antiderivative = 17.02

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x^5*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="giac")
```

output
```
((sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*a^2*d^4*p^5*sgn((sqrt
(b*x^2 + a)*d + c)*d - c*d) - (sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d +
 c)^p*a^2*c*d^4*p^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 19*(sqrt(b*x^2
+ a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*a^2*d^4*p^4*sgn((sqrt(b*x^2 + a)*d
 + c)*d - c*d) - 20*(sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d + c)^p*a^2*
c*d^4*p^4*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 2*(sqrt(b*x^2 + a)*d + c)
^4*(sqrt(b*x^2 + a)*d + c)^p*a*d^2*p^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d
) + 6*(sqrt(b*x^2 + a)*d + c)^3*(sqrt(b*x^2 + a)*d + c)^p*a*c*d^2*p^5*sgn(
(sqrt(b*x^2 + a)*d + c)*d - c*d) - 6*(sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2
 + a)*d + c)^p*a*c^2*d^2*p^5*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 2*(sqr
t(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d + c)^p*a*c^3*d^2*p^5*sgn((sqrt(b*x^
2 + a)*d + c)*d - c*d) + 137*(sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d
+ c)^p*a^2*d^4*p^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 155*(sqrt(b*x^2
+ a)*d + c)*(sqrt(b*x^2 + a)*d + c)^p*a^2*c*d^4*p^3*sgn((sqrt(b*x^2 + a)*d
 + c)*d - c*d) - 34*(sqrt(b*x^2 + a)*d + c)^4*(sqrt(b*x^2 + a)*d + c)^p*a*
d^2*p^4*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 108*(sqrt(b*x^2 + a)*d + c)
^3*(sqrt(b*x^2 + a)*d + c)^p*a*c*d^2*p^4*sgn((sqrt(b*x^2 + a)*d + c)*d - c
*d) - 114*(sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*a*c^2*d^2*p^
4*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 40*(sqrt(b*x^2 + a)*d + c)*(sqrt(
b*x^2 + a)*d + c)^p*a*c^3*d^2*p^4*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) ...
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int x^5\,\left(c+d\,\sqrt{b\,x^2+a}\right)^p\,dx$$

input
```
int(x^5*(c + d*(a + b*x^2)^(1/2))^p,x)
```

output
```
int(x^5*(c + d*(a + b*x^2)^(1/2))^p, x)
```

Reduce [F]

$$\int x^5\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int x^5\left(c+d\sqrt{b\,x^2+a}\right)^p dx$$

input
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int(x^5*(c+d*(b*x^2+a)^(1/2))^p,x)
```

3.291 $\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx=-\frac{c(c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{1+p}}{b^2d^4(1+p)}+\frac{(3c^2-ad^2)\left(c+d\sqrt{a+bx^2}\right)^{2+p}}{b^2d^4(2+p)}-\frac{3c\left(c+d\sqrt{a+bx^2}\right)^{3+p}}{b^2d^4(3+p)}+\frac{\left(c+d\sqrt{a+bx^2}\right)^{4+p}}{b^2d^4(4+p)}$$

output
```
-c*(-a*d^2+c^2)*(c+d*(b*x^2+a)^(1/2))^(p+1)/b^2/d^4/(p+1)+(-a*d^2+3*c^2)*(
c+d*(b*x^2+a)^(1/2))^(2+p)/b^2/d^4/(2+p)-3*c*(c+d*(b*x^2+a)^(1/2))^(3+p)/b
^2/d^4/(3+p)+(c+d*(b*x^2+a)^(1/2))^(4+p)/b^2/d^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$
$$=\frac{\left(c+d\sqrt{a+bx^2}\right)^{1+p}\left(-6c^3+6c^2d(1+p)\sqrt{a+bx^2}+d^3(3+4p+p^2)\sqrt{a+bx^2}(-2a+b(2+p)x^2)-cd^2\right.}{b^2d^4(1+p)(2+p)(3+p)(4+p)}$$

input
```
Integrate[x^3*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
((c + d*Sqrt[a + b*x^2])^(1 + p)*(-6*c^3 + 6*c^2*d*(1 + p)*Sqrt[a + b*x^2]
 + d^3*(3 + 4*p + p^2)*Sqrt[a + b*x^2]*(-2*a + b*(2 + p)*x^2) - c*d^2*(2*a
*(-3 + p + p^2) + 3*b*(2 + 3*p + p^2)*x^2)))/(b^2*d^4*(1 + p)*(2 + p)*(3 +
 p)*(4 + p))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7283, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d\sqrt{a+bx^2}+c\right)^p\,dx$$

$$\Big\downarrow \; 7283$$

$$\frac{1}{2}\int x^2\left(c+d\sqrt{bx^2+a}\right)^p\,dx^2$$

$$\Big\downarrow \; 896$$

$$\frac{\int bx^2\left(c+d\sqrt{bx^2+a}\right)^p\,d(bx^2+a)}{2b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -bx^2\left(c+d\sqrt{bx^2+a}\right)^p d(bx^2+a)}{2b^2}$$

$$\Big\downarrow \ 1732$$

$$-\frac{\int \sqrt{bx^2+a}(a-x^4)\left(c+d\sqrt{bx^2+a}\right)^p d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \ 522$$

$$-\frac{\int \left(\frac{(c^3-acd^2)\left(c+d\sqrt{bx^2+a}\right)^p}{d^3}+\frac{(ad^2-3c^2)\left(c+d\sqrt{bx^2+a}\right)^{p+1}}{d^3}+\frac{3c\left(c+d\sqrt{bx^2+a}\right)^{p+2}}{d^3}-\frac{\left(c+d\sqrt{bx^2+a}\right)^{p+3}}{d^3}\right) d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{c(c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{p+1}}{d^4(p+1)}-\frac{(3c^2-ad^2)\left(d\sqrt{a+bx^2}+c\right)^{p+2}}{d^4(p+2)}+\frac{3c\left(d\sqrt{a+bx^2}+c\right)^{p+3}}{d^4(p+3)}-\frac{\left(d\sqrt{a+bx^2}+c\right)^{p+4}}{d^4(p+4)}}{b^2}$$

input
```
Int[x^3*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
-(((c*(c^2 - a*d^2)*(c + d*Sqrt[a + b*x^2])^(1 + p))/(d^4*(1 + p)) - ((3*c
^2 - a*d^2)*(c + d*Sqrt[a + b*x^2])^(2 + p))/(d^4*(2 + p)) + (3*c*(c + d*S
qrt[a + b*x^2])^(3 + p))/(d^4*(3 + p)) - (c + d*Sqrt[a + b*x^2])^(4 + p)/(
d^4*(4 + p)))/b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^3 \left(c + d\sqrt{b\,x^2 + a}\right)^p dx$$

input
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(143) = 286$.

Time = 6.43 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.01

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx = -\frac{\left(6\,a^2d^4-12\,ac^2d^2-(b^2d^4p^3+6\,b^2d^4p^2+11\,b^2d^4p+6\,b^2d^4)x^4+6\,c^4+2\,(a^2d^4+ac^2d^2)p^2-(abd^4p^3\right.}{}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
-(6*a^2*d^4 - 12*a*c^2*d^2 - (b^2*d^4*p^3 + 6*b^2*d^4*p^2 + 11*b^2*d^4*p +
 6*b^2*d^4)*x^4 + 6*c^4 + 2*(a^2*d^4 + a*c^2*d^2)*p^2 - (a*b*d^4*p^3 + (4*
a*b*d^4 - 3*b*c^2*d^2)*p^2 + 3*(a*b*d^4 - b*c^2*d^2)*p)*x^2 + 4*(2*a^2*d^4
 - a*c^2*d^2)*p + (4*a*c*d^3*p^2 - (b*c*d^3*p^3 + 3*b*c*d^3*p^2 + 2*b*c*d^
3*p)*x^2 + 2*(5*a*c*d^3 - 3*c^3*d)*p)*sqrt(b*x^2 + a))*(sqrt(b*x^2 + a)*d
+ c)^p/(b^2*d^4*p^4 + 10*b^2*d^4*p^3 + 35*b^2*d^4*p^2 + 50*b^2*d^4*p + 24*
b^2*d^4)
```

Sympy [F]

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx = \int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$

input
```
integrate(x**3*(c+d*(b*x**2+a)**(1/2))**p,x)
```

output
```
Integral(x**3*(c + d*sqrt(a + b*x**2))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.34

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx=$$

$$-\frac{\frac{\left((bx^2+a)d^2(p+1)+\sqrt{bx^2+a}cdp-c^2\right)\left(\sqrt{bx^2+a}d+c\right)^pa}{(p^2+3p+2)d^2}-\frac{\left((p^3+6p^2+11p+6)\left(bx^2+a\right)^2d^4+(p^3+3p^2+2p)\left(bx^2+a\right)^{\frac{3}{2}}cd^3-3\left(bx^2+a\right)(p^2}{(p^4+10p^3+35p^2+50p+24)d^4}}{b^2}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
-(((b*x^2 + a)*d^2*(p + 1) + sqrt(b*x^2 + a)*c*d*p - c^2)*(sqrt(b*x^2 + a)
*d + c)^p*a/((p^2 + 3*p + 2)*d^2) - ((p^3 + 6*p^2 + 11*p + 6)*(b*x^2 + a)^
2*d^4 + (p^3 + 3*p^2 + 2*p)*(b*x^2 + a)^(3/2)*c*d^3 - 3*(b*x^2 + a)*(p^2 +
 p)*c^2*d^2 + 6*sqrt(b*x^2 + a)*c^3*d*p - 6*c^4)*(sqrt(b*x^2 + a)*d + c)^p
/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*d^4))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. 2(143) = 286.

Time = 0.19 (sec) , antiderivative size = 1396, normalized size of antiderivative = 9.25

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="giac")
```

output
```
-((sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*a*d^2*p^3*sgn((sqrt(
b*x^2 + a)*d + c)*d - c*d) - (sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d +
c)^p*a*c*d^2*p^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 8*(sqrt(b*x^2 + a)
*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*a*d^2*p^2*sgn((sqrt(b*x^2 + a)*d + c)*
d - c*d) - 9*(sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d + c)^p*a*c*d^2*p^2
*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - (sqrt(b*x^2 + a)*d + c)^4*(sqrt(b*
x^2 + a)*d + c)^p*p^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 3*(sqrt(b*x^2
 + a)*d + c)^3*(sqrt(b*x^2 + a)*d + c)^p*c*p^3*sgn((sqrt(b*x^2 + a)*d + c)
*d - c*d) - 3*(sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*c^2*p^3*
sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + (sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2
 + a)*d + c)^p*c^3*p^3*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 19*(sqrt(b*x
^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*a*d^2*p*sgn((sqrt(b*x^2 + a)*d
+ c)*d - c*d) - 26*(sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d + c)^p*a*c*d
^2*p*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 6*(sqrt(b*x^2 + a)*d + c)^4*(s
qrt(b*x^2 + a)*d + c)^p*p^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 21*(sqr
t(b*x^2 + a)*d + c)^3*(sqrt(b*x^2 + a)*d + c)^p*c*p^2*sgn((sqrt(b*x^2 + a)
*d + c)*d - c*d) - 24*(sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*
c^2*p^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 9*(sqrt(b*x^2 + a)*d + c)*(
sqrt(b*x^2 + a)*d + c)^p*c^3*p^2*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + 12
*(sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*a*d^2*sgn((sqrt(b*...
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int x^3\left(c+d\sqrt{b\,x^2+a}\right)^p\,dx$$

input
```
int(x^3*(c + d*(a + b*x^2)^(1/2))^p,x)
```

output
```
int(x^3*(c + d*(a + b*x^2)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.46

$$\int x^3\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$

$$= \frac{\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2\right)^p\left(-4\sqrt{b\,x^2+a}\,ac\,d^3p^2-10\sqrt{b\,x^2+a}\,ac\,d^3p+\cdots\right.}{}$$

input
```
int(x^3*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
((sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x + a*d +
b*d*x**2)**p*( - 4*sqrt(a + b*x**2)*a*c*d**3*p**2 - 10*sqrt(a + b*x**2)*a*
c*d**3*p + sqrt(a + b*x**2)*b*c*d**3*p**3*x**2 + 3*sqrt(a + b*x**2)*b*c*d*
*3*p**2*x**2 + 2*sqrt(a + b*x**2)*b*c*d**3*p*x**2 + 6*sqrt(a + b*x**2)*c**
3*d*p - 2*a**2*d**4*p**2 - 8*a**2*d**4*p - 6*a**2*d**4 + a*b*d**4*p**3*x**
2 + 4*a*b*d**4*p**2*x**2 + 3*a*b*d**4*p*x**2 - 2*a*c**2*d**2*p**2 + 4*a*c*
*2*d**2*p + 12*a*c**2*d**2 + b**2*d**4*p**3*x**4 + 6*b**2*d**4*p**2*x**4 +
 11*b**2*d**4*p*x**4 + 6*b**2*d**4*x**4 - 3*b*c**2*d**2*p**2*x**2 - 3*b*c*
*2*d**2*p*x**2 - 6*c**4))/((sqrt(a + b*x**2) + sqrt(b)*x)**p*b**2*d**4*(p*
*4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.292 $\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx=-\frac{c\left(c+d\sqrt{a+bx^2}\right)^{1+p}}{bd^2(1+p)}+\frac{\left(c+d\sqrt{a+bx^2}\right)^{2+p}}{bd^2(2+p)}$$

output
```
-c*(c+d*(b*x^2+a)^(1/2))^(p+1)/b/d^2/(p+1)+(c+d*(b*x^2+a)^(1/2))^(2+p)/b/d
^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\frac{\left(c+d\sqrt{a+bx^2}\right)^{1+p}\left(-c+d(1+p)\sqrt{a+bx^2}\right)}{bd^2(1+p)(2+p)}$$

input
```
Integrate[x*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
((c + d*Sqrt[a + b*x^2])^(1 + p)*(-c + d*(1 + p)*Sqrt[a + b*x^2]))/(b*d^2*
(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2024, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d\sqrt{a+bx^2}+c\right)^p dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \left(c+d\sqrt{bx^2+a}\right)^p d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{774}$$

$$\frac{\int \sqrt{bx^2+a}\left(c+d\sqrt{bx^2+a}\right)^p d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^{p+1}}{d}-\frac{c\left(c+d\sqrt{bx^2+a}\right)^p}{d}\right)d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(d\sqrt{a+bx^2}+c\right)^{p+2}}{d^2(p+2)}-\frac{c\left(d\sqrt{a+bx^2}+c\right)^{p+1}}{d^2(p+1)}}{b}$$

input
```
Int[x*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
(-((c*(c + d*Sqrt[a + b*x^2])^(1 + p))/(d^2*(1 + p))) + (c + d*Sqrt[a + b*
x^2])^(2 + p)/(d^2*(2 + p)))/b
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [F]

$$\int x\left(c+d\sqrt{b\,x^{2}+a}\right)^{p}dx$$

input
```
int(x*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int(x*(c+d*(b*x^2+a)^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx = \frac{\left(ad^2p+\sqrt{bx^2+a}cdp+ad^2+(bd^2p+bd^2)x^2-c^2\right)\left(\sqrt{bx^2+a}d+c\right)^p}{bd^2p^2+3\,bd^2p+2\,bd^2}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
(a*d^2*p + sqrt(b*x^2 + a)*c*d*p + a*d^2 + (b*d^2*p + b*d^2)*x^2 - c^2)*(s
qrt(b*x^2 + a)*d + c)^p/(b*d^2*p^2 + 3*b*d^2*p + 2*b*d^2)
```

Sympy [F]

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx = \int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$

input
```
integrate(x*(c+d*(b*x**2+a)**(1/2))**p,x)
```

output
```
Integral(x*(c + d*sqrt(a + b*x**2))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx = \frac{\left((bx^2+a)d^2(p+1)+\sqrt{bx^2+a}cdp-c^2\right)\left(\sqrt{bx^2+a}d+c\right)^p}{(p^2+3\,p+2)bd^2}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
((b*x^2 + a)*d^2*(p + 1) + sqrt(b*x^2 + a)*c*d*p - c^2)*(sqrt(b*x^2 + a)*d
 + c)^p/((p^2 + 3*p + 2)*b*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(61) = 122$.

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.57

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$
$$= \frac{\left(\sqrt{bx^2+a}d+c\right)^2\left(\sqrt{bx^2+a}d+c\right)^p dp\operatorname{sgn}\left(\left(\sqrt{bx^2+a}d+c\right)d-cd\right)-\left(\sqrt{bx^2+a}d+c\right)\left(\sqrt{bx^2+a}d+c\right)^p}{}$$

input
```
integrate(x*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="giac")
```

output
```
((sqrt(b*x^2 + a)*d + c)^2*(sqrt(b*x^2 + a)*d + c)^p*d*p*sgn((sqrt(b*x^2 +
 a)*d + c)*d - c*d) - (sqrt(b*x^2 + a)*d + c)*(sqrt(b*x^2 + a)*d + c)^p*c*
d*p*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) + (sqrt(b*x^2 + a)*d + c)^2*(sqrt
(b*x^2 + a)*d + c)^p*d*sgn((sqrt(b*x^2 + a)*d + c)*d - c*d) - 2*(sqrt(b*x^
2 + a)*d + c)*(sqrt(b*x^2 + a)*d + c)^p*c*d*sgn((sqrt(b*x^2 + a)*d + c)*d
- c*d))/((p^2 + 3*p + 2)*b*d^3)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx = \frac{\left(c+d\sqrt{b\,x^2+a}\right)^p\,\left(b\,x^2+a\right)\,{}_2\mathrm{F}_1\left(2,-p;\ 3;\ -\frac{d\sqrt{b\,x^2+a}}{c}\right)}{2\,b\,\left(\frac{d\sqrt{b\,x^2+a}}{c}+1\right)^p}$$

input
```
int(x*(c + d*(a + b*x^2)^(1/2))^p,x)
```

output
```
((c + d*(a + b*x^2)^(1/2))^p*(a + b*x^2)*hypergeom([2, -p], 3, -(d*(a + b*
x^2)^(1/2))/c))/(2*b*((d*(a + b*x^2)^(1/2))/c + 1)^p)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int x\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$
$$=\frac{\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,dx+\sqrt{b\,x^2+a}\,c+\sqrt{b}\,cx+ad+bd\,x^2\right)^p\left(\sqrt{b\,x^2+a}\,cdp+a\,d^2p+a\,d^2+b\,d^2p\,x^2+b\,d^2x\right.}{\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^p b\,d^2\left(p^2+3p+2\right)}$$

input
```
int(x*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
((sqrt(b)*sqrt(a + b*x**2)*d*x + sqrt(a + b*x**2)*c + sqrt(b)*c*x + a*d +
b*d*x**2)**p*(sqrt(a + b*x**2)*c*d*p + a*d**2*p + a*d**2 + b*d**2*p*x**2 +
 b*d**2*x**2 - c**2))/((sqrt(a + b*x**2) + sqrt(b)*x)**p*b*d**2*(p**2 + 3*
p + 2))
```

3.293 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx = -\frac{\left(c+d\sqrt{a+bx^2}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}\right)}{2\left(c-\sqrt{a}d\right)(1+p)} - \frac{\left(c+d\sqrt{a+bx^2}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)}{2\left(c+\sqrt{a}d\right)(1+p)}$$

output
```
-1/2*(c+d*(b*x^2+a)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],(c+d*(b*x^2+a)^(
1/2))/(c-a^(1/2)*d))/(c-a^(1/2)*d)/(p+1)-1/2*(c+d*(b*x^2+a)^(1/2))^(p+1)*h
ypergeom([1, p+1],[2+p],(c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))/(c+a^(1/2)*d)
/(p+1)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx = -\frac{\left(c+d\sqrt{a+bx^2}\right)^{1+p}\left(\left(c+\sqrt{a}d\right)\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}\right)+\left(c-\sqrt{a}d\right)\text{Hyper}\right.}{2\left(c-\sqrt{a}d\right)\left(c+\sqrt{a}d\right)(1+p)}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^p/x,x]
```

output
```
-1/2*((c + d*Sqrt[a + b*x^2])^(1 + p)*((c + Sqrt[a]*d)*Hypergeometric2F1[1
, 1 + p, 2 + p, (c + d*Sqrt[a + b*x^2])/(c - Sqrt[a]*d)] + (c - Sqrt[a]*d)
*Hypergeometric2F1[1, 1 + p, 2 + p, (c + d*Sqrt[a + b*x^2])/(c + Sqrt[a]*d
)]))/((c - Sqrt[a]*d)*(c + Sqrt[a]*d)*(1 + p))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7282, 896, 25, 1732, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^p}{x}dx$$

$$\Big\downarrow \text{ 7282}$$

$$\frac{1}{2}\int \frac{\left(c+d\sqrt{bx^2+a}\right)^p}{x^2}dx^2$$

$$\Big\downarrow \text{ 896}$$

$$\frac{1}{2}\int \frac{\left(c+d\sqrt{bx^2+a}\right)^p}{bx^2}d(bx^2+a)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int -\frac{\left(c+d\sqrt{bx^2+a}\right)^p}{bx^2}d(bx^2+a)$$

$$\Big\downarrow \text{1732}$$

$$-\int \frac{\sqrt{bx^2+a}\left(c+d\sqrt{bx^2+a}\right)^p}{a-x^4}d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{615}$$

$$-\int \left(\frac{\left(c+d\sqrt{bx^2+a}\right)^p}{2\left(-bx^2-a+\sqrt{a}\right)}-\frac{\left(c+d\sqrt{bx^2+a}\right)^p}{2\left(\sqrt{a}+\sqrt{bx^2+a}\right)}\right)d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d\sqrt{a+bx^2}+c\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{c+d\sqrt{bx^2+a}}{c-\sqrt{a}d}\right)}{2(p+1)\left(c-\sqrt{a}d\right)}-\frac{\left(d\sqrt{a+bx^2}+c\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{c+d\sqrt{bx^2+a}}{c+\sqrt{a}d}\right)}{2(p+1)\left(\sqrt{a}d+c\right)}$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^p/x,x]
```

output
```
-1/2*((c + d*Sqrt[a + b*x^2])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (
c + d*Sqrt[a + b*x^2])/(c - Sqrt[a]*d)])/((c - Sqrt[a]*d)*(1 + p)) - ((c +
 d*Sqrt[a + b*x^2])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (c + d*Sqrt
[a + b*x^2])/(c + Sqrt[a]*d)])/(2*(c + Sqrt[a]*d)*(1 + p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^{2}+a}\right)^{p}}{x}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx=\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**p/x,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x, x)
```

Giac [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx = \int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx = \int \frac{\left(c+d\sqrt{b\,x^2+a}\right)^p}{x}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^p/x,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x}\,dx = \int \frac{\left(\sqrt{b\,x^2+a}\,d+c\right)^p}{x}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x,x)
```

output
```
int((sqrt(a + b*x**2)*d + c)**p/x,x)
```

3.294 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\begin{aligned}\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx &= -\frac{\left(c+d\sqrt{a+bx^2}\right)^p}{2x^2} \\ &+ \frac{bd\left(c+d\sqrt{a+bx^2}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}\right)}{4\sqrt{a}\left(c-\sqrt{a}d\right)} \\ &- \frac{bd\left(c+d\sqrt{a+bx^2}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)}{4\sqrt{a}\left(c+\sqrt{a}d\right)}\end{aligned}$$

output
```
-1/2*(c+d*(b*x^2+a)^(1/2))^p/x^2+1/4*b*d*(c+d*(b*x^2+a)^(1/2))^p*hypergeom
([1, p],[p+1],(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d))/a^(1/2)/(c-a^(1/2)*d)-1
/4*b*d*(c+d*(b*x^2+a)^(1/2))^p*hypergeom([1, p],[p+1],(c+d*(b*x^2+a)^(1/2)
)/(c+a^(1/2)*d))/a^(1/2)/(c+a^(1/2)*d)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.19

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx$$
$$=\frac{\left(c+d\sqrt{a+bx^2}\right)^{1+p}\left(bd\left(c+\sqrt{a}d\right)^2px^2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}\right)+\left(-c+\sqrt{a}d\right)\right.}{4\sqrt{a}(1+}$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^p/x^3,x]
```

output
```
((c + d*Sqrt[a + b*x^2])^(1 + p)*(b*d*(c + Sqrt[a]*d)^2*p*x^2*Hypergeometr
ic2F1[1, 1 + p, 2 + p, (c + d*Sqrt[a + b*x^2])/(c - Sqrt[a]*d)] + (-c + Sq
rt[a]*d)*(2*Sqrt[a]*(c + Sqrt[a]*d)*(1 + p)*(c - d*Sqrt[a + b*x^2]) + b*d*
(c - Sqrt[a]*d)*p*x^2*Hypergeometric2F1[1, 1 + p, 2 + p, (c + d*Sqrt[a + b
*x^2])/(c + Sqrt[a]*d)])))/(4*Sqrt[a]*(1 + p)*(c^2*x - a*d^2*x)^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.53, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7283, 896, 1732, 593, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^p}{x^3}\,dx$$

$$\Big\downarrow \; 7283$$

$$\frac{1}{2}\int \frac{\left(c+d\sqrt{bx^2+a}\right)^p}{x^4}dx^2$$

$$\Big\downarrow \; 896$$

$$\frac{1}{2}b\int \frac{\left(c+d\sqrt{bx^2+a}\right)^p}{b^2x^4}d\left(bx^2+a\right)$$

$$\Big\downarrow \; 1732$$

$$b\int \frac{\sqrt{bx^2+a}\left(c+d\sqrt{bx^2+a}\right)^p}{\left(a-x^4\right)^2}d\sqrt{bx^2+a}$$

$$\Big\downarrow \; 593$$

$$b\left(\frac{\left(c-d\sqrt{a+bx^2}\right)\left(d\sqrt{a+bx^2}+c\right)^{p+1}}{2\left(a-x^4\right)\left(c^2-ad^2\right)}-\frac{d\int \frac{p\left(c-d\sqrt{bx^2+a}\right)\left(c+d\sqrt{bx^2+a}\right)^p}{a-x^4}d\sqrt{bx^2+a}}{2\left(c^2-ad^2\right)}\right)$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{\left(c-d\sqrt{a+bx^2}\right)\left(d\sqrt{a+bx^2}+c\right)^{p+1}}{2\left(a-x^4\right)\left(c^2-ad^2\right)}-\frac{dp\int \frac{\left(c-d\sqrt{bx^2+a}\right)\left(c+d\sqrt{bx^2+a}\right)^p}{a-x^4}d\sqrt{bx^2+a}}{2\left(c^2-ad^2\right)}\right)$$

$$\Big\downarrow \; 657$$

$$b\left(\frac{\left(c-d\sqrt{a+bx^2}\right)\left(d\sqrt{a+bx^2}+c\right)^{p+1}}{2\left(a-x^4\right)\left(c^2-ad^2\right)}-\frac{dp\int \left(\frac{\left(\sqrt{a}c-ad\right)\left(c+d\sqrt{bx^2+a}\right)^p}{2a\left(-bx^2-a+\sqrt{a}\right)}+\frac{\left(\sqrt{a}c+ad\right)\left(c+d\sqrt{bx^2+a}\right)^p}{2a\left(\sqrt{a}+\sqrt{bx^2+a}\right)}\right)d\sqrt{bx^2+a}}{2\left(c^2-ad^2\right)}\right)$$

$$\Big\downarrow \; 2009$$

$$b\left(\frac{\left(c-d\sqrt{a+bx^2}\right)\left(d\sqrt{a+bx^2}+c\right)^{p+1}}{2\left(a-x^4\right)\left(c^2-ad^2\right)}-\frac{dp\left(\frac{\left(c-\sqrt{a}d\right)\left(d\sqrt{a+bx^2}+c\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{c+d\sqrt{bx^2+a}}{c+\sqrt{a}d}\right)}{2\sqrt{a}(p+1)\left(\sqrt{a}d+c\right)}-\right.}{2\left(c^2-a\right.}\right.$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^p/x^3,x]
```

output
```
b*(((c - d*Sqrt[a + b*x^2])*(c + d*Sqrt[a + b*x^2])^(1 + p))/(2*(c^2 - a*d
^2)*(a - x^4)) - (d*p*(-1/2*((c + Sqrt[a]*d)*(c + d*Sqrt[a + b*x^2])^(1 +
p)*Hypergeometric2F1[1, 1 + p, 2 + p, (c + d*Sqrt[a + b*x^2])/(c - Sqrt[a]
*d)])/(Sqrt[a]*(c - Sqrt[a]*d)*(1 + p)) + ((c - Sqrt[a]*d)*(c + d*Sqrt[a +
 b*x^2])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (c + d*Sqrt[a + b*x^2]
)/(c + Sqrt[a]*d)])/(2*Sqrt[a]*(c + Sqrt[a]*d)*(1 + p))))/(2*(c^2 - a*d^2)
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{\left(c +d \sqrt{b \,x^{2}+a}\right)^{p}}{x^{3}}d x$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x^3,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x^3}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^3,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^p/x^3, x)
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx=\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**p/x**3,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**p/x**3, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x^3}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x^3}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^3,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx = \int \frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^p}{x^3}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^p/x^3,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^3}\,dx = \int \frac{\left(\sqrt{b\,x^2+a}\,d+c\right)^p}{x^3}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x^3,x)
```

output
```
int((sqrt(a + b*x**2)*d + c)**p/x**3,x)
```

3.295 $\int x^4 \left(c+d\sqrt{a+bx^2}\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 184

$$\int x^4 \left(c+d\sqrt{a+bx^2}\right)^p \, dx = \frac{1}{5}x^5\left(c+d\sqrt{a+bx^2}\right)^p - \frac{x^5\left(c+d\sqrt{a+bx^2}\right)^p \operatorname{AppellF1}\left(p,-\frac{5}{2},-\frac{5}{2},1+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d},\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)}{5\left(1-\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}\right)^{5/2}\left(1-\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)^{5/2}}$$

output
```
1/5*x^5*(c+d*(b*x^2+a)^(1/2))^p-1/5*x^5*(c+d*(b*x^2+a)^(1/2))^p*AppellF1(p
,-5/2,-5/2,p+1,(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d),(c+d*(b*x^2+a)^(1/2))/(
c+a^(1/2)*d))/(1-(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d))^(5/2)/(1-(c+d*(b*x^2
+a)^(1/2))/(c+a^(1/2)*d))^(5/2)
```

Mathematica [F]

$$\int x^4 \left(c + d\sqrt{a + bx^2}\right)^p dx = \int x^4 \left(c + d\sqrt{a + bx^2}\right)^p dx$$

input
```
Integrate[x^4*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
Integrate[x^4*(c + d*Sqrt[a + b*x^2])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d\sqrt{a + bx^2} + c\right)^p dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^4 \left(d\sqrt{a + bx^2} + c\right)^p dx$$

input
```
Int[x^4*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^4 \left(c + d\sqrt{b\,x^2 + a}\right)^p dx$$

input
```
int(x^4*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int(x^4*(c+d*(b*x^2+a)^(1/2))^p,x)
```

Fricas [F]

$$\int x^4 \left(c + d\sqrt{a + bx^2}\right)^p dx = \int \left(\sqrt{bx^2 + a}d + c\right)^p x^4 \, dx$$

input
```
integrate(x^4*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^p*x^4, x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^4 \left(c + d\sqrt{a + bx^2}\right)^p dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**4*(c+d*(b*x**2+a)**(1/2))**p,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^4 \left(c+d\sqrt{a+bx^2}\right)^p \, dx = \int \left(\sqrt{bx^2+a}d+c\right)^p x^4 \, dx$$

input
```
integrate(x^4*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p*x^4, x)
```

Giac [F]

$$\int x^4 \left(c+d\sqrt{a+bx^2}\right)^p \, dx = \int \left(\sqrt{bx^2+a}d+c\right)^p x^4 \, dx$$

input
```
integrate(x^4*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(c+d\sqrt{a+bx^2}\right)^p \, dx = \int x^4 \left(c+d\sqrt{b\,x^2+a}\right)^p \, dx$$

input
```
int(x^4*(c + d*(a + b*x^2)^(1/2))^p,x)
```

output
```
int(x^4*(c + d*(a + b*x^2)^(1/2))^p, x)
```

Reduce [F]

$$\int x^4\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int x^4\left(c+d\sqrt{b\,x^2+a}\right)^p dx$$

input
```
int(x^4*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int(x^4*(c+d*(b*x^2+a)^(1/2))^p,x)
```

3.296 $\int x^2 \left(c+d\sqrt{a+bx^2}\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 184

$$\int x^2 \left(c+d\sqrt{a+bx^2}\right)^p \, dx = \frac{1}{3}x^3\left(c+d\sqrt{a+bx^2}\right)^p - \frac{x^3\left(c+d\sqrt{a+bx^2}\right)^p \text{AppellF1}\left(p,-\frac{3}{2},-\frac{3}{2},1+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d},\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)}{3\left(1-\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}\right)^{3/2}\left(1-\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)^{3/2}}$$

output
```
1/3*x^3*(c+d*(b*x^2+a)^(1/2))^p-1/3*x^3*(c+d*(b*x^2+a)^(1/2))^p*AppellF1(p
,-3/2,-3/2,p+1,(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d),(c+d*(b*x^2+a)^(1/2))/(
c+a^(1/2)*d))/(1-(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d))^(3/2)/(1-(c+d*(b*x^2
+a)^(1/2))/(c+a^(1/2)*d))^(3/2)
```

Mathematica [F]

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int x^2\left(c+d\sqrt{a+bx^2}\right)^p\,dx$$

input
```
Integrate[x^2*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
Integrate[x^2*(c + d*Sqrt[a + b*x^2])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d\sqrt{a+bx^2}+c\right)^p\,dx$$

$$\Big\downarrow \; 7299$$

$$\int x^2\left(d\sqrt{a+bx^2}+c\right)^p\,dx$$

input
```
Int[x^2*(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^2 \left(c + d\sqrt{b\,x^2 + a}\right)^p dx$$

input
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^p,x)
```

Fricas [F]

$$\int x^2 \left(c + d\sqrt{a + bx^2}\right)^p dx = \int \left(\sqrt{bx^2 + a}d + c\right)^p x^2 dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^p*x^2, x)
```

Sympy [F]

$$\int x^2 \left(c + d\sqrt{a + bx^2}\right)^p dx = \int x^2 \left(c + d\sqrt{a + bx^2}\right)^p dx$$

input
```
integrate(x**2*(c+d*(b*x**2+a)**(1/2))**p,x)
```

output
```
Integral(x**2*(c + d*sqrt(a + b*x**2))**p, x)
```

Maxima [F]

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int\left(\sqrt{bx^2+a}d+c\right)^px^2\,dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int\left(\sqrt{bx^2+a}d+c\right)^px^2\,dx$$

input
```
integrate(x^2*(c+d*(b*x^2+a)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int x^2\left(c+d\sqrt{b\,x^2+a}\right)^p\,dx$$

input
```
int(x^2*(c + d*(a + b*x^2)^(1/2))^p,x)
```

output
```
int(x^2*(c + d*(a + b*x^2)^(1/2))^p, x)
```

Reduce [F]

$$\int x^2 \left(c + d\sqrt{a + bx^2}\right)^p dx = \int \left(\sqrt{b\,x^2 + a}\,d + c\right)^p x^2 dx$$

input
```
int(x^2*(c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int((sqrt(a + b*x**2)*d + c)**p*x**2,x)
```

3.297 $\int \left(c+d\sqrt{a+bx^2}\right)^p dx$

Optimal result

Integrand size = 17, antiderivative size = 175

$$\int \left(c+d\sqrt{a+bx^2}\right)^p dx = x\left(c+d\sqrt{a+bx^2}\right)^p - \frac{x\left(c+d\sqrt{a+bx^2}\right)^p \text{AppellF1}\left(p,-\frac{1}{2},-\frac{1}{2},1+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d},\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)}{\sqrt{1-\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}}\sqrt{1-\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}}$$

output

```
x*(c+d*(b*x^2+a)^(1/2))^p-x*(c+d*(b*x^2+a)^(1/2))^p*AppellF1(p,-1/2,-1/2,p
+1,(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d),(c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d)
)/(1-(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d))^(1/2)/(1-(c+d*(b*x^2+a)^(1/2))/(
c+a^(1/2)*d))^(1/2)
```

Mathematica [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^p \, dx = \int \left(c+d\sqrt{a+bx^2}\right)^p \, dx$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
Integrate[(c + d*Sqrt[a + b*x^2])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d\sqrt{a+bx^2}+c\right)^p \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \left(d\sqrt{a+bx^2}+c\right)^p dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \left(c+d\sqrt{b\,x^{2}+a}\right)^{p}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^p,x)
```

Fricas [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^p dx = \int \left(\sqrt{bx^2+a}d+c\right)^p dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^p, x)
```

Sympy [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^p dx = \int \left(c+d\sqrt{a+bx^2}\right)^p dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**p,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**p, x)
```

Maxima [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^p dx = \int \left(\sqrt{bx^2+a}d+c\right)^p dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p, x)
```

Giac [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^p dx = \int \left(\sqrt{bx^2+a}d+c\right)^p dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+d\sqrt{a+bx^2}\right)^p dx = \int \left(c+d\sqrt{b\,x^2+a}\right)^p dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^p,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^p, x)
```

Reduce [F]

$$\int \left(c+d\sqrt{a+bx^2}\right)^p\,dx=\int \left(\sqrt{b\,x^2+a}\,d+c\right)^p dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p,x)
```

output
```
int((sqrt(a + b*x**2)*d + c)**p,x)
```

3.298 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx = -\frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x} + \frac{\left(c+d\sqrt{a+bx^2}\right)^p\sqrt{1-\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}}\sqrt{1-\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}}\,\text{AppellF1}\left(p,\frac{1}{2},\frac{1}{2},1+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d},\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)}{x}$$

output
```
-(c+d*(b*x^2+a)^(1/2))^p/x+(c+d*(b*x^2+a)^(1/2))^p*(1-(c+d*(b*x^2+a)^(1/2)
)/(c-a^(1/2)*d))^(1/2)*(1-(c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(1/2)*Appel
lF1(p,1/2,1/2,p+1,(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d),(c+d*(b*x^2+a)^(1/2)
)/(c+a^(1/2)*d))/x
```

Mathematica [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx = \int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^p/x^2,x]
```

output
```
Integrate[(c + d*Sqrt[a + b*x^2])^p/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^p}{x^2}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^p}{x^2}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^2+a}\right)^p}{x^2}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x^2,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx=\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**p/x**2,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x^2}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx=\int \frac{\left(c+d\,\sqrt{b\,x^2+a}\right)^p}{x^2}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^p/x^2,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^2}\,dx=\int \frac{\left(\sqrt{b\,x^2+a}\,d+c\right)^p}{x^2}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x^2,x)
```

output
```
int((sqrt(a + b*x**2)*d + c)**p/x**2,x)
```

3.299 $\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 184

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx = -\frac{\left(c+d\sqrt{a+bx^2}\right)^p}{3x^3} + \frac{\left(c+d\sqrt{a+bx^2}\right)^p\left(1-\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d}\right)^{3/2}\left(1-\frac{c+d\sqrt{a+bx^2}}{c+\sqrt{a}d}\right)^{3/2}\text{AppellF1}\left(p,\frac{3}{2},\frac{3}{2},1+p,\frac{c+d\sqrt{a+bx^2}}{c-\sqrt{a}d},\frac{c+d\sqrt{a+bx}}{c+\sqrt{a}d}\right.}{3x^3}$$

output
```
-1/3*(c+d*(b*x^2+a)^(1/2))^p/x^3+1/3*(c+d*(b*x^2+a)^(1/2))^p*(1-(c+d*(b*x^
2+a)^(1/2))/(c-a^(1/2)*d))^(3/2)*(1-(c+d*(b*x^2+a)^(1/2))/(c+a^(1/2)*d))^(
3/2)*AppellF1(p,3/2,3/2,p+1,(c+d*(b*x^2+a)^(1/2))/(c-a^(1/2)*d),(c+d*(b*x^
2+a)^(1/2))/(c+a^(1/2)*d))/x^3
```

Mathematica [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx = \int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx$$

input
```
Integrate[(c + d*Sqrt[a + b*x^2])^p/x^4,x]
```

output
```
Integrate[(c + d*Sqrt[a + b*x^2])^p/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^p}{x^4}dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\left(d\sqrt{a+bx^2}+c\right)^p}{x^4}dx$$

input
```
Int[(c + d*Sqrt[a + b*x^2])^p/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+d\sqrt{b\,x^2+a}\right)^p}{x^4}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x^4,x)
```

output
```
int((c+d*(b*x^2+a)^(1/2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}\,d+c\right)^p}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^4,x, algorithm="fricas")
```

output
```
integral((sqrt(b*x^2 + a)*d + c)^p/x^4, x)
```

Sympy [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx = \int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx$$

input
```
integrate((c+d*(b*x**2+a)**(1/2))**p/x**4,x)
```

output
```
Integral((c + d*sqrt(a + b*x**2))**p/x**4, x)
```

Maxima [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}\,d+c\right)^p}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx=\int \frac{\left(\sqrt{bx^2+a}d+c\right)^p}{x^4}\,dx$$

input
```
integrate((c+d*(b*x^2+a)^(1/2))^p/x^4,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*d + c)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx=\int \frac{\left(c+d\sqrt{b\,x^2+a}\right)^p}{x^4}\,dx$$

input
```
int((c + d*(a + b*x^2)^(1/2))^p/x^4,x)
```

output
```
int((c + d*(a + b*x^2)^(1/2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(c+d\sqrt{a+bx^2}\right)^p}{x^4}\,dx=\int \frac{\left(\sqrt{b\,x^2+a}\,d+c\right)^p}{x^4}dx$$

input
```
int((c+d*(b*x^2+a)^(1/2))^p/x^4,x)
```

output
```
int((sqrt(a + b*x**2)*d + c)**p/x**4,x)
```

3.300 $\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx$

Optimal result

Integrand size = 19, antiderivative size = 262

$$\begin{aligned}\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx &= -\frac{bdp\left(bc-a\sqrt{c+\frac{d}{x}}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p x}{4c\left(a^2-b^2c\right)}+\frac{1}{2}\left(a+b\sqrt{c+\frac{d}{x}}\right)^p x^2\\ &+\frac{bd^2\left(a-b\sqrt{c}(2-p)\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a-b\sqrt{c}}\right)}{8\left(a-b\sqrt{c}\right)^2 c^{3/2}}\\ &-\frac{bd^2\left(a+b\sqrt{c}(2-p)\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a+b\sqrt{c}}\right)}{8\left(a+b\sqrt{c}\right)^2 c^{3/2}}\end{aligned}$$

output

```
-1/4*b*d*p*(b*c-a*(c+d/x)^(1/2))*(a+b*(c+d/x)^(1/2))^p*x/c/(-b^2*c+a^2)+1/
2*(a+b*(c+d/x)^(1/2))^p*x^2+1/8*b*d^2*(a-b*c^(1/2)*(2-p))*(a+b*(c+d/x)^(1/
2))^p*hypergeom([1, p],[p+1],(a+b*(c+d/x)^(1/2))/(a-b*c^(1/2)))/(a-b*c^(1/
2))^2/c^(3/2)-1/8*b*d^2*(a+b*c^(1/2)*(2-p))*(a+b*(c+d/x)^(1/2))^p*hypergeo
m([1, p],[p+1],(a+b*(c+d/x)^(1/2))/(a+b*c^(1/2)))/(a+b*c^(1/2))^2/c^(3/2)
```

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.15

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx$$

$$= \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\left(\frac{2d\left(-2ab^2c(-1+p)+b^3c(-2+p)\sqrt{c+\frac{d}{x}}+a^2bp\sqrt{c+\frac{d}{x}}\right)x}{a^2-b^2c}+4\left(ac-bc\sqrt{c+\frac{d}{x}}\right)x^2+\frac{bd^2p\left(\left(a+b\sqrt{c}\right)^3(\cdots\right.}{\cdots}\right.}{8c\left(a^2\cdots\right.}$$

input
```
Integrate[(a + b*Sqrt[c + d/x])^p*x,x]
```

output
```
((a + b*Sqrt[c + d/x])^(1 + p)*((2*d*(-2*a*b^2*c*(-1 + p) + b^3*c*(-2 + p)
*Sqrt[c + d/x] + a^2*b*p*Sqrt[c + d/x])*x)/(a^2 - b^2*c) + 4*(a*c - b*c*Sq
rt[c + d/x])*x^2 + (b*d^2*p*((a + b*Sqrt[c])^3*(a + b*Sqrt[c]*(-2 + p))*Hy
pergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d/x])/(a - b*Sqrt[c])] -
(a - b*Sqrt[c])^3*(a - b*Sqrt[c]*(-2 + p))*Hypergeometric2F1[1, 1 + p, 2 +
 p, (a + b*Sqrt[c + d/x])/(a + b*Sqrt[c])]))/((a - b*Sqrt[c])*(a + b*Sqrt[
c])*Sqrt[c]*(a^2 - b^2*c)*(1 + p))))/(8*c*(a^2 - b^2*c))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.52, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {7268, 25, 593, 686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$$

$$\Big\downarrow \; 7268$$

$$-2d^2\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\sqrt{c+\frac{d}{x}}x^3}{d^3}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \; 25$$

$$2d^2\int -\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\sqrt{c+\frac{d}{x}}x^3}{d^3}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \; 593$$

$$-2d^2\left(\frac{b\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\left(b\sqrt{c+\frac{d}{x}}(2-p)+ap\right)x^2}{d^2}d\sqrt{c+\frac{d}{x}}}{4\left(a^2-b^2c\right)}-\frac{x^2\left(a-b\sqrt{c+\frac{d}{x}}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{4d^2\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \; 686$$

$$-2d^2\left(\frac{b\left(-\frac{\int \frac{p\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\left(a\left(a^2-b^2c(3-2p)\right)+b\left(b^2c(2-p)-a^2p\right)\sqrt{c+\frac{d}{x}}\right)x}{d}d\sqrt{c+\frac{d}{x}}}{2c(a^2-b^2c)}-\frac{x\left(2abc(1-p)-\sqrt{c+\frac{d}{x}}(b^2c(2-p)-a^2p)\right)\left(a+b\sqrt{c+\frac{d}{x}}\right.}{2cd(a^2-b^2c)}\right.}{4\left(a^2-b^2c\right)}\right.$$

$$\Big\downarrow \; 25$$

$$-2d^2\left(\frac{b\left(\frac{\int -\frac{p\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\left(a\left(a^2-b^2c(3-2p)\right)+b\left(b^2c(2-p)-a^2p\right)\sqrt{c+\frac{d}{x}}\right)x}{d}d\sqrt{c+\frac{d}{x}}}{2c(a^2-b^2c)}-\frac{x\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\left(2abc(1-p)-\sqrt{c+\frac{d}{x}}(b^2c(2-p)-a\right.}{2cd(a^2-b^2c)}\right.}{4\left(a^2-b^2c\right)}\right.$$

$$\Big\downarrow \; 27$$

$$-2d^2\left(\frac{b\left(\frac{p\int -\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\left(a\left(a^2-b^2c(3-2p)\right)+b\left(b^2c(2-p)-a^2p\right)\sqrt{c+\frac{d}{x}}\right)x}{d}d\sqrt{c+\frac{d}{x}}}{2c(a^2-b^2c)}-\frac{x\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\left(2abc(1-p)-\sqrt{c+\frac{d}{x}}(b^2c(2-p)-\right.}{2cd(a^2-b^2c)}\right.}{4\left(a^2-b^2c\right)}\right.$$

$\downarrow$ 657

$$-2d^2\left(\frac{b\left(\frac{p\int\left(\frac{\left(a\sqrt{c}\left(a^2-b^2c(3-2p)\right)+bc\left(b^2c(2-p)-a^2p\right)\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{2c\left(\sqrt{c}-\sqrt{c+\frac{d}{x}}\right)}+\frac{\left(a\sqrt{c}\left(a^2-b^2c(3-2p)\right)-bc\left(b^2c(2-p)-a^2p\right)\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{2c\left(\sqrt{c}+\sqrt{c+\frac{d}{x}}\right)}\right)d\sqrt{c+\frac{d}{x}}}{2c(a^2-b^2c)}-\right.}{4\left(a^2-b^2c\right)}\right.$$

$\downarrow$ 2009

$$-2d^2\left(\frac{b\left(\frac{p\left(\frac{(a-b\sqrt{c})^2(a+b\sqrt{c}(2-p))\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{c+\frac{d}{x}}}{a+b\sqrt{c}}\right)}{2\sqrt{c}(p+1)(a+b\sqrt{c})}-\frac{(a+b\sqrt{c})^2(a-b\sqrt{c}(2-p))\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\text{Hyper}}{2\sqrt{c}(p+1)(a-}\right.}{2c(a^2-b^2c)}\right.}{4\left(a^2-b^2c\right)}\right.$$

input
```
Int[(a + b*Sqrt[c + d/x])^p*x,x]
```

output
```
-2*d^2*(-1/4*((a - b*Sqrt[c + d/x])*(a + b*Sqrt[c + d/x])^(1 + p)*x^2)/((a
^2 - b^2*c)*d^2) + (b*(-1/2*((a + b*Sqrt[c + d/x])^(1 + p)*(2*a*b*c*(1 - p
) - (b^2*c*(2 - p) - a^2*p)*Sqrt[c + d/x])*x)/(c*(a^2 - b^2*c)*d) + (p*(-1
/2*((a + b*Sqrt[c])^2*(a - b*Sqrt[c]*(2 - p))*(a + b*Sqrt[c + d/x])^(1 + p
)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d/x])/(a - b*Sqrt[c])
])/((a - b*Sqrt[c])*Sqrt[c]*(1 + p)) + ((a - b*Sqrt[c])^2*(a + b*Sqrt[c]*(
2 - p))*(a + b*Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (
a + b*Sqrt[c + d/x])/(a + b*Sqrt[c])])/(2*(a + b*Sqrt[c])*Sqrt[c]*(1 + p))
))/(2*c*(a^2 - b^2*c))))/(4*(a^2 - b^2*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p*x,x)
```

output
```
int((a+b*(c+d/x)^(1/2))^p*x,x)
```

Fricas [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx=\int \left(b\sqrt{c+\frac{d}{x}}+a\right)^p x\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p*x,x, algorithm="fricas")
```

output
```
integral((b*sqrt((c*x + d)/x) + a)^p*x, x)
```

Sympy [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx=\int x\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\,dx$$

input
```
integrate((a+b*(c+d/x)**(1/2))**p*x,x)
```

output
```
Integral(x*(a + b*sqrt(c + d/x))**p, x)
```

Maxima [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx=\int \left(b\sqrt{c+\frac{d}{x}}+a\right)^p x\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p*x,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p*x, x)
```

Giac [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx=\int \left(b\sqrt{c+\frac{d}{x}}+a\right)^p x\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p*x,x, algorithm="giac")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx=\int x\left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$$

input
```
int(x*(a + b*(c + d/x)^(1/2))^p,x)
```

output
```
int(x*(a + b*(c + d/x)^(1/2))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x\,dx=\int \frac{\left(\sqrt{cx+d}\,b+\sqrt{x}\,a\right)^p x}{x^{\frac{p}{2}}}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p*x,x)
```

output
```
int(((sqrt(c*x + d)*b + sqrt(x)*a)**p*x)/x**(p/2),x)
```

3.301 $\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$

Optimal result

Integrand size = 17, antiderivative size = 166

$$\begin{aligned}\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx &= \left(a+b\sqrt{c+\frac{d}{x}}\right)^p x \\ &- \frac{bd\left(a+b\sqrt{c+\frac{d}{x}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a-b\sqrt{c}}\right)}{2\left(a-b\sqrt{c}\right)\sqrt{c}} \\ &+ \frac{bd\left(a+b\sqrt{c+\frac{d}{x}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a+b\sqrt{c}}\right)}{2\left(a+b\sqrt{c}\right)\sqrt{c}}\end{aligned}$$

output
```
(a+b*(c+d/x)^(1/2))^p*x-1/2*b*d*(a+b*(c+d/x)^(1/2))^p*hypergeom([1, p],[p+
1],(a+b*(c+d/x)^(1/2))/(a-b*c^(1/2)))/(a-b*c^(1/2))/c^(1/2)+1/2*b*d*(a+b*(
c+d/x)^(1/2))^p*hypergeom([1, p],[p+1],(a+b*(c+d/x)^(1/2))/(a+b*c^(1/2)))/
(a+b*c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.17

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$$
$$=\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\left(-b\left(a+b\sqrt{c}\right)^2 dp\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right)\left(2\left(\right.\right.\right.}{2\sqrt{c}\left(a^2-\right.}$$

input
```
Integrate[(a + b*Sqrt[c + d/x])^p,x]
```

output
```
((a + b*Sqrt[c + d/x])^(1 + p)*(-(b*(a + b*Sqrt[c])^2*d*p*Hypergeometric2F
1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d/x])/(a - b*Sqrt[c])]) + (a - b*Sqrt[c
])*(2*(a + b*Sqrt[c])*Sqrt[c]*(1 + p)*(a - b*Sqrt[c + d/x])*x + b*(a - b*S
qrt[c])*d*p*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d/x])/(a +
b*Sqrt[c])])))/(2*Sqrt[c]*(a^2 - b^2*c)^2*(1 + p))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {7268, 593, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$$

$$\Big\downarrow \text{7268}$$

$$-2d\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\sqrt{c+\frac{d}{x}}x^2}{d^2}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{593}$$

$$-2d\left(-\frac{b\int -\frac{p\left(a-b\sqrt{c+\frac{d}{x}}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p x}{d}d\sqrt{c+\frac{d}{x}}}{2\left(a^2-b^2c\right)}-\frac{x\left(a-b\sqrt{c+\frac{d}{x}}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{2d\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$-2d\left(-\frac{bp\int -\frac{\left(a-b\sqrt{c+\frac{d}{x}}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p x}{d}d\sqrt{c+\frac{d}{x}}}{2\left(a^2-b^2c\right)}-\frac{x\left(a-b\sqrt{c+\frac{d}{x}}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{2d\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$-2d\left(-\frac{bp\int \left(\frac{\left(a\sqrt{c}-bc\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{2c\left(\sqrt{c}-\sqrt{c+\frac{d}{x}}\right)}+\frac{\left(\sqrt{c}a+bc\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{2c\left(\sqrt{c}+\sqrt{c+\frac{d}{x}}\right)}\right)d\sqrt{c+\frac{d}{x}}}{2\left(a^2-b^2c\right)}-\frac{x\left(a-b\sqrt{c+\frac{d}{x}}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{2d\left(a^2-b^2c\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2d\left(-\frac{bp\left(\frac{\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{c+\frac{d}{x}}}{a+b\sqrt{c}}\right)}{2\sqrt{c}(p+1)\left(a+b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c}\right)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\text{Hypergeometric2F1}\left(\right.}{2\sqrt{c}(p+1)\left(a-b\sqrt{c}\right)}\right.}{2\left(a^2-b^2c\right)}\right.$$

input
```
Int[(a + b*Sqrt[c + d/x])^p,x]
```

output
```
-2*d*(-1/2*((a - b*Sqrt[c + d/x])*(a + b*Sqrt[c + d/x])^(1 + p)*x)/((a^2 -
 b^2*c)*d) - (b*p*(-1/2*((a + b*Sqrt[c])*(a + b*Sqrt[c + d/x])^(1 + p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d/x])/(a - b*Sqrt[c])])/((
a - b*Sqrt[c])*Sqrt[c]*(1 + p)) + ((a - b*Sqrt[c])*(a + b*Sqrt[c + d/x])^(
1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d/x])/(a + b*Sqr
t[c])])/(2*(a + b*Sqrt[c])*Sqrt[c]*(1 + p))))/(2*(a^2 - b^2*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p,x)
```

output
```
int((a+b*(c+d/x)^(1/2))^p,x)
```

Fricas [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx = \int \left(b\sqrt{c+\frac{d}{x}}+a\right)^p dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral((b*sqrt((c*x + d)/x) + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx = \int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$$

input
```
integrate((a+b*(c+d/x)**(1/2))**p,x)
```

output
```
Integral((a + b*sqrt(c + d/x))**p, x)
```

Maxima [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx = \int \left(b\sqrt{c+\frac{d}{x}}+a\right)^p dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx = \int \left(b\sqrt{c+\frac{d}{x}}+a\right)^p dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx = \int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx$$

input
```
int((a + b*(c + d/x)^(1/2))^p,x)
```

output
```
int((a + b*(c + d/x)^(1/2))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p dx = \int \frac{\left(\sqrt{cx+d}\,b+\sqrt{x}\,a\right)^p}{x^{\frac{p}{2}}}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p,x)
```

output
```
int((sqrt(c*x + d)*b + sqrt(x)*a)**p/x**(p/2),x)
```

3.302 $\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx = \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)} + \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a+b\sqrt{c}}\right)}{\left(a+b\sqrt{c}\right)(1+p)}$$

output
```
(a+b*(c+d/x)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],(a+b*(c+d/x)^(1/2))/(a-
b*c^(1/2)))/(a-b*c^(1/2))/(p+1)+(a+b*(c+d/x)^(1/2))^(p+1)*hypergeom([1, p+
1],[2+p],(a+b*(c+d/x)^(1/2))/(a+b*c^(1/2)))/(a+b*c^(1/2))/(p+1)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx$$
$$=\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\left(\left(a+b\sqrt{c}\right)\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+\frac{d}{x}}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right)\text{Hypergeo}\right.}{\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c}\right)(1+p)}$$

input
```
Integrate[(a + b*Sqrt[c + d/x])^p/x,x]
```

output
```
((a + b*Sqrt[c + d/x])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1 + p
, 2 + p, (a + b*Sqrt[c + d/x])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hypergeo
metric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d/x])/(a + b*Sqrt[c])]))/((a -
b*Sqrt[c])*(a + b*Sqrt[c])*(1 + p))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7268, 25, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx$$

$$\Big\downarrow \; 7268$$

$$-2\int -\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\sqrt{c+\frac{d}{x}}x}{d}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \; 25$$

$$2\int -\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p \sqrt{c+\frac{d}{x}}x}{d}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{615}$$

$$2\int \left(\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{2\left(\sqrt{c}-\sqrt{c+\frac{d}{x}}\right)}-\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{2\left(\sqrt{c}+\sqrt{c+\frac{d}{x}}\right)}\right)d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{c+\frac{d}{x}}}{a-b\sqrt{c}}\right)}{2(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}\text{Hypergeomet}}{2(p+1)}\right.$$

input
```
Int[(a + b*Sqrt[c + d/x])^p/x,x]
```

output
```
-2*(-1/2*((a + b*Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
 (a + b*Sqrt[c + d/x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 + p)) - ((a +
 b*Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c
 + d/x])/(a + b*Sqrt[c])])/(2*(a + b*Sqrt[c])*(1 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x,x)
```

output
```
int((a+b*(c+d/x)^(1/2))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx = \int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*sqrt((c*x + d)/x) + a)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx = \int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx$$

input
```
integrate((a+b*(c+d/x)**(1/2))**p/x,x)
```

output
```
Integral((a + b*sqrt(c + d/x))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx=\int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx=\int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx=\int \frac{\left(a+b\,\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx$$

input
```
int((a + b*(c + d/x)^(1/2))^p/x,x)
```

output
```
int((a + b*(c + d/x)^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x}\,dx = \int \frac{\left(\sqrt{cx+d}\,b+\sqrt{x}\,a\right)^p}{x^{\frac{p}{2}}x}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x,x)
```

output
```
int((sqrt(c*x + d)*b + sqrt(x)*a)**p/(x**(p/2)*x),x)
```

3.303 $\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx=\frac{2a\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}}{b^2d(1+p)}-\frac{2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{2+p}}{b^2d(2+p)}$$

output
```
2*a*(a+b*(c+d/x)^(1/2))^(p+1)/b^2/d/(p+1)-2*(a+b*(c+d/x)^(1/2))^(2+p)/b^2/
d/(2+p)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx=\frac{2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\left(a-b(1+p)\sqrt{c+\frac{d}{x}}\right)}{b^2d(1+p)(2+p)}$$

input
```
Integrate[(a + b*Sqrt[c + d/x])^p/x^2,x]
```

output
```
(2*(a + b*Sqrt[c + d/x])^(1 + p)*(a - b*(1 + p)*Sqrt[c + d/x]))/(b^2*d*(1
+ p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7247, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{ 7247}$$

$$-\frac{\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p d\left(c+\frac{d}{x}\right)}{d}$$

$$\Big\downarrow \text{ 774}$$

$$-\frac{2\int \left(a+b\sqrt{c+\frac{d}{x}}\right)^p \sqrt{c+\frac{d}{x}}d\sqrt{c+\frac{d}{x}}}{d}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{2\int \left(\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{b}-\frac{a\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{b}\right)d\sqrt{c+\frac{d}{x}}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\left(\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{b^2(p+1)}\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d/x])^p/x^2,x]
```

output
```
(-2*(-((a*(a + b*Sqrt[c + d/x])^(1 + p))/(b^2*(1 + p))) + (a + b*Sqrt[c +
d/x])^(2 + p)/(b^2*(2 + p))))/d
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7247
```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Simp[q   Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fal
seQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x^2,x)
```

output
```
int((a+b*(c+d/x)^(1/2))^p/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx = -\frac{2\left(abpx\sqrt{\frac{cx+d}{x}}+b^2dp+b^2d+(b^2cp+b^2c-a^2)x\right)\left(b\sqrt{\frac{cx+d}{x}}+a\right)^p}{(b^2dp^2+3\,b^2dp+2\,b^2d)x}$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^2,x, algorithm="fricas")
```

output
```
-2*(a*b*p*x*sqrt((c*x + d)/x) + b^2*d*p + b^2*d + (b^2*c*p + b^2*c - a^2)*
x)*(b*sqrt((c*x + d)/x) + a)^p/((b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)*x)
```

Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx = \int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c+d/x)**(1/2))**p/x**2,x)
```

output
```
Integral((a + b*sqrt(c + d/x))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx = \int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(62) = 124$.

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.44

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx =$$

$$-\frac{2\left(\left(b\sqrt{\frac{cx+d}{x}}+a\right)^2\left(b\sqrt{\frac{cx+d}{x}}+a\right)^pp-\left(b\sqrt{\frac{cx+d}{x}}+a\right)\left(b\sqrt{\frac{cx+d}{x}}+a\right)^pap+\left(b\sqrt{\frac{cx+d}{x}}+a\right)^2\left(b\sqrt{\frac{cx+d}{x}}\right.\right.}{(p^2+3\,p+2)b^2d}$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^2,x, algorithm="giac")
```

output
```
-2*((b*sqrt((c*x + d)/x) + a)^2*(b*sqrt((c*x + d)/x) + a)^p*p - (b*sqrt((c
*x + d)/x) + a)*(b*sqrt((c*x + d)/x) + a)^p*a*p + (b*sqrt((c*x + d)/x) + a
)^2*(b*sqrt((c*x + d)/x) + a)^p - 2*(b*sqrt((c*x + d)/x) + a)*(b*sqrt((c*x
 + d)/x) + a)^p*a)/((p^2 + 3*p + 2)*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx = -\frac{\left(a+b\,\sqrt{c+\frac{d}{x}}\right)^p\,\left(c+\frac{d}{x}\right)\,{}_2\mathrm{F}_1\left(2,-p;\;3;\;-\frac{b\,\sqrt{c+\frac{d}{x}}}{a}\right)}{d\,\left(\frac{b\,\sqrt{c+\frac{d}{x}}}{a}+1\right)^p}$$

input
```
int((a + b*(c + d/x)^(1/2))^p/x^2,x)
```

output
```
-((a + b*(c + d/x)^(1/2))^p*(c + d/x)*hypergeom([2, -p], 3, -(b*(c + d/x)^
(1/2))/a))/(d*((b*(c + d/x)^(1/2))/a + 1)^p)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^2}\,dx = \int \frac{\left(\sqrt{cx+d}\,b+\sqrt{x}\,a\right)^p}{x^{\frac{p}{2}}x^2}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x^2,x)
```

output
```
int((sqrt(c*x + d)*b + sqrt(x)*a)**p/(x**(p/2)*x**2),x)
```

3.304 $\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx = \frac{2a(a^2-b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}}{b^4d^2(1+p)} - \frac{2(3a^2-b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{2+p}}{b^4d^2(2+p)} + \frac{6a\left(a+b\sqrt{c+\frac{d}{x}}\right)^{3+p}}{b^4d^2(3+p)} - \frac{2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{4+p}}{b^4d^2(4+p)}$$

output
```
2*a*(-b^2*c+a^2)*(a+b*(c+d/x)^(1/2))^(p+1)/b^4/d^2/(p+1)-2*(-b^2*c+3*a^2)*
(a+b*(c+d/x)^(1/2))^(2+p)/b^4/d^2/(2+p)+6*a*(a+b*(c+d/x)^(1/2))^(3+p)/b^4/
d^2/(3+p)-2*(a+b*(c+d/x)^(1/2))^(4+p)/b^4/d^2/(4+p)
```

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx$$
$$=\frac{2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\left(6a^3x-6a^2b(1+p)\sqrt{c+\frac{d}{x}}x-b^3(3+4p+p^2)\sqrt{c+\frac{d}{x}}(d(2+p)-2cx)+ab^2(3d(\right.}{b^4d^2(1+p)(2+p)(3+p)(4+p)x}$$

input
```
Integrate[(a + b*Sqrt[c + d/x])^p/x^3,x]
```

output
```
(2*(a + b*Sqrt[c + d/x])^(1 + p)*(6*a^3*x - 6*a^2*b*(1 + p)*Sqrt[c + d/x]*
x - b^3*(3 + 4*p + p^2)*Sqrt[c + d/x]*(d*(2 + p) - 2*c*x) + a*b^2*(3*d*(2
+ 3*p + p^2) + 2*c*(-3 + p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4
 + p)*x)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {7268, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx$$
$$\Big\downarrow \text{7268}$$
$$-\frac{2\int \frac{d\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\sqrt{c+\frac{d}{x}}}{x}d\sqrt{c+\frac{d}{x}}}{d^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{2\int -\frac{d\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\sqrt{c+\frac{d}{x}}}{x}d\sqrt{c+\frac{d}{x}}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$\frac{2\int \left(\frac{(a^3-ab^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{b^3}+\frac{(b^2c-3a^2)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{b^3}+\frac{3a\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+2}}{b^3}-\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+3}}{b^3}\right)d\sqrt{c+\frac{d}{x}}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{a(a^2-b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{b^4(p+1)}+\frac{(3a^2-b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+2}}{b^4(p+2)}-\frac{3a\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+4}}{b^4(p+4)}\right)}{d^2}$$

input
```
Int[(a + b*Sqrt[c + d/x])^p/x^3,x]
```

output
```
(-2*(-((a*(a^2 - b^2*c)*(a + b*Sqrt[c + d/x])^(1 + p))/(b^4*(1 + p))) + ((
3*a^2 - b^2*c)*(a + b*Sqrt[c + d/x])^(2 + p))/(b^4*(2 + p)) - (3*a*(a + b*
Sqrt[c + d/x])^(3 + p))/(b^4*(3 + p)) + (a + b*Sqrt[c + d/x])^(4 + p)/(b^4
*(4 + p))))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x^3,x)
```

output
```
int((a+b*(c+d/x)^(1/2))^p/x^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(145) = 290$.

Time = 0.91 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.02

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx = -\frac{2\left(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2-2\,(3\,b^4c^2-6\,a^2b^2c+3\,a^4+(b^4c^2+a^2b^2c)p^2+2\,(2\,b^4c^2-a\right.}{}$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^3,x, algorithm="fricas")
```

output
```
-2*(b^4*d^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2 - 2*(3*b^4*c^2
 - 6*a^2*b^2*c + 3*a^4 + (b^4*c^2 + a^2*b^2*c)*p^2 + 2*(2*b^4*c^2 - a^2*b^2
*c)*p)*x^2 + (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*b
^2)*d*p)*x - (2*(2*a*b^3*c*p^2 + (5*a*b^3*c - 3*a^3*b)*p)*x^2 - (a*b^3*d*p
^3 + 3*a*b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt((c*x + d)/x))*(b*sqrt((c*x + d)/
x) + a)^p/((b^4*d^2*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p +
 24*b^4*d^2)*x^2)
```

Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx=\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*(c+d/x)**(1/2))**p/x**3,x)
```

output
```
Integral((a + b*sqrt(c + d/x))**p/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx=\int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx=\int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx=\int \frac{\left(a+b\,\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx$$

input
```
int((a + b*(c + d/x)^(1/2))^p/x^3,x)
```

output
```
int((a + b*(c + d/x)^(1/2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^3}\,dx=\int \frac{\left(\sqrt{cx+d}\,b+\sqrt{x}\,a\right)^p}{x^{\frac{p}{2}}x^3}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x^3,x)
```

output
```
int((sqrt(c*x + d)*b + sqrt(x)*a)**p/(x**(p/2)*x**3),x)
```

3.305 $\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 254

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx = \frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}}{b^6d^3(1+p)} - \frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{2+p}}{b^6d^3(2+p)} + \frac{4a(5a^2-3b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{3+p}}{b^6d^3(3+p)} - \frac{4(5a^2-b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{4+p}}{b^6d^3(4+p)} + \frac{10a\left(a+b\sqrt{c+\frac{d}{x}}\right)^{5+p}}{b^6d^3(5+p)} - \frac{2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{6+p}}{b^6d^3(6+p)}$$

output
```
2*a*(-b^2*c+a^2)^2*(a+b*(c+d/x)^(1/2))^(p+1)/b^6/d^3/(p+1)-2*(b^4*c^2-6*a^
2*b^2*c+5*a^4)*(a+b*(c+d/x)^(1/2))^(2+p)/b^6/d^3/(2+p)+4*a*(-3*b^2*c+5*a^2
)*(a+b*(c+d/x)^(1/2))^(3+p)/b^6/d^3/(3+p)-4*(-b^2*c+5*a^2)*(a+b*(c+d/x)^(1
/2))^(4+p)/b^6/d^3/(4+p)+10*a*(a+b*(c+d/x)^(1/2))^(5+p)/b^6/d^3/(5+p)-2*(a
+b*(c+d/x)^(1/2))^(6+p)/b^6/d^3/(6+p)
```

Mathematica [A] (verified)

Time = 3.45 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.19

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx = \frac{2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{1+p}\left(120a^5x^2-120a^4b(1+p)\sqrt{c+\frac{d}{x}}x^2-4a^2b^3(1+p)\sqrt{c+\frac{d}{x}}x(5d(6+5p+p^2)+2c(\right.}{}$$

input
```
Integrate[(a + b*Sqrt[c + d/x])^p/x^4,x]
```

output
```
(2*(a + b*Sqrt[c + d/x])^(1 + p)*(120*a^5*x^2 - 120*a^4*b*(1 + p)*Sqrt[c +
 d/x]*x^2 - 4*a^2*b^3*(1 + p)*Sqrt[c + d/x]*x*(5*d*(6 + 5*p + p^2) + 2*c*(
-30 - 4*p + p^2)*x) + 12*a^3*b^2*x*(5*d*(2 + 3*p + p^2) + 4*c*(-5 + p + p^
2)*x) - b^5*(15 + 23*p + 9*p^2 + p^3)*Sqrt[c + d/x]*(d^2*(8 + 6*p + p^2) -
 4*c*d*(2 + p)*x + 8*c^2*x^2) + a*b^4*(5*d^2*(24 + 50*p + 35*p^2 + 10*p^3
+ p^4) + 4*c*d*(-30 - 43*p - 10*p^2 + 4*p^3 + p^4)*x - 8*c^2*(-15 + 10*p +
 12*p^2 + 2*p^3)*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(
6 + p)*x^2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7268, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{2\int \frac{d^2\left(a+b\sqrt{c+\frac{d}{x}}\right)^p\sqrt{c+\frac{d}{x}}}{x^2}d\sqrt{c+\frac{d}{x}}}{d^3}$$

$$\Big\downarrow \textbf{522}$$

$$-\frac{2\int \left(-\frac{a(a^2-b^2c)^2\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{b^5}+\frac{(5a^4-6b^2ca^2+b^4c^2)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{b^5}-\frac{2(5a^3-3ab^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+2}}{b^5}-\frac{2(b^2c-5a^2)\left(a+b\sqrt{\cdots}\right.}{b^5}\cdots\right.}{d^3}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{2\left(-\frac{a(a^2-b^2c)^2\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+1}}{b^6(p+1)}-\frac{2a(5a^2-3b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+3}}{b^6(p+3)}+\frac{2(5a^2-b^2c)\left(a+b\sqrt{c+\frac{d}{x}}\right)^{p+4}}{b^6(p+4)}+\frac{(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{\cdots}\right.}{b^6(p+2)}\cdots\right.}{d^3}$$

input
```
Int[(a + b*Sqrt[c + d/x])^p/x^4,x]
```

output
```
(-2*(-((a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d/x])^(1 + p))/(b^6*(1 + p))) +
((5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d/x])^(2 + p))/(b^6*(2 +
p)) - (2*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d/x])^(3 + p))/(b^6*(3 + p))
+ (2*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d/x])^(4 + p))/(b^6*(4 + p)) - (5*a*(
a + b*Sqrt[c + d/x])^(5 + p))/(b^6*(5 + p)) + (a + b*Sqrt[c + d/x])^(6 + p
)/(b^6*(6 + p))))/d^3
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x^4,x)
```

output
```
int((a+b*(c+d/x)^(1/2))^p/x^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(242) = 484.

Time = 2.81 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.86

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx = -\frac{2\left(b^6d^3p^5+15\,b^6d^3p^4+85\,b^6d^3p^3+225\,b^6d^3p^2+274\,b^6d^3p+120\,b^6d^3+8\,(15\,b^6c^3-45\,a^2b^4c^2+45\,a^4\right.}{}$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^4,x, algorithm="fricas")
```

output
```
-2*(b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*b^6*d^3*p^2 + 274*
b^6*d^3*p + 120*b^6*d^3 + 8*(15*b^6*c^3 - 45*a^2*b^4*c^2 + 45*a^4*b^2*c -
15*a^6 + (b^6*c^3 + 3*a^2*b^4*c^2)*p^3 + 3*(3*b^6*c^3 + 3*a^2*b^4*c^2 - 2*
a^4*b^2*c)*p^2 + (23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p)*x^3 - 4*((
b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23*b^6*c^2
 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)
*d*p)*x^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (41*b^6*c -
30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c - a^2*b^
4)*d^2*p)*x + (8*((3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 3*(7*a*b^5*c^2 - 3*a^3*b
^3*c)*p^2 + (33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p)*x^3 - 4*(2*a*b^5*c
*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3*b^3)*d*p^2 +
 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 +
35*a*b^5*d^2*p^3 + 50*a*b^5*d^2*p^2 + 24*a*b^5*d^2*p)*x)*sqrt((c*x + d)/x)
)*(b*sqrt((c*x + d)/x) + a)^p/((b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3
*p^4 + 735*b^6*d^3*p^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)*
x^3)
```

Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx=\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*(c+d/x)**(1/2))**p/x**4,x)
```

output
```
Integral((a + b*sqrt(c + d/x))**p/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx=\int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx = \int \frac{\left(b\sqrt{c+\frac{d}{x}}+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*(c+d/x)^(1/2))^p/x^4,x, algorithm="giac")
```

output
```
integrate((b*sqrt(c + d/x) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx = \int \frac{\left(a+b\,\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx$$

input
```
int((a + b*(c + d/x)^(1/2))^p/x^4,x)
```

output
```
int((a + b*(c + d/x)^(1/2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{c+\frac{d}{x}}\right)^p}{x^4}\,dx = \int \frac{\left(\sqrt{cx+d}\,b+\sqrt{x}\,a\right)^p}{x^{\frac{p}{2}}x^4}\,dx$$

input
```
int((a+b*(c+d/x)^(1/2))^p/x^4,x)
```

output
```
int((sqrt(c*x + d)*b + sqrt(x)*a)**p/(x**(p/2)*x**4),x)
```

3.306 $\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx$

Optimal result

Integrand size = 19, antiderivative size = 386

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx = \frac{d\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(b(a^2c(4-p)-b^2(2+p))-4a^3c\sqrt{c+\frac{d}{x}}+2ab^2(1+p)\sqrt{c+\frac{d}{x}}\right)(d+cx)}{4c\left(b^2-a^2c\right)^2\sqrt{c+\frac{d}{x}}} - \frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}(d+cx)^2}{2c\left(b^2-a^2c\right)} + \frac{bd^2p\left(3a\sqrt{c}-b(2+p)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b-a\sqrt{c}}\right)}{8\left(b-a\sqrt{c}\right)^3c^{3/2}(1+p)} + \frac{bd^2p\left(3a\sqrt{c}+b(2+p)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b+a\sqrt{c}}\right)}{8\left(b+a\sqrt{c}\right)^3c^{3/2}(1+p)}$$

output
```
1/4*d*(a+b/(c+d/x)^(1/2))^(p+1)*(b*(a^2*c*(4-p)-b^2*(2+p))-4*a^3*c*(c+d/x)
^(1/2)+2*a*b^2*(p+1)*(c+d/x)^(1/2))*(c*x+d)/c/(-a^2*c+b^2)^2/(c+d/x)^(1/2)
-1/2*(a-b/(c+d/x)^(1/2))*(a+b/(c+d/x)^(1/2))^(p+1)*(c*x+d)^2/c/(-a^2*c+b^2
)+1/8*b*d^2*p*(3*a*c^(1/2)-b*(2+p))*(a+b/(c+d/x)^(1/2))^(p+1)*hypergeom([1
, p+1],[2+p],-c^(1/2)*(a+b/(c+d/x)^(1/2))/(b-a*c^(1/2)))/(b-a*c^(1/2))^3/c
^(3/2)/(p+1)+1/8*b*d^2*p*(3*a*c^(1/2)+b*(2+p))*(a+b/(c+d/x)^(1/2))^(p+1)*h
ypergeom([1, p+1],[2+p],c^(1/2)*(a+b/(c+d/x)^(1/2))/(b+a*c^(1/2)))/(b+a*c^
(1/2))^3/c^(3/2)/(p+1)
```

Mathematica [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx=\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx$$

input
```
Integrate[(a + b/Sqrt[c + d/x])^p*x,x]
```

output
```
Integrate[(a + b/Sqrt[c + d/x])^p*x, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {7268, 25, 1894, 1803, 25, 602, 25, 27, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

$$\Big\downarrow \text{ 7268}$$

$$-2d^2\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \sqrt{c+\frac{d}{x}}x^3}{d^3}d\sqrt{c+\frac{d}{x}}$$

$\downarrow$ 25

$$2d^2\int -\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \sqrt{c+\frac{d}{x}}x^3}{d^3}d\sqrt{c+\frac{d}{x}}$$

$\downarrow$ 1894

$$2d^2\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(\frac{c}{c+\frac{d}{x}}-1\right)^3\left(c+\frac{d}{x}\right)^{5/2}}d\sqrt{c+\frac{d}{x}}$$

$\downarrow$ 1803

$$-2d^2\int -\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(c+\frac{d}{x}\right)^{3/2}}{\left(1-c\left(c+\frac{d}{x}\right)\right)^3}d\frac{1}{\sqrt{c+\frac{d}{x}}}$$

$\downarrow$ 25

$$2d^2\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(c+\frac{d}{x}\right)^{3/2}}{\left(1-c\left(c+\frac{d}{x}\right)\right)^3}d\frac{1}{\sqrt{c+\frac{d}{x}}}$$

$\downarrow$ 602

$$-2d^2\left(\frac{\int -\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(abp+\frac{4a^2c-b^2(p+2)}{\sqrt{c+\frac{d}{x}}}\right)}{c\left(1-c\left(c+\frac{d}{x}\right)\right)^2}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{4\left(b^2-a^2c\right)}+\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4c\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)^2}\right)$$

$\downarrow$ 25

$$-2d^2\left(\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4c\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)^2}-\frac{\int\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(abp+\frac{4a^2c-b^2(p+2)}{\sqrt{c+\frac{d}{x}}}\right)}{c\left(1-c\left(c+\frac{d}{x}\right)\right)^2}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{4\left(b^2-a^2c\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$-2d^2\left(\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4c\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)^2}-\frac{\int\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(abp+\frac{4a^2c-b^2(p+2)}{\sqrt{c+\frac{d}{x}}}\right)}{\left(1-c\left(c+\frac{d}{x}\right)\right)^2}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{4c\left(b^2-a^2c\right)}\right)$$

$$\Big\downarrow \text{686}$$

$$-2d^2\left(\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4c\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)^2}-\frac{\frac{\int\frac{bcp\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(a\left(3a^2c-b^2(2p+1)\right)-\frac{b\left(a^2c(4-p)-b^2(p+2)\right)}{\sqrt{c+\frac{d}{x}}}\right)}{1-c\left(c+\frac{d}{x}\right)}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{2c(b^2-a^2c)}-\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{}}{4c\left(b^2-a^2c\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$-2d^2\left(\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4c\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)^2}-\frac{\frac{bp\int\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(a\left(3a^2c-b^2(2p+1)\right)-\frac{b\left(a^2c(4-p)-b^2(p+2)\right)}{\sqrt{c+\frac{d}{x}}}\right)}{1-c\left(c+\frac{d}{x}\right)}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{2(b^2-a^2c)}-\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{}}{4c\left(b^2-a^2c\right)}\right.$$

$$\Big\downarrow \text{657}$$

$$-2d^2\left(\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4c\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)^2}-\frac{\frac{bp\int\left(\frac{\left(a\left(3a^2c-b^2(2p+1)\right)-\frac{b\left(a^2c(4-p)-b^2(p+2)\right)}{\sqrt{c}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{2\left(1-\frac{\sqrt{c}}{\sqrt{c+\frac{d}{x}}}\right)}+\frac{\left(\frac{b\left(a^2c(4-p)-b^2(p+2)\right)}{\sqrt{c}}\right.}{2}}{2(b^2-a^2c)}}{}\right.$$

$\downarrow$ 2009

$$-2d^2\left(\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4c\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)^2}-\frac{bp\left(\frac{(a\sqrt{c}+b)^2(3a\sqrt{c}-b(p+2))\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right.}{b-a\sqrt{c}}\right.}{2\sqrt{c}(p+1)(b-a\sqrt{c})}\right.}{2(}\right.$$

input

```
Int[(a + b/Sqrt[c + d/x])^p*x,x]
```

output
```
-2*d^2*(((a - b/Sqrt[c + d/x])*(a + b/Sqrt[c + d/x])^(1 + p))/(4*c*(b^2 -
a^2*c)*(1 - c*(c + d/x))^2) - (-1/2*((a + b/Sqrt[c + d/x])^(1 + p)*(2*a*(2
*a^2*c - b^2*(1 + p)) - (b*(a^2*c*(4 - p) - b^2*(2 + p)))/Sqrt[c + d/x]))/
((b^2 - a^2*c)*(1 - c*(c + d/x))) + (b*p*(((b + a*Sqrt[c])^2*(3*a*Sqrt[c]
- b*(2 + p))*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, -((Sqrt[c]*(a + b/Sqrt[c + d/x]))/(b - a*Sqrt[c]))])/(2*(b - a*Sqrt[c]
)*Sqrt[c]*(1 + p)) + ((b - a*Sqrt[c])^2*(3*a*Sqrt[c] + b*(2 + p))*(a + b/S
qrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (Sqrt[c]*(a + b/S
qrt[c + d/x]))/(b + a*Sqrt[c])])/(2*(b + a*Sqrt[c])*Sqrt[c]*(1 + p))))/(2*
(b^2 - a^2*c)))/(4*c*(b^2 - a^2*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 602
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomia
lRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
+ b*x^2, x], x, 1]}, Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^(p + 1)*((a*(d*e
 - c*f) + (b*c*e + a*d*f)*x)/(2*a*(p + 1)*(b*c^2 + a*d^2))), x] + Simp[1/(2
*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*x^2)^(p + 1)*ExpandToS
um[2*a*(p + 1)*(b*c^2 + a*d^2)*Qx + e*(b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3
)) - a*c*d*f*n + d*(b*c*e + a*d*f)*(n + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, d, n}, x] && IGtQ[m, 1] && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p*x,x)
```

output
```
int((a+b/(c+d/x)^(1/2))^p*x,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx=\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p*x,x, algorithm="fricas")
```

output
```
integral(x*((a*c*x + b*x*sqrt((c*x + d)/x) + a*d)/(c*x + d))^p, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx=\int x\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\,dx$$

input
```
integrate((a+b/(c+d/x)**(1/2))**p*x,x)
```

output
```
Integral(x*(a + b/sqrt(c + d/x))**p, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx=\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p*x,x, algorithm="maxima")
```

output
```
integrate((a + b/sqrt(c + d/x))^p*x, x)
```

Giac [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx=\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p*x,x, algorithm="giac")
```

output
```
integrate((a + b/sqrt(c + d/x))^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx=\int x\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\,dx$$

input
```
int(x*(a + b/(c + d/x)^(1/2))^p,x)
```

output
```
int(x*(a + b/(c + d/x)^(1/2))^p, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p x\,dx=\int \frac{\left(\sqrt{cx+d}\,a+\sqrt{x}\,b\right)^p x}{(cx+d)^{\frac{p}{2}}}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p*x,x)
```

output
```
int(((sqrt(c*x + d)*a + sqrt(x)*b)**p*x)/(c*x + d)**(p/2),x)
```

3.307 $\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$

Optimal result

Integrand size = 17, antiderivative size = 180

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx = \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p (d+cx)}{c} - \frac{bd\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b-a\sqrt{c}}\right)}{2\left(b-a\sqrt{c}\right)c} - \frac{bd\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b+a\sqrt{c}}\right)}{2\left(b+a\sqrt{c}\right)c}$$

output
```
(a+b/(c+d/x)^(1/2))^p*(c*x+d)/c-1/2*b*d*(a+b/(c+d/x)^(1/2))^p*hypergeom([1
, p],[p+1],-c^(1/2)*(a+b/(c+d/x)^(1/2))/(b-a*c^(1/2)))/(b-a*c^(1/2))/c-1/2
*b*d*(a+b/(c+d/x)^(1/2))^p*hypergeom([1, p],[p+1],c^(1/2)*(a+b/(c+d/x)^(1/
2))/(b+a*c^(1/2)))/(b+a*c^(1/2))/c
```

Mathematica [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx=\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

input
```
Integrate[(a + b/Sqrt[c + d/x])^p,x]
```

output
```
Integrate[(a + b/Sqrt[c + d/x])^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {7268, 1894, 1803, 593, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

$$\Big\downarrow \text{ 7268}$$

$$-2d\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \sqrt{c+\frac{d}{x}}x^2}{d^2}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{ 1894}$$

$$-2d\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(\frac{c}{c+\frac{d}{x}}-1\right)^2\left(c+\frac{d}{x}\right)^{3/2}}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{1803}$$

$$2d\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(1-c\left(c+\frac{d}{x}\right)\right)^2\sqrt{c+\frac{d}{x}}}d\frac{1}{\sqrt{c+\frac{d}{x}}}$$

$$\Big\downarrow \text{593}$$

$$2d\left(\frac{b\int \frac{p\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{1-c\left(c+\frac{d}{x}\right)}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{2\left(b^2-a^2c\right)}-\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{2\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2d\left(\frac{bp\int \frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{1-c\left(c+\frac{d}{x}\right)}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{2\left(b^2-a^2c\right)}-\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{2\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$2d\left(\frac{bp\int \left(\frac{\left(a-\frac{b}{\sqrt{c}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{2\left(1-\frac{\sqrt{c}}{\sqrt{c+\frac{d}{x}}}\right)}+\frac{\left(a+\frac{b}{\sqrt{c}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{2\left(\frac{\sqrt{c}}{\sqrt{c+\frac{d}{x}}}+1\right)}\right)d\frac{1}{\sqrt{c+\frac{d}{x}}}}{2\left(b^2-a^2c\right)}-\frac{\left(a-\frac{b}{\sqrt{c+\frac{d}{x}}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{2\left(b^2-a^2c\right)\left(1-c\left(c+\frac{d}{x}\right)\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2d\left(\frac{bp\left(\frac{\left(a+\frac{b}{\sqrt{c}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b-a\sqrt{c}}\right)}{2(p+1)(b-a\sqrt{c})}+\frac{\left(a-\frac{b}{\sqrt{c}}\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}\text{Hypergeometric2F1}}{2(p+1)(a\sqrt{c}+b)}\right.}{2\left(b^2-a^2c\right)}\right.$$

input
```
Int[(a + b/Sqrt[c + d/x])^p,x]
```

output
```
2*d*(-1/2*((a - b/Sqrt[c + d/x])*(a + b/Sqrt[c + d/x])^(1 + p))/((b^2 - a^
2*c)*(1 - c*(c + d/x))) + (b*p*(((a + b/Sqrt[c])*(a + b/Sqrt[c + d/x])^(1
+ p)*Hypergeometric2F1[1, 1 + p, 2 + p, -((Sqrt[c]*(a + b/Sqrt[c + d/x]))/
(b - a*Sqrt[c]))])/(2*(b - a*Sqrt[c])*(1 + p)) + ((a - b/Sqrt[c])*(a + b/S
qrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (Sqrt[c]*(a + b/S
qrt[c + d/x]))/(b + a*Sqrt[c])])/(2*(b + a*Sqrt[c])*(1 + p))))/(2*(b^2 - a
^2*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p,x)
```

output
```
int((a+b/(c+d/x)^(1/2))^p,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx = \int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral(((a*c*x + b*x*sqrt((c*x + d)/x) + a*d)/(c*x + d))^p, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx = \int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

input
```
integrate((a+b/(c+d/x)**(1/2))**p,x)
```

output
```
Integral((a + b/sqrt(c + d/x))**p, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx = \int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/sqrt(c + d/x))^p, x)
```

Giac [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx = \int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((a + b/sqrt(c + d/x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx = \int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx$$

input
```
int((a + b/(c + d/x)^(1/2))^p,x)
```

output
```
int((a + b/(c + d/x)^(1/2))^p, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p dx = \int \frac{\left(\sqrt{cx+d}\,a+\sqrt{x}\,b\right)^p}{(cx+d)^{\frac{p}{2}}}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p,x)
```

output
```
int((sqrt(c*x + d)*a + sqrt(x)*b)**p/(c*x + d)**(p/2),x)
```

3.308 $\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x} dx$

Optimal result

Integrand size = 21, antiderivative size = 222

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x} dx = -\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b-a\sqrt{c}}\right)}{\left(b-a\sqrt{c}\right)(1+p)} + \frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b+a\sqrt{c}}\right)}{\left(b+a\sqrt{c}\right)(1+p)} - \frac{2\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a\sqrt{c+\frac{d}{x}}}\right)}{a(1+p)}$$

output
```
-c^(1/2)*(a+b/(c+d/x)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],-c^(1/2)*(a+b/
(c+d/x)^(1/2))/(b-a*c^(1/2)))/(b-a*c^(1/2))/(p+1)+c^(1/2)*(a+b/(c+d/x)^(1/
2))^(p+1)*hypergeom([1, p+1],[2+p],c^(1/2)*(a+b/(c+d/x)^(1/2))/(b+a*c^(1/2
)))/(b+a*c^(1/2))/(p+1)-2*(a+b/(c+d/x)^(1/2))^(p+1)*hypergeom([1, p+1],[2+
p],1+b/a/(c+d/x)^(1/2))/a/(p+1)
```

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx =$$

$$-\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(\frac{\sqrt{c}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b-a\sqrt{c}}\right)}{b-a\sqrt{c}}-\frac{\sqrt{c}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b+a\sqrt{c}}\right.}{b+a\sqrt{c}}\right.}{1+p}$$

input
```
Integrate[(a + b/Sqrt[c + d/x])^p/x,x]
```

output
```
-(((a + b/Sqrt[c + d/x])^(1 + p)*((Sqrt[c]*Hypergeometric2F1[1, 1 + p, 2 +
 p, -((Sqrt[c]*(a + b/Sqrt[c + d/x]))/(b - a*Sqrt[c]))])/(b - a*Sqrt[c]) -
 (Sqrt[c]*Hypergeometric2F1[1, 1 + p, 2 + p, (Sqrt[c]*(a + b/Sqrt[c + d/x]
))/(b + a*Sqrt[c])])/(b + a*Sqrt[c]) + (2*Hypergeometric2F1[1, 1 + p, 2 +
p, 1 + b/(a*Sqrt[c + d/x])])/a))/(1 + p))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7268, 25, 1894, 1803, 25, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x} \, dx$$

$$\Big\downarrow \text{ 7268}$$

$$-2 \int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \sqrt{c+\frac{d}{x}}x}{d}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{ 25}$$

$$2 \int -\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \sqrt{c+\frac{d}{x}}x}{d}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{ 1894}$$

$$2 \int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(\frac{c}{c+\frac{d}{x}}-1\right)\sqrt{c+\frac{d}{x}}}d\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{ 1803}$$

$$-2 \int -\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(1-c\left(c+\frac{d}{x}\right)\right)\sqrt{c+\frac{d}{x}}}d\frac{1}{\sqrt{c+\frac{d}{x}}}$$

$$\Big\downarrow \text{ 25}$$

$$2 \int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(1-c\left(c+\frac{d}{x}\right)\right)\sqrt{c+\frac{d}{x}}}d\frac{1}{\sqrt{c+\frac{d}{x}}}$$

$$\Big\downarrow \text{615}$$

$$2\int \left(\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\sqrt{c+\frac{d}{x}}}-\frac{c\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(c\left(c+\frac{d}{x}\right)-1\right)\sqrt{c+\frac{d}{x}}}\right)d\frac{1}{\sqrt{c+\frac{d}{x}}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)}{b-a\sqrt{c}}\right)}{2(p+1)\left(b-a\sqrt{c}\right)}-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}\text{Hyperg}}{2}\right.$$

input
```
Int[(a + b/Sqrt[c + d/x])^p/x,x]
```

output
```
-2*((Sqrt[c]*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, -((Sqrt[c]*(a + b/Sqrt[c + d/x]))/(b - a*Sqrt[c]))])/(2*(b - a*Sqrt[c]
)*(1 + p)) - (Sqrt[c]*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, (Sqrt[c]*(a + b/Sqrt[c + d/x]))/(b + a*Sqrt[c])])/(2*(b + a*S
qrt[c])*(1 + p)) + ((a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 + b/(a*Sqrt[c + d/x])])/(a*(1 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x,x)
```

output
```
int((a+b/(c+d/x)^(1/2))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x,x, algorithm="fricas")
```

output
```
integral(((a*c*x + b*x*sqrt((c*x + d)/x) + a*d)/(c*x + d))^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d/x)**(1/2))**p/x,x)
```

output
```
Integral((a + b/sqrt(c + d/x))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x,x, algorithm="maxima")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x,x, algorithm="giac")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx$$

input
```
int((a + b/(c + d/x)^(1/2))^p/x,x)
```

output
```
int((a + b/(c + d/x)^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{cx+d}\,a+\sqrt{x}\,b\right)^p}{(cx+d)^{\frac{p}{2}}\,x}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x,x)
```

output
```
int((sqrt(c*x + d)*a + sqrt(x)*b)**p/((c*x + d)**(p/2)*x),x)
```

3.309 $\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx = -\frac{2b^2\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b}{a\sqrt{c+\frac{d}{x}}}\right)}{a^3d(1+p)}$$

output
```
-2*b^2*(a+b/(c+d/x)^(1/2))^(p+1)*hypergeom([3, p+1],[2+p],1+b/a/(c+d/x)^(1
/2))/a^3/d/(p+1)
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx = -\frac{2b^2\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b}{a\sqrt{c+\frac{d}{x}}}\right)}{a^3d(1+p)}$$

input
```
Integrate[(a + b/Sqrt[c + d/x])^p/x^2,x]
```

output
```
(-2*b^2*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1
 + b/(a*Sqrt[c + d/x])])/(a^3*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7247, 774, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{7247}$$

$$-\frac{\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p d\left(c+\frac{d}{x}\right)}{d}$$

$$\Big\downarrow \text{774}$$

$$-\frac{2\int \left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p \sqrt{c+\frac{d}{x}}d\sqrt{c+\frac{d}{x}}}{d}$$

$$\Big\downarrow \text{798}$$

$$\frac{2\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(c+\frac{d}{x}\right)^{3/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{d}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2b^2\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{b}{a\sqrt{c+\frac{d}{x}}}+1\right)}{a^3d(p+1)}$$

input
```
Int[(a + b/Sqrt[c + d/x])^p/x^2,x]
```

output
```
(-2*b^2*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1
 + b/(a*Sqrt[c + d/x])])/(a^3*d*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 7247
```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Simp[q   Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fal
seQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x^2,x)
```

output
```
int((a+b/(c+d/x)^(1/2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^2,x, algorithm="fricas")
```

output
```
integral(((a*c*x + b*x*sqrt((c*x + d)/x) + a*d)/(c*x + d))^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(c+d/x)**(1/2))**p/x**2,x)
```

output
```
Integral((a + b/sqrt(c + d/x))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx=\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(c+\frac{d}{x}\right)\,{}_2\mathrm{F}_1\left(2-p,-p;\;3-p;\;-\frac{a\sqrt{c+\frac{d}{x}}}{b}\right)}{d\,\left(\frac{p}{2}-1\right)\,\left(\frac{a\sqrt{c+\frac{d}{x}}}{b}+1\right)^p}$$

input
```
int((a + b/(c + d/x)^(1/2))^p/x^2,x)
```

output
```
((a + b/(c + d/x)^(1/2))^p*(c + d/x)*hypergeom([2 - p, -p], 3 - p, -(a*(c
+ d/x)^(1/2))/b))/(d*(p/2 - 1)*((a*(c + d/x)^(1/2))/b + 1)^p)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^2}\,dx=\int \frac{\left(\sqrt{cx+d}\,a+\sqrt{x}\,b\right)^p}{(cx+d)^{\frac{p}{2}}\,x^2}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x^2,x)
```

output
```
int((sqrt(c*x + d)*a + sqrt(x)*b)**p/((c*x + d)**(p/2)*x**2),x)
```

3.310 $\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3} dx = \frac{b(3-p)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(c+\frac{d}{x}\right)^{3/2}}{6a^2d^2} - \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(c+\frac{d}{x}\right)^2}{2ad^2} + \frac{b^2(12a^2c-b^2(6-5p+p^2))\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b}{a\sqrt{c+\frac{d}{x}}}\right)}{6a^5d^2(1+p)}$$

output
```
1/6*b*(3-p)*(a+b/(c+d/x)^(1/2))^(p+1)*(c+d/x)^(3/2)/a^2/d^2-1/2*(a+b/(c+d/
x)^(1/2))^(p+1)*(c+d/x)^2/a/d^2+1/6*b^2*(12*a^2*c-b^2*(p^2-5*p+6))*(a+b/(c
+d/x)^(1/2))^(p+1)*hypergeom([3, p+1],[2+p],1+b/a/(c+d/x)^(1/2))/a^5/d^2/(
p+1)
```

Mathematica [A] (verified)

Time = 3.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}\,dx = -\frac{2\left(-\frac{b^2c\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b}{a\sqrt{c+\frac{d}{x}}}\right)}{a^3(1+p)}+\frac{b^4\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(5,1+p,2+p,1+\frac{b}{a\sqrt{c+\frac{d}{x}}}\right)}{a^5(1+p)}\right)}{d^2}$$

input
```
Integrate[(a + b/Sqrt[c + d/x])^p/x^3,x]
```

output
```
(-2*(-((b^2*c*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2
+ p, 1 + b/(a*Sqrt[c + d/x])])/(a^3*(1 + p))) + (b^4*(a + b/Sqrt[c + d/x])
^(1 + p)*Hypergeometric2F1[5, 1 + p, 2 + p, 1 + b/(a*Sqrt[c + d/x])])/(a^5
*(1 + p))))/d^2
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {7268, 25, 1894, 1803, 25, 520, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}\,dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{2\int \frac{d\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\sqrt{c+\frac{d}{x}}}{x}d\sqrt{c+\frac{d}{x}}}{d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{2\int -\frac{d\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\sqrt{c+\frac{d}{x}}}{x}d\sqrt{c+\frac{d}{x}}}{d^2}$$

$$\Big\downarrow \ 1894$$

$$\frac{2\int \left(\frac{c}{c+\frac{d}{x}}-1\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(c+\frac{d}{x}\right)^{3/2}d\sqrt{c+\frac{d}{x}}}{d^2}$$

$$\Big\downarrow \ 1803$$

$$-\frac{2\int -\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(1-c\left(c+\frac{d}{x}\right)\right)}{\left(c+\frac{d}{x}\right)^{5/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{2\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(1-c\left(c+\frac{d}{x}\right)\right)}{\left(c+\frac{d}{x}\right)^{5/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{d^2}$$

$$\Big\downarrow \ 520$$

$$-\frac{2\left(\frac{\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(\frac{4ac}{\sqrt{c+\frac{d}{x}}}+b(3-p)\right)}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{4a}+\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4a\left(c+\frac{d}{x}\right)^2}\right)}{d^2}$$

$$\Big\downarrow \ 87$$

$$-\frac{2\left(\frac{\frac{\left(12a^2c-b^2(2-p)(3-p)\right)\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(c+\frac{d}{x}\right)^{3/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{3a}-\frac{b(3-p)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{3a\left(c+\frac{d}{x}\right)^{3/2}}}{4a}+\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4a\left(c+\frac{d}{x}\right)^2}\right)}{d^2}$$

$$\Big\downarrow \ 75$$

$$-\frac{2\left(\frac{-\frac{b^2\left(12a^2c-b^2(2-p)(3-p)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{b}{a\sqrt{c+\frac{d}{x}}}+1\right)}{3a^4(p+1)}-\frac{b(3-p)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{3a\left(c+\frac{d}{x}\right)^{3/2}}}{4a}+\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4a\left(c+\frac{d}{x}\right)^2}\right)}{d^2}$$

input
```
Int[(a + b/Sqrt[c + d/x])^p/x^3,x]
```

output
```
(-2*((a + b/Sqrt[c + d/x])^(1 + p)/(4*a*(c + d/x)^2) + (-1/3*(b*(3 - p)*(a
 + b/Sqrt[c + d/x])^(1 + p))/(a*(c + d/x)^(3/2)) - (b^2*(12*a^2*c - b^2*(2
 - p)*(3 - p))*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2
 + p, 1 + b/(a*Sqrt[c + d/x])])/(3*a^4*(1 + p)))/(4*a)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 520
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2)^p, e*x, x], R = Pol
ynomialRemainder[(a + b*x^2)^p, e*x, x]}, Simp[R*(e*x)^(m + 1)*((c + d*x)^(
n + 1)/((m + 1)*(e*c))), x] + Simp[1/((m + 1)*(e*c))   Int[(e*x)^(m + 1)*(c
 + d*x)^n*ExpandToSum[(m + 1)*(e*c)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[m, -1] &&  !IntegerQ[n]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x^3,x)
```

output
```
int((a+b/(c+d/x)^(1/2))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3} \, dx = \int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3} \, dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^3,x, algorithm="fricas")
```

output
```
integral(((a*c*x + b*x*sqrt((c*x + d)/x) + a*d)/(c*x + d))^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d/x)**(1/2))**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3} \, dx = \int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3} \, dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}\,dx$$

input
```
int((a + b/(c + d/x)^(1/2))^p/x^3,x)
```

output
```
int((a + b/(c + d/x)^(1/2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^3}\,dx=\int \frac{\left(\sqrt{cx+d}\,a+\sqrt{x}\,b\right)^p}{(cx+d)^{\frac{p}{2}}\,x^3}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x^3,x)
```

output
```
int((sqrt(c*x + d)*a + sqrt(x)*b)**p/((c*x + d)**(p/2)*x**3),x)
```

3.311 $\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 316

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx = -\frac{b(3-p)\left(60a^2c-b^2(20-9p+p^2)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(c+\frac{d}{x}\right)^{3/2}}{180a^4d^3}$$
$$+\frac{\left(60a^2c-b^2(20-9p+p^2)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(c+\frac{d}{x}\right)^2}{60a^3d^3}$$
$$+\frac{b(5-p)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(c+\frac{d}{x}\right)^{5/2}}{15a^2d^3}-\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\left(c+\frac{d}{x}\right)^3}{3ad^3}$$
$$-\frac{b^2\left(360a^4c^2-b^2(2-p)(3-p)\left(60a^2c-b^2(20-9p+p^2)\right)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{1+p}\text{Hypergeometric2F1}\left(3,1\right.}{180a^7d^3(1+p)}$$

output
```
-1/180*b*(3-p)*(60*a^2*c-b^2*(p^2-9*p+20))*(a+b/(c+d/x)^(1/2))^(p+1)*(c+d/
x)^(3/2)/a^4/d^3+1/60*(60*a^2*c-b^2*(p^2-9*p+20))*(a+b/(c+d/x)^(1/2))^(p+1
)*(c+d/x)^2/a^3/d^3+1/15*b*(5-p)*(a+b/(c+d/x)^(1/2))^(p+1)*(c+d/x)^(5/2)/a
^2/d^3-1/3*(a+b/(c+d/x)^(1/2))^(p+1)*(c+d/x)^3/a/d^3-1/180*b^2*(360*a^4*c^
2-b^2*(2-p)*(3-p)*(60*a^2*c-b^2*(p^2-9*p+20)))*(a+b/(c+d/x)^(1/2))^(p+1)*h
ypergeom([3, p+1],[2+p],1+b/a/(c+d/x)^(1/2))/a^7/d^3/(p+1)
```

Mathematica [A] (verified)

Time = 4.05 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.50

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx = -\frac{2b^2\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(b+a\sqrt{c+\frac{d}{x}}\right)\left(a^4c^2\,\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b}{a\sqrt{c+\frac{d}{x}}}\right)-2a^2b^2c\,\text{H}\right.}{a^7d^3(1-}$$

input
```
Integrate[(a + b/Sqrt[c + d/x])^p/x^4,x]
```

output
```
(-2*b^2*(a + b/Sqrt[c + d/x])^p*(b + a*Sqrt[c + d/x])*(a^4*c^2*Hypergeomet
ric2F1[3, 1 + p, 2 + p, 1 + b/(a*Sqrt[c + d/x])] - 2*a^2*b^2*c*Hypergeomet
ric2F1[5, 1 + p, 2 + p, 1 + b/(a*Sqrt[c + d/x])] + b^4*Hypergeometric2F1[7
, 1 + p, 2 + p, 1 + b/(a*Sqrt[c + d/x])]))/(a^7*d^3*(1 + p)*Sqrt[c + d/x])
```

Rubi [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {7268, 1894, 1803, 520, 2124, 25, 520, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx$$

$\Big\downarrow$ 7268

$$-\frac{2\int \frac{d^2\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\sqrt{c+\frac{d}{x}}}{x^2}d\sqrt{c+\frac{d}{x}}}{d^3}$$

$\Big\downarrow$ 1894

$$-\frac{2\int \left(\frac{c}{c+\frac{d}{x}}-1\right)^2\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(c+\frac{d}{x}\right)^{5/2}d\sqrt{c+\frac{d}{x}}}{d^3}$$

$\Big\downarrow$ 1803

$$\frac{2\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(1-c\left(c+\frac{d}{x}\right)\right)^2}{\left(c+\frac{d}{x}\right)^{7/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{d^3}$$

$\Big\downarrow$ 520

$$\frac{2\left(-\frac{\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(-6a\left(c+\frac{d}{x}\right)^{3/2}c^2+\frac{12ac}{\sqrt{c+\frac{d}{x}}}+b(5-p)\right)}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{6a}-\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{6a\left(c+\frac{d}{x}\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 2124

$$\frac{2\left(-\frac{-\frac{\int -\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(60ca^2-30c^2\left(c+\frac{d}{x}\right)a^2-b^2\left(p^2-9p+20\right)\right)}{\left(c+\frac{d}{x}\right)^{5/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{5a}-\frac{b(5-p)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{5a\left(c+\frac{d}{x}\right)^{5/2}}}{6a}-\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{6a\left(c+\frac{d}{x}\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 25

$$\frac{2\left(-\frac{\frac{\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(60ca^2-30c^2\left(c+\frac{d}{x}\right)a^2-b^2\left(p^2-9p+20\right)\right)}{\left(c+\frac{d}{x}\right)^{5/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{5a}-\frac{b(5-p)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{5a\left(c+\frac{d}{x}\right)^{5/2}}}{6a}-\frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{6a\left(c+\frac{d}{x}\right)^3}\right)}{d^3}$$

$\big\downarrow$ 520

$$\frac{2\left(-\frac{\frac{-\frac{\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p\left(\frac{120c^2a^3}{\sqrt{c+\frac{d}{x}}}+b(3-p)\left(60a^2c-b^2\left(p^2-9p+20\right)\right)\right)}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{4a}-\frac{\left(60a^2c-b^2\left(p^2-9p+20\right)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4a\left(c+\frac{d}{x}\right)^2}}{5a}-\frac{b(5-p)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{5a\left(c+\frac{d}{x}\right)^{5/2}}}{6a} \cdots\right)}{d^3}$$

$\big\downarrow$ 87

$$\frac{2\left(-\frac{\frac{-\frac{\frac{\left(360a^4c^2-b^2(2-p)(3-p)\left(60a^2c-b^2\left(p^2-9p+20\right)\right)\right)\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{\left(c+\frac{d}{x}\right)^{3/2}}d\frac{1}{\sqrt{c+\frac{d}{x}}}}{3a}-\frac{b(3-p)\left(60a^2c-b^2\left(p^2-9p+20\right)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{3a\left(c+\frac{d}{x}\right)^{3/2}}}{4a}-\frac{\left(60a^2c-b^2\left(p^2 \cdots\right.\right.}{\cdots}}{5a}\cdots}{6a}\cdots\right)}{d^3}$$

$\big\downarrow$ 75

$$\frac{2\left(-\frac{\frac{-\frac{\left(60a^2c-b^2\left(p^2-9p+20\right)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{4a\left(c+\frac{d}{x}\right)^2}-\frac{-\frac{b(3-p)\left(60a^2c-b^2\left(p^2-9p+20\right)\right)\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^{p+1}}{3a\left(c+\frac{d}{x}\right)^{3/2}}-\frac{b^2\left(360a^4c^2-b^2(2-p)(3-p)\left(60a^2c-b^2\left(p^2-9p+\cdots\right.\right.\right.}{\cdots}}{4a}}{5a}}{6a}\cdots\right.}{d^3}$$

input
```
Int[(a + b/Sqrt[c + d/x])^p/x^4,x]
```

output
```
(2*(-1/6*(a + b/Sqrt[c + d/x])^(1 + p)/(a*(c + d/x)^3) - (-1/5*(b*(5 - p)*
(a + b/Sqrt[c + d/x])^(1 + p))/(a*(c + d/x)^(5/2)) + (-1/4*((60*a^2*c - b^
2*(20 - 9*p + p^2))*(a + b/Sqrt[c + d/x])^(1 + p))/(a*(c + d/x)^2) - (-1/3
*(b*(3 - p)*(60*a^2*c - b^2*(20 - 9*p + p^2))*(a + b/Sqrt[c + d/x])^(1 + p
))/(a*(c + d/x)^(3/2)) - (b^2*(360*a^4*c^2 - b^2*(2 - p)*(3 - p)*(60*a^2*c
 - b^2*(20 - 9*p + p^2)))*(a + b/Sqrt[c + d/x])^(1 + p)*Hypergeometric2F1[
3, 1 + p, 2 + p, 1 + b/(a*Sqrt[c + d/x])])/(3*a^4*(1 + p)))/(4*a))/(5*a))/
(6*a)))/d^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 520
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2)^p, e*x, x], R = Pol
ynomialRemainder[(a + b*x^2)^p, e*x, x]}, Simp[R*(e*x)^(m + 1)*((c + d*x)^(
n + 1)/((m + 1)*(e*c))), x] + Simp[1/((m + 1)*(e*c))   Int[(e*x)^(m + 1)*(c
 + d*x)^n*ExpandToSum[(m + 1)*(e*c)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[m, -1] &&  !IntegerQ[n]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x^4,x)
```

output
```
int((a+b/(c+d/x)^(1/2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx = \int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^4,x, algorithm="fricas")
```

output
```
integral(((a*c*x + b*x*sqrt((c*x + d)/x) + a*d)/(c*x + d))^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d/x)**(1/2))**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx$$

input
```
integrate((a+b/(c+d/x)^(1/2))^p/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/sqrt(c + d/x))^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx$$

input
```
int((a + b/(c + d/x)^(1/2))^p/x^4,x)
```

output
```
int((a + b/(c + d/x)^(1/2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+\frac{d}{x}}}\right)^p}{x^4}\,dx = \int \frac{\left(\sqrt{cx+d}\,a+\sqrt{x}\,b\right)^p}{(cx+d)^{\frac{p}{2}}\,x^4}dx$$

input
```
int((a+b/(c+d/x)^(1/2))^p/x^4,x)
```

output
```
int((sqrt(c*x + d)*a + sqrt(x)*b)**p/((c*x + d)**(p/2)*x**4),x)
```

3.312 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

output
```
-48*(2+(1+x^(1/2))^(1/2))^(1/2)+88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(
1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1+x^(1/2))^(1/2))^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{105}\sqrt{2+\sqrt{1+\sqrt{x}}}\left(-280+76\sqrt{1+\sqrt{x}} + 3\left(-12+5\sqrt{1+\sqrt{x}}\right)\sqrt{x}\right)$$

input
```
Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

output
```
(8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*S
qrt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {7267, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{\sqrt{x}}{\sqrt{\sqrt{\sqrt{x}+1}+2}}d\sqrt{x}$$

$$\Big\downarrow \text{896}$$

$$2\int \frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+2}}d\left(\sqrt{x}+1\right)$$

$$\Big\downarrow \text{25}$$

$$-2\int -\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+2}}d\left(\sqrt{x}+1\right)$$

$$\Big\downarrow \text{1732}$$

$$-4\int \frac{(1-x)\sqrt[4]{x}}{\sqrt{\sqrt{x}+3}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{522}$$

$$-4\int \left(-\left(\sqrt{x}+3\right)^{5/2}+6\left(\sqrt{x}+3\right)^{3/2}-11\sqrt{\sqrt{x}+3}+\frac{6}{\sqrt{\sqrt{x}+3}}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$-4\left(-\frac{2}{7}\left(\sqrt{x}+3\right)^{7/2}+\frac{12}{5}\left(\sqrt{x}+3\right)^{5/2}-\frac{22}{3}\left(\sqrt{x}+3\right)^{3/2}+12\sqrt{\sqrt{x}+3}\right)$$

input
```
Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

output
```
-4*(12*Sqrt[3 + Sqrt[x]] - (22*(3 + Sqrt[x])^(3/2))/3 + (12*(3 + Sqrt[x])^
(5/2))/5 - (2*(3 + Sqrt[x])^(7/2))/7)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$-48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}$	54
default	$-48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}$	54

input
```
int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-48*(2+(1+x^(1/2))^(1/2))^(1/2)+88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(
1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1+x^(1/2))^(1/2))^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx=\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(s
qrt(x) + 1) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}}{7} - \frac{48\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}}{5} + \frac{88\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}}{3} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

output
```
8*(sqrt(sqrt(x) + 1) + 2)**(7/2)/7 - 48*(sqrt(sqrt(x) + 1) + 2)**(5/2)/5 +
 88*(sqrt(sqrt(x) + 1) + 2)**(3/2)/3 - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - \frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + \frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 8
8/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - 126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + 385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2}-8\sqrt{x}-7\right)\mathrm{sgn}\left(4\,x-3\right)}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2
) + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(
sgn(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

input
```
int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

output
```
int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$$
$$= \frac{8\sqrt{\sqrt{\sqrt{x}+1}+2}\left(15\sqrt{x}\,\sqrt{\sqrt{x}+1}+76\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)}{105}$$

input
```
int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x)
```

output
```
(8*sqrt(sqrt(sqrt(x) + 1) + 2)*(15*sqrt(x)*sqrt(sqrt(x) + 1) + 76*sqrt(sqr
t(x) + 1) - 36*sqrt(x) - 280))/105
```

3.313 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2} - \frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2} + \frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}$$

output
```
64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)+8/9*(2+(
4+x^(1/2))^(1/2))^(9/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8}{315}\sqrt{2+\sqrt{4+\sqrt{x}}}\left(-64\left(2+\sqrt{4+\sqrt{x}}\right) + 2\left(2+5\sqrt{4+\sqrt{x}}\right)\sqrt{x}+35x\right)$$

input
```
Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

output
```
(8*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*(-64*(2 + Sqrt[4 + Sqrt[x]]) + 2*(2 + 5*Sqr
t[4 + Sqrt[x]])*Sqrt[x] + 35*x))/315
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {7267, 896, 25, 1388, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{x}+4}+2}dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \sqrt{\sqrt{\sqrt{x}+4}+2}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{896}$$

$$2\int \sqrt{\sqrt[4]{x}+2}\sqrt{x}d\left(\sqrt{x}+4\right)$$

$$\Big\downarrow \text{25}$$

$$-2\int -\sqrt{\sqrt[4]{x}+2}\sqrt{x}d\left(\sqrt{x}+4\right)$$

$$\Big\downarrow \text{1388}$$

$$-2\int \left(2-\sqrt[4]{x}\right)\left(\sqrt[4]{x}+2\right)^{3/2}d\left(\sqrt{x}+4\right)$$

$$\Big\downarrow \text{900}$$

$$-4\int \left(-\sqrt{x}-2\right)\left(\sqrt{x}+6\right)^{3/2}\sqrt[4]{x}d\sqrt[4]{x}$$

$$\Big\downarrow \text{86}$$

$$-4\int \left(-\left(\sqrt{x}+6\right)^{7/2}+6\left(\sqrt{x}+6\right)^{5/2}-8\left(\sqrt{x}+6\right)^{3/2}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$-4\left(-\frac{2}{9}\left(\sqrt{x}+6\right)^{9/2}+\frac{12}{7}\left(\sqrt{x}+6\right)^{7/2}-\frac{16}{5}\left(\sqrt{x}+6\right)^{5/2}\right)$$

input
```
Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

output
```
-4*((-16*(6 + Sqrt[x])^(5/2))/5 + (12*(6 + Sqrt[x])^(7/2))/7 - (2*(6 + Sqr
t[x])^(9/2))/9)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.27

method	result	size
meijerg	$2x\,\mathrm{hypergeom}\left(\left[-\frac{1}{4}, \frac{1}{4}, 2\right], \left[\frac{1}{2}, 3\right], -\frac{\sqrt{x}}{4}\right)$	17
derivativedivides	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41
default	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41

input
```
int((2+(4+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*hypergeom([-1/4,1/4,2],[1/2,3],-1/4*x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35\,x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}$$

input
```
integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt
(sqrt(sqrt(x) + 4) + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(54) = 108$.

Time = 1.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.38

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = -\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi} - \frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} - \frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi} + \frac{64\sqrt{2}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} + \frac{128\sqrt{2}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

input
```
integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

output
```
-2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/
4)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamm
a(-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(
-1/4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) +
 4) + 2)*gamma(-1/4)*gamma(1/4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) +
 4) + 2)*gamma(-1/4)*gamma(1/4)/(315*pi)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}} - \frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}} + \frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

input
```
integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 6
4/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(40) = 80$.

Time = 0.20 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.19

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$$
$$= \frac{8}{315}\left(\left(35\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}} - 360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}} + 1512\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}} - 3360\left(\sqrt{\sqrt{x}+4}+\right.\right.\right.$$
$$- 15)$$

input
```
integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
8/315*((35*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 360*(sqrt(sqrt(x) + 4) + 2)^(7/
2) + 1512*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 3360*(sqrt(sqrt(x) + 4) + 2)^(3/
2) + 5040*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x)
- 79) + 18*(5*(sqrt(sqrt(x) + 4) + 2)^(7/2) - 42*(sqrt(sqrt(x) + 4) + 2)^(
5/2) + 140*(sqrt(sqrt(x) + 4) + 2)^(3/2) - 280*sqrt(sqrt(sqrt(x) + 4) + 2)
)*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 84*(3*(sqrt(sqrt(x) + 4) + 2)
^(5/2) - 20*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 60*sqrt(sqrt(sqrt(x) + 4) + 2)
)*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 840*((sqrt(sqrt(x) + 4) + 2)^
(3/2) - 6*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x)
- 79))*sgn(4*x - 15)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \int \sqrt{\sqrt{\sqrt{x}+4}+2}\,dx$$

input
```
int(((x^(1/2) + 4)^(1/2) + 2)^(1/2),x)
```

output
```
int(((x^(1/2) + 4)^(1/2) + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8\sqrt{\sqrt{\sqrt{x}+4}+2}\,\left(10\sqrt{x}\,\sqrt{\sqrt{x}+4}-64\sqrt{\sqrt{x}+4}+4\sqrt{x}+35x-128\right)}{315}$$

input
```
int((2+(4+x^(1/2))^(1/2))^(1/2),x)
```

output
```
(8*sqrt(sqrt(sqrt(x) + 4) + 2)*(10*sqrt(x)*sqrt(sqrt(x) + 4) - 64*sqrt(sqr
t(x) + 4) + 4*sqrt(x) + 35*x - 128))/315
```

3.314 $\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\frac{x^2\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},-\frac{1}{2},\frac{2+n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
1/2*x^2*(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(2/n,1/2,-1/2,(2+n
)/n,-d*x^n/c,-a*d*x^n/(a*c+b))/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [F]

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx$$

input
```
Integrate[x*Sqrt[a + b/(c + d*x^n)],x]
```

output
```
Integrate[x*Sqrt[a + b/(c + d*x^n)], x]
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{c+dx^n}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x\sqrt{\frac{ac+adx^n+b}{c+dx^n}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{x\sqrt{adx^n+b+ac}}{\sqrt{dx^n+c}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{x\sqrt{adx^n+b+ac}}{\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{x\sqrt{\frac{adx^n}{b+ac}+1}}{\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},-\frac{1}{2},\frac{n+2}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2\sqrt{\frac{adx^n}{ac+b}+1}}$$

input
```
Int[x*Sqrt[a + b/(c + d*x^n)],x]
```

output
```
(x^2*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*AppellF1[2/
n, 1/2, -1/2, (2 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(2*Sqrt[1
+ (a*d*x^n)/(b + a*c)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int x\sqrt{a+\frac{b}{c+d\,x^{n}}}dx$$

input
```
int(x*(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int(x*(a+b/(c+d*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b/(c+d*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\int x\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\,dx$$

input
```
integrate(x*(a+b/(c+d*x**n))**(1/2),x)
```

output
```
Integral(x*sqrt((a*c + a*d*x**n + b)/(c + d*x**n)), x)
```

Maxima [F]

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\int\sqrt{a+\frac{b}{dx^n+c}}x\,dx$$

input
```
integrate(x*(a+b/(c+d*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))*x, x)
```

Giac [F]

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\int\sqrt{a+\frac{b}{dx^n+c}}x\,dx$$

input
```
integrate(x*(a+b/(c+d*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\int x\,\sqrt{a+\frac{b}{c+d\,x^n}}\,dx$$

input
```
int(x*(a + b/(c + d*x^n))^(1/2),x)
```

output
```
int(x*(a + b/(c + d*x^n))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+\frac{b}{c+dx^n}}\,dx=\int\frac{\sqrt{x^nd+c}\,\sqrt{x^nad+ac+b}\,x}{x^nd+c}dx$$

input
```
int(x*(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*x)/(x**n*d + c),x)
```

3.315 $\int \sqrt{a+\frac{b}{c+dx^n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx=\frac{x\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
x*(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(1/n,1/2,-1/2,1+1/n,-d*x
^n/c,-a*d*x^n/(a*c+b))/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx=\int \sqrt{a+\frac{b}{c+dx^n}}\,dx$$

input
```
Integrate[Sqrt[a + b/(c + d*x^n)],x]
```

output
```
Integrate[Sqrt[a + b/(c + d*x^n)], x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2057, 2058, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{c+dx^n}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \sqrt{\frac{ac+adx^n+b}{c+dx^n}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{adx^n+b+ac}}{\sqrt{dx^n+c}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{adx^n+b+ac}}{\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{\frac{adx^n}{b+ac}+1}}{\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{\sqrt{\frac{adx^n}{ac+b}+1}}$$

input
```
Int[Sqrt[a + b/(c + d*x^n)],x]
```

output
```
(x*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*AppellF1[n^(-
1), 1/2, -1/2, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/Sqrt[1 +
 (a*d*x^n)/(b + a*c)]
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \sqrt{a+\frac{b}{c+d\,x^{n}}}dx$$

input
```
int((a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((a+b/(c+d*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c+d*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx = \int \sqrt{a+\frac{b}{c+dx^n}}\,dx$$

input
```
integrate((a+b/(c+d*x**n))**(1/2),x)
```

output
```
Integral(sqrt(a + b/(c + d*x**n)), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx = \int \sqrt{a+\frac{b}{dx^n+c}}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^n + c)), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx=\int \sqrt{a+\frac{b}{dx^n+c}}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx=\int \sqrt{a+\frac{b}{c+d\,x^n}}\,dx$$

input
```
int((a + b/(c + d*x^n))^(1/2),x)
```

output
```
int((a + b/(c + d*x^n))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{c+dx^n}}\,dx=\int \frac{\sqrt{x^n d+c}\,\sqrt{x^n a d+a c+b}}{x^n d+c}\,dx$$

input
```
int((a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*d + c),x)
```

3.316 $\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x} \, dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{a}}\right)}{n} - \frac{2\sqrt{b+ac}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{b+ac}}\right)}{\sqrt{c}n}$$

output
```
2*a^(1/2)*arctanh((a+b/(c+d*x^n))^(1/2)/a^(1/2))/n-2*(a*c+b)^(1/2)*arctanh
(c^(1/2)*(a+b/(c+d*x^n))^(1/2)/(a*c+b)^(1/2))/c^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x} \, dx = \frac{2\left(-\frac{\sqrt{-b-ac}\,\text{arctan}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}} + \sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^n)]/x,x]
```

output
```
(2*(-((Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)
])/Sqrt[-b - a*c]])/Sqrt[c]) + Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^n)/(c
 + d*x^n)]/Sqrt[a]]))/n
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2053, 2052, 25, 27, 383, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x} \, dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{\int x^{-n}\sqrt{\frac{adx^n+b+ac}{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{2052}$$

$$-\frac{2bd\int -\frac{x^{2n}}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bd\int \frac{x^{2n}}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{x^{2n}}{(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{383}$$

$$\frac{2b\left(\frac{a\int \frac{1}{a-x^{2n}}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}-\frac{(ac+b)\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}\right)}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{b}-\frac{(ac+b)\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}\right)}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{2b\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{b}-\frac{\sqrt{ac+b}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{ac+b}}\right)}{b\sqrt{c}}\right)}{n}$$

input
```
Int[Sqrt[a + b/(c + d*x^n)]/x,x]
```

output
```
(2*b*((Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]/Sqrt[a]])/b -
 (Sqrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/Sq
rt[b + a*c]])/(b*Sqrt[c])))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(71) = 142$.

Time = 0.67 (sec) , antiderivative size = 590, normalized size of antiderivative = 6.78

method	result
derivativedivides	$\sqrt{\frac{x^n ad+ac+b}{c+d\,x^n}}\,(c+d\,x^n)\left(-2\sqrt{a\,d^2}\,\ln\left(\left(2x^n acd+2a\,c^2+b\,x^n d+2\sqrt{(ac+b)c}\,\sqrt{x^{2n} a\,d^2+2x^n acd+a\,c^2+b\,x^n d+bc}+2bc\right)\right.\right.$
default	$\sqrt{\frac{x^n ad+ac+b}{c+d\,x^n}}\,(c+d\,x^n)\left(-2\sqrt{a\,d^2}\,\ln\left(\left(2x^n acd+2a\,c^2+b\,x^n d+2\sqrt{(ac+b)c}\,\sqrt{x^{2n} a\,d^2+2x^n acd+a\,c^2+b\,x^n d+bc}+2bc\right)\right.\right.$

input
```
int((a+b/(c+d*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/n*((x^n*a*d+a*c+b)/(c+d*x^n))^(1/2)*(c+d*x^n)*(-2*(a*d^2)^(1/2)*ln((2*
x^n*a*c*d+2*a*c^2+b*x^n*d+2*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a
*c^2+b*x^n*d+b*c)^(1/2)+2*b*c)/(x^n))*a*c^2+2*((a*c+b)*c)^(1/2)*ln(1/2*(2*
a*d^2*x^n+2*a*c*d+2*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*(a
*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*c*d-2*(a*d^2)^(1/2)*ln((2*x^n*a*c*d+2*a*
c^2+b*x^n*d+2*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b
*c)^(1/2)+2*b*c)/(x^n))*b*c+((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x^n+2*a*c*d+
2*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(
a*d^2)^(1/2))*b*d-((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((c+d*x^
n)*(x^n*a*d+a*c+b))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b*d+2*((x^n)^2
*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2
)-2*(a*d^2)^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/2)*((a*c+b)*c)^(1/2))/((c
+d*x^n)*(x^n*a*d+a*c+b))^(1/2)/c/(a*d^2)^(1/2)/((a*c+b)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 581, normalized size of antiderivative = 6.68

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x}\,dx$$

$$= \left[\frac{\sqrt{a}\log\left(2\,adx^n+2\,ac+2\,\left(\sqrt{a}dx^n+\sqrt{a}c\right)\sqrt{\frac{adx^n+ac+b}{dx^n+c}}+b\right)+\sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx^n+2\,bc-2\left(cdx^n\right.}{x}\right.}{n}\right.$$

$$-\frac{2\sqrt{-a}\arctan\left(\frac{\left(\sqrt{-a}dx^n+\sqrt{-a}c\right)\sqrt{\frac{adx^n+ac+b}{dx^n+c}}}{adx^n+ac+b}\right)-\sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx^n+2\,bc-2\left(cdx^n\sqrt{\frac{ac+b}{c}}+c^2\sqrt{\frac{ac+b}{c}}\right)\sqrt{}}{x^n}\right.}{n}$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
[(sqrt(a)*log(2*a*d*x^n + 2*a*c + 2*(sqrt(a)*d*x^n + sqrt(a)*c)*sqrt((a*d*
x^n + a*c + b)/(d*x^n + c)) + b) + sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*
c + b)*d*x^n + 2*b*c - 2*(c*d*x^n*sqrt((a*c + b)/c) + c^2*sqrt((a*c + b)/c
))*sqrt((a*d*x^n + a*c + b)/(d*x^n + c)))/x^n))/n, -(2*sqrt(-a)*arctan((sq
rt(-a)*d*x^n + sqrt(-a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n + c))/(a*d*x^n
+ a*c + b)) - sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*c + b)*d*x^n + 2*b*c
- 2*(c*d*x^n*sqrt((a*c + b)/c) + c^2*sqrt((a*c + b)/c))*sqrt((a*d*x^n + a*
c + b)/(d*x^n + c)))/x^n))/n, (2*sqrt(-(a*c + b)/c)*arctan(c*sqrt((a*d*x^n
 + a*c + b)/(d*x^n + c))*sqrt(-(a*c + b)/c)/(a*c + b)) + sqrt(a)*log(2*a*d
*x^n + 2*a*c + 2*(sqrt(a)*d*x^n + sqrt(a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x
^n + c)) + b))/n, 2*(sqrt(-(a*c + b)/c)*arctan(c*sqrt((a*d*x^n + a*c + b)/
(d*x^n + c))*sqrt(-(a*c + b)/c)/(a*c + b)) - sqrt(-a)*arctan((sqrt(-a)*d*x
^n + sqrt(-a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n + c))/(a*d*x^n + a*c + b)
))/n]
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^n}}}{x}\,dx = \int \frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{x}\,dx$$

input
```
integrate((a+b/(c+d*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*c + a*d*x**n + b)/(c + d*x**n))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^n}}}{x}\,dx = \int \frac{\sqrt{a + \frac{b}{dx^n+c}}}{x}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x} \, dx = \int \frac{\sqrt{a+\frac{b}{dx^n+c}}}{x} \, dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x} \, dx = \int \frac{\sqrt{a+\frac{b}{c+d\,x^n}}}{x} \, dx$$

input
```
int((a + b/(c + d*x^n))^(1/2)/x,x)
```

output
```
int((a + b/(c + d*x^n))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x} \, dx = \int \frac{\sqrt{x^n d+c}\,\sqrt{x^n a d+a c+b}}{x^n d x+c x} dx$$

input
```
int((a+b/(c+d*x^n))^(1/2)/x,x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*d*x + c*x),x)
```

3.317 $\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2} \, dx = -\frac{\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}} \operatorname{AppellF1}\left(-\frac{1}{n},\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{x\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
-(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(-1/n,1/2,-1/2,-(1-n)/n,-
d*x^n/c,-a*d*x^n/(a*c+b))/x/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2} \, dx = \int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2} \, dx$$

input
```
Integrate[Sqrt[a + b/(c + d*x^n)]/x^2,x]
```

output
```
Integrate[Sqrt[a + b/(c + d*x^n)]/x^2, x]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{x^2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{adx^n+b+ac}}{x^2\sqrt{dx^n+c}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{adx^n+b+ac}}{x^2\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{\frac{adx^n}{b+ac}+1}}{x^2\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\operatorname{AppellF1}\left(-\frac{1}{n},\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{x\sqrt{\frac{adx^n}{ac+b}+1}}$$

input
```
Int[Sqrt[a + b/(c + d*x^n)]/x^2,x]
```

output
```
-((Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*AppellF1[-n^(
-1), 1/2, -1/2, -((1 - n)/n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(x*Sq
rt[1 + (a*d*x^n)/(b + a*c)]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{a+\frac{b}{c+d\,x^{n}}}}{x^{2}}dx$$

input
```
int((a+b/(c+d*x^n))^(1/2)/x^2,x)
```

output
```
int((a+b/(c+d*x^n))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2}\,dx = \int \frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{x^2}\,dx$$

input
```
integrate((a+b/(c+d*x**n))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*c + a*d*x**n + b)/(c + d*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^n+c}}}{x^2}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^n+c}}}{x^2}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{c+d\,x^n}}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x^n))^(1/2)/x^2,x)
```

output
```
int((a + b/(c + d*x^n))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^2}\,dx = \int \frac{\sqrt{x^n d+c}\,\sqrt{x^n a d+a c+b}}{x^n d\,x^2+c\,x^2}dx$$

input
```
int((a+b/(c+d*x^n))^(1/2)/x^2,x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*d*x**2 + c*x**2),x)
```

3.318 $\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx = -\frac{\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2x^2\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
-1/2*(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(-2/n,1/2,-1/2,-(2-n)
/n,-d*x^n/c,-a*d*x^n/(a*c+b))/x^2/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx = \int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx$$

input
```
Integrate[Sqrt[a + b/(c + d*x^n)]/x^3,x]
```

output
```
Integrate[Sqrt[a + b/(c + d*x^n)]/x^3, x]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3} \, dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{x^3}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{adx^n+b+ac}}{x^3\sqrt{dx^n+c}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{adx^n+b+ac}}{x^3\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\sqrt{\frac{adx^n}{b+ac}+1}}{x^3\sqrt{\frac{dx^n}{c}+1}}dx}{\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\text{AppellF1}\left(-\frac{2}{n},\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2x^2\sqrt{\frac{adx^n}{ac+b}+1}}$$

input
```
Int[Sqrt[a + b/(c + d*x^n)]/x^3,x]
```

output
```
-1/2*(Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*AppellF1[-
2/n, 1/2, -1/2, -((2 - n)/n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(x^2*
Sqrt[1 + (a*d*x^n)/(b + a*c)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{a+\frac{b}{c+d\,x^{n}}}}{x^{3}}dx$$

input
```
int((a+b/(c+d*x^n))^(1/2)/x^3,x)
```

output
```
int((a+b/(c+d*x^n))^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n))**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^n+c}}}{x^3}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^n+c}}}{x^3}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^n + c))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{c+d\,x^n}}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^n))^(1/2)/x^3,x)
```

output
```
int((a + b/(c + d*x^n))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^n}}}{x^3}\,dx=\int \frac{\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+b}}{x^n d\,x^3+c\,x^3}dx$$

input
```
int((a+b/(c+d*x^n))^(1/2)/x^3,x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*d*x**3 + c*x**3),x)
```

3.319 $\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}dx=\frac{(b+ac)x^2\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}\,\text{AppellF1}\left(\frac{2}{n},\frac{3}{2},-\frac{3}{2},\frac{2+n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2c\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
1/2*(a*c+b)*x^2*(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(2/n,3/2,-
3/2,(2+n)/n,-d*x^n/c,-a*d*x^n/(a*c+b))/c/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(107) = 214.

Time = 12.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.05

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}dx=\frac{x^2\sqrt{\frac{b+ac+adx^n}{c+dx^n}}\left(4b+\frac{\sqrt{\frac{b+ac+adx^n}{b+ac}}\sqrt{1+\frac{dx^n}{c}}\left((b+ac)(2+n)(b(-4+n)+acn)\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{dx^n}{c},-\frac{ac}{b\cdots}\right.\right.}{(2+n)(b+a(c+dx^n)\cdots}\right.}{2cn}$$

input
```
Integrate[x*(a + b/(c + d*x^n))^(3/2),x]
```

output
```
(x^2*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*(4*b + (Sqrt[(b + a*c + a*d*x^n
)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*((b + a*c)*(2 + n)*(b*(-4 + n) + a*c*n)*A
ppellF1[2/n, 1/2, 1/2, (2 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] +
2*a*d*(-4*b + a*c*n)*x^n*AppellF1[(2 + n)/n, 1/2, 1/2, 2*(1 + n^(-1)), -((
d*x^n)/c), -((a*d*x^n)/(b + a*c))]))/((2 + n)*(b + a*(c + d*x^n)))))/(2*c*
n)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{x(adx^n+b+ac)^{3/2}}{(dx^n+c)^{3/2}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{x(adx^n+b+ac)^{3/2}}{\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{x\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}{\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\operatorname{AppellF1}\left(\frac{2}{n},\frac{3}{2},-\frac{3}{2},\frac{n+2}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2c\sqrt{\frac{adx^n}{ac+b}+1}}$$

input
```
Int[x*(a + b/(c + d*x^n))^(3/2),x]
```

output
```
((b + a*c)*x^2*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*A
ppellF1[2/n, 3/2, -3/2, (2 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/
(2*c*Sqrt[1 + (a*d*x^n)/(b + a*c)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int x\left(a+\frac{b}{c+d\,x^{n}}\right)^{\frac{3}{2}}dx$$

input
```
int(x*(a+b/(c+d*x^n))^(3/2),x)
```

output
```
int(x*(a+b/(c+d*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b/(c+d*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b/(c+d*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b/(c+d*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)*x, x)
```

Giac [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b/(c+d*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx=\int x\,\left(a+\frac{b}{c+d\,x^n}\right)^{3/2}\,dx$$

input
```
int(x*(a + b/(c + d*x^n))^(3/2),x)
```

output
```
int(x*(a + b/(c + d*x^n))^(3/2), x)
```

Reduce [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx=\text{too large to display}$$

input
```
int(x*(a+b/(c+d*x^n))^(3/2),x)
```

output

```
(4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*a**2*c*x**2 + 4*sqrt(x**n*d +
 c)*sqrt(x**n*a*d + a*c + b)*a*b*x**2 - 8*x**n*int((x**(2*n)*sqrt(x**n*d +
 c)*sqrt(x**n*a*d + a*c + b)*x)/(8*x**(3*n)*a**2*c*d**3 - x**(3*n)*a*b*d**
3*n + 4*x**(3*n)*a*b*d**3 + 24*x**(2*n)*a**2*c**2*d**2 - 3*x**(2*n)*a*b*c*
d**2*n + 20*x**(2*n)*a*b*c*d**2 - x**(2*n)*b**2*d**2*n + 4*x**(2*n)*b**2*d
**2 + 24*x**n*a**2*c**3*d - 3*x**n*a*b*c**2*d*n + 28*x**n*a*b*c**2*d - 2*x
**n*b**2*c*d*n + 8*x**n*b**2*c*d + 8*a**2*c**4 - a*b*c**3*n + 12*a*b*c**3
- b**2*c**2*n + 4*b**2*c**2),x)*a**3*b*c*d**3*n - 32*x**n*int((x**(2*n)*sq
rt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*x)/(8*x**(3*n)*a**2*c*d**3 - x**(3
*n)*a*b*d**3*n + 4*x**(3*n)*a*b*d**3 + 24*x**(2*n)*a**2*c**2*d**2 - 3*x**(
2*n)*a*b*c*d**2*n + 20*x**(2*n)*a*b*c*d**2 - x**(2*n)*b**2*d**2*n + 4*x**(
2*n)*b**2*d**2 + 24*x**n*a**2*c**3*d - 3*x**n*a*b*c**2*d*n + 28*x**n*a*b*c
**2*d - 2*x**n*b**2*c*d*n + 8*x**n*b**2*c*d + 8*a**2*c**4 - a*b*c**3*n + 1
2*a*b*c**3 - b**2*c**2*n + 4*b**2*c**2),x)*a**3*b*c*d**3 + x**n*int((x**(2
*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*x)/(8*x**(3*n)*a**2*c*d**3 -
 x**(3*n)*a*b*d**3*n + 4*x**(3*n)*a*b*d**3 + 24*x**(2*n)*a**2*c**2*d**2 -
3*x**(2*n)*a*b*c*d**2*n + 20*x**(2*n)*a*b*c*d**2 - x**(2*n)*b**2*d**2*n +
4*x**(2*n)*b**2*d**2 + 24*x**n*a**2*c**3*d - 3*x**n*a*b*c**2*d*n + 28*x**n
*a*b*c**2*d - 2*x**n*b**2*c*d*n + 8*x**n*b**2*c*d + 8*a**2*c**4 - a*b*c**3
*n + 12*a*b*c**3 - b**2*c**2*n + 4*b**2*c**2),x)*a**2*b**2*d**3*n**2 - ...
```

3.320 $\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx=\frac{(b+ac)x\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{c\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
(a*c+b)*x*(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(1/n,3/2,-3/2,1+
1/n,-d*x^n/c,-a*d*x^n/(a*c+b))/c/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 983 vs. 2(98) = 196.

Time = 2.93 (sec) , antiderivative size = 983, normalized size of antiderivative = 10.03

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx=\text{Too large to display}$$

input
```
Integrate[(a + b/(c + d*x^n))^(3/2),x]
```

output
```
(2*b*x*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/(c*n) + (x*Sqrt[(b + a*c + a
*d*x^n)/(c + d*x^n)]*(4*b^2*c*(b + a*c)*(1 + n)^2*AppellF1[n^(-1), 1/2, 1/
2, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] + 4*a*b*c^2*(b + a*c)
*(1 + n)^2*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n
)/(b + a*c))] - 2*b^2*c*(b + a*c)*n*(1 + n)^2*AppellF1[n^(-1), 1/2, 1/2, 1
 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] - 4*a*b*c^2*(b + a*c)*n*(
1 + n)^2*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/
(b + a*c))] - 2*a^2*c^3*(b + a*c)*n*(1 + n)^2*AppellF1[n^(-1), 1/2, 1/2, 1
 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] - 2*a*b*d*x^n*Sqrt[(b + a
*c + a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*AppellF1[1 + n^(-1), 1/2, 1/2
, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]*(a*c*d*n*x^n*AppellF1[
1 + n^(-1), 1/2, 3/2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] -
(b + a*c)*(-(d*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), -((d*x^n)/
c), -((a*d*x^n)/(b + a*c))]) + 2*c*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 +
n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])) + a^2*c*d*n*x^n*Sqrt[(b +
a*c + a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*AppellF1[1 + n^(-1), 1/2, 1/
2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]*(a*c*d*n*x^n*AppellF1
[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] -
 (b + a*c)*(-(d*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), -((d*x^n)
/c), -((a*d*x^n)/(b + a*c))]) + 2*c*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, ...
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2057, 2058, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx$$

$$\Big\downarrow \text{2057}$$

$$\int \left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2} dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int\frac{(adx^n+b+ac)^{3/2}}{(dx^n+c)^{3/2}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int\frac{(adx^n+b+ac)^{3/2}}{\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{937}$$

$$\frac{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int\frac{\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}{\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{c\sqrt{\frac{adx^n}{ac+b}+1}}$$

input
```
Int[(a + b/(c + d*x^n))^(3/2),x]
```

output
```
((b + a*c)*x*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*App
ellF1[n^(-1), 3/2, -3/2, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]
)/(c*Sqrt[1 + (a*d*x^n)/(b + a*c)])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \left(a+\frac{b}{c+d\,x^{n}}\right)^{\frac{3}{2}}dx$$

input
```
int((a+b/(c+d*x^n))^(3/2),x)
```

output
```
int((a+b/(c+d*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c+d*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2), x)
```

Giac [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx=\int \left(a+\frac{b}{c+d\,x^n}\right)^{3/2}\,dx$$

input
```
int((a + b/(c + d*x^n))^(3/2),x)
```

output
```
int((a + b/(c + d*x^n))^(3/2), x)
```

Reduce [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^{3/2}\,dx=\text{too large to display}$$

input
```
int((a+b/(c+d*x^n))^(3/2),x)
```

output

```
(4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*a**2*c*x + 4*sqrt(x**n*d + c)
*sqrt(x**n*a*d + a*c + b)*a*b*x - 4*x**n*int((x**(2*n)*sqrt(x**n*d + c)*sq
rt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**2*c*d**3 - x**(3*n)*a*b*d**3*n + 2*
x**(3*n)*a*b*d**3 + 12*x**(2*n)*a**2*c**2*d**2 - 3*x**(2*n)*a*b*c*d**2*n +
 10*x**(2*n)*a*b*c*d**2 - x**(2*n)*b**2*d**2*n + 2*x**(2*n)*b**2*d**2 + 12
*x**n*a**2*c**3*d - 3*x**n*a*b*c**2*d*n + 14*x**n*a*b*c**2*d - 2*x**n*b**2
*c*d*n + 4*x**n*b**2*c*d + 4*a**2*c**4 - a*b*c**3*n + 6*a*b*c**3 - b**2*c*
*2*n + 2*b**2*c**2),x)*a**3*b*c*d**3*n - 8*x**n*int((x**(2*n)*sqrt(x**n*d
+ c)*sqrt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**2*c*d**3 - x**(3*n)*a*b*d**3
*n + 2*x**(3*n)*a*b*d**3 + 12*x**(2*n)*a**2*c**2*d**2 - 3*x**(2*n)*a*b*c*d
**2*n + 10*x**(2*n)*a*b*c*d**2 - x**(2*n)*b**2*d**2*n + 2*x**(2*n)*b**2*d*
*2 + 12*x**n*a**2*c**3*d - 3*x**n*a*b*c**2*d*n + 14*x**n*a*b*c**2*d - 2*x*
*n*b**2*c*d*n + 4*x**n*b**2*c*d + 4*a**2*c**4 - a*b*c**3*n + 6*a*b*c**3 -
b**2*c**2*n + 2*b**2*c**2),x)*a**3*b*c*d**3 + x**n*int((x**(2*n)*sqrt(x**n
*d + c)*sqrt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**2*c*d**3 - x**(3*n)*a*b*d
**3*n + 2*x**(3*n)*a*b*d**3 + 12*x**(2*n)*a**2*c**2*d**2 - 3*x**(2*n)*a*b*
c*d**2*n + 10*x**(2*n)*a*b*c*d**2 - x**(2*n)*b**2*d**2*n + 2*x**(2*n)*b**2
*d**2 + 12*x**n*a**2*c**3*d - 3*x**n*a*b*c**2*d*n + 14*x**n*a*b*c**2*d - 2
*x**n*b**2*c*d*n + 4*x**n*b**2*c*d + 4*a**2*c**4 - a*b*c**3*n + 6*a*b*c**3
 - b**2*c**2*n + 2*b**2*c**2),x)*a**2*b**2*d**3*n**2 - 4*x**n*int((x**(...
```

3.321 $\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x} \, dx = \frac{2b\sqrt{a+\frac{b}{c+dx^n}}}{cn} + \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{a}}\right)}{n} - \frac{2(b+ac)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{b+ac}}\right)}{c^{3/2}n}$$

output
```
2*b*(a+b/(c+d*x^n))^(1/2)/c/n+2*a^(3/2)*arctanh((a+b/(c+d*x^n))^(1/2)/a^(1
/2))/n-2*(a*c+b)^(3/2)*arctanh(c^(1/2)*(a+b/(c+d*x^n))^(1/2)/(a*c+b)^(1/2)
)/c^(3/2)/n
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x} \, dx = \frac{2\left(b\sqrt{c}\sqrt{\frac{b+ac+adx^n}{c+dx^n}} + (-b-ac)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}{\sqrt{-b-ac}}\right) + a^{3/2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{b+a\cdots}{c\cdots}}}{\cdots}\right.\right.}{c^{3/2}n}$$

input
```
Integrate[(a + b/(c + d*x^n))^(3/2)/x,x]
```

output
```
(2*(b*Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)] + (-b - a*c)^(3/2)*Arc
Tan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/Sqrt[-b - a*c]] + a^(3
/2)*c^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]/Sqrt[a]]))/(c^(3
/2)*n)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2057, 2053, 2052, 25, 27, 381, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{\int x^{-n}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^{3/2}dx^n}{n}$$

$$\Big\downarrow \text{2052}$$

$$-\frac{2bd\int -\frac{x^{4n}}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bd\int \frac{x^{4n}}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{x^{4n}}{(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \; 381$$

$$\frac{2b\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{c}-\frac{\int \frac{a(b+ac)-(b+2ac)x^{2n}}{\left(a-x^{2n}\right)\left(-cx^{2n}+b+ac\right)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{c}\right)}{n}$$

$$\Big\downarrow \; 397$$

$$\frac{2b\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{c}-\frac{\frac{(ac+b)^2\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}-\frac{a^2c\int \frac{1}{a-x^{2n}}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}}{c}\right)}{n}$$

$$\Big\downarrow \; 219$$

$$\frac{2b\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{c}-\frac{\frac{(ac+b)^2\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}-\frac{a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{b}}{c}\right)}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{2b\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{c}-\frac{\frac{(ac+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{ac+b}}\right)}{b\sqrt{c}}-\frac{a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{b}}{c}\right)}{n}$$

input
```
Int[(a + b/(c + d*x^n))^(3/2)/x,x]
```

output
```
(2*b*(Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]/c - (-((a^(3/2)*c*ArcTanh[Sqrt
[(b + a*c + a*d*x^n)/(c + d*x^n)]/Sqrt[a]])/b) + ((b + a*c)^(3/2)*ArcTanh[
(Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/Sqrt[b + a*c]])/(b*Sqrt[c]
))/c))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2924 vs. $2(95) = 190$.

Time = 0.68 (sec) , antiderivative size = 2925, normalized size of antiderivative = 25.88

method	result	size
derivativedivides	Expression too large to display	2925
default	Expression too large to display	2925

input
```
int((a+b/(c+d*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/n*(-4*(a*d^2)^(1/2)*ln((2*x^n*a*c*d+2*a*c^2+b*x^n*d+2*((a*c+b)*c)^(1/2
)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)+2*b*c)/(x^n))*a*b*c^
2*d^2*(x^n)^2+2*ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((x^n)^2*a*d^2+2*x^n*a*c*d+a
*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2
)*a^2*c^4*d+ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2
+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^
2*d^3*(x^n)^2-ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((c+d*x^n)*(x^n*a*d+a*c+b))^(1
/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^2*d^3*(x^n)^2-4*
(a*d^2)^(1/2)*ln((2*x^n*a*c*d+2*a*c^2+b*x^n*d+2*((a*c+b)*c)^(1/2)*((x^n)^2
*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)+2*b*c)/(x^n))*a*b*c^4+2*(a*d^2
)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*((a*c+b)*c)^(1
/2)*a*c^3-6*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^(1
/2)*a*c^3+ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b
*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^2*
c^2*d-12*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/2)
*a*c^2*d*x^n-ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/
2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*((a*c+b)*c)^(1/2)*b^2*c^2*d+2*(a*d^2)
^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*((a*c+b)*c)^(1/
2)*b*c^2-2*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/
2)*b*c^2-6*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 723, normalized size of antiderivative = 6.40

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x}\,dx = \left[\frac{a^{\frac{3}{2}}c\log\left(2\,adx^n+2\,ac+2\left(\sqrt{a}dx^n+\sqrt{a}c\right)\sqrt{\frac{adx^n+ac+b}{dx^n+c}}+b\right)+(ac+b)\sqrt{\frac{ac+b}{c}}\log}{cn}\right.$$

$$-\frac{2\sqrt{-a}ac\arctan\left(\frac{\left(\sqrt{-a}dx^n+\sqrt{-a}c\right)\sqrt{\frac{adx^n+ac+b}{dx^n+c}}}{adx^n+ac+b}\right)-(ac+b)\sqrt{\frac{ac+b}{c}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx^n+2\,bc-2\left(cdx^n\sqrt{\frac{ac+b}{c}}+c^2\right.}{x^n}\right.}{cn}$$

$$-\frac{2\left(\sqrt{-a}ac\arctan\left(\frac{\left(\sqrt{-a}dx^n+\sqrt{-a}c\right)\sqrt{\frac{adx^n+ac+b}{dx^n+c}}}{adx^n+ac+b}\right)-(ac+b)\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{c\sqrt{\frac{adx^n+ac+b}{dx^n+c}}\sqrt{-\frac{ac+b}{c}}}{ac+b}\right)-b\sqrt{\frac{adx}{d}}\right.}{cn}$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
[(a^(3/2)*c*log(2*a*d*x^n + 2*a*c + 2*(sqrt(a)*d*x^n + sqrt(a)*c)*sqrt((a*
d*x^n + a*c + b)/(d*x^n + c)) + b) + (a*c + b)*sqrt((a*c + b)/c)*log(-(2*a
*c^2 + (2*a*c + b)*d*x^n + 2*b*c - 2*(c*d*x^n*sqrt((a*c + b)/c) + c^2*sqrt
((a*c + b)/c))*sqrt((a*d*x^n + a*c + b)/(d*x^n + c)))/x^n) + 2*b*sqrt((a*d
*x^n + a*c + b)/(d*x^n + c)))/(c*n), -(2*sqrt(-a)*a*c*arctan((sqrt(-a)*d*x
^n + sqrt(-a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n + c))/(a*d*x^n + a*c + b)
) - (a*c + b)*sqrt((a*c + b)/c)*log(-(2*a*c^2 + (2*a*c + b)*d*x^n + 2*b*c
- 2*(c*d*x^n*sqrt((a*c + b)/c) + c^2*sqrt((a*c + b)/c))*sqrt((a*d*x^n + a*
c + b)/(d*x^n + c)))/x^n) - 2*b*sqrt((a*d*x^n + a*c + b)/(d*x^n + c)))/(c*
n), (a^(3/2)*c*log(2*a*d*x^n + 2*a*c + 2*(sqrt(a)*d*x^n + sqrt(a)*c)*sqrt(
(a*d*x^n + a*c + b)/(d*x^n + c)) + b) + 2*(a*c + b)*sqrt(-(a*c + b)/c)*arc
tan(c*sqrt((a*d*x^n + a*c + b)/(d*x^n + c))*sqrt(-(a*c + b)/c)/(a*c + b))
+ 2*b*sqrt((a*d*x^n + a*c + b)/(d*x^n + c)))/(c*n), -2*(sqrt(-a)*a*c*arcta
n((sqrt(-a)*d*x^n + sqrt(-a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n + c))/(a*d
*x^n + a*c + b)) - (a*c + b)*sqrt(-(a*c + b)/c)*arctan(c*sqrt((a*d*x^n + a
*c + b)/(d*x^n + c))*sqrt(-(a*c + b)/c)/(a*c + b)) - b*sqrt((a*d*x^n + a*c
 + b)/(d*x^n + c)))/(c*n)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x}\,dx=\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b/(c+d*x**n))**(3/2)/x,x)
```

output
```
Integral(((a*c + a*d*x**n + b)/(c + d*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^{3/2}}{x}\,dx$$

input
```
int((a + b/(c + d*x^n))^(3/2)/x,x)
```

output
```
int((a + b/(c + d*x^n))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x}\,dx = \frac{-4\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+b}\,a^2c-4\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+b}\,ab+x^n\left(\int \frac{}{x^{3n}a\,d^3x+}\right.}{}$$

input
```
int((a+b/(c+d*x^n))^(3/2)/x,x)
```

output
```
( - 4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*a**2*c - 4*sqrt(x**n*d + c
)*sqrt(x**n*a*d + a*c + b)*a*b + x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(
x**n*a*d + a*c + b))/(x**(3*n)*a*d**3*x + 3*x**(2*n)*a*c*d**2*x + x**(2*n)
*b*d**2*x + 3*x**n*a*c**2*d*x + 2*x**n*b*c*d*x + a*c**3*x + b*c**2*x),x)*a
**2*b*d**3*n + x**n*int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3
*n)*a*d**3*x + 3*x**(2*n)*a*c*d**2*x + x**(2*n)*b*d**2*x + 3*x**n*a*c**2*d
*x + 2*x**n*b*c*d*x + a*c**3*x + b*c**2*x),x)*a**2*b*c**2*d*n + 2*x**n*int
((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a*d**3*x + 3*x**(2*
n)*a*c*d**2*x + x**(2*n)*b*d**2*x + 3*x**n*a*c**2*d*x + 2*x**n*b*c*d*x + a
*c**3*x + b*c**2*x),x)*a*b**2*c*d*n + x**n*int((sqrt(x**n*d + c)*sqrt(x**n
*a*d + a*c + b))/(x**(3*n)*a*d**3*x + 3*x**(2*n)*a*c*d**2*x + x**(2*n)*b*d
**2*x + 3*x**n*a*c**2*d*x + 2*x**n*b*c*d*x + a*c**3*x + b*c**2*x),x)*b**3*
d*n + int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a
*d**3*x + 3*x**(2*n)*a*c*d**2*x + x**(2*n)*b*d**2*x + 3*x**n*a*c**2*d*x +
2*x**n*b*c*d*x + a*c**3*x + b*c**2*x),x)*a**2*b*c*d**2*n + int((sqrt(x**n*
d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a*d**3*x + 3*x**(2*n)*a*c*d**2*
x + x**(2*n)*b*d**2*x + 3*x**n*a*c**2*d*x + 2*x**n*b*c*d*x + a*c**3*x + b*
c**2*x),x)*a**2*b*c**3*n + 2*int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b
))/(x**(3*n)*a*d**3*x + 3*x**(2*n)*a*c*d**2*x + x**(2*n)*b*d**2*x + 3*x**n
*a*c**2*d*x + 2*x**n*b*c*d*x + a*c**3*x + b*c**2*x),x)*a*b**2*c**2*n + ...
```

3.322 $\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2} \, dx = -\frac{(b+ac)\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}\operatorname{AppellF1}\left(-\frac{1}{n},\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{cx\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
-(a*c+b)*(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(-1/n,3/2,-3/2,-(
1-n)/n,-d*x^n/c,-a*d*x^n/(a*c+b))/c/x/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.55 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2} \, dx = \frac{\sqrt{\frac{b+ac+adx^n}{c+dx^n}}\left(2b+\frac{\sqrt{\frac{b+ac+adx^n}{b+ac}}\sqrt{1+\frac{dx^n}{c}}\left(-\left((b+ac)(-1+n)(acn+b(2+n))\right)\operatorname{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{-1+n}{n},- \cdots\right.\right.}{(-1+n)(b+a(c+\cdots}\right.}{cnx}$$

input
```
Integrate[(a + b/(c + d*x^n))^(3/2)/x^2,x]
```

output
```
(Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*(2*b + (Sqrt[(b + a*c + a*d*x^n)/(b
 + a*c)]*Sqrt[1 + (d*x^n)/c]*(-((b + a*c)*(-1 + n)*(a*c*n + b*(2 + n))*App
ellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]
) + a*d*(2*b + a*c*n)*x^n*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), -((d*
x^n)/c), -((a*d*x^n)/(b + a*c))]))/((-1 + n)*(b + a*(c + d*x^n)))))/(c*n*x
)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{(adx^n+b+ac)^{3/2}}{x^2(dx^n+c)^{3/2}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{(adx^n+b+ac)^{3/2}}{x^2\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}{x^2\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\text{AppellF1}\left(-\frac{1}{n},\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{cx\sqrt{\frac{adx^n}{ac+b}+1}}$$

input

```
Int[(a + b/(c + d*x^n))^(3/2)/x^2,x]
```

output

```
-(((b + a*c)*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*App
ellF1[-n^(-1), 3/2, -3/2, -((1 - n)/n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c
))])/(c*x*Sqrt[1 + (a*d*x^n)/(b + a*c)]))
```

Defintions of rubi rules used

rule 1012

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{c+d\,x^{n}}\right)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int((a+b/(c+d*x^n))^(3/2)/x^2,x)
```

output
```
int((a+b/(c+d*x^n))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2} \, dx = \int \frac{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2} \, dx = \int \frac{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2} \, dx = \int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^{3/2}}{x^2} \, dx$$

input
```
int((a + b/(c + d*x^n))^(3/2)/x^2,x)
```

output
```
int((a + b/(c + d*x^n))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^2}\,dx = \text{too large to display}$$

input
```
int((a+b/(c+d*x^n))^(3/2)/x^2,x)
```

output
```
( - 4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*a**2*c - 4*sqrt(x**n*d + c
)*sqrt(x**n*a*d + a*c + b)*a*b + 4*x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqr
t(x**n*a*d + a*c + b))/(4*x**(3*n)*a**2*c*d**3*x**2 + x**(3*n)*a*b*d**3*n*
x**2 + 2*x**(3*n)*a*b*d**3*x**2 + 12*x**(2*n)*a**2*c**2*d**2*x**2 + 3*x**(
2*n)*a*b*c*d**2*n*x**2 + 10*x**(2*n)*a*b*c*d**2*x**2 + x**(2*n)*b**2*d**2*
n*x**2 + 2*x**(2*n)*b**2*d**2*x**2 + 12*x**n*a**2*c**3*d*x**2 + 3*x**n*a*b
*c**2*d*n*x**2 + 14*x**n*a*b*c**2*d*x**2 + 2*x**n*b**2*c*d*n*x**2 + 4*x**n
*b**2*c*d*x**2 + 4*a**2*c**4*x**2 + a*b*c**3*n*x**2 + 6*a*b*c**3*x**2 + b*
*2*c**2*n*x**2 + 2*b**2*c**2*x**2),x)*a**3*b*c*d**3*n*x - 8*x**n*int((x**(
2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**2*c*d**3*x*
*2 + x**(3*n)*a*b*d**3*n*x**2 + 2*x**(3*n)*a*b*d**3*x**2 + 12*x**(2*n)*a**
2*c**2*d**2*x**2 + 3*x**(2*n)*a*b*c*d**2*n*x**2 + 10*x**(2*n)*a*b*c*d**2*x
**2 + x**(2*n)*b**2*d**2*n*x**2 + 2*x**(2*n)*b**2*d**2*x**2 + 12*x**n*a**2
*c**3*d*x**2 + 3*x**n*a*b*c**2*d*n*x**2 + 14*x**n*a*b*c**2*d*x**2 + 2*x**n
*b**2*c*d*n*x**2 + 4*x**n*b**2*c*d*x**2 + 4*a**2*c**4*x**2 + a*b*c**3*n*x*
*2 + 6*a*b*c**3*x**2 + b**2*c**2*n*x**2 + 2*b**2*c**2*x**2),x)*a**3*b*c*d*
*3*x + x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(4*x*
*(3*n)*a**2*c*d**3*x**2 + x**(3*n)*a*b*d**3*n*x**2 + 2*x**(3*n)*a*b*d**3*x
**2 + 12*x**(2*n)*a**2*c**2*d**2*x**2 + 3*x**(2*n)*a*b*c*d**2*n*x**2 + 10*
x**(2*n)*a*b*c*d**2*x**2 + x**(2*n)*b**2*d**2*n*x**2 + 2*x**(2*n)*b**2*...
```

3.323 $\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = -\frac{(b+ac)\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2cx^2\sqrt{1+\frac{adx^n}{b+ac}}}$$

output
```
-1/2*(a*c+b)*(1+d*x^n/c)^(1/2)*(a+b/(c+d*x^n))^(1/2)*AppellF1(-2/n,3/2,-3/
2,-(2-n)/n,-d*x^n/c,-a*d*x^n/(a*c+b))/c/x^2/(1+a*d*x^n/(a*c+b))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.60 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{\frac{b+ac+adx^n}{c+dx^n}}\left(4b+\frac{\sqrt{\frac{b+ac+adx^n}{b+ac}}\sqrt{1+\frac{dx^n}{c}}\left(-\left((b+ac)(-2+n)(acn+b(4+n))\right)\text{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{-2+n}{n},-\right.\right.}{(-2+n)(b+a(c+}\right.}{2cnx^2}$$

input
```
Integrate[(a + b/(c + d*x^n))^(3/2)/x^3,x]
```

output
```
(Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*(4*b + (Sqrt[(b + a*c + a*d*x^n)/(b
 + a*c)]*Sqrt[1 + (d*x^n)/c]*(-((b + a*c)*(-2 + n)*(a*c*n + b*(4 + n))*App
ellF1[-2/n, 1/2, 1/2, (-2 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]) +
 2*a*d*(4*b + a*c*n)*x^n*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, -((d*x^n)
/c), -((a*d*x^n)/(b + a*c))]))/((-2 + n)*(b + a*(c + d*x^n)))))/(2*c*n*x^2
)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{(adx^n+b+ac)^{3/2}}{x^3(dx^n+c)^{3/2}}dx}{\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{(adx^n+b+ac)^{3/2}}{x^3\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{ac+adx^n+b}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\int \frac{\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}{x^3\left(\frac{dx^n}{c}+1\right)^{3/2}}dx}{c\sqrt{\frac{adx^n}{ac+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2cx^2\sqrt{\frac{adx^n}{ac+b}+1}}$$

input
```
Int[(a + b/(c + d*x^n))^(3/2)/x^3,x]
```

output
```
-1/2*((b + a*c)*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]*
AppellF1[-2/n, 3/2, -3/2, -((2 - n)/n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c
))])/(c*x^2*Sqrt[1 + (a*d*x^n)/(b + a*c)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{c+d\,x^{n}}\right)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int((a+b/(c+d*x^n))^(3/2)/x^3,x)
```

output
```
int((a+b/(c+d*x^n))^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n))**(3/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^n))^(3/2)/x^3,x)
```

output
```
int((a + b/(c + d*x^n))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}{x^3}\,dx = \text{too large to display}$$

input

```
int((a+b/(c+d*x^n))^(3/2)/x^3,x)
```

output

```
( - 4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*a**2*c - 4*sqrt(x**n*d + c
)*sqrt(x**n*a*d + a*c + b)*a*b + 8*x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqr
t(x**n*a*d + a*c + b))/(8*x**(3*n)*a**2*c*d**3*x**3 + x**(3*n)*a*b*d**3*n*
x**3 + 4*x**(3*n)*a*b*d**3*x**3 + 24*x**(2*n)*a**2*c**2*d**2*x**3 + 3*x**(
2*n)*a*b*c*d**2*n*x**3 + 20*x**(2*n)*a*b*c*d**2*x**3 + x**(2*n)*b**2*d**2*
n*x**3 + 4*x**(2*n)*b**2*d**2*x**3 + 24*x**n*a**2*c**3*d*x**3 + 3*x**n*a*b
*c**2*d*n*x**3 + 28*x**n*a*b*c**2*d*x**3 + 2*x**n*b**2*c*d*n*x**3 + 8*x**n
*b**2*c*d*x**3 + 8*a**2*c**4*x**3 + a*b*c**3*n*x**3 + 12*a*b*c**3*x**3 + b
**2*c**2*n*x**3 + 4*b**2*c**2*x**3),x)*a**3*b*c*d**3*n*x**2 - 32*x**n*int(
(x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(8*x**(3*n)*a**2*c*d*
*3*x**3 + x**(3*n)*a*b*d**3*n*x**3 + 4*x**(3*n)*a*b*d**3*x**3 + 24*x**(2*n
)*a**2*c**2*d**2*x**3 + 3*x**(2*n)*a*b*c*d**2*n*x**3 + 20*x**(2*n)*a*b*c*d
**2*x**3 + x**(2*n)*b**2*d**2*n*x**3 + 4*x**(2*n)*b**2*d**2*x**3 + 24*x**n
*a**2*c**3*d*x**3 + 3*x**n*a*b*c**2*d*n*x**3 + 28*x**n*a*b*c**2*d*x**3 + 2
*x**n*b**2*c*d*n*x**3 + 8*x**n*b**2*c*d*x**3 + 8*a**2*c**4*x**3 + a*b*c**3
*n*x**3 + 12*a*b*c**3*x**3 + b**2*c**2*n*x**3 + 4*b**2*c**2*x**3),x)*a**3*
b*c*d**3*x**2 + x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c +
b))/(8*x**(3*n)*a**2*c*d**3*x**3 + x**(3*n)*a*b*d**3*n*x**3 + 4*x**(3*n)*a
*b*d**3*x**3 + 24*x**(2*n)*a**2*c**2*d**2*x**3 + 3*x**(2*n)*a*b*c*d**2*n*x
**3 + 20*x**(2*n)*a*b*c*d**2*x**3 + x**(2*n)*b**2*d**2*n*x**3 + 4*x**(2...
```

3.324 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\frac{x^2\sqrt{1+\frac{adx^n}{b+ac}}\,\text{AppellF1}\left(\frac{2}{n},-\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
1/2*x^2*(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(2/n,-1/2,1/2,(2+n)/n,-d*x^n/c,-
a*d*x^n/(a*c+b))/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx$$

input
```
Integrate[x/Sqrt[a + b/(c + d*x^n)],x]
```

output
```
Integrate[x/Sqrt[a + b/(c + d*x^n)], x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{x}{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{x\sqrt{dx^n+c}}{\sqrt{adx^n+b+ac}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{x\sqrt{\frac{dx^n}{c}+1}}{\sqrt{adx^n+b+ac}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{x\sqrt{\frac{dx^n}{c}+1}}{\sqrt{\frac{adx^n}{b+ac}+1}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^2\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(\frac{2}{n},-\frac{1}{2},\frac{1}{2},\frac{n+2}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[x/Sqrt[a + b/(c + d*x^n)],x]
```

output
```
(x^2*Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[2/n, -1/2, 1/2, (2 + n)/n, -((
d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(2*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n
)]*Sqrt[1 + (d*x^n)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+d\,x^{n}}}}dx$$

input
```
int(x/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int(x/(a+b/(c+d*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b/(c+d*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{x}{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}\,dx$$

input
```
integrate(x/(a+b/(c+d*x**n))**(1/2),x)
```

output
```
Integral(x/sqrt((a*c + a*d*x**n + b)/(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{x}{\sqrt{a+\frac{b}{dx^n+c}}}\,dx$$

input
```
integrate(x/(a+b/(c+d*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a + b/(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{dx^n+c}}}\,dx$$

input
```
integrate(x/(a+b/(c+d*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(a + b/(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{c+d\,x^n}}}\,dx$$

input
```
int(x/(a + b/(c + d*x^n))^(1/2),x)
```

output
```
int(x/(a + b/(c + d*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{\sqrt{x^nd+c}\,\sqrt{x^nad+ac+b}\,x}{x^nad+ac+b}\,dx$$

input
```
int(x/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*x)/(x**n*a*d + a*c + b),x)
```

3.325 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\frac{x\sqrt{1+\frac{adx^n}{b+ac}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
x*(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(1/n,-1/2,1/2,1+1/n,-d*x^n/c,-a*d*x^n/
(a*c+b))/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx$$

input
```
Integrate[1/Sqrt[a + b/(c + d*x^n)],x]
```

output
```
Integrate[1/Sqrt[a + b/(c + d*x^n)], x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2057, 2058, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{\sqrt{dx^n+c}}{\sqrt{adx^n+b+ac}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{\sqrt{\frac{dx^n}{c}+1}}{\sqrt{adx^n+b+ac}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{\sqrt{\frac{dx^n}{c}+1}}{\sqrt{\frac{adx^n}{b+ac}+1}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[1/Sqrt[a + b/(c + d*x^n)],x]
```

output
```
(x*Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[n^(-1), -1/2, 1/2, 1 + n^(-1), -
((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n
)]*Sqrt[1 + (d*x^n)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+d\,x^{n}}}}dx$$

input
```
int(1/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int(1/(a+b/(c+d*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/(c+d*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx$$

input
```
integrate(1/(a+b/(c+d*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b/(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}}\,dx$$

input
```
integrate(1/(a+b/(c+d*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}}\,dx$$

input
```
integrate(1/(a+b/(c+d*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a + b/(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{c+d\,x^n}}}\,dx$$

input
```
int(1/(a + b/(c + d*x^n))^(1/2),x)
```

output
```
int(1/(a + b/(c + d*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{\sqrt{x^nd+c}\,\sqrt{x^nad+ac+b}}{x^nad+ac+b}dx$$

input
```
int(1/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*a*d + a*c + b),x)
```

3.326 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{a}}\right)}{\sqrt{a}n}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{b+ac}}\right)}{\sqrt{b+acn}}$$

output
```
2*arctanh((a+b/(c+d*x^n))^(1/2)/a^(1/2))/a^(1/2)/n-2*c^(1/2)*arctanh(c^(1/
2)*(a+b/(c+d*x^n))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\frac{2\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}{\sqrt{-b-ac}}\right)}{\sqrt{-b-ac}}+\frac{\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{n}$$

input
```
Integrate[1/(x*Sqrt[a + b/(c + d*x^n)]),x]
```

output
```
(2*((Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/Sqrt[-
b - a*c]])/Sqrt[-b - a*c] + ArcTanh[Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]/
Sqrt[a]]/Sqrt[a]))/n
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2053, 2052, 25, 27, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{\int \frac{x^{-n}}{\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}dx^n}{n}$$

$$\Big\downarrow \text{2052}$$

$$-\frac{2bd\int -\frac{1}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bd\int \frac{1}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{1}{(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{303}$$

$$\frac{2b\left(\frac{\int \frac{1}{a-x^{2n}}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}-\frac{c\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}\right)}{n}$$

$\Big\downarrow$ 219

$$\frac{2b\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{c\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}\right)}{n}$$

$\Big\downarrow$ 221

$$\frac{2b\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{ac+b}}\right)}{b\sqrt{ac+b}}\right)}{n}$$

input
```
Int[1/(x*Sqrt[a + b/(c + d*x^n)]),x]
```

output
```
(2*b*(ArcTanh[Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]/Sqrt[a]]/(Sqrt[a]*b) -
 (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/Sqrt[b +
 a*c]])/(b*Sqrt[b + a*c])))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(71) = 142$.

Time = 0.86 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.84

method	result
derivativedivides	$-\frac{\sqrt{\frac{x^{n} a d+a c+b}{c+d\,x^{n}}}\,(c+d\,x^{n})\left(-2\sqrt{(ac+b)c}\,\ln\left(\frac{2a\,d^{2} x^{n}+2acd+2\sqrt{x^{2n} a\,d^{2}+2x^{n} acd+a\,c^{2}+b\,x^{n} d+bc}\,\sqrt{a\,d^{2}}+bd}{2\sqrt{a\,d^{2}}}\right)acd+2\sqrt{a\,d^{2}}\cdots}{\cdots}$
default	$-\frac{\sqrt{\frac{x^{n} a d+a c+b}{c+d\,x^{n}}}\,(c+d\,x^{n})\left(-2\sqrt{(ac+b)c}\,\ln\left(\frac{2a\,d^{2} x^{n}+2acd+2\sqrt{x^{2n} a\,d^{2}+2x^{n} acd+a\,c^{2}+b\,x^{n} d+bc}\,\sqrt{a\,d^{2}}+bd}{2\sqrt{a\,d^{2}}}\right)acd+2\sqrt{a\,d^{2}}\cdots}{\cdots}$

input
```
int(1/x/(a+b/(c+d*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/n*((x^n*a*d+a*c+b)/(c+d*x^n))^(1/2)*(c+d*x^n)*(-2*((a*c+b)*c)^(1/2)*l
n(1/2*(2*a*d^2*x^n+2*a*c*d+2*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)
^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*c*d+2*(a*d^2)^(1/2)*ln((2*x^n*a
*c*d+2*a*c^2+b*x^n*d+2*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+
b*x^n*d+b*c)^(1/2)+2*b*c)/(x^n))*a*c^2-((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x
^n+2*a*c*d+2*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(
1/2)+b*d)/(a*d^2)^(1/2))*b*d+2*(a*d^2)^(1/2)*ln((2*x^n*a*c*d+2*a*c^2+b*x^n
*d+2*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)
+2*b*c)/(x^n))*b*c-((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((c+d*x
^n)*(x^n*a*d+a*c+b))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b*d+2*(a*d^2)
^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/2)*((a*c+b)*c)^(1/2)-2*((x^n)^2*a*d^
2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)*((a*c+b)*c)^(1/2))/((
c+d*x^n)*(x^n*a*d+a*c+b))^(1/2)/(a*c+b)/(a*d^2)^(1/2)/((a*c+b)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 599, normalized size of antiderivative = 6.89

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \left[\frac{a\sqrt{\frac{c}{ac+b}}\log\left(-\frac{2\,ac^2+(2\,ac+b)dx^n+2\,bc-2\left((ac+b)dx^n\sqrt{\frac{c}{ac+b}}+(ac^2+bc)\sqrt{\frac{c}{ac+b}}\right)\sqrt{\frac{adx^n+ac+b}{dx^n+c}}}{x^n}\right)+\sqrt{a}\log\left(2\,adx^n+2\,ac\right.}{an}\right.$$

input
```
integrate(1/x/(a+b/(c+d*x^n))^(1/2),x, algorithm="fricas")
```

output
```
[(a*sqrt(c/(a*c + b))*log(-(2*a*c^2 + (2*a*c + b)*d*x^n + 2*b*c - 2*((a*c
+ b)*d*x^n*sqrt(c/(a*c + b)) + (a*c^2 + b*c)*sqrt(c/(a*c + b)))*sqrt((a*d*
x^n + a*c + b)/(d*x^n + c)))/x^n) + sqrt(a)*log(2*a*d*x^n + 2*a*c + 2*(sqr
t(a)*d*x^n + sqrt(a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n + c)) + b))/(a*n),
 (2*a*sqrt(-c/(a*c + b))*arctan(sqrt((a*d*x^n + a*c + b)/(d*x^n + c))*sqrt
(-c/(a*c + b))) + sqrt(a)*log(2*a*d*x^n + 2*a*c + 2*(sqrt(a)*d*x^n + sqrt(
a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n + c)) + b))/(a*n), (a*sqrt(c/(a*c +
b))*log(-(2*a*c^2 + (2*a*c + b)*d*x^n + 2*b*c - 2*((a*c + b)*d*x^n*sqrt(c/
(a*c + b)) + (a*c^2 + b*c)*sqrt(c/(a*c + b)))*sqrt((a*d*x^n + a*c + b)/(d*
x^n + c)))/x^n) - 2*sqrt(-a)*arctan((sqrt(-a)*d*x^n + sqrt(-a)*c)*sqrt((a*
d*x^n + a*c + b)/(d*x^n + c))/(a*d*x^n + a*c + b)))/(a*n), 2*(a*sqrt(-c/(a
*c + b))*arctan(sqrt((a*d*x^n + a*c + b)/(d*x^n + c))*sqrt(-c/(a*c + b)))
- sqrt(-a)*arctan((sqrt(-a)*d*x^n + sqrt(-a)*c)*sqrt((a*d*x^n + a*c + b)/(
d*x^n + c))/(a*d*x^n + a*c + b)))/(a*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{x\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}\,dx$$

input
```
integrate(1/x/(a+b/(c+d*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a*c + a*d*x**n + b)/(c + d*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}\,x}\,dx$$

input
```
integrate(1/x/(a+b/(c+d*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x^n + c))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}\,x}\,dx$$

input
```
integrate(1/x/(a+b/(c+d*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/(d*x^n + c))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{x\,\sqrt{a+\frac{b}{c+d\,x^n}}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x^n))^(1/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x^n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+b}}{x^n adx+acx+bx}\,dx$$

input
```
int(1/x/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*a*d*x + a*c*x + b*x)
,x)
```

3.327 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx = -\frac{\sqrt{1+\frac{adx^n}{b+ac}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{x\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
-(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(-1/n,-1/2,1/2,-(1-n)/n,-d*x^n/c,-a*d*x
^n/(a*c+b))/x/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + b/(c + d*x^n)]),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + b/(c + d*x^n)]), x]
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^2\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{\sqrt{dx^n+c}}{x^2\sqrt{adx^n+b+ac}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{\sqrt{\frac{dx^n}{c}+1}}{x^2\sqrt{adx^n+b+ac}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{\sqrt{\frac{dx^n}{c}+1}}{x^2\sqrt{\frac{adx^n}{b+ac}+1}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{x\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[1/(x^2*Sqrt[a + b/(c + d*x^n)]),x]
```

output
```
-((Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[-n^(-1), -1/2, 1/2, -((1 - n)/n)
, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(x*Sqrt[(b + a*c + a*d*x^n)/(c +
d*x^n)]*Sqrt[1 + (d*x^n)/c]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+d\,x^n}}}dx$$

input
```
int(1/x^2/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int(1/x^2/(a+b/(c+d*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b/(c+d*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{x^2\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(c+d*x**n))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a*c + a*d*x**n + b)/(c + d*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(c+d*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x^n + c))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(c+d*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/(d*x^n + c))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{x^2\sqrt{a+\frac{b}{c+d\,x^n}}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x^n))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x^n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+b}}{x^n ad\,x^2+ac\,x^2+b\,x^2}dx$$

input
```
int(1/x^2/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*a*d*x**2 + a*c*x**2
+ b*x**2),x)
```

3.328 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx=-\frac{\sqrt{1+\frac{adx^n}{b+ac}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2x^2\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
-1/2*(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(-2/n,-1/2,1/2,-(2-n)/n,-d*x^n/c,-a
*d*x^n/(a*c+b))/x^2/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx$$

input
```
Integrate[1/(x^3*Sqrt[a + b/(c + d*x^n)]),x]
```

output
```
Integrate[1/(x^3*Sqrt[a + b/(c + d*x^n)]), x]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^3\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{\sqrt{dx^n+c}}{x^3\sqrt{adx^n+b+ac}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{\sqrt{\frac{dx^n}{c}+1}}{x^3\sqrt{adx^n+b+ac}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{\sqrt{\frac{dx^n}{c}+1}}{x^3\sqrt{\frac{adx^n}{b+ac}+1}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2x^2\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[1/(x^3*Sqrt[a + b/(c + d*x^n)]),x]
```

output
```
-1/2*(Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[-2/n, -1/2, 1/2, -((2 - n)/n)
, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(x^2*Sqrt[(b + a*c + a*d*x^n)/(c
+ d*x^n)]*Sqrt[1 + (d*x^n)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+d\,x^n}}}dx$$

input
```
int(1/x^3/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int(1/x^3/(a+b/(c+d*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b/(c+d*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{x^3\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(c+d*x**n))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a*c + a*d*x**n + b)/(c + d*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(c+d*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x^n + c))*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^n+c}}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(c+d*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/(d*x^n + c))*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{1}{x^3\,\sqrt{a+\frac{b}{c+d\,x^n}}}\,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x^n))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x^n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^n}}}\,dx=\int \frac{\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+b}}{x^n ad\,x^3+ac\,x^3+b\,x^3}dx$$

input
```
int(1/x^3/(a+b/(c+d*x^n))^(1/2),x)
```

output
```
int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**n*a*d*x**3 + a*c*x**3
+ b*x**3),x)
```

3.329 $\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\frac{cx^2\sqrt{1+\frac{adx^n}{b+ac}}\,\text{AppellF1}\left(\frac{2}{n},-\frac{3}{2},\frac{3}{2},\frac{2+n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2(b+ac)\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
1/2*c*x^2*(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(2/n,-3/2,3/2,(2+n)/n,-d*x^n/c
,-a*d*x^n/(a*c+b))/(a*c+b)/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 228 vs. 2(107) = 214.

Time = 11.51 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.13

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\frac{x^2\sqrt{\frac{b+ac+adx^n}{c+dx^n}}\left(-4b(c+dx^n)+\frac{\sqrt{\frac{b+ac+adx^n}{b+ac}}\sqrt{1+\frac{dx^n}{c}}\left(c(2+n)(4b+acn)\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-d\right.\right.}{\cdots}\right.}{2a(b+ac)n\left(b+a\left(c+dx\right.\right.}$$

input
```
Integrate[x/(a + b/(c + d*x^n))^(3/2),x]
```

output
```
(x^2*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*(-4*b*(c + d*x^n) + (Sqrt[(b +
a*c + a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*(c*(2 + n)*(4*b + a*c*n)*App
ellF1[2/n, 1/2, 1/2, (2 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] + 2*
d*(a*c*n + b*(4 + n))*x^n*AppellF1[(2 + n)/n, 1/2, 1/2, 2*(1 + n^(-1)), -(
(d*x^n)/c), -((a*d*x^n)/(b + a*c))]))/(2 + n)))/(2*a*(b + a*c)*n*(b + a*(c
 + d*x^n)))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{x}{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{x(dx^n+c)^{3/2}}{(adx^n+b+ac)^{3/2}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{c\sqrt{ac+adx^n+b}\int \frac{x\left(\frac{dx^n}{c}+1\right)^{3/2}}{(adx^n+b+ac)^{3/2}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{c\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{x\left(\frac{dx^n}{c}+1\right)^{3/2}}{\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}dx}{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{cx^2\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(\frac{2}{n},-\frac{3}{2},\frac{3}{2},\frac{n+2}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[x/(a + b/(c + d*x^n))^(3/2),x]
```

output
```
(c*x^2*Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[2/n, -3/2, 3/2, (2 + n)/n, -
((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(2*(b + a*c)*Sqrt[(b + a*c + a*d*x^n
)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{x}{\left(a+\frac{b}{c+d\,x^{n}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x/(a+b/(c+d*x^n))^(3/2),x)
```

output
```
int(x/(a+b/(c+d*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b/(c+d*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{x}{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(c+d*x**n))**(3/2),x)
```

output
```
Integral(x/((a*c + a*d*x**n + b)/(c + d*x**n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{x}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(c+d*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a + b/(d*x^n + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{x}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(c+d*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(a + b/(d*x^n + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{x}{\left(a+\frac{b}{c+d\,x^n}\right)^{3/2}}\,dx$$

input
```
int(x/(a + b/(c + d*x^n))^(3/2),x)
```

output
```
int(x/(a + b/(c + d*x^n))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\text{too large to display}$$

input
```
int(x/(a+b/(c+d*x^n))^(3/2),x)
```

output
```
(4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*c*x**2 + 8*x**n*int((x**(2*n)
*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*x)/(8*x**(3*n)*a**3*c*d**3 + x*
*(3*n)*a**2*b*d**3*n + 4*x**(3*n)*a**2*b*d**3 + 24*x**(2*n)*a**3*c**2*d**2
 + 3*x**(2*n)*a**2*b*c*d**2*n + 28*x**(2*n)*a**2*b*c*d**2 + 2*x**(2*n)*a*b
**2*d**2*n + 8*x**(2*n)*a*b**2*d**2 + 24*x**n*a**3*c**3*d + 3*x**n*a**2*b*
c**2*d*n + 44*x**n*a**2*b*c**2*d + 4*x**n*a*b**2*c*d*n + 24*x**n*a*b**2*c*
d + x**n*b**3*d*n + 4*x**n*b**3*d + 8*a**3*c**4 + a**2*b*c**3*n + 20*a**2*
b*c**3 + 2*a*b**2*c**2*n + 16*a*b**2*c**2 + b**3*c*n + 4*b**3*c),x)*a**2*b
*c*d**3*n + 32*x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b
)*x)/(8*x**(3*n)*a**3*c*d**3 + x**(3*n)*a**2*b*d**3*n + 4*x**(3*n)*a**2*b*
d**3 + 24*x**(2*n)*a**3*c**2*d**2 + 3*x**(2*n)*a**2*b*c*d**2*n + 28*x**(2*
n)*a**2*b*c*d**2 + 2*x**(2*n)*a*b**2*d**2*n + 8*x**(2*n)*a*b**2*d**2 + 24*
x**n*a**3*c**3*d + 3*x**n*a**2*b*c**2*d*n + 44*x**n*a**2*b*c**2*d + 4*x**n
*a*b**2*c*d*n + 24*x**n*a*b**2*c*d + x**n*b**3*d*n + 4*x**n*b**3*d + 8*a**
3*c**4 + a**2*b*c**3*n + 20*a**2*b*c**3 + 2*a*b**2*c**2*n + 16*a*b**2*c**2
 + b**3*c*n + 4*b**3*c),x)*a**2*b*c*d**3 + x**n*int((x**(2*n)*sqrt(x**n*d
+ c)*sqrt(x**n*a*d + a*c + b)*x)/(8*x**(3*n)*a**3*c*d**3 + x**(3*n)*a**2*b
*d**3*n + 4*x**(3*n)*a**2*b*d**3 + 24*x**(2*n)*a**3*c**2*d**2 + 3*x**(2*n)
*a**2*b*c*d**2*n + 28*x**(2*n)*a**2*b*c*d**2 + 2*x**(2*n)*a*b**2*d**2*n +
8*x**(2*n)*a*b**2*d**2 + 24*x**n*a**3*c**3*d + 3*x**n*a**2*b*c**2*d*n +...
```

3.330 $\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}} dx = \frac{cx\sqrt{1+\frac{adx^n}{b+ac}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{(b+ac)\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
c*x*(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(1/n,-3/2,3/2,1+1/n,-d*x^n/c,-a*d*x^
n/(a*c+b))/(a*c+b)/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 784 vs. 2(98) = 196.

Time = 3.02 (sec) , antiderivative size = 784, normalized size of antiderivative = 8.00

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}} dx = \text{Too large to display}$$

input
```
Integrate[(a + b/(c + d*x^n))^(-3/2),x]
```

output
```
(-2*b*x*(c + d*x^n)*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/(a*(b + a*c)*n*
(b + a*(c + d*x^n))) + (x*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*(2*b*d*x^n
*Sqrt[(b + a*c + a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*AppellF1[1 + n^(-
1), 1/2, 1/2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] + b*d*n*x^
n*Sqrt[(b + a*c + a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*AppellF1[1 + n^(
-1), 1/2, 1/2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] + a*c*d*n
*x^n*Sqrt[(b + a*c + a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*AppellF1[1 +
n^(-1), 1/2, 1/2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))] + (4*b
*c^2*(b + a*c)*(1 + n)^2*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), -((d*x^n)/
c), -((a*d*x^n)/(b + a*c))])/(-(a*c*d*n*x^n*AppellF1[1 + n^(-1), 1/2, 3/2,
 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]) + (b + a*c)*(-(d*n*x^n
*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b +
 a*c))]) + 2*c*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), -((d*x^n)/c)
, -((a*d*x^n)/(b + a*c))])) + (2*a*c^3*(b + a*c)*n*(1 + n)^2*AppellF1[n^(-
1), 1/2, 1/2, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(-(a*c*d*
n*x^n*AppellF1[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), -((d*x^n)/c), -((a*d*x^n)
/(b + a*c))]) + (b + a*c)*(-(d*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^
(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]) + 2*c*(1 + n)*AppellF1[n^(-1)
, 1/2, 1/2, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]))))/(a*(b +
a*c)*n*(1 + n)*(b + a*(c + d*x^n)))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2057, 2058, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 2057}$$
$$\int \frac{1}{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{(dx^n+c)^{3/2}}{(adx^n+b+ac)^{3/2}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \textbf{937}$$

$$\frac{c\sqrt{ac+adx^n+b}\int \frac{\left(\frac{dx^n}{c}+1\right)^{3/2}}{(adx^n+b+ac)^{3/2}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \textbf{937}$$

$$\frac{c\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{\left(\frac{dx^n}{c}+1\right)^{3/2}}{\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}dx}{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \textbf{936}$$

$$\frac{cx\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[(a + b/(c + d*x^n))^(-3/2),x]
```

output
```
(c*x*Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[n^(-1), -3/2, 3/2, 1 + n^(-1),
 -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/((b + a*c)*Sqrt[(b + a*c + a*d*x^n
)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\left(a+\frac{b}{c+d\,x^{n}}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b/(c+d*x^n))^(3/2),x)
```

output
```
int(1/(a+b/(c+d*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/(c+d*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(c+d*x**n))**(3/2),x)
```

output
```
Integral((a + b/(c + d*x**n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(c+d*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(c+d*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{c+d\,x^n}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/(c + d*x^n))^(3/2),x)
```

output
```
int(1/(a + b/(c + d*x^n))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(1/(a+b/(c+d*x^n))^(3/2),x)
```

output
```
(4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*c*x + 4*x**n*int((x**(2*n)*sq
rt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**3*c*d**3 + x**(3*n
)*a**2*b*d**3*n + 2*x**(3*n)*a**2*b*d**3 + 12*x**(2*n)*a**3*c**2*d**2 + 3*
x**(2*n)*a**2*b*c*d**2*n + 14*x**(2*n)*a**2*b*c*d**2 + 2*x**(2*n)*a*b**2*d
**2*n + 4*x**(2*n)*a*b**2*d**2 + 12*x**n*a**3*c**3*d + 3*x**n*a**2*b*c**2*
d*n + 22*x**n*a**2*b*c**2*d + 4*x**n*a*b**2*c*d*n + 12*x**n*a*b**2*c*d + x
**n*b**3*d*n + 2*x**n*b**3*d + 4*a**3*c**4 + a**2*b*c**3*n + 10*a**2*b*c**
3 + 2*a*b**2*c**2*n + 8*a*b**2*c**2 + b**3*c*n + 2*b**3*c),x)*a**2*b*c*d**
3*n + 8*x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(4*x
**(3*n)*a**3*c*d**3 + x**(3*n)*a**2*b*d**3*n + 2*x**(3*n)*a**2*b*d**3 + 12
*x**(2*n)*a**3*c**2*d**2 + 3*x**(2*n)*a**2*b*c*d**2*n + 14*x**(2*n)*a**2*b
*c*d**2 + 2*x**(2*n)*a*b**2*d**2*n + 4*x**(2*n)*a*b**2*d**2 + 12*x**n*a**3
*c**3*d + 3*x**n*a**2*b*c**2*d*n + 22*x**n*a**2*b*c**2*d + 4*x**n*a*b**2*c
*d*n + 12*x**n*a*b**2*c*d + x**n*b**3*d*n + 2*x**n*b**3*d + 4*a**3*c**4 +
a**2*b*c**3*n + 10*a**2*b*c**3 + 2*a*b**2*c**2*n + 8*a*b**2*c**2 + b**3*c*
n + 2*b**3*c),x)*a**2*b*c*d**3 + x**n*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(
x**n*a*d + a*c + b))/(4*x**(3*n)*a**3*c*d**3 + x**(3*n)*a**2*b*d**3*n + 2*
x**(3*n)*a**2*b*d**3 + 12*x**(2*n)*a**3*c**2*d**2 + 3*x**(2*n)*a**2*b*c*d*
*2*n + 14*x**(2*n)*a**2*b*c*d**2 + 2*x**(2*n)*a*b**2*d**2*n + 4*x**(2*n)*a
*b**2*d**2 + 12*x**n*a**3*c**3*d + 3*x**n*a**2*b*c**2*d*n + 22*x**n*a**...
```

3.331 $\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=-\frac{2b}{a(b+ac)n\sqrt{a+\frac{b}{c+dx^n}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^n}}}{\sqrt{b+ac}}\right)}{(b+ac)^{3/2}n}$$

output
```
-2*b/a/(a*c+b)/n/(a+b/(c+d*x^n))^(1/2)+2*arctanh((a+b/(c+d*x^n))^(1/2)/a^(
1/2))/a^(3/2)/n-2*c^(3/2)*arctanh(c^(1/2)*(a+b/(c+d*x^n))^(1/2)/(a*c+b)^(1
/2))/(a*c+b)^(3/2)/n
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\frac{2\left(-\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}-\frac{c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^n}{c+dx^n}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{n}$$

input
```
Integrate[1/(x*(a + b/(c + d*x^n))^(3/2)),x]
```

output
```
(2*(-(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])) - (c^(3/2)*Ar
cTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)])/Sqrt[-b - a*c]])/(-b
- a*c)^(3/2) + ArcTanh[Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]/Sqrt[a]]/a^(3
/2)))/n
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2057, 2053, 2052, 25, 27, 382, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{\int \frac{x^{-n}}{\left(\frac{adx^n+b+ac}{dx^n+c}\right)^{3/2}}dx^n}{n}$$

$$\Big\downarrow \text{ 2052}$$

$$-\frac{2bd\int -\frac{x^{-2n}}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2bd\int \frac{x^{-2n}}{d(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b\int \frac{x^{-2n}}{(a-x^{2n})(-cx^{2n}+b+ac)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{n}$$

$$\Big\downarrow \; 382$$

$$\frac{2b\left(\frac{\int \frac{-cx^{2n}+b+2ac}{\left(a-x^{2n}\right)\left(-cx^{2n}+b+ac\right)}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{a(ac+b)}-\frac{x^{-n}}{a(ac+b)}\right)}{n}$$

$$\Big\downarrow \; 397$$

$$\frac{2b\left(\frac{\frac{(ac+b)\int \frac{1}{a-x^{2n}}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}-\frac{ac^2\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}}{a(ac+b)}-\frac{x^{-n}}{a(ac+b)}\right)}{n}$$

$$\Big\downarrow \; 219$$

$$\frac{2b\left(\frac{\frac{(ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{ac^2\int \frac{1}{-cx^{2n}+b+ac}d\sqrt{\frac{adx^n+b+ac}{dx^n+c}}}{b}}{a(ac+b)}-\frac{x^{-n}}{a(ac+b)}\right)}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{2b\left(\frac{\frac{(ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{ac^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}{\sqrt{ac+b}}\right)}{b\sqrt{ac+b}}}{a(ac+b)}-\frac{x^{-n}}{a(ac+b)}\right)}{n}$$

input
```
Int[1/(x*(a + b/(c + d*x^n))^(3/2)),x]
```

output
```
(2*b*(-(1/(a*(b + a*c)*x^n)) + (((b + a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^n
)/(c + d*x^n)]/Sqrt[a]])/(Sqrt[a]*b) - (a*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(b
 + a*c + a*d*x^n)/(c + d*x^n)])/Sqrt[b + a*c]])/(b*Sqrt[b + a*c]))/(a*(b +
 a*c))))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3398 vs. $2(102) = 204$.

Time = 0.86 (sec) , antiderivative size = 3399, normalized size of antiderivative = 28.32

method	result	size
derivativedivides	Expression too large to display	3399
default	Expression too large to display	3399

input
```
int(1/x/(a+b/(c+d*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/n*((x^n*a*d+a*c+b)/(c+d*x^n))^(1/2)*(c+d*x^n)/a*(-8*ln((2*x^n*a*c*d+2*
a*c^2+b*x^n*d+2*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d
+b*c)^(1/2)+2*b*c)/(x^n))*(a*d^2)^(1/2)*a^3*b*c^3*d*x^n+((a*c+b)*c)^(1/2)*
ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c
)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*b*c*d^3*(x^n)^2+3*((a*c+b)*c
)^(1/2)*ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/2)*(a
*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*b*c*d^3*(x^n)^2-2*ln((2*x^n*a*c*d+2*a*
c^2+b*x^n*d+2*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b
*c)^(1/2)+2*b*c)/(x^n))*(a*d^2)^(1/2)*a^3*b*c^2*d^2*(x^n)^2+10*((a*c+b)*c)
^(1/2)*ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/2)*(a*
d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c*d^2*x^n+2*((a*c+b)*c)^(1/2)*((x^n
)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)*a^3*c*d^2*(x^
n)^2-6*((a*c+b)*c)^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/2)*(a*d^2)^(1/2)*a
^3*c*d^2*(x^n)^2+6*((a*c+b)*c)^(1/2)*ln(1/2*(2*a*d^2*x^n+2*a*c*d+2*((x^n)^
2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1
/2))*a^3*b*c^2*d^2*x^n-8*((a*c+b)*c)^(1/2)*((c+d*x^n)*(x^n*a*d+a*c+b))^(1/
2)*(a*d^2)^(1/2)*a*b^2*d*x^n+4*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*
d+a*c^2+b*x^n*d+b*c)^(1/2)*(a*d^2)^(1/2)*a^3*c^2*d*x^n-4*ln((2*x^n*a*c*d+2
*a*c^2+b*x^n*d+2*((a*c+b)*c)^(1/2)*((x^n)^2*a*d^2+2*x^n*a*c*d+a*c^2+b*x^n*
d+b*c)^(1/2)+2*b*c)/(x^n))*(a*d^2)^(1/2)*a^2*b^2*c^2*d*x^n-12*((a*c+b)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(102) = 204$.

Time = 0.14 (sec) , antiderivative size = 1187, normalized size of antiderivative = 9.89

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b/(c+d*x^n))^(3/2),x, algorithm="fricas")
```

output

```
[(((a^2*c + a*b)*sqrt(a)*d*x^n + (a^2*c^2 + 2*a*b*c + b^2)*sqrt(a))*log(2*
a*d*x^n + 2*a*c + 2*(sqrt(a)*d*x^n + sqrt(a)*c)*sqrt((a*d*x^n + a*c + b)/(
d*x^n + c)) + b) + (a^3*c*d*x^n*sqrt(c/(a*c + b)) + (a^3*c^2 + a^2*b*c)*sq
rt(c/(a*c + b)))*log(-(2*a*c^2 + (2*a*c + b)*d*x^n + 2*b*c - 2*((a*c + b)*
d*x^n*sqrt(c/(a*c + b)) + (a*c^2 + b*c)*sqrt(c/(a*c + b)))*sqrt((a*d*x^n +
 a*c + b)/(d*x^n + c)))/x^n) - 2*(a*b*d*x^n + a*b*c)*sqrt((a*d*x^n + a*c +
 b)/(d*x^n + c)))/((a^4*c + a^3*b)*d*n*x^n + (a^4*c^2 + 2*a^3*b*c + a^2*b^
2)*n), (2*(a^3*c*d*x^n*sqrt(-c/(a*c + b)) + (a^3*c^2 + a^2*b*c)*sqrt(-c/(a
*c + b)))*arctan(sqrt((a*d*x^n + a*c + b)/(d*x^n + c))*sqrt(-c/(a*c + b)))
 + ((a^2*c + a*b)*sqrt(a)*d*x^n + (a^2*c^2 + 2*a*b*c + b^2)*sqrt(a))*log(2
*a*d*x^n + 2*a*c + 2*(sqrt(a)*d*x^n + sqrt(a)*c)*sqrt((a*d*x^n + a*c + b)/
(d*x^n + c)) + b) - 2*(a*b*d*x^n + a*b*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n
+ c)))/((a^4*c + a^3*b)*d*n*x^n + (a^4*c^2 + 2*a^3*b*c + a^2*b^2)*n), -(2*
((a^2*c + a*b)*sqrt(-a)*d*x^n + (a^2*c^2 + 2*a*b*c + b^2)*sqrt(-a))*arctan
((sqrt(-a)*d*x^n + sqrt(-a)*c)*sqrt((a*d*x^n + a*c + b)/(d*x^n + c))/(a*d*
x^n + a*c + b)) - (a^3*c*d*x^n*sqrt(c/(a*c + b)) + (a^3*c^2 + a^2*b*c)*sqr
t(c/(a*c + b)))*log(-(2*a*c^2 + (2*a*c + b)*d*x^n + 2*b*c - 2*((a*c + b)*d
*x^n*sqrt(c/(a*c + b)) + (a*c^2 + b*c)*sqrt(c/(a*c + b)))*sqrt((a*d*x^n +
a*c + b)/(d*x^n + c)))/x^n) + 2*(a*b*d*x^n + a*b*c)*sqrt((a*d*x^n + a*c +
b)/(d*x^n + c)))/((a^4*c + a^3*b)*d*n*x^n + (a^4*c^2 + 2*a^3*b*c + a^2*...
```

Sympy [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{1}{x\left(\frac{ac+adx^n+b}{c+dx^n}\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/x/(a+b/(c+d*x**n))**(3/2),x)
```

output

```
Integral(1/(x*((a*c + a*d*x**n + b)/(c + d*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b/(c+d*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x^n + c))^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b/(c+d*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a + b/(d*x^n + c))^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+\frac{b}{c+d\,x^n}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x^n))^(3/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x^n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \frac{4\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+b}\,c+x^n\left(\int \frac{x^{2n}\sqrt{x^n d+c}\,\sqrt{x^n ad+ac+}}{x^{3n}a^2d^3x+3x^{2n}a^2c\,d^2x+2x^{2n}ab\,d^2x+3x^n a^2c^2dx+4x^n abc}\right.}{}$$

input

```
int(1/x/(a+b/(c+d*x^n))^(3/2),x)
```

output

```
(4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*c + x**n*int((x**(2*n)*sqrt(x
**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a**2*d**3*x + 3*x**(2*n)*a*
*2*c*d**2*x + 2*x**(2*n)*a*b*d**2*x + 3*x**n*a**2*c**2*d*x + 4*x**n*a*b*c*
d*x + x**n*b**2*d*x + a**2*c**3*x + 2*a*b*c**2*x + b**2*c*x),x)*a*b*d**3*n
 + x**n*int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a**2*d**
3*x + 3*x**(2*n)*a**2*c*d**2*x + 2*x**(2*n)*a*b*d**2*x + 3*x**n*a**2*c**2*
d*x + 4*x**n*a*b*c*d*x + x**n*b**2*d*x + a**2*c**3*x + 2*a*b*c**2*x + b**2
*c*x),x)*a*b*c**2*d*n + int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c
 + b))/(x**(3*n)*a**2*d**3*x + 3*x**(2*n)*a**2*c*d**2*x + 2*x**(2*n)*a*b*d
**2*x + 3*x**n*a**2*c**2*d*x + 4*x**n*a*b*c*d*x + x**n*b**2*d*x + a**2*c**
3*x + 2*a*b*c**2*x + b**2*c*x),x)*a*b*c*d**2*n + int((x**(2*n)*sqrt(x**n*d
 + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a**2*d**3*x + 3*x**(2*n)*a**2*c*
d**2*x + 2*x**(2*n)*a*b*d**2*x + 3*x**n*a**2*c**2*d*x + 4*x**n*a*b*c*d*x +
 x**n*b**2*d*x + a**2*c**3*x + 2*a*b*c**2*x + b**2*c*x),x)*b**2*d**2*n + i
nt((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a**2*d**3*x + 3*x
**(2*n)*a**2*c*d**2*x + 2*x**(2*n)*a*b*d**2*x + 3*x**n*a**2*c**2*d*x + 4*x
**n*a*b*c*d*x + x**n*b**2*d*x + a**2*c**3*x + 2*a*b*c**2*x + b**2*c*x),x)*
a*b*c**3*n + int((sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(x**(3*n)*a**
2*d**3*x + 3*x**(2*n)*a**2*c*d**2*x + 2*x**(2*n)*a*b*d**2*x + 3*x**n*a**2*
c**2*d*x + 4*x**n*a*b*c*d*x + x**n*b**2*d*x + a**2*c**3*x + 2*a*b*c**2*...
```

3.332 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=-\frac{c\sqrt{1+\frac{adx^n}{b+ac}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{(b+ac)x\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
-c*(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(-1/n,-3/2,3/2,-(1-n)/n,-d*x^n/c,-a*d
*x^n/(a*c+b))/(a*c+b)/x/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(108) = 216.

Time = 2.50 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\frac{\sqrt{\frac{b+ac+adx^n}{c+dx^n}}\left(-2b+\frac{2b^2}{b+ac+adx^n}+\frac{a\sqrt{\frac{b+ac+adx^n}{b+ac}}\sqrt{1+\frac{dx^n}{c}}\left(-c(-1+n)(-2b+acn)\,\text{AppellF1}\left(-\frac{1}{n},\right.\right.}{\ldots}\right.}{a^2(b+ac)n\ldots}$$

input
```
Integrate[1/(x^2*(a + b/(c + d*x^n))^(3/2)),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*(-2*b + (2*b^2)/(b + a*c + a*d*x^n)
 + (a*Sqrt[(b + a*c + a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*(-(c*(-1 + n
)*(-2*b + a*c*n)*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, -((d*x^n)/c), -((
a*d*x^n)/(b + a*c))]) + d*(b*(-2 + n) + a*c*n)*x^n*AppellF1[(-1 + n)/n, 1/
2, 1/2, 2 - n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]))/((-1 + n)*(b +
 a*(c + d*x^n))))))/(a^2*(b + a*c)*n*x)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^n}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x^2 \left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{(dx^n+c)^{3/2}}{x^2(adx^n+b+ac)^{3/2}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{c\sqrt{ac+adx^n+b}\int \frac{\left(\frac{dx^n}{c}+1\right)^{3/2}}{x^2(adx^n+b+ac)^{3/2}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{c\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{\left(\frac{dx^n}{c}+1\right)^{3/2}}{x^2\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}dx}{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{c\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{x(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[1/(x^2*(a + b/(c + d*x^n))^(3/2)),x]
```

output
```
-((c*Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[-n^(-1), -3/2, 3/2, -((1 - n)/
n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/((b + a*c)*x*Sqrt[(b + a*c + a*
d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^2 \left(a + \frac{b}{c + d x^n}\right)^{\frac{3}{2}}} dx$$

input
```
int(1/x^2/(a+b/(c+d*x^n))^(3/2),x)
```

output
```
int(1/x^2/(a+b/(c+d*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \left(a + \frac{b}{c + dx^n}\right)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b/(c+d*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(\frac{ac+adx^n+b}{c+dx^n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(c+d*x**n))**(3/2),x)
```

output
```
Integral(1/(x**2*((a*c + a*d*x**n + b)/(c + d*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(c+d*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x^n + c))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(c+d*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a + b/(d*x^n + c))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(a+\frac{b}{c+d\,x^n}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x^n))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x^n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(1/x^2/(a+b/(c+d*x^n))^(3/2),x)
```

output

```
( - 4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*c - 4*x**n*int((x**(2*n)*s
qrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**3*c*d**3*x**2 - x
**(3*n)*a**2*b*d**3*n*x**2 + 2*x**(3*n)*a**2*b*d**3*x**2 + 12*x**(2*n)*a**
3*c**2*d**2*x**2 - 3*x**(2*n)*a**2*b*c*d**2*n*x**2 + 14*x**(2*n)*a**2*b*c*
d**2*x**2 - 2*x**(2*n)*a*b**2*d**2*n*x**2 + 4*x**(2*n)*a*b**2*d**2*x**2 +
12*x**n*a**3*c**3*d*x**2 - 3*x**n*a**2*b*c**2*d*n*x**2 + 22*x**n*a**2*b*c*
*2*d*x**2 - 4*x**n*a*b**2*c*d*n*x**2 + 12*x**n*a*b**2*c*d*x**2 - x**n*b**3
*d*n*x**2 + 2*x**n*b**3*d*x**2 + 4*a**3*c**4*x**2 - a**2*b*c**3*n*x**2 + 1
0*a**2*b*c**3*x**2 - 2*a*b**2*c**2*n*x**2 + 8*a*b**2*c**2*x**2 - b**3*c*n*
x**2 + 2*b**3*c*x**2),x)*a**2*b*c*d**3*n*x + 8*x**n*int((x**(2*n)*sqrt(x**
n*d + c)*sqrt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**3*c*d**3*x**2 - x**(3*n)
*a**2*b*d**3*n*x**2 + 2*x**(3*n)*a**2*b*d**3*x**2 + 12*x**(2*n)*a**3*c**2*
d**2*x**2 - 3*x**(2*n)*a**2*b*c*d**2*n*x**2 + 14*x**(2*n)*a**2*b*c*d**2*x*
*2 - 2*x**(2*n)*a*b**2*d**2*n*x**2 + 4*x**(2*n)*a*b**2*d**2*x**2 + 12*x**n
*a**3*c**3*d*x**2 - 3*x**n*a**2*b*c**2*d*n*x**2 + 22*x**n*a**2*b*c**2*d*x*
*2 - 4*x**n*a*b**2*c*d*n*x**2 + 12*x**n*a*b**2*c*d*x**2 - x**n*b**3*d*n*x*
*2 + 2*x**n*b**3*d*x**2 + 4*a**3*c**4*x**2 - a**2*b*c**3*n*x**2 + 10*a**2*
b*c**3*x**2 - 2*a*b**2*c**2*n*x**2 + 8*a*b**2*c**2*x**2 - b**3*c*n*x**2 +
2*b**3*c*x**2),x)*a**2*b*c*d**3*x + x**n*int((x**(2*n)*sqrt(x**n*d + c)*sq
rt(x**n*a*d + a*c + b))/(4*x**(3*n)*a**3*c*d**3*x**2 - x**(3*n)*a**2*b*...
```

3.333 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=-\frac{c\sqrt{1+\frac{adx^n}{b+ac}}\operatorname{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2(b+ac)x^2\sqrt{1+\frac{dx^n}{c}}\sqrt{a+\frac{b}{c+dx^n}}}$$

output
```
-1/2*c*(1+a*d*x^n/(a*c+b))^(1/2)*AppellF1(-2/n,-3/2,3/2,-(2-n)/n,-d*x^n/c,
-a*d*x^n/(a*c+b))/(a*c+b)/x^2/(1+d*x^n/c)^(1/2)/(a+b/(c+d*x^n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(110) = 220.

Time = 2.54 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.08

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx=\frac{\sqrt{\frac{b+ac+adx^n}{c+dx^n}}\left(-4b(c+dx^n)+\frac{\sqrt{\frac{b+ac+adx^n}{b+ac}}\sqrt{1+\frac{dx^n}{c}}\left(-c(-2+n)(-4b+acn)\operatorname{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\right.\right.}{}\right.}{2a(b+ac)nx^2\left(b+a\left(\right.\right.}$$

input
```
Integrate[1/(x^3*(a + b/(c + d*x^n))^(3/2)),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^n)/(c + d*x^n)]*(-4*b*(c + d*x^n) + (Sqrt[(b + a*c
+ a*d*x^n)/(b + a*c)]*Sqrt[1 + (d*x^n)/c]*(-(c*(-2 + n)*(-4*b + a*c*n)*App
ellF1[-2/n, 1/2, 1/2, (-2 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]) +
 2*d*(b*(-4 + n) + a*c*n)*x^n*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, -((d
*x^n)/c), -((a*d*x^n)/(b + a*c))]))/(-2 + n)))/(2*a*(b + a*c)*n*x^2*(b + a
*(c + d*x^n)))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^3 \left(\frac{ac+adx^n+b}{c+dx^n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^n+b}\int \frac{(dx^n+c)^{3/2}}{x^3(adx^n+b+ac)^{3/2}}dx}{\sqrt{c+dx^n}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{ac+adx^n+b}\int \frac{\left(\frac{dx^n}{c}+1\right)^{3/2}}{x^3(adx^n+b+ac)^{3/2}}dx}{\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{c\sqrt{\frac{adx^n}{ac+b}+1}\int \frac{\left(\frac{dx^n}{c}+1\right)^{3/2}}{x^3\left(\frac{adx^n}{b+ac}+1\right)^{3/2}}dx}{(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

$$\Big\downarrow \ 1012$$

$$-\frac{c\sqrt{\frac{adx^n}{ac+b}+1}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2x^2(ac+b)\sqrt{\frac{dx^n}{c}+1}\sqrt{\frac{ac+adx^n+b}{c+dx^n}}}$$

input
```
Int[1/(x^3*(a + b/(c + d*x^n))^(3/2)),x]
```

output
```
-1/2*(c*Sqrt[1 + (a*d*x^n)/(b + a*c)]*AppellF1[-2/n, -3/2, 3/2, -((2 - n)/
n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/((b + a*c)*x^2*Sqrt[(b + a*c +
a*d*x^n)/(c + d*x^n)]*Sqrt[1 + (d*x^n)/c])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^3 \left(a + \frac{b}{c + d\,x^n}\right)^{\frac{3}{2}}} dx$$

input
```
int(1/x^3/(a+b/(c+d*x^n))^(3/2),x)
```

output
```
int(1/x^3/(a+b/(c+d*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(a + \frac{b}{c + dx^n}\right)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b/(c+d*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left(\frac{ac+adx^n+b}{c+dx^n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(c+d*x**n))**(3/2),x)
```

output
```
Integral(1/(x**3*((a*c + a*d*x**n + b)/(c + d*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(c+d*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x^n + c))^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^n+c}\right)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(c+d*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a + b/(d*x^n + c))^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\, dx = \int \frac{1}{x^3 \left(a+\frac{b}{c+d\,x^n}\right)^{3/2}} \,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x^n))^(3/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x^n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^n}\right)^{3/2}}\, dx = \text{too large to display}$$

input
```
int(1/x^3/(a+b/(c+d*x^n))^(3/2),x)
```

output

```
( - 4*sqrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b)*c - 8*x**n*int((x**(2*n)*s
qrt(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(8*x**(3*n)*a**3*c*d**3*x**3 - x
**(3*n)*a**2*b*d**3*n*x**3 + 4*x**(3*n)*a**2*b*d**3*x**3 + 24*x**(2*n)*a**
3*c**2*d**2*x**3 - 3*x**(2*n)*a**2*b*c*d**2*n*x**3 + 28*x**(2*n)*a**2*b*c*
d**2*x**3 - 2*x**(2*n)*a*b**2*d**2*n*x**3 + 8*x**(2*n)*a*b**2*d**2*x**3 +
24*x**n*a**3*c**3*d*x**3 - 3*x**n*a**2*b*c**2*d*n*x**3 + 44*x**n*a**2*b*c*
*2*d*x**3 - 4*x**n*a*b**2*c*d*n*x**3 + 24*x**n*a*b**2*c*d*x**3 - x**n*b**3
*d*n*x**3 + 4*x**n*b**3*d*x**3 + 8*a**3*c**4*x**3 - a**2*b*c**3*n*x**3 + 2
0*a**2*b*c**3*x**3 - 2*a*b**2*c**2*n*x**3 + 16*a*b**2*c**2*x**3 - b**3*c*n
*x**3 + 4*b**3*c*x**3),x)*a**2*b*c*d**3*n*x**2 + 32*x**n*int((x**(2*n)*sqr
t(x**n*d + c)*sqrt(x**n*a*d + a*c + b))/(8*x**(3*n)*a**3*c*d**3*x**3 - x**
(3*n)*a**2*b*d**3*n*x**3 + 4*x**(3*n)*a**2*b*d**3*x**3 + 24*x**(2*n)*a**3*
c**2*d**2*x**3 - 3*x**(2*n)*a**2*b*c*d**2*n*x**3 + 28*x**(2*n)*a**2*b*c*d*
*2*x**3 - 2*x**(2*n)*a*b**2*d**2*n*x**3 + 8*x**(2*n)*a*b**2*d**2*x**3 + 24
*x**n*a**3*c**3*d*x**3 - 3*x**n*a**2*b*c**2*d*n*x**3 + 44*x**n*a**2*b*c**2
*d*x**3 - 4*x**n*a*b**2*c*d*n*x**3 + 24*x**n*a*b**2*c*d*x**3 - x**n*b**3*d
*n*x**3 + 4*x**n*b**3*d*x**3 + 8*a**3*c**4*x**3 - a**2*b*c**3*n*x**3 + 20*
a**2*b*c**3*x**3 - 2*a*b**2*c**2*n*x**3 + 16*a*b**2*c**2*x**3 - b**3*c*n*x
**3 + 4*b**3*c*x**3),x)*a**2*b*c*d**3*x**2 + x**n*int((x**(2*n)*sqrt(x**n*
d + c)*sqrt(x**n*a*d + a*c + b))/(8*x**(3*n)*a**3*c*d**3*x**3 - x**(3*n...
```

3.334 $\int x\left(a+\frac{b}{c+dx^n}\right)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\frac{1}{2}x^2\left(1+\frac{dx^n}{c}\right)^p\left(1+\frac{adx^n}{b+ac}\right)^{-p}\left(a+\frac{b}{c+dx^n}\right)^p\text{AppellF1}\left(\frac{2}{n},p,-p,\frac{2+n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)$$

output
```
1/2*x^2*(1+d*x^n/c)^p*(a+b/(c+d*x^n))^p*AppellF1(2/n,p,-p,(2+n)/n,-d*x^n/c
,-a*d*x^n/(a*c+b))/((1+a*d*x^n/(a*c+b))^p)
```

Mathematica [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int x\left(a+\frac{b}{c+dx^n}\right)^p\,dx$$

input
```
Integrate[x*(a + b/(c + d*x^n))^p,x]
```

output
```
Integrate[x*(a + b/(c + d*x^n))^p, x]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a+\frac{b}{c+dx^n}\right)^p \, dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int x \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \, dx$$

$$\Big\downarrow \text{ 2058}$$

$$(c+dx^n)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int x(dx^n+c)^{-p} (adx^n+b+ac)^p \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int x(adx^n+b+ac)^p \left(\frac{dx^n}{c}+1\right)^{-p} dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \left(\frac{adx^n}{ac+b}+1\right)^{-p} \int x \left(\frac{dx^n}{c}+1\right)^{-p} \left(\frac{adx^n}{b+ac}+1\right)^p dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{1}{2}x^2 \left(\frac{dx^n}{c}+1\right)^p \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \left(\frac{adx^n}{ac+b}+1\right)^{-p} \text{AppellF1}\left(\frac{2}{n},p,-p,\frac{n+2}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)$$

input
```
Int[x*(a + b/(c + d*x^n))^p,x]
```

output
```
(x^2*((b + a*c + a*d*x^n)/(c + d*x^n))^p*(1 + (d*x^n)/c)^p*AppellF1[2/n, p
, -p, (2 + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(2*(1 + (a*d*x^n)/
(b + a*c))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int x\left(a+\frac{b}{c+d\,x^{n}}\right)^{p}dx$$

input
```
int(x*(a+b/(c+d*x^n))^p,x)
```

output
```
int(x*(a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int\left(a+\frac{b}{dx^n+c}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral(x*((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int\left(a+\frac{b}{dx^n+c}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^p*x, x)
```

Giac [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p dx=\int \left(a+\frac{b}{dx^n+c}\right)^p x\,dx$$

input
```
integrate(x*(a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p dx=\int x\left(a+\frac{b}{c+d\,x^n}\right)^p dx$$

input
```
int(x*(a + b/(c + d*x^n))^p,x)
```

output
```
int(x*(a + b/(c + d*x^n))^p, x)
```

Reduce [F]

$$\int x\left(a+\frac{b}{c+dx^n}\right)^p dx=\int \frac{(x^n ad+ac+b)^p\,x}{(x^n d+c)^p}dx$$

input
```
int(x*(a+b/(c+d*x^n))^p,x)
```

output
```
int((((x**n*a*d + a*c + b)**p*x)/(x**n*d + c)**p,x)
```

3.335 $\int \left(a+\frac{b}{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \left(a+\frac{b}{c+dx^n}\right)^p \, dx = x\left(1+\frac{dx^n}{c}\right)^p \left(1+\frac{adx^n}{b+ac}\right)^{-p} \left(a+\frac{b}{c+dx^n}\right)^p \operatorname{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)$$

output
```
x*(1+d*x^n/c)^p*(a+b/(c+d*x^n))^p*AppellF1(1/n,p,-p,1+1/n,-d*x^n/c,-a*d*x^
n/(a*c+b))/((1+a*d*x^n/(a*c+b))^p)
```

Mathematica [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^p \, dx = \int \left(a+\frac{b}{c+dx^n}\right)^p \, dx$$

input
```
Integrate[(a + b/(c + d*x^n))^p,x]
```

output
```
Integrate[(a + b/(c + d*x^n))^p, x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2057, 2058, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{2057}$$

$$\int \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$(c+dx^n)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int (dx^n+c)^{-p} (adx^n+b+ac)^p dx$$

$$\Big\downarrow \text{937}$$

$$\left(\frac{dx^n}{c}+1\right)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int (adx^n+b+ac)^p \left(\frac{dx^n}{c}+1\right)^{-p} dx$$

$$\Big\downarrow \text{937}$$

$$\left(\frac{dx^n}{c}+1\right)^p \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \left(\frac{adx^n}{ac+b}+1\right)^{-p} \int \left(\frac{dx^n}{c}+1\right)^{-p} \left(\frac{adx^n}{b+ac}+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(\frac{dx^n}{c}+1\right)^p \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \left(\frac{adx^n}{ac+b}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)$$

input
```
Int[(a + b/(c + d*x^n))^p,x]
```

output
```
(x*((b + a*c + a*d*x^n)/(c + d*x^n))^p*(1 + (d*x^n)/c)^p*AppellF1[n^(-1),
p, -p, 1 + n^(-1), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(1 + (a*d*x^n)/(
b + a*c))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \left(a+\frac{b}{c+d\,x^{n}}\right)^{p}dx$$

input
```
int((a+b/(c+d*x^n))^p,x)
```

output
```
int((a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^p dx = \int \left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral(((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^n}\right)^p dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^p dx = \int \left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^p, x)
```

Giac [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int \left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int \left(a+\frac{b}{c+d\,x^n}\right)^p\,dx$$

input
```
int((a + b/(c + d*x^n))^p,x)
```

output
```
int((a + b/(c + d*x^n))^p, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int \frac{(x^n ad+ac+b)^p}{(x^n d+c)^p}\,dx$$

input
```
int((a+b/(c+d*x^n))^p,x)
```

output
```
int((x**n*a*d + a*c + b)**p/(x**n*d + c)**p,x)
```

3.336 $\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x} \, dx = -\frac{c\left(a+\frac{b}{c+dx^n}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx^n}\right)}{b+ac}\right)}{(b+ac)n(1+p)} + \frac{\left(a+\frac{b}{c+dx^n}\right)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a(c+dx^n)}\right)}{an(1+p)}$$

output
```
-c*(a+b/(c+d*x^n))^(p+1)*hypergeom([1, p+1],[2+p],c*(a+b/(c+d*x^n))/(a*c+b
))/(a*c+b)/n/(p+1)+(a+b/(c+d*x^n))^(p+1)*hypergeom([1, p+1],[2+p],1+b/a/(c
+d*x^n))/a/n/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx$$

input
```
Integrate[(a + b/(c + d*x^n))^p/x,x]
```

output
```
Integrate[(a + b/(c + d*x^n))^p/x, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p}{x}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{\int x^{-n}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx^n}{n}$$

input
```
Int[(a + b/(c + d*x^n))^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{c+d\,x^{n}}\right)^{p}}{x}dx$$

input
```
int((a+b/(c+d*x^n))^p/x,x)
```

output
```
int((a+b/(c+d*x^n))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x,x, algorithm="fricas")
```

output
```
integral(((a*d*x^n + a*c + b)/(d*x^n + c))^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx=\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d*x**n))**p/x,x)
```

output
```
Integral(((a*c + a*d*x**n + b)/(c + d*x**n))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx = \int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^p}{x}\,dx$$

input

```
int((a + b/(c + d*x^n))^p/x,x)
```

output

```
int((a + b/(c + d*x^n))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x}\,dx = \int \frac{(x^n ad+ac+b)^p}{(x^n d+c)^p\,x}dx$$

input

```
int((a+b/(c+d*x^n))^p/x,x)
```

output

```
int((x**n*a*d + a*c + b)**p/((x**n*d + c)**p*x),x)
```

3.337 $\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2} dx = -\frac{\left(1+\frac{dx^n}{c}\right)^p\left(1+\frac{adx^n}{b+ac}\right)^{-p}\left(a+\frac{b}{c+dx^n}\right)^p \operatorname{AppellF1}\left(-\frac{1}{n},p,-p,-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{x}$$

output
```
-(1+d*x^n/c)^p*(a+b/(c+d*x^n))^p*AppellF1(-1/n,p,-p,-(1-n)/n,-d*x^n/c,-a*d
*x^n/(a*c+b))/x/((1+a*d*x^n/(a*c+b))^p)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2} dx = \int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2} dx$$

input
```
Integrate[(a + b/(c + d*x^n))^p/x^2,x]
```

output
```
Integrate[(a + b/(c + d*x^n))^p/x^2, x]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{2058}$$

$$(c+dx^n)^p(ac+adx^n+b)^{-p}\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\int \frac{(dx^n+c)^{-p}(adx^n+b+ac)^p}{x^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p(ac+adx^n+b)^{-p}\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\int \frac{(adx^n+b+ac)^p\left(\frac{dx^n}{c}+1\right)^{-p}}{x^2}dx$$

$$\Big\downarrow \text{1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\left(\frac{adx^n}{ac+b}+1\right)^{-p}\int \frac{\left(\frac{dx^n}{c}+1\right)^{-p}\left(\frac{adx^n}{b+ac}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{dx^n}{c}+1\right)^p\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\left(\frac{adx^n}{ac+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{n},p,-p,-\frac{1-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{x}$$

input
```
Int[(a + b/(c + d*x^n))^p/x^2,x]
```

output
```
-((((b + a*c + a*d*x^n)/(c + d*x^n))^p*(1 + (d*x^n)/c)^p*AppellF1[-n^(-1),
 p, -p, -((1 - n)/n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(x*(1 + (a*d*
x^n)/(b + a*c))^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{c+d\,x^{n}}\right)^{p}}{x^{2}}dx$$

input
```
int((a+b/(c+d*x^n))^p/x^2,x)
```

output
```
int((a+b/(c+d*x^n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x^2,x, algorithm="fricas")
```

output
```
integral(((a*d*x^n + a*c + b)/(d*x^n + c))^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^p}{x^2}\,dx$$

input
```
int((a + b/(c + d*x^n))^p/x^2,x)
```

output
```
int((a + b/(c + d*x^n))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^2}\,dx=\int \frac{(x^n a d+ac+b)^p}{(x^n d+c)^p\,x^2}dx$$

input
```
int((a+b/(c+d*x^n))^p/x^2,x)
```

output
```
int((x**n*a*d + a*c + b)**p/((x**n*d + c)**p*x**2),x)
```

3.338 $\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3} dx = -\frac{\left(1+\frac{dx^n}{c}\right)^p \left(1+\frac{adx^n}{b+ac}\right)^{-p} \left(a+\frac{b}{c+dx^n}\right)^p \text{AppellF1}\left(-\frac{2}{n},p,-p,-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2x^2}$$

output
```
-1/2*(1+d*x^n/c)^p*(a+b/(c+d*x^n))^p*AppellF1(-2/n,p,-p,-(2-n)/n,-d*x^n/c,
-a*d*x^n/(a*c+b))/x^2/((1+a*d*x^n/(a*c+b))^p)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3} dx = \int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3} dx$$

input
```
Integrate[(a + b/(c + d*x^n))^p/x^3,x]
```

output
```
Integrate[(a + b/(c + d*x^n))^p/x^3, x]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p}{x^3}dx$$

$$\Big\downarrow \text{2058}$$

$$(c+dx^n)^p(ac+adx^n+b)^{-p}\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\int \frac{(dx^n+c)^{-p}(adx^n+b+ac)^p}{x^3}dx$$

$$\Big\downarrow \text{1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p(ac+adx^n+b)^{-p}\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\int \frac{(adx^n+b+ac)^p\left(\frac{dx^n}{c}+1\right)^{-p}}{x^3}dx$$

$$\Big\downarrow \text{1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\left(\frac{adx^n}{ac+b}+1\right)^{-p}\int \frac{\left(\frac{dx^n}{c}+1\right)^{-p}\left(\frac{adx^n}{b+ac}+1\right)^p}{x^3}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{dx^n}{c}+1\right)^p\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\left(\frac{adx^n}{ac+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{2}{n},p,-p,-\frac{2-n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{2x^2}$$

input
```
Int[(a + b/(c + d*x^n))^p/x^3,x]
```

output
```
-1/2*(((b + a*c + a*d*x^n)/(c + d*x^n))^p*(1 + (d*x^n)/c)^p*AppellF1[-2/n,
 p, -p, -((2 - n)/n), -((d*x^n)/c), -((a*d*x^n)/(b + a*c))])/(x^2*(1 + (a*
d*x^n)/(b + a*c))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^p}{x^3}dx$$

input
```
int((a+b/(c+d*x^n))^p/x^3,x)
```

output
```
int((a+b/(c+d*x^n))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x^3,x, algorithm="fricas")
```

output
```
integral(((a*d*x^n + a*c + b)/(d*x^n + c))^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n))**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3}\,dx = \int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x^3}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3}\,dx = \int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^p}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^n))^p/x^3,x)
```

output
```
int((a + b/(c + d*x^n))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{x^3}\,dx = \int \frac{(x^n ad+ac+b)^p}{(x^n d+c)^p\,x^3}dx$$

input
```
int((a+b/(c+d*x^n))^p/x^3,x)
```

output
```
int((x**n*a*d + a*c + b)**p/((x**n*d + c)**p*x**3),x)
```

3.339 $\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p \, dx = \frac{(ex)^{1+m}\left(1+\frac{dx^n}{c}\right)^p\left(1+\frac{adx^n}{b+ac}\right)^{-p}\left(a+\frac{b}{c+dx^n}\right)^p \text{AppellF1}\left(\frac{1+m}{n},p,-p,\frac{1+m+n}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(1+d*x^n/c)^p*(a+b/(c+d*x^n))^p*AppellF1((1+m)/n,p,-p,(1+m+n)/
n,-d*x^n/c,-a*d*x^n/(a*c+b))/e/(1+m)/((1+a*d*x^n/(a*c+b))^p)
```

Mathematica [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p \, dx = \int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b/(c + d*x^n))^p,x]
```

output
```
Integrate[(e*x)^m*(a + b/(c + d*x^n))^p, x]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2057, 2058, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p \, dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int (ex)^m \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \, dx$$

$$\Big\downarrow \text{ 2058}$$

$$(c+dx^n)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int (ex)^m (dx^n+c)^{-p} (adx^n+b+ac)^p \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int (ex)^m (adx^n+b+ac)^p \left(\frac{dx^n}{c}+1\right)^{-p} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(\frac{dx^n}{c}+1\right)^p \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \left(\frac{adx^n}{ac+b}+1\right)^{-p} \int (ex)^m \left(\frac{dx^n}{c}+1\right)^{-p} \left(\frac{adx^n}{b+ac}+1\right)^p \, dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(ex)^{m+1} \left(\frac{dx^n}{c}+1\right)^p \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \left(\frac{adx^n}{ac+b}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{n},p,-p,\frac{m+n+1}{n},-\frac{dx^n}{c},-\frac{adx^n}{b+ac}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b/(c + d*x^n))^p,x]
```

output
```
((e*x)^(1 + m)*((b + a*c + a*d*x^n)/(c + d*x^n))^p*(1 + (d*x^n)/c)^p*Appel
lF1[(1 + m)/n, p, -p, (1 + m + n)/n, -((d*x^n)/c), -((a*d*x^n)/(b + a*c))]
)/(e*(1 + m)*(1 + (a*d*x^n)/(b + a*c))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [F]

$$\int (ex)^m \left(a + \frac{b}{c + d\,x^n}\right)^p dx$$

input
```
int((e*x)^m*(a+b/(c+d*x^n))^p,x)
```

output
```
int((e*x)^m*(a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(a + b/(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p dx = \int (ex)^m \left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((e*x)^m*(a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(a + b/(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p dx = \int (e\,x)^m \left(a+\frac{b}{c+d\,x^n}\right)^p dx$$

input
```
int((e*x)^m*(a + b/(c + d*x^n))^p,x)
```

output
```
int((e*x)^m*(a + b/(c + d*x^n))^p, x)
```

Reduce [F]

$$\int (ex)^m \left(a+\frac{b}{c+dx^n}\right)^p dx = e^m \left(\int \frac{x^m (x^n ad+ac+b)^p}{(x^n d+c)^p} dx\right)$$

input
```
int((e*x)^m*(a+b/(c+d*x^n))^p,x)
```

output
```
e**m*int((x**m*(x**n*a*d + a*c + b)**p)/(x**n*d + c)**p,x)
```

3.340 $\int (ex)^{-1+3n}\left(a+b(c+dx^n)^2\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 311

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^2\right)^p\,dx$$
$$= -\frac{c(2+p)x^{-3n}(ex)^{3n}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{1+p}}{bd^3en(1+p)(3+2p)}$$
$$+\frac{x^{-2n}(ex)^{3n}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{1+p}}{bd^2en(3+2p)}$$
$$+\frac{2^p\left(a-bc^2(3+2p)\right)x^{-3n}(ex)^{3n}\left(\frac{\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n}{\sqrt{-a}}\right)^{-1-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{1+p}\text{Hypergeometric}}{\sqrt{-a}b^{3/2}d^3en(1+p)(3+2p)}$$

output

```
-c*(2+p)*(e*x)^(3*n)*(a+b*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^(p+1)/b/d^3/e/n/(
p+1)/(3+2*p)/(x^(3*n))+(e*x)^(3*n)*(a+b*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^(p+
1)/b/d^2/e/n/(3+2*p)/(x^(2*n))+2^p*(a-b*c^2*(3+2*p))*(e*x)^(3*n)*(((-a)^(1
/2)-b^(1/2)*c-b^(1/2)*d*x^n)/(-a)^(1/2))^(-1-p)*(a+b*c^2+2*b*c*d*x^n+b*d^2
*x^(2*n))^(p+1)*hypergeom([-p, p+1],[2+p],1/2*((-a)^(1/2)+b^(1/2)*c+b^(1/2
)*d*x^n)/(-a)^(1/2))/(-a)^(1/2)/b^(3/2)/d^3/e/n/(p+1)/(3+2*p)/(x^(3*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.60

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^2\right)^p\,dx$$
$$=\frac{(ex)^{3n}\left(\frac{\sqrt{-abd^2}-bd(c+dx^n)}{-bcd+\sqrt{-abd^2}}\right)^{-p}\left(\frac{\sqrt{-abd^2}+bd(c+dx^n)}{bcd+\sqrt{-abd^2}}\right)^{-p}\left(a+b(c+dx^n)^2\right)^p\operatorname{AppellF1}\left(3,-p,-p,4,-\frac{bd^2x^n}{bcd+\sqrt{-abd^2}}\right.}{3en}$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
((e*x)^(3*n)*(a + b*(c + d*x^n)^2)^p*AppellF1[3, -p, -p, 4, -((b*d^2*x^n)/
(b*c*d + Sqrt[-(a*b*d^2)])), (b*d^2*x^n)/(-(b*c*d) + Sqrt[-(a*b*d^2)])])/(
3*e*n*((Sqrt[-(a*b*d^2)] - b*d*(c + d*x^n))/(-(b*c*d) + Sqrt[-(a*b*d^2)]))
^p*((Sqrt[-(a*b*d^2)] + b*d*(c + d*x^n))/(b*c*d + Sqrt[-(a*b*d^2)]))^p)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2086, 1694, 1693, 1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}\left(a+b(c+dx^n)^2\right)^p\,dx$$

$$\Big\downarrow \text{2086}$$

$$\int (ex)^{3n-1}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1694}$$

$$\frac{x^{-3n}(ex)^{3n}\int x^{3n-1}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx}{e}$$

$$\Big\downarrow \text{1693}$$

$$\frac{x^{-3n}(ex)^{3n}\int x^{2n}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{en}$$

$$\Big\downarrow \; 1166$$

$$\frac{x^{-3n}(ex)^{3n}\left(\frac{\int -\left(\left(2bcd(p+2)x^n+bc^2+a\right)\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p\right)dx^n}{bd^2(2p+3)}+\frac{x^n\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}}{bd^2(2p+3)}\right)}{en}$$

$$\Big\downarrow \; 25$$

$$\frac{x^{-3n}(ex)^{3n}\left(\frac{x^n\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}}{bd^2(2p+3)}-\frac{\int\left(2bcd(p+2)x^n+bc^2+a\right)\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{bd^2(2p+3)}\right)}{en}$$

$$\Big\downarrow \; 1160$$

$$\frac{x^{-3n}(ex)^{3n}\left(\frac{x^n\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}}{bd^2(2p+3)}-\frac{\left(a-bc^2(2p+3)\right)\int\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n+\frac{c(p+2)\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}}{d(p+1)}}{bd^2(2p+3)}\right)}{en}$$

$$\Big\downarrow \; 1096$$

$$\frac{x^{-3n}(ex)^{3n}\left(\frac{x^n\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}}{bd^2(2p+3)}-\frac{\frac{c(p+2)\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}}{d(p+1)}-\frac{2^p\left(a-bc^2(2p+3)\right)\left(\frac{\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n}{\sqrt{-a}}\right)^{-p-1}\left(a+bc^2+2\right.}{}}{bd^2(2p+3)}\right.}{en}$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
((e*x)^(3*n)*((x^n*(a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^(1 + p))/(b*d
^2*(3 + 2*p)) - ((c*(2 + p)*(a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^(1 +
 p))/(d*(1 + p)) - (2^p*(a - b*c^2*(3 + 2*p))*((Sqrt[-a] - Sqrt[b]*c - Sqr
t[b]*d*x^n)/Sqrt[-a])^(-1 - p)*(a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^(
1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (Sqrt[-a] + Sqrt[b]*c + Sqrt[b]
*d*x^n)/(2*Sqrt[-a])])/(Sqrt[-a]*Sqrt[b]*d*(1 + p)))/(b*d^2*(3 + 2*p))))/(
e*n*x^(3*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 1694
```
Int[((d_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x
_Symbol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[
n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Maple [F]

$$\int (ex)^{-1+3n} \left(a+b(c+d\,x^n)^2\right)^p dx$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+3n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2b+a\right)^p (ex)^{3\,n-1} \, dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^(2*n) + 2*b*c*d*x^n + b*c^2 + a)^p*(e*x)^(3*n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} \left(a+b(c+dx^n)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+3*n)*(a+b*(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^2\right)^p\,dx=\int \left((dx^n+c)^2b+a\right)^p(ex)^{3\,n-1}\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(3*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^2\right)^p\,dx=\int \left((dx^n+c)^2b+a\right)^p(ex)^{3\,n-1}\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^2\right)^p\,dx=\int (e\,x)^{3\,n-1}\,\left(a+b\,\left(c+d\,x^n\right)^2\right)^p\,dx$$

input
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n)^2)^p,x)
```

output
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n)^2)^p, x)
```

Reduce [F]

$$\int (ex)^{-1+3n} \left(a+b(c+dx^n)^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
(e**(3*n)*(2*x**(3*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*b**
2*c*d**3*p**2 + 3*x**(3*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**
p*b**2*c*d**3*p + x**(3*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**
p*b**2*c*d**3 + 2*x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**
p*b**2*c**2*d**2*p**2 + x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c
**2)**p*b**2*c**2*d**2*p + 2*x**n*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*
c**2)**p*a*b*c*d*p**2 + 2*x**n*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**
2)**p*a*b*c*d*p - 2*x**n*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*
b**2*c**3*d*p - (x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*a**2*p -
(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*a**2 - (x**(2*n)*b*d**2 +
 2*x**n*b*c*d + a + b*c**2)**p*a*b*c**2*p + (x**(2*n)*b*d**2 + 2*x**n*b*c*
d + a + b*c**2)**p*b**2*c**4 + 8*int((x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b
*c*d + a + b*c**2)**p)/(4*x**(2*n)*b*d**2*p**2*x + 8*x**(2*n)*b*d**2*p*x +
 3*x**(2*n)*b*d**2*x + 8*x**n*b*c*d*p**2*x + 16*x**n*b*c*d*p*x + 6*x**n*b*
c*d*x + 4*a*p**2*x + 8*a*p*x + 3*a*x + 4*b*c**2*p**2*x + 8*b*c**2*p*x + 3*
b*c**2*x),x)*a**2*b*d**2*n*p**4 + 24*int((x**(2*n)*(x**(2*n)*b*d**2 + 2*x*
*n*b*c*d + a + b*c**2)**p)/(4*x**(2*n)*b*d**2*p**2*x + 8*x**(2*n)*b*d**2*p
*x + 3*x**(2*n)*b*d**2*x + 8*x**n*b*c*d*p**2*x + 16*x**n*b*c*d*p*x + 6*x**
n*b*c*d*x + 4*a*p**2*x + 8*a*p*x + 3*a*x + 4*b*c**2*p**2*x + 8*b*c**2*p*x
+ 3*b*c**2*x),x)*a**2*b*d**2*n*p**3 + 22*int((x**(2*n)*(x**(2*n)*b*d**2...
```

3.341 $\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \frac{x^{-2n}(ex)^{2n}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{1+p}}{2bd^2en(1+p)} + \frac{2^p c x^{-2n}(ex)^{2n}\left(\frac{\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n}{\sqrt{-a}}\right)^{-1-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,\right.}{\sqrt{-a}\sqrt{b}d^2en(1+p)}$$

output
```
1/2*(e*x)^(2*n)*(a+b*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^(p+1)/b/d^2/e/n/(p+1)/
(x^(2*n))+2^p*c*(e*x)^(2*n)*(((-a)^(1/2)-b^(1/2)*c-b^(1/2)*d*x^n)/(-a)^(1/
2))^(-1-p)*(a+b*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^(p+1)*hypergeom([-p, p+1],[
2+p],1/2*((-a)^(1/2)+b^(1/2)*c+b^(1/2)*d*x^n)/(-a)^(1/2))/(-a)^(1/2)/b^(1/
2)/d^2/e/n/(p+1)/(x^(2*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.84

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx$$
$$= \frac{(ex)^{2n}\left(\frac{\sqrt{-abd^2}-bd(c+dx^n)}{-bcd+\sqrt{-abd^2}}\right)^{-p}\left(\frac{\sqrt{-abd^2}+bd(c+dx^n)}{bcd+\sqrt{-abd^2}}\right)^{-p}\left(a+b(c+dx^n)^2\right)^p \text{AppellF1}\left(2,-p,-p,3,-\frac{bd^2x^n}{bcd+\sqrt{-abd}}\right.}{2en}$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
((e*x)^(2*n)*(a + b*(c + d*x^n)^2)^p*AppellF1[2, -p, -p, 3, -((b*d^2*x^n)/
(b*c*d + Sqrt[-(a*b*d^2)])), (b*d^2*x^n)/(-(b*c*d) + Sqrt[-(a*b*d^2)])])/(
2*e*n*((Sqrt[-(a*b*d^2)] - b*d*(c + d*x^n))/(-(b*c*d) + Sqrt[-(a*b*d^2)]))
^p*((Sqrt[-(a*b*d^2)] + b*d*(c + d*x^n))/(b*c*d + Sqrt[-(a*b*d^2)]))^p)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2086, 1694, 1693, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} \left(a+b(c+dx^n)^2\right)^p \, dx$$
$$\Big\downarrow \ 2086$$
$$\int (ex)^{2n-1} \left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p dx$$
$$\Big\downarrow \ 1694$$
$$\frac{x^{-2n}(ex)^{2n}\int x^{2n-1}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx}{e}$$
$$\Big\downarrow \ 1693$$

$$\frac{x^{-2n}(ex)^{2n}\int x^n\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{en}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x^{-2n}(ex)^{2n}\left(\frac{\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}}{2bd^2(p+1)}-\frac{c\int\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{d}\right)}{en}$$

$$\Big\downarrow \text{1096}$$

$$\frac{x^{-2n}(ex)^{2n}\left(\frac{c2^p\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1}\left(\frac{\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n}{\sqrt{-a}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{\sqrt{b}dx^n+\sqrt{b}c+\sqrt{-a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\sqrt{b}d^2(p+1)}+\frac{(a+bc^2\cdots}{}\right.}{en}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
((e*x)^(2*n)*((a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^(1 + p)/(2*b*d^2*(
1 + p)) + (2^p*c*((Sqrt[-a] - Sqrt[b]*c - Sqrt[b]*d*x^n)/Sqrt[-a])^(-1 - p
)*(a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^(1 + p)*Hypergeometric2F1[-p,
1 + p, 2 + p, (Sqrt[-a] + Sqrt[b]*c + Sqrt[b]*d*x^n)/(2*Sqrt[-a])])/(Sqrt[
-a]*Sqrt[b]*d^2*(1 + p))))/(e*n*x^(2*n))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 1694
```
Int[((d_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x
_Symbol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[
n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Maple [F]

$$\int (ex)^{-1+2n} \left(a+b(c+d\,x^{n})^{2}\right)^{p} dx$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2 b+a\right)^p (ex)^{2\,n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^(2*n) + 2*b*c*d*x^n + b*c^2 + a)^p*(e*x)^(2*n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2b+a\right)^p (ex)^{2n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(2*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2b+a\right)^p (ex)^{2n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int (e\,x)^{2\,n-1} \left(a+b\,(c+d\,x^n)^2\right)^p dx$$

input
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n)^2)^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n)^2)^p, x)
```

Reduce [F]

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^2\right)^p \, dx$$
$$= \frac{e^{2n}\Big(2x^{2n}(x^{2n}b\,d^2+2x^nbcd+a+b\,c^2)^p\,b\,d^2p+x^{2n}(x^{2n}b\,d^2+2x^nbcd+a+b\,c^2)^p\,b\,d^2+2x^n(x^{2n}b\,d^2+2x}{}$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
(e**(2*n)*(2*x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*b*d
**2*p + x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*b*d**2 +
 2*x**n*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*b*c*d*p - (x**(2*
n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*a - (x**(2*n)*b*d**2 + 2*x**n*b*
c*d + a + b*c**2)**p*b*c**2 + 8*int((x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*
c*d + a + b*c**2)**p)/(2*x**(2*n)*b*d**2*p*x + x**(2*n)*b*d**2*x + 4*x**n*
b*c*d*p*x + 2*x**n*b*c*d*x + 2*a*p*x + a*x + 2*b*c**2*p*x + b*c**2*x),x)*a
*b*d**2*n*p**3 + 12*int((x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*
c**2)**p)/(2*x**(2*n)*b*d**2*p*x + x**(2*n)*b*d**2*x + 4*x**n*b*c*d*p*x +
2*x**n*b*c*d*x + 2*a*p*x + a*x + 2*b*c**2*p*x + b*c**2*x),x)*a*b*d**2*n*p*
*2 + 4*int((x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p)/(2*
x**(2*n)*b*d**2*p*x + x**(2*n)*b*d**2*x + 4*x**n*b*c*d*p*x + 2*x**n*b*c*d*
x + 2*a*p*x + a*x + 2*b*c**2*p*x + b*c**2*x),x)*a*b*d**2*n*p))/(2*b*d**2*e
*n*(2*p**2 + 3*p + 1))
```

3.342 $\int (ex)^{-1+n}\left(a+b(c+dx^n)^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^2\right)^p\,dx = -\frac{2^p x^{-n}(ex)^n\left(\frac{\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n}{\sqrt{-a}}\right)^{-1-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+\right.}{\sqrt{-a}\sqrt{b}den(1+p)}$$

output
```
-2^p*(e*x)^n*(((-a)^(1/2)-b^(1/2)*c-b^(1/2)*d*x^n)/(-a)^(1/2))^(-1-p)*(a+b
*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^(p+1)*hypergeom([-p, p+1],[2+p],1/2*((-a)^
(1/2)+b^(1/2)*c+b^(1/2)*d*x^n)/(-a)^(1/2))/(-a)^(1/2)/b^(1/2)/d/e/n/(p+1)/
(x^n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^2\right)^p\,dx = \frac{x^{1-n}(ex)^{-1+n}(c+dx^n)\left(a+b(c+dx^n)^2\right)^p\left(1+\frac{b(c+dx^n)^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b(c+dx^n)^2}{a}\right)}{dn}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
(x^(1 - n)*(e*x)^(-1 + n)*(c + d*x^n)*(a + b*(c + d*x^n)^2)^p*Hypergeometr
ic2F1[1/2, -p, 3/2, -((b*(c + d*x^n)^2)/a)])/(d*n*(1 + (b*(c + d*x^n)^2)/a
)^p)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2086, 1694, 1690, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} \left(a+b(c+dx^n)^2\right)^p \, dx$$

$$\Big\downarrow \text{ 2086}$$

$$\int (ex)^{n-1} \left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p dx$$

$$\Big\downarrow \text{ 1694}$$

$$\frac{x^{-n}(ex)^n \int x^{n-1}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx}{e}$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{x^{-n}(ex)^n \int \left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{en}$$

$$\Big\downarrow \text{ 1096}$$

$$-\frac{2^p x^{-n}(ex)^n \left(\frac{\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n}{\sqrt{-a}}\right)^{-p-1} \left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^{p+1} \text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{\sqrt{b}c}{\cdots}\right.}{\sqrt{-a}\sqrt{b}den(p+1)}$$

input
```
Int[(e*x)^(-1 + n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
-((2^p*(e*x)^n*((Sqrt[-a] - Sqrt[b]*c - Sqrt[b]*d*x^n)/Sqrt[-a])^(-1 - p)*
(a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^(1 + p)*Hypergeometric2F1[-p, 1
+ p, 2 + p, (Sqrt[-a] + Sqrt[b]*c + Sqrt[b]*d*x^n)/(2*Sqrt[-a])])/(Sqrt[-a
]*Sqrt[b]*d*e*n*(1 + p)*x^n))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 1694
```
Int[((d_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x
_Symbol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[
n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Maple [F]

$$\int (ex)^{-1+n} \left(a + b(c + d\,x^n)^2\right)^p dx$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2 b+a\right)^p (ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^(2*n) + 2*b*c*d*x^n + b*c^2 + a)^p*(e*x)^(n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+n} \left(a+b(c+dx^n)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+n)*(a+b*(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2 b+a\right)^p (ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^2\right)^p\,dx=\int \left((dx^n+c)^2b+a\right)^p(ex)^{n-1}\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^2\right)^p\,dx=\int (e\,x)^{n-1}\left(a+b\,(c+d\,x^n)^2\right)^p\,dx$$

input
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n)^2)^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n)^2)^p, x)
```

Reduce [F]

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^2\right)^p\,dx$$
$$=\frac{e^n\left(x^n(x^{2n}b\,d^2+2x^nbcd+a+b\,c^2)^p\,bcd+(x^{2n}b\,d^2+2x^nbcd+a+b\,c^2)^p\,a+(x^{2n}b\,d^2+2x^nbcd+a+b\,c\right.}{}$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^2)^p,x)
```

output

```
(e**n*(x**n*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*b*c*d + (x**(
2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*a + (x**(2*n)*b*d**2 + 2*x**n*
b*c*d + a + b*c**2)**p*b*c**2 - 4*int((x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*
b*c*d + a + b*c**2)**p)/(2*x**(2*n)*b*d**2*p*x + x**(2*n)*b*d**2*x + 4*x**
n*b*c*d*p*x + 2*x**n*b*c*d*x + 2*a*p*x + a*x + 2*b*c**2*p*x + b*c**2*x),x)
*a*b*d**2*n*p**2 - 2*int((x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b
*c**2)**p)/(2*x**(2*n)*b*d**2*p*x + x**(2*n)*b*d**2*x + 4*x**n*b*c*d*p*x +
 2*x**n*b*c*d*x + 2*a*p*x + a*x + 2*b*c**2*p*x + b*c**2*x),x)*a*b*d**2*n*p
))/(b*c*d*e*n*(2*p + 1))
```

3.343 $\int \frac{\left(a+b(c+dx^n)^2\right)^p}{ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{\left(a+b(c+dx^n)^2\right)^p}{ex}\,dx = \frac{\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p \text{AppellF1}\left(-2p,-p\right.}{2enp}$$

output
```
1/2*(a+b*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^p*AppellF1(-2*p,-p,-p,1-2*p,((-a)^
(1/2)/b^(1/2)-c)/d/(x^n),-((-a)^(1/2)/b^(1/2)+c)/d/(x^n))/e/n/p/((-((-a)^(
1/2)-b^(1/2)*c-b^(1/2)*d*x^n)/b^(1/2)/d/(x^n))^p)/((((-a)^(1/2)+b^(1/2)*c+
b^(1/2)*d*x^n)/b^(1/2)/d/(x^n))^p)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b(c+dx^n)^2\right)^p}{ex}\,dx = \frac{\left(\frac{x^{-n}\left(-\sqrt{-abd^2}+bd(c+dx^n)\right)}{bd^2}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-abd^2}+bd(c+dx^n)\right)}{bd^2}\right)^{-p}\left(a+b(c+dx^n)^2\right)^p \text{AppellF1}\left(-2p,-p,-p,1-\right.}{2enp}$$

input
```
Integrate[(a + b*(c + d*x^n)^2)^p/(e*x),x]
```

output
```
((a + b*(c + d*x^n)^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -((b*c*d + Sqrt[-
(a*b*d^2)])/(b*d^2*x^n)), (-(b*c*d) + Sqrt[-(a*b*d^2)])/(b*d^2*x^n)])/(2*e
*n*p*((-Sqrt[-(a*b*d^2)] + b*d*(c + d*x^n))/(b*d^2*x^n))^p*((Sqrt[-(a*b*d^
2)] + b*d*(c + d*x^n))/(b*d^2*x^n))^p)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {27, 2086, 1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b(c+dx^n)^2\right)^p}{ex}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(b(dx^n+c)^2+a\right)^p}{x}dx}{e}$$

$$\Big\downarrow \ 2086$$

$$\frac{\int \frac{(2bcdx^n+bd^2x^{2n}+bc^2+a)^p}{x}dx}{e}$$

$$\Big\downarrow \ 1693$$

$$\frac{\int x^{-n}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{en}$$

$$\Big\downarrow \ 1178$$

$$-\frac{(x^{-n})^{2p}\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p\int (x^{-n})^{-2p-1}}{en}$$

$$\Big\downarrow \ 150$$

$$\frac{\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}(a+bc^2+2bcdx^n+bd^2x^{2n})^p\,\text{AppellF1}\left(-2p,-p,-p\right.}{2enp}$$

input
```
Int[(a + b*(c + d*x^n)^2)^p/(e*x),x]
```

output
```
((a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^p*AppellF1[-2*p, -p, -p, 1 - 2*
p, (Sqrt[-a]/Sqrt[b] - c)/(d*x^n), -((Sqrt[-a]/Sqrt[b] + c)/(d*x^n))])/(2*
e*n*p*(-((Sqrt[-a] - Sqrt[b]*c - Sqrt[b]*d*x^n)/(Sqrt[b]*d*x^n)))^p*((Sqrt
[-a] + Sqrt[b]*c + Sqrt[b]*d*x^n)/(Sqrt[b]*d*x^n))^p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Maple [F]

$$\int \frac{(a+b(c+d\,x^n)^2)^p}{ex}dx$$

input
```
int((a+b*(c+d*x^n)^2)^p/e/x,x)
```

output
```
int((a+b*(c+d*x^n)^2)^p/e/x,x)
```

Fricas [F]

$$\int \frac{(a+b(c+dx^n)^2)^p}{ex}\,dx=\int \frac{\left((dx^n+c)^2b+a\right)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^2)^p/e/x,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^(2*n) + 2*b*c*d*x^n + b*c^2 + a)^p/(e*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b(c+dx^n)^2)^p}{ex}\,dx=\text{Timed out}$$

input
```
integrate((a+b*(c+d*x**n)**2)**p/e/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b(c+dx^n)^2)^p}{ex}\,dx = \int \frac{((dx^n+c)^2b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^2)^p/e/x,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p/x, x)/e
```

Giac [F]

$$\int \frac{(a+b(c+dx^n)^2)^p}{ex}\,dx = \int \frac{((dx^n+c)^2b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^2)^p/e/x,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p/(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(c+dx^n)^2)^p}{ex}\,dx = \int \frac{(a+b\,(c+d\,x^n)^2)^p}{e\,x}\,dx$$

input
```
int((a + b*(c + d*x^n)^2)^p/(e*x),x)
```

output
```
int((a + b*(c + d*x^n)^2)^p/(e*x), x)
```

Reduce [F]

$$\int \frac{(a+b(c+dx^n)^2)^p}{ex}\,dx$$
$$= \frac{(x^{2n}b\,d^2+2x^nbcd+a+b\,c^2)^p+\left(\int \frac{(x^{2n}b\,d^2+2x^nbcd+a+b\,c^2)^p}{x^{2n}b\,d^2x+2x^nbcdx+ax+b\,c^2x}dx\right)anp+\left(\int \frac{(x^{2n}b\,d^2+2x^nbcd+a+b\,c^2)^p}{x^{2n}b\,d^2x+2x^nbcdx+ax+b\,c^2x}dx\right)b\,c^2}{enp}$$

input
```
int((a+b*(c+d*x^n)^2)^p/e/x,x)
```

output
```
((x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p + int((x**(2*n)*b*d**2 +
 2*x**n*b*c*d + a + b*c**2)**p/(x**(2*n)*b*d**2*x + 2*x**n*b*c*d*x + a*x +
 b*c**2*x),x)*a*n*p + int((x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p
/(x**(2*n)*b*d**2*x + 2*x**n*b*c*d*x + a*x + b*c**2*x),x)*b*c**2*n*p - int
((x**(2*n)*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p)/(x**(2*n)*b*d
**2*x + 2*x**n*b*c*d*x + a*x + b*c**2*x),x)*b*d**2*n*p)/(e*n*p)
```

3.344 $\int (ex)^{-1-n} \left(a+b(c+dx^n)^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^2\right)^p \, dx = -\frac{(ex)^{-n} \left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p} \left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p} (a+bc^2+2bcdx^n+bd^2x^{2n})^p \operatorname{AppellF1}}{en(1-2p)}$$

output
```
-(a+b*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^p*AppellF1(1-2*p,-p,-p,2-2*p,((-a)^(1
/2)/b^(1/2)-c)/d/(x^n),-((-a)^(1/2)/b^(1/2)+c)/d/(x^n))/e/n/(1-2*p)/((e*x)
^n)/((-((-a)^(1/2)-b^(1/2)*c-b^(1/2)*d*x^n)/b^(1/2)/d/(x^n))^p)/((((-a)^(1
/2)+b^(1/2)*c+b^(1/2)*d*x^n)/b^(1/2)/d/(x^n))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.74 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.88

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^2\right)^p \, dx = \frac{(ex)^{-n} \left(1+\frac{cx^{-n}}{d}+\frac{ax^{-n}}{\sqrt{-abd^2}}\right)^{-p} \left(\frac{x^{-n}\left(\sqrt{-abd^2}+bd(c+dx^n)\right)}{bd^2}\right)^{-p} \left(a+b(c+dx^n)^2\right)^p \operatorname{AppellF1}\left(1-2p,-p,-p\right.}{en(-1+2p)}$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
((a + b*(c + d*x^n)^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -((b*c*d + Sqr
t[-(a*b*d^2)])/(b*d^2*x^n)), (-(b*c*d) + Sqrt[-(a*b*d^2)])/(b*d^2*x^n)])/(
e*n*(-1 + 2*p)*(e*x)^n*(1 + c/(d*x^n) + a/(Sqrt[-(a*b*d^2)]*x^n))^p*((Sqrt
[-(a*b*d^2)] + b*d*(c + d*x^n))/(b*d^2*x^n))^p)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2086, 1694, 1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1} \left(a+b(c+dx^n)^2\right)^p dx$$

$$\Big\downarrow \text{ 2086}$$

$$\int (ex)^{-n-1} \left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p dx$$

$$\Big\downarrow \text{ 1694}$$

$$\frac{x^n(ex)^{-n}\int x^{-n-1}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx}{e}$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{x^n(ex)^{-n}\int x^{-2n}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{en}$$

$$\Big\downarrow \text{ 1178}$$

$$-\frac{(ex)^{-n}\left(x^{-n}\right)^{2p-1}\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p\int (x}{en}$$

$$\Big\downarrow \text{ 150}$$

$$-\frac{(ex)^{-n}\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p \text{AppellF1}\left(1-\right.}{en(1-2p)}$$

input
```
Int[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
-(((a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^p*AppellF1[1 - 2*p, -p, -p, 2
 - 2*p, (Sqrt[-a]/Sqrt[b] - c)/(d*x^n), -((Sqrt[-a]/Sqrt[b] + c)/(d*x^n))]
)/(e*n*(1 - 2*p)*(e*x)^n*(-((Sqrt[-a] - Sqrt[b]*c - Sqrt[b]*d*x^n)/(Sqrt[b
]*d*x^n)))^p*((Sqrt[-a] + Sqrt[b]*c + Sqrt[b]*d*x^n)/(Sqrt[b]*d*x^n))^p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 1694
```
Int[((d_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x
_Symbol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[
n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Maple [F]

$$\int (ex)^{-1-n} \left(a+b(c+d\,x^n)^2\right)^p dx$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^2\right)^p\,dx = \int \left((dx^n+c)^2b+a\right)^p (ex)^{-n-1}\,dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^(2*n) + 2*b*c*d*x^n + b*c^2 + a)^p*(e*x)^(-n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1-n)*(a+b*(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2b+a\right)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(-n - 1), x)
```

Giac [F]

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2b+a\right)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-n} \left(a + b(c + dx^n)^2\right)^p \, dx = \int \frac{\left(a + b\,(c + d\,x^n)^2\right)^p}{(e\,x)^{n+1}} \, dx$$

input
```
int((a + b*(c + d*x^n)^2)^p/(e*x)^(n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^2)^p/(e*x)^(n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-n} \left(a + b(c + dx^n)^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^2)^p,x)
```

output

```
( - x**n*(x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*d - 2*(x**(2*n)*
b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p*c*p + 2*(x**(2*n)*b*d**2 + 2*x**n*b
*c*d + a + b*c**2)**p*c + 4*x**n*int((x**(2*n)*b*d**2 + 2*x**n*b*c*d + a +
 b*c**2)**p/(x**(2*n)*a*b*d**2*p*x - x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c
**2*d**2*p*x - x**(2*n)*b**2*c**2*d**2*x + 2*x**n*a*b*c*d*p*x - 2*x**n*a*b
*c*d*x + 2*x**n*b**2*c**3*d*p*x - 2*x**n*b**2*c**3*d*x + a**2*p*x - a**2*x
 + 2*a*b*c**2*p*x - 2*a*b*c**2*x + b**2*c**4*p*x - b**2*c**4*x),x)*a*b*c**
2*d*n*p**3 - 8*x**n*int((x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p/(
x**(2*n)*a*b*d**2*p*x - x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c**2*d**2*p*x
- x**(2*n)*b**2*c**2*d**2*x + 2*x**n*a*b*c*d*p*x - 2*x**n*a*b*c*d*x + 2*x*
*n*b**2*c**3*d*p*x - 2*x**n*b**2*c**3*d*x + a**2*p*x - a**2*x + 2*a*b*c**2
*p*x - 2*a*b*c**2*x + b**2*c**4*p*x - b**2*c**4*x),x)*a*b*c**2*d*n*p**2 +
4*x**n*int((x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p/(x**(2*n)*a*b*
d**2*p*x - x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c**2*d**2*p*x - x**(2*n)*b*
*2*c**2*d**2*x + 2*x**n*a*b*c*d*p*x - 2*x**n*a*b*c*d*x + 2*x**n*b**2*c**3*
d*p*x - 2*x**n*b**2*c**3*d*x + a**2*p*x - a**2*x + 2*a*b*c**2*p*x - 2*a*b*
c**2*x + b**2*c**4*p*x - b**2*c**4*x),x)*a*b*c**2*d*n*p + 4*x**n*int((x**(
2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p/(x**(2*n)*a*b*d**2*p*x - x**(2
*n)*a*b*d**2*x + x**(2*n)*b**2*c**2*d**2*p*x - x**(2*n)*b**2*c**2*d**2*x +
 2*x**n*a*b*c*d*p*x - 2*x**n*a*b*c*d*x + 2*x**n*b**2*c**3*d*p*x - 2*x**...
```

3.345 $\int (ex)^{-1-2n}\left(a+b(c+dx^n)^2\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 210

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^2\right)^p\,dx = -\frac{(ex)^{-2n}\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}(a+bc^2+2bcdx^n+bd^2x^{2n})^p\,\mathrm{AppellF1}}{2en(1-p)}$$

output
```
-1/2*(a+b*c^2+2*b*c*d*x^n+b*d^2*x^(2*n))^p*AppellF1(2-2*p,-p,-p,3-2*p,((-a
)^(1/2)/b^(1/2)-c)/d/(x^n),-((-a)^(1/2)/b^(1/2)+c)/d/(x^n))/e/n/(1-p)/((e*
x)^(2*n))/((-((-a)^(1/2)-b^(1/2)*c-b^(1/2)*d*x^n)/b^(1/2)/d/(x^n))^p)/((((
-a)^(1/2)+b^(1/2)*c+b^(1/2)*d*x^n)/b^(1/2)/d/(x^n))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.93 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^2\right)^p\,dx = \frac{(ex)^{-2n}\left(1+\frac{cx^{-n}}{d}+\frac{ax^{-n}}{\sqrt{-abd^2}}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-abd^2}+bd(c+dx^n)\right)}{bd^2}\right)^{-p}\left(a+b(c+dx^n)^2\right)^p\,\mathrm{AppellF1}\left(2-2p,-p,-\right.}{2en(-1+p)}$$

input
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
((a + b*(c + d*x^n)^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -((b*c*d + Sqr
t[-(a*b*d^2)])/(b*d^2*x^n)), (-(b*c*d) + Sqrt[-(a*b*d^2)])/(b*d^2*x^n)])/(
2*e*n*(-1 + p)*(e*x)^(2*n)*(1 + c/(d*x^n) + a/(Sqrt[-(a*b*d^2)]*x^n))^p*((
Sqrt[-(a*b*d^2)] + b*d*(c + d*x^n))/(b*d^2*x^n))^p)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2086, 1694, 1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-2n-1} \left(a+b(c+dx^n)^2\right)^p dx$$

$$\Big\downarrow \; 2086$$

$$\int (ex)^{-2n-1} \left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p dx$$

$$\Big\downarrow \; 1694$$

$$\frac{x^{2n}(ex)^{-2n}\int x^{-2n-1}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx}{e}$$

$$\Big\downarrow \; 1693$$

$$\frac{x^{2n}(ex)^{-2n}\int x^{-3n}\left(2bcdx^n+bd^2x^{2n}+bc^2+a\right)^p dx^n}{en}$$

$$\Big\downarrow \; 1178$$

$$-\frac{x^{2n}(ex)^{-2n}\left(x^{-n}\right)^{2p}\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^n\right)}{\sqrt{b}d}\right)^{-p}\left(a+bc^2+2bcdx^n+bd^2x^{2n}\right)^p\int (}{en}$$

$$\Big\downarrow \; 150$$

$$-\frac{(ex)^{-2n}\left(-\frac{x^{-n}\left(\sqrt{-a}-\sqrt{b}c-\sqrt{b}dx^{n}\right)}{\sqrt{b}d}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{-a}+\sqrt{b}c+\sqrt{b}dx^{n}\right)}{\sqrt{b}d}\right)^{-p}(a+bc^{2}+2bcdx^{n}+bd^{2}x^{2n})^{p}\operatorname{AppellF1}\left(2-\right.}{2en(1-p)}$$

input
```
Int[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^2)^p,x]
```

output
```
-1/2*((a + b*c^2 + 2*b*c*d*x^n + b*d^2*x^(2*n))^p*AppellF1[2 - 2*p, -p, -p
, 3 - 2*p, (Sqrt[-a]/Sqrt[b] - c)/(d*x^n), -((Sqrt[-a]/Sqrt[b] + c)/(d*x^n
))])/(e*n*(1 - p)*(e*x)^(2*n)*(-((Sqrt[-a] - Sqrt[b]*c - Sqrt[b]*d*x^n)/(S
qrt[b]*d*x^n)))^p*((Sqrt[-a] + Sqrt[b]*c + Sqrt[b]*d*x^n)/(Sqrt[b]*d*x^n))
^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 1694
```
Int[((d_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x
_Symbol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[
n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Maple [F]

$$\int (ex)^{-1-2n} \left(a+b(c+d\,x^n)^2\right)^p dx$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \left((dx^n+c)^2b+a\right)^p (ex)^{-2\,n-1} \, dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^(2*n) + 2*b*c*d*x^n + b*c^2 + a)^p*(e*x)^(-2*n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1-2*n)*(a+b*(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^2\right)^p\,dx=\int \left((dx^n+c)^2b+a\right)^p(ex)^{-2\,n-1}\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(-2*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^2\right)^p\,dx=\int \left((dx^n+c)^2b+a\right)^p(ex)^{-2\,n-1}\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^2*b + a)^p*(e*x)^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \int \frac{\left(a+b\,(c+d\,x^n)^2\right)^p}{(e\,x)^{2\,n+1}} \, dx$$

input
```
int((a + b*(c + d*x^n)^2)^p/(e*x)^(2*n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^2)^p/(e*x)^(2*n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^2)^p,x)
```

output
```
( - (x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p + 2*x**(2*n)*int((x**
(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p/(x**(3*n)*a*b*d**2*p*x - 2*x*
*(3*n)*a*b*d**2*x + x**(3*n)*b**2*c**2*d**2*p*x - 2*x**(3*n)*b**2*c**2*d**
2*x + 2*x**(2*n)*a*b*c*d*p*x - 4*x**(2*n)*a*b*c*d*x + 2*x**(2*n)*b**2*c**3
*d*p*x - 4*x**(2*n)*b**2*c**3*d*x + x**n*a**2*p*x - 2*x**n*a**2*x + 2*x**n
*a*b*c**2*p*x - 4*x**n*a*b*c**2*x + x**n*b**2*c**4*p*x - 2*x**n*b**2*c**4*
x),x)*a*b*c*d*n*p**2 - 4*x**(2*n)*int((x**(2*n)*b*d**2 + 2*x**n*b*c*d + a
+ b*c**2)**p/(x**(3*n)*a*b*d**2*p*x - 2*x**(3*n)*a*b*d**2*x + x**(3*n)*b**
2*c**2*d**2*p*x - 2*x**(3*n)*b**2*c**2*d**2*x + 2*x**(2*n)*a*b*c*d*p*x - 4
*x**(2*n)*a*b*c*d*x + 2*x**(2*n)*b**2*c**3*d*p*x - 4*x**(2*n)*b**2*c**3*d*
x + x**n*a**2*p*x - 2*x**n*a**2*x + 2*x**n*a*b*c**2*p*x - 4*x**n*a*b*c**2*
x + x**n*b**2*c**4*p*x - 2*x**n*b**2*c**4*x),x)*a*b*c*d*n*p + 2*x**(2*n)*i
nt((x**(2*n)*b*d**2 + 2*x**n*b*c*d + a + b*c**2)**p/(x**(3*n)*a*b*d**2*p*x
 - 2*x**(3*n)*a*b*d**2*x + x**(3*n)*b**2*c**2*d**2*p*x - 2*x**(3*n)*b**2*c
**2*d**2*x + 2*x**(2*n)*a*b*c*d*p*x - 4*x**(2*n)*a*b*c*d*x + 2*x**(2*n)*b*
*2*c**3*d*p*x - 4*x**(2*n)*b**2*c**3*d*x + x**n*a**2*p*x - 2*x**n*a**2*x +
 2*x**n*a*b*c**2*p*x - 4*x**n*a*b*c**2*x + x**n*b**2*c**4*p*x - 2*x**n*b**
2*c**4*x),x)*b**2*c**3*d*n*p**2 - 4*x**(2*n)*int((x**(2*n)*b*d**2 + 2*x**n
*b*c*d + a + b*c**2)**p/(x**(3*n)*a*b*d**2*p*x - 2*x**(3*n)*a*b*d**2*x + x
**(3*n)*b**2*c**2*d**2*p*x - 2*x**(3*n)*b**2*c**2*d**2*x + 2*x**(2*n)*a...
```

3.346 $\int (ex)^{-1+3n} (a+b(c+dx^n))^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int (ex)^{-1+3n} (a+b(c+dx^n))^p \, dx = \frac{(a+bc)^2 x^{-3n} (ex)^{3n} (a+bc+bdx^n)^{1+p}}{b^3 d^3 en(1+p)} - \frac{2(a+bc) x^{-3n} (ex)^{3n} (a+bc+bdx^n)^{2+p}}{b^3 d^3 en(2+p)} + \frac{x^{-3n} (ex)^{3n} (a+bc+bdx^n)^{3+p}}{b^3 d^3 en(3+p)}$$

output
```
(b*c+a)^2*(e*x)^(3*n)*(a+b*c+b*d*x^n)^(p+1)/b^3/d^3/e/n/(p+1)/(x^(3*n))-2*
(b*c+a)*(e*x)^(3*n)*(a+b*c+b*d*x^n)^(2+p)/b^3/d^3/e/n/(2+p)/(x^(3*n))+(e*x
)^(3*n)*(a+b*c+b*d*x^n)^(3+p)/b^3/d^3/e/n/(3+p)/(x^(3*n))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(149) = 298.

Time = 0.64 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.65

$$\int (ex)^{-1+3n} (a+b(c+dx^n))^p \, dx$$
$$= \frac{x^{-3n} (ex)^{3n} \left(1+\frac{bdx^n}{a+bc}\right)^{-p} (a+b(c+dx^n))^p \left(2a^3 \left(-1+\left(\frac{a+bc+bdx^n}{a+bc}\right)^p\right) - 2a^2 b \left(dpx^n \left(\frac{a+bc+bdx^n}{a+bc}\right)^p - 3c(-1 \right.\right.}{}$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n))^p,x]
```

output
```
((e*x)^(3*n)*(a + b*(c + d*x^n))^p*(2*a^3*(-1 + ((a + b*c + b*d*x^n)/(a +
b*c))^p) - 2*a^2*b*(d*p*x^n*((a + b*c + b*d*x^n)/(a + b*c))^p - 3*c*(-1 +
((a + b*c + b*d*x^n)/(a + b*c))^p)) + a*b^2*(-4*c*d*p*x^n*((a + b*c + b*d*
x^n)/(a + b*c))^p + d^2*p*(1 + p)*x^(2*n)*((a + b*c + b*d*x^n)/(a + b*c))^
p + 6*c^2*(-1 + ((a + b*c + b*d*x^n)/(a + b*c))^p)) + b^3*(-2*c^2*d*p*x^n*
((a + b*c + b*d*x^n)/(a + b*c))^p + c*d^2*p*(1 + p)*x^(2*n)*((a + b*c + b*
d*x^n)/(a + b*c))^p + d^3*(2 + 3*p + p^2)*x^(3*n)*((a + b*c + b*d*x^n)/(a
+ b*c))^p + 2*c^3*(-1 + ((a + b*c + b*d*x^n)/(a + b*c))^p))))/(b^3*d^3*e*n
*(1 + p)*(2 + p)*(3 + p)*x^(3*n)*(1 + (b*d*x^n)/(a + b*c))^p)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2073, 800, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1} (a+b(c+dx^n))^p \, dx$$

$$\Big\downarrow \text{2073}$$

$$\int (ex)^{3n-1} (a+bc+bdx^n)^p dx$$

$$\Big\downarrow \text{800}$$

$$\frac{x^{-3n}(ex)^{3n} \int x^{3n-1}(bdx^n+a+bc)^p dx}{e}$$

$$\Big\downarrow \text{798}$$

$$\frac{x^{-3n}(ex)^{3n} \int x^{2n}(bdx^n+a+bc)^p dx^n}{en}$$

$$\Big\downarrow \text{53}$$

$$\frac{x^{-3n}(ex)^{3n} \int \left(\frac{(a+bc)^2(bdx^n+a+bc)^p}{b^2d^2} - \frac{2(a+bc)(bdx^n+a+bc)^{p+1}}{b^2d^2} + \frac{(bdx^n+a+bc)^{p+2}}{b^2d^2}\right) dx^n}{en}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{-3n}(ex)^{3n}\left(\frac{(a+bc)^2(a+bc+bdx^n)^{p+1}}{b^3d^3(p+1)}-\frac{2(a+bc)(a+bc+bdx^n)^{p+2}}{b^3d^3(p+2)}+\frac{(a+bc+bdx^n)^{p+3}}{b^3d^3(p+3)}\right)}{en}$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n))^p,x]
```

output
```
((e*x)^(3*n)*(((a + b*c)^2*(a + b*c + b*d*x^n)^(1 + p))/(b^3*d^3*(1 + p))
- (2*(a + b*c)*(a + b*c + b*d*x^n)^(2 + p))/(b^3*d^3*(2 + p)) + (a + b*c +
 b*d*x^n)^(3 + p)/(b^3*d^3*(3 + p))))/(e*n*x^(3*n))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Maple [F]

$$\int (ex)^{-1+3n}\left(a+b(c+d\,x^{n})\right)^{p}dx$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.49

$$\int (ex)^{-1+3n}(a+b(c+dx^n))^p\,dx = -\frac{(2\,(b^3c^2+2\,ab^2c+a^2b)de^{3\,n-1}px^n-(b^3d^3p^2+3\,b^3d^3p+2\,b^3d^3)e^{3\,n-1}x^{3\,n}-((b^3c+ab^2)d^2p^2+(b^3c+}{b^3d^3np^3+6\,b^3d^3np^2+11\,b^3d^3np+}$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
-(2*(b^3*c^2 + 2*a*b^2*c + a^2*b)*d*e^(3*n - 1)*p*x^n - (b^3*d^3*p^2 + 3*b
^3*d^3*p + 2*b^3*d^3)*e^(3*n - 1)*x^(3*n) - ((b^3*c + a*b^2)*d^2*p^2 + (b^
3*c + a*b^2)*d^2*p)*e^(3*n - 1)*x^(2*n) - 2*(b^3*c^3 + 3*a*b^2*c^2 + 3*a^2
*b*c + a^3)*e^(3*n - 1))*(b*d*x^n + b*c + a)^p/(b^3*d^3*n*p^3 + 6*b^3*d^3*
n*p^2 + 11*b^3*d^3*n*p + 6*b^3*d^3*n)
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b(c+dx^n))^p\,dx = \int (ex)^{3n-1}\,(a+bc+bdx^n)^p\,dx$$

input
```
integrate((e*x)**(-1+3*n)*(a+b*(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(3*n - 1)*(a + b*c + b*d*x**n)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.38

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)\right)^p\,dx$$
$$= \frac{((p^2+3\,p+2)b^3d^3e^{3\,n}x^{3\,n}+2\,b^3c^3e^{3\,n}+6\,ab^2c^2e^{3\,n}+6\,a^2bce^{3\,n}+2\,a^3e^{3\,n}+((p^2+p)b^3cd^2e^{3\,n}+(p^2+p)}{(p^3+6\,p^2+11\,p+6)b^3d^3en}$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*d^3*e^(3*n)*x^(3*n) + 2*b^3*c^3*e^(3*n) + 6*a*b^2*c^2
*e^(3*n) + 6*a^2*b*c*e^(3*n) + 2*a^3*e^(3*n) + ((p^2 + p)*b^3*c*d^2*e^(3*n
) + (p^2 + p)*a*b^2*d^2*e^(3*n))*x^(2*n) - 2*(b^3*c^2*d*e^(3*n)*p + 2*a*b^
2*c*d*e^(3*n)*p + a^2*b*d*e^(3*n)*p)*x^n)*(b*d*x^n + b*c + a)^p/((p^3 + 6*
p^2 + 11*p + 6)*b^3*d^3*e*n)
```

Giac [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)\right)^p\,dx=\int ((dx^n+c)b+a)^p(ex)^{3\,n-1}\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)*b + a)^p*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)\right)^p\,dx=\int (e\,x)^{3\,n-1}\,\left(a+b\,\left(c+d\,x^n\right)\right)^p\,dx$$

input
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n))^p,x)
```

output
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.44

$$\int (ex)^{-1+3n} (a+b(c+dx^n))^p \, dx$$
$$= \frac{e^{3n}(x^n bd+a+bc)^p \left(x^{3n}b^3d^3p^2+3x^{3n}b^3d^3p+2x^{3n}b^3d^3+x^{2n}a\,b^2d^2p^2+x^{2n}a\,b^2d^2p+x^{2n}b^3c\,d^2p^2+x^{2n}b\right.}{b^3d^3en\left(p^3+6p^2+11p+6\right.}$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n))^p,x)
```

output
```
(e**(3*n)*(x**n*b*d + a + b*c)**p*(x**(3*n)*b**3*d**3*p**2 + 3*x**(3*n)*b*
*3*d**3*p + 2*x**(3*n)*b**3*d**3 + x**(2*n)*a*b**2*d**2*p**2 + x**(2*n)*a*
b**2*d**2*p + x**(2*n)*b**3*c*d**2*p**2 + x**(2*n)*b**3*c*d**2*p - 2*x**n*
a**2*b*d*p - 4*x**n*a*b**2*c*d*p - 2*x**n*b**3*c**2*d*p + 2*a**3 + 6*a**2*
b*c + 6*a*b**2*c**2 + 2*b**3*c**3))/(b**3*d**3*e*n*(p**3 + 6*p**2 + 11*p +
 6))
```

3.347 $\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx = -\frac{(a+bc)x^{-2n}(ex)^{2n}(a+bc+bdx^n)^{1+p}}{b^2d^2en(1+p)} + \frac{x^{-2n}(ex)^{2n}(a+bc+bdx^n)^{2+p}}{b^2d^2en(2+p)}$$

output
```
-(b*c+a)*(e*x)^(2*n)*(a+b*c+b*d*x^n)^(p+1)/b^2/d^2/e/n/(p+1)/(x^(2*n))+(e*
x)^(2*n)*(a+b*c+b*d*x^n)^(2+p)/b^2/d^2/e/n/(2+p)/(x^(2*n))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(97) = 194.

Time = 0.54 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.57

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\left(1+\frac{bdx^n}{a+bc}\right)^{-p}(a+b(c+dx^n))^p\left(-a^2\left(-1+\left(\frac{a+bc+bdx^n}{a+bc}\right)^p\right)+ab\left(dpx^n\left(\frac{a+bc+bdx^n}{a+bc}\right)^p-2c\left(-1+\right.\right.\right.}{b^2d^2en(1+p)(}$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n))^p,x]
```

output
```
((e*x)^(2*n)*(a + b*(c + d*x^n))^p*(-(a^2*(-1 + ((a + b*c + b*d*x^n)/(a +
b*c))^p)) + a*b*(d*p*x^n*((a + b*c + b*d*x^n)/(a + b*c))^p - 2*c*(-1 + ((a
 + b*c + b*d*x^n)/(a + b*c))^p)) - b^2*(-(c*d*p*x^n*((a + b*c + b*d*x^n)/(
a + b*c))^p) - d^2*(1 + p)*x^(2*n)*((a + b*c + b*d*x^n)/(a + b*c))^p + c^2
*(-1 + ((a + b*c + b*d*x^n)/(a + b*c))^p))))/(b^2*d^2*e*n*(1 + p)*(2 + p)*
x^(2*n)*(1 + (b*d*x^n)/(a + b*c))^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2073, 800, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} \left(a+b(c+dx^n)\right)^p \, dx$$

$$\Big\downarrow \text{ 2073}$$

$$\int (ex)^{2n-1} (a+bc+bdx^n)^p dx$$

$$\Big\downarrow \text{ 800}$$

$$\frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(bdx^n+a+bc)^p dx}{e}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{x^{-2n}(ex)^{2n} \int x^n(bdx^n+a+bc)^p dx^n}{en}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{x^{-2n}(ex)^{2n} \int \left(\frac{(-a-bc)(bdx^n+a+bc)^p}{bd}+\frac{(bdx^n+a+bc)^{p+1}}{bd}\right) dx^n}{en}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{-2n}(ex)^{2n} \left(\frac{(a+bc+bdx^n)^{p+2}}{b^2d^2(p+2)}-\frac{(a+bc)(a+bc+bdx^n)^{p+1}}{b^2d^2(p+1)}\right)}{en}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n))^p,x]
```

output
```
((e*x)^(2*n)*(-(((a + b*c)*(a + b*c + b*d*x^n)^(1 + p))/(b^2*d^2*(1 + p)))
 + (a + b*c + b*d*x^n)^(2 + p)/(b^2*d^2*(2 + p))))/(e*n*x^(2*n))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Maple [F]

$$\int (ex)^{-1+2n} (a+b(c+d\,x^n))^p dx$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx = \frac{((b^2c+ab)de^{2n-1}px^n + (b^2d^2p+b^2d^2)e^{2n-1}x^{2n} - (b^2c^2+2abc+a^2)e^{2n-1})(bdx^n+bc+a)^p}{b^2d^2np^2+3b^2d^2np+2b^2d^2n}$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
((b^2*c + a*b)*d*e^(2*n - 1)*p*x^n + (b^2*d^2*p + b^2*d^2)*e^(2*n - 1)*x^(
2*n) - (b^2*c^2 + 2*a*b*c + a^2)*e^(2*n - 1))*(b*d*x^n + b*c + a)^p/(b^2*d
^2*n*p^2 + 3*b^2*d^2*n*p + 2*b^2*d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx = \int (ex)^{2n-1} (a+bc+bdx^n)^p \, dx$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(2*n - 1)*(a + b*c + b*d*x**n)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx$$
$$= \frac{(b^2d^2e^{2n}(p+1)x^{2n} - b^2c^2e^{2n} - 2abce^{2n} - a^2e^{2n} + (b^2cde^{2n}p + abde^{2n}p)x^n)(bdx^n + bc + a)^p}{(p^2+3p+2)b^2d^2en}$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
(b^2*d^2*e^(2*n)*(p + 1)*x^(2*n) - b^2*c^2*e^(2*n) - 2*a*b*c*e^(2*n) - a^2
*e^(2*n) + (b^2*c*d*e^(2*n)*p + a*b*d*e^(2*n)*p)*x^n)*(b*d*x^n + b*c + a)^
p/((p^2 + 3*p + 2)*b^2*d^2*e*n)
```

Giac [F]

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx = \int ((dx^n+c)b+a)^p (ex)^{2n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)*b + a)^p*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx = \int (e\,x)^{2\,n-1}\,(a+b\,(c+d\,x^n))^p \, dx$$

input
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int (ex)^{-1+2n} (a+b(c+dx^n))^p \, dx = \frac{e^{2n}(x^n bd+a+bc)^p (x^{2n}b^2d^2p+x^{2n}b^2d^2+x^n abdp+x^n b^2cdp-a^2-2abc-b^2c^2)}{b^2d^2en\,(p^2+3p+2)}$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n))^p,x)
```

output
```
(e**(2*n)*(x**n*b*d + a + b*c)**p*(x**(2*n)*b**2*d**2*p + x**(2*n)*b**2*d*
*2 + x**n*a*b*d*p + x**n*b**2*c*d*p - a**2 - 2*a*b*c - b**2*c**2))/(b**2*d
**2*e*n*(p**2 + 3*p + 2))
```

3.348 $\int (ex)^{-1+n} (a+b(c+dx^n))^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int (ex)^{-1+n} (a+b(c+dx^n))^p \, dx = \frac{x^{-n}(ex)^n (a+bc+bdx^n)^{1+p}}{bden(1+p)}$$

output
```
(e*x)^n*(a+b*c+b*d*x^n)^(p+1)/b/d/e/n/(p+1)/(x^n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int (ex)^{-1+n} (a+b(c+dx^n))^p \, dx = \frac{x^{1-n}(ex)^{-1+n} (a+b(c+dx^n))^{1+p}}{bdn(1+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*(c + d*x^n))^p,x]
```

output
```
(x^(1 - n)*(e*x)^(-1 + n)*(a + b*(c + d*x^n))^(1 + p))/(b*d*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2073, 800, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (a+b(c+dx^n))^p \, dx$$

$$\Big\downarrow \text{2073}$$

$$\int (ex)^{n-1} (a+bc+bdx^n)^p dx$$

$$\Big\downarrow \text{800}$$

$$\frac{x^{-n}(ex)^n \int x^{n-1}(bdx^n+a+bc)^p dx}{e}$$

$$\Big\downarrow \text{793}$$

$$\frac{x^{-n}(ex)^n (a+bc+bdx^n)^{p+1}}{bden(p+1)}$$

input
```
Int[(e*x)^(-1 + n)*(a + b*(c + d*x^n))^p,x]
```

output
```
((e*x)^n*(a + b*c + b*d*x^n)^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Maple [F]

$$\int (ex)^{-1+n} (a+b(c+d\,x^n))^p dx$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int (ex)^{-1+n} (a+b(c+dx^n))^p \, dx = \frac{(bde^{n-1}x^n + (bc+a)e^{n-1})(bdx^n + bc + a)^p}{bdnp + bdn}$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
(b*d*e^(n - 1)*x^n + (b*c + a)*e^(n - 1))*(b*d*x^n + b*c + a)^p/(b*d*n*p +
 b*d*n)
```

Sympy [F]

$$\int (ex)^{-1+n} (a+b(c+dx^n))^p \, dx = \int (ex)^{n-1} (a+bc+bdx^n)^p \, dx$$

input
```
integrate((e*x)**(-1+n)*(a+b*(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(n - 1)*(a + b*c + b*d*x**n)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int (ex)^{-1+n}(a+b(c+dx^n))^p\,dx=\frac{(bde^nx^n+bce^n+ae^n)(bdx^n+bc+a)^p}{bden(p+1)}$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
(b*d*e^n*x^n + b*c*e^n + a*e^n)*(b*d*x^n + b*c + a)^p/(b*d*e*n*(p + 1))
```

Giac [F]

$$\int (ex)^{-1+n}(a+b(c+dx^n))^p\,dx=\int ((dx^n+c)b+a)^p(ex)^{n-1}\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)*b + a)^p*(e*x)^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}(a+b(c+dx^n))^p\,dx=\int (e\,x)^{n-1}\,(a+b\,(c+d\,x^n))^p\,dx$$

input
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n))^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int (ex)^{-1+n} (a+b(c+dx^n))^p \, dx = \frac{e^n (x^n bd+a+bc)^p (x^n bd+a+bc)}{bden (p+1)}$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n))^p,x)
```

output
```
(e**n*(x**n*b*d + a + b*c)**p*(x**n*b*d + a + b*c))/(b*d*e*n*(p + 1))
```

3.349 $\int \frac{(a+b(c+dx^n))^p}{ex} dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{(a+b(c+dx^n))^p}{ex} dx = -\frac{(a+bc+bdx^n)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bdx^n}{a+bc}\right)}{(a+bc)en(1+p)}$$

output
```
-(a+b*c+b*d*x^n)^(p+1)*hypergeom([1, p+1],[2+p],1+b*d*x^n/(b*c+a))/(b*c+a)
/e/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{(a+b(c+dx^n))^p}{ex} dx = -\frac{(a+bc+bdx^n)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bdx^n}{a+bc}\right)}{(a+bc)en(1+p)}$$

input
```
Integrate[(a + b*(c + d*x^n))^p/(e*x),x]
```

output
```
-(((a + b*c + b*d*x^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*d
*x^n)/(a + b*c)])/((a + b*c)*e*n*(1 + p)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {27, 2073, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(c+dx^n))^p}{ex}dx$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{(a+b(dx^n+c))^p}{x}dx}{e}$$
$$\Big\downarrow \; 2073$$
$$\frac{\int \frac{(bdx^n+a+bc)^p}{x}dx}{e}$$
$$\Big\downarrow \; 798$$
$$\frac{\int x^{-n}(bdx^n+a+bc)^p\,dx^n}{en}$$
$$\Big\downarrow \; 75$$
$$-\frac{(a+bc+bdx^n)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bdx^n}{a+bc}+1\right)}{en(p+1)(a+bc)}$$

input
```
Int[(a + b*(c + d*x^n))^p/(e*x),x]
```

output
```
-(((a + b*c + b*d*x^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*d
*x^n)/(a + b*c)])/((a + b*c)*e*n*(1 + p)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Maple [F]

$$\int \frac{(a+b(c+d\,x^n))^p}{ex}dx$$

input
```
int((a+b*(c+d*x^n))^p/e/x,x)
```

output
```
int((a+b*(c+d*x^n))^p/e/x,x)
```

Fricas [F]

$$\int \frac{(a+b(c+dx^n))^p}{ex}\,dx=\int \frac{((dx^n+c)b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n))^p/e/x,x, algorithm="fricas")
```

output
```
integral((b*d*x^n + b*c + a)^p/(e*x), x)
```

Sympy [F]

$$\int \frac{(a+b(c+dx^n))^p}{ex}\,dx=\frac{\int \frac{(a+bc+bdx^n)^p}{x}\,dx}{e}$$

input
```
integrate((a+b*(c+d*x**n))**p/e/x,x)
```

output
```
Integral((a + b*c + b*d*x**n)**p/x, x)/e
```

Maxima [F]

$$\int \frac{(a+b(c+dx^n))^p}{ex}\,dx=\int \frac{((dx^n+c)b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n))^p/e/x,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)*b + a)^p/x, x)/e
```

Giac [F]

$$\int \frac{(a+b(c+dx^n))^p}{ex}\,dx=\int \frac{((dx^n+c)b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n))^p/e/x,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)*b + a)^p/(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(c+dx^n))^p}{ex}\,dx=\int \frac{(a+b\,(c+d\,x^n))^p}{e\,x}\,dx$$

input
```
int((a + b*(c + d*x^n))^p/(e*x),x)
```

output
```
int((a + b*(c + d*x^n))^p/(e*x), x)
```

Reduce [F]

$$\int \frac{(a+b(c+dx^n))^p}{ex}\,dx = \frac{(x^n bd+a+bc)^p+\left(\int \frac{(x^n bd+a+bc)^p}{x^n bdx+ax+bcx}dx\right)anp+\left(\int \frac{(x^n bd+a+bc)^p}{x^n bdx+ax+bcx}dx\right)bcnp}{enp}$$

input
```
int((a+b*(c+d*x^n))^p/e/x,x)
```

output
```
((x**n*b*d + a + b*c)**p + int((x**n*b*d + a + b*c)**p/(x**n*b*d*x + a*x +
 b*c*x),x)*a*n*p + int((x**n*b*d + a + b*c)**p/(x**n*b*d*x + a*x + b*c*x),
x)*b*c*n*p)/(e*n*p)
```

3.350 $\int (ex)^{-1-n} (a+b(c+dx^n))^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int (ex)^{-1-n} (a+b(c+dx^n))^p \, dx = \frac{bdx^n(ex)^{-n} (a+bc+bdx^n)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bdx^n}{a+bc}\right)}{(a+bc)^2en(1+p)}$$

output
```
b*d*x^n*(a+b*c+b*d*x^n)^(p+1)*hypergeom([2, p+1],[2+p],1+b*d*x^n/(b*c+a))/
(b*c+a)^2/e/n/(p+1)/((e*x)^n)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int (ex)^{-1-n} (a+b(c+dx^n))^p \, dx = \frac{x(ex)^{-1-n} \left(1+\frac{(a+bc)x^{-n}}{bd}\right)^{-p} (a+bc+bdx^n)^p \text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{(-a-bc)x^{-n}}{bd}\right)}{n(-1+p)}$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b*(c + d*x^n))^p,x]
```

output
```
(x*(e*x)^(-1 - n)*(a + b*c + b*d*x^n)^p*Hypergeometric2F1[1 - p, -p, 2 - p
, (-a - b*c)/(b*d*x^n)])/(n*(-1 + p)*(1 + (a + b*c)/(b*d*x^n))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2073, 800, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1} (a+b(c+dx^n))^p \, dx$$

$$\Big\downarrow \text{ 2073}$$

$$\int (ex)^{-n-1} (a+bc+bdx^n)^p dx$$

$$\Big\downarrow \text{ 800}$$

$$\frac{x^n(ex)^{-n} \int x^{-n-1}(bdx^n+a+bc)^p dx}{e}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{x^n(ex)^{-n} \int x^{-2n}(bdx^n+a+bc)^p dx^n}{en}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{bdx^n(ex)^{-n} (a+bc+bdx^n)^{p+1} \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bdx^n}{a+bc}+1\right)}{en(p+1)(a+bc)^2}$$

input
```
Int[(e*x)^(-1 - n)*(a + b*(c + d*x^n))^p,x]
```

output
```
(b*d*x^n*(a + b*c + b*d*x^n)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1
+ (b*d*x^n)/(a + b*c)])/((a + b*c)^2*e*n*(1 + p)*(e*x)^n)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Maple [F]

$$\int (ex)^{-1-n} (a+b(c+d\,x^n))^p dx$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1-n} (a + b(c + dx^n))^p \, dx = \int ((dx^n + c)b + a)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((b*d*x^n + b*c + a)^p*(e*x)^(-n - 1), x)
```

Sympy [F]

$$\int (ex)^{-1-n} (a + b(c + dx^n))^p \, dx = \int (ex)^{-n-1} (a + bc + bdx^n)^p \, dx$$

input
```
integrate((e*x)**(-1-n)*(a+b*(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(-n - 1)*(a + b*c + b*d*x**n)**p, x)
```

Maxima [F]

$$\int (ex)^{-1-n} (a + b(c + dx^n))^p \, dx = \int ((dx^n + c)b + a)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)*b + a)^p*(e*x)^(-n - 1), x)
```

Giac [F]

$$\int (ex)^{-1-n} (a + b(c + dx^n))^p \, dx = \int ((dx^n + c)b + a)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)*b + a)^p*(e*x)^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-n} (a + b(c + dx^n))^p \, dx = \int \frac{(a + b\,(c + d\,x^n))^p}{(e\,x)^{n+1}} \, dx$$

input
```
int((a + b*(c + d*x^n))^p/(e*x)^(n + 1),x)
```

output
```
int((a + b*(c + d*x^n))^p/(e*x)^(n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-n} (a + b(c + dx^n))^p \, dx = \frac{-(x^n bd + a + bc)^p + x^n \left(\int \frac{(x^n bd + a + bc)^p}{x^n bdx + ax + bcx} dx\right) bdnp}{x^n e^n en}$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n))^p,x)
```

output
```
( - (x**n*b*d + a + b*c)**p + x**n*int((x**n*b*d + a + b*c)**p/(x**n*b*d*x
 + a*x + b*c*x),x)*b*d*n*p)/(x**n*e**n*e*n)
```

3.351 $\int (ex)^{-1-2n} (a+b(c+dx^n))^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (ex)^{-1-2n} (a+b(c+dx^n))^p \, dx = -\frac{b^2 d^2 x^{2n} (ex)^{-2n} (a+bc+bdx^n)^{1+p} \text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bdx^n}{a+bc}\right)}{(a+bc)^3 en(1+p)}$$

output
```
-b^2*d^2*x^(2*n)*(a+b*c+b*d*x^n)^(p+1)*hypergeom([3, p+1],[2+p],1+b*d*x^n/
(b*c+a))/(b*c+a)^3/e/n/(p+1)/((e*x)^(2*n))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int (ex)^{-1-2n} (a+b(c+dx^n))^p \, dx = \frac{x(ex)^{-1-2n} \left(1+\frac{(a+bc)x^{-n}}{bd}\right)^{-p} (a+bc+bdx^n)^p \text{Hypergeometric2F1}\left(2-p,-p,3-p,\frac{(-a-bc)x^{-n}}{bd}\right)}{n(-2+p)}$$

input
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n))^p,x]
```

output
```
(x*(e*x)^(-1 - 2*n)*(a + b*c + b*d*x^n)^p*Hypergeometric2F1[2 - p, -p, 3 -
 p, (-a - b*c)/(b*d*x^n)])/(n*(-2 + p)*(1 + (a + b*c)/(b*d*x^n))^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2073, 800, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-2n-1}(a+b(c+dx^n))^p\,dx$$

$$\Big\downarrow \text{2073}$$

$$\int (ex)^{-2n-1}(a+bc+bdx^n)^p\,dx$$

$$\Big\downarrow \text{800}$$

$$\frac{x^{2n}(ex)^{-2n}\int x^{-2n-1}(bdx^n+a+bc)^p\,dx}{e}$$

$$\Big\downarrow \text{798}$$

$$\frac{x^{2n}(ex)^{-2n}\int x^{-3n}(bdx^n+a+bc)^p\,dx^n}{en}$$

$$\Big\downarrow \text{75}$$

$$-\frac{b^2d^2x^{2n}(ex)^{-2n}(a+bc+bdx^n)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{bdx^n}{a+bc}+1\right)}{en(p+1)(a+bc)^3}$$

input
```
Int[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n))^p,x]
```

output
```
-((b^2*d^2*x^(2*n)*(a + b*c + b*d*x^n)^(1 + p)*Hypergeometric2F1[3, 1 + p,
 2 + p, 1 + (b*d*x^n)/(a + b*c)])/((a + b*c)^3*e*n*(1 + p)*(e*x)^(2*n)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x
]
```

Maple [F]

$$\int (ex)^{-1-2n} (a+b(c+d\,x^n))^p dx$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1-2n} (a + b(c + dx^n))^p \, dx = \int ((dx^n + c)b + a)^p (ex)^{-2n-1} \, dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((b*d*x^n + b*c + a)^p*(e*x)^(-2*n - 1), x)
```

Sympy [F]

$$\int (ex)^{-1-2n} (a + b(c + dx^n))^p \, dx = \int (ex)^{-2n-1} (a + bc + bdx^n)^p \, dx$$

input
```
integrate((e*x)**(-1-2*n)*(a+b*(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(-2*n - 1)*(a + b*c + b*d*x**n)**p, x)
```

Maxima [F]

$$\int (ex)^{-1-2n} (a + b(c + dx^n))^p \, dx = \int ((dx^n + c)b + a)^p (ex)^{-2n-1} \, dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)*b + a)^p*(e*x)^(-2*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1-2n}(a+b(c+dx^n))^p\,dx=\int ((dx^n+c)b+a)^p(ex)^{-2n-1}\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)*b + a)^p*(e*x)^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}(a+b(c+dx^n))^p\,dx=\int \frac{(a+b\,(c+d\,x^n))^p}{(e\,x)^{2\,n+1}}\,dx$$

input
```
int((a + b*(c + d*x^n))^p/(e*x)^(2*n + 1),x)
```

output
```
int((a + b*(c + d*x^n))^p/(e*x)^(2*n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-2n}(a+b(c+dx^n))^p\,dx$$
$$=\frac{-x^n(x^nbd+a+bc)^p\,bdp-(x^nbd+a+bc)^p\,a-(x^nbd+a+bc)^p\,bc+x^{2n}\left(\int\frac{(x^nbd+a+bc)^p}{x^nabdx+x^nb^2cdx+a^2x+2abcx+b^2c}\right.}{}$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n))^p,x)
```

output

```
( - x**n*(x**n*b*d + a + b*c)**p*b*d*p - (x**n*b*d + a + b*c)**p*a - (x**n
*b*d + a + b*c)**p*b*c + x**(2*n)*int((x**n*b*d + a + b*c)**p/(x**n*a*b*d*
x + x**n*b**2*c*d*x + a**2*x + 2*a*b*c*x + b**2*c**2*x),x)*a*b**2*d**2*n*p
**2 - x**(2*n)*int((x**n*b*d + a + b*c)**p/(x**n*a*b*d*x + x**n*b**2*c*d*x
 + a**2*x + 2*a*b*c*x + b**2*c**2*x),x)*a*b**2*d**2*n*p + x**(2*n)*int((x*
*n*b*d + a + b*c)**p/(x**n*a*b*d*x + x**n*b**2*c*d*x + a**2*x + 2*a*b*c*x
+ b**2*c**2*x),x)*b**3*c*d**2*n*p**2 - x**(2*n)*int((x**n*b*d + a + b*c)**
p/(x**n*a*b*d*x + x**n*b**2*c*d*x + a**2*x + 2*a*b*c*x + b**2*c**2*x),x)*b
**3*c*d**2*n*p)/(2*x**(2*n)*e**(2*n)*e*n*(a + b*c))
```

3.352 $\int (ex)^{-1+3n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$\int (ex)^{-1+3n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx = -\frac{(6ac+b(2-p))x^{-3n}(ex)^{3n}(c+dx^n)^2\left(a+\frac{b}{c+dx^n}\right)^{1+p}}{6a^2d^3en} + \frac{x^{-3n}(ex)^{3n}(c+dx^n)^3\left(a+\frac{b}{c+dx^n}\right)^{1+p}}{3ad^3en} - \frac{b(6a^2c^2+b(6ac+b(2-p))(1-p))\,x^{-3n}(ex)^{3n}\left(a+\frac{b}{c+dx^n}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1\right.}{6a^4d^3en(1+p)}$$

output
```
-1/6*(6*a*c+b*(2-p))*(e*x)^(3*n)*(c+d*x^n)^2*(a+b/(c+d*x^n))^(p+1)/a^2/d^3
/e/n/(x^(3*n))+1/3*(e*x)^(3*n)*(c+d*x^n)^3*(a+b/(c+d*x^n))^(p+1)/a/d^3/e/n
/(x^(3*n))-1/6*b*(6*a^2*c^2+b*(6*a*c+b*(2-p))*(1-p))*(e*x)^(3*n)*(a+b/(c+d
*x^n))^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(c+d*x^n))/a^4/d^3/e/n/(p+1)/(
x^(3*n))
```

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{-1+3n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b/(c + d*x^n))^p,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b/(c + d*x^n))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}\left(a+\frac{b}{c+dx^n}\right)^p\,dx$$

$$\Big\downarrow \; 2057$$

$$\int (ex)^{3n-1}\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\,dx$$

$$\Big\downarrow \; 2054$$

$$x^{1-3n}(cx)^{3n-1}\int x^{3n-1}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p\,dx$$

$$\Big\downarrow \; 2053$$

$$\frac{x^{1-3n}(cx)^{3n-1}\int x^{2n}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p\,dx^n}{n}$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b/(c + d*x^n))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2054
```
Int[((f_)*(x_))^(m_)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)
^(n_.)))^(p_), x_Symbol] :> Simp[Simp[(c*x)^m/x^m]   Int[x^m*(e*((a + b*x^n
)/(c + d*x^n)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && Integ
erQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int (ex)^{-1+3n} \left(a + \frac{b}{c + d\,x^{n}}\right)^{p} dx$$

input
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+3n} \left(a + \frac{b}{c + dx^n}\right)^p dx = \int (ex)^{3\,n-1} \left(a + \frac{b}{dx^n + c}\right)^p dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(3*n - 1)*((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1+3*n)*(a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{3\,n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(3*n - 1)*(a + b/(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{3\,n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(3*n - 1)*(a + b/(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+\frac{b}{c+dx^n}\right)^p dx=\int (e\,x)^{3\,n-1}\left(a+\frac{b}{c+d\,x^n}\right)^p dx$$

input
```
int((e*x)^(3*n - 1)*(a + b/(c + d*x^n))^p,x)
```

output
```
int((e*x)^(3*n - 1)*(a + b/(c + d*x^n))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{c+dx^n}\right)^p dx=\frac{e^{3n}\left(\int \frac{x^{3n}(x^n ad+ac+b)^p}{(x^n d+c)^p x}dx\right)}{e}$$

input
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n))^p,x)
```

output
```
(e**(3*n)*int((x**(3*n)*(x**n*a*d + a*c + b)**p)/((x**n*d + c)**p*x),x))/e
```

3.353 $\int (ex)^{-1+2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int (ex)^{-1+2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\frac{x^{-2n}(ex)^{2n}\left(c+dx^n\right)^2\left(a+\frac{b}{c+dx^n}\right)^{1+p}}{2ad^2en}+\frac{b(b+2ac-bp)x^{-2n}(ex)^{2n}\left(a+\frac{b}{c+dx^n}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx^n)}\right)}{2a^3d^2en(1+p)}$$

output
```
1/2*(e*x)^(2*n)*(c+d*x^n)^2*(a+b/(c+d*x^n))^(p+1)/a/d^2/e/n/(x^(2*n))+1/2*
b*(2*a*c-b*p+b)*(e*x)^(2*n)*(a+b/(c+d*x^n))^(p+1)*hypergeom([2, p+1],[2+p]
,1+b/a/(c+d*x^n))/a^3/d^2/e/n/(p+1)/(x^(2*n))
```

Mathematica [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{-1+2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b/(c + d*x^n))^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b/(c + d*x^n))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1}\left(a+\frac{b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int (ex)^{2n-1}\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{ 2054}$$

$$x^{1-2n}(cx)^{2n-1}\int x^{2n-1}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{x^{1-2n}(cx)^{2n-1}\int x^n\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx^n}{n}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b/(c + d*x^n))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2054
```
Int[((f_)*(x_))^(m_)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)
^(n_.)))^(p_), x_Symbol] :> Simp[Simp[(c*x)^m/x^m]   Int[x^m*(e*((a + b*x^n
)/(c + d*x^n)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && Integ
erQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int (ex)^{-1+2n} \left(a + \frac{b}{c + d\, x^n}\right)^p dx$$

input
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+2n} \left(a + \frac{b}{c + dx^n}\right)^p dx = \int (ex)^{2n-1} \left(a + \frac{b}{dx^n + c}\right)^p dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(2*n - 1)*((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} \left(a + \frac{b}{c + dx^n}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+2*n)*(a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{2n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(a + b/(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{2n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(a + b/(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (e\,x)^{2n-1}\left(a+\frac{b}{c+d\,x^n}\right)^p\,dx$$

input
```
int((e*x)^(2*n - 1)*(a + b/(c + d*x^n))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b/(c + d*x^n))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+2n} \left(a + \frac{b}{c + dx^n}\right)^p dx = \frac{e^{2n} \left(\int \frac{x^{2n}(x^n ad + ac + b)^p}{(x^n d + c)^p x} dx\right)}{e}$$

input
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n))^p,x)
```

output
```
(e**(2*n)*int((x**(2*n)*(x**n*a*d + a*c + b)**p)/((x**n*d + c)**p*x),x))/e
```

3.354 $\int (ex)^{-1+n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int (ex)^{-1+n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx = -\frac{bx^{-n}(ex)^n \left(a+\frac{b}{c+dx^n}\right)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx^n)}\right)}{a^2den(1+p)}$$

output
```
-b*(e*x)^n*(a+b/(c+d*x^n))^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(c+d*x^n))
/a^2/d/e/n/(p+1)/(x^n)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int (ex)^{-1+n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx = -\frac{x^{1-n}(ex)^{-1+n}(c+dx^n)\left(a+\frac{b}{c+dx^n}\right)^p \left(1+\frac{a(c+dx^n)}{b}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{a(c+dx}{b}\right.}{dn(-1+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b/(c + d*x^n))^p,x]
```

output
```
-((x^(1 - n)*(e*x)^(-1 + n)*(c + d*x^n)*(a + b/(c + d*x^n))^p*Hypergeometr
ic2F1[1 - p, -p, 2 - p, -((a*(c + d*x^n))/b)])/(d*n*(-1 + p)*(1 + (a*(c +
d*x^n))/b)^p))
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2057, 2054, 2053, 7270, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} \left(a+\frac{b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{2057}$$

$$\int (ex)^{n-1} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{2054}$$

$$x^{1-n}(cx)^{n-1} \int x^{n-1} \left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{x^{1-n}(cx)^{n-1} \int \left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx^n}{n}$$

$$\Big\downarrow \text{7270}$$

$$\frac{x^{1-n}(cx)^{n-1} (c+dx^n)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int (dx^n+c)^{-p} (adx^n+b+ac)^p dx^n}{n}$$

$$\Big\downarrow \text{80}$$

$$\frac{x^{1-n}(cx)^{n-1} \left(-\frac{a(c+dx^n)}{b}\right)^p (ac+adx^n+b)^{-p} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p \int (adx^n+b+ac)^p \left(-\frac{adx^n}{b}-\frac{ac}{b}\right)^{-p} dx^n}{n}$$

$$\Big\downarrow \text{79}$$

$$\frac{x^{1-n}(cx)^{n-1}\left(ac+adx^n+b\right)\left(-\frac{a(c+dx^n)}{b}\right)^p\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{adx^n+b+ac}{b}\right)}{adn(p+1)}$$

input
```
Int[(e*x)^(-1 + n)*(a + b/(c + d*x^n))^p,x]
```

output
```
(x^(1 - n)*(c*x)^(-1 + n)*(-((a*(c + d*x^n))/b))^p*(b + a*c + a*d*x^n)*((b
 + a*c + a*d*x^n)/(c + d*x^n))^p*Hypergeometric2F1[p, 1 + p, 2 + p, (b + a
*c + a*d*x^n)/b])/(a*d*n*(1 + p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2054
```
Int[((f_)*(x_))^(m_)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)
^(n_.)))^(p_), x_Symbol] :> Simp[Simp[(c*x)^m/x^m]   Int[x^m*(e*((a + b*x^n
)/(c + d*x^n)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && Integ
erQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

Maple [F]

$$\int (ex)^{-1+n} \left(a+\frac{b}{c+d\,x^n}\right)^p dx$$

input
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n} \left(a+\frac{b}{c+dx^n}\right)^p dx = \int (ex)^{n-1} \left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(n - 1)*((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1+n)*(a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(n - 1)*(a + b/(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(a + b/(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} \left(a+\frac{b}{c+dx^n}\right)^p dx = \int (e\,x)^{n-1}\,\left(a+\frac{b}{c+d\,x^n}\right)^p dx$$

input
```
int((e*x)^(n - 1)*(a + b/(c + d*x^n))^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b/(c + d*x^n))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+n} \left(a+\frac{b}{c+dx^n}\right)^p dx = \frac{e^n \left(\int \frac{x^n (x^n ad+ac+b)^p}{(x^n d+c)^p x}dx\right)}{e}$$

input
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n))^p,x)
```

output
```
(e**n*int((x**n*(x**n*a*d + a*c + b)**p)/((x**n*d + c)**p*x),x))/e
```

3.355 $\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx = -\frac{c\left(a+\frac{b}{c+dx^n}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx^n}\right)}{b+ac}\right)}{(b+ac)en(1+p)} + \frac{\left(a+\frac{b}{c+dx^n}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a(c+dx^n)}\right)}{aen(1+p)}$$

output

```
-c*(a+b/(c+d*x^n))^(p+1)*hypergeom([1, p+1],[2+p],c*(a+b/(c+d*x^n))/(a*c+b
))/(a*c+b)/e/n/(p+1)+(a+b/(c+d*x^n))^(p+1)*hypergeom([1, p+1],[2+p],1+b/a/
(c+d*x^n))/a/e/n/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx$$

input
```
Integrate[(a + b/(c + d*x^n))^p/(e*x),x]
```

output
```
Integrate[(a + b/(c + d*x^n))^p/x, x]/e
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{x}dx}{e}$$

$$\Big\downarrow \ 2057$$

$$\frac{\int \frac{\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p}{x}dx}{e}$$

$$\Big\downarrow \ 2053$$

$$\frac{\int x^{-n}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx^n}{en}$$

input
```
Int[(a + b/(c + d*x^n))^p/(e*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{c+d\,x^{n}}\right)^{p}}{ex}dx$$

input
```
int((a+b/(c+d*x^n))^p/e/x,x)
```

output
```
int((a+b/(c+d*x^n))^p/e/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/e/x,x, algorithm="fricas")
```

output
```
integral(((a*d*x^n + a*c + b)/(d*x^n + c))^p/(e*x), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx=\frac{\int \frac{\left(\frac{ac}{c+dx^n}+\frac{adx^n}{c+dx^n}+\frac{b}{c+dx^n}\right)^p}{x}\,dx}{e}$$

input
```
integrate((a+b/(c+d*x**n))**p/e/x,x)
```

output
```
Integral((a*c/(c + d*x**n) + a*d*x**n/(c + d*x**n) + b/(c + d*x**n))**p/x,
 x)/e
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/e/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c))^p/x, x)/e
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{dx^n+c}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n))^p/e/x,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c))^p/(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^p}{e\,x}\,dx$$

input
```
int((a + b/(c + d*x^n))^p/(e*x),x)
```

output
```
int((a + b/(c + d*x^n))^p/(e*x), x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{c+dx^n}\right)^p}{ex}\,dx=\frac{\int \frac{(x^nad+ac+b)^p}{(x^nd+c)^p x}dx}{e}$$

input
```
int((a+b/(c+d*x^n))^p/e/x,x)
```

output
```
int((x**n*a*d + a*c + b)**p/((x**n*d + c)**p*x),x)/e
```

3.356 $\int (ex)^{-1-n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int (ex)^{-1-n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx = -\frac{bdx^n(ex)^{-n}\left(a+\frac{b}{c+dx^n}\right)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx^n}\right)}{b+ac}\right)}{(b+ac)^2en(1+p)}$$

output
```
-b*d*x^n*(a+b/(c+d*x^n))^(p+1)*hypergeom([2, p+1],[2+p],c*(a+b/(c+d*x^n))/
(a*c+b))/(a*c+b)^2/e/n/(p+1)/((e*x)^n)
```

Mathematica [F]

$$\int (ex)^{-1-n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx = \int (ex)^{-1-n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b/(c + d*x^n))^p,x]
```

output
```
Integrate[(e*x)^(-1 - n)*(a + b/(c + d*x^n))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1} \left(a+\frac{b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{2057}$$

$$\int (ex)^{-n-1} \left(\frac{ac+adx^n+b}{c+dx^n}\right)^p dx$$

$$\Big\downarrow \text{2054}$$

$$x^{n+1}(cx)^{-n-1}\int x^{-n-1}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{x^{n+1}(cx)^{-n-1}\int x^{-2n}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx^n}{n}$$

input
```
Int[(e*x)^(-1 - n)*(a + b/(c + d*x^n))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2054
```
Int[((f_)*(x_))^(m_)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)
^(n_.)))^(p_), x_Symbol] :> Simp[Simp[(c*x)^m/x^m]   Int[x^m*(e*((a + b*x^n
)/(c + d*x^n)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && Integ
erQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{c+d\,x^n}\right)^p dx$$

input
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{c+dx^n}\right)^p dx = \int (ex)^{-n-1}\left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(-n - 1)*((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-n}\left(a+\frac{b}{c+dx^n}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1-n)*(a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{-n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(-n - 1)*(a + b/(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{-n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(-n - 1)*(a + b/(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^p}{(e\,x)^{n+1}}\,dx$$

input
```
int((a + b/(c + d*x^n))^p/(e*x)^(n + 1),x)
```

output
```
int((a + b/(c + d*x^n))^p/(e*x)^(n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-n} \left(a + \frac{b}{c + dx^n}\right)^p dx = \frac{\int \frac{(x^n a d + a c + b)^p}{x^n (x^n d + c)^p x} dx}{e^n e}$$

input
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n))^p,x)
```

output
```
int((x**n*a*d + a*c + b)**p/(x**n*(x**n*d + c)**p*x),x)/(e**n*e)
```

3.357 $\int (ex)^{-1-2n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int (ex)^{-1-2n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx = -\frac{(ex)^{-2n}(c+dx^n)^2\left(a+\frac{b}{c+dx^n}\right)^{1+p}}{2c(b+ac)en} + \frac{bd^2(b+2ac+bp)x^{2n}(ex)^{-2n}\left(a+\frac{b}{c+dx^n}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c\left(a+\frac{b}{c+dx^n}\right)}{b+ac}\right)}{2c(b+ac)^3en(1+p)}$$

output
```
-1/2*(c+d*x^n)^2*(a+b/(c+d*x^n))^(p+1)/c/(a*c+b)/e/n/((e*x)^(2*n))+1/2*b*d
^2*(2*a*c+b*p+b)*x^(2*n)*(a+b/(c+d*x^n))^(p+1)*hypergeom([2, p+1],[2+p],c*
(a+b/(c+d*x^n))/(a*c+b))/c/(a*c+b)^3/e/n/(p+1)/((e*x)^(2*n))
```

Mathematica [F]

$$\int (ex)^{-1-2n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx = \int (ex)^{-1-2n} \left(a+\frac{b}{c+dx^n}\right)^p \, dx$$

input
```
Integrate[(e*x)^(-1 - 2*n)*(a + b/(c + d*x^n))^p,x]
```

output
```
Integrate[(e*x)^(-1 - 2*n)*(a + b/(c + d*x^n))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-2n-1}\left(a+\frac{b}{c+dx^n}\right)^p dx$$
$$\Big\downarrow \text{ 2057}$$
$$\int (ex)^{-2n-1}\left(\frac{ac+adx^n+b}{c+dx^n}\right)^p dx$$
$$\Big\downarrow \text{ 2054}$$
$$x^{2n+1}(cx)^{-2n-1}\int x^{-2n-1}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx$$
$$\Big\downarrow \text{ 2053}$$
$$\frac{x^{2n+1}(cx)^{-2n-1}\int x^{-3n}\left(\frac{adx^n+b+ac}{dx^n+c}\right)^p dx^n}{n}$$

input
```
Int[(e*x)^(-1 - 2*n)*(a + b/(c + d*x^n))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2054
```
Int[((f_)*(x_))^(m_)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)
^(n_.)))^(p_), x_Symbol] :> Simp[Simp[(c*x)^m/x^m]   Int[x^m*(e*((a + b*x^n
)/(c + d*x^n)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && Integ
erQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{c+d\,x^n}\right)^p dx$$

input
```
int((e*x)^(-1-2*n)*(a+b/(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1-2*n)*(a+b/(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{c+dx^n}\right)^p dx=\int (ex)^{-2\,n-1}\left(a+\frac{b}{dx^n+c}\right)^p dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b/(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(-2*n - 1)*((a*d*x^n + a*c + b)/(d*x^n + c))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+\frac{b}{c+dx^n}\right)^p dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1-2*n)*(a+b/(c+d*x**n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{-2n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b/(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(-2*n - 1)*(a + b/(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int (ex)^{-2n-1}\left(a+\frac{b}{dx^n+c}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b/(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(-2*n - 1)*(a + b/(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\int \frac{\left(a+\frac{b}{c+d\,x^n}\right)^p}{(e\,x)^{2\,n+1}}\,dx$$

input
```
int((a + b/(c + d*x^n))^p/(e*x)^(2*n + 1),x)
```

output
```
int((a + b/(c + d*x^n))^p/(e*x)^(2*n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{c+dx^n}\right)^p\,dx=\frac{\int \frac{(x^n ad+ac+b)^p}{x^{2n}(x^n d+c)^p x}dx}{e^{2n}e}$$

input
```
int((e*x)^(-1-2*n)*(a+b/(c+d*x^n))^p,x)
```

output
```
int((x**n*a*d + a*c + b)**p/(x**(2*n)*(x**n*d + c)**p*x),x)/(e**(2*n)*e)
```

3.358 $\int (ex)^{-1+3n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 238

$$\int (ex)^{-1+3n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p \, dx = \frac{x^{-3n}(ex)^{3n}(c+dx^n)^3\left(a+\frac{b}{(c+dx^n)^2}\right)^{1+p}}{3ad^3en}$$
$$+\frac{(3ac^2-b(1-2p))\,x^{-3n}(ex)^{3n}(c+dx^n)\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(1+\frac{b}{a(c+dx^n)^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-\right.}{3ad^3en}$$
$$+\frac{bcx^{-3n}(ex)^{3n}\left(a+\frac{b}{(c+dx^n)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx^n)^2}\right)}{a^2d^3en(1+p)}$$

output
```
1/3*(e*x)^(3*n)*(c+d*x^n)^3*(a+b/(c+d*x^n)^2)^(p+1)/a/d^3/e/n/(x^(3*n))+1/
3*(3*a*c^2-b*(1-2*p))*(e*x)^(3*n)*(c+d*x^n)*(a+b/(c+d*x^n)^2)^p*hypergeom(
[-1/2, -p],[1/2],-b/a/(c+d*x^n)^2)/a/d^3/e/n/(x^(3*n))/((1+b/a/(c+d*x^n)^2
)^p)+b*c*(e*x)^(3*n)*(a+b/(c+d*x^n)^2)^(p+1)*hypergeom([2, p+1],[2+p],1+b/
a/(c+d*x^n)^2)/a^2/d^3/e/n/(p+1)/(x^(3*n))
```

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{-1+3n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b/(c + d*x^n)^2)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.73 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {7273, 2089, 1804, 1802, 1291, 25, 27, 1269, 1118, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx$$

$$\Big\downarrow \ 7273$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{3n-1}(dx^n+c)^{-2p}\left(a(dx^n+c)^2+b\right)^p\,dx$$

$$\Big\downarrow \ 2089$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{3n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p\,dx$$

$$\Big\downarrow \ 1804$$

$$\frac{x^{-3n}(ex)^{3n}(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{3n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)}{e}$$

$$\Big\downarrow \ 1802$$

$$\frac{x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{2n}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p d}{en}$$

$\downarrow$ 1291

$$\frac{x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{\int -d^2(dx^n+c)^{-2p}(6acdx^n+3ac^2+b-2bp)\left(2acdx^n+ad^2x^{2n}+c\right.}{3ad^4}\right.}{en}$$

$\downarrow$ 25

$$\frac{x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(c+dx^n)^{1-2p}(ac^2+2acdx^n+ad^2x^{2n}+b)^{p+1}}{3ad^3}-\frac{\int d^2(dx^n+c)^{-}}{}\right.}{en}$$

$\downarrow$ 27

$$\frac{x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(c+dx^n)^{1-2p}(ac^2+2acdx^n+ad^2x^{2n}+b)^{p+1}}{3ad^3}-\frac{\int (dx^n+c)^{-2p}}{}\right.}{en}$$

$\downarrow$ 1269

$$\frac{x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(c+dx^n)^{1-2p}(ac^2+2acdx^n+ad^2x^{2n}+b)^{p+1}}{3ad^3}-\frac{6ac\int (dx^n+c)}{}\right.}{en}$$

$\downarrow$ 1118

$$\frac{x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(c+dx^n)^{1-2p}(ac^2+2acdx^n+ad^2x^{2n}+b)^{p+1}}{3ad^3}-\frac{\left(-3ac^2-2bp+b\right.}{}\right.}{en}$$

$\downarrow$ 279

$$\frac{x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(c+dx^n)^{1-2p}(ac^2+2acdx^n+ad^2x^{2n}+b)^{p+1}}{3ad^3}-\frac{\left(-3ac^2-2bp+\right.}{}\right.}{en}$$

$\downarrow$ 278

$$x^{-3n}(ex)^{3n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(c+dx^n)^{1-2p}\left(ac^2+2acdx^n+ad^2x^{2n}+b\right)^{p+1}}{3ad^3}-\frac{\left(-3ac^2-2bp+\right.}{}\right.$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
((e*x)^(3*n)*(c + d*x^n)^(2*p)*(a + b/(c + d*x^n)^2)^p*(((c + d*x^n)^(1 -
2*p)*(b + a*c^2 + 2*a*c*d*x^n + a*d^2*x^(2*n))^(1 + p))/(3*a*d^3) - (((b -
 3*a*c^2 - 2*b*p)*(c + d*x^n)^(1 - 2*p)*(b + a*x^(2*n))^p*Hypergeometric2F
1[(1 - 2*p)/2, -p, (3 - 2*p)/2, -((a*x^(2*n))/b)])/(d*(1 - 2*p)*(1 + (a*x^
(2*n))/b)^p) + (3*a*c*(c + d*x^n)^(2 - 2*p)*(b + a*x^(2*n))^p*Hypergeometr
ic2F1[1 - p, -p, 2 - p, -((a*x^(2*n))/b)])/(d*(1 - p)*(1 + (a*x^(2*n))/b)^
p))/(3*a*d^2)))/(e*n*x^(3*n)*(b + a*(c + d*x^n)^2)^p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1118
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[1/e   Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x,
d + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1291
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1804
```
Int[((f_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((
d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[f^IntPart[m]*((f*x)^FracPar
t[m]/x^FracPart[m])   Int[x^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[
Simplify[(m + 1)/n]]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [F]

$$\int (ex)^{-1+3n} \left(a+\frac{b}{(c+d\,x^n)^2}\right)^p dx$$

input
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+3n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = \int (ex)^{3\,n-1} \left(a+\frac{b}{(dx^n+c)^2}\right)^p dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(3*n - 1)*((a*d^2*x^(2*n) + 2*a*c*d*x^n + a*c^2 + b)/(d^2*x
^(2*n) + 2*c*d*x^n + c^2))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+3*n)*(a+b/(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+3n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = \int (ex)^{3n-1} \left(a+\frac{b}{(dx^n+c)^2}\right)^p dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(3*n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Giac [F]

$$\int (ex)^{-1+3n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = \int (ex)^{3n-1} \left(a+\frac{b}{(dx^n+c)^2}\right)^p dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(3*n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = \int (e\,x)^{3n-1} \left(a+\frac{b}{(c+d\,x^n)^2}\right)^p dx$$

input
```
int((e*x)^(3*n - 1)*(a + b/(c + d*x^n)^2)^p,x)
```

output
```
int((e*x)^(3*n - 1)*(a + b/(c + d*x^n)^2)^p, x)
```

Reduce [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = \frac{e^{3n}\left(\int \frac{x^{3n}(x^{2n}a\,d^2+2x^n acd+a\,c^2+b)^p}{(x^{2n}d^2+2x^n cd+c^2)^p x}dx\right)}{e}$$

input
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
(e**(3*n)*int((x**(3*n)*(x**(2*n)*a*d**2 + 2*x**n*a*c*d + a*c**2 + b)**p)/
((x**(2*n)*d**2 + 2*x**n*c*d + c**2)**p*x),x))/e
```

3.359 $\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx = -\frac{cx^{-2n}(ex)^{2n}(c+dx^n)\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(1+\frac{b}{a(c+dx^n)^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx^n)^2}\right)}{d^2en} - \frac{bx^{-2n}(ex)^{2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b}{a(c+dx^n)^2}\right)}{2a^2d^2en(1+p)}$$

output
```
-c*(e*x)^(2*n)*(c+d*x^n)*(a+b/(c+d*x^n)^2)^p*hypergeom([-1/2, -p],[1/2],-b
/a/(c+d*x^n)^2)/d^2/e/n/(x^(2*n))/((1+b/a/(c+d*x^n)^2)^p)-1/2*b*(e*x)^(2*n
)*(a+b/(c+d*x^n)^2)^(p+1)*hypergeom([2, p+1],[2+p],1+b/a/(c+d*x^n)^2)/a^2/
d^2/e/n/(p+1)/(x^(2*n))
```

Mathematica [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b/(c + d*x^n)^2)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.38 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {7273, 2089, 1804, 1802, 1269, 1118, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx$$

$$\Big\downarrow \text{ 7273}$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{2n-1}(dx^n+c)^{-2p}\left(a(dx^n+c)^2+b\right)^p\,dx$$

$$\Big\downarrow \text{ 2089}$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{2n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p\,dx$$

$$\Big\downarrow \text{ 1804}$$

$$\frac{x^{-2n}(ex)^{2n}(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{2n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^{}}{e}$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{x^{-2n}(ex)^{2n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^n(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx}{en}$$

$\Big\downarrow$ 1269

$$\frac{x^{-2n}(ex)^{2n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{\int(dx^n+c)^{1-2p}(2acdx^n+ad^2x^{2n}+ac^2+b)^p dx^n}{d}-\frac{c\int(dx^n+c)}{}\right.}{en}$$

$\Big\downarrow$ 1118

$$\frac{x^{-2n}(ex)^{2n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{\int(dx^n+c)^{1-2p}(ax^{2n}+b)^p d(dx^n+c)}{d^2}-\frac{c\int(dx^n+c)^{-2p}(ax^{2n}+}{d^2}\right.}{en}$$

$\Big\downarrow$ 279

$$\frac{x^{-2n}(ex)^{2n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(ax^{2n}+b)^p\left(\frac{ax^{2n}}{b}+1\right)^{-p}\int(dx^n+c)^{1-2p}\left(\frac{ax^{2n}}{b}+1\right)^p d(dx^n+c)}{d^2}\right.}{en}$$

$\Big\downarrow$ 278

$$\frac{x^{-2n}(ex)^{2n}\left(c+dx^n\right)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\left(\frac{(ax^{2n}+b)^p\left(\frac{ax^{2n}}{b}+1\right)^{-p}(c+dx^n)^{2-2p}\text{Hypergeometric2F1}\left(1-\right.}{2d^2(1-p)}\right.}{en}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
((e*x)^(2*n)*(c + d*x^n)^(2*p)*(a + b/(c + d*x^n)^2)^p*(-((c*(c + d*x^n)^(
1 - 2*p)*(b + a*x^(2*n))^p*Hypergeometric2F1[(1 - 2*p)/2, -p, (3 - 2*p)/2,
 -((a*x^(2*n))/b)])/(d^2*(1 - 2*p)*(1 + (a*x^(2*n))/b)^p)) + ((c + d*x^n)^
(2 - 2*p)*(b + a*x^(2*n))^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((a*x^(2*
n))/b)])/(2*d^2*(1 - p)*(1 + (a*x^(2*n))/b)^p)))/(e*n*x^(2*n)*(b + a*(c +
d*x^n)^2)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1118
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[1/e   Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x,
d + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1804
```
Int[((f_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((
d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[f^IntPart[m]*((f*x)^FracPar
t[m]/x^FracPart[m])   Int[x^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[
Simplify[(m + 1)/n]]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\left(c+d\,x^{n}\right)^{2}}\right)^{p}dx$$

input
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{2\,n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(2*n - 1)*((a*d^2*x^(2*n) + 2*a*c*d*x^n + a*c^2 + b)/(d^2*x
^(2*n) + 2*c*d*x^n + c^2))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1+2*n)*(a+b/(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{2n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{2n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (e\,x)^{2\,n-1}\,\left(a+\frac{b}{{\left(c+d\,x^n\right)}^2}\right)^p \,d x$$

input
```
int((e*x)^(2*n - 1)*(a + b/(c + d*x^n)^2)^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b/(c + d*x^n)^2)^p, x)
```

Reduce [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\frac{e^{2n}\left(\int \frac{x^{2n}\left(x^{2n}a\,d^2+2x^n acd+a\,c^2+b\right)^p}{\left(x^{2n}d^2+2x^n cd+c^2\right)^p x}d x\right)}{e}$$

input
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
(e**(2*n)*int((x**(2*n)*(x**(2*n)*a*d**2 + 2*x**n*a*c*d + a*c**2 + b)**p)/
((x**(2*n)*d**2 + 2*x**n*c*d + c**2)**p*x),x))/e
```

3.360 $\int (ex)^{-1+n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int (ex)^{-1+n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p \, dx = \frac{x^{-n}(ex)^n (c+dx^n) \left(a+\frac{b}{(c+dx^n)^2}\right)^p \left(1+\frac{b}{a(c+dx^n)^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx^n)^2}\right)}{den}$$

output
```
(e*x)^n*(c+d*x^n)*(a+b/(c+d*x^n)^2)^p*hypergeom([-1/2, -p],[1/2],-b/a/(c+d
*x^n)^2)/d/e/n/(x^n)/((1+b/a/(c+d*x^n)^2)^p)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int (ex)^{-1+n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p \, dx = \frac{x^{1-n}(ex)^{-1+n} (c+dx^n) \left(a+\frac{b}{(c+dx^n)^2}\right)^p \left(1+\frac{b}{a(c+dx^n)^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b}{a(c+dx^n)^2}\right)}{dn}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
(x^(1 - n)*(e*x)^(-1 + n)*(c + d*x^n)*(a + b/(c + d*x^n)^2)^p*Hypergeometr
ic2F1[-1/2, -p, 1/2, -(b/(a*(c + d*x^n)^2))])/(d*n*(1 + b/(a*(c + d*x^n)^2
))^p)
```

Rubi [A] (warning: unable to verify)

Time = 1.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {7273, 2089, 1804, 1798, 1118, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx$$

$$\Big\downarrow \text{7273}$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{n-1}(dx^n+c)^{-2p}\left(a(dx^n+c)^2+b\right)^p dx$$

$$\Big\downarrow \text{2089}$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx$$

$$\Big\downarrow \text{1804}$$

$$\frac{x^{-n}(ex)^n(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p d}{e}$$

$$\Big\downarrow \text{1798}$$

$$\frac{x^{-n}(ex)^n(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx^n}{en}$$

$$\Big\downarrow \text{1118}$$

$$\frac{x^{-n}(ex)^n\,(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (dx^n+c)^{-2p}\left(ax^{2n}+b\right)^p d(dx^n+c)}{den}$$

$$\Big\downarrow \text{279}$$

$$\frac{x^{-n}(ex)^n\left(ax^{2n}+b\right)^p\left(\frac{ax^{2n}}{b}+1\right)^{-p}(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (dx^n+c)^{-2p}\left(\frac{ax^{2n}}{b}+1\right)}{den}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{-n}(ex)^n\,(c+dx^n)\left(ax^{2n}+b\right)^p\left(\frac{ax^{2n}}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),-p,\frac{1}{2}(3-2p),-\frac{ax^{2n}}{b}\right)\left(a+\frac{b}{(c+dx}\right.}{den(1-2p)}$$

input
```
Int[(e*x)^(-1 + n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
((e*x)^n*(c + d*x^n)*(b + a*x^(2*n))^p*(a + b/(c + d*x^n)^2)^p*Hypergeomet
ric2F1[(1 - 2*p)/2, -p, (3 - 2*p)/2, -((a*x^(2*n))/b)])/(d*e*n*(1 - 2*p)*x
^n*(1 + (a*x^(2*n))/b)^p*(b + a*(c + d*x^n)^2)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1118
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[1/e   Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x,
d + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 1804
```
Int[((f_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((
d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[f^IntPart[m]*((f*x)^FracPar
t[m]/x^FracPart[m])   Int[x^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[
Simplify[(m + 1)/n]]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{(c+d\,x^n)^2}\right)^p dx$$

input
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(n - 1)*((a*d^2*x^(2*n) + 2*a*c*d*x^n + a*c^2 + b)/(d^2*x^(
2*n) + 2*c*d*x^n + c^2))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1+n)*(a+b/(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx=\int (ex)^{n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx=\int (e\,x)^{n-1}\left(a+\frac{b}{(c+d\,x^n)^2}\right)^p dx$$

input
```
int((e*x)^(n - 1)*(a + b/(c + d*x^n)^2)^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b/(c + d*x^n)^2)^p, x)
```

Reduce [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx=\frac{e^n\left(\int \frac{x^n(x^{2n}a\,d^2+2x^n acd+a\,c^2+b)^p}{(x^{2n}d^2+2x^n cd+c^2)^p x}dx\right)}{e}$$

input
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
(e**n*int((x**n*(x**(2*n)*a*d**2 + 2*x**n*a*c*d + a*c**2 + b)**p)/((x**(2*
n)*d**2 + 2*x**n*c*d + c**2)**p*x),x))/e
```

3.361 $\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 230

$$\begin{aligned}\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx &= -\frac{c\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(1+\frac{b}{a(c+dx^n)^2}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b}{a(c+dx^n)^2},\frac{c^2}{(c+dx^n)^2}\right)}{en\,(c+dx^n)}\\
&\quad-\frac{c^2\left(a+\frac{b}{(c+dx^n)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{c^2\left(a+\frac{b}{(c+dx^n)^2}\right)}{b+ac^2}\right)}{2\,(b+ac^2)\,en(1+p)}\\
&\quad+\frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a(c+dx^n)^2}\right)}{2aen(1+p)}\end{aligned}$$

output
```
-c*(a+b/(c+d*x^n)^2)^p*AppellF1(1/2,1,-p,3/2,c^2/(c+d*x^n)^2,-b/a/(c+d*x^n
)^2)/e/n/(c+d*x^n)/((1+b/a/(c+d*x^n)^2)^p)-1/2*c^2*(a+b/(c+d*x^n)^2)^(p+1)
*hypergeom([1, p+1],[2+p],c^2*(a+b/(c+d*x^n)^2)/(a*c^2+b))/(a*c^2+b)/e/n/(
p+1)+1/2*(a+b/(c+d*x^n)^2)^(p+1)*hypergeom([1, p+1],[2+p],1+b/a/(c+d*x^n)^
2)/a/e/n/(p+1)
```

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx$$

input
```
Integrate[(a + b/(c + d*x^n)^2)^p/(e*x),x]
```

output
```
Integrate[(a + b/(c + d*x^n)^2)^p/x, x]/e
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(a+\frac{b}{(dx^n+c)^2}\right)^p}{x}dx}{e}$$

$$\Big\downarrow \ 7273$$

$$\frac{(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int \frac{(dx^n+c)^{-2p}\left(a(dx^n+c)^2+b\right)^p}{x}dx}{e}$$

$$\Big\downarrow \ 2089$$

$$\frac{(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int \frac{(dx^n+c)^{-2p}(2acdx^n+ad^2x^{2n}+ac^2+b)^p}{x}dx}{e}$$

$$\Big\downarrow \ 1802$$

$$\frac{(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{-n}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx^n}{en}$$

$$\Big\downarrow \ 1292$$

$$\frac{(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{-n}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx^n}{en}$$

input
```
Int[(a + b/(c + d*x^n)^2)^p/(e*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{(c+d\,x^n)^2}\right)^p}{ex}dx$$

input
```
int((a+b/(c+d*x^n)^2)^p/e/x,x)
```

output
```
int((a+b/(c+d*x^n)^2)^p/e/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{(dx^n+c)^2}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n)^2)^p/e/x,x, algorithm="fricas")
```

output
```
integral(((a*d^2*x^(2*n) + 2*a*c*d*x^n + a*c^2 + b)/(d^2*x^(2*n) + 2*c*d*x
^n + c^2))^p/(e*x), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx=\frac{\int \frac{\left(\frac{ac^2}{c^2+2cdx^n+d^2x^{2n}}+\frac{2acdx^n}{c^2+2cdx^n+d^2x^{2n}}+\frac{ad^2x^{2n}}{c^2+2cdx^n+d^2x^{2n}}+\frac{b}{c^2+2cdx^n+d^2x^{2n}}\right)^p}{x}\,dx}{e}$$

input
```
integrate((a+b/(c+d*x**n)**2)**p/e/x,x)
```

output
```
Integral((a*c**2/(c**2 + 2*c*d*x**n + d**2*x**(2*n)) + 2*a*c*d*x**n/(c**2
+ 2*c*d*x**n + d**2*x**(2*n)) + a*d**2*x**(2*n)/(c**2 + 2*c*d*x**n + d**2*
x**(2*n)) + b/(c**2 + 2*c*d*x**n + d**2*x**(2*n)))**p/x, x)/e
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{(dx^n+c)^2}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n)^2)^p/e/x,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^n + c)^2)^p/x, x)/e
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{(dx^n+c)^2}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n)^2)^p/e/x,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^n + c)^2)^p/(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{(c+d\,x^n)^2}\right)^p}{e\,x}\,dx$$

input
```
int((a + b/(c + d*x^n)^2)^p/(e*x),x)
```

output
```
int((a + b/(c + d*x^n)^2)^p/(e*x), x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{(c+dx^n)^2}\right)^p}{ex}\,dx = \frac{\int \frac{(x^{2n}a\,d^2+2x^n acd+a\,c^2+b)^p}{(x^{2n}d^2+2x^n cd+c^2)^p x}dx}{e}$$

input
```
int((a+b/(c+d*x^n)^2)^p/e/x,x)
```

output
```
int((x**(2*n)*a*d**2 + 2*x**n*a*c*d + a*c**2 + b)**p/((x**(2*n)*d**2 + 2*x
**n*c*d + c**2)**p*x),x)/e
```

3.362 $\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 296

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = -\frac{dx^n(ex)^{-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(1+\frac{b}{a(c+dx^n)^2}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{b}{a(c+dx^n)^2},\frac{c^2}{(c+dx^n)^2}\right)}{en(c+dx^n)} - \frac{c^2dx^n(ex)^{-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(1+\frac{b}{a(c+dx^n)^2}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{b}{a(c+dx^n)^2},\frac{c^2}{(c+dx^n)^2}\right)}{3en(c+dx^n)^3} - \frac{bcdx^n(ex)^{-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{c^2\left(a+\frac{b}{(c+dx^n)^2}\right)}{b+ac^2}\right)}{(b+ac^2)^2\,en(1+p)}$$

output
```
-d*x^n*(a+b/(c+d*x^n)^2)^p*AppellF1(1/2,2,-p,3/2,c^2/(c+d*x^n)^2,-b/a/(c+d
*x^n)^2)/e/n/((e*x)^n)/(c+d*x^n)/((1+b/a/(c+d*x^n)^2)^p)-1/3*c^2*d*x^n*(a+
b/(c+d*x^n)^2)^p*AppellF1(3/2,2,-p,5/2,c^2/(c+d*x^n)^2,-b/a/(c+d*x^n)^2)/e
/n/((e*x)^n)/(c+d*x^n)^3/((1+b/a/(c+d*x^n)^2)^p)-b*c*d*x^n*(a+b/(c+d*x^n)^
2)^(p+1)*hypergeom([2, p+1],[2+p],c^2*(a+b/(c+d*x^n)^2)/(a*c^2+b))/(a*c^2+
b)^2/e/n/(p+1)/((e*x)^n)
```

Mathematica [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx=\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
Integrate[(e*x)^(-1 - n)*(a + b/(c + d*x^n)^2)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx$$

$$\Big\downarrow \text{ 7273}$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{-n-1}(dx^n+c)^{-2p}\left(a(dx^n+c)^2+b\right)^p dx$$

$$\Big\downarrow \text{ 2089}$$

$$(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int (ex)^{-n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx$$

$$\Big\downarrow \text{ 1804}$$

$$\frac{x^n(ex)^{-n}(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{-n-1}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p}{e}$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{x^n(ex)^{-n}(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{-2n}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx}{en}$$

$$\Big\downarrow \text{1292}$$

$$\frac{x^n(ex)^{-n}(c+dx^n)^{2p}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\left(a(c+dx^n)^2+b\right)^{-p}\int x^{-2n}(dx^n+c)^{-2p}\left(2acdx^n+ad^2x^{2n}+ac^2+b\right)^p dx}{en}$$

input
```
Int[(e*x)^(-1 - n)*(a + b/(c + d*x^n)^2)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1804
```
Int[((f_)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((
d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[f^IntPart[m]*((f*x)^FracPar
t[m]/x^FracPart[m])   Int[x^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[
Simplify[(m + 1)/n]]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+d\,x^n)^2}\right)^p dx$$

input
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n)^2)^p,x)
```

Fricas [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx=\int (ex)^{-n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(-n - 1)*((a*d^2*x^(2*n) + 2*a*c*d*x^n + a*c^2 + b)/(d^2*x^
(2*n) + 2*c*d*x^n + c^2))^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1-n)*(a+b/(c+d*x**n)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{-n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(-n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Giac [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int (ex)^{-n-1}\left(a+\frac{b}{(dx^n+c)^2}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n)^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(-n - 1)*(a + b/(d*x^n + c)^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-n}\left(a+\frac{b}{(c+dx^n)^2}\right)^p\,dx=\int \frac{\left(a+\frac{b}{(c+d\,x^n)^2}\right)^p}{(e\,x)^{n+1}}\,dx$$

input
```
int((a + b/(c + d*x^n)^2)^p/(e*x)^(n + 1),x)
```

output
```
int((a + b/(c + d*x^n)^2)^p/(e*x)^(n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-n} \left(a+\frac{b}{(c+dx^n)^2}\right)^p dx = \frac{\int \frac{(x^{2n}a\,d^2+2x^n acd+a\,c^2+b)^p}{x^n(x^{2n}d^2+2x^n cd+c^2)^p x}dx}{e^n e}$$

input
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n)^2)^p,x)
```

output
```
int((x**(2*n)*a*d**2 + 2*x**n*a*c*d + a*c**2 + b)**p/(x**n*(x**(2*n)*d**2
+ 2*x**n*c*d + c**2)**p*x),x)/(e**n*e)
```

3.363 $\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 319

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx = -\frac{2ax^{-3n}(ex)^{3n}\left(a+b(c+dx^n)^{3/2}\right)^{1+p}}{3b^2d^3en(1+p)(2+p)}$$
$$+\frac{2x^{-3n}(ex)^{3n}(c+dx^n)^{3/2}\left(a+b(c+dx^n)^{3/2}\right)^{1+p}}{3bd^3en(2+p)}$$
$$+\frac{c^2x^{-3n}(ex)^{3n}(c+dx^n)\left(a+b(c+dx^n)^{3/2}\right)^p\left(1+\frac{b(c+dx^n)^{3/2}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{b(c+dx^n)}{a}\right.}{d^3en}$$
$$-\frac{cx^{-3n}(ex)^{3n}(c+dx^n)^2\left(a+b(c+dx^n)^{3/2}\right)^p\left(1+\frac{b(c+dx^n)^{3/2}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{b(c+dx^n)}{a}\right.}{d^3en}$$

output
```
-2/3*a*(e*x)^(3*n)*(a+b*(c+d*x^n)^(3/2))^(p+1)/b^2/d^3/e/n/(p+1)/(2+p)/(x^
(3*n))+2/3*(e*x)^(3*n)*(c+d*x^n)^(3/2)*(a+b*(c+d*x^n)^(3/2))^(p+1)/b/d^3/e
/n/(2+p)/(x^(3*n))+c^2*(e*x)^(3*n)*(c+d*x^n)*(a+b*(c+d*x^n)^(3/2))^p*hyper
geom([2/3, -p],[5/3],-b*(c+d*x^n)^(3/2)/a)/d^3/e/n/(x^(3*n))/((1+b*(c+d*x^
n)^(3/2)/a)^p)-c*(e*x)^(3*n)*(c+d*x^n)^2*(a+b*(c+d*x^n)^(3/2))^p*hypergeom
([4/3, -p],[7/3],-b*(c+d*x^n)^(3/2)/a)/d^3/e/n/(x^(3*n))/((1+b*(c+d*x^n)^(
3/2)/a)^p)
```

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^(3/2))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx$$

$$\Big\downarrow \ 7299$$

$$\int (ex)^{3n-1}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+3n} \left(a + b(c + d\,x^n)^{\frac{3}{2}}\right)^p dx$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+3n} \left(a + b(c + dx^n)^{3/2}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} \left(a + b(c + dx^n)^{3/2}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+3*n)*(a+b*(c+d*x**n)**(3/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p(ex)^{3\,n-1}\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(3*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p(ex)^{3\,n-1}\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int (e\,x)^{3\,n-1}\,\left(a+b\,(c+d\,x^n)^{3/2}\right)^p\,dx$$

input
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n)^(3/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\frac{e^{3n}\left(\int \frac{x^{3n}\left(x^n\sqrt{x^n d+c}\,bd+\sqrt{x^n d+c}\,bc+a\right)^p}{x}dx\right)}{e}$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
(e**(3*n)*int((x**(3*n)*(x**n*sqrt(x**n*d + c)*b*d + sqrt(x**n*d + c)*b*c
+ a)**p)/x,x))/e
```

3.364 $\int (ex)^{-1+2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 198

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = -\frac{cx^{-2n}(ex)^{2n}(c+dx^n)\left(a+b(c+dx^n)^{3/2}\right)^p\left(1+\frac{b(c+dx^n)^{3/2}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{b(c+dx^n)^{3}}{a}\right.}{d^2en} + \frac{x^{-2n}(ex)^{2n}(c+dx^n)^2\left(a+b(c+dx^n)^{3/2}\right)^p\left(1+\frac{b(c+dx^n)^{3/2}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{b(c+dx^n)^{3/}}{a}\right.}{2d^2en}$$

output
```
-c*(e*x)^(2*n)*(c+d*x^n)*(a+b*(c+d*x^n)^(3/2))^p*hypergeom([2/3, -p],[5/3]
,-b*(c+d*x^n)^(3/2)/a)/d^2/e/n/(x^(2*n))/((1+b*(c+d*x^n)^(3/2)/a)^p)+1/2*(
e*x)^(2*n)*(c+d*x^n)^2*(a+b*(c+d*x^n)^(3/2))^p*hypergeom([4/3, -p],[7/3],-
b*(c+d*x^n)^(3/2)/a)/d^2/e/n/(x^(2*n))/((1+b*(c+d*x^n)^(3/2)/a)^p)
```

Mathematica [F]

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \int (ex)^{-1+2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^(3/2))^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{2n-1} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+2n}\left(a+b(c+d\,x^{n})^{\frac{3}{2}}\right)^{p}dx$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+2n}\left(a+b(c+dx^{n})^{3/2}\right)^{p}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+b(c+dx^{n})^{3/2}\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*(c+d*x**n)**(3/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+2n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p(ex)^{2\,n-1}\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(2*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p(ex)^{2\,n-1}\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int (e\,x)^{2\,n-1}\,\left(a+b\,(c+d\,x^n)^{3/2}\right)^p\,dx$$

input
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n)^(3/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \frac{e^{2n} \left(\int \frac{x^{2n} \left(x^n \sqrt{x^n d+c}\, bd+\sqrt{x^n d+c}\, bc+a\right)^p}{x} dx\right)}{e}$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
(e**(2*n)*int((x**(2*n)*(x**n*sqrt(x**n*d + c)*b*d + sqrt(x**n*d + c)*b*c
+ a)**p)/x,x))/e
```

3.365 $\int (ex)^{-1+n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\frac{x^{-n}(ex)^n\,(c+dx^n)\left(a+b(c+dx^n)^{3/2}\right)^p\left(1+\frac{b(c+dx^n)^{3/2}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\right.}{den}$$

output
```
(e*x)^n*(c+d*x^n)*(a+b*(c+d*x^n)^(3/2))^p*hypergeom([2/3, -p],[5/3],-b*(c+
d*x^n)^(3/2)/a)/d/e/n/(x^n)/((1+b*(c+d*x^n)^(3/2)/a)^p)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\frac{x^{1-n}(ex)^{-1+n}\,(c+dx^n)\left(a+b(c+dx^n)^{3/2}\right)^p\left(1+\frac{b(c+dx^n)^{3/2}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},\right.}{dn}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
(x^(1 - n)*(e*x)^(-1 + n)*(c + d*x^n)*(a + b*(c + d*x^n)^(3/2))^p*Hypergeo
metric2F1[2/3, -p, 5/3, -((b*(c + d*x^n)^(3/2))/a)])/(d*n*(1 + (b*(c + d*x
^n)^(3/2))/a)^p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} \left(a+b(c+dx^n)^{3/2}\right)^p dx$$

$$\Big\downarrow \; 7299$$

$$\int (ex)^{n-1} \left(a+b(c+dx^n)^{3/2}\right)^p dx$$

input
```
Int[(e*x)^(-1 + n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+n} \left(a+b(c+d\,x^n)^{\frac{3}{2}}\right)^p dx$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+n} \left(a + b(c + dx^n)^{3/2}\right)^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+n} \left(a + b(c + dx^n)^{3/2}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+n)*(a+b*(c+d*x**n)**(3/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+n} \left(a + b(c + dx^n)^{3/2}\right)^p \, dx = \int \left((dx^n + c)^{\frac{3}{2}} b + a\right)^p (ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p(ex)^{n-1}\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int (e\,x)^{n-1}\left(a+b\,(c+d\,x^n)^{3/2}\right)^p\,dx$$

input
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n)^(3/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\frac{e^n\left(\int \frac{x^n\left(x^n\sqrt{x^nd+c}\,bd+\sqrt{x^nd+c}\,bc+a\right)^p}{x}dx\right)}{e}$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
(e**n*int((x**n*(x**n*sqrt(x**n*d + c)*b*d + sqrt(x**n*d + c)*b*c + a)**p)
/x,x))/e
```

3.366 $\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex} \, dx = \frac{d\text{Int}\left(\frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{dx},x\right)}{e}$$

output
```
d*Defer(Int)((a+b*(c+d*x^n)^(3/2))^p/d/x,x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex} \, dx = \int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex} \, dx$$

input
```
Integrate[(a + b*(c + d*x^n)^(3/2))^p/(e*x),x]
```

output
```
Integrate[(a + b*(c + d*x^n)^(3/2))^p/x, x]/e
```

Rubi [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {27, 7282, 896, 25, 7267, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(b(dx^n+c)^{3/2}+a\right)^p}{x}dx}{e}$$

$$\Big\downarrow \; 7282$$

$$\frac{\int x^{-n}\left(b(dx^n+c)^{3/2}+a\right)^p dx^n}{en}$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{x^{-n}\left(b(dx^n+c)^{3/2}+a\right)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{x^{-n}\left(b(dx^n+c)^{3/2}+a\right)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow \; 7267$$

$$-\frac{2\int \frac{\sqrt{dx^n+c}(bx^{3n}+a)^p}{c-x^{2n}}d\sqrt{dx^n+c}}{en}$$

$$\Big\downarrow \; 7276$$

$$-\frac{2\int \left(\frac{(bx^{3n}+a)^p}{2(-dx^n-c+\sqrt{c})}-\frac{(bx^{3n}+a)^p}{2(\sqrt{c}+\sqrt{dx^n+c})}\right)d\sqrt{dx^n+c}}{en}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(\frac{1}{2}\int \frac{(bx^{3n}+a)^p}{-dx^n-c+\sqrt{c}}d\sqrt{dx^n+c}-\frac{1}{2}\int \frac{(bx^{3n}+a)^p}{\sqrt{c}+\sqrt{dx^n+c}}d\sqrt{dx^n+c}\right)}{en}$$

input
```
Int[(a + b*(c + d*x^n)^(3/2))^p/(e*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b(c+d\,x^{n})^{\frac{3}{2}}\right)^{p}}{ex}dx$$

input
```
int((a+b*(c+d*x^n)^(3/2))^p/e/x,x)
```

output
```
int((a+b*(c+d*x^n)^(3/2))^p/e/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+b(c+dx^{n})^{3/2}\right)^{p}}{ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c+d*x^n)^(3/2))^p/e/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(c+d*x**n)**(3/2))**p/e/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex}\,dx = \int \frac{\left((dx^n+c)^{\frac{3}{2}}b+a\right)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^(3/2))^p/e/x,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p/x, x)/e
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex}\,dx = \int \frac{\left((dx^n+c)^{\frac{3}{2}}b+a\right)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^(3/2))^p/e/x,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p/(e*x), x)
```

Mupad [N/A]

Not integrable

Time = 8.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex}\,dx=\int \frac{\left(a+b\,\left(c+d\,x^n\right)^{3/2}\right)^p}{e\,x}\,dx$$

input
```
int((a + b*(c + d*x^n)^(3/2))^p/(e*x),x)
```

output
```
int((a + b*(c + d*x^n)^(3/2))^p/(e*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+b(c+dx^n)^{3/2}\right)^p}{ex}\,dx=\frac{\int \frac{\left(x^n\sqrt{x^nd+c}\,bd+\sqrt{x^nd+c}\,bc+a\right)^p}{x}dx}{e}$$

input
```
int((a+b*(c+d*x^n)^(3/2))^p/e/x,x)
```

output
```
int((x**n*sqrt(x**n*d + c)*b*d + sqrt(x**n*d + c)*b*c + a)**p/x,x)/e
```

3.367 $\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \frac{d^2x^n(ex)^{-n}\text{Int}\left(\frac{x^{-1-n}\left(a+b(c+dx^n)^{3/2}\right)^p}{d^2},x\right)}{e}$$

output
```
d^2*x^n*Defer(Int)(x^(-1-n)*(a+b*(c+d*x^n)^(3/2))^p/d^2,x)/e/((e*x)^n)
```

Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
Integrate[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^(3/2))^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1} \left(a+b(c+dx^n)^{3/2}\right)^p dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{-n-1} \left(a+b(c+dx^n)^{3/2}\right)^p dx$$

input
```
Int[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int (ex)^{-1-n} \left(a+b(c+d\,x^n)^{\frac{3}{2}}\right)^p dx$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1-n)*(a+b*(c+d*x**n)**(3/2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p\,dx = \int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p (ex)^{-n-1}\,dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(-n - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(-n - 1), x)
```

Mupad [N/A]

Not integrable

Time = 8.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \int \frac{\left(a+b\,\left(c+d\,x^n\right)^{3/2}\right)^p}{\left(e\,x\right)^{n+1}} \, dx$$

input
```
int((a + b*(c + d*x^n)^(3/2))^p/(e*x)^(n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^(3/2))^p/(e*x)^(n + 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int (ex)^{-1-n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\frac{\int \frac{\left(x^n\sqrt{x^n d+c}\,bd+\sqrt{x^n d+c}\,bc+a\right)^p}{x^n x}dx}{e^n e}$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
int((x**n*sqrt(x**n*d + c)*b*d + sqrt(x**n*d + c)*b*c + a)**p/(x**n*x),x)/
(e**n*e)
```

3.368 $\int (ex)^{-1-2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \frac{d^3 x^{2n} (ex)^{-2n} \text{Int}\left(\frac{x^{-1-2n}\left(a+b(c+dx^n)^{3/2}\right)^p}{d^3}, x\right)}{e}$$

output
```
d^3*x^(2*n)*Defer(Int)(x^(-1-2*n)*(a+b*(c+d*x^n)^(3/2))^p/d^3,x)/e/((e*x)^
(2*n))
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx = \int (ex)^{-1-2n} \left(a+b(c+dx^n)^{3/2}\right)^p \, dx$$

input
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^(3/2))^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-2n-1} \left(a+b(c+dx^n)^{3/2}\right)^p dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{-2n-1} \left(a+b(c+dx^n)^{3/2}\right)^p dx$$

input
```
Int[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^(3/2))^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int (ex)^{-1-2n} \left(a+b(c+d\,x^n)^{\frac{3}{2}}\right)^p dx$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1-2*n)*(a+b*(c+d*x**n)**(3/2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p(ex)^{-2\,n-1}\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(-2*n - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^{3/2}\right)^p dx = \int \left((dx^n+c)^{\frac{3}{2}}b+a\right)^p (ex)^{-2\,n-1}\, dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(3/2))^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^(3/2)*b + a)^p*(e*x)^(-2*n - 1), x)
```

Mupad [N/A]

Not integrable

Time = 8.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^{3/2}\right)^p dx = \int \frac{\left(a+b\,(c+d\,x^n)^{3/2}\right)^p}{(e\,x)^{2\,n+1}}\, dx$$

input
```
int((a + b*(c + d*x^n)^(3/2))^p/(e*x)^(2*n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^(3/2))^p/(e*x)^(2*n + 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^{3/2}\right)^p\,dx=\frac{\int \frac{\left(x^n\sqrt{x^n d+c}\,bd+\sqrt{x^n d+c}\,bc+a\right)^p}{x^{2n}x}dx}{e^{2n}e}$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(3/2))^p,x)
```

output
```
int((x**n*sqrt(x**n*d + c)*b*d + sqrt(x**n*d + c)*b*c + a)**p/(x**(2*n)*x)
,x)/(e**(2*n)*e)
```

3.369 $\int (ex)^{-1+3n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 362

$$\begin{aligned}
\int (ex)^{-1+3n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx
&= -\frac{2a(a^2-b^2c)^2 x^{-3n}(ex)^{3n}\left(a+b\sqrt{c+dx^n}\right)^{1+p}}{b^6d^3en(1+p)} \\
&\quad + \frac{2(5a^4-6a^2b^2c+b^4c^2)\, x^{-3n}(ex)^{3n}\left(a+b\sqrt{c+dx^n}\right)^{2+p}}{b^6d^3en(2+p)} \\
&\quad - \frac{4a(5a^2-3b^2c)\, x^{-3n}(ex)^{3n}\left(a+b\sqrt{c+dx^n}\right)^{3+p}}{b^6d^3en(3+p)} \\
&\quad + \frac{4(5a^2-b^2c)\, x^{-3n}(ex)^{3n}\left(a+b\sqrt{c+dx^n}\right)^{4+p}}{b^6d^3en(4+p)} \\
&\quad - \frac{10ax^{-3n}(ex)^{3n}\left(a+b\sqrt{c+dx^n}\right)^{5+p}}{b^6d^3en(5+p)} + \frac{2x^{-3n}(ex)^{3n}\left(a+b\sqrt{c+dx^n}\right)^{6+p}}{b^6d^3en(6+p)}
\end{aligned}$$

output

```
-2*a*(-b^2*c+a^2)^2*(e*x)^(3*n)*(a+b*(c+d*x^n)^(1/2))^(p+1)/b^6/d^3/e/n/(p
+1)/(x^(3*n))+2*(b^4*c^2-6*a^2*b^2*c+5*a^4)*(e*x)^(3*n)*(a+b*(c+d*x^n)^(1/
2))^(2+p)/b^6/d^3/e/n/(2+p)/(x^(3*n))-4*a*(-3*b^2*c+5*a^2)*(e*x)^(3*n)*(a+
b*(c+d*x^n)^(1/2))^(3+p)/b^6/d^3/e/n/(3+p)/(x^(3*n))+4*(-b^2*c+5*a^2)*(e*x
)^(3*n)*(a+b*(c+d*x^n)^(1/2))^(4+p)/b^6/d^3/e/n/(4+p)/(x^(3*n))-10*a*(e*x)
^(3*n)*(a+b*(c+d*x^n)^(1/2))^(5+p)/b^6/d^3/e/n/(5+p)/(x^(3*n))+2*(e*x)^(3*
n)*(a+b*(c+d*x^n)^(1/2))^(6+p)/b^6/d^3/e/n/(6+p)/(x^(3*n))
```

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{-1+3n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Sqrt[c + d*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{3n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+3n} \left(a + b\sqrt{c + d\,x^n}\right)^p dx$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+3n} \left(a + b\sqrt{c + dx^n}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} \left(a + b\sqrt{c + dx^n}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+3*n)*(a+b*(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+3n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{3\,n-1}\left(\sqrt{dx^n+c}b+a\right)^p\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(3*n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1+3n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{3\,n-1}\left(\sqrt{dx^n+c}b+a\right)^p\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(3*n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (e\,x)^{3\,n-1}\,\left(a+b\,\sqrt{c+d\,x^n}\right)^p\,dx$$

input
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(3*n - 1)*(a + b*(c + d*x^n)^(1/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+3n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\frac{e^{3n}\left(\int\frac{x^{3n}\left(\sqrt{x^n d+c}\,b+a\right)^p}{x}dx\right)}{e}$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
(e**(3*n)*int((x**(3*n)*(sqrt(x**n*d + c)*b + a)**p)/x,x))/e
```

3.370 $\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=-\frac{2a(a^2-b^2c)\,x^{-2n}(ex)^{2n}\left(a+b\sqrt{c+dx^n}\right)^{1+p}}{b^4d^2en(1+p)}+\frac{2(3a^2-b^2c)\,x^{-2n}(ex)^{2n}\left(a+b\sqrt{c+dx^n}\right)^{2+p}}{b^4d^2en(2+p)}-\frac{6ax^{-2n}(ex)^{2n}\left(a+b\sqrt{c+dx^n}\right)^{3+p}}{b^4d^2en(3+p)}+\frac{2x^{-2n}(ex)^{2n}\left(a+b\sqrt{c+dx^n}\right)^{4+p}}{b^4d^2en(4+p)}$$

output
```
-2*a*(-b^2*c+a^2)*(e*x)^(2*n)*(a+b*(c+d*x^n)^(1/2))^(p+1)/b^4/d^2/e/n/(p+1
)/(x^(2*n))+2*(-b^2*c+3*a^2)*(e*x)^(2*n)*(a+b*(c+d*x^n)^(1/2))^(2+p)/b^4/d
^2/e/n/(2+p)/(x^(2*n))-6*a*(e*x)^(2*n)*(a+b*(c+d*x^n)^(1/2))^(3+p)/b^4/d^2
/e/n/(3+p)/(x^(2*n))+2*(e*x)^(2*n)*(a+b*(c+d*x^n)^(1/2))^(4+p)/b^4/d^2/e/n
/(4+p)/(x^(2*n))
```

Mathematica [F]

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Sqrt[c + d*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{2n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+d\,x^n}\right)^p\,dx$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{2n-1}\left(\sqrt{dx^n+c}b+a\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{2n-1}\left(\sqrt{dx^n+c}b+a\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (e\,x)^{2n-1}\left(a+b\,\sqrt{c+d\,x^n}\right)^p\,dx$$

input
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b*(c + d*x^n)^(1/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\frac{e^{2n}\left(\int \frac{x^{2n}\left(\sqrt{x^n d+c}\,b+a\right)^p}{x}dx\right)}{e}$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
(e**(2*n)*int((x**(2*n)*(sqrt(x**n*d + c)*b + a)**p)/x,x))/e
```

3.371 $\int (ex)^{-1+n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int (ex)^{-1+n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx = -\frac{2ax^{-n}(ex)^n \left(a+b\sqrt{c+dx^n}\right)^{1+p}}{b^2den(1+p)} + \frac{2x^{-n}(ex)^n \left(a+b\sqrt{c+dx^n}\right)^{2+p}}{b^2den(2+p)}$$

output
```
-2*a*(e*x)^n*(a+b*(c+d*x^n)^(1/2))^(p+1)/b^2/d/e/n/(p+1)/(x^n)+2*(e*x)^n*(
a+b*(c+d*x^n)^(1/2))^(2+p)/b^2/d/e/n/(2+p)/(x^n)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int (ex)^{-1+n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx = \frac{2x^{-n}(ex)^n \left(a+b\sqrt{c+dx^n}\right)^{1+p} \left(-a+b(1+p)\sqrt{c+dx^n}\right)}{b^2den(1+p)(2+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
(2*(e*x)^n*(a + b*Sqrt[c + d*x^n])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x^n]
))/(b^2*d*e*n*(1 + p)*(2 + p)*x^n)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$
$$\Big\downarrow \; 7299$$
$$\int (ex)^{n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 + n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+n}\left(a+b\sqrt{c+d\,x^n}\right)^p\,dx$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+n} \left(a + b\sqrt{c + dx^n}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+n} \left(a + b\sqrt{c + dx^n}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+n)*(a+b*(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+n} \left(a + b\sqrt{c + dx^n}\right)^p dx = \int (ex)^{n-1} \left(\sqrt{dx^n + c}b + a\right)^p dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{n-1}\left(\sqrt{dx^n+c}b+a\right)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (e\,x)^{n-1}\left(a+b\,\sqrt{c+d\,x^n}\right)^p\,dx$$

input
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b*(c + d*x^n)^(1/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\frac{e^n\left(\int \frac{x^n\left(\sqrt{x^n d+c}\,b+a\right)^p}{x}dx\right)}{e}$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
(e**n*int((x**n*(sqrt(x**n*d + c)*b + a)**p)/x,x))/e
```

3.372 $\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = -\frac{\left(a+b\sqrt{c+dx^n}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx^n}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)en(1+p)} - \frac{\left(a+b\sqrt{c+dx^n}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx^n}}{a+b\sqrt{c}}\right)}{\left(a+b\sqrt{c}\right)en(1+p)}$$

output
```
-(a+b*(c+d*x^n)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],(a+b*(c+d*x^n)^(1/2)
)/(a-b*c^(1/2)))/(a-b*c^(1/2))/e/n/(p+1)-(a+b*(c+d*x^n)^(1/2))^(p+1)*hyper
geom([1, p+1],[2+p],(a+b*(c+d*x^n)^(1/2))/(a+b*c^(1/2)))/(a+b*c^(1/2))/e/n
/(p+1)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = -\frac{\left(a+b\sqrt{c+dx^n}\right)^{1+p}\left(\left(a+b\sqrt{c}\right)\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx^n}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right)\text{Hyper}\right.}{\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c}\right)en(1+p)}$$

input
```
Integrate[(a + b*Sqrt[c + d*x^n])^p/(e*x),x]
```

output
```
-(((a + b*Sqrt[c + d*x^n])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1
 + p, 2 + p, (a + b*Sqrt[c + d*x^n])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hy
pergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x^n])/(a + b*Sqrt[c])])
)/((a - b*Sqrt[c])*(a + b*Sqrt[c])*e*n*(1 + p)))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {27, 7282, 896, 25, 1732, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(a+b\sqrt{dx^n+c}\right)^p}{x}dx}{e}$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int x^{-n}\left(a+b\sqrt{dx^n+c}\right)^p dx^n}{en}$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{x^{-n}\left(a+b\sqrt{dx^n+c}\right)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow 25$$

$$-\frac{\int -\frac{x^{-n}\left(a+b\sqrt{dx^n+c}\right)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow 1732$$

$$-\frac{2\int \frac{\sqrt{dx^n+c}\left(a+b\sqrt{dx^n+c}\right)^p}{c-x^{2n}}d\sqrt{dx^n+c}}{en}$$

$$\Big\downarrow 615$$

$$-\frac{2\int \left(\frac{\left(a+b\sqrt{dx^n+c}\right)^p}{2\left(-dx^n-c+\sqrt{c}\right)}-\frac{\left(a+b\sqrt{dx^n+c}\right)^p}{2\left(\sqrt{c}+\sqrt{dx^n+c}\right)}\right)d\sqrt{dx^n+c}}{en}$$

$$\Big\downarrow 2009$$

$$-\frac{2\left(\frac{\left(a+b\sqrt{c+dx^n}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{dx^n+c}}{a-b\sqrt{c}}\right)}{2(p+1)\left(a-b\sqrt{c}\right)}+\frac{\left(a+b\sqrt{c+dx^n}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{dx^n+c}}{a+b\sqrt{c}}\right)}{2(p+1)\left(a+b\sqrt{c}\right)}\right)}{en}$$

input

```
Int[(a + b*Sqrt[c + d*x^n])^p/(e*x),x]
```

output

```
(-2*(((a + b*Sqrt[c + d*x^n])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (
a + b*Sqrt[c + d*x^n])/(a - b*Sqrt[c])])/(2*(a - b*Sqrt[c])*(1 + p)) + ((a
 + b*Sqrt[c + d*x^n])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sq
rt[c + d*x^n])/(a + b*Sqrt[c])])/(2*(a + b*Sqrt[c])*(1 + p))))/(e*n)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{c+d\,x^{n}}\right)^{p}}{ex}dx$$

input
```
int((a+b*(c+d*x^n)^(1/2))^p/e/x,x)
```

output
```
int((a+b*(c+d*x^n)^(1/2))^p/e/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c+d*x^n)^(1/2))^p/e/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = \frac{\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{x}\,dx}{e}$$

input
```
integrate((a+b*(c+d*x**n)**(1/2))**p/e/x,x)
```

output
```
Integral((a + b*sqrt(c + d*x**n))**p/x, x)/e
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = \int \frac{\left(\sqrt{dx^n+c}\,b+a\right)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^(1/2))^p/e/x,x, algorithm="maxima")
```

output
```
integrate((sqrt(d*x^n + c)*b + a)^p/x, x)/e
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = \int \frac{\left(\sqrt{dx^n+c}b+a\right)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^(1/2))^p/e/x,x, algorithm="giac")
```

output
```
integrate((sqrt(d*x^n + c)*b + a)^p/(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = \int \frac{\left(a+b\,\sqrt{c+d\,x^n}\right)^p}{e\,x}\,dx$$

input
```
int((a + b*(c + d*x^n)^(1/2))^p/(e*x),x)
```

output
```
int((a + b*(c + d*x^n)^(1/2))^p/(e*x), x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{c+dx^n}\right)^p}{ex}\,dx = \frac{\int \frac{\left(\sqrt{x^n d+c}\,b+a\right)^p}{x}dx}{e}$$

input
```
int((a+b*(c+d*x^n)^(1/2))^p/e/x,x)
```

output
```
int((sqrt(x**n*d + c)*b + a)**p/x,x)/e
```

3.373 $\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^n}\right)^p dx$

Optimal result

Integrand size = 27, antiderivative size = 211

$$\begin{aligned}\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^n}\right)^p dx &= -\frac{(ex)^{-n}\left(a+b\sqrt{c+dx^n}\right)^p}{en} \\ &\quad + \frac{bdx^n(ex)^{-n}\left(a+b\sqrt{c+dx^n}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+dx^n}}{a-b\sqrt{c}}\right)}{2\left(a-b\sqrt{c}\right)\sqrt{c}en} \\ &\quad - \frac{bdx^n(ex)^{-n}\left(a+b\sqrt{c+dx^n}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+dx^n}}{a+b\sqrt{c}}\right)}{2\left(a+b\sqrt{c}\right)\sqrt{c}en}\end{aligned}$$

output
```
-(a+b*(c+d*x^n)^(1/2))^p/e/n/((e*x)^n)+1/2*b*d*x^n*(a+b*(c+d*x^n)^(1/2))^p
*hypergeom([1, p],[p+1],(a+b*(c+d*x^n)^(1/2))/(a-b*c^(1/2)))/(a-b*c^(1/2))
/c^(1/2)/e/n/((e*x)^n)-1/2*b*d*x^n*(a+b*(c+d*x^n)^(1/2))^p*hypergeom([1, p
],[p+1],(a+b*(c+d*x^n)^(1/2))/(a+b*c^(1/2)))/(a+b*c^(1/2))/c^(1/2)/e/n/((e
*x)^n)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01

$$\int (ex)^{-1-n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$
$$=\frac{(ex)^{-n}\left(a+b\sqrt{c+dx^n}\right)^{1+p}\left(b\left(a+b\sqrt{c}\right)^2dpx^n\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx^n}}{a-b\sqrt{c}}\right)-(a-}{2\sqrt{c}(}$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
((a + b*Sqrt[c + d*x^n])^(1 + p)*(b*(a + b*Sqrt[c])^2*d*p*x^n*Hypergeometr
ic2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x^n])/(a - b*Sqrt[c])] - (a - b*S
qrt[c])*(2*(a + b*Sqrt[c])*Sqrt[c]*(1 + p)*(a - b*Sqrt[c + d*x^n]) + b*(a
- b*Sqrt[c])*d*p*x^n*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*
x^n])/(a + b*Sqrt[c])])))/(2*Sqrt[c]*(a^2 - b^2*c)^2*e*n*(1 + p)*(e*x)^n)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{-n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 - n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1-n} \left(a+b\sqrt{c+d\,x^{n}}\right)^{p} dx$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^{n}}\right)^{p} dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^{n}}\right)^{p} dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1-n)*(a+b*(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^n}\right)^p dx = \int (ex)^{-n-1} \left(\sqrt{dx^n+c}b+a\right)^p dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(-n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^n}\right)^p dx = \int (ex)^{-n-1} \left(\sqrt{dx^n+c}b+a\right)^p dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(-n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^n}\right)^p dx = \int \frac{\left(a+b\sqrt{c+d\,x^n}\right)^p}{(e\,x)^{n+1}}\,dx$$

input
```
int((a + b*(c + d*x^n)^(1/2))^p/(e*x)^(n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^(1/2))^p/(e*x)^(n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx = \frac{\int \frac{\left(\sqrt{x^n d+c}\, b+a\right)^p}{x^n x} dx}{e^n e}$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
int((sqrt(x**n*d + c)*b + a)**p/(x**n*x),x)/(e**n*e)
```

3.374 $\int (ex)^{-1-2n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 323

$$\int (ex)^{-1-2n} \left(a+b\sqrt{c+dx^n}\right)^p \, dx = -\frac{(ex)^{-2n} \left(a+b\sqrt{c+dx^n}\right)^p}{2en} + \frac{bdpx^n(ex)^{-2n} \left(bc-a\sqrt{c+dx^n}\right) \left(a+b\sqrt{c+dx^n}\right)^p}{4c\,(a^2-b^2c)\,en} - \frac{bd^2\left(a-b\sqrt{c}(2-p)\right)x^{2n}(ex)^{-2n}\left(a+b\sqrt{c+dx^n}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+dx^n}}{a-b\sqrt{c}}\right)}{8\left(a-b\sqrt{c}\right)^2 c^{3/2} en} + \frac{bd^2\left(a+b\sqrt{c}(2-p)\right)x^{2n}(ex)^{-2n}\left(a+b\sqrt{c+dx^n}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{a+b\sqrt{c+dx^n}}{a+b\sqrt{c}}\right)}{8\left(a+b\sqrt{c}\right)^2 c^{3/2} en}$$

output

```
-1/2*(a+b*(c+d*x^n)^(1/2))^p/e/n/((e*x)^(2*n))+1/4*b*d*p*x^n*(b*c-a*(c+d*x
^n)^(1/2))*(a+b*(c+d*x^n)^(1/2))^p/c/(-b^2*c+a^2)/e/n/((e*x)^(2*n))-1/8*b*
d^2*(a-b*c^(1/2)*(2-p))*x^(2*n)*(a+b*(c+d*x^n)^(1/2))^p*hypergeom([1, p],[
p+1],(a+b*(c+d*x^n)^(1/2))/(a-b*c^(1/2)))/(a-b*c^(1/2))^2/c^(3/2)/e/n/((e*
x)^(2*n))+1/8*b*d^2*(a+b*c^(1/2)*(2-p))*x^(2*n)*(a+b*(c+d*x^n)^(1/2))^p*hy
pergeom([1, p],[p+1],(a+b*(c+d*x^n)^(1/2))/(a+b*c^(1/2)))/(a+b*c^(1/2))^2/
c^(3/2)/e/n/((e*x)^(2*n))
```

Mathematica [F]

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*Sqrt[c + d*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-2n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{-2n-1}\left(a+b\sqrt{c+dx^n}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 - 2*n)*(a + b*Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+d\,x^n}\right)^p\,dx$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1-2*n)*(a+b*(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p dx = \int (ex)^{-2n-1}\left(\sqrt{dx^n+c}b+a\right)^p dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(-2*n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p dx = \int (ex)^{-2n-1}\left(\sqrt{dx^n+c}b+a\right)^p dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(-2*n - 1)*(sqrt(d*x^n + c)*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p dx = \int \frac{\left(a+b\sqrt{c+d\,x^n}\right)^p}{(e\,x)^{2\,n+1}}\,dx$$

input
```
int((a + b*(c + d*x^n)^(1/2))^p/(e*x)^(2*n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^(1/2))^p/(e*x)^(2*n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-2n}\left(a+b\sqrt{c+dx^n}\right)^p\,dx=\frac{\int\frac{\left(\sqrt{x^nd+c}\,b+a\right)^p}{x^{2n}x}dx}{e^{2n}e}$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^(1/2))^p,x)
```

output
```
int((sqrt(x**n*d + c)*b + a)**p/(x**(2*n)*x),x)/(e**(2*n)*e)
```

3.375 $\int (ex)^{-1+3n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 406

$$\int (ex)^{-1+3n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \, dx = \frac{b(3-p)\left(60a^2c-b^2(20-9p+p^2)\right)x^{-3n}(ex)^{3n}(c+dx^n)^{3/2}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}}{180a^4d^3en} - \frac{\left(60a^2c-b^2(20-9p+p^2)\right)x^{-3n}(ex)^{3n}(c+dx^n)^2\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}}{60a^3d^3en} - \frac{b(5-p)x^{-3n}(ex)^{3n}(c+dx^n)^{5/2}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}}{15a^2d^3en} + \frac{x^{-3n}(ex)^{3n}(c+dx^n)^3\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}}{3ad^3en} + \frac{b^2\left(360a^4c^2-b^2(2-p)(3-p)\left(60a^2c-b^2(20-9p+p^2)\right)\right)x^{-3n}(ex)^{3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometr}}{180a^7d^3en(1+p)}$$

output
```
1/180*b*(3-p)*(60*a^2*c-b^2*(p^2-9*p+20))*(e*x)^(3*n)*(c+d*x^n)^(3/2)*(a+b
/(c+d*x^n)^(1/2))^(p+1)/a^4/d^3/e/n/(x^(3*n))-1/60*(60*a^2*c-b^2*(p^2-9*p+
20))*(e*x)^(3*n)*(c+d*x^n)^2*(a+b/(c+d*x^n)^(1/2))^(p+1)/a^3/d^3/e/n/(x^(3
*n))-1/15*b*(5-p)*(e*x)^(3*n)*(c+d*x^n)^(5/2)*(a+b/(c+d*x^n)^(1/2))^(p+1)/
a^2/d^3/e/n/(x^(3*n))+1/3*(e*x)^(3*n)*(c+d*x^n)^3*(a+b/(c+d*x^n)^(1/2))^(p
+1)/a/d^3/e/n/(x^(3*n))+1/180*b^2*(360*a^4*c^2-b^2*(2-p)*(3-p)*(60*a^2*c-b
^2*(p^2-9*p+20)))*(e*x)^(3*n)*(a+b/(c+d*x^n)^(1/2))^(p+1)*hypergeom([3, p+
1],[2+p],1+b/a/(c+d*x^n)^(1/2))/a^7/d^3/e/n/(p+1)/(x^(3*n))
```

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b/Sqrt[c + d*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx$$
$$\Big\downarrow \ 7299$$
$$\int (ex)^{3n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p dx$$

input
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+3*n)*(a+b/(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx=\int (ex)^{3n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(3*n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx=\int (ex)^{3n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(3*n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx=\int (e\,x)^{3\,n-1}\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p dx$$

input
```
int((e*x)^(3*n - 1)*(a + b/(c + d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(3*n - 1)*(a + b/(c + d*x^n)^(1/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+3n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \frac{e^{3n}\left(\int \frac{x^{3n}\left(\sqrt{x^n d+c}\,a+b\right)^p}{(x^n d+c)^{\frac{p}{2}}x}dx\right)}{e}$$

input
```
int((e*x)^(-1+3*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
(e**(3*n)*int((x**(3*n)*(sqrt(x**n*d + c)*a + b)**p)/((x**n*d + c)**(p/2)*
x),x))/e
```

3.376 $\int (ex)^{-1+2n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$

Optimal result

Integrand size = 27, antiderivative size = 222

$$\int (ex)^{-1+2n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = -\frac{b(3-p)x^{-2n}(ex)^{2n}(c+dx^n)^{3/2}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}}{6a^2d^2en} + \frac{x^{-2n}(ex)^{2n}(c+dx^n)^2\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}}{2ad^2en} - \frac{b^2(12a^2c-b^2(6-5p+p^2))\,x^{-2n}(ex)^{2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b}{a\sqrt{c}}\right.}{6a^5d^2en(1+p)}$$

output
```
-1/6*b*(3-p)*(e*x)^(2*n)*(c+d*x^n)^(3/2)*(a+b/(c+d*x^n)^(1/2))^(p+1)/a^2/d
^2/e/n/(x^(2*n))+1/2*(e*x)^(2*n)*(c+d*x^n)^2*(a+b/(c+d*x^n)^(1/2))^(p+1)/a
/d^2/e/n/(x^(2*n))-1/6*b^2*(12*a^2*c-b^2*(p^2-5*p+6))*(e*x)^(2*n)*(a+b/(c+
d*x^n)^(1/2))^(p+1)*hypergeom([3, p+1],[2+p],1+b/a/(c+d*x^n)^(1/2))/a^5/d^
2/e/n/(p+1)/(x^(2*n))
```

Mathematica [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx=\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b/Sqrt[c + d*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$

$$\Big\downarrow \; 7299$$

$$\int (ex)^{2n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p dx$$

input
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+2*n)*(a+b/(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{2n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{2n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (e\,x)^{2\,n-1}\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p\,dx$$

input
```
int((e*x)^(2*n - 1)*(a + b/(c + d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b/(c + d*x^n)^(1/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\frac{e^{2n}\left(\int\frac{x^{2n}\left(\sqrt{x^nd+c}\,a+b\right)^p}{\left(x^nd+c\right)^{\frac{p}{2}}x}dx\right)}{e}$$

input
```
int((e*x)^(-1+2*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
(e**(2*n)*int((x**(2*n)*(sqrt(x**n*d + c)*a + b)**p)/((x**n*d + c)**(p/2)*
x),x))/e
```

3.377 $\int (ex)^{-1+n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \frac{2b^2x^{-n}(ex)^n\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b}{a\sqrt{c+dx^n}}\right)}{a^3den(1+p)}$$

output
```
2*b^2*(e*x)^n*(a+b/(c+d*x^n)^(1/2))^(p+1)*hypergeom([3, p+1],[2+p],1+b/a/(
c+d*x^n)^(1/2))/a^3/d/e/n/(p+1)/(x^n)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.34

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = -\frac{2x^{-n}(ex)^n\left(c+dx^n\right)\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\left(1+\frac{a\sqrt{c+dx^n}}{b}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{a\sqrt{c+dx^n}}{b}\right)}{den(-2+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
(-2*(e*x)^n*(c + d*x^n)*(a + b/Sqrt[c + d*x^n])^p*Hypergeometric2F1[2 - p,
 -p, 3 - p, -((a*Sqrt[c + d*x^n])/b)])/(d*e*n*(-2 + p)*x^n*(1 + (a*Sqrt[c
+ d*x^n])/b)^p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$
$$\Big\downarrow \ 7299$$
$$\int (ex)^{n-1} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$

input
```
Int[(e*x)^(-1 + n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+n} \left(a+\frac{b}{\sqrt{c+d\,x^{n}}}\right)^{p}dx$$

input
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1+n)*(a+b/(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (e\,x)^{n-1}\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p\,dx$$

input
```
int((e*x)^(n - 1)*(a + b/(c + d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b/(c + d*x^n)^(1/2))^p, x)
```

Reduce [F]

$$\int (ex)^{-1+n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\frac{e^n\left(\int\frac{x^n\left(\sqrt{x^nd+c}\,a+b\right)^p}{(x^nd+c)^{\frac{p}{2}}x}dx\right)}{e}$$

input
```
int((e*x)^(-1+n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
(e**n*int((x**n*(sqrt(x**n*d + c)*a + b)**p)/((x**n*d + c)**(p/2)*x),x))/e
```

3.378 $\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 240

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx = \frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)}{b-a\sqrt{c}}\right)}{\left(b-a\sqrt{c}\right)en(1+p)} - \frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)}{b+a\sqrt{c}}\right)}{\left(b+a\sqrt{c}\right)en(1+p)} + \frac{2\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b}{a\sqrt{c+dx^n}}\right)}{aen(1+p)}$$

output
```
c^(1/2)*(a+b/(c+d*x^n)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],-c^(1/2)*(a+b
/(c+d*x^n)^(1/2))/(b-a*c^(1/2)))/(b-a*c^(1/2))/e/n/(p+1)-c^(1/2)*(a+b/(c+d
*x^n)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],c^(1/2)*(a+b/(c+d*x^n)^(1/2))/
(b+a*c^(1/2)))/(b+a*c^(1/2))/e/n/(p+1)+2*(a+b/(c+d*x^n)^(1/2))^(p+1)*hyper
geom([1, p+1],[2+p],1+b/a/(c+d*x^n)^(1/2))/a/e/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx = \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\left(1+\frac{a\sqrt{c+dx^n}}{b}\right)^{-p}\left(\left(1+\frac{a\sqrt{c+dx^n}}{b}\right)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{\left(a-\frac{b}{\sqrt{c}}\right)\sqrt{c+dx^n}}{b+a\sqrt{c+dx^n}}\right)+\right.}{}$$

input
```
Integrate[(a + b/Sqrt[c + d*x^n])^p/(e*x),x]
```

output
```
((a + b/Sqrt[c + d*x^n])^p*((1 + (a*Sqrt[c + d*x^n])/b)^p*Hypergeometric2F
1[1, -p, 1 - p, ((a - b/Sqrt[c])*Sqrt[c + d*x^n])/(b + a*Sqrt[c + d*x^n])]
 + (1 + (a*Sqrt[c + d*x^n])/b)^p*Hypergeometric2F1[1, -p, 1 - p, ((a + b/S
qrt[c])*Sqrt[c + d*x^n])/(b + a*Sqrt[c + d*x^n])] - 2*Hypergeometric2F1[-p
, -p, 1 - p, -((a*Sqrt[c + d*x^n])/b)]))/(e*n*p*(1 + (a*Sqrt[c + d*x^n])/b
)^p)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {27, 7282, 896, 25, 1776, 1803, 25, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{x}dx}{e}$$

$$\Big\downarrow \; 7282$$

$$\frac{\int x^{-n}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p dx^n}{en}$$

$$\Big\downarrow \ 896$$

$$\frac{\int \frac{x^{-n}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{x^{-n}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow \ 1776$$

$$-\frac{\int \frac{x^{-n}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{cx^{-n}-1}d(dx^n+c)}{en}$$

$$\Big\downarrow \ 1803$$

$$\frac{2\int -\frac{x^{-n}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{1-cx^{2n}}d\frac{1}{\sqrt{dx^n+c}}}{en}$$

$$\Big\downarrow \ 25$$

$$-\frac{2\int \frac{x^{-n}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{1-cx^{2n}}d\frac{1}{\sqrt{dx^n+c}}}{en}$$

$$\Big\downarrow \ 615$$

$$-\frac{2\int \left(x^{-n}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p-\frac{c\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{\sqrt{dx^n+c}(cx^{2n}-1)}\right)d\frac{1}{\sqrt{dx^n+c}}}{en}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)}{b-a\sqrt{c}}\right)}{2(p+1)\left(b-a\sqrt{c}\right)}-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{\sqrt{c}(}{}\right.}{2(p+1)\left(a\sqrt{c}+b\right)}\right.}{en}$$

input
```
Int[(a + b/Sqrt[c + d*x^n])^p/(e*x),x]
```

output
```
(2*((Sqrt[c]*(a + b/Sqrt[c + d*x^n])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
 + p, -((Sqrt[c]*(a + b/Sqrt[c + d*x^n]))/(b - a*Sqrt[c]))])/(2*(b - a*Sqr
t[c])*(1 + p)) - (Sqrt[c]*(a + b/Sqrt[c + d*x^n])^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, (Sqrt[c]*(a + b/Sqrt[c + d*x^n]))/(b + a*Sqrt[c])])/(2*
(b + a*Sqrt[c])*(1 + p)) + ((a + b/Sqrt[c + d*x^n])^(1 + p)*Hypergeometric
2F1[1, 1 + p, 2 + p, 1 + b/(a*Sqrt[c + d*x^n])])/(a*(1 + p))))/(e*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1776
```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+d\,x^{n}}}\right)^{p}}{ex}dx$$

input
```
int((a+b/(c+d*x^n)^(1/2))^p/e/x,x)
```

output
```
int((a+b/(c+d*x^n)^(1/2))^p/e/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c+d*x^n)^(1/2))^p/e/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx = \text{Timed out}$$

input
```
integrate((a+b/(c+d*x**n)**(1/2))**p/e/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx = \int \frac{\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n)^(1/2))^p/e/x,x, algorithm="maxima")
```

output
```
integrate((a + b/sqrt(d*x^n + c))^p/x, x)/e
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx = \int \frac{\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p}{ex}\,dx$$

input
```
integrate((a+b/(c+d*x^n)^(1/2))^p/e/x,x, algorithm="giac")
```

output
```
integrate((a + b/sqrt(d*x^n + c))^p/(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p}{e\,x}\,dx$$

input
```
int((a + b/(c + d*x^n)^(1/2))^p/(e*x),x)
```

output
```
int((a + b/(c + d*x^n)^(1/2))^p/(e*x), x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{ex}\,dx=\frac{\int \frac{\left(\sqrt{x^n d+c}\,a+b\right)^p}{\left(x^n d+c\right)^{\frac{p}{2}}x}dx}{e}$$

input
```
int((a+b/(c+d*x^n)^(1/2))^p/e/x,x)
```

output
```
int((sqrt(x**n*d + c)*a + b)**p/((x**n*d + c)**(p/2)*x),x)/e
```

3.379 $\int (ex)^{-1-n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$

Optimal result

Integrand size = 27, antiderivative size = 241

$$\int (ex)^{-1-n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = -\frac{dx^n(ex)^{-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p}{cen\left(1-\frac{c}{c+dx^n}\right)} + \frac{bdx^n(ex)^{-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)}{b-a\sqrt{c}}\right)}{2\left(b-a\sqrt{c}\right)cen} + \frac{bdx^n(ex)^{-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)}{b+a\sqrt{c}}\right)}{2\left(b+a\sqrt{c}\right)cen}$$

output
```
-d*x^n*(a+b/(c+d*x^n)^(1/2))^p/c/e/n/((e*x)^n)/(1-c/(c+d*x^n))+1/2*b*d*x^n
*(a+b/(c+d*x^n)^(1/2))^p*hypergeom([1, p],[p+1],-c^(1/2)*(a+b/(c+d*x^n)^(1
/2))/(b-a*c^(1/2)))/(b-a*c^(1/2))/c/e/n/((e*x)^n)+1/2*b*d*x^n*(a+b/(c+d*x^
n)^(1/2))^p*hypergeom([1, p],[p+1],c^(1/2)*(a+b/(c+d*x^n)^(1/2))/(b+a*c^(1
/2)))/(b+a*c^(1/2))/c/e/n/((e*x)^n)
```

Mathematica [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \int (ex)^{-1-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 - n)*(a + b/Sqrt[c + d*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{-n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$$

input
```
Int[(e*x)^(-1 - n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{\sqrt{c+d\,x^{n}}}\right)^{p}dx$$

input
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1-n)*(a+b/(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \, dx = \int (ex)^{-n-1} \left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(-n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1-n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \, dx = \int (ex)^{-n-1} \left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(-n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p \, dx = \int \frac{\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p}{(e\,x)^{n+1}} \, dx$$

input
```
int((a + b/(c + d*x^n)^(1/2))^p/(e*x)^(n + 1),x)
```

output
```
int((a + b/(c + d*x^n)^(1/2))^p/(e*x)^(n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\frac{\int \frac{\left(\sqrt{x^n d+c}\,a+b\right)^p}{x^n\left(x^n d+c\right)^{\frac{p}{2}}x}dx}{e^n e}$$

input
```
int((e*x)^(-1-n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
int((sqrt(x**n*d + c)*a + b)**p/(x**n*(x**n*d + c)**(p/2)*x),x)/(e**n*e)
```

3.380 $\int (ex)^{-1-2n} \left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx$

Optimal result

Integrand size = 27, antiderivative size = 483

$$\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p dx = \frac{d^2x^{2n}(ex)^{-2n}\left(a-\frac{b}{\sqrt{c+dx^n}}\right)\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}}{2c(b^2-a^2c)\,en\left(1-\frac{c}{c+dx^n}\right)^2}$$
$$-\frac{d^2x^{2n}(ex)^{-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\left(b(a^2c(4-p)-b^2(2+p))-4a^3c\sqrt{c+dx^n}+2ab^2(1+p)\sqrt{c+dx^n}\right)}{4c\,(b^2-a^2c)^2\,en\sqrt{c+dx^n}\left(1-\frac{c}{c+dx^n}\right)}$$
$$-\frac{bd^2p\left(3a\sqrt{c}-b(2+p)\right)x^{2n}(ex)^{-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,-\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+d}}\right.}{b-a\sqrt{c}}\right.}{8\left(b-a\sqrt{c}\right)^3c^{3/2}en(1+p)}$$
$$-\frac{bd^2p\left(3a\sqrt{c}+b(2+p)\right)x^{2n}(ex)^{-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{\sqrt{c}\left(a+\frac{b}{\sqrt{c+dx^n}}\right.}{b+a\sqrt{c}}\right.}{8\left(b+a\sqrt{c}\right)^3c^{3/2}en(1+p)}$$

output
```
1/2*d^2*x^(2*n)*(a-b/(c+d*x^n)^(1/2))*(a+b/(c+d*x^n)^(1/2))^(p+1)/c/(-a^2*
c+b^2)/e/n/((e*x)^(2*n))/(1-c/(c+d*x^n))^2-1/4*d^2*x^(2*n)*(a+b/(c+d*x^n)^
(1/2))^(p+1)*(b*(a^2*c*(4-p)-b^2*(2+p))-4*a^3*c*(c+d*x^n)^(1/2)+2*a*b^2*(p
+1)*(c+d*x^n)^(1/2))/c/(-a^2*c+b^2)^2/e/n/((e*x)^(2*n))/(c+d*x^n)^(1/2)/(1
-c/(c+d*x^n))-1/8*b*d^2*p*(3*a*c^(1/2)-b*(2+p))*x^(2*n)*(a+b/(c+d*x^n)^(1/
2))^(p+1)*hypergeom([1, p+1],[2+p],-c^(1/2)*(a+b/(c+d*x^n)^(1/2))/(b-a*c^(
1/2)))/(b-a*c^(1/2))^3/c^(3/2)/e/n/(p+1)/((e*x)^(2*n))-1/8*b*d^2*p*(3*a*c^
(1/2)+b*(2+p))*x^(2*n)*(a+b/(c+d*x^n)^(1/2))^(p+1)*hypergeom([1, p+1],[2+p
],c^(1/2)*(a+b/(c+d*x^n)^(1/2))/(b+a*c^(1/2)))/(b+a*c^(1/2))^3/c^(3/2)/e/n
/(p+1)/((e*x)^(2*n))
```

Mathematica [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx$$

input
```
Integrate[(e*x)^(-1 - 2*n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 - 2*n)*(a + b/Sqrt[c + d*x^n])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-2n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (ex)^{-2n-1}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx$$

input
```
Int[(e*x)^(-1 - 2*n)*(a + b/Sqrt[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1-2n} \left(a + \frac{b}{\sqrt{c + d\, x^n}}\right)^p dx$$

input
```
int((e*x)^(-1-2*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
int((e*x)^(-1-2*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{-1-2n} \left(a + \frac{b}{\sqrt{c + dx^n}}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(-1-2*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   do_a
lg_rde: unimplemented kernel
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1-2*n)*(a+b/(c+d*x**n)**(1/2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{-2n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(-2*n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int (ex)^{-2n-1}\left(a+\frac{b}{\sqrt{dx^n+c}}\right)^p\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b/(c+d*x^n)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(-2*n - 1)*(a + b/sqrt(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\int \frac{\left(a+\frac{b}{\sqrt{c+d\,x^n}}\right)^p}{(e\,x)^{2\,n+1}}\,dx$$

input
```
int((a + b/(c + d*x^n)^(1/2))^p/(e*x)^(2*n + 1),x)
```

output
```
int((a + b/(c + d*x^n)^(1/2))^p/(e*x)^(2*n + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-2n}\left(a+\frac{b}{\sqrt{c+dx^n}}\right)^p\,dx=\frac{\int \frac{\left(\sqrt{x^n d+c}\,a+b\right)^p}{x^{2n}(x^n d+c)^{\frac{p}{2}}x}dx}{e^{2n}e}$$

input
```
int((e*x)^(-1-2*n)*(a+b/(c+d*x^n)^(1/2))^p,x)
```

output
```
int((sqrt(x**n*d + c)*a + b)**p/(x**(2*n)*(x**n*d + c)**(p/2)*x),x)/(e**(2
*n)*e)
```

3.381 $\int (ex)^{-1+3n} (a+b(c+dx^n)^q)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\int (ex)^{-1+3n} (a+b(c+dx^n)^q)^p \, dx = \frac{c^2x^{-3n}(ex)^{3n}(c+dx^n)(a+b(c+dx^n)^q)^p\left(1+\frac{b(c+dx^n)^q}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1}{q},1+\frac{1}{q},-\frac{b(c+dx^n)^q}{a}\right)}{d^3en} - \frac{cx^{-3n}(ex)^{3n}(c+dx^n)^2(a+b(c+dx^n)^q)^p\left(1+\frac{b(c+dx^n)^q}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{2}{q},\frac{2+q}{q},-\frac{b(c+dx^n)^q}{a}\right)}{d^3en} + \frac{x^{-3n}(ex)^{3n}(c+dx^n)^3(a+b(c+dx^n)^q)^p\left(1+\frac{b(c+dx^n)^q}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{3}{q},\frac{3+q}{q},-\frac{b(c+dx^n)^q}{a}\right)}{3d^3en}$$

output
```
c^2*(e*x)^(3*n)*(c+d*x^n)*(a+b*(c+d*x^n)^q)^p*hypergeom([-p, 1/q],[1+1/q],
-b*(c+d*x^n)^q/a)/d^3/e/n/(x^(3*n))/((1+b*(c+d*x^n)^q/a)^p)-c*(e*x)^(3*n)*
(c+d*x^n)^2*(a+b*(c+d*x^n)^q)^p*hypergeom([-p, 2/q],[(2+q)/q],-b*(c+d*x^n)
^q/a)/d^3/e/n/(x^(3*n))/((1+b*(c+d*x^n)^q/a)^p)+1/3*(e*x)^(3*n)*(c+d*x^n)^
3*(a+b*(c+d*x^n)^q)^p*hypergeom([-p, 3/q],[(3+q)/q],-b*(c+d*x^n)^q/a)/d^3/
e/n/(x^(3*n))/((1+b*(c+d*x^n)^q/a)^p)
```

Mathematica [F]

$$\int (ex)^{-1+3n} (a + b(c + dx^n)^q)^p \, dx = \int (ex)^{-1+3n} (a + b(c + dx^n)^q)^p \, dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^q)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1} (a + b(c + dx^n)^q)^p \, dx$$

$$\Big\downarrow \; 7299$$

$$\int (ex)^{3n-1} (a + b(c + dx^n)^q)^p dx$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+3n} (a + b(c + d\,x^n)^q)^p dx$$

input
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^q)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+3n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{3\,n-1} \, dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="fricas")
```

output
```
integral((((d*x^n + c)^q*b + a)^p*(e*x)^(3*n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b(c + dx^n)^q)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+3*n)*(a+b*(c+d*x**n)**q)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+3n}(a+b(c+dx^n)^q)^p\,dx=\int ((dx^n+c)^q b+a)^p(ex)^{3n-1}\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(3*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+3n}(a+b(c+dx^n)^q)^p\,dx=\int ((dx^n+c)^q b+a)^p(ex)^{3n-1}\,dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}(a+b(c+dx^n)^q)^p\,dx=\int (a+b\,(c+d\,x^n)^q)^p\,(e\,x)^{3\,n-1}\,dx$$

input
```
int((a + b*(c + d*x^n)^q)^p*(e*x)^(3*n - 1),x)
```

output
```
int((a + b*(c + d*x^n)^q)^p*(e*x)^(3*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+3n}\left(a+b(c+dx^n)^q\right)^p\,dx = \text{too large to display}$$

input

```
int((e*x)^(-1+3*n)*(a+b*(c+d*x^n)^q)^p,x)
```

output

```
(e**(3*n)*(x**(3*n)*((x**n*d + c)**q*b + a)**p*d**3*p**2*q**2 + 3*x**(3*n)
*((x**n*d + c)**q*b + a)**p*d**3*p*q + 2*x**(3*n)*((x**n*d + c)**q*b + a)*
*p*d**3 + x**(2*n)*((x**n*d + c)**q*b + a)**p*c*d**2*p**2*q**2 + x**(2*n)*
((x**n*d + c)**q*b + a)**p*c*d**2*p*q - 2*x**n*((x**n*d + c)**q*b + a)**p*
c**2*d*p*q + 2*((x**n*d + c)**q*b + a)**p*c**3 + int((x**(3*n)*((x**n*d +
c)**q*b + a)**p)/((x**n*d + c)**q*b*p**3*q**3*x + 6*(x**n*d + c)**q*b*p**2
*q**2*x + 11*(x**n*d + c)**q*b*p*q*x + 6*(x**n*d + c)**q*b*x + a*p**3*q**3
*x + 6*a*p**2*q**2*x + 11*a*p*q*x + 6*a*x),x)*a*d**3*n*p**6*q**6 + 9*int((
x**(3*n)*((x**n*d + c)**q*b + a)**p)/((x**n*d + c)**q*b*p**3*q**3*x + 6*(x
**n*d + c)**q*b*p**2*q**2*x + 11*(x**n*d + c)**q*b*p*q*x + 6*(x**n*d + c)*
*q*b*x + a*p**3*q**3*x + 6*a*p**2*q**2*x + 11*a*p*q*x + 6*a*x),x)*a*d**3*n
*p**5*q**5 + 31*int((x**(3*n)*((x**n*d + c)**q*b + a)**p)/((x**n*d + c)**q
*b*p**3*q**3*x + 6*(x**n*d + c)**q*b*p**2*q**2*x + 11*(x**n*d + c)**q*b*p*
q*x + 6*(x**n*d + c)**q*b*x + a*p**3*q**3*x + 6*a*p**2*q**2*x + 11*a*p*q*x
 + 6*a*x),x)*a*d**3*n*p**4*q**4 + 51*int((x**(3*n)*((x**n*d + c)**q*b + a)
**p)/((x**n*d + c)**q*b*p**3*q**3*x + 6*(x**n*d + c)**q*b*p**2*q**2*x + 11
*(x**n*d + c)**q*b*p*q*x + 6*(x**n*d + c)**q*b*x + a*p**3*q**3*x + 6*a*p**
2*q**2*x + 11*a*p*q*x + 6*a*x),x)*a*d**3*n*p**3*q**3 + 40*int((x**(3*n)*((
x**n*d + c)**q*b + a)**p)/((x**n*d + c)**q*b*p**3*q**3*x + 6*(x**n*d + c)*
*q*b*p**2*q**2*x + 11*(x**n*d + c)**q*b*p*q*x + 6*(x**n*d + c)**q*b*x +...
```

3.382 $\int (ex)^{-1+2n} (a+b(c+dx^n)^q)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 194

$$\int (ex)^{-1+2n} (a+b(c+dx^n)^q)^p \, dx = -\frac{cx^{-2n}(ex)^{2n}(c+dx^n)(a+b(c+dx^n)^q)^p \left(1+\frac{b(c+dx^n)^q}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{1}{q},1+\frac{1}{q},-\frac{b(c+dx}{a}\right.}{d^2en} + \frac{x^{-2n}(ex)^{2n}(c+dx^n)^2(a+b(c+dx^n)^q)^p \left(1+\frac{b(c+dx^n)^q}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{2}{q},\frac{2+q}{q},-\frac{b(c+dx^n)}{a}\right.}{2d^2en}$$

output
```
-c*(e*x)^(2*n)*(c+d*x^n)*(a+b*(c+d*x^n)^q)^p*hypergeom([-p, 1/q],[1+1/q],-
b*(c+d*x^n)^q/a)/d^2/e/n/(x^(2*n))/((1+b*(c+d*x^n)^q/a)^p)+1/2*(e*x)^(2*n)
*(c+d*x^n)^2*(a+b*(c+d*x^n)^q)^p*hypergeom([-p, 2/q],[(2+q)/q],-b*(c+d*x^n
)^q/a)/d^2/e/n/(x^(2*n))/((1+b*(c+d*x^n)^q/a)^p)
```

Mathematica [F]

$$\int (ex)^{-1+2n}\,(a+b(c+dx^n)^q)^p\;dx=\int (ex)^{-1+2n}\,(a+b(c+dx^n)^q)^p\;dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^q)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1}\,(a+b(c+dx^n)^q)^p\;dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int (ex)^{2n-1}\,(a+b(c+dx^n)^q)^p\,dx$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+2n} (a + b(c + d\,x^n)^q)^p dx$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^q)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+2n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{2\,n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="fricas")
```

output
```
integral(((d*x^n + c)^q*b + a)^p*(e*x)^(2*n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} (a + b(c + dx^n)^q)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*(c+d*x**n)**q)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+2n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{2n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(2*n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+2n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{2n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} (a + b(c + dx^n)^q)^p \, dx = \int (a + b\,(c + d\,x^n)^q)^p\,(e\,x)^{2\,n-1} \, dx$$

input
```
int((a + b*(c + d*x^n)^q)^p*(e*x)^(2*n - 1),x)
```

output
```
int((a + b*(c + d*x^n)^q)^p*(e*x)^(2*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+2n} \left(a + b(c + dx^n)^q\right)^p \, dx = \text{Too large to display}$$

input
```
int((e*x)^(-1+2*n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
(e**(2*n)*(x**(2*n)*((x**n*d + c)**q*b + a)**p*d**2*p*q + x**(2*n)*((x**n*
d + c)**q*b + a)**p*d**2 + x**n*((x**n*d + c)**q*b + a)**p*c*d*p*q - ((x**
n*d + c)**q*b + a)**p*c**2 + int((x**(2*n)*((x**n*d + c)**q*b + a)**p)/((x
**n*d + c)**q*b*p**2*q**2*x + 3*(x**n*d + c)**q*b*p*q*x + 2*(x**n*d + c)**
q*b*x + a*p**2*q**2*x + 3*a*p*q*x + 2*a*x),x)*a*d**2*n*p**4*q**4 + 4*int((
x**(2*n)*((x**n*d + c)**q*b + a)**p)/((x**n*d + c)**q*b*p**2*q**2*x + 3*(x
**n*d + c)**q*b*p*q*x + 2*(x**n*d + c)**q*b*x + a*p**2*q**2*x + 3*a*p*q*x
+ 2*a*x),x)*a*d**2*n*p**3*q**3 + 5*int((x**(2*n)*((x**n*d + c)**q*b + a)**
p)/((x**n*d + c)**q*b*p**2*q**2*x + 3*(x**n*d + c)**q*b*p*q*x + 2*(x**n*d
+ c)**q*b*x + a*p**2*q**2*x + 3*a*p*q*x + 2*a*x),x)*a*d**2*n*p**2*q**2 + 2
*int((x**(2*n)*((x**n*d + c)**q*b + a)**p)/((x**n*d + c)**q*b*p**2*q**2*x
+ 3*(x**n*d + c)**q*b*p*q*x + 2*(x**n*d + c)**q*b*x + a*p**2*q**2*x + 3*a*
p*q*x + 2*a*x),x)*a*d**2*n*p*q - int((x**n*((x**n*d + c)**q*b + a)**p)/((x
**n*d + c)**q*b*p**2*q**2*x + 3*(x**n*d + c)**q*b*p*q*x + 2*(x**n*d + c)**
q*b*x + a*p**2*q**2*x + 3*a*p*q*x + 2*a*x),x)*a*c*d*n*p**3*q**3 - 3*int((x
**n*((x**n*d + c)**q*b + a)**p)/((x**n*d + c)**q*b*p**2*q**2*x + 3*(x**n*d
 + c)**q*b*p*q*x + 2*(x**n*d + c)**q*b*x + a*p**2*q**2*x + 3*a*p*q*x + 2*a
*x),x)*a*c*d*n*p**2*q**2 - 2*int((x**n*((x**n*d + c)**q*b + a)**p)/((x**n*
d + c)**q*b*p**2*q**2*x + 3*(x**n*d + c)**q*b*p*q*x + 2*(x**n*d + c)**q*b*
x + a*p**2*q**2*x + 3*a*p*q*x + 2*a*x),x)*a*c*d*n*p*q))/(d**2*e*n*(p**2...
```

3.383 $\int (ex)^{-1+n} \left(a+b(c+dx^n)^q\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int (ex)^{-1+n} \left(a+b(c+dx^n)^q\right)^p \, dx = \frac{x^{-n}(ex)^n \left(c+dx^n\right) \left(a+b(c+dx^n)^q\right)^p \left(1+\frac{b(c+dx^n)^q}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{1}{q},1+\frac{1}{q},-\frac{b(c+dx^n)^q}{a}\right)}{den}$$

output
```
(e*x)^n*(c+d*x^n)*(a+b*(c+d*x^n)^q)^p*hypergeom([-p, 1/q],[1+1/q],-b*(c+d*
x^n)^q/a)/d/e/n/(x^n)/((1+b*(c+d*x^n)^q/a)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int (ex)^{-1+n} \left(a+b(c+dx^n)^q\right)^p \, dx = \frac{x^{1-n}(ex)^{-1+n} \left(c+dx^n\right) \left(a+b(c+dx^n)^q\right)^p \left(1+\frac{b(c+dx^n)^q}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{1}{q},1+\frac{1}{q},-\frac{b(c+dx^n)^q}{a}\right)}{dn}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
(x^(1 - n)*(e*x)^(-1 + n)*(c + d*x^n)*(a + b*(c + d*x^n)^q)^p*Hypergeometr
ic2F1[-p, q^(-1), 1 + q^(-1), -((b*(c + d*x^n)^q)/a)])/(d*n*(1 + (b*(c + d
*x^n)^q)/a)^p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (a+b(c+dx^n)^q)^p \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int (ex)^{n-1} (a+b(c+dx^n)^q)^p dx$$

input
```
Int[(e*x)^(-1 + n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-1+n} (a+b(c+d\,x^n)^q)^p dx$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^q)^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="fricas")
```

output
```
integral(((d*x^n + c)^q*b + a)^p*(e*x)^(n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a + b(c + dx^n)^q)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1+n)*(a+b*(c+d*x**n)**q)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-1+n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(n - 1), x)
```

Giac [F]

$$\int (ex)^{-1+n} (a+b(c+dx^n)^q)^p \, dx = \int ((dx^n+c)^q b+a)^p (ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a+b(c+dx^n)^q)^p \, dx = \int \left(a+b\left(c+d\,x^n\right)^q\right)^p \left(e\,x\right)^{n-1} \, dx$$

input
```
int((a + b*(c + d*x^n)^q)^p*(e*x)^(n - 1),x)
```

output
```
int((a + b*(c + d*x^n)^q)^p*(e*x)^(n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+n} (a+b(c+dx^n)^q)^p \, dx = \frac{e^n \left(x^n((x^n d+c)^q b+a)^p d+((x^n d+c)^q b+a)^p c+\left(\int \frac{x^n((x^n d+c)^q b+a)^p}{(x^n d+c)^q bpqx+(x^n d+c)^q bx+apqx+ax} dx\right) adn\, p^2q^2+\left(\int\right.\right.}{den\,(pq+1)}$$

input
```
int((e*x)^(-1+n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
(e**n*(x**n*((x**n*d + c)**q*b + a)**p*d + ((x**n*d + c)**q*b + a)**p*c +
int((x**n*((x**n*d + c)**q*b + a)**p)/((x**n*d + c)**q*b*p*q*x + (x**n*d +
 c)**q*b*x + a*p*q*x + a*x),x)*a*d*n*p**2*q**2 + int((x**n*((x**n*d + c)**
q*b + a)**p)/((x**n*d + c)**q*b*p*q*x + (x**n*d + c)**q*b*x + a*p*q*x + a*
x),x)*a*d*n*p*q))/(d*e*n*(p*q + 1))
```

3.384 $\int \frac{(a+b(c+dx^n)^q)^p}{ex} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex} dx = \frac{d\text{Int}\left(\frac{(a+b(c+dx^n)^q)^p}{dx}, x\right)}{e}$$

output
```
d*Defer(Int)((a+b*(c+d*x^n)^q)^p/d/x,x)/e
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex} dx = \int \frac{(a+b(c+dx^n)^q)^p}{ex} dx$$

input
```
Integrate[(a + b*(c + d*x^n)^q)^p/(e*x),x]
```

output
```
Integrate[(a + b*(c + d*x^n)^q)^p/x, x]/e
```

Rubi [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {27, 7282, 896, 25, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex} \, dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(b(dx^n+c)^q+a)^p}{x}dx}{e}$$

$$\Big\downarrow \; 7282$$

$$\frac{\int x^{-n}(b(dx^n+c)^q+a)^p\,dx^n}{en}$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{x^{-n}(b(dx^n+c)^q+a)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{x^{-n}(b(dx^n+c)^q+a)^p}{d}d(dx^n+c)}{en}$$

$$\Big\downarrow \; 7299$$

$$-\frac{\int -\frac{x^{-n}(b(dx^n+c)^q+a)^p}{d}d(dx^n+c)}{en}$$

input
```
Int[(a + b*(c + d*x^n)^q)^p/(e*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b(c+d\,x^n)^q)^p}{ex}dx$$

input
```
int((a+b*(c+d*x^n)^q)^p/e/x,x)
```

output
```
int((a+b*(c+d*x^n)^q)^p/e/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex}\,dx = \int \frac{((dx^n+c)^q b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^q)^p/e/x,x, algorithm="fricas")
```

output
```
integral(((d*x^n + c)^q*b + a)^p/(e*x), x)
```

Sympy [N/A]

Not integrable

Time = 9.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex}\,dx = \frac{\int \frac{(a+b(c+dx^n)^q)^p}{x}\,dx}{e}$$

input
```
integrate((a+b*(c+d*x**n)**q)**p/e/x,x)
```

output
```
Integral((a + b*(c + d*x**n)**q)**p/x, x)/e
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex}\,dx = \int \frac{((dx^n+c)^q b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^q)^p/e/x,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p/x, x)/e
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex}\,dx=\int \frac{((dx^n+c)^q b+a)^p}{ex}\,dx$$

input
```
integrate((a+b*(c+d*x^n)^q)^p/e/x,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p/(e*x), x)
```

Mupad [N/A]

Not integrable

Time = 8.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex}\,dx=\int \frac{(a+b\,(c+d\,x^n)^q)^p}{e\,x}\,dx$$

input
```
int((a + b*(c + d*x^n)^q)^p/(e*x),x)
```

output
```
int((a + b*(c + d*x^n)^q)^p/(e*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 242, normalized size of antiderivative = 11.00

$$\int \frac{(a+b(c+dx^n)^q)^p}{ex}\,dx$$
$$= \frac{((x^n d+c)^q b+a)^p + \left(\int \frac{((x^n d+c)^q b+a)^p}{x^n(x^n d+c)^q bdx+(x^n d+c)^q bcx+x^n adx+acx}dx\right) acnpq + \left(\int \frac{(x^n d+c)^q((x^n d+c)^q b+a)^p}{x^n(x^n d+c)^q bdx+(x^n d+c)^q bcx+x^n adx}\right.}{enpq}$$

input
```
int((a+b*(c+d*x^n)^q)^p/e/x,x)
```

output
```
(((x**n*d + c)**q*b + a)**p + int(((x**n*d + c)**q*b + a)**p/(x**n*(x**n*d
 + c)**q*b*d*x + (x**n*d + c)**q*b*c*x + x**n*a*d*x + a*c*x),x)*a*c*n*p*q
+ int(((x**n*d + c)**q*((x**n*d + c)**q*b + a)**p)/(x**n*(x**n*d + c)**q*b
*d*x + (x**n*d + c)**q*b*c*x + x**n*a*d*x + a*c*x),x)*b*c*n*p*q + int((x**
n*((x**n*d + c)**q*b + a)**p)/(x**n*(x**n*d + c)**q*b*d*x + (x**n*d + c)**
q*b*c*x + x**n*a*d*x + a*c*x),x)*a*d*n*p*q)/(e*n*p*q)
```

3.385 $\int (ex)^{-1-n} \left(a+b(c+dx^n)^q\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^q\right)^p \, dx = \frac{d^2 x^n (ex)^{-n} \text{Int}\left(\frac{x^{-1-n}(a+b(c+dx^n)^q)^p}{d^2},x\right)}{e}$$

output
```
d^2*x^n*Defer(Int)(x^(-1-n)*(a+b*(c+d*x^n)^q)^p/d^2,x)/e/((e*x)^n)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-n} \left(a+b(c+dx^n)^q\right)^p \, dx = \int (ex)^{-1-n} \left(a+b(c+dx^n)^q\right)^p \, dx$$

input
```
Integrate[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
Integrate[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^q)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-n-1} (a+b(c+dx^n)^q)^p \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int (ex)^{-n-1} (a+b(c+dx^n)^q)^p dx$$

input
```
Int[(e*x)^(-1 - n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ex)^{-1-n} (a+b(c+d\,x^n)^q)^p dx$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^q)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="fricas")
```

output
```
integral(((d*x^n + c)^q*b + a)^p*(e*x)^(-n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-n} (a + b(c + dx^n)^q)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-1-n)*(a+b*(c+d*x**n)**q)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(-n - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-n} (a + b(c + dx^n)^q)^p \, dx = \int ((dx^n + c)^q b + a)^p (ex)^{-n-1} \, dx$$

input
```
integrate((e*x)^(-1-n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(-n - 1), x)
```

Mupad [N/A]

Not integrable

Time = 8.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-n} (a + b(c + dx^n)^q)^p \, dx = \int \frac{(a + b\,(c + d\,x^n)^q)^p}{(e\,x)^{n+1}} \, dx$$

input
```
int((a + b*(c + d*x^n)^q)^p/(e*x)^(n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^q)^p/(e*x)^(n + 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\int (ex)^{-1-n} (a + b(c + dx^n)^q)^p \, dx$$
$$= \frac{-((x^n d + c)^q b + a)^p + x^n \left(\int \frac{(x^n d+c)^q ((x^n d+c)^q b+a)^p}{x^n (x^n d+c)^q bdx+(x^n d+c)^q bcx+x^n adx+acx} dx \right) bdnpq}{x^n e^n en}$$

input
```
int((e*x)^(-1-n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
( - ((x**n*d + c)**q*b + a)**p + x**n*int(((x**n*d + c)**q*((x**n*d + c)**
q*b + a)**p)/(x**n*(x**n*d + c)**q*b*d*x + (x**n*d + c)**q*b*c*x + x**n*a*
d*x + a*c*x),x)*b*d*n*p*q)/(x**n*e**n*e*n)
```

3.386 $\int (ex)^{-1-2n} (a+b(c+dx^n)^q)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ex)^{-1-2n} (a+b(c+dx^n)^q)^p \, dx = \frac{d^3 x^{2n} (ex)^{-2n} \text{Int}\left(\frac{x^{-1-2n}(a+b(c+dx^n)^q)^p}{d^3}, x\right)}{e}$$

output
```
d^3*x^(2*n)*Defer(Int)(x^(-1-2*n)*(a+b*(c+d*x^n)^q)^p/d^3,x)/e/((e*x)^(2*n
))
```

Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-2n} (a+b(c+dx^n)^q)^p \, dx = \int (ex)^{-1-2n} (a+b(c+dx^n)^q)^p \, dx$$

input
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
Integrate[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^q)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-2n-1}(a+b(c+dx^n)^q)^p\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int (ex)^{-2n-1}(a+b(c+dx^n)^q)^p\,dx$$

input
```
Int[(e*x)^(-1 - 2*n)*(a + b*(c + d*x^n)^q)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ex)^{-1-2n}(a+b(c+d\,x^n)^q)^p dx$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^q)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^q\right)^p\,dx=\int ((dx^n+c)^q b+a)^p (ex)^{-2\,n-1}\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="fricas")
```

output
```
integral(((d*x^n + c)^q*b + a)^p*(e*x)^(-2*n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^q\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(-1-2*n)*(a+b*(c+d*x**n)**q)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-2n}\left(a+b(c+dx^n)^q\right)^p\,dx=\int ((dx^n+c)^q b+a)^p (ex)^{-2\,n-1}\,dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="maxima")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(-2*n - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ex)^{-1-2n} (a+b(c+dx^n)^q)^p \, dx = \int ((dx^n+c)^q b+a)^p (ex)^{-2\,n-1} \, dx$$

input
```
integrate((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^q)^p,x, algorithm="giac")
```

output
```
integrate(((d*x^n + c)^q*b + a)^p*(e*x)^(-2*n - 1), x)
```

Mupad [N/A]

Not integrable

Time = 9.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (ex)^{-1-2n} (a+b(c+dx^n)^q)^p \, dx = \int \frac{(a+b\,(c+d\,x^n)^q)^p}{(e\,x)^{2\,n+1}} \, dx$$

input
```
int((a + b*(c + d*x^n)^q)^p/(e*x)^(2*n + 1),x)
```

output
```
int((a + b*(c + d*x^n)^q)^p/(e*x)^(2*n + 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 397, normalized size of antiderivative = 15.88

$$\int (ex)^{-1-2n} \left(a+b(c+dx^n)^q\right)^p \, dx$$

$$= \frac{-x^n((x^nd+c)^q b+a)^p dpq - ((x^nd+c)^q b+a)^p c - x^{2n}\left(\int \frac{((x^nd+c)^q b+a)^p}{x^{2n}(x^nd+c)^q bdx+x^n(x^nd+c)^q bcx+x^{2n}adx+x^n acx}dx\right) ac}{}$$

input
```
int((e*x)^(-1-2*n)*(a+b*(c+d*x^n)^q)^p,x)
```

output
```
( - x**n*((x**n*d + c)**q*b + a)**p*d*p*q - ((x**n*d + c)**q*b + a)**p*c -
 x**(2*n)*int(((x**n*d + c)**q*b + a)**p/(x**(2*n)*(x**n*d + c)**q*b*d*x +
 x**n*(x**n*d + c)**q*b*c*x + x**(2*n)*a*d*x + x**n*a*c*x),x)*a*c*d*n*p*q
- x**(2*n)*int(((x**n*d + c)**q*b + a)**p/(x**n*(x**n*d + c)**q*b*d*x + (x
**n*d + c)**q*b*c*x + x**n*a*d*x + a*c*x),x)*a*d**2*n*p*q + x**(2*n)*int((
(x**n*d + c)**q*((x**n*d + c)**q*b + a)**p)/(x**n*(x**n*d + c)**q*b*d*x +
(x**n*d + c)**q*b*c*x + x**n*a*d*x + a*c*x),x)*b*d**2*n*p**2*q**2 - x**(2*
n)*int(((x**n*d + c)**q*((x**n*d + c)**q*b + a)**p)/(x**n*(x**n*d + c)**q*
b*d*x + (x**n*d + c)**q*b*c*x + x**n*a*d*x + a*c*x),x)*b*d**2*n*p*q)/(2*x*
*(2*n)*e**(2*n)*c*e*n)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral',
         'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
         'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file